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Preface

This, the 60th volume of the Transactions of the American Institute of Electrical Engineers, contains all approved technical-program papers and related discussions presented during the calendar year 1941, and all previously unpublished papers and discussions presented in 1940. They were presented at the following AIEE conventions and District meetings:

1. 1940 summer convention, Swampscott, Mass., June 24–28: 17 papers and related discussion; 24 papers and related discussion were published in the 1940 volume.
2. 1940 Pacific Coast convention, Los Angeles, Calif., August 27–30.
3. Middle Eastern District meeting, Cincinnati, Ohio, October 9–11, 1940.
4. 1941 winter convention, Philadelphia, Pa., January 27–31.
5. North Eastern District meeting, Rochester, N. Y., April 30–May 2, 1941.
6. 1941 summer convention, Toronto, Ont., Can. June 16–20.
7. 1941 Pacific Coast convention, Yellowstone National Park, August 27–29.
8. South West District meeting, St. Louis, Mo., October 8–10, 1941.
9. Southern District meeting, New Orleans, La., December 3–5, 1941.

In addition this volume contains:

1. Report "Telemetering, Supervisory Control, and Associated Circuits," prepared by a joint subcommittee of the AIEE committees on automatic stations, and instruments and measurements.
2. "Bibliography of Relay Literature, 1927–1939," prepared by the relay subcommittee of the AIEE committee on protective devices.
3. Report "Standard Basic Impulse Insulation Levels," prepared by the joint committee on co-ordination of insulation of AIEE, Edison Electric Institute, and National Electrical Manufacturers Association.
4. 1941 annual report of the AIEE board of directors.
5. A complete listing of AIEE officers and committees for 1941–42.

Under the present publication policy a large proportion of the content of this volume was given advance publication in the Transactions Sections of the monthly issues of Electrical Engineering during 1941, and extra sheets were printed at the same time for binding in this volume. The papers so published comprise pages 1–312 and 763–1050, inclusive; pages 313–762 and 1051–1410, inclusive, did not appear in Electrical Engineering, but were given advance publication in the June and December 1941 "Supplements to Electrical Engineering—Transactions Section," respectively.

Full correlation of all material in this volume has been accomplished through the medium of the multientry reference index beginning on page 1469. A reference to any of the several subject entries for any technical paper will lead directly to the paper and to any published discussion on that paper.

Statements and opinions given in papers and discussions published in Transactions are the expressions of contributors for which the American Institute of Electrical Engineers assumes no responsibility.

Errata

1. Page 114. Illustration designated Figure 3 should be Figure 4; illustration designated Figure 4 should be Figure 3.

Page 194, column 1, line 5. The statement ". . . at 38 kv . . ." should read ". . . at 88 kv . . ."

3. Page 194, Figure 4. The ordinate scale now reads $KVA = \frac{EI}{1000}$; should read $\frac{KVA}{1000}$.

4. Page 194, column 3, line 22. The word "minimum" should read "maximum."

A Push-Button-Tuned 50-Kw Broadcast Transmitter

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Synopsis: This paper describes the use of automatic push-button tuning on short-wave broadcast transmitters that are required to change from one frequency to another with the least possible interruption to programs. The tuning control circuits and the arrangement of the radio-frequency circuits of the transmitter are described.

THE effective use of high frequencies for international radiobroadcasting requires the transmitter to use a frequency which will place the strongest signal in the selected service area. Since the best frequency to use will vary throughout the day, and since it may be desirable to shift the signal from one area to another, a transmitter built for international broadcasting should be able to change frequencies easily and rapidly.

Ordinarily a shutdown period is necessary to retune the transmitter whenever the frequency is changed. In the past when it has been necessary to change frequencies rapidly, as for instance during the short interval between programs, it has been customary to use a second transmitter tuned to the new frequency and then shift transmitters. This cumbersome procedure can be avoided if the tuning process is speeded up by the use of automatic devices. Retuning the transmitter is then a matter of seconds and is accomplished by simply pushing a button.

Description of Transmitter

The 50-kw WLWO transmitter of The Crosley Corporation employs automatic push-button tuning and a description of this transmitter will illustrate the practicality of this method of tuning.

The transmitter is designed to operate on any one of eight different frequencies. Figure 1 is a circuit diagram of the radio-frequency section of this transmitter. The radio-frequency amplifier up to and including the one-kilowatt stage is mounted on one chassis. Since the coils and capacitors for the grid and plate circuits of these stages are small, a separate set of coils and capacitors is used for each frequency. These coils and capacitors are mounted on a drum which is rotated by a motor until the proper tuned circuits make contact with a set of fixed wipers. The eight crystals are mounted separately and are kept at operating temperature at all times. When the drum rotates to bring into position the coils for a particular frequency, it also makes connection to the crystal for that frequency.

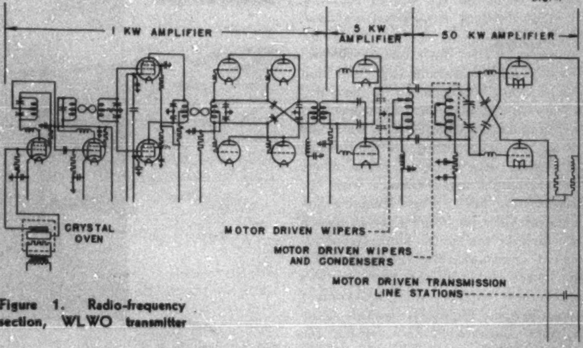


Figure 1. Radio-frequency section, WLWO transmitter

The plate coil of the 5-kw stage and grid coil of the 50-kw stage are too large to employ the preceding method of tuning. These circuits are tuned, therefore, by means of motor-driven wipers mounted on a shaft which runs through the center of the coil. The shaft is arranged to rotate the wipers in opposite directions. As the shaft turns the wipers slide along the turns of the coil, traveling from the middle toward each end. The wipers short-circuit a certain number of turns of the coil depending upon the frequency to be used. Figure 2 is a picture of the plate coil of the 5-kw stage. The rotating drum and the wipers on both the 5-kw tank circuit and the 50-kw grid circuit in addition to being motor driven may be rotated by a hand wheel mounted on the front of the transmitter cabinets. This arrangement is necessary to adjust the tuning system as described later in this paper.

The 50-kw amplifier employs a transmission-line tank circuit. This trans-

Figure 2. Five-kw tank coil

Paper 40-161, recommended by the AIEE committee on communication, and presented at the AIEE Middle Eastern District meeting, Cincinnati, Ohio, October 9–11, 1940. Manuscript submitted August 12, 1940; made available for preprinting September 10, 1940.

R. J. Rockwell, member IRE, and H. Lepple, associate member IRE are with WLW, Crosley Radio Corporation, Cincinnati, Ohio.

Figure 3. Transmission-line station

mission line is designed to present the proper load impedance to the final amplifier and also couple the antenna load to the transmitter. This is accomplished by switching across the transmission line at the proper point a capacitor of such size as to obtain the desired coupling. If eight different frequencies are used, there will be eight stations along the transmission line. Each station will consist of a group of vacuum capacitors that can be swung into contact with the transmission line by a motor. The capacitors are mounted on Mycalex arms which are swung from a horizontal position through 90 degrees to a vertical position where the capacitors make contact with the transmission line. When changing frequencies, the capacitors for the frequency on which the transmitter has been operating will swing away from the transmission line at the same time that the capacitors for the new frequency swing into contact with the line. Figure 3 is a picture of the WLWO transmission line showing the capacitor stations.

In addition to tuning the various tank circuits, it is necessary to match the antenna to the transmission line for each frequency. The circuits necessary for doing this are brought to contacts mounted on Mycalex plates. Wipers are rotated by a motor to select the circuits for the frequency to be used. Figure 4 shows the method of mounting these coils and the selecting mechanism. A portion of the coils have been removed to simplify the picture. This equipment is located in the antenna coupling house, some 200 yards from the transmitter building.

Tuning Control Circuits

From the preceding description it is evident that five operations must be completed to retune the transmitter. The drum on the 1-kw chassis must rotate to the proper set of coils, the wipers on the 5-kw tank coil and the wipers on the grid coil of the 50-kw stage must each move to a new position, a different capacitor station along the transmission line must swing into position, and the antenna must be matched to the transmission line.

Figure 4. Antenna matching mechanism

Each of these operations, except the tuning of the transmission line, is carried out by a rotating drum, a rotating switch, or a rotating wiper. The problem is to control these rotating devices so that they will turn through the shortest path to their new position and stop. To do this, three-phase motors and the motor-reversing circuit illustrated in figure 5 were used. *D* is a bakelite disk mounted on a shaft which is mechanically connected to the rotating drum or wiper arms. This disk has a pin protruding from its side which engages the switch arm as shown in the diagram. If this disk is turned clockwise the switch arm will take the position shown in *A* and the circuit will be completed through the upper spring contact to relay number 2. However, if the disk is turned counterclockwise, as shown in *B*, relay number 1 will be operated and the motor will be reversed.

If the disk is in position *A* when the switch in P_1 is closed, relay number 2 will operate and the motor, being connected to rotate the disk counterclockwise will turn the disk until it is in position *C* when the relay will be released and the motor will stop. Likewise, when the disk is in position *B* when P_1 is closed, the motor will rotate in the reverse direction turning the disk clockwise until position *C* is reached.

If it is desired to have the motor stop at any one of eight positions, eight of these disks will have to be mounted on the shaft. For eight positions the circuit will be as shown in figure 6. Each of the eight "disk-switches" will make connection to either relay number 1 or number 2 depending upon whether the wiper will have to rotate clockwise or counterclockwise to reach its position. To select one of the eight positions it is only neces-

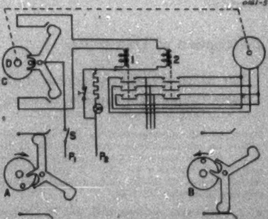


Figure 5. Tuning-motor reversing circuit

sary to close one of the eight switches S_1 through S_8, figure 6.

In applying the above system to WLWO it was necessary to control four motors in the above manner. To do this the circuit shown in figure 7 was devised. Eight frequency-selecting relays replace the switches S_1 through S_8 of figure 6. Each relay has a normally open contact for each of the motors that is to be controlled, that is, four normally open contacts. For simplicity only one tuning index and one transmission line station is shown connected in figure 7. The three tuning motors and the seven transmission line stations that are not shown are connected to the remaining normally open contacts in a similar manner. Each of the frequency relays may be "locked in" by pressing the proper frequency button as shown in figure 7. When one of these relays is "locked in", one of the disk-switches on each of the four motor index systems is energized and each of the four motors will rotate to its position and stop. It will be noted in figure 7 that the push buttons and the relay holding circuits are designed so that when a relay is "locked in" by pressing one of the buttons, the relay previously "locked in" is released. This arrangement makes it unnecessary to use an off button to release the previous frequency.

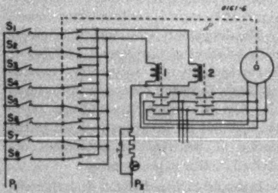
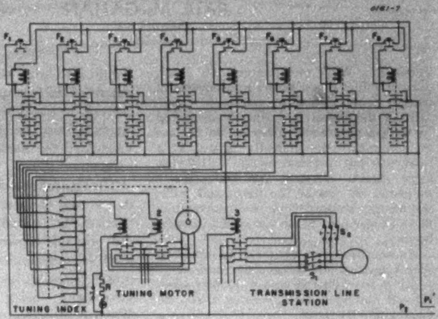


Figure 6. Eight-position motor control circuit

Figure 7 also shows the control circuits for the transmission line stations. Relay number 3 is used to reverse the motor phases. When this relay is not energized the phases are connected to rotate the capacitors away from the transmission line. A limit switch operated by a cam on the rotating capacitor arms stops the motor. When the relay is energized the motor is connected to swing the capacitors into contact with the transmission line and again the motor is stopped by a limit switch. Each motor is equipped with a magnetic brake which is needed to bring the motor to a quick stop after the power is disconnected.

Figure 7. Push-button-tuning circuit



Adjusting the Tuning System

Referring to figure 5 it will be seen that a resistor and a six-volt bulb are connected in P_2 of the relay circuit. A switch is shown short-circuiting the bulb and resistor. If this switch is opened and the "disk-switch" is in either position A or B the lamp will burn but the relay will not pull in because of the voltage drop across the lamp and resistor. If the disk is turned the lamp will go out when position C is reached. The bakelite disks shown in figure 5 are held by friction to the shaft which is connected to the wipers and are arranged so that they will slip when turned by hand.

The procedure for adjusting the tuning system is as follows: First, the short-circuiting switch is opened and one of the frequency buttons is pressed to lock in one of the frequency relays. Next, the transmitter is tuned manually. This is done by rotating the wipers by hand, using the hand wheels provided for this purpose, until resonance is reached. Then, the bakelite disk is rotated, slipping on its shaft while the wiper arms are held stationary, until the light goes out. The short-circuiting switch is then closed and the transmitter is ready to tune automatically to this frequency. The same procedure must be followed for each tuning motor and for each frequency. A different push button and its associated "disk-switch" will be used for each frequency. After each of the tuned circuits is adjusted as just described, the entire transmitter will retune to any one of the eight frequencies by simply pressing the proper button.

An Improved Type of D-C Wattmeter of the Shunted Type

PAUL MacGAHAN
MEMBER AIEE

Synopsis: While wattmeters for high current values of a-c power are universally used in connection with instrument transformers, d-c power is ordinarily measured by using separate ammeters and voltmeters on account of the difficulty in making a heavy-current d-c wattmeter. Although large direct currents are conveniently measured by use of the shunt and millivoltmeter, shunted-type d-c wattmeters have in the past been somewhat unsatisfactory.

An improved d-c wattmeter is described which corresponds in accuracy and sensitivity to the usual permanent-magnet moving-coil ammeter and operates from the usual ammeter shunt.

THE measurement of power in a d-c circuit is generally accomplished by the use of separate ammeters and voltmeters. This has been acceptable for most d-c conditions because a simple multiplication of amperes and volts gives the correct power. Furthermore, the construction of a direct-reading d-c wattmeter for large currents has heretofore involved great difficulties in construction and installation.

In certain cases where the voltage is variable, it would be distinctly preferable to read the watts directly. The instrument described is designed to fill this need.

D-c wattmeters for low values of current can be satisfactorily designed using the electrodynamic principle with a stationary coil carrying the entire current and a moving coil in the voltage circuit. The practical limitations of this construction are due to the difficulty in constructing the series coil for currents in excess of about 400 amperes capacity, as well as the matter of the heavy and expensive leads required to bring the current to the terminals of the instrument. A minor difficulty is to avoid residual magnetization errors because the heavy currents tend to magnetize the metal cases and internal magnetic shields.

The reason these difficulties do not exist in the case of measuring d-c amperes is because the permanent-magnet moving-coil principle requires a very low power level of operation, compared with the power taken by instrument coils of any instrument using other principles. Whereas an ordinary portable or switchboard-type electrodynamic wattmeter without iron cores requires a power level of the order of one or two watts to energize the current coil, the coils of corresponding permanent-magnet moving-coil movements of normal sensitivity take only 0.0004 watt and when one of the newer types of high coercive magnet steel is used, the power level may be much less.

It is this unique quality of the permanent-magnet moving-coil type instruments[1] which permits their use as shunted ammeters, with shunts having the usual standard drop of only 50 millivolts. In order to make a shunted-type coreless electrodynamic wattmeter for 10,000 amperes with the same ratio of copper to swamping resistance, and consequently the same temperature error as in the d'Arsonval ammeter, would require a shunt consuming at least five or six kilowatts.

It is not unusual to mount instruments on switchboards, 50 or more feet from the bus-bar structure. Such long leads still further increase the resistance, and consequently, the power required in the shunt.[2]

Another difficulty attending attempts to operate such wattmeters from ammeter shunts is that in order to obtain a sufficiently large current in the shunted-coil circuit, the resistance of the coil and of the leads must be made very low. Thus, any contact resistance in the connections introduces a variable which causes a considerable error in the readings.

However, such shunted-type electrodynamic wattmeters have been built, generally by the expedients of reducing the ampere turns of the series coils to subnormal values and obtaining a torque subnormal in value but yet sufficient to operate the movement by working the voltage coils much harder in proportion. This involves considerable compromise in the direction of allowing much higher temperature errors in both the moving-coil and current-coil circuits. The constants of one wattmeter of this construction are shown in column *A* of table I. Such designs can be regarded only as compromises.

The desirability of finding other means as a solution of the general problem of heavy-capacity d-c wattmeters became apparent from the data given in column *A*. Several other possibilities were studied and the investigations led to the design of the type of instrument described further in this paper.

It should be noted that a desirable element of the problem from an economic viewpoint was the necessity of utilizing to as great extent as possible, existing instrument mechanisms, because the probable applications are not so large as are those of other instruments; moreover, commercial instruments can be built economically only with the aid of expensive tool equipment. Uniformity in appearance with other switchboard instruments was another desideratum.

These considerations led to the belief that the most practical solution might lie in substituting an iron-core electromagnet for the usual permanent magnet of the

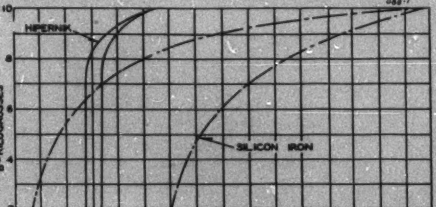


Figure 1. Hysteresis loop for Hipernik and high-silicon iron

Paper 40-88, recommended by the AIEE committee on instruments and measurements, and presented at the AIEE summer convention, Swampscott, Mass., June 24–28, 1940. Manuscript submitted July 10, 1939; made available for preprinting April 22, 1940.

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The author gratefully acknowledges the assistance given in this development by L. J. Lunas and D. A. Young of the Westinghouse meter engineering staff, by Doctor T. D. Yensen of the research department, Westinghouse Electric and Manufacturing Company, and by Doctor H. B. Brooks of the National Bureau of Standards.

[1] For all numbered references, see list at end of paper.

Table I. Constants Compared

	(A) Shunted Electrodynamic Wattmeter Without Iron Core	(B) Shunted Iron-Cored Wattmeter
Voltage-coil circuit (460 volts)		
Total resistance (ohms)	13,000	4,350
Resistance, copper coil (ohms)	65	375
Ratio, copper to total resistance	0.005	0.086
Current-coil circuit		
Resistance, current coil (ohms)	0.0055	0.27
Resistance, two shunt leads (ohms)	0.0073	0.12
Resistance, springs (ohms)		0.44
Swamping resistance* (ohms)	0.02	2.12
Total resistance, shunted current circuit (ohms)	0.0328	2.95
Current-circuit copper ratio	0.39	0.17
Full-scale current in coil (amperes)	6.2	0.017
Length, pair of leads (feet)	120.0	120.0
Cross section, leads (square centimeters)	157,380.0	9,700.0
Approximate cost of leads, compared with cost of instrument (per cent)	300	5
Full-load millivolts at shunt (per cent)	200	50
Full-load watts in shunt (10,000 amperes) (per cent)	2,000	500
Temperature influence† (per cent)	1.08	0.4
Weight of moving element (grams)	2.8	1.4
Full-scale torque (millimeter grams)	1.3	1.7
T/W ratio	0.46	1.2
Residual error by reversing polarity of current circuit (per cent)	2.0	0

* Low temperature coefficient wire in series with coil.

† Includes voltage and current circuit and effect of springs.

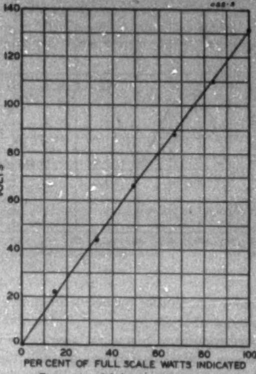

Figure 3. Voltage-error curve

Variable voltage, constant millivolts (amperes); average of increasing and decreasing voltage readings

conventional d-c ammeter, and winding the stationary coils of the electromagnet for excitation from the voltage of the circuit to be measured. This would bring the power-level requirement of the moving-coil circuit down to that of the permanent-magnet moving-coil type, thereby avoiding the difficulties exemplified in column *A* of table I. The problem of eliminating the residual magnetizing errors due to the usual iron-cored magnetic circuit remained to be solved before a commercially satisfactory instrument could be obtained.

Soon after the original development by Doctor Yensen[4] of the low-loss, low-residual forms of iron known as Hipernik, certain designers succeeded in greatly improving the accuracy of moving-iron ammeters or voltmeters for direct current by using this material for the vanes.

Hipernik is characterized by the almost total absence of magnetic coercive force. As indicated in the hysteresis curve, figure 1, the coercive field is of the low value of 0.04 to 0.06 oersted. As a comparison, the following values may be noted for other forms of steel or iron.

Cold rolled steel, annealed	2.0 oersteds
Armco (a very pure low-carbon steel)	1.0 oersted
Silicon steel	0.32 to 0.85 oersted

In the instrument herein described, use is made of nearly all the usual parts of the corresponding size d-c permanent-magnet moving-coil ammeter, with the exception that an electromagnet is used instead of the permanent magnet. The shape of the electromagnet is so designed as to be assembled in the place of the usual permanent magnet. Hipernik is used for the electromagnet core and also instead of the soft-iron pole pieces and cylindrical core ordinarily used in ammeters or voltmeters. The Hipernik is further annealed in hydrogen at 1,200 degrees centigrade after being machined. The moving element, complete with windings, pointer, springs, and swamping resistance, is exactly the same as that used in the ammeters.

By using the same flux density in the electromagnet as in a permanent magnet, the sensitivity of the current element is made approximately the same as that of the ammeter. This permits the use of the instrument in connection with the usual ammeter shunt, with the same conditions as to length of leads, temperature error compensation, and mechanical action. The ratio of torque to weight, and thus the "figure of merit," is also the same.[5]

The flux density for the working limits of voltage variation in the circuits on which the wattmeter is intended is chosen to correspond with the straightest portion of the magnetization curve of the Hipernik as shown in figure 2. This and the presence of the air gap result in the voltage influence being reduced to a minimum. Figure 3 shows the actual voltage errors of the instrument. The curve is drawn to the average of the ascending and descending values so as to distinguish from the residual errors.

The temperature error of the voltage circuit is reduced to permissible limits by the use of an external resistor of the usual low-temperature-coefficient type such as manganin, constantin, or similar alloy wire.

As in the permanent-magnet moving-coil type, the damping of the instrument is due to the eddy currents generated in the aluminum frame on which the moving coil is wound. As the flux varies with the voltages, the damping effect also varies but is so proportioned as to give the best damping at the average operating voltage. Thus the instrument is inclined to be overdamped at higher voltages and underdamped at low voltages. This, however, is not

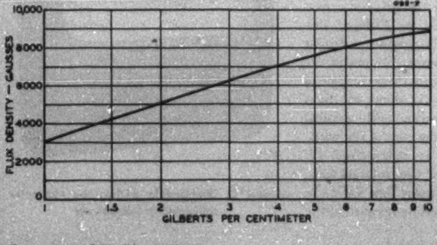

Figure 2. Magnetization curve for Hipernik

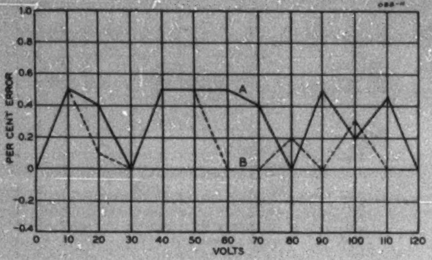

Figure 4. Residual errors—varying voltages, amperes constant

A—Normal polarity
B—Reverse polarity

wholly undesirable, because less damping is required at the lower parts of the scale corresponding in general to lower applied voltage and loads. Where desired, the damping can be increased readily by the well-known method of shunting the entire moving element by a stationary resistance coil. This is, however, at the expense of a corresponding increase of current in the leads which connect the instrument to the shunt.

Figure 4*A* shows the actual residual errors of the instrument on descending values of voltage, the ascending values being used as standard. Figure 4*B* is similar to figure 4*A* except that the polarities have been reversed. It should be noted that there are no residual errors due to variations or reversal of direction of the applied current. Voltage variations are, of course, less extreme in practice than current variations.

A further reduction in the over-all residual error to a value of plus or minus one-fourth per cent is possible by using the average of the increasing and decreasing voltage readings in calibrating the instrument. As errors of this order are less than the usual errors of temperature, friction, and scale marking in the ordinary high-grade switchboard instruments, the residual errors may be disregarded for practical purposes.

The polarity of the voltage does not ordinarily reverse in the course of the normal operation of a wattmeter and the terminals are marked for connection to a definite polarity as used for calibration. This is to avoid the slight additional residual error which would occur from reversal of the electromagnet flux.

It will be noted from the residual-error curve that this effect is not entirely eliminated at all parts of the voltage cycle. However, the performance in this respect compares very favorably with that of the best moving-iron ammeters or voltmeters and is considerably better than in the electrodynamic type described previously.

The completed instrument is shown in figure 5. A similar instrument of the rectangular type, with the scale calibrated in horsepower, is illustrated in figure 6. Figure 7 shows the internal construction of the instrument.

The constants and data of this new d-c wattmeter are given in column *B* of table I. In order to allow a direct comparison with the electrodynamic type d-c wattmeter, column *A*, an instrument of the same capacity and for the same length of leads, is tabulated.

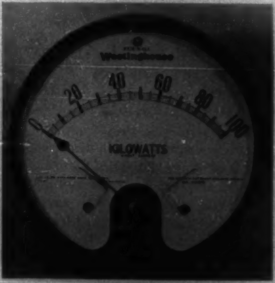

Figure 5. Round-pattern d-c wattmeter calibrated in kilowatts

While the instrument described in this paper is of the indicating switchboard type, satisfactory d-c recording or portable wattmeters could be built on the same principle.

A further study of the prior art, particularly as practiced in Europe, disclosed the existence of somewhat similar d-c iron-cored wattmeters. In one of the wattmeters described in these European references,[6,7] a magnetic shunt of high-coercive material is permanently applied to the pole pieces of the core of the wattmeter. This shunt is magnetized directly by the field of the voltage coils. As the voltage and main flux varies, a compensating effect results because of the high remanence of the shunting member which produces a remanent flux through the air gap. The strength and polarity of this remanent flux depend upon the previous magnetic condition of the electromagnet. Since this flux is of the opposite polarity to that of the residual magnetization of the core itself, the residual errors are reduced. During the course of development, this construction was tried, but the experiments failed to produce a satisfactory result.

Another form described in the above reference (Keinath, page 284) depends upon the use of a very short magnetic circuit instead of a compensating element, but the performance curves given show considerable residual error, because a nonresidual magnetic material, such as Hipernik was not available.

Application

In general, it is more desirable to indicate the power directly on a dial than to determine the power by simultaneous voltmeter and ammeter readings. This may even be the case where the voltage is fairly constant. But the more important application of the d-c wattmeter is to circuits in which the voltage varies between wide limits, either purposely for speed- or power-control reasons, or because of circuit conditions. The well-known "variable-voltage control," as introduced originally by H. Ward Leonard[2,3] is a typical example.

In this system, the speed of a motor is controlled by varying the voltage across its armature. This is accomplished by changing the field excitation and consequently the voltage of the generator which drives the motor, thus avoiding the losses and other disadvantages of direct rheostatic speed control.

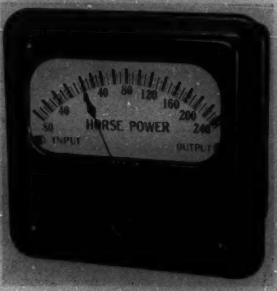

Figure 6. Rectangular-pattern d-c wattmeter calibrated in horsepower

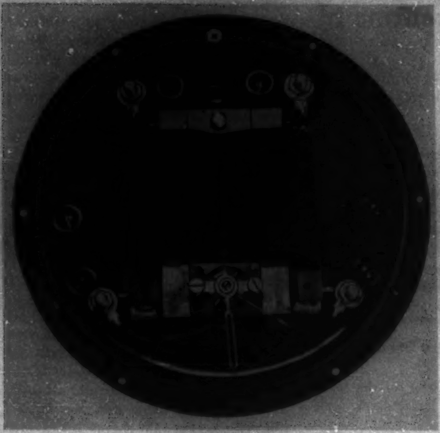

Figure 7. Internal view of d-c wattmeter

When such speed or power control is applied to certain functions, such as harbor dredges or other marine power equipment, it is important to have a direct indication of power at the switchboard or control station.

Another actual application is to power indication of electric dynamometer load sets for production testing of internal-combustion engines. On such applications it is generally preferred to calibrate the scale to read directly in horsepower instead of in kilowatts. The instrument illustrated in figure 6 was specially intended for this purpose.

Bibliography

1. **United States Patent 446,489, Edward Weston.** *Electric World*, November 17, 1889.

2. **H. Ward Leonard,** AIEE Transactions, volume 13, November 18, 1896.

3. **L. W. Markle,** *Electric Journal*, December 1938.

4. **Magnetic Properties of the Fifty Per Cent Iron-Nickel Alloy, T. D. Yensen.** *Journal* of the Franklin Institute, volume 199, 1925, page 333.

Permeability of Hipernik Reaches 167,000, T. D. Yensen. *Electric Journal*, volume 28, June 3, 1931, pages 386–8.

Hipernik and Other Iron-Nickel Alloys; Revolutionary Improvements Due to Heat Treating, T. D. Yensen. Presented American Society Steel Treating, January 14, 1932. Published in *Metal Progress*, volume 21, June 1932, pages 28–34, 70.

5. **Indicating-Instrument Quality, B. E. Lenehan and Paul MacGahan.** *Electric Journal*, volume 26, November 1929, pages 520–3.

J. H. Goss, *General Electric Review*, April 1933.

6. **G. Keinath,** *Die Technik Elektrischer Messgeräte*, volume 1, pages 269–98.

7. *Archiv für Technisches Messgeräte, J-741-I.*

8. **Edgecombe and Ockender,** *Industrial Electrical Instruments*, pages 237–41.

9. **Remote Indication of Direct Currents, F. S. Stickney.** *Electric Journal*, September 1938.

10. United States Patent 1,751,411, **Lamb and Kinnerd.**

11. United States Patent 1,976,498, **Hoare and Parker.**

12. **Drysdale,** *Journal* of IEE, January 1933, page 371.

Discussion

Discussion will be found in the 1941 annual Transactions volume and in the June 1941 Supplement to Electrical Engineering—Transactions Section.

Some Characteristics and Applications of Negative-Glow Lamps

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THE glow lamp is a modern electrical device employing the peculiar properties of various rare gases which when energized by the application of a voltage ionize and pass current. This ionization is accompanied by the production of light.

It has long been known that if sufficiently high voltage is applied to electrodes that are sealed within a bulb containing an inert gas (neon, for example) at the proper pressure, light is produced at the negative electrode or cathode. For this reason, glow lamps are sometimes called "negative-glow lamps" since all the glow takes place at the negative electrode. This is evident on direct current, but on alternating current the reversal of polarity is so rapid that both electrodes appear to glow.

This phenomenon, until recently, was always associated with high voltage. During recent years, however, methods have been developed for so sensitizing electrode surfaces that the voltage for initiating and maintaining a discharge in these rare gases has been reduced to as low as 60 volts on direct current and about 42 volts on alternating current.

Glow Lamps as Light-Producing Devices

With the advent of low-voltage lamps the use of glow lamps on commercial lighting circuits became practical. Various sizes and types of glow lamps are now commercially available ranging from about 0.25 watt to 3 watts in power consumption.

Neon, as used in glow lamps, has proved to be the most efficient of the rare gases as a light-producing medium. In standard commercial glow lamps the light output is about 0.025 candlepower per milliampere. Their normal brightness is approximately 0.8 candlepower per square inch of sensitized electrode surface.

Since the light output of glow lamps is not great, they of course find only limited use as sources of illumination; they are valuable, however, as signals, pilots, and indicators of the presence of potential and the consequent condition of the device to which this potential is applied.

Why Glow Lamps Are Used

There are many reasons for the selection of glow lamps for innumerable uses. Chief among these reasons are:

1. Low current consumption
2. Insignificant heat
3. Reliability
4. Long life
5. Wide voltage range
6. Ruggedness

The low current consumption is indicated by the current range which is from only 0.002 ampere for the smallest to 0.030 ampere for the largest lamp as now manufactured.

It is apparent that the amount of heat generated at the low wattages that glow lamps consume is insignificant.

These lamps are extremely dependable. Unlike other lamps, they do not suddenly fail at the end of some definite period, but rather, slowly decrease in light output during a period of some 3,000 hours. During this time, slow disintegration causes the walls of the bulb gradually to darken, and this is accompanied by an increasing cathode drop, so that after 3,000 hours use the light output has dropped to 70 per cent of its initial value.

This process continues until the lamp is no longer useful as a light-giving device. Unless the lamps are broken or subjected to a mechanical shock sufficiently violent to short-circuit the electrodes, they will not suddenly fail. This reliability recommends them for many unusual uses.

Voltage

Glow lamps are not affected by voltage variations to the same degree as filament lamps. Their light output varies in almost direct proportion to the current, while life varies, roughly, inversely as the square of the current. It is therefore apparent that ordinary fluctuations experienced on commercial circuits will have but a small influence on either the life or light output.

There are three separate and distinct voltages to be considered when discussing glow lamp voltage, namely:

1. Breakdown
2. Maintaining
3. Supply

Breakdown voltage is that voltage at which the gas becomes ionized and begins to pass current. Below this voltage the lamp cannot be made to light and will not pass current.

Maintaining voltage is the voltage at which the lamp will remain lighted after having started. On alternating current the maintaining voltage is practically the same as the breakdown voltage, while on direct current it is approximately 15 volts lower. There is, therefore, a considerable difference between the supply voltage and that at which the lamp will maintain. This difference is absorbed by a ballast resistor which serves to stabilize the lamp.

Volt-Ampere Characteristics

All gaseous-conduction lamps, of which the glow lamp is one, have a negative or "run away" characteristic. Because of this characteristic, if the lamp were connected directly across a source of voltage sufficiently high to ionize the gas, the current would immediately rise to such proportions as to destroy the lamp. It is

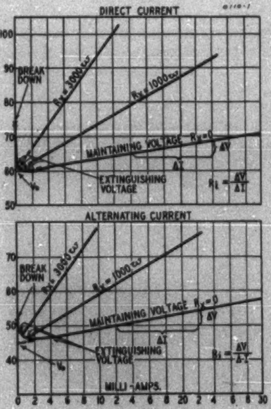

Figure 1. Characteristics of average S-14 two-watt lamp

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therefore essential that a limiting resistance be used in series with the lamp. In all lamps referred to here the resistor is built into the base. (See table III.)

The Electrical Characteristics of Negative-Glow Lamps

The negative-glow lamp, when carrying current, can be treated in an electrical circuit as the equivalent of a counter electromotive force with an internal resistance.

Thus for the purpose of calculation, the following symbols can be used:

Let

V = applied voltage
V_0 = the counter electromotive force
R_i = the internal resistance
R_x = the external resistance
I = the current in amperes

Note: R_x = the sum of all resistance outside of the lamp bulb and includes the resistance placed in the base, the resistance of the supply circuit, the meters used in testing, etc.

Then, the current in amperes is:

$$I = \frac{V - V_0}{R_i + R_x}$$

V_0 can accurately be determined from volt-ampere characteristics as the voltage at which the current is reduced to zero, but is approximately given by the minimum maintaining voltage or extinguishing voltage on either alternating current or direct current.

The internal resistance is mainly a function of electrode size, and secondly of the gas composition, pressure, and electrode construction. The exact value can be determined from the quotient of the change in maintaining voltage with corresponding change in discharge current. The a-c internal resistance is somewhat higher than d-c internal resistance, due to the high electrostatic capacity of the discharge in the cathode fall.

Before discharge occurs, that is, at voltages insufficient to cause breakdown, the lamp can be treated as a very high resistance and capacitance.

The Constants of Negative-Glow Lamps

Average values of the constants measured on standard a-c d-c neon glow lamps are included in table I. The external resistance indicated is the value to be used in series with the lamp to secure normal current at the specified alternating voltage. This is the resistance that is placed in lamps based with medium or candelabra screw bases.

A complete characteristic curve is shown and the graphical method of determining the constants of glow devices in figure 1.

Nonresistance Lamps

For some special uses, glow lamps are required without the resistance in the base. Lamps of this type are supplied mounted in double-contact bayonet bases, or bases of such design that they cannot be put into screw sockets, normally used on commercial circuits. With these lamps, it is necessary to use an external series resistor of a value equal to that normally built into the base of the corresponding lamp (see table III), or to operate these lamps in circuits whose natural impedance is sufficient to limit the lamp current to normal values.

Improper application of nonresistance lamps will result in the violent destruction of the lamp.

Glow lamps may be used on voltages many times greater than their normal rating, provided sufficient additional external resistance is used in series with the lamp. The proper value of external resistance to be used with standard lamps on various voltages is shown in table II, together with the proper wattage rating for the resistor. The wattage rating shown is in accordance with accepted practice for cool operation when used in enclosed devices. If used in open circulating air, a rating of approximately one-fourth these wattage ratings may be used.

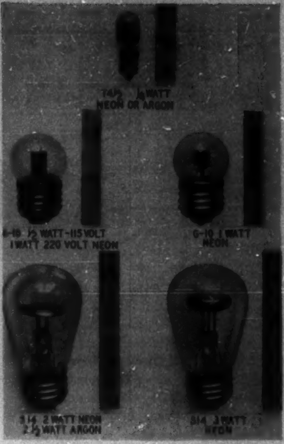


Figure 2

Example. If a one-quarter-watt 115-volt lamp is to be built into a housing or enclosure, and operated on a 300-volt circuit, the table indicates the use of a 93,000-ohm 1.5-watt resistor. However, if used in open air, a 0.5-watt resistor will suffice.

If lamps having no resistance in the base are operated on any of the voltages indicated in table II, the resistance value shown for a given voltage should be increased by the amount normally used in the base of the corresponding screw-base lamp as shown in table III.

Example. If, in the example given above, a nonresistance lamp were used, the external resistance value should be increased to 123,000 ohms.

Response Speed

The response speed of neon negative-glow lamps can be explained along three definite lines. First, the ignition or breakdown voltage of glow lamps varies with the duration of voltage application. This is because the discharge has a time lag, and the voltage required for breakdown increases the shorter the duration of application. It is known that the breakdown may increase 50 volts for a time application of 0.01 second and increases further for shorter time intervals.

Table I

Type Lamp	Supply Voltage	Breakdown Voltage	Minimum Maintaining Voltage	V_0	Ohms R_i	Ohms R_x
S-14-3 watt	120 a-c	53	50	43	310	2,200
	120 d-c	78	58	57	220	2,200
S-14-2 watt	120 a-c	52	49	45	420	3,500
	120 d-c	74	60	60	380	3,500
G-10-1 watt	120 a-c	45	44	43	450	5,900
	120 d-c	65	57	57	400	5,900
T-4½-¼ watt	120 a-c	57	56	44	2,600	30,000
	120 d-c	83	64	63	2,200	30,000
T-2	120 a-c	58	58			200,000
	120 d-c	82	62	60	8,500	200,000

Table II. External Resistor Values for Use on Various Voltages With Lamps Having Normal Resistance in Base

Line Voltage	150		200		250		300		350		400		450		500	
Lamp	Resistance (Ohms)	Wattage Rating	Resistance (Ohms)	Wattage Rating	Resistance (Ohms)	Wattage Rating	Resistance (Ohms)	Wattage Rating	Resistance (Ohms)	Wattage Rating	Resistance (Ohms)	Wattage Rating	Resistance (Ohms)	Wattage Rating	Resistance (Ohms)	Wattage Rating
*T*4½ 0.25 watt	17,500	0.5	42,000	0.5	68,000	1.0	93,000	1.5	120,000	1.5	150,000	2.0	170,000	2.0	200,000	3.0
*G*10 0.5 watt	9,000	0.5	21,000	1.5	33,000	2.0	45,000	3.0	58,000	3.0	70,000	5.0	82,000	5.0	95,000	5.0
*G*10—220 volts 1 watt							20,000	1.0	32,000	2.0	45,000	3.0	58,000	3.0	70,000	5.0
*G*10—110 volts 1 watt	3,000	1.5	7,000	4.0	11,000	5.0	15,000	10.0	20,000	10.0	24,000	10.0	28,000	20.0	32,000	20.0
S-14 2 watts	2,000	2.0	4,700	5.0	7,500	10.0	10,000	10.0	13,000	20.0	16,000	20.0	19,000	20.0	21,000	20.0
*S*14 3 watts	1,200	3.0	2,800	10.0	4,500	20.0	6,200	20.0	7,800	20.0	9,500	30.0	11,000	30.0	13,000	40.0

Second, the light-current response begins to lose a linear relationship in the neighborhood of 15,000 cycles. For certain uses, however, the current-light response has been used at much higher equivalent frequencies.

Third, after the discharge has been started, the lamp will restart at the maintaining voltage if the current is interrupted for a period not greater than 0.02 second; the normal breakdown is not reached until approximately 0.1 second elapses.

The Influence of Surrounding Light

Practically all glow lamps that start under 90 volts direct current have electrode surfaces so sensitive that they emit electrons photoelectrically when light falls on them. This current emission aids in reducing the breakdown of the lamp. For example, a standard glow lamp measured in the dark had a d-c breakdown of 75 volts and a maintaining voltage of 60 volts; measured in a normally lighted room the breakdown was 73 volts. The same lamp when placed several inches away from a 200-watt incandescent lamp burning at normal voltage broke down at 60 volts. Further increase in illumination did not alter the breakdown, but a sensitive galvanometer placed in the lamp circuit showed current changes with changes in illumination intensity.

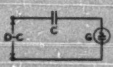


FIGURE 3

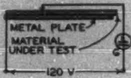


FIGURE 4

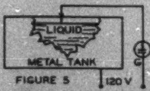


FIGURE 5

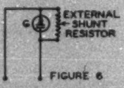


FIGURE 6

Figure 3. A glow lamp as an indicator of short circuits or high-resistance leaks in capacitors

Figure 4. A glow lamp used to test insulating paper, cloth, or other materials

Figure 5. A glow lamp used to check conductivity of liquids

Figure 6. Shunt resistors may be used to regulate the sensitivity of glow lamps

Effect of Temperature

Due to the sensitivity of the surface of the electrodes they will emit electrons even at normal room temperatures. When ambient temperatures become as high as 75 to 150 degrees centigrade the breakdown voltage is lowered to some extent. An effect of ambient temperatures of the range of 180 to 200 degrees centigrade is to evolve gases from the glass wall that poison the electrode surfaces. No appreciable effect of temperature can be observed in the range of 0 to 50 degrees centigrade.

The Nature of the Radiation From Glow Lamps

Neon negative-glow lamps radiate mainly in the red and yellow region of the spectrum; there is some radiation from the infrared that is invisible to the eye; a little radiation is spread from the yellow region into the violet. There is very little invisible radiation on the violet side between the violet and the transmission limit of the glass bulbs. The glass bulbs absorb whatever small amount of therapeutic ultraviolet radiation may be generated in the discharge.

Argon glow lamps, which consist of a mixture of gases, radiate mainly in the blue, violet, and near ultraviolet region. The negative glow appears blue-violet. The fact that there is strong radiation in the near ultraviolet region can be demonstrated by the fluorescent effects produced on uranium glass and many phosphorescent and fluorescent substances. The near ultraviolet radiation can be determined by other means too—for example, with a photo-cell, or by photographic means. Here again, the glass container absorbs any ultraviolet of therapeutic value, if it is generated. A recent trial of argon lamps in high-speed photographic printing machines has shown an enormous advantage over all other light sources.

Standard Lamps

A group of glow lamps is now available which covers a wide range of uses and

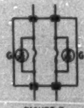


FIGURE 7

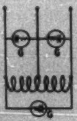


FIGURE 8

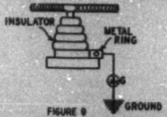


FIGURE 9

Figure 7. When a fuse blows the corresponding glow lamp lights

Figure 8. Serious unbalance is indicated by change in brightness or extinguishing of glow lamp

Figure 9. Capacitively connected to a high-voltage line the glow lamp indicates when line is alive

applications, as shown in figure 2 and the listing of pertinent data in table III.

Applications and Uses

The unique characteristics of the glow lamp lend themselves to many applications and uses which would be quite impossible with devices not possessing these characteristics. Its ability to produce light at low energy levels suggests its use as a signal or pilot in circuits where little current is available or where low current drain is desirable, as in radio circuits, "B"-battery-operated devices, and high-resistance circuits. It may be used in capacitively or inductively coupled circuits where little current may be drawn.

As a Test Lamp

The glow lamp makes an ideal all-around test lamp. It will indicate

1. Circuit alive—circuit continuity
2. By brilliancy, whether 110 or 220 volts
3. By flicker, if 25 cycle
4. Polarity (only negative glows on direct current)
5. Grounded neutral wire. Holding one lead in hand and touching separately each of three legs of a three-wire system with the other lead, the two live lines will cause a glow. Lamp will not light on grounded neutral.

High sensitivity suits the lamp to many high-resistance tests, such as checking leakage in capacitors (figure 3); leakage between tube elements; leaks in insulating material (figure 4); conductivity of liquids (figure 5) and many similar uses.

When the lamp proves to be too sensitive, showing up conditions which may not be particularly serious, the lamp sensitivity may be regulated by shunting a high resistance across it (figure 6).

As a Signal or Pilot Lamp

In many cases the lamp may be permanently connected in a production line or on a machine to indicate the occurrence of certain conditions. It might be used as an indicator of any of the conditions mentioned under the heading "As a Test Lamp".

As a low-current, dependable, long-life pilot lamp, the glow lamp is outstanding and has already been used extensively on all kinds of electrically heated or operated devices, as well as for exit lights, fire-station markers, and location signals for valves, switches, or other emergency control units.

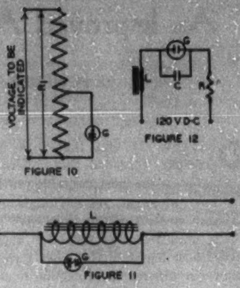
Some further pilot lamp applications are shown in the following sketches:

Figure 7. When fuse blows, the respective glow lamp will light.

Figure 8. As a control for polyphase lines. When one phase is overloaded, the respective glow lamp will dim or go out, depending on voltage drop.

Figure 9. To indicate that high-voltage lines are carrying current. The small capacity current will cause lamp to glow.

Figure 10. As a visual voltmeter. When desired voltage develops across R_1 the voltage across the lamp rises to the breakdown value causing it to light.



Figures 10–12

Many other adaptations may be made of the critical breakdown voltage of the glow lamp by connecting it across resistors, chokes, or current transformers so chosen that the normal drop across the lamp is below breakdown. Then, any condition which causes the voltage to reach the breakdown value will be indicated by a glow in the lamp. (Figure 11.)

As a Low-Frequency Generator

If a glow lamp is shunted across a capacitance and connected through a resistance to a d-c supply, the capacitor will charge up to the breakdown voltage and discharge through the lamp. By proper selection of the circuit constants, the lamp may be made to flash or in fact to oscillate at frequencies well up into the audio range. (Figure 12.) The addition of choke L_1 insures uniformity of oscillation and may vary from 1 to 30 henrys.

References

1. Gaseous Conduction Light From Low-Voltage Circuits, Moore. AIEE Transactions, volume 39, 1920, page 732.

2. Some Electrical Properties of Neon Lamps, Pearson and Anson. P.S.L. *Proc.*, volume 34, 1922, page 175.

Table III. Standard Glow Lamps

Watts	Volts	Starting Volts Below Minimum Marking on Base A-C	Starting Volts Below Minimum Marking on Base D-C	Current Rating Amperes at 120 Volts	Approx. Resistance in Base Ohms*	Bulb	Finish†	Screw Base	Service‡	Useful Life Hours at 120 Volts	Electrode Shape	Gas	Over-all Length (Inches)
3	115–125	60	25	0.030	2,200	*S*-14	Clear	Medium♯	A-c d-c	3,000	Round plates	Neon	$3^{9}/_{16}$
2	115–125	60	25	0.020	3,500	*S*-14	Clear	Medium♯	A-c d-c	3,000	Half round plates	Neon	$3^{9}/_{16}$
2	115–125	60	25	0.020	3,500	*S*-14	Sprayed red or yellow	Medium♯	A-c d-c	3,000	Half round plates	Neon	$3^{9}/_{16}$
$2^{1}/_{2}$	115–125	60	25	0.025	2,500	*S*-14	Clear only	Medium♯	A-c d-c	3,000	Half round plates	Argon	$3^{9}/_{16}$
$^{1}/_{2}$**	115–125	20		0.005	3,500	*G*-10	Clear	Medium♯	A-c only	3,000	Cylinder	Neon	$1^{63}/_{64}$
1	115–125	60	25	0.010	4,800	*G*-10	Clear	Medium♯	A-c d-c	3,000	Cylinder and helix	Neon	$1^{63}/_{64}$
$^{1}/_{4}$	115–125	60	25	0.002	30,000	*T*-$4^{1}/_{2}$	Clear only	Candelabra	A-c d-c	3,000	Hemisphere	Neon	$1^{1}/_{2}$
$^{1}/_{4}$	115–125	40		0.003	20,000	*T*-$4^{1}/_{2}$	Clear only	Candelabra	A-c only	1,000	Hemisphere	Argon	$1^{1}/_{2}$

* Lamps supplied without resistance only when equipped with double-contact bayonet base.

† Lamps listed in clear finish are available with sprayed red or yellow finish, except in the one-fourth-watt sizes.

‡ On direct current, only one electrode (negative) glows, but gives same candlepower as do both electrodes on alternating current.

** Supplied in one-watt size for 210–240-volt a-c or d-c service, medium screw base only.

♯ Available in skirted candelabra screw base.

Note: Normal light output is obtained at voltage range shown. Lamps will operate at reduced light output down to starting voltage shown.

An Improved A-C Pilot-Wire Relay

J. H. NEHER
MEMBER AIEE

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ASSOCIATE AIEE

DESPITE the fact that it is quite generally conceded that differential protection is the most desirable principle for line protection, yet few applications of relay systems of this type have been made until recently except for lines of very short length. The older forms of pilot-wire differential protection required three or four wires of low resistance; and even then, the systems were characterized by lack of sensitivity, and by high burden on the current transformers.

Because of the general availability of reliable communication facilities between power stations, d-c pilot-wire systems,[1] utilizing a single pair of telephone-type pilot wires, have been developed. These systems operate on the "directional comparison" principle, hence require a potential supply for the relays, and may be influenced by out-of-step conditions. The relays ordinarily used are not easily insulated to protect against induced voltages. While the requirement for a potential supply, the tendency to operate on out-of-step conditions, and the matter of relay insulation are not always influencing factors in the choice of a relay system, there are, of course, advantages in the use of a system which would not be so limited.

A differential system which uses a single telephone pair as a pilot circuit is definitely limited by the energy which can be transmitted over the pilot wires. This energy limit is far below that required by the older forms of differential protection; and some means must be found for making the single relay system operative on all types of faults. Such a relay system has been developed[2] in which these difficulties have been solved by the use of supersensitive relay elements and networks.

This paper describes a similar relay system in which one standard relay element is used at each terminal. It is operative over a single telephone pair and gives full protection for any type of fault without the need for a potential supply. In addition, there is adequate insulation between the relay element and the pilot wires. The relay is of the current product type, in which the energy required to operate the element is provided largely by the polarizing current; thus permitting the use of a relatively low energy level in the pilot-wire circuit.

In many respects this system is similar to the "Translay" system[3,4] used successfully in Europe for many years. Of its several improvements, an outstanding one is the use of the modern high-speed induction-cylinder relay element.

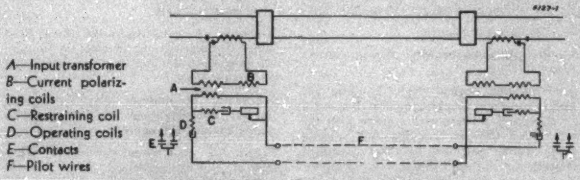

Figure 1. Single-phase schematic diagram of relay circuits

Basic Principles of the Pilot-Wire System

Fundamentally, the system is of the opposed-voltage type in which substantially no current flows in the pilot wires under normal, or through fault conditions.

Figure 1 shows the fundamental circuits on a single-phase basis. At each end of the line, the current transformer energizes the following two elements:

1. The polarizing coils of a wattmetric type of relay element.
2. The primary of the "input transformer" which is a reactor with a secondary winding.

Current flow in the primary of the input transformer causes a voltage to be induced in the secondary. This voltage is applied to a shunt-connected restraining circuit, and to the operating circuit which is in series with one of the pilot wires.

The current which flows in the restraining coil of the relay reacts with the polarizing coil current to produce a torque in the contact-opening direction.

The current which flows in the operating-coil circuit is dependent upon the voltages of the input transformers at both ends of the line. Under normal or through fault conditions, these voltages oppose each other, and practically no

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Paper 40-127, recommended by the AIEE committee on protective devices, and presented at the AIEE Pacific Coast convention, Los Angeles, Calif., August 27–30, 1940. Manuscript submitted June 20, 1940; made available for preprinting July 11, 1940.

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[1] For all numbered references see list at end of paper.

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3. NEON GLOW DISCHARGE LAMPS ON D.C., McGregor, Morris, Dowty, and Privett. *Anne d. Phys.*, 1922, page 625.

4. NEON GLOW DISCHARGE LAMPS ON A.C., Brockbank and Ryall. *Electrician*, volume 90, 1923, page 4.

5. PEAK VOLTAGE MEASUREMENTS BY MEANS OF NEON GLOW LAMPS, L. E. Ryall. *World Power* (England), May 1924.

6. DIFFUSION OF IONS IN NEON TUBES, Foulke and McCurdy. *Phys. Rev.*, volume 23, 1924, page 776.

7. EXPONENTIAL RISE OF DISCHARGE CURRENTS IN NEON, Foulke and McCurdy. *Phys. Rev.*, volume 23, 1924, page 776.

8. A VISUAL WARNING SIGNAL FOR THE ROENTGENOSCOPIC ROOM, F. W. Bishop. *Am. Journal of Roentgenology and Radium Therapy*, volume 23, 1930, number 3.

9. ALTE UND NEUE GLIMMLAMPENFORMEN, W. Pohle and H. Straehler. *Osram*, volume 5, 1935, number 8–9.

10. ANWENDUNGS-MÖGLICHKEITEN DER GLIMMLAMPE, Doctor Hans Ewest. *Osram*, Berlin, 1935.

11. A COLD CATHODE ARC DISCHARGE TUBE, Germeshausen and Edgerton. ELECTRICAL ENGINEERING, volume 55, 1936, page 790.

12. A NEW TYPE VOLUME INDICATOR, S. Read, Jr. *Jour.* S.M.P.E., April 1936.

13. VACUUM TUBES IMPROVE SELECTIVE RINGING, L. J. Stacy. *Bell Lab. Record*, volume 15, number 4, 1936, page 111.

14. COLD CATHODE GAS FILLED TUBES AS CIRCUIT ELEMENTS, S. B. Ingram. ELECTRICAL ENGINEERING, July 1939, page 342.

15. ON ELECTRONIC VOLTAGE STABILIZERS, Hunt and Hickman. *Rev. Sci. Inst.*, January 1939.

Discussion

Discussion will be found in the 1941 annual TRANSACTIONS volume and in the June 1941 SUPPLEMENT to ELECTRICAL ENGINEERING—TRANSACTIONS SECTION.

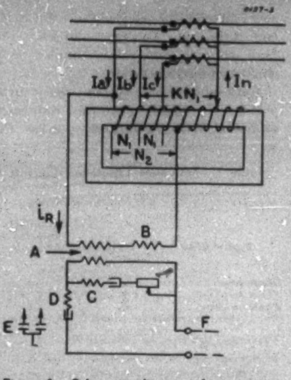


Figure 2. Schematic diagram of one termina showing connections of autotransformer

A, B, etc., same as figure 1

current flows in the pilot wires. With an internal fault, the voltages are additive; and the resultant current flowing in the operating circuit reacts with the polarizing current, and produces a torque in the contact-closing direction of sufficient magnitude to overcome the restraining torque. On an internal fault, the pilot-wire current is proportional to the sum of the fault currents entering each end.

The wattmetric type of element used is of the multipole induction-cylinder construction which has previously been described before the AIEE.[5] Three of the potential coils are used in the operating circuit, and the fourth furnishes the restraint. The four current polarizing coils are connected in series.

Purpose of Capacitors in Operating and Restraining Circuits

Since the torque of a wattmetric type of relay is proportional to the product of two currents and the sine of the angle between them, maximum torque occurs when the two currents are displaced by 90 degrees. The secondary voltage of the input transformer is displaced by 90 degrees from the current in the primary. Thus, for maximum torque, the currents flowing in the operating circuits must be in phase with the voltages producing them. For this purpose, capacitors are used in both the restraining and operating circuits; both circuits being resonant at rated frequency.

Besides producing a high-efficiency relay, other beneficial effects are obtained from the resonant operating circuit. The current which flows in the operating circuit, because of the electrostatic capacity of the pilot wires, is highly leading; and therefore, is almost in phase with the polarizing current. In consequence, substantially no operating torque is produced by pilot-wire capacity current. Hence, greater sensitivity is possible without interference by capacity current. The resonant circuit suppresses harmonic currents in the pilot wires; especially the higher noise-producing harmonics. Also, the effect of currents caused by switching transients is greatly reduced, since such currents usually contain a variety of harmonics. Thus, the pilot circuit is both free from outside interference, and at the same time, does not itself produce troublesome telephone interference. Since differences in current transformers usually involve a phase-angle shift as well as a change in the ratio, the voltages applied to the pilot wires may be out-of-phase during an external fault. However, the circulating current which flows through the pilot wires because of phase shift is at an angle unfavorable to the development of full relay torque.

Purpose of the Wattmetric Type of Relay Element

Since the operating torque of the relay element depends upon both the polarizing and the pilot-wire currents, most of the driving energy of the relay can be concentrated in the polarizing coils, thus making it possible to obtain a satisfactory torque level with a small amount of power transmitted over the pilot wires. Thus, special pilot wires of extremely low resistance are not required.

Open and Short-Circuited Pilot Wires

This pilot-wire system is subject to the disadvantage common to all pilot-wire systems, namely, incorrect operation when the pilot wires are either open-circuited or short-circuited. An open-circuited pilot wire will prevent tripping on an internal fault; whereas short-circuited pilot wires will cause tripping on external faults.

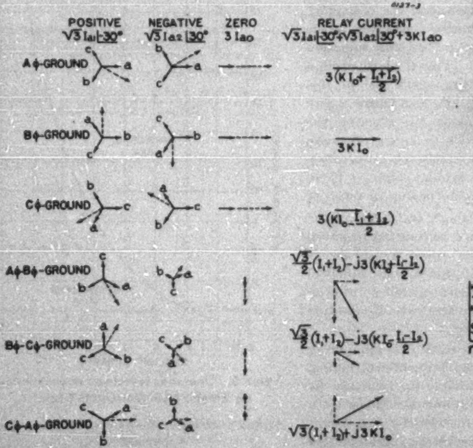
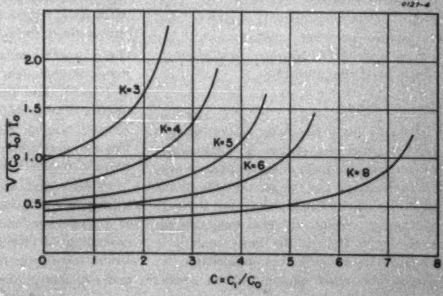

POSITIVE
$\sqrt{3}I_{a1}\angle{-30°}$
NEGATIVE
$\sqrt{3}I_{a2}\angle{30°}$
ZERO
$3I_{a0}$
RELAY CURRENT
$\sqrt{3}I_{a1}\angle{-30°}+\sqrt{3}I_{a2}\angle{30°}+3KI_{a0}$
Aϕ-GROUND
Bϕ-GROUND
Cϕ-GROUND
Aϕ-Bϕ-GROUND
Bϕ-Cϕ-GROUND
Cϕ-Aϕ-GROUND
a
b
c
$3(KI_0+\frac{I_1+I_2}{2})$
$3KI_0$
$3(KI_0-\frac{I_1+I_2}{2})$
$\frac{\sqrt{3}}{2}(I_1+I_2)-j3(KI_0+\frac{I_1-I_2}{2})$
$\frac{\sqrt{3}}{2}(I_1+I_2)-j3(KI_0-\frac{I_1-I_2}{2})$
$\sqrt{3}(I_1+I_2)+j3KI_0$

Figure 3. Vector diagrams of relay currents

Figure 4. Pickup current on phase C-ground fault (zero pilot-wire resistance)

Method of Obtaining Polyphase Protection

Thus far, the pilot-wire relay system has been considered on a single-phase basis only, in order to show that it operates in response to circulating current



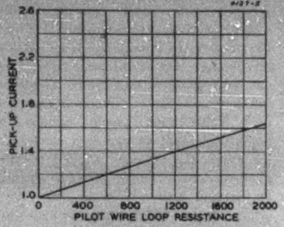


Figure 5. Effect of pilot-wire resistance on pickup current

Pickup current plotted as multiple of pickup current with zero pilot-wire resistance

in the pilot wires reacting with the polarizing current. In order that a single pair of relays can furnish protection for all phase and ground faults over a single pair of pilot wires, it is necessary to provide some means of impressing on the relay system a single electrical quantity for any type of fault.

It is often true that the simplest device also provides the most satisfactory performance. In this instance, a simple tapped autotransformer is the means of providing the single electrical quantity.

Figure 2 shows the connections to and from the autotransformer. The autotransformer is interposed between the current transformers and the remaining relay circuits. Considering the two left-hand sections of the autotransformer, it is obvious that its output, during phase faults, is proportional to $(I_a - I_c)$. During ground faults the residual current flows through the right-hand section, producing a component of output current which is proportional to I_n times factor K. K is a constant relating the turns in the phase and ground windings, and is adjustable by means of taps. The total output current of the autotransformer is therefore proportional to $I_a - I_c + KI_n$ (equation 3, appendix I).

Figure 3 shows vectorially the relations between positive, negative, and zero-phase-sequence currents, and the output current of the autotransformer on faults involving ground.

Blind Spots

When a pilot-wire relay system utilizes any network to obtain a single-phase output current from a three-phase source, there is always the danger that the relays at both ends of the line may fail to operate under certain internal fault conditions, even though the fault current is apparently of sufficient magnitude to exceed the relay pickup. Such conditions may be termed "blind spots". Any such relay system must be carefully analyzed to determine its limitations in this respect. In general, "blind spots" are possible only on faults involving ground.

With the relay system described above, one of the following conditions is necessary for a "blind spot" to occur:

1. A low autotransformer output current at *each* end of the line.
2. A low pilot-wire current.
3. An unfavorable phase angle between the pilot-wire and polarizing currents at *each* end of the line.

It is possible (although usually avoidable by properly selecting constant K) to have a low autotransformer output at one end of the line. However, with a solidly grounded neutral system, it is not possible to have a low autotransformer output at each end of the line simultaneously; nor are any of the other conditions possible.

In unusual instances, it may not be possible to avoid a low polarizing current at one end of the line. The relay at the other end of the line will operate, however, and after the circuit breaker at that end opens, the relay at the first end will operate sequentially, provided, of course, that there is sufficient current for operation on single-end-feed.

A low autotransformer output current at one end is caused by unequal distribution of positive and zero-phase-sequence currents. For example, it can be seen (figure 2) that with a C phase-ground fault, the current in phase C can divide in such a manner that I_n will flow in one direction, and the remainder of the current in phase C will flow in the opposite direction. (See also equation 29, appendix II.) The C phase-ground fault is the limiting condition in this respect (appendix II), and figure 4 provides a ready means for selecting the constant K in order to insure adequate torque for simultaneous relay operation with a C phase-ground fault. These curves show the relay pickup in terms of the geometric mean $(\sqrt{(c_0I_0)(I_0)})$ of the zero-phase-sequence current fed from one end (c_0I_0), and the total zero-phase-sequence current (I_0), for various values of K and $c = c_1/c_0$, where c_1 and c_0 are positive and zero-phase-sequence distribution factors, respectively.

In those cases in which it is apparently impossible to avoid sequential tripping, it is sometimes possible to eliminate the cascading of circuit breaker times by a simple arrangement whereby the operation of either pilot-wire relay immediately short-circuits the pilot-wire circuit at its end of the line. To the second relay, the fault then appears to be similar to a single-end-feed condition.

With an overhead system, the neutral points of which are all resistance grounded, it is theoretically possible for the relays at one or both ends to develop insufficient or negative torque on a C phase-A phase-ground fault. This can be noted from equation 35 given in appendix II. The large number of variables in this equation prevents a simple treatment such as has been given above in the case of a single-phase-to-ground fault. A study of this equation indicates, in general, that if simultaneous operation is obtained at both ends on all phase-to-phase faults, sufficient torque will be available to operate both relays under any practical case of resistance grounding, although sometimes the operation may be sequential.

Table I. Minimum Operating Currents

Type of Fault	Phases	CT Secondary Pick-Up Amperes
Three-phase	A-B-C	4
Phase-to-phase	A-B and B-C	7
Phase-to-phase	A-C	3.5
Single-phase-to-ground	C-ground**	1.0 to 3.5*

* Five taps to adjust for constant K.

** Pick-ups slightly less on phases A and B.

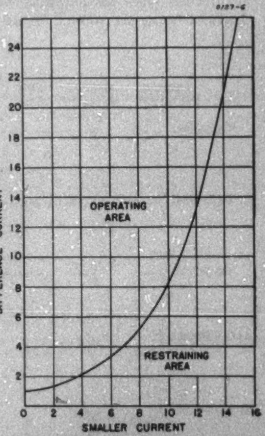


Figure 6. Operating-restraining characteristics on through faults (currents in phase)

Plotted as multiples of single-end-feed pickup current

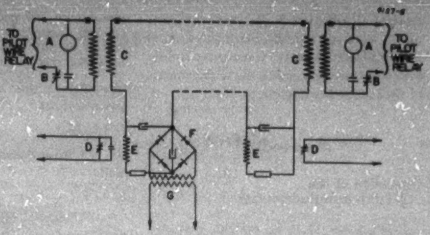


Figure 8. Schematic diagram of automatic supervisory equipment

A—Voltmeter
B—Switch
C—Insulating transformer
D—Contacts
E—Relay coil
F—Rectifier
G—Supply transformer

Characteristics of the Relay System

Minimum Operating Current

Based on single-end-feed, table I gives the minimum operating currents on various types of faults. These values are based on zero pilot-wire resistance. Since the operating coil current decreases as the pilot-wire resistance increases, the pickup current will increase. The effect of pilot-wire resistance upon pickup is shown in figure 5.

Operating-Restraining Characteristics

If saturation of the mixing transformers could be neglected, the slope of the operating-restraining curve on a through fault would be approximately 50 per cent; that is, differential current in per cent of the smaller of the restraining currents. However, saturation of the mixing transformer causes the slope to increase rapidly at the higher currents as shown in figure 6.

This characteristic permits the use of unmatched current transformers or burdens. Since the operating current varies with the resistance of the pilot wire, means are provided to adjust the restraining current so that substantially the same curve applies regardless of the pilot-wire resistance.

Current Transformer Burden

The three-phase burden is less than one volt-ampere per phase at five amperes. The single-phase-to-ground burden varies between 12 and 57 volt-amperes at five amperes, depending upon the ground tap (constant *K*) used.

Operating Time

The time-current curve is shown in figure 7. This curve is based upon single-end-feed. Operating times are slightly less when additional operating current is received over the pilot wires from the other end of the line.

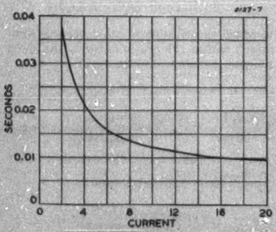


Figure 7. Operating time

Current plotted as multiple of minimum pickup with single-end-feed

Pilot-Wire Circuit

The reliability of any pilot-wire protective scheme is dependent upon continuity of the pilot channel. Interruptions may occur from electrical and mechanical causes which include the following:

1. Induction from power lines.
2. Difference in station ground potentials.
3. Lightning.
4. Contact with foreign objects.
5. Unauthorized tampering with pilot circuits.

Experience has indicated that the use of lead-sheath cable of sufficient insulation and mechanical strength practically eliminates trouble from the first four hazards. Elimination of interruptions caused by trouble-shooting, etc., can be accomplished by treating the pilot-wire circuit as any high-voltage power circuit, thus requiring clearance orders from the dispatcher. If the circuits are leased, co-ordination is necessary between the lessee and lessor.

Insulation stress caused by induction can be reduced by the use of different routes for the power lines and pilot cable.

The difference between station ground potentials is largely concentrated near the power station. If this potential difference is large, it may be economical to use two different levels of cable insulation strength, using the higher level close to the stations. In such instances, neutralizing transformers[8] can also be considered.

Whether the voltage is derived from induction, or rise in station ground potential, each relay is protected by means of an insulating transformer tested at over 10,000 volts between the pilot-wire winding and ground, and between the pilot-wire winding and the relay winding.

In addition to insulation problems, induction may be the cause of further difficulty, namely, incorrect operation because of a voltage difference between the pilot wires. If the pilot-wire circuit is the preferred twisted pair, this voltage difference does not have to be considered. However, if the pilot wire circuit is other than a twisted pair, it should be determined that the voltage between wires is less than five volts under maximum fault conditions.

Pilot-Wire Supervision

Since the pilot wires are such an important part of the relay system, manual pilot-wire checking equipment is furnished with each relay. The equipment consists of a voltmeter and a switch (*A* and *B* in figure 8). When the switch is operated, the voltage transmitted from the relay at the opposite end of the pilot wires is read. A positive reading indicates a sound operating circuit including both insulating transformers and the relay at the other end of the line. A zero voltage reading indicates a short circuit or open circuit, provided, of course, there is sufficient load current to produce a voltmeter deflection.

Voltmeter readings taken at the same time as the usual periodic readings should provide an ample check of the pilot circuit. However, some operators may desire a continuous automatic check of the pilot wires. The automatic supervisory equipment is shown schematically in figure 8. A small direct current is circulated continuously over the pilot wires. Sensitive polarized relays at each end indicate a short-circuited or an open-circuited pilot channel, and close alarm circuits at each station. The source of direct current is a rectifier supplied from a small transformer. Each relay is tested at 10,000 volts between the pilot-wire circuit and both low-voltage circuits, and between the pilot-wire circuit and the relay case.

Conclusion

In conclusion, it is felt that the relay system described, offering protection against all types of faults, and consisting of a relatively simple arrangement of standard components, constitutes a definite step toward meeting the requirements of the ideal relay scheme.[7]

Appendix I

Operation of the Mixing Autotransformer

From figure 2 it is evident that the ampere-turn balance in the mixing autotransformer will always be as follows:

$$N_2 i_R = N_1[I_a(1+K)+I_bK+I_c(K-1)] \quad (1)$$

Since

$$I_n = I_a+I_b+I_c \quad (2)$$

$$I_R = I_a-I_c+KI_n \quad (3)$$

where

$$I_R = \frac{N_2}{N_1}i_R \quad (4)$$

is the current fed to the relay element, referred to the input side of the mixing autotransformer.

Equation 3 may be resolved into its phase-sequence components as follows:

$$I_a = I_{a1}+I_{a2}+I_{a0} \quad (5)$$

$$\begin{aligned} I_c &= I_{c1}+I_{c2}+I_{c0} \\ &= I_{a1}\underline{/120^\circ}+I_{a2}\underline{/-120^\circ}+I_{a0} \end{aligned} \quad (6)$$

$$I_n = 3I_{a0} \quad (7)$$

$$I_R = \sqrt{3}I_{a1}\underline{/-30^\circ}+\sqrt{3}I_{a2}\underline{/30^\circ}+3KI_{a0} \quad (8)$$

The derivation of the currents fed to the relay elements under the various fault conditions will be made on the assumption that the positive and negative-phase-sequence networks are identical, and that the positive and zero-phase-sequence networks have the same phase angle, except as noted hereafter.

The resultant relay currents for all types of faults will be expressed in terms of three fundamental currents

$$I_P = \frac{\sqrt{3}E}{2Z_1} \quad (9)$$

which is the total phase-to-phase fault current

$$I_N = \frac{3E}{2Z_1+Z_0} \quad (10)$$

which is the total ground current for a single-phase-to-ground fault, or

$$I_{N'} = \frac{3E}{Z_1+2Z_0} \quad (11)$$

which is the corresponding total ground current on a two-phase-to-ground fault. In equations 9, 10, and 11, the voltage, currents, and impedances can be considered to be scalar quantities, since it has been assumed that Z_1 and Z_0 have the same phase angle.

The following expressions for the resultant relay currents on faults not involving ground are self-evident from equation 3. (C_1 and C_0 are the current distribution factors for the positive and zero-phase-sequence networks, respectively.)

(*a*). Three-phase and *C* phase-*A* phase faults

$$I_R+2c_1I_P \quad (12)$$

(*b*). *A* phase-*B* phase and *B* phase-*C* phase faults

$$I_R = c_1I_P \quad (13)$$

The derivation of the relay current on faults involving ground can best be made from equation 8 and the vector diagram shown in figure 3.

(*c*). *A* phase-ground fault

$$I_R = (Kc_0+c_1)I_N \quad (14)$$

(*d*). *B* phase-ground fault

$$I_R = Kc_0I_N \quad (15)$$

(*e*). *C* phase-ground fault

$$I_R = (Kc_0-c_1)I_N \quad (16)$$

In the case of two-phase-to-ground faults the following relationships have been employed which are justifiable in view of the assumption that $Z_2 = Z_1$. Here again, I_1 and I_2 are scalar quantities, and are the magnitudes of I_{a1} and I_{a2}, respectively.

$$I_1+I_2 = \frac{(Z_1+Z_0)E}{Z_1(Z_1+2Z_0)}+\frac{Z_0E}{Z_1(Z_1+2Z_0)} = \frac{E}{Z_1} \quad (17)$$

$$I_1-I_2 = \frac{E}{Z_1+2Z_0} \quad (18)$$

(*f*). *A* phase-*B* phase-ground fault

$$I_R = c_1I_P-j(Kc_0+c_1/2)I_N' \quad (19)$$

(*g*). *B* phase-*C* phase-ground fault

$$I_R = c_1I_P-j(Kc_0-c_1/2)I_{N'} \quad (20)$$

(*h*). *C* phase-*A* phase-ground fault

$$I_R = 2c_1I_P+jKc_0I_{N'} \quad (21)$$

Appendix II

Relay Torque Equations

From figure 1 and the discussion given in the text it is apparent that the torque developed by the relay in terms of the pick-up torque t can be expressed as:

$$T = \frac{I_R}{t}\left(\frac{N_1}{N_2}\right)^2 k\left[\frac{I_R-I_{R'}\cos\theta}{2R_o+R_p}-\frac{I_R}{3R_r}\right] \quad (22)$$

where

I_R = the current fed to the relay element at the end under consideration

$I_{R'}$ = the current fed to the relay element at the other end

θ = the phase angle between I_R' and I_R

k = transreactance of the pilot-wire input transformers

R_o = resistance of relay operating circuit

R_r = resistance of relay restraint circuit

R_p = resistance of pilot-wire loop

With the restraint circuit properly adjusted for 50 per cent slope, equation 22 becomes, with the proper values substituted:

$$T = \frac{I_R}{32.5F}[I_R-I_{R'}\cos\theta-I_R/3] \quad (23)$$

where the effect of the pilot-wire resistance is given by

$$F = 1+\frac{R_p}{1,140} \quad (24)$$

By the use of equation 9 through 16, and 19 through 21, equation 23 may be solved to give the torque developed on internal faults ($\theta = 180$ degrees) for the various phase and ground combinations. The expressions for pilot-wire currents $(I_R+I_{R'})$ are similar to the corresponding expressions for I_R with c_1 and c_0 made unity where they occur.

When I_R and $(I_R+I_{R'})$ have reactive components no torque can result from the cross-product terms.

(*a*). Three-phase and *C* phase-*A* phase faults

$$T = 4\frac{I_P^2}{32.5F}c_1(1-c_1/3) \quad (25)$$

(*b*). *A* phase-*B* phase and *B* phase-*C* phase faults

$$T = \frac{I_P^2}{32.5F}c_1(1-c_1/3) \quad (26)$$

(*c*). *A* phase-ground fault

$$T = \frac{I_N^2}{32.5F}(Kc_0+c_1)[K+1-(1/3)(Kc_0+c_1)] \quad (27)$$

(*d*). *B* phase-ground fault

$$T = \frac{I_N^2}{32.5F}Kc_0(K-Kc_0/3) \quad (28)$$

(*e*). *C* phase-ground fault

$$T = \frac{I_N^2}{32.5F}(Kc_0-c_1)[K-1-(1/3)(Kc_0-c_1)] \quad (29)$$

It is apparent here that reduced or negative torque can be obtained, depending upon the relative values of Kc_0 and c_1. Equation 29 can be rewritten in the following form:

$$T = \frac{I_N^2}{32.5F}c_0(1-c_0/3)\left[K\left(K-\frac{3-c_1}{3-c_0}\right)-\frac{c_1}{c_0}\left(K-\frac{3-c_1}{3-r_0}\right)\right] \quad (30)$$

in which the solution of the expression contained within the brackets forms the basis of figure 4 discussed in the text.

(*f*). *A* phase-*B* phase-ground fault

$$T = \frac{I_P^2}{32.5F}c_1(1-c_1/3)+\frac{1}{4}\frac{I_{N'}^2}{32.5F}(2Kc_0+c_1)\times [2K+1-(1/3)(2Kc_0+c_1)] \quad (31)$$

(*g*). *B* phase-*C* phase-ground fault

$$T = \frac{I_P^2}{32.5F}c_1(1-c_1/3)+\frac{1}{4}\frac{I_{N'}^2}{32.5F}(2Kc_0-c_1)\times [2K-1-(1/3)(2Kc_0-c_1)] \quad (32)$$

Here again, it is possible that a negative torque from the second half of equation 32 will offset the positive torque of the first part of the equation. It is apparent, however, that the relationships between Kc_0 and c_1 required to render equation 29 positive will also make the second part of equation 32 positive.

(*h*). *C* phase-*A* phase-ground fault

$$T = 4\frac{I_P^2}{32.5F}c_1(1-c_1/3) + \frac{I_{N'}^2}{32.5F}Kc_0(K-Kc_0/3) \qquad (33)$$

In the case of a two-phase-to-ground fault with the system grounded through a resistance, the previous assumption that Z_1 and Z_0 have the same phase angles is no longer valid. The introduction of an additional resistance component in Z_0 will advance the components containing $I_{N'}$ in phase position. This will increase the values of torque developed from equations 31 and 32 but may decrease it in the case of equation 33.

Although equation 21 was derived on the basis that I_P and $I_{N'}$ were scalar quantities, it can be shown that the equation is also valid when I_P and $I_{N'}$ are vectors. If the resistive and reactive components of Z_1 and Z_0 are R_1, X_1 and R_0, X_0 respectively, then the angle of advance of $I_{N'}$ is given by the following expression

$$\phi = \tan^{-1}\frac{X_1}{R_1} - \tan^{-1}\frac{X_1+2X_0}{R_1+2R_0} \qquad (34)$$

Equation 21 in appendix I becomes

$$I_R = 2c_1I_P - I_{N'}Kc_0 \sin\phi + jI_{N'}Kc_0\cos\phi \qquad (21a)$$

and equation 33 becomes

$$T = 4\frac{I_P^2}{32.5F}c_1(1-c_1/3) - \frac{2KI_PI_{N'}}{32.5F} \times [c_1(1-c_0/3) + c_0(1-c_1/3)]\sin\phi + \frac{I_{N'}^2}{32.5F}Kc_0(K-Kc_0/3) \qquad (35)$$

in which a negative torque term now appears.

Bibliography

1. RELAY SYSTEMS UTILIZING COMMUNICATION FACILITIES, J. H. Neher. ELECTRICAL ENGINEERING, March 1933, pages 162–8.

2. A SINGLE-ELEMENT DIFFERENTIAL PILOT-WIRE RELAY SYSTEM, E. L. Harder and M. A. Bostwick. *The Electric Journal*, November 1938, pages 443–8.

3. RECENT DEVELOPMENTS IN THE PROTECTION OF THREE-PHASE TRANSMISSION LINES AND FEEDERS, T. W. Ross and H. G. Bell. IEE *Journal*, volume 68, July 1930, pages 801–52.

4. THE METROPOLITAN-VICKERS "TRANSLAY", T. W. Ross. *Metropolitan-Vickers Gazette*, November 1929, pages 414–19.

5. A SINGLE-ELEMENT POLYPHASE DIRECTIONAL RELAY, A. J. McConnell. ELECTRICAL ENGINEERING (AIEE TRANSACTIONS), January 1937, pages 77–80.

6. PILOT-WIRE RELAY PROTECTION, E. E. George and W. R. Brownlee. ELECTRICAL ENGINEERING (AIEE TRANSACTIONS), November 1935, pages 1262–9.

7. RELAYING OF HIGH-VOLTAGE INTERCONNECTIONS, H. P. Sleeper. ELECTRICAL ENGINEERING, June 1933, pages 402–08.

Vario-Losser Circuits

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Synopsis: Methods are given for the design of circuits using "varistor" elements to secure specified variations of loss as a function of input signal. The types of circuits include compressors and expandors using shunt, series, and lattice networks. Formulas for steady-state transmission and distortion in terms of varistor characteristics are developed.

THE functions performed in the telephone plant by various automatic gain-control devices such as vogads and compandors have been described in a number of published articles. These devices have the common property that the gain or loss of a portion of the circuit is made to vary in accordance with changes in the transmitted signal. The term vario-losser is used to designate that part of the system in which the change in loss or gain actually occurs. Vario-lossers operating by virtue of the change in gain or plate impedance of vacuum tubes as a function of varying grid bias have been described elsewhere.[1,2,3] In this paper there are discussed the design considerations for certain simple vario-losser circuits which make use of "varistors," that is, nonlinear elements such as copper oxide and silicon carbide in which the resistance is a function of current or voltage, to furnish the desired variation in loss. These circuits have the advantage of low initial cost, stable properties, long life, and simple maintenance requirements.

It should be kept in mind that in general it is not the instantaneous value of the signal wave which controls the loss or gain of the vario-losser, but some more slowly varying quantity such as the "volume" or "envelope." Since "volume"[*] controlled circuits require control arrangements of a very high degree of complexity, which do not necessarily affect the vario-losser design, we shall limit our discussion to considerations of "envelope" control. The same considerations would of course enter into "volume" control design.[4]

In "envelope" control, a control current or voltage is commonly derived by rectifying the signal and sending the rectified output through a network (for example, a shunt capacitor and series resistance) which discriminates against the higher frequencies. For our purposes, we shall refer to a current or voltage obtained in this way as representing the envelope of the signal and designate its amplitude by capital letters *I* or *V*. The instantaneous values will be designated by lower case letters *i* and *v*. Furthermore we shall assume that the envelope varies at such a slow rate that what is under consideration is essentially the steady state.

The operating point of and hence the loss produced by the varistors is determined by the amount of current or voltage bias furnished by the control current, which is in turn fixed by the envelope. The range of variations of the instantaneous signal about the operating point is limited to a region in which the varistors are substantially linear. Under these conditions, the wave form of the signal is preserved, and we may say that the ratio of instantaneous signal output to instantaneous signal input is the same as that of envelope output to envelope input. As general equations therefore, we write

$$v_2/v_1 = V_2/V_1 = A \qquad (1)$$

where the subscripts 1 and 2 refer to input and output voltages, respectively and *A* is a voltage transfer ratio determined by the envelope. If the value of *A* is determined by V_1, the input voltage, the device is said to be *forward acting;* if *A* is determined by V_2, the device is said to be *backward acting*. The usual law of operation desired is that in which the output envelope varies as some power of the input envelope, that is:

$$V_2 = kV_1^n \qquad (2)$$

where *k* and *n* are constants. If *n* is less than unity, the device is called a *compressor*, since a given range of variation of input envelope produces a smaller range of variation in output envelope. When *n* is greater than unity, the device is called an *expandor*, since the range of variation of envelope is increased by the device.

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[1] For all numbered references, see list at end of paper.

[*] The term "volume" is used here to designate that quantity measured by the volume indicator.

The value of n is called the *control ratio.*[5] Obviously if n is equal to unity the device has a fixed loss or gain and does not operate as a vario-losser.

From (1) and (2),

$$A = kV_1^{n-1} \tag{3}$$

and

$$A = k^{1/n}V_2^{1-1/n} \tag{4}$$

Equation 3 may be regarded as the fundamental equation for forward acting vario-lossers, and 4 as the corresponding equation for backward acting vario-lossers. In applications to the design of specific circuits, however, it is more convenient to have the constant k appearing above expressed in terms of the loss or gain at some particular value of load, for example, the maximum operating value. We shall represent the reference condition by a zero subscript, that is, $A = A_0$, $V_1 = V_{10}$, $V_2 = V_{20}$, etc. Equations 3 and 4 then become

$$A = A_0(V_1/V_{10})^{n-1} \tag{3a}$$

$$A = A_0(V_2/V_{20})^{1-1/n} \tag{4a}$$

If the input and output impedances terminating the vario-losser are equal, the loss in decibels is 20 $\log_{10}(1/A_0)$ when the input voltage is V_{10} and the output voltage is V_{20}.

Some discussion should be inserted here concerning the actual current-voltage characteristics available in typical varistor elements. In order that a vario-losser may operate satisfactorily over the range of inputs desired, varistor elements must be provided whose resistances can be controlled in the desired manner over a similar range. Anticipating some of the results derived in the next section, dealing with specific circuit arrangements, we point out that a particularly suitable type of curve for varistors from the theoretical standpoint is that in which the current is an exponential function of the voltage over a considerable range. That such behavior can actually be realized to within small error except for low currents is indicated by the measured curve, figure 1, for copper oxide. Semilogarithmic co-ordinate paper is used, which yields a straight-line graph when the function plotted is exponential. Circuits in which the exact form of the varistor characteristic is not critical also exist; they will be discussed under the section on marginal detector types.

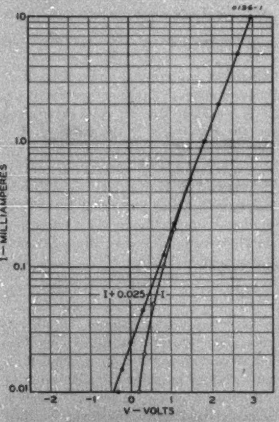


Figure 1. Current versus voltage for typical copper-oxide varistor

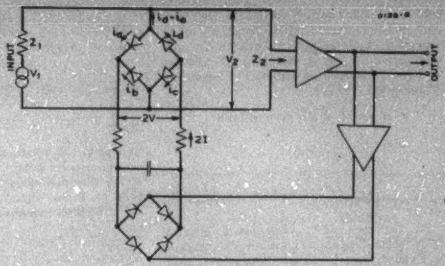


Figure 2. Shunt-type backward-acting vario-losser

It should be noted also that in supplying the bias derived from the envelope for controlling the varistor resistances, minimum complexity is secured if the biasing current or voltage is made linearly proportional to the envelope amplitude. It will appear from the discussion to follow, that the combination of a linear current relation with the exponential varistor characteristic is particularly favorable to compression or expansion in the ratio of 2:1. No such simple results are obtained with voltage-controlled devices. We shall therefore confine our treatment to current-controlled devices with the exception of one type of voltage-controlled vario-losser which has the interesting property that a continuous variation of the compression or expansion ratio may be secured.

The general problem is then that of determining the varistor characteristic to obtain the required exponent n with a suitable—generally a linear—rectifier.

Types of Vario-Losser Circuits

From the standpoint of circuit configuration, vario-lossers may be classified as either of the two-terminal or four-terminal type, depending on whether the varistor combination is inserted merely as an impedance element or as a general transmission network. The two-terminal class may be further subdivided into the series and shunt types. The four-terminal class will be represented in this discussion by the lattice type, which is sufficiently general for illustrative purposes.

Two-Terminal Networks

SHUNT TYPE

A shunt-type vario-losser may be constructed by shunting the signal path with nonlinear elements having low resistance compared to that of the remainder of the circuit. A typical circuit of the backward-acting type is shown in figure 2. A portion of the output voltage is amplified and rectified, the rectified current is transmitted through a capacitance-resistance network and then introduced into a varistor bridge. The pair of terminals of the bridge which are conjugate to the bias input are connected directly across the signal input circuit. If r represents the a-c resistance of the varistor bridge shunted across the signal circuit and Z_1, Z_2 the input and output signal circuit impedances, respectively

$$A = \frac{V_2}{V_1} = \frac{rZ_2}{Z_1Z_2 + r(Z_1+Z_2)} \tag{5}$$

If r is small compared with $Z_1Z_2/(Z_1+Z_2)$, the impedance of Z_1 and Z_2 in parallel, we have approximately

$$A = r/Z_1 \tag{6}$$

The resistance r varies with the bias current which is in turn proportional to V_2. Let I represent the bias current through and V the bias voltage across one varistor element. For simplicity the elements are assumed to be identical; it follows that r is also the a-c resistance of one element. Then

$$r = dV/dI \tag{7}$$

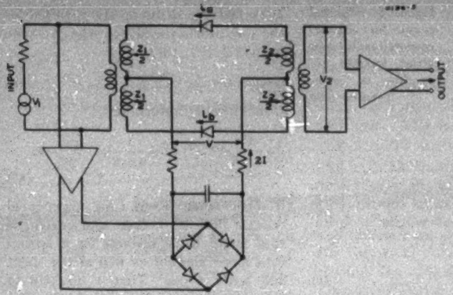


Figure 3. Series-type forward-acting vario-losser

If the impedance of the varistor bridge is low compared with the internal impedance of the rectifier-filter circuit, I is proportional to V_2 and may be written as

$$I = I_0 V_2 / V_{20} \tag{8}$$

Substituting the appropriate expression for A from (4a) in (6) and (7)

$$r = Z_1 A_0 (V_2/V_{20})^{1-1/n} = dV/dI \tag{9}$$

whence, substituting (8),

$$dV = r_0 (I/I_0)^{1-1/n} dI \tag{10}$$

Integrating, we thus obtain the general current-voltage relation for the typical varistor element, necessary to make the device satisfy (2),

$$V = \frac{r_0 I_0}{2-1/n}(I/I_0)^{2-1/n} + C$$
$$= V_0 (I/I_0)^{2-1/n} + C \tag{11}$$

The value of the integration constant C must be zero if zero bias is to be included in the operating range, since current and voltage vanish simultaneously. The particular case $n = {}^1/_2$, gives the simple solution of (10),

$$I = I_0 e^{(V-V_0)/r_0 I_0} \tag{12}$$

An exponential current-voltage relation is thus appropriate for a shunt-type backward-acting 2:1 compressor. Since (12) gives a nonzero value of current when the voltage is zero, it cannot be satisfied exactly by physical materials, but it can be made to hold with sufficient accuracy for all except small values of V.

Similarly, for a shunt-type forward-acting device, we substitute, $I = I_0 V_1 / V_{10}$ and make use of (3a). We then find

$$V = \frac{r_0 I_0}{n}(I/I_0)^n + C = V_0 (I/I_0)^n + C \tag{13}$$

as the ideal current-voltage curve, and as before the value of C is zero for physical elements. This equation indicates that a material in which the current varies as the mth power of the voltage is suitable for a forward acting shunt type m:1 compressor.

If two shunt type backward acting vario-lossers with exponential varistors such as described above are connected in tandem with a separating pad or amplifier between and equal bias currents proportional to V_2, the output envelope of the second, are supplied to each losser, we would have for the over-all combination,

$$A = b(r/Z_1)^3 \tag{14}$$

From (12), $r = \alpha/\mathrm{I}$, α = constant, and since $I = aV_2$,

$$A = \frac{br_0^3}{Z_1^3}(V_{20}/V_2)^3 \tag{15}$$

Comparing with (4a), then

$$n = 1/3,\ A_0 = br_0^3/Z_1^3 \tag{16}$$

This gives a practical method of constructing a 3:1 compressor using exponential varistors. It is also readily demonstrated that the output of the first losser is compressed in the ratio 3:2. In a similar manner, using more units, we may obtain other ratios.

SERIES TYPE

Figure 3 shows a forward-acting vario-losser of the series type. Let the total a-c resistance inserted in the circuit by the nonlinear branches be r. We readily calculate

$$A = V_2/V_1 = \frac{Z_2}{Z_1 + Z_2 + r} \tag{17}$$

If r is large compared with $Z_1 + Z_2$.

$$A = Z_2/r \tag{18}$$

From (3a),

$$r = \frac{Z_2}{A_0}(V_1/V_{10})^{1-n} = 2dV/dI \tag{19}$$

If

$$I = I_0 V_1 / V_{10} \tag{20}$$

$$2dV = r_0 (I/I_0)^{1-n} dI \tag{21}$$

Integrating

$$V = \frac{r_0}{2(2-n)}(I/I_0)^{2-n} +$$
$$C = V_0 (I/I_0)^{2-n} + C \tag{22}$$

The particular case $n = 2$ gives the simple solution of (21),

$$I = I_0 e^{2(V-V_0)/r_0 I_0} \tag{23}$$

An exponential current-voltage relation is thus found to be also appropriate for a series type forward acting 2:1 expandor.

For a backward acting series type vario-losser, we substitute $I = I_0 V_2 / V_{20}$, make use of (4a), and find

$$V = \frac{r_0 I_0}{2}(I/I_0)^{1/n} + C = V_0 \left(\frac{I}{I_0}\right)^{1/n} + \mathrm{C} \tag{24}$$

This indicates a method of realizing an n:1 expandor from varistor elements in which the current varies as the nth power of the voltage.

Four-Terminal Networks

LATTICE TYPE

The two-terminal networks just described have the defect that the impedance of the vario-losser network must be either small or large compared with the impedance of the signal circuit; hence the

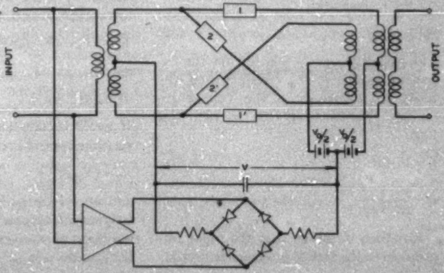


Figure 4. Lattice-type forward-acting vario-losser

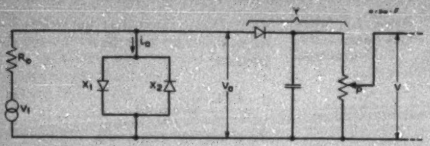


Figure 5. Control circuit for voltage-operated vario-losser giving variable expansion or compression ratio

minimum loss introduced may be excessive, and troublesome reflections caused by the mismatch may result. These difficulties may be avoided by using a lattice arrangement of varistors. In figure 4, the series varistor elements 1, 1′ have biasing voltage $(V_0+V)/2$ and a-c resistance r_1, and the shunt varistor elements 2, 2′ have biasing voltage $(V_0-V)/2$, and a-c resistance r_2. Here V_0 is a fixed direct voltage derived from a battery and V is a varying control component derived from the envelope of the signal wave. The input and output impedances of the signal circuit are assumed real and equal to R. From the equations of a lattice-type network:

$$I_1=\frac{V_1}{2}\left(\frac{1}{R+r_1}+\frac{1}{R+r_2}\right) \tag{25}$$

$$I_2=\frac{V_1}{2}\left(\frac{1}{R+r_1}-\frac{1}{R+r_2}\right) \tag{26}$$

where I_1 and I_2 are input and output currents respectively. If we let

$$r_1=Rf(V) \tag{27}$$

it follows that

$$r_2=Rf(-V) \tag{28}$$

Equations 25 and 26 then become

$$I_1=\frac{V_1}{2R}\left[\frac{1}{1+f(V)}+\frac{1}{1+f(-V)}\right] \tag{29}$$

$$I_2=\frac{V_1}{2R}\left[\frac{1}{1+f(V)}-\frac{1}{1+f(-V)}\right] \tag{30}$$

These equations permit of a useful simplification if

$$f(-V)=1/f(V) \tag{31}$$

a functional equation which is satisfied by the exponential,

$$f(V)=e^{-\alpha V} \tag{32}$$

The negative sign is selected for the exponent because in the varistors used, the resistance decreases as the voltage increases. Substituting (31) in (29) and (30),

$$I_1=V_1/2R \tag{33}$$

$$I_2=\frac{V_1}{2R}\,\frac{1-f(V)}{1+f(V)} \tag{34}$$

Equation 33 shows that the input impedance to the lattice is a constant resistance matching that of the signal circuit. When the relation (32) is substituted in (34), and the output voltage is introduced to compare with (2),

$$V_2=I_2R=\frac{V_1}{2}\tanh\frac{\alpha V}{2} \tag{35}$$

From (2), then

$$A=\frac{1}{2}\tanh\frac{\alpha V}{2} \tag{36}$$

If $\alpha V/2$ is small we have approximately,

$$A=\frac{\alpha V}{4}=A_0V/V_0 \tag{37}$$

If the device is forward-acting, we equate (3*a*) and (37), and get

$$V=V_0(V_1/V_{10})^{n-1} \tag{38}$$

We note that V is directly proportional to V_1 if $n=2$; hence a forward-acting lattice type gives a simple method of constructing a 2:1 expandor. If the device is backward acting, we equate (4*a*) and (37), giving

$$V=V_0(V_2/V_{20})^{1-1/n} \tag{39}$$

and V cannot be made directly proportional to V_2 except when n approaches infinity. The lattice type circuit is therefore best suited for a forward acting 2:1 expandor. A 2:1 compressor may however be realized by putting a 2:1 expandor in the negative feedback path of an amplifier. If an n:1 expandor is inserted in the feedback path in such a direction as to make the feedback negative, we have

$$V_g=V_1-\beta V_2^n \tag{40}$$

where V_g is the actual input voltage of the amplifier, and

$$V_2=\mu V_g=\mu V_1-\mu\beta V_2^n \tag{41}$$

or

$$V_2+\mu\beta V_2^n=\mu V_1 \tag{42}$$

If $\mu\beta V_2^n$ is made large compared with V_2, we have approximately,

$$V_2=(V_1/\beta)^{1/n} \tag{43}$$

and an n:1 compressor is secured. Similarly an n:1 expandor can be realized by inserting an n:1 compressor in the feedback path.

To find the characteristic required for the varistors in the lattice type, we write from (27), (32), (37),

$$r_1=dV/dI=Re^{-4A_0V/V_0} \tag{44}$$

from which by integration

$$I=\frac{V_0}{4A_0R}e^{4A_0V/V_0}+C=\frac{1}{4A_0R}(e^{4A_0V/V_0}-1) \tag{45}$$

if $I=0$ when $V=0$. An exponential relation between voltage and current is thus found to be suitable for a lattice-type losser element as well as the shunt and series types. That a function of form (45) can actually be realized over a wide range including bias values near zero is indicated by the curve of $I+0.025$ in figure 1.

It is to be noted from (36) that as V becomes large, the hyperbolic tangent approaches unity, and hence the voltage transfer ratio approaches a constant. The lattice may therefore be used as a *limited range expandor*, that is, to expand small inputs in the ratio of 2:1, while leaving large inputs unaffected. Likewise when placed in the feedback path of an amplifier, the lattice may be used to obtain a 2:1 *limited range compressor*, and a combination of these forms a *limited range compandor*.

Variable-Ratio Device

In figure 5 is shown a device differing from those previously discussed in that the control is on a voltage basis rather than current. The resistance of the varistor combination X_1, X_2 is made small at all times compared with the resistance R_0 and that of the rectifier-filter circuit Y. We therefore have approximately

$$i_a=\frac{V_1}{R_0} \tag{46}$$

Furthermore if the varistors have exponential current-voltage relationships, the voltage v_a across the varistor combination and the current i_a must satisfy the equation

$$|i_a|=\alpha e^{\beta|v_a|} \tag{47}$$

with the additional proviso that i_a is an odd function of v_a because of the oppositely poled pair of elements.* It follows that

$$|v_a|=\frac{1}{\beta}\log\frac{|i|}{\alpha}=\frac{1}{\beta}\log\frac{|v_1|}{\alpha R_0} \tag{48}$$

Now considering the rectifier-filter circuit Y, let the output voltage V be proportional to V_a, the envelope of v_a, and V_a be related to the envelope V_1 of the voltage v_1 in the same manner as v_a and v_1 are related by (48). Then

$$V=gV_a \tag{49}$$

where g varies between 0 and 1, as determined by the setting of the potentiometer P, and

$$V_a=\frac{1}{\beta}\log\frac{V_1}{\alpha R_0} \tag{50}$$

If the voltage V is used as bias on the varistor elements of a vario-losser circuit, with the varistor impedance high enough not to affect the relation between V and V_1 appreciably, we should have, assuming varistor elements which have the same exponential characteristic as (47),

$$i/r=dI/dV=d(\alpha e^{\beta V})dV$$
$$=\alpha\beta e^{\beta V}$$

Hence

$$r=\frac{1}{\alpha\beta}e^{-\beta V}=\frac{1}{\alpha\beta}e^{-\beta g V_a}$$
$$=\frac{1}{\alpha\beta}e^{-g\log\frac{V_1}{\alpha R_0}}=\frac{1}{\alpha\beta}\left(\frac{V_1}{\alpha R_0}\right)^{-g} \tag{51}$$

If the vario-losser is of the shunt type, we have from (6),

$$A=kV_1^{-g},\quad k=1/\alpha\beta Z_1(\alpha R_0)^{-g} \tag{52}$$

Comparing with (3) and (4), we then have the results that for a backward-acting shunt type, $n=1-g$; while for a forward-acting shunt type, $n=1/(1+g)$. Similarly, if the vario-losser is of the series type, we make use of (18) and find

$$A=kV_1^{g},\quad k=\alpha\beta Z_2(\alpha R_0)^{g} \tag{53}$$

whence for forward action, $n=1+g$, and for backward action, $n=1/(1-g)$. Since g may be varied continuously between the limits zero and unity, any positive value of n may theoretically be obtained by choosing from the above four types. The ranges of n available from each type are:

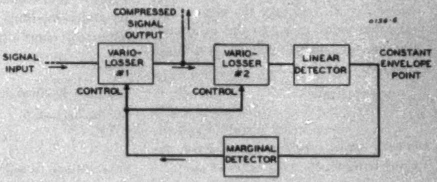
	n	Use
Shunt type, forward acting	0 to 1	Compressor
Shunt type, backward acting	$^1/_2$ to 1	Compressor
Series type, forward acting	1 to 2	Expandor
Series type, backward acting	1 to ∞	Expandor

Marginal-Detector Types

The losser circuits so far described require that the varistor elements have specific current-voltage characteristics, for example, the exponential form, in order to yield the desired mode of operation. It is possible to construct vario-losser circuits to obtain specified ratios of compression or expansion without requiring any property of the varistors other than that of resistance variable with bias. This is accomplished by introducing an auxiliary device, the so-called "marginal detector." A marginal detector may be defined for our purposes as a device which delivers no output when the input is below a certain fixed value, and increases its output very rapidly as the input is increased very slightly through the critical value. A vacuum-tube detector with grid biased negatively beyond the cut-off is an approximation to the ideal marginal detector. If the input of a marginal detector is taken from the output envelope of the signal (see figure 6) and the marginal detector output is used as bias on varistors in a shunt-type vario-losser, no effect occurs when the output is below the marginal value. When the marginal value is exceeded the loss in the shunt varistors increases rapidly with the result that the output cannot appreciably exceed the marginal value. Such an arrangement therefore acts as an envelope limiter.

Now consider the result of using two identical shunt losser circuits in succession as in figure 6 with the same biasing current supplied to each from the marginal detector. The envelope is evidently held constant at the output of the second losser whenever the input to the first exceeds a critical value. Since the two lossers are identical, each must have the same loss. Let x_0 represent the total loss in decibels in the two lossers when the critical input value is not reached. Then if the input to the first losser is x decibels above the critical value, the loss in each of the two lossers is $(x+x_0)/2$ decibels. The output of the first losser therefore increases by $x/2$ decibels when the input increases by x decibels and is therefore compressed in a ratio of 2:1. If three losser units are used instead of two, the output of the first is compressed 3:2 and the output of the second 3:1. In general if n units are used, the output of the mth unit is compressed by the ratio $n/(n-m)$.

Figure 6. 2:1 compressor of marginal-detector type



A 2:1 expandor may be realized by means of two lossers and a marginal detector, as shown in figure 7. The bias must in this case be supplied by the two lossers in such a way that the loss decreases in the first and increases in the second with the increasing bias. If the amount of decrease in the first is made equal to the amount of increase in the second, the output of the first losser is expanded in the ratio of 2:1.

The marginal detector circuits have the obvious advantage of not being limited by the types of varistor characteristics available, but this is offset by the added complication of the marginal detector itself, which may offer a troublesome adjustment and maintenance problem.

Distortion

It is of course necessary for satisfactory operation that the distortion of the signal resulting from transmission through the vario-losser path be held to a low value. Quantitative estimates of the distortion can be obtained by considering the case of a single-frequency input. The envelope then becomes a constant, and the bias applied to the typical varistor element consists of a direct current. The instantaneous voltage across a particular varistor element is the sum of the bias voltage and the voltage produced by the signal. Substitution of this instantaneous voltage in the current-voltage relation for the varistor element enables the approximate calculation of the distorted component of signal current.

We may illustrate by considering the shunt-type vario-losser circuit, figure 2. If the signal input is represented by

$$v_1=V_1\cos pt \tag{54}$$

the signal output is to a first approximation, also sinusoidal and of the form,

$$v_2=V_2\cos pt \tag{55}$$

If all four varistor elements are identical, each with the instantaneous current-voltage relation, $i=f(v)$,

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* This arrangement is necessary in the case of copper oxide. In the case of silicon carbide, the material itself would have the proper symmetry and one element would suffice.

$$i_a=i_c=f\left(V+\frac{1}{2}V_2\cos pt\right) \tag{56}$$

$$i_b=i_d=f\left(V-\frac{1}{2}V_2\cos pt\right) \tag{57}$$

$$i_d-i_a=f\left(V-\frac{1}{2}V_2\cos pt\right)-f\left(V+\frac{1}{2}V_2\cos pt\right)$$
$$=-f'(V)V_2\cos pt-\frac{f'''(V)}{3!}\frac{V_2^3}{4}\cos^3 pt-\ldots \tag{58}$$

The principal distortion component is thus a third harmonic

$$i_d-i_a=-\frac{f'''(V)}{24}V_2^3\cos^3 pt \tag{59}$$

This current may be considered to be generated in the varistor bridge and to divide between the impedances Z_1 and Z_2. No even harmonics appear in the output circuit because of the assumed balance of the bridge. The third harmonic voltage appearing across the output is

$$v_{23}=-\frac{Z_1Z_2f'''(V)V_2^3}{96(Z_1+Z_2)}\cos 3pt \tag{60}$$

The significance of the negative sign is that the phase of the third harmonic is such as to flatten the peak of a sine wave, which is to be expected since the shunting action of the varistors is greater for large amplitudes. It is convenient to assume $Z_1=Z_2$, and take the ratio of the maximum value of v_{23} to the maximum fundamental output voltage amplitude V_2, giving

$$\frac{V_{23}}{V_2}=\frac{Z_1f'''(V)V_2^2}{192} \tag{61}$$

For a series-type losser, we have, referring to figure 3,

$$i_a=f\left(V-\frac{V_1}{2}\cos pt\right) \tag{62}$$

$$i_b=f\left(V+\frac{V_1}{2}\cos pt\right) \tag{63}$$

$$i_b-i_a=f'(V)V_1\cos pt+\frac{V_1^3f'''(V)}{4.3!}\cos^3 pt+\ldots \tag{64}$$

$$V_{23}=\frac{V_1^3Z_2f'''(V)}{96}=\frac{f'''(V)}{96Z_2^3}r^3V_2^3 \tag{65}$$

$$\frac{V_{23}}{V_2}=\frac{f'''(V)}{12[f'(V)]^3}(V_2/Z_2)^2 \tag{66}$$

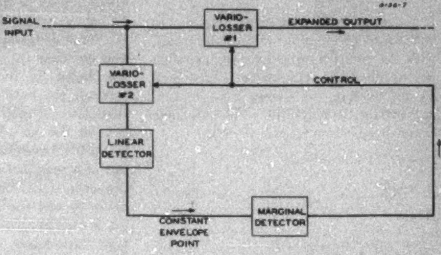

Figure 7. 2:1 expandor of marginal-detector type

The above principles may be illustrated by working out an example. Suppose it is desired to use copper-oxide varistors having the characteristic shown in figure 1 in a shunt-type vario-losser such as shown in figure 2 with third harmonic at least 30 decibels down on fundamental over a 30-decibel range of loss variation. Also given that the maximum amount of direct current per varistor is four milliamperes. It is required to find the maximum output voltage permissible and the impedance which should face the varistors. We assume $Z_1=Z_2$. The characteristic of figure 1 may be represented by the equation

$$I=0.025(e^{2V}-1) \tag{67}$$

where V is in volts and I in milliamperes. By differentiation, we find that the resistance in ohms is given by

$$r=2.10^4e^{-2V} \tag{68}$$

and that the third derivative is

$$f'''(V)=\frac{4}{r} \tag{69}$$

Substituting in equation 61, we find for the ratio of third harmonic to fundamental,

$$\frac{V_{23}}{V_2}=\frac{0.0208Z_1V_2^2}{r} \tag{70}$$

The greatest distortion occurs at the maximum load. Calling the maximum output voltage V_{20}, and noting that distortion 30 decibels down is equivalent to a voltage ratio of 0.0316, we write as one requirement:

$$\frac{Z_1V_{20}^2}{r_0}=1.53 \tag{71}$$

The required 30-decibel variation in loss is to be obtained as r is increased from the minimum permissible value specified by (71) to its maximum value, which occurs with zero bias. If we assume that the maximum value of r is large compared with Z_1, we see from (5) and (6) that the value of A varies from $^1/_2$ to r_0/Z_1. The total variation in loss is therefore 20 $\log_{10}$ $2r_0/Z_1$, and if this is to be 30 decibels we must have

$$r_0/Z_1=0.0158 \tag{72}$$

Substituting (72) in (71), we find $V_{20}=$ 0.155 volt. Since the bias current I_0 available for each varistor at this value of V_{20} is four milliamperes, the corresponding voltage V_0 across each varistor is, from figure 1, 2.5 volts, and from (68), the resistance r_0 is 132.5 ohms. From (72), the terminating impedance Z_1 should be 8,390 ohms.

References

1. The Compandor—An Aid Against Static in Radio Telephony, **R. C. Mathes** and **S. B. Wright.** *B.S.T.J.*, volume 13, 1934, number 3, 315–32.

2. A Vogad for Radiotelephone Circuits, **S. B. Wright, S. Doba,** and **A. C. Dickieson.** IRE *Proc.*, volume 27, 1939, number 4, 254–7.

3. Devices for Controlling Amplitude Characteristics of Telephonic Signals, **A. C. Norwine.** *B.S.T.J.*, volume 17, 1938, number 4, 539–54.

4. For a general discussion of other design considerations see Amplitude Range Control, **S. B. Wright.** *B.S.T.J.*, volume 17, 1938, number 4, 520–38.

Possibilities and Methods of Extending a Carrier Current Relay Channel to Other Uses

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CARRIER-CURRENT relaying is now well established as the most complete type of protection. It has had sufficient field experience to well justify the fundamental principles on which it is founded and the manner in which it has been worked out. As it often happens, the solution of one problem will lend itself to another similar problem. This seems to be the case of carrier-current relaying as it has provided the communication channel which might be available to solve other problems involving the transmission and reception of signaled information. This common solution to two problems naturally lessens the economic factors which relieves each problem of the strain of carrying the entire burden. Other problems involving the use of communication means which could be provided by the relay channel are: telemetering, supervisory control, load control, communication, remote tripping, etc. The addition of these other functions to the carrier channel, while not involving serious technical difficulties, nevertheless, requires the consideration of some problems. Methods are available and actual installations have been made involving various combinations of additional uses of the relay channel.

While the accommodation of several quantities on the carrier channel may seem to violate the old adage that two bodies cannot occupy the same space at the same time, nevertheless, the telephone industry has shown for a number of years that several separate sets of information-carrying quantities can occupy the same transmitting medium. There are two aspects of the problem of extending the functions of a carrier-current relay channel involving different degrees of complication. One method is to add functions but not require that they be carried on at exactly the same time. This way lessens the amount of apparatus required but introduces the necessity for compromises in the desired characteristics. There are a large number of combinations of functions which can be added to the relay function without materially adding to the equipment provided they are co-ordinated and compromised in the right manner. The manner in which the problem is solved is specific for each application. It is not the purpose of this paper to consider all of these combinations but merely to point out some of the possibilities.

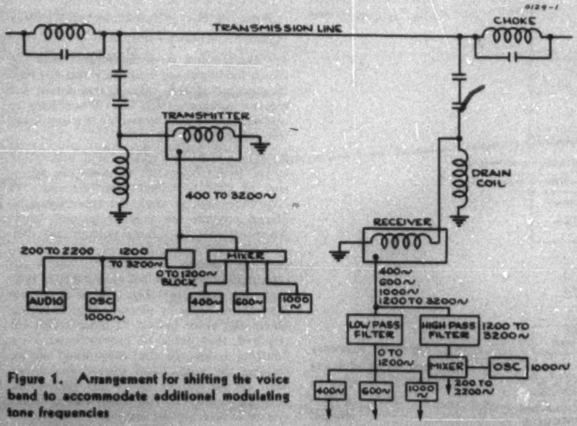

Figure 1. Arrangement for shifting the voice band to accommodate additional modulating tone frequencies

The other method assumes that all the functions performed by the carrier channel must be carried on concurrently. This means that additional equipment must be provided to render the functions entirely independent of one another and, of course, removes the necessity of imposing any restrictions or compromises.

Fundamental Problems

There are a number of fundamental requirements which must be considered in any scheme to extend the use of a relay channel. Since the primary object of the carrier channel in a relay system is to transmit information between the relays at the ends of the protected section, the relays must be capable at all times of starting and stopping the carrier signal regardless of whatever other use is being made of the carrier wave. Any other use of the carrier channel must be temporarily suspended during the occurrence of faults on the system. Ordinarily, this should not seriously interfere with the additional functions since with high-speed relays the time of interruption is small. The loss of a few telemetering impulses or a syllable of speech would hardly be considered serious. Even the interruption occasioned by slow-speed relays clearing a fault elsewhere on the system is not serious. This preferential use of the carrier channel by the relays can always be attained in systems where all of the carrier functions are entirely independent but in systems where compromises are made for the sake of simplicity there may be times when the relays are unable to take control of the carrier. An example is the case where no fault current flows at one end of the line. Here a telemetering impulse or telephone conversation could prevent tripping because there is no relay action at the zero current end to stop the carrier.

There are two ways in which the relay carrier channel can be extended to other uses.

1. The carrier wave itself may be used to carry the intelligence. This method requires the least amount of additional equipment but has limitations in the number of

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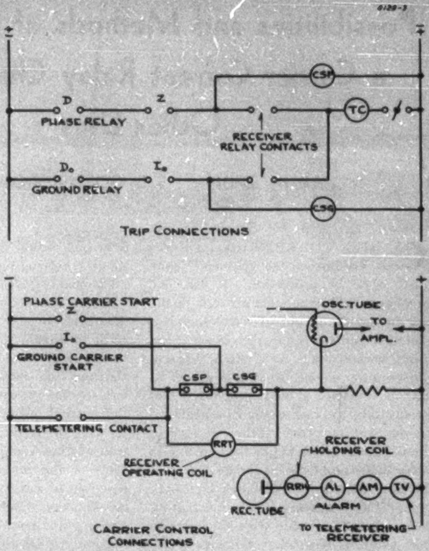
1. For all numbered references, see list at end of paper.

functions that can be handled. Telemetering can be added by merely "keying" the carrier to send out a set of impulses in which either the rate or the duration of the pulses are used to actuate the telemetering receiver. Communication alone can be easily added, since it requires merely that the carrier be held on continuously. However, communication and telemetering could not be accomplished simultaneously since the one function requires continuous and the other requires impulsed carrier. Furthermore, more than one telemetering quantity cannot be used, since the impulses would interfere.

In general, it may be said that if the carrier wave itself is employed, it can take care of only one additional function at a time. If the additional functions do not need to be performed at the same time, then several purposes can be served by the relay carrier channel. Thus, communication, telemetering, supervisory, and remote tripping could be accomplished.

The chief disadvantage of utilizing the carrier wave itself is, as was indicated previously, the possibility of preventing tripping when no fault current occurs at one terminal. In this case, the carrier, if initiated by communication, telemetering, etc., at the end where no fault current flows, cannot be stopped by the relays at this end.

Figure 3. Schematic diagram of a carrier current relay scheme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+
CSP
D
Z
PHASE RELAY
TC
RECEIVER RELAY CONTACTS
D₀
I₀
GROUND RELAY
CSG
TRIP CONNECTIONS
−
+
PHASE CARRIER START
Z
I₀
GROUND CARRIER START
OSC. TUBE
TO AMPL.
CSP
CSG
TELEMETERING CONTACT
RRT
RECEIVER OPERATING COIL
RECEIVER HOLDING COIL
REC. TUBE
RRH
AL
AM
TV
ALARM
CARRIER CONTROL CONNECTIONS
TO TELEMETERING RECEIVER

2. A number of tone or modulating frequencies may be imposed upon the carrier and each tone may be used to transmit a separate set of signals. If the tone frequencies are not inside the voice frequency band, communication may also be added. Also, if the relay signal is placed on one of the tones instead of the carrier itself, the difficulty experienced when no fault current flows initially at one terminal is entirely avoided. Assuming that communication is not desired, the number of functions which can be accommodated is limited only by the number of tone frequencies which can be placed in the audio band width which the carrier set will pass. With communication the number of functions is limited by the number of tones that can be placed outside an intelligible voice band but inside the band capable of being passed by the transmitter-receiver set.

The use of a tone frequency means that the carrier wave is modulated by one or more definite audio frequencies, say, 600 and 1,000 cycles. At the receiver end the carrier is demodulated and the tone frequencies are separated by filter circuits. It is obvious that each tone may be used to transmit signals entirely independent of all other tones and this system is, therefore, applicable where the number of additional functions cannot be accommodated by the carrier wave itself.

Tone Modulating System

Some of the technical aspects of the tone system are as follows:

1. There must be a minimum ratio between the impulsing frequency and the tone frequency. If this ratio is too small, the receiver apparatus will have a tendency to respond to the tone as well as the impulsing frequency.

2. There must be a wide enough separation in the tone frequencies to avoid the necessity of sharply tuned filter circuits. If communication is used, the tones must be placed outside the voice-frequency band. An intelligible voice band is from, say, 200 to 2,200 cycles and the band which can be ordinarily passed by the carrier set is approximately 200 to 3,200 cycles. It is thus seen that not more than one or two tone frequencies could be placed either below or above the voice band. If more tones are required, they could be provided either by "cutting holes" in the voice band or by shifting the voice band toward the upper limit of the carrier set. The process of "cutting holes" in the voice band would consist of filtering out say from 900 to 1,100

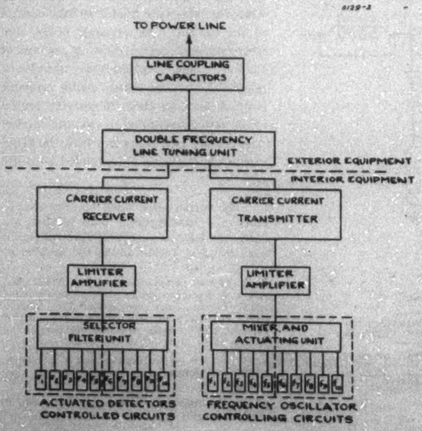

0129-2
TO POWER LINE
LINE COUPLING CAPACITORS
DOUBLE FREQUENCY LINE TUNING UNIT
EXTERIOR EQUIPMENT
INTERIOR EQUIPMENT
CARRIER CURRENT RECEIVER
CARRIER CURRENT TRANSMITTER
LIMITER AMPLIFIER
LIMITER AMPLIFIER
SELECTOR FILTER UNIT
MIXER AND ACTUATING UNIT
ACTUATED DETECTORS CONTROLLED CIRCUITS
FREQUENCY OSCILLATOR CONTROLLING CIRCUITS

Figure 2. Extensive use of modulating tone frequencies on a carrier current channel

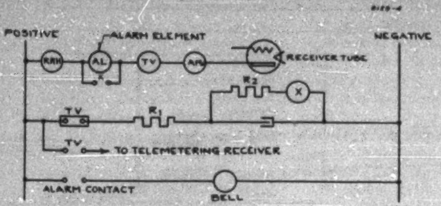


Figure 4. Arrangement to prevent alarm bell response on telemetering impulses

cycles from the voice input to the modulator and then supplying a 1,000-cycle note from an audio oscillator. The absence of the 900 to 1,100-cycle notes would have some tendency to impair the speech quality and, of course, if too many of these holes were cut, the speech would become too poor for intelligible use.

Another method for accommodating speech and tone frequencies would be to shift the voice frequency toward the high end of the receiver band. Assuming 200 to 2,200 cycles sufficient for clear speech, this could be shifted to 1,200 to 3,200 cycles by adding 1,000 cycles. This would give a band of say 100 to 1,200 cycles for tone frequencies. Possible tones of 100, 130, 170, 220, 286, 370, 480, 620, 810, 1,050 could be accommodated with a 30 per cent separation below 1,200 cycles. Figure 1 shows schematically the arrangement needed to accomplish this system. The speech input is heterodyned with the output of a 1,000-cycle oscillator and the sum frequency is passed into the carrier set modulator. A high-pass filter cuts off all frequencies below 1,200 cycles. Separate oscillators producing different tones also feed through a mixing device to regulate the amplitude into the carrier set. At the receiving end frequencies up to 1,200 cycles are first filtered out, leaving the voice band 1,200 to 3,200 cycles from which 1,000 cycles is subtracted by heterodyne action with a 1,000-cycle oscillator. The output of the low-pass filter is further filtered into the individual tones.

To our knowledge, no application of this voice-shift system has yet been made in this country, but it has been used in France.[1] The chief object here is to point out the engineering aspects involved in connection with a very extensive additional use of a carrier relay channel. Figure 2 shows an extensive use of tone frequencies on a carrier-current channel on which communication is not desired.

Telemetering

It is the purpose here to discuss some of the specific problems which occur when the telemetering function is added to the relay carrier channel. There are two types of impulse telemetering systems. One takes account of the rate of the impulses and the other operates on the duration and spacing of impulses. The problems discussed here are based on the system utilizing the rate of impulses. The same problems could, no doubt, be solved in a similar manner for the impulse duration principle. Also, it should be understood that each application of telemetering to a relay carrier channel may involve specific problems requiring a solution applying only to that application and it is not the intention here to cover all of the problems which might arise. Furthermore, no attempt is made here to discuss telemetering systems themselves.

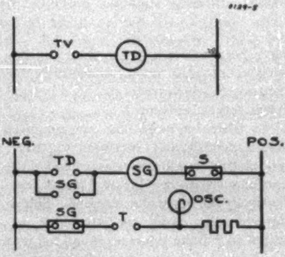


Figure 5. Scheme to stop telemetering from the receiver end

Figure 3 shows schematically a relay carrier scheme.[2] The carrier is initiated by the closure of the impedance or overcurrent contacts, *Z* or I_0, which impresses negative potential on the oscillator cathode. Carrier is stopped when the directional and impedance contacts, *D* and *Z*, close to energize the *CSP* coil which in turn opens the *CSP* contact, thus returning the cathode to positive. Carrier is started and stopped in a similar manner by the action of the ground relay contacts, D_0 and I_0. The telemetering contact is placed in parallel with the *Z* carrier start contact and it is obvious that with this connection the relay contacts can start and stop carrier regardless of what position the telemetering contact is in. At the receiving terminal a relay is placed in the plate circuit of the receiver tube and is actuated by each of the carrier pulses. The contacts of this relay are connected to actuate the telemetering receiver.

At both the sending and receiving terminals there is an alarm relay, *AL* whose function is to close contacts when a carrier signal is received and ring an alarm bell. The primary purpose of this alarm bell is to enable the operators at each end of the line to signal each other when it is desired to test the transmission and reception of carrier or to communicate by telephone. Obviously, if telemetering signals are being transmitted, it would be

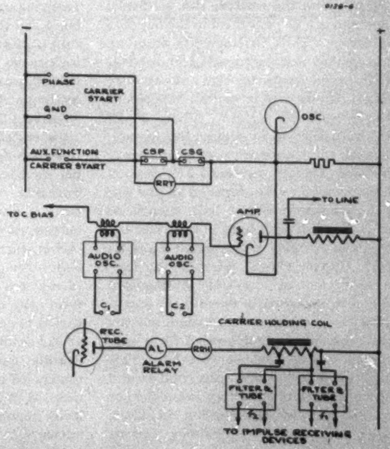
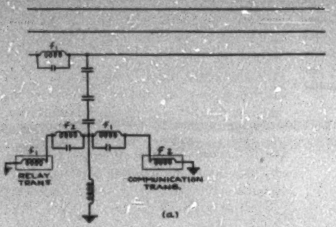
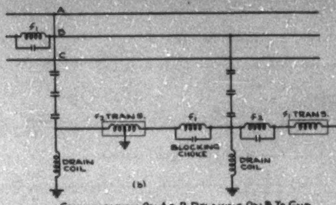


Figure 6. Schematic diagram of modulating tone frequencies

Figure 7



Relaying & Communication One Phase Wire To Ground

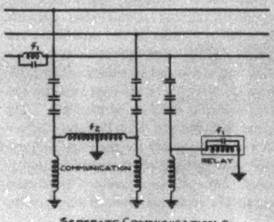


Communication On A & B Relaying On B To Gnd.

confusing to ring the alarm bell on each signal. Of course, a simple solution is to disconnect the bell, but this prevents the bell from performing its normal signaling function.

An arrangement to prevent the alarm relay from operating on telemetering impulses but permit it to operate when it is desired to signal is shown in figure 4. During the reception of telemetering signals the alarm relay coil, *AL*, is short-circuited by the *X* contact operated by the *X* relay which is energized through the resistors, R_1 and R_2, and the *TV* back contact. The *TV* relay operates on each carrier impulse and opens its back contact, but the *X* coil is energized by the capacitor discharge until the *TV* contact closes again. When it is desired to operate the alarm bell the carrier is held on at the remote station for approximately one second. This opens the *TV* contact and after the capacitor has discharged, the *X* relays drops out, thus allowing the alarm relay, *AL*, to pick up on the carrier signal.

Another problem which arises is the necessity of stopping the transmission of signals, from the receiver end, so that a function can be performed in the opposite direction. Thus, telemetering from an unattended station to an attended station may be stopped so that a supervisory signal may be sent to control breakers at the unattended station. Figure 5 depicts a scheme to accomplish this feature. During the transmission of telemetering impulses the relay contacts, *TV*, close on each impulse. The duration of the impulse, however, is not long enough to allow the time delay relay, *TD*, to pick up, thus the *SG* relay is de-energized and its back contact closed, allowing the telemetering contact, *T*, to transmit impulses. When it is desired to operate the supervisory from the attended station, the carrier push button is depressed for a long enough time to close the *TD* relay. This picks up the *SG* which seals itself in and opens its back contact to prevent further transmission of telemetering impulses. After the desired supervisory actions have been performed, the final supervisory action is to open momentarily the contact, *S*, which resets the *SG* relay and telemetering is resumed.



Seperate Communication & Relaying Channels
(c)

To telemeter more than one quantity simultaneously, it is necessary to "key", modulating tone frequencies instead of the carrier wave itself. Figure 6 shows schematically the apparatus required. Simple vacuum-tube audio oscillators are adjusted to definite frequencies and fed into the grid of the amplifier tube. At the receiving end the carrier wave is demodulated and the plate circuit of the receiver tube contains the tone frequencies. These frequencies are separated and fed to the telemetering receiver devices. At the sending end the audio oscillators are impulsed by the contacts, C_1 and C_2, which are operated by the telemeter transmitter. If signals are sent continuously, the carrier wave must, of course, be held on continuously. If the signaling is intermittent, it would be possible to have the telemetering transmitter also operate the carrier start contact.

Another principle which has been used to enable a single carrier channel to perform more functions and avoid complication is to store the telemetering impulses at the sending end and transmit only at definite intervals. The impulses are merely fed into a mechanical counter and then at regular intervals the accumulated impulses in the counter are transmitted.

Supervisory and Remote Tripping

There are numerous combinations of supervisory, telemetering, and remote tripping which can be applied to a relay carrier-current channel. For instance, the telemetering can be made one point on the supervisory control.

Where supervisory control is installed, it is generally important enough to demand a good pilot circuit. Certain troubles with conventional pilot circuits have brought out the fact that a high-voltage line is not only reliable as a power distributor, but is also just as reliable as a pilot wire. For this reason carrier-current channels over the high-voltage line itself are more and more being used for supervisory control.

In general, supervisory control requires the transmission of signals in both directions. However, these signals do not need to be sent simultaneously, and therefore, a single-frequency relay carrier channel is all that is required. It is possible to combine the supervisory system with other functions without complication by making the other functions merely points on the supervisory system.

A relay carrier channel provides an excellent means of tripping a distant breaker when the fault current is too small to actuate the protective relays. A practical example is the case where a transformer bank is included as part of the high-voltage line with no high-voltage breaker. The problem here is to protect the transformer against internal faults which may not draw enough current from the distant end to actuate the line relays. With a carrier channel available, sensitive differential protection can be applied to the transformer, and the tripping of the distant breaker can be initiated by the transmission of a carrier signal. Of course, the local low-voltage breaker is tripped directly by the differential relays. One company in New England has installed this remote tripping scheme on a carrier-protected line which includes a transformer bank at one end and has had successful results. The differential relays trip the local low-voltage breaker and also cause a coded signal to be sent via the carrier to trip the breaker at the other end of the line. The use of a coded set of carrier impulses eliminates possible trippings on stray interference. The transmission and reception of the code signal required about one-third second. Faster action could be obtained by using one or more modulating tone frequencies instead of applying impulses to the carrier wave itself.

Communication

In applying communication to a relaying carrier-current channel, it should be realized that the carrier channel is primarily for relaying purposes and was not designed as a communication channel. A high-quality communication channel requires a number of characteristics which a relay carrier transmitter-receiver does not possess. When the communication requirements as to quality, distance, number of talking points, etc., become important, it is usually more economical and satisfactory to divorce the communication from the relays by providing a separate channel. In some cases, it is possible to obtain the channel separation but still make some of the apparatus serve both relays and communication.

However, for moderate communication requirements, the relay carrier equipment lends itself admirably to communication purposes. One relay carrier transmitter-receiver has a built-in modulator for this purpose. The type of communication ordinarily provided by a relay carrier channel is known as the "point-to-point-push-to-talk" method. That is, an operator at one station first signals the operator at another station by means of the carrier push button and the alarm bell. After contact has been made, the operator wishing to talk presses a button on the handset to cause carrier transmission. After he is through, the operator at the other end of the line must depress his handset button in order to reply.

Communication on a relay carrier channel is necessarily limited to point-to-point because the relay scheme requires that each line section operate on a frequency of its own, isolated from other sections. Communication through a line sectionalizing point means that the carrier must be by-passed around the station. Obviously, a relay carrier channel cannot be handled this way. It has been suggested that several line sections be coupled together for communication on the relay carrier channel by repeating the voice frequencies around the substation. Such an audio by-pass can be arranged, but a complication introduced is the cascading of the receiver distortion. With several sections coupled in this manner the speech would be unintelligible. Not more than two line sections can be coupled together and still retain a fair quality of speech.

If the importance of the communication function requires operation from several locations, selective calling, and other telephone requirements, then it becomes more economical to provide a separate communication channel and a communication carrier set rather than attempt to adapt a relay carrier channel to this purpose.

In providing an additional carrier channel for communication, some savings may be affected on higher-voltage lines by using a common coupling capacitor to connect the two carrier channels to the high-voltage line. Both relay and communication channels can be on one phase-to-ground, or the communication can be phase-to-phase and the relay channel phase-to-ground. Figure 7*a* shows relaying and communication on one phase-to-ground using one coupling capacitor. No line chokes are shown for the communication frequency, since they are not necessary unless the distances and number of line paths over which the carrier can divide are too great. Figure 7*b* shows the communication phase-to-phase and the relay carrier channel phase-to-ground. Here it is necessary to provide one extra coupling capacitor for the communication. Phase-to-phase communication is more efficient and encounters a lower noise level than phase-to-ground coupling and is generally used where the communication covers a large area.

Conclusions

1. Definite economies can be effected by using a relay carrier channel for other uses. These additional uses will sometimes justify the expense of a relay carrier-current installation.
2. Other functions can be added without much complication, if it is not necessary to perform them simultaneously.
3. Simultaneous performance of two or more functions requires that modulating tone frequencies be provided.
4. The addition of telephone communication limits the number of modulating tone frequencies available for other functions. It is possible to shift the voice-frequency band to accommodate more tone frequencies.
5. If more extensive telephone communication than "point-to-point" is desired, it should be placed on a separate carrier channel.
6. Two separate carrier channels can utilize common coupling capacitors.

Bibliography

1. Perfectionnements aux Systemes de Telecommunication par Courants Porteurs sur Lignes à Haute Tension, Parrisier Publication Conference Internationale des Grands Reseaux Electriques à Haute Tension, 1939.

2. A New High-Speed Distance Carrier-Pilot Relay System, E. L. Harder, B. E. Lenehan, and S. L. Goldsborough. AIEE Transactions, 1938 (January section) page 5.

Discussion

Discussion will be found in the 1941 annual Transactions volume and in the June 1941 Supplement to Electrical Engineering—Transactions Section.

Power-Factor Testing of Transformer Insulation

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MEMBER AIEE

Synopsis: This paper points out a method for analyzing the results obtained from dissipation-factor tests of transformer insulation. Experimental data, obtained both from transformer tests, extending over the past three years, and from tests of sample insulation, indicate a definite relation between resistance and capacitance, with varying conditions of temperature and moisture. This relation furnishes an additional means for determining the condition of insulation when considered with other accepted methods.

The physical arrangement of the transformer windings, and the type of test equipment used, affect the tests and should be considered when interpreting results.

THE term, "power-factor testing" of insulation during the past several years has come into wide use. However, it is felt that the use of the dissipation factor is better because it can be more readily determined and is more convenient to use. Dissipation factor is the cotangent of the angle between the voltage and current while power factor is the cosine of the angle.

Equivalent Insulation Circuits

The electrical characteristics of insulation can be represented in the simplest form, for any given frequency, by an equivalent circuit consisting of a single lumped resistance and a single lumped capacitance. The lumped resistance and capacitance may be arranged either in series or in parallel (see figures 1 and 2). However, for this analysis the parallel equivalent circuit has certain advantages over the series circuit and will, therefore, be used throughout.

Since capacitance bridges used for insulation testing read capacitance in terms of the series equivalent circuit, it is necessary to convert these readings to the corresponding resistance and capacitance components of the parallel equivalent circuit. Table II shows the operations necessary to obtain the equivalent parallel resistance and capacitance from the readings of several types of commercial instruments. It will be found that for low values of dissipation factor, the parallel capacitance is approximately equal to the series capacitance. As the dissipation factor increases, however, the difference between the parallel capacitance and the series capacitance increases so that a conversion is necessary for the larger values of dissipation factor.

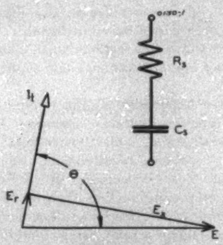

Figure 1. Equivalent series circuit of the insulation

$$\text{Dissipation factor} = \frac{2\pi f C_s R_s}{10^6}$$

When $f = 60$ cycles, C in micromicrofarads, R in megohms:

$$\text{Dissipation factor} = \frac{C_s R_s}{2{,}650}$$

Physical Arrangement of Windings

Transformer windings are grouped in various arrangements depending upon the kilovolt-ampere rating, the voltage and the impedance requirements, and whether of shell- or core-type construction.

Figures 3, 4, and 5 show typical winding arrangements or groupings for two winding transformers and the equivalent circuits of such transformers as composed of three major parts as follows:

1. Insulation between high-voltage winding and ground designated by subscript *a*.

2. Insulation between high-voltage winding and low-voltage winding designated by subscript *b*.

3. Insulation between low-voltage winding and ground designated by subscript *c*.

It is recognized that resolving the insulation structure into these very simple forms does not always give a complete picture but this division is used because of its simplicity.

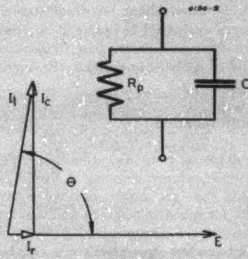

Figure 2. Equivalent parallel circuit of the insulation

$$\text{Dissipation factor} = \frac{10^6}{2\pi f C_p R_p}$$

When $f = 60$ cycles, C in micromicrofarads, R in megohms:

$$\text{Dissipation factor} = \frac{2{,}650}{C_p R_p}$$

If the measuring instrument used contains a guard circuit these individual parts of the insulation may be tested directly.

When a guard circuit is not available, it is then necessary to conduct the following three tests:

Test *A*—High-voltage winding to low-voltage winding and ground.

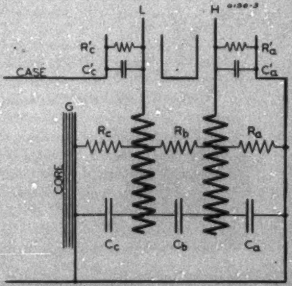

Figure 3. Schematic diagram of a two-winding transformer having core-low-high coil grouping

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Paper 40-130, recommended by the AIEE committee on electrical machinery, and presented at the AIEE Pacific Coast convention, Los Angeles, Calif., August 27–30, 1940. Manuscript submitted July 1, 1940; made available for preprinting July 12, 1940.

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Table I. Separated Capacitance and Resistance Components for Equivalent Circuits of Figures 3, 4, and 5

From Tests A, B, and G	For Figure 3	For Figure 4	For Figure 5	Part of Insulation
$\frac{1}{2}(C_A+C_G-C_B)=$	C_a+C_a'	C_a'	C_a+C_a'	High voltage to ground
$\frac{1}{2}(C_A+C_B-C_G)=$	C_b	C_b	C_b	High voltage to low voltage
$\frac{1}{2}(C_B+C_G-C_A)=$	C_c+C_c'	C_c+C_c'	C_c'	Low voltage to ground
$\frac{1}{2}\left(\frac{1}{R_A}+\frac{1}{R_G}-\frac{1}{R_B}\right)=$	$\frac{1}{R_a}+\frac{1}{R_a'}$	$\frac{1}{R_a'}$	$\frac{1}{R_a}+\frac{1}{R_a'}$	High voltage to ground
$\frac{1}{2}\left(\frac{1}{R_A}+\frac{1}{R_B}-\frac{1}{R_G}\right)=$	$\frac{1}{R_b}$	$\frac{1}{R_b}$	$\frac{1}{R_b}$	High voltage to low voltage
$\frac{1}{2}\left(\frac{1}{R_B}+\frac{1}{R_G}-\frac{1}{R_A}\right)=$	$\frac{1}{R_c}+\frac{1}{R_c'}$	$\frac{1}{R_c}+\frac{1}{R_c'}$	$\frac{1}{R_c'}$	Low voltage to ground

Test *B*—Low-voltage winding to high-voltage winding and ground.

Test *G*—High-voltage winding and low-voltage winding to ground.

From these tests, the values of *R* and *C* of the separate parts of the insulation may be calculated using the relations shown in tables I and II.

Table I shows that the coil grouping materially affects the results and must therefore be considered in the analysis. It will be noted that the capacitance and the resistance components of the high-voltage winding to ground test of figure 4 is a measure of the high-voltage bushings, leads, and terminal boards to ground rather than high-voltage winding to ground. Such cases can usually be detected from the abnormally low values of capacitance for the high voltage winding. A similar case develops in the low-voltage-to-ground reading of figure 5. The circuit, in such cases, may be composed almost entirely of leakage resistance to ground and may have a high dissipation factor without indicating an abnormal condition.

Measuring Equipment

Available testing equipment for conducting dissipation factor tests is of two general types:

1. Instruments which measure the volts applied, the charging current and the loss.
2. Capacitance bridges which measure the equivalent series capacitance and the dissipation factor.

Table II shows the conversion to equivalent parallel capacitance, parallel resistance, and dissipation factor for each type of instrument. This conversion must be recognized in order to place all data on the same basis of parallel resistance and parallel capacitance.

Results

The results of more than three years of testing have definitely indicated that insulation of low moisture content shows a reduction in its a-c resistance with increasing temperature, the capacity part of the circuit remaining practically constant. For wet insulation, on the other hand, increasing temperature has been found to produce a considerable increase in capacitance as well as a reduction in the a-c resistance.

Dry insulation, therefore, has the highest value of a-c resistance and the lowest value of capacitance. As moisture is absorbed, the first change appears in a reduction of the resistance. Further increase in moisture content at constant temperature increases the capacitance along with a more gradual reduction in resistance.

Figure 6 shows this relation, and the predicted change in dissipation factor, at constant temperature, for varying amounts of moisture.

Figure 7 shows the relation between *R* and *C*, for both wet and dry insulation, over the temperature range of 20 to 75 degrees centigrade. This curve is an attempt to show the approximate limits for thoroughly dry insulation and for insulation containing approximately 40 per cent moisture.

Figure 8 shows the relations between temperature and dissipation factor for wet and for dry insulation. It is interesting to note that the dissipation factor for dry insulation increases for increasing temperature faster than for wet insulation. This means that for a known dissipation factor at one temperature, it is not possible to predict what it will be at another temperature, unless the insulation is known to be dry.

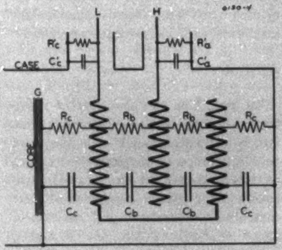

Figure 4. Schematic diagram of a two-winding transformer having core-low-high-low coil grouping

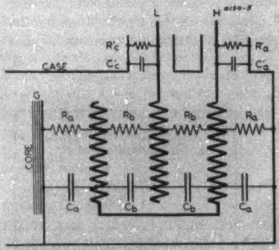

Figure 5. Schematic diagram of a two-winding transformer having core-high-low-high coil grouping

Table II. Conversion to Equivalent Parallel Circuit Constants for 60-Cycle Tests

Method Used	Instrument Reads	R_p	C_p	Dissipation Factor
1. Loss	*E*, *I*, and *W*	$\frac{E^2}{W}$	$\frac{2,650}{R_p \times DF}$	$\cot \cos^{-1}\frac{W}{E \times I}$
2. Capacitance bridge	C_s and DF	$\frac{2,650}{C_p \times DF}$	$C_s \times \sin^2\theta$	From bridge

Note: Express *E* in kilovolts, *I* in milliamperes, *W* in watts, *R* in megohms, *C* in micromicrofarads, *θ* in degrees (power-factor angle).

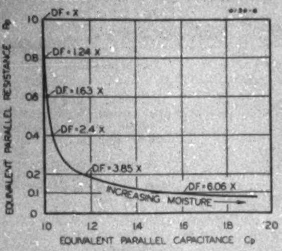

Figure 6. Relation between R_p and C_p for constant temperature and increasing moisture content

Figure 9 shows the changes of R and C for wet and dry insulation as a function of temperature. These characteristics furnish the means for determining the condition of the insulation from the moisture standpoint, since the capacitance component for wet insulation increases much more rapidly than for dry insulation with increasing temperature.

Analysis

To determine the condition of transformer insulation where high dissipation factors are encountered, it is suggested that in addition to the usual checks, such as tests on the oil and Megger tests, dissipation-factor tests be conducted at more than one temperature, and that these tests be separated into the R and C values of the component parts. If the tests do not show an appreciable increase in C for the higher temperature readings, the insulation is relatively moisture free, while if C shows a considerable increase for any one of the separated values, the presence of distributed moisture in that part of the insulation is probably indicated. Corrective measures then consist of either drying the insulation or of reconditioning by other means.

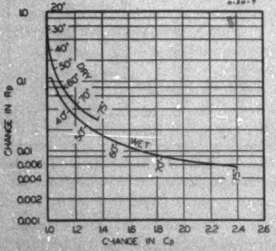

Figure 7. Relation between R_p and C_p for constant moisture content and increasing temperature

Several cases have presented themselves of transformers having high dissipation factors which when tested at two different temperatures did not show an increase in capacitance at the higher temperature. To check this condition, the units were placed in a drying oven for more than 400 hours, part of the time under vacuum. During this time the capacitance did not change and the cause of the high dissipation factor was traced to the transformer oil. When the oil was replaced, the dissipation factor tested normal.

In another case tests conducted on a series of small transformers of the same design, showed variations in the dissipation factor. The winding arrangement of these units was similar to figure 3. When the test results were analyzed all of the variations appeared in the R_b and R_c values, both of which are made up of solid insulation. The R_a value, principally a test of the oil, remained constant. Since these variations were only in the resistance values of the solid insulation, it would indicate slight differences in the drying-out procedure.

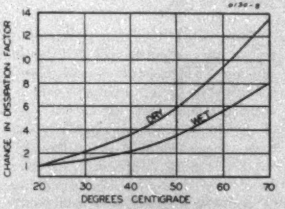

Figure 8. Change of dissipation factor with temperature

An analysis of dry-out runs made in oil have shown, by separating the values, that parts of the insulation sometimes have more moisture in them after the dry-out than they had before, although the over-all dissipation factor was lower. This would evidently indicate a migration of the moisture from one part of the winding to another and that the dry-out run had not been completed.

Cases where the resistance is abnormally low, without a corresponding increase in capacitance, indicate a local low resistance path rather than distributed moisture.

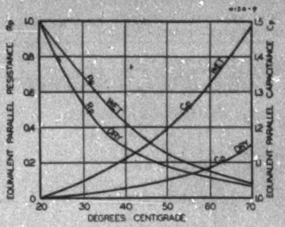

Figure 9. Change of R_p and C_p with temperature

It is recommended that when high voltages are used for conducting these tests, the test voltage be less than the rated voltage of the winding. If higher than rated voltages are used, additional losses, such as for example, the corona loss in the bushings or other parts, may affect the readings and obscure the true condition of the insulation itself.

From this, it is suggested that the results of tests conducted at more than one voltage may be used to detect the presence of corona.

Conclusions

Tests have indicated that when properly applied, dissipation-factor testing, in connection with other accepted tests, may be useful in determining the condition of insulation and sometimes the location of the defective part may be actually determined. It is also possible to determine whether the high dissipation factor is due to moisture or to other cases, which drying out would not improve.

No conclusions have, as yet, been reached regarding the extent to which the resistance may decrease without an appreciable increase in the capacitance before the insulation is considered unsafe from a moisture standpoint. For example: assuming a dissipation factor of X (figure 6), it will be seen that the resistance may drop to 0.4 of its dry value, at the same temperature, with only a slight increase in C, but with an increase to $2.4X$ in the dissipation factor. A change of this order may not be objectionable as it still lies in the vertical part of the curve, indicating only a small increase in moisture content for the higher dissipation factor.

Discussion

Discussion will be found in the 1941 annual TRANSACTIONS volume and in the June 1941 SUPPLEMENT to ELECTRICAL ENGINEERING—TRANSACTIONS SECTION.

Investigation of Starting Requirements of a 660-Horsepower Locomotive Diesel Engine

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PRIOR to 1926 the starting of railway-type Diesel engines was generally accomplished by the use of compressed air. This method of starting had certain disadvantages in locomotive operation, and in an effort to overcome them, a trial installation was made of an electric starting system. This involved the application of a 56-cell lead-acid type storage battery and the addition of a special series winding in the main generator of the locomotive, through which the battery current was passed when starting, causing the generator to function as a series-wound motor. Since that time this method has been used successfully on practically all Diesel-electric locomotives.

Much of the application work in succeeding years was done on an empirical basis; more recently it was felt that a definite analysis of the conditions governing starting should be made on a railway-type Diesel engine. Tests were desired that would permit determination of (*a*) the maximum torque required to turn the engine over against compression, (*b*) the torque and speed at the moment the engine begins to fire, and (*c*) the elapsed time from the closing of the cranking power circuit until the engine begins to fire, for the conditions of a cold, a normally hot, and an abnormally hot engine.

Equipment

The major items of equipment used for the test were an American Locomotive Company 660-horsepower 700-rpm six-cylinder four-cycle, 12$^{1}/_{2}$-inch by 13-inch Diesel engine, a direct-connected General Electric *GT*-551 traction generator, and a 56-cell *MVMT*-13 Exide Ironclad battery to supply power to the generator for cranking the engine.

Some of the technical difficulties involved in making an investigation of this nature may be readily anticipated, permitting appropriate steps to be taken for their successful solution, while the existence of still other problems may not even be suspected until after a large amount of test data have been worked up and carefully analyzed. The determination of the torque delivered by the cranking motor to the engine, and the engine speed, fall into the former classification.

Because of the pulsating nature of the compression torque of a Diesel engine, it is to be expected that the cranking torque and the speed during cranking will also be pulsating in nature. In the case of a six-cylinder four-stroke-cycle engine, there are three compression strokes per revolution, resulting, at 100 revolutions per minute, in a pulsation frequency of 300 cycles per minute or 5 cycles per second. Recording meters are entirely unsuited for making measurements of this nature because of the high inertia of their moving systems and because of the limitations in the performance of their recording stylus on the chart. The conventional multielement permanent-magnet oscillograph is the ideal measuring and recording instrument for the purpose.

Having determined the necessity of using an oscillograph in the investigation, we must consider what quantities to measure and record. In this respect, the determination of the torque input to the Diesel engine presents the greatest problem. It would be most desirable to determine and record it directly by means of one of the various strain measuring methods, but unfortunately this cannot be done because of the closeness and rigidity of coupling between the generator and engine. In view of this, the most practical method seemed to be the use of the generator itself as a dynamometer. Since it is connected as a simple series-excited motor during the engine cranking period, it is necessary to measure only a single electrical quantity, current input, to permit a quite accurate determination of the torque developed. The few minor loads supplied by the battery while cranking the engine are entirely insignificant compared to its input to the generator; hence the addition of a battery voltage trace to the oscillograph record will permit determination of the battery volt-ampere discharge characteristic. Recording the armature voltage of a small separately excited shunt generator, direct connected to the engine generator set, was chosen as the best and most convenient method of determining engine speed. A 60-cycle voltage wave was also recorded to serve as the basis for all time measurements. Thus, four oscillograph elements were required and arranged to indicate:

A—Battery current
B—Battery voltage
C—Generator speed
D—Time (60-cycle wave)

Test Procedure

Preparatory to the first cranking test, the Diesel engine was warmed up to average operating temperature as indicated by a radiator water temperature of 120 degrees Fahrenheit. Then, after a shutdown of approximately five minutes and at a prearranged signal from the oscillograph operator, the engine starting button was pressed. This start button closed magnetically operated contactors between the battery and the generator, and was held in, maintaining this connection, until the engine was audibly running under its own power. This procedure was followed for each of the four succeeding starts, each time using four cells less of the battery. Oscillograph records designated as tests 1, 2, 3, 4, and 5 in the following tabulations were taken using 56, 52, 48, 44, and 40 cells of battery, respectively.

Following this series of normal-engine-temperature starts, the battery was once more restored to its usual 56-cell connection, and the engine operated under load with radiator fans shut down until an engine water temperature of 180 degrees Fahrenheit was obtained. An oscillograph record designated as test 6 was then taken in the manner described above except that the shutdown preceding the start was of only three minutes instead of five minutes duration, in order to prevent too great a loss in engine temperature, which would have defeated the attempt to obtain the cranking characteristics of the engine while abnormally hot.

Following completion of the day's testing, the water was drained from the engine and radiator system and the

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locomotive moved out of doors and left to cool down. During the night the outdoor temperature varied between 12 and 22 degrees Fahrenheit. The engine and generator were completely exposed to the wind and weather during this time, as the hood which normally covers this apparatus had not yet been installed. In the morning the locomotive was pushed indoors, the radiator system filled with cold water, and the oscillograph reconnected before proceeding with the cold series of tests. These were made in the same manner as the normal-engine-temperature tests, except 8 cells of battery were cut out between starts, oscillograph records designated as tests 7, 8, and 9 being taken using 56, 48, and 40 cells of battery respectively.

In addition to the oscillograph records, readings were taken from time to time, of radiator-water temperature, lubricating-oil temperature, also battery-electrolyte temperature and specific gravity.

Results

The nine oscillograms taken under conditions as just described have yielded the data shown in table I, which shows the mechanical quantities of torque and speed which these tests were undertaken to determine, also the electrical quantities of amperes (from which the basic uncorrected torque figures are obtained) and volts. Unfortunately, the existence of a calibration error in oscillogram number 1 rendered a portion of its data inaccurate, which accounts for the blank spaces in the tabulation.

The oscillogram of test number 4 reproduced in figure 1 is typical of the records obtained. Various explanatory notes and lines have been added to the original record to aid in tying it in with the values entered in the table of data just mentioned.

However, before attempting a step-by-step interpretation of how the desired information can be obtained from the traces on such an oscillogram, it is desirable to consider for a moment the sequence of events as they take place during the cranking period:

1. As the engine stops preceding the start under consideration, it rocks back and forth several times, finally coming to rest near one of three equally spaced angular positions as determined by the action of the various cylinders which operate in turn as air motors and air compressors.

2. From this position of rest, the engine is rapidly accelerated when the cranking circuit is closed. As the first compression stroke will be only a partial one, the maximum compression torque will be less than usual, and the electrical torque of the starting motor may readily be sufficient to continue accelerating the engine as it turns through the position corresponding to the first peak of torque demand.

3. As the first piston goes over dead center, the air motor action in this cylinder aids the starting motor in further acceleration of the engine.

4. As the second compression peak is approached (first full compression stroke), the rotating system decelerates and its stored kinetic energy is released, thus aiding the starting motor. Deceleration must of course cease and acceleration again take place if the engine is to keep on turning over. At this particular instant of zero acceleration, when the rotating inertia effects are zero, the net torque developed by the starting motor is all delivered to the Diesel engine. The tests indicate that this is the maximum torque output required of the starting motor by the Diesel engine at any time during the starting period.

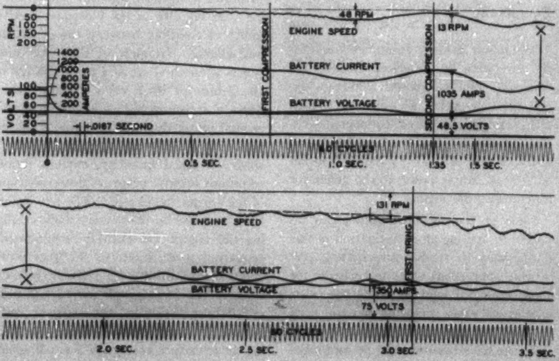
5. The governor and fuel injection system used on this engine are arranged so that a gear pump in the governor must build up a sufficient pressure and flow of oil to operate its power piston before any fuel can be injected into the engine cylinders. Observations indicate that when starting from rest after several minutes shutdown, the engine must turn over approximately three complete revolutions to obtain a fuel rack position equal to that required at idling speed and no load, which corresponds approximately to the minimum charge of fuel which the engine can fire successfully. If the engine speed at the end of three revolutions is sufficient to heat the charge of air undergoing compression to the igniting point of the fuel oil, the engine will start. If the blow-by, slowness of compression, coolness of the engine, or radiation losses are such as to prevent the rise of the combustion air up to the ignition temperature of the fuel by the time three revolutions have been made, additional time, or additional time and cranking speed, will be required before the engine can fire and run under its own power.

From the foregoing it should be clear that the observed speed at which the engine first begins to fire does not necessarily bear any relation to the minimum speed at which the engine could fire under the particular temperature conditions. When the engine does begin to fire, its speed rises at an increasing rate with a corresponding change in the rate at which the motor current decreases. This change in rate of decrease of the motor current is found to be the best indication on the oscillograph records of the particular compression stroke which resulted in the first explosion.

The three pulsations per revolution which occur in all the quantities it is desired to measure make necessary the establishment of some arbitrary method of determining their value at the time of firing. For the purpose of this investigation, the average value during the one-third revolution preceding the first explosion will be considered as the value of that quantity at firing.

The conversion of cranking-motor current as read from the battery-current trace on the oscillograms to motor torque is accomplished by reference to figure 2. This is the type of curve sheet commonly prepared to cover the cranking characteristics of a railway-type Diesel-driven generator. In the present discussion, consideration need be given only to the torque-versus-ampere curve. To determine the torque delivered to the Diesel engine at the time of firing, a correction must be made to the value read from the curve to allow for the torque developed by the motor, but which is absorbed internally in accelerating its rotor. The value of this accelerating torque is obtained by first determining the average

Figure 1. Oscillogram showing engine cranking and starting



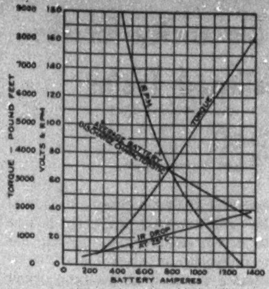


Figure 2. Engine cranking characteristics of GT-551 traction generator with 56 cells of MVA-13 Exide battery

rate of change of speed at the time of firing and applying the formula

$$T=K\left(\frac{\text{rpm}}{\text{second}}\right), \text{ where}$$

T = torque in pound feet absorbed by rotor inertia

K = a constant having a value of 12, being in the general case $WR^2\times\frac{2\pi}{60\times32.2}$

$\left(\frac{\text{rpm}}{\text{second}}\right)$ = average acceleration at time of firing expressed in revolutions per minute per second

On some of these tests this correction factor turns out to be a very appreciable percentage of the total torque developed by the starting motor.

After this discussion of some of the significant events leading up to the first explosion and a few words regarding the method of determining specific voltage, current, speed, and torque values from the ever-changing values recorded on the oscillograms, we have reached a point where the story told by figure 1 becomes much more understandable.

On this record, time progresses from left to right in the conventional manner. The lower trace is the 60-cycle timing wave, having a time of 0.0167 second between successive upward peaks. Immediately above it is the straight horizontal zero reference line of the battery-voltage element, which shows a maximum deflection corresponding to 95 volts immediately prior to the closing of the starting contactors. The horizontal line immediately above the battery-voltage zero line is the battery-current zero line. Both elements deflect upward for increasing values. However, in the case of the tachometer generator's armature voltage trace at the top of the film, the deflection has been reversed so that a downward trend indicates increased engine speed. Elapsed time measurements will all be referred to the instant that battery current starts to flow.

For convenience, a small amount of overlap has been allowed on the upper and lower portions of the oscillogram as reproduced, the reference line X-X being shown on both portions of the picture. The upper portion shows three compression strokes (one partial and two full) while the lower portion shows 11 compression strokes. Thus the entire record covers approximately $4^1/_2$ revolutions, with an elapsed time of less than four seconds.

The exact location of the peak torque point on the first partial compression stroke is not obvious from a casual inspection of the oscillogram, but may be located accurately by integrating the area between the speed trace and its zero reference line. It will be noted that, following this first compression stroke, the angular velocity of the engine momentarily reaches a value equivalent to 48 rpm, but drops back to 13 rpm at the maximum torque point of the first full compression stroke. At this instant the record shows the battery voltage to be 46.5 volts, and the current 1,035 amperes, with 1.35 seconds elapsed time. By referring to figure 2 we find 1,035 amperes corresponds to 5,450 pound-feet torque.

Having passed over the maximum torque point of the first full compression stroke, the torque required for succeeding compression strokes is, as indicated previously, materially reduced by virtue of the energy recovered from the expansion of the air previously compressed. Because of the series motor characteristics of the starting motor, the system accelerates quite rapidly, with a relatively uniform dropping off in current. A smooth curve drawn through the peaks of the current trace shows a decided break following the ninth full compression stroke. Thus it is on the ninth full compression stroke or approximately 3.1 revolutions from rest, and after 3.10 seconds elapsed time, that the first explosion took place. Although the starting contactors were held closed for approximately 0.9 second after this first explosion, they could have been opened sooner, and the engine would have continued to run under its own power, accelerating up to the idling speed for which its governor was adjusted.

Averaging the traces of the current, voltage, and speed elements for the one-third of a revolution preceding the time of firing gives values of 350 amperes, 75 volts, and 131 rpm, corresponding to the figures appearing in table I. The average current of 350 amperes corresponds to 790 pound-feet torque as read from the curve of figure 2. As previously explained, this must be corrected for the torque used in accelerating the generator armature. A line drawn to represent the "average" speed of the engine prior to firing has a slope corresponding to approximately 24 rpm per second. Applying the formula $T=12$ (rpm/second) results in a value of 290 pound-feet for the average armature accelerating torque. It is only the difference between 790 pound-feet and 290 pound feet, or 500 pound-feet of torque, that is available for cranking the engine.

Conclusions

In 1930, when this particular size Diesel engine was first being applied to railway

Table I. Summary of Information Obtained From Oscillograph Tests

Battery			At Second Compression				At Firing Speed					
Number of Test	Number of Cells	Temp. of Circ. Water (Deg F)	Battery Volts	Battery Amp	Torque (Lb Ft)	Speed (RPM)	Battery Volts	Battery Amp	Torque (Lb Ft)	Speed (RPM)	Elapsed Time (Sec)	Revolutions to Fire
1	56	120	56			33	96			171	2.72	3.8
2	52	120	57	1,050	5,600	30	88	420	800	138	2.42	2.8
3	48	120	53	1,040	5,500	25	80.5	410	900	130	2.76	3.1
4	44	120	46.5	1,035	5,450	13	75	350	500	131	3.10	3.1
5	40	120	41.5	1,030	5,450	0	67	355	600	106	4.66	2.8
6	56	180	55	1,200	7,400	10	98	400	600	160	2.67	3.2
7	56	56	57.5	1,040	5,500	21	82	565	1,800	101	2.72	3.1
8	48	53	44.5	1,100	5,970	8	70.5	540	1,500	93	3.00	2.8
9	40	53	35.5	1,050	5,600	0	60.5	460	1,200	89	5.48	3.8

use, its cranking requirements were given as 4,500 pound-feet breakaway torque and 1,500 pound-feet torque at 140 rpm firing speed. An equipment designed to meet these specifications was found inadequate, and in 1931 the requirements were altered to 6,400 pound-feet breakaway torque and 1,500 pound-feet torque at 80 rpm firing speed. As a result of these tests, it seems that the cranking requirements of the engine are still understated, and that at least 7,000 pound-feet torque at standstill and at least 2,000 pound-feet torque at 80 rpm firing speed should be provided. The excellent service record of the electrical cranking equipment used on these engines during the past eight years is most certainly due in part to the leeway provided by the manufacturers over the engine builder's stated requirements. It may be of interest to compare the torque rating of this engine, which is 4,952 pound-feet, with the 7,000 pound-feet or more of cranking torque required under certain conditions.

Past practice in applying electrical starting to Diesel engines has universally been based on the assumption that the breakaway torque of the cold engine represented the maximum torque requirement. This series of tests indicates that the torque required to turn a hot Diesel engine through its first full compression stroke may exceed that necessary for all other operating conditions. While no true breakaway torque requirements of this engine were obtained, the ease with which it started on test number 7 and the accumulated service experience on a large number of similar equipments tends to confirm this new point of view. Aluminum pistons were used in the engine under test. It is to be expected that the use of cast-iron pistons might materially change starting conditions.

Electrical equipment for engine cranking could most certainly be more efficiently designed and applied if additional information on the actual starting requirements of large Diesel engines were available. Special caution must be used if an attempt is made to apply data derived from these tests to other large engines differing from it as regards number of cylinders, size of cylinders, or combustion cycle.

Discussion

Discussion will be found in the 1941 annual TRANSACTIONS volume and in the June 1941 SUPPLEMENT to ELECTRICAL ENGINEERING—TRANSACTIONS SECTION.

The Application of Electricity for the Auxiliaries of Railroad Trains

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Synopsis: Although the application of electricity for the auxiliaries of railroad trains has not been as spectacular as its use for traction purposes this has increased in a remarkable manner since its first application for electric-lighting purposes about 60 years ago.[1] This application was confined largely to lighting purposes until the last few years when streamlined high-speed passenger trains and air-conditioned cars have ushered in a new era of railroad transportation. This paper points out the increased use of electricity for auxiliaries other than those in electric and Diesel-electric locomotives.

First Uses of Electricity on Cars

THE earliest applications were for lighting purposes only and were of the following systems:

First. The head end system, consisting of a steam engine generator or steam turbogenerator, together with several sets of storage battery. This furnished lights to an entire train.

Second. The axle-generator system, consisting of a generator belted to one of the car axles, combined with a storage battery. This furnished lights to the individual car on which it was applied.

Third. The straight storage system, consisting of several storage batteries on a train, the batteries being charged at terminals.

In addition to these, numerous other experiments were tried, even including a windmill-driven generator attached to the front of a steam locomotive.

Head-End System

The "first" method was called the "head end" system of lighting, so called due to the turbogenerator being located in a baggage car at the head end of the train. With the head-end system several storage batteries were distributed in the train and the nominal voltage was either 110 or 64 volts. The generator was operated at scheduled times with most of the battery charging done in the daytime to prevent excessive lamp burnouts due to overvoltage or else an adjustable resistance was used in the lamp circuit during charging periods. Cars were equipped with an overhead train line consisting usually of three 4/0 cables with two of the cables cross connected at the rear of the train in order to give more uniform lighting voltage throughout the train by means of a three-wire loop system.

In earlier head-end installations a generator of about 25-kw capacity supplied energy for electric lights and battery charging for an entire train but the advent of air conditioning has increased the electric load to such an extent that electric lighting is now only a minor part of the total electric load.

Some of the shorter Diesel-driven streamlined trains use a nominal 64-volt generator belted to the main Diesel engine but the longer trains use one or more auxiliary Diesel-engine-driven alternators which furnish 220-volt 60-cycle three-phase power for the train. This voltage was used as 220 volts has been adopted by the American railroads as the standard a-c voltage for station and coach-yard standby connections for air-conditioned cars.

However, especially where a certain amount of electric heating is used, this voltage necessitates unusually heavy train lines so that for trains of more than 10 or 12 cars a higher voltage would be more economical. The next step would be to 460-volt three-phase generators or, if much electric heating is used on the train, 2,300-volt three-phase generators would undoubtedly prove more economical.

This use of head-end power is feasible only with articulated trains or others that are operated as a unit and on account of the increased flexibility the greater number of passenger cars have their own individual axle generators and batteries. As a result the head-end system has been abandoned for other trains with the exception of certain suburban trains where the turbogenerator is located on the locomotive. The usual practice on such suburban trains is to omit the storage batter-

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1. For all numbered references, see list at end of paper.

ies and operate the turbogenerator whenever any lights are required in the train.

Axle Generators

For non-air-conditioned cars the axle generators have generally been of from two to four kilowatt capacity, using 32-volt batteries of from 250 to 350 ampere-hour capacity.

For air-conditioned cars axle generators, 15 and 20 kw capacity, are in common use with electromechanical air conditioning systems, with 1,000-ampere-hour batteries at 32 volts, almost the minimum size. A nominal voltage of 32 is in general use although certain roads are using 64 volts to reduce the size of cables and other current-carrying equipment.

For the smaller axle generators, such as four kilowatt, flat rubber belts have been the general standard although a number of vee belts and a few chain drives are in use. For 15- or 20-kw axle generators either gear drives or combination vee-belt and gear drives are universally used, the large majority of the generators being suspended from the body of the car and driven through splined propeller shaft. However, a number of generators are truck suspended and driven either by vee belts or spur gears.

Successful air conditioning of cars has been much more difficult than air conditioning of buildings due to the low narrow rooms, the limited space available for equipment, and the limited power supply. Much care has been used in developing air-conditioning control and air distribution in order to provide better temperature conditions and to reduce objectionable drafts.

Power Supply for Air Conditioning

Various methods are used to drive the Freon compressor when the Freon compression system of air conditioning is employed.

One system uses a d-c motor, often in the same frame with a 220-volt three-phase motor for station or yard use.

In a second system the compressor is driven from the axle through an electric slip clutch which limits the compressor to a predetermined maximum speed. Here again an electric motor is often added to take over the compressor load automatically whenever the car speed is low or the car is standing still, this motor being sometimes so controlled that it becomes a generator when the car speed reaches a certain point, thus making a separate axle generator unnecessary.

A third system uses an automatically controlled propane-gas engine for driving the compressor.

The first two of these systems of course, take power from the locomotive to operate the compressor whereas the third system does not.

Two other systems of car air conditioning are in general use, one being the steam-jet system using steam from the locomotive and the other the ice-activated system. All systems use electric motors for driving blowers and fans inside the car, the steam-jet system uses them for pumps and condenser fans, and the ice-activated system for the ice-water pump.

Use of Diesel and Propane-Gas Engines

Four cars have been equipped with individual Diesel-engine-driven alternators which furnish power for air conditioning, lighting, electric heating, etc. Here hot water from the Diesel-engine jacket water system is also used for heating, no steam heat being used. The Diesel engine and alternator are mounted on a sliding carriage so that they can be pulled out for convenient maintenance. When in normal position they are enclosed in a steel box to quiet the engine noise. The Diesel engine is operated whenever the car is in service. The engine is started and stopped by push buttons located inside of the car and generator voltage regulation is entirely automatic. The transition between car heating and car cooling is also performed automatically.

One or more cars have been equipped with a Diesel engine driving both an air-conditioning compressor and an electric generator.

A number of cars have also been equipped with an automatically controlled propane-gas engine generator set, the majority of these being on cars using a propane-gas engine for air conditioning also.

Locomotive Lighting

In contrast with the large increase in the electric load of passenger cars the electric load of modern steam locomotives is really less than it was on the earlier installations. This is due to the higher efficiency of incandescent headlight lamps over that of the arc headlights which were used at first. A 250-watt headlight is the usual size and a steam turbogenerator with a capacity of 500 to 1,000 watts at 32 volts direct current is quite generally used as the source of power supply.

The addition of train-control equipment has increased the electric load on locomotives equipped with this but this increase has been less than the decrease due to the substitution of incandescent lamps for arc lamps.

Progress in Car Equipment

The improvements that have been brought about in the art of supplying electricity for car use have been primarily to supply more reliable power and second to supply enough power in recent years to meet the ever-increasing demands.

Axle generators and their control were improved year after year as experience brought out their weak points. Moving parts of voltage regulators were counterbalanced to protect them from sudden movement of the car, flat belts and pulleys were improved to give the belts longer life and prevent excessive belt slippage in freezing weather. Vee-belt drives and gear drives were developed to give more positive driving conditions with larger power requirements. Voltage for lights and certain motors and control has been made more uniform through lamp-voltage regulators of the carbon-pile type. Thermostatic temperature regulation is now practically universal for air-conditioned cars, both for heating and cooling.

Electric lighting in cars is being constantly improved and higher intensity provided. Lighting fixtures have been improved, perhaps occasionally with more regard to artistic lighting than to furnishing the proper intensity efficiently.

Indirect lighting has been used in many cars with good results. Lenses of both the prismatic and the "bull's eye" type are being used on lighting fixtures to distribute the light properly. Night lighting has advanced from reduced-voltage dimming arrangements to the use of small auxiliary blue lights and the occasional use of under-seat aisle lights. Many coaches now have reading lights on the lower side of the bag racks, individually switched at the passengers' will. Sleeping-car berth lights have been decidedly improved recently and special attention has been given to mirror lighting in dressing rooms. Electric lamps are of course, being continually improved and today a number of special lamps are used where the type of fixture requires it.

Fluorescent-lighted cars appeared in 1939 and a number of them are now in service. The effect produced by fluorescent lighting is good as well as novel and these lamps offer a means of improving the lighting of cars due to their high efficiency. Due to the limited amount of electricity available on passenger cars it has always been more difficult to furnish good lighting on cars than it is in buildings, consequently any improved light source is welcomed by car lighting engineers. However, with the exception of the few cars equipped with a-c generators, the common use of low-voltage direct current for car lighting has complicated the use of fluorescent lamps and made their over-all efficiency lower than it would be were a-c power available. In some installations a small motor-alternator has been used to provide the a-c power and in others a number of small vibrators have been used. Now the lamp manufacturers have introduced fluorescent lamps and auxiliaries that will operate on 64 volts direct current. One or more test installations have been made and where this voltage is available it is unnecessary to convert it to alternating current.

Ten years ago 700 watts was an average electric load for a railroad coach whereas now 2,500 watts is not considered excessive for lighting in addition to an air-conditioning load of around ten kilowatts. In other words it is not unusual for a present day coach to have over 16 times the electric load of a coach ten years ago while the addition of modern dressing and smoking rooms has decreased its passenger-carrying capacity.

There have been a few installations of electric refrigeration on dining cars but their use has not been extended as refrigerators must be kept in service when the car is standing in the coach yard between trips and this involves a serious drain on the car battery. Electric drinking water coolers are however, now frequently installed.

Electric cooking equipment is used on a small scale, that is for grilles, toasters, coffee makers, etc., but so far as we are aware there has been no completely equipped dining car as the electric range and electric water heaters would have a connected load of about 87 kw and the cost of providing power for such a load is not warranted. Consequently coal in some form is still the usual fuel for cooking, with a few installations of oil and of propane gas.

Some recent dining cars however, have motor-driven blowers to furnish draft for the range and have electric dish washers and garbage grinders.

Other miscellaneous electric devices on modern cars are radios, either a-c receivers with a motor generator set or inverted rotary converter or else a-c receivers of the automotive type. In fact radios have been in common use on the western roads for over ten years and a few are in use in the east.

In a number of cases one receiving set operates loud speakers in other cars or even on an entire streamlined train and there are a number of installations of radio-phonographs.

The introduction of electric razors has caused a demand for 110-volt a-c outlets in dressing rooms. This voltage has generally been obtained by the use of vibrating inverters.

The need of improved air brakes for high-speed trains has caused the use there of electric control in connection with the air-brake system. With such control the brakes can be applied at the same instant on every car in the train. Automatic track-sanding devices and devices to prevent wheel sliding with heavy brake applications are sometimes added.

This equipment does not require much electricity for its operation but does necessitate much additional wiring and equipment on the train.

Station and Coach-Yard Facilities

Station and coach-yard requirements have naturally been greatly increased due to the increased use of electricity on cars. Battery-charging facilities at terminals have always been necessary with axle-generator cars. Formerly the capacity of such battery-charging plants was determined by the needs in adverse winter weather when batteries would become exhausted due to excessive belt slippage.

The present use on air-conditioned cars of batteries having a capacity three or more times than formerly used has changed the heavy battery-charging season to the hot summer season and in many cases has made the former facilities inadequate.

As the majority of cars air conditioned with the Freon compression system have 220-volt three-phase motors for use in terminals this has required many installations there of a-c receptacles, each with sufficient capacity to furnish power for a 15-horsepower motor. Incidentally this motor frequently operates the d-c compressor motor also as a generator to furnish sufficient direct current to carry the blower motors, electric lights, etc., on the car.

To avoid the expense of installing additional high-capacity battery-charging circuits at a terminal already equipped with a-c receptacles, portable motor generator sets or rectifiers are in quite common use. Generally these are enclosed units mounted on rubber-tired wheels so that they can be moved to the side of a car when needed. A line of small portable gas-engine-driven generators is also on the market for use where a-c power is not available.

Passengers will undoubtedly continue to demand higher intensity of electric lighting in passenger cars just as they continue to use higher intensity in their homes and offices.

Conclusion

A recent committee report of the Association of American Railroads[2] estimates the possible total load of a railroad car as follows:

Lighting, etc.	5.0 kw
Air conditioning	10.0 kw
Overhead heat	15.0 kw
Water heating	10.0 kw
Battery charging	5.0 kw
Total	45.0 kw

With steam as the common source of heating for passenger cars there is not likely to be much electric heating used, especially on cars equipped with axle generator so that the committee report is undoubtedly correct in predicting that 30 kw is undoubtedly the limit to the size of future axle generators.

References

1. Report of Committee on History of Railway Electric Train Lighting, Patrick Kennedy, committee chairman. Transcript of *Proceedings* of the Association of Car Lighting Engineers, first annual convention, 1908, pages 23–5.

2. Report of Committee on Car Electrical Equipment, L. J. Verbarg, committee chairman. 1939 *Proceedings* of the Association of American Railroads, Electrical Section, Mechanical Division, pages 125–213.

Discussion

Discussion will be found in the 1941 annual Transactions volume and in the June 1941 Supplement to Electrical Engineering—Transactions Section.

A New High-Speed Thermal Wattmeter

JOHN H. MILLER
MEMBER AIEE

Synopsis: The thermal converter discussed in this paper is of the type which produces at its output terminals a millivoltage proportional to the polyphase power connected to its input terminals, the connections being made as for a wattmeter. High-speed response is secured through the use of a new circuit network wherein the thermocouples function also as heaters. Response time of the order of one-half second is obtained. The use of nickel-alloy transformer cores saturating at overloads broadens the overload capacity of the thermal unit itself. Thermal converters of this type are widely used in the summation of power.

THE measurement of various electrical quantities through the medium of heat has intrigued engineers for many years. Instruments indicating through the mechanical expansion of an electrically heated member were among the very first available to the engineer. However, difficulties in compensation for ambient temperature changes and the lack of perfectly elastic materials at the higher temperatures have caused such devices to be replaced by others not subject to such deficiencies.

The thermal ammeter, with a conventional, permanent-magnet moving-coil movement actuated by the potential developed by a thermocouple in thermal contact with the heated member, has no such inherent disadvantages and may be completely compensated against the effect of ambient temperature variations. Through the use of noble-metal alloys for heater and couple, long-time stability is assured. Such instruments are considered as standard for the measurement of current at frequencies above the conventional design range of electromagnetic devices. Mention should also be made here of thermal converters in vacuum for low currents and those having thin tubular heaters for heavy currents at the higher frequencies, all expanding the range of usefulness of the general principle. Through the use of suitable low-range thermal instruments of this type and series impedances of suitable characteristics the accurate measurement of potential becomes possible even at the higher frequencies.

Theory

The measurement of power through thermal means is nearly as old as the measurement of current. Two heated units are used, one of which is heated by the algebraic sum of two derived currents and the other, by their difference. One of the derived currents a, is in phase with and proportional to the circuit current; the other, b, is in phase with and proportional to the potential. The current in one heater is then

$$\sqrt{a^2+2ab\cos\varphi+b^2}$$

and in the other is

$$\sqrt{a^2-2ab\cos\varphi+b^2}$$

where φ is the phase angle between current and voltage. The quantity of heat produced in the two heaters, assuming the resistance to remain constant, is I^2R, or

$$R(a^2+2ab\cos\varphi+b^2)$$

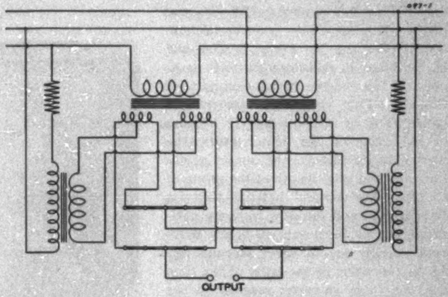
and

$$R(a^2-2ab\cos\varphi+b^2)$$

respectively. This heat is lost largely by conduction and convection with very little lost by radiation, all by reason of heater design. Goodwin,[16] shows that under such conditions, and for the relatively small temperature rise of less than 200 degrees centigrade used here, the temperature rise of the heated member is very closely proportional to the quantity of heat produced in it. If an indication can be obtained proportional to the *difference* between the temperature of the two heaters, the squared terms cancel, and an indication results proportional to $4R\,ab\cos\varphi$, or to power.

The derived currents are usually obtained through transformers. Current transformers of conventional design or modified to meet circuit requirements may be used. Potential transformers are also required to give a current in the network which is a function of the true voltage but of the same order of magnitude as the full-load current from the current transformer for optimum operation of the thermocouple heater combination. Resistance in the potential circuit may be added in the primary or secondary for adjustment and to maintain the derived current truly representative of the potential.

Figure 1. Complete schematic circuit of polyphase thermal converters



Prior Devices

The Lincoln demand meter functions on this principle, the deflection of thermally responsive bimetallic spirals associated with the two heaters and coupled in opposition giving the final indication. Another form of device disclosed by Lincoln,[15] and functioning on the difference of pressures developed in hydrostatic chambers associated with the heaters is representative of another method of securing the mechanism for indicating a temperature difference. Both of these methods are slow in response because of the relatively great thermal capacity of the systems and find their greatest field of usefulness in the art of demand metering where their inherent slow logarithmic response is useful.

The use of thermocouples thermally connected to the two heaters and con-

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16. For all numbered references, see list at end of paper.

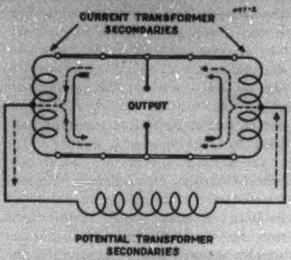
CURRENT TRANSFORMER SECONDARIES
OUTPUT
POTENTIAL TRANSFORMER SECONDARIES

Figure 2. Schematic diagram of transformer seconderies and thermocouples in one phase

nected in opposition to a d-c millivoltmeter was appreciated early in this development as indicated by a drawing in one of Lincoln's patents. The idea was further developed into commercial form some years ago by Perry Borden and H. S. Baker in collaboration with engineers of the Lincoln Meter Company of Canada. This converter, however, is rather slow in response largely because of the mass of the heater units, the surrounding structure, the couple insulation, and the total length of the group of couples. For demand metering this was satisfactory, and the slow response mitigated against burn-outs on heavy overloads, again due to sheer thermal lag. As an indication of the order of the time lag, one such converter reaches 90 per cent of its final deflection in 11 seconds or 99 per cent in 22 seconds, practically final deflection. It might also be noted that the internal resistance of the d-c millivolt circuit has generally been high, of the order of 35 ohms.

For polyphase power, two such systems may have their d-c millivolt outputs connected in series to give an output which is a function of true three-wire polyphase power and for three-phase four-wire service, three single-phase systems may have their output added. For totalizing power in independent systems which may or may not be in phase or, indeed, of the same frequency, the millivolt output of several polyphase converters may be added, with the input to the several units isolated by means of transformers.

The New Device

In the practical application of prior devices the slow speed of response has frequently prevented proper records or indications of phenomena acting at a moderately rapid rate. With a response time for 90 per cent of 11 seconds, only rather slow changes can be shown. Occasionally there have been requirements for much more rapid response, of the order of one or two seconds, which were simply impossible of attainment because of the inherent lag in the produced direct potential. A device with much more rapid response was, therefore, indicated as being most desirable and quite essential for the indication of material system power changes happening within a few seconds.

Another difficulty with the prior devices is their relatively high resistance. While largely used for totalizing in conjunction with potentiometer-type indicating or recording devices, in some instances simple indicating instruments are deemed suitable for the application. However, with the normal output of 50 millivolts at full rating of the prior devices, with an internal resistance of 35 ohms a maximum of only 18 microwatts could be supplied to an instrument. Even though several such units were totalized, the energy is still very small if it must be transmitted over any considerable length of line. It was indicated, therefore, that a new device should have a much lower internal resistance as respects the output circuit, and, if practicable, a higher millivolt value. As evidence that this has been accomplished, the new thermal converter produces 100 millivolts with an internal resistance in the millivolt circuit of 1.8 ohms so that 1,400 microwatts may be taken from it at full rating. This increase of about 75 to 1 in total output to the indicating or recording device, allows for the use of indicating instruments with better performance characteristics over considerable line lengths; the device will also actuate a direct-writing d-c recording instrument although not at any great distance.

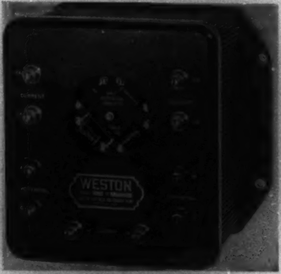

Figure 3. Assembled thermal converter with plate removed showing resistors for field adjustment

The two desirable features of more rapid response and lower resistance would seem to be possible if the thermocouples and heaters could be more closely associated to secure better heat transfer; the mass of the thermocouples might then be reduced and the resistance of the couples themselves also materially lowered.

In all of this preliminary study it was also obvious that the overload capacity was important and should not be reduced in the improved design. Disregarding for the moment the apparent inconsistency of rapid response and high overload capacity, it is obvious that for a short time constant the heaters and associated thermocouples must be made small in mass. Such heater and thermocouple assemblies as used in ammeters, with temperature compensation, would serve except for the fact that in these the couple is welded directly to the heater, and in the conventional, previously used circuits for watt converters these welds would result in cross-connected short circuits. While couples have been attached to heaters with insulation between, the assembly is usually difficult and fragile if reasonable response speed is obtained.

The Circuit

A study was therefore initiated as to the possibility of using conductively connected couples and heaters and evolving a suitable circuit network. Figure 1 shows such a circuit where the thermocouples proper also act as heaters, somewhat as in the bridge-type converters for

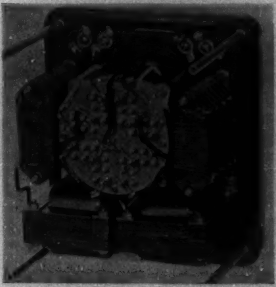

Figure 4. Interior assembly of polyphase thermal converter

thermal milliammeters. The thermocouple wires shown solid are of one material, those in outline of another alloy. The wires are of the same size and the same resistance, but their thermoelectromotive force is considerable with respect to each other. The small circles represent the studs to which the couples are attached and which function as the cold ends of the thermocouples, being bolted to a heavy brass plate with thin mica insulation. This arrangement gives excellent heat flow from the studs to the brass plate which serves as a thermal base line. The temperature rise of the thermo junctions between these studs, above the temperature of the studs and the brass plate, is a direct function of the I^2R loss in them, and without regard to the ambient temperature. And the millivoltage produced by each thermo junction is also a direct function of the I^2R loss except for the slight curvature of less than one per cent in the millivolt versus temperature relation of the materials selected and over the temperature range actually used.

For simplicity in the schematic diagrams, two couples only are shown on each side or a total of eight in all for one phase; in the final polyphase device to obtain the full 100 millivolts, a total of 40 couples are used, 20 in each phase with 5 in each of the four legs of the bridge. It should be noted that the thermally produced potentials of the several couples in each arm are truly additive since each couple is brought back to a pair of studs representing the cold ends with the center junction at a higher temperature.

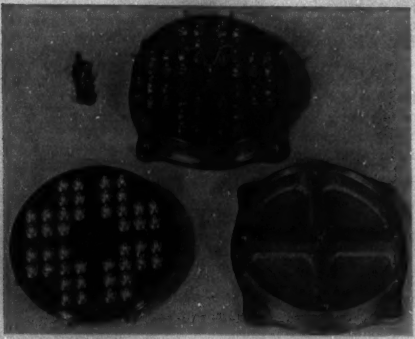

Figure 5. Details of the thermocouple group

Figure 2 shows a schematic diagram of the transformer secondaries and couples alone of one phase only, and it will be noted that the output terminals are symmetrical and at zero alternating potential with respect to each other from either transformer. The upper set of couples carries the sum of the two currents and the lower, the difference; hence, the upper set will be at a higher temperature than the lower. If the couple wires change materially in resistance, that is, have a high temperature coefficient of resistance, the current from the potential transformer will no longer be divided equally in both branches. This, however, may be largely prevented through the use of alloys having low temperature coefficients of resistance as compared with the coefficients of pure metals.

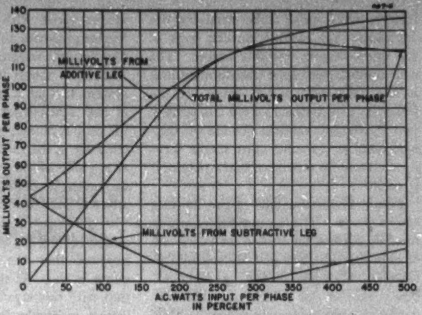

Figure 6. Total millivolts output for one phase and millivolts from additive and subtractive groups individually

Analyzing the circuit with respect to the current transformer, it will be noted that, for a given primary and secondary current, the net secondary ampere turns remain constant irrespective of the value of the current from the potential transformer. The assembly of current transformers with the two independent secondaries, each of which is center tapped, is relatively simple and introduces no difficulties.

General Description

Such an assembly, using thermocouples operating at around one ampere, has a very fast response and will actually come to 99 per cent deflection in 0.5 second. However, this fast response would make the assembly particularly vulnerable to short-time overloads when used in a power plant or in a power system for totalizing.

Recourse has, therefore, been had to the use of nickel-iron alloy cores (Hypernik, Permalloy, Mumetal), all of which have a high permeability at low densities but which saturate quite rapidly thereafter. Such materials, if used for cores in current transformers and where toroidal lamina are used without joints, will allow for the design of current transformers which are amply accurate for the purpose, both as to ratio and phase angle, from no load to full load, with a very small drop for full load at 50 per cent power factor and thereafter will saturate rapidly and give a high degree of protection against overload. This, of course, is predicated on a fixed load and a fixed frequency; the fixed load in this instance is the thermocouple converter proper, and the device can be designed for the frequency with which it will be used. The final design has been so proportioned that the current transformer will be within one-half of one per cent of the five-ampere value up to five amperes primary current, will be within two per cent at ten amperes as representative of low power factor and at higher currents will rise very slowly.

Figure 3 shows an assembled polyphase Thermoverter and figure 4 shows the interior assembly. The toroidal nickel-iron-alloy-core current transformers which are of rather small size are contained in a molded housing back of the thermocouple assembly proper. The conventional shell-type transformers seen in the lower part of the figure are the potential transformers for the two phases; just above them on each side are the conventional wire-wound mica sheets serving as the series resistance in the potential circuits.

Figure 5 shows details of the thermocouple group. The thermocouples, which function also as the heated members, are of noble-metal alloys and are soldered to the relatively heavy studs as can be noted. The chamber on which the plate is mounted contains four chambers so that the heated air in the *A*-phase summation group for example, does not circulate in the portion containing the subtraction group and raise its temperature. This construction minimizes position errors although these are still of the order of one per cent from vertical to horizontal position, and the device should be mounted in a vertical plane as calibrated.

At full load, 500 watts per phase at 115 volts, the potential losses are about 7.4 watts for 7.5 volt-amperes and 0.8 watt for 0.8 volt-ampere in each current circuit. The output is 100 millivolts from a circuit having a resistance of 1.8 ohms.

Figure 6 shows the millivolt output of a single phase plotted against per cent load as well as the individual millivolt values from the addition and subtraction groups. Note that the subtraction group gives a millivolt value going through zero and again rising, this applying only at unity power factor. At other power factors this value will not go to zero, but the total millivoltage will still be truly representative of the total power.

Field Adjustments

The four small resistance units which are normally covered by a plate serve for field adjustments. The two upper spools are in series with the potential-transformer primaries and will allow for about five per cent adjustment of total output. The lower spools form a potentiometer system so that a fixed proportion only of the total output may be taken from the output binding posts. This is of importance in the summation of the output of a group of thermal converters where the millivolt output of each must represent a definite ratio to the primary power; with different transformer ratios the output millivolts of each unit may be so adjusted that the true summation of power is had. It should be pointed out that with this arrangement the effective output resistance from the converter is increased, and summation must be by some means drawing no current as with an automatic recording potentiometer.

Accuracy

An accuracy of better than one per cent of full-load output, 100 millivolts, is had up to 125 per cent of full-load volt-amperes; up to 200 per cent full-load volt-amperes the error is less than two per cent, again in terms of the 100-millivolt output. It should be noted that the errors at the higher volt-ampere loads are not power factor errors as such but are evidence of the beginning of saturation in the current transformers. At zero power factor the output is zero and at 50 per cent power factor no discernible power factor error as such exists.

No claim is made to high precision in a device of this sort. The effort has been rather to produce a device of moderate accuracy for use in totalizing and in conjunction with switchboard instruments with ample output and inherent overload protection.

Conclusion

With a d-c millivoltmeter connected to the output of this new converter, and a conventional polyphase wattmeter in series with the converter to a rapidly varying load, it is interesting to watch the more rapid response of the d-c instrument and converter as compared to the conventional polyphase wattmeter.

The combination of rapid response and low output resistance would indicate a larger field of usefulness for the thermal watt converter.

Bibliography

1. United States Patent 425,260, April 8, 1890, Milton E. Thompson.

2. British Patent 15,618, 1897, M. B. Field.

3. German Patent 11,721, 1899, R. Bauch.

4. British Patent 2,948, November 4, 1904, William D. B. Duddell.

5. French Patent 435,290, February 26, 1912, M. E. Fauvin.

6. United States Patent 1,156,412, October 12, 1915, P. M. Lincoln.

7. Rates and Rate Making, P. M. Lincoln. AIEE Transactions, volume 34, October 1915.

8. United States Patent 1,199,444, September 26, 1916, Thomas W. Varley.

9. British Patent 119,388, 1918, E. Fawsett.

10. United States Patent 1,765,563, June 24, 1930, Perry A. Borden, H. S. Baker, and L. A. Paine.

11. E. Fawsett, *Electrician* (London), volume 88, January 20, 1922, page 71.

12. The Character of the Thermal Storage Demand Meter, P. M. Lincoln. AIEE Transactions, volume 37, 1918, page 189.

13. Totalizing the Load of Four Stations, Perry A. Borden, *Electrical News* (Toronto), volume 35, December 1, 1926, pages 35–8.

14. Totalizing of Electric System Loads, P. M. Lincoln. AIEE Transactions, volume 48, 1929, page 775.

15. Improved Electrothermic Instrument, P. M. Lincoln. Electrical Engineering (AIEE Transactions), 1935, page 474.

16. The Compensated Thermocouple Ammeter, W. N. Goodwin, Jr. Electrical Engineering (AIEE Transactions), 1936, page 23.

Discussion

Discussion will be found in the 1941 annual Transactions volume and in the June 1941 Supplement to Electrical Engineering—Transactions Section.

A Comparison of the Relative Efficiency of the Schafer and Pole-Top Methods of Artificial Respiration

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THE pole-top method of artificial respiration was devised by E. W. Oesterreich[1] of the Duquesne Light Company, Pittsburgh, Pa., to give prompt aid in case of electric shock occurring while men are working on poles of power lines. When a worker receives an electric shock while working on a pole minutes elapse before he can be lowered to the ground and resuscitation started by the prone-pressure method. In the field, the elapsed time is seldom less than five minutes[2] and in many cases it is considerably longer. The pole-top method of resuscitation was developed to reduce this period. Tests have shown that an operator can reach a man working alone on a pole and start squeezing his abdomen within approximately one minute.[3] If there are two men on the pole and one receives a shock the elapsed time before the first pressure is applied is much shorter. While an operator is applying the pole-top method, other members of the line crew make preparation for lowering the victim to the ground where the prone-pressure method may be applied, should it prove necessary.

The results obtained in the field with the pole-top method were excellent, and in 1939 E. W. Oesterreich[3] reported 14 successful resuscitations out of 16 applications. This is an exceptionally fine record when compared with the reports of cases where artificial respiration was not started until the victims had been lowered to the ground.

At the request of the Duquesne Light Company a comprehensive study of the pole-top method was started in the fall of 1939 at The Johns Hopkins University. This is a joint investigation in which its schools of engineering, hygiene, and medicine are co-operating.

This paper presents a report of the first stage of the investigation and deals only with a comparison of the relative amounts of air moved by the standard Schafer prone-pressure method, the modified Schafer method, and the pole-top method.

Standard Schafer Method

The prone-pressure method of resuscitation was developed by E. A. Schafer[4] in 1907 and has been used widely. In applying this method the subject is placed on his abdomen, with one arm extended and the other bent at the elbow to support the face. The operator kneels, straddling one or both of the subject's thighs, and places his hands on the small of the back. With the arms held straight, the operator swings slowly forward and brings his weight to bear upon the patient. Then the operator swings back quickly so as to remove the pressure entirely. The hands are, however, kept in position on the subject's back. The operation is repeated 12 to 15 times a minute.

Modified Schafer Method

In the modified Schafer method of artificial respiration as employed in this investigation, the "snap-off"[5] method of removing the hands from the subject's back was used. The procedure is similar in every way to that described under the standard prone-pressure method except that the operator releases the pressure suddenly by rolling the palms of his hands outward and thus causing them to slip off the subject's back to the ground.

Pole-Top Method

At the present time the recommended procedure in the pole-top method[3] is as follows:

"1. The first man to reach the victim clears the body from electrical contact permitting the body to hang from the safety strap alongside the pole if possible.

"2. The rescuer then takes a position below the victim on the pole and after placing his safety strap around the pole, proceeds to work his way up with one leg of the victim on either side of his safety strap, and with the body of the victim between himself and the pole. When the safety strap is as high on the pole as can be reached, the weight of the victim's body is taken in a straddle position on the safety strap of the operator.

"3. The mouth of the victim is cleared of foreign substances, the tongue is pulled out and the head pushed forward toward the pole.

"4. The operator then encircles the waist of the victim (under arms) placing both hands, one from each side, on the abdomen of the victim, thumbs below the lower ribs and fingers touching.

Figure 1. Pole-top method of artificial respiration

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The authors wish to take this opportunity of thanking the Duquesne Light Company of Pittsburgh for funds for carrying on this research and the Consolidated Gas Electric Light and Power Company of Baltimore for furnishing equipment.

[1] For all numbered references, see list at end of paper.

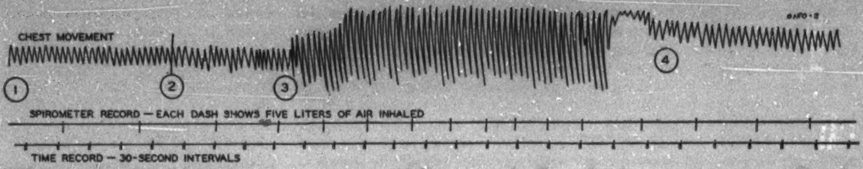

Figure 2. Record of Schafer prone-pressure test

(1) First control. (2) Second control. Operator straddling subject's thigh. (3) Artificial respiration. Note increased chest movement. (4) Final control

> "5. The operator applies a constricting action with his arms and hands, obtaining a compression force in an upward direction on the upper abdominal region. At the finish of the stroke the hands should be cupped with the fingers depressing the abdomen under the breast bone. The pressure is then released and periodically reapplied at a frequency of 12–15 per minute until the victim regains consciousness or the resuscitation is to be discontinued."

Subjects

In carrying out this investigation volunteers were chosen from among the students and the staffs of the University. The 15 men tested ranged in age from 20 to 60 years and in weight from 125 to 211 pounds. Each subject is designated by a roman numeral.

All three methods of artificial respiration were applied to each subject. Repeat readings were also made on each man. The time interval that elapsed between successive observations of the same individual varied from five days to one or more weeks.

The results reported in this paper were obtained from men who possessed the ability to relax completely and to place themselves entirely in the hands of the operator. No drugs or anaesthetics were used in the work. Every effort was made to assure the subjects and to convince them by preliminary experiments and trials that there was nothing to fear and that all that was necessary on their part was passiveness. The apparatus was so arranged that they were not cognizant of the results obtained.

Apparatus

The subject breathed through a gas mask connected with a spirometer or gas meter which measured the amount of air inhaled. Each revolution of the spirometer corresponded to five liters of air. The amplitude of movement of the chest and the rate of respiration were recorded on a smoked drum by the change or pressure set up in a pneumograph strapped around the subject's chest. Each revolution of the spirometer and a time mark at 30-second intervals were recorded simultaneously with the record of the respirations. The drum was driven by a clockwork mechanism.

Experimental Procedure

In general, the experimental procedure employed with the Schafer methods was also used in the study of the pole-top method.

In the prone-pressure tests the subject lay face down on a thin mattress which was placed on the floor. With the gas mask on and the pneumograph in place the subject rested for at least ten minutes or until his breathing was regular and normal. The smoked drum was then started and a normal record of the man's breathing taken. Then the operator took his position straddling the subject's thigh. A second or control record of the respiration was taken after a short interval in order to determine if the operator had disturbed the normal rate and amplitude of the subject's breathing. If the two records were similar, then the operator proceeded to apply the designated method of artificial respiration. A complete record of the man's respirations and of the amount of air moved was taken during the period of artificial respiration, which was never continued for more than five minutes so as not to tire the subject.[6] Following the application of artificial respiration, a third control record was taken.

A section of a standard pole approximately ten feet in length was mounted in the laboratory. This was used in the pole-top method studies. The subject was seated on a safety belt which passed around the pole. The ends of the belt were held by a special pipe support in a position which simulated an operator holding the subject. This support was developed so as to relieve the operator of the work of supporting the subject's weight during the period of relaxation and rest.

The subject seated himself on this belt, and with the gas mask and pneumograph in place, rested until his breathing became regular and normal. This usually required about ten minutes. A short record of the subject's breathing was taken, then the operator climbed the pole and took the weight of the subject on his own safety belt. The pipe support was moved out of the way. Several minutes were then allowed to pass and a second or control record of the subject's breathing was taken when respiration had again become uniform and normal. The operator then applied the pole-top method of resuscitation for from three to five minutes. A continuous record was taken on the smoked drum during this period. This

Table I. Records of Four Tests on Subject I, Pole-Top Method of Artificial Respiration

Date of Test (1940)	Operator Holding Relaxed Subject: 10 Liters of Air: Time to Move: Min.	Sec.	No. of Resp.	Rate Resp. Per Min.	C.C. Air Per Resp.	Subject Receiving Artificial Respiration: 10 Liters of Air: Time to Move: Min.	Sec.	No. of Resp.	Rate Resp Per Min.	C.C. Air Per Resp.
2/1	1	20	19	14.6	526	1	00	11	11.0	909
2/8	1	20	19	14.6	526	0	55	13	14.1	769
2/16	1	25	21	14.8	476	1	5	9	8.3	1111
3/15	1	35	21	13.3	476	1	15	12	9.6	833
Av.	1	25		14.3	501	1	4		10.8	906

record was followed by a third control record while the operator still supported the subject. The operator then climbed down and the subject was again supported by the safety belt attached to the pipe support. In this position a final control record of normal breathing was taken.

A photograph of a subject on the pole with the operator in position and about to apply artificial respiration is shown in figure 1.

One of the records taken on the smoked drum is shown in figure 2. The record consists of three parts. The upper saw-toothed line is a record of the movement of the subject's chest. The increased movement during the period of artificial respiration may be seen clearly. Each dash in the middle line indicates that the subject has inhaled five liters of air. The increase in the amount of air moved during the application of artificial respiration is shown by the closer spacing of the vertical dashes. Each break in the bottom line records the passage of 30 seconds of time.

Three different individuals served as operators who gave artificial respiration during this investigation. Most of the work, however, fell to the lot of one experienced operator. The results obtained by different operators on the same subjects check remarkably well and give evidence of the uniformity of the work and of the passiveness of the subjects. Not only was the subject ignorant of the results obtained, but the operator was also in such a position as to prevent his seeing the record during the period of the application of artificial respiration.

Results

The results are given in tabular form. A small amount of resistance was introduced by the subject's breathing through the mask and the spirometer. Measurements of the variations in pressure in the mask were made with a water manomometer. They were found to amount to less than one inch of water pressure. No corrections were made for these variations nor were corrections made for changes in barometric pressure or temperature of ambient air.

VARIATIONS BETWEEN TESTS OF SAME INDIVIDUAL

As stated, two or more records were made on each individual by each of the three methods of artificial respiration studied. In table I the results for four separate records on subject I using the pole-top method are given. Two different operators were used on this subject.

Table II. Rate of Respiration Per Minute During Control and Artificial Respiration Periods

			Control			Artificial Respiration		
Subject	Age	Weight (Lbs)	Std. Sch. Lying Prone	Mod. Sch. Lying Prone	Pole-Top Sitting on Belt	Std. Schafer	Mod. Schafer	Pole-Top Method
I	60	155	13.2	10.8	14.3	9.1	11.5	10.8
II	54	175	15.3	14.3	16.3	12.0	11.0	10.8
III	42	134	14.5	11.1	16.9	13.7	9.6	7.0
IV	36	140	10.4	10.6	12.0	8.9	9.2	10.3
V	34	188	15.7	17.1	14.7	8.5	10.6	8.3
VI	24	125	13.7	13.6	14.3	8.6	11.4	7.5
VII	27	145	16.6	12.4	15.6	10.5	8.7	8.2
VIII	31	160	12.8	12.8	19.2	15.3	11.7	10.4
IX	27	211	12.0	12.0	12.0	12.3	12.0	10.9
X	20	148	9.0	9.0	8.1	8.1	6.6	8.1
XI	20	193	12.4	12.4	18.0	13.3	12.2	13.8
XII	49	130	11.2	11.2	9.4	5.3	6.5	4.7
XIII	28	140	15.3	15.3	15.1	14.5	13.3	10.9
XIV	38	160	12.8	12.8	13.7	11.6	10.1	12.9
XV	21	140	17.8	22.2	23.0	14.0	18.1	20.1
Av.			13.5	13.2	14.9	11.1	10.8	10.3

Table III. Cubic Centimeters of Air Moved Per Respiration

	Control Period Subject Relaxed			Subject Receiving Artificial Respiration		
	Lying Prone		Sitting on Belt			
Subject	Std. Sch.	Mod. Sch.	Pole-Top	Std. Sch.	Mod. Sch.	Pole-Top
I	463	410	501	635	839	906
II	528	505	509	729	972	1,485
III	576	576	526	611	1,340	1,340
IV	646	572	607	955	1,010	1,111
V	445	495	501	913	955	1,270
VI	403	467	451	769	646	1,429
VII	417	635	557	1,389	1,625	2,500
VIII	635	635	396	607	749	801
IX	667	667	714	625	714	1,000
X	662	662	742	1,111	1,270	1,667
XI	667	667	574	679	1,056	1,056
XII	555	555	742	1,500	1,715	2,250
XIII	488	466	500	572	628	837
XIV	487	487	530	850	932	1,459
XV	528	400	526	1,080	972	1,340
Av.	544	546	558	868	1,028	1,363

The data illustrate the differences found in repeated tests on the same individual.

RATES OF RESPIRATION

In table II are given the rates of respiration with the subject relaxed and lying prone in both the standard and modified Schafer methods and with the subject relaxed and sitting on the operator's belt in the pole-top method. In addition, the rate of breathing for all three methods of artificial respiration is also given. The age and weight of each subject is included in table II.

Lying prone in a relaxed posture the average rate of respiration was practically identical for the standard and modified Schafer methods. With the subjects sitting on the operator's belt the normal rate of respiration is somewhat higher as would be expected. The average rate of application of artificial respiration was substantially the same, namely, 11 times per minute, in both of the prone-pressure methods, and at a slightly less tempo, about 10 per minute, in the pole-top method.

AIR MOVED PER RESPIRATION

Table III gives the cubic centimeters of air moved per inspiration for the subjects relaxed in the prone and sitting positions and for all three methods of artificial respiration studied. The air moved per respiration was naturally found to be the same for the two tests where the subjects were lying on the mattress. In the sitting position, it was slightly higher than for the prone position.

The artificial respiration results are given in the last three columns of table III. In the standard Schafer method the average was 868 ± 51 cubic centimeters, and in the modified Schafer method 1,028 ± 57 cubic centimeters. The pole-top method gave an average 1,363 ± 85 cubic centimeters per inhalation. The

Behavior of Point Gaps at 60 Cycles

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VALUES of arc-over voltages for point gaps in air have often been considered rather inconsistent and difficult to reproduce with reasonable accuracy. Accordingly, a study was made to determine, first, if certain types of electrodes might give more consistent results than others and, if so, under what conditions, and, second, whether or not it would be possible to derive a formula which would take into account disturbing factors such as humidity. The results of this study are summarized in this paper, which, the author hopes, may be a small contribution toward a better understanding of arc-over phenomena.

Equipment

The gaps tested were set up vertically in the center of a specially built cylindrical chamber, 12 feet high and 12 feet in diameter, both the floor and ceiling of the chamber being made conducting in order to minimize the disturbances from surrounding objects on the field around the gap. One electrode was grounded, the other one being connected to the high-voltage side of a 350-kv 350-kva testing transformer. The air humidity in the chamber was controlled by the introduction of water vapor.

The electrodes studied are classified as follows:

Electrode Number	Description
1	$^{9}/_{16}$-inch round rod ended by a 15-degree cone
2	$^{9}/_{16}$-inch round rod ended by a section perpendicular to its axis
3	$^{9}/_{16}$-inch round rod ended by a 90-degree cone
4	$^{9}/_{16}$-inch round rod ended by a 120-degree cone
5	$^{1}/_{2}$-inch square rod ended by a section perpendicular to its axis
6	$^{9}/_{16}$-inch round rod ended by a hemisphere
7	Loud-tone phonograph needles

All the electrodes except the phonograph needles were made of brass.

To avoid erratic results due to oxidation and burning of the electrodes, an ultrahigh-speed circuit breaker was built and placed in the circuit. The arc current was actually interrupted by this breaker the first time it went through zero after the arc started—that is, in one quarter of a cycle or approximately $^{4}/_{1,000}$ second. This eliminated almost completely the burning of the points; and the pitting of the electrodes after more than 30 spark-overs with this breaker in the circuit was much less noticeable than that corresponding to one arc-over interrupted by a five- or six-cycle circuit breaker. It is the writer's belief that, without the use of this circuit breaker, the results herein described could not have been obtained. With a conventional type of breaker, it would have been necessary to change the points after each arc-over, a very lengthy and tedious task considering that, in order to derive the conclusions found, about 6,500 values of arc-over have been observed. A brief description of this high-speed breaker is given in appendix.

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The author is presenting in this paper a summary of the work he did at the H. J. Ryan high-voltage laboratory of Stanford University for his doctoral dissertation while he was a fellowship student of the Belgian-American Educational Foundation. He wishes to take this opportunity to express his appreciation to both Stanford University and the Belgian-American Educational Foundation for the facilities extended to him, and his sincere gratitude to Doctor J. S. Carroll for the aid and helpful suggestions given him.

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plus-minus values given above are the probable error in the arithmetical mean and were calculated by the method of least squares.[7] A study of table III shows clearly that for every individual tested the amount of air moved per respiration in the pole-top method was equal or greater than that for either of the other two methods studied. The rate of respiration is, however, slightly less in the pole-top method than in the others. (See table II.)

It is also clear from the data that all three methods produce adequate ventilation of the lungs. In many of the subjects at the termination of artificial respiration there was a diminished concentration of carbon dioxide and an increased concentration of oxygen in the blood. The respiratory centers in the brain were not stimulated to activity and a brief period of apnea resulted. Soon normal breathing was resumed. This is clearly shown in the record in figure 2 where there is a brief pause in the chest movement following artificial respiration.

While in general, the modified Schafer method, where the hands are "snapped-off" the subject's back, moves more air than the standard prone-pressure method, it would, if continued for any length of time, tend to remove the skin from the subject's back.

TIME TO MOVE TEN LITERS OF AIR

A very interesting method of studying the results is to compare the time required for the subject to breathe ten liters of air. When this was done, the average times are found to be as follows:

	Minutes	Seconds
Control period—lying prone	1	22
Control period—sitting on belt	1	12
Standard Schafer method	1	2
Modified Schafer method	0	54
Pole-top method	0	43

The above comparison shows clearly that all three methods of artificial respiration studied are efficient and that the actual volume of air moved by their use in a given time is comparable to the amount that the subject would breathe under normal conditions.

Conclusion

It is evident that the ventilation of the lungs obtained by the pole-top method is adequate and that the subject receives sufficient oxygen to satisfy his needs.

Bibliography

1. USING VITAL MOMENTS IN SAVING A LIFE. NELA *Bulletin*, April 1932, page 228.

2. REMARKS ON POLE-TOP RESUSCITATION, C. K. Drinker. National Safety Congress and Exposition, October 1939.

3. THE POLE-TOP METHOD OF RESUSCITATING LINEMEN AFTER ELECTRIC SHOCK, E. W. Oesterreich. Pennsylvania Electric Association, March 1, 1939.

4. HARVEY SOCIETY LECTURES, E. A. Schafer, page 223, 1907–08.

5. FIRST AID TEXT BOOK, American Red Cross, 1937 edition, page 113.

6. ADVENTURES IN RESPIRATION (a book), Y. Henderson, 1938 edition, page 273.

7. METHOD OF LEAST SQUARES (a book), M. Merriman. 1911 edition, page 70.

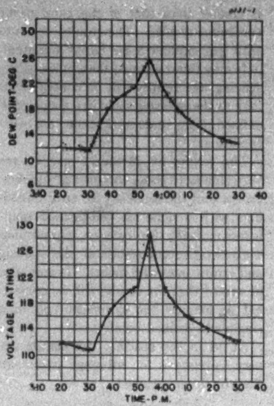


Figure 1. Variation of humidity and corresponding variation of arc-over voltage as a function of time

Gap number 3; spacing 12 inches; barometric pressure 29.9 inches of mercury

Apparatus

The voltage was measured directly from the voltmeter coil of the testing transformer. It must be noted that the purpose of this study was not to calibrate the point gaps, but to determine the relative effect of various factors on their arc-over voltages. For relative values only, the accuracy of these measurements may be considered as being better than 0.5 per cent.

The measurement of humidity required elaborate equipment. It was necessary to know, at all times, the value of absolute humidity inside the test chamber. To do this, a semiautomatic dew-point hygrometer was built according to the following principle:

Light emitted from a constant source was reflected onto a photoelectric cell by a highly polished nickel surface. If dew forms on the nickel surface, the amount of light reflected, and hence the current in the photoelectric cell, decreases. If the nickel surface is soldered on a wall of a small copper vessel, its temperature will be controllable by changing the proportions of a mixture of ice cold and hot water flowing through the vessel. This change can be achieved by the variation of current in the photoelectric cell, setting into action a suitable system of relays, motor, and valve in the hot-water circuit, to increase the temperature of the mixture as soon as dew is formed and to decrease it when dew has disappeared.

The temperature of the polished surface does not, however, respond immediately to the change of current in the photoelectric cell; to prevent any error that might occur from overshooting, a special device has been arranged to give exactly the time at which the dew forms or disappears on the polished surface. The temperature is measured by thermocouple; the polished nickel, being soldered on a sheet of copper, is used as the warm junction, the cold junction being immersed in melting ice. The resulting electromotive force is applied directly to a galvanometer, and readings are taken both when the dew forms and when it disappears, the average being taken as the true dew point. Tests made to determine the accuracy of this semiautomatic dew-point hygrometer showed that, under the most adverse conditions, the error in the dew-point temperature does not exceed 0.2 degree centigrade.

Tests and Procedure

The behavior of the following gaps has been studied:

Gap Number	Electrode Number High Voltage	Ground
1	1	1
2	2	2
3	3	3
4	4	4
5	5	5
6	6	6
7	7	7
8	5	1

Spacings ranging from 6 to 30 inches were studied, except with gap number 7, for which the maximum spacing studied was 12 inches.

All the tests were made by varying the absolute humidity from approximately 10 to 25 millimeters of mercury. The air density was practically constant during the tests. It was not possible, however, to prevent a slight increase in temperature when the humidity was increased artificially by the introduction of water vapor in the test chamber. The resulting change in air density never amounted to more than two per cent.

Since the humidity inside the test chamber was varied continuously, the dew point was recorded as a function of time, while the value of arc-over voltage was observed every 30 seconds. The results are plotted as a function of time, and the average curves drawn (sample curves are given in figure 1). From these curves, the relation between the humidity and the voltage may be easily deduced. They also make it easy to determine under what conditions the readings are erratic, and also if they must be considered as low or high compared to the average.

Results

The results obtained will now be discussed by considering: first, the influence of the shape of the electrodes on the arc-over voltage; second, the combined influence of both the shape of the electrodes and the absolute humidity on the behavior of the gap; third, the effect of humidity for each type of electrode; and, finally, some of the other factors affecting the arc-over voltage.

1. Influence of the Shape of the Electrodes

Table I gives, in kilovolts effective, the arc-over voltages observed at a humidity of 15 millimeters of mercury for the different types of electrodes studied.

It is seen that the arc-over voltage for gap number 2 is 60 per cent greater at 18 inches and 35 per cent greater at 24 inches than that of any other gap. The arc-over could not be observed at 30 inches, but it is more than 20 per cent greater than that of the other gaps (over 360 kv). With that exception, all values observed at 18, 24, and 30 inches check within less than five per cent. Since no correction has been made for air density—the influence of which may amount to one or two per cent—and since, at the same air density, variations of a few per cent may occur for the same type of gap, it may be

Table I

Spacings (Inches) Gap Number	6	12	18	24	30
1	60.1	117.5	173	236.8	298.7
2	117.5	205	283	320	
3	61.5	119.2	181.5	239.5	302.5
4	66.3	119	177.5	244.3	297
5	85	136	179.6	235.8	296.5
6	72	120	174.2	237	298
7	61.2	116.3			
8	63.8	115	173.5	229.5	293.8

stated that, at these spacings, all the gaps studied in these tests give approximately the same arc-over voltage, except gap number 2.

The results at 12 inches spacing check within four per cent for all types of electrodes except two, and may thus also be said to be identical. The two exceptions are gap number 2, with an arc-over voltage 70 per cent higher, and gap number 5, which gives results approximately 15 per cent higher, than the others.

The arc-over voltage at six inches spacing is much more influenced by the shape of the electrodes; gaps numbers 1, 3, and 7 give the same value, and as the result for gap number 8 is only 4 per cent higher it may thus still be considered the same. The other results are higher; for gap number 4 by 12 per cent; for gap number 6 by 20 per cent; for gap number 5 by 40 per cent; and for gap number 2 by 95 per cent.

These results show that, as was expected, the smaller the spacing, the greater the influence of the shape of the electrodes.

2. Erratic Nature of Point Gaps

Gap number 2 will not be considered in the following discussion, because it gives erratic results at all spacings studied, and its arc-over voltage is always much higher than that of gaps formed by the other electrodes. The results obtained for the other gaps are shown in figure 2, *a* to *e*, and explained hereafter.

At six inches spacing, gap number 1 gives consistent results when the electrodes are clean. When they are burned, however, the readings are erratic and appreciably higher than those of clean points. The same effect has been observed for the phonograph needles, where a slight burning of the tip tends to make the results erratic and to increase the average arc-over value; however, at high humidity, the readings stay consistent even with the points slightly burned. Gap number 3, when clean, gives consistent results, while gaps 4, 5, and 6 are erratic; their arc-over voltage is greater than that of the gaps giving consistent results. The results obtained with gap number 8 are fairly consistent.

It may thus be stated that, at six inches spacing, the results will be consistent as long as the electrodes end in a single well-defined point, and erratic under the other conditions.

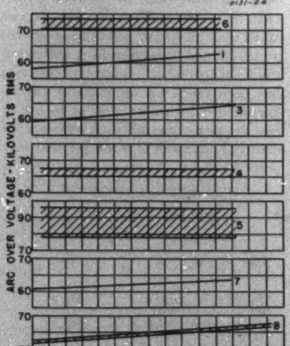
The results at 12 inches spacing are very similar to those obtained at 6 inches, except that the tendency for erratic readings with a none-too-well-defined point is less marked; for instance, gap number 4 gives consistent results at this spacing and erratic ones at 6 inches. It may also be stated that the results will be consistent as long as the electrodes end in a single well-defined point.

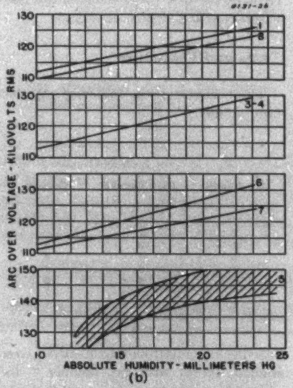
At 18 inches spacing, the results are very consistent for all gaps under all the conditions studied.

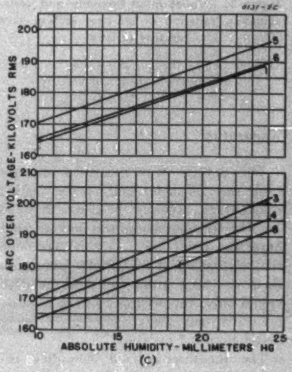
Figure 2, a to e. Variation of the arc-over voltage as a function of absolute humidity for the gaps studied at the following spacings:

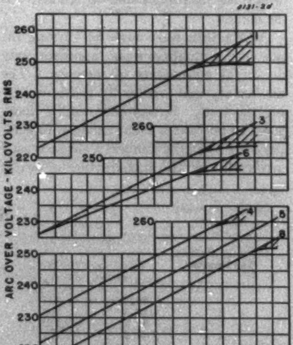
(*a*)— 6 inches
(*b*)—12 inches
(*c*)—18 inches
(*d*)—24 inches
(*e*)—30 inches

The number on each curve corresponds to the gap number as given in this paper









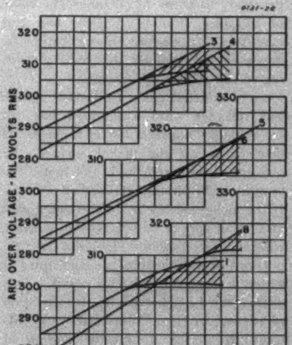


Table II. Values of Arc-Over Voltage in RMS Kilovolts as a Function of Spacing and Absolute Humidity for the Gaps Studied

Gap Number	Humidity (Mm Hg)	Spacing (Inches)				
		6	12	18	24	30
1	10	58.1	112	164	223.3	284.7
	15	60.1	117.5	173	236.8	298.7
	20	62.1	122.8	182	250.2	(305.6)
2	10	116.6	200.5	277.5		
	15	117.5	205	283		
	20	118.6	(208)	288.5		
3	10	59.4	113	170.2	226	289.3
	15	61.5	119.2	181.5	239.5	302.5
	20	63.6	125.5	192.9	253	315.8
4	10	(66.3)	112.7	167.5	231.2	282.5
	15	(66.3)	119	177.5	244.3	297
	20	(66.3)	125.5	187.5	257.5	311.5
5	10	(85)	?	170.5	222	281.7
	15	(85)	(136)	179.6	235.8	296.5
	20	(85)	(144.5)	189	249.5	311.2
6	10	(72)	112.8	165.4	226	285
	15	(72)	120	174.2	237	298
	20	(72)	127	183	247.2	311
7	10	60.9*	111.4			
	15	62.1*	116.3			
	20	63.2*	121			
8	10	62	109.8	163.5	215.6	277.2
	15	63.8	115	173.5	229.5	293.8
	20	65.5	120.2	183.5	243.2	310.5

Note: Values in brackets have not been used for computing the relative influence of humidity.

* These values correspond to a spacing of $6^{3}/_{16}$ inches.

At 24 inches spacing, gap number 5 is the only one to give consistent results for all the conditions studied. With all the other electrodes, the results are consistent below a certain value of humidity, which will be referred to as "critical humidity". Above that value, they are always very erratic; the maximum readings fall on the extrapolated curve of voltage-humidity, but the average is notably decreased. Besides, it seems that, if the humidity is increased much farther, the voltage stays practically constant. The critical value of humidity appears to be approximately 20 millimeters of mercury for gaps numbers 1, 3, 4, and 6, and 23 millimeters of mercury for gap number 8.

The results at 30 inches spacing are entirely similar to those obtained at 24 inches; gap number 5 gives consistent results under all the conditions tested, while the others show a critical humidity, beyond which the average is decreased below the values expected from extrapolation. The critical value of humidity is approximately 17 millimeters of mercury for gaps numbers 1, 3, 4, and 6, and 20 millimeters of mercury for gap number 8.

The conclusion, therefore, is that, in order to obtain consistent results with the rod diameter used and at spacings between 6 and 18 inches, clean electrodes ending in a single well-defined point must be used. At spacings between 18 and 30 inches, however, a type of electrode offering several sharp points—such as a square rod ended by a plane section—should be used to obtain consistent results under all humidity conditions. Finally, a composite gap such as gap number 8 shows characteristics similar to those of gap number 1 at small spacings, while at large spacings, its behavior approaches that of gap number 5.

3. Influence of Humidity

For all the electrodes studied, the arc-over voltage increases linearly with the absolute humidity. There are, however, two exceptions. At six inches spacing, the arc-over voltage of the electrodes which give very erratic results does not seem to be influenced by humidity. At large spacings, when the humidity is greater than its critical value, the proportionality law holds true only if the maximum instead of the average readings are considered; and, for still higher values of humidity, the arc-over voltage seems to become independent of humidity.

Since the function of arc-over voltage versus humidity is linear, it was easy for comparison to compute the value of "relative influence of humidity". The author has called relative influence of humidity the percentage of increase in arc-over voltage corresponding to an increase of one millimeter of mercury of water-vapor pressure. It is thus given by:

$$I_h = 10\,\frac{V_{20} - V_{10}}{V_{15}} \text{ in per cent}$$

where V_{10}, V_{15}, and V_{20} are the arc-over voltages corresponding respectively to values of absolute humidity of 10, 15, and 20 millimeters of mercury. The values of I_h, obtained from tables II and III (see appendix), are plotted on figure 3, *a* and *b*.

It is seen immediately that the influence of humidity is far from being constant at all spacings. It attains a maximum of approximately 1.2 per cent per millimeter of mercury for spacings between 18 and 24 inches. On the other hand, the values observed may be considered as practically independent of the type of electrodes—with the exception of those observed with gap number 2.

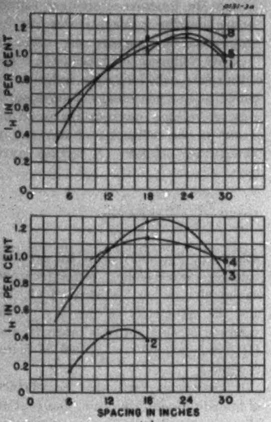
4. Other Factors Affecting the Arc-Over Voltage

Some observations made showed—as had already been observed—that the point gaps arc-over voltage increases with air density. However, no systematic study of that factor was made during these tests.

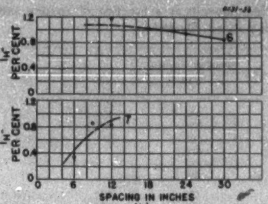
Some other factors as yet unknown have a decided influence on the point gaps arc-over voltages: differences as high as three per cent were observed from one day to another—sometimes even on the same day when the air of the test chamber had been disturbed—even if the temperature, barometric pressure, and humidity were exactly the same. It is supposed that these factors are either vapors or impurities such as dust parti-

Table III. Values of Relative Influence of Humidity I_h in Per Cent as a Function of Spacing for the Gaps Studied

Gap Number	Spacing (Inches)				
	6	12	18	24	30
1	0.67	0.92	1.04	1.14	0.94
2	0.17	0.44	0.39		
3	0.68	1.05	1.25	1.12	0.88
4		1.07	1.13	1.08	0.98
5			1.03	1.17	1.00
6		1.18	1.01	0.91	0.87
7	0.37	0.82			
8	0.55	0.90	1.15	1.20	1.13



(a)



(b)

Figure 3, a and b. Relative influence of humidity I_h as a function of spacing for the gaps studied

The number on each curve corresponds to the gap number as given in this paper

cles in the air. This assumption is based on two observations made with gap number 1, at six inches spacing. The introduction of small quantities of ether vapor in the test chamber increased the arc-over voltage by approximately 17 per cent. A noticeable increase in voltage was also observed after introduction in the test chamber of vapors from moist wood. Lack of time unfortunately prevented any systematic research along these lines.

Conclusions

The results obtained from approximately 6,500 point gaps arc-overs under varying conditions may be summarized as follows:

1. The voltage at all spacings up to 30 inches increases linearly with an increase in absolute humidity.

2. The percentage of increase in voltage per millimeter of mercury increase in water vapor pressure is not constant at all spacings:

(a). It attains a maximum value of 1.2 per cent per millimeter of mercury for spacings between 18 and 24 inches;

(b). It is practically the same for all the electrodes studied in these tests.

3. At spacings of 18 inches or more, the arc-over voltage is practically the same for all the electrodes studied, provided they are ended by a finite number of points.

4. The readings are erratic at small spacings if the electrodes do not end in a single well-defined point or if that point becomes burned, thus offering a rough surface. The average is then greater than that obtained for a clean point.

5. Except for the electrodes number 5 ($^{1}/_{2}$-inch square rod ended by a section perpendicular to its axis) the readings are erratic at large spacings for high values of humidity. The average is then smaller than would be expected by extrapolation of the results obtained at lower humidity.

6. Under all the other conditions studied, the results were very consistent.

7. It is possible, by using two different types of electrodes, to obtain a gap partaking of the advantages of each electrode of which it is made.

8. Impurities or other vapors in the air affect the arc-over voltage.

Appendix

High-Speed Circuit Breaker

The ultrahigh-speed circuit breaker used in this experiment consisted primarily of a vertical plunger-type electromagnet, bearing three separate windings. The plunger had a displacement of approximately three-eighths inch, and acted on two sets of moving contacts of the breaker. It was rigidly connected to a pair of current-carrying contacts, and, during the last one-fourth inch of its travel, it opened a pair of arcing contacts. This ensured a total gap of one-half inch, quite sufficient to break a 2,000-volt circuit under oil.

The main winding carried the total primary current of the high-voltage transformer; the second winding created a flux exactly opposed to that produced by the exciting current of the transformer flowing in the first winding; and the third winding merely acted as a hold-open device.

The main winding was so designed that the pull on the plunger averaged about 80 times the weight of the plunger during the first one-fourth cycle of the arc-over current. At the end of this one-fourth cycle, the current went through zero, since it starts on the crest of the voltage wave and is practically in phase with the voltage. At the same time, the current carrying contacts had separated about $^{3}/_{16}$ inch, and the arcing contacts were beginning to move at such a velocity that the insulation between them and the stationary contacts was building up faster than the recovery voltage, thus preventing the arc from being established.

The purpose of the second winding was to prevent any pull on the plunger that might be caused by the exciting current, the value of which was approximately $^{1}/_{30}$ of the arc-over current. Besides, it created the only flux keeping the contacts open between the interruption of the arc-over current and the establishment of the current in the third winding.

The third winding was necessary because the contacts were not designed to close the circuit and had to be kept open until the generator voltage was brought down to zero, thus preventing any rush of exciting current while the contacts were closing. A suitable system of relays was set up so that after the breaker had opened the circuit, current started flowing in the third winding, and by manual operation of a single switch, the generator voltage was brought down to zero, the breaker closed, and the transformer voltage built up to about 90 per cent of the expected arc-over value.

The operation of this breaker proved to be very satisfactory, since practically no arcing was visible, even when interrupting currents of 300 amperes rms at 2,000 volts.

Bibliography

1. INFLUENCE OF HUMIDITY ON SURGE FLASHOVER, F. D. Fielder. *Elec. J.*, July 1932.

2. NORMAL-FREQUENCY ARC-OVER VALUES OF INSULATORS AS AFFECTED BY SIZE AND HUMIDITY, H. A. Frey and K. A. Hawley. AIEE TRANSACTIONS, September 1932.

3. INSULATOR SPARKOVER, W. L. Lloyd, Jr. AIEE TRANSACTIONS, September 1932.

4. FLASHOVER VOLTAGES OF INSULATORS AND GAPS, Committee Report. ELECTRICAL ENGINEERING (AIEE TRANSACTIONS), volume 53, 1934, page 882.

5. IMPULSE AND 60-CYCLE STRENGTH OF AIR, P. L. Bellaschi and W. L. Teague. ELECTRICAL ENGINEERING (AIEE TRANSACTIONS), volume 53, 1934, page 1638.

The Effect of Overexciting Transformers on System Voltage Wave Shapes and Power Factor

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It is well known that the voltage and current wave shapes of transmission and distribution circuits are distorted more or less from a true fundamental-frequency sinusoid. Several years ago extensive tests were conducted, measuring the wave shapes for different types and voltages of circuits.[1] There were however no definite conclusions drawn concerning the origin or improvement of these wave distortions.

Measurements by the authors of harmonics on various systems in the country, as well as those measurements referred to, show in general that the harmonic voltages increase in magnitude as the measurements are made farther and farther from the generating stations. This indicates that probably transformer exciting currents have more effect on system wave shapes than is usually given credit.

The nontriple odd-harmonic frequency components of exciting currents of all the transformers in a system are supplied by generators, load circuits, or system capacitances. The ohmic values of the load and capacitive circuit impedances are usually many times those of the circuits which include the generating points or similar large rotating machines. Hence, practically all of the nontriple-frequency components of exciting current for transformers can be said to be supplied by the generators.

With this thought in mind one can appreciate that transformers in outlying districts may have to draw their exciting current through a great deal of impedance. Current going through impedance naturally produces voltages in accordance with Ohm's law so it can well be expected that under the proper conditions the harmonic content of these exciting currents could have a measurable effect on system wave shapes. Further, it is well known that in general, due to inadequate regulating equipment, and other reasons, certain transformers are overexcited in order to deliver the desired voltages at the load. It is, however, not so well recognized that this overexcitation greatly magnifies these system harmonics and reduces the system power factor. This reduction in power factor results in less boost in voltage than is expected with the final result being *excessive* overexcitation in certain areas, with the attendant decreased power factor, increased losses, and increased harmonic content of the voltage and current waves. It was the desire to determine more or less quantitatively what effect this overexcitation of transformers might have on system wave shape and power factor that prompted this study.

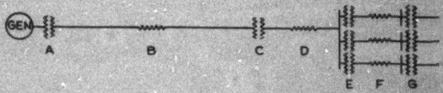

Figure 1. Diagram of the hypothetical system analyzed

Refer to table I for explanation of symbols

Experience and measurements show in general that the third-harmonic voltages and currents are relatively small. This is because nearly all three-phase transformer banks have one winding connected delta, which permits the triple-frequency currents to circulate thus preventing them from flowing beyond the first delta winding. The only triple-frequency voltage then that gets out on the system is that which represents the impedance drop in the lines to the first delta and the delta windings themselves of the third harmonic currents. Experience shows that the fifth harmonic is generally the largest single harmonic component in the current and voltage waves. It is appreciated that the magnitude of various harmonics varies considerably in any given system, but in general it has been found that the fifth is the largest component. Admittedly, in isolated cases motors or generators generate higher frequencies known as "slot ripples", which may need special attention, but this phase of the subject is not studied.

In order, therefore, to assign definite objectives to the problem, it was decided to study the effect of transformer overexcitation in a sample system on the fifth-harmonic voltage and power factor and determine what practical methods were available for improving these conditions if and when they required it.

General Method of Attack

The hypothetical system represented by figure 1 was chosen for study. Average typical circuit constants and transformer ratings were assumed as listed in table I. This circuit, with each transformer represented by its equivalent with a harmonic generator in its magnetizing branch for supplying harmonic current (refer to figure 2 and appendix) was set up on the a-c network analyzer. With this arrangement, the circuit constants, the magnitude of harmonic current flow for different values of excitation, and the ratio between substation and distribution transformer capacity could be readily varied.

With a predetermined excitation on the transformers, the magnitude of the fifth-harmonic current required was made to flow by adjusting the voltage of the harmonic generator in each transformer circuit. With these adjustments made, the fifth-harmonic voltage was then read at any desired point in the system. This procedure was carried out for normal and 110 per cent excitation on all transformers.

This type of test assumes a pure sine wave of fundamental voltage is applied to each transformer, which is not true in the strictest sense of the word since the reactance from the voltage source to each transformer results in small harmonic voltages being applied instead of a pure sine wave. This error is small for the value of voltages encountered hence the effect is neglected.

Conclusions

1. A small negligible value of fifth-harmonic voltage can be expected to appear in transmission and distribution circuits as a

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[1] For all numbered references, see list at end of paper.

result of the flow of fifth-harmonic components of exciting currents required by normally excited transformers.

2. This fifth-harmonic voltage is more than doubled when the excitation on the majority of the transformers in a system is increased ten per cent.

3. The over-all system power factor can be materially reduced by exciting currents resulting from *excessive* overexcitation of the majority of the transformers.

4. Since the advantages of operating at the present established levels of meter-box voltages cannot be denied, distribution systems utilizing a large proportion of the older designs of distribution transformers requires more careful operation and regulation in order to not excessively overexcite the transformers.

5. The use of capacitors and reactors in series for the dual pupose of power-factor and wave-form improvement provides a simple, effective, and economically sound method of coping with these two problems presented by the overexcitation of transformers.

6. The replacement of obsolete distribution transformers with up-to-date built-for-the-service equipment would reduce these harmonic voltages, excessive exciting currents and losses, caused by overexcitation. This point should be more closely scrutinized by distribution engineers, particularly in view of the recent advances made in the art of building distribution transformers.[1]

7. The objectionable conditions arising from the overexcitation of transformers should further encourage the movement attempting to standardize circuit and appliance voltages so that further increases in transformer voltages will be unnecessary.

Transformer and Circuit Data of System in Figure 1

The system analyzed and represented by figure 1, does not necessarily represent any particular system in existence but it does represent in substance the equivalent amount of transformers and lines that might be used in transmitting and distributing a given block of power.

The power transformers in the line are assumed to be of 100 per cent capacity while each of the three step-down substation transformers are taken as 50 per cent capacity. Diversity in feeder loads usually requires at least this ratio of capacities. The complete circuit data including ratings and reactances of the various lines and transformers, and the per cent exciting currents used are listed in table I. For purposes of this study, it is assumed that an average normal density of 70 kilolines per square inch is representative for the distribution transformers and a density of 85 kilolines for the power transformers.

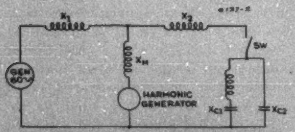

Figure 2. Diagram of equivalent transformer circuit with harmonic generator in the magnetizing branch for supplying the required harmonic currents. The switch connects a series reactor-capacitor combination for improving wave form and power factor—refer appendix

In figure 3 are plotted the data of a harmonic analysis of the exciting current of a representative iron that might be used for transformers. Thus, for the different values of excitation studied the per cent harmonic content of the exciting current can be obtained. It should be noted that these data give the per cent harmonic content of the total exciting current. Since it is the *total* amperes of harmonic current flowing that determines the voltage, these percentage values must be co-ordinated with the actual magnitudes, of exciting current. A certain transformer under an overvoltage condition might require 10 per cent total exciting current having 10 per cent fifth harmonic while another could require 4 per cent total current but having 20 per cent fifth. Thus the one having a *less percentage* of fifth may actually have *more amperes* of fifth harmonic.

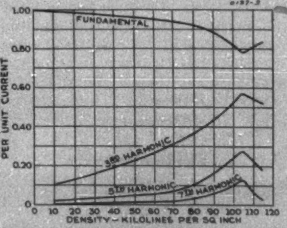

Figure 3. Harmonic analysis of the exciting current of a representative sample of iron that might be used for transformers

Data plotted as per unit of total exciting current against density

Normal Excitation on All Transformers

This study involved the determination of the fifth-harmonic voltages in the hypothetical system laid out in figure 1 for normal excitation on all transformers. This is indeed somewhat of a hypothetical condition, since never would a system be found that had normal voltage applied to all transformers. However it does have the academic value of showing to what extent normal-excitation exciting currents would generate harmonic voltages in such a system.

The voltages that the prescribed values of harmonic current caused to appear in the test system are shown plotted by the solid lines in figure 4. Curve *A* represents the conditions when the ratio of distribution to substation capacity is unity. Here it is seen that the fifth-harmonic voltage gradually rises from zero at the generator to approximately 0.02 per unit in the distribution area. The solid curves labeled *B* and *C* show the voltages when the distribution-transformer capacity ratio was increased to 2 and 3 respectively. Here it is seen that the harmonic voltage increases slightly for these two conditions. These figures are representative of the lower values of fifth-harmonic voltages measured in actual systems.

1.1 Per Unit Excitation on All Transformers

The next consideration was that of determining the fifth-harmonic voltages when all the system transformers were overexcited ten per cent. Again, the reader is cautioned that this does not necessarily represent a practical case, but it does show, when compared with the results of the previous case how the harmonic voltages can be expected to increase when the transformers are operated above their normal densities.

The exciting currents for this condition of excitation, for the two types of transformers are listed in table I.

Cases *A*, *B*, and *C* were analyzed for this condition just as they were for normal excitation. The dashed curves in figure 4 show the fifth-harmonic voltages measured for the three different values of distribution transformer capacity. These data show that the harmonic voltages are more than doubled over the values obtained for the condition of normal excitations on all transformers.

In an actual system, these harmonic voltages are usually amplified more or less by the transmission line and transformer capacitances, so it is not uncommon to find voltages much higher than those shown in figure 4. Also, the magnetizing current required by induction motors and other "iron core" loads have their effect in increasing these voltages. In fact all loads with nonlinear impedance characteristics contribute to this phenomenon.

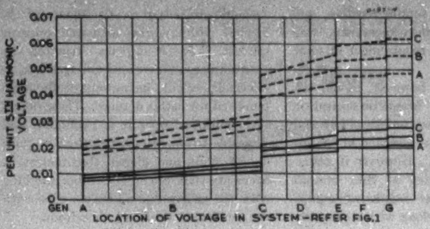

Figure 4. Magnitude of fifth harmonic voltage appearing in the hypothetical system studied

Solid curves—Normal excitation on all transformers

Dashed curves—1.1 per unit excitation on all transformers

Curves	A	B	C
Ratio of Distribution Substation Capacity	1	2	3

The authors have studied test data taken on two systems that exhibited unusually high fifth-harmonic voltages. In these tests it was found that a general lowering of the voltage levels by approximately 10 per cent, resulted in a decrease of 50 per cent in the fifth-harmonic voltages.

On another occasion, severe overcurrents were being experienced due to the circuit being in *partial* resonance with capacitance. It was found that a decrease of only 4 per cent in the voltage feeding the affected area reduced the harmonic condition sufficiently to cure the difficulty.

Reduction of Harmonic Voltages Arising From Exciting Currents

In certain areas the application of shunt capacitors has been somewhat impeded due to these harmonic voltages. Resonance has occurred on occasion resulting in excessive fuse blowing. Also an occasional case of low-frequency induction in communication circuits has been experienced. Almost without exception these cases have been controlled by the fifth-harmonic or 300-cycle component. Therefore it is believed that if a method of reducing this component to a negligible amount was realized, the broad problem could be considered solved.

Use of Capacitors. Since capacitors have more or less exposed this harmonic condition in modern distribution circuits during the last few years, it seems paradoxical that they could be used to suppress these harmonic voltages. Such a method however is available and attention is directed to it in this paper since it is believed to be not very well known in this country. It has however been used successfully for several years in Japan[3] in the larger capacitor installations.

The scheme consists of putting a reactor in series with a group of capacitors. This reactor is proportioned in respect to the capacitors, so that the combination is in or near resonance with the fifth harmonic. The fifth-harmonic voltage is then effectively short-circuited at the point of application with the improvised resonant shunt. In fact it can be shown that it is reduced to a negligible amount in the surrounding area for a considerable radius. The amount of harmonic current that flows into this "harmonic short circuit" is equal to the voltage at that point before the equipment was applied divided by the system impedance viewed from that point. In general this current will not be so much since the harmonic voltage isn't large to begin with and further the system reactance is large, being increased by the order of the harmonic. It is interesting to reflect on the possibilities of this scheme when carried to a limit. If a sufficient number of these "harmonic short circuits" or resonant shunts were applied to a system, practically all of the fifth-harmonic current could be made to flow in short localized paths from the transformers to the shunts, thus practically eliminating the harmonic voltages from appearing, as described in the body of the paper.

A very important point to remember in this respect is that the normal, 60-cycle power-factor improvement afforded by the capacitors is not reduced by this series reactor. Since approximately a four per cent reactor is required for this application, the 60-cycle voltage on the capacitors is raised by this amount and the net available corrective kilovolt-amperes to the system is likewise increased by four per cent. If it is desirable from other considerations to practically eliminate the flow of harmonic current instead of the harmonic voltage, the reactor can be so proportioned to make a harmonic wave trap (equation 5 in appendix) out of the capacitor bank, and as before the normal 60-cycle correction afforded by the capacitors is increased by the reactor. Thus it is seen that power-factor and waveform improvements can be obtained simultaneously by a single master stroke.

Reduction of Transformer Excitations

Reducing excessive overexcitations and providing the proper regulating equipment for voltage control would, of course, be in order where it could be justified. No doubt there are many transmission and substation transformers that could be operated at much lower densities if the distribution circuits were properly regulated.

The distribution transformers however, in general, cannot have their voltages reduced without accepting certain losses in revenue and a reduction in the quality of service rendered. As described in more detail below under power factor, it is well known that in the last several years the transformer secondary voltages have been creeping up from 110 to 125 volts and even higher. Obviously many older-type transformers originally designed for 110 volts are now operating 10 per cent and even 15 per cent overexcited. In view of these trends, it is believed that a policy of more closely scrutinizing the replacement of obsolete equipment with up-to-date designed-for-the-service apparatus would prove a sound investment.

Table I

Equipment Identification Refer Figure 1	Type of Equipment	Per Unit Rating	Per Unit Reactance on Own Base	Per Unit Exciting Current on Own Base: Normal Excitation	Per Unit Exciting Current on Own Base: 1.1 Per Unit Excitation
Generator	Sine-wave voltage source	Infinite	0		
A	Transformer—step up	1.0	0.08	0.045	0.08
B	High-voltage transmission line	1.0	0.05		
C	Transformer—step down	1.0	0.08	0.045	0.08
D	Medium voltage distribution	1.0	0.05		
E	Three substation transformers	0.5 ea.	0.05 ea.	0.04	0.07
F	Three lines—four-kv feeders	0.5 ea.	0.05 ea.		
G	Transformer equivalent of total connected distribution transformers, Case *A*	0.5	0.025	0.02	0.03
	Case *B*	1.0	0.025	0.02	0.03
	Case *C*	1.5	0.025	0.02	0.03

Cases *A*, *B*, and *C* differ only in the ratio of connected distribution to substation transformer capacity, of which the ratios are 1, 2, and 3, respectively.

SYSTEM POWER FACTOR AND OVEREXCITATION OF TRANSFORMERS

The past several years has seen in general a marked decrease in over-all system power factor. The deleterious effects of poor power factors coupled with increased loads upon system capacity and voltage are well known. The answer to the questions inquiring into the cause of this decrease in power factor has generally been "motorized appliances, air conditioning, gaseous-tube lighting, etc." These factors beyond a doubt have had their effect, but it is interesting to speculate on the effect of transformer magnetizing currents, especially when the transformers are overexcited.

Not so long ago it was the practice to serve a customer with 110 volts, or its equivalent at a higher level. Later the standard was considered to be 115, so taps were changed and general levels raised to accommodate this change in philosophy. During the last few years this meter-box voltage has risen to 120 which represents approximately a ten per cent increase over the original 110. In fact in many places it is necessary to hold 125 volts or even higher at the transformer in order that the load be served at rated voltage. During the evolution of this practice of gradually raising the voltage, new transformers designed for the voltage were of course being added to accommodate load growth, but, most of the older equipment obviously remained on the system. Thousands of kilovolt-amperes of transformers on most systems today that were built to operate at a normal density corresponding to 110 or 115 volts, are actually being operated at or above 120 volts. Transformers in general can stand this duty because they are designed for it, but obviously they must draw larger magnetizing currents to accommodate the increased flux requirements.

The effect on the magnitude of the total exciting current required by all the transformers in the circuit of figure 1 due to overexciting the transformers ten per cent is clearly seen by the figures in table II. Here it is seen that the exciting-current requirements are practically doubled, for the overexcited condition. For the case at hand, reactive kilovolt-amperes approximating 40 per cent of the system rating may be required merely to excite the transformers. These figures may of course be off a little quantitatively, but they are believed to be sufficiently accurate qualitatively to show that system power factor can be materially reduced by operating transformers above their normal densities. It is well known that many times a generator must be kept on the line merely to supply reactive, and the overexcited transformers may well be the cause of the requirement in some cases.

The transformer, however, should not be indicted for this. Magnetic saturation is a fundamental characteristic of iron and must be more or less accepted in the interests of economy. However if transformers were operated at their normal densities and proper regulating equipment were applied along with a properly designed distribution system, these exigencies would not arise. Admittedly this is a Utopia impossible to attain, but it is believed that with these facts known and understood, more attention to the problem can be intelligently and profitably applied. Magnetic saturation and magnetizing reactive kilovolt-amperes is something with which the engineer must live—hence it behooves him to treat it with respect and design his systems accordingly. It is believed that the replacement of lots of these older, overexcited distribution transformers with up-to-date designs and/or the use of appropriate regulating equipment could be justified from the standpoint of decreased losses and decreased magnetizing currents alone. One should not conclude from this discussion that power-factor improvement be attained by reducing voltages, but the over-all condition can be greatly improved by properly regulating the voltage. In this respect the authors are acquainted with one distribution system in which voltage regulators were systematically applied and much to the surprise of the engineer, the power factor coincidentally was improved from 70 to 85 per cent.

Table II

Total Per Unit Exciting Current Required by Transformers in Circuit of Figure 1 for Various Excitations and Ratios of Distribution to Substation Transformer Capacity. Kilovolt-Ampere Base = 1.0

Capacity ratio	1/1	2/1	3/1
Normal excitation	0.18	0.21	0.24
1.1 per unit excitation	0.31	0.35	0.40

Reference to the preceding section on reduction of harmonic voltages should be made for further discussion of reactive control. In this section the use of capacitors is described for improving wave form distortions due to saturation and at the same time provide leading kilovolt-amperes for power-factor improvement. It is interesting at this point to note that since shunt capacitors increase their kilovolt-ampere output as the square of the applied voltage, the increased exciting current kilovolt-amperes required by transformers at increased voltages can, to a certain extent, be automatically compensated for in this manner. These points are mentioned since it is profitable and desirable to operate distribution systems at 120 volts and higher, thus making it necessary that the engineer be provided with the proper equipment to cope with the conditions resulting therefrom.

Appendix. Use of an Equivalent Circuit for Determining Flow of Harmonic Magnetizing Currents

Consider the circuit shown in figure 2 which is the equivalent diagram of a transformer connected to a pure sine-wave voltage source with a harmonic generator in the magnetizing branch for furnishing the harmonic magnetizing currents. The circuit connected by the switch represents a set of capacitors, x_{c1}, in series with a reactor x_s and a set of capacitors, x_{c2}, with no reactor.

Consider the switch open, and the transformer excited from the pure 60-cycle source. Since there is magnetic saturation in the x_m branch, certain harmonic currents will flow and these can be represented as being driven through the system by a harmonic generator located in the x_m branch. Then letting

i_n = harmonic current
x_1 = 60-cycle reactance of system up to transformer plus transformer primary reactance
x_2 = same on secondary side
n = order of harmonic

the harmonic voltage at the switch can be written as

$$e_n = i_n n x_1 \tag{1}$$

This reasoning assumes that the magnitude of i_n is determined solely by the applied 60-cycle excitation sine wave of voltage and neglects the infinitesimal amount that flows due to the appearance of a small harmonic voltage at the transformer terminals.

Assume further that the reactor x_s and the capacitor x_{c1} are proportioned so that

$$n x_s = x_{c1}/n \tag{2}$$

then the harmonic current meets zero impedance when the switch is closed.

Closing the switch results in the flow of i_n' through the switch of the magnitude

$$i_n' = i_n \frac{n x_1}{n(x_1 + x_2)} \tag{3}$$

and substituting equation 1

$$i_n' = \frac{e_n}{n(x_1 + x_2)} \tag{4}$$

Thus, to determine the readjustment of the flow of the harmonic current due to a resonant shunt or a "harmonic short circuit" it is merely necessary to divide the voltage at the point of application by the

Testing of Elevator Buffers by Electrical Means

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THE purpose of an elevator buffer is to stop the elevator in event it reaches the bottom of its hoistway out of control. To prevent injury to its occupants, the rate of stopping must not exceed $2^1/_2$ times gravity or about 80 feet per second per second for more than $^1/_{25}$ second. The tests on a buffer should be such that the velocity at which the elevator strikes the buffer and the rate of deceleration at every point in the piston stroke may be determined. If the buffer contains oil, it is important for the designer to know also the oil pressure at every point in the stroke.

Methods Formerly Used

The test method found most satisfactory up to this time employs a platen, driven directly from the buffer piston or by the elevator, on which a tuning-fork-driven stylus records the distance traveled by the platen during successive cycles of the fork vibrations. From this record the velocity and deceleration are calculated. A modified steam-engine type of pressure indicator operates another stylus on the same platen to record the varying oil pressure.

In methods previously used, the tuning fork vibrates with constant frequency while the platen velocity varies with that of the car. This arrangement produces a record on which the highest velocity of fall produces the fewest cycles per inch on the record. With the electrical method described, the highest velocity of fall is recorded with the greatest number of cycles per inch of record and the accuracy of the calculations of retardation are best therefore when the velocity of fall is highest. Inasmuch as dangerous deceleration rates are more likely to occur early rather than late in the stroke of a buffer reasonably well designed, it is better to achieve accuracy of measurement at the higher rather than at the lower velocities.

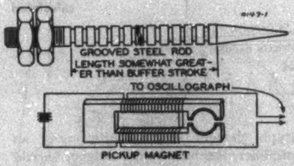

Figure 1. Pickup magnet and grooved iron inductor rod

Determination of Velocity and Deceleration

To determine the striking velocity of the elevator car, a magnetic pickup coil and grooved iron rod shown in figure 1, is used. The rod is mounted vertically on the car, attached by its upper end so that as it falls with the car it passes through the bore of the pickup magnet which is mounted on one of the guide rails. The magnet has an exciting winding of about 200 turns of number 18 magnet wire and also a secondary coil of about the same number of turns which is connected to a high-sensitivity galvanometer element of an oscillograph. As the rod drops through the bore of the pickup magnet, the changes in permeance of its magnetic circuit, caused by the grooves in the rod, vary the magnetic flux and induce in the secondary coil an alternating voltage, which is recorded by the oscillograph. One six-volt storage battery serves to excite the pickup magnet. A similar rod and pickup coil may also be used on the buffer piston itself if the performance of the piston must be known.

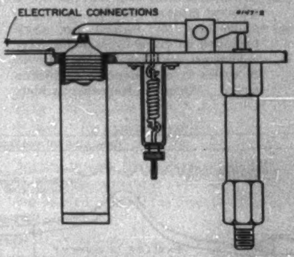

Figure 2. Carbon-pile oil-pressure gauge

A 60-cycle voltage wave on the same oscillogram as the varying-frequency alternating voltages of the pickup coils is of convenience in timing.

In calculating the velocity of the car at any point in its fall the following formula may be used:

$$V=\frac{5NL\times m_{60}}{m_r}$$

in which

V = velocity of fall in feet per second
N = number of cycles scaled on pickup record on oscillogram

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driving point impedance of the circuit viewed from that point. This fundamental principle has been used for years by engineers in making short-circuit calculations. It can be shown, within the assumptions, that this method of calculating the flow of short-circuited harmonic current in a complicated network, is correct. The demonstration above for a simple circuit is given in the interests of clarity.

It is interesting to note that if x_s is proportioned so that

$$nx_s=x_{c1}/n+x_{c3}/n \tag{5}$$

a wave trap is formed by the network, which presents infinite (neglecting losses) impedance to the harmonic thus preventing the flow of harmonic current.

Either of these reactor schemes can be used in capacitor applications depending upon the objective. The condition imposed by equation 2 reduces the line harmonic voltage to zero whereas the conditions represented by equation 5 reduces the line harmonic current to zero.

References

1. SURVEY OF POWER SYSTEM WAVE SHAPES, Engineering Reports of the Joint Subcommittee on Development and Research, volume 2, pages 180–290.

2. THE WOUND-CORE DISTRIBUTION TRANSFORMER, E. D. Treanor. AIEE TRANSACTIONS. 1938 (November section).

3. ON THE PHASE ADVANCEMENT AND THE WAVE FORM IMPROVEMENT DUE TO STATIC CONDENSER, K. Kitagawa and I. Tanaka. *Journal* of the Institute of Electrical Engineers of Japan, volume 56, number 532, 1936.

IMPROVEMENT OF VOLTAGE WAVE FORM BY STATIC CONDENSERS FOR POWER FACTOR IMPROVEMENT, K. Kitagawa and I. Tanaka. *Electrotechnical Journal*, August 1937.

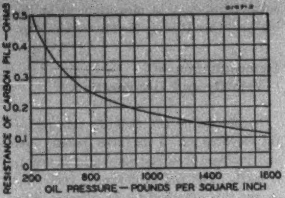

Figure 3. Typical carbon-pile pressure calibration curve

L = distance in inches between centers of adjacent grooves on rod

m_{60} = millimeters (or inches) per cycle measured on the 60-cycle timing wave

m_r = millimeters (or inches) measured over N cycles at chosen point on pickup record. When the frequency is high, three or four cycles should be used. When the frequency is low, two cycles are sufficient.

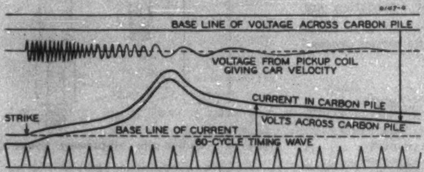

Figure 4. Typical oscillogram from which the performance of the buffer is calculated

To obtain the curve of deceleration versus inches of stroke it is necessary to plot a curve of velocity versus time from figure 5. The slope of this curve at any point is then the rate of change of velocity from which the curve of deceleration of figure 5 is plotted.

Measurement of Oil Pressure

The oil-pressure gauge consists of an oil-pressure cylinder and a carbon pile mounted together with a rocking beam connecting them as in figure 2. The carbon pile consists of 42 carbon disks each of which is $^1/_8$ inch thick and $1^1/_2$ inches in diameter. The disks were ground to a true surface on each side as far as possible. The disk pile is enclosed in a paper tube and inserted in a brass tube shown in figure 2. The rocking beam by which the pressure at the piston is transmitted to the carbon pile has a lever arm ratio of $1^1/_2$ inches to $4^1/_2$ inches. Because the long end of the beam moves only $^1/_{16}$ inch with the greatest oil pressure encountered, inertia effects are reduced to a satisfactory minimum.

The carbon pile is calibrated in resistance versus pressure. The current through the carbon pile and the voltage across it are both recorded on the oscillograph film so that the resistance may be calculated and the pressure determined for any point in the piston stroke. A calibration curve is shown in figure 3. Figure 4 shows a sketch of a typical oscillogram. Figure 5 shows the final curves of performance of the buffer.

Difficulties to Be Avoided

To obtain sufficient voltage in the pickup magnet coil to operate the oscillograph element without interposing an amplifier, the hole in the magnet should not exceed three-fourths inch in diameter when a five-eighths-inch grooved rod is used. Grooves one-eighth inch long, one-eighth inch deep and spaced three eighths inch apart were found quite satisfactory.

If there is no guide provided for the suspended rod, the impact of the car on the buffer piston may cause the rod to deflect and strike the pickup magnet instead of its hole. A long brass point fitted to the rod attached to the car proved helpful in guiding that rod through the pickup magnet in its fall. A fiber bushing in the pickup magnet hold (not shown in figure 1) was found helpful in guiding the rod. It was also found advisable to mount the magnet so that it was free to move laterally to follow any swinging of the rod.

In comparison with electric strain gauges recently developed the carbon pile has the advantage of requiring no amplifier or source of high-frequency voltage for oscillographic recording.

Summary

The curves of figure 5 are typical of those obtainable by the electrical apparatus described. Inasmuch as the oil-pressure indicator was connected to the bottom of the buffer cylinder, there was a short time interval between the instant of maximum pressure as shown on the pressure gauge and the maximum deceleration. This time interval is that required for the high-pressure wave to travel through the oil from the piston to the gauge. This difference in time demonstrates the elasticity of the light transformer oil used.

The accuracy of this method is satisfactory for all existing test codes.

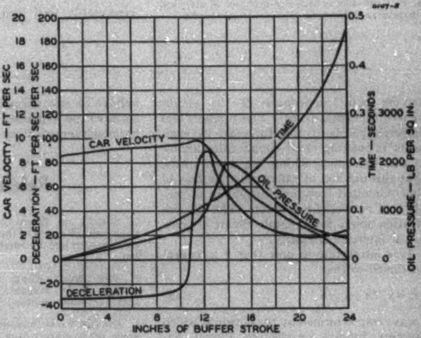

Figure 5. Typical performance curves of a buffer with poor workmanship or of poor design

Single-Phase Induction-Motor Performance Calculation

F. S. HIMEBROOK
MEMBER AIEE

IN the manufacture of electrical machines it is always desirable to calculate the performance of a machine before its construction to determine whether or not it will meet the specifications. The usual methods of predetermining performance involve the use of either a circle diagram or an equivalent circuit and since either of these methods involves considerable work there are always possibilities that errors may be introduced by slight mistakes in construction, measuring, etc. It was believed that if a method could be developed by which the quantities necessary for performance calculation could be given in the form of a set of curves considerable time might be saved provided that the accuracy so obtained was of the same degree as that given by other methods.

This paper describes the development of a method which may be used to plot, as families of curves, the quantities necessary for the performance calculation of single-phase induction motors. Consideration is first given to a circle diagram which is known to give accurate results when used to calculate the performance of motors of this type. It is then shown that all of the quantities involved in the calculation of performance are dependent upon certain ratios of the fundamental motor constants and not upon the absolute values of the constants. This means that it is not necessary to draw a new circle diagram for each motor and any diagram may be used in any number of cases as long as the fundamental relations are held constant. Thus if a number of diagrams are made for different ratios of motor constants the data required for performance determination may be evaluated and plotted in the form of curves which may be used to take the place of the circle diagrams. Such curves are at least as accurate as the diagrams they replace because slight errors in reading values become apparent when plotted and tend to be either minimized or averaged out.

Besides the motor characteristics in the usual operating range it is always desirable to know the value of the maximum torque. It was found that the information required for the calculation of the maximum torque might be determined by making use of the diagrams previously described and a method of plotting the necessary quantities in the form of curves is given.

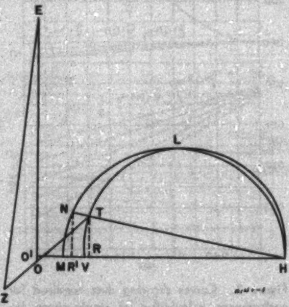

Figure 1. Circle diagram used in determining performance of single-phase induction motors

Extension of the methods given for the calculation of the performance of polyphase motors is shown to be possible and is discussed briefly.

The Circle Diagram

The circle diagram used (figure 1) is the one given by Branson[1] in his paper which explains the operation of the single-phase motor by the cross-field theory. The methods of constructing this diagram and of using it are included in appendices I and II.

By reference to appendix I it will be seen that since $O'H$ is a constant and since $O'M$ is a variable dependent only upon K_r (defined in appendix I), that the value of MH and the semicircle $MNLH$ are also functions of only K_r. Thus, for any particular value of K_r this part of the diagram is fixed.

In a like manner, the value of MV is determined by the value of K_r and point V located accordingly. The value of LH is dependent upon both K_r and the ratio r_2/X but for any particular values of these two quantities the point L is definitely located. Because points V, L, and H are all located and because all three points must lie on the circumference of one circle the center of this circle is located at the intersection of the perpendicular bisectors of lines VL and LH. After the center has been located arc VLH may be swung.

Actually the line OO' is not constant, as assumed in appendix I, but varies with the motor under consideration. However the assumed value is sufficiently close to the average value calculated for normal motors that the error introduced by this assumption is very slight. Having located point O the directions of lines OT are defined.

By definition

$$OZ = (S_1 r_1 / S_s) \times OT \qquad (1)$$

and by substituting equivalent values for S_1 and S_s we may change this to the form

$$OZ = \frac{(E/10X)}{(E/OE)} \times r_1 \times OT = \frac{(OE=10)}{10X} \times r_1 \times OT = \frac{r_1}{X} \times OT \qquad (2)$$

giving a solution for the length OZ which is a direct function of r_1/X.

Thus it is seen that the entire diagram is dependent only upon K_r, r_2/X and r_1/X and that the performance of all motors having the same values for these quantities will be identical. Since this is true it is evident that with the use of only a relatively small number of diagrams it is possible to determine the performance of a wide variety of motors. We may also proceed one step farther and plot the values of OT, MT, etc., in the form of curves and so have all of the information required for performance calculation in a convenient form. In carrying out this work for a given value of K_r it will be found convenient to plot the values of OT, MT, etc., against the length of MN since this value remains constant for a given value of K_r regardless of the ratios r_1/X and r_2/X.

This method of plotting the curves may be seen by reference to figures 2 and 3. In some cases all of the values for the

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[1] For all numbered references, see list at end of paper.

ranges of r_2/X considered do not differ appreciably and so only one value has been shown. For example the values of *TR* as shown all lie within about 0.01 inch of the value shown and since errors in measurement might have been at least ±0.01 inch it was not believed necessary to attempt to show any more than one curve.

In calculating the performance the procedure followed is that outlined in appendix II.

Performance Calculation Using Curves

In the final analysis the utility of a method lies in the accuracy of the results obtained. The performance of a one-fourth-horsepower single-phase induction motor was calculated by the use of the curves described herein and the methods described in appendix II, and is shown with the actual test data for this motor in table I. Examination of this table shows that the calculated and measured results do not differ widely in any respect.

Determination of Maximum Torque

It was known that the point corresponding to maximum torque lay on arc *VTL* (figure 1) somewhere beyond, but near to, the center of arc *VTL*. To locate this point precisely would have required the choosing of several points, calculating the data for each point, and then finding the desired point by the process of elimination. It was found, experimentally, that the point corresponding to the maximum torque lay almost midway between the center of arc *VTL* and the point of tangency between the arc *VTL* and a line drawn parallel to *ML*. The point found in this manner located the point of maximum torque closely enough for all practical purposes.

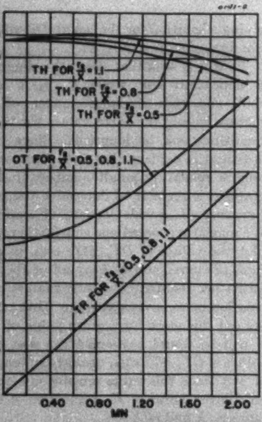

Figure 2. Curves showing data required for performance calculation of single-phase induction motors. (K_r = 0.93)

The values of *OT*, *MT*, etc., corresponding to the point of maximum torque may also be read and plotted in the form of curves. It was found convenient to plot these values against K_r as shown in figures 4 to 7, inclusive.

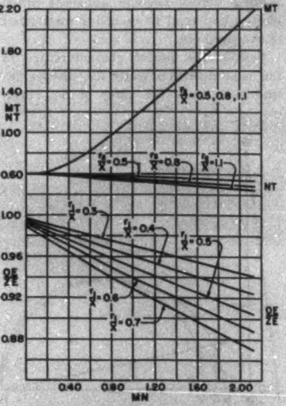

Figure 3. Curves showing data required for performance calculation of single-phase induction motors. (K_r = 0.93)

The locus of the current vector in an ideal motor follows a circular path but this ideal is deviated from in the case of an actual motor. Usually the circular locus is accurate for points in the normal operating range but some deviation is often noted when calculating the maximum torque. It has been found[1] that the maximum torque as obtained by a brake test is only about 0.92 of the calculated value and so it is necessary to multiply the calculated torque by 0.92 to secure the value to be expected in an actual motor. Aside from this, calculation of the maximum torque is carried out in the same way as for any other point. (See appendix II.)

When the calculation was made upon the motor previously considered the indicated maximum torque was found to check closely (see table I) with the measured value.

Extension of Method to Polyphase Motors

The same diagram may be used to derive a similar set of curves for polyphase motors. For this work *MH* is again equal to $10K_r$ and the length *OO′* becomes 0.05 inch for all practical purposes. The inner circle *VTLH* does not exist and point *T* lies on *MNLH*. For use with polyphase motors the values of *OT*, *MT*, *TR*, *OZ*, and *ZE* are required. These correspond to *ON*, *MN*, *NR′*, *OZ*, and *ZE* of the single-phase diagram (figure 1).

Thus without a great deal of additional work a set of polyphase curves might be obtained. However, these curves are not believed to be in as usable a form as the "*a-b-c*" curves described by Branson.[2]

Discussion

It was assumed, in deriving the material contained herein, that the core loss *OO′* could be given as 0.10 inch in all work and that the ratio of the saturation factors in the main and cross fields was unity. Some such assumption was required or a complete set of curves would have been required for each value of core loss and each ratio of saturation factors. A number of tests were computed and indicated that these assumptions might readily be made. In all use to date these assumptions have been found sufficiently accurate to warrant their use.

The ranges of values of K_r, r_1/X, and r_2/X shown in the sample curves are believed great enough to cover most ordinary motors. As special cases are encountered their data may be recorded and used to build up a more extensive set of curves.

Table I. Comparison of Measured and Calculated Characteristics of a One-Fourth-Horsepower Single-Phase Induction Motor

Quantity	Measured	Calculated
r_1	1.29 ohms	1.30 ohms
r_2	2.51 ohms	2.41 ohms
X	3.60 ohms	3.60 ohms
X_0	51.2 ohms	50.2 ohms
K_r	0.929	0.932
Full-load speed	1,747 rpm	1,737 rpm
Full-load current	4.82 amp	4.81 amp
Full-load efficiency	64.3 per cent	61.5 per cent
Full-load power factor	56.7 per cent	57.2 per cent
Full-load input	292 watts	302 watts
Maximum torque—in per centrated load torque	310	304

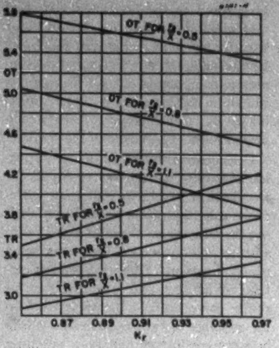

Figure 4

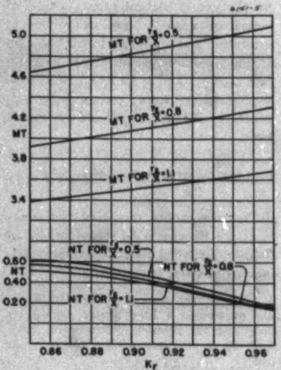

Figure 5

The results obtained by the use of the curves indicate that when the original work is carefully carried out that considerable time may be saved and results are obtained which are as accurate as might usually be obtained by the construction of a special diagram for each motor. In addition, when the curves are plotted any but the most trivial errors are immediately made evident because any errors (except consistent errors) are shown by irregularities in plotting.

Appendix I

Construction of Circle Diagram

Before constructing the circle diagram (figure 1) it is necessary to know the values of the following quantities:

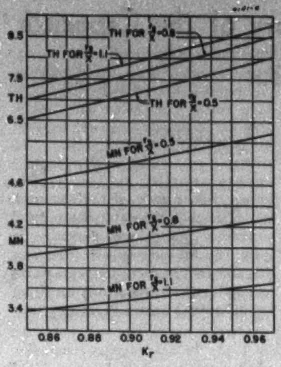

Figure 6

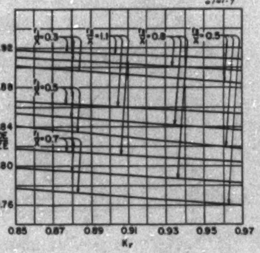

Figure 7

Figures 4–7. Curves showing data required for calculation of the maximum torque which may be expected of a single-phase induction motor

r_1=stator resistance, ohms
r_2=rotor resistance, ohms, referred to the stator

Both r_1 and r_2 represent the resistances when the motor is at its normal operating temperature. For a motor designed for a 40-degree centigrade rise, the hot resistance is 1.15 (cold resistance).

X=total leakage reactance, ohms
X_0=no load reactance, ohms

$$K_p = \frac{\text{permeance of mutual path}}{\text{permeance of mutual and primary leakage paths in parallel}}$$

$$K_s = \frac{\text{permeance of mutual path}}{\text{permeance of mutual and secondary leakage paths in parallel}}$$

$K_r = K_pK_s = (X_0 - X)/X_0$
E=impressed electromotive force, volts

To construct the circle diagram proceed in the following manner. Lay off the horizontal line $O'H$=10 inches. (Any length may be used but 10 inches is probably the most convenient. The line OE which is passed perpendicularly through $O'H$ at O' is also made equal to 10 inches for convenience.)

Calculate the voltage and current scales.

S_e=voltage scale, volts per inch, $(=E/10)$
S_1=stator current scale, amperes per inch, $(=E/10X)$
S_2=rotor current scale, amperes per inch, $(=S_1/K_p)$

Lay off

$$\left.\begin{matrix} i_m = E/X_0 \\ O'M = E/S_1X_0 \end{matrix}\right\} = 10(1-K_r)$$

Using MH as a diameter, construct the semicircle $MNLH$.

$$i_{ms} = \frac{i_mK_r}{2-K_r} \times \frac{\text{saturation factor (cross field)}}{\text{saturation factor (main field)}}$$

$$\text{Calculate } MV = \frac{E}{S_1X_0} \times \frac{K_r}{2-K_r} \times \frac{\text{saturation factor (cross field)}}{\text{saturation factor (main field)}}$$

where the saturation factor is defined as

$$\frac{\text{total ampere turns required}}{\text{ampere turns required for air gap alone}}$$

(The saturation factors for the data shown in the curves have, for convenience, been assumed equal. This is practically true for most motors.)

Locate the locked point L. The distance $LH = MH \sin^{-1} (\tan r_2/X)$

Construct a circle passing through V, L, and H

Construct OO'=0.10 inch (OO' is the main field iron loss current and is equal to main field power loss/ES_1. Its value in most normal motors lies very close to 0.10 inch)

From O lay off OE=10 inches

Choose various points T, draw lines NTH and OT, extending the lines OT to the left of OE a distance $OZ = \{(S_1r_1)/S_e\} \times OT$

Draw lines ZE.

Appendix II

Calculation of Performance Data

For each point T the following procedure is used.

Secure curves (figures 2 and 3) for desired value at $K\gamma$. Select values of MN; read values of TH, OT, and TR for proper r_2/X from figure 2; read values of MT and NT for proper value of r_2/X and OE/ZE for the proper value of r_1/X from figure 3.

1. OT—from figure 2 for proper value of K_r
2. MT—from figure 3 for proper value of K_r
3. MN—selected value, figures 2 and 3
4. NT—from figure 3 for proper value of K_r
5. TH—from figure 2 for proper value of K_r
6. TR—from figure 2 for proper value of K_r

The Cross-Field Theory of the Capacitor Motor

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ANALYSES of the capacitor motor operation date back to the 1920's, a pioneer effort being that of Brüderlin[1] in Europe. Work by Bloch,[2] Biermanns,[3] and others antedates that of Morrill[4] in this country whose analysis has been widely used. All of these have been based on the double revolving field theory, being extensions of that theory to the usual single-phase induction motor.

The cross-field theory has been successfully applied to plain single-phase induction motor analyses by Sumec,[5] Arnold and LaCour,[6] Steinmetz,[7] West,[8] and others, but heretofore no application of this theory has been made to the capacitor motor. The present paper is an exposition of such an analysis.

For the present purpose only two fundamental variations appear in the treatments of the cross-field theory. These concern the speed electromotive forces, built up by rotation of the rotor conductors through the rotor leakage fluxes. To elaborate on this idea, one may take the view that for the same saturation when locked and running, the machine leakage reactance is independent of rotation and that no revolving harmonic fields are present. Of the usual components of leakage flux, namely, slot, zigzag, end, and differential, the stator slot and end connection leakage are stationary in space. By the same reasoning the rotor slot and end connection leakage fluxes rotate with the rotor in space; the zigzag and differential leakage fluxes vary their distribution more or less with different rotor positions, although tacitly an average is assumed. Arnold and West imply that more accurate results are obtained if the assumption is made that the rotor leakage fluxes stand still in space, as opposed to the Steinmetz view that they move with rotation. As a working basis for the former viewpoint which will be used here, the slot, zigzag and end connection components of rotor leakage flux will all be considered as stationary, making up X_2. The differential leakage component should be charged to the stator.

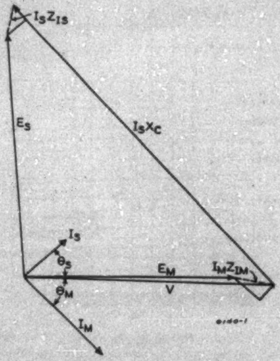

Figure 1. Vector diagram showing stator currents and voltage components

Considerable difference of opinion exists regarding the relative merits of the two classical theories for explaining single-phase induction-motor phenomena. It is not the intention of the present writers to advance the cross-field theory as being wholly superior. Each method has certain advantages and defects. For instance the cross-field theory is blind to the existence of two separate skin effects in rotor conductors and to a pulsating torque superposed on a steady torque. (This latter difficulty could be avoided by the use of instantaneous instead of vector quantities.) But differences in saturation and core losses in the two axes are much easier to visualize with the cross-field theory. And as for its application to the capacitor motor, certain other advantages will be developed and contrasted later.

Fundamental Relationships

The relationship on which the equations are based can be stated briefly as follows (assuming that the stator windings are in space quadrature):

1. The voltage applied to the main winding overcomes a local stator impedance drop and the counter electromotive force built up by the mutual flux of the main axis.

2. The stator current is the resultant of a load component reflected by transformer action from the rotor and a main-axis magnetizing current which may or may not contain an iron loss component.

3. The voltage applied to the quadrature winding (called here the start winding) overcomes the local stator impedance drop of that winding, the reactance or impedance

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[1] For all numbered references, see list at end of paper.

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7. *OE/ZE*—from figure 3 for proper value of K_r
8. $MN \times r_2/X$
9. $NT \times r_2/X$
10. $TH-(8)$

11. $$\text{rpm} = \sqrt{\frac{(10)}{(5)}} \times \text{synchronous speed (approximately)}$$
$$= \sqrt{\frac{(10)^2+(9)^2}{(5)\times(10)}} \times \text{synchronous speed (exact formula)}$$

12. Primary amperes $=(1)\times(7)\times S_1$
13. Secondary amperes $=(2)\times(7)\times S_2$
14. Secondary copper loss (cross field) $=(i_{ms}/K_p)^2 \times r_2 \times \{(11)/\text{syn. rpm} \times (7)\}^2$
15. Secondary copper loss (main field) $=(13)^2 \times r_2$
16. Iron loss (cross field) $=0.45$ (polyphase iron loss)
17. Friction and windage loss = previously determined
18. Iron loss (main field) $=0.50$ (polyphase iron loss)
19. Primary copper loss $=(12)^2 \times r_1$
20. Secondary input $=(7)^2 \times (6) \times E \times S_1$
21. Total input $=(18)+(19)+(20)$
22. Total losses $=(14)+(15)+(16)+(17)+(18)+(19)$
23. Output $=(21)-(22)$
24. Torque (ounce-feet) $=112.7 \times (23)/(11)$
25. Efficiency $=(23)/(21)$
26. Input, volt-amperes $=(12) \times E$
27. Power factor $=(21)/(26)$

Bibliography

1. SINGLE-PHASE INDUCTION MOTORS, W. J. Branson. AIEE TRANSACTIONS, volume 31, 1912, pages 1749–87.

2. POLYPHASE INDUCTION MOTOR PERFORMANCE CALCULATION, W. J. Branson. AIEE TRANSACTIONS, volume 49, 1930, pages 319–28.

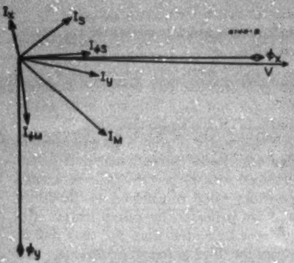

Figure 2. The stator current components and the flux vectors

drop of the capacitor in series with the winding, and the counter electromotive force built up by the mutual flux of the cross axis.

4. The stator current of the cross axis is the resultant of a load component reflected by transformer action from the rotor, and a start or cross axis magnetizing current similar to that of the main winding, but usually differing in magnitude.

5. In the rotor, the main or y-axis current results from two electromotive forces:

(*a*). The transformer voltage, opposite in phase to the stator counter electromotive force but of the same magnitude if the rotor constants are all considered in main winding terms.

(*b*). A speed voltage, resulting from the rotor conductors cutting the cross-axis mutual flux, and (as suggested by West and Arnold) the leakage flux of the rotor as well. By Kirchoff's law the vector sum of these voltages and the rotor impedance drop caused by the main axis current, equals zero.

6. In the start or x-axis of the rotor, two electromotive forces, similar to those listed above, are present:

(*a*). A transformer voltage opposite in phase to the cross-axis counter electromotive force but different in magnitude by the ratio of transformation between start winding and rotor, or start and main windings.

(*b*). A speed voltage resulting from cutting the main winding mutual and rotor leakage fluxes.

The vector sum of these voltages and the rotor impedance drop of the cross-axis rotor current equals zero.

The four fundamental equations are then (see list of symbols):

Main axis: Stator:

$$Z_M(I_M-I_y)+I_MZ_{1M}=V \tag{1}$$

Start axis: Stator:

$$Z_S(I_S-I_X)+I_S(Z_{1S}+Z_C)=V \tag{2}$$

X-axis: Rotor:

$$\frac{Z_S}{a}(I_S-I_X)+jS[-Z_M(I_M-I_y)+jI_yX_2]=aZ_2I_x \tag{3}$$

Y-axis: Rotor:

$$-Z_M(I_M-I_y)+jS\left[-\frac{Z_S}{a}(I_S-I_X)+jaI_xX_2\right]=-I_yZ_2 \tag{4}$$

These equations are extensions of West's[8] equations for the cross-field theory applied to the plain single-phase induction motor. The terms I_yX_2 and aI_xX_2 in equations 3 and 4, respectively, represent the added effect on the speed electromotive forces of cutting the rotor leakage fluxes. If these are dropped, the fundamental equations reduce to a form which Steinmetz's single-phase motor equations would yield.

By substitution, the stator components of the rotor currents, I_y and I_x, can be eliminated in these four equations, leaving two in terms of the two components of stator current. For further convenience in obtaining a final solution by determinants, the coefficients of these currents can be represented by the A, B, and C constants as shown below. Then:

$$A_1I_M+B_1I_S=C_1 \tag{5}$$

$$A_2I_M+B_2I_S=C_2 \tag{6}$$

where:

$$A_1=S\left[jZ_{1M}-X_2\left(1+\frac{Z_{1M}}{Z_M}\right)\right] \tag{7}$$

$$B_1=-\left[aZ_2+(Z_{1S}+Z_C)\left(\frac{1}{a}+a\frac{Z_2}{Z_S}\right)\right] \tag{8}$$

$$C_1=V\left[S\left(j-\frac{X_2}{Z_M}\right)-\left(\frac{1}{a}+a\frac{Z_2}{Z_S}\right)\right] \tag{9}$$

$$A_2=\left[Z_{1M}\left(1+\frac{Z_2}{Z_M}\right)+Z_2\right] \tag{10}$$

$$B_2=S\left[-aX_2+(Z_{1S}+Z_C)\left(j/a-a\frac{X_2}{Z_S}\right)\right] \tag{11}$$

$$C_2=V\left[\left(1+\frac{Z_2}{Z_M}\right)+S\left(j/a-a\frac{X^2}{Z_S}\right)\right] \tag{12}$$

Then by determinants:

$$I_M=\frac{C_1B_2-C_2B_1}{A_1B_2-A_2B_1} \tag{13}$$

$$I_S=\frac{A_1C_2-A_2C_1}{A_1B_2-A_2B_1} \tag{14}$$

From (1) and (2),

$$I_y=I_M\left(1+\frac{Z_{1M}}{Z_M}\right)-\frac{V}{Z_M} \tag{15}$$

$$I_x=I_S\left(\frac{Z_S+Z_{1S}+Z_C}{Z_S}\right)-\frac{V}{Z_S} \tag{16}$$

For rotor-locked conditions, (13) and (14), reduce to

$$I_M=\frac{V}{Z_{1M}+\left(\dfrac{Z_2}{1+\dfrac{Z_2}{Z_M}}\right)} \tag{17}$$

$$I_S=\frac{V}{(Z_{1S}+Z_C)+\dfrac{aZ_2}{\dfrac{1}{a}+a\dfrac{Z_2}{Z_S}}} \tag{18}$$

The counter electromotive forces in the separate windings are:

$$E_M=V-I_MZ_{1M} \tag{19}$$

$$E_S=V-I_S(Z_{1S}+Z_C) \tag{20}$$

The rotor input, or the power transferred across the gap, is the sum:

$$E_MI_y\cos(E_M, I_y)+E_SI_x\cos(E_S, I_x) \tag{21}$$

The power converted to mechanical form:

$$\text{Power across gap by (21)}-(I_y^2+a^2I_x^2)r_2=ST \tag{22}$$

$$\text{Net output}=\text{power converted to mechanical form}-(F\,\&\,W) \tag{23}$$

The torque in synchronous watts, (T) is the sum:

$$aE_MI_x\sin(E_M, I_x)+\frac{E_S}{a}I_y\sin(E_S, I_y) \tag{24}$$

The stator input is the sum:

$$VI_M\cos\theta_M+VI_S\cos\theta_S \tag{25}$$

The stator losses:

$$I_M^2r_{1M}=\text{copper loss in the main winding} \tag{26}$$

$$I_S^2(r_{1S}+r_C)=\text{copper loss in start winding circuit} \tag{27}$$

$$\frac{E^2M}{r_e}+\frac{E_S^2}{r_{eS}}=\text{iron losses} \tag{28}$$

The rotor losses:

$$I_y^2r_2=\text{copper loss in the }y\text{-axis} \tag{29}$$

$$a^2I_x^2r_2=\text{copper loss in the }x\text{-axis} \tag{30}$$

Some attention should be given to the matter of iron losses. In the double-field theory of Morrill, the iron losses are neg-

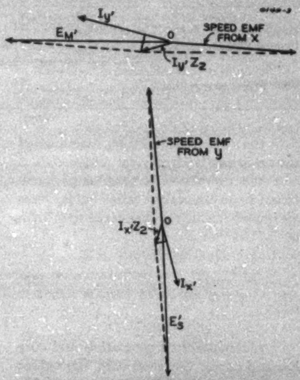

Figure 3. The rotor current and voltage vectors

lected in the equations, but good results are claimed for a correction based on consideration of these losses as external to the motor. In the use of the revolving-field analyses it has been the experience of the present writers that a lack of agreement may result between calculated and test values, particularly in the start winding under running conditions. This may be due to saturation, harmonics, or both, corrections for which are not readily introduced in the equations.

If desired, the iron losses can be considered in the symmetrical-component method of Lyon and Kingsley,[9] or in the method of Punga[10] which rests on the polyphase motor. The work of Y. Hasumi[11] on the symmetrical-component basis indicates a very simple method of introducing the iron losses in Morrill's equations by using for forward and backward impedance, respectively, the terms:

$$Z_f=\frac{1}{\frac{1}{r_0}+\frac{1}{jX_m}+\frac{1}{r_{2/s}+jX_2}}=R_f+jX_f \qquad (31)$$

$$Z_b=\frac{1}{\frac{1}{r_0}+\frac{1}{jX_m}+\frac{1}{r_{2/2-s}+jX_2}}=R_b+jX_b \qquad (32)$$

In any case the labor involved is considerable, the efforts to reduce it, such as by the method of Krondl,[12] or by tabulated form sheets and curves, have met with partial success.

In the method used here with the cross-field theory the magnetizing reactance of the main axis is jX_M. The addition of a core-loss resistance in that axis, transferred to an equivalent series value, gives an impedance of the magnetizing branch as

$$Z_M=\frac{1}{\frac{1}{r_0}+\frac{1}{jX_M}} \qquad (33)$$

Table I. Motor Running With a Slip of Four Per Cent

Method	I_M	I_S	Input and Iron	$I_M^2r_{1M}$	$I_S^2\cdot(r_C+r_{1S})$	Rotor Copper	Friction and Windage	Net Output
Morrill	$2.47\overline{\backslash 59°3'}$	$0.948\underline{/38°46'}$	221+24	12.3	14.5	11.4	13	177
Lyon & Kingsley	$2.57\overline{\backslash 55°48'}$	$0.935\underline{/38°40'}$	239	13.3	14.1	11.8	13	165
Punga	$2.57\overline{\backslash 56°30'}$	$0.948\underline{/40°50'}$	234	13.3	14.5	10.9	13	162
Cross-field	$2.64\overline{\backslash 58°36'}$	$0.954\underline{/39°0'}$	233+24	14.1	14.7	13.2	13	177 to 179.8

If widely different saturation and core losses occur in the cross-axis, the values should be modified but the magnetizing impedances in the two axes take the same form; thus:

$$Z_S=\frac{1}{\frac{1}{r_{0s}}+\frac{1}{jX_s}} \qquad (34)$$

Neglecting the iron losses, with the idea of considering them later as external to the motor, results in the reduction of Z_M and Z_S to jX_M and jX_S, respectively.

Symbols

V = applied volts. Used as reference vector
E_M = the counter electromotive force in the main axis
$Z_{1M}=r_{1M}+jX_{1M}$ = the impedance (local) of the main stator winding
Z_2 = the rotor impedance in main winding terms (local) $=r_2+jX_2$
E_S = the counter electromotive force of the start or capacitor winding
$Z_{1S}=r_{1S}+jX_{1S}$ = the local impedance of the stator start winding
$Z_C=r_C-jX_C$ = capacitor impedance
I_M = the stator current, main axis
I_S = the stator current, start axis
$I_{\phi M}$ = the magnetizing current, main axis
$I_{\phi S}$ = the magnetizing current, start axis
I_y = the stator component of the rotor current, main axis
I_x = the stator component of the rotor current, start axis
Z_S = the magnetizing impedance in the S-axis
Z_M = the magnetizing impedance in the M-axis

$$Z_S=\frac{1}{\frac{1}{r_{0S}}+\frac{1}{jX_S}}$$

$$Z_M=\frac{1}{\frac{1}{r_0}+\frac{1}{jX_M}}$$

$a=\dfrac{\text{Effective start winding turns}}{\text{Effective main winding turns}}$

$S=\dfrac{\text{Actual rpm}}{\text{Synchronous rpm}}$

s = slip

Vector Diagrams

One advantage of this analysis rests in the graphical picture which can readily be obtained for the physical relationships by vector diagrams. Figure 1 indicates the stator voltage components, as well as the usual positions of the stator currents.

In figure 2 are shown the current components and the flux vectors. The vectors $I_{\phi S}$ and $I_{\phi M}$ indicate magnetizing currents for the respective axis.

Rotor vector relationships are shown in figure 3. Rotor currents and voltages are reversed with respect to their respective positions when transferred to the stator and are indicated by prime terms. Experience indicates that I_X' may fall in either the third or fourth quadrants. The vectors shown here represent the solution of equations 3 and 4.

Summary and Comparative Results

A number of examples have been calculated by this cross-field analysis, one of which is shown in the appendix. The final data are tabulated in tables I and II along with results on the same motor as obtained through other analyses. The method throws light on the phenomena from another point of view and a favorable degree of accuracy is indicated with a competitive ease (or difficulty) of application.

To the engineer who has become thoroughly familiar with the calculation of capacitor-motor performance by the double-field theory, using convenient tabulations of equations, the use of this cross-field analysis will probably appear long and complex. However, considerable experience in handling these equations has led the writers to the belief that for time involved and complexity of calculation there is little to choose between the double-revolving and the cross-field theories. Advantages, then, for this analysis can be found chiefly in the fact that different saturation effects in the two axes can be considered more easily and that the equations result in vector diagrams which are significant in showing the effect of various design changes. This is particularly true in dealing with the conditions necessary for balanced "two-phase" operation, although detailed derivations for this application will not be carried out here. With this analysis a revolving field will exist when:

$$jI_y'=I_X' \qquad (35)$$

$$jE_M'=\frac{E_S'}{a} \qquad (36)$$

Either of these relations is sufficient as indicating that the fluxes are equal and in quadrature. With the rotor currents in terms of the main winding (35) becomes:

Table II. Starting Conditions

Method	I_M	I_S	Input and Iron	$I_M^2 r_{1M}$	$I_S^2\cdot(r_C+r_{1S})$	Starting Torque (Synchronous Watts)
Morrill	$14.2\overline{\diagdown 40^\circ 50'}$	$6.30\underline{/27^\circ 55'}$	1,796+24	406	401	755
Lyon & Kingsley	$14.0\overline{\diagdown 40^\circ 50'}$	$6.20\underline{/27^\circ 50'}$	1,750	398	390	728
Punga	$14.2\overline{\diagdown 40^\circ 45'}$	$6.28\underline{/27^\circ 40'}$	1,790	406	400	708
Cross-field	$14.2\overline{\diagdown 40^\circ 0'}$	$6.25\underline{/27^\circ 50'}$	1,800+24	408	396	759

$$jI_y = aI_X \qquad (37)$$

Also, in main winding terms:

$$jI_M = aI_S \qquad (38)$$

When this is used the total revolving magnetomotive forces are indicated as being equal and in quadrature

The problem of obtaining a true revolving field has also been discussed by Specht[12] and Trickey[14] in this country. Some interesting relationships developed by Punga,[10] Krondl,[11] and Wolf[13] do not seem to be generally known. For example when stator main and start windings have equal copper and result in equal saturation, a true revolving field is obtained at some prescribed slip s when

$a = \tan\phi$ (Punga and Krondl)

and

$X_C = (a^2+ja)Z$ (Punga)

wherein:

ϕ = the power factor angle of a two-phase motor with both phases like the main and operating at a slip s

Z = the machine impedance of phase M at a prescribed slip s, for a two-phase motor

Z_0 = the machine impedance similar to Z above, but at a slip of 2-s

All of these quantities can be obtained from the equivalent network, other equations, or the circle diagram of a two-phase motor.

Trickey's formulas for balanced condition are somewhat longer than those pertaining to the analyses quoted above although they have greater generality. Punga indicates an interesting relationship, in that under the assumptions of equal copper and saturation, I_M becomes zero when for some given value of slip, the capacitor impedance is fixed thus:

$$Z_C = -\frac{Z_0(a^2-ja)+Z(a^2+ja)}{2}$$

No one familiar with the process of designing capacitor motors can be thoroughly satisfied with the present analytical or design tools available to the designer. No convenient expression has been derived for the speed-torque curve nor for the maximum torque that can be developed. The lack of such equations as well as simplified forms of many other relationships, has proved a serious handicap. In the long run such developments result only from the examination of all available theories and viewpoints. It is the hope of the present writers that the cross-field analysis may be a step forward in aiding more useful analyses to be developed.

Appendix I

To illustrate the application of the cross-field theory an example will be worked out using the same motor for which constants are given in Morrill's[4] paper. The values for this four-pole one-fourth horsepower 110-volt motor are:

$r_{1M} = 2.02$
$r_{1S} = 5.12\times 1.18^2 = 7.13$
$r_2 = 4.12$
$r_C = 9.00$
$X_{1M} = 2.79$
$X_{1S} = 2.31\times 1.18^2 = 3.22$
$X_2 = 2.12$
$X_C(\text{running}) = -172.0$
$a = 1.18$
$X_M = 66.8$
$X_S = 92.9$

Friction and windage = 13 watts
Iron losses = 24 watts

On any given motor these constants can be obtained by the usual test methods applicable to the ordinary single-phase motor, using each winding independently. Their calculation from a design sheet involves the usual processes pertaining to the single-phase motor found in standard works on design. It is necessary to recall that the machine acts as a transformer in each of its two axes and that consideration must be given (in calculations of rotor constants) to the winding to which these constants are referred. Furthermore, as noted in this example, magnetizing reactance, and rotor resistance and leakage reactance have twice the values by the cross-field theory that they would display for the double revolving field method.

In this example, the use of an equivalent iron loss resistance will be neglected and hence Z_M and Z_S reduce to the respective values of jX_M and jX_S.

Performance will be calculated at a slip of 0.04.

From:
(7) $A_1 = -4.80+j2.00$
(8) $-B_1 = 10.14-j145.9$
(10) $A_2 = 6.367+j4.876$
(11) $B_2 = 139.1+j13.55$
(9) $C_1 = -96.2+j114.7$
(12) $C_2 = 113.5+j85.6$
(13) $I_M = 2.64\overline{\diagdown 58^\circ 36'}$
(14) $I_S = 0.954\underline{/39^\circ 0'}$
(15) $I_y = 1.55\overline{\diagdown 28^\circ 30'}$
(16) $I_x = 0.757\underline{/131^\circ 25'}$

The magnetizing current ($I\phi_M$) in the main axis $= I_M - I_y$
$= 0.015 - j1.51$

In the cross-axis ($I\phi_S$): $I_S - I_x = 1.241 + j0.034$

(19) $E_M = 102.67\underline{/0^\circ 19'}$
(20) $E_S = 113.5\underline{/91^\circ 16'}$

The rotor input:
$E_M I_y \cos(E_M, I_y) = 102.67\times 1.55\times \cos(28^\circ 30' + 0^\circ 19')$
= 140 watts from the main axis
$E_S I_X \cos(E_S, I_x) = 113.5\times 0.757\times \cos(131^\circ\ 25' - 91^\circ 16')$
= 66 watts from the cross axis

Total input to rotor = 206 watts
The rotor losses:

$I_y^2 r_2 = 1.55^2\times 4.12$ or 9.9 watts
$a^2 I_x^2 r_2 = 1.18^2\times 0.757^2\times 4.12$ or 3.3 watts
Friction and windage = 13.0 watts
Total rotor losses......26.2
Net output = 206 − 26.2 or 179.8 watts

The stator input:

$VI_M \cos\theta_M = 110\times 2.64\times \cos 58^\circ 36'$
= 151 watts
$VI_S \cos\theta_S = 110\times 0.954\times \cos 39^\circ$
= 82 watts
Total input = 233 watts

The stator losses:

$I_M^2 r_{1M} = 2.64^2\times 2.02$ or 14.1 watts
$I_S^2(r_{1S}+r_C) = 0.954^2(7.13+9)$ or 14.7 watts
Total stator losses = 28.8

Note that iron losses have been neglected throughout rather than calculated as in equation 28 and considered with the stator losses.

Output from input − losses = 233 − 28.8 − 26.2 or 178 watts.

This checks fairly well with rotor output of 179.8 watts, as calculated.

The torque from (24),

$1.18\times 102.67\times 0.757\times \sin 131^\circ 6' = 69.0$ synchronous watts

$\frac{113.5}{1.18}\times 1.55\times \sin 119^\circ 46' = 129.0$ synchronous watts

Total torque, in synchronous watts = 198.0

$$\text{Ounce feet} = \frac{112.7 \text{ synchronous watts}}{\text{synchronous rpm}}$$

The power converted to mechanical form $= ST$
$= 0.96\times 198$ or 190 watts
The output = 190 − 13 (friction and windage) = 177 watts

Some Insulator Designs Require Special Features to Insure Radio Quietness

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Synopsis: The suggested requirements for radio-free pin-type, switch and bus, and suspension insulators, if acceptable, indicate and justify from a factory testing and production viewpoint, the following conclusions as regards existing standard designs:

1. For distribution voltages up to 4,400 or even 5,500 volts, the small one-piece pin-type insulators used require no "radio-proofing", even on solidly grounded pins.

2. If there is 6 to 12 inches of crossarm in series with small one-piece pin-type insulators on low-voltage distribution lines up to 13 kv, no "radio-proofing" is needed on the insulators.

3. Standard untreated pin-type insulators used on 17 to 69-kv distribution and transmission lines have radio noise-influence voltages in the order of several thousand microvolts at a test potential about ten per cent above the usual line-to-ground voltages. "Radio-proofed" pin-type insulators, readily meeting reasonable requirements for freedom from radio interference, are available for use on lines in this voltage range.

4. Standard cemented pin-and-cap switch and bus insulators up to the 69-kv rating, with the exception of the 66-*S*-class, readily meet the suggested requirements for freedom from radio interference without any special "radio-proofing" treatment; they are inherently quiet.

5. Standard switch and bus insulators for the 66-*S*-class and the 115-kv and higher-voltage classes may require "radio-proofing" or other precautions to meet the suggested abnormally low requirements. These insulators with slight changes can be made to meet these requirements readily.

6. Regular cemented pin-and-cap suspension insulators of modern manufacture readily meet the suggested requirements for freedom from radio interference without any form of "radio-proofing"; they are inherently quiet.

RADIO interference originating on electrical power lines and substations has been studied extensively during the past 10 or 15 years by both the utility engineers and the manufacturers of equipment and materials used in construction. While it is the responsibility of the former to utilize properly the materials available for building their lines, they cannot construct a line or substation which is better than any of its component parts. Therefore, at least part of the responsibility for radio-free power lines and substations is passed on to the manufacturers of materials used in their construction.

Primarily, a power line or substation consists of several conductors which are mechanically held in place and electrically isolated from ground and from each other by means of insulators. The several conductors have a potential with respect to each other and also with respect to ground. One end of an insulator is at the potential of the attached conductor while the other end of the insulator may be at ground potential, or it may have supplementary insulation to ground such as a wood crossarm or a section of a wood pole.

In a three-phase system with a grounded neutral the maximum voltage which can appear across an insulator is the line-to-line voltage divided by 1.732. With wood in series with an insulator to ground, the voltage across the insulator itself may be somewhat less than indicated above. In a three-phase system with an ungrounded neutral an accidental grounding of one of the conductors may place the full line-to-line voltage across the insulators of the other two phases. But this is an abnormal condition which would occur only rarely and then would undoubtedly be corrected as soon as possible. Therefore, the greatest voltage which would normally appear across an insulator in a three-phase system is 0.577 of the line-to-line voltage. For convenience, this voltage is often called the "operating voltage" of the insulator.

Insulators as a Source of Radio Interference

If a power line or substation is to be free from radio interference, it is essential

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Thus by three different approaches outputs of 179.8, 178.9, and 177 watts are obtained. The discrepancies are largely due to slide rule error as a ten-inch slide rule was used throughout.

With a starting capacitor of 3 ohms resistance and 14.5 ohms reactance, the starting conditions can be calculated as shown below:

$$\text{From (17)}\quad I_M = 14.2\diagdown\!\overline{40^\circ 0'}$$

$$\text{(18)}\quad I_S = 6.25\underline{/27^\circ 50'}$$

A convenient method of determining I_y and I_z is to substitute the numerical values of I_M and I_S in equations 1 and 2 respectively.

Then,

$$I_z = \frac{j78.6\, I_S}{4.86 + j81.1} \text{ or } 6.06\underline{/31^\circ 20'}$$

This is in terms of S winding; in terms of M:

$$I_z = 1.18 \times 6.06 \text{ or } 7.15 \text{ amperes}$$

Similarly:

$$I_y = \frac{j66.8 I_M}{4.12 + j68.92} \text{ or } 13.8\diagdown\!\overline{36^\circ 34'} \text{ amperes}$$

The counter electromotive force

$$E_M = jX_M(I_M - I_y)$$
$$= 63.6\diagdown\!\overline{10^\circ 47'}$$
$$E_S = jX_S(I_S - I_z)$$
$$= 38.8\overline{/57^\circ 0'}$$

The starting torque:

$$aE_M I_z \sin\,(E_M,\ I_z) = 1.18 \times 63.6 \times 6.06 \sin 42^\circ 7' \text{ or } 306$$

$$\frac{E_S}{a} I_y \sin\,(E_S,\ I_y) = \frac{38.8}{1.18}\, 13.8 \sin 93^\circ 34' \text{ or } 453$$

In synchronous watts: 306 + 453 or 759

References

1. Die Induktionsmaschine als Phasenumformer, Brüderlin. *Arbeiten elektrotechnischen Inst. Karlsruhe*, volume 4, 1920.

2. Die Ortskurven der graphischen Wechselstromtechnik (a book), O. Bloch. Rascher and Company, Zürich.

3. Ueber einen Einphasen-Induktions-Motor, J. Biermanns. *Arch. für Elek.*, 1926, page 519.

4. The Revolving Field Theory of the Capacitor Motor, W. J. Morrill. AIEE Transactions, April 1929.

5. Der Einphasige Induktionsmotor in Graphisch-Analytischer Darstellung, J. K. Sumec. *Zeitschrift für Elektrotechnik*, September 6, 1903.

6. Die Wechselstromtechnik (a book), E. Arnold. Volume 5, part 2, Julius Springer, Berlin.

7. Theory and Calculation of Electrical Apparatus (a book), C. P. Steinmetz. McGraw-Hill Book Company.

8. The Cross-Field Theory of Alternating Current Machines, H. R. West. AIEE Transactions, volume 45, 1926.

9. Analysis of Unsymmetrical Machines, W. V. Lyon and Charles Kingsley, Jr. Electrical Engineering (AIEE Transactions), May 1936.

10. Das Stromvektordiagram des Einphasenmotors mit Kapazität in der Hilfsphase, F. Punga. *Elektrotechniche Zeitschrift*, March 30 and April 13, 1933.

Graphische Behandlung des Einphasen-Induktions-motor mit Kapazität und Hilfsphase, F. Punga. *Arch. für Elek.*, volume 18, 1927, page 267.

11. Theory of the Capacitor Motor, Y. Hasumi. JIEE (Japan), 1934.

12. Berechnung von Einphasen-Kondensator-Motoren, M. Krondl. Zürich-Oerlikon, Bulletin number 6, XXV Year, 1934, pages 143–7.

13. The Fundamental Theory of the Capacitor Motor, H. C. Specht. AIEE Transactions, April 1929.

14. Design of Capacitor Motors for Balanced Operation, P. H. Trickey. AIEE Transactions, September 1932.

15. Der Einphasen-Induktionsmotor mit Kondaenstoren in der Hilfsphase, Hans Wolf. *Arch. für Elek.*, volume 23, 1930, pages 459–84.

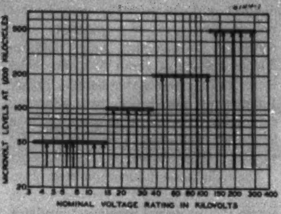


Figure 1. Proposed maximum radio noise-influence voltage levels for "radio-free" insulators

that the insulators used in its construction will not cause disturbance under normal operating conditions. On the other hand, it must not be assumed that the use of "radio-free" insulators will prevent radio disturbance from being generated by the other components used in construction.

The realization that certain types of insulators have been responsible for part of the radio noise emanating from power lines and substations has been the incentive for progressive insulator manufacturers to develop insulators which cannot cause radio disturbances when properly placed in service. Since the high-voltage pin-type insulator was the principal offender, the pioneer investigations were made on this type. (See "Behavior of High-Voltage Insulators on Radio Test," by Charles J. Miller, Jr., in *Electrical World*, March 25, 1939, pages 45–8.)

Fundamentally, the radio interference originating from insulators is caused by an electrically overstressed condition of the air adjacent to certain regions of the insulator. Fortunately, overstressed air, besides giving rise to radio interference, makes its presence known by a visual phenomenon commonly called corona, or corona discharge. This visual corona enabled the pioneer radio-interference investigators to locate the regions of electrical overstress on the pin-type insulator. It was found that these principal regions were at the conductor and tie wire and in the pin hole where thin layers of air were being subjected to intense electric fields. The obvious solution to this problem was to eliminate the air from these critical regions or to alter the design of the insulator in such a way that the intense dielectric field of the insulator between the pin and the conductor and tie wire existed wholly within the dielectric medium of the insulator and not across these layers of air.

Methods of Radio-Proofing Insulators

A conduction paint was developed which could be brushed around the head of the pin-type insulator and in the pin hole. This conduction paint altered the electric field in the critical regions and reduced the noise level of the insulator by many times. However, this paint was too temporary a remedy to be a final solution of the problem. Progressive manufacturers were not satisfied with a temporary remedy and continued the search for a permanent treatment which would last the life of the insulator.

The pin-hole problem was readily solved by cementing a zinc thimble or metal insert into the pin hole. The cement, being a semiconducting medium readily transfers the flux lines from the inner surface of the porcelain to the metal thimble.

The top of the insulator presented a bigger problem and numerous devices were used to attain the desired end. Certain designs were brought out with cemented metal hardware on top of the insulator. Some of these designs have excellent radio properties. Other designs have metallized coatings bonded to the critical area of the insulator by various means. The most recent designs utilize conducting glazes in the critical regions. This type of treatment is being very widely adopted on pin-type insulators because of its satisfactory characteristics and ease of application.

The EEI-NEMA-RMA Measuring Circuit

Along with the advancement of the art of "radio-proofing" insulators came the development of a factory means of measuring the radio-interference properties of equipment and insulators. In the early days of the art, each manufacturer devised his own method of testing and, consequently, when the time came to compare results, it was found that there was no common yardstick for measuring radio interference.

In the meantime the Edison Electric Institute, National Electric Manufacturers Association, and Radio Manufacturers Association were faced with the problem of measuring all sorts of radio interference from all types of equipment. To study the problem in general, the three organizations appointed a committee which came to be known as the "Joint Co-ordination Committee on Radio Reception of EEI, NEMA, and RMA." This committee devised a method of measuring radio noise and established various standards for these measurements. A pamphlet entitled "Methods of Measuring Radio Noise" was published in December 1935 by the joint committee. The latest publication by this committee (EEI Publication No. *G*9—NEMA Publication No. 107—RMA Engineering Bulletin No. 32) is entitled "Methods of Measuring Radio Noise, 1940" and was published in February 1940.

The joint committee devised a circuit to be used for high-voltage devices. This circuit was adopted by leading insulator manufacturers in 1937 and has been the recognized method of test since. (See "Establishes Method of Measuring Radio Interference," by Charles J. Miller, Jr.,

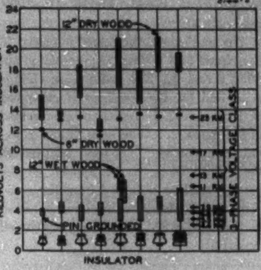


Figure 2. Kilovolts necessary to produce a radio noise-influence voltage of 50 microvolts on small one-piece pin-type insulators with the pin grounded and with wood in series with the insulator

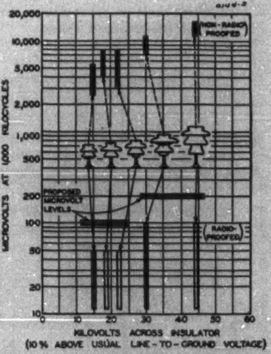


Figure 3. Radio noise-influence voltages from regular pin-type insulators at a test voltage about ten per cent above the usual line-to-ground voltage

in *Electrical World*, March 11, 1939, pages 48–50.)

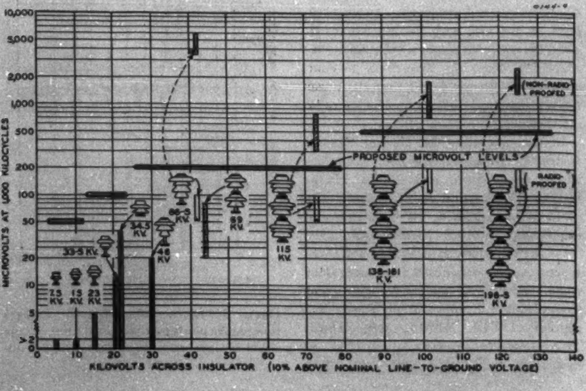
The insulator to be tested is placed in the circuit and a 60-cycle potential is applied across it. The radio noise voltage generated by the insulator is measured with the radio noise meter and is expressed in "microvolts." The term "radio noise-influence voltage" has been adopted as the official name to be applied to the voltage reading obtained from the radio noise meter when the specimen is tested in the EEI-NEMA-RMA recommended circuit.

Requirements for Radio-Free Insulators

To be free of radio interference, insulators must meet certain requirements when tested in the EEI-NEMA-RMA circuit. Based on field experience and known characteristics of available "radio-free" insulators, the majority of manufacturers of high-voltage insulators have suggested radio noise-influence voltage levels which will insure freedom from interference of various classes of insulators under service conditions.

Radio interference originating on a power line may be transferred to a nearby antenna by direct coupling between the power line and antenna, or it may be carried an appreciable distance along a low-voltage distribution line and then be transferred to a receiving antenna. (See "Sources of Radio Interference and How to Eliminate Them," by Paul M. Ross, in *Electric Light and Power*, September 1939, pages 38–9.) In the latter case, coupling must exist between the power line and the low-voltage distribution line and also between the distribution line and the antenna.

Figure 4. Radio noise-influence voltages from regular pin-and-cap switch and bus insulators at a test voltage about ten per cent above the nominal line-to-ground voltage



The farther the distribution line or the antenna from the power line, the smaller is the transfer of radio interference. In general, the higher the voltage of a power line, the greater is its distance from low-voltage distribution lines and antennas. Accordingly, it seems quite reasonable to select a maximum permissible microvolt radio noise level in keeping with the voltage of the line.

Low-voltage distribution lines up to the 13-kv class cover widespread areas and are more or less closely coupled to distribution networks supplying individual homes. It is felt that a maximum radio noise-influence voltage of about 50 microvolts for insulators used on these lines will insure freedom from radio noise provided the radio noise-influence voltages of all other components of the line are also below equally satisfactory levels. Obviously, disturbance on a power line arising from sources other than insulators cannot be blamed on the insulators, nor can "radio-free" insulators suppress radio interference arising from these other sources.

Power lines in the 17 to 34.5-kv classes are, in general, not as closely coupled to the distribution networks and, therefore, a maximum radio noise-influence voltage of about 100 microvolts seems to be a reasonable level. In the range from 46 to 115-kv classes, a level of 200 microvolts should provide adequate protection from radio interference. Transmission lines in the 138 to 287-kv classes seldom parallel lower-voltage lines for any appreciable distance and are generally more or less remote from areas of population. Accordingly, a maximum radio noise-influence voltage of about 500 microvolts seems to be a reasonable figure. Figure 1 illustrates graphically these proposed maximum radio noise-influence voltage levels for radio-free insulators for the various voltage classes.

The radio noise-influence voltage of an insulator is dependent upon the voltage across it. In actual service an insulator is subjected to the line-to-ground or operating voltage. Every transmission line has a "nominal" voltage rating which may be slightly different than the actual voltage. Furthermore, the actual voltage on a transmission line may be higher at the sending end than at the receiving end. To compensate for possible voltage variations from the nominal value, it seems quite logical to subject the insulator to a test voltage about 10 per cent higher than the nominal line-to-ground voltage when testing it in the EEI-NEMA-RMA circuit.

With a standard method for measuring the radio noise-influence voltage produced by an insulator and with a set of requirements which must be fulfilled by a "radio-free" insulator, the insulator manufacturer has at his disposal a radio-interference yardstick with which he can judge the merits of the insulators he manufactures.

Radio Noise-Influence Voltages of Various Types of Insulators

Having a definite yardstick for measuring the radio interference properties of insulators, a survey of available insulators of all types provides an interesting study. Briefly, such a study leads to the conclusion that certain types of insulators which have been manufactured for years require no special "radio-proofing" to meet the suggested requirements for freedom from radio interference; they are inherently quiet.

Conducting glazes provide effective "radio-proofing" on pin-type insulators. However, it does not necessarily follow that conducting glazes are required on all designs and classes or that they provide the only effective means of "radio-proofing" an insulator. For example, pin-type insulators which are "radio-proofed" with metallized coatings or cemented hardware are available.

SMALL ONE-PIECE PIN-TYPE INSULATORS

Small one-piece pin-type insulators are commonly used on low-voltage distribution circuits. In many cases these insulators are mounted on wood crossarms. This provides a section of wood in series with the insulator to ground or between phases. The voltage divides in proportion to the impedance of the insulator and that of the section of wood, thereby reducing the potential across the insulator from the operating voltage to a lower value. This means that the radio noise-influence voltage to ground from a given insulator will be lower with a section of wood in series with it than with the pin grounded.

A "radio-proof" power line is one which does not interfere with radio reception in nearby homes. It makes no difference how the "radio-proofing" is accomplished so long as the desired results are attained. Small "radio-proofed" pin-type insulators are available for even the lowest-voltage lines. However, before proceeding with the application of "radio-proofed" insulators to these voltage lines, it might be well to stop for a moment and consider the limitations of the old reliable "non-radio-proofed" insulators used for years on the lower-voltage lines.

Figure 2 shows the kilovolts across the insulator necessary to produce a radio noise-influence voltage of 50 microvolts on several different sizes of insulators. Since no two insulators of even the same size give exactly the same results on test, the bands in which all of the test values fall are given. With the pin grounded, the 50-microvolt level falls between 3 and 6 kv for all of the insulators.

Numerous tests were made with 12 inches of dry wood in series with the insulator and a wide range of kilovolt levels was found, ranging from about 12 to 21 kv. A single test was made on each size insulator with 6 inches of dry wood in series with the insulator and the kilovolt levels ranged from 11.5 to 13.5 kv.

Tests were made on one size insulator with 12 inches of very damp wood in series with it. Two samples of wood crossarm were used. These crossarms had been lying outside the laboratory all winter and accordingly were very wet inside, probably much more so than crossarms in service on poles. In spite of this, these crossarms added 2 or 3 kv to the 50-microvolt level voltage.

In any power line, the voltage across the insulator is determined by the operating voltage of the line. In a three-phase system, the voltage across the insulator is generally considered to be the line-to-line voltage divided by the $\sqrt{3}$. It is this voltage which may cause radio interference from the insulator. In figure 2 the line-to-neutral voltage is shown for several nominal operating voltages.

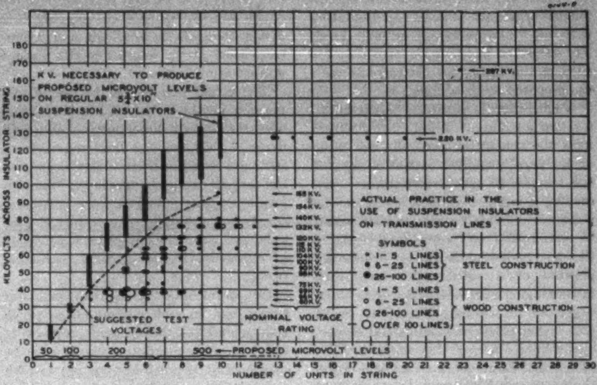


Figure 5. A comparison between the voltages applied to suspension insulator strings in actual practice and voltages necessary to produce the proposed microvolt levels

Up to and including 4,400-volt lines, not a single insulator would produce as much as 50 microvolts of radio interference even with the pins bonded. Accordingly, the ordinary plain insulator used for years is entirely adequate as regards radio interference and nothing can be gained by using "radio-proofed" insulators on 4,400-volt and lower-voltage power lines.

When there is 6 to 12 inches of dry crossarm in series with the insulator, the 50-microvolt level occurs at such a high voltage that even 13 and 17-kv lines are free from radio interference with the ordinary plain insulator. The actual improvement provided by the wood depends almost entirely upon its moisture content. Real wet wood, a condition existing for infrequent and short periods, may provide only a few kilovolts improvement. In figure 2, wet wood adds from 2 to 4 kv to the 50-microvolt level voltage, but this is enough to assure freedom from radio disturbance on 7.5-kv lines. Crossarms in service exposed to the air would probably never be as wet as the arms tested. Accordingly, it seems reasonable to expect that under average conditions 12 inches of wood in series with the insulator will assure freedom of radio interference from power lines operating at 7.5 kv to ground (13 kv, three phase) and below without the necessity of using "radio-proofed" insulators.

High-Voltage Pin-Type Insulators

The standard pin-type insulators used on 17 to 69-kv distribution lines have radio noise-influence voltages in the order of several thousand microvolts at a test voltage ten per cent above the nominal line-to-ground voltage for which the insulators are commonly used. Figure 3 shows the range of radio noise-influence voltages for five of the popular sizes of pin-type insulators. The proposed microvolt levels are also shown. Obviously, the conventional pin-type insulator cannot qualify as a radio-free pin-type insulator.

Dry wood crossarms in series with these insulators will reduce the noise level a marked degree but in general the reduction is not sufficient to approach the suggested microvolt levels. The only practical permanent solution for the radio-interference problem on the higher-voltage distribution lines is to use "radio-proofed" pin-type insulators which have been manufactured for the purpose. Several types of "radio-proofed" insulators which readily meet the suggested requirements are available. The use of conduction paint or other temporary means of suppression offer an opportunity for satisfactory treatment of existing lines where reinsulation by improved designs cannot be justified.

Switch and Bus Insulators

Generally accepted apparatus voltage ratings have been assigned to the various sizes of switch and bus insulators. The radio noise-influence voltages measured on the various switch and bus insulators at a 60-cycle test voltage about ten per cent above the voltage rating divided by the $\sqrt{3}$ are shown by the bands in figure

4. These are the results of test on the regular cemented pin-and-cap insulators with no "radio-proofing" of any kind. The suggested microvolt levels shown for each of the insulators corresponds with the proposed levels of figure 1.

A substation is generally restricted to a rather small area. Lines leading to and from the substation generally are more or less poorly coupled with it as regards the transfer of radio interference. Accordingly, higher microvolt levels may be permissible within the station than would be permissible on power lines running through residential districts. Other components of a substation which tend toward higher microvolt levels than those suggested for insulators seem to prove that higher levels can and should be permitted in substations. The ease with which switch and bus insulators can be brought to the suggested microvolt levels appears to be the only justification for these ultraconservative values being considered.

A study of figure 4 reveals the important fact that every one of the switch and bus insulators up to and including the 69-kv class, with the exception of the four-part single-unit 66-*S*-kv insulator, has a radio noise-influence voltage which is less than the suggested allowable value for "radio-free" insulators. This means that regular switch and bus insulators up to the 69-kv class, excepting the 66-*S*-class, do not need any special "radio-proofing" treatment to make them quiet; they are inherently so.

The 66-*S*-class insulator definitely requires a "radio-proofing" treatment if the suggested microvolt level is to be met. It has been successfully "radio-proofed" by slight revisions and its radio noise-influence voltage is well below the permissible 200 microvolts at a test voltage 10 per cent above the nominal line-to ground voltage. These revised designs are now the standard products of leading manufacturers.

The stacking insulators used in the 115-kv class and above give radio-influence voltages which exceed the proposed microvolt levels. To meet these suggested levels, slight revisions have been necessary to "radio-proof" these insulators in manufacture.

Suspension Insulators

No definite voltage ratings have been assigned to suspension insulators. In fact a survey of general practice over the entire country reveals an amazing lack of consistency in the application of various suspension string lengths to the various voltage classifications.

The actual practice in the use of suspension insulators on 612 transmission lines throughout the country is shown in figure 5. Of the 612 lines represented, 291 have wood in their construction and 321 are all-steel construction. This information was gathered from published charts and tabulations in technical journals, and, though incomplete, it probably is representative of existing construction.

By considering the shortest insulator string used in each voltage class, which also represents the highest voltage to which a given insulator string may be subjected, it is possible to assign to each string length a maximum microvolt level which is consistent with the proposed levels of figure 1. The proposed microvolt levels assigned to each string length are indicated at the bottom of figure 5.

The kilovolts necessary to produce the proposed microvolt levels for the different string lengths of regular cemented pin-and-cap suspension insulators of modern manufacture are shown by the bands in figure 5. These values are given only for string lengths up to ten units as this was the upper limit of testing equipment available at the time the tests were made. Higher-voltage equipment is now available.

In every case, the minimum kilovolts necessary to produce the proposed microvolt level is well above the highest voltage to which the given string would be subjected in service. In other words, at the highest operating voltage the radio noise-influence voltages from standard unaltered and untreated suspension insulators are less than the proposed microvolt levels for freedom from radio interference. Accordingly, it seems unnecessary to apply special treatment to standard cemented pin-and-cap suspension insulators; they are inherently "radio-free."

It is possible to assign to each string length a 60-cycle test voltage which represents the severest conditions to which the string may be subjected. To be consistent the test voltage assigned to each string length should bear some relation to other string lengths. One such set of assigned values is connected by a curve in figure 5. This curve is more or less the upper envelope of the points representing the suspension insulator applications. There seems to be no reason for placing this curve much higher as this would increase the severity of the radio test on suspension insulators and it would benefit no one.

In spite of the excellent radio characteristics of suspension insulators, conduction glaze has been applied both at the cap and in the pin hole. The cap treatment has increased the visual corona voltages materially. The treatment in the pin-hole area, however, has not proved as successful as at the cap, and in long strings the corona voltage at the pin of the line unit is in general lower than the corona voltage for the corresponding nontreated suspension insulators. The radio noise-influence voltages are generally about the same for the treated and nontreated insulators but there seems to be a tendency for the treated insulators to come lower. The failure of the conduction glaze treatment to provide the improvement found in its other applications is probably due to the configuration of the dielectric field in the pin hole and the inability of the conducting glaze, which must necessarily be applied only to the surface of the porcelain, to alter this field sufficiently to relieve the stress on the adjacent air.

On practically all lines, there are conductor and hardware members operating at high voltages, including clamps or other means for attaching the conductors to the insulators, reinforcing devices such as armor rods, vibration dampers, and similar special items. On a "radio-free" transmission line there must be no disturbances originating from the hardware. When such disturbances occur, they are caused by electrically over-stressed air adjacent to sharp corners or edges of the hardware. Such sources of noise generate a great amount of radio disturbances. Laboratory tests show that when such a disturbance starts, the radio noise-influence voltage jumps abruptly to several hundred microvolts and a small increase in the 60-cycle test voltage is accompanied by a big increase of the radio noise. It is essential in "radio-proofing" a transmission line to be sure that the hardware is adequately designed as well as the insulators.

Performance Calculations on Repulsion Motors

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Synopsis: A method of calculating the performance of repulsion motors is developed for the purpose of predetermining the torques and currents, especially for repulsion-start, induction-run motors during the starting period. The method is based on the cross-field theory presented by H. R. West. It is developed along similar lines to the single-phase calculation method presented by C. G. Veinott,[2] and is intended for use in connection with it. The paper presents also the results of investigation showing the effects of varying the several motor constants, and a method of determining the effect of brush resistance.

IN a paper presented a number of years ago, H. R. West developed methods of calculating the performance of single-phase induction and repulsion motors, based on the cross-field theory. In 1932, C. G. Veinott further developed the method for single-phase motors, from the point of view of routine calculation, and presented a calculation sheet for this purpose which is now in use in the engineering departments of many motor companies.

It is the purpose of this paper to develop the repulsion-motor method in the same manner, and to offer a calculation sheet of the same general form as that presented by Mr. Veinott. It has been developed particularly for use in conjunction with it in the design of repulsion-start, induction-run motors, but may, of course, be used for ordinary repulsion-run motors.

Repulsion-Motor-Performance Calculations

The circuits of the repulsion motor may be represented by figure 1. The development of the method consists in writing the equations of voltages in the three circuits of the motor in accordance with Kirchhoff's law, and solving these equations for the motor currents and torque. The problem then becomes one of algebraic transformation into a form most suitable for routine solution. This particular derivation has had several important objectives. The first, of course, is speed. Repulsion motors are usually fractional horsepower, and unless the calculation is quickly and easily made, it is more economical to rewind a sample motor once or twice. The second objective is simplicity and clarity. It often happens that repulsion-motor designs come at infrequent intervals in the course of designing many other types, usually split-phase or polyphase. It is desirable that the designer be able to proceed immediately into a calculation without any preliminary "brushing up" on the method. Another objective in this case is the possibility of calculating torques and speeds without calculating currents or inputs. This is desirable with repulsion-start, induction-run motors, in obtaining the starting and pull-up torques.

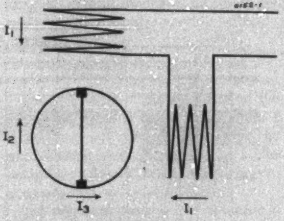

Figure 1. Schematic diagram of the repulsion motor

As will be shown later, one of the characteristics of the repulsion motor is that its starting torque is a maximum with no current in the coils short-circuited by the brushes. This would occur with infinite brush resistance. The method has been so arranged that a simple calculation with infinite brush resistance is easily made as a check on the loss of torque due to the short-circuit current.

All of the assumptions and details in the papers of Mr. West and Mr. Veinott hold for this method also. The angle of hysteretic lag between the flux and the magnetomotive force is assumed to be zero. Sinusoidal voltages and currents are assumed. The flux is assumed proportional to the current and magnetomotive force.

The method involves choosing a value of speed, and calculating the performance at that speed. The currents and torques are calculated independently of each other. Although it is possible by this method to obtain a vector solution for the primary current, and from the power component derive the watts input, it was felt as in the case of Mr. Veinott's calculation sheet that a more accurate value would be obtained by adding the losses to the output.

Due to the most frequent use of this method being at the lower speeds and greater currents, it was decided to subtract the iron loss from the output in the same manner as the friction loss.

There is one point in the calculation where it is assumed that the primary and secondary reactances are equal, that is, $x_1 = x_3$. This occurs in calculating the constant H_1,

for $x_1 \neq x_2$,

$$H_1 = \frac{r_1}{X_0}\left(1 + \frac{r_2}{r_3}\right) + \frac{r_2}{X_0}\left(\frac{X_m + x_1}{X_m + x_2}\right)$$

for $x_1 = x_2$,

$$H_1 = \frac{r_1}{X_0}\left(1 + \frac{r_2}{r_3}\right) + \frac{r_2}{X_0}$$

For all ordinary work the second formula is used, but if desired the first one may be followed. Figure 2 shows the small change in performance when the reactances are not equal.

Figure 12 shows the form of the calculation sheet and an illustrative example.

The Effect of Changing the Motor Constants

A number of calculations have been made showing the effect of changing the different constants of the motor. Figure 2 has already been mentioned showing the effect of transferring the reactance from stator to rotor or vice versa. Figure 3 shows the effect of increasing or decreasing the total leakage reactance. The improvement in a motor with increased numbers of slots is largely the result of decreased reactance.

The speed-torque curves of figure 4 show little change with variation of the open-circuit reactance, the greatest change being in the current and power factor. However, it will be noticed that as X_0 decreases, there is finally a reduc-

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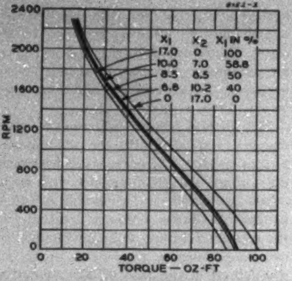

Figure 2. Effect of distribution of leakage reactance between primary and secondary

tion in torque, and it will be found that as the size of the motor decreases or the number of poles becomes greater, this effect will become quite prominent. This is the result of the increased primary impedance drop which reduces the voltage remaining to supply the air-gap flux.

No curves are given for the effect of changing the motor turns, for the result is the same as that obtained by varying the applied voltage, that is, the torques vary as the square of the voltage. Also, any change in the primary resistance has much the same effect, that is, the impedance drop varies and with it the remaining useful voltage.

Figure 5 showing the effect of secondary resistance is somewhat different than for the induction motor, for even at standstill the torque is decreased with increased rotor resistance.

Figure 6 illustrates several interesting points. As will be shown later, r_2 is the resistance of the useful part of the armature, which is made up of a number of coils in series with the brushes. r_3 is the resistance of the short circuits caused by the brushes, and is usually made up of one coil only, in series with two halves of a brush. The several short-circuited coils have the effect of operating in parallel. Figures 6 and 7 show very clearly that the higher r_3, the more desirable the speed-torque characteristics. Whereas, figure 5 shows that r_2 should be decreased for improvement. The problem then is one of increasing r_3 without at the same time increasing r_2. To do this by increasing the resistance of the brush itself is not advantageous beyond a certain point, for although r_3 may be increasing much faster than r_2, a point will be reached when any further increase in r_2 will result in too low pull-up torque. The usual method is to increase the number of coils and commutator bars, which makes the brush resistance a smaller percentage of r_2, and a greater percentage of r_3. It will be noted that r_3 has comparatively little effect on the torque at 75 per cent of synchronous speed, which is the usual operating speed of the short-circuiter, while effecting the starting torque almost directly.

A point of academic interest is obtained with r_2 very low and equal to r_3. Under these conditions, the motor has no starting torque, and operates exactly like a single-phase induction motor. This abnormal condition would be obtained with very wide low-resistance brushes, or with double the regular number of brushes, spaced 45 electrical degrees apart.

It might be noted at this point, that r_3 inherently varies with commutator position as the number of coils short-circuited by the brush changes. For instance, suppose the brush thickness equals one commutator bar. In this case, when the brush is exactly over the bar no coils are short-circuited, and r_3 is infinite. When the brush is touching two bars, there is one coil short-circuited under each brush. Thus, r_3 varies from some definite value to infinity, as the rotor passes through one commutator bar pitch. This is the usual case, but supposing the brush is a little wider and can touch three bars at some positions. In this case, r_3 will vary from the worst or lowest value of the narrow brush, to a value which is only half as much.

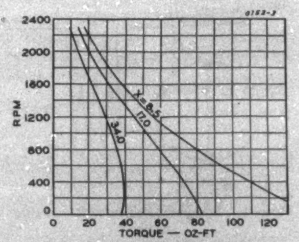

Figure 3. Effect of leakage reactance

The most outstanding variable in the repulsion motor is the brush shift. Figures 8 and 9 illustrate the effect on the starting and running torques, respectively. It would, of course, be desirable to operate at the point of maximum starting torque. However, the actual dimensions of the brush shift are not very large, particularly in four-, six-, and eight-pole motors, and to be safe under conditions of manufacturing variations, the brush shift is usually chosen on that side of the peak value corresponding to more shift. This side of the curve is less steep, and if the brush location is not exactly correct, the starting torque will not decrease as much.

Brush Resistance

There is no difficulty in calculating the primary resistance and the reactances of the repulsion motor, but it might be well to discuss the calculation of the secondary resistances r_2 and r_3. One of the difficulties in the design of repulsion motors has been the calculation of that part of r_2 and r_3 caused by the contact resistance of the brushes.

$$r_{2I}=\left[\frac{CKw_m}{2\,tpc\,Kw_{2I}}\right]^2\frac{r_c}{N_c} \tag{1}$$

$$r_2=\left[\frac{CKw_m}{2\,tpc\,Kw_{2R}N_c/a}\right]^2\frac{\dfrac{r_cN_c}{a}+\dfrac{r_b4}{A_b}}{a} \tag{2}$$

$$r_3=\left[\frac{CKw_m}{2\,tpc\,Kw_m}\right]^2\frac{r_c+\dfrac{r_b4}{A_b}}{a} \tag{3}$$

where

r_{2I} = rotor resistance referred to the primary when all commutator bars are short-circuited together, and operating as an induction motor
r_2 = rotor resistance as a repulsion motor
r_3 = rotor resistance of the short-circuited coils
r_c = resistance per coil of rotor
r_b = brush contact resistivity, see figure 10
CKw_m = total effective series conductors of primary
tpc = turns per coil in rotor
N_c = number of coils on rotor
a = number of parallel paths in rotor
Kw_{2I} = rotor winding factor (pitch times distribution factor) with commutator short-circuited

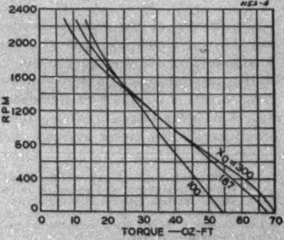

Figure 4. Effect of X_0, the "open-circuit reactance"

Kw_{2r} = rotor winding factor operating repulsion
Kw_3 = winding factor of short-circuited coils
A_b = brush area (minus mica area) in square inches

Equation 1 gives the value of rotor resistance r_{2i} to be used for calculation when the motor is operating as an induction motor with all the commutator bars short-circuited. Equation 2 should be used for the rotor resistance r_2 when operating repulsion, and equation 3 should be used for r_3. It can be shown that for uniform coil rotors, Kw_{2i} and Kw_{2r} are equal, and it may then be noted that when the brush resistance is equal to zero, r_{2i} equals r_2. It is, therefore, convenient to obtain the value r_2 from the formula given below.

$$\underset{\text{repulsion}}{r_2} = \underset{\text{induction}}{r_{2i}} \frac{R_A+R_B}{R_A} \qquad (4)$$

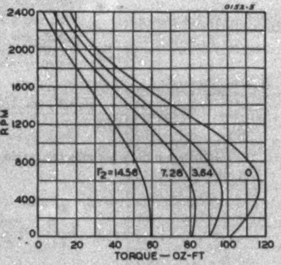

Figure 5. Effect of rotor resistance

where

$$R_A = \frac{r_c N_c}{a} \text{ and } R_B = \frac{r_b 4}{A_b}$$

The value for r_3 may be obtained in a similar manner.

$$r_3 = r_{2i}\left[\left(\frac{r_c+R_B}{r_c}\right)\frac{N_c}{a}\,\frac{DF_{2i}}{DF_3}\right] \qquad (5)$$

where

DF_{2i} = distribution factor when operating induction (approximately 0.637)
DF_3 = distribution factor for short-circuited coils (usually 1.0)

Figure 10 shows the effect of current density on the brush resistance. To be exact, it would be necessary to use a different value of r_3 corresponding to the current density, for each point of a calculated speed-torque curve. In practice, r_3 is usually high enough so that an average value may be used for the whole curve. It will be found that the brush current densities for most repulsion-start induction-run motors are very high reaching several hundred amperes per square inch during the starting period.

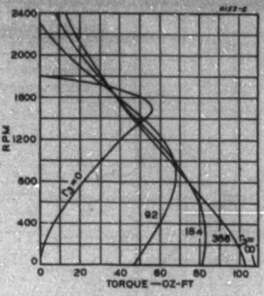

Figure 6. Effect of brush short-circuit resistance

Conclusions and Acknowledgments

The derivation of the equations of the calculation sheet is given in appendix I along with an example illustrating its use. Derivations relating to the motor constants and brush resistances are given in appendix II.

The brush resistance curve of figure 10 is from data furnished by the National Carbon Company.

Partial List of Symbols

(All secondary values are referred to the primary.)

E = applied voltage
I_1 = line or primary current
I_2 = secondary current in main field
I_3 = secondary current in cross field (brush short-circuit current)
r_1 = primary resistance
r_{2i} = rotor resistance as induction motor
r_2 = resistance of main field-rotor circuit
r_3 = resistance of cross-field rotor circuit (brush short-circuit resistance)
X_m = mutual or magnetizing reactance of stator and rotor windings
x_1 = leakage reactance of primary
x_2 = leakage reactance of each rotor circuit
f = frequency
S = speed as a fraction of synchronism
A = angle of brush shift in electrical degrees
Φ_t = flux in transformer axis (main field)
Φ_f = flux in field axis (cross field)

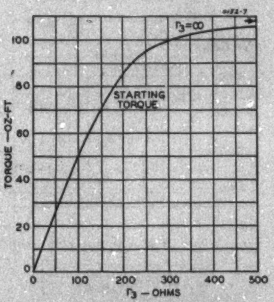

Figure 7. Effect of brush short-circuit resistance on starting torque

Appendix I

Derivation of Repulsion Motor Performance Equations

The circuits of a repulsion motor may be represented schematically by the diagram of figure 1. If A is the angle of brush shift in electrical degrees, the equations for the three circuits may be set up according to Kirchhoff's law. Mr. West[1] writes these equations as given below:

$$E = jX_m(I_1\cos^2 A - I_2\cos A) + jX_m(I_1\sin^2 A - I_3\sin A) + (r_1+jx_1)I_1$$

$$0 = -jX_m(I_1\cos A - I_2) + (r_2+jx_2)I_2 + SX_mI_1\sin A - S(X_m+x_2)I_3$$

$$0 = -jX_m(I_1\sin A - I_3) + (r_3+jx_2)I_3 - SX_m(I_1\cos A - I_2) + Sx_2I_2$$

Solving these equations for the three currents,

$$I_1 = E \times \frac{[r_2r_3 - (1-S^2)(X_m+x_2)^2 + j(r_2+r_3)(X_m+x_2)]}{U'+jW'}$$

$$I_2 = EX_m \times \frac{[-Sr_3\sin A - (1-S^2)(X_m+x_2)\cos A + jr_3\cos A]}{U'+jW'}$$

$$I_3 = EX_m \times \frac{[Sr_2\cos A - (1-S^2)(X_m+x_2)\sin A + jr_2\sin A]}{U'+jW'}$$

where

$$U' = -r_3X_m^2\cos^2 A - r_2X_m^2\sin^2 A - (r_2+r_3)(x_1+x_2)X_m - (r_2+r_3)x_1x_2 + r_1r_2r_3 - (1-S^2)r_1(X_m+x_2)^2$$

$$W' = r_1(r_2+r_3)(X_m+x_2) + r_2r_3(X_m+x_1) + S(r_3-r_2)X_m^2\sin A\cos A - (1-S^2)\times(X_m+x_2)[X_m(x_1+x_2)+x_1x_2]$$

Torque in synchronous watts =

$$\frac{E^2X_m^2}{U'^2+W'^2}\left\{(r_3^2-r_2^2)(X_m+x_2)\sin A\cos A + (r_3\sin^2 A + r_2\cos^2 A)[-Sr_2r_3 + S(1-S^2)\times(X_m+x_2)^2]\right\}$$

It is advisable, at this point, to divide the numerator and denominator of the current equations by $(X_m+x_2)^2$ and the torque equation by $(X_m+x_2)^4$.

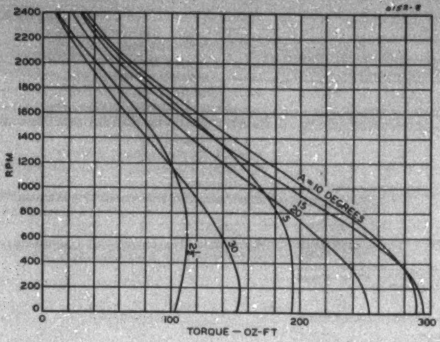

Figure 8. Effect of brush shift on speed-torque curves

A = Brush shift in electrical degrees

Then

$$I_1=E\frac{\dfrac{r_2r_3}{(X_m+x_2)^2}-\dfrac{(1-S^2)(X_m+x_2)^2}{(X_m+x_2)^2}+j\dfrac{(r_2+r_3)(X_m+x_2)}{(X_m+x_2)^2}}{U+jW}$$

$$I_2=\frac{EX_m}{(X_m+x_2)}\,\frac{\dfrac{-Sr_3\sin A}{X_m+x_2}-(1-S^2)\dfrac{X_m+x_2}{X_m+x_2}\times\cos A+j\dfrac{r_3}{X_m+x_2}\cos A}{U+jW}$$

$$I_3=\frac{EX_m}{(X_m+x_2)}\,\frac{\dfrac{Sr_3}{X_m+x_1}\cos A-(1-S^2)\times\dfrac{X_m+x_2}{X_m+x_2}\sin A+j\dfrac{r_3}{X_m+x_2}\sin A}{U+jW}$$

$$T=\frac{\dfrac{E^2X_m^2}{(X_m+x_2)^2}}{U^2+W^2}\left\{\frac{(r_2^2-r_3^2)(X_m+x_2)}{(X_m+x_2)^2}\times\sin A\cos A+[r_2\sin^2 A+r_3\cos^2 A]\times\left[-S\frac{r_2r_3}{(X_m+x_2)^2}+S(1-S^2)\frac{(X_m+x_2)^2}{(X_m+x_2)^2}\right]\right\}$$

where

$$U=-r_2\frac{X_m^2}{(X_m+x_2)^2}\cos^2 A-r_3\frac{X_m^2}{(X_m+x_2)^2}\times\sin^2 A-\frac{(r_2+r_3)(x_1+x_2)X_m}{(X_m+x_2)^2}-\frac{(r_2+r_3)x_1x_2}{(X_m+x_2)^2}+\frac{r_1r_2r_3}{(X_m+x_2)^2}-(1-S^2)r_1\frac{(X_m+x_2)^2}{(X_m+x_2)^2}$$

$$W=\frac{r_1(r_2+r_3)(X_m+x_2)}{(X_m+x_2)^2}+\frac{r_2r_3(X_m+x_1)}{(X_m+x_2)^2}+\frac{S(r_3+r_2)X_m^2\sin A\cos A}{(X_m+x_2)^2}-(1-S^2)\times\frac{(X_m+x_2)}{(X_m+x_2)^2}[X_m(x_1+x_2)+x_1x_2]$$

$$\Phi_f=\frac{\sqrt{2}I_2(X_m+x_2)}{2\pi fN10^{-8}}\left[\frac{X_m}{(X_m+x_2)}\frac{I_1}{I_2}\sin A-1\right]$$

$$\Phi_t=\frac{\sqrt{2}I_3(X_m+x_2)}{2\pi fN10^{-8}}\left[\frac{X_m}{(X_m+x_2)}\frac{I_1}{I_3}\cos A-1\right]$$

From this point on, we will depart from Mr. West's derivation, and method of attack. For simplicity of calculation, and so that the designer may know just how the motor losses are distributed, we will calculate the magnitudes alone of the currents, and obtain the input from the sum of output and losses. In order to simplify the above equations, let us arbitrarily define

$$X_0=X_m+x_2$$

$$K_p=\frac{X_m}{X_m+x_2}=\frac{X_m}{X_0}$$

$$K_r=K_p^2=\frac{X_m^2}{X_0^2}=\frac{X_m^2}{(X_m+x_2)^2}$$

$$I_1=E\frac{\dfrac{r_2}{X_0}\dfrac{r_3}{X_0}-(1-S^2)+j\dfrac{(r_2+r_3)}{X_0}}{U+jW}$$

$$I_2=EK_p\,\frac{-S\dfrac{r_3}{X_0}\sin A-(1-S^2)\cos A+j\dfrac{r_3}{X_0}\cos A}{U+jW}$$

$$I_3=EK_p\,\frac{S\dfrac{r_3}{X_0}\cos A-(1-S^2)\sin A+j\dfrac{r_3}{X_0}\sin A}{U+jW}$$

$$T=\overline{EK_p}^2\left\{\frac{\dfrac{(\sqrt{r_3^2-r_2^2})^2}{X_0}\sin A\cos A+[r_2\sin^2 A+r_3\cos^2 A]\times\left[-S\dfrac{r_2r_3}{X_0X_0}+S(1-S^2)\right]}{U^2+W^2}\right\}$$

$$U=-r_2K_r\cos^2 A-r_3K_r\sin^2 A-\frac{(r_2+r_3}{X_0}\left[\frac{x_1X_m+x_2X_m+x_1x_2}{(X_m+x_2)}\right]+\frac{r_1r_2r_3}{X_0X_0}-(1-S^2)r_1$$

$$W=\frac{r_1(r_2+r_3)}{X_0}+\frac{r_2r_3}{X_0}\frac{(X_m+x_1)}{(X_m+x_2)}+S(r_3-r_2)K_r\sin A\cos A-(1-S^2)\left[\frac{x_1X_m+x_2X_m+x_1x_2}{X_m+x_2}\right]$$

$$\Phi_{f\max}=I_2X_0\left(\frac{I_1}{I_2}K_p\sin A-1\right)\frac{45\times10^6}{fCK_w}$$

$$\Phi_{t\max}=I_3X_0\left(\frac{I_1}{I_3}K_p\cos A-1\right)\frac{45\times10^6}{fCK_w}$$

(It must be remembered that the currents are vectors in these equations.)

Let us now define

$$i_m=\frac{E}{X_0}$$

$$X=x_1+x_2K_p=\frac{x_1X_m+x_2X_m+x_1x_2}{X_m+x_2}$$

Then

$$I_1=\frac{i_mr_2\dfrac{r_3}{X_0}-E(1-S^2)+ji_m(r_2+r_3)}{U+jW}$$

$$I_2=\frac{-Si_mr_3K_p\sin A-(1-S^2)EK_p\cos A+ji_mr_3K_p\cos A}{U+jW}$$

$$I_3=\frac{Si_mr_3K_p\cos A-(1-S^2)EK_p\sin A+ji_mr_3K_p\sin A}{U+jW}$$

The torque equation is not affected by these last simplifications.

$$U=-(1-S^2)r_1-r_2K_r\cos^2 A-r_3K_r\sin^2 A-\frac{r_2+r_3}{X_0}X+\frac{r_1r_2r_3}{X_0^2}$$

$$W=-(1-S^2)X+S(r_3-r_2)K_r\sin A\cos A+r_1\frac{r_2+r_3}{X_0}+\frac{r_2r_3(X_m+x_1)}{X_0(X_m+x_2)}$$

In motors of normal proportions, r_3 is very large, and often as a check, a calculation is made with r_3 increased to infinity. This is a convenient way to tell whether the brush resistance is as large as it ought to be. Let us divide both numerator and denominator of the current equations by $-r_3$ and the torque equation by $(-r_3)^2$. This will cause the terms affected by the useless brush short-circuit current, to become smaller and smaller as r_3 approaches infinity.

$$U=(1-S^2)\frac{r_1}{r_3}+\frac{r_2}{r_3}K_r\cos^2 A+K_r\sin^2 A+\frac{X}{X_0}\left(1+\frac{r_2}{r_3}\right)-\frac{r_1r_2}{X_0X_0}$$

$$W=(1-S^2)\frac{X}{r_3}-S\left(1-\frac{r_2}{r_3}\right)K_r\sin A\cos A-\frac{r_1}{X_0}\left(1+\frac{r_2}{r_3}\right)-\frac{r_2}{X_0}\frac{(X_m+x_1)}{(X_m+x_2)}$$

$$I_1=\frac{(1-S^2)\dfrac{E}{r_3}-\dfrac{i_mr_2}{X_0}-ji_m\left(1+\dfrac{r_2}{r_3}\right)}{U+jW}$$

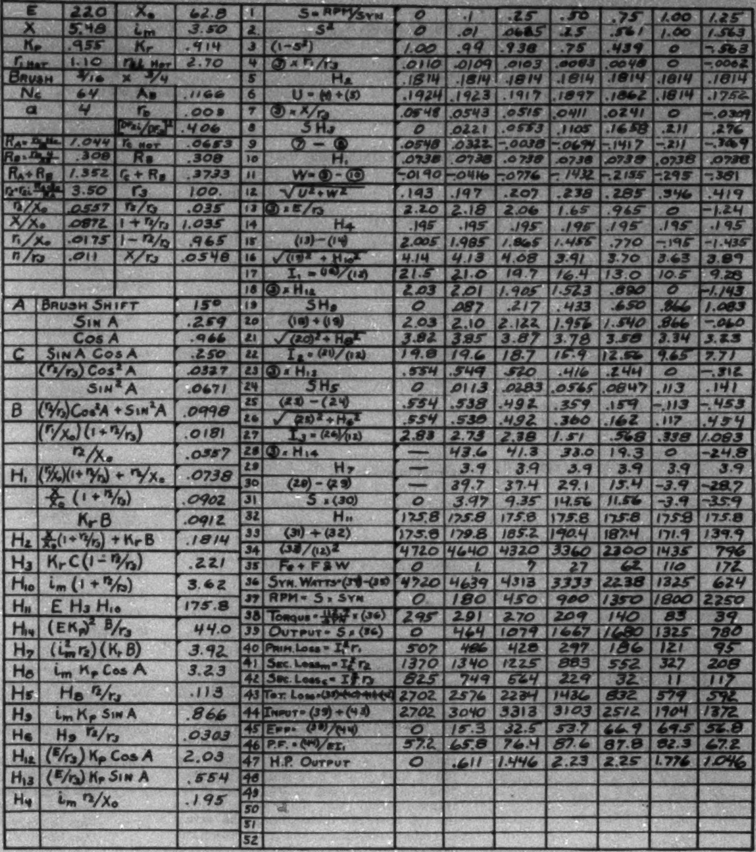

Figure 12. Calculation sheet for repulsion motors

E	220	X_0	62.8
X	5.48	i_m	3.50
K_p	.955	K_r	.914
r_1 HOT	1.10	r_{2L} HOT	2.70
BRUSH	3/16	× 3/4	
N_c	64	A_B	.1166
a	4	r_b	.008
		$[D_{F2i}/D_{F2}]^2$	.406
$R_A =$ [illegible]	1.044	r_c HOT	.0653
$R_B =$ [illegible]	.308	R_B	.308
$R_A + R_B$	1.352	$r_c + R_B$	.3733
$r_2 = r_{2i}$ [illegible]	3.50	r_3	100.
r_2/X_0	.0557	r_2/r_3	.035
X/X_0	.0872	$1 + r_2/r_3$	1.035
r_1/X_0	.0175	$1 - r_2/r_3$	.965
r_1/r_3	.011	X/r_3	.0548

A	BRUSH SHIFT	15°
	SIN A	.259
	COS A	.966
C	SIN A COS A	.250
	$(r_2/r_3)\cos^2 A$	.0327
	$\sin^2 A$	.0671
B	$(r_2/r_3)\cos^2 A + \sin^2 A$	.0998
	$(r_1/X_0)(1 + r_2/r_3)$	.0181
	r_2/X_0	.0557
H_1	$(r_1/X_0)(1 + r_2/r_3) + r_2/X_0$	.0738
	$\frac{X}{X_0}(1 + r_2/r_3)$	.0902
	$K_r B$	.0912
H_2	$\frac{X}{X_0}(1 + r_2/r_3) + K_r B$	.1814
H_3	$K_r C(1 - r_2/r_3)$	.221
H_{10}	$i_m(1 + r_2/r_3)$	3.62
H_{11}	$E H_3 H_{10}$	175.8
H_{14}	$(E K_p)^2 B/r_3$	44.0
H_7	$(i_m^2 r_2)(K_r B)$	3.92
H_8	$i_m K_p \cos A$	3.23
H_5	$H_8 r_2/r_3$	.113
H_9	$i_m K_p \sin A$	.866
H_6	$H_9 r_2/r_3$	.0303
H_{12}	$(E/r_3) K_p \cos A$	2.03
H_{13}	$(E/r_3) K_p \sin A$	.554
H_4	$i_m r_2/X_0$	.195

1	S = RPM/SYN	0	.1	.25	.50	.75	1.00	1.25
2	S^2	0	.01	.0625	.25	.561	1.00	1.563
3	$(1-S^2)$	1.00	.99	.938	.75	.439	0	-.563
4	③ × r_1/r_3	.0110	.0109	.0103	.0083	.0048	0	-.0062
5	H_2	.1814	.1814	.1814	.1814	.1814	.1814	.1814
6	U = (4) + (5)	.1924	.1923	.1917	.1897	.1862	.1814	.1752
7	③ × X/r_3	.0548	.0543	.0515	.0411	.0241	0	-.0309
8	$S H_3$	0	.0221	.0553	.1105	.1658	.211	.276
9	⑦ − ⑧	.0548	.0322	-.0038	-.0694	-.1417	-.211	-.369
10	H_1	.0738	.0738	.0738	.0738	.0738	.0738	.0738
11	W = ⑨ − ⑩	-.0190	-.0416	-.0776	-.1432	-.2155	-.295	-.381
12	$\sqrt{U^2 + W^2}$	.193	.197	.207	.238	.285	.346	.419
13	③ × E/r_3	2.20	2.18	2.06	1.65	.965	0	-1.24
14	H_4	.195	.195	.195	.195	.195	.195	.195
15	(13) − (14)	2.005	1.985	1.865	1.455	.770	-.195	-1.435
16	$\sqrt{(15)^2 + H_{10}^2}$	4.14	4.13	4.08	3.91	3.70	3.63	3.89
17	$I_1 = (16)/(12)$	21.5	21.0	19.7	16.4	13.0	10.5	9.28
18	③ × H_{12}	2.03	2.01	1.905	1.523	.890	0	-1.143
19	$S H_9$	0	.087	.217	.433	.650	.866	1.083
20	(18) + (19)	2.03	2.10	2.122	1.956	1.540	.866	-.060
21	$\sqrt{(20)^2 + H_8^2}$	3.82	3.85	3.87	3.78	3.58	3.34	3.23
22	$I_2 = (21)/(12)$	19.8	19.6	18.7	15.9	12.56	9.65	7.71
23	③ × H_{13}	.554	.549	.520	.416	.244	0	-.312
24	$S H_5$	0	.0113	.0283	.0565	.0847	.113	.141
25	(23) − (24)	.554	.538	.492	.359	.159	-.113	-.453
26	$\sqrt{(25)^2 + H_6^2}$	.554	.538	.492	.360	.162	.117	.454
27	$I_3 = (26)/(12)$	2.83	2.73	2.38	1.51	.568	.338	1.083
28	③ × H_{14}	—	43.6	41.3	33.0	19.3	0	-24.8
29	H_7	—	3.9	3.9	3.9	3.9	3.9	3.9
30	(28) − (29)	—	39.7	37.4	29.1	15.4	-3.9	-28.7
31	S × (30)	0	3.97	9.35	14.56	11.56	-3.9	-35.9
32	H_{11}	175.8	175.8	175.8	175.8	175.8	175.8	175.8
33	(31) + (32)	175.8	179.8	185.2	190.4	187.4	171.9	139.9
34	$(33)/(12)^2$	4720	4640	4320	3360	2300	1435	796
35	Fe + F & W	0	1.	7	27	62	110	172
36	SYN. WATTS = (34) − (35)	4720	4639	4313	3333	2238	1325	624
37	RPM = S × SYN	0	180	450	900	1350	1800	2250
38	TORQUE = $\frac{112.7}{SYN}$ × (36)	295	291	270	209	140	83	39
39	OUTPUT = S × (36)	0	464	1079	1667	1680	1325	780
40	PRIM. LOSS = $I_1^2 r_1$	507	486	428	297	186	121	95
41	SEC. $\text{LOSS}_m = I_2^2 r_2$	1370	1340	1225	883	552	327	208
42	SEC. $\text{LOSS}_c = I_3^2 r_3$	825	749	564	229	32	11	117
43	TOT. LOSS = (35)+(40)+(41)+(42)	2702	2576	2234	1436	832	579	592
44	INPUT = (39) + (43)	2702	3040	3313	3103	2512	1904	1372
45	EFF. = (39)/(44)	0	15.3	32.5	53.7	66.9	69.5	56.8
46	P.F. = $(44)/EI_1$	57.2	65.8	76.4	87.6	87.8	82.3	67.2
47	H.P. OUTPUT	0	.611	1.446	2.23	2.25	1.776	1.046
48								
49								
50								
51								
52								

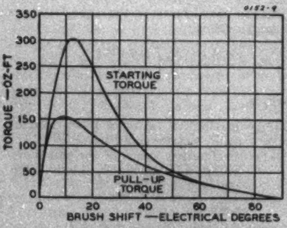

Figure 9. Effect of brush shift on starting and pull-up torques

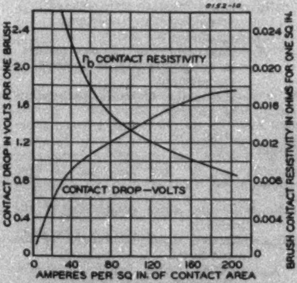

Figure 10. Typical brush resistance and volt-ampere curves

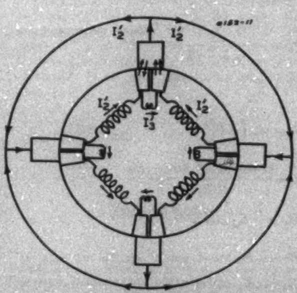

Figure 11. Schematic diagram of secondary circuit of repulsion motor

$$I_2=\frac{(1-S^2)\dfrac{E}{r_3}K_p\cos A+Si_mK_p\sin A-ji_mK_p\cos A}{U+jW}$$

$$I_3=\frac{(1-S^2)\dfrac{E}{r_3}K_p\sin A-Si_m\dfrac{r_2}{r_3}K_p\cos A-ji_m\dfrac{r_2}{r_3}K_p\sin A}{U+jW}$$

$$T=\frac{S\left\{(1-S^2)\left[\dfrac{\overline{EK_p}^2}{r_3}\left(\dfrac{r_2}{r_3}\cos^2 A+\sin^2 A\right)\right]-\left(\dfrac{EK_p}{X_0}\right)^2 r_2\left(\dfrac{r_2}{r_3}\cos^2 A+\sin^2 A\right)\right\}}{U^2+W^2}+\frac{\dfrac{\overline{EK_p}^2}{X_0}\sin A\cos A\left[1-\left(\dfrac{r_2}{r_3}\right)^2\right]}{U^2+W^2}$$

For purposes of calculation, let us arrange the terms in each equation into groups, those terms which are a function of speed in one group, and those which are independent of speed in another group.

$$U=(1-S^2)\frac{r_1}{r_3}+H_2$$

where

$$H_2=K_rB+\frac{X}{X_0}\left(1+\frac{r_2}{r_3}\right)-\frac{r_1r_2}{X_0X_0}$$

(This last term $\frac{r_1r_2}{X_0X_0}$ is usually negligible)

$$B=\frac{r_2}{r_3}\cos^2 A+\sin^2 A$$

$$W=(1-S^2)\frac{X}{r_3}-SH-H_1$$

$$H_3=K_r\left(1-\frac{r_2}{r_3}\right)C$$

$$C=\sin A\cos A$$

$$H_1=\frac{r_1}{X_0}\left(1+\frac{r_2}{r_3}\right)+\frac{r_2}{X_0}\frac{(X_m+x_1)}{(X_m+x_2)}$$

(The term $\frac{(X_m+x_1)}{(X_m+x_2)}$ can usually be taken as unity)

$$I_1=\frac{(1-S^2)\dfrac{E}{r_3}-H_4-jH_{10}}{U+jW}$$

$$H_4=\frac{i_mr_2}{X_0}$$

$$H_{10}=i_m\left(1+\frac{r_2}{r_3}\right)$$

$$I_2=\frac{(1-S^2)H_{12}+SH_8-jH_9}{U+jW}$$

$$H_{12}=\frac{E}{r_3}K_p\cos A$$

$$H_8=i_mK_p\sin A$$

$$H_9=i_mK_p\cos A$$

$$I_3=\frac{(1-S^2)H_{13}-SH_5-jH_6}{U+jW}$$

$$H_{13}=\frac{E}{r_3}K_p\sin A$$

$$H_5=i_m\frac{r_2}{r_3}K_p\cos A$$

$$H_6=i_m\frac{r_2}{r_3}K_p\sin A$$

Torque in synchronous watts =

$$\frac{S[(1-S^2)H_{14}-H_7]+H_{11}}{U^2+W^2}$$

$$H_7=i_m^2r_2K_rB$$

$$H_{11}=EH_{10}H_8$$

$$H_{14}=\overline{EK_p}^2\frac{B}{r_3}$$

Output = S times torque in synchronous watts

Torque in ounce-feet = $\frac{112.7}{\text{synchronous rpm}}$ times torque in synchronous watts

By arranging the calculation of these terms in proper sequence, a simple and straightforward calculation sheet may be obtained. Such a calculation sheet is shown in figure 12.

Appendix II

Derivation of the Equations of Rotor Resistance

The secondary resistance of the armature of a repulsion motor must be calculated for two separate conditions when operating repulsion, and for a third condition when operating induction in a repulsion-start, induction-run motor. In all cases, the problem is to obtain the effective secondary resistance, referred to the primary, of a motor which for this problem may be considered as a transformer. Since the resistances of a transformer may be transferred from primary to secondary or vice versa by multiplying by the square of the transformation ratio, the problem becomes one of correctly determining the actual secondary resistance, and the transformation ratio.

Let

"ratio" = transformation ratio squared
r_s = actual secondary resistance
r_c = resistance per coil
N_c = total number of coils on armature
r_2 = secondary resistance referred to the primary
a = the number of parallel paths in the secondary

$$\text{"ratio"}=\frac{(CK_{wm})^2}{(CK_{ws})^2}$$

where

CK_{wm} = effective series conductors of the primary
CK_{ws} = effective series conductors of the secondary
K_w = winding factor (pitch times distribution factor)

Then

r_2 = "ratio" times r_s

CASE I

Repulsion-start, induction-run motor operating induction with all bars of the commutator short-circuited.

$r_{s1}=r_c/N_c$
$CK_{ws}=2\,tpc\,K_{ws1}$ (K_{ws1} is usually close to 0.637)
tpc = turns per coil on the armature

CASE II

Normal repulsion motor, r_2 = resistance of main phase or useful part of the armature.

$$r_{s2}=\frac{\dfrac{r_cN_c}{a}+\dfrac{r_b4}{A_b}}{a}$$

where

N_c/a = number of coils between brushes
r_b = resistance per square inch of brush contact
A_b = brush contact area in square inches (minus that area of the brush in contact with mica)
$CK_{ws2}=\dfrac{2\,tpc\,N_cK_{ws2}}{a}$ (K_{ws2} is usually close to 0.637)

It can be shown that if r_b equal zero, $r_{s1}=r_{s2}$. Thus

$$r_2=r_{s1}\frac{R_A+R_B}{R_A}$$

Where

$$R_A=\frac{r_cN_c}{a}\text{ and }R_B=\frac{r_b4}{A_b}$$

CASE III

Normal repulsion motor, r_3 = secondary resistance of the short-circuited coils.

$$r_{s3}=\frac{r_c+\dfrac{r_b4}{A_b}}{a}$$

$CK_{ws3}=2\,tpc\,K_{ws3}$ (K_{ws3} is usually equal to the armature chord factor, for with the brush spanning two bars only, and the one coil connected to these bars in one slot only, the distribution factor will be unity)

A convenient method calculating r_3 is to write the formula in terms of r_{s1}

$$r_3=r_{s1}\frac{(K_{ws1})^2}{(K_{ws3})^2}\frac{r_c+R_B}{r_c}\frac{N_c}{a}$$

But the chord factor is the same for either case, therefore

$$r_3=r_{s1}\left\{\frac{r_c+R_B}{r_c}\frac{N_c}{a}\left[\frac{DF_{s1}}{DF_3}\right]^2\right\}$$

where

DF_{s1} = distribution factor of short-circuited rotor, usually about 0.637
DF_3 = distribution factor of the coil short-circuited by the brush, usually unity

$$DF=\frac{\sin n\theta/2}{n\sin\theta/2}$$

where

n = number of coils
θ = angle between coils in electrical degrees

Performance Calculations on Capacitor Motors

The Revolving-Field Theory

P. H. TRICKEY
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Synopsis: A method is presented whereby the equations of capacitor motor performance calculation as developed by Mr. W. J. Morrill,[1] are arranged in a calculation sheet or form for use in routine design work.

IN 1929, W. J. Morrill presented his classic paper on "The Revolving Field Theory of the Capacitor Motor", giving the complete equations for calculating the performance and torque of a capacitor motor at any value of slip.

The equations are straightforward, but involve vector operations, and are rather slow for routine design work. Since 1929, many engineers have used the method, and have devised different methods of doing the many numerical operations necessary to obtain a final solution. The methods have varied from one extreme to the other, some engineers using the equations directly as written making each vector operation as necessary, and others working up a calculation sheet where all operations were numerical.

Although engineers as a rule are perfectly capable of handling vector operations, most of those involved in motor design work, feel that any great number of vector operations in the midst of routine work, brings considerable risk of errors, and they like to avoid vectors where possible. The writer has developed a calculation sheet for using these formulas, where all the operations except the last in calculating the currents are performed without vectors. This seems to be a fairly satisfactory compromise between the difficulties of either extreme, and has worked out quite well in practice.

Calculation Sheet for Mr. Morrill's Equations

Figures 1 and 2 show pages 1 and 2 of the calculation method. As in Mr. Morrill's paper, the method consists of determining the currents in each branch circuit and their angles with the voltage. With these values, the torque, output, and losses may be determined. Figure 1 is taken up with the calculation of currents, and figure 2 with calculation of torque and performance. The first part of figure 1 covers the calculation of the apparent resistance and reactance of the secondary circuit to the forward and backward fields.

It will be noted that many of the terms and expressions in the equations are of very similar form. Advantage has been taken of this to arrange the sheet so that the operations are performed in two columns in such a way that the numbering of the operations is nearly the same. That is, operations 1, 2, 3, 4, etc. are exactly the same in form as operations 101, 102, 103, and 104. Upon reaching operation 13 and 113, it becomes necessary to cross from one column to the other occasionally, but the same essential similarity between the two columns is kept.

Iron Loss

A slightly different method of accounting for iron loss is used than that of Mr. Morrill. Mr. Morrill's equations neglect iron loss and then correct for it by adding it to the friction and windage losses as a reduction in output.

The writer has found that a closer comparison between calculated values and tested values may be obtained, if a slightly different method of iron loss correction is used. As with Mr. Morrill, the vector operations of items 35, and 37 give fictitious values of current which have been obtained while neglecting iron loss entirely. These fictitious values are used in calculating the internal torque and output. The friction, windage, and odd-frequency iron losses are then sub-

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Brush Contact Resistance

The equations for secondary resistance contain a term which is a function of the contact drop at the brush and commutator. The resistance of the contact is determined by a number of things. First, the type of brush, second the current density, and in addition to these factors it is affected by the speed and temperature of the commutator. A typical brush resistance curve as a function of current density is shown in figure 10. It is furnished through the courtesy of the National Carbon Company. It will be noted that the curve has been extended beyond the conditions of normal continuous operation. This is done because motors designed for repulsion operation during the starting period only, have small brushes, and abnormally high current densities. These densities are likely to be between 100 and 300 amperes per square inch at the standstill condition.

As mentioned above, in order to be exact, it would be necessary to use a different value of brush resistance for each point of a calculated speed-torque curve. However, for practical cases, it is usually sufficiently accurate, to use the starting current density throughout the entire range of the caclulation.

Actual Secondary Currents

Figure 11 illustrates the conditions occurring at the brush, with the useful currents entering the brush from adjacent commutator bars, and the short-circuit current entering from one bar and leaving by way of the next bar. It may happen in some cases, that the short-circuit current is even greater than the useful current. For convenience, the actual values of the currents are given below.

$$I_2' = I_2 \frac{CK_{wm}}{CK_{w2}} \frac{1}{a}$$

$$I_3' = I_3 \frac{CK_{wm}}{CK_{w3}} \frac{1}{a}$$

Reactance Voltage

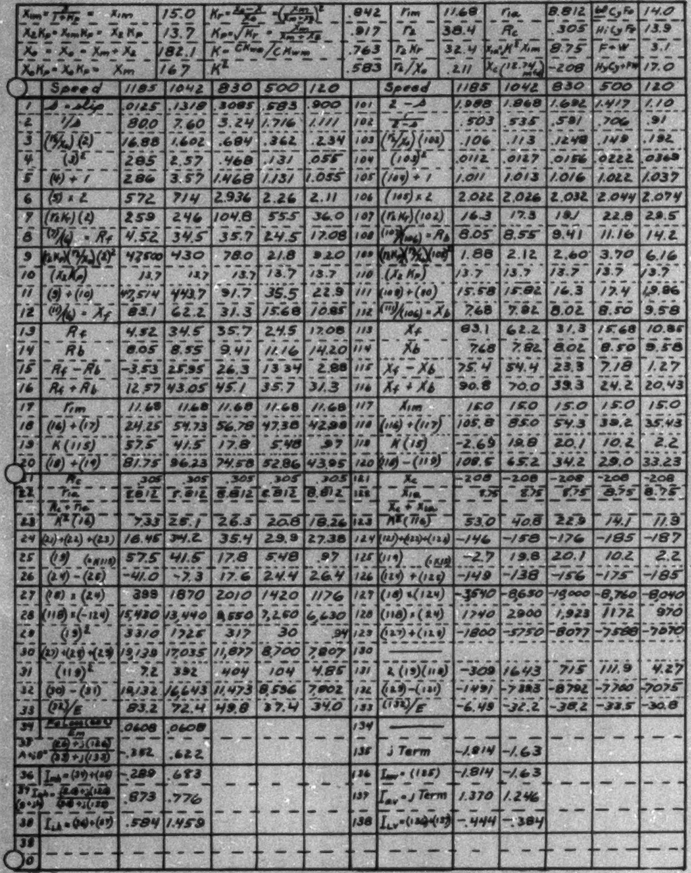
Since r_3 is so much greater than the reactance of the short-circuited coils, the commutation voltage referred to the primary is simply $I_3 r_3$. However, the actual voltage commutated by the brush must be obtained from the transformation ratio, that is,

$$E_c = I_3 r_3 \frac{2tpcK_{w3}a}{CK_{wm}}$$

Bibliography

1. The Cross-Field Theory of Alternating-Current Machines, H. R. West. AIEE Transactions, February 1926, page 466.

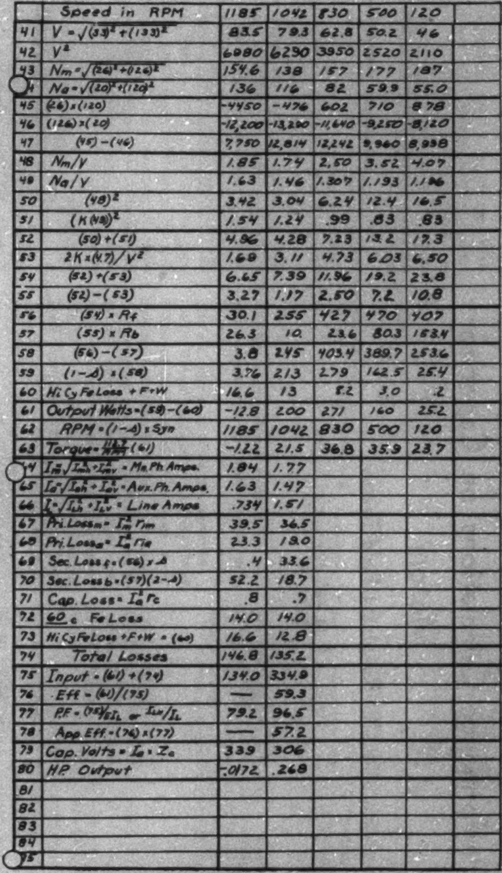
2. Performance Calculations on Induction Motors, C. G. Veinott. AIEE Transactions, September 1932, page 743.



$X_{im}=\frac{X}{1+K_p}=X_{im}$	15.0	$K_r=\frac{X_0-X}{X_0}=\left(\frac{X_m}{X_m+X_2}\right)^2$	.842	r_{im}	11.68	r_{ia}	8.812	60 Cy Fe	14.0
$X_2K_p=X_{im}K_p=X_2K_p$	13.7	$K_p=\sqrt{K_r}=\frac{X_m}{X_m+X_2}$	.917	r_2	38.4	R_c	.305	Hi Cy Fe	13.9
$X_0=X_0=X_m+X_2$	182.1	$K=CK_{wa}/CK_{wm}$	.763	r_2K_r	32.4	$X_{ia}=K^2X_{im}$	8.75	F+W	3.1
$X_0K_p=X_0K_p=X_m$	167	K^2	.583	r_2/X_0	.211	$X_c\left(\frac{12.74}{mfd}\right)$	-208	HyCy+FW	17.0

	Speed	1185	1042	830	500	120		Speed	1185	1042	830	500	120
1	s = slip	.0125	.1318	.3085	.583	.900	101	$2-s$	1.988	1.868	1.692	1.417	1.10
2	$1/s$	80.0	7.60	3.24	1.716	1.111	102	$\frac{1}{2-s}$	.503	.535	.581	.706	.91
3	$(r_2/X_0)(2)$	16.88	1.602	.684	.362	.234	103	$(r_2/X_0)(102)$	.106	.113	.1248	.149	.192
4	$(3)^2$	285	2.57	.468	.131	.055	104	$(103)^2$	.0112	.0127	.0156	.0222	.0369
5	$(4)+1$	286	3.57	1.468	1.131	1.055	105	$(104)+1$	1.011	1.013	1.016	1.022	1.037
6	$(5)\times 2$	572	714	2.936	2.26	2.11	106	$(105)\times 2$	2.022	2.026	2.032	2.044	2.074
7	$(r_2K_r)(2)$	259	246	104.8	55.5	36.0	107	$(r_2K_r)(102)$	16.3	17.3	18.1	22.8	29.5
8	$(7)/(6)=R_f$	4.52	34.5	35.7	24.5	17.08	108	$(107)/(106)=R_b$	8.05	8.55	9.41	11.16	14.2
9	$(r_2K_r)(r_2/X_0)(2)^2$	47500	430	78.0	21.8	9.20	109	$(r_2K_r)(r_2/X_0)(102)^2$	1.88	2.12	2.60	3.70	6.16
10	(X_2K_p)	13.7	13.7	13.7	13.7	13.7	110	(X_2K_p)	13.7	13.7	13.7	13.7	13.7
11	$(9)+(10)$	47,514	443.7	91.7	35.5	22.9	111	$(109)+(110)$	15.58	15.82	16.3	17.4	19.86
12	$(11)/(6)=X_f$	83.1	62.2	31.3	15.68	10.85	112	$(111)/(106)=X_b$	7.68	7.82	8.02	8.50	9.58
13	R_f	4.52	34.5	35.7	24.5	17.08	113	X_f	83.1	62.2	31.3	15.68	10.85
14	R_b	8.05	8.55	9.41	11.16	14.20	114	X_b	7.68	7.82	8.02	8.50	9.58
15	R_f-R_b	-3.53	25.95	26.3	13.34	2.88	115	X_f-X_b	75.4	54.4	23.3	7.18	1.27
16	R_f+R_b	12.57	43.05	45.1	35.7	31.3	116	X_f+X_b	90.8	70.0	39.3	24.2	20.43
17	r_{im}	11.68	11.68	11.68	11.68	11.68	117	X_{im}	15.0	15.0	15.0	15.0	15.0
18	$(16)+(17)$	24.25	54.73	56.78	47.38	42.98	118	$(116)+(117)$	105.8	85.0	54.3	39.2	35.43
19	$K(115)$	57.5	41.5	17.8	5.48	.97	119	$K(15)$	-2.69	19.8	20.1	10.2	2.2
20	$(18)+(19)$	81.75	96.23	74.58	52.86	43.95	120	$(118)-(119)$	108.5	65.2	34.2	29.0	33.23
21	R_c	.305	.305	.305	.305	.305	121	X_c	-208	-208	-208	-208	-208
22	r_{ia} R_c+r_{ia}	8.812	8.812	8.812	8.812	8.812	122	X_{ia} X_c+X_{ia}	8.75	8.75	8.75	8.75	8.75
23	$K^2(16)$	7.33	25.1	26.3	20.8	18.26	123	$K^2(116)$	53.0	40.8	22.9	14.1	11.9
24	$(21)+(22)+(23)$	16.45	34.2	35.4	29.9	27.38	124	$(121)+(122)+(123)$	-146	-158	-176	-185	-187
25	(19) $(=K115)$	57.5	41.5	17.8	5.48	.97	125	(119) $(=K15)$	-2.7	19.8	20.1	10.2	2.2
26	$(24)-(25)$	-41.0	-7.3	17.6	24.4	26.4	126	$(124)+(125)$	-149	-138	-156	-175	-185
27	$(18)\times(24)$	399	1870	2010	1420	1176	127	$(18)\times(124)$	-3540	-8,650	-10,000	-8,760	-8,040
28	$(118)\times(-124)$	15,430	13,440	9,550	7,260	6,630	128	$(118)\times(24)$	1740	2900	1,923	1172	970
29	$(19)^2$	3310	1725	317	30	.94	129	$(127)+(128)$	-1800	-5750	-8077	-7588	-7070
30	$(27)+(28)+(29)$	19,139	17,035	11,877	8,700	7,807	130	——					
31	$(119)^2$	7.2	392	404	104	4.85	131	$2(19)(119)$	-309	1643	715	111.9	4.27
32	$(30)-(31)$	19,132	16,643	11,473	8,596	7,802	132	$(129)-(131)$	-1491	-7393	-8792	-7700	-7075
33	$(32)/E$	83.2	72.4	49.8	37.4	34.0	133	$(132)/E$	-6.49	-32.2	-38.2	-33.5	-30.8
34	$\frac{\text{Fe Loss}(60\sim)}{E_m}$	.0608	.0608				134	——					
35	$A+jB=\frac{(26)+j(126)}{(33)+j(133)}$	-.352	.622				135	j Term	-1.814	-1.63			
36	$I_{mh}=(34)+(35)$	-.289	.683				136	$I_{mv}=(135)$	-1.814	-1.63			
37	$I_{ah}=\frac{(24)+j(124)}{(20)+j(120)}$	.873	.776				137	$I_{av}=j$ Term	1.370	1.246			
38	$I_{Lh}=(36)+(37)$	.584	1.459				138	$I_{Lv}=(136)+(137)$	-.444	-.384			
39													
40													

Figure 1. Calculation sheet for capacitor motors, page 1



	Speed in RPM	1185	1042	830	500	120	
41	$V=\sqrt{(33)^2+(133)^2}$	83.5	79.3	62.8	50.2	46	
42	V^2	6980	6290	3950	2520	2110	
43	$N_m=\sqrt{(26)^2+(126)^2}$	154.6	138	157	177	187	
44	$N_a=\sqrt{(20)^2+(120)^2}$	136	116	82	59.9	55.0	
45	$(26)\times(120)$	-4450	-476	602	710	878	
46	$(126)\times(20)$	-12,200	-13,290	-11,640	-9,250	-8,120	
47	$(45)-(46)$	7,750	12,814	12,242	9,960	8,998	
48	N_m/V	1.85	1.74	2.50	3.52	4.07	
49	N_a/V	1.63	1.46	1.307	1.193	1.196	
50	$(48)^2$	3.42	3.04	6.24	12.4	16.5	
51	$(K(49))^2$	1.54	1.24	.99	.83	.83	
52	$(50)+(51)$	4.96	4.28	7.23	13.2	17.3	
53	$2K\times(47)/V^2$	1.69	3.11	4.73	6.03	6.50	
54	$(52)+(53)$	6.65	7.39	11.96	19.2	23.8	
55	$(52)-(53)$	3.27	1.17	2.50	7.2	10.8	
56	$(54)\times R_f$	30.1	255	427	470	407	
57	$(55)\times R_b$	26.3	10.	23.6	80.3	153.4	
58	$(56)-(57)$	3.8	245	403.4	389.7	253.6	
59	$(1-s)\times(58)$	3.76	213	279	162.5	25.4	
60	Hi Cy Fe Loss + F+W	16.6	13	8.2	3.0	.2	
61	Output Watts = (59) − (60)	-12.8	200	271	160	25.2	
62	RPM = $(1-s)\times$ Syn	1185	1042	830	500	120	
63	Torque = $\frac{112.7}{RPM}(61)$	-1.22	21.5	36.8	35.9	23.7	
64	$I_m=\sqrt{I_{mh}^2+I_{mv}^2}$ = Mn. Ph. Amps.	1.84	1.77				
65	$I_a=\sqrt{I_{ah}^2+I_{av}^2}$ = Aux. Ph. Amps.	1.63	1.47				
66	$I_L=\sqrt{I_{Lh}^2+I_{Lv}^2}$ = Line Amps	.734	1.51				
67	Pri. Loss$_m$ = $I_m^2 r_{im}$	39.5	36.5				
68	Pri. Loss$_a$ = $I_a^2 r_{ia}$	23.3	19.0				
69	Sec. Loss$_f$ = $(56)\times s$	.4	33.6				
70	Sec. Loss$_b$ = $(57)(2-s)$	52.2	18.7				
71	Cap. Loss = $I_a^2 r_c$	.8	.7				
72	60 c Fe Loss	14.0	14.0				
73	Hi Cy Fe Loss + F+W = (60)	16.6	12.8				
74	Total Losses	146.8	135.2				
75	Input = (61) + (74)	134.0	334.9				
76	Eff = (61)/(75)	——	59.3				
77	P.F. = $(75)/EI_L$ or I_{Lh}/I_L	79.2	96.5				
78	App. Eff. = (76) × (77)	——	57.2				
79	Cap. Volts = $I_a \times Z_c$	339	306				
80	H.P. Output	-.0172	.268				
81							
82							
83							
84							
85							

Figure 2. Calculation sheet for capacitor motors, page 2

tracted to give the useful output. A close approximation to the true current is obtained by adding to the in-phase component of the main winding current a value of in-phase current obtained by dividing the fundamental-frequency iron loss by the voltage.

This method accounting for the iron loss is based on the fact that the fundamental-frequency iron losses are normally supplied by the stator and the odd-frequency losses of the surfaces and tooth pulsations are supplied by the rotating member with a corresponding loss of torque.

Performance Calculation

Credit should be given to C. G. Veinott of the Westinghouse Electric and Manufacturing Company for the essential form of figure 2. It should be noted that the fictitious values of the currents are used for calculating the torque and the corrected values for calculating the stator copper losses. The odd-frequency iron losses are subtracted from the gross output along with the friction and windage.

It is believed that more accuracy is secured by obtaining the input from the output plus losses rather than by multiplying the voltage and the in-phase component of the line current. However, it is usually wise to check the two methods against each other.

Appendix

It is well at this point to definitely state the object of the following derivations. It is simply this: Starting with Mr. Morrill's equations, the object is to develop a simple routine calculation form which will give the performance of a capacitor motor at any speed when the necessary impedances or motor constants are known. The desired form is one with which results of practical accuracy are obtained with a minimum amount of calculation time, and the least likelihood of errors. From the point of view of routine work, it is also desirable that the work may be easily resumed after interruptions such as telephone calls, interviews, etc.

Since the design engineer who works on single-phase motors is very likely to design split-phase, capacitor-start, repulsion, and capacitor-run motors, the symbols and form of the calculation sheet have been arranged in accordance with the single-phase performance method of C. G. Veinott[2] and with the repulsion motor method of the writer now in process of publication by the AIEE.

The three fundamental equations of Mr. Morrill are as follows:

$$I_m=E_m\left\{\frac{[R_c+a^2(R_{1a}+R_f+R_b)]+j[X_c+a^2(X_{1a}+X_f+X_b)]+ja[(R_f-R_b)+j(X_f-X_b)]}{\{[R_c+a^2(R_{1a}+R_f+R_b)]+j[X_c+a^2(X_{1a}+X_f+X_b)]\}\{(R_{1m}+R_f+R_b)+j(X_{1m}+X_f+X_b)\}-a^2[(R_f-R_b)+j(X_f-X_b)]^2}\right\} \qquad (122)$$

$$I_s=E_m\left\{\frac{[(R_{1m}+R_f+R_b)+j(X_{1m}+X_f+X_b)]-ja[(R_f-R_b)+j(X_f-X_b)]}{\{[R_c+a^2(R_{1a}+R_f+R_b)]+j[X_c+a^2(X_{1a}+X_f+X_b)]\}\{(R_{1m}+R_f+R_b)+j(X_{1m}+X_f+X_b)\}-a^2[(R_f-R_b)+j(X_f-X_b)]^2}\right\} \qquad (123)$$

$$\text{Torque(average)}=[I_m{}^2+a^2I_s{}^2][R_f-R_b]+2a[Ah-Bg][R_f+R_b] \qquad (8)$$

where

$I_m=A+jB$ and $I_s=g+jh$

The rearrangement consists of the steps:

A. Rewriting the equations in the symbols of Mr. Veinott's paper.

B. Rewriting the equations for the forward and backward impedances.

C. Obtaining the current terms $A+jB$ and $g+jh$ in quotient form.

$$A+jB=\frac{(26)+j(126)}{(33)+j(133)}$$

and

$$g+jh=\frac{(20)+j(120)}{(33)+j(133)}$$

D. Rearranging the torque equation so that the components $(26)+j(126)$ and $(20)+j(120)$ may be used instead of $A+jB$ and $g+jh$. This allows the calculation of torque without any vector operations.

E. Adding an in-phase iron loss component to $A+jB$ to correct for stator iron loss.

F. Addition of phase currents to obtain the line current.

A. REWRITING THE EQUATIONS IN THE SYMBOLS OF MR. VEINOTT'S PAPER. (SEE TABLE I)

Let us arbitrarily define

$$X_0=X_m+x_2,\quad K_p=X_m/(X_m+x_2),\quad K_r=K_p{}^2=X_m{}^2/X_0{}^2$$

Substitute these values and divide both numerator and denominator by $X_0^2/4$

$$R_f=\frac{1}{2}\,\frac{(r_2K_r)1/s}{[(r_2/X_0)(1/s)]^2+1}$$

R_b becomes the same equation with $(2-s)$ in place of s

$$R_b=\frac{1}{2}\,\frac{(r_2K_r)\,1/(2-s)}{[(r_2/X_0)(1/(2-s))]^2+1}$$

Mr. Morrill writes

$$X_f=\frac{X_m[(R_2/s)^2+X_2(X_2+X_m)]}{(R_2/s)^2+(X_2+X_m)^2}=$$

Rewritten in symbols of Mr. Veinott

$$\frac{(X_m/2)[(r_2/2s)^2+x_2(x_2+X_m)/4]}{(r_2/2s)^2+(x_2+X_m)^2/4}$$

Table I

New Symbol	Old Symbol	Definition
K	a	$\dfrac{\text{Total effective series conductors in auxiliary phase}}{\text{Total effective series conductors in main phase}}$
r_{1a}	a^2R_{1a}	Primary resistance of auxiliary phase
x_{1a}	a^2X_{1a}	Primary leakage reactance of auxiliary phase
r_{1m}	R_{1m}	Primary resistance of main phase
r_2	$2R_2$	Total secondary resistance referred to primary of main phase
x_{1m}	X_{1m}	Primary leakage reactance of main phase
x_2	$2X_2$	Total secondary leakage reactance referred to primary of main phase
X_m	$2X_m$	Total magnetizing reactance of main phase
s	S	Slip in per unit of synchronism
I_a	I_s	Auxiliary phase current

$$I_m=E_m\left\{\frac{[R_c+r_{1a}+K^2(R_f+R_b)]+j[X_c+x_{1a}+K^2(X_f+X_b)]+jK[(R_f-R_b)+j(X_f-X_b)]}{\{[R_c+r_{1a}+K^2(R_f+R_b)]+j[X_c+x_{1a}+K^2(X_f+X_b)]\}\{(r_{1m}+R_f+R_b)+j(x_{1m}+X_f+X_b)\}-K^2[(R_f-R_b)+j(X_f-X_b)]^2}\right\}$$

$$I_a=E_m\left\{\frac{[(r_{1m}+R_f+R_b)+j(x_{1m}+X_f+X_b)]-jK[(R_f-R_b)+j(X_f-X_b)]}{\{[R_c+r_{1a}+K^2(R_f+R_b)]+j[X_c+x_{1a}+K^2(X_f+X_b)]\}\{(r_{1m}+R_f+R_b)+j(x_{1m}+X_f+X_b)\}-K^2[(R_f-R_b)+j(X_f-X_b)]^2}\right\}$$

$$T_{\text{avg}}=(I_m{}^2+K^2I_a{}^2)(R_f-R_b)+2K(Ah-Bg)(R_f+R_b)$$

B. REWRITING THE FORWARD AND BACKWARD IMPEDANCES

Mr. Morrill's equation

$$R_f=\frac{X_m{}^2\,R_2/s}{(R_2/s)^2+(X_2+X_m)^2}=$$

Rewritten in symbols of Mr. Veinott

$$\frac{(X_m/2)^2\,(r_2/2)(1/s)}{(r_2/2)^2(1/s)^2+\left(\dfrac{x_2+X_m}{2}\right)^2}$$

Substitute $X_0=X_m+x_2$, $K_p=X_m/(X_m+x_2)$, divide both numerator and denominator by $X_0/4$, and rearrange

$$X_f=\frac{1}{2}\,\frac{x_2K_p+(r_2K_p)(r_2/X_0)(1/s)^2}{[(r_2/X_0)(1/s)]^2+1}$$

and

$$X_b=\frac{1}{2}\,\frac{x_2K_p+(r_2K_p)(r_2/X_0)(1/(2-s))^2}{[(r_2/X_0)(1/(2-s))]^2+1}$$

C. OBTAINING A+jB AND g+jh IN QUOTIENT FORM

It will be noted from the equations of I_m and I_a, that many of the groups of terms are the same. Let us rewrite them using a symbol for each group. For simplicity let us use for symbols the item numbers of the final calculation sheets figures 1 and 2. Our purpose is to obtain

$$A+jB=\frac{(26)+j(126)}{(33)+j(133)}$$

and

$$g+jh=\frac{(20)+j(120)}{(33)+j(133)}$$

Then, assigning a number symbol to each term, we obtain

$$A+jB=E_m\times\left\{\frac{(24)+j(124)+jK[(15)+j(115)]}{\{(24)+j(124)\}\{(18)+j(118)\}-K^2[(15)+j(115)]^2}\right\}$$

$$g+jh=E_m\times\left\{\frac{(18)+j(118)-jK[(15)+j(115)]}{\{(24)+j(124)\}\{(18)+j(118)\}-K^2[(15)+j(115)]^2}\right\}$$

Simplifying to obtain one real and one quadrature term in both numerator and denominator,

$$A+jB=E_m\times\left\{\frac{[(24)-K(115)]+j[(124)+K(15)]}{[(24)(18)-(124)(118)-K^2(15)^2+K^2(115)^2]+j[(18)(124)+(118)(24)-2(K15)(K115)]}\right\}$$

$$g+jh=E_m\times\left\{\frac{[(18)+K(115)]+j[(118)-K(15)]}{[(24)(18)-(124)(118)-K^2(15)^2+K^2(115)^2]+j[(18)(124)+(118)(24)-2(K15)(K115)]}\right\}$$

Assigning number symbols to the above groups of terms,

$$A+jB=E_m\frac{(26)+j(126)}{(32)+j(132)}$$

and

$$g+jh=E_m\frac{(20)+j(120)}{(32)+j(132)}$$

Dividing numerator and denominator by E_m gives the desired result

$$A+jB=\frac{(26)+j(126)}{(33)+j(133)}$$

and

$$g+jh=\frac{(20)+j(120)}{(33)+j(133)}$$

D. REARRANGING THE TORQUE EQUATION

$$T=(I_m^2+K^2I_a^2)(R_f-R_b)+2K(Ah-Bg)(R_f+R_b)$$

In order to avoid making the vector operation involved in obtaining A, B, g, and h, let

$$N_m=\sqrt{(26)^2+(126)^2}$$

$$N_a=\sqrt{(20)^2+(120)^2}$$

$$V=\sqrt{(33)^2+(133)^2}$$

Also since $(Ah-Bg)$ is the quadrature term obtained by multiplying $(g+jh)$ by the conjugate of $(A+jB)$, performing this operation on both right and left sides of the equations gives us an expression for $(Ah-Bg)$.

$$(A-jB)(g+jh)=(Ag+Bh)+j(Ah-Bg)$$

$$\frac{(26)-j(126)}{(33)+j(133)}\times\frac{(20)+j(120)}{(33)+j(133)}=\frac{[(26)(20)+(126)(120)]-j[(26)(120)-(126)(20)]}{(33)^2+(133)^2}$$

Thus

$$Ah-Bg=\frac{(26)(120)-(126)(20)}{V^2}$$

and

$$T=[(N_m/V)^2+(KN_a/V)^2](R_f-R_b)+2K\left[\frac{(26)(120)-(126)(20)}{V^2}\right](R_f+R_b)$$

This allows the calculation of torque without any vector operations, which is particularly desirable in calculating speed-torque curves, pull-up torque of capacitor-start motors, or in designing two- or three-speed fan motors.

E. CORRECTING FOR PRIMARY IRON LOSS

Although the fundamental frequency iron loss is supplied by the primary, and the odd-frequency iron loss by the rotor, it is not possible to include the stator iron loss in the primary part of the revolving field equations without making them too complicated. If the total iron loss is subtracted from the output similar to the friction and windage losses, it will be found in certain motors, particularly those designed for speed change driving low-horsepower fans, that the calculated useful torque will not agree with the tested value. The writer has had considerable success in approximating the true condition by simply adding an in-phase current component to the main phase current to account for the stator iron loss. This seems to agree with test much closer than adding half to the main phase and half to the auxiliary phase. The final main phase current is then

$$A+\frac{\text{fundamental-frequency iron loss}}{E_m}+jB$$

The auxiliary phase current is $g+jh$, and the line current is the vector sum.

In figure 1 the fundamental-frequency iron loss is denoted by the symbol "60 Cy Fe" and the odd-frequency iron loss by the symbol "Hi Cy Fe". $F+W$ is used as the symbol for friction and windage.

As a matter of interest, the motor used as an example for figure 1 is a one-fourth horsepower 230-volt 60-cycle six-pole single-value capacitor motor designed for three-speed unit-heater fan service to be used with an autotransformer for speed control.

Bibliography

1. THE REVOLVING FIELD THEORY OF THE CAPACITOR MOTOR, W. J. Morrill. AIEE TRANSACTIONS, April 1929, pages 614–29.

2. PERFORMANCE CALCULATIONS ON INDUCTION MOTORS, C. G. Veinott. AIEE TRANSACTIONS, September 1932, pages 743–55.

A New Current-Limiting Fuse

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Synopsis: Increasing concentration of power combined with the desire for more compact switchgear assemblies has created a demand for small, totally enclosed, high-interrupting-capacity fuses suitable for potential and operating transformer protection. The current-limiting fuse meets these requirements for it is inherently a high-interrupting-capacity fuse of low current rating and can be totally enclosed in a small space.

The theory and construction of previous current-limiting fuses are briefly described. The theory and construction of a new current-limiting fuse is given. The new fuse is simple in theory and construction. Its fundamental elements can be separated, studied, and tested, making it possible to predict an unusual degree of reliability and freedom from voltage surges. Test results confirm the reliability, high interrupting capacity, and freedom from voltage surges of the new fuse.

THE development and adoption of the first current-limiting fuses were prompted by the need for high interrupting ability. The other characteristics of the fuse were merely incidental and were not major factors in the wide acceptance of the current-limiting fuse in Europe. To limit the current and thereby limit the amount of energy liberated in the fuse was one way of obtaining high interrupting ability. In fact, before the advent of boric acid, as an interrupting medium, the current-limiting fuse was the only fuse that could safely be applied to many of the systems then in existence. During 1931 current-limiting fuses were imported, tested, and studied for possible adoption in this country where high interrupting ability was required. These fuses had the desirable features of high interrupting ability in a relatively small size and did not expel arc gases or other material during operation. However, the limited range of current ratings available in the fuse so restricted the scope of its application that its commercial development for use in this country was abandoned. Further development evolved the boric acid fuse with its high interrupting ability, totally enclosed features, and wide range of current ratings.

The increased use of metal-enclosed switchgear has placed a premium on space. The amount of room required for a low-interrupting-ability fuse with current-limiting resistor or a standard power fuse for the protection of potential or small operating transformers appears to be all out of proportion to the size of the apparatus being protected and the load handled. It should be remembered, however, that the real duty of the fuse is to protect the system rather than the apparatus. There is a need, therefore, for a fuse having the small size and characteristics of the current-limiting fuse for connecting small loads to high-power systems as a supplement to power fuses that are used for heavier capacity loads.

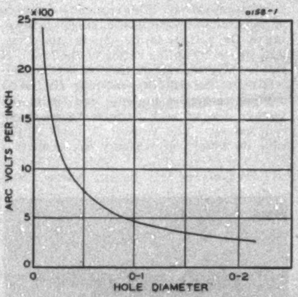


Figure 1. Characteristics of arcs in small holes

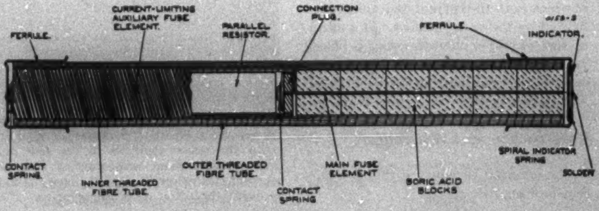
At the present time there are two types of current-limiting fuses available. One type consists of a fuse and resistor in series enclosed in a common cartridge. The resistor limits the current during the interruption of the fuse. The required characteristics of a resistor to carry load current and to limit the current on short circuit are directly opposed. To satisfactorily combine both characteristics in one resistor requires a relatively large resistor that necessitates a bulky cartridge.

The other type of current-limiting fuse consists of a silver wire spirally wound on a ceramic core and surrounded by inert crystals.[1,4] This type of fuse has had an excellent record in Europe for over a decade and in this country for two years. Characteristics of this fuse believed capable of improvement are:

1. It has a high melting temperature and, therefore, if made in higher ampere ratings would have a high operating temperature.
2. It offers no flexibility in obtaining time-current characteristics to meet various applications.
3. On operations at currents slightly above the melting point of the wire the operation may not be as reliable as in a spring-operated fuse.
4. The reliability of the fuse depends on the characteristics of the inert crystals used, therefore, a reliable source of supply and very rigid inspection are necessary to maintain the quality of the fuse.

Early tests on various forms of current-limiting fuses revealed high voltage surges during current limitation. Small variations in construction would result in widely different surge voltages. In order for a fuse to limit the current a voltage must be produced in the fuse equal to or greater than the voltage available in the circuit at the time of current limitation. Also the voltage produced must not rise to a value that might cause flashover or otherwise endanger the circuit on which it is applied. It is evident, therefore, that in order for a current-limiting fuse to be

Figure 2. Sectional view of new-type current-limiting fuse



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1. For all numbered references, see list at end of paper.

reliable the voltage rise during current limitation must be accurately controlled and kept within a definite range. A further requirement of reliability is that the means for controlling the voltage rise should not be critical or susceptible to the small variations in size or material that are liable to occur in commercial production.

Investigations on arcing characteristics revealed that arcs drawn in small holes in solid material gave a high arc voltage that increased very rapidly as the size of the hole was reduced.[3] This characteristic is shown by the curve in figure 1. By utilizing a very small hole, a very high arc voltage and therefore a very definite current-limiting action can be obtained. The smaller the hole, the higher the arc voltage and the more reliable the current-limiting action will be, disregarding any possible danger to the system from the high voltage.

Figure 3. Photograph of new current-limiting fuse

To limit the voltage on any device it is merely necessary to by-pass the current through a suitable resistor. This resistor may have a valve characteristic to limit the current at normal voltage and permit much higher currents to flow at higher voltages. If the fuse wire in a very small bore is paralleled with such a resistor, the basis is formed for a current-limiting fuse with a very definite and reliable current-limiting element and a very definite and reliable voltage-limiting element. A greater degree of reliability can be attained in such a device than in a device in which both the current and voltage are limited by the same element. If a clean-up fuse of proper characteristics is placed in series with such a combination, a complete current-limiting fuse is obtained. Such a fuse is shown in figures 2 and 3.

Construction and Operation

The essential elements of this new current-limiting fuse are:

1. *The Auxiliary or Current-Limiting Fuse Element.* This consists of a silver wire spirally wound on a multipitch threaded fiber tube which is screwed into a similarly threaded tube that forms the outer shell of the fuse holder. Fiber was selected for this purpose due to its gas-evolving properties and its mechanical strength although numerous tests have not demonstrated the need for any great mechanical strength in these parts. With gas-evolving material around the fuse element the arc voltage will be higher than with nongas-evolving materials. Furthermore, residual arc currents will be interrupted quickly with no possibility of a low-resistance path due to incandescent surfaces. The clearance left in the bottom of the thread after assembly provides a small hole slightly larger than the diameter of the wire, in which arc restriction occurs. On assembly a special cement is applied to the threads to form a seal between threads. Even with loose fitting parts the arc voltage produced in the restricted hole is many times the voltage required for current transfer to the parallel resistor rod.

2. *The Parallel Resistor Rod or Voltage-Limiting Element.* This element is a silicon-carbide rod of proper characteristics depending on the voltage rating, the current rating, and the interrupting rating of the fuse. It has a valve characteristic that becomes effective at currents approaching the interrupting rating of the fuse and limits the voltage rise at such currents to relatively safe values. This resistor is inside of the tube on which the auxiliary fuse element is wound. Connection is made to it by means of contact springs. No special precautions are necessary in making contact with the resistor, as it passes current only during arc interruption, when full line voltage is impressed across the fuse.

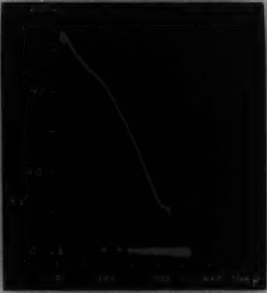

Figure 4. Dynamic volt-ampere characteristic of auxiliary fuse on 500-ampere 10x100-microsecond current wave

3. *The Main Fuse Element.* This element, the clean-up element, is connected in series with the other elements of the fuse and serves as the final interrupter of all overcurrents that flow through the fuse. It also provides isolation after interruption. The fuse element operates in a small hole through solid boric acid. The excellent performance of boric acid in fuses both on low and high currents has been described in previous publications and has been demonstrated by many years in commercial use.[2] In the new current-limiting fuse this interrupting element is subjected to very easy service since it is called upon to open either extremely low currents or moderately low currents at a high power factor. The main fuse element itself is of the low-temperature type, can be designed to give any desired time-current characteristics, and is spring operated to insure a positive gap on low-current operations. The spring that separates the fuse element also operates a suitable indicator that provides visual indication of a blown fuse.

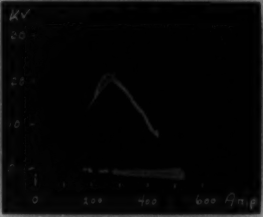

Figure 5. Dynamic volt-ampere characteristic of auxiliary fuse with parallel resistor rod on 500-ampere 10x100-microsecond current wave

The action of the fuse on low currents is not the same as on high currents due to the main fuse element being calibrated to melt at a lower current than the auxiliary fuse element. This was done to prevent any appreciable temperature rise of the auxiliary silver fuse wire and eliminate the possibility of the auxiliary fuse being melted before the main fuse. On low values of current the main fuse element may melt and interrupt the current without the auxiliary fuse wire being melted. In a fuse designed for potential-transformer protection this will happen on all currents below approximately 80 amperes. On high-current operation, the auxiliary fuse will melt on the initial rise in current and transfer the current to the parallel resistor rod. The thermal capacity of this rod is sufficient to permit a half cycle of current to pass through it without an excessive temperature rise. At the end of the first half cycle after current transfer the main fuse will interrupt the circuit.

Theory and Calculations

The operating characteristics of the current and voltage-limiting elements of this fuse are of particular importance. Calculations can readily be made from which the performance of the device may be predicted.

When a short circuit occurs the current rises in the auxiliary fuse element with practically no limiting influence until the fuse element melts. At this point voltage rises rapidly to a high arc voltage and the current is limited and transferred to the resistor rod.

The maximum value which the current may reach when the fuse melts may be readily computed. Since the temperature rise of the fuse wire equals

$$\frac{\text{energy input}}{\text{mass} \times \text{specific heat}}$$

an infinitesimal temperature rise,

$$dT = \frac{i^2R(1+aT)dt}{Mk} \tag{1}$$

where

T = temperature, degrees centigrade
i = amperes
t = seconds
R = fuse resistance at zero degrees
a = temperature coefficient of resistance
M = mass of fuse wire, grams
k = watt seconds per gram per degree centigrade

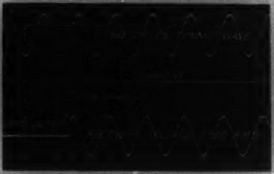

Figure 6. Operation of current-limiting fuse on a 2.3-kv system, with 20,000-ampere available short-circuit capacity

The maximum rate of current rise occurs on symmetrical short circuits. Under such conditions we may, for simplicity and without appreciable error, assume that the current in the fuse wire up to the melting point is linear and equal to:

$$i = 377I_mt \tag{2}$$

where I_m = maximum value of available 60-cycle short-circuit current.

We may then rewrite equation 1 as follows:

$$\frac{dT}{R(1+aT)} = \frac{(377)^2I_m^2}{Mk}t^2dt \tag{3}$$

Integrating equation 3, taking T between the limits of the initial fuse temperature, T_i, and the melting temperature, T_m, gives:

$$\frac{1}{Ra}\log_e\frac{1+aT_m}{1+aT_i} = \frac{(377)^2I_m^2}{Mk}\frac{t_m^3}{3} \tag{4}$$

whence time to the melting point,

$$t_m = \sqrt[3]{377I_m \cdot \frac{3Mk}{Ra}\log_e\frac{1+aT_m}{1+aT_i}} \tag{5}$$

Substituting this value of t in equation 2, gives the current at time of melting.

$$i_m = \sqrt[3]{377\,I_m \cdot \frac{3Mk}{Ra}\log_e\frac{1+aT_m}{1+aT_i}} \tag{6}$$

Whence

$$i_m = K\sqrt[3]{I_m}$$

This indicates that as the available system short-circuit current rises, the maximum value of the initial rush of short-circuit current in the fuse increases only as the cube root of the available short-circuit current. Thus, if the available short circuit increases from 10,000 to 80,000 amperes, the maximum value to which current can rise in the fuse will only double.

This is important since the initial voltage surge caused by the blowing of the fuse cannot exceed the product of the maximum initial fusing current times the resistance of the parallel resistor rod. It is, therefore, possible to design this fuse so as not to exceed any specified surge voltage. Since ordinary switching operations frequently cause voltage surges of twice the peak system voltage it is evident that the fuse may safely be designed to have twice peak value as an upper limit of surge voltage when the parallel fuse melts. Thus, if the fuse is designed to give a maximum surge voltage on a 10,000-ampere system, equal to peak voltage, then, since the surge varies as the cube root of the available system current, a short circuit on an 80,000-ampere system can be interrupted without exceeding a surge of twice the peak voltage. This indicates that a fuse of this type can cover a wide range of short-circuit currents without subjecting connected apparatus to excessive voltage surges.

As previously mentioned, it is further possible to make use of valve characteristics of the material in the parallel resistor rod, thus still further extending the range to which the fuse may be applied without excessive voltage surges.

Test Results

Important features of this fuse are the current-limiting fuse element and the voltage-limiting parallel resistor. The high arc voltage produced in the current-limiting fuse element and the effect of the parallel resistor in limiting that voltage are well illustrated in the oscillograms shown in figures 4 and 5. The tests represented by these oscillograms were made on a surge generator. Figure 4 shows the dynamic volt-ampere characteristics of the auxiliary fuse alone, while figure 5 shows the same characteristic of the auxiliary fuse and parallel resistor.

Numerous tests were made during the development of this fuse. Figures 6 and 7 show typical oscillograms of short-circuit interruptions on the fuse as finally perfected.

In figure 6 an interruption is shown on a 20,000-ampere 2,300-volt circuit. The fuse used in this test was similar to the illustration shown in figure 3, but of much smaller size, having an over-all length of only $4^5/_8$ inches. In this test it may be noted that the initial rush of current was limited at 175 amperes, while the surge voltage barely exceeded the peak value of the restored voltage.

Figure 7 shows an interruption on a 20,000-ampere 13.2-kv circuit. The fuse used in this test was as shown in figures

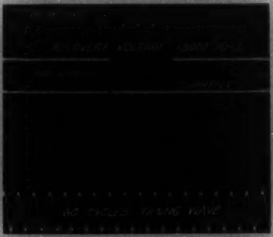

Figure 7. Operation of current-limiting fuse on a 13.2-kv system with 20,000-ampere available short-circuit capacity

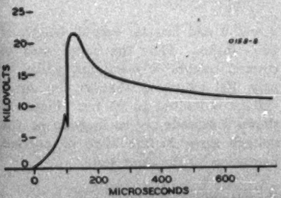

Figure 8. Cathode-ray oscillogram of voltage surge occurring on interruption shown in figure 7

An Electrical Governor

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THE advantages obtained from the application of electrical methods of control to all classes of machinery, suggested to inventors the idea of regulating the speed of prime movers by electrical means. Frequent attempts have been made to accomplish this result. Many of these governors were very ingenious, but for one reason or another they never came into general use.

The electrical governor described in this paper is the result of several years effort to develop a practical method of electrically controlling and regulating the speed of hydraulic turbines.

Briefly, the method utilizes a form of frequency meter which measures variations in the frequency (corresponding to changes in speed) of an alternating current supplied by the generator driven by the turbine to be governed. This meter, through suitable control equipment, moves a pilot valve, which in turn controls the operating engine that opens or closes the wicket gates of the turbine.

An application of the scheme has been made to a 3,000-horsepower turbine operating under a head of 55 feet. The present installation is entirely experimental, the equipment being "home-made" and crude but the results obtained have been quite encouraging and indicate that the method has interesting possibilities.

A very steady speed is maintained at no load and synchronizing is done without difficulty with the governor in control of the unit. Operation under load is quite satisfactory, the governor responding promptly to small fractions of a cycle change in frequency (60 cycle). Changes in the loading of the machine are easily made, the governor responding instantly to such adjustments. Loads requiring gate openings of 80 to 90 per cent have been suddenly dropped by opening the generator circuit breaker. The governor closed the gates promptly and there was no tendency to "hunt" about the no-load position. Sudden accessions of load such as large "across the line" motors, are followed instantly by the opening of the turbine gates with no hunting whatever. The unit has been started from standstill many times by electrical remote control and when approximate normal speed was reached, placed under governor control by throwing a switch.

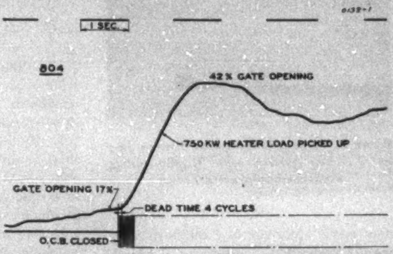

Figure 1. 750-kw load applied

Some of the advantages of electrical governing are:

(*a*). Small inertia of the control element.

(*b*). Increased sensitivity.

(*c*). Elimination of long rods, cables, bell cranks, etc., required for the compensation devices of fly-ball governors.

(*d*). Since, with electrical control, the force operating the pilot valve can be made large, and is independent of the amount of the speed change, friction in this valve has little or no effect.

(*e*). The possibility of mounting the control element at any convenient location, such as on the main switchboard.

(*f*). The valve controlling the admission of pressure fluid to the servomotor can be mounted immediately on this cylinder thus reducing the amount of piping and releasing valuable floor space ordinarily required for governors.

(*g*). Ease of starting and stopping the turbine by remote control. This permits *complete* control of the unit by the switchboard operator.

(*h*). Ease of applying frequency and load control methods.

(*i*). By means of simple test equipment, the sensitivity of the control element (frequency meter) of an electrical governor can be checked. A corresponding check of a fly-ball device is more difficult and complicated.

The writer is of the opinion that a description of the device and an account of the progress made up to the present, may prove of interest to engineers in general, and more particularly to those who are intimately concerned with governing problems.

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Frank I. Morgan is maintenance engineer, Pennsylvania Water and Power Company, Baltimore, Md.

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2 and 3 and has an over-all length of $12^1/_2$ inches. Here the initial rush of current reaches a maximum value of only 400 amperes, whereas the surge voltage is limited to 23 kv. Figure 8 shows a cathode-ray oscillogram of the voltage surge at the time of current limitation for the same test.

Summary

1. A new type of current-limiting fuse has been developed and tested.

2. The construction of the fuse is such that the voltage surge as well as the current surge during interruption is limited to definite and predetermined values.

3. Tests on high-capacity systems have not revealed the limitations of the fuse. Indications are that the fuse has an exceedingly high interrupting ability.

4. The design of the fuse is flexible. Time-current characteristics can be obtained as desired.

5. The fuses may be applied to the protection of potential transformers and other apparatus of small power consumption. They are especially applicable to use in enclosed switchgear.

Bibliography

1. Hochspannungs-Hochleistungs-Trennsicherungen, *E.T.Z.*, August 27, 1931, page 1122.

2. The Boric-Acid Fuse, A. P. Strom and H. L. Rawlins. AIEE Transactions, volume 51, p. 1020.

3. The Expulsion Fuse, J. Slepian and C. L. Denault. AIEE Transactions, volume 51, 1932, page 157.

4. The Current-Limiting Power Fuse, D. C. Prince and E. A. Williams, Jr. AIEE Transactions, volume 58, 1939, pages 11–14.

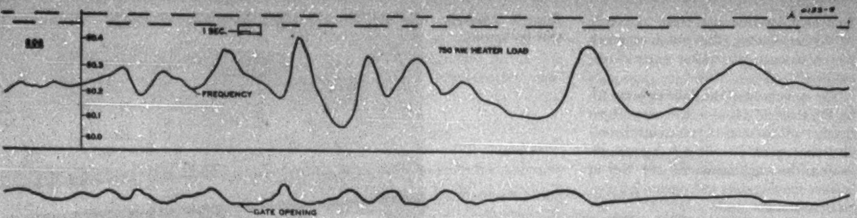


Figure 2. Frequency-gate movement

Preliminary Tests

The three tracings of oscillograms shown are a record of the performance of the governor during preliminary tests made in January and May 1937. These records are submitted, not as an indication of what can ultimately be accomplished, but simply to show the progress made up to the time of these tests.

The load on the unit during these tests was a purely resistance one amounting to 750 kw and it was possible to throw it on and off the unit at will.

Figure 1 (oscillogram number 804) shows the operation of the governor when a load of 750 kw was suddenly thrown on the unit. The governor started to open the gates in the remarkably short time of 4 cycles (60-cycle basis).

Figure 2 (oscillogram number 806) shows frequency and gate opening variations while the unit was carrying the load mentioned above.

Figure 3 (oscillogram number 808) gives the result of dropping load tests, showing the action of the governor when the load of 750 kw was dropped by opening the generator circuit breaker. The time required for the governor to start closing the gates varied from six to nine cycles.

Subsequent to the tests described in the foregoing, the apparatus has been applied to a double runner 13,500-horsepower turbine. This unit is equipped with a standard fly-ball governor of an old design. The control of its pilot valve was transferred from the flyballs to the electrical governor. The speed regulation, after the change, was very satisfactory and far superior to that obtained with the flyballs in control.

Operating Principle

An instrument sensitive to frequency variations and having two parallel electrical circuits, one branch containing resistance and the other reactance, is used as the control element. The change in impedance of the reactance with frequency is balanced by a corresponding change in resistance so that the currents flowing in the two branches of the circuit are maintained equal in value. The required changes in resistance are obtained by means of variable rheostats. Being a "null" method, the action of the control element is unaffected by changes in voltage within wide limits.

Referring to the schematic diagram, figure 4, *A* is a Kelvin balance employed as the frequency-responsive element, the moving coils of which are provided with a contact 27. This contact engages with either of the stationary contacts, 28 or 29. A resistance 10, is connected to one set of coils 3, and a reactance 11, to the other set of coils 2, of the Kelvin balance. Solenoids 24 and 25 actuate the walking beam 22, to which is connected the valve 18, admitting pressure fluid to either end of the "servomotor" cylinder 1. Rheostats 12 and 13 provide "compensation", the movable contact of rheostat 12 being connected to the servomotor piston rod by lever 30. The movable contact of rheostat 13 is connected to lever 30 by rod 33 and springs 31. A dashpot is provided to delay the movement of the contact of rheostat 13.

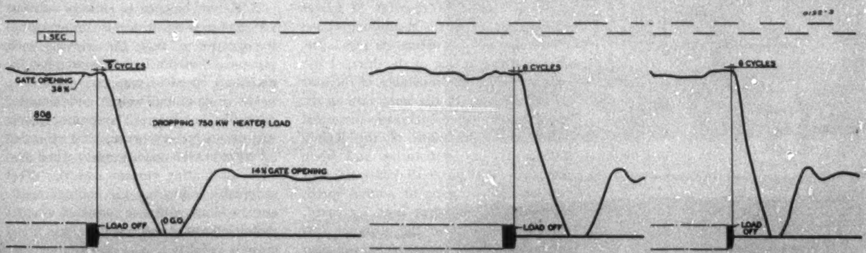
"Inherent speed drop" (also called "degree of irregularity," or the decrease in speed between no load and full load necessary for parallel operation) is adjusted by changing the location of the pivot point of rod 33 on the lever of rheostat 13.

The speed maintained by the governor is adjusted by means of rheostat 14.

Operation

Assuming normal frequency and steady load conditions, the currents flowing through the two sets of coils of the Kelvin balance are equal and the moving coils are in equilibrium, contact 27 being midway between contacts 28 and 29. Now, if the load on the unit increases, the speed will drop, lowering the frequency. Because of the lowered frequency, the reactance of 11 is decreased permitting more current to flow through coils 2 of the balance, upsetting the equilibrium of the moving coils. Contacts 27 and 29 close, energizing solenoid 24 which moves walking beam 22 and valve 18. Pressure

Figure 3. Dropping load tests



fluid then flows through pipes 20 and 21 to the servomotor, the piston of which moves to open the wicket gates of the turbine.

The movement of the walking beam 22, by the solenoid 24, also increases the resistance of rheostat 15 sufficiently to reduce the current in coils 2, to normal, restoring the equilibrium of the Kelvin balance moving coils and separating contacts 27 and 29.

The opening of the wicket gates by the servomotor increases the resistance of rheostat 12 by moving lever 30. This additional increase of resistance in series with coils 2 unbalances the moving coils in the opposite direction, closing contacts 27 and 28 to energize solenoid 25, which moves walking beam 22. This results in return of valve 18 toward its neutral position, and at the same time, decreases the resistance of rheostat 15. It should be noted that the movement of valve 18 back to its neutral position is simultaneous with the opening of the gates by the servomotor and the increase in resistance of rheostat 12.

When the resistance of rheostat 12 has been increased by an amount equivalent to the decrease in impedance of reactance 11 and the increase in resistance of rheostat 15 by the initial movement of the walking beam, the equilibrium of the Kelvin balance is again restored and valve 18 has been brought back to its neutral position, stopping movement of the gates.

The wicket gates have now been opened to increase the amount of water to the turbine, but because of the inertia of the water, its flow does not immediately increase. During the time the gates are being opened to the new load position and until the flow of the water increases sufficiently, the additional load demand is met by energy stored in the revolving parts of the turbine and generator which is released by a decrease in speed. As the flow of water increases through the greater gate opening, the speed of the unit returns toward normal. Unless provision is made to prevent it, this increase in speed (and frequency) toward normal would upset the equilibrium of the Kelvin balance—which has been balanced for the lowered speed of the unit by the change in resistance of rheostat 12—and cause a closing movement of the gates. This is avoided through the action of springs 31, rheostat 13, and the dashpot. (Second compensation.)

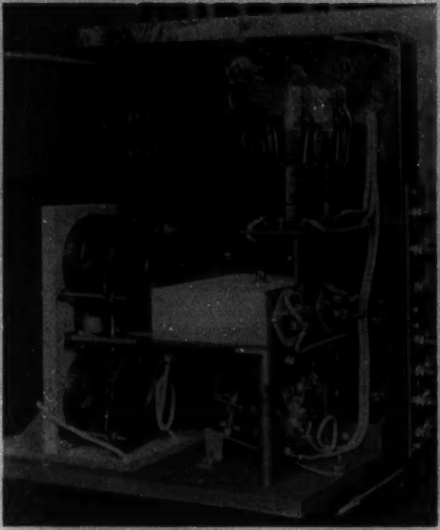


Figure 5. Magnetic vane instrument

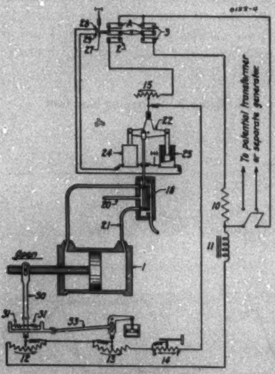


Figure 4. Schematic diagram

The tension of the spring 31 has been increased by the movement of lever 30, and the contact of rheostat 13 moves slowly, because of the restraining action of the dashpot, to reduce its resistance. Provided the setting of the dashpot bypass is correct, the resistance of rheostat 13 will decrease at the same rate as the speed (and frequency) increases to normal value, the equilibrium of the Kelvin balance will not be disturbed and, when movement of rheostat 13 ceases, the unit will be again running at normal speed, although the load has been increased. Circuit conditions of the governor (assuming isochronous adjustment) will then be exactly as they were before the change of load occurred.

It is apparent that increasing the resistance of rheostat 14, has the same effect as an increase in impedance, of reactor 11, resulting from an increase in speed (and frequency) of the turbine. Such an increase of resistance will cause the governor to close the turbine gates by a definite amount and lower the turbine speed.

Conversely, a decrease in resistance of rheostat 14 will cause the governor to open the turbine gates and increase the speed of the turbine.

The action of the governor on a decrease in load is, practically, the reverse of that described in the foregoing.

Control Instrument

A Kelvin balance is entirely satisfactory as far as sensitivity is concerned but it was found that the moving coils possessed too much inertia to give the necessary speed of response, even when made of minimum weight and mounted in jeweled bearings. The contact system was unsatisfactory because the vibration inherent in such an instrument when used with alternating current had the effect of greatly increasing the contact resistance. Many different forms of contact mounting were tried but none was found entirely reliable.

A magnetic vane instrument, shown in figure 5, was constructed and substituted for the Kelvin balance. The moving element of this instrument consists of a small aluminum shaft fitted with steel pivots, on which two light annealed steel vanes are mounted. This shaft is supported in jeweled bearings. Each vane is surrounded by a coil, one of which is connected in series with resistance 10, the other being connected in series with reactance 11. The vanes are placed on the shaft at an angle of approximately 80 degrees with respect to one another so that, with current flowing through the two coils, they tend to move in opposite directions and the shaft assumes a position depending on the relative strengths of the currents in the two coils. The moving element is dampened by a small oil dashpot. A small mirror is mounted on the shaft midway between the ends.

Figure 6. Compensation rheostats

Light from a 21 candle-power automobile head lamp is reflected from the mirror to two photoelectric cells which are connected to the grids of two vacuum tubes. The two coils of a differential relay of the D'Arsonval type are connected in the plate circuits of the tubes. The contacts of this relay control the solenoids 24 and 25 which operate valve 18.

The use of a lamp and vacuum tubes naturally raises the question of reliability since that feature is of paramount importance in a governor.

Records of lamp life, in continuous telemeter service, have shown that the life of such lamps (6–8 volts) when operated at 4 volts, varied from 12,000 to 19,000 hours.

An effective means of insuring reliability is the use of a double filament lamp (number 2331) and two sets of vacuum tubes. A simple one-contact relay operates to connect the second set of tubes and the second lamp filament in the event any one filament fails in the first set. If desired an alarm contact could be added to the relay.

Power Requirements

The magnetic vane instrument requires about 30 volt-amperes for its operation including the 21 candle-power lamp and the filaments of the vacuum tubes. This indicates that the load on a potential transformer for this type of governor would be quite negligible and makes feasible the use of a small magneto for this purpose if it were considered undesirable, from the standpoint of reliability, to use potential transformers.

Governor Action on Loss of Load

When a heavy load is suddenly dropped from a hydraulic turbine, the speed immediately increases and the governor promptly closes the gates. Because that part of the compensation device (second compensation) which contains the time delay feature acts slowly, the speed reaches its highest value and decreases toward normal before the time delay element has returned to the point where it can function properly for the gate opening required to maintain normal no-load speed. As a result, when the speed decreases sufficiently to cause the flyballs to start opening the gates, the second compensation cannot operate properly and the gates are opened far beyond the point necessary to maintain normal speed at no load. The turbine overspeeds, causing the flyballs to again close the gates completely. During each interval of gate closure, the second compensation approaches the position where it can function properly. Until it reaches this point, these gate opening and closing cycles repeat at intervals with gradually decreasing amplitude. The effect, for the time being, is much the same as that which would be obtained if the second compensation feature were omitted.

This objectionable operation has been remedied in one well-known make of governor by means of a cam which increases the action of the spring which opposes the centrifugal force of the flyballs. This cam begins to be effective when the gates, on closing, reach the point which admits sufficient water to operate the turbine at about normal speed with no load. The cam increases the "regulation" of the governor at no load, i.e., it adjusts the flyballs so that the speed maintained at no load is considerably higher than that at full load. In other words, the cam provides additional compensation for a limited range of gate movement. In this way, as the turbine

Figure 7. Pilot-valve solenoids

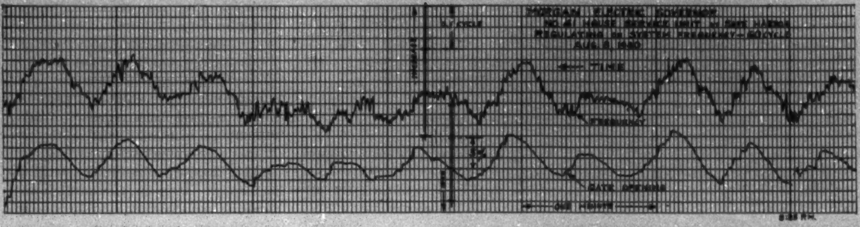


Figure 8. Record of test made August 8, 1940

slows down, the flyballs start to open the gates sooner than they would had no adjustment been made, the downward trend of the speed is arrested earlier and the second compensation functions properly. In addition, the opening movement of the gate is stopped at a point at which the readjusted speed will be maintained.

In at least two other makes of governors, this surging of gate movement and speed after dropping heavy loads is avoided by the provision of a bypass for the dashpot, which is opened automatically when quick gate movements beyond a certain amount occur.

In the governor described above, a bar is used to connect the movable levers of the compensation rheostats 12 and 13. This bar is slotted at the end connected to the lever of rheostat 13, as can be seen in figure 6. The purpose of this bar is the return of the lever of rheostat 13, on large, quick gate movements to a point where the second compensation can function properly. The slot allows the second compensation to operate normally on small gate movements or large gate movements which do not take place rapidly. This method of overcoming the faulty operation mentioned above has the advantage that no change is made in the regulation of the governor.

Load Limit and Safety Devices

Limitation of the load carried by the unit can be accomplished by arranging a stop, operated by an electromagnet, to prevent movement of walking beam 22 and valve 18 to cause further opening of the gates. The electromagnet would be energized through suitably arranged contacts to be closed by lever 30, these contacts to be adjustable so that the gate opening could be limited to any degree desired. This arrangement would not interfere with the closing of the gates at any time.

Emergency shutdown of the unit could be accomplished by the use of a relay, which when de-energized, would release a spring to move the pilot valve, independently of solenoids 24 and 25, in the direction to close the turbine gates. The relay releasing the spring could be operated by over-speed, over or undervoltage, decrease in governor fluid pressure below a definite value, etc.

Conclusion

No claim is made that the method of governing described in the foregoing is the ultimate solution of the problem of regulating the speed of hydraulic turbines. It is readily apparent that the results obtained, so far, with this experimental set-up, are more or less academic in value, and that the real worth of the method can only be determined by considerable operating experience which will bring out the weak points and emphasize those which are of advantage.

One possibility which may be deserving of consideration is the ease with which this equipment can be applied to obsolete or antiquated pressure operated governors and thereby secure the sensitivity and satisfactory operation of a modern governor without the expense of replacing the entire installation.

A High-Power Oilless Circuit Interrupter Using Water

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MEMBER AIEE

THE oil circuit breaker has been for many years the standard type of circuit interrupter for heavy-duty power-house service. Since the introduction of modern arc-rupturing devices into these circuit breakers, the occurrence of serious trouble attributable directly to oil switchgear failure has been rare. However, the appearance on the market of various types of oilless circuit breakers of considerable interrupting capacity has encouraged the idea that the complete elimination of oil in power house switchgear may be both desirable and practicable. Further experimental investigation and development activity on the part of switchgear manufacturers seems to indicate that currents of 60,000 amperes or more at 13,200 volts can be interrupted satisfactorily in circuit breakers using either air or water as the arc-quenching medium. This paper describes the construction and operating characteristics of a high-power water circuit breaker.

Water Circuit Breakers

Interrupters using water as the extinguishing medium have a history of some 15 years, and are now used fairly extensively in moderate interrupting capacities, particularly in Germany.[1] In the form designated as "expansion breaker", the arc is drawn in a confined chamber where the generation of steam and gas quickly builds up a high pressure. The chamber is usually constructed of one or more sections resiliently mounted so that passages to a condensing chamber are opened up by the pressure, as indicated in figure 1. It is claimed that the deionization of the arc space is greatly accelerated by the cooling effect accompanying the sudden expansion of the steam. In any case, it is certain that the arc is subjected to a turbulent blast of steam and gas as these arc products make their escape laterally or longitudinally from the recesses in the interrupting chamber. The combined effect of cooling and turbulence causes dielectric strength to be built up with sufficient rapidity that the arc will not restrike.

Since even distilled water is a relatively poor dielectric the moving contact must be lifted clear of the water, and in many designs moved completely out of the chamber so as to interpose a series air break. The condensation of a large proportion of the vapors considerably reduces the external demonstration as compared with an oil breaker, the operation being somewhat quieter than an air switch. Factors which prevent this device from fulfilling the requirements of an ideal breaker include contamination and loss of water, and the possibility of corrosion of metal parts and deterioration of the insulation exposed to water or steam. Nevertheless, these problems are not at all insuperable, and high-capacity water breakers of the type described have been made experimentally for ratings up to 1,500,000 kva at 15 kv.

Water-Breaker Construction

Investigations on the breakdown of water under electric stress, by J. Slepian, C. L. Denault, and A. P. Strom,[2] indicate the desirability of not only removing voltage completely from the interrupting chamber immediately after opening the circuit, but also closing the circuit by an air switch rather than in the water. The experimental water-breaker pole unit shown in figure 2 is arranged with a series disconnect switch and also a main paralleling switch for continuous-current-carrying ratings of 1,200 amperes and above. The pole-unit mechanism is arranged to give the following sequence of operation:

Closing

1. Arcing contacts close in water.
2. Series disconnect switch completes the circuit in air.
3. Main switch closes, paralleling the arcing contacts and disconnect switch.

Opening

1. Main switch opens, diverting the current through the arcing contacts.
2. Arcing contacts open, extinguishing the arc in the water chamber.
3. Series disconnect switch opens, removing potential from the interrupter.

The different positions of the breaker contacts are illustrated in figure 3.

Although condensation reduces the amount of gas vented to the outside of the breaker, heavy local pressure impulses created in the neighborhood of the arc, when interrupting very high currents require exceptionally strong construction of the main body of the interrupter. Figure 4 is a cross section of the interrupting chamber. Overlapping inside and outside bronze cylinders *A* hold a heavy insulating tube *B* in compression and shear to develop maximum strength. Special attention has been given to the problem of shock absorption by providing a large air volume above the water level and also by utilizing the compressibility of air pockets sealed in rubber *C* located in the removable bottom bronze bell cover *D*.

The stationary arcing contact assembly consists of a cluster of fingers *E* while the arcing contact itself is made of a strong rustproof alloy rod *F* with a tip of arc-resisting material. This contact moves in and out of a chamber formed by three large blocks *G* of special insulating material with a centrally located water trapping pocket in each. A hollow spacer *H* above these blocks engages a rubber ring *I*. Pressure under the lowest block raises the whole arcing chamber assembly, compressing this rubber ring and at the same time opening the vent *J*.

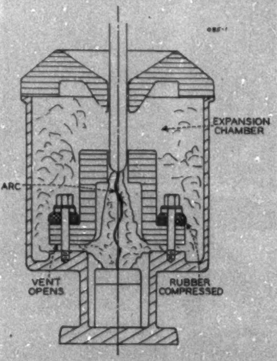

Figure 1. Siemens expansion breaker for low voltage

Paper 40-85, recommended by the AIEE committee on protective devices, and presented at the AIEE summer convention, Swampscott, Mass., June 24–28, 1940. Manuscript submitted November 13, 1939; made available for preprinting April 23, 1940.

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Among those of the author's associates who have aided in the development of this water breaker he wishes to acknowledge particularly the contributions of A. J. A. Peterson and A. H. Bakken in creating a workable mechanical design and the untiring efforts of B. P. Baker in following through the construction and testing of the experimental pole unit.

[1] For all numbered references, see list at end of paper.

Table I. Interrupting Test on Experimental Water Breaker

13,200 Volts, Single Phase, 60 Cycles

Test	Current (RMS Amperes)	Interrupting Time (Cycles)	Arc Duration (Cycles)
Series A			
1	28,000	4.7	1.7
2	47,500	4.5	1.5
3	34,500	4.8	1.8
4	61,000	4.5	1.5
Series B			
1	9,100	4.3	1.4
2	7,500	4.3	1.5
3	25,000	4.4	1.6
4	27,500	4.4	1.6
5	27,500	4.3	1.5
6	26,000	4.2	1.4
7	51,000	3.9	1.2
8	29,000	4.2	1.4
9	44,000	4.3	1.4
10	35,000	4.6	1.7
11	58,000	4.2	1.3
Series C			
1	1,420	5.1	2.1
2	4,750	4.7	1.7
3	4,500	4.3	1.4
4	5,100	4.4	1.6
5	4,900	4.2	1.3
6	34,000	4.2	1.2
7	20,700	4.3	1.3
8	37,000	4.3	1.3
Series D			
1	50	4.2	1.3
2	50	3.8	0.9
3	150	3.2	0.3
4	150	3.5	0.6
5	290	4.5	1.6
6	295	4.5	1.6
7	620	4.4	1.5
8	600	4.5	1.6
9	910	4.2	1.3
10	810	3.7	0.8

Figure 2. 15-kv water breaker experimental pole unit set up for interrupting tests

While research work by Kesselring and others in Germany might seem to have shown that arcs can be extinguished by the cooling action accompanying sudden expansion of saturated vapors surrounding an arc, it is not believed that this high-power water breaker operates in quite the same manner. For instance, no attempt is made to synchronize the opening of the vent and resultant vapor expansion with the instant of current zero. When interrupting a high-current arc, the vent *J* begins to open during the first half cycle of arcing. As the contact tip draws the arc *K* into each successive water pocket, more steam and dissociated gases are formed which turbulently mix into the arc stream on their way to the vent *J*. It should be noted further that the upward displacement of the interrupting elements *G* and *H* as the rubber ring *I* is compressed reduces the volume of the space between the upper block and the contact guide. This results in a quantity of water being forced in a longitudinal flow down along the contact and into the arc stream, tending to sweep the arc products downward and away from the contact tip. The deionization produced by the turbulent flow of the relatively cool water vapor and steam through the arc space is so effective that at the first or second current zero after the arc is drawn into the bottom of the arcing chamber, the dielectric strength builds up with sufficient rapidity that reignition cannot take place and the arc is extinguished.

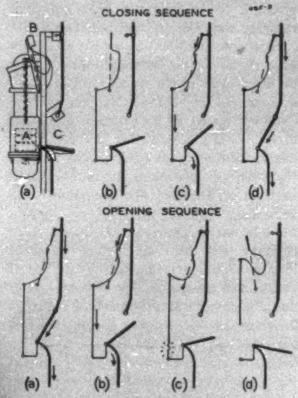


Figure 3. Switching sequence of water-breaker pole unit

A—Arcing break in water chamber
B—Series air disconnect
C—Main paralleling switch

Closing sequence:
(a) Open, (*b*) arcing contacts closed, (*c*) disconnect switch closed, (*d*) main contacts closed

Opening sequence:
(*a*) Closed, (*b*) main contacts opening, (*c*) arcing contacts opening, (*d*) disconnect switch open

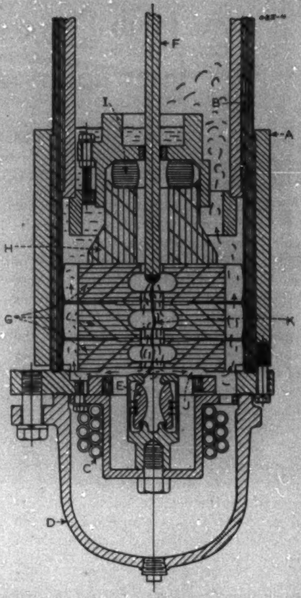


Figure 4. Cross section of interrupting chamber

A—Micarta tube
B—Bronze cylinders
C—Air-filled rubber
D—Bottom bell cover
E—Finger contacts
F—Arcing contact rod
G—Micarta blocks
H—Hollow spacer
I—Rubber ring
J—Arcing chamber vent

When interrupting lower currents the valve-type vent *J* opens only a very small amount, maintaining fairly high pressure in the arcing chamber. This keeps the gas bubble small, so that effective deionization is still obtained and the arcing time is only slightly longer than at high currents. By using only the one vent at the bottom instead of at several points along the arc stream all of the gas is

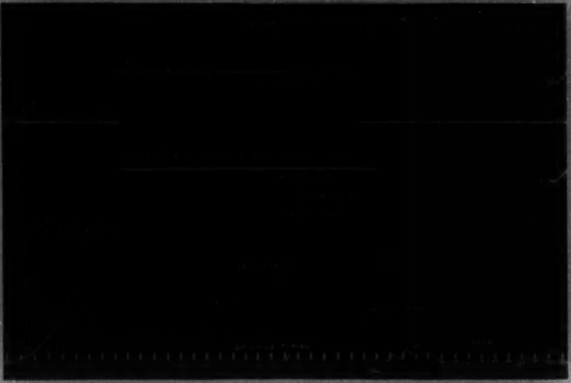


Figure 6. Oscillogram of low-current interruption at 13,200 volts

forced through the restricted arc passage and the deionizing action thus intensified.

The steam and gas from the vents are directed through a condensing labyrinth, shown in figure 5, and the permanent gases such as hydrogen and carbon monoxide that remain after the water vapor is removed pass out of a small port to the atmosphere. The amount of vented gas is estimated to be only about ten per cent of the total quantity generated by the arc, thus making possible almost complete enclosure of the interrupter. Water and steam that may be forced out around the upper contact guide is not thrown out of the breaker but only up into the mechanism chamber. Leakage from the chamber is avoided by providing a close-fitting bushing for the rotating shaft which operates the arcing and disconnect contacts.

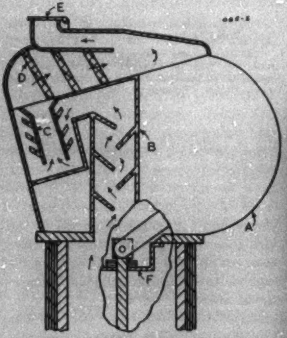


Figure 5. Steam condensing and venting channels

A—Mechanism chamber
B—Baffled passage beside mechanism chamber
C—Centrifugal separator
D—Metal screens
E—Hinged cover
F—Upper contact guide

Test Results

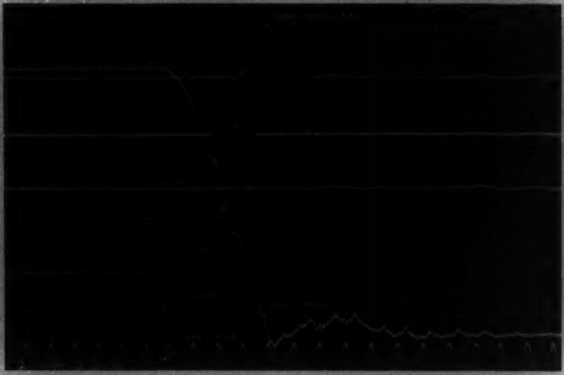
In table I are recorded a few of the results from the many series of interrupting tests on this water-breaker pole unit. A current range of 50 to 61,000 amperes at 13,200 volts is covered in these tests, the arcing time averaging close to 1½ cycles for all currents. Oscillograms of representative opening and close-open interruptions are shown in figures 6, 7, 8, and 9. These show that for only the last one or two half-cycles of this time is there appreciable arc voltage, indicating effective action as soon as flow through the arcing chamber is established. The closing of high currents on the air-disconnect-switch contacts is accomplished with very little arcing due to the high-speed snap action. On the opening stroke, this switch shows a small spark when clearing the very low residual current conducted through the water in the arcing chamber immediately following the interruption of the main arc. The burning at the main contact and fingers is found to be very moderate considering the length of the current path through the arcing contact and disconnect switch to which the short-circuit current must be transferred by the opening of the main paralleling switch.

Distilled water is used in the breaker, samples from the interrupter usually measuring about 100,000 ohm-centimeters just after filling. The tests indicate that 100 or more low- and medium-current interruptions can be made before the resistivity will go to a value which indicates the need for water replacement. From four to six short circuits in the range from 40,000 to 60,000 amperes can be interrupted without changing the water in the breaker. The interrupting effectiveness of this water breaker design is clearly demonstrated by the fact that although the full operating line voltage was applied across a single-pole unit, the arc was quickly extinguished after separating the arcing contacts only a few inches in the single interrupting chamber.

Figure 7. Oscillogram of close-open test interrupting 29,000 amperes at 13,200 volts

Figure 8 (above). Oscillogram of opening test interrupting 58,000 amperes at 13,200 volts

Figure 9 (below). Oscillogram of opening test interrupting 61,000 amperes at 13,200 volts



Advantageous Design Features

Features of particular interest in the design of the water breaker just described are as follows:

(*a*). The bottom bell *D*, figure 4, is readily removable for access to the stationary contact structure without disturbing the mounting of the pole unit.

(*b*). Because of the top enclosure of the pole unit, very little water or gas can escape into the surrounding air when the breaker is opening short circuits. The outlet for gas is designed to strain moisture from it before it reaches the atmosphere.

(*c*). No porcelain is used in the construction of this breaker.

(*d*). The main current-carrying loop being at the rear of the pole unit is of minimum length, and the severe mechanical strains caused by short-circuit currents which occur in many designs are not present in this one.

Conclusions

Tests on a high-power water breaker have shown that water can be used as an arc-extinguishing medium with results comparable to an oil circuit breaker. With proper precautions in the design which take into account the relatively low dielectric strength of water and the possibilities of corrosion, the water breaker becomes very interesting. As further experience is obtained it may find a place in the field of oilless switchgear for central-station service in America as it has in Europe.

Bibliography

1. THE EXPANSION BREAKER, F. Kesselring. *VDI*, volume 78, March 1934, pages 293–7.

2. THE DIELECTRIC STRENGTH OF WATER IN RELATION TO USE IN CIRCUIT INTERRUPTERS, J. Slepian, C. L. Denault, and A. P. Strom. AIEE TRANSACTIONS. Volume 60, 1941.

A Power-System Governor Sensitive to Frequency and Load

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Synopsis: This paper describes a new electrical control system with which a standard power-system governor may be made sensitive to system load as well as frequency. A proposed scheme of application is presented which is believed to permit electrical control of a standard governor instead of mechanical control, without any sacrifice of reliability.

THE frequency of a power system is governed by controlling the power input to the prime movers of the system. Consider, for example, a typical power system in which one hydraulically-driven synchronous generator is governing the system frequency. Since the remaining generating units are operating at constant loads, the power output of the governing unit must be regulated so that the unit will assume any variations of load on the system. Simultaneously, this unit is controlled to hold the system frequency at a constant value.

Figure 1 is the schematic representation of a typical governing system of a hydro-electric generating unit. The power output of this unit is regulated by varying the opening of the needle valve to control the flow of water through the turbine This valve is operated by oil pressure through a control valve and piston, as shown in figure 2. The speed-sensitive element in this governing system is a centrifugal governor, driven from the turbine shaft, which operates the control valve by pivoting the beam about point P.

To meet changing system load, the load level of the governor is varied by the electrically-operated load control. The load control is operated through a contact-making frequency meter, connected to the system, which periodically resets the motor to change the relationship between the elevation of point P and the opening of the needle valve. The governor operation is thereby made isochronous.

It is now pointed out that the governing system just described is actuated by turbine speed and system frequency. A definite time lag must then exist in its operation to compensate for system load changes, since the rate of change of frequency which results from a differential between power generated and power demanded must exist for a definite period of time before frequency and turbine speed change sufficiently to actuate the governor. It is the purpose of this paper to describe a new governing system which is sensitive to both frequency and load.

Operating Principle

Figure 3 is a schematic representation of the proposed governing system. The turbine governor is actuated by an electronic frequency controller, through the medium of a balanced solenoid. This frequency controller provides two distinct components of control to make the governor sensitive to both frequency and load. The first component is derived from the pair of d-c currents I_A and I_B as functions of α, the rate of change of frequency in cycles per second per second. The second component is derived from the pair of d-c currents I_C and I_D as functions of F, the frequency in cycles per second. The controlling currents I_A, I_B, I_C, and I_D are obtained through the use of vacuum-tube circuits in the frequency controller. The characteristic operating curves are shown in figure 3. Current values are indicated in milliamperes. The solenoid plunger is then displaced from its central position in proportion to the current unbalance between I_A+I_C and I_B+I_D. An unbalance of I_A and I_B then actuates

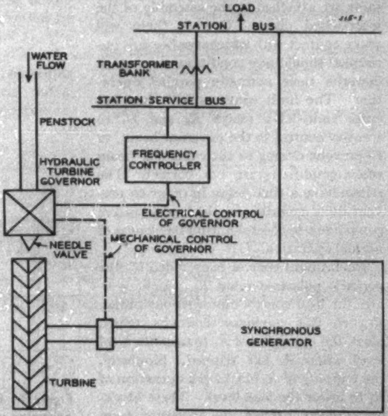

Figure 1. Governing schematic of hydroelectric generating unit

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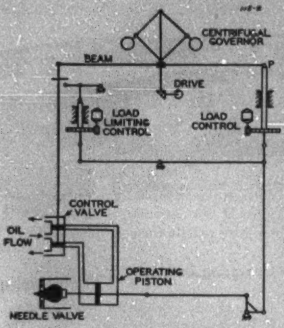

CENTRIFUGAL GOVERNOR
BEAM
P
DRIVE
LOAD LIMITING CONTROL
LOAD CONTROL
CONTROL VALVE
OIL FLOW
OPERATING PISTON
NEEDLE VALVE

Figure 2. Hydraulic turbine governor schematic

the governor instantaneously whenever equilibrium is disturbed between power generated and power demanded. An unbalance of I_C and I_D actuates the governor whenever frequency deviates from its normal value.

The balanced solenoid operates the governor control valve through the conventional mechanical linkage, similar to that shown in figure 2. However, the standard centrifugal governor is retained on a separate beam. The controller beam and the governor beam have a common pivot at one end. At the other end, there is an interlocking pivot arrangement which couples to either one beam or the other. The solenoid M is energized to transfer *from* the governor beam *to* the controller beam. Signal lights S indicate which beam is in use. Four limit contacts are attached to the assembly of the interlocked pivots. One of these will make contact with whichever beam is uncoupled should any condition arise which disturbs their normally parallel operation. The limit contacts will then operate time-delay relays R_3 and R_4 to transfer control to the governor beam, or to prevent closing to the controller beam when conditions are not correct. The relays have a time delay in order to prevent their operation by the load-sensitive component of control during the course of normal operation.

Isochronous control is provided by the periodic polarized relay R_2, which operates the load-control motor through relay R_1. R_3 has auxiliary contacts which block the operation of R_1 to raise the load level when R_3 has tripped. Similarly, the tripping of R_4 blocks the operation of R_1 to lower the load level. These blocking circuits protect the governor from faulty operation of the load control. As indicated in figure 3, automatic time control may be coupled in at relay R_2.

Due to the use of balanced circuits throughout the frequency controller, it is possible to place a balance relay R_6 in the common-current return to terminal E. Any fault conditions within the controller will change the current in R_6, thereby tripping R_6 instantaneously to transfer control to the governor beam. By shorting out R_5, the tripping of R_6 also prevents possible erroneous operation of the load control. Control is automatically transferred to the controller beam when the current through R_6 returns to normal, provided that the transfer circuit has not been opened by either R_3 or R_4.

As outlined above, the control system is protected against faults within the controller by balance relay R_6. If, for any reason, R_6 should fail to operate, either R_3 or R_4 will transfer control to the governor beam. Relays R_3 and R_4 provide a continuous check of the calibration of the frequency controller against the centrifugal governor.

For the sake of simplicity, the load-limiting control is not indicated in figure 3. See figure 2.

The operation of the control system for a sudden increase of load is indicated in figure 4. The curve I_D-I_C represents frequency deviation, and curve I_B-I_A represents rate of change of frequency. The solenoid operation is represented by the sum of these two curves, $-I_A+I_B-I_C+I_D$. The shaded area A for time T represents the net improvement in control that this proposed control system provides over and above that of existing governing systems. The operation of the load-control motor is indicated at the bottom of the figure.

Solenoid Characteristics

In figure 5 are shown the operating characteristics of the balanced solenoid. The ordinates D represent the displacement of the plunger from its central position. The force-versus-displacement curves for each coil change with the frequency applied to the controller. Thus, for an applied frequency of 60.5 cycles per second, the force curves of the two coils intersect at P_3, whose ordinate is the displacement assumed by the plunger. When the frequency applied to the controller is 60 cycles, the force curves intersect at P_4, where the plunger displacement D is equal to zero.

If a differential force dF is applied to the plunger, it can move through a displacement D_2, as indicated. This differential force may represent either the threshold force of the valve linkage or a

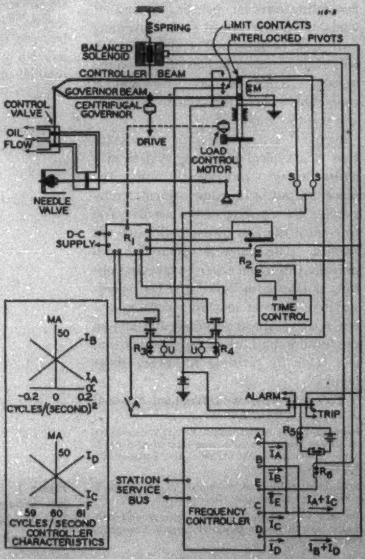

SPRING
LIMIT CONTACTS
INTERLOCKED PIVOTS
BALANCED SOLENOID
CONTROLLER BEAM
GOVERNOR BEAM
M
CONTROL VALVE
CENTRIFUGAL GOVERNOR
OIL FLOW
DRIVE
LOAD CONTROL MOTOR
S
S
NEEDLE VALVE
D-C SUPPLY
R1
R2
TIME CONTROL
R3
U
U
R4
A
ALARM
TRIP
R5
R6
A
B
E
C
D
IA
IB
IE
IC
ID
IA+IC
IB+ID
STATION SERVICE BUS
FREQUENCY CONTROLLER
MA
50
IB
IA
α
-0.2
0
0.2
CYCLES/(SECOND)²
MA
50
ID
IC
F
59
60
61
CYCLES/SECOND
CONTROLLER CHARACTERISTICS

Figure 3. Proposed governing system

force tending to produce oscillation. In figure 6, which indicates the equilibrant forces of the centrifugal governor, it can be seen that the same differential force *dF* produces a relatively greater displacement D_1. Therefore, in comparing a centrifugal governor with a balanced solenoid operating in the same force range, the threshold limit of operation and the magnitude of possible oscillation of the solenoid is about one-half that of the former.

Curve *A* in figure 6 represents the restraining force of the centrifugal governor. The operating displacement is determined by its point of intersection with the lifting-force curve corresponding to the system frequency and turbine speed.

Controller Circuit

Figure 7 is the schematic circuit diagram of the frequency controller. System frequency is supplied to the controller through transformer T_1. A capacitance C_1 is connected in the circuit of one secondary winding, and an inductance L_1 is connected in the other. As frequency increases, the a-c current through R_1 decreases, while that through R_2 increases. The magnitudes of the a-c voltages appearing across R_1 and R_2 are converted into correspondingly proportional d-c currents in R_9 and R_{10}. This conversion is accomplished by vacuum tubes V_1, V_2, V_3, V_4, V_5, V_6, V_7, and V_8. Phase-shifting circuits C_2–R_3–C_3 and L_2–R_4–L_3 convert the single-phase voltage across R_1 to four-phase star voltages at the grids of tubes V_1, V_2, V_3, and V_4. Since the tubes are biased to cut-off, their action is that of rectification. The phase displacement of the plate-current pulses is such as to provide a nearly uniform current in R_9. Circuit C_6–L_6–C_7 filters out the remaining current ripple. The function of V_5, V_6, V_7, and V_8 is similar to that described above for V_1, V_2, V_3, and V_4, except that frequency response is inverse. Polyphase rectification is employed since it is essential that the d-c currents in R_9 and R_{10} should have a minimum ripple. Also, in order to minimize reactive transients, it is desirable to use polyphase rectification in order that circuits C_6–L_6–C_7 and C_8–L_7–C_9 may have a low reactance.

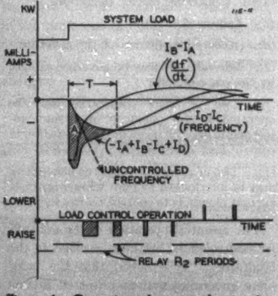


Figure 4. Operation of proposed governing system for abrupt load increase

The d-c voltage across R_9 and R_{10} is a function of the frequency applied to T_1. This voltage is center-tapped by R_{11} and R_{12}, and applied to the grids of V_9 and V_{10}. The plate currents flow through two windings of transformer T_2 and R_{15} and R_{16}. The voltage across R_{15} and R_{16} is applied to V_{15} and V_{16} to provide currents I_C and I_D, as shown in figure 3.

A voltage appears across the secondary winding of T_2 when the plate currents of V_9 and V_{10} are changing with respect to time. This voltage is therefore a function of the rate of change of system frequency with respect to time. It is amplified by V_{11} and V_{12}, and converted by V_{13} and V_{14} into currents I_A and I_B, also as in figure 3.

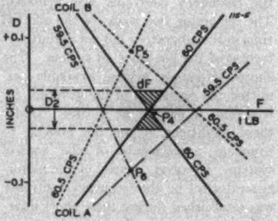


Figure 5. Equilibrant forces of balanced solenoid

An alternative scheme for obtaining the load-sensitive component of control is the use of two capacitors and two resistors instead of the transformer T_2. This replacement of transformer T_2 is indicated in figure 8. A d-c current flows through R_{29} and R_{30} in response to a rate of change of potential across R_{15} and R_{16}. The voltage drop across R_{29} is then applied to the grid of V_{12}, while that across R_{30} is applied to V_{11}.

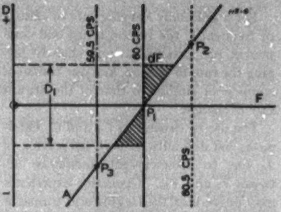


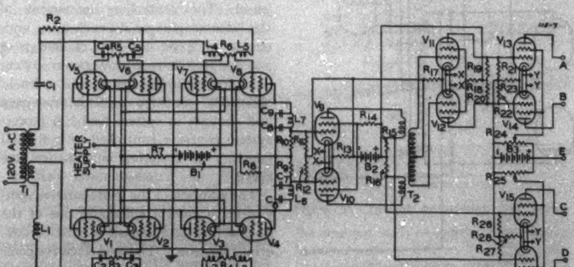
Figure 6. Equilibrant forces of centrifugal governor

The response of the load-sensitive component of control is not strictly accurate when capacitors are used instead of the transformer. Thus, for a suddenly-applied frequency rate of change, the voltage response at the grid of V_{11} is:

$e = k_1k_2R_{30}C + k_3R_{30}e^{-t/rC}$
k_1 = time constant of applied rate of change of frequency
k_2 = voltage—frequency constant across R_{15} and R_{16}
k_3 = constant depending upon capacitor voltage conditions
C = series capacitance of C_{10} and C_{11}
$r = R_{29} + R_{30}$
t = time in seconds

It will be observed that the first term is directly proportional to the frequency rate of change. The second term is a capacitive transient during the initial period of the change while voltages are seeking their proper relationships. During tests, this transient was observed to be of the order of one-half second.

Figure 7. Schematic of frequency controller



For a gradually varying rate of change of frequency, the response has one component proportional to and slightly lagging the rate of change, and one transient component similar to that of the abrupt rate of change.

The use of capacitors C_{10} and C_{11} eliminates the difficulty of core saturation of T_3 due to plate-current unbalance between V_9 and V_{10}. Operating experience may prove that the lagging and transient components present with the use of capacitors have no deleterious effect upon the quality of the frequency control.

The characteristics of the controller are readily variable. While maintaining the same normal current, the slopes of the I_A and I_B curves may be varied by adjusting R_{19} and R_{20}. Similarly, the slopes of the I_C and I_D curves may be varied with R_{15} and R_{16}. The current level of I_A and I_B is controlled by R_{24}. That of I_C and I_D is controlled by R_{25}. The normal operating points of the controller are independent of voltage variations at T_1. Supply-voltage variations affect only the slopes of the characteristic curves. The variation in slope is approximately proportional to the voltage variation. However, the effect of such variations is negligible.

Tests

The solenoid-operated control valve used in testing the frequency controller is shown in figure 9. The over-all length of this valve is seven inches. The operating piston, or servomotor, was operated by oil flowing through the two valve ports near the center of the valve. The component parts of the valve are shown in figure 10. This photograph shows the rear view of the brass valve cylinder. Oil under pressure was supplied to the center connection. Oil discharged from the operating piston passed through the outer two connections. The soft-steel valve plunger moves longitudinally through the two solenoid coils. The two center flanges match the valve ports shown in figure 9. Small diagonal holes were drilled through the flanges in order to revolve the plunger by the reaction of oil passing through. This rotation of the plunger maintained its freedom of movement. The two coils were connected to the frequency controller. Through the medium of the solenoid valve, the controller then controlled a piston-operated rheostat in the field circuit of a d-c shunt motor. This motor drove a 15-kva synchronous generator, from which the frequency controller was operated. The remaining two weeks of the allotted time in the laboratory did not permit perfection of the governor oil system. The simple oil-supply system available permitted the operation of the governor for less than a minute at a time. Under these conditions, it was not possible to complete adjustments of the control system. However, control of the motor-generator set by the frequency controller was established during these short intervals.

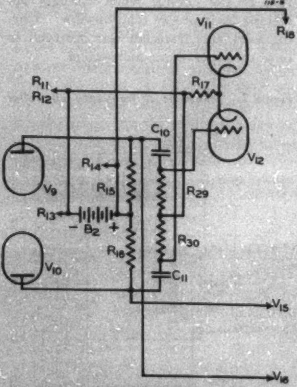


Figure 8. Alternative load-sensitive circuit for substitution in figure 7

Conclusions

A new electronic system of frequency control has been developed in the laboratory. It is believed that this control system can be adapted to standard hydraulic governors now in use.

The electronic frequency controller described in this paper provides movement of a solenoid plunger comparable to the controlling movement of a fly-ball governor. The controlling movement of a fly-ball governor depends upon change of speed. The controlling movement of the solenoid plunger depends upon both change of speed and rate of change of speed. It follows that a power-system governor controlled by such a solenoid should be sensitive to both frequency and changes in system load. A power-system governor controlled by a fly-ball governor is sensitive only to turbine speed.

The addition of the load-sensitive component of control eliminates one of the several time lags present in existing governing systems. The load-sensitive component of the solenoid motion actuates the governor to oppose hunting swings.

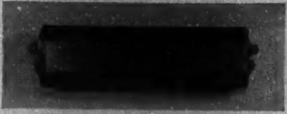


Figure 9. Solenoid-operated control valve used in tests

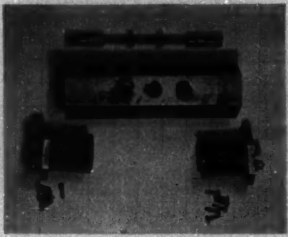


Figure 10. Component parts of control valve

It is believed that the solenoid described in this paper can be substituted, with no sacrifice of reliability, for the standard fly-ball governor head, provided that the latter is retained in a stand-by capacity.

The scheme of controlling machines in accordance with both speed and rate of change of speed is not new, as inertia governors have been used for many years in some applications other than electric power systems. It is believed, however, that the electronic control system herein proposed has the following advantages over the mechanical inertia governor:

(*a*). Response to rate of change of frequency separately adjustable during operation.

(*b*). Response to frequency deviation separately adjustable during operation.

(*c*). Increased sensitivity.

(*d*). Reduced oscillation of the governor.

(*e*). Several units may be governed at once by the one controller.

(*f*). Two or more controllers may be operated on the system at once, since adjustments may be made during operation to control load division.

(*g*). Two or more controllers on one system may be interlocked by pilot wires or carrier-current channels to control tie-line loads.

The essential operation of the controller and solenoid has been verified in the laboratory. It is hoped that the complete governing system proposed in this paper may be submitted to the test of operating experience in the near future.

Automatic Printing Ammeter

T. G. LE CLAIR
FELLOW AIEE

Synopsis: A new instrument has been developed which will automatically read and record in numerals, at predetermined time intervals, the ampere loading on 50 different electric circuits. This instrument also adds the readings on groups of circuits and prints these totals in columns beside the ampere load readings. The paper outlines briefly the reasons for developing such a device. The paper also describes, with the aid of schematic diagrams, the process by which the instrument obtains and prints all the measurements and totals on a single "log sheet."

Need for Compact Ammeter Records

THE rating of most electrical equipment is on an ampere basis. To determine when additional capacity in lines, transformers, or generators, is required on a power system, it is necessary to have an accurate record of past ammeter loads. For maximum usefulness, these ammeter records should be as compact and legible as possible for ready determination of the shape of the load curves and rate of growth.

In large power stations it is common practice for an operator to note and record on a "log sheet" the ammeter readings on all outgoing transmission lines at frequent intervals. When several of these outgoing lines supply the same load it is necessary to totalize the loads on all lines in a given group in order to estimate the load growth on the combined group. Figure 1 illustrates a typical group of substations fed by several lines from the same generating station. When 50 to 100 or more lines emanate from the same station the instrument readings must be recorded with a minimum of effort and in the most usable form. Recording instruments would be suitable for this purpose if their record could be compact enough.

Substations such as substation *A* in figure 1 may frequently be of an unattended type. In such a substation it is desirable to have ammeter records in convenient and compact form of the loading on transformers and distribution feeders. It is also desirable to obtain these records automatically, to avoid the expense of an attendant.

Available Types of Instruments

At present a number of types of instruments are available which can be used for obtaining compact records automatically. The most common type, of course, is the ordinary graphic instrument with one instrument installed on each circuit. These instruments are available with charts for various periods from one day up. There is also available a graphic instrument for ten circuits which, instead of drawing a load curve, puts on the graph paper a series of characters so that the line connecting the same characters is the equivalent of a graphic record. For totalizing the readings of several circuits, equipment is available by which either amperes or watts, a-c, can be converted to a d-c voltage, and the d-c voltages from several of these converters connected in series on a d-c instrument for a totalized reading.

In Chicago unattended substations the practice has been to use graphic instruments for ammeter records. In generating stations compact recordings of line readings are obtained by having an operator read a series of indicating instruments and record these readings on a log sheet in rows and columns. The usual arrangement is that each column represents the readings on a given circuit and each row is the series of readings taken at a given time. An increasing need was felt to relieve the operator of this burden in large stations, or to obtain these readings in substations where an operator is not on hand for 24 hours a day. This need resulted in the development of a measuring combination with the following basic principle:

Fundamental Principle of New Instrument

A single accurate measuring device is constructed which is automatically connected in turn to each of the circuits to be measured. The single measuring device is used in combination with a printer which records on a log sheet figures which are a measure of the magnitude of the current. The column position of a given reading on the log sheet signifies the circuit on which the reading was taken. The row position of the reading on the log sheet signifies the time when the reading was taken. The measuring device itself utilizes a balancing relay in which the current to be measured is balanced against the current from a standardizing source which is co-ordinated electrically to the printing device.

The totalizing of the readings is accomplished by means of additional mechanism which prints the total figures beside the readings which have been totalized. The new instrument eliminates the human error, both in the accuracy of the readings and in the timing of the periodic readings.

Laboratory Development

A simple experimental instrument was built first to show that the measuring principle would work. Later a full size model was made to record all the ammeter readings in one supervisory control substation. The complete model is shown in the photograph, figure 2. This model was constructed in the laboratory from various available standard parts. Recording is done on an ordinary typewriter on which the typewriter keys are operated by solenoids rather than by human fingers. The operating solenoids can be seen in the photograph directly under the typewriter keyboard at the top of the instrument cabinet. A sample of the record sheet obtained in this machine is

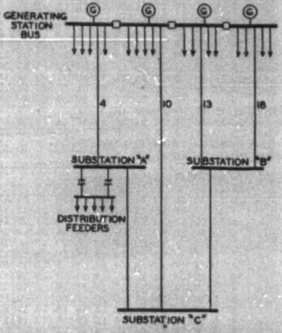


Figure 1. Typical transmission line group

The maximum loads on individual transmission lines may occur at different times due to different load characteristics on the three different substations

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Paper 41-3, recommended by the AIEE committee on instruments and measurements and presented at the AIEE winter convention, Philadelphia, Pa., January 27-31, 1941. Manuscript submitted March 30, 1940; made available for preprinting October 26, 1940.

T. G. Le Clair is supervising development engineer for the Commonwealth Edison Company, Chicago, Ill.

The writer wishes to express his appreciation to Mr. H. M. Schaibly, whose quick perceptions and nimble fingers turned sketches and diagrams into a working machine. Appreciation is also due to Mr. J. R. Harrington and his associates for reducing the laboratory product to a flexible standardized design.

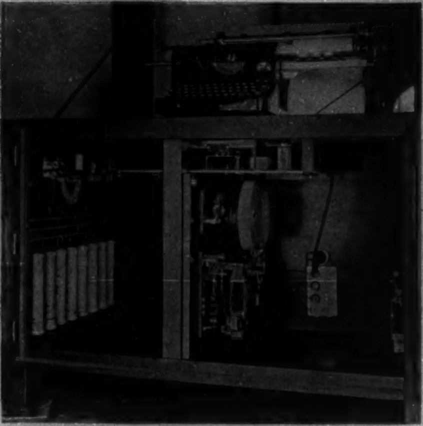


Figure 2. The first full-size automatic printing ammeter

This multicircuit printing ammeter, built in the laboratory, was the first one to record the loads on all transformers and outgoing 4,000-volt feeders of a complete substation

shown in figure 3. On this log sheet the first 12 columns of figures are the ammeter readings on each of the three phases of the four transformer banks. The remaining columns are the ammeter readings on the three phases of the outgoing 4,000 volt feeders. The spare circuit and circuit number 30 were not carrying load on the day this log sheet was taken. The figures on the log sheet are the ammeter readings in tens of amperes.

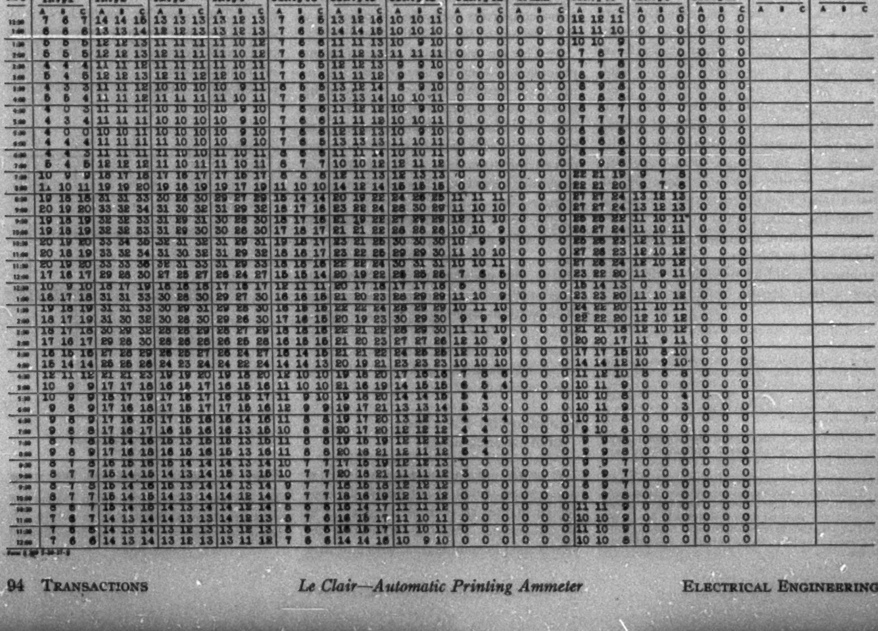
This machine, which has been in constant service for about three years, has given very satisfactory results and is adequate except for its home-made appearance. A detailed description of this printing ammeter and its method of operation will not be given in this paper because the principles are similar to those of the newer model which is appreciably more complete. The home-made instrument records the ammeter readings on 36 circuits and takes these readings every 30 minutes. At the end of the day, the log sheet for that day, in triplicate, is ejected from the machine automatically, and the log sheet for the following day is inserted ready for use.

Improved Design

After the laboratory model had been given a thorough trial in commercial operation, it was considered desirable to install a similar instrument in a generating

Figure 3. Typical log sheet from first full-size printing ammeter

This is a photograph of an actual log sheet showing the instrument readings on a typical day on all the circuits in an unattended substation



Daily Record of Ampere Loads on 4-Kv Circuits and Transformers

GOOSE ISLAND Substation Date 3/1/40

CIR. #	TR.#1			TR.#2			TR.#3			TR.#4			CIR.#43			CIR.#45			CIR.#41			CIR.#42			SPARE			CIR.#48			CIR.#3			CIR.#30								
	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
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10:30	20	19	20	33	34	35	32	31	32	31	29	31	19	18	17	23	21	25	30	30	30	10	9	9	0	0	0	26	26	23	12	11	12	0	0	0						
11:00	20	18	19	33	32	34	31	30	32	31	29	32	18	18	17	23	22	25	29	29	30	11	10	10	0	0	0	27	28	23	12	10	12	0	0	0						
11:30	20	19	20	33	33	35	32	31	33	31	29	33	18	18	18	22	22	24	30	31	31	10	10	11	0	0	0	27	28	24	12	10	12	0	0	0						
12:00	17	16	17	29	28	30	27	25	27	26	24	27	15	15	14	20	19	22	28	25	26	7	6	5	0	0	0	23	22	20	11	9	11	0	0	0						
12:30	10	9	10	18	17	19	18	18	18	17	18	17	12	11	11	20	17	18	17	17	18	6	0	0	0	0	0	15	14	13	0	0	0	0	0	0						
1:00	18	17	18	31	31	33	30	28	30	29	27	30	16	16	15	21	20	23	28	29	29	11	10	9	0	0	0	23	23	20	11	10	12	0	0	0						
1:30	19	18	19	31	31	33	30	29	31	29	28	30	18	18	16	22	22	24	28	29	29	10	11	10	0	0	0	24	22	20	11	10	11	0	0	0						
2:00	18	17	19	31	30	32	30	28	30	29	26	30	17	16	15	20	19	23	30	29	30	9	9	9	0	0	0	22	22	20	12	10	12	0	0	0						
2:30	18	17	18	30	29	32	28	28	29	28	27	29	18	18	16	22	21	22	28	29	30	11	11	10	0	0	0	21	21	18	12	10	12	0	0	0						
3:00	17	16	17	29	28	30	28	26	28	26	25	28	16	16	16	21	20	23	27	27	26	12	10	9	0	0	0	20	20	17	11	9	11	0	0	0						
3:30	16	15	16	27	28	29	26	25	27	26	24	27	16	14	15	21	21	22	24	25	25	12	10	10	0	0	0	17	17	15	10	8	10	0	0	0						
4:00	15	14	14	25	25	26	24	23	24	24	22	24	14	14	13	20	19	21	23	23	23	10	10	10	0	0	0	14	14	12	10	9	10	0	0	0						
4:30	12	11	12	21	21	23	19	19	20	19	18	20	12	10	10	19	18	20	17	16	16	7	6	6	0	0	0	11	12	10	8	6	8	0	0	0						
5:00	10	9	9	17	17	18	16	15	17	16	15	16	11	10	10	21	16	19	14	15	15	6	5	4	0	0	0	10	11	9	0	0	0	0	0	0						
5:30	10	9	9	18	17	19	17	16	17	16	16	17	11	9	10	19	18	20	14	14	16	5	4	0	0	0	0	10	10	8	0	0	4	0	0	0						
6:00	9	8	9	17	16	18	17	15	17	17	16	16	12	9	9	19	17	21	13	13	14	5	3	0	0	0	0	10	11	9	0	0	3	0	0	0						
6:30	9	8	9	17	18	18	17	15	16	16	14	16	11	8	8	19	17	20	13	12	13	4	4	4	0	0	0	10	10	8	0	0	0	0	0	0						
7:00	9	8	8	17	16	17	16	15	16	16	13	16	10	8	8	20	17	20	12	12	12	4	4	0	0	0	0	9	10	6	0	0	0	0	0	0						
7:30	8	7	8	16	14	16	16	13	16	16	13	15	11	8	8	19	16	19	12	12	12	5	4	0	0	0	0	9	9	8	0	0	0	0	0	0						
8:00	9	8	9	17	16	18	16	15	16	15	13	16	11	8	8	20	18	21	12	11	12	6	4	0	0	0	0	9	9	8	0	0	0	0	0	0						
8:30	8	7	8	16	15	16	16	14	14	14	13	14	10	7	7	17	15	19	12	12	13	0	0	0	0	0	0	9	9	7	0	0	0	0	0	0						
9:00	8	7	7	15	14	15	14	13	14	15	13	15	10	7	7	20	18	21	11	11	12	3	0	0	0	0	0	9	9	7	0	0	0	0	0	0						
9:30	8	7	8	16	14	16	16	13	14	14	13	14	9	7	7	18	18	18	12	12	12	0	0	0	0	0	0	8	9	7	0	0	0	0	0	0						
10:00	8	7	7	15	14	15	14	13	14	14	12	14	9	7	7	18	16	19	12	11	12	0	0	0	0	0	0	8	9	8	0	0	0	0	0	0						
10:30	8	8	7	15	14	15	14	13	14	14	12	14	8	8	8	16	14	17	11	11	12	0	0	0	0	0	0	11	11	9	0	0	0	0	0	0						
11:00	7	8	7	14	13	14	14	13	13	14	12	13	8	7	6	16	15	17	11	10	11	0	0	0	0	0	0	10	10	9	0	0	0	0	0	0						
11:30	7	6	5	14	13	14	13	12	13	13	12	13	6	6	6	16	15	17	11	10	11	0	0	0	0	0	0	11	10	9	0	0	0	0	0	0						
12:00	7	6	6	14	13	14	13	12	13	13	11	12	7	6	6	14	14	16	10	9	10	0	0	0	0	0	0	10	10	8	0	0	0	0	0	0						

station to record the ampere measurements on 45 outgoing transmission lines. However, in this station the indicating instruments are arranged in the same order on the switchboard as are the line switches on the busses in the station. The operator arranges the columns on his log sheet in the same order to avoid errors. Furthermore, the 45 lines are divided into ten different groups. The lines to a given group of load are not on adjacent switch positions, but are scattered throughout the station, as illustrated in figure 1, in order to diversify the supply to the load. This makes it necessary for the planning engineer, in using the log sheet, to look at scattered columns in order to totalize the loads in each group. Therefore, the design of the instrument was expanded to include a totalizing mechanism which records the total of the loads on the lines of each group in addition to the individual line loads.

The design was also increased in flexibility so that it would be a relatively simple matter to change the order in which the instrument readings appear on the log sheet or to change the groups of readings included in a given total. This was accomplished by the use of a plugging board arranged so that the secondary current circuits of transmission lines may be connected to the instrument in any order or grouping desired without changing any of the permanent wiring in the instrument. This is very convenient as new transmission lines are installed, or lines are cut over from one substation group to another, at intervals of a few months.

The new design also includes a printer which is designed as a part of the combined unit replacing the more cumbersome typewriter used in the early model. The result of this development is the instrument shown in the photograph, figure 4.

Parts of the Measuring Equipment

The measuring equipment in its finished form is shown in the photograph, figure 4. Figure 4*a* is a front view with the covers removed to show all of the operating parts of the equipment. Figure 4*b* is a rear view also with the cover removed, showing test switches. Each circuit to be measured is connected to a pair of these test switches, arranged so that the current circuit can be short-circuited and disconnected without disturbing any of the wiring within the instrument. The individual parts of the unit can be identified by reference to figure 5 which shows each element identified by name in the same physical position which it occupies in the photograph, figure 4*a*. For ease in describing the parts in the following method of operation, it is assumed that the current transformer ratios are 1:1 and that the instrument has a full scale reading of 100 amperes.

A contact-making clock is used to start the sequence of operations to obtain readings. This contact-making clock may be either a simple device making contact every 15, 30, or 60 minutes, or it may be set up with a definite program to take readings at variable intervals.

The master sequence switch is a rotary control switch used to determine the order in which the various functions of the instrument are performed. Each step in the process of taking measurements is controlled by this master sequence switch.

The circuit selector switch is a 50-point rotary switch with each of the circuits which are to be measured connected to one of the points or positions. A common connection leads from this switch to the measuring part of the equipment. Each circuit to be measured is normally short-circuited on itself in this selector switch but not connected to any other circuit. The circuit selector switch is used to connect each circuit in turn to the remainder of the equipment.

The balancing relay has two cores pulling at opposite ends of a balancing beam. The coil on one core is connected in turn to each circuit to be measured through the circuit selector switch; the current in this element is the current flowing in the measured circuit. The other core has two coils; each of these coils is supplied a variable current which is adjusted until the relay comes to a balance.

The measuring transformer is connected to a constant voltage a-c source and has two secondary windings to supply current to the two coils on the one core of the balancing relay through measuring switches.

The tens measuring switch is a rotary switch used to select steps on a resistor in the circuit from the measuring transformer to one coil of the balancing relay. Each step taken by this measuring switch increases the current in one coil of the balancing relay by ten amperes. Separate contacts on this measuring switch are connected to corresponding solenoid stops in the printing mechanism to select figures corresponding to the tens of amperes in the circuit.

The units measuring switch is similar to the tens measuring switch except that each step that this rotary switch takes varies the current in its corresponding coil of the balancing relay by one ampere instead of ten amperes. Separate con-

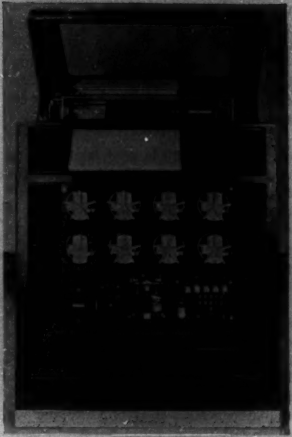


Figure 4a. Front view of standard model automatic printing ammeter

Covers have been removed to show mechanism. Printed forms feed from the front compartment through the printer mechanism at top. The typed figures on the log sheet are continuously visible through the glass cover when operating

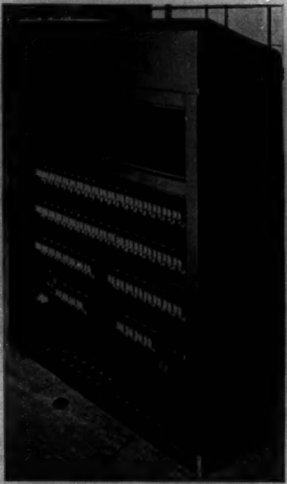


Figure 4b. Back view of standard model automatic printing ammeter

Covers have been removed to show test switches, one pair for each circuit to be measured. Completed log sheets drop into the compartment above the test switches tacts in this rotary switch are also connected to the solenoid stops in the printing mechanism.

The totalizing sequence switch is similar to the master sequence switch except that it controls the order of procedure for the printing of totals only.

Three totalizing rotary switches are used. One switch is used for totalizing units, the second one for tens, and the third for hundreds of amperes. Their method of operation is similar to that of an ordinary odometer. Each totalizing switch has contacts connected to the points in the printing mechanism to select the correct figures to be printed.

The printer consists essentially of a small type wheel which has numerals engraved on its periphery. This type wheel is caused to rotate after the instrument has struck a balance. Solenoid stops are used to position the type wheel so that the proper number is opposite the log sheet. The type wheel is then forcibly pressed against the paper and the carriage of the type wheel is shifted to the right one space ready for the next printing operation.

A number of control buttons are mounted on a panel in front of the printer. These buttons allow the operator to type line numbers at the top of the log sheet. There are also buttons for other manual control operations.

Method of Operation

An exact description of all the connections in the instrument and the method of operation would be unnecessarily long. An abbreviated description will show the essential steps in the operation without the detailed connections. The method of operation can be traced by reference to figure 6 in connection with the description. Figure 6 is a schematic or block diagram which shows each major element in relation to its place in the sequence of operations rather than in its physical position.

The numbers in parentheses inserted in the following paragraphs refer to corresponding numbers in circles in figure 6. The numbers represent the order of the steps in which the various parts of the equipment are brought into operation.

Assume that all circuits are dead and all rotary switches are in their normal or home positions. (1). The contact-making clock closes its contacts to start the taking of a reading. Through the operation of this clock and the master sequence switch the circuit selector switch connects the first circuit to be measured with one coil of the balancing relay.

The units measuring switch is first set in a position which supplies nine amperes in one coil of the balancing relay so that, if the current in the circuit to be measured is less than nine amperes, the balancing relay will trip. However, if the current in the circuit to be measured is greater than nine amperes, nothing happens.

(2). The master sequence switch through the balancing relay next starts the tens measuring switch to rotate. Each step that this switch takes increases the current in the corresponding coil of the balancing relay by 10 amperes, so that the corresponding total pull on the relay is successively 9 amperes, 19 amperes, 29 amperes, etc., until the balancing relay trips. For example, assume the current to be measured is 36 amperes. In this case the relay trips in the third position when the pull of 39 amperes overcomes the pull of 36 amperes. The tens totalizing switch is driven synchronously with the tens measuring switch, but no connection is made from the totalizing switch to the printer at this time. (3). The tripping of the balancing relay completes a circuit through the second group of contacts on the tens measuring switch to select the correct solenoid stop on the printing mechanism to print on a log sheet the number corresponding to the tens of amperes measured (in the example, the number 3).

(4). The master sequence switch transfers the operation to the units measuring switch. The units measuring switch is first placed in the position of zero current in the units coil so that the balancing relay then has a total current less than that in the circuit to be measured. (In the example 30 amperes.) The units measuring switch then rotates, increasing the current in its corresponding coil one ampere at each step until the balancing relay again trips. The units totalizing switch also operates synchronously with the units measuring switch but does not operate the printer. (5). The tripping of the balance relay then selects through the separate contacts of the units measur-

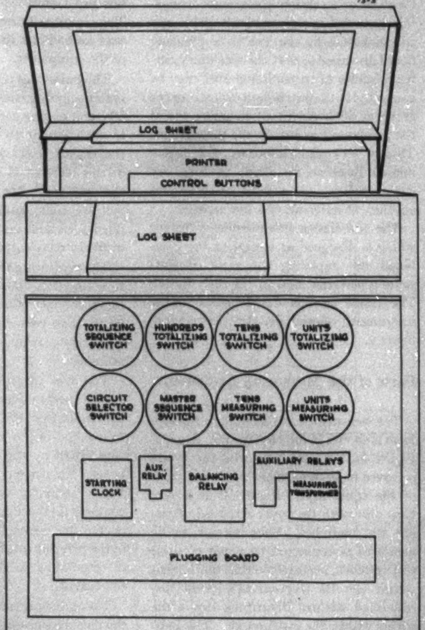


Figure 5. Outline sketch of printing ammeter

Outline sketch of front view in figure 4a on which each unit of the equipment has been labeled for identification

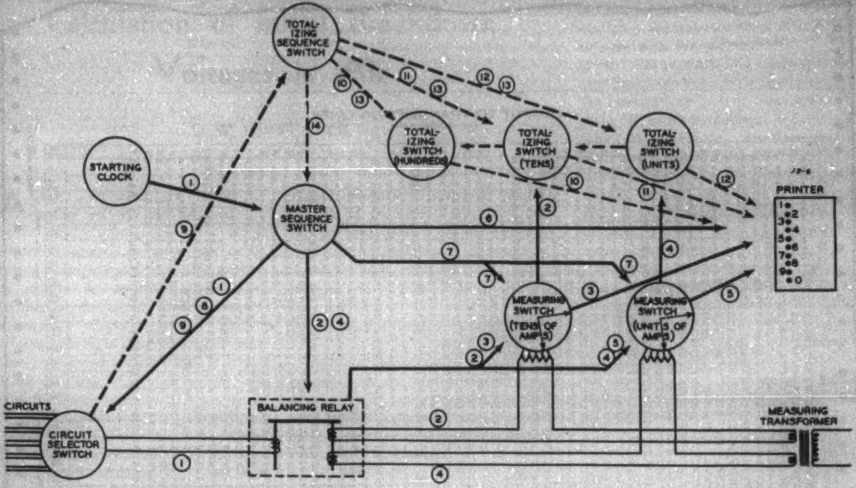


Figure 6. Block diagram for operating sequences of the instrument

Same elements are shown as in figure 5, except that elements are shown in relation to their sequence in the operation of the instrument and not in their physical position. Small numbers in circles refer to the order in which the steps are taken in the process of recording instrument readings. These numbers correspond to numbers in the description in the text

ing switch the correct solenoid stop on the printing mechanism to print the figure corresponding to the units of amperes measured (in the example, the number 6). (6). Then the master sequence switch operates the printer to space the carriage ready for the next reading.

(7). The master sequence switch next restores the tens and units measuring switches to the home position. However, the totalizing rotary switches are not restored to the home position at this time, but remain as set by the first reading.

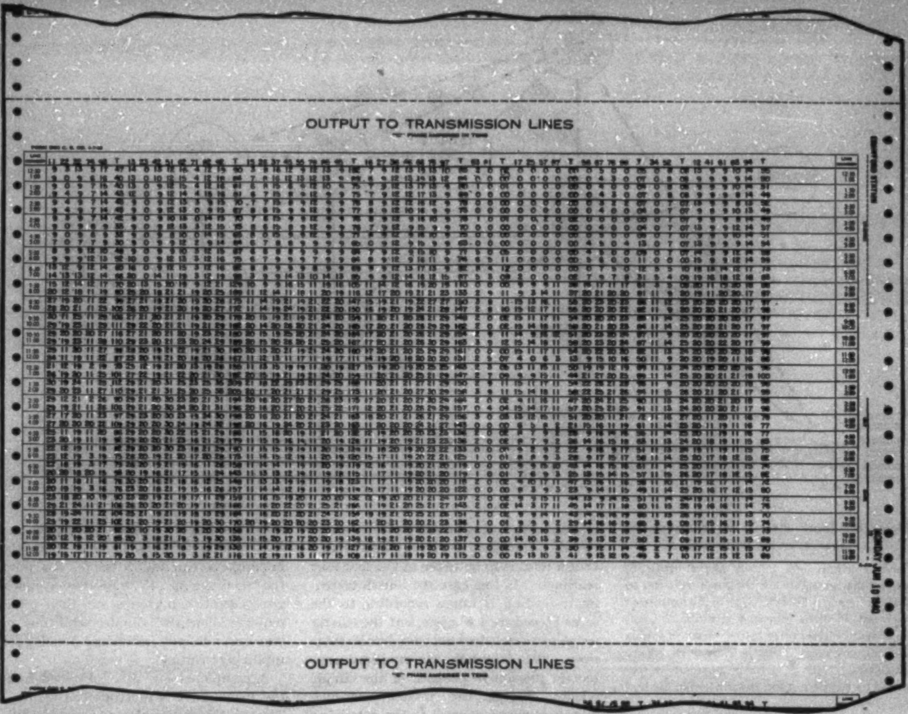
The steps just described complete the cycle of operation for taking and printing the instrument reading for one circuit. (8). The master sequence switch next advances the circuit selector switch one step in order to disconnect the first measured circuit from the balancing relay and connect the second circuit to be measured to the balancing relay. The operation of measuring and printing is repeated for the second circuit exactly as for the first.

The circuit selector switch is equipped with separate contacts by means of which a total reading can be taken after the instrument reading on any predetermined circuit. For example, it may be assumed that a total is to be taken of the first four readings. In this case the fourth instrument reading is taken according to the same procedure as before, but the master sequence switch does not complete its rotation. (9). Through the auxiliary contact in the fourth position of the circuit selector switch the master sequence switch transfers control to the totalizing sequence switch. The totalizing sequence switch then connects to the printer in turn the (10) hundreds, (11) tens, and (12) units totalizing switches to record the positions of the dials, which correspond to the total of the figures printed for the first four instrument readings. (13). The totalizing sequence switch then returns the three totalizing switches to the normal or home position and (14) transfers control back to the master sequence switch to complete its travel and go on with the next reading.

Additional equipment which is relatively simple and not shown on the block diagram is used to return the printer carriage to the home position, advance the log sheet one line for the next reading, and de-energize the instrument at the end of a complete set of readings. Other auxiliary equipment is also used to eject one log sheet from the printing ammeter and insert a new one at the end of a specified period when the log sheet is filled.

This description of the sequence of operation may create an illusion as to the length of time required for a complete set of readings. Actually the relays and switches operate much faster than the human eye can follow them. To print the readings of 45 transmission lines, grouped into ten groups, and their totals requires about two minutes total time, or two seconds per individual reading or totalizing printing.

A completed log sheet obtained from the instrument is illustrated in figure 7. The printing mechanism types across this log sheet every 30 minutes and at the end of the reading advances the sheet automatically to the next line. The right-hand and left-hand columns of figures on the printed form are the hours of the day at which each row of instrument readings was recorded. The typed numbers in the top row are the numbers of the outgoing transmission lines and the numbers below these column headings are the ampere readings in ten's on each line. The letter "T" above a column represents the column of totals for the group of line readings directly to the left of this column. For example, the first total is of the ampere readings on line numbers 11, 22, 32, 74, and 93. The picture shows also the top of the form for the following day. When the sheet is advanced automatically at the end of the day, the next form is ready for the first reading at 12:30 a.m. and the log for the day can be torn off at the perforations. In unattended substations the continuous forms can accumulate for several days in the compartment back of the printer until an operator arrives to remove them.



OUTPUT TO TRANSMISSION LINES

OUTPUT TO TRANSMISSION LINES

Figure 7. Typical log sheet obtained from the automatic printing ammeter

Photograph shows the log sheet for one day with the bottom edge of the log sheet for the preceding day and the top of the sheet for the following day above and below the perforated lines on the continuous form. Column headings are the numbers of the outgoing transmission lines

The complete printing ammeter contains a number of relay contacts. However, not many of these contacts are involved in a particular operation. The printing ammeter shown in figure 2 has been in regular operation for about three years and has been nearly free of trouble since the initial period when it was subject to frequent tampering for the benefit of curious visitors. The instrument shown in figure 4 has been in service nearly a year. It appears that the maintenance costs on one printing ammeter for a multiplicity of lines will compare very favorably with the maintenance cost on a number of indicating ammeters, one for each line.

Fields of Usefulness

One of the major fields in which this device is very useful is in automatic or supervisory control substations. In such substations the installed cost of this multiple-circuit instrument should approximate the installed cost of individual instruments on each circuit. For such installations a traveling operator has only one instrument from which to remove records. The single record sheet is much more convenient to handle, mail, and file, than a large number of graphic instrument charts containing similar information.

In generating stations or larger substations which are attended, the elimination of the human element results in a more accurate and more legible record. All readings are taken on scheduled time as determined by the clock contacts whether or not the operator is interrupted by the telephone or other operating duties. In busier galleries there may be a saving of labor.

The instrument can be made in more portable form so that it can be used for special tests or for periodic routine load checks if desired. The plugging board, similar to a telephone switchboard, makes it relatively simple to connect circuits in any desired order for special test purposes.

For any of these applications, the instrument is appreciably more accurate than the standard switchboard type of instrument. In a device such as this where only one measuring element is used for a multiplicity of circuits, it is economical to have not only a more accurate, but a more rugged measuring element. The laboratory model of the instrument has been in service about three years, during which time the measuring element has been dismantled and put back together for check, and it has not required any recalibration to maintain an accuracy of plus or minus one per cent.

Where records are used in more than one office for various purposes, the instru-

Calculation of Initial Breakdown Voltages in Air

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ALTHOUGH spark-over and corona starting voltages can be predetermined satisfactorily in cases for which empirical formulas or curves are available, a more generally applicable method is needed. An approach to such a method is through a modification of Townsend's theory[1,1a] which, though possibly lacking in rigor, suffices in many cases for the accurate calculation of breakdown voltages. Calculations of this kind by Schumann[2] in 1923 for air in nonuniform fields were not entirely successful because of inadequate knowledge of the Townsend coefficients for air, a deficiency now remedied.[3,4,5] Loeb has discussed the modified Townsend theory at length and has applied it quantitatively to nitrogen in uniform fields.[6,7,8a]

The present paper outlines the theory and shows that the initial breakdown voltage may be calculated using two quantities characteristic of the gas for cases where the field, if nonuniform, converges toward the cathode. Formulas expressing one of the characteristic quantities for air, the Townsend coefficient α, are assembled, and a formula for the second quantity is deduced from an analysis of published spark-over gradients for plane gaps. The method is applied to two cases and the results are shown to be in agreement with experiment

Fundamental Considerations

Electrical breakdown of a gas depends primarily on the presence of free electrons. Their concentration, normally very small, is determined by the relative activity of the ionizing agents, radioactive emanations for instance, and the effects which cause them to disappear. Besides recombining directly with positive ions, the free electrons may be swept away by an applied field, and in air they may form negative ions by attachment to neutral oxygen molecules.

In an electric field, the electrons may gain sufficient energy to excite or ionize molecules in their paths. When the gradient exceeds 15 kilovolts per centimeter in atmospheric air, the electrons are more likely to ionize than to become attached to oxygen molecules. Then each electron as it travels toward the anode continues to make ionizing collisions, liberating more electrons which in turn multiply similarly. A single electron set free by an external agent can thus initiate an *electron avalanche* consisting of a cluster of electrons which grows rapidly as it advances and which leaves a trail of positive ions. With the field strength occurring at breakdown, an avalanche has a velocity of the order of 10^7 centimeters per second, while the positive ions left in the trail move back toward the cathode with but a hundredth of this speed. The negative ions resulting from attachment of electrons to molecules and the positive ions cannot gain sufficient energy to ionize by collision in fields attained at breakdown. The ions generated by a single avalanche disappear quickly, causing merely a small pulse of current, and the passage of a single or even many separate avalanches from cathode to anode does not necessarily result in breakdown.

There are, however, *secondary mechanisms* by which an avalanche can cause the liberation outside itself of electrons which may start new avalanches. If each avalanche initiated causes the release of at least one secondary electron in a favorable location, a self-sustaining discharge current, that is breakdown, can result from the liberation of a single electron by an external agent. The exact nature of the secondary mechanisms active in given circumstances remains a matter for speculation[8a] but a few generalizations are safe.

Two kinds of breakdown are recognized which, though they may merge, are distinct as to secondary mechanisms believed to be active. One, the *cathode initiated type*, occurs in the corona around negative electrodes and in the normal or static spark-over of short gaps. In it, positive ion bombardment or radiation from molecules excited by the avalanche causes emission of secondary electrons from the cathode surface. The other type, of which the lightning leader stroke is a manifestation, is the *positive streamer*, and occurs in the corona around positive electrodes, in the normal spark-over of long gaps, and in short gaps when more than the initial breakdown voltage is very suddenly applied. The secondary mechanism for this type is more complicated and probably involves photoelectric action in the gas and intensification of the

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D. W. VER PLANCK is assistant professor of electrical engineering, Yale University, New Haven, Conn., but is on leave of absence during 1940–41 for work in the Navy Department, Washington, D. C.

[1] For all numbered references, see list at end of paper.

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ment will produce four or five good carbon copies. In the Chicago substation three copies of the record are made, one for the operating department, one for the substation load record, and one for the distribution feeder load record. In the generating station four copies are made.

The printer need not be installed in the same cabinet with the measuring equipment. It is practicable to locate the measuring equipment near the control wiring and have the printer make the record sheet in a different room.

The present instrument is confined to ampere readings. However, it will probably be extended to other quantities such as volts or watts when the need arises.

The totalizer element of the instrument was a relatively inexpensive addition. It proved to be a valuable time saver for planning engineers because it eliminated the need of totalizing the figures in scattered columns. The hour of maximum load is obtained by scanning the total column only, and load analysis is usually made for only the maximum load.

Conclusion

This new automatic printing ammeter produces a compact record of the load on any number of circuits up to 50, with readings arranged in the form of a column for each circuit and a line for each hour of the day. The load record also includes totals for groups of lines. The following are incidental benefits derived from the use of the instrument: greater accuracy, greater legibility, duplicate records, and uniform timing of readings.

field by the space charge of successive avalanches.

This paper is concerned only with the calculation of the voltage at which cathode initiated breakdown occurs, and the question of whether the initial breakdown is a complete spark-over or a stable corona discharge is not considered.

The Townsend Coefficient α

The number of electrons n comprising an avalanche increases with distance s along the path at a rate proportional to the number already present. Thus

$$\frac{dn}{ds}=\alpha n \qquad (1)$$

where α, the Townsend coefficient, is dependent on the field strength as well as the density and kind of gas. The path of the avalanche is a line of electrostatic force if the gas density is high enough so that the electron mean free path is small compared to the radius of curvature of the field line, a condition likely to be true in practice. For an avalanche initiated by a single electron from the cathode, $s=0$, integration along a line of electrostatic force gives for the number of electrons when the avalanche reaches the anode, $s=S$,

$$n=\epsilon^{\int_0^S \alpha ds} \qquad (2)$$

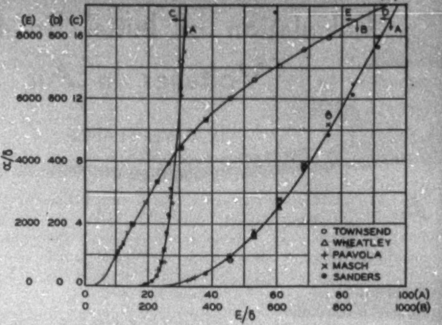
where α through its dependence on field strength is a function of s. The number of positive ions left in the trail of the avalanche is practically equal to n, although accurately it is one less since the initiating electron was emitted from the cathode.

If the avalanche grows large enough, the space charge of the ions causes the field to vary within the avalanche so that α is not the same for all of the electrons, and equation 2 is no longer true. However, certain calculations,[9] too lengthy to be included here, indicate that the field distortion caused by a single avalanche is negligibly small in a wide range of practical cases.

The method of measuring α, originated by Townsend,[1b] is based on the application of equation 2 to the continual succession of avalanches initiated by a current of photoelectrons emitted from the cathode of a parallel plane gap. Townsend found that α is related to field strength E and gas density δ by $\alpha/\delta=F(E/\delta)$ where the function F depends on the kind of gas.[1c]

Townsend,[1c,2b] Wheatley,[2b] Paavola,[4a] Masch,[5] and Sanders[3] have determined α for air with the results shown in figure 1. In this paper E is in kilovolts per centimeter and δ is the density relative to that at 760 millimeters and 25 degrees centigrade, units more convenient for calculations on atmospheric discharges than those employed by physicists, which are respectively volts per centimeter and millimeters of mercury. Density is used rather than pressure because mean free path, the significant factor, depends on density in general but on pressure only at constant temperature.

Figure 1. The Townsend coefficient α for air



The curves in figure 1 are plotted using the following empirical formulas which serve to express α for air in the ranges of E/δ indicated.

$$\left.\begin{array}{l}
\text{Range I.}\quad 15<E/\delta<25.8\\
\alpha/\delta=a_1\epsilon^{b_1E/\delta}\\
\quad a_1=1.67\times10^{-5}\\
\quad b_1=0.47\\
\text{Range II.}\quad 25.8<E/\delta<78.7\\
\alpha/\delta=a_2(E/\delta-b_2)^2\\
\quad a_2=0.166\\
\quad b_2=21.5\\
\text{Range III.}\quad 78.7<E/\delta<200\\
\alpha/\delta=a_3(E/\delta-b_3)\\
\quad a_3=19.0\\
\quad b\;=50.0\\
\text{Range IV.}\quad 200<E/\delta<467\\
\alpha/\delta=a_4\epsilon^{-b_4\delta/E}\\
\quad a_4=1.08\times10^4\\
\quad b_4=266\\
\text{Range V.}\quad E/\delta>467\\
\alpha/\delta=a_5(E/\delta)^{1/2}-b_5\\
\quad a_5=322\\
\quad b_5=850
\end{array}\right\} \qquad (3)$$

Formulas for α similar in form to the first, second, fourth, and fifth of these have been given by Sanders,[3] Paavola,[4b] Townsend,[1d] and Loeb[6a] respectively, but here the constants have been adjusted for agreement with all of the data. The formulas result in curves tangent to each other at the given limits, so that for convenience in calculation the limits may be shifted slightly without serious loss of accuracy.

For $E/\delta<15$ it is assumed that $\alpha/\delta=0$; it is certainly less than 0.02, the value at $E/\delta=15$.

Cathode Initiated Breakdown

In cathode initiated breakdown a single electron released by an external ionizing agent starts an avalanche from the cathode surface. As this avalanche crosses the gap, the secondary mechanisms come into play and if they release at least one electron from the cathode, one or more new avalanches may follow. When the secondary mechanisms are sufficiently active to assure the continued repetition of this process without further action by the external agent, breakdown results.

One of the two important mechanisms by which an avalanche can cause electron emission from the cathode is through bombardment by the positive ions left in the trail. The probability that a positive ion striking the cathode will cause an electron to be released is a function of the kind of surface and the energy of the impinging positive ion. The mean free path being small, the energy for a given gas is proportional directly to the gradient E_c at the cathode surface and inversely to the density of the gas; that is, to the ratio E_c/δ. The probability that a released electron will escape diffusion back to the cathode or attachment to a neutral molecule and be able to start an avalanche also depends on E_c/δ.[4b] Thus the number N of positive ions which must strike the cathode to assure the initiation of one secondary avalanche is a function of E_c/δ and is independent of gap geometry. The condition for breakdown is that the number of positive ions left in the trail of the original avalanche must equal or exceed the number N; that is, with the aid of equation 2,

$$\epsilon^{\int_0^S \alpha ds}\geq N \qquad (4)$$

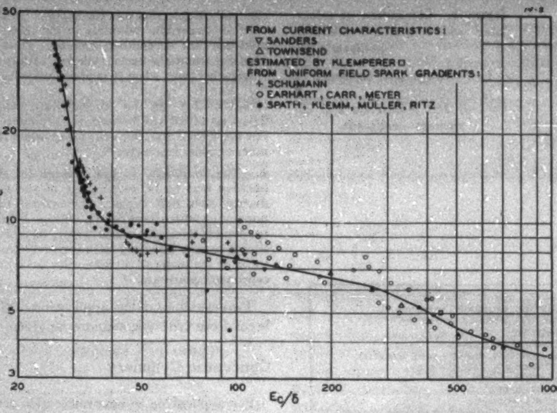


Figure 2. Log$_e$ N for air

a criterion given in the literature in various equivalent forms.[16,8c,8a]

The other important mechanism for the release of secondary electrons at the cathode is through photoelectric action of radiation from molecules excited by the avalanche.[8d] It will now be shown that equation 4 applies approximately for this process also. The number of molecules excited by the impact of electrons is roughly proportional to the number ionized,[8d] hence the total radiation from the trail of an avalanche is proportional to the number of positive ions formed, exp $(\int_0^S \alpha ds)$. Radiation reaching the cathode surface has its greatest intensity near the starting point of the avalanche, this being nearest to all parts of the trail because the avalanche path, a line of force, necessarily starts normal to the cathode surface and in practical cases never curves sharply. For a given cathode gradient and avalanche size, the intensity of radiation at the cathode should depend on the variation of gradient with distance from the cathode because this would affect the distribution of excited molecules along the avalanche trail. However, this effect of gap geometry is probably not great and is neglected here. The number of electrons released per unit of radiation reaching the cathode depends on the properties of the surface, and as before the probability that an electron will escape attachment or diffusion back to the cathode and be able to start a new avalanche depends on E_c/δ. Thus, as with positive ion bombardment, the probability that a secondary avalanche will be initiated is a function of E_c/δ and is proportional to the number of positive ions formed by the original avalanche, and consequently equation 4 is also the criterion for breakdown by the photoelectric mechanism.

With either mechanism the secondary avalanches are most likely to be initiated near the starting point of the original avalanche, for this part of the cathode is both where the positive ions strike and where the radiation is most intense. The two mechanisms may act jointly but evidence that the time lag for spark formation in uniform fields is less than the gap transit time for a positive ion makes the photoelectric mechanism more likely,[8e] although for corona on small negative wires, positive ion bombardment probably predominates.[7]

The Number N for Air

The criterion 4 implies that breakdown occurs when exp $(\int_0^S \alpha ds)$, the number of electrons in an avalanche which has crossed the gap, equals or exceeds N, a number characteristic of the gas and dependent on the cathode gradient to density ratio E_c/δ, but independent of the type of field.

To obtain a knowledge of N for air, equation 4 was applied as the criterion for a spark in uniform field, when if the gap length is S the relation

$$\log_e N = \alpha S \quad (5)$$

follows at once. Here the right member can be determined for any value of E/δ with the aid of the empirical formulas 3 for α/δ and abundant information in the literature relating the quantities $S\delta$ and E/δ for sparks in uniform fields. In this way N can be determined for a wide range of E_c/δ, since in this case E_c and E are identical. The data used were the well known curve by Schumann[30] based on a survey of the work done prior to 1922; results by Earhart,[8d] Meyer,[29] and Carr[1f] which fall outside the range of Schumann's curve; and results obtained since 1922 by Spath,[10] Klemm,[11a] Müller,[12a] and Ritz.[13]

In figure 2 log$_e$ N has been plotted rather than N because it is the former that is desired for general use. While the points show considerable dispersion, a mean curve can be determined with fair certainty. This curve is expressed in the range $26<E_c/\delta<250$ by the empirical formula

$$\log_e N = 10.0(25\delta/E_c)^{0.3} + 43.0(25\delta/E_c)^{9.6} \quad (6)$$

The last term is negligible when $E_c/\delta>60$.

When positive ion bombardment is the predominant secondary mechanism, the reciprocal of N is identical with the probability γ that a positive ion impact will release an electron from the cathode. A few values of γ for air obtained from prebreakdown current characteristics are reported by Sanders,[2] and a few others may be calculated from similar data by Townsend.[16] Points obtained from these and from an estimate of γ attributed to Klemperer[14] are included in figure 2.

Application of the Breakdown Criterion

For the calculation of initial breakdown voltages the criterion 4 is more convenient in the equivalent form

$$\int_0^S \alpha ds \geq \log_e N \quad (7)$$

The greater the inequality, the more nearly certain is breakdown, but since the left member turns out to be a function which increases very rapidly with applied voltage, the equality may be taken for practical purposes as the criterion for breakdown to be certain.

The left member of equation 7 depends on the field intensity E at every point along a line of force between the electrodes, and the right on the intensity E_c at the cathode end of this line. The field intensity, given by electrostatic theory, is proportional at every point to the voltage between electrodes if the field distortion caused by single avalanches is neglected and if no other space charges are present. Thus equation 7 gives the voltage implicitly or, if desired, the gradient at any point such as the cathode, at which breakdown will occur along the line of force considered. The breakdown voltage for

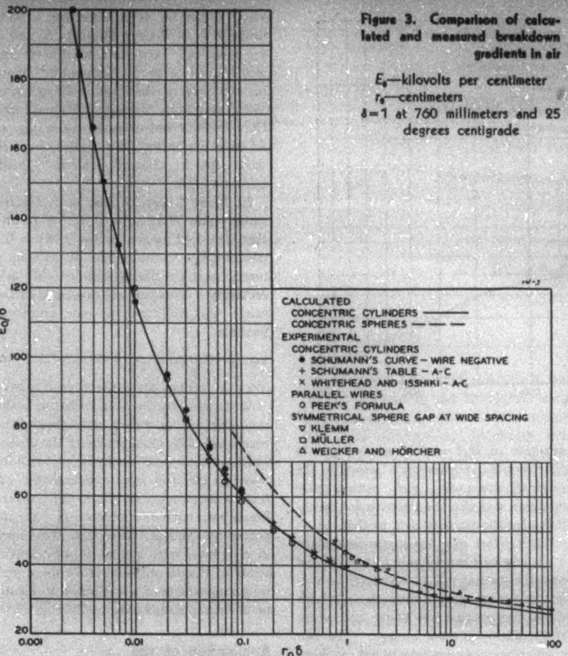


Figure 3. Comparison of calculated and measured breakdown gradients in air

E_0—kilovolts per centimeter
r_0—centimeters
$\delta = 1$ at 760 millimeters and 25 degrees centigrade

a given gap is found from the line of force for which equation 7 gives the lowest voltage. In many practical cases the proper line can be selected by inspection as the shortest one.

The integration corresponds to the entire growth of an avalanche, and consequently in general is from the cathode $s=0$ along a line of force, to the anode $s=S$. However, in air if the field diverges from the cathode to such an extent that E/δ becomes less than 15 before the anode is reached, the upper limit may be taken at the point where $E/\delta=15$, because $\alpha=0$ for $E/\delta<15$.

Special consideration is necessary if the field converges again beyond a region where $E/\delta<15$. At the lower field intensities, $E/\delta<15$, the electrons are more likely to become attached to oxygen molecules than they are to ionize by collision. With no field applied, the electrons survive on the average during the motion of thermal agitation only about ten collisions with oxygen molecules before becoming attached.[18] Moreover, the electrons do not become detached from negative ions when E/δ is less than about 70.[8b] Then since negative ions do not ionize by collision in fields normally attained, an avalanche having passed through a region of appreciable extent where $E/\delta<15$ would not grow further unless it passed into a region where $E/\delta>70$. In such cases equation 7 is not directly applicable.

The breakdown criterion, equation 7, is applicable to impulse as well as to direct voltages, provided that:

1. An adequate supply of initiating electrons is maintained by the external ionizing agents.

2. The applied voltage does not change appreciably during the extremely brief time of transit of the avalanche. In the case of sphere-gaps the impulse breakdown voltage is the same as the static value if the time from the zero of voltage to the crest exceeds about two microseconds.[16]

The criterion also gives the crest value of alternating voltage at which breakdown will occur with a given electrode considered as cathode, provided in addition that:

3. Breakdown has not already occurred during the previous half cycle when the electrode in question was the anode. In the case of concentric cylinders in air there is evidence that breakdown occurs at a lower gradient when the inner cylinder, radius r_0, is negative than when it is positive for $r_0\delta<0.01$ and for $r_0\delta>0.19$ and at least up to $r_0\delta=7.0$.[2f,17,18] In the intermediate range $0.01<r_0\delta<0.19$ the breakdown gradient is not more than four per cent lower when the inner cylinder is positive.[2f]

4. The frequency is low enough for the positive ions to be swept clear of the gap during each half cycle. The critical frequency at which breakdown voltage begins to depart from its static value has been shown to vary inversely with gap length and to be of the order of 50 kilocycles for a one centimeter plane gap.[19]

Two examples of the application of the breakdown criterion will now be given.

Concentric Cylinders

For application to concentric cylinders the inner of which is negative, the breakdown criterion, equation 7, is expressed in terms of the radius r, the co-ordinate along a line of force. Thus

$$\int_{r_0}^{r_1} \alpha dr = \log_e N \tag{8}$$

where r_0 and r_1 are respectively the radii of the inner and outer cylinders. The right member is a function of the cathode gradient which in this case is the gradient E_0 at the inner cylinder.

Because α is a function of the field intensity E, it is convenient to change the variable of integration from r to E by means of the relation $E=E_0r_0/r$ for the field between concentric cylinders. Equation 8 then becomes

$$\int_{E_0}^{E_0r_0/r_1} \alpha(-E_0r_0E^{-2}dE) = \log_e N \tag{9}$$

Then

$$r_0 = \frac{\log_e N}{E_0 \int_{E_0r_0/r_1}^{E_0} \alpha E^{-2} dE} \tag{10}$$

which for any given ratio of radii, r_0/r_1, relates the radius r_0 of the inner cylinder to the gradient E_0 at which a negative discharge will start.

The integral can be evaluated graphically as the area under a curve derived directly from α figure 1, or analytically, using the empirical formulas 3 for α as is done in appendix *A*.

An important case is where the outer cylinder is quite large compared to the inner. When it is large enough so that at its surface $E/\delta<15$, that is $r_1>(E_0r_0/15\delta)$, α is zero in the space adjacent to the outer cylinder and the integration can be started at $E=15\delta$ instead of at $E=E_0r_0\div r_1$, making the result independent of r_1.

In this case when the integrals from appendix *A* and numerical coefficients from the formulas 3 and 6 are substituted in equation 10 and the result arranged for easy computation, there is obtained for E_0/δ between 25.8 and 78.7

$$r_0\delta=\frac{10.0\left(\frac{25\delta}{E_0}\right)^{1.2}+43.0\left(\frac{25\delta}{E_0}\right)^{10.6}}{0.226+103.75\left(\frac{E_0}{25\delta}-1\right)-178.45\times\log_e\left(\frac{E_0}{25\delta}\right)+76.73\left(1-\frac{25\delta}{E_0}\right)} \quad (11)$$

and for E_0/δ between 78.7 and 200

$$r_0\delta=\frac{7.92\left(\frac{80\delta}{E_0}\right)^{1.2}}{236.8+1{,}520\log_e\left(\frac{E_0}{80\delta}\right)-950\left(1-\frac{80\delta}{E_0}\right)} \quad (12)$$

The results of theory and of experiment are compared in figure 3. The solid curve is a plot of equations 11 and 12. The experimental points are from a curve by Schumann,[3f] based on work of Farwell, Schaeffers, Whitehead and Brown, and Whitehead and Lee; from a table by Schumann[3g] based on researches by himself, Peterson, and Peek; from empirical formulas by Whitehead and Isshiki[17]; and from the empirical formula for parallel wires by Peek.[20] Excepting the last, all of the data are for concentric cylinders, and for either a negative inner cylinder or alternating voltage. The greatest departure of any point from the curve is about four per cent, and at that place there are other points on the opposite side. Thus the theoretical curve is in substantial agreement with experiment throughout a range of $r_0\delta$, the extremes of which are in a ratio of about 6,000 to 1.

An empirical formula often used to correlate corona starting gradients for cylinders is of the type

$$E_0/\delta=A+B(r_0\delta)^{-1/2} \quad (13)$$

where *A* and *B* are constants which have been given various values.[17,20] A plot of E_0/δ against $(r_0\delta)^{-1/2}$ as calculated from equations 11 and 12 shows some deviation from the form of equation 13, for as shown in figure 4, instead of having a constant value *B*, the slope first decreases considerably and then increases very slightly. Equation 13 thus does not agree with the theory although it may serve as a very convenient approximation over limited ranges.

Concentric Spheres

The breakdown criterion, equation 7, written for concentric spheres is identical with equation 8 except that *r* is the spherical instead of the cylindrical radius. Letting E_0 be the gradient at the surface of the inner (negative) sphere of radius r_0, the gradient at radius *r* is given by $E=E_0(r_0/r)^2$ from which it follows that

$$dr=-\frac{r_0E_0^{1/2}dE}{2E^{3/2}} \quad (14)$$

Substituting this in equation 8 and solving for r_0:

$$r_0=\frac{2\log_e N}{E_0^{1/2}\int_{15\delta}^{E_0}\alpha E^{-3/2}dE} \quad (15)$$

In making the lower limit of the integral 15δ, it is assumed that the region of ionization does not extend to the outer (positive) electrode.

The integral must be separated into parts corresponding to the ranges of α formulas 3, and when the integrations are carried out as shown in appendix *B*, there results for the case where E_0/δ is between 25.8 and 78.7

$$r_0\delta=\frac{10.0\left(\frac{25\delta}{E_0}\right)^{0.7}+43.0\left(\frac{25\delta}{E_0}\right)^{10.1}}{0.1049+34.58\left[\left(\frac{E_0}{25\delta}\right)^{3/2}-1\right]-178.45\times\left[\left(\frac{E_0}{25\delta}\right)^{1/2}-1\right]+76.73\left[1-\left(\frac{25\delta}{E_0}\right)^{1/2}\right]} \quad (16)$$

This result is plotted as the dashed curve in figure 3. The experimental points shown, taken from curves presented by Klemm,[11b] Müller,[12b] and Weicker and Hörcher,[21] are the limiting breakdown gradients approached at wide spacings in the symmetrical gaps between equal spheres at potentials equally above and below ground.

The justification for comparing results measured for symmetrical sphere-gaps with those calculated for concentric spheres, as well as for including experimental points for parallel wires with the curve calculated for concentric cylinders, is as follows:

1. As the spacing of the symmetrical gap is increased, the field in the region surrounding the negative electrode where the ionization occurs approaches that for concentric electrodes.
2. The electrons passing into mid gap where the field is weak become attached to neutral molecules forming negative ions which cannot ionize by collision as they go on into the intense field around the positive electrode.

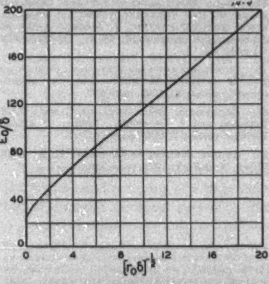


Figure 4. Calculated results for concentric cylinders tested for agreement with empirical formula of the type $E_0/\delta=A+B(r_0\delta)^{-1/2}$

Conclusion

The initial breakdown voltages for two distinct types of nonuniform field have been calculated accurately by means of the breakdown criterion, equation 7, and the data contained in formulas 3 and 6, for as shown in figure 3 the results are in excellent agreement with experiment. In addition to the two examples given here, the method is being applied to breakdown between sharp edged electrodes, and it is hoped to present the results when the experimental confirmation is complete.

For the successful application of the method it is fortunate that the voltage calculated from equation 7 is relatively insensitive to changes in *N* because this quantity depends on mechanisms whose exact nature is somewhat obscure. Consequently the agreement of theory and experiment shown in figure 3 is not offered as proof of the correctness of the physical picture used in developing the method. There is evidence[8a] that in plane gaps where $S\delta>0.3$, which corresponds approximately to $E_0/\delta<35$, breakdown is not cathode initiated as assumed here but proceeds by streamer formation. However, as may be seen in the lower right hand portion of figure 3, the method yields correct results when $E_0/\delta<35$ in spite of the possible lack of rigor in this range.

Finally it should be noted that while the data presented pertain to breakdown in air, the method is general and applicable to any gas for which the characteristic quantities α and *N* are known.

Appendix A—Integration for Concentric Cylinders

The integral in equation 10 must be separated into parts corresponding to the ranges in which the formulas 3 for α apply. The extreme limits of integration are the gradients at the outer and inner cylinders, while the intermediate limits of the partial integrations are the limits of the ranges of α.

The number of ranges which must be included depends on the case considered.

When the gradient at the outer cylinder is less than 15δ and unless the inner cylinder is extremely large, the integration must go completely through range I and terminate in ranges II or III. For range I

$$a_1\delta\int_{15\delta}^{25.8\delta}\epsilon^{b_1E/\delta}E^{-2}dE=-a_1\epsilon^{b_1E/\delta}\delta/E\Big]_{15\delta}^{25.8\delta}+$$

$$b_1a_1\int_{15\delta}^{25.8\delta}\epsilon^{b_1E/\delta}E^{-1}dE$$

$$=-a_1\epsilon^{b_1E/\delta}\delta/E\Big]_{15\delta}^{25.8\delta}+$$

$$b_1a_1[\overline{\mathrm{Ei}}(25.8b_1)-\overline{\mathrm{Ei}}(15b_1)] \qquad (17)$$

where the exponential integral function $\overline{\mathrm{Ei}}$ is defined and tabulated by Jahnke and Emde.[22a]

The integration in range II is

$$a_2\delta\int_{25.8\delta}^{E_0}(E/\delta-b_2)^2E^{-2}dE=$$

$$a_2[E/\delta-2b_2\log_\epsilon(E/\delta)-b_2^2\delta/E]_{25.8\delta}^{E_0} \qquad (18)$$

For very small inner cylinders the integration must go beyond range II. Then the upper limit of 18 becomes 78.7δ instead of E_0 and integration into range III is necessary.

$$a_3\delta\int_{78.7\delta}^{E_0}(E/\delta-b_3)E^{-2}dE=$$

$$a_3[\log_\epsilon(E/\delta)+b_3\delta/E]_{78.7\delta}^{E_0} \qquad (19)$$

Appendix B—Integration for Concentric Spheres

The integral in equation 15 must be separated into parts corresponding to the ranges in which the formulas 3 for α apply. Treating only the case where E_0 is in range II, the entire contribution of range I is

$$a_1\delta\int_{15\delta}^{25.8\delta}\epsilon^{b_1E/\delta}E^{-3/2}dE=$$

$$-2a_1\delta\epsilon^{b_1E/\delta}E^{-1/2}\Big]_{15\delta}^{25.8\delta}+$$

$$2a_1b_1\int_{15\delta}^{25.8\delta}\epsilon^{b_1E/\delta}E^{-1/2}dE$$

$$=-2a_1\delta\epsilon^{b_1E/\delta}E^{-1/2}\Big]_{15\delta}^{25.8\delta}+4a_1(b_1\delta)^{1/2}\times$$

$$\int_{\sqrt{15b_1}}^{\sqrt{25.8b_1}}\epsilon^{Z^2}dZ \qquad (20)$$

where the last integral is obtained through the substitution $(b_1E/\delta)=Z^2$. Since values of this integral are available[22b] only for the range of limits 0 to 2, while the limits here are approximately 2.7 and 3.5, it is necessary to resort to a series[23] for computation.

For range II:

$$a_2\delta\int_{25.8\delta}^{E_0}(E/\delta-b_2)^2E^{-3/2}dE=$$

$$a_2\delta^{1/2}\Big[(2/3)(E/\delta)^{3/2}-4b_2(E/\delta)^{1/2}-$$

$$2b_2^2(\delta/E)^{1/2}\Big]_{25.8\delta}^{E_0} \qquad (21)$$

References

1. Electricity in Gases (a book), J. S. Townsend. Clarendon Press, Oxford, 1915: (*a*) articles 185, 225, 293; (*b*) article 193; (*c*) article 198; (*d*) article 206; (*e*) article 225; (*f*) page 360.

2. Elektrische Durchbruchfeldstärke von Gasen (a book), W. O. Schumann. Springer, Berlin, 1923: (*a*) pages 170–232; (*b*) page 123; (*c*) page 26; (*d*) page 44; (*e*) page 56; (*f*) page 86; (*g*) page 85.

3. Measurement of the Townsend Coefficients for Ionization by Collision, F. H. Sanders. *Physical Review*, volume 44, 1933, pages 1020–4.

4. Der dunkle Entladungsvorstrom in Luft von Atmosphärendruck, M. Paavola. *Archiv für Elektrotechnik*, volume 22, 1929: (*a*) page 453; (*b*) page 456.

5. Über Elektronenionisierung von Stickstoff, Sauerstoff und Luft bei geringen und hohen Drucken, K. Masch. *Archiv für Elektrotechnik*, volume 26, 1932, page 593.

6. The Problem of the Mechanism of Static Spark Discharge, L. B. Loeb. *Reviews of Modern Physics*, volume 8, 1936: (*a*) pages 276–9; (*b*) pages 268–71; (*c*) pages 272–6; (*d*) page 268; (*e*) pages 284–9.

7. Fundamental Mechanisms Which Determine the Starting Potentials of the Low-Pressure Corona Discharge, L. B. Loeb. *Journal of Applied Physics*, volume 8, 1937, pages 495–6.

8. Fundamental Processes of Electrical Discharges in Gases (a book), L. B. Loeb. Wiley, New York, 1939: (*a*) chapter X; (*b*) page 291.

9. Calculation of Initial Breakdown Voltages in Air (a dissertation), D. W. Ver Planck. Unpublished, Library, Yale University, New Haven, Conn.

10. Über die Durchschlagseigenschaften von Transformatorenölen, W. Spath. *Archiv für Elektrotechnik*, volume 12, 1923, page 336.

11. Anfangsspannung und Durchbruchfeldstärke von Kugelelektroden bei Gleichspannung, A. Klemm. *Archiv für Elektrotechnik*, volume 12, 1923: (*a*) page 565; (*b*) page 571.

12. Über Durchbruchfeldstärke, Anfangsspannung und Funkenspannung bei Wechselstrom von 500 Per/sec, F. Müller. *Archiv für Elektrotechnik*, volume 13, 1924: (*a*) page 482; (*b*) page 483.

13. Durchschlagfeldstärke des homogenen Feldes in Luft, H. Ritz. *Archiv für Elektrotechnik*, volume 26, 1932, page 227.

14. Electrical Discharges in Gases, K. T. Compton and I. Langmuir. *Reviews of Modern Physics*, volume 2, 1930, page 181.

15. Atomic Structure (a book), L. B. Loeb. Wiley, New York, 1938, page 329.

16. Sphere-Gap Characteristics on Very Short Impulses, P. L. Bellaschi and W. L. Teague. *Electric Journal*, volume 32, 1935, pages 120–3.

17. The Corona Voltmeter, J. B. Whitehead and T. Isshiki. AIEE Transactions, volume 39, 1920, pages 1057–1110.

18. Der elektrische Durchschlag von Luft zwischen konzentrischen Zylindern, E. Uhlmann. *Archiv für Elektrotechnik*, volume 23, 1929, pages 326–7.

19. Die Durchschlagsenkung bei Hochfrequenz, H. Böcker. *Archiv für Elektrotechnik*, volume 31, 1937, pages 166–79.

20. Dielectric Phenomena in High Voltage Engineering (a book), F. W. Peek. McGraw-Hill Publishing Company, New York, 1920, page 46.

21. Grundlagen zu neuen Eichtafeln für Kugelfunkenstrecken, W. Weicker and W. Hörcher. *Elektrotechnische Zeitschrift*, volume 59, 1938, figure 7, page 1070.

22. Tables of Functions (a book), E. Jahnke and F. Emde. Teubner, Berlin, third edition, 1938: (*a*) pages 1–9; (*b*) page 32.

23. On the Numerical Value of $\int_0^h e^{x^2}dx$, H. G. Dawson. *Proceedings* of the London Mathematical Society, volume 29, 1898, pages 519–22.

List of Symbols

In all cases electric field strengths are in kilovolts per centimeter and lengths are in centimeters.

$a_1 \ldots a_6$,
$b_1 \ldots b_6$ = Empirical factors in formulas 3 for α
E = Field strength
E_c = Field strength at the cathode surface
E_0 = Breakdown gradient at inner cylinder or sphere
F = Function relating α/δ to E/δ; expressed for air by formulas 3
N = Number of electrons in an avalanche which has crossed the gap at the breakdown voltage, a function of E_c/δ expressed for air by the empirical formula 6
n = Number of electrons in an avalanche
r = Radial co-ordinate
r_0 = Radius of inner cylinder or sphere
r_1 = Radius of outer cylinder or sphere
S = Distance from cathode to anode along the avalanche path, a line of force
s = Co-ordinate of avalanche center measured from the cathode along a line of force
α = The Townsend coefficient for ionization by electron impact; the number of electrons added to an avalanche per centimeter of travel per electron already present (centimeters)$^{-1}$
γ = The probability that a positive ion striking the cathode will release an electron
δ = Relative air density; unity for 760 millimeters and 25 degrees centigrade

Picture Transmission by Submarine Cable

J. W. MILNOR
FELLOW AIEE

Synopsis: The direct transmission of pictorial matter over transatlantic submarine cable has become practical through the development of some special types of networks and amplifiers. These are required because of the rapid increase of the circuit attenuation with frequency, and in order to override certain characteristic electrical interference to which the cable is subject. A summary is presented of the fundamental theory which governs facsimile and telegraph transmission, and the methods are shown by which this is applied to meet the conditions of cable picture working.

AN efficient system for transmission of facsimile matter from London to New York by submarine cable was made available to the public in April 1939. This provides an important advantage over radio sending of pictorial subjects, in that the cable system is not subject to unpredictable variations in the transmitting medium. But while the path from sender to receiver is inherently constant with time, the cable possesses characteristics which provide some unusual problems of transmission. The measures which were adopted to insure an undistorted likeness of the original and to transmit the intelligence at a maximum rate within the limited frequency spectrum available through the cable, appear to be sufficiently unique to justify a description at some length.

This new system should not be confused with the "Bartlane" system of picture sending which was applied to cables for several years. In the Bartlane system the picture was analyzed into small unit squares, and a code character was set up corresponding to each unit. The characters were sent by means of the equipment ordinarily serving for telegraph transmission, and were decoded, usually automatically, at the receiving end. That system was inherently slow and made inefficient use of the available cable capacity as compared with the arrangement described herein.

Description of Cable Circuit

The transatlantic cable which is utilized for picture transmission, indicated in figure 1, extends from Penzance, England to a repeater point at Bay Roberts, Newfoundland and thence to Rockaway, New York. Conductors in lead covered cable complete the circuit from London, England through Bristol to Penzance, and from Rockaway to New York City. The submarine cable contains a central copper conductor which is provided with a continuous wrapping of permalloy tape for magnetic loading, and which in turn is covered with gutta percha for insulation.

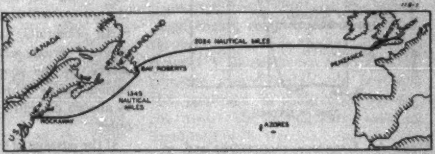
For picture transmission there are inserted repeaters of the electronic type, together with suitable signal shaping networks, at each of the above named points. The repeaters replace those in part mechanical which are in service when the cable is in telegraph operation.

In figure 2 is shown the electrical loss in each portion of the circuit, plotted against frequency.

Fundamental Transmission Requirements

The usual practice in picture transmission is to make use of a carrier current in the path from sending to receiving terminals. However, any carrier arrangement is wasteful of the available frequency spectrum, and in the case of submarine cable working it was especially necessary to minimize any waste. Therefore a carrier is not used in the cable in the system under discussion; instead the currents correspond directly with the shades of the original picture, varying from black, which produces maximum negative current in the cable, through gray to white which corresponds to maximum positive current.

Figure 1. Route of 1926 loaded submarine cable used for picture transmission



The transmission of a picture involves the sending and receiving of a large number of transient electrical currents. But every transient can be considered as the sum of a series of sinusoidal waves, and it is far more convenient to discuss the phenomena in terms of the ability of the system to transmit alternating current in a band of frequencies. In this case we are interested in the transmission from sender to receiver of all frequencies from near zero to about 110 cycles per second.

There is a definite upper limit of frequency which may be passed through a submarine cable. The cable, like radio, is subject to static interference, and because of the rapid increase in cable attenuation with frequency as shown in figure 2, the signal-to-noise ratio rapidly falls as the frequency is increased. In this cable the ratio becomes so low at about 110 cycles that currents at frequencies above this value tend to be lost in the interference. The receiving equipment is arranged to cut off shortly above this frequency in order to minimize as far as practicable the effects of outside interference.

It also is necessary to arrange the system so that the receiving equipment is insensitive to direct current or to alternating currents of very low frequency, otherwise it would be unworkable from time to time because of the earth potentials which affect ocean cables. Amounts of five volts or more occur al-

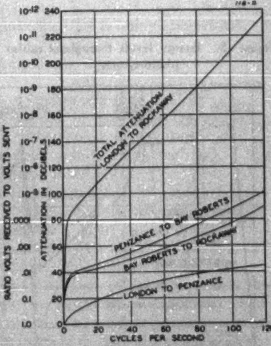


Figure 2. Attenuation of cable sections

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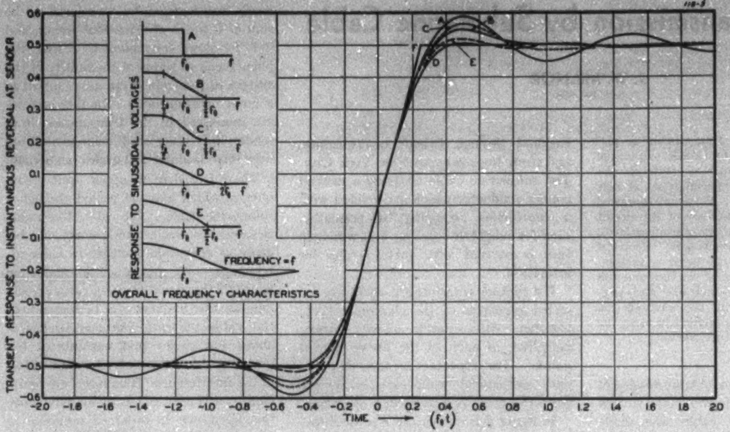


Figure 3. Arrival curves

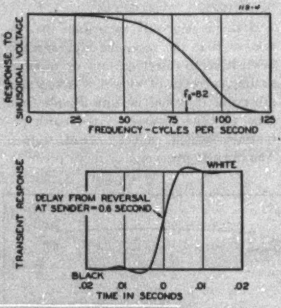
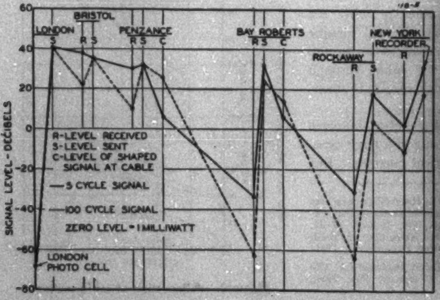


Figure 4. Frequency characteristic and arrival curve obtained over London-New York cablephoto system

Figure 5. Energy levels throughout cablephoto system



most daily, and at times when abnormal conditions occur on the surface of the sun, the voltages are much higher. Values as high as 500 volts have been recorded, although such amounts occur only at long intervals. Earth potentials may be considered to be very low frequency alternating voltages, the frequency ranging from perhaps one cycle per hour to two cycles per minute. The receiving picture system is made immune to such earth potentials, except at the very occasional periods when they are unusually severe, by the shaping networks at the various points, which are arranged largely to absorb currents at frequencies of this order.

If the passage of direct currents through the cable were desirable, their transmission would in fact have been difficult because of practical considerations in the design of the amplifiers, and because of the necessity of separating from the cables the high voltage batteries used as plate supply in power amplifiers, in order to prevent the possibility of damage to cable insulation. The condensers used for the purpose have the unavoidable effect of removing most current components of the lowest frequencies.

Any arrangement of telegraph or of picture transmission in which low frequencies and direct currents are not received would be inoperable unless some arrangement were provided to compensate for the missing components of the signal. In the present case this is accomplished

Figure 6. Cablephoto transmitter at London. Photograph sent by cable to New York

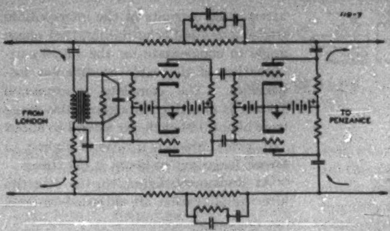


Figure 7. Repeater at Bristol

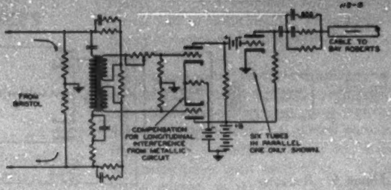

FROM BRISTOL
COMPENSATION FOR LONGITUDINAL INTERFERENCE FROM METALLIC CIRCUIT
SIX TUBES IN PARALLEL ONE ONLY SHOWN
CABLE TO BAY ROBERTS

Figure 8 (right). Repeater at Penzance

by an automatic adjustment of the received signal at the end of each scanning line in the picture. At that moment it is arranged to reduce to zero (corresponding to neutral gray in the picture) the current at the sending station. At the corresponding time there is cut into the circuit at the receiving station a set of equipment which measures the amount of current being received and automatically compensates for the current level if it departs from normal. Thus the next scanning line is started with the received level at its correct value.

In its passage through the system, the facsimile signal must be amplified sufficiently to overcome the cable attenuation at the highest frequency that is utilized, in addition to the unavoidable loss of energy in various elements of the equipment. The total available amplification from the sending photocell to the receiving lamp is approximately 420 decibels.

The shaping networks at various points perform the functions discussed in the past few paragraphs, and in addition they discriminate between different frequencies in a manner to compensate with some precision for their large difference in attenuation and phase shift in the cable. For example, a current at five cycles is reduced as much as 109 decibels, as compared with that at 100 cycles, by the shaping networks.

Theory of Signal Shaping

The combined effect of the shaping networks at all points determines the faithfulness of the final product and the efficiency of utilization of the available frequency spectrum. The problem is parallel to that encountered in telegraphy in endeavoring to transmit a maximum quantity of material within a given range of the frequency spectrum.

When a reversal from black to white occurs at the sending transmitter, the effect reaches the distant recorder in the form of an "arrival" voltage, rounded somewhat and possibly otherwise distorted from the shape of the current as sent. Each possible arrival curve corresponds to a definite "frequency characteristic" of the entire transmission system, which is the ratio of received to sent voltage plotted as a function of frequency, assuming that the transmitter has been replaced by a generator of sinusoidal currents. The choice of the frequency characteristic markedly influences the efficiency of the system.

The frequency characteristic comprises in general both an amplitude function and a phase function. An arrival curve is not received at the same instant that the reversal occurs at the transmitter; instead there is a slight delay, and that lag corresponds to a change in phase angle which is proportional to the frequency. Such a phase change is not of importance as affecting the efficiency. If such phase changes only are present the arrival curve is symmetrical. Basic theory* shows that any phase change other than in proportion to the frequency, which introduces components in the frequency characteristic in quadrature to the main component, causes the arrival curve to be nonsymmetrical. Such a phase response makes less effective use of the cable capacity.

Since the attenuation of a cable increases rapidly with frequency, and the higher frequencies are increasingly difficult to utilize, it might be expected that the preferred over-all frequency characteristic would be one which cuts off sharply at the upper end of the useful range. Investigation, however, shows that such a characteristic is associated with an arrival curve which oscillates to an objectionable extent both before and after the main reversal.

Plots of a number of symmetrical arrival curves, with their corresponding frequency characteristics, are illustrated in figure 3**. The characteristics have been so chosen (with equal area under each) that the maximum slope of each arrival curve is the same. The amount of resolution in the direction of scanning is approximately proportional to the mean frequency f_0. The effect of an abrupt cutoff of the characteristic is shown in curve *A*.

Entire freedom from oscillations in the arrival curve requires the use of a characteristic which with increasing frequency decreases quite gradually to zero, and which therefore is quite wasteful of cable capacity. Curve *D* is an example of such a characteristic. However, a desirable compromise between the efficiency of cable utilization and the appearance of the resulting picture is obtained by permitting a moderate overthrow and oscillation in the arrival curve, which is not objectionable and may be of advantage in accenting the lines of the image.

Frequency characteristics precisely like those just shown, and with the correct phase angles, could hardly be obtained through the use of equipment which it is practical to assemble, but fairly close approaches should be feasible providing that networks of sufficient complexity are used. The arrival curve and the amplitude of the frequency characteristic actually obtained over the cablephoto system are shown in figure 4. Phase angle changes, other than those proportional to frequency, are small, and the arrival curve therefore is nearly symmetrical.

Amplifying Equipment

A number of unique problems were encountered in providing the necessary amplification and signal shaping at various points along the route. The manner in which the signal level varies from point to point is indicated in figure 5.

The transmitter in London, figure 6, contains a light shutter ahead of the photocell which breaks the beam at the rate of 1,350 times per second. The output of the photocell in the form of a carrier is amplified, demodulated, again amplified and impressed upon the line in the form of a signal in which the various voltages from maximum negative to

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*SUBMARINE CABLE TELEGRAPHY, J. W. Milnor AIEE TRANSACTIONS, February, 1922. Note equation 10 in the appendix, which served for computations of the arrival curves included herein.

**In this figure the frequency characteristics *C*, *D*, and *E* are portions of sine or cosine curves as shown. The characteristic *F* is of the form $2f_0/\pi f \sin \pi f/2f_0$.

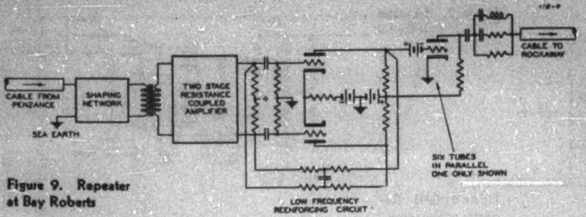
CABLE FROM PENZANCE
SHAPING NETWORK
SEA EARTH
TWO STAGE RESISTANCE COUPLED AMPLIFIER
SIX TUBES IN PARALLEL ONE ONLY SHOWN
LOW FREQUENCY REENFORCING CIRCUIT
CABLE TO ROCKAWAY

Figure 9. Repeater at Bay Roberts

maximum positive correspond to the shades from black to white. The wires which are available from London to Bristol and from Bristol to Penzance, set up as a metallic pair, are capable of handling frequencies somewhat above 120 cycles per second, but are not suitable for carrier working.

At Bristol the currents are passed through a type of repeater, figure 7, which in the interest of simplicity is arranged to amplify the higher frequency components only. The lower frequency components are by-passed around the amplifier.

At Penzance, figure 8, the currents are received from a metallic pair and after amplification are sent into a submarine cable which makes use of ground return. The metallic pair, while relatively free from interference between the wires, is subject to serious longitudinal interference. The problem of passing into a ground-return circuit the wide frequency range of the needed currents, while avoiding the longitudinal interference, is met by the use of a push-pull amplifier including a degenerating arrangement which renders it insensitive to longitudinal voltages, while amplifying the transverse signal voltages from the incoming pair.

The transformers and the condensers which were placed in circuit at certain points because of practical considerations, have the unavoidable effect of largely reducing some of the desirable frequency components in the signal, particularly those components below about 0.4 cycle per second. To correct for this loss there is inserted in the repeater at Bay Roberts, figure 9, and again in that at Rockaway, a network which through feed-back action serves to re-enforce the lowest frequencies, while having no influence upon the higher ones.

Shaping networks in addition to those just described are included at all points except at the terminals for the purpose of attenuating currents at the intermediate frequencies and controlling the phase angle to compensate for the varying attenuation of the cable at different frequencies. The complete transmission system is effective throughout the range from about 0.04 cycle per second to about 110 cycles, which is a ratio of highest to lowest frequency of about 2,750 times.

At Rockaway, figure 10, the currents after amplification are adjusted in level automatically in the manner previously described, at the end of each scanning line, in order to compensate for the absence of signal components below about 0.04 cycle per second. At that instant, because of the momentary reduction to zero of the output from London, the output of the Rockaway amplifier should also be zero. The relays in the automatic corrector are caused to vibrate rapidly at that time. If any difference in potential then exists across the amplifier, the grid of one of its tubes is adjusted quickly until the difference in potential is removed.

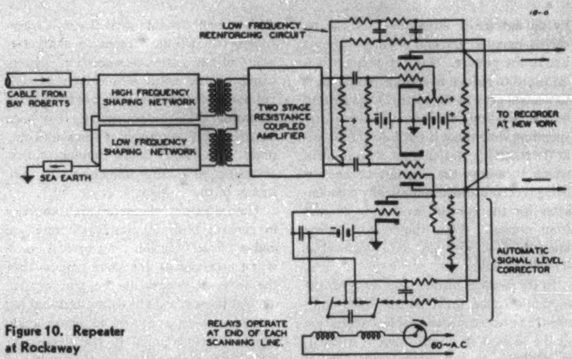
LOW FREQUENCY REENFORCING CIRCUIT
CABLE FROM BAY ROBERTS
HIGH FREQUENCY SHAPING NETWORK
LOW FREQUENCY SHAPING NETWORK
SEA EARTH
TWO STAGE RESISTANCE COUPLED AMPLIFIER
TO RECORDER AT NEW YORK
AUTOMATIC SIGNAL LEVEL CORRECTOR
RELAYS OPERATE AT END OF EACH SCANNING LINE.
60~A.C

Figure 10. Repeater at Rockaway

The output of the repeater at Rockaway is passed on to New York, and is there again amplified and impressed upon a gas-filled lamp which serves to expose the receiving photographic film.

Times Wide World

Figure 11. Trafalgar Square. Photograph transmitted by cable from London to New York

Results

Typical photographs transmitted by cable are shown in figures 6 and 11. These were transmitted at the rate of 2.2 square inches per minute, with scanning at 60 lines per inch. The resolution in the direction of scanning is especially fine, and is sufficient to warrant an increase either in the number of scanning lines per inch or a decrease in the time of transmission of a given area of picture.

The Audio Noise of Transformers

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MEMBER AIEE

Synopsis: When transformers are installed near inhabited districts, their noise must be limited to unobjectionable values. Consequently, means of predetermining noise levels from the design constants of the transformer and means of noise reduction become necessary. It has been determined that the audio noise of transformers originates in the change in dimensions (magnetostriction) produced in the core steel by the alternating magnetic flux. The vibration caused by the change in dimensions is transmitted to the air through the tank walls. The basic noise may be increased due to the resonant vibration of mechanical parts. Theoretical relations for the sound produced have been derived. These relations are checked by tests on commercial transformers.

Introduction

When power transformers are installed in locations remote from residences or other buildings used constantly by human beings, audio noise created by them is not a problem. However, as loads increase it becomes desirable to place larger transformers closer to inhabited districts. Large transformers can create sufficient noise as to make their presence in inhabited districts objectionable. In addition, the public is becoming more noise conscious and insistent on the elimination of objectionable noise of any kind.

Consequently, the calculation of the noise of a transformer from its design constants in order to predict the amount of the noise and a study of means of noise reduction becomes a practical necessity.

Origin of Noise of Transformers

Audio noise of transformers originates in the change in dimensions produced in the core steel when it is magnetized.

When iron is magnetized, its dimensions change due to the magnetization. Usually, the iron lengthens parallel to the flux lines and decreases in dimensions at right angles to the flux lines. The change in dimensions is extremely small. For example, a typical transformer punching 100 inches long will increase in length 0.0002 inches when the transformer is excited at full voltage.

Figure 1 shows percentage changes in length for typical transformer steel. Although small, the change in dimensions sets up sound waves in the manner common whenever rapid movement of a solid (in this case the steel) in contact with an elastic medium (air or oil) occurs. The intensity of the sound (in watts per square centimeter or similar units) is proportional to the square of the amplitude of vibration. When the magnetizing force is alternating as in a transformer, the sound waves issue continuously. Such is the source of noise in a transformer. Its origin is the vibration of the iron core caused by the change in dimensions commonly called magnetostriction.

Measurement of Noise*

A method of predetermining the noise of a transformer from its design is the first step in designing a quiet transformer. This requires, in turn, a satisfactory means of measuring noise and convenient units for expressing the amount of noise.

Noise can be expressed in a wide variety of ways. A fairly complete description would be a curve of watts per square centimeter versus time. It is obvious that such a curve does not lend itself to ready comparison and analysis. Recurrent sound may be expressed as the sum of its various harmonics each expressed in terms of its maximum amplitude. Such an expression is useful for sound analysis and is frequently used for this purpose in noise reduction. It may be obtained by recording the curve of intensity versus time and analyzing it into its harmonics, or by the more usual process of obtaining each harmonic separately by a harmonic analyzer.

However, such measurements do not result in the simplicity necessary for convenient general use. It is highly desirable to have a method of measurement of noise or sound intensity which can be expressed in terms of a single unit.

Such measurement is complicated by the fact that the human ear which is the final arbiter of noise has a different response for each different frequency of sound and in addition, the comparative response varies with the amplitude of the sound.

To take account of the variation in response of the ear to the various frequencies, the measurement must give weighted values to the intensities of the various frequencies. Standard curves for such weighting have been established. Using these curves as a base, the common method in use today for giving an over-all expression of the amount of noise is to express it in decibels. For example, a particular sound is said to have an intensity of 60 decibels.

The decibel is really the measure of the ratio of two amounts of power. Since the human ear responds to an extremely wide range of sound intensities, the decibel offers a convenient means of expression of sound intensity. The decibel is so expressed mathematically that the sound intensity increases ten times for each ten decibels increase in sound level. Mathematically the sound level in decibels =

$$10 \log_{10} \frac{I}{I_0}$$

where I_0 is the reference intensity for the scale in watts per square centimeter or similar units and I is the measured intensity in the same units. The reference level I_0 in common use at present is 10^{-16} watts per square centimeter which at a temperature of 20 degrees centigrade and a pressure of 760 millimeters of mercury corresponds to a pressure of 0.000204 dynes per square centimeter for plane or spherical free waves. Expressed in terms of the pressure, the sound level in decibels, $\text{db} = 20 \log \frac{P}{P_0}$ where P_0 is the reference sound pressure in dynes per square

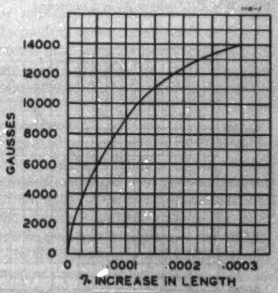

Figure 1. Typical magnetostriction curve of high-silicon transformer steel

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* Further information can be found in the American Tentative Standards Z24.1, Z24.2, and Z24.3, and AIEE test code No. 520.

centimeter or similar units and P is the measured sound pressure in the same units. Suitable instruments have been developed for measuring the over-all sound level in decibels. Other instruments have been developed for analyzing a sound into its harmonics. At the present time these are not precision instruments in the same sense that the more common electrical measuring instruments are. However, they are sufficiently accurate to be extremely useful in sound level determination. One of the obstacles has been the difficulty of calibration so that readings with the same meter or other meters calibrated at different times will be the same.

Theoretical Variations of Sound With Weight

Based on these methods of expressing and measuring sound, theoretical relations for the noise of transformers are derived. It is general experience that the noise of a transformer varies with its size and that large transformers are usually more noisy than small transformers.

It is also well known that the noise of a given transformer increases as the applied voltage is increased. That is, the sound level of a transformer increases as the induction in the iron is increased.

According to the standards which have been established for measuring transformer noise, the sound intensity is measured one foot away from the major sound producing surface. Consequently the sound intensity measured is practically the sound intensity at the surface of the sound transmitting member.

The first relation derived is that between sound intensity and the weight of the transformer core. For transformers of similar proportions the core weight, W, is proportional to the length of the core L, raised to the third power.

Thus $W=K_1L^3$ where K_1 is a constant

When the transformers considered have the same induction in the iron core, the noise intensity, I, at the outside of the transformer is proportional to the square of the core length because of the general law that the noise intensity is proportional to the square of the amplitude of vibration and because the change in dimensions due to magnetostriction is proportional to the core length.

Thus $I=K_2L^2$ where K_2 is a constant

$$L=\frac{(W)^{1/3}}{(K_1)^{1/3}}$$

Substituting for L in: $I=K_2L^2$

$$I=K_3W^{2/3} \text{ where } K_3=\frac{(K_2)}{(K_1)^{2/3}}$$

or the noise intensity is proportional to the weight of the core raised to the two-thirds power when the induction in the iron is constant.

Since the sound level in decibels, db by definition, equals

$$10\log_{10}\frac{I}{I_0};\ \text{db}=10\log_{10}\frac{(K_3W^{2/3})}{(I_0)}$$

$$\text{db}=10\log_{10}\frac{(K_3)}{(I_0)}+\frac{20}{3}\log_{10}W$$

Where $K_4=10\log_{10}\frac{(K_3)}{(I_0)}=$ a constant when the reference intensity I_0 is constant

$$\text{db}=K_4+\frac{20}{3}\log_{10}W$$

or the noise level in decibels is equal to a constant plus 6.67 times $\log_{10}$ (weight).

From this relation, it is evident that changing the reference level does not change the slope of the curve, but merely the value of K_4.

Theoretical Variation of Sound With Induction

The second relation derived is that between sound intensity due to magnetostriction and the maximum induction in the iron. The curve of change in length due to magnetostriction (figure 1) is obtained from tests on small samples of iron tested one sheet at a time in a device for determining the change in length due to magnetization of the sheet.

If S is the per cent increase in length due to magnetostriction, and K is a suitable factor, the sound intensity, I, for a given transformer is equal to KS^2. If the relative proportions of the different harmonics of the sound to each other remain the same with variation in S, K will be a constant. This derivation applies to all cases where K may be assumed to be constant with permissible error.

If S_0 is the per cent increase in length for reference level of induction B_0, the sound intensity, I_0, at the reference level is equal to KS_0^2.

Since the sound level in decibels, db by definition equals $10\log_{10}\frac{I}{I_0}$;

$$\text{db}=10\times\log_{10}\frac{S^2}{S_0^2}=20\log_{10}\frac{S}{S_0}.$$

If in figure 1, the reference level is chosen as 12,000 gausses $S_0=0.000177$ per cent, and $\text{db}=20\log_{10}\frac{S}{0.000177}$.

By substituting values of S from figure 1, values for plotting curve A of figure 2 are obtained. Curve A of figure 2 shows the change in sound level in decibels, due to magnetostriction when the maximum induction in the iron changes.

Tested Variation of Sound With Induction

Curve B of figure 2 was obtained by testing with a sound level meter the sound intensity of complete commercial transformers. The values of decibels read on the meter were corrected to a reference level of 12,000 gausses by subtracting the db at 12,000 gausses from the values at other inductions. It will be noted that the agreement between curves A and B of figure 2 is very close. This is an indication of the correctness of the theoretical analysis and in addition an indication that the origin of the noise was the magnetostriction of the iron with at most a negligible quantity of noise from other sources.

If steel having different magnetostriction characteristics were used, curves similar to figure 2 for the different steel would be used for sound intensity determinations. For certain cases, accurate determination of curve A is involved but curve B can always be obtained directly from sound intensity measurements.

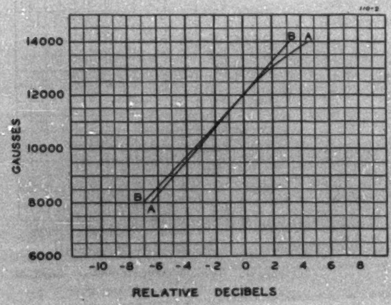

Figure 2. Variation of sound intensity with induction

Curve A—From magnetostriction tests on steel

Curve B—From sound level tests on complete transformers

Tested Variation of Sound Level With Core Weight

In order to check the correctness of the derived relation between noise in db and the core weight, a large number of sound level tests made on commercial transformer both in the shop and in the field were compared with calculated values.

Most of the tests were made at normal flux density for the transformers and in order to compare the values, they were reduced to a common flux density of 12,000 gausses. The values plotted in figure 3 are the sound intensity corrected to an induction of 12,000 gausses and the actual weight of the iron core in per cent of a common base.

The figure of 12,000 gausses to which the sound levels were corrected was chosen as being near the average induction of typical transformers. The correction was made by the use of figure 2. If W is the total weight of the core, W_1 is the weight of the part of the core considered, and its induction during the noise test was B_1, the value from figure 2 corresponding to B_1 was F_1, and the correction in db due to the part of the core considered was C_1, then $C_1=\left(\frac{(F_1W_1)}{W}\right)$. These corrections are obtained for the various parts of the core and added algebraically. In order to correct the sound reading obtained by test to the sound level corresponding to 12,000 gausses for plotting, the correction is subtracted algebraically from the test value. This method of making the corrections, is of course, an approximation but for the small variation in flux density which existed, the error introduced is negligible.

For example, a transformer has a tested noise level of 62 decibels. The weight of the core steel, W_1, is 2,000 pounds; B_1 its induction is 12,500 gausses; the weight, W_2, of the yoke steel is 3,000 pounds, its induction B_2 is 11,000 gausses. From figure 2, F_1 on curve A corresponding to 12,000 gausses is 1.0, F_2 corresponding to 11,000 gausses is −1.7.

$$C_1=\frac{F_1W_1}{W}=1.0\times\frac{2,000}{5,000}=0.4$$

$$C_2=\frac{F_2W_2}{W}=1.7\times\frac{3,000}{5,000}=-1.0$$

$$C_1+C_2=0.4-1.0=-0.6$$

The sound level of the transformer corrected to 12,000 gausses is $62-(-0.6)=$ 62.6 decibels.

This is the value which would be plotted in figure 3. Values obtained in this manner were plotted and the curves of figure 3 were obtained.

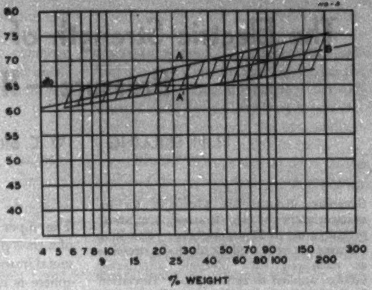

Figure 3. Variation of sound intensity with weight of core

Band *A*—*A'* indicates test values

Curve *B*—Theoretical curve

To obtain quantitative results from the data given for a particular transformer, the procedure is to obtain the decibel value for the transformer from figure 3 corresponding to the total weight. This is the decibels of the transformer at one foot provided all the iron were at an induction of 12,000 gausses. If the induction is at some other value, a correction for induction is made using figure 3.

EXAMPLE

A transformer with the following characteristics is assumed:

Core weight	30 per cent of the base weight
Yoke weight	50 per cent of the base weight
Total core weight	80 per cent of the base weight
Core induction	12,800 gausses
Yoke induction	11,500 gausses

The decibels corresponding to 80 per cent of the base weight on figure 3 is 69.3. From figure 2, curve A, the corrective decibels for the core =

$$\frac{30}{80}\times 1.6=0.6$$

The corrective decibels for the yoke = $50/80\times -0.9=-0.56$

The corrected decibels $=69.3+0.6-0.56=$ 69.34 decibels

The tested values as corrected for induction are shown by the shaded area between the lines A and A'. The straight line B is drawn through the approximate center of the band with a slope so that the sound intensity varies as the two-thirds power of the core weight. The slope of this line is the same as the slope of the shaded area within the limits of experimental error.

The weights in figure 3 are given as a percentage of an arbitrarily chosen base weight. The same curve can be used for other proportions or forms of construction after the correct base weight has been determined experimentally. The band width of the curves of figure 3 is due partly to variation in the magnetostriction of the steel itself. Some of the high points outside the band were probably due to resonance phenomena involving mechanical vibration of some part of the transformer.

Measurements on identical transformers extended over a large part of the band width. Physical proportions of the transformer also affect this sound level.

It was found that radiators had little effect on the sound level when the sound level was measured at one foot from the major sound producing surface in accordance with the latest standards. This is not wholly unexpected because the sound travels through oil and metal with little attenuation, and the sound wave transmitted is extremely complex.

In general, transformers with higher exciting currents, compared with identical transformers produced higher sound levels, but additional investigation is required before general conclusions on this subject can be offered with confidence.

Reduction of Noise

The reduction of the sound level of transformers is most effectively secured by reducing the induction in the iron. To produce quiet transformers, it is necessary that parts be so constructed that reasonant vibration is negligible. No successful method of reducing the transmission of sound to the air has been developed so far.

Generally most of the objectionable sound is borne by the air. In cases where sound is conducted through the supports, sound isolating devices may be used but the solution is difficult owing to the large low frequency components of the sound.

In the future, noise reduction through steel of lower magnetostriction may be expected.

The most variable quantity at present is the magnetostriction inherent in the

The Electric Strength of Air at High Pressure—II

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Synopsis: Results are given of an investigation of the electric strength of air at pressures up to 21 atmospheres. Sparking voltages are presented for air under pressure as measured with 60-cycle alternating voltage applied to sphere gaps. Deviation from the linear pressure-voltage relation that exists at low pressures is discussed, and certain characteristics of sparking in compressed air are noted.

A STUDY of the electric strength of air under pressure was reported three years ago.[1] Because of increased interest in the subject this study has been extended, and parts of the work have been repeated with a higher degree of accuracy. Data thus obtained are of interest in the design of air-blast circuit-breakers and other types of compressed-air switches, gas-insulated transformers, power cables operating under pressure, gas-filled radio transmitting condensers, etc.

The present paper presents data relating to sparking between spherical electrodes in air, the air pressure ranging from 1 to 21 atmospheres (0 to 300 pounds per square inch gauge pressure). Results obtained with spherical electrodes may be applied with reasonable accuracy to any spark gap in which the electric field is substantially uniform. The results of this investigation are therefore applicable to sparking between the parallel plates of condensers, electrostatic shields of high-voltage apparatus, and other smooth surfaces. Breakdown between points, and at sharp edges, and between small wires, is discussed in the earlier paper.[1]

Apparatus

The spherical electrodes of the test gap, mounted in their supporting framework, are shown in figure 1. The spheres shown are approximately 20 millimeters or slightly less than one inch in diameter. The upper sphere is supported by means of a glass plate which insulates it from the metal framework. The stem of the upper sphere is electrically connected, through a spring, to the metal rod at the axis of the two glass cones near the top of the picture. These cones, clamped back-to-back, constitute the high-voltage bushing for the pressure tank in which testing is done. The axial rod through the cones serves as the high-voltage lead into the tank, and a connecting wire from the top of the rod to the high-voltage transformer may be seen in the picture. The heavy metal disk between the glass cones becomes the head of the pressure cylinder when the apparatus is assembled.

The lower sphere is adjustable vertically by means of a micrometer mechanism. The sphere may be raised or lowered without rotation. Springs are provided to prevent error due to backlash in the screw or bearings. To adjust the gap between spheres to its desired value the lower sphere is raised until it touches the upper sphere. Contact is indicated by a neon glow lamp that is connected in series with the gap while making this adjustment. The lower sphere is then lowered to its proper position as indicated by the micrometer calibration. Micrometer settings are read to 0.0001 inch.

The supporting framework for the gap is quite rigid. It is subjected to no stress other than its own weight, and the gap length is adjusted in the same position in which electrical testing is done. In particular, the supporting framework is so designed that changes of pressure of the gas do not change the length of the spark gap, as would occur if the spheres were supported from opposite ends of the tank. Temperature changes are not large, and since all major parts of the supporting framework are of the same material (steel) the gap length is not appreciably affected by expansion or contraction.

Dimensions and clearances are such that the gap is essentially a standard sphere spark gap as defined by AIEE standards.

For testing in gases other than air at atmospheric pressure the assembly shown in figure 1 is inserted in a pressure chamber as shown in figure 2. The cylinder, a 29-inch length of 10-inch steel oil well casing, is closed at top and bottom by heads of cast steel that are clamped together by 12 bolts as shown. Provision is made to admit gas to the test chamber through a pipe at the top, and two electric circuits other than the high-voltage circuit are led into the chamber by means of spark plugs. One of these circuits is used for a lamp to illuminate the interior of the chamber, and the second serves to operate a fan, to supply a source of ultraviolet radiation, and for other purposes as required. A shaft carried through a stuffing-box in the upper head allows adjustments of the spark-gap length to be made while the apparatus is assembled and under pressure. There are two small glass windows in the wall of the chamber.

High voltage is supplied to the test gap within the pressure chamber from either of two transformers, one rated 110 to 33,000 volts, 15 kva, and the other 220 to 300,000 volts, 300 kva. The latter transformer is equipped with a tertiary voltmeter winding, which is used for voltage measurement. When the 33 kv transformer is used, voltage is measured across the primary winding and the output voltage is obtained by means of the transformer turn ratio. A crest voltmeter is

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[1] For all numbered references, see list at end of paper.

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steel. Consequently noise can be reduced by proper selection of the steel.

Conclusions

1. Transformer noise originates principally in the magnetostriction of the core iron.
2. Transformer noise can be calculated from the design constants using as a base magnetostriction tests of the core iron and the other relations given.
3. The most effective methods of reducing basic transformer noise are by reducing the induction in the iron or by reducing the magnetostriction of the iron.
4. The sound intensity produced by similar transformers with the same induction in the core, varies as the two-thirds power of the weight.
5. The sound intensity produced by a transformer with core iron having normal magnetostriction varies as the square of the magnetostriction. This relation can be used to compare the sound intensities produced at various inductions or to compare the sound levels of various core steels.

used for all measurements to eliminate error resulting from nonsinusoidal wave form. The over-all accuracy of voltage measurement was checked by comparison of measured values of sparking voltage between 20-millimeter spheres with AIEE standard values; the difference was less than 0.5 kv up to a reading of 40 kv, the highest voltage compared.

Resistance is used in the high-voltage connection to limit current in the test gap to one-half ampere. When sparking occurs the flow of current is interrupted at the end of the first quarter-cycle of 60-cycle current by a high-speed circuit breaker in the transformer primary circuit.

Voltage is continuously adjustable over the entire range of the transformers by means of a tap-changing transformer and an induction-type voltage regulator in the transformer primary circuit. During certain tests the transformer power is supplied from a synchronous sine-wave motor-generator set; in this case it is possible to adjust voltage by changing the generator field current as well as by means of the transformer and regulator.

Procedure

A generally uniform method of testing for the sparking voltage was followed. The spheres between which sparking was to take place were mounted in the micrometer head within the pressure chamber after having been cleaned and polished. Various methods of polishing were used, and no difference was found between polishing with rouge or metal polish, by hand or by motor-driven buffing wheel. In one case a motor-driven wire brush was used on steel spheres, and although it altered the appearance from a mirror surface to a matte surface it had no effect on the electrical behavior. The presence of an oil film on the spheres made no apparent difference in sparking voltage; however it is difficult to be sure that in any case the spheres were entirely free from oil because of the probability of oil being carried into the test chamber from either an air compressor or a storage cylinder.

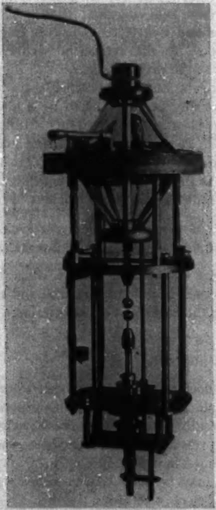

Figure 1. The sphere spark gap and supporting framework

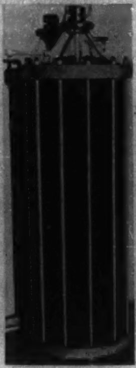

Figure 2. The test chamber containing air under pressure

When an air compressor was used the air was cooled and filtered before being admitted to the test chamber. Despite the filtering, particles of oil of extremely small size were carried into the test chamber. Although the presence of the resulting haze of oil particles within the chamber had no noticeable effect on the sparking voltage, its presence resulted in a black oily deposit on the sphere surfaces after continued sparking. This was avoided by compressing the air several hours before it was used, thereby allowing the oil particles to settle out; when this procedure was followed the compressed air was perfectly clear and free from haze.

Before measuring sparking voltage the gap was adjusted to the desired length. A zero setting of the micrometer was obtained by means of a neon glow lamp, as described above. The micrometer setting was read when the lamp lighted as the spheres were being brought together and when it went out as they were separated. The difference between the two readings was normally less than 0.0001 inch.

Sixty-cycle voltage was applied to the gap, and the voltage was raised gradually until a spark occurred. The time required to raise voltage to sparking value varied from a few seconds to about half a minute, and sometimes was extended over several minutes. In some cases the speed of application of voltage was an important factor, as will be discussed.

At each particular spacing and pressure a number of readings of sparking voltage were measured and recorded. The number of readings depended upon circumstances, and particularly upon the uniformity of the measurements. If the first eight or ten sparks were at substantially the same voltage (within two or three per cent) they were considered satisfactory, but if there were radical variations the test was continued until 20, or 40, or as many as 100 sparking voltages were recorded. Either the pressure or the spacing was then changed and another series of readings was made.

Several series of tests were made with a quartz-tube mercury-arc lamp so located within the pressure chamber that its light fell on the spheres and passed through the gap between the spheres. It is impossible to say that its presence made any significant difference in the results, either in the value of voltage or in consistency of results. Use of the lamp was not continued.

Interpretation of Data

An *average* of the several voltage readings at any given spacing and pressure is not adequate to represent the results. There is apparent in much of the data a distinct trend:[4] the first sparking voltages are low and succeeding ones are higher in such a way that they appear to approach a limit. After four or five sparking voltages have been measured in rapid succession (less than a minute apart) the value of sparking voltage becomes stabilized. There is frequently a gradual rise of ten per cent or more in sparking voltage from the first spark to the fourth or fifth, while random variations of sparking voltage thereafter are not likely to vary more than one or two per cent from the trend value. If the series of sparking measurements is interrupted for five or ten minutes, however,

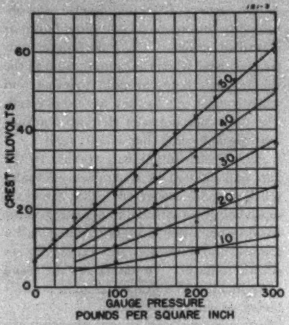

Figure 3. Sparking in air between brass spheres. Spark gap length as noted on curves in thousandths of an inch

and is then recommenced, it is probable that the first few sparks after interruption will occur at slightly reduced voltage and the stabilized value will have to be re-established. Occasionally, also, there is a break from the stabilized sparking voltage for no apparent reason; a spark will occur several per cent below the trend voltage, and successive sparks will be consecutively higher until the stabilized value is again reached.

An upward trend at the beginning of a series of sparks was found in most voltage measurements in which the voltage was greater than about 30 kv; that is, in case of either high pressure or relatively long spark gap. There was no such trend observed at atmospheric pressure, or in most measurements below about 20 kv. Under no circumstances was there any appearance of a downward trend; the voltage readings were either quite consistent or else the erratic readings were below the stabilized value. The amount of variation between the stabilized value of voltage and the lowest sparking voltages was of the order of magnitude of ten per cent.

The stabilized value of voltage is reproducible from time to time; an average voltage value, on the other hand, cannot readily be repeated and is not particularly significant because it depends on the number of measurements at a given setting of the gap and on the history of sparking of the gap. Therefore the stabilized value of voltage has been accepted as the best value to use in representing test results in this paper. It must be recognized that a reasonable engineering value of sparking voltage is about ten per cent below the values here given because of the likelihood of occasional sparks at voltages below the stabilized value.

To a slight extent the sparking voltage at high pressure was dependent upon time of application of voltage. The stable trend value was found by raising voltage at such a rate that sparks were produced at the rate of one or two per minute. If, after the stable condition had been established, voltage two or three per cent lower than the stable sparking value was applied continuously, sparking would usually occur after some time. The time that elapsed before sparking in such a case might be from a few seconds to ten minutes. If the applied voltage was as much as five per cent below the trend value, sparking might be delayed 30 minutes. Applied voltages ten per cent or more below the trend value did not produce sparking regardless of time. To insure steady voltage supply for these tests, power was taken from a motor-generator set.

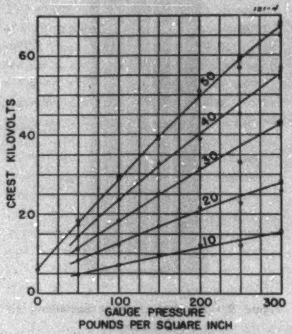

Figure 4. Sparking in air between steel spheres. Spark gap length as noted on curves in thousandths of an inch

It is believed that the rising trend of sparking voltage and the time lag of sparking are associated phenomena, for they appeared under the same conditions.

Results

Results are shown in figures 3 and 4. Each indicated point represents a stabilized value determined from a series of individual voltage readings. About 2,000 individual readings of sparking voltage are represented by these sets of curves.

The data of figure 4 may be presented in another form which is more compact, although less convenient for reference. This is done by plotting the sparking voltage as a function of the product of the independent variables, pressure, and spacing between electrodes. This product may be designated *PS*; the pressure *P* is most conveniently measured in atmospheres, and the spacing between electrodes, *S*, may be in inches.

According to the law for sparking commonly known as Paschen's law[3,4,5], voltage at breakdown should be a function of the product *PS*. Numerically, Paschen's law may be expressed[3] in the above units as

$$V = 75.4PS + 1.7 \text{ kilovolts} \qquad (1)$$

If Paschen's law could be extrapolated from the low pressure range, below one atmosphere, for which it was determined, to cover the many-times greater range up to 21 atmospheres, the experimental points from figure 3 should all lie on a single straight line when plotted against the product *PS*. That this is approximately true may be seen in figure 5, in which all experimental points could be represented by a single straight line with an error not exceeding ten per cent. There is, however, a consistent deviation that corresponds to the drooping curvature of the lines of figure 3, and sparking voltages at high pressure are lower than sparking voltages for the same value of *PS* at low pressure.

In earlier papers[1] it has been suggested that this systematic deviation might result from field emission; that is, it might be caused by electrons drawn from the cathode by the extremely high electric field at the cathode surface. If this is so the deviation from Paschen's law could reasonably be expected to be a function of the electric field strength.[3] Inspection of figure 5 suggests the following modified expression for sparking voltage, in which V/S is the electric field strength between electrodes, and a is a coefficient to be determined from the data:

$$V = 76.0PS(1 - aV/S) + 1.7 \text{ kilovolts} \qquad (2)$$

Solving for sparking voltage, discarding a term of negligible importance, and evaluating the coefficient a, one obtains:

$$V = \frac{76.0PS}{1 + 0.009P} + 1.7 \text{ kilovolts} \qquad (3)$$

The coefficient has here been evaluated to make equation 3 agree with experimental results obtained with *brass* electrodes. The pressure *P* is in atmospheres and the spacing *S* in inches.

It may be seen that results obtained with steel electrodes (figure 4) differ slightly from results with brass electrodes (figure 3). Sparking voltages at pressures less than about 50 pounds per square

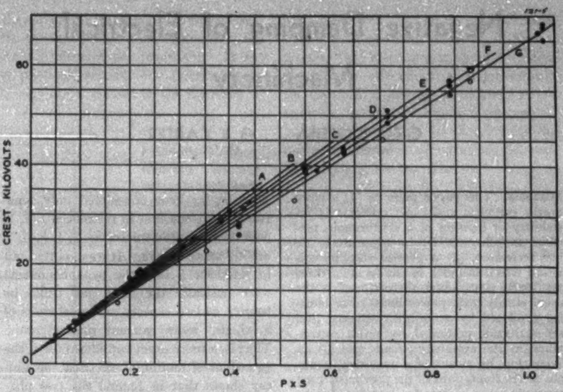

Figure 5. Sparking in air between brass spheres

P, absolute air pressure in atmospheres; *S*, spark gap length in inches. Points shown are experimental, lines are computed from equation 3

Line *A* and points marked ◐ are for 0 pounds per square inch pressure

Line *B* and points marked ⊕ are for 50 pounds per square inch pressure

Line *C* and points marked ⊙ are for 100 pounds per square inch pressure

Line *D* and points marked ⊖ are for 150 pounds per square inch pressure

Line *E* and points marked ● are for 200 pounds per square inch pressure

Line *F* and points marked ○ are for 250 pounds per square inch pressure

Line *G* and points marked ⊗ are for 300 pounds per square inch pressure

inch are the same for steel and brass, and both agree with Paschen's law. At higher pressures the deviation from Paschen's law is greater for steel than for brass. If the deviation is assumed to be a result of field emission, it is not surprising that it should be different with different electrode materials. A difference between sparking voltage with iron and copper electrodes was noted in an earlier paper by one of the authors;[1] although it was not at that time known to be significant, it now appears comparable to the difference between steel and brass shown by figures 3 and 4.

Values of sparking voltage were measured between aluminum spheres, also. Results using aluminum electrodes in air were inclined to be erratic, some sparking voltages being as high as the values obtained with brass spheres while others were lower than results with steel. Repeated sparking between aluminum spheres in air tended to reduce the sparking voltage, the inference being that a chemical change of the electrode surfaces was taking place. (Aluminum electrodes give consistent results, comparable with brass electrodes, when used in nitrogen.)

To describe results obtained with *steel* electrodes an expression of the form of equation 3 may be used:

$$V = \frac{76.0PS}{1+0.019P} + 1.7 \text{ kilovolts} \qquad (4)$$

The coefficient in the denominator is here adjusted to give as good agreement as possible with the data of figure 4; agreement is within about ten per cent.

Although this expression for steel electrodes was derived to fit the experimental results obtained by the authors, it gives excellent agreement with sparking voltages measured by A. H. Howell.[5] Howell's electrodes were "turned from ordinary cold rolled steel." His results, although obtained with direct current from a Van de Graaff generator, appear to extend the range of equation 4 to pressures as high as 600 pounds per square inch and values of the product *PS* as great as ten.

Equation 4 also gives reasonably good agreement with results obtained by R. C. Buehl[6] with 60-cycle alternating voltage in nitrogen up to pressures of 140 atmospheres (2,060 pounds per square inch) and *PS* of seven. Agreement is within ten per cent for most of Buehl's data.

It is, furthermore, evident that Paschen's law may be considered a special case of equations 3 or 4 that applies with good accuracy when the air pressure is below about five atmospheres. Since Paschen's law has been experimentally substantiated for values of *PS* as low as 0.05 for pressures below atmospheric, it follows that equations 3 and 4 are accurate in that region also.

These considerations indicate the useful range of application of equations 3 and 4.

References

1. THE ELECTRIC STRENGTH OF AIR AT HIGH PRESSURE, H. H. Skilling. AIEE TRANSACTIONS, volume 58, 1939 (Apil section), pages 161–5.

2. Discussion of reference 1 by D. W. Ver Planck.

3. ELECTRICITY IN GASES, J. S. Townsend. Oxford Press, London, 1915.

4. FUNDAMENTAL PROCESSES OF ELECTRICAL DISCHARGE IN GASES, L. B. Loeb. John Wiley and Sons, New York, 1939.

5. BREAKDOWN STUDIES IN COMPRESSED GASES, A. H. Howell. AIEE TRANSACTIONS, volume 58, 1939 (May section), pages 193–204.

6. Discussion of reference 5 by R. C. Buehl.

See references 1 and 5 for complete bibliography.

Negative Damping of Electrical Machinery

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Synopsis: This paper presents (1) a more nearly complete and exact criterion of instability or hunting of a synchronous machine as influenced by its armature or tie line resistance, (2) a general stability criterion which includes in one formula three previously separately treated cases: the usual steady state power limit, rotor hunting produced by armature resistance, and self-excitation produced by series capacitance in the armature circuit, and (3) an indication, by means of a numerical example, of the limitations of the previously used approximate criterion for hunting due to armature resistance.

IN PREVIOUS treatments, hunting, self-excitation, and loss of synchronism have been considered only separately, neglecting their mutual interactions. The present paper, on the other hand, considers their mutual effects and thus provides a general analysis by reference to which the limitations and proper fields of application of the approximate formulas may be found. Application of the complete criterion given is however rather laborious, and it is not suggested for general use in place of the approximate forms.

Introduction

When synchronous machines are tied together through impedances having rather high resistance components, and when there is insufficient rotor damping, the machines tend to hunt. Also, when a machine is connected to a capacitance load or to a system through a series capacitor there is exhibited under certain conditions a tendency to self-excite. Both of these phenomena have been treated extensively in electrical engineering literature, and the conditions under which one or the other form of operation is to be expected are reasonably well known. However, the two phenomena have so far been considered only separately, e.g., the first in references 1, 2, 3, and the second in references 3, 4, 5, 6, 7, among many others. It was pointed out in reference 3 that the two phenomena were basically the same and could be treated as two aspects or special cases of a single, more general characteristic. That is, a more exact consideration of the equations of motion of electrical machinery shows that in general the two phenomena are mutually dependent and in fact inseparable, and that it is only because of the relatively large differences in the orders of magnitudes of the principal controlling features of each that they can be approximately separated. Since all previous treatments have considered them separately and have consequently neglected their mutual interactions, it was felt that an investigation should be made to find out just what these interactions might be. The results of such an investigation, comprising a quantitative study of the complete equations of motion of a simple electromechanical system, are presented in this paper.

Preliminary Considerations

Before proceeding to the combining of self-excitation and hunting into a single form (here called negative damping), the simpler case of a system which can show only hunting was first studied, since it has been found that the behavior of this system according to the equations used here is somewhat different from that according to the approximate criteria previously given.[1]

A single synchronous machine with only one rotor winding, connected to an infinite bus, may become unstable at small generator angles or at any motor angle if the armature resistance is too high, and at large generator angles if the armature resistance is too low. Both concepts are well known, but the latter may be amplified by pointing out that at large generator angles and high resistances the system, although stable, has only a negligible synchronizing torque and load capacity.

Figures 1 and 2 show the results of stability calculations made for this case. The regions of stable operation were determined by the application of Routh's[11] stability criterion to equation 6 of the appendix. Figure 1 is for comparatively low values of armature resistance; in this figure, the solid line represents the stability limit for unit field excitation, the dashed line that for one and a half times unit excitation. For comparison, the hunting criterion $r \leqq x_q \tan \delta$, of reference 1, is also shown, as a dot-and-dash line for generator angles.

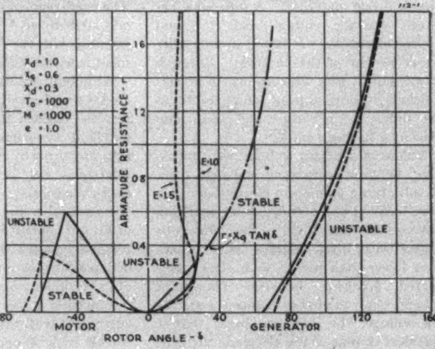

Figure 1. Synchronous machine connected to infinite bus—stability limits—effect of armature resistance r and field excitation E

It is seen that the approximate criterion is optimistic for small angles and pessimistic for larger generator angles; for the larger motor angles (from about 30 degrees to the steady state boundary) the approximate criterion $r = x_q \tan \delta$ agrees fairly closely with the more nearly exact criterion used here. At light loads figure 1 indicates that $r \leqq x_d' \tan \delta$ would be a

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[1] For all numbered references, see list at end of paper.

better criterion of stability than $r \leqq x_q \tan \delta$, at least for the value of field time constant $T_0=1{,}000$ studied. The more nearly exact criterion shows no instability due to high armature resistance for generator angles of more than about 30 degrees. This is not in conflict with experience, since those cases of hunting which have actually been encountered have usually occurred only at light load. Another check is obtained by noting that the amount of armature resistance required to produce negative damping at light loads decreases as the field excitation is increased, which is consistent with the observation that the severity of hunting is usually increased by increased excitation. The outer boundaries of figure 1 are found to check the steady state stability limits determined by the usual steady state power-angle relations.[12]

Figure 2 is of only theoretical interest, as the resistances involved are so high that the machine armature is practically open circuited.

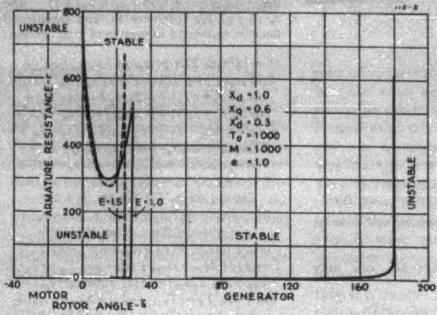

Figure 2. Synchronous machine connected to infinite bus—stability limits—effect of armature resistance r and field excitation E at high values of r

Negative Damping With Series Capacitors

Figure 3 shows some of the results of stability calculations made for the general case of a synchronous machine connected to a bus through a series capacitor. Equation 4 of the appendix gives the equation for this case. In a general way the correspondence of this stability boundary to those which would be obtained by the previous approximate methods is that first, the left hand rising portion of the curve corresponds roughly to the lower portion of the curve for $M=\infty$ of figure 1, reference 5, and second, the right hand decreasing portion corresponds to the boundary which would be found by application of the approximate criterion for hunting if the capacitive reactance x_c is subtracted from the quadrature axis reactance x_q to form an effective net reactance. As in the previous section, the criterion $r \leqq (x_d'-x_c) \tan \delta$ (shown in the figure by a dashed line) forms a better, in fact a rather good, criterion.

Figure 3 indicates that instability may occur if the resistance is too small and if the resistance is too large. An amortisseur on the machine rotor will increase the maximum allowable armature resistance so much as practically to remove any upper (right hand in figure 3) boundary. On the other hand the allowable minimum resistance will also be increased, as indicated by figure 4. Figure 4 has been calculated by the equations of reference 5, and may be compared to figure 1, reference 5, to show the effect of the amortisseur. The fact that an amortisseur does not tend to suppress the negative damping introduced by series capacitance need not be alarming however, as figure 5 shows that resistance in shunt with the capacitor is still effective in preventing the instability, just as was found in reference 3, figure 18. Figure 5 was calculated from equation 1d of reference 3, using an extension of equations 2d and 3d to include the amortisseur.[10]

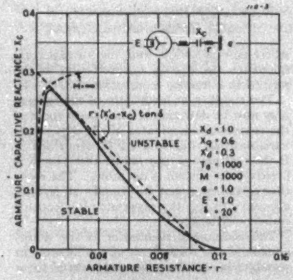

Figure 3. Synchronous machine connected to infinite bus through armature series capacitance—stability limits at rotor angle $\delta=20$ degrees

Conclusions

1. A more complete and exact criterion of instability of a synchronous machine as influenced by armature resistance has been derived by application of Routh's test to the system performance equations.

2. A general criterion of instability has been derived which includes three previously separately treated cases: the normal steady state power limit, rotor hunting produced by armature resistance, and self-excitation produced by series capacitance in the armature circuit.

3. The limitations of the previously used approximate criterion for hunting due to armature resistance have been determined.

4. The effect on the stability limit of series capacitance in the armature circuit has been studied by the more complete criterion and shown to agree fairly well with the approximate criteria for appropriate limiting conditions.

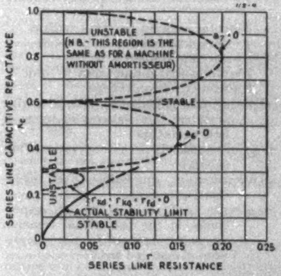

Figure 4. Negative damping of synchronous machines with series capacitors—effect of amortisseur. Series resistance required for stable operation

$x_d=1.0$ $x_d'=0.3$ $x_d''=0.218$
$r_{kd}=0.02$ $T_0=1{,}000$, $\omega=1.0$, $\alpha=0$
$x_q=0.6$ $x_q''=0.311$ $r_{kq}=0.04$
($a_7=0$, $a_6=0$, etc., are approximate criteria)

Appendix

Nomenclature

r = armature circuit resistance
δ = angle between rotor axis and axis of stator voltage

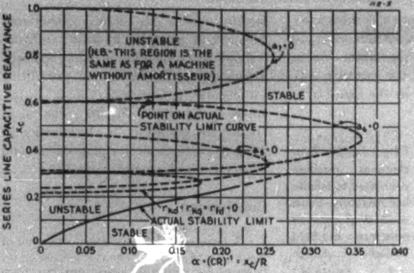

Figure 5. Negative damping of synchronous machines with series capacitors—effect of amortisseur. Shunt resistance required for stable operation

$x_d=1.0$ $x_d'=0.3$ $x_d''=0.218$
$r_{kd}=0.02$ $T_0=1{,}000,\ \omega=1.0,\ r=0$
$x_q=0.6$ $x_q''=0.311$ $r_{kq}=0.04$
($a_7=0$, $a_6=0$, etc., are approximate criteria)

x_d = direct axis synchronous reactance
x_q = quadrature axis synchronous reactance
x_d' = direct axis transient reactance
x_d'' = direct axis subtransient reactance
x_q'' = quadrature axis subtransient reactance
T_0 = field open circuit time constant
x_c = armature circuit capacitive reactance
R = resistance in parallel with x_c
$\alpha=x_c/R$
M = rotor inertia factor in radians
e = stator voltage
i = stator current
ψ = stator flux linkages
(subscripts d and q indicate direct and quadrature axis components, respectively)
θ = rotor angle in electrical radians
p = time derivative operator $=d/dt$
$(p\theta)=\omega$ = rotor angular velocity
T = mechanical torque input to rotor
E = field excitation voltage
$G(p)$ = mutual impedance operator giving armature flux linkages in terms of field excitation
$x_d(p)$ = armature direct axis operational impedance
$x_q(p)$ = armature quadrature axis operational impedance
a_7, a_6 = the coefficients of p^0, p^1, respectively, in the operational common denominators used in the calculations of figures 4 and 5

Analysis

In this appendix the equations of a synchronous or induction machine with a capacitor and resistor in parallel in the armature circuit are extended to include the effect of rotor inertia during small changes (e.g., oscillations) of rotor speed. The fundamental equations for this case have been derived in references 5, 8, and 9, and are:

$$\left.\begin{aligned}
&(p+\alpha)[e_d-p\psi_d+ri_d+\psi_q(p\theta)]+x_ci_d-\\
&\qquad(p\theta)[e_q-p\psi_q+ri_q-\psi_d(p\theta)]=0\\
&(p+\alpha)[e_q-p\psi_q+ri_q-\psi_d(p\theta)]+x_ci_q+\\
&\qquad(p\theta)[e_d-p\psi_d+ri_d+\psi_q(p\theta)]=0\\
&T=\psi_di_q-\psi_qi_d+Mp^2\theta
\end{aligned}\right\}\quad(1)$$

where

$$e_d=e\sin\delta\qquad \psi_d=G(p)E-x_d(p)i_d$$
$$e_q=e\cos\delta\qquad \psi_q=-x_q(p)i_q$$

and the other symbols are as defined in reference 8 and in the nomenclature. From equations 1, by taking small changes in e, E, i_d, i_q, and $\delta(\Delta\theta=\Delta\delta)$, one may obtain:

$$\left.\begin{aligned}
&[(p+\alpha)(px_d(p)+r)+x_c-\omega_0^2x_d(p)]\times\\
&\Delta i_d-[(p+\alpha)\omega_0x_q(p)+\omega_0(px_q(p)+r)]\times\\
&\Delta i_q+\{[(p+\alpha)\psi_{q0}+2\omega_0\psi_{d0}-e_{q0}-ri_{q0}]\times\\
&p+(p+\alpha)e_{q0}+\omega_0e_{d0}\}\,\Delta\delta=[-(p+\alpha)\times\\
&\sin\delta_0+\omega_0\cos\delta_0]\,\Delta e+[(p+\alpha)p-\omega_0^2]\times\\
&\qquad\qquad G(p)\,\Delta E\\
&[(p+\alpha)\omega_0x_d(p)+\omega_0(px_d(p)+r)]\,\Delta i_d+\\
&[(p+\alpha)(px_q(p)+r)+x_c-\omega_0^2x_q(p)]\,\Delta i_q+\\
&\{[-(p+\alpha)\psi_{d0}+2\omega_0\psi_{q0}+e_{d0}+ri_{d0}]p-\\
&(p+\alpha)e_{d0}+\omega_0e_{q0}\}\,\Delta\delta=[-(p+\alpha)\cos\delta_0-\\
&\omega_0\sin\delta_0]\,\Delta e+(2p+\alpha)\omega_0G(p)\,\Delta E\\
&-[\psi_{q0}+i_{q0}x_d(p)]\,\Delta i_d+[\psi_{d0}+i_{d0}x_q(p)]\times\\
&\qquad\Delta i_q+Mp^2\,\Delta\delta=\Delta T-i_{q0}G(p)\,\Delta E
\end{aligned}\right\}\quad(2)$$

where $\omega_0=(p\theta)_0=1.0$, and the steady state quantities ψ_{d0}, ψ_{q0}, i_{d0}, i_{q0} are found as described in reference 5, equations 32, 33, 34.

The stability or instability of the system may now be determined from the p roots of the common denominator of the dependent variables Δi_d, Δi_q, and $\Delta\delta$ in equations 2. If the real part of any one of these roots is positive, the system is unstable. The roots themselves need not be found; it is only necessary to test the expression by Routh's stability criterion.[11]

The common denominator is the determinant of the coefficients of the dependent variables in equations (2), and is equal to:

$$\begin{aligned}
D=\{&[(p+\alpha)(px_d(p)+r)+x_c-\omega_0^2x_d(p)]\times\\
&[(p+\alpha)(px_q(p)+r)+x_c-\omega_0^2x_q(p)]+\\
&[(p+\alpha)\omega_0x_d(p)+\omega_0(px_d(p)+r)]\times\\
&[(p+\alpha)\omega_0x_q(p)+\omega_0(px_q(p)+r)]\}\,Mp^2+\\
&[\![\{[(p+\alpha)\psi_{q0}+2\omega_0\psi_{d0}-e_{q0}-ri_{q0}]p+\\
&(p+\alpha)e_{q0}+\omega_0e_{d0}\}[(p+\alpha)\omega_0x_d(p)+\\
&\omega_0(px_d(p)+r)]-\{[-(p+\alpha)\psi_{d0}+\\
&2\omega_0\psi_{q0}+e_{d0}+ri_{d0}]p-(p+\alpha)e_{d0}+\\
&\omega_0e_{q0}\}[(p+\alpha)(px_d(p)+r)+\\
&x_c-\omega_0^2x_d(p)]]\!][\psi_{d0}+i_{d0}x_q(p)]+\\
&[\![[(p+\alpha)\omega_0x_q(p)+\omega_0(px_q(p)+r)]\times\\
&\{[-(p+\alpha)\psi_{d0}+2\omega_0\psi_{q0}+e_{d0}+ri_{d0}]p-\\
&(p+\alpha)e_{d0}+\omega_0e_{q0}\}+[(p+\alpha)(px_q(p)+r)+\\
&x_c-\omega_0^2x_q(p)]\{[(p+\alpha)\psi_{q0}+2\omega_0\psi_{d0}-\\
&e_{q0}-ri_{q0}]p+(p+\alpha)e_{q0}+\omega_0e_{d0}\}]\!]\times\\
&[\psi_{q0}+i_{q0}x_d(p)]
\end{aligned}\quad(3)$$

With $M=\infty$, equation 3 reduces to that for the previously treated case of constant rotor speed (reference 3, appendix D). Also if the machine is initially unexcited, so that $\psi_0=i_0=0$, the regions of stable operation are the same as before. In the general case, however, the performance will be affected by the operating angle, machine inertia, and excitations.

For a machine with only one rotor winding, $x_d(p)=(x_d'T_0p+x_d)/(T_0p+1)$ and $x_q(p)=x_q$ (see reference 8), and equation 3 may be expanded in powers of p to become:

$$\begin{aligned}
D'=\{&p^5x_qx_d'T_0+p^4[x_qx_d+rT_0(x_q+x_d')+\\
&2\alpha T_0x_qx_d']+p^3[(2x_qx_d'+x_c(x_q+x_d'))T_0+\\
&r(x_d+x_q+rT_0)+\alpha(2x_dx_q+\alpha x_qx_d'T_0)+\\
&2\alpha rT_0(x_q+x_d')]+p^2[2x_dx_q+x_c(x_d+x_q)+\\
&r(T_0(2x_c+x_q+x_d')+r)+\alpha(T_0x_c(x_q+x_d')+\\
&2T_0x_qx_d'+\alpha x_dx_q)+\alpha r(2x_d+2x_q+2T_0r+\\
&T_0\alpha(x_q+x_d'))]+p[T_0(x_c-x_q)(x_c-x_d')+\\
&r(x_d+x_q+2x_c+rT_0)+\alpha(x_c(x_d+x_q)+\\
&2x_dx_q+\alpha x_qx_d'T_0)+\alpha r(2x_cT_0+2r+\\
&\alpha(x_d+x_q)+\alpha rT_0)]+[(x_d-x_c)(x_q-x_c)+\\
&r^2+\alpha^2x_dx_q+2\alpha rx_c+(\alpha r)^2]\}\,Mp^2+\\
&(\psi_{d0}+i_{d0}x_q)\{p^5(x_d'T_0\psi_{d0})+p^4[-x_d'T_0\times\\
&(-\alpha\psi_{d0}+2\psi_{q0}+ri_{d0})+(x_d+\alpha x_d'T_0+rT_0)\times\\
&\psi_{d0}+2x_d'T_0\psi_{q0}]+p^3[x_d'T_0(\alpha e_{d0}-e_{q0})-\\
&(x_d+\alpha x_d'T_0+rT_0)(-\alpha\psi_{d0}+2\psi_{q0}+ri_{d0})+\\
&(\alpha x_d+r+\alpha rT_0-x_d'T_0+x_cT_0)\psi_{d0}+\\
&2x_d'T_0(\alpha\psi_{q0}+2\psi_{d0}-ri_{q0})+(2x_d+rT_0+\\
&\alpha x_d'T_0)\psi_{q0}]+p^2[(x_d+\alpha x_d'T_0+rT_0)(\alpha e_{d0}-\\
&e_{q0})+(2x_d+rT_0+\alpha x_d'T_0)(\alpha\psi_{q0}+2\psi_{d0}-\\
&ri_{q0})-(\alpha x_d+r+\alpha rT_0-x_d'T_0+x_cT_0)\times\\
&(-\alpha\psi_{d0}+2\psi_{q0}+ri_{d0})+(r+\alpha x_d)\psi_{q0}+(x_c-\\
&x_d+\alpha r)\psi_{d0}+2x_d'T_0(\alpha e_{q0}+e_{d0})]+p[(\alpha x_d+\\
&r+\alpha rT_0-x_d'T_0+x_cT_0)(\alpha e_{d0}-e_{q0})+(2x_d+\\
&rT_0+\alpha x_d'T_0)(\alpha e_{q0}+e_{d0})-(x_c-x_d+\alpha r)\times\\
&(-\alpha\psi_{d0}+2\psi_{q0}+ri_{d0})+(r+\alpha x_d)\times\\
&(\alpha\psi_{q0}+2\psi_{d0}-ri_{q0})]+[(x_c-x_d+\alpha r)\times\\
&(\alpha e_{d0}-e_{q0})+(r+\alpha x_d)(\alpha e_{q0}+e_{d0})]\}+\\
&[(\psi_{q0}T_0+i_{q0}x_d'T_0)p+(\psi_{q0}+i_{q0}x_d)]\times\\
&\{p^4(\psi_{q0}x_q)+p^3[\psi_{q0}(\alpha x_q+r)+(\alpha\psi_{q0}+\\
&2\psi_{d0}-ri_{q0})x_q-\psi_{d0}2x_q]+p^2[\psi_{q0}(r\alpha+\\
&x_c-x_q)+(\alpha\psi_{q0}+2\psi_{d0}-ri_{q0})(\alpha x_q+r)+\\
&(\alpha e_{q0}+e_{d0})x_q-\psi_{d0}(\alpha x_q+r)+(-\alpha\psi_{d0}+\\
&2\psi_{q0}+ri_{d0})2x_q]+p[(\alpha\psi_{q0}+2\psi_{d0}-ri_{q0})\times\\
&(r\alpha+x_c-x_q)+(\alpha e_{q0}+e_{d0})(\alpha x_q+r)+\\
&(-\alpha\psi_{d0}+2\psi_{q0}+ri_{d0})(\alpha x_q+r)-\\
&(\alpha e_{d0}-e_{q0})2x_q]+[(\alpha e_{q0}+e_{d0})(r\alpha+x_c-x_q)-\\
&(\alpha e_{d0}-e_{q0})(\alpha x_q+r)]\}
\end{aligned}\quad(4)$$

Note that the coefficient of Mp^2 in this equation is the Δ' of equation 4d, appendix D, of reference 3.

Equation 4 may be used to find the effect of rotor inertia on negative damping in the region of series capacitor applications where the machine is connected to an excited bus through a capacitor. For a capacitance load, as for transmission line capacitance, e_{d0} and e_{q0} should be taken as zero, in which case the effect of rotor oscillations on the borderline of stability is believed to be negligible, as there are no synchronizing torques.

For the special case of no armature circuit capacitance ($x_c=0$ or $\alpha=\infty$), the performance equations 2 become

$$\left.\begin{aligned}
&-(px_d(p)+r)\,\Delta i_d+\omega_0x_q(p)\,\Delta i_q-\\
&\qquad(\psi_{q0}p+e_{q0})\,\Delta\delta=\sin\delta_0\,\Delta e-pG(p)\,\Delta E\\
&-\omega_0x_d(p)\,\Delta i_d-(px_q(p)+r)\,\Delta i_q+\\
&(\psi_{d0}p+e_{d0})\,\Delta\delta=+\cos\delta_0\,\Delta e-\omega_0G(p)\,\Delta E\\
&-(\psi_{q0}+i_{q0}x_d(p))\,\Delta i_d+(\psi_{d0}+i_{d0}x_q(p))\times\\
&\qquad\Delta i_q+Mp^2\,\Delta\delta=\Delta T-i_{q0}G(p)\,\Delta E
\end{aligned}\right\}\quad(5)$$

Hollow Pipes of Relatively Small Dimensions

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Summary

SEVERAL cross sectional shapes for hollow-pipe transmission lines are described that provide lower operating frequencies for given outside dimensions than do the simple shapes heretofore proposed. The theory for one such line, the "septate coaxial cable" is derived, and experiments are reported. Cavity resonator embodying these principles are also described.

Introduction

The hollow-pipe type of conductor[1,2] for ultrahigh-frequency electromagnetic energy, although possessing several desirable features, has the disadvantage that its transverse dimensions are relatively large; namely, of magnitude comparable to the wave length. The object of this paper is to lessen this encumbrance by describing pipes that have transverse dimensions several times smaller than those of the simple forms previously proposed.*

The principle underlying the improved conductors derives from the properties of the $H_{0,1}$-waves[3] in a pipe of flat rectangular cross section, figure 1*A*. In this case, the critical wave length λ_0, i.e. the wave length above which no transmission takes place, is independent of d and depends on l only ($\lambda_0 = 2l$). By bending or folding the pipe transversely, keeping the separation between the top and bottom sides unchanged, a hollow conductor of smaller over-all dimensions may be obtained that has the same general internal configuration as the rectangular pipe. The deformation of the cross section may take place in a number of ways, several of which are shown in figure 1*B* to *D*. In the illustrations, the corresponding dimension l is indicated by a dotted line. In each of the deformed conductors there is a type of wave with a critical frequency approximately equal to $2l$, but the over-all dimensions of the deformed pipe are considerably smaller than those of 1*A* or of a circular pipe of the same critical value.

The conductor 1*B* may be thought of as a concentric line in which the center conductor is connected to and supported from the outer conductor along its entire length by a conducting radial rib or strip. Dielectric supports and spacers are unnecessary and the system is a uniconductor. Transmission in this particular example of the deformed pipes, which is here termed the *septate coaxial cable*, has been analyzed theoretically. Experiments have been made on it and on the partitioned rectangular type of 1*D*. The remainder of the paper presents the more important results of this research.

Theory of the Septate Coaxial Cable

The cylindrical co-ordinate system θ, r, x, in which x is the longitudinal co-ordinate, is indicated in figure 1*C*. A variation of all field components with the time t and with x of the form $e^{-hx+i\omega t}$ will be assumed, where h is the propagation constant, $\omega = 2\pi$ times frequency and $i = \sqrt{-1}$. The propagation constant for a pipe with a perfectly conducting wall is $i\beta$, where β is the phase constant. The attenuation constant α will not be given in this paper; although laborious, it may be calculated by known methods. The interior of the pipe has the dielectric constant ϵ, which is $10^{-11}/36\pi$ farads per centimeter for vacuum, air, and other gases that are primarily of interest for practical applications, and the permeability μ which is $4\pi \times 10^{-9}$ henries per centimeter for vacuum or gas.

The analysis begins with the wave equation in cylindrical co-ordinates, which is used to obtain solutions for the longitudinal field components E_x or H_x:

$$\left.\begin{matrix} E_x \\ H_x \end{matrix}\right\} = \sin(k\theta + C_3)\left\{C_1 J_k\left(r\sqrt{h^2+\left(\frac{\omega}{c}\right)^2}\right) + C_2 Y_k\left(r\sqrt{h^2+\left(\frac{\omega}{c}\right)^2}\right)\right\} e^{-hx+i\omega t} \qquad (1)$$

J_k and Y_k are Bessel functions of first and second kind, respectively, and of order k; the constant $C_1, \ldots C_3$ and the quantities k and h are determined from

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Much of the analytical and the experimental work of this paper was carried out in the Master's Thesis "Electromagnetic Waves in Longitudinally Partitioned Hollow Pipes," by H. Schaevitz, M.I.T., 1938.

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[1] For all numbered references, see list at end of paper.

* It is known that the dimensions of an ordinary hollow-pipe conductor may be reduced by filling its interior with a substance of dielectric constant substantially greater than unity. This method, however, introduces large losses because of the imperfectness of available dielectrics. The type of cables described herein accomplish the same result without incurring such dielectric losses. They will also effect a further reduction of dimensions in the dielectric filled pipes.

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and equation 3 reduces to

$$\begin{aligned} D = Mp^2[(px_d(p)+r)(px_q(p)+r)+ \\ \omega_0^2 x_q(p)x_d(p)] + (\psi_{d0} + i_{d0}x_q(p)) \times \\ [(px_d(p)+r)(\psi_{d0}p+e_{d0})+ \\ \omega_0 x_d(p)(\psi_{q0}p+e_{q0})] + (\psi_{q0}+i_{q0}x_d(p)) \times \\ [-\omega_0 x_q(p)(\psi_{d0}p+e_{d0})+ \\ (px_q(p)+r)(\psi_{q0}p+e_{q0})] \end{aligned} \qquad (6)$$

References

1. STABILITY OF SYNCHRONOUS MACHINES—EFFECT OF ARMATURE CIRCUIT RESISTANCE, C. A. Nickle and C. A. Pierce. AIEE TRANSACTIONS, volume 49, 1930, page 338.

2. EFFECT OF ARMATURE RESISTANCE UPON HUNTING OF SYNCHRONOUS MACHINES, C. F. Wagner. AIEE TRANSACTIONS, volume 49, 1930, page 1011.

3. ANALYSIS OF SERIES CAPACITOR APPLICATION PROBLEMS, J. W. Butler and C. Concordia. AIEE TRANSACTIONS, volume 56, 1937 (August section), page 975.

4. TRANSIENT PHENOMENA OF AN ALTERNATOR UPON CONDENSIVE LOAD, M. Takahashi. Researches of the Electrotechnical Laboratory, Tokyo, Japan, number 350, August 1933.

5. TWO-REACTION THEORY OF SYNCHRONOUS MACHINES, S. B. Crary. AIEE TRANSACTIONS, volume 56, 1937 (January section), pages 27–31.

6. Discussion of H. C. Stanley's paper "An Analysis of the Induction Machine," C. Concordia. AIEE TRANSACTIONS, volume 57, 1938, page 756.

7. TRANSIENT ANALYSIS OF ALTERNATING CURRENT MACHINERY, Y. H. Ku. *Journal of Electrical Engineering*, China, volume 3, number 4, August 1932, page 179.

8. TWO-REACTION THEORY OF SYNCHRONOUS MACHINES—GENERALIZED METHOD OF ANALYSIS—PART I, R. H. Park. AIEE TRANSACTIONS, volume 48, 1929, page 716.

9. TWO-REACTION THEORY OF SYNCHRONOUS MACHINES WITH ANY BALANCED TERMINAL IMPEDANCE, C. Concordia. AIEE TRANSACTIONS, volume 56, 1937 (September section), pages 1124–7.

10. THE OPERATIONAL IMPEDANCES OF A SYNCHRONOUS MACHINE, M. L. Waring and S. B. Crary. *General Electric Review*, volume 35, November 1932, pages 578–82.

11. ADVANCED RIGID DYNAMICS, E. J. Routh. Fourth edition, 1884, Macmillan and Company, New York, N. Y., page 170.

12. SYNCHRONOUS MACHINES II—STEADY-STATE POWER-ANGLE CHARACTERISTICS, R. E. Doherty and C. A. Nickle. AIEE TRANSACTIONS, volume 45, 1926, pages 927–42.

boundary conditions. The other components of field are obtained from (1) by means of Maxwell's equations. Two different kinds of solutions exist: one, in which $H_z=0$, corresponding to hollow-pipe waves of the E type; and the other, in which $E_z=0$, corresponding to hollow-pipe waves of the H type. In each instance, the boundary conditions for a perfectly conducting wall, i.e., the vanishing of the tangential component of electric field on all surfaces of the conductor, are next imposed on the solutions. A transcendental equation for the propagation constant h results, the solution for which allows the critical frequency, wave length and the other transmission properties to be calculated.

E$_{n,m}$-WAVES

We place $H_z=0$ and derive the other field components from E_z in (1). The boundary condition requires $E_z=0$ for $\theta=0$, θ_0 and $r=a$, b. We find that $C_3=0$ and $k=n\pi/\theta_0$, where $n=1, 2 \ldots$ and that the field is given by:

$$\left.\begin{aligned}
E_z &= \sin k\theta\cdot G\cdot e^{-hz+i\omega t}\\
E_\theta &= \frac{-hk}{r\left[h^2+\left(\frac{\omega}{c}\right)^2\right]}\cos k\theta\cdot G\cdot e^{-hz+i\omega t}\\
E_r &= \frac{-h}{h^2+\left(\frac{\omega}{c}\right)^2}\sin k\theta\cdot\frac{dG}{dr}\cdot e^{-hz+i\omega t}\\
H_\theta &= \frac{i\omega\epsilon}{h^2+\left(\frac{\omega}{c}\right)^2}\sin k\theta\cdot\frac{dG}{dr}\cdot e^{-hz+i\omega t}\\
H_r &= \frac{i\omega\epsilon k}{r\left[h^2+\left(\frac{\omega}{c}\right)^2\right]}\cos k\theta\cdot G\cdot e^{-hz+i\omega t}\\
H_z &= 0\\
&\text{where}\\
G &= C_1J_k\left(r\sqrt{h_2+\left(\frac{\omega}{c}\right)^2}\right)+\\
&\qquad C_2Y_k\left(r\sqrt{h^2+\left(\frac{\omega}{c}\right)^2}\right)
\end{aligned}\right\}\quad(2)$$

The transcendental equation for h is:

$$\frac{J_k\left(a\sqrt{h^2+\left(\frac{\omega}{c}\right)^2}\right)}{Y_k\left(a\sqrt{h^2+\left(\frac{\omega}{c}\right)^2}\right)}=\frac{J_k\left(b\sqrt{h^2+\left(\frac{\omega}{c}\right)^2}\right)}{Y_k\left(b\sqrt{h^2+\left(\frac{\omega}{c}\right)^2}\right)}\quad(3)$$

This equation may be solved approximately by means of McMahon's series[4] to get β, where $h=i\beta$:

$$\beta=\sqrt{\left(\frac{\omega}{c}\right)^2-\left[\frac{m\pi}{b-a}+\frac{(4\pi^2n^2-\theta_0^2)(b-a)}{8abm\pi\theta_0^2}+\ldots\right]^2}\quad(4)$$

where $m=1, 2, 3 \ldots\ldots$

The critical frequency is the root of the equation $\beta=0$:

$$f_0=\frac{c}{2\pi}\left[\frac{m\pi}{b-a}+\frac{(4\pi^2n^2-\theta_0^2)(b-a)}{8abm\pi\theta_0^2}+\ldots\right]\quad(5)$$

The transmission characteristics may be found in the usual way: Wave length in pipe $=\lambda_p=\frac{2\pi}{\beta}$; phase velocity $=v_p=\frac{\omega}{\beta}$; group velocity $=v_g=\frac{d\omega}{d\beta}$.

In the septate coaxial cable, $\theta_0=2\pi$. For the special case of $n=1$, k becomes 1/2 and J_k and Y_k reduce to the following form:

$$\left.\begin{aligned}
J_{1/2}(x) &= \sqrt{\frac{2}{\pi x}}\sin x\\
Y_{1/2}(x) &= \sqrt{\frac{2}{\pi x}}\cos x\\
&\text{where}\\
x &= r\sqrt{h^2+\left(\frac{\omega}{c}\right)^2}
\end{aligned}\right\}\quad(6)$$

Consequently, the phase constant becomes

$$\left.\begin{aligned}
\beta &= \sqrt{\left(\frac{\omega}{c}\right)^2-\left(\frac{m\pi}{b-a}\right)^2}\\
&\text{and}\\
\lambda_0 &= \frac{2(b-a)}{m}
\end{aligned}\right\}\quad(7)$$

For $m=1$, the critical wave length is equal to twice the radial distance between the outer and inner circular walls; this result bears a striking similarity to the H$_{1,0}$-wave in a rectangular pipe.

The electric intensity in the transverse cross section is given by:

E$_{1,m}$-WAVE

$$\left.\begin{aligned}
E_r &=\\
&\frac{C_4}{\sqrt{r}}\sin\frac{\theta}{2}\left[\frac{m\pi}{b-a}\cos\left(\frac{m\pi}{b-a}r-C_5\right)-\right.\\
&\qquad\left.\frac{1}{2r}\sin\left(\frac{m}{b-a}r-C_5\right)\right]e^{-hz+i\omega t}\\
E_\theta &= \frac{C_4}{2r\sqrt{r}}\times\\
&\qquad\cos\frac{\theta}{2}\sin\left(\frac{m\pi}{b-a}r+C_5\right)e^{-hz+i\omega t}
\end{aligned}\right\}\quad(8)$$

where C_4 and C_5 are new constants.

Sketches of the field configurations of the $E_{1,1}$-wave and the $E_{2,1}$-wave in the transverse section are shown in figure 2. It is apparent that higher order E-waves may be considered as built up from $E_{1,1}$-waves. We observe that in the $E_{2,1}$-wave a second septum may be placed within the pipe at $\theta=\pi$, providing two independent and separate hollow-pipe channels having the same critical frequency as that of the $E_{2,1}$-wave in the septate coaxial line. This idea has been suggested by Schelkunoff.[5] Figure 3 illustrates the relations between the ratio of the critical wave length to the radius of the outer tube, λ_0/b, and the angular dimension θ_0; evidently the longest critical wave length is afforded by $\theta_0=2\pi$, that is, by the septate coaxial line. The critical wave length increases with increasing b/a, as is also illustrated in figure 3.

H$_{n,m}$-WAVES

We place $E_z=0$ and derive the remaining field components from H_z in (1), obtaining:

$$\left.\begin{aligned}
H_z &= \cos\left(\frac{n\pi}{\theta_0}\theta\right)\cdot G\cdot e^{-hz+i\omega t}\\
H_\theta &= \frac{h\frac{n\pi}{\theta_0}}{r\left[h^2+\left(\frac{\omega}{c}\right)^2\right]}\sin\left(\frac{n\pi}{\theta_0}\theta\right)\cdot G\cdot e^{-hz+i\omega t}\\
H_r &= \frac{-h}{h^2+\left(\frac{\omega}{c}\right)^2}\cos\left(\frac{n\pi}{\theta_0}\theta\right)\frac{dG}{dr}\cdot e^{-hz+i\omega t}\\
E_\theta &= \frac{i\omega\mu}{h^2+\left(\frac{\omega}{c}\right)^2}\cos\left(\frac{n\pi}{\theta_0}\theta\right)\cdot\frac{dG}{dr}\cdot e^{-hz+i\omega t}\\
E_r &= \frac{i\omega\mu\frac{n\pi}{\theta_0}}{r\left[h^2+\left(\frac{\omega}{c}\right)^2\right]}\sin\left(\frac{n\pi}{\theta_0}\theta\right)\cdot G\cdot e^{-hz+i\omega t}
\end{aligned}\right\}\quad(9)$$

where h and G have the same meaning as before. The transcendental equation for h is now

$$\frac{J_k'\left(a\sqrt{h^2+\left(\frac{\omega}{c}\right)^2}\right)}{Y_k'\left(a\sqrt{h^2+\left(\frac{\omega}{c}\right)^2}\right)}=\frac{J_k'\left(b\sqrt{h^2+\left(\frac{\omega}{c}\right)^2}\right)}{Y_k'\left(b\sqrt{h^2+\left(\frac{\omega}{c}\right)^2}\right)}\quad(10)$$

The approximate solution of (10) provides the values:

$$\left.\begin{aligned}
\beta &= \sqrt{\left(\frac{\omega}{c}\right)^2-\left[\frac{m}{b-a}+\frac{(4\pi^2n^2+3\theta_0^2)(b-a)}{8abm\pi\theta_0^2}+\ldots\ldots\right]^2}\\
f_0 &= \frac{c}{2\pi}\left[\frac{m\pi}{b-a}+\frac{(4\pi^2n^2+3\theta_0^2)(b-a)}{8abm\pi\theta_0^2}+\ldots\ldots\right]
\end{aligned}\right\}\quad(11)$$

A sketch of the field configuration of the H$_{0,1}$-wave is reproduced in figure 2. In this wave, there is only one component of electric intensity, namely in the angular direction, and this component varies approximately as a half-sinusoid radially. A comparison of (11) with (4) and (5) shows that the critical frequency has the same functional form as that for the *E*-wave.

$H_{1,0}$-WAVES

If we let $\theta_0 \to 0$ as $a \to b \to \infty$, the cross section of the pipe approaches a rectangle, and consequently there must be a wave in the septate coaxial cable which will degenerate into the $H_{n,0}$-wave in the rectangular pipe. Thus there is reason to believe that an approximate solution may be obtained for $m=0$ even though (1) is not strictly valid for this value of *m*. Calculations are possible by graphical methods for the special case $n=1$ and $\theta_0=2\pi$. For this case $k=1/2$, and

$$G=\sqrt{\frac{2}{\pi r\sqrt{h^2+\left(\frac{\omega}{c}\right)^2}}}\left[C_1\sin\left(r\sqrt{h^2+\left(\frac{\omega}{c}\right)^2}\right)-C_2\cos\left(r\sqrt{h^2+\left(\frac{\omega}{c}\right)^2}\right)\right] \quad (12)$$

After applying the boundary condition $\frac{dG}{dr}=0$ for $r=a$, b and simplifying we obtain the following transcendental equation for *h:*

$$\tan\left[(b-a)\sqrt{h^2+\left(\frac{\omega}{c}\right)^2}\right]=\frac{2(b-a)\sqrt{h^2+\left(\frac{\omega}{c}\right)^2}}{1+4ab\left[h^2+\left(\frac{\omega}{c}\right)^2\right]} \quad (13)$$

This equation has been solved by graphical means for specific ratios of *a* and *b*. The constant β and λ_0 can be found in the usual way.

The importance of this wave is that it has the lowest critical frequency of any wave in a septate coaxial line. To illustrate this point, let us consider the case in which $a \to b$. Equation 13 then becomes simply

$$\sqrt{h^2+\left(\frac{\omega}{c}\right)^2}=\frac{1}{2b}$$

i.e.

$$\beta=\sqrt{\left(\frac{\omega}{c}\right)^2-\left(\frac{1}{2b}\right)^2} \quad (14)$$

and the corresponding critical wave length is:

$$\lambda_0=2\cdot 2\pi b \quad (15)$$

that is, the critical wave length is twice the circumference of the pipe. The longest critical wave length in a rectangular pipe is twice the width of the pipe, and the longest critical wave length in a circular pipe is 3.41 times the radius. Thus, *the limiting critical wave length of the septate line is greater than that of a circular hollow*

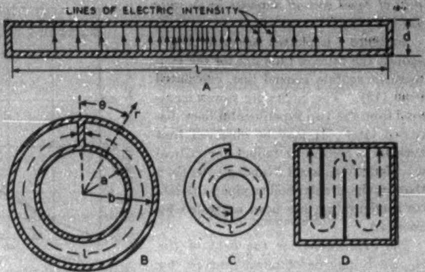


Figure 1. Transverse cross sections of several shapes of hollow pipes; (A) flat rectangular, (B) septate coaxial, (C) spiral, (D) folded. The critical frequencies are approximately the same for each pipe

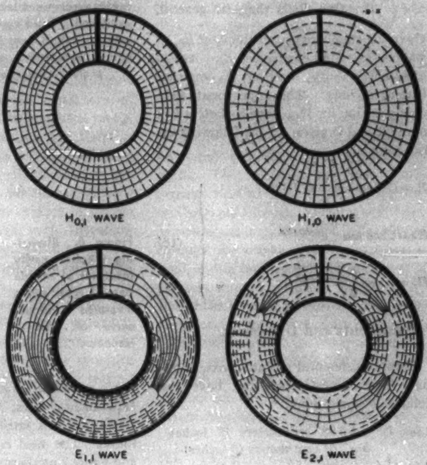
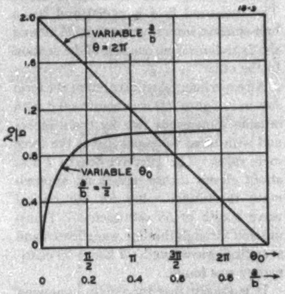


Figure 2. Configuration of the electric field intensity for four types of waves in a septate coaxial cable

Figure 3 (below). $E_{1,1}$ wave. Relative critical wave length λ_0/b versus a/b in a septate coaxial cable ($\theta_0=2\pi$), and λ_0/b versus θ_0 for a/b = 1/2



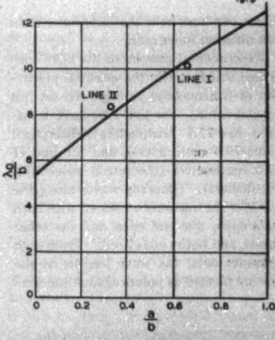


Figure 4. $H_{1,0}$ wave in septate coaxial cable. Relative critical wave length λ_0/b versus a/b. The small circles indicate measured values from two experimental cables

pipe of equal over-all diameter by the ratio $4\pi/3.41 = 3.68$.

In figure 4 is plotted the ratio of λ_0/b against the ratio of radii a/b for the $H_{1,\theta}$-wave in septate coaxial line, calculated from (13). The circles are values measured from the two experimental lines discussed later. It may be observed that even for a zero inner radius, the critical wave length is about 1.5 times greater than that for the circular hollow pipe; this latter configuration has been suggested by Sonoda.[6]

The expressions for the field of the $H_{1,\theta}$-wave may be obtained from (9) by substituting the appropriate values for $h^2+\left(\frac{\omega}{c}\right)^2$. One finds that, in general, there is a small angular component E_θ, as well as the radial component E_r. As b/a approaches unity, however, E_θ vanishes leaving E_r only. This approximate field configuration for $H_{1,\theta}$-wave is sketched in figure 2 and is given by the following expressions:

$$\left.\begin{aligned} E_r &= C2i\omega\mu b \sin\frac{\theta}{2}\, e^{-hz+i\omega t} \\ H_\theta &= C2hb \sin\frac{\theta}{2}\, e^{-hz+i\omega t} \\ H_z &= C \cos\frac{\theta}{2}\, e^{-hz+i\omega t} \end{aligned}\right\} \qquad (16)$$

Experiments and Discussion

Two experimental septate coaxial lines have been investigated that have the following dimensions:

Line I: $a = 5.08$ centimeters (2 inches)
$b = 7.62$ centimeters (3 inches)

Line II: $a = 2.54$ centimeters (1 inch)
$b = 7.62$ centimeters (3 inches)

These two lines have the same outer radii but different inner radii.

Experimental measurements have confirmed satisfactorily the essential properties of transmission through the septate coaxial cable. For line I we have measured $\lambda_0 = 77.5$ centimeters (theoretical value 79.5 centimeters), and for line II 63.5 centimeters (theoretical value 61.3 centimeters). Standing waves along the inside of the line were measured with both ends open, one end open and the other closed, and both ends closed. From such measurements, the wave lengths on the line are plotted as points against the free-space wave lengths in figure 5 for the two experimental lines; the solid curves are theoretical. A satisfactory agreement is found. The field configuration of (16) and figure 2 has been verified by measurements with a small "probe" antenna. The measured field is radial and has a half-sinusoid intensity distribution around the 2π radians from one side of the septum to the other.

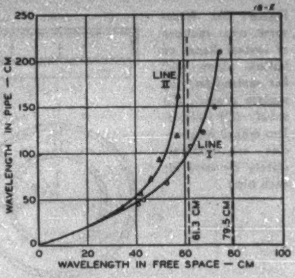


Figure 5. Wave length in two septate coaxial cables versus wave length in free space of the excitation. Solid curves give the theoretical values; dotted lines indicate the theoretical critical wave length, and the circles and triangles represent experimental values

An experimental septate line of the form shown in figure 1*D* was constructed with outside dimensions two by three inches and with three vanes parallel to the three inch sides. The effective length l was about eleven inches, giving an approximate theoretical value for the critical wave length of 55 centimeters. Transmission through this line was effected and a critical wave length of about 50 centimeters was found.

It is known that the cavity contained within a section of hollow pipe closed at both ends manifests pronounced resonance characteristics.[1,3] A highly effective or "high Q" resonator for use in conventional circuits is provided by such a cavity when a connection to the cavity is made by inserting a short length of wire into the cavity through a small hole. Septate lines are readily adapted to this use and offer the advantage of a relatively low resonant frequency for given over-all dimensions. Figure 6 shows a resonator of this kind and a measured response curve obtained in a cavity established in the experimental line II between transverse sheet-metal annuli. Improved construction would improve markedly the sharpness of resonance. The sketch of figure 6 will also serve to show one method of connecting a septate-line terminal to conventional sending or receiving apparatus. Another cavity resonator based on the same principles and providing, too, a relatively long resonant wave length comprises two concentric metal spheroids connected together by a single radial metal rod, which may also serve as a supporting member for the inner spheroid.

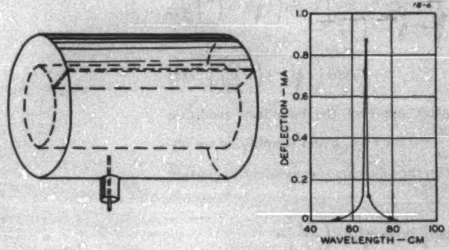


Figure 6. Illustrating a hollow cavity resonator of septate coaxial type and a measured response curve of such a resonator

Bibliography

1. G. C. Southworth, *Bell System Technical Journal*, volume 15, April 1936, pages 284–309.

2. W. L. Barrow, IRE *Proceedings*, volume 24, 1936, pages 1298–1328.

3. L. J. Chu and W. L. Barrow, IRE *Proceedings*, volume 26, 1938, pages 1520–55.

4. ON THE ROOTS OF THE BESSEL AND CERTAIN RELATED FUNCTIONS, J. McMahon. *Annals of Math.*, volume 9, 1894–95, page 23.

5. S. A. Schelkunoff, Brit. Pat. 464,185; and IRE *Proceedings*, volume 25, November (1937), pages 1457–92. U. S. Pat. 2,199,083.

6. S. Sonoda, *Electrotechnical Journal* (Japan), volume 1, number 7, pages 214–15.

Impulse and 60-Cycle Characteristics of Driven Grounds

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FELLOW AIEE

I. Introduction

DRIVEN grounds are important in electric power transmission and distribution. In fact, they comprise one of the essential elements in the art of lightning protection. Yet, to this day, the value of protection derived from grounds under actual operating conditions of lightning discharge is difficult to state in full quantitative measure. And the reason for this situation lies partly in the lack of fundamental knowledge of the impulse characteristics of driven grounds. In part, the difficulty also is due to the complex factors that inherently make up driven grounds and ground systems.

A comprehensive survey of principles and methods on driven grounds was issued in 1918 by the Bureau of Standards.[1] Further contributions have appeared since, some presenting new developments and findings,[2] others dealing on theoretical aspects of the problem,[3] and a third group bearing on related questions.[4] In recent years, progress has been centered on the immediate field of application. Here the emphasis has been to obtain effective service with such methods of grounding as lend themselves particularly to economical installation. For instance, a common practice with some utilities nowadays is to drive rods to considerable depths, even down to bedrock, so as to attain the lowest measurable resistance. A recognized practice for securing low-resistance grounds is also to place a sufficient number of electrodes in parallel (multiple grounds). Still another expedient is that of reducing the resistivity of the soil immediately surrounding the electrode by suitable treatment with common salt (NaCl) or other conducting solution. All these developments have been based largely either on 60-cycle values or on closely similar methods of testing. And from the measured 60-cycle values plus what experience would ensue there have been established present practices.

It is a fact that the 60-cycle methods of measurement have the great merit of simplicity[5,6] and it is conceded that experience can often rationalize methods of evaluating performance that are not altogether fundamental. Yet the need of establishing the performance of grounds under the actual service conditions of lightning discharge has long been recognized. In this connection, the tests by H. W. Towne[7] in 1928 and more recently an investigation by H. Norinder[8] has contributed to point out the significance of the impulse characteristics of driven grounds—even though both investigations are somewhat limited in scope considering the extent and practical importance of the subject.

Those versed in the art know that the impulse resistance of a driven ground is below the corresponding megger or 60-cycle measured values. The practice usually followed, to consider the megger values as a basis for design, is also a recognition of this fact. There are other factors which affect design and economy of installation, but these need not be gone into at this time. An immediate fundamental need is for impulse data on actual driven grounds, under impulse currents well approaching lightning conditions. The results of the investigation presented in this paper should therefore be timely and helpful to the art of lightning protection.

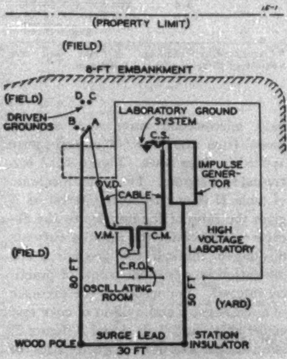

Figure 1. Test arrangement

CRO—Cathode-ray oscillograph
VM—Voltage measurement
VD—Voltage divider
CM—Current measurement
CS—Current shunt

II. Test Arrangement

The Sharon High-Voltage Laboratory is located in the open. This is a favorable circumstance that has facilitated the investigation, for the grounds can be driven in the natural soil within easy reach of a well-equipped impulse laboratory. The arrangement that was used is shown in figure 1.

The impulse generator and the instruments are grounded to the common, low-resistance grid of the laboratory, which measured with the megger 0.9 ohm to earth. The voltages of the driven grounds were recorded at the cathode-ray oscillograph through a voltage divider. Likewise the impulse currents discharged through the driven grounds were measured at the oscillograph, by means of a suitable shunt inserted in the grounded end of the impulse generator. The technique of testing and measurement complies essentially with the recognized methods described in AIEE Standards No. 4 (1940) and in the literature. Other details on the test arrangement will be apparent from figure 1 and the following.

III. Physical Characteristics of Driven Grounds

Four one-inch diameter steel rods, designated as *A*, *B*, *C*, and *D* in figures 1 and 2, were driven in the natural soil outside the laboratory in 1933. They have been in the soil ever since. Two of the rods were driven nine feet in the earth, the other two struck rocks at seven and one-half feet and could not be driven deeper. Early in 1940, after this investigation, two of the rods were removed for inspection. There was no evidence of rusting of the metal at the surface in contact with the soil.

The soil of the grounds is a combination of shale and clay with a mixture of gravel and sand. The earth is normally moist in this location. The physical nature of the soil became more apparent from the examination of a six-foot diameter ten-foot deep hole that was dug in May 1940 near the laboratory at about 25 feet from the location of the grounds. First, sev-

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1. For all numbered references, see list at end of paper.

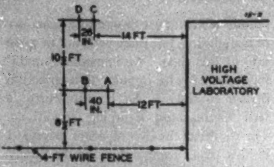


Figure 2. Physical characteristics of driven grounds: A, B, C, and D

Length of rods—ten feet
Diameter of rods—one inch
Taper of end in ground—four inches
Metal (rods)—steel
Depth in ground, *A* and *B*—108 inches
Depth in ground, *C* and *D**—90 inches
Rods driven in 1933

Class of soil—shale and clay with mixture of gravel and sand. Earth naturally moist. Rods in natural soil. Coke cinder on ground surface along fence, extending to rod *A*. Natural surface of soil elsewhere

**C* and *D* struck rocks

eral inches of vegetation soil was removed. Then a combination of clay, sand, and gravel followed to a depth of five feet. In fact a strata of about a foot consisted of a compact layer of gravel and rocks. From five feet down to ten feet, the bottom of the hole, the soil consisted of a thick blue clay. In digging the hole, water started seeping in at a depth of three feet. A rod consisting of multiple sections was driven straight down in the field nearby. It struck solid bedrock at 30 feet. Still other details of the terrain and adjacent objects will be apparent from figures 1 and 2. In short, the class and the nature of the soil are quite typical of conditions that are encountered in Western Pennsylvania and also in other sections of the country.

IV. Tests

(*A*). First series. These tests were completed in November 1939 before the ground had frozen at all. The winter in 1939–1940 got under way late in Western Pennsylvania. Sixty-cycle values for the four driven grounds and for combinations of them are summed up in table I. The voltmeter-ammeter method with sinusoidal waves was applied in these and later tests. Currents ranged from one to five amperes and in one case up to ten amperes. Within these limits the resistance is found to decrease two to three per cent with increase in the current.

Table I. Sixty-Cycle Measurements of Grounds

(Data of Tests 11/27/39)

Driven Grounds	Ammeter (Amps)	Voltmeter* (Volts)	Resistance (Ohms)
A alone	1.01	43.4	43.0
	2.00	86.0	43.0
	2.98	127.2	42.7
	4.00	170.0	42.5
	4.50	193.0	42.1
			Avg. = 42.66
B alone	4.75	189.5	39.9
	4.00	160.5	40.1
	3.01	120.5	40.0
	2.03	81.2	40.0
	1.02	41.5	40.7
			Avg. = 40.1
C alone	4.98	178.5	35.8
	4.30	190.5	35.9
	4.00	142.2	35.5
	3.01	107.0	35.6
	2.13	76.7	36.0
	1.03	37.6	36.4
			Avg. = 35.87
D alone	1.01	37.5	37.2
	2.12	77.3	36.5
	3.01	107.8	35.8
	4.02	143.3	35.7
	5.0	178.0	35.6
	5.35	190.2	35.9
			Avg. = 36.12
A and B in parallel	2.54	66.6	25.9
	3.04	77.1	25.4
	3.98	102.3	25.7
	5.00	129.2	25.9
	6.00	154.0	25.6
	7.00	178.5	25.5
			Avg. = 25.66
C and D in parallel	5.32	190.2	36.1
	4.96	176.0	35.4
	4.03	142.5	35.4
	3.04	107.3	35.3
	2.14	76.7	35.8
	1.03	37.0	35.9
			Avg. = 35.65
A, B, C, and D in parallel	1.28	21.7	16.95
	2.01	33.0	16.45
	2.90	49.5	16.53
	3.97	65.0	16.40
	5.0	82.0	16.40
	6.1	100.0	16.40
	7.1	115.8	16.30
	8.6	139.8	16.30
	10.0	163.5	16.35
			Avg. = 16.45

* Corresponding readings of rms and avg voltmeters all agree within 1.5 per cent.

The results of the impulse tests which were made at this same time are summarized in table II. The footnote describes the arrangement of the generator. A 6x13 microsecond wave and currents (crest) from 2,000 to 8,000 amperes were applied in these tests. Figure 3-*AF* is a typical oscillogram. The seventh column of table II is of particular interest as it gives the ratio of the impulse to the 60-cycle resistance for the impulse currents discharged through the grounds. The tabulation reveals other points of practical interest. For instance, the presence of a metal fence and a fill-in of coke and cinder adjacent to rods *A* and *B* (figure 2) affected the impulse characteristics of these grounds, particularly at the higher currents. This illustrates the various considerations that require attention when establishing grounds.

(*B*). Second series. The second series of tests was made in January 1940. By this time the ground had frozen at the surface six or eight inches and the temperature had dropped. The resistance had accordingly increased about ten per cent. Through the assistance of the local utility (Pennsylvania Power Company), megger readings of the grounds were taken. These with the 60-cycle values are reported in table III.

The impulse data are summarized in table IV. In these tests, the generator was set for a 12x50 microsecond wave and currents (crest) from 700 to 6,500 amperes were available. Typical oscillograms of the current and the voltage are shown in *AX* and *BJ* of figure 3. In the next to the last column of table IV is reported the ratio of the impulse to the 60-cycle resistance for these tests. Again when the two grounds adjacent to the fence, particularly rod *A*, were tested at the higher currents, surface flashover and apparent breakdown in the earth took place. The oscillograms of *BT* (figure 3) illustrate this effect.

(*C*). Other tests. Sixty-cycle check tests on the grounds were made in the summer (1940). They are reported in table III. The data in this table thus show the seasonal variation of the ground resistances.

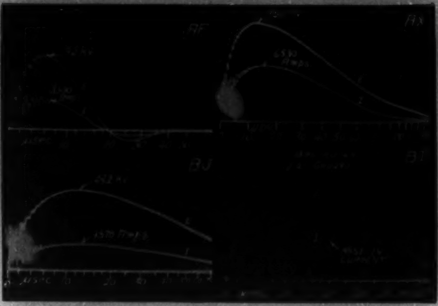
V. Impulse and 60-Cycle Characteristics

As shown in figure 4, a definite relation holds between the ratio of the impulse to the 60-cycle resistance and the impulse current. The values plotted are for the tests on grounds *C*, *D*, *B*, and the combination *C* and *D* (tables II and IV). The test data are for 6x13 microsecond and 12x50 microsecond waves, but it appears reasonable that the curve applies for impulses over a wider range of waves as encountered in the field. It should be noted that successive applications or tests repeated at different times did not affect the ratio, other than an amount due to inherent variations.

In rounded figures the ratio for these grounds at 1,000, 2,500, 5,000 and 10,000 amperes decreases respectively to the values 0.85, 0.75, 0.65, and 0.55. At higher currents, as 20,000 amperes—provided the potentials developed can be sustained by the grounds, from extrapolation of the curve the ratio does not seem to come materially below 0.50. A lightning stroke to a distribution or a transmission circuit or to a lightning-rod system in general is likely to discharge to earth through more than one path to ground. Therefore the range of currents in figure 4 covers field conditions from a traveling surge to a direct stroke of lightning.

Two factors in particular governed the low impulse breakdown from rod *A* to the adjacent wire fence. Rod *A* was driven eight and one-half feet from the fence, but what seems to have been even a greater influencing factor was the presence of a fill-in of coke and cinders on the ground layer from the fence to the rod. The relative proximity to rod *A* affected rod *B* somewhat, but only at the higher currents. Rods *C* and *D* on the other hand were amply removed from adjacent bodies and were well surrounded by homogeneous and natural soil, with the result that they sustained the impulse voltages up to the top limit of the test. The performance of rod *A* illustrates emphatically that other considerations govern the installation of grounds besides those dictated by 60-cycle measurements alone.

Figure 3. Typical oscillograms of impulse voltages (E) and currents (I)



The 60-cycle resistance for the various parallel combinations of the four rods (table III) is approximately the amount expected from calculation[3] or from previous studies.[1] For example, *A* and *B* in parallel calculates 29 ohms, *B* and *D* 24.5 ohms and *A*, *B*, *C*, and *D* 18.5 ohms. Rods *C* and *D* in parallel show no reduction due in part to the close proximity of the electrodes and possibly to other factors residing in the earth. However, grounds *C* and *D* were good grounds to test with as these were in natural and homogeneous earth, uninfluenced by extraneous bodies and least affected by seasonal variations such as result from changes of the temperature and moisture content in the earth (table III).

For parallel grounds liberally spaced and of normal characteristics, the ratio of impulse to 60-cycle resistance for the individual grounds should hold adequately close for the parallel combination when applied on the basis of the current per rod. The only data available to corroborate this point are the impulse tests on *B* and *D* in parallel (table IV).

A characteristic fundamentally significant is the variation of the resistance of grounds during the impulse discharge. For instance, from the voltage and current oscillograms of *BJ* in figure 3, the resistance from ten to fifty microseconds is found to average 31.5 ohms within about one per cent. Other typical oscillograms are analyzed in table V. The grounds of this investigation exhibited markedly constant resistance, usually within a ten per cent variation, over the part of the discharge that can be considered of more practical significance, i.e., for currents above 50 per cent crest. The relatively constant value of the impulse resistance is apparent also from figure 4 which shows that for a given

Table II. Impulse Measurements of Grounds*

(Date of Tests 11/28/39)

	Impulse Values						
Driven Grounds	Cathode Ray Oscillogram	Wave Form**	Measured Ground Crest Values Kv	Measured Ground Crest Values Amps	Impulse Resistance (Ohms)	Ratio of Impulse to 60-Cycle Resistance	Comments
C alone...	AD	Current wave 6x13 MS Voltage wave 6x13 MS	166..	7,680....	21.6....	0.60	Refer table I for avg. 60-cycle resistance values. For typical oscillogram, see AF in figure 3
	AE		131..	5,500....	23.9....	0.66	
	AF		92..	3,500....	26.0....	0.73	
	AG		56..	2,050....	27.5....	0.77	
D alone...	W-X	Ditto	59..	2,240....	26.5....	0.73	Ditto
	Y		93..	3,330....	27.9....	0.78	
	Z-AA		95..	3,330....	28.5....	0.79	
	AB		132..	6,020....	21.9....	0.61	
	AC		166..	7,680....	21.6....	0.60	
C and D in parallel..	AH	Ditto	58..	2,100....	27.6....	0.77	Ditto
	AI		92..	3,520....	26.2....	0.73	
	AJ-AK		133..	4,800....	27.6....	0.77	
	AL		166..	7,050....	23.8....	0.67	
B alone...	AM	Ditto	159..	7,050....	22.6....	0.56	Collapse of voltage near crest on oscillograms AM and AN due to breakdown in the ground. Current in AM built up to 8,070 amps
	AN		119..	4,870....	24.4....	0.61	
	AO		100..	3,200....	31.3....	0.78	
	AP		61..	2,100....	29.2....	0.73	
A alone...	F-G	Oscillatory wave, 35 MS period	52..	11,400....		....	Low impulse resistance caused by breakdown in ground to wire fence. Adjacent ground and wire fence a factor
	M-N		51..	8,060....		....	
	K-L		43..	4,670....		....	
	I-J		33..	2,750....		....	
A, B, C, and D in parallel	B-C	Oscillatory wave, 35 MS period	53..	11,300....		....	Impulse resistance much similar to values of A alone. Adjacent ground and wire fence a factor
	Q-R		46..	8,330....		....	
	S-T		39..	4,920....		....	
	U-V		31..	3,070....		....	

* Impulse-generator arrangement consisted of ten, 100 kv, 1.5, microfarad capacitor banks in series. Effective series capacitance and inductance of test circuit were respectively 0.15, microfarad and 210, microhenrys. No series resistance inserted in generator (figure 1).

** Positive polarity.

Table III. Sixty-Cycle and Megger Measurements of Grounds

Driven Grounds	Average 60-Cycle Resistance* (Ohms)	Average Megger Resistance** (Ohms)	Average 60-Cycle Resistance*** (Ohms)
A(alone)	46.8	49	33.2
B(alone)	44.1	45	31.3
C(alone)	39.1	39.5	35.7
D(alone)	38.8	38.5	32.0
A and B (in parallel)	28.0	28	
C and D (in parallel)	39.0	39	
B and D (in parallel)	22.4		
A, B, C, and D (in parallel)	17.8	18	

* Measurements by voltmeter-ammeter method, as in table I. Date of tests 1/15/40.

** Measurements recorded with G. J. Biddle Megger. Date of tests 1/31/40.

*** Measurements by voltmeter-ammeter method as in table I. Date of tests 7/22/40.

Table IV. Impulse Measurements of Grounds*

(Date of Tests 1/2/40)

Driven Grounds	Impulse Values						Comments
	Cathode Ray Oscillogram	Wave Form***	Measured Ground Crest Values: Kv	Measured Ground Crest Values: Amps	Impulse Resistance (Ohms)	Ratio of Impulse to 60-Cycle Resistance	
D alone....	AQ	Current wave 13x48 MS Voltage wave 12x50 MS	24.0	735	32.6	0.85	Refer table II for avg. 60-cycle and Megger resistance values.** For typical oscillogram, see AX in figure 3
	AR		47.7	1,505	31.7	0.82	
	AS		70	2,300	30.4	0.79	
	AT		91	3,140	29.0	0.75	
	AU		110	4,160	26.4	0.69	
	AV		132	5,310	24.9	0.65	
	AW		153	6,340	24.2	0.63	
	AX		151	6,530	23.1	0.60	
	AY		24.4	672	36.3	0.94	
	BE		23.7	672	35.2	0.91	
	BF		89.6	3,390	26.4	0.69	
	BG		129	5,560	23.2	0.60	
	BK		47.6	1,600	29.9	0.78	
C alone....	BB	Ditto	109	4,420	24.7	0.63	For typical oscillogram see BJ in figure 3
	BC		69.8	2,340	29.8	0.76	
	BD		24.4	640	38.2	0.97	
	BH		125.5	5,250	23.9	0.61	
	BI		80.3	3,240	27.5	0.70	
	BJ		49.2	1,570	31.4	0.80	
C and D in parallel	BL	Ditto	23.7	737	32.1	0.82	Ditto
	BM		47.4	1,600	29.6	0.76	
	BN		68.5	2,240	30.6	0.79	
	BO		89.2	3,330	26.8	0.69	
	BP		112	4,160	26.9	0.69	
	BQ		131	5,500	23.8	0.61	
	BR		150	6,780	22.1	0.57	
B alone....	BS	Current wave 10x60 MS Voltage wave 11x60 MS	....	....	....	....	Collapse of voltage on front and crest of oscillograms BS, BT, and BU. See oscillogram BT, figure 3. In these tests the ground around the rod was blown up and sparking on fence noted
	BT		134	5,000	26.8	0.60	
	BU		114	4,100	27.8	0.62	
	BV		93.6	2,880	32.5	0.72	
	BW		71.8	1,950	36.8	0.82	
	BX		49	1,310	37.4	0.83	
	BY		25	660	38.0	0.85	
	BZ		25	660	38.0	0.85	
B and D in parallel	CA	Current and voltage waves 15x44 MS	20.4	1,090	18.8	0.84	Ditto
	CB		41.5	2,270	18.3	0.82	
	CC		60.5	3,200	18.9	0.84	
	CD		78.0	4,480	17.4	0.78	

* In these tests the impulse generator arrangement consisted of three, 100 kv, 5 microfarad capacitor banks in series capacitance and inductance of test circuit were respectively 1.667 microfarad and 210 microhenrys. No series resistance inserted in generator (figure 1).

** Avg megger readings (table III) used to calculate ratio in column 7 above.

*** Positive polarity.

current (crest) of the characteristic the ratio does not increase more than 15 per cent at a current (crest) half that value. It is of interest to note that the crest voltage developed precedes the crest current one or two microseconds. As the current recedes on the tail of the wave, the resistance increases (table V). A similar analysis at the lower currents on the front was not possible on account of the superimposed oscillations present in this part of the oscillograms.

The process of the variation of resistance with the current discharge becomes more apparent from the ratio of the impulse to the 60-cycle resistance during the discharge, when referred to the characteristic of figure 4. The ratios of typical oscillograms (table V) are plotted in figure 5 along with the characteristic. Consider oscillograms *AW* and *BR*. As indicated by the arrows in figure 5, as the current rises on the front and approaches crest, the ratio follows the characteristic in a downward trend. When the current reaches crest and then recedes on the tail, the ratio retraces back under the curve traced for the rising part of the current. With further drop of the current the ratio continues to rise, following the characteristic upward. As the case of oscillogram *AS* shows, impulse currents of lower amplitude likewise trace paths that follow near along the characteristic.

While the nature of the oscillograms does not permit complete analysis from the beginning of the front (first few microseconds), yet it is clear from the foregoing that the mechanism of the impulse current discharge for the grounds tested is as follows. The ratio of the impulse to the 60-cycle resistance as the current rises on the front follows in good measure the characteristic (figure 4), down to the value corresponding to the crest current which lies on the characteristic. As the current recedes on the tail the ratio traces a loop under the characteristic for an impulse of high current. Apparently ionized paths or similar effects established by the discharge account for the relatively constant resistance at currents of the discharge above fifty per cent of the crest. As the current drops further, the ratio swings upward with the characteristic. As the lowest current values are reached, a residual voltage may even be present (see *AX* in figure 3).

Though the voltage of driven grounds is usually considered as a resistance drop, some inductance drop is present also. In most cases, however, the inductance of the ground proper is practically unimportant, unless the current rises or recedes unusually abruptly or in the case rods are driven to considerable depth. For each of the driven rods employed in this investigation, the inductance is approximately three microhenrys and even then only a part is effective since the current diminishes along the rod. The time constant (L/R) of the grounds is then less than 0.1 microsecond. Thus for a current rising exponentially to crest in five microseconds, the ohmic drop (Ri) at 2.5

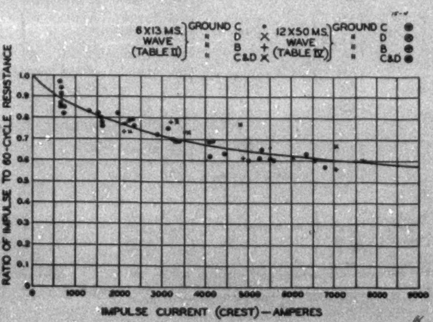


Figure 4. Characteristic of driven grounds: ratio of impulse to 60-cycle resistance versus impulse current (crest)

microseconds is 97 per cent of the impedance drop $(Ri+Ldi/dt)$ and as the wave progresses beyond the two merge together since the inductive drop practically disappears. On the other hand it should be amply clear that the inductive effect of leads or in the tower structure that connect the ground proper to the point of incidence of the lightning stroke may be a factor of primary importance in lightning protection.

Table V. Resistance (R) of Grounds at Various Times (T) of Impulse Discharge

T (MS)	E (Kv)	I (Amps)	R (Ohms)	T (MS)	E (Kv)	I (Amps)	R (Ohms)
Driven Ground D (alone)							
Oscillograms AS				**Oscillograms AW**			
4	46.8	1600	29.3	4	108.8	4160	26.1
6	58.5	1990	29.4	6	133	5115	26.0
8	65.8	2180	30.2	8	143.8	5640	25.5
10	69.5	2270	30.6	10	150.5	6080	24.8
12	70.5	2300	30.7	12	153.8	6200	24.8
16	70	2280	30.7	16	152.1	6270	24.3
20	66.9	2160	31.0	20	146.2	6140	23.8
30	56.8	1820	31.2	30	123	5310	23.2
40	46.8	1470	31.8	40	97.8	4030	24.3
50	38.4	1200	32.0	50	76.0	2880	26.4
60	33.4	1025	32.6	60	63.5	2240	28.4
				Oscillograms AX			
				20	145	6400	22.6
				40	96.1	4160	23.1
				60	58.5	2120	27.8
				80	36	960	37.4
				100	24	545	44.0
Driven Ground C (alone)				**Driven Grounds B and D (in parallel)**			
Oscillograms BB				**Oscillograms CD**			
4	76	3010	25.2	4	46.6	2820	16.5
6	93.3	3585	26.0	8	68.8	4040	17.0
8	102.6	4100	25.0	10	74.5	4350	17.1
10	108.3	4225	25.6	12	77.8	4510	17.2
12	109	4415	24.7	16	79.0	4580	17.3
16	107	4415	24.4	20	75.5	4480	16.9
20	102.2	4255	24.0	30	62.1	3520	17.7
30	86.1	3715	23.2	40	45.8	2560	17.9
40	67	2880	23.2	50	32.0	1600	20.0
50	52.9	2240	23.6	60	24.4	1153	21.2
60	43.5	1665	26.2				
Driven Grounds C and D (in parallel)							
Oscillograms BR				**Oscillograms BN**			
4	107	4410	24.3	4	46.8	1538	30.4
8	142.5	6010	23.7	8	64.5	2080	31.0
10	148.5	6400	23.2	10	68.5	2210	31.0
12	150	6720	22.4	12	68.8	2240	30.7
16	147.8	6700	21.8	16	67.1	2210	30.4
20	141.8	6720	21.1	20	63.8	2115	30.2
30	117	5760	20.3	30	54.8	1890	29.0
40	90.1	4350	20.7	40	44.8	1508	29.5
50	68.1	3200	21.3	50	35.8	1150	31.0
60	55.4	2430	22.8	60	30.8	960	32.0
70	43.4	1760	24.7	70	26.1	800	32.6

VII. Discussion of Results

The characteristic of figure 4 applies for a soil of medium resistivity, that is a soil consisting largely of clay with some gravel, etc. The same rods driven in other soils would likely be governed by curves of the same general character, but of different slope. Towne in his investigation[7] on 0.84-inch diameter pipes driven into loose, gravelly soil (artificial fill) obtained ratios from 0.80 to 0.34 for currents (30 x 120 microsecond wave) well under 1,000 amperes. A single ten-foot driven-pipe ground which measured 82 ohms on 60-cycle had a resistance of 50 ohms at 660 amperes—that is a ratio of 0.61. McEachron, Hemstreet, and Rudge in their field tests[8] reported for a pipe driven into the ground, which measured 180 ohms with the megger, a ratio of 0.67 at 550 amperes—the highest impulse current applied. For a 1.3 inch pipe driven four feet into natural soil consisting largely of clay, Norinder found a ratio of 0.76 corresponding to a current of approximately 1,000 amperes. It is of interest to note that in his other tests[8] where the impulse resistance was determined for both positive and negative polarities, the difference found practically was small. In this investigation (tables II and IV) positive impulse currents were applied. In short, it is clear that characteristic curves similar to figure 4 for other typical soils such as sand, etc., obviously are desirable.

Future investigations on high and low resistivity soils may furthermore shed light also on the physical processes that determine the decrease in resistance with increase in current. The present explanation, borne out by this and the investigations cited, is that "the decrease in resistance is probably due to high-resistance contacts between the more conducting particles of soil, at or near the surface of the electrode, which are bridged over by arcs or sparks when sufficient voltage is applied." For instance, the resistivity of the soil at the low 60-cycle currents for grounds *C* and *D* (table III) in the order of 7,500 ohms per cubic centimeter. On the basis of this resistivity the voltage gradient at the rod, when discharging an impulse current of 10,000 amperes (crest), would be $10{,}000 \times 7{,}500/\pi \times 90 \times 6.45$, i.e., 41 kilovolts per centimeter. It is doubtful whether grounds could sustain gradients of this magnitude at all without breakdown. As a matter of fact, from figure 4, the average resistivity of the soil at 10,000 amperes is 0.55 the 60-cycle value to which there corresponds a gradient of 22.5 kilovolts per centimeter. Actually the effective resistivity of the soil at the rod is even less, and the voltage gradient at the rod is correspondingly lower. Studies of the gradients developed should contribute to a better understanding of driven grounds.

Another factor which determines the characteristic of grounds is the size, form, and arrangement of the electrode. Besides rods and pipes, other common types

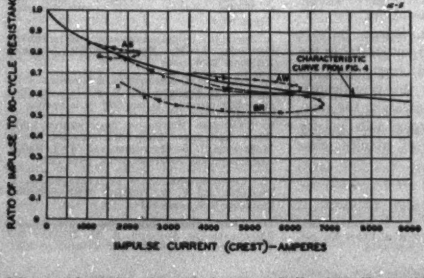


Figure 5. Variation of "impulse resistance to 60-cycle resistance" ratio during impulse current discharge compared with characteristic (figure 4)

Minimum Insulation Level for Lightning Protection of Medium-Voltage Lines

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LIGHTNING protection of wood pole lines is a problem of wide interest among operating companies. It is therefore timely to cite the experience of the Philadelphia Electric Company during the six year period, 1935–40, in successfully renovizing its 13 and 33 kv wood pole lines to reduce trouble.

In 1935 this company was faced with the problem of determining how to materially reduce lightning trouble on a variety of pole top configurations at a minimum of expense. It had been shown by Andrews and Stroup[1] that wood insulation could be employed effectively. However, no information was available to indicate what reduction in trouble could be obtained for various increases in insulation. The high ridge pin configuration proposed by these authors, while reported to be very effective, could not be applied to many of the existing line structures without making major changes in construction at considerable expense. There was therefore need for an investigation into the degree of protection afforded by lesser amounts of insulation. With this problem in mind, the author set out to make a detailed investigation of the performance of the various types of structures and to correlate their insulation strength with their rate of trouble.

It was discovered from these correlations that substantial reductions in trouble occurred with increases in insulation strength up to a certain level and that equal increases above this level were much less effective. It was evident, therefore, that the greatest reduction in trouble, at least cost, could be obtained by improving those structures which were below the indicated level. As the result of these findings, minimum insulation levels were adopted for both the 13 and 33 kv systems and used as a basis for renovizing the lines. The levels adopted were somewhat below the insulation strength of the high ridge pin construction proposed by Andrews and Stroup.

Besides establishing a minimum insulation level on which to base construction practice, the data obtained were also found very helpful in predicting what reduction in trouble might be obtained from construction changes and thereby aided in justifying the expenditure required.

This paper describes the procedure followed in determining the susceptibility to lightning trouble of pole line structures of various insulation strengths, indicates what changes were made to structures below the required minimum level and shows what reduction in trouble was obtained by these changes on the 13 and 33 kv systems.

Description of Systems

The 13 kv lines investigated were those in Philadelphia. They are three wire three-phase circuits with the neutral grounded through a four ohm resistor at the generating stations and through a grounding transformer at one substation. Although operated as radial circuits, they form a network over the city with numerous breakdowns to each other through pole top air break switches. These lines have a total length of 125 circuit miles, and an average length per circuit of 2.6 miles.

The 33 kv lines are in the suburban territory covering an area of approximately 1,400 square miles surrounding Philadelphia. They are three wire three-phase circuits, with the neutral solidly

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[1] For all numbered references, see list at end of paper.

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of grounds which serve to conduct lightning currents into the earth, are the footing of transmission-line towers and of other structures such as are found at substations, etc. In general, it appears from the preceding considerations and the test data that electrodes which contribute inherently to lower the current density in the soil are likely to influence the characteristic in a direction towards a more constant resistance with increase in current. The practical usefulness and basic importance in establishing characteristic curves for typical forms of ground electrodes need not be emphasized. Through such an approach the problems presented from grounds can be rationalized more readily and the value derived from grounds assessed in a more quantitative measure. An effective ground means greater safety in protection and an improved ground may mean reduced insulation or insulation clearances with the resulting benefit of reduced costs.

VII. Summary

This investigation presents test data on the impulse and 60-cycle characteristics of common rods driven in natural soil (largely clay composition). It sums up and analyzes the results in the characteristic curve of the ratio of impulse to 60-cycle resistance for impulse currents that represent conditions ranging from a traveling surge to direct strokes of lightning. The basic reasons for the performance of grounds to impulse currents as they do are pointed out insofar as the experimental data and observations permit to do so. From this investigation the desirability of establishing the characteristic for other typical soils and for other common types of grounds (electrodes) is apparent.

References

1. GROUND CONNECTIONS FOR ELECTRICAL SYSTEMS, O. S. Peters. Technological Paper No. 108 of the Bureau of Standards, June 1918.

2. PRACTICAL ASPECTS OF EARTHING, E. Fawssett, H. W. Grimmit, G. F. Shotter, and H. G. Taylor. *Journal* of the Institution of Electrical Engineers, October, 1940, page 357.

3. CALCULATION OF RESISTANCES TO GROUND, H. B. Dwight. AIEE TRANSACTIONS, volume 55, 1936 (December section), page 1319.

4. EARTH RESISTIVITY AND GEOLOGICAL STRUCTURE, R. H. Gard. AIEE TRANSACTIONS, volume 54, 1935 (November section), page 1153.

5. TESTING LIGHTNING ARRESTER GROUNDS, E. Beck. AIEE Lightning Reference Book, page 381.

6. Various bulletins and trade publications on ground rods and ground testing: Anaconda, James C. Biddle Company, Borden Electric Company, Copperweld.

7. IMPULSE CHARACTERISTICS OF DRIVEN GROUNDS, H. M. Towne. AIEE Lightning Reference Book, page 259.

8. THE IMPULSE RESISTANCE OF EARTHING CONDUCTORS, H. Norinder and R. Nordell. CIGRÉ paper 302, presented at the 1939 Paris meeting.

9. TRAVELING WAVES ON TRANSMISSION LINES WITH ARTIFICIAL LIGHTNING SURGES, K. B. McEachron, J. G. Hemstreet, and W. J. Rudge. AIEE Lightning Reference Book, page 722.

10. COUNTERPOISE TESTS AT TRAFFORD, C. L. Fortescue and F. D. Fielder. AIEE Lightning Reference Book, page 1314.

11. CHARTS SHOW GROUND ROD DEPTH FOR ANY RESISTANCE—IN ADVANCE, R. M. Schahfer and W. H. Knutz. *Electrical World*, October 19, 1940.

grounded at two points centrally located on the system. These lines are primarily connecting links between the suburban 33 kv-4 kv substations and have a total length of 460 circuit miles and an average length per circuit of 14.2 miles.

Basis for Protection

Experience on the lines of the Philadelphia Electric Company had shown that power arcs resulting from lightning flashovers were the principal source of trouble and that lightning itself seldom caused more than slight splintering of the pole or crossarm. It was concluded, therefore, that a good basis for protection would be one which minimized the chances for power arc formation. The method adopted was to lengthen the initial arc-over paths along the pole and crossarm by increasing the insulation between phases and between phase and ground.

In determining the insulation strength of the various types of structures it appeared desirable from the above experience to base it on factors directly influencing power arc formation. Two factors which appeared predominant were the length of insulation in the arc-over path and the 60 cycle voltage across the path. Accordingly, these terms were used as a basis for measurement of insulation strength.

Measurement of Insulation Length

At the beginning of this investigation, field surveys were conducted which showed that both the 13 and 33 kv systems had a large variety of pole top arrangements, resulting in a wide range of insulation lengths along the arc-over paths between phases and between phase and ground. It was therefore necessary to make all comparisons of power arc trouble on the basis of individual structures rather than on miles of line, as is frequently done. Accordingly detailed information was obtained for each pole indicating such things as length of crossarm, type of insulators and braces, location of line wires, guys, ground wires, and other pertinent data. From these data the length of insulation in the arc-over path between phases and between phase and ground was determined for each structure. In all, approximately 20,000 poles were investigated.

In each case the arc-over path was considered as being along the surface of the pole or crossarm rather than directly between wires in air. The length of insulation in the arc-over path was taken as the sum of the porcelain and wood insulation lengths. Parts of the crossarm or pole shunted by metallic hardware were excluded. The insulation length assigned to each pin type insulator was its wet arc-over distance as given by the manufacturer's catalog, while for a string of suspension insulators their insulation length was taken to be the wet arc-over of one unit plus the distance between centers of successive additional units. This was done because this distance is the length of the arc-over path after the lightning has flashed over the string.

The measurement of length of insulation in the arc-over path is illustrated in figures 1 and 2. In figure 1 the total length of insulation from phase *A* to phase *B* is 84 inches, and from phase *A* to ground *G* is 49 inches. The wet arc-over distance for each 33 kv pin type insulator is six inches. In figure 2 the suspension insulators are ten inches in diameter, have a distance between centers of $5^3/_4$ inches, and a wet arc-over per unit of approximately four inches. Adding the four inch wet arc-over distance to four times $5^3/_4$ inches (the distance between centers of insulators) gives a total arc-over distance of 27 inches for the five-unit string. This added to the 24 inches of wood on the pole gives a phase to ground insulation length of 51 inches.

Unit of Insulation Strength

For simplicity, it was found desirable to combine the two factors, insulation length and 60 cycle voltage, into a common unit of insulation strength called "inches per kv." This unit is simply the length of insulation in the arc-over path divided by the operating voltage. When determining the phase to phase insulation strength the length of insulation between phases is divided by the line to line voltage, whereas for phase to ground strength the length of insulation between phase and ground is divided by the line to ground voltage. Referring to figure 2, the insulation strength between phases *A* and *B* is 3.1 inches per kv, or 102 inches divided by the line to line voltage, 33 kv. Between phase *A* and the guy it is 2.7 inches per kv, or 51 inches divided by line to ground voltage, 19 kv. For this structure the minimum is 2.7 inches per kv. All comparisons in this investigation were based on the minimum insulation strength of each structure expressed in inches per kv, irrespective of whether it was phase to phase or phase to ground. This was done because experience had shown that phase to phase as well as phase to ground strengths are important in the control of power arc formation.

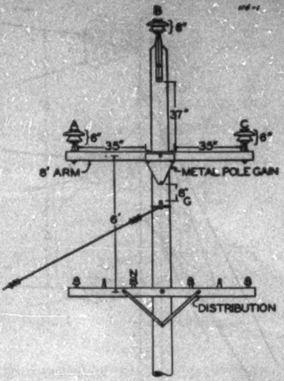


Figure 1. 33 kv high ridge pin structure

	Insulation Length Inches	Insulation Strength Inches Per Kv
A to *B*	84	2.6
A to C	82	2.5
A to G	49	2.6
B to G	51	2.7

Using "inches per kv" as the basis for measurement has the advantage over using lightning flashover data of employing two readily measurable factors, insulation length and 60 cycle voltage, which directly govern the tendency for formation of power arcs following lightning flashover. Taking these factors into account appears to meet in part the need expressed by Andrews[2] in 1938 for data on factors influencing power-follow in addition to lightning flashover data. Factors such as magnitude of short circuit current, circuit tripping time, wind and rain also have some influence, but inclusion of them was beyond the scope of this investigation. There is still need for more study of the influence of all these factors on power arc formation.

It should also be appreciated that the results given here are based on empirical data and that investigations under controlled laboratory conditions with lightning surges superimposed on the 60 cycle voltages would be of considerable value.

Minimum Insulation Level

Having determined the minimum insulation strength in "inches per kv" for each structure from field surveys, the next step was to classify structures according to their strength. The annual rate of trouble experienced in power arc faults per 100 circuit structures was then determined for each group and a susceptibility curve

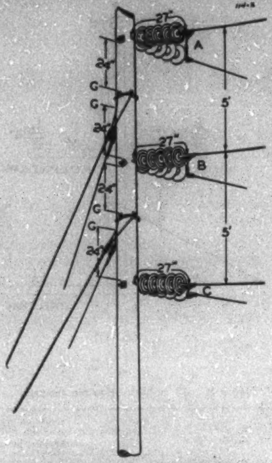


Figure 2. 33 kv corner pole structure

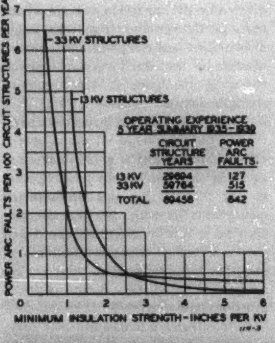
	Insulation Length Inches	Insulation Strength Inches Per Kv
A to *B*	102	3.1
A to G	51	2.7

plotted for each system of the two voltage classes studied. Susceptibility curves based on the composite results for the five year period 1935–39 are shown in figure 3. These indicate that as the insulation strength increases up to a certain level, the rate of power arc faults decreases quite rapidly, whereas further increases beyond this level are much less effective. The fact that the 13 and 33 kv curves do not parallel each other is probably due to the dissimilarity of the two systems in regard to location, number of structures in a given insulation strength class, storm exposure, and other variables referred to in the preceding section of this paper. The significant thing is that both curves show the same general trend and permit selection of a minimum insulation level on which to base construction practice.

Based on susceptibility curves developed earlier in this investigation and similar to those in figure 3, a minimum insulation level of two inches per kv was adopted for the 33 kv system and 3.5 inches per kv for the 13 kv system. These points are just below the knee of the curves. Also, much of the improvement work could be carried on fairly economically up to these levels with substantial reductions in trouble. A level above two inches per kv for 33 kv lines would have been difficult to obtain on many existing structures without replacing the present poles with larger ones. Also, a level above 3.5 inches per kv for 13 kv lines would have required increasing the insulation strength on a large number of low ridge pin poles which were already 3.5 inches per kv, and which had been changed from flat top to this configuration prior to this investigation. The principal aim of the improvement program was to raise the insulation strength of structures which were below the adopted minimum to that value or better. Just how much insulation strength is required for immunity to power arc trouble is not known. However, thus far with 2,300 structure years of experience no trouble has occurred on 13 kv high ridge pin configuration in Philadelphia having a minimum insulation strength of from 6.5 to 7 inches per kv. For the 33 kv system there appears to be no practical value of insulation which will entirely prevent power arc trouble.

Besides being of value in adopting a minimum insulation level the susceptibility curves provide a means for predicting what reduction in power arc faults may be expected by increasing insulation strength from one level to another. Such predictions are of value in determining whether proposed changes on the lines may be economically justified. Being based on five years' experience on a large number of circuit structures, the curves in figure 3 are believed to closely approximate the average annual performance that may be expected for various levels of insulation strength.

In the development of the susceptibility curves power arc faults were selected on the following basis. Any evidence of power arc damage to wires or insulators on a given structure of a single circuit line was classed as a single case of power arc trouble, irrespective of the number of phases involved. Similarly evidence of power arc damage to two circuits on the same structure was classed as two cases of trouble. This classification was necessary because both single and twin circuit structures were included in the analysis. For the same reason it was necessary to place the rate of trouble on a "circuit structure" basis rather than just on a "structure" basis. Thus the number of circuit structures on a single circuit line was taken to equal the number of poles in that line, whereas the number of circuit structures in a twin circuit line was double the number of poles.



OPERATING EXPERIENCE
5 YEAR SUMMARY 1935–1939

	CIRCUIT STRUCTURE YEARS	POWER ARC FAULTS
13 KV	29694	127
33 KV	59764	515
TOTAL	89458	642

Figure 3. Susceptibility of 13 and 33 kv aerial circuit structures to power arc damage following lightning flashover

Line Improvements

By the close of the 1940 lightning season, the 13 kv line improvement work in Philadelphia had been virtually completed, with approximately 95 per cent of the circuit structures having an insulation strength of 3.5 inches per kv or better. Similarly, improvements to the 33 kv lines had brought about 75 per cent of that system up to two inches per kv or better.

The principal changes on the 13 kv lines consisted of (*a*) removal, relocation or insulation of guys, (*b*) replacement of metal with wood braces, (*c*) use of porcelain or wood sections in the horizontal and vertical operating bars of gang operated air break switches, (*d*) rebuilding of dead-ends and corners, and (*e*) changeover from low ridge pin to high ridge pin configuration by lowering the crossarm or use of wood pole extensions. These changes were made to obtain additional wood insulation between phases and between phase and ground.

As already mentioned, much of the 13 kv construction in Philadelphia had been changed over from flat top to low ridge pin, figure 4, prior to this investigation. However, where pole space permitted lowering the crossarm two to four feet at little cost, this was done to obtain high ridge pin construction. In replacing old poles and in building new lines, the four foot high ridge pin configuration has been used. This construction gives approximately equal insulation strength between phases *A* and *B*, *B* and *C*, and *C* and *A*. It also has about the same dimensions as the 33 kv high ridge pin shown in figure 1 and permits ready conversion to 33 kv operation by merely changing to larger insulators. High ridge pin configuration has been applied to a number of 13 kv pole top switches in Philadelphia.

Changes on the 33 kv system were similar to those cited above. However, much of the improvement to this system has been obtained through relocation or removal of guys and removal of overhead and vertical ground wires. Whatever benefit may have been obtained from the overhead ground wires was nullified by their iron supports and vertical ground leads shunting out the wood pole insulation and thus permitting a ready path for formation of power arcs. Breakage of the overhead ground wires in storms was also another cause for their removal. Pole gains of the type shown in figure 1 have been used extensively to replace metal braces. Isolated groups of steel towers in the wood pole lines at such places as railroad crossings have been improved by replacing metal arms with wood or by adding suspension insulator units up to a total of seven per string. Pole top switches with steel supporting frames and steel operating rods have been particularly susceptible to trouble. Many of these have been rebuilt, using wood supporting members and wood sections in the operating bar. A new design 33 kv vertical switch using post type insulators mounted on two rotating wood uprights is shown in figure 5. This has four feet separation between phases and an insulation strength of two inches per kv. Also on new lines, construction of the type shown in figures 1 and 2 is being used.

Because guys have been one of the chief sources of lightning trouble on both the 13 and 33 kv systems, it is well to consider some of the remedies and certain conditions that should be looked for when trying to obtain additional wood insulation. In most instances improvement has been obtained by lowering or raising guys to the position on the pole which provides the desired amount of wood insulation without sacrificing mechanical strength. Often side guys were found attached to stub poles, which in turn were guyed to ground. Separation of the guys on the stub pole in the manner shown in figure 6 was frequently done to gain wood insulation. This diagram shows an air clearance of nine inches between the guys on the stub so that lightning flashover would be more likely to go across the 15 inches of wood pole rather than across the air gap. If flashover should occur across the air gap, the power arc which followed would therefore be shorter and not so readily interrupted as if flashover had originated across the 15 inches of wood. As a basis for determining desirable air clearance for guys, one inch of air path was taken approximately to equal two inches of wood on an impulse strength basis. Where practical, therefore, an effort was made to have the air gap distance a little greater than half the length of wood insulation.

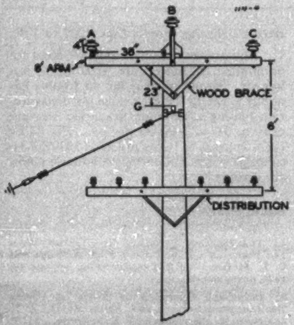


Figure 4. 13 kv low ridge pin structure

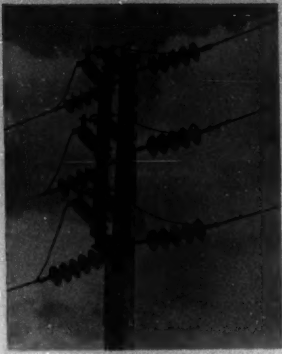


Figure 5. 33 kv vertical switch

Guys were sometimes found too close to line wires, or were brought down at such a sharp angle as to shunt out much of the wood pole insulation by passing too close to a through bolt holding the line suspension insulators as in the case of corner poles. Also, guys which were supposedly insulated from ground were found close to or in contact with lightning arrester ground wires, pipes and conduits on the pole, or to other grounded objects. Tail guys used in dead-ending the three phases of a line were found to converge on a common point of attachment. This condition permitted phase to phase short circuits through the guys when flashover of the dead-end insulators occurred and was remedied by obtaining vertical separation of the three guys on the attachment pole. Where guys could not be effectively relocated, wood guy strain insulators equipped with arcing horns have been used in a number of instances.

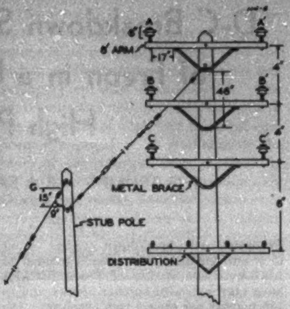


Figure 6. 33 kv twin circuit structure

In this study the conventional porcelain guy insulators were assumed to provide no insulation strength because of their small wet arc-over distance.

Performance

While the susceptibility curves indicate what reduction in rate of trouble can be obtained on individual structures through increases in insulation strength, it is of interest to note what over-all decrease in line interruptions and power arc faults has been obtained through the various changes that have been made.

On the 13 kv system in Philadelphia line interruptions during lightning storms decreased from 55 per 100 circuit miles in 1935 to 14 in 1940, a reduction of 75 per cent. Similarly, 33 kv line interruptions decreased 62 per cent, or from 45 to 17 per 100 circuit miles from 1937 to 1940, in which period almost all of the improvement work was done. While variations in the severity of the lightning seasons affect such comparisons, it is believed that the reductions in line interruptions given above were largely due to the line improvements.

Approximately the same percentage reduction in power arc faults as in line interruptions were obtained. On the 33 kv lines cases of pole splintering without accompanying power arc trouble showed an appreciable increase during the period of line renovation. This was expected and further confirms the results showing the value of increased insulation in reducing power arc trouble.

There has been no apparent increase in

D-C Breakdown Strength of Air and of Freon in a Uniform Field at High Pressures

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Synopsis: D-c breakdown studies of air in a uniform field have been extended to 1,000 kv. D-c studies of Freon (CCl_2F_2) have been extended to over 350 kv and to 135 pounds per square inch absolute. The departure from Paschen's law at the higher values of pressure times gap is noted. The mechanisms which account for the higher insulating strength of such gaseous compounds as CCl_2F_2 are outlined and their possibilities and limitations in the insulation of high-voltage apparatus are briefly discussed.

Introduction

IT has long been recognized[1] that the insulating strength of air and other gases in a uniform field increases with the pressure. The classical analysis of the voltage-breakdown mechanism and its dependence on pressure and other properties of the gas was made by Townsend in 1903.[2] This explanation was based on the ionization of the interelectrode gas by both electron and positive-ion collision—a process which Townsend showed would become cumulative when the positive ions produced by the electrons moving toward the anode under the influence of the field become capable, by their ionizing collisions, of producing an even greater number of new electrons. Since that time, Townsend's second coefficient β has been reinterpreted to include various other mechanisms which contribute regeneratively to the charged particles in the interelectrode space. Among these are photoelectric emission, secondary electron emission due to positive-ion impact at the cathode, photo-ionization in the gas, and ionization by excited atoms and by multiple collisions. In addition, it has become evident that any explanation of high-pressure breakdown must also take into account the several processes of ion removal and the important influence of space charge and the myriads of tiny self-healing electron avalanches in modifying the electric field.[3] Recognition of these various ion-producing, ion-removing, and field-modifying mechanisms has clarified the general picture of breakdown in gases but at the same time has shown it to be too intricate for full quantitative description. Because of this complexity, knowledge of gaseous insulation continues to depend to a large extent on the experimental determination of the breakdown strength and pre-breakdown behavior of the various gases as a function of pressure and electrode geometry, and on the investigation—both under simplified and under practical conditions—of the relative importance of the several mechanisms involved in gaseous breakdown. This paper extends d-c breakdown studies in air and in Freon 12 (dichlorodifluoromethane) at high pressures and in a uniform field to higher voltages than have been reported previously, and indicates reasons for the observed departures from Paschen's law and for the relatively high insulating strength of the Freon.

Apparatus

The investigation of the breakdown of air at voltages up to 1,000 kv was made with a pressure-insulated electrostatic generator[4] developed at the Institute for the production of high-voltage X rays for medical use. The uniform gap was established by a fixed electrode of polished steel* attached to the high-voltage terminal and a similar but movable electrode connected to the tank. The diameter of these electrodes was six inches, and their shape was such that in the central gap area the electric field was a maximum and substantially uniform for gaps up to one and one-half inches. The voltage was measured by a generating voltmeter[5] with an estimated error of less than two per cent.

Most of the studies of the breakdown of Freon were made in a 13-inch-diameter

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1. For all numbered references, see list at end of paper.

* The several grades of steel, stainless steel, brass, and aluminum give about the same results as electrode materials, the breakdown voltage at higher pressure and gradient being dependent primarily on the smoothness and freedom from contamination of the surfaces rather than on the nature of the metal itself.

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substation lightning trouble on the 13 and 33 kv systems due to the increase in line insulation.

Summary and Conclusions

1. On the 13 and 33 kv wood pole lines of the Philadelphia Electric Company power arcs following lightning flashover were the principal source of trouble, while lightning itself seldom caused more than slight splintering of the pole or crossarm.

2. The chances of a power arc forming after lightning flashover depend upon the 60 cycle voltage and length of insulation. These factors were combined into a common unit of insulation strength called "inches per kv."

3. Increases in insulation strength up to approximately two inches per kv for 33 kv lines and 3.5 inches per kv for 13 kv lines were found to be much more effective in reducing trouble than increases beyond these limits.

4. These limits were adopted by the Philadelphia Electric Company as a practical minimum of insulation strength on which to base construction practice in renovizing its lines.

5. Improvements to obtain the minimum insulation strength of 3.5 inches per kv or better on the 13 kv system in Philadelphia are believed largely responsible for a 75 per cent reduction in line interruptions during lightning storms from 1935 to 1940.

6. Similar improvements to obtain the minimum insulation strength of two inches per kv or better on the 33 kv system are believed largely responsible for a 62 per cent reduction in line interruptions during lightning storms from 1937 to 1940.

7. The amount of insulation required for immunity to power arc trouble is not known. However, thus far no trouble has occurred on 13 kv high ridge pin configuration having a minimum insulation strength of from 6.5 to 7 inches per kv. For 33 kv there appears to be no practical value of insulation which will entirely eliminate power arc trouble.

References

1. TRANSMISSION RESEARCH AND DESIGN WITH THE FIELD AS A LABORATORY, F. E. Andrews and C. L. Stroup. AIEE TRANSACTIONS, volume 49, 1930, pages 959–68.

2. LIGHTNING STRENGTH OF WOOD IN POWER TRANSMISSION STRUCTURES, Philip Sporn, J. T. Lusignan, Jr., discussion by F. E. Andrews. AIEE TRANSACTIONS, volume 57, 1938 (July section), page 410.

steel pressure tank[6] provided with a Textolite high-voltage bushing for 500 kv. External heaters were applied to the tank so that vapor pressures up to 140 pounds per square inch absolute could be maintained. The high-voltage source was a 500-kv constant-potential electrostatic generator of the Van de Graaff type. The voltage was measured with an error of less than three per cent, using a calibrated spiral-line type of high resistance.

Insulating Strength of Compressed Air in a Uniform Field

Figure 1 gives the d-c breakdown voltage of dry air in a uniform field for a range of pressures from 1 to 13 atmospheres and electrode separations up to one and one-half inches. This figure also gives similar data taken in the same apparatus on the breakdown of Freon at pressures up to 45 pounds per square inch absolute, and for voltages up to 500 kv. These Freon data are discussed later in this report. It is to be observed that at each pressure the voltage increases linearly with electrode separation. Since both the voltage and the product of pressure times gap are substantially higher than any heretofore reported, the data on compressed-air insulation are examined with reference to Paschen's law.

It was stated by Paschen[7] and it followed from Townsend's and subsequent analyses that for a given gas and a uniform field the sparking voltage is a function of the product of the interelectrode gap and the gas pressure. The more general similarity law enunciated 50 years later declares the sparking potential of a gas to remain unchanged if the physical dimensions of the electrodes and gap are changed in the inverse ratio to the change in pressure. Paschen's law has been checked at pressures ranging from the critical pressure (corresponding to minimum sparking potential) to many atmospheres, and experimenters have been generally agreed that Paschen's law holds up to gas pressures of ten atmospheres.[8] Some experimenters have measured the linear increase in the breakdown voltage across a uniform gap with gas pressure, and concluded Paschen's law to hold until departure from the linearity was found. It must be pointed out, however, that Paschen's law does not predict this linearity of voltage and pressure, though this linearity does exist over a considerable pressure range. Paschen's law predicts for a uniform field the constancy of the breakdown potential when the product of the interelectrode gap and the gas pressure is constant. In figure 2 the data in figure 1 are replotted to show the variation with pressure of the d-c breakdown voltage across a uniform field in air for constant values of *pd* from 10 to 240 pounds per inch. (One pound per inch = 131 millimeters Hg×centimeter.) It is seen that the relative voltage change with pressure is greater at the higher values of *pd*. The bulk of the recorded data lies in the realm in which Paschen's law fails, so that a slight reduction of voltage with pressure is shown even for the lowest value of *pd*.

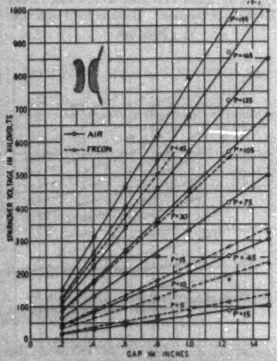

Figure 1. Increase of d-c spark-over voltage of air and of Freon 12 with electrode separation at several absolute pressures in pounds per square inch. Uniform field

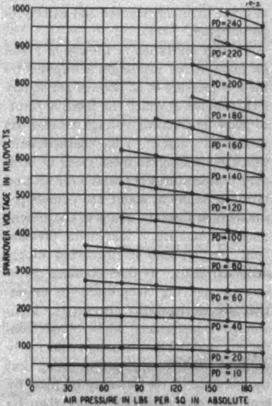

Figure 2. The spark-over voltage in air as a function of pressure for constant values of pressure times gap in pounds per inch

The decrease in the breakdown voltage with increasing pressure at the higher values of *pd* is not unexpected, since the higher gradients favor photoelectric and secondary emission at the cathode. The contribution of the electrode surfaces to the ionization by high field emission may become appreciable at gradients of the order of 10^6 volts per inch, particularly for surfaces that have been inadequately cleaned and conditioned by sparkover or other discharges. Moreover, at higher pressures the reduced mobility and diffusivity of the ions in the interelectrode gap increase the field distortion due to space charge and hence upset the ideal considerations upon which Paschen's law is based. Analysis of other data indicates that departure from Paschen's law begins at values of *pd* of about two pounds per inch. Differences in the experimental values of *pd* at which Paschen's law has been observed to fail are probably due largely to variations in electrode conditioning, which has a marked effect on the breakdown value, as well as to variations in the experimental arrangements such as size, shape, and smoothness of the electrodes, proximity of the chamber walls, and voltage wave shape.

Insulating Strength of Freon at High Pressures

That certain compounds in the gaseous state, notably those which contain chlorine, fluorine, or other electronegative atoms, have a higher insulating strength than air or the other permanent gases, has been known since the investigations of Natterer and others in 1898. A number of studies have been made of the insulating properties of Freon 12[9]—a gas which was developed primarily for use as a refrigerant. As seen from figure 1, Freon can insulate a given d-c voltage at about one-third the pressure required with air. It was felt desirable to extend the breakdown studies of Freon to higher pressures and voltages, since the full advantage of such special compounds over air is realized only if they can be used at essentially the same pressures. Figure 3 shows the breakdown strength of Freon in a uniform field for pressures from 5 to 135 pounds per square inch absolute, and with constant potentials up to 350 kv. Since the vapor pressure of this compound at 70 degrees Fahrenheit is about 85 pounds per square inch absolute, the higher pressures were obtained by heating the chamber the required amount. Care was necessary to prevent condensation of the gas on the electrodes, such condensation lowering the voltages which could otherwise be obtained. A small amount of preliminary sparking was found desirable to condition the electrode surfaces, but the decomposition of the Freon resulting from sparking gradually led to sufficient deposition of carbon on the electrodes to lower the breakdown voltage. This effect was most noticeable at the higher pressures, in which the energy of the discharges was greatest. In general it was found that the behavior of Freon was more erratic the higher the pressure, probably because of the changing nature of the electrode surfaces through carbon deposition and the increasingly high field emission from these surfaces. However, with uniform fields the breakdown gradient continued to increase with pressure, as is shown by the curve in figure 4 for a 0.2-inch gap. Gradients of over 1,200 kv per inch were insulated at 135 pounds per square inch absolute, and there was evidence that with careful electrode conditioning the values indicated by this curve might be increased by about 15 per cent. Figure 5 shows the dependence of the breakdown potential of Freon upon pressure for constant values of pressure times gap. The departure from Paschen's law, as indicated by the slope of these curves, begins when *pd* is greater than two pounds per inch and increases with the higher values. That this departure progresses more rapidly than in the case of air is probably due in part to the influence of inadequately conditioned electrode surfaces.

Reasons for the Higher Insulating Strength of Freon

The higher insulating strength of Freon and similar compounds, as compared with air and other permanent gases, may be ascribed primarily to two causes. The one is the relatively large number of inelastic impacts which an electron may make in such gases[10] with consequent loss of energy and ionizing power; the other is the electron affinity of the electronegative particles in the gas, such as the chlorine and fluorine atoms. This results in the removal of free electrons from the gas with the accompanying formation of relatively inert negative ions. By the former mechanism, such electrons as are created by ionization in the gas are retarded in their accumulation of ionizing energy by the frequent loss of energy on impact. Through the second mechanism the electrons, which are the most effective ionizing agents, are rendered impotent by attachment to heavy electronegative particles.

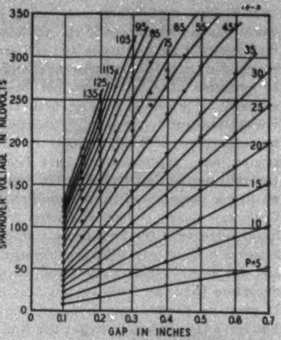

Figure 3. Increase of the d-c spark-over voltage of Freon 12 with electrode separation for absolute pressures from 5 pounds to 135 pounds per square inch. Uniform field

An obstacle to the immediate widespread use of Freon or other such compounds is the gradual decomposition of the gas under the influence of corona and sparkover. In the experimental use of Freon in the pressure-insulated electrostatic generator, although some decomposition of the gas occurred, no impairment of the insulating strength of the gas or of the solid insulation in the chamber, and very little corrosion of the metals was observed after about ten hours. Since there is evidence, however, that the long-time effects of this chemical instability may be serious, this problem and methods of removing the decomposition products of Freon are being studied and will be reported later.

Another limitation to the use of Freon for high-voltage insulation is imposed by the low pressure of 55 pounds per square inch absolute at which the positive point-to-plane breakdown voltage reaches its maximum value.[11] This characteristic, which is common to most gases at ordinary temperatures but which for air and nitrogen occurs at about 160 pounds per square inch absolute, seems at the present time to establish a maximum pressure limit for applications involving highly distorted fields. It thus becomes necessary in applying compressed-gas insulation to electrostatic generators, high-voltage transformers, high-voltage condensers, and the like, to secure sufficiently uniform fields throughout in order that the pressure of the gas may be fixed by economic considerations rather than by an inherent limitation in its insulating properties.

When it is considered that in many high-voltage applications, such as the insulation of electrostatic generators, any increase in the dielectric strength of the gas is reflected (in the limit) as the cube in reduced volume of apparatus, the great practical importance of such superior insulating gases as Freon is realized. It is not unlikely that compounds will become available which show even greater chemical stability than Freon, and at the same time have higher vapor pressures at room temperature. Such gaseous compounds, whose use for insulation is already in a preliminary commercial stage,[12] may ultimately supplant the permanent gases and even liquid dielectrics for the insulation of high-voltage apparatus.

Acknowledgments

The authors take pleasure in acknowledging the assistance in this work of Mr. A. T. Norton, Jr., as well as of members of the WPA Medical Radiation group. The development of the pressure-insulated X-ray generator was made possible

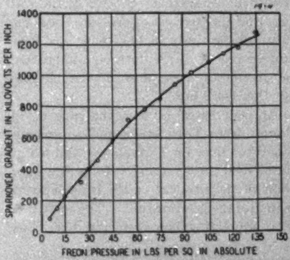

Figure 4. Spark-over gradient in Freon 12 as a function of pressure. Uniform field with 0.2-inch gap

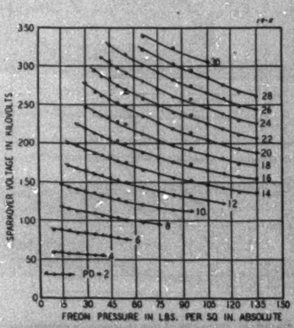

Figure 5. Spark-over voltage of Freon 12 as a function of pressure for several values of pressure times gap in pounds per inch

A Five-Figure Table of the Bessel Function $I_n(x)$

HERBERT B. DWIGHT
FELLOW AIEE

THE modified Bessel function of the first kind, $I_n(x)$, where x is a real quantity, is of widespread application, but it is of particular interest to electrical engineers, since the current and voltage for a traveling wave on an electric line can be expressed as a series of these functions.

A table of these functions was published in the Report of the British Association for the Advancement of Science, 1889, pages 29–32, and has been reprinted in "Bessel Functions" by Gray, Mathews, and MacRobert. While a large number of significant figures were given, the arguments were 0.2, 0.4, 0.6, etc.* The interpolation, as used in engineering work, is somewhat easier and more according to a decimal system when arguments 0.1, 0.2, 0.3, etc., are used. Accordingly, additional values for a five-figure table have been computed, and the enlarged five-figure table is given in this paper.

In a number such as $0.0^{8}26053$ in the table, the figure 8 represents the number of zeros before the 2.

The work of computing the new values for this table was done by students under the National Youth Administration at the Massachusetts Institute of Technology, those doing a substantial amount of work including A. L. Carpentier, D. R. Erb, S. Grunes, L. A. King, H. A. Lent, R. P. Mork, R. T. Parry, and B. H. Sexauer.

The values in the table were computed to more decimal places than have been tabulated, and the computations were checked. As a further precaution against errors, columns of differences of the final table were computed and examined for consistency, to a large extent by R. P. Mork, who used the equation

$$\Delta_4 = A - 4B + 6C - 4D + E$$

for the computation of the fourth difference Δ_4 with a calculating machine, where A, B, C, D, and E are any five successive values of $I_n(x)$. This was found to save time and work. See also the introduction to "Tables of the Exponential Function e^x," by WPA of the City of New York, A. N. Lowan, Technical Director. The use of columns of differences is a very searching method of exposing errors in a table.

More than half of the new values were computed by an interpolation method, from the original table mentioned in the second paragraph. Interpolation coefficients given in the paper "Tables and Methods of Extending Tables for Interpolation Without Differences," by George Rutledge and Prescott Crout, *Journal of Mathematics and Physics*, volume 9, 1930, were employed. These, when used with a calculating machine, were found to save considerable work, as no pencil work was needed and no columns of differences of the original table were required. For $x = 5.7$ and 5.9, Gregory-Newton interpolation coefficients were used, and this gave a comparison of the amount of work by the different methods. For the higher orders, the values were proportionately far apart and the power series in x was used, as follows:

$$I_n(x) = \frac{\left(\frac{1}{2}x\right)^n}{n!}\left[1 + \frac{\left(\frac{1}{2}x\right)^2}{1(n+1)} + \frac{\left(\frac{1}{2}x\right)^4}{1\times 2(n+1)(n+2)} + \dots\right]$$

Equations for traveling waves on an electric line, which involve Bessel functions of the type $I_n(x)$, are given on pages 161 and 163 of chapter VII of "Principles of Electric Power Transmission," by L. F. Woodruff, and a numerical example is given on pages 163 to 165.

Acknowledgments are here made for valuable help and suggestions from Professors R. D. Douglass, P. Franklin, and L. F. Woodruff.

(For table, see next page.)

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* Large tables of $I_1(x)$ and $I_0(x)$ were given in the Report of the British Association in 1893 and 1896.

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by a generous grant from the Godfrey M. Hyams Trust.

References

1. ON SOME ELEMENTARY LAWS OF ELECTRICITY, W. Snow Harris. *Philosophical* Transactions of the Royal Society, 1834, pages 213–45.

2. THE GENESIS OF IONS BY THE MOTION OF POSITIVE IONS AND A THEORY OF THE SPARKING POTENTIAL, John S. Townsend. *Philosophical Magazine*, volume 6, 1903, pages 598–618.

3. MECHANISM OF SPARK DISCHARGE IN AIR AT ATMOSPHERIC PRESSURE, I, II, L. B. Loeb and S. M. Meek. *Journal of Applied Physics*, volume 11, 1940, pages 438–47, 459–74. See bibliography.

4. A COMPACT PRESSURE-INSULATED ELECTROSTATIC X-RAY GENERATOR, J. G. Trump and R. J. Van de Graaff. *Physical Review*, volume 55, 1939, pages 1160–5.

5. GENERATING VOLTMETER FOR PRESSURE-INSULATED HIGH-VOLTAGE SOURCES, J. G. Trump, F. J. Safford, and R. J. Van de Graaff. *Review of Scientific Instruments*, volume 11, 1940, pages 54–6.

6. BREAKDOWN STUDIES IN COMPRESSED GASES, A. H. Howell. AIEE TRANSACTIONS, volume 58, 1939 (May Section), pages 193–204. See bibliography.

7. ÜBER DIE ZUM FUNKENÜBERGANG IN LUFT, WASSERSTOFF UND KOHLENSÄURE BEI VERSCHIEDENEN DRUCKEN ERFORDERLICHE POTENTIALDIFFERENZ, Friedrich Paschen. *Wiedemann-Annalen der Physik*, volume 37, 1889, pages 69–96.

8. CONDUCTION OF ELECTRICITY THROUGH GASES (a book), J. J. Thomson and G. P. Thomson. Cambridge University Press, third edition, volume 2, 1933, 608 pages, page 490.

9. DIELECTRIC STRENGTH OF INSULATING FLUIDS, E. E. Charleton and F. S. Cooper. *General Electric Review*, volume 40, 1937, pages 438–42. See bibliography.

10. R. F. Baker and J. T. Tate. *Physical Review*, volume 53, 1938, page 683. An abstract.

11. EFFECT OF PRESSURE ON POSITIVE POINT-TO-PLANE DISCHARGE IN N_2, O_2, CO_2, SO_2, SF_6, CCl_2F_2, A, He, AND H_2, H. C. Pollock and F. S. Cooper. *Physical Review*, volume 56, 1939, pages 170–5.

12. MILLION-VOLT X-RAY OUTPUT, E. E. Charleton, W. F. Westendorp, L. E. Dempster, and G. Hotaling. *Journal of Applied Physics*, volume 10, 1939, pages 374–85.

Table of the Bessel Function $I_n(x)$

x	$I_0(x)$	$I_1(x)$	$I_2(x)$	$I_3(x)$	$I_4(x)$	$I_5(x)$	$I_6(x)$	$I_7(x)$	$I_8(x)$	$I_9(x)$	$I_{10}(x)$	$I_{11}(x)$
0	1.0000	0	0	0	0	0	0	0	0	0	0	0
0.1	1.0025	0.050063	$0.0^{2}12510$	$0.0^{4}20846$	$0.0^{6}26055$	$0.0^{8}26053$	$0.0^{10}21700$	$0.0^{12}15506$	$0.0^{15}96908$	$0.0^{17}53836$	$0.0^{19}26918$	$0.0^{21}12235$
0.2	1.0100	0.10050	$0.0^{2}50167$	$0.0^{3}16708$	$0.0^{5}41750$	$0.0^{7}83472$	$0.0^{8}13909$	$0.0^{10}19866$	$0.0^{12}24829$	$0.0^{14}27585$	$0.0^{16}27582$	$0.0^{18}25073$
0.3	1.0226	0.15169	0.011335	$0.0^{3}56567$	$0.0^{4}21189$	$0.0^{6}63519$	$0.0^{7}15871$	$0.0^{9}33996$	$0.0^{11}63723$	$0.0^{12}10618$	$0.0^{14}15923$	$0.0^{16}21710$
0.4	1.0404	0.20403	0.020268	$0.0^{2}13467$	$0.0^{4}67202$	$0.0^{5}26845$	$0.0^{7}89398$	$0.0^{8}25524$	$0.0^{10}63775$	$0.0^{11}14166$	$0.0^{13}28321$	$0.0^{15}51478$
0.5	1.0635	0.25789	0.031906	$0.0^{2}26451$	$0.0^{3}16481$	$0.0^{5}82232$	$0.0^{6}34212$	$0.0^{7}12205$	$0.0^{9}38108$	$0.0^{10}10578$	$0.0^{12}26430$	$0.0^{14}60041$
0.6	1.0920	0.31370	0.046365	$0.0^{2}46022$	$0.0^{3}34362$	$0.0^{4}20556$	$0.0^{5}10256$	$0.0^{7}43883$	$0.0^{8}16436$	$0.0^{10}54731$	$0.0^{11}16406$	$0.0^{13}44713$
0.7	1.1263	0.37188	0.063790	$0.0^{2}73674$	$0.0^{3}64074$	$0.0^{4}44670$	$0.0^{5}25982$	$0.0^{6}12963$	$0.0^{8}56615$	$0.0^{9}21987$	$0.0^{11}76869$	$0.0^{12}24436$
0.8	1.1665	0.43286	0.084353	0.011100	$0.0^{2}11013$	$0.0^{4}87635$	$0.0^{5}58202$	$0.0^{6}33164$	$0.0^{7}16545$	$0.0^{9}73404$	$0.0^{10}29319$	$0.0^{11}10649$
0.9	1.2130	0.49713	0.10826	0.015972	$0.0^{2}17790$	$0.0^{3}15904$	$0.0^{4}11871$	$0.0^{6}76039$	$0.0^{7}42652$	$0.0^{8}21278$	$0.0^{10}95576$	$0.0^{11}39040$
1.0	1.2661	0.56516	0.13575	0.022168	$0.0^{2}27371$	$0.0^{3}27146$	$0.0^{4}22489$	$0.0^{5}15992$	$0.0^{7}99606$	$0.0^{8}55184$	$0.0^{9}27529$	$0.0^{10}12490$
1.1	1.3262	0.63749	0.16709	0.029801	$0.0^{2}40493$	$0.0^{3}44101$	$0.0^{4}40138$	$0.0^{5}31369$	$0.0^{6}21476$	$0.0^{7}13080$	$0.0^{9}71745$	$0.0^{10}35791$
1.2	1.3937	0.71468	0.20260	0.039359	$0.0^{2}58007$	$0.0^{3}68789$	$0.0^{4}68209$	$0.0^{5}58093$	$0.0^{6}43354$	$0.0^{7}28788$	$0.0^{8}17216$	$0.0^{10}93653$
1.3	1.4693	0.79733	0.24262	0.050815	$0.0^{2}80888$	$0.0^{2}10371$	$0.0^{3}11124$	$0.0^{4}10253$	$0.0^{6}82818$	$0.0^{7}59535$	$0.0^{8}38550$	$0.0^{9}22707$
1.4	1.5534	0.88609	0.28755	0.064522	0.011026	$0.0^{2}15191$	$0.0^{3}17520$	$0.0^{4}17369$	$0.0^{5}15095$	$0.0^{6}11678$	$0.0^{8}81382$	$0.0^{9}51598$
1.5	1.6467	0.98167	0.33783	0.080774	0.014738	$0.0^{2}21706$	$0.0^{3}26777$	$0.0^{4}28406$	$0.0^{5}26426$	$0.0^{6}21885$	$0.0^{7}16331$	$0.0^{8}11088$
1.6	1.7500	1.0848	0.39397	0.099892	0.019371	$0.0^{2}30356$	$0.0^{3}39874$	$0.0^{4}45060$	$0.0^{5}44666$	$0.0^{6}39424$	$0.0^{7}31358$	$0.0^{8}22696$
1.7	1.8640	1.1963	0.45650	0.12223	0.025080	$0.0^{2}41665$	$0.0^{3}58039$	$0.0^{4}69587$	$0.0^{5}73208$	$0.0^{6}68593$	$0.0^{7}57926$	$0.0^{8}44518$
1.8	1.9896	1.3172	0.52604	0.14819	0.032077	$0.0^{2}56248$	$0.0^{3}82798$	$0.0^{3}10495$	$0.0^{4}11677$	$0.0^{5}11574$	$0.0^{6}10841$	$0.0^{8}84091$
1.9	2.1277	1.4482	0.60327	0.17820	0.040545	$0.0^{2}74830$	$0.0^{2}11603$	$0.0^{3}15500$	$0.0^{4}18180$	$0.0^{5}19001$	$0.0^{6}17905$	$0.0^{7}15359$
2.0	2.2796	1.5906	0.68895	0.21274	0.050729	$0.0^{2}98257$	$0.0^{2}16002$	$0.0^{3}22464$	$0.0^{4}27699$	$0.0^{5}30442$	$0.0^{6}30170$	$0.0^{7}27222$
2.1	2.4463	1.7455	0.78390	0.25235	0.062895	0.012751	$0.0^{2}21754$	$0.0^{3}32011$	$0.0^{4}41388$	$0.0^{5}47707$	$0.0^{6}49600$	$0.0^{7}46956$
2.2	2.6291	1.9141	0.88906	0.29763	0.077345	0.016374	$0.0^{2}29195$	$0.0^{3}44923$	$0.0^{4}60761$	$0.0^{5}73288$	$0.0^{6}79748$	$0.0^{7}79029$
2.3	2.8296	2.0978	1.0054	0.34922	0.094415	0.020825	$0.0^{2}38722$	$0.0^{3}62173$	$0.0^{4}87785$	$0.0^{4}11056$	$0.0^{5}12565$	$0.0^{6}13007$
2.4	3.0493	2.2981	1.1342	0.40787	0.11448	0.026257	$0.0^{2}50814$	$0.0^{3}84966$	$0.0^{3}12499$	$0.0^{4}16406$	$0.0^{5}19436$	$0.0^{6}20976$
2.5	3.2898	2.5167	1.2765	0.47437	0.13798	0.032843	$0.0^{2}66033$	$0.0^{2}11478$	$0.0^{3}17560$	$0.0^{4}23978$	$0.0^{5}29557$	$0.0^{6}33199$
2.6	3.5533	2.7554	1.4337	0.54963	0.16537	0.040786	$0.0^{2}85045$	$0.0^{2}15342$	$0.0^{3}24368$	$0.0^{4}34560$	$0.0^{5}44256$	$0.0^{6}51648$
2.7	3.8417	3.0161	1.6075	0.63463	0.19721	0.050313	0.010863	$0.0^{2}20300$	$0.0^{3}33437$	$0.0^{4}49175$	$0.0^{5}65319$	$0.0^{6}79085$
2.8	4.1573	3.3011	1.7994	0.73048	0.23408	0.061686	0.013772	$0.0^{2}26636$	$0.0^{3}45403$	$0.0^{4}69146$	$0.0^{5}95134$	$0.0^{5}11933$
2.9	4.5027	3.6126	2.0113	0.83841	0.27666	0.075204	0.017337	$0.0^{2}34648$	$0.0^{3}61056$	$0.0^{4}96163$	$0.0^{4}13686$	$0.0^{5}17762$
3.0	4.8808	3.9534	2.2452	0.95975	0.32571	0.091206	0.021684	$0.0^{2}44721$	$0.0^{3}81370$	$0.0^{3}13237$	$0.0^{4}19464$	$0.0^{5}26104$
3.1	5.2945	4.3262	2.5034	1.0960	0.38205	0.11008	0.026957	$0.0^{2}57309$	$0.0^{2}10754$	$0.0^{3}18049$	$0.0^{4}27388$	$0.0^{5}37913$
3.2	5.7472	4.7343	2.7883	1.2489	0.44665	0.13226	0.033325	$0.0^{2}72948$	$0.0^{2}14102$	$0.0^{3}24391$	$0.0^{4}38155$	$0.0^{5}54459$
3.3	6.2426	5.1810	3.1027	1.4202	0.52054	0.15825	0.040982	$0.0^{2}92274$	$0.0^{2}18357$	$0.0^{3}32690$	$0.0^{4}52660$	$0.0^{5}77421$
3.4	6.7848	5.6701	3.4495	1.6119	0.60490	0.18861	0.050153	0.011604	$0.0^{2}23734$	$0.0^{3}43470$	$0.0^{4}72046$	$0.0^{4}10900$
3.5	7.3782	6.2058	3.8320	1.8264	0.70105	0.22398	0.061096	0.014512	$0.0^{2}30489$	$0.0^{3}57384$	$0.0^{4}97761$	$0.0^{4}15207$
3.6	8.0277	6.7927	4.2540	2.0661	0.81046	0.26509	0.074109	0.018055	$0.0^{2}38932$	$0.0^{3}75232$	$0.0^{3}13163$	$0.0^{4}21034$
3.7	8.7386	7.4357	4.7193	2.3338	0.93475	0.31273	0.089532	0.022356	$0.0^{2}49431$	$0.0^{3}97992$	$0.0^{3}17595$	$0.0^{4}28859$
3.8	9.5169	8.1404	5.2325	2.6326	1.0758	0.36784	0.10778	0.027554	$0.0^{2}62427$	$0.0^{2}12686$	$0.0^{3}23357$	$0.0^{4}39293$
3.9	10.369	8.9128	5.7983	2.9658	1.2355	0.43145	0.12923	0.033815	$0.0^{2}78443$	$0.0^{2}16329$	$0.0^{3}30805$	$0.0^{4}53115$
4.0	11.302	9.7595	6.4222	3.3373	1.4163	0.50472	0.15446	0.041330	$0.0^{2}98099$	$0.0^{2}20903$	$0.0^{3}40379$	$0.0^{4}71306$
4.1	12.324	10.688	7.1100	3.7511	1.6206	0.58809	0.18404	0.050322	0.012213	$0.0^{2}26620$	$0.0^{3}52620$	$0.0^{4}95115$
4.2	13.442	11.706	7.8684	4.2120	1.8513	0.68571	0.21863	0.061048	0.015140	$0.0^{2}33734$	$0.0^{3}68194$	$0.0^{3}12609$
4.3	14.668	12.822	8.7043	4.7249	2.1115	0.79656	0.25899	0.073805	0.018692	$0.0^{2}42552$	$0.0^{3}87914$	$0.0^{3}16618$
4.4	16.010	14.046	9.6258	5.2955	2.4046	0.92342	0.30598	0.088939	0.022980	$0.0^{2}53438$	$0.0^{2}11277$	$0.0^{3}21779$
4.5	17.481	15.389	10.642	5.9301	2.7347	1.0684	0.36057	0.10684	0.028170	$0.0^{2}66826$	$0.0^{2}14397$	$0.0^{3}28393$
4.6	19.093	16.863	11.761	6.6355	3.1060	1.2338	0.42389	0.12798	0.034400	$0.0^{2}83235$	$0.0^{2}18297$	$0.0^{3}36829$
4.7	20.858	18.479	12.995	7.4195	3.5233	1.4223	0.49719	0.15286	0.041869	0.010328	$0.0^{2}23153$	$0.0^{3}47541$
4.8	22.794	20.253	14.355	8.2903	3.9921	1.6369	0.58191	0.18210	0.050798	0.012768	$0.0^{2}29178$	$0.0^{3}61087$
4.9	24.915	22.199	15.854	9.2575	4.5182	1.8809	0.67967	0.21638	0.061448	0.015730	$0.0^{2}36625$	$0.0^{3}78148$
5.0	27.240	24.336	17.506	10.331	5.1082	2.1580	0.79229	0.25649	0.074117	0.019316	$0.0^{2}45800$	$0.0^{3}99554$
5.1	29.789	26.680	19.326	11.523	5.7697	2.4724	0.92185	0.30334	0.089153	0.023643	$0.0^{2}57069$	$0.0^{2}12631$
5.2	32.584	29.254	21.332	12.845	6.5106	2.8288	1.0707	0.35796	0.10696	0.028852	$0.0^{2}70864$	$0.0^{2}15965$
5.3	35.648	32.080	23.542	14.312	7.3402	3.2324	1.2415	0.42152	0.12800	0.035107	$0.0^{2}87705$	$0.0^{2}20104$
5.4	39.009	35.182	25.978	15.939	8.2686	3.6890	1.4371	0.49538	0.15281	0.042598	0.010820	$0.0^{2}25226$
5.5	42.695	38.588	28.663	17.743	9.3070	4.2052	1.6611	0.58106	0.18202	0.051550	0.013309	$0.0^{2}31545$
5.6	46.738	42.328	31.620	19.742	10.468	4.7884	1.9171	0.68031	0.21633	0.062225	0.016322	$0.0^{2}39319$
5.7	51.173	46.436	34.879	21.959	11.765	5.4466	2.2095	0.79511	0.25657	0.074925	0.019961	$0.0^{2}48854$
5.8	56.038	50.946	38.470	24.415	13.214	6.1890	2.5430	0.92771	0.30367	0.090004	0.024346	$0.0^{2}60519$
5.9	61.377	55.900	42.427	27.136	14.831	7.0250	2.9230	1.0807	0.35872	0.10787	0.029617	$0.0^{2}74750$
6.0	67.234	61.342	46.787	30.151	16.637	7.9685	3.3558	1.2569	0.42297	0.12901	0.035940	$0.0^{2}92070$

The Service Factor Rating of Arc-Welding Generators and Transformers

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Synopsis: The primary purpose of rating an electrical machine is to give the user an idea of what he may expect from the unit which he purchases.

It is the purpose of this paper to propose a method of rating arc-welding generators and transformers which will give a truer picture of their working ability than is afforded by the present one-hour rating. Such a method of rating will be beneficial both to the customer who buys and uses arc welders and to the manufacturers who produce and market the units. The user will be better able to select a machine which will do the job to be performed, and at the same time he will be assured that he is not purchasing a machine unduly large for his requirements.

The present custom is to give welding transformers and generators a one-hour rating, specifying the current they can deliver for a one-hour period, starting cold, without exceeding the permissible temperature rise. The plan here proposed is to give them a current rating indicative of their normal operating capacity, or short-time welding ability; and an additional service factor rating, indicative of their continuous current capacity, as limited by thermal considerations.

All electric apparatus has these two major limitations on its output, one a "size" limit expressed by breakdown torque, commutation limit, or voltage drop; and the other a thermal limit expressed by the degrees temperature rise permissible for the type of insulation used. These two limits are usually quite independent of each other, so that no single number, such as a one-hour rating, can fully describe the usability of the apparatus. The proposed service factor rating gives both of these limits, and therefore gives the user the data for applying the apparatus to a variety of duty cycles.

In conclusion, it is suggested that welding generators and transformers be designed for a service-factor rating of 75 per cent, since this corresponds best to typical welding duty cycles. It is also proposed that the standard AIEE values of temperature rise by resistance for continuous rated machines, 60 degrees centigrade for class *A*, or 80 degrees centigrade for class *B* insulation, be recognized as the limiting values in continuous operation at the service-factor rating.

Rating for Welding Service

BEFORE proposing a method of rating, it is advisable to consider the purpose of rating. "The rating of a machine or other equipment is a set of performance characteristics which are subject to verification by test under specified conditions and which by mutual agreement between buyer and seller may serve as a basis of specifications and contracts covering the purchase and sale of the machine or other equipment. The rating is used by the manufacturer in the design and fabrication of the equipment and is used by the purchaser as the basis for application; therefore, the terms in which the rating is stated should be definite, easily verified, of such a nature as to permit intelligent use of the equipment, and as far as possible inclusive of usual conditions as found in practice."[1]

A common sense approach to the problem requires consideration of three different factors:

1. A rating suited to typical welding duty cycles, indicating electrical and thermal limitations.

2. A method of temperature measurement that will give the best indication of actual insulation temperature.

3. Selection of temperature-rise limits to assure satisfactory life of the machine as well as effective use of material in its construction.

In general, there are two types of duty to which an arc-welding generator or transformer may be subjected: Hand welding duty, and automatic welding duty. Because of the infinite variety of hand welding jobs, the duty cycle of such work varies from below 25 per cent up to as high as 75 per cent or 80 per cent of the total elapsed time in the case of production line work. In the case of intermittent welding done in automatic installations, the duty cycles may equal those in the upper brackets of hand welding operation. In a small percentage of cases such as pipe welding and other long seam welding, the length of the arc time may be so long that the application requires a continuous rated machine.

Standards[1] for rating arc-welding transformers and generators now specify temperature rise to be measured by operating the machine at rated current continuously for one hour (one-half hour for the smaller sizes), starting cold. There is also included a statement that transformers should be capable of carrying 75 per cent of rated current continuously. The emphasis has been placed on the one-hour rating and most purchasers, even those having strict specifications of their own, have paid little attention to the continuous capacity, but have required test only at the one-hour rating.

Disadvantages of a Short-Time Rating

In order to illustrate clearly the shortcomings of a short-time rating for machines which must carry cyclic loads, we have borrowed two hydraulic analogy diagrams from L. E. Hildebrand's paper on "Duty Cycles and Motor Rating."[2]

Figures 1 and 2 present hydraulic analogies of temperature rises in two electrical machines. Water flowing into the tank represents motor losses—the smaller stream represents no load losses and the larger stream stands for additional losses under load. The height of water in the tank is analogous to temperature rise, while the capacity of the tank is analogous to the heat storage capacity of the machine. The outlets at the bottom of the tank illustrate heat dissipation.

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1. For all numbered references, see list at end of paper.

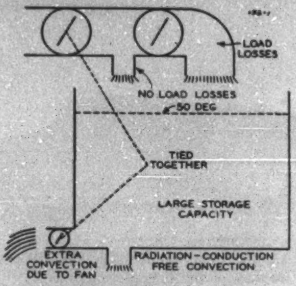

Figure 1. Hydraulic analogy

Short-time rated machines
Poorly ventilated

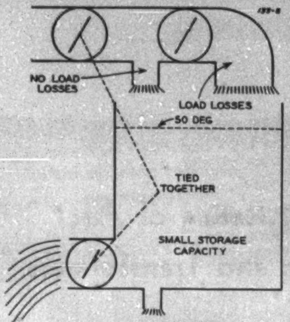

Figure 2. Hydraulic analogy

Short-time rated machines
Well ventilated

In these two diagrams the losses in the two machines are the same. However, in the machine shown in figure 1 we find that the heat dissipating ability as illustrated by the size of the outlet is small, while the thermal capacity of the machine as indicated by the volume of the tank is large; so that in the period of one hour the level of the temperature will just reach a point of 50 degrees rise.

In the case of the diagram in figure 2, we find a machine with a much smaller thermal capacity which is illustrated as a smaller tank. It is only as a result of the large heat dissipating ability as illustrated by the large outlet pipe that this machine is able to run for a period of one hour without exceeding the temperature rise of 50 degrees. Thus, here we have two machines both of which will meet identically the same one-hour load rating specification. But if we were to hold these two machines on a duty-cycle basis all day, we would find a great difference in their operation. The machine represented by the diagram in figure 1 has small heat dissipating ability; therefore, during periods of no load, heat is dissipated from this machine very slowly. During a day's welding, we find that the level of temperature rise continues to climb and may far exceed that obtained on the constant load during the one-hour period.

On the other hand, as soon as the load is removed from the machine represented by the diagram in figure 2, we find that heat is drained rapidly from that machine as a result of the high heat dissipating ability of that unit. With heat being thus quickly and efficiently removed, this unit can run all day on a duty-cycle load and will likely not even reach the level of temperature rise indicated at the end of a one-hour period.

Figure 3 illustrates these points in terms of specific transformer designs. In the figure, curve B_1 is the heating curve of a transformer with a one-hour rating. This same transformer carrying welding load will heat as in curve B_2, if designed with adequate cooling.

However, a transformer could be built with insufficient cooling which would match the one-hour heating curve B_1 but would heat under welding load as in curve B_3. Such a transformer would run considerably over its rated temperature rise on a long-continued load, such as a half a day's work. A transformer designed with heavy copper coils at low current density without adequate ventilating ducts, or a transformer with windings wound close on the core to allow coil heat to be absorbed by the slowly heating core, would meet the one-hour temperature rise but would run hot on actual welding service.

On the other hand, a transformer can be built with heating curves similar to A_1 and A_2. This transformer would reach rated temperature rise in only one-half hour on the short-time test at continuous rated current (curve A_1), but would operate well under rated temperature rise on all-day welding duty as shown by curve A_2. Also, its quick cooling after removal of load (end of curve), would compensate for its quicker heating time by reducing the total time at high temperature.

It can thus be seen that a properly designed transformer rated for a half-hour short-time test may be better than a poorly designed unit which meets the one-hour test. This is the most important reason why a short-time rating may be meaningless. The short-time rating is artificial, does not take into account the important consideration of all-day operation, and does not tell the operator what he can obtain in actual service without injury to the transformer.

Another drawback to the short-time rating is the difficulty of obtaining accurate tests. The problems involved in starting cold, adjusting load accurately and quickly, and timing accurately are serious and make the tested temperature rise obtained doubtful, especially if thermometers are used.

Continuous Ratings

One alternative method of rating is to rate the welding generator or transformer in terms of its maximum continuous current obtainable without exceeding rated temperature rise. This is done to a considerable extent in Europe. Continuous rating suffers from the disadvantage that it does not tell the operator what current he can obtain while welding—always more than the continuous current—since all welding operations are inherently discontinuous loads.

Typical Duty Cycles

In manual welding there is no such thing as a continuous load. In hand welding the maximum time on without interruption is short, about two minutes, the time for one stick of electrode to be consumed. After one electrode is consumed the operator must raise his hood, remove the end of the consumed electrode, insert the new electrode, position the electrode, lower the hood, and restart the arc. This operation consumes time at least equal to ten per cent of the arc time. However, there are always other operations required, such as change or reposition work, chip slag, add parts, change current, rest and delay.

From a careful study of many types of hand welding operations, the writers have come to the following conclusions:

> Many welding duty cycles average about 25 per cent.
>
> On highly repetitive work the duty cycle may rise to about 50 per cent. One such cycle is shown in table I in this paper, and is followed by a similar but simplified cycle, table II, which was used as the basis of a heating test. The curve of temperature rise on this test is shown in figure 4. This particular test was made on a large fan-cooled welding transformer.
>
> Exceptional welding jobs have been found with as high as 72 per cent duty. One such is shown in table III. This shows a time study of a job requiring 63 minutes total. However, there was permitted the welding operator an additional 15 per cent time for rest and delay not shown in the table

which would reduce the duty considerably. It seems probable that 70 per cent approaches the maximum duty cycle which can be obtained in long-continued manual welding.

Other Factors Affecting Heating

Since most arc-welding transformers and generators are furnished in standard ratings and are not built to the exact rating needed for a particular job, the factors of overload and underload should also be considered. The overload range of arc-welding transformers is not very large, generally about 25 per cent. Further overload is prevented by the limit of reactance or other type of current control. Here again proper information on the duty-cycle curve in the hands of the operator would be of value, permitting him to keep within safe temperatures by controlling the duty cycle on those jobs where the overload range is used.

It can safely be expected that in the vast majority of applications the machine will be operated at less than rated current. The range of current obtainable at welding voltages generally goes down to 20 per cent of the rated current in order that one machine may be capable of doing a wide variety of work. In many jobs it is necessary to change current several times to accommodate the different sizes of electrodes required to weld an assembly. However, it is necessary to rate the generator or transformer on a reasonably high duty cycle at its rated current for the rating implies that the equipment may be used for the vast majority of welding operations at that current. Nor is it fair to make the standard duty cycle so low as to deprive the user of any increased life he may obtain when using the machine at less than rated current.

Table I. Typical Heavy Duty Cycles—Hand Arc Welding

Minutes on	Minutes off
	1.93
1.85	0.49
1.85	0.49
1.85	0.60
1.17	3.33
2.16	0.49
0.61	1.68
0.26	0.35
Total 9.75	9.36
Duty cycle 51%	

Table II. Typical Heavy Duty Cycles—Hand Welding

Minutes on	Minutes off
	2
2	0.5
2	0.5
2	0.5
1	3.5
2.5	0.5
0.5	1.5
0.75	1.25
Total 10.75	10.25
Duty cycle 51%	

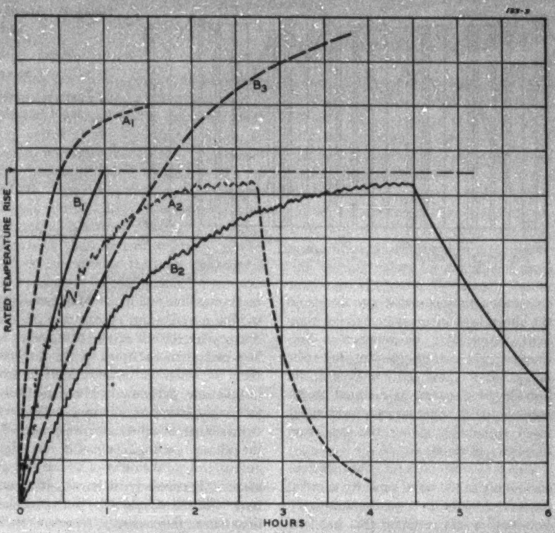

Figure 3. Heating curve

Arc-welding transformer
A—Transformer with low heat storage (half-hour rated)
A_1—Continuous at rated current
A_2—At 60 per cent welding duty cycle
B—Transformer with high heat storage (one-hour rated)
B_1—Continuous at rated current
B_2—At 60 per cent duty cycle (well ventilated)
B_3—Average at 60 per cent duty cycle (poorly ventilated)

Suggested Duty-Cycle Standard

When all the above factors are taken into consideration, the authors feel that a standard duty cycle for rating arc-welding generators and transformers should be 60 per cent. A machine thus rated would carry the vast majority of hand welding operations without exceeding rated temperature. Also, since most welding operations do not exceed 50 per cent time on, a reasonable margin is left for operation at some overload. A maximum time on of six minutes out of ten would insure adequate heat storage capacity.

There is precedent for a duty-cycle rating in welding. All resistance-welding transformers are now rated on a duty-cycle basis, the standard being 50 per cent.

The most comparable test would be of course, a duty-cycle test with the generator or transformer continuously excited and alternately loaded for six minutes and open-circuited for four minutes. This test would result in a saw-tooth shaped temperature curve. Such a test is rather difficult to make due to the problem of accurate timing, but it is provided for in the VDE (German) motor standards. Special arrangements might have to be made to test welders where output current is controlled by taps, rather than by adjustable magnetic means, in order to make sure that each winding is properly heated.

Service-Factor Rating

On the assumption that the temperature rise varies as the square of the load current, a duty cycle with 60 per cent time on would give the same heating as a test with $\sqrt{0.60}=0.775$ current continuously. For the sake of simplicity, therefore, it is suggested that the standard temperature-rise test should consist of a continuous heat run at 75 per cent of rated current, carried on until the temperature rise becomes constant. This test will definitely fix the capacity of the machine on a 100 per cent duty cycle, or

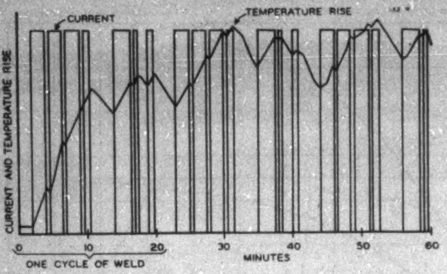

Figure 4. Heating curve

Arc-welding transformer

(51½ per cent welding duty cycle)

continuous service, and it also avoids all the difficulties inherent in making transient temperature measurements. Furthermore, the temperature-rise limit to be prescribed for this continuous test will logically be the same as dictated by experience for all other types of continuous rated apparatus, giving the maximum simplicity in standards.

This 0.75 ratio of permissible continuous output to the rated capacity is called the service factor, as it corresponds to the definition of service factor that has long been used for general-purpose motors. Such motors have 40-degrees-centigrade rise at their rating, and a service factor of 1.15, corresponding on the average to the increased load at the 50-degrees-centigrade rise value permissible for continuous rated apparatus in specific applications.[3]

It has already been suggested by L. E. Hildebrand that service factor ratings of less than unity should be used for high torque motors on intermittent duty cycles, instead of the customary short-time ratings.[2] The same reasoning outlined in the preceding paragraphs applies to a wide variety of apparatus in intermittent use. In each case the nameplate rating should be indicative of the safe allowable load for a particular kind of duty, and the service factor indicates the permissible continuous, or rms, load.

On the several grounds of convenience, testing accuracy, and versatility of application, therefore, the service-factor rating method is suggested as preferable for welding transformers and generators; and the specific value of 75 per cent service factor is suggested as most suitable for usual arc-welding service.

Automatic Welding

All the foregoing discussion refers to hand welding, that is, welding with the electrode holder in a man's hand. No attempt will be made in this paper to recommend duty-cycle ratings for automatic machine welding. Many machine welding installations operate at very low duty cycles since it often takes longer to load and align the work in the machine than to make the weld. Many other installations, particularly where one transformer is switched alternately between two welding machines, one welding while the other is loading, operate at very high duty cycles, on the order of 80 per cent or higher. Fortunately, it is not so important to have a standard for such installations since they usually represent such large investments as to justify individual study and application.

Temperature-Rise Measurement

The aim of temperature-rise measurement is to determine the hottest temperature reached by the insulation, in order that long life of the machine may be assured.

The three general methods of temperature measurement are: The measurement of the temperatures upon external parts of windings or core by thermometer or thermocouple, measurement from the cold to hot resistance change of a winding, or the use of embedded detectors.

As a result of the demands of the industry for easily portable arc-welding sets, the trend of generator designs is to small machines which are very well ventilated. The fact that these units are so well ventilated by an air blast means that there is, of necessity, a greater thermal gradient from the inside of the coil winding to the external surface of the insulation. This means that surface temperatures vary widely from internal temperatures and that the extent of this variation is influenced considerably by the method of ventilation employed.

It is very difficult to get accurate transformer temperatures by thermometer or thermocouple unless thermocouples are actually built into the winding. In most natural draft transformers, particularly those with heavy coils, a thermometer or thermocouple on the outside of the coil may read 20 to 30 per cent lower than the average temperature rise of the winding. The rise by resistance method, of course, becomes mandatory in oil or synthetic fluid-immersed transformers since it is then practically impossible to insert thermometers or thermocouples.

While the embedded detector method is of unquestioned desirability in heat flow tests, its use in commercial testing is pretty much out of the question.

A combination of the inadequacy of surface temperature measurements with the difficulty of making embedded detector measurements leads us to the measurement of temperature rise by the resistance method. By the resistance method, it is possible to obtain a true average winding temperature regardless of whether the coil is exposed to a blast of cooling air or whether it is well wrapped up in electrical and thermal insulating material. Although this method will, of course, not give an actual hot-spot temperature indication, yet it will be a much more reliable indication of the hot-spot temperature than any surface temperature measurement.[4]

As an aid in attaining a true indication of temperature rise, we will find that the adoption of the duty-cycle or service-factor method of rating is of considerable help. With short-time periods of test the rate of rise of internal temperatures may be so great that measurements of a final surface temperature are likely to be very inconsistent and consequently therefore, also inaccurate. While it is a fact that a true sportsman will not shoot at a standing bird, fortunately there is no sporting or ethical principle which requires the builders of arc-welding equipment to attempt to measure temperature rise "on the wing." With the adoption of the service-factor method of rating welders it will be possible to allow the temperature of the machine to reach a state of equilibrium and thus allow accurate measurement. The combination of service-factor rating with temperature rise by resistance measurement will assure us that we do not have our heads in the sand but that we will actually know the average temperature rise of the windings in our arc-welding machines. To know the true values of temperature rise is of unquestioned advantage for both the user and the manufacturer of welding machines. The manufacturer can know that he is making optimum use of all of the material within the machine without exceeding safe temperature limits; while the purchaser will be assured of long life machines giving trouble-free operation.

Limiting Temperature Rise

The two important factors governing the life of insulation are maximum temperature and the length of time the insulation is hot.

The maximum permissible operating temperature of the insulation is a function of the type of insulation, both fibrous materials and impregnating compounds. The general subject of insulation characteristics is too long to discuss here, so for the purpose of this paper it will be assumed that the commonly accepted standards[4] will insure satisfactory life of insulation, and that these rises should not be exceeded except for very short intervals of time.

For a given temperature, the life of insulation is reduced more or less directly with time; that is, if a transformer were hot only one-fifth of the time, its insulation could be expected to last five times as long as one hot continuously. For class *A* insulation it has been shown[5] that within limits the life of insulation is reduced roughly 50 per cent for each 8- to 12-degree increase in temperature. Therefore, the life reduction, in aging units, can be approximately estimated in the form of an exponential function of temperature, multiplied by the length of time the temperature persists in the insulation. For class *B* insulation, such as is used in most arc-welding transformers, the rate of aging has not yet been determined; however, it undoubtedly follows some similar law but probably with a higher temperature increment for a given aging.[6]

From the foregoing factors it will be seen that the shapes of the heating and cooling curves of a welder affect insulation life. For example, a welder which heats slowly may have the advantage of slow heating offset by cooling more slowly and thereby keeping its insulation hot longer than a welder which heats and cools quickly.

Since the present NEMA standards for welding generators require measurement of temperature rise by thermometer, it is pertinent to inquire what the corresponding values of rise by resistance are for actual machines now in use. As pointed out previously, and recognized in the AIEE "General Principles Upon Which Temperature Limits Are Based in the Rating of Electrical Machinery and Apparatus," there is a considerably greater difference between the two methods of measurement for highly rated direct-current machines than the ten-degree-centigrade value usually assumed for class *A* insulation. With the forced draft ventilation commonly in use on arc-welder sets, the thermal gradient between the inside windings and the outside of the coils is so great that the actual excess of the rise by resistance over that by thermometer is often 20 degrees centigrade or more.

Depending upon the thickness of insulation over the windings and the intensity of the blast of ventilating air, it is a matter of test record that temperatures measured by the resistance method range from 17 to 42 degrees higher than the values read by thermometer or thermocouple at the outer surface of the coils.

In view of the intermittent character of hand welding, and the satisfactory service records of welders having these relatively high internal temperatures under rated load conditions, it is permissible to set considerably higher limits for temperature rise for short-time rated than for continuous-rated machines. For example, the latest NEMA welding standards[1] permit 100-degrees-centigrade rise by thermometer, or 125 degrees centigrade by resistance, for "class *BB*" insulated welders rated on a half-hour or one-hour basis. Presumably this insulation is similar to the special class *B* insulation used in railway motors, for which 120-degrees-centigrade rise by resistance is permitted.[7] However, as previously indicated, the typical welding duty cycle of 60 per cent time on and 40 per cent time off with a time cycle of only a few minutes, results in no greater operating temperature than would be caused by a continuous load of about 75 per cent of the short-time rated amperes. And, to assure the service life normally expected of electrical apparatus, this average operating temperature should not exceed the standard limiting values for continuous-rated machines, which are: For class *A* insulation, 50 degrees centigrade by thermometer or 60 degrees centigrade by resistance, and for class *B* insulation, 70 degrees centigrade by thermometer or 80 degrees centigrade by resistance.[12] Thus, the 75 per cent service-factor rating method with the standard continuous-temperature limits is a logica outcome of the recent trends in welding transformers and generators.

Table III. Typical Heavy Duty Cycles—Hand Welding

Minutes on	Minutes off	Minutes on	Minutes off	Minutes on	Minutes off
	2.07	1.24	1.72	0.66	2.92
1.54	0.19	0.90	0.19	2.70	0.19
1.54	0.19	0.90	0.19	0.34	0.19
1.54	0.19	0.90	0.19	0.34	0.19
1.54	0.19	0.90	1.32	0.34	0.19
1.54	0.19	0.90	0.19	0.34	0.19
1.54	0.19	0.90	0.19	0.34	0.19
1.54	0.19	0.90	0.19	0.34	0.19
1.54	0.19	0.90	0.19	0.34	0.19
1.54	0.19	0.90	0.19	0.34	0.19
1.54	0.19	0.90	0.19	0.34	0.60
1.54	0.19	1.24	0.85	2.37	0.44
1.54	0.19	0.66	0.19	2.37	0.18
1.54	0.70	0.66	0.19		
1.24	0.19	0.66	0.19		
Total				45.88	17.26
Duty cycle				72.7%	

Conclusions

After consideration of all these factors, it is recommended that welding generators and transformers be given a nameplate rating representative of their expected normal welding currents, as at present, and in addition a service-factor rating of 75 per cent. Acceptance tests would then include a continuous test run at 0.75 of the nameplate amperes without exceeding a limiting temperature rise by resistance of 60 degrees centigrade for class *A* insulation or 80 degrees centigrade for class *B* insulation; or corresponding values of 50-degrees-centigrade and 70-degrees-centigrade rise if the thermometer method of measurement is used.

References

1. ELECTRIC ARC-WELDING MACHINE AND ELECTRODE STANDARDS, NEMA, Publication No. 40-62, October 1940.

2. DUTY CYCLES AND MOTOR RATING, L. E. Hildebrand. AIEE TRANSACTIONS, volume 58, 1939 (September section), pages 478–83.

3. RATING OF GENERAL PURPOSE INDUCTION MOTORS, P. L. Alger and T. C. Johnson. AIEE TRANSACTIONS, volume 58, 1939 (September section), pages 445–59.

4. GENERAL PRINCIPLES UPON WHICH TEMPERATURE LIMITS ARE BASED IN THE RATING OF ELECTRICAL MACHINERY AND APPARATUS, AIEE STANDARDS No. 1, June 1940.

5. LOADING TRANSFORMERS BY TEMPERATURE, V. M. Montsinger. AIEE TRANSACTIONS, volume 49, April 1930, pages 776–92.

6. APPLICATION OF CLASS B INSULATION TO AUXILIARY TYPE D-C MOTORS IN SEVERE DUTY SERVICE, F. A. Compton, Jr. AIEE TRANSACTIONS, volume 59, 1940, pages 828–34.

7. AMERICAN TENTATIVE STANDARDS FOR RAILWAY MOTORS AND OTHER ROTATING ELECTRICAL MACHINERY, AIEE STANDARDS No. 11, March 1937.

Engineering Requirements for Program Transmission Circuits

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Synopsis: Present-day program networks are reviewed from the standpoints of engineering, design, and operation as developed to meet the needs of the broadcasters. The factors requiring consideration in the further development of program networks in anticipation of future needs are also discussed. The presentation of the paper is supplemented by a demonstration of the quality obtainable by transmission over various types of telephone facilities.

Introduction

THE growth of radiobroadcasting to the magnitude of a major national industry within the last 20 years has been accompanied by the development of a nation-wide system of wire-line networks interconnecting hundreds of broadcasting stations. Papers have been presented before this Institute from time to time[1,2,3] describing the types of plant used for these networks and discussing important features of their design and operation. With these 20 years of experience as a background, it should now be of interest to review how the various requirements of broadcasting have influenced the development of the networks and to consider some of the factors which have determined the point to which transmission and operating features have so far been carried.

Simply stated, broadcasting is a means by which sounds originated at one place are reproduced simultaneously to large numbers of listeners distributed over wide areas. The simplest possible radiobroadcasting system would consist of a microphone, a radiobroadcast transmitter and some radio receiving sets. Such a system could serve only the listeners within the comparatively limited service area of the transmitter. To serve the whole nation many transmitters must be established about the country. Furthermore the most desirable sources of program are not usually in the neighborhood of the transmitter to which a particular listener can tune, since talent tends to be concentrated in certain parts of the country, and special events of interest may occur anywhere. To give a true country-wide service so that every listener can hear the programs he enjoys wherever they may originate, a supplementary transmission system must be provided, interconnecting the many studios and broadcasting stations. The wire networks that perform this function comprise the subject matter of this paper.

The present extent of the wire-line facilities which are associated with the major portion of these networks is indicated in figure 1. The width of the lines on this chart has been made proportional to the numbers of circuits in the various sections. The total length of these circuits is in excess of 110,000 miles, and it is not unusual for a program originating at some point on a network to traverse more than 7,000 miles of circuit before being broadcast by the most remote station.

The requirements which the program networks must meet are in the final analysis determined largely by the needs of the broadcasters. The objective of a program network service is to meet these needs in as complete and prompt a manner as possible consistent with reasonable cost. With this objective in mind, it is necessary in planning the plant to consider not only the day-to-day needs, but the possible future needs as well. The importance of this may be appreciated when it is considered that plant provided today for program transmission service will need to be adaptable to the service requirements 10 or 20 years hence. As a result of such planning, cables and equipment installed 5, 10, and 15 years ago, meet present-day requirements, and with some rearrangements, will take care of those likely to develop tomorrow.

The detailed planning of program transmission circuits requires consideration of:

1. The numbers of circuits likely to be required, section by section, over each route;
2. The provisions for reliability, flexibility, operation, and supervision essential to a high-grade network service;
3. The transmission requirements, or electrical characteristics, necessary to achieve a natural reproduction of the program.

These three general classes of requirements will be considered in order.

Number of Circuits Required

The circuits which have been established on a full-time basis for continuing use form the backbone of the program networks. Even for these circuits, however, permanence is relative since frequent extensions and rearrangements are made to meet changing requirements of the broadcasters. Aside from these full-time circuits there are intermittent requirements occasioned by special events and other short-period needs of the broadcasters, some of which involve networks almost as extensive as the full-time networks. In addition reliability of service requires provision for rerouting the networks in the event of trouble. Figure 2 shows the year-by-year growth in the operated mileage of program circuits for the period 1926 to 1940. Of the more than 110,000 miles of circuits shown for 1940, about 45,000 miles have been provided for the short-period services and as stand-by facilities for protection. In addition to these, there are still other circuits, normally assigned to other services, which are arranged to be readily adaptable to program service to supplement the reserve facilities maintained on a full-time basis.

The time interval necessarily accompanying any extensive construction project makes it necessary to engineer plant considerably in advance of actual service requirements to meet, not only the expected growth, but also the changes in network routing. Figure 3 shows for two typical sections along major routes the variations in requirements for full-time network circuits resulting from growth and rearrangements required by the broadcasters. While, in planning to meet these rapid variations in circuit requirements, advantage can be taken of some latitude which exists in the choice of routes for occasional services and protection facilities, the task of balancing the provision of circuits against requirements is an entertaining and at times difficult one for the circuit engineer.

Operating Requirements

Considering for a moment the variety of programs originating at many different

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[1] For all numbered references, see list at end of paper.

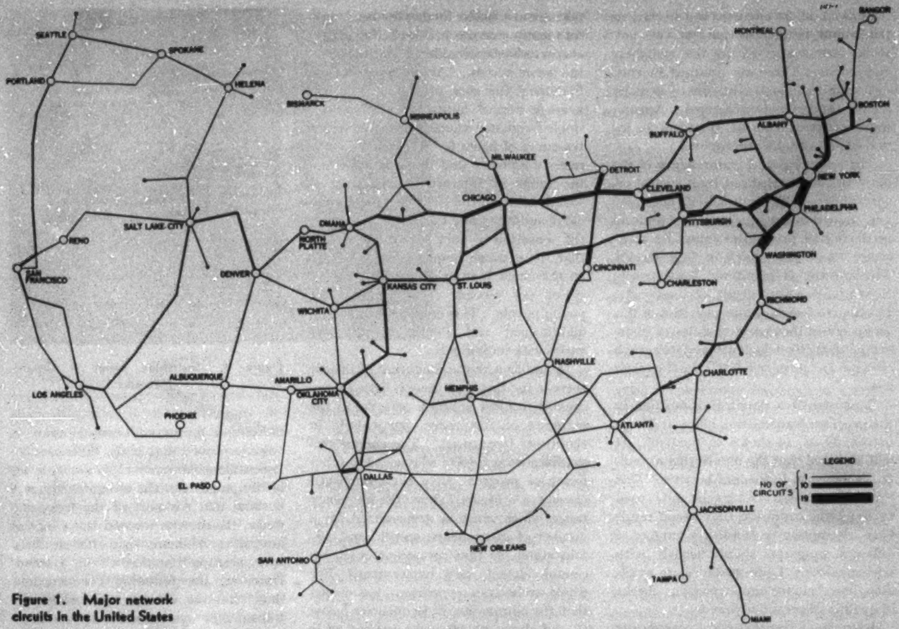
SEATTLE
SPOKANE
PORTLAND
HELENA
BISMARCK
MINNEAPOLIS
MILWAUKEE
DETROIT
MONTREAL
BANGOR
BUFFALO
ALBANY
BOSTON
NEW YORK
CLEVELAND
PHILADELPHIA
PITTSBURGH
WASHINGTON
CHICAGO
OMAHA
NORTH PLATTE
SALT LAKE CITY
RENO
SAN FRANCISCO
DENVER
KANSAS CITY
ST. LOUIS
CINCINNATI
CHARLESTON
RICHMOND
WICHITA
NASHVILLE
CHARLOTTE
LOS ANGELES
ALBUQUERQUE
AMARILLO
OKLAHOMA CITY
MEMPHIS
ATLANTA
PHOENIX
EL PASO
DALLAS
LEGEND
NO OF CIRCUITS
1
10
19
JACKSONVILLE
NEW ORLEANS
SAN ANTONIO
TAMPA
MIAMI

Figure 1. Major network circuits in the United States

points that can be heard on any home radio set in the course of an evening without once changing the tuning, it will be apparent that minute-to-minute rearrangements of an established interconnecting network must be possible. For example, studios have to be changed from receiving to originating, sections of the network have to be made to transmit first in one direction and then in the other, and branches have to be connected and disconnected. These changes in the network have to be made in the few seconds elapsing between the close of one program and the start of the next on the receipt of selected cue words or sounds. Even during the course of a single program, switches or reversals may have to be made to change the originating point temporarily. To provide for these rapid changes, special operation and special switching and reversing equipment are required at many points along the network. Much of this equipment is under remote control from selected points.

The greater portion of the switching of program circuits is done at about 25 points throughout the country on the major networks. On the average more than 25,000 switching operations per month are performed at these 25 points. During the busy hours of any typical evening there may be something over 500 men on duty at all of the offices about the networks.

At points where switching requirements are simple, the switching equipment consists merely of a few keys. At the larger points where the switching requirements are complex, the switching equipment consists of elaborate relay and control arrangements. These are so designed that it is possible to set up in advance the circuit combinations required for the ensuing program period without disturbing the programs in progress. The actual switching operation takes place at the instant the monitoring attendants signal the receipt of the last of selected cues, and not before then. This type of arrangement affords a maximum of protection against error, as it is possible to check the presetting for the next switch or make a last minute change if necessary any time before the switch has been made.

Figure 4 shows a picture of such a switching arrangement in use at Omaha, Nebraska for one broadcasting company.

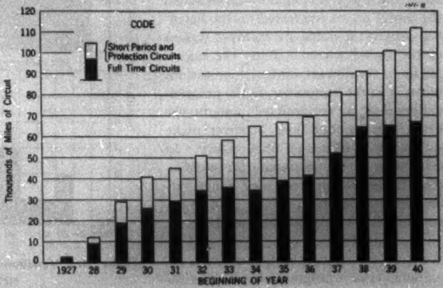

Figure 2. Growth in mileage of major network circuits

At this point 13 circuits used in various trunk and branch sections of two networks are connected to the switching equipment. These are grouped in various combinations to take care of as many as five simultaneous programs. A maximum of five cues might, therefore, be involved in a switch at this point.

The operation and maintenance of the networks are carried out by a special organization under centralized authority and trained in the application of uniform methods and procedures found by experience to be productive of best results. Transmission is monitored continuously at strategic points about the networks. In order to facilitate the activities of this group many thousands of miles of intercommunicating telephone and telegraph circuits are provided full time for their use.

A picture of a monitoring position in the program transmission office at Washington, D. C. is shown in figure 5. It will be noted that the monitoring attendant is using an individual headset. This is of a special high fidelity type and is used to avoid the confusion that would result from attempting to monitor a number of different programs simultaneously with loud-speakers. Loud-speakers are available, however, for supplementary checks of quality whenever required.

Accurate transmission measuring equipment is necessary at the various operating points about the networks to insure satisfactory transmission maintenance results.

Transmission Requirements

The general transmission requirement for a broadcasting system is that the program material be transmitted with a high degree of naturalness. Although the exact determination of the transmission characteristics which would accomplish this involves many considerations it will be assumed in this discussion that an ideal transmission system is one in which the sound waves impressed on the listeners' ears in the home are an exact replica of the sound waves striking the microphone in the distant studio. Limitations inherent in the human ear, in the program material to be transmitted, and in the usual listening conditions, however, make such ideal transmission unnecessary. In expressing the requirements for satisfactory transmission, frequency range, attenuation distortion, delay distortion, nonlinearity and noise are used as indices of quality.

Before taking up the transmission requirements of a program circuit, it is important to consider further the fundamental factors that are involved in fixing the characteristics considered desirable for the entire system. According to Harvey Fletcher,[4] the zone of audibility of the average normal human ear for pure or single frequency sounds is the area within the curve of figure 6. The abscissas represent frequency and the ordinates show the range of intensity recognizable as sound, between the lower limit or threshold of audibility and the upper limit where the sensation of pain is felt. It is seen that the extreme frequency range shown on the chart is from about 20 to 20,000 cycles per second. This range is for young people. It is considerably less for middle-aged and elderly people, and varies with individuals.

In addition to the limitation of the ear there is the fact that there is little energy present in most program material in the extremes of this range, particularly in the upper frequencies. The energy versus frequency spectra of music and other forms of program have been published elsewhere.[5] Figure 7 shows the frequency range which must be transmitted for a number of instruments, speech, and certain noises, so that competent observers cannot detect any impairment.[6] For whole orchestras, experiment has shown that the elimination of frequencies below 40 and above 15,000 cycles per second is undetectable.[4] If the upper limit of the transmitted frequencies is lowered from 15,000 cycles, the impairment is at first barely detectable but increases at an accelerating rate. When the limit is materially lower than 8,000 cycles, the loss is readily apparent to many people.

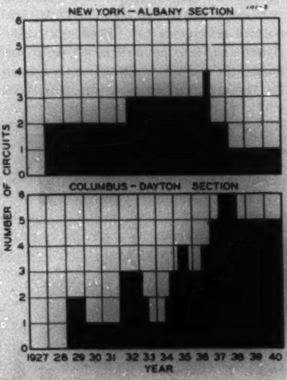

Figure 3. Variations in full-time program circuits

Figure 4. Switching panel at Omaha, Nebraska

Another important consideration is volume range—that is the difference between the maximum and minimum levels of the program. The ordinates of figure 6 show that for part of the frequency range, the ear can respond to a range of intensities of more than 120 decibels, with perhaps 100 decibels as a mean. However, the following considerations show that the volume range which the transmission system needs to accommodate is considerably narrower than the intensity range to which the ear can respond.

In the first place, the range of program volumes to which the ear can respond is much less than the range of single-frequency intensities shown by the curve. Program waves are in general very irregular in shape, and even at constant volume contain large and small peaks differing in amplitude by many decibels. The range between the volume at which the highest peaks reach the maximum instantaneous intensity which the ear can tolerate and the volume at which the smallest peaks are just above the threshold of audibility is therefore less by a number of decibels than the intensity range of the ear as measured by single frequencies.

In the second place, the volume range of the usual program material has definite limits. Measurements have shown that a large symphony orchestra produces a maximum volume range of about 70 decibels.[4] The volume range of most other types of program is considerably less than this, for example, being only about 25 to 30 decibles for dance music and as little as about 15 decibels for much of the dialogue of actors in radio drama.

In the third place, the usual listening

Figure 5. Monitoring position at Washington, D. C.

conditions impose a definite limit on the useful volume range. The loudest passages in the music of a symphony orchestra correspond to a sound level of about +95 decibels at a point, say one-third the way back in an auditorium, but most people in their homes prefer a level which is lower than this by 5 to 10 decibels. Figure 8 shows the results of an extensive survey[7] of acoustic room noise in homes. It will be noted that the average noise level is +43 decibels on the sound-level scale, and that few homes are quieter than +30 decibels even in the suburbs. The signal-to-noise range inherent in the listening conditions, and allowing nothing for the noise contributed by the transmission system or the room noise where the program is being produced, is therefore seen to be somewhere between 45 and 65 decibels. There is therefore, no advantage to the listener in providing a permissible volume range materially wider than this in the transmission system.

The above discussion applies primarily to the transmission of symphonic and similar high-grade program material. Much program material is less exacting in its requirements, but on the other hand some sound effects such as the tearing of paper require the reproduction of higher frequencies for complete naturalness.

With these broad considerations in mind, the requirements of high-quality program circuits may now be taken up. As has been noted, the program network is but one part of the over-all broadcasting system which, in addition, includes microphones and studio equipment, radio transmitters, and the home receivers with their loud-speakers. It may be taken as a goal for the program networks that their transmission be nearly enough distortionless so that the over-all performance in regard to naturalness of reproduction will not be limited by them.

To meet such a requirement for short program circuits having only one or two sections is not difficult technically, and does not in general require costly types of plant. However, the vast country-wide program networks are made up of many sections of circuits in tandem, which as mentioned before may total in some cases as much as 7,000 miles. This makes it necessary to design and operate the individual circuits to very close limits so that the cumulative discrepancies in the whole network will not exceed tolerable values; and to consider carefully the types of plant employed lest by virtue of sheer numbers of units involved, the total cost be out of line with the over-all grade of service being given the listener. These two conflicting factors are important ones in the consideration of transmission requirements for networks. The determination of the practical working characteristics of program networks involves a consideration not only of the physical and cost factors discussed above but also of such other factors as cost of studios, broadcast transmitters and receivers, and the limitations of the frequency allocations of broadcast stations.

From the standpoint of frequency band the consideration of all factors has resulted in the major present-day program networks being set up to transmit a frequency band with an upper limit of about 5,000 cycles. All program facilities installed in the last ten years or so, however, have been designed to be adaptable to the future transmission of frequencies up to 8,000 cycles. Operation on an 8,000-cycle basis, however, requires the release of additional frequency space now occupied by other services in much of the plant and a general readjustment of the program-circuit characteristics. In 1933, experimental wire circuits were set up between Philadelphia and Washington to transmit frequency bands up to 15,000 cycles. These were employed in a demonstration of stereophonic transmission and reproduction of music.[4] Studio-transmitter loops have been provided to transmit wider frequency bands than the 5,000 cycles currently provided on the nation-wide networks. At the present time, many of the studio-transmitter loops are being set up to transmit bands up to 15,000 cycles. A demonstration will be given at the close of the paper of the transmission of programs over cable circuits about 1,200 miles in length with frequency bands extending to 15,000, 8,000, and 5,000 cycles. The 5,000-cycle circuit is of the type in present commercial use. The 8,000-cycle circuit is of a type to which much of the present program plant can readily be modified. The 15,000-cycle circuit consists of a standard carrier system to which has been added program terminal equipment now under development.

In the consideration of transmission requirements for program circuits other than nominal frequency band, the varia-

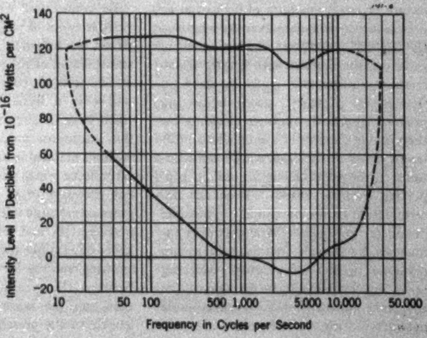

Figure 6. Limits of audible sound

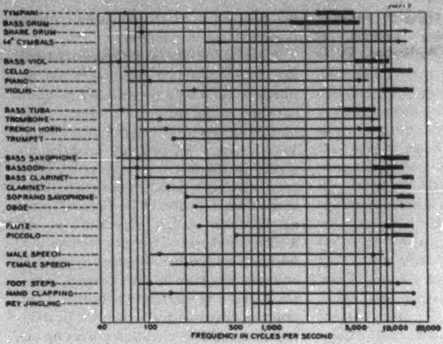

Figure 7. Audible frequency ranges of music, speech, and noise

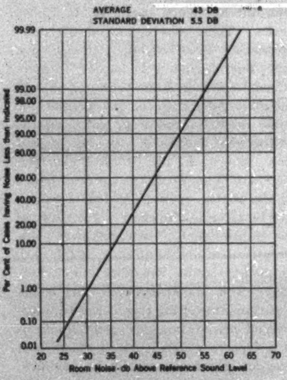

Figure 8. Residence room noise with radio sets silent

tion in performance with length and type of circuit is important, since the factors tending to impair transmission are in the nature of small amounts of distortion or noise which accumulate over the length of the circuit. If these effects varied in some definite manner with length, transmission requirements could be fixed on that basis. However, good engineering practice frequently requires choosing for the various sections of a long circuit, different types of facilities whose contributions to the total effects are not in proportion to their length. Even the determination of the maximum permissible distortion and noise on a circuit is influenced by outside factors such as are involved in the broadcasters weighing operating flexibility and cost against the frequency of occurrence of unfavorable network routings and the number of stations affected. For example, in order to secure operating flexibility with a minimum of total network mileage, most of the networks employ the so-called "round robin" principle for a part of the network. In this arrangement the circuit follows a route from station to station forming a continuous loop which returns to its starting point. This naturally results in increased circuit mileage between the program source and the more distant listeners with an attendant increase in undesired transmission effects. For these reasons no exact or specific transmission requirements can be stated for even the over-all performance of program transmission service.

Volume Range

The permissible volume range for a program circuit is determined by the maximum volume which can be transmitted as limited by nonlinear distortion or cross talk, and the minimum volume which can be transmitted without impairment from the noise present on the circuit.

In connection with their design the various types of program circuit are subjected to listening tests in which the transmission of program over a long loop of the circuit is compared with transmission over a local distortionless circuit. Each type of circuit thus is rated as to the maximum volume it can transmit without noticeable distortion. The highest volume which can be permitted without excessive cross talk into other program or message telephone circuits is also investigated, and whichever limit is the lower determines the maximum allowable working volume for service. The range between the maximum permissible volume and the noise level on very long lengths of the present program circuits is about 45 or 50 decibels, except under some conditions on certain open-wire sections. On the individual links making up the long circuit, the range is 10 or 20 decibels greater than this.

Attenuation and Delay Distortion

Another important consideration is the amount of attenuation and delay (or phase) distortion to be permitted within the transmitted frequency band. It is the practice to equip program circuits with adjustable attentuation equalizers. By means of these once the desired frequency band has been chosen the deviation in attenuation at any frequency within that band, compared with that at 1,000 cycles, can be adjusted within close limits. On very long circuits, however, experience has shown that even with automatic regulating features and careful operation residual variations which may amount to several decibels may develop as a result of changing temperature and other conditions. These variations are kept within tolerable limits by readjustment of the equalizers from time to time.

Associated with the attenuation distortion is another effect detrimental to program quality, namely, differences in time of transmission for different frequency components of the signal. In practice circuits tend to have a lower velocity of transmission near the edges of the frequency band than in the middle portions. This results in frequency components near the edges being delayed as compared to the middle portions of the band. This difference in time of transmission is called delay distortion of the circuit. Careful listening tests have shown that it becomes noticeable if, at the highest transmitted frequency, the delay is more than eight milliseconds greater than at 1,000 cycles, and if, at 100 cycles, it is more than about 15 milliseconds greater than at 1,000 cycles. It is controlled by careful attention to the design of loading systems, amplifiers, repeating coils, and all other elements of the circuit. Since such small amounts of over-all delay distortion are detectable and since networks frequently have 100 or more amplifiers in tandem between an originating point and the broadcasting stations on the more distant portions of the networks, it is necessary that the delay distortion of all individual components of a network be held to exceedingly close limits. Accumulations of residual delay distortion which cannot be entirely eliminated in design are reduced by the use of delay equalizers along the circuits when they are set up.

Prime-Mover Speed Governors for Interconnected Systems

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Synopsis: System operating requirements, from the standpoint of frequency and tie-line loading, are continually becoming more rigorous and receiving more widespread attention. Supplementary controls have been developed to assist in the solution of these problems—but the speed governors of the prime movers still constitute the backbone of system control.

This paper outlines the general problems encountered; gives definitions of terms for both steam and hydro governors and discusses the performance characteristics of these two general classes of prime movers. As a result of discussions with several operating groups certain definite conclusions were reached for prime-mover governor characteristics on the larger systems, viz., dead band should be as small as practicable; uniform incremental regulation desirable; adjustable regulation not necessary; similar rates of response not necessary and accurate response to supplementary control desirable. These conclusions are supported by the analytical work presented in the companion paper.[1]

A summary of the reasoning supporting these conclusions under the subjects of stability, dead band, regulation, response, short circuits, and tie-line swings is included.

Introduction

DURING the past several years the interconnected power system has come into being and has expanded rapidly to the point where some of these systems now comprise several million kilovolt-amperes of connected capacity. Many problems have appeared and of these, two important and closely associated ones, speed and load control, have continued to receive increased attention. In most cases large portions of these systems are tied together by tie lines of relatively small capacity and as a result power flow across the ties must be carefully controlled.

Frequency control is being widely used in varying degrees of refinement and automatic tie-line control is being used in more and more locations.[1,12,13,14] The control associated with these two problems is usually applied in the form of relatively slow supplementary adjustments to the normal action of the basic speed governors, which still constitute the backbone of system control. It is logical, then, as system operation becomes more refined and the requirements more rigorous, that more attention[2,3,4] be given to the characteristics of the speed governing mechanisms. During this same period special instruments[8,9,10] for accurately measuring small changes in system frequency have been developed and these instruments have been used to advantage by several groups in analyzing governor performance.

The manufacturers of power generating equipment endeavor to equip their prime movers with suitable governing equipment.[5,6] Also, in the analysis of power system performance it is essential to appreciate the influence of all associated apparatus, including speed governors[11,15] on system performance. On this basis, the general problems associated with speed governing were discussed with several operating groups, and with the evidence procured so far, we have reached certain definite conclusions as to what characteristics prime mover governors should have to meet the operating requirements on the larger power systems.

Definition of Terms

There is a surprising lack of a common language for this subject and thus, before proceeding further a brief definition of terminology used *in this paper* will be in order. Two sections in the ASME—Power Test Codes for Governors and Committee on Industrial Instruments and Regulators—each have a set of definitions and this is no attempt to

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The authors wish to express appreciation to the representatives of the operating companies who contributed during the several discussions on this subject and to acknowledge the assistance of Messrs. C. Concordia, S. B. Crary, and E. E. Parker of the General Electric Company in the preparation of this paper.

[1] For all numbered references, see list at end of paper.

Conclusion

From this discussion it is seen that the program networks are comprised of many parts, each of which must meet exacting requirements in order that over-all results will be satisfactory. It is seen that equally important with transmission are the requirements for plant flexibility, adequate reserves, uniform practices, and centralized supervision of the networks.

The features discussed have been those found desirable for present-day network service. As indicated earlier, consideration of the needs of the future as well as those of the present is an essential feature of the design and engineering of the plant for program-network service. As a result of having done this it will be possible to provide with present plant, and with new plant currently being installed, adequate network facilities as the broadcasting art develops toward higher standards of performance. With the past experience as a guide, it appears that there should be no fundamental difficulty in meeting all reasonable requirements, always remembering that in the long run, requirements and costs bear definite relations to each other.

References

1. TELEPHONE CIRCUITS FOR PROGRAM TRANSMISSION, F. A. Cowan. AIEE TRANSACTIONS, volume 48, number 3, July 1929, pages 1045–9.

2. LONG DISTANCE CABLE CIRCUIT FOR PROGRAM TRANSMISSION, A. B. Clark and C. W. Green. *Bell System Technical Journal*, volume 9, number 3, July 1930, pages 567–94.

3. WIDE BAND OPEN WIRE PROGRAM SYSTEM, H. S. Hamilton. ELECTRICAL ENGINEERING, volume 53, number 4, April 1934, pages 550–62.

4. SYMPOSIUM ON WIRE TRANSMISSION OF SYMPHONIC MUSIC AND ITS REPRODUCTION IN AUDITORY PERSPECTIVE, ELECTRICAL ENGINEERING, volume 53, number 1, January 1934, pages 9–32, 214–19:

Basic Requirements, Harvey Fletcher.

Physical Factors, J. C. Steinberg and W. B. Snow.

Loud Speakers and Microphones, E. C. Wente and A. L. Thuras.

Amplifiers, E. O. Scriven.

Transmission Lines, H. A. Affel, R. W. Chesnut, and R. N. Mills.

System Adaptation, E. H. Bedell and Iden Kerney.

5. ABSOLUTE AMPLITUDE AND SPECTRA OF CERTAIN MUSICAL INSTRUMENTS AND ORCHESTRAS, L. J. Sivian, H. K. Dunn, and S. D. White. *Journal Acoustical Society of America*, volume II, January 1931, pages 330–71.

6. AUDIBLE FREQUENCY RANGE OF MUSIC, SPEECH AND NOISE, W. B. Snow. *Journal Acoustical Society of America*, volume III, July 1934, pages 155–6.

7. Two papers: ROOM NOISE AT TELEPHONE LOCATIONS, D. F. Seacord. ELECTRICAL ENGINEERING, volume 58, number 6, June 1939, pages 255–7, and volume 59, number 6, June 1940, pages 232–4.

arbitrate any differences, or establish new terms. A final decision on terminology will be reached through mutual agreement between representatives of manufacturers and operating groups both in the AIEE and ASME as well as other contributing groups.

Speed governor shall include the speed responsive element which positions a member in terms of speed. In figure 1 this is schematically represented by a conventional spring-restrained centrifugal flyball mechanism, but an oil pump working against a spring loaded diaphragm or a resonant circuit producing a torque motor effect will perform the same function.

Control mechanism shall include all linkages, pilot valves, dashpots, operating cylinders, cam shaft, cams and valves necessary to control the prime mover input in terms of speed governor position. In figure 1 a single pilot valve is shown between the speed governor and the operating cylinder. As the required pilot-valve size increases, a larger and more powerful speed governor is required. Instead of this, a relay mechanism or amplifier consisting of an intermediate pilot valve and piston is often interposed between the speed governor and the main pilot valve.

Governing mechanism shall include both the speed governor and control mechanism all as shown schematically in figure 1.

Over-all regulation is the sustained per cent change in speed of a prime mover for full load change from an initial speed of 100 per cent.

The regulation of the mechanism shown in figure 1 may be changed by varying the ratio of linkages, but there is always a lower limit for regulation below which stable operation cannot be obtained. This limit is determined by system conditions as well as by governor mechanism characteristics—but other conditions being similar, faster mechanisms permit smaller regulation with stability.*

Operating piston times for steam units are in the order of one second or less—whereas servo-motor times in hydro units are four seconds or greater to satisfy hydraulic conditions. Thus, if this simple mechanism in figure 1 were employed in a hydro governor, the regulation required for stability would be so large that other operating requirements would not be satisfied.

The manufacturers of hydro governors have solved this problem very neatly by employing droop correction as shown in figure 2. For sudden changes the dashpot acts as a rigid connection—giving broad transient regulation for stability, but after some time delay, the action of the dashpot and springs produces a much smaller steady-state regulation. In the figure, zero steady-state regulation is shown, but for paralleling purposes, the necessary amount of over-all steady-state regulation is obtained by positioning the spring supports in terms of operating cylinder position. Thus, droop (the remaining droop after the correction has operated) and over-all regulation are synonymous.

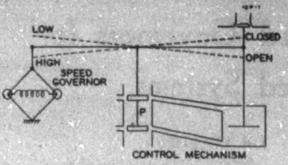

Figure 1. Schematic diagram of speed governing mechanism

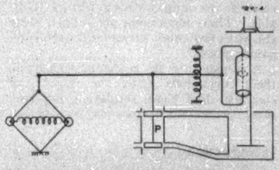

Figure 2. Schematic diagram of speed governing mechanism with compensating dashpot for droop correction, arranged for zero droop

Incremental regulation is the slope of the speed-kilowatt curve passing through 100 per cent speed at the load in question, expressed in per cent of rated speed.

Unfortunately the uniform characteristic shown in figure 4 is not so easy to obtain and requires proper co-ordination in the design of both the speed-position characteristic of the governor and the flow-travel curve of the control mechanism. In the case of a hydro unit, the increase in flow for increased gate openings tends to fall off at the larger gate openings. In this respect most steam turbines have several inlet valves, each having its flow-travel characteristic. A comparison is shown in figure 3 where the dotted curve applies to a hydro unit and the solid curve to a four-valve steam unit.

Mechanisms as in figure 2 have both transient and steady-state incremental regulation. As far as governor influence is concerned, load division between units on a system is determined initially by transient incremental regulation and subsequently by the steady-state incremental regulation. Since most load changes on the larger systems are relatively slow, steady-state incremental regulation is the significant characteristic.

Dead band is the magnitude of the sustained speed change within which change of prime mover input will not be produced. Figure 4 shows two governors with equal dead band, one having broad and the other narrow over-all regulation. It may be seen that the governor having broader regulation will hold its load within a smaller range for a constant frequency and for the same frequency dead band, than the governor with narrow regulation. This dead band is a measure of the sensitivity of the mechanism. A method of simultaneously recording speed on one axis and mechanism movement on the other axis has been developed using apparatus similar to that described in reference 8. When proper interpretation of such records can be made, this method should prove useful in determining the over-all dead band of governing mechanisms while in service.

Supplementary adjustment—Any means employed to adjust externally the prime mover input.

Conclusions

The following conclusions, which apply to the requirements of both steam and hydro units, have been arrived at after having considered operating requirements, nature of load changes on large systems, reliability of operation, simplicity in mechanisms, and ease of maintenance.

1. Dead band should be as small as practicable.
2. Uniform incremental regulation is desirable but not essential.
3. Adjustable regulation is not necessary.
4. Similarity of rates of response is not necessary.
5. Accurate response for supplementary adjustment is desirable.
6. Indications of all adjustments are desirable.

A brief review of the reasons supporting the above conclusions follows:

STABILITY

One of the first and axiomatic requirements is that the mechanism be stable** at all times. A stable governor is one which does not by itself produce or excite an oscillation. A governor with a high degree of, or with good stability, is effective in quickly reducing the magnitude of an oscillation produced by any disturbance. Small dead band, broad incremental regulation, both transient and steady-state, and small time lags all contribute to stability and are largely under the control of the designer. Generally, if the unit is stable for synchronizing, it will be still more stable when connected to a system due to the synchronizing and damping torque action provided by the system. For the same reason a governing mechanism which is poor when operated alone may give no trouble when it becomes part of a system already equipped with good governors.*

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*Figure 3, reference 7.

**Figures 1–12 inclusive, reference 7.

At the same time a single machine or a number of machines behaving as a group is influenced to a large extent by the damping characteristic of the load, i.e., the change of load per unit of speed change, other conditions being constant.**

Dead Band

Small dead band (small over-all insensitivity) contributes to stability, maintains accurate loads at fixed frequency and gives accurate response to frequency variations. (See figure 4.) Study of this characteristic has convinced us that it is the over-all dead band, rather than dead band of some particular portion of the mechanism, that is the significant factor.†

In a few instances there has been evidence to show that if insensitive governors control the major portion of the system, speed variations may be aggravated by either unnecessary movement of the mechanisms or by insufficient movement caused by the dead band allowing a greater change in frequency before the proper input correction is obtained. These conditions can be overcome by improved design in available mechanisms.

Present evidence indicates that over-all dead band should be made as small as possible just so long as the effort is not carried to the point of requiring undue expense in manufacture, and of requiring undue maintenance in the field to keep that degree of performance. Operating groups can, by tests and analyses, assist the governor designers in establishing the proper economic limit.

Incremental Regulation

The incremental regulation in the normal operating range should be as uniform as is consistent with economical design of the mechanism and efficient turbine operation. By making the incremental regulation as uniform as possible and as large as necessary, stability margin at all valve points is assured and more uniform response to supplementary adjustment is obtained. If the improved uniformity is obtained entirely through mechanism design, more powerful operating cylinders may be necessary or more complicated mechanisms may be required. If the improved uniformity is obtained by increased overlapping in the valve openings, the turbine efficiency is reduced at the valve openings.

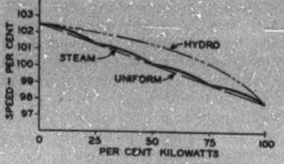

Figure 3. Typical speed-load characteristics for steam and hydro units. Slope of curves at any point is incremental regulation

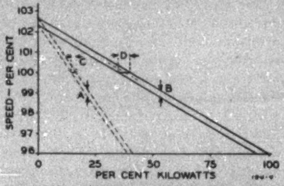

Figure 4. Speed-load curves showing dead band. A and B—Possible speed variation within dead band. C and D—Possible load variation within dead band

Incremental regulation should be as broad as is consistent with reasonable system speed variations. Broad regulation improves stability, contributes to most rapid settling down after a disturbance, and minimizes any tendency for hunting which might be caused by dead band.

Regulation

For steam turbines the maximum over-all regulation which can be tolerated and still meet the present overspeed and emergency trip requirements is about six per cent, and the minimum over-all regulation for stability is in the order of two or three per cent without droop correction. For this reason it is believed that some reasonable over-all regulation in this range can be used for most steam turbine mechanisms, thus avoiding the added complication of adjustable regulation. An advantage in operating on small over-all regulation might be to reduce slightly the duty on the supplementary corrective devices now employed, but it appears as though there is a good chance of creating a wider frequency fringe by attempting to operate the units of an entire system on small over-all regulation, because of the reduced degree of stability obtained.‡

As previously mentioned, hydro governors ordinarily use a large transient regulation in order to obtain stability and droop correction to obtain reasonably small steady-state regulation. Adjustment of this droop is readily obtained. Where adjustable regulation for steam turbines is considered desirable, it may be provided in two ways; first, with a simple mechanism similar to figure 1 where the linkages are adjustable over the two to six per cent range; second, by adding speed droop correction, figure 2, which permits over-all regulations as low as zero, on one machine.

For smaller systems the per cent load changes relative to system capacity are usually larger and maintaining speed is more of a problem. In these cases governing mechanisms which can, if desired, be operated at zero steady-state regulation have a distinct field of usefulness. It is believed, however, that while mechanisms are being built to meet these special conditions on smaller systems, the conditions on the larger systems do not demand such flexibility in speed governing mechanisms.

Rate of Response

The present rates of response of steam and hydro governors are widely different, and this results in a corresponding redistribution of power flow subsequent to load changes. Usually, the sudden load changes on most large systems are relatively small and the major load changes occur at a rate which can be followed by even the slowest mechanisms. In an interconnected system, all active governors respond to all load changes regardless of location in proportion to their ability, and in principle it is the function of supplementary control (manual or automatic, or both) finally to allocate a load change in any area to generation in the same area. Any redistribution of load due to different governor characteristics is so masked by real load changes that it is not considered important to modify designs in order to match characteristics.

Matching characteristics of steam and hydro units would mean slowing down steam turbine governor response to approach that dictated by hydraulic limitations on hydro units, whereas actually in the interest of limiting overspeeds and improving stability it appears desirable to make steam governor response as rapid as is possible without complicating the mechanism. However, in this regard there is no necessity of attempting to make governors which are responsive to rate of change of speed, e.g., inertia governors, since overspeed and stability requirements can both be adequately met with a pure speed responsive mechanism.

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*Figure 11, reference 7.

**Figures 3 and 4, reference 7.

†Table 1, reference 7.

‡Figures 3 and 14, reference 7.

Terminal frequency has been suggested to provide speed indication for the governor. This method has no advantages so far as performance is concerned over using shaft speed, but it does have at least two distinct disadvantages, both based on the probability that during out-of-step conditions, it may not indicate the actual speed of the unit. As a result, first, in the case of a turbine generator, a separate pre-emergency device to prevent tripping the emergency on overspeed would be required; second, the unit is not intelligently helped to pull back into step. The use of terminal drive has been generally limited to small hydro machines in locations where these disadvantages are not important.

System Short Circuits

In transient stability calculations of systems (studies of the ability of generators to remain in synchronism after a disturbance), governor performance is usually neglected and this assumption gives only slightly pessimistic results for the amount of power which can be transmitted through a short-circuit disturbance.

Fast switching is being commonly used and with breaker speeds now commercially available, stability during short circuits at practical loadings for transmission lines is possible, so that very little gain can be expected through improved governor performance during this short interval. The governing mechanism will do its share if it assists in damping out the subsequent synchronous oscillation.

Out-of-Step Conditions

Out-of-step operation has occurred usually as a result of too slow clearing of a severe fault. When the fault is finally cleared, the generator or station is running faster than the rest of the system. If hydro units are involved, large overspeeds may occur, and system relays will probably trip on overcurrents before the units will resynchronize themselves, even if supplementary means are employed.

Steam units, on the other hand, have faster governing mechanisms, much better induction torque characteristics due to the solid rotor generator construction and a stronger electrical tie to the system and load. Therefore, the units settle down soon at a low overspeed. This situation alone gives a good opportunity for self-resynchronizing. As a supplementary measure, increasing the field strength is probably one of the easiest and best assurances of resynchronizing, although some have advocated reducing the input torque.[18]

Tie-Line Load Swings

The normal tie-line power fluctuations due to sudden load changes cannot be appreciably reduced by governor operation, regardless of rate of response, since the initial power flow is largely determined by the system and tie-line impedances, later modified by inertias and finally determined by governor action. Gradual load changes will be divided according to composite governor characteristics of various portions of the system and later modified by supplementary control.

In the steady-state condition most tie lines have a sustained power oscillation at the natural frequency of the interconnected groups. These power oscillations are usually a small percentage of the tie-line capacities and alone represent no problem. With these power oscillations there is associated a speed oscillation of small magnitude. Experience and our studies[7] indicate that the characteristic of these swings may be influenced by governor action, but usually should be within reasonably satisfactory limits with ordinary governor performance. In those cases where the governors tend to excite unnecessary power oscillations, proper adjustment of the governor characteristics, for example, broadening the regulation, should eliminate the tendency.

Adjustment and Indications

Supplementary adjustment is provided on all speed governors and may be used for either manual or automatic correction. This adjustment should be arranged so that reasonably uniform response is assured throughout the operating range.

Gate position indication has been available on hydro units for some time and indication of operating piston position on steam units, with valve opening points marked, has been suggested. Gate indication on a hydro unit is often more useful than a wattmeter, particularly on a variable head installation and on a steam unit it is desirable to avoid operation at partial valve openings. With the more recent turbines a recorder is being provided which records speed of the unit prior to synchronizing and which records cam shaft position after synchronizing. This arrangement gives a complete record of the manner in which the machine has been brought up to speed and loaded.

Load-limiting features are furnished on most turbines and are still being used to some extent. This feature is particularly useful in maintaining a fixed load on a machine connected to a system where the frequency is allowed to vary through wide limits. On most large systems, today, frequency is normally held within a 0.1-cycle band by means of supplementary control. Any machine equipped with a five per cent speed governor will change load by only three per cent in response to such a frequency change and most steam operators agree that this is essentially fixed load, for machines carrying a reasonable load. In general, the frequency fringe on a system is reduced by increasing the percentage of machines on governor control, and to this end load limit as such may be advantageously discarded except as a back-up measure to prevent severe overloads on boilers and associated equipment during system emergencies.

Summary

The conclusions which have already been enumerated and discussed apply to governing mechanisms of prime movers on the larger systems. For smaller systems governing mechanisms which can, if desired, be operated at zero steady-state regulation, have a distinct field of usefulness. For governing mechanisms on the large systems, emphasis should be placed on obtaining uniform incremental regulation, accurate response for supplementary control, and small dead band. The experience and judgment of the operating groups will be of assistance in determining the economic limit, beyond which the design cannot be justifiably carried.

Bibliography

1. Frequency, Time, and Tie-Line Load Control on Interconnected Systems, **Philip Sporn and V. M. Marquis.** *Electrical World*, March 12, 1932, page 495 and April 2, 1932, page 618.

2. Experiences in System Speed Regulation, **Otto Holden.** ASME *Transactions*, volume 62, April 1940, number 3, page 237.

3. The Field of System Governing, **Albion Davis.** ASME *Transactions*, volume 62, April 1940, number 3, page 207.

4. Regulation of System Load and Frequency, **Herbert Estrada and H. A. Dryer.** ASME *Transactions*, volume 62, April 1940, number 3, page 221.

5. Constant System Speed and the Steam-Turbine Governor, **A. F. Schwendner.** ASME *Transactions*, volume 62, April 1940, number 3, page 199.

6. Steam-Turbine Governors, **R. J. Caughey.** ASME *Transactions*, volume 62, April 1940, number 3, page 191.

7. Effect of Prime-Mover Speed Governor Characteristics on Power-Frequency Variations and Tie-Line Power Swings, **C. Concordia,**

Analysis of Short-Circuit Oscillograms

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Summary

METHODS of analyzing the oscillograms of short circuit currents of synchronous machines, in accordance with the two reaction theory, are presented in this paper. Differences between the results found here and the more generally accepted equations, based on symmetrical components, are pointed out.

Introduction

The analysis of short circuit oscillograms to obtain synchronous machine reactances is usually made by means of the equations developed from symmetrical component theories. These equations have been employed by previous writers on the subject and have been incorporated in published standards. Because symmetrical component methods usually start from the assumption of sinusoidal currents of single frequency in the armature, and because it is known that short circuit currents depart appreciably from the simple sinusoidal form, it is desirable to know the effect of removing the assumption of sinusoidal currents on the equations used in the analysis of short circuit oscillograms.

The two reaction theory of synchronous machines, as developed by Doherty and Nickle[1,2], permits a calculation of the harmonics of armature current. The method has been widely used to determine operating characteristics of machines; but the application to the calculation of machine quantities from oscillograms of short circuit currents is not well known. It is the purpose of this paper to show how the two reaction theory applies in such a determination of reactances.

The equations of armature current as previously published are restated here. From these equations, expressions for the envelopes of the currents are written. These envelopes have certain slight limitations which will be mentioned later, but, except in rare cases, they may be drawn readily on the oscillogram. The equations of the envelopes permit a simple statement of the relations between the voltages, currents and reactances.

The two reaction theory has been checked by test within the limits of the assumptions under which it was derived. The results herein described have about the same limitations and the same degree of applicability. While it is true that the presence of saturation, resistance and nonsinusoidal windings cause appreciable departures from the case of the ideal machine here treated, knowledge of the characteristics of the ideal machine is basal to an understanding of the effects of these complicating factors.

Three-Phase Short Circuit

Doherty and Nickle[2] give the equation of the armature current after a sudden three phase short circuit at the terminals of the machine. It may be shown that the current in one phase, when the effect of an amortisseur is included, is defined by the following equation.

$$i=\frac{e_0F_3}{x_d''}\cos(t-\alpha)-\frac{e_0(x_d''+x_q'')}{2x_d''x_q''}A_3\cos\alpha+\frac{e_0(x_d''-x_q'')}{2x_d''x_q''}A_3\cos(2t-\alpha)\text{ times normal peak current}\quad(1)$$

where

i = per unit current at any time t.

e_0 = per unit voltage before fault (peak value).

t = per unit time, $t=0$ at time of fault. One per unit time is required for the machine to progress through one electrical radian.

α = electrical angle between centerline of armature and centerline of field at the time of the fault.

x_d = per unit direct axis synchronous reactance.

x_d' = per unit direct axis transient reactance.

x_d'' = per unit direct axis subtransient reactance.

x_q'' = per unit quadrature axis subtransient reactance.

σ_a = decrement factor of the armature.

σ_f' = decrement factor of the field, transient.

σ_f'' = decrement factor of the field, subtransient.

$A_3=\epsilon^{-\sigma_a t}$

$$F_3=x_d''\left[\frac{1}{x_d}+\left(\frac{1}{x_d'}-\frac{1}{x_d}\right)\epsilon^{-\sigma_f't}+\left(\frac{1}{x_d''}-\frac{1}{x_d'}\right)\epsilon^{-\sigma_f''t}\right]$$

Consider the case, $\alpha=0$, which gives a fully offset wave.

Subject to conditions stated later, two equations representing the envelope may be written as follows:

$$\text{Envelope line number 1}=\frac{-F_3-A_3}{x_d''}e_0\quad(2)$$

$$\text{Envelope line number 2}=\frac{F_3-A_3}{x_d''}e_0\quad(3)$$

It may readily be verified that the slopes and values of equation 2 at $t=\pi+2n\pi$ are identical with the slopes and values of equation 1 at $t=\pi+2n\pi$. Similarly, the slopes and values of equations 3 and 1 are equal at $t=0+2n\pi$. Therefore, unless there is more than one maximum per half cycle, they are the envelopes as usually defined. (This latter condition will be discussed later.)

The difference obtained by subtracting equation 2 from equation 3 gives the width of the envelope,

$$\text{width of envelope}=\frac{2F_3e_0}{x_d''}\text{ (in terms of normal peak current)}\quad(4)$$

The d-c component of the current is given by the second term of equation 1. For a fully offset wave,

$$\text{d-c component}=\frac{A_3e_0(x_d''+x_q'')}{2x_d''x_q''}\quad(5)$$

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[1] For all numbered references, see list at end of paper.

S. B. Crary, and E. E. Parker. AIEE Transactions, volume 60, 1941.

8. Oscillograph Analyzes Governor Performance, J. E. Allen. *Power*, November 1934, page 610.

9. Photo-Electric Recorder, C. W. LaPierre. AIEE Transactions, volume 51, 1932, page 221.

10. Applications of the Photo-Electric Recorder, W. L. Carson. *General Electric Review* volume 39, April 1936, number 4, page 189.

11. Overvoltage on Transmission Systems Caused by Dropping Load, E. J. Burnham. AIEE Transactions, volume 51, 1932, page 105.

12. Automatic Frequency Control, Robert Brandt. *Electrical World*, February 1923.

13. Phase Angle Control of System Interconnections, R. E. Pierce and B. W. Hamilton. AIEE Transactions, volume 58, 1939 (March section), pages 83–91.

14. Tie-Line Control With Frequency Bias, E. L. White. *Electrical West*, volume 80, June 1938, page 132.

15. Load Dump Tests Made on Colfax Turbines, T. E. Purcell. *Power*, volume 69, January and June 1929, pages 590 and 627.

16. Synchronization of Machines in Disturbances of Stability, P. S. Shdanow. Conférence Internationale des Grands Reseaux Electriques, volume 1, 1935, page 110.

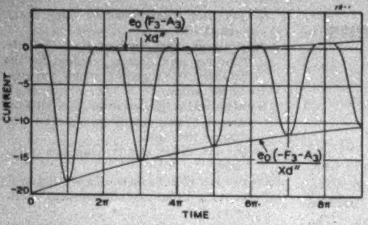

Figure 1. Three-phase fault current and envelope, equations 1, 2, 3

$\alpha=0$
$x_d''=0.10$ per unit
$x_q''/x_d''=3.0$
$x_d'=0.15$ per unit
$x_d=1.00$ per unit
$e_0=1.0$ per unit
$\sigma_a=0.0265$ per unit
$\sigma_f''=0.106$ per unit
$\sigma_f'=0.0076$ per unit

This component is often considered to be traced by the midpoint of the envelope lines. That this is not the case is evidenced by averaging equations 2 and 3, which determines the midpoint:

$$\text{Median line between envelope lines} = \frac{A_3 e_0}{x_d''} \tag{6}$$

A study of equation 4, together with the expression given for F directly after equation 1, shows that the envelope width at $t=0$, when extrapolated from the subtransient region, is $2e_0/x_d''$; when extrapolated from the transient region it is $2e_0/x_d'$. These relations have been shown only for the case of $\alpha=0$, giving a fully displaced wave. They also hold at $\alpha=180$, but not for a fault at any other point in the cycle, unless $x_d''=x_q''$.

If the rates of change of the decrement factors, A_3 and F_3, in equation 1 (for $\alpha=0$) are assumed to be zero (in commercial machines they are small compared to the rates of change of the sinusoidal terms) the conditions for maxima and minima of current are:

$$\sin t=0 \tag{7}$$

$$\cos t=\frac{x_q''}{x_q''-x_d''}\,\frac{F_3}{2A_3} \tag{8}$$

The roots of equation 7, substituted into equation 1, with $\alpha=0$, yield the envelope lines, equations 2 and 3. Equation 2 is derived by setting $t=\pi, 3\pi, 5\pi$, etc.; equation 3, by setting $t=0, 2\pi, 4\pi$, etc. These envelopes have previously been shown to pass through the maximum and minimum points (at 0, 180, 360 deg., etc.) of the current wave, and to have the same rates of change as the current at these points.

Roots of equation 8 exist only when

$$|2A_3(x_q''-x_d'')|>|F_3 x_q''| \tag{9}$$

It is apparent that when $x_q''-x_d''$ is small, only one maximum exists per half cycle. Under such conditions the envelope may be drawn on the oscillogram in the usual way and equations 2 and 3 will represent the envelope.

When the inequality (9) is satisfied, equation 8 can be solved and additional maxima are introduced. However, the envelope represented by equations 2 and 3 can still be drawn, and the relations derived from these equations still hold. The manner of constructing the envelope for such conditions is indicated in figure 1.

Calculation of peak currents may be made from the above equations. Consider the first negative peak of figure 1. It is apparent that the maximum current occurs just previous to the point of tangency with the envelope (the 180° point) and that for most purposes this peak value may be approximated by the value at 180°. From equation 1 or 2, this value is

$$\text{peak current}=e_0\,\frac{A_3+F_3}{x_d''}\ \text{times normal peak current} \tag{10}$$

Disregarding the small increase in this value mentioned above, the value of peak current given in equation 10 can be exceeded only if x_d'' is appreciably greater than x_q''. Under this condition the sign of the second harmonic is reversed and the double maximum which is shown in figure 1, near the 360° point, occurs near the 180° point. An examination of figures 1, 2, and 3 shows how the change of sign on the second harmonic component (see equation 1) changes the position of the double maximum. The condition, $x_d''>x_q''$, is uncommon in commercial machines and need not be considered further.

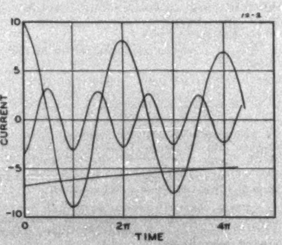

Figure 2. Components of figure 1

The above discussion and equations are valid only for the case $\alpha=0$ (or 180°), which results in a fully offset wave. For a fault at any other point in the cycle, no similar simple relations have been written. However, the extent to which the wave shape changes with change of α is indicated in figure 4. The envelope lines for the case, $x_q''=x_d''$, (where envelope width is $2F_3/x_d''$) are shown to indicate the amount of variation from the simple case which may be expected.

Single-Phase Case

Nickle, Pierce and Henderson[3] give the equation of armature current for single phase short circuit at the machine terminals:

$$i=\frac{3e_0}{k}\,\frac{F_1\cos(t+\alpha)-A_1}{H} \tag{11}$$

or,

$$i=\frac{3e_0}{2k}\,\frac{2F_1O}{x_d''+x_2}-\frac{A_1}{x_2}E \tag{12}$$

where

$k=1$ for L-N, 3 for L-L fault

$H=(x_d''+x_q'')+(x_d''-x_q'')\cos 2(t+\alpha)$

$A_1=\epsilon^{-at}\cos\alpha$

$$F_1=\frac{x_d''+x_2}{x_d+x_2}+\frac{(x_d''+x_2)(x_d-x_d')}{(x_d+x_2)(x_d'+x_2)}\epsilon^{-\sigma_f't}+\frac{x_d'-x_d''}{x_d'+x_2}\epsilon^{-\sigma_f''t}$$

$O=\cos(t+\alpha)+b\cos 3(t+\alpha)+b^2\times\cos 5(t+\alpha)+\ldots$

$E=1+2b\cos 2(t+\alpha)+2b^2\cos 4(t+\alpha)+\ldots$

$$b=\frac{x_2-x_d''}{x_2+x_d''}=\frac{\sqrt{x_q''}-\sqrt{x_d''}}{\sqrt{x_q''}+\sqrt{x_d''}}$$

$x_2=\sqrt{x_d''x_q''}$

For a line-to-neutral fault use $x_d''+\frac{x_0}{2}$ for x_d'' etc.

Similar to the three phase case, the equation of the envelope of the single phase fault current is

$$\text{Envelope line number 1}=\frac{3e_0}{k}\left[\frac{-F_1-A_1}{2x_d''}\right] \tag{13}$$

$$\text{Envelope line number 2}=\left[\frac{3e_0}{k}\,\frac{F_1-A_1}{2x_d''}\right] \tag{14}$$

$$\text{The width of the envelope is }\frac{3e_0}{k}\,\frac{F_1}{x_d''} \tag{15}$$

The extrapolation of the subtransient part of the envelope to $t=0$ yields $\frac{3e_0}{kx_d''}$ times normal peak current. The extrapolation of the transient part of the envelope to $t=0$ yields $\frac{3e_0}{kx_d''}\cdot\frac{(x_d''+x_2)}{(x_d'+x_2)}$.

Similar to the three phase case, unless

$$-1 \leq \left[\frac{A}{F}+\sqrt{\frac{A^2}{F^2}+\frac{x_q''}{x_d''-x_q''}}\right] \leq 1 \text{ and is real} \tag{16}$$

the envelope of equations 13 and 14 is drawn in the usual way. If these conditions do exist, there will be more than one maximum per half cycle, but the envelope lines may still be drawn tangent at the 0° and 180° points, similar to figure 1, which treats the three phase case.

The peak value of the single phase fault current may be written from equation 13. The absolute peak value occurs when the wave is fully offset

$$\text{peak value of current} = \frac{3e_0}{k}\,\frac{F_1+A_1}{2x_d''} \text{ times normal peak value} \tag{17}$$

Certain limitations must be applied to equation 17 similar to those applied to equation 10 which governs the three phase case. Equation 17 can be used with confidence where $x_d''<x_q''$; however, where x_d'' is appreciably greater than x_q'', the absolute peak value of current will occur before the 180° point (assuming a totally offset wave) and may be appreciably greater than the value given by equation 17. This possibility is pointed out but not investigated further because of the improbable nature of the condition.

Conclusions

1. In a single phase line-to-line short circuit the width of the envelope (in terms of normal peak value of current) extrapolated to the time of the fault is $\frac{\sqrt{3}e_0}{x_d''}$. The reactance, x_2, does not appear as it does in the symmetrical component method of analysis, $(2\sqrt{3}e_0)/(x_d''+x_2)$. In a single phase line-to-neutral short circuit the extrapolated value of the envelope width is $3e_0/(x_d''+x_0/2)$.

2. An extrapolation to the time of the fault of the transient part of the envelope of a line-to-line fault shows the width of the envelope to be

$$\frac{\sqrt{3}e_0}{x_d''}\,\frac{x_d''+x_2}{x_d'+x_2}$$

The above relations developed from the two reaction theory indicate that it is not possible to determine x_2 from the subtransient part of the envelope of the current of a line-to-line fault. Any attempt to use the transient part of the envelope to find x_2 is very liable to errors arising from errors in the determination of x_d'' and x_d'.

Similarly, unless x_0 is very large, its accurate determination from the envelope of current of a line-to-neutral fault is not practicable.

3. The peak current on a line-to-line fault is $\sqrt{3}e_0\frac{F_1+A_1}{2x_d''}$ times normal peak current, where A_1 and F_1 are decrement factors, equal to unity at the time of the fault.

The peak current of a line-to-neutral fault is $3e_0\frac{F_1+A_1}{2x_d''+x_0}$.

It is of interest to note that x_2 does not enter in the above expressions as it does in the equations resulting from the use of symmetrical component methods. In every case where $x_2>x_d''$ the symmetrical component equations apparently calculate too low a value of peak current.

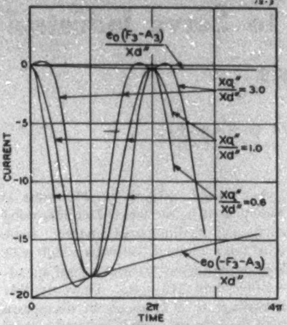

Figure 3. Change of wave shape with change of x_q''/x_d''

Other constants same as in figure 1

4. In an ideal machine, when the armature current in a three phase short circuit is fully offset, the envelope width at the time of the fault is $\frac{2e_0}{x_d''}$ times normal peak current when extrapolated from the subtransient region and $\frac{2e_0}{x_d'}$ when extrapolated from the transient region. In general, this is not true when the fault is made at any position of the rotor other than that which gives a fully offset wave.

In most commercial machines the ratios of machine constants are such that the degree of offset is of little practical importance in the determination of reactance from oscillograms of three phase short circuits. However, in view of the fact that several published statements prefer a symmetrical current wave, the above conclusions preferring a fully offset wave are of interest.

The conclusions given here are dependent on a prescribed method of drawing the envelope, which is explained in the body of the paper. In the majority of commercial machines this fact presents no great limitations.

The equations presented above are dependent on the assumption of an ideal machine: negligible resistance; negligible saturation; sinusoidal windings; and a sinusoidal variation of armature subtransient reactance with rotor position. The presence of resistance alters the phase relationship between the current harmonics, although it does not usually cause impedances to be appreciably greater than the associated reactances in short circuits at the machine terminals. Saturation may be partially treated by employing saturated values of the reactances; but the wave form distortion due to the changing values of reactance during the fault and the nonsinusoidal distribution of windings may cause appreciable departures from the calculated wave shape, and thereby vitiate in part the conclusions of this paper. However that may be, a more thorough understanding of the ideal machine relationships should be an aid to more accurate determination of synchronous machine constants from test.

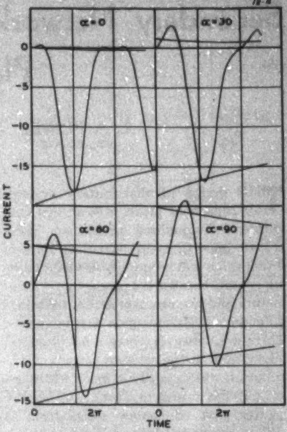

Figure 4. Change of wave shape with change of α

Other constants same as figure 1. See text for description of "envelope" lines

Bibliography

1. Synchronous Machines, Part IV, Doherty and Nickle. AIEE Transactions, volume 47, number 2, 1928, page 514.

2. Synchronous Machines, Part V, Doherty and Nickle. AIEE Transactions, volume 49, number 2, 1930, page 700.

3. Single Phase Short-Circuit Torque of a Synchronous Machine, Nickle, Pierce, and Henderson. AIEE Transactions, volume 51, number 4, 1932, page 966.

Secondary Networks to Serve Industrial Plants

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THE design of distribution systems for industrial plants will always constitute individualized problems. There is little hope that any particular form of distribution will be applicable to all industrial plants because the processes of manufacture and the arrangement of buildings differ widely depending on circumstances. There are, however, certain fundamental advantages to the network system of distribution which justify a much wider use than it has found to date. By far the greater part of factory distribution is still of the radial type and the network system must compete with the radial system in economy, reliability, and flexibility. The purpose of this paper is to show that it can do so successfully.

Assuming that the product of the factory is such that a relatively large number of small and medium-sized motors are required rather than a few large ones, it will be found that the physical location in the factory of the various machines is not of such importance in the layout of the distribution scheme as the total connected load, the diversity factor and the over-all power factor, and it is on its ability to take advantage of diversity that some of the economy of the network system rests.

The information available on the load density of factories is quite fragmentary. For the more common products of manufacture it varies from 5 to 30 kw per 1,000 square feet, 10 to 20 being the value mostly encountered, although in exceptional cases it may go as high as 100 kw. Load density should not be confused with the rating of the installed motors and utilization devices. The diversity between the various loads will considerably reduce this figure, and the "demand-factor," that is, the ratio of the maximum demand over a period of time to the sum of the utilization device ratings, which is of significance in determining the transformer capacity to be installed, may be 50 per cent or less.

A nominal voltage of 440 volts is widely used in the larger factories using small and medium-sized motors. Some mills have adopted 550 volts which with long radial feeds naturally has an advantage over 440 volts. However, this advantage to a large extent disappears with the network system and 440 volts is preferable. Experience with 208-volt metropolitan networks indicates that motors of 750 horsepower and even larger can readily be started up on a 440-volt network without disturbance.

It is desirable that the 440 volts be maintained at the motor locations without much deviation either up or down, and since the regulation in the network is better than in most existing radial distribution systems, the voltage at the supply source should not be much higher than 440 volts.

In the conventional 208-volt city network, the lighting is carried out at 120 volts with the units connected between the outside conductors and neutral. With 440 volts maintained on the suggested factory network the same possibility exists if 250-volt lighting can be used with safety. The type of lighting that might be operated successfully on 250 volts would include primarily the high-intensity mercury lamps of 250- and 400-watt sizes in high-bay reflectors (figure 1), and for intermediate mounting heights the larger fluorescent lamps.

It is generally found necessary in modern manufacturing methods to change tool machinery and processes around in the factory from time to time to meet market conditions, but it is also true that such changes cannot be of a radical nature as the manufacturing possibilities are limited by the type and size of the building. It is, therefore, advisable in designing the distribution system to make the power available at a number of points conveniently spaced throughout the factory building without too much regard to the detailed location of the individual tools.

The network is well adapted to supplying power to such points, and where the total load is 500 kva or more a network, either a distributed or spot network, will frequently be justified. As the total load increases the network becomes more decisively economical.

It is convenient to divide networks into spot networks and distributed networks. A spot network consists of a bus supplied through two or more network transformers and protectors over two or more primary feeders as shown in figure 2. It may be used in the smaller plants or large commercial buildings where the demand is relatively light and distances short. The distributed network on the other hand comprises a secondary grid, the junction points of which are supplied through network transformers and protectors from two or more primary feeders, as shown in figure 3. There is practically no limit to the load that can be carried on such a grid, and spot networks grow into conventional networks as the load increases.

It is not possible to lay down any definite rules regarding the number of supply points and secondary ties that should be included in factory networks, nor the best

Figure 1. General lighting at an average intensity of 25 foot-candles supplied by three 40-watt daylight fluorescent lamps in each luminaire

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size of copper to use. This will depend on the load density and arrangement of the manufacturing aisles in the factory buildings. It can be said, however, that when high current-carrying capacity is required in the secondary mains it is preferable to limit the size of conductor and use two or more smaller conductors in parallel instead of a single large conductor. For example, two 4/0 conductors in parallel provide more carrying capacity than a 500,000-circular-mil conductor and also provide better voltage regulation for a given current at the power factors usually prevailing in distribution systems. Furthermore, in case of a fault in one of these parallel cables, the limiters will isolate it without breaking the network.

The ease with which increasing loads or rearrangement of existing loads in the plant can be taken care of is one of the outstanding advantages of the network system. The addition of a transformer at a junction point of the grid can be made at any time without interruption of service, the relatively small self-contained transformer units with the network protector mounted on it requiring very little space.

Further advantages of the system are reliability, good regulation, and high economy. Reliability is improved by virtue of the fact that the grid is supplied from all the feeders supplying the factory and through all the transformers. The loss of a feeder or transformer would, therefore, pass unnoticed as far as the factory operations are concerned. The regulation is better and the disturbances due to starting up motors are less because of the multiplicity of paths over which the current flows.

In most factories supplied through radial systems of distribution it is usual to put in a substation at some convenient location with one or more transformer banks stepping down the voltage from the power company's primary distribution voltage. A full capacity primary circuit breaker and a 440-volt distribution bus are provided in the substation from which a number of radial feeders are run to the distribution centers in the factory through suitable circuit breakers. The disadvantages of this scheme are obvious.

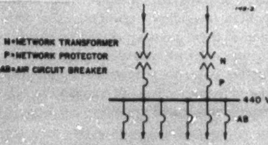
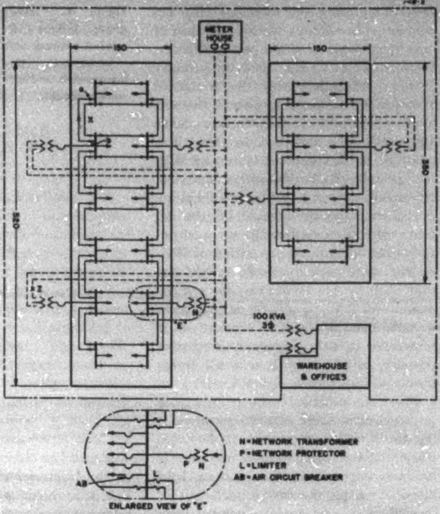


Figure 2. **Spot network**

Figure 3. **Distributed network for factory buildings**



All the breakers must have high rupturing capacity, the low-voltage, that is, the heavy-copper, feeders are of considerable length before reaching the utilization points and maximum economy is obtained only if it is known beforehand exactly what the power requirements are likely to be. Experience has shown that this assumption cannot be made and consequently the transformer capacity installed is usually very much greater than necessary.

A network for a typical factory consisting of two manufacturing and assembly buildings and a five-story office and warehouse building is shown in figure 3. The supply from the utility is over two cables at 13,200 volts, 60 cycles. The maximum demand is 2,500 kva. For taking care of the shop load six transformers of 450 kva, 13,200/440 volts, equipped with primary change-over switches and secondary protectors are provided, and for the warehouse load two 100-kva, 13,200/208-volt, network transformers.

The transformers may be standard oil-filled transformers with network protectors mounted on them, similar to those used in metropolitan distribution systems. In this case they will be placed outside of the factory buildings, either in the open or in vaults. In many cases a more economical installation can be made with air-cooled transformers placed inside the factory buildings, the transformers having mounted at one end an open-type network protector and on the other end a primary switch, the entire unit being enclosed in a single-ventilated housing as shown in figure 4. The network protector of each transformer would be connected to a short bus from which the utilization devices would be supplied through air circuit breakers. These busses would be interconnected among themselves throughout the factory by means of twin cables so as to form the network as indicated in figure 3.

The cardinal principle of the 120/208-volt network system is that secondary faults are burnt clear by the fault current. Experience has shown that while this is entirely desirable and feasible with the low voltages and heavy amperages used in metropolitan distribution, it must be modified for application to factory distribution. At 440 volts secondary faults are not self clearing and it becomes necessary to use protective fuses or limiters at each end of each secondary conductor to definitely clear faults and limit the damage to that section of cable.

With the arrangement proposed it will be seen that an electrical fault anywhere in the plant will leave the major operations undisturbed. The failure of a consuming device will be cleared by its own breaker. A fault on one of the cables forming the network, for instance at *X*, will cause the limiters or fuses *a* and *b* to blow to isolate the fault. These are the only two limiters to carry the full-fault current and they will, therefore, clear before any other limiter opens. A failure of the transformer or of the primary feeder, for instance at Z, will cause the primary breaker associated with the feeder or transformer to open and also all the network protectors on that feeder. Since the network is designed so that there must be no interruption of service with one primary feeder out of commission, this means that with two feeders the spare transformer capacity would be 100 per cent; with three feeders the spare capacity would be 50 per cent; with four feeders the spare capacity would be 33.3 per cent, and so on. With two feeders the amount of spare capacity is so great as to become uneconomical. Therefore, it is preferable to use a manual throw-over primary switch on each network transformer as indicated in figure 3.

There is a practical limit to the saving in transformer spare capacity by increasing the number of primary feeders, because with increasing number of feeders it becomes impossible to get uniform load division among the transformers left in service when one feeder is out. Also, when more than six feeders are used it is frequently necessary to consider that two feeders may be out of service at once; consequently, seven or eight feeders may require more transformer capacity than four or five feeders. For very large plants this is a very practical consideration and it may dictate the use of two networks in preference to a single network.

The quality of service, that is, reliability and voltage regulation, rendered by the secondary network is better than that provided by any other a-c distribution system. Whether the network is economical in a particular case depends on the degree of reliability required, the amount of flexibility required to meet load growth or change, and the load density and total load to be served.

Whenever it is necessary to provide greater reliability than can be obtained from a simple radial system the network will always be economical, particularly for relatively large loads. To get improved reliability in a radial distribution system it is necessary to use duplicate feeders, manual or automatic throw-over switches or breaker, load-transfer switches, sectionalizing switches or fuses, or other schemes for reducing the duration of outages. For faults on a primary feeder or in a transformer even these special radial schemes do no more than reduce the duration of the outage to a momentary interruption of service, which in many factories may not be permissible. Faults in main secondary circuits will interrupt service to all the load on that circuit until the fault can be repaired. The secondary network eliminates outages due to primary faults and most secondary faults will not interrupt service to any load. The switching equipment required in either manual or automatic radial schemes to provide duplicate power supply is expensive.

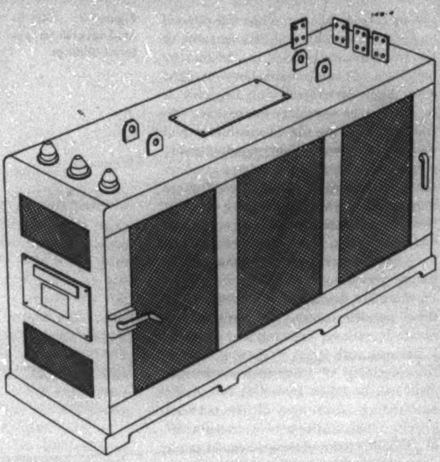


Figure 4. Three-phase indoor air-cooled network unit—transformer, primary switch, and network protector

The flexibility of the secondary network is an economic advantage. Changes to provide for load growth or changes of load concentration from one part of a plant to another usually can be made by simply installing or moving transformers to meet new load conditions. In the radial scheme any changes in load may necessitate not only changing the transformer capacity but also changing a considerable amount of secondary circuits. Since changes in processes and machines frequently are made in industrial plants, this characteristic of the network system will represent a large saving in many cases.

A comparison of the initial cost of a radial system and of a network system was made for the factory shown in figure 3. In the case of the radial system of distribution a small substation was provided at the upper end of the property comprising a transformer bank of four single-phase units of 833 kva each and primary switchgear consisting of two 500,000-kva, 15-kv oil-circuit breakers with necessary control panels. On the secondary side of the transformer bank a breaker was installed in the leads between the transformer and the 440-volt secondary bus. Five 440-volt radial feeders supply distributing points suitably located in the factory buildings.

In the case of the network system of distribution the primary 13,200-volt cables are brought to the six 440-volt network transformers of 450-kva each. Normally three transformers are connected to one cable and three to the other. In case of a fault on one primary cable the three transformers on the sound cable will be overloaded for the short period of time necessary to operate the primary switches to remove the three transformers from the fault cable and put them onto the sound cable.

With this layout it was found that the network system of distribution showed a saving of approximately 20 per cent over the radial system. It should be noted that in both systems primary circuit breakers were included at the point where the primary cables enter the property. With the network system where a group of primary feeders supply one or more industrial plant networks, it is questionable if these primary breakers are necessary in view of experience with power supply to large commercial buildings from metropolitan secondary networks. The omission of these primary circuit breakers on factory network systems would materially further lower the investment required for the network system as compared to the comparable radial system.

An Electron Microscope for Practical Laboratory Service

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DIRECT observation of extremely minute objects is of great value to both science and industry. Consequently a great deal of time and effort has been spent developing and perfecting optical instruments for making such observations. The modern light microscope is capable of rendering visible detail which is as fine as 0.00002 centimeter. Since the limit of resolving power of the normal eye is between 0.01 and 0.02 centimeter, the maximum useful magnification with these instruments is 1,000 to 2,000 diameters. Where ultraviolet light is used, the useful magnification can be increased to about 3,000 diameters.

The limits discussed above are not due to imperfections in the optical systems or lenses, but are fundamental in nature. It has been shown quite generally that diffraction effects at the object determine the least separation between points or lines on the object which can be resolved. On the basis of this limitation, the resolving power (for lines) at the object is given by

$$d = \frac{0.5\,\lambda}{\mu \sin \alpha}$$

where λ is the wave length of the radiation illuminating the object; μ the index of refraction of the medium in which the object is imbedded, and α the half angle of the cone of light entering the optical system. For visible light the smallest wave length is about 4,000 angstrom units, and the maximum index for the fluid immersing the object is 1.7; therefore, since $\sin \alpha$ cannot be greater than unity, the minimum distinguishable distance is seen to be about 1,200 angstrom units.

From the foregoing it is evident that to extend the range of observation, it is necessary either to find an immersion substance which has a very much greater index of refraction, or to illuminate the object with rays having a much shorter wave length. No known optical medium has an index of refraction greater than two or three, and no liquid suitable for use with an immersion objective has an index greater than the figure used above, namely, 1.7. Therefore, there is little hope of achieving high resolution in this direction. Electromagnetic radiation in the form of X rays and gamma rays has a wave length much shorter than that of visible and ultraviolet light used at present in microscopy. However, due to the fact that lenses or reflectors cannot be made for this portion of the spectrum, advances in this direction are also barred.

For many years the problem of observing appreciably finer detail appeared to be incapable of solution. The search, however, had been restricted almost entirely to the realm of optics, whereas actually the solution lay in an entirely different field—electronics. A little less than 15 years ago it was found that the path of electrons in electric and magnetic fields could be described in terms which are analytically equivalent to those of optics. By means of this electron optical equivalence, it was shown that axially symmetric electric or magnetic fields had the properties of optical lenses, and, consequently, that it is possible to form electron images in the same way that light images can be formed.

At about the same time, the discovery was made that any material particle in motion had associated with it a characteristic wave length. For electrons this wave length, in terms of their velocity expressed in electron volts, is given by the following relation:

$$\lambda = \sqrt{\frac{150}{V}} \text{ angstrom units}$$

On the basis of this equation, electrons moving with a velocity corresponding to 60 kv have an effective wave length of 0.05 angstrom unit, or only about 1/100,000 that of light. In other words, suitably designed electronic systems employing these high-speed electrons should be capable of extremely high resolving power.

These two concepts led directly and logically to the idea of an electron microscope based on the same principles used in an optical microscope. Such an instrument employs condenser, objective, and projection lenses, performing the same functions as the corresponding elements in the light microscope, but the lenses, instead of being made of glass, are formed by axially symmetric electric or magnetic fields. During the past 10 years, instruments of this type have been investigated in detail in various parts of the world. As a result, the electron microscope has been developed to a point where it is capable of a useful magnification nearly two orders of magnitude greater than the ordinary light microscope. Until recently, the development has been in the hands of physicists, and the instruments built were designed almost entirely for the purpose of studying the microscope itself. However, the microscope has now progressed beyond this stage, and has become a research tool of great potential value.

The problem of realizing this potential value is one for engineers, and as such, its solution was undertaken in the RCA Research Laboratories at Camden, New Jersey. Starting from the basic physical principles, it was necessary to work out the design of an instrument which would be sufficiently simple in operation and rugged in construction to be operated by the average worker in any scientific or industrial research laboratory, and would not require the supervision of a trained physicist. An RCA microscope, developed by L. Marton,[1] which fulfills in part the requirements of a generally practical microscope, was described in an earlier number of ELECTRICAL ENGINEERING.[2] This microscope, although very satisfactory from the standpoint of resolving power, proved to be somewhat difficult to operate and very critical in adjustment and alignment. Furthermore, the instrument and associated equipment occupied a large amount of space.

Work on the electron microscope was continued in order to overcome the objectionable features mentioned above. As a result, an instrument has now been developed by J. Hillier which is completely self-contained, occupies only a small amount of space, and is so simple to adjust and operate that it can be readily handled by any capable laboratory technician.[3] In addition, the microscope incorporates an improved type of magnetic objective lens which increases its resolving power for specimens supported in the manner described below.

Figure 1 is a photograph of the complete microscope. The unit includes not only the microscope itself, but also the regulated power supplies providing the 60-kv over-all voltage and the current for

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[1] For all numbered references, see list at end of paper.

Figure 1. RCA electron microscope

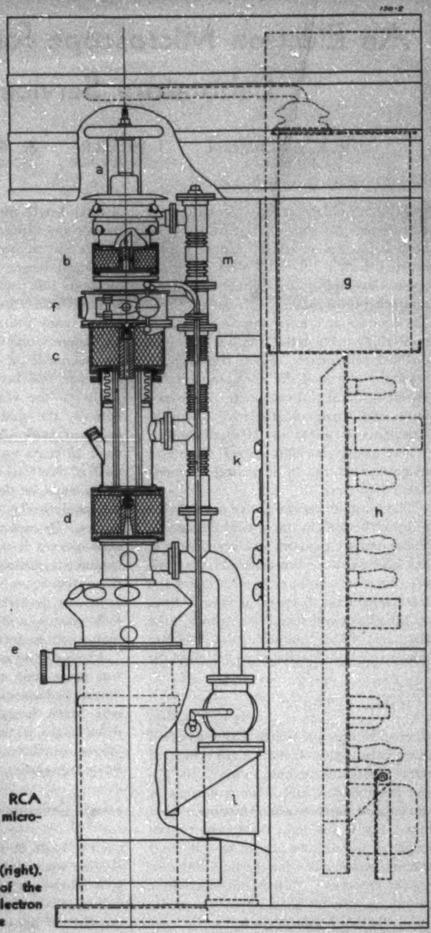


Figure 2 (right). Diagram of the RCA electron microscope

the three magnetic lens coils. The oil diffusion pump for maintaining the required vacuum within the instrument is also contained in the unit.

A simplified drawing is given in figure 2, which shows the construction of the microscope. The electrons used in the imaging process are supplied from the electron gun (*a*), containing a thermionic cathode which is maintained at 60-kv negative with respect to ground, and a final anode at ground potential. Between the two are disposed the electrodes required for governing the electron paths. The electrons leaving the gun have their full velocity, corresponding to 60 kv.

The condenser lens (*b*), which consists of an iron-clad coil with pole pieces shaped to give the required magnetic field, causes the electrons to converge upon the specimen held a few centimeters below it. The condenser, like the condenser lens of a light microscope, can be used to control the angular aperture of the illumination at the specimen.

After passing through the specimen, the electrons enter the objective lens (*c*). This lens deflects the electrons leaving the specimen in such a way as to focus them into a magnified intermediate image of the specimen. This image is formed directly above the projection lens (*d*). The objective, like the condenser, is an armored coil, but the pole pieces are, of course, different in design to meet the requirements of this element. The design of the object lens is very important, and the success or failure of an electron microscope depends to a large extent upon the skill and accuracy with which it is designed and constructed.

The final image is formed from the por-

Figure 3. Air lock and object chamber

tion of the intermediate image which passes through the projection lens (*d*) and is reimaged in the plane of the observing screen or photographic plate at (*e*). The magnification of the intermediate image is about 100 diameters, and the magnification by the projection lens can be varied between 20 and 300 diameters; therefore the total magnification can be controlled over the range between 2,000 and 30,000 diameters. The maximum electronic magnification of the microscope is two or three times less than the greatest useful magnification corresponding to the resolving power of the instrument, the full useful magnification, for reasons that are discussed below, being obtained by additional photographic enlargement. When visual observation of the specimen is required, the final image is allowed to fall on a fluorescent screen. This screen can be raised out of the way to permit the image to impinge upon a photographic plate for a permanent record, or for more detailed examination.

Since electrons will not travel freely through air, the entire electron optical path of the microscope must be under vacuum, that is, at a pressure of about 10^{-4} millimeters Hg. All permanent joints are brazed or soldered so that they are vacuum-tight without the aid of sealing compounds. The demountable joints, such as those between the electron lenses and the microscope chambers, are sealed with Neoprene, and are arranged in such a way that the alignment is maintained by metal-to-metal contacts rather than by the metal-to-plastic contacts. At certain points in the microscope, parts must be movable with respect to one another in order to permit alignment. Such parts are jointed by flexible metal bellows so the adjustments may be made while the microscope is under operating conditions. No greased joints are used in the microscope.

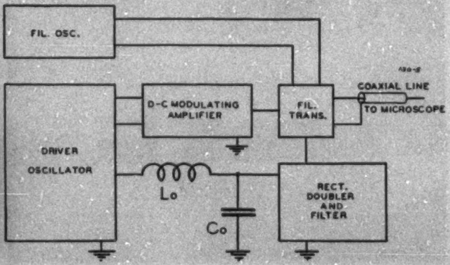
If it were necessary to repump the entire microscope every time a specimen or photographic plate is changed, it would greatly curtail the speed of operation of the instrument. Consequently air locks are provided which permit making these changes without breaking the main vacuum. The air lock for the object chamber (*e*) is shown in figure 3. A handle at the rear of the chamber raises the object from its position over the objective, moves it into the air lock, and closes the latter off from the main chamber of the microscope. A second handle admits air to the lock and opens it so that the object may be removed. When the object is replaced, the second handle closes the air lock, connects it with an auxiliary pump for a preliminary exhaust, and then seals it off. With the first handle, the specimen is moved back into the microscope to its original position.

The photographic chamber, which is shown in figure 4, is arranged with a similar air lock. The photographic plate used with the microscope is about ten by two inches in size. The long plate permits making a number of exposures on the same plate. An adjustable mask governs the width of the pictures.

The microscope pumping system con-

Figure 4. Photographic chamber

Figure 5. Block diagram of high-voltage supply



sists of a three-stage oil diffusion pump (*l*), in figure 2, which is connected to the main chamber of the instrument through the manifold (*m*). This pump is backed up by a mechanical fore pump. A second auxiliary mechanical pump is used to exhaust the two air lock chambers.

Figure 6. Rear of microscope showing power supply and regulator units

It can be shown that the focal length of a magnetic electron lens depends upon the current through the lens coil and the velocity of the electrons themselves. This means that the high-voltage supply and the lens current sources must be very free from fluctuations. In order to obtain a resolving power of 10^{-7} centimeters it is found that the following stabilities are required:

Over-all microscope voltage	0.015 per cent
Objective current	0.0075 per cent
Projection lens current	0.068 per cent
Condenser lens current	0.1 per cent

The problem of obtaining 60 kv with the required degree of constancy in the small space available is both difficult and interesting. The high-voltage supply, which was designed by A. W. Vance, employs radio-frequency power rather than the conventional 60-cycle power to actuate the step-up transformer unit.[8] This results in a number of advantages. A low-loss resonant coil can be used which occupies only a small volume and requires only a small exciting power. Much smaller condensers can be used for a given allowable amount of ripple. A regulator operating on the low-voltage side of a rectifier is limited in its speed of regulation by the frequency of the power supplied. At radio frequency, this limitation becomes unimportant. Finally, stray fields, which are difficult to shield at 60 cycles, are easily shielded when radio frequency is used. It is, therefore, possible to mount the power supply close to the microscope as an integral part of the unit.

A block diagram of the high-voltage unit is given in figure 5. The basic circuit consists essentially of a high-Q resonant net which presents series resonance to the driving oscillator, and is parallel resonant to the high-voltage output rectifiers. The resonant circuit consists of coil L_0 and the effective lumped capacity C_0, which is made up of the rectifier capacitance and stray capacities. To obtain a given output voltage, the exciting power required from the oscillator, since the load is very small, depends upon the Q of the circuit and the resonant impedance of the high-voltage coil. Great care must be used in the design of the coil and in the selection of operating frequency in order to obtain a high Q and maximum impedance. The actual frequency selected is 32 kilocycles, and the Q of the resonant circuit is about 200. The exciting power required to produce the desired power is about 150 watts, and the maximum load in the neighborhood of 30 watts.

Since the Q of the resonant circuit is high, the driving frequency from the oscillator must be exactly correct if large inefficiencies are to be avoided. In order to maintain the correct driving frequency, the frequency oscillator is controlled by the resonant circuit itself. The output voltage of the power supply is governed by the amplitude of the current from the oscillator. This, in turn, is varied by the screen-grid voltage applied to the oscillator output tubes. By controlling the screen voltage, not only can the output voltage be set at any desired value, but also the constancy of the voltage can be governed.

The rectifier across the resonant circuit is arranged to give voltage doubling. The type of doubler used permits grounding one side of the high-voltage coil, thus permitting the use of a simple resonant circuit rather than a tuned transformer.

Figure 7. Preparation of supporting films for specimens

Figure 8. Micrograph of pseudomonas aeruginosa bacteria. Magnification 14,000 diameters

Figure 9. Micrograph of slightly polymerized vinyl chloride. Magnification 77,000 diameters

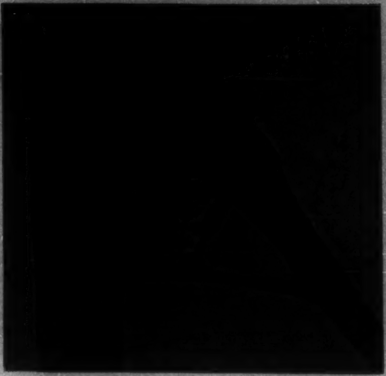


Figure 10 (right). Micrograph of polymerized vinyl chloride. Magnification 100,000 diameters

The high-frequency ripple is eliminated from the high-voltage output by means of a tuned filter. Low-frequency disturbances are removed by comparing a fraction of the output voltage obtained from a special divider with a standard voltage,[4] and controlling the oscillator screens by means of a d-c amplifier. Such a feedback regulator, when certain necessary precautions are effected, is capable of extreme constancy. The measured voltage fluctuation over a period of 30 minutes is less than 0.002 per cent in the actual microscope power supply.

The current supplies for the three lens coils are controlled by very sensitive regulators. These regulators make use of well-known principles, and will not be discussed here.[4] By careful design and the use of a number of special refinements, these current sources have a constancy of better than 0.002 per cent for the objective lens, 0.004 per cent for the projection lens, and 0.02 per cent for the condenser lens.

The complete power supply is assembled in a cabinet which forms the back part of the microscope unit. The lens current sources and the oscillators are located at (*e*) in figure 2. The high-voltage coil, rectifiers, and filters are immersed in the oil tank (*f*). Short, corona-proof leads carry the high voltage to the gun of the microscope. All power supplies are completely interlocked to insure absolute safety, and protective relays are arranged to open the high-voltage circuit in event that vacuum conditions within the microscope are not right. Figure 6 is a photograph of the rear of the instrument with the door to the power-supply section open so that the general arrangement of the equipment can be seen.

The operation of the instrument is simple in the extreme. The operator seated in the observers' position can reach the electrical controls and thus adjust overall voltage, magnification, or focus, while watching the image. In addition, the specimen can be moved in two directions in a horizontal plane, so that any portion of the object may be brought into the field of the microscope. Six ports are provided for viewing the final image, and, in order to permit initial orientation of the specimen, three small windows allow direct observation of the intermediate image.

To record the image photographically, it is only necessary to move the plate into position under the fluorescent screen, and then to raise the screen so that the electron image falls on the plate. The fluorescent screen is designed to serve as the shutter for timing the exposure. It is expedient when making electron micrographs to use as low an electronic magnification as possible, and to obtain the

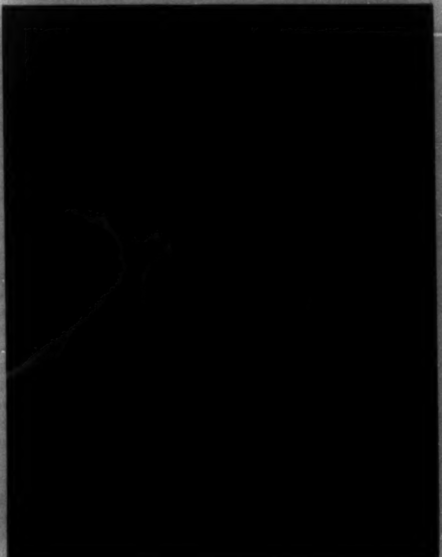


Figure 11. Micrograph of commercial chemical. Magnification 150,000 diameters

Displacement and Diffusion in Fluid-Flow Arc Extinction

JOSEPH SLEPIAN
FELLOW AIEE

Synopsis: The displacement and diffusion theories of arc extinction in flowing fluids are defined and contrasted. The question as to the possibility according to hydrodynamical principles of the formation of the wedge required by the displacement theory is considered and decided in the negative. Photographic evidence is also found to contradict the displacement theory. How turbulence will multiply diffusive effects is considered, and an estimate of 100fold multiplication is given for conditions existing in gas blast breakers. This enhancement of diffusion by turbulence is found to make the diffusion theory adequate for accounting for the circuit interrupting capacities of arcs in gas blast circuit breakers.

I. Introduction

THE important large effect which motion of the circumambient fluid has upon the circuit interrupting ability of an a-c arc has long been known, but there are still widely diverging theories as to how this effect comes about. Slepian,[1] in discussing the influence of gas motion in self-generating gas blast devices such as oil circuit breakers and expulsion fuses stressed the effect of turbulence in causing a more rapid mixing of hot and cold gas volumes, thus speeding the spread of heat, and a more rapid mixing of highly ionized with unionized gas volumes, thus speeding the dilution of the ions into larger volumes. The theory here, then seems to have been that the function of the gas motion is to intensify or enhance diffusive effects, and more particularly, thermal conduction and ion diffusion. This type of theory which makes the powerful arc extinguishing effect of the fluid flow depend entirely upon its creating conditions where diffusive effects are greatly enhanced shall be called in this paper the "diffusion theory".

A radically different theory of the extinguishing effect of the fluid flow was advocated by Prince,[2,3] which shall be called in this paper the "displacement theory". This theory states that at current zero (or perhaps at a moment very near to current zero), the physical continuity of the arc is broken at some obvious geometrical point, by the interposition of a small quantity of *cool, un-ionized*, and *dielectrically completely sound* fluid. From this moment on, the dielectric strength of the arc space is completely given by the dielectric strength of this interposed fluid. The relative velocity of the two ends of this interposed fluid is believed to be one or sometimes two times the mean velocity of the fluid in that neighborhood. Thus the thickness of the interposed fluid grows rapidly, being given by the product, vt, of the fluid velocity and the time. The rate of growth of the total dielectric strength of this interposed fluid in volts per second, will be given by Xv, where X is the intrinsic dielectric strength of the interposed fluid in volts per centimeter, and v is the fluid velocity in centimeters per second. Arc extinction thus is said to depend on whether or not this rate of recovery of dielectric strength of the interposed fluid exceeds the so-called "recovery rate" of the circuit. That is the arc will be extinguished if

$$Xv > \frac{dV}{dt}$$

where $\frac{dV}{dt}$ is the circuit "recovery rate" in volts per second.

Apart from whether the "displacement theory" is correct or not, it has the incontestable advantage that it leads directly to an attractively simple equation, so that the designer or user of a circuit breaker who accepts this theory may have the comforting assurance that the breaker performance has been "calculated". Hap-

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[1] For all numbered references, see list at end of paper.

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full useful magnification by photographic enlargement of the plates. In general, a photographic enlargement of six to ten times can be made without bringing out the grain of the emulsion. Therefore, if the total useful magnification required is 100,000 diameters, the electronic magnification will be 10,000 to 16,000. The advantages of this procedure are that it decreases the exposure time, reduces the bombardment to which the object must be submitted, and increases the useful field of view of the microscope.

The specimens themselves are mounted somewhat differently from those used with a light microscope. Glass slides would be completely opaque to electrons; therefore the objects to be examined are mounted on extremely thin cellulose films, supported by a fine wire mesh. The method of preparing one of these supporting films, which are only about 0.000001-centimeter thick, is illustrated in figure 7. The material or particles to be studied are usually suspended in water or some other suitable liquid, and a drop of the suspension is placed on the film. After the liquid has evaporated, the specimen is ready for use.

The utility of this instrument is almost unlimited, and it finds applications in many fields of scientific and industrial research. Two fields, chosen at random, serve to illustrate its value. In bacteriology and biology it opens up new and entirely unexplored realms, for with this microscope microorganisms such as viruses, which cannot even be seen with an optical instrument, can be studied in detail. Figure 8 is a typical micrograph of pseudomonas aeruginosa bacteria. Much of the structure seen in this photograph is well below the resolving power of an ordinary microscope. In physical chemistry it can be used to study colloids, point reactions, the structure of thin films, and even large organic molecules. To illustrate this application, micrographs of slightly polymerized and polymerized vinyl chloride films are shown in figures 9 and 10. The final photograph illustrates a commercial chemical product. Structure of less than 50 angstrom units can be resolved in this micrograph.

References

1. A New Electron Microscope, L. Marton. *Phys. Rev.*, volume 58, number 1 (July 1, 1940), pages 57–60.

2. Electrons Extend the Range, V. K. Zworykin. Electrical Engineering, volume 59, November 1940, pages 441–3.

3. Recent Developments in the Electron Microscope, J. Hillier and A. W. Vance. *Proc. I.R.E.*, 1941.

4. Stable Power Supplies for Electron Microscopes, A. W. Vance. *RCA Review* (January 1941) volume 5, pages 293-300.

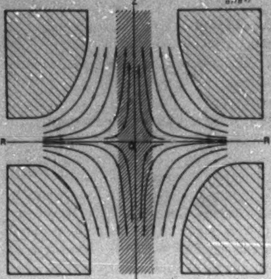


Figure 1. Ideal flow in a double nozzle

pily, however, designers of breakers "calculated" in this way have not dispensed with thoroughgoing high power laboratory tests, so that the performance of such breakers rests on a more solid foundation than the "calculation".

European investigators have advocated both the diffusion and displacement theories. O. Mayr[4] quite definitely makes diffusive effects the dominating factor in determining arc extinction, but Kesselring and Koppelmann[5] principally because they are not able to calculate a sufficiently large interrupting effect on the basis of diffusion, espouse the displacement theory. However, in their calculations which at best can only be regarded as giving orders of magnitude, because of the uncertainty of our present knowledge of the fundamental processes going on in the rapidly changing arc, Kesselring and Koppelmann underestimate completely the great enhancement of diffusive effects arising from even only a moderate degree of turbulence.

Holm, Kirschstein, and Koppelmann[6] speak of the effect of gas motion and its turbulence in increasing the thermal losses from the arc during the current conduction period, but do not take into account the effect of turbulence in increasing ion loss by enhanced diffusion. Also, surprisingly, although they seem to recognize the importance of turbulence during the conduction period, in the "currentless pause" (stromlose pause) near current zero they use for the thermal conductivity and ion diffusion coefficient numbers appropriate to a gas at rest, and overlook the great increase in these quantities arising from the gas motion. Thus they greatly underestimate the voltage interrupting capacity of an arc of small section at current zero.

These last authors also speak of the possibility of a cutting of the physical continuity of the arc, but state at once that such cutting cannot happen, *loc. cit.*, page 84, but that rather a configuration favorable for rapid diffusion occurs where one might naively suggest a cutting. With this exception, none of the references just mentioned question whether accepted hydrodynamical principles will permit the formation of a growing wedge of sound fluid interrupting the continuity of the arc.

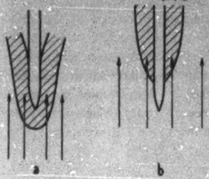


Figure 2 (left). Wedge formation at sharp splitter in ideal fluid

II. Impossibility of Wedge Formation in Interior of Ideal Fluid—Diffusion Absent

To see clearly why hydrodynamical principles forbid the formation of an interrupting wedge, consider the flow of an ideal fluid pictured in figure 1. This portrays a flow corresponding to the double nozzle discussed by Kesselring and Koppelmann. We choose an ideal fluid, that is, one with zero viscosity, because we wish to exclude diffusion effects, and viscosity is a consequence of diffusion, namely the lateral diffusion of molecules from faster moving regions into slower moving regions, and vice versa. For simplicity in calculations let us assume the fluid incompressible. A little thought soon convinces one that compressibility does not alter the conclusions reached.

In figure 1 we have fluid flowing inward radially through a circumferential opening, and axially out along the *Z* axes through a double nozzle. At the time $t=0$, we fix our attention upon the column of fluid shown shaded, which might be the arc configuration near current zero. However, to keep the flow considered simple, let us assume that the shaded column of fluid has the same density as the rest of the fluid, and not a much lower density as in the high temperature arc. We may perhaps regard the shaded column of fluid marked with a nondiffusing ink giving a negligible density change in the fluid.

One might imagine from inspection of the figure, that as the flow proceeds, the shaded column would be broken or cut in two, and that presently the two newly formed ends would be receding from each

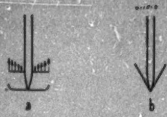


Figure 3. Bending of an arc around a gas-forming splitter

other with twice the axial fluid velocity through the nozzles. However, a little calculation shows that this is not at all the case. In the neighborhood, *O*, of the geometrical center of the figure, the axial velocity is given by

$$v_z = Az \tag{1}$$

where z is the distance along the axis from *O*, and *A* is a constant, and the radial velocity, is given by

$$v_r = \frac{dr}{dt} = -\frac{1}{2}Ar \tag{2}$$

where r is the distance from the central axis. These equations will hold approximately out to distances approaching the nozzle dimensions.

The bounding cylindrical surface of the shaded region will be described by the equation,

$$r = r_0 \tag{3}$$

at $t=0$.

The subsequent motion of this bounding surface will be obtained by integrating equation 2. We obtain at once,

$$r = r_0 e^{-\frac{1}{2}At} \tag{4}$$

The cylindrical surface remains cylindrical, but its radius approaches zero at a constantly diminishing rate. There is no formation of a surface with two nappes; no interrupting wedge appears. The length of this cylindrical column of constantly diminishing radius will be comparable with the nozzle dimensions.

To get some quantitative ideas of the rate of thinning of the cylindrical column, let the axial velocity of the fluid be $3 \cdot 10^4$ centimeters per second at 1 centimeter from *O* then $A = 3 \cdot 10^4$. Then equation 4 becomes

$$r = r_0 e^{-\frac{t}{66.7 \times 10^{-6}}} = r_0 2^{-\frac{t}{45.2 \times 10^{-6}}} \tag{5}$$

We see that the column has its radius halved each 45.2 microseconds, but the radius never becomes actually zero.

If we assume no diffusion takes place, the density of the ink marking the thinning column remains unchanged. Applying the ideas developed here to the arc, we see that pure streaming, without diffusive effects, will thin the arc, but will not change its temperature or ion density, and therefore will not change its dielectric strength per unit length.

III. Enhanced Diffusion Responsible for Interrupting Effectiveness of Axial Flow

The preceding section has made clear that without diffusion (of which thermal conductivity and viscosity are consequences), axial streaming of the fluid as in a double nozzle can have no interrupting effect upon the alternating current arc near current zero. All it can do is to cause the radius of the arc stream to shrink asymptotically to zero, with no change in ion density or temperature of the arc space.

However, it is evident that this effect of the motion of the fluid in reducing the arc diameter can play an important part in increasing greatly the interrupting effect of such diffusive forces as are intrinsically present. The time constant for diffusion from a cylindrical region, that is, for example the time for the ion density to be reduced to one-half, or the time for the temperature to fall to one-half, varies as the square of cylinder diameter. Thus, in the numerical example given in the preceding section, the rate of ion loss, and the rate of cooling will multiply by 4 every 45.2 microseconds, during the interrupting period, due to the reducing arc diameter.

However, this is far from the whole story. As Kesselring and Koppelmann[5] have shown, taking the small arc diameters such as are actually observed at current zero in the double nozzle flow, ordinary diffusion and thermal conduction are still not great enough to account for the observed interrupting effect. The high interrupting capacity comes because there is inevitably a high degree of turbulence in the axial flow through the arc.

In the example discussed in the preceding section and pictured in figure 1, the velocity in the incompressible ideal fluid varies in space according to equations 1 and 2. By Bernouilli's principle, the pressure in the fluid will vary in space according to the equation

$$p_0 = p + \frac{1}{2}\rho(v_z^2 + v_r^2) \qquad (6)$$

where p_0 is the pressure at 0, the point of zero velocity. At each point, the velocity is determined by the fall in pressure to that point, but it should be observed that for a given pressure distribution, the velocity will vary inversely with the square root of the density of the fluid.

But now suppose that the shaded cylinder of figure 1, instead of having the same density as the rest of the fluid, has a density, like that of an arc, only $^1/_{20}$ or less, of the density of the surrounding fluid. The pressure distribution must remain continuous. Hence at any particular distance from O, the velocity in the arc would need to be $\sqrt{20}$ or more times the velocity of the adjacent cool and more dense fluid. There would have to be then a sharp discontinuity in fluid velocity, and enormous shearing stresses at the arc boundary due to viscosity. What happens under such conditions is that the smooth streamline flow portrayed in figure 1, is replaced by an intensely turbulent flow through the arc, the kinetic energy of the turbulence adding itself to the kinetic energy of the average motion to give a total velocity head compatible with Bernouilli's equation 6.

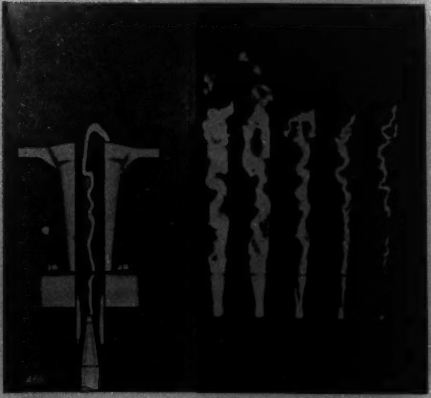
As we shall see in section VI, this high degree of turbulence functions to further greatly increase the rate of cooling and deionization of the arc stream at current zero, and probably fully accounts for large interrupting capacity of the axial flow.

IV. The Cross Blast—Wedge Formation at a Splitter

Compressed air circuit breakers have been recently introduced[7] in which the arc is blown transversely against several splitters made of gas forming material. Let us examine what may happen to an arc column at such a sharp splitter.

Under the influence of the fluid motion, the arc is lapped round the sharp splitter in a thin column as current zero is approached as in figure 2*a*. What does the further motion of the fluid do to this column? Photographic records show that the thickness of this column is too great for simple diffusion or thermal conductivity to account for the high interrupting capacity actually found. Does the moving fluid cause the column to be cut, and introduce between the cut ends a longer and longer path through good dielectric? Or does the fluid motion so enhance diffusive effects that even the relatively thick long column cools and deionizes quickly enough to have a high interrupting capacity?

If the fluid is ideal, with zero viscosity (and therefore also with zero thermal conductivity and zero ion diffusion), the answer is certainly that the arc column is cut as in figure 2*b*, with the recovered dielectric strength equal to that of the path through the introduced fluid between the cut ends of the arc column. The displacement theory then will apply. But



Lichtbogen vor dem Erlöschen
Gleiche Aufnahme, weicher kopiert.
Fortgesetzte Emission von Ladungsträgern
aus der glühenden Elektrode

Figures 4 (left) and 5 (right). High-speed motion pictures by Biermanns

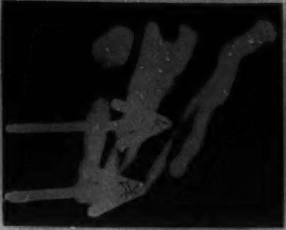


Figure 6. Arc against splitter, by Prince

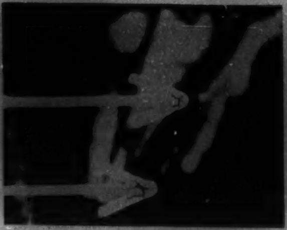


Figure 7. Arc against splitter, $16^2/_3$ microseconds after figure 6

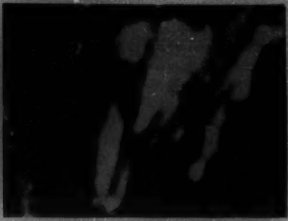


Figure 8. Arc against splitter, $16^2/_3$ microseconds after figure 7

the fluid will need to flow with its full velocity parallel to and right up to the splitter wall.

However, actual fluids cannot flow in this manner. Even with very small viscosity, the fluid velocity next to the splitter wall must be zero. Otherwise infinite shearing stresses would be exerted on the fluid and wall. A layer is formed next to the splitter wall, in which the fluid velocity varies from zero at the wall, to the full streaming velocity away from the wall. In this boundary layer, the shearing stresses arising from viscosity are large compared with the inertial forces.

With homogeneous fluid, the boundary layer starts with zero thickness at the splitter edge, and grows in thickness upstream from the edge. In this first part of the boundary layer, the flow is laminar, but at some point far enough upstream the flow in the boundary layer becomes turbulent, and the boundary layer thickness then increases more rapidly.

For air, at high velocity, and with a well designed splitter, such as an air-foil, the boundary layer a few centimeters upstream is only a few thousandths or a hundredth of a centimeter thick. We might imagine then that figure 2 could be corrected by drawing such a thin boundary layer joining the two receding ends of the cut column. Since so thin a boundary layer would cool and deionize exceedingly rapidly, this correction would not seriously change the conclusions of the displacement theory.

However, in the actual circuit breaker, we do not have a homogeneous fluid flowing past the splitter, but the density of the arc column is only one-twentieth that of the flowing air. We may expect then that the boundary layer will start in a turbulent state from the very beginning at the splitter edge. Still further complicating the situation is that under the heat of the arc, the splitter material gives off gas at a high rate, this gas evolution continuing right through current zero. This gas projected out from the wall will certainly widen the boundary layer enormously, and make it wide even at the beginning of the splitter. But with a wide boundary layer, the displacement theory fails completely since a wedge of fresh fluid interrupting the arc growing with the full streaming velocity of the fluid cannot form. The arrows in figure 3*a* are intended to show by their length the mean velocity distribution in the boundary layer. We fix our attention on the horizontal line just touching the splitter edge, and which we suppose moves with the fluid. This line may represent a bounding surface of an arc column near current zero. A finite time later the line will look as in figure 3*b*. Because of the zero velocity at the splitter surface, the line is bent, but not cut.

V. Photographic Evidence Concerning Displacement Versus Enhanced Diffusion

High-speed motion pictures taken of arcs being extinguished in axial and cross blasts offer valuable evidence for determining whether the displacement or enhanced diffusion theory is correct. J. Biermanns[8] shows pictures taken at 5,000 per second of an arc blown through a single nozzle. Figures 4 and 5 are examples taken from Biermanns' paper. Extinction seems to have taken place after the fourth picture in figure 5. If the displacement theory were correct we would expect to see in the fifth picture a completely dark break in the continuity of the luminous arc, and this break should be at most $200 \cdot 10^{-6}$ second, the time interval between pictures, multiplied by the probable velocity, about $3 \cdot 10^{4}$ cm, or about 6 cm. Actually, in the fifth picture of the lower half of figure 6, there is a length of about this magnitude which is darker than the rest, but it is not completely dark, and also there are in it several splotches which are more intensely luminous. If this darker length does represent the growing wedge introduced into the arc according to the displacement theory, we must conclude that this wedge far from being fresh dielectrically sound fluid, is considerably contaminated by inmixing of ionized and radiating gas from the arc itself. This contamination, which is evidence of considerable turbulence, should be extremely great when the wedge is short, thus making it impossible to speak at all of a short wedge of nearly fresh dielectrically sound fluid. It is only after the "wedge" is of considerable length that it can have decontaminated itself so that it can be regarded as being of dielectrically sound material. This decontamination, however, according to section II can proceed only from diffusion, so that we are driven inevitably to the diffusion theory according to which a considerable length of the arc column finds itself in a condition so favorable for rapid diffusion that it recovers dielectric strength sufficiently rapidly to effect circuit interruption.

Comparison of the upper part of figure 5 with the lower which was obtained from a photographic print with much less contrast, also gives evidence of diffusion much intensified by turbulence. The sharply defined intensely luminous arc column in upper figure 5 is revealed in lower figure 5 as surrounded by less luminous gas which must have diffused rapidly out of the arc column.

Prince[7] has shown some extraordinary pictures taken at 60,000 per second of a circuit interruption by an arc in a cross air blast against fiber splitters. Three of these successive pictures, which are believed by the present author to include the moment of circuit interruption are reproduced here as figures 6, 7, and 8. In figure 6, the arrows introduced by this author point out the two-inch length of arc which he believes to have been responsible for the circuit interruption. This length of arc lies squeezed up against the splitter surface. At the splitter surface itself, the mean velocity of the gas in the arc parallel to the surface must be zero. At the other boundary of the arc, the non-luminous air is moving with very high, probably acoustic velocity. The velocity gradient is then very large in this arc. Also, there is a very sharp change in gas density at the boundary of the arc. Lastly, gas from the decomposing splitter is being projected perpendicularly into the arc column. All these factors must create an intense turbulence, which must so enhance diffusion that these two inches of arc are cooled and deionized extraordinarily rapidly.

In any case, figure 7 shows that after only $16\frac{2}{3}$ microseconds the two inches have lost most of their luminosity, and in figure 8, after another $16\frac{2}{3}$ microseconds the luminosity has almost completely disappeared. If we assume acoustic velocity of $3 \cdot 10^4$ cm per second, the fluid displacement in $16\frac{2}{3}$ microseconds would be only $\frac{1}{2}$ cm. This is very small compared to the two-inch length of arc column which seems to be the first less luminous break to make its appearance in the succession of pictures. Thus these high speed pictures of Prince support strongly the enhanced diffusion theory as against the displacement theory.

VI. Turbulence and Enhanced Diffusion

Appreciation of the fact that fluid motion enhances diffusion is as widespread as the custom of accelerating the sweetening of coffee by stirring with a spoon. Recent developments in the theory of turbulence, however, now make it possible to give a rough numerical measure to this degree of enhancement.

When a sufficiently small dimensioned measuring instrument is used, the velocity at a given point in a steadily streaming fluid is found to be not constant, but to have very rapid variations in magnitude of an apparently random character. Thus figure 9 shows the velocity reading obtained by Dryden[9] in a wind tunnel with a hot wire anemometer made of Wollaston wire, 0.008 mm (0.0003 in.) in diameter. This rapidly varying deviation, u, from the mean velocity U is called the turbulent velocity, and its root mean square with respect to time, $\sqrt{\overline{u^2}}$, is called the intensity of the turbulence. The ratio of the turbulence intensity to the mean velocity in figure 9, according to Dryden is as much as nearly 0.01, even in a wind tunnel where every effort is made to eliminate turbulence as far as possible. In the cases discussed in the previous section the ratio of the turbulence intensity to the mean velocity must be very much greater. One, or an even greater number, would seem to be a not unreasonable figure for this ratio. We may then certainly take 10,000 cm per second as a conservative estimate of the intensity of turbulence in the arc columns placed in rapidly streaming fluids in the examples discussed in the previous sections.

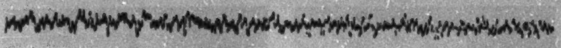


Figure 9. Turbulence in a wind tunnel, by Dryden

Turbulence of wind tunnel, $\sqrt{\overline{u^2}}/U = 0.0085$. Time = 0.4 second approximately. Relative amplification = 64

The turbulence may perhaps be pictured as a large number of small eddies or vortices superimposed on the average smooth motion of the fluid. These eddies are of constantly changing size, and velocities, but in a steady state of flow show a statistical constancy. To reveal the turbulent motion, the velocity measuring instrument must have dimensions small compared to the geometrical dimensions of the individual eddies.

This picture suggests how turbulence may bring about diffusion. A very small portion of the fluid upon which we fix our attention, takes part in the motion of an eddy. A moment later, however, it is part of another eddy, having a different center. It can thus wander from eddy to eddy, off the average flow lines of the fluid. But such motion we call diffusion.

A great similarity will be noted between this picture, and the picture of diffusion in a stationary gas offered by the kinetic theory of gases. According to this kinetic theory, even for a gas at rest, the individual molecules have velocities which are random in direction and magnitude. The mean random speed, c, of the molecules is of the order of acoustic velocity of the gas, or for normal air $3 \cdot 10^4$ cm per second.

The motion of a molecule moving in any particular direction will be arrested and changed into other random directions by collisions with other molecules. Thus the individual molecule will wander about in space in a random manner. This random wandering is molecular diffusion.

The speed with which a particular molecule wanders away from any particular fixed point in space will depend on the average absolute speed of the molecule, c, but it will also depend on the average distance the molecule goes before making a collision which changes the direction of its motion, that is the mean free path λ. The diffusion coefficient for the molecule, according to elementary kinetic theory is given by

$$D_m = \frac{1}{3} c\lambda \qquad (7)$$

In looking for an analogous expression to estimate the diffusion coefficient arising from turbulence, we already have a factor which would seem to correspond to the molecular speed, c, namely the turbulence intensity $\sqrt{\overline{u^2}}$. What will correspond to the mean free path λ? Most plausibly, the average eddy size!

G. I. Taylor[10] has recently shown how from measurements of the turbulent velocity, u, a length may be determined of just this character. Taylor considered the correlation coefficient between the turbulent velocities, u_1 and u_2 at two nearby points in the moving fluid. The correlation coefficient is defined as

$$R = \frac{\overline{u_1 u_2}}{\sqrt{\overline{u_1^2}}\,\sqrt{\overline{u_2^2}}} \qquad (8)$$

The dashes above the functions of time in equation 8 signify that the average with respect to time of the functions must be taken.

It may be shown mathematically that if two functions u_1, u_2, have a correlation coefficient R equal to unity, then the two functions must be completely identical in form, that is $u_1 = ku_2$, where k is a constant. On the other hand, if the two functions are completely unrelated, then u_1 is as likely at any moment to differ in algebraic sign from u_2 as to agree with it, and $\overline{u_1 u_2}$ and therefore R will be zero.

If the two points, 1, 2, being considered stayed always on the same eddy, then $u_1 = ku_2$, and $R = 1$. On the other hand if the points 1, 2, are always on different eddies, then u_1 and u_2 are completely unrelated, and the correlation will be zero, $R = 0$. The actual correlation R for two points, which will generally have a value between 1 and 0, may be regarded as measuring the fraction of the time the two points are on the same eddy.

The correlation coefficient may be directly measured by the use of appropriate electrical circuits used in conjunction with very fine hot wire anemometers. Figure 10 shows the correlation coefficient obtained by Dryden[9] as a function of distance of the second point from the first in turbulence produced in a wind tunnel by screens of different mesh sizes. As might be expected when the two points are very close together, the correlation coefficient is unity, and when they are far apart it is zero. The distance in which the correlation coefficient drops to zero gives a measure of the average eddy size.

Taylor[10] defined a turbulence scale factor

$$L = \int_0^\infty R\,ds \qquad (9)$$

where s is the separation of the two points. L may be regarded as the average eddy size, and should be the analogy of the mean free path λ, in the kinetic theory of molecular diffusion. From figure 10, we see that in the turbulence measured by Dryden, the scale factor is of the same order of magnitude as the mesh sizes of the screens producing the turbulences.

Leaning on the analogy with the kinetic theory of molecular diffusion in gases at rest, and referring to equation 7, we arrive at the strong expectation that the effective diffusion coefficient, D_t, due to the turbulence should be given by

$$D_t = B\sqrt{\overline{u^2}}\,L \qquad (10)$$

where B is a constant not very small compared to unity. The ratio of D_t to D_m may then be expected to be of the order of $\sqrt{\overline{u^2}}L/c\,\lambda$. We have just seen that c is of the order of acoustic velocity, and that in practical gas blast circuit breakers $\sqrt{\overline{u^2}}$ is also of the order of 10^4 cm/second λ, under normal gas conditions is of the order of 10^{-5} cm. What magnitude shall be taken for L?

Referring again to figure 10, which shows that the turbulence scale factor has about the same magnitude as the scale of the physical structure responsible for the turbulence we are led to expect that in the gas blast circuit breaker, L, will be about the same as the dimension of the arc section. Near current zero, in the axial flow breaker, this is about 10^{-1} cm, and for the cross blast breaker, with gas forming splitters, it is considerably larger. We find, then, with respect to order of magnitude,

$$D_t/D_m = B\sqrt{\overline{u^2}}L/{}^1/_3c\lambda$$
$$= 10^4\cdot 10^{-1}/10^4\cdot 10^{-5} = 10^4 \qquad (11)$$

Thus we see that under the conditions occurring in gas blast breakers the intense turbulence brings about an enormous multiplication of the effective diffusion coefficient. Even allowing for all the looseness in dealing only with orders of magnitude, we see that a 100fold enhancement of diffusion effects is not an unreasonable expectation.

This means that all the factors contributing to arc extinction, such as ion diffusion and cooling by thermal conduction are also multiplied 100fold. The difficulty then which Kesselring and Koppelmann[4] experience in adequately accounting for the interrupting capacity on the basis of diffusive effects alone, disappears.

VII. Summary

1. The displacement and diffusion theories of a-c arc extinction in rapidly flowing fluids are contrasted.

2. The formation, in the interior of a streaming perfect fluid, of the wedge required by the displacement theory is shown to be impossible according to accepted hydrodynamical theory.

3. The formation, at the edge of a gas forming splitter in a cross blast, of the wedge required by the displacement theory is shown to be impossible according to accepted hydrodynamical theory.

4. High speed motion pictures of arcs extinguished in gas blasts are shown to contradict the displacement theory and support the diffusion theory.

5. Rough calculations are given which show that the turbulence which may be expected to exist in the extinguishing arc in gas blast breaker, may multiply diffusive effects by 100 or more.

6. This multiplication of diffusive effects is stated to be sufficient to make the diffusion theory adequate for accounting for the high interrupting capacity of gas blast circuit breakers.

References

1. J. Slepian, AIEE Transactions, volume 49, 1930, pages 421–30; *Journal* of the Franklin Institute, volume 214, 1932, pages 413–42.

2. D. C. Prince and W. F. Skeats, AIEE Transactions, volume 50, 1931, pages 506–12.

3. D. C. Prince, AIEE Transactions, volume 51, 1932, pages 166–70.

4. O. Mayr, *E. T. Z.*, volume 53, 1932, pages 75–81; *Bull. Assoc. Suisse Elect.* volume 32, 1932, pages 605–10.

5. F. Kesselring and F. Koppelmann, *Arch. f. Elektrotech.*, volume 30, 1936, pages 71–108.

6. R. Holm, B. Kirschstein, and F. Koppelmann, *Wiss. veroffent. Siemens Konzern*, volume 13, number 2, 1934, pages 63–86.

7. D. C. Prince J. A. Henley, and W. K. Rankin; L. R. Ludwig and R. H. Nau, AIEE Transactions, volume 59, 1940 (September section), pages 510–22.

8. J. Biermanns, *Bull. Assoc. Suisse Elect.* volume 23, 1932, page 588.

9. H. L. Dryden, *Journal* of the Aeronautical Society, volume 6, 1939, pages 85–9.

10. G. I. Taylor, *Proceedings* of the Royal Society (London), volume 151, 1935, pages 421–78.

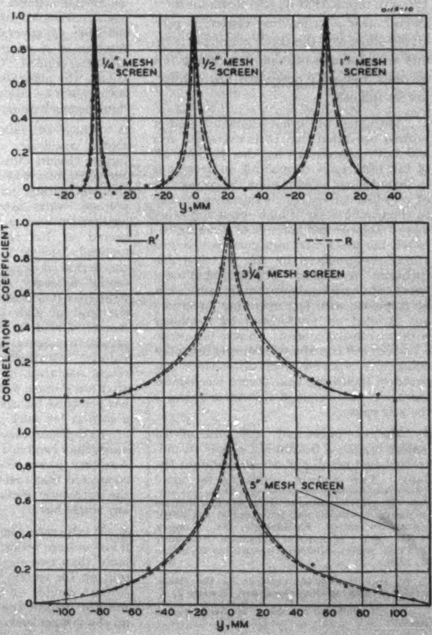


Figure 10. Curves showing variation of correlation coefficient with the cross-stream separation of the hot wires behind screens in wind tunnel, by Dryden

System Stability

F. W. GAY
FELLOW AIEE

Synopsis: That the specter of a major system shutdown is prominent in the thoughts of many operators is evidenced by the 1939 symposium, covering methods of modernization of station-switching facilities, and the 1940 symposium, discussing means of restoring service to systems after a major shutdown. In this paper, system and apparatus performances are studied, during the period between the time allotted for the functioning of the high-speed relay protection, and the point where the system becomes so unstable that a major shutdown occurs. With a view to the further improvement of system operation, the paper studies:

1. Phenomena preceding and during instability;
2. Method of lengthening the period between the fault and instability;
3. A local high-speed back-up relay scheme;
4. Means of automatically and quickly stabilizing an unstable system; and
5. In case all of the above fail, progressive isolation of the system by zones selected by degree of low voltage.

THE length of time to reach the critical period of instability for severe types of faults, that is, a three-phase short circuit near a generating center, is relatively short and the three major phenomena which come into play within a few seconds, following persistent low voltage, are as follows:

1. Machines, which have demonstrated in normal operation that their overspeed trip will not operate following the throwing off of full load, have tripped off on overspeed in three to four seconds upon the occurrence of very low voltage. This appears to be due to the combination of high speed, following loss of voltage and hence load, hunting between machines, and abnormal mechanical vibrations produced by the electrical disturbance. A variation in the setting of overspeed trips may be a contributing cause, that is, machines with fast operating governors are speeded up (motorized) by machines with poor speed control. In rare instances, a turbine will trip almost instantly upon the occurrence of an electrical disturbance, apparently indicating that severe mechanical vibration, as a result of this disturbance, is the sole cause.

2. Modern pulverized fuel and oil-fed boilers require a continuous supply of pulverized fuel or oil and air to maintain the flame. Low voltage, persisting for more than a few seconds is very liable to upset the carefully balanced supply of fuel and air and to result in flame extinguishment. The boiler must then be restarted by hand to avoid a possible explosion. Temporary loss of steaming capacity results. Low voltage, persisting a still longer time, may cause circulating pumps and other vital auxiliaries to fail.

3. Generating stations, or even individual machines, may pull out of step and overspeed, resulting in heavy crosscurrents between generators and a further drop in the already low voltage.

Relief from the above conditions may be had as follows:

1. It is believed that a major portion of all generators on a system will be found to have overspeed trips which may have their trip latches forced back into position when running at speeds well over normal. On two modern machines, turning at 3,600 rpm and having overspeed latches, it was found possible to force back the overspeed latch which tripped at 110 per cent upon a reduction to 105.5 per cent and 103 per cent speed respectively. It should be possible, on most machines, to force back the overspeed trip and automatically return the machine to governor operation within a fraction of a minute after the overspeed latch has tripped. On many 3,600-rpm machines, the manufacturer requires that the turbine not be run at speed without steam flow for much more than 60 seconds.

2. Where switching facilities permit, essential auxiliaries may have their transformer circuits closely connected to one of the generators which may be tripped from the bus during periods of persistent low voltage, while other parallel sources of auxiliary power are simultaneously disconnected so that this generator will automatically become a house generator during the period of persistent low voltage. Essential auxiliary electric motors, serving powdered fuel boilers, should be of ample size and of high pull-out torque. Such motors should be capable of maintaining satisfactory operation down to 60 per cent voltage. An alternate method is to provide relays operating to clear the station bus after low voltage (below 60 per cent normal) has persisted for several seconds. Clearing a station bus may involve resynchronizing after the disturbance has passed and in such a case may involve a temporary loss of load. Auxiliary power transformers must be so connected that this load is not dropped by the automatic withdrawal from service of any single bus.

3. It is becoming generally recognized that, if low voltage persists on a system for much more than two seconds, it is necessary to split up the system into a number of parts so that trouble on one part of the system will not involve the others. This splitting up of a system leads to certain consequences which must be cared for by the operation of the system continuously in preparation for such a split-up; otherwise, some of the parts will have an excess of generating capacity while other parts will be deficient in generating capacity. That particular section of the system, on which the disturbance originated, may be short of generator capacity following the disturbance. It may, therefore, be necessary to drop load temporarily, either manually or automatically. Such reduction in load can probably best be accomplished by automatically reversing the normal function of feeder-voltage regulators and load-ratio control transformers so that, with a reduction of frequency below approximately 58 cycles, there will be a reduction in service voltage to the limit of this regulating equipment's capacity to buck down voltage. This low voltage will be maintained until normal frequency is closely approached.

Whenever a careful study is made following a system disturbance, it is found that communication facilities, which are adequate for normal operation, are inadequate to permit quick restoration of service following the disturbance. It is believed that eventually it may be necessary to install an entirely different type of communication system if it is desired to handle emergencies efficiently. One such system is suggested as follows:

The head operator at the time of the disturbance shall act as a broadcaster giving orders only, and his broadcast shall be audible to all parties who may have an interest in the orders given. His lieutenants will receive information from the field and set up this information in the form of a load dispatcher's board so that the head operator can visually note the actual instant condition of the system. The head operator's orders will carry to every generating station and switching station operator simultaneously and will include, at least, the turbine room operator, the boiler room operator, and the pump room operator in the generating stations so that each of these men will instantly know when their equipment is, or is about to be called on for increased effort. Any company executive or engineer can be plugged into this circuit and can follow the course of the trouble and the restoration of normal conditions.

Twenty important generators in the Public Service Electric and Gas Company system are, or soon will be, equipped with field adjusters. These field adjusters are inoperative during normal conditions, allowing machine-field strengths to be maintained by hand operation at or above stable values, and allowing reactive volt-amperes as well as watts to be allocated by the load dispatcher. Upon the occurrence of low voltage from any cause, these field adjusters operate instantly to add approximately one-third of normal field current to the field of each generator. Upon the return of normal conditions, the fields are automatically returned to their predisturbance value. The amount

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1. For all numbered references, see list at end of paper.

of field current added to each generator is chosen so that no injury should result to any field, even if the high-field current is maintained for many minutes. An alarm warns the operator of high-field current and a volt-ampere (ohm) chart indicates meter readings which must not be exceeded.

Persistent low voltage may so reduce the pull-out torque of a generator that it is unable to transmit to the connected low-voltage system the entire output of its connected turbine which is being supplied with a full quota of steam. The unit will speed up until the governor reduces the steam supply to balance the reduced load and the generator will then continue to run out of step as an asynchronous generator. If the unit is equipped with a field adjuster, it should not trip on overspeed.

The Public Service Electric and Gas Company are equipping five turbine-generators with resynchronizing governors which it is hoped, will overcome this difficulty. Associated with each of these governors is an out-of-step relay. This relay determines an out-of-step condition by reason of the simultaneous occurrence of heavy current and pulsating kilowatts. Upon the occurrence of such phenomena, the relays proceed to close the steam valve and this closing operation continues until the out-of-step relay determines that the generator has resynchronized. (This relay does not alter the setting of the governor, but closes the steam valve against the governor.) When the relay determines that the generator has resynchronized, it returns the unit to the governor by backing off slowly so that the rate of steam supply is changed relatively slowly.

The amount of time used to reload the turbine is proportional to the amount of time taken to resynchronize, that is, the rate of change of load is constant.

Instability is a function of voltage loss. It follows quickly on the heels of a severe voltage drop and more slowly for lesser voltage drops. The following is Mr. Dean's conclusion as stated at last winter's convention, and is a good summation of experience in this respect:

> "It is interesting to note that the index upon which the necessity for emergency switching turns is the voltmeter reading. It has been found from experience that under fault condition, the behavior of current indicating devices is often so erratic as to make them undependable *and that persisting low voltage is about the only reliable criterion.*"

Figures 1 and 2 illustrate a suggested method of back-up relaying for a utility system, based entirely on low voltage. In figure 1, curve *T* shows the characteristics of a simple timing relay which operates to close its trip contacts for a voltage on any phase to ground below approximately 60 per cent of normal and its associated timer closes the trip contacts after an elapsed time of 32 cycles.

Curves *A* and *B* show the characteristics of timing relays which comprise a voltage element on each phase, operating to close the contacts of a timing element on that phase, for phase-to-ground voltage on any phase below a desired minimum. The timing element comprises a watt-hour meter and a rectifier operating across a resistance to normally charge a battery through this watt-hour meter.

A battery, allocated to each phase, is continuously charged from phase-to-ground voltage on that phase through a step-down transformer and full-wave rectifier. A watt-hour meter in this battery charging circuit is connected to operate in reverse for normal charging. A high-speed register on this watt-hour meter turns in reverse against a stop. For phase-to-ground voltages below a desired critical value, the battery discharges through the resistor and at a rate proportional to the drop in voltage below the desired critical value. During the battery-discharge period, the watt-hour meter turns its high-speed register to close the tripping contacts. The rate of closure is proportional to the drop in voltage below the desired critical value.

The multitrip relay opens the main battery-discharge circuit through the potentiometer resistance of the timing relay and the timing relay slowly resets. This circuit is also held open by the voltage element that operates the timing element described above. The reclosing of this circuit follows the reclosing of the multitrip relay only after the re-energizing of the cleared circuit.

For a meter adjusted according to curve *A*, the transformer and resistance taps are chosen so that the battery is discharging for phase-to-ground voltages below 40 per cent of normal and, when adjusted according to curve *B*, the battery is discharging for voltages below 80 per cent of normal.

A suggested use of these relays is illustrated in figure 2 wherein a system, *Y*, is connected to the *NY* bus of station *N*, having short-circuit capacity on the bus *NY* of 1,000,000 kva with system *Y* alone connected; that is, a per-unit reactance for system *Y* of ten per cent, based on 100,000-kva unit. The bus *NY* is shown connected to the red and blue busses of station *N* by transformers *T*-5 and *T*-6. These transformers are of the double-wound secondary type and have an equivalent primary negative reactance of minus 15 per cent (per unit of 100,000 kva) and an individual secondary reactance of plus 60 per cent. One of the secondaries of each transformer is shown connected to the red *N* bus and the other connected to the blue *N* bus. Additional stations *O*, *P*, and *Q* are shown with *N* and each have a red and blue bus. Load on each red bus is carried by transformers such as *NRT*, *ORT*, etc., each feeding a low-voltage load bus, *L*-1, and load on each blue bus is carried by trans-

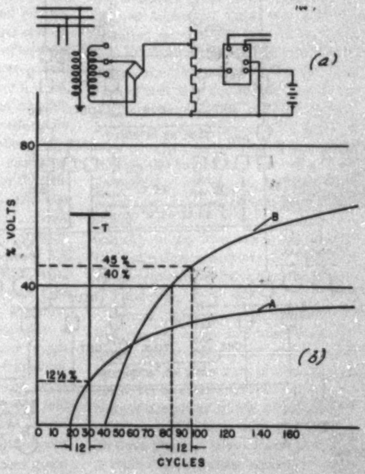

Figure 1a. Diagram of timing element in suggested relay scheme b. voltage-time curves

Curve *A*—Timing element set for 20 cycles (maximum closing speed); range adjusted for operation between zero and 40 per cent volts

Curve *B*—Timing element set for 40 cycles (maximum closing speed); range adjusted for operation between zero and 80 per cent volts formers such as *NBT*, *OBT*, etc., each feeding a low-voltage load bus, *L*-2. The *L*-1 bus is shown connected to the red bus of each station through a transformer having a per-unit reactance of 20 per cent and to a tie bus in common with *L*-2 through 20 per cent reactors. The *L*-2 low-voltage bus is similarly connected to the blue bus and the tie bus. At stations *P* and *Q* two 100,000-kw generators (125,000 kva) are shown, each connected to the red and blue bus through a two-winding secondary transformer having equivalent negative primary reactance of 15 per cent, and a secondary winding reactance of 60 per cent for each winding alone. One secondary of each transformer is shown connected to the red bus and the other secondary to the blue bus. The arrangement shown is identical at station *P* and at station *Q*.

If, now, it is assumed that a three-phase short circuit occurs on the terminals of the power transformer connecting the high-voltage blue bus to the *L*-2 low-voltage bus at station *P* (short-circuit *X*, figure 2) and further, that the relays on this short-circuited transformer bank fail to operate, then a three-phase short circuit will remain at *X* on the blue bus *P* until the *PBT* transformer breaker is opened. (This fault will also remain on low-voltage load bus *L*-2.) The short circuit will, in effect, be a bus short circuit. To this bus, as well as to all other busses, is shown connected a relay of the *A* type, figure 1. With zero voltage on the blue bus at station *P*, the type *A* relay battery will discharge its maximum current through the current coils of the wattmeter-relay element and the high-speed register will turn at maximum speed to close the tripping contacts. As shown on curve *A*, figure 1, these contacts will be closed at approximately 20 cycles after the short circuit occurs, and since the high-voltage breaker on the transformer feeding the *L*-2 low-voltage bus has not operated, all high-voltage breakers connected to the *P* bus will be tripped and the *P* blue bus will be killed within the next 12 cycles. If pilot wire or carrier current is available, the breakers at both ends of lines leaving the blue bus at station *P* may and should be tripped. Voltage-timing relays *T* are shown for the reactor tie busses connecting load busses *L*-1 and *L*-2 at each station. With a short circuit at *X*, figure 2, as shown, the voltage on the tie-reactor busses at stations *N*, *O*, *P*, and *Q* may be less than 60 per cent normal and, if so, these relays start functioning to trip the *L*-1 reactor breakers so that the red and blue busses at stations *N*, *O*, *P*, and *Q* may not be connected through the low-voltage reactor busses. With a short circuit at *X*, as shown, the blue bus at *P* will be removed from circuit before relays *T* operate and only the *T* relay at station *P* will function, due to the fault on its connected transformer bank.

A study of curve *A* will show that, for the arrangement shown, if blue busses at stations *O* and *Q* have a voltage in excess of 12.5 per cent of normal, there will be no danger of the type *A* relays on these busses beating the type *A* relays on the *P* bus. (See dotted lines in figure 1.) With the arrangement as shown, the voltage on the red bus during the short at

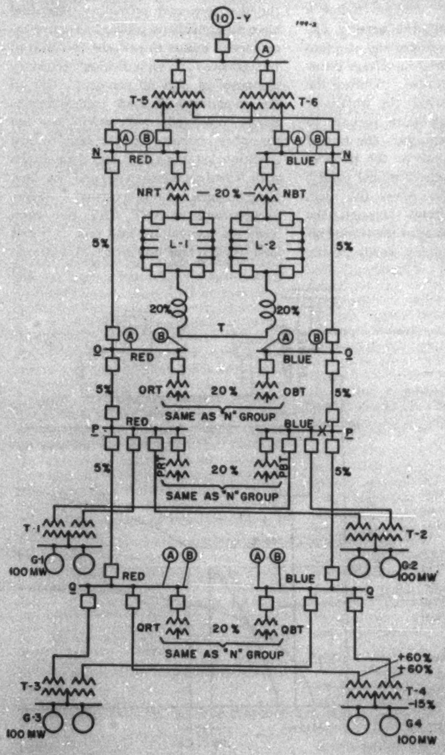

Figure 2. System layout

Employing three-winding transformers, each having a per-unit through impedance of 15 per cent (100,000-kva unit) and minus 15 per cent equivalent negative primary impedance, i.e., 45 per cent impedance through one high-voltage winding

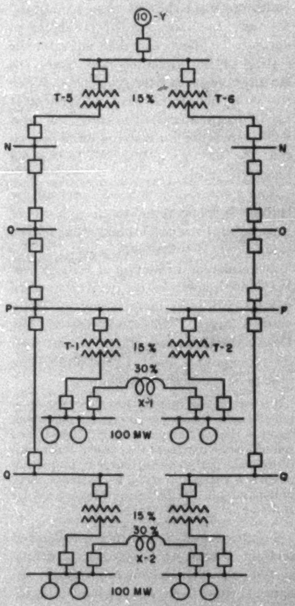

Figure 3. System layout

Similar to figure 2 but employing two-winding transformers each having a per-unit through impedance of 15 per cent (100,000-kva unit)

X will be over 100 per cent and on the blue bus at stations *O* and *Q* there will be a safe margin of voltage over the 12.5 per cent minimum required. The only load that will be lost will be that of the *L*-2 bus, connected to the faulted transformer bank. This bus may be reenergized immediately through the reactor circuits. A short circuit on any high-voltage bus should be cleared by this back-up relay scheme without danger of killing any adjacent bus. Curves *A* and *B* show that, as voltages, during a fault, become higher, the selectivity improves.

The use of relays with a *B* setting are required for protection against persistent low voltage originating outside the high-voltage circuit of the system under consideration as, for instance, trouble on the interconnected system *Y*. If the trouble is sufficiently serious to endanger service on the red and blue bus, the *B* relays at station *N* will function and are connected to trip the oil-circuit breakers of transformers *T*-5 and *T*-6. Again, if the generators or turbines at, for instance, station *Q*, should be in serious trouble, the *B* relays at station *Q* will function to open the output breakers of these generators.

No brief is held for the operating time used in this discussion except that the operating times closely approach the minimum. Longer times will obviously give greater selectivity.

The advantages of a relay scheme similar to the above are as follows:

1. Any generating station bus which has a voltage on it sufficiently low to cause hazard to auxiliary power-station equipment will be removed from service by the back-up relays in a period so short that the functioning of the auxiliary power equipment will not be seriously disturbed.

Note: With this system, auxiliary power transformers must be connected to generator circuits before these circuits reach the station busses, since station busses may be automatically killed in case of a bus fault.

2. A bus at any generating station will be removed from service so quickly that there will be no danger of unloading generators with consequent tripping of overspeed relays and temporary loss of capacity on the system.

3. The low voltage will exist for so short a time that system stability will not be in danger.

Figure 3 differs from figure 2, described above, in showing a more usual transformer arrangement at generating stations *P* and *Q* and interconnecting station *N*. All load circuits *L*-1 and *L*-2 are omitted in figure 3 in the interest of simplicity, as are the red and blue designations. A 100,000-kw (0.8 power factor) generator is shown connected directly to the primary of each of two 125,000-kva transformer banks, *T*-1, *T*-2, *T*-3, and *T*-4 and the transformer primaries at each station, as at *P* and *Q*, are shown connected together through reactors having a per-unit value of 30 per cent (100,000-kva unit). The six transformers, *T*-1, *T*-2, *T*-3, *T*-4, *T*-5, and *T*-6 in figure 2 and in figure 3, are shown to have a straight through reactance to balanced loads of 15 per cent; however, the two-winding transformers shown in figure 2 have an equivalent negative primary reactance of minus 15 per cent and a positive reactance of 60 per cent for each of the high-voltage windings singly so that the reactance from any individual generator to either the red or the blue bus will be 45 per cent for the transformer and, adding the generator subtransient reactance of 10 per cent, 55 per cent total to an individual bus. For balanced loading, the reactance from an individual generator to both busses will be 15 per cent for the transformer plus 10 per cent for the generator, or a total of 25 per cent, and will be the same for both figures 2 and 3. If, in figures 2 and 3, a badly unbalanced condition is considered, that is, 60 per cent load on each of the blue busses against 40 per cent load on each of the red busses as shown, and the generators are loaded to capacity and there is no interconnection load, we find that the total load on the blue bus will be 240 mw and 180 mvar (300 mva) and on the red bus 160 mw and 120 mvar (200 mva). Each transformer shown in figure 2 will be equally loaded, that is, it will have 125 mva on its primary winding and 75 mva on one secondary winding and 50 mva on the other secondary winding. The total loss in each transformer in figure 2 will be approximately 102 per cent of normal, indicating that a small amount of additional copper will have to be added for both high-voltage windings above that required for a balanced load condition. In contrast, in figure 3 it is seen that the two transformers connected to the blue bus must take the maximum load, in both their primary and secondary windings, that is, the transformers will carry 120 per cent of the normal generator rating and will have 144 per cent load loss. The transformers in figure 3 must, therefore, have a continuous-carrying capacity of 150,000 kva per bank. In other words, the single-winding secondary transformers, shown in figure 3, must be something more than 15 per cent larger than the double-winding secondary transformers shown in figure 2. Furthermore, with the instantaneous reactances given, should there be a fault at *X*, as shown on the blue bus for station *P*, the voltage of the generator connected to transformer *T*-2 will be approximately 82 per cent at the instant of fault, while the corresponding voltage at generator terminals in figure 3 will be approximately 63 per cent.

Assuming that all generators are of the modern type, equipped with field adjusters and having a sufficiently high-exciter ceiling voltage to maintain approximately 200,000 kva per generator through the fault, the sustained voltage, with the arrangement shown in figure 2, will be approximately 90 per cent for each of the generators connected to the red and blue bus in station *P*, and 48 per cent for the generator connected to the blue bus in the arrangement shown in figure 3. Furthermore, since the blue bus at station *P* is short-circuited, synchronizing power for generator *G*-2 is through reactor *X*-1 to generator *G*-1 only. It is obvious that generator *G*-2, shown in figure 3, will fall out of step and run above synchronism in the assumed case of failure, that is, relays fail to clear the faulted transformer bank *PBT*, feeding load bus *L*-2 so that the blue bus at station *P* remains short-circuited.

In figure 2, both generators *G*-1 and *G*-2 are equally affected by the fault at *X* and also both generators are connected to the red bus and will not pull out with respect to each other or be unloaded and run over speed. Furthermore, the voltage of the red bus will be over 100 per cent of normal and there will be a powerful synchronizing tie to generator *G*-3 and *G*-4 at station *Q* over the line connecting the red busses at these two stations. Phantom current will flow through the line connecting the blue bus at station *P* with the blue bus at station *Q*. This current will flow due to the mutual inductance between the two high-voltage windings of the transformers *T*-1 and *T*-2. This phantom current will flow directly through the faulted blue bus but not into the fault and will serve to maintain a low impedance between generators *G*-1 and *G*-2 on the one hand, and generators *G*-3 and *G*-4 on the other hand, and between the individual generators *G*-1 and *G*-2 and between the individual generators *G*-3 and *G*-4.

Several years ago, a small model system was set up to determine if the impedance between two machines connected by two parallel transmission circuits could be maintained substantially constant while either of the connecting circuits were simultaneously grounded on all three phases. It was found that, with a relatively enormous equivalent nega-

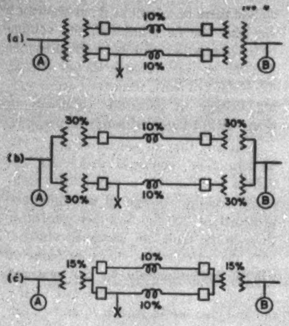

Figure 4

(a)—Simple two-circuit transmission system employing terminal transformers, each having two separately insulated high-voltage windings. Each transmission circuit served by an individual high-voltage winding at each end

(b)—Same as (a) above but each transmission circuit having its own transformer at each end

(c)—Same as (a) above but a single transformer at either end serving both transmission circuits at that end

tive primary impedance in the transformers at both ends of the lines, either connecting line could be short-circuited and the voltage on the nonshort-circuited line would be instantly doubled. The power would flow over the nonfaulted line at double voltage and a phantom power current of substantially equal value flowed over the short-circuited line but did not enter the short circuit.

As the equivalent primary negative impedance of the transformers at both ends of a dual circuit transmission is reduced by design from a relatively enormous value to zero, the effect of a single-line short circuit on the through impedance will be found to change, having substantially no effect for very high negative and double the impedance for no negative.

The value of equivalent-negative primary impedance in reducing fault currents has been well covered in the technical press but the value of such negative impedance in maintaining a stable system seems to have been neglected.

In order to better assure the reliability of large capacity, closely coupled systems, it is believed that operators and manufacturers must find some solution to the factors causing instability outlined in this paper. While the experiences described cover those of one system only and the remedies suggested have had very limited or no field trial, it is hoped that after further development they will prove of substantial value.

Appendix

A two-circuit transmission system was set up on the circuit analyzer to determine the characteristics of a double-wound high-voltage transformer bank having various amounts of equivalent primary negative impedance.

Figure 4 (*a*) is a one-line diagram showing a hypothetical system *A* having infinite capacity connected to a like system *B* by a transmission circuit comprising two transmission lines, each having ten per cent reactance (100,000-kva unit) and being serviced by a transformer at each end, having a single low-voltage winding and two high-voltage windings. Each transmission line is served at each end by an individual high-voltage winding.

Figure 4 (*b*) shows a somewhat similar circuit connecting the two systems *A* and *B*. In this arrangement individual transformers are used in each transmission circuit, each transformer having twice the through impedance of the transformer shown in (*a*) above, that is 30 per cent.

Figure 4 (*c*) shows the same two systems, *A* and *B*, connected through individual transformers to the same two circuits. It is obvious that in normal operation, the performance of the three systems, *a*, *b*, and *c* will be identical, since the through reactances are the same and resistances are neglected.

If a three-phase fault is applied at *X* in each of these systems, it will not be possible to transmit any power between the system *A* and the system *B* with the arrangement shown at (*c*), while with the arrangement (*b*) the power transmitted between system *A* and system *B*, with the three-phase fault existing, will be only one-half the amount that may be transmitted without the fault. (The current flowing from systems *A* and *B* into the fault is of no consequence since these systems are assumed to have infinite capacity.)

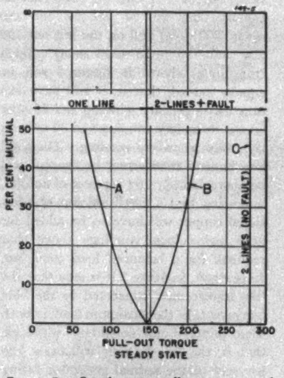

Figure 5. Steady-state pull-out torque of system shown in figure 4a

(a)—Variation of steady-state pull-out torque with per cent mutual for a single line in circuit, i.e., the faulted line cleared at both ends

(b)—Variation of steady-state pull-out torque with per cent mutual for two-line operation and a three-phase fault on one line at *X*

(c)—Variation of steady-state pull-out torque with per cent mutual for two-line operation

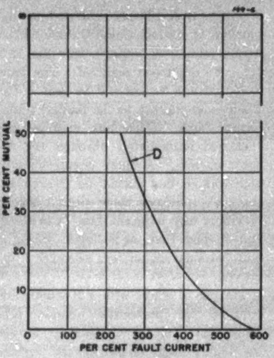

Figure 6. Variation of fault current with per cent mutual

In arrangement (*a*), if the equivalent negative primary impedance (per cent mutual) in the transformers is a maximum by design, then the current that will flow into fault *X* will be negligible, and the transmitting characteristics of the circuit shown in (*a*) will be the same whether the three-phase fault at *X* is present or clear. If the fault is on, the voltage on the nonfaulted line will be 200 per cent. The current flowing in the faulted line will be identical to that flowing in the unfaulted line. This current is called a phantom current and is induced in the faulted line by the mutual impedance in the transformer circuits. As the mutual impedance is diminished, the volts on the sound line during the fault will diminish and the phantom current in the faulted line diminishes.

Figure 5 shows the characteristics of circuit (*a*). The steady state pull-out torque in per-unit (100,000-kw unit) will be approximately 280 per cent, with both lines in circuit and without a fault. This pull-out torque will be a constant, regardless of the amount of equivalent primary negative impedance (per cent mutual) built into the terminal transformers, so long as there is no fault on either transmission circuit. If however, there is a fault on one circuit, the steady-state pull-out torque is shown by curve *B*. This pull-out torque will reduce with a reduction in equivalent negative primary impedance and for zero negative the circuit is the equivalent of (*b*), and the steady state pull-out torque will, of course, be one-half of what it is with an unfaulted line. If now the faulted circuit is opened at both ends to clear the fault, the transmission will be left with a single circuit and

The Water-Cooled Steel-Tank Rectifier Corrosion Problem

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Synopsis: The first mercury-arc rectifiers utilized glass tubes and consequently the physical size and electrical capacity was limited. The efforts to increase the available output led to the development of water-cooled steel-tank vacuum chambers after many years of research and the solution of many problems. This paper is primarily concerned with the solution of the problem presented by corrosion. Corrosion is essentially the returning of a metal to its original state and the problem consists in finding a means to retard the return sufficiently so that corrosion will not be a limiting factor in the life of rectifiers.

Analysis of the problem indicates that pitting corrosion causes the greater damage and is largely a function of the character and condition of the metal, the quantity of electricity passing, and also that it is self-propagative. The rate of corrosion is affected by the temperature of the cooling water, the chemical constituency of the water, and potential differences between points on the surface of the rectifier tank.

The first efforts in the solution of the corrosion problems were directed toward finding a protective coating to isolate the steel tank from the cooling water. Various paints, varnishes and lacquers, and enamels were investigated, as well as sprayed metal, but were not found satisfactory. Chemical treatment of the water was then investigated and sodium chromate found to give excellent results. The action of this chemical is reviewed and both laboratory and field evidence of its effectiveness submitted, followed by a discussion of the various factors to be considered in the application of this chemical to prevent corrosion.

Introduction

WITH few exceptions, all machines for the generation and conversion of electrical energy have been mechanical in nature. During the early part of the present century, Cooper-Hewitt and Steinmetz presented to the electrical world a conversion unit for transforming alternating current to direct current which was nonmechanical, but electronic in nature. This conversion unit known to us as the glass-bulb mercury-arc rectifier, has since its introduction won wide acclaim as an efficient, practical converter. Because of the fragility of glass, the physical size and current rating of the mercury-in-glass rectifier has been essentially limited. The desirable features of this particular type of converter are as follows:

(*a*). High operating efficiency
(*b*). Silent operation
(*c*). Absence of moving parts
(*d*). Comparatively long life

Studies were soon begun to increase the current rating, operating potential, and naturally its physical size. As it was impractical to increase the physical size of the mercury-in-glass rectifiers, a substitute for glass was sought. This investigation which was carried on by several groups, both in the United States and abroad, resulted in the adoption of the steel-tank vacuum chamber. Subsequent investigations indicated that by surrounding the steel vacuum chamber with a water jacket, the heat losses[1] could be effectively removed and the rating of the rectifier considerably increased. This transition from the glass to steel vacuum chamber and the increase in rectifier rating required several years of research and necessitated the solution of many problems before this new rectifier was available for service in the field. It is the purpose of this paper to explain the steps taken to overcome one of these problems, namely the corrosion problem, which became evident during the early days of the water-cooled steel-tank rectifier field performance. While this subject does not pertain to the phenomena of rectification, nevertheless, its immediate solution was necessary in order to give the assurance that the life of the water-cooled steel tank rectifier would not be limited by corrosion. This assurance would make its future manufacture and application economically justifiable.

The Corrosion Problem

Water when available in sufficient quantities is one of the most practical and economical mediums for controlling the temperature of machines. Likewise, iron is one of the most practical and economical metals for the fabrication of machines.

The application of water in contact with iron presents to the manufacturer and operator of electrical apparatus, the so-called "corrosion problem." This problem presents itself in varying degrees, occasionally of no consequence, and then again if not retarded, considerable damage results. This damage may appear as loss of good will, interrupted service, or expensive repairs. The performance or the life of electrical or mechanical apparatus of the present day should not be determined by the corroding away of the water-cooled iron parts.

Because of the many different theories regarding its nature, corrosion is generally studied from an academic point of view. When one studies the many possible causes for the corrosion of iron, one can readily appreciate why the early students of the corrosion problem, asked themselves, why ferrous metals survive corrosion at all.

Corrosion of metals is essentially the reverse of smelting, or in other words, corrosion is the returning of a metal to its original ore state. As corrosion in the theoretical sense cannot be entirely stopped, nevertheless, we must find means to retard it sufficiently so that corrosion will in no way be a factor to impair the performance or limit the expected life of apparatus, especially where water is used as a coolant.

The temperature of steel-tank rectifiers of early manufacture were controlled by tap water cooling, using whatever water was available for the purpose. In locations where water from city mains was not available, wells were provided near the station. As the quantity of water available was the criterion rather than the quality, many of the waters corrosive in nature were used as a coolant. The more corrosive waters attacked the iron parts of the rectifier and subsequently advanced stages of corrosion

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1. For all numbered references, see list at end of paper.

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with this single circuit, the one-line steady state pull-out torque will vary inversely as the negative impedance (per cent mutual), diminishing with an increase in such impedance in accordance with curve *A*. The current in the fault *X* will vary with equivalent primary negative impedance having approximately 580 per cent value for zero negative impedance (per cent mutual), and diminishing along the curve *D* to have a zero value for an infinite negative impedance (figure 6).

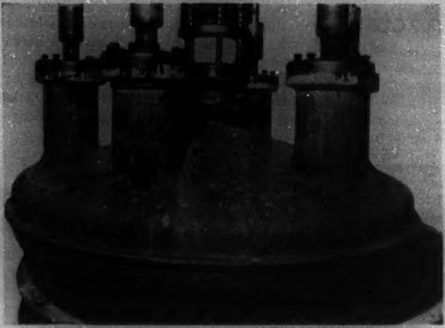

Figure 1. Tubercles on mercury-arc-rectifier vacuum chamber

were noted, some instances resulting in vacuum leaks in the rectifier or water leaks in the water jacket.

The Corrosion Study

Examination of the corroded parts of various rectifiers indicated that pitting corrosion rather than the so-called general corrosion was causing the greater damage. In "general corrosion" the anodic and cathodic areas which characterize all corrosion attack by water on iron are evenly distributed and of such fine grain, so to speak, that the net attack is fairly uniform and so slow as to be unimportant. This effect is largely a function of the character and condition of the metal. No way is known to insure that corrosion attack shall be confined to this type. "Pitting corrosion" occurs when positive anodic areas are set up surrounded by relatively large cathodic areas. This may result from patches of mill scale or other foreign matter or from inhomogeneity of the metal, as at welded junctions. It is the usual situation in fabricated structures since anodic areas are essentially areas of oxygen starvation. The resultant pitting is self-promoting: The condition once established tends to become worse. Figure 1 shows the presence of tubercles of iron oxide resulting primarily from pitting corrosion. These tubercles are small mounds which grow in size in proportion to the quantity of electricity passing between the covered portion acting as anode and the surrounding metallic surface acting as cathode, and are generally found at welded junctions, breaks in mill scale or at a point where a foreign substance is attached to the iron. From within the tubercle or pit anodically produced iron salts diffusing outward meet a relatively alkaline water from the cathodic area whereby the iron is precipitated forming mounds of black magnetic iron oxide, which further obstruct the access of oxygen. Being loose and not adherent, they do not constitute a protective film. Deposits of this nature, when not removed and the pits repaired or thoroughly cleaned, ultimately cause perforation of the water jacket. Tubercles shown in figure 1, occurred at the hottest point and on a welded surface. Figure 2 shows the surface of a rectifier water jacket before cleaning. Many tubercles and much water sediment is evident, with a fertile field for corrosion. Figure 3 shows the cleaned surface, with the pits in the iron jacket plainly visible. It is readily seen that such disintegration if allowed to continue will reduce the active life of apparatus. The application of sodium chromate even in this advanced stage will stop corrosion and make corrosion no longer the limiting factor of the machine's life. This preliminary study taking into consideration the several variables all conducive to corrosion, placed the rectifier corrosion problem in a category of its own. These variables may be enumerated as follows:

The rectifier water temperatures are variable, often ranging from tap water temperature to boiling water temperature.

The waters in different localities also differ in chemical composition.

During operation the steel tank rectifier has inherent differences in potential between points on its surface.

The rate at which corrosion occurs, is greatly increased with increase in temperature. This is due mainly to the fact that both the electrical conductivity of an electrolyte and the rate of chemical action increase with temperature. It is for this reason that water-cooled parts of rectifiers operating in excess of 50 degrees centigrade corrode at a considerably higher rate than those operating at lower temperatures.

It is of interest to note the differences in chemical constituents of the various waters used as rectifier coolants throughout the United States. These waters are classed as lake waters, river waters, mountain spring waters, and local well waters. The following table shows the extremes, in parts per million (ppm) of the constituents in these various waters:

	Maximum	Minimum
Total hardness ($CaCO_3$)	370	3
Total dissolved solids	353	19
Silicon (Si)	39	1.6
Iron (Fe)	1.4	0
Calcium (Ca)	107	0.8
Magnesium (Mg)	25	0.2
Sodium (Na) Potassium (K)	74	1.2
Bicarbonate (HCO_3)	289	7.3
Sulphate (SO_4)	120	1.7
Chloride (Cl)	44	0.7
Nitrate (NO_3)	25	0

The excess of some of the constituents listed in the above table are harmless, while the excess and combination of others provide fertile grounds for corrosion.

The presence of calcium carbonate is nearly universal. Where it is not associated with an excess of free carbonic acid it tends to precipitate along with the rust on iron surfaces forming a dense scale which resists further corrosive attack. In fortunate cases, the accumulation of this scale is so slow that it does not cause troublesome interference with heat transfer. Where, on the other hand, an excess of free carbonic acid is present deposition of calcium carbonate is prevented and the water becomes "aggressive" toward iron.

The first step in the search for a remedy of the rectifier cooling system-corrosion problem was naturally to find a protective coating which would perfectly isolate the iron parts of the rectifier from the water of the cooling system and thus prevent corrosion.[3] Asphaltum, metallic and metallic oxide paints recommended for their corrosive-preventive merits were studied. Varnishes and various lacquers were investigated. The corrosion-resisting properties of sprayed molten metals and their combinations were likewise investigated. Both air drying and baked vitreous enamels applied to iron surfaces were given vigorous tests.

The tests revealed that the binder in many of the air-drying applications would dissolve in warm water, while others would dry sufficiently porous to permit corrosion to develop.

It was also found that sprayed metal

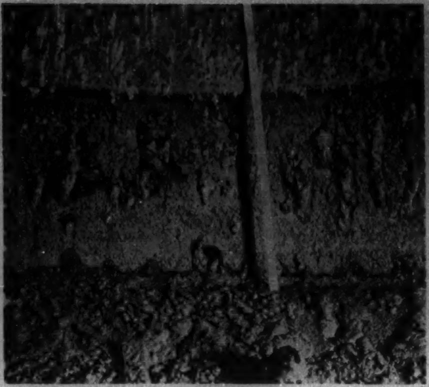

Figure 2. Tubercles on mercury-arc-rectifier water jacket

Figure 3. Electrolytic corrosion on mercury-arc-rectifier water jacket

would not make a sufficiently homogeneous surface to prevent water from reaching the iron. Similarly it was discovered that metals near the alkali end of the electromotive-force series would not long withstand the solvent action of the water. Therefore the effective life of metals of this nature would depend on the thickness of the coating.

Enameled surfaces in iron were found not sufficiently homogeneous to retard electrolytic corrosion. The application of vitreous enamels in preventing corrosion of rectifier surfaces was found very detrimental because any microscopic hole or crack in the enamel surface would take the total electrolytic current for the section and thus in a comparatively short time sufficient disintegration would occur to cause a vacuum leak.

Following the negative results of this initial investigation, it was decided to study methods of chemically treating the circulating water. For this test commercial corrosion-preventive liquids, oils, and many miscellaneous chemicals were studied. These were studied at constant and varying temperatures for their general and electrolytic corrosion-retarding qualities.

The study revealed several more or less promising treatments of the water for retarding general corrosion under the operating conditions of power rectifiers, but only one, a soluble chromate, $Na_2CrO_4 10H_2O$, effectively controlled both general corrosion and the intensified action due to electrolysis.

Sodium dichromate $Na_2Cr_2O_7 2H_2O$, is usually used to supply the chromate for a corrosion-inhibiting solution, because this salt is readily obtainable and is of low cost. Many natural waters are sufficiently alkaline to convert this to the chromate. Where this is not the case it is advisable to add the requisite alkali. This does not materially affect the anticorrosive action of the chromate, but may prevent a rapid loss of the chromate from certain unwanted side reactions due to impurities in the water.

As an example of the effectiveness of a sodium chromate-treated coolant, the photograph figure 4 is submitted, showing three iron electrodes, all immersed in their respective containers for 12 days. The electrodes were immersed in separate containers, *A*—the positive electrode of a cell containing Schenectady tap water, *B*—the positive electrode of a cell containing a one-half per cent solution of sodium chromate using Schenectady tap water. A five-volt potential was impressed between the electrodes of the two cells. Electrode *C* was placed in a container of Schenectady tap water, with no potential impressed. Electrode *C* is an example of the so-called general corrosion, a fairly uniform covering of reddish-brown iron oxide. No tubercles or concentrated corrosion is noted on this specimen. Electrode *A* shows the effect of electrolytic disintegration, much of the iron having been removed by the effect of the few milliamperes of current. As a contrast, electrode *B*, which was immersed in a sodium chromate solution shows on microscopic examination no evidence of either the so-called general or concentrated form of electrolytic corrosion. It is interesting to note that by virtue of the relatively high electrical conductivity of the sodium chromate solution, approximately ten times as much current was passed between the electrodes of this cell in comparison with electrodes of the cell containing tap water and electrode *A*. Subsequent tests where the cell current was increased several hundred per cent, indicated that even with this unusual current, the one-half per cent solution of sodium chromate was capable of inhibiting electrolytic corrosion.

This passivating action of the chromate is somewhat complex. Recent research[3] indicates that it is not a simple oxidizing action. Other powerful oxidants such as the permanganates and

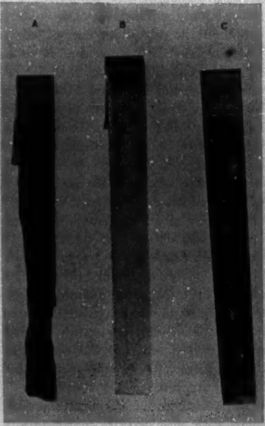

Figure 4. Three iron samples after corrosion test

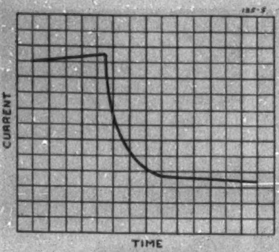

Figure 5. Current characteristic during growth of protective chromium film

hydrogen peroxide produce no such effect. The effectiveness of the protective oxide film produced, appears to be due not only to the simple production of a film of iron oxide but also to the inclusion of chromic oxide in the precipitate or rust formed in the simple corrosion reaction. This so modifies it that a dense and adherent protective film results rather than a loose nonprotective rust or tubercle. Of these factors the inclusion of Cr_2O_3, is probably the most important as the other action is also common to other oxidizing agents which have less or no protective effect.

The growth of the protecting chromate film is an interesting phenomenon, as shown in figure 5. With the deposition current as abscissa and time as ordinate we see what normally occurs in a few seconds or a few minutes, depending upon the chromate concentration and the existing difference in potential. The formation of the film takes place quickly, and then the deposition current which generally is of the nature of a few milliamperes, gradually recedes as the thickness of the film increases. Ultimately, the current reaches a minimum value. Laboratory cell tests made to determine the proper concentration of sodium chromate indicated, as would be expected, electrolytic corrosion at the positive terminal when untreated water is used. By the addition of a small amount of chromate it was noted that the effects of electrolytic corrosion became more pronounced. With each subsequent small increase up to a certain amount, the effects of electrolytic corrosion increased proportionally. When the critical amount was exceeded, it was noted that the evidence of electrolytic corrosion abruptly stopped.

The resistance of chemical solutions varies with their nature, concentration, and temperature, the higher-temperature solutions being the better electrical conductors. The chromate solution of the concentration used as the rectifier coolant is many times the better electrical conductor than is the average tap water. This fact is shown in the curves figure 6, where the electrical conductivity of Schenectady tap water and sodium chromate solution are shown at increasing temperatures. These relative curves show the trend of their respective conductivities as measured with one-sixteenth-inch diameter wire electrodes at a distance of one inch, and with current values approximating those found passing between different parts of the rectifier cooling system. The important consideration of these curves is that ample lengths of insulating coolant conductors must be provided between points of different electrical potential; so that the amount of leakage current will be negligible.

The application of chromate as a corrosion inhibitor is by no means new. Solutions containing chromate have been used for many years in the retarding of the general corrosion of iron.[4] The application of a sodium chromate coolant to specifically retard pitting corrosion in the steel-tank power rectifier, apparently is a new field of service.

The Applications of Sodium Chromate

The laboratory tests revealed that a one-half per cent solution of sodium dichromate was sufficient for the average water used as a rectifier coolant. This strength of solution has been found to be sufficient to inhibit corrosion when the solution is made up of any waters of potable purity. Where waters are available which are of exceptional purity, then it is possible to reduce the percentage of solution. A safe solution percentage can be determined definitely by the electrochemical means, explained below. Naturally the safest solution is made using distilled water, where the absence of foreign chemicals which would tend to react with the sodium dichromate and thus limit its life as a corrosion inhibitor is assured. Solutions made up of potable waters containing an excessive amount of chloride will require the full amount of sodium dichromate in order to obtain the full corrosion protection of the dichromate solution.

Before installing anticorrosive treatment in rectifier cooling systems, it is essential that all metal surfaces be perfectly clean. The coolant is made by adding a sufficient amount of the orange-colored sodium dichromate crystals of technical purity, to the water of the cooling system. A one-half per cent solution requires 4.2 pounds of the crystals to every 100 gallons of cooling water. The crystals are readily dissolved in cold or warm water and as soon as added to the system the coolant should be circulated.

The following method of checking the effectiveness of any given treated water has been found convenient and practical. A sample of the solution is placed in a glass bottle provided with two iron-wire electrodes spaced about one inch apart. Impress approximately five volts d-c potential across the electrodes. Should the solution be weak, a grayish-colored formation or precipitate will become visible within 5 to 30 minutes about the positive electrode, indicating anodic attack which may also be seen on close inspection of the electrode. The solution should then be strengthened and the test again repeated. Should the solution be of sufficient strength to retard corrosion, no precipitate will form around the electrodes. In order to eliminate any possible external influence, it is desirable that all such tests be made at a specific solution temperature maintaining the same size of bottle and dimensions for the electrodes.

Field Experience With Sodium Dichromate

Sodium dichromate has been used as a corrosion inhibitor in the steel-tank power rectifier since 1930. Since this time, all rectifiers in which the inhibitor solution was properly maintained have been free from both the general and concentrated pitting corrosion. Prior to the application of sodium dichromate, the temperature of practically all steel tank rectifiers, was controlled by tap water cooling. At the present time only 12 per cent of the rectifiers have tap-water temperature control, the remaining 88 per cent are provided with recirculating cooling sys-

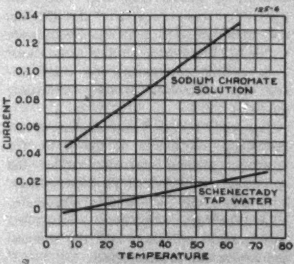

Figure 6. Electrical conductivity versus temperature

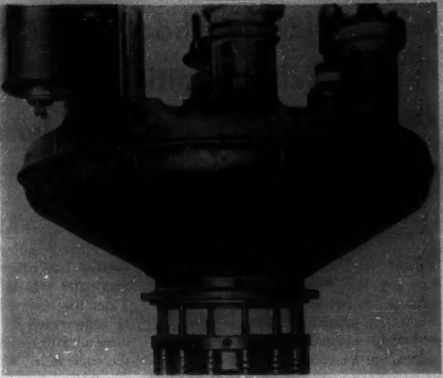

Figure 7. General Electric mercury-arc rectifier vacuum chamber after eleven months' operation with sodium dichromate coolant in place of tap water showing corrosion arrested

tems and treated water. Of the rectifiers originally operating with tap water cooling, approximately 25 per cent have experienced corrosion trouble terminating in vacuum leaks.

The life of a sodium dichromate solution depends upon the amount of make-up water added to the closed circulating system, and also on the chemical nature of the raw water. Our experience indicates that some solutions using mountain waters which are essentially pure, have a life in excess of three years. Well waters which are potable but have a higher impurity content, when made into an inhibiting solution have a life ranging upward from nine months. As already indicated, it has been found that the addition of small amounts of caustic soda ($NaOH$) to the chromate solution would increase its effective life. The function of the soda is to convert the inhibitor to a neutral chromate. A proportion of 1.5 ($NaOH$) to 5 ($Na_2Cr_2O_7 2H_2O$) ordinary crystalline dichromate, is sufficient in most cases. Slightly more ($NaOH$) than this proportion will do no harm. It has been found desirable to make periodic electrochemical examinations of the efficiency of an inhibiting solution; this can be done monthly or semiannually, depending upon the chemical nature of the water used, and the amount of water added for make-up.

Figure 7 shows the rectifier tank indicated in figure 1, but following the application of sodium dichromate. This rectifier had previously operated on a closed cooling system using untreated water, during which time the carbuncles, some shown in figure 1 had produced the pitting indicated. Following the first 11-month application, no new indications of pitting were evident and the vacuum chamber was in a clean condition as indicated in figure 7. Since this photograph was taken, this rectifier has been operating continually for the past six years without corrosion troubles.

Various Types of Cooling Systems

Sodium dichromate coolant is equally as well adapted to either water-to-water or water-to-air type of temperature regulators. The application of either type depends naturally on the particular installation. Locations such as the railway subways, where a comparatively high ambient temperature prevails at all times, the water-to-water temperature regulator is the most satisfactory. In such cases where the prevailing ambient temperature is slightly below the operating temperature of the rectifier, water-to-air temperature regulators can be applied.

Water has been found to be the most practical coolant where the heat to be removed is relatively high. In apparatus where the watts per square inch are relatively low, forced air, or circulating oil may be utilized. For certain applications, such as the cooling of rectifier mercury-condensation pumps, tap water cooling is the most practical. Here the amount of heat to be removed is small, the rate of water flow is low and the apparatus to be cooled is relatively small. The apparatus being small, makes it possible to confine the water cooling system in a copper jacket or copper cooling coils.

Low Ambient or Freezing Protection

Water cooling systems utilizing the recirculating principle can be protected from freezing by maintaining a higher temperature by means of jacket water heaters or by the use of a nonfreezing solution. Both systems have been found practical. Where tap water cooling is used, it may be necessary to lag the water piping sufficiently to prevent freezing, and insure that a constant water flow is maintained.

Maintenance of Cooling Systems

All cooling systems require a certain degree of maintenance, depending upon the nature or chemical content of the water used.

Closed cooling systems utilizing sodium dichromate as corrosion inhibitor have been found to require the least attention. Many systems have operated for several years without the necessity of cleaning.

Tap water-cooled apparatus must be descaled frequently to prevent water stoppage. Here again the frequency of cleaning will depend upon the amount of mineral water in solution and the amount of matter in suspension, both of which will eventually clog the water system. Copper water piping is no exception. It also readily scales, but naturally will not corrode as iron pipe does. Water containing excessive amounts of chlorine has increased corrosion effect on iron piping. Water containing much algii, or manganese dioxide, makes frequent cleaning of the water system necessary.

Electrical Insulation of Water Systems

In order to insulate a water-cooled apparatus or a part of the apparatus from parts of higher or lower potential, the coolant is conducted through insulating tubing such as rubber hose or rigid tubing. The two ends of an insulating tubing containing a coolant in general form the terminals of an electrolytic cell. It is natural that electrolytic disintegration will be expected at the positive terminal with electrolytic deposition on the negative end. In order to retard the accelerating form of corrosion in the cooling system, so-called "electrolytic leads" have been applied which consist of a long metallic rod of positive potential which gradually disintegrates whenever a difference of potential exists in between the ends of the insulating tubing. The disintegration of the metallic rod prevents the metal ends of the insulating tubing from disintegrating and will last several months or years, depending upon the prevailing

A New Method for Introducing Relaxed Initial Conditions in Transient Problems

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IN the calculation of transients in linear electrical and mechanical systems by the classical methods of differential equations, the constants of integration which arise in the solution are ordinarily evaluated from the initial values of the dependent variable and its derivatives.[1,2] The process of finding the initial derivatives is sometimes difficult for even rather simple systems, and the difficulties are particularly pronounced when the system of equations has undergone a change of variable to remove variable coefficients, as for example in Park's equations for the synchronous machine.[3] The purpose of this paper is to present a new method by which the initial derivatives can be found for an initially relaxed system (all currents in inductances and all charges on condensers initially zero) by a manipulation of the final differential equation itself. The solution for a nonrelaxed system can be found from the relaxed solution by superposition or by means of a systematic change in the original differential equations themselves, as is commonly done in operational calculus.[1,4,5]

The Differential and Integral Operators

The procedure in determining initial derivatives directly from the differential equation is based on the properties of the differential operator p (sometimes designated instead by the symbol D) and the integral operator $1/p$. It is shown in elementary treatments of linear constant-coefficient differential equations that if the derivative dy/dt be written py, where p indicates the operation d/dt, then p has many of the manipulative properties of an algebraic coefficient. Thus the second derivative d^2y/dt^2 can be represented by $p(py)$ or p^2y, and the integral with respect to time $\int y\,dt$, being the operation which cancels the first derivative, can be written symbolically $(1/p)y$. Similarly the iterated integral $\int\int..\int_n y dt$ is written $(1/p^n)y$.

In an initially relaxed system, all currents and voltages are zero for negative time. It is therefore possible in such a system to interpret the indefinite integral operator $1/p$ to mean the definite integral from 0– to t. The assignment of these limits does not interfere with the previous definition of $1/p$ as the indefinite integral, for the variable upper limit t provides this indefinite integral, and the lower limit merely supplies the proper constant of integration.

Examine next the integral $\int_{0-}^{t} y\,dt = (1/p)y$ as the upper limit of integration t approaches zero. It is evident from elementary integral calculus that under this condition the value of the integral approaches zero also, provided that y is not infinite at $t=0$. Thus we can write the useful relation

$$\lim_{t\to 0}\frac{1}{p}y=0 \tag{1}$$

which holds whenever the system is without energy at $t=0$, provided that y is finite at this time.

It will be observed that it is the quantity $(1/p)y$ and not $(1/p)$ itself which is zero at zero time. This is because $1/p$ is an operator and not merely a multiplier. The distinction will save much confusion, since if we wrote $1/p=0$ we should feel obliged to write $p=\infty$ at $t=0$. This, of course, is not correct, and will be avoided by observing that $(p)y$ is not the reciprocal of $(1/p)y$, and hence is not necessarily infinite when $(1/p)y$ is equal to zero.

The Problem

In the usual circuit problem a differential equation is set up for each mesh of the network, using Kirchhoff's laws. By expressing all of the derivatives and integrals in operator form and by manipulating the operator p as an algebraic coefficient, algebraic methods of solving simultaneous equations can be used to eliminate all of the dependent variables (e.g., currents) except the desired one. In doing this care must be exercised not to take any indicated derivatives but instead to retain all p's in operator form. By convention all operators are written to the left of the currents and voltages on which they operate. The resulting differential equation in terms of the desired variable is of the general form:

$$\phi(p)i=F_1(p)e_1(t)+F_2(p)e_2(t)+\,.\,. \tag{2}$$

in which

$e_1(t)$ is the voltage (in general a function of time) impressed in mesh number 1
$e_2(t)$ is the voltage impressed in mesh number 2, etc.
$\phi(p)$, $F_1(p)$, $F_2(p)$, etc., are polynomials in p

The polynomial $\phi(p)$ is generally as high or higher in degree of p than $F_1(p)$, $F_2(p)$, etc. If $\phi(p)$ is lower in degree of p than some $F(p)$, as is the case when solving for the current in a circuit in which a complete path through condensers exists between input terminals, the response i may contain an infinite impulse at $t=0$, and equation 1 will no longer hold. This difficulty can be eliminated by rewriting the differential equation in terms of charge q, where $pq=i$, and later taking the derivative of the solution for q to find the form of i.[1]

It is known from differential-equation theory that for any given system of differential equations there is one and only one set of solutions which is zero for negative time, and hence for any dependent variable in the system there is one and only one set of initial derivatives

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[1] For all numbered references, see list at end of paper.

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difference of potential and the nature of the coolant.

References

1. MERCURY-VAPOR PRESSURE CONTROL OF MERCURY-ARC RECTIFIERS, E. J. Remscheid. *General Electric Review*, December 1938.

2. RETARDING CORROSION IN THE WATER SYSTEMS OF POWER RECTIFIERS, Charles Van Brunt and Emil J. Remscheid.

3. THE PASSIVITY OF METALS, PART VII; THE SPECIFIC FUNCTION OF CHROMATES, T. P. Hoar and U. R. Evans. *Journal of the Chemical Society*, part II, 1932, pages 2456–81.

4. METALLIC CORROSION PASSIVITY AND PROTECTION, U. R. Evans, page 324.

which satisfy the condition of initial relaxation. We now desire to obtain directly from the differential equation 2 the values of these derivatives of the dependent variable i as t approaches zero from the positive side. In general, if $\phi(p)$ is of the nth degree, it is sufficient to know the initial values of the dependent variable and its first $n-1$ derivatives.

The Procedure

Since we know from the discussion in a preceding section that for an initially relaxed nonimpulsive system all integrals of current and voltage must be zero at $t=0$, it is advantageous to first put the differential equation 2 into integral form by dividing through by p to the highest power found in $\phi(p)$.* It will then be found that by making t and the various integrals $1/p$, $1/p^2$, etc., equal to zero everywhere in the integral equation, the initial value of current, $i(0)$, can be found immediately. Here t is considered to approach zero from the positive side.

To find the initial value of the first derivative of current, $i'(0)$, take the first derivative of the integral equation in a symbolic manner by multiplying through by p, take all indicated derivatives (denoting them by primes where necessary), and again set t and the various integrals $1/p$, $1/p^2$, etc., equal to zero everywhere. Solve for i', which will be di/dt at $t=0$ or $i'(0)$, where t is considered to approach zero from the positive side.

To obtain the initial value of the second derivative, multiply the integral equation through by p^2 and repeat the process. This procedure can be continued indefinitely until the required number of initial derivatives has been found. The process must be a successive one since each derivative will be expressed in terms of lower derivatives.

Since the method requires only the formation of the integral equation and the application of the fact that in an initially relaxed nonimpulsive system the various integrals of current and voltage are zero at $t=0$, it is simpler to apply the process to each problem separately without attempting to derive formulas.

Examples

(1) An A-C Circuit

The differential equations for the circuit shown in figure 1 can be written as:

$$(R+1/pC)i_1-(1/pC)i_2=E\sin\omega t$$
$$(-1/pC)i_1+(1/pC+pL)i_2=0 \qquad (3)$$

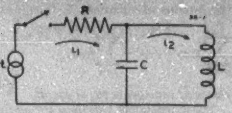

Figure 1. Circuit of example number 1

The elimination of i_2 by algebraic methods yields:

$$\left(RLp^2+\frac{L}{C}p+\frac{R}{C}\right)i_1=\left(Lp^2+\frac{1}{C}\right)E\sin\omega t \qquad (4)$$

The steady-state current can be found by the usual complex impedance method:

$$i_{ss}=\frac{E}{|Z|}\sin(\omega t-\phi)$$

where

$$Z=R+j\frac{\omega L}{1-\omega^2LC}$$

and

$$\phi=\tan^{-1}\frac{\omega L}{R(1-\omega^2LC)}$$

If the roots of the characteristic equation $(m^2RL+mL/C+R/C)=0$ are $m=-a_1,\ -a_2$, the total current is given by the expression:

$$i_1=\frac{E}{|Z|}\sin(\omega t-\phi)+C_1\epsilon^{-a_1t}+C_2\epsilon^{-a_2t} \qquad (5)$$

where C_1 and C_2 are the constants of integration to be determined from the initial conditions.

To find the relaxed initial derivatives, return to differential equation 4 and rewrite it as an integral equation by dividing through by p to the highest power found on the left side, p^2. This yields:

$$\left(RL+\frac{L}{C}\frac{1}{p}+\frac{R}{C}\frac{1}{p^2}\right)i=\left(L+\frac{1}{C}\frac{1}{p^2}\right)E\sin\omega t \qquad (6)$$

For the initial value of i, set t and the integrals $1/p$ and $1/p^2$ equal to zero in the integral equation 6. Then we have

$$(RL+0+0)i=(L+0)E\sin(0)$$

or

$$i(0)=0$$

For the initial value of di/dt, take the first derivative of the integral equation 6 by multiplying through by p. Then, taking the indicated derivatives and setting t and the various integrals equal to zero, we obtain:

$$RLi'(0)+\frac{L}{C}i(0)+0=LE\omega\cos(0)+0$$

or, since $i(0)=0$,

$$i'(0)=\frac{\omega E}{R}$$

Substituting these two conditions for time $t=0$ into the solution 5 and its first derivative, two equations in terms of the Cs are obtained:

$$C_1+C_2=\frac{E}{|Z|}\sin\phi$$

$$a_1C_1+a_2C_2=\omega E\left[\frac{\cos\phi}{|Z|}-\frac{1}{R}\right]$$

When the equations are solved simultaneously for the Cs, and these substituted into equation 5, the result is the complete solution for an initially relaxed system:

$$i_1=\frac{E}{|Z|}\left[\sin(\omega t-\phi)+\frac{(R\omega\cos\phi-Ra_2\sin\phi-\omega|Z|)\epsilon^{-a_1t}+(Ra_1\sin\phi-R\omega\cos\phi+\omega|Z|)\epsilon^{-a_2t}}{R(a_1-a_2)}\right]$$

(2) Solution of an Oscillatory Circuit

The circuit shown in figure 2 is one in which the initial value of the first derivative of input current is very difficult to find by ordinary means, and where the operational solution becomes rather laborious because of complex quantities which must be rationalized.

The differential equation for the input current can be shown to be:

$$\left[p^2L(R_1+R_2)+p\left(R_1R_2+\frac{L}{C}\right)+\frac{R_1}{C}\right]i=\left(p^2L+pR_2+\frac{1}{C}\right)E \qquad (7)$$

If the circuit is oscillatory a convenient form of the solution is:

$$i=\frac{E}{R_1}+\epsilon^{-at}(A\sin\omega t+B\cos\omega t) \qquad (8)$$

where

$$a=\frac{R_1R_2+L/C}{2L(R_1+R_2)}$$

$$\omega=\sqrt{\frac{R_1}{LC(R_1+R_2)}-\frac{(R_1R_2+L/C)^2}{4L^2(R_1+R_2)^2}}$$

To find the successive initial derivatives, first obtain the integral equation by dividing differential equation 7 through by p^2. Set the various integrals equal to zero and obtain

$$[L(R_1+R_2)+0+0]i=(L+0+0)E$$

or

$$i(0)=\frac{E}{R_1+R_2}$$

To find the initial value of the first derivative, multiply the integral equation through by p, take the indicated derivatives, and set the various integrals

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*For a proof of the method, see the appendix.

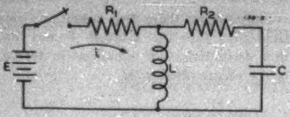

Figure 2. Circuit of example number 2

equal to zero. This yields

$$L(R_1+R_2)i'(0)+(R_1R_2+L/C)i(0)+0=\\ 0+R_2E+0$$

and since the value of $i(0)$ has just been found, we can solve for the desired $i'(0)$, obtaining:

$$i'(0)=\frac{E(R_2^2-L/C)}{L(R_1+R_2)^2}$$

If these values of i and i' are substituted into the solution 8 and its first derivative at $t=0$, we find that

$$A=\frac{E}{2\omega L(R_1+R_2)^2}\left[R_2^2-\frac{L}{CR_1}(2R_1+R_2)\right]$$

$$B=\frac{-ER_2}{R_1(R_1+R_2)}$$

The complete solution for relaxed initial conditions can now be written from equation 8 by substitution of these values for A and B. The solution has been obtained in a simple and direct manner, and has avoided a rather laborious manipulation of complex numbers.

Appendix—Proof of Method of Determining Initial Derivatives

Consider the system of differential equations:

$$\sum_{j=1}^{n}(a_{ij}p+b_{ij})x_j=f_i(t),\quad i=1,2..n \tag{9}$$

Let this system be written in terms of continuous variables, for example if current is discontinuous, write the equations in terms of charge. Where second derivatives occur, introduce a new variable z such that:

$$px_j-z_j=0 \tag{10}$$

The second derivative p^2x_j can then be written pz_j, and equation 10 will form part of system 9.

Since the system is linear, the various terms $f(t)$ can be introduced one at a time, and the total solution found by the superposition of the separate solutions. It is therefore sufficient to prove that the method is correct with only one $f(t)$ present. Hence let:

$$\begin{aligned} f_i(t)&=f(t) \quad \text{for } i=1\\ &=0 \quad \text{for } i\neq 1 \end{aligned} \tag{11}$$

First find the initial values of the derivatives of x_k by solving the system of equations 9 for time $t=0$. To do this, transpose the b-terms of equations 9, thus obtaining:

$$\sum_{j=1}^{n}a_{1j}x_j'=f_1(t)-\sum_{j=1}^{n}b_{1j}x_j \tag{12}$$

where x_j' represents px_j or dx_j/dt.

The independent variables are continuous, so $x_j(0+)=0$. Let $t=0$ in equations 12 and solve the system for $x_k'(0+)$. Then:

$$x_k'(0+)=\frac{M_{1k}}{D}f(0+) \tag{13}$$

where D is the determinant of the a's, and M_{1k} is the minor obtained by striking out the first row and kth column of this determinant and attaching the sign $(-1)^{k+1}$.

Differentiate equations 12 once, set $t=0$, substitute equation 13, and solve for $x_k''(0+)$, which is d^2x_k/dt^2 at $t=0+$.

$$x_k''(0+)=\frac{M_{1k}}{D}f'(0+)-\\ \frac{f(0+)}{D^2}\sum_{i=1}^{n}\sum_{j=1}^{n}b_{ij}M_{ik}M_{1j} \tag{14}$$

This process can be continued indefinitely to find the initial values of higher derivatives.

It will now be shown that the same results are obtained by means of the new method. Return to the original system of equations 9 and eliminate all of the dependent variables except x_k. This yields:

$$(A_0p^n+A_1p^{n-1}+\ldots+A_n)x_k=\\ (B_1p^{n-1}+B_2p^{n-2}+\ldots+B_n)f(t) \tag{15}$$

where

$$A_0=D$$

$$A_1=\sum_{i=1}^{n}\sum_{j=1}^{n}b_{ij}M_{ij} \tag{16}$$

$$\vdots$$

$$B_1=M_{1k}$$

$$B_2=\sum_{i=2}^{n}\sum_{\substack{j=1\\ j\neq k}}^{n}b_{ij}M_{1i,jk}$$

$$\vdots$$

in which $M_{1i,jk}$ represents the second minor of the determinant D obtained by striking out the first and ith rows and the jth and kth columns, and attaching the sign $(-1)^{1+i+j+k}$ for $j>k$ and $(-1)^{i+j+k}$ for $j<k$.

Applying the new method of determining initial values of derivatives to equation 15, we find that:

$$x_k(0+)=0 \tag{17}$$

$$x_k'(0+)=\frac{M_{1k}}{D}f(0+) \tag{18}$$

$$x_k''(0+)=\frac{M_{1k}}{D}f'(0+)+\\ \frac{f(0+)}{D^2}\left[\sum_{i=2}^{n}\sum_{\substack{j=1\\ j\neq k}}^{n}b_{ij}DM_{1i,jk}-\right.\\ \left.\sum_{i=1}^{n}\sum_{j=1}^{n}b_{ij}M_{ij}M_{1k}\right] \tag{19}$$

Equation 17 is correct by virtue of the continuity of the x's, equation 18 checks with equation 13 as obtained by the other method, and comparison of equation 14 with equation 19 shows that the following condition must be satisfied:

$$\sum_{i=1}^{n}\sum_{j=1}^{n}b_{ij}M_{ik}M_{1j}=\\ \sum_{i=1}^{n}\sum_{j=1}^{n}b_{ij}M_{ij}M_{1k}-\\ \sum_{i=2}^{n}\sum_{\substack{j=1\\ j\neq k}}^{n}b_{ij}DM_{1i,jk} \tag{20}$$

The coefficients of the b's must be separately equal. Because of the conditions on the last summation, three cases must be distinguished:

Case I. i = 1, j unrestricted

The coefficients of b_{1j} as obtained from equation 20 are:

$$M_{1k}M_{1j}=M_{1j}M_{1k}$$

which is an identity.

Case II. i unrestricted, j = k

The coefficients of b_{ik} in equation 20 are:

$$M_{ik}M_{1k}=M_{ik}M_{1k}$$

Case III. i ≠ 1, j ≠ k

The coefficients of b_{ij} yield the following relation which must be satisfied:

$$M_{ik}M_{1k}-M_{ik}M_{1j}=D\cdot M_{1i,jk} \tag{21}$$

The left side of equation 21 can be written as an nth order determinant, which, when multiplied by the determinant D rearranged with the former kth and jth columns as the first and second columns, and with the former ith row shifted to the second row, yields the following result:

$$\begin{vmatrix} a_{1k} & a_{1j} & a_{11} & \ldots & a_{1n}\\ a_{ik} & a_{ij} & a_{i1} & \ldots & a_{in}\\ a_{2k} & 2_{2j} & a_{21} & \ldots & a_{2n}\\ \vdots & \vdots & \vdots & & \vdots\\ \cdot & \cdot & \cdot & & \cdot\\ a_{nk} & a_{nj} & a_{n1} & \ldots & a_{nn} \end{vmatrix} \cdot \begin{vmatrix} M_{1k} & M_{ik} & 0 & 0 & \ldots 0\\ M_{1j} & M_{ij} & 0 & 0 & \ldots 0\\ M_{11} & M_{i1} & 1 & 0 & \ldots 0\\ M_{12} & M_{i2} & 0 & 1 & \ldots 0\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ M_{1n} & M_{in} & 0 & 0 & \ldots 1 \end{vmatrix} =\\ \begin{vmatrix} D & 0 & a_{11} & \ldots\ldots & a_{1n}\\ 0 & D & a_{i1} & \ldots\ldots & a_{in}\\ 0 & 0 & a_{21} & \ldots\ldots & a_{2n}\\ \vdots & \vdots & \vdots & & \vdots\\ \vdots & \vdots & \vdots & & \vdots\\ 0 & 0 & a_{n1} & \ldots\ldots & a_{nn} \end{vmatrix}$$

The left side is equal to $(-1)^{i+j+k+1}D[M_{ij}M_{1k}-M_{ik}M_{1j}]$ for $j>k$ and to $(-1)^{i+j+k}D[M_{ij}M_{1k}-M_{ik}M_{1j}]$ for $j<k$. The right side, when expanded, is found to be equal to $(-1)^{i+j+k+1}D^2\cdot M_{1i,jk}$ for $j>k$ and to $(-1)^{i+j+k}D^2\cdot M_{1i,jk}$ for $j<k$, so therefore:

$$M_{ij}M_{1k}-M_{ik}M_{1j}=D\cdot M_{1i,jk}$$

which was to be proved.

If the differential equation 15 had been written in terms of a variable with a simple discontinuity at $t=0$, say in terms of the first derivative of x_k, each term on the right side of equation 15 would have been raised once in power of p. The method will yield the proper initial values in this case also, for the initial value of the new variable will be equal to the first derivative of the old, and so on. The method is therefore applicable to variables with a simple discontinuity at $t=0$, as well as to variables which are continuous everywhere.

References

1. Operational Circuit Analysis (a book), Vannevar Bush. John Wiley and Sons, pages 48–55 and 291–334.

2. Communication Networks (a book), E. A. Guillemin. McGraw-Hill Book Company, volume I, chapter VI.

3. Two-Reaction Theory of Synchronous Machines, Part I, Generalized Method of Analysis, R. H. Park. AIEE Transactions, volume 48, July 1929, pages 716–30.

4. Operational Methods in Mathematical Physics (a book), H. Jeffreys. Cambridge Tract in Mathematics and Mathematical Physics No. 23, 1927, Cambridge University Press, chapter 1.

5. A Simple Proof and Extension of Heaviside Operational Calculus, Balth. Van der Pol. *Philosophical Magazine*, vii, 1929, page 1153.

The Varioplex—a New Development in Telegraphy

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MEMBER AIEE

Synopsis: The varioplex method of telegraphic operation provides each pair of stations connected together by it an ever-ready two-way channel for the exchange of traffic, which occupies a band width of zero when idle and of a variable width when busy, depending upon extent of simultaneous usage by other connected stations. It possesses certain advantages over, or in supplement to, other forms of manual and machine switching or repeatering. Adapted to the use of private subscribers through telemeter service, it is finding extensive application in the telegraph plant.

A GOOD deal of current effort in the telegraph engineering field is being directed toward the solution of problems of trunking and switching. Basically, these problems arise from the circumstance that important trunking centers, like Boston, Atlanta, or Los Angeles, have tributary to them most of the less important telegraph offices in their respective surrounding areas. That this is so with respect to the many branch telegraph offices and tie-line connections which dot the larger American cities requires no explanation; the growth of commerce along the railroads has made it no less true of cities and towns many miles from trunking centers. The existence of fanned-out local telegraph circuits around Boston as a hub, to take that city as an example, is inherent in the commercial relationship of most New England towns to its dominating city. It can be appreciated, then, why the Boston office, in years past, has acted not only as the terminal for a considerable day-to-day traffic between Taunton and Boston but also as the relay point for the less frequent messages between Taunton and New York. This situation exists the country over. Of recent years, improvements in the transmission qualities of these tributary circuits have made it possible to terminate them, almost universally, in printing telegraph equipment.

Between the larger cities for years there have existed the more heavily loaded trunk wires, over which has flowed traffic manually fed to them by the tributary networks. Through the use of synchronously-operating multiplex distributors, the capacity of these intercity trunk facilities has been made conveniently divisible into units called traffic channels, of an output adapted to the capabilities of operators. These channels operate at speeds of from 60 to 75 words per minute. The gradual standardization of printer equipment, the stabilization of unit capacities, and the adaptability of printer circuits to end-on repeatering have together extended an open invitation to engineers to increase the number of trunk channels regularly connected, or temporarily connected on demand, to other trunk channels and to fanned-out tributary trunks and tie lines, thus markedly increasing the number of pairs of cities, pairs of towns, of branch offices, and of customers, directly tied together by traffic channels.

In creating such through channels between distant terminals, recourse has been had to end-on switching of a variety of types of circuit constituting the intervening geographical circuit sections. Some have been of start-stop, others of the multiplex time-division, or sector, type; some have been routed over duplexed ground-return physical conductors, others over metallic loops, or over composite or superimposed d-c pairs, or over a-c carriers; and either with or without electromechanical storage en route.

The type of switching adopted for increasing the length and number of these standard-width traffic channels has been influenced in part by the holding time of the connections. Where day-by-day connection of intercity trunks has been desirable, fork-series connection of multiplex channels,[1] and extended start-stop channels,[2] have produced efficient results. Where telegraph-talk connections have had to be quickly set up and pulled down[3] in a manner similar to that employed in connecting long-distance telephone calls, the general pattern of toll switching[4] has been followed. Where the bulk of the traffic is single-shot, forms of dial machine switching[5] have proved advantageous. In the admixture of all types of traffic in a large office installation, the manual switching of reperforators[6] has been successful. All these methods have been, and still are, significant in telegraphy as it is being developed in the United States today.

The Varioplex Concept

Whatever the form of switching, switched telegraph channels, up until the present time, have had in common the characteristic of occupying a frequency band of constant width, whether busy or idle. Although not standardized, such bands have been of the order of 20 cycles per second in width, accommodating traffic at a rate, representatively, of 65 words a minute.

Against the foregoing background of progress, Philo Holcomb, Jr., the author of a companion paper[7] before this convention, originally projected the concept of, and devised the circuits for, a telegraphic "lane" for traffic, of variable, rather than of constant, width—a lane whose width occupied the full capacity of a geographical circuit section at times when that lane alone was being accommodated—a lane whose width adjusted itself automatically to occupy only its proportionate share of a component circuit section at times when that circuit section was simultaneously accommodating other lanes of traffic—and, most important of all, a lane for traffic whose

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[1] For all numbered references, see list at end of paper.

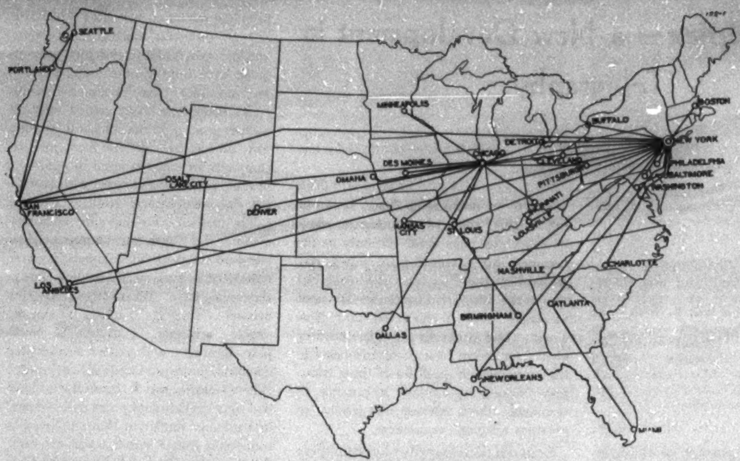

Figure 1. Map of varioplex systems in United States

width decreased to zero, automatically, whenever no traffic happened to be passing over the lane.

It was at once obvious that with such an expanding, contracting, variable-width lane, it would be practicable, ultimately, to set up a multiplicity of direct connections theretofore considered out of the question because of light loads. For the lane width now having been reduced to zero when idle, there remained no necessity to deny to two telegraph offices or to two tie-line customers or subscribers a direct lane for traffic merely because the load offered happened to be small in relation to the total capacity of a 20-cycle-per-second channel of multiplex over which it might be switched.

An important part of the concept was that the effect of switching thus secured was to be accomplished, not manually as by a plug-and-jack switchboard, nor automatically as by dialing a number, but by the expedient of providing that the two terminals should always remain interconnected through the system but completely "inert" until traffic was offered. The fundamental switched informational unit was to be, not the duration of a toll call, not the length of a series of messages or even of one telegram, but the amount of information transmitted by the operator without pause, be that the extreme length of a press dispatch or, at the other extreme, the length of one word, a numeral, or even a solitary letter.

Applications

The original concepts and circuits were developed by the Western Union engineering department and the systems given the name of varioplex. The map, figure 1, shows the extent of application of such systems, and the photograph, figure 2, depicts the installation of 4 out of the 16 systems terminating in the main telegraph office in New York City. The 39 systems operate over 92 channels of standard duplex multiplexing equipment and provide 736 unidirectional lanes, or 368 two-directional "subchannels" as they are referred to, of communication.

A typical varioplex system is that set up between Los Angeles and San Francisco, consisting of 27 subchannels. Twenty-two of the 27 subchannels provide direct two-way printer operation between 22 pairs of San Francisco and Los Angeles branch offices or subscribers' tie lines, the business between which hitherto would have had to be carried on 22 individual wires or manually relayed in San Francisco and again in Los Angeles. The remaining five subchannels provide Los Angeles Western Union with direct connection to offices in Seattle, Chicago, Sacramento, Oakland, and Reno, through flexible linkage at San Francisco. In the case of the Seattle connection, the Los Angeles-San Francisco varioplex subchannel is connected to a San Francisco-Seattle varioplex subchannel; a similar connection is made at San Francisco on a varioplex subchannel going through to Chicago. In the case of Sacramento and Oakland, physical wires carrying start-stop teleprinter signals between those cities and San Francisco are repeatered separately at San Francisco into the two varioplex subchannels to Los Angeles. In the case of Reno, a multiplex channel

Figure 2. Four of 16 systems of varioplex apparatus in New York main office

between Reno and San Francisco is coupled to the Los Angeles varioplex subchannel at San Francisco.

Capacity

Sixty-six words per minute has been adopted as the standard multiplex channel speed for varioplex systems. The total capacities of such systems are, therefore, 66, 132, 198, and 264 words per minute in each direction, depending upon whether the varioplex system is carried over one, two, three, or four channels of multiplex. The Los Angeles-San Francisco system referred to is of 264 words capacity.

Some idea of the flexibility of varioplex systems may be gained by consideration of the equipment of three multiplex channels on a physical wire (or two unidirectional channels of audio-frequency telegraph carrier) between New York and St. Louis. One of the channels is retained in use as a duplex printer trunk; the second carries a three-subchannel varioplex system; the third carries a six-subchannel varioplex. By way of comparison, the Chicago-Los Angeles varioplex of six subchannels occupies the combined capacity of all three multiplex channels.

The type of varioplex system chosen and the number, type, and traffic loading of the subchannels assigned to it, are matters tied up with the time, frequency, and duration of the occurrence of peaks of simultaneous demand for facilities. The maximum number of subchannels now assigned to a varioplex system in service is 36. It is thought that this figure by no means represents the upper limit of lightly loaded subchannels which ultimately will economically be so assigned. If all 36, or all 48, 99, or whatever the upper limit may prove to be—if all the connected load were to demand simultaneous use of the circuit, the speed per channel would be very low indeed, the total to be divided among all being only, say, 264 words per minute. In practice, however, the maximum simultaneous demand has been recorded at only nine subchannels on this 36 subchannel system; each channel was operating at 29 words per minute during the period of maximum demand. The usual condition of simultaneous demand has been found to lie between two and five channels on this 36 subchannel system. Since all subchannels enjoy full speed of 66 words per minute on this circuit until the fifth subchannel cuts in, it may be said that only occasionally is the speed ever reduced from that figure, and in such cases only down to 53 words per minute for brief intervals.

When, because too large a number of varioplex subchannels are in simultaneous use their respective speeds are unreasonably low, the obvious cure is to reassign the subchannels. Examination into the causes of abnormal simultaneous usage has often pointed the way to corrective action. Low subchannel speed is not a defect of the varioplexing process, for the situation on the superseded multiplex under the conditions of manual operation and with the same concentrated fall of traffic would be even worse. For a given fall of traffic the varioplex packs the informational content more compactly into the available transmission band width than does any other form of telegraphy. If a multiplex circuit between New York and Seattle has a capacity of 100 messages per hour and satisfactorily handles all the traffic on a manual-relay basis between the New York and Seattle areas, experience has pointed to the conclusion that all the offices in those areas could, if desired, be interconnected by varioplex over the long haul and there would be no excessive demand peak, and no unreasonably low channel speeds would be encountered. In fact, it would be found that more offices and more traffic could be added to the connected load without encountering trouble.

Occasional reduction of speed is noticeable to a receiving operator watching a printer working on a varioplex subchannel. From the viewpoint of the sending operator, however, there is no difference between varioplex and teleprinter transmission. The line from the sending printer on each operating table is connected to an individual distributor and automatic perforator combination at the varioplex terminal, shown in figure 3. This combination converts the seven unit start-stop signals from the sending printer keyboard to a perforated paper tape in the usual five-unit multiplex code. The sending line and distributor-perforator combination are continuously available to the operating table so that the sending operator may send at any time at the full teleprinter speed of 65 words per minute. Thus is met the requirement of a continuously available full-speed channel for the transmission of traffic from the operating position. Regardless of the speed variations that may exist in transmitting the characters over the trunk channels, the full capacity and efficiency of the sending operator may be utilized whenever traffic is on hand for transmission.

Advantages

Regardless of the extent to which the varioplex has already been installed, its full potentialities in increasing the efficiency of utilization of trunk circuits and equipment may not yet have been fully realized. A telegraph informational transmission band is essentially very narrow—of the order of 20 cycles per second or less, as pointed out before. To assign a physical wire to its transmission is an unconscionable waste from an engineer's standpoint. To assign to it a carrier with comparatively wide intercarrier spacing is only slightly more pardonable. Interlocking multiplexed channels are more efficient as to band width occupied, but even here the fixed width of each of the channels admits of no conformation either to the human limitations of the operators or to the vagaries of the fall of traffic. Surveys of trunk groups with a mixture of physical, derived, and carrier circuits employed in diverse services have shown an unused capacity as high as 70 per cent on facilities considered "full." Divided into units of capacity of 60 to 70 words per minute, there were, literally, no facilities to be spared in the groups under survey. Only by varioplexing all facilities in such circuit groups can 100 per cent utilization be approached. One great contribution of the varioplex to the telegraph art is that its flexibility makes available for traffic many of the heretofore idle cracks and corners of the telegraph wire-plant capacity.

A second important advantage of the varioplex is that no matter how heavily congested one or more of the circuit sections may be over which the lane for traffic is connected, transmission of the first character of every message initiated is started immediately and completed forthwith, from origin to destination, as soon as the keyboard is manipulated. There is no testing for idle channels, no busy signals are received, no time is wasted in handling dial-switching impulses, there is no additive delay due to such impulses traversing the chain of connection, no waiting for a particularly long message to clear out of the way. Since all varioplex connections are, in essence, perpetually connected two-way circuits, there is no waiting involved on the part of the sending office for a quick acknowledgment of receipt on the part of the receiving office.

A third major advantage which the varioplex contributes to the art is the compounding of possibilities of elimination of manual relaying of telegraphic information. While other modern meth-

Figure 3. Racks containing equipment for nine type C varioplex subchannels

ods of switching channels accomplish a similar purpose, their scope has been limited by the requirement that, to justify economical switching, the switched informational unit must contain a practical minimum number of words. Elimination of such an economic minimum through the varioplex principle throws open a wide field of development for switching, with all its demonstrated advantages over manual relaying. The number of operations and the amount of supervision are reduced—so are investment, operating maintenance, and inspection costs of belt conveyors, branch-office pneumatic tubes, printers, perforators, typewriters, and stationery. So too are routing and servicing costs cut down. Because varioplex repeaters require much less room than corresponding channels for manual relaying, there are additional savings in rental items.

A fourth advantage of the varioplex is, in a sense, a derivative of the other three in their application to the use of public telegraph facilities by private communication systems. By employing varioplex subchannels, users need no longer choose between the continuous utilization of a full standard-width communication channel on the one hand and a discontinuous service on the other. Varioplex connections between two private customers afford them teleprinter intercommunication at any time without the necessity of ordering a circuit to be made up and without any impulse to hurry or terminate a conversation through lapse of time alone. Opportunity is thus afforded for deliberation between questions and answers and for consulting files, assistants, and authorities. To utilize a varioplex subchannel for this type of service it is necessary only to present a sufficient volume of traffic to pay the costs of providing the subchannel equipment; the charge may then be based on volume rates for the traffic actually handled. A metering instrument incorporating a character counter has been developed for such an application, and the varioplex facility is now widely used by subscribers under the name of telemeter service.

Conclusion

The increase in the efficiency of utilization of trunk-circuit capacity effected by the use of varioplex equipment is an obvious and worth-while contribution to telegraph progress. The dissociation of subchannels for point-to-point communication from the use of trunk-circuit capacity except when traffic is actually being transmitted is reflected in a greater return on investment in lines and terminal equipment. In laying out point-to-point channels for the handling of commercial telegraph traffic operating engineers need no longer be bound by former limitations as to traffic volumes. To establish a direct channel of communication, a volume of traffic sufficient to justify the use of a fixed amount of trunk-circuit capacity represented by at least one full trunk channel has in the past been necessary. Assuming that the total traffic between the areas concerned justifies the use of varioplex equipment as many individual subchannels as needed can be set up, it being necessary for the loads on these individual subchannels to be sufficient to justify only the subchannel operating equipment and the equipment for connecting it into the varioplex system. This possibility is further multiplied through the interconnection of subchannels from two or more systems to effect direct connections between areas not directly served by individual varioplex systems.

Experience is now beginning to reveal the optimum economic usage of methods of machine-switching, reperforation, and varioplex. Many interesting combinations are possible. Long-haul, intercity varioplex channels may be dialed by various branch offices in the city of origin, and fanned out by manually-switched reperforators in the cities of destination. Thus is indicated a material increase in the number of direct connections between telegraph offices and a resulting better telegraph service to the American public.

References

1. Telegraph Traffic Engineering, H. Mason and C. J. Walbran. AIEE Transactions, volume 46, February 1927, page 379.

2. Annual Report of the Committee on Communication. AIEE Transactions, volume 50, 1931, pages 1488–94.

3. The 1-A Teletypewriter Switchboard, A. A. Burgess. *Bell Laboratories Record*, volume 16, October 1937, pages 34–8.

4. A General Switching Plan for Telephone Toll Service, H. S. Osborne. AIEE Transactions, volume 49, October 1930, pages 1549–57.

5. Some Recent Developments in Teleprinter Switching Systems, P. J. Clemens. *Electrical Communication*, volume 16, January 1938, pages 263–75.

6. Recent Developments in Telegraph Switching, F. E. d'Humy and H. L. Browne. AIEE Transactions, volume 59, February 1940, pages 71–8.

7. Basic Principles of the Varioplex Telegraph, Philo Holcomb, Jr. AIEE Transactions, volume 60, 1941.

Power Supply for Resistance-Welding Machines—III

Factory Wiring for Resistance Welders

AIEE COMMITTEE ON ELECTRIC WELDING
Subcommittee on Power Supply for Welding Operations*

Purpose of Report

AN adequate power supply is one of the requisites of present-day, high-production resistance welding. A major part of the power supply system of any industrial plant lies within the plant itself and consists of the plant wiring and transformer substations. It is the purpose of this report to show how to properly lay out such a plant distribution system for serving welders. Much of the fundamental data included in the tables and charts is not new but is included in order that the layout engineer will have complete information available in one compact report and will not have to refer to various articles and handbooks, copies of which may not be readily available.

General Considerations

The first report of this series, "Guide to Good Electrical Performance of Resistance Welding Machines," pointed out the need for keeping welder kilovolt-ampere inputs as low as possible for given welding current outputs, and the next step consists of providing adequate supply facilities for handling these inputs. Because of the high currents involved, transformers should be located as close to the welders as possible, keeping the secondary bus and feeder runs short. In many cases it will be found advantageous to use transformers containing a liquid that will not burn rather than oil in order that they can be located adjacent to the welding machines rather than at some relatively remote outdoor or vault location. With the load power factor generally falling between 20 and 40 per cent, it is essential that the reactance of the power supply system be kept at a minimum. Generous copper cross section in welding feeders is usually less important than proper spacing and arrangement of the conductors for minimum reactance.

The first question to be answered is: "For what voltage drop shall the supply system be designed?" No specific answer can be given which will apply to all cases and, as pointed out in the preceding report, satisfactory results may be obtained with 15 per cent regulation in one case and 5 per cent may be too much in another. However, in general with the usual class of production resistance welding involving low and medium carbon steels, satisfactory welding will result if the voltage drop to a point on the supply system common to two or more machines does not exceed ten per cent for the largest machine of the group. This statement is based on the assumption that there are relatively few large machines in the group being served (say, not over five or ten machines) and that, while two of them may occasionally operate at the same time, the chances of three or more operating together is extremely remote. Thus, if the machines are adjusted for perfect welding with a ten per cent voltage drop in their supply circuit, the welds will still be good when two machines operate together causing nearly 20 per cent drop. If three or more happen to hit together, poor welds or rejects may result. Care should be exercised in adjusting machines, with the setting made under highest voltage and maximum current conditions, just short of burning the weld so that under minimum voltage conditions a good weld will still result.

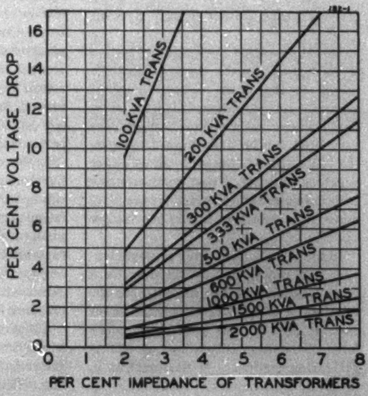

Figure 1. Welding supply—transformer regulation

Per cent voltage drop per 1,000 amperes of 30 per cent power factor, single-phase welding current

1. For 480-volt, single-phase transformer bank, use chart values of per cent voltage drop
2. For 480-volt, three-phase transformer bank, multiply per cent voltage drop by 2
3. For 240-volt, single-phase transformer bank, multiply per cent voltage drop by 1/2
4. For 240-volt, three-phase transformer bank, use chart values of per cent voltage drop

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1. For all numbered references, see list at end of paper.

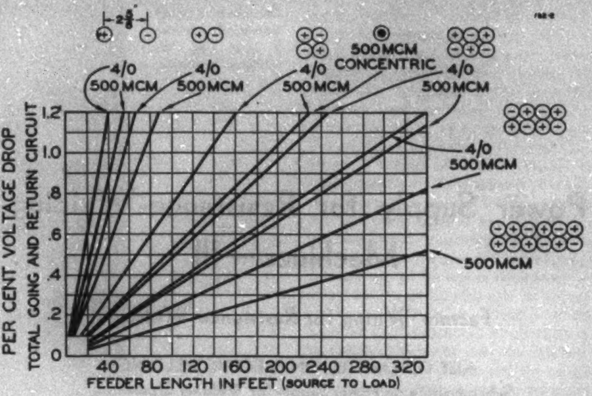

Figure 2. Welding supply—wire feeder regulation

Per cent voltage drop per 1,000 amperes of 30 per cent power factor, single-phase welding current

1. For 480-volt feeder, use chart values of per cent voltage drop
2. For 240-volt feeder, multiply per cent voltage drop by 2

Experience indicates that this general rule of ten per cent drop for the largest machine will result in a good workable installation in most cases although it should be realized there will be exceptions. The procedure outlined in the following sections will follow the ten-per-cent rule with some indication as to how to determine those installations which will require special consideration as exceptions to the general rule.

Where even less than ten per cent regulation can be provided at about equal cost, such as obtained by close spacing of feeders, this obviously should be done. The ten-per-cent figure is set only as an approximate allowable upper limit and any improvement below this value is all to the good and makes for that much better welding conditions.

Interlocking can be resorted to under some conditions to insure not more than one or two machines operating at the same time. This is an inexpensive practice where only a few large machines are involved and in most cases production speed is not materially reduced.

While the current-carrying capacity of the supply feeders and transformers must receive consideration, if provision is made to keep the voltage drop within the necessary limits, there usually will be sufficient attendant thermal or carrying capacity.

Voltage Drop

For convenience, the supply system can be broken up into three parts:

1. High-voltage power supply system (2,400 volts or higher)
2. Step-down transformer bank
3. Low-voltage bus or feeder (240 or 480 volts)

High-Voltage Power Supply System

The high-voltage power supply system in most cases is the power company's service to the plant. Where the plant generates its own power, it is typified by the generator bus system. In most cases very little can be done toward reducing the voltage drop in the high-voltage power supply system at a cost commensurate with the expenditure needed for the same percentage reduction in the low-voltage system.

In the case of service from a power company, the amount of voltage drop for a given welding load is dependent upon the relative location of the industrial plant with respect to the power company's large distribution substations as well as generating stations. If the plant is in an established industrial area served by large substations and not too far from large generating stations, the voltage drop in the high-voltage system will be a negligible factor. However, if the plant has been located in a rural or small-town area far removed from the power company's main generating stations, as is becoming more common under the present-day trend toward decentralization of industry in the search for low manufacturing costs and reduced taxes, the high-voltage system regulation may loom up as a very troublesome factor. Any plant in which resistance welding is extensively used should carefully investigate this matter of high-voltage system regulation when contemplating moving the plant to any other location with less adequate power facilities for welding.

In the case of private power generation, the voltage drop at the generator bus will be inversely proportional to the total rating of generators feeding the bus. The small and moderately sized plant will experience considerable difficulty in handling any but the smaller-sized welding machines without encountering lamp flicker problems in their plant and office lighting.

The first step in designing the plant wiring and bus layout for welding is to determine the high-voltage system regulation. This can be obtained from the local power company by giving them the maximum kilovolt-amperes and power factor that the largest machine will draw from the power system. Subtract the figure obtained from the total allowable ten per cent and the remainder is that which can be allowed in the step-down bank and secondary or low-voltage bus system. For instance, assume the largest welder is rated 300 kva and will deliver a maximum welding current of 100,000 amperes with a high secondary open circuit voltage of 8 volts. The approximate maximum kilovolt-ampere input (neglecting exciting current) is $(100,000 \times 8)/1,000 = 800$ kva. From the power company representative it is learned that 800 kva at an estimated 30 per cent power factor will cause four per cent drop in the power company service to the transformer bank at the customer. This subtracted from ten per cent leaves six per cent which can be used up in the step-down transformers and low-voltage bus system.

Step-Down Transformer Bank

On the assumption that the remaining allowable six per cent drop can be divided about equally between the step-down transformer bank and low-voltage bus system, it is necessary to select the proper size transformer which will limit the drop to about three per cent.

Welders can either be served from a single-phase transformer bank or a three-phase bank, and where the welding load is large enough to justify a separate transformer bank for welding load only, it will be found advantageous and more economical of capacity to use a single-phase system. The load is single phase and it will remain single phase no matter

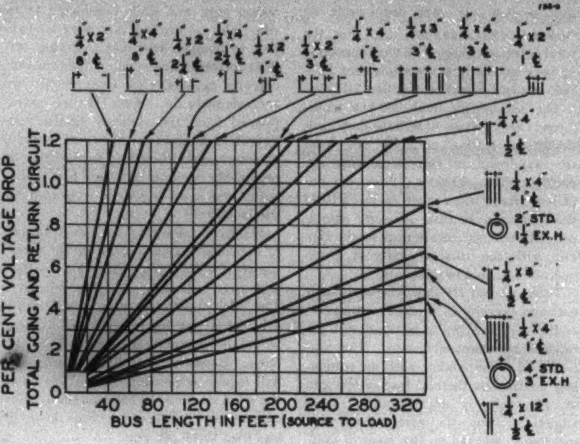

Figure 3. Welding supply—bus regulation

Per cent voltage drop per 1,000 amperes of 30 per cent power factor, single-phase welding current

1. For 480-volt bus, use chart values of per cent voltage drop
2. For 240-volt bus, multiply per cent voltage drop by 2

how the transformers ahead of it are connected. Sometimes it is desirable to spread the individual single-phase loads over the three-phase wires for more economical utilization of the supply lines, but unless there are large numbers of machines connected, the necessity for balancing the load is not important. When there are many machines, they are usually scattered around in different parts of the plant so that several step-down banks become necessary at different locations. In this case individual single-phase banks can be connected to separate phases and thus bring about some semblance of balance on the supply wires.

A given kva of single-phase load fed from a three-phase transformer bank will cause just twice as much transformer voltage drop as the same kva load fed from the same capacity in transformers connected as a single-phase bank. It is thus obvious that, where the transformer voltage drop must be kept at a minimum, it is highly desirable and more economical to use single-phase, step-down banks of transformers.

In case the welding load is relatively small in comparison with the motor and other power load of the plant, it can readily be served from the same three-phase bank serving the power load. Individual plant and service conditions thus dictate whether single-phase or three-phase banks are the logical selection. Irrespective of whether the welding load is served from single-phase transformers or from three-phase transformers in combination with the motor and power load, the lighting load should usually be handled separately in order to prevent flicker difficulties.

Figure 1 shows per cent voltage drop per 1,000 amperes of 30 per cent power factor welding current through transformer banks of different sizes and impedance. Plant practice may determine the size and impedance of individual transformers and the number of units to connect in parallel can be determined from the chart. For instance, assume that 300-kva, 3$^1/_2$-per-cent-impedance transformers are used by this particular plant in its step-down banks. A load of 800 kva represents 1,670 amperes at 480 volts. From figure 1: 1,670 amperes through a 300-kva, 3$^1/_2$-per-cent-impedance transformer causes 1.67 times 5.6 per cent = 9.4 per cent = voltage drop. Therefore, three transformers in parallel would have to be used to stay within the predetermined limit of three per cent drop. In some cases a special low-impedance transformer may be economically justified. For example, if a two-per-cent-impedance transformer is purchased, the nearest size transformer would be 500 kva which would allow 3.2 per cent drop. This might be cheaper than the three 300-kva transformers but would have the definite disadvantage of not being usable interchangeably at other plant locations. If a plant has sufficient welding load, it sometimes pays to standardize on special low-impedance transformers for welding only and not attempt to make them interchangeable with transformers used in power and lighting banks.

Low-Voltage Bus or Feeder

There is now only three per cent remaining out of the original allotted ten per cent drop. It is found that the bus or feeder must be 300 feet in length to reach the two machines located farthest from the transformer bank.

Figures 2 and 3 show the per cent voltage drop per 1,000 amperes of 30 per cent power factor welding current for different types and sizes of bus and wire construction. Figure 2 is for various wire arrangements and figure 3 for different bus arrangements. Some plant engineers prefer one class of construction and some the other. The bus can be made up on the job or it can be purchased, prefabricated in unit lengths either with or without enclosures.

Usually if the welders are in a fairly compact group but at some appreciable distance from the transformer bank, some arrangement of wire feeder works out to advantage. However, where the machines are scattered and frequent taps must be made to the supply wires or bus, a bus system is perhaps more advantageous in that frequent connections to it are readily made.

From the charts it is found that two 500,000-circ-mil wires per leg in conduit would handle 800 kva with 2.6 per cent drop at the 300-foot distance. (800 kva = 1,670 amperes. From the chart the drop for 150 feet = 0.78 per cent; for 300 feet and 1,670 amperes the voltage drop equals 0.78 per cent times 2 times 1.67 = 2.6 per cent.) For the bus construction a bus consisting of two $^1/_4$-inch by 4-inch copper bars on one-inch

Table I. Equivalent Continuous Loading of Typical Welding Machine Installations

Types of Welding	Equivalent Continuous Load Expressed in Per Cent of Sum of Name-Plate Ratings
Spot, projection, flash	5 to 10 (most installations) 10 to 20 (very few installations)
Hydromatic, butt	10 to 20
Seam	25 to 70

centers would be suitable from a voltage-drop standpoint.

It is possible that either of these selections may have sufficient current-carrying capacity for handling the equivalent steady or thermal load of the group of welders. However, if it is found that more carrying capacity is needed, other configurations are shown with equal or better voltage-drop characteristics.

Equivalent Continuous Thermal Loading

There are several ways in which the equivalent continuous thermal loading of a group of welders can be determined. It might be reasoned that the quickest and simplest method would be to add the name-plate kilovolt-ampere ratings of the individual welding transformers, and then apply a suitable diversity factor for group operation. The welding transformers are rated on a 50-per-cent duty-cycle basis and, consequently, if all of the machines are operated continuously right up to the thermal capacity of their transformers, the equivalent continuous thermal loading would be $\sqrt{0.50}$ or 70.7 per cent of the sum of the name-plate ratings. Thus, if there were ten 300-kva welders, the supply system on this basis should have a carrying capacity of 70.7 per cent of 3,000 kva, or 2,100 kva less the diversity factor correction.

In actual practice, however, most machines, except perhaps some seam welders, operate well below their maximum thermal capacity, because the size of transformer in the machine is dictated by the required welding current output rather than by thermal considerations.

Tests of many installations show that the values given in table I are more clearly representative of actual field conditions. Thus, if the machines are predominately spot, projection, or flash welders, a supply system with a rating of 15 per cent of the sum of the name-plate ratings of the machines would, in most cases, be adequate. Thus, taking the ten 300-kva welders, the supply system should be good for about 15 per cent of 3,000 kva, or 450 kva.

A second check should be made based upon the actual kilovolt-ampere loads of the welders and their operating duty cycles. Assume that each of the ten 300-kva machines was on a production schedule of 100 welds per hour with maximum loads of 800 kva for 10 cycles per weld. The corresponding duty cycle of each machine is: $(10\times100)/(60\times60\times60)=0.46$ per cent. The equivalent continuous load corresponding with 800 kva at 0.46 per cent duty cycle is: $\sqrt{0.0046}\times800$ kva $=54$ kva. For the total of ten machines, if all were operating at peak production, the equivalent kilovolt-amperes would be $10\times54=540$ kva.

The selection of three 300-kva transformers on the basis of voltage-drop considerations is thus perfectly safe from the standpoint of current-carrying capacity. The 500-kva, 2-per-cent-impedance transformer might be just a little small from a thermal capacity standpoint for ten machines in peak production but, with the usual diversity of group operation, should be adequate.

Tables II and III show the equivalent carrying capacity in continuous amperes of the various types of bus and wire arrangements commonly used. The ampere load corresponding with an equivalent continuous thermal load of 500 kva at 480 volts is approximately 1,000 amperes. Thus, the wire construction previously selected on the basis of voltage drop is not large enough from the standpoint of current-carrying capacity. A wire grouping consisting of four 500,000-circ-mil wires per leg (item 11, table II) is rated about 1,000 amperes and should be adequate for the job. This wire arrangement will have only 1.2 per cent voltage drop for an 800-kva load swing, thus being well within the three per cent allowed.

It should be emphasized here that the current-carrying capacities given in table II for six or more total wires in a trough or raceway enclosure are estimated values only and while considered to be reasonable approximations, they are subject to change whenever verifying test data become available. The current-carrying capacities for most of the bus arrangements in table III are extrapolated from test data of similar configurations both in the open and in metal enclosures and should, consequently, be reasonably accurate although here again, actual test data would be welcomed by the industry.

Similarly, table III shows several types of bus construction well within the three-per-cent voltage-drop allowance which will have adequate carrying capacity. The original selection of one $^1/_4$-inch by 4-inch bar per leg on one-inch centers would be adequate if mounted in the open. However, most bus installations will require enclosures and on this basis the selection would be between the heavier current-carrying busses having drops of less than three per cent.

Chance of Simultaneous Welds

Designing for a total of ten per cent voltage drop for the operation of the largest machine produces satisfactory welding conditions only when three or more machines do not happen to operate at the same instant. Based upon the general law of averages, the expectancy of simultaneous welds can be readily calculated. For the sake of simplicity, it is assumed that even if only a single

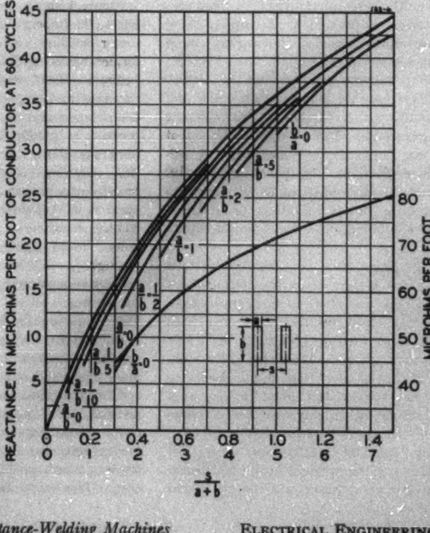

Figure 4. Reactance of strap conductors at 60 cycles

Table II. Welding Supply—Wire Feeder Data. Calculated Data Per 100 Feet of Distance From Source to Load (200 Feet of Circuit)

Item	Wires Per Leg	Spacing (See Figure 2)	Size	Area Square Inches	Approximate Current-Carrying Capacity Copper 60 Deg C Ambient 30 Deg C (Amperes)	Weight (Pounds)	D-C Resistance 25 Deg C (Ohms)	Reactance (Ohms)	Voltage Drop Per 1,000 Amperes 30 Per Cent Power Factor	
									Volts	Per Cent at 480 Volts
1	1	$2^{5}/_{8}$-inch centers	4/0	0.166	225[1]	131	0.01	0.012	14.3	2.98
2	1	$2^{5}/_{8}$-inch centers	500,000 circ-mil	0.393	400[1]	308	0.0043	0.0098	10.7	2.22
3	1	Wires laced together	4/0	0.166	210[2]	131	0.01	0.0059	8.72	1.82
4	1	Ditto	500,000 circ-mil	0.393	350[2]	308	0.0043	0.0056	6.63	1.38
5	2	Ditto	4/0	0.332	400[2]	261	0.0051	0.0022	3.63	0.76
6	1	Concentric cable	500,000 circ-mil	0.393	300[5]	308	0.0043	0.0013	2.53	0.53
7	2	Wires laced together	500,000 circ-mil	0.786	650[3]	616	0.0022	0.002	2.54	0.53
8	3	Ditto	4/0	0.498	500[3]	392	0.0034	0.0014	2.36	0.49
9	4	Ditto	4/0	0.664	600[3]	522	0.0026	0.001	1.72	0.36
10	3	Ditto	500,000 circ-mil	1.18	850[3]	925	0.0014	0.0013	1.63	0.34
11	4	Ditto	500,000 circ-mil	1.58	1,000[3]	1,230	0.0011	0.00089	1.17	0.24
12	6	Ditto	500,000 circ-mil	2.36	1,300[3]	1,850	0.00072	0.0006	0.79	0.17

NOTES: 1. Code limit.
2. In conduit, wires laced together.
3. In trough, wires laced together (estimated values—no test data known).
4. Reactance is slightly higher when wire is in magnetic trough or conduit.
5. In conduit (estimated value—no test data known).

cycle of one weld overlaps another, this would constitute a simultaneous weld. Obviously, except for very short-time welds (two to three cycles), a single cycle overlap should not prove serious and this assumption thus introduces a considerable factor of safety.

T = total time per weld per machine, including "off" and loading time as well as actual weld time

t = actual weld time per machine

E = number of machines

$\frac{T}{tE} \times \frac{T}{t(E-1)}$ = interval in terms of number of welds between operations in which *two* machines make a weld at the same time including any slight overlapping of actual welding time

$\frac{T}{tE} \times \frac{T}{t(E-1)} \times \frac{T}{t(E-2)}$ = interval in terms of number of welds between operations in which *three* machines make a weld at the same time including any slight overlapping of welding time

In usual production welding if rejects due to simultaneous welds do not occur oftener than once per hour, which would be one weld out of every 5,000 or 10,000, conditions would probably be considered highly satisfactory. Table IV shows various combinations of machines, welding time, hourly production, and corresponding interval of time between expected simultaneous welds. For instance, 20 machines with three-cycle welding can produce 200 welds per hour per machine with an expectancy of three simultaneous welds and three rejects every 1.7 hours. With the production stepped up to 300 welds per hour per machine, the resultant expected interference might be such that it would be desirable to either split the machines into two groups or design for a five per cent instead of ten per cent voltage drop in the supply system.

The last column shows total time divided by the actual weld time of all large machines on the bus. As long as this value is greater than 10 or 20, satisfactory welding conditions should generally result on a supply system with ten per cent voltage drop, as the chances of three simultaneous welds occurring oftener than once per hour is rather remote. (For exceptions see last paragraph.) It should be remembered that these considerations apply only to the large machines. Innumerable small machines can be served from the same transformer bank and bus system along with the five or ten large machines, upon which the distribution system design is based, without any further consideration except that of thermal carrying capacity of the system.

It might not be amiss in this connection to point out another frequently encountered cause of poor welding in addition to that of excessive voltage regulation. There is a tendency on the part of many users of welding equipment to increase the welding speeds far beyond the capacity of the equipment to make good welds. As a result, a certain percentage of such welds are inferior. The designer then specifies a greater number of welds in an attempt to assure himself of a sufficient number of good welds to produce a satisfactory joint. The production department must then make these welds at an even faster rate of speed to maintain costs, resulting in an even greater number of inferior welds.

This vicious circle of events can be eliminated by insistence upon good welding throughout, resulting in fewer welds per joint, which in turn permits welding speeds commensurate with good welding. This results in better voltage regulation by reducing the chances of simultaneous or interfering welds on any given feeder line.

There may be some classes of welding where every weld must be perfect and there must be no chance of even a few undetected poor welds passing inspection. In these cases it may be desirable to design for closer regulation. However, even under these conditions, the extra investment needed for reduced regulation should be balanced against the cost of necessary recorders and indicators for detecting welds made under improper conditions of voltage, current, and time

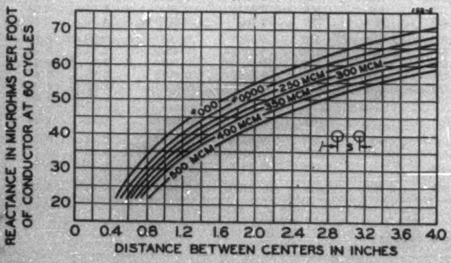

Figure 5. Reactance of standard strand cables at 60 cycles

It might be more economical to invest in the recorders and indicators than in excessive feeder and transformer capacity which would only be needed once or twice a day when three or more machines might happen to hit together.

Example. Ten 300-kva machines, each machine producing 100 welds per hour of 10 cycles duration each:

$$T=\frac{60\times 60\times 60}{100}=2{,}160 \text{ cycles}$$

$t=10$ cycles

$E=10$ machines

$$\frac{T}{tE}\times\frac{T}{t(E-1)}=\frac{2{,}160}{10\times 10}\times\frac{2{,}160}{10\times 9}=518 \text{ welds}$$

between operations in which *two* machines make a weld at the same time

Total output = 1,000 per hour

Two welds together every (518×60)/1,000 = 31 minutes

$$\frac{T}{tE}\times\frac{T}{t(E-1)}\times\frac{T}{t(E-2)}=\frac{2{,}160}{10\times 10}\times\frac{2{,}160}{10\times 9}\times\frac{2{,}160}{10\times 8}=14{,}000$$

welds between operations in which *three* machines make a weld at the same time

Three welds together every 14,000/1,000 = 14 hours

Total time divided by weld time of all welders = (60×60×60) / (10×10×100) = 21.6

Interlaced Bus Calculations

The reactance values of the various wire groupings and bus-bar configurations shown in figures 2 and 3 and in tables II and III were calculated by means of the following short-cut method, utilizing the charts of reactance for a pair of conductors (either for conductors, figure 4, or standard strand cables, figure 5).[1] In this way it is possible to calculate the reactance of an interlaced bus or cable system with a reasonable degree of accuracy in a few minutes, as compared with the more laborious method based upon the fundamental relations of mutual and self-inductance which at best requires several hours.

This short-cut method of calculation is incorporated as part of this report in order that quick calculations can be made for other wire or bus configurations than those shown in the tables.

The basic methods and curves used here are not new, but were published in U. S. Bureau of Standards Scientific Paper No. 281 by Francis B. Silsbee, and also in *The Electric Journal* of June 1919 in an article entitled "Reactance Values for Rectangular Conductors" by H. B. Dwight. However, it was felt desirable to bring together all of the necessary data for the calculation of welder bus reactance and to show by actual examples the use of these data for the calculation of interlaced conductors.

The more general types of welder busses which are commonly used are: (1) four or more stranded insulated conductors interlaced and clamped together, (2) interlaced copper bars, and (3) concentric tubes. Figures 4 and 5 show the total reactance, self minus mutual, between two conductors. The same curves can be used in the following manner when more than two conductors are involved and there is an even number of conductors. Consider an interlaced bus consisting of six conductors, three going and three returning, as shown in item 15, table III, arranged as follows:

(+1) (−4) (+2) (−5) (+3) (−6)

Since the average drop of the going circuit equals the average drop of the return circuit, it is only necessary to find the drop in the going circuit and multiply it by two to determine the total circuit drop. If, as a first approximation, it is assumed that the current divides evenly between all of the going conductors, the reactance of any going conductor is the sum of the reactances of all the return conductors to the given going conductor minus the reactance of the remaining two going conductors to the given going conductor. Dividing the sum of the reactances of the going conductors by three gives the average going reactance per conductor and dividing this average by three (since it is assumed that only one-third of the current went through each conductor) gives the going circuit reactance. Multiplying this value by two gives the total circuit (going plus return) reactance.

The reactance of each going conductor can be written as follows:

$$X_1=X_{14}+X_{15}+X_{16}-X_{12}-X_{13}$$
$$X_2=X_{24}+X_{25}+X_{26}-X_{21}-X_{23}$$
$$X_3=X_{34}+X_{35}+X_{36}-X_{31}-X_{32}$$

The total circuit reactance (going plus return) is:

$$X_T=\frac{2}{9}(X_1+X_2+X_3)$$

It should be pointed out that, although this method is approximate, it is ac-

Table III. Welding Supply—Bus-Bar and Tubing Data. Calculated Data Per 100 Feet of Distance From Source to Load (200 Feet of Circuit

Item	Bars Per Leg	Spacing (See Figure 3)	Size (Inches)	Area Square Inches	Approximate Current-Carrying Capacity 35-Deg C Rise (Amperes)		Weight (Pounds)	D-C Resistance 25 Deg C (Ohms)	Reactance (Ohms)[3]	Voltage Drop Per 1,000 Amperes 30 Per Cent Power Factor	
					Open	Housing[2]				Volts	Per Cent at 480 Volts
1	1	8" centers	1/4x2	0.5	750	490	386	0.0033	0.0126	13.0	2.71
2	1	8" centers	1/4x4	1.0	1,400	950	772	0.0017	0.0098	9.84	2.05
3	1	2 1/4" centers	1/4x2	0.5	750	490	386	0.0033	0.0072	7.86	1.64
4	1	2 1/4" centers	1/4x4	1.0	1,400	950	772	0.0017	0.0048	5.07	1.06
5	1	1" centers	1/4x2	0.5	720	450	386	0.0033	0.0041	4.91	1.02
6	2	3" centers interlaced	1/4x2	1.0	1,400	950	772	0.0017	0.0039	4.19	0.87
7	1	1" centers	1/4x4	1.0	1,370	900	772	0.0017	0.0025	2.88	0.60
8	2x2	3" centers interlaced	1/4x3	3.0	3,600	2,600[4]	2,315	0.00056	0.0028	2.84	0.59
9	2	3" centers interlaced	1/4x4	2.0	2,600	1,800[4]	1,544	0.00084	0.0026	2.73	0.57
10	2	1" centers interlaced	1/4x2	1.0	1,350	870	772	0.0017	0.0018	2.25	0.47
11	1	1/2" centers	1/4x4	1.0	1,370	900	772	0.0017	0.0014	1.83	0.38
12	2	1" centers interlaced	1/4x4	2.0	2,500	1,700[4]	1,544	0.00084	0.0011	1.27	0.26
13	1	2" Standard tube around 1 1/4 extra heavy tube		1.09 0.89	800		765	0.0017	0.00078	1.25	0.26
14	1	1/2" centers	1/4x8	2.0	2,600	1,800[4]	1,544	0.00084	0.00076	0.98	0.20
15	3	1" centers interlaced	1/4x4	3.0	3,500	2,400[4]	2,315	0.00056	0.0007	0.83	0.17
16	1	4" Standard tube around 3 extra heavy tube		3.18 3.05	2,000		2,410	0.00054	0.0005	0.64	0.13
17	1	1/2" centers	1/4x12	3.0	3,600	2,500[4]	2,315	0.00056	0.0005	0.64	0.13

Notes: 1. Extrapolated from test data.
2. Housing material, size, and color affect rating to marked degree.
3. Reactance may be as much as 5 per cent to 20 per cent higher if bus is in magnetic housing.
4. Nonmagnetic housing.

Table IV. Chance of Simultaneous or Interfering Welds

Number of Machines	Weld Time (Cycles)	Welds Per Hour Per Machine	Total Output Per Hour	Time Interval Between Simultaneous Welds: Two Welds Together	Time Interval Between Simultaneous Welds: Three Welds Together	Number of Good Welds for Every Weld in Which Three Hit Together	Total Time Divided by Weld Time of All Welders
5	3	100	500	52 hr	12,500 hr	6,250,000	144
5	3	200	1,000	6.5 hr	777 hr	777,000	72
5	3	300	1,500	1.9 hr	154 hr	231,000	48
5	10	100	500	4.2 hr	336 hr	168,000	43
5	10	200	1,000	35 min	21 hr	21,000	22
5	10	300	1,500	10 min	4.2 hr	6,200	14
3	30	100	300	2.9 hr	207 hr	62,200	24
3	30	200	600	22 min	13 hr	7,760	12
3	30	300	900	6 min	2.6 hr	2,310	8
5	30	100	500	31 min	12.4 hr	6,220	14
5	30	200	1,000	4 min	47 min*	778	7
10	3	100	1,000	5.8 hr	518 hr	518,000	72
10	3	200	2,000	43 min	32.4 hr	65,000	36
10	3	300	3,000	13 min	6.4 hr	19,200	24
10	10	100	1,000	31 min	14 hr	14,000	22
10	10	200	2,000	4 min	52 min*	1,743	11
10	10	300	3,000	1 min	10 min*	520	7
10	30	100	1,000	3 min	28 min*	467	7
20	3	100	2,000	41 min	27 hr	54,000	36
20	3	200	4,000	5 min	1.7 hr	6,840	18
20	3	300	6,000	2 min	20 min*	2,020	12
20	10	100	2,000	4 min	44 min*	1,480	11

*Note: On border line of unsatisfactory operating conditions if supply system is designed for ten per cent voltage drop for operation of largest machine. For these conditions supply system should probably be designed for about five per cent voltage drop.

curate within ten per cent which is good enough for most practical calculations.

Numerical Example 1. Determine the reactance of an interlaced bus 1,000 feet long, consisting of 1/4-inch by 4-inch copper bars spaced one-inch center-to-center as shown in table III, item 15. The distance between the center lines of each conductor and all of the other conductors should be calculated and the reactances corresponding to these distances should be obtained from figure 4.

For example, the distance between the center lines of conductors 1 and 4 is $1'' \times \frac{S}{a+b} = \frac{1''}{{}^1/_4{}'' + 4''} = 0.235$ and the corresponding reactance is 0.0125 ohm per 1,000 feet. The remaining reactances can be found in a similar manner.

$$X_1 = 0.0125 + 0.0290 + 0.0392 - 0.0220 - 0.0347 = 0.0240$$

$$X_2 = 0.0125 + 0.0125 + 0.0290 - 0.0220 - 0.0220 = 0.0100$$

$$X_3 = 0.0290 + 0.0125 + 0.0125 - 0.0347 - 0.0125 = -0.0027$$

$$X_T = \frac{2}{9}(X_1 + X_2 + X_3) = \frac{2}{9}(0.0249 + 0.0100 - 0.0027) = 0.00696 \text{ ohm (going and return)}$$

The same method can be used on stranded cables, except that the reactance values are taken from figure 5.

Numerical Example 2. Determine the reactance of six 250,000-circ-mil interlaced standard-strand, rubber-insulated cables 1,000 feet long and arranged similar to item 8, table II, arranged as follows:

(+1) (−4) (+2)
(−6) (+3) (−5)

The copper diameter is 0.576 inch, the rubber thickness is 0.094 inch, and the braid thickness is 0.020 inch, giving a total diameter of 0.804 inch. The distance between the center lines of each conductor and all of the other conductors should be calculated and the corresponding reactances should be obtained from figure 5.

The distance between the center lines of conductors 1 and 3 is $\sqrt{0.804''+0.804''} =$ 1.138 inches and the corresponding reactance is 0.0368 ohm per 1,000 feet. The remaining reactances can be found in a similar manner.

$$X_1 = 0.0288 + 0.0288 + 0.0473 - 0.0448 - 0.0368 = 0.0233$$

$$X_2 = 0.0288 + 0.0473 + 0.0288 - 0.0448 - 0.0368 = 0.0233$$

$$X_3 = 0.0288 + 0.0288 + 0.0288 - 0.0368 - 0.0368 = 0.0128$$

$$X_T = \frac{2}{9}(X_1 + X_2 + X_3) = \frac{2}{9}(0.0233 + 0.0233 + 0.0128) = 0.0132 \text{ ohm (going and return)}$$

The reactance of concentric tubular busses is easily expressible in a formula. Let the outer radius of the outer tube be r_4, the inner radius of the outer tube be r_3, the outer radius of the inner tube be r_2, and the inner radius of the inner tube be r_1. The reactance then can be expressed as:

$$X = 11.49 \; 4.606\left(\log_{10}\frac{r_3}{r_2} + \frac{r_1^4}{(r_2^2-r_1^2)^2}\log_{10}\frac{r_2}{r_1} + \frac{r_4^4}{(r_4^2-r_3^2)^2}\log_{10}\frac{r_4}{r_3}\right) - \left.\frac{r_2^2+r_1^2}{2(r_2^2-r_1^2)} - \frac{r_4^2+r_3^2}{2(r_4^2-r_3^2)}\right] \times 10^{-3}$$

where X is in ohms per 1,000 feet of bus (total going and return circuit)

Feeder Data for Small Installations

In cases of small installations of welders served at low voltage (240 or 480 volts) by the power company the plant wiring will generally consist of wires pulled in conduit. It is usually unnecessary to lace the going and return wires together for minimum spacing before pulling in conduit, as the natural positioning of the wires in the conduit will give close enough spacing.

Table V gives the impedances of circuits and current-carrying capacities of relatively small wires suitable for small installations.

TYPICAL EXAMPLES

Example 1. Small Job-Welding Shop

Machines:
One 150-kva seam welder
Four 20-kva spot welders

Maximum kilovolt-ampere load swing of largest machine:
1,500 amperes at 240 volts = 360 kva

Table V.—Impedance and Carrying Capacity of Conductors in One Conduit

Wire Size	Conduit Size (Inches)	Resistance One Conductor Per 1,000 Feet (Ohm)	Reactance One Conductor Per 1,000 Feet (Ohm)	Current-Carrying Capacity (Amperes) Copper, 60 Deg C Ambient, 30 Deg C
2	1¼	0.156	0.040	107
0	1½	0.098	0.041	139
2/0	2	0.078	0.038	160
3/0	2	0.062	0.036	182
4/0	2	0.049	0.038	210

Regulation for 360 kva:	Per Cent
1. Supply system (4,800 volts)	3.0
2. 200-kva, single-phase, 4,800/240-volt, 3-per-cent, *Z* transformer	5.5
3. 100 feet two 4/0 wires per leg in conduit	2.3
Total	10.8

Equivalent continuous thermal loading:
0.7×150 kva =105
0.1×4×20 kva= 8
113 kva=470 amperes at 240 volts

Example 2. Small Job-Welding Shop

Machines:
Three 50-kva spot welders
Two 150-kva press welders
One 200-kva projection welder

Maximum kilovolt-ampere load swing of largest machine:
2,100 amperes at 240 volts=500 kva

Regulation for 500 kva:	Per Cent
1. Supply system (4,800 volts)	4.1
2. 500-kva, single-phase, 4,800/240-volt, 3-per-cent, *Z* transformer	2.9
3. 150 feet one $^1/_4$-inch by 4-inch bar per leg, 1-inch centers	3.7
Total	10.6

Equivalent continuous thermal loading:
10 per cent×650 kva=65 kva=270 amperes at 240 volts

Example 3. Small Manufacturing Plant

Machines:
Five 250-kva press welders
Two 50-kva spot welders
Eight 30-kva spot welders

Maximum kilovolt-ampere load swing of largest machine:
825 amperes at 485 volts=400 kva

Regulation for 400 kva:	Per Cent
1. Supply system (24,000 volts)	1.1
2. 700-kva, single-phase, 24,000/485-volt, 13-per-cent, *Z* transformers	7.4
3. 150 feet one $^1/_4$-inch by 2-inch bar per leg, $2^1/_4$-inch centers	2.0
Total	10.5

Equivalent continuous thermal loading:
Practically all three-cycle welding so use about five per cent of sum of connected load 0.05×1,590 kva=80 kva=165 amperes at 485 volts

Example 4. Small Manufacturing Plant

Machines:
Two 150-kva projection welders (automatic feed)
One 75-kva spot welder
One 50-kva spot welder
Eleven 25-kva portable welders

Maximum kilovolt-ampere load swing of largest machine:
850 amperes at 240 volts=205 kva

Regulation for 205 kva:	Per Cent
1. Supply system (4,800 volts)	2.7
2. 600-kva, three-phase, 4,800/240-volt, 5-per-cent, *Z* power transformer bank	3.4
3. 170 feet two 4/0 wires per leg in conduit	2.6
Total	8.7

Equivalent continuous thermal loading:
150-kva projection welder
2,000 welds per hour—3-cycle welds
Duty cycle=(2,000×3)/(60×60×60)=2.77 per cent
200 kva at 2.77 per cent duty cycle=34 kva continuous
Two machines=2×34=68 kva continuous
Remaining machines, 10 per cent×400=40 kva
Total, 108 kva

Example 5. Large Manufacturing Plant

Location *A*. Connect to Phase *AB*

Machines:
Three 550-kva butt welders
One 500-kva projection welder
One 400-kva projection welder
Three 150-kva projection welders

Maximum kilovolt-ampere load swing of largest machine:
2,700 amperes at 480 volts=1,300 kva

Regulation for 1,300 kva:	Per Cent
1. Supply system (4,800 volts)	5.6
2. Three 500-kva, single-phase, 4,800/480-volt, $3^1/_2$-per-cent, *Z* transformers	3.0
3. 125 feet two $^1/_4$-inch by 4-inch bars per leg, interlaced $2^1/_4$-inch centers	1.7
Total	10.3

Equivalent continuous thermal loading:
550-kva butt welder
200 welds per hour—30-cycle welds
Duty cycle=(200×30)/(60×60×60)=2.77 per cent
1,300 kva at 2.77 per cent duty cycle=216 kva continuous
Three machines=3×216=648 kva continuous
Remaining machines, 10 per cent × 1,350=135 kva
Total, 783 kva=1,670 amperes at 480 volts

Location *B*. Connect to Phase *BC*

Machines:
40 hydromatic welders, 150 to 300 kva each
Total connected load=7,000 kva

Maximum kilovolt-ampere load swing of largest machine:
2,100 amperes at 480 volts=1,000 kva

Regulation for 1,000 kva:	Per Cent
1. Supply system (4,800 volts)	4.1
2. Two 500-kva, single-phase, 4,800/480-volt, $3^1/_2$-per-cent, *Z* transformers	3.5
3. 200 feet two $^1/_4$-inch by 4-inch bars per leg, interlaced $2^1/_4$-inch centers	1.9
Total	9.5

Equivalent continuous thermal loading:
15 per cent×7,000 kva=1,050 kva=2,200 amperes at 480 volts

Location *C*. Connect to Phase *CA*

Machines:
Two 700-kva butt welders
Two 500-kva butt welders

Maximum kilovolt-ampere load swing of largest machine:
3,500 amperes at 480 volts=1,670 kva

Regulation for 1,670 kva:	Per Cent
1. Supply system (4,800 volts)	6.9
2. Three 500-kva, single-phase, 4,800/480-volt, $3^1/_2$-per-cent, *Z* transformers	4.0
3. 150 feet two double $^1/_4$-inch by 3-inch bars per leg, interlaced 3-inch centers	3.1
Total	14.0

The two 700-kva machines were interlocked so that they could not be operated together and the two 500-kva machines were also interlocked.

Equivalent continuous thermal loading:
700-kva welder
100 welds per hour—$2^1/_2$ second welds
Duty cycle=(100×$2^1/_2$×60)/(60×60×60)=6.95 per cent
1,670 kva at 6.95 per cent duty cycle=440 kva
Two machines=2×440=880 kva
500-kva welder
100 welds per hour—2 second welds
Duty cycle=(100×2×60)/(60×60×60)=5.55 per cent
1,200 kva at 5.55 per cent duty cycle=283 kva
Two machines=2×283=566 kva
Total, 880+566=1,446 kva

Reference

1. REACTANCE CHART FOR WELDER BUS PROBLEMS, R. C. Bouse and J. D. Scarbrough. *Electrical World*, volume 115, No. 10, March 8, 1941 page 106.

Discussion

Discussion will be found in the 1941 annual TRANSACTIONS volume and in the December 1941 SUPPLEMENT to ELECTRICAL ENGINEERING—TRANSACTIONS SECTION.

A Conserved-Pressure Air-Blast Circuit Breaker for High-Voltage Service

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Synopsis: This paper describes the design and construction of a 138-kv, 1,500,000-kva air-blast circuit breaker. The construction is made possible by the use of a new interrupting device which utilizes the axial blast nozzle together with a "conserved pressure" chamber. The chamber receives the exhaust from the interrupting nozzle and contains the movable contact of the breaker in its open position. By means of a spring-loaded relief valve the pressure and dielectric strength of the air in the chamber are maintained at a high level.

The pressure in the chamber is at the same time less than 50 per cent of the pressure above the nozzle restriction so that the air flow through the interrupting nozzle is unimpaired, being the same as through the same nozzle discharging to atmospheric pressure. As a consequence, the conserved pressure breaker is found to have the same ability to interrupt current as a conventional axial nozzle but to be capable of interrupting circuits of much higher voltage.

The 138-kv three-phase breaker consists of two interrupting units in series per pole with a disconnecting device included as part of the breaker. Operation of the breaker is entirely pneumatic, opening and closing being performed by energizing the electrically operated valve.

Interrupting Unit

TWO distinct types of air-blast interrupting units have previously been described, the cross blast,[1,2,4] in which air is directed across the arc formed in the path of the separating contacts, and the axial blast in which air flows through an orifice in the direction of the moving contacts.

Data[1] published at the 1940 midwinter convention showed that the European axial blast unit required a volume of air for interruption that was proportional to the maximum current interrupted. As a consequence such a breaker would require excessive quantities of air if it were built in the ratings found in this country. The problem of interrupting these high currents, of as much as 100,000 amperes, was solved by the introduction of the cross-blast unit which makes very economical use of air.

For high-voltage service, however, the currents to be interrupted are relatively light and the interruption problem is one of voltage magnitude. Under these conditions, the axial nozzle shows more promise for it is naturally a better insulating structure than the cross-blast breaker. Since it is cylindrical it may be turned from a solid piece of insulating material and it can be readily accommodated inside of a porcelain insulator.

However, the straight axial nozzle was found to have limited voltage-interrupting capacity and it was through a study of these limits that the conserved pressure interrupter was developed. The conserved pressure breaker is a solution to the problem of interrupting relatively light currents at high voltage. Its development and a direct comparison between it and two axial nozzles will be described.

A simple form of the axial-blast interrupting unit which has been used in Europe is illustrated by figure 1. In this design high-pressure air flows around the cylindrical stationary contact and expands to the atmosphere through the nozzle from which the moving contact has been withdrawn. The moving contact is drawn out into the open air and in the fully open position comes to rest in a region of atmospheric pressure.

Figure 2 illustrates the variation of the dielectric strength of air as a function of pressure. As a consequence of this variation, the dielectric strength of the air at the fixed contact of the breaker is five or six times as great as at the moving contact whereas, depending upon the interrupter configuration, the voltage gradient in the low-pressure area may be as high as in the high-pressure region. As the voltage rises, after the current has been momentarily interrupted by the air blast, the moving contact then provides a starting point for an electrical breakdown between the contacts.

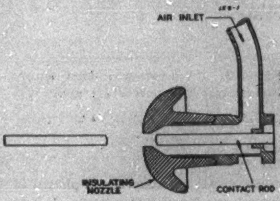

Figure 1. European high-voltage air-blast nozzle

This dielectric breakdown between the contacts can be avoided by a nozzle configuration of the type shown in figure 3*A*. Such a nozzle extends the region of high pressure—high dielectric-strength air to such a length that the contact spacing can withstand a high-applied voltage in spite of the fact that the air at the end of the moving contact is overstressed.

The interrupting ability of this long

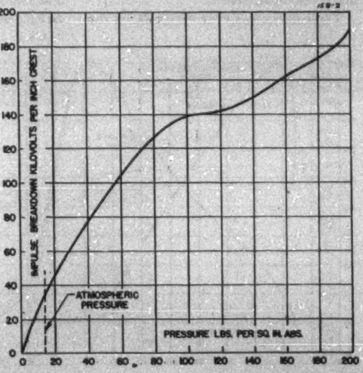

Figure 2. Impulse breakdown of a two-centimeter rod gap in air as a function of pressure. Temperature 25 degrees centigrade

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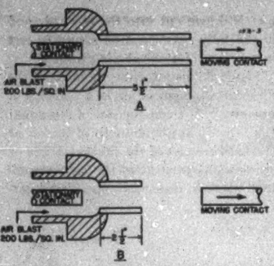
Paper 41-58, recommended by the AIEE committee on protective devices, and presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941. Manuscript submitted November 28, 1940; made available for preprinting December 13, 1940.

R. M. Bennett is in the circuit breaker development division of the General Electric Company, Philadelphia, Pa.; W. R. Rankin is also employed by this company.

[1] For all numbered references, see list at end of paper.



AXIAL NOZZLES

Figure 3. High-voltage air-blast nozzle. Inlet pressure 200 pounds per square inch

Exhaust pressure—atmospheric. Orifice diameter 1¼ inches

nozzle, in kilovolt-amperes, is plotted in figure 4 as a function of voltage. It is to be noted that although the breaker can interrupt a relatively large kilovolt-ampere at 38 kv, it has a definite current limit, so that, at lower voltages, where a larger current must be interrupted for a given kilovolt-ampere, the breaker fails. That is, the large volumes of ionized gas produced by large currents cannot be discharged by the long-restricted nozzle with sufficient rapidity to permit interruption.

The nozzle of figure 3*B* represents another extreme. It consists of little more than an orifice and is consequently capable of interrupting considerable current at low voltage. See figure 4. However, after interruption of the current at high voltages, progressive dielectric breakdown proceeds from the overstressed air at the tip of the moving contact back toward the orifice to such an extent that the high-pressure air becomes overstressed and it too breaks down.

These difficulties were overcome by the conserved pressure design which permits the moving contact to remain in a region of high-pressure air after the contacts have separated.

Figure 5 is a schematic representation of the breaker. High-pressure air is supplied from below, flows around the stationary contact and expands through an orifice to a chamber surrounding the moving contact. Pressure in this chamber is maintained at approximately 100 pounds per square inch by means of a spring-loaded relief valve. Provided the pressure in the chamber does not rise above approximately 53 per cent of the pressure above the orifice, the velocity and density of the air blast are unaffected by the pressure. It would therefore be expected that the performance of the conserved pressure breaker would be similar to that of the short orifice with the exception that it could interrupt much higher voltages. The dashed curve of figure 4 confirms this fact.

Breaker Construction

Each phase of the 138-kv air-blast circuit breaker consists of two conserved pressure-type interrupting units in series. The breaker which is rated 1,500,000 kva is supplied with air at 350 pounds per square inch.

The schematic drawing figure 6 shows the general arrangement of a single pole of the breaker. See figure 7. The essential parts consist of a main-blast valve for admitting air to the interrupters, an interrupting unit, a disconnect, and a mechanism for operating the disconnect.

The three-phase breaker has a main-air blast valve for each phase of the breaker. These are of the same type as the valves used in the 15-kv air-blast breakers described by Messrs. Strang and Boisseau at the 1940 midwinter AIEE convention.[3] They are poppet-type valves actuated by a piston which in turn is operated by air from a solenoid-operated differential pilot valve. The valves have an over-all operating time of two cycles from the time the solenoid coil is energized until the valve is completely open.

The interrupting contacts are separated by the action of the main air blast and are returned to the closed position by springs as soon as the air blast is turned off. Consequently a disconnect is provided, the function of which is to provide a clear air break after interruption is complete and before the interrupting contacts return to their closed position.

Closing of the breaker is performed by closing the disconnect. The closing speed of the breaker is not materially affected by current magnitudes in excess of the minimum breaker rating. This is accounted for by the fact that in the 138-kv breaker, the maximum rated current is a relatively light 9,500 amperes so that the magnetic forces are small compared with the inertia of the disconnect assembly.

The interrupting units are entirely pneumatic in operation, and require no mechanical connection with ground. The main air blast acts on a piston which is integral with the moving contact to move the upper contact upward separating the contacts.

When the movable contact has been withdrawn from the nozzle restriction the air blast which follows sweeps the contact space free of ionized gas at a current zero. The 138-kv breaker uses two interrupters in series, the total contact motion of each being eight inches. The interrupting contacts are returned by springs to the closed position as soon as the blast valve closes.

The interrupting nozzle is turned from a block of insulating material similar to fiber in possessing the property of becoming volatile at moderate temperatures. In the presence of an arc this property prevents the deposit of carbon on the surface of the nozzle and at the same time the gas which is evolved prevents the nozzle surface from reaching the high temperatures at which many otherwise

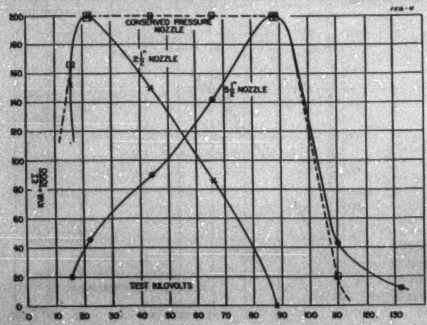


Figure 4. Interrupting ability of high-voltage air-blast nozzles. Inlet pressure 200 pounds per square inch. Maximum recovery rate 2,500 volts per microsecond

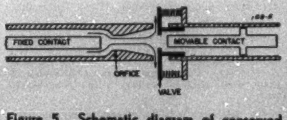


Figure 5. Schematic diagram of conserved pressure-interrupting unit

Inlet pressure 200 pounds per square inch. Orifice diameter 1¼ inches

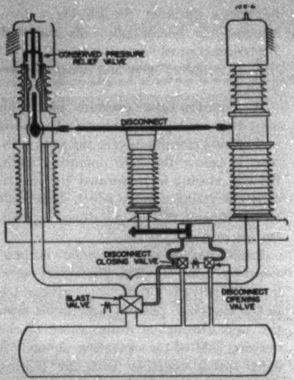
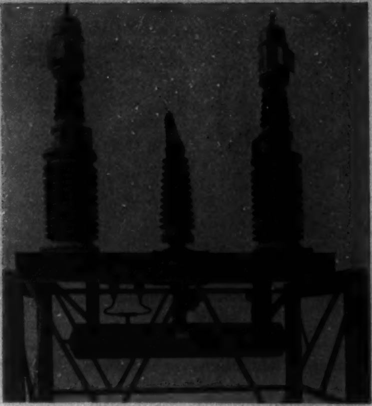


Figure 6. Schematic diagram of a single pole of the 138-kv, 1,500,000-kva air-blast circuit breaker

Figure 7. A single pole of the 138-kv, 1,500,000-kva air-blast circuit breaker



good insulators become conductors. The nozzle is contained in a herkolite tube which supports the upper contact piston and the conserved pressure-relief valve. The tube in turn is contained in a porcelain insulator.

The disconnect is also supported by a porcelain insulator which was specially developed for the purpose. The porcelain instead of having a straight creepage path inside, requiring that it be oil-filled and sealed, is made of a number of sections fused together. Each section is shaped like a cylinder closed at one end. When the sections are stacked up to form the insulator the closed ends of the sections become solid partitions across the inside of the insulator.

The disconnect arm engages long sliding stationary contacts by means of a

Table I. Single-Phase Opening and Closing-Opening Tests of AR-30-150, 138-Kv, 1,500,000-Kva Air-Blast Circuit Breaker

Test Number	Service	Single-Phase (Kilovolts)	RMS Amperes		Trip Impulse to Interruption (Cycles)	Recovery Voltage		Air Pressure in Tank
			Maximum Loop Closing	Initial in Arc		Rate of Rise	Peak Value (Kilovolts)	
1	O	132		40	5.2	590	290	300
2	O	132		43	4.9	630	320	300
3	O	132		470	5.7	1,420	320	300
4	O	132		720	5.7	1,600	360	300
5	O	132		890	6.0	2,000	290	300
6	O	132		1,150	5.5	2,000	290	300
7	O	132		1,280	6.5	2,200	310	300
8	O	132		1,420	5.7	2,400	280	300
9	O	88		750	4.9	1,300	200	300
10	O	88		1,330	5.7	2,100	220	300
11	O	88		1,880	5.5	2,400	180	300
12	O	88		2,100	5.5	2,500	190	300
13	O	66		2,900	5.3	2,400	110	300
14	O	44		3,800	5.4	1,900	82	300
15	O	44		4,400	4.8	1,900	75	300
16	O	22		6,900	5.0	1,000	36	300
17	O	22		7,700	5.2	1,100	38	300
18	O	22		9,700	5.5	1,100	33	300
19	O	22		10,700	5.5	1,100	33	300
20	O	22		9,500	5.5	710	30	300
21	O	132		80	4.4	1,410	373	300
22	O	132		670	5.2	3,100	312	300
23	O	132		1,100	5.5	4,100	261	300
24	O	132		1,500	5.0	4,500	261	300
25	CO	132	61	61	4.5	430	230	350
26	CO	132	690	660	5.5	1,700	300	350
27	CO	132	1,180	970	5.0	1,900	240	350
28	CO	132	1,490	1,190	5.0	2,000	250	350
29	CO	132	1,810	1,390	5.2	2,100	240	350
30	CO	132	2,300	1,610	5.2	2,300	250	350
31	CO	88	84	84	4.3	780	210	300
32	CO	88	3,500	2,600	5.0	2,500	150	300
33	CO	88	5,300	3,600	5.0	2,500	192	300
34	CO	44	4,800	3,900	5.4	1,800	83	300
35	CO	44	4,900	4,300	4.7	1,800	74	300
36	CO	22	10,200	7,500	4.8	930	36	300
37	CO	22	11,300	8,100	4.7	980	37	300
38	CO	22	10,000	8,500	5.2	1,100	37	300
39	CO	22	13,600	9,500	5.2	1,000	33	300

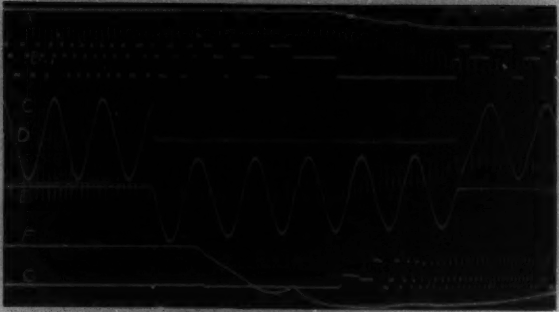


Figure 8. Test number 30. Table I. Single-phase closing-opening operation at 132 kv

Curve *A*—Disconnect trip impulse
Curve *B*—Disconnect travel—0.67 inch per step
Curve *C*—Pressure in interrupting chamber
Curve *D*—Voltage at breaker terminals
Curve *E*—Current through breaker
Curve *F*—Air-blast trip-coil current
Curve *G*—Interrupting contact travel—0.1 inch per step

loose-hinged blade which twists as it disengages. The contacts are so shaped that the arc formed during closing burns heavy contact tips rather than the contact surfaces. Both opening and closing of the disconnect is performed by a double-ended pneumatic piston which acts to rotate, about a vertical axis, the whole assembly, consisting of disconnect arm and insulator.

Air Auxiliaries

Air is supplied to the breaker tank from a conventional electric motor-driven storage and compressor unit. To eliminate the possibility of moisture reaching the breaker a compression cycle is used to dry the air.

Air is compressed to 1,000 pounds per square inch, cooled so that moisture is condensed in the high-pressure chamber, and is expanded to 350 pounds per square inch. After expansion the dew point of the air is approximately 80 degrees Fahrenheit below the temperature of the ambient air.

The control for the expansion valves is by means of Bourdon-type pressure switches. The control is arranged so that no air expands from the 1,000-pound tank to the 350-pound tank unless the compressor has brought the high-pressure tank up to full pressure. Thus no air can flow into the 350-pound tank unless it has gone through the complete compression cycle.

Operation

A breaker-opening operation is performed by electrically energizing the three-blast valve solenoids in parallel. The three valves are pneumatically interconnected so that all three main blast valves can be opened by any one of the solenoid-operated pilot valves. Opening of the blast valves initiates two functions. First, an air blast is supplied to the interrupting units to separate the contacts and interrupt the arc. Second, air is supplied to the disconnect control to initiate opening of the disconnects. The disconnect control is pneumatically interlocked with the blast valves so that the disconnect cannot operate unless all three blast valves have opened. If all three blast valves open, air is admitted to a differentially-operated pilot valve which in turn admits air directly from the tank to the disconnect operating piston. Separation of the disconnect contacts is timed to take place after the arc has been interrupted. Once the opening operation has been initiated the closing control solenoid is electrically disconnected. Electrical control is also provided so that the disconnect operation can be prevented.

A Bourdon-type pressure switch is used to open both the electrical opening and closing circuits in case the tank pressure is below the safe operating limit. Breaker closing is performed by electrically energizing a valve which admits air to the closing side of the disconnect operating piston.

Rapid reclosing is provided as indicated by figure 9. The disconnect opening pilot valve is a three-way valve which in the energized position admits air from the tank through a dump valve to the opening side of the operating piston. A light spring together with air flowing through the dump valve in the direction of the operating cylinder holds the dump valve closed.

When the pilot valve is de-energized, however, it opens a passage, from the back of the dump valve to the atmosphere, permitting the dump valve to open and exhaust the air from the opening side of the disconnect operating piston. Trip-free operation is provided by using the same valve arrangement for the closing side of the disconnect.

A complete closing-opening operation of the breaker takes place in the following order. Starting with the interrupting contacts closed and the disconnect open,

1. Disconnect solenoid energized.
2. Disconnect closes.
3. Disconnect solenoid de-energized.
4. Blast-valve solenoid energized.
5. Blast valve opens.
6. Disconnect opening pilot energized by air blast.
7. Interrupting contacts parted by air blast and arc interrupted.
8. Disconnect opens.
9. Blast valve de-energized.
10. Interrupting contacts return to closed position.

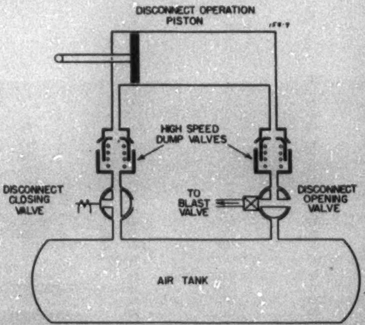


Figure 9. Disconnect operating mechanism

A New Air Circuit Breaker With 250,000-Kva Interrupting Capacity

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Synopsis: Heretofore it has not been commercially feasible to apply air-type breakers of the 5,000-volt class to circuits capable of producing short-circuit currents equivalent to 250,000 kva. However the continued demand for air circuit breakers of higher interrupting ratings has stimulated the development of a new breaker suitable for these applications. This achievement has been accomplished through magnetically enhanced diffusion of ionized particles, establishing dielectric strength in a gaseous region which was previously highly conducting. The interrupter consists essentially of an arc chamber of laterally spaced refractory plates embodying V-shaped slots and a magnetic circuit which imposes an intense magnetic field transverse to the slotted plates during the arcing period.

The theory of arc interruption by this new air breaker is considered. Design details are described and illustrated. Complete sets of test results are given, with representative oscillograms. The test results are discussed, and it is demonstrated that the new breaker is applicable to modern metal-clad switchgear.

I. Introduction

THE advantages of air-type power switching equipment are becoming increasingly apparent to the electrical industry. The result is a general desire to be able to supply air circuit breakers over the entire range of applications which have become established by the use of oil-type switching equipment. In response to the demand to fill in the gaps in the established interrupting ratings with air circuit breakers, a new self-contained unit suitable for an interrupting rating of 250,000 kva at five kv has been developed.

Early in 1938 development of principles somewhat different from those of the original "De-ion" air breakers[1] resulted in devices having very high-current interrupting ability at moderate voltages.[2] Currents as high as 120,000 amperes at 750 volts were interrupted. The principle of operation is a combination of arc construction and rapid gas motion. The gas motion is produced by virtue of a self-induced magnetic field which imparts a net upward component of velocity to the conducting electrons. The arcing time is definitely limited to one-half cycle or less.

The scope of this principle of magnetic deionization was expanded to devices having 150,000-kva interrupting ratings at 2,500 and 5,000 volts.[3] Continuation of the development has resulted in still further application of magnetically enhanced diffusion of ionized particles. The following sections are devoted to describing the newly developed air circuit breaker having an interrupting rating of 250,000 kva at 4,000 to 5,000 volts.

II. Design of the Interrupter

In addition to meeting the current carrying and interrupting requirements, an air circuit breaker should have a space factor comparable to the similarly rated oil breaker. The problem is then one of suitably drawing and enclosing the arc and controlling the ionized gases and other by-products of arc interruption. It has been found that definite advantages exist in utilizing a relatively short arc and permitting the current of the arc in interruption to go to its normal zero in cycle—whereat intensive deionization is provided to establish dielectric strength. The method of magnetic deionization possesses this type of arc characteristic.[3]

An extended series of tests, table I, made on a standard 150,000-kva 5,000-volt breaker demonstrated that there was a wide margin of safety above the 150,000-kva rating. Hence the logical program was to continue development of this design.

It had been indicated that the voltage which could be interrupted was very nearly proportional to the number of refractory plates in the arc chamber. Further, the amount of current that could be interrupted was a function of the plate width and thickness and of the vent space between the plates. Hence for the five-kilovolt ratings, two refractory sections or twice the number of insulating plates were necessary. In order to bring the arc uniformly into the V-shaped slots of all the plates the structure was supported at sufficient height above the arcing contacts to permit the use of sloping arc horns. See figure 1. The added arc length and distance of arc travel required greater venting to exhaust and deionize the hot gases. The vents between the refractory plates were made more effective by using wider and longer plates and by stacking them farther apart. A space was provided intermediate to the two refractory sections to release the initial gas pressure and to stabilize the arc during current interruption. Figure 2 is a picture of the two sections of refractory plates taken from a 250,000-kva 5-kv arc chamber.

During short-circuit operation of the magnetic "De-ion" breaker an arc is drawn between the separating arcing contacts, and is looped upward, magnetically, into the V-shaped slots. By transfer of the arc to the panel-end horn (figure 1),

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[1] For all numbered references, see list at end of paper.

Interrupting Tests

The 138-kv, 1,500,000-kva breaker is required to interrupt 6,300 amperes at rated voltage which is approximately 80 kv line to neutral. The maximum current rating is 9,500 amperes.

Table I is a series of single-phase interrupting tests performed on a single pole of the breaker. The currents interrupted range from light currents, similar to line-charging currents, to a maximum of 10,700 amperes.

Rates of rise of recovery voltage up to 4,500 volts per microsecond have been interrupted at 132 kv.

Figure 8 is a magnetic oscillogram of the closing-opening operation of test number 30.

References

1. THE CROSS-AIR-BLAST CIRCUIT BREAKER, D. C. Prince, J. A. Henley, and W. K. Rankin. AIEE TRANSACTIONS, volume 59, 1940 (September section), pages 510–17.

2. A NEW 15-KV PNEUMATIC CIRCUIT INTERRUPTER, L. R. Ludwig, H. L. Rawlins, and B. P. Baker. AIEE TRANSACTIONS, volume 59, 1940 (September section), pages 528–33.

3. DESIGN AND CONSTRUCTION OF HIGH-CAPACITY AIR-BLAST CIRCUIT BREAKERS, H. E. Strang and A. C. Boisseau. AIEE TRANSACTIONS, volume 59, 1940 (September section), pages 522–7.

4. MEDIUM-CAPACITY AIR-BLAST CIRCUIT BREAKERS FOR METAL-CLAD SWITCHGEAR, R. M. Bennett and B. W. Wyman. AIEE TRANSACTIONS, volume 60, 1941.

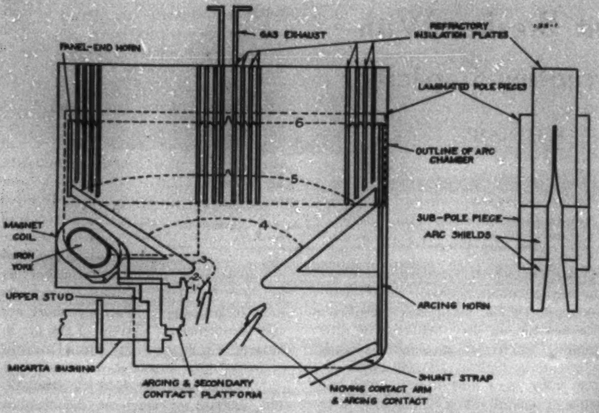


Figure 1. A diagrammatic sketch showing partial sections of the 5,000-volt magnetic "De-ion" interrupter. The dotted lines 1, 2, 3, 4, 5, and 6 indicate progressive positions in arc interruption

the magnet coil is switched into the circuit, thus imposing a strong transverse magnetic field. The upward motion of the arc is impeded by the refractory plates; however, a net upward component of velocity is imparted to the conducting electrons. They in turn bombard gas particles and produce a rapid flow of gas perpendicular to the arc. This reaction enhances the diffusion of the ionized gas particles and continuously causes the arc to ionize fresh gas in considerable quantities. The deionizing action is particularly prevalent near the edges of plates, and it continues in a sufficient degree to establish high dielectric strength during the period of relative low recovery voltage following a current zero.

By this method of magnetically enhanced diffusion of ionized gas particles the effectiveness and life of the interrupter are preserved. The long series of tests listed in tables II and III are typical runs which have not produced excessive wear on either the arc chamber plates or breaker contacts.

III. The Breaker Mechanism

The contact system and electromagnetic operating mechanism of the magnetic "De-ion" breakers are specially designed for high speed. Fast operation promotes both better contact protection and one-half cycle arcing time for currents of short-circuit values. Low inertia of the movable contact is obtained by a pivotally mounted contact arm upon which a pair of secondary and arcing contacts are rigidly attached to protect the main contact member. This main contact member is a solid copper block having heavy silver-inlay contacts. The arcing and secondary contacts are a silver-tungsten cintered mixture which is highly resistant to arcing. Figure 3 is a picture of a beaker with its contacts in open position.

A standard three-pole 5,000-volt breaker is shown in figure 4. The barrier structure is a plain interphase arrangement pivotally mounted as a unit at the

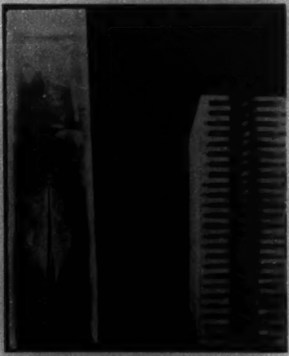


Figure 2. Two ceramic sections taken from an arc chamber after a series of 19 interrupting tests at 3,830 volts restored, single phase, and currents equivalent to 192,000 to 306,000 kva three phase

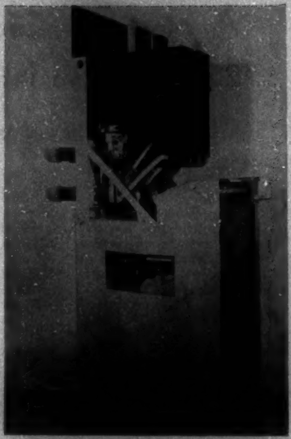


Figure 3. A 5-kv 2,000-ampere magnetic "De-ion" breaker with the unit interphase barrier and left-pole arc chamber removed to show the contacts and magnet coil

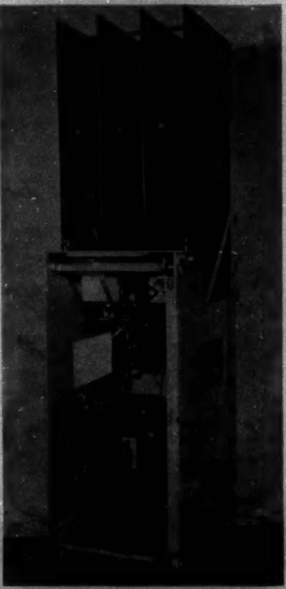


Figure 4. Complete draw-out type of magnetic "De-ion" breaker for five-kilovolt service

lower front corner. The unit is simply removed by pulling the top forward and lifting the assembly from its pivots.

The arc chambers are removable units supported by the laminated pole pieces. An individual chute assembly is shown in figure 5. The connectors of the arcing horns are automatically engaged when the arc chamber is pushed back into position on the laminated pole pieces.

The trip-free operating mechanism is mounted in a horizontal position immediately below the lower studs and moving contacts. It is isolated from the live parts by a steel barrier, and actuates the contacts through suitable insulating push rods. See figure 3.

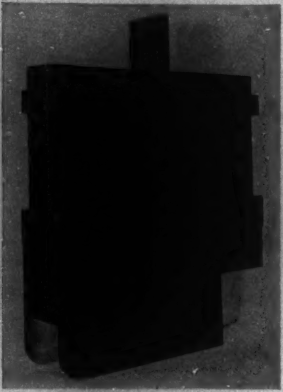


Figure 5. The arc-chamber unit of a magnetic "De-ion" breaker

An air dashpot is directly coupled to the operating lever of the mechanism. The volume of air handled is sufficient to absorb the kinetic energy of the opening contacts at both no-load and short-circuit capacity, thus bringing the contacts to a gentle rest without impairing the opening speed for approximately 50 per cent of the contact travel. The air dashpot is clearly shown in figure 4.

The magnetic "De-ion" breaker is designed to meet the requirements of modern metal-clad switchgear. Figure 6 illustrates the standard arrangement of the entire line of magnetic "De-ion" breakers for 2.5 to 5 kv, 600 to 2,000 amperes and up to 250,000-kva interrupting rating at 5 kv. The width of the 100,000- and 150,000-kva 3-pole breakers is 21³/₈ inches, and that of the 250,000-kva 5-kv unit is 31 inches.

Figure 6. Horizontal draw-out metal enclosed switchgear unit with a magnetic "De-ion" breaker for 2.5 to 5 kv, 600 to 2,000 amperes and up to 250,000-kva interrupting rating

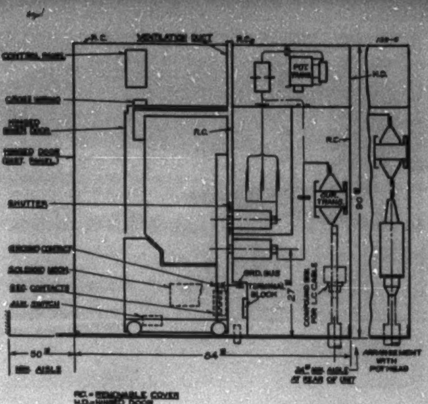


Table I. Typical Data From Interrupting Tests on a Single Pole of a 5,000-Volt 150,000-Kva Magnetic "De-ion" Breaker

Test Number	Test Voltage	RMS Amp. Interrupted	Arcing Time Cycles	Equivalent 3-φ Mva
1	3,300	27,000	1/2	154
2	3,500	28,500	1/2+	173
3	3,700	19,200	1/2	123
4	3,900	17,500	1/2−	118
5	4,100	18,600	1/2	132
6	4,300	25,000	1/2	186
7	4,500	18,600	1/2	145
8	4,700	19,400	1/2	158
9	3,000	10,000	1/2	52
10	4,900	18,000	1/2−	152
11	4,900	15,300	1/2	130
12	4,900	16,700	1/2−	142
13	5,000	16,000	1/2	139
14	5,000	600	4	
15	5,000	550	4	
16	5,000	380	5	
17	5,000	400	5	
18	5,000	360	5	
19	3,000	13,300	1/2	69
20	3,500	13,900	1/2−	84
21	4,000	13,300	1/2	92
22	4,300	16,500	1/2	123
23	4,500	14,500	1/2	113
24	4,700	15,800	1/2−	128
25	4,900	17,800	1/2	151
26*	5,000	16,800	3/4	146
27	5,000	18,600	1/2+	161
28*	5,000	16,000	1/2	139
29	5,000	16,000	1/2−	139
30	5,000	19,000	1/2	165
31	5,000	20,000	1/2+	173
32	5,000	19,400	1/2	168
33	4,000	19,400	1/2−	134
34*	4,000	18,800	1/2−	130
35*	4,000	20,500	1/2	144
36	4,000	22,600	1/2+	157
37	4,000	26,000	1/2	180
38	4,000	30,500	1/2−	211
39	4,000	26,500	1/2+	184
40	4,000	24,000	1/2−	166
41	4,000	26,000	1/2+	180
42	4,000	33,500	1/2	232
43	4,000	21,800	1/2−	151
44	4,000	23,500	1/2	163
45	4,000	30,500	1/2−	211
46	4,000	26,500	1/2	184
47*	4,000	24,000	1/2	166
48	4,200	37,000	1/2	269
49	4,200	31,300	1/2+	228

*Oscillograms of these tests are shown in figure 7.

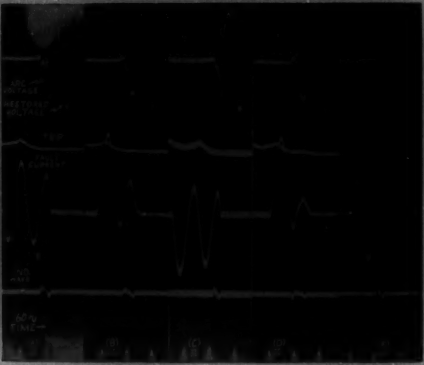
Current interrupted, amperes

16,800 16,000 18,800 20,500 24,000

Restored volts

4,600 4,700 3,750 3,640 3,800

Figure 7. Representative oscillograms of tests listed in table I



Current interrupted, amperes

24,800 39,900 25,900 36,800 25,900

Restored volts

3,830 3,800 3,830 3,890 3,950

Figure 8. Representative oscillograms of tests listed in table II

IV. Test Results

Extended series of tests were made during the development of the 250,000-kva 5-kv magnetic "De-ion" breaker. In addition to demonstrating ample margin for an interrupting rating of 250,000-kva at 5 kv, these numerous tests have shown the continued effectiveness of the breaker when applied to repetitious duty at and above the interrupting rating. Indeed, the wear on the contacts and arc chamber parts is small even after many operations, and the functioning of the breaker is in no way impaired.

Table I illustrates a typical run of 49 consecutive interrupting tests made on a single pole of a 5-kv 150,000-kva breaker without service to the breaker or arc chamber. The short-circuit values ranged from load currents to 37,000 amperes (equivalent to 291,000 kva, 3-phase). Figure 7 is a picture of five oscillograms taken from the tests listed in table I.

A short run of 19 interrupting tests with currents as high as 306,000 kva is indicated in table II. Figure 8 shows representative oscillograms from the group. These tests were made on a 5-kv 250,000-kva breaker, causing very little wear upon the arc chamber parts. The two individual ceramic sections of this chute are shown in figure 2.

A series of 34 short-circuit tests were made on a similar 5-kv 250,000-kva arc chamber. These tests are listed in table III, and the starred data are illustrated by the oscillograms of figure 9. The data indicate considerable margin above the interrupting rating of 250,000 kva.

Table II. Typical Data From Interrupting Tests on a Single Pole of a 5,000-Volt 250,000-Kva Magnetic "De-ion" Breaker

Test Number	Test Voltage	RMS Amp. Interrupted (Thousands)	Arcing Time Cycles	Equivalent 3-φ Mva
1	4,000	29.0	1/2−	201
2	4,300	33.6	1/2	250
3	4,300	35.0	1/2+	261
4	4,300	28.0	1/2	209
5	4,300	31.2	1/2−	232
6	4,300	24.8	1/2	185
7	4,300	37.8	1/2−	282
8*	4,300	24.8	1/2	185
9	4,300	27.4	1/2	204
10*	4,300	39.9	1/2+	297
11	4,300	24.8	1/2+	185
12	4,300	24.8	1/2	185
13	4,300	26.4	1/2+	197
14*	4,300	25.9	1/2−	193
15*	4,300	36.8	1/2+	274
16	4,300	27.4	3/4	204
17	4,300	26.4	3/4	197
18	4,300	27.4	1/2−	204
19*	4,300	25.9	1/2−	193

*Oscillograms of these tests are shown in figure 8.

During another series of tests on a single pole of a 5-kv 250,000-kva breaker two consecutive shots of 300,000 kva 3-phase were obtained. The severity of these interruptions did not stress the interrupter unduly. The oscillograms are shown in figure 10.

Numerous interrupting tests have been made at lower current values with five kilovolts across a single pole. These low currents were obtained by connecting an air-cored reactor of negligible resistance directly in series with the breaker and a 20,000-kva generator. At 1.6 amperes the arc was interrupted in less than one-half cycle. The arcing time-current characteristic rises gradually to about 22 cycles at 14 to 20 amperes and recedes to 4 cycles at 600 amperes. At currents above several thousand amperes the arcing time is only one-half cycle.

The magnetic "De-ion" breaker is applicable to circuits requiring repetitious switching. No-load *CO* life tests have been made in excess of 25,000 operations without repair to any part. At the close of the tests there was no indication of wear or strain on the breaker which would require overhauling or replacement of any major item.

V. Conclusions

The principle of magnetically enhanced diffusion of ionized particles for establish-

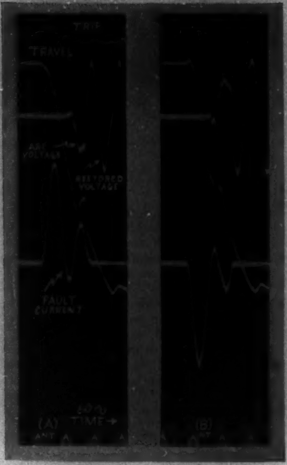
Figure 9. Oscillograms of tests indicated in table III

Current interrupted, amperes
31,000 40,300 28,400 29,400 41,300
Restored volts
4,030 3,800 4,000 4,150 3,920

Table III. Extended Series of Interrupting Tests on a Single Pole of a 5,000-Volt 250,000-Kva Magnetic "De-ion" Breaker

Test Number	Test Voltage	RMS Amp. Interrupted (Thousands)	Arcing Time Cycles	Equivalent 3-φ Mva
1	4,000	36.2	3/4	250
2	4,300	41.3	1/2−	308
3*	4,300	31.0	3/4	231
4	4,300	32.6	1/2−	243
5	4,300	29.0	3/4	216
6	4,300	33.6	3/4	250
7	4,300	37.2	3/4	277
8	4,300	38.8	3/4	290
9	4,300	33.1	1/2−	247
10	4,300	26.9	3/4	200
11	4,300	24.8	1/2	185
12	4,300	25.8	1/2−	194
13	4,300	23.3	1/2+	174
14	4,300	31.0	3/4	231
15	4,300	25.8	3/4	192
16	4,300	41.3	1/2+	308
17	4,300	33.6	3/4	250
18	4,300	40.3	1/2−	301
19	4,300	31.0	1/2−	231
20*	4,300	40.3	3/4	301
21	4,300	28.4	1 −	212
22	4,300	28.4	1	212
23	4,300	24.3	1/2	181
24	4,300	30.0	1/2−	224
25	4,300	36.2	1/2−	270
26	4,300	39.8	1 −	295
27	4,300	29.5	3/4	220
28	4,300	37.2	1/2−	277
29*	4,300	28.4	1/2+	212
30	4,300	41.3	1/2−	308
31*	4,300	29.4	3/4	219
32	4,300	31.0	1−	231
33	4,300	28.4	1/2	212
34*	4,300	41.3	3/4	308

*Oscillograms of these tests are shown in figure 9.



Current interrupted, amperes
40,000 40,000
Restored volts
4,020 4,020

Figure 10. Two consecutive high-current interruptions obtained with a 5-kv 250,000-kva magnetic "De-ion" breaker

ing dielectric strength in a previously high conducting region has been successfully applied to an air circuit breaker. Extended series of interrupting tests with currents as high as 40,000 amperes and restored circuit voltages of 4,020 volts across a single pole indicate a conservative interrupting rating of 250,000 kva. The magnetic "De-ion" air circuit breaker is suitable for severe repetitious duty, and also meets the requirements of modern metal-clad switchgear.

References

1. THEORY OF THE "DE-ION" CIRCUIT BREAKER, J. Slepian. AIEE TRANSACTIONS, volume 48, 1929, page 523.

2. A NEW HIGH-CAPACITY AIR BREAKER, L. R. Ludwig and G. G. Grissinger. AIEE TRANSACTIONS, volume 58 (August section), 1939, page 414.

3. MAGNETIC "DE-ION" AIR BREAKER FOR 2,500–5,000 VOLTS, L. R. Ludwig and R. H. Nau. AIEE TRANSACTIONS, volume 59 (September section), 1940, page 518.

Magnetic Fields in Watt-Hour Meters

Effects of Wave Form on the Registration of Single-Phase Watt-Hour Meters

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Synopsis: The growing use of electrical equipment which causes distortion in the supply voltage and current has caused increased concern regarding the performance of watt-hour meters used to meter the energy required by such equipment. The watt-hour meter is designed and adjusted, in practice, to operate on practically distortionless current and voltage.

An analysis of the factors involved and an experimental study of meter registration under definitely controlled conditions show that watt-hour meters should perform satisfactorily unless the degree of distortion is large. These controlled conditions included wide variations in the distortion of potential and current both individually and together. The characteristics of meters of recent design, of all the major manufacturers, were studied experimentally.

Introduction

THIS study of the effects of variation in wave form on the registration of a watt-hour meter is a part of a general study of the magnetic conditions which exist in such meters during their operation. Data which are presented for illustration of these effects are to be considered more qualitatively than quantitatively since they apply directly to particular types of meters of recent design. The order of magnitude of the effects of variations in wave form can be inferred in any case except perhaps for meters of very early design.

Single-phase meters only are considered here. The influence of the interaction of the several elements of polyphase meters is to complicate relationships introduced by a variation in wave form and depends upon the position of the elements relative to each other. Since the general effect of harmonics is found to be small, the relatively small influence of one element on another in a multidisk meter should give rise to effects of but minor importance so far as they are caused by harmonics.

Because of the necessary differences between the designs of elements of multidisk and single-disk polyphase meters, the effects of wave-form variations on the performance of single-disk meters are being given separate study.

Meters of recent designs, although made by various manufacturers are much more nearly alike, fundamentally, in their magnetic circuits than those formerly manufactured. Many differences in detail are still present and although the net result is, in each case, a meter with satisfactory performance characteristics, variations may be noted in the performances of the meters when used in circuits where abnormal wave forms occur in voltage or current.

In addition to differences in the proportions of parts of the magnetic circuits of the meters, design details which influence the flux distribution and variation in a meter and consequently the performance characteristics under abnormal loads are the lagging circuit and the compensations for friction, temperature change, and overload. No attempt is made to evaluate separately the influence of each component part, their collective influence only being noted. Even with meters of the same design, differences among them which are not evident under normal conditions may become noticeable under abnormal conditions.

Discussion

The operation of a watt-hour meter depends upon the interaction of the fluxes produced by the component magnetomotive forces of the potential and current elements and the disk currents produced by the changing of those fluxes. The changes in these fluxes may be in either magnitude or position or both. The distribution of these fluxes in the meter and their variations in magnitude with time are important factors in the determination of the disk torque upon which the meter registration depends. Differences in both the space distribution of the fluxes with time for the various meters are determined to a large degree by the differences in the details in the designs of the meters. Although the details of element design have some influence, the time variation of the flux is determined largely by the external conditions in the circuit. The design of the magnetic circuit, with associated electrical circuits, practically determines the space distribution of the flux. Because of the variable timing of the magnetomotive forces of the potential and current elements the space distribution of their resultant flux is variable and hence is a function of external conditions as well as the magnetic design. In a like manner, the time variation of the resultant flux depends upon the variations in magnetic reluctance introduced by the magnetic design as well as the magnetomotive force variation dictated by external conditions. The materials of construction and their physical properties will have marked influences on the flux and hence on the characteristics of the meter.

It is the purpose, here, to discuss the effects of variations in the wave form of current and voltage, as fixed externally, on the registration of watt-hour meters.

Generally, the net conditions existing in and around any circuit where currents and voltage are nonsinusoidal can be considered to depend upon all components, fundamental and harmonics, present. This dependence may be difficult to determine unless linear relations exist. However, in most instances in power circuits, if harmonics are present to an important degree it is because of nonlinear response of iron in magnetic circuits to magnetization. In such cases the third and fifth harmonics are usually most prominent and except in instances of very high magnetic densities are the only ones of importance. Frequently only the third harmonic is sufficiently large to have much influence. For this reason, an experimental study of the effects of one major harmonic (the third) should give an understanding of the effects to be expected in practical cases.

In any case, net meter-disk torque should be produced only when voltage and current of the same frequency are used because in such cases only is there a net transfer of energy. This applies to harmonic components as well as to the fundamental. Experimentally it has been found that this is not true in all cases with practical meters. Net torque is produced even though voltages and currents of different frequencies are used. However, a distinction must be made between potentials and currents and the meter fluxes that result from them. The presence of iron in the magnetic circuits of the meters with its nonlinear response

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to magnetizing forces will give rise to time variations in the fluxes of harmonics which are not present in the magnetizing forces. These harmonics may give rise to torque components which with reference to the torque produced by the fundamentals may be positive or negative. Figures 2–5 are reproductions of oscillograms showing the presence of such harmonics in the variation of the air-gap flux in a representative meter of recent design with only fundamental potential and current variations.

In certain single-phase watt-hour meters the cores of the current and potential elements are placed as shown in figure 1 relative to the disk. The points indicated along a center line of these cores designate positions at which the rates of variation of flux density are shown in figures 2, 3, 4, and 5. These rates of variation are for conditions specified and correspond to actual conditions of operation with sine waves of potential and current.

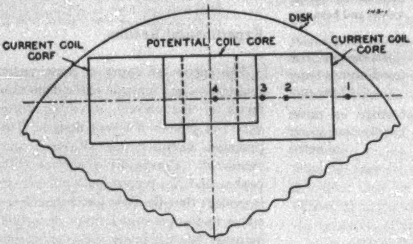

Figure 1. Plan—magnetic core of meter showing survey points

It will be noted that with sinusoidal potential impressed alone (figure 2), that the variations at all points approximate the sinusoidal. A careful analysis of these variations, however, shows that several harmonic components of small magnitude are present. The magnitudes at the various points differ widely. These conditions are to be expected since the flux linkages with the potential coil windings must vary in such a manner as to produce an induced voltage differing from the (sinusoidal) impressed voltage by only the resistance drop. This, with an iron core, requires that the induced voltage (and hence variation of flux) must contain harmonic components because of the harmonic content of the exciting current. These flux harmonics are of small magnitude relative to the fundamental as the curves of figure 2 indicate. The variations in amplitudes among the points are a result of the distribution of core material.

For the condition of the current coil alone, excited by sinusoidal currents (figure 3), the rates of variation of flux density at the various points are definitely nonsinusoidal. The nonlinear response of the iron to magnetizing force introduces harmonics of considerable magnitude. The influence of core distribution is evident.

With normal excitation for both potential and current elements, the resultant flux variation is not a superposition of those produced by each element acting independently. Portions of the magnetic circuit are influenced by both windings and having a nonlinear response, as stated before, will have resultant magnetic conditions which are not proportional to the component magnetizing forces. The conditions at the various points used to illustrate this are shown for two power factors in figures 4 and 5. In these cases the time relationship between the variations in flux density at the different points is not indicated in these figures.

Because of the difference in timing of the magnetomotive forces of the potential and current elements and the distribution of magnetic core material, the distribution of the resulting flux is not fixed. The variation includes both magnitude and position. Because of this shifting position and magnitude, a variable torque, tending to drive the disk, is produced. This variable torque can be considered as the resultant of many component torques, some positive, some negative. These components can be considered as produced by the fundamentals and harmonics in the space distribution and time variation of the air-gap flux as in any motor.

Although the net instantaneous torque in the case of sinusoidal potential and current is not uniform the single-phase watt-hour meter can be adjusted to give satisfactory registration characteristics. With external circuit conditions such that nonsinusoidal voltage or current or both exist, the proportions of the component

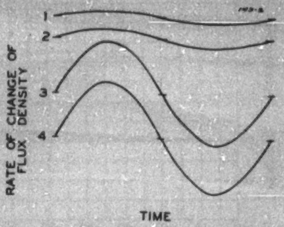

Figure 2. Cyclic rate of change of flux density for rated sinusoidal potential excitation only

Numbers on curves identify the survey points of figure 1

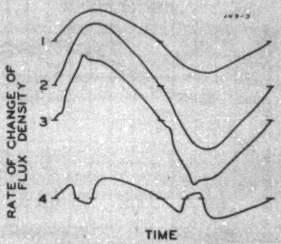

Figure 3. Cyclic rate of change of flux density for rated sinusoidal current excitation only

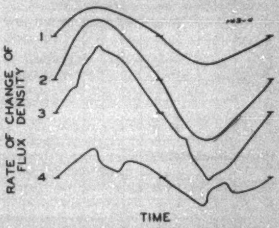

Figure 4. Cyclic rate of change of flux density for both rated sinusoidal potential and rated current excitation at unity power factor

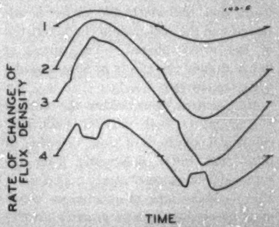

Figure 5. Cyclic rate of change of flux density for both rated sinusoidal potential and rated current excitation at 80 per cent power-factor lag

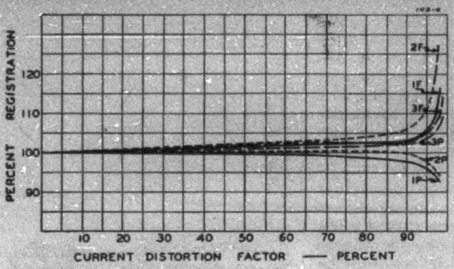

Figure 6. Per cent registration as a function of current-distortion factor for a resistance load with rated sinusoidal potential and rated current (effective value)

Numbers on curves identify manufacturer of meters

Curves *F*—Flat-topped current waves

Curves *P*—Peaked current waves

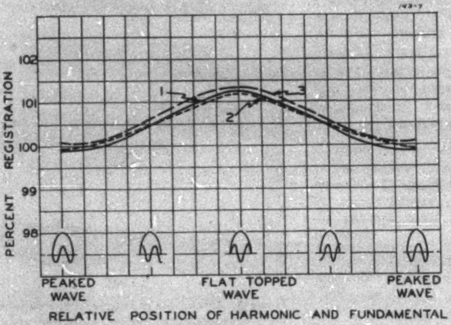

Figure 7. Per cent registration as a function of relative time position of fundamental and harmonic

Sinusoidal potential, constant power, current-distortion factor of 40 per cent

Numbers on curves identify manufacturer of meters

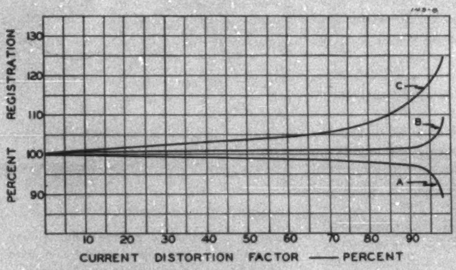

Figure 8. Per cent registration as a function of current-distortion factor for sinusoidal potential and peaked current waves

Curves for meter number 1 at rated current

Curve *A*—Fundamental current in phase with fundamental voltage

Curve *B*—Fundamental current lags fundamental voltage by 37 degrees

Curve *C*—Fundamental current lags fundamental voltage by 60 degrees

torques caused by harmonics will differ from that for which the meter is adjusted and the registration characteristics of the meter altered. The curves of figure 6 show the effects of nonsinusoidal waves of current.

In practical cases, either the voltage or current or both may be essentially sinusoidal functions of time, or either or both may differ sufficiently from the sinusoidal to warrant study. The probable combinations of conditions which may be encountered in practice are (1) both sinusoidal, (2) potential sinusoidal with current nonsinusoidal, (3) both nonsinusoidal. Since the effects of nonsinusoidal conditions are under consideration the base for comparison is taken as the combination (1) namely, the current and potential both sinusoidal.

For cases of nonsinusoidal conditions, varying degrees of distortion as well as various time relationships between harmonic components and the fundamental are considered. For purposes of establishing a measurable effect the distortion was limited to that caused by the presence of a third harmonic. Similar effects should be noted for other harmonics. It will be realized that as the number of harmonic components increases, the number of possible time relationships between the components increases beyond possible experimental study.

The term "distortion factor" is defined as "the ratio of the effective value of the residue after the elimination of the fundamental to the effective value of the original wave." (A.S.A. C-42-1932 section 10.95.430.)

For the case of nonsinusoidal variation of both potential and current an unlimited number of combinations are possible depending upon circuit conditions. Several examples are given, the conditions being noted with the curves showing experimental results.

Experimental Results

The curves of figure 6 show meter registration as a function of the distortion factor of the current wave for three makes of meters of recent design. The potential variation for all cases was sinusoidal. Curves are given for both peaked and flat-topped wave forms. It is evident that the disk speed should decrease as the per cent current distortion increases because under such conditions, the average power decreases. However, for all cases, the meter registration was computed as the ratio of actual disk speed to the disk speed for sinusoidal potential and current conditions giving the same average power. Although the large registration error, indicated on the curves, exists at large per cent values of current distortion, loads of this character are seldom experienced in practice.

For large values of distortion factor, the registrations of the meters with peaked currents differ greatly from those for flat-topped currents. These two wave forms can be considered as limiting cases of the time relationship between the fundamental and third harmonic components of current. For intermediate relationships of the components, intermediate values of the effects would be expected. Figure 7 shows the influence of the time relationship of the components for a constant-load and a constant-current distortion factor of 40 per cent.

Curves are shown for the three makes of meters as in figure 6. It can be noted that for all meters, flat-topped current waves gave the greatest meter registra-

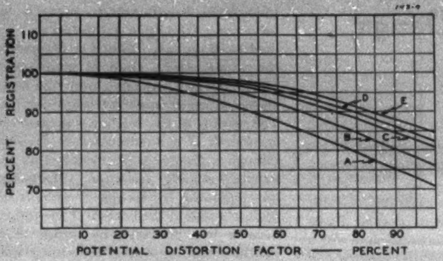

Figure 9. Per cent registration as a function of potential distortion factor for nonsinusoidal potential and current waves

Curves for meter number 1 at several resistance loads for peaked wave form

Curve *A*—20 per cent load
Curve *B*—50 per cent load
Curve *C*—100 per cent load
Curve *D*—200 per cent load
Curve *E*—300 per cent load

tion with a sine wave of potential and that all meters showed about the same performance.

The performance of the meters differed widely for large values of current distortion factor, although all meters were adjusted to give correct registration with zero-distortion factor. Harmonic components in the currents in the current-element windings, through magnetic coupling between those windings and circuits in the details of design, give rise to harmonic currents in those circuits not in exact phase opposition to those in the current coils. These harmonic components may produce torques which have an appreciable magnitude relative to the fundamental torque. It was noted during the tests that 100 per cent current-distortion factor with the potential element unexcited some meters crept backward rapidly, some forward rapidly, and some did not creep. With the potential coil short-circuited or excited, disk movement was reduced in magnitude. At large values of distortion factor the summation of the torques due to harmonics became an important factor in the total torque.

All test meters responded similarly to variations in the phase angle between the fundamental components of potential and current. Figure 8 shows meter registration as a function of distortion factor for peaked waves of current and for three values of fundamental phase angle. The voltage and current were rated values.

For conditions of distortion in both potential and current waves, an unlimited number of combinations are possible. If the load is noninductive the distortion of both the waves should be the same. Under these conditions the time variations of the magnetomotive forces caused by both the potential and current circuits of the meter contain corresponding harmonic components. It should be noted, however, that the presence of corresponding harmonic components in the magnetomotive forces does not imply the presence of harmonic components in the fluxes having the same relative relationships as those of the magnetomotive forces.

Because of the higher frequency, the potential flux per volt for a harmonic is less than that for a fundamental. The variations in the air-gap flux will then be the resultant of components of different relative magnitudes than the respective component voltages. The effects of such conditions on the registration of a typical meter at various per cent loads and for peaked waves are shown in figure 9. Experimental results, using flat-topped waves, differed but slightly from those for peaked waves. In no case was this difference more than one-half per cent. With resistance loading, a reversal or change of any component in the voltage was accompanied by a corresponding change in the current component of that frequency. The net result of the change on the registration of the meters was of no practical magnitude. The flux density at which modern meter cores operate is low enough to make the effects of changing the maximum flux density (caused by change in wave form) small.

When the load circuit is inductive, the distortion factors of potential and current differ. One example of such loading gave results shown in figure 10. In this case, load resistance and inductance were of such values as to produce a fundamental phase displacement of approximately 37 degrees (0.8 power factor) the fundamental current lagging the voltage. Peaked waves of potential were used. The load power was constant.

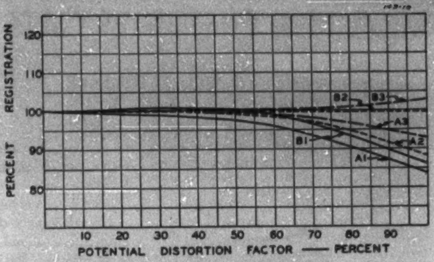

Figure 10. Per cent registration as a function of potential distortion factor for nonsinusoidal potential and current waves

Peaked wave form, two inductive loads
Fundamental current lags fundamental voltage by 37 degrees
Numbers on curves identify manufacturer of meters

Curves *A*— 25 per cent load
Curves *B*—150 per cent load

Conclusions

1. Meter registration errors, as the result of wave distortion, for properly adjusted meters, usually will not exceed the limits of permissible calibration error unless the current-distortion factor is greater than 30 per cent.

2. Indicating instruments are recommended for use in the testing of watt-hour meters when the distortion factor of the current exceeds 50 per cent.

3. No definite statement can be made regarding the effects of distortion on meter registration if the potential distortion is the result of impedance drops caused by distorted currents since the time relationships of the harmonic components depend on circuit conditions.

4. Because of the differences in the space and time variations of the fluxes which would be produced by fundamental and harmonic components of potential, current or both, the net registration of a meter is not the summation of the registrations that the several harmonic components of potential and current would produce, each acting alone.

5. As a result of observations made during the tests it is believed that minor differences in the adjustments required to give satisfactory registration by meters of the same model may cause noticeable differences in the effect of wave distortion on the characteristics of the meters.

6. In the majority of meter applications where minor distortion of current or potential exists the watt-hour meter of recent design measures the energy satisfactorily if adjusted within the limits of commercial accuracy under customary testing conditions.

A New Principle of Cable Design

WM. A. DEL MAR
FELLOW AIEE

Synopsis: Study of cable behavior on load-cycle tests led to the conclusion that instability of the insulation results from its deformation, leaving ionizable voids in the inner layers, where deterioration starts.

This led to the conception of a cable in which the inner and outer layers of paper would have different moduli of elasticity, the outer papers having the higher modulus so that during the cooling period of a load cycle, the outer papers would compress the inner ones and so reduce or eliminate voids therein.

The tension of application of the tapes is the same for all layers and does not depart from ordinary practice.

It was found that cables made on this principle maintained a remarkable stability of power factor under load-cycle conditions.

Finally, a study of cable impregnants showed that an essential characteristic is stability under electrical discharges. An impregnant was developed consisting of a paraffin base oil with a definite proportion of a new synthetic derivative of wood rosin, which possesses this characteristic in a marked degree. This, used in combination with the above-described structure gives the maximum stability we have been able to attain.

IT HAS been the general belief that oil-filled paper cables differ from solid-type paper cables in that the former have a perfect radial and longitudinal flow and ebb of oil throughout the insulation during heat cycles whereas in the latter, the oil flows but does not ebb, perfectly, leaving ionizable voids in the depth of the insulation.

It is undoubtedly true that, in the oil-filled cable, the oil flow is reversible but our researches show that in the solid-type cable, there is little flow of oil out of the insulation at ordinary operating temperatures and, therefore, substantially no trouble to be expected from its imperfect ebb. The view is expressed herein that the real difference between an oil-filled cable and one of solid type is that, in the former, the oil is of such low viscosity that it can flow out and back again under pressure too small to deform the paper permanently, whereas in solid-type cables, the oil viscosity is so great that very high pressures are created by the expanding oil which tend to stretch the paper permanently, creating gaps between tapes which become vacuous when the oil contracts.

This is a useful conception of cable operation and a design of cable based upon it is described, the principle of which is to improve the effective elasticity of the mass of paper by using successive layers of increasing modulus of elasticity, so that each layer will be held in compression by circumjacent layers of greater modulus.

By this means the insulation of solid-type cable acquires some measure of the reversibility characteristic of oil-filled and pressure cables but without adding in any way to the dimensions or complications of the cable or its accessories.

The construction now used consists merely in the use of two layers of paper, the inner comprising three-fourths of the thickness, and the outer, one-fourth of the thickness of the insulation, the paper of the outer layer having a modulus of elasticity 15 per cent to 20 per cent greater than that of the inner layers.

Relative Imperviousness of Impregnated Paper to Viscous Oil Flow

Tests were made to establish the critical viscosity at which oil can move through insulation without exerting enough pressure to put undue stress on the paper.

Typical test results are shown in figure 1. It will be noted that with viscosities above 1,000 to 2,000 seconds there is but little pressure transmission through the insulation. Furthermore, in an operating cable, the oil temperature in the middle of the insulation will be considerably less than at the conductor, so that compound does not flow until the oil at the conductor surface has attained a considerably higher temperature than would be suggested by the data on figure 1.

Elastic Properties of Cable Paper

A study of the elastic properties of impregnated-paper cable was undertaken in order to better appreciate the stresses and strains arising from cyclical loading of cables. A cabinet was constructed capable of holding in vertical suspension a strip of paper one meter in length, changes in length being measured by means of a steel scale graduated in hundredths of an inch. Glued to the end of the paper was a piece of transparent celluloid which extended over the scale and upon which was scratched a hairline. The reading of the hairline was made to one-fourth of a hundredth-inch division through a glass door by means of an instrument-reading telescope. Humidity was regulated by means of anhydrous acids in a dish at the bottom of the tower. Tension was produced by means of a Chatillon spring scale. Humidity was measured by means of a calibrated Taylor humidiguide. The appearance of the setup is shown by figure 2.

It was found that impregnated-cable paper has peculiar physical properties, which would be likely to influence its behavior in a cable. The stress-strain characteristic of a typical cable paper is shown by figure 3. Different papers, however, have different slopes of curve, corresponding to different moduli. Although the graph is a straight line up to about 5,000 pounds per square inch, the paper is not perfectly elastic, within that range, but has a permanent set after each cycle of stretching, the amount of set rising with the tension. The relation between elongation and permanent set was found to be of the form shown by figure 4.

Armed with these data, the next step was to calculate the paper tensions that would result from oil expansion. This was started with the generally accepted idea that this expansion would cause the tape to stretch, thereby separating the layer sufficiently to permit the expanding oil to flow freely outward toward the sheath. The accepted idea was that in cooling, the elasticity of the paper would cause it to close in upon the contracting oil, even to the extent of the outer layers cutting off some of the oil before it could return to the interior of the cable.

It was found, on the contrary, that the paper did not stretch sufficiently to create the necessary oil passages between layers, but that, with the calculated stresses, the permanent set was sufficient to leave voids in cooling.

The obvious step to be taken to improve the quality of cable was to use a truly elastic paper, but we could not find a paper which showed true elasticity when impregnated. All papers tested showed more or less permanent set.

It was noted, however, that different papers had different moduli of elasticity and the idea presented itself of placing layers of paper of high modulus over layers of low modulus so that as the inner papers were stretched by the expanding

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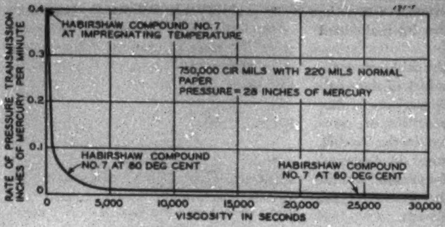

Figure 1. Relative imperviousness of impregnated-paper cable to viscous oil under pressure

Viscosities above 3,500 seconds which could not be measured on the Saybolt universal viscosimeter were measured by rate of penetration through single sheets calibrating the sheets with an oil of known viscosity

oil, they would meet resistance by the outer layers and in cooling, the pressure exerted by these outer layers would help to restore the inner layers to their original condition.

In order to test the correctness of this theory, cables were made with two grades of paper, the inner layers being made with the normal paper and the outer layers with paper having about 20 per cent greater modulus of elasticity. These cables were tested in comparison with cables of normal construction by means of load-cycle life tests.

Accelerated Aging Tests

The best available data on the correlation of accelerated aging with natural aging were those of the Commonwealth Edison Company published by D. W. Roper, AIEE TRANSACTIONS, volume 52, 1933, page 1028 and by Halperin and Betzer, AIEE TRANSACTIONS, volume 55, 1936, page 1074.

In order to ensure that our tests should be equally correlated with service conditions we decided to make our tests at the same maximum stress, as in the Halperin-Betzer tests, namely 208 volts per mil and at a conductor temperature five degrees centigrade over the normal operating temperature. When meter corrections were taken into account, our actual maximum stress was 203 volts per mil. The cable type selected for test was a 4/0 AWG, 19 strand, single-conductor cable with 220 mils of insulation and 63 mils lead sheath.

Heating current was maintained for eight hours per day, five days per week. Voltage was maintained the full 24 hours and over weekends, except for intervals while measurements were being made.

Figure 2. Test chamber for modulus determination

Roper reported that tests made under these conditions gave failure of similar nature to those occurring in service and that it would be unnecessary to carry the tests to failure as the rising power factor, ionization and hot-spot temperature would differentiate between cables of different qualities in the course of from two to seven weeks, i.e., 10 to 35 load cycles.

Our preliminary work showed that this generalization was also applicable to our tests.

> Poorly impregnated cables, i.e., cables drained of part of their oil, were eliminated in two weeks, i.e., 10 or 12 load cycles.
>
> Questionable cables were eliminated in four weeks, i.e., 20 load cycles.
>
> Good cables, as then made, lasted at least five or six weeks, i.e., 25 to 30 load cycles.

Roper found that whereas tests made at $2^1/_2$ times operating voltage, corresponding to a maximum stress of 208 volts per mil were indicative of service life, those made at three times operating voltage, corresponding to a maximum stress of 250 volts per mil, had no such significance but, on the contrary, were quite erratic.

The suggestion that tests be discontinued before failure, was a fruitful one as it not only enabled more tests to be made in a given time, but also permitted examination of the cable where failure seemed imminent and before destruction of the evidence occurred. The tapes from a typical arrested failure on a cable of ordinary type is shown in figure 5.

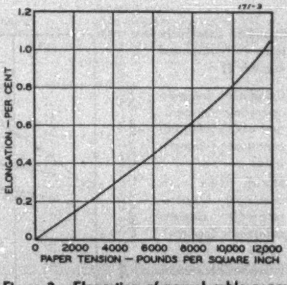

Figure 3. Elongation of normal cable paper impregnated

As the tests were to be discontinued when the cables showed signs of instability, it was necessary to establish arbitrary limits indicative of this condition.

The Commonwealth Edison limits and those we adopted were as follows:

	Commonwealth Edison Company	Habirshaw
Power factor at 85 degrees centigrade		2.00
Rise of power factor at high temperature of cycle (per cent)	0.75	
Rise of ionization (per cent)	0.50	0.50
Hot-spot temperature rise (degrees centigrade)	8	8

Each test was made on a set of four cables impregnated at the same time, each 15 feet long and equipped with pressure-tight terminals. The initial power factors on these four supposedly identical cables were nearly equal but not identical. As, however, they tended to equalize after a few load cycles, it seemed fairer to set the limit at a certain final power factor rather than a definite rise from the start. A value of two per cent was chosen as giving results close to those of the Commonwealth Edison Company.

The first load-cycle tests were made with two sets of cables, one made up with 160 mils of normal or type *N* paper surrounded by 60 mils of high modulus or type *S* paper, a construction referred to for convenience as "Titebilt," the other

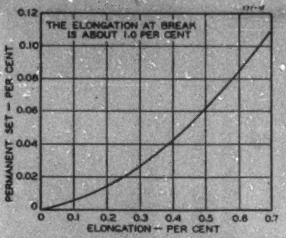

Figure 4. Permanent set of normal cable paper impregnated

made with the same total thickness of paper as ordinarily used, i.e., 50 mils of paper, selected for high density, covered with 170 mils of type *N* paper. The average number of cycles required to reach the limits set were as follows:

Set Number 1

Construction	Power Factor	Ionization	Hot Spot	Breakdown
Ordinary	25	25	22	25
Titebilt	60	82	64	82

Another series of tests was made, adding, however, a set of cables made entirely of type *S* paper. It was expected that if the elasticity theory were correct, the cables made of this kind of paper should be inferior to the Titebilt cable. This was confirmed by the following results:

Construction	Average Number of Load Cycles to Reach Power-Factor Limit Set Number 2
Ordinary construction	30
220 mils of type *S* paper	42
Titebilt	69

The progress of power-factor rise in the Titebilt and ordinary cables for set number 2 is shown in figure 6. It will be noted, that the Titebilt cable shows a smoothly rising curve which contrasts remarkably with the erratic curve of the ordinary cable. The power-factor curve of the Titebilt cable did not show any final upturn until two or three cycles before failure.

The above tests were confirmed by two more sets of tests as follows:

	Load Cycles to Reach Power-Factor Limit	
Construction	Set Number 3	Set Number 4
Ordinary	27	29
Titebilt	72	75+

Set number 4 was stopped when the Titebilt cable had shown its undoubted superiority.

After all these tests the cables were examined and a large amount of data collected. After the stability of power factor, the first feature noted was the complete absence of dendrites in the Titebilt cables. Cable wax formed in cables of both types, but less frequently and in smaller amounts in the Titebilt type. The exact data are as follows:

Construction	Cables With Dendrites (Per Cent)	Cables With Wax (Per Cent)
Ordinary	36	57
Titebilt	0	25

No gas was found in the tape-edge channels of the Titebilt cables whereas many of the other cables showed varying amounts of gas.

The average ionization of cables of the ordinary type was 0.069 per cent at the end of the life tests whereas that of Titebilt cables was 0.058 per cent in spite of the fact that the latter lasted more than twice as many cycles on life test.

Four lengths of Titebilt cable constituting set number 5 were tested on load cycle at 243 volts per mil, maximum stress which, as indicated above, is too high to yield results comparable with operating conditions. Two of these developed slight dendrites in the *outer* half of the insulation. The dendrites in ordinary cable always appeared in the inner half of the insulation. This shows that the inner part of the insulation, which carries the highest electrical stress at operating voltages, remains tighter in the Titebilt cable, affording an interesting proof of the Titebilt principle. The same peculiarity has since been noted in Titebilt cables tested to breakdown in accordance with AEIC specifications.

In addition to these five recorded sets of tests, there were numerous preliminary tests and tests to obtain specific design data, all of which substantiated the Titebilt design.

Belted Cable

So far, all the discussion has related to single-conductor cable, although obviously, the Titebilt idea is equally applicable to multiple-conductor shielded cable. The principle is not so easily applied to the belted cables, but considerable benefit has been found to result

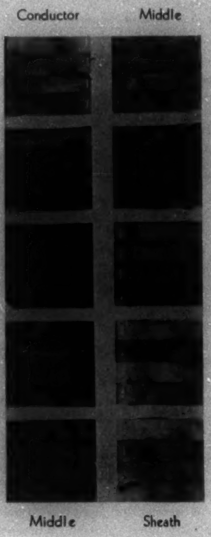

Figure 5. Arrested failure of ordinary cable

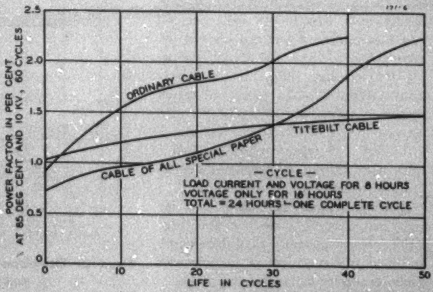

Figure 6. Life-cycle tests (compound number 7)

Conductor maximum operating temperature = 85 degrees centigrade (approximate)

Sheath maximum operating temperature = 60 degrees centigrade (approximate)

Room temperature = 25 degrees centigrade (approximate)

Figure 7. Three-phase load-cycle test equipment

from using the Titebilt composite construction for the conductor insulation.

For instance, one such test was on cable having an AEIC rating of 12,000 volts. It was tested at 24,000 volts; i.e., at twice rated voltage, but the copper temperature was brought up to 85 degrees centigrade. After 30 cycles the Titebilt cable power factor at 85 degrees centigrade rose from 1.01 to 1.06 per cent, whereas that of the old type cable rose from 1.04 to 1.22 per cent. The layout for this test is shown in figure 7.

In general, these tests showed that Titebilt belted cable was consistently superior to the old type cable but by a smaller margin than in the case of the single-conductor and shielded cables.

Compound Number 8

The tests recorded above were all made with compound number 7, a preponderantly paraffin base oil with 15 per cent wood rosin. The following tests were made with cables of the Titebilt construction, using compound number 8 consisting of the same oil with ten per cent of a synthetic derivative of rosin, namely hydrogenated wood rosin.

The type of test on single-conductor cables, described above, proved to be quite inadequate for cables made in this way. It will be remembered that while ordinary cables attained the two per cent power factor (85 degrees centigrade) limit in from 25 to 30 load cycles and the original Titebilt cables in from 60 to 75 load cycles, there was no indication that Titebilt cable with compound number 8 would attain the limit within two or three times that number. In fact, the test seemed to have no effect on the power factor of the cable.

The compound itself, tested, in a new type of bombardment cell, shown in figure 8, gave remarkable results. In preparing a sample of oil for test, the assembled discharge cell is placed on a water bath held at 100 degrees centigrade, and is then evacuated to an absolute mercury pressure of 0.1 millimeter, the residual being air. A sample of oil weighing 25 grams is poured into the funnel of the oil inlet, from which it is slowly drawn into the discharge chamber and degassed. After degassing the apparatus is allowed to stand for one hour at room temperature. At this time, a potential of 9,000 volts, 60 cycles, is applied to the test cell from a gas-sign transformer, and continued for 1,000 minutes. The relative oil stabilities are determined by change of power factor as measured in a Davidson power-factor cell. Whereas in this cell compound number 7 ended the test with a power factor of 4.0 per cent at 85 degrees centigrade, compound number 8 underwent the same ordeal and ended with a power factor of 0.4 per cent. This compound mixes with joint oils, in all proportions, without appreciable rise of power factor.

Table I. Load-Cycle Test of Titebilt Cable

Three-Conductor, 500,000-Circular-Mil, Type *H*, 122 Mils of Insulation (Rated at 7,000 Volts), 117 Mils of Lead

	Load Cycles		Hours			
Week*	Number of Cycles	Maximum Temperature (Degrees Centigrade)	Heating	Cooling	Power Factor at 60 Degrees Centigrade (Per Cent)	Ionization Per Cent at Room Temperature
Standard test						
Start					0.39	0.01
1	2	60	7	17	0.39	0.04
	3	80	7	17		
2	1	60	7	17	0.39	0.00
	4	100	7	17		
3	2	60	7	17	0.40	0.01
	3	115	7	17		
4	5	60	7	17	0.39	0.00
Supplemental test						
5	1	60	48	9		
	1	85	15	9		
	2	100	15	9		
	1	115	15	9		
	1	60	15	48	0.40	0.00

*Saturdays and Sundays, after each week of test, the cables carried 24 kv but no load current.

Ionizations which were reported as having small negative values are entered as zero.

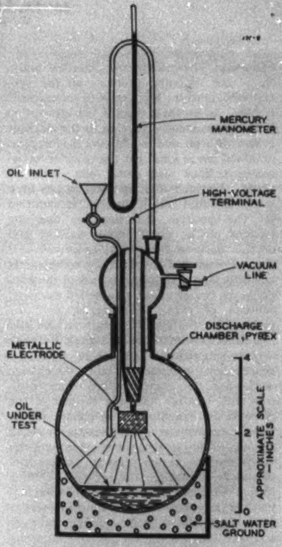

Figure 8. Merrell discharge cell for oil-stability tests

At this stage of our work Mr. Halperin called our attention to a new type of load-cycle test which he had developed that was calculated to be more destructive than the old and we decided to test a three-conductor shielded cable of the new design. This test procedure is described in his 1939 AIEE paper. Like the earlier one, it uses heating and cooling periods with voltage but instead of a single upper temperature of 85 degrees centigrade, it includes cycles at conductor temperatures of 60, 80, 100, and 115 degrees centigrade at twice operating voltage, corresponding to an average stress of 114 volts per mil and a maximum stress estimated at about 195 volts per mil, i.e., substantially the same as in the earlier tests.

The main criteria of stability are the change in the ionization factor at room temperature and in the power factor at 24 kv and 60 degrees centigrade, so that a cycle at that temperature was made after each of the groups of cycles at higher temperatures.

Positive and Negative Damping in Synchronous Machines

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Synopsis: The straight mathematical analysis of the damping problem on the synchronous machine leads to very complicated results. From these results, the conception of the phenomena occurring in the machine is very difficult and in addition the formulas obtained are so long that the calculation becomes tedious and errors can easily be made. In this paper, an attempt has been made to find a simple physical explanation for the positive and negative damping torques of the synchronous machine and on this basis to derive simpler formulas for the calculation of the damping torques. It has been found that the torque of the synchronous machine, as a double-fed machine, consists of three components, two dependent on the speed of the machine and the third dependent on the angle between the rotating field and the rotor. When the synchronous machine oscillates, the change of the first two components produces positive and negative damping torques of the synchronous machine. The change of the third component produces the synchronizing torque. The positive damping torque depends mainly on the primary voltage and secondary resistance; the negative damping torque depends mainly on the excitation and the primary resistance. This knowledge made it possible to derive a simple and very accurate formula for the positive damping torque as well as a simple approximate formula for negative damping torque. On the basis of these formulas, the influence of the two axes of the machine on the damping can be easily found. A comparison between the results obtained by using the simple formulas and the long accurate formulas as well as a comparison between calculation and test on a small machine have been carried through.

I. Simple Equations for Determining the Damping and Synchronizing Torque

FOR the purpose mentioned, a symmetrical machine will be considered first, both parts of which have polyphase windings carrying currents of different frequencies f_1 and f_2. The frequency f_1 as well as the corresponding voltage E_{L1} are assumed to be constant. All quantities and constants involved will be reduced to terms of this side of the machine. As shown in appendix the torque of such a machine is equal to (all values in per unit notation)

$$T=[e_1{}^2sr_2-e_2{}^2r_1+e_1e_2\sqrt{l_1{}^2+m_1{}^2}\times \sin(\delta_0+\alpha)]\frac{1}{l^2+m^2} \qquad (1)$$

l_1, m_1, l, and m are functions of the slip $s=f_2/f_1$.

Thus the torque consists of three components. The first two components are dependent on the speed of the machine, the third component is dependent on the speed of the machine as well as on the angle between the both line voltages e_1 and e_2. The first component is the well-known torque of the normal induction machine.

The synchronous machine is a special case of the double-fed machine, with $f_2=0$ and $e_2=I_2r_2$.

The three components of the torque T (equation 1) will change their value when the double-fed machine oscillates. To the change of the first two components will correspond a damping torque, to the change of the third component will correspond a synchronizing torque.

Thus the damping torque consists of two components, one positive and the other negative. The positive component is proportional to the square of the primary voltage and is dependent on the secondary resistance; it is equal to zero, when the secondary resistance is equal to zero. The negative component is proportional to the square of the secondary

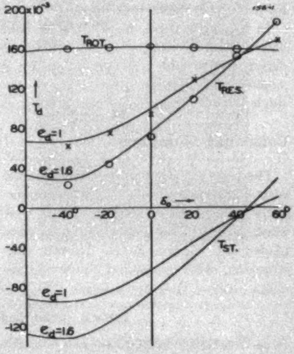

Figure 1. Both axes

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[1] For all numbered references, see list at end of paper.

A test of this kind on Titebilt cable with compound number 8 is described below.

Shielded Three-Conductor Cable

A cable of the following construction was made, having an AEIC rating of 7,000 volts and tested at 24,000 volts in accordance with procedure described above.

Three-conductor 500,000 circular mils
Insulation 122 mils $\begin{cases}\text{type } N\text{, 90 mils}\\ \text{type } S\text{, 30 mils}\end{cases}$
Impregnant, compound number 8
Shield, three-mil copper with intercalated paper
Fillers, paper
Binder, five-mil bronze
Lead, copper bearing, 117 mils
Length of test piece, 25 feet under lead
Position of ends during test, horizontal
Condition during test, pressure tight

The cycles of load are given in table I with the power factors at 60 degrees centigrade at the end of each week. It will be noted that the new Titebilt cable, showed substantially no change in either power factor or ionization. Two cables were subjected to the above test and a third one to the first four weeks only, with the same result. A 115-foot sample of the same cable was subjected, at Chicago, to the same type of test with practically the same results.

Similar tests are recorded by Halperin, AIEE TRANSACTIONS, volume 58, 1939, page 536, where it is shown that standard 12,000-volt ($^9/_{64}$ inch by $^5/_{64}$ inch) belted cables made in 1937 and 1938 show power-factor increases of 19 to 300 per cent of the original values and ionization rises from substantially zero to 0.5 per cent, absolute values.

While Titebilt cable was developed along the line of scientific utilization of mechanical properties of paper, the disposition of the papers planned on that basis may, and probably does, lead to a better arrangement from other points of view, which I hope to express on some subsequent occasion.

voltage and is dependent on the primary resistance; it is equal to zero, when the primary resistance is equal to zero or when the secondary voltage (in terms of the synchronous machine, the excitation) is equal to zero. The positive component of the damping torque will be called rotor-damping while the negative component will be called stator-damping.

It is now possible to derive a simple formula for the rotor-damping of the synchronous machine (1 and 3). It is (for the symbol s. List of Symbols)

$$T_{\text{rot.}}=\frac{e^2}{Z}\left[\frac{r_{2d}/s_p}{Z_{sd}^2}\sin^2\delta_0+\frac{r_{Dq}/s_p}{Z_{sq}^2}\cos^2\delta_0\right] \quad (2)$$

where

$$s_p=\frac{Z}{\omega}\quad \tau_{1d}=\frac{x_1}{x_{ad}}\quad \tau_{1q}=\frac{x_1}{x_{aq}} \quad (3)$$

$$x_{sd}=x_1+(1+\tau_{1d})x_{2d}\quad x_{sq}=x_1+(1+\tau_{1q})x_{Dq} \quad (4)$$

$$Z_{sd}^2=[(1+\tau_{1d})r_{2d}/s_p]^2+x_{sd}^2$$
$$Z_{sq}^2=[(1+\tau_{1q})r_{Dq}/s_p]^2+x_{sq}^2 \quad (5)$$

The rotor damping of the asynchronous machine is determined directly by the slope of the speed-torque curve which is represented by the first component of the equation 1. The rotor damping of the synchronous machine is dependent on this slope at $s=0$ and in addition is modified by the angle δ_0 which determines the power of the synchronous machine.

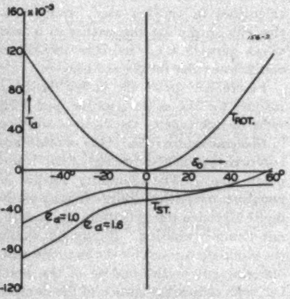

Figure 2. Direct axis

The result given by the equation 2 will be compared with that obtained by Doherty and Nickle in their "Synchronous Machines, Part III"[2] where r_1 is assumed to be equal to zero. The comparison will be made for a 48-pole machine of 2,500 kva, 6,000 volts, 50 cycles, but in order to make the negative damping more noticeable, it will be assumed that this machine has an abnormally high stator resistance, and that it operates on 1,000 volts, 8 cycles per second. Under these conditions, the constants are:

$r_1=0.20$ $\quad r_{2d}=0.022$
$x_1=0.116$ $\quad x_{2d}=0.301$
$x_{ad}=0.855$ $\quad r_{Dq}=0.078$
$x_{aq}=0.519$ $\quad x_{Dq}=0.08$

Further, it is assumed that $Z=5.03$; i.e., $s_p=0.10$.

The curve $T_{\text{rot.}}$ in the upper part of figure 1 was calculated by using the formula contained in said paper, the circles were calculated by using equation 2. The agreement is very satisfactory, as has been found on many other occasions.

Unfortunately, it is not possible to find a similarly simple and accurate formula for the stator (negative) damping. On the basis of the second term of the equation 1 and with support of calculations using the exact formulas it is possible, however, to derive an approximate formula that gives satisfactory results for the usual range of δ_0($\delta_0=+35$ degrees to -35 degrees) and for excitations corresponding to $e_d=1.0$ to 1.6.

When using the second term of the equation 1 it has to be considered that in the synchronous machine the voltage e_2 corresponds to the ohmic drop of the secondary current and that this current multiplied by the reactance of the armature reaction is equal to the electromotive force induced in the stator. Thus the main quantities of the second term of the equation 1 are involved. The influence of the angle δ_0 upon the stator damping is taken into account by two factors k_1 and k_2 based on the exact formulas. Thus, the approximate formula for the stator-damping is:

$$T_{\text{st.}}=-\frac{e_2}{Z}\frac{r_1}{\left(\frac{x_{ad}+x_{aq}}{2}\right)^2}\times$$
$$\left[\left(\frac{e_d}{e}\right)^2\frac{(r_{2d}/s_p)^2}{Z_{sd}^2}k_1+\frac{(r_{Dq}/s_p)^2}{Z_{sq}^2}k_2\right] \quad (6)$$

For a generator:

$k_1=1-\tan^2\delta_0 \qquad k_2=1-\sin 2\delta_0$

and for a motor:

$k_1=1+\tan^2\delta_0 \qquad k_2=1-\sin\dfrac{\delta_0}{2}$

The values of the resultant damping torque $T_{d\,\text{res.}}=T_{\text{rot.}}+T_{\text{st.}}$ given by the equations 2 and 6 will be compared with those obtained by Park[4] on the assumption $x_{ad}=x_{fd}$ and $x_{aDq}=x_{aq}$. As in the comparison with Doherty and Nickle, it is further assumed that there is only one winding in the direct axis.

In the figure 1 the curves $T_{\text{res.}}$ are the torques obtained by using Park, the crosses and circles are values calculated from the equations 2 and 6. The agreement is satisfactory. It should be remarked that the damping torques obtained by Nickle and Pierce[5] for the running as a *motor* at the heavier loads are about 30 per cent higher than those obtained by Park; i.e., in this range the method of Nickle and Pierce gives too small values for the negative damping.

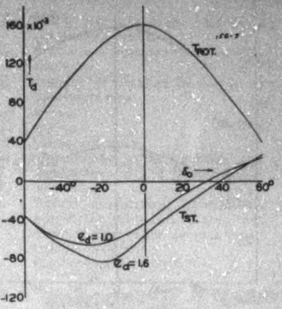

Figure 3. Quadrature axis

In the bottom part of the figure 1 is shown separately the stator (negative) damping torque. The negative damping is much larger when the machine operates as a motor than when it operates as a generator. For the same (absolute) value of the angle δ_0 the resultant damping for the motor is much lower than for the generator. The negative damping torque increases with increasing excitation. This influence of the excitation is also larger for the motor than for the generator The rotor-damping of the synchronous machine is positive for any angle δ_0, while in the normal induction machine the rotor-damping can also be negative. In the normal range of δ_0 the stator-damping of the synchronous machine is always negative, however, at large *positive* values of δ_0 it can become positive.

The figures 2 and 3 show the rotor-damping as well as the stator-damping for both axes separately. For small values of δ_0 the negative-damping in the direct axis is greater than the positive damping, while in the quadrature axis the negative damping is at any angle δ_0 smaller than the positive damping.

The curve T_d of the figure 4 gives the comparison between the results obtained by Park and those obtained by using the equations 2 and 6 for the same machine as above, but at 50 cycles, $s_p=0.027$ and a primary resistance 70 per cent higher than normal. The agreement between both methods of calculation is satisfac-

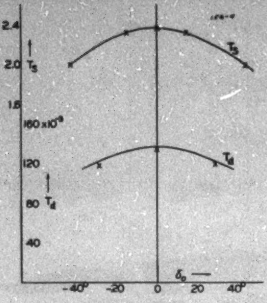

Figure 4

tory. The shape of this curve T_d is typical for the standard larger generators having a low-resistance damper winding. Here the influence of the negative damping is small, and the damping torque can be calculated by using equation 2 only.

The mathematical analysis used in deriving an expression for the damping torque also leads to the synchronizing torque. Since the equations for the synchronizing torque derived in the papers mentioned above (2, 4, and 5) are very complicated, a simple equation for the synchronizing torque was derived that corresponds to equation 2.[11] This equation is

$$T_s=\frac{ee_d}{x_d}\cos\delta_0+e^2\frac{x_d-x_q}{x_dx_q}\cos 2\delta_0+ e^2\left[\frac{x_{td}}{(1+\tau_{1d})Z_{sd}^2}\sin^2\delta_0+ \frac{x_{tq}}{(1+\tau_{1q})Z_{sq}^2}\cos^2\delta_0\right] \quad (7)$$

The results given by this equation will be compared with those obtained by Doherty and Nickle.[3] The curve T_s in figure 4 shows the comparison for $e_d=1.6$. The agreement is very good, as has been found on other occasions. The correctness of equation 7 is proved by the results of experimental determinations of the natural frequency.

II. Tests to Determine the Damping Torque

There are very few tests published for determining the damping torque of a synchronous machine while the machine is hunting. Some valuable data are given by F. A. Hamilton.[10] Systematic tests, suggested by the author of this paper, were made by R. Kuenmich at the University of Stuttgart.[9] A small 4-pole, 1.6-kva, 50-cycle synchronous machine having an external field system was used in making these tests. The primary resistance was equal to 0.072; the material of the damper winding was copper. The damping torques are given in figure 5*B* for the machine without damper winding and in figure 5*A* for the machine with damper winding. Both diagrams reveal that the damping torque increases with increasing load and that it decreases with increasing excitation. It could not be expected, however, that this influence of the excitation would be appreciable in the large machines used by Hamilton in making his tests.

It follows from figure 5*B* that for the point $P=0.2$ kw and $i_f=1.66$ amperes, the torque $T_d=-14.1\times10^{-3}$. By using Park, the calculated torque under these conditions is found to be $T_d=-12.1\times10^{-3}$; by using the equations 2 and 6, it is found to be $T_d=-12.4\times10^{-3}$. The agreement is satisfactory, if it is considered that the accuracy of the tests is from ±10 to ±20 per cent. Figure 5*A* gives for the point $P=1.6$ kw and $i_f=1.66$ amperes, the torque $T_d=9.3\times10^{-3}$. By using Park the calculated torque is found to be $T_d=23.7\times10^{-3}$; by using the equations 2 and 6, it is found to be $T_d=20.6\times10^{-3}$. The calculation does not check with the test. This is only partly a result of the limited accuracy of the tests. The main reason for the discrepancy is the difficulty of properly determining the constants of the machine, especially in the present case, where the saturation is high (the ratio of no-load AT to gap AT being equal 1.57). In addition, the frequency of the oscillations is high and therefore the assumptions for the derivation of the formulas are not fully justified. The papers of Linville[7] and Waring and Crary[8] were used in calculating the constants. Only the constants of the damper winding were calculated in another way; namely, in the way shown by the author in a previous paper[12] (i.e., by introducing an equivalent value for the *y* of Linville).

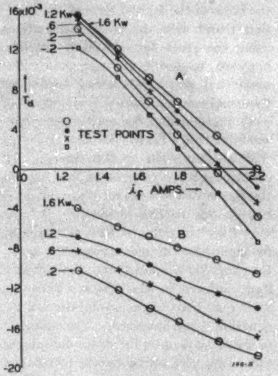

Figure 5

A—Damping torque with damper winding

B—Damping torque without damper winding

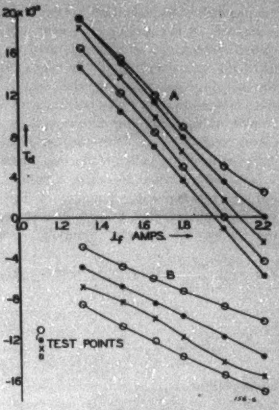

Figure 6

A—One ring open in the middle of the pole

B—One ring open in the middle of the interpolar space

Figure 6*A* shows the tested damping torques for the same machine, but with reduced influence of the damper winding in the quadrature axis. One of the rings has been cut in the middle of the interpolar space. The damping torque is negative over the entire range, as in figure 5*B* which refers to the machine without damper winding. Figure 6*B* shows the damping torques for the machine with one ring cut in the middle of the pole; i.e., with reduced influence of the damper winding in the direct axis. It is worth noticing that the torques are now larger than in figure 5*A* which refers to the machine with normal squirrel-cage damping. The reason for this can be seen from figure 2. The tested machine has a small angle δ_0 ($\delta_0=11.5°$ at 1.6 kw) and a small angle δ_0 may cause the negative damping in the direct axis to be stronger than the positive damping. In some cases (for example, when the machine is used for electric ship propulsion) it may be useful to cut the ring in the middle of the pole for this would reduce

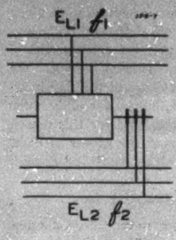

Figure 7

the short-circuit current without appreciably changing the damping torque.

Appendix

For the machine as per figure 7, it will be assumed that the stator voltage E_{L1} and the stator frequency f_1 are constant. All values will be reduced to terms of the stator. If the rotor rotates in the direction of the field produced by the stator currents:

$$n=\frac{120(f_1-f_2)}{p}=\frac{120f_1(1-s)}{p} \qquad s=\frac{f_2}{f_1} \tag{1}$$

Assuming that the iron losses are negligible and that the vectors E_{L1} and E_{L2} are displaced by an angle δ, it is found that

$$\mathbf{Z}_1=r_1+jx_1 \qquad \mathbf{Z}_2=r_2+jsx_2 \tag{2}$$

$$E_{L1}=-\mathbf{E}_1+\mathbf{I}_1\mathbf{Z}_1 \quad E_{L2}e^{-j\delta}=-s\mathbf{E}_1+\mathbf{I}_2\mathbf{Z}_2 \tag{3}$$

$$\mathbf{I}_m=\mathbf{I}_1+\mathbf{I}_2=j\mathbf{E}_1\frac{1}{x_m} \tag{4}$$

The author has shown in a previous paper[13] that it is useful to apply the "superposition principle" in treating the double-fed machine. With this principle, the stator current will consist of two parts, one produced by the stator voltage on the assumption that the rotor is short-circuited, the other produced by the rotor voltage on the assumption that the stator is short-circuited. The same is valid with reference to the rotor current. Thus

$$\mathbf{I}_1=\mathbf{I}_{11}+\mathbf{I}_{12} \qquad \mathbf{I}_2=\mathbf{I}_{21}+\mathbf{I}_{22} \tag{5}$$

From the equations 2 to 5 follow the four current components:

Stator:

$$\mathbf{I}_{11}=E_{L1}\frac{f+jk}{l+jm}$$

$$\mathbf{I}_{12}=-E_{L2}\frac{\cos\delta-j\sin\delta}{l+jm} \tag{6}$$

Rotor:

$$\mathbf{I}_{21}=-E_{L1}\frac{s}{l+jm}$$

$$\mathbf{I}_{22}=E_{L2}\frac{(\cos\delta-j\sin\delta)\left(1+\tau_1-j\dfrac{r_1}{x_m}\right)}{l+jm} \tag{7}$$

where

$$\tau_1=\frac{x_1}{x_m},\ \tau_2=\frac{x_2}{x_m},\ f=(1+\tau_2),\ \ k=-\frac{r_2}{s}\frac{1}{x_m} \tag{8}$$

$$l=\left[(1+\tau_2)r_1+(1+\tau_1)\frac{r_2}{s}\right]\times s$$

$$m=\left[x_1+(1+\tau_1)x_2-\frac{r_2}{s}\frac{r_1}{x_m}\right]\times s \tag{9}$$

The torque is given by the equation

$$T=E_{L1}\bullet\mathbf{I}_1-\mathbf{I}_1^2r_1=j\mathbf{I}_1\bullet\mathbf{I}_2x_m \tag{10}$$

The dot (•) indicates that the scalar product of the two vectors has to be taken. Thus

$$T=j\mathbf{I}_{11}\bullet\mathbf{I}_{21}x_m+j\mathbf{I}_{11}\bullet\mathbf{I}_{22}x_m+j\mathbf{I}_{12}\bullet\mathbf{I}_{21}x_m+ \\ j\mathbf{I}_{12}\bullet\mathbf{I}_{22}x_m \tag{11}$$

By evaluating these four scalar products and using the abbreviations

$$l_1=\left[(1+\tau_2)r_1-(1+\tau_1)\frac{r_2}{s}\right]\times s$$

$$m_1=x_1+(1+\tau_1)x_2+\frac{r_2}{s}\frac{r_1}{x_m} \tag{12}$$

$$\tan\alpha=\frac{l_1}{m_1} \tag{13}$$

the equation 1 will be obtained.

List of Symbols

e_1, e_2	terminal voltage
E_L	line voltage
E_I	induced electromotive force
f_1, f_2	line frequency
f_p	frequency of the oscillation
I	current
I_m	magnetizing current
n	revolutions per minute
p	number of poles
r_1, r_2	resistance
r_{2d}	resultant secondary resistance in the direct axis
r_{Dq}	resistance of the quadrature-axis damper winding
s	slip
s_p	$\dfrac{Z}{\omega}$
T	torque
T_d	damping torque per radian angular displacement per second
x_{2d}	resultant secondary leakage reactance in the direct axis
x_{Dq}	leakage reactance of the quadrature-axis damper winding
x_{ad}	reactance of armature reaction in the direct axis
x_{aq}	reactance of armature reaction in the quadrature axis
x_d	x_1+x_{ad}
x_q	x_1+x_{aq}
x_1, x_2	leakage reactance
x_m	reactance due to the main flux
Z	angular frequency of the oscillation
$\mathbf{Z}$	impedance
δ_0	angular displacement between the axis of the rotor and rotating field corresponding to the average load
τ	leakage coefficient
ω	angular frequency of the line

Bibliography

1. DAMPING TORQUE AND DESIGN OF THE DAMPER WINDING OF SYNCHRONOUS MACHINES, **M. M. Liwschitz.** *Arch. fuer Elektrotechnik*, volume 10, 1921, pages 96–132.

2. SYNCHRONOUS MACHINES, Part III, **R. E. Doherty and C. A. Nickle.** AIEE TRANSACTIONS, volume 46, 1927, pages 1–14.

3. EFFECT OF THE DAMPER WINDING ON THE NATURAL FREQUENCY OF THE SYNCHRONOUS MACHINE, **M. M. Liwschitz.** *Arch. fuer Elektrotechnik*, volume 20, 1928.

4. TWO REACTION THEORY OF SYNCHRONOUS MACHINES, **R. H. Park.** AIEE TRANSACTIONS, volume 48, 1929, pages 716–27.

5. STABILITY OF SYNCHRONOUS MACHINES, **C. A. Nickle and C. A. Pierce.** AIEE TRANSACTIONS, volume 49, 1930, pages 338–50.

6. EFFECT OF ARMATURE RESISTANCE ON THE HUNTING OF SYNCHRONOUS MACHINES, **C. F. Wagner.** AIEE TRANSACTIONS, volume 49, 1930, pages 1011–24.

7. STARTING PERFORMANCE OF SALIENT-POLE SYNCHRONOUS MOTORS, **T. M. Linville.** AIEE TRANSACTIONS, volume 49, 1930, pages 531–45.

8. TESTS TO DETERMINE THE EFFECT OF A DAMPER WINDING ON A SYNCHRONOUS MACHINE WHILE HUNTING, **R. Kuenmich.** Doctorate Dissertation, University of Stuttgart, 1930.

9. THE OPERATIONAL IMPEDANCES OF A SYNCHRONOUS MACHINE, **M. L. Waring and S. B. Crary.** *General Electric Review*, volume 35, 1932.

10. FIELD TEST TO DETERMINE THE DAMPING CHARACTERISTICS OF SYNCHRONOUS GENERATORS, **F. A. Hamilton.** AIEE TRANSACTIONS, volume 51, 1932, pages 775–9.

11. SYNCHRONIZING TORQUE OF THE SYNCHRONOUS MACHINE, **M. M. Liwschitz.** Scient. Public. of the Siemens Concern, volume 12, 1933.

12. STARTING PERFORMANCE OF SALIENT-POLE SYNCHRONOUS MACHINES, **M. M. Liwschitz.** AIEE TRANSACTIONS, volume 59, 1940, pages 913–19.

13. APPLICATION OF THE SUPERPOSITION PRINCIPLE TO THE ANALYSIS OF CONTROL PROBLEMS OF THE ASYNCHRONOUS MACHINE, **M. M. Liwschitz.** Scient. Public. of Siemens Concern, volume 8, 1930.

14. THE THEORY OF THE DOUBLE-FED INDUCTION MACHINE, **W. T. Kassianow.** *Electrotechnik und Maschinenbau*, volume 50, 1932, pages 453–7.

Control of the Switching Surge Voltages Produced by the Current-Limiting Power Fuse

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Synopsis: A new transient condition unique to fuses of the current-limiting type is described and it is recommended that these fuses be designed to inherently control switching surge voltage magnitudes to meet the following conditions:

1. The transient recovery voltage should not exceed the insulation level of rotating machines $\sqrt{2}(2E+1{,}000)$ where E is the rated terminal-to-terminal rms voltage of the lowest rated machine involved.

2. The transient recovery voltage should preferably not exceed the 60-cycle crest spark-over voltage of associated lightning arresters.

3. If the transient recovery voltage does exceed the 60-cycle crest spark-over voltage of associated arresters, the interrupting rating of the fuse should be determined and assigned to limit the arrester current to a safe magnitude.

These conditions may be met by current-limiting fuses employing conducting elements having two or more sections of different diameters. Such fuses have been designed and are now commercially available. Typical oscillograms illustrating the operation of these fuses are included.

Introduction

It is generally recognized that most circuit-interrupting devices may be expected to produce a transient switching surge voltage having a magnitude appreciably in excess of the normal crest voltage. The magnitude of this transient voltage is a function of the characteristics of both the interrupting device and the circuit interrupted. To this background of existing and recognized transient voltage conditions may be added a relatively new phenomenon; the switching surge voltage (transient recovery voltage) of the current-limiting fuse.[1,2] This type of fuse tends to interrupt short-circuit currents so quickly that unless the interruption is properly controlled, the transient recovery voltage may reach a very high magnitude. With proper control, the transient voltage can be held within allowable safe limits.

The switching surge voltage of a current-limiting fuse differs from that of other interrupting devices in that the energy associated with the surge may be considerably greater. This difference results from the fact that the fuse limits the current by rapidly inserting arc resistance into the circuit at the instant the current-responsive element volatilizes. Since the high-resistance steady-state arc voltage of the fuse is practically equal to open-circuit voltage, the "recovery" occurs at the inception of the arc and *before* the interruption of the short-circuit current. With other interrupting devices, the voltage recovers at the end of the arcing period and *after* the interruption of the short-circuit current. Thus, the switching surge voltage of the fuse is associated with a definite value of dynamic short-circuit current and for this reason a new conception of the effects of transient recovery voltage must be considered.

The purpose of this paper is:

1. To call attention to the nature of a new switching transient phenomenon.
2. To point out the desirability of controlling the new switching transient to prevent damage to associated equipment.
3. To describe a method of designing current-limiting fuses with effective means of controlling switching surge voltages.

Permissible Limits for Switching Surge Voltages

Permissible limits for switching surge voltages should be based on several factors, one of which is the insulation level of rotating machines. Transformers and similar apparatus are considered to have an impulse ratio in the order of two or more, whereas rotating machines should be considered to have an impulse ratio only slightly greater than unity.[3] For

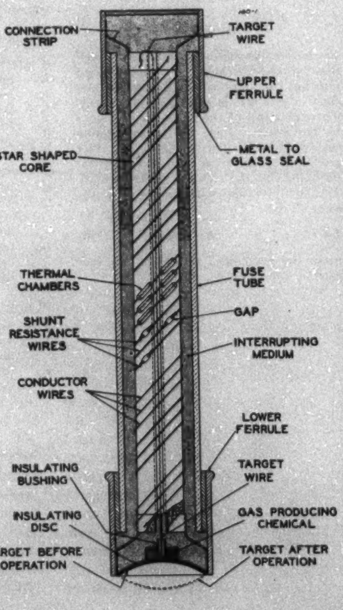

Figure 1. Current-limiting fuse with shunt-resistance wires

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The authors wish to acknowledge the parallel work of several European companies along the lines described in this paper. The contribution of K. A. Lohausen, Allgemeine Elektricitäts-Gesellschaft, is particularly noteworthy.

[1] For all numbered references, see list at end of paper.

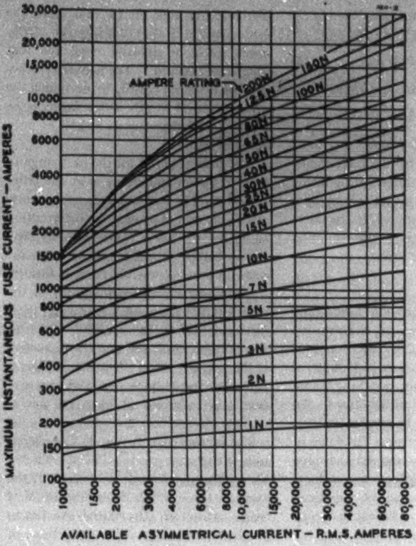

Figure 2. Maximum instantaneous current permitted by a line of current-limiting type fuses

the purpose of selecting permissible limits of switching surge voltage, safe practice will be adhered to by considering the crest value of the machine's one-minute high-potential test voltage as the maximum value of allowable transient voltage. The first logical limitation is then $\sqrt{2}(2E+1{,}000)$ where E is the rated terminal-to-terminal rms voltage of the lowest rated machine connected to the circuit in which the fuse is to be applied. For example, the limit for a 5,000-volt fuse is based on $E=4{,}000$ since this is the lowest rated machine in the 5,000-volt class. Since all other apparatus appears to have a higher insulation level than rotating machines, this limit should be entirely safe from a consideration of insulation alone.

A second and less obvious limit is imposed by the possibility of the operation of lightning arresters during voltage surges which would not damage insulation. The minimum 60-cycle spark-over voltage of an arrester is 1.5 times its maximum permissible line-to-ground voltage (proposed revision AIEE Standard 28) and its average spark-over voltage is somewhat higher. It is obviously desirable to minimize the number of arrester operations in all cases and to eliminate unnecessary operations wherever possible. Therefore, it becomes apparent that a desirable limit for switching surge crest voltages is 1.5 times the maximum crest rating of arresters used on the circuits for which the fuses are designed.

It has been found practical to design current-limiting fuses, as described later, to inherently limit switching surge voltages to magnitudes below the crest of the 60-cycle spark-over voltage of arresters which are rated above the voltage rating of the fuse. However, good engineering practice dictates the application of current-limiting fuses capable of interrupting line-to-line voltage whereas operating experience justifies the use of lower rated arresters on solidly grounded neutral systems. Therefore, since a line-to-line rated fuse may be expected to produce a switching surge voltage somewhat higher than the crest of normal line-to-line voltage, the associated line-to-neutral arrester is subject to spark over during every fuse operation. This condition does not exist when the fuses and arresters are *both* applied on the basis of line-to-line voltage.

When an arrester sparks over while a current-limiting fuse is interrupting a circuit, the two devices divide the dynamic short-circuit current in inverse proportion to their respective resistances. As previously pointed out, this current may be considerably higher than that ordinarily associated with switching surge voltages and its duration may be in the order of 1,000 to 1,600 microseconds. The ability of an arrester to pass current for relatively long periods of time is limited by economic considerations and this limit is recognized in AIEE Standard No. 28. It therefore becomes advisable to determine and assign interrupting ratings to fuses in a way that the short-circuit currents permitted by the fuses, when arresters are subject to spark over, are limited to values such that the arresters may safely pass their share of current without damage to the arresters.

From the considerations discussed above, it may be concluded that fuses of the current-limiting type should be so designed that switching surge voltages are inherently controlled to meet the following conditions:

1. The transient recovery voltage should not exceed the insulation level of rotating machines $\sqrt{2}(2E+1{,}000)$ where E is the rated terminal-to-terminal rms voltage of the lowest rated machine involved.
2. The transient recovery voltage should preferably not exceed the 60-cycle crest spark-over voltage of associated lightning arresters.
3. If the transient recovery voltage does exceed the 60-cycle crest spark-over voltage of associated arresters, the interrupting rating of the fuse should be determined and assigned to limit the arrester current to a safe magnitude.

Designing Current-Limiting Fuses to Control Switching Surge Voltages

One method of controlling transient recovery voltages to meet specified limits has been used for two years in commercially available fuses. It consists essentially, figure 1, of resistance wires, in parallel with the conducting wires, through which the energy of the surge is shunted and absorbed. The currents which the shunt-resistance wires must handle are shown as the ordinates of the curves of figure 2. The practical application of this principle appears to be limited to fuses rated approximately ten amperes and below as the required ability of the shunt-resistance wires to absorb surge energy increases rapidly as the continuous-current ratings of the fuses increase. Since the shunt-resistance wires occupy space and serve no useful purpose during normal current conditions, it is obvious that the fuses may be made smaller and less costly by combining the voltage limiting and current-carrying functions. This desirable result may be obtained by controlling the growth of arc resistance during the interrupting process. The conducting elements, schematically shown in figure 3, form a basis for the reliable control of arc resistance.

The resistance of an arc is a function of its diameter, length, and the magnitude of current in the arc. The voltage which appears across the fuse at any instant during current interruption is the product of the arc resistance and arc current at that instant. Neglecting other circuit resistance which may be present, this condition may be expected by the fundamental equation:

$$e = ri + L\frac{di}{dt} \qquad (1)$$

where

e = generated voltage
r = arc resistance
i = arc current
L = circuit inductance
t = time

Therefore, it is apparent that the rate and amount of arc resistance inserted in the circuit determines the rate of rise and magnitude of the transient recovery voltage.

When the available short-circuit current is of such magnitude that the current is appreciably limited by the fuse, it has been found empirically that the arc resistance of the fuse at the instant of surge voltage crest, is directly proportional to the arc length. The arc length at any instant is in turn equal to the length of the conducting element which has been volatilized. The number and diameter of the conducting elements volatilized also influence the arc resistance, at the instant of surge voltage crest, as indicated by the following empirical equation:

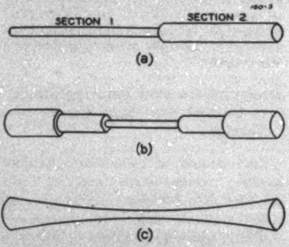

Figure 3. Conducting elements (schematic) showing use of progressive steps in cross section

$$r = [K_1 - (K_2 + K_3d^2)\sqrt{n}]l \qquad (2)$$

where

r = arc resistance at instant of surge voltage crest
d = diameter of conducting elements
n = number of conducting elements
l = arc length
K_1 K_2 K_3 = constants

The surge voltage crest may then be expressed as:

$$V = ir = i[K_1 - (K_2 + K_3d^2)\sqrt{n}]l \qquad (3)$$

Where i is the instantaneous current at surge voltage crest as given by the curves of figure 2.

Since the values of d and n are fixed within narrow limits by the continuous-current rating and the interrupting rating of any given fuse[4] and since the maximum value of i is also determined by the interrupting rating of the given fuse, equation 3 reduces to the simple form:

$$V = Kl \qquad (4)$$

The accuracy of equation 4 has been established by hundreds of comparative interrupting tests. This equation provides the basis for a new and effective method of controlling switching surge voltages of current-limiting fuses. To hold the crest voltage to a value V_1, it is only necessary to determine the value of K for the fuse rating concerned and solve equation 4 for the length of the conducting element, l_1, which may be vaporized at the time V_1 occurs. The length of conducting element so determined may be, and in most cases is, less than the minimum length required to safely interrupt the circuit. Therefore, it is necessary to provide additional conducting element length which, when vaporized, will produce sufficient additional arc resistance to effect interruption.

To prevent the crest voltage produced by the added length of element from superimposing itself on the first crest, the vaporization of the added length of element must be delayed until the first surge has declined. The necessary delay may be obtained readily by designing the added element length with a larger cross section than the original length as indicated in figure 3*a*.

The time required for the first surge to reach its crest and decay is in the order of 100 to 150 microseconds with short-circuit current at or near the interrupting rating of the fuse. This fact has been established by hundreds of short-circuit tests on fuses of various current ratings. Typical cathode-ray oscillograms of the transient voltage conditions during these tests are shown in figure 4. It may therefore be concluded that the diameters of the two sections of conducting element, figure 3*a*, should be so designed that section 2 is vaporized at least 150 microseconds after section 1.

The factors contributing to the magnitude of the surge produced by section 2 of the conducting element are essentially the same as those which determine the first surge. However, the initial conditions for the second surge differ from those of the first surge in that the short-circuit current has declined to a lower value and the initial voltage is at least equal to the normal frequency voltage at that instant. With these exceptions, the maximum length of section 2 may be determined from equation 4 with the proper evaluation of constants. The optimum element design is that which gives the minimum

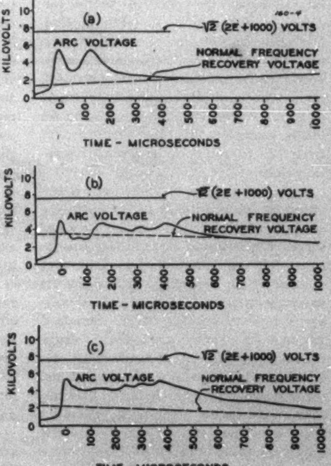

Figure 4. Typical oscillograms of controlled transient recovery voltage produced by current-limiting fuses

(*a*)—Fuse rating—2,500 volts, 10 amperes
Test voltage—2,500 volts
Available rms current—21,000 amperes
Phase angle of generator voltage at time of first surge crest—24 degrees

(*b*)—Fuse rating—2,500 volts, 40 amperes
Test voltage—2,500 volts
Available rms current—23,000 amperes
Phase angle of generator voltage at time of first surge crest—90 degrees

(*c*)—Fuse rating—2,500 volts, 100 amperes
Test voltage—2,500 volts
Available rms current—27,000 amperes
Phase angle of generator voltage at time of first surge crest—145 degrees

A Vertical-Flow Outdoor Compressed-Air Breaker

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1. Introduction

THE elimination of oil in indoor circuit breakers, particularly for powerhouse use, has been insistently demanded to reduce fire hazard. In an effort to meet these requirements, various types of breakers not containing oil have been built and studied. Self-contained breakers of the "De-ion" type[1] have been available for several years, and the potentialities of water breakers[2] have been carefully investigated. Compressed-air breakers[3] for this service have been successfully developed, particularly to meet high kilovolt-ampere interrupting requirements, and the circuit-rupturing ability of these devices has been so amazingly efficient that experimental breakers using this principle have now been built and tested for high-voltage outdoor application.

2. Interruption With Compressed Air

Compressed-air breakers for interrupting medium powers have been in use in Europe for ten years, but these devices have not been available in America until recently. The longitudinal flow-type interrupter, generally used in Europe, was investigated and found to be very satisfactory for moderate values of current, but its inherent limitations made it impractical for the medium and high kilovolt-ampere requirements at powerhouse voltages. On the contrary, an interrupter using a blast of compressed air and insulating splitters disposed perpendicular to the arc, was found to be very effective and not subject to the same arc-current limitations. Structurally, it was also found well adapted for use with high-current carrying contacts.

Just one year ago, the $1^{1}/_{2}$-million kva, 15-kv compressed air circuit breaker[3] of the transverse-flow type was reported, and oscillograms of single-phase operations at 13.2 kv up to 65,300 amperes rms were shown. Since then, the same $1^{1}/_{2}$-million kva breaker has been successfully demonstrated at 13.2 kv, 3 phase up to 86,500 amperes, and this type of interrupter is now being built into a $2^{1}/_{2}$-million kva, 15-kv breaker for 4,000 amperes continuous current-carrying capacity. Currents in excess of 100,000 amperes can be interrupted with this breaker using tank pressures not greater than 150 pounds per square inch.

The theory of this interrupting process in breakers of this type has been studied. The effectiveness of the cross-blast, splitter-type interrupter appears to depend on, first, the fact that a number of interrupting elements or splitters can easily be used in series, and second, the use of a gas-forming material such as fiber for the splitter. The function of the gas-forming splitter elements is largely to introduce turbulence into the arc stream to enhance the deionization by diffusion, following a current zero. Slepian[4] has described this theory in detail.

As breaker voltages are increased from 15 kv or 34 kv, the problem of insulation becomes of increasing significance and it becomes more difficult to design a satisfactory structure which operates on the cross-blast principle. Fortunately, the currents to be interrupted decrease as the voltage increases but not to a sufficient extent so that plain metallic nozzle interrupters can be used with moderate air pressure. Consequently, for the higher voltages it has been necessary to develop a new type of interrupting element which lends itself to vertical porcelain-clad breaker construction and provides high-interrupting ability.

3. Vertical-Flow Compressed-Air Interrupter

The reasons cited for the success of the cross-blast interrupter suggest that a satisfactory form of device can be built for higher voltages by using fiber disks horizontally disposed. These disks have central orifices through which the arc is drawn and through which a vertical flow of air passes to effect the interruption. One or more of such elements can be placed in series, and a dual interrupter of this type is shown in figure 1. It consists of two fiber disks with a metal insert in the center. These inserts have centrally located holes which form the orifices. Between these orifice disks another

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[1] For all numbered references, see list at end of paper.

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voltage at any instant during interruption. This condition exists, other factors being equal, when both surges reach the same crest magnitude.

The desired limit of switching surge voltage cannot always be met, and successful interruption obtained, by the simple two-section conducting element. In such cases, it is necessary to extend the same design principles to include multisection conducting elements similar to that shown in figure 3*b*. Carried to its logical extreme, this procedure would result in conducting elements characterized by a smooth taper as indicated in figure 3*c*. However, from practical economic and manufacturing considerations, these tapered elements cannot yet be justified.

Conclusion

The control of fuse switching surge voltages by the use of conducting elements with uniform sections of different diameters has proved both practical and flexible. Current-limiting fuses employing this principle have been designed and are now commercially available. Comprehensive tests have shown that these fuses consistently limit the switching surge voltages to magnitudes within the acceptable levels given in the first part of this paper.

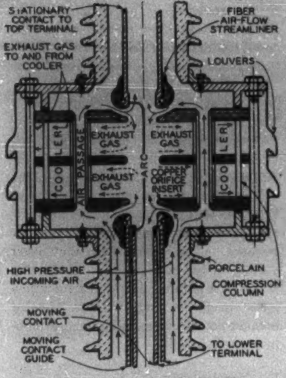
References

1. THE CURRENT LIMITING POWER FUSE, D. C. Prince and E. A. Williams, Jr. AIEE TRANSACTIONS, volume 58, 1939 (January section), page 11.

2. A NEW CURRENT LIMITING FUSE, H. L. Rawlins, A. P. Strom, and H. W. Graybill. AIEE TRANSACTIONS, volume 60, 1941 (February section), pages 77-80.

3. PROTECTION OF ROTATING A-C MACHINES AGAINST TRAVELING WAVE VOLTAGES DUE TO LIGHTNING, W. J. Rudge, Jr., R. W. Wieseman, and W. W. Lewis. AIEE TRANSACTIONS, volume 52, June 1933, page 434.

4. Discussion of the paper, THE CURRENT LIMITING POWER FUSE, D. C. Prince and E. A. Williams, Jr. AIEE TRANSACTIONS, volume 58, 1939 (January section), page 16.


184-1a
STATIONARY CONTACT TO TOP TERMINAL
EXHAUST GAS TO AND FROM COOLER
FIBER AIR-FLOW STREAMLINER
LOUVERS
COOLER
AIR PASSAGE
EXHAUST GAS
EXHAUST GAS
ARC
EXHAUST GAS
COPPER ORIFICE INSERT
COOLER
HIGH PRESSURE INCOMING AIR
PORCELAIN
COMPRESSION COLUMN
MOVING CONTACT
MOVING CONTACT GUIDE
TO LOWER TERMINAL

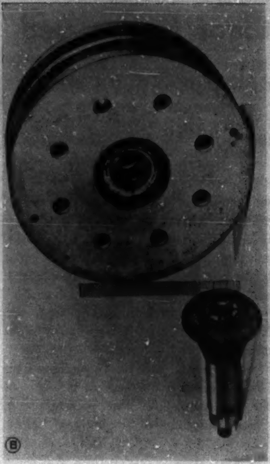


Figure 1. The interrupting element of a 138-kv vertical-flow outdoor compressed-air circuit breaker

(Above) Schematic diagram showing air flow

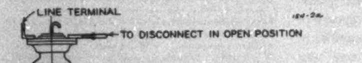
(Right) Photograph of interrupter and contact after heavy duty tests shown in tables I and II

insulating plate is introduced which restrains the random motion of the arc in the exhaust gas. The interrupter is symmetrical about this insulating plate so that the stationary contact and the upper orifice form one working unit. Likewise, the moving contact and the lower orifice plate form another. The contact members are hollow to permit a portion of the air blast to pass through them. The metal inserts in the fiber disks are used to lessen erosion and enlargement of the hole in the disks.

The air blast passes through the porcelain below the interrupter where part of it escapes through the lower working unit. The remainder passes through the tubes in the interrupter to the upper porcelain from which it escapes through the upper working unit. The air, as shown in figure 1, approaches the arc in a radial direction from between the orifice plate and the streamliner of the contact. Upon reaching the arcing space, it turns and flows vertically along the arc stream, some of it escaping through the contact, while the greater portion escapes through the orifice plate.

It has been found experimentally that this combination is better than a solid conductor and an orifice, or two orifices with equal discharge area. The unsymmetrical flow of air serves to eliminate a "dead spot" in the center which is not readily deionized, and likewise the arc terminal is carried into the hollow of the contact, so that the flow of air carries the metallic vapor from the arc terminal out into the vent instead of into the restricted portion of the arc path where the growth of dielectric is the greatest. The smaller quantity of gas which is discharged through the contacts escapes out the top of the breaker and into the mechanism box. The greater amount which passes through the orifices escapes radially between the orifice plates and is cooled by the screening placed in the annular space formed by the two orifice plates, the air tubes, and the pin supports.

This very simple arrangement gives in reality two interrupting or working combinations in series. The first, consisting of the stationary contact and the upper orifice, which becomes effective with the first third of the contact motion.


184-2a
LINE TERMINAL
TO DISCONNECT IN OPEN POSITION
INTERRUPTER
LINE TERMINAL
DISCONNECT
POLE UNIT OPERATING MECHANISM
MECHANICAL TIE BETWEEN MECHANISM AND AUXILIARY SWITCH
CURRENT TRANSFORMER
AIR BLAST COLUMN
INSULATION TO GROUND
OPERATING AIR COLUMNS
VALVES
TANK
RELAYS AND CONTROL PANEL

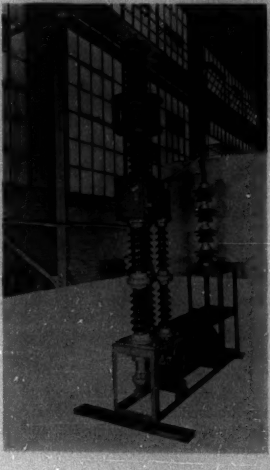


Figure 2. Single-pole 138-kv vertical-flow outdoor compressed-air circuit breaker

(Left) Schematic diagram

(Right) Photograph of breaker mounted in the high power laboratory

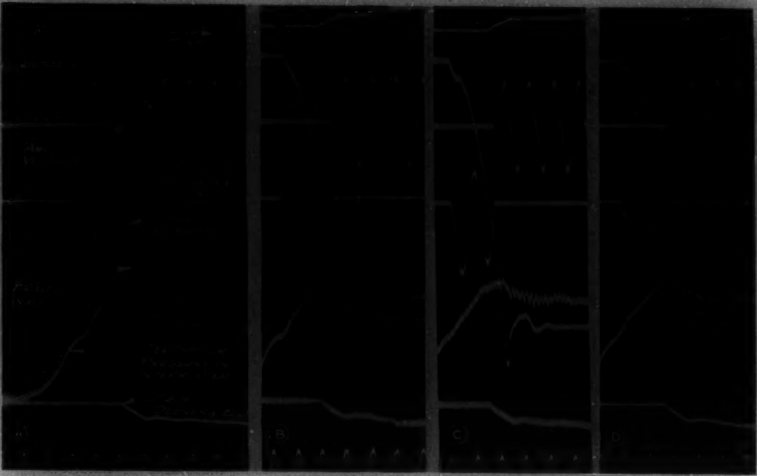


Figure 3. Magnetic oscillograms from table I. Current and voltage values noted are rms values per AIEE Standards

A—Fault current, 5,840 amperes recovery voltage, 123 kv

B—2,520 amperes, 123 kv

C—7,150 amperes, 123 kv

D—8,900 amperes, 123 kv

The second, consisting of the moving contact and the lower orifice, which becomes effective during the last third of the contact motion. In practically every interruption, with currents, voltages, and voltage-recovery rate typical of commercial applications arc extinction is obtained in the first working unit.

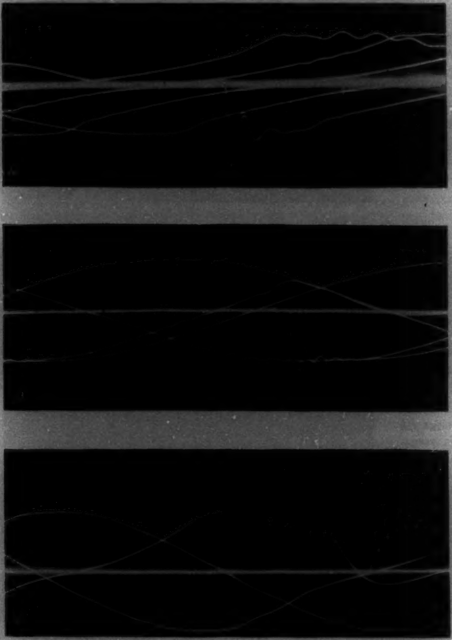


Figure 4. Cathode-ray oscillograms corresponding to those in figure 3

A—Fault current 2,520 amperes rms. Restored voltage 123 kv. Oscillatory frequency 2,600 cycles per second. Recovery rate 1,750 volts per microsecond. Maximum instantaneous voltage 296 kv

B—Fault current 5,840 amperes rms. Restored voltage 122 kv. Oscillatory frequency 3,660 cycles per second. Recovery rate 2,310 volts per microsecond. Maximum instantaneous voltage 264 kv

C—Fault current 8,900 amperes rms. Restored voltage 123 kv. Oscillatory frequency 4,310 cycles per second. Recovery rate 2,970 volts per microsecond. Maximum instantaneous voltage 295 kv

4. 138-Kv Compressed-Air Breaker

Figure 2 shows both a photograph and a schematic diagram of a single-pole unit 138-kv experimental breaker. The usual air storage tank is mounted in a simple frame which also houses the electrical relaying equipment, blast valve, and valves to supply air for opening and closing the breaker. The air-blast valve is mechanically interlocked with the opening valves so that the contacts cannot part without sufficient air for arc extinction. The operating valves are also interlocked pneumatically so that the opening operation always takes preference over the closing operation. The blast valve located adjacent to the tank discharges into a hollow porcelain insulator which carries the air direct to the interrupter chamber with a minimum of length and turns. The operating mechanism is at line potential when the isolating disconnect switch is closed and engaging the current transformer or the top of a post insulator which forms the one line terminal. The other line terminal is at the top of the breaker. The interrupting element is halfway both electrically and mechanically between the top terminal and the operating mechanism. When the breaker is in the open position, the disconnect removes potential from the interrupter and short-circuits it by engaging suitable clips at the

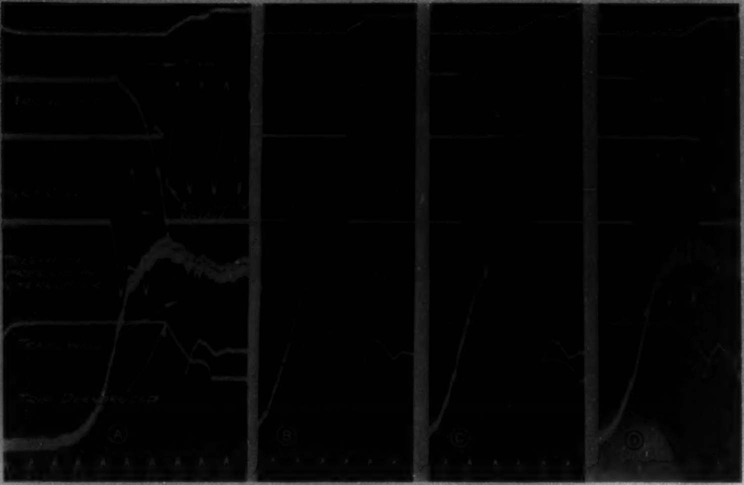


Figure 5. Magnetic oscillograms from table II. Current and voltage values noted are rms values per AIEE Standards

A—Fault current, 2,800 amperes. Recovery voltage, 65 kv

B—8,100 amperes, 64 kv

C—11,200 amperes, 63 kv

D—13,900 amperes, 62 kv

top. The disconnect is mechanically interconnected to the interrupting contact and both are operated by an air piston moving in a cylinder located in the pole-unit mechanism box. Air for operating this piston is transmitted from the tank through valves at ground potential, and thence through hollow porcelain insulators, one for closing and one for opening. These two porcelains with the third and larger one which carries the air blast from the tank to the interrupter, form a supporting tripod and the insulation to ground. The mechanical linkage between the operating cylinder, interrupting contacts, and disconnecting member are such that in normal service the disconnect opens last on an opening operation and closes first on a closing operation.

The interrupting contact consists of a simple rod which slides within two tulip-type contacts, one supported from the mechanism box and the other from the top of the breaker, both extending toward the interrupting element which is centrally located. The arc is drawn from the upper stationary contact, downwardly through the interrupter, where it is subjected to the air blast.

The breaker is ideal for high-speed reclosing duty because of efficient disposal of ionized gases from the interrupter and the high mechanical speed of the air-operating mechanism. In case of reclosure, the disconnecting switch is not opened; the contacts are simply parted in the air blast, the arc is interrupted and the contacts are reclosed before the air blast is turned off.

This type of construction provides porcelain insulation to ground of the current-carrying parts and air isolation between terminals when the breaker and disconnect are in the open position. There are no heavy operating rods with potential across them, since the entire mechanical effort is transmitted through the medium of air flow in the porcelain columns. Contacts, interrupter, and mechanisms are very accessible and easily maintained. Provision for a porcelain-

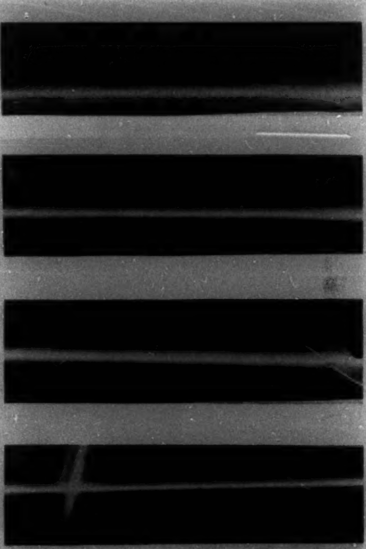


Figure 6. Cathode-ray oscillograms corresponding to those of figure 5

A—Fault current 2,800 amperes rms; restored voltage 65 kv. Oscillatory frequency 3,270 cycles per second; recovery rate 1,225 volts per microsecond. Maximum instantaneous voltage 157 kv

B—Fault current 8,100 amperes rms; restored voltage 64 kv. Oscillatory frequency 4,900 cycles per second; recovery rate 1,830 volts per microsecond. Maximum instantaneous voltage 124 kv

C—Fault current 11,200 amperes rms; restored voltage 63 kv. Oscillatory frequency 6,600 cycles per second; recovery rate 2,440 volts per microsecond. Maximum instantaneous voltage 135 kv

D—Fault current 13,900 amperes rms; restored voltage 62 kv. Oscillatory frequency 9,780 cycles per second; recovery rate 3,570 volts per microsecond. Maximum instantaneous voltage 135 kv

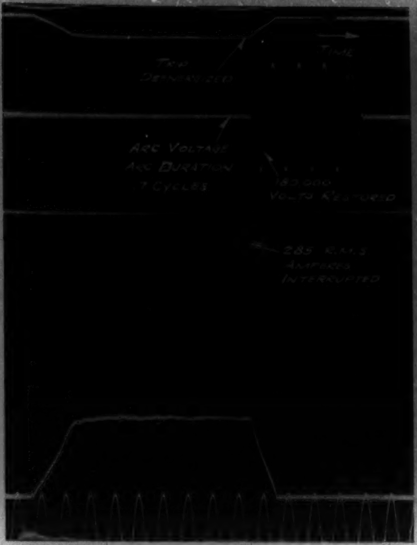


Figure 7. Magnetic oscillogram of 138-kv single-pole unit interrupting low current at 180 kv

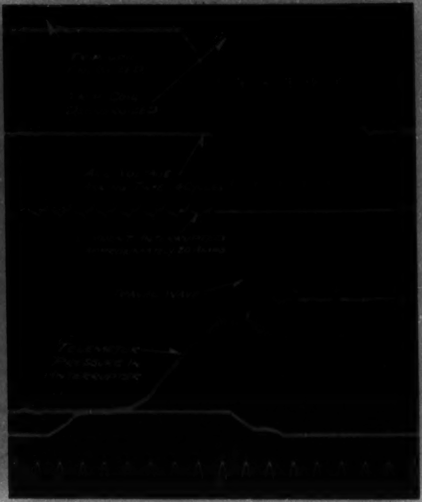


Figure 8. Typical interruption of transformer magnetizing current

clad current transformer is made on the disconnect end of the breaker. The pole units of a three-phase breaker are separate and distinct, and may be arranged for individual phase operation if desired. On the other hand, co-ordination of the three poles is possible by interlocking the air valves in obvious ways.

5. Experimental Test Results

Tables I and II show the results of two typical series of single-phase tests on the single-pole unit, 138-kv compressed air circuit breaker shown in figures 1 and 2. The highest of these tests represents power equivalent to slightly in excess of 2,000,000 kva on the basis of an ungrounded three-phase 138-kv system. The current on this film (figure 3) was unsymmetrical and thus contained considerable d-c component. On the basis that 127 kv is the line-to-neutral voltage of a grounded neutral system, the three-phase power equivalent is over 3,250,000 kva.[5,6]

Table I shows the results of a series of 18 tests at 132-kv applied voltage on the single-pole unit, 138-kv breaker at current values between 270 and 8,900 amperes. The restored voltage varied from 132 kv to 121 kv. The air pressure in the storage tank of the breaker varied from 210 to 230 pounds per square inch. The arcing time varied from 0.6 to 2.1 cycles. This variation indicates that in many of the operations even at 132 kv, the interruption was completed in the upper working unit without waiting for the arc to be drawn into the lower one. Figure 3 shows three typical oscillograms; one at 2,520, 5,840, and 8,900 amperes respectively. From the travel wave it is apparent that the 8,900 amperes was interrupted in the upper working unit which consists of one orifice and one contact, spaced less than one inch apart. Figure 4 shows the three cathode-ray oscillograms corresponding to the magnetic oscillograms in figure 3. The highest oscillatory frequency shown is 4,310 cycles per second, which occurred on the 8,900-ampere test. This corresponds to a recovery rate of 2,970 volts per microsecond. However, in table I frequencies as high as 4,620 cycles per second, or 3,180 volts per microsecond are recorded.

Table II shows the results of a series of tests at 66 kv and current values varying from 2,800 amperes to 13,900 amperes. Again all tests were single phase on the single-pole 138-kv unit shown in figure 2. The air pressure in the breaker storage tank varied from 210 to 230 pounds per square inch. The arcing time varied from 0.7 to 1.2 cycles. In all cases the circuit was interrupted without assistance from the lower working unit. Figure 5 shows the magnetic oscillograms for tests 1, 3, 5, and 7 of table II. Figure 6 shows the cathode-ray oscillograms corresponding to the magnetic oscillograms in figure 5. The oscillatory frequency of the 66-kv circuit was considerably higher than that of the 132-kv circuit. As recorded, these values varied from 3,270 to 9,780 cycles per second. This frequency resulted in a recovery rate which varied from 1,225 to 3,570 volts per microsecond, which is slightly in excess of that obtained on the 132-kv circuit when adjusted for minimum impedance.

Figure 7 shows the magnetic oscillogram of the upper working unit of the 138-kv breaker interrupting low current at 180 kv. Also, successful interruption of low values of current on a highly inductive circuit has been obtained up to 264 kv, but not until the arc was drawn into the second or lower working unit.

Figure 8 shows the magnetic oscillogram of the 138-kv pole unit interrupting the magnetizing current of two single-phase power transformers in parallel, each connected for 13.2 to 132 kv, 3.3 per cent impedance and 33,333 kva, giving a total current for both transformers of approximately 20 amperes. The circuit consisted of a 13.2-kv genera-

Table I. Opening Tests on an Experimental Compressed-Air Circuit Breaker

132-Kv Initial Voltage Applied to a Single-Pole Unit of a 138-Kv Circuit Breaker, 60 Cycles

Test Number	Recovery Voltage (Kv)	Amperes Interrupted	Tank Pressure	Arcing Time (Cycles)	Oscillatory Frequency (Cycles Per Sec.)	Test Transient Recovery Rate (Volts Per Microsec.)	Circuit Transient Recovery Rate (Volts Per Microsec.)
1	132	270	210	0.9			
2	130	910	210	0.8			
3	128	1,140	230	1.0			
4	124	1,340	230	1.0			
5	127	2,160	210	0.6			
6	126	2,290	230	0.6	2,430	1,700	1,750
7	123	2,520	230	1.2	2,600	1,750	1,880
8	123	3,200	230	2.1	2,170	1,865	2,000
9	123	3,300	230	1.4			
10	122	4,200	230	1.8	3,550	1,960	2,120
11	123	4,230	210	1.2			
12	122	4,570	230	0.8	3,280	2,010	2,170
13	123	5,600	220	1.2			
14	122	5,840	230	1.2	3,660	2,310	2,500
15	123	5,900	220	1.9	4,060	2,490	2,670
16	123	7,150	230	0.8	4,620	3,180	3,420
17	121	8,250	230	2.0	4,380	2,620	2,860
18	123	8,900	230	0.8	4,310	2,970	3,190

Table II. Opening Tests on an Experimental Compressed-Air Circuit Breaker

66-Kv Initial Voltage Applied to a Single-Pole Unit of 138-Kv Circuit Breakers, 60 Cycles

Test Number	RMS Amperes Interrupted	Cycles of Arcing	Restored Voltage	Air Pressure in Tank (Pounds Per Square Foot)	Circuit Oscillatory Frequency (Cycles Per Sec.)	Test Transient Recovery Rate (Volts Per Microsec.)	Circuit Transient Recovery Rate (Volts Per Microsec.)
1	2,800	0.7	65	210	3,270	1,225	1,243
2	5,500	0.8	65	210			
3	8,100	0.7	64	210	4,900	1,830	1,890
4	10,000	1.2	64	210			
5	11,200	1.0	63	210	6,600	2,440	2,550
6	12,200	0.8	64	210			
7	13,900	1.2	62	230	9,780	3,570	3,800

tor feeding a 13.2 to 132-kv transformer. In series with this transformer was the test breaker and the 132-kv windings of the unloaded transformers just described. Tests were made both with one and with two transformers for the load. In all cases the current was interrupted promptly in about one-half cycle, as shown in figure 8.

6. Conclusions

The general acceptance in this country of compressed air as an interrupting medium for low-voltage heavy-duty powerhouse breakers has stimulated interest in its application to outdoor high-voltage breakers.

Experimental evidence has shown that while the continuous current-carrying requirements and heavy interrupting duty of powerhouse breakers dictate a design using splitters and transverse flow, the vertical-flow type of interrupter may be employed to advantage for porcelain-clad outdoor high-voltage breakers.

Tests on a new type of vertical-flow interrupter indicate that it can be developed to handle the maximum kilovolt-ampere requirements at 69 kv and above. A comparison of table I and table II suggests that if approximately 14,000 amperes at 68 kv and 5,370 volts per microsecond can be successfully handled on one-half of the interrupter, currents considerably in excess of 8,900 amperes at 132 kv may be expected to carry this type of interrupter far into the future kilovolt-ampere requirements.

Bibliography

1. Structural Development of the De-ion Circuit Breaker up to 15,000 Volts, R. C. Dickinson and B. P. Baker. AIEE Transactions, volume 48, 1929, page 528.

2. A High Power Oilless Circuit Interrupter Using Water, W. M. Leeds. AIEE Transactions, volume 60, 1941 (February section), pages 85–8.

3. New 15-Kv Pneumatic Circuit Interrupter, L. R. Ludwig, H. L. Rawlins, B. P. Baker. AIEE Transactions, volume 59, 1940 (September section), page 528.

4. Displacement and Diffusion in Fluid Flow Arc Extinction, J. Slepian. AIEE Transactions, volume 60, 1941 (April section), pages 162–7.

5. Multiple-Grid Breakers for High Voltage Service, J. B. MacNeill and A. W. Hill. AIEE Transactions, volume 58, 1939 (August section), pages 427–31.

6. The Vertical Flow Interrupter and Its Application to Oil-Poor Circuit Breakers, B. P. Baker. AIEE Transactions, volume 60, 1941.

Development of the Glow-Switch

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THE glow-switch[1] is an automatic switch for starting fluorescent lamps.[2] It is unique in that it is a thermal switch without a heater element, the heat being supplied by a glow discharge. The glow-switch functions as a time-delay relay yet it is a single-element switch with only two leads. At one value of applied voltage its resistance becomes a fraction of an ohm, while at a voltage only slightly lower, its resistance is measured in megohms. Before proceeding with the story of the development of the glow-switch, it is in order to discuss briefly the task which it performs.

Once the discharge is started, a fluorescent lamp can be operated from an a-c supply whose voltage is approximately twice lamp voltage. The lamp must be operated in series with an impedance, usually an inductance, to limit the current.[3] The most commonly used circuit is shown in figure 1. A voltage considerably higher than the supply voltage is necessary to start the discharge. The discharge should not be started until after the lamp cathodes have been heated to a temperature at which they can emit the necessary supply of electrons without entailing excessive positive ion bombardment. The glow-switch serves to automatically connect the lamp cathodes in series until they are heated to operating temperature, and then to break the connection, and subject the lamp to a transient voltage sufficiently high to start the discharge.

The glow-switch, figure 5, consists of a glass bulb, filled with an inert gas and containing a pair of contacts operated by thermostatic bimetal. Initially the contacts are open and the supply voltage appears across the switch. The supply voltage is sufficient to establish in the switch a glow discharge which heats the bimetal and causes the contacts to close. The current passed by the glow discharge is only a few milliamperes and does not cause any appreciable heating of the lamp cathodes, but when the contacts close, the current is limited only by the series inductance and the resistance of the cathodes. The cathode-heating current is approximately $1^1/_3$ times normal lamp current and it quickly heats the cathodes to operating temperature. While the contacts are closed, the glow discharge is extinguished so that the bimetal cools and after a short-time delay, during which the lamp cathodes are heated, the contacts open. Opening of the contacts suddenly interrupts the current and induces in the series inductance a transient voltage sufficiently high to start the arc through the lamp. After the lamp is started the cathodes are self-heating and the voltage across the lamp is insufficient to re-establish the glow discharge in the switch. The contacts remain open and the switch consumes no energy. If for any reason the lamp does not start the first time the contacts open, the cycle is quickly repeated.

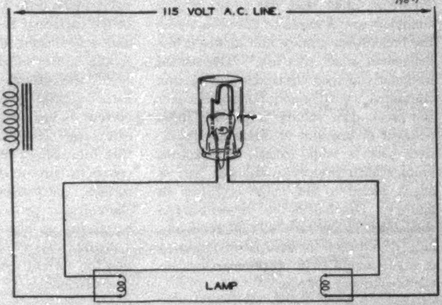

Figure 1. Fluorescent lamp circuit

The requirements that must be met by the glow-switch are as follows:

(1). The breakdown voltage for the glow discharge must be less than supply voltage and greater than lamp voltage.

(2). The contacts must close quickly.

(3). The contacts must remain closed for a period of time sufficient to allow the lamp cathodes to reach operating temperature.

(4). The switch must interrupt the current quickly so that the transient voltage induced in the series inductance will be sufficiently high to start the lamp.

(5). The switch must be small and inexpensive.

Breakdown Voltage

The breakdown voltage, or the voltage at which the glow discharge starts, must be less than the lowest supply voltage with which the glow-switch is to be used, and must be safely above peak lamp voltage so that the switch will not glow and the contacts will not close after the lamp starts. The *T*-12, 15-watt, the *T*-8, 15-watt, and the *T*-12, 20-watt lamps operate at 48, 56, and 62 volts respectively, and all three lamps are for operation from a supply of 118 volts ± 10 per cent. The same glow-switch is used with all three lamps. The voltage wave form for a fluorescent lamp is nonsinusoidal, and at low current the voltage is abnormally high. The peak voltage for a 20-watt lamp operating from minimum allowable supply voltage is equal the peak value of a sinusoidal voltage of 70 volts. Accordingly, the minimum breakdown voltage was set at 75 volts alternating current, and the voltage necessary to start the glow discharge and cause the contacts to close in a reasonable time was set at 105 volts.

It was found that neon containing a small amount of argon, used with electrodes coated with magnesium, gave a minimum breakdown voltage just greater than the minimum allowable limit. The breakdown voltage should not vary greatly with respect to gas pressure, gas mixture, or electrode separation, so that reasonable tolerances can be allowed in manufacture, and so that breakdown voltage will be relatively independent of pressure changes due to gas cleanup during the life of the switch. Curve *A* of

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Credit for development of the glow-switch to its present form is shared by D. S. Gustin, W. E. Carpenter and the author, all of whom are members of the engineering department, vapor lamp development section, of the Westinghouse lamp division at Bloomfield, N. J.

[1] For all numbered references, see list at end of paper.

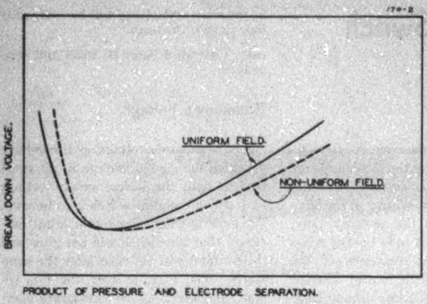

Figure 2. Break-down voltage

With respect to gas pressure and electrode separation for (*A*) parallel plane electrodes, and (*B*) irregularly shaped electrodes (Slepian)

figure 2 shows how breakdown voltage varies with respect to the product of gas pressure and electrode separation for a glow discharge between plane parallel electrodes.[4] Curve *B* shows the effect of a nonuniform field obtained by using electrodes of irregular shape. Fortunately, a nonuniform field makes the break-down voltage less dependent upon the product of pressure and electrode separation at values greater than that corresponding to minimum breakdown voltage. Figure 3 shows that the addition of a small amount of argon to the neon lowers the breakdown voltage and makes it less dependent upon pressure.[5] The use of irregularly shaped electrodes and of neon containing a small amount of argon makes the breakdown voltage practically independent of pressure or electrode separation over a wide range. Breakdown voltage with respect to the per cent of argon added to the neon[5] is shown in figure 4. One-half of one per cent argon was added to the neon so that relatively large deviation from the specified amount of argon would have no serious effect on the breakdown voltage.

Fast Closing of the Contacts

The contacts can be made to close quickly by the use of a sensitive bimetal element, by using closely spaced contacts, by using a bimetal element with minimum heat capacity, and by producing a high-current density at the surface of the bimetal when it serves as cathode for the glow discharge. Sensitive bimetal and close contact spacing result in closure of the contacts upon a slight rise in temperature. The temperature of the bimetal necessary for closing the contacts should be as high as 100 degrees centigrade, so that operation will be relatively independent of room temperature. A sensitive bimetal element is necessarily long and thin, and consequently, the force per unit deflection of the contact is small. If the bimetal element is sensitive and the contact spacing is small, there will be little force available to pull the contacts apart when they stick. The bimetal should be short and only as thick as necessary to obtain the required rigidity so that the heat capacity will be low. The most desirable method of increasing speed of operation is that of increasing the current density.

For a normal glow discharge the cathode-voltage drop and the current density at the surface of the cathode are constant, and the current density is proportional to the square of the gas pressure. The size of the cathode spot increases, but the cathode drop remains constant as the current is increased until the cathode is completely covered by the glow. Further increase in current demands an increase in current density at the cathode surface, and causes an increase in cathode-voltage drop. Such a condition represents an abnormal glow discharge.

Slepian[4] has given the following equation for current density for an abnormal glow discharge:

$$J=J_0+K(E-E_0)^2P^2 \qquad (1)$$

$$J_0=BP^2 \qquad (2)$$

where J is current density; J_0, normal current density; E, cathode drop, E_0, normal cathode drop; P, gas pressure; and B and K are constants. This equation shows that to cause rapid closing of the contacts by producing a high-current density, the gas pressure must be high, E_0 must be as low as permissible, and E must be high. For a neon-argon mixture in a nonuniform field, and PD greater than the critical value, E_0 is practically equal the breakdown voltage, the value of which has already been determined. E is equal the supply voltage minus the IZ drop in the inductance and the lamp cathodes. Obviously, E will be a maximum only if the current drawn by the glow discharge is a minimum. A high-current density and yet a small current can be obtained only by the use of a small cathode-surface area. Then, for rapid closing of the contacts, the gas pressure must be high, and the bimetal element must be as small as is practical from the standpoint of manufacture.

Time Delay

The contacts must remain closed for a period of time sufficient to allow the lamp cathodes to reach operating temperature. For 15- and 20-watt fluorescent lamps this time is approximately one second. The time delay is produced by the inevitable sticking of the contacts and by nonuniform heating of the bimetal, which results from the use of high gas pressure.

Transient Voltage for Starting the Lamp

When the contacts open, there must be induced in the series inductance a transient voltage of sufficient magnitude and duration to start the lamp. It is impos-

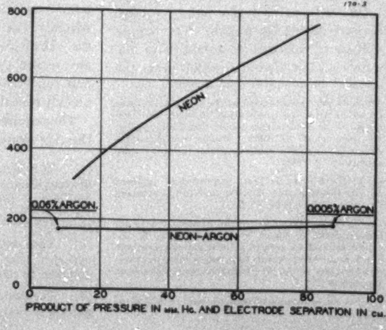

Figure 3. Break-down voltage

With respect to gas pressure and electrode separation for pure neon and for a mixture of neon and argon (Penning) sible to interrupt a current through an inductance instantaneously. For an instant after the contacts open, the inductance forces the current to continue flowing at a value practically equal to that of the instant before the contacts opened. The current continues to flow and decreases in magnitude as the energy stored in the magnetic field of the series inductance is dissipated. Then, until the lamp starts, the voltage across the glow-switch and across the lamp is the voltage corresponding to the current flowing through the switch whose contacts are open. Current can continue flowing through the switch after the contacts separate because of arcing at the contacts and re-establishment of the glow discharge. As the contacts separate, the current is carried by one point on each contact and the temperature of these points reaches the value which Slepian[6] has shown to be equal

$$T = \frac{E^2}{33.5kp} \tag{3}$$

where k is the thermal conductivity of the contact material (calories/centimeters squared/degrees centigrade/centimeters) and p is the electrical resistivity of the contact material (ohms/centimeters cubed). Copper contacts interrupting a one-volt circuit reach a temperature of approximately 4,000 degrees. Obviously, there is no metallic contact material available which will not melt and vaporize at the voltage to be used. Arcing at the contacts is supported chiefly by thermionic emission from the hot points and by thermal ionization of the hot vapor emitted from the points. Arcing can be kept at a minimum by using contact material which evolves a minimum amount of vapor. Tungsten makes an ideal contact because of its high work function, high melting point, low vapor pressure even in the molten state, and high resistance when hot. The current due to arcing at the tungsten contacts is negligible compared with that due to re-establishment of the glow discharge. Therefore, the transient voltage is practically equal the voltage across the glow discharge corresponding to the instantaneous value of current through the circuit.

Figure 5. The glow-switch, assembly of glow-switch and capacitor for suppressing radio interference, and the complete starter unit

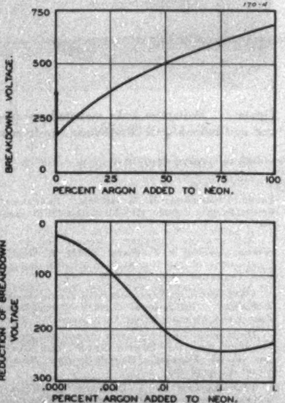

Figure 4. Minimum breakdown voltage for neon with respect to the percentage of added argon (Penning)

Rearrangement of equation 1 shows that the voltage across the glow discharge is equal

$$E = E_0 + (J - J_0)^{1/2}\frac{K}{P} \tag{4}$$

where

$$J = \frac{i}{A} \tag{5}$$

Equation 4 shows that the transient voltage can be made high by the use of low gas pressure. It has already been shown that the pressure must be high if the contacts are to close quickly. A compromise must be made. The pressure should be made only as high as is necessary for the contacts to close in a reasonable length of time; say, one or two seconds. Even so, the transient voltage is not always high enough to insure reliable starting of the lamp. Since E_0 and J_0 have already been fixed, the only other expedient is to increase the value of J.

Equation 5 shows that J can be made large by reducing the value of A. Fortunately, as has already been shown, A, the surface area of the cathode for the glow discharge, should be small to permit quick closing of the contacts. With the bimetal as small as is practical from the standpoint of manufacture, its surface area is still too large to insure reliable starting of the lamp. It is necessary further to increase the value of J by using only one strip of bimetal and letting the other electrode for the glow discharge consist of only a short lead wire with a contact welded to it. The surface area of this short lead wire and contact can easily be made very small. Of course, the highest transient voltage is produced only if at the instant the contacts separate, the current is flowing in the direction that will cause the small surface area electrode to serve as cathode for the glow discharge. The fact that there is only a 50 per cent probability that the highest transient voltage will be produced is not considered objectionable because, if the voltage is insufficient to start the lamp, the energy stored in the series inductance is instantly dissipated in the glow switch, and the contacts immediately reclose to repeat the cycle. The value of the series inductance has little effect upon the magnitude of the transient voltage but does affect its duration. Equations 4 and 5 are only approximately true[7] because the transient voltage varies too rapidly for the glow discharge to reach equilibrium. However, they show what steps must be taken to improve the characteristics of the switch.

Conclusion

It has been shown that for proper operating characteristics, the glow-switch necessarily must be small. The small size and simple construction of the switch as shown in figure 5 lend it to economical production on automatic machinery.

A glow-switch will start a lamp under normal conditions approximately a mil-

The Protective Link

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Synopsis: The protective link is a device recently developed for internal mounting in transformer of a wide variety of types and ratings. Its most common application, however, is in distribution transformers where it serves to disconnect the transformer in case of internal fault. Its action in arc extinction depends on a number of factors including a novel arrangement of the lower terminal which allows it to be expelled from the link when heavy currents are interrupted.

Although simple and compact, the link is a very efficient arc-quenching device, and has a remarkably high-circuit interrupting capacity combined with high speed of operation. Power arcs of more than 250,000-kva single phase have been interrupted in one-half cycle.

Introduction

SOME form of circuit-opening means has been used with transformers from the very beginning of the a-c system for the purpose of protecting the line or system against an extended outage in case of damage to an individual transformer. Economic factors in general have dictated the kind or type or the extent of the protection. Large important substations are protected with circuit breakers and differential-transformer protection to remove quickly a faulted transformer. Less important substations are protected by circuit breakers actuated by overcurrent or ground relays, and finally where circuit breakers cannot be justified from the standpoint of cost, high-voltage fuses are used to protect smaller substations, transformer banks, and individual transformers. Recently protective links have been applied to transformers for a similar purpose. They have been used on a wide range of ratings, but it is in the distribution range where they have found most of their application up to the present time.

In this application they are used in conjunction with circuit breakers to permit loading the transformer to its maximum safe copper temperature regardless of the amount or duration of load. The link alone, though designed for long-time lag, will not serve this purpose, because of the extremely long time constant of the transformer. The circuit breaker, on the low voltage side, follows this characteristic of the transformer and effectively prevents it from burnout because of overload. The protective link, on the high voltage side, protects the line from outage which might occur in case of failure within the transformer. In this function, its construction and mounting provide a number of inherent advantages. It is electrically protected against false operation by the transformer breaker on the one side and by sectionalizing or line breakers on the other. It is mechanically strong, and resistant to the action of transformer oil. Because of its internal mounting it is not exposed to weathering, and because of its electrical connection at the transformer bushing it is not subjected to lightning surges. These factors are especially important on the smaller ratings where the physical size of the fusible element is necessarily very small.

Description of the Protective Link

The appearance of the link as used in distribution transformers is shown in figure 1. This shows a core and coil assembly with a protective link mounted at each side of the porcelain tap-changer block. When assembled in the transformer, the high-voltage bushing lead runs directly to the top terminal on the link, thus placing the link electrically ahead of any fault which may occur either in the coil or the tap changer. The link itself is shown in section in the drawing of figure 2. It is simple in construction, consisting essentially of a fusible element enclosed in an insulating tube with a fixed and a movable terminal. The movable terminal, mounted at the

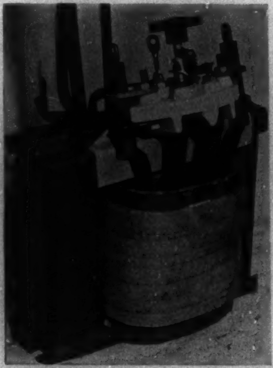

Figure 1. Protective links mounted on core and coil assembly for distribution transformers

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lion times, so that its life ordinarily should be very long. However, when a lamp fails, the switch keeps operating in an attempt to start it. Under failed lamp conditions, the life of the switch is approximately 100 hours.

In addition to starting lamps under conditions which insure good lamp life, the glow-switch has a number of advantages over other types of starters. It is operative over a wide range of current and compared with other types of lamp starters, it has a very long life under failed lamp conditions. Because the glow-switch is a two-wire device, it reduces wiring to a minimum. Simplified wiring permits placement of the glow-switch in a removable capsule which fits into an extension of the fluorescent lamp socket. Field results with the glow-switch, since its introduction two years ago, have been very satisfactory, and at present there are approximately six million glow-switches in use.

Bibliography

1. An Electronic Switch for Starting Fluorescent Lamps, R. F. Hays. *Electronics*, May 1940, page 14.

2. An Electronic Relay for Initiating Vapor Discharge, E. C. Dench. Electrical Engineering, volume 59, November 1940, page 461.

3. Alternating Current Circuits for Fluorescent Lamps, R. F. Hays and D. S. Gustin. *Transactions* of the Illuminating Engineering Society, December 1940, page 939.

4. Conduction of Electricity in Gases (book), J. Slepian. Educational Dept., Westinghouse Electric and Manufacturing. Co., Course No. 38.

5. Uber den Einflub sehr geringer Beimischungen auf die Zundspannung der Edelgase, F. M. Penning. *Zeitschrift Fuer Physik*, volume 46, page 335.

6. Temperature of a Contact, J. Slepian. Journal of the AIEE, volume 45, October 1926, page 930.

7. Dynamic Characteristics of Glow Discharge Tubes, H. J. Reich and W. A. Depp. *Journal of Applied Physics*, volume 9, June 1938, page 421.

lower end of the tube, is one of the new features of this design, and plays an important part in extinguishing the arc, as explained below. The fusible element consists of a mechanically strong wire which is chemically neutral in transformer oil and resistant to corrosion; these properties are essential to insure stability of the fusing characteristic over a long period of time. The fusible element is composed of copper, or, for the smaller size, a high-resistance copper alloy. The fusing temperature is much in excess of the boiling temperature of the oil. This is an important characteristic, for it tends to reduce the damage to the link caused by momentary overcurrent, and provides a means for control of the fusing-time curve of the link as described below.

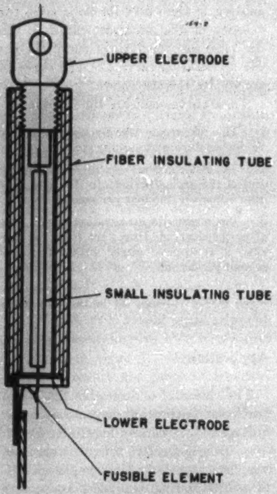


Figure 2. Section through protective link

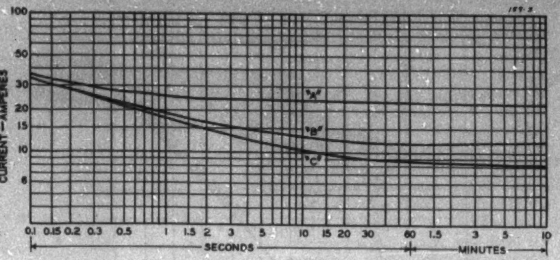


Figure 3. Fusing characteristic of protective links

(*a*)—Bare conductor in oil
(*b*)—Small insulated tube over conductor
(*c*)—Small insulated tube over conductor and gas entrapped within link

Surrounding the fusible element (figure 2) is a small insulating tube, the purpose of which is to retard the flow of heat from the wire. When a current slightly above the minimum fusing value is passed through the wire, its temperature is increased depending upon the rate of heat generation in the conductor and the rate of heat dissipation into the oil. With the current constant, the temperature increases approximately exponentially until the boiling temperature of the oil is reached. The chemical change in the oil under these conditions greatly increases the rate of heat dissipation of the oil. As a result, little further change in temperature results until the distillation gases sufficiently blanket the conductor. The temperature again increases approximately exponentially until the fusing point is reached.

If a bare conductor placed in the oil were to be used as a protective link, the gases distilled from the oil would escape freely and the current-time fusing characteristic of the conductor would be quite flat as shown in curve *A*, figure 3. By surrounding the link with a small insulating tube, the distillation gases quickly displace the oil in the tube and it then effectively insulates the conductor and retards the heat flow. This reduces the current and the time required to fuse the conductor for all ranges of current and time beyond that where all heat is stored in the conductor. An example of the reduction in current is shown in curve *B*, figure 3. By controlling the blanketing action of the distillation gases, the shape of the current-time curve can be adjusted to the desired shape to co-ordinate most effectively with other current-limiting devices on the system. The maximum blanketing occurs when the conductor is entirely surrounded by a gas (curve *C*, figure 3).

With the present knowledge of the impulse characteristics of transformer insulation and with modern lightning protective devices arranged for balanced lightning protection, major insulation failures from lightning have been practically eliminated. Therefore, line-to-line or line-to-ground failures of transformers in service, which would make it necessary for the protective links to open the full short-circuit current of the system, are uncommon. To insure successful operation, however, the protective links should be selected with sufficient arc-interrupting capacity to interrupt the current resulting from a short circuit across the terminal board of the transformers.

This, as mentioned, is not the usual condition. One of the most usual causes of transformer failure is short circuit in one of the windings between turns or between coils. Under these conditions, the fault current builds up spasmodically with a rush of current as each new element of the winding is involved in the failure. Since a considerable portion of the trans-

Figure 4. Oscillogram of protective link operation showing

(*a*)—Voltage across link
(*b*)—Current through link
(*c*)—Timing wave
(*d*)—Position indicator wave

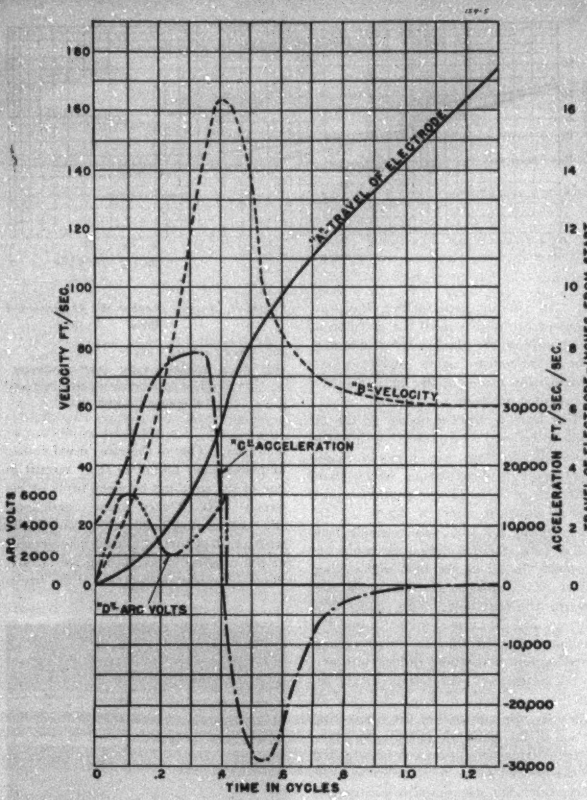


Figure 5. Analysis of electrode travel during arc interruption

former-winding impedance remains in series with the fault, the current builds up to a value sufficient to blow the protective link but does not approach system short-circuit proportions.

At the time of trouble in actual service, when the current reaches a value sufficient to fuse the link, an arc is formed in the oil inside the fiber-insulating tube. A high pressure is immediately developed inside the tube which increases the arc-voltage drop and aids in quenching the arc. This high pressure drives the lower electrode violently from the tube as if by explosive action. The arc trailing after it passes from the high-pressure area inside the tube to an area of comparatively low pressure outside the tube. This is an area of violent turbulence which is effective in arc quenching, principally because of the diffusion of the oil into the gas bubble surrounding the arc. Finally, the movable electrode moves rapidly through the oil stream tending to "neck" the arc and quench it.

Interruption of fault current occurs usually at the first current zero after the arc has been struck. The rapid movement of the free terminal through the oil builds up a layer of fresh oil between the moving electrode and the gas bubble after current zero, which effectively prevents restriking. Mounting the link in a vertical position so that the free electrode moves downward aids in this respect since the gas bubble is displaced upward by the pressure of the oil.

In order to investigate the action which takes place during the operation of the protective link, a special test was set up. A link was made in which the lower electrode was carried on a light but rigid arm. This arm was pivoted at one end, to allow the electrode to swing down freely as it does when attached to a flexible cable. A commutator device was mounted on the arm to permit recording of its position on an oscillograph.

The action of the link in interrupting the arc and a conception of the speed and the forces involved can be obtained from the results of this test as shown in figures 4 and 5. Figure 4 is an oscillogram of an interruption of a circuit connected for 1,500 amperes and 22,000 volts. It shows the normal operation of clearing the circuit in one-half cycle, and also shows very clearly the eight make-and-break points obtained by the commutator (curve *D*, figure 4).

From these data, a curve of position of the electrode against time is obtained, as shown in figure 5, curve *A*. Graphical analysis of this curve for slope gives the velocity of the electrode, curve *B*. In the same way, analysis of curve *B* gives the acceleration curve *C*. The curve of arc voltage *D* is also shown.

These curves indicate the following:

1. The electrode travels approximately six inches during the arcing period.
2. The maximum velocity is reached at the end of the arcing period; in this case it is approximately 165 feet per second.
3. The maximum acceleration occurs just after the electrode leaves the tube; in this case it is approximately 40,000 feet per second per second.
4. The electrode travels approximately eight inches through oil in the half cycle following the extinction of the arc.

Application

The choice of a protective link for a particular application depends upon the voltage and the current to be interrupted. The simple link shown in figure 1 has been supplied extensively to a completely self-protected distribution transformer, and other distribution transformers exposed to severe lightning conditions. Its interrupting ability is adequate for practically all applications for transformers up to 100 kva and 15,000 volts. These links will interrupt successfully 12,000 to 20,000-kva single phase in the above voltage range.

For larger units in the range of approximately 20,000 to 75,000-kva single-phase interrupting capacity, a longer and stronger fiber tube is necessary. A protective link of this type is shown in figure 6. The general construction remains the same. It will be noted, however, that the lower electrode is not attached to the tube, but is suspended from the fixed

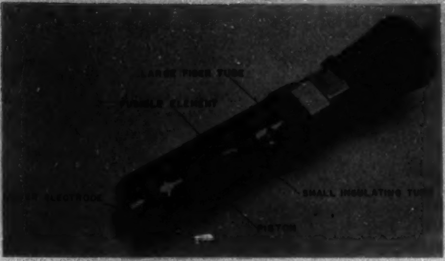


Figure 6. Protective link for application to power transformer

electrode. A piston of insulating material is used to transmit the mechanical impulse to the movable electrode. Protective links of this rating have been applied to completely self-protected power transformers in sizes up to 3,000 kva and 33,000 volts.

Another interesting application is to subway distribution transformers. Links with an arc-interrupting capacity of 10,000 amperes were applied to 2,400-volt subway transformers. These transformers were equipped with thermally-operated breakers in the secondary, properly coordinated with the protective links and arranged to protect the units against burnout from overload and secondary short circuit. The use of the protective links together with a disconnecting-type, high-voltage cable connection made the unit self-contained and very easy to install.

For application to still higher capacity circuits, the protective link is modified as shown in figure 7. Means are provided to reduce the pressure produced inside the tube and to increase the arc-quenching action at low currents and high voltages. The reduction in pressure is accomplished by enlarging the space above the main insulating tube and mounting inside it a shock-absorbing device. This device consists of a thin-walled cylindrical ring filled with gas under atmospheric pressure. Under high-current operation, the walls of the gas container collapse and cushion the pressure wave, thereby reducing the mechanical shock and the internal pressure in the device.

Interruption of low currents at high voltage is accomplished by means of the small auxiliary fiber tube. This tube creates a high-pressure zone immediately surrounding the arc, making it effective for currents as low as the exciting current of the transformer. At high current this tube is ruptured.

Links of the type as shown in figure 7 are applied to transformers with voltage ratings up to 69,000 volts. They have been tested at power levels of more than 250,000-kva single phase. An example of this is shown on the oscillogram of figure 8.

Field Experience

Within the last few years many thousands of protective links have been placed in service in distribution transformers. Some of these have had occasion to operate either because of a defect in the winding causing coil failure, or because of low electrical clearance at some point causing breakdown on surge. In most cases the lower electrode was driven from the end of the tube. Occasionally, when the current was only slightly above the minimum required to fuse, the arc was quenched before the electrode was dislodged. Usually the electrode was found to be driven out violently; in fact the evidence of the violence of this action is some indication of the magnitude of the current interrupted. But even when severe mechanical damage was done, the link served to clear the circuit, and the small amount of burning on the electrodes showed that the arc was extinguished quickly.

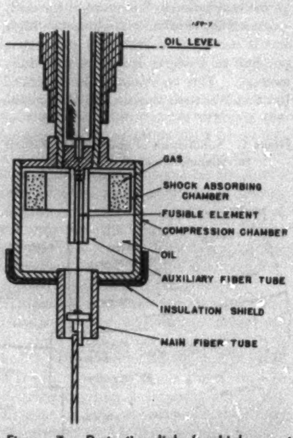


Figure 7. Protective link for high-current and high-voltage interrupting capacity

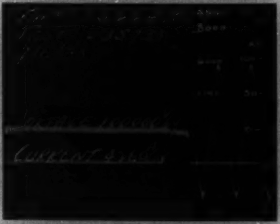


Figure 8. Oscillogram of protective link operation for link shown in figure 7

A case of operation on power transformer has occurred. Links were applied to the high-voltage side of a 33,000 to 4,160 volt, 1,000-kva 3-phase unit, and a breaker was connected on the low-voltage side. This breaker was rated at 5 kv, and protected by 33-kv links connected in series. This was considered desirable because high- and low-voltage lines were carried on the same pole. Shortly after this unit was put into operation, a wind storm blew off a sheet metal roof from a barn which struck the line, broke the high-voltage wires and caused them to fall across the low-voltage line. The breaker opened, but was not able to clear the high-voltage arc, which burned across the open contacts until the protective links cleared the circuit. Two of the links on the low-voltage side and one on the high-voltage side were blown during this disturbance. Except for the burning of the breaker contacts and adjacent parts, no damage was done to the unit. After reloading the links, the transformer was put back on the line and has since functioned satisfactorily.

Variations of Atmospheric Temperature With Altitude in the United States

H. W. TENNEY
MEMBER AIEE

IT has long been recognized that with increase in altitude the density of air decreases and influences the voltage rating and temperature rise of electrical apparatus. However, different standards for apparatus do not give the same weight to these factors. Examination of Weather Bureau data shows that there is a decrease in average atmospheric temperature with altitude, which should compensate to some extent for the influence of the decrease in air density on the rate of cooling at higher altitudes. This led to a study of temperature records of the United States for the purpose of presenting a summary of the variation of mean temperature with altitude.

The Weather Bureau of the United States is to be complimented on the fund of data which it has collected and the usable form in which it is available. The chief source of data used in this paper is the 1938 issue of Climatological Data for the United States.[1] In this issue there are temperature data from 3,119 stations for the year 1938 and the departure of the annual mean temperature from the mean of the temperature for all years of record. Consequently, by algebraic subtraction of the departure of the mean for 1938 from the annual mean, the mean temperature for the period of record is obtained.

The daily maximum and minimum temperatures are read from properly exposed thermometers at the co-operative observation station. At the first-order Weather Bureau stations thermograph charts are taken, giving continuous record of temperature variations. The records made by co-operative observers of the Weather Bureau, which are used largely in preparing charts and drafts, are made with standard maximum and minimum thermometers exposed in approved shelters, usually at an elevation of five feet above the ground. The true average daily temperature corresponds closely to the average of 24 hourly observations, but as several other combinations of hourly values give averages that differ but little from the true daily average some one of these is generally used to reduce observational work. The combination—

$$\frac{(7\text{ a.m.}+2\text{ p.m.}+9\text{ p.m.}+9\text{ p.m.})}{4}$$

gives a value which differs only slightly from the true daily average, and

$$\frac{(\text{sunrise}+2\text{ p.m.}+9\text{ p.m.})}{3}$$

also gives fairly accurate results. The formula

$$\frac{(\text{maximum}+\text{minimum})}{2}$$

is easy of application and very satisfactory when dependable maximum and minimum thermometers are used and properly exposed. The mean of the daily extremes is, as a rule, slightly too high, but it usually does not vary more than one-half of a degree from the true daily average. This combination is employed by the Weather Bureau to obtain the average daily temperature, and the data for the accompanying charts and diagrams were compiled by its use.[2]

A superficial analysis of some data indicated that the variation of temperature with altitude at the same general altitude differed with the watershed, as, for example, the eastern and western watersheds of the Allegheny Mountains in Pennsylvania. Therefore, in this analysis the country was divided into six districts as shown on the map, figure 1, as follows:

District No. 2—North Central United States (Mississippi Watershed).

District No. 3—Northern Rocky Mountains and Western Seaboard.

District No. 4—Southern Atlantic Seaboard.

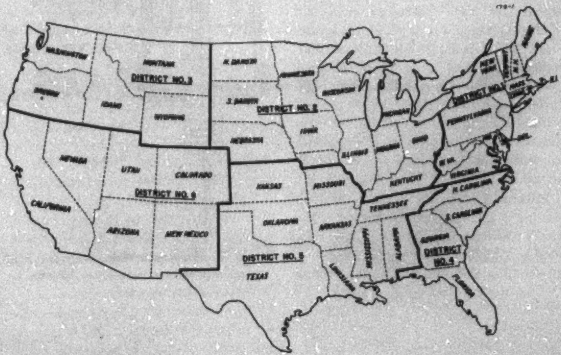
District No. 5—South Central United States (Mississippi Valley).

District No. 6—Southern Rocky Mountains.

In order to simplify the analytical work, the border lines of the districts, in most instances, have followed the border lines of the several states included in each. While this makes for odd-shaped districts, a check on one district in which the northern and southern border lines were made to parallel the lines of latitude showed that the complexion of the results was not materially changed.

The mean variation of temperature with altitude for the entire United States for all years of record for 3,119 Weather Bureau stations, is shown in figure 2. The slope of this curve indicates a decrease in average temperature of approximately one degree for every 330-foot increase in altitude. This curve of temperature with altitude is not a straight line and the increment of elevation per degree of average temperature will vary. In a report from the United States Weather Bureau,[2] it is stated that the

Figure 1. Subdivision of country into districts for temperature-altitude analysis



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The author acknowledges the able assistance of C. S. Hague, Jr., and R. C. Fowler in analyzing the vast amount of Weather Bureau data, which form the basis for the curves presented in this paper.

1. For all numbered references, see list at end of paper.

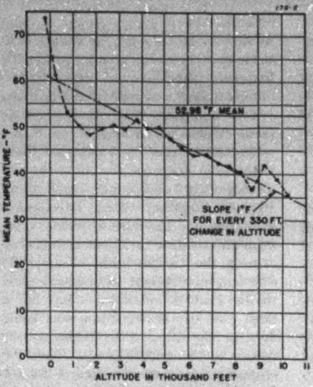

Figure 2. Variation of mean temperature with altitude for the entire United States

(Temperatures are mean for all years of record based on data from U. S. Weather Bureau reports)

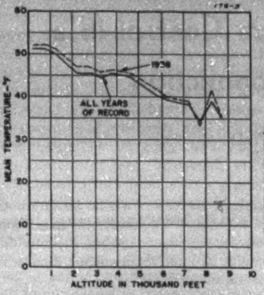

Figure 3. Comparison with altitude of mean temperatures for the year 1938 and mean temperatures for all years of record

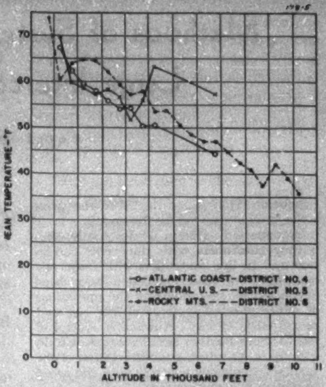

Figure 5. Variation of temperature with altitude for the southern districts of the United States

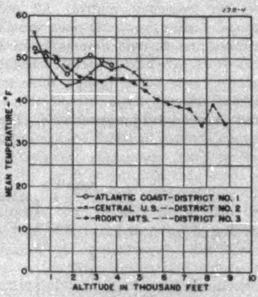

Figure 4. Variation of mean temperature with altitude for the northern districts of the United States

average decrease in temperature with increase in altitude in free air is about one degree for each 330 feet, but the rate varies with the season of the year and also is much affected by local conditions. It is more rapid in summer than in winter and is greatest during the warmer hours of the day. A study made of temperature variation with altitude made in England,[3,4] determined by sounding balloons, showed a decrease in temperature of one degree Fahrenheit for every 300-foot increase in altitude up to approximately seven miles, beyond which there is no further reduction in temperature.

The length of time records have been kept at the individual observation stations varies from 2 to 159 years. Through the use of mean temperatures for varying lengths of time the influence of periodic changes of temperature may be introduced. A method of extending short-term records has been suggested for the purpose of developing a satisfactory comparison between prevailing temperatures of two stations.[4] This may be done by comparing the mean temperature at the short-term station with the mean for a similar period of a similarly located long-term station and assuming that the same relationship would exist between the long-term means of the two stations. This procedure was not followed out in this paper, because it would multiply the analytical work tremendously and the increase of accuracy that might be expected would not warrant it. This is demonstrated in figure 3, which shows two curves of variation of temperature with altitude. One is the mean temperature for the year 1938 and the other is the mean temperature of all years of record. The year 1938 was a warm year and the annual mean temperatures for the entire country were above normal. However, in comparing the mean temperatures of one year with the mean temperatures for all years of record, it is seen that the curves are similar in shape and the difference between them is not great.

Figure 4 shows the variation of mean temperature for all years of record for the three northern districts numbers 1, 2, and 3, of the United States, and in figure 5 the same curves for the three southern districts numbers 4, 5, and 6 are shown. In order to interpret these clearly a brief description of how the data were worked up will be helpful. The mean temperatures for all years of record for all Weather Bureau stations were first calculated from the annual mean of 1938 and the deviation of this mean from the mean of all years of record.[1] It was then arbitrarily decided that the temperature data would be analyzed on the basis of 500-foot increments in change of altitude. The mean temperature of all stations located at altitudes between 0 and 500 feet, 501 and 1,000 feet, etc., were averaged and plotted halfway between the 0 and 500, 501 and 1,000, etc., ordinates on the curves. Analysis of the districts, using smaller increments for altitude, may influence the shape of the curve slightly, but will not seriously affect the general considerations involved.

Referring to figure 4, it is seen that the temperature decreases quite rapidly from 0–2,000 feet. There is then a reversal in the slope of the curve and the temperature increases gradually up to 3,000 or 3,500 feet. The mean temperature again falls off with increased altitude, but at a slower rate than in the first part of the curve. The curve of the Northern Rocky Mountain district seems to become erratic at the high altitudes. This is probably explained by the fact that there is only one Weather Bureau station in the ranges of altitude of 7,500–8,000 and 8,000–8,500 and only two in the range of 8,500 to 9,000. In ranges where only one or two stations are located unusual local conditions are not averaged out This is shown even more clearly in figure 5 in the curve for South Central United States for the ranges of altitude of 4,000–4,500 feet and 6,500–7,000 feet. These points are also for only one Weather Bureau station. The data for the point in the 4,000–4,500 range were taken at Alpine, Texas, and extend over a period of 16 years. Therefore, any extreme annual

Methods of System Control in a Large Interconnection

EARLE WILD
MEMBER AIEE

Synopsis: This paper describes the principles used in controlling the load in a large interconnected system. Automatic frequency controllers with time-error correction and tie-line controllers with frequency bias and time-error correction have been developed for co-ordinating the operations of interconnected systems of any size and geographical extent. Automatic control of two or more stations simultaneously within a group is described, and the relation of the auxiliary frequency controllers to the speed governors is also discussed.

Introduction

THE interconnection of many contiguous power systems into large co-ordinated groups of several million kilowatts has made imperative the development of control apparatus of great variety and refinement. The intended purpose of these interconnections required that economies of generation be maintained and that designated power flows be held steady. The geographical extent and the variety of relationships between the many properties made it necessary to use control equipment which would function in widely separated locations and with a minimum of direction. Increases in the size and number of large variable loads indicated the need for closer speed control. The final object of the control apparatus, therefore, was to maintain the most constant frequency and thus permit each local group to operate its individual system as local requirements dictate for maximum economy.

Basic System of Control

The original system of control consisted in giving one central group responsibility for the frequency control. All other groups starting from this point were interconnected through one or more tie lines which were held at definite scheduled loads. These other groups thus were varying their own generation locally, to meet their own load requirements. Theoretically, the frequency-control stations would only be following the variations in the load of the central group. Automatic frequency controllers and automatic tie-line controllers were used for this purpose.

Two objections arose to this simple plan. When two, three, or sometimes more of the tie lines departed from schedule in the same direction, the frequency-control station was unable to restore the frequency without an excessive load change. The other objection came from the continual variations in frequency causing the tie-line loads to vary. It was not only difficult to maintain these tie-line loads at scheduled value, but a large part of the regulation was being done in opposition to the frequency correction of the central group. Thus the original simple system of control did not function as well as intended.

Tie Line Biased by Frequency

These difficulties led to the development of the tie-line biased-by-frequency method of automatic control, which is

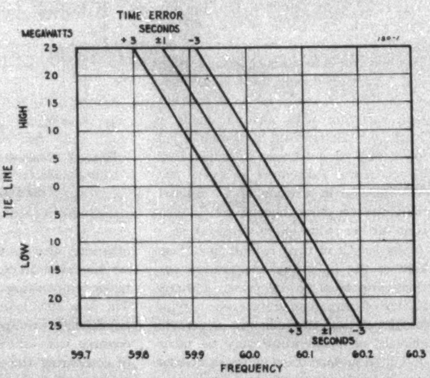

Figure 1. Characteristic curves of a tie-line controller

Shows tie-line load departures from normal, for corresponding frequency and time-error deviations

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condition should be fairly well averaged out. The displacement can only be explained by unusual local conditions. On the other hand, the point in the 6,500–7,000-foot range is for the Weather Bureau Station at Mount Locke, where the period of record is only four years. Here a year of unusual temperature conditions might have affected the mean temperature some but probably not sufficiently to bring this point within the more or less expected trend of the curve.

Examination of figure 5 shows changes in slope at different points along the three curves, but these do not appear in the same range of altitudes as was the case with the three northern districts. As would be expected, the mean temperatures at the lower altitudes are higher than for similar altitudes in the northern districts. However, the curves for the northern and southern districts approach each other at the higher altitudes.

As seen from figure 2 the retardation and inversion of the temperature-lapse rate supports the general practice of making no correction for ratings for altitudes up to 3,300 feet (1,000 meters). Data are presented here from which a basic study may be made to determine the limits at which standard apparatus may be used at higher altitudes.

References

1. Climatological Data of the United States for 1938 from the United States Weather Bureau, Department of Commerce, Washington, D. C.

2. ATLAS OF AMERICAN AGRICULTURE, Part 2, Climate, Section B, Temperature, Sunshine and Wind, Joseph B. Kincer, senior meteorologist, United States Weather Bureau, Department of Commerce Washington, D. C.

3. SIR JOHN MOORE'S METEOROLOGY (book).

4. ELEMENTS OF HYDROLOGY (book), A. F. Meyer.

being used extensively in large interconnections. With it, the fundamental plan of regional control of loads is continued, but instead of holding tie-line loads at constant value, they are permitted to deviate from schedule whenever the frequency varies and in a direction to relieve the burden of the frequency-controlling center. The amount the tie-line load is varied, commonly known as *bias*, is a function of the frequency effect on load and the action of the speed governors of the turbines. This relationship will be described in a later section.

This bias principle permits tie-line loads to be varied in a relieving direction and be restored with a minimum change in generation in each group. It also assures the frequency-regulating center that several tie lines will be biased correctly for each one causing a frequency departure from normal. Thus any number of groups may be interconnected under this plan without increasing the burden of the central group.

The performance on several of these tie lines has been checked by means of counters, which register the deviations from normal. These show that most groups hold to scheduled tie-line loads about 70 per cent of the time, cause frequency deviations about 10 per cent of the time, and vary in a direction to relieve the frequency deviations 20 per cent of the time. Actually, each group on bias is assisting, in direction at least, twice as much as it is contributing to frequency deviations.

Frequency Effect on Load

Before the correct amount of bias could be determined for any system it was necessary to know the effect of frequency changes on the load. Studies were made in the Chicago system to determine the amount of load change for a given frequency change. It was found, as a very general rule, that one-tenth-cycle frequency departure resulted in approximately a one per cent change in system load. This figure is somewhat less than has been observed on some large interconnections, but has been checked many times since the staged tests were made, whenever important intersystem tie lines opened automatically under heavy load and there were sudden frequency changes. This figure takes into account the average response of a composite group of governors on the turbines affected, as they are found in normal service and without unusual attention being given to increasing their sensitivity or to block them.

This figure also includes some effect of voltage changes. Whenever the frequency departs from normal there is a change in voltage, due to change in the speed of the shaft-driven exciters on the main turbines. A test in one large load zone, which is mostly industrial in character, shows that whenever the voltage drops one per cent there is a $1^1/_2$ per cent drop in customer load. However, when the frequency drops there is not a general drop in voltage proportional to the drop in speed, as the automatic voltage regulators provide correction for at least 50 per cent of the load. Probably the largest effect of minor frequency changes comes both from a change in speed in the industrial motor load and the action of the more sensitive speed governors.

Tie-Line Bias Setting

The tie-line bias setting used is the one which will permit the tie-line load to change an amount equal to the group-load change for any given frequency deviation. As the frequency rises the load increases, but the generation remains constant except for the small decrease caused by the speed governors. The tie line supplies the difference and thus tends to oppose the frequency rise. When the frequency is restored to normal the tie-line load is returned to schedule without adjusting the generation. Thus whenever the frequency varies one-tenth cycle, the tie-line load is changed an amount equal to one per cent of the total generated load. This bias is extended proportionally for other departures from the normal of 60 cycles.

Time-Error Bias

There are times, however, after the frequency has departed from normal for a considerable period, when it is necessary to arbitrarily offset the frequency in the opposite direction in order to correct an integrated frequency or time error. If a great many of the tie lines in an interconnection are being operated on the tie-line bias principle, it becomes very difficult for the centrally-located frequency-control station to offset the frequency manually. It was desirable, therefore, to add time-error bias to each of the frequency-biased tie lines. Whenever the time departs from normal a time-error indicator, therefore, similarly causes a departure in tie line in such a direction as to offset the frequency and correct the time error.

Figure 1 is a chart of the relation of tie-line load to frequency and electric time error in a controller. All three of these factors must be satisfied, or the controller will change the system load until the tie-line load reaches a balance point. Figure 2 shows the changes in tie-line load as the frequency and time-error deviate from normal on a large group.

Description of Controllers

The equipment used in the control consists of the following apparatus:

1. Frequency controller with an electric time-error correction.
2. Tie-line controller.
3. Tie-line controller biased by frequency.
4. Tie-line controller biased by frequency and with electric time-error correction.
5. Cascading settings of tie-line controllers when used in series.
6. Devices for controlling the load on two turbines in separate stations.

The tie-line controller biased by frequency and with electric time-error correction, and the frequency controller with electric time-error correction will be described. Figure 3 shows a diagram of the electrical circuit of a typical tie-line controller, which has both frequency bias and time-error correction. The apparatus, as commonly used for this purpose, consists of a frequency bridge which is tuned for 60 cycles. One corner of the bridge has been opened and extended to a slide wire mounted in the recording tie-line meter. The zero or center point of this slide wire has a front "setter" and is placed at the scheduled tie-line point, wherever that may be from hour to hour. The movable contact of this slide wire is operated by the tie-line recorder carriage. Whenever the tie line is at the scheduled point, the contact is in the center of the slide wire and the frequency bridge remains balanced. If the tie line is displaced to one side or the other of the zero point, more resistance is inserted in one leg of the frequency bridge than the other, calling for a change in frequency in order to restore the balance of the galvanometer circuit or the correction of the tie-line load. Similarly, a second corner of the frequency bridge has been opened and extended to a slide wire in a time-error indicator. This time-error indicator consists of two electric motors, one driven by the system frequency and the other driven by the exact frequency of 60 cycles obtained from a crystal oscillator time standard. The speed of these two motors is compared through differential gearing, and any resulting displacement operates the pointer on the slide wire of the time-error indicator to change appropriately the balance point on the frequency bridge.

In order to regulate a system within plus or minus one second over widely scattered controllers requires that highly accurate time standards be provided. Some of the larger cities have sources of astronomical time, but these are not generally available over a wide interconnection. It has been agreed, in this interconnection, to use the primary frequency standard of the crystal oscillator type as a source of true time. These have an error of one part in two million and may be readily compared with observatory time as received from Arlington. Even assuming an error in setting these oscillators at widely scattered points, it will not be difficult to keep the time-error indicators within a quarter of a second of each other. The amount of tie-line departure that might be introduced by an error of a quarter of a second in the time standard would, at the most, be only one or two megawatts in any group.

The apparatus in the frequency-controlling centers is similar to the tie-line controlling apparatus described above, except that there is no tie-line slide wire. The bridge is continuously tuned for 60 cycles, except when the time-error indicator is displaced from its center point.

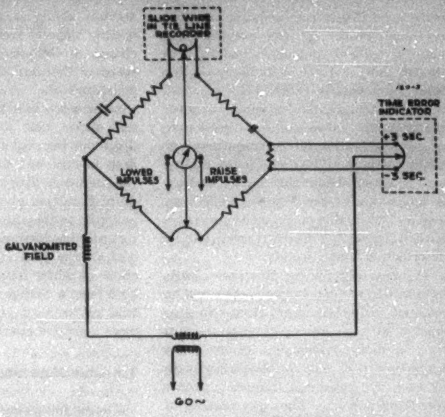

Figure 3. Circuit diagram of a tie-line controller, with frequency bias and time-error correction

The galvanometer is a Leeds and Northrup contact-making type, in which there are wiping cams which make contact with the "raise" or "lower" circuits once every two seconds. The duration of these contacts is proportional to the amount the frequency has deviated from the calibration value of the frequency bridge.

Rate of Load Response

The galvanometer circuit is unbalanced any time one of the three characteristics is varied from the normal value. If two of the characteristics are abnormal, the galvanometer is still further unbalanced unless these two are in opposition.

Comparison of these counterinfluences in the frequency bridge is very important. For example, if the tie line is off schedule and the frequency has not departed from normal, indicating that the load in some other group of companies is off schedule in the opposite direction, the controller sends out very short corrective impulses and it takes considerable time for this tie-line error to be corrected. Similarly, the other zone, whose generation is displaced from the correct value in the opposite direction, would be making a very slow correction and the load of the frequency-controlling center would not need to be disturbed. If, on the other hand, were the tie line to be off schedule in the same direction as the frequency, indicating that this group of companies is producing the frequency departure, the controller would send out very long impulses and cause the generation to change very rapidly at the point which was most responsible for the frequency departure.

The unbalancing of the bridge by time error is a much more gradual influence and requires, likewise, a much longer period for the time correction.

The controllers in the large group have been set to give a maximum correction to tie-line load of 20 megawatts in 30 seconds. The frequency-controller group has about one-half this rate of response, and the smaller groups are decreased to one megawatt a minute.

Cascading of Controllers

Several controllers may be installed in series along an extended transmission link, as shown in figure 4. The one most remote from the frequency-control center is given a bias setting proportional to the

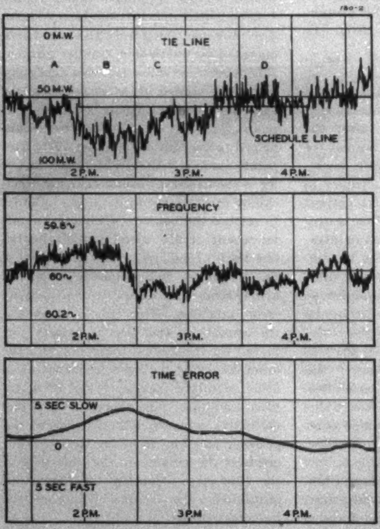

Figure 2. A tie-line load curve showing variations from schedule, effected by frequency and time-error biases

The tie-line load is (*A*) high with low frequency, (*B*) increasing with slow time error, (*C*) decreasing with high frequency but continued time error, (*D*) low with high frequency

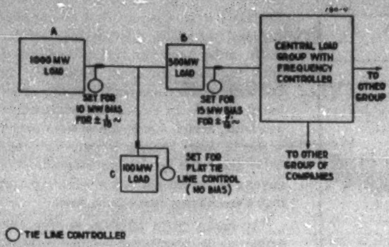
190-4
A
1000 MW LOAD
B
300MW LOAD
CENTRAL LOAD GROUP WITH FREQUENCY CONTROLLER
TO OTHER GROUP
SET FOR 10 MW BIAS FOR ± 1/10 ~
SET FOR 15 MW BIAS FOR ± 1/10 ~
C
100 MW LOAD
SET FOR FLAT TIE LINE CONTROL (NO BIAS)
TO OTHER GROUP OF COMPANIES
TIE LINE CONTROLLER

Figure 4. Cascaded settings of three tie-line controllers on a transmission system

Bias setting of group *B* includes bias of group *A*, but does not include group *C*, a small system operating on flat tie-line control

size of its system. The next controller toward the frequency-controlling center is given a setting which includes its own proportional bias and the bias of the controller in back of it. If there are controllers in smaller companies which are on flat tie-line settings or which are held manually at a constant value, these, of course, are not taken into account in determining the bias setting. The controller which is nearest to the frequency-controlling center and having the highest bias may actually be operating in one of the smaller companies or one of the smaller generating stations. If the controller in the more remote company should fail to effect its tie-line departure, the controller more centrally located would then be called on for a contribution out of proportion to the size of the latter company. This is prevented in two ways: (1) By a suitable load-limiting band on the generators which restrict the departure to a proper value, and (2) by limiting the rate of response of the smaller controller so that the larger company will have sufficient time in which to make its correction before the smaller company has made an extended departure from its normal generation.

In one large interconnected system there are now 60 companies operating in parallel. These have generally adopted this principle of frequency and tie-line control. There are 17 of these controllers operating in parallel, 2 of them regulating the frequency only, and the other 15 on one or another form of tie-line control. The smaller companies are generally on flat tie-line control unless they are in series with a larger group which is on bias. One has installed time-error correction, and several others are now providing time-error correction equipment. With this method of control all local problems of generation are handled locally. Responsibility for frequency departures or inadequate generation is indicated at its source and corrected. The physical distance between companies or groups of companies is not a limitation, and new groups may be added to the interconnection and perform satisfactorily by adding a new controller without requiring changes in any of the existing facilities.

New interchange contracts which are made locally between any groups of companies do not call for changes in the apparatus, as it is adaptable to almost any power-flow requirements. The only precaution necessary is that the frequency bridges must be reasonably accurately tuned for 60 cycles. Where time standards are provided this is very simply checked, and where there are no standards, other apparatus, such as tuning forks, can be used, which will permit checking the frequency bridge accurately enough for load-control purposes.

Simultaneous Automatic Control of Two Stations

Once a company or group of companies have installed a controller, the problem arises as to the method to be used in utilizing the controlling impulses. The load and the number of generating stations have frequently grown to a point where it is not practical to assign the regulating responsibility to one station. This has been overcome in the Chicago area by the installation of one controller, which is sending its correcting impulses to two generating stations simultaneously. This appears to be the best arrangement in any company or group of companies which are operating under a pooling arrangement, and the control of the load on the local transmission system is not necessary.

After the controlling impulses are directed to two stations, it is desirable that the turbine selected for the automatic control in each station respond in similar amounts to the identical impulses. These turbines may be of different size and have entirely different governor characteristics. Two devices have been used for introducing these impulses into the governors of the respective turbines. One of these consists of a rheostat which operates a torque motor attached to the pilot cylinder of the governor, and makes a small and controlled load change. The two rheostats may be easily timed to travel at the same rate in the two stations, and by control of the necessary torque they can be made to make similar load changes, except at valve-opening points. These valve-opening points are traversed manually. This device, acting on the pilot valve, is not only faster in response than the synchronizing motor, but it avoids any inertia or loose play which is present in governor linkage. The rheostat also acts as its own load-limiting device.

The other apparatus which has been developed consists of an instantaneous wattmeter of the Kelvin balance or electronic type, in which the small driving motor in the meter is used to drive the carriage of the meter from the impulses of the controller. The impulses are also sent to the synchronizing motor, and if the turbine makes a larger load change than required for the length of impulses being used, the wattmeter becomes unbalanced and interrupts any subsequent impulses

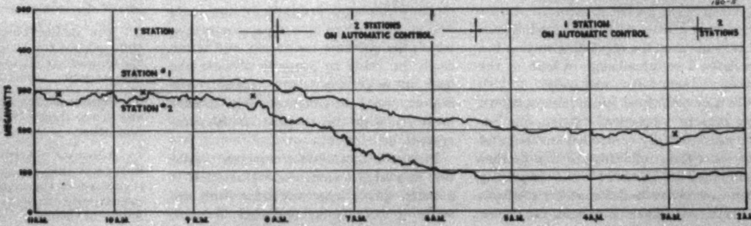

Figure 5. Load curves of two stations equipped for simultaneous automatic control

Amount of load variations at (X) increases sharply when only one station is on automatic regulation

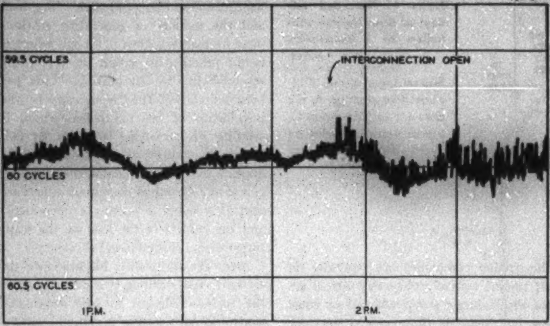

Figure 6. Frequency curve of a large system, showing effect of interconnections on width of frequency variations

to the turbine. The speed of the driving motors in the two wattmeters can similarly be timed together.

Need for More Automatic Control of Stations

In any station which has automatic load control, the variation in its load can be analyzed to come from two sources: One, the variations in the load of the system as it changes from minute to minute, and the other the irregularities in generation of the nonautomatic stations. These latter variations are caused largely by the inability of these stations to follow adequately the system-load changes. Frequently it has been found that some of the largest burden on the automatically-controlled station comes from the simultaneous delayed action on two or three other stations within the same group.

In the Chicago system an attempt was made many years ago to assist all stations in following the load trend by having a system totalizer in every operating gallery, which would indicate the total system load at all times. Each operator was then given a schedule which indicated the portion of the system total load assigned to that station. Although this method has proved quite workable, no operator is able to follow manually the trend as fast as the automatic station does, and so after each large load change it is found that the regulating station has to make a counterchange of load as the other stations follow manually.

In a system of, let us say, five stations, one may be a base-load station, another the automatically-controlled station, and the other three following the trend either manually, as observed on a totalizing meter, or under load dispatcher's orders. If these latter three, or at least the one or two of them making the largest load variations, could make their change in load before the full-load change of the automatic station has been completed, the burden on the automatic station would be very sharply reduced.

The experience in Chicago with two stations operating simultaneously (figure 5) indicates that the burden on either one is less than half what it would be if only one were on automatic control. This indication has proved so definite that steps are now being taken to extend the automatic control to practically all the major generating stations. The so-called base-load stations during the minimum hours become the variable-load stations on the system, and at these hours take over the automatic regulation.

Relation of Controllers to Governors

The function of the auxiliary speed device or frequency controller, as it is commonly called, does not supersede that of the speed governor. The speed governor must continue to maintain instantaneous frequency, but in maintaining this frequency it distributes the load in a manner that is often not desired. The controller, on the other hand, is relatively slow in its action, but it does give droop correction, which the governor cannot do, and it can easily be made to properly allocate the load in any proportions desired. The importance of the governor seems to vary inversely with the size of the electrical system.

Figure 6 shows the frequency variation in a large system while it was part of a much larger interconnection and also while it was being operated isolated. Certainly it is apparent that the smaller systems, especially if supplying variable loads, must have the turbines equipped with the best governors available. In the large interconnections, on the other hand, there is an enormous inertia effect available, which holds the system speed within a very narrow band. This has led to the belief in some quarters that the governors should not be too sensitive and may even have a small dead band.

The load-control problem is generally rather easily handled in a small isolated system. The large interconnection-load control is much more difficult. Although the instantaneous frequency varies but little from minute to minute, it does drift over the longer period of hours or more, over a much wider band than is found in the small systems. This is a load-control problem, which must be handled by the controllers or by supplementary manual action.

The manufacturers have proposed that governors be built with a sensitivity of 0.04 per cent and a per cent regulation up to two per cent. These are undoubtedly desirable on the smaller systems, but on the large interconnections governors with a sensitivity of 0.08 per cent and from four per cent to six per cent regulation would be satisfactory. Where flat tie-line regulation is used, the action of too responsive governors is contrary to the action called for by the controllers. With the tie-line bias principle, however, more responsive governor action could easily be accommodated by increasing the amount of bias setting. The problem of modernizing enough governors in a large interconnection, where they may run up to 1,000 in number, so that a few will not be carrying a large portion of the regulation, is very difficult, and also the benefits to be derived from such a program are probably quite limited.

Conclusions

1. A centrally located frequency control with automatic tie-line controllers maintaining the proper loads locally, has proved to be a practical method of holding steady the frequency in large interconnections, regardless of size or numbers of interconnections.

2. The tie-line biased-by-frequency principle relieves the regulating burden on both the frequency control and the tie-line regulation stations.

3. Several of the variable-load stations in any group should be sharing the automatic regulation.

4. Adequate governors are necessary, but it appears that the sensitivity and per cent regulation of these governors may vary inversely with the size of the systems and the nature of the load.

Modern Relaying for A-C Secondary Network Systems

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ASSOCIATE AIEE

Synopsis: This paper outlines the application of a new three-phase, single-element induction cylinder network master relay which uses line-to-line connected potential coils. Previous network master relays were three-phase, three-element and employed line-to-neutral connected potential coils.

The new relay with line-to-line connected potential coils produces far greater tripping torque during unbalanced primary feeder faults than the older relays with line-to-neutral connected potential coils. Examples are given to show the advantage of the new relay.

Modern relays have materially improved tripping characteristics which extend the range of highly leading and lagging currents over which the relays will operate selectively at normal balanced three-phase voltages.

The mechanical improvements of the network relays are discussed.

Introduction

DURING the past decade there have been many improvements in relays for a-c secondary network systems. There have been no basic changes in the functions performed by these relays. However, the introduction of a new single-element, three-phase induction cylinder relay with line-to-line connected potential coils and improved electrical and mechanical features, represents a basic change in the design and performance of these relays. The major improvements have been made in tripping performance and mechanical construction.

In the conventional *a-c secondary network system,*[1] figure 1, a secondary grid is fed by a number of network transformers which are supplied from a substation through two or more high-voltage feeders. The function of the network protector, located on the secondary of each transformer, is to disconnect automatically all transformers, on a faulty feeder, from the secondary grid. Hence, the grid will remain energized and continue to supply all loads from the healthy primary feeders. After the faulty feeder has been restored to normal and re-energized, it is the second function of the network protector to automatically reconnect the transformers on that feeder to the secondary grid.

The tripping of the network protectors is performed by a three-phase power directional relay, commonly called the "master relay," and the reclosing is performed by the same three-phase relay in conjunction with a single-phase "phasing relay."

Secondary network systems are now being more generally applied to areas which are relatively *lightly loaded.*[2] Installations in these areas are characterized by wide spacing of transformers, and relatively high secondary grid impedance, and consequently *lower fault voltages and currents* during primary feeder faults than are generally encountered in heavy-density systems which are more closely coupled. Hence, it is more difficult for the master relay to operate to trip the network protector during primary feeder faults in light-density than in heavy-density secondary network systems.

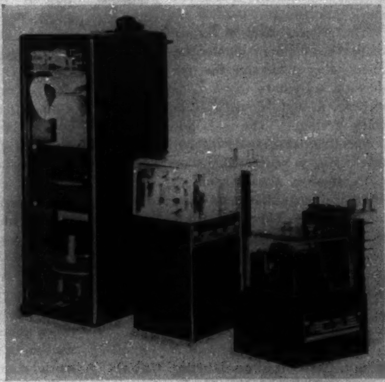

Figure 2. Three types of three-phase network master relays

Left—Three-element induction disk relay

Center—Three-element induction cylinder relay

Right—Single-element induction cylinder relay

New Single-Element Master Relay

The new single-element three-phase master relay, figure 2 (right) overcomes this difficulty first because on three-phase primary feeder faults it produces more torque in proportion to the weight of the moving element than does any other master relay. Hence, there is a greater margin between the total torque and the friction torque which increases the useful torque available for operating the relay.

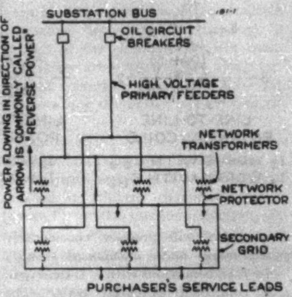
SUBSTATION BUS
OIL CIRCUIT BREAKERS
HIGH VOLTAGE PRIMARY FEEDERS
NETWORK TRANSFORMERS
NETWORK PROTECTOR
SECONDARY GRID
PURCHASER'S SERVICE LEADS
POWER FLOWING IN DIRECTION OF ARROW IS COMMONLY CALLED "REVERSE POWER"

Figure 1. Schematic diagram of a-c secondary network system

Further improvement is obtained by using *line-to-line* (*L-L*) *instead of line-to-neutral* (*L-N*) *potential coil connections.* This parallels transmission line relay practice where most directional relays use *L-L* connected potential coils. A

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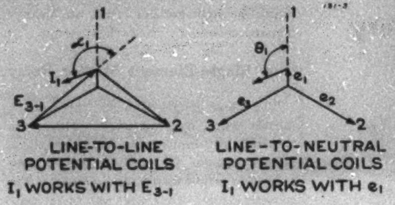
[1] For all numbered references, see list at end of paper.


1
3
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E_{3-1}
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LINE-TO-LINE POTENTIAL COILS
I_1 WORKS WITH E_{3-1}
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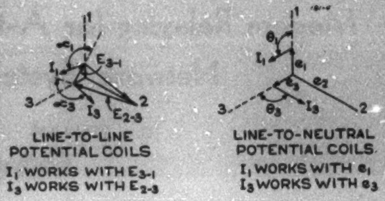
e_1
e_3
e_2
3
2
LINE-TO-NEUTRAL POTENTIAL COILS
I_1 WORKS WITH e_1

Figure 3 (left). Vector diagram of line-to-ground primary feeder fault

Figure 4 (right). Vector diagram of double line-to-ground primary feeder fault

Both wye-connected high-voltage windings in network transformer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LINE-TO-LINE POTENTIAL COILS
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I_3 WORKS WITH E_{2-3}
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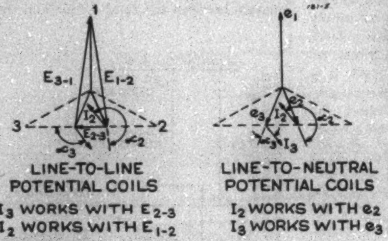
LINE-TO-NEUTRAL POTENTIAL COILS
I_1 WORKS WITH e_1
I_3 WORKS WITH e_3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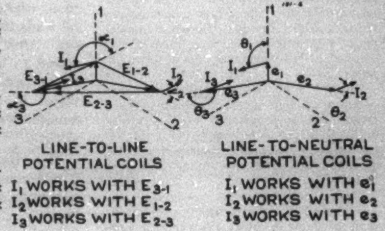
LINE-TO-LINE POTENTIAL COILS
I_3 WORKS WITH E_{2-3}
I_2 WORKS WITH E_{1-2}
e_1
e_3
e_2
I_2
I_3
LINE-TO-NEUTRAL POTENTIAL COILS
I_2 WORKS WITH e_2
I_3 WORKS WITH e_3

Figures 5 and 6. Vector diagrams of line-to-line primary feeder faults

(Left). Wye-connected high-voltage winding in network transformer

(Right). Delta-connected high-voltage winding in network transformer


1
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2
LINE-TO-LINE POTENTIAL COILS
I_1 WORKS WITH E_{3-1}
I_2 WORKS WITH E_{1-2}
I_3 WORKS WITH E_{2-3}
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θ_2
3
2
LINE-TO-NEUTRAL POTENTIAL COILS
I_1 WORKS WITH e_1
I_2 WORKS WITH e_2
I_3 WORKS WITH e_3

given relay will produce considerably *greater torque under unbalanced primary feeder fault* conditions with *L-L* than with *L-N* connected potential coils. The torque produced during three-phase primary feeder faults or under balanced three-phase conditions is the same for a given relay whether *L-L* or *L-N* potential coil connections are used.

The greater ratio of torque to weight of moving element makes the single-element relay attractive for use in heavy-density secondary network systems as well as in light-density secondary network systems. The *L-L* potential coil connections are advantageous for master relays used in heavy-density networks because there is always some reduction in voltage when unbalanced primary feeder faults occur. Therefore, the new single-element master relay, although originally developed for light-density network systems, is now applied to heavy-duty network protectors for heavy-density networks. The following discussion applies to network systems in general.

Relay Performance During Unbalanced Primary Feeder Faults

The advantage of the three-phase single-element relay with *L-L* connected potential coils can be readily seen by referring to figure 3 and table I. In table I comparisons are made at three different voltages. Figure 3 is a vector diagram of the voltages existing at the relay for *a line-to-ground primary feeder fault on a wye-connected transformer*. There is fault current I_1 in only one phase of the system, and hence only one phase of the master relay. With the older relays, with *L-N* connected potential coils, this current I_1 reacts with line-to-neutral voltage e_1 which is very small and the power factor is very low. With the new three-phase single-element relay with *L-L* connected potential coils, the torque is considerably greater because the same fault current I_1 reacts with line-to-line voltage E_{3-1} which is far greater in proportion to normal than e_1. Furthermore, the fault current is closer to the maximum torque position of the *L-L* relay, which further increases its torque.

Other possible unbalanced primary feeder faults are: (1) double line-to-ground, and (2) line-to-line faults. The actual torques for two similar relays with *L-N* and *L-L* connected potential coils are compared in table II.

The vector diagrams in figures 4, 5, and 6 show pictorially the relation between the fault voltages and currents in the two relays employing *L-L* or *L-N* potential connections.

The increased torque of the new relay during unbalanced primary feeder faults provides more positive operation and also increases the speed of operation of the relays, table III.

Hence, it permits more latitude in selection of network-protector fuses without the probability of a fuse blowing on unbalanced primary feeder faults before the master relay trips the network protector.

The greater torque of this relay has proved particularly advantageous where higher reverse power settings are required to prevent "pumping" of the network protector during normal operation. These higher settings may actually prevent a relay with *L-N* potential coils from tripping during certain unbalanced primary feeder faults. The greater torque produced by the single-element relay with *L-L* connected potential coils permits greater reverse power settings without the relay failing to trip on unbalanced primary feeder faults.[3]

Singe-Phase Switching of Primary Feeders

In some installations light-density networks are fed from overhead 4-kv wye-connected primary feeders with *single-phase switching* at the source or single-phase fuses between the source and the network transformers. A single phase-to-ground primary feeder fault may open only the faulty phase at the substation or at the fuse. Then there is reverse power on one phase being fed from the secondary grid through the transformer. This produces a tripping torque on one phase of the master relay. The other two phases may be supplying load to the network grid and this would produce a nontripping torque in the other two phases of the master relay. If the relay is to trip the protector the torque produced by the fault current and voltage in one phase must overcome the torque produced

Table I. Comparison of Relays for Line-to-Ground Fault, Figure 3

Fault Current (I_1)* (Per Cent)	Potential Coil Voltage Which Operates With (I_1)*	Angle Between Fault Current and Maximum Torque Angle of Relay (Degrees)	Per Cent of Normal Torque**		Ratio of Relay Torque L-L to L-N
			L-L Relay	L-N Relay	
200	10% e_1	75		1.7	15.30
200	58% E_{3-1}	47	26		
500	25% e_1	75		11	6.1
500	71% E_{3-1}	55	67		
1,400	70% e_1	75		81	1.70
1,400	85% E_{3-1}	51	139		

*Fault current and potential coil voltage are in per cent of rated current and voltage of associated network transformer.

**Normal torque is the torque produced by the relay with all three phases energized with rated voltage and rated current at the angle of maximum torque.

Table II. Comparison of Master Relays for Various Types of Unbalanced Primary Feeder Faults, Figures 4, 5, and 6

Basis: Fault Voltage Equals 25 Per Cent of Normal. Power Factor of Fault Current 75 Degrees Lagging

Type of Fault	Transformer Primary Connection	Per Cent of Normal Torque		Ratio of L-L to L-N Relay Torques
		L-L Relay	L-N Relay	
Double line-to-ground	Wye	$15.1\frac{I^*}{I_n}$	$4.2\frac{I}{I_n}$	3.6
Line-to-line	Wye	$24.8\frac{I}{I_n}$	$3.7\frac{I}{I_n}$	6.7
Line-to-line	Delta	$21.4\frac{I}{I_n}$	$3.2\frac{I}{I_n}$	6.7

* $\frac{I}{I_n}$ = Ratio fault current to rated current of network relay.

Table III. Comparative Operating Times of Similar Relays on Single Phase-to-Ground Primary Feeder Faults, Figure 3, With L-L and L-N Connected Potential Coils

Fault Voltage 25 Per Cent Normal and Power Factor of Fault Current 75 Degrees

Fault Current In Per Cent of Normal	Operating Time in Seconds	
	L-L Relay	L-N Relay
500	0.16	0.9

by the load current in the other two phases. The increased tripping torque of the single-element relay with *L-L* potential coil connections will be beneficial in these cases. Single-phase switching on primary feeders to secondary networks *is to be avoided wherever possible.*

Relay Connection

The basic differences in *L-L* and *L-N* potential coil connections are shown in figure 7. In this figure the *L-L* connections are shown to involve three potentials for the sake of simplicity. The actual connections of the new three-phase single-element master relay[4] are shown in figure 8. This relay uses two sets of potential coils in open delta.

Improved Tripping Characteristics

In addition to tripping on primary feeder faults as outlined above, network master relays must trip on very low values of reverse power flow (i.e., power flow from the grid to the network) such as transformer-core loss and cable-charging current, to enable clearing a primary feeder by merely opening the station breaker.

Since the relay must function on *very small power reversals* involving very highly leading cable charging current I_c and lagging transformer exciting current I_m, the ideal tripping characteristic would be that of a true wattmeter or curve *a-a'*, figure 9.

Practically, it is not possible to build a relay which has a truly wattmeteric characteristic because of the saturation of the iron used in the magnetic structures of the relay and the current transformers. The effect of saturation is to produce a characteristic like that of *c-c'*, figure 9. Currents through the protector represented by vectors terminating to the left of the characteristic will cause the master relay to trip the protector. Current vectors terminating to the right will not cause the master relay to trip the protector.

Improvements in the *quality of iron* available and refinements of the design of the *magnetic structures* of the relay have resulted in tripping characteristic *b-b'* (figure 9) for the modern induction cylinder single-element and three-element, three-phase master relays as compared with characteristic *c-c'* for early types of induction disk master relays.

The newer induction cylinder master relays with straighter characteristics enable more universal application as they permit selective operation with larger highly leading and lagging currents. The characteristics of earlier induction disk master relays were so bent that it would

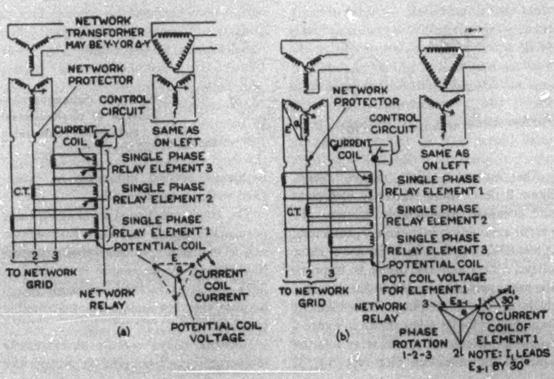
NETWORK TRANSFORMER MAY BE Y-Y OR Δ-Y
NETWORK PROTECTOR
CONTROL CIRCUIT
CURRENT COIL
SAME AS ON LEFT
SINGLE PHASE RELAY ELEMENT 3
SINGLE PHASE RELAY ELEMENT 2
SINGLE PHASE RELAY ELEMENT 1
C.T.
POTENTIAL COIL
1
2
3
TO NETWORK GRID
NETWORK RELAY
E
e
CURRENT COIL CURRENT
POTENTIAL COIL VOLTAGE
(a)
E
e
NETWORK PROTECTOR
CONTROL CIRCUIT
CURRENT COIL
SAME AS ON LEFT
SINGLE PHASE RELAY ELEMENT 1
SINGLE PHASE RELAY ELEMENT 2
SINGLE PHASE RELAY ELEMENT 3
C.T.
POTENTIAL COIL
POT. COIL VOLTAGE FOR ELEMENT 1
1
2
3
TO NETWORK GRID
NETWORK RELAY
3
E_{3-1}
1
I_1
30°
TO CURRENT COIL OF ELEMENT 1
PHASE ROTATION 1-2-3
2
NOTE: I_1 LEADS E_{3-1} BY 30°
(b)

Figure 7. Schematic diagram of network master relay connections

(*a*)—Line-to-neutral connected potential coils

(*b*)—Line-to-line connected potential coils

Table IV. Comparison of Mechanical and Electrical Characteristics of Two Types of Older Three-Phase, Three-Element Master Relays and the New Single-Element, Three-Phase Master Relay

Type of Relay	Watts Input (Per Cent)	Torque Per Watt (Per Cent)	Ratio Torque to Weight of Moving Element (Per Cent)	Volume (Per Cent)
Three-phase, three-element induction disk relay (old)	100	100	100	100
Three-phase, three-element induction cylinder relay (old)	45	320	209	42
Three-phase single-element induction cylinder relay (new)	26	465	231	33

not be to the right of both I_c and I_m. Hence, it was necessary to adjust the characteristics of many of these relays to meet varying conditions and to restrict their application to locations where cable-charging currents were negligible.

Mechanical Improvements

The mechanical improvements of network relays can be seen in figure 2 and table IV where the principal characteristics were given.

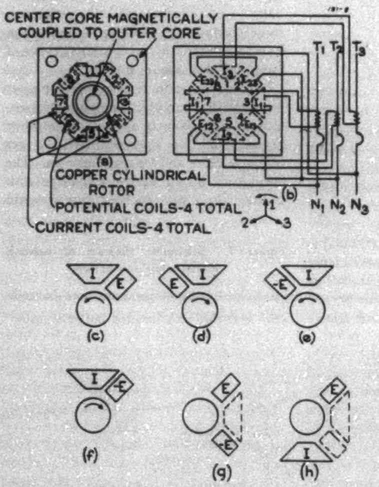

Figure 8. Schematic connections of single-element three-phase master relay

The induction disk master relays, figure 2 (left), were relatively inefficient and occupied considerable volume. To permit a reduction in the size of protectors, it was desirable to reduce the size of these relays. This and the desire to improve the relay performance led to the development of the three-phase three-element induction cylinder relay shown in figure 2 (center). Electrically, this relay was the same as the induction disk type, i.e., it had three independent single-phase elements all on a common shaft and had *L-N* connected potential coils. The new single-element relay is a further improvement of the induction cylinder design in that all three phases are combined in one element with only one rotor.

Network relay adjustments have been simplified in that individual phase adjustments are not required. *Drag magnets* which were a source of maintenance on the older relays have been eliminated. Contacts have been developed that will withstand several thousands of operations without maintenance.

The above-mentioned refinements have been made mainly to *reduce maintenance* and to enable *longer time between inspection periods.* Maintenance is a particularly important factor in network relays because they are mounted in inaccessible locations.

Network relays operate at extremely low torques when tripping on transformer exciting current. Hence, friction plays an important part in the performance of these relays. The single-element master relay has a much lighter moving element; hence, the *friction force is lower*, and consequently operation is more positive. Induction cylinder relays have balanced magnetic circuits and are inherently free from vertical vibration. Hence the major source of bearing wear has been eliminated.

Modern network relays operate at a *lower temperature rise*, i.e., about 40 degrees centigrade rise over ambient around relay, to allow for higher ambients encountered inside higher capacity submersible network protectors now being built, without unduly shortening the life of the relay coils.

Summary

A new three-phase, single-element induction cylinder relay has been introduced which, although functionally the same as previous relays in so far as network-protector control is concerned, has:

1. Line-to-line connected potential coils instead of line-to-neutral connected potential coils which provides a far greater tripping torque during unbalanced primary feeder faults.

2. This is particularly advantageous in light-density network systems where fault currents and voltages are lower than in heavy-density systems which are more closely coupled. The line-to-line connected potential coils offer advantages in heavy-density systems too.

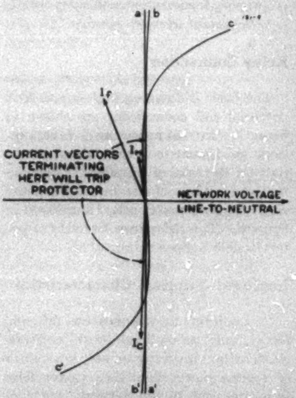

Figure 9. Network master relay tripping characteristics

I_c—Cable charging current
I_f—Short-circuit current
I_M—Transformer exciting current

Five Years' Experience With Ultrahigh-Speed Reclosing of High-Voltage Transmission Lines

PHILIP SPORN
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MEMBER AIEE

Synopsis: The data covering five years' operating experience with 33 ultrahigh-speed reclosing breaker installations mainly on a large interconnected and integrated high-tension system are presented and analyzed, and conclusions based on this experience are drawn. Out of 71 cases of flashover cited, 65 reclosures were successful and 6 cases of reclosure resulted in restrikes, a record of 91.5 per cent successful reclosures. The conclusions drawn are that not only is actual performance better than appeared possible when the development was first made but that further improvements are possible by further reducing breaker time.

I. Introduction

THE basic principles of ultrahigh-speed reclosing of high-voltage transmission lines have been previously presented[1] in a paper before the Institute. Likewise, operating experience with the original high-speed reclosing equipment first developed and with a modification of that equipment were presented[2] two years ago. However, the data available at that time covered only three sections (six terminals) of high-voltage transmission lines and a period of only three years, in one section of which the equipment was in service for only one year and another the equipment was in service for only a portion of one year, so that operating experience with only 15 cases of reclosure could be cited and discussed. Since then, however, the application of ultrarapid reclosure has not only been widely extended but a great many more operations have been experienced. These now constitute a set of data that is extensive enough to be able to furnish a basis for some well-founded conclusions. It is the purpose of this paper to present and analyze these data and draw such conclusions therefrom as are warranted and pertinent.

II. System on Which Installed

The system on which all terminal equipments with the exception of four (representing two line sections) have been installed is the central system of the American Gas and Electric Company. This is shown diagrammatically in figure 2. This system is an integrated system operating in seven states and comprising altogether 1,455 miles of 132-kv transmission, substantially all of which is of double-circuit steel-tower construction with a total, as of the date of writing, of 2,483 miles of circuit. It serves a load whose last-recorded maximum one-hour integrated peak was 997,000 kw. On this system the power supply, that is, power generation and power transmission to load centers, is integrated and co-ordinated to the highest degree. The system, which is a predominantly steam-electric system, nevertheless has a substantial portion of hydro capacity. Besides this, extensive and important foreign interconnections exist on several portions of the system through which diversity and other exchanges are carried out with adjacent systems. This, coupled with the diversities in load and, to some extent, of time over the system, makes imperative frequent and in many cases sudden changes in power flow over many of the important transmission lines on the system. Besides that, the availability of economical supplies of hydro in portions of the system and the concentration of generation by steam-electric stations at the points where combined optimum fuel and water conditions prevail, make economically desirable and necessary the transfer of heavy flows of power, at one time or another, over most of the transmission lines on the system. Under these conditions, the importance of continuity of supply, if the entire system is to operate in accordance with predetermined principles of economy, is apparent. Hence, the first and so far the widest application for ultrahigh-speed reclosing has been found on that system.

The four remaining terminals not installed on the central system are on the double-circuit Deepwater-Atlantic City lines of the South Jersey system. These two circuits are installed on 132,000-volt towers and insulated for that voltage but are operating at present at a voltage of 66 kv.

The first installation using the experimental equipment was on the Fort Wayne-Deer Creek circuit in Indiana. As will be noted from the diagram, since then 27 additional terminals (13 line sections) on the system have been installed covering a total circuit mileage of 655 miles. These have been extended progressively as the need for them arose and as the operating advantages or necessities were indicated.

III. Operating Experience

A summary of the total operating experience since the first installation is given in table I. Referring to that table it will be seen that taking the two systems which the record covers, a total of 33 terminals involving 16 line sections have been covered, and that these 16 line sections cover approximately 840 circuit miles of line, all but 126 of which operate at 132,000 volts. In the period May 1936 to November 1940, covering substantially five lightning years, 72 cases

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[1] For all numbered references, see list at end of paper.

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3. The tripping characteristics have been improved to permit operation at normal voltage over a wider range of highly lagging and highly leading currents.

4. There have been many mechanical improvements in network relays which have improved their performance and reduced maintenance.

References

1. LOW VOLTAGE A-C NETWORKS, D. K. Blake. *General Electric Review*, February to November 1928, inclusive.

WHEN TO START A NETWORK, D. K. Blake. *Electrical World*, volume 95, February 15, 1930, page 347.

HOW TO START A NETWORK, D. K. Blake. *Electrical World*, volume 96, July 19, 1930, page 115.

2. WHERE TO USE LIGHT-DENSITY NETWORKS, D. L. Beeman. *Distribution*, July 1939, pages 10–11.

3. LIMITATIONS ON SETTINGS OF NETWORK RELAYS, D. L. Beeman. *Distribution*, January 1940, page 13.

4. A SINGLE-ELEMENT POLYPHASE DIRECTIONAL RELAY, A. J. McConnell. AIEE TRANSACTIONS, volume 56, 1937 (January section), pages 77–80 and 113.

of flashover resulting in line opening occurred. Of these, 63 cases were single-line trouble and 9 cases were two-line trouble. Out of the same number of 72 cases of flashover, 65 reclosures were successful and 7 cases of reclosure resulted in restrikes. This is a record of 9.7 per cent unsuccessful reclosures; but in this connection it needs to be pointed out that in one of these cases permanent physical damage had previously resulted to a tower and that under such conditions obviously successful reclosure was a physical impossibility since nothing but a permanent outage could clear the trouble. Disregarding, therefore, this one case, the total unsuccessful reclosures were six, representing a percentage slightly over eight per cent. An interesting thing in this connection, too, is that in not a single case did the equipment fail mechanically, that is, the mechanical performance was in every case perfect and reclosure was carried through and was successful in every case where the circuit condition permitted re-energization of the line. Table I shows clearly, too, the rapid accumulation of experience with the year 1940. It will be noted that the percentage of restrikes or unsuccessful reclosures for 1940 was again slightly over eight per cent, the same figure as for the total five-year period.

The table further shows the distribution of faults between phase to ground, phase to phase and ground, and three phase. Of the 71 cases of flashover, 55 cases were phase-to-ground faults on one line with 53 successful reclosures and only two restrikes which were probably due to multiple lightning stroke as explained in previous paper.[3] Three cases of phase-to-phase and ground faults on one line occurred, all of which reclosed successfully. Five cases of three-phase faults on one line occurred with three successful reclosures and two restrikes. Three cases of phase-to-ground faults on two lines occurred, all of which reclosed successfully. Four cases of phase-to-phase and ground faults on two lines occurred with three successful reclosures and one restrike. One case of three-phase fault on two lines occurred which restruck.

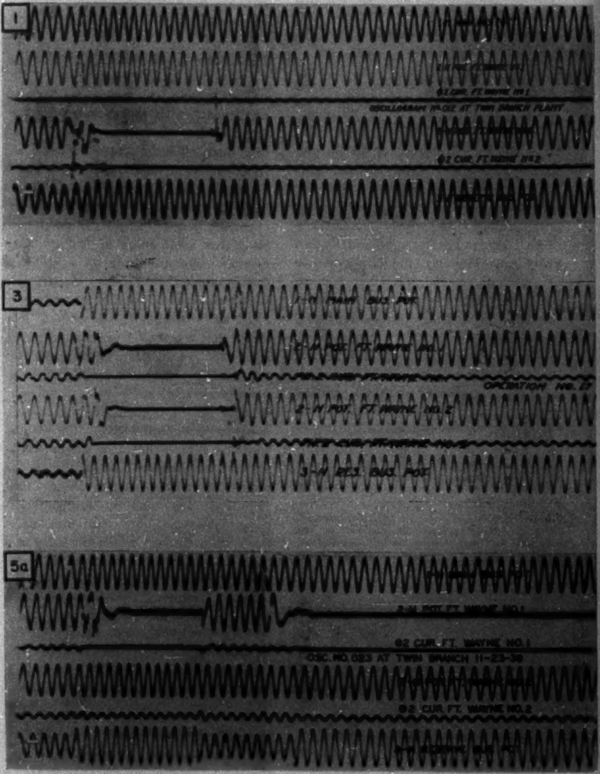
IV. Some Typical Oscillograms and Their Significance

In figure 1 are shown five sets of oscillographic records taken at Twin Branch station showing system performance for five typical cases of trouble on the system. The top and bottom traces on all oscillograms represent phase 1-to-neutral and phase 3-to-neutral voltages respectively on 132-kv bus at Twin Branch station. The second and fourth traces from the top indicate phase 2-to-neutral voltages of number 1 and number 2 Twin Branch-Fort Wayne lines respectively taken on the line side of the breakers at Twin Branch station. The third and fifth traces from the top represent phase 2 line currents on number 1 and number 2 Twin Branch-Fort Wayne lines respectively.

Oscillogram number 1 is a record of a lightning flashover on number 2 Twin Branch-Fort Wayne line involving phase 3 to ground that occurred at 9:34 p.m. on July 22, 1939. The fault was cleared in approximately $6^1/_2$ cycles and after a line de-energization of 12 cycles, reclosure was entirely successful.

Oscillogram number 2 is a record of a lightning flashover on number 1 Twin Branch-Fort Wayne line involving phase 1 to ground that occurred at 2:21 p.m. on June 18, 1940. The fault was cleared in approximately eight cycles and after a line de-energization of seven cycles, the line was re-energized from one end and three cycles later the breaker at the other end reclosed. The difference in reclosing times of the two breakers was due to the experimental adjustment of breaker-contact operating speed which existed at this time. With this adjustment the breaker contacts reached a point five-eighths inch from contact "make" position after 15 cycles and hesitated at this point, completing the stroke at 18 cycles. It is believed that when the five-eighths inch point was reached, the breaker at one end of the line arced across its contacts, thus energizing the line. As soon as the line was energized, most of the potential

Figure 1. Oscillograms of actual operation on the 132-kv transmission system of the central system

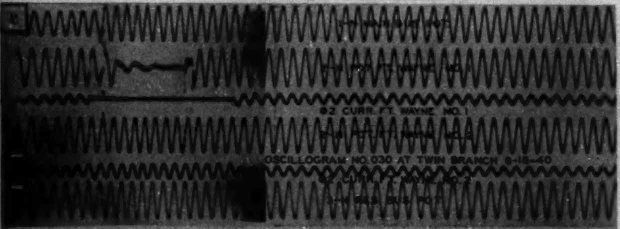


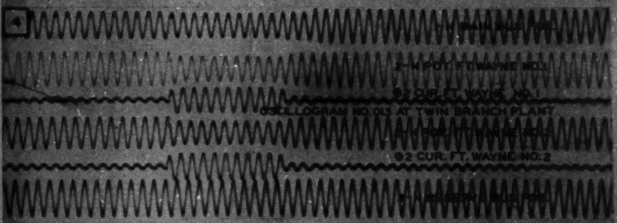
across the contacts of the breaker at the other end disappeared and the circuit was therefore not closed until electrical contact was finally made by the second breaker at 18 cycles.

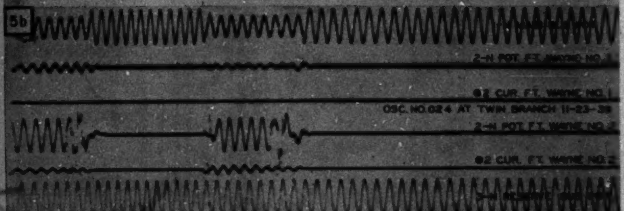
Oscillogram number 3 is a record of lightning flashover occurring simultaneously on number 1 and number 2 Twin Branch-Fort Wayne lines involving phases 1 and 3 to ground that occurred at 12:50 a.m. on July 8, 1939. Both lines were opened at both ends in approximately $6^1/_2$ cycles after the inception of the fault and after both lines had been de-energized for $11^1/_2$ cycles, reclosure of all four breakers on both lines was entirely successful.

Oscillogram number 4 is a record of lightning flashover occurring simultaneously on Fort Wayne-Deer Creek and Fort Wayne-Delaware lines involving phases 1 and 2 to ground that occurred at 12:13 a.m. on July 23, 1939. At this time the Fort Wayne-Deer Creek line only was equipped with ultrahigh-speed reclosing equipment, which line reclosed successfully. The Fort Wayne-Delaware line was only protected with instantaneous overcurrent relays in conjunction with time-delay overcurrent relays. Under these conditions, it is to be noted that both lines opened up at both ends $10^1/_2$ cycles after the inception of the fault.

Oscillogram number 5 is a record taken when mechanical damage occurred to a steel tower on number 1 and number 2 Twin Branch-Fort Wayne lines at 9:54 p.m. on November 23, 1939. From the oscillogram it will be noted that a fault involving phase 3 to ground occurred first on number 1 line, which opened at both ends $7^1/_2$ cycles after the inception of the fault and reclosed after de-energization of the line for ten cycles and reopened due to permanent trouble on the line section. Approximately 30 cycles later number 2 line became involved in the trouble with phase 1 going to ground, opening both ends after $6^1/_2$ cycles and reclosing after line had been de-energized 12 cycles when it retripped on account of permanent trouble on the line section.







V. Additional Installations

The accumulated operating experience has been so beneficial and the results obtained so much greater than even the authors originally anticipated that as the continuity problem and the problem of reliability of supply had to be met on other sections of the system, ultrarapid reclosure was extended and is still being extended today. Thus at the present time there are 12 additional terminals in process of installation covering additional 6 line sections with a total 224.2 miles of 132-kv circuit. These are clearly indicated in the diagram figure 2. Other points on the central system, and on the other two systems of the American Gas and Electric Company where the same general problem is being studied, are being seriously considered for equipment with ultrarapid reclosure on the terminal breakers and there is no doubt in the authors' minds that such additional applications will be found desirable and/or necessary. Particularly is attention being given to medium-voltage lines in the subtransmission or distribution class such as those of 23,000-volt and 33,000-volt rating. Here, however, the problem that still has to be solved is the one of providing an economical reclosing mechanism and a relay arrangement that will combine the basic elements of one cycle differential operation without the expense involved in the present system. While neither of these two problems has been entirely worked out the present indications are that both can and will be.

VI. Conclusions

Based upon the five years' experience studied and discussed here the authors believe that the following conclusions are clearly indicated:

1. In general on a high-voltage system properly insulated and provided with ground-wire protection at least 90 per cent successful reclosure can be expected by the use of ultrarapid reclosing. In the case of the two systems cited and discussed, out of 72 operations 71 were apparently due to lightning and of these 65 reclosures were successful. This is a record of better than 91 per cent successful operation and compares with the previously indicated figure of not less than 75 per cent successful operation.

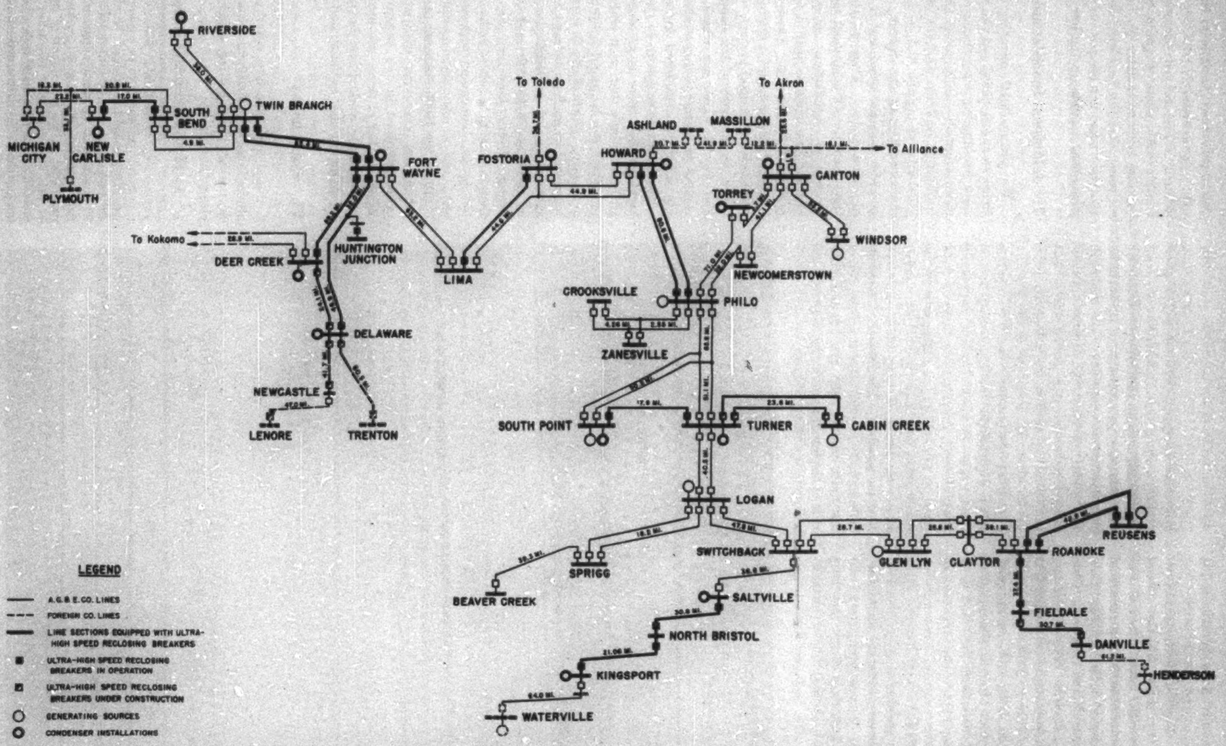

RIVERSIDE
TWIN BRANCH
SOUTH BEND
MICHIGAN CITY
NEW CARLISLE
PLYMOUTH
19.3 MI.
20.9 MI.
23.2 MI.
17.0 MI.
4.9 MI.
FORT WAYNE
To Kokomo
28.9 MI.
DEER CREEK
HUNTINGTON JUNCTION
DELAWARE
NEWCASTLE
47.0 MI.
LENORE
TRENTON
LIMA
To Toledo
FOSTORIA
44.9 MI.
HOWARD
ASHLAND
MASSILLON
20.7 MI.
41.9 MI.
19.2 MI.
To Akron
18.1 MI.
To Alliance
CANTON
TORREY
WINDSOR
NEWCOMERSTOWN
PHILO
CROOKSVILLE
4.26 MI.
2.55 MI.
ZANESVILLE
SOUTH POINT
17.9 MI.
TURNER
23.6 MI.
CABIN CREEK
40.5 MI.
LOGAN
18.2 MI.
47.8 MI.
39.3 MI.
SPRIGG
BEAVER CREEK
SWITCHBACK
36.6 MI.
SALTVILLE
30.9 MI.
NORTH BRISTOL
21.06 MI.
KINGSPORT
94.0 MI.
WATERVILLE
28.7 MI.
GLEN LYN
28.8 MI.
CLAYTOR
38.1 MI.
ROANOKE
42.9 MI.
REUSENS
FIELDALE
30.7 MI.
DANVILLE
61.2 MI.
HENDERSON
LEGEND
A.G.& E.CO. LINES
FOREIGN CO. LINES
LINE SECTIONS EQUIPPED WITH ULTRA-HIGH SPEED RECLOSING BREAKERS
ULTRA-HIGH SPEED RECLOSING BREAKERS IN OPERATION
ULTRA-HIGH SPEED RECLOSING BREAKERS UNDER CONSTRUCTION
GENERATING SOURCES
CONDENSER INSTALLATIONS

Figure 2. Ultrahigh-speed reclosing oil circuit breaker installations on the 132-kv transmission system of the central system of the American Gas and Electric Company

Table I. Summary of Actual Operations of 132-Kv Ultrahigh-Speed Reclosing Oil Circuit Breakers, May 1936 to November 1940

Line	Line Voltage (Kv)	Length of Line (Circuit Miles)	Number of Terminals Involved	Date of Installation	Ultrahigh-Speed Oil Circuit Breaker Operations																		Types of Faults																	
					Number of Operations						Number of Successful Reclosures						Number of Restrikes						Phase to Ground						Phase to Phase to Ground						Three Phase					
					1936	1937	1938	1939	1940	Total	1936	1937	1938	1939	1940	Total	1936	1937	1938	1939	1940	Total	1936	1937	1938	1939	1940	Total	1936	1937	1938	1939	1940	Total	1936	1937	1938	1939	1940	Total
Fort Wayne-Deer Creek	132	50.2	2	5/17/36	4	5	3	1	3	16	3	4	3	1	3	14	1	1				2	4	5	3	1	3	16												
South Bend-New Carlisle	132	17.0	2	1/29/38			1	1		2			1	1		2									1	1		2												
No. 1 Twin Branch-Fort Wayne	132	65.4	2	8/31/38				1	2	3				1	2	3										1	2	3												
No. 2 Twin Branch-Fort Wayne	132	65.4	2	7/25/38			2	3	1	6			2	3	1	6									2	3	1	6												
Nos. 1 and 2 Twin Branch-Fort Wayne	132		4	8/31/38				2		2				1		1				1		1										1		1				1		1
Fort Wayne-Delaware	132	77.9	3	5/26/40					1	1					1	1											1	1												
Saltville-North Bristol	132	30.8	2	3/22/40					6	6					6	6											3	3					1	1					2	2
North Bristol-Kingsport	132	21.1	2	3/20/40					6	6					4	4					2	2					3	3											3	3
Roanoke-Fieldale	132	37.35	2	4/28/40					12	12					12	12											10	10					2	2						
No. 1 Roanoke-Reusens	132	42.89	2	5/ 1/40																																				
No. 2 Roanoke-Reusens	132	42.89	2	5/ 1/40					3	3					3	3											3	3												
Nos. 1 and 2 Roanoke-Reusens	132		4	5/ 1/40					6	6					4	4					2	2					3	3					3	3						
No. 1 Philo-Howard	132	80.6	2	5/ 2/40					3	3					3	3											3	3												
No. 2 Philo-Howard	132	80.6	2	5/20/40					3	3					3	3											3	3												
No. 1 and No. 2 Philo-Howard	132		4	5/20/40																																				
Fostoria-Lima	132	44.6	2	10/3/40																																				
No. 1 Deepwater-Atlantic City	66	63.1	2	6/27/40																																				
No. 2 Deepwater-Atlantic City	66	63.1	2	6/27/40					2	2					2	2											2	2												
Nos. 1 and No. 2 Deepwater-Atlantic City	66		4	6/27/40					1	1					1	1																	1	1						
Turner-South Point	132	47.94	2	8/ 1/40																																				
Total		839.87	33		4	5	6	8	49	72	3	4	6	7	45	65	1	1	..	1	4	7	4	5	6	6	37	58	..	..	..	1	7	8	..	..	..	1	5	6

An Improved Polyphase Directional Relay

BERT V. HOARD
MEMBER AIEE

THE inductor-loop principle of operation has been employed successfully for many years in building single-phase directional relays of the high-speed type. This principle of operation has now been extended to obtain a high-speed polyphase directional relay of a radically different type of construction than has been employed heretofore, which has such features as simplicity, compactness, low burden, high sensitivity, and nonbounce contacts.

The trend toward high-speed operation for all types of protective-relay applications has developed a need for such a relay. This is particularly true in directional-comparison schemes of pilot relaying. Other applications include the use of a polyphase directional element with instantaneous overcurrent or impedance relays and for these applications high speed in the directional element is needed for proper co-ordination.

The principle of operation of the inductor-loop element is that of a conductor carrying current in a magnetic field. The inherent efficiency and simplicity of the magnetic circuit combined with low inertia of the inductor loop makes possible a directional-relay design having a high torque-to-weight ratio which is essential if sensitive, high-speed operation is to be obtained.

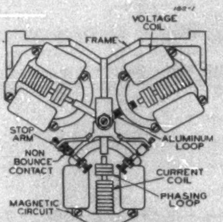


Figure 1. Diagram of electromagnets, loops, and contacts

The arrangement of the components of such a relay element designed for three-phase operation is shown schematically in figure 1. It consists of three electromagnets and three loops around a vertical shaft. Each loop is mounted on the shaft with its outer side located in an air gap within the electromagnet. The voltage coils, one on each of the outer two legs of each electromagnet, have a voltage applied which induces a large current in the loop by transformer action. Current flowing through the current coil, which is located on the center leg of the electromagnet, produces an air-gap flux which interacts with the loop current in the outer side of the loop and causes rotation of the shaft in a direction corresponding to the flow of alternating current power. If the direction of current flow reverses, the torque and the direction of rotation of the shaft reverse. A photograph of the relay is shown in figure 2.

The phase-angle characteristic of the inductor-loop electromagnet is essentially that of a watt element, although maximum torque is obtained when the applied current is slightly lagging the voltage. In the voltage circuit the loop can be considered as the secondary load of a transformer which has a high resistance compared to its reactance, so that the loop current lags the primary voltage by an angle of about 15 degrees. In the current circuit the air-gap flux is in phase with the applied current. When loop-current and air-gap flux are in phase, maximum torque is obtained, hence this condition is fulfilled when the applied current lags the voltage by 15 degrees.

One loop and its associated electromagnet make up one phase of the three-phase relay, and a delta voltage and a star current are applied. For actual installations, the 90-degree connection is normally used for polyphase directional relays because during phase-to-phase faults more sensitive operation will be obtained, since the two elements carrying fault current have the two unfaulted

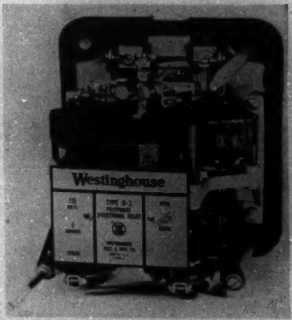


Figure 2. Photograph of H-3 relay with glass cover removed

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2. The probability of restrike on double-circuit lines of the type existing on the central system in case of double-circuit flashovers is approximately four times as great as in the case of single-circuit flashovers. Thus, in the case of the two systems described, out of eight cases of two-line trouble (omitting the one due to permanent disability) two restrikes occurred or a restrike record of 25 per cent. On the balance of 63 cases of one-line flashover, four restrikes occurred or a restrike record of 6.5 per cent.

3. Ultrarapid reclosure has definitely established itself as an indispensable tool in high-tension integrated power systems. A definite start has been made in the direction of extending it to lower voltage lines. Further development here is needed and will depend upon the successful removal of the economic limitation to the extension of this application to such lower voltage and, therefore, lesser capacity lines.

4. There is reason to believe that the present 8.5 per cent unsuccessful operation can be reduced and particularly that the percentage of 25 per cent failure to reclose successfully on two-line faults can be reduced. The means for carrying this out so far indicated are as follows:

(*a*). Reducing the total percentage of two-line flashovers by further increasing the speed of clearing of the fault.

(*b*). Reducing the percentage of restrikes by speeding up the clearing time and thus permitting a corresponding increased waiting time before reclosure is attempted.

References

1. ULTRA-HIGH SPEED RECLOSING OF HIGH VOLTAGE TRANSMISSION LINES, Philip Sporn and D. C. Prince. AIEE TRANSACTIONS, volume 56, 1937, pages 81–90.

2. EXPERIENCE WITH ULTRA-HIGH SPEED RECLOSING OF HIGH VOLTAGE TRANSMISSION LINES, Philip Sporn and C. A. Muller. AIEE TRANSACTIONS, volume 58, 1939, pages 625–31.

delta voltages applied to them, respectively. Using this connection, one of the electromagnets will have line current from phase *a* and the delta voltage opposite phase *a* applied. Thus, for unity power factor in the system, the current through the relay electromagnet is leading the applied voltage by 90 degrees, and for a fault which causes the system current to lag by 45 degrees the relay current leads the applied voltage by 45 degrees.

For the 90-degree connection the relay phase-angle characteristic should, therefore, be shifted so that maximum torque is obtained when the relay current leads the relay voltage by 45 degrees. A series resistance-phase shifter, similar to that commonly used in the induction-disk type relay cannot be used for the inductor-loop relay, because the latter has a voltage circuit which is mostly resistance. However, a series capacitor of the proper size will produce the 45-degree phase-angle characteristic as shown in the diagram of figure 3*a*.

A simpler phase shifter and one that is better from an operating and maintenance standpoint is the use of a phasing loop on the magnetic circuit in front of the current coil as shown in figure 1. The phasing loop consists of three heavy rectangular-shaped copper washers and causes the air-gap flux to lag behind the applied relay current, so that the current must be advanced in phase position to bring the loop-current and air-gap flux back in phase and obtain maximum torque. This type of phase shifter is used on the relay to obtain a 45-degree characteristic as shown in figure 3*b*. To obtain the same magnitude of air-gap flux and relay sensitivity, a larger number of turns are used on the current coil when phasing loops are used. Even with the increased number of turns required, the burden of the current circuit is very low as shown subsequently.

The instantaneous torque in any one electromagnet of the relay is produced by the instantaneous products of voltage (loop current) and current (air-gap flux) in the electromagnet, which is a double-frequency pulsating torque. Figures 4*a* and 4*b* show this torque respectively for phase angles when the average torque is a maximum and when it is zero. The torques in the other two electromagnets are also pulsating as shown, respectively, in figures 4*c* and 4*d*; however, they are displaced 120 degrees in time phase from each other and when all three instantaneous values are added up, the result is a uniform nonpulsating torque. Hence, under normal conditions with the three applied currents and voltage balanced there is no tendency for these torques to cause vibration. During unbalanced faults, in general, there will be a combination of a balanced torque and a pulsating torque on the moving element and the pulsating torque will tend to produce some vibration. However, this vibration has been minimized through proper moving element and contact design which keeps the contacts positively closed when the unbalanced torque is present.

The contact travel for any type of high-speed relay is necessarily small as one means of obtaining high speed. For this relay, due to its extremely high ratio of torque to inertia, the normal travel of 0.030 inch is greater than has been customarily used in the past for similar relays. Nonbounce contacts are used, which consist of a small enclosed cylinder partially filled with tungsten powder mounted on the end of a flexible spring. When the cylinder strikes the rigid stationary contacts, the particles of tungsten slide over each other and absorb the energy of impact. After the contact spring has been slightly deflected, the adjusting screw in the stop arm (figure 1) strikes the moving contact cylinder and then the full torque of the relay is transmitted to the moving and fixed contacts. Hence, the contacts act as the moving-element stop and full contact pressure is assured at all times. Connection is made to the moving contact by means of a very flexible copper-conducting strip.

Among the advantages obtained because of the loop type of construction is that large air gaps can be used between iron and loop which makes the element easy to inspect and eliminates the possibility of air-gap friction from foreign materials lodging in the gap. The efficiency of the design is such that very small burdens per phase are imposed. The voltage circuit requires 4.0 volt-amperes per phase at 115 volts and the cur-

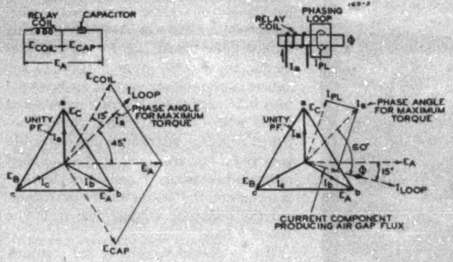

Figure 3. Vector diagram for phase (a) electromagnet using 90-degree connection

(*a*)—Using capacitor phase shifter in potential circuit to obtain maximum torque in relay when system-fault current lags by 45 degrees

(*b*)—Using phasing-loop phase shifter in current circuit, to obtain maximum torque in relay when fault current lags 45 degrees

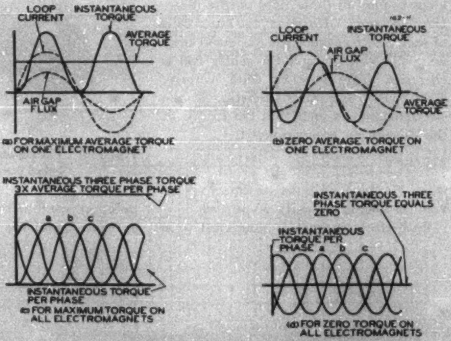

Figure 4. Torques produced in polyphase directional relay

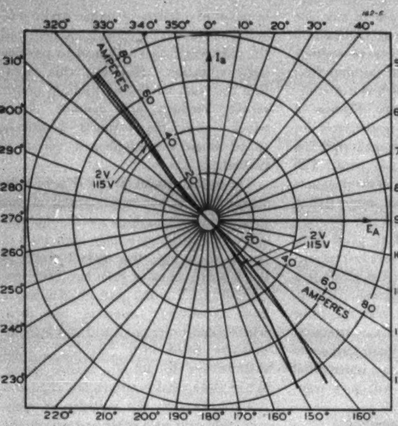

Figure 5. Phase-angle curves at 115 and 2 volts

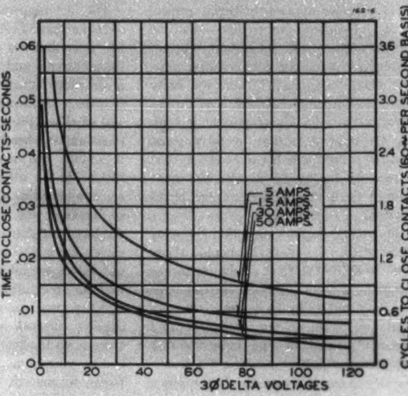

Figure 6. Timing curves of type H-3 directional relay

rent circuit, which also includes the phasing loops, requires 1.6 volt-amperes per phase at five amperes. The sensitivity of the relay is excellent, being less than one volt over a current range of 2.5 amperes to 100 amperes at the maximum torque phase angle. From the phase-angle curves shown in figure 5 it will be seen that the relay has adequate directional characteristics even at a very low voltage and high current.

Timing curves for the relay taken with a 0.030-inch contact gap are shown in figure 6. It will be seen that at 65 volts and above 15 amperes the timing is less than 0.6 cycle on a 60-cycle basis.

The directional relay is also made with a voltage-restraint element as shown in figure 7. The restraint element is mounted below the directional element and consists of three electromagnets and loops similar to those used on the directional element. However, the outer coils of each electromagnet are energized from one delta voltage, and the center coil is energized from another delta voltage. The phase-angle characteristic of the electromagnet is such that the torque of the element is proportional to:

$$E_1E_2 \sin \theta$$

where θ is the angle between the two delta voltages and is normally 60 degrees.

Each electromagnet is connected to a different combination of delta voltages so that a uniform restraint torque and balanced burdens are obtained. During a three-phase fault the voltage-restraint torque is reduced in proportion to the square of the voltage, so that for low voltages it is negligible. During unbalanced faults when any one delta voltage becomes zero, the restraint is automatically removed, because one of the applied voltages becomes zero on two of the electromagnets, and on the third electromagnet the collapsed voltage has made the sine of θ zero.

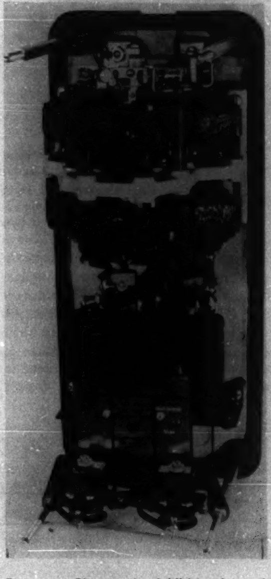

Figure 7. Photograph of HV-3 relay with glass cover removed

If it is desired to remove the restraint during a fault, it can easily be accomplished by opening any one supply lead to the restraint element. This applies single-phase voltage to all three electromagnets and the sine of θ becomes zero on all, so that there is no torque on any of them. The restraint is such that it requires ten amperes at normal voltage at the maximum torque phase angle of the directional relay to overcome the restraint torque.

In conclusion, the new polyphase inductor-loop directional relay has all the desirable characteristics inherent in the induction-disk type relay, with the additional ones of compactness, low burden, high sensitivity, and high-speed operation, which is difficult to obtain with the polyphase induction-disk type. With these outstanding advantages, it is believed that the new relay will replace the polyphase induction-disk type where formerly used and will be considered as an important advancement in relaying art.

Experience With Preventive Lightning Protection on Transmission Lines

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MEMBER AIEE

Synopsis: Footing resistances for transmission line towers must be low for good operation, but it appears that no benefit results from reducing them below a certain limiting value characteristic of a given construction. This is because flashover can apparently be caused not only by excessive footing resistance drop, but also by the reactive drop accompanying rapid rates of current rise when tower currents are large.

The records seem to show that overhead ground wires provide better shielding than has been believed. Overhead ground wires and tower footing resistances co-ordinated with line insulation provide preventive protection which is highly efficient.

Operating experience indicates that lightning outages can be predicted fairly well from simple probability calculations.

I. Introduction

THE art of protecting transmission lines against lightning has advanced considerably in the last decade, but there are some phases of the problem which are still obscure. In analyzing the experience of the Pennsylvania Water and Power Company, which operates steel tower transmission lines at 66 to 220 kv, certain trends have appeared which are believed to be significant. It is the purpose of this paper to bring these findings to the attention of those interested in lightning protection in order that the apparent trends may be widely investigated and their validity established or disproved. The interpretation of results as given here, therefore, in no sense should be considered as final, but only as bases for further study.

Preventive lightning protection for transmission lines, as the term implies, is designed to prevent flashovers entirely as contrasted to remedial measures which are designed to minimize the effects of lightning. In this paper, preventive lightning protection is used in a special sense as the designation for the mounting of overhead ground wires on the same steel towers as the power conductors and the co-ordination of tower footing resistances with line insulation. Tower footing resistances on the lines referred to here have been almost exclusively reduced by the installation of buried counterpoise conductors of various lengths and number.

II. Extent of Experience

The Pennsylvania Water and Power Company first began to try preventive lightning protection on its transmission lines in 1931 with the construction of the Safe Harbor-Westport 220-kv line.[1] Since then two new lines have been constructed and five old ones improved using the same basic principles, but with variation in details.

In 1934, surge-crest ammeter links[2] were installed on the Safe Harbor-Westport-Takoma 220-kv line and since then their use has been gradually expanded until in 1939, 2,279 transmission towers on 318 miles of line have been equipped. These devices have permitted a relatively close check to be made of the efficiency of the lightning protection.

Surge-crest ammeter links have also been placed on 21 banks of 66-kv lightning arresters, two banks of 132-kv arresters, and one bank of 220-kv arresters.

III. Distribution of Current in Tower Members

For several years, there has been a total of 87 towers equipped with a pair of surge-crest ammeter links on each of the four corner or leg members of each tower. Since 1934, 209 sets of records have been obtained from these installations, of which 98 records are from towers believed to have been struck by lightning or adjacent to struck spans. The purpose of these observations has been to determine the error entering into the assumption that the tower current is four times the current in one corner leg.

The results of this study are given in table I. The sum of the four indications in the corner legs has been taken as the approximately correct tower current. The deviation from it of four times each of the four individual leg currents has been obtained. The deviations for each group of tower records in 5,000-ampere increments of tower current have been averaged in table I.

Although relatively few readings have been obtained when the tower currents were of the larger magnitudes, from table I it appears that the indications obtained on towers having a bracket on only one leg can be assumed to be fairly good approximations when the currents are in

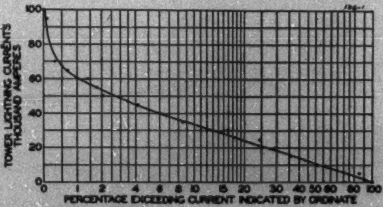

Figure 1. Distribution of 1,201 currents measured in towers believed struck by lightning (1934–39)

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1. For all numbered references, see list at end of paper.

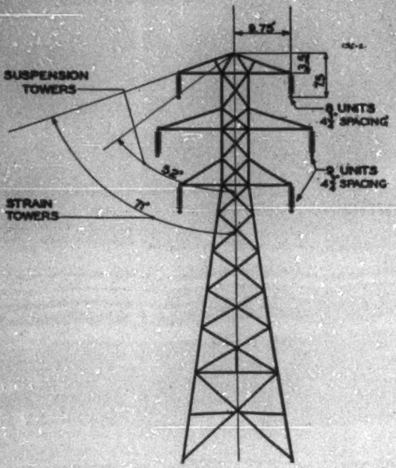

Figure 2. Minimum shielding zones required of single overhead ground wire on suspension and strain towers of the Holtwood–Coatesville 66-kv line

the upper range, that is, greater than about 40,000 amperes.

In 45, or 22 per cent of the 209 sets of observations, the magnetic links on all legs of a tower did not have the same polarity of magnetization. It is believed that records of this type are due to successive strokes of opposite polarity, occurring either close together or with an appreciable elapsed time between them. As they have been found only in the lower current ranges, it is not likely that the large deviations in indication which they cause greatly affect the analyses given here.

It has been suggested that tower currents, as taken from records obtained in the four corner leg members of a tower, be multiplied by a factor ranging from about 1.3 to 2 to account for the current passing through the cross braces of the tower.[3] Because of the uncertainty of the value of the factor,[4] and the unequal distribution of lightning current in tower legs as shown in table I, none has been used in the data reported here.

IV. Distribution of Tower Currents of Various Magnitudes

In the period of 1934–39, a total of 1,201 tower currents has been recorded where it is believed that lightning struck. The tower current in each case was recorded as four times the current indicated in one of the four tower legs, or as the sum of the four indicated leg currents, depending upon the number of SCA links installed at the respective tower. The data have been plotted in figure 1.

V. Distribution of Lightning Strokes Along Lines

The records show that the 1,201 strokes which have terminated on the lines of the system have been well distributed. For instance, on the Safe Harbor-Perryville 132-kv line there were not over three consecutive towers which were not struck during the five-year period of the study. On the Safe Harbor-Westport-Takoma 220-kv line where the stroke records have been kept for six years, five is the maximum number of consecutive towers where no strokes have occurred. Although a considerable length of a line may not be struck during the first observation year, in general, it has been found that during succeeding years the maximum length of unstruck line is reduced to a few spans.

VI. Lightning-Stroke Frequency

In table II, the variations in frequency of occurrence of lightning are apparent. It will be noticed that the number of recorded strokes to a line will vary in the ratio of almost as much as three to one from year to year. During a year when the number of strokes to one line may be relatively high, those on another may be very low, as for instance, in the years 1935 and 1938. The average number of strokes per hundred miles of line per year is 126 for rights-of-way having only one line. This value cannot as yet be considered as final because the averages for the individual lines still deviate from it by as much as 29 per cent.

VII. Flashovers

Examination of the lightning current records of the Holtwood-Coatesville 66-kv line obtained during the three years before the installation of counterpoise re-

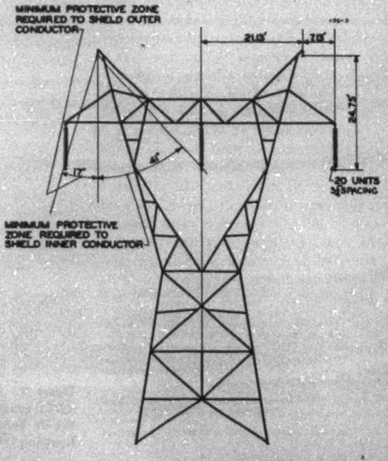
MINIMUM PROTECTIVE ZONE REQUIRED TO SHIELD OUTER CONDUCTOR
21.0'
7.0'
24.75'
45°
17°
20 UNITS 5¾ SPACING
MINIMUM PROTECTIVE ZONE REQUIRED TO SHIELD INNER CONDUCTOR

Figure 3. Minimum shielding required of overhead ground wires on 220-kv towers

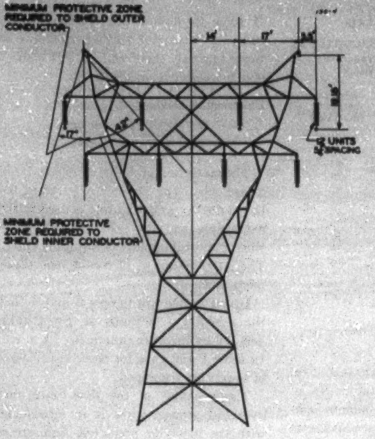
MINIMUM PROTECTIVE ZONE REQUIRED TO SHIELD OUTER CONDUCTOR
14'
17'
3.5'
10.5'
17°
43°
12 UNITS 5¾ SPACING
MINIMUM PROTECTIVE ZONE REQUIRED TO SHIELD INNER CONDUCTOR

Figure 4 (left). Minimum shielding required of overhead ground wires on 132-kv towers

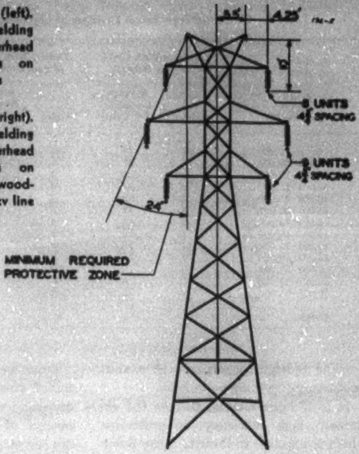

Figure 5 (right). Minimum shielding required of overhead ground wires on towers of Holtwood-York 66-kv line

veals no regular behavior. There were 21 single- and double-circuit tripouts during 1935, 20 during 1936, and 17 during 1937. The individual effects of poor grounding, insulation, and shielding are indeterminate from the data. However, after the installation of counterpoise and the improvement of insulation, the relative importance of the several protection factors became more apparent. The Holtwood-Coatesville line is the only line of the company on which surge-crest ammeter links were placed prior to the installation of counterpoise.

Data on the cases since 1934 in which flashover occurred on lines provided with preventive protection are given in table III.

It will be noticed that in six of the seven cases the indicated tower currents are greater than 50,000 amperes and in five of the cases they are of the order of 60,000 amperes. The latter group suggests that currents around 60,000 amperes are just above the maximum that can be carried without flashover by towers of the type used on the 66-kv lines. In 1937, a tower of the York line carried an indicated current of 63,200 amperes without flashover, which is the only case where a tower of the 66-kv lines carried current in excess of 50,000 amperes without flashover occurring. The tower footing resistance was seven ohms in this case, giving a calculated tower potential of 440 kv, 45 per cent of the normal safe maximum, or flashover level.

In the case of tower 99 of the Coatesville line in table III, it appears that the top conductor of circuit 13 was struck by lightning, causing the flashover. Tower 99 is a suspension tower.

These data on flashovers seem to show that there is a limiting current which towers can carry, irrespective of measured footing resistance. Due to recognized errors in the method of measurement of tower current, the results are not accurate, but a trend is indicated. For five out of six instances, flashovers occurred on 66-kv towers when tower currents were indicated in the order of 60,000 amperes. On the 132-kv line, the flashover recorded in table III was the only one which occurred in three cases where currents were in excess of 50,000 amperes. It is not clear whether the flashover was due to a combination of circumstances, the ground resistance of the counterpoise being relatively high within 300 feet of the tower (about $16^1/_2$ ohms) and a tall tower structure, or due to inadequate shielding. The available data seem to favor the first explanation.

The present-day theory[4] of the behavior of circuits carrying lightning surge currents indicates that there are two voltage effects present. One is the resistance drop which is at a maximum when the surge current is maximum, and the other is the reactance drop which is at its maximum when the current is changing most rapidly. These maxima obviously cannot occur at the same instant, but the maximum circuit voltage will occur somewhere in the interval between the instants of the maxima of the two component drop voltages. Either of the two may predominate, or their effects may be of equal importance, depending upon circuit conditions. From the flashover data presented here, it appears that when conditions are such as to give an indicated tower current of about 60,000 amperes, the rate of current change is sufficiently great to make the reactance component of drop appreciable. For towers, grounding, and insulation of the type used on the 66-kv lines considered, it apparently becomes predominant to the extent of almost always causing flashover. For towers, grounding, and insulation used on the 132-kv line, the reactive effect is appreciable, but apparently becomes of consequence only when tower heights and footing resistances are relatively great.

If future experience bears out the trend which now appears, there will be no advantage in reducing footing resistances below six or seven ohms for the type of 66-kv towers discussed here. Other types of construction will probably be found to have other values for the lowest worth-while limit of footing resistance.

VIII. Efficiency of Protection

The efficiency of a protective scheme can be computed by taking the ratio of the number of times strokes did not cause interruptions to the number of strokes. The efficiencies of the lightning protection during the years that the lightning current study has been in progress on the several lines are given in table IV. The true efficiencies may be slightly higher than those given in table IV because of

Table I. Deviation From Equal Division of Lightning Current in Tower Legs

Tower Current Range (Amperes)	Towers With Links of One Polarity		Towers With Links of Both Polarities		All Towers in Range	
	Cases	Average Deviation (Per Cent)	Cases	Average Deviation (Per Cent)	Cases	Average Deviation (Per Cent)
1– 5,000	61	73	35	90	96	80
5,001–10,000	45	27	7	83	52	35
10,001–15,000	26	23	2	119	28	29
15,001–20,000	9	32	0		9	32
20,001–25,000	7	32	0		7	32
25,001–30,000	8	17	0		8	17
30,001–35,000	2	5	0		2	5
35,001–40,000	1	10	1	89	2	50
40,001–45,000	2	15	0		2	15
45,001–50,000	1	14	0		1	14
50,001–55,000	0		0		0	
55,001–60,000	1	6	0		1	6
60,001–65,000	1	6	0		1	6
Totals	164		45		209	

possible multiple records or weak records on the surge-crest ammeter links.

It is of some interest to note the extremely high efficiency of protection which is required to limit to a low figure the number of tripouts over a period of years.

IX. Efficiency of Shielding

The relative merits of one and two overhead ground wires and of the many possible positions in which they may be placed have been studied by many investigators at great length theoretically, and in the laboratory.[6] From the lightning current data which have been collected, it is possible to evaluate the efficiency under operating conditions of the various arrangements used on the lines to which reference is made here.

For an overhead ground wire to shield a line conductor from direct strokes, the line conductor must lie within the protective zone of the ground wire. In figures 2, 3, 4, and 5 are indicated the minimum protective shielding zones that the respective overhead ground wires must provide. Where a single overhead ground wire has been installed, the required protective zone is wider for a strain tower than for a suspension tower, as shown in figure 2. This is because the conductor is at the height of the cross arm on a strain tower, and below the cross arm on a suspension tower by the length of an insulator string. Where two overhead ground wires have been installed on the newer lines of the company, the overhead ground wires have been elevated at strain towers to maintain the same clearances above line conductors as is provided at suspension towers. The protective zones required on strain and suspension towers having such construction are, therefore, the same. (See figures 3, 4, and 5.)

From the number of flashovers and recorded strokes to the several classes of transmission lines, the minimum efficiencies of the shielding arrangements can be computed. The number of flashovers which have occurred are known accurately for practical purposes from the tripouts which have occurred. The recorded strokes to a line may be slightly less than the actual number due to more than one stroke affecting a set of links between inspections and calibrations. Ignoring some strokes because of the weak magnetic effect produced will also affect the number of reported strokes in the same manner.

When a flashover occurs, there may be some question of the cause being inadequate shielding or high-tower potential. If some flashovers are believed to be caused by excessive footing resistance or tower current, there will be less chargeable to imperfect shielding.

Efficiency of shielding =

$$100\left(1-\frac{\text{outages}}{\text{strokes}}\right)\text{per cent}$$

The maximum probable efficiency is obtained when the number of outages believed due to poor shielding is used in the expression, and the minimum efficiency when the total number of outages is used. The latter figure is, of course, the pessimistic one.

It can be seen in figures 3 and 4 that the shielding conditions on the 220-kv and 132-kv lines are practically identical. In table V the data for these two classes of towers are combined.

The behavior of the lines with two overhead ground wires is in agreement with the behavior predicted from surge tests on miniature models, but that of the line with one overhead ground wire is much better. In the latter case the shielding on the suspension towers is theoretically superior to that on the strain towers, but their behavior during the period of study has been practically the same.

X. Comparison of Calculated Probable Outages and Actual Outages

Some years ago Lewis and Foust[7] suggested that, as a result of their studies,

Table II. Strokes to Lines

Line	Length in Miles	Year	Recorded Strokes		Strokes Per 100 Miles	
			In Year	Average	In Year	Average
Safe Harbor-Westport-Takoma 220 kv	91.8	1935	115		125	
		1936	108		118	
		1937	87		95	
		1938	54		59	
		1939	118		129	
				96		105
Safe Harbor-Riverside 220 kv	50.5	1938	46		91	
		1939	45		89	
				46		90
Safe Harbor-Perryville 132 kv	31.5	1935	27		86	
	31.5	1936	69		219	
	31.5	1937	44		140	
	33.8	1938	56		166	
	33.8	1939	53		157	
				50		154
Holtwood-York 66 kv	22.8	1936	31		136	
		1937	41		180	
		1938	15		66	
		1939	22		97	
				27		120
Holtwood - Coatesville 66 kv	29.4	1936	30		102	
		1937	25		85	
		1938	68		232	
		1939	45		153	
				42		143
Holtwood - Baltimore 66 kv (2 lines) and Philadelphia Road-Gunpowder 110 kv	40 (of right-of-way)	1939	66		165	
				66		165

Average strokes per 100 miles of line per year for all lines except the last in the list is 126.

flashovers of shielded transmission lines only occur when tower potentials exceed the insulation strength of the given line. The tower potential in each case was taken as the product of tower footing resistance and tower lightning current. Based on this theory a method was suggested several years ago by the present writer for calculating the probable outages of shielded transmission lines.[8] In table VI a comparison is made between the actual flashovers experienced on the several lines and the calculated behavior.

For all but one of the lines, the actual performance has not been as good as the estimated. The record of flashovers given in table III shows that flashovers can be caused by tower currents less than those predicted from the theory based on tower footing resistance drop.

As the original empirical Lewis and Foust rule,[7] the data used in the probability calculations, and the records found when flashovers occurred are all based on the same measurement method, any discrepancies between calculated and actual outages do not seem to be attributable to neglecting the current carried by tower cross braces. The discrepancies are, therefore, thought to result from the impedance drop in the tower structure and footings being appreciable.

To make the probability calculations conform to field experience, the theory must be modified to state that flashover does not occur unless the product of tower current and tower footing resistance exceed the insulation level of the line, or unless the tower current exceeds the characteristic safe current for the given tower and grounding arrangement. The characteristic safe current seems to be around 60,000 amperes for the 66-kv lines. From the circumstances attending the single flashover which occurred on the 132-kv line, from the relative insulation strengths of the 66- and 132-kv lines, and from other data, it is estimated that the characteristic safe current of this 132-kv line is probably around 90,000 to 100,000 amperes. Experience with the 220-kv lines seems to indicate that the safe characteristic current of the towers, grounding, and insulation used on these lines probably is in excess of those occurring in nature. The only flashover experienced on the 220-kv lines occurred in 1933 at a tower having a footing resistance of 25 ohms. This was before the lightning current studies were begun, but the flashover can be accounted for on the basis of the original Lewis and Foust theory by assuming a tower current of 100,000 amperes, a value which has been measured several times in the field.

Table III. Flashovers on Transmission Lines Provided With Preventive Lightning Protection

Line and Tower	Year	Tower Current in Amperes	Tower Footing Resistance in Ohms	Tower Height in Feet	Tower Potential (I×R) (Kv)	Ratio of Tower Potential to Flashover Level
Holtwood-York 63 (66 kv)	1937	58,400	13.8	75	805	0.82
Holtwood-York 145 (66 kv)	1937	60,000	19	76	1140	1.16
Holtwood-Coatesville 40 (66 kv)	1938	64,400	17.2	73	1108	1.30
Holtwood-Coatesville 97 (66 kv)	1939	60,200	5.4	83	325	0.38
Holtwood-Coatesville 99 (66 kv)	1939	13,400	8.5	73	115	0.14
Holtwood-Baltimore 130–56 (66 kv)	1939	62,800	3.6	67	225	0.26
Safe Harbor-Perryville 15 (132 kv)	1939	52,600	12.2	137	640	0.41

Table IV. Efficiencies of Preventive Lightning Protection

Line	Years of Study	Lightning Outages*	Recorded Strokes	Efficiency of Protection (Per Cent)
Safe Harbor-Westport-Takoma 220 kv	5	0**	482	100**
Safe Harbor-Riverside 220 kv	2	0	91	100
Safe Harbor-Perryville 132 kv	5	1	249	99.6
Holtwood-York 66 kv	4	2	109	98.2
Holtwood-Coatesville 66 kv				
Before improvements	2†	37	55	33
After improvements	2	3	113	97.4
Holtwood-Baltimore 66 kv (Two lines on one right-of-way)	1	1‡	65	98.4

*Single, double, and triple circuit tripouts.

**A flashover of this line occurred in 1933 before the lightning current studies were begun. The estimated efficiency for the entire eight years of operation is:

$$100\left(1-\frac{1}{8/5\times 482}\right)=99.9 \text{ per cent}$$

†Stroke records are complete for only two years (1936 and 1937).

‡Not included is one tripout which occurred before the installation of counterpoise was completed in 1930.

The probable outages for the lines have been recalculated in table VII using the estimated safe characteristic tower currents given above as the maxima that can be carried under any circumstances. The calculations may possibly be further refined in the future when tower and footing impedances become evaluated and their effect in combination with footing resistance is determined quantitatively.

XI. Lightning Arrester Records

Surge-crest ammeter links have been mounted on the lightning arresters at Highlandtown substation in Baltimore since 1935. These arresters are connected to the four 25-cycle, 66-kv circuits of the two Holtwood-Baltimore lines. Each year since their installation, except 1936, they have indicated some current in at least three of the four arrester banks. The maximum current recorded was 2,600 amperes in 1937.

Similarly, arresters connected to other 66-kv circuits have carried small currents from time to time. The largest current recorded was 7,400 amperes in 1939 in the arresters connected to circuit 83 at Violet Hill substation in York. Circuit 83 is an unshielded wood-pole line of the Metropolitan Edison Company connecting through the high-tension bus at the substation to the Holtwood-York 66-kv line.

It is interesting and of some importance to note that during three seasons of study, no indications have been obtained of discharges through any of the four banks of arresters connected to circuits protected by two overhead ground wires and buried counterpoise systems.

XII. Summary

1. Because of the large number of observations, 3,289, which have been made during the six years of study of lightning currents, some confidence can be felt that the data are fairly representative of the several effects of lightning. Some uncertainties still exist in the data, as for instance the effect of tower cross braces on the measurement of tower currents.

2. The distribution curve of the various magnitudes of currents in towers believed

Table V. Efficiencies of Shielding

Type of Line	Years of Record	Number of Strokes	Number of Outages	Estimated Strokes to Conductors	Probable Efficiencies of Shielding Minimum (Per Cent)	Probable Efficiencies of Shielding Maximum (Per Cent)
220 kv (2 lines) (figure 3)	7*	573	0*	0		
132 kv (figure 4)	5	249	1	0		
	12	822	1	0	99.9	100
66 kv (figure 2) (1 overhead ground wire)						
Strain towers	2	30	1	0	97	100
Suspension towers	2	83	2	1	98	99
66 kv (figure 5) (2 overhead ground wires)	4	109	2	0	98	100

*Includes only years of lightning current study. See table IV.

to have been struck by lightning has been established from 1,201 records. The curve can be used satisfactorily for purposes where the error due to cross braces is not objectionable. Considerable confidence can be placed in the general shape of the curve.

3. Over a period of years it appears that all sections of the transmission lines of the Pennsylvania Water and Power Company are struck by lightning, there being no localities that are immune.

4. The average number of strokes to the transmission lines studied is 126 per 100 miles per year. The actual number of strokes to a given transmission line, however, may vary in the ratio of as much as three to one from year to year. This variation emphasizes the need for examining lightning experience records for a period of several years before any conclusions are reached about lightning phenomena.

5. Analysis of the flashovers which have occurred on lines having overhead ground wires and buried counterpoise systems indicates that flashovers can be caused not only by the ground resistance drop being greater than the insulation strength, but also by the reactive drop due to the rapid rates of current rise which seem to accompany heavy tower currents. On the observed 66-kv lines this latter phenomenon seems to be of critical magnitude when the indicated tower currents have been around 60,000 amperes.

Table VI. Comparison of Actual and Probable Outages

Probable Outages Calculated on Basis of Lewis and Foust Theory as Originally Presented[7]

Line	Years of Record	Lightning Outages Per Year Actual	Lightning Outages Per Year Probable
Safe Harbor-Westport-Takoma 220 kv	8	0.13	0.014
Safe Harbor-Riverside 220 kv	2	0	0*
Safe Harbor-Perryville 132 kv	5	0.2	0.0078
Holtwood-York 66 kv	4	0.5	0.29
Holtwood - Coatesville 66 kv	2	1.5	0.68
Holtwood - Baltimore 66 kv (Two lines on one right-of-way)	1	1	0.2

* Does not include section 1.03 miles in length where there are no overhead ground wires.

6. Because of this latter phenomenon it appears that no advantage accrues from reducing tower footing resistances on such lines below approximately six ohms. This figure allows for suspected inaccuracies in the measurement of tower lightning currents and footing resistances. It is thought to be desirable that this value of footing resistance be obtained within not more than 200 or 300 feet of the tower base.

7. Operating experience shows that the arrangement of the two overhead ground wires on the newer lines of the company provides practically perfect shielding, that is, probably within less than 0.1 per cent. The experience with a single overhead ground wire is superior to that predicted from laboratory models. The record shows the shielding to be effective for about 98 or 99 per cent of the strokes contacting the Holtwood-Coatesville 66-kv line.

8. Preventive protection embodying the use of overhead ground wires and buried counterpoise installations co-ordinated with line insulation has been shown to be very effective. Since this work has been carried out completely on the 220-kv lines it has been 100 per cent effective. It has been 99.6 per cent effective on the 132-kv line and about 98 per cent effective on the 66-kv lines.

9. Discrepancies were found in the comparison of actual and calculated probable lightning outages. These have been traced to the phenomenon of a maximum surge current which a given construction can apparently withstand irrespective of the measured tower footing resistance.

10. The lightning arresters connected to transmission lines provided with effective preventive protection are seldom called upon to carry lightning discharges, and then they are apparently of small magnitudes.

References

1. Safe Harbor-Westport 230 Kv Transmission Line, Edwin Hansson. Electrical Engineering, volume 51, December 1932, pages 834-8.

2. Direct Measurement of Surge Currents, C. M. Foust and J. T. Henderson. AIEE Transactions, volume 54, 1935 (April section), pages 373-8.

3. Lightning Investigation on a 220-Kv System—III, Edgar Bell. AIEE Transactions, volume 59, 1940, pages 822-8.

4. Lightning Investigation on Transmission Lines—VII, W. W. Lewis and C. M. Foust. AIEE Transactions, volume 59, 1940 (April section), pages 227-32.

5. Theory and Tests of the Counterpoise, L. V. Bewley. AIEE Transactions, volume 53, 1934 (August section), pages 1163-72.

6. Shielding of Transmission Lines, C. F. Wagner, G. D. McCann, and G. L. MacLane, Jr. AIEE Transactions, volume 60, 1941.

7. Lightning Investigation on Transmission Lines—IV, W. W. Lewis and C. M. Foust. Electrical Engineering, volume 53, 1934 (August section), pages 1180-6.

8. Probable Outages of Shielded Transmission Lines, S. K. Waldorf. AIEE Transactions, volume 56, 1937 (May section), pages 597-600.

Table VII. Comparison of Probable and Actual Outages

Probable Outages Calculated on Basis of the Suggested Modification of the Lewis and Foust Theory

Line	Estimated Safe Maximum Current	Lightning Outages Per Year Actual	Lightning Outages Per Year Probable
Safe Harbor-Westport-Takoma 220 kv	Greater than 150,000 A	0.13	0.014
Safe Harbor-Riverside 220 kv	Greater than 150,000 A	0	0*
Safe Harbor-Perryville 132 kv	90,000 A	0.2	0.05
Holtwood-York 66 kv	60,000 A	0.5	0.58
Holtwood-Coatesville 66 kv	60,000 A	1.5	1.3**
Holtwood-Baltimore 66 kv (Two lines on one right-of-way)	60,000 A	1	1

*Does not include 1.03 miles section where there are no overhead ground wires.

**Includes allowance for shielding being only 99 per cent efficient. (See section IX.)

High-Voltage Bushings Designed to Meet Modern Service

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MEMBER AIEE

H. L. RORDEN
MEMBER AIEE

Introduction

BUSHINGS are a very important part of over-all system insulation. Since all of the power transmitted from station to station and station to consumer passes through a great many bushings the majority of which are a part of vital equipment such as transformers, breakers, etc., it is immediately apparent that system-service continuity depends, to a large degree, upon satisfactory bushing operation. Early bushing designers were handicapped by a lack of operating history and high-voltage testing facilities. However, in spite of these handicaps they developed some sound designs as evidenced by the many 25- to 30-year-old bushings in satisfactory service today. In these early bushings porcelain was one of the principal dielectrics.

Comprehensive studies of porcelain oil-filled bushings have been made to improve both the electrical and mechanical characteristics. As a result of these studies a better understanding of the internal and external electrostatic field conditions has resulted, and a new method of voltage grading developed. Mechanical improvements have been made which not only simplify construction, but stabilize the electrical characteristics by maintaining the over-all assembly close to its original condition. These developments will be discussed here.

Control of Electrical Characteristics in Bushing Design

Design Requirements

From an electrical standpoint, a bushing must have adequate internal insulation to provide a safe margin between internal puncture and external flashover. There also should be no corona formation at operating voltage and radio interference must be kept at a minimum. The over-all electrical characteristics of a bushing should be stabilized in so far as they affect operating performance. A design which has excellent electrical characteristics in the laboratory, but is subject to change under operating conditions, is undesirable if these changes adversely affect the performance. Extensive tests have been made in the laboratory and operating records have been carefully checked to guard against this possibility.

A bushing must provide sufficient insulation in two ways. First, it must have the required external flashover voltage commensurate with its rating and the insulation level of associated equipment, and secondly, the internal puncture strength must have a satisfactory margin over the external flashover. Since external flashover of a bushing is through a surrounding medium, such as the air around the upper or weather end, full consideration must be given the characteristics of this medium to obtain the balanced design necessary for stable electrical performance.

Porcelain oil-filled bushings use insulating materials of differing dielectric constants or flux-carrying capacities. With insulation in series there is a tendency for those of lower dielectric constants to have imposed relatively greater portions of the total voltage. Accordingly, it is essential that high localized stress be avoided in those insulating zones having low dielectric constants. This applies to the medium surrounding the bushing, as well as the insulation within it.

Means of Controlling Characteristics

In order to determine the effectiveness of voltage grading or electrostatic field-controlling arrangements a series of impulse and 60-cycle studies were made. Many assemblies were tested, but only representative types having practically the same striking distance will be discussed here. These are shown in figure 1.

In figure 2 are shown photographs of corona due to 60-cycle overstress on the assemblies of figure 1. Figure 3 shows photographs of the field patterns around these same assemblies as indicated by corona from impulse-suppressed discharges. Complete electrical data on the above assemblies, along with voltages applied in the various photographs, are given in table I.

In figure 1*a* is shown a porcelain tube with a metal cap and flange, but *without* a central conductor. Appreciable 60-cycle corona can be seen at the cap in figure 2*a*, while a uniform field pattern exists in figure 3*a* for both polarities. This assembly is included to show the radical change in the field when a conductor is passed through the center of the tube. Figure 1*b* is the same as figure 1*a* except *with* a central conductor added. Heavy 60-cycle overstress is apparent in figure 2*b* immediately above the metal flange and figure 3*b* shows the field overstress due to suppressed discharges.

The difference between the positive and negative field patterns of figure 3 is in accord with observations made on the study of dielectric fields existing between various types of electrodes in air.[1] It is possible to control these stresses to some degree by the shape and arrangement of electrodes and by various methods of grading. The central conductor of figure 1*b* has changed the electrostatic field so that the air at the edge of the flange is highly overstressed. This overstress is due to the low dielectric constant or flux-carrying capacity of air as compared to porcelain. The air space between the central conductor and the porcelain is subject to overstress for the same reason. These two regions are the cause of unstable flashover and radio-interference characteristics in this type of bushing.

In contrast with figure 2*a* there is no overstress at the cap in figure 2*b*. The central conductor shields the cap since most of the electrostatic flux lines emanating from the flange terminate on the central conductor well below the cap. Figure 4 shows the approximate field condition for figure 1*b*. Initial overstress starts in the air immediately above the flange at *A*. As the voltage is increased, the air becomes ionized, forming a conducting envelope over the porcelain surface to a point where flashover occurs at a rather low voltage.

In figure 1*c* is illustrated a typical solid porcelain bushing. Figures 2*c* and 3*c* show its 60-cycle and impulse corona patterns, which are similar to those of figures 2*b* and 3*b*. The performance of the figure 1*c* design can be improved somewhat by changing the porcelain configuration.

Oil-filled bushing designs readily adapt themselves to the elimination of localized overstress. For example figure 1*d* is a

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T. F. Brandt and H. L. Rorden are development engineers for the Ohio Brass Company, Barberton, Ohio.

[1] For numbered reference, see end of paper.

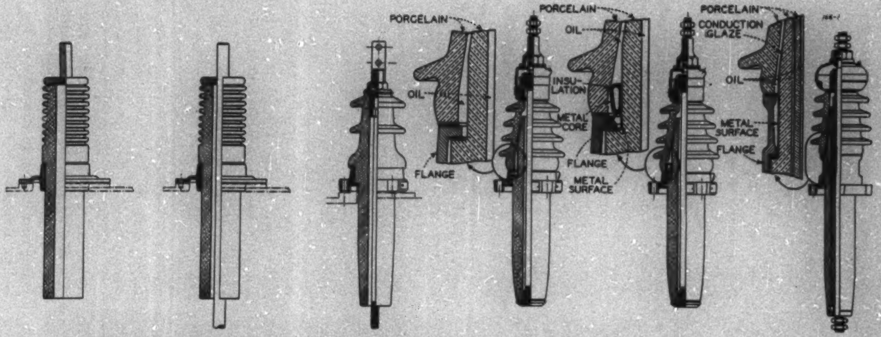
PORCELAIN
OIL
FLANGE
PORCELAIN
OIL
INSU-LATION
METAL CORE
FLANGE
METAL SURFACE
PORCELAIN
CONDUCTION GLAZE
OIL
METAL SURFACE
FLANGE

Figure 1. Details of bushing construction

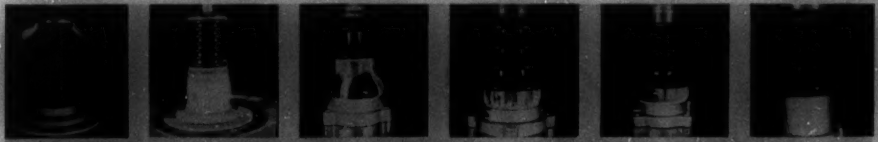

Figure 2. Corona due to 60-cycle overstress

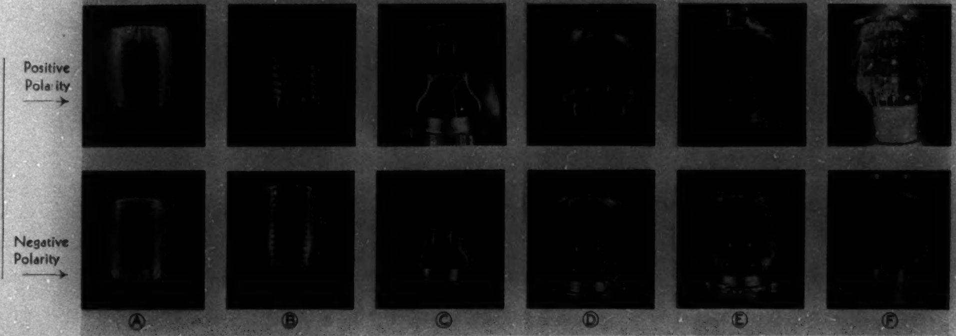
Positive Polarity
Negative Polarity
A
B
C
D
E
F

Figure 3. Corona due to suppressed impulse discharge

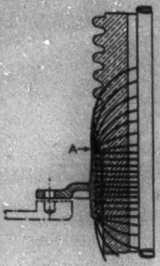

Figure 4. Field pattern for figure 1b

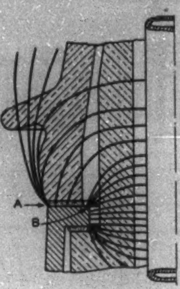

Figure 5. Field pattern for figure 1d

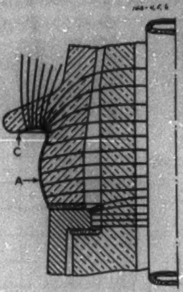

Figure 6. Field pattern for figure 1d near flashover

simple oil-filled assembly with no grading means other than its porcelain configuration. The 60-cycle corona in figure 2*d* shows the heavy overstress immediately above the flange. The approximate field condition around the flange area of the figure 1*d* design below the point of excessive corona is shown by figure 5. The inside air between the conductor and the porcelain has been replaced by a combination of oil and wrapped insulation, thereby preventing overstress in this area. However, no effort has been made to control the field at the mounting flange, thereby resulting in two highly stressed regions. One is at *B*, figure 5, inside the bushing where the flange terminates in the oil; the other is at *A*, the outer exposed edge of the flange. The voltage gradient at *B* is higher than at *A*. However, since the dielectric strength of the oil at *B* is appreciably higher than that of the air at *A*, overstress appears first at *A* at a lower voltage gradient.

This overstressing at *A* establishes a conducting envelope of ionized air with increasing voltage. It does not progress up the bushing as far as on figure 2*b* but stops under the first petticoat. This is due to the conducting envelope assuming a form of grading means, as shown by figure 6. This same condition is shown in figure 2*c* where overstress stopped abruptly under the porcelain petticoat. This has not been generally appreciated and many bushings have been made in both the solid and oil-filled types with the region from *A* to *C*, figure 6, unstable, thereby involving possible radio interference.

Figure 1*e* is similar to figure 1*d* except that a capacity grading means in the form of an insulation-wrapped metal core is incorporated. The metal core and the metalizing on the porcelain tube are electrically connected to the flange. Since the wrapped metal core extends up inside the bushing, it partially shields the outside exposed edge of the mounting flange. At the same time it reduces the voltage stress in the oil at the inside edge of the flange and at the top edge of the metalizing on the porcelain tube. Covering the metal with insulation is essential for proper field control. Tests on various forms of uninsulated metal grading means have proved them to be relatively ineffective.

The approximate field condition of figure 1*e* is shown by figure 7. The metalized, ground potential surface on the porcelain shorts out the highly stressed oil at *B* in figure 5. The point *A* is not completely shielded since it goes into corona as shown by figure 2*e*. However, the corona is materially reduced from that shown on figure 2*d*. In table I, figures 1*d* and 1*e* are seen to have practically the same 60-cycle flashover although an appreciable difference exists in the radio-interference values. This is because at flashover voltage the external field control set up by the conducting envelope of corona in figure 2*d* and the grading means of figure 2*e* are equally effective in so far as flashover voltage is concerned.

In the lower operating voltage bushings, namely 69 kv and below, cover opening diameters have been somewhat restricted. The problem here has been to develop the most efficient methods of using the available space. Various voltage grading arrangements have been used, but most of them require space which can be used better for effective insulation between the central conductor and ground. A newly developed method, not requiring this, is a form of resistance grading by means of conduction glaze applied directly to the porcelain surface. Figure 8 shows the approximate field of the figure 1*f* design which utilizes this resistance grading.

Figure 7, the type used in figure 1*e*, can be called an auxiliary means since it is not integral with the porcelain. Figure 9 is an effective field-controlling arrangement used on present high-voltage bushing designs of ratings, 92 kv and above. This type, formed integral with the porcelain, is very effective where space permits its use. The types in figures 7 and 9 function by means of their configuration and the voltage drop through a dielectric. Figure 8 functions because of the voltage drop through a high resistance. There must be enough voltage drop along the conduction-glazed surface from *D* to *E* to prevent overstressing at both *D*, the

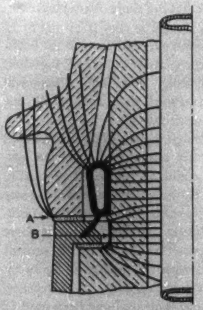

Figure 7. Field pattern for figure 1e

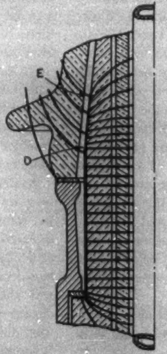

Figure 8. Field pattern for figure 1f

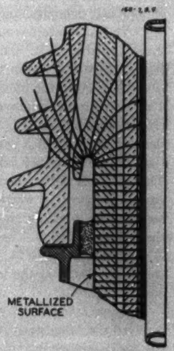

Figure 9. Field pattern for figure 11

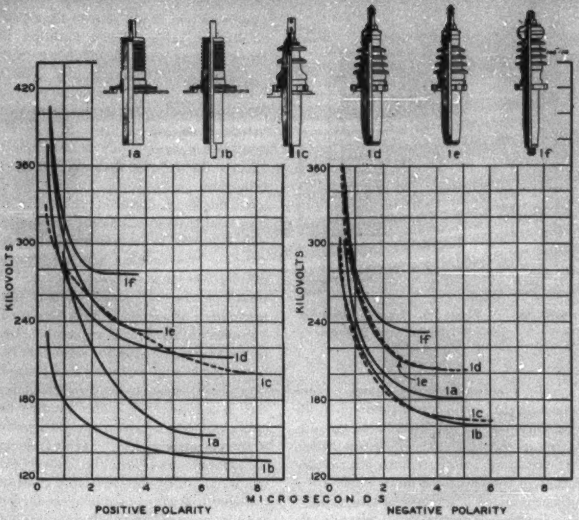

Figure 10. Volt-time characteristics of assemblies in figure 1

end of the metalizing, and at *E*. This means that the resistance of the conduction glaze region from *D* to *E* must be maintained within definite limits by careful manufacturing control. Likewise, the conduction glaze must have permanently stable characteristics.

Figure 1*f* illustrates a modern bushing using resistance grading. Figure 2*f* shows a very light glow between the petticoats with 120 kv, 60 cycle applied on the bushing. This shows the absence of localized overstress outside the bushing. The radio interference (table I) indicates the absence of the internal overstress. Figure 8 shows the field condition discussed in the previous paragraph. This construction improves the electrical characteristics materially by approaching the uniform field of a sphere gap rather than the concentrated field of a rod gap. This is particularly emphasized by the curves in figure 10 where this design is seen to have a rather high and flat volt-time characteristic. The other extreme is the design of figure 2*b* whose poor field condition results in the lowest volt-time curve of figure 10.

Bushings rated 92 kv and above have received the same study, and figure 11 is the one developed for 115-kv service. It is provided with capacitance grading of the type shown in figure 9, and gives proportionately the same performance as figure 1*f*. Since these modern designs are stable at voltages considerably in excess of their rating, it is safe to assume that their characteristics will remain unchanged under normal operating conditions.

Mechanical Improvements

Excellent electrical properties are of little value if the mechanical design does not maintain the bushing at or near its original condition for a long period of time. This requires, primarily, sturdy construction with efficient gasket joints and one that will maintain a stable condition of the oil. Figure 11 shows a modern bushing having these characteristics.

An early design is shown by figure 12 where all of the external parts of the bushing were completely cemented together. If gasket leaks or other mechanical troubles developed, there arose a difficult maintenance problem.

A later design, shown in figure 13, could be more easily maintained since the gasket joints could be tightened or readily replaced. However, the large metal parts cemented to porcelain presented occasional problems. In the first place there was the possible chance of a cement change with time where joint *A*, figure 13, was directly exposed to the weather. Weatherproof coatings have been used to prevent cement deterioration, but under extreme conditions they have been found wanting. Also a differential thermal expansion existed between the metals and porcelain. Special treatment of the cement joints to compensate for thermal expansion has also been tried. There are many thousands of large oil-filled bushings such as those of figures 12 and 13 in service which have given a good account of themselves, but gasket leaks and damaged porcelain are not uncommon.

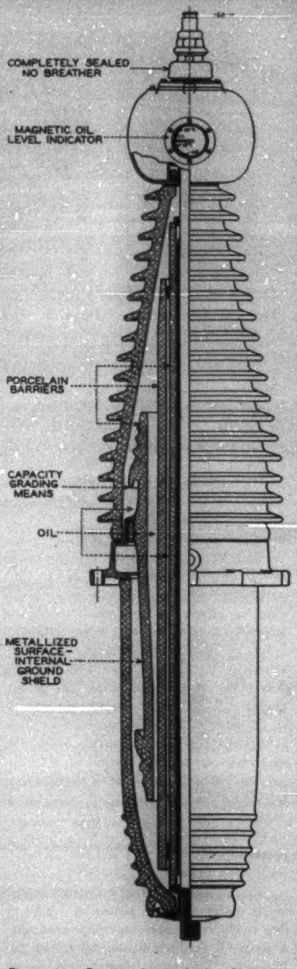
COMPLETELY SEALED NO BREATHER
MAGNETIC OIL LEVEL INDICATOR
PORCELAIN BARRIERS
CAPACITY GRADING MEANS
OIL
METALLIZED SURFACE-INTERNAL-GROUND SHIELD

Figure 11. Present design, high-voltage porcelain oil-filled bushing

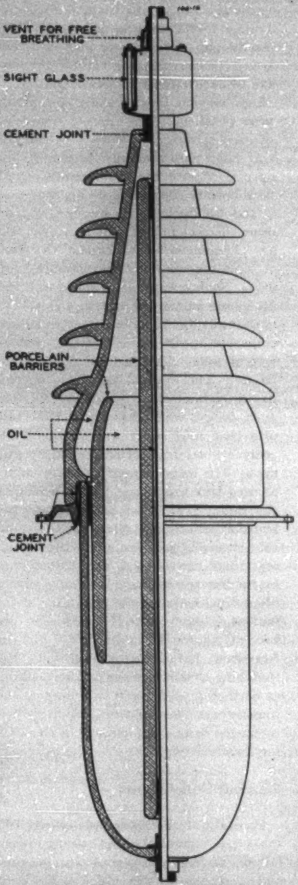
166-15
VENT FOR FREE BREATHING
SIGHT GLASS
CEMENT JOINT
PORCELAIN BARRIERS
OIL
CEMENT JOINT

Figure 12. High-voltage porcelain oil-filled bushing of early design (1920)

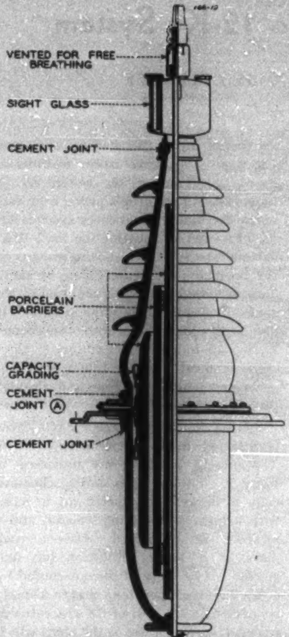
166-13
VENTED FOR FREE BREATHING
SIGHT GLASS
CEMENT JOINT
PORCELAIN BARRIERS
CAPACITY GRADING
CEMENT JOINT Ⓐ
CEMENT JOINT

Figure 13. High-voltage porcelain oil-filled bushing of ten years ago

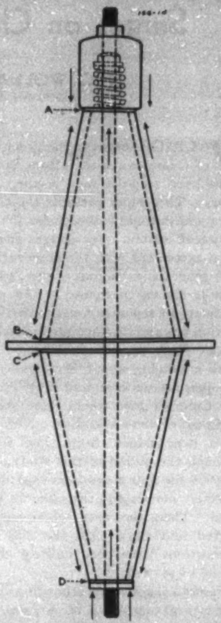

Figure 14. Schematic view of compression assembly showing how gaskets are kept under compression

Several years ago the full compression-assembled bushing was introduced. This produced a very rugged assembly, eliminated external cement joints, and kept the gasket joints permanently oil-tight. It is a well-established fact that most gasket materials kept under constant pressure between mating surfaces remain oil-tight indefinitely. Formerly, gaskets had to keep themselves tight. To illustrate this, when the gaskets in bushings such as figure 13 were originally tightened, the material, due to its initial resilience, exerted a strong back pressure against the mating surfaces. However, this back pressure for practically all gasket materials diminishes with time. In addition some shrink or take a permanent set. In any event, in a relatively short time the gasket joint is not as effective as when first made.

The full-compression assembly shown

Table I

Assembly Figure Number	Corona Due to 60-Cycle Overstress		Corona Due to Impulse Suppressed Discharge		60-Cycle Flashover (Kv RMS)		Critical Flashover $1\frac{1}{2} \times 40$-Microsecond Wave Kv (Crest)		Radio Interference Voltage on Bushing for 100-Microvolt Level Kv (60 Cycle RMS)
	Kv (RMS) on Assembly	Photo Figure Number	Kv (Crest) on Assembly*	Photo Figure Number	Dry	Wet	Pos.	Neg.	
1*a*	89	2*a*	377	3*a*	91	72	153	182	41.2
1*b*	75	2*b*	205	3*b*	77	65	133	161	20.1
1*c*	81	2*c*	315	3*c*	95	74	202	165	21.2
1*d*	81	2*d*	350	3*d*	115	78	211	204	21.2
1*e*	81	2*e*	350	3*e*	119	88	234	206	44.7
1*f*	118	2*f*	422	3*f*	131	93	276	233	118.0

*Wave is "chopped" by parallel rod gap, allowing voltage considerably above critical flashover value.

in figure 11 compensates for all gasket variations since the mating parts are continually acted upon by the compression spring in the oil reservoir. This is shown schematically in figure 14 which indicates how the compression loading of the spring is transmitted to the various gasket surfaces at *A*, *B*, *C*, and *D*. The compression spring must have adequate "pickup" or deflection so that any shrinkage of the gaskets will not release the spring enough to affect its loading appreciably.

With dependable gasket joints now available, it is possible to have a completely sealed bushing. This has been done over an entire range of oil-filled porcelain bushings to secure a stable oil condition. Moisture, oxygen, and sunlight are all detrimental to insulating oil, but by completely isolating the oil from them, its dielectric properties remain practically constant. Where it is desired to check the oil level occasionally, especially in the larger bushings, a magnetic oil level indicator is used. This affords a definite indicating means without exposing the bushing oil to sunlight, moisture laden air, etc.

In order to keep the pressure in the bushing within reasonable limits, adequate space in the top oil reservoir shown must be provided for oil expansion due to temperature changes. The compression spring arrangement must be adequate to keep the gasket joints tight against the various pressures in the bushing.

Reference

1. The Control Gap for Lightning Protection, **Ralph Higgins** and **H. L. Rorden**. AIEE Transactions, volume 55, 1936 (September section), pages 1029–34.

Surges on Chicago 12-Kv System

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VOLTAGE surges on the 12-kv system of Commonwealth Edison Company have been recorded and studied for 20 years. This paper presents the Chicago data and outlines a theory for the explanation of system overvoltages that is in good agreement with the observations.

A geographical layout of the 12-kv system is shown in figure 1. At present there are six separate zones consisting of a station bus and its associated radial 12-kv lines. Each zone has about 50 or 60 lines averaging about three miles long. All lines are underground cable except in the Calumet zone where there are some sections of overhead line. New zones have been added from time to time through the period of this study. Prior to 1928 the four zones then existing were directly connected through 12-kv tie lines. These ties were subsequently removed and since 1928 the only interconnections have been indirect through the 66-kv system.

Figure 2 is a schematic one-line diagram of a typical bus layout for any one of the six generating or distributing stations in Chicago. The two basic features of every 12-kv zone in the system are (1) independent feed of bulk power from each of the two independent 66-kv systems and (2) provision for splitting the local 12-kv system into two independent systems by the operation of sectionalizing breakers. Two types of grounding are used in the zones involved in the surge studies. In one type a derived neutral transformer having an inductive impedance of three ohms is used. In the other type, one generator neutral is grounded through three ohms of resistance.

Spark-gap installations have been maintained on the system since 1919. The present locations are indicated in figure 1. The device is shown schematically in figure 3. A sphere gap in series with a fuse, a protecting resistor, and a recorder is connected between each phase of a 12-kv substation bus and ground. The surge of current caused by a gap flashover operates a simple solenoid connected to the pen of the clock-driven recorder, leaving a jog on the chart which shows when the overvoltage occurred and which phase it was on. The magnitude of the overvoltage remains unknown, except that the breakdown of the gap establishes its lower limit. The gap settings have been adjusted from time to time through the 20 years of the study to cover the range 1.85 times normal crest voltage to 3.5 times normal crest voltage. As the gap settings were increased there was a corresponding decrease in the number of overvoltages recorded, until at 3.5 times normal crest voltage no records were obtained. The time of occurrence of each spark-gap operation is carefully correlated with the daily log of switching operations and the oscillograph records of abnormal conditions in the system.

The oscillograph records provide another distinct line of attack on overvoltage phenomena by facilitating the study of simultaneous or "sympathetic" failures. Such occurrences consist of a failure in one part of the system, followed almost immediately by one or more failures elsewhere in the same system, perhaps several miles distant from the original failure. The original failure is referred to throughout this paper as the primary failure while the one or more failures that may occur almost simultaneously are referred to as secondary failures. The terms primary and secondary as used here should not be confused with the same terms used to designate two transformer-coupled circuits. Questions that naturally arise are: (*a*) Is the secondary failure caused by a brief transient or by the increase in 60-cycle voltage on two phases incidental to the grounding of the faulted phase? (*b*) If transients are present, what is their nature? (*c*) Are the secondary failures associated with the switching that clears the original failure, or do they precede such switching? The routine high-speed records, supplemented by staged tests, have assisted in elucidating these matters.

General Conclusions

From the studies of records of over 700 spark-gap operations, 54 simultaneous failures of lines or apparatus in service, and 15 staged tests in which faults were thrown on the system, the following conclusions have been drawn:

1. The maximum magnitude of overvoltages on this system is about 3.5 times normal crest voltage.

2. The overvoltages of highest magnitude are associated with system failures and are best understood by use of the "cancellation-wave" theory; the highest transients occur at the inception of the fault, not during the switching that clears the fault.

3. Overvoltages associated with routine switching are amenable to the "cancellation-wave" theory, but are usually of smaller magnitude than those associated with failures and are therefore of less interest.

4. Most secondary failures in simultaneous failures are caused by the overvoltages mentioned in conclusion 2.

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5. The overvoltages on the Chicago 12-kv system are essentially power-frequency phenomena.

6. When an overvoltage condition is set up, it is propagated throughout all parts of the system that have direct metallic connections, but is strictly confined to such metallically connected parts.

7. Secondary failures occur only in insulation that is deteriorated to the point where it will not withstand (*a*) 60-cycle overvoltage of value 1.73 times normal crest to ground applied for the duration of a system fault to ground, (*b*) a few cycles of transient overvoltage having a maximum magnitude of 3.5 times normal crest to ground.

8. Zones having a derived neutral ground, which is essentially inductive, have more and higher transient overvoltages than zones having the generator neutral grounded through resistance.

9. The number and magnitude of overvoltages recorded in the zone having some sections of 12-kv overhead line, excluding overvoltages due to lightning and unknown causes, were about the same as for the zones in which all the lines were underground.

Spark-Gap Data

The data on system overvoltages obtained in the period 1919–28, when all the 12-kv zones were directly interconnected, are shown in figure 4. A few surges exceeded 3.2 times normal, which was the highest gap setting used in this period. The recording devices were needle gaps. Similar data for sphere gaps (one-inch spheres) are given in figure 5 for the period 1932–38, when the zones were not directly interconnected. A few overvoltages exceeded three times normal, but none exceeded 3.5 times normal. The latter setting was maintained at all spark-gap installations for about $1^1/_2$ years. From this it appears that the maximum overvoltages on the system do not exceed 3.5 times normal. The needle-gap and sphere-gap data both show that overvoltages associated with failures are usually higher than overvoltages associated with routine switching.

The correlation of overvoltages with failures is shown in figure 6. About 40 per cent of all faults are accompanied by overvoltages of twice normal or greater. Only about six per cent of the faults are accompanied by overvoltages exceeding three times normal.

Comparing figures 4 and 5, it is seen that more overvoltages of a given magnitude occurred during 1919–28, when needle gaps were used, than during 1932–38, when sphere gaps were used. The reasons are threefold:

(*a*). The zones were directly interconnected in 1919–28, so that a system disturbance might leave a record at every spark-gap installation, instead of at only one or two as was the case for 1932–38.

(*b*). There were many more failures, hence many more chances of having system overvoltages, in the 1919–28 period than in 1932–38.

(*c*). Needle-gaps have a greater tendency than sphere gaps to flash over at less than the nominal setting as a result of ionization and accumulation of dust in the gap.

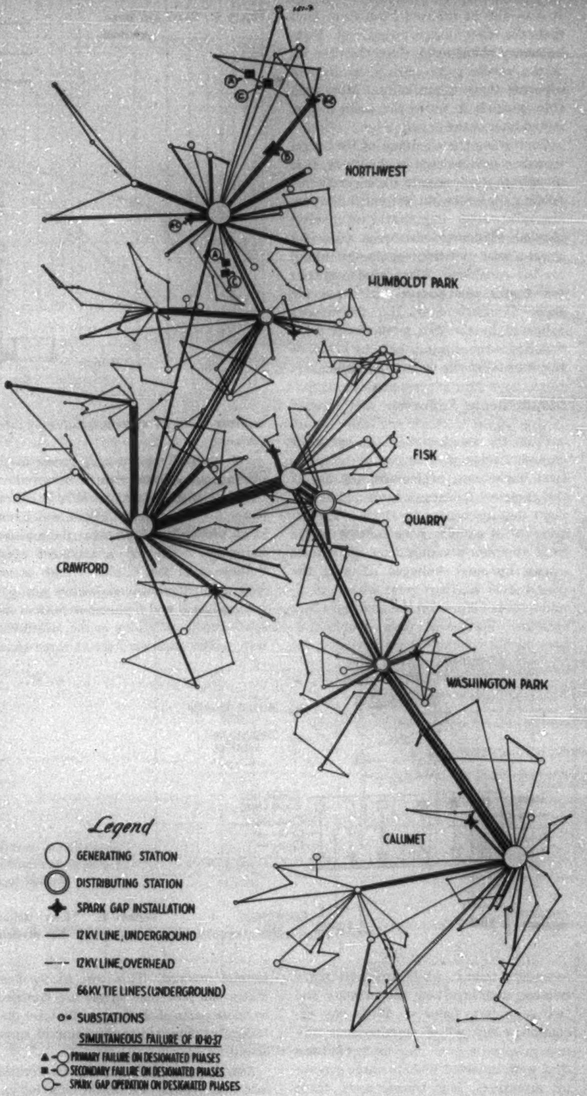

NORTHWEST
HUMBOLDT PARK
FISK
QUARRY
CRAWFORD
WASHINGTON PARK
CALUMET
A
B
C
Legend
GENERATING STATION
DISTRIBUTING STATION
SPARK GAP INSTALLATION
12 KV. LINE, UNDERGROUND
12 KV. LINE, OVERHEAD
66 KV. TIE LINES (UNDERGROUND)
SUBSTATIONS
SIMULTANEOUS FAILURE OF 10-10-37
PRIMARY FAILURE ON DESIGNATED PHASES
SECONDARY FAILURE ON DESIGNATED PHASES
SPARK GAP OPERATION ON DESIGNATED PHASES

Figure 1. Chicago 12-kv system

It may also be inferred from these facts that the overvoltages were not high-frequency phenomena, since the time lag of the needle gaps apparently did not suppress them appreciably. More reliable evidence to prove the point is presented later in the paper.

Concerning the accuracy of the surge-recording devices used in the study, it is thought that sphere gaps are very reliable, needle gaps somewhat less so, and klydonographs rather inaccurate. Capacity-coupled klydonographs were tried for about a year, but the records were found to be worthless. The klydonographs were connected in parallel with the needle gaps. Comparison of the magnitudes indicated by the two methods for each recorded surge showed that the klydonograph installations were not reliable.

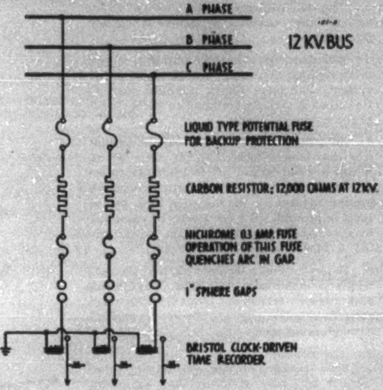

A PHASE
B PHASE
C PHASE
12 KV. BUS
LIQUID TYPE POTENTIAL FUSE FOR BACKUP PROTECTION
CARBON RESISTOR; 12,000 OHMS AT 12 KV.
NICHROME 0.3 AMP. FUSE OPERATION OF THIS FUSE QUENCHES ARC IN GAP
1" SPHERE GAPS
BRISTOL CLOCK-DRIVEN TIME RECORDER

Figure 3. Spark-gap surge recorder

Simultaneous Failures

From the standpoint of the operating engineer, surge phenomena are of practical importance principally because of the electrical failures of lines and equipment that are caused by them. The occurrence of simultaneous failures is the most common manifestation of troubles caused by surge voltages. During the period 1923–39, there were 54 sets of simultaneous failures on the Chicago 12-kv systems. Forty-nine were double failures, each involving one primary and one secondary failure, while five were triple failures, each involving one primary and two secondary failures. With one exception, every set of simultaneous failures involved a line. Apparatus failures that were involved include three generator armatures, four transformers, three reactors, three oil switches, and two bus structures.

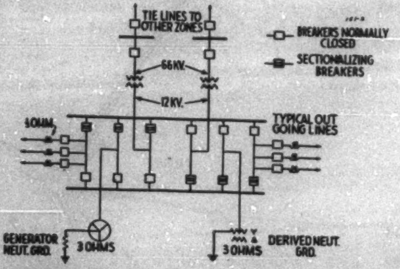


Figure 2. 12-kv station bus diagram

During the period under consideration there were 1,240 primary failures on the Chicago 12-kv systems. Of these failures, 4.35 per cent caused simultaneous failures to occur, thus increasing the total number of 12-kv system failures by 4.84 per cent.

An especially interesting group of simultaneous failures, which occurred in the Northwest zone, is illustrated on figure 1. The primary failure, which was in all probability *B* phase at the location indicated, originated a transient overvoltage on *A* and *C* phases that caused four distinct additional failures and left a record on *A* and *C* phases at both of the spark-gap installations in the Northwest zone. The gaps were set at three times normal voltage. This case nicely illustrates the fact that surges are confined to those parts of the system that are metallically connected to the point of surge origin.

Simultaneous failures may be divided into four types based on the time lag existing between the inception of the primary and secondary failure. These are:

1. Secondary failure occurs within one or two cycles of the primary failure's inception, i.e., simultaneous with the initial surge caused by the primary failure.

2. Secondary failure occurs during the steady-state fault period of the primary failure.

3. Secondary failure occurs simultaneous with the clearing of the primary failure.

4. Secondary failure occurs a definite length of time after the clearing of the primary failure.

No oscillograph records of type 3 failures have been obtained in Chicago. It is reasonable to assume, however, that such failures are possible, and so this type has been included. Most simultaneous failures appear to be of type 1. This is inferred from the fact that the oscillograph records usually show both failures existing at the start of the record, the start being about 0.1 second (six cycles) after the inception of the primary failure. Many records of this type have been obtained. Type 2 failures are fairly common, while type 4 failures are rather rare. Type 2 is illustrated by the oscillogram in figure 7. On this record, a *B*-phase-to-ground cable fault, located three miles from the station, existed from *P* to *Q*. *B*-phase voltage is depressed while *A* and *C* voltages are above normal. At *Q* the secondary fault occurred on *C* phase to ground in another cable one-fourth mile from the station. For three cycles, *Q* to *R*, both faults existed simultaneously. At *R*, the primary fault cleared, leaving the *C*-phase fault on the system. A few records of type 4 simultaneous failures have been obtained on the 12-kv system, but unfortunately these records are not suitable for reproduction. An entirely similar case is, however, available from the records of the 9-kv 25-cycle system and is shown in figure 8. The primary fault was on *A* phase, *S* to *T*. This fault cleared at *T* and the system was normal

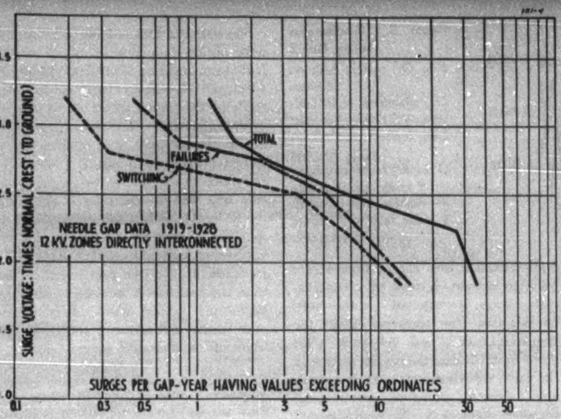


Figure 4. Surge data for 12-kv system, 1919–28

for 1.6 seconds, T to U. At U, the secondary fault occurred on B phase to ground, continuing to V, at which point it was cleared from the system. This interesting record clearly shows that a very definite time lag may occur between the period during which insulation is overstressed and the actual time of its electrical breakdown.

Staged Tests

It is easier to grasp the significance of the test data if the postulated theory of surge production is first understood. The discussion will be confined to surges caused by single line-to-ground faults, which are the highest occurring on the Chicago 12-kv system. The overvoltage resulting from the fault may properly be considered in three parts, as follows.

A. The "Make" Transient

If the initiation of the fault is a simple "make" operation involving no intermittent arcing, then on the approximation that all circuit elements are linear the effect is precisely as though the fault were replaced by a single-phase voltage generator of zero impedance, generating a cancellation voltage that is at all times exactly equal and opposite to the unfaulted steady-state voltage at the fault. The actual voltages and currents in all parts of the network may be obtained by superposing on the unfaulted steady-state condition the transient and ensuring steady state of the network as fed by the cancellation generator.

Figure 9 shows a schematic three-phase diagram of the 12-kv system at a generating station. There are n outgoing lines, each equipped with one-half-ohm reactors. The simplification of zero interphase coupling is introduced and the n conductors and reactors on each phase are grouped to give single parallel equivalents.

A fault is applied on C phase of one of the lines. The cancellation voltage, which may be thought of as a traveling wave if the fault is fairly remote from the station, travels into the station, where it initiates a complex transient whose frequencies are determined mainly by the inductance of the faulted line's reactor and the total system capacitance to ground. Let E be the normal phase-to-ground peak voltage. The theoretical limiting value for the cancellation transient at the bus, assuming infinite neutral impedance, is $4E$ if there are no outgoing lines other than the faulted line, and $2E$ if there are a large number of outgoing lines. This latter value is reduced slightly by the presence of a neutral grounding impedance of a few ohms, the reduction being greater with a resistance ground than with an inductance ground. In any case, the transient resulting from the incoming wave appears almost symmetrically on all three phases of the system. The resulting system overvoltage is the superposition of this cancellation wave and the normal steady state. It is reasonable to expect that for the Chicago

Figure 5. Surge data for 12-kv system, 1932–38

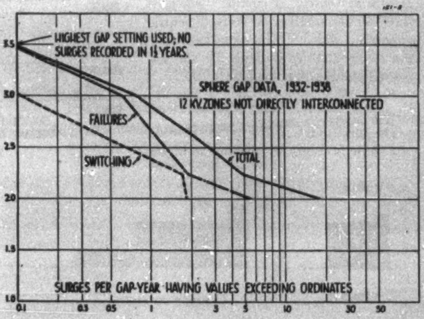
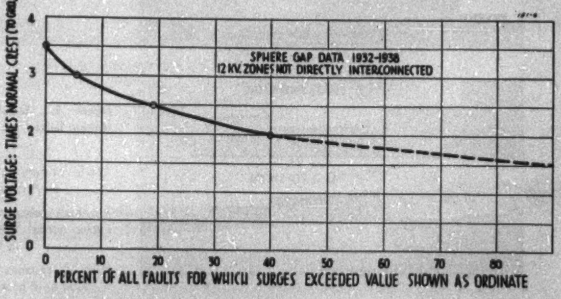


Figure 6 (below). Correlation of surges and failures



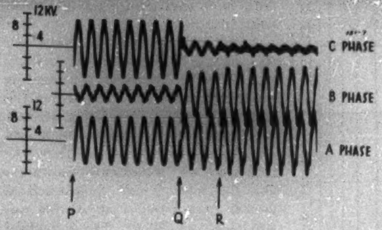


Figure 7. Simultaneous failure, type 2

P to *Q*: primary fault on *B* phase

Q: secondary fault initiated on *C* phase of another line; *B*-phase fault still on first line

Q to *R*: faults coexisting on *B* phase and *C* phase

R: *B*-phase fault cleared; *C*-phase fault still on but cleared later

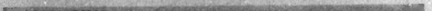
system this surge, which constitutes the first portion or "make" transient of the system overvoltage, should not exceed 3.5*E*. This value is obtained by adding a maximum expected cancellation transient of 2.5*E* to the normal steady-state value of *E*.

B. The Steady-State Fault Condition

After the transient of the applied cancellation wave has died out, the network conditions may be pictured as the sum of the normal steady state and the steady state of the cancellation generator. More simply, however, the conditions are obtained directly by the methods of symmetrical components. It is easily verified that for Chicago conditions with generator neutral grounding, which is three ohms resistive, or derived neutral grounding, which is three ohms inductive, the steady-state overvoltage on the unfaulted phases will not exceed $\sqrt{3}E$.

C. The "Break" Transient

The surge that results from the clearing of a fault is obtained precisely as in the fault initiation case except that a cancellation current, instead of a cancellation voltage, is "injected" at the fault. In this case, the transient effects the transition from a peak voltage of $\sqrt{3}E$ down to *E* whereas the reverse is true in the fault initiation case. It is reasonable to expect, therefore, greater total transient voltages on the fault "make" than on the break, assuming in all cases no restriking of the arc.



Some complete records of 12-kv surges were obtained in a series of 15 staged fault tests made at Northwest station in Chicago. The faults were thrown directly on the system, which was in normal everyday service. In each of the tests, a line-to-ground fault was applied by a special breaker to *A* phase of one outgoing line on the line side of the one-half-ohm line reactor. In some of the tests the 12-kv station bus was sectionalized to split the Northwest zone 12-kv system into two entirely independent parts (refer to figure 2). In the remainder of the tests the two busses were tied, which is the normal system setup. Four cycles prior to the application of the fault, the following oscillographs were initiated:

1. Cathode-ray oscillograph capacitatively coupled to *C* phase of the 12-kv station bus. Since the faults were applied on *A* phase, the transient overvoltages would appear on *C* and *B* phases. A film speed of six inches per cycle was used.

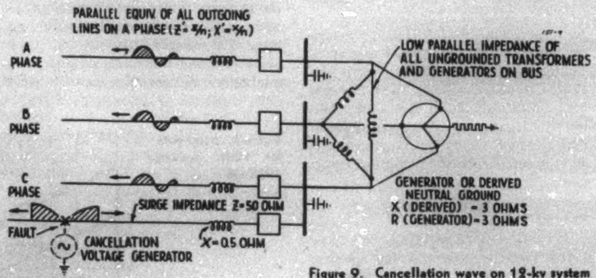

PARALLEL EQUIV. OF ALL OUTGOING LINES ON A PHASE (Z'=Z/n; X'=X/n)
A PHASE
B PHASE
C PHASE
SURGE IMPEDANCE Z=50 OHM
FAULT
CANCELLATION VOLTAGE GENERATOR
X=0.5 OHM
LOW PARALLEL IMPEDANCE OF ALL UNGROUNDED TRANSFORMERS AND GENERATORS ON BUS
GENERATOR OR DERIVED NEUTRAL GROUND
X (DERIVED) = 3 OHMS
R (GENERATOR)=3 OHMS

Figure 9. Cancellation wave on 12-kv system

2. Magnetic oscillograph to record *A*, *B*, and *C* phase potentials on both sections of the 12-kv station bus.

3. Magnetic oscillograph to record the current in the system neutral and *A*-phase fault current.

The *C*-phase potentials recorded by the cathode-ray oscillograph in two of the tests are shown in figure 10. The *A*-phase fault in these two cases consisted of a one-inch arc in air between copper bars. Eight cycles after the initiation of the faults they were cleared by the line breaker. Under each of the *C*-phase potential records, there is entered the cancellation potential, which starts at the time of fault and is equal and opposite to the normal potential on *A* phase. The transient resulting from the application of the cancellation wave is seen to last about one cycle. The first portion of the transient, corresponding to the first half cycle of the natural frequency of this transient, is seen to correlate, as it should,

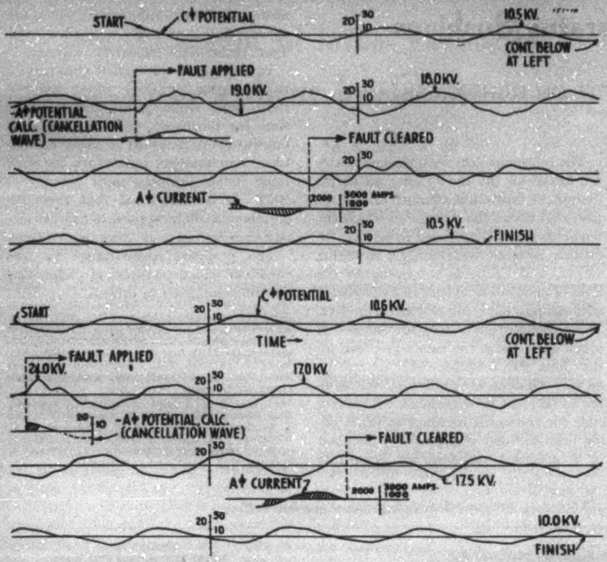


Figure 10. Oscillograms of staged fault tests

a (above)—12-kv zone sectionalized
b (below)—12-kv zone not sectionalized

exactly in both magnitude and polarity with the cancellation potential at the time of fault.

For the test in figure 10*a* the 12-kv bus was sectionalized, resulting in a higher transient frequency than in 10*b* where the systems were tied. The transient frequency did not exceed 500 cycles per second in any of the tests. The absence of higher frequencies, for the detection of which the cathode-ray oscillograph was well adapted, checks the conclusion derived from the spark-gap studies.

The overvoltage during the steady state of the fault is seen to be very close to $\sqrt{3}E$ as expected.

The transient overvoltage accompanying the clearing of the fault was less than $\sqrt{3}E$ in both cases shown in figure 10. Intermittent arcing is not indicated by any of the records obtained. The fault current during the break period is indicated under both oscillograms of figure 10. As is to be expected, the break occurred near a current zero.

The results of the remaining 13 fault tests are in accordance with the two cases described. In no case did the break transient give voltages in excess of $\sqrt{3}E$ and in no case was the faulted steady-state overvoltage in excess of $\sqrt{3}E$. In 5 of the 15 tests a surge voltage of between 24 and 26 kv ($2.27E$ to $2.45E$) was measured within one cycle of the initiation of the fault. In all of these cases the grounding was of the derived neutral type. In four tests with generator (resistance) grounding, the maximum surge potential did not exceed $2E$.

Summary

The highest transient overvoltages on the Chicago 12-kv system occur at the inception of single line-to-ground faults. They may be thought of as resulting from the superposition of a "cancellation wave" on the normal steady-state voltage of the system. The transient is not a high-frequency phenomenon and is not generated by an "intermittent-arcing" mechanism. The maximum crest values reached are in the neighborhood of $3.5E$ and persist for only the first cycle or two of the fault. For the remainder of the fault, including the clearance switching, the maximum overvoltage is $\sqrt{3}E$. These overvoltages may cause secondary (simultaneous) failures in any part of the connected system where the insulation is unable to withstand them for the short times they persist.

Report on Apparatus Bushings

AIEE JOINT COMMITTEE ON BUSHINGS

THE AIEE joint committee on bushings presented a report at the 1940 winter convention in which the committee proposed uniform requirements for the electrical characteristics of outdoor apparatus bushings. Since that time the committee has continued its work and has now completed a tabulation of proposed requirements for indoor and outdoor apparatus bushings rated 2,500 volts and higher.

The scope of the committee includes bushings for circuit breakers, transformers, regulators, and similar apparatus. It does not include wall and roof bushings potheads, and insulators for back-connected disconnecting switches.

The recommendations of the committee are based on the theory that an ample margin of insulation strength should be provided above the voltage to which the bushings may be subjected in service, including dynamic overvoltages, switching surges, and lightning surges up to the level of protection which can be provided with modern lightning protective devices. The impulse characteristics suggested are in accord with the values proposed by the joint AIEE-EEI-NEMA committee on co-ordination of insulation. All tests are to be made in accordance with a test code for apparatus bushings which is being prepared and which will be included in a proposed standard for apparatus bushings. The recommended values are expressed in terms of "withstand voltages" rather than "flashover voltages."

In order to avoid conflict with certain small indoor bushings which will not meet these requirements and to allow time for modifying the design of such bushings it is proposed to delay making these requirements mandatory for such small indoor bushings until January 1, 1943. It is understood that these requirements will be applied to any new design or redesigns made prior to that time.

The proposed requirements for the electrical characteristics of apparatus bushings are given in table I.

Paper 41-76, recommended by the AIEE committees on protective devices, electrical machinery, and power transmission and distribution, and presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941. Manuscript submitted November 18, 1940; made available for preprinting December 18, 1940.

Personnel of AIEE joint committee on bushings: R. T. Henry, *chairman;* F. S. Brown, J. B. Clem, I. W. Gross, L. H. Hill, J. T. Lusignan, A. C. Monteith, J. R. North, E. Peipho, M. S. Oldacre, A. J. A. Peterson, R. E. Pierce, C. B. Springer, P. C. Turk, F. J. Vogel, and L. Wetherill.

Table I. Proposed Standard Withstand Test Voltages for Apparatus Bushings

Insulation Classification[a] (Kv)	Low-Frequency Test RMS Kv[b]					Impulse Test 1.5x40-Microsecond Full Wave Crest Kv[b,d]		
	Outdoor Bushings					Outdoor Bushings		
	Large Apparatus[e]		Small Apparatus[f]		Indoor[g] Bushings			
	1 Min Dry	10 Sec Wet	1 Min Dry	10 Sec Wet	1 Min[c] Dry	Large[e] Apparatus	Small[f] Apparatus	Indoor[g] Bushings
1.2			10	6			30	
2.5	21	20	15	13	20	60	45	45
5.0	27	24	21	20	24	75	60	60
8.7	35	30	27	24	30	95	75	75
15	50	45	35	30	45[h]	110	95	110[h]
23	70	60	70	60	60	150	150	150
34.5	95	80	95	80	80	200	200	200
46	120	100	120	100		250	250	
69	175	145				350		
92	225	190				450		
115	280	230				550		
138	335	275				650		
161	385	315				750		
196	465	385				900		
230	545	445				1,050		
287.5	680	555				1,300		
345	810	665				1,550		

NOTES: [a] Bushings of a given insulation classification are in general recommended for apparatus having a rating up to and including the insulation classification of the bushing and may be used for apparatus of a higher voltage rating when adequate for the particular application.

[b] All values are withstand test values without negative tolerance.

[c] Wet test values are not assigned to indoor bushings.

[d] Either positive or negative waves may be used—whichever gives the lower value.

[e] Bushings for use in large apparatus are those intended for use in transformers rated above 500 kva, outdoor circuit breakers, and other apparatus of corresponding importance.

[f] Bushings for use in small apparatus are those intended for use in transformers rated 500 kva and less and other apparatus of corresponding importance.

[g] Bushings for use in indoor apparatus are those intended for use in indoor-type circuit breakers, instrument transformers, and other indoor apparatus except dry-type instrument transformers, air-cooled transformers, air-cooled regulators, and bushings used primarily for mechanical protection of insulated cable leads.

[h] Bushings for small indoor apparatus may be supplied to withstand a low-frequency test of 38 kv and an impulse test of 95 kv.

The Detection of Initial Failure in High-Voltage Insulation

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Introduction

IN a series of accelerated life tests[1] on laboratory samples of impregnated-paper insulation of the type commonly used in high-voltage cables, certain methods have been employed for limiting the breakdown current and thereby the attendant burning of the insulation. In this way it has been found possible to obtain partial failures in which the breakdown is limited to only a portion of the insulation wall, say one-half or one-third of the total number of layers, the remaining layers being apparently unaffected. In these partial failures, the burning is very slight and it is possible to obtain a very good idea as to the location of the initial point of failure and the nature of its subsequent progress. With one of the methods used it has been possible on occasion to limit the breakdown to only two layers of paper and to study the extension of the failure through adjacent layers. The method gives great promise for the study of the mechanism of breakdown under laboratory conditions. In this paper we are reporting some of the preliminary results of a more extended program of study still under way.

Probably the condition most conducive to the beginnings of failure under normal conditions of service is the presence of gas bubbles, pockets, or layers within the body of the insulation. Potential gradients at which ionization can begin in such gas pockets or layers are often exceeded in equipment for only moderately high voltages. Gas layers in the insulation circuit may be caused by imperfect impregnation, or as the result of bending or other mechanical strains in the normal processes of manufacture, transportation, and installation. In other cases gas spaces may result from the temperature variations attendant upon load changes. The accelerated life tests already referred to have indicated still a further cause of gas liberation, namely high-voltage stress on the oil layers or other free oil in impregnated-paper insulation, such as the channels between successive turns of paper tape. Study of the partial failures referred to clearly indicates that initial failure usually starts in the oil channel and is accompanied by the evolution of gas.

Under alternating stress gaseous ionization and other initial discharges occur first on the crests of the alternate half waves. This gives the discharges an impulse character so that in conjunction with the capacitance and inductance of the high-voltage circuit the discharges take on an oscillatory character. This fact has afforded a simple means of detecting the presence of such discharges and has been utilized almost exclusively by those who have studied the phenomenon.

Ripples on the crests of a-c voltage waves due to gas discharges were apparently first shown by Edward Bennett,[2] using the electromagnetic oscillograph. With the development of the cathode-ray oscilloscope, visual observation of the ripples is a simple matter, and this instrument is commonly used as a qualitative detector in all studies of this character. The earlier studies of Paine[3] and coworkers are well known. The possibility of more sensitive and quantitative studies has been shown by Arman and Starr,[4] the distinguishing characteristic of whose work is a bridge, filter, and amplifier circuit free from disturbance due to local discharges in the high-voltage circuit and extraneous to the specimen under study.

Measuring Equipment

In the present work the method of Arman and Starr, with certain modifications and extensions, has been used for the detection of initial internal gaseous discharge and the quantitative study of the subsequent stages of the discharge, first on impregnated-paper insulation originally containing no free gas, and also on several types of insulating wall with gas pockets and oil channels of controlled dimensions.

BRIDGES

Two types of bridge have been used. The first for voltages up to 30 kv was the modified Schering bridge due to Whitehead.[5] This bridge was in constant use for the measurement of power factor in the series of accelerated life studies on impregnated-paper insulation of cable type already referred to. At regular intervals in the life runs of these specimens the detection circuit of the present research was connected to the Schering bridge, following the periodic measurement of 60-cycle power factor in the life runs. In this way it was possible to fix within a few hours the time at which internal ionization began and in some measure to correlate the progress of internal ionization with changes in 60-cycle power factor and applied voltage.

For closer study of internal discharges, it appeared desirable to work with thinner specimens in which the critical gradients would be reached with the specimen continuously in the bridge circuit. Accordingly the simplified form of Schering bridge shown in figure 1 was constructed. C_x is the specimen and C_a is the air capacitor. Auxiliary studies indicated that up to ten kilovolts, the limit of bridge operation, the high-voltage circuit including the air capacitor was free from internal discharges.

BRIDGE TRANSFORMER

The detection equipment was connected to either of the bridges through a high-quality bridge transformer, with electrostatic shielding between the pri-

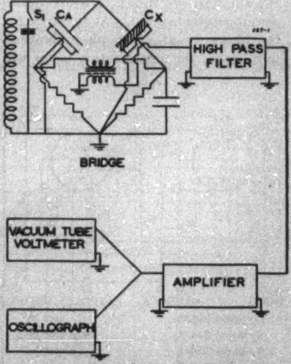

Figure 1. Discharge bridge circuit and connection of apparatus

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[1] For all numbered references, see list at end of paper.

mary and secondary windings, both of which were capacitively balanced with respect to ground. This permitted one end of the output winding of the transformer to be grounded without disturbing the symmetry of the bridge circuit. This is important because it permits the grounding of one side of the whole amplifying and detecting circuit, thereby permitting that the whole circuit be enclosed in a grounded shield. Circuit changes on the output side of the transformer thus have no influence on the balance of the bridge. The voltage ratio of the transformer was measured between 1 and 50 kilocycles and found to be closely uniform up to about seven kilocycles, and to decrease somewhat beyond. The over-all impedance ratio of the transformer based on its turn ratio was 4.4 to 1 which brought about an impedance match between the bridge and the high-pass filter.

FILTER

A filter having three full sections was adopted. The cutoff frequency, 3,600 cycles, was chosen because it was low enough to pass the band of oscillations due to internal discharges and high enough to reject any high harmonics of the 60-cycle charging current. The terminal impedance at each end was 5,500 ohms. Impedance matching at the input end of the filter was accomplished by the bridge output transformer. Impedance matching with the amplifier was effected by placing a noninductive resistor of 5,500 ohms across the filter.

AMPLIFIER

A resistance-capacitance coupled amplifier was inserted between the filter and the detector (see figure 1). Details of the amplifier are shown in figure 2 with circuit constants given. At 10,000 cycles, the frequency at which all calibrations were made, the amplification factor was 4,700. As a function of the input voltage saturation took place at an input voltage of 5.5 millivolts. Consequently all readings recorded were obtained with input voltage to the amplifier of five millivolts or less.

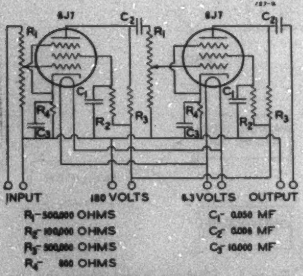


Figure 2. Amplifier

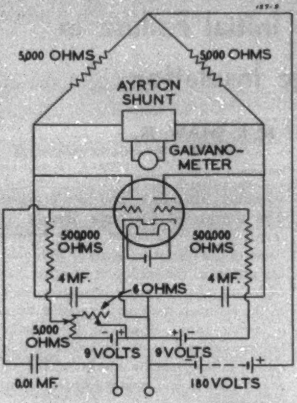


Figure 3. Circuit diagram of balanced vacuum tube voltmeter using type 6N7 tube

DETECTORS

A standard model of a well-known make of cathode-ray oscilloscope was used for qualitative measurements. An amplifier of gain 40 was incorporated in the oscilloscope. This, in conjunction with the external amplifier made it possible to realize an over-all gain of 40×4,700, or 188,000 at 10,000 cycles. This instrument is extremely useful, permitting ready and rapid visual observation of conditions obtaining in the circuit.

For quantitative measurements a balanced vacuum tube voltmeter as indicated in figure 3 was used. This arrangement is independent of small changes of battery voltage and the combination of duplex tube, decade resistance boxes, and Ayrton shunt for the galvanometer, results in a very sensitive quantitative instrument. The calibration of the meter with galvanometer of 120 megohms sensitivity with various settings of the shunt was found to hold over the range 60 cycles to 50 kilocycles. The voltage required for one centimeter deflection at maximum sensitivity was 13,000 microvolts. The over-all amplification factor between the input of the bridge transformer and the output of the amplifier as function of frequency may be had from the data in table I. At ten kilocycles the over-all amplification factor was 1,830. Hence a voltage of 13,000/1,830 or 7.1 microvolts gave a galvanometer deflection of one centimeter. In figure 4 the voltage for one centimeter deflection is plotted over the range 1 to 50 kilocycles.

Tests and Calibration of Equipment

TEN-KILOVOLT DISCHARGE DETECTION BRIDGE

A first necessary step in the use of this method is to balance the bridge for minimum effect in the detection apparatus, of local high-frequency discharges external to the specimen itself. Accordingly a small discharge gap consisting of a few layers of varnished cambric between two brass electrodes was connected between the bridge and the high side of the supply transformer, as shown in figure 1. The primary voltage is increased until discharge occurs in the gap, the latter being adjusted for discharge at relatively low voltage. The detecting circuit is then connected using the oscilloscope as indicator. The bridge is then balanced so as to bring to a minimum the signal on the screen of the oscilloscope. This minimum could be made practically zero, even though the ionization in the gap was strong enough to be audible several feet away.

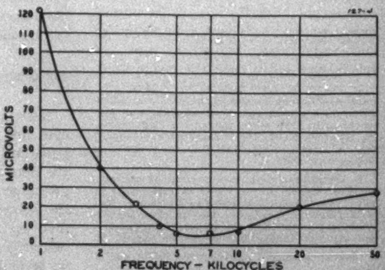


Figure 4. Voltage necessary at bridge terminals to cause one centimeter galvanometer deflection

In this method it is to be noted that stress is applied to the specimen under test at 60 cycles but that only a fraction of the total discharge current over a comparatively narrow band of frequencies reaches the detecting circuit. This fraction passes through the several elements of the complete circuit before reaching the final detecting instrument. It seemed advisable, therefore, to test the over-all sensitivity and accuracy of the equipment by comparison with the known properties of the breakdown strength of air in thin layers as related to pressure, through Paschen's law. The apparatus was a guarded capacitor consisting of a layer of air backed by a layer of glass for preventing complete breakdown. For the range of pressure between 15 centimeters and 76 centimeters Hg, the voltage stresses for initial ionization, as determined by the equipment, and as computed from Paschen's law are shown in figure 5 for the temperatures indicated. In figure 6, the critical stress as a function of the electrode spacing is shown in the solid curve; for comparison, the experimental results of Dubsky[6] on thin air films, are also plotted.

30-Kv Schering Bridge

The large physical size of this bridge resulted in enough pickup of extraneous high-frequency disturbance to serve as a guide in the preliminary balance without recourse to a discharge gap. The effect of the external discharges was readily balanced out by adjustment of the resistance arms and the variable capacitor. In this condition the bridge could be used for the study of initial discharges in specimens of impregnated-paper cable insulation on life test and at higher voltage gradients in these life specimens than possible with the ten-kilovolt bridge.

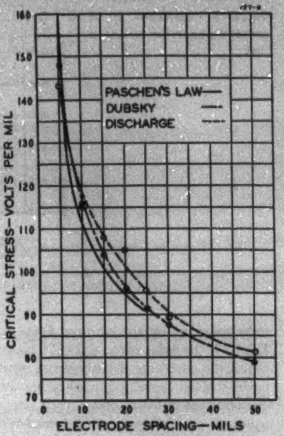


Figure 6. Critical stress for gaseous discharge between parallel plates as detected by discharge bridge and as function of separation of plates

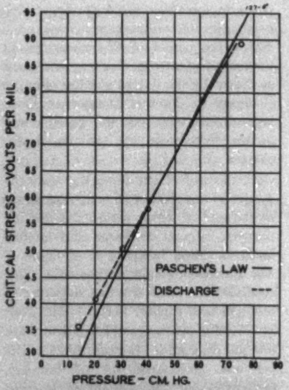


Figure 5. Critical stress for gaseous discharge between parallel plates as detected by discharge bridge and as function of pressure

Experimental Results

The method was applied to several types of insulation as follows: (*A*) laboratory specimens of impregnated paper as found in high-voltage cables, and originally free from gas bubbles or pockets; (*B*) single oil channels as found in cable insulation; (*C*) channels and bubbles of air of known dimensions, and subject to visual observation. The general method of test was the determination of the voltage gradient at which internal gaseous discharge begins and the increase in the "volume" of the latter with increasing gradient. At an early point it was found that the elapsed time of application of stress may have an important influence on both the critical discharge stress and on the "volume" of discharge at sustained high stress. The readings of the detecting instrument are obviously proportional to an integrated value of a product of the number of discharge areas and the intensity of discharge current in each area. In what follows the word "volume" has been used to indicate this integrated product.

(A). Cable Insulation

In the accelerated life tests on standard types of impregnated-paper insulation, the test samples consisted of 16 layers of 0.005-inch wood pulp paper one inch wide, spiraled cable fashion on one-inch diameter smooth brass tubes with $33^{1}/_{3}$ per cent overlay and a butt spacing of approximately $^{1}/_{64}$ inch. These samples were dried and impregnated with a low-viscosity high-grade cable oil in the usual way, and tested to break down under the impregnating oil. The accelerated step-up life test started at an average stress of 400 volts per mil on the specimen and the applied stress was increased by 3.12 per cent every four hours, thus giving a geometric increase such as to bring the stress to 700 volts per mil within three days. Under this program, the life of the specimen was from 60 to 80 hours, and the breakdown gradients from 600 to 800 volts per mil, at 50 to 65 kv on the test transformer.[1]

At four-hour intervals the life test was interrupted for the measurement of power factor over the range 100 to 400 volts per mil. Following the power-factor measurement, the galvanometer was cut out of

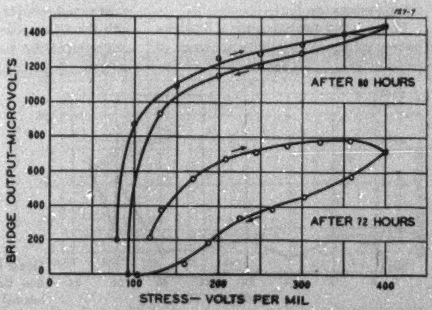


Figure 7. Ionization in impregnated-paper insulation

the Schering bridge and the discharge equipment connected. The bridge was then balanced for a negligible minimum reading of the discharge equipment due to external discharges in the high-voltage wiring, up to a voltage somewhat below that at which internal discharges begin in the sample. The equipment was then ready for observation.

First observation was made on the specimen after it had been on test for 24 hours reaching a stress of 467 volts per mil. No internal discharge was found at this point nor at subsequent intervals of 12 hours up to 60 hours with a final stress of 618 volts per mil. Positive readings of internal discharge were first observed after the specimen had been on test for 72 hours with a final stress of 679 volts per mil. In these observations it is to be recalled that the specimen was removed from the life test voltage, power factor and internal discharge being measured over the range 100 to 400 volts per mil.

The relation between internal discharge and stress, finally found after 72 hours of test, is shown in the lower curve of figure 7. As will be seen the volume of discharge increases with the stress, tending, however, to a maximum value and then to a somewhat lower value at the maximum stress of 400 volts per mil. The volume of discharge on decreasing stress is seen to be markedly lower than for increasing stress In another test, the stress was held constant at 400 volts per mil and it was found that the volume of discharge decreased with time and ultimately ceased entirely. This indicates that when internal discharge occurs, it will continue at substantially lower values of applied stress. However, there is a lower value of stress at which there will be a decrease and final disappearance of the volume of internal discharge. This decrease is probably due to gas absorption and oil sealing, or perhaps to internal burning or carbonization, short-circuiting the gas spaces involved.

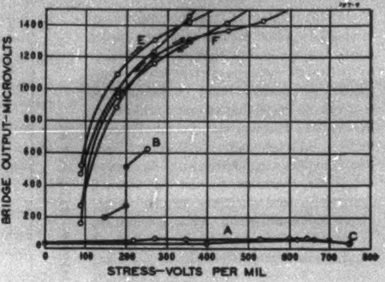
Power-factor measurements at 60 cycles corresponding to the above sequence of tests are given on the curve of figure 8. Point 7 is the value at the end of 72 hours before the discharge measurements were made. After sustained application at 400 volts per mil with cessation of internal discharge, the specimen was put back on high-voltage life test for one hour, then removed, and tested again for power factor and internal discharge. The power-factor value is given at point 8 and there was no evidence of internal discharge. The specimen was then returned to life test for eight hours with a final stress of 722 volts per mil, and during this interval two power-factor readings indicated the increase shown beyond point 8 in figure 8. The internal discharge now followed the upper curve of figure 7, which has the same general shape but greater magnitude than the earlier curve. The influence of time is still present though not so marked. It appears probable that owing to the higher stress, deterioration and new internal discharges are increasing more rapidly than the decreases shown in the tests at lower stress. On return to life test after these measurements, the specimen failed within two hours.

In order to be independent of the above life tests and to reach higher stresses at lower over-all voltage, similar specimens with only half the thickness of wall, were constructed, impregnated, and tested with the 30-kv Schering bridge. In this arrangement the stress could be carried to 800 volts per mil and internal discharge measurements made at all stresses up to this figure without interruption.

The specimen was first carried slowly up to 700 volts per mil with the result shown in curve *A* of figure 9. The level of discharge due to external influence is seen to be very low and uniform in the whole range of stress. After 12 hours rest, the power-factor stress relation shown by curve *A* of figure 10 was taken. The increase in power factor thus found seems to indicate that internal discharges are taking place in the specimen. None such were found however until after ten minutes' application of the maximum stress of 700 volts per mil, at which point a violent indication of internal discharge was found. The voltage was immediately removed and the specimen given a ten-minute rest. It was then found that internal discharge started at 100 volts per mil, with a further increase as indicated by curve *B* of figure 9. A power-factor observation taken at this point indicated an increase of value, and during the period of this measurement the volume of internal discharge increased as shown by the dotted portion of curve *B*, figure 9. Further increase of stress beyond 200 volts per mil gave a still further increase in the volume of discharge.

At the end of the foregoing test, the specimen was given a 70-hour rest period at the end of which period the relation between internal discharge and applied stress is shown in curve *C* of figure 9. The specimen is apparently in the same condition as when it was first subjected to test, perhaps slightly better. One hour at 700 volts per mil gave no indication of internal discharge. Increase to 750 volts per mil, maintained for 55 minutes, brought about the renewal of internal discharge. The voltage was immediately reduced to zero and then gradually reapplied, discharge beginning at 620 volts

Figure 9. Ionization in impregnated-paper half-wall specimen



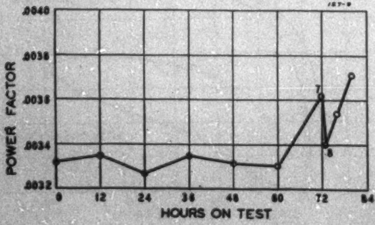


Figure 8. Power-factor variation before and after initial internal discharge

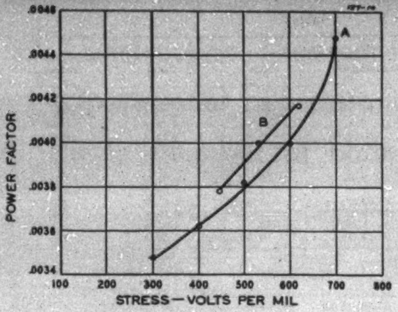


Figure 10. Power factor of half-wall specimen

per mil. The power factor had also increased (see curve *B* of figure 10).

Two discharge stress runs were then made, with a short-time interval between runs. The results are shown in curves *E* and *F* of figure 9. Note that the volume of discharge in the second run was less than in the first, suggesting a slight recovery in the interval between runs.

After another 12-hour rest period, the specimen was again found to be free of internal discharge after 700 volts per mil, maintained for eight hours. The stress was then increased to 750 volts per mil when internal discharge began immediately, but the specimen failed before a quantitative measurement could be taken.

(B). Oil Channels in Small Specimens

In order to investigate the value of stress at which internal discharges begin in an oil channel in impregnated paper, a group of two- and three-layer impregnated-paper specimens were prepared and tested between horizontal, parallel, polished brass plates. The three-layer specimens consisted of two sheets of impregnated paper and a third sheet in which a slit was made. In the two-layer specimens, one layer was continuous and the other contained a slit. The slits were the same width (0.0156 inch) as the oil channels in the life test specimen referred to above; their length was three inches. Drying and impregnation were in accordance with three-layer and two-layer specimens referred to above.

Preliminary tests were made with three-layer and two-layer specimens of continuous sheets of paper, and it was found that these could be carried to 1,100 and to 1,200 volts per mil respectively, and maintained for six hours without evidence of internal discharge.

For testing the specimens containing slit papers forming oil channels, the following schedule was used: (*a*) 15 minutes at 1,000 volts per mil in the oil channel (ratio of dielectric constants, impregnated paper to oil: 2); (*b*) three per cent per minute increase in stress to 1,600 volts per mil in channel; (*c*) five per cent increase in stress at five-minute intervals from 1,600 volts per mil to incipient failure.

This accelerated schedule is necessary to eliminate the influence of time found in the earlier studies. In two two-layer specimens, initial internal discharge was found at 1,530 and at 1,860 volts per mil in the oil channel; in the three-layer specimens the values found were 1,600 and 1,630 volts per mil. These values are somewhat higher than those reported by Whitehead[1] which were in the range 1,350 to 1,550 volts per mil. In the latter case, however, the specimens were under high stress a much longer time. In the present case all of the incipient discharges eventually produced punctures originating in the oil channel.

(C). Tests on Air Bubbles in Oil Channels

A special test cell permitted the insertion of air bubbles in the oil channel in a slit in impregnated paper. Visual observation was possible through the upper electrode, consisting of a salt solution in a tray with plate glass bottom. Special provisions permitted the operation of the cell up to ten kilovolts without disturbance in the detecting circuit due to discharges from the guard circuit.

With paper in place, the whole apparatus was evacuated to one millimeter Hg at 40 degrees centigrade for two hours, at which point all gas has ceased to come out of the oil. A test was then made at low stress which showed the absence of discharges. The upper electrode was then carefully moved to one side, exposing the channel. An air bubble was then introduced by means of an eye dropper, and the electrode then brought back into place.

The gaseous discharge as a function of applied stress was measured on bubbles of lengths, 47.2, 98.5, and 132 mils, and width of 15.6 mils. The quantitative results for bubbles of the same length varied somewhat but certain characteristics were invariably repeated. Three curves, for temperature 40 degrees centigrade are shown in figure 11. For shorter bubbles, the discharge in general increases with increasing stress and decreases with decreasing stress. For the longer bubbles the conditions become erratic and no definite behavior is indicated which may be correlated with the results on the cable specimens. Using the results on the shorter bubbles as a measure, it is suggested that the initial discharges in the cable specimens begin bubbles less than 0.1 inch in length, and 0.03 inch in width.

Discussion

The peculiar relationships between internal discharge, stress, and time noted in connection with the cable specimens may perhaps be best explained in terms of the frequent observation that first evidences of breakdown occur in the oil

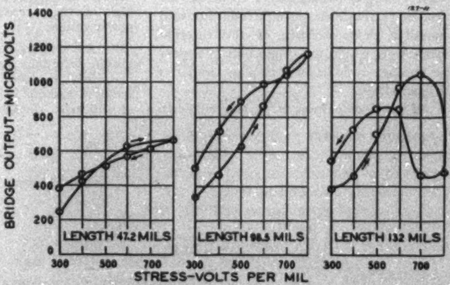


Figure 11. Ionization in air bubbles

Table I

Frequency (Kilocycles)	Loss or Gain* (Decibels)				Amplification Factor
	Amplifier	Transformer	Filter	Total	
1	+74.0	− 3.4	−30.0	+40.6	107
2	+74.0	− 3.4	−20.0	+50.6	338
3	+74.0	− 3.4	−15.0	+55.6	600
4	+73.8	− 3.4	− 8.0	+62.4	1,300
5	+73.8	− 3.4	− 4.2	+66.2	2,040
7	+73.5	− 4.2	− 2.3	+67.0	2,240
10	+73.3	− 8.0	0	+65.3	1,830
15	+72.2	−11.3	− 1.4	+59.5	920
20	+70.8	−12.0	− 3.0	+55.8	620
30	+69.3	− 9.6	− 4.0	+55.7	606
40	+67.2	− 7.6	− 4.6	+55.0	561
50	+66.0	− 7.6	− 5.0	+53.4	470

*+ indicates a gain; − indicates a loss.

channels and that the initial stages of this breakdown are accompanied by the liberation of gas. At initial discharge in the oil, corona formation results in the evolution of gas and polymerization. Once a bubble of gas is formed, it may be ionized at a value of stress much lower than that causing the original discharge. The gas bubble, however, is not stable and may be reabsorbed into the oil. This is responsible for the variation with time of the intensity or volume of discharge, and also for the complete recovery following a long rest period. The healing effect of time without stress is particularly noticeable in the thicker cable specimens on life test at higher stresses. Interruption of the high stress and subsequent measurement of power factor at very much lower stress provides opportunity for healing action due to gas absorption. It also probably accounts for the steady decrease of internal discharge even when stress is applied to the specimen, when this stress is substantially lower than that at which the internal discharge was first initiated.

The peculiar loops showing decreases of volume of discharge with increasing stress and vice versa noticed in the impregnated paper and air bubble tests may perhaps be explained as due to variations in pressure in gas pockets. A more or less rapid increase in pressure causing a reduction in the volume of discharge is followed by a slow release of pressure with corresponding increase in the volume of discharge.

The increase in the strength of the insulation with successive discharge cycles may possibly be explained by the process of polymerization. The wax which replaces the oil has high dielectric strength and prevents this particular spot from ionizing again. With increasing stress, discharge will first occur at a different point. It also appears possible that surface discharges, tree patterns, and the like, may at times result in the short circuit of a gas pocket, thus reducing the ionizing stress and causing a reduction and possible disappearance of the internal discharge, which may then only reappear in another cycle in another place, and at a higher stress.

Of particular interest is the evidence that the time of application of stress is an important factor in fixing the value of stress at which discharge appears; for example in the thin-wall cable specimens, the stress was taken up to 700 volts per mil without immediate evidence of discharge. Moreover at this high value of stress a considerable time was required before internal discharge appeared.

The tests on oil channels indicate that for short times the strength of these channels is quite high and probably very close to that of the oil alone. The lower values reported by Whitehead show the influence of time due perhaps, as suggested by him, to contact with the paper. The peculiar results found in the observation on air bubbles may perhaps be due to temperature, to saturation conditions in the gas, or to subsequent changes in volume or pressure in the gas. Further study is in progress, notably as to the time element, both under stress, and after its removal.

Conclusions

(1). Internal discharges occur in impregnated-paper insulation well in advance of failure and once initiated continue at stresses much below the breakdown values commonly attributed to this type of insulation.

The stresses at which initial discharges begin in insulation originally free of gas bubbles are from seven to eight times as high as the stresses usually obtaining in power cables.

(2). An improved method for detecting initial internal discharges in insulating materials is described. Calibrated in terms of gaseous ionization in thin air films of different thicknesses and at different pressures, it is found to detect initial discharge and its variation in accordance with Paschen's law.

(3). The seat of incipient failure in the samples of impregnated-paper insulation here studied is the oil channel between successive turns of paper tape. An initial discharge is followed by the evolution of gas which is ionized on successive alternating wave crests; the resulting oscillations are utilized for detecting the initial discharge and as a measure of the subsequent volume of discharge.

(4). Specimens stressed to the point where initial discharge begins may be brought back to apparently normal condition by allowing them to stand without stress for a period of time. This is attributed to the absorption of gas by the oil.

(5). A plus or minus variation with time under stress below the critical value, may occur in the volume of internal discharge. This is attributed to changes in pressure, or in volume of ionized gas, or to the absorption of gas by the oil.

Bibliography

1. The Dielectric Strength and Life of Impregnated-Paper Insulation, Part I, **J. B. Whitehead.** AIEE Transactions, volume 59, 1940, pages 715–20.

2. An Oscillograph Study of Corona, **Edward Bennett.** AIEE Transactions, volume 32, 1913, page 1787.

3. Bulletin No. 260, **J. T. Tykociner, H. A. Brown, and E. B. Paine.** Engineering Experiment Station, University of Illinois, 1933, page 6.

4. The Measurement of Discharges in Dielectrics, **A. N. Arman and A. T. Starr.** *Journal* IEE, volume 79, July 1936, page 67.

5. Influence of Residual Air and Moisture in Impregnated-Paper Insulation, **J. B. Whitehead and F. Hamburger.** AIEE Transactions, volume 47, 1928, page 314.

6. The Dielectric Strength of Air Films Entrapped in Solid Insulation and a Practical Application of the Problem for Alternator Coils and Cables, **F. Dubsky.** AIEE Transactions, volume 38, 1919, pages 537–58.

A Construction Theorem for Evaluating Operational Expressions Having a Finite Number of Different Roots

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ASSOCIATE AIEE

IN the course of the past 25 years Heaviside's direct operational method of solving differential equations has, to a large extent, been replaced by solutions involving functional transformations. While mathematically sound this trend has resulted in placing a powerful analytical tool far above the mathematical equipment of the average engineer. With the exception of the celebrated expansion theorem or the method of partial fractions no *direct* method appears to have been discovered by which the average engineer or engineering student could solve even relatively simple differential equations by Heaviside's calculus. Several expansion theorems have been developed[1,2,3,4] to cover cases where repeated roots are present but none of these have been set up in a form which could be used by undergraduates. It does not appear that any direct general procedure has ever been suggested to cover the usual type of asymptotic solutions which appear in problems in electrical engineering.

It is the purpose of this paper to develop a direct method for evaluating operational expressions involving a finite number of different roots. For rational expressions the theory is based upon the usual procedure in obtaining the particular integrals of ordinary differential equations with constant coefficients. The procedure is heuristically extended to irrational operators by analogy so that its greatest utility lies in the field of asymptotic solutions.

The term "construction" has been applied to this theorem because the operations to be performed are only indicated as contrasted with the general solutions which characterize the "expansion type" theorems.

In order that this paper may be of value to engineers who are not familiar with integration in the complex plane no mathematically rigorous proof will be given. An outline for such a proof is presented in the last section.

Definitions and Fundamental Operations

Consider the steps necessary in solving for the particular integral of an ordinary linear differential equation with constant coefficients. As an example take the equation

$$(D+a)i=\epsilon^{bt}t \tag{1}$$

where D is defined as the usual derivative operator of differential equations d/dt and t is the independent variable time. i is the dependent variable and is usually the current in amperes for electrical work. a and b are constants and $b>-a$. To solve for the steady-state current, or particular integral, divide both sides of the equation by $(D+a)$

$$i_s=\frac{1}{(D+a)}\epsilon^{bt}t \tag{2}$$

The exponential can now be "shifted" from right to left.

$$i_s=\epsilon^{bt}\frac{1}{(D+a+b)}t \tag{3}$$

The next step is to expand the denominator in ascending powers of D by division or by the binomial theorem

$$i_s=\frac{\epsilon^{bt}}{a+b}\left(1-\frac{D}{(a+b)}+\frac{D^2}{(a+b)^2}\cdots\right)t \tag{4}$$

Performing the operations indicated

$$i_s=\frac{\epsilon^{bt}}{a+b}\left(t-\frac{1}{a+b}\right) \tag{5}$$

which is the steady-state solution for equation 1 Notice that no arbitrary constants appear and that the process involves differentiation but no integration. This is important since the usual Heaviside operator p can be substituted in place of D. $1/p$ is to mean $\int_0^t dt$. In order to identify this general process a new operator will be defined.

Definition of a Bar Operator: Two bars placed over an operator $\overline{\overline{(p+a)}}^n$ *are to indicate the complete operation of expanding the operator in ascending powers of p by the binomial theorem and performing the indicated differentiations. n may be a positive or negative real number including fractions. If this operator operates on a linear combination of powers of t multiplied by an exponential the first operation is always a "shift" of the exponential.*

By the use of the bar operator the result shown in equation 5 can be expressed as

$$i_s=\overline{\overline{(p+a)}}^{-1}\epsilon^{bt}t \tag{6}$$

Notice that equation 6 without the double bar might be confused with the equivalent complete operational solution which would include a transient term not indicated by (6).

These bar operators have several properties which are of interest when used with the fundamental and well-known operational expression

$$\frac{p}{(p+a)^n}1=\frac{\epsilon^{-at}t^{n-1}}{\Gamma(n)} \tag{7}$$

n may be a positive integer, proper or improper fraction and a is a constant, real or complex. These properties are easily proved so that no proof will be given here.

1. Bar operators are commutative. That is, they may be used in any order to bring about the same result.

$$\overline{\overline{(p+b)}}^m\,\overline{\overline{(p+c)}}^r\left[\frac{\epsilon^{-at}}{\Gamma(n)}t^{n-1}\right]=\overline{\overline{(p+c)}}^r\,\overline{\overline{(p+b)}}^m\left[\frac{\epsilon^{-at}}{\Gamma(n)}t^{n-1}\right] \tag{8}$$

Where $a\neq b\neq c$ and r and m are positive.

2. Reciprocal bar operators perform reciprocal operations.

$$\overline{\overline{(p+b)}}^n\left[\overline{\overline{(p+b)}}^{-n}\frac{\epsilon^{-at}}{\Gamma(n)}t^{n-1}\right]=\frac{\epsilon^{-at}}{\Gamma(n)}t^{n-1} \tag{9}$$

3. In the case of "equal roots"

$$\overline{\overline{(p+a)}}^m\left[\frac{\epsilon^{-at}}{\Gamma(n)}t^{n-1}\right]=\frac{\epsilon^{-at}t^{n-m-1}}{\Gamma(n-m)} \tag{10}$$

m and n are both positive integers or fractions. If $(n-m)$ is a negative integer or zero the gamma function of this number is infinite and the result is zero.

4. The case of unequal roots

$$\overline{\overline{(p+b)}}^m\left[\frac{\epsilon^{-at}}{\Gamma(n)}t^{n-1}\right]=\epsilon^{-at}(b-a)^m\overline{\overline{\left(1+\frac{p}{b-a}\right)}}^m\frac{t^{n-1}}{\Gamma(n)} \tag{11}$$

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[1] For all numbered references, see list at end of paper.

n is positive but m may be positive, negative, or a fraction. $a \neq b$.

5. If $b=0$ and $n=1$ in case 4 the result is

$$\overline{(p)}^{m}\epsilon^{-at}=(-a)^{m}\epsilon^{-at} \tag{12}$$

For integral positive powers of m the bar operator behaves as an ordinary differential operator. If m is a fraction or has a negative value, however, the results are not the same as for the equivalent operational procedure. Purely imaginary terms obtained in this way must be set equal to zero after the final result has been obtained.

Since the bar operators behave in exactly the same manner for fractional values of n as for integral values a "fractional power" differential equation may be set up as easily as an "integral power" differential equation. As an example

$$\overline{(p+a)}^{1/2}i=\epsilon^{bt}t \tag{13}$$

and the steady-state solution is, as in equation 6

$$i_s=\overline{(p+a)}^{-1/2}\epsilon^{bt}t \tag{14}$$

It should be pointed out that these "fractional power" differential equations have been set up by analogy with "integral power" differential equations; a purely formal procedure.

Operational Differential Equations

An operational differential equation is to be defined for use in this paper as any one of the equivalent "fractional" or integral power differential equations which may be set up from an operational expression such as

$$i=\frac{p}{(p+a_1)^{n_1}(p+a_2)^{n_2}(p+a_3)^{n_3}\ldots(p+a_k)^{n_k}}1 \tag{15}$$

No loss in generality will result in the following theory if only three roots are considered since the process may obviously be extended to cover any number of roots. Consider the operational expression

$$i=\frac{p}{(p+a_1)^{n_1}(p+a_2)^{n_2}(p+a_3)^{n_3}}1 \tag{16}$$

Using the operator in the numerator in turn with each of the root functions in the denominator to form an expression corresponding to equation 7 the three most easily recognizable operational differential equations are

$$\overline{(p+a_2)}^{n_2}\overline{(p+a_3)}^{n_3}i=\frac{\epsilon^{-a_1t}t^{n_1-1}}{\Gamma(n_1)} \tag{17a}$$

$$\overline{(p+a_1)}^{n_1}\overline{(p+a_3)}^{n_3}i=\frac{\epsilon^{-a_2t}t^{n_2-1}}{\Gamma(n_2)} \tag{17b}$$

$$\overline{(p+a_1)}^{n_1}\overline{(p+a_2)}^{n_2}i=\frac{\epsilon^{-a_3t}t^{n_3-1}}{\Gamma(n_3)} \tag{17c}$$

The only restrictions are that $a_1 \neq a_2 \neq a_3$ and n_1, n_2, and n_3 are positive integers, proper or improper fractions. Notice that if the n's are positive integers the bars could be removed from the bar operators on the left-hand sides of equations 17 since they would then be ordinary differential equations any one of which could be represented operationally by (16).

It is now to be proved that in the case of integral positive values of n the unique solution for equations 17 consists of the sum of the particular integrals of (17*a*), (17*b*), and (17*c*). Actually these operations may be extended to fractional values of n but the proof to be given here is not mathematically rigorous. In this case the result usually includes an asymptotic series.

The following expression for $i(t)$ will satisfy equations 17.

$$\begin{aligned}i=&\frac{1}{(p+a_2)^{n_2}(p+a_3)^{n_3}}\frac{\epsilon^{-a_1t}t^{n_1-1}}{\Gamma(n_1)}+\\&\frac{1}{(p+a_1)^{n_1}(p+a_3)^{n_3}}\frac{\epsilon^{-a_2t}t^{n_2-1}}{\Gamma(n_2)}+\\&\frac{1}{(p+a_1)^{n_1}(p+a_2)^{n_2}}\frac{\epsilon^{-a_3t}t^{n_3-1}}{\Gamma(n_3)}\end{aligned} \tag{18}$$

since when (18) is substituted back into, say (17*a*)

$$\begin{aligned}&\overline{(p+a_2)}^{n_2}\overline{(p+a_3)}^{n_3}\times\\&\quad\left[\frac{1}{(p+a_2)^{n_2}(p+a_3)^{n_3}}\frac{\epsilon^{-a_1t}t^{n_1-1}}{\Gamma(n_1)}\right]+\\&\overline{(p+a_2)}^{n_2}\overline{(p+a_3)}^{n_3}\times\\&\quad\left[\frac{1}{(p+a_1)^{n_1}(p+a_3)^{n_3}}\frac{\epsilon^{-a_2t}t^{n_2-1}}{\Gamma(n_2)}\right]+\\&\overline{(p+a_2)}^{n_2}\overline{(p+a_3)}^{n_3}\times\\&\quad\left[\frac{1}{(p+a_1)^{n_1}(p+a_2)^{n_2}}\frac{\epsilon^{-a_3t}t^{n_3-1}}{\Gamma(n_3)}\right]=\\&\qquad\qquad\frac{\epsilon^{-a_1t}t^{n_1-1}}{\Gamma(n_1)}\end{aligned} \tag{19}$$

The first term of (19) gives the same time function by (9) while the next two terms become zero since they involve expressions having "equal roots" as in (10). Equivalent results are obtained if (18) is substituted in equations 17*b* and 17*c*. That is, the solution (18) satisfies each of the operational differential equations 17*a*, 17*b*, and 17*c*. In terms of ordinary differential equation theory the first unit of (18) is the particular integral of (17*a*) and the second and third units together form the complementary function with constants of integration already evaluated. Thus the particular integral of one of the equations 17 forms a part of the complementary function for each of the other equations. Since the three original equations are known to have identical operational solutions equation 18 is always a unique evaluation of the operational expression (16). If the n's are positive integers the result (18) is always in a closed form. If (18) is extended to fractional values of n the result usually contains one or more asymptotic series.

The Construction Theorem

The process outlined in the last section can obviously be extended to cover any expression of the form (15) so that the equivalent time function would be given by:

$$i=\sum_{i=1}^{k}\frac{1}{\prod\limits_{j=1\ j\neq i}^{k}(p+a_j)^{n_j}}\frac{\epsilon^{-a_it}t^{n_i-1}}{\Gamma(n_i)} \tag{20}$$

The denominator of the bar operator of (20) is the same as that of (15) except that $(p+a_i)^{n_i}$ is omitted.

If the original operational expression (15) had a constant in the numerator instead of the operator p equation 20 would represent the solution for $pi(t)$. The final solution for $i(t)$ could be easily obtained, except in the case where (20) represents an asymptotic series, by operating on the result with p^{-1}. If the original expression had a numerator $F(p)$ consisting of a polynomial in integral powers of p equation 20 could be operated on with $F(p)$ to give the new solution. The integration must be performed before the differentiations or the net result of dividing $F(p)$ by p applied to equation 20.

The result may be given in the form of a "construction theorem" to evaluate operational expressions.

$$\begin{aligned}&\frac{F(p)}{\prod\limits_{i=1}^{k}(p+a_i)^{n_i}}1=\frac{F(p)}{p}\times\\&\quad\left[\sum_{i=1}^{k}\frac{1}{\prod\limits_{j=1\ j\neq i}^{k}(p+a_j)^{n_j}}\frac{\epsilon^{-a_it}t^{n_i-1}}{\Gamma(n_i)}\right]\end{aligned} \tag{21}$$

The restrictions for using this theorem are:

1. Functions having equal roots must be combined so that no two roots are identical including zero. The roots may be real or complex. (Two roots which are the same except for the sign are different roots.)

2. $F(p)$ is a polynomial containing only constants or constants multiplied by integral positive powers of p. In cases where the network is impulsive $F(p)$ will contain higher powers of p than the denominator. The result will not, in general, contain the impulsive terms which may be obtained in the usual manner.[8]

3. n_i is a positive integer, positive or negative proper or improper fraction.

4. In cases where the final result contains purely imaginary terms (as, for instance, in the case of $\sqrt{p}\epsilon^{-at} = \sqrt{-a}\epsilon^{-at}$ it is necessary to specify that such terms be dropped after all operations have been performed.

5. The theorem cannot be used in this from when $F(p)/p$ would operate on such terms as t^m where $m \leqq -1$. This restriction is necessary to prevent the "integration" of an asymptotic series.

The construction theorem in this form is more useful than the more complete form to be given which removes the last restriction. An operational expression which contains a fractional power root function in the numerator can usually be cleared of such terms by the usual algebraic procedure.

The last restriction may be removed by writing the left-hand side of equation 21 so as to segregate the zero root function. If $a_k = 0$ and $n_k = q$ then

$$\frac{F(p)}{p^q \prod_{i=1}^{k-1}(p+a_i)^{n_i}} 1 =$$

$$F(p)\left[\frac{p}{p^{q+1}\prod_{i=1}^{k-1}(p+a_i)^{n_i}}\right]1 =$$

$$F(p)\left\{\sum_{i=1}^{k-1}\frac{1}{p^{q+1}\prod_{j=1,\, j\neq i}^{k-1}(p+a_j)^{n_j}}\frac{\epsilon^{-a_i t}t^{n_i-1}}{\Gamma(n_i)} + \frac{1}{\prod_{i=1}^{k-1}(p+a_i)^{n_i}}\frac{t^q}{\Gamma(q+1)}\right\} \quad (22)$$

q is a positive integer or fraction. In this case $a_i \neq 0$. Multiplying both numerator and denominator by p arranges the expression inside the square brackets in the form of equation 15 so that the solution is given by the last line which is equivalent to equation 21. This final form of the construction theorem appears to be perfectly general under the limitations imposed by the first four restrictions. Only differentiations are involved which may be performed on any time function which may appear.

Illustrative Example

Evaluate the operational expression

$$i = \frac{p+c}{(p+a)(p+b)} 1 \quad (23)$$

Using the first form of the construction theorem (21) $F(p) = p+c$ and (23) can be written

$$i = \left[\frac{p+c}{p}\right]\left[\frac{p}{(p+a)(p+b)} 1\right] \quad (24a)$$

Two of the fundamental forms (7) may be recognized in (24*a*)

$$i = \left[1+\frac{c}{p}\right]\left[\frac{1}{(p+b)}\left\{\frac{p}{(p+a)} 1\right\} + \frac{1}{(p+a)}\left\{\frac{p}{(p+b)} 1\right\}\right] \quad (24b)$$

From (7)

$$\frac{p}{(p+a)} 1 = \epsilon^{-at}$$

and

$$\frac{p}{(p+b)} 1 = \epsilon^{-bt}$$

and (24*b*) becomes

$$i = \left[1+\frac{c}{p}\right]\left[\frac{1}{(p+b)}\epsilon^{-at} + \frac{1}{(p+a)}\epsilon^{-bt}\right] \quad (24c)$$

Shifting the exponentials

$$i = \left[1+\frac{c}{p}\right]\left[\epsilon^{-at}\frac{1}{b-a} + \epsilon^{-bt}\frac{1}{a-b}\right] \quad (24d)$$

Performing the integration and rearranging the final result is

$$i = \frac{c}{ab} + \frac{1}{b-a}\left[\left(\frac{a-c}{a}\right)\epsilon^{-at} + \left(\frac{c-b}{b}\right)\epsilon^{-bt}\right] \quad (24e)$$

Cable With Terminal Inductance

One of the interesting problems originally solved by Heaviside[6] is the case of a unit voltage applied to an infinite cable with terminal inductance. The operational expression giving the voltage at the head end of the cable is of the form

$$V_0 = \frac{1}{1+(bp)^{3/2}} 1 \quad (25)$$

where b is a positive constant. Write this as

$$V_0 = \frac{1-(bp)^{3/2}}{1-(bp)^3} 1$$

$$= \frac{1}{b^3}\left\{b^{3/2}p \times \left[\frac{p}{\sqrt{p}\left(p-\frac{1}{b}\right)\left(p+\frac{1-j\sqrt{3}}{2b}\right)\left(p+\frac{1+j\sqrt{3}}{2b}\right)}\right] - \frac{1}{p} \times \left[\frac{p}{\left(p-\frac{1}{b}\right)\left(p+\frac{1-j\sqrt{3}}{2b}\right)\left(p+\frac{1+j\sqrt{3}}{2b}\right)}\right]\right\} 1 \quad (26)$$

The construction theorem may now be used for both terms.

$$V_0 = b^{3/2}\left[\frac{p}{\sqrt{p}\left(p-\frac{1}{b}\right)\left(p+\frac{1-j\sqrt{3}}{2b}\right)} \times \epsilon^{-\left(\frac{1+j\sqrt{3}}{2b}\right)t} + \frac{p}{\sqrt{p}\left(p-\frac{1}{b}\right)\left(p+\frac{1+j\sqrt{3}}{2b}\right)}\epsilon^{-\left(\frac{1-j\sqrt{3}}{2b}\right)t} + \frac{p}{\sqrt{p}\left(p+\frac{1+j\sqrt{3}}{2b}\right)\left(p+\frac{1-j\sqrt{3}}{2b}\right)}\epsilon^{\frac{1}{b}t}\right] - \frac{b^{3/2}}{1-(bp)^3}\left\{-\frac{1}{2\sqrt{\pi}}t^{-3/2}\right\} - \frac{1}{b^3 p}\left[\frac{1}{\left(p-\frac{1}{b}\right)\left(p+\frac{1-j\sqrt{3}}{2b}\right)}\epsilon^{-\left(\frac{1+j\sqrt{3}}{2b}\right)t} + \frac{1}{\left(p-\frac{1}{b}\right)\left(p+\frac{1+j\sqrt{3}}{2b}\right)}\epsilon^{-\left(\frac{1-j\sqrt{3}}{2b}\right)t} + \frac{1}{\left(p+\frac{1+j\sqrt{3}}{2b}\right)\left(p+\frac{1-j\sqrt{3}}{2b}\right)}\epsilon^{\frac{1}{b}t}\right] \quad (27)$$

When the indicated operations of (27) are carried through and the terms collected

$$V_0 = 1 - \frac{4}{3}\epsilon^{-\frac{1}{2b}t}\cos\frac{\sqrt{3}t}{2b} + \frac{1}{2\sqrt{\pi}}\left(\frac{b}{t}\right)^{3/2} \times \left[1 - 1\cdot3\cdot5\cdot7\left(\frac{b}{2t}\right)^3 + 1\cdot3\cdot5\cdot7\cdot9\cdot11\cdot13\left(\frac{b}{2t}\right)^6 \ldots\ldots\right] \quad (28)$$

The final result[5] as given by equation 28 checks that of Heaviside obtained by "advanced ideas" not stated.

This example is sometimes given as a case in which "Heaviside's method" fails, although he obtained the correct solution. As Heaviside himself made no general statement of his methods it might be inferred that he was at least partially aware of the relationship between his own work and ordinary differential equations. He makes the statement that,[7] "sometimes a series of differentiations may equivalently replace a series of integrations." It is unfortunate that he did not develop this relationship more fully.

Sinusoidal Electromotive Force Applied to an Infinite Cable

Another well-known problem is to find the entering current when a sinusoidal electromotive force is applied to an infinite cable. The operational equation is of the form

$$i = \sqrt{p}\,\sin\omega t 1 \quad (29)$$

which is a combination of

$$\sqrt{p}\,1=\frac{1}{\sqrt{\pi t}}$$

and

$$\frac{\omega p}{p^2+\omega^2}1=\sin\omega t$$

Using the construction theorem

$$i=\omega p\left[\frac{1}{\sqrt{p}\,(p-j\omega)}\epsilon^{-j\omega t}+\frac{1}{\sqrt{p}\,(p+j\omega)}\epsilon^{-j\omega t}+\frac{1}{\omega^2+p^2}\frac{1}{\sqrt{\pi t}}\right] \qquad (30)$$

and the final result is:

$$i=\sqrt{\frac{\omega}{2}}(\sin\omega t+\cos\omega t)-\frac{1}{\sqrt{\pi t}}\left(\frac{1}{(2\omega t)}-\frac{1\cdot3\cdot5}{(2\omega t)^3}+\frac{1\cdot3\cdot5\cdot7\cdot9}{(2\omega t)^5}\cdots\right) \qquad (31)$$

One of the advantages of using this method of attack on operational expressions is the ease of checking the result obtained. Equation 29 may be set up in two equivalent operational differential equations:

$$(p^2+\omega^2)i=\frac{\omega p^2}{\sqrt{p}}1=-\frac{\omega}{2\sqrt{\pi}}t^{-3/2} \qquad (32)$$

and

$$\sqrt{p}\,i=\frac{\omega p^2}{p^2+\omega^2}1=\omega\cos\omega t \qquad (33)$$

If (31) is substituted in (32) the first term of (31) becomes zero since the second derivative of this term is the term itself multiplied by $-\omega^2$. All of the series terms will cancel out except the term on the right of (32). If (31) is substituted in (33) the first term of (31) becomes $\omega\cos\omega t$ and the odd-half power terms of (31) are reduced to zero as in equation 10.

Conclusions

The construction theorem which has been developed is not always the most direct method of solving every operational expression for which it is applicable. In the case of fractional power root functions the result will, in general, contain one or more asymptotic series. In particular if a certain solution is desired in terms of a convergent series of fractional powers of the variable t the result must be obtained by some other method. Several of the more obvious advantages in using the theorem may be listed as follows:

(*a*). The theorem brings out the relationship between ordinary differential equations and Heaviside's calculus. This results in the concept of "equivalent operational differential equations" which is far more important than the development of a fixed routine for the solution of individual operational expressions.

(*b*). The theorem involves no exceptions or complicated tests which must be applied before it is used.

(*c*). The necessary mathematical manipulations are easily learned since the main prerequisites are a knowledge of the binomial theorem and the ability to differentiate simple functions of time.

(*d*). Operational expressions which might otherwise be evaluated by Heaviside's expansion theorem can, more often than not, be evaluated by inspection.

(*e*). The final result may be easily checked by substituting results into the original equations.

Outline for a Rigorous Mathematical Proof of the Construction Theorem

To prove the construction theorem it is necessary and sufficient to show that the operations indicated by the key equation 18 can be substantiated by an equivalent procedure using the Bromwich[8] contour integral in the complex plane (*a*) for integral positive values of n and (*b*) for fractional positive values of n.

The symbols and definitions to be used in this discussion are given by reference 8.

Equation 16 becomes

$$i=\frac{1}{2\pi j}\int_{-j\infty+c}^{+j\infty+c}\times\frac{\epsilon^{zt}}{(z+a_1)^{n_1}(z+a_2)^{n_2}(z+a_3)^{n_3}}dz \qquad (34)$$

1. Bromwich contour integral 1 is equivalent to Bromwich contour integral 2 for (34) since for large values of z the finite values of a may be set equal to zero and the proof is given by reference 8, pages 58–61.

2. The final result for (34) can now be obtained by taking the sum of the contributions of the individual roots along Bromwich integral 2. To evaluate the contribution of one root, say $-a_1$, first change the variable to $z'=z+a_1$. This is equivalent to the shifting operation indicated for ϵ^{-a_1t} in equation 18. Expand the bracket terms containing a_2 and a_3 by the binomial theorem in ascending powers of z' separately and then perform the indicated multiplications. The resulting series is analytic for values of z' less than the least radius of convergence of the two original series but not for large values of z'.

The contribution of each root is now in the form of a series of terms of the type

$$\frac{\epsilon^{-a_1t}}{2\pi j}\int_{Br_1}\frac{\epsilon^{z't}}{z'^{r+1}}dz'=\frac{\epsilon^{-a_1t}t^r}{\Gamma(1+r)} \qquad (35)$$

good for all values of r. Proof for (35) is indicated on pages 76–8 of reference 8. Term by term this can be shown to be equivalent to the differentiating process indicated by the first unit of equation 18.

4. In the case of integral positive values of n the contribution of the two straight lines of Bromwich integral 2 for any particular root in an expression such as (34) is zero, so that (35) includes only the integral around the circle whose radius can be made as small as necessary to make the series expansion convergent. For integral powers of n equation 18 always gives the evaluation of expression 16 in closed form.

5. In the case of fractional powers of n the series expansions do not represent the original functions beyond the least radius of convergence of the two series. The expression to be integrated along the straight lines of Bromwich integral 2 does not converge except close to the new origin. However, for large values of t most of the integral is obtained in the neighborhood of the origin, a process which usually results in an asymptotic series.

In integrating along Bromwich integral 2 for fractional powers of n it sometimes happens that one root appears in the "tail" of the integration path for the other. In this case the weaker root may be dropped for an asymptotic solution. This is equivalent to the restriction 4 for the construction theorem. The contributions obtained in this way coincide with those given by the construction theorem.

References

1. Über eine Formel von Heaviside zur Berechnung von Einschaltvorgänge, **K. W. Wagner.** *Archiv. f. Elek.*, volume 4, 1916, pages 159–93.

2. The Cauchy-Heaviside Expansion Formula and the Boltzmann-Hopkinson Principle of Superposition, **F. D. Murnaghan.** American Mathematical Society Bulletin, volume 33, 1927, pages 81–9.

3. On a General Expansion Theorem for the Transient Oscillations of a Connected System, **John R. Carson.** *Physical Review*, volume 10, series 2 (1917), pages 217–25.

4. A Generalized Infinite Integral Theorem, **M. G. Malti.** AIEE Transactions, volume 54, 1935 (November section), pages 1222–7.

5. Operational Circuit Analysis (a book), **V. Bush.** John Wiley and Sons, 1937.

6. Electromagnetic Theory (a book), **Oliver Heaviside.** London, 1899, page 42.

7. Electromagnetic Theory (a book), **Oliver Heaviside.** London, 1899, page 303.

8. Complex Variable and Operational Calculus With Technical Applications (a book), **N. W. McLachlan.** Cambridge University Press, 1939.

A Study of Sound Levels of Transformers

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I. Introduction

MUCH has been accomplished in the past few years in understanding the problem of transformer noise. It has been definitely established that magnetostriction discovered by Joule in 1842 is the principal source of transformer noise. The various harmonic vibrations generated in the core by magnetostriction have been analyzed. Methods of calculating sound levels in terms of the standards prepared by the American Standards Association have been developed. A large number of modes in which different core structures may vibrate have been determined experimentally and analytically. As a result, methods for determining the correct design proportions of the core have been developed so that the natural modes of vibration may not coincide with the harmonic vibrations generated by magnetostriction. These methods have been successfully applied to actual designs of transformers including the largest ratings.

This paper will serve as a partial report of the research and experimental work which has been accomplished in the laboratories of the company with which the writer is associated.

II. Transformer Noise

Until recent years it had been assumed generally that the main cause of transformer noise was the magnetic pull between the laminations at their joints. In order to determine the correctness of this assumption, an investigation was started in 1932 by experimenting with an iron ring without gaps or joints. This ring was made up of laminations, 18 inches outside diameter, 8½ inches inside diameter, built up to five inches. The laminations were clamped between heavy maple blocks; they could be energized by means of one turn through the center opening of the ring or more turns wound uniformly around the ring. When energized with alternating current, the ring emitted the characteristic "hum" heard from any transformer as soon as it is energized. The "hum" or noise increased as the flux density was increased. With a sine wave of voltage applied to the exciting winding and with a fixed number of ampere turns, the noise level was the same whether the ring was energized by means of one or several turns.

From these tests it was apparent that the joints in the core and mechanical dissymmetries between the energizing winding and the core could not account for the noise emitted from a well-built transformer.

In measuring the vibrations parallel with the plane of the laminations at different points around the periphery of the ring, it was found that the ring as a whole expanded with a very pronounced vibration of twice the applied frequency. These peripheral vibrations were all in phase or, in other words, the diameter of the ring was pulsating with an amplitude which was found to depend upon the flux density in the ring. It was only possible to explain these vibrations as a phenomenon of magnetostriction. The unit elongations calculated from these vibrations were of the expected order based on a knowledge of the magnetostriction.

Typical vibration measurements at the fundamental vibrational frequency taken on a rectangular core are shown in figure 1. It will be noticed that the deflections at the different flux densities are approximately proportional to the length of the various core members. The deflections shown occur simultaneously, that is, they are in phase. Vibrations as symmetrical as shown in figure 1[1] are obtained only when the laminations are straight and uniformly clamped. By changing the method of clamping, the deflections along a certain member may change considerably. The core may, for instance, vibrate as shown in figure 2.

The 120-cycle sound intensities calculated from the deflections shown in figures 1 and 2 correspond closely to those measured with a sound level meter with the microphone located approximately one-half inch from the core. The 120-cycle sound level of this core is, therefore, practically entirely accounted for by magnetostriction. The relation between magnetostriction and induction (B) is generally such that the core vibrates also at harmonically related higher frequencies. Both odd and even harmonic vibrations are produced. In figure 3 is shown such vibrations measured at one end of a large rectangular core. With the apparatus available, it was possible to measure vibrations up to the tenth harmonic. In this figure the vibrations are plotted as velocities. The amplitudes of the vibrations are in inverse ratio to the frequencies.

The number and magnitude of these higher harmonic vibrations depend upon the nature of the steel, the flux density, and the linear dimensions of the core. Any particular grade of steel has, therefore, a characteristic noise spectrum for each flux density. Such a spectrum is shown in figure 4 for steel at a flux density $B=15{,}000$ gausses. The ordinates are plotted in decibels.

The core of any transformer is in itself a structure which may vibrate in many modes, depending upon its degree of freedom. When the laminations of a core as shown in figure 1 are clamped between heavy end frames, the core is practically free to swing only in a direction parallel with the plane of the laminations. A large number of modes of vibration are possible in this plane, each mode occurring at a certain frequency. This form of core and end frame construction is common in shell-type transformers. If the core is not clamped between heavy end frames and is standing upright as in core-type transformers, it has several degrees of freedom, that is, it may vibrate in several planes, and in numerous modes in each plane. In either case, it is possible that the core structure may be in mechanical resonance with one or more of the frequencies generated in the core due to magnetostriction. The amplitudes of vibration at such frequencies will, therefore, be in excess of the forced vibrations caused by magnetostriction.

In figure 5 are shown some typical vibration measurements made on a shell-type transformer which was energized at a constant induction at different frequencies. In this test the core was not clamped tight, and the bottom and top end frames were, therefore, more or less loosely coupled with the core. The vibrations were measured at five points along the vertical center line of the long side as shown.

The upper end frame showed resonance at approximately 100 cycles (figure 5, point 5a); above 130 cycles it was driven

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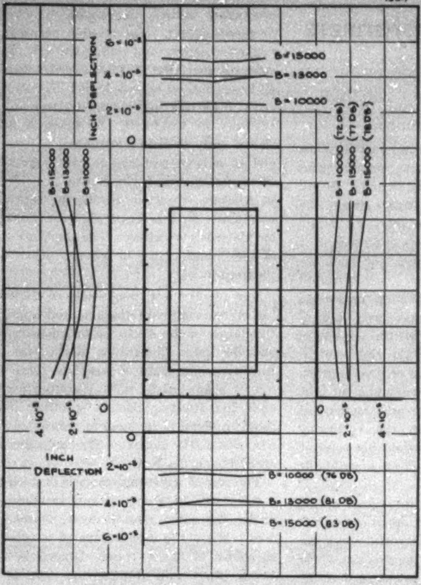


Figure 1. 120-cycle core deflections parallel with the plane of laminations at 60-cycle excitation

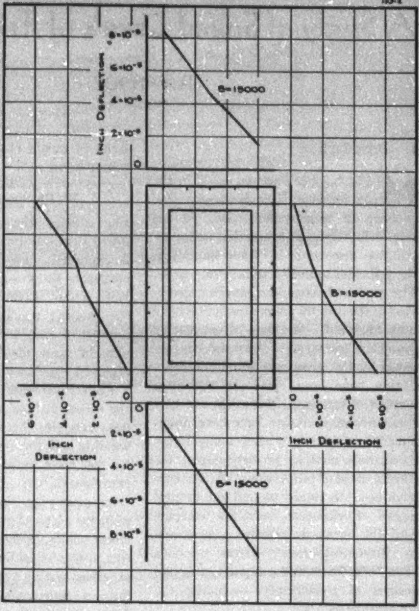


Figure 2. 120-cycle core deflections parallel with the plane of laminations at 60-cycle excitation

by the core which was in resonance at 136 cycles as shown in figure 5 (points 5*b*, 5*c*, and 5*d*). The calculated deflection of the core, due to magnetostriction, was approximately 0.05 mil double amplitude, and it will be noted that at 136 cycles, the core vibrated with an amplitude 16 times that corresponding to the magnetostriction. This means that the noise level on this side of the core was approximately 24 decibels higher than normal at this particular flux density. From figure 5, point 5*e*, it will be seen that the lower end frame is in resonance at approximately 102 cycles and starts to be driven by the core when the frequency was above 132 cycles.

By increasing the coupling between the core and the end frames, that is, by clamping the structures together, the vibrations in the range from 100 to 130 cycles will be somewhat increased. It is, therefore, evident that it is desirable that the core and the end frames have as nearly as possible the same natural frequencies. It will also be noted from these figures, that the band of partial resonance extends about 20 to 25 cycles on either side of the resonance frequency.

The mode of vibration at the resonance frequency of 136 cycles was determined by the use of two equally sensitive vibration pickups connected in series so that their voltages add. The mode was of the nature shown in figure 6*a*. Two of the opposite sides are deflecting outward at the instant the other two sides are deflecting inward. This is the lowest possible mode for a box structure which has practically only one degree of freedom. The exciting frequency of the supply at the 136 cycles vibrational frequency was 68 cycles. The same mode would have occurred at 136 cycles if the core had been energized at 34 cycles. In this case, the core would have been in resonance with the second harmonic vibration due to magnetostriction. But, because of clamping in the core and since for this particular steel the second harmonic driving force is less than for the fundamental, the amplitude of the 136-cycle vibration would have been less.

If the frequency had been further increased above 136 cycles, another resonance point would have occurred at approximately 292 cycles. The mode of vibration at this frequency would have been as shown in figure 6*b*. Here all four sides are simultaneously deflecting outward. This mode requires a rather rigid joint at the corners and is, therefore, not always excited.

At still higher frequencies more complicated modes are excited. The different sides will then vibrate in odd number of half cycles.

Figure 7 shows the consecutive modes of vibration which occur in a three-phase shell-type transformer.

The frequencies of these modes depend upon the dimensions of the core and the modulus of elasticity of the steel used. They may be calculated from a formula of the form

$$f=K\frac{W}{l_1^2} \qquad (1)$$

where

K = a function of the ratio l_1/l_2 and the modulus of elasticity
W = width of the laminations
l_1 = mean length of the longest side
l_2 = mean length of the short side

From experimental data it has been found advisable to design the core of

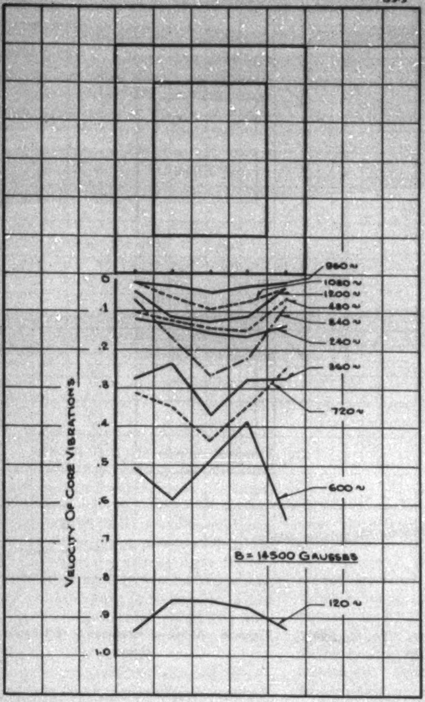


Figure 3. Velocity of core vibrations parallel with the plane of laminations at 60-cycle excitation

such dimensions that its natural frequencies are separated approximately 25 to 30 cycles from any of the harmonic frequencies generated in the core by the excitation frequency.

The end frames for shell-type transformers are more or less complicated box structures and their natural frequencies may be determined along the same lines as indicated for the core.

As previously stated, the core-type transformer as usually constructed is free to vibrate in several planes. The modes of vibration parallel with the plane of the laminations may be of the same nature as the modes described for box structures, and may be determined by means of equation 1. Such a mode for a single-phase core-type transformer is shown in figure 8. In this particular core the mechanical resonance frequency was 294 cycles. This mode would, therefore, not be excited by any of the frequencies generated in the core at 60-cycle excitation. The amplitudes in the center of the legs are approximately 19 times that corresponding to the magnetostriction at this particular flux density. On the other hand, the upright core represents two or more columns with a load at their ends which are free to vibrate in planes principally parallel and perpendicular to the laminations. The core may, therefore, also vibrate in modes as shown in figures 9 and 10, in a plane parallel with the laminations or in modes as shown in figure 11, in a plane perpendicular to the plane of laminations. In figure 11, the elongations of the legs in the direction of their length are restrained by inertia forces due to the presence of large masses on their ends, and they bow out. Some of these modes are rather critical and difficult to duplicate under apparently similar conditions. They are apparently influenced by the mounting of the core on its foundation.

With resonance eliminated, by proper selection of the core dimensions, the amplitude (A) at the fundamental vibrational frequency of any member of the core may be calculated from the magnetostriction characteristics of the steel. For a core member of length (L), the amplitude at each end equals

$$A=\frac{\Delta L}{L}\times\frac{L}{2} \tag{2}$$

where $\Delta L/L$ = elongation per unit length due to magnetostriction. The corresponding physical intensity of the sound wave may be calculated by means of the fundamental equation for a plane sound wave, emitted from a large plane vibrating surface.

The maximum excess pressure in such a wave equals

$$P=V\times d_0\times A\times 2\pi f \tag{3}$$

where V = velocity of wave, d_0 = mean density, A = maximum amplitude of the wave, and f = frequency.

When V is in centimeters per second

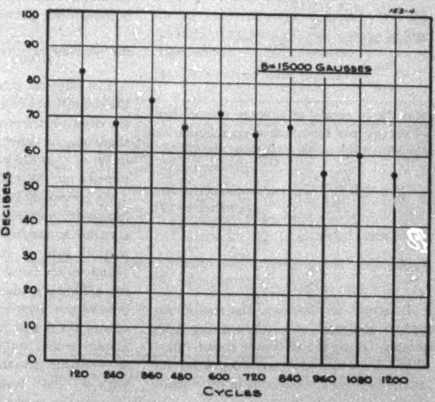


Figure 4. Noise spectrum of transformer steel at 60-cycle excitation

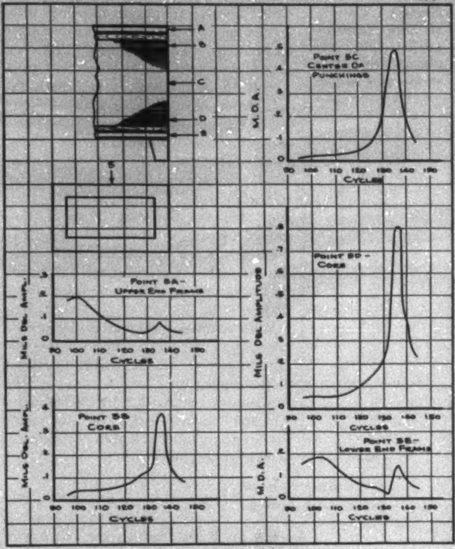


Figure 5. Resonance curve of shell-type transformer core. Core not clamped

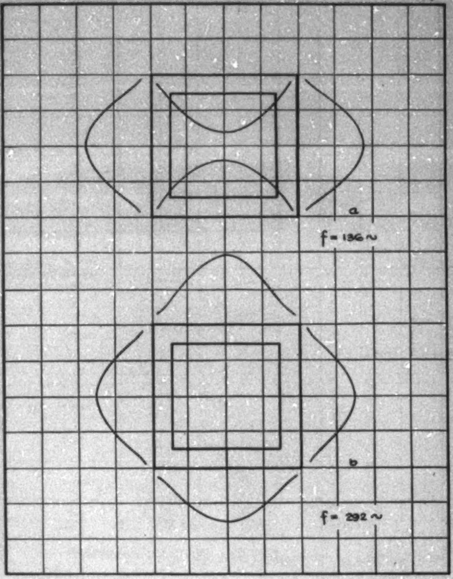


Figure 6. Mode of vibrations of rectangular cores

(=33,300 for air), d_0 in gram per cubic centimeter (=0.0012 for air), the rms excess sound pressure in dynes per square centimeter equals

$$P_{\text{rms}} = 177 \times A_{\text{cm}} \times f \tag{4}$$

or if A is in inches

$$P_{\text{rms}} = 452 \times A_{\text{in}} \times f \tag{5}$$

The average energy density of such an acoustic wave equals

$$W = 2\pi^2 A^2 f^2 d_0 = \frac{P_{\text{rms}}^2}{d_0 V^2} \text{ gram per cubic centimeter} \tag{6}$$

and the intensity or average rate of flow of energy per unit area perpendicular to the direction of propagation, equals

$$I = W \times V = \frac{P_{\text{rms}}^2}{d_0 V} \text{ ergs per second per square centimeter} \tag{7}$$

For a wave in air

$$I = 2.5 \times 10^{-9} \times P_{\text{rms}}^2 \text{ watts per square centimeter} \tag{8}$$

In sound terminology, the decibel expresses the ratio of any given sound intensity (I) to the reference sound intensity. The standard reference level for intensity level comparisons is 10^{-16} watts per square centimeter. The decibel is defined as

$$\text{db} = 10 \log_{10} \frac{I}{10^{-16}} \tag{9}$$

The decibel also expresses the ratio of any given sound level pressure (P) to the reference pressure. The standard reference level for pressure level comparisons is 0.0002 dyne per square centimeter. Since the intensity is proportional to the square of the pressure, we have

$$\text{db} = 20 \log_{10} \frac{P}{0.0002} \tag{10}$$

The physical intensity, in decibels, of the sound wave emitted at the fundamental vibrational frequency of the core may, therefore, be determined by substituting in (9) or (10) the values for I or P determined from (8) or (5).

As previously stated, a large number of harmonic vibrations are produced in the core due to magnetostriction. The arithmetical sum of the intensities, of the sound waves caused by these vibrations, for ordinary grades of silicon steel, is in general not more than one decibel in excess of the sound intensity of the fundamental sound wave. The sound intensity measured with a sound level meter with a "flat" frequency response will, therefore, approximately equal the intensity of the fundamental sound wave calculated as indicated above. This value will be designated as db_F. The intensities of the individual harmonics may also be weighted according to any of the recognized standard equal loudness contours for pure tones. For transformers it is the standard practice to refer all sound level measurements to the 40-decibel equal loudness contour. This value may be measured directly with a modern sound level meter, and will be designated as db_{40}.

The difference between the sound measured with the "flat" and the 40-decibel frequency response will give a fair indication of the quality or the harmonic components of the sound measured. This difference may, therefore, be designated "harmonic index" (h.i.), or

$$\text{h.i.} = \text{db}_F - \text{db}_{40} \tag{11}$$

Where the h.i. is large it is an indication that the sound contains large components of low-frequency tones, whereas a small h.i. indicates that higher frequency tones are predominating.

The harmonic index of a certain grade

of steel as a function of the induction is shown in figure 12.

The sound level of the core may now be estimated as follows. The amplitude of the core vibrations at fundamental vibrational frequency is determined from the magnetostriction of the steel, the length of the magnetic circuit involved, and the flux density at which the core is worked. From this amplitude the excess pressure of the sound wave in dynes per square centimeter and the corresponding decibel sound intensity is calculated (db_p). In order to obtain the sound level corresponding to the 40-decibel frequency response, the "harmonic index" corresponding to the flux density at which the core is worked should be subtracted from the above sound intensity, or the sound level of the core is approximately

$$db = db_p - h.i. \qquad (12)$$

Numerous tests on different cores and at different flux densities have shown close agreement between the calculated and measured sound levels. However, because of the large number of assumptions made in these calculations and since these assumptions are only approximations, caution should be made in their use. The method of calculation would, for instance, have to be changed in case the steel has a sound spectrum widely different from that commonly used in present transformers.

It is well known that the load currents give rise to magnetic forces around and between the high- and low-voltage coils of a transformer. These forces will produce vibrations of the coils with double the applied frequency. Higher harmonics are in general not produced. The construction of the coils is in general such that they are relatively poor sound radiators. In a properly designed transformer, the vibrations produced by the coils will, therefore, contribute relatively little to the total noise level of the transformer when measured on the 40-decibel frequency response.

When the core and coils are placed in their tank and immersed in oil, their motion is transmitted through the oil to the tank wall and from there emitted as sound waves to the surrounding air. For most of the vibrations generated in the core the oil may be considered an incompressible fluid. This means that the oil will act as an added mass to the tank wall and only a small amount of energy at the higher frequencies will be transmitted through the oil and the tank wall as a true wave motion. The tank wall, bottom, and cover are, therefore, forced to vibrate as diaphragms. In general, these diaphragmatic vibrations are not produced in the tank wall as a whole, at any frequency. The tank is a relatively complex structure. Radiators, potheads, switch cabinets, etc., may be welded to the tank wall. Its surface is, therefore, broken up in a large number of smaller areas, vibrating with different phases and magnitudes. Provided no parts of the tank surface are in resonance with any of the exciting frequencies produced by the core, the resulting vibrations are forced. Their magnitudes may be smaller than that of the core. The reduction depends upon the height and thickness of the tank wall, the distance between the core and tank wall, the relative density of the material of the tank wall and that of the oil.

Methods for predetermining the amount of attenuation are being studied, but the problem is very complex and empirical constants are used at present.

Assuming the oil to act as an incompressible fluid, the average "harmonic index" of the tank wall vibrations will correspond closely to the average "harmonic index" observed on the core itself. For example, on one of the largest transformers built, the average "harmonic index" of the core was 9.1 decibels when measured out of its tank. When the transformer was placed in its tank, the average h.i. measured 9.6 decibels, indicating relatively little change in the harmonic vibrations of the core and the tank wall. A large number of similar

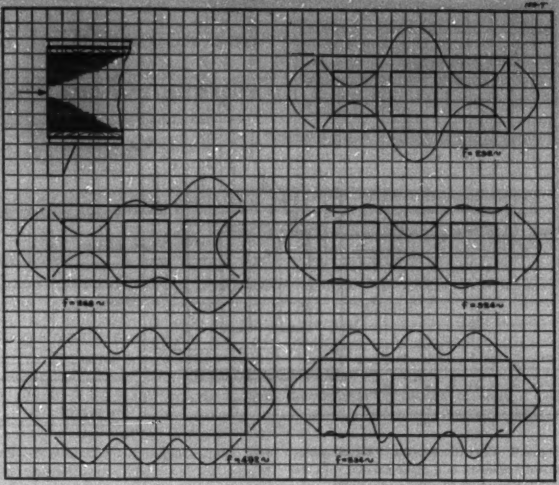


Figure 7. Phase relations of vibrations measured on center line of three-phase shell-type transformer

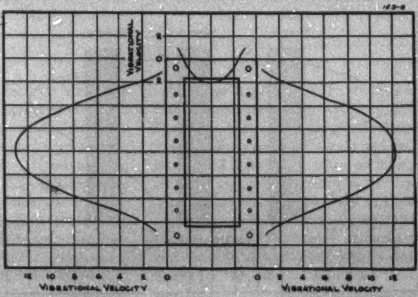


Figure 8. Magnitude and phase relations of 294-cycle vibrations parallel to the plane of laminations

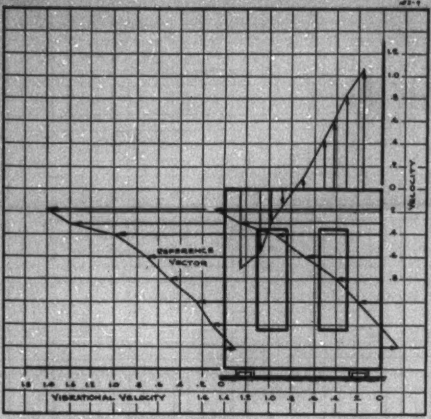


Figure 9. Core vibrations at 124-cycle vibrational frequency in a plane parallel with the laminations of a three-phase core

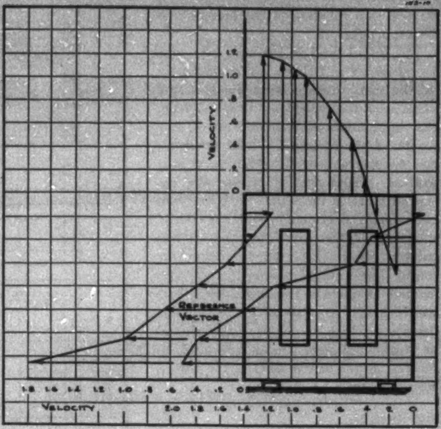


Figure 10. Core vibrations at 116-cycle vibrational frequency in a plane parallel with the laminations of a three-phase core

tests on a variety of transformers have given similar results. Where a large discrepancy has been observed it has been an indication of resonance in some parts of the tank wall or improper mounting of the transformer in its tank. The tank wall or segments of the tank wall may have free vibrations of their own at a large number of frequencies and modes. If such vibrations are in resonance with any of the core vibrations, they may in general be reduced by relatively simple means, such as braces anchored at suitable points.

Cooling tubes, radiators, etc., may complicate the motions of the tank wall, but in themselves these parts are relatively poor sound radiators for sounds of the frequencies involved in transformer operation. They contribute, therefore, relatively little to the total sound level of the transformer.

The vibrations of the tank may produce two kinds of sound, when the transformer is installed in its final location:

1. A direct air-borne sound, radiated from the tank wall, cover, radiators, etc.

2. A sound transmitted through the transformer foundation to the wall or other structural work of the building in or near where the transformer is installed.

Of these two sources, number 1 is, in general, the most important and also the most difficult to insulate against. Air-borne sounds must be "sealed in." This may be accomplished by means of sound-absorbing walls surrounding the transformer, but this is a remedy which seldom can be used.

Source number 2 may be of importance in certain installations where large areas may be set in vibration by transmission of vibrations through materials like steel or concrete which transmit vibrations much more readily than air. Such vibrations may be reduced by elastic support between the transformer base and

Figure 11. Phase relations of 116-cycle vibrations perpendicular to the plane of laminations of a three-phase core

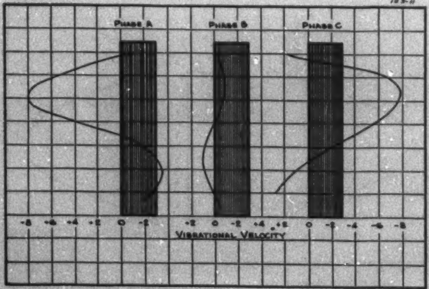
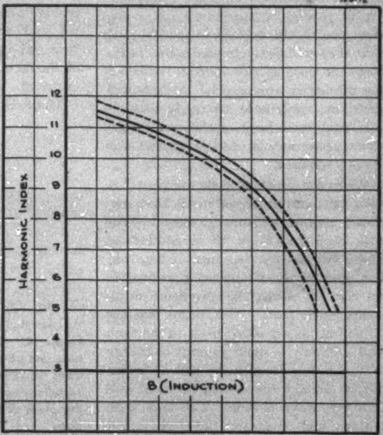


Figure 12 (right). Harmonic index of transformer steel corresponding to 40-decibel frequency response



Suppression of Magnetic Vibration and Noise of Two-Pole Turbine Generators

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THE constant striving for improvement in over-all economy in the generation of electric power by steam has resulted in the last three or four years in a rather rapid increase in the capacity of steam turbine generators built to operate at 3,600 rpm. One of the factors contributing to this growth was the progress made in the use of high-pressure and high-temperature steam which led to the application of a number of such turbines for superposition on existing comparatively low-pressure steam plants, as well as for straight through condensing operation. The operating experience with such recently developed 3,600-rpm generators ranging in capacity up to 66,667 kva has been unusually good. The troubles which have been experienced, either mechanical or electrical, have not been of a serious nature.

While 3,600-rpm generators have been built for many years, the double-frequency vibration and noise produced by them was not until recent years of much moment or concern. The comparatively recent machines with which we are here concerned are large as compared to the earlier ones and have in many cases produced an objectionable amount of 120-cycle noise and vibration, the noise being often an intense humming of all-pervading character, seeming to go all through the plant. The intensity of the noise, at some points, is materially affected by reflection from various smooth surfaces, being particularly loud where the reflected sound wave reinforces the original.

In addition, the actual amplitude of the mechanical vibration has been sometimes sufficient to result in severe vibration of many parts of the station structure or equipment such as beams, brackets, floor plates, housings or parts of them, even exciter brush arm brackets, latch mechanisms, or perhaps delicate relay parts. This is particularly so if the natural period of vibration of some beam or part is in resonance or near resonance with the vibration source.

Early investigations of the 120-cycle vibration and noise showed that they resulted when field excitation was applied to the generator and that in fact they were two aspects of the same disturbance. This definitely indicated that the cause was magnetic. The disturbance, as has been pointed out in a previous paper,[1] is due to the magnetic forces of a two-pole field rotor bringing about a distortion of the stator into an elliptical shape, the distortion revolving in synchronism with the rotor. Since each pole of the rotor pulls the stator radially inward, any point on the stator core vibrates at a frequency double that of rotation, or at a frequency of 120 cycles for a 3,600-rpm machine.

As is well known, the magnetic force of attraction between the stator and rotor at any given point varies as the square of the flux density at that point. Figure 1 has been plotted in terms of magnetic force or pull as determined from the flux distribution typical of a two-pole machine.

While the magnetic vibration has not been severe enough to have any apparent effect on the life of the stator winding or other parts, nevertheless, the noise has in many instances been so intense as to be very annoying to live with, which indicated that there was a need for some means of suppression or mitigation of this noise and vibration and led to the undertaking of a careful analysis of the problem. The presentation of the various phases of this analysis, together with a description of the development of an apparently complete cure for the double-frequency magnetic vibration nuisance by means of a simple form of resilient core mounting, is the subject of this paper.

Observations on Magnetic Vibration

Since the double-frequency vibration and noise are caused by distortion of the generator stator structure, the amount of this distortion is of primary importance. The frequency being high, special equipment is required for accurate measurement. Readings of vibration amplitude taken with an optical amplitude indicator on a representative large two-pole machine of early design are shown in figure 2. These were taken radially on the surface of the stator, and the plot shows the variation in amplitude along the length of the machine. The same curve applies to either side or the top of the generator, or at any angle in between, for *radial* amplitudes. With excitation removed, this vibration disappears, although there is usually a residual vibration at running speed frequency, transmitted from the rotor, which is small if the machine is well balanced. The residual vibration in this case is shown by the dotted curve.

The variation of this double-frequency

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[1] For all numbered references, see list at end of paper.

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the foundation. This elastic support should be correctly designed, otherwise its effect may be very detrimental. The elastic support should have a natural frequency considerably below the lowest impressed vibrational frequency produced by the transformer. Spring supports are, in general, not practicable. There are on the market special high-grade rubber compounds which do not lose their resilience appreciably with time.

The problem of transformer sound and methods for its reduction is far from being solved. With the exception of reduction in magnetostriction, no means have been found by which the sound level may be economically reduced. Continued research is being done on this problem. The company with which the writer is associated has a large soundproof room which is used to facilitate such investigations. This room has inside dimensions of 24 feet by 25 feet and 22 feet height. It is of a double-room construction. The outer room is of eight-inch brick walls with a seven-inch thick concrete roof. The inner room is completely insulated from the outer room. Its walls and ceiling are constructed of sound-absorbing material particularly effective for frequencies in the range from 100 to 1,000 cycles. A seven-inch air space separates the two rooms. The average attenuation is of the order of 55 decibels, which is sufficient for the work for which the room was designed.

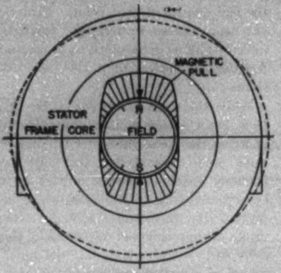


Figure 1. Magnetic pull of a typical two-pole a-c generator

amplitude with speed is shown in figure 3. It may be noted that, as speed is reduced below 3,600 rpm, with constant excitation, the amplitude drops off somewhat, but levels off to a definite value for zero speed. The shape of this curve is similar to part of a resonance curve, and suggests that if the speed could be carried considerably beyond 3,600 rpm, the amplitude would increase greatly, perhaps reaching a peak value at some higher speed.

Analytical Approach to Problem

Consideration of data such as are shown in these figures suggested an analytical approach to the problem to account for the observed vibration, along the following lines:

(1). Evaluation of the magnetic forces acting radially in the air gap between rotor and stator;

(2). Calculation of the deformation of the stator structure by these forces, considered as acting statically, that is, with the rotor at rest;

(3). Estimation of a resonance "build-up" factor (ratio of running amplitude to static deformation) based upon the relationship of the frequency of the magnetic forces at running speed to the natural frequency of the stator vibrating as a cylindrical ring with a four-node pattern.

Tests on Stiffness of Stator Elements

The magnetic forces were found to be rather large, equivalent roughly to two concentrated opposite radial loads on the stator of the order of 50,000 to 100,000 pounds each in large machines. The static deformation of the stator frame by such forces, assuming at first that the core laminations had no stiffness of their own, was calculated by well-known ring formulas[2,3] to be far greater than the observed vibration. It seemed therefore that the laminations must contribute a large share of the total stiffness of the stator. As a check on this point, deflection tests were made on a full-sized stator frame of greater than average stiffness before and after the laminations were assembled, with results as shown in figure 4. The frame was set on the factory floor and loaded with heavy rotor forgings laid on top of it on wooden chocks, and the deflection measured by means of dial indicators set to register the change in the vertical diameter. Upon removal of the loads, the indicators all returned to zero. It will be noted that with the heaviest load, the bare frame deflected 10.5 mils, as compared with 1.3 mils after assembly of the core.

In anticipation of these results, and to settle the question as to whether this great stiffness was a property of the tightly pressed core itself or depended upon assembly of the core in the frame, a ring had been made of some of these same laminations pressed together with rectangular steel bars as shown in figure 5. This ring was also subjected to deflection tests, by loading it across a diameter with a hydraulic jack, and measuring the deformation with dial indicators in much the same way as on the frame. It was found that this ring of punchings was quite elastic; that it would take loads up to several times the normal magnetic load, and return to its original shape; and that it was exceedingly stiff, sufficiently so to account fully for the difference in stiffness between the bare frame and the stacked frame. Also, while the ring stiffness varied somewhat with the tightness of clamping, this effect was comparatively slight so long as the laminations were reasonably tight. In short, the core was found to be a cylinder of great strength and stiffness independent of the generator frame.

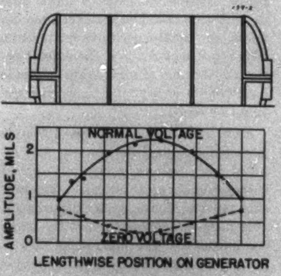


Figure 2. Distribution of vibration on stator of a two-pole generator

Tests to Determine Natural Frequency of Core

Other rings of punchings were tested with similar results. Further tests were made on them to determine the natural frequency as a cylindrical ring vibrating radially with a four-node pattern. The two types of tests, for static stiffness and for natural frequency, were correlated, and it was found possible to calculate the natural frequency of a core structure with reasonable accuracy. In all cases, it was found to be high, of the order of 160 to 200 cycles in the largest machines, and higher in smaller machines.

Use of Resonance Curve in Calculating Magnetic Vibration

This information made it possible to carry through with the three steps suggested for the calculation of the amplitude of magnetic vibration of two-pole generators in general. The stiffness of the frame usually being small in comparison with that of the core, little error results from leaving the frame entirely out of the calculations. The first two steps, to determine the magnetic forces and the resulting static deflections of the cylindrical ring of punchings, are carried out by well-known methods that do not need to be repeated here. The third step makes use of a resonance curve such as that shown in figure 6. Here amplitude is plotted against the frequency of the deflecting force. For zero frequency, i.e., a steadily applied force, the amplitude is shown as 1.0, which represents the static deflection. With increasing frequency, the amplitude increases and reaches a peak value at resonance, when the frequency of the deflecting force equals the natural frequency of the setup. The frequency scale is also on a per-unit basis, with 1.0 representing resonance. For higher frequencies beyond resonance, the amplitude decreases rapidly. This relationship between amplitude and frequency is rigorous[4,5] for any simple system in which the disturbing force remains constant in magnitude

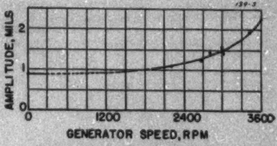


Figure 3. Variation of magnetic vibration with speed

as the frequency varies. It has been assumed to apply to a cylindrical ring. Damping has been neglected; if taken into account it would reduce materially the amplitudes near resonance, as shown by a sample dotted curve. However, the four-node natural frequencies of the generator cores were found to be so far above the impressed frequency of 120 cycles that damping can be neglected. The use of the curve is fairly obvious. It merely shows a "build-up" factor, representing the increase of the running amplitude above the static deflection. For example, if the natural frequency of the stator core is determined as 200 cycles the ratio of impressed frequency to natural frequency is 120/200 or 0.6; and the "build-up" factor is 1.56.

Improvement Obtained by Increased Radial Depth of Core

Calculations were made along these lines for all large 3,600 rpm generators made by the General Electric Company. Agreement between the calculations and the maximum of test observations similar to those plotted in figure 2 was very satisfactory. It was obvious from the results that considerable reduction in the vibration could be obtained by the use of punchings of increased radial depth. The greater stiffness of such "deep core" machines helps in two ways: (1) it reduces the "static" deformation; and (2) it increases the natural frequency so that the "build-up" factor is also reduced. Deep-core construction has been used in quite a number of generators in the past two years, with a gratifying reduction in the double-frequency disturbance. The ratio of improvement in the largest machines has been of the order of two to one.

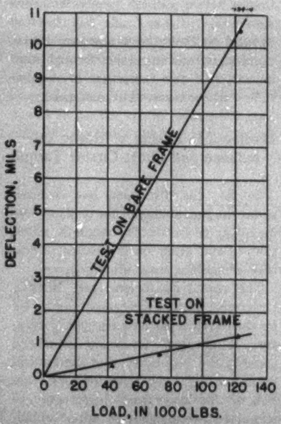
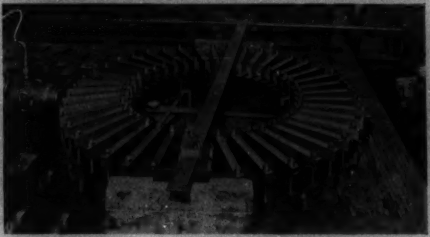


Figure 4. Results of deflection tests on stator frame

Consideration of Resilient Mounting of Core to Isolate Vibration

Although the improvement obtained by the simple expedient of deepening the core was considered to be sufficient to remove most of the nuisance effect of the noise and vibration in all cases where station acoustics are reasonably good and mechanical resonance is avoided in adjacent structures, nevertheless it seemed desirable to see what could be done to eliminate the magnetic disturbance almost entirely. A suggestion to this end had been made by L. P. Grobel, shortly after the first large 3,600-rpm generators came to test. His proposal was to isolate the vibration by mounting the core on spring members within the stator frame. This seemed like a rather radical departure, but the tests described in preceding paragraphs demonstrated that the tightly pressed laminated cores had great strength and stiffness of their own, and indicated that the isolation idea was an inviting possibility. A number of complications, however, had to be faced. First, should the spring members provide radial flexibility only, and be rigid tangentially; or should they be flexible both ways? Second, if the frame is not attached rigidly to the core, its own four-node natural frequency will determine its response to the stimulation coming through the spring members, and must be taken into account. Third, the core should apparently be mounted in such a way that it will not have natural frequencies for bodily motions which would cause it to respond unduly to the running speed frequency disturbances caused by unbalance of the rotor. Fourth, the core mounting must be designed to withstand the severe torsional effect of short circuits. A complete treatment of these matters is entirely beyond the scope of this paper. They will be briefly discussed, however, in the next few paragraphs.

Figure 5. Setup for deflection tests on ring of core laminations



Tangential Displacements of Cylindrical Rings

A tangentially rigid but radially flexible type of mounting was suggested independently by several engineers within the General Electric Company, and was given serious consideration. Its effectiveness would apparently depend upon the amount of tangential vibration accompanying the radial vibration at the point of attachment, at the outside diameter of the core, or farther out if a double frame should be used. It is not at once obvious that there should be any tangential motion at all, although it was pointed out long ago by Lord Rayleigh[8] that in the case of a *thin* circular ring, elliptical distortion or four-node vibration resulted in a tangential displacement of the nodal points equal to just 50 per cent of the maximum radial displacement. This is illustrated in figure 7, from which the reason for the tangential displacement may be seen. The distance around a quadrant of the deflected ring, from one 45-degree point to the next, is obviously greater across the major axis than across the minor axis, so that, unless the ring is forcibly stretched in two quadrants and shortened in the other two, the "nodal" points at 45 degrees must move tangentially.

Two-pole generator cores, however, are not thin rings, but relatively deep ones. An investigation was made to determine how the tangential displacement might vary with the ring depth. The curves in figure 8 are drawn to illustrate some of the results of an analysis made by Doctor H. Poritsky and H. D. Snively. These show that deep rings have a tangential displacement of more

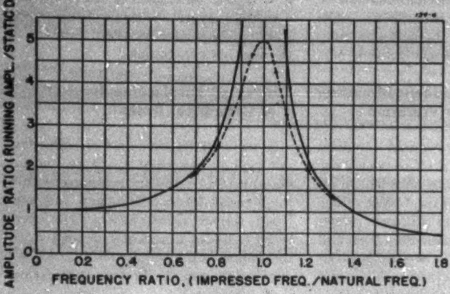


Figure 6. Basic resonance curve

than 50 per cent of the radial displacement, at the inside diameter; but less than 50 per cent at the outside. The exact values depend upon the depth ratio (outside diameter)/(inside diameter). Amplitude observations during the tests for the natural frequency of rings of punchings, described in a previous paragraph, gave an excellent check on these curves, as shown by the *X* points (figure 8). It seems that for two-pole generator cores of normal proportions, with a ratio of outside to inside diameter (over the slots) of 1.5 to 1.6, the tangential vibration on the outside diameter is likely to be about 20 per cent or more of the radial vibration. Now, the frame may also be considered as a deep ring, with a ratio of outside to inside diameter of the same order of magnitude as the core. Consequently, the tangential vibration at the *inside* diameter of the frame is likely to be about 80 per cent of its radial vibration. Thus, if the point of attachment for a tangentially rigid core mounting is at the outside diameter of the core, something like 20 per cent of the radial vibration of the core would be tied in rigidly with 80 per cent of the radial vibration of the frame. The result would be a reduction of about four to one in the radial vibration at the surface of the frame. Such an improvement would be quite worth while, but it was felt that a higher ratio of reduction would be desirable.

Importance of Natural Frequency of Stator Frame

Consideration of a resilient mounting having both radial and tangential flexibility indicated that an improvement ratio of 10 or 20 to 1, or even higher, could be obtained by suitable design of the spring members and the frame. In brief, the spring members must be relatively flexible, and the frame relatively stiff. The static performance of such a combination is readily calculated. The vibrational loading of the frame is computed as the product of the amplitude of the core and the gradient of the spring members. This neglects the yielding of the frame, which is intended to be negligibly small anyway. Both tangential and radial loading must be taken into account. However, the running performance depends upon the natural frequency of the frame, in the same manner as the running amplitude of the core depends upon its natural frequency. Strictly, the core, spring members, and frame form a compound elastic system with two natural frequencies, both of which are different from the natural frequencies of the core and frame considered separately. The behavior of such a system has been carefully worked out by methods described in chapter V of reference 4. With the aid of some simplifying assumptions, however, calculation of the running performance to a close degree of approximation can be made, using the ordinary basic resonance curve of figure 6. The effect of the frame and spring members on the stiffness, natural frequency, and running amplitude of the core is neglected; but a correction to the natural frequency of the frame due to the spring members may be required, especially if the frame frequency is low, below 120 cycles. The static deformation of the frame, which would be produced by the combined radial and tangential loading due to the *running* amplitude of the core acting on the spring members is calculated, and multiplied by the "build-up" factor based upon the natural frequency of the frame.

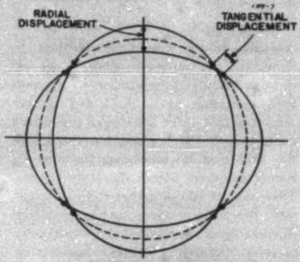


Figure 7. Sketch illustrating tangential displacements for a thin ring

For this purpose, it is of course necessary to know the four-node natural frequency of the frame. To date, it has been necessary to use test information on this point. Test equipment available for vibrating turbine wheels and buckets has been used with the addition of some small but powerful a-c magnets, which are set to pull diametrically across the bore of the frame under test. Even the largest and heaviest frames are set into vibration in this way, and by varying the frequency of the a-c supply to the magnets, the resonant frequency of the frame is readily determined. In general, it is felt that a high natural frequency (above 120 cycles per second) is desirable for frames to be used with resiliently-mounted cores, for the reason that it avoids a "critical speed" or resonance peak of frame vibration which would otherwise occur below running speed, if the generator is brought up or down with excitation. It is undoubtedly practical, however, to use resilient mountings with frames having natural frequencies below 120 cycles, if the frame can be well damped to limit the momentary vibration encountered in passing through resonance, or if the excitation is reduced sufficiently or removed at that speed.

Mechanical Effects of Rotor Unbalance and Short-Circuit Torque

So far, the discussion has centered about the functioning of the resilient core mounting to isolate the magnetic vibration. Now, attention is directed to possible difficulties from other sources.

The core, end flanges, and winding structure, taken together, comprise by far the heaviest part of a two-pole generator. If "floated" on spring members, there is the possibility that mechanical resonance might take place for bodily motion of this heavy mass in response to stimulation coming from rotor unbalance. Considered as a simple mechanical system with the frame support on the foundation assumed to be rigid, the obvious solution is to make the natural frequencies of the core on the spring members low for both translation, i.e., "straight bouncing," and for rotation about an axis at right angles to the shaft, which can be called "rocking." Actually, the foundation is never rigid; for large machines it is far from being so, especially in the horizontal plane. The result is a complicated elastic system, comprising the rotor, the frame, and the spring-mounted core (not to mention the turbine), all mounted on a foundation of considerable mass and with flexibility both for horizontal and vertical motion. Such systems have been investigated by E. H. Hull, using a novel experimental technique described in a recent paper.[7] He finds a whole series of resonance speeds, depending upon the critical speeds of the rotor (as calculated for rigid bearing supports) and the natural frequencies of the spring-mounted core, as well as the mass and flexibility of the foundation. Fortunately, it seems to be possible to control these resonance speeds, for ordinary values of shaft criticals and foundation characteristics, by making the natural frequencies of the spring-mounted core rather low, say, of the order of 1,200–1,500 vibrations per minute. Also, none of them involve much vibration if the machine is properly balanced.

Special consideration must also be given to the severe transient short-circuit torques characteristic of turbine generators, in order that the spring members shall not be overstressed. Instantaneous peak values of the applied torque, although they may be many times normal rated torque, are ordinarily not high enough to cause concern in this respect. Their magnitude is usually of such an order as would about take the gravity load off one foot of the generator and double it on the other. Since the considerable inertia of the core is interposed between the short-circuit torque, applied at the air gap, and the spring members outside the core, the stresses in the spring members are not a direct function of the applied torque alone but depend upon the torsional displacement of the core relative to the frame. This displacement can be calculated by methods[8,9] previously developed for analysis of short-circuit torques at the couplings of turbine-generator sets.

These methods are rather well known, but a few remarks about them might be in order here. The simplest approach considers the torques as suddenly applied, but neglects the decrements, and assumes a rigid foundation such that the torsional system has only one natural frequency. On this basis, the maximum displacement may be computed as the sum of a series of components of displacement which occur at the applied torque frequencies (principally 60 and 120 cycles) and also at the torsional natural frequency of the system. Each of these components is a function of the frequency ratio (impressed frequency/natural frequency), in a manner very similar to the basic resonance curve of figure 6. However, it appears advantageous in this case to keep the torsional natural frequency well below the lowest frequency of the impressed torque. When foundation flexibility is taken into account, the system becomes a compound one with two natural frequencies. The analysis becomes considerably more elaborate but has been worked out fully on the basis of neglecting the mechanical damping and the torque decrements. These assumptions, of course, are on the safe side, but before this study of short-circuit torques can be considered complete, these factors must be taken into account. In general, however, it is found that foundation flexibility reduces the maximum displacement of the core relative to the frame below that which would occur on a rigid foundation.

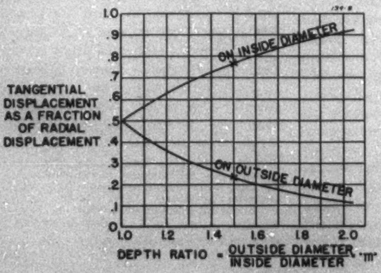


Figure 8. Tangential displacements in deep rings

Description of Resilient Core Mounting as Built

A resilient core mounting and a nonresonant stator frame were designed along these lines for a 31,250-kva air-cooled generator. A photograph of the frame is included as figure 9, and figure 10 gives a close-up view of the assembled spring members. These are twin sets of spring bars of rectangular section, attached by bolting and welding to the bore of the frame and arranged in three groups lengthwise. The long dovetail key bars on which the core is stacked, are attached by body-bound studs to the centers of the spring bars, which provide a sturdy tie between the core and the frame, rigid lengthwise but with some flexibility both in the radial and tangential directions. The spring bars are really beams with fixed ends, and with heavy bosses at their centers where they are bolted to the dovetail key bars and also at their ends where they are attached to the frame, so that there are no moving joints to wear out. Fatigue seems to be completely out of range for these bars, as the alternating component of stress is of the order of two per cent of the elastic limit of the material, superposed upon steady gravity stresses of about 20 to 25 per cent. And fatigue needs to be out of range, as at 120 cycles per second these bars must undergo more than 10,000,000 cycles of duty per day. The frame was also specially designed, to have a high four-node natural frequency, well above 120 cycles. The test value on the bare frame, without end shields, was 201 cycles. On this basis, the expected performance of the resilient core mounting was a reduction of the magnetic vibration from core to frame of about 30 to 1. When this machine came to test at the factory, it met our expectations fully. In fact, no 120-cycle vibration could be detected at the middle of the frame, by touch or by

Figure 9. Frame of 31,250-kva generator with resilient core mounting

Figure 10. Close-up view to show spring bars in 31,250-kva generator

Figure 11. Turbine unit with 31,250-kva generator having resilient core mounting, Westport plant, Consolidated Gas Electric Light and Power Company of Baltimore, Md.

means of optical vibration indicators, Frahm vibrating reed tachometers, or a vibration-velocity pickup device connected to a cathode-ray oscillograph. The core was reached by means of rods inserted through small holes drilled in the frame, and showed about 1.3 mils of magnetic vibration. But it was impossible for a man standing alongside the generator to tell, by touch or by hearing, when excitation was applied or removed.

Operating Results

This 31,250-kva air-cooled generator, as built with resilient core mounting and nonresonant stator frame, was installed at the Westport plant of the Consolidated Gas Electric Light and Power Company of Baltimore, Maryland, and went into service on August 10, 1940. Figure 11 shows this generator and driving turbine as installed at Baltimore.

The turbine driving this generator was built for operation at a throttle pressure of 1,250-pounds gauge 900 degrees Fahrenheit, being supplied with steam from two high-pressure boilers. The turbine exhausts into the 200-pound steam mains of the earlier Westport plant but with piping arranged for specific superposing if desired on two 25,000-kva 200-pound regenerative condensing units, which were the last 200-pound units installed at Westport.

The operation of this superposed 3,600-rpm unit has well demonstrated the value of the new core mounting for suppression of 120-cycle core noise and vibration, the operating results being quite in agreement with the factory tests. In actual service, a man standing alongside of the machine could not detect by touch or hearing the application or removal of field excitation.

Other machines incorporating the improved type of core mounting are now being built, and it is expected that this feature will be applied to many 3,600-rpm generators of the future.

References

1. Special Problems of Two-Pole Turbine Generators, C. M. Laffoon and B. A. Rose. AIEE Transactions, volume 59, 1940 (January section), pages 30–3.

2. The Design of Large Electrical Machine Frames, Ralph Poole. *Journal* of the Institute of Electrical Engineers (England), volume 77, 1935, pages 176–88.

3. Strength of Materials (book), S. Timoshenko. D. Van Nostrand Company, part II (2nd volume), 1930, pages 434–8.

4. Vibration Prevention in Engineering (book), A. L. Kimball. John Wiley and Sons, 1932, page 30.

5. Vibration Problems in Engineering (book), S. Timoshenko. D. Van Nostrand Company, 2nd edition, 1937, pages 14–16.

6. Theory of Sound (book), Lord Rayleigh. MacMillan and Company, 2nd edition, volume 1, 1894, page 388.

7. The Effect of Foundation Stiffness on the Resonant Frequencies of Rotating Machines, E. H. Hull. *Journal of Applied Mechanics*, volume 8, 1941.

8. Single Phase Short Circuit Torque of a Synchronous Machine, Nickle, Pierce, and Henderson. AIEE Transactions, volume 51, 1932, pages 967–73.

9. Transient Torques in Synchronous Machines, M. Stone and L. A. Kilgore. AIEE Transactions, volume 52, 1933, pages 945–53.

High-Potential Testing Equipment for Quantity Production

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Introduction

MANUFACTURERS of electrical apparatus and appliances are aware of the value of high-potential insulation tests and have endeavored constantly to improve testing facilities and methods. Practical circumstances very early gave rise to the practice of applying a short-time test voltage substantially above the normal excitation of the apparatus or appliance. Because of the availability and simplicity of amplitude control, alternating voltage of ordinary power frequency has been used and the test voltage levels have been developed through operational experience. Occasionally circumstances have made the use of power frequency impractical; hence higher frequencies and direct voltage have been resorted to. Also, surge voltage tests have been used increasingly in an effort to simulate transient voltages which occur in field circuits.

For large apparatus and for high-voltage equipment the dielectric testing sets have been standardized and the testing procedure has been generally established. In this field a generous amount of time can be allowed for conducting tests, because the quantity of apparatus manufactured is comparatively low.

Because of the large quantity of small apparatus and low-voltage equipment now manufactured, high-potential testing must be co-ordinated with production circumstances where rates of 20 or 30 complete dielectric tests per minute are commonly encountered. Accordingly, the recognized practice of gradually raising and lowering the test voltage is entirely impractical from the standpoint of time consumed. Consequently many of the factors which have been so carefully considered in the high-voltage field have been neglected in low-voltage testing.

The purpose of this paper is to point out that the simplicity of low-voltage testing does not obviate surge voltages and that the same general fundamental factors established for high-voltage testing also apply to low-voltage testing. In this paper an attempt has been made to evaluate the surge voltages in terms of their effect on the breakdown of insulation. First, a cathode-ray oscillograph study was made to determine the magnitude of the surges. Then a number of comparison tests were made on a selected insulation to determine the comparative breakdown strength with and without influence of the surges.

As a result of these investigations it was found that the magnitude of the surge voltage varied considerably depending on the method used for limiting the short-circuit current and on the characteristics of the dielectric. In some instances, surges of 100 per cent above the normal test voltage were measured. In a controlled test, the surges reduced the apparent breakdown voltage of a specific insulation by roughly 25 per cent. It has been found that a properly co-ordinated high-potential transformer and a resistance potentiometer control forms a suitable system for testing low-voltage apparatus. The system is relatively free from surge voltages and it does not interfere with high-speed testing. This method also has led indirectly to the development of a cathode-ray fault detector which has been very useful in analyzing the physical characteristics of the dielectric and the type of failures that occur.

Present Practice

The testing equipment for this low-voltage apparatus can be obtained and installed with comparative ease in any production line. The test voltage can be conveniently controlled and easily applied. Usually, some means of limiting the short-circuit current is employed in order to minimize the material damage to the apparatus or appliance in case the dielectric fails. A pair of insulated electrodes, called stickers, permanently connected to the high-voltage terminals of the high-potential transformer, is commonly used to apply the test voltage between the windings and the frame or ground of the apparatus under test. In some cases, however, two or more units may be connected in parallel to the high-potential testing transformer which is then energized for a brief period of time. In either case, surge voltages are liable to occur in the high-voltage circuit which casts some doubt on the effective value of the test voltage.

Switching Surges

It is common knowledge that the interruption of the excitation circuit of a transformer will produce a transient voltage across its terminals. The amplitude of the impulse is proportional to the rate of change of flux in the magnetic circuit, and it is dependent on the position on the

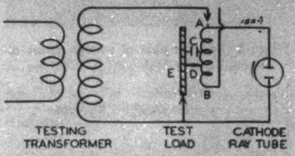

Figure 1. Equivalent high-potential testing circuit

A—Arc
B—Winding of apparatus
C—Capacitance of winding to ground
D—Resistance of insulation
E—Frame of apparatus (ground)

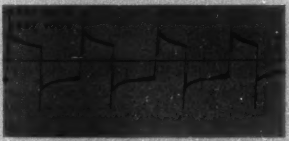

Figure 2. Oscillogram of voltage across dielectric during high-voltage switching

Transformer controlled by variable-voltage autotransformer
Normal crest voltage—3,530 volts

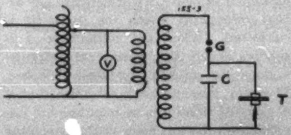

Figure 3. Test circuit for evaluating effect of switching surges on dielectric strength of paper

G—Sphere gap
C—Capacitor equivalent to the capacitance of low-voltage apparatus
T—Test sample

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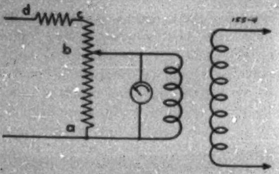

Figure 4. Potentiometer-controlled testing transformer

Minimum series resistance—*d-c*
Maximum parallel resistance—*ca*

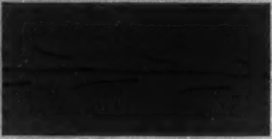

Figure 5. Oscillogram of voltage across dielectric during high-voltage switching

Transformer controlled by potentiometer normal crest voltage 3,530 volts

flux-time curve at which the circuit is opened. Whenever high-potential testing methods employ low-voltage switching, it is obvious that the dielectric circuit will be subjected to an indefinite abnormal voltage. Furthermore, the maximum amplitude of the surge may occur when the transformer is de-energized at the conclusion of the test. The dielectric may fail at this final instant and if it does, it may not be detected unless a subsequent test is made.

High-voltage switching is employed every time the stickers are applied and removed from the apparatus. An arc occurs when the stickers are applied and another when they are removed. The testing circuit, consisting of resistance, inductance, and capacitance, may have excessive transient voltages induced in or applied to all parts of the circuit during this arcing period.

A cathode-ray oscillograph connected as shown in figure 1 will indicate the actual voltage to which the dielectric is subjected. When the contact at *A* is solidly closed the normal transformer voltage is applied to the dielectric. However, when an arc is established at *A* a series of transients or impulses are impressed on the test piece at the beginning of each one-half cycle. The oscillogram in figure 2 shows this condition for a common type of testing circuit. This shows a crest voltage of over 5,000 volts whereas the steady-state test voltage had a crest value of 3,500 volts. This is somewhat equivalent to superposing an impulse on the power frequency. It has been observed in the past that under these conditions a flashover of an insulation may not be followed by a power arc, particularly when the total voltage amplitude is near the flashover value for that insulation. Under the somewhat similar conditions as illustrated by figure 2, the insulation may be partially damaged, by a pinhole puncture, for example, which has not been followed up by a power arc. This may be a sufficient cause for a future failure at a lower test voltage. This experience indicates that it is desirable to maintain a sinusoidal voltage on the test piece throughout the entire test period.

Field Data

In order to obtain some field data on the effect of arcing surges some special tests on fractional-horsepower motor armatures were conducted with a pair of stickers connected in parallel with the regular production line. The armatures were tested at 1,500 volts, but would normally withstand 2,500 volts. The test pair of stickers was connected to an armature and an arc maintained between one sticker and the winding for a period of time. Five consecutive armatures were broken down on the production line during this test, although a voltmeter still indicated an rms test voltage of 1,500 volts.

Table I. Effect of Surges on Dielectric Strength of Paper

Sample Number	Breakdown Voltage—No Arcing (Voltmeter Readings)	Breakdown Voltage Arcing to Simulate Removal of Stickers (Voltmeter Readings)
1	1,330	1,170
2	1,670	1,330
3	1,700	1,280
4	1,860	1,375
5	1,430	1,265
6	1,460	1,170
7	1,400	1,230
8	1,670	1,330
9	1,530	1,350
10	1,400	1,200
11	1,370	1,140
12	1,310	1,150
13	1,370	1,170
14	1,400	1,220
15	1,670	1,330
16	1,700	1,310

In another instance, a plate-glass condenser was used as a dielectric load and during the arcing period a sufficiently high-surge voltage was established to flash over the one-fourth-inch plate glass of the condenser, although the test voltage as indicated on an instrument remained at 2,500 volts. The condenser had a sinusoidal flashover value of 6,000 volts.

A test circuit shown in figure 3 was established to measure the effective breakdown value of the surges initiated by an arc in the high-voltage circuit. The sphere gap *G* represents the stickers and it was used merely to control the arc and not to measure voltage. Past experience has indicated that the capacitance of the dielectric of low-voltage apparatus and appliances varies from 0.0005 to 0.002 microfarad. A 0.001-microfarad capacitor, chosen as a representative load, was connected in series with the gap so that the voltage across it is comparable to that across the dielectric of the apparatus in actual practice. A transformer having about three per cent reactance was used for the source of high potential. Several selected samples of an insulation, consisting of four layers of 0.0005 condenser paper, were used as test specimens and connected directly across the capacitor. Each specimen was large enough to obtain two breakdown tests, one when a sinusoidal voltage was applied and the other when surges were produced by the arc. The first test was obtained by closing the sphere gap *G* and raising the primary voltage very slowly by means of a variable-voltage autotransformer until breakdown occurred. The second test was obtained by setting the primary voltage at some low value and opening the gap *G* until the arc was extinguished. (This action simulates the removal of stickers from the apparatus.) The sup-

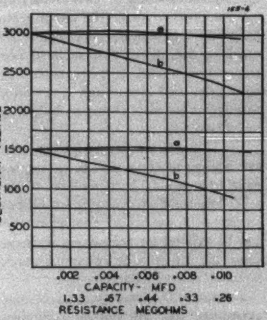

Figure 6. Voltage-regulation curve of 0.25-kva 3,000-volt potentiometer-controlled transformer

a—Capacitance load
b—Resistance load

ply voltage was raised in small increments and the procedure repeated until the specimen broke down and in each case the rms voltmeter readings were recorded. The results, shown in table I, indicate that the breakdown voltage is 20 per cent lower when the surges are present or in other words the dielectric is subjected to a higher level than the voltmeter indicates.

Surge voltages varying from 25 per cent to 100 per cent above the test voltage are commonly found in testing installations employing stickers, and utilizing reactance to limit the short-circuit current. Even when reactance is not deliberately introduced to limit the current, and when a testing transformer of normal design with less than five per cent reactance is used, the surges are still present. They may not be severe for many types of apparatus where the factor of safety is large. Furthermore, the degree of damage the surge can create in the insulation has not been conclusively established. The serious factor involved, however, is the uncertainty of the actual voltage to which the dielectric is subjected.

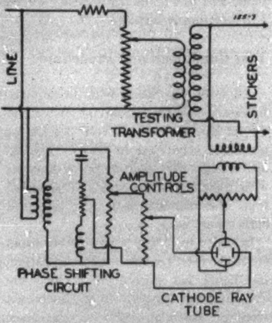

Figure 7. Circuit diagram for potentiometer-controlled testing transformer and cathode-ray fault detector

Methods of Reducing Surge Voltages

Resistance inserted in the high-voltage circuit in series with the test load will greatly reduce the surges appearing across the dielectric. In one case it was found that 50,000 ohms in the secondary of a 0.25-kva low-reactance transformer reduced the surges to a negligible amount. An equivalent resistance on the low-voltage side of the transformer similarly reduced the surges. A circuit shown in figure 4 has been found to be relatively free from all types of surges caused by low- or high-voltage switching, because the parallel resistance tends to absorb the energy of the low-voltage switching.

The effectiveness of the potentiometer control was demonstrated in the following manner. A transformer with a rating of 0.25 kva with three per cent reactance and a secondary voltage of 2,500 volts with 110 volts on the primary was supplied from a variable-voltage autotransformer. Surge voltages initiated by low-voltage and high-voltage switching were measured across a 0.003-microfarad capacitor load. The value of the surge most often obtained is given in table II as a percentage of the open-circuit secondary peak voltage.

Table II. Surge Voltages in a Circuit Controlled by a Variable-Voltage Autotransformer

Normal Secondary Voltage	Surges in Per Cent of Normal Secondary Peak Voltage: Primary Switching	Secondary Switching
2,500	200	250
1,500	200	200
900	200	200

The variable-voltage autotransformer then was replaced by a potentiometer, or a resistance control as in figure 4 and the switching repeated. The results are shown in table III. A typical oscillogram of the high-voltage switching surge is shown in figure 5, and compares with figure 2 for the previous case. Satisfactory operation is thus assured with the potentiometer control if the series portion is limited to some predetermined minimum value of resistance and also if the parallel portion is limited to some maximum value.

Any type of testing circuit will supply a limited load when some means of current limitation is employed. Figure 6 shows the voltage-load curves for a potentiometer-controlled circuit. The capacitance of the dielectric circuit does not have a great effect on the regulation, curve *a*, especially in the range encountered in low-voltage apparatus. The resistance of the dielectric, however, does cause an appreciable regulation, curve *b*. Therefore, if it is desired to maintain a regulation of less than three per cent the resistance of the dielectric should not be lower than three megohms.

The high-voltage wave form can be maintained reasonably sinusoidal. If the series resistor alone is used, the normal exciting current required by the transformer causes an appreciable voltage distortion. If, however, the current through the resistor *ab* in figure 4 is approximately three times the value of exciting current, the wave form is acceptable, with a harmonic content of less than five per cent. The amount of distortion, of course, depends materially on the degree of saturation in the transformer core, but if this is maintained at a reasonable value, a good high-voltage wave form can be maintained.

Figure 8. Lissajous figure of test voltage

a—Open circuit
b—Resistance load

Figure 9. Lissajous figure of test voltage

a—Open circuit
b—Capacitance load

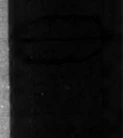

Figure 10. Lissajous figure of test voltage with corona in insulation

Figure 11. Lissajous figure of test voltage when arc is jumping through insulation

Figure 12. Lissajous figure of test voltage

a—Open circuit
b—Short circuit

Fault Detection

The arrangement of resistors just discussed permits an easy detection of short

Table III. Surge Voltages in a Circuit Controlled by a Potentiometer

Normal Secondary Voltage	Series Resistance (Ohms)	Parallel Resistance (Ohms)	Surges in Per Cent of Normal Secondary Voltage: Primary Switching	Secondary Switching
2,500	73	250	104	100
1,500	420	250	103	100

circuits because the voltage on both sides of the transformer drops in accordance with the severity of the failure in the dielectric. If the organic dielectric materials are moist, due to high humidity, the conduction current will be high, and the transformer regulation will cause the voltage to drop. Usually a high-leakage current will not destroy the insulation, but it will cause a definite indication of some abnormal condition.

A cathode-ray oscilloscope has proved to be a very useful tool in connection with the testing circuit shown in figure 4 for analyzing the type of faults that may occur in dielectrics.

Cathode-Ray Oscilloscope Fault Detector

Figure 7 shows the circuit diagram of the dielectric testing transformer, potentiometer, and cathode-ray circuit. The adjustable phase-shifting circuit permits the voltage on the horizontal plates to be shifted 90 degrees from the test voltage. The amplitude of voltage on both the horizontal and vertical plates can be adjusted so that the image on the cathode-ray tube screen can be made to appear as a circle as shown in figure 8*a* for the normal test voltage. When voltage is applied to a test sample through the stickers, the image will remain unchanged if the dielectric is perfect. If the insulation contains excessive moisture there will be a high value of conduction current, hence the voltage on the vertical plates will drop and the image on the cathode-ray tube will change to an ellipse as in figure 8*b*.

A high-capacitance current flowing through the dielectric, will cause a phase shift in the test voltage, hence the image will appear as an ellipse with an inclined axis as in figure 9*b*. Corona streamers in the dielectric will cause the image to appear as shown in figure 10, while an arc jumping across or through the insulation will change the image to figure 11. A complete short circuit produces a horizontal line as in figure 12*b*.

Conclusion

The data presented in this paper is by no means conclusive. It does not infer that all types of high-potential testing equipment are dangerous to insulating material, nor does it prove that all testing methods are improper. It has brought out the possibility that indefinite surge voltages may exist and that these surges may cause some damage to the insulation. These hazards arise, however, only when some fundamental principle has been violated, such as low- and high-voltage switching. The use of resistance for reducing surge voltages is not new. It has been shown, however, that either low-voltage or high-voltage switching may be employed without creating dangerous surges when the resistance is properly used. Thus, high-speed testing may be conducted to meet any production requirement and at the same time a definite predetermined test voltage can be maintained. There may be other methods of accomplishing the same result, but the resistance control seems to be the most expedient.

The cathode-ray fault detector is a very useful tool in connection with the resistance-controlled transformer. It enables one to determine by a glance, the wave form and magnitude of the test voltage, some of the physical properties of the insulation, and whether or not streamers or corona are present. The voltage which a material will withstand can be readily determined, and unseen flashovers can be detected by careful observation of the screen. If the power arc follows the flashover either momentarily or continuously, it too can be observed. The cathode-ray fault detector may not be practical to use in high-quantity production because of the longer time consumed in making the observations, but it is very useful in laboratory or other work where time is not so important.

Arc-Backs in Ignitrons in Series

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Synopsis: Theory based on the random occurrence of arc-backs indicates that the arc-back rate of ignitrons in series should be very low compared with the arc-back rate of the individual ignitron operating alone on its share of the voltage. To check these ideas, experiments were carried out with high-current ignitrons (350 amperes average per anode) operating in series with voltages balanced and unbalanced, and individually. The results of these tests are given and discussed.

Introduction

THE ignitron is a single-anode mercury-vapor rectifier with a mercury-pool cathode and a stationary electrode, called the ignitor, partially immersed into the cathode pool. A current of a few amperes at about 100 volts passed through the ignitor initiates the arc once each cycle and at any time desired in the cycle. The use of ignitrons with the usual rectifier-transformer connections gives a power-converting system with all the advantages of grid-controlled rectifiers at higher efficiencies, since no grids are required near the anode for the control of load.

In the last few years the use of ignitrons commercially has been greatly accelerated and single assemblies up to 3,000-kw capacity at 600 volts are in service.

Research work continues on the still baffling problem of arc-back. Accepting the arc-back rates as obtained on commercial tubes, however, it seemed desirable to check whether the lower arc-back rate for ignitrons in series, as predicted by the theory of random arc-back could be really obtained.

Two ignitrons were tested in series and individually, at water temperatures abnormally high, and without baffles and shields, so that arc-back rates were high enough for conclusions to be drawn in not too excessive a time. It was found that the two ignitrons in series under these conditions would operate with an average of 13.75 hours per arc-back, whereas an individual ignitron at *half the voltage* would operate with an average of only 0.0442 hours between arc-backs. The calculated value was 16.4 hours per arc-back for the two ignitrons in series.

Test Conditions and Procedure

All tests were made with single-phase full-wave connection from 540-kva transformer bank to two 375-volt, 300-kw d-c machines connected in series. The ignitron tanks were eight inches in diameter with 6-inch graphite anodes $2^{3}/_{4}$ inches from the mercury pool. No grids or baffles were used.

In order to obtain a reasonably rapid arc-back rate for the series operation, and hence not excessively prolong the test, the water temperature was increased

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until the arc-back rate was a value which would give rates for series operation of roughly one arc-back per day. It was found that at 700-amperes load and a cooling water-temperature range of 55 begrees to 67 degrees centigrade the arc-dack rate was satisfactory for the test. A current of 700-amperes load is 350 amperes per anode and is not too high for a 6-inch diameter graphite anode.

With conditions as stated above, the ignitrons were operated at 300 volts direct current, one at a time, until ten arc-backs had occurred for each ignitron. These data established the arc-back rate for the individual ignitron for 300-volt operation. Next, the tubes were connected in series and operated balanced (6,000 ohms across each ignitron) or unbalanced (1,000 ohms across one and 75,000 ohms across the other) at 600 volts. After completion of this series test, one ignitron was again operated at 300 volts to check if arc-back rate had changed during the experiment. During this test rates were also obtained for 300 volts by counting breakdowns, on one of the two ignitrons by means of a cathode-ray oscillograph, when the tubes were operating in series at 600 volts with voltages balanced.

The arc-back rate for 600-volt operation of *one* ignitron only was obtained by unbalancing the tubes in series with parallel unequal resistances and counting the breakdowns on the tube which carried almost all of the inverse voltage. From these measured rates at 300 and 600 volts, the arc-back rate for series operation was calculated with the assumption that the arc-back lasts for the whole half-cycle at which it occurs.

Results

The data as obtained in the experiment are compiled in table I. Several counts for the individual ignitrons were made of various durations varying from 5 to 38 minutes. The total time for counting is about the same for the 300- and 600-volt tests. From these data we find that the average time between arc-backs for ignitrons in series is 16.4 hours, when calculated from the arc-back rates of the individual ignitrons, whereas the measured average time between arc-backs is 13.75 hours. These values are for the balanced condition and check reasonably well. For the unbalanced condition, the values are seven hours as measured, and four hours as calculated. The method of calculation is described in the next section.

Discussion

When two ignitrons of the same design are connected in series, the inverse voltage will generally not divide equally between them at all times during the inverse cycle. This unequal distribution of voltage occurs only during the de-ionization time and varies from cycle to cycle. From oscillograms the inverse voltage varied from 600 to 266 volts, immediately after transition, on a single ignitron, in various cycles. This varying unbalance exists, to a continually decreasing extent, for approximately 40 degrees after transition.

To calculate the arc-back rate for ignitrons in series from the arc-back rates of the individual ignitrons, it was assumed that the arc-back cause in the individual tube exists only for the cycle considered, and that complete breakdown of the ignitrons in series will occur, if with an arc-back on one ignitron, one arc-back takes place in the same half-cycle on the other ignitron.

The calculation for the average time between arc-backs for series operation was made as follows:

If

t_3 = average time in seconds between arc-backs at 300 volts, for one ignitron
t_6 = average time in seconds between arc-backs at 600 volts, for one ignitron
f = frequency in cycles per second of power source
t_s = average time in *hours* between arc-backs for series operation

then

$\frac{t_6}{2}$ = average time for one or the other of the two ignitrons when operated in series on 600 volts

After an arc-back occurs in one ignitron, the voltage on the other anode is increased to 600 volts and its individual arc-back rate is increased. The time in cycles between arc-backs under this new condition is for one ignitron only, since we are considering that two in series are used, and we have t_3f as the number of cycles between arc-backs. The product of the above factors gives the average over-all arc-back free time for two ignitrons connected in series and in terms of hours.

$$T=\frac{ft_3t_6}{2\times 3,600}\text{ hours}$$

and for 60-cycle power source

$$T=\frac{t_3t_6}{120}$$

hours between arc-backs for the balanced condition. This equation was used to determine the arc-back free time for series operation after t_3 and t_6 had been measured.

For the unbalanced case, t_6 is used twice and since the voltage before an arc-back is practically zero on one of the

Table I

Ignitron	Resistance Across Other Ignitron	Volts	Amperes	Time of Count in Minutes	Number of Arc-Backs	Average Time Between Arc-Backs in Seconds T
3*A*48-2	None	300	700	21	6	210
3*A*48-1	None	300	700	25	7	214
3*A*48-2	None	300	700	38	10	228
3*A*48-2	6,000	300	700	19	26	46
3*A*48-1	6,000	300	700	10	5	120
3*A*48-2	6,000	300	700	10	4	150
	6,000	300	700	8	6	80
3*A*48-2	1,000	300	700	15	4	225
For 300 volts average time between arc-backs = 159 seconds						
3*A*48-2	6,000	600	700	14.5	207	4.2
3*A*48-1	6,000	600	700	15	61	14.8
3*A*48-1	1,000	600	700	14	100	8.4
	1,000	600	700	8	72	6.67
3*A*48-2	1,000	600	700	32	66	29.0
3*A*48-1	1,000	600	700	19	136	8.4
3*A*48-1	1,000	600	700	5	0	Water 33 degrees to 40 degrees centigrade
3*A*48-2	1,000	600	700	7	23	18.3
3*A*48-1	1,000	600	700	7	20	21.0
3*A*48-1	1,000	600	700	10	31	19.4
3*A*48-1	1,000	600	700	16	72	13.3
3*A*48-2	1,000	600	700	12	24	30.0
Average time between arc-backs at 600 volts = 15.8 seconds						

NOTES: Measured average time between arc-backs for two ignitrons in series and balanced was 13.75 hours (Test time was 55 hours.) Calculated from data:

$$\frac{159\times 15.8}{120}=21\text{ hours}$$

Calculated with first three values of table I omitted:

$$\frac{124\times 15.8}{120}=16.4\text{ hours}$$

Measured average time between arc-backs for two ignitrons in series and unbalanced was 7 hours. (Test time was 21 hours.)

Calculated from data:

$$\frac{15.8\times 15.8}{60}=4\text{ hours}$$

Electrolytic Process of Scale Removal From Steel

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MEMBER AIEE

ELECTROLYTIC action is by no means new or unfamiliar. It is as old as electricity itself and has been evidenced in many ways throughout the entire application and use of electricity. One of the early applications of electrolytic action pertaining to metals was employed in the refining of copper and similar metals. Also, electrolytic action was employed in the plating of metals on surfaces requiring a high finish or protective coating, which, of necessity, should be very thin. This application is best evidenced in nickel plating and chromium plating which has been widely used for a number of years.

Electrolytic action has been in general use in steel mills and other industries for cleaning steel sheets, for example, the hot caustic electrolytic scrubbing equipment for removing oil and dirt from cold-rolled sheets. This application of electrolytic action is used in conjunction with mechanical scrubbing and brushing rolls for cleaning the sheets, the electrolytic action being a primary and the scrubbing action the secondary means for obtaining this result. It, therefore, may seem strange that this very familiar and well-known principle of electrolytic action which has been in wide and rather diversified use for so long a time has not been applied to pickling or the removal of scale from steel in a sulphuric acid bath.

The answer to this, we believe, is due to the physical and mechanical difficulties encountered when trying to apply electrolytic action in a sulphuric acid bath. There are many physical difficulties encountered when a combination of electrolytic action and sulphuric acid pickling action are combined. This can be readily understood when it is realized that electrolytic action in combination with sulphuric acid is decidedly destructive to any current-carrying metal such as nickel, copper, aluminum, or steel. Also, sulphuric acid is an active agent in decomposing some of these materials, particularly steel. Therefore, the reluctance in applying electrolytic action to the pickling of steel which involves the use of sulphuric acid, has, no doubt, largely been due to these physical difficulties encountered rather than a lack of appreciation of the benefits derived from such an installation.

Development

In the processing of flat-rolled steel, it is necessary to remove the mill scale and oxide from the entire surface of the steel which is usually produced in coils of various widths and lengths. To remove the mill scale from these coils of steel, the normal procedure is to run the strip steel through a continuous pickling line which consists of a series of tanks arranged in tandem, filled with approximately a ten per cent solution of sulphuric acid and provided with the necessary mechanism to permit the strip steel to be uncoiled and fed through the pickling line in a continuous strip, the strip being delivered at the finishing end with the scale removed by the action of the sulphuric acid in which it has been submerged. Any practical application of electrolytic action which might be used in augmenting or improving the pickling action described above, should be applied to the existing continuous pickling line and would take the position of being additional apparatus applied to this already standard equipment.

On account of the physical difficulties mentioned above, it would therefore follow that considerable experimentation would be required to develop a feasible working assembly of equipment that would have commercial application in a continuous pickling line. With this viewpoint in mind, a number of years were spent in experimenting and developing this electrolytic equipment. Early experiments were made to determine the feasibility of using either direct or alternating current, and it was found that practically equally satisfactory results could be obtained with either of these currents.

Another problem was the method of applying the current to the sheet of steel which was to be pickled, bearing in mind that since this sheet steel is being continuously fed through the pickling line, it would be necessary that any current-carrying device that made contact with the strip would have to be in the form of a roller or some similar contacting device. This, in itself, presents quite a problem, as the contacting medium would be submerged in a sulphuric acid bath and would also be carrying electric current. This naturally would result in the contact

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1. For numbered reference, see end of paper.

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tubes, it is not necessary to divide by two, as was done above. The arc-back free time for the unbalanced case will be

$$T=\frac{t_at_b}{60}\text{ hours}$$

Following table I the calculations are shown for the two cases. In the balanced case two values are given; namely, 21 hours and 16.4 hours. In obtaining the latter value, the first three values of table I were omitted. It is believed that they are high because the time ignitrons were "off the line" after each arc-back is an appreciable fraction of the operating time. This allows the tubes to cool somewhat, and higher times may, therefore, be measured. Also the unbalance due to ionization variation made these three tests of uncertain value.

When the ignitrons were connected in series and the parallel resistance across an ignitron was 1,000 ohms, an arc-back in an individual ignitron, as determined by the cathode-ray oscillograph, would continue to the end of the half-cycle at which it started. However, with 6,000 ohms an arc-back would often clear before the end of the half-cycle. This instability is characteristic of arcs when the current is limited.

Conclusions

Experiments verify that the arc-back rates of ignitrons operated in series can be calculated from their individual arc-back rates. This indicates an advantage in direct series operation of ignitrons for high voltage, rather than placing ignitrons with their associated transformers in series.

material being eaten away rather rapidly by the electrolytic action and, in most cases, would result in depositing a coating of this material on the strip steel which is to be pickled.

From experimentation, it was found that the amount of current necessary to obtain proper electrolytic action in the removal of scale was from 100 amperes to 150 amperes per square foot of the strip steel being pickled. This large amount of current, if applied directly to the steel strip, would require contact rolls at frequent intervals throughout the length of the steel strip to equally distribute the current over the entire surface of the steel. Due to the difficulty encountered in applying the current in this way, it was decided to place current-carrying electrodes in the pickle bath, these electrodes being spaced on either side of the steel strip top and bottom, the strip allowed to pass through the sulphuric acid pickling bath between these electrodes. Then, by applying alternating current to these electrodes, the current would continuously pass back and forth between the electrodes and, in so doing, would pass through the sheet, in this way setting up an electrolytic action on the sheet materially assisting in the removal of the scale. This scheme was adopted and top and bottom electrodes, spaced apart, were installed in the pickling tanks and means provided for carrying the strip continuously through the tank between these electrodes.

The next consideration was the proper material to use in forming these electrodes. Considerable experimentation was carried on in this matter and, after trying out many different materials, it was found the most satisfactory material was graphite slabs which could be made up into electrodes of any required size by the assembly of these slabs into a unit of the required dimensions. Each pickling tank is approximately 60 feet long and of a width sufficient to allow the passage of the widest steel strip, which means that these tanks, in some cases, are eight feet wide. Since it was important that the electrolytic current be, as near as possible, uniformly distributed throughout the entire length of the submerged sheet, it was decided to install a number of electrodes in each tank rather than a single top and bottom electrode which would extend the entire active length of the tank. Consequently, these electrodes were designed to a dimension that would correspond to the width of the tank and allow a multiple number of electrodes to be installed throughout the length of the tank. This resulted in a final selection of six top and six bottom electrodes being installed in each pickling tank. The actual dimensions of these electrodes, in our case, were about four feet two inches wide and six feet six inches long and were installed in a pickling line that was designed for accommodating a maximum width steel strip of 44 inches.

To further insure uniform distribution of current, means were provided for bringing current into each electrode at four points. Each pickling tank was supplied with current from two transformers, each transformer feeding one-half of the electrodes in a tank. This results in a minimum length of current-carrying conductors between the transformer and the electrodes. This is an important factor to be considered when dealing with current flow of this large amount and low voltage.

The voltage required to pass the amount of current necessary through the pickling solution varies with the strength of the sulphuric acid and the content of the iron sulphate in the solution. Therefore, transformers were provided with a five-point tap changer on the primary side which would give a secondary voltage range from 15 volts down to 7 volts. The transformers were also provided with three segregated secondary windings, each winding connected direct to one top and one bottom electrode. This was done to further insure the uniform distribution of the current to the strip steel being pickled. The lower or seven-volt tap of the transformers is used when the pickling bath is first put in use, which means that there is at least ten per cent sulphuric acid solution and no iron sulphate. As the pickling action progresses, the acid is consumed resulting in a gradual lowering of the acid in the solution. Also, since the pickling action will remove a small amount of steel from the sheet, along with the scale, there will be gradual build-up of iron sulphate in the solution. These two conditions result in increasing the electrical resistance of the pickling solution which necessitates increasing the voltage to compensate for the increased resistance. This accounts for the various voltage taps mentioned above.

It is necessary to provide some means for controlling the current supply to the electrodes which is synchronized with the feeding of the strip through the pickling tanks and so connected that the electrolytic power is on only during the operation of the pickling lines and automatically shut off each time the line is stopped. This is necessary to prevent the pickling action continuing while the strip is not in motion. If this precaution was not taken, it would result in the strip being overpickled each time the line is momentarily stopped. In addition to this automatic control, there is also provided a manual control on the pickle line operator's control pulpit board.

Electrolytic action is most active after the sulphuric acid solution has penetrated through the film of scale and has reached the steel surface of the sheet. For this reason, the first tank in a continuous pickling line is not provided with electrodes, this tank being used as a conditioning tank which will allow the acid to penetrate through the film of scale. The electrodes are installed in the second and third tanks only in which the entire electrolytic action is applied and where the major part of the scale is removed. The fourth or last tank in the continuous pickling line is used only as a clean-up tank which will remove any remaining particles of scale still adhering to the steel strip.

The terminals connecting to the top and bottom electrodes carry a heavy current and it is necessary that they be well insulated to prevent current leakage and electrolytic action on these conductors. The insulating material used should be able to stand up under the action of hot sulphuric acid and acid vapor. Good grades of rubber will give very acceptable service in acid or vapor, with temperatures not exceeding 125 degrees Fahrenheit. The temperature of the pickling solution in a continuous pickling line is approximately 190 degrees Fahrenheit. At this temperature, the life of rubber is greatly reduced, therefore it is not entirely satisfactory in this service.

After considerable experimentation, we found that a glass fabric tape, impregnated with one of the synthetic insulating compounds such as Koroseal would give very satisfactory results, and this is the type of insulation that has been employed. In some of these cases, rubber is first applied to the conductors and then the glass fabric tape applied over the rubber. In others, it is applied directly to the conductor, and both installations have proved entirely satisfactory.

The steel strip forms a catenary in each pickling tank being supported on rollers in the end of each tank and allowed to dip down into the pickling solution. When electrodes are installed above the strip, it is necessary to provide some means to prevent the steel from coming in contact with the top electrodes as it travels through the tanks. Hold-down rollers were installed at each end of the electrodes in each tank. These rollers are required to operate submerged in the acid solution and are subjected to the combined effect of hot sulphuric acid, iron sulphate, electrolytic action, and mechanical wear and shock. Materials that will stand up indefinitely in sulphuric acid, such as bronze and monel metal, will be dissipated in a very few hours when subjected to electrolytic action. Bearings have to operate submerged with no lubrication other than that of the hot sulphuric acid. These conditions are rather severe for any material that would provide a satisfactory hold-down roller.

However, a roller assembly has been developed which stands up well and gives long life and service. This consists of a roller with an extended shaft made from either carbon or a dense, compact compound using a resin base and asbestos binder. To the shaft of the roller are fitted Pyrex glass bearings. The rollers are held in position in the tanks by means of steel pipe arms which are covered with rubber, around which is applied an outer wrapping of glass fabric tape which has been impregnated with a synthetic compound. The glass bearing is fitted into a bearing housing supported by these pipes, the housing being made from the same material as the roller. This type of roller has given very satisfactory service during several years of use.

Operation

An electrolytic pickling unit, in our case, is installed on one of three continuous pickling lines.[1] This particular line is one that normally handles coils of comparatively thin gauge, narrow width, and considerable length. Due to this condition a greater speed is required on this line to produce a given tonnage in a given time than is required in the case of the other pickling lines which handle wider and heavier materials. This is the reason for the selection of this particular line to be equipped with electrolytic equipment.

The handling of coils in the feed end of a pickle line including leveling, shearing, and welding or stitching, requires a given amount of time regardless of the length of the coil. Therefore, a long coil lends itself more readily to the use of electrolytic pickling equipment than a short coil. For example, assuming a normal pickling speed of 100 feet per minute and a coil length of 200 feet: in this case, it will require two minutes for the entire coil to pass through the pickling bath. If two minutes are also required to place a new coil in the feeding equipment and perform the necessary operations such as leveling, shearing, and welding, the time of the two operations balance and there is no necessity for installing electrolytic pickling equipment in order to increase the pickling speed which is limited by the feeding operations. On the other hand, assume that the coils are available twice the length specified, or 400 feet and since the feeding time would be the same regardless of the length of the coil, electrolytic equipment could be installed to advantage, as the feeding operation would permit, in this case, a pickling speed as high as 200 feet per minute. Therefore, it is readily seen that the length of coil available has an important bearing on the advisability of applying electrolytic equipment in order to obtain additional pickling speed.

In any continuous pickling line there will be found a large variation in pickling speeds. A given line may, at times, operate at a speed of 120 feet per minute and at other times may be able to remove the scale and do a perfect pickling job on the steel at 200 feet per minute. This wide range of pickling speed is due to a number of factors, such as coiling the steel hot, or somewhat cooled before coiling in the hot mill, coils stacked hot and allowed to cool before pickling, or coils being pickled while still fairly hot, high phosphorus or high carbon steel. There are also other factors that affect the pickling speed, such as the strength of the sulphuric acid solution in all or part of the tanks, and the presence or lack of iron sulphate in the pickling tanks. Any one of these factors has an effect on the speed of pickling and when a combination of a number of these factors is present, a low pickling speed will result, and ideal conditions will naturally produce the higher pickling speeds. Therefore, these conditions also have to be given consideration in considering any means relative to increasing the pickling speed of a continuous pickling line.

Any increase in speed of pickling due to electrolytic action over what can be obtained without it results in a direct increase of tonnage from the line. This tonnage is realized with no increase in labor or overhead. The only additional cost of this tonnage is the interest on investment of electrolytic equipment, its maintenance, and the cost of the current consumed. Our experience over a number of years shows that we obtain an average increase of 35 per cent in tonnage with the electrolytic equipment over what would be obtained in this same pickling line without the use of electrolytic. The electric load for the electrolytic pickling equipment is 600 kw.

When considering the interest on investment, the maintenance, and the cost of the current consumed, it is found that the cost of pickling chargeable directly to the additional tonnage that is realized due to the operation of electrolytic equipment is considerably lower than the normal cost of pickling a ton of steel without electrolytic equipment. Therefore, under the proper conditions, electrolytic equipment shows a decided saving.

Electrolytic equipment installed on a continuous pickling line will increase the speed of the line regardless of the type of scale, the amount of scale-breaking equipment, or the grade of steel being pickled. To realize the full advantage of this increase in speed and to actually secure an increase in tonnage, the pickling line must be able to operate in the time required for a coil to pass through the pickling bath at the higher speed that can be obtained from the use of electrolytic action. Under this condition an electrolytic installation will show a good return on investment. If this condition does not exist, the time saved by the use of electrolytic equipment will be offset by the delay due to preparing a new coil for feeding into the pickling line.

The handling and feeding of coils into pickling lines is constantly being speeded up. More efficient coil handling and processing equipment is being provided by the equipment manufacturers. The time required to weld and trim coils is also being steadily decreased, all of which results in the cutting down of the time required to deliver the coil to the pickling line. This creates a condition receptive to any means that will result in a faster pickling of strip steel and electrolytic pickling equipment has a most feasible application under these conditions as a definite means of securing increased production from a pickling line.

Reference

1. For complete description see ELECTROLYTIC PICKLING OF STRIP STEEL, H. W. Neblett. Association of Iron and Steel Engineers Yearly Proceedings for 1939, page 188.

Enclosed Bus-Bar Electrical Distribution Systems for Industrial Plants

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Introduction

ENCLOSED bus-bar distribution systems have come into common usage to meet the needs of industry for the greater availability of electrical power in production areas. Beginning in 1927 the automotive industry pioneered the application of enclosed bus-bar systems as a solution to the frequent recurring problem of modifying the electrical distribution system incidental to mass production changes. Since that time there has been an increased utilization of bus-bar systems by all branches of industry.

It is the purpose of the author to present in this paper a review of the fundamental electrical characteristics of such systems, discuss and describe some typical applications, and to emphasize the need for further fundamental research to meet the growing demands of industry.

The nature of industrial application requiring economy, safety, and compactness, brought about commercial designs using formed metal enclosures, porcelain or cold-molded insulating supports, and closely spaced copper and aluminum bus bars. The common industrial voltages for which commercial designs are being manufactured are: 120/240 volts, single phase, and 120/208 volts, three phase, four wire, for light; and up to 600 volts, three phase, for power.

The industrial applications of enclosed bus bars are for main feeders generally in current ratings up to 4,000 amperes, and for obtaining convenient outlets for power in current ratings, from 125 to 1,000 amperes. Both types of products are classified as "busways" in the National Electric Code, but in this paper the two types will be differentiated by the terms "feeder busways" and "plug-in busways."

While enclosing bus bars in switchboard structures, bus-bar cell construction in central stations, and interconnecting busses between equipment has been common practice for years the experience and empirical data and studies made of these bus-bar applications were not as helpful as anticipated. This is explained by the fact that these designs were of rectangular bus bars on comparatively wide spacing, and in large enclosures.

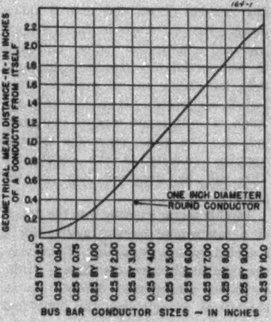

Figure 1. Geometrical mean distance of a bus bar from itself

As in all electrical distribution systems the important factors to be considered in planning a busway system are: voltage regulation, temperature rise, and mechanical design. The fundamental approach to the application of busways requires a determination of the impedance per unit length and current-carrying capacity of the selected design.

Voltage Regulation

Knowing the characteristics of a unit length of a selected design, the specified voltage regulation of a busway system, as for other distribution systems, is obtained by the usual series and parallel circuit connections.

Unfortunately, at present, specific unit length characteristics of busway systems are not as available in literature as for cable systems. One of the factors which contributes to this situation is that in meeting the increasingly exact service requirements of industry, the manufacturers have continued a process of evolution which in itself has not given encouragement to the publishing of tables and data. A second contributing factor is that in the tendency to closer and closer bar-to-bar spacing, skin and proximity effects become of increasing importance and must be studied more carefully than in the past, if exact resistance and reactance values are to be made available, for the multitude of conductor combinations now required by industry. Bar face-to-face spacings of the order of a fraction of an inch are not unlikely.

Assuming a uniform current distribution in each bar, that is neglecting skin and proximity effect, which has been rather common practice, the inductance of a rectangular bus-bar system may be determined from the scientific papers of E. B. Rosa and F. W. Grover.[3,4] This work gives the approximate self inductance of a straight cylindrical wire as:

$$L_s = 2(l)\left(\log_e \frac{2l}{\rho} - 1 + \frac{\mu}{4}\right) \qquad (1)$$

And the approximate mutual inductance between two parallel cylindrical wires as:

$$L_m = 2(l)\left(\log_e \frac{2l}{d} - 1 + \frac{d}{l}\right) \qquad (2)$$

And the approximate net total inductance per wire of a two-wire transmission line as:

$$L = 2(l)\left(\log_e \frac{d}{\rho} + \frac{\mu}{4} - \frac{d}{l}\right) \qquad (3)$$

where

l = length of wire
d = center-to-center separation of wires
ρ = radius of wire
μ = permeability of medium

When the length (l) of the wire is great in comparison with the spacing (d) the last term (d/l) of this equation may be neglected.

For a nonmagnetic cylindrical tube of infinitesimal thickness or for high-frequency circuits, where no magnetic field exists within the conductor then the second last term ($\mu/4$) may also be neglected. This condition is not always met in industrial power distribution practice and the ($\mu/4$) term (with $\mu = 1$) should remain in the equation. This term represents the self inductance due to the flux within the conductor itself and is a factor of considerable relative importance on close spacings.

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1. For all numbered references, see list at end of paper.

When other than a straight cylindrical wire is employed as for example the rectangular conductor in a busway, it is convenient to use modifying factors to convert the dimensions of the conductor arrangements into that of an equivalent cylindrical wire system in order to utilize the above fundamental inductance relationships.

The most acceptable modifying factors in use at the present are those based upon the concept of geometrical mean distances introduced by Clerk Maxwell.[1] Later F. W. Grover[4] published tables and data for calculating the inductance of rectangular conductors, in which the following definitions appear. "The geometrical mean distance (gmd) of a single area is that distance whose natural logarithm is equal to the average of the natural logarithm of the distances of all points of the area from each other." For a rectangular area of thickness a and width b an approximate expression of self-geometrical mean distance is:

$$R=0.2235(a+b) \tag{4}$$

where R = self-geometrical mean distance.

"The geometrical mean distance of two areas is that distance whose logarithm is the average of the logarithms of the distances of all the points of one area from all the points of the other." If D equals the geometrical mean distance of two areas and d the center-to-center distance between these areas then:

$$D=kd \tag{5}$$

or

$$\log_e D=\log_e k+\log_e d \tag{6}$$

Value of the $\log_e k$ for equal parallel rectangular areas are given in tables by Grover. Doctor O. R. Schurig[13] further simplified the solution of mutual geometrical mean distance by a graph published in his article on the calculation of inductance for rectangular bar conductors.

If the current distribution within each conductor is uniform these geometrical mean distance factors, as discussed by Rosa and Grover, can be substituted in equation 3, R for ρ, and D for d, as follows:

$$L=2(l)\left(\log_e \frac{D}{R}\right)\times 10^{-6} \text{ millihenrys per centimeter} \tag{7}$$

Note that $\mu/4$ turn does not appear in equation 7 since the flux within the conductor is taken into account in the determination of the gmd, and nonmagnetic conductors are assumed. C. C. Levy[13] discussed this point in his article "Calculation of Inductance and Current".

Examples of the geometrical mean distance factors R and D for a range of one-fourth-inch copper bus bars versus conductor spacing are given in figure 1 and figure 2 respectively. Applying these factors to equation 7 the calculated 60-cycle reactance for a wide range of one-fourth-inch bar widths is shown in figure 3. The values in figure 3 are for uniform current distribution.

It is well known, however, that the current distribution within large conductors is not uniform and therefore the geometrical mean distance factor modifications are not strictly accurate. If the current is crowded to the ends and sides of a rectangular conductor due to skin effect and if the crowding effect is made asymmetrical due to proximity effect, then this current distribution pattern is important in the determination of the flux linkages of self and mutual inductance. Further research should be encouraged to determine factors applicable to commercial bus structures, particularly for closely spaced multiple interleaved conductors. It is not only the value of inductance that is important but also the effect upon the current-carrying capacity of the conductors. This will be discussed later in the paper.

Another factor entering into the design of a commercial busway system for a specified voltage regulation is the voltage unbalance between the phases. If the conductor arrangement differs from an equilateral triangle then the individual phase voltages must be calculated and if necessary the phase conductors transposed at intervals along the busway. The problem is often identical to that of a flat transmission line. This unbalance in the induced voltages of the respective conductors is due to two factors: difference in reactance between the respective phases and the 120-degree displacement of the current in the three phases. Fortunately, it is a simple matter to make transposition in most modern busway systems and thereby attain balanced voltages.

In figure 4 are shown phase-voltage readings along an early wide-spaced (five-inch center to center of bars) high reactance, flat facewise configuration busway installation before and after transpositions were made. Present-day closely spaced designs would produce the same relative characteristics, but with voltages of much improved regulation.

Power-factor correction static condensers are often located along the lines of a busway system to take full advantage of improved voltage regulations along with power-factor correction.

In these early systems as well as in many installations of today, reactance plays a dominant part in the impedance drop, while as spacings are made closer, the resistance component becomes increasingly important and may even itself become dominant. Here again, the importance of skin and proximity effect enters into voltage regulation calculations, this time by increasing the a-c resistance.

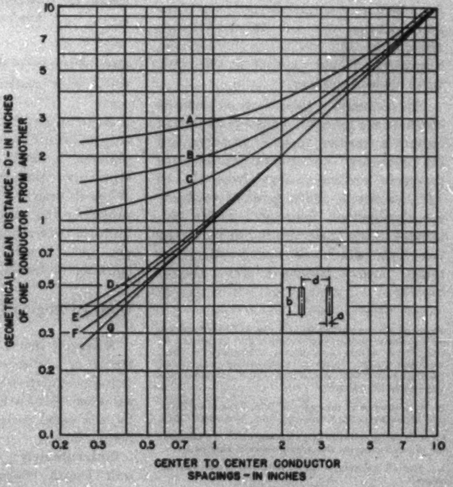

Figure 2. Mutual geometrical mean distance versus spacing for two rectangular bus bars

Designation of bus-bar sizes on curves, all bar dimensions in inches:

A—0.25 by 10
B—0.25 by 6
C—0.25 by 4
D—0.25 by 1
E—0.25 by 0.75
F—0.25 by 0.50
G—0.25 by 0.25

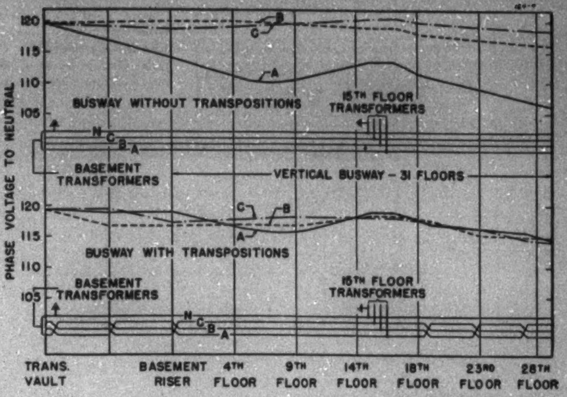

Figure 4. Voltage regulation of the respective phases along a three-phase, four-wire wide-spaced early type busway design before and after making transpositions

Current-Carrying Capacity

While skin effect causes a higher density in the outside periphery of a conductor, proximity effect has the tendency to crowd the current to the outside when currents flow in the same direction in adjacent conductors and to the inside when currents flow in the opposite direction in adjacent conductors. In certain cases, therefore, proximity effect may in combination with skin effect, actually cause a better distribution of current and, therefore, a lower a-c resistance than results from skin effect alone. On the other hand, these two effects more often combine to produce a considerably greater a-c resistance. Tests have shown this increase in resistance to be very significant. A more comprehensive knowledge of these combined effects will prove beneficial in the accurate establishment of the current ratings of bus-bar systems for a given temperature rise. Limited investigations seem to point to the importance of symmetry where several conductors are connected in parallel.

When bus bars are spaced closely, phases connected in parallel and interleaved as in figure 5, more research and study will be required before an accurate prediction can be made of the system's voltage regulation, power loss, and current-carrying capacity.

It is present commercial practice to rate bus systems arbitrarily on the basis of 1,000 amperes per square inch, despite the fact that this system of rating does not result in a uniform temperature rise for the respective sizes at rated load. Progress is being made, however, toward a more accurate rating of the bus systems through the co-operation of manufacturers with the underwriters. In the meantime, the manufacturers simply provide the necessary capacity to carry the load for the specified temperature rise.

Another and very important factor in the modern busway system is the design of its protective enclosure. The present practice is to make these enclosures of steel for ratings below 1,500 amperes, and of nonmagnetic material for high-current ratings.

Housings of nonmagnetic materials are often preferred to steel for certain high-current rating designs because of the high-flux densities and eddy currents induced in the steel housings which give rise to excessive temperature and voltage drops.

Various means are employed for ventilating the housing and cooling the enclosure by convection, but in general, the enclosed system will not have as high a current-carrying capacity for a given temperature rise as will bars in the open air. Industry, of course, realizes this

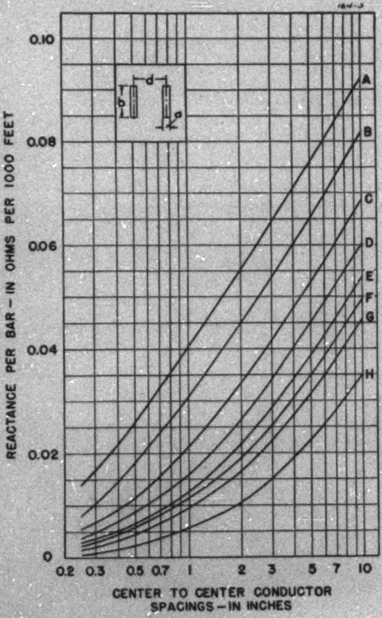

Figure 3. Reactance per bar versus spacing for two rectangular bus bars

Designation of bus-bar sizes on curves, all bar dimensions in inches:

A—0.1875 by 0.625
B—0.25 by 1
C—0.25 by 2
D—0.25 by 3
E—0.25 by 4
F—0.25 by 5
G—0.25 by 6
H—0.25 by 10

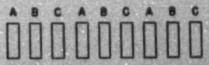

Figure 5. An example of parallel interlaced three-phase rectangular bus-bar conductors

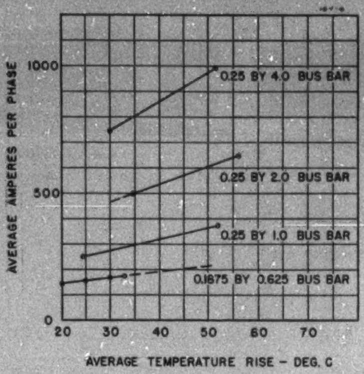

Figure 6. Test data on current versus temperature rise of three-phase, three-bar, 60-cycle plug-in busways. Bus bars of copper. Dimensions in inches

situation and accepts the lower capacity of the enclosed system because of its obvious other advantages. The actual rating of an enclosed system in the final analysis, probably, will be based upon tests of the particular design. In general, it is the same problem as encountered in the operation of a number of cables in a duct enclosure. An example of the carrying capacity of some typical busway systems is shown in figure 6.

Applications

It is interesting that the earliest industrial electrical distribution systems had open feeders extending the length of the plant, with ceiling-mounted link-fuse protection of tapoffs for individual power loads, similar to the fundamental concept of modern busway systems. The fire and personal injury hazards of the materials then available brought about the alternative practice of radiating conduit and wire feeders from a main switchboard or from several subdistribution centers.

The radial system met industry's needs through the early decades of this century, but with the advent of mass production, individual motor drives, expanding and changing load centers, the static nature of the radial system often resulted in overloaded feeders and equipment, poor voltage regulation, and costly rearrangement expenditures. A system which would provide the same convenience for power in industry as plug receptacles do in the modern home, became industry's demand.

Plug-in Busways

This need of industry for a greater convenience of power is being met by manufacturers with a system of power and light plug-in devices which engage bus bars supported within a metal enclosure.

The complete system for an industrial plant usually consists of a number of overhead parallel busway runs located according to machinery concentrations, plant conditions, and column centers. These bus structures are usually located overhead and supported by building structural members. An exception to this method of supporting the busway is shown in figure 10 which is a photograph of an installation in a high bay crane area where suspension means of support could not be used.

Connections are made to the busway circuits by the insertion of combination plug-in switch and protective devices at such points as to provide the most economical branch circuit layout. A section of a typical busway run with plug-in device in position is shown in figure 8. The arrangement of bus bars and construction details of a typical busway type of enclosure with its insulator assembly is shown in figure 7. Branch circuits are extended from the plug-in protective device by the customary wiring method to the individual power equipment drives or distribution centers. Such an example is shown by the photograph in figure 11.

The growing need for the installation of static condensers to correct for low power factor has led to the design of plug-in capacitors which correct for power factor and improve the voltage regulation on the busway. The location of these capacitors, of course, depends upon the characteristics and distribution of the load, but as a rule, a location near the load end of the busway provides best results. A photograph of a typical

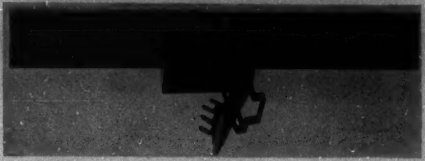

Figure 8. Protective device engaged to plug-in busway through one of the enclosure outlets

Load conduits are extended from knockouts in protective devices

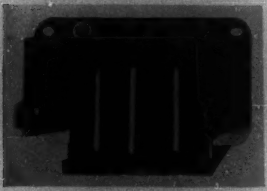

Figure 7. End view commercial design plug-in busway

1/4- by 4-inch rounded edge bars. Insulator one-piece porcelain. Enclosure assembly of formed steel sheets

Figure 9. Combination plug-in protective device and capacitor

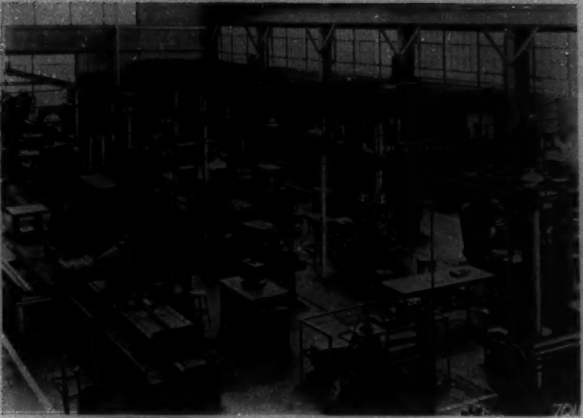

Figure 10. Plug-in busway system supported by iron pipe uprights spaced on 20-foot centers for supplying power to motors in high bay crane area

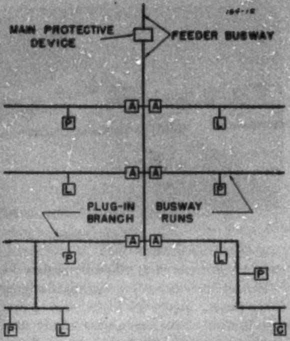

Figure 12. Schematic layout of a busway system frequently used in industrial plants

Symbols indicate protective devices for the following applications:

P—Power loads
L—Light loads
A—Branch plug-in busway
C—Capacitors

plug-in capacitor is shown in figure 9.

The common industrial application of busway plug-in systems is to 440-volt, three-phase, three-wire, 60-cycle a-c power distribution, although there is an increasing acceptance of the plug-in busway for three-phase, four-wire combination light and power systems. Where light is supplied from a busway it is general practice to extend conduit and wire circuits from plug-in equipment to lighting distribution panel boards or transformers. Where there is a need for an even more flexible supply of power to portable tools, such as along an assembly line, it is the practice to install a trolley collecting device supported on a track which is an integral part of the bus-bar enclosure.

Figure 11. A close-up view of layout, figure 10, showing protective devices and branch motor circuits

Plug-in busways may be supplied with power directly from distribution centers, by conduit and cable wiring methods, or by a feeder type of busway.

Feeder Busway

Feeder busways differ from the plug-in type in that they are generally of higher current-carrying capacity and are not provided with plug-in outlets. The usual method of tapping off loads from feeder busways is by bus-bar connections or conduit and wire. The conduit and wire branch feeder connections are generally made through metal enclosures attached at the joint sections.

A layout of a typical feeder and plug-in busway system is shown in figure 12. In this figure a section of feeder busway extends from the transformer supply to the main protective device. From the protective device the feeder busway extends the full length of the plant to supply the individual plug-in busway branch runs. Because of the high-capacity requirements of feeder busways it is often necessary to arrange many bus bars in parallel and to interleave the phase conductors to obtain a low reactance. Such a feeder busway is shown by the photograph in figure 13. A multitude of arrangements may result in the application of feeder busways to and from

Figure 13 (left). Commercial design of interlaced feeder busway with steel backplate and ventilated aluminum enclosing channel

switchboard, as is typified by figure 14.

Feeder busway systems, particularly for three-phase, four-wire light and power distribution, have been successfully used as risers in buildings having many floors. The load requirements of each floor are tapped off the busway and distributed through panel boards or plug-in busway.

A comprehensive treatment of the general problem of welding applications as reported by the AIEE subcommittee on power supply for welding applications,[16] is published in the May 1940 issue of Electrical Engineering. This activity has stimulated the development of feeder busways to provide designs for improved voltage regulation on welder service. These improved designs will be characterized by close phase-to-phase spacing of bus bars and convenient methods of extending conduit and wire feeders from the busway to welding machines.

Figure 14. Example of feeder busway extending from a main distribution switchboard

Bibliography

1. Electricity and Magnetism, Clerk Maxwell. Volume II (1873).

2. Absolute Measurements in Electricity and Magnetism, A. Gray. Volume II (1893).

3. On the G.M.D. of Rectangular Areas and the Calculation of Self-Inductance, E. B. Rosa. Bulletin of the Bureau of Standards, volume 3, 1907.

4. Formulas and Tables for the Calculation of Mutual and Self-Inductance, E. B. Rosa and F. W. Grover. Bulletin of the Bureau of Standards, volume 8, 1912.

5. Experimental Researches on Skin Effect in Conductors, A. E. Kennelly, F. A. Laws, and P. H. Pierce. AIEE Transactions, volume 34, 1915, pages 1953–2018.

6. Skin Effect of a Return Circuit of Two Adjacent Strap Conductors, H. B. Dwight. *The Electric Journal*, April 1916, page 157.

7. The Reactance of Strap Conductors, H. B. Dwight. *Electrical Review and Western Electrician*, June 1917.

8. Skin Effect in Tubular and Flat Conductors, H. B. Dwight. AIEE Transactions, volume 37, 1918, pages 1379–1400.

9. Reactance Values for Rectangular Conductors, H. B. Dwight. *The Electric Journal*, volume 16, June 1919.

10. Untersuchungen uber Stromverdrangung in rechteckigen Querschnitten, H. Schwenkhagen. *Archiv für Electrotechnik*, volume 17, January 31, 1927, pages 537–89.

11. Current-Carrying Capacity of Busbars, H. W. Papst. *Electrical World*, September 21, 1929.

12. Calculation of Inductance and Current Distribution in Low-Voltage Connections to Electric Furnaces, C. C. Levy. AIEE Transactions, volume 51, 1932, page 903.

13. Engineering Calculation of Inductance and Reactance for Rectangular Bar Conductors, O. R. Schurig. *General Electric Review*, May 1933.

14. Reactance of Busbars. H. B. Dwight and T. K. Wang. AIEE Transactions, volume 57, 1938, pages 762–5.

15. Temperature Rise of Busbars, H. B. Dwight, G. W. Andrew, and H. W. Tileston, Jr. *General Electric Review*, May 1940.

16. Power Supply for Resistance-Welding Machines, AIEE Subcommittee on Power Supply for Welding Operations. AIEE Transactions, volume 59, 1940 (May section), pages 306–20.

Lead Storage Batteries in the Transportation Field

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Synopsis: A review of storage battery applications in the railroad, motor bus, and aircraft transportation fields. Historical notes are recorded of events in the development of gas and Diesel-electric rail cars and Diesel-electric locomotives, especially as those events relate to storage batteries. Similar treatment is given to other battery applications—car lighting, air conditioning, busses, and aircraft. The changes made in the construction of batteries for these services are described. Physical and electrical characteristics are emphasized where they have a bearing on the application.

Introduction

ELECTRIC starting of automobile engines is taken for granted until you have to crank one by hand. The drivers of 24,000,000 automobiles and 4,000,000 trucks which operate today, have little appreciation of the job done by storage batteries. Probably only those engineers intimately associated with the application of storage batteries in the transportation industry know of the degree to which storage batteries have performed much larger jobs. For example, they are used on all commercial air transports; they furnish the power to crank gas and Diesel engines on over 1,500 rail cars and locomotives, and on over 138,000 busses. Storage batteries also help to electrically air condition over 12,000 railway passenger cars, and they insure power supply to 40,000 signals used by the railroads for safe operation of trains. There are few who are familiar with the changes in design which were necessary to fit storage batteries to the peculiar requirements of certain transportation applications.

I. Railroads

A. ENGINE CRANKING

1. *History.* At the beginning of the century a self-propelled rail car was introduced in which the electric driving motors received their power from a storage battery operating in parallel with a generator driven by a gasoline engine. The storage battery took care of peak loads and the generator furnished the average power required.

The forerunner of the present-day gas-electric rail car using variable voltage control appeared first in 1905. The engine was started by compressed air. About 1910, a small six-volt storage battery was added to furnish ignition current.

In 1905 a mechanically driven car known as the McKeen car was also introduced. Following Mr. Kettering's success of starting automobile engines by an electric motor in 1911, the idea was expanded to include the larger size gas engines then being applied to rail cars. Usually a 12-volt 120–160 ampere-hour storage battery served for cranking and lighting purposes.

There was a decided lull in the development of rail cars between 1912 and 1924. In the latter year an improved design of gas-electric rail car was introduced in which a 32-volt 215 ampere-hour storage battery was used for engine cranking, car lighting, and generator excitation. Thereafter a rapid expansion in the number of gas-electric rail cars took place. In Europe, on the other hand, gasoline was expensive and difficult to obtain, so that greater efforts were directed toward Diesel-powered rail cars.

In 1923 a Diesel locomotive, employing electric drive, was built in Europe. Starting windings were provided on the generator and the Diesel engine was cranked by the storage battery in this manner for the first time.

In February 1924 a 300-horsepower Diesel electric locomotive was placed in service in the United States. The engine was started by compressed air and a 16-cell 108 ampere-hour storage battery furnished current for excitation, control, and lighting. Two years later a 64-cell battery of the same capacity was added for engine cranking. Soon thereafter the cranking, excitation, control, and lighting circuits were connected to a common 56-cell battery.

In 1933 the Diesel-electric locomotive entered the high-speed passenger train field with a single 600-horsepower engine using a 64-volt 425 ampere-hour battery. Some of the present 6,000-horsepower locomotives are composed of three cab units operated by multiple-unit control. Each cab unit has two 1,000-horsepower engines and a 64-volt 425 ampere-hour battery.

In the early days of electric cranking of engines, it was the practice of locomotive manufacturers to use empirical formulas to determine the torques required to "breakover" and "fire" Diesel engines. By combining this information with the characteristics of the generator starting winding and a battery, it was possible to arrive at a suitable size of storage battery. Experience taught them that a battery sufficiently large to perform the cranking duty would adequately take care of the other electrical duties assigned to the battery. As engines grew larger it was desirable to re-examine the cranking requirements. Oscillograms were obtained of the starting conditions and the information used in a manner similar to that described in a paper delivered before AIEE at its 1940 summer convention in Swampscott, Mass.

2. *Adaptation of Storage Batteries.* Although the eight-hour battery capacity rating is traditional, it is of little value in Diesel cranking service, because, in this service, the battery must (1) "breakaway" the engine from standstill and (2) quickly bring it up to "firing" speed.

Condition (1) is essentially one of torque, and torque is proportional to amperes. The high breakaway torque has, in practice, required batteries to discharge at rates up to approximately their one-minute rate. To provide the necessary high-current rates, it is desirable to keep the internal resistance of the battery at a minimum and the current-carrying parts of adequate cross section. Condition (2) involves delivery of current for a reduced torque and at a voltage that will give speeds somewhat in excess of the minimum speed required for firing in order that the engine may fire quickly.

In order to satisfy both conditions, it has been found advisable to use multiple current-carrying posts, and to use copper inserts in these terminal posts and in the lead intercell connectors. Also cells were arranged in the trays in such a way as to make the intercell connectors as short as possible and the connectors were lead-burned to the terminal posts in order to reduce contact drop.

Figure 1 shows a typical characteristic of a battery used to crank a 900-horse-

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1. For all numbered references, see list at end of paper.

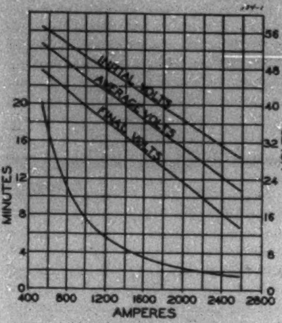

Figure 1. Discharge characteristic of a typical cranking battery

power Diesel engine. Figure 2 shows the accumulated horsepower of Diesel-electric locomotives in the United States.

B. Car Lighting and Air Conditioning

1. History. The first attempts at car lighting by electricity were made in 1881 and continued in an experimental stage for about ten years before practical results were obtained. The Planté type of storage battery which had been developed in 1860 was the first to meet car lighting requirements successfully. The Faure or pasted-plate type of battery was developed about 1881, but was not used in car lighting service until many years later because the earlier designs were not rugged enough. In the early types of car lighting batteries the elements were installed in alloy containers, two containers being grouped in a wooden tray. Porcelain rests supported the elements at a sufficient height from the bottom of the cell to provide space for sediment accumulation. Later, rubber jars were substituted for the alloy containers. In either case alloy or rubber covers were used.

Railroads operated well-equipped battery repair shops and regularly examined car lighting batteries, removing sediment, changing separators, and renewing plates when necessary. It was not uncommon to obtain a life of 20 years from negative plates, requiring three or four sets of positive plates. Gradually this practice of rebuilding batteries was discouraged by the railroads in favor of using a "single life" battery whose separators and plates would wear uniformly. The trend was definitely away from the Planté type and toward the lighter weight pasted-plate type battery.

In 1927 the first air-conditioned railway passenger car, a Pullman sleeping car, was placed in experimental service. Three years later the idea was tried by several railroads, and it was enthusiastically accepted by the traveling public. The rapid expansion of air conditioning on railroads in the United States is shown in figure 3.

Five different types of air-conditioning systems are used by the railroads. They are distinguished chiefly by the manner in which refrigeration is produced, and are separated as (1) ice activated, (2) mechanical compression with the compressor driven by an electric motor (electromechanical), (3) steam ejector, (4) mechanical compression with the compressor driven directly from car axle (direct mechanical), and (5) mechanical compression with the compressor driven by an internal combustion engine. The refrigeration capacities available for railway passenger cars vary from three to nine tons per 24 hours.

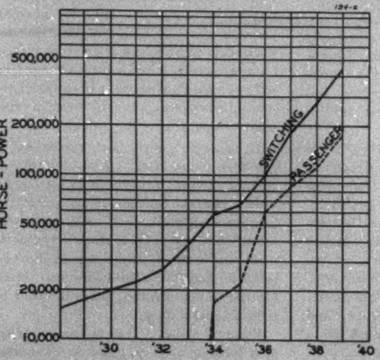

Figure 2. Cumulative horsepower of Diesel-electric locomotives in U.S.A.

Before describing the increased power requirements brought about by the addition of air conditioning, it will be helpful to refer to table I which shows what the Association of American Railroads set as a standard for storage battery and generator capacities when applied to *car lighting* service. The AAR is undertaking a study to make comparable recommendations for combined air conditioning and car lighting service. During the past three years better illumination has been emphasized and has been adopted in varying degrees by different railroads. Attractive interiors were provided on newly designed cars with new and improved types of lighting fixtures which meant further increases in intensity at the lighting source. In some of the latest cars the lighting load has reached a total of 170 amperes at 32 volts.

These heavier lighting loads plus any additional amount demanded by air-conditioning equipment are taken from the storage battery during periods of a train run when the train is operating below the cut-in speed of the generator and, in some cases, when the car is being precooled prior to a run. This was responsible for increasing the size of the storage battery to from two to four times that previously used in car lighting service alone. The capacity of generators was also increased to correspond with the increase in load. Experience indicates that a minimum generator capacity of 166 per cent of the connected load should be provided. While this ratio may seem somewhat high for new cars in long runs or express service, it is only a matter of time when such new cars are relegated to local or semilocal service in which higher generating capacity is desirable.

2. Air-Conditioning Power Requirements. The electric load in an ice activated system consists of 0.5 kw to the cooling coil fan and 0.75 kw to the cold water circulating pump or a total of approximately 38 amperes at 32 volts. In the majority of cases, one four-kilowatt axle-driven generator has sufficed. Referring to table II, it will be noted that the 600 ampere-hour size battery is the most popular. However, the lighting load and the type of car has influenced the choice of this capacity more than has the air-conditioning load.

The greatest range in power requirements occurs in the electromechanical system of air conditioning. Refrigeration capacities vary from 3.5 to 9.6 tons. In the vicinity of the six-ton rating, the total power requirements may vary from 8.5 kw to 12 kw. The motor driving the compressor requires approximately 93 per cent of this total, the remainder being consumed by evaporator fans, control, and in some cases, condenser fans. Comparisons made of various manufacturers' equipment rated around six tons showed that an average of 1.75 kw per ton of refrigeration was required, or a total of 10.5 kw at six tons. This represents an average increase above car lighting requirements of about 330 amperes at 32 volts. Generator capacities were raised to 15 kw, then to 20 kw, but it was not practical to increase the storage battery capacity correspondingly. Space and weight are limited and the problem became one of how much capacity could be installed in the standard AAR battery box. The electromechanical system of air conditioning imposed the greatest demands on the storage battery and therefore provided the greatest influence in the change of storage battery design. One of the large eastern railroads, a pioneer in the use of the electromechanical system, set 1,000 ampere-hours capacity (32 volts) of storage battery as a goal for manufacturers to strive to build and install within the standard compartment.

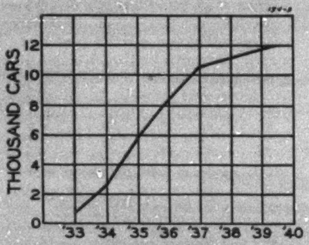

Figure 3. Air-conditioned railway passenger cars in U.S.A.

The power required in the steam ejector system is second to that of the electromechanical system. The total load amounts to approximately 104 amperes at 32 volts and consists of the condenser fan, condenser pump, cooling coil fan, and cold water pump. Referring to table II, it will be noted that the majority of installations use approximately 850 ampere-hour storage batteries. Generator capacities for this type of system vary from $7^1/_2$ kw to 10 kw, the latter size being found in most cases.

The direct mechanical system has the lowest electrical requirement and therefore the least effect on the size of battery installed. The 550 ampere-hour capacity was the most popular because one of the largest operators of air-conditioned cars chose it as their standard. It is estimated that the bulk of installations from 1938 to date used 600 ampere-hour batteries.

One manufacturer has developed an internal combustion engine which uses bottled propane for fuel and can be adapted to drive either a compressor or a generator. As an engine generator it removes the load from the car axle and there is no change in electric power used. When the engine is directly connected to the compressor, however, the electric power requirements drop to that of the direct mechanical system. Most of the installations of this type of system have been made on railroads in the Middle West and the preponderance of 450 ampere-hour storage batteries coincide with the standard size of car lighting battery of two large users.

3. Storage Battery Developments. The life of Faure or pasted plates depends on the cohesion of the active material and its retention by the grid structure. These are influenced by the porosity of the active material and by the shape of the grid section. Thus a soft porous plate will give higher capacity in its early life, but has a tendency to shed the active material. Hard dense plates do not give as much capacity initially, but are slower to "soften" with work and age, and consequently will give much longer life. It is of prime importance that plates be uniform because they will be "teamed" as groups in a battery which must work and wear evenly. To accomplish uniformity manufacturers must control closely not only the quality of materials used but also the physical conditions of manufacture, such as temperature, time and humidity, that exist while making and processing the plates.

A retainer or envelope is placed about the positive plate for a twofold purpose: (1) to hold the oxides in place mechanically and in contact with the grid, and (2) to delay oxidation of the separators. In one case the retainer has been a thin sheet of finely slotted rubber. In another case a modified Faure plate was made by enveloping a series of grid pencils with cylinders of active paste material and each cylinder encased in a finely slotted rubber tube. In a third case the retainer has been a thin mat of fibrous material usually made of spun glass.

Plate separators act as electrolytic diaphragms which permit current to pass freely between the plates but prevent any migration of dislodged active material from one plate to another of opposite polarity which might bridge and cause a short circuit. In some respects wood is an excellent material for separator use but its disadvantage lies in its inability to withstand high temperatures. In 1933 a new separator was developed from liquid latex, or the milk of rubber as the base. This material when compounded with certain other materials and vulcanized forms a microporous diaphragm which successfully withstands oxidation and heat. In another case a similar separator was obtained by vulcanizing a mixture of crude rubber and sulfur with certain other materials, the latter being dissolved out to form the minute pores.

The trays used for car lighting batteries were usually made from oak, maple, or long-leaf yellow pinewood, painted with an asphaltum paint and then immersed in a molten wax compound. Hard rubber jars held the elements and electrolyte, and the wood trays became the supporting structure for two or more jars. Such an assembly occupied more space and weighed more than a self-supporting container. Another disadvantage occurred when high temperatures melted the wax coating and exposed the wood to decay. A self-supporting rubber monobloc container was developed in 1935. This container saves space, is not subject to decay, and possesses a remarkable quality of toughness. It is handmade from rubber compounds which contain relatively high percentages of rubber. One type of container is made up of two dissimilar rubber compounds—one resistant to heat, the other resistant to shock—calendered together and handwrapped about mandrels, and then vulcanized. There are now over 50,000 of these monobloc containers in service.

Table I. AAR Recommended Battery and Generator Capacities for Railway Car Lighting

Class of Service	Amperes Load	Battery Capacity Range Ampere-Hours Min	Battery Capacity Range Ampere-Hours Max	Generator Capacity (Kw)
Official, sleeping, parlor, mail, and dining cars having heavy lighting loads or severe operating conditions	50	350	600	4
Mail, dining cars, etc., where the lighting loads are not so heavy or the operating conditions so severe	35	300	350	3
Coaches, passenger-baggage cars, baggage cars under average operating conditions	25	200	300	2
Same cars as next above operating in slow speed branch line or local service	25	150	225	2

Table II. Storage Battery Installations in Air-Conditioning Service

Air-Conditioning System	Ampere-Hour Capacity Range		Electrical System (Volts)	Installations Made in						Nine-Year Summary	
				1933*	1934	1935	1936	1937	1938	Total	Per Cent
Ice											
		300	110	0	0	0	3	3	0	6	0.2
	250	350	32	6	29	35	29	44	0	143	4.1
	375	450	32	25	50	314	360	31	1	781	22.5
		500	32	108	101	112	146	34	3	504	14.5
		600	32	97	538	511	594	165	13	1,918	55.1
	750	900	32	55	15	25	9	5	2	111	3.2
	1,000	1,200	32	6	2	0	0	4	0	12	0.4
Total				297	735	997	1,141	286	19	3,475	31
Compressor driven from an electric motor											
		300	110	0	7	30	47	26	5	115	5.4
		450	64	0	11	7	0	0	0	18	0.8
		500	64	0	0	60	50	75	50	235	11.1
		600	64	0	6	57	62	104	8	237	11.2
	750	800	32	10	33	20	16	0	0	79	3.8
	900	1,000	32	188	292	486	174	282	3	1,425	67.2
	1,200	1,250	32	0	0	4	0	0	2	6	0.2
		2,000	32	3	3	0	1	0	0	7	0.3
Total				201	352	664	350	487	68	2,122	19
Steam ejector											
	550	600	32	2	20	57	78	31	109	297	15.6
	700	800	32	3	37	35	72	40	6	193	10.2
	850	875	32	0	132	407	145	327	152	1,163	61.2
	900	950	32	33	2	10	12	72	1	130	6.8
		1,000	32	5	6	28	14	35	20	108	5.7
		1,200	32	0	0	0	0	10	0	10	0.5
Total				43	197	537	321	515	288	1,901	17
Compressor driven mechanically from car axle											
	300	350	32	0	6	118	9	30	4	167	5.0
	375	450	32	33	74	17	74	88	0	286	8.5
		450	64	0	4	0	0	0	0	4	0.1
		500	32	12	19	61	30	75	0	197	5.9
		550	32	0	395	882	505	363	54	2,199	65.9
		600	32	61	1	4	14	91	62	233	6.9
		600	64	0	32	0	0	0	0	32	1.0
	800	900	32	0	0	21	3	63	63	150	4.5
		1,000	32	0	62	2	0	3	0	67	2.1
		1,250	32	0	0	0	0	0	2	2	0.1
		1,800	32	0	0	1	0	0	0	1	
Total				106	593	1,106	635	713	185	3,338	30
Compressor driven from internal combustion engine											
	200	400	32	0	0	1	1	36	11	49	15.4
	450	600	32	1	0	3	57	146	29	236	74.3
	750	850	32	0	1	13	0	8	0	22	6.9
	900	1,000	32	0	0	1	3	6	1	11	3.4
Total				1	1	18	61	196	41	318	3
Grand total				648	1,878	3,322	2,508	2,197	601	11,154	100

*This also includes all installations made prior to 1933.

In order to minimize IR drop, number 0000 cable composed of upward of 5,000 strands of wire is used to connect adjacent battery trays where 800 to 1,000 ampere-hour batteries are used. Extra flexible cable is required for the larger batteries which practically fill available compartment space. One alternative for reducing the large cable requirement has been adopted by several railroads when the system voltage was doubled to 64- and 32-cell 500 ampere-hour or 600 ampere-hour storage batteries used. In another case one railroad equipped 115 cars with 110-volt lighting and air-conditioning systems.

Table III compares a 300 ampere-hour Planté car lighting battery of 1920 with a typical car lighting battery as used today. Today the railroad can obtain a 300 ampere-hour car lighting battery that will occupy one-third the volume of the 300 ampere-hour Planté battery used in 1920; it will weigh only 40 per cent of the 1920 battery; it will cost only 70 per cent as much, and will have a longer life expectancy.

For heavy duty air-conditioning service, it is now possible to install a 16-cell 1,000 ampere-hour battery in the same size compartment which formerly held 16 cells of a 400 to 450 ampere-hour battery. The increased cycling of the battery together with the heavier loads mean higher operating temperatures which are now permissible because of the present development of plate separators. Tightly packed elements permit today's storage batteries to operate throughout their life without mechanical failure of the separators which otherwise might be caused by vibration. Compared with the 1920 300 ampere-hour car lighting battery, the railroad can obtain today an air-conditioning battery of $3^1/_2$ times the capacity installed in the same size compartment, weighing only 25 per cent more, costing only 70 per cent more, and, when used on a load ten times greater, having a life expectancy equal to the 1920 battery.

C. STREET RAILWAY PCC CARS

Several papers have been presented to the Institute on "Presidents Conference Committee" street railway cars. Today there are about 1,500 of these PCC cars in service in this country, each of which is equipped with a 16-cell 55 ampere-hour storage battery. The battery is connected across a voltage-regulated motor-generator set. The loads on the battery circuit include magnetic track brakes, control, step treadle equipment, headlight, stop lights, marker lights, passenger signal system, instrument panel, and sometimes fare box lighting. The total load may reach a maximum of 75 to 80 amperes while magnetic track brakes are energized.

II. Motor Busses

Passenger busses are classified along with passenger automobiles and trucks as automotive equipment. While technical discussions on this subject are normally conducted by SAE, a few remarks on the application of storage batteries to busses may be appropriate in view of their widespread use for commercial transportation.

The year 1911 marked an important milestone in the development of automotive equipment, for it was in that year the electric starting motor for gasoline engines was introduced. Prior to that time all automotive engines were cranked by hand. A campaign for better illumination of bus interiors brought greater lighting loads and in order to minimize voltage drop, a 12-volt circuit was standardized. Battery sizes increased with the change in lighting load from 13 plates to 17 plates and in some cases up to 25 plates. Bus battery plates were thicker than those used in automobile batteries because busses operated in more continuous service and under more adverse conditions.

In railway, marine, and stationary battery applications it is customary to rate the batteries on an eight-hour basis, whereas in bus service it is customary to rate them on the four-hour basis. The capacity of bus batteries was at one time selected on the basis of a six-hour reserve for the operating load exclusive of the starting motor. From experience it was found that a size capable of supplying this reserve was ample for any starting conditions normally encountered. Today the electric loads on busses reach values of 65 amperes at 12 volts and the size of battery recommended is one that will carry the load for four hours.

Fuel economies associated with Diesel engine operation helped to bring about the development of the small high-speed Diesel for bus and truck service. Two six-cell bus-type batteries are connected in series for 24-volt starting and then paralleled for 12-volt lighting.

In 1935, trolley busses made their appearance with 12-volt storage batteries used for control and lighting purposes to avoid high trolley voltages within the bus.

III. Aircraft

Military planes have been responsible for most developments of aircraft electric equipment. The Liberty motor in service during World War I was the first to use a storage battery for ignition. The battery weighed 11 pounds and contained four cells with seven plates per cell.

By the end of that war, military planes were being equipped with direct drive starters. The cranking batteries were usually six-cell, 20, 35, or 70 ampere-hours capacity (five-hour rating). Similar equipment was used by mail and commercial planes about 1925.

The inertia starter, brought out in 1925 plus an improved design of generator, hastened the adoption of electric starting of aircraft engines. Because of the importance of weight, very thin pasted-plate batteries were developed for aircraft service. The plates used in the first Army plane batteries were only 0.05 inch thick. As airplane flights became longer, swifter, and more severe on auxiliary equipment, it was found desirable in 1930 to increase the plate thickness by 50 per cent.

Table III. Comparison of 1920 and 1940 Typical Car Lighting Batteries

	1920	1940
8-hour capacity	300 A.H.	300 A.H.
No. cells per tray	2	4
Tray material	Wood	Rubber monobloc
Tray dimensions		
Length	$24\frac{3}{4}$"	19
Width	11"	$9\frac{3}{4}$"
Height	$19\frac{1}{4}$"	$19\frac{3}{4}$"
No. trays per battery	8	4
Total battery volume	24.26 cu. ft.	8.5 cu. ft.
Total battery weight	2,600 pounds	1,010 pounds
Purchase price	100%	70%
Expected life	5 years	7 years

It is interesting to note that a modern passenger airplane's intermittent load on a 12-volt circuit may total 225 amperes. During a $5\frac{3}{4}$-hour night flight in the winter, a typical example of the electric load would be about 490 ampere-hours at 12 volts normally supplied by two 50-ampere generators in parallel. The six-cell 13-plate battery used in this example weighs 78 pounds, has a capacity of 88 ampere-hours (five-hour), and occupies a space 14 by 7 by 11 inches high. It would be capable of carrying 150 amperes for 18 minutes or 200 amperes for 12 minutes.

The large 21-passenger skyliners have two 850-horsepower engines, two storage batteries like the one described above, and means to use either battery or both connected in parallel. The latest Stratoliners have four 1,150-horsepower engines and two 24-volt 50 ampere-hour batteries. The very large engines are using the direct drive starter again rather than the inertia starter. To conserve battery capacity, the heavy duty cranking prior to a plane's departure from an airport, is taken care of by a portable ground-cranking battery which is wheeled under the plane.

Future Developments

Thought and study is being devoted to heating railway passenger cars electrically. Air conditioning includes the proper control of temperature and humidity of the air during winter months as well as summer months. At least three manufacturers are now developing engine-driven equipment that will heat, light, and air condition a car. On one system it is planned to generate 220 volts three-phase, alternating current for heating and air-conditioning purposes. A 64-volt 150 ampere-hour battery will be used to crank the engine, provide the control circuit energy, and light the car. The battery will be charged from the exciter. The adoption of 64 volts direct current for lighting permits the use of a recent development—the fluorescent tube which operates at 60 volts direct current from the battery circuit without transformation of energy. Thus the engine may be shut down temporarily without affecting the source of illuminating power.

In view of the large quantities of rubber materials now used in the production of lead storage batteries, some concern is felt for the continuity of crude rubber supplies. While numerous synthetic rubber substitutes are now being marketed, little enthusiasm can be expressed for any one of them at this time, because experience has taught us to be wary of any possible unhappy union of storage battery electrolyte with an element from the synthetic material. Research is now directed toward the use of some of the newly developed plastics, and it is predicted that they will find increased use in the storage battery industry.

References

1. The Gas Electric Car—Characteristics and Operating Features, L. C. Josephs, Jr. *Proceedings* of Association of Railway Electrical Engineers, October 1931, page 237.

2. Self-Propelled Railway Passenger Cars, S. T. Dodd. *General Electric Review*, 1913, page 637.

3. Engineering Report on Air Conditioning of Railroad Passenger Cars, prepared by Division of Equipment Research, Association of American Railroads, April 15, 1937.

4. Statistics on Generator and Storage Battery Capacities Used on Air Conditioned Cars obtained from the following issues of *Railway Electrical Engineer:* February 1934; December 1934; March 1936; February 1937; February 1938; April 1939.

5. Investigation of Starting Requirements of a 660 Horsepower Locomotive Diesel Engine, J. C. Davidson and R. Lamborn. AIEE Transactions, volume 60, 1941 (January section), pages 31–4.

Bushing and Associated Insulation Testing by the Power-Factor Method

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I. Introduction

THE periodic cleaning, inspection, and testing of equipment plays a very important part in obtaining service continuity. The cost of the labor and equipment involved in locating and eliminating defective apparatus by the above procedure may well be charged to that hypothetical account known as "preventive maintenance." The economics of the procedure, however, must be governed by the results obtained.

For many years, bushings and associated insulation such as the internal parts of circuit breakers and windings of instrument and power transformers were tested by methods which were frequently unreliable, inadequate or of a destructive nature. During the past ten years the a-c test known as the power-factor method of insulation testing has gradually been replacing the former methods used for periodic checkup. It is also gaining recognition as one of the requirements in the acceptance tests made on certain types of new equipment.

The results obtained by Philadelphia Electric Company, using the power-factor test method for approximately eight years, has warranted the continuation of a periodic test program. A few bushing failures still occur but they are confined mostly to the low-impulse type of design and cannot be anticipated by any nondestructive type of test. A centralized bushing repair shop has provided the means for salvaging bushings and other equipment at a reasonable cost, for years of useful life. All equipment rejected by test has, upon investigation, disclosed faulty insulation which warranted its removal from service.

II. Scope of Testing

The power-factor method of insulation testing was first used by this company in 1933, and was confined primarily to outdoor oil circuit breaker bushings. The Doble type *I* tester was used for this work and the results of the first three years' work has been previously summarized.[1]

In 1936, a second type *I* tester was placed in service and the range of testing was broadened to include power and instrument transformers and lightning arresters. With the further development of test equipment by the Doble Company, the two type *I* sets were replaced in 1939, by an *I*-10 and an *I*-30-150 set. A type *U* set was also obtained for use in the bushing repair shop in 1938.

The power-factor testing outlined for each year is as follows:

(*a*). Outdoor oil circuit breakers.

(*b*). Power transformers.

(*c*). Instrument transformers, alternated every other year with station-type lightning arresters.

(*d*). Cold collar and/or hot collar tests on all bushings containing a compound filler.

(*e*). Hot-guard test on all draw-lead type transformer bushings.

(*f*). Oil samples on all power transformers.

(*g*). Oil samples on all oil circuit breakers and instrument transformers as necessitated by over-all test results.

(*h*). Spare bushings.

(*i*). Follow-up field tests as required.

(*j*). Indoor oil circuit breakers during winter months.

(*k*). Follow-up and acceptance tests on all equipment reconditioned in the shop (by the shop force).

(*l*). Acceptance tests on all new bushings.

(*m*). Base checks on all new power transformers.

III. Field Testing

The technique of field testing has been very thoroughly covered in previously published articles,[2–7] listed in the bibliography and will, therefore, not be repeated.

It requires about two years' work to completely cover the outdoor equipment on the Philadelphia Electric Company system. However, the most important locations are tested yearly. A tentative schedule allocating the work is made up each year as a guide for the field test men. Due to its larger capacity, the *I*-30-150 set is used for practically all power transformer tests.

In order to insure proper follow-up of rejected equipment, a 3- by 5-inch card is used for each piece of equipment tested (bushings are included with the circuit breaker) and the condition of the equipment is noted as satisfactory, follow-up, or rejected. These cards are sent daily to the superintendent in charge of the location where the testing is being done. A summary sheet of all rejected equipment is made up weekly and forwarded to the general superintendent of the respective division. When the testing at a specific location has been completed, a copy of each data sheet, approved by the supervisor in charge of the tests, is forwarded to the superintendent of that location for reference and file.

Field testing is seldom carried on when the ambient temperature is lower than five degrees centigrade or when the humidity is high enough to cause considerable surface leakage. The rating schedule for bushings and instrument transformers at 20 degrees centigrade ambient temperature is given in table I.

An arbitrary value of five per cent maximum at 40 degrees centigrade has been established for power transformers although it is still a moot question as to the satisfactory condition of the winding insulation when lower than one per cent especially when subjected to a dry-out run. The current and watts loss readings must also be carefully analyzed and due consideration given to the type of insulation and winding construction before the unit is accepted or rejected.

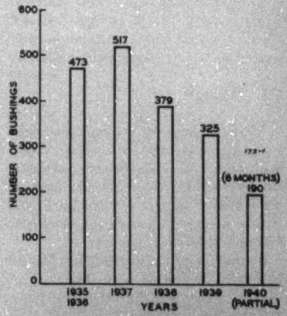

Figure 1. Bushings reconditioned in the shop. Total for 5 1/2 years = 1,884

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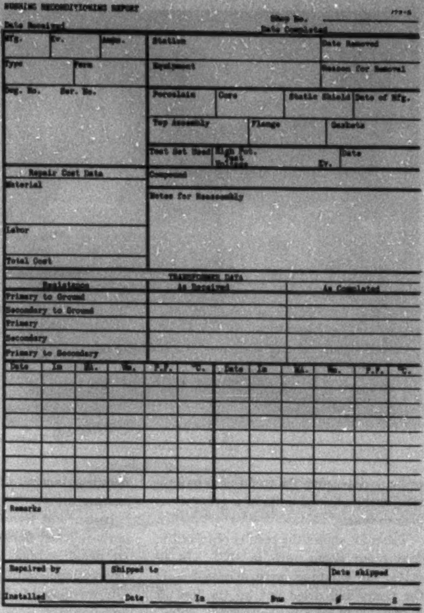
[1] For all numbered references, see list at end of paper.



BUSHING RECONDITIONING REPORT

Shop No. __________

Date Received

Date Completed

Mfg.	Kv.	Amps.	Station	Date Removed
Type	Form		Equipment	Reason for Removal
Dwg. No.	Ser. No.		Porcelain Core Static Shield Date of Mfg.	
			Top Assembly Flange Gaskets	
			Test Set Used High Pot. Test Voltage Kv. Date	
Repair Cost Data			Compound	
Material			Notes for Reassembly	
Labor				
Total Cost				

TRANSFORMER DATA

Resistance	As Received	As Completed
Primary to Ground		
Secondary to Ground		
Primary		
Secondary		
Primary to Secondary		

Date	In	MA.	Wa.	P.F.	°C.	Date	In	MA.	Wa.	P.F.	°C.

Remarks

Repaired by ____ Shipped to ____ Date shipped ____

Installed ________ Date ____ In ______ Bus ____ ø ____ S ____

Figure 2. Bushing reconditioning report

Considerable work has been done since 1937, in the investigation of station-type lightning arresters such as the oxide film, type *BO;* auto-valve, type *SV* (disk and porous block construction); and Thyrite, types *LAIB* and *LAIY.* Arbitrary values obtained by comparison and internal investigation of units are being used rather successfully but it is hoped that compilation of further data will offer a more direct solution of the problem.

Wood members, insulators, and the porcelain-tube type of wall bushings are analyzed on the basis of current and watts loss data rather than by power factor. Satisfactory oil is found to be one per cent or under. Above that value other tests, such as dielectric, acidity, color, etc., are made before rejection.

IV. Test Results

A. Bushings (Field Test)

For the three-year period 1933–35 inclusive, about 6,000 bushings were tested. Since that time approximately 4,000 bushings are tested in the field each year. Table II indicates that 1,493 bushings of various types and voltages have been rejected in the past five years. This represents about 7.5 per cent of the total number tested. It should be noted that the rejection of indoor bushings during the past four years accounts for approximately 30 per cent of the total number rejected. The rating schedule was arbitrarily lowered for the indoor bushings on the basis that they would only be tested at four- or five-year intervals.

Table III presents a breakdown of the reasons for rejection. Power factor accounts for 65 per cent of the total and compares with the results of the 1933–35 period[1] of 63.8 per cent. However, in order to make a fair comparison, the indoor bushings must be deducted which would then approximate 35 per cent instead of 65 per cent. This indicates that the trend is downward which should be expected in order to justify the expenditures made in replacement and reconditioning. Moisture accounts for more than 90 per cent of the rejections by power factor.

The 17 per cent rejection due to oil migration and compound leaks is confined largely to transformer bushings and is usually determined by inspection rather than test. The "miscellaneous rejections" is somewhat high (14 per cent) due to placing a number of multipiece porcelain bushings which were rebuilt with single-piece porcelain, in this classification. The percentage of miscellaneous rejections would be in the order of three per cent or less if this modernization item were omitted.

B. Bushings (Shop Test)

The bushing reconditioning shop was started in 1935. Figure 1 shows the number of bushings reconditioned since that time. The total of 1,884 consists of all types of bushings ranging from 7.5 kv to 73 kv. The first year was devoted to the training of personnel, experimentation with materials and methods, building special racks, providing special tools, etc. Larger ovens were designed and installed in 1936, so that large-scale production did not get under way until 1937. During the period 1935–37, it was necessary to assign a field man for shop testing several days each week. This decreased the amount of outdoor testing possible and at the same time proved to be inadequate to meet shop requirements. In 1938, a Doble type *U* set was provided. The shop men now average about 3,000 tests (follow-up, acceptance, etc.) per year.

All bushings (15 kv to 73 kv) after being reconditioned are subjected to a one-minute hold voltage test using the values recommended by the manufacturer for present-day design. This test has resulted in the elimination of a few older type bushings which probably would have failed in service due to faults which could not be determined by a nondestructive test.

Figure 2 is the form developed for detailing the shop history of bushings and instrument transformers. Figures 3 and 4 show the front and rear view of the type *U* tester used in the shop.

C. Transformers

During the past four years, 493 power transformers have been tested. This group of transformers covered a voltage range from 13.2/2.3 kv to 220/66/13.2 kv and a capacity range of 100 kva to 43,333 kva single phase and 1,000 kva to 20,000 kva three phase. All low-voltage windings (2.3 kv and 4 kv) are tested at 5 kv, the higher voltage windings at 10 kv. From the test data obtained, the transformers have been divided into three groups as follows:

Per cent power factor at 40 degrees centigrade: under 5.0, 274; 5.1 to 10.0, 136; over 10, 83.

This grouping is used as a follow-up in

Figure 3. Front view of type U tester used in reconditioning shop

Figure 4. Rear view of type U tester used in reconditioning shop

order to determine the trend of deterioration in the various units of a group from year to year.

Table IV shows the results obtained in drying out seven units in 1937. In all cases, the windings were removed from the tanks and dried out by the hot air method at 85 degrees centigrade. Subsequent tests to date on these transformers have shown that the "as left" readings have remained stabilized. The bushings were reconditioned in the shop and are included in both the "as found" and "as left" readings. Moisture was the contributing factor to the high pow in each case.

Instrument transformers and metering units rejected during the past five years are listed in table V. These units were reconditioned in the shop and again moisture was found to be the major cause for rejection.

D. MISCELLANEOUS

Several hundred defective supporting insulators have been removed from service either as a direct result of being found by test or by the inspection of equipment by the field test man.

Wood cross braces, *A* frames, lift rods, etc., have been reconditioned due to field investigation and rejection.

Power-factor test on oil samples is used as one of the criteria for recommending reconditioning.

Testing of lightning arrester units by power factor has resulted in the reconditioning of several hundred units due to

Table I A. Bushing Power-Factor Rating Schedule

Bushing Condition	Per Cent Power-Factor Limits for Various Type Bushings at 20 Degrees Centigrade				
	Kv	Condenser	Oil-Filled	Compound	Porcelain*
Satisfactory	15–37	0–3.0	0–3.5	0–3.5	0–3.5
Satisfactory	73–230	0–2.0	0–2.0	0–2.5	0–3.5
Follow-up	15–37	3.0–5.0	3.5–5.0	3.5–5.0	3.5–6.0
Follow-up	73–230	2.0–3.0	2.0–3.0	2.5–4.0	3.5–5.0
Replace	15–37	Over 5.0	Over 5.0	Over 5.0	Over 6.0
Replace	73–230	Over 3.0	Over 3.0	Over 4.0	Over 5.0

* Current and watts readings are of more value than power factor for this type of bushing.

Application of the above values may be somewhat arbitrary based on type of bushing construction, service requirements, and analysis experience. Approximately eight years of use has proved their reliability.

Table I B. Instrument Transformer Power-Factor Rating Schedule

	Per Cent Power-Factor Limits at 20 Degrees Centigrade				Per Cent Power-Factor Limits at 20 Degrees Centigrade		
Type P.T.	Satisfactory	Follow-Up	Replace	Type C.T.	Satisfactory	Follow-Up	Replace
E-15	5.0	5.0–7.0	Over 7.0	*K*-31	3.0	3.0–5.0	Over 5.0
E-115	5.0	5.0–7.0	Over 7.0	*K*-32	4.0	4.0–6.0	Over 6.0
E-16	3.5	3.5–5.0	Over 5.0	*K*-61	3.0	3.0–5.0	Over 5.0
E-116	3.5	3.5–5.0	Over 5.0	*K*-204	2.5	2.5–4.0	Over 4.0
P	3.5	3.5–5.0	Over 5.0	*OA*	4.0	4.0–6.0	Over 6.0
(34.5 kv) *VC*	3.5	3.5–5.0	Over 5.0	*O*-37	3.0	3.0–5.0	Over 5.0
(73 kv) *VC*	2.5	2.5–4.0	Over 4.0	*KE* (meter unit)	2.5	2.5–4.0	Over 4.0

broken castings, defective seals, corroded internal parts, etc.

V. Bushing Failures

Moisture was responsible for 69 per cent of the 48 bushing failures occurring between 1932 and 1940, as shown in figure 5. Entrance to the bushing was gained in most cases through defective gaskets or cement seals. In 1935, 12 of the 15 failures were caused by defective gaskets. This led to the program of replacing the top gaskets on nearly 1,000 bushings in place as previously reported.[1] Of the total failures (48), 27 per cent had never been tested and 6 per cent had been recommended for replacement previous to failure. The type *SI* compound-filled bushing was involved in 69 per cent of the failures with the condenser type accounting for 23 per cent. While it is not expected that failures will be eliminated entirely, it is definitely realized from the experience gained during the past eight years that many incipient failures were removed from the system.

Very few follow-up tests are made since the reconditioning shop was established. A bushing is now recommended for reconditioning if it does not pass the "satisfactory" rating. This eliminates the possibility of a failure between tests because the rate of deterioration is an unknown factor and also provides more time for productive effort on that equipment yet to be tested.

VI. Conclusions

From the experience gained and the data compiled in eight years of testing and reconditioning, the following conclusions may be logically presented:

1. The power-factor test has provided an adequate nondestructive method for determining insulation deterioration.
2. It is satisfactory in the results obtained and therefore justifies its continuation.
3. Due to the unknown factor "rate of deterioration" the period for testing of equipment at important locations should not exceed one year.
4. The major factor in insulation deterioration is moisture.
5. The rating schedules used for various types of equipment have proved satis-

Table II. Number of Bushings Rejected (1936 to 1940)

		Condenser	Oil-Filled	(SI) Compound	Porcelain	Total
1936	O.C.B.	119*	1	130	4	254
	trans.	38†	6	142	24†	210
1937	O.C.B. (outdoor)	35	33**	32	7	107
	O.C.B. (indoor)	32			1	33
	trans.	3		70	4	77
1938	O.C.B. (outdoor)	52		108	1	161
	O.C.B. (indoor)	252				252
	trans.	2		39	32	73
1939	O.C.B. (outdoor)	10	2	24		36
	O.C.B. (indoor)	156		5		161
	trans.	1	1	26	7	35
1940 (partial)	O.C.B.	8	2	57		67
	trans.	7		12	8	27
Total		715	45	645	88	1,493
Per cent of total rejected		48	3	43	6	

*Includes a change from multipiece to single-piece porcelain.

**Includes a rebuild of 24 obsolete type bushings.

†Includes 10 condenser and 18 porcelain bushings damaged by fire.

Table III. Reasons for Rejection (1936 to 1940)

		Power Factor	Oil Migration and Compound Leaks	Broken Porcelain	Miscellaneous*	Total
1936		286	86	43	49	464
1937	outdoor	113	55	9	7	184
	indoor	32		1		33
1938	outdoor	79	53	3	99	234
	indoor	252				252
1939	outdoor	26	22	2	21	71
	indoor	156	5			161
1940 (partial)		25	34	4	31	94
Total		969	255	62	207	1,493
Per cent of total rejected		65	17	4	14	

*Includes voids, low compound level, defective cement and litharge seals, defective gaskets, carbonized compound, certain modernization, etc.

Table IV. Results Obtained in Drying Out Seven Power Transformers, Oil Conservator Type

Case Number		Kva	Voltage	Phase	Oil		Winding Power Factor at 40 Degrees Centigrade		
					Temperature Degrees Centigrade	Power Factor	High+Low	High	Low
1	as found	333	13.2/4	1	0	0.2	46.8	20.8	40.5
	as left				15	New	2.94	2.07	3.98
2	as found	5,000	33/4	3	12	1.7	23.4	5.2	24.3
	as left					New	3.44	1.14	2.46
3	as found	3,353	33/13.2	1	30	0.8	30.6	12.4	23.4
	as left				27	New	3.3	0.7	2.8
4	as found	3,333	33/13.2	1	30	0.9	34.3	12.5	26.8
	as left				27	New	2.8	0+	3.1
5	as found	833	13.2/2.3	1	25	10.9	36.4	32.3	27.1
	as left				30	New	5.7	2.5	3.5
6	as found	833	13.2/2.3	1	25	11.3	39.4	33.0	30.1
	as left				30	New	4.9	1.75	3.7
7	as found	833	13.2/2.3	1	25	4.6	9.0	13.4	8.8
	as left				30	New	5.2	3.5	3.6

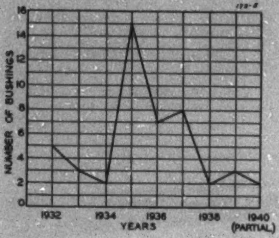


Figure 5. Bushing failures

Cause:	
Moisture	69 per cent
External	23 per cent
Impulse	6 per cent
Internal pressure	2 per cent

factory in that the investigation of all rejected equipment has disclosed faults warranting its removal from service.

6. A close inspection should be made of all seals on bushings namely, gaskets, cement, litharge, etc., for defective seals have accounted for the largest number of failures.

7. Faulty equipment should be removed from service immediately and in extreme cases, equipment held out of service until such replacement is made.

8. Reconditioning of equipment may be accomplished at a reasonable cost and at the same time provide years of useful life to equipment that might otherwise prove to be a costly emergency repair job.

9. A centralized shop, where reconditioning is being done by the power company, provides the most economic and reliable method of reconditioning insulation.

Table V. Instrument Transformers Rejected by Power Factor (1936 to 1940)

	Type P.T.	Number Rejected	Type C.T.	Number Rejected
1936	*VC*	17	*OA*	2
	P	6	*K*-32	5
	E-15	4	*K*-61	2
	E-116	8	*K*-204	1
			KE (meter unit)	2
	Total	35		12
1937	*VC*	9	*K*-32	2
	P	2		
	E-15	2		
	E-16	16		
	E-116	10		
	Total	39		2
1938	*VC*	6	*K*-204	1
	E-115	2		
	E-116	1		
	Total	9		1
1939	*E*-15	2	*K*-32	1
1940 (partial)	*VC*	2	*K*-204	1
	E-116	13		
	Total	15		1

10. Reconditioned equipment in service is showing no increase in power factor in subsequent field tests.

11. The experience gained has provided the means for a more logical consideration of new design.

12. The economics involved in this type of testing cannot be evaluated, but from the standpoint of "preventive maintenance," service continuity, and customer satisfaction, it is unquestionably justified.

Bibliography

1. BUSHING P.F. TESTS SHOW FAILURE CAUSES, E. L. Schlottere. *Electrical World*, March 28, 1936, pages 34–7.

2. TESTING BUSHINGS AND INSULATION BY POWER FACTOR METHOD, I. W. Gross and H. E. Turner. *Electrical World*, January 13, 1934, pages 68–73, and January 20, 1934, pages 111–16.

3. PORTABLE SCHERING BRIDGE FOR FIELD TESTS, C. F. Hill, T. R. Watts, and G. A. Burr. AIEE TRANSACTIONS, volume 53, 1934 (January section), pages 176–82.

4. Discussions of above paper—AIEE TRANSACTIONS, volume 53, 1934 (April section), pages 618–22.

5. FIELD TESTING OF BUSHINGS AND TRANSFORMER INSULATION, I. W. Gross. AIEE TRANSACTIONS, volume 57, 1938 (October section), pages 589–96.

6. POWER FACTOR TESTING OF TRANSFORMER INSULATION, Frank C. Doble and G. H. Browning. *Electrical World*, December 2, 1939, pages 49–50.

7. HIGH VOLTAGE BUSHINGS TESTED IN PLACE, C. F. Benham. *Electrical World*, September 28, 1935, pages 29–30. 77.

Shielding of Transmission Lines

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MODERN theories of direct-stroke protection premise that the ground wires are so located as to intercept the stroke and provide perfect shielding. In spite of the fundamental importance of this question there still exists considerable doubt as to the correct position of the ground wires relative to the transmission conductors. Two avenues of approach are suggested for the attack of this question; first, the collection of statistical information regarding actual line performance, and second, the use of laboratory models. Line performance is, after all, the final criterion. However, it is difficult to isolate the shielding effect from other factors which may produce outage due to lightning. Studies with models eliminate this difficulty but always contain the element of doubt as to whether the laboratory conditions are sufficiently representative of actuality as to justify general conclusions. Perhaps the best course, that which is attempted in this paper, is to try both and co-ordinate the results so obtained.

Previous Investigations

Models have previously been used for investigating both the protective value of horizontal ground wires and vertical masts[1–9] for shielding transmission lines and other vulnerable objects such as substations, buildings, and oil storage tanks. In one of the earliest investigations with ground wires, Peek[3] states that "for a single wire, the ground is never hit nearer the projection of the wire than about four times its height above ground." Peek[4] further expands upon this point by stating that from model tests upon rods any object is protected if it lies between ground and within the surface generated by a segment of a circle whose center is the cloud height and is tangent to the ground and passes through the tip of the rod. This assumes that all the strokes occur to the object nearest to the origin of the stroke.[10] Upon this assumption a set of curves can be constructed as shown in figure 1 for guidance in the location of the ground wires with respect to the conductors. In this figure their relative location is specified in terms of the protective angle, the angle between the vertical and a line through the conductor and ground wire. The consideration of minimum sparkover distance just enunciated according to figure 1 indicates, for a cloud height of 1,000 feet and a ground wire height of 100 feet, a protective angle of about 65 degrees which is essentially constant over the practical range of conductor and ground-wire separations.

No data of any consequence such as tests on models of different scales have been presented in an attempt to show that model work is applicable to actual size systems although Peek in a reply to the discussion of his 1926 paper stated that "tests were made, however, over a wide range of scales with substantially the same results. In other words, for a given arrangement and ratio of rod and cloud height the results were independent of the scale." No data to substantiate this statement have been published.

Zalesski,[5] using impulse waves and continuous voltages, concluded that a positive cloud represented by a horizontal cylindrical electrode should be used for test work since it gave more pessimistic results. He determined the worst position of the electrode and with this position studied the necessary protective rod configuration to give complete protection to transmission lines and substations. Akopian[6] likewise limited his investigations mostly to vertical rods and agrees with Zalesski that positive impulse waves should be used in model tests. He departed from previous investigators by representing the cloud by means of a point electrode.

Schwaiger[7] in 1937 followed the philosophy that the conductors, if they are to be protected, must lie within the shaded areas shown in figure 2.

In 1939 Matthias and Burkhardtsmaier[8,9] presented results of investigations relating to shielding of transmission lines and other objects. They, however, premised that natural lightning of both polarities is adequately simulated by laboratory sparks of corresponding polarity. They used only one model scale which gave a reduction of 100:1. An argument was presented to justify the statement that for model work the protective value of a ground wire is not altered as the ratio of the cloud height to ground wire height is increased beyond a certain value. The effect of ground was represented by dry sand. They presented data which indicate a protective angle of the order of 23 degrees for a positive stroke and 31 degrees for a negative stroke.

Fortescue and Conwell[11] from practical considerations gained from experience on actual systems, advocated a protective angle of not more than 20 degrees.

Additional work appeared to be desirable to reconcile, if possible, the divergent views of different investigators. In the present investigation the authors have studied the mechanism of laboratory sparks and have shown that the propagation of leader streamers is quite closely simulated by positive waves of slow rates of rise. Arguments are presented to show that for practical purposes model investigations can be carried out with $1^1/_2$x40-microsecond positive impulses. Other points investigated are the effects of overvoltage, atmospheric conditions, scale of models, and height of cloud. After establishing the fundamentals of model investigations, considerations affecting construction practice such as the location of ground wires, topography of terrain, and soil resistivity, were studied. These questions were then co-ordinated with the results of actual system performance.

Mechanism of Natural Lightning

POLARITY OF STROKE

A knowledge of the characteristics of natural lightning is a prerequisite to an attempt to simulate its effect in the laboratory on a model scale. The present theories of charge formation are still controversial; suffice it for the present purpose to observe that it is concerned in some manner with the reactions between an upward movement of air, usually in the head of the storm, and a downward motion of water droplets. The droplets in the lower portion of the cloud are usually charged negatively and those in the upper portion, positively. This is borne out by the fact that 90 to 95 per cent of the strokes to transmission lines lower negative charge, by the measurement of electrical gradient at the ground in the vicinity of charged clouds[12] and by Simpson and Scrace's[13] measure-

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[1] For all numbered references, see list at end of paper.

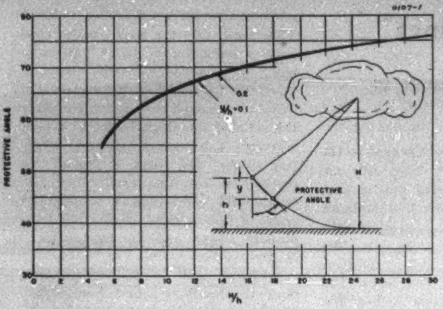


Figure 1. Protective angle based upon the clearance arc[10]

ments of electrical gradients within storm clouds by means of free balloons. In addition, it appears that the charge density is nonhomogeneous and is distributed in zones or charge centers.

Rate of Charge Accumulation and Ground Gradients

The rate at which the charge accumulates is relatively slow as evidenced by the measurements of ground gradients by Wilson[12] and others. Figure 3 is a typical record obtained by Wilson which shows both the magnitude and manner of variation of this quantity. The division between the solid black and shaded areas indicates the magnitude of the gradient according to the scale on the left-hand side. Sudden discontinuities such as that at *A* and *B* represent the destruction of a portion of the gradient as the result of a lightning stroke. Immediately after the occurrence of a stroke the regenerative processes within the cloud begin to re-establish the field at the rate indicated by the rate of change of the gradient. It can be seen that in general the curve is exponential in character and requires a time of the order of several seconds before the charging process attains a substantially constant value. The prominence at 14 hours 12 minutes 30 seconds was produced by the measuring device to establish the zero line and is not a record of change in gradient due to a stroke. The fine weather gradient[13] at the surface of the earth is of the order of one volt per centimeter but during thunderstorms it may reach several hundred volts per centimeter.

Mechanism of Lightning Stroke

The knowledge regarding the mechanism of the actual discharge is due almost entirely to the work of Schonland[14] and his associates in South Africa. Approximately half of all lightning flashes are made up of a number of separate discharges or strokes which, for the region that can be photographed between cloud and earth, travel along the same path. The time interval between successive strokes may vary between 0.0005 to 0.5 second and the total duration may be a second, and in a few cases even longer. Infrequently the number constituting a single flash might be as high as 40.

Each stroke when resolved presents a rather complicated picture. During the process of charge formation or as a result of some local condition, such as vertical air convection currents, the electric field near the base of the cloud may rise to such an extent as to exceed the value which air can withstand. The discharge is then initiated at that point and propagates earthward at a velocity of the order of 1/20 of one per cent of that of light. The current associated with this so-called pilot streamer is small, of the order of a few hundred amperes and its luminosity is very low. As it proceeds, the pilot or leader streamer is accompanied by points of luminescence which move in steps or jumps, which give rise to the term "stepped leader". Each of these steps is about 50 yards in length with fairly regular pauses. This process usually involves a fresh direction of travel after each pause, and these changes in direction are responsible for the well-known zigzag nature of lightning. Branching of the leader streamer also occurs, the branching being outward toward the earth.

When one of these stepped streamers strikes the ground an intense streamer of very great luminosity is observed to travel from the ground to the cloud. The velocity of the "return" streamer is quite high, being of the order of 10 per cent of that of light. Its current is also very high, being of the order of 10,000 to 200,000 amperes, values which have come to be associated popularly with lightning.

From figure 4 it can be seen that as the leader streamer proceeds it lowers negative charge from the cloud and distributes it throughout the antenna-like system of streamers. This lowering of negative charge may consume a time of the order of 10,000 microseconds. As the stepped leader strikes the earth this same charge is lowered to earth at a rate dependent upon the rate of propagation of the return streamers, a process which requires a time of the order of 50 microseconds to reach the cloud. Thus the same charge which had been lowered and distributed over the streamer network in a time of the order of 10,000 microseconds is then discharged to ground in a time of the order of 50 microseconds. This accounts for the large difference between the pilot streamer and the return streamer currents.

Only a portion of the cloud charge is involved in this part of the mechanism. After the lapse of 50 or 100 microseconds the charge in the region between the cloud and ground is discharged and the stroke current at the ground decreases to a relatively small value. Thereafter the stroke current is determined largely by the rate at which leader streamers propagate into the cloud and tap more charge in that particular region.

After some time one of the leader streamers propagating into the interior may meet a similar leader emanating from another charge center. In this case the charge from the second charge center is discharged through the original lightning channel as shown in figure 4 and the same process is repeated except for the fact that the leader streamer is not branched and stepped. After the

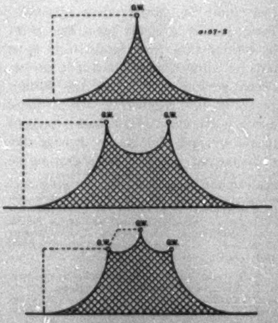


Figure 2. Protective zones of one or more ground wires[7]

charge from the second center has been dissipated, the same process may be repeated for the third and subsequent charge centers.

McEachron[16] has shown that in the discharge of lightning to a very tall building, the initial streamer usually proceeds upward from the building in the form of a stepped leader. This stepped leader then develops into what he terms a continuing stroke—a stroke of long duration and relatively small current. The subsequent strokes are similar to those which follow the first stroke of discharges that propagate from cloud to earth. From the published evidence and some small evidence obtained by the authors, the phenomenon of leader streamers from the earth appears to be characteristic, in the main, only of discharges to tall structures. Discharges to lower structures seem to lack ground streamers of any great length which rise to meet the pilot leader descending from the clouds. Schonland in a letter to the authors regarding his lightning investigations states that "it must be remembered that the country in which we work consists of rolling hills and valleys, so that the base of the discharge is often obscured, there must, however, be a large number of cases in which the full length of the discharge was recorded by the cameras and we have seen no evidence of any extensive leader discharge from ground. Such ground leaders as do occur are comparatively short, for otherwise we should have detected them."

Small streamers may and probably do exist which in the case of strokes to transmission structures rise from the tower or conductors. In general, however, the evidence available seems to point to the fact that in most cases the streamers, if any, are small.

The factors which control the path of the initial leader determine the point at which it strikes the line and therefore the shielding characteristics of an overhead ground wire. Upon completing the initial streamer the path is determined thereafter even to the extent of multiple strokes, all of which follow the same path blazed by the first discharge.

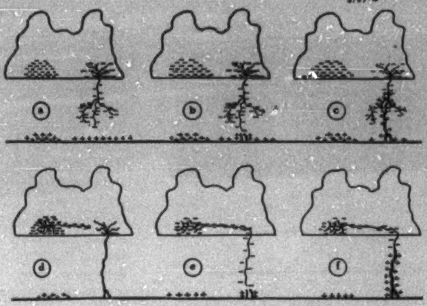


Figure 4. Propagation of natural lightning

Charge

According to Schonland,[15] the quantity of electricity conveyed by a lightning discharge varies between 2 and 100 coulombs with an average value of about 20, but McEachron[16] in his continuing strokes has measured a value as high as 164 coulombs.

Cloud Heights

A search of the literature reveals very little definite data regarding the height at which the stroke may be said to originate. Simpson and Scrace[13] from a limited number of cases obtained in England estimate charge centers as occurring as low as 1,500 feet and as high as 30,000 feet. Of course, the origin of the stroke may not coincide with the charge center but may lie between the charge center and the base. E. J. Minser, chief meteorologist of the Transcontinental and Western Air, may be quoted as authority that in his experience the altitude of the base of low-level thunderclouds frequently lies between 500 and 1,000 feet. He further states[17] that his studies show that the majority of lightning discharges were found to have occurred in the cumulus clouds of the shower type and that strokes to ground occur most frequently from clouds having the lower altitudes. Data in possession of the United States Weather Bureau indicate cumulonimbus clouds as having a mean ceiling of 5,500 feet with some of them as low as 600 to 700 feet. Thunderstorms for which the ceiling is practically zero are also reported at times. Cases for which the storm clouds actually envelop mountains rising from a plain are quite common.

In view of such data as the authors have been able to obtain it is believed that an altitude of 1,000 feet for the origin of the stroke is sufficiently representative and pessimistic for use in this paper.

Figure 3. Measurement by C. T. R. Wilson[13] of ground gradient during thunderstorms

Fundamentals of Model Tests

Allibone and Meek[18,19] have demonstrated certain similarities between the mechanism of natural lightning and that of long arcs obtained in the laboratory. It was considered advisable to extend the study of the mechanism of such laboratory arcs to determine the effects of their characteristics upon the protective performance of transmission line ground wires and diverters.

Description of Apparatus

A 3,000,000-volt 0.003-microfarad surge generator was used as a voltage source for the experiments. Various types of electrodes were used to represent the cloud, and for most of the work a smooth conducting plane represented the earth's surface upon which was placed the model to be tested.

In order to vary the wave shape of the voltage over a wide range, two different types of circuits were employed. The conventional surge-generator circuit with a resistance in parallel with the gap was used for waves with relatively steep fronts such as the $1^1/_2$x40 wave. As was done by previous investigators,[18] for

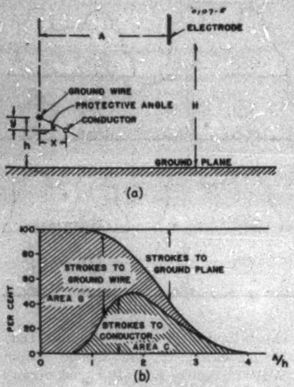


Figure 5. Symbols utilized with stroke distribution curves

waves of much smaller rates of rise this parallel resistance was removed and a high resistance placed in series with the gap. It was then necessary to use a capacity divider with the cathode-ray oscillograph for measuring the voltage. Since, as will be shown later, the shunt capacity of this circuit has a very pronounced effect upon the voltage wave shape its capacity had to be as small as possible. For this reason a short length of wire, about two feet long, was used as an antenna. The voltage of the ground plane was used as a reference and that of the electrode specified with respect to it.

A high-speed camera was constructed for the study of the visual characteristics of the laboratory stroke. It consists of a drum 13 inches in diameter which rotates at 3,600 rpm and is enclosed in a light-tight case. Thirty-five-millimeter film is placed on this drum and rotates past a stationary lens in a direction perpendicular to the direction of propagation of the stroke. It was found that satisfactory photographs could be obtained of the leader mechanism of arcs produced by voltages of slow rates of rise with an ordinary f/1.8 lens and Eastman Super XX film. Typical photographs of arcs made with this camera are shown in figure 6. The rotational speed of the film is such that time intervals as small as one microsecond can be measured. Since in all cases the time for the return stroke of the arc is much less than this, the image of the return stroke can be taken as the reference path for measuring the progress of the initial leaders. The camera will not photograph satisfactorily the leader mechanism of arcs produced with the standard $1^1/_2$x40 wave. This is due not only to the high speed of propagation of the leaders but also to the great luminosity of the return stroke. For this reason high-speed camera studies were restricted to the slower waves.

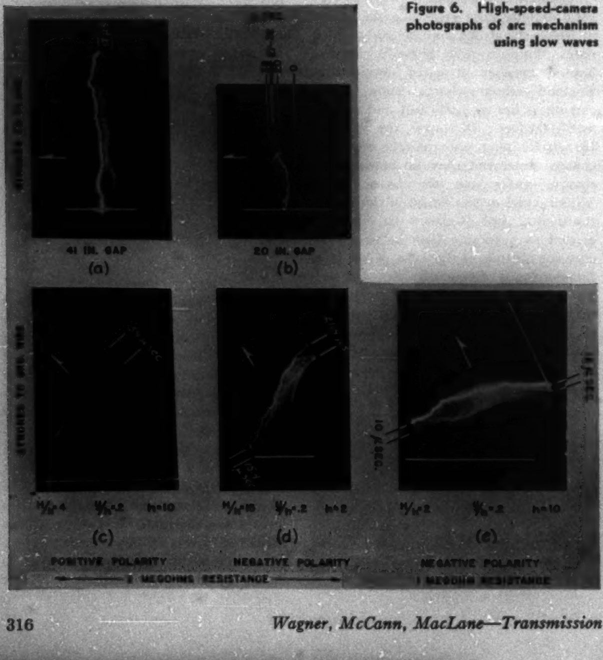


Figure 6. High-speed-camera photographs of arc mechanism using slow waves

Still photography was also employed for studying the visual characteristics of the arcs. Such pictures not only show the shape of the path of the arcs but also something of the character of the leader mechanism. For this reason they are particularly valuable for arcs which cannot be recorded by the rotating camera. In figure 8 are shown still photographs of strokes taken with the conductor and ground wire in place. They were taken in a direction parallel to the wires and thus, as nearly as possible, perpendicular to the arc path. Those showing more than one arc were taken by exposing the same film to a number of strokes taken in succession under the same conditions in order to show the variable character of the paths.

Distribution Curves

In this paper the lightning protection performance of a particular model configuration is based primarily upon distribution curves of the type shown in figure 5. The fundamental tests on ground-wire protection were made with one ground wire and one conductor as shown. The positions are specified by the following dimensions: h, the height of the ground wire above the ground plane; y, the vertical distance of the ground wire above the conductor; and x, its horizontal distance from the conductor. H specifies the vertical distance of the electrode representing the cloud source of the stroke above the plane (or cloud height) and A its horizontal distance from the ground wire, from the same side as that upon which the conductor is located. All dimensions are specified in inches. For a given configuration (value of h, x, y, and H) it is desired to determine the terminating point of strokes for various positions of the electrode corresponding to various values of A.

When the cloud electrode is directly above the ground wire all strokes terminate on the ground wire. As A is increased a position will be reached, if the conductor is not fully protected, for which the strokes divide between the ground wire and the conductor. Since this is a statistical phenomenon sufficient strokes should be observed, 50 is usually enough, so that a percentage division of the strokes between the ground wire and conductor can be determined. As A is further increased some of the strokes go to the plane and finally a position will be reached where all of the strokes go to the plane. The distribution curve of figure 5*b* is obtained in this manner. A/h is used as the abscissa in these curves instead of A so that models of different scale dimensions are directly comparable. Frequently cases occurred for which strokes terminated at more than one place as shown in figures 8*d* and 8*e*. Any strokes of this type which terminated on the conductor were counted as strokes to the conductor.

Because of the slow rate of charge formation in nature a better representation of the conditions of natural lightning is obtained if the minimum flashover voltage is used. Unless stated otherwise, all distribution curves were obtained by readjusting the voltage as A was varied to obtain the minimum spark-over value.

GAP AND PLANE

The slowest waves used were obtained with a series resistance of two megohms, which gave a wave rising to crest of the order of 50 microseconds. For this case it was not possible to obtain spark-over between point and plane for values of H greater than about 50 inches for positive polarity and 30 inches for negative polarity. However, when using the $1^1/_2$x40 wave spark-over could be obtained for spacings as high as 160 inches. For such a spacing a ground plane at least 20 feet square is necessary to eliminate edge effects. The difficulty of obtaining a smooth surface of such size was solved by the formation of a salt-water plane on the laboratory floor. Tests made with a dry metal plane, then with a metal plane covered with water, and lastly, with a water plane alone in which a network of wires was laid below the water surface to further improve its conductivity, showed no essential difference. The salt-water plane was employed for most of the tests.

MECHANISM OF SPARK

The first tests on arc mechanism consisted of determining the characteristics of arcs of both polarities for a point-to-plane gap. Some of the results of these tests confirm the previous work of Allibone and Meek and will be discussed here primarily from the standpoint of lightning protection. High-speed camera records obtained with a minimum flashover voltage and a very flow front using the surge generator circuit with a two-megohm series resistance, are shown in figures 6*a* and 6*b*. Examination of these two figures shows that in the case of negative polarity, leaders start from both the cloud and ground and meet about midway in the gap. However, with positive polarity there is only a positive downward leader. This characteristic difference has been pointed out by Allibone and Meek[18] and others to be due to the fact that a positive leader will propagate at a lower gradient than a negative leader. For this same reason the voltage required to cause spark-over of the positive point-to-plane gap is much lower than for the negative point-to-plane gap.

The negative downward leader for the case of the negative-polarity point electrode was found in most cases to be stepped in character as shown in figure 6*b*. The negative leader starts from the electrode at a very high velocity, too high to be measured with the high-speed camera, but dies out after progressing a short distance. Before the occurrence of the final streamer there may be several leaders of this incomplete character, each progressing further than the previous one. The final leader also moves at this relatively high speed until it has reached a point somewhat farther than the previous leader, then its velocity decreases rapidly and it moves over the final portion of its path at a fairly uniform speed, of the order of $^1/_{100}$th of a per cent of the speed of light. The positive leader which rises from the plane moves continuously at a fairly uniform velocity of the same low order of magnitude.

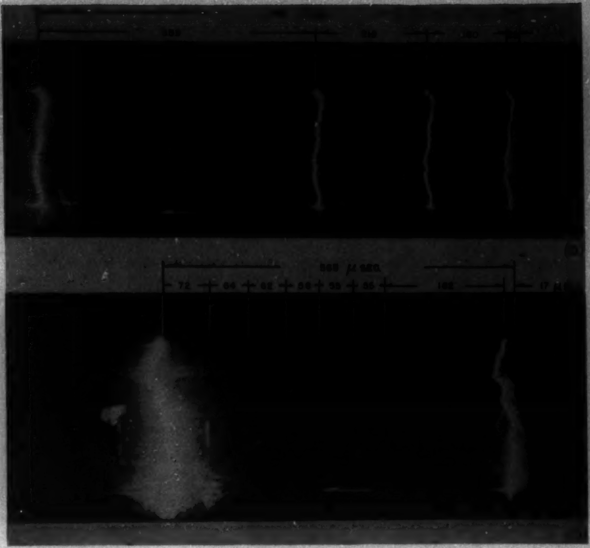


Figure 7. Laboratory multiple strokes

For the case of positive polarity the downward positive leader is sometimes also stepped and exhibits the same characteristics as the negative leader. Quite frequently, however, cases are found where it moves continuously, starting at a relatively slow velocity from the point electrode and increasing its velocity until it reaches a critical point in the gap, then proceeding for the rest of its distance at a uniform velocity of the order of magnitude of $^1/_{100}$th of a per cent of the speed of light.

Similar tests made with the ground wire and conductor in place are shown in figures 6*c* and 6*d*. In this case it is seen that for negative polarity the upward positive leader is the more predominant one, there being only a short negative downward leader. For positive polarity the reverse is the case, there is only a very short upward leader. The character of the positive leaders for

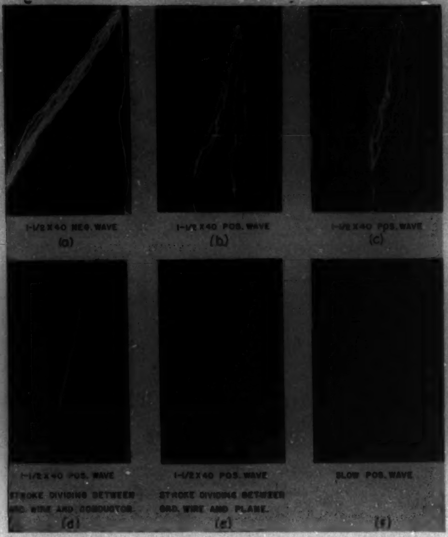


Figure 8. Still photographs showing character of arc paths

the two polarities are similar to those for the point-to-plane gap.

The relative development of the two initial leaders is thought to play a dominant part in determining the distribution curves and it is felt that this is one of the most important characteristics which must be correctly simulated in the model. As will be recalled, the discharge of natural lightning, which is usually negative, is accompanied by very little streamer formation from the earth. Peculiarly, this is more closely represented in the laboratory by a positive discharge which likewise has relatively small leaders emanating from the ground plane. Negative strokes in the laboratory, on the other hand, have quite large streamers from the plane. The difference between the character of the negative stroke in the laboratory and in natural lightning in this regard is probably due to the relative difference in the field gradients at the ground in the two cases. It is thought that these gradients are much higher in the model due to the fact that it is not possible to get breakdown of a point-to-plane gap without a certain amount of corona streamer formation from the point electrode. This increases the effective size of the electrode and thus the field gradient at the ground for a given voltage. Further, this tends to decrease the field at the upper electrode and increase the voltage necessary to cause the leaders to progress downward. Due to this fact it is thought that the ratio of the cloud potentials in the case of natural lightning to the model potentials is considerably less than the ratio of their dimensions. These differences may be increased further by the lower breakdown strength of air at the high altitudes at which the stroke originates in natural lightning as compared to that at the earth.

One factor which is influenced greatly by the character of the wave shape is the duration of the luminosity of the leaders and the current which flows from the electrode into the leader. The capacity of the point-to-plane gap is relatively small so that if no additional capacity is in parallel with it the regulation of the voltage at the gap is very poor when a large series resistance is used. At the instant the leader starts from the electrode enough current is drawn through the series resistance to cause the voltage to drop. Under such conditions of regulation the current flows into the leader only a very short time after its start from the electrode. The leader propagates at a relatively low velocity and its luminosity dies out rapidly behind the leader tip. Thus the leader photographs on the rotating camera film as a narrow line. This is shown in figure 6*d*. In the case of natural lightning it is thought probable that a considerable voltage drop may exist in the lightning channel during the propagation of the downward leader. This is caused by the recombination of free charges behind the leader tip. For the laboratory arcs this condition is greatest for the above type of discharge due not only to the fact that a negligible amount of current flows into the leader to re-establish its conductivity, but also due to the drop in the series resistor.

As the regulation of the voltage is improved by the addition of a parallel capacitor the current flowing into the leader and the luminosity of the leader persists for a longer time. The record shown in figure 6*a* was taken with a sphere gap in parallel with the point-to-plane gap. By varying the spacing of the sphere gap

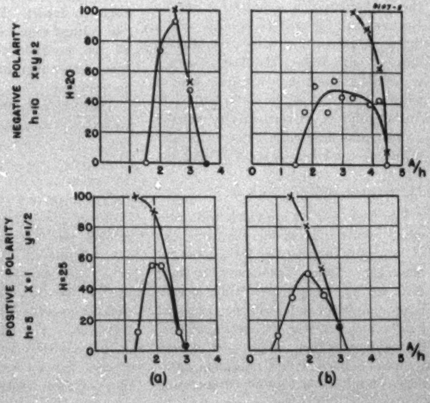


Figure 9. Effect of wave shape

(*a*)—1½x40

(*b*)—Slow rate of rise

it served as a variable capacitor. The increased duration of the leader luminosity is evidenced by the wide band of lower luminosity to the right of the return stroke. This record also shows a greater amount of branching of the leader than in the case shown in figure 6*c*.

By the addition of such capacitance it is also possible to produce multi strokes having many of the characteristics of those occurring in natural lightning. Allibone and Meek produced such strokes in this way. In figure 7*a* is shown a series of repetitive strokes produced in this manner. By varying the magnitude and wave shape of the applied voltage and the parallel capacity the number of repetitive strokes, the time between them, and the magnitude of the stroke current can be varied over a wide range. By this method it is possible to study the recovery characteristics of air. In figure 7*b* is shown a case in which there are only two strokes 565 microseconds apart. This record is of interest because it shows the presence of a stepped leader for the second stroke.

Table I. Outages Due to Lightning for Lines Without Ground Wires

Line	Kv	Length of Line	Years Covered	Total Outages	Outages Per 100 Miles of Line Per Year	Reference
Interstate Power Company Clinton-Dubuque	66	54	6	277	87	Wisco and Monteith" Experience With Protector Tubes", *Electric Journal*, August 1938
Pennsylvania Water and Power Holtwood-York	66	23	11	213	84	Hansson, "Design, Construction, and Operation of a Lightning-Proof Transmission Line," BEI Publication No. *F6*
16-*A*	110	79.4	6		24	Sporn, "Lightning Performance of 110 to 165 Kv Transmission Lines", AIEE TRANSACTIONS, 1939, page 294
20-*A*	140	148.0	10		5.6	
23-*BB*	132	39.8	8		37.2	
31-*B*	132	111.0	1		0	
34-*K*	110	38.3	10		61	
34-*O*	110	104.5	10		22.8	
Above 100 kv—steel		673	1929		21	Great Lakes Division of NELA, 1929–30 report of overhead systems committee
		560	1930		14	
60 to 100 kv—wood		394	1929		26.6	1930–31 report of engineering section—transmission-line operating records 1930
60 to 100 kv—steel		481	1929		23.5	
		738	1930		25.9	
30 to 60 kv—wood		3,475	1929		34.6	
		420	1930		30.4	
30 to 60 kv—steel		165	1929		17.6	
		128	1930		14.2	

As stated previously, the process of charge generation in actual clouds is relatively slow. One would expect that the laboratory representation of this condition would be most closely simulated by a continuous voltage. This condition is difficult of attainment but can be approximated by the use of waves of very slow rates of rise. As will be shown subsequently, this is not a very important factor. As the time to crest of the wave is decreased or the crest magnitude of the voltage is increased the speed of propagation of the leader increases. The velocity of positive leaders of continuous character increases uniformly. For leaders of stepped character the number of stepped leaders decreases rapidly. The effective velocity of the negative leader increases somewhat faster than that for the positive. High-speed camera records, obtained when the rate of rise of voltage is gradually increased by decreasing the series resistance, show this effect. Figure 6*e* is a record of a negative stroke to the ground wire when a series resistance of one megohm is used. This shows a negative leader bridging about one-third of the gap as compared with the much shorter leader of figure 6*d*.

The effect of waves of very steep fronts such as those of standard $1^{1}/_{2}$x40 waves, can be studied by still photographs. The direction of branching of leaders of both polarities is always in the direction of its propagation. By this means it is known that a particular leader progressed at least as far as the last branching point. As the leaders approach each other they branch out, and frequently several meet simultaneously, the return strokes occurring along all of them. This forms several parallel paths over a short distance of the arc at the junction point of the two leaders and provides another method for determining the meeting point of the leaders. Both of these characteristics can be seen on the photographs of figure 8.

The examination of many such still pictures shows that for the $1^{1}/_{2}$x40 wave the essential characteristics of the point-to-plane gap alone and with the ground wire and conductor in place are the same as with the slower waves. However, the relative development of the negative leader, although rather variable, is usually somewhat greater. This is most noticeable in the case of positive polarity strokes for which the negative upward leader is found to have maximum lengths as high as one-sixth the gap spacing for point-to-plane gaps and one-fifth for

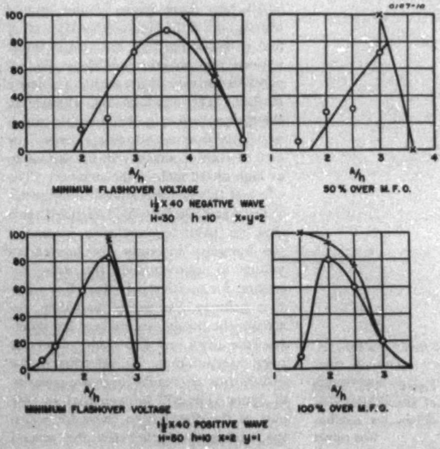


Figure 10. Effect of overvoltage on distribution curves

point-to-wire gaps for a ratio of *H/h* of five to one. The corona streamer formation at the cloud electrode was found to be considerably greater for the $1^1/_2$x40 wave than for the very slow one. This additional effect tends to increase the field gradient at the plane and thus the relative development of upward leaders.

Table II. Experience of Pennsylvania Water and Power Company

Line	Kv	Mile-Years of Operation for Data Considered	Protective Angle (Degrees)	y/h	Strokes to Line Per 100 Miles Per Year	Per Cent Strokes to Conductor	Strokes to Conductor Per 100 Miles Per Year
Safe Harbor-Westport and Safe Harbor-Riverside	230	837	16	0.29	106	0	0
Safe Harbor-Perryville	132	158	17	0.18	153	0.4	0.6*
Holtwood-Coatesville	69	59	52 and 71	0.11	192	1.75	3.4**

* One flashover in five years. ** Two flashovers in two years.

Wave Shape and Distribution Curves

Results of studies to determine the effect of the character of the applied voltage upon the distribution curves for both positive and negative strokes are shown in figures 9 and 10. Figure 9 shows the effect of wave shape and figure 10 that of overvoltage. From these results it is seen that the nature of the stroke voltage influences greatly the curves for negative polarity strokes and has little effect on those for positive strokes. The greatest discrepancy caused by varying voltage conditions for negative strokes is in the variation of the position of the portion of the distribution curve formed by the division of strokes between the wires and the ground plane. The slower the rate of rise of voltage the larger the value of *A/h* before strokes start to go to the plane. Thus a greater proportion of strokes will go to both the ground wire and the conductor and the conductor is less protected for this type of wave.

The effect of voltage upon the character of the distribution curves can be explained by the nature of their leader mechanism. The spark-over characteristics of the point-to-plane gap and the point-to-wire gap are more nearly equal for positive-polarity sparks than for negative ones. This is due to the fact that the positive-polarity spark is initiated by a positive downward leader traveling practically all the way across the gap for both gap configurations. The time lag and the voltage required for spark-over are more nearly the same for positive polarity and little affected by the character of the voltage. The somewhat larger upward leader in the case of the $1^1/_2$x40 wave has no appreciable effect on the distribution curves. However, for negative-polarity strokes the point-to-wire gap is initiated by an upward positive leader which progresses most of the gap length, while for a point-to-plane gap the leaders progress simultaneously from both electrodes toward the center of the gap. For this reason the time lag of the point-to-plane gap is less than that for the point-to-wire gap. However, the point-to-plane gap requires a higher voltage for spark-over for the same spacing. Thus voltages of slower rates of rise will break down the point-to-wire gap more readily.

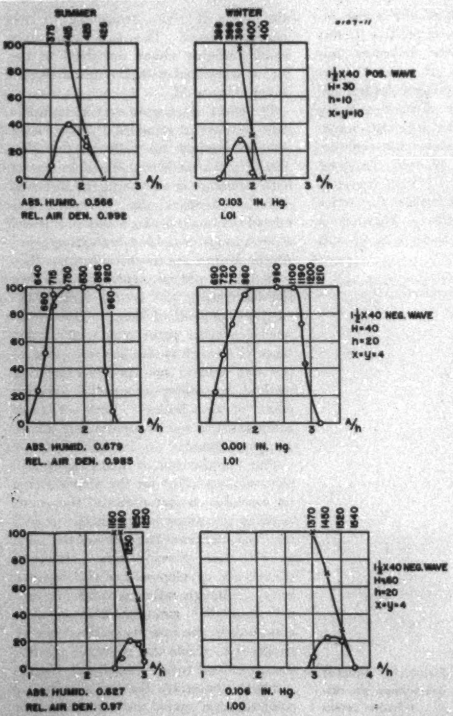


Figure 11. Effect of atmospheric conditions on distribution curves

Weather Conditions

All tests were made in the natural atmosphere of the high-voltage laboratory and it was found that data taken under various atmospheric conditions did not check in the case of the negative-polarity strokes. The predominant change in the atmosphere is in the humidity which may vary from an absolute value as low as 0.01 inch of mercury in the winter to as high as 0.6 inch in the summer. The effect of this factor is shown in figure 11.

Data published by Y. Ishiguro[20] show that for point-to-plane gaps increasing the humidity increases the breakdown voltage in approximately the same proportion for positive polarity. For negative polarity, increasing humidity decreases the breakdown voltage for point-to-plane gaps and has little effect for point-to-point gaps. In referring to the distribution curves for negative polarity of figure 11 it will be seen that in that part of the distribution curve for which the strokes divide between the ground

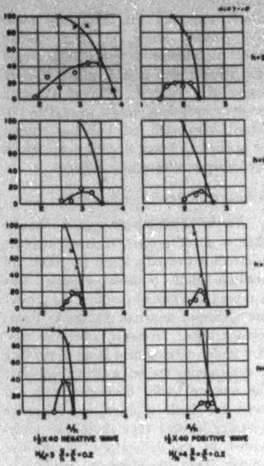


Figure 12. Effect of changing scale

wire and conductor the breakdown voltage is not influenced to any considerable extent by humidity but where the strokes divide between the wires and plane increasing humidity decreases the breakdown voltage. Applying Ishiguro's data indicates that there should be no substantial change when the distribution occurs between wires but where the distribution occurs between wires and plane a greater proportion of the strokes should go to the plane at higher humidity. This has the effect of moving the distribution curve to the left in this region. In the case of positive polarity the breakdown voltage of both point-to-wire and point-to-plane gaps are affected only slightly and therefore one would expect little effect upon the distribution curves as is borne out by the results of figure 11 and a considerable amount of other unpresented data.

GEOMETRIC SIMILITUDE

Tests were made to determine the effect of geometric similitude. In figure 12 is shown some of the data in which the scale dimensions were varied over a range of eight to one. The results of these tests indicate that the range of scales considered has no very great effect upon the distribution curves for positive strokes. The very small effect that may be present is in a direction to make the protective value less for small scales, which would produce pessimistic results in model work. There is, however, a very definite effect for negative strokes. As the scale is increased, and thus the spacing between the cloud electrode and ground, the range of A/h for which shots go to the wires decreases and approaches the results obtained with positive strokes.

FACTORS AFFECTING ARC PATH

Examination of the still pictures of figure 8 show how variable the path of the arc is even though conditions are maintained as nearly constant as possible. As the leader tip progresses it is apparently very unstable and easily affected by its own irregularities. For this reason it follows a tortuous path. The paths of the negative polarity strokes are much straighter and more closely grouped than those of positive strokes, while the paths of the arcs of slow voltage waves are more spread out and more irregular than those of the $1^1/_2$x40 waves.

In considering these factors it is convenient to segregate the field at the tip of the leader into two parts: that produced by charges remote from the tip including the effect of the leader from the opposite electrode; which will be called the directive field; and that produced by the ionization phenomena taking place in the region of the leader tip. The irregularities at the tip cause one arc to have a totally different path from another and possibly a different terminating point; thus the division of strokes in distribution curves. The first component of the field has, however, a very definite directive effect in influencing the ionization process at the leader tip and an appreciable upward leader influences the downward one.

The greater irregularity of arcs produced by slow waves is probably due to the fact that since the breakdown voltage is less the directive field is weaker and has less directive effect. It thus approaches more closely the case of natural lightning. By taking a large number of shots in obtaining the distribution curves a statistical average is obtained of the effect of the factors which influence the path of the arc. The fact that wave shape has no effect upon the distribution curves for positive polarity indicates that on the average the actual magnitude of the directive field is not important. It is merely its relative value throughout the region in which the arc might progress that determines the average result.

The greater upward leaders in the tests with the $1^1/_2$x40 waves over those in the tests with slow waves had no effect upon the distribution curves. Arguing from these results, the extremely small upward leaders that may occur with natural lightning should be insignificant in determining the terminating point of the stroke.

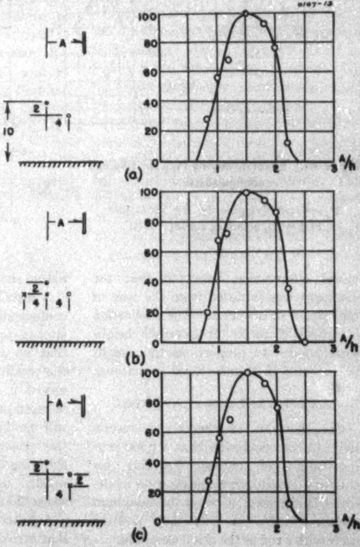


Figure 13. Effect of the presence of other conductors on the protection of the most exposed conductor

$1^1/_2$x40 positive wave, $H/h=3$, $y/h=2$

So far leader development and distribution curves have been considered for a rod electrode only. It was felt that perhaps the shape of the electrode might have a certain directive effect upon the leader in its vicinity and thus influence the distribution curves. To test this point a special electrode was made by soldering a small wire one-half inch long to a ten-inch sphere. With the wire directed downward from a height of 30 inches a distribution curve was obtained for $h=10$, $x=4$, and $y=2$. This electrode gave the same distribution curve as a rod electrode and visual observation indicated that the discharge was initiated from the wire of the sphere in all cases. The proportion of sphere diameter to electrode height corresponded to proportions that might be expected in thundercloud formations.

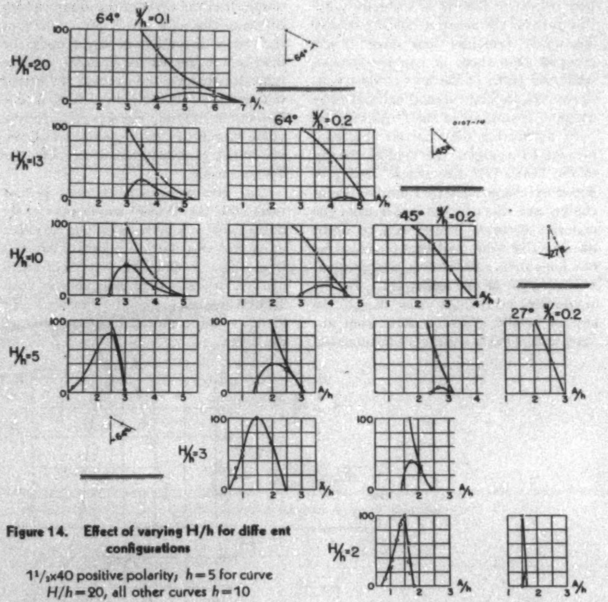


Figure 14. Effect of varying H/h for different configurations

$1^{1}/_{2}$x40 positive polarity; $h=5$ for curve $H/h=20$, all other curves $h=10$

Final Choice of Laboratory Model

Consideration of the above results leads to the conclusion that a good representation of natural lightning for studying ground wire shielding on model scales is obtained by using the minimum flashover value of a $1^{1}/_{2}$x40 positive wave with a rod as the cloud electrode.

It must be remembered, however, that any results which can be deduced with models can only be of a statistical nature. The distribution of charge within a thundercloud is likely to be unsymmetrical about the point of origin of the discharge and will influence to some extent the shielding value of the ground wires. In open, level country this dissymmetry should, however, be eliminated by the statistical averaging of a large number of strokes. Where the prevailing storms in a locality originate from the same direction this may be true but, in general, any effect of this nature should be very small.

A distribution curve obtained for any configuration which shows no area for strokes to a conductor does not mean that no strokes will ever terminate on the conductor. If enough shots are observed it will be found that eventually in most practical configurations a stroke will finally strike the conductor. For this reason isolated records of natural lightning which show strokes terminating within certain regions of protective space should not be overemphasized.

Another important consideration is that strokes to very tall buildings more closely approach the laboratory negative polarity stroke in their relative leader development. Since it is found that strokes with a predominate upward leader give entirely different shielding results conclusions drawn from data obtained from such tall buildings cannot be applied to lower structures for which the leader development is different.

Attractive Effect of Polarity

Allibone[19] from laboratory tests showed that a small projection from a plane electrode attracted a larger proportion of negative than positive strokes. This was used as a basis to explain the greater prevalence of negative strokes to transmission lines. However, since the relative leader development is all controlling in determining the point struck and since the relative leader development is the same for both positive and negative lightning strokes they should be represented in the laboratory by positive waves. This consideration would appear to throw doubt upon this explanation for the prevalence of negative strokes to transmission lines.

Results of Model Tests

Wire Size

Correct proportioning of the model wires to those in actual practice resulted in destruction of the wire for each shot. Recourse was therefore had to larger conductors. Distribution curves made with wires of diameters varying from 0.008 inch to 0.09 inch showed that wire size has negligible effect for positive polarity. Tests made with negative polarity however as may be expected are not so consistent.

Grounding of Conductors

The speed of leader development is so slow that it is thought that both the ground wire and conductor, except for the dynamic voltage, remain at essentially ground potential during the development of the leaders. However, to verify this point the effect of having both the ground wire and conductor grounded was compared with the case for which the ground wire was grounded but the conductor connected to ground at each end through 500-ohm resistors to represent the surge impedance of the line. Little difference was found for these two cases. Other tests in which the conductor was insulated gave somewhat better protection. The most pessimistic results are obtained by grounding the conductor, a condition under which all subsequent tests were made.

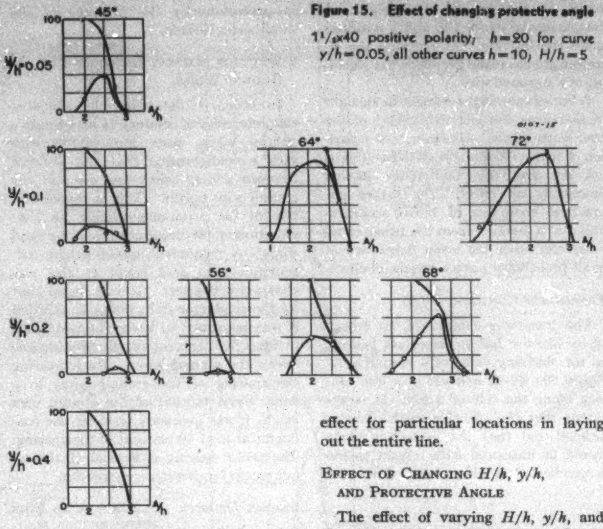


Figure 15. Effect of changing protective angle

$1\frac{1}{2}$x40 positive polarity; $h=20$ for curve $y/h=0.05$, all other curves $h=10$; $H/h=5$

Presence of Other Conductors

Actual transmission-line construction involves more complicated conductor configurations than the simple ground wire and single-conductor arrangement so far discussed. To determine the effect of the presence of other conductors upon the general problem of shielding, tests were made upon several arrangements of ground wire and conductor. Figure 13 shows a portion of the results of these tests in the form of distribution curves. It was found that the added conductors of *b* and *c* which are indicated by x's were not stricken as the electrode was moved to the right of the ground wire. In addition, the distribution curves are not influenced by the presence of the added conductors. Additional ground wires likewise had little effect. The following data are, therefore, confined to the single-ground-wire and single-conductor configurations.

Towers

The point from which insulators are supported on steel suspension towers is directly above the conductor and naturally tends to protect it. Tests show, however, that the extent to which this is effective is unimportant in level terrain. In mountainous conditions particular towers supporting spans having large sags might prove to be advantageous from this viewpoint. It would be unwise, however, to take advantage of this effect for particular locations in laying out the entire line.

Effect of Changing H/h, y/h, and Protective Angle

The effect of varying H/h, y/h, and the protective angle are shown by the distribution curves of figures 14 and 15. In figure 5 areas *G* and *C* are proportional to the total strokes to the ground wire and total strokes to the conductor, respectively. The ratio area *C*/(area *C*+ area *G*) then represents the proportion of the total strokes to the structure which strike the conductor. Data of the character shown in figures 14 and 15 supplied information from which the curves shown in figures 16, 17, and 18 were plotted. Figure 16 shows that the strokes to conductor decrease continuously as H/h is increased. This result is in direct variance with that obtained by Matthias[5,6] who concluded that above a value of H/h of 7 the proportion of strokes which strike the conductor is constant. Figure 17 is interesting in showing the general effect of changing y/h while keeping the protective angle and H/h constant. Figure 18 shows how the strokes to conductor vary as the protective angle is altered for different values of y/h.

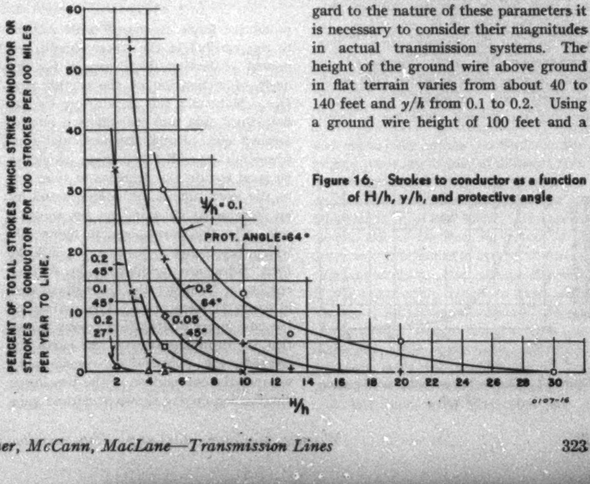


Figure 16. Strokes to conductor as a function of H/h, y/h, and protective angle

The curves of figure 1 were plotted on the theory that the stroke strikes the nearest object, that is, either all the strokes terminate on the ground wire, all on the conductor, or all on the plane. For H/h equal to 5 the protective angle is 55 degrees and is essentially independent of y/h. Figure 18 emphasizes that the desired configuration is dependent to a very great extent upon the degree of shielding to be expected. Thus for 99 per cent shielding the protective angle varies between 46 degrees and 60 degrees as y/h varies between 0.05 and 0.2 and if y/h is 0.1 the protective angle varies between 51 degrees and 72 degrees as the shielding varies between 99 per cent and 95 per cent.

The safe location of a conductor has frequently been described as any position within a volume enclosed by two plane surfaces intersecting the ground wire and making an angle of 45 degrees with the vertical. The curves of figures 17 and 18 show that these surfaces should not be plane but curved concave upward in a manner similar to that shown in figure 2 by Schwaiger. However, the curves by Schwaiger result in protective angles which are entirely too small.

In order to orient one's self with regard to the nature of these parameters it is necessary to consider their magnitudes in actual transmission systems. The height of the ground wire above ground in flat terrain varies from about 40 to 140 feet and y/h from 0.1 to 0.2. Using a ground wire height of 100 feet and a cloud height of 1,000 feet, gives a value of H/h equal to 10. Examination of figure 16 indicates that for these conditions and y/h equal to 0.2, and a protective angle of 64 degrees, 5 per cent of the strokes strike the conductor. It must be understood, however, that these conditions are not met in practice except in flat country with very low soil resistivity. In rough, rugged, and hilly country advantage is usually made of locating towers on prominences so that the effective height of the ground wire above ground is considerably in excess of that at the tower. This fact prompts the use of a smaller value of H/h and y/h than just mentioned, both of which decrease the protection afforded by the ground wire. For example, if h is doubled the corresponding values of H/h and y/h would then become 5.0 and 0.1, respectively, and from figure 16 it can be seen that 30 per cent of the strokes would strike the conductor.

Soil Resistivity and Counterpoise

Soil resistivity might likewise affect the appropriate value of h to use. The propagation of the initial leader of natural lightning is so slow that for ordinary soil conductivity the surface of the earth is probably maintained at essentially the same potential. For dry, sandy, or rocky soil this consideration does not apply and the equivalent depth of earth plane should be lower than the actual surface of the earth. Matthias[9] in his model work took as a representative case a layer of dry sand of approximately the same depth as that of the ground-wire height. While this may be typical in so far as resistivity of the soil and geometric similitude is concerned it does not follow that the voltage gradients in the earth are in proportion. In order to show the general nature of this phenomenon the tests illustrated in figure 19 were made in which high earth resistivity was simulated by simply lowering the ground plane by a depth equal to the ground-wire height. A comparison of figure 19a and b indicates quite conclusively that lowering the effective ground plane decreases the shielding of the ground wire.

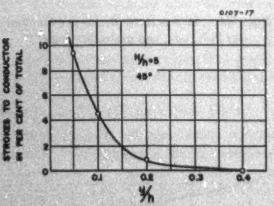


Figure 17. Strokes to conductor as a function of y/h

A buried counterpoise might be thought of as raising the effective height of the true water plane. However, a comparison of figure 19b and c shows that this is not the case, the counterpoise having practically no effect. The rather impractical condition of buried counterpoises at a distance from the tower equal to several times the tower height would in all probability have a beneficial effect.

Transverse Contour

The transverse contour of the topography likewise has an important bearing on the shielding value of a ground wire. Figure 20a shows a tower of a line running along the side of a hill. It is apparent that the effective earth plane is inclined and that the protective angle should be measured with respect to the perpendicular to this plane. Thus the protective angle measured with respect to the vertical of the tower should, for perfect protection, be increased by the angle of inclination of the earth's surface. Naturally this is additive on the downward side and substractive on the upward side. But if the particular configuration provided adequate shielding on level terrain the decrease in shielding on the lower side is not counteracted by an increase in shielding on the upward side. For the case shown in figure 20b which represents a tower on a hillock, both sides are additive. To further emphasize this fact the tests illustrated in figure 21 were made. Comparison of the two upper distribution curves shows that the dropping away of the earth decreased the protection considerably whereas the comparison of the two lower distribution curves shows a marked gain in protection by the presence of the neighboring hillside.

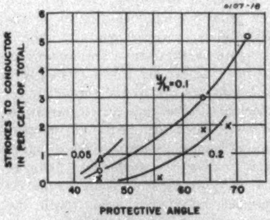


Figure 18. Strokes to conductor as a function of protective angle for H/h = 5

Conductor Midway Between Two Ground Wires

In order to determine the shielding characteristics of conductors between two ground wires, tests were made with such a configuration that the distance between ground wires was one-half the ground wire height. This is representative of the maximum spacing for construction employing conventional ground wires. A pessimistic cloud height corresponding to H/h equal to five was chosen for the test. It was found that for the conductor to be perfectly shielded it was necessary to lower the conductor at least 15 per cent of the spacing between the ground wires. By increasing the spacing of the ground wires to a value equal to that of the ground wire height it was necessary to lower the conductor at least 10 per cent of the spacing. The latter spacing is typical of the requirement for diverter construction.

Stroke Density and Strokes to Line

In an effort to apply a quantitative measure to the effectiveness of ground-wire protection a knowledge of the number of strokes to the line, S, per hundred miles per year is required. As shown in the appendix this quantity in terms of the stroke density, D, in number per square mile per year, is equal to $Dh/26.4$ (area G + area C). From a knowledge of the distribution curves which were obtained for a wide variation of h, H, x, and y, the quantity S/Dh may be plotted as shown in figure 22a. The strokes S are thus proportional to D and are a function of H and h. In figure 22b these data are replotted with S/D as the ordinate and H as the abscissa for h equal to 50, 75, and 100 feet. It is interesting to observe that for a given stroke density the strokes to the line increase with the height.

Tests with two ground wires separated varying distances up to one-half the ground-wire height showed little difference in the total area representing strokes to the structure. Thus there should be little difference between the total strokes to a line with a single or double ground wire or between a single- or double-circuit line. In what follows strokes or outages will be based on a line rather than a circuit basis.

Operating Experience

To correlate the laboratory data with the performance of actual systems some

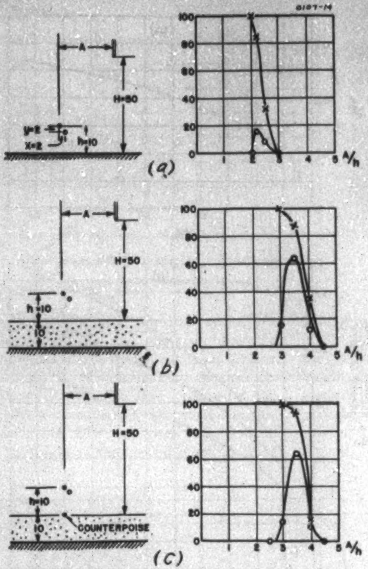
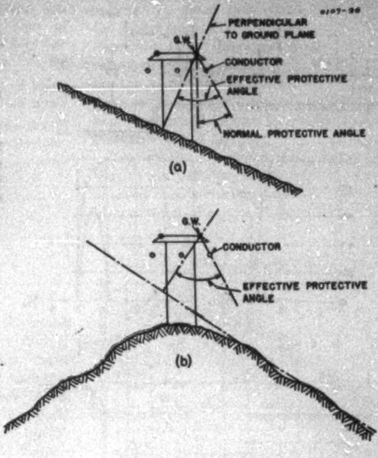


Figure 19 (left). Effect of high soil resistivity upon protective value of ground wires

1½ x 40 positive polarity

Figure 20 (right). Cases for which lateral contour requires smaller protective angle


0107-20
PERPENDICULAR TO GROUND PLANE
G.W.
CONDUCTOR
EFFECTIVE PROTECTIVE ANGLE
NORMAL PROTECTIVE ANGLE
(a)
G.W.
CONDUCTOR
EFFECTIVE PROTECTIVE ANGLE
(b)

published data are available and, in addition, a number of utilities have made available to the authors certain unpublished data.

STROKES TO LINE

E. Hansson and S. K. Waldorf, from data obtained with magnetic links between 1935 and 1938 upon 703 mile-years of operation of lines of the Pennsylvania Water and Power Company, indicate an average of 116 strokes per 100 miles per year. Bell[21] from data accumulated between 1926 and 1933 on the portion of the Wallenpaupack-Siegfried line (without ground wires), obtained a value of 82.8 flashed towers per 100 miles per year. This figure is probably lower than the actual number of strokes to the line as some of the strokes may not have left their markings. In the same period data for the same line indicated an average of 25 tripouts per 100 miles per year. A comparison between these figures shows that the frequency of lightning strokes should be considerably in excess, something of the order of three times, the number of outages. Table I is a compilation of outage data for a number of high-voltage lines without ground wires. The weighted average of these lines lies between 30 and 35. Applying the factor three to these values results in an estimated value of 100 strokes per 100 miles per year. It must, of course, be remembered that this value is very much of an approximation and will vary with location and other factors. The ordinate of figure 16 which expresses the per cent of strokes which strike the protected conductor can also express numerically the number of strokes to the conductor on a basis of 100 strokes to line per 100 miles per year.

STROKES TO CONDUCTOR AND OUTAGE DATA

The next step is to verify to what extent the data of figure 16 represent the performance of actual system operation. Before this is attempted it is well to point out some of the difficulties encountered in this task. It will be observed first that the performance to be expected of any configuration will depend to an enormous extent upon the particular assumption of cloud height or H/h. As pointed out previously, very little data are available upon this point. Cloud heights may and do vary with geographical location and local topography. Along with the variation of storm numbers per year at different locations there also exists variations in range of these heights. Two similar lines at different locations might therefore have entirely different performance records. The variation of shielding with H/h as shown in figure 16 therefore anticipates that difficulties might be present in the comparison of system performance.

In forming an opinion of what is adequate protection the criterion will be adopted that the strokes to conductor should be less than one per 100 miles per year. For a value of H/h equal to 10 the curves of figure 16 indicate that a protective angle of 60 degrees is adequate for a value of y/h equal to 0.2 and 50 degrees for y/h equal to 0.1.

Further obstacles are encountered when an attempt is made to verify the extent to which these data represent the performance of actual systems. It is difficult to isolate the effect of shielding alone. Even for systems of very low tower-footing resistance the question still arises as to whether flashovers might not be due to currents of high rates of rise for which the tower surge impedance and strokes to midspan become important. Thunderstorms are usually accompanied by wind and since the phase conductors and ground wires are usually of different type and sagged differently the protective angle may be considerably greater at midspan than at the tower. Of course, on one side of the line the increment of angle would be additive and on the other side subtractive. The outage function is not linear with respect to angle and, therefore, the increased angle will cause a much greater outage increase than the decreased angle a decrease. However, in some cases the smaller sag of the ground wire may completely annul this condition and even give the line a

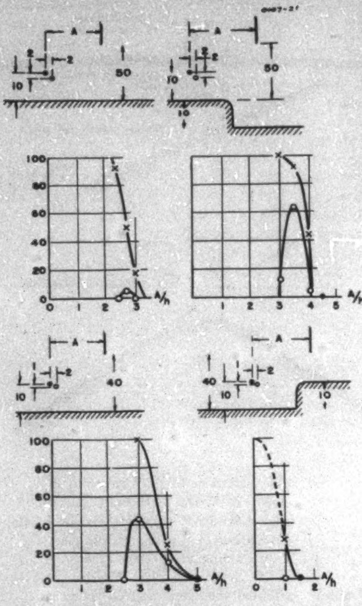
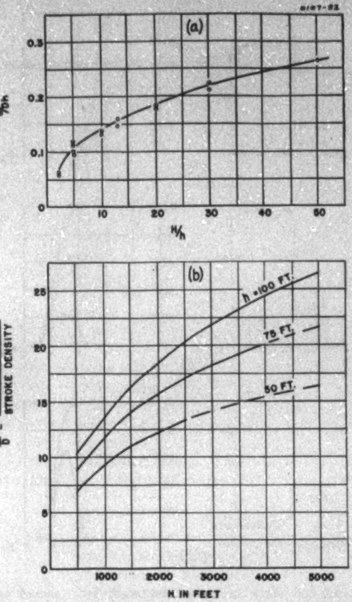


Figure 21 (left). Effect of uneven ground plane

$1\frac{1}{2}\times40$ positive polarity

Figure 22 (right). Strokes to transmission line as influenced by cloud and ground-wire height



smaller protective angle at midspan.

Hansson and Waldorf have supplied the authors with the data tabulated in table II. It will be observed that the Safe Harbor lines to Westport, Riverside, and Perryville, all have protective angles well within the values set by the curves as being adequate and the operating experience verifies this result. The Holtwood-Coatesville line, being nearer the limits set above, offers a better opportunity to check the laboratory results. Inspection of the curves in figure 16 for H/h equal to 10 and y/h equal to 0.1 shows that the per cent of strokes which strike the conductor is of the correct order of magnitude, the curves being somewhat pessimistic.

The Fort Wayne-Marion-Muncie line of the American Gas and Electric Corporation is interesting in that it has very low tower-footing resistances, the average being about 2.5 ohms. The protective angle is 50 degrees and y/h is 0.11. Data obtained from I. W. Gross giving operating results over a five-year period, representing 400 mile-years of operation, shows 23 flashovers which is equivalent to 5.7 per 100 miles of line per year. As stated previously, the curves of figure 16 for H/h equal to 10, y/h equal to 0.1, and protective angle of 50 degrees give one stroke per 100 miles per year, a value which is somewhat low with respect to the actual performance. It is probable that the change in angular position occasioned by wind conditions is an important factor in this case since the line is so close to the critical angle.

A number of utilities have maintained rather accurate records of the resistance of each individual tower and the particular ones upon which flashovers occurred. By dividing the towers into classes according to their resistances and sorting the flashovers into the same classes it is possible, by weighting the flashovers according to the number of towers in the class, to plot a curve of flashovers as a function of tower-footing resistances. This has been done for data relating to flashovers and outages for several companies. The intercept with the ordinate for resistance equal to zero should represent the effect of lack of shielding, tower impedance drop, and possibly the effect of ground-wire impedance for strokes to midspan. Figure 23 represents data of this type obtained for a 132-kv system in flat country of low soil resistivity. Data of this nature are of necessity somewhat random in character and it is difficult to determine the curve to be drawn through the plotted points. The significant fact is that the ordinate intercept for resistance zero is zero or very small. This would indicate not only satisfactory shielding but also that the effect of tower surge impedance is negligible. Published data[22] for the Victoria Falls-Transvaal Power Company, plotted in figure 24, shows a similar result. This line also transverses relatively flat country. The results of these two systems check the data from the model work.

Similar data supplied by I. W. Gross applies to the system of the American Gas and Electric Corporation and is plotted in figure 25. Accepting the result of the two previous curves that the effect of tower surge impedance is not of any great importance, the y-intercept of 12 flashovers per 100 miles per year must be attributed to poor shielding. This number contrasts with about one per 100 miles per year for y/h equal to 0.1 and a protective angle of 45 degrees as dictated by figure 16. For the same time for which these curves were obtained the total flashovers of 521 corresponded to 371 tripouts. Applying this same ratio to those due to lack of shielding provides a value equal to nine tripouts per 100 miles per year. A large portion of the lines involved in this group traversed mountainous country in which the effect

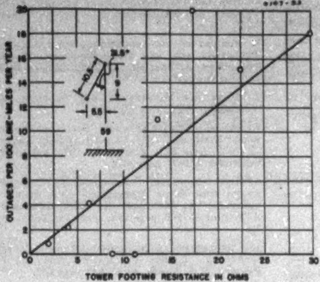


Figure 23. Lightning outage record of lines of 132-kv system in flat country

1,470 mile-years of single and double circuit

$x/h = 0.093$, $y/h = 0.152$, protective angle = 31.5 degrees

of transverse slope, the decrease of y/h across valleys and canyons, and the lowering of the equivalent ground plane might be of considerable importance. It is also possible that because of the mountainous terrain, the cloud heights, and thus H/h, are lower. In general, the height of the ground wires on these lines is larger than for the lines previously considered, so that for a given cloud height H/h is smaller which likewise tends to decrease the shielding. Perhaps in view of these factors the agreement is not as bad as might first appear. If the disagreement is to be attributed to these causes, then it follows that the nominal protective angle and y/h at the towers must be corrected in some manner to take these factors into consideration.

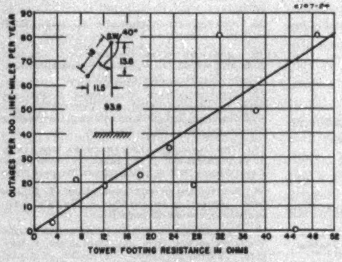


Figure 24. Lightning outage record of lines of Victoria Falls-Transvaal Power Company

132 kv, 444 mile-years of double circuit

$x/h = 0.12$, $y/h = 0.15$, protective angle = 40 degrees

The lightning and insulator subcommittee of the AIEE power transmission and distribution committee under the chairmanship of Philip Sporn submitted a report[23] in 1938 relating to the lightning performance of lines between 110 kv and 165 kv. A number of the lines tabulated in that report were selected for which the average tower-footing resistance was less than ten ohms. Figure 26 shows the outage data of these lines plotted against their protective angles. Some of the towers in this group must have had a footing resistance considerably in excess of 10 ohms which might explain some of the outages. In drawing this curve weight was given to the points which represented the largest circuit-years of experience. Information to plot the point marked by an x was furnished by W. Cronin, for the system of the Indianapolis Power and Light Company. The maximum tower-footing resistance for this system is 11 ohms. The curve indicates that satisfactory shielding is obtained for angles less than about 35 degrees. If an allowance of 10 or 20 degrees were provided for soil resistivity, lowering of effective ground plane across canyons and swinging of ground wires and phase conductors, a substantial check with laboratory results would be indicated.

Résumé and Conclusion

The relative development of the leaders preceding the actual discharge of the lightning stroke is the most important part of the mechanism which must be correctly simulated in model work when studying the shielding problem. In this regard natural lightning, both from positive and negative charge sources, is most closely simulated in the laboratory by making a point electrode positive with respect to a plane upon which is located the model transmission line to be tested. Positive-polarity discharges in the laboratory are more consistent in reproducing the same distribution curves and in obeying the laws of geometric similitude and also are less affected by weather conditions.

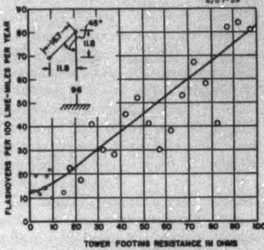


Figure 25. Lightning flashover record of lines of American Gas and Electric Company

132 kv, 2,426 mile-years of double circuit $x/h = 0.12$, $y/h = 0.12$, protective angle = 45 degrees

The results of model tests indicate that to limit the strokes to one per cent of the strokes to the line for a cloud height of ten times the ground wire height a protective angle of 50 degrees is necessary for y/h equal to 0.1 and 60 degrees for y/h equal to 0.2. These values have been checked substantially by operating experience upon lines in level country. Other factors enter the problem, chief of which are:

1. Cloud height. Strokes to the protected conductor decrease with increasing H in the manner shown in figure 16.
2. Soil resistivity and topography between spans. The effect of high soil resistivity is to lower the effective ground plane. A depression between spans such as a canyon or valley has a similar effect. In lowering the effective plane both y/h and H/h decrease which tend to increase the outages.
3. Transverse contour. The slope of the surface of the earth perpendicular to the transmission decreases the effective protective angle by the angle of the slope.
4. Wind. The relative deflection of ground wires and phase conductors under the wind conditions prevailing at the time of a storm may produce a very significant increase in the protective angle.

If a 20-degree tolerance is allowed for such uncertain factors as the above, a recommended value of protective angle at the tower of 30 degrees is obtained. This should prove adequate for all practical values of y/h.

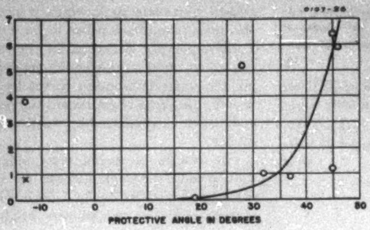


Figure 26. Lightning outage record as a function of protective angle for a number of lines whose average tower-footing resistance is less than ten ohms

Supporting data are given in the tabulation below

Lines	Mile-Years of Records	Average Resistance	Protective Angle	y/h	Distance Between Ground Wire and Conductor at Zero Swing	Outages Per 100 Miles Per Year
8A, 8B	1,016	3.8	−13	0.12	13.0	3.8
25B	840	6.9	19	0.23	21.7	0.07
15F	216	10.0	28	0.11	11.7	5.2
15G, H, I	780	9.6	32	0.10	10.6	1.0
1A	656	7.9	37	0.13	14.8	0.9
23J, K, P R, U, II	2,807	3.8	45	0.12	15.4	7.2
23GG	144	6.9	45	0.13	17.7	1.2
23LL	148	5.0	46	0.13	17.2	5.9
	372	5.0	−13	0.11	11.3	0.8

Appendix

Assume a flat earth surface of infinite extent and a uniform stroke density. If D represents the density of strokes per square mile per year, then from figure 5*b* it can be seen that the strokes that emanate from an element of cloud, dA in width and one mile in length is DdA. However, if A is measured in feet, this quantity becomes $D/5{,}280\ dA$, or $Dh/5{,}280\ d(A/h)$, where h is in feet. If n represents the fraction (not the percentage) of total strokes, as obtained from a distribution curve for a particular configuration and cloud height, that strike both conductors and ground wires, then the total strokes from element dA which strike the line is $Dh/5{,}280$ or $d(A/h)$. The total strokes to the line is then

$$S=\frac{Dh}{5{,}280}\int_{-}^{+} nd\left(\frac{A}{h}\right)$$

$$=\frac{2Dh}{5{,}280}\int_{-}^{+} nd\left(\frac{A}{h}\right)$$

Reduced to strokes per 100 miles this becomes

$$S=\frac{Dh}{26.4}\int_{0}^{+} nd\left(\frac{A}{h}\right)$$

The integral is simply the area under the distribution upon one side of the line.

Bibliography

1. Lightning and Other Transients on Transmission Lines, **F. W. Peek, Jr.** AIEE Transactions, volume 43, October 1924, page 1205.

2. Lightning Protection for Oil Storage Tanks and Reservoirs, **Sorensen, Hamilton, and Hayward.** AIEE Transactions, volume 47, January 1928, pages 164–80.

3. A Study of Lightning Rods and Cages With Special Reference to Oil Tanks, **F. S. Peek, Jr.** AIEE Transactions, volume 45, 1926, pages 1131–44.

4. Dielectric Phenomena in High-Voltage Engineering (a book), **F. W. Peek, Jr.** McGraw-Hill Book Company, Inc., 1929.

5. Recherches sur Modeles Relatives à la Protection des Lignes et des Sousstations Contre les Descharges de la Foudre, **A. M. Zaleski.** CIGRE, Session 1935, bulletin number 317.

6. Recherches de Laboratorie sur les Zones Protegees par des Paradoudres à Lignes Multiples, **A. Akopian.** CIGRE, Session 1937, bulletin number 328.

7. Uber den Schutzwert der Erdsiele, **A. Schwaiger.** *ETZ*, May 13, 1937.

8. Modell versuche uber Blitzeinschlage, **Adolph Matthias.** *ETZ*, 58 Jahrg, Heft 32, August 12, 1937, page 881; August 26, 1937, page 928; September 9, 1937, page 973.

9. Der Schutzraum von Blitzfeng-Vorrichtungen und Seine Ermittlung dur Modell versuche, **A. Matthias** and **Burkhardtsmaier.** *ETZ*, June 8, 1939, page 681; June 15, 1939, page 720.

10. Protection of Transmission Lines Against Lightning; Theory and Calculations, **L. V. Bewley.** *G. E. Review*, April 1937, page 180.

11. Lightning Discharges and Line Protective Measures, **C. L. Fortescue** and **R. N. Conwell.** AIEE Transactions, volume 50, September 1931, page 1090. Also Lightning Reference Book, page 801.

12. Investigations on Lightning Discharges and on the Electrical Field of Thunderstorms, **C. T. R. Wilson.** *Phil. Trans. Roy. Soc.*, Sec. A, volume 221, 1920, page 73.

13. The Distribution of Electricity on Thunderclouds, **Sir George Simpson** and **F. J. Scrace.** *Proceedings* of Royal Society, Sec. A, No. 906, volume 161, August 1937, page 309.

14. The Lightning Discharge (a book), **B. F. Schonland.** The Clarendon Press, Oxford, England.

15. Atmospheric Electricity (a book), **B. F. Schonland.** Methuen & Co., Ltd., London.

16. Lightning to the Empire State Building, **K. B. McEachron.** *Journal* of the Franklin Institute, volume 277, No. 2, February 1939.

17. Meteorological Conditions Associated With Aircraft, Lightning Discharges and Atmospherics, **E. J. Minser.** *Journal Aeronautical Science*, volume 7, No. 2, December 1939.

18. The Development of the Spark Discharge, **T. E. Allibone** and **J. M. Meek.** *Proceedings* of Royal Society, Sec. A, 1938, volume 166, page 97.

19. Mechanism of the Long Spark, **T. E. Allibone.** *Journal of the Institute of Electrical Engineers*, 1938, volume 82, page 513.

20. Effect of Humidity on Impulse Flashover Voltages of Rod Gaps and Insulators, **Y. Ishiguro.** *Institute of Electrical Engineers*, of Japan, volume 3, Number 7, July 1939, page 144.

21. Lightning Investigation on a 220-Kv System, **Edgar Bell.** Electrical Engineering (AIEE Transactions), 1934, pages 1184–94.

22. An Analysis of the Lightning Faulting Characteristics of the 132-Kv Lines of the Victoria Falls and Transvaal Power Company, Limited, **M. C. Rendell** and **H. D. Goff.** Trans. of the South African Institute of Elec. Engr., volume 24, Part XI.

23. Lightning Performance of 110 to 165 Kv Transmission Lines, AIEE Lightning and Insulator Subcommittee. AIEE Transactions, volume 58, 1939 (June section), page 294.

Abnormal Voltage Conditions in Three-Phase Systems Produced by Single-Phase Switching

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IT has been recognized for some time that in three-phase circuits, high voltages may cause damage to equipment following the blowing of a fuse or the nonsimultaneous opening or closing of switch contacts.[1–5] Motors on such circuits may reverse their direction of rotation. Field observations of the reversal of direction of rotation of motors have been reproduced in the laboratory, and the results reported.[6] Failures of lightning arresters, under conditions involving blowing of fuses or the opening of switches, appear to have occurred because of high sustained fundamental - frequency overvoltages above the arrester ratings. Cases of the breakage of a line conductor with resulting abnormal voltages have been noted. Abnormal voltage conditions have been observed on potential transformers.[7] The V.D.E. Standard,[8] in effect since October 1, 1925, points out the danger of using fuses and single-pole disconnecting switches under certain conditions, and states that the breakage of a conductor, where the end on the system side falls to ground, may give sustained voltages to ground on the portion of the conductor severed from the main system as high as $3\sqrt{3}$ times normal line-to-neutral voltage.

With the renewed interest in fuses at high voltages, and the possibility of using single-pole switches, the problem of determining the conditions under which these devices may be safely used in circuits supplying ungrounded transformer banks has attained new importance. This paper gives the results of an investigation to determine the conditions under which high voltages may occur with one and two conductors open, with and without faults, and suggests rules for the application of fuses and single-pole switches. Interpreting this reference to single-pole switches very broadly, one may include any type of switching device, such as single-pole disconnecting switches, oil circuit reclosers, horn gap switches, oil circuit breakers, etc. The interval of time between the closing or opening of the first and last phases may vary widely depending upon the type of switching device and also upon the circuit being energized or de-energized. For example, in cases of fuse operation, one phase or two phases may be open for a relatively long time. On the other hand, in the case of an oil circuit breaker, the interval of time during which one contact would be closed and two opened, for example, would be very short. The time required for the transition to the condition of sustained voltages depends upon the circuit constants and the characteristics of the interrupting device. It may vary from a few cycles to a relatively long period of several seconds. The phenomena to be discussed in this paper may be associated with any interrupting device which has characteristics such that the interval of time between the opening or closing of the first and last phases is of sufficient length to permit the ultimate steady state voltage conditions to be attained.

The phenomena encountered may be summarized as follows: When one conductor of a three-phase circuit supplying an ungrounded, unloaded transformer bank is open, there is a path for currents from the closed conductors through the exciting impedance of the bank and the capacitance between conductors to the open conductor, and thence to ground through the capacitance-to-ground of the open conductor. There is a similar path with two conductors open. When the three-phase system is grounded, the path is completed through the system grounded neutrals; if ungrounded, through the capacitance-to-ground on the system side of the opening. With the inductive reactance of the transformer bank paralleled by capacitance between conductors in series with line capacitance to ground, high voltage to ground at the transformer bank terminal or terminals of the open phase or phases will result for certain ratios of the transformer exciting reactances to the line capacitive reactances. The capacitances of the transformer bank, bushings, and bus structure, with no intervening length of line, may be sufficient to produce high voltages on small banks, such as potential transformers. The magnitude of the overvoltage is a function of the ratio of the line capacitive reactance to the transformer magnetizing reactance; or, approximately, the length of line left connected between the break and the transformer for a given transformer kilovolt-ampere and circuit voltage. The higher the voltage and the smaller the transformer kilovolt-amperes, the more likely are high voltages to occur for a given line length.

The solidly grounded and isolated-neutral systems were studied in detail. Systems with neutrals grounded through resistance or through reactance (including the ground fault neutralizer) were not included as part of this investigation. Single-phase, and three-phase shell-type and core-type transformers, with different primary and secondary connections, and the effect of one and of two open conductors, with and without faults to ground, are included in the study.

Basis of Study

Since the highest voltages are apt to occur with the unloaded or lightly loaded transformer bank, the study was made with the bank unloaded. Voltage conditions were obtained with transformer banks made up of three single-phase units and with three-phase transformers of both the shell-type and core-type, the windings on the line side and also the secondaries being connected alternately in delta and wye for each case.

Figure 1*a* gives a one-line diagram of the system studied, which consists of a three-phase power source supplying a transmission circuit and transformer bank through fuses or single-pole switches. It will be assumed that the three-phase system is large relative to the kilovolt-ampere rating of the bank, and therefore can be represented by an equivalent generator with balanced line-to-line voltages and negligible positive- and negative-sequence impedances. If the system is grounded, the equivalent generator will also have balanced line-to-ground voltages and its zero-sequence impedance will be

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[1] For all numbered references, see list at end of paper.

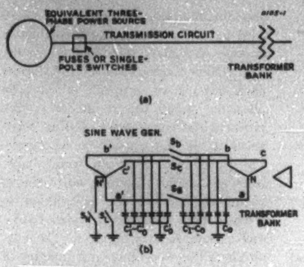


Figure 1 (left). System studied

(*a*)—One-line diagram

(*b*)—Miniature system equivalent circuit

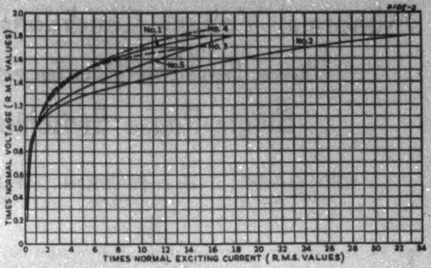


Figure 2. Transformer saturation curves used in making calculations and in the miniature system

1—Miniature system single-phase transformers—135 rms volts normal—1,185 ohms normal magnetizing impedance

2—Assumed curve having more saturation than curve 1. Used in making calculations to determine the effect of this factor

3—Miniature system three-phase core-type transformer—158 rms volts line to line normal wye—840 ohms per phase normal magnetizing impedance

4—Miniature system three-phase shell-type transformer—144 rms volts line to line normal wye—520 ohms per phase normal magnetizing impedance

5—Miniature system special transformers 110 rms volts normal—13,200 ohms normal magnetizing impedance. Single-phase transformers

negligible. Figure 1*b* gives the miniature system[8] equivalent circuit for figure 1*a*, with which the study was made. The impedance of the sine-wave generator is very low relative to the exciting reactance of the unloaded transformer bank and the line capacitive reactances used, so that balanced line-to-line voltages are applied to the circuit at $a'b'c'$ under all conditions. The system becomes a grounded one with balanced line-to-ground voltages when switch S_N is closed. Resistance and reactance are neglected in the transmission circuit since they are insignificant relative to the capacitive reactances for the lengths of line under discussion. C_1' and C_0' represent the positive and zero-sequence capacitances, respectively, on the system side of the fuses or single-pole switches; C_1 and C_0, the positive and zero-sequence capacitances of the circuit supplying the ungrounded, unloaded transformer bank on the transformer side, which in figure 1*b* is shown as wye connected on the line side and delta connected on the load side.

To obtain various degrees of saturation for the miniature system transformer banks, it was only necessary to vary the normal operating voltage level in the miniature system. In obtaining the results herein summarized, the miniature system single-phase transformers were operated at a normal voltage corresponding to the saturation curve shown by curve 1 of figure 2. Curve 2 of figure 2 gives a transformer saturation curve with a slightly higher degree of saturation than 1 of the miniature system. Curve 5, for the specially built miniature system transformers, gives an intermediate value of saturation and because of its higher impedance, made it possible to cover a greater range of system constants. These curves are thought to be fairly typical of power transformer saturation. The saturation curves of the three-phase core-type and shell-type transformers of the miniature system are given by curves 3 and 4, respectively, of figure 2. These units are reproductions to scale of typical power transformers. All saturation curves give the ratio of the applied sinusoidal rms voltage to the exciting rms current, with voltage and current expressed in terms of normal voltage and normal exciting current, respectively.

Cases Studied

I. POWER SOURCE GROUNDED—LOAD TRANSFORMERS UNGROUNDED

A. One phase open—no fault.

1. Single-phase load transformers.

(*a*). Delta-connected on line side, wye- or delta-connected on load side.

(*b*). Wye-connected on line side, delta-connected on load side.

(*c*). Wye-connected on line side, wye-connected on load side.

2. Three-phase load transformers.

(*a*). Delta-connected on line side, wye-connected on load side.

1. Core type.
2. Shell type.

(*b*). Wye-connected on line side, delta-connected on load side.

1. Core type.
2. Shell type.

B. Two phases open—no fault.

Same as for *A*.

II. POWER SOURCE UNGROUNDED—LOAD TRANSFORMERS UNGROUNDED

A. One phase open—no fault.

1. Single-phase load transformers.

(*a*). Delta-connected on line side, wye- or delta-connected on load side.

B. One conductor open—fault-to-ground on system side of open conductor.

1. Single-phase load transformers.

(*a*). Delta-connected on line side, wye- or delta-connected on load side.

(*b*). Wye-connected on line side, delta-connected on load side.

(*c*). Wye-connected on line side, wye-connected on load side.

C. One conductor open—line-to-ground fault on load side of open conductor.

1. Single-phase transformers.

(*a*). Delta-connected on line side, wye- or delta-connected on load side.

Results

Curves showing the voltages to ground of the open phase or phases as a function of the system constants were obtained from the miniature system by varying the capacitances, C_1 and C_0, but keeping their ratio constant. Data were obtained for $C_1=C_0$ and $C_1=2C_0$ for the solidly grounded system. Calculated curves were obtained by the methods given in the appendix.

In order to demonstrate the effect of various factors in producing overvoltages and phase reversal, case 1 will be discussed first in detail. Throughout the following discussion, x_m is the magnetizing reactance of the transformer bank on

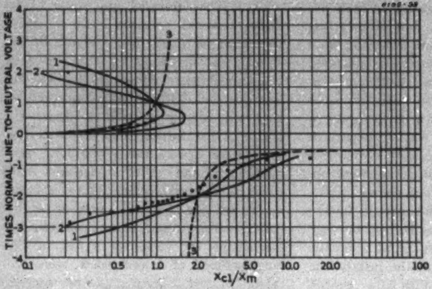


Figure 3a. Steady-state maximum voltages on open phase with one conductor open. No fault on system. Power source solidly grounded

$C_1=C_0$. Transformers delta-connected on line side and delta- or wye-connected on load side

Curve 1—Saturation curve 1, figure 2

Curve 2—Saturation curve 2, figure 2

Curve 3—No saturation

O—Test points—miniature system with saturation curve 1, figure 2

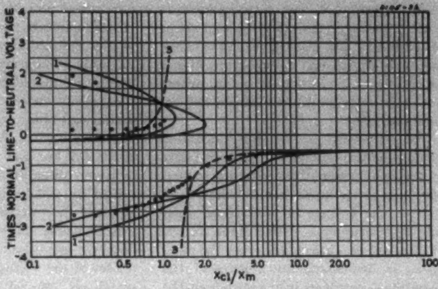


Figure 3b. Steady-state maximum voltages on open phase with one conductor open. No fault on system. Power source solidly grounded

$C_1=2C_0$. Transformers delta-connected on line side and delta- or wye-connected on load side

Curve 1—Saturation curve 1, figure 2

Curve 2—Saturation curve 2, figure 2

Curve 3—No saturation

O—Test points—miniature system with saturation curve 1, figure 2

the load side of the open conductor at normal voltage. In more general terms, it is the equivalent magnetizing reactance of the total transformer kilovolt-amperes on the load side of the fuse or single-pole switch location. Thus, the single-load transformer of figure 1*b* may be considered as the equivalent of several smaller transformers all on the load side of the open conductor. Likewise, the positive- and zero-sequence capacitive reactances, x_{c1} and x_{c0}, respectively, should be interpreted in a more general manner to correspond to total positive and zero-sequence capacitances C_1 and C_0 on the load side of the open conductor. Thus the results assume much broader significance and are much more widely applicable.

I. SOURCE GROUNDED—A. ONE PHASE OPEN—NO FAULT

1. *Single-Phase Transformers*

(*a*). *Delta-Connected on Line Side, Wye- or Delta-Connected on Load Side.* The curves of figure 3*a* give the steady-state maximum peak voltage of the open conductor on the transformer side of the opening in terms of normal line-to-neutral peak voltage with the transformer bank delta-connected on the line side, and wye- or delta-connected on the load side, versus x_{c1}/x_m for $C_0=C_1$. Curves 1, 2, and 3 are calculated by methods given in the appendix. Curve 1 gives calculated voltages using saturation curve 1 of figure 2, which corresponds to the saturation of the miniature transformers.

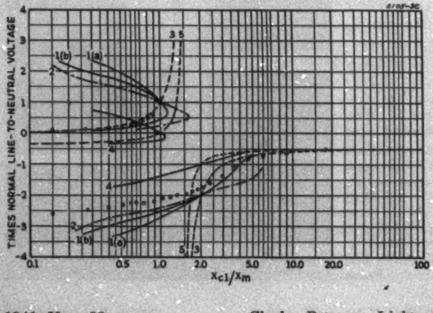


Figure 3c (lower left). Steady-state maximum voltages on open phase with one conductor open. No fault on system. Power source solidly grounded

$C_1=C_0$. Transformers wye-connected on line side, wye- and delta-connected on load side

Curve 1—Saturation curve 1, figure 2. (*a*) Wye-wye, (*b*) wye-delta

Curve 2—Saturation curve 2, figure 2. Wye-wye

Curve 3—No saturation

Curve 4—Neutral displacement saturation curve 1, figure 2. Connection wye-wye

Curve 5—Neutral displacement no saturation

O—Test points—miniature system with saturation curve 1, figure 2. Connection wye-delta

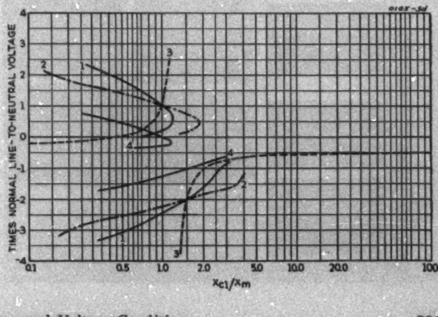


Figure 3d. Calculated steady-state maximum voltages on open phase with one conductor open. No fault on system. Power source solidly grounded

$C_1=2C_0$. Transformers wye-connected on line side and delta-connected on load side

Curve 1—Saturation curve 1, figure 2

Curve 2—Saturation curve 2, figure 2

Curve 3—No saturation

Curve 4—Neutral displacement saturation curve 1, figure 2

Points obtained by test on the miniature system are indicated. The general shape of calculated and test curves is the same, although calculated voltages are higher except for large ratios of x_{c1}/x_m where both calculated and test voltages are less than normal.

The reference vector for calculated voltages is the voltage of the open phase on the system side of the opening, so that with no capacitance in the circuit, or $x_{c1}=\infty$, the voltage on the transformer side of the opening is -0.5 and the voltage at the load is single-phase. As the ratio x_{c1}/x_m is decreased, the voltage of the open phase increases in magnitude in the negative direction and there is phase reversal at the load. At $x_{c1}/x_m=2$ for all calculated curves and about 1.5 for the test curve, the voltage-to-ground of the open phase is -2 and the line-to-line voltages at the load are balanced normal voltages of reverse phase rotation. (For the calculated curves, $V_a=-2$, $V_b=-{}^1/_2-j\sqrt{3}/2$, $V_c=-{}^1/_2+j\sqrt{3}/2$). As x_{c1}/x_m is further decreased the voltage-to-ground of the open phase continues to increase and remains negative on both calculated and test curves; but before $x_{c1}/x_m=1$ is reached, two other steady-state values of voltage are possible as indicated by the calculated curve. Both are positive. Only one of these other indicated values was obtained in test, the lower of the two, until x_{c1}/x_m was decreased to about 0.2 but for values of $x_{c1}/x_m=0.2$ or lower, three different values were obtained by test. Figure 4 shows oscillograms of the voltages and currents for the three possible stable conditions, any one of which may occur, depending upon the phases of the voltages at the instant the switch is opened. With the switch opened at random a number of times, the high negative voltage or the low positive occurred more often than the medium positive voltage.

Curve 2 of figure 3*a* was calculated, using transformer saturation curve 2 of figure 2. Curve 3 is for no saturation.

Figure 3*b* is similar to figure 3*a* except that $C_1=2C_0$. The voltages above normal of figure 3*b* are lower than those of figure 3*a* for the same values of x_{c1}/x_m.

(*b*). *Wye-Connected on Line Side, Delta-Connected on Load Side,* (*c*) *Wye-Connected on Load Side.* Figure 3*c* for $C_1=C_0$ and figure 3*d* for $C_1=2C_0$, with the transformers wye-connected on the line side and delta- or wye-connected on the load side, are similar to figures 3*a* and *b*. Test values from the miniature system for $C_1=C_0$ and a wye-delta connection do not differ materially from those for the transformer delta-connected on the line side except that only two of the stable voltage conditions indicated by calculation for values of x_{c1}/x_m in the neighborhood of 1 and below, were obtained in test. Figure 5 shows oscillograms of the voltages for the two stable conditions obtained at $x_{c1}/x_m=0.15$, with $C_1=C_0$. No tests were made for $C_1=2C_0$.

2. *Three-Phase Transformers*

(*a*). *Delta-Connected on the Line Side, Wye-Connected on the Load Side.* Figure 6*a* gives test values of the voltage-to-ground of the open phase for both core-type and shell-type transformers, delta-connected on the line side and wye-con-

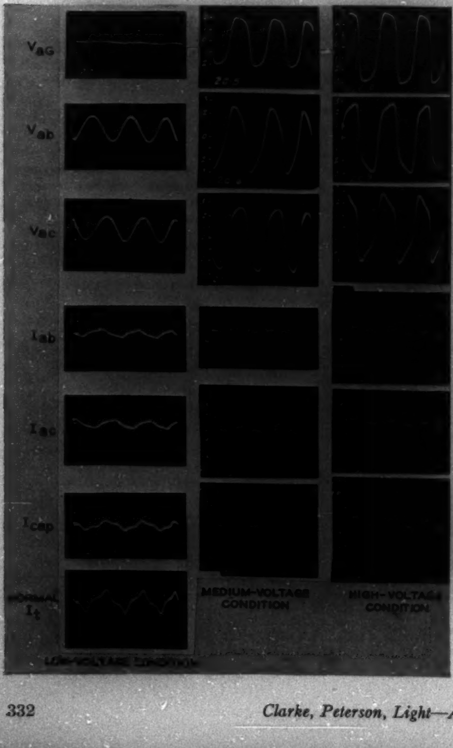


Figure 4 (left). Delta-connected load transformer; $C_0=C_1$; 135 volts line-to-line; $x_{c1}+x_m=0.212$

Calibration: Current—Times normal crest transformer exciting current. Voltage—Times normal line-to-neutral crest voltage

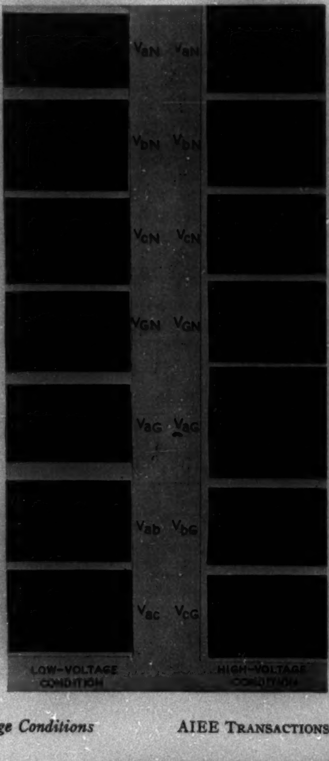


Figure 5. Wye-connected load transformer; $C_0=C_1$; 120 volts line-to-line; $x_{c1}\div x_m=0.15$

Calibration: Times normal line-to-neutral crest voltage

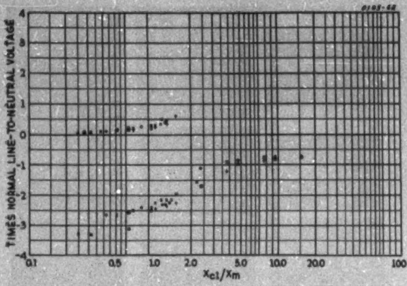


Figure 6a. Steady-state voltages on open phase with one conductor open; no fault on system; power source grounded; $C_1 = C_0$; three-phase load transformer, connected line delta—load wye, ungrounded

Normal line-line voltage = 91 volts for core-type, 83 volts for shell-type

x—Shell-type; middle leg unopened

+—Shell-type; end leg unopened

O—Core-type; middle leg unopened

□—Core-type; end leg unopened

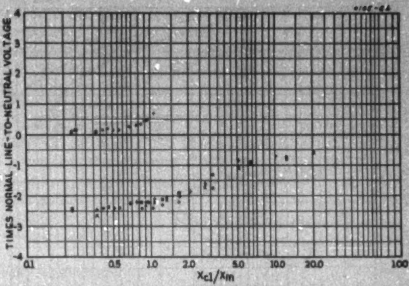


Figure 6b. Steady-state voltages on open phase with one conductor open; no fault on system; power source grounded; $C_1 = C_0$; three-phase load transformer connected line wye—load delta, ungrounded

Normal line-line voltage = 79 volts for core-type, 72 volts for shell-type

O—Core-type; middle leg opened

□—Core-type; end leg opened

x—Shell-type; middle leg opened

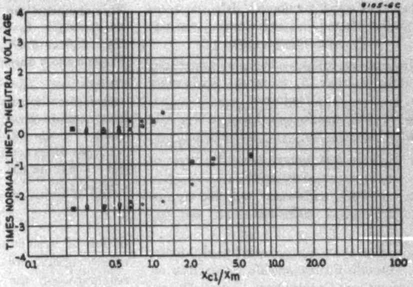
+—Shell-type, end leg opened

Figure 6c (below). Steady-state voltages on open phase with one conductor open; no fault on system; power source grounded; $C_1 = 2C_0$; three-phase load transformer connected line wye—load delta, ungrounded

Normal line-line voltage = 79 volts for core-type

O—Core-type; middle leg opened

□—Core-type; end leg opened



nected on the load side, obtained from the miniature system with saturation curves 3 and 4, respectively, of figure 2 and with $C_1 = C_0$. Calculated values of voltage for saturation curves 3 and 4 of figure 2 are not given, but they would not differ materially from those for saturation curve 1 of figure 2, given by curve 1 of figure 3*a*. In general, the curves of 6*a* agree with those of 3*a*, except that the medium voltage of the three calculated voltage conditions indicated by calculations for values of x_{c1}/x_m in the neighborhood of 1 and below, does not appear. Instead, for x_{c1}/x_m lower than 0.7, higher voltages than indicated by calculations appear, giving three stable voltage conditions. These high voltages have pronounced subharmonics[10,11] and therefore could not be predicted by the methods used. However, they are but little higher than the calculated voltages of curve 1, figure 3*a*.

(*b*). *Wye-Connected on Line Side, Delta-Connected on the Load Side.* Figures 6*b* for $C_1 = C_0$ and 6*c* for $C_1 = 2C_0$ are similar to figure 6*a* except that the transformers are wye-connected on the line side and delta-connected on the load side. Only two stable voltage conditions were obtained for values of x_{c1}/x_m in the vicinity of 1 and below. The voltages of figure 6*b* are substantially the same as those of figure 3*c* for the same values of x_{c1}/x_m.

B. Two Phases Open—No Fault

While numerous factors were investigated in connection with this condition as outlined in the summary of cases studied, detailed results will not be shown. Instead, a composite curve showing the range of values obtained for all of the conditions studied using single-phase transformers is shown in figure 7. The narrow range of values obtained indicates that the transformer connection and its construction (core, shell, or single phase) have little effect on the overvoltages obtained. Figure 8 shows some typical oscillograms for one case with single-phase transformers.

II. Power Source Ungrounded—Load Transformers Ungrounded—

A. One Phase Open—No Fault

1. *Single-Phase Transformers—(a) Delta-Connected on the Line Side, Wye- or Delta-Connected on the Load Side*

Figure 9 gives the calculated voltages-to-ground of the open phase on the load side of the opening in per unit of normal peak line-to-neutral voltage, with x_{c1}/x_m as abscissa and x_{c0}'/x_{c1} as parameter, for $C_1 = C_0$, and the voltage of the open conductor on the system side as reference vector. Test points from the miniature system with saturation curve 1 of figure 2 for various values of x_{c0}'/x_{c1} are indicated. The voltages are independent of x_{c1}'.

B. One Conductor Open—Fault-to-Ground on the System Side of the Open Conductor—Single-Phase Transformers

Figure 10 shows the range of voltages obtained for the various transformer connections. Here again the transformer connections make little difference except for very low ratios of x_{c1}/x_m. The wye-delta connection tends to reduce the overvoltages somewhat in this region.

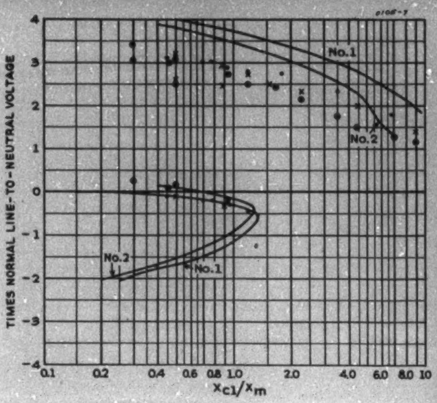


Figure 7. Steady-state voltages on open phase with two conductors open. No fault on system. Power source solidly grounded

Curve 1—Calculated using curve 1, figure 2 for $C_1 = C_0$ and transformers delta-connected on line side and wye-connected on load side

Curve 2—Same but $C_1 = 2C_0$

Test points as follows—Miniature system using saturation curve 1, figure 2. Single-phase transformers

●—$C_1 = C_0$, transformers delta line wye load

◐—$C_1 = 2C_0$, transformers delta line wye load

×—$C_1 = C_0$, transformers wye line wye or delta load

⊗—$C_1 = 2C_0$, transformers wye line wye or delta load

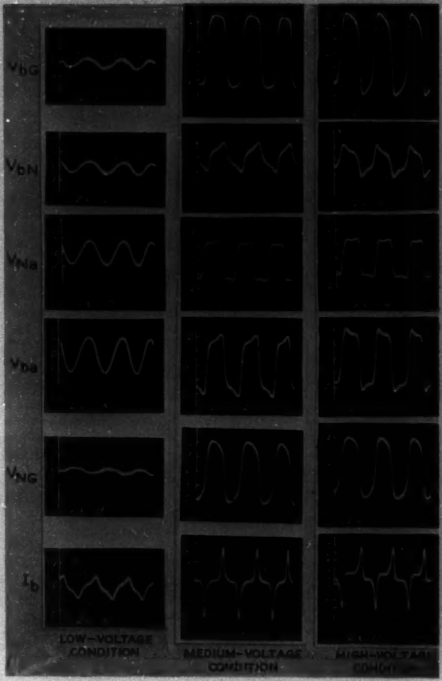


Figure 8 (right). Wye-connected load transformer, $C_0 = C_1$, $x_{c1}/x_m = 0.89$. Phases b and c open

Calibrations: Current—Times normal crest transformer exciting current. Voltage—Times normal line-to-neutral crest voltage

C. One Conductor Open—Line-to-Ground Fault on the Load Side of the Open Conductor—Single-Phase Transformers Delta-Connected on Line Side—Wye- or Delta-Connected on Load Side

Figure 11 shows the voltages obtained for this assumed condition. It should be noted that the severity of this system from the standpoint of overvoltages can be defined for all practical purposes by the ratio of total zero-sequence capacitive reactance to transformer magnetizing reactance. The ratio of $x_{c0'}/x_{c0}$ has relatively little effect.

The basic circuit which produces the high voltages shown for this condition is somewhat more complicated than those circuits investigated in the other cases. The principal elements of the circuit are a capacitance in series with a parallel circuit composed of another capacitance and a saturable reactance. These three elements can be identified with the zero-sequence capacitance on the system side of the open conductor, a parallel combination of system and load-side zero-sequence capacitances, and the magnetizing reactance of the transformer respectively shown in figure 1*b*.

If the total capacitive reactance of the system (capacitance on both sides of the opening) is large, no high voltages are produced. As this reactance is decreased, voltages are increased. Only one voltage value is obtained until the ratio of total zero sequence system capacitive reactance to magnetizing reactance, $x_{(c0+c0')}/x_m$, reaches a value of about 5.0. Below this value, three stable voltage conditions were obtained in some cases. As the capacitive reactance was decreased, increasingly higher maximum voltages were obtained. With a relatively low-loss transformer (curve 1 of figure 2), the high-voltage condition could be obtained for values of $x_{(c0+c0')}/x_m$ down to 0.03, although the low-voltage condition occurred more often. In other words, the higher the voltage, the less would be the probability of reaching that voltage.

With the smaller transformers (curve 5 of figure 2) which had slightly higher losses, the high-voltage condition did not occur for values of $x_{(c0+c0')}/x_m$ below 0.2, approximately. This elimination refers only to the steady-state condition, however. In many cases, long transient conditions were observed, which could be quite hazardous to insulation or connected apparatus. It is important to bear this in mind in connection with the protective criterion offered in the latter part of this paper.

Figure 11 indicates that phenomena such as this can take place in a relatively large percentage of ungrounded systems. Perhaps the occurrence of faults followed by the blowing of fuses may account for much of the heretofore obscure transient phenomena occurring in isolated-neutral systems.

Discussion

The manner of connecting the windings of an ungrounded transformer bank, whether delta-delta, delta-wye, wye-wye, or wye-delta, does not materially affect the magnitudes of the voltages-to-ground obtained on the open phases except in the very high voltage region. The voltages-to-ground of the open phases obtained with three-phase transformers of both core-type and shell-type are approximately the same as those obtained with banks of single-phase units, with the same saturation curve, for corresponding values of x_{c1}/x_m.

The method of calculating sustained voltages-to-ground when one or two conductors of the three-phase circuit supplying an unloaded, ungrounded transformer bank are open, gives values which approximately check tests made on the miniature system. Except where the calculated voltages are less than normal, the method gives values, which, with a few exceptions in the very high voltage region, are too high. The method can therefore be considered to give conservative values.

While test points agree quite well with calculated results, it may be well to point out that for some of the conditions which gave rise to very high voltages, it was possible to obtain slight variations in the wave shape depending upon the initiating conditions. For instance, the high voltage might, for one opening of the switch, be essentially of fundamental frequency. For a second opening of the switch, it might be essentially of the same magnitude, but contain definite subharmonics.[10,11] Some of these subharmonics observed were of very low frequency.

For the ground system or the isolated system effectively grounded through its capacitance to ground, a fault to ground on the open conductor, or conductors, on the system side of the opening, or openings, does not affect the sustained voltages at the transformer bank terminals. A fault-to-ground on the transformer side of the opening reduces the voltage there to zero. The highest voltages are obtained when two conductors are open and there is no fault. Such a condition can exist when a line-to-line fault is cleared by the blowing of two fuses; and also on a circuit controlled by single-pole switches when one phase is closed before the other two.

If the peak value of the sustained voltage-to-ground of the open conductor on the load side of the opening in a solidly grounded system with ungrounded transformer bank is to be kept below 1.73 times normal line-to-neutral peak voltage, the approximate length of line, *l*, between the opening and the transformer bank should not exceed a value which can be approximately determined from the normal voltage of the line and the rated kilovolt-amperes and exciting current of the transformers.

The positive-sequence capacity susceptance of overhead transmission circuits, $2\pi fC_1$, in micromhos per mile at 60 cycles varies from about 5 to 7, depending upon the diameter of the conductors and the spacing between them. Since these, in turn, depend upon voltage, $2\pi fC_1$ at a given voltage and frequency can be estimated with fair accuracy. The following values will be taken as typical at 60 cycles:

Kv	$2\pi fC_1$ Micromhos Per Mile
230–115	5.2
69–34.5	5.5
34.5–13.8	6
Below 13.8	6.5

The zero-sequence capacitance of overhead transmission circuits is less than the positive, but usually greater than one-half the positive.

From figure 7 it is seen that calculated voltages in the region of 1.73 times normal line-to-neutral voltage are greater than test voltages. Allowing for this, and with $C_1 > C_0 > C_1/2$, voltages above 1.73 times normal will probably not occur with transformers of the usual degree of saturation if the ratio x_{c1}/x_m is 6 or greater.

With a criterion of $x_{c1}/x_m = 6$,

$$x_{c1}/x_m = I_m x_{c1} = I_m \frac{10^6}{2\pi fC_1 l} \times \frac{\text{kva}}{\text{kv}^2 10^3} = 6$$

$$\therefore l = I_m \frac{\text{kva } 10^3}{\text{kv}^2} / 6(2\pi fC_1) \qquad (1)$$

where

l = allowable length of line between fuses or single-pole switches and load transformer bank

kva = rated kilovolt-amperes of the transformer bank

Figure 9. Steady-state maximum voltages on open phase on load side with one conductor open. No fault on system. Power source ungrounded

$C_1 = C_0$. Transformers delta-connected on line side and wye-connected on load side

x_{c0}'/x_{c1} = 0 = +
= 0.1 = Δ
= 0.5 = □
= 1.0 = ○
= 10 = x
Test points—Miniature system with saturation curve 1, figure 2

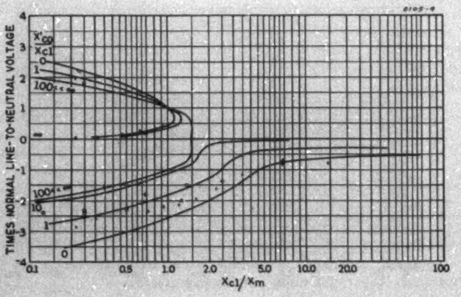


Figure 10. Maximum steady-state voltages to ground with one conductor open. Line-ground fault on source side of open conductors. System neutral isolated. $C_1 = C_0$. These results are independent of C_1' and C_0'. No load on system

Single-phase transformers, delta-connected on line side—delta- or wye- on load side:

Curve 1—Calculated voltages based on saturation curve 1, figure 2

Curve 2—Calculated voltages based on saturation curve 2 of figure 2

Curve 3—Calculated voltages based on no saturation

● Test points—Miniature system saturation curve 1, figure 2

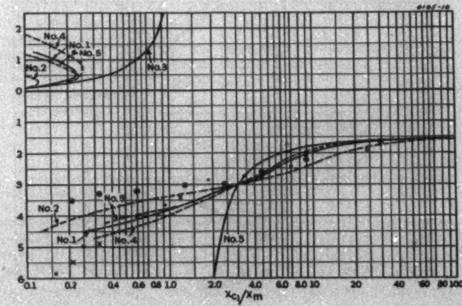
Single-phase transformers, wye-connected on line side:

Curve 4—Calculated voltages based on saturation curve 1 of figure 2 with wye-connected load side

Curve 5—Same as 4 but delta-connected load side

⊙—Test points—miniature system—saturation curve 1, figure 2—delta-connected load side

x—Same but wye-connected load side



kv = normal line-to-line voltage of the circuit in kilovolts

I_m = average per unit rms exciting current at normal voltage

$2\pi f C_1$ = positive-sequence capacitive susceptance of the circuit in micromhos per mile

The transformer exciting current I_m lies between 1 per cent and 7 per cent, with 3.5 per cent not an unusual value. Figure 12, drawn with $I_m = 3.5$ per cent, gives the approximate maximum allowable length of line at various voltages supplying a transformer bank of given kilovolt-ampere rating and 3.5 per cent average rms magnetizing current which can be operated by fuses or single-pole switches without the risk of voltages-to-ground exceeding line-to-line voltages when two conductors are open. This allowable length of line varies directly as the per unit rms exciting current of the bank and inversely as the ratio x_{c1}/x_m selected. For an rms exciting current of 7 per cent, the allowable lengths of line would be doubled; for an exciting current of 1 per cent, they would be only about one-third those of figure 12.

Since for some cases, the maximum allowable length of line is very short, it may be necessary under such conditions to take into account the internal capacitance of the transformer bank itself. Generally this will not have to be done. For cases requiring this refinement, representative values of internal capacitance of power transformers are given in a convenient form by L. V. Bewley.[12] In figure 8 of his paper he gives the capacitance between windings in magnetomotive force per phase which may be considered as the capacitance to ground, as the capacitance of the low-voltage winding to ground is usually larger, while the capacitance of the high-voltage winding to ground is small. The largest values of capacitance given are on the order of 0.01 microfarad which would be equivalent to one mile of line, or less under most conditions. For the higher voltages, the internal capacitance would be equivalent to considerably less than a mile of overhead line. Bushing capacitance, being very small relatively, can be neglected.

In case the transmission circuit contains some cable, its equivalent in terms of miles of overhead line may be used. For that purpose, figure 13 has been prepared which makes it possible to evaluate this equivalent readily. This figure has been reproduced from reference 13.

For the ungrounded system, it is important to note that the criterion for the solidly grounded system is adequately conservative for conditions not involving faults. See figure 9. However, should a permanent fault occur on the system side, the criterion must be changed if voltages are to be limited to maximum values of $\sqrt{3}$ times normal. Figure 10 shows that a minimum ratio of $x_{c1}/x_m = 25.0$ approximately is required. This criterion alone is not sufficient, however. If a permanent fault on the load side is assumed, then the total zero sequence capacitance of the system must be very large or else very small in order to prevent overvoltages. See figure 11. Therefore, in addition to imposing the criterion of $x_{c1}/x_m \geq 25.0$, restrictions must also be imposed on the ratio of $x_{c0'}/x_m$.

From figure 11, the ratio of $x_{(c0+c0')}/x_m$ should be less than approximately 0.04 or greater than a value varying from 4.7 to 25.0 approximately, depending upon the ratio $x_{c0'}/x_{c0}$. In the light of our first restriction on x_{c1}/x_m, this can be conservatively interpreted to impose the restriction that $x_{c0'}/x_m$ must be less than 0.04 or greater than 40.0 approximately.

Figure 12 can be used to interpret the first criterion in terms of total transformer kilovolt-amperes, system voltage, and maximum allowable lengths of line on the transformer side of the open conductor location. This figure is based on a ratio of $x_{c1}/x_m = 6.0$ and since for the isolated system this must be increased to approximately 25.0, it is necessary to divide the miles of line shown by approximately 4.0 in order to conform to the required criterion.

In figure 14 the second criterion, imposing the restrictions on the lengths of line on the system side of the open conductor, is shown. This curve shows that overvoltages may be expected as a result of faults and subsequent fuse blowing for wide ranges of system constants.

In applying these criteria, it should be pointed out that these are approximate values. In some cases it may be necessary to make a more detailed calculation. Exact line configuration, exact magnetizing reactance, and more detailed knowledge concerning the particular transformer saturation characteristics may be necessary. However, the curves serve to indicate in general the range of system constants which can give rise to high

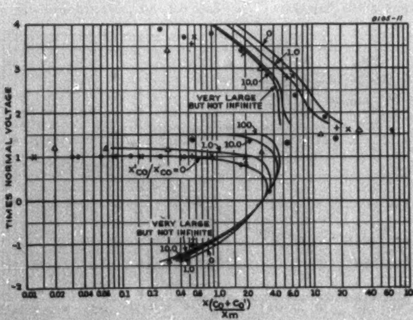


Figure 11. Isolated-neutral-system steady-state voltages on open phase with one conductor open. Fault to ground on load side of open conductor curves calculated using saturation curve 1 of figure 2

⊙—Test points for $x_{c0}'/x_{c0} = 0$ and saturation curve 1, figure 2

Test points—Saturation curve 5, figure 2

×—$x_{c0}'/x_{c0} = 0$

+—$x_{c0}'/x_{c0} = 0.1$

Δ—$x_{c0}'/x_{c0} = 1.0$

⊗—$x_{c0}'/x_{c0} = 10.0$

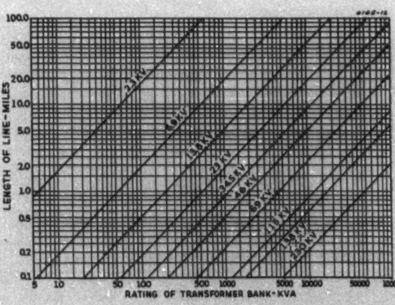


Figure 12. Solidly grounded system—approximate maximum allowable length of line between fuses or single-pole switches and load transformer to limit line-to-ground voltage on open phase to $\sqrt{3}$ times normal, with two conductors open

$x_{c1}/x_m = 6$

$I_m = 3.5$ per cent rms average exciting current

NOTE: Allowable length of line varies directly with rms average exciting current

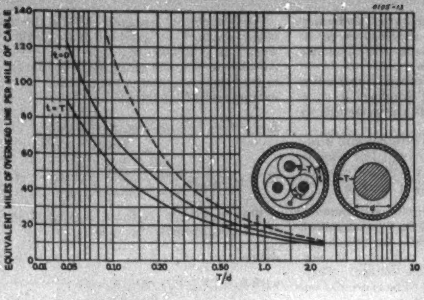


Figure 13. Overhead line capacitance equivalent of cables

Dashed curve—Single conductor

Solid curves—Three conductor

For paper insulated cables assumed dielectric constant 3.8

For varnished cambric or rubber insulation multiply by 1.35

Overhead line equivalent based on positive-sequence capacity susceptance of 6.0 micromhos per mile

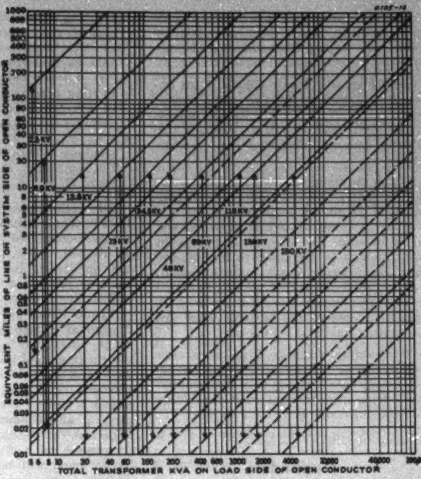


Figure 14. Isolated neutral system—approximate range of equivalent line lengths on system side of fuse or single-pole switch location which may cause voltages greater than $\sqrt{3}$ times normal with one conductor open and a single line-to-ground fault on the load side

x_{c0}'/x_m must be greater than 40.0 or less than 0.04

I_m=3.5 per cent rms average exciting current

NOTE: Allowable length of line varies directly with rms average exciting current

NOTE: This criterion alone is not sufficient for elimination of overvoltages. The criterion of figure 12 must be met also. Maximum permissible miles of line in figure 12 should be divided by 4.0 for the isolated-neutral system

overvoltages in isolated-neutral systems. Under lightly loaded conditions, it would seem that a relatively high percentage of isolated-neutral systems are subject to overvoltages as a result of fuse blowing or single-pole switching under permanent fault conditions.

Obviously, there are other combinations of faults and open conductors possible in isolated systems which might tend to shift somewhat the range of constants causing high voltages. In distribution circuits involving single-phase feeders, a single blown fuse can sometimes cause overvoltages if the system constants are of certain values. Generally, such cases can be interpreted in terms of three-phase constants such that some one of the numerous curves included in this paper can be used directly in determining the overvoltages possible.

The effect of load on the transformers is to reduce the voltages on the open phases obtained with the transformers unloaded. With the transformers loaded and phase a open, V_a, the voltage of the open phase on the transformer side of the opening referred to the voltage on the system side, will have a quadrature component in addition to the positive or negative in-phase component. If the in-phase component is negative and greater than -0.5, V_{a2} will be greater than V_{a1} and there will be phase reversal. Running motors, however, will not slow down if the difference between the positive- and negative-sequence torques is equal to that required to maintain rotation in the positive direction, but if it is less, they will slow down and stop. They will then reverse their direction of rotation if the negative sequence torque is great enough to overcome the positive sequence torque and provide the torque required to start the motors from rest. With a motor running, the positive sequence torque is greater than the negative for equal positive and negative sequence voltages. At standstill, these torques are equal for equal voltages.

The conditions under which loaded motors will reverse their direction of rotation, will depend upon the characteristics of the motors, their loads and the voltages at their terminals, the latter being influenced by the presence of the motors and other loads on the transformers. Since reversal of the direction of rotation has occurred in practice following the opening of a conductor, it is reasonable to assume that the higher the negative voltage produced on the open phase with the transformer unloaded, the more apt is reversal of the direction of rotation of motors to occur.

Conclusions

1. In a three-phase power system, high sustained voltages may in some cases result from the opening of one or two conductors which separates ungrounded transformers from the rest of the system.

2. The transformer connections (wye or delta) and construction (single-phase, or three-phase core- or shell-type) make little difference in the magnitudes of voltage or in the regions in which high voltages are encountered.

3. High overvoltages can be eliminated in grounded systems by grounding the neutrals of the load transformers.

4. The regions within which abnormal voltages are obtained, are considerably extended by the effects of transformer saturation.

5. The regions wherein voltages in solidly grounded systems in excess of rated line-to-line voltages occur, and therefore of importance in the application of lightning arresters, are for the usual cases confined to ratios of x_{c1}/x_m less than three for one conductor open, and less than six for two conductors open.

6. To avoid the possibility of obtaining unduly high abnormal voltages in solidly grounded systems, single-pole switches or fuses should not be used where the length of line, or equivalent capacitance, between the

break and ungrounded transformers supplying the load is greater than that given by equation 1. Figure 12 can be used directly when the rms exciting current is 3.5 per cent.

7. The highest voltages appear in an ungrounded system with the simultaneous occurrence of an open conductor and line-to-ground fault, such as might correspond to the breaking and falling to ground of a line conductor.

8. From the standpoint of open conductors without faults, the criterion given in conclusion 6 is always conservative for ungrounded systems. However, if the criteria are to include the effects of permanent faults, then it is necessary to impose two approximate requirements, namely, x_{c1}/x_m must be greater than 25.0, and x_{c0}'/x_m must be either less than 0.04 or greater than 40.0. Figures 12 and 14 can be used when the rms exciting current is 3.5 per cent.

9. The voltages arising from open conductors can be calculated with reasonable accuracy by consideration only of the fundamental-frequency components of voltage and current as outlined in the appendix.

Appendix

Analytic Determination of the Sustained Voltage to Ground on Open Conductors in a Circuit Supplying an Unloaded Transformer Bank

Using figures 1*b*, the following additional assumptions will be made:

1. The sustained voltages to be determined are substantially sinusoidal voltages of fundamental frequency.

2. The effective exciting reactance of transformer windings to be used in calculating fundamental-frequency voltages can be determined from the saturation curve of the transformer, determined by applying sinusoidal voltages. The exciting reactance varies with the magnitude of the impressed voltage, but at any voltage, it is the ratio of the rms voltage to the rms current.

3. Resistance in the transformers as well as in the transmission circuit can be neglected.

I. POWER SOURCE SOLIDLY GROUNDED

A—Phase a Open, No Fault

Referring to figure 1*b*, switches S_N, S_b, S_c are closed and S_a and S_L open. No voltage is applied to phase *a* of the three-phase circuit supplying the transformer bank. The voltages to ground, V_b and V_c, applied to phases *b* and *c* in per unit of normal line-to-neutral voltage, with the applied voltage of phase *a* as reference vector, are

$$V_b=-{}^1/_2-j\sqrt{3}/2;\ V_c=-{}^1/_2+j\sqrt{3}/2 \quad (2)$$

Let

V_a = voltage to ground of phase *a* on transformer side of opening in per unit of normal line-to-neutral voltage

V_N = voltage to ground of neutral of wye-connected transformer bank in per unit of normal line-to-neutral voltage

x_m = per unit effective transformer exciting reactance of any winding at normal voltage, either wye or delta-connected

x_{ab}, x_{ac}, x_{bc} = per unit effective transformer exciting reactances of windings *ab*, *ac*, *bc* of delta-connected bank corresponding to voltages across them

x_a, x_b, x_c = per unit effective transformer exciting reactances of windings *a*, *b*, *c* of wye-connected bank corresponding to voltages across them

x_{c1}', x_{c0}', x_{c1}, x_{c0} = per unit positive- and zero-sequence system and circuit line capacitive reactances, respectively, corresponding to C_1', C_0', C_1 and C_0 of figure 1*b*

All reactances are in per unit on a common kilovolt-ampere base. Effective reactances of delta-connected transformer windings, as defined above, are expressed in per unit on normal line-to-line voltage. They are to be multiplied by 3 to be expressed on normal line-to-neutral voltage.

V_b and V_c can be resolved into two components of voltage in quadrature with each other, $-{}^1/_2$ and $-j\sqrt{3}$. The former sends current to ground through phases *b* and *c* in parallel in series with phase *a*. The latter produces equal and opposite currents in phases *b* and *c*. Since the circuit is symmetrical about phase *a* and resistance is neglected, the voltages between neutral and terminals *b* and *c* of a wye-connected transformer bank will be equal in magnitude. Therefore the impedances x_b and x_c are equal. In a delta-connected bank, the voltages between *a* and *b* and *a* and *c* will be equal in magnitude, and therefore x_{ab} and x_{ac} are equal. The component $-j\sqrt{3}$ will produce no voltage at terminal *a*. With resistance neglected, V_a can be positive or negative but will have no quadrature component. In a wye-connected bank, the neutral will likewise have no quadrature component of voltage.

Let

I_a = total current flowing from phase *a* on the transformer side of the opening to ground

I_t = current flowing from transformer bank terminal *a*

x_t = effective exciting reactance of the transformer bank to I_t

With delta-connected primary windings and secondary windings either wye- or delta-connected,

$$x_t=3x_{ab}/2 \quad (3)$$

With wye-connected primary and secondary windings,

$$x_t=x_a+x_b/2 \quad (4)$$

With wye-connected primary windings and delta-connected secondary windings, neglecting leakage reactance, the sum of the three voltages to neutral of the wye must be zero.

$$\therefore V_{aN}=2\ (\text{real part of } V_{Nb})$$

If I_Δ is the circulating current in the delta, assumed opposite to I_t in phase *a*,

$$x_a(I_t-I_\Delta)=2x_b(I_t/2+I_\Delta) \text{ and}$$

$$\therefore I_\Delta=I_t(x_a-x_b)/(x_a+2x_b)$$

The voltage drop through the bank, from *b* to *a* is

$$j(I_t/2+I_\Delta)x_b+j(I_t-I_\Delta)x_a=jI_t[(9/2)x_ax_b\div(x_a+2x_b)]$$

$$\therefore x_t=(9/2)x_ax_b/(x_a+2x_b) \quad (5)$$

The capacitive reactance of the three-phase shunt, C_1-C_0, which is in parallel with x_t is $(3/2)x_{c0}x_{c1}/(x_{c0}-x_{c1})$. The two parallel paths are in series with the capacitance C_0 between phase *a* and ground. I_a is independent of C_1' and C_0'.

$$I_a=-0.5/[-jx_{c0}+j3x_tx_{c1}x_{c0}/\{3x_{c1}x_{c0}-2x_t(x_{c0}-x_{c1})\}] \quad (6)$$

$$V_a=-jx_{c0}I_a=\frac{3x_{c0}x_{c1}-2x_t(x_{c0}-x_{c1})}{2x_t(x_{c1}+2x_{c0})-6x_{c0}x_{c1}} \quad (7)$$

With saturation neglected, $x_t=3x_m/2$ in (3), (4), or (5) and (7) becomes

$$\text{with } x_{c0}=x_{c1},\ V_a=x_{c1}/x_m/(3-2x_{c1}/x_m) \quad (8)$$

$$\text{with } x_{c0}=2x_{c1},\ V_a=(2x_{c1}/x_m-1)\div(5-4x_{c1}/x_m) \quad (9)$$

V_a, calculated from (8), is the ordinate of curves 3 of figures 3*a* and 3*c*; calculated from (9), the ordinate of curves 3 of figures 3*b* and 3*d*.

Effect of Transformer Saturation. (*a*). Delta-connected primary windings, delta- or wye-connected secondary windings. Replacing x_t in (7) by $3x_{ab}/2$, and dividing both numerator and denominator by $3x_{ab}x_{c1}$,

$$V_a=[(x_{c1}/x_{ab})(x_{c0}/x_{c1})-(x_{c0}/x_{c1}-1)]\div[(1+2x_{c0}/x_{c1})-(2x_{c1}/x_{ab})(x_{c0}/x_{c1})] \quad (10)$$

$$|V_{ab}|=\frac{1}{\sqrt{3}}\left|\left(-\frac{1}{2}-V_a\right)-j\frac{\sqrt{3}}{2}\right| =\frac{1}{\sqrt{3}}\sqrt{\left(\frac{1}{2}+V_a\right)^2+\frac{3}{4}} \text{ per unit normal line-to-line voltage} \quad (11)$$

The following procedure can be used to obtain a graph of V_a versus x_{c1}/x_m, when the ratio of x_{c0}/x_{c1} is known.

1. Calculate V_a for an assumed ratio of x_{c1}/x_{ab} from (10).

2. Calculate $|V_{ab}|$ corresponding to V_a from (11).

3. From the given transformer saturation curve, calculate x_{ab}/x_m, the ratio of the reactance at voltage $|V_{ab}|$ to the reactance at normal voltage.

4. x_{c1}/x_m corresponding to V_a is the product of x_{c1}/x_{ab} and x_{ab}/x_m.

Illustration: Let $x_{c0}=2x_{c1}$. Performing the four steps of the procedure given above, indicating each by number:

(1) Assuming $x_{c1}/x_{ab}=1.42$, from (10), $V_a=-2.71$

(2) From (11), $|V_{ab}|=\frac{1}{\sqrt{3}}\sqrt{(-2.21)^2+0.75}=1.37$ per unit of normal line-to-line voltage

(3) From saturation curve 2 of figure 2,

$$x_{ab}/x_m=\frac{\text{rms voltage}}{\text{rms exciting current}}=1.37/8.0=0.172$$

(4) $x_{c1}/x_m=1.42\times0.172=0.244$

The point, $V_a=-2.71$, $x_{c1}/x_m=0.244$, is plotted in curve 2 of figure 3*b*. For no saturation, $x_{ab}/x_m=1$ and $x_{c1}/x_m=1.42$. The point $V_a=-2.71$, $x_{c1}/x_m=1.42$ is plotted in curve 3 of figure 3*b*.

(*b*). Wye-connected primary and secondary windings. The current, I_t, through the wye-connected bank produces voltage drops between b and N, and N and a in the ratio $x_b/2$ to x_a. Replacing x_t in (7) by $x_a+x_b/2$ and dividing numerator and denominator by x_{c1}/x_m,

$$V_a=\frac{(3x_{c1}/x_m)(x_{c0}/x_{c1})+(2x_a/x_m+x_b/x_m)(1-x_{c0}/_{c1})}{(2x_a/x_m+x_b/x_m)(1+2x_{c0}/x_{c1})-6(x_{c1}/x_m)(x_{c0}/x_{c1})} \quad (12)$$

The real part of

$$V_{Nb}=\pm\sqrt{|V_{Nb}|^2-3/4} \quad (13)$$

$$V_{aN}/(x_a/x_m)=\pm\frac{2}{(x_b/x_m)}\sqrt{|V_{bN}|^2-3/4} \quad (14)$$

$$V_a=V_b-V_{Nb}-V_{aN}=-\frac{1}{2}-V_{Nb}\text{ real}-V_{aN} \quad (15)$$

A graph of V_a versus x_{c1}/x_m for a given ratio of x_{c0} to x_{c1} can be obtained by the following procedure:

1. Assume $|V_{Nb}|$, the magnitude of the voltage between transformer terminal b and neutral in per unit of normal. From the given transformer saturation curve, obtain x_b/x_m corresponding to $|V_{Nb}|$.

2. Calculate the real part of V_{Nb} from (13).

3. Obtain the ratio $V_{aN}/(x_a/x_m)$ from (14), and from the given transformer saturation curve find values of $|V_{aN}|$ and (x_a+x_m) which are in this ratio. The sign of V_{aN} is the same as that of the real part of V_{Nb}.

4. Calculate V_a from (15).

5. Knowing V_a, calculate x_{c1}/x_m from the implicit equation (12), after replacing x_{c0}/x_{c1} by its given value.

(*c*). Wye-connected primary and delta-connected secondary windings. This case differs from the preceding in that $x_t=(9/2)\times x_ax_b/(x_a+2x_b)$ and

$$V_{aN}=2V_{Nb}\text{ real} \quad (16)$$

The procedure to determine a graph of V_a versus x_{c1}/x_m is similar to that of case (*b*) except that (16) is used to determine V_{aN}, and x_a/x_m corresponding to $V_{aN}|$ is obtained from the transformer saturation curve. V_a is calculated from (15). With V_a determined, the corresponding value of x_{c1}/x_m can be calculated from (7) if x_t is replaced by its value from (5).

B—Phases b and c Open, No fault

Referring to figure 1*b*, switches S_N and S_a are closed and S_b, S_c, and S_L open. Case *B* differs from case *A*, in that the only voltage applied to the circuit supplying the transformer bank is $V_a=1$; and I_a, the current from phase *a*, flows to ground through C_0 of phases *b* and *c* in parallel. Making these two changes, with V_a as reference vector,

$$V_b=V_c=I_a(-jx_{c0}/2)=-\frac{3x_{c1}x_{c0}-2x_t(x_{c0}-x_{c1})}{2x_t(2x_{c1}+x_{c0})-3x_{c1}x_{c0}} \quad (17)$$

(*a*). *Delta-Connected Primary Windings, Delta- or Wye-Connected Secondary Windings.* Replacing x_t in (17) by $3x_{ab}/2$,

$$V_b=V_c=-\frac{(x_{c1}/x_{ab})(x_{c0}/x_{c1})-(x_{c0}/x_{c1}-1)}{(2+x_{c0}/x_{c1})-(x_{c0}/x_{c1})(x_{c1}/x_{ab})} \quad (18)$$

$$|V_{ba}|=\frac{1}{\sqrt{3}}|V_a-V_b|=\frac{1}{\sqrt{3}}|1-V_b|\text{ times normal line-to-line voltage} \quad (19)$$

Using (18) and (19) instead of (10) and (11), respectively, the procedure for determining a graph of V_b or V_c versus x_{c1}/x_m for a given ratio x_{c0}/x_{c1} is analogous to that for one open conductor.

Wye-Connected Primary Windings. Secondary Windings (b) Wye-Connected, (c) Delta-Connected. The procedure for obtaining a graph of V_b or V_c versus x_{c1}/x_m for a known ratio, x_{c0}/x_{c1}, is analogous to that used for one open conductor, except there is no quadrature component of voltage in V_{Nb}, and equation (17) is used instead of (7).

II. Power Source Ungrounded

If the capacitance to ground of the power source with ungrounded neutral is large relative to that of the transmission circuit and bank, the power source can be considered to have its neutral effectively grounded. Case II then becomes case I. When this is not the case, the zero-sequence capacitance of the power source, C_0', in figure 1*b* must be considered. Under the assumption of balanced line-to-line voltages at $a'b'c'$, the positive-sequence capacitance, C_1', has no effect upon the resulting voltages.

The procedure used in case II is analogous to that given for case I. The voltage which sends current to ground is 1.5 times normal line-to-neutral voltage applied in a loop circuit in which the ground can be considered a single point.

References

1. Theory of Abnormal Line-to-Neutral Transformer Voltages, **C. W. LaPierre.** AIEE Transactions, March 1931, page 328.

2. Physical Nature of Neutral Instability, **A. Boyajian and O. P. McCarty.** AIEE Transactions, March 1931, page 317.

3. Inversion of System Resulting From Switching Operations, **A. Boyajian and W. J. Rudge.** *General Electric Review*, July 1931, page 436.

4. Mathematical Analysis of Nonlinear Circuits—Part I, **A. Boyajian.** *General Electric Review*, September 1931, page 531. Part II, *General Electric Review*, December 1931, page 752.

5. Elektrische Schaltvorgänge (a book), **R. Rüdenberg.** Third edition.

6. Talk by **F. A. Hamilton, Jr.**, at a meeting of the AIEE Toronto Section reported in *Electrical News and Engineering*, November 1, 1937.

7. Experiences With Grounded-Neutral, Y-Connected Potential Transformers on Ungrounded Systems, **C. T. Weller.** AIEE Transactions, March 1931, page 299.

8. Leitsätze für den Schutz elektrischer Anlagen gegen Überspannungen, *VDE*, 0145/1933, page 242.

9. An Electric Circuit Transient Analyzer, **H. A. Peterson.** *General Electric Review*, September 1939.

10. Subharmonics in Circuits Containing Iron-Cored Reactors, **Irven Travis and C. N. Weygandt.** AIEE Transactions, 1938 (August section).

11. Analysis of Series Capacitor Application Problems, **J. W. Butler and C. Concordia.** Electrical Engineering (AIEE Transactions), August 1937.

12. Equivalent Circuits of Transformers and Reactors to Switching Surges, **L. V. Bewley.** AIEE Transactions, volume 58, 1939, pages 797–802.

13. Protector Tubes for Power Systems, **H. A. Peterson, W. J. Rudge, A. C. Monteith, and L. R. Ludwig.** AIEE Transactions, volume 59, 1940 (May section).

Arcing Faults in Power Systems

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It has for many years been recognized that voltage surges of considerable magnitude could be produced under certain system conditions by switching operations or by arcing faults. Such voltage surges have been studied in some detail in the past, and various theories have been developed to explain their occurrence and to predict their magnitude. The theories were in most cases sound, but the prediction of surge magnitude was on a rather insecure basis.

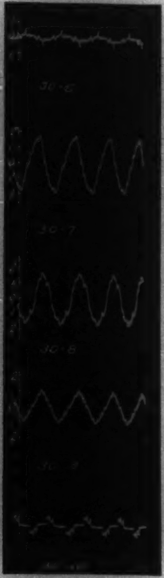


Figure 1. Sustained arcing ground (type 1) on reactance-grounded system

Within the past few years the study of the effect of switching operations in electric circuits has been greatly facilitated by the development of miniature circuits together with equipment for producing regularly recurrent switching and for indicating the resultant surge voltages and currents. This development has made it possible to re-examine on a more exact and physically consistent basis the various theories of intermittent switching and arcing, and to extend very much the range of possibilities investigated. Two recent papers[1,2] have presented preliminary results obtained by this new method of attack on the problem.

Reference 1 presented data on a particular form of arc restriking over a range of system parameters, and a discussion of two other forms of restriking. Reference 2 presented data on four types of arc restriking for a particular system. The present paper extends the work begun in these two references, and presents:

(*a*). Data on several types of arc restriking over a considerable range of system parameters,

(*b*). Data on the effects of various possible sequences of restriking,

(*c*). Data on the effects of various changes in system parameters and in system configuration,

(*d*). Data on the effects of fault and system resistances, and

(*e*). A discussion of the general effect of various methods of system neutral grounding on voltage surges, based on the studies which have been made.

Throughout this study the circuit-interrupting device enters only in so far as its assumed dielectric recovery characteristic determines the point at which restriking takes place, the resulting recovery voltage being otherwise completely determined by the circuit characteristics. Certain types of interrupting devices may modify this resultant recovery characteristic, and therefore the form and magnitude of the surge voltage, considerably.

The miniature circuit used in obtaining most of the data presented is described in reference 5.

Results

(*A*) Five types of arc restriking were studied, and are discussed in turn below.

Type 1. Continuously sustained arc restriking of one phase to ground at the voltage level which results in the highest voltages on the unfaulted phases, the restriking voltage being the same for either polarity. Such restriking may approximate the arcing conditions over an insulator string in the open air, wherein the restriking voltage might wander more or less slowly over the range which can be sustained, as the arc path wanders. In general, the highest restriking voltage which can be sustained gives the highest voltages on the unfaulted phases.

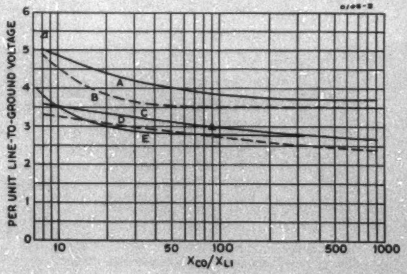
Figure 1 shows the phase and neutral voltages to ground, together with the fault current, for one case studied. In this case successive restriking of alternating polarity took place at half-cycle intervals, while in some of the other cases studied the arc restruck two or three times with the same polarity in each half cycle. In all cases restriking took place whenever the recovery voltage reached the predetermined restrike voltage level, regardless of polarity.

Figures 2–6 give the maximum voltages appearing on either unfaulted phase for restriking of type 1. Curves *C* and *D* of figure 2, for systems with isolated neutral, may be compared with curve *E* to show that these sustained voltages are of the same order of magnitude as the transient voltages appearing on applica-

Figure 2. Maximum sustained sound phase voltage for arcing ground on isolated neutral system of figure 16

A—Type 2, $C_1=C_0$
B—Type 2, $C_1=2C_0$
C—Type 1, $C_1=C_0$
D—Type 1, $C_1=2C_0$
E—Type 1 or 2, application of solid fault, $C_1=C_0$

Points △, 100 miles of artificial line



Paper 40-106, recommended by the AIEE committee on power transmission and distribution, and presented at the AIEE summer convention, Swampscott, Mass., June 24–28, 1940. Manuscript submitted October 24, 1939; made available for preprinting May 10, 1940.

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[1] For all numbered references see list at end of paper.

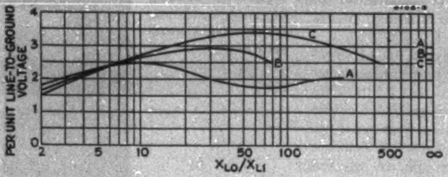


Figure 3. Maximum sustained sound phase voltage for arcing ground of type 1—reactance-grounded system of figure 16—$C_1=C_0$

A—$X_{c0}/X_{l1}=90$
B—$X_{c0}/X_{l1}=280$
C—$X_{c0}/X_{l1}=1,120$
(Ground fault neutralizer at $X_{L0}/X_{L1}=X_{c0}/X_{L1}$)

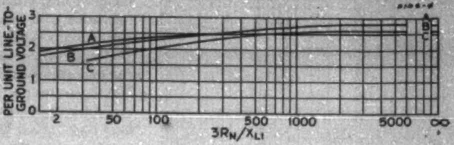


Figure 4. Maximum sustained sound phase voltage for arcing ground of type 1—resistance-grounded system of figure 16—$C_1=C_0$

A—$X_{c0}/X_{l1}=90$
B—$X_{c0}/X_{l1}=280$
C—$X_{c0}/X_{l1}=1,120$

tion of a solid fault. Figure 5, for reactance-grounded systems with positive-sequence capacitance C_1 equal to twice the zero-sequence capacitance C_0, shows that for the smaller ratios of zero-sequence reactance X_{l0} to positive sequence reactance X_{l1}, ($X_{l0}/X_{l1}<5$), the voltage on the faulted phase is higher than those on the unfaulted phases. Figures 3–6 indicate that for this type of restriking (*a*) the voltages appearing on systems grounded through reactance are usually greater than those on systems grounded through resistance of the same ohmic value except near the ground-fault-neutralizer reactance, (*b*) the voltages are greater with $C_1=C_0$ than with $C_1=2C_0$, and (*c*) the voltages on resistance-grounded systems are more or less independent of the system natural frequencies in the range studied.

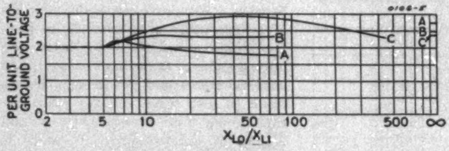


Figure 5. Maximum sustained sound phase voltage for arcing ground of type 1—reactance-grounded system of figure 16—$C_1=2C_0$

A—$X_{c0}/X_{l1}=90$
B—$X_{c0}/X_{l1}=280$
C—$X_{c0}/X_{l1}=1,120$
Dashed line—voltage on faulted phase
(Ground fault neutralizer at $X_{L0}/X_{L1}=X_{c0}/X_{L1}$)

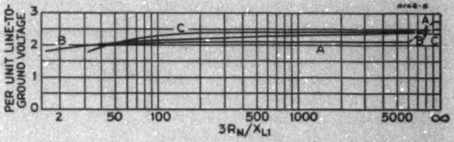


Figure 6. Maximum sustained sound phase voltage for arcing ground of type 1—resistance-grounded system of figure 16—$C_1=2C_0$

A—$X_{c0}/X_{l1}=90$
B—$X_{c0}/X_{l1}=280$
C—$X_{c0}/X_{l1}=1,120$

Type 2. Continuously sustained arc restriking at the voltage levels which result in the highest voltages on the unfaulted phases, without regard to the equality of the restriking voltage of either polarity. This form of restriking has been studied as a convenient representation of possible irregularities in the restriking characteristics. That is, as this particular form of irregularity either increases or does not affect the maximum attainable voltages, there is a basis for concluding that transient variations in the restriking characteristics either are or are not important respectively. Comparison of curves *A* and *B* with curves *C* and *D* in figure 2 (isolated-neutral system) shows a considerable increase in voltage due to inequality of the negative and positive restriking voltage.

The data of figures 2–6 were obtained with an arcing circuit having an inherent dielectric strength recovery characteristic given approximately by the relation:

$$\text{Breakdown voltage} = V(1-e^{-t/T})$$

where

V = maximum breakdown voltage
t = time in microseconds
T = 2,400 microseconds

Thus, if successive restriking occurred several times in a half cycle the restriking voltages of the various restrikes were not all exactly alike, depending on the length of the interval from a current zero to the subsequent restrike. Several check points were taken with the time constant T made very small so that the restriking voltage was truly constant, and little difference could be observed in the magnitudes of the unfaulted phase voltage.

Type 3. Sustained arc restriking of one phase to ground at regular time intervals, in this case at half-cycle intervals. Data on this form of restriking are presented as an indication of what may occur with extremely random, rapid, and large variations in the restriking or flashover voltage. Such restriking is presumably unlikely to occur in an actual system, but could occur under unusual circumstances. The data are included here principally to show that the form of the restriking characteristic, as well as the circuit characteristics, is very important, particularly for systems with isolated neutral. (See figures 7 and 8 for the results obtained. Figure 9 illustrates the building up process leading up to the ultimate sustained maximum voltages of this type.)

Type 4. Transient restriking after interruption of a solid line-to-ground fault. This approximates conditions which may arise on interruption of a faulted conductor by a circuit-interrupting device as described in reference 1. (See figures 10 and 11.)

The sequence of switching operations in figures 10 and 11 is as follows. First, the fundamental-frequency fault current flowing in the arc of the circuit breaker is interrupted at a current zero. The normal recovery voltage then appears across the switch contacts and the arc restrikes at some point on this recovery voltage wave. The consequent restrike current is interrupted at its first current zero, a recovery voltage again appears, restriking takes place a second time, the restrike current is again interrupted at its first cur-

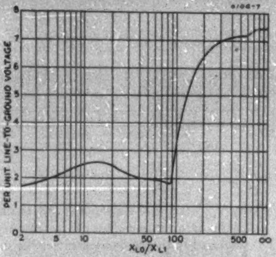


Figure 7. Maximum sustained sound phase voltage for arcing ground of type 3—reactance-grounded system—100-mile line

$C_1=1.6C_0$—0.5 ohm/mile in line neutral—$X_{c0}/X_{l1}=89$
(Ground fault neutralizer at $X_{L0}/X_{L1}=X_{c0}/X_{L1}$)

rent zero, and the final recovery voltage appears. The points at which the restrikes occur are set by trial so as to give the maximum voltage on any phase to ground.

Type 5. Figures 12, 13, and 14 are for the same general type of restriking considered in figures 10 and 11, with the exception that the fault is initially cleared at its first current zero rather than after transient natural frequency oscillations have died out, and illustrate the fact that for circuits of low natural frequency (that is, large inductance and/or capacitance) the most probable manner in which restriking takes place tends to become rather less obvious and thus more difficult to specify, and is moreover increasingly important. For example, one might take as a guide to the selection of a restriking sequence the principle that the dielectric recovery characteristic of the interrupting device never decreases but always either increases or at least remains constant as a function of time. Then restriking could never occur as in figure 13 just after a crest of any oscillatory component of the recovery voltage (that is, when the voltage is decreasing in magnitude), but could occur either before or at a crest as in figure 12. On the other hand, restriking would not necessarily have to occur at or before such a crest, if a later higher crest of either polarity provided a suitable opportunity. It must not be inferred that a decrease of effective voltage breakdown strength with time after a circuit interruption is impossible. In fact, there have been some indications that such action may occur in circuit breakers, especially if their construction and operation allows the dielectric strength to decrease momentarily at any point (for example, by the formation of a bubble of ionized gas) as the contacts part.

Comparison of figures 12, 13, 14, shows the importance of the restriking sequence and illustrates the facts that (*a*) the highest voltages require in general a rather long time (about three or four cycles) to build up, and (*b*) a monotonic dielectric recovery characteristic (figure 12) imposes an appreciable limitation on the magnitude of the voltage surges attainable. (Compare also figures 2–6 with figures 7 and 8 to illustrate this point). It is of interest to compare these results with those of figure 12 (for the one line case) of reference 2, as the systems studied in figures 12–14 of the present paper were very similar, the principal difference being a more exact representation of the line. (See figure 15.)

For most of the systems and types of arcing studied in this paper the maximum surge voltages occur with isolated-neutral systems, minimum points occur with systems grounded either solidly or through ground fault neutralizers, and a second maximum point occurs at a value of neutral reactance equal to about:

(*a*). One-ninth of the ground-fault-neutralizer value for arcing of types 1 and 2,

(*b*). One-sixth of the ground-fault-neutralizer value for arcing of type 3, and

(*c*). One-fiftieth of the ground-fault-neutralizer value for arcing of types 4 and 5.

This second maximum point will occur at greater fractions of the ground-fault-neutralizer value for systems having very low ratios of X_{c0}/X_{l1} (that is, low natural frequencies), for example, in figure 12, reference 2 it occurred at about one-third. In this case, any of the smaller fractions listed as *a*, *b*, *c* above correspond to ratios of zero-sequence reactance X_{l0} to positive-sequence reactance X_{l1} below the critical ratios of figure 5, reference 1, and consequently would not be expected to result in high surge voltages, while the fraction one-third corresponds to a ratio X_{l0}/X_{l1} of approximately 16, which is above the critical ratio.

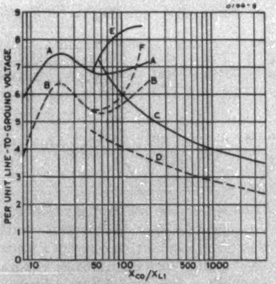
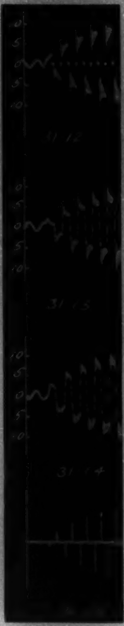


Figure 8. Maximum sustained voltage for arcing ground of type 3—isolated-neutral system of figure 16—$C_1=1.6C_0$

A—Sound phase voltage, 100-mile line
B—Neutral voltage 100-mile line
C—Sound phase voltage, lumped capacitance
D—Neutral voltage, lumped capacitance
E—Sound phase voltage, decreasing line length with X_{l1} constant
F—Neutral voltage, decreasing line length with X_{l1} constant

Figure 9. Build-up of arcing ground (type 3) on isolated-neutral system



(*B*) In addition to the comparison of the effects of several types of arc restriking, the figures also show the effects of various factors in the system configuration as follows:

1. Distribution of system capacitance. The curves of figures 2–6 are for a system as shown in figure 16. Figure 2, for a system with isolated neutral, shows also voltages obtained when the lumped capacitance of figure 16 is replaced by a 100-mile transmission line. It is seen that for a rather large source reactance ($X_{c0}/X_{l1}=8.3$) the surge voltages with the line are greater than with lumped capacitance, while for a smaller source reactance ($X_{c0}/X_{l1}=90$) the reverse is true. (The ratio of zero-sequence

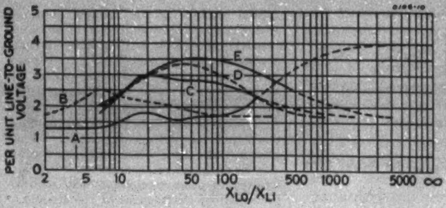


Figure 10. Maximum phase-to-ground voltage for restriking of type 4—reactance-grounded system of figure 16—$C_1 = 1.6C_0$

A—$X_{c0}/X_{l1} = 90$, 100-mile line
B—$X_{c0}/X_{l1} = 300$, 30-mile line
C—$X_{c0}/X_{l1} = 900$, 10-mile line
D—$X_{c0}/X_{l1} = 1{,}800$, lumped capacitance
E—$X_{c0}/X_{l1} = 3{,}900$, lumped capacitance
(Ground fault neutralizer at $X_{L0}/X_{L1} = X_{c0}/X_{L1}$)

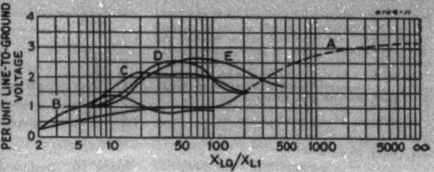


Figure 11. Maximum neutral-to-ground voltage for restriking of type 4—reactance-grounded system of figure 16—$C_1 = 1.6C_0$

A—$X_{c0}/X_{l1} = 90$, 100-mile line
B—$X_{c0}/X_{l1} = 300$, 30-mile line
C—$X_{c0}/X_{l1} = 900$, 10-mile line
D—$X_{c0}/X_{l1} = 1{,}800$, lumped capacitance
E—$X_{c0}/X_{l1} = 3{,}900$, lumped capacitance
(Ground fault neutralizer at $X_{L0}/X_{L1} = X_{c0}/X_{L1}$)

capacitive reactance to positive-sequence inductive reactance was determined without considering the line inductance.) These results suggest that X_{c0}/X_{l1} and C_1/C_0 are not the only factors affecting the phenomena and that therefore an attempt to generalize very much from results obtained from a study of restriking in a simple lumped constant system may sometimes lead to erroneous results. While the trend of the curves of voltage as a function of X_{c0}/X_{l1} given in figure 2 appears to be correct, the maximum voltages possible in an actual system may vary considerably on either side of these curves, depending on system connections.

3. Ratio of positive-sequence capacitance C_1 to zero-sequence capacitance C_0. Figure 2 and comparison of figures 3 and 5 and figures 4 and 6 show that higher voltages are in general obtained for smaller ratios of C_1/C_0. From a different point of view, this confirms the conclusion of reference 1 that systems of lower C_1/C_0 require in general lower values of neutral impedance as far as transient voltage surges are concerned.

It must be pointed out that the effective ratio C_1/C_0 may be less than 1, not only for generators as mentioned in reference 1, but also for transmission lines and systems in general. That is, there is presented to the fault an effective ratio C_1/C_0 which depends on its location along the line and also on the magnitude of the neutral impedance. For example, for a fault at one end of a single unloaded line supplied at the other end from a power source whose reactance is small compared to that of the line, one might expect very roughly on the basis of an equivalent π line an effective ratio C_1/C_0 of about half of 1.6, or 0.8, if the source is ungrounded. This ratio is obtained as follows. The ratio C_1/C_0 for a transmission line as ordinarily defined neglecting line inductance is about 1.6. However, in the positive-sequence circuit viewed from the fault, half of the total positive-sequence capacitance of the equivalent π line is shunted at the source end by the low source reactance, and so may not be very effective as far as disturbances produced by the fault are concerned. On the other hand, in the zero-sequence circuit there is no inductance shunting the part of the capacitance at the source end, if the neutral is isolated. Thus the total zero-sequence capacitance may be effective. If the source is grounded through an impedance the effective capacitance ratio is correspondingly increased, since the neutral impedance is now in shunt with the line capacitance at the source end in the zero-sequence circuit. These considerations bring to light two points:

(*a*). While it is true that the system performance is completely determined by the positive, negative, and zero-sequence networks viewed from the point of fault, one must be cautious in the interpretation and use of the conclusion which apparently follows, that it makes no difference whether, for example, a high ratio of X_{l0}/X_{l1} results from inherent properties of the generating network or from an impedance inserted in the neutral, since insertion of such an impedance may change the effective ratio C_1/C_0 for a given system.

(*b*). The effective capacitance ratio and the capacitance distribution are in general intimately related,

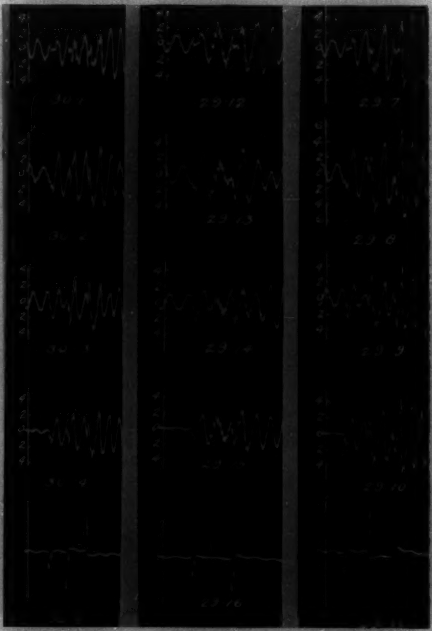


Figure 12 Figure 13 Figure 14

Figures 12, 13, 14. Various restriking sequences (type 5) following interruption of transient line-to-ground fault current—reactance-grounded system of figure 15

so that it may not be possible to specify the probable transient performance by merely stating an over-all system capacitance ratio and noting whether the system capacitance should be classed as lumped or distributed. Instead the actual system configuration may have to be described, and in some cases studied by miniature circuit representation, to obtain a good estimate of its performance.

The voltages of figures 2–8, 10, and 11 may thus be exceeded for systems with the effective value of C_1 less than C_0. The greatest differences may be expected with isolated-neutral systems. Voltages as high as 16 times normal have been obtained for certain system configurations with restriking of type 3 and with $X_{c0}/X_{l1}2\cong 00$. (Compare figure 8, where $C_1/C_0=1.6$ and the corresponding voltage magnitude is 5.2 times normal.) In the limit one approaches the condition of negligible effective C_1 for a "... fault supplied from a rather extensive system through a small and more or less concentrated single reactance .... This reactance may be a transformer or reactor, or even a short length of transmission line or cable".[1] The qualifying words "small" and "short" should here be interpreted as indicating (*a*) a sufficiently high natural frequency with the fault applied, and (*b*) a comparison to the reactance beyond (from the fault) the principal system capacitance. In case of line-to-line faults this latter reactance is the system negative sequence reactance and is seldom large; in case of ground faults, it is the reactance to ground and in the isolated neutral case is infinitely large. Perhaps cable should not be included in the quotation above, because of its usual high ratio of resistance to reactance.

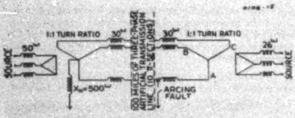


Figure 15. System diagram for figures 12, 13, 14

This discussion of capacitance ratio would be incomplete without mention of the case of a small system, as for example an industrial plant, supplied through a transformer from a large power system. Here the effective ratio of C_1/C_0 may be considerably greater than unity for faults near the supply bus if the supply transformer reactance is small compared to the power system reactance, and thus the allowable neutral reactance will be high. Faults sufficiently far out along feeders so that the supply system reactance becomes small compared to the transformer and feeder reactance revert to the case of C_1/C_0 near unity and therefore require consideration of a proper neutral reactance as indicated in reference 1.

3. Fault resistance. Figure 17 shows the effect of fault resistance in reducing overvoltage caused by arcing grounds of type 2. Figure 18 similarly shows the effect for restriking of type 4. Both figures are for systems with isolated neutral.

The data of figure 17 are for the system of figure 16. A similar study in which 100 miles of artificial transmission line was substituted for the lumped capacitance of figure 16 has shown that fault resistance may at first slightly increase the voltages if:

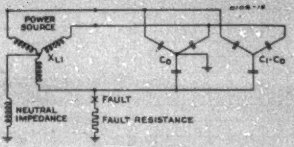


Figure 16. System diagram for figures 1–11 and 17–20

(*a*). X_{c0}/X_{l1} is sufficiently large, for example, 90, and

(*b*). the line has sufficient resistance, for example, $R_1/X_1\cong 0.37$ and $R_0/X_0\cong 1.1$ for the line.

However, a high ratio of X_{c0}/X_{l1} and high line resistance in themselves reduce the voltages, and the voltage with a high line resistance is always less than that with a low resistance.

4. Nonlinear resistor in shunt with neutral reactor. It has been pointed out[1] that resistance in shunt with neutral reactance is beneficial as far as surges resulting from restriking of type 4 are concerned, and that a resistance sufficiently small to produce critical damping of the ground circuit is more than sufficient to prevent successive restriking. It is evident that the shunt resistor can be a nonlinear element such as Thyrite, and figure 19 shows the effect of the degree *n* of nonlinearity on the amount of resistance required. The ratio of required resistance *R* at normal leg voltage to critical damping resistance R_c for a system with high neutral reactance X_{ln} and high natural frequencies is shown to increase as the nonlinearity increases.

5. A gapped resistor such as a lightning arrester can also be used. Figure 20 shows a typical example of the voltages obtained with a reactance-grounded system and with restriking of type 4, as affected by the gap spark-over setting and Thyrite resistance in series with the gap. The ground circuit critical damping value is indicated on the abscissa as $R_c\cong 900$ ohms.

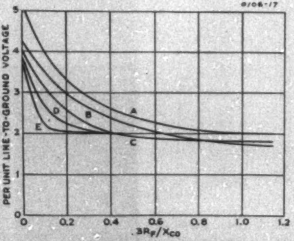


Figure 17. Effect of fault resistance R_F on maximum sound phase voltage for arcing ground of type 2—isolated-neutral system of figure 16—$C_1=C_0$

A—$X_{c0}/X_{l1}=8.2$
B—$X_{c0}/X_{l1}=29$
C—$X_{c0}/X_{l1}=89$
D—$X_{c0}/X_{l1}=290$
E—$X_{c0}/X_{l1}=890$

An arrester is seen to be effective in holding down the surge voltage magnitude. Thus, even in those cases where the surge energy may be sufficient to cause arrester damage, damage to other equipment may still be prevented by the presence of the arrester.

Discussion of the Effects of Neutral Impedance

The results of this and previous papers show that there are many factors which affect the choice of a proper method of grounding, and that all of them must be carefully considered if the best over-all method is to be obtained. These remarks are concerned only with the effects of neutral impedance on voltage surges produced by switching or by arcing faults. Several kinds of switching phenomena may occur, each kind being somewhat differently affected by the method of grounding. For one kind a certain critical value of neutral reactance may be specified, above which it is undesirable to go, for this kind also, resistance grounding is much better and permits much greater impedances to be used. For a second kind, the lower the value of neutral impedance the better, regardless of whether the impedance is reactance or resistance, that is, it is only the magnitude of the neutral impedance which is of importance. For a third kind of switching surge, the surges may increase equally in severity with either neutral reactance or resistance up to a certain point, above which an increase in neutral reactance decreases the severity of the surge while an increase in neutral resistance continues to increase the severity of the surge. A fourth kind is affected very little by neutral impedance.

The first kind of voltage surge includes types 4 and 5 of this paper. Studies have indicated[1] that it is possible to prevent the building up of excessive voltages due to such restriking by:

1. Keeping the reactance under the critical value shown by figure 5 of reference 1,
2. Using a ground fault neutralizer for the neutral reactor,
3. Shunting the reactor by a suitable resistor.

Even if the neutral reactance is obtained by means of a small grounding transformer rather than by a neutral reactor, it may still be possible to use an effective shunt resistor in those cases

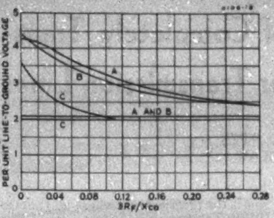


Figure 18. Effect of fault resistance R_F on maximum sound phase voltage for restriking of type 4—isolated-neutral system of figure 16

A—$X_{c0}/X_{l1}=89$, lumped capacitance
B—$X_{c0}/X_{l1}=89$, 100-mile line
C—$X_{c0}/X_{l1}=890$, lumped capacitance

wherein a nearby large transformer is in operation, by putting a resistor in the neutral-to-ground circuit of the large transformer. If the reactance of the large transformer is sufficiently small compared to that of the small grounding transformer, the resistor will be effectively in parallel with the neutral reactance in the zero-sequence circuit. Any case for which such a scheme is proposed should be studied to find out in the first place whether or not it is feasible when the transformer sizes and system connections are considered, and in the second place the required resistance.

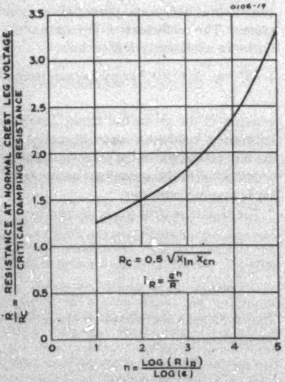


Figure 19. Effect of degree n of nonlinearity on resistance required in shunt with neutral reactor to prevent successive restriking of type 4—$C_1=C_0$

R = shunt resistance at normal leg voltage
R_c = critical damping resistance for ground circuit
X_{ln} = inductive reactance of ground circuit $=X_{l0}/3$ for a three-phase system
X_{cn} = capacitive reactance of ground circuit $=X_{c0}/3$ for a three-phase system
i_R, e = resistor current and voltage

The second kind of voltage surge may arise from restriking of the arc in the unfaulted phase or phases, during the interruption of a double-line-to-ground fault or a solid single-line-to-ground fault, if the unfaulted phase carries principally a line charging current or other capacitance load.[2] For this condition there is no particular critical value of impedance, and studies similar to those of reference 2 have shown very little difference between the performance of resistance or reactance grounding.

The third kind of voltage surge may arise from (*a*) restriking in the unfaulted phases during the interruption of an arcing single-line-to-ground fault if the unfaulted phase involved carries principally a capacitance current, or (*b*) from the arcing ground itself. The points of minimum voltage occur with solid grounding and with a ground fault neutralizer. The minimum at the ground-fault-neutralizer value in the voltage caused by restriking in the unfaulted phases occurs because there should be very little occasion to interrupt an arcing single-line-to-ground fault in systems equipped with ground fault neutralizers.

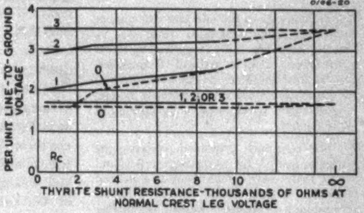


Figure 20. Effect of gapped Thyrite in shunt with neutral reactor on maximum sound phase voltage for restriking of type 4—reactance-grounded system of figure 16

$X_{l0}=557$ ohms
$X_{l1}=13.9$ ohms
$X_{c0}=53{,}000$ ohms
$X_{c1}=33{,}100$ ohms
$X_{l0}/X_{l1}=40$ ohms
$X_{c0}/X_{l1}=3{,}820$ ohms
R_c = ground circuit critical damping resistance
Upper four curves—two restrikes
Lower two curves—no restrikes
Numbers on curves are per unit breakdown voltage of gap

The fourth kind may arise in the interruption of the line charging current of an unfaulted line. Here also, if a rather large portion of the total system capacitance is being switched off, the surges may be somewhat reduced when a ground fault neutralizer is used.[2]

In these last two kinds of switching surges, the effects of neutral impedance in general become smaller as the amount of capacitance being switched off becomes a smaller portion of the total system capacitance.

Other factors which should be considered in evaluating the importance of these surges are the energy content and relative probability of occurrence of the various kinds. For example, if a high voltage appearing on a line due to any of the last three kinds causes a lightning arrester to function, damage to the arrester will not result unless the surge energy (which may be principally determined by the capacitance or length of line involved) is sufficiently great. On the other hand, surges of the first kind are known to have caused arrester damage and breaker failure. The interruption of line charging current is also known to require in many cases rather long clearing times, and in some cases to result in high surge voltages. In at least one case it has caused the operation of protector tubes. Some studies of the clearing of faulted lines[4] have shown restriking only in the faulted phase.

No mention has been made here of the usual voltage transients occurring on application of solid faults, as these have been treated in reference 3. It is emphasized that this discussion is very general and that a great many exceptions can be taken in particular instances.

Conclusions

The principal conclusions to be drawn from the work done so far are:

1. The results obtained cannot always be generalized to make them applicable to all systems. Certain general principles and effects have been discussed in order to show the trends in voltage surge severity which may be expected, but in particular instances it may be advisable to study the system in question (possibly by means of miniature circuit representation), rather than to estimate the probable effects from general data. This is particularly true in evaluating quantitatively the effect of

lightning arresters or other resistors in reducing the magnitude of voltage surges.

2. The effects of the distribution of capacitance within a power system are most pronounced in isolated neutral systems and with arc restriking of type 3.

3. For most of the systems and types of arcing studied, resistance grounding results in lower switching surge voltages than reactance grounding except at, and in the region immediately below, the ground fault neutralizer impedance.

4. For either reactance or resistance-grounded systems the voltages caused by switching surges are not greatly different from those arising from simple application or removal of a solid fault.

5. For isolated-neutral systems, voltages very much higher than those arising from simple application or removal of a solid fault may occur with certain types of arc restriking.

6. In many cases the effective capacitance ratio C_1/C_0 and reactance ratio X_{t0}/X_{t1} viewed from the fault, together with the approximate circuit natural frequencies, can be used as guides to the evaluation of the probable severity of switching surges; it is necessary in all cases to consider all of these factors.

7. For reactance grounded systems the voltages caused by switching surges and arcing faults have a maximum at a neutral reactance of from about $^1/_{10}$ to $^1/_3$ of the ground-fault-neutralizer value, depending on the type of arc restriking and on the circuit natural frequencies.

8. From the standpoint of switching surges and arcing faults, methods of system grounding which should be satisfactory are:

(*a*). Solid grounding.

(*b*). Grounding through moderate values of resistance.

(*c*). Grounding through a ground-fault-neutralizer, with lightning-arrester protection, and

(*d*) Grounding through moderate values of reactance, if a study of the system in question, based on the considerations treated above, indicates that unduly high surges will not result.

If a very low neutral reactance is used (for example, if the resulting ratio of X_{t0}/X_{t1} is less than about 5) a detailed study should not be necessary.

9. For the interruption of open lines, circuit breakers should be used which have a minimum tendency to restrike.

References

1. Effect of Restriking on Recovery Voltage, C. Concordia and W. F. Skeats. AIEE Transactions, volume 58, 1939 (August section).

2. Power-System Transients Caused by Switching and Faults, R. D. Evans, A. C. Monteith, and R. L. Witzke. AIEE Transactions, volume 58, 1939 (August section).

3. Overvoltages During Power-System Faults, Edith Clarke, S. B. Crary, and H. A. Peterson. AIEE Transactions, volume 58, 1939 (August section).

4. Discussion by P. A. Jeanne, AIEE Transactions, volume 58, 1939 (August section), page 395.

5. An Electric Circuit Transient Analyzer, H. A. Peterson. *General Electric Review*, volume 42, September 1939, page 394.

Transient Analysis of Completely Transposed Multiconductor Transmission Lines

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Synopsis: Starting with the general equations of the completely transposed *n*-conductor transmission system expressed in matrix form, the equations are subjected to a similarity transformation. This transformation reduces the general equations to a set of *n* independent equations. These equations are then subjected to a Laplacian transformation and solved subject to the impression of arbitrary electromotive forces at a certain point of the lines. Expressions are obtained for the resulting traveling waves of voltage and current and the velocities of propagation are discussed.

CONSIDERABLE effort has been expended in the transient analysis of multiconductor transmission lines.[1–4] It is the purpose of this paper to extend the theory of the practical case of perfect transposition. The effect of resistance and leakage conductance is considered in the general case. The analysis is greatly facilitated by the use of matrix algebra and the Laplacian transformation. The elements of matrix algebra and the Laplacian transformation presented in references 5 and 6 will be assumed in the discussion.

I. The General Equations

Let us consider a system of *n* long, parallel, widely separated, cylindrical conductors over an equipotential ground plane. It may be shown[1] that the equations governing the distribution of potentials and currents of the system are:

$$-\frac{\partial}{\partial x}(e)=[R](i)+[L]\frac{\partial}{\partial t}(i) \qquad (1)$$

$$-\frac{\partial}{\partial x}(i)=[G](e)+[C]\frac{\partial}{\partial t}(e) \qquad (2)$$

where (e) and (i) are columnar matrices whose elements $e_r(x, t)$ and $i_r(x, t)$, $r=1, 2, \ldots n$ are the potentials to ground and the currents of the several conductors respectively.

The case to be considered in this discussion is the practical case where all the conductors are identical and perfectly transposed with respect to each other and to the ground plane. In such a case we have:

$$[R]=\begin{Vmatrix} R_0 & \cdots & \cdots & 0 \\ 0 & R_0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & \cdots & R_0 \end{Vmatrix} \qquad (3)$$

where R_0 is the series resistance of each conductor per unit length. $[L]$ is a square matrix of the *n*th order of the form:

$$[L]=\begin{Vmatrix} L_0 & L & L \ldots\ldots & L \\ L & L_0 & L \ldots\ldots & L \\ \cdots & \cdots & \cdots & \cdots \\ L & L & L \ldots\ldots & L_0 \end{Vmatrix} \qquad (4)$$

where L_0 is the self-induction coefficient of the various conductors defined by:

$$L_0=[^1/_2+2\log_e(2h/r)]10^{-9} \text{ henry per centimeter} \qquad (5)$$

r is the radius of the conductors and h is the geometric mean height of the conductors above the zero potential ground plane. The coefficients L are mutual inductance coefficients defined by:

$$L=2[\log_e(a/b)]10^{-9} \text{ henry per centimeter} \qquad (6)$$

where a is the geometric mean distance between a conductor and the image of another conductor in the process of transposition. b is the geometric mean spacing between conductors.

The square matrix $[G]$ is defined by:

$$[G]=\begin{Vmatrix} G_0 & G & G \ldots\ldots & G \\ G & G_0 & G \ldots\ldots & G \\ \cdots & \cdots & \cdots & \cdots \\ G & G & G \ldots\ldots & G_0 \end{Vmatrix} \qquad (7)$$

The coefficients are defined by the following equations:

$$G_0=g_0+(n-1)g \qquad (8)$$

$$G=-g \qquad (9)$$

g_0 is the leakage conductance per unit length to ground of the several conductors. g is the leakage conductance per unit length between the conductors.

The matrix $[C]$ is a square matrix of the *n*th order and has the form:

$$[C]=\begin{Vmatrix} C_0 & C & C \ldots\ldots & C \\ C & C_0 & C \ldots\ldots & C \\ \cdots & \cdots & \cdots & \cdots \\ C & C & C \ldots\ldots & C_0 \end{Vmatrix} \qquad (10)$$

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[1] For all numbered references, see list at end of paper.

The $[C]$ matrix is related to an $[S]$ matrix of the form:

$$[S]=\begin{Vmatrix} S_0 & S & S\ldots\ldots S \\ S & S_0 & S\ldots\ldots S \\ \ldots & \ldots & \ldots\ldots\ldots \\ S & S & S\ldots\ldots S_0 \end{Vmatrix} \tag{11}$$

called the elastance matrix by the following equation:

$$[C]=[S]^{-1} \tag{12}$$

The coefficients of the elastance matrix are defined by:

$S_0=18\times10^{11}\log_e(2h/r)$ darafs per centimeter (13)

$S=18\times10^{11}\log_e(a/b)$ darafs per centimeter (14)

where the quantities h, r, a, and b have been defined above.

II. Discussion of the Line Parameters

The formulation of the n transmission line problem by the matrix equations 1 and 2 is a generalization of the engineering treatment of the single-line problem. The complexity of the n conductor problem is such that an analysis based on the Maxwell field equations is practically impossible, hence a simplified treatment based on the Maxwell coefficients of capacity and induction must be undertaken. The question of the validity of such a procedure naturally arises. A qualitative discussion of the principles involved is helpful in making clear the approximations which are introduced.

(*a*). The Coefficients of Induction

The coefficients of self- and mutual-induction as given by equations 5 and 6 are based on the following assumptions:

1. The neglection of radiation from the system. This amounts to saying that we are dealing with a quasi-stationary process and that the displacement current term in the field equations may be neglected.

2. The mean spacing of the conductors is so great that we may neglect the "proximity effect". This effect would tend to disturb the symmetrical distribution of current in the wires on which the calculation of the induction coefficients is based.

3. The "skin effect" is neglected. The factor of $^1/_2$ in the coefficients of self-induction in equation 5 is obtained on the basis of uniform current distribution inside the conductors. This is correct for solid conductors only for the case of zero frequency. At high frequencies or for traveling waves, the skin effect is so great that there cannot be any penetration of the current into the conductors. Hence the factor of $^1/_2$ must be neglected in such cases. In the "dissipationless" case of no conductor resistance, the factor $^1/_2$ must be neglected at all frequencies since in this case there is no penetration of current into the wires.

(*b*). The Coefficients of Capacitance

The coefficients of capacitance vary so little with frequency that they may be considered constant even with an appreciable amount of conductor resistance present. Although theoretically the capacitance matrix is given very accurately by the reciprocal of the elastance matrix whose elements are specified by equations 13 and 14, yet the capacitance parameters do vary with frequency or steepness of wave front slightly. This is because the theory does not include the effect of insulators, pins, and crossarms which are essential to the support of the lines. Metal pins introduce additional lumped capacitances at the supports. The resultant circuit is quite complex and may be approximately represented by a ladder network. There is some evidence that the capacitance coefficients are functions of the potentials of the conductors when the line potentials are above the critical corona potentials, however it has been found sufficient to take into account the effect of corona by modifying the leakage coefficients.

It may be also noted that in the case of an imperfectly conducting earth, the height h to ground of equation 13 differs from that of equation 5 which must be taken as the height above an equivalent conducting mass which represents the effect of the ground.

(*c*). The Resistance Coefficients

The theoretical variation of the resistance parameters with the steepness of the front of traveling current waves is due to changes in the distribution of internal flux linkages in the conductors. The transient skin effect tends to crowd the current towards the surface in all radial directions. This tends toward an infinite surface density of current for very steep wave fronts. This effect, therefore, tends to increase the conductor resistance with frequency over its direct current value.

(*d*). The Leakage Parameters

Below the critical corona voltage, the leakage parameters may be determined by measurement and regarded as constant. They of course depend greatly on the condition of the insulation and whether the weather is dry or wet. Above the disruptive corona point, the leakage parameters may be regarded as functions of the line potentials. It is customary to simplify the analysis by assuming certain magnitudes of the leakage coefficients that will give the same energy loss per unit length as the actual corona discharge determined by experiment. The problem is one of the utmost complexity and the best that can be done is to assume certain values for the leakage parameters that appear reasonable.

III. Transformation of the Canonical Equations

The canonical equations 1 and 2 will now be transformed by means of the nonsingular, symmetric matrix $[A]$ defined by:

$$[A]=[A_{rs}] \tag{15}$$

where

$r=1, 2, \ldots n \quad s=1, 2, \ldots n$

and

$$A_{rs}=a^{-(r-1)(s-1)} \tag{16}$$

$$a=\epsilon^{j2\pi/n} \tag{17}$$

This transformation matrix $[A]$ occupies a central position in the theory of symmetrical components. The various rows of $[A]$ are the sequence operators of Fortescue.

The transformation matrix $[A]$, is thus a square matrix of the nth order with the important properties that:

$$[A]'=[A] \tag{18}$$

where $[A]'$ is the transposed matrix of $[A]$.

$$[A]^{-1}=\frac{1}{n}[\text{conj } A] \tag{19}$$

where $[A]^{-1}$ is the inverse of $[A]$ and $[\text{conj } A]$ is the conjugate of $[A]$. Let us now premultiply equations 1 and 2 by $[A]^{-1}$, we thus obtain:

$$-\frac{\partial}{\partial x}[A]^{-1}(e)=[A]^{-1}[R][A][A]^{-1}(i)+[A]^{-1}[L][A][A]^{-1}\frac{\partial}{\partial t}(i) \tag{20}$$

$$-\frac{\partial}{\partial x}[A]^{-1}(i)=[A]^{-1}[G][A][A]^{-1}(e)+[A]^{-1}[C][A][A]^{-1}\frac{\partial}{\partial t}(e) \tag{21}$$

Now, if we define:

$$(e)_s=[A]^{-1}(e)=(e_{rs}) \quad r=1, 2, \ldots n \tag{22}$$

$$(i)_s=[A]^{-1}(i)=(i_{rs}) \quad r=1, 2, \ldots n \tag{23}$$

$$[R]_s=[A]^{-1}[R][A]=\begin{Vmatrix} R_0 & 0 & 0\ldots 0 \\ 0 & R_0 & 0\ldots 0 \\ \ldots & \ldots & \ldots \\ 0 & 0 & 0\ldots R_0 \end{Vmatrix} \tag{24}$$

$$[L]_s=[A]^{-1}[L][A]=\begin{Vmatrix} [L_0+(n-1)L] & 0\ldots\ldots & 0 \\ 0 & (L_0-L)\ldots & 0 \\ \ldots & \ldots & \ldots \\ 0 & 0 & (L_0-L) \end{Vmatrix} \tag{25}$$

$$[G]_s=[A]^{-1}[G][A]=\begin{Vmatrix}[G_0+(n-1)G] & 0\ldots\ldots & 0\\ 0 & (G_0-G)\ldots\ldots & 0\\ \ldots\ldots & \ldots\ldots & \ldots\ldots\\ 0 & 0 & (G_0-G)\end{Vmatrix} \qquad (26)$$

$$[C]_s=[A]^{-1}[C][A]=\begin{Vmatrix}[C_0+(n-1)C] & 0\ldots\ldots & 0\\ 0 & (C_0-C)\ldots\ldots & 0\\ \ldots\ldots & \ldots\ldots & \ldots\ldots\\ 0 & 0 & (C_0-C)\end{Vmatrix} \qquad (27)$$

Then we may write (20) and (21) in the form:

$$-\frac{\partial}{\partial x}(e)_s=[R]_s(i)_s+[L]_s\frac{\partial}{\partial t}(i)_s \qquad (28)$$

$$-\frac{\partial}{\partial x}(i)_s=[R]_s(e)_s+[C]_s\frac{\partial}{\partial t}(e)_s \qquad (29)$$

In view of the diagonal form of the matrices $[R]_s$, $[L]_s$, $[G]_s$, and $[C]_s$, the matrix equations 28 and 29 separate into the scalar equations:

$$-\frac{\partial e}{\partial x}1s=R_0i_{1s}+[L_0+(n-1)L]\frac{\partial i}{\partial t}1s \qquad (30)$$

$$-\frac{\partial i}{\partial x}1s=[G_0+(n-1)G]e_{1s}+[C_0+(n-1)C]\frac{\partial e}{\partial t}1s \qquad (31)$$

and

$$-\frac{\partial e}{\partial x}rs=R_0i_{rs}+(L_0-L)\frac{\partial i}{\partial t}rs \qquad r=2,3,\ldots n \qquad (32)$$

$$-\frac{\partial i}{\partial x}rs=(G_0-G)e_{rs}+(C_0-C)\frac{\partial e}{\partial t}rs \qquad (33)$$

IV. Introduction of a Laplacian Transformation

Following the methods of reference 6, part III, let us introduce the quantities $E_{rs}(x, p)$ and $I_{rs}(x, p)$ by means of the equations:

$$E_{rs}(x,p)=p\int_0^\infty e^{-pt}e_{rs}(x,t)dt \qquad r=1,2,\ldots n \qquad (34)$$

$$I_{rs}(x,p)=p\int_0^\infty e^{-pt}i_{rs}(x,t)dt \qquad r=1,2,\ldots n \qquad (35)$$

Or using the symbolic notation of Van der Pol, we write:

$$E_{rs}(x,p)\div e_{rs}(x,t) \qquad (36)$$

$$I_{rs}(x,p)\div i_{rs}(x,t) \qquad (37)$$

for the Laplacian transforms of e_{rs} and i_{rs}.

The equations 30–33 are of the type:

$$-\frac{\partial e}{\partial x}(x,t)=Ri(x,t)+L\frac{\partial i}{\partial t}(x,t) \qquad (38)$$

$$-\frac{\partial i}{\partial x}(x,t)=Ge(x,t)+C\frac{\partial e}{\partial t}(x,t) \qquad (39)$$

Following the methods of part III*B* of reference 6, it is easy to show that the equations satisfied by the transforms of $e(x, t)$ and $i(x, t)$ of (38) and (39) are of the type:

$$-dE/dx=RI+LpI \qquad (40)$$

$$-dI/dx=GE+CpE \qquad (41)$$

provided we assume no initial distribution of potential or current along the line. If we let:

$$e(0,t)\div E_0(p) \text{ at } x=0 \qquad (42)$$

$$i(0,t)\div I_0(p) \text{ at } x=0 \qquad (43)$$

the solution of the simultaneous equations 40 and 41 may be easily shown to be:

$$E(x,p)=E_0(p)\cosh(ax)-ZI_0(p)\sinh(ax)/a \qquad (44)$$

$$I(x,p)=I_0(p)\cosh(ax)-E_0(p)\sinh(ax)/Z_0 \qquad (45)$$

where:

$$Z=Lp+R \qquad (46)$$

$$Y=Cp+G \qquad (47)$$

$$a=(ZY)^{1/2} \qquad (48)$$

$$Z_0=(Z/Y)^{1/2} \qquad (49)$$

In the absence of impressed current at $x=0$, we have $i(0, t)=0$ and hence the above equations reduce to:

$$E(x,p)=E_0(p)\cosh(ax) \qquad (50)$$

$$I(x,p)=-E_0(p)\sinh(ax)/Z_0 \qquad (51)$$

Returning now to the equations 30–33 we have as a solution for the transforms:

$$E_{1s}(x,p)=E_{1s}^0(p)\cosh(a_1x) \qquad (52)$$

$$I_{1s}(x,p)=-E_{1s}^0(p)\sinh(a_1x)/Z_{01} \qquad (53)$$

where $E_{1s}^0(p)$ is the transform of the symmetrical component of voltage at the point $x=0$. An explicit equation for it will be given in a later section, and:

$$a_1=\sqrt{([L_0+(n-1)L]p+R_0)\times([C_0+(n-1)C]p+[G_0+(n-1)G])} \qquad (54)$$

$$Z_{01}=\sqrt{([L_0+(n-1)L]p+R_0)\div([C_0+(n-1)C]p+[G_0+(n-1)G])} \qquad (55)$$

and also:

$$E_{rs}(x,p)=E_{rs}^0(p)\cosh(a_2x) \qquad (56)$$

$$I_{rs}(x,p)=-E_{rs}^0(p)\sinh(a_2x)/Z_{02} \qquad r=2,3,\ldots n \qquad (57)$$

$$a_2=\sqrt{[(L_0-L)p+R_0][(C_0-C)p+(G_0-G)]} \qquad (58)$$

$$Z_{02}=\sqrt{[(L_0-L)p+R_0]/[(C_0-C)p+(G_0-G)]} \qquad (59)$$

The quantities $E_{rs}^0(p)$ are the transforms of the e_{rs} quantities at $x=0$.

V. The Dissipationless Case

(*a*). THE VOLTAGE WAVES

We shall first consider the ideal case of no resistance or leakage. This is the dissipationless case in which no energy is lost in the system.

For this case we have:

$$R_0=G_0=G=O \qquad (60)$$

We therefore have:

$$a_1=p\sqrt{[L_0+(n-1)L][C_0+(n-1)C]}=pb_1 \qquad (61)$$

$$E_{1s}(x,p)=E_{1s}^0(p)[e^{pb_1x}+e^{-pb_1x}]/2 \qquad (62)$$

In view of one of the basic theorems in the theory of the Laplacian transformation (reference 6, part I), which may be stated in the form:

$$\text{If } g(p)\div h(t) \qquad (63)$$

then

$$e^{-kp}g(p)\div\begin{cases}0 & t<k \quad k>0\\ h(t-k) & t>k\end{cases} \qquad (64)$$

it is seen that the term $e_{1s}(x, t)$ is composed of two waves, one traveling in the positive x direction and another one in the negative x direction. The positive wave has the magnitude:

$$e_{1s}^0(t-b_1x)/2=e_{1s}^+ \qquad (65)$$

and the negative wave has the magnitude:

$$e_{1s}^0(t+b_1x)/2=e_{1s}^- \qquad (66)$$

the velocity of propagation of the waves is:

$$v=1/b_1=\{[L_0+(n-1)L][C_0+(n-1)C]\}^{-1/2} \qquad (67)$$

In the case of the no-loss line there is no penetration of the current into the conductor and hence there is no internal magnetic field. The factor of $1/2$ in equation 5 is not present since this is due to the internal magnetic field in the conductors. We thus must have:

$$L_0=2\times10^{-9}\log_e(2h/r) \text{ henry per centimeter} \qquad (68)$$

Comparing this with (13) we see that we may write:

$$L_0=S_0/v_0^2 \qquad (69)$$

where v_0 is the velocity of light in a vacuum and equals 3×10^{10} centimeters per second.

Comparing (6) and (14) we see that we may also write:

$$L=S/v_0^2 \qquad (70)$$

From the relation 12 it may be shown that:

$$C_0=[S_0+(n-2)S]/(S_0-S)[S_0+(n-1)S] \qquad (71)$$

and

$$C=S(S-S_0)^{n-2}/(S_0-S)^{n-1}[S_0+(n-1)S] \tag{72}$$

Substituting the relations (69), (70), (71), and (72) into (67) we obtain:

$$v=v_0 \tag{73}$$

This shows that the forward and backward waves of equations 65 and 66 are propagated with the velocity of light.

It may be similarly shown that the group of waves whose transforms are given by the equations 56 are also propagated with the velocity of light in this dissipationless case.

(*b*). The Current Waves

From the equation 53 we see that we have current waves associated with the voltage waves. We have:

$$i_{1s}^+=e_{1s}^0(t-x/v_0)/2Z_{01} \tag{74}$$

for the forward traveling wave along the positive x direction and

$$i_{1s}^-=-e_{1s}^0(t+x/v_0)/2Z_{01} \tag{75}$$

for the negative traveling wave of current. For the dissipationless case we have:

$$Z_{01}=\{[L_0+(n-1)L]/[C_0+(n-1)C]\}^{1/2} \tag{76}$$

The equations 57 give a set of forward and backward traveling waves of current of the form:

$$i_{rs}^+=e_{rs}(t-x/v_0)/2Z_{02} \quad r=2,3,\ldots n \tag{77}$$

in the positive x direction and:

$$i_{rs}^-=-e_{rs}^0(t-x/v_0)/2Z_{02} \quad r=2,3,\ldots n \tag{78}$$

in the negative x direction. Where:

$$Z_{02}=[(L_0-L)/(C_0-C)]^{1/2} \tag{79}$$

Let us introduce the matrix:

$$(e^+)_s={}^1/_2(e_{rs}^+) \quad r=1,2,\ldots \tag{80}$$

the elements of this matrix are the symmetrical components of the traveling waves of voltage in the positive direction of the system.

The actual positive traveling waves of the system are given by the elements of the matrix:

$$(e^+)=[A]^{-1}(e^+)_s \tag{81}$$

Now if the elements of the matrix (e^0) defined by:

$$(e^0)=e_r^0(t) \quad r=1,2,\ldots n \tag{82}$$

are the impressed voltages on the lines at the point $x=0$, then the symmetrical components of these voltages are given by:

$$(e^0)_s=[A]^{-1}(e^0) \tag{83}$$

Now in view of (80) we may also write:

$$(e^+)_s=\{e^0(t-x/v_0)\}_s/2 \tag{84}$$

and hence:

$$(e^+)_s=\frac{1}{2}[A]^{-1}\{e^0(t-x/v_0)\} \tag{85}$$

By (83) the actual positive traveling waves of voltage are given by:

$$\begin{aligned}(e^+)&=\frac{1}{2}[A][A]^{-1}\{e^0(t-x/v_0)\}\\&=\frac{1}{2}\{e^0(t-x/v_0)\}\end{aligned} \tag{86}$$

Hence we see that in the perfectly transposed line the impressed electromotive forces are propagated in both directions with the velocity of light without distortion or interference provided there is no dissipation.

Let us now define the matrix $(i^+)_s$ whose elements are the symmetrical components of the current waves traveling in the positive x direction by the equation:

$$(i^+)_s=\begin{Vmatrix}1/Z_{01} & 0 & 0\ldots.0\\ 0 & 1/Z_{02} & 0\ldots.0\\ \cdots & \cdots & \cdots\\ 0 & 0 & 0\ldots.1/Z_{02}\end{Vmatrix}(e^+)_s \tag{87}$$

This equation follows from (74), (77), and (80). If we let:

$$[u]=\begin{Vmatrix}1/Z_{01} & 0 & 0\ldots\ldots0\\ 0 & 1/Z_{02} & 0\ldots\ldots0\\ \cdots & \cdots & \cdots & \cdots\\ 0 & 0 & 0 & 1/Z_{02}\end{Vmatrix} \tag{88}$$

then the actual current waves traveling in the positive direction are given by the elements of the matrix:

$$(i^+)=\frac{1}{2}[A][u][A]^{-1}\{e^0(t-x/v_0)\} \tag{89}$$

It is thus apparent that in the case of no dissipation, impressing potential waves at a point of a transmission line results in the propagation of voltage and current waves in the positive and negative directions. These waves are all propagated with the velocity of light and are given by equations 86 and 89.

VI. The General Case (Dissipation Included)

Let us introduce the notation:

$$L_1=L_0+(n-1)L \tag{90}$$

$$C_1=C_0+(n-1)C \tag{91}$$

$$G_1=G_0+(n-1)G \tag{92}$$

$$L_2=(L_0-L) \tag{93}$$

$$C_2=(C_0-C) \tag{94}$$

$$G_2=(G_0-G) \tag{95}$$

Then we may write (54), (55), (58), and (59) in the form:

$$a_1=[(L_1p+R_0)(C_1p+G_1)]^{1/2} \tag{96}$$

$$Z_{01}=[(L_1p+R_0)/(C_1p+G_1)]^{1/2} \tag{97}$$

$$a_2=[(L_2p+R_0)(C_2p+G_2)]^{1/2} \tag{98}$$

$$Z_{02}=[(L_2p+R_0)/(C_2p+G_2)]^{1/2} \tag{99}$$

The equation 52 for the transform of the symmetrical component voltage waves may be written in the form:

$$E_{1s}(x,p)=E_{1s}^0(p)[e^{a_1x}+e^{-a_1x}]/2 \tag{52}$$

We can therefore write:

$$E_{1s}^+(x,p)=E_{1s}^0(p)e^{-a_1x}/2 \tag{100}$$

for the transform of the wave moving in the positive x direction. Similarly we have for the negative wave, the equation:

$$E_{1s}^-(x,p)=E_{1s}^0(p)e^{+a_1x}/2 \tag{101}$$

The transforms of the equation 100 must be computed. Since

$$e_{1s}^0(t)\div E_{1s}^0(p) \tag{102}$$

and $e_{1s}^0(t)$ is given explicitly from the known impressed voltages at $x=0$ as given by (83), we may compute the inverse transform.

To do this, we may use the Faltung theorem of the Laplacian transformation theory (reference 6, part I). This theorem states that if

$$g_1(p)\div h_1(t) \tag{103}$$

and

$$g(_2p)\div h_2(t) \tag{104}$$

then

$$g_1g_2/p\div\int_0^t h_1(u)h_2(t-u)du \tag{105}$$

In a similar manner, the expression (53) for the transform of the currents may be written in the form:

$$I_{1s}(x,p)=E^0(x,p)[e^{-a_1x}-e^{a_1x}]/2Z_{01} \tag{53}$$

and we can write:

$$I_{1s}^+(x,p)=E_{1s}^0(p)e^{-a_1x}/2Z_{01} \tag{106}$$

for the transform of the current wave moving in the positive x direction and:

$$I_{1s}^-(x,p)=-E_{1s}^0(p)e^{a_1x}/2Z_{01} \tag{107}$$

for the current wave moving in the negative x direction.

In a similar manner, from a consideration of (56) and (57) we may write:

$$E_{rs}^+(x,p)=E_{rs}^0(p)e^{-a_2x}/2 \quad r=2,3,\ldots n \tag{108}$$

and

$$I_{rs}^+(x,p)=E_{rs}^0(p)e^{-a_2x}/2 \quad r=2,3,\ldots n \tag{109}$$

Since by symmetry, the negative waves behave in the same manner as the positive waves, we shall restrict our attention to the latter.

A. APPLICATION OF CONSTANT ELECTROMOTIVE FORCES

1. *The Voltage Waves*

Let us consider the application of n constant electromotive forces to the various conductors at $x=0$ and $t=0$, and let it be required to determine the magnitude of the resultant waves of voltage which result from the application of these electromotive forces. Let the magnitudes of the constant electromotive forces be given by the elements of the matrix:

$$(e^0)=(e_r) \qquad r=1, 2, \ldots n \tag{82}$$

Now we have:

$$(e^0)_s=[A]^{-1}(e^0) \tag{83}$$

From (36) it follows that:

$$E_{rs}{}^0 = e_{rs}{}^0 \qquad r=1, 2, 3, \ldots n \tag{110}$$

since the e_t terms are constants with respect to time. We must now obtain the inverse transform of the term e^{-ax}, where:

$$a=[(Lp+R)(Cp+G)]^{1/2} \tag{111}$$

This inverse transform is given in reference 7, table II, and in our notation we have:

$$e^{-ax} \div \begin{cases} 0 & t<x/v \\ e^{-rx/v}+\dfrac{sx}{v}\displaystyle\int_{x/v}^{t}\dfrac{e^{-ru}I_1\left(s\sqrt{u^2-x^2/v^2}\right)du}{\sqrt{u^2-x^2/v^2}} & t>x/v \end{cases} \tag{112}$$

where:

$$r=(R/L+G/C)/2 \tag{113}$$

$$s=(R/L-G/C)/2 \tag{114}$$

$$v=(LC)^{-1/2} \tag{115}$$

where $I_1(y)$ is the modified Bessel function of the first order. If we introduce the notation:

$$B_1(x, t) \div e^{-a_1x} \tag{116}$$

and

$$B_2(x, t) \div e^{-a_2x} \tag{117}$$

Then we have the set of equations:

$$\left.\begin{array}{l} e_{1s}{}^+=B_1(x, t)e_{1s}{}^0/2 \\ e_{rs}{}^+=B_2(x, t)e_{rs}{}^0/2 \cdots\cdots\cdots\cdots\cdots\cdots \end{array}\right\} \tag{118}$$

Defining the square matrix $[B]$ by the equation:

$$[B]=\left\|\begin{matrix} B_1 & 0 \ldots\ldots\ldots\ldots & 0 \\ 0 & B_2 \ldots\ldots\ldots\ldots & 0 \\ \cdots & \cdots\cdots\cdots\cdots\cdots & \cdots \\ 0 & 0 \ldots\ldots\ldots\ldots & B_2 \end{matrix}\right\| \tag{119}$$

then we may write the set (118) in the form:

$$(e^+)_s=\frac{1}{2}[B](e^0)_s \tag{120}$$

Or, by using (83) and premultiplying (120) by $[A]$ we have:

$$(e^+)=\frac{1}{2}[A][B][A]^{-1}(e^0) \tag{121}$$

This equation gives explicitly the values of the actual forward waves in terms of the constant applied electromotive forces at $x=0$.

2. *The Velocity of Propagation*

From the form of the expression 112 it is seen that the positive waves are composed of component waves having *two* velocities of propagation v_1 and v_2 given by the equations:

$$v_1=[L_0+(n-1)L][C_0+(n-1)C]^{-1/2} \tag{122}$$

and

$$v_2=[(L_0-L)(C_0-C)]^{-1/2} \tag{123}$$

In the case with dissipation, there is penetration of current into the line and hence the term L_0 does not have the value given by (68) but one intermediate between (68) and (5) depending on the transient skin effect. The velocities v_1 and v_2 are, therefore, less than the velocity of light and differ from each other.

3. *The Current Waves*

To obtain the positive current waves, we must consider the inverse transforms of (106) and (109). These inverse transforms depend on the inverse transform of an expression of the form: e^{-ax}/Z_0 where a has the value given by (111) and Z_0 is given by:

$$Z_0=[(Lp+R)/(Cp+G)]^{1/2} \tag{124}$$

From reference 7, table II we obtain:

$$e^{-ax}/Z_0 \div \begin{cases} 0 & t<x/v \\ \dfrac{e^{-rx/v}}{Z_0}+\dfrac{1}{k}\displaystyle\int_{x/v}^{t}e^{-ru}\left[\dfrac{su}{z}I_1(sz)-sI_0(sz)\right]du & t>x/v \end{cases} \tag{125}$$

where r, s, and v have the values given by (113) and (115) and we have:

$$k=(L/C)^{1/2} \tag{126}$$

$$z=(u^2-x^2/v^2)^{1/2} \tag{127}$$

Now if we introduce the notation:

$$H_1(x, t) \div e^{-ax_1}/Z_{01} \tag{128}$$

$$H_2(x, t) \div e^{-ax_2}/Z_{02} \tag{129}$$

then, in view of (106), (109), and (110), we may write equations 130.

$$\left.\begin{array}{l} i_{1s}{}^+=\dfrac{1}{2}H_1(x, t)e_{1s}{}^0 \\ i_{2s}{}^+=\dfrac{1}{2}H_2(x, t)e_{2s}{}^0 \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ i_{ns}{}^+=\dfrac{1}{2}H_2(x, t)e_{ns}{}^0 \end{array}\right\} \tag{130}$$

If we introduce the square matrix $[H]$ defined by:

$$[H]=\left\|\begin{matrix} H_1 & 0 & 0 \ldots\ldots\ldots & 0 \\ 0 & H_2 & 0 \ldots\ldots\ldots & 0 \\ \cdots & \cdots & \cdots\cdots\cdots\cdots & \cdots \\ 0 & 0 & 0 \ldots\ldots\ldots & H_2 \end{matrix}\right\| \tag{131}$$

the set of equations 130 may then be written concisely in the form:

$$(i^+)=\frac{1}{2}[A][H][A]^{-1}(e^0) \tag{132}$$

This equation gives explicitly the values of the various forward waves of current in the n conductors in terms of the constant applied electromotive forces at $x=0$. From (125) it is seen that the current waves are composed of two sets of waves having the velocities given by (122).

B. THE APPLICATION OF VARIABLE ELECTROMOTIVE FORCES

In the above section, the method of computing the voltage and current waves resulting from the application of constant electromotive forces at $x=0$ and $t=0$ was given. If the applied electromotive forces are variable, with respect to time, the procedure must be modified.

The calculation of the effect produced by the application of variable electromotive forces is most easily carried out by the use of the Faltung theorem (105). To do this, we make use of the inverse transform (reference 7, table II)

$$pe^{-ax} \div \begin{cases} 0 & t<x/v \\ \delta(t-x/v)e^{-rx/v}+\dfrac{sx}{vz}e^{-rt}I_1(sz) & t>x/v \end{cases} \tag{134}$$

where a, r, s, v, and z are given by (111), (113), (114), (115), and (127) respectively and $\delta(t)$ has the property:

$$\int_{-\infty}^{+\infty}F(t)\delta(t-a)dt=F(a) \tag{135}$$

where $F(t)$ is an arbitrary function of t. We may also use the transform (136).

Linear Transformations in Three-Phase Circuits

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Synopsis: A general method of finding linear transformations to eliminate the coupling between substitutive networks is developed. The method is compared with that of symmetrical components and modified symmetrical components and its practical applicability is discussed.

SINCE the introduction of substituted variables in the analysis of polyphase networks by Stockvis,[1] in 1914 and Fortescue[2] in 1918, there has been a great deal of interest in the use of linear substitutions in the analysis of unbalanced three-phase systems. The substituted variables have been of many different types. Fortescue's method of symmetrical components was followed by the method of direct and quadrature axis quantities of Park,[3] and by the "modified symmetrical components" of Clarke.[4]

In a recent paper by Kimbark,[5] another set of substituted variables which are related to symmetrical components and to modified symmetrical components was presented. In the discussion of Kimbark's paper by Boyajian, Helwith, and Stigant, the power of matrix algebra in linear transformations was demonstrated.

A study of the literature shows that the use of substituted variables is of advantage provided that the three-phase system under consideration has certain symmetry so that the derived systems are not coupled. It is the purpose of this paper to present a general method by which the appropriate linear transformation may be found so that the coupling between the substitutive networks is eliminated.

I. Three-Phase Network With Grounded Neutral

To fix the ideas in question, consider an apparatus with four terminals as shown in figure 1. Three of the terminals are to be connected to the lines of a three-phase source and the fourth to the grounded neutral of the system.

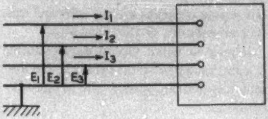

Figure 1

Let the complex line currents be I_1, I_2, and I_3, and the complex line-to-ground voltages E_1, E_2, E_3 be measured as shown in figure 1.

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[1] For all numbered references, see list at end of paper.

$$pe^{-ax}/Z_0 \div \begin{cases} 0 & t<x/v \\ \frac{1}{k}e^{-rx/v}\delta(t-x/v)+\frac{1}{k}e^{-rt}\left[\frac{st}{z}I_1(sz)-sI_0(sz)\right] & t>x/v \end{cases} \tag{136}$$

where the various constants are defined in section *A* above. Let us now introduce the notation:

$$pe^{-a_1x}\div M_1(x,t) \tag{137}$$

$$pe^{-a_2x}\div M_2(x,t) \tag{138}$$

and introducing the square matrix:

$$[M(x,t)]=\left\|\begin{matrix} M_1(x,t) & 0\ldots\ldots.0 & \\ 0 & M_2(x,t)\ldots.0 & \\ \cdots & \cdots & \cdots \\ 0 & 0 & M_2(x,t)\end{matrix}\right\| \tag{139}$$

then it is easy to show by an application of (105) that the positive waves of voltage produced by the application of variable electromotive forces at $x=0$ and $t=0$ that are the elements of the matrix 82 are given explicitly by:

$$(e^+)=\frac{1}{2}\int_0^t [A][M(x,u)][A]^{-1}(e^0(t-u))du \tag{140}$$

In a similar manner, if we introduce the notation:

$$pe^{-a_1x}/Z_{01}\div N_1(x,t) \tag{141}$$

$$pe^{-a_2x}/Z_{02}\div N_2(x,t) \tag{142}$$

and the square matrix:

$$[N(x,t)]=\left\|\begin{matrix} N_1(x,t) & 0 & 0\ldots..0 \\ 0 & N_2(x,t) & 0\ldots..0 \\ \cdots & \cdots & \cdots \\ 0 & 0 & 0\ldots N_2(x,t)\end{matrix}\right\| \tag{143}$$

we have explicitly:

$$(i^+)=\frac{1}{2}\int_0^t [A][N(x,u)][A]^{-1}(e^0(t-u))du \tag{144}$$

an equation giving the various traveling waves of current produced by the application of the variable electromotive forces on the several lines.

VII. Conclusion

By the application of a symmetrical-component transformation and a Laplacian transformation to the canonical equations of the completely transposed *n*-wire transmission line, formal expressions have been obtained for the current and voltage waves on the system initiated by the application of arbitrary electromotive forces on the conductors at an arbitrary point on the lines.

The expressions obtained are simple and lend themselves readily to numerical computation. The simplicity of the analysis when matrix algebra and the Laplacian transformation are employed may be noted. It is hoped that the solution of this important technical problem by these methods will recommend them for the solution of similar problems whose difficulty may be termed "mechanical" rather than mathematical.

VIII. Bibliography

1. TRAVELING WAVES ON TRANSMISSION LINES (a book), **L. V. Bewley.** John Wiley.

2. INDUKTIONSWIRKUNGEN VON WANDERWELLEN IN NACHBARLEITUNGEN, **K. W. Wagner.** *ETZ*, 1914.

3. TRAVELING WAVES ON TRANSMISSION SYSTEMS, **L. V. Bewley.** AIEE TRANSACTIONS, volume 50, 1931.

4. MATRIX THEORY OF MULTICONDUCTOR TRANSMISSION LINES, **L. A. Pipes.** *Phil. Mag.*, series 7, volume 24, pages 97–113.

5. MATRICES IN ENGINEERING, **L. A. Pipes.** ELECTRICAL ENGINEERING (AIEE TRANSACTIONS), September 1937, pages 1177–90.

6. THE OPERATIONAL CALCULUS, Parts I, II, and III, **L. A. Pipes.** *Journal of Applied Physics*, volume 10, March 1939, pages 172–80; April 1939, pages 258–64; May 1939, pages 301–11.

7. FOURIER INTEGRALS FOR PRACTICAL APPLICATIONS, **G. A. Campbell and R. M. Foster.** Bell Telephone System Monograph *B*-584, September 1931.

Then, by Kirchhoff's laws we may write the three voltage equations:

$$\left.\begin{aligned}E_1&=Z_{11}I_1+Z_{12}I_2+Z_{13}I_3\\E_2&=Z_{21}I_1+Z_{22}I_2+Z_{23}I_3\\E_3&=Z_{31}I_1+Z_{32}I_2+Z_{33}I_3\end{aligned}\right\}\qquad(1)$$

The complex numbers Z_{ij} are the self- and mutual impedances of the apparatus, and may be determined by measurement. The set of equations, (1), may be written concisely in the matrix form:

$$(E)=[Z](I) \qquad (2)$$

Where (E) is a column matrix whose elements are the voltages E_1, E_2, E_3, (I) is a column matrix whose elements are the line currents, and $[Z]$ is a square matrix whose elements are the self- and mutual impedances of the apparatus.

It may be noted that if the apparatus is a stationary bilateral network, the mutual impedances satisfy the reciprocity relations:

$$Z_{ij}=Z_{ji} \qquad (3)$$

and hence the impedance matrix, $[Z]$, satisfies the equation:

$$[Z]=[Z]' \qquad (4)$$

where $[Z]'$ is the transpose of the impedance matrix, $[Z]$.

In this case, the matrix $[Z]$ contains at most only six distinct coefficients. If the network is completely symmetrical, that is, the three phase impedances are equal and the six mutual impedances are also equal, then the coefficients merge into two in number, and we may write the impedance matrix in the form:

$$[Z]=\begin{Vmatrix}Z_0 & Z & Z\\ Z & Z_0 & Z\\ Z & Z & Z_0\end{Vmatrix} \qquad (5)$$

In the case of symmetrical rotating machines, the impedance matrix takes the form:

$$[Z]=\begin{Vmatrix}Z_0 & Z & Z_1\\ Z_1 & Z_0 & Z\\ Z & Z_1 & Z_0\end{Vmatrix} \qquad (6)$$

and is not symmetrical.

In the general three-phase problem it is required to determine the currents in the system when the elements of the impedance matrix and the voltages are known. This may always be done since $[Z]$ is a nonsingular matrix and has an inverse.

If we premultiply the equation 2 by $[Z]^{-1}$, the inverse of $[Z]$, we have:

$$(I)=[Z]^{-1}(E) \qquad (7)$$

an explicit solution for the currents.

The matrix $[Z]^{-1}$ has the value:

$$[Z]^{-1}=\frac{1}{D}\begin{Vmatrix}(Z_{22}Z_{33}-Z_{23}Z_{32}) & (Z_{32}Z_{13}-Z_{12}Z_{33}) & (Z_{12}Z_{23}-Z_{13}Z_{22})\\ (Z_{23}Z_{31}-Z_{21}Z_{33}) & (Z_{11}Z_{33}-Z_{13}Z_{31}) & (Z_{13}Z_{21}-Z_{11}Z_{23})\\ (Z_{21}Z_{32}-Z_{22}Z_{31}) & (Z_{12}Z_{31}-Z_{11}Z_{32}) & (Z_{11}Z_{22}-Z_{12}Z_{21})\end{Vmatrix} \qquad (8)$$

where D is the determinant:

$$D=\begin{vmatrix}Z_{11} & Z_{12} & Z_{13}\\ Z_{21} & Z_{22} & Z_{23}\\ Z_{31} & Z_{32} & Z_{33}\end{vmatrix} \qquad (9)$$

It is thus possible, by the use of the matrix $[Z]^{-1}$ in equation 7, to solve immediately for the currents I_1, I_2, and I_3 in the general case. This procedure, in general, is quite laborious and it is natural to ask if there exists an alternative procedure that will effect the solution more simply.

II. The General Linear Transformation

The general theory of linear transformations will now be presented and it will be apparent that the methods of Fortescue, Clarke, Kimbark, etc., are special cases of a general linear transformation. To fix the general method, the equations for the general three-phase network with grounded neutral will be considered.

Let the current and voltage matrices (I) and (E) be related to transformed current and voltage matrices $(I)_t$ and $(E)_t$ by the following matrix equations.

$$[B]^{-1}(I)=(I)_t \qquad (10)$$

$$[A](E)=(E)_t \qquad (11)$$

Where the matrix $(I)_t$ is a column matrix whose elements I_{1t}, I_{2t}, I_{3t} are the transformed currents of the currents I_1, I_2, I_3 with respect to the linear transformation (10). $[B]^{-1}$ is a nonsingular matrix whose elements are complex numbers. In a similar manner, the matrix $(E)_t$ is a column matrix whose elements E_{1t}, E_{2t}, E_{3t} are the transformed voltages of the voltages E_1, E_2, E_3, with respect to the linear transformation (11). $[A]$ is a nonsingular matrix whose elements are complex numbers.

Substituting these values of (I) and (E) from (10) and (11) into equation 2 one obtains:

$$[A][Z][B](I)_t=(E)_t \qquad (12)$$

Now if we let:

$$[Z]_t=[A][Z][B] \qquad (13)$$

then we can write equation 12 in the form:

$$[Z]_t(I)_t=(E)_t \qquad (14)$$

and we have an equation of the same form as (2) in the transformed voltages and currents.

If we let,

$$[A]=\begin{Vmatrix}A_{11} & A_{12} & A_{13}\\ A_{21} & A_{22} & A_{23}\\ A_{31} & A_{32} & A_{33}\end{Vmatrix} \qquad (15)$$

$$[B]=\begin{Vmatrix}B_{11} & B_{12} & B_{13}\\ B_{21} & B_{22} & B_{23}\\ B_{31} & B_{32} & B_{33}\end{Vmatrix} \qquad (16)$$

$$[Z]=\begin{Vmatrix}Z_{11t} & Z_{12t} & Z_{13t}\\ Z_{21t} & Z_{22t} & Z_{23t}\\ Z_{31t} & Z_{32t} & Z_{33t}\end{Vmatrix} \qquad (17)$$

then, the general element Z_{mnt} of the matrix $[Z]_t$ is given by:

$$Z_{mnt}=\sum_{r=1}^{r=3}\sum_{k=1}^{k=3}A_{mk}Z_{kr}B_{rn} \qquad (18)$$

as may be seen by direct matrix multiplication.

The whole point of introducing the linear transformations (10) and (11) is to choose the elements of the matrices $[A]$ and $[B]$ so that the matrix $[Z]_t$ will be simpler than the original matrix $[Z]$. If the elements of $[A]$ and $[B]$ are so chosen that the matrix $[Z]_t$ has the diagonal form:

$$[Z]_t=\begin{Vmatrix}Z_{11t} & 0 & 0\\ 0 & Z_{22t} & 0\\ 0 & 0 & Z_{33t}\end{Vmatrix} \qquad (19)$$

then the matrix equation 14 reduces to the set of three independent equations:

$$\left.\begin{aligned}Z_{11t}I_{1t}&=E_{1t}\\Z_{22t}I_{2t}&=E_{2t}\\Z_{33t}I_{3t}&=E_{3t}\end{aligned}\right\}\qquad(20)$$

and we may solve for the transformed currents most simply. The actual currents of the system are obtained by:

$$(I)=[B](I)_t \qquad (21)$$

Before discussing the general question of reducing the matrix $[Z]$ to the diagonal form (19) by the transformation (13), it is well to consider the special types of transformations which have been used in practice in the light of the general theory.

III. Symmetrical Components

The special type of linear transformation which is used in the theory of symmetrical components is effected by a nonsingular, symmetric matrix $[S]$ defined by the equation:

$$[S]=\begin{Vmatrix}1 & 1 & 1\\ 1 & a^2 & a\\ 1 & a & a^2\end{Vmatrix} \qquad (22)$$

where,

$$a=e^{2\pi j/3} \qquad (23)$$

From the definition of $[S]$ it is easy to show that $[S]$ has the following properties:

$$[S]^{-1}=\frac{1}{3}[\text{conj } S]=\frac{1}{3}[\overline{S}] \tag{24}$$

where [conj S] is the conjugate of $[S]$.

$$[S]'=[S] \tag{25}$$

where $[S]'$ is the transpose of $[S]$.

In the theory of symmetrical components, the equations:

$$[S]^{-1}(I)=(I)_s \tag{26}$$

$$[S]^{-1}(E)=(E)_s \tag{27}$$

replace the equations 10 and 11. Where $(I)_s$ and $(E)_s$ are matrices whose elements are symmetrical components of current and voltage respectively. Comparing these two sets of equations, it may be seen that the matrix $[S]$ is related to the matrices $[A]$ and $[B]$ of the general transformation of section II by the equations:

$$[S]=[B] \tag{28}$$

$$[S]^{-1}=[A] \tag{29}$$

The transformed matrix (13) is now given by:

$$[Z]_s=[S]^{-1}[Z][S]=\frac{1}{3}[\overline{S}][Z][S] \tag{30}$$

By direct matrix multiplication, one obtains:

$$[Z]_s=\begin{Vmatrix} Z_{11s} & Z_{12s} & Z_{13s} \\ Z_{21s} & Z_{22s} & Z_{23s} \\ Z_{31s} & Z_{32s} & Z_{33s} \end{Vmatrix}\frac{1}{3} \tag{31}$$

where:

$$\left.\begin{aligned}
Z_{11s}&=Z_{11}+Z_{21}+Z_{31}+Z_{12}+Z_{22}+\\ &\quad Z_{32}+Z_{13}+Z_{23}+Z_{33}\\
Z_{12s}&=Z_{11}+Z_{21}+Z_{31}+a^2Z_{12}+a^2Z_{22}+\\ &\quad a^2Z_{32}+a(Z_{13}+Z_{23}+Z_{33})Z_{33}\\
Z_{13s}&=Z_{11}+Z_{21}+Z_{31}+a(Z_{12}+Z_{22}+\\ &\quad Z_{32})+a^2(Z_{13}+Z_{23}+Z_{33})\\
Z_{21s}&=Z_{11}+Z_{12}+Z_{13}+a(Z_{21}+Z_{22}+\\ &\quad Z_{23})+a^2(Z_{31}+Z_{32}+Z_{33})\\
Z_{22s}&=Z_{11}+Z_{22}+Z_{33}+a(Z_{21}+Z_{32}+\\ &\quad Z_{13})+a^2(Z_{31}+Z_{12}+Z_{23})\\
Z_{23s}&=Z_{11}+Z_{22}+Z_{33}+a(Z_{31}+Z_{12}+\\ &\quad Z_{23})+a^2(Z_{21}+Z_{32}+Z_{13})\\
Z_{31s}&=Z_{11}+Z_{12}+Z_{13}+a(Z_{31}+Z_{32}+\\ &\quad Z_{33})+a^2(Z_{21}+Z_{22}+Z_{23})\\
Z_{32s}&=Z_{11}+Z_{22}+Z_{33}+a(Z_{31}+Z_{12}+\\ &\quad Z_{13})+a^2(Z_{21}+Z_{12}+Z_{33})\\
Z_{33s}&=Z_{11}+Z_{22}+Z_{33}+a(Z_{21}+Z_{12}+\\ &\quad Z_{23})+a^2(Z_{21}+Z_{32}+Z_{13})
\end{aligned}\right\} \tag{32}$$

It is thus seen that if the matrix $[Z]$ is the general matrix, the transformed impedance matrix $[Z]_s$ is far more complicated than the original one.

Simplification in Cases of Symmetry

From the general form of (31) it is evident that the method of symmetrical components effects no simplification if the system exhibits no symmetry. However, if the original matrix $[Z]$ has the perfectly symmetrical form:

$$[Z]=\begin{Vmatrix} Z_0 & Z & Z \\ Z & Z_0 & Z \\ Z & Z & Z_0 \end{Vmatrix} \tag{33}$$

then $[Z]_s$ has the diagonal form:

$$[Z]_s=\begin{Vmatrix} (Z_0+2Z) & 0 & 0 \\ 0 & (Z_0-Z) & 0 \\ 0 & 0 & (Z_0-Z) \end{Vmatrix} \tag{34}$$

and the several sequence voltages and currents are independent. Since most balanced polyphase systems have this type of symmetry, this accounts for the great simplification introduced in the analysis of these systems by the method of symmetrical components.

Another important type of matrix that may be diagonalized by a transformation to symmetrical components is the matrix:

$$[Z]=\begin{Vmatrix} Z_0 & Z_1 & Z_2 \\ Z_2 & Z_0 & Z_1 \\ Z_1 & Z_2 & Z_0 \end{Vmatrix} \tag{35}$$

This case occurs in the theory of three phase synchronous and induction machines with smooth air gaps. In this case, the matrix $[Z]_s$ has the form:

$$[Z]_s=\begin{Vmatrix} (Z_0+Z_1+Z_2) & 0 & 0 \\ 0 & (Z_0+a^2Z_1+aZ_2) & 0 \\ 0 & 0 & (Z_0+aZ_1+a^2Z_2) \end{Vmatrix} \tag{36}$$

and a simplification is introduced by the method of symmetrical components.

IV. Modified Symmetrical Components

Clarke,[4] defines another set of substituted variables which she has named "modified symmetrical components". They may be obtained from the general transformations (10) and (11) by choosing the matrix $[B]^{-1}$ to be:

$$[B]^{-1}=\begin{Vmatrix} 1/3 & 1/3 & 1/3 \\ 2/3 & -1/3 & -1/3 \\ 0 & 1/2 & -1/2 \end{Vmatrix} \tag{37}$$

$$[A]=[B]^{-1} \tag{38}$$

In this case the transformed currents and voltages have the form:

$$(I)_t=\begin{Vmatrix} I_0 \\ I_\alpha \\ I_\beta \end{Vmatrix} \tag{39}$$

$$(E)_t=\begin{Vmatrix} E_0 \\ E_\alpha \\ E_\beta \end{Vmatrix} \tag{40}$$

The matrix $[A]$ has the inverse:

$$[A]^{-1}=\begin{Vmatrix} 1 & 1 & 0 \\ 1 & -1/2 & \sqrt{3}/2 \\ 1 & -1/2 & -\sqrt{3}/3 \end{Vmatrix}=[B] \tag{41}$$

The transformed impedance matrix is given by (13) and by matrix multiplication we obtain:

$$[Z]_t=\begin{Vmatrix} Z_{11t} & Z_{12t} & Z_{13t} \\ Z_{21t} & Z_{22t} & Z_{23t} \\ Z_{31t} & Z_{32t} & Z_{33t} \end{Vmatrix} \tag{42}$$

where for this case the elements of $[Z]_t$ are found by direct multiplication to be:

$$\left.\begin{aligned}
Z_{11t}&={}^1/_3(Z_{11}+Z_{21}+Z_{31}+Z_{12}+Z_{22}+\\ &\quad Z_{32}+Z_{13}+Z_{23}+Z_{33})\\
Z_{12t}&={}^1/_3(Z_{11}+Z_{21}+Z_{31})-{}^1/_6(Z_{12}+\\ &\quad Z_{22}+Z_{32})-{}^1/_6(Z_{13}+Z_{23}+Z_{33})\\
Z_{13t}&=\frac{1}{2\sqrt{3}}(Z_{12}+Z_{22}+Z_{32})-\\ &\quad \frac{1}{2\sqrt{3}}(Z_{13}+Z_{23}+Z_{33})\\
Z_{21t}&={}^2/_3(Z_{11}+Z_{12}+Z_{13})-{}^1/_3(Z_{21}+\\ &\quad Z_{31}+Z_{22}+Z_{32}+Z_{23}+Z_{33})\\
Z_{22t}&=\left(\frac{2}{3}Z_{11}-\frac{1}{3}Z_{21}-\frac{1}{3}Z_{31}\right)-\\ &\quad \frac{1}{2}\left(\frac{2}{3}Z_{12}-\frac{1}{3}Z_{22}-\frac{1}{3}Z_{32}\right)-\\ &\quad \frac{1}{2}\left(\frac{2}{3}Z_{13}-\frac{1}{3}Z_{23}-\frac{1}{3}Z_{33}\right)\\
Z_{23t}&=\frac{\sqrt{3}}{2}\left(\frac{2}{3}Z_{12}-\frac{1}{3}Z_{22}-\frac{1}{3}Z_{32}\right)-\\ &\quad \frac{\sqrt{3}}{2}\left(\frac{2}{3}Z_{13}-\frac{1}{3}Z_{23}-\frac{1}{3}Z_{33}\right)\\
Z_{31t}&={}^1/_2(Z_{21}+Z_{22}+Z_{23}-Z_{31}-\\ &\quad Z_{32}-Z_{33})\\
Z_{32t}&={}^1/_2(Z_{21}-Z_{31})+\\ &\quad {}^1/_4(Z_{32}-Z_{22}+Z_{33}-Z_{23})\\
Z_{33t}&=\frac{\sqrt{3}}{4}(Z_{22}-Z_{32}-Z_{23}+Z_{33})
\end{aligned}\right\} \tag{43}$$

Simplification in the Case of Symmetry

If there is symmetry about phase 1, that is, if the following relations are satisfied:

$$Z_{22}=Z_{33},\ Z_{12}=Z_{21}=Z_{13}=Z_{31},\ Z_{23}=Z_{32} \tag{44}$$

then the transformed matrix reduces to:

$$[Z]_t=\begin{Vmatrix} Z_{11t} & Z_{12t} & 0 \\ Z_{21t} & Z_{22t} & 0 \\ 0 & 0 & Z_{33t} \end{Vmatrix} \tag{45}$$

the values of the elements of (45) are given by the equations 43 provided use is made of the symmetry conditions (44).

It is thus seen that if there is symmetry about phase 1 the transformed matrix is simplified but not diagonalized. The coupling between the transformed or sequence circuits is reduced.

V. Two-Phase Co-ordinates

Kimbark[5] has introduced a slightly different linear transformation than that used by Clarke and calls the transformed voltages and currents "two-phase co-ordinates". This transformation is effected by giving the following values to the elements of the matrices $[A]$ and $[B]$ of the general linear transformation (10) and (11).

$$[A]=\begin{Vmatrix} {}^{2}/_{3} & -{}^{1}/_{3} & -{}^{1}/_{3} \\ 0 & -1/\sqrt{3} & 1/\sqrt{3} \\ {}^{1}/_{3} & {}^{1}/_{3} & {}^{1}/_{3} \end{Vmatrix} \qquad (46)$$

$$[B]^{-1}=\begin{Vmatrix} {}^{2}/_{3} & -{}^{1}/_{3} & -{}^{1}/_{3} \\ 0 & -1/\sqrt{3} & 1/\sqrt{3} \\ {}^{2}/_{3} & {}^{2}/_{3} & {}^{2}/_{3} \end{Vmatrix} \qquad (47)$$

The transformed currents and voltages are given by:

$$(I)_t=\begin{Vmatrix} I_x \\ I_y \\ I_z \end{Vmatrix} \qquad (48)$$

$$(E)_t=\begin{Vmatrix} E_x \\ E_y \\ E_z \end{Vmatrix} \qquad (49)$$

The transformation matrices have the inverses:

$$[A]^{-1}=\begin{Vmatrix} 1 & 0 & 1 \\ -\frac{1}{2} & \frac{-\sqrt{3}}{2} & 1 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \end{Vmatrix} \qquad (50)$$

$$[B]=\begin{Vmatrix} 1 & 0 & \frac{1}{2} \\ -\frac{1}{2} & \frac{-\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{1}{2} \end{Vmatrix} \qquad (51)$$

By direct multiplication, the transformed impedance matrix is found to be:

$$[Z]_t=\begin{Vmatrix} Z_{11t} & Z_{12t} & Z_{13t} \\ Z_{21t} & Z_{22t} & Z_{23t} \\ Z_{31t} & Z_{32t} & Z_{33t} \end{Vmatrix} \qquad (52)$$

where in this case the elements of the transformed matrix are given by:

$$\left.\begin{aligned}
Z_{11t}&=\frac{1}{6}(4Z_{11}-2Z_{12}-2Z_{13}-2Z_{21}+ \\ &\qquad Z_{22}+Z_{23}-2Z_{31}+Z_{32}+Z_{33}) \\
Z_{12t}&=\frac{1}{2\sqrt{3}}(2Z_{13}-2Z_{12}+Z_{22}-Z_{23}+ \\ &\qquad Z_{32}-Z_{33}) \\
Z_{13t}&=\frac{1}{6}(2Z_{11}+2Z_{12}+2Z_{13}-Z_{21}- \\ &\qquad Z_{22}-Z_{23}-Z_{31}-Z_{32}-Z_{33}) \\
Z_{21t}&=\frac{1}{2\sqrt{3}}(Z_{22}-2Z_{21}+Z_{23}+2Z_{31}- \\ &\qquad Z_{32}-Z_{33}) \\
Z_{22t}&=Z_{22}-Z_{23}-Z_{32}+Z_{33} \\
Z_{23t}&=\frac{1}{2\sqrt{3}}(Z_{31}+Z_{32}+Z_{33}-Z_{21}- \\ &\qquad Z_{22}-Z_{23}) \\
Z_{31t}&=\frac{1}{6}(2Z_{11}-Z_{12}-Z_{13}+2Z_{21}-Z_{22}- \\ &\qquad Z_{23}+2Z_{31}-Z_{32}-Z_{33}) \\
Z_{32t}&=\frac{1}{2\sqrt{3}}(Z_{13}-Z_{12}-Z_{22}+Z_{23}- \\ &\qquad Z_{32}+Z_{33}) \\
Z_{33t}&=\frac{1}{6}(Z_{11}+Z_{12}+Z_{13}+Z_{21}+Z_{22}+ \\ &\qquad Z_{23}+Z_{31}+Z_{32}+Z_{33})
\end{aligned}\right\} \qquad (53)$$

This is the general form of the transformed matrix. In the special case of symmetry about phase 1, we have:

$$Z_{22}=Z_{33},\ Z_{12}=Z_{21}=Z_{13}=Z_{31},\ Z_{23}=Z_{32} \qquad (54)$$

In this symmetrical case, the transformed impedance matrix reduces to:

$$[Z]_t=\begin{Vmatrix} \frac{1}{3}(2Z_{11}-4Z_{12}+Z_{22}+Z_{23}) & 0 & \frac{1}{6}(Z_{11}+Z_{12}-Z_{22}-Z_{23}) \\ 0 & (Z_{22}-Z_{23}) & 0 \\ \frac{1}{3}(Z_{11}+Z_{12}-Z_{22}-Z_{23}) & 0 & \frac{1}{3}\left(\frac{1}{2}Z_{11}+2Z_{12}+Z_{22}+Z_{23}\right) \end{Vmatrix} \qquad (55)$$

We note that as in the case of modified symmetrical components the coupling is reduced by this linear transformation in the case of symmetry about phase 1.

VI. The Reduction of a Square Matrix to the Diagonal Form

The above brief discussion of the various linear transformations which have been used in the analysis of networks reveals the fact that these transformations are introduced in order that the transformed matrix will have a simpler form than the original matrix. In case that the transformed matrix has the diagonal form, then the various transformed voltages and currents are related to one another in a most simple manner.

The reduction of given matrices to the diagonal form is one of the fundamental problems of matrix algebra. In order to understand the problem it is essential to know what is meant by equivalent matrices in a certain number field, F.

EQUIVALENT MATRICES IN A FIELD

The matrix $[Z]$ whose elements are complex numbers is equivalent in the field of complex numbers to the matrix $[Z]_t$ if nonsingular matrices $[A]$ and $[B]$ exist so that:

$$[Z]_t=[A][Z][B] \qquad (13)$$

where the elements of $[A]$, $[Z]$, $[B]$, and therefore of $[Z]_t$ all belong to the field of complex numbers.

For a given matrix $[Z]$ in the complex number field, F, the problem is to discover non-singular matrices $[A]$ and $[B]$ which will make the equivalent matrix $[Z]_t$ as simple as possible, a diagonal form being the most desirable.

H. J. S. Smith[6] has demonstrated that given a square matrix $[Z]$ of n rows and columns, there exists a diagonal matrix:

$$[A][Z][B]=\begin{Vmatrix} a_1 & 0 & 0 & 0 \ldots\ldots\ldots \\ 0 & a_2 & 0 & 0 \ldots\ldots\ldots \\ \ldots & \ldots & \ldots & \ldots\ldots\ldots\ldots \\ 0 & 0 & 0 & 0\ a_r \ldots\ldots \\ 0 & 0 & 0 & 0 \ldots\ldots 0 \end{Vmatrix} \qquad (56)$$

that has r nonzero elements a_i isolated in the diagonal, and that is equivalent to $[Z]$ in the complex number field, F. The set of r numbers a_i are rational functions of the elements Z_{ij} of Z.

The proof of this theorem consists of operating successively on $[Z]$ with matrices $[A]_i$ and $[B]_i$ so chosen that the matrices $[A]_1[Z][B]_1$, $[A]_2[A]_1[Z][B]_1\times[B]_2$, etc., assume more and more the diagonal form.

FUNDAMENTAL OPERATIONS

The operations which will be used to diagonalize the matrix $[Z]$ will be of the following fundamental types:

(*a*). *Interchange of Rows and Columns*

Consider the identity:

$$\begin{Vmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{Vmatrix} \begin{Vmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{Vmatrix} = \begin{Vmatrix} Z_{21} & Z_{22} & Z_{23} \\ Z_{11} & Z_{12} & Z_{13} \\ Z_{31} & Z_{32} & Z_{33} \end{Vmatrix} \qquad (57)$$

This shows that the effect of premultiplying $[Z]$ by a unit matrix in which the first two rows have been interchanged is to interchange the same two rows in $[Z]$.

Let $U_{(ij)}$ denote the unit matrix in which the ith and jth rows have been interchanged without disturbing the remaining $(n-2)$ rows. By direct multiplication it may be shown that the effect of multiplying $[Z]$ by such a matrix will be:

$$\left.\begin{array}{l} U_{(ij)}[Z]\text{: interchanges row}_i\text{ and row}_j \\ [Z]U_{(ij)}\text{: interchanges column}_i\text{ and} \\ \qquad \text{column}_j \\ U_{(ij)}[Z]U_{(ij)}\text{: interchanges both rows} \\ \qquad \text{and columns } i,j \end{array}\right\} \qquad (58)$$

(*b*). *Linear Combinations of Rows or Columns*

If we consider the identity:

$$\begin{Vmatrix} 1 & 0 & 0 \\ 0 & 1 & k \\ 0 & 0 & 1 \end{Vmatrix} \begin{Vmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{Vmatrix} =$$

$$\begin{Vmatrix} Z_{11} & Z_{12} & Z_{13} \\ (Z_{21}+kZ_{31}) & (Z_{22}+kZ_{32}) & (Z_{23}+kZ_{33}) \\ Z_{31} & Z_{32} & Z_{33} \end{Vmatrix} \qquad (59)$$

we see that premultiplication of $[Z]$ by a unit matrix supplemented by an element k in the indicated position has the effect of giving the operation $\text{row}_2+k\ \text{row}_3$.

In general, let:

$$[H]=U+(k)_{ij} \quad (i\neq j) \qquad (60)$$

denote the unit matrix with an element k inserted in the (ij)th position. Then it can be seen that:

$$\left.\begin{array}{l}[H][Z]:\text{row}_i+k\ \text{row}_j\\ [Z][H]:\text{col}_j+k\ \text{col}_i\end{array}\right\}\qquad(61)$$

It may be noted that transformations of the type (a) and (b) involve pre- and postmultiplication by matrices whose determinants are equal to unity. Transformations of this type are unimodular transformations.

(c). *Multiplication of Rows or Columns*

In operations of type (b), the element k had been inserted in the diagonal of U where $k\neq-1$, then we would have:

$$[H]=U+(k)_{ii}\qquad(62)$$

$$H=1+k\qquad(63)$$

If

$$h=1+k\qquad(64)$$

the rules of operation for this matrix are:

$$\begin{array}{l}[H][Z]:h\ \text{row}_i\\ [Z][H]:h\ \text{col}_i\end{array}\qquad(65)$$

VII. Diagonalizing a Symmetric Matrix

As an example of the general procedure of section VI let the problem of reducing the impedance of a static three-phase system to the equivalent diagonal form be considered.

Static networks have the property that their impedance matrices are symmetrical, that is, $Z_{ij}=Z_{ji}$. The problem in this case is to reduce the matrix:

$$[Z]=\begin{Vmatrix}Z_{11}&Z_{12}&Z_{13}\\Z_{21}&Z_{22}&Z_{23}\\Z_{31}&Z_{32}&Z_{33}\end{Vmatrix}\qquad(66)$$

to the diagonal form by finding the proper transformation matrices $[A]$ and $[B]$.

As a first step in the procedure, let us write:

$$n_{ij}=Z_{ij}/Z_{11}\qquad(67)$$

since the elements of the $[Z]$ matrix are complex numbers, n_{ij} will, in general, be a complex number. In terms of the numbers n_{ij} we may write:

$$[Z]=Z_{11}\begin{Vmatrix}1&n_{12}&n_{13}\\n_{21}&n_{22}&n_{23}\\n_{31}&n_{32}&n_{33}\end{Vmatrix}\qquad(68)$$

It is evident that this matrix may be diagonalized by operations involving linear combinations of rows and columns as discussed in section VI part b.

We have as a first step:

$$\begin{Vmatrix}1&0&0\\-n_{12}&1&0\\0&0&1\end{Vmatrix}\begin{Vmatrix}1&n_{12}&n_{13}\\n_{12}&n_{22}&n_{23}\\n_{13}&n_{23}&n_{33}\end{Vmatrix}Z_{11}=$$

$$Z_{11}\begin{Vmatrix}1&n_{12}&n_{13}\\0&(n_{22}-n_{12}^2)&(n_{23}-n_{13}n_{12})\\n_{13}&n_{23}&n_{33}\end{Vmatrix}\qquad(69)$$

The next step is to premultiply this matrix by one that will remove the factor n_{13} in the first column. Then the resulting matrix is postmultiplied by matrices that will remove the factors n_{12} and n_{13} in the first row. These matrices may then be multiplied together in the proper order. At this stage we have:

$$Z_{11}\begin{Vmatrix}1&0&0\\-n_{12}&1&0\\-n_{13}&0&1\end{Vmatrix}\begin{Vmatrix}1&n_{12}&n_{13}\\n_{12}&n_{22}&n_{23}\\n_{13}&n_{23}&n_{33}\end{Vmatrix}\begin{Vmatrix}1&-n_{12}&-n_{13}\\0&1&0\\0&0&1\end{Vmatrix}=$$

$$Z_{11}\begin{Vmatrix}1&0&0\\0&(n_{22}-n_{12}^2)&(n_{23}-n_{13}n_{12})\\0&(n_{23}-n_{13}n_{12})&(n_{33}-n_{13}^2)\end{Vmatrix}\qquad(70)$$

Further diagonalizing may be accomplished in the same manner and we finally obtain:

$$[A][Z][B]=Z_{11}\begin{Vmatrix}1&0&0\\0&(n_{22}-n_{12}^2)&0\\0&0&[k(n_{23}-n_{13})+(n_{33}-n_{13}^2)]\end{Vmatrix}\qquad(71)$$

where:

$$[A]=\begin{Vmatrix}1&0&0\\-n_{12}&1&0\\-(kn_{12}+n_{13})&k&1\end{Vmatrix}\qquad(72)$$

$$[B]=\begin{Vmatrix}1&-n_{12}&-(kn_{12}+n_{13})\\0&1&k\\0&0&1\end{Vmatrix}\qquad(73)$$

$$k=(n_{12}n_{13}-n_{23})/(n_{22}-n_{12}^2)\qquad(74)$$

It may be noted that in this case, $[B]=[A]'$. The matrices $[A]$ and $[B]$ have the inverses:

$$[A]^{-1}=\begin{Vmatrix}1&0&0\\n_{12}&1&0\\n_{13}&-k&1\end{Vmatrix}\qquad(75)$$

$$[B]^{-1}=\begin{Vmatrix}1&n_{12}&n_{13}\\0&1&-k\\0&0&1\end{Vmatrix}\qquad(76)$$

Unlike the special choice of the matrices $[A]$ and $[B]$ used in the theory of symmetrical components and modified symmetrical components, the matrices required to diagonalize the general symmetric impedance matrix $[Z]$ contains elements that depend on the various elements of $[Z]$. This is unavoidable. However the numbers n_{ij} and k may be easily computed and the matrix under consideration diagonalized. It may be noted that the procedure of diagonalizing a given matrix is not unique but may be carried out in many ways.

The general procedure outlined in part VI is always applicable and advantage may be taken of the symmetry of the matrix under consideration.

VIII. Diagonalizing the General Unsymmetric Matrix

As a last example of the general procedure let the problem of diagonalizing the general unsymmetric impedance matrix of a three-phase system be considered. Specifically, the problem is to find matrices $[A]$ and $[B]$ that will reduce the general matrix:

$$[Z]=\begin{Vmatrix}Z_{11}&Z_{12}&Z_{13}\\Z_{21}&Z_{22}&Z_{23}\\Z_{31}&Z_{32}&Z_{33}\end{Vmatrix}\qquad Z_{ij}\neq Z_{ji}\qquad(77)$$

to the diagonal form. By the same method used in section VII, it may be shown that the matrices:

$$[A]=\begin{Vmatrix}1&0&0\\-n_{21}&1&0\\-(k_1n_{21}+n_{31})&k_1&1\end{Vmatrix}\qquad(78)$$

$$[B]=\begin{Vmatrix}1&-n_{12}&-(k_2n_{12}+n_{13})\\0&1&k_2\\0&0&1\end{Vmatrix}\qquad(79)$$

$$k_1=(n_{12}n_{31}-n_{32})/(n_{22}-n_{12}n_{21})\qquad(80)$$

$$k_2=(n_{21}n_{13}-n_{23})/(n_{22}-n_{12}n_{21})\qquad(81)$$

Diagonalize the matrix $[Z]$ in the form:

$$[A][Z][B]=Z_{11}\begin{Vmatrix}1&0&0\\0&(n_{22}-n_{12}n_{21})&0\\0&0&[k_1(n_{23}-n_{12}n_{13})+(n_{33}-n_{13}n_{31})]\end{Vmatrix}\qquad(82)$$

The complex numbers n_{ij} have the same significance that they have in section VII.

IX. Transformation of the Vector Volt-Ampere Input

The vector volt-ampere input P, in terms of the original voltages and currents of the system is defined by the equation:

$$P=(I)'(\text{conj }E)\qquad(83)$$

where $(I)'$ is the transpose of the current matrix (I) and (conj E) is the conjugate of the voltage matrix.

Substituting for (I) and (E) the values given in terms of $(I)_t$ and $(E)_t$ by the general transformation equations, (10) and (11), we have:

$$P=([B](I)_t)'[\text{conj }A]^{-1}(\text{conj }E)_t\qquad(84)$$

but by a fundamental theorem of matrix algebra we have:

$$([B](I)_t)'=(I)_t'[B]'\qquad(85)$$

Hence we have:

$$P=(I)_t'[B]'[\text{conj } A]^{-1}(\text{conj } E)_t \qquad (86)$$

Now, if:

$$[B]'=[\text{conj } A] \qquad (87)$$

then:

$$P=(I)_t'(\text{conj } E)_t=P_t \qquad (88)$$

and the vector volt-ampere input is invariant to the linear transformation. The relation (87) is not in general satisfied and the equation (86) gives the proper result in all cases.

X. Illustrative Example

Consider a three phase system whose impedance matrix has the value:

$$[Z]=\begin{Vmatrix}(4.84+j1.30) & (3.48+j2.00) & (1.81+j.846)\\ (8.95+j.945) & (9.98+j.690) & (2.97+j.416)\\ (15.05+j5.48) & (5.65+j5.65) & (5.36+j4.50)\end{Vmatrix} \qquad (89)$$

Let the voltage matrix of the system be:

$$(E)=\begin{Vmatrix}100\\ 80\;\underline{/100^\circ}\\ 70\;\underline{/200^\circ}\end{Vmatrix} \qquad (90)$$

Let it be required to determine the proper transformation that will diagonalize $[Z]$ and hence make the various sequence networks independent of one another. Let the resulting three phase currents of the system be determined.

From equations 78, 79, 80, 81, and 82, the following matrices are calculated.

$$[A]=\begin{Vmatrix}1 & 0 & 0\\ 1.8\;\underline{/171} & 1 & 0\\ 5.02\underline{/161^\circ 40'} & 1.345\underline{/-40^\circ 35'} & 1\end{Vmatrix} \qquad (91)$$

$$[B]=\begin{Vmatrix}1 & .8\underline{/195^\circ} & .552\underline{/187^\circ 10'}\\ 0 & 1 & .192\underline{/-14^\circ 42'}\\ 0 & 0 & 1\end{Vmatrix} \qquad (92)$$

The transformed impedance matrix is diagonalized by these two matrices and has the value:

$$5\underline{/15^\circ}\begin{Vmatrix}1 & 0 & 0\\ 0 & .754\underline{/44^\circ 52'} & 0\\ 0 & 0 & .758\underline{/-77^\circ 53'}\end{Vmatrix}=[Z_t] \qquad (93)$$

The transformed voltage matrix is given by (11) and has the value:

$$(E)_t=\begin{Vmatrix}100\\ 224\underline{/151^\circ 50'}\\ 536\underline{/155^\circ}\end{Vmatrix} \qquad (94)$$

In the transformed system, the sequence voltages and currents are independent because of the diagonal form of the impedance matrix. The matrix equation $[Z]_t(I)_t=(E)_t$ reduces in this case to the three scalar equations:

$$\left.\begin{matrix}(5\underline{/15^\circ})I_{1t}=100\\ (5\underline{/15^\circ})(.754\underline{/44^\circ 52'})I_{2t}=224\underline{/151^\circ 50'}\\ (5\underline{/15^\circ})(.758\underline{/-77^\circ 53'})I_{3t}=536\underline{/155^\circ}\end{matrix}\right\} \qquad (95)$$

These equations may be solved for the sequence currents by division and the transformed current matrix is found to be:

$$(I)_t=\begin{Vmatrix}20\underline{/-15^\circ}\\ 59.5\underline{/91^\circ 58'}\\ 141\underline{/217^\circ 53'}\end{Vmatrix} \qquad (96)$$

The phase currents are then obtained by the equation $(I)=[B](I)_t$ and we have:

$$(I)=\begin{Vmatrix}88.5\underline{/2^\circ 50'}\\ 55.7\underline{/119^\circ}\\ 141\underline{/217^\circ 53'}\end{Vmatrix} \qquad (97)$$

This example illustrates how a general unsymmetrical matrix may be diagonalized by the method of section VIII. As in the case of symmetrical components, the sequence networks are independent.

XI. Conclusion

The general method presented in this paper is a generalization of the method of symmetrical components, modified symmetrical components, etc., in that by its use the proper transformation matrix may be determined to make the various sequence currents and voltages of a general system independent. It therefore appears that by the use of this method a large class of problems that cannot be solved by the use of symmetrical components because of lack of symmetry may be solved in a manner very similar to that of symmetrical components.

Bibliography

1. PRODUCTION OF TRIPLE-FREQUENCY ELECTROMOTIVE FORCES IN THREE-PHASE ALTERNATORS BY UNBALANCED LOADS, *Compt. rend.*, volume 159, 1914, pages 46–9.

2. METHOD OF SYMMETRICAL CO-ORDINATES APPLIED TO THE SOLUTION OF POLYPHASE NETWORKS, **C. L. Fortescue.** AIEE TRANSACTIONS, volume 37, 1918, pages 1027–1140.

3. TWO-REACTION THEORY OF SYNCHRONOUS MACHINES, Part I, **R. H. Park.** AIEE TRANSACTIONS, volume 48, July 1929, pages 716–27.

4. PROBLEMS SOLVED BY MODIFIED SYMMETRICAL COMPONENTS, **Edith Clarke.** *General Electric Review*, volume 41, November, December 1938, pages 488–94, 545–9.

5. TWO-PHASE CO-ORDINATES OF A THREE-PHASE CIRCUIT, **E. W. Kimbark.** AIEE TRANSACTIONS, volume 58, 1939, pages 894–910.

6. AN INTRODUCTION TO THE THEORY OF CANONICAL MATRICES, **H. W. Turnbull and A. C. Aitken.** Blackie and Sons.

High-Voltage D-C Point Discharges

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MEMBER AIEE

Synopsis: D-c corona discharges from pointed and sharp-edged conductors have been found to give rise to severe radio interference on aircraft under certain precipitation conditions. Laboratory studies, employing a point-to-plane arrangement, have shown that these discharges are greatly influenced not only by voltage and polarity but also by the geometry of the points.

Three distinct types of positive and two of negative corona have been found. One of these types, that occurring on sharp, slender points, is essentially common to both polarities and is free from radio interference at all frequencies up to at least several megacycles. Those discharges common to rather bluntly-backed points extend out some distance from the electrodes and are made up, in general, of a steady ion-migration component plus a series, orderly or otherwise, of ionic bursts. The latter phenomena, through an impulse excitation mechanism, give rise to the radio interference.

A number of oscillograms were taken with the Duddell-type oscillograph showing the simultaneous current and radio-interference characteristics of the various types of point discharges. These oscillograms, taken with carefully constructed amplifiers, show the steady component of the discharge to be free from radio interference. Each impulsive burst, however, gives rise to a burst of radio output. Cathode-ray oscillograms show these current bursts to be typical double-exponential impulses of very steep wave front, and in general, of short duration. High resistance in the discharge circuit near the ionization point has a suppressing action on the impulses and greatly reduces the accompanying interference. Further reduction can be effected by the addition of inductance in the discharge circuit.

AN extensive investigation of precipitation-static interference phenomena in aircraft radio led to the conclusion that high-voltage corona discharges were

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1. For all numbered references, see list at end of paper.

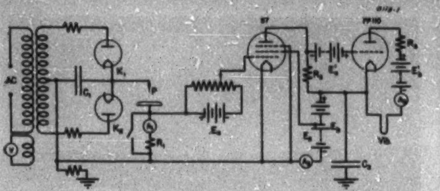


Figure 1. Resistance-coupled linear amplifier for oscillographic study of high-voltage d-c point discharges

A_1—150 microamperes
A_2—5 milliamperes
A_3—250 milliamperes
C_1—1.0 microfarad
E_b—315 volts
E_b'—125 volts
E_c—6 volts
E_c'—90 volts
E_s—113 volts
R_1—5,000–25,000 ohms
R_2—0.5 megohm
R_3—200 ohms
C_2—0.25 microfarad

responsible for the major part of this interference.[1,2,7] It was observed that these discharges were of a d-c nature and that they occurred from sharp points and edges in the vicinity of the extremities of all-metal airplanes. In an attempt to analyze the erratic character of the observed disturbances and to devise means for controlling the discharges, the investigation outlined in this paper was undertaken.

It was found that the familiar point-to-plane gap which has been used by other investigators[3] gave excellent results and that all of the different types of radio interference observed in actual flight could be produced by it under accurately controlled conditions in the laboratory. By obtaining a qualitative record on the Duddell oscillograph, it was possible to study the character of, and correlation between, the discharges and accompanying radio disturbances. The cathode-ray oscillograph indicated the true instantaneous characteristics of the discharges and made possible a thorough analysis of the basic mechanism.

Experimental Study

Early in the investigation, it was learned that the type of discharge obtained was dependent very largely on the geometry of the point employed, and that the radio interference accompanying the ionization was very definitely a function of the discharge characteristics. The maximum potentials required were of the order of 50 kv direct current, and it was essential that the voltage ripple be reduced to a minimum. The currents involved were of the order of microamperes and, since it was desired to record by means of the Duddell oscillograph both the discharge current and the accompanying radio interference, it was necessary that a current amplifier be employed. In recording the radio interference on this oscillograph, it was necessary only to provide an impedance-matching transformer between the low-frequency output of the radio set and the vibrator circuit.

The general test circuit used is given in figure 1. A full-wave, high-vacuum rectifier was employed for the high-voltage generator, and a loading capacitance of one microfarad was used to reduce the voltage ripple. The output voltage of the rectifier was determined by sphere gaps used within their proper polarity range and correlated with the voltmeter-coil instrument, designated as *V* on the diagram.

A two-stage linear vacuum-tube amplifier employing a type 57 and a *GE* type *FP* 110 tube was constructed. This is a resistance-coupled amplifier, and by means of a proper setting of the bias voltage E_0, it was possible to utilize the amplifier over its entire range for either positive or negative polarities on the point *P*. It was convenient, by connecting the deflector-plates of a cathode-ray oscillograph across R_1, R_2, or R_3, to obtain wave-form and amplitude checks against the indications of the Duddell oscillograph. These comparisons indicated that the latter oscillograph was responding with a satisfactory degree of accuracy. In the case of steep-front impulses of current or radio output, this oscillograph gave somewhat erroneous peak indications, and the vibrators displayed a minor tendency to fall into damped oscillations at their natural period. These characteristics, however, were not at all confusing.

The capacitance C_2 was necessary to prevent feedback oscillations in the amplifier circuit.

To examine the radio interference created by discharges from the point *P* to a plane, the conductor connecting this plane to the short-circuiting switch around A_1 and R_1 was carried once around the loop antenna of a Western Electric radio field-strength measuring set tuned for approximately 900 kilocycles.[4] The position of this coupling loop was so adjusted that no difference in radio output was observed when R_1 was introduced or short-circuited. The coupling was sufficiently loose to prevent the oscillations set up in the antenna circuit of the field strength set from feeding back into the current amplifier and giving erroneous indications in the amplifier output.

Types of Discharges

As the point discharge studies progressed, it was observed that when the point is positive with respect to the ground plane, there are three distinct types of discharges to be obtained. These discharge types are dependent on the geometry of the point employed and are illustrated in figure 2. Type 1 is characteristic of the discharge obtained throughout the range of voltages available, from a sharp, slender point. In total darkness, a small bead of ionization is visible at the end of the point and a tapering sheath of ionization extends for a short distance back from the point. An extremely interesting characteristic of this type of discharge is that no normal-frequency radio interference accompanies it.

The discharge indicated as type 2 is characteristic of that obtained from points

Figure 2. D-c corona discharges

Electrodes: point to plane
Polarity: positive
Spacing: 10.2 centimeters
Voltage: 44.0 kv

Type 1—Point: sharp, slender; current: 42.5 microamperes

Type 2—Point: number 0 steel needle; current: 48.0 microamperes

Type 3—Point: 60-degree conical; current: 41.5 microamperes

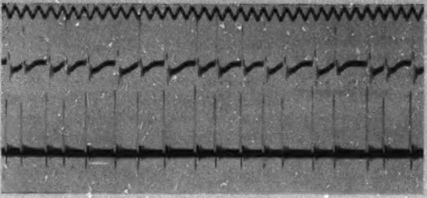


Figure 3. D-c corona discharge characteristics

Electrodes: 60-degree point to plane; spacing 10.2 centimeters

Point positive, plane grounded

$E = 15.2$ kv, $I = 2.0$ microamperes; upper, radio noise; lower, current with zero line; timing wave: one kilocycle

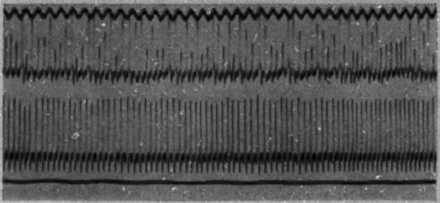


Figure 4. D-c corona discharge characteristics

Electrodes: 60-degree point to plane; spacing 10.2 centimeters

Point positive, plane grounded

$E = 38.0$ kv, $I = 28.3$ microamperes; upper, radio noise; lower, current with zero line; timing wave: one kilocycle

which have a rather acute angle of backing. That is, in general, if the angle made by the surface of the point with its center line is of the order of 30 degrees, and the point is carefully polished, this type of discharge will almost always occur. It may be described as a long, sharp ionized needle extending a considerable distance out from the point, and the current accompanying it for a given voltage is characteristically high. The radio interference produced by this type of discharge is severe and is characteristically harsh.

The type of discharge practically always obtained from a rough point or one which has a rather blunt backing, such as a conical point with sides making an angle of 60 degrees with the center line, is that illustrated as type 3 in figure 2. In this case, there is a typical bead of ionization adjacent to the point proper, and then a rather slender stem extending below the bead for a short distance before the discharge blossoms into a broad plume. The current drawn by this type of discharge is in general the lowest of the three and the radio interference produced has a characteristic musical note. The pitch of this note may run from the order of several thousand cycles per second down to less than 100 cycles per second, and is exactly similar to that described as being observed under certain conditions in airplane flight tests. It would appear, then, that at least type 2 and type 3 discharges occur from airplanes in flight under certain storm conditions.

OSCILLOGRAMS

Oscillographic records of point-discharge currents and the accompanying radio disturbances bring out a number of interesting and fundamental characteristics. The oscillogram of figure 3 shows the discharge current and radio noise characteristics of a 60-degree positive point, spaced 10.2 centimeters from the ground plane and operating at a potential of 15.2 kv. The average current, as indicated by a D'Arsonval instrument, was 2.0 microamperes. The scales were such that the current zero line blends with the actual current wave between the impulse peaks. The timing wave employed was 1,000 cycles per second, and the average frequency of the current impulses was approximately 500 cycles per second.

It is important to note that the radio noise surges occur exactly in synchronism with the current impulses, there being no radio output during the intervals of steady current flow. These current impulses, when viewed on the screen of a cathode-ray oscillograph employing a high-speed sweep, are very similar to those of an impulse generator. Figure 13 contains typical cathode-ray oscillograms of both the positive and the negative discharges. The individual steep-front impulses superimposed upon a steady direct current are shown in detail. It is evident that these impulses create the interference response in the radio receiving equipment. This subject will be treated further in the section, "Nature of Radio Interference Arising From Corona Discharges". The actual positive discharges used for the above oscillograms were of type 3.

When the voltage on the same point was increased to 38 kv, the current increased to 28.3 microamperes, and the radio noise and current appeared as is shown in the oscillogram of figure 4. The individual impulse character is still present in the current wave, but the impulses are spaced much more closely, the frequency now being of the order of 2,200 cycles per second. The radio interference, although somewhat irregular in character has the same general frequency

Figure 5. D-c corona discharge characteristics

Electrodes: 45-degree point to plane; spacing 10.2 centimeters

Point positive, plane grounded

E ‖ 43.5 kv, $I = 42$–47 microamperes; upper, radio noise; lower, current with zero line; timing wave: one kilocycle

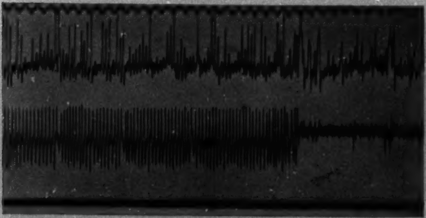
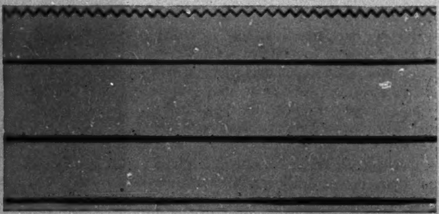


Figure 6. D-c corona discharge characteristics

Electrodes: slender, sharp point to plane; spacing 10.2 centimeters; point positive, plane grounded; $E = 43.2$ kv, $I = 40.0$ microamperes; upper, radio noise; lower, current with zero line; timing wave: one kilocycle



Type $1'$—Polarity, negative; current, 49.6 microamperes

Type 3—Polarity, positive; current, 41.5 microamperes

Figure 7. D-c corona discharges

Electrodes: 60-degree point to plane
Spacing: 10.2 centimeters
Voltage: 44.0 kv

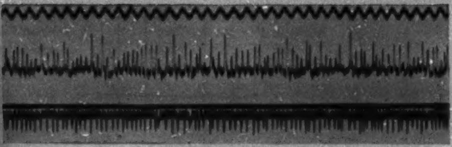


Figure 8. D-c corona discharge characteristics

Electrodes: 60-degree point to plane; spacing 10.2 centimeters

Point negative, plane grounded

$E=16.6$ kv, $I=3.0$ microamperes; upper, radio noise; lower, current with zero line; timing wave: one kilocycle

as the impulses in the discharge current. The slightly wavy zero line was due to a parasitic voltage picked up by the highly sensitive amplifier employed in the work.

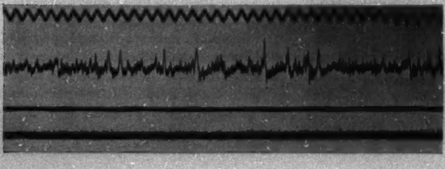
When the electrode was changed from a 60-degree to a 45-degree point, the discharges obtained were somewhat unstable in that they would occasionally shift back and forth from type 2 to type 3. The 45-degree point is on the border line between the geometry which consistently gives the type 3 discharge and that which gives type 2. The oscillogram of figure 5 was taken with this point operating at 43.5 kv to the ground plane. At the moment the exposure was made, an observer reported that the discharge changed from type 3 to type 2, and the record of this change is given on the oscillogram. The typical high-frequency succession of current impulses produced by the type 3 discharge is indicated in the first part of the oscillogram. On three occasions, however, there were breaks indicating that a change in discharge type was about to occur. Near the end of the oscillogram this change did occur, and the irregular current wave and associated radio interference typical of the type 2 discharge are shown in the last one-third of the record. It is indicated that there was a distinct change in the character of the radio interference in that the definite periodicity of the output impulses was largely gone. It is also of interest that the current increased from 42 microamperes during the type 3 discharge to 47 microamperes during the type 2 discharge.

Figure 9. D-c corona discharge characteristics

Electrodes: 60-degree point to plane; spacing 10.2 centimeters

Point negative, plane grounded

$E=36.2$ kv, $I=28.3$ microamperes; upper, radio noise; lower, current with zero line; timing wave: one kilocycle



It has been stated that the type 1 discharge which is obtained from sharp, slender positive points is free from radio interference. This characteristic is illustrated by the oscillogram of figure 6. The radio output is shown to have contained no interference whatever, while the current wave shows a number of very small impulses. These disturbances were evidently not sufficiently great to create any interference in the radio receiving set when tuned for normal frequencies.

After the earliest types of points had been subjected to a rather complete study under positive polarity conditions, the high-voltage generator was reversed and the studies were repeated for negative polarities. It was soon learned that all types of points, when of negative polarity, give discharges having essentially the same appearance. They seem similar to the type obtained for a positive sharp, slender point. A comparison between the type $1'$ discharge obtained on negative polarity and the type 3 obtained when positive, for the same identical point, is given in figure 7. It will be noted that the glow at the point of the electrode is very intense and restricted in the negative case, and that the current is approximately 20 per cent higher than for the same voltage when positive. Microscopically, the type $1'$ discharge negative is different from the type 1 positive when the negative discharge is obtained on anything but the sharp, slender point. On the bluntly backed point, the negative discharge appears to be a very small but intense, localized plume of ionization in contrast to the very much larger but less intense type 3 discharge obtained when the same type of point is of positive polarity. As brought out later, there is a very fundamental difference between the negative discharges from sharp, slender points and those from the bluntly-backed type of points.

The radio interference produced by the negative discharges was erratic in its character for all types of points except the sharp, slender one illustrated in figure 2. In this case, again, no perceptible interference was produced within the range of voltages available for the test.

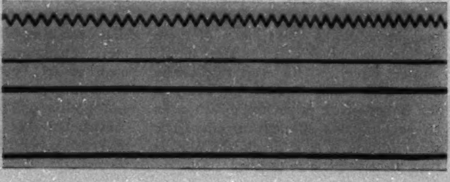
The oscillogram of figure 8 compares with that of figure 3 with the difference that the former is negative and the latter is positive. The currents and voltages were of the same order of magnitude and identical points were used for both. It is shown that the negative discharge current, in common with the positive as illustrated in figure 3, is made up of a con-

Figure 10. D-C corona discharge characteristics

Electrodes: slender, sharp point to plane; spacing 10.2 centimeters

Point negative, plane grounded

$E=38.5$ kv, $I=39.0$ microamperes; upper, radio noise; lower, current with zero line; timing wave: one kilocycle



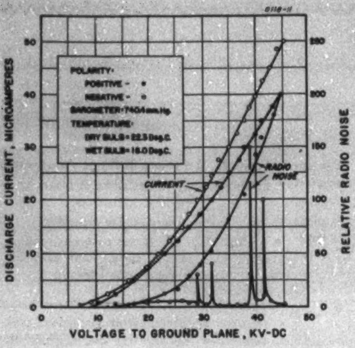


Figure 11 (left). D-c point discharge characteristics

Electrodes: 30-degree conical point to plane; spacing: 10.2 centimeters

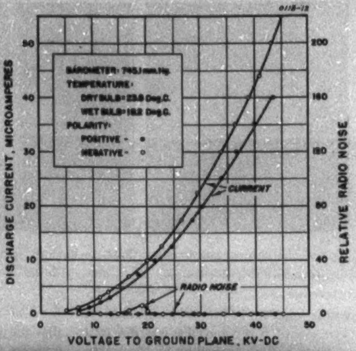


Figure 12 (right). D-c point discharge characteristics

Electrodes: sharp, slender point to plane; spacing: 10.2 centimeters

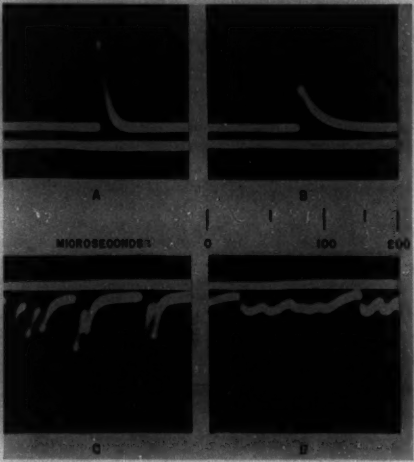


Figure 13. Cathode-ray oscillograms of point discharge currents

Electrodes: 60-degree point to plane; spacing 10.2 centimeters

Polarity: *A* and *B*, point positive; *C* and *D*, point negative

Current, average: *A* and *B*, eight microamperes

C and *D*, ten microamperes

Discharge resistor: *A* and *C*, zero

B and *D*, ten megohms

tinuous value with a superimposed high-frequency series of small impulses. The frequency of these impulses is of the order of 2,000 cycles per second as compared with approximately 500 cycles per second in the positive case. The amplitude of the negative impulses is considerably less than that of the positive. Again, the radio interference is well synchronized with the current impulses although somewhat less regularly so than in the positive case.

For the very low discharge currents, it was observed occasionally that in the negative case also an irregular musical note was produced by the interference in the radio receiver. When the voltage is increased to higher values, the negative impulses lose their individual distinction. In the case of most types of points, the interference becomes very erratic sometimes rising to high values and then dropping off to practically nothing only to rise again, perhaps to intermediate values, as the voltage is slowly changed.

The oscillogram of figure 9 compares with that of figure 4 in that the points and discharge currents were identical with polarities reversed. In the positive case, the radio interference peaks were much higher than in the negative case although the latter condition seemed to give the more irregular interference. Close observation will bring out the fact that the current wave in the negative case is made up of a continuous value plus a heterogeneous succession of current impulses. A great many of these impulses will be observed to start in the positive direction. In other words, the average current is negative, but many of the impulses are in such a direction as momentarily to reduce the current rather than increase it as is practically always the case with positive polarity. This reversal phenomenon was particularly evident for the larger values of discharge current when the oscillograph screens were observed visually. The cathode-ray oscillogram of figure 13*C* indicates the erratic character of the negative discharges for currents well above the threshold value.

In the experimental work with negative discharges, the 60-degree point was practically the only one that could be depended on to give finite radio-interference characteristics of any regularity whatever. Furthermore, points that were made of steel or copper were rapidly altered in their characteristics by oxidation and positive-ion bombardment. Points constructed of tungsten gave much better results for operation over long periods of time.

An oscillogram of the sharp, slender point operating at 39 microamperes negative is shown in figure 10. This oscillogram compares with that shown in figure 6, the difference being only in polarity. Again, there is no perceptible radio interference and the current contains no apparent ripple. It should be explained that the slight appearance of radio interference was due to an extraneous disturbing factor, foreign to the test setup.

Characteristic Curves

A series of discharge-current and radio-interference data was taken on each of several different types of electrodes. Typical curves obtained from these data are given in figures 11 and 12. The discharge characteristics of a 30-degree conical point to plane are shown in figure 11.

It is indicated that the negative currents were considerably higher throughout the entire range than the positive, and that the negative radio interference was very erratic. The character of discharge occurring during this test was type 1′ for the negative, and throughout most of the range, a combination of types 2 and 3 for the positive.

The very interesting and useful discharge characteristics of the sharp, slender point are given in figure 12. It is shown that the currents for this electrode were the highest of all the different points tested, but the radio interference, with the exception of one very weak disturbance at a relatively low voltage, was zero throughout the entire range. During practically all of the tests with this particular point, no interference was obtained on either polarity over the entire voltage range. The data indicating a small amount of interference were included on this particular curve because on one occasion a minor disturbance did occur in that region.

No attempt was made to formulate a mathematical law of the discharge current as functions of voltage, spacing, and point configuration. Such a study is sufficiently involved to warrant special consideration. For both polarities and for all of the different types of points, it is indicated, however, that the discharge currents in the region near the threshold voltages are an approximate parabolic function of the voltage above the critical disruptive value. For the higher voltages, these currents approach a linear function of the point potential. It is indicated also that the sharpness of the point, as well as the type of discharge, has a definite bearing on the currents. As the points are made sharper, that is, as the included angle becomes less, the discharge current for both polarities becomes greater. For all of the conditions employed in these tests, the negative currents were observed consistently to be higher than the positive.

Mechanism of Discharge

The foregoing curves and oscillograms point clearly to the nature and mechanism of high-voltage, d-c discharges from points. Again, as in the case of a-c corona discharges from conductors, space-charge phenomena are controlling factors in the point discharge mechanism.[3,5,6]

When the discharge electrode is positive, and of a blunt-backed type such as the 60-degree conical point, a general ion migration seems to account for a continuous flow of current between the point and the ground plane. Positive streamers, composed of electron avalanches, periodically project into space from the point, creating positive space charges which momentarily reduce the gradients and block further ionization until they have become reduced in intensity to the point where succeeding streamers can develop. Since the entire point is somewhat symmetrical, creating a localized region of high electric field concentration surrounded by a relatively large background shield, the phenomena repeat with definite regularity. The frequency of repetition as well as the intensity of the streamers depends largely upon the voltage and, therefore, upon the current being discharged from the point. These individual positive streamers are responsible for the current impulses superimposed upon the otherwise relatively steady flow of current.

In the case of the sharper points with a less effective backing, the space charges do not build up with such definite regularity, and the positive streamers are of a more random character. The orderly repetition of current impulses, therefore, does not occur.

Surrounding the end of the extremely sharp point having a long slender shank, the positive space charge seems to build up to a stable equilibrium and remain there over a wide range of discharge currents. This space charge, with its internal gradient falling off rapidly from the point outward, blocks off the development of any positive streamers and, hence, no current impulses occur.

In the case of the negative polarity, the positive-ion space charges are established in the region of high potential gradients and tend to move in toward the point of discharge. As they approach it closely, the voltage gradients build up to the point where further impulsive discharges occur. This mechanism results in the extraction of large numbers of electrons from the electrode and its vicinity. For low values of negative discharge currents, these electrons move outward with relatively low velocities and create, momentarily, a neutral or even negative space surrounding the point. As more remote ions move in, the positive space charge is again established, and the process is repeated. These phenomena result in the negative current impulses appearing in the circuit between the plane and ground (see figure 13*C*). However, at the higher voltages, the clouds of electrons are projected with much higher velocities and result in the rapid creation of high positive space charges which actually block off, momentarily, the passage of more electrons out into the interelectrode space. Hence, the current appearing in the ground circuit tends to drop off abruptly as each of these high positive space charges is created, thus resulting in apparent positive impulses on the negative current flow. As these positive space charges move in toward the ionization point, breakdown occurs with great abruptness and, hence, the fronts of the current impulses are exceedingly steep although the impulses themselves are not large in magnitude. Professor Loeb and his associates have done considerable work on the physical nature of these ionization phenomena and have studied certain phases of the subject in great detail.[8]

Two of the most interesting characteristics brought out by the above tests are as follows:

1. A sharp, slender point can be caused to discharge positive or negative electricity, in amounts up to at least 40 or 50 microamperes per point, without creating any appreciable radio interference at frequencies up to several megacycles.
2. Rough points, which microscopically are sharp points with rather blunt backing, produce the most severe negative radio interference and also produce the musical-note type of positive interference.

Nature of Radio Interference Arising From Corona Discharges

The nature of d-c point discharges as brought out by the oscillographic study outlined in the previous section gave a very definite clue to the mechanism of the radio interference produced by such discharges. It was evident that the disturbances were created by the steep-wave-front current impulses. Particularly in the low-current positive discharge studies, it was obvious that each impulse was accompanied by a heavy shock in the radio.

The essentials of the circuit employed in these tests were such that the energy input to the radio receiver was through an inductive coupling between the circuit carrying point-discharge current and the parallel-resonance antenna circuit of the receiver. The impulsive currents, in flowing to ground through such an inductively coupled system, induce transient voltages in the antenna circuit of the radio receiver. These voltages drive the tuned antenna circuit into oscillation at its natural period, and since each resultant oscillation has a definite decrement due to circuit losses, the high-frequency disturbance carried into the radio-frequency amplifiers effectively is modulated and appears in the output circuit as a surge of energy. The intensity of this surge depends on the amplitude of the oscillation induced in the antenna circuit by the steep-wave-front current impulse. Since the induced voltage is equal to $M \cdot di/dt$, the steepness of wave front of the current impulse, as well as its magnitude, influences this voltage. A current impulse of high amplitude and steep wave front, therefore, would be expected to induce a severe oscillation in the tuned antenna circuit of the radio receiver and a resultant heavy surge in the output.

In many cases of radio disturbances produced by point discharges, the coupling of the discharge point, and of the space charge surrounding that point, to the radio antenna is capacitive. Here again the individual current impulses occurring in the discharge circuit produce similar, although much smaller, current impulses in the primary of the antenna-circuit coupling coil. The same mechanism, therefore, applies in this case as in the former with respect to the transient excitation of the tuned radio-frequency circuits.

IMPULSE EXCITATION OF OSCILLATORY CIRCUITS

It is evident that in every practical case, the coupling of the radio receiver to the discharge circuit is such as to cause steep-wave-front currents to pass through the primary of the antenna coupling transformer when similar currents appear in the corona discharge circuit. The subject resolves, therefore, into a study of the impulse excitation of oscillatory circuits. This problem has been studied analytically and is reported elsewhere.[7] A brief summary of the findings will be given here:

1. The steepness of front and amplitude of the impinging impulse have a direct influence on the magnitude of the oscillation induced in the tuned circuit.

2. The steepness of descent of the tail of the impulse, if it occurs at approximately one-half period of the resonant circuit, has a direct influence on the amplitude of the oscillation.

3. The area under the impulse-current wave for the first one-fourth period of the oscillatory circuit has a direct bearing on the magnitude of the induced oscillation.

4. Resistance in the discharge circuit, since it cuts down the amplitude of the current impulse and therefore the average rate of rise of the wave from zero to crest, reduces the amplitude of the induced oscillation. Figure 13 illustrates this property oscillographically. Since resistance does not greatly change the initial rate of rise of the front of the impulse, its effect is most pronounced for the lower oscillation frequencies.

5. Inductance in the discharge circuit, since it reduces the steepness of front of the current impulse, tends to cut down the amplitude of the induced oscillation.

The above summary indicates that not only resistance but also inductance in the high-voltage point-discharge circuits should be effective in reducing the impulse excitation of the tuned antenna circuit of the radio receiver. This analysis again indicates that the type of radio interference produced by these high-voltage discharges is of impulse excitation character, the only high-frequency oscillations appearing in general being those set up in the antenna circuit of the radio receiver, at the frequency for which it is tuned, by the impinging impulse.[*] It is evident, therefore, that any operation upon the interfering circuit which will eliminate or will lessen the amplitude and reduce the steepness of wave front of the individual current impulses will result in improved radio interference characteristics. Continuous, heavy ionization at the discharge point such as can be produced by the projection of an ionized liquid, by relatively low-voltage d-c arcs or by high-voltage, radio-frequency arcs, will result in the elimination of current impulses and their accompanying radio disturbances. The performance of such devices under high-velocity air movement conditions is uncertain.

From the above analysis, it is further evident that as the resonance frequency of the tuned input circuit of the radio receiver is increased to the high and ultrahigh-frequency regions, the amplitude of the induced oscillations created by the current impulses will decrease to a marked degree. For a given radio-receiver sensitivity, then, much less interference would be expected from the high-voltage point discharges in the high and ultrahigh-frequency regions than in the normal and low-frequency bands. Recent experience with ultrahigh-frequency equipment has borne out this prediction.

The influence of discharge resistors and inductors on the radio-interference characteristics of point discharges was examined experimentally, and typical results on a 50-degree conical point indicated that a series resistance of two megohms would produce a reduction of over 50 per cent. A further 50 per cent decrease in the remaining interference followed the introduction of a 37-milihenry inductor. This inductor was not of a particularly good radio-frequency type, but it did produce a definite improvement in the interference characteristics.

It follows then that the introduction of large amounts of resistance and inductance, with a minimum of distributed or shunt capacitance, in the high-voltage circuit as close as possible to the discharge point will result in minimized radio disturbances. Certain precautions, however, must be employed in using these resistors and inductors. The voltage of the point may be expected to change abruptly with each impulsive discharge when resistors and inductors are incorporated in the discharge circuit. To prevent these sudden changes of voltage from inducing current impulses in the radio antennas, some form of shielding about the point might be necessary.

Conclusion

It has been shown that there are at least three distinct types of positive corona discharges from pointed electrodes, and two less sharply defined types of negative discharges. All of these discharges, with the exception of one type which is common to both positive and negative polarities and occurs only on sharp, slender points, give rise to an impulse excitation type of interference in nearby normal-frequency radio-receiving equipment. The silent type of point offers excellent possibilities for the development of an airplane precipitation-static charge-dissipating device. Because of its fundamental mechanism, it seems probable that point-discharge interference will not exist in any appreciable magnitude in receiving equipment operating at the ultrahigh frequencies. Further studies are under way in the ultrahigh-frequency field and in the development of interference suppressors.

Bibliography

1. PRECIPITATION-STATIC INTERFERENCE ON AIRCRAFT AND AT GROUND STATIONS, **H. M. Hucke.** *Proceedings* of Institute of Radio Engineers, volume 27, May 1939.

2. AIRCRAFT PRECIPITATION-STATIC RADIO INTERFERENCE, **R. C. Starr.** AIEE TRANSACTIONS, volume 60, 1941, pages 356–64.

3. MECHANISM OF THE POSITIVE POINT-TO-PLANE CORONA IN AIR AT ATMOSPHERIC PRESSURE, **G. W. Trichel.** *The Physical Review*, volume 55, number 4, February 15, 1939.

4. PORTABLE RECEIVING SETS FOR MEASURING FIELD STRENGTH, **Axel Jensen.** *Proceedings* of Institute of Radio Engineers, volume 14, June 1926.

5. THE LAW OF CORONA AND THE DIELECTRIC STRENGTH OF AIR—IV, **F. W. Peek, Jr.** AIEE TRANSACTIONS, volume 46, 1927.

6. SOME CHARACTERISTICS OF A-C CONDUCTOR CORONA, **F. O. McMillan.** AIEE TRANSACTIONS, volume 54, 1935 (March section), pages 282–92.

7. PRECIPITATION-STATIC RADIO INTERFERENCE PHENOMENA ORIGINATING ON AIRCRAFT, **R. C. Starr.** Bulletin number 10, 1939, Engineering Experiment Station, Oregon State College.

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[*] In certain cases, parts of the circuit carrying the discharge currents are driven into complex, damped high-frequency oscillations and interfering energy is radiated directly. This mechanism, however, is believed to be the exception rather than the rule, particularly in direct-current corona phenomena.

Aircraft Precipitation-Static Radio Interference

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Synopsis: Radio reception at normal frequencies on board high-speed, all-metal aircraft suffers severe interference under certain precipitation conditions. Flight research has shown that under these same conditions, an airplane will accumulate a relatively heavy electric charge. The resulting potential difference between this charged object and its immediate surroundings creates voltage gradients of sufficient magnitude on pointed and sharp-edged extremities of the craft to produce corona discharges. Local radio disturbances of varying character and severe intensity accompany these discharges.

Several different mechanisms were found to be involved in the production of the discharges, and currents of magnitudes greater than one milliampere were observed under severe conditions from a single point. Numerous graphic records were taken in flight showing the correlation between the charge and discharge currents at various points on the airplane. These records also illustrated the rapidly fluctuating magnitude of the discharges and indicated that their directions reversed rapidly under certain space-gradient conditions.

Ground tests involving the research airplane mounted on insulating supports and charged by means of a high-voltage rectifier, indicated that the radio interference and discharge currents observed in flight could be duplicated under controlled conditions on the ground. Further ground and laboratory tests verified and explained the recognized efficiency of the shielded radio loop in improving to a marked degree the signal-to-noise ratio in the presence of corona discharge interference.

The mechanism of the interference was studied intensively in the laboratory. Artificial airplane discharging devices involving trailing and projecting conductors terminating in special points and containing interference-suppressing resistors and inductors were developed. A simple but effective trailing discharge conductor to be projected and used in severe precipitation-static emergencies has been adopted by at least one large air transport company.

One of the elements essential to the success of modern scheduled aerial transportation is a reliable system of radio communication. Continuous contact with strategically located ground stations enables aircraft flight personnel to maintain accurate courses and to exchange important weather and dispatching information. As a further service, still in an experimental stage, the radio is making instrument or "blind" landing operations highly successful. The application of radio aids to aerial navigation is covered fully elsewhere.[1,2]

During normal weather conditions, these aids are successful in their operation, but at times when most needed, during heavy weather with low visibility, they may become erratic or entirely useless. Flight research has shown that certain precipitation conditions accompanying atmospheric storms cause large, high-speed, all-metal airplanes to become charged electrically and, subsequently, to discharge to the surrounding atmosphere producing local radio disturbances of severe intensity. It has been found that these charges are accumulated by a number of different mechanisms, and that the discharge is a process of maintaining the potential of the airplane at a value not greatly different from that of its surroundings.

Tests in flight and on the ground have indicated that these discharges are the same as those which can be created by high, direct voltages impressed upon point electrodes. An extensive study was made of the nature of these discharges and of means available for controlling them.

It is suggested that a systematic electrical discharging arrangement can be installed on airplanes to control effectively the accumulated charges and reduce the radio interference resulting from their dissipation to the atmosphere. In the form of a trailing wire, such discharging equipment has been installed on all of the transport airplanes of at least one of our major air lines. This trailing wire is so mounted that it can be projected into its normal position when severe static conditions develop and can be replaced economically when destroyed by contact with the ground during landing operations. Although this device is very simple and somewhat limited in its ability to discharge the airplane, it has, nevertheless, given a good account of itself in a number of different emergencies.

Radio Interference

In heavy weather, particularly in violent squalls and in air mass fronts, it is especially important that the airways radio ranges and marker beacons be operative. Under these conditions local cross winds of severe intensity may tend to drive the airplane far from its course unless the range signals can be received satisfactorily. Unfortunately, such weather conditions are usually accompanied by some form of precipitation. There may be snow, rain, dust, or frost crystals in the air. Experience has shown that an airplane, particularly of the all-metal variety, flying at speeds materially above 100 miles per hour into such precipitation, in general has all but its ultra-high-frequency radio reception very seriously impaired. The interference may appear in any one of, or in a combination of several different forms.

It has been known since the early days of radio communication that electrical discharges from conductors, either of low or high voltage, will produce local interference in receiving equipment. The making or breaking of a low-voltage circuit will produce a shock disturbance in the receiver. Low-voltage vibrating-type interrupters and machine commutators may produce such a rapid succession of these shock disturbances as to create a musical note in the receiver. High-voltage corona discharges on conductors and insulators are known to create serious radio interference.[3] On the airplane the high-voltage ignition system of the engine must be shielded very carefully to avoid the introduction of heavy shock interference into the receiving equipment.

Some of the radio interference created by an airplane flying at high speed into an area of precipitation is of such a type as to be susceptible to improvement by noise-suppressor circuits.[4] When the airplane is in a light snow of relatively large individual flakes, the interference created is generally of a typical low-frequency vibrator type. Such interference could be removed by noise-suppressor circuits. However, when the flight is through most forms of precipitation of normal density, the interference created is of such a continuous character that no noise-suppressor circuit at present avail-

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1. For all numbered references, see list at end of paper.

able has any beneficial effect. The interference ranges in nature from a harsh, rasping sound in the head phones to a varying, high-pitched musical note. The intensity frequently becomes so high as to make necessary the reduction of receiver gain to a point where the normal radio range signals in the absence of interference, could be read only a very short distance from the transmitter.

Such disturbing conditions may exist for only brief periods during the passage through a precipitation area or, in the event of flight along an air mass front, they may exist for a matter of 30 minutes or more. Under such conditions, the normal-frequency radio range cannot be read, and no reception of voice communication from the ground or other airplanes is possible. The flight personnel must, therefore, resort to dead reckoning to maintain the course at a time when the radio navigational aids are most essential.

Precipitation-Static Flight Research

As airplanes became larger and speeds became higher, the interference problem became more acute. A number of accidents were attributable directly to airplanes being off-course due to their having lost the radio range under storm conditions. Having then encountered unreported squalls for which no correction could be made, they were driven unknowingly into dangerous terrain. The hazard was recognized by pilots as well as by administrative personnel and efforts were started to attempt to solve the problem. At first it was thought that the interference was due to charged particles of precipitation impinging upon the exposed radio antennas. In an effort to eliminate this difficulty, the antenna conductors were covered with rubber insulation, but no improvement was obtained.

It was reported by a number of pilots that under certain cold, dry-snow and frost-crystal conditions, the airplane windshields had been seen to glow with a brush discharge. At times these brushes were reported to become so severe as to give the appearance of the windshield sparking over completely. Under these same conditions, the rubber deicers on the leading edges of the airplane wings were observed to glow with brush discharges, and at times to appear practically to spark completely across. The tips of the propellers were observed to glow at times with a typical Saint Elmo's fire.

These reports definitely established the existence of an electrical charging mechanism which was causing insulated areas on the airplane to attain very high potentials with respect to the ship itself. If that were the case, it was reasonable to assume that by the same mechanism the entire airplane was being charged to potentials considerably above that of the surrounding atmosphere. It was further evident, since the insulated surfaces such as windshields and rubber deicers were being charged to potentials considerably above those of the airplane proper, that the potential of the ship was being held down by some form of discharge mechanism. In other words, it appeared that the windshields and deicers were discharging to the ship, adding to the charge it was receiving by similar action, and the whole was discharging to the atmosphere. If such were the case, at least one source of the observed radio interference was the d-c brush or corona discharge from sharp extremities of the airplane to the atmosphere.

Figure 1. Interior view of precipitation-static research airplane showing synchronized graphic-recording milliammeters and current amplifiers

Recognizing this possibility, one of the large air transport organizations led a co-operative, systematic study of the problem. A regular ten-passenger, two-motor Boeing type 247-*D* transport airplane was withdrawn from passenger service and made available as a research ship. A number of the seats were removed and a test bench for scientific instruments was installed in their place. A partial view of the arrangement is shown in figure 1.

The exterior of the plane was equipped with a number of insulated charge- and discharge-measuring electrodes. Those which gave the most interesting results were as follows:

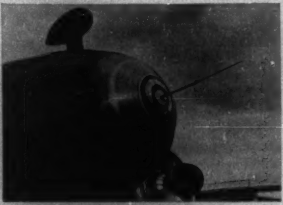


Figure 2. Nose electrode and nose plate installed on Boeing type 247-D airplane for precipitation-static research

1. A short rod projecting from the side of the fuselage directly behind the left propeller tips.
2. A sphere supported below the fuselage and behind the Pitot stub.
3. A 30-inch pointed rod projecting from the nose of the plane ahead into undisturbed air.
4. An insulated plate in the form of an annular disk, conforming to the streamlined shape of the nose of the plane.
5. Three pointed rods, each approximately 12 inches long, projecting from each wing tip.
6. A 30-inch pointed retractable rod extending rearward from the tail of the ship.

The nose electrode and nose plate are illustrated in figure 2. Each of the various electrodes was well insulated from the airplane and was connected through conductors to a terminal board beneath the test bench inside the cabin.

The instruments used in determining the currents of charge and discharge, and the polarity and character of these quantities were as follows:

1. Two highly-sensitive vacuum-tube electrometers.

2. A cathode-ray oscillograph.

3. Three synchronized graphic-recording milliammeters, driven by a three-element vacuum-tube ammeter.

4. A standard air-transport-beacon receiver.

5. A special Bell Telephone Laboratories receiver equipped with noise-suppressing circuit.

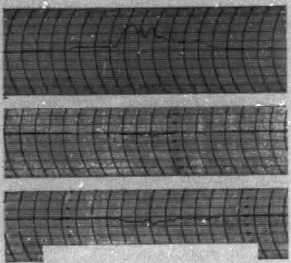


Figure 3. Airplane discharge currents; nose plate, nose and tail electrodes

Upper—Nose plate; center—nose electrode; lower—tail

Precipitation: dry snow

Scales: upper 0.1, others 10 microamperes per major division; eight units per minute; plane negative for negative deflections; Boeing 247-*D*

6. Various microammeters and auxiliary equipment.

The graphic recording instruments from which the most orderly data were obtained, together with their driving vacuum-tube ammeter, are shown in figure 1.

The airplane was also equipped with an intercommunication telephone system which could be connected to the output of a radio receiving set being driven by any one of the following antennas:

1. The standard "belly V" beacon receiver antenna.
2. The short-wave transmitting antenna.
3. A shielded loop projecting beneath the fuselage of the plane and located approximately on a line between the two engines.
4. A shielded loop in a streamlined housing above the nose of the airplane.
5. A diagonal antenna extending from one end of the horizontal fin to a position approximately midway of the fuselage of the ship.
6. An interchangeable shielded loop or crossed dipole projecting forward from the nose of the ship.

Neither of the last-named antennas could be employed at the time the nose electrode and nose plate were in use. For a time a straight-wire antenna enclosed in a nonconducting tube coated with a high-resistance conducting compound was installed beneath and parallel to the fuselage of the airplane.

In the early work an attempt was made to observe the charge and discharge currents on indicating instruments and to correlate these currents with the radio interference observed on the different antennas. It was soon learned, however, that the conditions change so very rapidly that it was not possible to take notes fast enough to obtain an accurate record.

Before many tests had been made, it was observed that all of the wire antennas gave approximately similar results. The one exception was the antenna extending toward the rear of the ship and this one was found to have a consistently higher noise pickup property than the others. It was also observed that the shielded loop antennas, in agreement with previous tests, gave a noticeably higher signal-to-noise pickup ratio than any of the wire antennas.

Since the standard "belly V" beacon receiver antenna gave results essentially as good as any of the wire antennas, it was used as the comparison standard throughout most of the tests. It was found that the interference picked up by this antenna became severe as soon as any of the pointed electrodes began to pass an appreciable current. When the electrode currents exceeded the order of 5 to 15 microamperes, the interference in the "belly V" became so intense as to make impossible the reception of radio-range signals even when relatively near the range transmitter.

The discharge interference picked up by the shielded loop antennas was noticeably much less than that picked up by the "belly V", but when the electrode currents approached orders of magnitude of 50 microamperes or more, the loop interference became excessive.

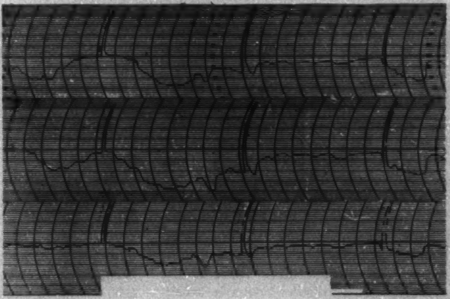
CHARGE AND DISCHARGE RECORDS

In order to determine something of the mechanism of the airplane charge and discharge phenomena, the graphic recording milliammeters were so connected as to obtain simultaneous records of three currents. For some of the tests, the nose plate, nose and tail electrodes were used, and for other tests the wing tip and tail electrodes were employed.

A great number of records were obtained by flights in different types of precipitation in various cloud formations, and examples of these records are given in figures 3, 4, 5, and 6. The record of figure 3 is low in intensity but is typical of those obtained under dry snow conditions. It is indicated that the nose plate charged negatively, whereas the nose and tail pointed electrodes discharged positively. That is, in the case of the nose-plate record the airplane was positive with respect to the plate, while in the case of the nose and tail electrodes, the ship was negative with respect to the points. Since no appreciable ionization could be taking place from the nose plate, it was obvious that this surface was charged negatively by the precipitation in which the airplane was flying. The points, however, since they would readily admit of ionization, were discharging this negative electricity into the atmosphere. This was a typical mechanism frequently observed in relatively stable dry-snow and frost-crystal areas.

A typical case of the airplane discharging heavily from all electrodes on which measurements were being taken is recorded in figure 4. The very consistent character of the negative electricity discharge from both wing tips and from tail electrode is shown. It will be noted that the change of intensity of the dis-

Figure 4. Airplane discharge currents; wing-tip and tail electrodes



Upper—right wing; center—left wing; lower—tail

Precipitation: wet snow

Scales: ten microamperes per major division, eight units per minute; plane negative for negative deflections; Boeing 247-*D* charge from any one electrode was accompanied by a similar change in the discharges from the other two electrodes. It is obvious that the ship was being charged negatively very strongly by some action of the precipitation in which it was flying. In this case the precipitation was heavy snow. The uniform vertical deflections present at periodic intervals in each graph were created by a special timing device and represent one-minute intervals. The radio interference accompanying the discharge in this case was very severe and of harsh character.

Many of the records indicated high longitudinal or transverse voltage gradients across the airplane. The ionization electrodes on the extremities of the ship under these conditions indicated currents flowing from nose to tail or from wing to wing. Such conditions were usually transitory, and sudden reversals of gradient frequently occurred.

An interesting condition of high transverse gradients is illustrated by the record in figure 5. In this case, the right wing was in an area which was very strongly negative with respect to that surrounding the end of the left wing. Transverse currents of considerable magnitude were flowing from wing tip to wing tip as a result of this short-circuiting action. Near the end of the record a slight reversal of the gradient occurred. It is interesting also that in this case the tail electrode was relatively inactive. Its tendency was to alternate somewhat between negative and positive discharges of relatively weak magnitude.

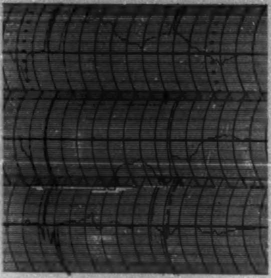


Figure 5. Airplane discharge currents; wing-tip and tail electrodes

Upper—Right wing; center—left wing; lower—tail

Precipitation: light, dry snow

Scales: Ten microamperes per major división, eight units per minute; plane negative for negative deflections; Boeing 247-*D*

A condition different from any other so far discussed is brought out in the record of figure 6. In this case the nose plate was receiving positive electricity at a time when both the nose and tail electrodes were discharging negatively. A period of longitudinal gradient reversals affecting only the ionization points also is brought out in this record. The most interesting feature, however, is that the ionization electrodes were indicating heavy negative discharges at a time when exposed surfaces, represented by the nose plate, were receiving positive electricity. This condition indicated charging mechanisms that were apparently considerably different from those in the other records. The precipitation into which the airplane was flying was evidently either neutral or positively charged, or the impact of the air foils with this precipitation resulted in a positive charge. The ship, however, was being charged negatively as indicated by the negative discharges. This leads to the belief that the propeller action in breaking up the heavy snowflakes was such as to create a resultant negative charge on the airplane even though some positive charge was being accumulated by contact of the air foils with the charged snow flakes.

A careful study of the discharge records leads to several rather definite and other somewhat obscure conclusions. It is well known that cloud masses can become heavily charged by some type of thermal activity similar to that described by Simpson.[8] When an airplane flies into one of these charged areas, it must gradually attain the potential of that area, first by electric induction and later by actual contact with the charged precipitation. A potential gained in such a manner would place the ship at the same potential as the region in which it is flying, and hence would result in no localized voltage gradients that would produce corona discharges.

It is evident then that the contact with and breaking up of particles of precipitation by the air foils and propellers results in charging the plane to a potential different from that of the particles themselves. The extreme windshield and deicer potentials observed at times indicate that contact with and breaking up of dry-snow and frost-crystal particles results in extremely high charge accumulations. The record of figure 6 indicates that the action of the air foils of the propellers may be such as to produce negative charges on the airplane from positively charged or neutral precipitation.

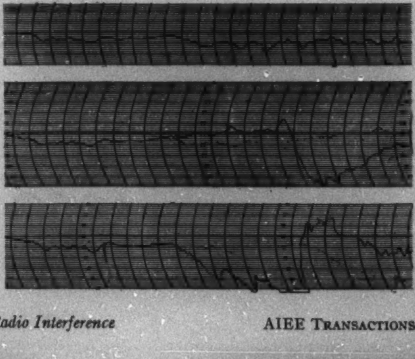
In large storm areas, where the precipitation is rather general and where the vertical air currents are not pronounced, a reasonably uniform charging condition may exist. At such times the airplane will attain a high negative potential with respect to the surrounding atmosphere and will discharge continuously. However, in squally conditions, notably in heat clouds, the charged areas are not at all uniform. The airplane may, therefore, fly from an area of low charge to one of much higher potential in a very brief period of time and, in so doing, the ship must be brought to the potential of the new region. This change will result in heavy corona discharges of short duration between the plane and the higher-potential region into which it is flying. Subsequently, as the ship leaves this high-potential region and enters one of lower potential, the reverse operation will take place, that is, the ship will discharge its higher voltage to the lower potential region. Most of the high longitudinal gradients illustrated in the records were due to conditions of this character. The so-called "crying" or musical-note static in the radio receivers was observed to accompany these particular conditions, and it will be shown in a companion paper "High-Voltage D-C Point Discharges" that positive corona dis-

Figure 6. Airplane discharge currents; nose plate, nose and tail electrodes

Upper—nose plate; center—nose electrode; lower—tail

Precipitation: heavy snow, center of cloud

Scales: upper 0.1, others 4 microamperes per major division; eight units per minute; plane negative for negative deflections; Boeing 247-*D*



charges under many conditions characteristically produce this type of interference. Under certain rather critical conditions, the negative discharges will also produce the musical note interference.

It was found that the discharge currents under practically all flight conditions fluctuate very rapidly. Some of the observations indicated transverse currents as great as one milliampere, and discharges from the tail electrode approaching the same order of magnitude were recorded on a few rather extreme occasions.

The characteristic Saint Elmo's fire discharge, so frequently reported as being observed during the hours of darkness at the propeller tips, is undoubtedly due to two elements. The first is that these propeller tips are very sharp, and since they extend out a considerable distance from the shielding influence of the fuselage and wings, they constitute excellent discharge points. The second is that due to the very high peripheral speeds of these propellers, as well as their pitch, a low-pressure region exists near the tips. Since ionization occurs much more readily at low absolute pressures than in the normal atmosphere, the discharges would take place in these regions at reduced potentials.

The influence of size and speed of aircraft upon the charging action observed in precipitation areas and hence the influence of these same factors on the radio interference created is pronounced. The larger and faster craft, since their frontal areas are larger and since their speeds are greater, make contact with more particles of precipitation per unit time than the smaller ships. The result is a definitely higher charge and discharge rate together with a greater volume of interference. The broad wing spans and great axial lengths of the larger ships cause them to span greater areas and hence to create greater disturbances in the regions of high voltage gradients.

Jets of water directed into the rapidly revolving propellers with the airplane at rest caused charges to accumulate on electrodes located in the slip stream. These charges were either positive or negative depending on the salt content of the water. It appears, therefore, that this charging action is similar to that obtaining when falling drops of water are subjected to a rising jet of air.[6,7]

Since various areas on airplanes experience such pronounced charging actions under certain storm conditions, it is evident that very thorough bonding should be practiced between all conducting parts of the airplane.

Any exposed insulating surfaces such as the windshields, the deicers, insulating housings over radio loops and the exposed landing wheel tires, will, under certain conditions as outlined previously, accumulate electric charges which will dissipate themselves in the form of sparks to nearby metal supports. This condition can be alleviated only by causing the surfaces of the offending elements to provide a leakage path to nearby bonded metal parts. It is interesting to observe that the ice caps, which under certain precipitation conditions will form on the leading edges of the air foils, can have charges developed on their exposed surfaces. Since the ice accumulated at high altitude forms a very excellent insulator, the ice cap creates, in effect, a localized insulated area similar to the windshields and plain rubber deicer boots.

High-Voltage D-C Ground Tests

The rapidly fluctuating character of the voltages encountered in the flight tests indicated the desirability of making tests on the airplane with more accurately controlled voltages. It was felt that flight conditions might be simulated by mounting the plane on insulators and charging it to a sufficiently high direct potential to cause exposed points to go into corona. Information had been obtained on the magnitudes of discharge currents to be expected under storm conditions and the type of radio interference created had been observed carefully. It was felt that if these same conditions could be developed on the ground under accurately controlled voltages, much more consistent results could be obtained.

In line with these thoughts, the research airplane was placed in a large hangar and mounted on high-voltage insulating supports. Adequate insulation and clearances were provided to prevent the appearance of brush discharges from the landing gear and wheels toward the concrete floor, or from the extremities of wings, tail surfaces, and propellers. An improvised high-voltage rectifier, utilizing X-ray equipment, was assembled and adjusted to give a maximum potential of approximately 100,000 volts to ground.

With a potential of the order of 50,000 volts applied to the airplane, conducting surfaces connected to ground were brought into close proximity to the various discharge points mounted on the airplane. Observers inside the cabin noted that currents of the same order of magnitude could be drawn from these points by this arrangement as were obtained in flight. They furthermore observed that the radio interference created by such discharges was of exactly the same character as that which had been obtained during the flight tests.

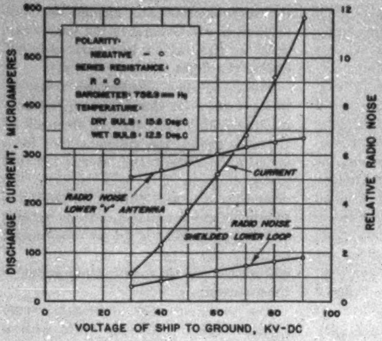
Following these preliminary tests, arrangements were made to suspend conducting points from the tail electrode of the ship toward the concrete floor for a series of discharge tests. It was felt that if the concrete floor were used as the ground plane, the currents would distribute so widely and return to the generator through such a broad conducting medium as to prevent the creation of disturbances in the radio antennas due to these currents.

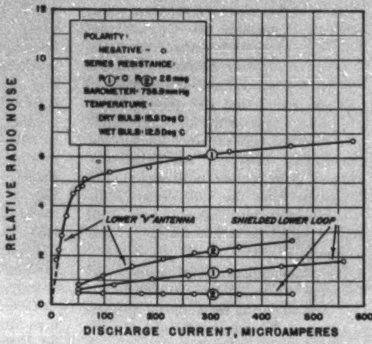
For the first series of tests, a 0.25-inch steel rod was sharpened with a rather long taper and was suspended 11 inches from the concrete floor. High-voltage ignition conductor was used for the support in order to eliminate the possibility of corona discharges from the conductor itself. The tail electrode was well covered with rubber insulating tape to prevent it from going into corona. Observations made during the hours of darkness indicated that no visible discharges were taking place from any part of the plane other than the point where the discharge was desired.

A local signal generator, modulated at a frequency of approximately 1,000 cycles per second, was employed to create a background carrier wave for which the beacon receiver was tuned. It was so located, together with the gain setting of the radio receiver, as to produce a rectified output in the headphone circuit of 0.5 microampere. For all tests, the receiver gain setting was kept at approximately the same point, and a signal of 0.5 microampere was obtained by so locating the local signal generator as to produce the desired output.

The results of the rod-point tests were somewhat erratic, but they checked the flight-test data very satisfactorily. It had been observed in storm flying that the shielded loop had much better signal-to-noise pickup characteristics than the open antennas, and here again exactly the same characteristics were exhibited.

It was felt that the rather large backing of the rod point was producing a shielding action that was influencing its characteristics at the higher voltages. Accordingly, a 7.5-inch length of tungsten wire with a diameter of 0.007 inch was suspended from the same support with the spacing reduced to three inches. The polarity of the ship remained negative, and the same local signal generator output was maintained as used previously. The results of the tests on this electrode





Figures 7 (left) and 8 (right). Airplane discharge characteristics

Discharge electrode: vertical wire 7.5 inches long, 0.007 inch diameter, 3 inches from concrete floor

Background noise: local signal generator, 0.5

are shown in figures 7 and 8. The maximum currents were of the order of 580 microamperes, and the radio interference as obtained from the lower or "belly V" antenna was severe throughout the entire voltage range. The shielded loop in this case also gave considerable interference for the higher current values, tapering down to relatively low interference at the lower voltages. Again, it was observed that the shielded loop provided a very favorable signal-to-noise ratio over the open antennas.

A resistance of 28 megohms was then connected in series with the discharge point and further tests were made. The data obtained with this arrangement are given in figure 8. It is shown that the insertion of resistance reduced the interference for a given discharge current to a marked degree. The interference pickup of the shielded loop was actually reduced to zero throughout the entire range of currents observed. Although some interference was still obtained from the open antenna, its performance was greatly improved.

These results immediately suggested the possibility of using in flight some type of trailing discharge conductor. The flight tests had indicated that the interference problem could be solved only by preventing the charging of the airplane to potentials greatly different from those of the surrounding atmosphere, or by dissipating these charges without disturbing the radio receiver. It was obvious that if the potential of the aircraft, by means of some discharging device, could be maintained at a value near that of the surrounding atmosphere, then no corona discharges could take place except in the event of passing through high-gradient regions where the airplane is effectively short-circuiting high differences of potential. Since these latter conditions are only transient in their character, they are not particularly objectionable. This trailing-wire discharger, with high-resistance noise suppressors, has been tried in flight and found to be very beneficial under most conditions.

At least one large air transport corporation has installed such trailing wire devices on all of its transport airplanes and has found them to be of material assistance in combating the precipitation-static problem. In this case, the trailing wires are contained in small cartridges and can be released from the tail of the airplane when severe static conditions arise. Normally, they are totally enclosed and, hence, are not subject to mechanical fatiguing and failure when not in use.

After a satisfactory amount of negative-polarity data had been obtained, the rectifier was reversed and provisions were made for obtaining positive-discharge data. Because of the very much greater sparking distance of point-to-plane electrodes with point positive than with point negative, it was necessary to increase the spacing between the end of the fine tungsten wire and the concrete floor from 3 inches to 7.5 inches. Otherwise, the setup employed was exactly the same as for the negative tests. The data obtained from these tests indicated some rather peculiar characteristics. Reference to the curves of figure 9 will show that as the current was raised slowly from zero to 110 microamperes, the interference received by all antennas was practically zero. Then, very suddenly the interference jumped to a high value, and as the current was further increased, continued on to still higher values. Observations of the discharge point indicated that prior to the sudden burst of interference the discharge was in the form of a very fine point of localized ionization. At the instant the interference started, a long ionized streamer was observed to burst out from the end of the discharge electrode. When the voltage was slowly reduced, the interference remained fairly high on down to low values of discharge current. These characteristics brought out the fact that the presence of ionization streamers was responsible to a considerable extent for the radio interference produced by the positive corona discharges. This phase of the subject is treated in detail in the companion paper, "High-Voltage D-C Point Discharges".

Again, the insertion of a discharge resistance of 28 megohms resulted in a marked improvement in the radio noise characteristics. Curve 2 (solid) of figure 9 illustrates the improvement obtained by the use of resistance. This curve is for decaying currents and represents ionized-streamer discharges from the highest to the lowest values. Curve 2 (dotted) indicates that the resistance completely eliminated the noise pickup of the shielded loop.

Co-ordinated Laboratory and Analytical Studies

The data obtained from the ground tests on the research airplane emphasized the necessity for additional basic studies in three different fields. The work to be covered was as follows:

1. The character of high-voltage d-c point discharges.
2. The mechanism of radio interference as produced by corona discharges from point electrodes and the possibility of suppressing this interference.
3. The fundamental characteristics of receiving antenna, particularly the shielded loop.

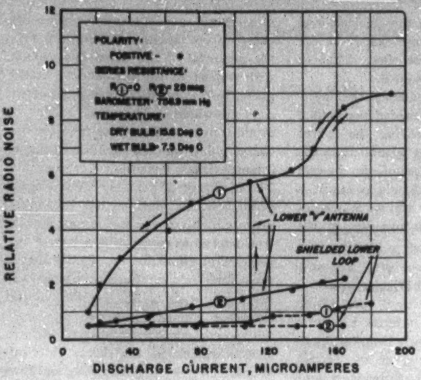


Figure 9. Airplane discharge characteristics

Discharge electrode: vertical wire 7.5 inches long, 0.007 inch diameter, 7.5 inches from concrete floor

Background noise: local signal generator, 0.5

The first of these subjects was studied in considerable detail and is reported in the companion paper referred to above. Figures 10, *a* and *b*, illustrate typical results of this work. For the type of point and the current employed in this case, the discharges of both polarities created the typical musical-note interference. The audio frequencies were much higher for the negative than for the positive discharge. It is shown that in both cases the discharge current is composed of a small ion, migration component plus a series of superimposed impulses of extremely steep wave front.

The bursts of radio interference are shown to coincide exactly with the fronts of these superimposed impulsive discharges. The radio disturbance is evidently the result of the tuned input circuit of the receiver being set into oscillation at its resonant frequency by the extremely steep-wave-front impulses. It is, therefore, a special case of the impulse excitation of oscillatory circuits. The interference mechanism falls in the classification of the second subject enumerated above and is treated in detail in the paper on point discharges.

Of particular interest was the finding that pointed electrodes with rather blunt backing and those of approximately hemispherical section, in general gave severe radio interference of widely varying character for both polarities of discharge. It was found, however, that sharp, slender points, that is, points having a terminal diameter of the order of 0.003 inch and having a very small included angle, produced a type of discharge on both polarities, and for all values of currents up to at least 50 microamperes, that was totally free from any radio interference in receivers operating up to several megacycles. This characteristic offers an extremely useful possibility for the development of improved airplane discharging systems. This matter will be treated in detail in a later section.

The third subject has a direct bearing on the aircraft problem and will be discussed in the following section.

Radio Receiving Antennas

Energy abstracting devices for radio-receiving equipment fall into three general classes:

1. The extended-conductor or open-wire antenna.
2. The simple loop antenna.
3. The shielded loop antenna.

The extended-wire type responds to the traveling electromagnetic wave and is also sensitive to any electric field in its vicinity. The true wave of electromagnetic radiation is composed of equal electric and magnetic components, and the voltage induced in the open antenna is essentially the same as that created in space by the passage of the wave. Induction fields, however, preponderantly electric or magnetic, can be caused to exist but their space coverage is relatively very limited. An extended-wire antenna in the presence of these induction fields will respond to both types (appendix III of reference 5).

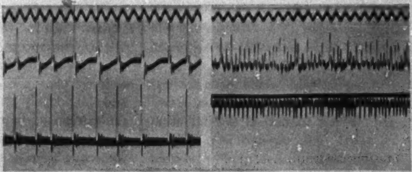
The unshielded loop antenna may be of either a balanced or unbalanced type. In the latter case, the loop will respond to both magnetic and electric fields. The magnetic response is essentially by transformer action. The electric response is due principally to charging currents flowing through the loop circuit to ground.

The balanced loop responds well to magnetic excitation, again by transformer action, but is very insensitive to electric excitation. In this case, the charging currents due to electric fields flow to ground equally in both directions through the loop, thus effectively cancelling any pickup which might result. If the midpoint of this balanced loop is grounded, a similar cancelling of electric-field pickup will occur.

The shielded radio loop, when shielding is composed of a discontinuous metallic tube surrounding the turns of the loop and is balanced to ground, is responsive only to the magnetic field. In this case, any changing electric field terminating on the loop produces currents which are conducted to ground in practically equal magnitudes and in opposite directions around the two parts of the shield. Consequently, no potential is induced in the loop proper due to these charging currents flowing to ground. Any improvement in the shielding design which will tend to maintain a balance between the charging currents flowing to ground through the two sides of the shield, will be beneficial in its results.

The Shielded Loop Antenna

It was brought out very definitely in the sections on flight research and ground tests that the shielded loop possessed marked advantages over the extended-wire antenna in its ability to discriminate between the desirable signal and undesirable "static" interference. This feature has been known for several years, particularly as regards effectiveness with certain types of static, and it has been employed



A—Point positive; $E = 15.2$ kv, $I = 2.0$ microamperes *B*—Point negative; $E = 16.6$ kv, $I = 3.0$ microamperes

Figure 10. D-c corona discharge characteristics

Electrodes: 60-degree point to plane; spacing 10.2 centimeters

Upper, radio noise; lower, current with zero line; timing wave: one kilocycle advantageously in a number of fields. Its effectiveness in the attenuation of precipitation static on aircraft is due to the fact that this static interference originates fundamentally as a high-voltage low-current discharge phenomenon. The electric fields associated with these discharges are necessarily intense while the magnetic fields, due to the relatively feeble character of the currents, are very weak.

It has been shown that the discharge currents are composed of steady values with superimposed double-exponential impulses. Such currents can be represented by a series of multiple-frequency waves. These waves will produce true electromagnetic radiations. However, since the a-c components are very small, the radiation fields, which are functions of the currents, necessarily will be weak. The electric induction fields, on the other hand, are relatively intense because the voltages involved are of the order of several thousand, and the distances from the source of the interference to the antennas are not great. As a matter of fact, for the normal radio frequencies, these distances are only of the order of a small fraction of a wave length. An open antenna should, therefore, respond strongly to the electric fields in the vicinity of the corona discharges, and weakly to the magnetic fields accompanying these discharges (appendix III of reference 5).

The properly shielded loop, however, as has been shown in the previous section, rejects the electric induction interference and accepts only the magnetic field. This magnetic excitation has essentially two components (appendix IV of reference 5). The first is that of induction which varies directly with the current and inversely as the square of the distance from the source. The second is that of radiation which not only varies directly with the current but also with frequency, and inversely as the first power of the distance.

In the case of the normal beacon receivers, the wave length is of the order of 1,000 meters, and the distance of equality between the magnetic components of the induction and radiation fields is approximately 159 meters. At distances of the order of 10 to 50 feet from the source of interference, the radiation component of the magnetic field would then be expected to have a value of the order of two to ten per cent of that of the induction component. The shielded loop will respond to this resultant magnetic field even though it is weak, and at short distances, the response will be preponderantly to the induction component. The strength of interference pickup in the shielded loop should then vary almost exactly as the inverse square of these short distances from the interference source.

The problem of reducing the interference pickup of the shielded loop is then primarily one of removing the source of disturbance as far as possible from the loop, taking advantage of the inverse square law, and of reducing as much as possible the amplitude and wave front of the individual current impulses in the discharges.

Control of Electrical Discharges From Aircraft

Since it is apparently impossible to prevent the accumulation of electric charges during flight through certain storm conditions, corona discharges from extremities of the aircraft must of necessity occur when the potential differences between these extremities and the surrounding atmosphere become sufficiently high. It has been shown that these discharges will produce interference in the normal-frequency radio receiving equipment on board the aircraft unless they are so controlled as to prevent the appearance of random, steep-wave-front current impulses.

A conductor trailing or projecting from the tail of the airplane and terminating in one or more sharp, slender points would be expected to discharge considerable energy to the region immediately surrounding the points. It has been shown that such sharp, slender points produce discharges, regardless of polarity, which for low and intermediate values of current are free from the disturbing current impulses.[5] To guard against interference from other types of discharges which might form in the vicinity of these points, it would be well to supply the trailing or projecting conductors with resistive and inductive wave-front suppressors.

For extremely large airplanes and for conditions of flight through severe storm areas, a single trailing conductor does not provide adequate discharging capacity. In those cases it would be desirable to have point dischargers of the sharp, slender variety, with resistive and inductive wave-front suppressors, on the wing tips and probably also on the nose of the airplane. The wing tip electrodes, and also possibly other discharge points at the extremities of the vertical and horizontal fins, might be of either a trailing or projecting type. Other sharp projecting edges and points on exposed parts of the airplane then should be carefully removed.

The end to be worked toward is that of allowing sufficient energy to be discharged from the controlled points to keep the potential of the plane low enough, with respect to the surrounding atmosphere, to prevent discharges from occurring elsewhere. This end might be attained not only by means of simple discharge points but also by the creation of intense primary ionization at these points. The discharge of a highly ionized liquid at the points, a relatively low-voltage d-c arc or a high-voltage, radio-frequency arc would serve to suppress the current impulses and eliminate the accompanying radio interference. Such procedures are necessarily more or less complicated and should be avoided if possible.

Exposed, normally insulating surfaces such as windshields and deicer boots can be made electrically conducting, and thus the accumulation of sufficient charge on such surfaces to cause sparking over to metal supports can be prevented.

Conclusion

Some of the results of this investigation already have been incorporated in emergency discharging devices installed on the transport airplanes of at least one of our major commercial airlines. The results obtained from this rather simple device have been very gratifying. The incorporation on a test airplane of all the numerous results and suggestions outlined in this paper would entail a great amount of time and expense. The problem eventually must be solved if the progress of aerial navigation is to continue. It is to be hoped that through active cooperation of all interested agencies this solution will be reached in the not-too-distant future.

Bibliography

1. Electrical Engineers' Handbook, **Pender and McIlwain.** Volume 5, Communications and Electronics; John Wiley and Sons, Inc., New York City.

2. Radio Engineering Handbook, **K. Henney.** Second edition; McGraw-Hill Book Company, New York City.

3. Radio Interference From Insulator Corona, **F. O. McMillan.** AIEE Transactions, volume 51, January 1932.

4. A Noise Silencing I. F. Circuit for Super Heterodyne Receivers, **J. J. Lamb,** QST, February 1936, volume 20, number 2.

5. Precipitation-Static Radio Interference Phenomena Originating on Aircraft, **E. C. Starr.** Bulletin Number 10, 1939, Engineering Experiment Station, Oregon State College.

6. High-Voltage D-C Point Discharges, **E. C. Starr.** AIEE Transactions, volume 60, 1941, pages 356–62.

7. **G. C. Simpson.** *Philosophical Transactions,* Royal Society, London, A-209, 1909.

8. Precipitation-Static Interference on Aircraft and at Ground Stations, **H. M. Hucke.** *Proceedings* of Institute of Radio Engineers, volume 27, May 1939.

A Wide-Band Square-Wave Generator

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Synopsis: A square-wave generator is described which covers a very wide frequency range, is compact, self-contained, and adaptable to many other uses besides the investigation of transient response. The instrument uses a positive-bias type multivibrator. The operation of the multivibrator and clipping circuits are explained. Approximate formulas for the multivibrator frequency and pulse width are derived and discussed and its increased stability against interference explained. Methods and fields for application are then discussed.

FOR many purposes in communication work a square-wave generator delivering "square" voltage waves of variable frequency and "pulse width", is a valuable instrument. It can be used for testing and adjusting of complete telegraph, telephone, radio, and television channels, or of part of such channels, such as filters or amplifiers. The considerable distortion, produced in a square wave, by even small amounts of phase distortion makes it an ideal instrument for use in testing and aligning television circuits. In power circuits it can be used to determine the response of lines, transformers, and other components to unit step voltages.

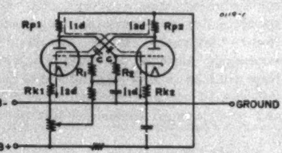

Figure 1. Positive-bias multivibrator

In all these cases the response can be viewed directly and thus immediate correlation with the results of unit step calculations is obtained. Given a wide frequency range of the square-wave generator and an independent scanning frequency in the oscillograph, it is possible to study either coarse or fine structure of the response. The constant repetition of the phenomenon permits photographic registration.

The following describes a square-wave generator, developed for these purposes. It has a very wide frequency range (approximately 2 to 200,000 cycles per second) a variable "pulse width" and is completely self-contained.

Description

A multivibrator is used as the oscillator in this instrument, its operation will be discussed in the next section. The output of this multivibrator is "clipped" in a following limiter stage and delivered into a low impedance (1,000 ohms) by a buffer tube. The anode voltage for these circuits is provided by a built-in, regulated power supply.

The multivibrator is of the positive-bias type, in which a variable positive bias, applied to the grid resistors, controls the frequency. This circuit offers the advantage of giving a very wide variation in frequency for the same set of circuit constants, while it greatly reduces the change of pulse width with frequency in each range. Furthermore, at higher values of the positive bias a much greater stability against external interference is obtained. The use of a regulated power supply results in a high stability against frequency drift, a representative figure being 13 cycles per second in 13,000 cycles per second over a ten-minute period, measured after the instrument had reached temperature equilibrium.

Due to the fact that a multivibrator is used, this square-wave generator can be synchronized from low-level external signals, which can be applied to a low-impedance input. The synchronization can be in a 1:1 ratio, but the instrument will also synchronize in multiple or submultiple ratios. These properties make it especially suitable for measurements of deflection linearity in television and oscilloscope time bases. Addition of a discharge circuit makes the instrument into a saw-tooth generator for oscilloscopes, while by cascading several square-wave generators, frequency division or multiplying chains (electronic gears) can easily be built up.

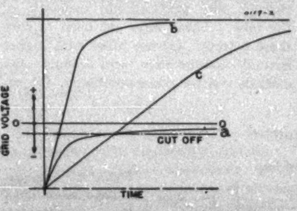

Figure 2. Multivibrator grid characteristics

Operation

The operation of "normal" multivibrators has been described by several authors and it can thus be treated very briefly. Fundamentally it is a two-stage resistance-capacity coupled amplifier, in which the input and output are directly interconnected. Oscillations will thus occur in the circuit. The frequency of these is chiefly determined by the *R* and *C* values used in the grid circuits of each tube; the flywheel effect, which carries the circuit over its dead points, is provided by the unavoidable inductances in the circuit wiring and the grid input capacities of the tubes. This oscillation produces such high voltages that, in alternation, the grid circuit of one of the tubes is pushed beyond its cut-off value and the amplification in this tube is reduced to zero. Therefore, the picture of a two-stage amplifier in which in- and output are interconnected, holds only during the very short "flip-over" or reversal time when both tubes amplify.

At every "flip-over" of the multivibrator operation, the grids are charged to a high negative voltage. This charge then leaks off along an exponential till the grid voltage attains cutoff. The tube then becomes conducting and the multivibrator again reverses or "flips over".

Let us now connect the grids to a point of positive bias, as shown in figure 1. Then—in the absence of grid and plate current in the tube—the grid would discharge along another exponential, until it had reached the higher positive voltage to which it was biased. As can be seen from figure 2, curves *a* and *b*, the part of the exponential between the starting point and the intersection with the cutoff voltage, constitutes a smaller part of the total voltage variation as we increase the positive bias. This results in two effects: (*a*) the time required for the grid to attain the cutoff value is reduced as the positive bias is increased, (*b*) the difference between this part of the

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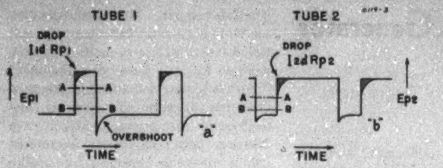


Figure 3. Plate-voltage wave forms in unsymmetrical multivibrator

exponential and a straight line is decreased, that is, the steepness of the grid-voltage curve is increased at the cutoff point.

The first effect results in an increase in frequency with increased bias, the second one results in reduced sensitivity to interference with increased bias. Both effects will be discussed in more detail. The frequency range that can be obtained with any one set of circuit components is large; it is in the order of 10:1. Thus, with two ranges for every 10:1 increase in frequency, a wide overlap of the ranges can be obtained.

This type of frequency control has the practical advantage of being a d-c control and as the frequency control does not carry any signal currents it can be placed at any distance from the oscillator circuit.

The voltages appearing on the multivibrator plates have the forms shown by figures 3*a* and 3*b*. It is easy to see that at least two clipping operations are necessary to obtain a square output from these waveforms. First the part above *AA* has to be clipped off. For a fixed frequency this can easily be done. For a variable frequency however, we find that the curved part occupies a varying fraction of the peak-to-peak swing and this fraction can easily go up to 70 per cent. To ensure a good clipping action under these conditions we have to either reduce the frequency range per set of circuit constants; introduce more than two stages of clipping; or have an instrument that is very critical in adjustment.

All these objections were overcome by inserting resistors in the cathodes of each multivibrator tube. These cathode resistances are shown in figure 1. The wave form on these resistors is as shown in figures 4*a* and 4*b*. A single clipping operation which removes the part above *CC* would convert this into a square wave, but the polarity of the signal makes it impossible to obtain this clipping by pushing the grid of a tube beyond cutoff. This was circumvented by connecting the cathodes of the clipper and one of the multivibrator tubes together and grounding the clipper grid.

Adjustment of the clipping point was done by changing the screen-grid voltage. This "upside down" operation of the clipper has proved very satisfactory. It provides a positive pulse in the anode circuit; this can be directly fed to the grid of the buffer tube. The interstage coupling was designed for wide-band operation and carefully adjusted for low-frequency phase shift. A high-frequency "peaking" inductance can be added to good advantage for equalizing the high-frequency to the midband response.

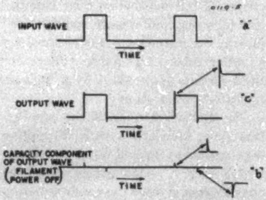


Figure 5. Wave forms in cathode output circuit

The buffer circuit is of the conventional cathode output type and delivers five volts in 1,000 ohms. At high frequencies "peaking" distortions which produce overshoots of the edges, can occur in such a buffer circuit due to the coupling through the grid-cathode capacity of the tube. Three ways are open to cure these defects: (*a*) use tubes with higher mutual conductance and reduce the cathode resistor correspondingly, (*b*) reduce the driving voltage as much as is compatible with a reasonable safety margin (for instance 30 per cent) for clipping, (*c*) put some parallel capacity in the driving circuit to reduce the slope of the driving wave form. Using these methods good wave forms are obtainable even at high frequencies.

At frequencies near the top frequency of operation where a high number of harmonics cannot be produced by the multivibrator, the output wave form rounds off, as is to be expected.

In order to increase the frequency range in which these effects are unnoticeable, proper care should be taken to obtain good high-frequency response in the multivibrator circuit. This includes short leads, reduction of the loop area of the circuits, and reduction of the capacity between grid and plate circuits to the same tube.

Multivibrator Frequency

In connection with the positive-bias-type multivibrator, it is interesting to investigate which factors determine the multivibrator frequency and pulse width. It is well known that a rigorous analysis requires solution of the nonlinear differential equation, as treated by van der Pol (*Phil. Mag.*, volume 11, 1926, page 478, and other places).

Some of the important properties of a multivibrator can, however, be determined without recourse to a nonlinear treatment. While the results necessarily must be an approximation, they will lead to an understanding of the influence of circuit constants, tube gain and grid bias on frequency and pulse width. As a by-product they give a fairly simple formula for the multivibrator frequency which seems to be a much closer approximation than the handbook formula: $f=1/rc$

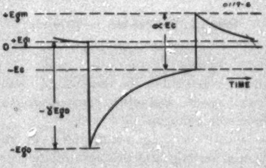


Figure 6. Grid-voltage wave form of multivibrator

We introduce the following symbols, which are partly illustrated in figure 6

E_g = instantaneous grid voltage
$+E_a$ = anode supply voltage

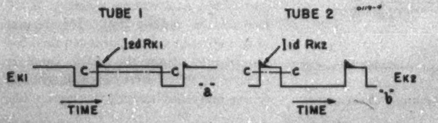


Figure 4. Cathode-voltage wave forms in unsymmetrical multivibrator

$+E_{gb}$ = grid (bias) supply voltage
$+E_{gm}$ = initial (or maximum) positive grid voltage after a positive "flip-over"
$+E_{gt}$ = positive grid voltage just before a negative "flip-over"
$-E_c$ = cutoff voltage of the amplifier tube
$-E_{g0}$ = initial negative grid voltage after negative "flip-over"
$\alpha = \dfrac{E_{gm}+E_c}{E_c}$ = "overswing ratio" in positive direction
$\gamma = \dfrac{E_{gt}+E_{g0}}{E_{g0}}$ = "overswing ratio" in negative direction
$\beta = E_{gb}/E_a$ = bias ratio
G = absolute value of the stage gain if one of the multivibrator tubes is used as a straight amplifier
$r = R_g+R_p$ = total discharge resistance (for most practical cases we may put $r=R_g$)
C = grid to plate coupling capacitor

In the region where the grid voltage is below cutoff, it is given by:

$$E_g = E_{gb}-(E_{gb}+E_{g0})\epsilon^{-\frac{t}{rc}} \tag{1}$$

"Flip-over" occurs as soon as the grid obtains the cutoff value, thus when:

$$E_g = -E_c \tag{2}$$

The cutoff voltage (in triodes) depends on the anode voltage and the amplification factor μ of the tube and is given to good approximation by:

$$E_c = +\frac{E_a}{\mu} \tag{3}$$

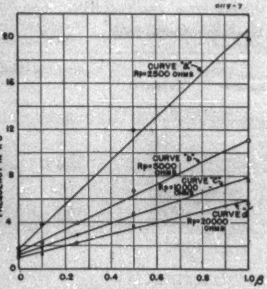


Figure 7. Frequency characteristics of positive-bias multivibrator

The initial grid swing is determined by the grid swing in the preceding tube, which is αE_c and by the gain G from this grid into the next grid. On this next grid it is $-\gamma E_{g0}$ by definition. Thus in a linear system we would have:

$$-\gamma E_{g0} = -G\alpha E_c \tag{4}$$

Now assume that the gain during the "flip-over" period is constant and equal to G and that the ratio $\delta=\alpha/\gamma$ is also a constant.

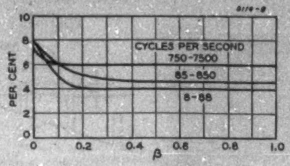


Figure 8. Pulse-width characteristics of positive-bias multivibrator

These simplifying assumptions reduce the problem from a nonlinear to a linear one. As mentioned above, we know that this constitutes an error because in at least the regions near cutoff or those of small positive grid bias, the tubes are nonlinear. Consequently we will never be able to determine the ratio δ, the effective gain or the relation of anode voltage and frequency from our calculation, but the result does give us an insight in the effect of different parameters on frequency and pulse width.

Now, substituting (3) in (4) we find:

$$E_{g0} = \delta/\mu G E_a \tag{4a}$$

Substituting $E_{gb}=\beta E_a$ (from the definitions) and equation 4*a* into (1) this becomes:

$$E_g = \beta E_a - \beta E_a\left(1+\frac{\delta G}{\mu\beta}\right)\epsilon^{-\frac{t}{rc}} \tag{5}$$

"Flip-over" occurs when E_g attains the value

$$E_g = -E_c = -\frac{E_a}{\mu}$$

Substituting this into (5) E_a drops from the formula and we can determine the time t_0 between two trippings. From it, we find for the time during which the grid remains below cutoff in a symmetrical multivibrator:

$$t_0 = rC\cdot \log_e\left(\frac{\delta G+\mu\beta}{1+\mu\beta}\right) \tag{6}$$

where $\log_e$ represents the natural logarithm and for the frequency of the multivibrator:

$$f_0 = \frac{1}{2\cdot rC\cdot \log_e\left(\dfrac{\delta G+\mu\beta}{1+\mu\beta}\right)} \tag{6a}$$

To obtain a real, positive value for t_0, the effective gain, including overshoot, must exceed unity. Decreasing gain leads to increased frequency. This is in accordance with the general behavior of multivibrators and we might thus expect a better approximation from (6) than from the handbook formula which does not contain the gain or the bias at all. The results given in tables I and II for the case of zero grid bias check this, but secondary effects may under circumstances reverse these conditions.

The absence of E_a from (6) shows agreement with the fact, that in most of the actual multivibrators tested, its influence on the frequency rapidly decreases for values of E_a above approximately 100 volts.

As explained, the ratio δ cannot be determined from the analysis. From observations, made on a number of multivibrators, it seems justified to assume a range of approximately 1.0 to 3.0, with a most probable value of 1.50.

Special Cases

We now consider special cases of formula (6).

1. For zero value of the bias ratio we find:

$$f_0 = \frac{1}{2\cdot rC\cdot \log_e(\delta G)} \tag{6b}$$

2. For very small values of β, for which $\mu\beta << 1$ the development for $\log_e(a+x)$ gives:

$$f_0 = \frac{1}{2\cdot rC\cdot \log_e(\delta G)}\left\{1+\frac{\mu\beta}{\delta G}\,\frac{\delta G-1}{\log_e(\delta G)}\right\} \tag{6c}$$

3. For high values of β we soon approach the case where the positive bias voltage is large compared to the initial grid voltage. We then operate on the steep initial part of the exponential and can stop the development for $\epsilon^{-t/rc}$ after the second term, thus:

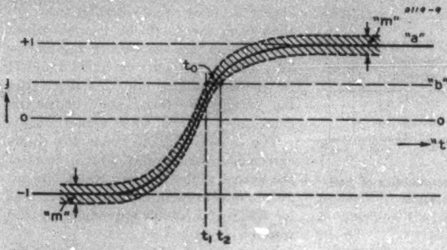
$$f_0 = \frac{1}{2\cdot rC}\cdot\frac{1+\mu\beta}{\delta G-1} \tag{6d}$$

We see that for higher values of the bias ratio the multivibrator frequency should vary linearly with the bias ratio. From (6*c*) and (6*d*) we can derive the steepness $\delta f/\delta\beta$ of the frequency characteristics for low and high bias ratio, thus:

$$\left(\frac{\delta f}{\delta\beta}\right)_{lo-\beta} = \frac{\mu}{2rC}\cdot\frac{1-\dfrac{1}{\delta G}}{\log_e{}^2(\delta G)} \tag{6e}$$

$$\left(\frac{\delta f}{\delta\beta}\right)_{hi-\beta} = \frac{\mu}{2rC}\cdot\frac{1}{\delta G-1} \tag{6f}$$

Comparing the steepnesses of (6*e*) and (6*f*) for different values of δG we see that for values ranging from 1.3 to 10 the ratio between the two steepnesses varies from 1.02:1 to 1.50:1. Thus there is only a small difference in the steepness between low and high values of the bias ratio. We might therefore expect that the frequency will vary nearly linearly, not only over the range of high values of the



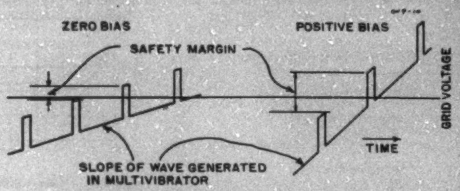


Figure 9 (left). Influence of interference on time of "tripping"

Figure 10 (above). Grid-voltage wave forms in electronic-gear operation

bias ratio, but throughout the whole range.

Reference to figure 7*a*, *b*, *c*, which gives some representative cases, shows that this conclusion is very well confirmed by the experiment. Thus formula (6) has allowed us to correctly deduce the character of the frequency versus bias characteristic. This linear relation of frequency and bias opens a wide range of applications for the positive bias type multivibrator.

Pulse Width

In many cases it is important to have a wave of variable frequency but approximately constant pulse width. With the preceding formulas we determine the pulse width as defined by

$$\lambda = \frac{t_{01}}{t_{01} - t_{02}} \tag{7}$$

where t_{01} and t_{02} represent the times during which grids 1 and 2 have been driven below cutoff.

Assume that the multivibrator uses two tubes of the same type and that their plate loads, and therefore their gains are equal. We then derive from (6)

$$t_{0k} = r_k C_k \cdot \log_e \left(\frac{\delta G + \mu\beta}{1 + \mu\beta} \right) \tag{7a}$$

where k can be 1 or 2. Substituting this in (7) we find:

$$\lambda = \frac{r_1 C_1}{r_1 C_1 + r_2 C_2} \tag{7b}$$

and deduce that the pulse width should be independent of the bias-setting and thus independent of the frequency. Figure 8 shows that this checks reasonably well; the change in pulse width for the three multivibrators shown, amounts to 20–50 per cent of the initial value, for frequency changes in a ratio of 10:1. Nearly all of this change occurs at low values of β ($\beta < 0.20$).

Interference

Consider an electrical circuit, which performs a certain operation as soon as the driving voltage or current in a certain control circuit attains a prescribed "critical" value. Let the voltage or current be a recurrent, periodic phenomenon which can be either impressed from outside or generated in the circuit itself.

In the absence of interference the circuit will then operate at exactly equal time intervals, that is, at constant frequency.

The multivibrator is an example of the second (oscillator) type circuit which provides its own driving voltage and has the tube cutoff as the critical value.

In figure 9 curve *a* represents the control-circuit driving voltage or current; *y* is its instantaneous value expressed as a fraction of its peak to peak swing; the center between peaks is used as zero or reference level; *b* is the critical value, and t_0 the time at which the operation is started.

Tables I, II, III: Comparison of Previous Formula (F_0) for Multivibrator Frequency With New Formula (f_0) and Observed Frequency (f)

I. Zero Bias—Test Multivibrator

Gain	$F_0 = \frac{1}{rC}$	$f_0 = \frac{1}{2rC \log_e (\delta G)}$	f observed	Error (Per Cent) In F_0	Error (Per Cent) In f_0
3.5	3,900	1,170	1,800	+120	−35
7.0	3,900	820	1,540	+160	−45
12.0	3,900	680	1,300	+200	−48
17.0	3,900	600	1,150	+220	−48

II. Zero Bias—Square-Wave Generator—Narrow Pulse

Range	rC	$F_0 = \frac{1}{rC}$	$f_0 = \frac{1}{2rC \log_e (\delta G)}$	f observed	Error (Per Cent) In F_0	Error (Per Cent) In f_0
1	0.5	3.6	1.05	1.2	+200	−13
2	0.1	18	5.3	5.5	+230	−4
3	0.03	60	17.5	16	+270	+10
4	0.01	180	53	50	+250	+6
5	0.004	450	132	150	+200	−12
6	0.0013	1,400	410	420	+230	−2
7	0.00045	4,000	1,180	1,400	+190	−16
8	0.00015	12,000	3,500	4,200	+190	−17
9	0.00004	45,000	13,200	14,000	+200	−6

III. Maximum Positive Bias—Square-Wave Generator—Narrow Pulse

Range	rC	$F_0 = \frac{1}{rc}$	$f_0 = \frac{(1+\mu\beta)}{2rC(\delta G - 1)}$	f observed	Error in f_0 (Per Cent)
1	0.5	3.6	6.3	8	−21
2	0.1	18	33	36	−8
3	0.03	60	105	125	−16
4	0.01	180	330	380	−14
5	0.004	450	790	1,100	−30
6	0.0013	1,400	2,450	2,900	−15
7	0.00045	4,000	7,000	10,000	−30
8	0.00015	12,000	21,000	30,000	−30
9	0.00004	45,000	79,000	135,000	−40

Figure 12. General Electric square-wave generator

Now consider the case where interference is present. Let the peak-to-peak value of the interference be a fraction m of the ptp swing, it may have any arbitrary wave form. Thus the effective driving voltage may have any value within the shaded area in figure 9. The operation may thus be started at any moment between t_1 and t_2 and the time of operation has an "uncertainty region" $\Delta t = t_2 - t_1$. Defining the slope of the undisturbed driving voltage as $s = dy/dt$, we find from the figure $s = dy/dt = m/\Delta t$ and

$$\Delta t = \frac{m}{s} \tag{8}$$

This relation is well known in telegraph communications (to the author's knowledge it was first clearly stated by Doctor H. Salinger, Documents, *CCIT*, 1926, pages 7–9) and shows that the uncertainty region is inversely proportional to the slope.

As can be seen from figure 2, curves *a* and *c*, the application of positive bias to a multivibrator will considerably increase the steepness of the grid voltage curve at the intersection with the cutoff value; for high bias values the steepness of the cutoff value practically equals the initial steepness. Thus the sensitivity to interference should be reduced. To check this an erratic interference was applied to a zero-bias multivibrator and increased in amplitude till the uncertainty zone covered 50 per cent of one cycle. This means that the operation had become so irregular that a fundamental frequency could no longer be recognized. Changing the same multivibrator to positive bias and readjusting the grid capacitors to obtain the same frequency the uncertainty zone was reduced to 8 per cent for the same amount of interference voltage.

This increased stability is particularly important to television systems where the multivibrator may be used as a time base. The increased steepness will also be an advantage in maintaining the proper frequency division at high-frequency ratios, as they may be encountered in pulse generators for television. Figure 10 demonstrates this point by showing the increase in safety margin obtained by increasing the slope of the grid characteristic.

Circuit Components

Further analysis leads to some practical conclusions on the choice of circuit components.

To obtain a large frequency range, the grid-leak resistors should be chosen as high as is compatible with safe operation of the tube. Failure to observe this, will—through grid rectification—result in a lowering of the voltage on the grid decoupling capacitor which counteracts the increase in bias.

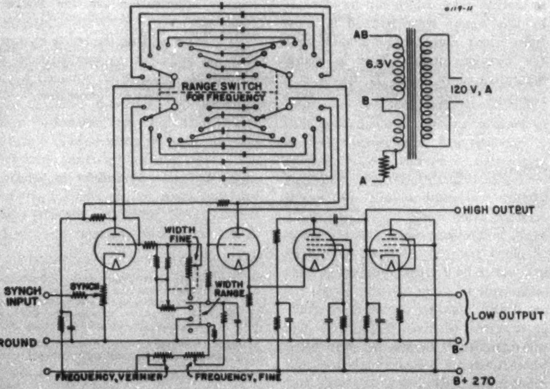


Figure 11. Circuit diagram of square-wave generator

Equal grid capacitors should be used if independence of the pulse width from the frequency is desired. During the "flipping over" part of the cycle, the grid coupling capacitor is connected between the plate of one tube and the low-impedance of the conducting grid-to-cathode path of the other tube. Its impedance determines the voltage transferred to the grids and if these capacitors are unequal, the ratio of the grid peak voltage will change with frequency. This will result in changing pulse width.

General Data

The whole instrument is mounted on a standard 19-inch relay rack panel. The panel height is $8^3/_4$ inches, the depth behind the panel is 13 inches. The controls on the front panel are: synchronizing amplitude, width (fine), frequency-range, width-range, frequency (fine), frequency (vernier).

The diagram is given in figure 11, the outside appearance is shown in figure 12.

Application

There is such a variety of problems for which the square-wave-generator can and has been used, that we cannot attempt to give an adequate review of them within the scope of this paper. Rather than that we will give some typical examples (two on amplifiers, one on transmission lines) from which the advantages of the method can easily be seen.

These advantages become clear if we realize, that the transient response, resulting from a unit-step voltage, in-

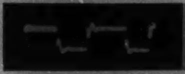


Figure 13. Response of LCR network—low damping

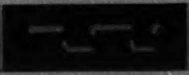


Figure 14. Response of LCR network—medium damping

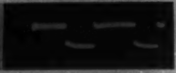


Figure 15. Response of LCR network—high damping

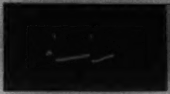


Figure 16. Response of network with low-frequency phase shift

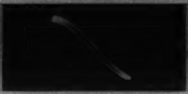


Figure 17. Lissajous figure for the network of figure 6

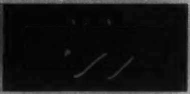


Figure 18. Effect of overload in amplifier on square-wave response

volves the complete amplitude and phase response of the circuit in consideration. Thus one single measurement with square waves can give us an amount of information, which we could otherwise only obtain by the laborious process of determining the complete amplitude and phase response curves as a function of frequency.

As a first example we show some oscillograms of the square-wave response of a wide-band amplifier, in whose plate circuit a series connection of L, C, and R was branched across the plate resistor. In order to demonstrate the results which can be obtained with even very simple equipment, a standard three-inch oscilloscope and a normal camera was used when these and the following oscillograms were photographed. The use of a larger oscilloscope will, of course, give the advantage of finer detail in the oscillogram.

In figure 13 the response is shown when a square wave of approximately 50 per cent width and a frequency of approximately 10 kilocycles is applied to the circuit. We can directly deduce from this one measurement that an oscillatory transient of approximately 60 kilocycles is present in the system. In any actual problem this result would tell us where to improve the circuit. Figures 14 and 15 show the effect of increasing the series resistance in this *LCR* branch.

In figure 16 the use of the square-wave generator to detect low-frequency phase shift is demonstrated. The presence of this phase-shift is shown by the "sloping off" of the top part of the narrow pulse and by the base line or "floor" of the pulse being at different levels before and after occurrence of this narrow pulse. Analysis will show that sensitivity of measuring phase which can be obtained by this square wave method is many times larger than that obtainable with the Lissajous figure. This is clearly demonstrated in figure 17 which is a Lissajous ellipse for the same phase shift as was used in figure 16 and for approximately equal deflection amplitudes on the oscilloscope screen. As can be seen the ratio between short and long axes of the ellipse is considerably smaller than that between the dropping-off of the pulse top and the pulse height. Figure 18 demonstrates what happens if overload occurs somewhere in the amplifier. In this case the leading and trailing edges of the pulse have different magnitude. This effect can, for instance, be caused by grid current on the amplifier, which grid current occurs at a swing of positive polarity on this grid and is absent during a negative swing. The responses to positive and negative unit-step voltages are then different as result of the rectification occurring in the grid.

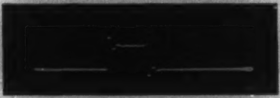


Figure 19. Response of improperly terminated transmission line

Square-wave input in transmission line terminated in pure resistance 25X characteristic impedance; observed at end of line

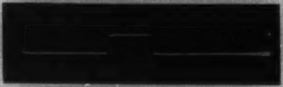


Figure 20. Response of properly terminated transmission line

As case 19, but terminated in characteristic impedance

In figure 19 an example is given of the transients set up in a transmission line, if it is not terminated in its characteristic impedance. The stepping up of the voltage due to excessive termination resistance is clearly visible. Using a decade box, the proper termination was immediately found by adjusting it until the square wave of figure 20 was observed. Determination of this characteristic impedance would have taken considerably more time by conventional methods and not yielded the comprehensive information on the transient response.

It is expected that the above examples may be sufficient to demonstrate the advantages that can be obtained by the use of the square wave generator.

Harmonic-Current-Restrained Relays for Transformer Differential Protection

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ASSOCIATE AIEE

DIFFERENTIAL protection, employing percentage-differential relays, has[1] long been the accepted method of fault protection for large power transformers. This method, illustrated by the schematic diagram, figure 1, is quite satisfactory in most respects but is subject to false tripping on the transient magnetizing-inrush current which flows when the transformer is energized. This current, since it flows in only one winding of the transformer, causes a differential current to flow which, to an ordinary relay, "looks" the same as the current due to an internal fault. One method of preventing operation on magnetizing inrush in the past has been to use slow-speed induction-type relays with long time and high current settings. This is undesirable because of the slow-speed, insensitive protection it affords on internal faults. Other methods, applied to high-speed relays, involve desensitizing the relay during a time period covering the maximum inrush transient controlled by auxiliary relays operating on the transformer voltage.[2] These methods have the disadvantage that they usually require the purchase of additional potential transformers, involve one or more auxiliary relays with the attendant circuit complication, and seriously reduce the sensitivity of the protection against faults present when the transformer is energized or which occur during the inrush period. In this paper a new type relay is described which, using the principle of harmonic-current restraint,[3] is able to distinguish between the differential current due to an internal fault and that due to magnetizing inrush by their difference in wave form, operating with high speed on the fault current and being restrained from operating by the magnetizing-inrush current. Being entirely current-operated, this relay does not require the purchase of potential transformers. It is entirely self-contained and the only moving part is the single relay armature which operates the tripping contacts. Minimum desensitization during the inrush period is obtained, depending on the relative magnitudes of the inrush and the fault currents.

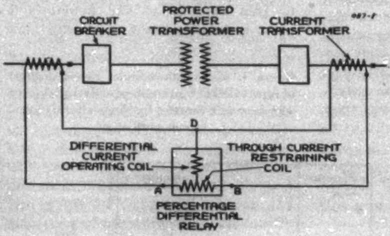
CIRCUIT BREAKER
PROTECTED POWER TRANSFORMER
CURRENT TRANSFORMER
987-F
D
DIFFERENTIAL CURRENT OPERATING COIL
THROUGH CURRENT RESTRAINING COIL
A
B
PERCENTAGE DIFFERENTIAL RELAY

Figure 1. Schematic one-line circuit diagram showing percentage-differential relay protection of a two-winding power transformer

Current may also flow in the differential circuit when no internal fault is present, due to saturation of the current transformers on heavy through faults causing ratio breakdown of differing amounts in the two transformers. The tendency to operate on this current is successfully restrained by the through-current restraint of the ordinary percentage-differential relay. Since the wave form of this current is nonsinusoidal it might seem that harmonic restraint could be used to prevent operation on it, too, and the through-current restraint eliminated. While the harmonic restraint does aid in this, and while it might be used alone in some cases, in general the through-current restraint is needed for two reasons: first, nearly all large modern power transformers are equipped with load-ratio control. As the transformer ratio is changed the ratio between the primary and secondary current-transformer ratios can be made to match it at only one point of the tap-changing range. At other points an unbalance current will flow in the differential relay. On normal load and on low-current through faults this unbalance current will have essentially sine wave form, hence the harmonic restraint cannot distinguish it from the current due to a small internal fault. Second, in many cases, particularly on wye-delta transformer banks, it is necessary to connect at least one set of the current transformer secondaries in delta. On balanced three-phase through faults, and to some extent on other faults, the third-harmonic component will circulate in the delta and will not appear in the differential relay current. This does not occur for the second, fourth, and fifth harmonics, so that adequate restraint can be obtained on differential currents containing liberal proportions of one or more of these. This effect does not occur even for the third harmonic when the current transformer secondaries are all wye-connected.

Figure 2. Type HDD11A percentage-differential relay with harmonic-current restraint for two-winding transformer protection

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Paper 40-87, recommended by the AIEE committee on protective devices, and presented at the AIEE summer convention, Swampscott, Mass., June 24–28, 1940. Manuscript submitted November 8, 1939; made available for preprinting June 3, 1940.

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[1] For all numbered references, see list at end of paper.

Harmonics

Any current of nonsinusoidal wave form ordinarily encountered in electric circuits may be considered as being composed of a direct-current component plus a number of sine-wave components of different frequencies; one of the fundamental line frequency of the circuit and the others harmonic multiples—two, three, four, five, etc., times the fundamental frequency.[4] The relative magnitudes and phase positions of the fundamental and the various harmonic components determine the wave form. Harmonic analyses of typical differential-current wave forms due to magnetizing inrush and due to current transformer saturation on heavy symmetrical through-fault current are shown in table I. These are expressed in terms of relative crest values with the fundamental component taken as 100 per cent.

The current - transformer - saturation wave analyzed, being due to a symmetrical fault current, is itself symmetrical and hence contains only odd-numbered harmonics. The wave for an offset fault current would be asymmetrical, containing direct current and even-numbered harmonics as well as the odd ones.

Table I. Harmonic Wave Form Analysis

	Typical Differential Current Wave Due to:	
Wave Component	Magnetizing Inrush (Per Cent)	Current Transformer Saturation (Per Cent)
Fundamental	100.0	100.0
Direct current	57.7	0.0
Second harmonic	63.0	0.0
Third harmonic	26.8	37.4
Fourth harmonic	5.1	0.0
Fifth harmonic	4.1	16.7
Sixth harmonic	3.7	0.0
Seventh harmonic	2.4	7.8

Briefly speaking, in the harmonic-restrained relay, the fundamental-frequency-current component tends to operate the relay while the harmonic components tend to restrain it. Operation occurs on waves having less than the percentage of harmonics for which the relay is set and is restrained on waves having more than this amount.

Relay Circuits

A percentage-differential relay with harmonic restraint, intended for the protection of a two-winding transformer, is shown in figure 2. The internal circuits of this relay are shown schematically by the diagram, figure 3. Referring to this diagram, it will be seen that the current enters the relay through two small auxiliary current-transformers mounted inside the relay case. One of these has its primary coil connected in the differential circuit and the other has two primary coils, one of which is connected in each of the main current-transformer circuits. These two coils are wound in such relative direction that their magnetizing effects are additive on through-fault currents. The coils on both transformers are tapped to allow approximate matching of the power-line current-transformer ratios. The tap leads are brought out to a double tap-block by means of which the number of turns through which the current flows is changed simultaneously on both transformers when the tap plug is moved from one position to another, maintaining a constant ratio and hence a constant through-current per cent slope characteristic.

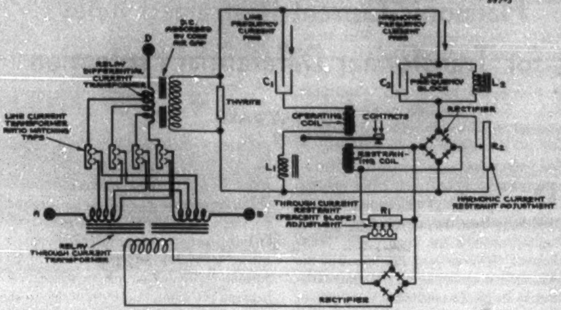
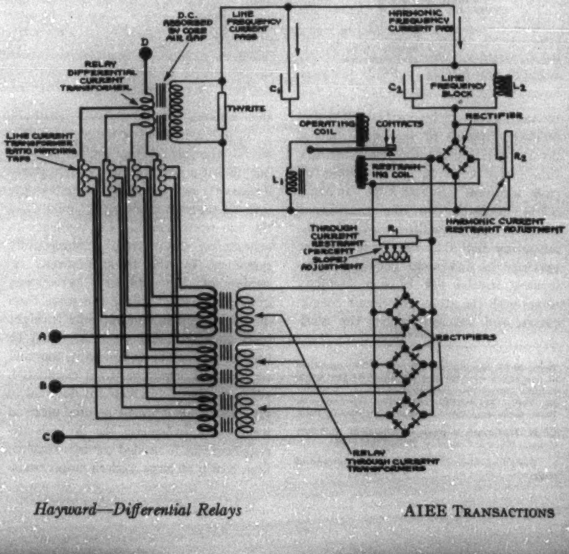

a87-3
D
D.C. ABSORBED BY CORE AIR GAP
LINE FREQUENCY CURRENT PASS
HARMONIC FREQUENCY CURRENT PASS
RELAY DIFFERENTIAL CURRENT TRANSFORMER
C1
C2
LINE FREQUENCY BLOCK
L2
THYRITE
LINE CURRENT TRANSFORMER RATIO MATCHING TAPS
OPERATING COIL
CONTACTS
RECTIFIER
RESTRAINING COIL
R2
L1
A
B
THROUGH CURRENT RESTRAINT (PERCENT SLOPE) ADJUSTMENT
R1
HARMONIC CURRENT RESTRAINT ADJUSTMENT
RELAY THROUGH CURRENT TRANSFORMER
RECTIFIER

Figure 3. Schematic internal circuit diagram of type HDD11A percentage-differential relay with harmonic restraint for two-winding transformer protection

Figure 4. Schematic internal circuit diagram of type HDD11B percentage-differential relay with harmonic restraint for three-winding transformer protection


D
D.C. ABSORBED BY CORE AIR GAP
LINE FREQUENCY CURRENT PASS
HARMONIC FREQUENCY CURRENT PASS
RELAY DIFFERENTIAL CURRENT TRANSFORMER
C1
THYRITE
C2
LINE FREQUENCY BLOCK
L2
OPERATING COIL
CONTACTS
RECTIFIER
LINE CURRENT TRANSFORMER RATIO MATCHING TAPS
RESTRAINING COIL
R2
L1
HARMONIC CURRENT RESTRAINT ADJUSTMENT
THROUGH CURRENT RESTRAINT (PERCENT SLOPE) ADJUSTMENT
R1
A
RECTIFIERS
B
C
RELAY THROUGH CURRENT TRANSFORMERS

Across the secondary of the differential-current auxiliary transformer two parallel circuits supplying the relay element are connected. One of these circuits includes the operating coil of the relay and is tuned to pass currents of the fundamental power-system frequency and to offer high

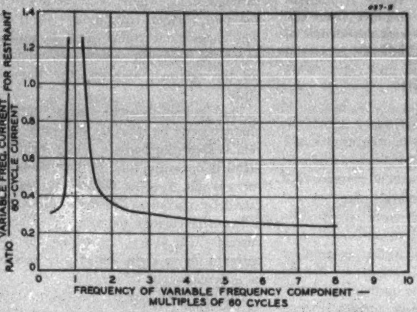


Figure 5. Harmonic-restraint versus frequency characteristic of harmonic-restraint relays

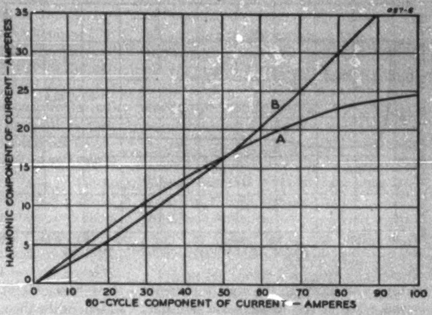


Figure 6. Harmonic-restraint versus current characteristics of harmonic-restraint relays

Curve A—Magnetizing-inrush type wave having predominant second harmonic

Curve B—Current-transformer-saturation type wave having predominant third harmonic

impedance to currents of other frequencies. The other circuit, including the restraining coil of the relay, is tuned to block system-frequency currents and to pass currents of harmonic frequencies. This circuit also includes a rectifier for smoothing the restraint magnet pull. The per cent of harmonic current required is adjusted by means of the shunting resistor R_3.

The d-c component present in the differential current is blocked by the differential-current transformer which is equipped with an air-gap core for this purpose. Only a small part of it appears in the secondary circuit and this divides between the operating and restraint circuits, producing little effect either to operate or to restrain the relay. Elimination of the d-c component prevents the saturation of the reactors L_1 and L_2 with consequent detuning of the filter circuits which would occur if it were allowed to flow.

A Thyrite resistor connected across the relay differential-current transformer secondary serves to limit the peak value of momentary voltage "kicks" which sometimes occur due to shock current excitation of the tuned circuits. These voltage peaks, if not limited, would be likely to puncture the rectifiers or capacitors. The time duration of the voltage peaks is so short and, due to the nonlinear resistance characteristic of the Thyrite, the current passed through it by the lower voltages present at other times is so small that the operation of the relay is not appreciably affected.

The secondary of the through-current transformer also supplies current to the restraining coil through a second rectifier. This transformer also has an air-gap core for elimination of the d-c component. A shunting resistor R_1 with taps brought out to a tap block provides adjustment of the per cent slope of the through-current restraint characteristic.

For three-winding power-transformer protection, a relay having the through-current restraint circuits modified as shown in figure 4 is provided. Three independent through-current-restraint auxiliary transformers, one for each of the three line-current-transformer circuits, are provided. Each auxiliary transformer has its secondary winding connected through an independent rectifier to the restraining coil of the relay. This arrangement allows complete freedom as to location of power sources and loads on the three windings of the power transformer, maintaining a constant per cent slope independent of the relative directions of current flow in the three windings.

Construction and Characteristics

The two-winding transformer relay, type *HDD11A*, is mounted in the standard $5^1/_2$ by 16-inch universal relay case, as shown in figure 2. The three-winding transformer relay, type *HDD11B*, is also mounted in the $5^1/_2$ by 16-inch case and has the same external appearance except that where, in the two-winding relay, two horizontal rows of tap positions are provided for matching the ratios of the current transformers in the two winding circuits, three rows are provided in the three-winding relay. Eight current taps are provided in each relay having nominal ratings of 2.9, 3.2, 3.5, 3.8, 4.2, 4.6, 5.0, and 8.7 amperes.

The relay element is of hinged-armature construction. Two electrically-separate single circuit-closing contacts are provided, with an electrically-operated target in series with one of them.

The differential burden imposed on the current transformers by the 60-cycle relay is 16.0 volt-amperes, 12.5 watts, measured at 5.0 amperes on the 5.0 ampere tap. The through-current burden on each current transformer is 1.2 volt-amperes, 1.0 watts measured in the same way. These burdens vary inversely as the square of the tap current rating for other taps but are substantially independent of the per cent slope setting.

The minimum pickup on sine-wave rated-frequency current, is 1.0 ampere. This is with the tap plugs in the 5.0 ampere and 25 per cent slope positions. This value varies directly with the tap current rating and is slightly higher for the 40 per cent slope, lower for the 15 per cent slope positions.

The harmonic-restraint versus frequency characteristic curve in figure 5 illustrates the selective action of the relay filter circuits. This curve was obtained by mixing two currents in the relay, one of 60-cycle frequency and the other of a frequency which was held at a number of different values varying over a range of from 25 to 480 cycles. At each point the variable-frequency current was held at ten amperes and the 60-cycle current increased until the relay operated.

The harmonic-restraint versus current characteristics in figure 6 were taken under conditions more closely simulating those of actual service. Curve *A* was taken with a current wave approximating the magnetizing-inrush form. This current was obtained from a small saturating reactor, polarized with direct current in another winding. Sine-wave 60-cycle

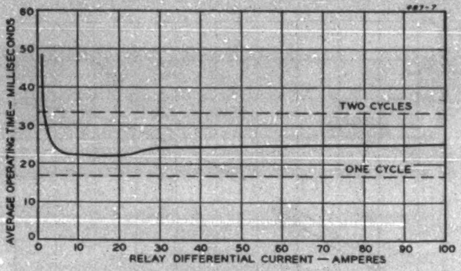


Figure 7. Operating-time versus current characteristic of harmonic - restraint relays

current was added to the reactor current in the relay in the same phase position as the 60-cycle component of the reactor current. The relative proportions of the two currents were adjusted to obtain a series of values at which the relay operated. The magnitudes of the fundamental and second-harmonic components were read with a special harmonic current meter. These two components were plotted against each other to obtain the curve. An attempt was made to operate the reactor in a saturation range giving as little third and higher harmonics as possible in order to isolate the effect of the second harmonic. The magnitude of the third harmonic measured as a check did not exceed eight per cent of the fundamental.

Curve *B* was obtained using the same reactor without the d-c polarization. This gave a current wave of the current-transformer-saturation type. In this case the magnitude of the predominant third harmonic was read and plotted against the fundamental.

The falling off of curve *A* as well as the rise in curve *B* is due to the exciting current taken by the relay differential-current transformer. This exciting current has a third harmonic component which increases in relative magnitude at the higher currents. In the case of the magnetizing-inrush or predominant-second-harmonic type wave used for curve *A* this exciting current subtracting from the differential current flowing in the primary of the relay differential current transformer leaves a third harmonic of reversed phase position in the secondary circuit which combines with the second harmonic in producing restraint. Hence, as the magnitude of the differential current is increased a decreasing proportion of second harmonic is needed in it to produce restraint. In the case of the current-transformer-saturation or predominant-third-harmonic type wave used for curve *B*, the third harmonic in the exciting current is in the same phase position as the third harmonic in the primary current so that when the exciting current is subtracted a reduced proportion of harmonic is present in the secondary circuit. This makes it necessary to have an increasing proportion of third harmonic in the differential current fed into the relay in order to obtain the constant proportion required to restrain the relay in the secondary circuit of the differential current transformer.

In the application of the relay the falling off of curve *A* is of no consequence since adequate restraint is provided for all magnitudes of magnetizing-inrush-current waves. The rise in curve *B* is definitely advantageous in that it gives a greater operating margin on high internal-fault currents, which contain a certain amount of harmonics due to current-transformer saturation. The percentage of harmonics present in the differential current due to an internal fault, where the currents from the two current transformers add, is less, of course, than in the case of the differential current due to a through fault, where the currents from the two current transformers subtract. The harmonic-restraint curve *B* lies between the harmonic percentages obtained for the internal fault and the through fault.

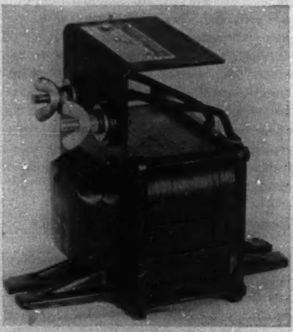


Figure 8. Portable saturating test reactor for adjusting harmonic-restraint setting of harmonic-restraint relays

The operating-time versus current characteristic on 60-cycle sine-wave current is shown in figure 7. The slight increase in operating time for the higher currents is due to some self-restraint introduced by partial saturation of the relay differential current transformer at high current. The average operating time for currents above eight amperes is seen to be approximately $1^1/_2$ cycles.

The through-current restraint slope characteristics are not shown since they are quite similar to those of other percentage differential relays. Taps for 15, 25, and 40 per cent slope are provided.

A means of checking the unbalanced differential current flowing in the relay on normal load is provided by two terminals, not shown in figures 3 and 4, connected to the secondary leads of the relay internal differential-current transformer. A high-resistance voltmeter momentarily connected to these terminals gives an indication of any unbalance current without disturbing the current connections of the relay.

Tests and Adjustments

The relay may be completely tested and adjusted on the switchboard by means of simple and readily portable equipment. The test circuits used for checking the minimum pickup current and the per cent slope or through-current restraint are the same as for any other percentage-differential relay and need not be described here.

For checking the harmonic-restraint setting of the relay a special saturating reactor is used. This reactor, illustrated in figure 8, is readily portable since it weighs only nine pounds. In use, the reactor is simply connected in series with the relay terminals to a 115-volt source of the rated frequency. The steady-state current passed has an rms value of ten amperes and contains a third-harmonic component whose magnitude is 25 per cent of that of the fundamental-frequency component. Under steady-state conditions, no even harmonics are present and higher odd harmonics are present only in negligibly small amounts. Normal line voltage variations do not appreciably affect the harmonic percentage of the reactor current because of a special core construction which holds the percentage constant within 1 per cent over a range of 105 to

125 volts, although the current magnitude varies. Variation in the harmonic percentage due to changes in the relay burden when the current transformer ratio-matching taps or the per cent-slope taps are changed is negligible. With the reactor current flowing in the relay, the harmonic-restraint adjusting resistor (*R*2 in figures 3 and 4) is varied until the relay armature is just on the verge of picking up.

Performance Tests

As "the proof of a pudding is in the eating", so the proof of a relay is in the quality of its performance under actual service conditions. A number of oscillograms illustrating the performance of the type *HDD*11*A* harmonic-restrained percentage-differential relay tested under conditions closely simulating those met in actual service are shown in figures 9*A*, *B*, *C*, *D*, and *E*. The only departure of any significance from typical service conditions in these tests lies in the size of the transformer used. The transformer to which the differential protection was applied in these tests was a single-phase 333-kva General Electric distribution-type transformer; type *H*, form *D*, 13,800/460 volts, 60 cycles. High-speed differential protection would not ordinarily be applied to a transformer as small as this. The only important effect this has on the results, however, is in the decay rate of the magnetizing-inrush current. In large transformers, the inrush current falls off at a much slower rate than it does in small transformers, in many instances remaining at values in excess of full-load current for a period of several seconds. The rate of decay in the transformer used, however, was sufficiently slow to illustrate the performance of the relay. The slower rate obtained with a larger transformer would only serve to make the need for harmonic restraint and its advantages over other methods of preventing false operation on magnetizing inrush more evident.

A type *KC*-58, 20/5-ampere current transformer was used on the 13,800-volt side of the transformer and a type *JW*-1, 600/5-ampere current transformer on the 460-volt side. These ratios were chosen so that the normal load secondary currents would exactly match. The current-transformer ratio-matching tap plugs on the relay were both placed in the 5.0-ampere position. For the first tests the through-current restraint tap plug was placed in the 25 per cent slope position and the burden on each current transformer, including leads and oscillograph shunts, was 1.0 ohm resistance.

Figure 9*A* shows a magnetizing-inrush test with the transformer being energized from the 13,800-volt side. No contact operation is indicated by vibrator number 6 although the crest of the first current peak measures 65 amperes and the pickup setting of the relay on sine wave current is 1.0 ampere rms.

Figure 9*B* shows a similar test in which the circuit breaker was closed near the

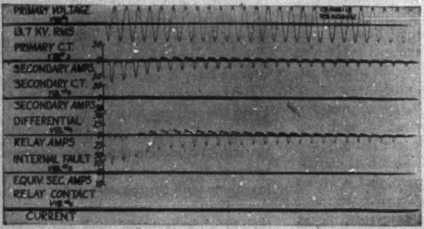


Figure 9. Oscillograms of performance tests of type HDD11A transformer percentage-differential relay with harmonic restraint

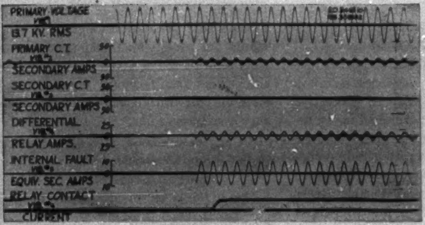
Oscillogram *A* (left)—Transformer magnetizing inrush

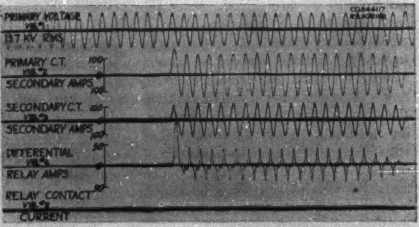
Oscillogram *B* (center left)—Low-current internal fault

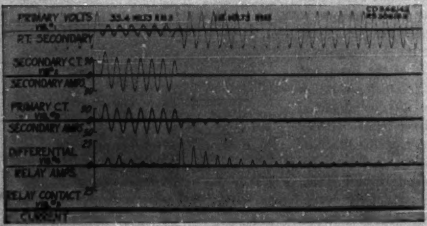
Oscillogram *C* (bottom left)—Magnetizing inrush due to recovery of transformer voltage after clearing of a through fault

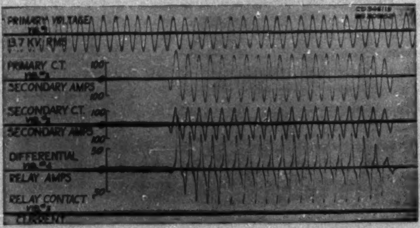
Oscillogram *D* (top right)—High-current through fault

Oscillogram *E* (bottom right)—High-current through fault with ten per cent ratio unbalance









crest of the voltage wave so that almost no inrush current was obtained. Six and one-half cycles after the transformer was energized a resistance load, simulating an internal fault, was connected across the 460-volt winding of the transformer, between the transformer and the 600/5-ampere current transformer. This load drew a current equivalent to 5.5 amperes rms in the current-transformer secondary circuit. Vibrator number 6 shows operation of the relay contacts in $1^1/_4$ cycles.

Figure 9*C* shows a test of the relay on the magnetizing-inrush current caused by the recovery of the transformer voltage following the clearing of a through fault. For this test, the transformer was supplied with power from the 460-volt side through another smaller transformer, the impedance of which simulated the impedance of a long transmission line. A short circuit was applied to the 13,800-volt side (now the secondary side) of the protected transformer, just outside of the 20/5-ampere current transformer. As shown by vibrator number 1, the primary voltage of the transformer as measured on the secondary side of a 460/115-volt potential transformer dropped from its open-circuit value of 112 volts to 33.4 volts when the fault was applied. Although the fault current was only seven times the rating of the current transformers and the total burden of each current transformer was only 0.5-ohm resistance, an appreciable differential current, probably due to residual magnetization in one of the current transformers, is shown by vibrator number 4 during the fault period. When the fault was cleared by operation of an oil circuit breaker on the 13,800-volt side, the transformer voltage returned to normal causing a magnetizing-inrush current to flow. Although lower in magnitude than the inrush current obtained when energizing an initially "dead" transformer, it is still high enough to cause the operation of relays not equipped to prevent it. The harmonic-restraint relay was restrained from operating in this test as is demonstrated by vibrator number 5.

Figure 9*D* shows a heavy through-fault test. The transformer was again supplied from the 13,800-volt side and a short circuit was applied to the 460-volt side just outside the 600/5-ampere current transformer. The current obtained was approximately 18 times the normal ratings of the current transformers, each of which had a total burden of 1.0 ohm resistance. For this test the through-current restraint tap plug was placed in the 15 per cent slope position, although the usual practice on other type percentage-differential relays is to use 25 per cent slope or more for transformer protection. Vibrator number 5 shows that the relay was restrained from operating although the 600/5-ampere current transformer broke down in ratio considerably more than did the 20/5-ampere one, resulting in a large differential current. The wave shape of the differential current, however, indicates the presence of a considerable third-harmonic component which made it possible for the harmonic-restraint feature of the relay to aid the through-current restraint in preventing false operation.

Figure 9*E* shows an even more severe test of the ability of the relay to restrain itself from operating on through faults. The differential system was unbalanced ten per cent by changing the transformer primary tap from the 13,800-volt position to the 12,420-volt position. In addition, this change resulted in an increase in the fault current to approximately 20 times the current transformer ratings. The already faltering 600/5-ampere current transformer was further embarrassed by increasing its burden to 1.5 ohms resistance while that of the 20/5-ampere current transformer was reduced to 0.5 ohm resistance. The resulting differential current is much greater, as would be expected, but owing to the harmonic restraint no relay operation resulted although only 15 per cent through-current restraint was used. It is obvious from the magnitude of the differential current that a higher percentage of through-current restraint would be required if it were used alone, without the aid of the harmonic restraint.

Application—Selection of Current Transformers

As with other high-speed protective relays, care should be exercised in the selection of current transformers for use with *HDD*-type relays to obtain types and ratings giving adequate performance during the transient period immediately following initiation of a fault. This is particularly important where the application is near the generators because of the slow decay rate of the d-c component in offset-wave fault currents obtained there. The time-constant for the d-c component increased with the size of the generator, reaching values of 0.25 to 0.35 second for the largest ones. The d-c component of the fault current will have this slow decay rate for a fault directly at the machine terminals. As the electrical distance to the fault increases the intervening transmission lines, cables, reactors, transformers, bus bars, etc., introduce resistance into the fault-current circuit at a more rapid rate than inductance so that the time-constant decreases rapidly at first, more slowly afterward, reaching values of 0.02 second or lower at the distribution points.

The d-c component requires much more flux in the current-transformer core for its faithful reproduction in the secondary circuit than does the a-c component, nearly 100 times as much for a fully offset 60-cycle wave having a 0.25-second d-c time constant. The cores of ordinary current transformers become saturated in attempting to carry this flux and ratio breakdown occurs. Under extreme conditions the breakdown may be almost complete. In the secondary differential circuit, then, the current transformer does not effectively distinguish between an internal and an external fault and no relay can be expected to give correct performance. While harmonic restraint, particularly when combined with through-current restraint, is an effective factor in promoting correct performance on values of current transformer saturation approaching this extreme, still it is not a "cure-all" and there is a limit beyond which a relay so equipped will fail. Current transformers should be selected to avoid this extreme saturation.

Standard current transformers, particularly the bushing types, are in general satisfactory for applications where the maximum fault current does not exceed 20 times the current-transformer rating and the d-c time constant does not exceed 0.05 second. Where the application is made to the protection of a transformer located near generation longer time constants are usually met and special consideration should be given the selection of the current transformers. Special large-sized current transformers may be necessary in extreme cases.

References

1. Relay Handbook and Supplement, National Electric Light Association, 1926 and 1931 editions, pages 607 and 1158.

Ratio-Differential Relay Protection, H. P. Sleeper. *Electrical World*, October 22, 1927, pages 827–31.

Relay Protection for Power Transformers, J. W. Dodge. *General Electric Review*, May 1929, pages 268–72.

Percentage-Differential Transformer Protection, R. E. Cordray. Electrical Engineering, May 1931, pages 361–3.

2. Preventing False Operation of Differential Relays, R. E. Cordray. *Electrical World*, July 25, 1931, pages 160–1.

3. Harmonic-Current-Restrained Relays for Differential Protection, L. F. Kennedy and C. D. Hayward. AIEE Transactions, volume 57, 1938 (May section), pages 262–71.

4. Theory of Alternating-Current Wave Forms (a book), Philip Kemp. Instruments Publishing Company, 1935.

Medium-Capacity Air-Blast Circuit Breakers for Metal-Clad Switchgear

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Synopsis: The cross-blast principle of arc interruption has been shown to provide maximum air economy; consequently this method of interruption is well suited for use in an air-blast breaker that must conform to the rigid space requirements and small phase spacings encountered in medium-capacity metal-clad equipment.

It is shown that proper co-ordination of the operating mechanism, air supply, interrupting chamber, and resistor make possible a design to meet these requirements.

The principle of operation is discussed and it is shown how a resistance is utilized to provide further air economy.

THE trend in the design of modern switchgear has for some years been in the direction of increased compactness and reliability. A marked advance in this development was the advent of the oil-blast principle[1] of circuit interruption, by which a stream of oil was made to flow across the arc path in a confined arcing chamber in such a manner as to sweep the ionized gases from between the contacts. The use of the oil-blast principle made possible drastic size reductions and increased the available ratings for metal-clad switchgear. A complete line of low-oil-content short-interrupting-time oil circuit breakers utilizing this oil-blast principle, rated from 25,000- to 500,000-kva interrupting capacity, were developed for vertical-lift metal-clad switchgear.

Recent years have seen a strong demand for oilless circuit breakers resulting from growing appreciation of their superiority.

As recent papers[2,3,4] have pointed out, air circuit breakers possess certain inherent advantages over oil breakers, which can be obtained only by the complete elimination of oil as an interrupting medium. These advantages, which arise from the nature of air circuit breakers, include freedom from oil fire and explosion hazards, cleanliness, and ease of inspection, and perhaps most important of all, greatly lowered maintenance due to the fact that it is no longer necessary to drain and renew the interrupting medium. This is particularly advantageous in breakers where the interrupting medium also acts as the insulation medium and is deteriorated by the action of the arc.

These benefits arising from the use of air circuit breakers are complementary to the well-known advantages of metal-clad gear; consequently, it became highly desirable to develop a medium-capacity air circuit breaker for use in metal-clad switchgear.

In a paper presented at the 1940 AIEE winter convention,[3] by D. C. Prince, J. A. Henley, and W. K. Rankin it was shown that the European designs for medium voltages, using radial-type nozzles, require excessive amounts of air to meet the requirements of American practice and they described the application of the cross-blast principle to overcome this difficulty. At the same time H. E. Strang and A. C. Boisseau[4] showed how this principle is applied in the construction of the high-capacity breakers for 15-kv service.

These breakers which are of the cubicle type are available for service in ratings from 500,000 kva to the highest capacity required at 15 kv.

Oilless circuit breakers demanded for metal-clad installation are of 250,000- and 500,000-kva rating. In the development of these air-blast breakers, the cross-blast principle of circuit interruption, characterized by the economical use of compressed air, has made possible the design of an extremely compact unit.

Requirements

Obviously any air circuit breaker that is expected to compete with the well-accepted oil circuit breaker must not only offer new features that are desirable but must also have ratings comparable to and be in every way as reliable and

Figure 1. Air-blast breaker shown in raised position in metal-clad enclosure 15 kv, 1,200 amperes

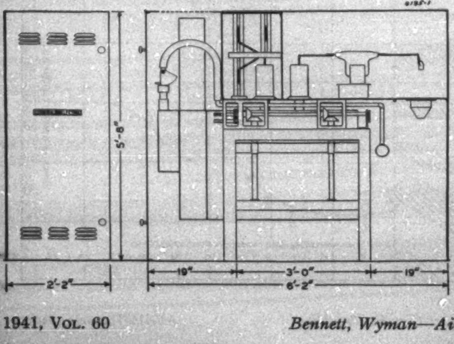
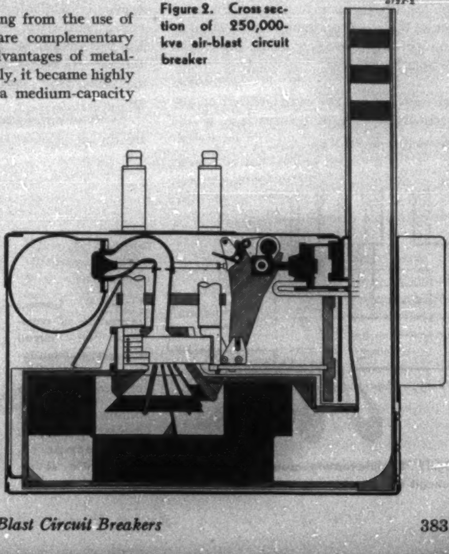


Figure 2. Cross section of 250,000-kva air-blast circuit breaker



Paper 40-125, recommended by the AIEE committee on protective devices, and presented at the AIEE summer convention, Swampscott, Mass., June 24-28, 1940. Manuscript submitted April 25, 1940; made available for preprinting May 16, 1940.

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1. For all numbered references, see list at end of paper.

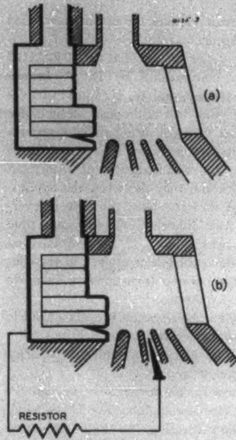

Figure 3. Interrupting chamber of air-blast circuit breaker

(*a*)—Without shunt resistor

(*b*)—With shunt resistor

efficient as its predecessor. To be acceptable for metal-clad service, this air-blast breaker must also be comparable in size to oil circuit breakers of the same rating. This means, for example, that a 250,000-kva circuit breaker must be accommodated by a 26-inch metal-clad structure with six-inch phase spacings for the circuit breaker bushings. Figure 1 shows the space available for the 250,000-kva air circuit breaker in the metal-clad compartment.

Inasmuch as nearly all switchgear manufacturers have standardized on the vertical-lift type of construction as the best possible arrangement and as providing maximum space for potheads and cable termination and for space conservation, it was highly desirable that an air-blast circuit breaker for metal-clad switchgear retain this feature. This type of construction requires the use of vertical bushings located at the top of the breaker.

One further well-recognized requirement of such a breaker is that all flame emission during interruption at full capacity must be eliminated.

Theory of Interruption

In order to meet the foregoing requirements, particularly with respect to size, a very careful co-ordination of the various parts of the breaker was necessary. Briefly, the breaker must consist of five parts, a control valve, a mechanism, an air tank, an interrupting chamber, and a space for disposing of the arc products. The compact arrangement of these parts in the completed breaker is shown by figure 2.

It has been recognized that the dimensions of the air tank, interrupting chamber, and muffling structure are all interrelated. This relationship will be briefly described.

Figure 3*a* shows the interrupting chamber of a typical cross-blast breaker. It is the purpose of this chamber to permit the arc, which is formed between the separating contacts, to be blown into a series of loops as in figure 4.

Except at the current zero the arc path is continuous but at the current zero no energy is available to further ionize gas, consequently the high-velocity air parts the arc stream at the foreward edge of the barriers and sweeps the remaining arc segments away into the exhaust structure.

As these arc segments are swept away by the air blast at the foreward edges of the barriers, they are replaced by fresh dielectric. The rate at which dielectric strength is inserted between the contacts depends upon the dielectric strength and velocity of the air stream, and the number of points at which fresh dielectric separates the arc stream. As the current reaches zero, however, the voltage across the breaker contacts begins to rise at a rate that is dependent upon the constants of the particular circuit being interrupted[5] and it is widely recognized that the rate at which this voltage appears across the breaker contacts is a measure of the severity of the interrupting duty imposed upon the breaker.[3]

For successful interruption, the dielectric strength inserted by the air stream between the terminals of the circuit breaker must increase at a rate greater than the rate of rise of recovery voltage. Thus if the air velocity is 0.01 inch per microsecond and the dielectric strength of air is 50,000 volts per inch, the rate of rise of dielectric strength of a single break is 50,000×0.01 or 500 volts per microsecond. That is, the rate of rise of dielectric strength between the breaker contacts is given by multiplying the dielectric strength of the air by the air velocity and by the number of breaks in the arc stream. At the pressure differences used, this velocity is very nearly constant at the velocity of sound. But the dielectric strength of this air is almost directly proportional to the pressure; consequently, the rate of rise of dielectric strength is proportional to the product of the air pressure and the number of breaks in the arc stream.

Consider now the air consumption of the device; with the velocity constant, the quantity of air discharged depends only upon the air pressure and the number and size of the air passages between the barriers. Consequently, the air consumption is directly proportional to the

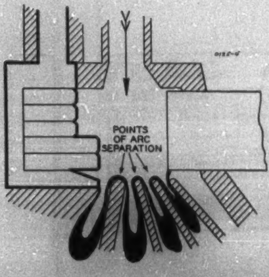

Figure 4. Interrupting chamber of air-blast circuit breaker showing configuration of arc

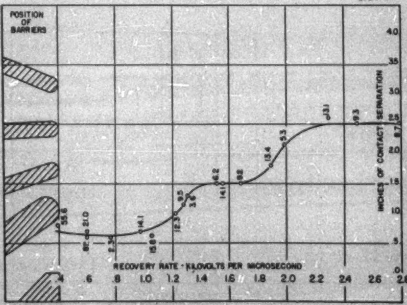

Figure 5. Contact separation of cross-blast air circuit breaker at interruption versus rate of rise of recovery voltage

Numbers indicate current interrupted in kiloamperes. Tests made at reduced air pressure rate of increase of dielectric strength required of the breaker. This rate of increase of dielectric strength must, of course, always exceed the rate of rise of voltage at the breaker terminals after a current zero which is purely a function of the circuit in which the breaker is employed. Therefore, it is evident that, with the number of air passages fixed by the rate of increase of dielectric strength required, the only way of achieving air economy is to reduce the size of each of the air passages through which the arc is blown to form a loop. (See figure 4.) However, it has been found that these openings are fixed by the amount of current to be interrupted. That is, they must be sufficiently great in area to permit the formation of a loop in the arc before current zero. If the openings are too small, the arc remains straight across the barriers up to the current zero. In this case the arc products cannot be forced out through the restrictions between the barriers before the recovery voltage rises and causes re-establishment of the arc.

The importance of the dimensions of these air passages is well illustrated by comparison of the curves in figures 5 and 6. Figure 5 shows that adequate air passage dimensions make the operation of the interrupting unit literally independent of current within its range, but dependent only upon the recovery rate of the circuit involved. Each point shown in figure 5 is the average of a large number of tests and represents the number of inches of contact separation required to interrupt the indicated rate of rise of recovery voltage under conditions of a single value of reduced tank pressure. The number associated with each point is the current interrupted in kiloamperes. Note that the points form a curve without any regard to the current magnitudes shown. It may be noted that a distinct flattening of the curve occurs at each barrier indicating that as each additional barrier is introduced into the arc path, permitting the formation of a new break in the arc stream, there occurs a definite increase in the rate of rise of recovery voltage that may be interrupted.

Figure 6 represents a series of tests in which the openings between the barriers were too small and were partially choked by the higher currents so that displacement of the arc products was delayed and additional breaks in the arc stream were required to produce interruption.

It has been indicated that the volume of air required and consequently the size of the air tank, is directly proportional to the rate of rise of recovery voltage.

Figure 8. Gas collected from one phase of 250,000-kva air-blast circuit breaker in rubber balloon fastened to exhaust tube

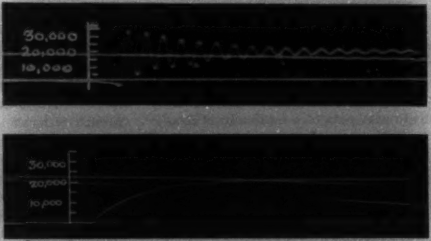

Figure 7. Cathode-ray oscillograms of recovery voltage after interruption

(*a*)—Without shunt resistor

(*b*)—With shunt resistor

In order to provide an adequate factor of safety for application on any circuit, the cross-blast breaker must have sufficient air passages and be supplied with sufficient air to interrupt the highest recovery rates encountered in practice. However, if the recovery rate of the circuit can be modified by a shunt impedance (3) and held at a known low value a smaller number of air passages are required and great air economy can be achieved. It is imperative to obtain maximum air economy in order that the air tank may be sufficiently small to fulfill the space requirements of the breaker.

To keep the air consumption of this breaker low then, the rate of rise of the recovery voltage has been limited by the use of a shunt resistor which reduces this rate to a maximum of 500 volts per microsecond regardless of normal circuit rates of rise.

Figure 3*b* shows the interrupting element and resistor connection employed.

With such an arrangement the recovery voltage appearing across the contacts after the circuit has been interrupted, breaks down the gap between the blade contact and the resistor electrode inserting the resistance between the contacts. The small current flowing through the resistance is interrupted after further movement of the blade contact which places a barrier between the resistance electrode and the moving contact. This

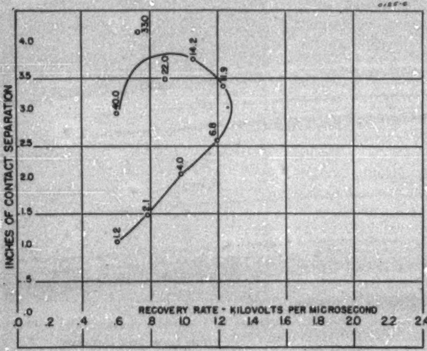

Figure 6. Contact separation of cross-blast air circuit breaker at interruption versus rate of rise of recovery voltage

Air passages between barriers too small

Numbers indicate current interrupted in kiloamperes. Tests made at reduced air pressure

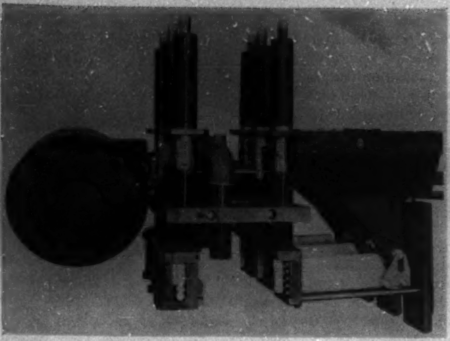

Figure 9 (left). Mechanism and contact arrangement of air-blast circuit breaker for metal-clad switchgear

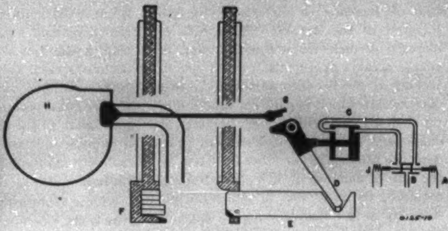

Figure 10. Schematic diagram of circuit-breaker mechanism

small resistor current is easily interrupted as it is nearly in phase with the applied voltage so that the rate of rise of the resistor circuit voltage is very low.

Figure 7 shows two cathode-ray oscillograms of the recovery voltage after interruption, both with and without a shunt resistor. In the first the rate of rise of recovery voltage is determined entirely by the external circuit constants and overshoots to a maximum of nearly twice the normal 60-cycle peak voltage. In the second, is shown an interruption of the same external circuit but with a shunt resistor added to the breaker. The rate of rise of recovery voltage in the latter case is reduced from 2,700 to less than 100 volts per microsecond and does not rise above its 60-cycle crest. This feature is of additional importance on circuits where switching transients are dangerous.

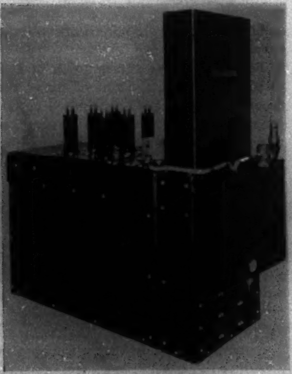

Figure 11. Complete metal-enclosed air-blast circuit breaker

In the process of interruption, considerable hot gas is produced and must be cooled and removed from the arc chute. This cooling is accomplished by two methods: by mixing the gas with cool air from the air blast and by driving it between closely spaced copper cooling plates which present a large surface to the air stream. After thorough cooling inside the arc chute, the gases are carried through exhaust tubes and delivered outside the metal-clad structure.

These cool gases have been collected in a rubber balloon fitted to the top of the exhaust tube for the purpose of gas analysis and a photograph of such a test (figure 8) at 36,000 amperes indicates the remarkably small amount of air and gas released from one phase during interruption.

Mechanism

The size of the mechanism of the air blast breaker is dependent largely upon the mass and speed required of the moving contact and the magnitude of the magnetic forces encountered in operation. The mechanism is pneumatically operated and its compactness and strength are well illustrated by the photograph of figure 9.

The mechanism consists of a two-way control valve that supplies air to two double-acting operating pistons connected directly to the operating shaft which in turn is directly connected to the blade-type contacts through operating arms of insulating material. The crankshaft also opens and closes the main air-blast valve at a predetermined position in the stroke of the blade contacts. This streamlined air-blast valve releases a blast of compressed air into a distributing manifold connected through insulating air tubes to the interrupting chamber of each of the arc chutes.

The action of the breaker can be seen from the schematic diagram in figure 10. With the breaker in the open position as shown, energizing closing coil *J* opens the control valve *B* to admit compressed air to the closing side of piston *C*. This operates the insulated arm *D* which moves the blade type contact *E* to engage the stationary finger contacts *F*. The moving blade makes contact first with the arc-resisting tungsten-silver arcing contacts. The opening of air-blast valve *H* by cam *G* shortly before the contacts meet allows a blast of compressed air to be directed between the contacts. The control valve is "sealed in" the closing position until the breaker is fully closed at which time it is released by a mechanically operated switch.

With the breaker in the closed position, energizing the closing coil *A* opens the control valve *B* to admit compressed air

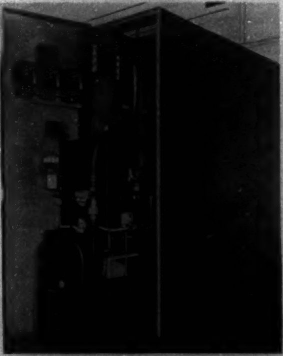

Figure 12. Air-blast breaker being removed from metal-clad enclosure

Table I

Test No.	Duty*	Test Kilovolts	Current Amperes Max. Phase: Max. Loop Peak Value	Current Amperes Max. Phase: Max. Loop RMS Value	Current Amperes Max. Phase: Initial in Arc RMS	Kva Three-Phase	Normal Circuit Recovery Rate Volts Per μ Sec	Cycles From Trip Impulse to Interruption	Arc Length (Cycles)
1	O	4.2	950	570	570	4,100	220	3.3	0.4
2	O	4.2	7,700	4,500	3,800	28,000	460	3.3	0.4
3	O	4.2	23,000	13,700	11,600	84,000	1,080	3.3	0.4
4	O	4.2	74,000	43,000	34,000	250,000	850	3.9	0.6
5	O	4.2	86,000	52,000	34,000	250,000	660	4.0	0.7
6	CO	4.4	1,040	620	620	4,700	190	3.9	0.3
7	CO	4.4	8,200	4,900	4,200	32,000	590	3.9	0.3
8	CO	4.4	25,000	15,000	12,700	97,000	840	4.4	0.6
9	CO	4.4	96,000	57,000	38,000	290,000	980	5.5	0.6
10	CO	4.4	94,000	56,000	37,000	280,000	850	5.6	0.7
11	CO	4.4	91,000	55,000	35,000	270,000	780	4.7	0.7

* O—Opening; CO—Closing-opening.

to the opening side of piston *C*, thereby opening the blade contacts and the blast valve *H* to admit a blast of compressed air across the contacts effecting interruption.

It will be noted from the mechanism photograph and the schematic diagram that the blade motion is horizontal and at right angles to the vertical studs. The magnetic forces on the moving contact due to the flow of current are, therefore, downward at right angles to the motion of the blade contact. This arrangement makes the closing speed of the breaker independent of the short-circuit current and eliminates any possibility of stalling the mechanism during closing.

One of the advantages of the air-blast breaker is the relatively small contact erosion experienced.[4] This characteristic is the result of a number of factors. The high closing and opening speeds attainable mean that the arc duration is very short, but equally important is the use of an air blast during closing as well as opening. It is found that the increased dielectric strength of the compressed air prevents the formation of an arc until the contacts are very nearly closed and in addition the air blast directs the arc to the arcing fingers and provides excellent cooling. The combination of these features has reduced contact erosion in air-blast breakers to a negligible amount, providing considerable improvement over oil breakers.

When the breaker is in the open position, the blade contacts are entirely withdrawn from the arc chute and act as internal disconnectors providing a clear air break between the contacts.

Removal of the interphase barriers permits a full view of these contacts, their condition being a good indication of the condition of the stationary finger-type contacts as well. If the blade contacts show any appreciable erosion, the stationary finger contacts can be easily taken out for inspection without removing the arc chutes.

The circuit breaker is completely enclosed in metal as shown by the photograph in figure 11. Each phase of the breaker is isolated by interphase barriers of high dielectric strength. The whole circuit breaker is arranged to be as easily removable from the metal-clad cabinet as the familiar oil breaker, figure 12 being a photograph showing the breaker disconnected and partially removed. Figure 14 is a photograph of metal-clad switchgear incorporating air-

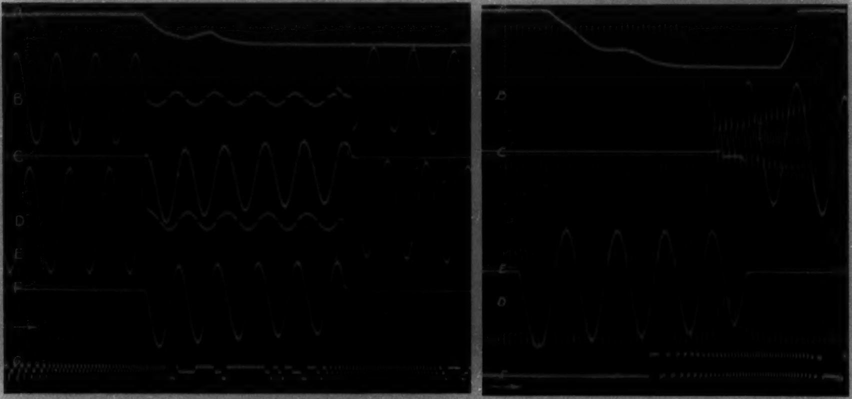

Figure 13. Magnetic oscillograms of typical operations

(a)—Two phases of a typical three-phase closing-opening operation at 4,400 volts, closing current 96,000 amperes peak, 57,000 amperes rms. Interrupted current, 38,000 amperes rms, initial in arc, 290,000 kva

(b)—One phase of a typical three-phase interruption at 14,500 volts, 12,600 amperes rms, initial in arc, 315,000 kva

Table II

Test No.	Duty*	Test Kilovolts	Current Amperes Max. Phase			Kva Three-Phase	Normal Circuit Recovery Rate Volts Per μ Sec	Cycles From Trip Impulse to Interruption	Arc Length (Cycles)
			Max. Loop						
			Peak Value	RMS Value	Initial in Arc RMS				
1	O	14.5	3,800	2,300	2,200	55,000	240	3.7	0.4
2	O	14.5	11,800	7,000	6,200	156,000	1,180	4.2	0.8
3	O	14.5	13,600	8,400	8,100	200,000	1,520	3.9	0.4
4	O	14.5	25,000	14,500	14,500	360,000	1,690	4.2	0.7
5	CO	14.5	30,000	18,000	12,100	300,000	1,360	5.5	1.4
6	O	14.5	2,200	1,480	1,400	35,000	670	4.2	0.8
7	O	14.5	7,400	4,700	3,900	98,000	710	4.3	0.6
8	O	14.5	10,000	5,800	5,200	131,000	1,080	3.9	0.3
9	O	14.5	12,600	7,900	7,400	186,000	1,480	5.0	0.9
10	O	14.5	31,000	18,500	15,800	400,000	1,600	4.5	0.8
11	O	14.5	25,000	14,800	11,700	290,000	1,520	4.3	0.7
12	O	14.5	22,000	12,600	11,000	280,000	1,540	4.3	0.6

* O—Opening; CO—Closing-opening.

blast circuit breakers.

Table I is a set of three-phase interrupting tests made for the representatives of the American Gas and Electric Company. They have been pioneers in the application of the air-blast circuit breaker for metal-clad service and now have eight in operation.

Figure 13*a* shows the operation of the 250,000-kva air-blast breaker closing against a peak current of 96,000 amperes, 57,000 amperes rms, and interrupting 290,000 kva at 4,400 volts.

Table II is a set of three-phase tests showing the operation of the 250,000-kva air-blast breaker at 14,500 volts and figure 13*b* shows the interruption of 315,000 kva at 14,500 volts.

Conclusion

It should be pointed out that the adaptability and flexibility of the air-blast principle of circuit interruption which was used in developing the highest capacity circuit breakers for 15-kv service[4] has also been used as the solution of the problem of a 250,000- and 500,000-kva oilless circuit breaker for metal-clad service. The air-blast principle also shows promise of being readily applicable to high-voltage outdoor circuit breakers if sufficient interest should develop.

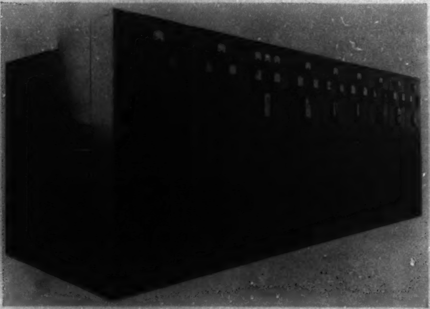

Figure 14. Metal-clad switchgear using air-blast circuit breakers

References

1. The Oil-Blast Circuit Breaker, D. C. Prince and W. F. Skeats. AIEE Transactions, June 1931, page 506.

2. "Magne-Blast" Air Circuit Breakers for 5,000-Volt Service, E. W. Boehne and L. J. Linde. AIEE Transactions, 1940 (April section), pages 202–08.

3. The Cross-Air-Blast Circuit Breaker, D. C. Prince, J. A. Henley, and W. K. Rankin. AIEE Transactions, volume 59, 1940 (September section), pages 510–17.

4. Design and Construction of High-Capacity Circuit Breakers, H. E. Strang and A. C. Boisseau. AIEE Transactions, volume 59, 1940 (September section), pages 522–7.

5. Circuit-Breaker Recovery Voltages—Magnitudes and Rates of Rise, R. H. Park and W. F. Skeats. AIEE Transactions, March 1931, page 204.

Dielectric Strength of Water in Relation to Use in Circuit Interrupters

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Synopsis: This paper describes the behavior of water as a dielectric, in particular with reference to its use in water circuit breakers.

Water is generally considered to have the properties of an electrical conductor rather than those of an insulator. There are, however, a few applications where the dielectric rather than the conductive properties of water are of importance, as, for example, in its use as the interrupting liquid in circuit breakers. Here the water between electrodes must withstand full circuit voltage during the closing operation until the electrodes are in metallic contact, and also in the opening operation following extinction of the arc, until some auxiliary switch in series can open.

Since there appears to be very little published information available on the subject, further knowledge of the behavior of water as a dielectric seemed to be of considerable interest and importance. To secure this information, a study of the subject was undertaken, the results of which will be reported in this paper.

Experimental data are presented which indicate that electric breakdown between electrodes immersed in water is, in general, determined by the temperature rise produced by the leakage current in the water. Breakdown appears to occur when steam generation begins. Calculation of time to cause temperature rises to 100 degrees centigrade in the water agree closely with observed time between applied voltage and breakdown.

When surge voltages of such short duration as to cause negligible heating are applied to electrodes submerged in water, the dielectric strength of water is found to be of the same order of magnitude as for insulating oils.

In comparing the properties of insulating liquids commonly used as dielectrics, with water, the most outstanding difference is the very much greater conductivity of water. The following table shows a comparison of the approximate conductivities of water and commonly used circuit breaker oils:

	Ohms Per Centimeter Cube	Conductivity Compared to Oil
Insulating oil	10^{16}	1
Good distilled water	5×10^{6}	2 billion
Tap water	5×10^{4}	200 billion

From the above table it is evident that, with equal electric gradients, water will absorb from 2 billion to 200 billion times as much heat energy due to current leakage through the liquid dielectric, as insulating oil. Since this thermal effect is of an entirely different order of magnitude in the water, it might be expected to play an important part in breakdown of water, whereas in oil or other highly insulating liquids, its effect would be negligible. Our investigations clearly demonstrate that this thermal effect is the determining factor in breakdown where sustained voltages are applied, both with high-resistivity distilled water and with comparatively low-resistivity tap water.

Testing Procedure and Apparatus

Since the comparatively low resistivity of even distilled water results in an appreciable current flow between submerged electrodes during a dielectric test, standard dielectric test transformers of small capacity cannot be used because of excessive regulation. The large 27,000-kva testing transformers of our Trafford laboratory were, therefore, used as a voltage source.

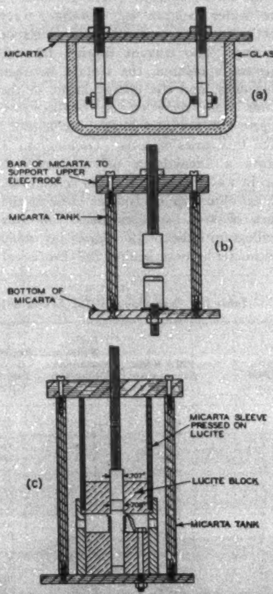

Figure 1. Apparatus used for dielectric tests on water

In making the tests, water of known resistivity was placed in a container which also held a pair of adjustable electrodes. Figure 1 shows three such vessels. The electrodes used were 1.5-inch sphere gaps, $^3/_4$-inch rod gaps with an edge radius of $^3/_{64}$ inch, and rod gaps 0.707 inch diameter, with very slight edge radius. The rod gaps on which the majority of tests were made were faced with copper "elkonite" to prevent burning by the current flow following breakdown.

Since the presence of a thermal effect would be evidenced by a delayed breakdown, a cathode-ray oscilloscope was connected in parallel with the gap being tested, so that the number of half cycles of applied voltage on the gap before breakdown occurred could be visually observed on the cathode-ray-tube screen. For the longer times a calibrated backup breaker was used to estimate the time of applied voltage. Magnetic oscillograms were also taken in many of the tests where sufficient accuracy could not be obtained by the above methods.

Breakdown in Water With 1.5-Inch-Diameter Sphere Gaps

In this series of tests, 1.5-inch sphere gaps were mounted in a horizontal plane in the glass bowl container, and submerged in distilled water with a resistivity of 125,000 ohms per centimeter cube. The gap length was varied, voltage being kept constant at 14,000 rms volts. It was found that with gap lengths in excess of 0.375 inch, breakdown did not occur for well over 20 half cycles (60-cycle circuit) after voltage was impressed. As the gap length was further shortened, the time delay between application of voltage and breakdown continued to decrease, in a manner as shown in table I. These results are shown graphically in the curves of figure 2. The last two tests in table I give re-

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Table I. Breakdown in Distilled Water Between 1.5-Inch-Diameter Spherical Electrodes

Test Number	Voltage Across Gap	Gap Length (Inches)	Water Resistivity (Ohms/Cm Cube)	Resistance Between Electrodes	Half Cycles Before Breakdown	Computed Calories Input Per Cubic Centimeter*
A	14,000	0.5	125,000	7,600	20	77
B	14,000	0.375	125,000	7,600	20	123
C	14,000	0.25	125,000	5,900	10	154
D	14,000	0.3125	125,000		20	198
E	14,000	0.1875	125,000		5	138
F	14,000	0.125	125,000		$1^{1}/_{2}$	92.5
G	4,670	0.125	125,000		20	137
H	4,670	0.0625	158,000	3,600	6	173

* Formula used in computing is $H=\frac{2E^2\cdot n}{504\rho l^2}$ calories per cubic centimeter; E = rms volts, n = half cycles, ρ = initial resistivity, ohms per centimeter cube, l = gap length, centimeters.

sults for the condition of voltage reduced from 14,000 to 4,600 volts. It may be noted here that there is a similar time lag to breakdown except that the time is now much longer than at the higher voltage.

Table II shows similar data (also shown graphically in figure 3) but for comparatively low-resistivity water. In this series of tests there was some drop in voltage in the current limiting reactor, and hence the column of voltage across the gap has been computed from the known constants of the circuit. In addition to the constant voltage condition, a series of tests was also made keeping gap length constant at 0.25 inch, and varying the impressed voltage.

Breakdown Between 0.75-Inch-Diameter Rod Gaps With $^{3}/_{64}$-Inch Edge Radius

These tests were similar in nature to those described above for the sphere gaps. The container, however, was of Micarta (shown in figure 1) and the electrodes were in a vertical plane.

Since it was observed in the first series of tests that the resistivity of water is extremely sensitive to temperature changes, careful observations of water temperatures and water resistivities were taken throughout the tests.

Data for the rod-gap tests using distilled water are presented in table III, and similar data for low resistivity water are given in table IV. Graphical results from these data are shown in figures 4 and 5.

Breakdown of Water With a Uniform Potential Gradient

In the tests described above, the field between the electrodes is evidently not absolutely uniform. For the sphere gap at $^{3}/_{8}$-inch separation, the maximum gradient is approximately 13 per cent higher than the minimum gradient. The nonuniformity increases with increasing gap length. Hence, especially for the longer gaps, there is some error in assuming a uniform gradient along the shortest filament of water between electrodes.

In order to observe breakdown in water with practically uniform potential gradient, another test cup was constructed as shown in figure 1*c*. In this cup electrodes having a diameter of 0.707 inch were placed in a Lucite cylinder of 0.709 inch internal bore, the test cup being so arranged that current can flow only through the water between the electrodes in the Lucite cylinder. The gradient on the water column is, therefore, very nearly uniform, regardless of electrode spacing. The current flowing through this water column, the voltage between electrodes, and the energy input to the water were recorded on oscillograms. Figure 8 shows three typical oscillograms from this series of tests. Oscillogram *A* shows a breakdown on a 0.125-inch gap in water at 19.8 degrees centigrade, 90,000 ohms per centimeter cube at the start of test. Measurements on this oscillogram indicate 79 calories per cubic centimeter input as against the theoretical value of 80.2. Oscillogram *B* shows a breakdown on a 0.5-inch gap in water of initial temperature of 17.5 degrees centigrade and 96,000 ohms per centimeter cube initial resistivity. The wattmeter record shows a total input at breakdown of 119 calories per cubic centimeter as against the theoretical 82.5 calories to start boiling. The oscillogram shows, however, that the current actually began to decrease after an input of 96 calories per cubic centimeter. This appears to indicate that minute bubbles of steam formed throughout the water volume, having the effect of decreasing the cross-sectional area of the water column. Pressure was undoubtedly also built up within the electrode volume since the lower contact was held in place by a spring pressure

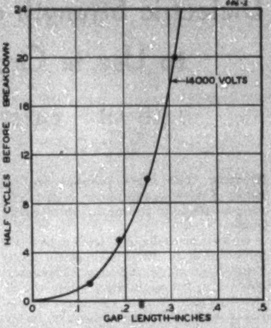

Figure 2. Time delay in breakdown of distilled water between 1.5-inch spherical electrodes

Initial water resistivity about 125,000 ohms per centimeter cube. Applied voltage, 14,000 rms volts

Table II. Breakdown of Tap Water Between 1.5-Inch-Diameter Spherical Electrodes

Test Number	RMS Volts Across Gap (Calculated)	Initial Water Resistivity (Ohms/Cm Cube)	Resistance Between Electrodes (Ohms)	Gap Length (Inches)	Half Cycles Before Breakdown	Computed Calories Input Per Cubic Centimeter*
I	3,800	4,700	300	0.0625	1	480
J	3,800	4,700	300	0.125	1	121
K	3,600	4,700	240	0.250	4.5	136
L	3,800	4,700	296	0.50	20	151
M	8,800	4,700	296	0.50	2.5	102
N	8,800	4,700	296	0.75	7	125
O	8,800	4,700	296	1.00	10	102
P	8,800	3,960	296	1.50	15	85
Q	8,900	4,500	305	0.25	1	172
R	7,300	4,500	294	0.25	1	116
S	6,150	4,500	235	0.25	1.5	124
T	4,700	4,500	235	0.25	3	145
U	3,680	4,500	294	0.25	9	265
V	3,300	4,500	235	0.25	8	190
W	1,870	4,500	235	0.25	20	152
X	2,800	4,200	235	0.25	10	183

* Formula given in table I has been used in these calculations.

Table III. Breakdown of Distilled Water Between 0.75-Inch-Diameter Rod Gaps With 3/64-Inch Edge Radius

Test Number	Gap Length (Inches)	RMS Volts on Gap	Initial Water (Ohms/Cm Cube)	Resistance Between Electrodes (Ohms)	Temperature Before Test (Deg)	Temperature After Test (Deg)	n Half Cycles Before Breakdown	* Computed Input H (Calories Per Cc at Breakdown)	* Computed Half Cycles to Reach Boiling Point
FH	0.125	14,000	116,000 at 21°	4,850	20.0	20.0	0.5	20.5	1.25
FI	0.125	14,000					0.5	20.5	1.25
FJ	0.125	14,000					0.5	20.5	1.25
FK	0.250	14,000	94,000 at 20°	5,680	19.8		5.5	117	4.0
FL	0.250	14,000					3.5	63	4.0
FM	0.250	14,000		5,460		21.0	6	177	4.0
FN	0.3125	14,000	88,500 at 21°	5,915	20.8		10	226	6.0
FO	0.3125	14,000					9	209	6.0
FP	0.3125	14,000		5,570		22.2	10	226	6.0
FQ	0.375	14,000	80,000 at 21°	5,600	21.7		11	151	7.8
FR	0.375	14,000		5,280		23.0	11	151	7.8
FS	0.4375	14,000	79,900 at 22°	5,600	22.5		11	86	10.4
FT	0.4375	14,000					Ap. 13	117	10.4
FU	0.4375	14,000		5,280		24.0	13	117	10.4
FW	0.50	14,000	73,800 at 22.5°	5,390	24.0		15	103	12.8
FX	0.50	14,000					Ap. 15	103	12.8
FY	0.50	14,000		5,090		27.0	Ap. 20	183	12.8
FZ	0.625	14,000	64,300 at 25°	5,190	26.0	33.0	Ap. 60		17.5
GA	0.75	14,000	110,000 at 18.5° to 71,300 at 25.5°	9,900 6,340	18.0	27.2	Ap. 240	136	18
GB	1.00	14,000	71,300 at 25.5° to 34,000 at 39°	6,970 3,190	26.3	43.2	Ap. 240		

* Formula used

$$H = (10.5 + T_i)\left(\epsilon^{\frac{0.00307 E^2 V n}{504 \rho l^2}} - 1\right) \text{ calories per cubic centimeter}$$

V = viscosity in millipoises
E = rms volts
n = half cycles
l = gap length in centimeters
ρ = ohms per centimeter cube

of approximately nine pounds per square inch. The additional heat input can, therefore, at least partially be accounted for by the increased boiling temperature due to pressure substantially higher than atmospheric.

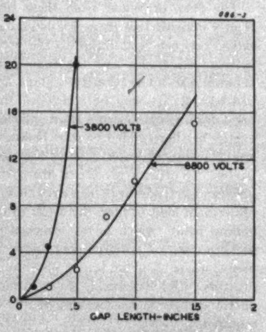

Figure 3. Time delay in breakdown of tap water between 1.5-inch spherical electrodes

Initial water resistivity about 4,700 ohms per centimeter cube. Applied voltages 3,800 and 8,800 rms volts

The current decrease seen in figure 8*B* is typical of long gaps, whereas immediate breakdown at the first appearance of gas seems to occur on short gaps as indicated on oscillogram *A*.

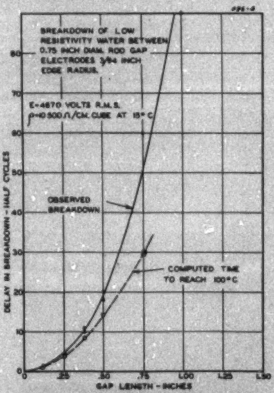

Figure 4. Comparison of observed and computed data for time delay in breakdown of low-resistivity water

On very short gaps with high applied gradients, breakdown occasionally occurs before boiling begins. Oscillogram *C* illustrates this case. Here 7,500 rms volts were applied to a 1/16-inch gap, the initial water conditions being 19.5 degrees centigrade, 115,000 ohms per centimeter cube. The oscillogram indicates that breakdown occurred near the first voltage peak after an input of 65 calories per cubic centimeter. Another test on a 1/16-inch gap broke down after an input of 41.5 calories per cubic centimeter. One-eighth-inch-long gaps with 13.8 kv applied also occasionally broke down before sufficient energy to cause boiling had been absorbed. In these cases, breakdown in water appears to be similar to breakdown in oil.

Breakdown of Gaps Having Air and Water in Series

In tests on water breakers, breakdown through the water has been observed on some occasions, during the closing operation, often resulting in considerable explosive effects. Such breakdowns appear to occur when the moving contact is close to, but not yet submerged below the water surface. A probable explanation of this behavior is, that as the contact approaches the water surface, the high dielectric constant and conductivity of the water compared to the similar quantities for the series air gap results in practically the full circuit voltage being applied across the air gap. The over-

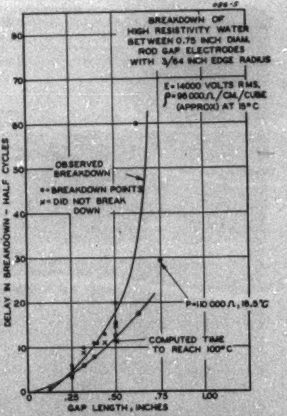

Figure 5. Comparison of observed and computed data for time of delay in breakdown of high-resistivity water

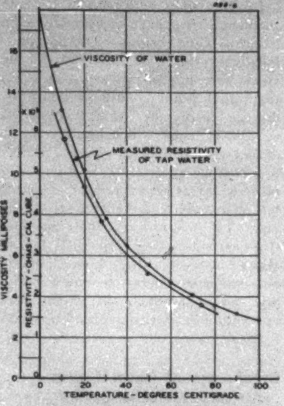

Figure 6. Comparison of resistivity versus temperature and of viscosity versus temperature curves for water

stressed air breaks down and arcs to the water, whence the current is conducted to the submerged electrode. However, after the air is ionized its effective dielectric constant approaches infinity, and hence there is an electrostatic force tending to draw the arc into the liquid. This action quickly forces the liquid out of the path between electrodes, completing the breakdown. For this reason it is evident that breakdown is particularly likely in closing operations as the contact approaches the water surface, or in opening operations after the contact has been drawn out of the water.

To study breakdown of gaps having air and water in series the tests recorded in table V were made. In this table, tests *BO* and *BN* show results with a 1.5-inch total gap. Test *BN* with both electrodes submerged broke down in two half cycles, whereas *BO* with a $^1/_{16}$-inch series air gap took three half cycles. While this seems contrary to the above theory, it should be remembered that a finite time is required to draw the arc down through the water to the submerged electrode, and it is quite possible that for conditions where the submerged gap breakdown is already very short, breakdown time with series air and water gaps may be, as indicated in these tests, as long as with water alone.

That this is not generally the case, however, is shown by tests *BW*, *BX*, and *CC*, where the breakdown time with series air gaps is seen to be less than half as long as with submerged electrodes. These tests, as well as tests *BQ*, *BP*, *BR*, and *BU* show, furthermore, that the time interval between application of voltage and breakdown with an air gap in series is independent of the air gap length. When, however, the air gap is increased to such length that it alone can withstand the full circuit voltage, breakdown does not occur at all. This condition is indicated in test *BU*.

In test *BY*, a wire screen was placed on the water surface to prevent the ionized path from being drawn down through the water. Breakdown rose to seven half cycles, comparable with the submerged electrodes. *BZ* is a similar test, but in this case a brass plate with a central hole was placed at the water surface. A wire was connected to this brass plate and brought to within 0.1 inch of the electrode above water, so that initial breakdown would occur to the wire rather than to the water surface. Here also the breakdown time was seven half cycles, comparable to the time with submerged electrodes.

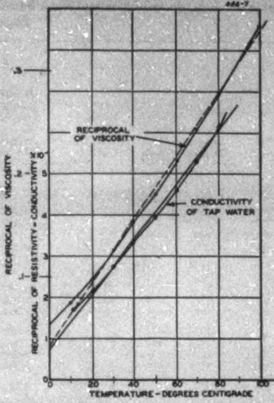

Figure 7. Comparison of conductivity versus temperature and of the reciprocal of the viscosity versus temperature curves for water

Table IV. Breakdown of Low-Resistivity Water Between 0.75-Inch-Diameter Rod Gaps With $^3/_{64}$-Inch Edge Radius

Test Number	Gap Length (Inches)	RMS Volts on Gap	Initial Water Ohms/Cm Cube	Resistance Between Electrodes (Ohms)	Temperature Before Test (Deg C)	Temperature After Test (Deg C)	Half Cycles Before Breakdown	* Computed Calories Per Cc Input at Breakdown	* Computed Half Cycles to Reach Boiling Point
GC	0.125	4,600	10,500 at 15°	480	15.5		1.5	207	1.0
GD	0.125	4,600		476			1+	88.5	1.0
GE	0.250	4,600	9,660 at 16.5°	616	16.5		4.5	115	3.8
GF	0.250	4,600		598			4.5	117	3.8
GG	0.375	4,600	9,350 at 17.2°	702	17.2		8	75	8.5
GH	0.375	4,600					11	143	8.5
GI	0.375	4,600					10	120	8.5
GJ	0.375	4,600		673			10	123	8.5
GK	0.500	4,600	8,850 at 19.5°	732	19.5		15	89	14.1
GL	0.500	4,600					Ap. 18	131	14.1
GM	0.500	4,600					15	94	14.1
GN	0.500	4,600		672			Ap. 20	171	14.1
GO	0.750	4,600	8,550 at 23°	881 691	23.2		Ap. 60	350	30
GP	1.00	4,600	8,070 at 26.5°	798	22.5	28.5	Ap. 120	265	
			to 7,100 at 29.5°	740	27.6	31.8			
GQ	1.5	4,600	9,600 at 17°	1,145	16.0	66.0	1,080		
			to 5,025 at 46°	657					

* Same formula used to compute *H*, as given in table III.

Discussion of Results

The data presented from these tests clearly indicate that there is a definite relation between heat input to the water between electrodes and the time delay between the application of voltage and breakdown. It may be concluded that breakdown usually occurs when sufficient energy has been added to start boiling in any filament of water joining the electrodes. In order to produce a steam path between electrodes it is necessary to supply enough heat to raise the water to the boiling point, which is, at atmospheric pressure $(100-T_i)$ calories per cubic centimeter, where T_i is the initial water temperature. For pressures above atmospheric slightly higher heat input will be required.

The heat input per cubic centimeter to a filament of water between parallel electrodes is

$$\frac{E^2t}{4.2\rho l^2} \text{ calories}$$

Table V. Breakdown of 0.75-Inch Diameter Rod Gaps With Water and Air in Series Between Electrodes

Test Number	Air Gap Length (Inches)	Total Gap Length (Inches)	Impressed RMS Volts	Approximate Initial Resistivity (Ohms/Cm Cube)	Half Cycles Delay in Breakdown	Remarks
BN	0.0	1.5	14,000	5,550	2	Contacts submerged
BO	0.0625	1.5	14,000	5,550	3	
BQ	0.0625	2.5	14,000	5,550	3	
BP	0.125	2.5	14,000	5,550	4	
BR	0.375	2.5	14,000	5,550	4	
BU	0.625	2.5	14,000	5,550	Held 20	
BW	0.0	3.5	14,000	5,080	9	Contacts submerged
BX	0.375	3.5	14,000	5,080	4	
CC	0.125	3.5	14,000	4,900	3	
BY	0.25	3.5	14,000	5,080	7	Wire screen on water surface
BZ	0.125	3.5	14,000	5,080	7	Brass disk on surface (wire gap to top electrode)

where E = volts, t = seconds, ρ = ohms per centimeter cube; l = gap length in centimeters. Expressing t as $n/120$ where n is the number of half cycles of 60-cycle applied voltage before breakdown occurs,

$$\text{Heat input per cubic centimeter} = \frac{E^2n}{504\rho l^2}$$

The condition under which this heat input causes boiling to begin is

$$\frac{E^2n}{504\rho l^2} = 100 - T_i$$

Solving for n,

$$n = \frac{(100 - T_i)(504\ \rho l^2)}{E^2}$$

This is the number of half cycles that the water should withstand voltage if breakdown occurs when steam begins to form. On the other hand, the experimentally determined value of n, the number of half cycles delay to breakdown, may be substituted in the formula to determine whether temperature rise thus obtained equals $100 - T_i$.

Checks of this kind have been made for the data with spherical electrodes of table I. The results are presented in column 6 of that table. It should be noted here that $\frac{\rho}{2}$ has been used in these computations as the average water resistivity. This is half of the initial value, which is a reasonable average between the values at the initial temperature and the boiling point.

These calculations show that for practically all tests the input to cause boiling is of the same order of magnitude as the computed energy input per cubic centimeter to the water between electrodes.

It was observed that the resistivity of water falls very rapidly with increase in temperature, the resistivity-temperature curve being approximately a displaced rectangular hyperbola. On comparing this curve with the viscosity-temperature curve for water (figure 6) it may be seen that the two are practically identical in shape. This similarity indicates that change in water resistivity with temperature results from a change of mobility of the ions as the viscosity varies, the total number of ions remaining practically constant. This relation affords a simple method of determining change of resistivity with temperature since the resistivity is proportional to the viscosity, a quantity for which accurate tables are available.

In deriving a more accurate expression for relating the variables of the heat input equation given above, it is convenient to make use of the conductivity-temperature curve for water. As shown in figure 7, this curve proves to be almost linear and may thus be expressed $C = A + BT$. We may now substitute $A + BT$ for $1/\rho$ in the heat input equation, thus taking into account the variation of resistivity with temperature. From the previously derived equation, calories per cubic centimeter input for a temperature change, dt, is

$$dT = \frac{E^2dt}{\rho l^2\,4.2} = \frac{E^2}{4.2l^2}(A + BT)\,dt$$

whence

$$\frac{dT}{A + BT} = \frac{E^2}{4.2l^2}\,dt$$

This equation can readily be solved, whence T may be expressed as a function of t. When this is done the calories input per cubic centimeter is found to be

$$H = \left(\frac{A}{B} + T_i\right)(\epsilon^{Kt} - 1) \text{ calories per cubic centimeter}$$

where

A = zero degree intercept of conductivity-temperature curve
B = slope of conductivity-temperature curve
$K = \frac{(BE^2)}{4.2l^2}$
T_i = temperature at $t = 0$
t = seconds
E = rms volts
l = gap length—centimeters

The above equation, substituting approximate constants, may be reduced to the following form:

$$H = (10.5 + T_i)\left(\epsilon\frac{0.00307E^2Vn}{504\rho l^2} - 1\right)$$

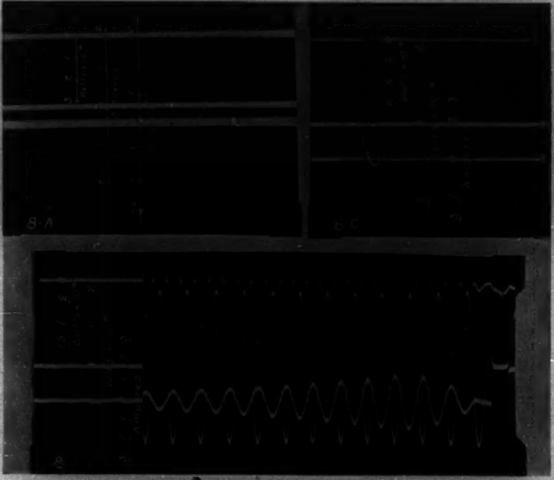

Figure 8 (right). Oscillograms showing breakdown in water with uniform potential gradients between electrodes

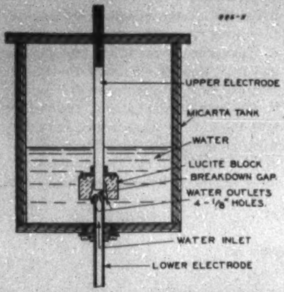

Figure 9. Apparatus used in studies of breakdown in rapidly flowing water

where

V = initial viscosity in millipoises (see figure 6)
ρ = initial ohms per centimeter cube
n = half cycles (60-cycle circuit)
ϵ = Naperian log base

Solving for n,

$$n=\left(\log_\epsilon\frac{110.5}{10.5+T_i}\right)\left(\frac{504l^2\rho}{0.00307E^2V}\right)$$

Using this equation, the number of half cycles that a given gap in water can withstand applied voltage before breakdown occurs may be calculated. These calculations should apply most accurately for those tests in which potential gradient was maintained constant.

Checking calculated and observed values for the test on a 0.125 inch gap corresponding to oscillogram figure 8*A* gives calculated time to breakdown 1.31 half cycles; observed from oscillogram, 1.30 half cycles. Checking again for the test on a 0.5-inch gap shown on oscillogram figure 8*B* gives calculated time to start boiling = 17.4 half cycles; observed from oscillogram = 19 half cycles. However, if for *H* the value 96 calories as actually measured on the oscillogram, is used instead of 82.5, the theoretical value, then the calculated time is 18.9 half cycles as compared to the observed value of 19 halves. In this case it should be noted that while boiling appears to have begun at 19 half cycles, breakdown did not actually occur until 25 half cycles had elapsed.

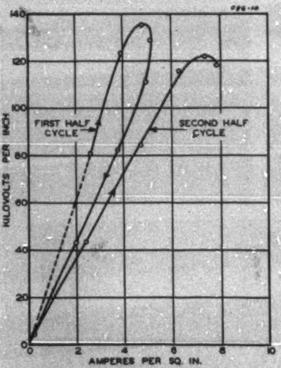

Figure 10. Volt-ampere curve for test shown in oscillogram A of figure 8

Gap length 0.125 inch

Further comparison of observed and calculated values of time to breakdown with a uniform potential gradient are given in table VI. In this table are given also the calories per cubic centimeter at breakdown as observed from the oscillographic records. These agree well with $(100-T_i)$, the theoretical value required for boiling at atmospheric pressure. If pressure rises above atmospheric as it probably did in tests that took several half cycles to reach the boiling point, a heat input in excess of $(100-T_i)$ per cubic centimeter is required.

It thus appears that close agreement between calculated and observed values of time to reach the boiling point can be attained where the potential gradient in the water is uniform, and that this also agrees well with the time of breakdown. For gaps in which the gradient is only approximately constant, it is found that the time to breakdown may still be estimated by calculation using the formula given above. The agreement of such calculations with observed times of breakdown may be seen from the tabulation of observed and computed values of two sets of data for 0.75 inch rod gaps as recorded in tables III and IV and shown graphically in curves of figures 4 and 5.

Figure 10 shows a volt-ampere curve plotted from the oscillogram shown in figure 8*A*. It may be observed that there is a steady decrease in resistance. The change in resistance appears to be a function of temperature, and at least approximately independent of the gradient on the water.

Breakdown in Water With Rapid Flow Across the Gap

Since breakdown in water appears to occur following a sufficient heat input to cause vaporization, it seemed probable that the electric breakdown of a gap could be prevented if a sufficiently high

Table VI. Breakdown in Water With Uniform Potential Gradients. Rod-Gap Electrodes 0.707-Inch Diameter

Test Number	Gap Length (Inches)	Kilovolts on Gap	Initial Water Ohms/Cm Cube	Initial Water Temperature	Observed Half Cycles Before Breakdown	(a) Computed Half Cycles to Reach Boiling Point	Calories Per Cubic Centimeter From Oscillogram	
GR	0.25	17.7	176,000	16.5	9.3	8.0	110	
GS	0.25	13.3	121,000	13.75	6.3	5.7	110	
GU	0.0962	10.6	130,000	14.75	1.3	1.45	95	
GV	0.1875	12.6	112,000	19.5	3.1	3.3	91	
GW	0.125	11.4	110,000	19.5	1.25	1.76	73	
GX	0.125	10.7	113,000	19.5	1.7	2.0	75	
GY	0.0625	7.5	115,000	19.5	0.5	1.1	65	(*d*)
GZ	0.3125	13.6	137,000	19.5	9.5	10	77	
HA	0.3175	13.5	112,000	19.75	9.8	8.2	94	
HB	0.375	13.7	126,000	19.0	(*b*)13–16	12.9	(*c*)103–119	
HC	0.375	13.6	108,000	19.0	13–17	11.2	93–120	
HD	0.4375	13.5	114,000	18.5	18–23	16	98–121	
HE	0.50	13.7	96,000	17.5	19–24	17.4	96–119	
HF	0.5625	13.6	93,500	17.75	25–32	21.5	94–120	
HG	0.625	13.5	98,500	17.5	31–45	29	101–123	
HH	0.75	13.5	94,000	17.5	42–67	38	98–123	
HI	0.75	13.5	78,000	17.0	36–51	32.4	98–121	
HJ	0.125	8.7	40,000	17.8	0.6	1.1	40	(*d*)
HK	0.125	9.83	58,000	17.8	1.3	1.27	62.5	
HL	0.1875	11.6	62,000	17.8	2.2	2.18	64	
HM	0.125	11.0	84,000	22.8	0.5	1.45		(*d*)
HN	0.125	11.6	106,000	23.8	0.5	1.63		(*d*)
HO	0.125	12.0	90,000	19.8	1.3	1.31	79	
HP	0.1875	13.0	100,000	19.5	3.5	2.84	77	
HQ	0.1875	12.65	118,000	19.4	4.1	3.5	92	
HR	0.0625	7.1	72,000	19.0	0.5	0.8	41.5	(*d*)

(*a*). Formula used, $n=\left(\log_\epsilon\frac{110.5}{10.5+T_i}\right)\left(\frac{504\rho l^2}{0.00307E^2V}\right)$.

(*b*). Half cycles to apparent boiling point, and half cycles to breakdown.

(*c*). Calories input per cubic centimeter at apparent boiling point and input at breakdown.

(*d*). Breakdown occurs with less than 100-T_i calories per cubic centimeter in these tests.

velocity of water flow were maintained across the gap so that the water fails to reach the boiling point during the time it remains between electrodes.

To verify this conclusion, an apparatus as shown in figure 9 was constructed to cause rapid flow of water between the faces of a pair of electrodes. The gap used consisted of one-inch-diameter rods with $^3/_{64}$-inch edge radius. The one electrode through which the water emerged had a $^3/_8$-inch central hole.

This device was first tested under conditions similar to those which had held voltage for six or seven half cycles with stationary water between the electrodes, except that sufficient flow was provided that the gap should now, according to calculations, hold voltage indefinitely. However, breakdown occurred almost immediately upon application of voltage. A consideration of the flow conditions present in the arrangement showed that this was to be expected. The outward radial flow of liquid in the gap resulted in a maximum velocity of liquid at the center of the gap with deceleration toward the outer edge. Since the pressure at the outside was atmospheric, the pressure near the center must have been less than atmospheric. The fact that breakdown occurred immediately indicates that a very low pressure, probably lower than the vapor pressure of water, actually existed near the center of the gap.

These results thus serve to further confirm the theory that breakdown occurs at the boiling point of the water between electrodes. It also further emphasizes the fact that in flow devices great care in design must be used to prevent flow conditions such that reduced liquid pressures result at points of dielectric stress. This is true for oil devices as well as for water, although in the case of oil the chief factor causing decrease in dielectric strength with reduced pressures, results not from vaporization of the oil itself, but from release of gases dissolved in the oil.

In order to eliminate the low-pressure conditions which had caused breakdown in this gap, the electrodes were enclosed in a block of Lucite containing a small cavity surrounding the gap. The discharge path for the water from this cavity consisted of four one-eighth-inch holes, the area of which was considerably smaller than the minimum restriction at other points in the path of flow. Thus the pressure at all points between electrodes was maintained higher than atmospheric.

Under these conditions it was possible to hold indefinitely 9,340 volts on a 50-mil gap with a flow of 0.463 cubic inch per half cycle. This flow was sufficient to change the water between electrodes in 0.084 half cycle or to change all the water in the enclosing cavity in 0.8 half cycle. The computed time to reach 100 degrees for this condition with stationary water is about 0.6 half cycle. In another test we succeeded in holding 14 kv indefinitely on a 0.125-inch gap with a flow of 0.424 cubic inches per half cycle. This flow would change the water between electrodes once every 0.23 half cycle and would change the water in the cavity surrounding electrodes once every 1.0 half cycle. The computed time to reach 100 degrees in stationary water is about 1.5 half cycles.

It may thus be seen that breakdown can be prevented by sufficient flow of water past the electrodes. In these tests a flow somewhat greater than might be expected from calculations was required to prevent breakdown. It should be remembered, however, that even after the water has been moved out of the gap it is still being subjected to a considerable dielectric stress, and energy continues to be absorbed until it is discharged from the cavity around the gap. It is also probable that a percentage of the original water in the cavity still remains therein after a quantity of water equal to the volume of the cavity has been discharged from it. The fact that the flow required to prevent breakdown is of the right order of magnitude to evacuate the gap and surrounding cavity before boiling can begin is, therefore, another confirmation of the theory that breakdown occurs when the water reaches the boiling point.

Surge Tests on Water

In order to observe the voltage required to cause rupture of distilled water having a resistivity in the order of 50,000 ohms per cubic centimeter, a few tests were made using impressed surge voltages of such short duration that the heating of the water between electrodes was negligible. Under these conditions the following results were obtained:

Gap Length (Inches)	Surge Breakdown for Water (Kilovolts)	Surge Breakdown—Insulating Oil (Kilovolts)
0.1	58.5	
0.050	42.9	60.0
0.025	34.4	48.7

These tests indicate that water has approximately 70 per cent as great dielectric strength as oil on surge voltages.

Conclusions

1. Dielectric tests on water indicate that breakdown is generally due to thermal effects and does not occur until a filament of water between electrodes has been almost completely vaporized.

2. An exception to this rule occurs when gradients on the water exceed approximately 125,000 rms volts per inch. Breakdown then appears to occur before boiling, although generally not until considerable heat has been absorbed.

3. An equation has been derived for calculating the time for the water in a given gap to reach the boiling point. This time is found also to be approximately the time of breakdown of the gap.

4. Long gaps withstand voltage somewhat longer than the computed time to reach the boiling point. This is due in part to the loss of heat to contacts and surrounding water, and in part to formation of minute bubbles in the interelectrode space which tend to reduce the rate of heat input.

5. Gaps with air and water in series may, under certain conditions, break down faster than the same gap completely submerged in water.

6. Short gaps in water can withstand voltages far in excess of those resulting in breakdown on stationary water if the liquid between electrodes is rapidly renewed by flow between the contact faces.

Superposition Methods for Calculating Effects of Additions to Power Systems

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Synopsis: Outline of a group of methods that will permit calculation of all the common changes on power-system networks by superposition methods having as a basis the representation of loads as fixed currents. Investigation of the errors involved, and comparison with results obtained by assuming constant-impedance loads, when applied to the average power-system networks in operation today.

On a growing power system it frequently becomes necessary to calculate the effect of the addition of certain loads or lines to the existing system. With the present status of power systems, the calculation of additions is a much more common problem than the calculation of complete new networks. These calculations usually consist of determining the load division in old and new circuits and the voltage at certain principal supply points or substation busses. More specifically, loads in amperes are usually more important than the kilowatt and reactive kilovolt-ampere values. The need for this information has long been recognized and is evidenced today by the increasing attention which is being given to a-c boards.

The calculation of currents and voltages in networks has received a great deal of study in years past and hence the known mathematical methods of approach are quite varied. Among the better known ones are the direct application of Kirchoff's laws or of Maxwell's equations of cyclical currents, which lead to a solution of a number of simultaneous equations, or the use of the familiar impedance network transformations, such as the delta-star, star-mesh and the like, which permit simplification of complex networks until they can be handled by more direct methods. Other less known methods are based on the principal of superposition, on Thevenin's theorem, and on trial and error methods which depend largely on experience and ingenuity in handling for a reasonably direct solution.

Practically all of the direct methods that are commonly used are based on the representation of all lines and loads as fixed impedances and the networks are considered as complete new networks in which nothing is known of the current division in the various lines resulting from any existing loads. A solution by these methods involves all line and load impedances for each alternative set of connections under consideration. In a-c networks, where it is necessary to deal almost entirely with complex quantities, the solution of networks made up of even a moderate number of loads and lines becomes a long and tedious process. Moreover, the work required to solve a network increases in a much greater proportion than the increase in the number of impedances involved. For this reason, any method which would permit the elimination of some of the impedances from consideration would greatly simplify any solution.

It is the purpose of this paper to describe a set of simplified methods which can be used to calculate the effects of any of the changes or additions that are commonly made on a power network, to show where they can be applied to better advantage than any other method, and to show the limits of accuracy that can be expected for the common types of loads and lines.

Basis for Methods

It is assumed first, that all load currents on a system remain fixed in magnitude and in phase angle with respect to the voltage at one point on the system regardless of what changes are made in connections and in voltages at the various loads. For many very common types of lines and loads, this assumption simulates actual conditions much more closely than the usual representation of loads as fixed impedances. Since the load currents remain fixed during a given change, they need not be dealt with in calculating the effects of that change and the problem is thereby simplified to a great extent. The methods are actually a solution for changes in currents and voltages in parts of a network of lines which can be added to the originals to give corrected values and can in general be classed as superposition methods. Thevenin's theorem is the basis for some of the methods.

To compare the assumption of fixed load current as against fixed load impedance and determine how closely each of them represent actual conditions, it is necessary to examine the various types of loads that are commonly supplied by transmission and subtransmission lines of the average power system. In so far as the effects of voltage changes on loads are concerned, the loads can be divided roughly into three principal groups.

1. Loads for which the value of kilovolt-amperes is constant regardless of supply voltage change, and for which, therefore, an increase in supply voltage will cause a proportionate decrease in current. Examples of this type load are loads supplied through automatic voltage regulators and synchronous motor loads on which the fields are controlled to maintain constant power factor.

2. Loads for which the power component is constant regardless of voltage change and for which the reactive kilovolt-amperes increases with an increase in voltage. This is true in general of motor loads and series type street-lighting circuits supplied through constant-current transformers.

3. Loads for which an increase in voltage produces an increase in current. This is true of lighting load, resistance-type heaters, and similar loads.

Table I

	Group 1	Group 2	Group 3	Assume Fixed Impedance	Assume Fixed Current
Kilowatts	Constant	Constant	$KE^{1.6}$ to KE^2	KE^2	KE
Reactive kilovolt-amperes	Constant	KE to KE^3	KE^2	KE^2	KE

On power systems in general the amount of load in either of the first two groups is considerably larger than in the third group, this being particularly true of the Cincinnati system. Table I gives the relationships of kilowatts and reactive kilovolt-amperes to voltage in all of the above groups, as well as for the theoretical cases in which fixed impedance as well as

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fixed current loads are assumed for calculating purposes.

Since the first two groups are the predominant groups, it becomes apparent that assuming the loads to draw fixed currents will give results which are at least as accurate and in most cases much more accurate than when the loads are assumed to be fixed impedances. Since the changes in voltage that are obtained by any one program of additions rarely exceed five per cent, the changes in load currents will rarely exceed that value. The change of power factor of the load in group 2 with change of load voltage tends to correct for the shift in phase angle of the load voltages on networks where the impedance angle of the lines is greater than the impedance angle of the loads. This condition exists on practically all power-system networks. The board-type ammeters and voltmeters from which the original readings are taken do not have a reliable accuracy of better than one per cent and in many cases not better than two per cent.

Description of Methods

Having indicated that the accuracy of the original assumptions is within accepted limits, it remains now to show the methods themselves and just how they simplify the calculation of additions to networks. There are four types of additions or changes to a system that are commonly made:

Case 1. The addition of a new load or the removal of an existing one from the system.

Case 2. The addition of a new circuit paralleling or partially paralleling existing circuits.

Case 3. A change in the impedance of a part of one of the existing circuits, such as might be made by changing conductor size.

Case 4. Changing of transformer or tap changer tap settings.

On an adequately metered system, it is possible to obtain the kilowatts and reactive kilovolt-amperes on the various circuits as well as for the principal loads, and the voltages on the principal busses for any existing setup. From these values, a comparatively small number of calculations will give the power and reactive components of currents as well as the voltage drops on various circuits expressed in terms of complex quantities. From these values all currents and voltages can be readily calculated with respect to one reference, for which the main supply bus voltage is usually chosen. Beginning, then, with this known set of conditions, changes in system connections will cause changes in the currents carried by the different lines. These changes in currents can be calculated for any of the above-mentioned four changes without considering the existing loads in any way. Direct additions of currents then give the new set of results.

For the purpose of illustrating the methods a simple form of transmission system is assumed in which the phase angles of all line and load impedances are assumed to be equal so that it will not be necessary to deal with complex quantities. The original values have no subscripts, the corrections are indicated by prime (′) and the corrected values by double prime (″).

Case 1. Addition of a Load

Figure 1 shows a single load of 210 amperes at C supplied by two parallel transmission circuits of nine ohms each, which, of course, divide the load equally and a new load of 126 amperes, which is to be added at A where the original voltage drop is 315 volts. The effect of load A can be calculated independently of the load at C and then the voltage drops and currents caused by the two loads added.

The load divides in the various circuits between bus E and point A inversely as the impedances of the paths regardless of the presence of other loads being supplied by the system. The impedance of A is:

$Z_a'' = \frac{3\times 15}{18} = 2.5$ ohms and the changes in currents and voltages become:

$E_{ea}' = Z_a'' I_n = 2.5\times 126 = 315$ volts additional drop

$I_{ea}' = \frac{E_{ea}'}{Z_{ea}} = \frac{315}{3} = 105$ amperes additional

$I_{eca}' = \frac{E_{ea}'}{Z_{eca}} = \frac{315}{15} = 21$ amperes additional

$E_{ec}' = Z_{ec} I_{eca}' = 9\times 21 = 189$ volts additional drop

$E_{ca}' = Z_{ca} I_{ca}' = 6\times 21 = 126$ volts additional drop

Corrected values of current and voltage are:

$$I_{ea}'' = I_{ea}' + I_{ea} = 105 + 105 = 210 \text{ amperes}$$
$$E_{ea}'' = E_{ea}' + E_{ea} = 315 + 315 = 630 \text{ volts}$$
$$I_{ac}'' = I_{ac} + I_{ac}' = 105 - 21 = 84 \text{ amperes}$$
$$E_{ec}'' = E_{ec} + E_{ec}' = 945 + 189 = 1{,}134 \text{ volts}$$
$$I_{ed}'' = I_{ed} + I_{ed}' = 105 + 21 = 126 \text{ amperes}$$
$$E_{ed}'' = I_{ed}'' Z_{ed} = 126\times 6 = 756 \text{ volts}$$

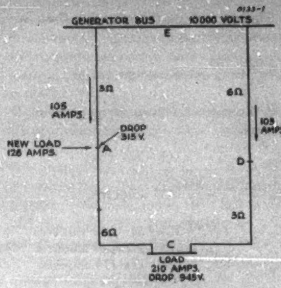

Figure 1. Addition of a new load

Case 2. Addition of a New Parallel Circuit

Figure 2 shows the original system with corrected currents and voltage drops resulting from the addition of the load at point A. The currents and voltages are such that Kirchoff's laws are again completely satisfied. A new circuit (Z_n) of three ohms is to be added from bus E to point D where the voltage drop is 756 volts. Therefore, with the new circuit connected to bus E and open at D the voltage between the new and old circuits at D will be 756 volts. When the lines are tied together at D, this voltage will cause current to flow in the new circuit and additional currents in the existing circuits until Kirchoff's laws are again satisfied.

The loop on which this voltage acts is made up of the new line connected in series with all other paths, which are in parallel between point D and bus E. The two other paths between point D and bus E are the 6-ohm line DE and the 12-ohm line $DCAE$, which when connected in parallel give

$$Z_{ed}\ (\text{total}) = \frac{12\times 6}{12+6} = 4 \text{ ohms}$$

and the impedance of the total loop, made up of this impedance in series with the new three-ohm line becomes $Z_L = Z_{ed} + Z_n =$ seven ohms.

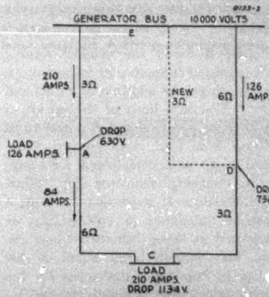

Figure 2. Addition of a new line

The current in the new line then becomes

$$I_n=\frac{E_d}{Z_L}=\frac{756}{7}=108 \text{ amperes}$$

and the correct voltage drop at point *D* becomes

$$E_{ed}''=Z_nI_n=3\times108=324 \text{ volts}$$

$$I_{ed}''=\frac{E_{ed}''}{Z_{ed}}=\frac{324}{6}=54 \text{ amperes}$$

$$I_{dc}''=I_n+I_{ed}''=108+54=162 \text{ amperes}$$
$$I_{ac}''=I_c-I_{dc}''=210-162=48 \text{ amperes}$$
$$I_{ea}''=I_a+I_{ac}''=126+48=174 \text{ amperes}$$

The corrected values of voltage are:

$$E_{dc}''=Z_{dc}I_{dc}''=3\times162=486 \text{ volts}$$
$$E_{ea}''=Z_{ea}I_{ea}''=3\times174=522 \text{ volts}$$
$$E_{ac}''=Z_{ac}I_{ac}''=6\times48=288 \text{ volts}$$
$$E_{ec}''=E_{ea}''+E_{ac}''=522+288=810 \text{ volts}$$

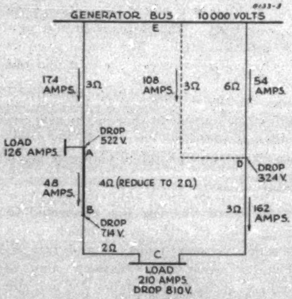

Figure 3. Change of impedance of a line

So that after adding these new superposed currents to the original values Kirchoff's laws are again satisfied. Since there are no changes in currents to the loads, this method can be applied when any number of loads are supplied at any number of points on the loop.

CASE 3. CHANGE OF IMPEDANCE OF A LINE

Figure 3 shows the original system with corrected currents and voltage drops resulting from the addition of a load and a line. The impedance of the section of line between points *A* and *B* is now to be reduced from four ohms to two ohms, which might be accomplished, for example, by changing conductor size. When this impedance is reduced two ohms, a fictitious unbalance in voltage can be considered as being introduced into the loop, this voltage being equal to the change in impedance multiplied by the current which was originally carried by that section of line, or $E_u=Z_rI_{ab}=2\times48=96$ volts. This voltage then divided by the new impedance of the entire loop gives the change in currents necessary to again satisfy Kirchoff's equations of current and voltage. When the impedance of a section is reduced, the current in that section increases and when the impedance of a section is increased, the current in that section will always be reduced. The added current in section *AB* then becomes:

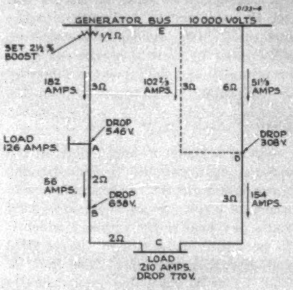

Figure 4. Change of transformer taps or regulator tap settings

$$I_L=\frac{E_u}{Z_L}=\frac{96}{12}=8 \text{ amperes}$$

and the corrected current in section *AB* is

$$I_{ac}''=I_{ac}+I_L=48+8=56 \text{ amperes}$$

also

$$I_{ea}''=I_{ea}+I_L=174+8=182 \text{ amperes}$$
$$I_{dc}''=I_{dc}+I_L=162-8=154 \text{ amperes}$$
$$I_n''=I_n+{}^2/_3I_L=108-(8\times{}^2/_3)=102^2/_3 \text{ amperes}$$
$$I_{ed}''=I_{ed}+1/3I_L=54-8/3=51^1/_3 \text{ amperes}$$

and the voltages are

$$\begin{aligned}E_{ea}''&=I_{ea}''Z_{ea}=182\times3=546 \text{ volts}\\ E_{ac}''&=I_{ac}''Z_{ac}=56\times4=\underline{224} \text{ volts}\\ E_c''&=\phantom{I_{ac}''Z_{ac}=56\times4=}770 \text{ volts}\\ E_{ed}''&=I_{ed}''Z_{ed}=51^1/_3\times6=308 \text{ volts}\\ E_{dc}''&=I_{dc}''Z_{dc}=154\times3=\underline{462} \text{ volts}\\ E_c''&=\phantom{I_{dc}''Z_{dc}=154\times3=}770 \text{ volts}\end{aligned}$$

CASE 4. CHANGING TRANSFORMER TAPS OR REGULATOR TAPS OR SETTINGS

Again figure 4 gives the corrected results obtained by changing a section of the line impedance and adding the necessary circulating current to again balance all currents and voltages. For the purpose of illustrating this fourth case, a step regulator at bus *E* in line *EA*, which is set to boost the line voltage there by $2^1/_2$ per cent or 250 volts, is assumed.

This is handled in the same way as the third case with the 250 volts acting on the impedance of the entire loop, except that in this case there is an increase in impedance in the loop instead of a reduction. The total impedance around the loop is now 12.5 ohms and, therefore, the changes in currents become 20 amperes. Corrections of the currents and voltages are made as in case 3. The only difference between the two cases is that the change of taps will produce this circulating current regardless of how much load is being carried, whereas in the case of the change of impedance in case 3, the change in current around the entire loop depends on how much load is being carried by the line section where the impedance change is made.

Accuracy of Methods

The foregoing examples, which were used merely to illustrate the methods, do not take into account the changes of phase angles of the load voltages with respect to the bus voltage. In the majority of power networks, the phase angles of the line impedance are greater than the phase angles of the load impedances and, therefore, the load voltages will lag the bus voltage. Any change on the system that causes an increase in voltage drop usually causes an increase in the phase angle between the load and bus voltage and in some cases introduces considerable error.

The simple case of a single load at the end of a line shown in figure 5, to which a second load equal to the first is added, will serve to show the effects of making changes on some of the common types of circuits serving the common types of loads.

Three cases, for which vector diagrams are shown in figure 6, are considered. In each case, the original load current is 100 amperes and the in-phase component of voltage drop is five per cent. In each case, the currents and voltages, obtained by calculation assuming both fixed impedance and fixed current, are shown as well as the values for loads made up entirely of motors as in group 2, those drawing a fixed kilovolt-ampere load as in group 1, and those made up of half load in group 1 and half in group 2. The reactive load current in group 2 is assumed to vary directly as the voltage. All values are referred to the voltage E_R. The errors that result from the two assumptions of fixed current and fixed impedance for each of the load groups are given in table II. Errors of voltage are expressed as a percentage of bus voltage and errors in current in

Table II. Percentage Errors Introduced for Various Loads by Assuming Constant I and Constant Z (Figures 5and 6)

Type Load	(a) Constant		(b) Constant		(c) Constant	
	I	Z	I	Z	I	Z
Group 2 **(Kw constant)** **(Rkva = Ke²)**						
I_W (power)	2.8	5.7	2.5	7.6	2.5	8.5
I_X (reactive)	−3.7	0.2	−0.4	0.6	3.2	1.3
I_T (total)	−1.0	4.1	1.6	6.2	3.9	8.1
E (load)	−0.1	0.4	0.1	0.4	0.6	0.6
1/2 Group 1 **1/2 Group 2**						
I_W (power)	3.2	6.1	2.6	7.6	2.6	8.5
I_X (reactive)	0.3	4.1	2.6	3.6	5.5	3.7
I_T (total)	2.3	7.2	3.8	8.3	4.8	9.2
E (load)	0.2	0.7	0.4	0.7	0.9	0.9
Group 1 **(Kva constant)** **Regulated**						
I_W (power)	3.5	6.2	2.9	7.8	2.7	8.6
I_X (reactive)	4.7	8.3	5.9	6.8	8.0	6.2
I_T (total)	5.8	10.6	5.9	10.4	6.2	10.6
E (load)	0.5	1.1	0.8	1.1	1.2	1.3

terms of the correct value of total current in each case.

These examples involving voltage changes of more than five per cent at the load and voltage drops of more than ten per cent were chosen as representing very close to extreme conditions. Usually when the drop reaches ten per cent, it is necessary to make some change to provide good service and the change that is made does not usually reduce voltage drop more than five per cent for the total load. Hence the examples represent maximum conditions resulting in maximum errors. The examples show the errors that result when only a load is added to a system, but the addition of a line that will cause the same change in load voltage will give errors of the same magnitude.

In the first case (*a*), the phase angles of line and original load impedances are equal and, hence, there is no quadrature component of voltage or change of phase angle except where there are loads of the motor type. The load power factor is 60 per cent. This case represents one extreme, but is not very common on power systems.

The second case (*b*), is very common on networks made up of moderate-capacity circuits of voltages of 33 kv and less and on some circuits up to 66 kv. The original load power factor at the generator bus is 80 per cent and the ratio of R/X is $^1/_2$, so that the difference of the phase angles of the line and load impedances is 26.7 degrees.

The third case (*c*), is also rather common and is given here because it represents the other extreme beyond which the fixed current assumption leads to errors that definitely limit the use of these methods to special cases. The errors in voltage drop in this case are about the same as the errors introduced by assuming fixed impedances for loads, although the errors in current are still a good deal smaller than when the fixed impedances are assumed. The original load power factor at the generator bus is about 90 per cent and difference in phase angles of the line and load impedance is 45 degrees.

In all of the above cases, the error in total current is never more than a fraction of one per cent greater than the change in load voltage. Since the average cases are represented by cases (*b*) and (*c*), and the regulated type of loads are the most common loads, the presence of unregulated motor loads or loads of the resistance type will reduce both the magnitude and phase-angle errors of the load currents and voltages. The change in average load current on a network supplying a number of loads is always less than the maximum and, therefore, the errors will be smaller than those in the above table.

In the event that the errors in load currents or voltages are considered too great, which will be true of networks supplying regulated loads more than of any other, corrections can be applied to the load currents and the network recalculated. When corrections have to be made, the work is prolonged considerably, but one set of corrections will again give much closer results than when one set of corrections are made using constant impedance loads and can usually be more easily made. When changes in high-capacity, high-voltage circuits, are involved, it is necessary to deal with line impedances having a ratio R/X as small as one-sixth and loads of practically unity power factor. Calculations of such cases will result in very large errors unless some method employing successive corrections is used.

On many systems it is common practice to maintain a constant voltage on some major substation bus remote from the generating plant instead of at the generator bus. This substation is usually a central switching point for a number of high-capacity lines and control of voltage at that point gives some measure of voltage control over a large part of the system. When this is true, the voltage of this substation bus can be taken as a reference for calculating purposes. The variation of load voltages will be smaller taken with reference to this bus than when referred to the generator bus and the errors involved will be correspondingly reduced.

This practice is simulated in the previous examples of figure 5 by assuming that constant voltage is maintained at some point on the line between the load and generator rather than on the generator bus itself. The maximum change in load voltage may be only two or three per cent instead of five per cent previously considered and with these conditions the superposition methods can be extended to more of the high-reactance lines with high power factor loads without excessive error.

Typical Network Solutions

The procedure for the complete solution of an addition to an actual network and the results expected can be demonstrated with the network of figure 7 as an example.

The diagram represents a loop of 13.2-kv overhead lines supplying a number of loads to which a new line *EF* is to be added from the generator bus to a substation at *F*. Simultaneous readings of the original kilowatts and reactive kilovolt-amperes given in the following table were obtained at the points indicated:

	Kilowatts	Reactive Kilovolt-Amperes
Line *EA* at *E*	3,200	2,290
Load *A* at *A*	230	220
Load *B* at *B*	2,590	1,490
Line *BC* at *B*	220	190
Load *C* at *C*	430	180
Lines *EH* at *E*	7,310	5,250
Line *HG* at *H*	3,590	1,710
Load *G* at *G*	1,080	170
Load *D* at *D*	2,380	1,490

Using the kilowatt and reactive kilovolt-ampere readings in the table and

Table III (Figure 7)

Currents	Original Network	Network With Line EF Added and Loads of		
		Constant Impedance	Constant Current	Constant Kva
EA	140-j100=172.0	94.9-j61.7=113.2	89.8-j61.8 =109.0	87.4-j56.8 =104.2
A	10-j10	10.1-j10	10-j10	10-j9.9
AB	130-j90 =158.1	84.8-j51.7=99.3	79.8-j51.8 = 95.1	77.4-j46.9 = 90.5
B	120-j80	126.7-j80.2	120-j80	117 -j73.6
BC	10-j10 = 14.1	41.9-j28.5 = 50.7	40.2-j28.2 = 49.1	39.6-j26.7 = 47.8
C	20-j10	21.5-j9.9	20-j10	19.2-j8.8
DC	10-j0 = 10.0	63.4-j38.4 = 74.1	60.2-j38.2 = 71.3	58.8-j35.5 = 68.6
D	110-j80	120.6-j80.9	110-j80	105.6-j71
FD	120-j80 =144.2	184 -j119.3=219.3	170.2-j118.2=207.2	164.7-j106.5=196.1
EF		238.4-j153.1=283.3	223.3-j152.4=270.3	217.6-j139.2=260.2
GF	120-j80 =144.2	-54.4+j33.8 = 64.0	-53.1+j34.2 = 63.2	-52.9+j32.7 = 62.2
G	50-j10	52.1-j9.1	50-j10	48.4-j8.4
HG	170-j90 =192.4	-2.3+j24.7 = 24.8	-3.1+j24.2 = 24.4	-4.5+j24.3 = 24.7
H	150-j140	157.4-j140.5	150-j140	149.1-j132.6
EH	320-j230=394.0	155.1-j115.8=193.6	146.9-j115.8=187.0	144.6-j108.3=180.6
Total	460-j330	488.4-j330.6	460-j330	449.6-j304.3
	$566.1\sqrt{35.7}$ p.f. 0.81	$589.8\sqrt{34.1}$ p.f. 0.83	$566.1\sqrt{35.7}$ p.f. 0.81	$542.9\sqrt{34.1}$ p.f. 0.83
Voltages				
E	7,620-j0 =$7{,}620\underline{/0}$			
EA	134+j158	83+j108	83+j102	77+j99
A	7,486-j158=7,488	7,537-j108=7,538	7,537-j102=7,538	7,543-j99 =7,544
AB	593+j301	361+j210	351+j191	328+j193
B	6,893-j459=6,908	7,176-j318=7,183	7,186-j293=7,192	7,215-j292=7,221
BC	27+j11	-88-j57	-87-j54	-82-j54
C	6,866-j470=6,882	7,263-j261=7,268	7,273-j239=7,277	7,297-j238=7,302
DC	4+j8	56+j35	55+j33	52+j33
D	6,872-j460=6,887	7,320-j226=7,323	7,328-j206=7,331	7,349-j205=7,352
FD	120+j76	180+j118	175+j106	162+j106
EF	628+j384	120+j108	117+j100	109+j99
GF	172+j76	-75-j36	-75-j35	-73-j35
G	7,164-j308=7,171	7,424-j144=7,426	7,428-j135=7,429	7,438-j134=7,439
HG	52+j16	-6+j4	-6+j4	-6+j4
H	7,216-j292=7,222	7,419-j140=7,421	7,422-j131=7,423	7,432-j130=7,433
EH	404+j292	201+j140	198+j131	188+j130

Per Cent Voltage	Original Network		Network With Line EF Added and Loads of					
			Constant Impedance		Constant Current		Constant Kva	
	Drop	Reg.	Drop	Reg.	Drop	Reg.	Drop	Reg.
A	1.8+j2.1	1.7	1.1+j1.4	1.1	1.1+j1.3	1.1	1.0+j1.3	1.0
B	9.6+j6.0	9.4	5.8+j4.2	5.7	5.7+j3.9	5.6	5.3+j3.8	5.2
C	9.9+j6.1	9.7	4.7+j3.4	4.6	4.6+j3.2	4.5	4.3+j3.1	4.2
D	9.8+j6.0	9.6	3.9+j3.0	3.9	3.8+j2.7	3.8	3.6+j2.7	3.5
E	0		0		0		0	
F	8.2+j5.1	8.1	1.6+j1.4	1.6	1.5+j1.3	1.6	1.4+j1.3	1.4
G	6.0+j4.1	5.9	2.6+j1.9	2.5	2.5+j1.8	2.5	2.4+j1.8	2.4
H	5.3+j3.8	5.2	2.6+j1.8	2.6	2.6+j1.7	2.6	2.5+j1.7	2.5

beginning at the generator bus, line current and voltage drop to the next load were calculated. Knowing the voltage at the next load, the load current there was calculated and referred in phase angle to the generator bus voltage and this process repeated until all of the currents and voltages on the original loop were known with reference to the generator bus voltage. These are the values shown on the diagram. The solution for all new currents and voltages after the new line is added then follows the method applied to figure 2.

Table III shows all values that applied to the original network, as well as three complete solutions based on the assumption that the loads acted as fixed values of impedance, current, and kilovolt-amperes respectively. The errors that result when all of the loads are regulated and calculations are made by methods assuming constant current and constant impedance respectively, are shown in table IV with voltage expressed in terms of generator bus voltage and current errors in terms of total corrected currents.

The relatively high average power factor of all the loads (84–85 per cent) on this network and the large voltage change which is a maximum at point *F* (6.7 per cent) make this example a rather severe test. The results when constant-current loads are assumed are within acceptable practical limits and are more accurate and much more easily obtained than when constant impedance is assumed.

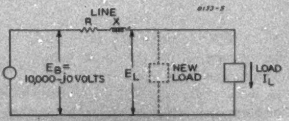

Figure 5. Single load at end of a circuit with identical new load to be added

A small amount of unregulated load, which is present on all networks, will bring the actual results closer to the calculated figures. The loop shown in figure 7 is a close approximation of an actual setup supplying a predominantly residential load, all of which was regulated. In spite of this fact, there were a small number of unregulated customer loads at points *A*, *C*, and *G*. Also during peak loads some of the regulators at stations *B* and *D* operated at full boost and probably allowed some drop in load voltage. With motor loads then at *A*, *C*, and *G*, and with 20 per cent of the load from stations *B* and *D* originally operating at a voltage two per cent below normal, the corrected total current to the entire network after the line addition would be 451 − j309.5, or 547 amperes. The percentage errors in the corresponding values obtained by superposition methods would be 1.6, 3.7, and 3.5 per cent respectively of the total current of 547 amperes.

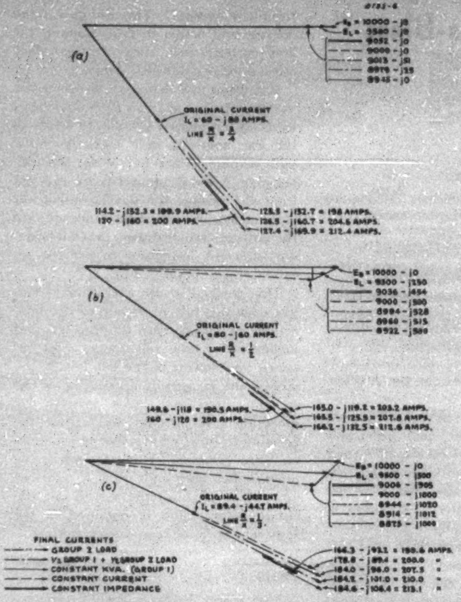
0135-6
(a)
E_B = 10000 - j0
E_L = 9500 - j0
9052 - j0
9000 - j0
9013 - j51
8979 - j35
8945 - j0
ORIGINAL CURRENT
I_L = 60 - j80 AMPS.
LINE $\frac{R}{X} = \frac{3}{4}$
114.2 - j152.3 = 189.9 AMPS.
120 - j160 = 200 AMPS.
125.5 - j152.7 = 198 AMPS.
126.5 - j160.7 = 204.6 AMPS.
127.4 - j169.9 = 212.4 AMPS.
(b)
E_B = 10000 - j0
E_L = 9500 - j250
9036 - j454
9000 - j500
8994 - j528
8960 - j515
8922 - j500
ORIGINAL CURRENT
I_L = 80 - j60 AMPS.
LINE $\frac{R}{X} = \frac{1}{2}$
148.6 - j118 = 190.5 AMPS.
160 - j120 = 200 AMPS.
165.0 - j119.2 = 203.2 AMPS.
165.5 - j125.9 = 207.8 AMPS.
166.2 - j132.5 = 212.6 AMPS.
(c)
E_B = 10000 - j0
E_L = 9500 - j500
9006 - j905
9000 - j1000
8944 - j1020
8914 - j1012
8875 - j1000
ORIGINAL CURRENT
I_L = 89.4 - j44.7 AMPS.
LINE $\frac{R}{X} = \frac{1}{3}$.
FINAL CURRENTS
GROUP 2 LOAD
½ GROUP 1 + ½ GROUP 2 LOAD
CONSTANT KVA. (GROUP 1)
CONSTANT CURRENT
CONSTANT IMPEDANCE
166.3 - j93.1 = 190.6 AMPS.
178.8 - j89.4 = 200.0 "
184.0 - j96.0 = 207.5 "
184.2 - j101.0 = 210.0 "
184.6 - j106.4 = 213.1 "

Figure 6. Analysis of the effect of adding load to the end of a circuit carrying a load identical to the added load

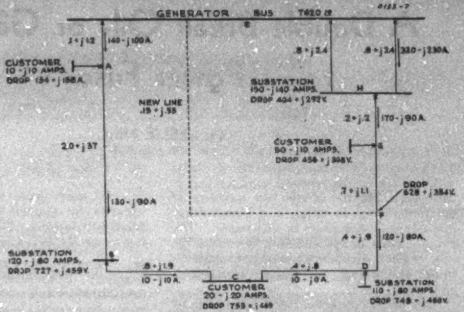
GENERATOR BUS 7620 ∠0
0135-7
E
1 + j1.2
140 - j100A.
CUSTOMER
10 - j10 AMPS.
DROP 134 + j188A.
A
NEW LINE
.15 + j.55
2.0 + j37
130 - j90A
SUBSTATION
120 - j80 AMPS.
DROP 727 + j459V.
B
.8 + j1.9
10 - j10A.
C
CUSTOMER
20 - j20 AMPS.
DROP 753 + j469
.4 + j.8
10 - j0A.
D
SUBSTATION
110 - j80 AMPS.
DROP 748 + j468V.
.8 + j2.4
.8 + j2.4
320 - j230A.
SUBSTATION
190 - j140 AMPS.
DROP 404 + j292V.
H
.2 + j.2
170 - j90A.
CUSTOMER
50 - j10 AMPS.
DROP 456 + j306V.
G
.7 + j1.1
DROP
628 + j384V.
F
.4 + j.9
120 - j80A.

Figure 7. Typical solution for the addition of a new line to an existing network

Table IV. Errors in Per Cent With Regulated Load (Kilovolt-Amperes Constant) (Figure 7)

	Constant Current Assumed				Constant Impedance Assumed			
	I_W	I_X	I_T	E	I_W	I_X	I_T	E
EA	2.3	4.8	4.6	*A*0.1	7.2	4.7	8.6	*A*0.1
AB	2.6	5.4	5.1	*B*0.4	8.2	5.3	9.7	*B*0.5
BC	1.3	3.1	2.7	*C*0.3	4.8	3.8	6.1	*C*0.4
CD	2.0	3.9	3.9	*D*0.3	6.7	4.2	8.0	*D*0.4
FD	2.8	6.0	5.6	*F*0.1	9.8	6.5	11.8	*F*0.1
EF	2.2	5.1	3.9	*G*0.1	8.0	5.3	8.9	*G*0.2
GF	0.3	2.4	1.6	*H*0.1	2.4	1.8	2.9	*H*0.2
HG	5.7	0.4	1.2		8.9	1.6	0.4	
EH	1.3	4.2	3.5		5.8	4.2	7.2	
Total	1.9	4.7	4.3		7.1	4.8	8.6	

Conclusions

The methods outlined here can be applied with a good degree of accuracy to many a-c networks, with the range of phase angles of both line and load impedances commonly found in operation. When they are used to calculate the effect of line changes that make up a part of a system between generating plants, other variables are introduced. The voltages on generating plant busses can often be assumed at a certain magnitude, but changes in phase angle introduce other factors that are difficult to handle by this method. When, however, the line change or addition makes up only a small part of the tie capacity between generating plants, the changes in angle between generators will be negligible and superposition can be applied. This is true of a large number of 13.2-kv and 33-kv lines on the Cincinnati Gas and Electric Company system where the capacity of one circuit is small compared to the total capacity of paralleled 66-kv and 13.2-kv lines.

It is recognized that the methods are not universally applicable to all problems, and that they are most useful for calculating additions to existing systems. On most subtransmission and distribution networks where a variety of types of loads are supplied, superposition methods will give a more accurate result than any of the others, unless a method of trial and error is used. There is no straightforward method of calculating exactly the general network, except where all loads act purely as fixed impedances and this is rarely found on power systems. Superposition methods are very direct where they can be applied and can be checked frequently in the course of one set of calculations. The reasons for the better accuracy of the superposition methods are:

(*a*). The original assumption that loads act as fixed currents simulates actual average conditions more closely than any other assumption.

(*b*). Calculations are made only of changes in values rather than of whole values.

(*c*). Fewer impedances are handled since the load values are omitted.

The method depends principally on getting a reliable set of load and voltage readings on the principal lines and loads, so that a complete operating condition can be set up before any changes are made. When, however, these data are available, these methods have a wide field of application on the moderate capacity circuits of the average power system. Many solutions are possible without making corrections because of the relatively small errors introduced by these methods and when errors become too great, the methods are well suited for making these corrections in many instances.

Bibliography

1. ELECTRIC CIRCUITS, THEORY AND APPLICATION (a book), O. G. C. Dahl. Volume 1.

2. CALCULATING SHORT-CIRCUIT CURRENTS IN NETWORKS, R. D. Evans and C. L. Fortescue. *Electric Journal*, 1919.

3. SOLUTION OF ELECTRICAL NETWORKS BY SUCCESSIVE APPROXIMATIONS, Lawrence L. Smith. Bulletin No. 299, University of Illinois Engineering Experiment Station.

A Double-Break Oil- or Gas-Blast Power Fuse

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ASSOCIATE AIEE

Synopsis: The use of two fuse elements of different rating, in series, in a high-voltage fuse device is described in this paper. The two elements are placed in a tube and the tube is filled with an arc-extinguishing substance and sealed; one end with an expellable plug.

In operation the lower rated element melts first on an overcurrent and draws an arc; the energy from this arc is used to build up a pressure and forces the sealing plug out which establishes a second arc. Then the pressure in the tube blasts the arc extinguishing substance into the second arc and interrupts the circuit.

For convenience, the circuit-interrupting part of the device is built in the form of a cartridge refill for insertion in a fuse tube or holder. Seven years of field experience have been obtained and is reported in this paper. The device has been applied on circuits up to 60 kv.

About seven years ago, operating and field experience of the company with which the writer is associated, established a need for a dry-type fuse that would be interchangeable with a liquid-filled spring-actuated fuse which was in general use on the overhead distribution lines.

The specifications as set up by the operating staff were as follows:

(*a*). The fuse was to be dry type and easily refillable in the field.

(*b*). The fuse tube must be glass and not an organic material.

(*c*). It must be easily observed when blown.

(*d*). It must not expel long arcs either upward or downward when interrupting a circuit.

(*e*). It must not expel molten metal or fuse element parts.

(*f*). It must have sufficient interrupting capacity for general distribution lines and small substations; the tentative rating being set at 100,000 kva at 12,500 volts.

(*g*). It must carry its rated current continuously, if necessary, without deterioration and blow at a reasonable overcurrent.

A survey of the available fuse devices on the market did not yield any that would meet the above specification so authorization was given for the development of a device.

The writer, being interested in the circuit-interrupting art, had previously experimented with and developed various fuse devices* so the opportunity was given for the field application of the principle of circuit interruption used in the double-break fuse.

All of the requirements had been previously met in the experimental devices except (*b*) calling for the use of a glass tube on a dry-type expulsion fuse device. It was desired to use a glass tube for ease of visibility and to avoid the problem of carbonization from leakage current that would ensue if an organic tube were used.

Principle of Operation

Referring to figure 1 it will be seen that the two fuse elements, *A* and *B*, are in series with the tubular metallic conductor *F* between them. The element *A*, being lower rated than element *B*, fuses first and establishes a primary arc on overcurrent. The heat from this primary arc decomposes the arc-extinguishing substance surrounding element *A*, generating a gas which builds up a pressure in chamber *I*. The pressure thus obtained is directed against the conductor *G* and sealing member *J*, moving them outward and placing a tension on element *B*. When the tension exceeds the strength of element *B*, which is also weakened by the heat produced by overcurrent, it is easily pulled apart, causing a secondary arc.

With the establishment of the secondary arc in the orifice or mixing tube *K*, the gases previously generated and the arc-extinguishing substance are blasted into this space at a high velocity and in a turbulent manner. This causes a rapid deionization action to take place at the first current zero after the secondary arc has been established.

Expulsion and deionization are shown in figure 2, the gas blast from chamber I expelling all ionized gas and products of the arc, resulting in a clean deionized path through orifice *K*. When the sealing plug *J* and conductor *G* are expelled from the orifice, the flexible conductor *H* is carried away from the refill and into the bottom trap chamber, thus introducing sufficient insulation to prevent reestablishment of the circuit.

This method of arc extinction is employed in this fuse regardless of the magnitude of the current interrupted. An increment of time will always exist between the blowing of the two fusible elements, the increment decreasing as the current increases. Arcs created by moderate overcurrents will cause the pressure to build up at a slower rate than on large currents. However, when the pressure has been built up to a sufficient amount to establish the second arc, the circuit interruption is obtained in the same manner as on large overcurrents.

Tests

The actual functioning of the two arcs in series in a fuse tube on a 12,000-volt circuit was studied by suitably placing the two fuse elements within a heavy-walled glass tube and taking high-speed moving pictures. This showed beyond any question that the interaction between the primary and secondary arcs was as theoretically anticipated.

The distance actually required for the extinction of an electric arc is surprisingly small for a given voltage, if the space between the two electrodes is rapidly changed from a conduction path to a nonconducting path at the a-c zero. The method used to change the space between the two electrodes from conducting to nonconducting in this fuse device, does not depend upon the decomposition of the wall of the fuse chamber, under the heat of the arc, as in the typical expulsion fuse. But instead, the arc-extinguishing material surrounds both fusible elements and also fills the chambers, thus obtaining a quicker and more efficient deionizing action.

The length of the orifice *K* (figures 1 and 2) varied from three-quarters to $2^{1}/_{2}$ inches in the cartridge refills tested on a 12,000-volt circuit. The circuit interruption was entirely successful at any of the lengths within that range and no apparent difference in arc duration

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Paper 40-134, recommended by the AIEE committee on protective devices, and presented at the AIEE Pacific Coast convention, Los Angeles, Calif., August 27–30, 1940. Manuscript submitted June 10, 1940; made available for preprinting July 18, 1940.

The author wishes to express his appreciation for the co-operation and helpful suggestions furnished by many individuals in the Pacific Gas and Electric Company. Especial appreciation is due the East Bay division for their willingness to be the first to place a sufficient number of the overhead dry-type devices in service, to obtain the initial field service, and the San Francisco division for their willingness to place the initial oil-filled fuses in service.

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*Patent Number 2,094,012 and Reissue 20,450.

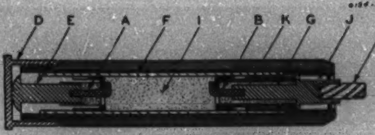

Figure 1. Cross section of the cartridge unit

The two fuse sections *A* and *B* are in series with the metal case *F* acting as the conductor between the fuse sections

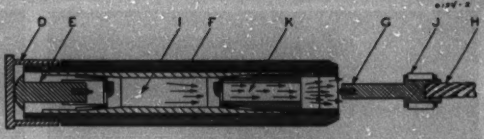

Figure 2. Cross section of the cartridge unit after a circuit interruption showing sealing plug being expelled from the end of the cartridge

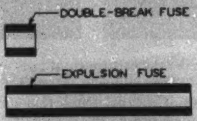

Figure 3. Approximate tube lengths required to interrupt a given voltage and current for the double-break fuse and the plain open-ended expulsion fuse

for a given current could be determined.

Figure 3 illustrates the relative distances required to successfully interrupt a given voltage and current by the double-break fuse and by the plain open-ended expulsion fuse.

When experiments were first made on the double-break fuse device, a spring was attached to the lower end of the conductor at the sealing plug *J*, figure 1, and extended to the lower ferrule. This spring was to aid in circuit interruption at moderate overcurrents. On heavy currents the expelled parts would be driven down into the spring and bent in such shapes as to prove, without question, the expelled members endeavored to drive the spring, rather than the movement being actuated by the spring. This led to the conclusion that the spring retarded the movement of the parting conductor rather than aid it.

Experiments were then made with the spring eliminated and it was found that the circuit was interrupted satisfactorily on moderate overload currents just sufficent to melt the *A* fuse element. The experiments were first conducted at 12,000 volts and later extended to 22,000, 39,000, and 62,000 volts.

It was thought by some engineers that two fuses in series would operate too slowly and that the blowing time between the two could not be co-ordinated so as to obtain the desired interaction of one arc upon the other. Sufficient tests have now been made and experience has been obtained to show that the interaction between the two fuse elements of different ratings, in a fuse tube, may be co-ordinated to a fairly high degree of accuracy.

It is readily recognized, from a study of the design shown in figures 1 and 2, that the pressure wave must travel from one end of the tube to the other end before the expellable plug is blown outward. It was also recognized that a granular material which would pack would not allow the pressure wave to pass, so tests were made to determine what the grading of the material should be.

The first material used had a large percentage of fines passing through a 200-mesh Tyler screen. This was soon found to pack and not allow the pressure wave to pass through at a rate sufficiently fast to prevent bursting of the tubes on heavy currents.

A coarser material was experimented with and at the present time the most suitable fineness for the granular material has been graded to pass a 60- and to be retained on a 100-mesh Tyler screen. This fineness is used in all the cartridges except the 14-inch length where the material is graded to pass a 60- and be retained on an 80-mesh screen. The percentage of voids in the −60+100 material is about 45 per cent and about 50 per cent for the −60+80 material.

Tables I and II give the arc duration, within the 14- and 7-inch cartridges, after the *A* fuse element melts and until the circuit is interrupted. This is for the "dry type" cartridge filled with a granular arc-extinguishing material. The total arc duration is divided into the primary arc time and the secondary arc time. The primary arc time represents the period required for the first arc to decompose a sufficient amount of the material, build up the pressure and blow the sealing member outward. The secondary arc time is that period required to interrupt the circuit after the second arc is started by the expulsion action of the sealing member being blown outward.

These data are obtained from the inspection of oscillograms taken during tests. On some of the oscillograms the point at which the second arc starts may be obtained by observing the rapid increase in the arc voltage but in other oscillograms this point is very difficult to distinguish. Where the total arc duration is of the order of one-half cycle or less, it was not practical to separate the two arc periods.

Figure 4 shows an example of an oscillogram taken of a double-break fuse interrupting 2,600 amperes at 12,500 volts,

Table I. Arc Duration Within 14-Inch Double-Break Fuse Cartridges

Dry Granular Arc Extinguishing Material

Test Number	Rating of "A" Fuse (Amp)	Voltage Across Fuse Cartridge	Current Interrupted RMS Amperes	Arc Time (Cycles) (After Fuse Element Melts)		
				Total	Primary	Secondary
1	2	60,000	12.6	96	No oscillogram	
2	2	60,000	22.4	40	No oscillogram	
3	2	60,000	40	15	No oscillogram	
4	2	60,000	48	20	No oscillogram	
5	2	60,000	170	7.5	7.3	0.20
6	40	62,500	310	10.5	9.0	1.50
7	40	62,500	520	4.75	4.5	0.25
8	100	62,500	800	4.5	4.0	0.50
9	40	62,500	920	3.5	3.0	0.50
10	5	36,000	64	15.0	14.7	0.30
11	25	36,000	400	4.5	4.25	0.25
12	40	12,500	750	3.25	2.75	0.50
13	40	12,500	1,100	2.75	2.50	0.25
14	40	12,500	1,500	2.0	1.65	0.35
15	2	12,500	1,920	2.4	1.90	0.50
16	25	12,500	2,240	1.85	1.60	0.25
17	100	12,500	2,560	1.40	0.90	0.50
18	100	12,500	3,100	1.25	1.00	0.25
19	100	12,500	3,680	1.20	0.85	0.35
20	45	12,500	4,000	0.75	0.60	0.15
21	100	12,500	4,080	1.00	0.70	0.30

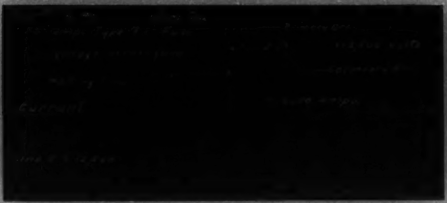

Figure 4. Oscillogram showing melting time of fuse element, primary and secondary arc time in a double-break fuse cartridge

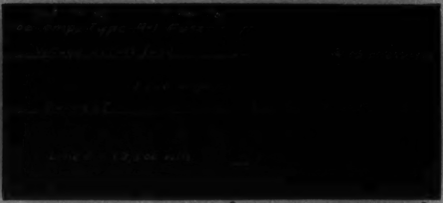

Figure 5. Oscillogram having melting time of fuse element, primary and secondary arc time all within one current loop in a double-break fuse cartridge

This film was selected to show the primary arc time of one cycle and then a secondary arc time of 0.45 cycle. It required two full cycles to melt the fuse. In figure 5 it will be seen that it is difficult to distinguish the periods of arcing. By an approximation, the primary arcing time may be taken as 0.35 cycle and the secondary time as 0.10 cycle. The melting time of the fuse in this test was 0.10 cycle.

The average values of arcing time for the 7- and 14-inch cartridges are shown graphically in figure 6. The time required for the primary arc to build up sufficient pressure to expell the sealing member and start the secondary arc is the difference between curves *A* and *C* for the 14-inch and curves *B* and *D* for the 7-inch cartridges.

The data for curves *A* and *C* were obtained on fuses rated from 5 to 100 amperes and the data for curves *B* and *D* were obtained on fuses rated from 2 to 200 amperes.

During the experimental tests and subsequent check tests on manufactured samples it was not found that the variation in the size of the *A* fuse (up to 200-ampere rating) had any appreciable effect on the arc duration or the circuit-interrupting ability of the device. It was definitely determined that the fuse device would not perform satisfactorily in "reverse", that is, with the *B* fuse smaller than the *A* fuse.

It has been interesting to inspect the interior of the refill cartridges after blowing, and to find practically no burning of the parts up to the higher currents tested, due to the decrease in the arcing time as the values of current increased. It has also been of interest to find that the *B* fuse in a great many cases had not been entirely burned away during tests where the current reached values as high as 2,500 amperes at 12,000 volts. This is a practical indication of the rapidity of the circuit interruption after the second arc is established when it is considered the *B* fuse is a lead-alloy wire about one-half inch long.

Experiments were made to determine the most suitable pressures that the primary arc should generate before the sealing member is expelled from the cartridge. The pressures could cover a fairly wide range and circuit interruption was satisfactory for moderate overcurrents and heavy short circuits. The range of the pressures varies from 50 to 90 pounds per square inch and applies to the short or long cartridges.

In addition to the circuit-interrupting tests covered in this paper, the double-break fuse met all the other requirements that a high-class device should meet such as:

(*a*). Radio interference on the various mountings.

(*b*). Current-carrying requirement that it should carry its rating.

(*c*). Freedom from corona.

(*d*). Exposure under the most adverse weather conditions.

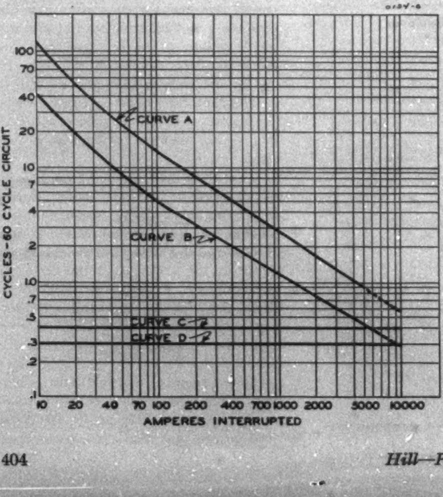

Figure 6. Arc duration within double-break fuse cartridges

Curve *A*—Total arc time—after fuse element melts—14-inch cartridges

Curve *B*—Total arc time—after fuse element melts—7-inch cartridges

Curve *C*—Average secondary arc time—14-inch cartridges

Curve *D*—Average secondary arc time—7-inch cartridges

Difference between curves *A* and *C* and *B* and *D* is time required for primary arc to build up pressure

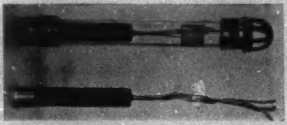

Figure 7. Dry-type double-break cartridge refill fuse in glass tube $11^1/_2$ inches center to center used on distribution circuits up to 22 kv

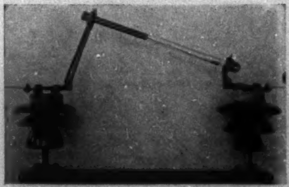

Figure 8. Dry-type double-break cartridge refill fuse in glass tube—used on 60-kv distribution lines—30 inches center to center for horizontal mounting

Figure 9. Interior of oil-filled cutout box used on 12-kv underground before modernization—using organic insulation and plain expulsion fuse

Moving pictures (64 frames per second) were taken of a large percentage of the circuit-interrupting tests and it was interesting to note the relatively small amount of arc gases emitted from the cartridges. There were seven tests at 62,500 volts where there was practically no visible arc and only two tests noted where there were more than two frames of visible arc duration in the pictures.

The greatest distance the arc was expelled out of the end of the tubes during any of the tests was two feet and most of the pictures show only two to six inches of arc out of the end of the fuse tube.

The tests were performed with power from various sources as follows:

(*a*). A three-phase 12,000-volt cable about five miles in length, utilizing iron-core and air-core reactors for limiting the current. Total power available from the line was approximately 100,000 kva.

(*b*). Synchronous condensers of 20,000 kva and 40,000 kva capacity.

(*c*). Water-wheel-driven generators of 15,000 kva and 12,500 kva capacity feeding through a transformer bank of 6,600/66,000 volts and 12,750 kva capacity.

Adaptation to Field Service

Figure 7 shows a dry-type double-break cartridge refill fuse used on distribution circuits up to 22 kv. The ferrules are two inches across the flats and are 11$^1/_2$-inch clip centers. A Pyrex glass tube is used instead of the usual organic insulating tube. One cartridge refill is shown inserted in the glass tube through the top ferrule, the cap of which acts as a clamping contact. Along side the fuse tube is a spare cartridge refill ready for insertion. The blown refill cartridges are accumulated in a central warehouse and then the internal fuse elements and extinguishing material are renewed. These fuses are built in sizes from 2 to 200 amperes.

Screwed onto the bottom ferrule is a perforated trap chamber which acts as a contact clamping means between the divided ends of the flexible conductor and the ferrule. The bottom trap chamber catches the flexible conductor and expelled parts from the cartridge and also aids in cooling any hot gases which may be expelled during blowing of the fuse. No tools are necessary for insertion or removal of the cartridge refill.

Figure 8 is the adaptation of the double-break cartridge refill fuse to 60-kv service. The fuse tube as shown is 30 inches, center to center of the clips, and the hardware is for horizontal mounting. The extended arm on one of the insulators is to place the closed end of the tube in a raised position to prevent rain from being blown in during a storm. The extended arm would be removed if the fuse tube were to be mounted in a vertical position; in this case the fuse tube is parallel with the channel-iron base. This fuse is built in sizes from 5 to 100 amperes.

The 60-kv fuse has been given a long-time fusing characteristic so that it is adaptable to transformer banks having reclosing oil circuit breakers on the low-voltage side.

Double-Break Principle Applied to Oil Fuse

The question of what fuse to use became quite a problem about five years ago when it was decided to modernize an oil-filled enclosed underground fuse switch that had been pioneered and developed in 1922 by the Great Western Power Company's system* in San Francisco. The interior of one of these original cutout boxes is shown as figure 9. The operating arms were of organic material and the fuse was of the open-end expulsion type with a fuse wire extending throughout its whole elngth, and fastened to the end ferrules.

The circuit-interrupting ability of this plain expulsion fuse, in an oil-filled cutout box, was very limited and the pressures set up in the box became quite high on relatively low currents so it was decided, in the modernization process, to install a higher-capacity fuse and also porcelain insulation on the operating arms and supports for the fuse terminals.

Figure 10 shows the interior of the improved cutout box with porcelain insulation and the new fuse. The fuse tube and refill cartridge are shown as figure 11. The cartridge utilizes the same principle of circuit interruption as the dry-type powder-filled cartridge shown in figures 1 and 2 except that oil is used as the filler inasmuch as the cutout box is filled with oil for insulation and for interrupting a load current when used as a manually operated switch.

It will be noted that the refill cartridge does not have an outer insulating tube surrounding the metal pressure tube *F* as it was found that the oil column surrounding this refill cartridge, when in-

Table II. Arc Duration Within Seven-Inch Double-Break Fuse Cartridges

Dry Granular Arc-Extinguishing Material (60-Cycle Circuit)

Test Number	Rating of "A" Fuse (Amp)	Voltage Across Fuse Cartridge	Current Interrupted RMS Amperes	Arc Time (Cycles) (After Fuse Element Melts) Total	Primary	Secondary
22	2	12,500	4.3	120.0	No oscillogram	
23	2	12,500	6.8	62.0	No oscillogram	
24	2	12,500	11.0	32.0	No oscillogram	
25	2	12,500	27.5	25.0	No oscillogram	
26	2	12,500	55	10.2	No oscillogram	
27	2	12,500	107	4.2	3.70	0.50
28	2	12,500	213	3.0	2.65	0.35
29	100	12,500	410	2.0	1.50	0.50
30	100	12,500	640	3.0	2.75	0.25
31	200	12,500	750	1.0	0.50	0.50
32	100	12,500	880	2.0	1.70	0.30
33	100	12,500	1,280	1.0	0.80	0.20
34	200	12,500	1,500	0.75	0.50	0.25
35	100	12,500	1,920	0.45	0.30	0.15
36	150	12,500	2,160	1.00	0.50	0.50
37	200	12,500	3,920	0.50	0.25	0.25
38	200	12,500	3,600	0.80	0.45	0.35
39	100	12,500	3,680	0.45	0.25	0.20
40	100	12,500	3,800	0.90	0.70	0.20
41	100	12,500	2,600	1.45	1.00	0.45
42	100	12,500	5,500	0.45	0.35	0.10
43	200	12,500	4,090	0.65	?	?
44	100	14,500	7,680	0.27	?	?
45	100	2,600	8,100	0.33	?	?
46	100	2,600	9,960	0.32	?	?
47	100	2,600	10,200	0.41	?	?
48	100	2,600	11,000	0.45	?	?

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*Now consolidated with the Pacific Gas and Electric Company system.

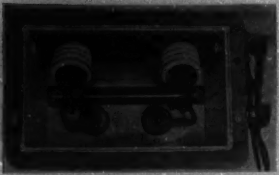

Figure 10. Interior of 12,000-volt oil-filled cutout box, after modernization using double-break oil-blast fuse and porcelain insulation

stalled in the fuse tube, had sufficient insulation to prevent flashback of the circuit around the insulating bushing on the expulsion end of the cartridge.

Table III gives circuit-interrupting test data of the double-break oil-filled fuse in an enclosed oil-fuse switch. These data cover a range of current from 5.6 amperes to 11,000 amperes at 12,000 volts. The tests were made on the design shown in figure 10. It was of interest to note that the arc time of the primary fuse *A* could not be separated from the secondary fuse *B* on the oscillograms as could be done in some of the oscillograms of the tests on the powder-filled devices. The maximum total arc time was only 0.6 cycle and the average total arc time was only 0.30 cycle on currents from 200 to 11,000 amperes. During the experimental tests, the maximum arc time was also only 0.6 cycle on currents as low as 160 amperes. Oscillograms were not taken during tests on the small currents of the order of five amperes, but the total time, including the melting of the fuse element, and the arc time was only a few cycles. The *A* fuse element in these tests was a number 40 advance wire.

Figure 12 gives the maximum pressures attained for interrupting various currents, at 12,000 volts, when the circuit is interrupted with a plain expulsion fuse and when interrupted with a double-break oil-blast fuse in a similar cutout box and similar circuit conditions. The double-break fuse interrupted approximately seven times as much current as the plain expulsion fuse, for a given pressure generated in the cutout box. It was of interest to note that there was no damage to the refill cartridge or fuse tube on currents as high as 11,000 amperes at 12,000 volts. The lid of the cutout box was not damaged until a current of 11,000 amperes was reached.

Table III. Circuit-Interrupting Tests of Double-Break Oil-Filled Cartridge Refill

Fuse in Enclosed Cutout Box—Tests at 60 Cycles

Voltage Across Fuse	Rating of "A" Fuse (Amp)	Current RMS Amp	Total Interrupting Time (Cycles)	Total Arc Time (Cycles)	Maximum Pressure (Lb Per Sq In.)	Remarks
11,700	No. 40 advance	5.6	15	?*	Not taken	Cleared O.K.—no damage to refill cartridge—fuse tube—or cutout box—no oil blown out
11,700	No. 40 advance	5.6	9.6	?*		
11,700	2	9.6	17.8	5.8		
11,700	2	9.6	20.5	8.5		
11,700	5	22.0	9.0	3.0		
11,700	40	200	12.0	0.50		
11,700	40	300	8.4	0.40		
12,200	40	416	2.4	0.35		
12,200	40	1,120	0.6	0.30	30	
12,200	40	1,120	0.5	0.25	30	
12,200	40	1,600	0.5	0.25	35	
12,200	40	1,680	0.55	0.25	30	
12,300	40	1,760	0.65	0.25	40	
12,300	40	2,720	0.50	0.25	40	
12,300	40	2,880	0.38	0.21	30	
12,300	40	3,040	0.40	0.25	40	
12,250	40	3,040	0.35	0.20	30	
12,300	40	3,840	0.50	0.27	66	
12,300	40	3,840	0.50	0.30	78	
12,300	40	3,040	0.35	0.20	30	
12,300	40	4,320	0.50	0.32	40	
12,300	100	4,300	0.50	0.25	66	
12,000	50	1,700	0.45	0.25	28	
12,000	50	2,800	0.45	0.30	52	
12,000	50	4,100	0.41	0.30	52	
12,000	50	11,000	0.70	0.60	168	Cleared O.K.—no damage to refill cartridge or tube—cover of cutout box bulged upward about one-fourth inch—about one-fourth of oil blown out
Cutout box repaired—more oil added—test resumed						
12,000	150	5,200	1.0	0.50	59	Cleared O.K.—no damage to refill cartridge or tube—one-half inch oil blown out—cover did not fit well
12,000	150	8,800	0.75	0.55	104	Ditto—but two inches oil blown out

*No oscillogram taken; total interrupting time obtained by synchronous timer.

These oil-filled cutout boxes and the double-break oil fuse were developed quite hurriedly to meet a situation in San Francisco where a large number of 11,000-volt overhead lines had to be placed underground. The circuit-interrupting requirement was set at 10,000 amperes 12,000 volts. The experimental work, test on factory models, the final engineering drawings, and the first delivery were accomplished in approximately 90 days.

Field Experience

There have been about seven years' field experience on the dry-type double-break cartridge refill fuse, with a glass tube, on 12- and 22-kv service. This type is shown as figure 7. A trial batch of about 100 were first installed and watched rather closely. Within one year, enough circuit interruptions had occurred to show the design was functioning satisfactorily and up to the present time only very small minor changes have been made in the mechanical structure.

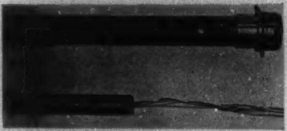

Figure 11. Oil-filled double-break fuse with refill cartridge used in 12,000-volt enclosed oil cutout box—oil film between refill cartridge and insulating tube prevents flashback during circuit interruption

Several thousand are now in service on the distribution lines and small substations. Examination of the refills, after blowing, show that practically 100 per cent may be used again by renewing the fuse elements and the arc-extinguishing material. This results in a very economical arrangement. The service men like the refill feature as it saves space in their truck. A few glass tubes may be carried and enough refills to meet the service requirement.

The fuses, with the glass tubes, were first restricted to circuits having a maximum short-circuit current of 4,000 amperes at 12 kv but are now installed on distribution circuits where reactors limit the maximum short-circuit current to approximately 7,000 amperes.

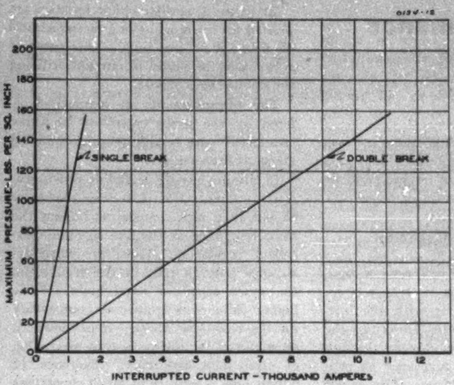

Figure 12. Maximum pressures obtained using plain expulsion fuse and double-break fuse in an enclosed oil-filled cutout box

There has been about four years' experience on the 60-kv fuse shown as figure 8. About 200 have been installed to date and cartridge refills are returned to the central warehouse for renewal, the same as the refills for the lower-voltage devices.

About 700 of the double-break oil-blast fuses have been used in new enclosed oil-filled cutout boxes in the 12-kv underground distribution service to date. The blown cartridges are returned to the central warehouse for renewal of the fuse elements and refilling with oil. The cartridges are then returned for service, in the field, the same as the dry type.

Additional Experimental Work

When it is analyzed that the circuit interruption takes place within the cartridge of the double-break fuse, it is realized the glass tube is simply a container and a means of connection to the circuit. Experiments have been made with the dry-type double-break 60-kv cartridge mounted without a glass tube and utilizing a stainless-steel retracting spring arm to give an air break clearance after the fuse blows. This device is shown in figures 13*a* and 13*b* and has been tested up to 400 amperes at 60 kv and 4,000 amperes at 12 kv. Moving pictures show a remarkably small amount of arc gases emitted and a relatively small amount of "mushrooming" back around the end of the cartridge. If this structural arrangement proves successful in service, it will make a very economical device and at the same time have desirable circuit-interrupting characteristics.

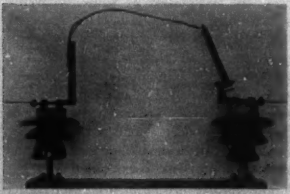

Figure 13a. Experimental double-break cartridge fuse mounted without glass tube—utilizing a stainless-steel retracting spring arm

The remarkable short arc time characteristics of the double-break oil-filled fuse has led to the consideration of using a noninflammable liquid in the type now using dry granular arc extinguishing material. Some tests have been made and practical difficulties will have to be overcome, such as sealing, etc., but if adopted, we would have the novel feature of a liquid-filled "dry type".

The requirements as originally set up for the devices for our distribution lines may be classed as an intermediate design as to interrupting duty, costs, etc. If a higher interrupting capacity should be desired, detail structural features of the double-break oil or gas-blast fuse may be changed to meet the requirements.

Conclusions

The various modifications of the double-break fuse, shown in this paper, have met the service need and requirements as set up for our overhead and underground distribution lines and they have proved to be high-class economical fuse devices.

When arc-extinguishing substances are confined and then released at a predetermined pressure, as in the double-break fuse, the volts interrupted per inch are much higher than in fuse devices such as open-ended expulsion fuses. The ratio of length, for a given circuit condition, may vary from 1:2 to 1:7 depending upon the device used for comparison.

The distance the arc gases are expelled and the volume of gases expelled from the double-break fuse have been found to be considerably less than other types of expulsion-fuse devices, when tested under the same circuit conditions.

The duration of the arc time, after the second arc is established, has been found to be of the order of one-half cycle or less for all voltages up to the highest tested which was 62,500. Thus the expelled arc gases, during circuit interruption, are of the same general magnitude for a given current, regardless of the circuit voltage.

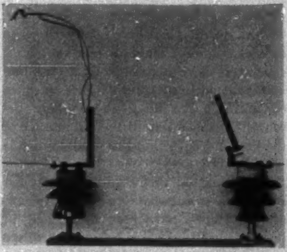

Figure 13b. Experimental double-break fuse, without glass tube—retracting spring arm giving air clearance after blowing

The total arc time of the double-break oil-blast fuse (that is, the duration of the primary arc to build up the pressure and the duration of the secondary arc to interrupt the circuit) is the shortest of any expulsion fuse device tested by the writer, for a given circuit condition.

The total arc time of the dry-type double-break fuse, using a granular filler, is comparable to the type of expulsion fuse devices utilizing large springs to aid in circuit interruption except on currents where the pressure generated by the primary arc is required to break the second fuse mechanically in order to establish the second arc.

Tests and field experience have proved that glass tubes may be used as fuse tubes in the dry-type double-break devices where the circuit interruption takes place entirely within the refill cartridge inserted within the glass tube.

Capacitors and Circuit Performance

J. W. BUTLER
MEMBER AIEE

CONSIDERABLE literature has appeared in the technical press during the past few years describing and analyzing the application of capacitors. Naturally, each author has treated the particular phase of the subject that best accomplished his objectives. It is felt, however, that a large majority of the potential users, and probably a good many of the actual users of capacitors, do not fully appreciate the change effected in the fundamental circuit characteristics when capacitors are applied. And the literature referred to does not seem to supply the demand for this type of information. This opinion has evolved through answering such queries as: "What should I use here, shunt or series capacitors?" "Should I put on shunt capacitors or regulators to fix up my bad voltage condition?" and "I guess a series capacitor is the thing to use here because it *balances* out all the inductance of the line and with that out, the voltage is bound to be good."

It is admitted that conditions may exist, in a somewhat restricted sense, wherein the alternatives mentioned in the two questions could be used, or where a series capacitor would perform as postulated. However, a knowledge of the conditions inspiring the inquiries showed that the effect on circuit performance of the suggested applications of capacitors was not thoroughly understood.

This paper analyzes the performance of circuits with different types of capacitor applications. By circuit performance is meant: voltage regulation, voltage level, voltage control, load division, power factor, and energy losses. These points are analyzed and discussed as they are influenced by both series and shunt capacitors. The field of application of capacitors, in distribution and transmission circuits is fairly well indicated in this paper, as far as circuit performance is concerned. The economic picture obviously will have its influence in a specific solution, and the information offered herein should make it easier for one to evaluate the results he expects to obtain in the way of circuit performance from the installation of capacitors.

Part A—The Series Capacitor

The series capacitor is simply a *negative* (capacitive) reactance, which is inserted directly in series with the line. Conventional definitions describe the effect of lagging current flowing through a *positive* (inductive) reactance as producing a drop in voltage. This effect is recognized as the usual garden variety of voltage drop experienced where current flows through distribution feeders, transformers, feeder reactors, etc. Conversely, when lagging current flows through a *negative* reactance, such as a series capacitor, a corresponding rise in voltage is obtained. It is this rise in voltage provided by the series capacitor which compensates more or less for the regulation and power-factor reductions effected by the inductive reactance of the line, transformers, etc. in the circuit.

1. Effect on Regulation

Since a series capacitor is a *negative* reactance, it is apparent that its judicious use can materially reduce the effective, or equivalent, impedance of a transmission or distribution circuit. With a reduction in the circuit impedance, it naturally follows that there will be less voltage variation or voltage regulation experienced when power is transmitted over the circuit. The effectiveness of a series capacitor in improving regulation may be clearly appreciated by merely calculating the regulation of a given circuit for various load power factors as the reactance is reduced to zero or even made negative. Such calculations, for demonstration purposes, have been made for two circuits and the results are shown plotted in figures 1 and 2.

In figure 1*a*, the dashed curve gives the regulation for a line having an impedance of $0.05+j0.20$ per unit, for various power factors from 0.6 to 1.0. This ratio of x to r may be found in circuits utilizing the larger wire sizes. Improvements in regulation obtained by reducing the reactance of the circuit to zero then to $-j0.05$ are shown in figures 1*b* and 1*c*, respectively.

In figure 2, similar characteristics are plotted for a circuit having an impedance of $0.15+j0.05$ per unit. This ratio of x to r may be found in circuits utilizing the smaller sizes of wire.

The improvements in regulation offered by the series capacitor for these two circuits are readily seen by a casual glance at these curves.

In circuits having negligible resistance, it is apparent that it is only necessary to put in a series capacitor having the same reactance as the line to obtain excellent regulation. However, when resistance plays a part in the regulation, the series capacitor must have a little higher value of reactance to compensate for the resistance drop. It is shown in appendix *A*, equation 7, that the value of equivalent reactance (reactance of circuit plus reactance of series capacitor) to give zero regulation is a definite function of the circuit resistance and load power factor. This relation is plotted in figure 3 which enables the ready determination of the equivalent reactance and hence the reactance of the series capacitor when the line resistance and load power factor are known. For example, suppose the resistance r' is 0.1 per unit, and the power factor is 0.7, from the curve it is read that x' must be -0.114 to have zero regulation. Then x_c will be 0.114 plus the reactance of the circuit, whatever its value might be.

It is interesting to note that there is a definite limit in power factor, for a given value of resistance at which zero regulation can be obtained. For instance, where $r'=0.1$, zero regulation cannot be obtained where the power factor exceeds 0.9. This limitation, however, has no practical significance at the present because such over-compensation of the line would make regulation very critical to changes in power factor.

Since a series capacitor is a *lumped* negative reactance, it produces all of the rise in voltage immediately at its location. Abrupt changes in voltage of this character, although in the opposite direction, are recognized as those occurring at a feeder reactor. It is thus seen that the voltage on the *load* side of the series capacitor is all that is affected. The voltage on the source side is unchanged; hence the so-called "balancing out the reactance" is effective on the load side of the capacitor only. This attribute may or may not be desirable, depending upon the local conditions, as explained later.

The voltage profile of a certain feeder with and without a series capacitor is shown in figure 4. It should be noted that the voltages between the source

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and the equipment is unaffected by the series capacitor, while the voltages beyond the installation are greatly improved. The series capacitor acts exactly like a branch or feeder regulator in this respect.

2. Effect on Power Factor

A series capacitor at rated load provides power-factor improvement to the same degree as the same amount of capactive kilovolt-amperes does, connected in shunt. It is interesting to note in this respect that a series capacitor accomplishes power factor improvement by providing an out-of-phase component of voltage as compared to the shunt capacitor accomplishing the same improvement with an out-of-phase component of current. These points are demonstrated by the vector diagrams of figure 5. In figure 5*b*, it is seen that the voltage across the series capacitor, E_c, acts in a way as to reduce the angle between the load current and the voltage on the source side of the capacitor. Hence the power factor on the source side is reduced since the cosine of this angle is the power factor. This point should be considered when predicting the performance of cascaded series capacitors that may be used in long distribution feeders. It should be noted here that this power-factor reduction does not release thermal capacity nor reduce line losses until the current has passed through a regulator where the proper transformations are made. However, voltage capacity of the line and regulating range of the regulator is released, which many times are the limiting factors in overhead lines.

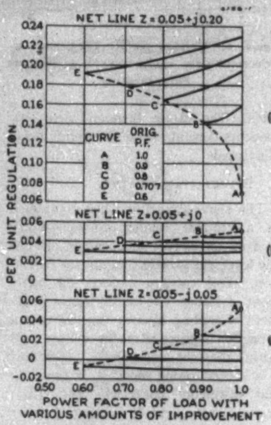

Figure 1. Voltage regulation

Dashed curves give regulations of different circuits for constant kilovolt-ampere load at different power factors. Solid curves give regulations for same conditions as various initial load power factors are improved by unswitched shunt capacitors. The net line impedance of *b* and *c* represents initial line with appropriate series capacitor

In the ordinary application for voltage improvement, sufficient capacitive kilovolt-amperes is not used to effect a sizable power-factor improvement. Nevertheless there are loads of such a nature as to allow power-factor improvement with series capacitors to be economically and automatically accomplished. This method in brief involves putting in a series capacitive reactance equal to the inductive reactance of the load and bus work. Then the current that flows is limited only by the load resistance—hence the circuit power factor is always unity. Thus reactive kilovolt-amperes, or power-factor improvement, is automatically provided strictly as a function of the load with no moving parts or switching, as would be required with shunt capacitors. So far these applications have been limited to welding and resistance-furnace loads. These applications are discussed later.

3. Effect on Voltage Level

Voltage level is generally referred to as describing the average voltage over a line from the substation to the end of the line. The series capacitor can raise this voltage level only insofar as it is effected by the voltage drop in the line itself. The series capacitor provides a voltage boost proportional to its reactance and the load current passing through it. This characteristic causes the voltage improvement or boost to appear independently of the magnitude of the circuit voltage; that is, the substation voltage of the circuit does not influence the voltage rise provided by the series capacitor. This means that the series capacitor cannot correct for substation voltage variations that occur independently of the load on the substation. In the practical case, however, this is not usually interpreted as a serious limitation, but it should be recognized when the series capacitor is being installed for regulating voltage for the daily load cycle, instead of regulators.

4. Applications of the Series Capacitor

(*a*). *For Improving Voltage Regulation.* Fluctuating, intermittent, and suddenly applied loads corresponding to motor starting, log sawing, resistance welding, etc., cause voltage fluctuations that can be and are very objectionable. It has been to circuits of this type that the series capacitor has been mostly applied up to the present time. These rapid voltage changes are automatically and instantaneously corrected by the series capacitor. The series capacitor in providing these instantaneous, automatic voltage improvements as a function of the load and power factor holds a unique position in its field. The voltage improvement offered for this type of application is demonstrated in the curves of figure 4. From these curves it is seen that a very good voltage condition exists throughout the line beyond the series capacitor during normal load conditions as well as when the motor is started which was causing bad voltage fluctuations.

Since a series capacitor provides an *abrupt* voltage rise and not a gradual rise in the circuit, and further since only certain regions of the circuit can obtain substantially zero regulation, it follows that under certain circuit conditions the solution offered by the series capacitor may not be acceptable. For example, the voltage of a long feeder supplying a reasonably uniform distributed load along its length and a large fluctuating load, such as a saw or rolling mill at the end, would be difficult to fix up with a single

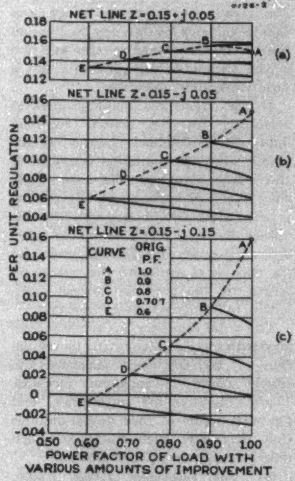

Figure 2. Same as figure 1 except line impedances are changed

series capacitor. The conditions existing in this type of application are shown in the curves of figure 6. Here it is seen that the consumers on the source side of the capacitor get no relief while those on the load side adjacent to the capacitor get a sudden rise in voltage which may still cause light flicker. The regulation then decreases gradually toward the end of the line until a point is reached where the line impedance plus the series capacitor reactance would be of such a value as to give substantial zero regulation. The better application is obviously the one wherein the load is taken from the circuit where the minimum regulation can be obtained or, saying it another way, a circuit wherein the acceptable low values of regulation can be obtained in the region on the circuit where most of the load is served. Since a series capacitor acts exactly as a regulator in this respect, a good preliminary test as to the feasibility of a proposed application is the question "Would the problem be satisfactorily solved by an induction or step regulator that could automatically and instantaneously assume its new position as dictated by the new load condition?" If the answer to this question is yes, then it is very probable that a series capacitor is applicable.

If the regulation problem involves correcting for the normal everyday light-load to full-load changes on the circuit, then the limitation described in the above paragraph may not apply since the load increases or changes are gradual which do not cause objectionable voltage flickers. There have been a few series capacitor installations made for the purpose of regulating the voltage required by the everyday normal light-load to full-load changes on the circuit.[1]

The series capacitor has recently been used with success in improving the regulation of high-frequency generators to supply power to induction furnaces. These generators are single-phase, and the very nature of their design dictates that they have high reactance which reflects itself in very poor regulation.

(*b*). *For Improving Power Factor.* The use of series capacitors for this service has been limited so far to welding and resistance-furnace installations. Applications of this nature have been reported by Marbury-Levy[2] and Levoy.[3] In each of these applications the series capacitor is tuned to the reactance of the bus work, furnace, etc., or the welding transformer, and its secondary circuit, thus forming a substantially unity-power-factor circuit. The supply voltage is, of course, reduced from the value used before the installation of the capacitor since the current is substantially resistance limited. It is felt that these types of applications have a great future, since the use of electricity is playing a larger and larger part in welding and metal reduction processes.

(*c*). *For Improving Load Division in Parallel Circuits.* The operation of parallel circuits having widely different impedances may become unsatisfactory from the standpoint of load division. The series capacitor can be used to alter the impedance of one or more of the circuits in order to secure the desired division of current. In fact, it was to this use that the first series capacitor was put in transmission circuits.[4] It is of interest to note further that it is possible to apply series capacitors in two parallel circuits to obtain a prescribed division of load and at the same time obtain substantially zero regulation at the receiving end. An application of this type recently arose which involved a voltage flicker and load division problem resulting from an arc-furnace load being supplied through an underground cable and an overhead line.

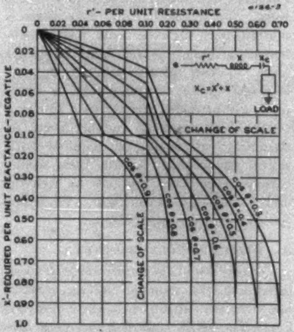

Figure 3. Curves for determining necessary reactance to give zero regulation as a function of line resistance and load power factor

Part B—The Shunt Capacitor

The so-called shunt capacitor is simply a capacitive reactance connected on an electric circuit in shunt with the loads on the circuit. This piece of equipment can be considered merely as another load on the circuit with its own peculiarities or as a change that has been made in the impedance of the line supplying the other loads. The choice between these two concepts depends upon the points being illustrated.

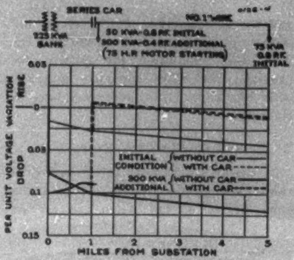

Figure 4. Voltage profile—illustrating improvement from application of a five-ohm series capacitor

1. EFFECT ON REGULATION

The shunt capacitor when considered as a *constant* leading kilovolt-ampere load on the circuit, independent of voltage level, has no effect on voltage regulation. This concept neglects the increment of voltage rise caused by the increment of leading kilovolt-amperes that a capacitor requires when the voltage rises during the light-load periods. Since this is a second order effect, it is generally and justifiably neglected. The effect, however, may actually be that of either increasing or decreasing the regulation, depending upon the amount of resistance in line. Since this effect is fully treated in part *C* of the paper, wherein the effect of using shunt and series capacitors together is discussed and analyzed, it will not be repeated here.

2. EFFECT ON POWER FACTOR

The shunt capacitor is fundamentally a power-factor improver. Any benefits that can be derived in a system by improving the power factor can, therefore, be realized by properly utilizing the shunt capacitor. The several particular responses of a system through the improvement of power factor will be discussed below in the application section, and since the fundamentals of power-factor improvement are so well known, they will be only briefly outlined here.

The vector diagrams of figures 5*a* and 5*c* indicate the manner in which shunt capacitor current or kilovolt-amperes combines with the load current or kilovolt-amperes to effect an improvement in power factor. The equations defining these diagrams are given in appendix *C*. In order that the reader may readily obtain a concept as to the power-factor-improving ability of capacitive kilovolt-

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1. For all numbered references see list at end of paper.

amperes for different load power factors, the curves of figure 7 have been plotted from equation 17. These curves show the relation between the capacitive kilovolt-amperes added at any original power factor to the improved power factor. For example, the heavy line (figure 7) shows the improvement in power factor of a 0.6 power factor load as capacitive kilovolt-amperes is added. In order to improve to 0.9 power factor, 0.5 per unit of the original load kilovolt-amperes in capacitive kilovolt-amperes, must be added. These curves are the usual data plotted in slightly different form.

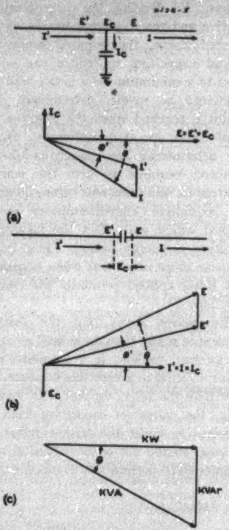

Figure 5. Vector diagrams

Illustrating relation of:

(a)—Voltage, current, and power factor with shunt capacitors

(b)—Voltage, current, and power factor with series capacitors

(c)—Kilowatts, kilovars, and kilovolt-amperes for the general case

3. Effect on Voltage Level

When studying the effects of the shunt capacitor in raising the circuit voltage, the concept of considering the capacitor as a fixed load of leading power factor proves helpful. Neglecting resistance in the circuit, the voltage rise caused by I amperes of leading current flowing through X ohms of inductive reactance is IX volts. If the line reactance is expressed in per unit, with the capacitive kilovolt-amperes as a base, this rise in voltage is simply equal to the per unit reactance. For instance, if a 180-kva capacitor bank is connected on a four-kv three-phase circuit so that its current flows through two ohms of reactance, what will be the voltage rise? Changing the two ohms to per unit, on 180-kva base by equation 14, there is

$$X = \frac{2(180)}{(4)^2 1{,}000} = 0.0225$$

which says that a rise of 0.0225 per unit of normal voltage will result when base current (that corresponding to 180 kva) flows.

Since the line reactance is independent of any other loads on the circuit, and since the capacitor current is also independent (this neglects small variations in current due to the voltage changes caused by other loads on the circuit) of the other loads, it follows for practical purposes that the voltage rise effected by capacitors is a constant and fixed value. This point should be kept in mind when applying capacitors for purposes of voltage improvement in order to alleviate objectionable voltages during light load.

In order to demonstrate clearly this type of voltage improvement, the curves in figure 8 have been prepared. A feeder is taken that supplies five equal loads distributed along its length and the secondary voltage conditions obtaining from three methods of operation are illustrated.

Original Condition. The regulator is connected to give ten per cent boost and no buck. At full load the voltage is ten per cent high at the substation and ten per cent low at the end. At no load the voltage is normal over the complete circuit. The regulation, as defined in this paper, varies from ten per cent at either end to zero one-third the distance out from the substation.

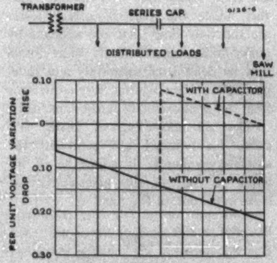

Figure 6. Voltage profile

Illustrating voltage conditions caused by fluctuating load at the end of a line. The voltage served to distributed loads may or may not be satisfactory, while voltage at the load is good

With Shunt Capacitors. By adding sufficient shunt capacitors at the end to obtain a ten per cent rise, the regulator can be connected to operate over a range of five per cent buck or boost. Then at full load the regulator is in full boost at five per cent, and the voltage profile is as illustrated in the curves, with the voltage up at the end five per cent above the original condition. At no load the voltage is five per cent below normal at the source and it is five per cent above normal at the far end. The regulation again varies from ten per cent at either end to zero one-third the way out on the feeder.

With Line Boosters or Taps on the Distribution Transformers. The permanent voltage boosts provided by the shunt capacitors at each load location may be simulated by line boosters (autotransformers) appropriately located or by taps on the distribution transformers. In the case at hand, each transformer is set on a tap to give a $2^1/_2$ per cent boost to the secondary voltage and again the regulator can be reconnected to operate over a ±5 per cent range.

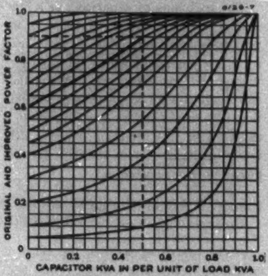

Figure 7. Power-factor improvement—curves for determining necessary per-unit capacitive kilovolt-amperes to effect a prescribed improvement

Example—Original power factor = 0.6, desired power factor = 0.9, required capacitive kilovolt-amperes = 0.5 per unit of original load kilovolt-amperes

Comparing these results with those of the shunt capacitor, it is seen that

(a). The average voltages over the line under all conditions of load are the same.

(b). The regulations at any point in the line are the same.

(c). The regulators operate over identical ranges.

There is no doubt that a better choice of taps or location of capacitors could be made to effect a better over-all practical voltage condition than that shown, but this case is merely given to illustrate the principle under discussion. Capacitors are valuable and useful in performing this type of voltage improvement because taps are seldom available on transformers in areas requiring improvement. Even if taps were available, a practical complication may result through the necessity of properly setting taps on transformers where transfers, changes, or additions are being made. Then, of course, it should not be forgotten that capacitors offer many other well-known benefits through power-factor improvement that transformer taps and boosters cannot provide. This example is cited in an attempt to portray clearly these voltage improvements in terms that are so familiar to distribution engineers. If this picture were kept in mind when applying capacitors, overvoltages at light load would not be a problem.

The switching of capacitors as a function of voltage, current, or time, for improving voltage regulation is being used by a few engineers, where the light-load voltages become excessive. This method of voltage control seems to offer possibilities, and in the future great progress is expected in this direction, since it definitely increases system stability and improves voltage regulation.

EFFECT OF RESISTANCE ON VOLTAGE RISE

In the preceding section, the effect of resistance in limiting voltage rise is neglected. This point is briefly investigated in the interests of clarity and completeness. The following expression for the sending-end voltage, including resistance, is derived in appendix *A*:

$$e_s = [e_r^2 + (iz')^2 + 2e_r i(r' \cos \Theta + x' \sin \Theta)]^{1/2}$$

Defining the voltage rise V_r as the difference between e_s and e_r with e_r normal, and changing the constants to per unit on the capacitor kilovolt-ampere base, the following expression is obtained for zero leading power factor

$$V_r = (1.0 + x'^2 + r'^2 - 2x')^{1/2} - 1.0$$

In order to evaluate the effect of resistance in this expression, a three-phase four-kilovolt circuit one mile long with 100 capacitive-kilovolt-ampere connected at its end is taken. The voltage rise for various wire sizes, all having an equivalent spacing of 36 inches is shown plotted in figure 9. It is found that the resistance has no perceptible effect up to wire size number 1, and that the effect is negligible up to and including number 4 wire. The curve was carried on out to number 10 wire in order to complete the picture and it is seen that resistance begins to show its effect around number 7 or number 8 wire.

Since the per-unit impedance constants in the above equation for V_r vary directly with the length of the line and the capacitive kilovolt-amperes, the results in figure 9 may be considered as applying to 100 capacitive kilovolt-ampere miles. This may be 100 capacitive kilovolt-amperes on one mile, 50 capacitive kilovolt-amperes on two miles, etc. With this concept, the rise attributable to any given length of line and capacitive kilovolt-ampere in question may be obtained by merely dividing the capacitive kilovolt-ampere miles by 100 and multiplying it by the per unit rise given by the curve in figure 9, for the size of wire in question.

4. EFFECT ON RESISTANCE LOSSES

The resistance loss in a circuit is equal to the product of the resistance and the current squared. The current, in turn, is equal to the r.s.s. (root of the sum of the squares) value of the reactive and active components of current. Since the loss is a function of the current squared, it follows that the loss due to the two components of current may be reckoned separately—thus

$$\begin{aligned}\text{Total loss} &= R[(I \cos \Theta)^2 + (I \sin \Theta)^2] \\ &= \text{active-current losses plus reactive-current losses} \\ &= W_a + W_r\end{aligned}$$

This concept in figuring losses is useful since the active-current losses are not affected by power factor and these losses represent the absolute minimum to which the total losses may be reduced. Further, since the reactive-current losses are the only ones that are affected by power-factor improvement, these losses may be dealt with separately and independently, when figuring loss reductions from the application of shunt capacitors.

Since the active-current losses, W_a, cannot be reduced, and represent the absolute minimum, it might be of interest to express the reactive-current losses in per unit of W_a, thus

$$\frac{W_r}{W_a} = \frac{R(I \sin \Theta)^2}{R(I \cos \Theta)^2} = \tan^2 \Theta$$

This ratio tells directly what fraction of the absolute minimum circuit loss is being produced by the reactive current. This relation is shown plotted in figure 10, from which a visual picture may be readily obtained of the loss-producing ability of reactive current as a function of power factor.

In a practical case, the total energy loss, that is, the kilowatt-hour loss over a given period, is the figure that is usually of major importance. Therefore, the load cycle, the daily load pattern, must be considered in order to get an accurate picture. Since the losses vary as the square of the load, the rms value of the load current over a given period should be determined for accurate loss calculations. By dividing the current into its active and reactive components, as suggested above, the required amount of shunt capacitors to reduce the energy loss to a minimum for a given load pattern can be readily determined. This point is reached when the reactive current losses are at a minimum. Hence, by determining the capacitive kilovolt-amperes required to give the reactive current its minimum rms value, the value of capacitive kilovolt-amperes is obtained which will effect the largest reduction in energy losses. These points become more important when capacitors are being applied primarily for loss reductions.

Experience shows that the reactive current in many systems as well as in the component feeders, is reasonably constant, at least to a very much greater degree than the active current. In view of this, the value of capacitive kilovolt-amperes to effect the largest reduction in losses should not be as critical as may be inferred from the above discussion.

5. APPLICATIONS

The applications of shunt capacitors are so numerous and varied and they are proved in by so many different points of view, that this paper will merely mention, for completeness sake, the major fields of application.

(*a*). *For Releasing Capacity.* The releasing of capacity in cables, substations, generators, etc., where current loading is the limiting factor, seems to have been the basis for the installation of a major part of shunt capacitors applied to central station systems in this country.[5] It is the general rule that a steam turbine has sufficient kilowatt capacity to allow the generator to deliver its full kilovolt-ampere output at unity power factor. This is, however, many times impossible to accomplish in an emergency because the generator may be loaded up with reactive. The shunt capacitor has proved invaluable on several occasions by supplying this reactive and leaving the generator free to grind out kilowatts.

Such applications for releasing system capacity in general and postponing large investments in substation equipment are somewhat obvious and with the advances in manufacturing and design technique that have been and will be accomplished, the capacitor promises to become a major piece of system equipment.

(*b*). *For Reducing Power Costs.* Many power companies today have rate structures that effectively pay a bonus to those consumers improving their load power-factor to within certain prescribed limits. This policy began at about the turn of the century, at which time the effects on system performance of reactive kilovolt-amperes were being realized.

The following extracts are taken from the *Proceedings* of the 30th convention of NELA held in Washington, D. C., June 1907, pages 81–3:

> "With the rapid increase in the industrial use of induction motors and in arc lighting by alternating current, the troubles incident to low power factor, in systems not designed for it, have become unfortunately familiar."
>
> "Low power factor has two serious results: it limits the capacity of the electrical part of the system by loading it up with unproductive current—current for which no revenue is obtained; and it means poor voltage regulation."
>
> "The rating of electrical apparatus depends primarily on the heating produced by the load, and this heating depends entirely on the current. The kilowatt load, therefore, unless the corresponding power-factor is taken into account, is not a measure of the actual load on the apparatus."
>
> "The second injurious effect of low power factor is poor voltage regulation. This effect is well known and requires no extended explanation."

These injurious effects of reactive kilovolt-amperes being realized by the public utilities and the resultant power-factor clauses in rate structures having been put into effect, have made it possible for capacitors to effect untold savings to industrial consumers of electricity. A few reports on installations of this nature are listed at the end of the paper.[6]

(*c*). *For Reducing Energy Losses.* The reduction of energy losses has in many instances been the deciding factor in justifying shunt capacitors. Independent companies purchasing energy can place a definite evaluation on line losses, which makes it merely a matter of arithmetic to figure the savings. Operating companies, however, find this procedure a little more elusive, since line-loss evaluations vary in different parts of the system and even then personal opinions play such an important role.

Power-factor improvement, however, is accompanied by a definite reduction in transmission and distribution losses, and each individual case must be treated on its own merits.

(*d*). *For Improving Voltage.* Many users of shunt capacitors have reported voltage improvements of sufficient magnitude to justify capacitors on that basis alone. This, of course, depends upon the specific circumstances surrounding the case and the evaluation of the improvement, but the fact remains, capacitors can and do effect voltage improvements that are accompanied by many other well-known related benefits, which generally make their installation very attractive.

The use of capacitors switched in banks for regulating high-line voltages instead of synchronous condensers is another field which has only been touched in this country. On the continent this application has attained major proportions and it has promise of developing in this country in the near future.

Part C—The Series and Shunt Capacitors Together

The shunt capacitor is so popularly used today that more likely than not the application of a series capacitor would be useful to a circuit that already has some shunt capacitors on it. The response of this type of circuit to load changes would, therefore, be useful.

1. Method of Analysis

The effects of a series capacitor on circuit performance are usually calculated by considering it merely as a change in the impedance of the line through which the load is supplied. This same concept may be used to an advantage to accurately predict the voltage regulation of a circuit that utilizes shunt capacitors or a combination of series and shunt capacitors. Referring to figure 11, the single-line diagram of a feeder with a series capacitor, a shunt capacitor, and a load that may be connected, is seen. As far as the load is concerned, its current is supplied through an equivalent impedance, the characteristics of which are determined by the series and shunt capacitors, and the actual line impedance. In circuit analysis this equivalent impedance is called the *driving point* impedance[7] as viewed from the load. In the case of a single shunt capacitor bank, located on the receiving end of a line, the equivalent line impedance is merely the parallel value of capacitive reactance and the line impedance. This concept enables the regulation of a circuit consisting of any network of capacitors or line impedances, to be readily figured for a given load, by merely determining the driving point impedance as viewed from the load then using this *single* impedance in any standard regulation formula. The general equations defining the performance of the circuit in figure 11 employing this concept are derived in the appendices.

2. Effect on Regulation

Two different types of circuits were analyzed quantitatively with and without series and/or shunt capacitors in order that the points under discussion could be clearly illustrated.

Without Series Capacitors. The regulation of a circuit having an impedance of $0.05+j0.20$ per unit is shown in figure 1*a*. These values could represent the impedance of a circuit utilizing one of the larger sizes of wire, such as 4/0, and the substation transformers. The regulation of this circuit when delivering loads of various power factors and with different amounts of power-factor improvement is shown. For instance, full load (base kilovolt-ampere) at 0.60 power-factor uncorrected, curve *E*, figure 1, would give 0.19 per unit regulation, and if sufficient capacitive kilovolt-amperes were added in shunt to improve the power factor to 1.0, the regulation would be 0.23 per unit. The increase in regulation being due to the shunt capacitors increasing the *equivalent* impedance over the actual impedance of the line. In probably more familiar terms, the increase in regulation is due to the increment in leading current required by the capacitors when the voltage rises as the load is removed. The effect is somewhat magnified from the average case because the per-unit line reactance is high, meaning the circuit is heavily loaded, and the power factor is low.

Considering the other circuit analyzed attention is directed to figure 2*a*. The regulations of a circuit having an impedance of $0.15+j0.05$ per unit, are shown for the same loads as the case above. The regulations for full load and 0.6 power factor is 0.133 per unit while the regulation is reduced to 0.125 per unit when sufficient capacitive kilovolt-amperes is added to bring the power factor up to 1.0. In this instance the regulation was improved by the addition of shunt capacitors.

With Series Capacitors. The effect of adding a 0.20 per unit series capacitor in improving the circuit regulation is illustrated in figure 1*b*. The regulation is practically constant for any initial load power factor, regardless of the degree of improvement. The effect of the leading current of the shunt capacitor in increasing the regulation as in figure 1*a* has been eliminated, because the rise through the line reactance is entirely compensated by the drop through the series capacitor. By adding still more series capacitance, a circuit having the characteristics plotted in figure 1*c* is obtained. The characteristics of the load as well as the line dictate what value of series capacitance is used.

Again, referring to figure 2, the improvements offered by the series capacitor in the high-resistance circuit are illustrated. Very good regulations are obtained for the lower-power-factor loads, for all degrees of improvement. In fact, the addition of shunt capacitors improves the regulation by approximately 0.02 per unit for all loads of 0.8 power factor and below, when the net line reactance is made slightly negative by the series capacitor.

It should be noted that the series capacitor in *all* cases improves regulation, and the presence of shunt capacitors for improving power factor further improves the regulation when the line reactance is made slightly negative by the series capacitor. In these curves the improved power factor given by the abscissas considers only the improvement given by the shunt capacitors and not the small additional improvement given by the series capacitor.

The per-unit resistance of the line in figure 1 would be increased by three, or to 0.15, if it carried three times its present load. Thus, the circuit in figure 1 could be made to have regulations of those in figure 2 at three times the present load with the appropriate series capacitor. And this could be quite practical if the power factor was 0.8 or lower. This example merely indicates the immense possibilities the series capacitor promises in conjunction with the shunt capacitor in releasing line capacity where voltage drop is the limiting factor.

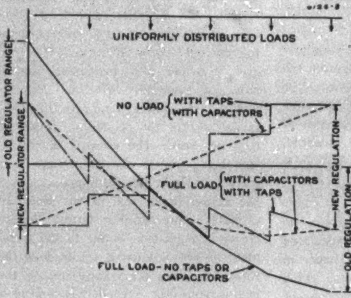

Figure 8. Voltage profile—illustrating similarity of shunt capacitors and transformer taps in facilitating voltage control

Appendix A. Voltage Regulation

The basic voltage equation for the circuit represented in figure 11*a* may be written in vector form as,

$$E_s = IZ' + E_r \qquad (1)$$

and in complex notation

$$E_s = i(\cos\Theta - j\sin\Theta)(r' + jx') + e_r \qquad (2)$$

where

E_s = sending end voltage
e_r = receiving end voltage (reference)
r' = equivalent line resistance (see appendix *B*)
x' = equivalent line reactance
Θ = power factor angle of load current referred to receiver (load) voltage
i = magnitude of load current

Solving for the magnitude of E_s there is

$$e_s = [(e_r + ir'\cos\Theta + ix'\sin\Theta)^2 + (ix'\cos\Theta - ir'\sin\Theta)^2]^{1/2}$$
$$= [e_r^2 + (iz)^2 + 2e_r i(r'\cos\Theta + x'\sin\Theta)]^{1/2} \qquad (3)$$

Voltage regulation is generally defined as the per unit rise in voltage when going from full load to no load, thus

$$R = \frac{e_s - e_r}{e_r} \qquad (4)$$

R = per unit regulation

And since normal voltage is usually that supplied at normal or full load, a simplified equation for regulation may be obtained by defining all constants in per unit on the load base. Then $e_r = 1.0 = i$ and by substituting equation 4 in 3 there is

$$R = [1 + (Z')^2 + 2(r'\cos\Theta + x'\sin\Theta)]^{1/2} - 1.0 \qquad (5)$$

This equation is plotted in figures 1 and 2 to show the effects on regulation of varying the constants of the circuit shown in figure 11*d*. Full explanation of these curves is given in the body of the paper.

Equating equation 5 to zero and solving for x', the value of equivalent line reactance (appendix *B*) is found that will give zero regulation for a given power factor.

$$R = [1 + (z')^2 + 2(r'\cos\Theta + x'\sin\Theta)]^{1/2} - 1.0 = 0 \qquad (6)$$

Solving for x' there is

$$x' = -\sin\Theta + \sqrt{1 - (\cos\Theta + r)^2} \qquad (7)$$

In the ordinary series capacitor application (figure 1*b*)

$$x' = x - x_c \qquad (8)$$

Hence for this case

$$x_c = x + \sin\Theta - \sqrt{1 - (\cos\Theta + r)^2} \qquad (9)$$

From equation 9 the per unit series capacitance can be found that will give zero regulation for any given power factor and circuit resistance within certain limits.

For instance, equating equation 7 to zero, it is found that zero regulation cannot be obtained when

$$r' > 1 - \cos\Theta \qquad (10)$$

Thus, equation 10 gives the theoretical limit of obtaining zero regulation in a circuit by using a series capacitor. This relation clearly shows that as the power factor approaches unity, the resistance must approach zero in order for zero regulation to obtain.

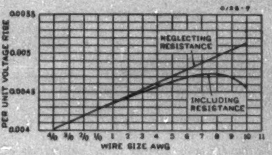

Figure 9 (right). Curves showing effect of line resistance on voltage rise

Conductor spacing—36-inch equivalent, circuit voltage—4 kv, capacitive kilovolt-amperes —100, length of line—one mile (100 capacitive-kilovolt-ampere miles)

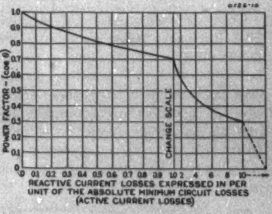

Figure 10 (right). Reactive-current copper losses

Appendix B. Equivalent Line Impedance

In appendix *A*, the term equivalent line impedance Z' is used. This is merely the driving point impedance of the circuit as viewed from the load terminals. The driv-

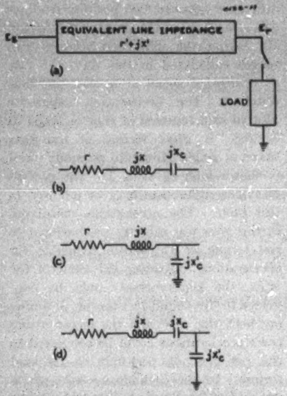

Figure 11. Schematic diagram of a load connected to its voltage source through an equivalent line impedance, which may be composed of:

(b)—Line plus series capacitor
(c)—Line plus shunt capacitor
(d)—Line plus series and shunt capacitors

ing point impedance is defined as the impedance to the flow of current when voltage is applied at the point in question with all other terminals of the network grounded.[6]

The circuit shown in figure 11*b* has an equivalent impedance of

$$Z'=r+j(x-x_c) \tag{11}$$

Thus

$$r'=r$$
$$x'=x-x_c$$

In figure 11*c* the equivalent impedance is the parallel value of the shunt capacitive reactance x_c' and the actual line impedance, thus

$$Z'=\frac{-jx_c(r+jx)}{r+j(x-x_c')}$$
$$=\frac{x_c^2r}{r^2+(x-x_c')^2}+j\frac{x_c[-x(x-x_c')-r^2]}{r^2+(x-x_c')^2} \tag{12}$$
$$=r'+jx' \tag{13}$$

where r' and x' correspond to the real and imaginary terms respectively of equation 12.

Similar expression may be deduced for any network comprising any number of voltage sources, side branches, or capacitor connections. In general network theory, the real and imaginary terms of the driving point impedance are identified as the resistance and reactance components, respectively, of the *equivalent* impedance spoken of in the body of the paper.

Appendix C. Capacitors and Power Factor

A capacitor operating at rated load represents leading kilovolt-amperes in the circuit equal to its rating. This leading kilovolt-amperes subtracted from the lagging kilovolt-amperes in the circuit results in a net value which, when combined with the kilowatts in the circuit, determines the power factor. This is true for *either* series or shunt capacitors and the relation may be shown by the vector diagram of figure 5*c*, from which the expression for the improved power factor may be derived as

$$\cos\Theta'=\frac{\cos\Theta}{[(\cos\Theta)^2+(\sin\Theta-c)^2]^{1/2}}$$
$$=\frac{\cos\Theta}{(1+c^2-2c\sin\Theta)^{1/2}} \tag{14}$$

where

$\cos\Theta$ = original power factor
$\cos\Theta'$ = improved power factor
c = per unit capacitive kilovolt-amperes in the circuit

An expression for $\cos\Theta'$, which is simpler for computing purposes is

$$\cos\Theta'=\cos\left(\tan^{-1}\frac{\sin\Theta-c}{\cos\Theta}\right) \tag{15}$$

since the answer may be read directly from a table of trigonometric functions. This relation is shown plotted in figure 7 with $\cos\Theta$ as the parameter. These curves may be used for determining the power-factor improvement effected by either shunt or series capacitors.

Appendix D. A Note on Per-Unit Constants

The principle of expressing circuit constants in per unit has been described many times in detail, hence only the equations of conversion will be given here.

Let Ω = the impedance constant in ohms. Then in per unit

$$Z=\frac{\Omega\text{ kva}}{(\text{kv})^2 1{,}000} \tag{16}$$

where

kva = base kva (arbitrary)
kv = kilovolt rating of circuit link in question

Shunt Capacitors. To express the reactance of a certain amount of shunt capacitor kilovolt-ampere in per unit, it is merely necessary to divide the base kilovolt-amperes by the capacitive kilovolt-amperes, since the capacitive reactance is equal to unity on its own base, and per-unit reactance varies directly with the base.

Thus

$$x_c'=\frac{\text{base kva}}{\text{capacitive kva}} \tag{17}$$

Series Capacitors. The per-unit reactance of a series capacitor may be obtained from equation 16 if the ohmic value is known. If the voltage across the capacitor at rated (base kva) load is e_c, then the per-unit reactance is

$$\left.\begin{array}{l} x_c=\dfrac{e_c}{e_r} \\ e_r=L-N\text{ receiver voltage} \end{array}\right\} \tag{18}$$

Or, since base current flows through the capacitor, multiplying numerator and denominator of equation 18 by the current there is obtained

$$x_c=\frac{\text{capactive kva}}{\text{base kva}} \tag{19}$$

References

1. SERIES CAPACITORS REGULATE RURAL VOLTAGES ECONOMICALLY, **C. C. Moler** *Electrical World*, September 23, 1939.

2. SERIES CAPACITOR REGULATES FURNACE POWER FACTOR, **R. E. Marbury** and **C. C. Levy.** *Electrical World*, April 23, 1938, page 62.

3. POWER-FACTOR CORRECTION OF RESISTANCE-WELDING MACHINES BY SERIES CAPACITORS, **L. G. Levoy, Jr.** AIEE TRANSACTIONS, volume 59, 1940, pages 1002–09.

4. SERIES CAPACITOR INSTALLATION AT BALLSTON, N. Y., **E. K. Shelton.** *G. E. Review*, volume 31, August 1928, pages 432–4.

5. SHUNT CAPACITORS ON DISTRIBUTION FEEDERS, **G. P. Gamble** and **F. M. Starr.** *G. E. Review*, October 1936, pages 466–74.

CAPACITORS PROVE OUT IN DECADE OF SERVICE, **P. H. Van Horn** and **W. S. Hill.** *Electrical World*, March 28, 1936.

SHUNT CAPACITORS REDUCE KVA LOADS, **C. M. Lytle** and **S. H. Pollock.** *Electric Light and Power*, November 1937.

SEVENTEEN SYSTEMS REPORT SMOOTH CAPACITOR PERFORMANCE, **M. C. Miller.** *Electrical World*, January 27, 1940.

6. THE APPLICATION OF CAPACITORS TO INDUSTRIAL PLANTS, **C. E. H. Von Sothen.** AIEE TRANSACTIONS, volume 57, 1938, pages 505–09.

POWER FACTOR CORRECTIONS IN INDUSTRIAL PLANTS, **C. T. Pearce.** *Electric Journal*, February 1938.

CAPACITORS EASE PEAK ON MUNICIPAL PLANTS, **G. L. Hoddy** and **M. J. Mylott.** *Electrical World*, August 12, 1939.

7. PROGRESS IN STUDY OF SYSTEM STABILITY—APPENDIX III, **I. H. Summers** and **J. B. McClure.** AIEE TRANSACTIONS, volume 49, 1930, page 132.

Some Acceleration Tests on Large Synchronous Pump Motors

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ACCELERATION tests on motors may yield very interesting and valuable information. These tests show the behavior of the motor in that region between zero speed and full speed through which each motor must pass every time it is started. The usual tests do not cover this range. In fact, it is difficult or impossible to obtain the actual motor characteristics in this range on many motors by any other means. The accelerating characteristics of a motor determine the margin by which that motor is able to start and bring its load up to speed in a reasonable length of time under the least favorable conditions which might be encountered. Acceleration tests may also be used in checking theories of operation, correctness of design constants, the presence of any peculiarities in the performance curves, and, in the case of induction and synchronous machines, any tendency to lock in at a subsynchronous speed. Where the motor has been installed and connected to the load, the characteristics of the load will affect the curves obtained. Tests made under these conditions may be of special value, since they represent operating practice.

The main tests reported in this paper were made on the large synchronous pump motors used on the Metropolitan Water District[1,2] aqueduct which is to supply Colorado River water to southern California. These motors include the largest machines of this type ever built and represent the latest designs produced by three leading American manufacturers. The tests were performed in the field, after the motors had been installed, with the pumps connected, and with normal operating conditions existing. Preliminary tests were performed on small laboratory motors in order to check the test methods and equipment. Some of the results obtained in these preliminary tests are included in this paper.

Methods

The methods of test and analysis used in these experiments were similar to those described in a previous paper by the author.[3] One important change in equipment was made. In the tests reported in the present paper, stroboscopic observations were used. This change permitted the tests to be performed even at 100 per cent of rated voltage instead of the very low voltages used in the previous work.

Briefly the test procedure and equipment may be described as follows. An accurately calibrated scale was attached to the motor shaft so that the angular position of the rotor could be determined to a small fraction of a degree. This scale was illuminated by light pulses from the mercury-vapor lamp of a stroboscope, flashing 60 times per second, and a photographic record of the position of the scale was obtained. The camera was made for use with a cathode-ray oscillograph, and had a high-speed lens but no shutter. Thirty-five-millimeter motion-picture film was moved past this lens at a uniform speed by a small motor. The light pulses were of such short duration that sharp pictures were obtained even though both the film and the scale photographed were moving. In this way 60 records of rotor position were obtained each second of the test. Small calibrated disks driven by synchronous motors were photographed simultaneously for the purpose of checking the test equipment.

The records thus obtained were analyzed by numerical differentiation. The original readings gave rotor position as a function of time. Since the intervals of time between flashes were uniform, the difference between two position readings was a measure of the speed of the motor. By taking the difference between two speeds calculated in this way, the rate of change of speed or acceleration was obtained. The acceleration could be plotted as a function of time or speed as desired. A short section of the data taken at Eagle Mountain pumping plant is shown later in table I. The process of taking differences may be followed in this table. The acceleration calculated in this way was directly proportional to the torque of the motor available for acceleration. Knowing the value of the WR^2, the accelerations could be converted to the pound-feet torque, or knowing both the WR^2 and the motor rating these accelerations could be converted to the per cent or per unit of full-load torque. In the preliminary tests where the motors were tested without load, the acceleration was proportional to the motor torque minus only the small friction and windage loss torques. In the main tests where the motors were tested with the loads connected, the acceleration was proportional to the motor torque minus the friction, windage, and load torques.

An analysis of this type is possible only if certain requirements have been met. The stroboscope must produce light pulses at exactly equal time intervals or corrections for inequalities must be applied. A timing disk having 360 divisions and driven by a four-pole synchronous motor will permit timing errors of $^{1}/_{1,000}$ part of a cycle to be detected providing that disk is read to 0.1 division. In this work timing errors were corrected only where those errors were greater than twice this value. Several runs were made in which no corrections of this type were required. The determination of the timing error may be complicated by the hunting of the timing-disk motor. Hunting will occur on the starting of the main motor if the timing disk and main motors are connected to the same supply lines. Where these motors were supplied from separate transformer banks in the preliminary tests, no hunting was found.

The calibration marks on the scale of the main motor must accurately show the position of the rotor. Inaccuracies may be found to be due to the improper centering of this scale on the motor shaft or due to flexibility between the rotor and the scale. Both of these conditions were found to exist in certain of the preliminary tests. In one set of tests the center of the calibrations on the 24-inch-diameter disk used and the center of ro-

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The writer wishes to thank James M. Gaylord, chief electrical engineer of the Metropolitan Water District, for permitting these tests to be made. Also thanks are extended to those other men of the district, especially the operating staffs of the plants, for their help in the performance of the tests. The writer wishes to call particular attention to the part played by C. N. Williamson, a senior student in electrical engineering at the University of California. Mr. Williamson worked with the writer both during the preliminary tests when the method was being developed, and also when these methods were applied to the large pump motors. He has spent many hours helping in the performance, calculation, and analysis of these tests.

[1] **For numbered references, see list at end of paper.**

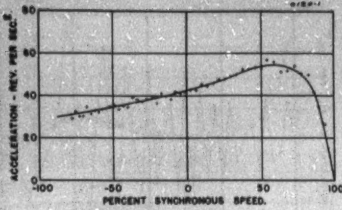


Figure 1. Acceleration-speed curve of a five-horsepower 1,200-rpm wound-rotor induction motor

Rotor circuits short-circuited. Rated voltage applied

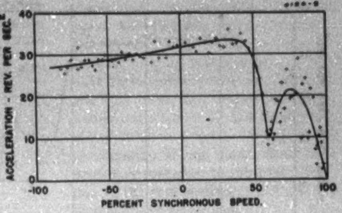


Figure 2 (right). Acceleration-speed curve of same wound-rotor induction motor with 0.75 ohm resistance added to one rotor phase

80 per cent of rated voltage applied

tation were found to be 0.03 inch apart. It was necessary to correct for this error if the best results were desired. The sudden application of power to the test motor was found to set up torsional vibrations in the motor shaft. The period of these oscillations was found to conform to the natural period of the rotor mass, the mass of the large disk, and the resilience of the connecting shaft. No correction for this effect was necessary in the tests reported, but the need for rigid connections was definitely shown.

Preliminary Tests

Two motors were used in the preliminary tests. One was a 5-horsepower wound-rotor induction motor, and the other was a 15-horsepower synchronous motor. Both machines were six pole, 60 cycle, three phase. The induction motor was tested with the rotor circuits short-circuited, and with resistance added to one rotor phase. Adding resistance to only one of the rotor phases produced the desired results of adding a cusp to the speed-torque curve. The presence of the cusp was desired since these preliminary tests were made to check the equipment, and, the more complicated the speed-torque curve, the greater the faith in the tests, providing the results of duplicate tests could be made to check. The synchronous motor was also tested under two conditions. The first tests were made with the field circuit open. The second tests were made with the field circuit short-circuited. A large bakelite disk with 500 graduations accurately engraved around the rim was mounted on the shaft of the test motor, and served as the scale giving rotor position. This disk was the only load on the test motors. The windage load of the disk was quite small, but the inertia of the disk was comparable with that of the rotor of the induction motor tested.

Power for these tests was taken from the lines of the Pacific Gas and Electric Company through transformer banks having more than ample capacity. The fluctuations in the voltage caused by the starting of these motors ranged from about three to six per cent. Voltages of 60, 80, and 100 per cent of rated value were used. The motors were first started in the reverse direction and run for several minutes until the bearings were warm. The timing motor and the stroboscope were next started. When everything was running smoothly, the main motor was reversed and the film drive motor started by the operation of a three-pole, double-throw switch. The test motor stopped, reversed, and came up to full speed in from one to three seconds.

The first run on the induction motor was made with the rotor circuit short-circuited and with full voltage applied to the stator. Following this test, runs were made with a resistance of 0.75 ohm added to one phase of the rotor circuit of the same motor. A scattering of the calculated points appeared in these tests. This scattering appeared in every run made under these conditions, and although one test might show that at a certain speed the torque was high, a second test under identical conditions might show the torque low at that same speed. The scattering was noted to be extremely bad as the motor approached synchronous speed. Vibration was suspected, but the same erratic behavior was found even after the motor had been bolted down to a heavy bed plate. Then it was observed that the indicated torques followed cyclic pulsations having a frequency equal to twice slip frequency. The unsymmetrical electric circuit of the rotor did not permit the flux to move along the rotor face with uniform resistance. Curves were drawn showing the torque as a function of time and, from these, mean values were found. These mean values checked on successive runs. The speed-torque curve of this motor, started at full voltage and with the rotor circuits short-circuited, is shown in figure 1. The speed-torque curve of the same motor, started at 80 per cent of rated voltage and with resistance added to one rotor phase, is shown in figure 2. The scattering of the points may be seen in this second figure.

The tests on the 15-horsepower synchronous motor followed closely the procedure of the induction-motor tests. The first runs were made with the field circuit open and the speed-torque curve of figure 3 was obtained. The voltage used in this test was 80 per cent of rated value. There is a small discontinuity in the curve at zero speed. This is due to the reversal of the torque due to bearing friction as the motor reversed direction of rotation. The effect was exaggerated in this motor because, being a special laboratory machine, it had the stator cradled for use as a dynamometer

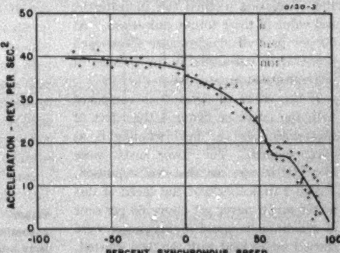


Figure 3. Acceleration-speed curve of a 15-horsepower 1,200-rpm synchronous motor

Field winding open. 80 per cent of rated voltage applied

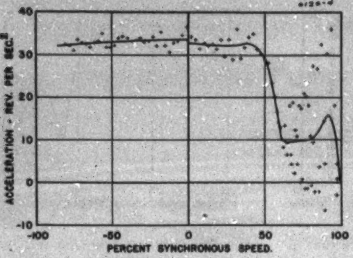
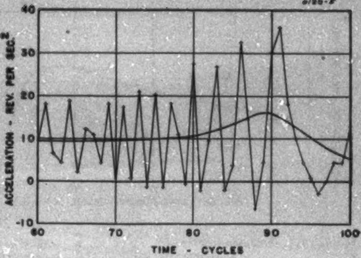


Figure 4 (left). Acceleration - speed curve of same 15-horsepower synchronous motor

Field winding short-circuited. 80 per cent of rated voltage applied

Figure 5. Section of the acceleration-time curve of the test shown in figure 4



or for the physical displacement of the stator windings relative to those of another machine on the same shaft, and therefore this motor had four sleeve-type bearings. This effect was not noted in the induction motor tests since that motor had ball bearings.

The torque pulsations were particularly pronounced when the synchronous motor was started with the field circuit short-circuited. Noise and vibration during the starting period were also found to be greater than in the previous tests. The magnitude of these torque pulsations may be seen by reference to figure 4 where the speed-torque curve is shown. The great scattering of the points above 60 per cent of synchronous speed made it impossible to draw a curve having any reasonable accuracy unless some further information was obtained. These acceleration points have been plotted to a time scale instead of speed in figure 5. The cyclic character of the pulsations is evident in this figure. The median curve may be drawn on this plot with reasonable accuracy. A second test under identical conditions gave the same median curve. Because the main scattering of the calculated points lies above 60 per cent of synchronous speed in all these tests does not signify that the torque pulsations exist only above this speed. They undoubtedly exist throughout the entire range of the test. It is only at speeds approaching synchronism that the calculation method can indicate the full value of these torque pulsations. At 50 per cent of synchronous speed each acceleration calculated is the average of one complete torque cycle.

If the curve of figure 3 is compared with the curve of figure 4 the effect of short-circuiting the field winding is at once evident. The two tests were identical except for this one condition. Figure 4 indicates that the speed of this motor might never get above 60 per cent of synchronous speed if this motor were started under load with the field shorted. In normal laboratory use this motor is directly connected to a second machine of the same design and a 20-horsepower d-c motor. With these other machines as load the motor will not come up to speed unless at least 70 per cent of rated voltage is applied to its terminals.

Tests on the Large Motors

There are three sizes of motors on the main pumps of the aqueduct system. The largest size is rated 12,500 horsepower at 450 rpm. Three of these motors are installed at Eagle Mountain pumping plant and three more are installed at Hayfield pumping plant. One of the motors at Eagle Mountain was given an acceleration test. The next motors in point of size are rated 9,000 horsepower at 400 rpm. Intake pumping plant and Gene pumping plant each have three of these motors at the present time. Two acceleration tests were made on one of the motors at Gene. A short section of the deceleration curve from full speed of this motor was obtained for the condition of a completely closed pump discharge valve. The smallest motors on the main aqueduct line are located at Iron Mountain pumping plant where each of the three motors has a rating of 4,300 horsepower at 300 rpm. An acceleration test was made on one of these motors and a deceleration test made on the same motor. The deceleration test was started with the motor fully loaded, pumping at full capacity against its normal head. All the above motors are vertical shaft, synchronous motors. They are rated at 6,900 volts, three phase, and 95 per cent power factor lead at full load.

The pump motors received their power from generators at Boulder Dam assigned to the exclusive use of the Metropolitan Water District. All the motors on this project, including the large synchronous pump motors, are started directly across the line in regular operation. Naturally, the voltage at the motor terminals could not be held constant when such large motors were started in this manner. It was known before these tests were made that the frequency also varied slightly during the starting of these large motors. The curves and calculations shown in this paper include no corrections for these variations in voltage and frequency. . The accelerations shown in the curves are those obtained under regular operating conditions.

The proper operation of the stroboscope and the timing disk demanded a source of power which was constant in voltage and in frequency. Fortunately the pumping plants had been equipped with constant-frequency sets. Each of

Figure 6. Arrangement of apparatus for main tests on the large pump motors

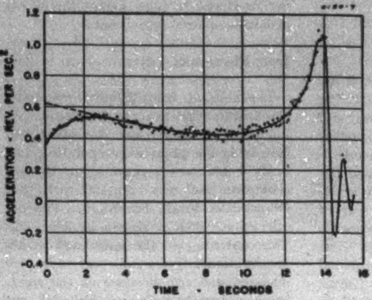


Figure 7. Acceleration-time curve of a 12,500-horsepower 450-rpm synchronous motor

Eagle Mountain pumping plant

these sets consisted of a single-phase alternator of one kilowatt capacity driven by a d-c motor from the station battery. The alternator was operated in synchronism with a temperature-controlled tuning fork. These constant-frequency sources were used in the tests. A second synchronously driven timing disk was operated from Boulder frequency. By readings taken from the two timing disks, Boulder frequency could be determined.

Because of local conditions existing at the pumping plants some modifications of the test equipment and procedure were necessary. The most difficult problem in performing the tests was in finding a method of attaching an accurately calibrated scale to the motor so that the rotor position could be determined. This problem was solved by attaching a steel tape with metric divisions around the couplings between the motors and the pumps. The couplings on the three motors tested ranged from 32.5 to 36 inches in diameter. Since the tape was calibrated in millimeters over the entire length, there were 2,593 to 2,873 divisions per revolution. Rotor position was estimated to 0.1 division. The installation of the tape proved quite simple. The coupling circumference was accurately measured with the tape. Using tape mending rivets, the tape was then riveted into a loop having a circumference about 0.3 millimeter less than that of the coupling. The tape was stretched over the coupling and remained firmly in place due to friction alone. A photograph of the equipment as it was assembled for test is shown in figure 6.

Eagle Mountain Test

The Eagle Mountain test was performed on number 1 motor with the other two motors in the plant shut down. The motor was line started according to normal operating practice, brought up to speed, and synchronized with the line using the regular automatic equipment installed in the plant. Recording was started a few seconds before the line switch was closed and was continued until after the field closing breaker had operated. The final curve of acceleration against time for this motor is shown in figure 7. All the calculated points, after the necessary corrections had been made, are indicated in this figure. The motor nearly locked into synchronism with reversed polarity. The reverse torque and the pulling into step after slipping one pole are indicated in the figure. Because of the length of time of the test only every fifth film was read. This procedure in calculation makes each calculated point the average of the conditions during five cycles. Much of the scattering of the points due to pole saliency has therefore been eliminated. During the test Boulder frequency dropped from 60.0 to 59.2 cycles.

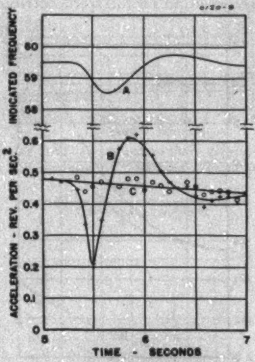


Figure 8. Enlarged section of figure 7 showing corrections

A—Curve of indicated Boulder frequency
B—Original curve drawn through test points
C—Final curve drawn through corrected test points

At three points along the curve as originally drawn there were cusps. These centered at about 7, 21, and 37 per cent of synchronous speed. After careful consideration it was found that these cusps were caused by the improper operation of the test equipment and did not represent the performance of the main motor. The data taken from the two timing disks indicated that the frequency of Boulder power had fluctuated in an altogether unexpected fashion. These fluctuations seemed much too rapid for such a system, and, it would seem from the readings, that Boulder frequency had momentarily increased due to the starting of this large motor. The only logical conclusion was that the apparent fluctuations of Boulder frequency were in reality only the hunting of the "constant frequency generator" relative to the tuning fork with which it was synchronized. The closing of the main line switch had momentarily depressed the voltage of the station battery, and this had started some hunting of the motor generator set. From theoretical considerations the correction required would be equal to:

$$\text{Correction} = -\frac{V}{60}\frac{df}{dt}$$

Where V is the velocity of the main motor in scale divisions per unit of reading time (in this case five cycles), and df/dt is the rate of change of indicated frequency per unit of reading time. With the application of these corrections, the three cusps in the acceleration curve

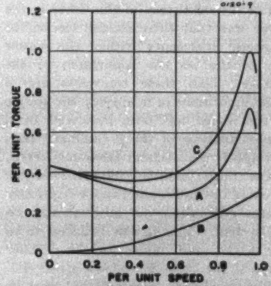


Figure 9. Speed-torque curves, Eagle Mountain

A—Accelerating torque from test
B—Hydraulic load of pump
C—Air-gap torque of motor

were entirely eliminated. Figure 8 shows the section of the original curve in the region of the largest cusp with both the original and corrected points plotted. Also shown in the figure is the curve of indicated Boulder frequency.

The left-hand end of the curve had a somewhat unexpected bend. From the electrical theory of the induction motor, the curve should follow the dotted extention instead of taking a decided downward plunge. At first this bend was thought to be due to the lack of proper lubrication of the thrust bearings of the machine. The motor had been standing idle for several hours with the entire weight of the rotor and pump on the thrust bearing. Although the bearing was immersed in oil, a proper oil film could not have existed until the motion of the rotor had swept in new oil to replace that squeezed out by the long standing.

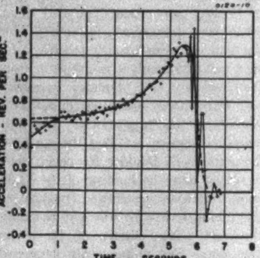


Figure 10. Acceleration-time curve of a 4,300-horsepower 300-rpm synchronous motor

Iron Mountain pumping plant

The second theory of this bend in the curve was that there existed electric or magnetic transients within the motor due to the sudden application of the voltage. That these transients are of little importance in modifying the torque of the motor has been indicated by a careful analysis of the preliminary tests and also by subsequent laboratory tests.

The third possible cause lies in the action of the automatic voltage regulators on the generators and other motors on the system. This is now believed to be one of the main causes for the peculiar shape of the curve at the low-speed end.

The accelerations calculated may be converted to per unit torques using the rating of the motor and the WR^2 values of the motor and pump. The WR^2 value has been taken from published tabulations.[4]

Unit torque is equivalent to

$$\frac{26{,}900\times \text{hp}}{N\times WR^2} \text{ revolutions per second}^2$$

where N is the synchronous speed of the motor. Since this motor had a rating of 12,500 horsepower at 450 rpm, and had a WR^2 of 506,000 pound-feet squared, unit torque corresponds to an acceleration of 1.48 revolutions per second per second. Only a part of the torque developed was used in acceleration. Part of the motor torque was used in overcoming the hydraulic load of the pump. The pump load could have been determined by making a deceleration test under identical conditions. Instead the values of motor output for full-speed operation at zero discharge were used in these calculations. The pumps at this plant require a motor output of 30.6 per cent of full load to drive them at zero discharge. For this condition the torque required varies as the square of the speed. In figure 9, curve A, is shown the data of figure 7, converted to torque and replotted using per unit speed as the abscissa instead of time. The drop off at the left-hand end of the curve has been ignored and the dotted extension used. Curve B of this figure shows the pump torque. Curve C is the sum of curves A and B and represents the air-gap torque of the motor. A curve of this type can only be obtained by a modified squirrel-cage design. The rotor bars of this motor were T shaped. This special shape and the composite structure of the bar produced to some extent the effects of double squirrel-cage design.

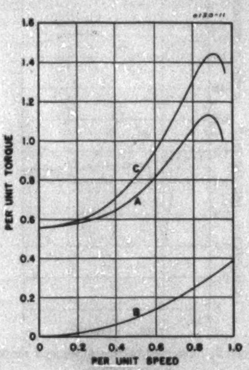


Figure 11. Speed-torque curves, Iron Mountain

A—Accelerating torque from test
B—Hydraulic load of pump
C—Air-gap torque of motor

Iron Mountain Tests

The tests at Iron Mountain pumping plant were the next to be made. At the time of these tests the other two motors in the plant were operating normally. The test procedure on the acceleration test was identical with that followed at Eagle Mountain. The acceleration curve is shown in figure 10. The scattering of the calculated points due to torque pulsations was very pronounced and can be seen at the right-hand end of the curve. Although not shown by this curve, the motor pulled into step on the reluctance torque due to salient-pole construction. It then fell out of step and slipped a pole, again locking into step with the application of the d-c field. During the starting period the frequency dropped from 59.74 to 59.46 cycles.

The same bend at the beginning end of the curve was found in this test as was found in the test at Eagle Mountain. The bend was not as pronounced however. This motor had about one-third the horsepower rating of the one at Eagle Mountain. It would therefore not effect the line as much, nor would it require as much regulator action. Also, the other two motors in this plant were operating at the time this motor was started and helped to maintain voltage. Differences in the design of the two thrust bearings may also have entered into this difference.

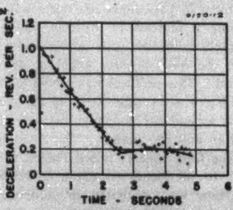


Figure 12. Deceleration-time curve of the same 4,300-horsepower synchronous motor

Motor tripped from the line while pumping under full load. Iron Mountain pumping plant

This motor had a rating of 4,300 horsepower at 300 rpm, with a WR^2 value of 339,000 pound-feet squared. The acceleration corresponding to full-load torque was 1.14 revolutions per second per second. The torque required to drive the pump at full speed with zero discharge was 0.384 per unit. The per unit accelerating torques, pump torques, and air gap torques have been plotted in figure 11. The conductors used in the pole face windings of this motor were round rods, and the curve shape obtained corresponds to that expected with this design of rotor bar.

After the motor was up to speed and was pumping water in normal fashion with the discharge valve wide open, a deceleration test was made. Upon depressing an emergency stop button the motor was immediately disconnected from the power lines, and the closing of the main discharge valve was started. The specifications on the discharge valve called for a rapid initial partial closing followed by a much slower final closing. The design accepted was to close off 95 per cent of the valve area in three seconds.[4] The deceleration curve shown in figure 12 would indicate that the high-speed part of the closing had been completed in 2.7 seconds. In these 2.7 seconds the motor had dropped to 73 per cent of synchronous speed. There was a slight rise in the decelerating torque between three and four seconds after the motor was tripped from the line. The cause of this rise is probably the surge in pipe-line pressure due to the sudden checking of the flow when this pump was shut down. Only the initial part of the deceleration test was recorded. After the pump valve had closed the deceleration was very low, and the motor and pump continued to turn for several minutes.

Gene Tests

The last motor tested was at Gene pumping plant. At the time of the tests the other two motors in the plant were shut down. Since a leakage test was in progress at one of the reservoirs, it was desired that no water should be pumped. The main discharge valve from the pump was blocked so that it could not open, and as a further precaution against an appreciable flow being pumped through the eight-inch by-pass, the motor was tripped from the line immediately after the field closing breaker had operated. Except for these modifications, the test procedure was indentical with the procedures followed in the other tests. Two acceleration tests were made. The curve obtained on the first of these tests is shown in figure 13. The field breaker closed at about 8.7 seconds and then just before 9 seconds the motor was tripped from the line. The film drive motor was also stopped at about this time and only a few readings were possible beyond this point. Some hunting of the constant-frequency set was noted and the data were corrected for this variation. The corrections were small in these tests because the hunting was not severe. The frequency dropped from 59.90 to 59.22 cycles during the starting period.

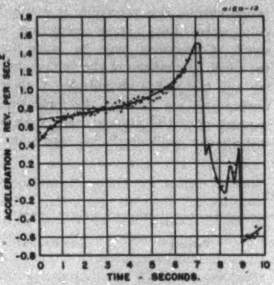


Figure 13. Acceleration-time curve of a 9,000-horsepower 400-rpm synchronous motor

Gene pumping plant

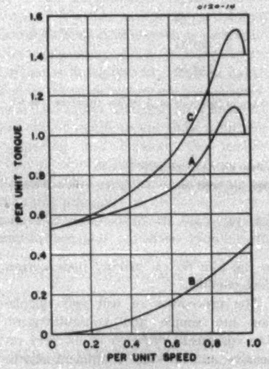


Figure 14. Speed-torque curves. Gene plant

A—Accelerating torque from test
B—Hydraulic load of pump
C—Air-gap torque of motor

The bend at the beginning of the curve was more pronounced in this test than in the test at Iron Mountain, and less pronounced than in the test at Eagle Mountain. If it were assumed that the bends were caused largely by voltage regulator action this would be the expected result. The horsepower ratings of the motors were in the same order as the prominence of the bends.

In the three tests for which the curves are reported, each motor had stood for several hours without being turned over. In the repeat test made at Gene pumping plant the motor was only allowed to remain stopped for a few seconds after coming to rest from the first test before being restarted. The starting acceleration recorded in the second test showed a small but definite increase over that recorded in the first test. This difference disappeared in about one-half second. If this difference were due to a difference in voltage conditions for the two tests, the effects should have been evident for a longer period. It has therefore been assumed that this difference in recorded accelerations was due to a change in thrust-bearing lubrication.

This motor had a rating of 9,000 horsepower at 400 rpm, and a WR^2 value of 468,000 pound-feet squared. The acceleration corresponding to full-load torque was 1.29 revolutions per second per second. The torque required to drive the pump at full speed with zero discharge was 0.45 per unit. The per unit accelerating torque, pump torque, and air-gap torque have been plotted in figure 14. The pole-face windings of this motor were similar to those of the 4,300-horsepower motor at Iron Mountain, and the curve shapes are also similar.

Further analysis of the data obtained has shown that in the main tests there was an apparent rapid fluctuation of Boulder frequency as each large motor approached synchronous speed. The motor speed at which this occurred was in each test 95 to 96 per cent of synchronous speed. There seemed to be no apparent reason for any hunting of the constant-frequency sets, and the period of this fluctuation was very much shorter than that of these sets. The natural period of the timing-disk motors was approximately one-sixth second, and this corresponded exactly to the fluctuation recorded for Boulder frequency. It seemed logical to blame this apparent fluctuation on errors of these timing disks if some cause for the starting of the hunting could be found. The torque pulsations of the main motor caused by the salient-pole construction have a period of one-sixth second for a slip of five per cent. If the torque exerted on the rotor by the stator is to vary, then the power component of the current to the stator must vary in the same way. If this varying power component is supplied through a reactive link, transmission line or transformers, then the angle between the generator voltage and the receiver voltage must vary with the power component. The result is a frequency modulation with the receiving end frequency varying above and below the generator frequency. The frequency modulation was present throughout each start. It was apparent only when resonance with the timing-disk period was approached.

Conclusions

The test methods outlined in this paper should find considerable use in the commercial and research fields. While these tests cannot replace the well-established brake and dynamometer tests for certain determinations, they may be used to obtain other data which is difficult to obtain in any other way. Any motor may be tested by this method providing a properly calibrated scale may be attached to it. Both vertical- and horizontal-shaft motors may be tested with the same equipment. There seems to be no limit to the machine size or rating.

Another advantage of this method is that the tests may be made at any voltage up to and including rated voltage. The excess heating usually present in full-voltage tests is not encountered due to the very short testing periods. A common procedure is to make a test at one voltage and then, by applying correction factors, to assume that the corrected results correspond to some other voltage. Since curves taken at different voltages do not have identical shapes due to saturation effects, this procedure is justified only where the voltage correction is small. Corrections may be made for the small fluctuations of the voltage during any test if the magnitudes of these fluctuations are known. In some subsequent tests the voltage trace from a cathode ray oscillograph has been added to the film record making these corrections possible.

Table I

Timing Disk	Boulder Disk	Rotor Position	Velocity*	Acceleration†
10.3	270.0	1,462.74		
			96.20	
190.5	81.5	1,366.54		0.86
			97.06	
10.5	252.7	1,269.48		0.88
			97.94‡	
190.5	63.9	1,458.84		0.84
			98.78	
10.4	235.5	1,360.06		0.92
			99.70	
190.5	47.3	1,260.36		0.83
			100.53	
10.5	218.7	1,447.13		0.91
			101.44	
190.6	30.4	1,345.69		0.92
			102.36	
10.5	202.2	1,243.33		

* The motor had 16 poles and the position readings were taken at intervals of 5 cycles, therefore, the 60-cycle synchronous speed of this motor was

$$287.3 \times \frac{2 \times 5}{16} = 179.56$$

† To get the acceleration in revolutions per second, these values must be multiplied by 0.50.

‡ Direct subtraction is impossible when the end of the tape has been passed during the time interval. The velocity indicated for this point is

$$287.30 + 1260.48 - 1458.84 = 97.94$$

The procedures of test and calculation are simple and straightforward. The equipment required is not very expensive, and the same equipment may be used for machines of all sizes. This equipment is not bulky, and it therefore can be easily moved should tests in remote places be desired. The time required for the making of the tests is simply that needed for the setting up and taking down of the apparatus. The actual running time is so small as to be negligible, ranging in these tests from less than one second to about 16 seconds. The photographic records must be developed, read, and calculated. This part of the work usually consumes the most time. The reading, calculating, and plotting of the data for figure 1 required between two and three hours. This was a short test of about one second running time, and no corrections were applied. Where extensive corrections must be applied due to irregularities in the operation of the test equipment, or where other factors must be considered, the time required will naturally increase.

The accuracy of the results will depend upon the accuracy with which the scale has been calibrated and read, and the extent to which the data have been corrected. Careful attention to details affecting the operation of the test, combined with accurate reading of the data from the films, produce results which need no corrections, except those due to voltage fluctuations should they be desired. The accuracy of the resulting curves from such a test should be within two per cent.

Table I is included to show the method of determination used for velocities and accelerations. The first three columns show the readings taken from the film record. The fourth column shows the calculated velocities and the fifth the calculated accelerations. The data used are from the test at Eagle Mountain pumping plant and cover the region between 8.5 and 9 seconds of that test. In the overlap of the tape used for indicating the rotor position the marking 1,210.00 centimeters was directly over 1,497.30. The tape length was therefore 287.30 centimeters.

References

1. THE PUMPING SYSTEM OF THE COLORADO RIVER AQUEDUCT, J. M. Gaylord. AIEE TRANSACTIONS, March 1939, volume 58, pages 112–23.

2. PUMP MOTORS AT EAGLE MOUNTAIN READY FOR SERVICE. ELECTRICAL ENGINEERING, November 1939, page 484.

3. AN APPLICATION OF DECELERATION TEST METHODS TO THE DETERMINATION OF INDUCTION-MOTOR PERFORMANCE, R. W. Ager. AIEE TRANSACTIONS, volume 58, 1939 (February section), pages 72–6.

4. PUMP DISCHARGES VALVES ON THE COLORADO RIVER AQUEDUCT, R. M. Peabody. Paper presented at ASME semiannual meeting in San Francisco, Calif., July 10–15, 1939. Not yet published.

A Generalized Equivalent Circuit in the Theory of Polyphase Commutator Motors

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MEMBER AIEE

THE demand for speed variation and power-factor compensation in a-c motors led to the development of the polyphase commutator types. In their simplest form these motors consist of two windings; a stator winding which is identical with that of the polyphase induction motor and a rotor winding of the closed type provided with a commutator and similar to the winding of a d-c armature. The field and armature windings are connected in series or shunt and give load characteristics similar to their d-c counterparts. Of these two characteristics the shunt is the more valuable in practice. Unfortunately this a-c motor runs at a constant speed and to secure speed variation it is necessary to inject an electromotive force onto the brushes. Variation of this electromotive force causes a change in the no-load speed. This may be shown simply by letting e_2 be the voltage induced in a rotor phase at standstill, and if e_b is the electromotive force impressed on the brushes then the rotational electromotive force e_r must be equal to the vector sum e_2+e_b. Now $e_r=K\phi n$ where n = revolutions per second, ϕ is the flux per pole and K a proportionality factor. So

$$e_r=e_2+e_b$$

or

$$n=e_2\left[1+\frac{e_b}{e_2}\right]\frac{1}{K\phi}=\frac{K_1\phi f}{K\phi}\left[1+\frac{e_b}{}\right]$$

$$\therefore n\propto f\left[1+\frac{e_b}{e_2}\right]$$

that is, by variation of e_b the speed n will be changed. This entails the use of an induction regulator to secure smooth speed variation. In the majority of practical applications this solution is uneconomical. The present-day method is to use what are termed commutator induction motors which essentially consist of an induction motor and phase converter accommodated within one machine. This type in common with other classes of a-c commutator motor is finding an ever-increasing application in practice.

The theory of these motors has of course been developed by specialists who assume prior knowledge of the subject and as the treatment lacks uniformity both as regards notation and vector representation it makes the matter difficult both to follow and correlate the performance features of the various types of field connections. This has naturally led to this class of motor not being given its due consideration.* When one compares this "single motor" treatment with that of the d-c machines, the benefits of a general theory are apparent. In the case of the d-c machine, the effect of changes in the field connections are taken care of by minor adjustments in the general theory. Of course such a theory does not deal with some details, such as commutation for example. Since all engineers are familiar with equivalent circuit in induction motor work and as analytical analysis is used rather than graphical methods, this circuit would appear to be a suitable connecting link between the various types of motor. The more so seeing that the commutator motor must operate on the same principles as the induction motor. The essentials of the theory of the transformer and induction motor with respect to the development of their equivalent circuits will now be reviewed briefly.

1. Theory of the Transformer and the Induction Motor

The performance of an air-cored transformer with a load of impedance Z on the secondary side is specified by the following vector equations:

$$V_1=Z_1I_1+j\omega MI_2$$

and

$$0=(Z_2+Z)I_2+j\omega MI_1$$

where V_1 is the applied voltage, I_1 and I_2 are the currents in the windings of impedance Z_1, Z_2 and $j\omega M$. In textbooks it is shown that the impedance from the supply side is $Z_1+\frac{\omega^2M^2}{Z_2+Z}$; whilst from the secondary side the impedance is $-\frac{j}{\omega M}[\omega^2M^2+Z_1(Z_2+Z)]$.

The introduction of an iron core, as is well known, provides two possible paths for the flux, the one through iron and the other mainly through air. The inductances of the windings may now be written as L_1+l_1 and L_2+l_2 where L denotes the inductance due to useful flux and l the inductance of the leakage flux. Thus the impedances now become

$$Z_1=R_1+j\omega l_1+j\omega L_1$$

and

$$Z_2=R_2+j\omega l_2+j\omega L_2$$

Under these conditions we have $M^2=L_1L_2$. The impedance of the transformer from the primary side is approximately

$$Z_1'=R_1+j\omega l_1+\frac{L_1}{L_2}(R_2+j\omega l_2+Z)$$

$$=\left[R_1+\left(\frac{T_1}{T_2}\right)^2R_2\right]+ j\omega\left[l_1+\left(\frac{T_1}{T_2}\right)^2l_2\right]+\left(\frac{T_1}{T_2}\right)^2Z$$

Likewise the impedance on the secondary side now approximates to,

$$Z_2'=\frac{T_2}{T_1}[R_1+j\omega l_1]+\frac{T_1}{T_2}[R_2+j\omega l_2+Z]$$

Thus both reactances are now defined solely by the leakage reactances. The approximations made involve ignoring the no-load current.

As the currents are now formulated by $I_1=V/Z_1'$ and $I_2=V/Z_2'$ respectively, the ratio of the currents is obtained as $\frac{I_1}{I_2}=\frac{T_2}{T_1}$. Inspection of the expression for the impedance Z_1' reveals that it may be imitated by two impedances connected in series as shown in figure 1 which is the equivalent circuit for the transformer. The resistance and leakage reactance of the secondary are transferred to the primary side on multiplication by $\left(\frac{T_1}{T_2}\right)^2$. This ratio is known as the transformation ratio. For our purposes this statement needs to be amplified. For the case of

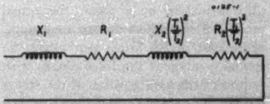
X_1
R_1
$X_2\left(\frac{T_1}{T_2}\right)^2$
$R_2\left(\frac{T_1}{T_2}\right)^2$

Figure 1. Equivalent circuit for a transformer: all quantities referred to primary side

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* Since presenting the paper, the author's attention has been called to a paper by Kostenko and Sawalischin dealing with this subject. (See reference 4.)

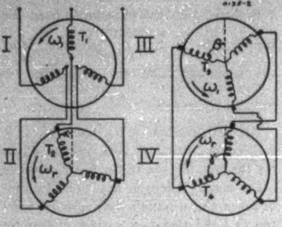

Figure 2. Diagram of connections for the generalized motor

I and III—Stator windings
II and IV—Rotor windings

the transformer it is seen that we may write

$$\left(\frac{T_1}{T_2}\right)^2=\frac{E_1}{E_2}\div\frac{I_1}{I_2}$$

or we have the transformation ratio as

$$\frac{\text{Primary induced volts}}{\text{Secondary induced volts}}\div\frac{\text{current in primary to balance secondary current}}{\text{secondary current}}$$

Passing to the case of the induction motor, some modifications are required due to rotation of the secondary. Here the secondary voltage varies with slip so that E_2 is replaced by sE_2 and the frequency of the currents and voltage in the rotor is of slip frequency. The secondary reactance is now sX_2 and the ratio is $\frac{1}{s}\left(\frac{T_1}{T_2}\right)^2$. The equivalent circuit is shown in figure 3 where the omission of the no-load current is rectified by a shunt admittance g_0-jb_0 connected across the voltage corresponding to that due to the air-gap flux.

2. Commutator Motors

Since commutator motors act on the same general principles as the induction motor, the above ideas can be applied with modifications due to certain constructional differences. In a commutator motor the position of the phases is fixed by the brush gear and as the latter is stationary the position of the rotor phases is fixed in space. Rotation of the armature merely alters the individual conductors making up a phase, the actual number of conductors therein is constant. Movement of the brush gear alters the spatial position of the rotor phases with respect to the fixed position of the stator phases. The electromotive force and current between brushes are now of supply frequency. This allows the rotor brushes to be connected to the supply and as we have seen in the case of a shunt motor allows a voltage suitable for speed variation to be impressed thereon. This should be compared with the induction motor where the phase connections rotate and the rotor phases consist of the same conductors at all times. In both cases the amplitude of the generated electromotive force will vary with slip.

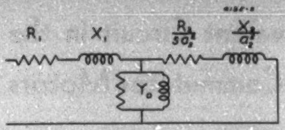

Figure 3. Equivalent circuit for an induction motor

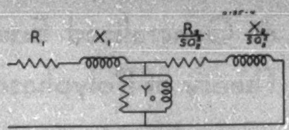

Figure 4. Equivalent circuit for motor with armature, brushes short-circuited

A further feature introduced by the commutator is that the magnetic axis of a rotor phase is fixed by the brush axis, exactly as for the d-c machine. So movement of the brush axis causes the rotor axis to move with respect to the stator axis. Since the air gap magnetomotive force is the resultant of the stator and rotor magnetomotive forces the air-gap flux will be moved by brush movement. This alters the time relationship between the induced stator and rotor electromotive forces.

In the subsequent discussion certain assumptions will be made. These are:

1. All voltages and currents are sinusoidal. In certain currents there is some deviation from the sine wave.
2. The relationship between current and flux is linear and saturation effects are neglected.
3. The reactance of the commutator winding is given by ωL; the effect of single-phase fields and the like is neglected.
4. Though all quantities are considered, some may be of minor importance in certain classes of motor. These would be detected readily when the constants of a given motor are measured.

3. Generalized Machine

A review of the possible combinations of the windings used in commutator motors leads to a general machine shown in figure 2. Here there are two windings accommodated on both the stator and the rotor. These windings are labelled I, II, III, and IV and they are connected up in pairs in series through brushes bearing on the commutators. The position of any winding may be altered without alteration to the main theory, thus allowing any machine to be discussed. For a series motor, for example, windings I and II would be used, whilst for a shunt motor, windings I and IV could be considered.

Assuming that the mesh-connected rotor is replaced by its equivalent star winding, the effective turns in each phase are T_1, T_2, T_3, and T_4. Let the angular displacement between the axes of the various windings with respect to the stator axis be α, β, and γ as shown. Then for the windings (I) and (II) carrying a current I_1 the resultant magnetomotive force per phase will be given by $\boldsymbol{A}_a=I_1[T_1+T_2\epsilon^{j\alpha}]$. Likewise in windings III and IV the resultant magnetomotive force per phase is given by $\boldsymbol{A}_b=I_2\times[T_3\epsilon^{j\beta}+T_4\epsilon^{j\gamma}]$.

The expression $\epsilon^{j\alpha}$ means that the vector I_1T_2 is advanced in space by an angle α on the vector I_1T_1. This applies to the displacement shown in figure 2. When the brushes are moved against the direction of rotation of the motor then this vector is retarded and given by $I_1T_2\epsilon^{-j\alpha}$. As the windings are mutually coupled the ampere turns must balance giving $\boldsymbol{A}_a=\boldsymbol{A}_b$, when the effect of the magnetizing ampere-turns are neglected. Or

$$\frac{I_2}{I_1}=[T_1+T_2\epsilon^{j\alpha}]\div[T_3\epsilon^{j\beta}+T_4\epsilon^{j\gamma}]$$

Denoting $\frac{T_2}{T_1}$ by a_2, $\frac{T_3}{T_1}$ by a_3 and $\frac{T_4}{T_1}$ by a_4

then

$$\frac{I_2}{I_1}=\frac{1+a_2\epsilon^{j\alpha}}{a_3\epsilon^{j\beta}+a_4\epsilon^{j\gamma}}$$

This is defined as the current ratio and is denoted by A_1.

Since the flux is assumed to be rotating counterclockwise, the brush shift shown causes the rotor electromotive force to appear later in time phase by an angle equal to the brush shift. Thus for windings I and II the resultant electromotive force $E_1=e_1+e_2\epsilon^{-j\alpha}$, where e_1 and e_2 are the electromotive forces at standstill. At any speed this may be written as

$$E_1=4.44\phi f[T_1+sT_2\epsilon^{-j\alpha}]\times10^{-8}$$

For windings III and IV the resultant electromotive force is

$$E_2=4.44\phi f[T_3\epsilon^{-j\beta}+sT_4\epsilon^{-j\gamma}]\times10^{-8}$$

Thus the voltage ratio is

$$\frac{E_1}{E_2}=\frac{1+sa_3\epsilon^{-j\alpha}}{a_3\epsilon^{-j\beta}+sa_4\epsilon^{-j\gamma}}=A_2\text{ say}$$

The complete transformation ratio is now given as $A_1\cdot A_2$. In appendix I it is shown that this expression is a complex quantity and can be written as $a+jb$. On denoting the total secondary impedance by R_2+jX_2, it is obvious on transfer to the primary side that the equivalent secondary resistance becomes R_2a-X_2b and the equivalent secondary reactance is R_2b+X_2a. Here R_2 must include the whole secondary resistance with an allowance for brush drop, whilst X_2 includes the reactance of all components of the secondary. In certain cases some of the reactances may be expressed in terms of each other.

In appendix II some examples are given in deriving the equivalent circuits for various classes of motor with shunt characteristics. In general these expressions are complicated but may be split into terms involving slip designated as the resistance and reactance of the main circuit. Others are independent of slip and may be grouped as admittances. This arrangement is arbitrary. For the case of the induction motor with frequency converter in the secondary, the main circuit resistance and reactance are given as

$$\frac{R_2}{a_3^2}\,\frac{1}{s+K\cos\gamma}$$

and

$$\frac{X_2}{a_3^2}\cdot\frac{1}{s+K\cos\gamma}$$

respectively. The no-load admittance is given as

$$(X_2-jR_2)\,\frac{a_3^2}{Z_2^2}\cdot K\sin\gamma$$

For practical purposes this may be written as

$$-j\,\frac{a^2}{R_2}K\sin\gamma=-jB$$

since X_2 is generally small. As the value of B is negative it is capacitive. This is the component of the circuit which ensures improvement of the power factor. Its action depends on the brush shift angle as shown.

4. Analysis of the Motor Performance

As for the induction motor, when the equivalent circuit is determined the analysis of motor performance proceeds by the usual methods. In appendix III an analysis of the Osnos motor is indicated. A full analysis may be found in the references given.

In general it is seen that the equivalent circuit of the motors discussed may be reduced to:

1. A shunt circuit carrying an almost constant current.

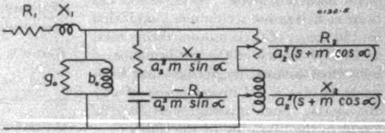

Figure 5. Equivalent circuit of a shunt motor

2. A main circuit whose components consist of a constant resistance (R_1), a constant reactance (X_1), a variable resistance (R_2), and a variable reactance (X_2).

In appendix IV the equation for such a circuit is given as

$$x^2+y^2-2Ax-2By=0$$

This is the equation of a circle provided that A and B are constants. The values of A and B are given by

$$A=\frac{V}{2X_1\left[1-\frac{R_1}{R_2}\cdot\frac{X_2}{X_1}\right]}$$

and

$$B=\frac{V}{2R_1\left[-\frac{X_1}{X_2}\cdot\frac{R_2}{R_1}\right]}$$

So the test for the locus of the current to be a circle is given by:

$$\frac{\text{Variable reactance }(X_2)}{\text{Variable resistance }(R_2)}=\text{constant for a given brush position}$$

To illustrate we will consider the induction motor with frequency converter. From appendix II we have the circuit constants as

$$R_A=\frac{r_3+r_4}{a_3^2}\cdot\frac{1}{s+K\cos\gamma}=R_2$$

and

$$X_A=\frac{sx_3+x_4}{a_3^2}\cdot\frac{1}{s+K\cos\gamma}=\frac{x_3}{a_3^2}+\frac{x_4-x_3K\cos\gamma}{a_3^2(s+K\cos\gamma)}$$

Hence

$$X_2=\frac{x_4-x_3K\cos\gamma}{a_3^2(s+K\cos\gamma)}$$

Hence

$$\frac{X_2}{R_2}=\frac{x_4-x_3K\cos\gamma}{r_3+r_4}=\text{a constant for any given brush angle}$$

Again for the Osnos motor the value of the ratio becomes approximately

$$\frac{X_2}{R_2}=-\frac{XK\cos\beta}{R_2}$$

which is also independent of slip. Thus the locus of the current is a circle. It is to be noted that there is a circle for each brush position. In appendix III it is demonstrated that for the Osnos motor the locus of the centers of the circles is an ellipse. The center of this ellipse corresponds to the center of the equivalent induction motor. For the Schräge motor for any fixed value of β the locus of the centers is a line parallel to the applied voltage axis. If in this motor the brush gear may be rotated as a whole then for each brush angle β the loci of the centers are lines parallel to the voltage axis. In actual tests it is found that the major axis of the ellipse does not lie on the x axis but at a small angle thereto. The question of construction of load, and torque lines has not yet been considered.

Figures 6 and 7. Induction motor with frequency converter showing the true and approximate equivalent circuits

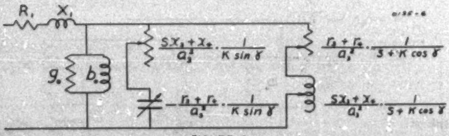
R₁
X₁
g₀
b₀
sx₃ + x₄ / a₃² · 1 / K sin γ
− r₃ + r₄ / a₃² · 1 / K sin γ
r₃ + r₄ / a₃² · 1 / s + K cos γ
sx₃ + x₄ / a₃² · 1 / s + K cos γ

FIGURE 6

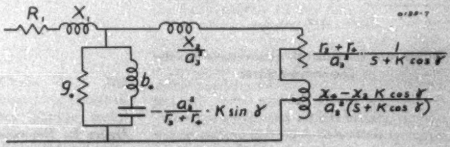
R₁
X₁
X₃ / a₃²
g₀
b₀
− a₃² / r₃ + r₄ · K sin γ
r₃ + r₄ / a₃² · 1 / s + K cos γ
x₄ − x₃ K cos γ / a₃²(s + K cos γ)

FIGURE 7

Conclusion

A general equivalent circuit for commutator motors has been developed. The constants of this circuit are based on a determination of the transformation ratio from the expressions for the magnetomotive force and voltage. For this class of motor, where the voltage vector equation method entails a consideration of time and space phase relationships, this method has the advantage that these are obtained from a preliminary consideration of the relative positions of the windings. Throughout subsequent analytical work all quantities are in their correct phase relationship to each other.

In common with other machines once the equivalent circuit has been developed, the performance of the motor is obtained by standard methods as used in the case of the induction motor. As the object has been to determine a common foundation for the theory of these motors, rather than a detailed theory of the performance of individual motors, no attempt has been made to give a full investigation of any one motor. Such investigations may be found in the literature of the subject.

List of Symbols

A_1 = current ratio = (secondary current)/primary current = I_2/I_1.
A_2 = voltage ratio = (primary voltage)/secondary voltage = E_1/E_2
B, b = susceptance of equivalent circuit
E_1 = electromotive force induced in primary phase windings
E_2 = electromotive force induced in secondary phase windings
f = supply frequency
G, g = conductance of equivalent circuit

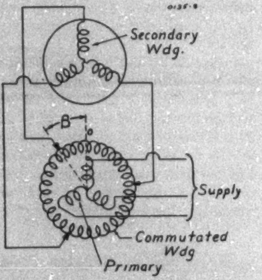

Figure 8. Diagram of connections of the Osnos-type motor

(In the case of the Schräge motor the star point is opened and the ends of the secondary windings terminated in brushes bearing on the commutator)

K = a ratio; for three winding motors = a_3/a_4
m = K/a_2 for shunt motor; for Osnos motor = $K \cos \beta$
n = $K \sin \beta$ (Osnos)
n_0 = synchronous speed
Q = α/π (Schräge motor)
R_1 = total resistance of primary windings per phase
R_2 = total resistance of other windings per phase; if there is a commutator, the resistance must include an allowance for the brush resistance (R_A). R_2 is a variable resistance
$r_1r_2r_3r_4$ = resistances per phase reduced to equivalent star values.
s = slip
T = effective turns per phase, that is, actual turns times the winding factor. All windings reduced to an equivalent star winding. The subscripts denote the actual member
V = applied voltage
X_1X_2 = total leakage reactances of primary and other circuits. X_2 variable
$x_1x_2x_3x_4$ = leakage reactances per phase at supply frequency
α, β, γ = space angles of windings (figure 2)
ϕ = flux

Appendix I

Determination of the Transformation Ratio (A)

The transformation ratio is

$$\frac{\text{Primary induced voltage}}{\text{Secondary induced voltage}} \times \frac{\text{secondary current}}{\text{primary current to balance the secondary current}}$$

Here it is

$$\frac{1+sa_2\epsilon^{-j\alpha}}{a_3\epsilon^{-j\beta}+sa_4\epsilon^{-j\gamma}} \times \frac{1+a_2\epsilon^{j\alpha}}{a_3\epsilon^{j\beta}+a_4\epsilon^{j\gamma}}$$

On rationalizing this becomes:

$$\frac{(1+a_2\epsilon^{j\alpha})(1+sa_2\epsilon^{-j\alpha})(a_3\epsilon^{j\beta}+sa_4\epsilon^{j\gamma})(a_3\epsilon^{-j\beta}+a_4\epsilon^{-j\gamma})}{(a_3^2+2a_3a_4 \cos (\gamma-\beta)+a_4^2)(a_3^2+2sa_3a_4 \cos (\gamma-\beta)+s^2a_4^2)} =$$

$$\frac{[1+sa_2^2+a_2(\epsilon^{j\alpha}+s\epsilon^{-j\alpha})][a_3^2+sa_4^2+a_3a_4(s\epsilon^{j(\gamma-\beta)}+\epsilon^{-j(\gamma-\beta)})]}{[a_3^2+2a_3a_4 \cos (\gamma-\beta)+a_4^2][a_3^2+2sa_3a_4 \cos (\gamma-\beta)+s^2a_4^2]}$$

For a three-winding type motor, winding II is omitted, that is, $T_2=0$ and so $a_2=0$. Making this substitution the ratio now becomes:

$$\frac{a_3^2+sa_4^2+a_3a_4(s\epsilon^{j(\gamma-\beta)}+\epsilon^{-j(\gamma-\beta)})}{(a_3^2+2a_3a_4 \cos (\gamma-\beta)+a_4^2)(a_3^2+2sa_3a_4 \cos (\gamma-\beta)+s^2a_4^2)}$$

With a secondary resistance of the form R_2+jX_2, the value of the equivalent resistance is given by:

$$R=r_1+\frac{R_2[a_3^2+sa_4^2+a_3a_4(1+s) \cos (\gamma-\beta)]+X_2a_3a_4(1-s) \sin (\gamma-\beta)}{[a_3^2+2a_3a_4 \cos (\gamma-\beta)+a_4^2]\cdot[a_3^2+2sa_3a_4 \cos (\gamma-\beta)+s^2a_4^2]}$$

and the equivalent reactance is:

$$X=x_1+\frac{X_2[a_3^2+sa_4^2+a_3a_4(1+s) \cos (\gamma-\beta)]-R_2a_3a_4(1-s) \sin (\gamma-\beta)}{[a_3^2+2a_3a_4 \cos (\gamma-\beta)+a_4^2][a_3^2+2sa_3a_4 \cos (\gamma-\beta)+s^2a_4^2]}$$

For this class of motor, let the arrangement of the windings be as shown in figure 8, then R_2 is the total resistance of the secondary winding, that is, the resistances of windings T_3 and T_4 plus an allowance for the contact resistance of the brushes. Since this latter depends on the current in the brushes, an average value will be used to simplify calculations. Also X_2 is the total leakage reactance of these windings comprising x_3 the reactance of the commutated winding T_3 and sx_4 the reactance of T_4. The frequency of the currents in this latter circuit is of slip frequency.

As the windings T_1 and T_3 are located in the same slots the two windings are mutually coupled and so x_3 may be expressed in terms of x_1. The coefficient of coupling will of

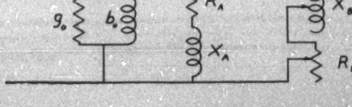

Figure 9. The equivalent circuit of the Osnos motor

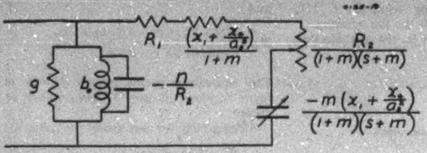
R_1
$\frac{(x_1+\frac{x_4}{a_1^2})}{1+m}$
$\frac{R_2}{(1+m)(s+m)}$
g
b_0
$-\frac{n}{R_2}$
$\frac{-m(x_1+\frac{x_4}{a_1^2})}{(1+m)(s+m)}$

Figure 10. The simplified diagram of connections of the Osnos motor

course depend on the dimensions of the slot. Here in the interest of simplicity this coefficient is taken as unity.

The mutual flux linking both windings is due to the resultant ampere-turns of these two windings and may be expressed as $I_1T_1-T_2T_3\epsilon^{j\beta}$. Hence the reactive drop in the primary due to the presence of the tertiary winding becomes

$$x_1I_1[1-a_3I_3\epsilon^{j\beta}/I_1]$$

The last term gives the alteration due to the mutual effect of the current in the commutated winding. The voltage drop is thus given by

$$V_1=I_1\left[Z_1-jx_{13}\frac{a_3}{a_3+a_4\epsilon^{-j\beta}}\right]$$

where x_{13} represents the mutual effect. On expansion we obtain the impedance as

$$Z_1'=\left[r_1-x_{13}\frac{a_3a_4\sin\beta}{a_3^2+2a_3a_4\cos\beta+a_4^2}\right]+ j\left[x_1-x_{13}\frac{a_3(a_3+a_4\cos\beta)}{a_3^2+2a_3a_4\cos\beta+a_4^2}\right]$$

This gives the effective resistance and reactance of the primary winding. In practice it is found that $x_{13}a_3a_4\sin\beta$ is very small and so may be neglected. Also x_{13} is almost equal to x_1. Thus we approximate as

$$R_1\cong r_1$$

and

$$X\cong\frac{x_1}{1+K\cos\beta}$$

For the commutated winding the effective ampere-turns are likewise reduced by

$$-\frac{I_1T_1}{I_3T_3}\epsilon^{-j\beta}$$

so the voltage equation becomes

$$V_3=I_3\left[Z_3-j\frac{x_{31}}{a_3}(a_4\epsilon^{-j\beta}+a_3)\right]$$

The impedance of that winding is now given by

$$Z_3'=\left[r_3+x_{31}\frac{\sin\beta}{K}\right]+ j[x_3-x_{31}(1+1/K\cos\beta)]$$

$$Z_3'\cong\left[r_3+x_3\frac{\sin\beta}{K}\right]-j\left[x_3\frac{\cos\beta}{K}\right]$$

Appendix II

Determination of Equivalent Circuits for Motors With Shunt Characteristics

(a). INDUCTION MOTOR

The windings requisite are I and IV without brush gear. Thus the frequency in the rotor circuit is of slip frequency, and we obtain:

$$A_1=\frac{I_4}{I_1}=\frac{T_1}{T_4}$$

and

$$A_2=\frac{E_1}{E_2}=\frac{T_1}{sT_4}.$$

giving a transformation ratio of $\frac{1}{sa^2}$. The equivalent circuit is shown in figure 3.

(b). A POLYPHASE MOTOR WITH ARMATURE, THE BRUSHES SHORT-CIRCUITED

The frequency in the rotor circuit is now of supply frequency. Using windings (1) and (2) with the stator connection removed we have:

$$A_1=\frac{I_2}{I_1}=\frac{T_1}{T_2\epsilon^{j\alpha}}$$

and

$$A_2=\frac{E_1}{E_2}=\frac{T_1}{sT_2\epsilon^{-j\alpha}}$$

So

$$A=\frac{1}{sa^2}$$

The equivalent resistance and leakage reactance are now given by

$$\left[r_1+\frac{R_2}{sa^2}\right] \text{ and } \left[x_1+\frac{X_2}{sa^2}\right]$$

respectively.

The equivalent circuit is now as shown in figure 4. For this motor there is no improvement in power factor due to brush shift.

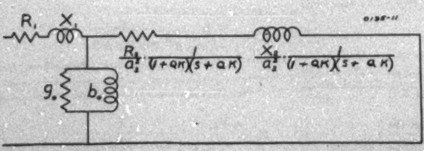
R_1
X_1
$\frac{R_2}{a_2^2}\cdot\frac{1}{(1+aK)(s+aK)}$
$\frac{X_2}{a_2^2}\cdot\frac{1}{(1+aK)(s+aK)}$
g_0
b_0

Figure 11. The equivalent circuit of the Schräge motor arranged for speed change only

(c). SHUNT MOTOR

For a simple shunt motor with its armature connected to the supply through a variable-ratio transformer, the electromotive force impressed on the brushes will be KV or approximately KE_1. Thus with windings I and II, the series connection being eliminated, the current ratio is

$$\frac{I_2}{I_1}=\frac{1}{a_2\epsilon^{j\alpha}}$$

The standstill voltage in the rotor

$$e_2=e_1\frac{T_2}{T_1}=e_1a_2$$

The voltage ratio is

$$\frac{E_1}{E_2}=\frac{e_1}{e_1[K+sa_2\epsilon^{-j\alpha}]}$$

Thus the complete ratio becomes

$$\frac{1}{a_2^2(s+m\epsilon^{j\alpha}}$$

where $m=\frac{K}{a_2}$

The referred secondary impedance is

$$\frac{R_2+jX_2}{a_2^2(s+m\epsilon^{j\alpha})}$$

Rationalizing and by the methods of the next section this becomes

$$R_A=\frac{R_2}{a_2^2}\cdot\frac{1}{s+m\cos\alpha}$$

and

$$X_A=\frac{X_2}{a_2^2}\cdot\frac{1}{s+m\cos\alpha}$$

$$R_B=\frac{X_2}{a_2^2}\cdot\frac{1}{m\sin\alpha}$$

and

$$X_B=-\frac{R_2}{a_2^2}\cdot\frac{1}{m\sin\alpha}$$

The equivalent circuit is shown in figure 5. It will be noted that this circuit does not show the total supply current drawn by the motor but only that taken through the stator.

(d). INDUCTION MOTOR WITH FREQUENCY CONVERTER

This arrangement uses two separate machines, an induction motor which is windings I and III and a rotary phase advancer which would be winding IV. The voltage generated in the phase advancer is due to the flux set up by its own current.

We will assume that this voltage is given by $e_4=Ke_3\epsilon^{j\gamma}$ where K is a constant. The total voltage in the whole secondary circuit is now given as $se_3+Ke_3\epsilon^{j\gamma}=e_2$ say where e_3 is the standstill voltage in the rotor circuit. So

$$\frac{E_1}{E_3}=\frac{e_1}{e_3(s+K\epsilon^{j\gamma})}=\frac{1}{a_3(s+K\epsilon^{j\gamma})}$$

The current ratio is given by

$$\frac{I_3}{I_1}=\frac{T_1}{T_3}=\frac{1}{a_3}$$

Hence the transformation ratio is

$$\frac{1}{a_3^2(s+Ke^{j\gamma})}$$

The referred secondary impedance

$$Z_2'=\frac{R_3+jX_3}{a_3^2(s+Ke^{j\gamma})}$$

where $R_3=r_3+r_4$ and $X_2=sx_3+x_4$, and r_4 and x_4 are the resistance and reactance of the phase-advancer armature. Rationalizing the equivalent impedance becomes

$$Z_2'=\frac{R_2^2}{a_3^2}(1+\tan^2\phi)\frac{1}{[R_3(s+K\cos\gamma)+X_3K\sin\gamma]-j[X_2(s+K\cos\gamma)-R_3K\sin\gamma]}$$

where

$$\tan\phi=\frac{X_3}{R_2}$$

This expression gives Z_2' in the form $\dfrac{1}{G-jB}$

where

$$G=\frac{a_3^2\cos^2\phi}{R_2^2}[R_3(s+K\cos\gamma)+X.K\sin\gamma]$$

and

$$B=\frac{a_3^2\cos^2\phi}{R_2^2}[X_2(s+K\cos\gamma)-R_3K\sin\gamma]$$

These expressions may be further split into $G=G_A+G_B$ and $B=B_A+B_B$. By usual methods, the corresponding resistances and reactances are:

$$R_A=\frac{r_3+r_4}{a_3^2}\,\frac{1}{s+K\cos\gamma}$$

and

$$X_A=\frac{sx_3+x_4}{a_3^2}\,\frac{1}{s+K\cos\gamma}$$

and

$$R_B=\frac{sx_3+x_4}{a_3^2}\cdot\frac{1}{K\sin\gamma}$$

and

$$X_B=-\frac{r_3+r_4}{a_3^2}\,\frac{1}{K\sin\gamma}$$

The equivalent circuit now consists of three parallel circuits as shown in figure 6. Alternatively the last terms may be left as a conductance and a susceptance and connected in parallel across the no-load branch. In practice it is found that R_B is small and may be omitted. The circuit now becomes that shown in figure 7. The phase advancing feature is clearly shown as the condensive susceptance $-\dfrac{a_3^2K\sin\gamma}{r_3+r_4}$.

(e). THE THREE-WINDING MOTORS

This type includes such well-known motors as the Osnos, Schräge, Torda, etc. For the Osnos motor the diagram of connections is shown in figure 8. The primary winding is carried on the rotor and in the same slots is the lap winding. The secondary winding is carried on the stator. This machine may be simulated by using windings I, III, and IV. Winding III is the lap winding.

The voltage induced in the lap winding is now given by

$$e_3=e_1\frac{T_3}{T_1}\epsilon^{-j\beta}$$

The voltage ratio now becomes

$$\frac{E_1}{E_2}=\frac{e_1}{e_1(sa_4+a_3\epsilon^{-j\beta})}=\frac{1}{a_4(s+K\epsilon^{-j\beta})}$$

The current ratio is now given as

$$\frac{1}{a_4(1+K\epsilon^{j\beta})}$$

and the transformation ratio as

$$\frac{1}{a_4^2(1+K\epsilon^{j\beta})(s+K\epsilon^{-j\beta})}$$

The referred secondary impedance is now given as

$$Z_2'=\frac{R_2+jX_2}{a_4^2(1+K\epsilon^{j\beta})(s+K\epsilon^{-j\beta})}$$

Using the values of R_2 and X_2 as given in appendix I we have

$$Z_2'=\frac{(R_2+x_3\sin\beta/K)+j(sx_4-x_3\cos\beta/K)}{a_4^2(1+K\epsilon^{j\beta})(s+K\epsilon^{-j\beta})}$$

$$=\frac{jx_4}{a_4^2(1+K\epsilon^{j\beta})}+\frac{(R_2+x_3\sin\beta/K)-j(Kx_4\epsilon^{-j\beta}+x_3\cos\beta/K)}{a_4^2(1+K\epsilon^{j\beta})(s+K\epsilon^{-j\beta})}$$

The numerator of the second term on expansion becomes

$$\left(R_2+\frac{x_3}{K}\sin\beta-Kx_4\cos\beta\right)-j\left(Kx_4\cos\beta+\frac{x_3}{K}\cos\beta\right)$$

This we will denote by $R+jX$. The second term of the expression for Z_2' may be written as,

$$Z_2''=\frac{R^2\sec^2\phi}{a_4^2}\left[\frac{1}{\{R(s+K^2+(1+s)K\cos\beta)+(s-1)XK\sin\beta\}-j\{X(s+K^2+(1+s)K\cos\beta)-(s-1)RK\sin\beta\}}\right.$$

where $\tan\phi=\dfrac{X}{R}$

We may thus segregate into terms dependent on slip and terms independent thereon and so obtain:

$$R_A=\frac{R(K+\cos\beta)-X\sin\beta}{a_4^2K(1+2K\cos\beta+K^2)}$$

and

$$X_A=\frac{X(K+\cos\beta)+R\sin\beta}{a_4^2K(1+2K\cos\beta+K^2)}$$

$$R_B=\frac{R(1+K\cos\beta)+XK\sin\beta}{a_4^2s(1+2K\cos\beta+K^2)}$$

and

$$X_B=\frac{X(1+K\cos\beta)-RK\sin\beta}{a_4^2s(1+2K\cos\beta+K^2)}$$

The equivalent circuit is now shown in figure 9 where R and X have values as above.

Alternatively we may group the terms as:

$$R_A=\frac{R(s+K^2+sK\cos\beta+K\cos\beta)+sXK\sin\beta}{a_4^2[(s+K^2+sK\cos\beta+K\cos\beta)^2+s^2K^2\sin^2\beta]}$$

$$X_A=\frac{X(s+K^2+sK\cos\beta+K\cos\beta)-sRK\sin\beta}{a_4^2[(s+K^2+sK\cos\beta+K\cos\beta)^2+s^2K^2\sin^2\beta]}$$

and

$$R_B=-\frac{X}{K\sin\beta}\quad\text{and}\quad X_B=\frac{R}{K\sin\beta}$$

As further analytical development is difficult we will make certain simplifying assumptions.

Since $sK\sin\beta$ is small we will neglect terms involving that quantity. Then we have

$$R_A=\frac{R_2}{a_4^2}\,\frac{1}{(s+K^2+sK\cos\beta+K\cos\beta)}\cong\frac{R_2}{a_4^2}\,\frac{1}{(s+K\cos\beta)(1+K\cos\beta)}$$

and

$$X_A\cong\frac{X}{a_4^2}\,\frac{1}{(s+K\cos\beta)(1+K\cos\beta)}$$

The term R_B is also negligibly small and so we have

$$R_B=0\quad\text{and}\quad X_B\cong\frac{R_2}{K\sin\beta}$$

Or in terms of susceptance we have

$$-B=\frac{K\sin\beta}{R_2}$$

This branch will be connected across the short circuit representing the variation of the no-load susceptance due to brush shift and as it is capacitive it will improve the power factor of the motor.

The parameters of the main circuit are approximately:

$$R_1\cong r_1$$

$$X_1=\frac{x_1}{1+K\cos\beta}+\frac{x_4/a_4^2}{1+K\cos\beta}=\frac{X}{1+K\cos\beta}\ \text{say}$$

$$R_2=\frac{r_3+r_4}{a_4^2}\cdot\frac{1}{(s+K\cos\beta)(1+K\cos\beta)}$$

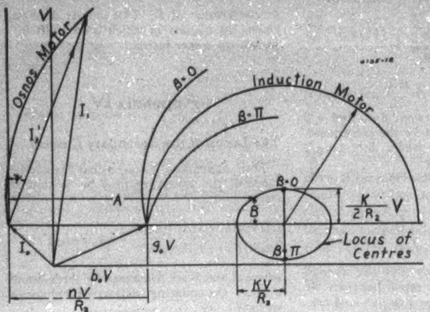

Figure 12. Circle diagram for the Osnos motor

$$X_2=-\frac{\left(x_1+\frac{x_4}{a_4^2}\right)K\cos\beta}{(s+K\cos\beta)(1+K\cos\beta)}$$

$$=-\frac{XK\cos\beta}{(s+K\cos\beta)(1+K\cos\beta)}$$

Substitution of $m=K\cos\beta$ and $n=K\sin\beta$ reduce the above expression to those shown in figure 10. This figure corresponds to the simplified circuit given by Rudra.

(f). Schräge Motor

This motor is identical in arrangement to the Osnos motor excepting that the phases of the secondary winding (4) are not connected in star but each end of a winding is terminated by brushes. Both sets of the brushes are arranged to rotate by means of a handwheel and worm gear. A variable voltage is taken off the commutator and is injected into the secondary winding. Unless arrangements are made to rotate the brush gear as a whole, the power factor of the motor is not compensated.

Let the brushes of a phase be separated by an angle of 2α, then the number of conductors between brushes will be given by $\alpha/\pi T_3=QT_3$ say. If e_3 is the diametric electromotive force of the lap winding then the electromotive force due to the lap winding is given by Qe_3. The resistance of this winding will be specified by Qr_4 and the reactance by Q^2x_4.

Assuming that the axes of all windings coincide with that of the stator winding, then the electromotive force of the whole secondary circuit is given by

$$se_4+Qe_3=e_4(s+KQ)$$

where

$$K=a_3/a_4$$

The voltage ratio now becomes

$$\frac{e_1}{e_4(s+KQ)}=\frac{1}{a_4(s+KQ)}$$

The current ratio is

$$\frac{1}{a_4(1+KQ)}$$

The transformation ratio now becomes

$$\frac{1}{a_4^2(1+KQ)(s+KQ)}$$

The equivalent resistance and reactance of the motor now becomes

$$R_e=r_1+\frac{R_2}{a_4^2}\cdot\frac{1}{(1+KQ)(s+KQ)}$$

and

$$X_e=x_1+\frac{X_2}{a_4^2}\,\frac{1}{(1+KQ)(s+KQ)}$$

Here

$$R_2=r_4+Qr_3+r_\Delta \text{ and } X_2=sx_4+Q^2x_3$$

where r_Δ is a term to account for the drop at the brush contacts. The equivalent circuit for the motor is shown in figure 11.

As for the Osnos motor we may express x_3 in terms of x_1. On substituting $\beta=0$ in the expression obtained for the Osnos motor we get $r_1=r_1$, $x_1=\frac{x_1}{1+KQ}$, $r_3=r_3$, and $x_3=-x_1a_3a_4/Q$. There is no change in the value of the equivalent resistance R_e as given above but the equivalent reactance now becomes,

$$X_e=\frac{x_1}{1+KQ}+\frac{sx_4-x_{13}a_3a_4Q}{a_4^2(1+KQ)(s+KQ)}$$

$$X_e=\frac{x_1+x_4/a_4^2}{1+KQ}-\frac{x_{13}a_3a_4Q+x_4KQ}{a_4^2(1+KQ)(s+KQ)}$$

$$=\frac{X}{1+KQ}-\frac{Qa_3a_4X}{a_4^2(1+KQ)(s+KQ)}$$

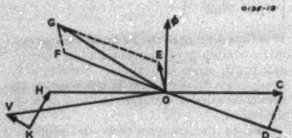

Figure 13. Vector diagram for the induction motor

$OB=I_2$
$OC=sE_1=E_2$
$CD=sX_2I_2=$ secondary leakage reactance drop
$OD=R_2\ I_2=$ secondary resistance drop
$OE=I_0=$ no-load current
$OF=I_1=-I_2$
$OG=I=I_1+I_0$
$OH=E_1$
$KH=IX_1$
$VK=IR_1$
$OV=V=$ applied volts

Appendix III

Analysis of the Performance of a Motor

When the parameters of the equivalent circuit are known, the analysis of the performance of a motor proceeds along the well-known lines of that of an induction motor as given in many textbooks. To illustrate we take the case of the Schräge motor. Since there is no compensation of power factor and the no-load current is almost the same for all brush positions we will neglect the no-load current.

Thus the input per phase of the motor is given by $VI\cos\phi=I_1^2R_e$. Substituting for R_e we have the input as

$$I_1^2\left[R_1+\frac{R_2}{a_4^2(1+KQ)(s+KQ)}\right]$$

The stator loss is given as $I_1^2R_1$ and the rotor copper loss as $I_2^2R_2$; on subtracting these the output per phase of the motor is obtained as

$$I_2^2\left[\frac{1-s}{s+KQ}\right]R_2$$

The torque in watts is given by

$$\frac{1}{2\pi n_0}\cdot\frac{I_2^2R_2}{S+KQ}$$

where n_0 is the synchronous speed. The starting torque is given by

$$\frac{1}{2\pi n_0}\cdot\frac{I_2^2R_2}{1+KQ}$$

The power factor of the motor is given by

$$\cos\left[\tan^{-1}\frac{X_1+X_2/\Delta}{R_1+R_2/\Delta}\right]$$

where

$$\Delta=a_4^2(1+KQ)(s+KQ)$$

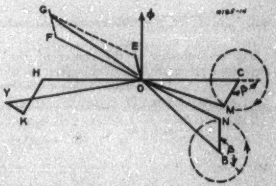

Figure 14. Simplified vector diagram for the Osnos motor

$CM=K\times$ electromotive force in commutator winding
$OM=$ resultant secondary electromotive force $=Z_2'$
$NB=KI_2$
$ON=I_2+KI_2=$ primary value of $I_2=-OF$

When both sets of brushes are on the same commutator segment, that is, $Q=0$ the motor is a simple induction motor. On substitution of $Q=0$ in the above expressions it is seen that they reduce to those of the induction motor.

The co-ordinates of the center of the circle diagram are given as

$$X_c=\frac{V}{2X}\,\frac{R_2'(1+KQ)}{R_2'+R_1QK(1+KQ)}$$

and

$$Y_c=\frac{V}{2}\cdot\frac{KQ(1+KQ)}{R_2'+R_1QK(1+KQ)}$$

The radius is

$$\frac{VR_2'(1+KQ)}{2[R_2'+R_1QK(1+QK)]}\left[\frac{1}{X_2^2}+\left(\frac{KQ}{R_2}\right)^2\right]^{1/2}$$

where

$$R_2'=R_2/a_4^2$$

On substitution $KQ=-p$ these results check with those given by Arnold.

Writing

$$\tan\theta=\frac{Y_c}{X_c}=KQ\frac{X}{R_2'}=-p\frac{X}{R_2'}$$

it is seen that for negative values of p the center of the circle is below that for the induction motor ($p=0$) whilst for positive values the center is above.

For the Osnos motor the motor performance may be analyzed in a similar fashion. Neglecting the no-load input and using the simplified equivalent circuit (figure 10) the input per phase is $I_1^2R_e$. Subtracting the losses due to the stator resistance, the input to the rotor is obtained as

$$I_2^2(1+m)^2\frac{R_2}{a_4^2(1+m)(s+m)}=I_2^2\frac{R_2}{a_4^2}\cdot\frac{1+m}{s+m}$$

Subtracting the losses in the rotor we have the input watts as $I_2^2\frac{R_2}{a_4^2}\cdot\frac{1+m}{s+m}$.

The torque in watts is

$$\frac{I_2^2}{2\pi n_0}\,\frac{R_2}{a_4^2}\cdot\frac{1}{s+K\cos\beta}$$

whilst the starting torque is

$$\frac{I_2^2}{2\pi n_0}\cdot\frac{R_2}{a_4^2}\cdot\frac{1}{1+K\cos\beta}$$

The co-ordinates of the center of the circle for a given brush angle are given by

$$X_c=\frac{V}{2X}\cdot\frac{1+m}{1+\frac{R_1}{R_2'}m(1+m)}$$

and

$$Y_c=\frac{V}{2R_2}\cdot\frac{m(1+m)}{1+\frac{R_1}{R_2'}m(1+m)}$$

To simplify these expressions we assume the design is such that the equivalent rotor resistance is equal to that of the primary as for the case in an induction motor. That is, $R_2'=R_1(1+m)$

So

$$X_c=\frac{V}{2X}\quad\text{and}\quad Y_c=\frac{V}{2R_2}m$$

For this motor $\tan\theta=\frac{mX}{R_2}$ and when m is positive, that is, between $\beta=0$ and $\pi/2$ the center is above that for the corresponding induction motor, whilst for $\beta=\pi/2$ to π the center is below.

In this motor the no-load current is given by

$$I_0=V\left[g_0-j\left(b_0-\frac{K\sin\beta}{R_2}\right)\right]$$

This expression shows that the no-load current varies with the brush position. It is seen that with $\beta=0$ or π the no-load current reduces to that of the induction motor namely $V(g_0-jb_0)$. Further when $\sin\beta=b_0R_2'/K$ the no-load current is at its minimum value and is in time phase with the applied voltage. For the usual machine constants this value of β is about 30 degrees or 150 degrees. For a value of β between these two limits the no-load current is leading. Thus the locus of the no-load current with a variable β is a line at right angles to the voltage axis and at a distance Vg_0 from the origin.

The locus of the centers of the approximate circles may be derived as follows:

At $\beta=0$ or $\beta=\pi$ as noted above the no-load currents are identical but Y_c now varies. At $\beta=0$, $Y_c=VK/2R_2'$ whilst at $\beta=\pi$, $Y_{c\pi}=-VK/2R_2'$. Thus the distance between these two centers is VK/R_2'. At $\beta=\pi/2$ and $\beta=3\pi/2$, Y_c is now zero for both brush positions. For all brush angles, X_c is a constant and is measured from the end of the no-load current vector. Here it is seen that the no-load currents differ by $2VK/R_2'$. So the distance between these two centers is $2VK/R_2'$. At any angle it is seen that the co-ordinates of the center measured from the center of the equivalent induction motor are given by

$x=VK\ \sin\beta/R_2'$ and
$y=VK\ \cos\beta/2R_2'$.

Thus it is seen that the locus of centers gives an ellipse where axes are VK/R_2' and $VK/2R_2'$ respectively. This is shown in figure 12.

It is of interest to compare the vector diagram of the polyphase induction motor with that of the three-winding Osnos motor. On the diagrams shown all quantities are given in terms of the primary or for a ratio of 1:1. In order to obtain the true values in any circuit the vector must be multiplied by the appropriate transformation factor. Thus in figure 13 representing the diagram for the induction motor the true value of the secondary current $=T_1/T_4$ times the value shown, the true value of the secondary electromotive force $=T_4/T_1$ times vector and the true value of the resistance is $(T_4/T_1)^2$ times the value shown on the diagram.

In figure 14 is shown a simplified vector diagram for the Osnos motor. Here MC is the electromotive force in the commutated winding referred to the primary side. The position of this vector is varied with brush position. Also BN represents the current in the lap winding and $OB+BN=ON=$ primary value of I. This simple diagram shows the manner in which brush shift improves the power factor.

Appendix IV

The Locus of the Secondary Current

From appendix I it is seen that the admittance of the secondary may be written in the form

$$\frac{(A_0+jA_0')+s(A_1+jA_1')}{(B_0+jB_0')+s(B_1+jB_1')}=x+jy\ \text{(say)}$$

where A and B are constants for given brush angles. On equating reals and imaginaries we have

$$s(B_1x-B_1'y-A_1)+B_0x-B_0'y-A_0=0$$

and

$$s(B_1y+B_1'x-A_1')+B_0y+B_0'x-A_0'=0$$

Eliminating the variable s, we have

$$(B_1x-B_1'y-A_1)(B_0y+B_0'x-A_0')-(B_0x-B_0'y-A_0)(B_1y+B_1'x-A_1')=0$$

or

$$x^2+y^2-x\left[\frac{A_1B_0'+A_0'B_1-B_0A_1'-A_0B_1'}{B_1B_0'-B_0B_1'}\right]+y\left[\frac{A_0'B_1'-A_1B_0-B_0'A_1'+A_0B_1}{B_1B_0'-B_0B_1'}\right]+\frac{A_1A_0'-A_0A_1'}{B_1B_0'-B_0B_1'}=0$$

This is the equation of a circle with center at

$$X_c=\frac{A_1B_0'+A_0'B_1-B_0A_1'-A_0B_1'}{B_1B_0'-B_0B_1'}$$

$$Y_c=\frac{A_1B_0-A_0'B_1'+B_0'A_1'-A_0B_1}{B_1B_0'-B_0B_1'}$$

The radius is given by

$$\left[\frac{A_1A_0'-A_0A_1'}{B_1B_0'-B_0B_1'}\right]^{1/2}$$

It is thus seen that the locus of the current under conditions of constant applied voltage and brush angle is a circle. Varying the brush angle causes a shift in the position of the centers and a variation in the length of the radius.

Due to the complications we adopt a method given by Rudra and Walker. In figure 12 let x, y be the co-ordinates of P such that $OP=I=$the load component of the primary current.

$\theta=$the angle between I and V the impressed electromotive force
$G=$the conductance of the main circuit
$B=$the susceptance thereof

Then

$$VB=I\sin\theta=x$$

and

$$VG=I\cos\theta=y$$

But the reactance $= \frac{B}{B^2+G^2} = \frac{V^2B}{V^2B^2+V^2G^2}$

$$= \frac{xV}{x^2+y^2}$$

Likewise the resistance is $= \frac{yV}{x^2+y^2}$

Now the resistance and reactance of the general equivalent circuit may be written as R_1+R_2 and X_1+X_2 where R_1, X_1 are constant terms belonging to the primary and R_2, X_2 are variable terms of the secondary circuit.

Thus

$$\frac{xV}{x^2+y^2} = X_1+X_2$$

and

$$\frac{yV}{x^2+y^2} = R_1+R_2$$

or

$$\frac{xV-(x^2+y^2)X_1}{x^2+y^2} = X_2$$

and

$$\frac{yV-(x^2+y^2)R_1}{x^2+y^2} = R_2$$

$$\therefore \frac{yV-(x^2+y^2)R_1}{xV-(x^2+y^2)X_1} = \frac{R_2}{X_2}$$

This becomes, after some work,

$$(x^2+y^2) - y\frac{VX_2}{X_2R_1-X_1R_2} + x\frac{VR_2}{X_1R_2-R_1X_2} = 0$$

or

$$x^2+y^2-2By-2Ax=0$$

This is the equation for a circle, provided that A and B are constant. The center of the circle is given by $X_c = A$ and $Y_c = B$ and the radius by $\sqrt{A^2+B^2}$.

Bibliography

Textbooks

1. Die Wechselstromkommutator Maschinen, (a book), **Arnold, La Cour, and Fraenckel.** Volume 5, part 2, Springer, Berlin, 1914.

2. The Induction Motor (a book), **Behrend.** McGraw-Hill, 1921.

Papers

3. Commutation in A-C Commutator Motors, **Lamme.** AIEE Transactions, volume 39, 1920, page 249.

On Series Commutator Motor

4. Circle Diagrams of Polyphase Commutator Machines by the Method of "General Transformer", M. Kostenko and D. Sawalischin. *Elektrotechnik und Maschinenbau*, volume 49 February 8, 1931, page 101.

5. **Rudenberg,** *E.T.Z.* volume 31, 1910, page 1181; volume 32, 1911, page 233.

6. **Karapetoff,** AIEE Transactions, volume 38, 1918, page 347.

Shunt Motor

7. **Blondel,** *L'Eclairage Electrique*, April 25, 1903.

8. **Bragstad,** *E.T.Z.*, 1903, page 368.

9. **Altes,** AIEE Transactions, volume 37, part 1, 1918, page 295.

10. **Ridsdale,** *Electrician* (London), April 15 and following, 1921.

11. Polyphase Commutator Motors and Their Applications, **Shuttleworth.** *Journal* IEE, volume 53, 1915, page 439.

Compensated Commutation Induction Motors

12. Heyland Machine as Motor and as Generator, **Adams.** AIEE Transactions, volume 21, 1904, page 519.

13. Phase Compensation for Induction Motors, **Osnos.** *Elektrotechnische Zeitschrift*, volume 23, 1902, page 919.

14. The New General-Purpose A-C Motor **Weischel.** AIEE Transactions, volume 44, 1925' page 7.

15. The Compensated Induction Motor, **Torda.** *Electrical World*, volume 86, 1925, page 1205.

16. Performance of Phase Advancers, **Rudra and Walker.** *Journal* IEE, volume 70, 1931, page 365.

17. **Arnold,** *Journal* IEE, volume 64, 1926, page 1143.

Braking Magnets for Watt-Hour Meters

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THE function of a braking magnet for a watt-hour meter is to produce a retarding torque acting to balance the electromagnetic driving torque. This retarding torque must be strictly proportional to the velocity of the disk for the watt-hour meter to register accurately. This is readily seen from a consideration of the fundamental watt-hour meter equation.[1]

This is probably the most exacting application for a permanent magnet due not only to the squared relationship between the flux[1,2] and braking torque but also the fact that watt-hour meters are the cash registers for the gigantic electrical industry. They are expected to operate accurately under all conditions indoors and outdoors for periods of five to ten years without servicing.

It is the purpose of this article to discuss suitable test methods for determining characteristics of permanent magnets and to review actual test data that have been obtained on magnets of various types and materials. A careful investigation is outlined showing the relationship of coercive force and the ability of the magnet to withstand demagnetizing fields. An attempt is made to formulate the requirements of an ideal watt-hour meter braking magnet and to illustrate how a designer might make a logical choice from materials that are now available.

A relatively small number of permanent magnet materials have been actually used for braking magnets for watt-hour meters. During the past ten years a number of new and interesting permanent magnet materials have been developed and made commercially available. Table I shows a list of some of these materials which includes both those that have been used and those that might conceivably be used at the present time.

Test Methods

Extreme accuracy of test is needed in determining braking magnet performance for reasons referred to above. As watt-hour meters are installed for service periods ranging from five to ten years, the magnet must remain constant within narrow limits over these periods if the watt-hour meter is to retain its initial calibration. Change in meter magnet strength as a function of time is referred to as aging of the magnet. Such aging may be either positive or negative, depending upon the shape of the magnet, the magnetic material, and its condition of stabilization. It is highly desirable to accurately determine aging trends in order to minimize the time required to judge aging characteristics. Furthermore, as will be shown later, in some magnetic materials aging is extremely small. It is, therefore, essential to develop testing methods that are accurate to within a few hundredths of one per cent. The obtaining of such accuracy is obviously outside the range of ordinary magnetic measurements.

Several methods[2] have been used for determining the permanency of braking magnets. These may be classified as follows:

1. Watt-Hour Meter Method

This method consists of placing magnets in actual watt-hour meters and observing the change of registration as a function of time. It has been shown that unless this method is carried out with extreme care, it is subject to variations

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[1] For all numbered references, see list at end of paper.

Table I

Material Number	Material	Typical Composition	Coercive Force	Residual Induction B_r	BXH Maximum	Mechanical Properties	Commercial Methods of Fabrication
1	Tungsten magnet steel	5 W, 1% C, bal. Fe	70	10,300	320,000	Hard-strong	Hot forge, punch, machine
2	Chromium magnet steel	3.5 Cr, 1% C, bal. Fe	65	9,000	295,000	Hard-strong	Hot forge, punch, cast, machine
3	10% Co magnet steel	2.44 W, 10 Co, 1.0 C, bal. Fe, Cr 1.4	100	9,000		Hard-strong	Hot forge, punch, cast, machine
4	36% Co magnet steel	36% Co, 3.5 Cr, 3 W, bal. Fe, 1% C	210	9,000	930,000	Hard-strong	Hot forge, punch, cast, machine
5	Comol magnet alloy	12 Co, 17 Mo, bal. Fe	245	10,300	1,100,000	Hard-strong	Hot forge, punch, cast, machine
6	Alnico I	12 Al, 20 Ni, 5 Co, bal. Fe	400	7,100	1,330,000	Hard-brittle	Cast, sinter, grind
7	Alnico II	10 Al, 17 Ni, 12.5 Co, 6 Cu, bal. Fe	540	7,200	1,650,000	Hard-brittle	Cast, sinter, grind
8	Alnico III	12 Al, 25 Ni, bal. Fe	400	7,100	1,330,000	Hard-brittle	Cast, sinter, grind
9	Alnico IV	12 Al, 28 Ni, 5 Co, bal. Fe	700	5,200	1,250,000	Hard-brittle	Cast, sinter, grind
10	Fe-Ni-Cu alloy*	20 Fe, 20 Ni, 60 Cu	450	5,300	1,070,000	Ductile-malleable	Cold roll, machine, punch
11	Co-Ni-Cu alloy	41 Co, 24 Ni, 35 Cu	440	5,300	993,000	Ductile-malleable	Cold roll, machine, punch, cast
12	Ni-Ti-Co-FeΔ	16 Ni, 11 Ti, 28 Co, bal. Fe	830	7,500	Not given	Hard-brittle	Cast, sinter, grind
13	Sintered oxide	30.1 Fe_2O_3, 43.6 Fe_3O_4, 26.3 Co_2O_3	860	2,100	770,000	Soft-brittle	Sinter, grind, machine

*Fe-Ni-Cu alloy must be cold worked to develop magnetic properties.

ΔU. S. Patent 2,105,653 Honda.

of several tenths of one per cent, which definitely limits its usefulness.[3]

2. Standard Fluxmeter Method

A standard magnetic method of measuring the total flux of a magnet consists of employing the well-known arrangement of a search coil and a flux-measuring galvanometer. Inaccuracies of two to five per cent are to be expected when using this method, and even under the most carefully controlled conditions, an inaccuracy in the order of one per cent is still to be expected. This method of test measurement was used in obtaining data shown in this article only in connection with the tests on bar magnets. In this study relatively large changes in flux were being measured and a more precise method was not necessary.

3. Precision Flux Test Method

Figure 1 shows an arrangement for measuring the total air-gap flux of a braking magnet. This arrangement has a precision of 0.02 per cent; that is, the readings on a given magnet can be repeated to within this limit. The absolute value of the total flux of the magnet is not, of course, known to anything like this precision. The object of this test is to detect changes and not to measure absolute values so that it is sufficient to measure and record changes from the initial measurement.

In operation the magnet to be measured is placed in a locating fixture and a search coil is dropped under the action of gravity through the magnet air-gap. At the instant the search coil cuts the flux in the magnet gap, the coil assembly actuates a switch which reverses a direct current flowing in the primary of the mutual inductance.

The voltage induced in the secondary

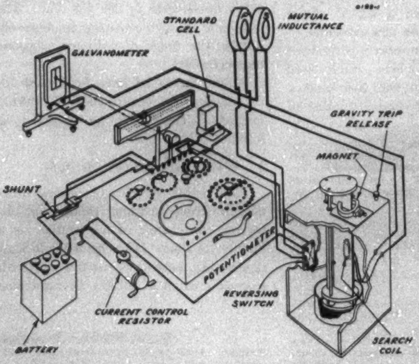


Figure 1. Precision flux-measuring equipment for permanent magnets

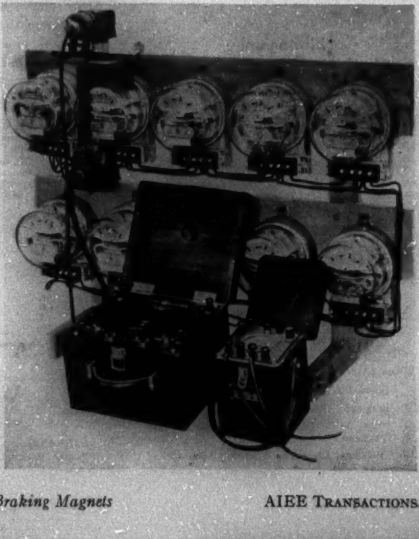


Figure 2 (right). Precision watt-hour meter equipment for measuring the permanency of braking magnets of the mutual inductance opposes the voltage induced in the search coil. A flux galvanometer is in series with the search coil and the secondary of the inductance. If the direct current flowing in the primary of the mutual inductance is adjusted to the correct value, the galvanometer will not deflect thereby indicating a true balance. The direct current is accurately measured with the potentiometer, and this evaluates the flux of the permanent magnet. All data shown on flux measurements were obtained using this method with the exception of those on bar magnets.

4. Precision Watt-Hour Meter Method

The total flux of a permanent magnet is not necessarily an accurate indication of the permanency of the magnet from the standpoint of its ability to produce braking torque. It is conceivable that a redistribution of the total flux over the pole face of the magnet might occur. It is also necessary to measure over-all braking strengths in determining the constancy of the complete magnet assembly. Figure 2 shows a refinement of method number 1 which has a precision within 0.03 per cent based on using the average of a group of ten magnets. This method differs fundamentally from method number 1 in that the human error is eliminated by the use of a photoelectric control and a reference standard of high accuracy is employed to which the portable standard watt-hour meter is compared.[3] This method of test was employed in obtaining data on all actual braking strengths of magnets.

Test Results

For many years the standard material used in the United States for watt-hour meter braking magnets has been chromium steel (material number 2, table I). Stabilizing processes have been developed which make these magnets very permanent, and they have given a high degree of satisfaction in service. It is of interest to study some of the facts that have been learned from operating experience, and research on chromium steel as they present an excellent background against which to judge and analyze new materials for this application.

1. Aging Phenomena

For the purpose of discussion in this paper certain terms are used which for a clear understanding should be definitely defined. The term, "aging" is applied to any change in the total flux or magnetic braking strength of a permanent magnet over a period of time.

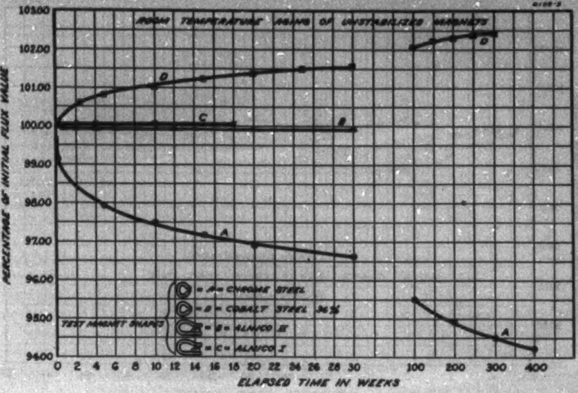


Figure 3. Room-temperature aging of unstabilized magnets

For the purpose of analysis, aging may be divided into two classifications:

(*a*). *Metallurgical Aging.* Applied to changes in total flux or braking strength with time due to metallurgical changes in the material.

(*b*). *Magnetic Aging.* Applied to changes in total flux or braking strength with time due to the magnetic state of the magnet excluding metallurgical changes.

It is also convenient in discussing aging to speak of "natural aging" and "artificial aging."

(*a*). *Natural Aging.* Any changes resulting over a period of time without any severe or unfavorable artificial conditions being present.

(*b*). *Artificial Aging.* Any changes resulting from the application of severe artificial conditions such as extremes in temperature.

Treatments that are given magnets to control aging are referred to as "stabilizing treatments." As used in this discussion, stabilization may be defined as:

Stabilization. The term, "stabilization" refers to certain treatments to reduce or eliminate subsequent aging of magnets.

In figure 3 are shown the aging characteristics of unstabilized magnets made from chromium steel, cobalt steel, and two types of Alnico.

Metallurgical Aging. The aging characteristics shown are largely due to the degree of metallurgical stability of the material, and it is quite obvious that in the case of curves *A* and *D* some form of stabilizing treatment is needed. A large amount of work involving heat treatments at various temperatures and varying lengths of time has shown that for the commonly used chromium steel a low temperature anneal is very effective in eliminating metallurgical aging provided the temperature and time are carefully controlled.

For chromium steel this stabilization treatment has been carried out on magnets both before and after magnetization and careful checks by the precision flux testing method have shown conclusively that the effect is independent of the magnetization of the magnet and must, therefore, be entirely metallurgical in nature.

Curve *D* indicates that unstabilized cobalt magnets tend to increase in flux with time. This phenomenon has not been further investigated since there has been no application of cobalt steel in

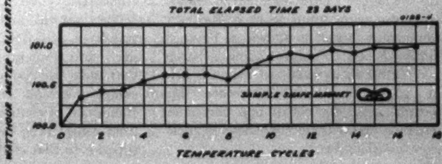


Figure 4. Cyclic aging of unstabilized Alnico II magnets

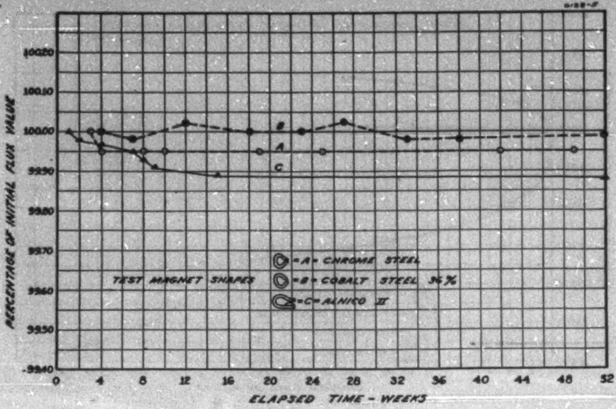


Figure 5. Room-temperature aging of stabilized magnets

watt-hour meter magnets in this country.

Magnetic Aging. In chromium steel magnets magnetic aging is a much smaller factor than is metallurgical aging. A suitable stabilizing treatment for chromium steel to practically eliminate magnetic aging has been found to be a temperature cycle, after the magnet has been fully magnetized and demagnetized an appropriate amount, that will take the magnet well above and below any temperature to which it will be subsequently subjected. As a result of such a temperature cycle, the magnet flux may change as much as 0.75 per cent. If the magnet is given a second and a third temperature cycle identical with the first, slight additional changes in the flux will be noted, although the first cycle produces almost the complete change. This stabilizing effect is entirely destroyed if the magnet is demagnetized after the temperature cycle and must be repeated if stability against temperature cycles is to be again obtained. This clearly indicates that this is a matter of magnetic aging and not metallurgical aging.

This clean-cut distinction between the two forms of aging does not necessarily exist with all forms of magnetic materials. In the newer materials such as Alnico, magnetic aging seems to be of the most importance, since as shown in figure 3, curves *B* and *C*, natural aging is barely perceptible in unstabilized magnets held at a constant ambient temperature. The natural aging for cobalt and chromium magnets under these conditions is, however, considerable.

Careful tests, however, under conditions of varying temperatures indicate that Alnico does require very careful stabilization treatments. This is shown by figure 4 which is the cyclic temperature aging of unstabilized magnets made from Alnico II.

Figure 5 shows the natural aging under constant temperature (22 degrees centigrade) of a group of stabilized magnets of chromium steel, cobalt steel, and Alnico, similar to those shown in figure 3. Figure 6 shows the cyclic temperature aging of stabilized chromium steel and Alnico II.

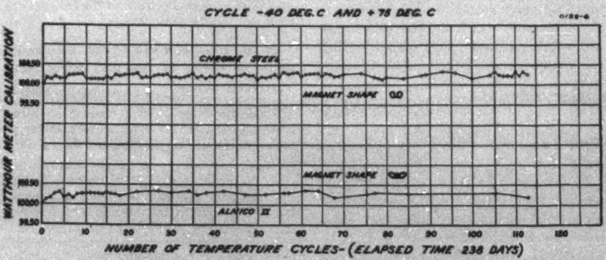


Figure 6. Cyclic temperature aging of stabilized magnets

The natural and artificial aging of stabilized Alnico II magnets and stabilized chromium magnets show definitely that these two types of magnets are fully comparable and satisfactory from the standpoint of continued stability. Stabilized Alnico II is also shown to be much superior to that of the unstabilized magnets shown by figure 4 when subjected to cyclic temperature aging.

It is of importance to note that the present practice in the United States of mounting watt-hour meters out-of-doors makes doubly important cyclic temperature stability. This installation practice has developed very largely within the past ten years. Prior to this watt-hour meters were mounted in protected or semi-protected locations.

2. Miscellaneous Conditions

Several points are of particular interest in regard to their effect on aging:

(*a*). If a magnet assembly of two chromium steel magnets is made up of one strong magnet and one weak magnet, it is of interest to note that the assembly is practically as stable as when two magnets of equal strength are paired.

(*b*). The normal method of demagnetizing a magnet from the fully magnetized state is to subject it to a demagnetizing field supplied by a coil of wire large in diameter in comparison to the size of the magnet. The magnet is placed approximately in the center of the coil and an a-c current is raised from "0" to a maximum and returned to "0." This same effect can, of course, be obtained by holding the current in the coil always at the maximum value but by moving the magnet from a distance into the center of the coil and out again. The direction of application of the demagnetizing field with respect to the magnet is important. Figure 7 shows that if the direction of the field is normal to plane of the magnet and parallel to the watt-hour meter disk, the magnet will show approximately one-sixth of the magnetic aging that will occur if the field is applied in the plane of the magnet and normal to the disk.

(*c*). In service braking magnets may be subjected to demagnetizing fields resulting from transient surges produced by switching or lightning currents flowing in the coils of the watt-hour meter or in adjacent leads. If a magnet is partially demagnetized by a high speed surge, such as would result from lightning, it is of interest to note whether or not the stability of the magnet is affected. Figure 8 shows the results of measurements made on chromium steel and Alnico II braking magnets when subjected to certain surges and then checked by the precision watt-hour meter method. It will be seen that the magnets are comparatively stable after this treatment.

Effect of External Fields

The degree of demagnetization of a braking magnet resulting from a given switching surge or lightning discharge through the watt-hour meter coils or in adjacent conductors is important. This is largely a matter of the fundamental properties of the permanent magnet material. It should be kept in mind in considering the effect of external fields that whether or not a magnet should withstand such fields is closely tied in with installation practice, particularly the location of adjacent conductors which may carry heavy currents or transient surges.

In carrying out a series of measurements to evaluate the relationship between fundamental magnetic properties and the effect of external fields, it was desirable to work with magnetic shapes that were simple. For this reason bar magnets were chosen. Their dimensions and other pertinent data are shown in table II.

All of the bar magnets were fully magnetized and then placed in a Helmholtz coil arrangement with the bar magnet placed lengthwise on the centerline of the coil system midway between the coils. The size of the coils and the number of turns used were always the same for both coils but were varied for the direct current, 60-cycle and fast surge conditions.

Use was made of the method of design devised by K. L. Scott[4] in choosing the dimensions of the bar magnets. The sintered oxide magnet sample was longer than this method of design indicated it should be. The samples of chromium steel, Alnico, and cobalt steel were, however, more nearly in accord with Scott's method. It should be recognized that the data obtained with these bar magnets are not directly applicable to watt-hour meter braking magnets quantitatively, although they are representative qualitatively.

In arriving at the values of demagnetizing force for each of the wave shapes employed, it should be noted that the maximum value of the wave was used.

Figures 9 and 10 show that the demagnetization of the test magnet by either a d-c field or a 60-cycle field is a function of the coercive force of the material. It is of interest to note the permanent magnet materials of widely different compositions fall on smooth curves relating coercive force as determined from a d-c demagnetization curve, and the percentage of the original flux remaining following the application of a demagnetizing field of given strength.

Table II

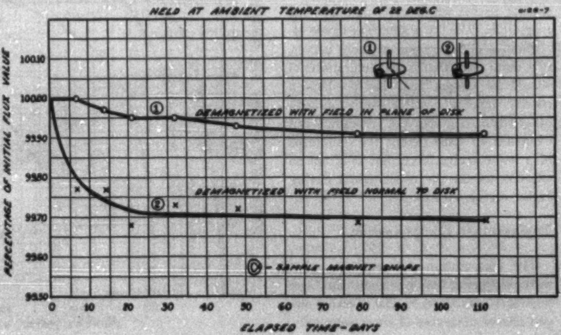
Material	3% (Min) Chromium Steel	36% Cobalt Steel	Alnico II	Sintered Oxide
Cross section (cms²)	1.85	1.62	2.84	1.32
Length (cms)	22.1	12.9	5.2	6.2
B_r (residual induc.)	9,200	10,000	7,200	2,100
H_c (coercive force)	68	210	540	860
Maximum B	13,050	17,300	12,400	5,300
Maximum permeability	92.5	37.8	9.6	
Specific resistance microhms/cm/cm sq.	43.7	73.9	59.7	225×10^6
Density lbs/cu in	0.270	0.294	0.255	0.097
Rockwell hardness	C-60	C-62	C-53	

Figure 11 shows the effect of surges that are very much faster and compare in nature with those resulting from fields of lightning discharge currents. A 15–45 microsecond surge means that the time from initiation of the surge to the crest value of the surge was 15 microseconds and the time from initiation to half value on the tail of the wave was 45 microseconds. A number of surges of different values were used in this work.

A fourth material is shown on this curve which is nonmetallic in nature and as shown in table I is composed of oxides of iron and cobalt. The electrical resistance of this material as shown in table II is extremely high as compared to the other magnetic materials. Differing as this material does in composition, it is seen that it falls on smooth curves relating coercive force and percentage of initial flux remaining after the application of a demagnetizing surge of a given peak value.

In dealing with surges of high speed, it is possible to achieve a measure of protection by encasing the magnet in a protective coating of a highly conductive material, for example, copper. With the simple bar magnet where it is practical to encase the entire magnet, this shielding is quite effective and comes about due to the eddy currents induced in this highly conducting coating. The shielding serves to limit the penetration of the demagnetizing field. Figure 11, curve *A*, shows the gain made by completely encasing the chromium steel bar magnet sample in a two-mil coating of copper. It will be noted that the points on the chromium steel curve are effectively moved up the curve by the copper plate responding as would a material of higher coercive force. It should be noted, of course, that this protective coating would give added surge protection to the other materials even those of high coercive force such as Alnico and sintered oxide. This protection in practice is not necessary except for low coercive force materials such as chromium steel. For very heavy demagnetizing fields, this coating appears to be more effective than for fields of lower intensity. This shielding in actual practice is reduced in effectiveness due to the inability to build up a protective coating of sufficient thickness in exposed air gaps of the magnet.

Figure 7. Effect of demagnetization with the field applied in different directions for chromium-steel watt-hour meter magnets



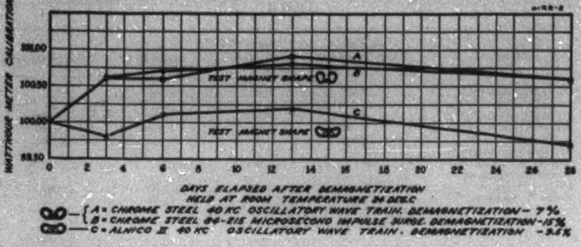


Figure 8. Stability of watt-hour meter magnets following surge demagnetization

Figure 12 shows an interesting effect that is of importance in arriving at the actual degree of demagnetization resulting from a surge. It is seen that repeated surges of like shape and crest value produce increasingly greater demagnetizing effects. This repeated surge effect has some connection with the conductivity of the magnet material as has been borne out by test results on the sintered oxide material. Due to the high electrical resistance of this material as shown by table II, it has very little self-shielding under surge conditions so that the initial surge produces practically the same effect as would be produced by an infinite number of surges of the same magnitude. This is not the case with chromium steel which is successfully demagnetized by a measurable amount up to 14 surges. The results in figure 11 are based on a single surge of the magnitude shown, whereas figure 13 takes into account the accumulative surge effect and is based on the ultimate demagnetization after 15 surges. It will be noted that figure 13 has been plotted for but one wave shape (15–45 microseconds). The relation between coercive force and percentage of original flux remaining still falls on smooth curves which differ in shape and magnitude from those shown in figure 11.

From a consideration of these curves, it is quite clear that a magnetic material should be chosen that has as high a coercive force as possible consistent with other required properties. Shielding can overcome to some extent the handicaps of a low coercive force, but it is not so effective as the use of materials having higher coercive forces.

Temperature Compensation

The temperature compensation of a watt-hour meter braking magnet has been previously described.[5] The effective air-gap flux of a magnet changes with temperature, and as has been pointed out, the change in flux is reflected in the accuracy of registration of the watt-hour meter. Other errors which are usually small in magnitude may be present in a given watt-hour meter, and it may be convenient to compensate the braking magnet to correct for these changes also. Figure 14 shows the results of ambient temperature variation on a modern single-phase watt-hour meter with an uncompensated chromium steel magnet, an uncompensated Alnico II magnet, a compensated chromium steel magnet and a compensated Alnico II magnet. It will be noted that the meter registration with the uncompensated chromium steel magnet and the uncompensated Alnico magnet is practically the same indicating that both materials have the same temperature coefficient.

Temperature compensation of braking magnets is very important for watt-hour meters installed outside of buildings. The errors that would result without compensation as shown by figure 14 may easily reach 2.5 per cent.

Specifications

From the foregoing it is evident that the designer has many problems to consider in selecting the materials and in the actual design of a watt-hour meter braking magnet. It has been found useful to formulate the following specifications in organizing the available data and arriving at the best possible design

1. Stability

The magnet should be stable under conditions of:

(*a*). Normal temperature operation

(*b*). Cyclic temperature operation over the range service conditions impose

(*c*). Stray fields, direct current or alternating current, of the magnitudes they will be subjected to ranging from commercial frequencies to lightning surges

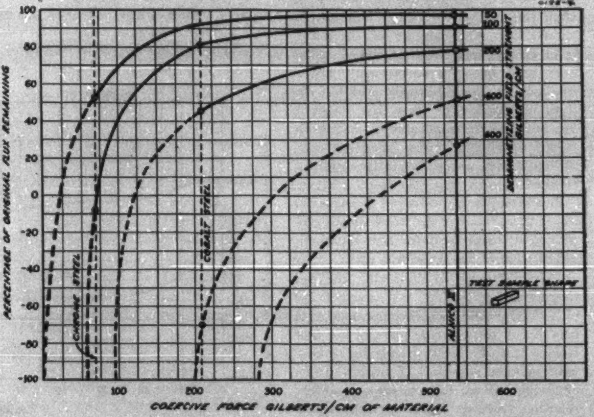
2. Compensation

The magnet must maintain constant braking irrespective of temperature change over the range of operating temperatures. If desirable this compensation may be increased to allow for errors in other parts of the meter.

3. Adjustments

The magnet should be provided with means of adjusting the damping force over a sufficient range to adapt it to the meter with which it is to be used. The adjustment should be smooth, permanent, and easy to use.

Figure 9. Demagnetization due to a d-c field as a function of coercive force



4. Economic Considerations

The size, weight, and cost of the magnet should be such as to make its use economically feasible.

5. Ease of Fabrication

The ease of fabrication of the material is important from the standpoint of uniformity, performance, and in keeping down variations between the magnets in large quantities.

6. Control of Magnet Quality

Complete control of the materials and processes is essential in order to produce high quality magnets in the numbers required.

Discussion of Available Materials

These specifications furnish an excellent basis for evaluating the materials in table I.

As has been previously shown, the requirement that the braking magnet have high inherent resistance to demagnetizing fields, particularly those resulting from lightning surges, can best be met by using a permanent magnet material of high coercive force (H_c).

The second basic magnetic property affecting the choice of material is the residual induction (B_r). It should be remembered that the cross section of a given magnet to produce a definite braking force will vary approximately inversely as the square of value of residual induction (B_r). The required length will vary approximately directly with the value of coercive force (H_c).

Based on these considerations, the outstanding materials in table I are the Alnicos, material No. 12, and sintered oxide, material No. 13.

Of these materials Alnico II represents the best balance, as it has high coercive force coupled with a good value of residual induction (B_r). Material No. 12 is of interest but is, as far as is known, not commercially available, the data having been taken from a United States patent as noted.

Material No. 13, composed of sintered oxides of cobalt and iron has a high coercive force but a low value of residual induction (B_r). The material is also very weak mechanically. These two limitations make it unattractive in comparison with Alnico II.

Actual Design

In considering materials and design of magnets, figure 15 is of interest as it shows in chronological order the watt-hour meter magnets made by one manufacturer from 1889 to 1940. The progress in design and utilization of materials can

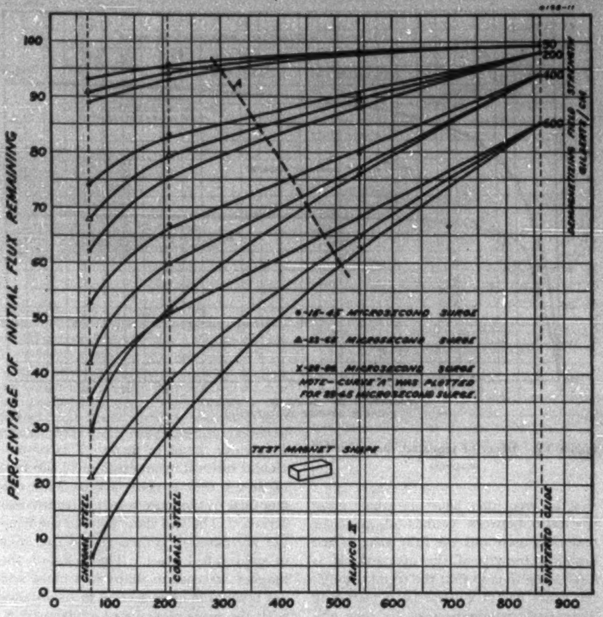
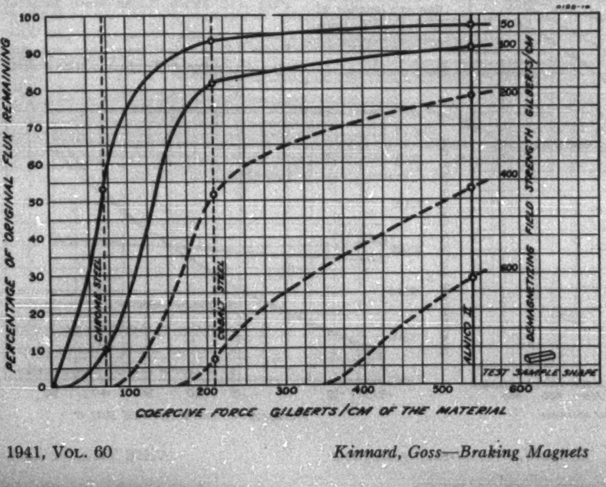


Figure 11. Surge demagnetization as a function of coercive force

Note—Curve *A* was obtained by subjecting the chromium-steel bar magnet with a 0.002-inch thick copper coat to the same surges as were given the magnet without the coat. Compare curve *A* with the vertical line labeled "chrome steel"

Figure 10. Demagnetization due to a 60-cycle field as a function of coercive force



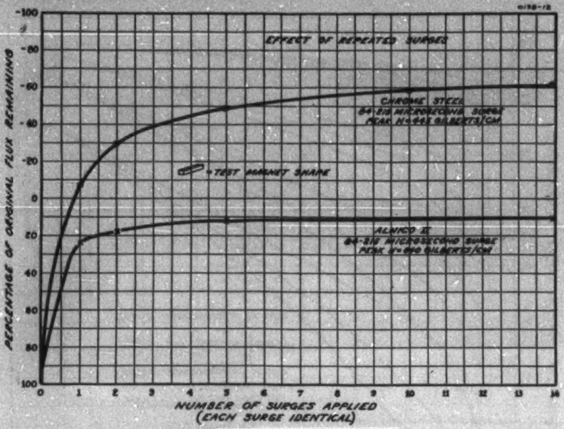


Figure 12. Effect of repeated surges on bar magnets

best be seen from figure 16 which gives the ratio between pounds of magnetic materials used and the fully magnetized braking strength of the magnet. It is interesting to note that the trend through the years has in general been markedly downward, culminating in the very efficient Alnico design shown in No. 10. The braking strength in figure 16 is measured in arbitrary units. For example, a single phase watt-hour meter requires a braking magnet of 515 points.

This magnet No. 10, which is shown in greater detail in figure 17, is of particular interest as an example of the type of design that is necessary to make efficient use of the more recent magnetic materials. The two short poles of the magnet are made of Alnico II and the yoke of cold rolled steel. The poles of the magnet are cast to shape in a mold and are tapered in cross section in accordance with the principles of design given by S. Evershed.[6] The steel yoke provides the return paths for the working flux and gives a rigid assembly with permanent air-gap dimensions. It also provides an excellent shield against demagnetizing fields of the lower frequencies.

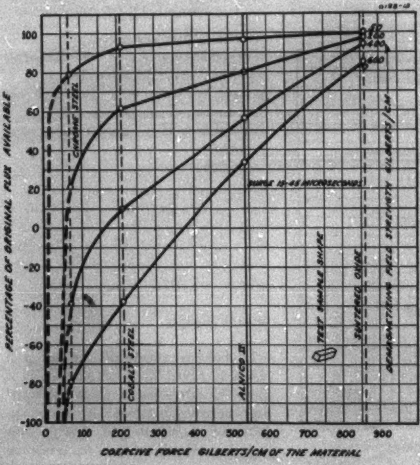


Figure 13. Ultimate surge demagnetization as a function of coercive force

A magnet of the type represented by No. 10 satisfactorily meets all requirements for a braking magnet. A high degree of stability against aging can be accomplished by proper heat treatments as shown by the lower curve in figure 6. High resistance to demagnetizing fields is inherent in this magnet because of the use of Alnico II, and temperature compensation can be satisfactorily provided as is seen from figure 14.

Due to the small size of such a braking magnet, it readily lends itself to great flexibility as far as interchangeability is concerned. It can easily fit into the space occupied by prior types of magnets and with the high magnetic energy available from the new materials, can readily be designed to have equivalent or greater braking force than the older magnets.

Summary and Conclusions

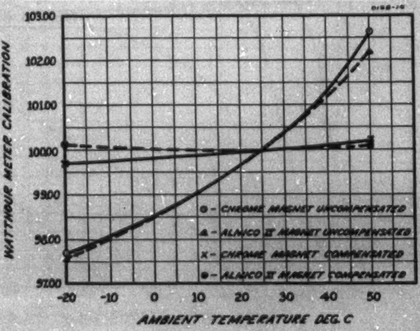
1. It has been shown that accurate methods of measuring changes in the effective flux and braking strength of watt-hour meter magnets are necessary in order to evaluate their characteristics properly. Methods of measurements have been described that are much more accurate than ordinary magnetic measurements and which can be depended upon to detect those changes to within 0.02 to 0.03 per cent.

2. In general, there seem to be two types of aging in connection with permanent magnets which may be classified as:

(*a*). Metallurgical aging
(*b*). Magnetic aging

For chromium steel magnets, metallurgical aging is of more importance whereas for Alnico magnets, the type of aging described as "magnetic" is the principal factor that must be guarded against by suitable stabilizing treatment. It has been shown that unstabilized magnets of chromium steel, cobalt steel, and Alnico exhibit some form of

Figure 14. Ambient temperature characteristics of a modern single-phase watt-hour meter



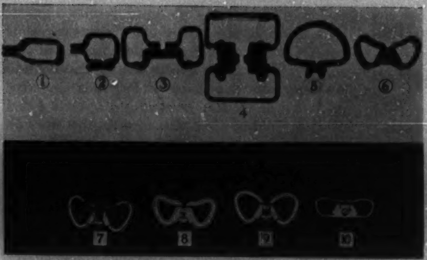
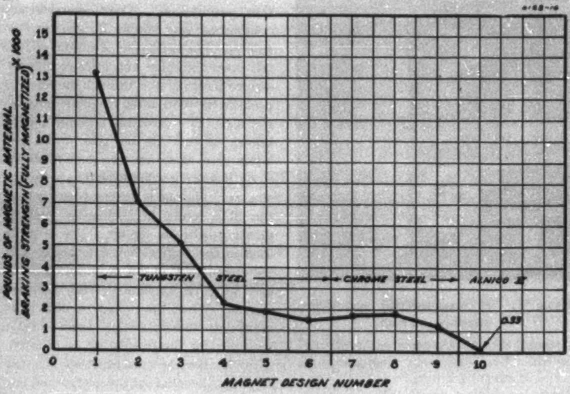


Figure 15. Historical review of watt-hour meter magnets of one manufacturer

aging. Chromium steel and Alnico can, however, be suitably stabilized, and it is probable that cobalt steel and other materials can also be stabilized by proper treatments.

3. The ability of a magnet to resist demagnetization by fields varying from those caused by direct currents to lightning surges is shown to be a function of the coercive force of the magnetic materials. This relationship is shown quantitatively on standard bar magnets, and the great improvement that can be expected for watt-hour meter magnet application by the selection of high coercive material such as Alnico II is clearly shown by experimental data.

Figure 16. Weight of permanent-magnet material used to produce unit damping strength of the magnet designs shown in figure 15



High conductivity coatings such as copper are somewhat effective in protecting a magnet against high speed surges. When applied to the lower coercive materials such as chromium steel, the protection obtained is not so effective as is the use of a higher coercive force material such as Alnico.

4. Temperature compensation preferably by means of the conventional magnetic shunt must be applied to magnets made from Alnico the same as has been the practice with those made from chromium steel. It is shown that about the same degree of compensation is necessary and that this can be accomplished satisfactorily.

5. After the analysis of important characteristics, definite specifications are drawn up to assist in designing the best possible meter braking magnet and in choosing the best available material.

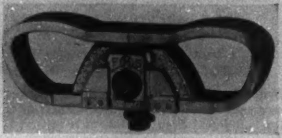


Figure 17. A modern watt-hour meter braking magnet using Alnico II

6. A design of a modern watt-hour meter braking magnet using Alnico II material has been shown to closely meet the specifications of an ideal magnet. The magnet is especially outstanding in regard to resistance to demagnetizing fields and possesses a high degree of permanency.

References

1. Development of a Modern Watt-Hour Meter, **I. F. Kinnard** and **H. E. Trekell.** Electrical Engineering, January 1937, pages 172–9.

2. Aluminum-Nickel-Cobalt Brake Magnets for Watt-Hour Meters, **S. Green.** AIEE Transactions, volume 58, 1939, pages 791–6.

3. A Precision Rotating Standard for the Measurement of Kilowatt-Hours, **J. H. Goss** and **A. Hansen, Jr.** AIEE Transactions, volume 59, 1940 (July section), pages 412–15.

4. Magnet Steels and Permanent Magnets, **K. L. Scott.** AIEE Transactions, volume 51, June 1932, pages 410–17.

5. Temperature Errors in Induction Watt-Hour Meters, **I. F. Kinnard** and **H. T. Faus.** AIEE Transactions, volume 44, 1925, pages 275–87.

6. Permanent Magnets in Theory and Practice, **S. Evershed.** *Journal* of the Institution of Electrical Engineers, volume 58, September 1920, pages 780–837.

7. Alnico—Its Properties and Possibilities, **J. Q. Adams.** *General Electric Review*, volume 41, December 1938, pages 518–22.

8. The Output and Optimum Design of Permanent Magnets Subjected to Demagnetizing Forces, **A. J. Hornfeck** and **R. F. Edgar.** AIEE Transactions, volume 59, 1940, pages 1017–24.

9. Permanent Magnets, **R. F. Edgar.** *General Electric Review*, October 1935, pages 466–9.

The Vertical-Flow Interrupter and Its Application to "Oil-Poor" Circuit Breakers

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Synopsis: The historical background of the development of the vertical-flow interrupter for "oil-poor" circuit breakers is reviewed. A description of the interrupter and its theory of operation is given. The application of two of these interrupters to a 138-kv $1^{1}/_{2}$-million-kva "oil-poor" porcelain-clad circuit breaker is shown, together with laboratory test results.

A review is given of important field tests on a 138-kv single-pole unit, tested line to ground on a large 220-kv operating system up to approximately $2^{1}/_{4}$ million three-phase kva. Magnetic and cathode-ray oscillograms of these tests are shown.

OUTDOOR oil breakers with metal tanks have long been the accepted standard in this country, and they have met operating requirements satisfactorily except for a possible demand for lower oil content which would simplify maintenance and reduce fire hazard. The experimental development of an "oil-poor" breaker has been undertaken to determine the merits of this type of device. "Oil-poor" design naturally dictates porcelain-clad construction similar in type to that used in Europe. Conventional interrupters used in dead-tank breakers are not ideally adapted either electrically or mechanically to porcelain structures. It has therefore, been necessary to develop a new type of interrupter.

The conception of a controlled arc confined between corrugated walls of insulating material and subjected to the relative motion of an insulating fluid has proved effective, and the desire to retain the general principle has led to the construction of the vertical flow interrupter which will be described. Its physical characteristics and the small amount of arc energy which it liberates make it especially well adapted to porcelain enclosures.

Theory of Arc Extinction and Operation of the Vertical Flow Interrupters

In any interrupting device, the extinction of an a-c arc following a current zero depends upon the ability of the interrupter to transform the arcing space from a conductor to a good insulator before the circuit is capable of restoring voltage to the breaker terminals.[1] The time available for this voltage rise is a very small fraction of a cycle, usually so small that the insertion of finite objects, such as solid insulation or even liquids across the arc path is most difficult.[2] The conductivity of the arc is determined by the density of ionization in it during the time the current is flowing. Compton, Slepian, Saha and others have pointed out that the chief source of ionization is thermal and thus depends on high temperatures. Cooling the arc stream permits a greater rate of deionization. The rate of cooling depends upon the configuration and construction of the arc stream, that is its diameter and the distance from any point of greatest ionization to a relative cool object. It is thus apparent that when an arc stream is interspersed with cool volumes of gas, liquid, vapor, or un-ionized groups of molecules, these not only act as cooling centers but nuclei to which ions may migrate and become eliminated by diffusion and recombination.

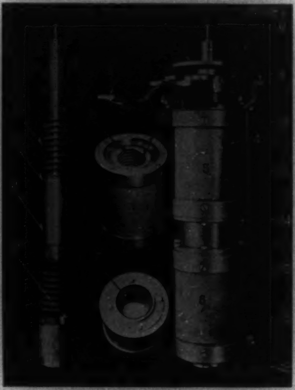

Figure 1. A two-interrupter vertical-flow assembly and parts

Assembly at right. Moving element (plunger) at left. Two views into interrupter well

From this very general review of arc-extinction theory it is apparent that an uncontrolled arc in an open body of oil or in an interrupter which allows the arc to surround itself with the relatively quiet atmosphere of a gas bubble, is going to be very slow in changing from a conducting state to that of a good insulator.

This theory suggests the necessity of limiting the random motion of an arc by surrounding it with corrugated insulating walls, and subjecting it to relative motion between it and a suitable fluid. There are two general means of obtaining relative motion between the arc and the fluid, first to hold the liquid in position and move the arc through it and second to hold the arc stationary and force the fluid past the arc. The former was accomplished very simply by stacking insulating plates with slots of various widths one upon the other and interspersing V-shaped pieces of iron through the stack to produce the motivating magnetic force on the arc. This structure is commercially known as the De-ion grid.[3,4] In the latter case there are two general directions of moving a fluid relative to an arc stream. The one is the cross-blast where the oil is forced in a direction perpendicular to the arc stream. The other is used in the vertical-flow interrupter and requires a flow of oil longitudinally to and in close proximity with the arc stream. When the motion of the oil is parallel to the arc stream, there is little tendency for the moving oil to interfere with the flow of current other than at zero, thus keeping the arc energy and gas generation low. At the same time the oil rapidly removes the excess gas from around the arc as fast as it is generated, so that as the current approaches its zero value the last little filament of arc may be regarded as passing through a small hole or chimney in the oil. Since the oil is under considerable pressure and adequate means is provided for the small amount of generated gas to escape freely, it may be assumed that the wall of oil surrounding the gas moves toward the arc as the arc space contracts with the

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[1] For all numbered references, see list at end of paper.

decrease of current in the cyclic variation. Thus, when the diameter of the hole through the oil becomes small, especially near the current zero, the process of deionization by diffusion to the surrounding cool oil is greatly speeded up. Soon after the current reaches its zero value, and before the generated voltage can shift from the circuit inductance to the arc terminals, the ionization in the remaining gas is reduced to a very small value. Since the re-ignition or breakdown voltage through this last thread of gas varies inversely with the degree of ionization in the gas and directly with the absolute pressure on the gas, it is possible to interrupt a large number of volts per inch of arc length without danger of breaking the continuity of the arc path just preceding the normal current zero. This avoids any rapid increase in arc voltage and the subsequent voltage surges.

Description of the Vertical-Flow Interrupter

Figures 1 and 2 show a 138-kv interrupting element consisting of two vertical-flow units in series. Each interrupter consists of an insulating tube or cylinder with metal fittings at either end. The upper end of the cylinder forms a chamber into which the stationary contacts are built and is vented to the outside. The lower end is lined with fiber washers of different inside diameters, so as to form a corrugated well into which the moving contact and its corrugated follower is plunged upon opening. The follower completely fills the well except for a small opening on one side in which the arc is drawn. Upon opening, the metal cylinder just below the moving contact moves out of the well as the contact and follower move in. The area of this cylinder is less than that of the follower, therefore, as the arc is drawn down into the well, the oil in the annular space below the contact is forced vertically up past the contact through the small corrugated passage and made to flow parallel to and surrounding the arc, as previously discussed in the theory. The two interrupters are similar and when mounted in series form a single stationary assembly and a single moving assembly. When the breaker is closed the conducting sections of the moving and stationary assemblies are staggered to form a circuit from one end to the other. When the breaker is open they move into a position opposite each other. As many of these interrupters may be stacked one upon the other as required by the voltage rating of the breaker.

Breaker Structure

Figures 3 and 4 show the general construction of the single-pole-unit 138-kv vertical-flow "oil-poor" circuit breaker. It consists of two separate porcelain-clad compartments. The lower one is the insulation to ground and houses the current transformer. The upper one is the insulation between terminals and contains the interrupting elements and the oil disconnect.

The current transformer is supported on the floor of the lower compartment, the low-voltage terminals coming out through small bushings in the base and the high-voltage leads extending upward into the metal casing between the two porcelains. One of these leads comes out through the metal casing and forms one breaker terminal. The other is attached to the disconnecting member which on closing engages the interrupting element.

The interrupting element consisting of two vertical-flow interrupters in series is suspended from the top metal casing which forms the other breaker terminal.

The moving contacts are mounted on an axially located shaft which runs through the center of the interrupters and is biased toward the open position by springs located in the top metal casing.

In closing, the disconnecting member rises from the metal casing between the porcelains, moves about nine inches through oil where it engages the moving element of the interrupters, and carries it to the closed circuit position.

In opening, this sequence is reversed by first obtaining full separation of the interrupting contacts and then removing potential from them by inserting the disconnect gap.

The disconnecting gap is capable of withstanding full potential and is so arranged that a breakdown between ter-

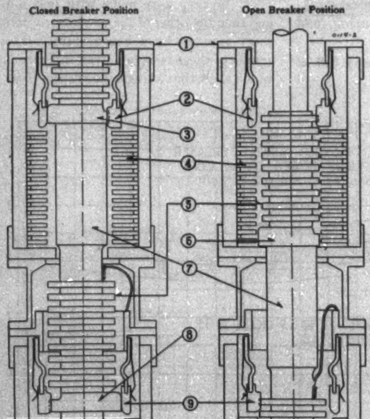


Figure 2. Section view of interrupters

- 1—Upper terminal connects here
- 2—Upper fixed contacts
- 3—Upper movable contact (closed position)
- 4—Corrugated wells (insulation)
- 5—Corrugated plungers (insulation)
- 6—Upper movable contact (open position)
- 7—Metal cylinder carrying upper contact
- 8—Lower movable contact (closed position)
- 9—Lower fixed contacts
- 10—Point of contact with disconnecting member

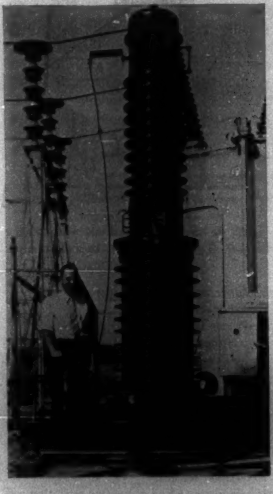


Figure 3. Single-pole "oil-poor" 138-kv vertical-flow circuit breaker set up for test in the Laguna Bell substation, Southern California Edison Company

minals must be a puncture through the oil in the disconnect gap, or a creepage across the entire insulation between terminals and not involving the interrupters.

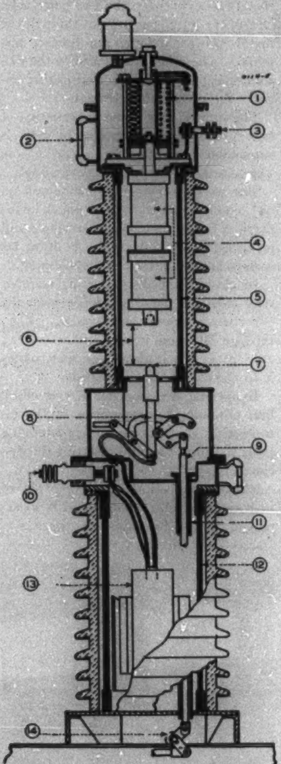


Figure 4. Schematic sectional view of the breaker (figure 3) shown in full opened position

1—Opening accelerating springs
2—Oil level
3—Power terminal
4—Interrupters
5—Insulating supporting tube
6—Oil disconnecting gap between terminals, breaker open
7—Disconnecting member
8—Pole unit mechanism
9—Insulating operating rod for both opening and closing
10—Power terminal
11—Insulating tube surrounding operating rod
12—Insulating supporting tube
13—Oil-insulated current transformer
14—Oiltight stuffing gland around torsion shaft

With this construction no external disconnecting switch is required to remove voltage from the breaker.

The disconnecting member is operated by a set of straight-line levers located in the center metal casing, which in turn are operated by an insulating pull rod extending down through the current transformer compartment to a torsion shaft. This torsion shaft extends through a stuffing box and is operated through cranks and pull rods by an electric mechanism. The pull rod which passes through the transformer compartment is surrounded by a small insulating tube which connects the center metal casing with the metal base housing the torsion shaft. The oil in the upper porcelain which is used for interrupting can, therefore, be drained through the tube surrounding the pull rod and is entirely separated from the oil in the lower porcelain or current-transformer compartment.

The pole unit is separable in the center, allowing each compartment to be handled individually without disturbing the oil in the transformer case.

The interrupting element can be withdrawn from the top by removing the dome and removing four hanger bolts. The dome is held in position by springs which allow it to act as a pop valve in case of sustained arcing.

The entire structure is very rigid, consisting of metal base, central metal casing, and top metal housing. Between these parts are heavy Micarta tubes, one concentric with and inside of each porcelain. These tubes take all the mechanical shock, hold the porcelain in compression, and are capable of supporting the structure, retaining the oil and otherwise rendering the breaker operable even though the porcelains were broken or removed.

Interrupting Characteristics—Laboratory Tests

At the present it is impossible to obtain sufficient power in the most modern laboratories to make direct tests on high-power breakers up to their full ratings. Yet, there are several ways of approximating such tests and obtaining the interrupting characteristics of a circuit breaker.

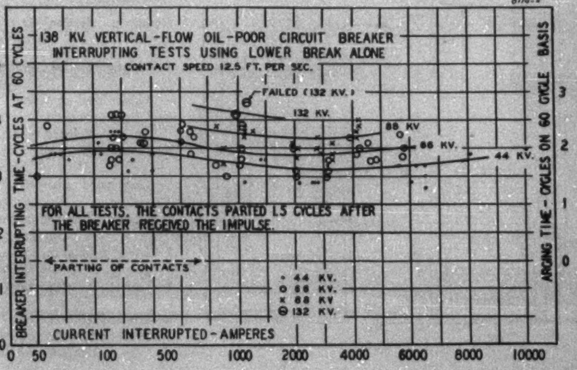
The synthetic method of using a high value of current at a reduced voltage and then at the instant of arc extinction applying a higher transient voltage to give the equivalent of a high current and voltage, is sometimes used.[5,6]

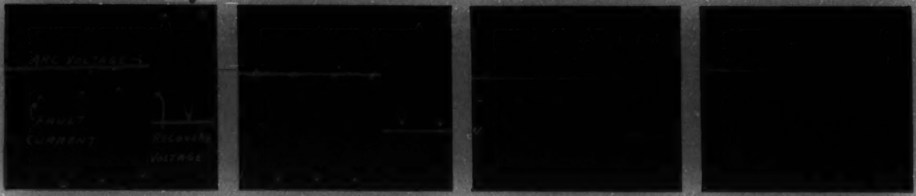
Sometimes the interrupting characteristics of a device are obtained by testing over the entire range of available current at some given voltage and then choose some suitable current, and at this value of current determining the voltage limitation. The proper interpretation of these results may lead to correct conclusions, but much depends upon the characteristics of the device itself.

Whenever it is possible to subdivide the interrupter into its elemental units and test each to its limit, greatest certainty of results is obtained.[7] This method is ideally applicable to the vertical-flow breaker for 138 kv.

Tests on the 138-kv vertical-flow "oil-poor" breaker were made with first one and then the other of the two interrupters short-circuited by a heavy shunt. The results of these tests are given in figure 5, which show that a single interrupter (one-

Figure 5. Interrupting results using half of a pole unit





10,300 amperes 44 kv

5,700 amperes 66 kv

4,100 amperes 88 kv

1,050 amperes 132 kv

Figure 6. Representative oscillograms from data of figure 5

half of a two-interrupter assembly) can handle all values of current up to over 10,000 amperes at 44 kv; slightly less than 6,000 amperes at 66 kv; and over 4,000 amperes at 88 kv. Successful operations were also obtained at 1,000 amperes and 132 kv. The interrupting time is very constant over the entire current range and increases only slightly with increase in voltage. Since the arc is confined to the corrugated channel in which it is drawn and subjected only to the longitudinal flow of oil, which does not interfere greatly with the flow of current except near the zero value, the arc energy and external demonstration were very small. The oscillograms shown in figure 6 are representative of those from which the results of figure 5 were obtained, and show the almost negligible arc voltage so essential in "oil-poor" breaker construction.

If the arc voltage, arc energy, and therefore the mechanical shock are very small for one interrupter, it is reasonable to expect two interrupters in series to function satisfactorily and be capable of handling twice the kilovolt-amperes of one, providing the voltage is reasonably well divided between the two. In this breaker the greatest unbalance occurs with the top terminal grounded and the lower one made high, in which 40 per cent of the voltage appears on the top interrupter and 60 per cent on the lower one. With this impractical connection, a complete series of tests at both 132 and 88 kv was made. With the terminals reversed, where the division of voltage was perfect, no difference in the performance of the breaker could be detected.

Figure 7 shows the results of a number of interrupting tests on the single-pole 138-kv unit covering the current range up to 5,500 amperes at 88 kv and 3,000 amperes at 132 kv.

Figure 8 shows a similar curve for closing-opening tests at 88 and 132 kv on the 138-kv single-pole unit. It should be noted that the opening speed for the closing-opening tests was 11.7 feet per second, while the opening speed for the opening test was 9.3 feet per second.

Figure 9 shows two typical oscillograms on closing-opening tests, one at 88 kv and one at 132 kv.

All of the laboratory tests reported above were made on the 60-cycle basis.

Field Tests

Although the laboratory tests on this circuit breaker were rigorous and uniformly successful, it remains for field tests to verify conclusions and establish confidence in any entirely new design. This opportunity was offered through the courtesy of the Southern California Edison Company, who operate one of the largest 220-kv systems in the world. They permitted short-circuiting the heart of this system under load conditions up to 2,200,000 kva three phase equivalent at their Laguna Bell substation. During October of 1938, the single-pole 138-kv $1^{1}/_{2}$-million-kva vertical-flow "oil-poor" breaker was mounted near the center of of the 220-kv switchyard as shown in figure 3. The upper end or terminal was connected to one line of the 220-kv system and the other terminal was grounded as shown. The voltage across the pole unit was, therefore, approximately 124 kv.

Table I shows the results of the ten short-circuit tests made at powers varying from 200,000 to 2,200,000 kva calculated as a three-phase equivalent. At each power setting one opening and one closing-opening test were made. The current range extended from 540 amperes to 5,900 amperes rms measured at the point of contact separation and takes into consideration the d-c component of the asymmetrical waves.

Figure 10 shows several magnetic-oscil-

Figure 7. Opening performance of a 138-kv single-pole unit on laboratory tests at 88 kv and 132 kv. Also on 220-kv field circuit tests, line-to-ground

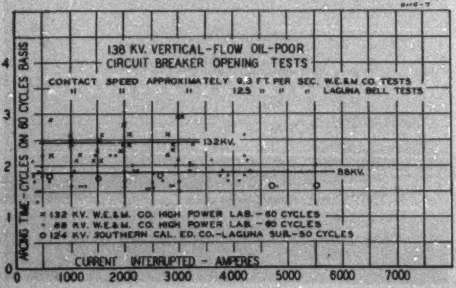


Figure 8. Closing–opening performance of a 138-kv single-pole unit on laboratory tests at 88 kv and 132 kv. Also on 220-kv field circuit tests, line-to-ground

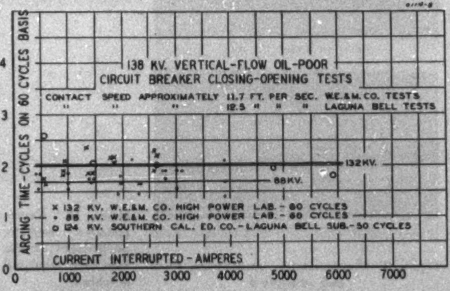


Table I. Westinghouse Vertical-Flow "Oil-Poor" Circuit Breaker 138-Kv Single-Pole Line-to-Ground Tests on Southern California Edison Company 220-Kv System Laguna Bell Substation—October 9 and 10, 1938

Test Number	Operation*	RMS Amp Interrupted	Kva Interrupted Three-Phase Basis	Kv Across Breaker	Time 50-Cycle Basis: Short-Circuit Time	Time 50-Cycle Basis: Breaker Time	Time 50-Cycle Basis: Arcing Time	Approx. Contact Separation at Arc Extinction (Inches)	Voltage Dip on Customer's System (Per Cent)	Oil Test Kv	Maximum Instantaneous Restored Voltage Per Cent of Crest
2 *OP*	O	1,430	530,000	124	3.5	3.2	1.4	3³/₄	Trace	New	
2*AOP*	CO	1,450	540,000	124	3.9	3.6	1.7	4³/₄	1		
3 *OP*	O	2,620	975,000	124	3.6	3.3	1.5	4	3		
3*AOP*	CO	2,640	980,000	124	4.0	3.6	1.7	4³/₄	3		
4 *OP*	O	4,700	1,750,000	124	3.4	3.1	1.3	3¹/₄	3		
4*AOP*	CO	4,800	1,790,000	124	3.9	3.6	1.7	4³/₄	3	27	
October 16, 1938											
1 *OP*	O	600	224,000	124	3.6	3.3	1.5	4	Trace	New	200
1*AOP*	CO	540	200,000	124	4.4	4.1	2.2	5³/₄	Trace		180
5 *OP*	O	5,500	2,050,000	124	3.4	3.1	1.3	3¹/₄	7		110
5*AOP*	CO	5,900	2,200,000	124	3.8	3.5	1.6	4¹/₂	7	27	115

In all CO operations the moving contact came into engagement with the stationary, even though the full 1¹/₈ inches of the contact lap was not reached before the motion was reversed.

*"O" indicates opening; "CO" indicates closing-opening.

logram records of these tests. Figure 11 shows the accompanying cathode-ray oscillograms. They indicate that for the lower-power tests the maximum instantaneous restored voltage reached 200 per cent of crest of the 50-cycle wave appearing across the breaker. The natural frequency of this oscillation was about 1,800 and 1,900 cycles per second. Its natural decrement reduced its crest value to that of the 50-cycle wave in approximately ¹/₄₀₀ of a second. This did not produce any evidence of voltage stress on the system.

For the highest concentration of power in which feeders from several stations were used, each seemed to have its own oscillatory frequency, resulting in a composite, with a maximum instantaneous value of 115 per cent above crest voltage and a natural frequency of the predominating harmonic of 700 to 800 cycles per second.

The total short-circuit duration varied from 3.4 to 3.6 cycles (50 per second) for the opening tests and from 3.8 to 4.4 cycles (50 per second) for the closing-opening tests. The relay time varied from 0.3 to 0.4 cycles (50 per second) and the time from rise of trip coil current to part of contacts varied from 1.8 to 1.9

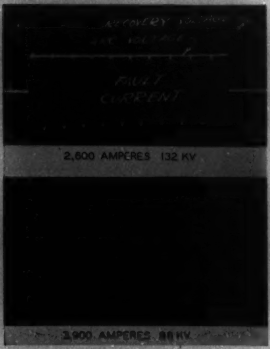


Figure 9. Representative oscillograms from data of figure 8

Figure 10. Representative magnetic oscillograms of the field tests tabulated in table I

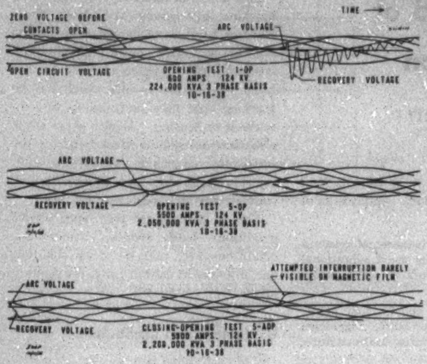


Figure 11. Cathode-ray oscillograms corresponding to the magnetic oscillograms of figure 10

cycles (50 per second), thus leaving an arcing time for the opening tests of from 1.3 to 1.5 cycles and a closing-opening arcing time from 1.6 to 2.2 cycles.

The increase in arcing time of the closing-opening tests over the opening tests does not appear consistent with results obtained in the laboratory where the breaker was allowed to close in and latch before opening. While the contacts closed far enough to touch and carry the current before reversing, they did not go entirely closed and thus the contact speed over the arcing period may have been slightly low. Even though they did not slide into full engagement which consists of over an inch of lap, enough contact was made to demonstrate their ability to close in on a short circuit and carry current if time delay relays were used.

The arcing time as a function of current for this set of tests is plotted in figures 7 and 8. The points are superimposed on the laboratory results, to show the consistency of operation over the entire range of current. The contact speed for the field tests was 12.5 feet per second while for the laboratory tests it varied from 9.3 to 11.7 feet per second. Also, the time from rise of trip coil current to part of contacts was 2.3 cycles for the field tests, whereas it was slightly less than 2 cycles for the laboratory tests.

The contact separation at the point of arc extinction varied from $3^1/_4$ inches to $5^3/_4$ inches. Therefore, for the two breaks in series the maximum volts interrupted per inch of contact separation or inch of arc length (since there is no looping of the arc) is almost 20,000 volts.

Recording meters placed on the customer's system indicated a very low dip in the system voltage, with no loss of synchronous load.

Preceding the first series of six tests and the second series of four tests, the interrupting compartment of the breaker was filled with clean oil (approximately 60 gallons). After the tests very little carbon was visible either in the oil or on the interrupters. The oil tested 27 kv with a standard test cup.

The contact resistance was measured before and after each series of two tests, consisting of an opening and a closing-opening test, and found to remain constant at 0.00086 ohm. Inspection of the breaker after the tests also indicated that the contacts as well as all other parts were in excellent condition.

Throughout the entire series of tests the external demonstration was negligible, no jump, vibration nor evidence of high internal pressures. On the heaviest power interruptions there was only a slight whiff of smoke to be seen with not a trace of soot or oil around the opening of the muffler.

Again, it is a privilege to thank the engineers of the Southern California Edison Company for their co-operation in making possible such extensive tests on a circuit breaker in which they may have only an academic interest.

No description of this development would be complete without a word of appreciation for the assistance and co-operation given in this development by the engineering department and by Doctor J. Slepian and his research group.

Conclusions

The vertical-flow interrupter possesses many desirable characteristics. Intensive testing under laboratory conditions, supplemented by field tests on a heavy concentration of power, indicates that within the range of currents encountered in American practice from 132 kv and upward its performance may prove to be very acceptable.

References

1. Extinction of an A-C Arc, Doctor J. Slepian. AIEE Transactions, volume 47, 1928.
2. Displacement and Diffusion in Fluid-Flow Arc Extinction, J. Slepian, AIEE Transactions, volume 60, 1341 (April section), pages 162–7.
3. Extinction of a Long A-C Arc, Doctor J. Slepian. AIEE Transactions, volume 49, 1930.
4. Use of Oil in Arc Rupture, B. P. Baker and H. M. Wilcox. AIEE Transactions, volume 49, 1930.
5. Special Tests on Impulse Circuit Breakers, W. F. Skeats. Electrical Engineering (AIEE Transactions), June 1936.
6. Discussion of Skeats paper by Doctor J. Slepian. Electrical Engineering (AIEE Transactions), November 1936, page 1256.
7. Circuit Breakers for Boulder Dam Line, H. M. Wilcox and W. M. Leeds. Electrical Engineering (AIEE Transactions), volume 55, page 626.

The "Black Band" Method of Commutation Observation

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Synopsis: Good commutation is essential for the reliable performance of d-c machines with low maintenance costs. As an aid to the commutation adjustment of machines in the factory, or in the field, a systematic test procedure has been found useful on large machines to indicate certain features of the machine's adjustment, notably the commutating field strength and brush position. This information is obtained directly from curves known as "black bands" or "buck and boost curves" plotted from test observations. The significance and interpretation of these curves in terms of machine adjustments, differential between commutation-reactance voltage and voltage induced by interpole flux are presented and results of observations on machines for special service (variable-speed motors and variable-voltage generators) are also given in this paper. Included are typically shaped "black bands," obtained in practice. The black-band method has proved to be an extremely powerful and useful tool in commutation adjustment and it is hoped that this discussion of it will lead to an increasing use of the method with a corresponding furtherance of knowledge about it, thus enlarging its scope of usefulness.

The Commutation Problem

ONE of the most important problems in the satisfactory operation of d-c machines is that of obtaining successful commutation. This problem has always been one requiring considerable attention because of the numerous factors influencing commutation, many of which are difficult to evaluate. A detailed discussion of these many factors is beyond the scope of this paper, but a brief tabulation of them here will be an aid to an appreciation of the part black-band tests play in the solution of the commutation problem. The factors may be divided into four main groups:

(1). Mechanical condition of rotor
(2). Brush conditions
(3). Atmospheric conditions
(4). Electrical and magnetic conditions

Breaking each of these groups up and naming a few of the many components of each, we have:

Under (1) mechanical trueness of commutator surface at operating speed, vibrational tendencies of rotor, condition of mica between commutator bars.

Under (2) brush top and bottom angle, brush-grade material, pressure, alignment, spacing around commutator, and condition and type of springs.

Under (3) humidity and temperature, presence of oil, oil vapors, paint, acid fumes, abrasive or chemically active dust, or steam in atmosphere.

Under (4) brush position, commutating field strength, commutating pole tip and gap.

The black-band method of commutation observation is important because it gives a measure of the commutating ability of a d-c machine. In other words, the black-band method is a sort of "commutation meter" which gives a more accurate picture of the commutation performance of a machine than is obtainable by other methods. Analysis of black-band observations give a measure of both the overload and transient load-commutating ability of a d-c machine, as well as indicating certain features of the machine's adjustment and design such as brush position, commutating field strength, and commutating zone width.

Significance of the Black-Band Method of Commutation Observation

This method of judging commutation performance derives its name from the zone of variable commutating field strength at any load, in which commutation will be sparkless or black. This zone is obtained by first opposing (bucking) then aiding (boosting) the load current flowing through the commutating field coils at any value of load on the machine, and plotting curves against load of amperes boost and buck at which light general sparking just begins. The resulting curves represent a band between the limits of which commutation is black, hence the name "black band." A black band on an ideal machine from a commutation performance standpoint is shown in figure 1.

It is not to be inferred, however, that a machine must operate at all loads within its black band, as successful commutation does not necessarily mean perfectly black commutation. A machine which commutates successfully is one which has no sparking that may be injurious to commutator or brushes. Hence, operation of a machine outside its black band may be satisfactory as long as the sparking is not injurious and does not cause excessive commutator and brush maintenance.

This is particularly true since service conditions—load swings, atmospheric conditions, etc., may be so variable that one adjustment cannot fulfill the requirements of all conditions.

Usefulness and Limitations

Black bands are useful both in adjusting new machines for proper commutation, and in determining what the difficulty may be with machines in service which are giving commutation trouble. They are far from infallible, and because of the many variable factors involved, occasionally a band is obtained which seems difficult to interpret in the light of present theory on this subject. Certain generalizations can be made, however, and more knowledge about them should be obtained as time goes on.

One of the limitations of black bands is that integrity of observation and recording is required to judge equally both upper and lower limits of the band, to assure determination of the true band center and proper band width. Since the band limits are taken by eye, they may tend to vary with observers. Another limitation is that it is sometimes difficult to determine the band limit if sparking is not uniform on all the brushes. This cannot be considered strictly a limitation since it is possible, if circumstances warrant, to set up a commutation grading system whereby the sparking condition of each brush is weighted. On machines with large numbers of brushes, however, this procedure is usually not practicable. If a single stud on a machine or a few brushes on a stud are limiting the band width, investigation should be made to correct the condition which is faulty about this stud, and eliminate its early sparking.

For a band to present a true picture, so that a very similar band could be obtained if observations were made a second time with conditions unchanged, the commutator film should be well established and stable; and the brushes must be well fitted to the commutator, have a good polish, and be free from any tend-

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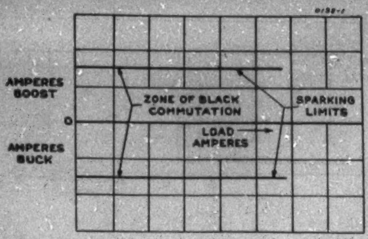

Figure 1 (left). Ideal black band

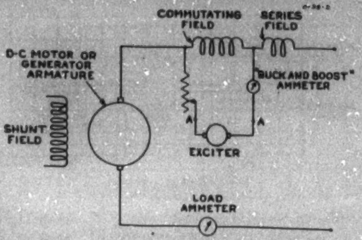

Figure 2 (right). Connections for taking black band

ency to vibrate or to change position in their holders ("take a multiple seat"). For design comparison of machines it is possible to partially control this commutator surface and brush condition by the use of a light application of brush seater on the commutator just before taking the black-band readings. This is not desirable for final adjustment, of course, where the band should be taken with the commutator and brush conditions as nearly as possible the same as those the machine will have in actual service.

Instructions for Taking Black Bands

In order to take a black band on a machine, it is necessary to vary both the load and the commutating field strength. For variation in load, the machine may be loaded on a water box or connected "pump back" if duplicate or similar units are available. For varying the commutating field strength a variable-voltage power supply of sufficient capacity is connected across the commutating field winding (and compensating or pole-face winding if one is present and it is interconnected with the commutating field winding) of the machine under observation. The most convenient type of variable-voltage supply is an exciter, the armature of which is connected through a water box or other suitable variable resistance to the commutating field terminals as shown in figure 2. The buck-and-boost current can then be varied both by the resistance in the armature circuit of the exciter and by field control of the exciter.

A convenient method of determining whether the ammeter is connected to read "buck" or "boost" is to disconnect the exciter lines at *A* in figure 2 and, with a low value of load on the machine, short the lines from the commutating field together, thus shunting some current around the commutating field. If the ammeter reads up scale, it is connected to read "buck" current.

Load should never be quickly removed from a machine which has buck or boost on it, as this may result in severe sparking and damage to the brushes and commutator.

The determination of the band limit at any load is a matter requiring some judgment on the part of the observer. If one or two brushes begin to spark slightly, and the buck-or-boost current can be increased considerably before otherwise general sparking begins without excessive sparking on the part of these few brushes, it would probably be best not to give much weight to these brushes in determining the black-band limit. When taking bands on machines with compara-

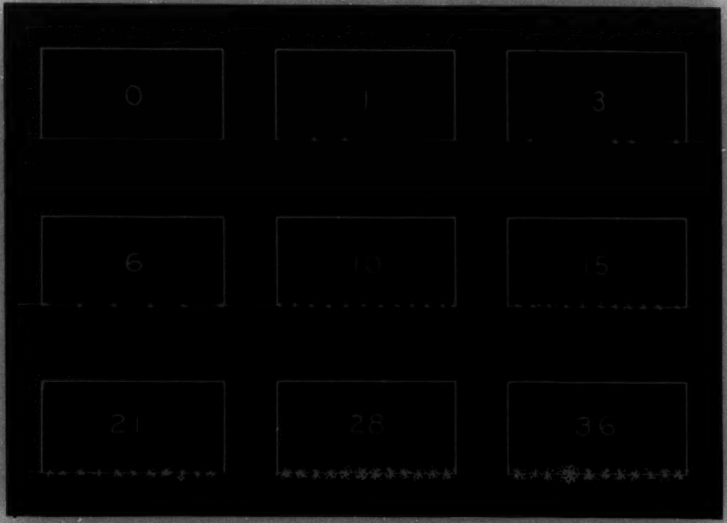

Figure 3. Sparking chart

(*a*)—Each diagram represents one brush

(*b*)—Numbers indicate relative energy or burning power of the sparks and may be added and averaged for all the brushes and brush holders of a machine

(*c*)—Streamers just begin to extend from the edge of the brush in condition 10 and increase in length for more severe conditions

(*d*)—In condition 36 a very hot spot slowly travels across brush and burns back toe of brush

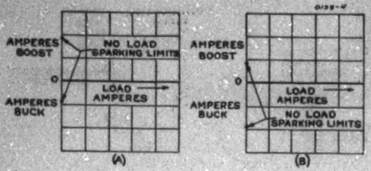

Figure 4 (left). Effect of brush position on no-load points of black-band curves

(*A*)—Brushes ahead of neutral, motor
Brushes behind neutral, generator

(*B*)—Brushes behind neutral, motor
Brushes ahead of neutral, generator

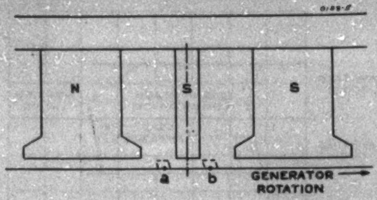

Figure 5 (right)

tively few brushes, a note should be made on the curve sheet regarding the sparking condition of each brush at the band limit decided upon.

The above-described method of obtaining band limits will probably be satisfactory for most cases, but it may not suffice for some conditions of machine load, speed, or voltage where the black band may be either very narrow or even nonexistent. For these conditions a refined method of accurately determining black-band limits, or of finding a band of minimum sparking (useful for adjustment of a commutating field strength and brush position where there is no black band) is necessary. If need for such a method arises, the following commutation grading system may be used:

Figure 3 illustrates various degrees of sparking to which arbitrary numerical values have been assigned representing as nearly as possible the amount of energy in the sparks or the amount of burning which the sparks will produce on the commutator. This grading system has been used very successfully for several years in building motors for transportation service in the General Electric Company. Each individual brush should be graded for a given set of conditions, i.e., for a particular load and particular amount of buck or boost. The average of these numerical values will fairly represent the over-all sparking of the machine. When there is no black band, it is desirable to traverse the whole range of buck and boost from fairly bad sparking at one extreme to similarly bad sparking at the other extreme. Such traverses should be made consistently with either falling commutating field strength or rising commutating field strength in order to avoid discrepancies in results due to hysteresis in the magnetic circuits. In making a traverse the range of buck and boost is divided into a suitable number of steps and each step is held until all the brushes have been graded. The process is repeated for each step and the results averaged and plotted. From these data the condition of best commutation can be determined accurately even if there is no black band. Such points of best commutation for various loads on the machine will line up as a continuation of the center line of a black band and a line connecting them can be used in a similar way to that in which a black band is used.

A good black band may be obtained by "bucking and boosting" the commutating field at but four or five load points including no load on a generator (running light load on a motor) and the maximum load at which the machine may be run long enough to get black-band points.

Theoretical Effects of Various Factors on Black Bands

The adjustment of a machine, principally its brush position and commutating field strength, as well as some design features, such as the width of neutral zone, commutating pole gap, and tip width, have certain theoretical effects on the black band. A knowledge of these effects is essential to the interpretation of the band.

1. Brush Position

Consider a brush ahead of neutral on a generator as at *b* in figure 5. Commutation is aided by flux from the main south pole, hence the amount of *buck* which may be applied to the commutating field before sparking begins at no load is greater than the amount of *boost* which may be applied, see figure 4*B*. If the brush is behind neutral, as at *a* figure 5 the main north pole flux *hinders* the commutating pole flux hence more *boost* than *buck* may be applied as in figure 4*A*. Similar reasoning is gone through for motor action with opposite results since the commutating pole shown in figure 5 becomes a north pole for a motor. With brushes on electrical neutral the black band is symmetrical about the zero point at no load. This is borne out by black bands taken on many reversing-type motors, whose brushes must be on electrical neutral (brush position giving same full-load speed with fixed shunt-field excitation, both rotations). They have shown the no-load points symmetrical about the zero axis.

2. Commutating Field Strength

A tilted black band with no-load points centered, as shown by the solid lines in figure 6 indicates brushes on electrical neutral but too weak a commutating field since the amount of boost which can be applied increases uniformly with load. Conversely, a downward tilted black band as shown by the dotted curve indicates too strong a field. This can be swung into the desired horizontal position, as shown by the dashed curve, by shunting a proper amount of current from

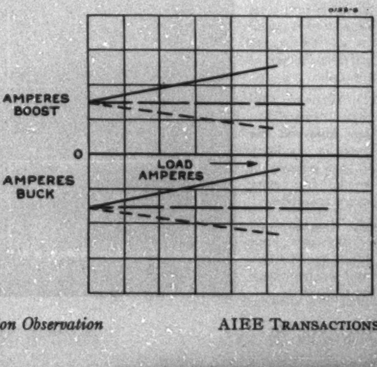

Figure 6. Effect of commutating field strength on black bands

Solid-line band—commutating field weak

Dotted-line band—commutating field strong

Dashed-line band—commutating field correct the commutating field, or reducing the commutating field strength by substitution of nonmagnetic for magnetic shims under the head of the commutating pole.

If the black band is horizontal but displaced upward for a motor, as shown in figure 7, it indicates a proper commutating field adjustment because it is horizontal but the brushes ahead of neutral as indicated by the displacement of the no-load points, see figure 4*A*. Such a band might continue out to a very high value of load, as discussed later. This may also be tilted by changing the commutating field strength and can be centered by shifting brushes.

3. Black-Band Width and Closure

The wider the black band, the greater may be the variation in commutating field strength from normal before visible sparking occurs. A wide band is desirable, obviously at all loads at which the machine will operate, so as to have black commutation during transient load periods, when the commutating field flux lags the armature current slightly due to hysteresis and eddy current effects in the magnetic circuit. A wide band also gives greater margin for changes in commutator film, brush riding conditions, etc., without getting into commutation difficulty.

Under normal commutator film and brush riding conditions, it is felt that the distribution of the neutral zone flux used for commutation (this may consist of commutating pole and main pole fringing flux) is one of the most important factors determining the width of the black band and its closure with load. How the commutating pole-tip width or gap length affects band width is not clearly known except that there are optimum conditions for both and any divergence therefrom makes the black band narrower than normal.

A black band of normal width based on experience with machines above 300 horsepower appears to have a total spread of boost plus buck in per cent of rated load amperes at rated full-load current about in accordance with the following table:

Commutator Peripheral Speed (Feet Per Minute)	Commutating Field Current, Boost Plus Buck at Full Load in Per Cent of Rated Full-Load Current
Machines 300 to about 1,000 horsepower	
1,000	10 to 20
5,000	1.5 to 5
Larger machines	
1,500	5 to 10
5,000	1 to 4

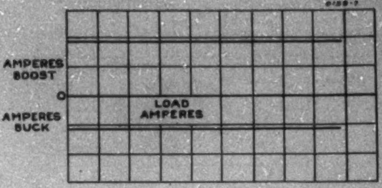

Figure 7. Black band for a motor with correct commutating field strength but brushes ahead of neutral

Data for this table were obtained from black bands taken on 30 variable-speed motors at both base and top speeds. A scatter curve of band width against commutator peripheral speed was plotted, and the limits indicated were obtained.

4. Differential Reactance Voltage

The black-band curve is of assistance in studying a quantity called differential reactance voltage. This voltage is defined as the difference between the commutation reactance voltage (the voltage in the coils undergoing commutation by virtue of the current changing from a positive to negative value through the inductive armature coils) and the counter voltage induced in these coils by the commutating pole flux, to promote the change of current. In an ideal machine, the differential voltage is zero at all loads, since the magnitude and space or time distribution of commutating voltage is such as to completely cancel the reactance voltage at each point in the commutating zone. If the differential voltage is not zero, because these conditions are not met, it varies directly with load. To show this, assume that at full load on a machine the average voltage induced in the coils undergoing commutation is 90 per cent of the average reactance voltage. The average differential reactance voltage is, therefore, 10 per cent. Both of these voltages are proportional to load current (if the commutating pole circuit is not saturated), so at half load the commutating voltage would be 45 per cent and the reactance voltage 50 per cent with the differential reactance voltage 5 per cent or half of what it was at full load. The effect of main-pole fringing flux is to produce a differential reactance voltage proportional to it at any value of load.

The relationship between this differential reactance voltage and load may be obtained by drawing on the black-band curve a line dividing the band, as nearly as possible, into two symmetrical halves, as line *a–b*, figure 8*A*. In this case the brushes are on neutral, so point *a* of the line is at the origin. Since the brushes are on neutral, there is negligible pole-fringing flux in the commutating zone and the differential reactance voltage is proportional to load. If the brushes are

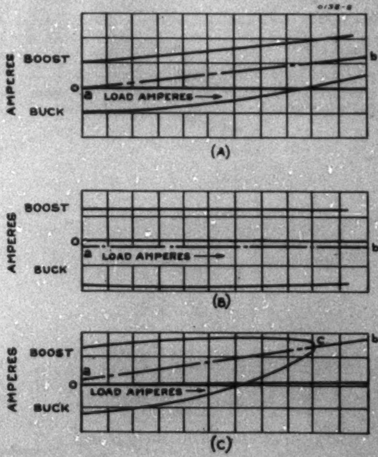

Figure 8. Reactance-voltage lines for various black bands

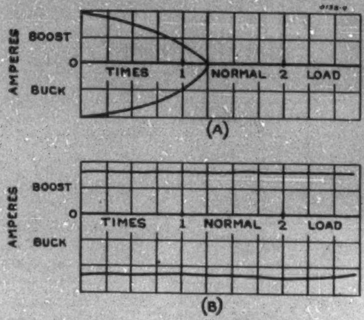

Figure 9 (left). Effect of slight brush shift (B) on some motor bands such as (A)

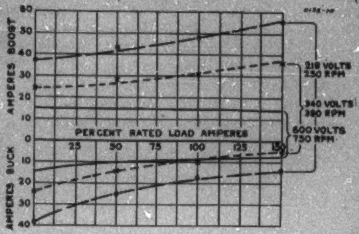

Figure 10 (right). Black-band curves on 1,000-horsepower 250/750-rpm 600-volt reversing motor showing effect of speed with constant main-pole flux

off neutral, point *a* of the line which strikes the no-load ordinate midway between the band limits is not on the origin but is displaced as in figure 8*B*. In this case the differential reactance voltage line is seen to be a constant distance away from the zero axis; this is because the differential reactance voltage is being caused by main-pole flux which does not vary with load.

It is interesting to note that the differential reactance voltage line coincides with the line of best commutation at loads beyond the point of closure of the black band, as line $c-b$ on figure 8*C*.

Observations in Practice

The use of black bands for observing and recording commutation performance for several years has led to certain useful practical conclusions regarding their interpretation and value in adjusting machines.

1. Band Width and Closure

The ideal black band is one that is symmetrical about the zero axis of buck and boost, and is wide open out to a large value of load, as shown in figure 1. It is not always desirable, however, to set the brushes on electrical neutral to get no-load band symmetry, as this sometimes causes a band to close up early as in figure 9*A*. Often on a motor a slight backward shift of the brushes causes 9*A* to become 9*B*, which is, of course, a better band from the standpoint of overload commutation and band width. If, on a reversing motor (where brushes must be set on electrical neutral) the overload commutation is required to be better than that shown by figure 9*A*, it is necessary to improve such a band by commutating field adjustment, commutating pole-gap change, etc.

The closing of a well-centered band under heavy load may be due to the mechanical problem of commutating or even conducting large currents. Greater heating at high load may affect brushes, commutator, and the resistance of armature coils undergoing commutation. Closing may also be due to differential reactance voltage increasing with load until at heavy loads the brush is no longer able to absorb it.

It has been observed that on some d-c motors a spark persists on the "leading" or "entering" edge of the brush regardless of the commutating field strength (value of buck or boost), and that this sparking could be obviated only by a narrower commutating pole tip or a decrease in commutating pole gap. "Trailing" or "leaving" edge sparking does not appear to be critical of commutating pole-tip width unless the rotation of the machine is such that the *last* conductor to commutate in a slot is a *top* conductor.

2. Commutating Pole-Circuit Saturation

A sudden tipping up and closing of the black band at a large value of load may indicate saturation in the commutating pole circuit, or brush characteristics associated with current density. To find which of these causes may be giving such a band, it is customary to remove half the brushes on the machine and take another band out to half the maximum load current to which the first band was taken. This permits the same brush-current density for the band, but the commutating pole-saturation effect should be absent if that is the trouble. This test is not conclusive, however, because of the possibility that the tipping up of the band is caused by changed flux-leakage conditions at the higher loads. As a further check, a commutating pole-saturation curve with the machine under actual load may be taken, using a fluxmeter to measure commutating pole flux.

Special Observations

To obtain further information concerning black-band characteristics on machines under varying operating conditions,

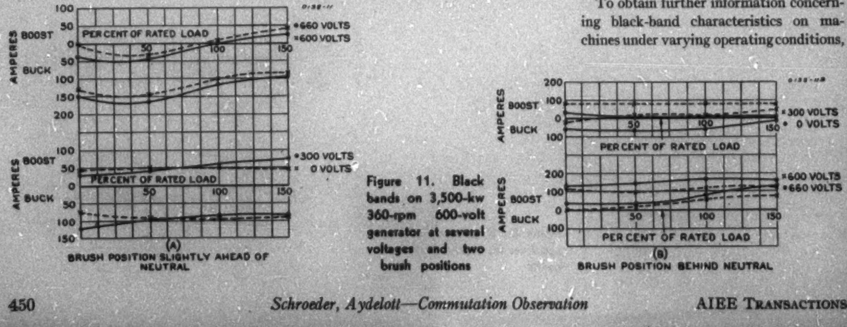

Figure 11. Black bands on 3,500-kw 360-rpm 600-volt generator at several voltages and two brush positions

a series of special tests were made. Two of these tests, one on a variable-speed motor and one on a variable-voltage generator, are here discussed.

1. Band Width and Tilt Versus Speed, Variable-Speed Motors

It has been found that the black band of a variable-speed motor is always narrower at top than at base speed. Further it is necessary on variable-speed motors to adjust the commutating field a little strong at top speed so that it will be only slightly weak at base speed; in other words, the effect of the commutating field, all other things being equal, increases with speed.

To determine whether or not a change in main-field fringing flux was partly responsible for these conditions, a test was recently made on a 1,000-horsepower 250/750-rpm 600-volt motor. As this was a reversing motor, brushes were on electrical neutral. Black bands were taken at 250, 390, and 750 rpm by varying the terminal voltage, thus keeping the same main-field flux condition for each speed. To hold constant flux at all speeds, it was necessary to hold normal top-speed field current, in order not to exceed rated voltage at top speed. The bands are shown in figure 10. The band at top speed was narrower than the one at base speed, and the band shown for 390 rpm and 340 volts was almost identical with an earlier band taken at 390 rpm and 600 volts. Hence, it appears that with brushes on electrical neutral, main-field fringing flux has little effect on black-band width. It will be seen that the commutating field strength, which was about right for 750 rpm, apparently decreased with speed (tipping up of black band increased).

2. Effect of Brush Shift on Generator Bands at Various Voltages

To show the effect of shifting the brushes on a generator off neutral on black bands taken at various generator terminal voltages, such bands were taken on a 3,500-kw 360-rpm 600-volt generator designed for Ward Leonard control. The bands are for generator action, two brush positions, 660, 600, 300, and 0-volts terminal. Figure 11*A* is for brushes ahead of neutral, and figure 11*B* for brushes behind neutral.

Figure 11*A* shows that with decreasing generator terminal voltage the no-load points of the band move more and more in the boost direction, until at zero voltage the band is nearly centered. The brushes are under the influence of fringing flux from a main pole of polarity like that of their commutating pole and as the main flux (generator terminal voltage) decreases this influence decreases also until at zero main-field flux the band is nearly centered in spite of the brushes being off neutral. Figure 11*B* shows the same effect in the opposite direction for the brushes behind neutral. Again at zero-terminal voltage the no-load band points are nearly centered.

As a result of these tests, it has been suggested that the proper way in which to adjust a machine too large to be loaded in test is to set the brushes for no-load band symmetry at rated voltage, then adjust the commutating field for a centered band at normal load current with the machine driven as a generator, terminals short-circuited.

Conclusion

When the black-band method is used in the systematic adjustment of a machine for best commutation, it removes much of the uncertainty and variability of commutation. While the black band indicates directly only brush position and commutating-zone magnetic conditions, it also indicates indirectly, some information regarding stability of commutator film and friction, a phase of the commutation problem which must still be solved largely by experience.

Increased Capacity and Interconnections of the Columbia Gas and Electric Corporation

C. W. DeFOREST
FELLOW AIEE

Introduction

THE electrical properties operated by the Columbia system are located in the southwestern quadrant of Ohio and in northern Kentucky. These properties are shown in figure 1. The actual operations are carried on by the Dayton group, and the Cincinnati group.

The principal loads served are in Cincinnati, in or near Dayton, Ohio, and in the industrial towns of the Great Miami Valley, lying between Cincinnati and Dayton. In the period from 1918 through 1939, the load has grown from 70,000 kw to 360,000 kw. To meet this increase in load, there was in progress almost continuously a program of increase in generating capacity and interconnections. The program is shown on the chart of figure 2. The growth of summer load in late years has increased the required reserve capacity and somewhat accelerated the whole expansion program.

Transmission and substations to serve the load were developed along with the additions to capacity. This development was influenced by operating experiences which included system instability, bus faults, and floods.

Generating Stations

In 1916 the principal generating stations of the area were Third Street and Fourth Street stations in Dayton, the Plum Street station in Cincinnati, and Newport station in Newport, Kentucky. The rapid growth of load during the war required increase of generating capacity in both localities. The Dayton and Cincinnati stations obtained their condensing water from the Miami-Erie Canal. This canal was ultimately to be abandoned so that it was necessary to provide not only for additional load, but to plan new stations on new sites to provide for the ultimate loss of these existing plants.

Millers Ford Station

Wolf Creek, the Stillwater River, and the Mad River, a spring fed stream, flow into the Miami River at Dayton and, therefore, the Miami River offered a good supply of cooling water. Railroad facilities were available at the point where the Big Four Railroad crossed the Miami

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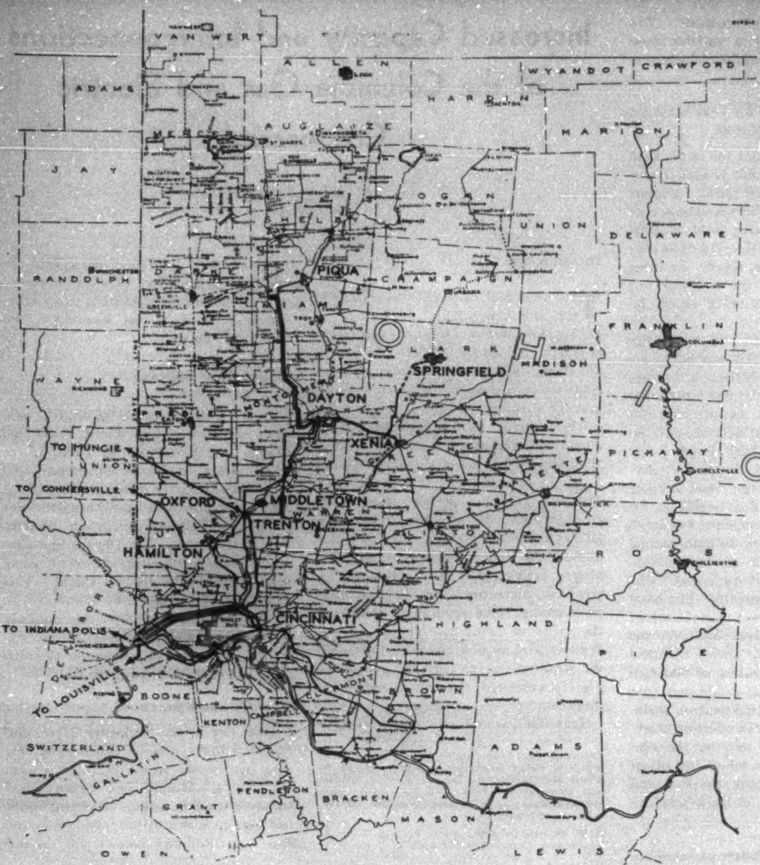
VAN WERT
ALLEN
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PIQUA
RANDOLPH
DARKE
CHAMPAIGN
MIAMI
FRANKLIN
CLARK
MADISON
SPRINGFIELD
WAYNE
PREBLE
MONTGOMERY
DAYTON
XENIA
GREENE
TO MUNCIE
UNION
PICKAWAY
FAYETTE
TO CONNERSVILLE
OXFORD
MIDDLETOWN
WARREN
TRENTON
CLINTON
HAMILTON
ROSS
CINCINNATI
TO INDIANAPOLIS
HIGHLAND
DEARBORN
PIKE
TO LOUISVILLE
BOONE
CLERMONT
BROWN
KENTON
CAMPBELL
SWITZERLAND
ADAMS
GALLATIN
GRANT
PENDLETON
BRACKEN
MASON
OWEN
LEWIS

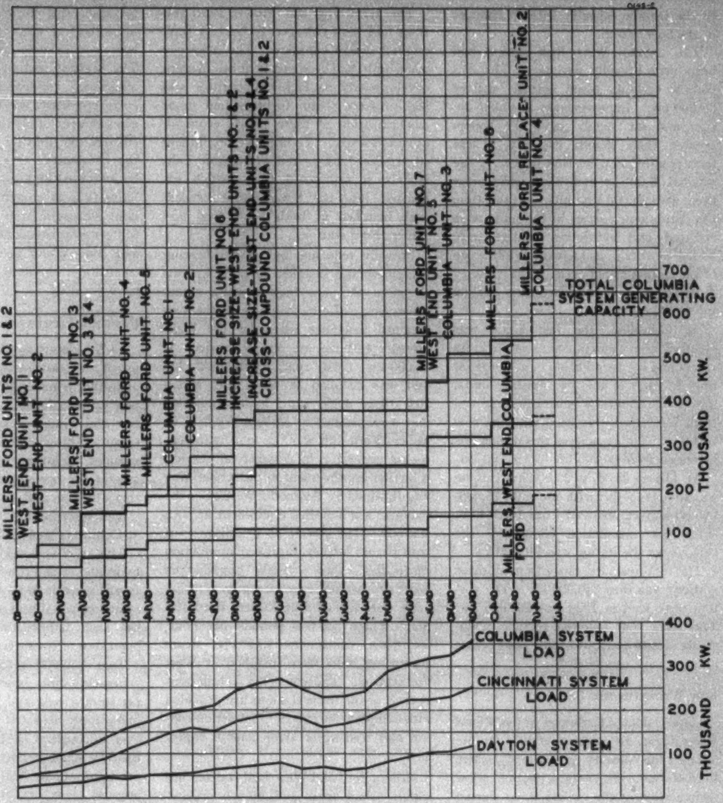
Figure 1. Electrical properties of the Columbia Gas and Electric Corporation

River at the south city limits of Dayton. The site was close to the load center and was within the Miami conservancy district which provided protection against flood of the river. It was decided to develop the new Millers Ford station at this point. The ultimate development was considered for 100,000 kw, with an initial installation of two 12,500-kw units. The steam conditions were 225 pounds per square inch and 550 degrees Fahrenheit. Construction was started in 1916 and the first two units were put in operation in 1918. From this time until 1924 three 20,000-kw units were installed to bring the total installed capacity of the station to 85,000 kw.

West End Station

In the Cincinnati area, the Ohio River provides an ample supply of good water for power-station condensers. Rail facilities are available on the Ohio side of the river from the east end of Cincinnati to the Indiana state line on the west. The river itself was available for transportation of coal from the West Virginia fields. The company owned property along the river upon which was located an artificial gas plant. This plant had been rendered obsolete by the introduction of natural gas to this region. The site, near the load center, approximately one mile southwest of the center of the business district, was chosen for the location of the new West End station. The station was designed for an ultimate capacity of 100,000 kw in four 25,000-kw units. Two units were installed in the initial installation. The steam conditions were 600 degrees Fahrenheit and 260 pounds per square inch. Construction was started in 1916. A principal problem in the design of this station was to allow for operation through the wide range of river elevations. Government records give flood stages as far back as 1832. The most severe flood on record had occurred in 1884 when the river reached a stage of 71.1 feet. The design level of West End station was set at one foot above this maximum flood stage. However, the design was such

Figure 2. Increase in load and generating capacity of the Columbia Gas and Electric Corporation



that by doing some emergency work, the plant could operate through floods rising several feet above this level. Condensing water must be handled through the entire range of levels. The most economical operation is obtained by locating the condensers at the low water level of the river. To accomplish this, condenser-well construction was used. This was one of the earliest stations to be so constructed. The first unit was completed in 1918, the second a few months later in 1919. The fourth unit was put in operation at West End in 1921. This brought the station to the 100,000-kw capacity for which it was originally designed.

Columbia Station

. The Cincinnati company then sought a new site for capacity to meet its continually growing load. Extensive studies were made of a mine-mouth steam plant, of southern mountain hydro sites, and of a new Ohio River steam plant. Transmission studies were made and the economics of each project thoroughly investigated. However the final analysis resulted in the selection of the site of the present Columbia station which is on the Ohio River, 20 miles downstream from Cincinnati. This location had practically unlimited space and facilities for future development. Coal could be received and handled conveniently by rail or barge. The river provided an excellent condensing water supply. Located as it is at the Indiana state line, it served to establish the territory of this system to that point. Transmission lines could be brought out of the station in any direction desired. ~~The station coal~~ storage, crib house, and discharge and intake tunnels were designed for an ultimate capacity of 320,000 kw. The initial installation consisted of two 45,000-kw units. The first unit was completed in the fall of 1925 and the second in the spring of 1926. These were tandem compound units operating on a reheat cycle with inlet steam at 725 degrees Fahrenheit and 600 pounds per square inch. The boiler plant of this station was the first to be initially designed for pulverized coal fuel.

Further Increases

The next increase in generating capacity came in 1928 and 1929. This program involved all three generating stations. All three stations had an excess of boiler capacity and it was possible to add to the turbine capacity without changes in the boiler room.

The original West End station had been very liberally designed. The 25,000-kw turbines could actually deliver 35,000 kw. Improvements in stoker design and air supply gave the boilers greater than design capacity. These facts resulted in a decision to remove the original 25,000-kw generators and replace them with 36,000-kw units. At this time the diaphragm nozzles in the units were changed and extraction heaters were added to obtain an increase in efficiency of the station-steam cycle. The capacity at this station was increased from 100,000 kw to 144,000 kw.

At Columbia, the installation was changed from tandem compound to cross compound by the addition of a 16,000-kw unit to each of the two existing units. Two of the 25,000-kw generators removed from West End were used with the new turbines. The capacity of this station was increased from 90,000 kw to 126,000 kw.

At Millers Ford, a 25,000-kw turbine was added together with one of the 25,000-kw generators removed from West End. With this installation completed, the capacity of the station was 110,000 kw.

The total installed capacity in all three stations was then 380,000 kw. The system peak load in 1929 was 261,000 kw. The years that followed brought a decrease of load and it was some years before new capacity was needed.

Low Water in the Miami

Unprecedented low water in the Miami River during the summer of 1934 greatly reduced the capacity available at Millers Ford station. To create a pool and allow recirculation of the cooling water during low flow, a dam was built in the river during the following year. It was thought that the pool temperatures would rise above that of the river. However, when operated, it was found that the cutoff piling of the dam which extended 13 feet below the river bed forced ground water up into the pool and dropped the temperature to five degrees Fahrenheit or more below that of the river flow. The full capacity of the station was restored by the dam.

Modernization of Old Plants

The growth of load in 1934 and 1935 indicated a need for an increase in capacity. Construction undertaken in the latter year had as its major objectives the improvement in efficiency of the production facilities and the elimination of the existing hazards in the switch rooms of the generating stations.

The West End and Millers Ford stations had been enlarged to beyond their original design capacity. The efficiency when built, had been excellent but modern developments in high-temperature and high-pressure steam turbines had made modernization of these plants economically attractive. To improve production efficiencies 1,200-pound 900-degree Fahrenheit superimposed units which exhausted into the low-pressure headers were installed at both West End and Millers Ford stations.

Pressure reducing and desuperheating equipment was installed at both stations to make the new high-pressure steam generating capacity available to the low-pressure machines whenever the high-pressure turbines are out of service. A 36,000-kw, 3,600 rpm, hydrogen-cooled unit was installed at West End, and a 30,000-kw, 3,600 rpm, hydrogen-cooled unit was installed at Millers Ford. The Millers Ford unit was the first hydrogen-cooled generator in commercial service. A further improvement in plant operation was obtained by the modernization of the coal and ash handling equipment at both stations. The new boiler installations at both plants further improved production efficiencies. Two 375,000 pounds per hour, dry-bottom type, steam generating units were installed at Millers Ford station. Three 350,000 pounds per hour, open-pass, continuous slag-type steam generating units were installed at West End station. The new boiler installations at each of the three stations are provided with complete automatic combustion control and feed-water level control equipment.

Millers Ford and West End stations are within urban areas and fly-ash precipitators were consequently installed at these stations. Cottrell-type precipitators operating on single-wave rectification were installed.

After the 1929 generator changes, in which the capacity of the units at West End station were increased from 25,000 to 36,000 kw, the original condenser design did not permit peak capacity at times of maximum river temperature. This was corrected in 1936 by a redesign of each of the condensers. These units are of the vertical tube, two-pass type, and as originally installed, were provided with a preheat section to recover the heat loss of subcooling of the condensate. A hydraulic jet air-ejector system was installed for air removal. In the modernization work, which was carried on simultaneously with the installation of the high-pressure capacity, an open-type deaerating heater was installed in the hot wells for deaerating the condensate and makeup water, as well as for recovering the heat loss in subcooling the condensate. The preheat section was converted into additional condenser service, and the entire condenser was retubed and the tube arrangement redesigned to allow for efficient distribution of steam throughout the condenser. This was accomplished by increased and improved steam laning. Summer peak capacities of each unit have been increased from about 31,000 kw to full generator capacity by these changes, and, even under peak river-temperature conditions, excellent vacuum conditions have resulted.

Further Increases at Columbia

In the depression years the increase in use of air conditioning, fans, refrigeration, and other home appliances had developed new summer loads to a point where summer and winter peaks were substantially equal. This tended to eliminate the summer maintenance period and made greater reserves necessary. To meet this situation and to provide for the continued system-load growth the third unit at Columbia was made necessary.

The unit is a 65,000-kw, 1,800 rpm, hydrogen-cooled condensing-type turbine. It was not felt to be economical to install a superimposed unit at this station due to the high temperature and pressure of operation of the original units.

Two 350,000 pounds-per-hour steam generating units, operating at 675 pounds per square inch, and 910 degrees Fahrenheit supply steam to the unit. A crossover through a desuperheater connects the new steam header to the 600 pounds-per-square-inch, 725 degrees Fahrenheit original plant header. The new boilers are essentially duplicates of West End steam generating units.

The West End and Millers Ford units were put in service during 1937. The Columbia unit was finished in 1938.

Ohio River Flood

About the middle of January 1937, flood waters were rising in the Ohio River. By the 21st of the month, it was evident that a flood of unprecedented magnitude might occur. Construction crews at work on the installation of the new units at Columbia and West End stations, together with plant maintenance crews of the Cincinnati Gas and Electric Company, were immediately put to work on emergency floodproofing of the stations. This work consisted in bulkheading doors and windows, providing additional pumps to remove seepage into the stations and, as the water rose higher, in the temporary location of station power-auxiliary switching and transformer equipment.

As the flood waters rose, difficulty too was experienced in handling coal at both plants. Despite the tremendous effort to maintain service, the stations were forced out of service when the river rose to a point where it was no longer possible to keep the water out. Columbia station was flooded when the water rose to 68.9 feet which was 4.7 feet above the greatest previous flood, which occurred in 1884. West End station remained in service until the river rose to 78.3 feet or 7.2 feet above the 1884 flood. The ultimate rise was to a level of 73.8 feet at Columbia station and 79.99 feet at West End station.

Before the flood had reached its crest work was started on designs for floodproofing the stations. The work of floodproofing was carried along with the completion of Unit No. 5 at West End station, and Unit No. 3 at Columbia station. The floor and wall structures were reinforced to withstand the greater hydrostatic head. Auxiliary switch rooms were removed to high levels. Means of handling coal during high water were added. Auxiliary flood pumps were installed. Power and control cables were raised to levels above the floodproofing level or replaced with waterproof cable. Where normal operations required doors or openings below the floodproofing level, marine type, bulkhead doors were installed so that they could be sealed during floods. At the time of the flood, the substructure for Unit No. 3 had not been long in progress. It was possible to redesign the substructure for the new design level, and this was done immediately.

The new design level for flood at West End station is now 7.4 feet above the crest of the 1937 flood, and for Columbia station the levels above that flood are 7.4 feet for Units Nos. 1 and 2, and 9.4 feet for the new Unit No. 3.

During the flood, the great value of interconnection was demonstrated. No capacity was available at the Cincinnati stations from January 25 to February 1. At no time were any essential loads disconnected. Power was delivered to Cincinnati from Dayton, and over the Indiana interconnections.

Current Program

In the Dayton area, the need for new capacity continued. Early in 1939, Unit No. 8, a duplicate of Unit No. 7, was started at Millers Ford station. The installation was recently completed. The capacity of Millers Ford station is now 170,000 kw.

In anticipation of increased load due to the development of armament loads, a program of increase in capacity is again in progress. At Millers Ford station, Unit No. 2, a 12,500-kw unit will be removed. With modern condensers and changes in steam extraction, it is possible to replace this unit with a modern and more efficient 30,000-kw unit. At Columbia station, Unit No. 4, a 65,000-kw, hydrogen-cooled unit, a duplicate of Unit No. 3, is under construction.

When this program is complete, the capacity of the load-center plants will be, at Millers Ford, 187,500 kw, and at West End, 180,000 kw. The total capacity in the system will then be 623,500 kw.

Interconnections

The Integrated System

The first interconnection between companies which later formed a part of the Columbia system was installed in 1913. This interconnection was between Plum Street station in Cincinnati and the Newport station of the company serving the Kentucky suburbs. The second interconnection was made in 1923 with the Dayton Power and Light Company which was not then a part of the Columbia system. The Cincinnati company was in need of capacity and had contracts with the Dayton company, which resulted in the installation of Unit No. 5 at Millers Ford station so that excess capacity there could be used by the Cincinnati company until the Columbia units could be placed in service. To deliver the capacity to Cincinnati, the systems were interconnected by two 66-kv circuits from Millers Ford station to Elmwood substation in Cincinnati. These circuits followed the general course of the Miami River from Cincinnati to Millers Ford station. The lines were completed in 1923.

With the acquisition of the Dayton properties the two companies with their interconnections became the present integrated electrical system of the Columbia Gas and Electric Corporation.

Interconnection With Other Systems

In 1924 the first interconnection with an outside system was made. This, a 66-kv circuit, was run from a new substation at Hamilton, Ohio, on the Cincinnati system, to the Indiana state line where it joined a circuit from Connersville, Indiana, on the system of the Public Service Company of Indiana.

A few years later the Indiana company was in further need of capacity. The additional capacity was delivered over the 132-kv circuit constructed in 1927, from Columbia station of the Cincinnati company to the Indiana company's Lenore substation near Indianapolis.

At the same time companies adjoining the Dayton company were in need of capacity. To supply the Ohio Edison Company at Springfield, a 66-kv circuit was built from Millers Ford to their Mad River station. A double-circuit line, built for 66 kv, but operated at 33 kv, was built from Millers Ford to the substation of the Greenville Power and Light Company at Covington, Ohio, to meet the needs of that company. The Dayton company's interconnections were completed in 1929.

In 1929, the decision was made to interconnect the Cincinnati company with the Louisville Gas and Electric Company. After satisfactory negotiations had been completed, work was started on the 132-kv line. This interconnection was completed in 1930. The operation of this circuit has been of great value to both companies. Virtually continuous use is made of the circuit for transfer of economy energy between systems. During light-load periods dump hydro-generated energy is delivered from Louisville to Cincinnati and during heavy load periods and at times when the hydro plant is out, low-cost Cincinnati steam-generated energy displaces generation in the Louisville system's higher cost peak-load plants.

For some time, there had been under discussion interconnection of this system and the American Gas and Electric system. An agreement was recently reached to construct a 132-kv circuit from Trenton substation of the Cincinnati company to Delaware substation of the Indiana General Service Company at Muncie, Indiana. This interconnection is now under construction.

The principal purposes of this interconnection are:

1. Reduction in required reserve generating capacity.
2. Insurance to service if a major source of supply is lost.
3. Savings and profits resulting from trading in economy energy.
4. Possibilities of staggering construction of generating capacity by short-time purchases and sales of capacity.

In all, three 132-kv circuits with a capacity of approximately 60,000 kva each will interconnect the Cincinnati system with the systems of adjacent states. The two Indiana systems in turn connect to the great interconnected systems of this region. A second circuit can be installed on the double-circuit steel tower lines to the Indiana companies. With completion of present construction, the interconnection capacity of the Cincinnati system at 132 kv will be approximately 180,000 kva and by addition of the second circuit on the two tower lines this capacity can be increased to 300,000 kva. In addition the capacity in the 66-kv transmission between the Cincinnati and Dayton groups makes capacity of either system available to the other.

Transmission and Substations

EARLY DEVELOPMENTS

When West End and Millers Ford stations were first started, it became necessary to develop transmission systems to connect the new stations to the existing stations and substations.

In the Cincinnati area, the existing Plum Street and Newport stations generated at 4,500 volts. To interconnect these stations, transformers at both stations, and two circuits operating at 11,000 volts had been installed in 1913. Similarly, circuits had been constructed to several substations in the suburban areas. These circuits provided a nucleus for a transmission system out of West End station. As 13.2 kv was rapidly becoming the standard voltage for generation and the existing transmission could be operated at that voltage it was selected for the new generators.

The new conduit systems for the transmission were constructed in 1916 and 1917. At that time, this was a very modern and most elaborate system. Ducts were installed from West End station to five substations. The trench length of the system was approximately 100,000 feet. Of this 8,000 feet were in the flood area near the station. At the time it was thought that this portion of the system must be watertight. The manholes had a heavy outer lid and an inner lid, with a gasket and clamping bars, to prevent the entrance of surface water. Also it was necessary to cross under many railroad tracks, gas and water mains. This required an unusual depth and strength of structure. The average trench depth here was 16 feet. The manholes were also of unusual depth, one manhole had a depth of 32 feet. The concrete duct system itself was steel reinforced and in section area approximately 250 per cent that of a modern system with an equal number of ducts. This transmission system continued to develop until 1923 in the form of radial feeders to substations.

In the Dayton system a new duct system was built and cables for operation at 6,600 volts were installed from Millers Ford station to Third and Fourth Street stations. This system continued to develop at this voltage for several years, but it soon was evident that a higher voltage would be more economical. The first step in this direction was made to increase the capacity of the circuits to Third Street station. Autotransformers were installed at Millers Ford and Third Street stations, and the operating voltage of the cables was changed from 6,600 volts to 12,000 volts.

To supply territory surrounding Dayton and Cincinnati 33-kv transmission systems were developed. In the Dayton area, this transmission was built in the form of a belt line around the city with radial taps to the surrounding area.

THE 66-KV SYSTEM

In 1924 and 1925 while Columbia station was under construction extensive additions were being made to the Cincinnati transmission system and substations. To deliver the power to the Cincinnati area, two double-circuit, 66-kv steel tower transmission lines were constructed from Columbia station to a new 66-kv switching station, Hartwell Terminal substation, to the north of the city. Two new step-down substations, Mitchell Avenue and Oakley, were erected. The 66-kv Dayton-Cincinnati interconnection circuits were brought into the Hartwell Terminal substation. The existing 66-kv circuits were used to supply Elmwood substation from Hartwell. A double-circuit steel tower line was built from the terminal substation to each of the new Oakley and Mitchell Avenue substations. A 66-kv substation was built at Trenton to supply the load in the Middletown and Franklin areas.

OPERATING EXPERIENCES

The 66-kv transmission plan as originally conceived intended that one pair of the Columbia-Hartwell circuits should be brought into each bus section at Hartwell substation and the busses there were to be operated with the bus tie open. It was intended that ultimately two 66-kv circuits from Hartwell through substations would tie to West End station. Without these ties in service it was found that the transmission from the step-down stations to West End would not satisfactorily divide load if the bus sections at Hartwell were operated with the bus-tie breaker open. The system was then operated with the busses at Hartwell tied together normally.

The development of the 66-kv system marked a transition in the Cincinnati system operation. Heretofore, the system had cable transmission as the main supply to load. With the new 66-kv transmission new protection problems for faults on the 66-kv system arose. Lightning storms became a major concern. While it had not yet become evident, a system-stability problem had been created. In the Cincinnati system two short circuits occurred which influenced the design of the system transmission and substations. In 1930 a bus fault caused by a circuit-breaker failure at West End station destroyed a length of bus supplying five switch positions. In 1932 a lightning-arrester failure caused a three-phase short circuit of such duration that system instability resulted. The loss of load at Columbia station during the fault caused the turbines to be tripped by their overspeed devices. It was necessary to drop load in Cincinnati to avoid complete loss of load at West End station.

Stability studies were made to determine the most economical solution to this problem. One of the principal causes of the instability was the large portion of the system load transmitted from Columbia to the Cincinnati substations. By reducing the proportion of load carried at Columbia it was possible for the existing protective equipment to clear 66-kv faults in time to prevent instability. This restriction is no longer necessary as the present transmission system is adequate to maintain stability under all normal conditions.

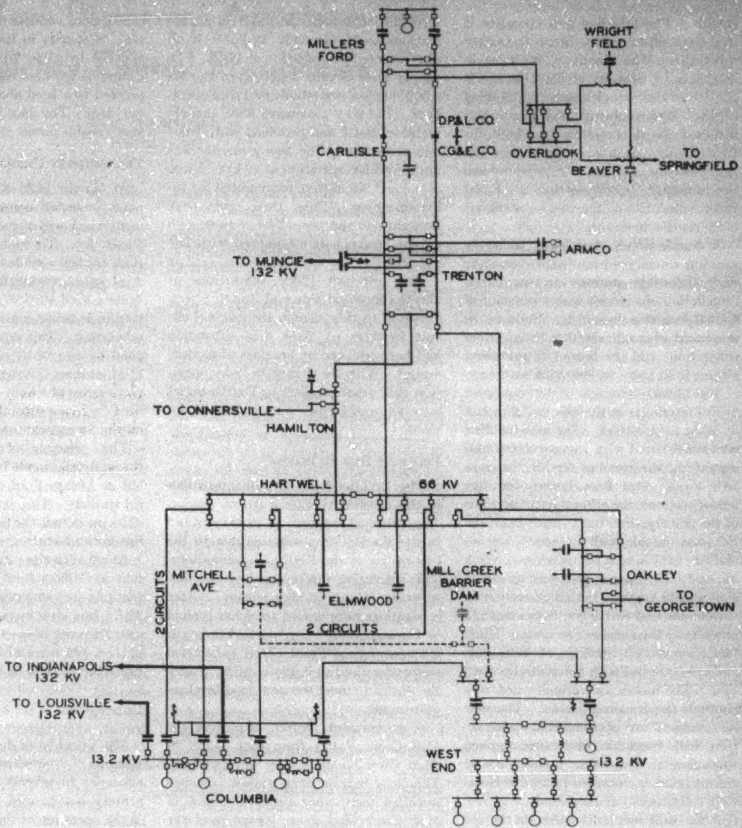
REMEDIAL MEASURES

The original bus at Columbia station was a separated phase structure and was provided with ground-fault bus protection. Following the bus fault at West End similar protection was applied at that station and the principal 13.2-kv substations. Also, the circuit breakers at West End station were equipped with an improved type of contact. With the development of the De-ion grid the circuit breakers were again rebuilt. De-ion grids were also installed in circuit breakers at the principal 66-kv and 13.2-kv substations.

Early operating experience had indicated the need for protective equipment which would quickly disconnect short circuits from the system. To this end transmission circuits were arranged in pairs and balanced protection applied to all principal circuits. With the growth of the system it became more evident that the system as a whole should be fundamentally designed to prevent long outages to any major portion of the system.

An opportunity to attack this problem came with the addition of Unit No. 3 at

Figure 3. High-voltage transmission and interconnections of the Columbia Gas and Electric Corporation


MILLERS FORD
WRIGHT FIELD
DP&L CO.
C.G.&E. CO.
CARLISLE
OVERLOOK
BEAVER
TO SPRINGFIELD
ARMCO
TO MUNCIE 132 KV
TRENTON
TO CONNERSVILLE
HAMILTON
HARTWELL
66 KV
MITCHELL AVE
ELMWOOD
MILL CREEK BARRIER DAM
OAKLEY
TO GEORGETOWN
2 CIRCUITS
2 CIRCUITS
TO INDIANAPOLIS 132 KV
TO LOUISVILLE 132 KV
13.2 KV
COLUMBIA
WEST END
13.2 KV

Columbia station. It was necessary to provide additional transmission to the 66-kv system and also substations to step down to the 13.2-kv load. It was desired that the new transmission should accomplish three things.

1. Increase capacity out of Columbia station.
2. Solve the stability problem.
3. Provide greater reliability to the 66-kv system as a whole.

Among the hazards to service were certain busses where short circuits could disrupt the system. Also, certain important busses were tied by one breaker, where, should a short circuit occur, both busses would be lost.

The solution finally adopted had two fundamentals. (1) The 66-kv transmission would be developed into two halves as had been originally intended. (2) When the two halves were tied together at generating stations or at substations the bus tie would be through two circuit breakers in series. Employment of these principles allowed confinement of major bus faults to one-half of the 66-kv system and confinement of bus outages to one section only.

One of the first projects in this direction was to rebuild Hartwell substation to reduce the hazard existing there. As it was then operated, a fault on the bus could completely separate all three generating stations. The station was rearranged into two structures with wide physical separation. The steel structures and circuit breakers were insulated from ground- and bus-fault protection applied to each bus structure and the bus-tie section separately.

A development of the original 66-kv transmission plan was started. The present development together with authorized construction now in progress is shown in figure 3. The plan includes the construction of two step-down transformer installations at Front Street substation located at West End station. The substation will consist of two physically separate structures. The 13.2-kv side of each transformer is to be tied to separate busses at West End station. As planned each structure will have two circuits, one circuit to Columbia station, the other to a suburban Cincinnati 66-kv substation. The first bus now complete is tied by a 66-kv cable circuit to Oakley substation. The circuit, eight miles in length, has a capacity of 60,000 kva. It consists of three single-conductor oil-filled cables. While only one cable circuit was installed the duct system was built for two circuits. It was intended that the spare duct be used for transmission from the ultimate second section at Front Street substation to the 66-kv system out of Hartwell substation.

A double-circuit steel tower transmission line connects Front Street substation with Columbia station. In the initial installation one circuit with a capacity of 60,000 kva was installed. Work to be done next year will complete Front Street substation, and the second transmission circuit from there to Columbia station.

The 13.2-kv terminals of the step-down banks terminate in the new switch house at West End station. The new building was constructed with a vertical fire wall separating the two bus sections on each two floors. One floor houses the two feeder sections, the other the two sections of the new equalizer bus. Each bus room has one outside wall in which are installed explosion-type windows which automatically open in the event an explosion should occur. Each bus section in the old and new switch houses ties through reactors to the equalizer sections. These ties have circuit breakers at both ends and are equipped with differential protection. The busses are normally tied only through the equalizer busses. The new bus sections are of metal-clad construction and bus-fault protection applied throughout. The same double bus-tie circuit-breaker arrangement used at Hartwell substation was employed.

A fire wall was built to divide the old switch house into two bus rooms and each room equipped for explosion-pressure relief. All the old and new bus rooms were equipped with emergency exhaust fans.

Along with the installation of Unit No. 7 at Millers Ford a program of modernization of the switch galleries was started. The old switches and busses were in concrete cells. They were designed for operation at 6,600 volts. At this time it was evident that considerable gain in transmission capacity to the Dayton substations could be made available if all were changed to operation at 12 kv. To supply these circuits a portion of the old 6,600-volt bus was rebuilt for 12-kv operation. The new structures were entirely metal enclosed and provided with fault-bus protection. The new generator was purchased for operation at 12 kv. Units No. 5 and No. 6 were reconnected for 12-kv operation. These three units were brought into the new 12-kv section. All circuit breakers were equipped with Deion grids.

The principal 12-kv substations at Dayton are West Side and Fourth Street. Previous to this change the original circuit breakers at West Side substation had been replaced by breakers of modern design. With the change to 12-kv operation the circuit breakers at Fourth Street were replaced by new and modern equipment.

PROJECTS DUE TO FLOOD

The 1937 flood put several substations in the Cincinnati system out of service. The principal outages to customers not in the flooded area occurred due to the loss of one of the Kentucky substations. This substation was replaced by two new substations built on high ground. Other substations were moved to higher ground or the equipment raised to higher levels. Construction of Front Street substation was under way but it was possible to alter the design to meet the new flood level requirements.

A government project to protect the Mill Creek Valley from back water of Ohio River floods is soon to be started. This is a barrier dam project. Pumps installed there must pump the flow of Mill Creek plus some sewage over the dam during floods in the Ohio River. The pump load, a maximum of approximately 50,000 kw, will be supplied by two 66-kv circuits. A steel tower line will be built from Front Street substation through the dam to Mitchell Avenue substation. These are two of the most reliable busses in the 66-kv system. Mitchell Avenue and Front Street are in the heart of this system. Three generating stations and two of the major interconnections connect directly to this system. Capacity in Indiana would not be affected by Ohio River floods and the Columbia system stations are flood-proofed to a level above that of the barrier dam. The dam is thus assured two very reliable power sources.

TRANSMISSION CHANGES AT DAYTON

At Wright Field the government proposes to install aeronautical test equipment which will increase the load there to 50,000 kw. To supply this load extensions are being made to the 66-kv system. A second 25,000-kva bank was installed at Millers Ford station. A new 66-kv substation is being constructed at Overlook substation. This substation will be supplied by two 66-kv circuits from Millers Ford station. Wright Field will have two sources of supply at 66 kv, one direct from Overlook substation, the other a tap off the Springfield interconnection.

The principle of tying the busses through two circuit breakers was carried out at Millers Ford and Overlook 66-kv substations. The protection is slightly different in that the tie section and circuit breakers are in the line protection.

Along with the installation of the new unit at Millers Ford station further extensions are being made to the metal-clad 12-kv bus structures. During the next year the work of completely replacing the old bus structures with a metal-clad 12-kv protected bus will be finished.

Conclusion

The principle of developing generating stations of approximately equal size and efficiency in several locations near load centers has proved highly satisfactory in the operation of this system.

The advantages of interconnection have been recognized in this system as far back as 1913 and it is the intent to continue to develop interconnection to its full usefulness.

The short-circuit protection of major busses and fire-wall provisions in stations and substations have become standard practice. In addition water spray or carbon dioxide fire extinguishing systems are applied to all major equipment.

Short-Wave Diathermy Apparatus and Frequency-Control Possibilities

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THE general technique of applying short-wave diathermy has become firmly entrenched in the therapeutic repertoire of the medical profession during the past eight years. The clinical effectiveness of this particular physical treatment, along with the commercial development of apparatus suitable for applying this form of energy at an economical cost, has given it wide usage. In spite of the general acceptance of short-wave diathermy in medicine there seems to be no universally accepted theory regarding the action of the electrical fields or currents in the body tissues themselves. In this country the most widely held theory is that the heating effects created in the tissues by the currents account for all the therapeutical results.[1,2] In Europe there is a school of physicians which holds that athermic treatments are clinically effective.[3]

Long-wave diathermy (contrasted with short-wave diathermy) employing frequencies of the order of 1,000,000 cycles per second has been known and used by doctors for the past 40 years. In this technique the high-frequency currents are applied to human tissue through metal electrodes in direct contact with the patient. The heat generated in the tissues is determined by the length of the treatment, t, the current, I, flowing through them, and their effective resistance, R, as defined by $Q = I^2Rt$. The deep tissues are heated as well as the superficial ones. From the patient's standpoint, the treatment tolerance is determined by the highest-created sensible temperature. Because the skin has a greater resistance than other tissues of the body and is the seat of nerve terminals sensitive to heat, the energy introduced in the patient is largely determined by the heating of the skin. Average healthy adults can tolerate approximately 100 milliamperes per square inch.

Applicators

With the advent of short-wave diathermy apparatus, the energy could be conveniently applied capacitively through insulated electrodes or applicators. The higher frequencies from 10,000,000 to 50,000,000 cycles per second permit the electrodes to be insulated and either placed directly on the patient or spaced a few, or even several, inches away. Furthermore, satisfactory treatment levels of energy do not cause excessive voltages between the patient and the electrodes.

Capacitive applicators have become known to the medical profession as of two different types, pads or cuffs, and spaced-plate applicators (figure 1). Pad or cuff applicators are supported by the patient and may be placed either directly against the patient or separated from him by layers of felt or toweling. These applicators are used at any of the frequencies from 10,000,000 to 50,000,000 cycles per second. Spaced-plate applicators are supported on arms and separated from the patient by air. Spacings up to about six inches are used. The generators for these applicators oscillate in the higher frequency ranges above 25,000,000 cycles per second in order to keep the voltage gradients within safe operable limits.

With short-wave diathermy machines it also became possible to apply energy electromagnetically to human tissue.[4] This is accomplished by wrapping an insulated cable of convenient length and diameter around the part to be treated and passing high-frequency current through the cable. The field set up by the current induces eddy currents within the tissues. An advantage of this method of treatment is that the deeper-lying tissues, having lower resistance than the skin, are heated proportionately more than the more highly resistant skin tissues. Fixed coil configurations mounted in air-spaced bakelite covers, called drums or disks, and fastened to convenient adjustable arms have proved a very flexible, safe, and convenient method of applying treatment energy (figure 2).

Generator

The short-wave diathermy generator consists essentially of a high-frequency, self-excited oscillator circuit using suitable vacuum tubes and being either self-rectified or using a rectified power supply. Most of the models marketed to the medical profession have a maximum output ranging from 200 watts to 600 watts. The patient circuit may be directly, electrostatically, or magnetically coupled. Depending on the type of coupling, the power is controlled in several ways. There may be a variable reactance (usually a capacitor) in the patient circuit; the input energy to the oscillating tank circuit may be controlled; or the coupling to the patient circuit may be varied. Many times a combination of these is used. The relative power output, furthermore, may be indicated either by a radio-frequency ammeter in the patient circuit, or by a milliammeter reading the input plate current to the tubes, or by a differential milliammeter reading the subtractive difference between the average plate and the average grid currents taken by the tubes, or by an auxiliary radio-frequency wattmeter circuit.[5]

Figure 3 shows the circuit diagram of a typical short-wave diathermy machine in which the patient circuit is inductively coupled and power output is controlled by a variable capacitor in the patient circuit. The power indicator is coupled magnetically to a tapped coil which carries average plate and grid current in such a manner that under idling conditions, i.e., no power expended in the patient circuit, the net voltage impressed on the indicator is zero. When the unit is fully loaded the instrument reads full scale. It is realized, of course, that in such an oscillator the input plate current consists of half-wave pulses of the power frequency. The meter then responds to the a-c components in the plate and grid currents. Linked magnetically to this metering circuit is an armature set to operate contacts when the load exceeds a predetermined value. The opening of the contacts inserts in the circuit cathode bias which automatically reduces the input into the tube and thus prevents damage should the difference in plate and grid current exceed a predetermined value.

Patient-Circuit Impedance

The external patient-circuit impedance varies over an exceedingly wide range. Not only does it vary from one treatment condition to the next, but it will vary even

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1. For all numbered references, see list at end of paper.

Figure 1. Capacitive applicators

A (left). Treatment of left arm with pad and cuff

B. Spaced-plate treatment of respiratory tract

several hundred per cent with movements of the patient during the course of the treatment. It is this characteristic which sharply differentiates short-wave diathermy engineering from conventional radio transmitting circuit practice. It is necessary in the design of the generator and applicators that adequate treatment power always be available for any accepted technique of treating. The reactance of the external patient circuit may be capacitive, inductive, or zero. In turn, the applicators, the length of the applicator leads, their proximity to each other, and the frequency, affect the value of the reactance presented to the generator. In general, the reactance of the path in the patient himself is negligible compared with the reactance of the electrode patient path. This results from the high average dielectric constant of human tissue, approximately 78 for all except fat which is about 13. Where capacitive or magnetic coupling and a variable capacitor to control power is used, the patient-circuit constants must be so chosen that the range of reactances of the external circuit can be adequately compensated to allow sufficient energy to flow.

The resistance of the external circuit is determined by the frequency, the type of applicator, and its configuration. The actual path within the patient, however, influences the resistance considerably. For example, a pair of pads placed on the front and back of the chest under certain conditions would present seven ohms resistance to the generator. If the pads were then placed with the same spacing, lead arrangement, etc., under the foot and on the knee, the resistance would be approximately 50 ohms. Table I shows typical values of patient-circuit resistances encountered in practice.

The conventional manufacturing practice has been to use fixed coupling of such an amount that satisfactory treatment energy can be transferred to the patient under conditions having the greatest resistance. This procedure has simplified the control of the generator. It is obvious that for treatment conditions having low resistances the generator will be overcoupled. In such situations, with a change in patient-circuit reactance, the reflection of the patient-circuit impedance in the oscillator may cause considerable frequency shift (15 per cent or more). Furthermore, excessive oscillator loading may occur unless the unit is equipped with satisfactory automatic circuit protection.

Figure 2. Electromagnetic applicators

A (left). Neck treatment with induction cable
B(below). Shoulder treatment with curved drum

Interference

It is generally recognized that short-wave diathermy machines may interfere with radio communications.[6] The possible and alleged interference caused by these machines has been and is being studied from many angles by the groups primarily interested, The Federal Communications Commission, The American Medical Association, the Physical Therapy Manufacturers' Group, communication interests, The National Electrical Manufacturers' Association, and The Radio Manufacturers' Association.[6,7,8,9,10] To date no definite agreement has been reached by all the interested parties. It is generally agreed, however, that the interference at distances greater than a few hundred feet is caused by radiation primarily from the electrodes, patient, and associated leads. Reber and Richardson[6] have shown the field strength under actual treatment conditions to be given by $v=v_0/d^k$ where v is the field strength of the signal in microvolts per meter, v_0 is field strength at a distance of one mile, d is distance in miles from the source, and k ranges from two to three. v_0 may vary from 1 to 50 microvolts per meter. When the machine is used without a dissipative load, i.e., no patient, v_0 may be several hundred microvolts per meter. They also have reported the average diathermy signal to be of the order of one microvolt per meter.[7,8] Other tests have been conducted and the results given much publicity, but the machines

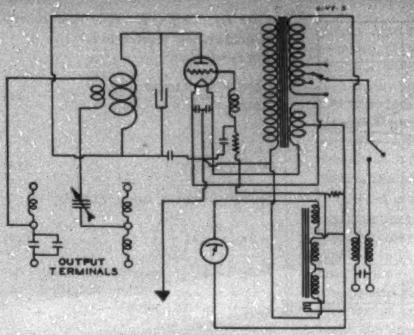

Figure 3. Single-dial-controlled, tuned-plate tuned-grid self-rectifying diathermy circuit. Note the three pairs of terminals in the patient circuit

were not treating under practical conditions. Instead, they were operating without a dissipative patient load in which signals had been detected over long distances.[11] It is held by some that the technical solution of providing adequate shielding for treatment rooms would not prove a practical economical one. It does appear, however, that standards regarding the following points may be adopted.

1. Nominal frequencies to be used.
2. Type of emission.
3. Maximum power output.
4. Amount of harmonic radiation.
5. Effective shielding of internal circuit.
6. Elimination of radiation from power supply.
7 Frequency stability.

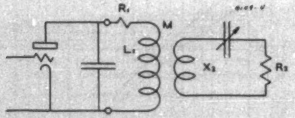

Figure 4. High-frequency portion of diathermy circuit

On the first six of the above points all the interested groups are in general agreement that a technically and economically satisfactory solution could be adopted.

Frequency Stability

The problem of frequency stability, on the other hand, is one of the most important of the seven, since the degree of stability attained will directly affect the cost and service requirements to the doctor, and indirectly affect the cost of adequate medical attention to the public. Because of the expense and complication of master-controlled oscillator circuits, the following analysis was undertaken to determine the degree of frequency stability possible with simple, conventional, self-excited, oscillator circuits.

Figure 4 shows the oscillator tank circuit and coupled patient circuit. Figure 5 shows the equivalence of this circuit.

$$Z_{00}=R_1+j\omega L_1+\frac{\omega^2M^2}{Z_2} \tag{1}$$

$$=R_1+\frac{\omega^2M^2R_2}{R_2^2+X_2^2}+j\omega(L_1+\Delta L) \tag{2}$$

where

$$\Delta L=\frac{-\omega^2M^2X_2}{R_2^2+X_2^2} \tag{3}$$

is the reflected inductance in the tank circuit and

$$R_R=\frac{\omega^2M^2R_2}{R_2^2+X_2^2} \tag{4}$$

is the reflected resistance in the tank circuit.

For a small change, ΔL, in L_1, the percentage change in resonant frequency is given by

$$\frac{\Delta f}{f}=\frac{1}{2}\frac{\Delta L}{L_1} \tag{5}$$

Substituting (3) in (5)

$$\frac{\Delta f}{f}=\frac{1}{2}\frac{\omega^2M^2X_2}{\omega L_1(R_2^2+X_2^2)} \tag{6}$$

and will be plus or minus, depending on the value of X_2.

The output power in the patient circuit is

$$I_T^2R_R=I_T^2\frac{\omega^2M^2R_2}{R_2^2+X_2^2} \tag{7}$$

It is to be noted from figure 6 that, as the patient circuit is tuned toward resonance from the inductive reactance side, the frequency deviation increases the oscillator frequency. At low values of R_2, as the tuning capacitor is further decreased in capacity below the value $X_2=R_2$, the frequency goes down, decreasing X_2 further, which decreases the frequency more, and so on until the oscillator stabilizes itself by operating at a negative-frequency deviation with the patient circuit having capacitive reactance. The output under such stabilized conditions may be more or less than the output under the conditions just before the frequency changed rapidly. For fixed coupling it is obvious from figure 6 that for low values of R_2 it may be impossible to reflect enough resistance into the tank coil to obtain maximum output from the oscillator. Practical use is made of this property in diathermy apparatus. The generators are critically coupled for the high-resistance loads and the above frequency instability is adapted to prevent overload of the tube when low-resistance paths are treated.

Maximum Frequency Deviation for Fixed Maximum Output Power

Assume that a generator is arranged with a fixed magnetic coupling between the tank circuit and the patient circuit. The problem is to determine the maximum frequency deviation for any patient-circuit resistance with the limitation that the maximum power dissipated in the coupled circuit under any condition will be fixed. It is understood that the power in the output circuit is controlled by varying its reactance. The problem is solved logically under three headings, i.e., when the resistance in the patient circuit is critically coupled, when the maximum frequency deviation occurs below the fixed output value, and when the maximum frequency deviation occurs at the fixed maximum output.

$$\omega^2M^2=R_LR_c \tag{8}$$

where R_c is the critical secondary resistance such that $R_R=R_L$, the reflected resistance, just loads the oscillator to full load when $X_2=0$.

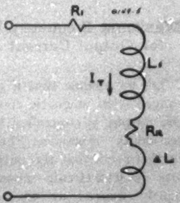

Figure 5. Equivalent circuit shows frequency variation is dependent on ΔL

Let

$$R_2 = kR_c$$

k may vary theoretically from 0 to ∞.

Case I. Critical coupling. $k=1$.

It is seen that $\Delta f/f$ is a maximum at $X_2 = R_2 = R_c$ and from (6) that

$$\frac{\Delta f}{f} = \frac{1}{4Q} \tag{9}$$

where Q is $\omega L_1/R_L$.

Case II. Maximum frequency deviation below maximum output. $k \geqq 0.5$.

In general $\Delta f/f$ is maximum when $X_2 = R_2 = kR_c$. For certain values of k, X_2 will be greater than kR_c, since its value is determined by the maximum load on the oscillator.

Let the reflected resistance term of (4) equal R_L. Solving for X_2 to satisfy this condition,

$$X_2 = R_c\sqrt{k(1-k)} \tag{10}$$

which determines the value of X_2 where maximum frequency deviation occurs, X_2 being limited by the output of the oscillator. $X_2 = kR_c$ determines where maximum frequency deviation occurs before the oscillator is fully loaded. At $k = 0.5$ both of these conditions are satisfied. For $k=0.5$,

$$\frac{\Delta f}{f} = \frac{1}{k}\,\frac{1}{4Q} \tag{11}$$

$$= \frac{1}{k} \quad \text{[maximum frequency deviation for critical coupling]}$$

Case III. Maximum frequency deviation at the maximum output. $k \leqq 0.5$.

Substituting (10) in (6), then

$$\frac{\Delta f}{f} = \frac{1}{4Q}\,2\sqrt{\frac{1}{k}-1} \tag{12}$$

$$= 2\sqrt{\frac{1}{k}-1} \quad \text{[maximum frequency deviation for critical coupling]}$$

Figure 7 shows that it is possible for the frequency to vary 20 times the critical coupled deviation when $k=0.01$. If the maximum power output is reduced 50 per cent the possible frequency variation at $k=0.01$ is reduced only 25 per cent.

Maximum Frequency Deviation for Output Current Limited

Let us assume that a generator is arranged with magnetic coupling and that the output is controlled by varying the secondary reactance. Let us assume, furthermore, that the output current for any patient-circuit impedance is arbitrarily limited at the current which would flow at the critically coupled condition for the given patient-circuit resistance. Then, this analysis, too, is divided into three parts, i.e., the critical coupled condition, the condition where the current limitation determines the maximum frequency deviation, and the condition where the current limitation has no effect in determining the maximum frequency deviation.

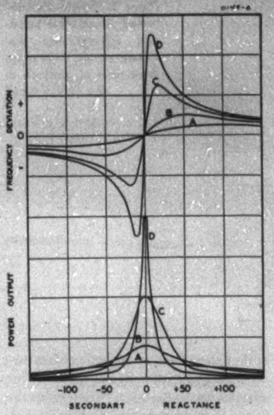

Figure 6. Variation of frequency and power with secondary reactance X_2

A—R_2 100 ohms
B—R_2 50 ohms
C—R_2 20 ohms
D—R_2 10 ohms

Case I. Critical coupled. $k=1$.

Here $X_2 = kR_2 = R_c$ and

$$I_2 = \frac{\omega M I_T}{\sqrt{2}R_c} = \frac{I_c}{\sqrt{2}} \tag{13}$$

and

$$\frac{\Delta f}{f} = \frac{1}{4Q} \tag{14}$$

Case II. $k \geqq \sqrt{2}$.

The value of k where I_2 just equals I_c and at the same time

$$X_2 = kR_c$$

is

$$\frac{I_2}{I_c} = \frac{1}{\sqrt{2k}}$$

and

$$k = \sqrt{2} \tag{15}$$

Therefore for $k \geqq \sqrt{2}$

$$\frac{\Delta f}{f} = \frac{1}{k}\left(\frac{1}{4Q}\right) \tag{16}$$

Case III. $k \leqq \sqrt{2}$. I_2 is now considered deliberately limited to $I_2 = I_c$.

For $I_2 = I_c$ then

$$\frac{\omega^2 M^2 I_T{}^2}{X_2{}^2 + k^2 R_c{}^2} = \frac{\omega^2 M^2 I_T{}^2}{R_c{}^2} \tag{17}$$

and

$$X_2 = R_c\sqrt{1-k^2} \tag{18}$$

Substituting (18) and (8) in (6)

$$\frac{\Delta f}{f} = 2\sqrt{1-k^2}\left(\frac{1}{4Q}\right) \tag{19}$$

$$= 2\sqrt{1-k^2} \quad \text{[maximum frequency deviation at critical coupling]}$$

It is seen from figure 8 that when X_2 in the patient circuit is limited so that I_2 does not exceed I_c, the maximum frequency deviation will not exceed double the deviation possible at critical coupling. A reduction of ten per cent in the maximum output current reduces the possible frequency variation approximately 20 per cent at $k=0.01$. The output is inversely proportional for k greater than one and directly proportional for k less than one.

Tank-Circuit Efficiency

In the above discussion it is to be remembered that the Q considered is that due to reflected secondary resistance in the tank circuit. The operating Q of the

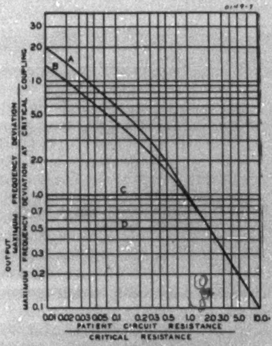

Figure 7. Possible frequency departure and power output versus patient-circuit resistance for fixed coupling and limited output conditions

A—Frequency departure for 100 per cent output
B—Frequency departure for 50 per cent output
C—Output limited to 100 per cent
D—Output limited to 50 per cent

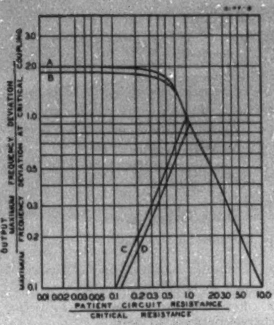

Figure 8. Possible frequency departure and power output versus patient-circuit resistance for limited output current

A—Frequency departure for 100 per cent I_c
B—Frequency departure for 90 per cent I_c
C—Power output for 100 per cent I_c
D—Power output for 90 per cent I_c

tank circuit, however, is determined by the reflected resistance, the resistance of the tank coil and condenser, and the equivalent resistance representing the grid-circuit losses. In a self-excited oscillator the grid-circuit losses vary with the load on the oscillator, being higher at no load than at full load.

The tank-circuit efficiency, η, is given by the ratio of R_L to $R_{F.L.}$

Where

$R_{F.L.}$ is the sum of the resistances enumerated above
$Q_{F.L.}$ is the full-load Q of the tank circuit
$Q_{N.L.}$ is the Q of the tank circuit at the idling condition, i.e., no power flow in the patient circuit
a is the ratio of the grid-circuit loss at no load to the grid-circuit loss at full load
b is the ratio of grid-circuit loss at full load to the coil loss at full load

In practical oscillating circuits having values of Q over 100 an average value of $\left(\frac{1+b}{1+ab}\right)$ is 0.94.

The tank circuit efficiency, η, is

$$\eta=\left[1-\left(\frac{1+b}{1+ab}\right)\frac{Q_{F.L.}}{Q_{N.L.}}\right]100 \text{ per cent}$$

and the frequency deviation at critical coupling is

$$\frac{\Delta f}{f}=\frac{1}{4Q_{F.L.}}(\eta)$$

In figure 9 it is evident that by using full-load tank circuit Q's of the order of 100 or more it is possible to hold the frequency deviation due to patient-circuit coupling to less than plus or minus 0.2 per cent when the short-wave generator is critically loaded. This amount of circuit stability is attainable with conventional circuits, tubes, and equipment, as currently used in short-wave diathermy apparatus with some sacrifice in maximum attainable power output.

Critical coupling can be obtained for each patient-circuit load by providing variable coupling between tank and patient circuit. Simple instructions to the user of the equipment, insisting on tuning the patient circuit to resonance and using the lowest value of coupling compatible with the power necessary for treatment, could possibly keep the frequency swing, due to the patient-circuit loading, at or near zero. If no control or indicator is on the generator other than a plate-current limiter, it is possible for the frequency to vary as shown in figure 7. An output current limiter arranged to limit current to I_c for any coupling would hold a possible frequency variation, as shown in figure 8, to within twice that of the critical coupled load.

From the foregoing analysis and discussion one can see how the frequency swing of diathermy apparatus may be limited to within 0.25 per cent for any possible patient-circuit condition. Two channels, one in the 10- to 20-megacycle range, the other preferably harmonically related in the 20- to 40-megacycle range, with a slightly greater than the abovementioned frequency variation, should prove adequate from a medical, manufacturing, and commercial standpoint. There can be no doubt that this procedure would reduce the problem of diathermy interference to a negligible amount as compared to the avowed present situation where any given machine is operated on a frequency partly determined by the whim of some manufacturer, and partly by the condition of its use. In the light of radio communication practice the above suggested frequency tolerance seems quite large. However, when it is considered that all short-wave diathermy apparatus may be operated on the allocations suggested above, it would appear that such a solution should stand on its own merits.

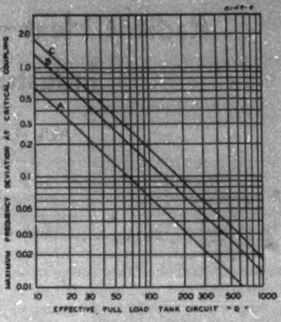

Figure 9. Per cent frequency departure versus effective full-load tank circuit Q

A—Tank circuit efficiency—25 per cent
B—Tank circuit efficiency—50 per cent
C—Tank circuit efficiency—75 per cent

References

1. ELECTRICITY IN MEDICINE, R. E. Williams. ELECTRICAL ENGINEERING. volume 57, June 1938, page 243

2. COUNCIL ON PHYSICAL THERAPY, John S. Coulter M.D. *Journal* of American Medical Association, volume 106, January 18, 1936, pages 209–14.

3. FOUNDATIONS AND THERAPEUTIC RESULTS OF NEARLY ATHERMIC SHORT WAVE THERAPY, Paul Liebesney. *Strahlentherapie*, volume 57, December 12, 1936, page 615.

4. BURSITIS — X - RAYS — HIGH FREQUENCY,

Table I. Typical Patient-Circuit Resistances Encountered in Practice

Type Applicators	Tissues Treated	Leads	Electrode Spacing	Frequency (Megacycles)	Resistance (Ohms)
Pads 6x8 inches	A.P. thigh*	54"	1/4"	13	11
6x8 inches	A.P. thigh	54"	3/4"	13	7
Pads 6x8 inches	Foot to foot	54"	1/4"	13	104
6x8 inches	Foot to foot	54"	3/4"	13	38
Pads 6x8 inches	Pelvis	54"	1/4"	13	12
6x8 inches	Pelvis	54"	3/4"	13	7
Cable 2 1/2 turns pancake coil 8-inch diameter	Back	12'	1/4"	13	20
Cable 1 large loop	Neck, chest, pelvis	12"	1/4"	13	3
Drum flat	Ant. chest**	30"	0"	13	12
Drum flat	Ant. chest	30"	1/2"	13	4
Plates 7-inch disks	A.P. chest	30"	1/2"	39	6
	A.P. chest	30"	2"	39	3

*A.P.—Anterior posterior.

**Ant.—Anterior.

Radiant Heat—a Full-Fledged Industrial Tool

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Introduction

ALTHOUGH the great majority of commercial heat sources deliver an appreciable portion of their energy in the infrared spectrum, most industrial heating processes now rely on heat transfer by convection. With the intermediate heating of the air between the source and the point of useful work, two or more energy transfers result with consequent loss in speed of transfer and efficiency while the available radiant energy is largely converted into convected heat by the heat transfer system before it is usefully employed.

The ability of radiant energy to heat objects apparently without regard to the surrounding air temperature has been known to man since his first exposure to the sun. The wave lengths responsible for this phenomenon have since been found to be those longer than visible radiation and because they are nearest the red end of the spectrum the name infrared or near red has been applied.

Infrared heating differs from other methods of heating primarily in that the energy is delivered directly from the source to the intended work in radiant form. Consequent convection from parts at elevated temperature is then a secondary rather than an intended result.

Temperature Investigation

In convection oven practice heat transfer is directly proportional to the temperature differential between the working surface and the surrounding air. Air movement helps eliminate the insulating influence of gaseous films adjacent to the work and thus directly increases the heat transfer. The rate of temperature rise inherently diminishes as the temperature of the work approaches that of the surrounding air, and when a state of equilibrium is reached no further useful heat transfer is effected. The temperature is also influenced by the mass, conductivity, and specific heat of the material.

In the direct transfer of radiant heat, the rate of temperature rise is directly dependent on the ability of the surface to absorb the energy and the concentration or density of the energy received. Mass, specific heat, and conductivity will affect the temperature rise and the uniformity of temperature within the material being heated. As in convection heating, however, the mean temperature finally attained is solely a matter of equilibrium but in radiant energy practice this occurs when the rate of heat loss is equivalent to the rate of energy input. Air temperature which is then not important in the original heat transfer becomes important because of its influence on losses since the temperature differential is customarily in reverse relation to that for convection practice. This provides inherent advantages for numerous types of applications while on others the heating cycle may be completed well before the equilibrium state is reached thus providing high operating efficiency. Since a knowledge of the heat flow as effected by these influencing factors is fundamentally essential to the design of radiant heat ovens, research studies have been undertaken to determine how known laws of thermodynamics might be suitably applied to radiant heating.

The equipment used for these studies included a 60-kw laboratory oven employing Thermalamp sources capable of providing uniform energy concentrations up to ten watts per square inch. Temperature measurements were made with a special Leeds and Northrup recording potentiometer balancing a thermocouple circuit once every second. Energy measurements were made with a special instrument developed for the purpose comprising a thermocouple and milliammeter circuit employing a black body receiver in an evacuated envelope to avoid convection effects. The test objects were mounted on a conveyor operating through the center of the oven illustrated with the various instruments in figure 1.

Radiation transferred from the source to the work may be diverted into:

1. Useful energy absorbed and conducted through the work.
2. Reflected energy losses due to surface characteristics of the work.
3. Convected energy losses to the surrounding air.
4. Re-radiation as may result due to the elevated temperature.

All tests were made from an initial air temperature of 80 degrees Fahrenheit. Losses by re-radiation were held to a negligible level and consequently not separated from other losses through the use of a test procedure in which the experimental panels were surrounded by surfaces at equivalent or higher temperatures. To further assure accuracy in the determination of constants for verifying theoretical conclusions, the reflected losses were held to a minimum by the use of black surfaces. Convected heat then accounted for the principal losses from the test panels.

With the working surfaces and surrounding air at the same initial temperature in the test procedure employed, the *initial* rate of temperature rise would be dependent only upon the radiant energy absorbed per unit area, specific heat, and mass of the material since the convected losses are momentarily negligible.

The heat in British thermal units necessary to raise any thin object through

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1. For all numbered references, see list at end of paper.

W. R. Whitney. *General Electric Review*, volume 38, number 2, February 1935, pages 70–6.

5. Dosimetry in Short Wave Therapy, E. Mittelmann. *Archives of Physical Therapy*, volume 18, October 1937, page 613.

6. Final Report on Investigation of Radio Interference by Short Wave Diathermy to American Congress of Physical Therapy and Physical Therapy Manufacturers Group, Reber and Richardson. April 1940.

7. Noise Levels on Short Waves, K. G. Jansky. *Proceedings* of the Institute of Radio Engineers, December 1937, pages 1527–30.

8. Report to Federal Communications Commission, T. G. Miller of E.R.P. June 1938.

9. Conference at Harkness Theatre, Columbia University, New York, N. Y., January 9, 1939.

10. The Medical Viewpoint of Interference to Radio Reception by Electro-Medical Equipment, *Archives of Physical Therapy*, volume 20, May 1939, pages 261–9.

11. Interference Source Discovered, *Electronics*, February 1936, page 19.

Figure 1. Experimental equipment for radiant heat investigations

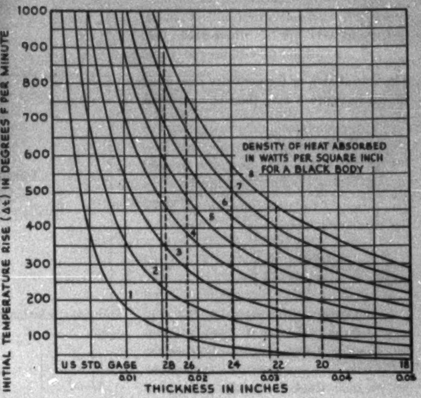

Figure 2. Initial rate of temperature rise for thin steel panels at various energy absorption rates

Δt degrees Fahrenheit per minute is then given by

$$Q = sm\Delta t \tag{1}$$

where s is the specific heat, m the mass, and Q the British thermal units per minute. As 3,413 Btu are equivalent to one kilowatt-hour, one British thermal unit per minute equals 17.58 watts. The relation between watts and British thermal units per minute is then

$$\text{watts} = 17.58Q = 17.58s(\rho Ad)\Delta t \tag{2}$$

where ρ is the density in pounds per cubic inch, A the area in square inches, and d the thickness in inches. For steel, the equation for watts per square inch as a function of d and Δt becomes

$$w = 0.55d\Delta t \tag{3}$$

For a given set of conditions this relation indicates that the initial rate of temperature rise is directly proportional to the watts per square inch input. This conclusion was confirmed experimentally and figure 2 shows the initial rate of temperature rise which may be expected at various energy absorption rates for thin steel panels with a true black body surface. The initial rate of temperature rise for a nonblack body may then be obtained from the product of the black body temperature rise and its reflection factor for the infrared spectrum. This reflection factor is discussed later in the paper.

These data would also hold for heavier masses except that the surfaces are initially heated more rapidly than would be indicated by the relation of total masses as expressed in (2) and (3). In practice such temperature acceleration would be greatly diminished after the first few seconds of exposure even for thin plates, due to the increasing rate of convection losses which constitute the next phase of this investigation.

In the interest of economy convection losses are held to a minimum in commercial practice. To simulate this condition in the test procedure, draft shields were used to limit air circulation to natural convection. According to Newton's law of cooling, the rate of heat loss is directly proportional to the difference in temperature of the work and the surrounding medium. This is expressed mathematically as:

$$\frac{dt}{d\theta} = k(t - t_a) \qquad \text{where } k < 0 \tag{4}$$

The temperature of the material is represented by t, time in minutes by θ, and air temperature by t_a. Separating the variables and forming the integral

$$\int_{t_0}^{t} \frac{dt}{t - t_a} = \int_0^{\theta} k\,d\theta \tag{5}$$

The limits of integration are determined by the mass cooling from t_0 to a temperature, t, at time θ. Integration of (5) gives

$$\log_e \frac{t - t_a}{t_0 - t_a} = k\theta \tag{6}$$

or solving for t to obtain the theoretical time-temperature cooling curve

$$t = t_a + (t_0 - t_a)e^{k\theta} \qquad \text{where } k < 0 \tag{7}$$

This equation is approximated by experimental data shown in the cooling region of figure 3.

k is most easily determined directly from the differential equation 4 by obtaining two values of $dt/d\theta$ and corresponding values of t from the experimental time-temperature curves, and eliminating t_a.

$$k = \left[\left(\frac{dt}{d\theta}\right)_1 - \left(\frac{dt}{d\theta}\right)_2\right] / (t_1 - t_2) \tag{8}$$

The mean temperature of a thin air film adjacent to the work can now be calculated with more accuracy than experimental measurements will permit. This calculation is made either by the original differential equation 4 or by solving for t_a in (6).

Having established the time-temperature relations for heating and cooling, the combined effect for actual determinations of temperature rise becomes the next objective. If the work were radiated uniformly and no heat losses were present, the rate of temperature rise would be proportional to the energy supplied.

$$\frac{dt}{d\theta} = h \tag{9}$$

h is directly proportional to the watts per square inch and may be expressed as:

$$h = \frac{rw}{17.58s\rho d} \tag{10}$$

where w is the energy received in watts per square inch and r the ratio of watts per square inch absorbed to watts per square inch received.

Since the temperature rise is not a linear function of time because of convection losses, equation 9 must be modified.

$$\frac{dt}{d\theta} = h + k(t - t_a) \qquad \text{where } k < 0 \tag{11}$$

Separating the variables and integrating from temperature t_a when $\theta = 0$ to t at time θ

$$\int_{t_a}^{t} \frac{dt}{h + k(t - t_a)} = \int_0^{\theta} d\theta \tag{12}$$

$$\log_e \frac{h+k(t-t_a)}{h} = k\theta \qquad (13)$$

Solving for t gives the theoretical time-temperature curve for heating:

$$t = t_a + \frac{h}{k}(e^{k\theta}-1) \qquad \text{where } k<0 \quad (14)$$

The initial rate of temperature rise, h, as shown in figure 2 may then be expressed as:

$$h = \frac{k(t-t_a)}{e^{k\theta}-1} \qquad (15)$$

The ratio of h to Δt discussed in (2) gives the absorption factor

$$r = \frac{h}{\Delta t} = 17.58 \frac{s\rho dk}{w}\left[\frac{t-t_a}{e^{k\theta}-1}\right] \qquad (16)$$

A comparison of theoretical and experimental results is shown in figure 3. For this example a heat density of 7.0 watts per square inch was used on a steel panel 0.0174 inch thick having a coating of black polymerizing paint. Substituting the proper values in equation 8, k is found to be -1.342. Calculating h from equation 15, a value of 640 degrees per minute is determined. The theoretical curve obtained from these values, using equations 7 and 14 shows a variation from the experimental results which can no doubt be attributed to temperature variations in the air adjacent to the test panels during the laboratory procedure. At lower heat densities the results show even higher accuracy thus establishing their usefulness for industrial engineering calculations.

The reflection losses may now be obtained from equation 16 by determining the initial rate of temperature rise (Δt) from figure 2. This value is found to be 710 degrees per minute giving an absorption factor of 0.9 or reflection losses amounting to ten per cent.

The amount of energy usefully employed to produce an intended working temperature may now be determined from the ratio of the effective energy required per square inch to attain the temperature, $sm(t-t_a)$, to the total energy absorbed during time, θ, required for the temperature change. This may be expressed in British thermal units as $rw\theta \div 17.58$. Using η for this ratio, the equation becomes

$$\eta = 17.58 sm(t-t_a)/rw\theta \qquad (17)$$

Expressing t as a function of the time by means of equation 13 and substituting

$$\eta = 17.58 \frac{smh}{krw}\left[\frac{e^{k\theta}-1}{\theta}\right] \qquad \text{where } k<0 \quad (18)$$

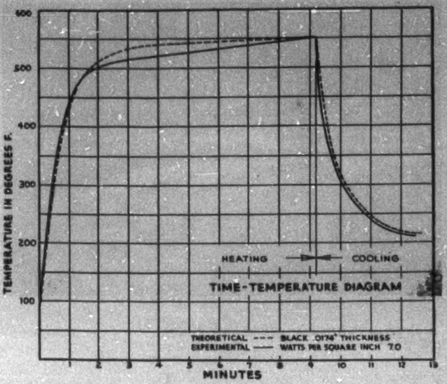

Figure 3. Typical radiant heat time-temperature diagram for a painted steel panel

Using the relation $rw = 17.58\, smh$ (18) becomes

$$\eta = \frac{1}{k}\left[\frac{e^{k\theta}-1}{\theta}\right] \qquad (19)$$

From this equation it is shown that the relation of useful energy retained in the work during the heating cycle to convection and re-radiation losses is a function of time. This is demonstrated graphically for a typical set of operating conditions in figure 4. The work in each case consisted of 24-gauge cold-rolled steel painted with a glossy black finish which reflected ten per cent of the radiant energy received. During this part of the investigation, the oven was allowed to reach an equilibrium air temperature for each of the heat densities employed. The steel was raised from an initial temperature of 80 degrees and the conditions as previously described were such that very little loss might be expected by re-radiation. Whether or not this is the case, no attempt has been made to separate these losses from convected heat as they are valueless. Since figure 4 only accounts for energy that has been absorbed by the work, the complete allocation of energy delivered to the panel surface for a desired working temperature of 300 degrees is shown in table I.

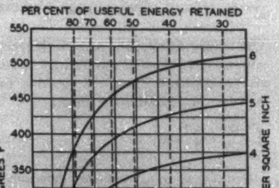

Figure 4. Relation of energy absorbed to useful energy needed for heating a 24-gauge steel panel having an infrared reflection factor of ten per cent

Table I. Relative Energy Distribution at Various Heat Densities for Raising a Typical Black Body to 300 Degrees Fahrenheit

Watts Per Square Inch Received	Allocation of Energy in Per Cent Useful Heat Retained	Reflection Losses	Convection and Re-radiation
3	25	10	65
4	65	10	25
5	76	10	14
6	85	10	5

The per cent of useful energy is not to be construed as an efficiency factor in the sense that it may be compared to efficiencies for other types of ovens since:

1. Energy that may be required to maintain a given working temperature during the heating cycle is not included.

2. The results obtainable at a given temperature may be quite different, particularly for vaporization processes due to a more advantageous rate of heat transfer.

3. Loss before and after the working

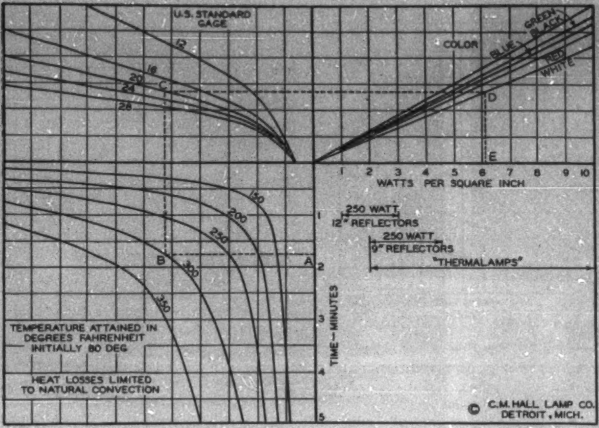

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Figure 5. Chart for computing time or heat density required to obtain desired temperatures under various conditions of mass and color

If the heat density required to obtain a given temperature in a given time is to be determined, the procedure is as follows: Starting with the given time (for example $1^3/_4$ minutes, at *A*) proceed clockwise following the dashed line successively to the desired temperature (for example 300 degrees, at *B*), to the gauge of sheet steel (for example 20 gauge, at *C*), to color (for example white, at *D*), and finally to *E*, giving the required watts per square inch. If the time required to reach a certain temperature with a known watts per square inch is to be determined, the foregoing process is carried out in reverse fashion, starting at *E* and proceding counterclockwise to *A*

cycle may be virtually eliminated by the instantaneous energy response.

4. Useful energy need not be consumed heating extraneous objects or parts as in the case of nondirectional heating.

5. No account is made of the energy consumed other than that actually reaching the working surface.

6. Convected heat from the working surface and radiant energy sources may also be usefully employed to increase working temperatures.

A comparison of the temperature limits indicated in figures 3 and 4 will demonstrate the importance of the air temperature surrounding the work. The results indicate the relation which frequently exists between uncontrolled experimental conditions and the higher temperatures normally resulting in production practice. The important conclusion to be reached is that within the safe limits of the materials entering an oven, the higher the heat density applied, the quicker will the desired temperature be attained and the greater will be the efficiency of the heat transfer.

The commercial importance of designing ovens for rapid, efficient heat transfer is thus not only shown to be advantageous from a heating standpoint but the consequent time saving affords many practical advantages. Figure 5 graphically demonstrates the time-temperature relations resulting from this investigation for various colors and gauges of sheet steel when subjected to heat densities within commercially available limits. The chart may either be used to calculate needed heat densities for practical installation conditions or to determine the necessary time for obtaining a given temperature at predetermined heat densities.

Production of Energy

Highly efficient sources for producing infrared energy have been available to the electrical industry since Edison's invention of the incandescent lamp, early models of which produced 15 to 20 times as much radiation in the infrared spectrum as in the visible. Although light production has meanwhile been the objective, incandescent lamps today provide many advantages as sources of radiant heat when operated at slightly reduced filament temperatures. With the filament operated in an atmosphere free from oxygen they provide an extremely efficient, long life source having virtually instantaneous response. Very little heat is lost by convection or conduction and the extremely small radiating surface makes for highly efficient control of the useful energy.

Incandescent sources possess the inherent characteristic of producing energy over an extremely broad range of wave lengths, the limits of which are dependent on filament temperature. Since the filament temperature is easily controlled, the effect of various wave lengths on materials which might be subjected to radiant heat is of great interest. Water films, for example, are excellent transmitters of energy at wave lengths below 11,000 angstrom units. Pigments commercially used for paint finishes vary considerably in their absorption characteristics to energy from 7,000 to 30,000 angstrom units. The result is of very little consequence, however, where the pigment is applied on a heat absorbent surface since paint films are customarily of such characteristics that the temperature response is virtually identical, whether the energy is converted from radient to physical form within the film or by conduction from the underlying surface.

Shorter wave lengths approaching the visible spectrum may be reflected, particularly by light colored objects, although the degree of reflection is in no way comparable with that for visible energy. Since energy losses by reflection are of important consequence to the effi-

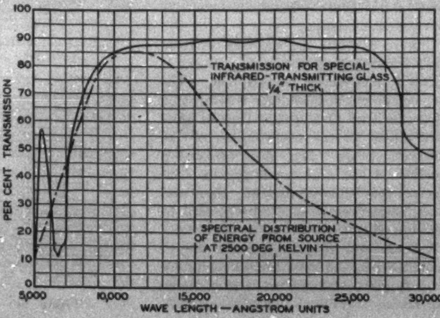


Figure 6. Energy distribution from a radiant heat source and characteristics of an infrared transmitting glass

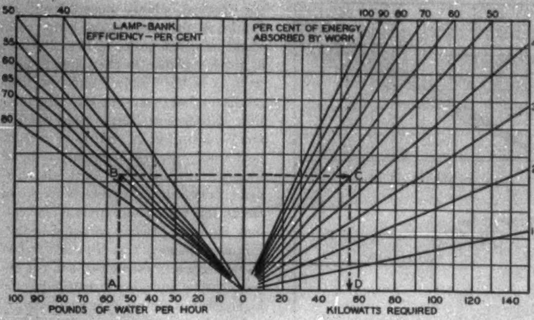


Figure 7. Graphical kilowatt computation for water evaporation

Starting with a given weight of water (for example, 55 pounds per hour, at *A*) proceed clockwise: to lamp-bank efficiency (for example, 65 per cent, at *B*); then to absorption per cent (for example, 50 per cent, at *C*); then to final answer, *D* (in this case, 55 kw). Chart is based on initial water temperature of 70 degrees Fahrenheit

ciency of heat transfer, it is essential that energy subject to reflection be held to a minimum. Conversely the maximum amount of energy must be produced in wave lengths sufficiently short to be efficiently transmitted by the glass enclosing envelope.

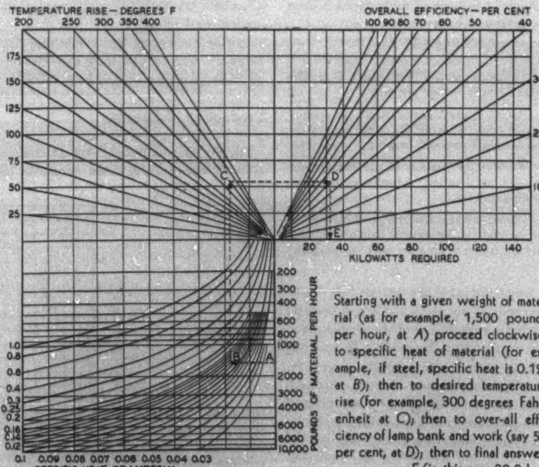
A further consideration is the fact that the surface area of filaments required to radiate equivalent energy is inversely proportional to the fourth power of the absolute temperature. Reductions in operating temperature may then impair the optical efficiency of the system although longer life and reduced glare will result. Qualitative research covering other possible aspects of wave length and energy production are in the formative stages and it is not unlikely that some wave bands may be found more effective for one type of work than another.

Figure 6 shows the spectral distribution of energy from a source operating at 2,500 degrees Kelvin (slightly over 4,000 degrees Fahrenheit) and the transmission characteristics of a special type of glass well suited to the transmission of the infrared energy. This glass converts a portion of the visible energy into convected heat thus minimizing glare and when employed as a lens, may be used to provide advantageous optical control.

Carbon and tungsten sources are in common use. Carbon filaments sublimate more rapidly, however, resulting in faster blackening of the enclosing envelope with consequent reduction in effective life. Tungsten sources are commercially available in larger sizes, facilitating higher heat densities and reducing the amount of associated equipment required for installation. Since filaments operating at 2,500 degrees Kelvin or lower temperatures are materially larger and the rate of sublimation appreciably slower than that for lighting service lamps, the practical life is customarily not directly dependent upon hours of filament service. Lamp blackening in later life may make it essential to replace sources whose usefulness is not otherwise terminated by mechanical failure or electrolysis within the lamp. Such failures are ordinarily not to be construed as being representative of defective equipment as the latter may occur after an appreciable service life when the operating circuit is interrupted. The rapid collapse of the induced field resulting from high current densities in adjacent supply wiring is believed to be responsible. New types of enclosed equipment prevent hazard from such failure. Other problems also encountered in the early periods of development from vibration, high temperatures, and humidity are now quite unlikely with new hard glass sources and proper auxiliary equipment.

Application of Radiant Heat

Radiant heating principles may eventually be employed in many types of heating service. For the present at least, the type of equipment covered by this paper is primarily limited to the low temperature heating of physical objects. The practical temperature limit with present-day equipments is approximately 600 degrees Fahrenheit and the equipment is made to operate at virtually any of the standard industrial distribution voltages. Lamp ratings vary from 250 to 1,000 watts and consequent heat densities may be provided up to approximately ten watts per square inch. Both open and enclosed type reflectors are available with aluminum or gold employed as the essential reflecting surface. Virgin gold provides the highest known reflection of infrared energy. Unfortunately, it has been found to fuse readily with other metals, particularly when subjected to



Starting with a given weight of material (as for example, 1,500 pounds per hour, at *A*) proceed clockwise: to specific heat of material (for example, if steel, specific heat is 0.12, at *B*); then to desired temperature rise (for example, 300 degrees Fahrenheit at *C*); then to over-all efficiency of lamp bank and work (say 50 per cent, at *D*); then to final answer, *E* (in this case 32.8 kw)

Figure 8. Graphical kilowatt computation for heating physical objects

Figure 9. Typical high-heat-density radiant oven—bakes a weather-resistant finish on heavy-gauge parts in 3½ minutes

Figure 10. Typical low-heat-density radiant oven installation

temperatures characteristic in heating service. In this service its initial efficiency of 97 per cent has been found to depreciate to that for the base metal on which it is applied within periods of from one to two years. Aluminum in properly oxidized form, is slightly less efficient initially than gold but particularly when employed in a controlled atmosphere will retain its initial characteristics at fairly high temperature for an almost indefinite period.

The great majority of applications for radiant heating not only require a knowledge of thermodynamics but also of optics, chemical and electrical engineering. Having developed from the field of lighting where the adaptability of the human eye is made to compensate for the many conditions presented, it is, for example, not to be construed that similar flexibility may be enjoyed in the curing of a paint finish intended to withstand weather conditions varying from summer heat to winter cold.

Fortunately for radiant heating, baking processes involving oxidation are very much on the decline. Oils are being replaced by synthetics which will cure by polymerization (combining of double valence atoms to produce an expanded molecular structure) as rapidly as the heat transfer can be effected. Unlike slower oxidizing processes which may continue over an indefinite period if initially incomplete, generally intended results with polymerization require appreciable accuracy both as to time and temperature of bake. Even in the simple case of dehydration processes, competition from generally lower cost fuels compels the careful study of evaporation rate with consequent adjustment in the density of energy applied throughout the operation.

With this type of heating, temperature control, oven adaptability, and the full realization of instantaneous-directional heating advantages, are within the province of the electrical engineer. Numerous types of circuits and interlocked features of operation make it possible to apply heat where and as desired. The ease with which electricity can be controlled as a fuel for industrial heating may well compensate for its higher cost.

Having produced the invisible rays responsible for this type of heating, there is the problem of their proper delivery to the working surfaces to insure consistent and uniform temperatures. This problem is often complicated by the many shapes and sizes of work which an oven may be required to handle. New optical principles have had to be introduced to avoid serious variations in heat density over a practical range of working distances from the equipment and applications involving materials of low thermal conductivity have necessitated improvements in the uniformity of heat distribution.

Most heat sources deliver an appreciable part of their energy in the infrared spectrum although the wave lengths and proportion of convected heat will vary with the temperature of the source. Since the primary difference between radiant heating and other types is in the method of heat delivery, the needed heat capacity of an oven may be determined in the same manner that it would for any other type of heating. For example, if a dehydration operation is to be performed, the liquid must first be raised to a point where its vapor pressure equals or slightly exceeds that of the surrounding atmosphere, and the additional heat of evaporation supplied. In general, water evaporation with radiant heating takes place when the accompanying mass temperature is as low as 160 degrees Fahrenheit, under normal atmospheric pressure. This apparently low temperature, resulting during the withdrawal of the heat of vaporization ordinarily does not exist throughout the entire cycle, thus indicating pressure variations in vapor balance which favor application of radiant heat at the wet end of drying process. Figure 7 shows a graphical means for determining the approximate kilowatt capacity needed for evaporating water at various rates. Similarly the required energy for raising the temperature of a solid may be expressed by:

$$\text{kw} = sM\eta T/3{,}413 \tag{20}$$

where M equals mass per hour, T the time in hours, and η the efficiency. Figure 8 provides a graphical means for determining the required kilowatt capacity of an oven in accord with this formula when the over-all efficiency of heat transfer is known. As previously shown, efficiency varies with the duration of the heating cycle as well as being effected by the many physical factors of oven design. Acknowledgment for the use of charts 7 and 8 is given to the Illuminating Engineering Society and to their author, Howard Haynes.

The several considerations to be made in connection with the proper application of radiant heat emphasize the need for engineering co-ordination. High rates of heat transfer combined with the proper chemistry of materials may in some cases produce phenomenal results. On the other hand, many applications may prove to be entirely impractical while some may or may not be as economical as other methods of heating due to individual conditions.

In principle radiant heating is sound. Its future must rest on its specific merits and the suitability of application which it receives. At the present stage of commercial practice, experimental trials are to be recommended in connection with each application. The resulting oven may employ one or more heat densities for most efficient operation as the controlled rate of heat transfer is the distinguishing feature of radiant energy heating. Figure 9 illustrates a typical high-heat-density installation of 96 kilowatts before draft shields and ventilating equipment have been installed. Figure 10 shows a completed low-heat-density installation of $22^1/_2$ kilowatts.

Advantages and Uses

Numerous advantages may be cited for radiant energy heating in addition to the controlled rate of heat transfer. The high temperature sources provide virtually an instantaneous source of heat, thus eliminating the customary warm-up time for convection ovens. Loss of heat during the warm-up period as well as the consequent loss after each period of operation is also eliminated. Because air temperatures are not raised to comparable convection oven levels, working conditions are frequently more comfortable. The elimination of fumes resulting from combustion fuels also contributes to improved working conditions and in some cases more satisfactory results.

The first cost is invariably low, inasmuch as the structure may be limited to a skeleton frame without insulated walls, doors, or temperature control equipment. Draft shields are ordinarily employed to minimize convection losses and under some circumstances, exhaust hoods may be required above the ovens to remove objectionable fumes. Working temperatures will consistently follow the intended time-temperature response if proper maintenance, voltage regulation, and uniform air conditions are assumed. The semipermanent construction allows the utmost flexibility for plant changes and the proper arrangement of electrical circuits frequently makes for advantageous conversion of the oven to intermittently accommodate varying types or sizes of work. The light-weight construction also permits hanging ovens from the ceiling allowing the free use of floor space below.

Since radiation can be effectively controlled, the energy may be directed or confined to a desired heat pattern or zone of operation. Commercial equipments are available providing various heat patterns so that the utilization efficiency may be of a very high order. This feature also makes possible the design of multiple heat density ovens so that the most effective heat concentration may be applied in each stage of a process and the total cycle consequently reduced to a minimum time. With convection practice this would necessitate sectionalized oven construction, at inherently greater cost.

The direct energy transfer without intermediate heating of steam or air not only facilitates high efficiency but the increased rate with which temperature changes may be effected invariably provides substantial savings in time. Where heating operations are mechanized this simultaneously reflects a corresponding saving in space thus facilitating automatic handling. Many industrial finishes are today being baked in less than two minutes indicating that the proper combination of polymerizing materials and heat application may reduce the time for such baking processes to a small fraction of that characteristic in current practice.

While the source temperatures used in radiant heating are substantially higher than temperatures with combustion or other customary heat sources, the enclosed construction assures relatively low surface temperatures. With a second enclosing envelope employed for optical control and breakage contingencies, the resulting safety from fire or other hazards is of an extremely high order as compared with many other types of ovens.

The effect of humidity in a radiant heat oven becomes practically negligible since the principal mass of air in the oven is at an appreciably lower temperature than that immediately in contact with the work. As a result the air goes through a continuous cycle of heating and cooling; absorbing the vapors when heated in contact with the work and allowing them to condense in colder portions of the oven. This affords a decided advantage for dehydration applications as the convection losses may be materially reduced by a lower rate of air change than would be necessary with convection drying.

At present radiant heat is being employed primarily for the low temperature heating of metals and plastics, the baking of paints and insulating varnishes, and a great variety of dehydration operations. Plans are now in process for extending it into many other fields, particularly where chemical operations may be accelerated. Fuel costs may or may not be directly comparable depending on individual conditions, as one or more of the other operating advantages may produce savings that will very well justify a far more extensive use of electricity in the field of low temperature heating.

Few developments in the last decade have been so revolutionary or have offered such promise for future expansion of the electrical industry as this one which had its inception for production application in the Ford Motor Company. Due credit is accordingly acknowledged to the work of that company and to the many people in the electrical industry who have co-operated in bringing this development forward to a place of respect as a significant tool for industrial use. The author wishes also, to thank his associate F. M. Tiller for his co-operation in the investigation.

Nomenclature

A = area in square inches
d = thickness in inches
h = constant expressing rate of temperature rise without effect of convection
k = constant for convection heat loss in Newton's equation
m = mass in pounds
M = pounds per hour
r = absorption factor
Q = heat in British thermal units per minute
s = specific heat
t = temperature, degrees Fahrenheit
t_0 = temperature of solid before cooling in degrees Fahrenheit

The Application of Ignitrons to Resistance-Welding Control

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SINCE the development of the ignitron several papers have been presented which pertain to the theory of operation[1,2,5] and explain its contribution to the recent advancement in the field of resistance-welding control.[7,8,9,12] The object of this paper is to review the ignitron characteristics and to point out the operating principles, which should be considered in order to insure successful applications.

In the field of resistance-welding control the ignitron has proved that it possesses a combination of characteristics such as high-current carrying capacity, efficiency, ease of control, reliability, instantaneous operation, no moving parts, quietness, and long life, unequaled by any other device. Therefore a brief description of the design and operation of the ignitron may be desired by those who have not closely followed its development or realized its usefulness as exemplified by its expansion of the field of resistance-welding control.

Construction

Figure 1 is a photograph of a typical water-cooled ignitron, the cross section of which is shown by figure 2. It essentially consists of an evacuated vessel containing a mercury-pool cathode, a graphite anode, and an ignitor for initiating an arc discharge between the cathode and anode at any desired instant during the time that the anode is positive. In this particular design the envelope consists of two concentric cylinders completely closed at the top and bottom and having water inlet and outlet connections, thus serving as the container and the water cooling jacket. A heavy rectangular bar is welded below and perpendicular to the lower header, and thus serves as a means for mounting the tube in an upright position and as the external connection through a large contact area to the mercury-pool cathode within. The top of the tube has a fernico seal supporting a large graphite anode and its external connection is made of heavy copper cable. The ignitor consists of a conically shaped rod of semiconductive material dipping into the mercury pool. It is supported and insulated by a fernico seal extending through the mercury pool and mounted on the header, and through this seal extends the external connection to the ignitor. The tube is exhausted to an extremely low gas pressure and permanently sealed.

Operation

When an electric current of a few amperes and a few microseconds duration is passed through the ignitor into the mercury pool, a cathode spot is formed on the mercury-pool surface. If the anode is positive when the ignitor is energized, a mercury arc is established between the cathode and anode, and persists until the anode current is reduced to zero either by decreasing the anode voltage to a very low value or by means of some external current-interrupting device. The positive-anode voltage may then be re-established and the tube will remain nonconductive until the ignitor is again energized. Since in a-c applications the anode current is naturally reduced to zero at the end of each conducting half-cycle the ignitor control may be regained during the negative half-cycle of anode voltage.

Characteristics

The ignitron can thus be briefly defined as a three-element electron tube having as one electrode a mercury pool provided with a means of instantaneously forming a cathode spot, such that the tube is inherently a controlled mercury-arc rectifier. Since the ignitor provides a means of instantaneously rendering the cathode electron emitting when desired, it avoids the necessity of maintaining a holding electrode during periods of negative-anode voltage and contributes to the ability of the tube to withstand relatively high inverse voltages without the development of cathode spots on the anode which might result in arc-back and reverse current.

The mercury pool has the ability to emit an electronic current of several thousand amperes and to withstand current emission far in excess of the tube average current rating without detrimental effects to the mercury-cathode element which, while being consumed, is replenished by mercury condensation. The current rating is restricted by other considerations. The cathode-anode arc voltage is substantially constant throughout the rated current range and being only about 15 volts contributes to high efficiency, even when the tube is employed in low-voltage circuits.

Application to Resistance Welding

Resistance welding is the process of raising portions of metallic parts to a welding temperature by virtue of the heat developed by an electric current flowing through the resistance of the portions of the metals to constitute the weld. It essentially consists of passing an electric current through metallic parts to be assembled under conditions such that the point of contact between the parts forms the major part of the circuit resistance, thus localizing the heat and producing the desired weld. This usually requires a current of several thousand amperes and

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1. For all numbered references, see list at end of paper.

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t_a = air temperature in degrees Fahrenheit
Δt = temperature change in degrees Fahrenheit per minute
T = time in hours
w = watts per square inch received at surface of work
η = per cent of useful energy retained
θ = time in minutes
ρ = density in pounds per cubic inch

Bibliography

1. Drying With Near Infra-Red Radiation, L. S. Ickis, Jr. and Howard Haynes. *General Electric Review*, volume 42, number 4, April 1939.

2. Paint Baking With Near Infra-Red, H. J. Bennett and Howard Haynes. *Chemical and Metallurgical Engineering*, February 1940.

3. Invisible Rays Build Visible Profits, Hugh Archer. *Electric Light and Power*, May 1940.

4. Infra-Red Heating Up to Date, Paul H. Goodell. Great Lakes Power Club, June 1940.

5. The Use of Radiant Energy for the Application of Heat, Howard Haynes. *Illuminating Engineering*, volume 36, January 1941, pages 61-78.

6. Modern Gold Plating—Part 6, Joseph B. Cushner. *Product Finishing*, February 1941.

requires clamping the work under considerable pressure between water-cooled electrodes, either stationary or revolving. The magnitude and the duration of the welding current must be independently controllable with a high degree of precision in order to produce welds of the best quality.

When high-quality welds are desired, and in all cases where they are of vital importance, the large welding-current requirement is met by connecting ignitron tubes in the primary of a welding transformer which transforms the energy to a low voltage and higher current as required. Since the ignitron is inherently a controlled mercury-arc rectifier, its application to the control of alternating currents necessitates connecting two ignitrons in inverse-parallel as shown in figures 3, 4, and 5. By means of the ignitor the current through either ignitron may be initiated at the desired instant during the half-cycles that the anode voltage of the ignitron is positive and the reversal of the anode voltage offers the opportunity to interrupt the current at the zero point of the current wave and to regain control during each half-cycle that the anode is negative. Firing the ignitrons at the power factor of the load* furnishes full half-cycles and a sine wave form of load current. Retarding the firing beyond the power-factor angle produces portions of half-cycles of load current and provides a convenient means of controlling the effective value. This is referred to as heat control by the phase-shift method. Thus, two ignitrons connected in inverse-parallel or back-to-back, being instantaneous in operation and capable of carrying several thousand amperes of alternating current, serve as an inertia-less high-speed accurate contactor, actually an electronic contactor with heat control, to meet the exacting requirements of resistance welding. By means of this control transients are eliminated and better welds are made with less line regulation, less heating in the welding transformer, and with longer electrode life.

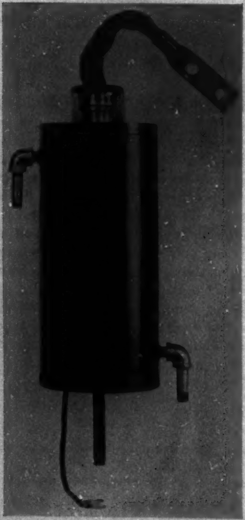

Figure 1. Type FG-258A water-cooled ignitron

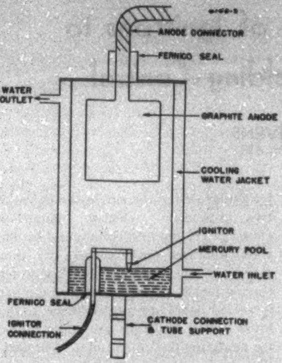

Figure 2. Schematic cross section of ignitron shown in figure 1

The ability of ignitrons to pass large currents with instantaneous response, in conjunction with the development of thyratron precision timing controls, has made large welding currents as short as one cycle practical and short accurate timing periods have greatly improved the technique of resistance welding, speeded up production methods, and extended the field of resistance welding to highly conductive nonferrous metals having critical welding temperature ranges.

The application of thyratron precision timing controls and ignitrons to resistance welding includes entirely too large a variety of circuits to be more than mentioned here. They include applications of spot, pulsation, and seam welding controls, to spot welding, projection welding, butt welding, flash welding, and seam welding, which have been adequately covered by preceding papers.[9,10,11]

Precision Ignitron Control

Thyratrons are more suitable than ignitrons as measuring devices and therefore in cases where precision timing and firing are required, they are used for the more delicate operating functions and in response to low energy voltage controls furnish ample ignitor current to insure firing the ignitrons in a manner similar to the use of a sensitive relay for controlling a large contactor or oil-circuit breaker. The circuits shown in figures 4 and 5 embody most of the operating principles which must be considered in the applications of ignitrons. Figure 4 for precision control of welding current will first be considered. The time of conduction of the ignitrons is controlled by the firing thyratrons in the ignitor circuits. The firing thyratrons are controlled by associated precision timing circuits which determine the duration of the weld, the time between welds, the number of welds, etc. This circuit provides a convenient method of heat control by the phase-shift method. Considering a half-cycle when line $L2$ of the power supply is positive, the full-line voltage is applied as the anode voltage for the firing thyratron 1 and the ignitron A until the firing thyratron 1 is rendered conductive by the associated control. At the instant of firing, the ignitor current is chiefly determined by the instantaneous value of line voltage, the ignitor resistance, the value of the current-limiting anode resistor $R1$, and the welding transformer load. An arc is initiated in the ignitron applying substantially full-line voltage to the load and reducing the firing thyratron anode voltage to such a low value that the ignitor current is extinguished. The load current then continues in a natural manner until it attempts to pass through zero, its continuance then being similarly dependent upon the conductivity of firing tube 2 and ignitron B. If the firing point is adjusted to occur at the power factor of the load the first half-cycle of current will always be of normal wave form (steady-state value) for the full heat condition. This corresponds to the wave form of current shown by the solid trace in figure 6. Preventing firing in advance

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*In this paper references to the power factor of the load mean the power-factor angle where the welding-transformer primary current passes through zero during steady-state conditions with the secondary closed on the work to be welded. This power-factor angle is intermediate to the corresponding power-factor angles of the exciting current and load-current components of the primary current. The accepted practice is to initially energize the welding transformer at the natural zero point of the primary current wave (except where phase control is used) which results in negligible transient in the primary current.

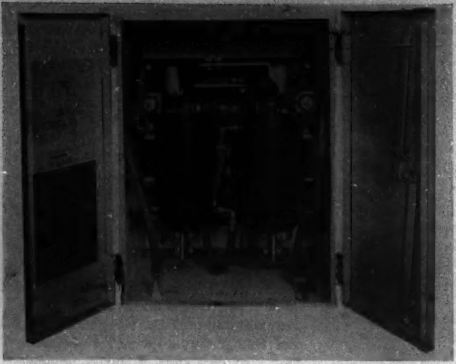


Figure 3. Ignitron contactor

of this point avoids the possibility of beginning the weld with a transient starting current with the first cycle having a magnitude and time duration exceeding that of the normal sustained value. If the controlled firing occurs later than the natural power factor of the load, the wave form of current and welder voltage will be as shown by the dotted current trace and cross-hatched portion of voltage shown in figure 6, resulting in a reduced heat condition. In any case the welding current is interrupted near the natural zero point of the current wave.

The operating principles which must be considered in order to insure successful operation of ignitrons can most easily be explained by referring to the wave forms of currents and voltages in the component circuits. An understanding of the wave forms existing during normal operation will assist in checking correct application and in detecting cases of abnormal operation which may be caused either by misapplication or defective tubes. Observing the form of circuit voltages by means of a cathode-ray oscillograph is the most satisfactory method of checking the operation. Due to the strong-stray magnetic field near a welding transformer and welder during operation, and due to the high sensitivity of the cathode-ray oscillograph, it is usually necessary to place the oscillograph somewhat distant from the welder and in the position for minimum response to the magnetic field.

The wave forms for different voltages and currents are drawn separately in figures 7*a*, 7*b*, and 7*c*, which show the conditions corresponding to full heat, medium heat, and minimum heat, respectively. These wave forms are shown for a lagging power-factor load which is usually between 60 degrees and 85 degrees for a welding transformer and in which case a full half-cycle of current can be conveniently initiated by virtue of the fact that considerable line voltage is available at the desired instant of firing.

Furthermore, the firing tube operates into a more resistive load than the ignitron welder load and the full heat firing point corresponds to a retarded firing angle for the thyratron. This facilitates obtaining a steep wave front of ignitor current which assists in quickly starting the ignitron. However the minimum heat adjustment for reliable starting is limited because of the low voltage near the end of the half-cycle. The minimum instantaneous ignitor voltage required for reliable starting is considered as being about 200 volts. To this must be added the normal 15-volt drop in the firing thyratron and the voltage drop across the anode current-limiting resistor. The recommended value of anode current-limiting resistance is four ohms for 440- or 550-volt supplies, and one ohm for a 220-volt supply. The instantaneous starting current required for reliable operation of a normal ignitron averages about 20 amperes, but it is recommended that a minimum of about 40 amperes be furnished. The minimum heat adjustment should therefore be restricted from firing beyond the 143-degree point of a 440-volt supply voltage or the 128-degree point of a 220-volt supply voltage.

Considering the limitation imposed by a 220-volt supply, with an 85-degree (0.087) power-factor load, the 128-degree point on the voltage wave corresponds to 43 degrees angular delay in firing. This reduces the effective current to about 28 per cent of the full value or the heat to about 8 per cent of full heat. A 60-degree (0.5) power-factor load permits an angular delay of 68 degrees and reduces the current to about 24 per cent or the heat to about 5.8 per cent. Firing at the 143-degree point of the voltage wave as permitted by a 440-volt supply reduces the current to about 13 per cent (about 1.7 per cent heat) in the case of an 85-degree power-factor load, and reduces the current to about 11 per cent (about 1.2 per cent heat) in the case of a 60-degree power-factor load.

It should be observed that firing at the 128-degree point on the voltage wave (or firing at the 143-degree point on the voltage wave) causes a little greater reduction in heat in the case of a 60-degree power-factor load than in the case of an 85-degree power-factor load. The phase-shift circuit used for heat control is such that the full heat adjustment prevents retarding the firing angle as far on a high power-factor load as it permits when adjusted for a low power-factor load. Consequently, if the full heat adjustment is correctly made the minimum current setting will be substantially independent of the power factor of the load considering the usual range of power factors of resistance welders. Therefore, control panels of the most recent design have heat control dials precalibrated from 20 per cent to 100 per cent current which provides a range from 4 per cent to 100 per cent heat. However, when operating from a 220-volt supply, current settings less than

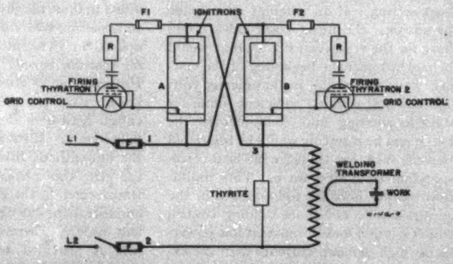


Figure 4. Schematic diagram showing precision control of ignitrons

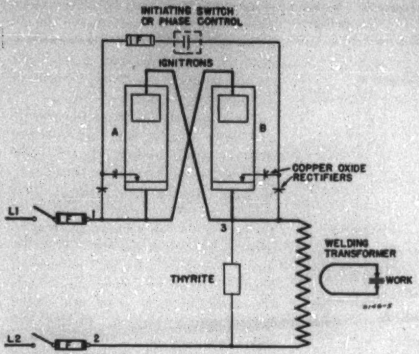


Figure 5. Schematic diagram of Ignitron contactor

40 per cent (16 per cent heat) are not recommended.

When operating at reduced current there are always zero-current gaps in the current wave as shown by the upper traces in figures 7*b* and 7*c* but the wave form of the individual current impulses or portions of half-cycles is appreciably affected by the nature of the load, that is whether the load is resistive, inductive, or capacitive, and also whether the combinations of resistance, inductance, and capacitance are effectively in parallel or in series. The series characteristic nearly always prevails in resistance-welding control circuits. In the case of the low heat condition—figure 7*c*—the effect of the inductive portion of the load is distinctly noticeable. As the load is made more resistive the current wave form naturally more nearly approaches the wave form of the voltage applied to the load or welding transformer. If the firing point for a pure resistance load were the same as shown in figure 7*a* for full heat for a given inductive load, the load voltage wave form would consist of only the first and third segments of voltage shown for the case illustrated. The current wave forms would be substantially similar and this would obviously correspond to a reduced heat setting. If an attempt were made to advance the firing point toward full heat for the resistive load, a point would be reached near the beginning of the voltage wave where the instantaneous voltage has not attained sufficient value for reliable starting and consequently the maximum heat setting would be restricted to some value less than the full heat corresponding to a sine wave. This limitation usually is of no consequence in the application to resistance-welding control which is noted for intermittent low power-factor high-demand currents with the exception of small bench welders which having high power factors and requiring comparatively small currents may require supplementary shunt loading.

A low power-factor load is of course objectionable from the power-supply viewpoint and therefore power-factor correction to unity power factor or even a leading power factor is sometimes applied. Therefore, in such cases the series capacitor method of power-factor correction is employed.[13] This introduces the difficulty of firing the ignitrons for full heat on unity power-factor load at which instant there is no line voltage available for firing. The difficulty is readily overcome by precharging the capacitor which thus provides a voltage for firing at the zero point of the line voltage wave. Controls may be used which insure that an even number of half-cycles of current are supplied to the load and since the current is naturally interrupted at the zero point on the wave the series capacitor is left charged with the correct polarity for initiating the next weld.

Under normal conditions the firing thyratron is only required to furnish an ignitor current of very short duration once each cycle at the instant that ignition is desired. An anode resistor is provided to limit the thyratron anode current to a permissible value for instantaneous operation. The medium time required for ignition is only a few microseconds. However, due consideration must be given to the possibility that a short-circuited ignitor or defective ignitron may fail to start. If the ignitron fails to start, the hot-cathode thyratron is required to energize the primary of the welding transformer to the extent limited by the anode resistor during the remainder of the half-cycle. Therefore a three-ampere fuse is provided to protect the firing thyratron and serve notice in event that the demand on the thyratron is excessive. Normally the ignitor current has a sliver-like wave shape as shown by the lower solid traces in figures 7*a*, 7*b*, and 7*c*. Its wave form can be readily determined by observing the wave shape of the voltage drop across the firing tube anode resistor. If the ignitron is hard starting it will be revealed by a voltage trace showing that the firing tube conducts current for a time longer than for normal instantaneous ignition, as shown by the dotted traces in figure 7*a*. In the extreme case where the ignitor is energized but the ignitron fails to fire at all, either the anode fuse will blow or the trace of firing tube anode current will approach the wave form of the line voltage during the period of conduction. In such cases it should be determined whether the ignition failure is due to a defective ignitron (for example, an ignitron contaminated by foreign gas or having a wet ignitor) or due to misapplication such as attempting to operate a load drawing insufficient current for ignition or attempting to fire the ignitor with insufficient voltage such as might occur at full heat on a resistive load or at minimum heat on a reactive load. Tests should not be made while the secondary of the welding transformer is open-circuited because the exciting current may not be sufficient to provide reliable starting of the ignitrons and may exceed the rating of the firing tubes. After ignition the load current must also be sufficient to maintain the cathode-to-anode discharge in the ignitron after the ignitron is started. The subject of supplementary loading to improve the ignitron operation for lightly loaded welders will be dealt with later.

On the other hand the absence of any trace at all of voltage across the firing tube anode resistor would show that no ignitor current flows but this should be most readily noted by observing that the firing tube does not glow. Such an occurrence might be due to the ignitron being tilted so that the ignitor fails to contact the mercury pool or due to the firing tube anode fuse being open.

It is always desirable, and sometimes necessary, to prevent initiating the welding current in advance of the power-factor angle of the welding transformer in order to avoid transient currents. The power factor is usually about 70 degrees. Therefore, ignitron controls provided with synchronous precision timers but without heat control are adjusted at the factory to initiate the welding current at 70 degrees power factor. Ignitron controls provided with heat control (phase control) are adjusted to prevent advancing the full heat setting ahead of 70 degrees power-factor angle. The effect of a transient starting current produced by firing ahead of the power-factor angle depends upon whether it is applied to spot welding or seam welding and upon whether or not it is associated with phase control, the results being quite different. In the case of spot welding without phase control, since the transient accompanying synchronous starting is consistent and since the welder secondary circuit is opened after each weld, the transient in some cases may be tolerated. When phase control using peaked firing signals is provided, firing in advance of the power-factor angle cannot be tolerated for the following reasons: The first half-cycle of transient starting current will have a duration greater than 180 degrees, and will prevent initiating the next half-cycle of opposite polarity at the 180-degree point when the next firing peak occurs. Therefore, when the first half-cycle of current decreases to zero the peaked firing signal for the second half-cycle has already occurred and there is no ignitor excitation for the corresponding ignitron with the result that only positive half-cycles of current are furnished to the welding transformer and furthermore the starting transients may be accumulative. It is recommended that all installations be checked, and readjusted if necessary, to insure that there is no possibility of initiating the welding current in advance of the power-factor angle. This can most satisfactorily be accomplished by observing the wave form of the voltage across either the ignitrons or the primary of the welding transformer by means of a cathode-ray oscillograph and in consideration of the following.

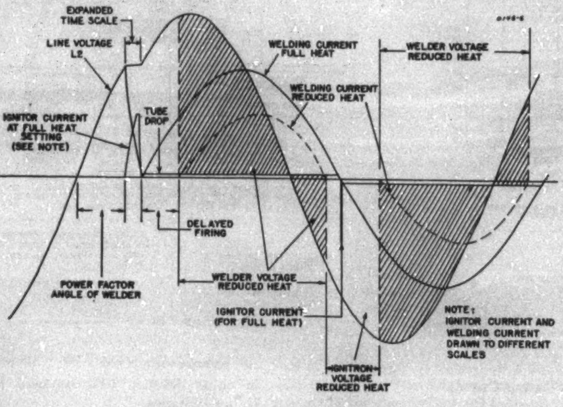


Figure 6. Current and voltage wave forms under phase-controlled conditions

If heat control is not provided it is merely necessary to observe whether or not the first few cycles of welding current are balanced. This can most conveniently be done by using a short-timing axis. In the case of heat control the wave forms should be observed. For example, considering the transformer primary voltage wave form as illustrated in figures 7*a*, 7*b*, and 7*c* when the heat control is advanced toward full heat the gap representing zero-primary voltage should decrease but not disappear until the full heat setting is reached. Conversely, as the heat control is advanced the line voltage across the ignitrons should not disappear until the full heat setting is reached. It will likely be observed that satisfactory operation can be still obtained if the phase control is advanced to the extent that the cathode ray barely shows a trace of line voltage during current transfer in the ignitrons, but it is recommended that the full heat setting be limited to allow a narrow trace of line voltage across the ignitrons during the current transfer (or a small gap for zero voltage across the primary of the welding transformer) to take care of variations in power factor of the welding transformer secondary circuit due to change of the welder throat dimensions, changes in amount of magnetic material in the throat, etc.

A few amperes of current are required to maintain cathode-to-anode conduction in an ignitron. When the anode current decreases below a few amperes the vapor-arc discharge may suddenly become extinguished. Therefore a resistance or thyrite unit is always connected across the primary of the welding transformer to suppress the generation of high voltage due to the sudden interruption of current.

The large supply current required by resistance welders usually causes considerable voltage regulation at the welder. The cathodes of the firing thyratrons have an appreciable thermal capacity for maintaining emission during reductions in cathode supply voltage of short duration, but long spot welds or seam welding accompanied by considerable voltage regulation may tend to shorten the life of the firing thyratrons on account of reduced filament voltage, and should be avoided by obtaining the control voltage from a location nearer the source.

Supplementary Loading

It is considered that the minimum current that can satisfactorily be controlled by ignitron tubes is 40 amperes. For currents less than 40 amperes, a shunt resistor or reactor must be connected across the primary of the welding transformer. The amount of the additional loading required depends to some extent on whether the shunt load is mostly resistive or reactive. That is, if the resultant load is made more resistive the full heat firing point for the welder will correspond to a retarded firing angle for the shunt-resistive load and the ignitor starting current can build up more rapidly than in the case where the shunt load is reactive. Therefore, a resistive load is preferable and a 25-ampere resistor load is considered adequate.

Ignitron Contactor

In some resistance-welding control applications it may be felt that the cost of thyratron synchronous precision timing control is not warranted but the use of ignitron contactors with their high speed, quiet operation, and low maintenance is desirable. The ignitron-contactor circuit shown in figure 5 is being used for such applications with the timing provided by some auxiliary device, such as a cam or vacuum-tube timer.

One convenient method of energizing the ignitrons is to connect them in series back-to-back across the ignitron cathode-anode circuits which are connected in inverse-parallel. A fuse is located in the

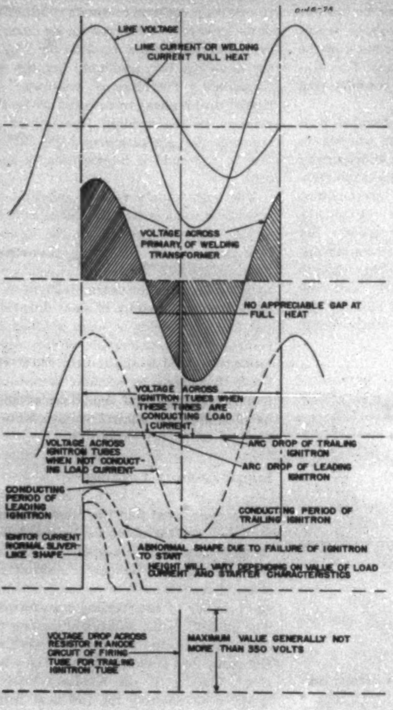
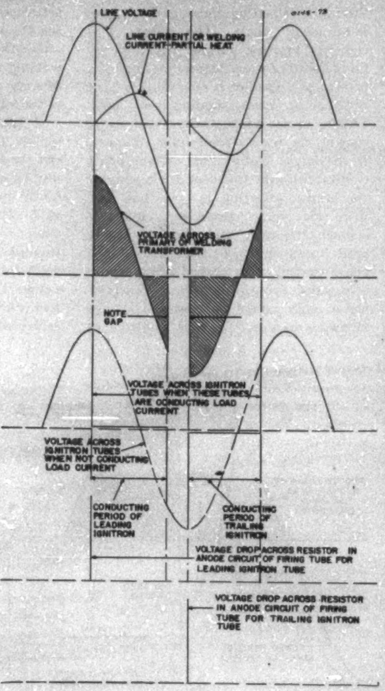

LINE VOLTAGE
LINE CURRENT OR WELDING CURRENT FULL HEAT
VOLTAGE ACROSS PRIMARY OF WELDING TRANSFORMER
NO APPRECIABLE GAP AT FULL HEAT
VOLTAGE ACROSS IGNITRON TUBES WHEN THESE TUBES ARE CONDUCTING LOAD CURRENT
VOLTAGE ACROSS IGNITRON TUBES WHEN NOT CONDUCTING LOAD CURRENT
ARC DROP OF TRAILING IGNITRON
ARC DROP OF LEADING IGNITRON
CONDUCTING PERIOD OF LEADING IGNITRON
CONDUCTING PERIOD OF TRAILING IGNITRON
IGNITOR CURRENT NORMAL SLIVER-LIKE SHAPE
ABNORMAL SHAPE DUE TO FAILURE OF IGNITRON TO START
HEIGHT WILL VARY DEPENDING ON VALUE OF LOAD CURRENT AND STARTER CHARACTERISTICS
VOLTAGE DROP ACROSS RESISTOR IN ANODE CIRCUIT OF FIRING TUBE FOR TRAILING IGNITRON TUBE
MAXIMUM VALUE GENERALLY NOT MORE THAN 350 VOLTS

Figure 7a (left). Wave forms corresponding to full heat

Figure 7b (right). Wave forms corresponding to medium heat


LINE VOLTAGE
LINE CURRENT OR WELDING CURRENT-PARTIAL HEAT
VOLTAGE ACROSS PRIMARY OF WELDING TRANSFORMER
NOTE GAP
VOLTAGE ACROSS IGNITRON TUBES WHEN THESE TUBES ARE CONDUCTING LOAD CURRENT
VOLTAGE ACROSS IGNITRON TUBES WHEN NOT CONDUCTING LOAD CURRENT
CONDUCTING PERIOD OF LEADING IGNITRON
CONDUCTING PERIOD OF TRAILING IGNITRON
VOLTAGE DROP ACROSS RESISTOR IN ANODE CIRCUIT OF FIRING TUBE FOR LEADING IGNITRON TUBE
VOLTAGE DROP ACROSS RESISTOR IN ANODE CIRCUIT OF FIRING TUBE FOR TRAILING IGNITRON TUBE

series ignitor circuit which also includes the initiating switch. This would necessitate one ignitor carrying reverse current during the instant that the other ignitor passes forward current for the ignitron having a positive voltage during the half-cycle considered. It is necessary to avoid passing reverse currents through ignitors and therefore copper-oxide rectifiers are provided to by-pass the reverse current. A low value of resistance usually four ohms directly in series with each ignitor contributes to the shunting effect of the copper-oxide rectifier for reverse currents. Using another rectifier instead of the resistor in series with the ignitor so that the rectifier readily passes forward ignitor current but tends to block reverse current reduces the ignitor current substantially to zero and improves ignitor life. If phase control is desired, the phase-control unit can be conveniently connected directly in series with the external initiating switch. The ignitron contactor with phase control prevents high-transient starting currents.

Rating Curves

Ignitron tubes are designed in various sizes having current ratings closely following a preferred number series. As previously stated their continuous-current rating is chiefly limited by the maximum allowable mercury-vapor pressure caused by arc heating—the permissible electron emission of the cathode being far in excess of this value. However, the greatest benefits of ignitron characteristics are obtained in their application to intermittent, short-time, high-current demands as required for resistance welding. In rating the ignitron the average current, effective current, and peak currents must be considered and the main criteria are to limit the current density accompanied by a high instantaneous mercury-vapor pressure and to limit the degree of ionization at the instant the anode voltage becomes negative. These limitations must be recognized or conditions leading to "shoot through"* and "arc back"** may occur, thus permitting current conduction in either direction unaccompanied by ignitor excitation.

In comparison, the intermittent current rating of a cable or a welding transformer can be determined from its continuous-current rating and the duty cycle, with consideration of its ability to radiate heat during short-time intervals. (In these applications where the load is intermittent, cable size is usually determined by its effect on voltage regulation, instead of the heating.) For example, if a conductor is capable of carrying a given current continuously, it is capable of carrying an intermittent current $I_t = I_c/\sqrt{D}$ where D is the duty cycle or percentage of time that the current flows. Since the ignitron operates at a substantially constant voltage drop, the heat loss is more nearly proportional to the current instead of the current squared. Intermittent current ratings tend to follow this characteristic except that other factors must be considered. In order to specify the

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*Loss of control in forward direction.

**Loss of control in reverse direction.

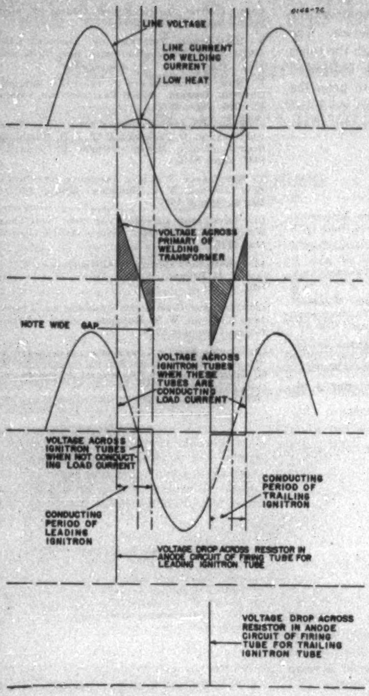
0145-7C
LINE VOLTAGE
LINE CURRENT OR WELDING CURRENT
LOW HEAT
VOLTAGE ACROSS PRIMARY OF WELDING TRANSFORMER
NOTE WIDE GAP
VOLTAGE ACROSS IGNITRON TUBES WHEN THESE TUBES ARE CONDUCTING LOAD CURRENT
VOLTAGE ACROSS IGNITRON TUBES WHEN NOT CONDUCTING LOAD CURRENT
CONDUCTING PERIOD OF TRAILING IGNITRON
CONDUCTING PERIOD OF LEADING IGNITRON
VOLTAGE DROP ACROSS RESISTOR IN ANODE CIRCUIT OF FIRING TUBE FOR LEADING IGNITRON TUBE
VOLTAGE DROP ACROSS RESISTOR IN ANODE CIRCUIT OF FIRING TUBE FOR TRAILING IGNITRON TUBE

Figure 7c. Wave forms corresponding to low heat

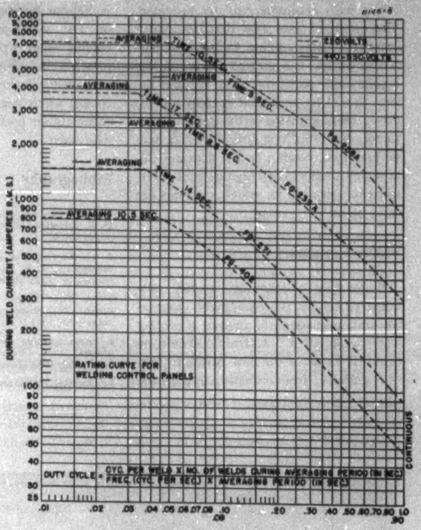

Figure 8 (above). Rating curves for resistance-welding control panels

current rating of an ignitron it is necessary to provide rating curves as shown in figure 8 which have been established after due consideration of the many determining factors involved. These curves give the permissible effective demand current (rms current during the weld) for different duty cycles or percentage of the time that the ignitrons are conducting. The rating curves are given for resistance-welding control panels in which case two ignitrons are connected in inverse-parallel. The duty cycle can be conveniently calculated by means of the formula given. Referring to the curve for the FG-258A ignitrons it will be observed that they will conduct 900 amperes continuously or 5,300 amperes at nine per cent duty cycle when operating from either 220-, 440-, or 550-volt power supplies. It is considered advisable to limit the current to this value when operating at lower duty cycles from 440 or 550-volt power supplies, but when operating from a 220-volt supply a current of 7,000 amperes is permissible if the duty cycle is less than five per cent, as is frequently the case in high kilovolt-ampere spot welding. Of course, this is the current which may be directly controlled through the primary of the welding transformer, the secondary current may be several times this value (depending upon the turns ratio), and in some cases is as high as 100,000 amperes or more. When determining the duty cycle there is a maximum allowable averaging time which depends upon the supply voltage and type of tubes as specified on the rating sheet. This averaging time limits the duration of the weld or the spot length at any current to a value which will not produce excessive internal tube temperature at the end of the spot.

The maximum permissible spot length at a given value of current can be readily determined from the tube-rating curve by multiplying the corresponding duty cycle by the averaging time of the tube. For example, considering the FG-258A ignitrons a 220-volt power supply and 5,000-amperes demand current, the maximum permissible spot length is 0.1 by ten or one second, in which case the interval between welds must not be less than nine seconds. That is, the duty cycle corresponding to a given value of current times the rated averaging time of the tube in cycles is equal to the maximum permissible number of conducting cycles during the averaging time. This applies to spot welding, pulsation welding, or seam welding. Considering a case of seam welding at 60-per cent duty cycle (for example, three cycles on, two cycles off, three cycles on, etc.), the type FG-258A ignitrons will allow a demand current of 1,500 amperes at either 220, 440, or 550 volts. The maximum permissible "on period" or the maximum spot length at 1,500 amperes is limited to six seconds during a ten-second interval when operating from a 220-volt supply, or three seconds during a five-second interval when operating from a 440- or 550-volt supply.

As previously explained, copper-oxide rectifiers are used in the ignitor circuits of ignitron contactors for the purpose of preventing reverse ignitor currents. The copper-oxide rectifiers can be designed with sufficient elements in series-parallel that they will allow the full rating of the ignitrons to be utilized. However, economic considerations necessitate a study of the unusual operating condition to which the rectifiers are subjected, in order to insure that rectifiers having adequate rating with minimum cost are provided. The results of tests show that low-ignitor currents, corresponding to low-load currents, impose the greatest duty on the copper-oxide rectifiers. The rectifier design should be such that it does not increase this limitation of the ignitron contactor, except at long duty cycles as may be justified. Such a design does not impose any limitation to high-current intermittent operation for which ignitrons are most useful.

Water Protection

Water-flow switches are usually provided to render the equipment inoperative unless an adequate amount of cooling water is provided. A strainer should be installed ahead of the flow switch to prevent sediment from collecting in the tube jackets or interfering with the operation of the water-protective device. The cooling water is passed through the tubes in series and should enter the bottom of each water jacket in order to keep the coolest water near the mercury pool and eliminate air pockets which might otherwise occur.

Bibliography

1. Ignitron—A New Controlled Rectifier, **D. D. Knowles.** *Electronics*, volume 6, June 1933, pages 164–6.

2. A New Method of Starting an Arc, **J. Slepian and R. R. Ludwig.** AIEE Transactions, volume 52, 1933, page 693.

3. A New Timer for Resistance Welding, **Ralph N. Stoddard.** AIEE Transactions, volume 53, 1934 (October section), pages 1366–70.

4. Applications of Electron Tubes in Industry, **D. E. Chambers.** AIEE Transactions, volume 54, 1935 (January section), pages 82–92.

5. Theory of Immersion Arc Rectifier, **J. M. Cage.** *General Electric Review*, volume 38, number 10, October 1935, pages 464–5.

6. High-Speed Pictures of Mercury Arc Spots, **H. W. Lord.** *Electronics*, May 1936, page 11.

7. New Developments in Ignitron Welding Control, **Dawson.** AIEE Transactions, volume 55, 1936, pages 1371–8.

8. Sealed-Off Ignitrons for Resistance Welding Control, **D. Packard and W. J. H. Hutchings.** *General Electric Review*, volume 40, February 1937, pages 93–7.

9. Resistance Welding Improved by Thyratron Control, **W. C. Hutchins.** *General Electric Review*, March 1937.

10. Thyratron Control for Spot and Seam Welding, **H. L. Palmer.** *General Electric Review*, May 1937.

11. Thyratron Control for Resistance Welding, **H. L. Palmer.** *General Electric Review*, July 1937.

12. Ignitron Contactor Control of Resistance Welding, **W. C. Hutchins.** *General Electric Review*, December 1939.

13. Power Factor Correction of Resistance-Welding Machines by Series Capacitors, **L. G. Levoy, Jr.** AIEE Transactions, volume 59, 1940, pages 1002–09.

Fault Voltage Drop and Impedance at Short-Circuit Currents in Low-Voltage Circuits

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MEMBER AIEE

Synopsis: In the paper are presented new data on (1) arc-voltage drop and current-limiting effects for arc faults in bar-conductor structures, (2) current-limiting effect of insulated-cable arc faults, (3) cable impedance in an iron conduit, and (4) impedance of low-voltage switchgear circuit assemblies, derived from tests at short-circuit currents in circuits with open-circuit line-to-line voltages of 208–600 volts at 60 cycles. The currents ranged from less than 10,000 amperes to values as high as 100,000 amperes rms in some cases.

The new data dispel several illusions with respect to accidental faults in low-voltage bus structures and other low-voltage gear. Contrary to inferences frequently drawn from previously published data, the paper shows that

(*a*). Faults in the typical low-voltage bus structures tested are not self-clearing.

(*b*). Fault arcs do not show the drastic current-limiting effect which they are frequently assumed to exert upon short-circuit currents. Instances are shown where actual fault currents reach 100 per cent of the rms symmetrical values calculated for *bolted* faults.

This study shows a self-clearing tendency for representative types of accidental faults in cable, but it was found that a large current-limiting action prior to current interruption cannot be depended on.

The new data, supplementing existing impedance data, definitely indicate the possibility of reasonably accurate calculation of short-circuit currents in low-voltage circuits. The magnitude of fault currents actually obtained shows the fallacy of applying arbitrary and large discount factors to calculated short-circuit current values or of establishing arbitrary "maximum" values of current obtainable in low-voltage circuits and systems.

Introduction

THE impedance determining the magnitude of short-circuit currents in low-voltage distribution circuits (600 volts and below) includes (1) that of power sources, lines, and step-down transformers, and (2) that of conductors, circuit breakers, control equipment, and other devices in the distribution circuit up to the point of short circuit. While it has been well known that the impedances of the various connections and devices in the distribution circuit often have an appreciable reducing effect upon the short-circuit current, different calculating procedures have been used to take into account their effect upon the magnitude of short-circuit currents. One of the least known factors affecting the magnitude of short-circuit currents for various kinds of arc faults has been their voltage drop and current-limiting action.

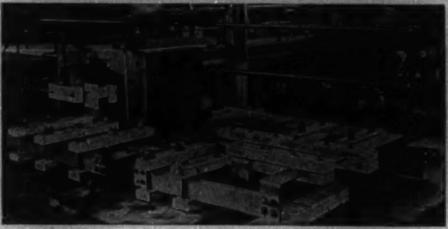

Figure 1. Metal-enclosed bus assembly with connections for tests of fault voltage across short-circuit arcs

Shows upper bus and lower bus at right angles to each other. Connections to lower bus removed. Tests were made with opening at power-supply end of enclosure, as shown

A method often used is that of applying a fixed reduction factor to the current calculated on the basis of the impedance from the power source up to and including the step-down transformer. Stieber[1] gives an approximate rule whereby the combined drop due to arc voltage at an arc fault and lead resistance is taken as approximately 50 per cent of the line voltage in 440-volt circuits and advocates allowing for these drops by a 50 per cent reduction of the current calculated on the basis indicated above.

Another arbitrary short-cut method is that of using a fixed maximum value of short-circuit current for all kinds and sizes of circuits. Fixed values ranging from 20,000 to 40,000 amperes have been suggested as representative estimates of the maximum probable short-circuit current in circuits of 600 volts and less. Stieber[1] concludes that "most of the larger 440-volt installations can be readily handled by breakers having momentary ratings and interrupting capacity of 20,000 amperes; in a few special cases 40,000-ampere ratings are required."

Proponents of the 50 per cent reduction factor and of the fixed values of maximum short-circuit current have cited the following tests by Papst[2]: In a circuit fed from a 20,000-kva generator through a low-resistance, 3,000-kva step-down transformer, in which the calculated short-circuit current at the low-voltage terminals of the transformer was 200,000 amperes, a maximum short-circuit current of 68,500 amperes was obtained in a bus remote from other conductors when the short circuit was initiated by a copper bar, hinged to one of the busses, falling upon the other bus bar. The copper bar was quickly repelled and interrupted the circuit in the first half cycle.

Other tests[3] using the same power source gave a maximum peak current of 24,000 amperes when a screwdriver was dropped across the same bus bars. Furthermore, a current of 17,000 amperes crest was obtained when a copper wire 0.075 inch diameter, was dropped across the busses. In both of these cases also the short circuit was interrupted at the fault within the first half cycle.

However, for short circuits close to the step-down transformer, the impedance of the metallic connections and devices from the transformer to the point of short circuit is ordinarily far too small to account for anything like a 50 per cent reduction of current, or a maximum value

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1. For all numbered references, see list at end of paper.

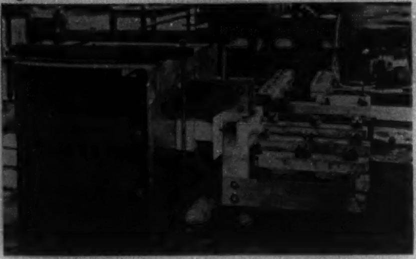

Figure 2. Rear end of metal-enclosed bus assembly for measurements of fault voltage across short-circuit arcs

Rear wall of sheet metal enclosure, normally closed, was removed after tests to show interior of bus assembly. Upper bars were sprung from their end supports by the action of short-circuit forces. Ends of upper bars badly burned in the course of arcing tests

of current limited to, say, 20,000 or even 40,000 amperes in the case of larger 440- or 550-volt installations. The short-cut methods mentioned, therefore, cannot generally be relied on to give correct predictions of short-circuit current for bolted short circuits or for faults nearly equivalent thereto when the short circuit is close to the power-supply mains. Trencham and Mortlock[3] concluded from an analysis of distribution-circuit impedances, that a fixed reduction factor applied to currents calculated from generator and transformer reactances alone cannot be used generally because of the large variations in conductor impedances and other impedance values to be considered.

Furthermore, the large current-limiting effects—obtained when objects, which are readily repelled, are dropped across bare conductors without an enclosure and remote from other live bar conductors—could not be expected in typical low-voltage structures where the presence of connection bars and the use of relatively small spacings result in congested assemblies similar to those described below. Slepian and Strom[4] found that short arcs between bare copper bars one-fourth-inch apart have arc voltages ranging from only 40 to 100 volts, at currents from 2,500 to 6,400 rms amperes.

Arc-fault characteristics of low-voltage cables have been investigated by Kehoe,[9] by Sutherland and MacCorkle,[10] by Slepian and Strom,[4] and by others, with particular reference to burn-off and reignition characteristics. Little conclusive information has been available, however, on the current-limiting effect of cable-fault arcs in circuits with available short-circuit currents in excess of 20,000 amperes.

In order to provide data on arc-voltage drop and current-limiting effect of arc faults in typical low-voltage structures and in cables, as well as more complete data on impedance of conductors and devices up to the point of fault, a series of fault-arc tests and impedance tests at short-circuit currents was made including the following items:

Fault-voltage drop for arc faults of various kinds

(*a*). **Between bare conductors in a metal enclosure, using both double-bus and single-bus assemblies.**

(*b*). **In three-conductor 350,000-circular-mil cables, not enclosed.**

Impedance of a three-conductor 350,000-circular-mil cable in an iron conduit.

Impedance of low-voltage switchgear circuit assemblies, of protective and control equipment.

In these tests the open-circuit line-to-line voltage ranged from 208 to 600 rms volts, at 60 cycles.

It is the purpose of this paper to present the more significant results of these investigations.

I. Fault Voltages and Current-Limiting Effect for Short-Circuit Arcs in Bar-Conductor Structure in Metal Enclosure

The metal-enclosed bus-bar assembly of figures 1 and 2 was used having bare copper bars four inches by one-fourth inch in a three-phase circuit with face-to-face bars spaced two inches on centers. One series of tests was run with two sets of bars, one set $1^3/_4$ inches above and at right angles to the other set of bars, as shown in figure 3. Both sets of three-phase bars were connected to the power-supply conductors through rigidly supported bars, so that both sets of bars would be simultaneously and continuously energized with voltage for the duration of each test, as shown in figure 4.

The purpose of the tests was to obtain information on arc-voltage and current-limiting effect of faults that would roughly correspond to nonbolted faults in typical low-voltage bar-conductor structures. Light wires, 0.045-inch diameter bare copper, and bare 250,000-circular-mil stranded cable, loosely placed across the bars, were used to initiate the faults, as shown in figure 3. The impedance of the metallic circuit was largely reactive, comprising a generator, adjustable reactors, bare copper connections and transformers. Open-circuit line-to-line voltages ranging from 300 to 600 volts rms were applied to the test assembly. Further details pertaining to test conditions are given in the appendix.

The following results were found, as indicated by the tabulated summaries, tables I and II, and by the curves of

Figure 3. Arrangement of bars and of means for initiating arc faults at short-circuit currents in metal enclosure

Bars 4 inches by 1/4 inch spaced 2 inches on centers. Enclosure (not shown) 36 inches long, 24 inches high, 20 inches wide. Supporting members and braces of molded material, not shown. Connections to test bars not shown. Shaded areas indicate parts of bars within enclosure. Short circuits initiated (*a*) by doubled bare copper wire 0.045 inch diameter about 4 inches long, (*b*) by bare 250,000-circular-mil copper cable about 2 feet long

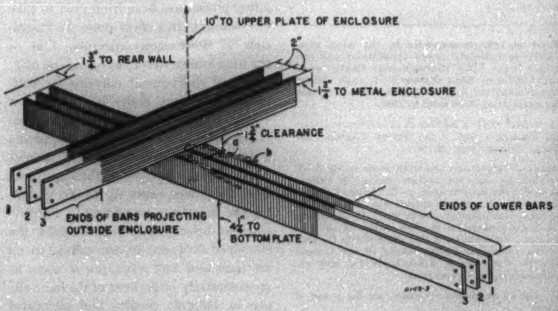
10" TO UPPER PLATE OF ENCLOSURE
$1\frac{3}{4}$" TO REAR WALL
2"
$1\frac{3}{4}$" TO METAL ENCLOSURE
$1\frac{3}{4}$" CLEARANCE
a
b
1
2
3
ENDS OF BARS PROJECTING OUTSIDE ENCLOSURE
$4\frac{1}{4}$" TO BOTTOM PLATE
ENDS OF LOWER BARS
3
2
1

Table I. Results of Fault-Voltage Tests for Short-Circuit Arcs in Metal-Enclosed Bus Structure

Test Number	Fault Initiated by	Open-Circuit Line-to-Line Voltage RMS	Available Current RMS Sym. Amp.	Range of Total Fault Currents (Average) Amp. Crest	Range of Line-to-Neutral Fault Voltage Volts Crest
Arc faults with double bus in metal enclosure, faults started at lower bars					
14–15	No. 17 wire doubled*	300	17,800	9,200– 22,000	97–194 84–180
16	No. 17 wire doubled *	600		19,200– 30,300	122–179
17	250,000-circular-mil cable**	300	17,800	5,600– 22,700	100–181
18	250,000-circular-mil cable**	300	43,000	30,000– 53,000	120–186
20	250,000-circular-mil cable**	600	98,000	97,000–141,000	112–150
Single-bus faults in metal enclosure, upper bars only used					
26	250,000-circular-mil cable***	300	43,000	24,000– 38,200	159–223
27	250,000-circular-mil cable***	600	98,000	54,000–126,000	220–348

*Doubled wires of No. 17 copper laid across bars 1 and 2 of lower bus. Test box included both upper and lower bars.

**250,000-circular-mil copper stranded cable laid across lower bars. Test box included both upper and lower bars.

***250,000-circular-mil copper stranded cable laid across upper bars. Test box included one set of bars (upper) only.

figures 6, 7, and 8 (for an outline of the procedure used in evaluating the results, see the appendix):

(*A*). Of particular interest are the minimum values of arc voltage, as follows:

(1). For faults initiated in the lower bus in the double-bus assembly, the minimum arc voltage was in the vicinity of 100 volts crest, line-to-neutral value, average per phase, for three-phase faults, at 1$^{3}/_{4}$ inches minimum conductor clearance, for a line-current range from 10,000 to over 120,000 amperes crest.

At 300 volts open-circuit voltage, one or two arc-voltage values below 100 volts crest were obtained; while at 600 volts open-circuit voltage the lowest value was about 10–20 per cent in excess of 100 volts crest. However, sufficient tests were not made for each voltage to indicate conclusively whether there is a material difference between the minimum arc voltage in 600-volt circuits and that in 300-volt circuits.

It made no material difference in the magnitude of the minimum arc voltage, or in the range of arc voltage, whether the arc was initiated by the small wire or by the 250,000-circular-mil cable.

(2). For faults in the single-bus assembly, the minimum arc voltage was of the order of 160 volts crest, line-to-neutral value, for currents up to 40,000 amperes crest at 300 volts open-circuit voltage, and was 220 volts crest or higher, line-to-neutral value, at 600 volts open-circuit voltage, at currents from 50,000 to 120,000 amperes crest.

(*B*). The faults were not self-clearing, but endured till the station breaker interrupted the circuit elsewhere after six to ten cycles.

(*C*). The current-limiting action in the enclosed-bus arc tests in question is summarized in table II, showing in the double-bus assembly, values from 72 to 100 per cent of the available short-circuit current (the latter determined for bolted short circuit); and in the single-bus assembly, values from 57 to 90 per cent of the available current.

(*D*). Although the faults started as single-phase line-to-line faults, they usually developed into three-phase short circuits within less than a half cycle and continued as such.

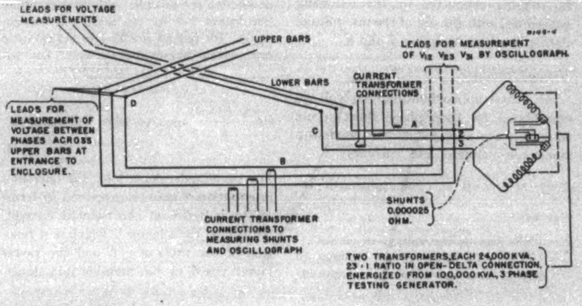
LEADS FOR VOLTAGE MEASUREMENTS
UPPER BARS
LOWER BARS
LEADS FOR MEASUREMENT OF V_{12} V_{23} V_{31} BY OSCILLOGRAPH.
CURRENT TRANSFORMER CONNECTIONS
LEADS FOR MEASUREMENT OF VOLTAGE BETWEEN PHASES ACROSS UPPER BARS AT ENTRANCE TO ENCLOSURE.
A
B
C
D
1
2
3
SHUNTS 0.000025 OHM.
CURRENT TRANSFORMER CONNECTIONS TO MEASURING SHUNTS AND OSCILLOGRAPH
TWO TRANSFORMERS,EACH 24,000 KVA., 23 : 1 RATIO IN OPEN-DELTA CONNECTION, ENERGIZED FROM 100,000 KVA, 3 PHASE TESTING GENERATOR.
0149-4

Figure 4. Diagram of connections for arc voltage

Tests of bar-conductor assembly in metal enclosure

Table II. Current-Limiting Effect of Three-Phase Arc Faults in Enclosed Bus Structure

1 Test Number	2 RMS Line Current Range, Amp. Av. for Three Phases, the Higher Value Occurring in First or Second Half Cycle	3 RMS Line Current After Several Cycles (Av. for Three Phases)	4 Available RMS Sym. Current Amp.***	5 Per Cent of Available Current Obtained in Fault Tests Based on Higher Value, Column 2	6 Based on Value, Column 3
(a) Faults initiated by loose No. 17 wire or by loose 250,000-circular-mil cable in double-bus assembly					
14*	6,500–14,000	12,900 after 2 cycles	17,800	79	72
15*	8,000–15,700	13,200 after 6 cycles	17,800	88	74
18*	21,300–37,500	31,000 after 4 cycles	43,100	87	72
20**	69,000–99,000	82,000 after 3 cycles	98,600	100	83
(b) Faults initiated by loose 250,000-circular-mil cable in single-bus assembly					
26*	17,000–27,000	24,600 after 6 cycles	43,100	63	57
27**	38,000–89,000	58,600 after 5 cycles	98,600	90	60

NOTE: In column 6 the per cent of available current attained is based on the current after several cycles (column 3), so as to exclude any d-c component that may have been present in the initial current (higher value column 2).

*300 volts open-circuit rms line-to-line voltage.

**600 volts open-circuit rms line-to-line voltage.

***Determined by bolted three-phase fault test.

PROCEDURE FOR ESTIMATING SHORT-CIRCUIT CURRENTS IN CIRCUITS WITH ARC FAULTS

The arc voltages for different kinds of faults in low-voltage circuits will vary depending on

The conductor spacing between phases,
The spacing between adjacent circuits,
The type of construction, whether open or enclosed,
The clearances to ground and to enclosing walls,
The type of conductors, whether bare or insulated,
The manner in which the fault is initiated,
The character of the enclosing walls, etc.

It will not be attempted to consider all of these in detail. Some of the fault conditions of particular interest in low-voltage switchgear construction are those caused by conductors dropping across phases in the following types of structures:

(*a*). Heavy-current bar-conductor assemblies with low to minimum spacings, metal enclosed, with two or more bar-conductor circuits in close proximity to each other.

(*b*). Bar-conductor circuits, remote from other live conductors, metal enclosed.

Less severe types of faults than those for (*a*) and (*b*) above may occur in open bar-conductor circuits remote from live conductors, and may give rise to short-circuit currents with more pronounced current-limiting effects.

When low to minimum clearances are used in structures of types (*a*) and (*b*) above, short-circuit currents due to objects (which are capable of being repelled or destroyed) bridging across phases may be roughly estimated by the following procedure, with the aid of the arc-voltage data given here in figures 7 and 8.

Basis of formulas:

As a first approximation, the voltage equation 1 for three-phase short-circuit including arc may be written assuming sine-wave voltages and currents:

$$E-v-ri-jix=0 \quad \text{rms volts to neutral} \quad (1)$$

Symbols:

E = 0.58 × (line-to-line voltage of circuit)
v = 0.71 × (crest value of arc voltage to neutral from figure 7 or 8, as required)
i = rms symmetrical short-circuit current, amperes
$j=\sqrt{-1}$
r = resistance of metallic circuit, ohms per phase, as determined for bolted fault
x = reactance of metallic circuit, ohms per phase, to the point of fault

Table III. Comparison of Calculated and Measured Values of Short-Circuit Currents for Arc Faults in Enclosed Double-Bus Assembly

Test Number	Rms Line Voltage Open Circuit	Arc-Voltage Crest Value From Test Line to Neutral (Volts)	Metallic Circuit Reactance × Ohms	Short-Circuit Current RMS Sym. Amperes Calculated	Short-Circuit Current RMS Sym. Amperes Test
18	300	146	0.00402	35,000	37,500
20	600	128	0.00352	95,000	99,000

Formulas for current, rms amperes per phase

$$i=-\frac{rv}{(r^2+x^2)}\times\left(1-\sqrt{1+\frac{(r^2+x^2)(E^2-v^2)}{r^2v^2}}\right) \quad (2)$$

The general formula 2 reduces to the simpler formula 3 when the resistance r of the metallic circuit is negligible.

Figure 5. Oscillogram of line-to-line arc voltages and total line currents

Shows arc voltages *A, C, E,* and total line currents *B, D, F,* for 3-phase short circuit initiated by 250,000-circular-mil bare copper cable placed across bare copper busses in double-bus test structure with metal enclosure (see figures 1–4 for test setup and connections). For test 18 at 300-volts rms 60-cycle open-circuit line-to-line voltage. For numerical values, see figure 6

$$i=\frac{\sqrt{E^2-v^2}}{x} \text{ amperes, when } r \text{ is negligible} \quad (3)$$

For simplified calculations, the current i in equation 2 may be expressed in terms of the bolted-fault short-circuit current, multiplied by a factor p which is a function of the ratio of v/E and the power factor $\cos\theta$ of the metallic impedance.

With the aid of curves of the factor p for particular values of the voltage ratio v/E and of $\cos\theta$, the calculation of short-circuit currents for arc faults becomes a simple procedure.

Results of calculations are given in table III. In these calculations the arc voltages observed in the tests were used together with the metallic circuit reactance based on bolted-fault tests. Calculated and observed short-circuit current values for the arc faults are in good agreement, considering that the test values include at least a small amount of d-c component. The calculated values obtained with the assumed sinusoidal form factor of 0.71 for the arc voltage are probably somewhat high since the arc voltage is generally flat-topped. Possibly a form factor of 0.8 or 0.85 for the arc voltages would be more nearly correct. The calculations of table III were made with a form factor of 0.71.

II. Arc-Fault Tests in Three-Conductor Cable

Short sections, approximately 4.5 feet long, of a three-conductor 350,000-circular-mil cable with varnished-cloth insulation, were provided with short-circuiting means including a number 17 short-circuiting wire placed in a hole (0.14 inch diameter) connecting two conductors, a twist drill (0.14 inch diameter) connecting two conductors, and $1^1/_4$-inch diame-

Table IV

Test No. 28—Cable-Fault Test on 350,000-Circular-Mil 3-Conductor Cable, Short-Circuited Between Phases 1 and 2 by a No. 17 Copper Wire Placed in a Drilled Hole $^1/_8$ Inch Diameter

Open-circuit line-to-line voltage:	300 volts rms
Available 3-phase short-circuit current:	43,100 rms sym. amp.
Peak current in cable fault: (triangular peak current limited by fault resistance)	14,000 amp. crest, in phases 1 and 2 only
Duration of short circuit:	0.006 sec.
Arc voltage in mid-region of short-circuit half cycle:	420 volts, mean value of flat top
Peak voltage at instant of current peak:	More than 800 volts (probably low due to magnetic oscillograph)

Results of test: The fault cleared itself with a fizzing sound. Contents of hole blown clear. All phases and sheath were found to be insulated from one another by test with a ringer. The same cable was used in the next test with a drill placed in the same hole

Crest current attained: 32 per cent of available rms symmetrical current

Table V

Test No. 29—Cable-Fault Test on 350,000-Circular-Mil 3-Conductor Cable. The Same Piece Tested in No. 28 Was Used Driving an 0.14-Inch Diameter Twist Drill Into Hole, Short-Circuiting Conductors 1 and 2. Voltage Was Applied to the Cable With the Drill in Place

Open-circuit line-to-line voltage:	600 volts rms
Available 3-phase short-circuit current:	98,600 amp. rms sym.
Peak current in cable fault: (minor current loop)	16,000 amp. crest in phases 1 and 2 only
Duration of short circuit	0.002 sec.
NOTE: This test not indicative of current-limiting action of fault since the circuit was closed on a minor loop of current of less than one-fourth-cycle total duration without abrupt current limitation, the maximum peak voltage across the fault being less than half of the phase-to-phase crest voltage	
Arc voltage (see note above):	200 volts max. peak
Results of test: The fault cleared itself with a fizzing sound. The drill was ejected from the hole	
Crest current attained: 16 per cent of available rms symmetrical current	

Table VI

Test No. 30—Cable-Fault Test on 350,000-Circular-Mil, 3-Conductor Cable. Short Circuit Initiated by Ramming a 1$^1/_4$-Inch Drill Resting Against the Cable, Into a Prepared Hole Between Phases 1 and 2 by a 20-Pound Weight Dropped on the Drill From a Height of 1 Foot Above It, After the Voltage Had Been Applied Across the 3 Conductors

Open-circuit, line-to-line voltage:	300 volts rms
Available 3-phase short-circuit current:	43,100 amp. rms sym.
Fault-current observations: Two faults occurred, the initial fault taking place at the instant voltage was applied to the circuit on account of very light contact or actual breakdown of gap. The fault cleared itself after 1.2 half cycle. The second fault occurred 21 cycles later when the drill was rammed into the hole by the weight and cleared itself after 1.3 half cycle	
First fault:	
Peak current	44,000 amp. crest in phases 1 and 2 only
Duration of current flow	1.2 half cycle
Arc voltage at current peak	Greater than 460 volts crest (off film)
Second fault:	
Peak current	52,000 amp. crest in phases 1 and 2 only
Duration of current flow	1.3 half cycle
Arc voltage at current peak	440 volts crest
Results of test: Two reports were heard. The drill was ejected, and the hole in the cable was materially enlarged by the explosions. Both faults were self-clearing	
Highest crest current attained:	120 per cent of available rms symmetrical current

ter spikes rammed into a prepared 1$^1/_4$-inch diameter hole between two of the conductors by a dropping weight, the latter short circuit resembling a pickaxe type of fault.

With three-phase voltage, of 300 or 600 rms line-to-line open-circuit volts, applied to one end of the samples from the generator and transformer equipment used in the other tests described here (see appendix), short circuits with available currents ranging from 43,000 to 98,000 rms symmetrical amperes (based on three-phase bolted fault tests) were produced, and the results summarized in tables IV–VII were obtained. They show:

(*a*). That a short circuit initiated by a fine wire in a small hole between phases limited the crest current to 14,000 amperes or 32 per cent of the available rms symmetrical current, with 300 volts open-circuit voltage,

(*b*). that short circuits initiated by forcing a 1$^1/_4$-inch diameter spike between two phase conductors resembling a pickaxe type of fault, gave currents ranging from 44,000 to 86,000 amperes crest or from 56 per cent to 120 per cent of the available rms symmetrical current,

(*c*). that the faults so obtained were self-clearing, interrupting the current within one or two half cycles,

(*d*). that short circuits initiated as two-conductor, phase-to-phase faults did not involve the third conductor, i.e., were interrupted as two-conductor faults.

III. Cable Impedance Tests

A. SHORT-CIRCUIT IMPEDANCE OF THREE-CONDUCTOR 350,000-CIRCULAR-MIL CABLE IN IRON CONDUIT

In these tests three-phase power was supplied to the three-conductor cable in a four-inch iron conduit, 100 feet long. At the far end of the cable a bolted three-phase short circuit was made. The test currents were in the vicinity of 15,000 amperes rms symmetrical at 60 cycles.

The cable and conduit specifications are stated in the appendix.

A comparison of test values of cable impedance in the iron conduit, with the calculated cable impedance in air, given in table IX, shows that the impedance in the conduit by test was 15 per cent higher than the calculated value for nonmagnetic surroundings. Similarly, the resistance increase and the reactance increase, due to the iron, by test were roughly 18 per cent and 12 per cent, respectively, over the calculated air values. The increases due to the magnetic conduit in these short-circuit tests, were within the range of values found in Brieger's[8] tests at normal load currents for three single-conductor cables held compactly and symmetrically in a triangle by suitable bonding within the iron conduit.

For single-conductor cables loosely placed within a magnetic conduit, greater increments of reactance and resistance, above the values for minimum triangular spacing in air, will be obtained, on account of the spreading of the conductors under action of the short-circuit forces.

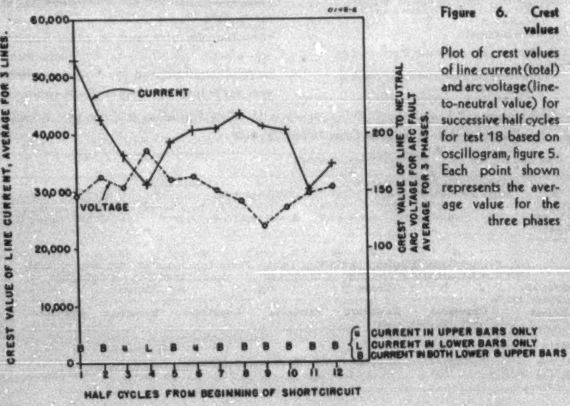

Figure 6. Crest values

Plot of crest values of line current (total) and arc voltage (line-to-neutral value) for successive half cycles for test 18 based on oscillogram, figure 5. Each point shown represents the average value for the three phases

B. CHECK TESTS OF IMPEDANCE OF THREE-CONDUCTOR CABLE IN AIR

The same three-conductor 350,000-circular-mil cable was also given a series of three-phase short-circuit impedance tests *in air*, with a bolted short circuit provided at the end of the 103-foot length of cable. The test results at currents of 3,800–15,000 amperes are tabulated in table X and are compared with calculated values based on the cable dimensions. From the table, it is seen that the air impedance values of the three-conductor cable by test agree closely with the calculated values, the differences being below five per cent for both resistance and reactance.

It is therefore concluded, that, as expected, the air impedance of the cables in question calculated from the available data applies with good accuracy at short-circuit currents in the absence of magnetic surroundings.

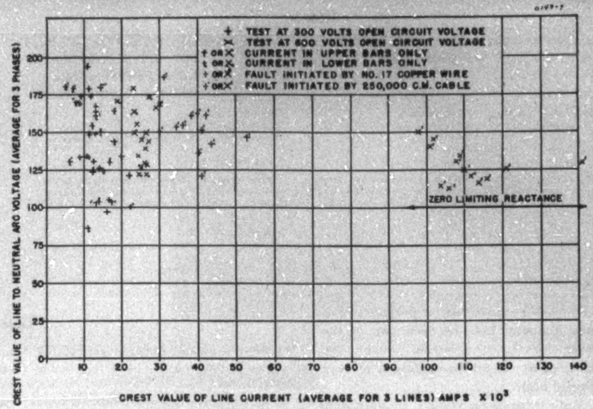

Figure 7. Crest values

Arc voltage versus short-circuit current for 3-phase 60-cycle short circuits initiated by loose conductors placed across bare copper busses in double-bus test structure with metal enclosure (see figures 1–4 for test setup and connections)

Table VII

Test No. 31—Cable-Fault Test on 350,000-Circular-Mil 3-Conductor Cable. Cable Hole Between Phases 2 and 3 Was Prepared Similar to That of Test 30 (Table VI). Short Circuit Was Initiated, as in the Preceding Test, by Ramming a Spike, Resembling the Point of a Pickaxe, Resting Against the Hole, Into the Cavity by a 20-Pound Weight Dropped on the Spike, From a Height of About 1 Foot, After the Voltage Had Been Applied to the Cable

Open-circuit line-to-line voltage:	600 volts rms
Available short-circuit current:	98,600 amp. rms sym.
Fault-current observations: The spike made its first contact with phases 2 and 3 after the circuit had been energized with voltage for 102 cycles. The fault cleared itself within one half cycle. Another fault occurred 15 cycles later, and lasted about one cycle, clearing itself. The explosion at the fault severed conductor 3 at the point of fault. After a further interval of 18 cycles, a third fault occurred between phases 2 and 3, which cleared itself within a half cycle. No further fault occurred during the remaining 16 cycles of the test	
First fault:	
Peak current	68,000 amp. crest, in phases 2 and 3 only
Duration of current flow	0.7 half cycle
Arc voltage at current peak	In excess of 850 volts crest; very sharp peak, not measurable accurately
Second fault:	
Peak current first half cycle	86,000 amp. crest, in phases 2 and 3 only
Peak current second half cycle	55,000 amp. crest, in phases 2 and 3 only
Arc voltage	No measurement
Duration of current flow	1.8 half cycle
Third fault:	
Peak current	73,000 amp. crest, in phases 2 and 3 only
Duration of current flow	0.8 half cycle
Arc voltage	No measurement
Results of test: Cable burst, phase conductor 1 remaining intact, conductor 2 grounded to sheath and was badly burned, conductor 3 separated	
Crest current attained:	From 56–87 per cent of available rms symmetrical current

IV. Impedance of Switchgear Conductor Assemblies and of Protective and Control Equipment

As one of the steps in preparing impedance data for engineering calculations of short-circuit currents in low-voltage circuits, various impedance tests were made to verify existing data and to supplement them. Accordingly, tests, as well as calculations in so far as practicable, were made for the impedance of various bar conductors in different arrangements, of air-circuit breakers, lever switches, current transformers and of other control equipment. Sections of complete commercial assemblies of low-voltage switchgear including a variety of these elements

Table VIII. Impedance of Three-Conductor Cable in Iron Conduit

Data and Results for One Set of Measurements

	2 RMS Line Currents	3 RMS Line Currents	4 RMS Line-to-Line Voltages at Input Terminals of Cable	5 RMS Line-to-Line Voltages at Input Terminals of Cable	6 Average Phase Impedance of 103-Foot Three-Conductor Cable at Five Degrees Centigrade (Ohms)	7 Average Phase Impedance of 103-Foot Three-Conductor Cable at Five Degrees Centigrade (Ohms)	8 Average Phase Impedance of 103-Foot Three-Conductor Cable at Five Degrees Centigrade (Ohms)	9 Phase Impedance of 1,000 Feet in Steel Conduit at 25 Degrees Centigrade (Ohms)	10 Phase Impedance of 1,000 Feet in Steel Conduit at 25 Degrees Centigrade (Ohms)	11 Phase Impedance of 1,000 Feet in Steel Conduit at 25 Degrees Centigrade (Ohms)
1 Test Number	Line in Which Current Was Measured	Magnitude and Phase of Current	Lines Between Which Voltage Was Measured	Magnitude and Phase of Voltage	Impedance	Resistance	Reactance	Impedance	Resistance	Reactance
	1	16 000/120	1–2	140/12						
2	2	15 600/0	2–3	141/258	0.00528/43	0.00386	0.00360	0.0534/41	0.0404	0.0349
	3	14 600/241	3–1	141/138						

were also tested at short-circuit currents, for over-all impedance determination.

As an example of the results of tests of assembled circuits, the data of table XI on the impedance of a section of metal-enclosed switchgear are given, the test values having been derived from oscillograph records. The circuit used was that of figure 9 comprising the following elements:

Item	Description	Length (Inches)	Center Spacing Between Phases (Inches)
a	2—5x$^1/_4$	24	6.5
b	2—5x$^1/_4$	14	6.5
c	2—5x$^1/_4$	23	9.5
d	2,000 amp air circuit breaker		
e	2—5x$^1/_4$	8	11
f	2—5x$^1/_4$	8	11
*g**	2—5x$^1/_4$	18	10
h	2—5x$^1/_4$	13$^1/_2$	7.25
i	2—5x$^1/_4$	40	7.25
j	copper angle 4x4x$^3/_8$	130	7.25

*Bars placed edge to edge in section *g*; face-to-face arrangement used in other sections.

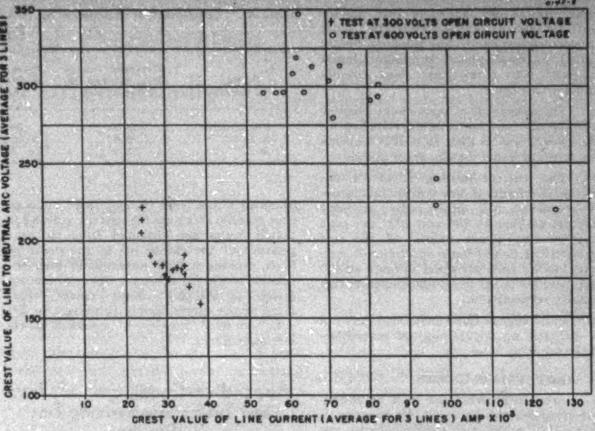

Figure 8. Crest values

Arc voltage versus short-circuit current in enclosed single-bus test structure. Short circuits initiated by loose conductors placed across bare copper busses

Calculated reactance values, also given in the table, are within ten per cent of the test values for the major portions of the assembly, the differences being somewhat higher for the smaller parts of the circuit.

The method and data used in the bar-conductor reactance calculations have been previously published.[5,6,7] As a first approximation in reactance calculations for bar assemblies with bends and loops, portions of the circuit having the same spacing and bar arrangement may be treated as if they were straight. Their combined reactance is then roughly equal to the sum of their separate lengths times the reactance per unit length for similar bars in a long straight circuit. The approximation results in reactance values somewhat too high (particularly when there are many short lengths between bends), but generally close enough for engineering calculations. To simplify calculations, curves of reactance per unit length for different bar arrangements may be drawn.

Table IX. Comparison of Impedance Results From Tests on 350,000-Circular-Mil Three-Conductor Cable in Four-Inch Iron Conduit With Calculated Data for the Same Cable in Air

1 Condition	2 Source From Which Value Is Obtained	3 Impedance Per 1,000 Feet at 25 Degrees Centigrade (Ohms) Impedance	4 Resistance	5 Reactance
Cable in iron conduit	Grand average of all tests	0.0538	0.039	0.037
Cable in air	Calculated from dimensions	0.0467	0.033	0.033
Ratio $\frac{\text{Cable in iron conduit, test}}{\text{Cable in air, calculated}}$		1.15	1.18	1.12

Table X. Results of Impedance Tests on Three-Conductor Cable in Air and Comparison With Calculated Values

1 Test Number	2 Location of Cable	3 RMS Line-to-Line Open-Circuit Voltage	4 Average RMS Phase Current for Three Phases	5 Average 60-Cycle Cable Impedance Per Phase for 103 Feet, Ohms Impedance	6 Resistance	7 Reactance
9	In air	100	3,800	0.0046		
10	In air	300	9,200	0.0045	0.0029	0.0035
11	In air	300	8,400	0.0045	0.0032	0.0032
13	In air	600	13,000	0.0048	0.0032	0.00355
Average impedance in air				0.00460	0.00310	0.00341
				Ohms Per 1,000 Feet		
Impedance of 1,000 feet at 25 degrees centigrade in air based on average *test* values above				0.0464/46	0.0325	0.0331
Impedance of 1,000 feet at 25 degrees centigrade in air *calculated* from dimensions				0.0467/45	0.033	0.033

In calculating the values of table XI the approximation described was used with the average conductor spacing between phases for each portion of the circuit, except that the circuit-breaker reactance, item *d*, was determined by separate test. The resistance of the bar conductors in question is negligible compared with their reactance and was therefore not included in the calculations.

In the current-transformer tests, not reported here, both wound and bar types were included. For the bar types the reactance at heavy short-circuit currents approached roughly that of the primary bar conductors.

With the necessary impedance data available, calculations of the impedance of low-voltage conductor and device assemblies are readily made and give results sufficiently close for short-circuit current calculations.

Summary and Conclusions

A. ARC FAULTS IN METAL-ENCLOSED BUS STRUCTURES TYPICAL OF MODERN LOW-VOLTAGE SWITCHGEAR

(1). The voltage drop at arc faults was found to be within the range from approximately 100 to over 300 volts crest, line-to-neutral value, at 300 volts and 600 volts rms open-circuit line-to-line voltages. Faults started as single-phase line-to-line faults developed into three-phase short circuits within a half cycle.

(2). The magnitude of short-circuit current taken by these arc faults at 600 volts line-to-line voltage was as high as 99,000 amperes rms, average value for the three phases.

(3). The current-limiting effect of accidental arc faults in low-voltage bus structures may be of a minor order, arc-fault currents as high as 88[*] and 100[*] per cent (average three-phase values) of the rms symmetrical bolted-fault short-circuit currents having been obtained in tests at 300 volts and 600 volts line-to-line, open-circuit voltage, respectively.

(4). Fault arcs in these structures may not be self-clearing within normal protective-device clearing time.

B. ARC FAULTS IN CABLES

(1). Arc faults produced in a three-conductor cable, not enclosed, by piercing it with a $1^{1}/_{4}$-inch diameter spike between two conductors gave currents as high as 80 per cent (approximate rms value) of the rms symmetrical bolted-fault short-circuit current, indicating that a large current-limiting effect (in excess of 20 per cent with respect to the rms symmetrical bolted-fault current) cannot be depended on.

(2). Short circuits caused by piercing cables with lighter metallic members resulted in currents limited to values well below half of the rms symmetrical bolted-fault short-circuit current.

(3). All of the cable faults in question were self-clearing within one or two half cycles.

C. CABLE IMPEDANCE

The impedance of a three-conductor 350,000-circular-mil cable in a four-inch iron conduit was 15 per cent higher than that obtained in nonmagnetic surroundings.

D. IMPEDANCE CALCULATIONS FOR SHORT-CIRCUIT CURRENT DETERMINATION

(1). Calculations of impedance for assembled switchgear circuits are in substantial agreement with test values at short-circuit currents.

(2). With the necessary impedance data available, calculations of the impedance of low-voltage conductor and device assemblies are readily made.

(3). The impedance of low-voltage circuits and the magnitude of short-circuit currents obtained in the tests reported here indicate that reliable short-circuit current values for bolted faults are obtained from the impedance of the circuit up to the point of fault, making it both unnecessary and unreliable to use (*a*) arbitrary discount factors applied to the short-circuit current calculated at the power-supply terminals, or (*b*) arbitrary fixed "maximum" values of short-circuit currents.

Table XI. Comparison of Impedance by Test With Calculated Reactance for Circuit Assembly Figure 9

Portion of Circuit	Measured Voltage Drop, Average Per Phase (Volts)	Average Current Per Phase (Amperes)	Average Impedance Per Phase by Test-Ohms	Calculated Average Reactance Per Phase-Ohms
a–h inclusive	18.5	39,200	0.00047	0.00056
A-B	16.0	22,800	0.00070	0.00072
B-C	2.9	5,900	0.00050	0.00049

Appendix—Conditions of Tests and Tables Summarizing Test Results

I. Fault-Arc Tests in Enclosed-Bar Structure

All the essential test conditions and details of test structure are shown in figures 1, 2, 3, and 4, except the dimensions of the metal enclosure which were

36 inches long, in the direction of the lower bars,
24 inches high,
20 inches wide, in the direction of the upper bars.

The sheet-steel walls, approximately one-eighth-inch thick, enclosed the bar assembly on five of the six sides, the sixth side, i.e., the side at which the lower bars entered the box, being left open. Braces of molded insulating material were used to support the bars in the box. The clearance between the unconnected ends of the bars and the metal walls was $1^{3}/_{4}$ inches, the metal enclosure being isolated from the electrical circuit, except that voltage-measuring devices were connected from the box to the bus bars.

Currents were measured not only in the power-supply conductors carrying the total line current, but also in each of the two branch connections *A* and *B* to the upper bars and lower bars respectively. By this means the duration of current flow in each of the busses could be determined.

EVALUATION OF ARC VOLTAGE AND CURRENT

Since the variations of line-to-line arc voltages were found to be relatively small, as indicated in the oscillograph record, figure 5, the mean[‡] crest value (in the center region of the half wave) was read for each half cycle of line-to-line arc voltage for which three-phase fault currents were flowing. In this manner three arc-voltage crest values—one for each phase—were obtained and were averaged for each of the successive half cycles. These voltages were then converted into line-to-neutral values by the approximate procedure of dividing by 1.732.

Crest values of current for the corresponding half cycles were then measured from the oscillograms, averaging in each case the three values for the three lines, using total line currents, regardless of whether there was current flowing in one or in both groups of bars.

A succession of crest values of fault voltage and of line current derived from the oscillogram of figure 5 is plotted for each half cycle in figure 6. From data similar to these the volt-ampere plot of figures 7 and 8 for the arc faults in the metal-enclosed bus assembly was prepared. A tabular summary of the fault voltage results is shown in table I. From these figures were derived the results for current-limiting effect in table II.

II. Cable-Fault Tests in Three-Conductor 350,000-Circular-Mil Cable

The cable tested was of the following description:

Conductor cross section, 350,000 circular mils
37 strands, each of 0.097 inch diameter bare copper, per phase
Conductor insulation, $^{7}/_{32}$ inch varnished cloth
Belt, $^{4}/_{32}$ inch varnished cloth
Lead sheath, $^{4}/_{32}$ inch thick

In the tests its sheath was isolated from the lines of the low-voltage test circuit. The essential test conditions are included with the test results in tables IV–VII.

III. Cable-Impedance Tests

The 350,000-circular-mil, lead-covered cable was of the same kind as that used in the cable-fault tests described above. It was 103 feet long, 100 feet of which was within a standard electrical conduit (iron) 4 inches inside diameter, and 4.5 inches outside diameter.

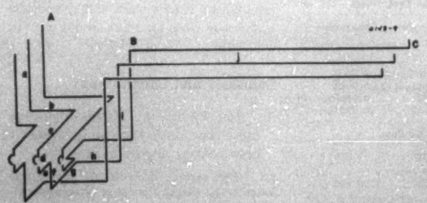

Figure 9. Circuit diagram of a section of switchgear circuit assembly

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[*] For convenience, the average value, per phase, of the initial rms three-phase unsymmetrical short-circuit current for bolted faults may be taken as 1.25 times the rms symmetrical value. If the per cent values of arc-fault current are taken with respect to the average unsymmetrical bolted-fault current, as here approximated, they become reduced by the factor 1.25, or 70 and 80 per cent respectively.

[‡] Averaging minor arc-voltage fluctuations occurring in the center region of the half wave.

Two New Methods of Accelerating Electric Motors Automatically

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Synopsis: The first of the two methods discussed in this paper is particularly applicable to a-c slip-ring motors. Series-resonance circuits responsive to the variation in rotor frequency with motor speed provide a means of accelerating the motor automatically, while a series-parallel resonance circuit similarly connected to the rotor can be used to protect the motor when "plugged" or suddenly reversed. The sensitive response of the latter circuit within a narrow frequency range makes possible its use as a "zero-speed" switch.

The second method has a more general application in that it can be used with any type of motor. A neon tube timing circuit measures the time intervals between the closures of accelerating contactors. One "timer" can be used for all steps of acceleration, with or without separate time adjustment. For a-c motor control, the introduction of a current transformer and a rectifying circuit makes possible the obtaining of time intervals which are increased automatically with increase in motor load. Such a scheme is often referred to as "time-current" acceleration in order to distinguish it from "constant-time" acceleration where the time interval is independent of the load.

Introduction

If full voltage is applied to an electric motor at rest the current will be excessively large, and there is a likelihood that the resultant forces will damage both the motor and the mechanism connecting it to the load. This is true of d-c motors in particular and, to a lesser degree, of a-c motors. It is necessary, therefore, to limit the "inrush" current during the starting period, and for d-c motors this is done by inserting resistors in series with the armature. Series resistors, series reactors or autotransformers are connected in the primary circuit for a-c motors, unless the a-c motor is of the wound-rotor induction type in which case it is customary to limit the primary current by connecting resistors in the secondary or rotor circuit external to the motor itself.

As the motor attains speed a counter electromotive force is generated which reduces the current, and if adequate torque is to be maintained the current-limiting means must be gradually removed from the circuit. This may be done either manually, by means of the usual drum or face-plate type of controller, or automatically, by means of relays controlling electromagnetic contactors. For many years the trend has been more and more toward fully automatic magnetic controllers, so that manual controllers are now rarely used in steel mills and similar industries.

The following is a brief review of some of the common automatic methods now in use.

(1). Counter-Electromotive-Force Method

When a d-c motor is accelerated, the back-electromotive force induced at the armature terminals is a function of the speed of the motor. If a series of relays are connected in parallel across the armature, the voltage applied to these relays will increase as the motor accelerates. If each relay is designed to operate at a definite voltage, it is obvious that such a scheme provides a means of short-circuiting the current-limiting resistance in successive steps in accordance with the speed of the motor. This method is generally confined to d-c motor control.

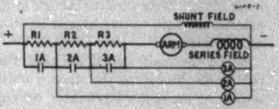

Figure 1. Counter-electromotive-force method

Figure 1 shows how the counter elec-

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Oscillograph measurements were made of line current and of the three line-to-line voltages at the power-supply end of the cable. In all tests the lead sheath was kept isolated from the circuit.

Method of Evaluating Impedance

The impedance magnitude was determined from the ratio of

$$\frac{\text{(average rms line-to-line voltage for 3 phases)}}{1.732\ \text{(average rms line current for 3 lines)}}$$

the line-to-line voltages being measured at the input end of the cable. Phase angles of voltages and currents were measured with respect to the phase of a reference current. The impedance-phase angle, used for evaluating the resistance and reactance components, was then obtained from the average phase-angle difference between line-to-line voltages and corresponding line currents, adding 30 degrees to allow for the phase difference between line-to-line and line-to-neutral values.

In these tests there were no appreciable distortions of wave shape, so that the rms values could be calculated by dividing the measured crest values by 1.414.

Impedance values of the three-conductor cable in the iron conduit and the test data on which they were based are given in table VIII for one set of measurements. The cable impedance per 1,000 feet, in the last three columns was derived from the test values, for 103 feet of cable in columns 7, 8, and 9.

In table IX, the reactance for the cable in air was calculated from the dimensions of the conductors and their spacing using the conventional formula as given in handbooks.

References

1. Four Hundred and Forty-Volt Distribution Systems for Industrial Plants, **T. E. Stieber.** *Electric Journal*, August 1938, pages 311–14.

2. Fault Currents Low With Heavy Power Capacity, **H. W. Papst.** *Electrical World*, volume 102, 1933, pages 208–09.

3. Circuit-Breaker Selection for Medium Voltage Power Circuits, **H. Trencham and J. R. Mortlock.** *BTH Activities*, volume 14, May-June 1938, pages 85–91.

4. Arcs in Low-Voltage A-C Networks, **J. Slepian and A. P. Strom.** AIEE Transactions, volume 50, 1931, pages 847–52.

5. Reactance Values for Rectangular Conductors, **H. B. Dwight.** *Electric Journal*, volume 16, June 1919, pages 255–6.

6. Engineering Calculation of Inductance and Reactance for Rectangular Bar Conductors, **O. R. Schurig.** *General Electric Review*, volume 36, May, 1933, pages 228–31.

7. Calculation of Inductance and Current Distribution in Low-Voltage Connections to Electric Furnaces, **C. C. Levy.** AIEE Transactions, volume 51, December 1932, pages 903–13.

8. Impedance of Three-Phase Secondary Mains in Nonmetallic and Iron Conduits, **L. Brieger.** *Edison Electric Institute Bulletin*, volume 6, 1938, pages 61–6.

9. Underground A-C Network Distribution for Central-Station Systems, **A. H. Kehoe.** AIEE Transactions, volume 43, 1924, pages 844–53.

10. Burn-Off Characteristics of A-C Low-Voltage Network Cables, **G. Sutherland and D. S. MacCorkle.** AIEE Transactions, volume 50, 1931, pages 831–44.

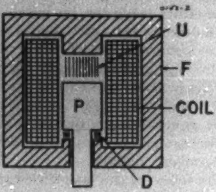


Figure 2. Series lock-out relay

tromotive force method is applied to a compound-wound motor. It will be noted that the voltage across the relays $1A$, $2A$, $3A$ during the accelerating period is $V-RI$, where V = line volts, I = armature current, and R = resistance between the positive side of the line and the relay connection. This permits the use of accelerating relays having almost identical characteristics.

When the main switch is closed, current flows through all the starting resistance. As the motor accelerates, the current decreases and the voltages applied to the relays increase until relay $1A$ closes, short-circuiting the first step of resistance R_1. The current increases immediately, applying greater torque to the motor and causing further acceleration. Again as the motor gains speed, the voltage across relay $2A$ increases until it closes, short-circuiting resistance R_2. This is repeated until the last acceleration contactor closes and the motor is connected directly across the line.

(2). Series Lock-Out Method

The series lock-out method has essentially the same characteristics as the counter electromotive force method inasmuch as each acceleration contactor closes after the motor has reached a definite speed. The accelerating relay itself merits description.

Referring to figure 2, the action of the relay is as follows. For very heavy current, the narrow portion of the plunger P is magnetically saturated so that considerable flux crosses the air-gap D. This causes a downward pull which, when added to the weight of the plunger, overcomes the upward pull through the air-gap U. As the current decreases, less flux crosses the air-gap D and the upward pull becomes sufficient to lift the plunger. Thus the relay remains open for heavy currents, closes for lower values, and opens again for still lower values.

Figure 3 shows the application of such a relay to a d-c motor. When the main line switch closes, current flows through the armature, limiting resistors and the first accelerating relay, $1A$. The relay ampere-turns are such that the plunger is magnetically saturated and the relay is held open. When the current falls to a definite value the plunger is pulled up, moving a contact-making mechanism which short-circuits the first resistance step R_1. This causes the current to pass through both $1A$ and $2A$ relay coils, relay $1A$ remains closed, but $2A$ will not close until the current again falls to its operating value. The operation is repeated until relay $3A$ closes, short-circuiting all the resistance as well as the series coils in all the relays. Relays $1A$ and $2A$ open, but $3A$ remains closed because of its additional shunt winding connected across the motor.

A similar relay has been used for automatic acceleration of a-c motors, placing it either in the primary or secondary circuit. Difficulties inherent in a-c relay design make it not nearly so satisfactory as the corresponding d-c relay.

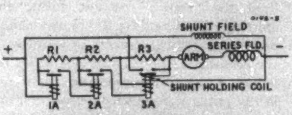


Figure 3. Series lock-out method

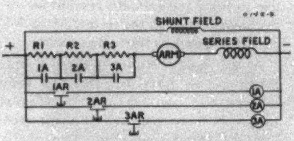


Figure 4. Constant-time method. Dashpot relays

The above methods are particularly suited to motors connected to heavy inertial loads, since the acceleration contactors will not close too quickly at any time. That is, the time interval between closure of acceleration contactors depends entirely upon the load. It often happens, however, especially in steel mills, that motors are overloaded with frictional loads and if any of the above types of control were used, the motor might remain "stalled" indefinitely. For such conditions it is necessary to provide "forced acceleration" so that the acceleration contactors will ultimately close, irrespective of the load.

Another disadvantage of the series lock-out method appears frequently with very lightly loaded motors, since the motor speeds up more rapidly than the accelerators close so that by the time the last acceleration relays are ready to close, the current has fallen to such a low value that the relays will not operate. This would leave some or all of the series resistance in the circuit.

(3). "Constant-Time" Method

The "constant-time" method gives acceleration intervals which are independent of the load on the motor. All that is necessary is some electrical or mechanical means of delaying the closure or opening of electrical contacts. Many schemes are now in use, such as dashpots or escapement mechanisms connected mechanically to the acceleration contactors, or motor-driven cams which operate a series of contacts in sequence.

The application of individual dashpot relays, mechanically connected to the accelerating contactors, is shown in figure 4. The $1AR$ relay is connected to the main line contactor, the $2AR$ relay to the $1A$ contactor, and the $3AR$ relay to the $2A$ contactor. When the main line contactor (not shown in the diagram) is open it holds the $1AR$ relay plunger down against a compression spring. Upon closure of the contactor the plunger is forced upward against the resistance offered by the fluid within the dashpot. After a definite time interval the plunger reaches its upper position and closes the relay contacts which permit the acceleration contactor $1A$ to close. The operation is repeated with each of the other relays in sequence until all of the current-limiting resistance has been short-circuited.

One decided disadvantage of the "constant-time" method becomes apparent when very heavy loads are encountered, in that the acceleration contactors will close before the motor attains any appreciable speed and the resulting excessive currents may cause considerable damage. Nevertheless, "constant-time" accelera-

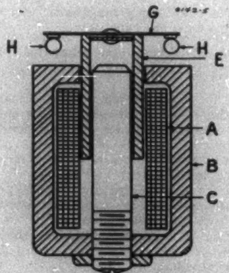


Figure 5. Time-current relay

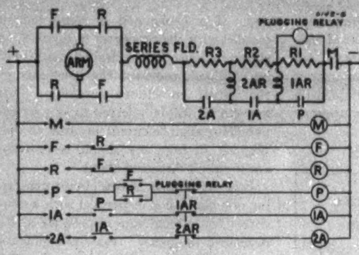


Figure 6. Time-current method. Plugging, reversing controller

tion has found favor because of its simplicity and the fact that it provides "forced acceleration." It has the added advantage of being equally applicable to a-c and d-c motors.

(4). "Time-Current" Method

In the "time-current" method, as the name implies, the acceleration intervals are modified by the current flowing in the motor. This method, which so far has been used in the control of d-c motors only, has all the advantages of the "constant-time" method, along with the added advantage that the time intervals between acceleration steps are automatically increased with increase in load.

A rather interesting example of a "time-current" relay is shown in figure 5. It is fundamentally a series relay with normally closed contacts. The iron structure *B* surrounding coil *A* provides a path for the magnetic flux. An iron core *C* passes through the center of the coil, its position being adjustable by means of screw threads. A movable aluminum sleeve *E* is suspended between the core and the coil. A bridge contact arm *G*, which normally rests on the two fixed contacts *H*, is fastened to the upper end of the aluminum sleeve. The initial inrush of current through the coil, which is connected in series with the motor armature, causes the aluminum sleeve to be repelled upward out of the coil. (The sleeve behaves like a short-circuited secondary winding and, by Lenz' law, the currents set up in it produce a magnetic field in opposition to that produced by the coil.) As long as the current is rapidly changing, the aluminum sleeve will experience an upward thrust, but as the current approaches a steady, or nearly steady, state the sleeve begins to fall under the action of gravity. As it falls, it cuts the main magnetic flux between the frame and the upper end of the core and therefore will fall at a rate which depends upon the current flowing in the coil: the higher the current, the slower the rate. In this way the length of time the relay contacts remain open depends upon the load on the motor. The relay will always close its contacts, however, to provide "forced acceleration."

Figure 6 shows a d-c plugging, reversing controller in which "time-current" relays of the above design are used. If the master is moved to the point 2*A* in a forward direction, contactors *M* and *F* will close immediately. With the motor initially at rest, the closure of *F* completes a circuit to the plugging contactor coil *P* through the plugging relay contacts, which do not open under these conditions. When *P* closes, its normally open auxiliary contacts close to provide a circuit to the 1*A* contactor. Contactor 1*A* cannot close, however, until the acceleration relay 1*AR* closes. When 1*A* closes, resistance R_2 is short-circuited and current flows through relay 2*AR*, causing it to open its contacts. After a second time interval contactor 2*A* will close, applying full voltage to the motor.

The plugging relay has a shunt winding connected across the plugging resistance R_1. Upon sudden reversal of the voltage applied to the motor when running in the forward direction, the voltage across R_1 becomes greater than it was when starting the motor from rest, so that the plugging relay opens its normally closed contacts and prevents the closure of the plugging contactor *P* until the motor has been brought to rest. In this way additional protective resistance is added to the armature circuit during the period of excessively high voltage, that is, when the motor is "plugged."

In the foregoing, acceleration methods which utilize the variation in frequency of rotor currents with speed of a wound-rotor induction motor or which provide "time-current" acceleration of a-c motors have not been considered. The new methods about to be described cover these two cases.

Method I—Automatic Acceleration of Wound-Rotor Induction Motors

It can be shown readily that the voltage across the external phase resistance R_E connected to a three-phase wound-rotor induction motor is given by

$$V_2 = \frac{sE_2R_E}{\sqrt{3}\sqrt{s^2X_0^2+(R+R_E)^2}} \tag{1}$$

where

s = rotor slip
E_2 = open-circuit rotor voltage across rings
X_0 = standstill rotor reactance per phase
R = rotor resistance per phase

Figure 7 shows how this voltage varies with slip for different values of R_E. In plotting the curves, the constants in equation 1 were those of a typical 20-horsepower, 220-volt motor in which R = 0.1 ohm, X_0 = 0.5 ohm and E_2 = 220 volts. For lower frequencies (higher motor speeds) the voltage is nearly proportional to the slip. This is particularly true for

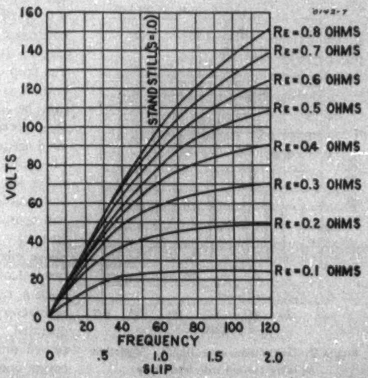


Figure 7. Variation in rotor voltage with frequency

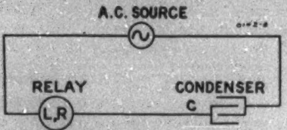


Figure 8. Series resonance circuit

greater values of R_E. In practice R_E is large when the motor is being plugged or started from rest and is small only when it has gained considerable speed. Thus for practical purposes the proportionality between V_2 and s will hold approximately over the entire range of frequencies ($s=0$ to 2). This proportionality will be assumed in the following discussion of relay circuits which might be used to provide automatic acceleration.

Consider a circuit consisting of a simple electromagnetic relay, the impedance of which is predominantly reactance. If such a relay is connected to a source of alternating potential difference proportional in magnitude to its frequency, the current in the relay will remain practically constant and independent of frequency. Obviously such a relay would not be responsive to motor speed and hence would be unsuitable for acceleration purposes.

If, however, a condenser is connected in series with the relay as in figure 8, the impedance of the circuit will be given by

$$|Z|=\sqrt{\left(\omega L-\frac{1}{\omega C}\right)^2+R^2} \tag{2}$$

where L and R are the inductance and resistance of the relay, C the capacity of the condenser, and $\omega=2\pi\times$frequency. When $\omega L=1/\omega C$ the impedance is a minimum and the circuit is said to be in resonance with the supply frequency. For frequencies greater than that of resonance ($\omega L>1/\omega C$) the impedance will be determined mainly by the reactance of the relay, while for lower frequencies ($\omega L<1/\omega C$) the impedance is approximated by the capacitance of the condenser. In the region close to resonance the resistance R determines the impedance.

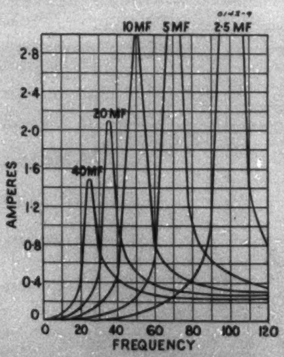


Figure 9. Series resonance circuit. Variation in relay current with frequency

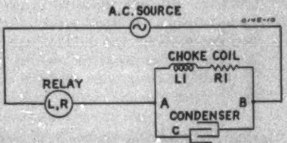


Figure 10. Series-parallel resonance circuit

If a voltage having a magnitude proportional to its frequency is applied to the above circuit, the current will increase as the square of the frequency for frequencies well below that of resonance, reaching a maximum at the point of resonance. Thereafter the current will decrease, approaching a steady value. Figure 9 shows this variation in current with frequency for different values of C. In plotting these curves a voltage equal to 1.5 times the frequency was employed and L and R were assumed equal to 1.0 henry and 25.0 ohms respectively.

By connecting a series of such relay circuits across the secondary resistance of a wound-rotor induction motor, it is possible to provide automatic acceleration at intervals determined by the motor speed. Similarly the circuit may be used to provide plugging protection in the following manner. Using a relay having a "pick-up" current twice its "drop-out" value and supposing the drop-out current to be 0.13 ampere, then with a 2.5-microfarad condenser in the circuit the relay would pick up at 75 cycles or higher frequency and drop out at 60 cycles. If closer operation is required the condenser size may be increased slightly to bring the steep portion of the curve nearer the desired frequency of operation. The applied voltage may be reduced accordingly.

For some motor-control applications it is desirable to have a relay means which will operate within still closer limits of frequency than that provided by the above circuit, and which will be independent of small changes in line voltage. Such a relay means would prove more satisfactory than any of the mechanically operated "zero-speed" switches now employed and might be used to give accurate operation in plugging a motor to rest. If a choke coil is connected in parallel with the condenser in figure 8, as shown in figure 10, the desired result may be obtained, provided the constants of the circuit are chosen carefully. This new circuit will be referred to as a series-parallel resonance circuit in order to distinguish it from the simple series-resonance circuit discussed above.

It is well known that a circuit consisting of a choke coil in parallel with a condenser has a maximum impedance at its resonance frequency. For "plugging-to-stop" service and for zero-speed relay purposes a rapid change in relay current must occur in the region of line frequency, therefore the line frequency f_0 might be chosen as the resonance frequency of the parallel or "tank" circuit AB in figure 10. Using the equation derived from this condition, the necessary relationship between the constants which will give a minimum impedance to the total circuit at a frequency just above f_0 may be approximated as follows:

Admittance of "tank" circuit $AB=$

$$\frac{R_1-j\omega L_1}{R_1^2+\omega^2L_1^2}+j\omega C \tag{3}$$

Impedance of "tank" circuit $AB=$

$$\frac{(R_1^2+\omega^2L_1^2)(R_1-j[\omega CR_1^2+\omega^2L_1^2C-\omega L_1])}{R_1^2+(\omega CR_1^2+\omega^2L_1^2C-\omega L_1)^2}$$

Impedance of total circuit $Z=R+j\omega L+$ (impedance of AB)

whence

$$|Z|=\sqrt{\left[R+\frac{R_1(R_1^2+\omega^2L_1^2)}{R_1^2+(\omega CR_1^2+\omega^2L_1^2C-\omega L_1)^2}\right]^2+\left[\omega L-\frac{(R_1^2+\omega^2L_1^2)(\omega CR_1^2+\omega^2L_1^2C-\omega L_1)}{R_1^2+(\omega CR_1^2+\omega^2L_1^2C-\omega L_1)^2}\right]^2} \tag{4}$$

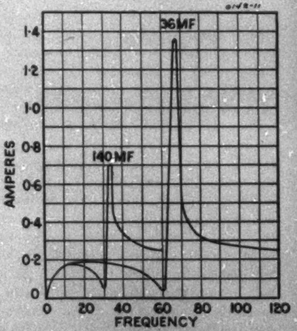


Figure 11. Series-parallel resonance circuit. Variation in relay current with frequency

Since a choke coil can be designed in which R_1 is much less than $\omega_0 L_1$ where $\omega_0 = 2\pi f_0$ and f_0 is as low as 30 cycles, equation 3 may be replaced by the approximation

$$\text{Admittance of } AB \approx \frac{R_1}{\omega^2 L_1^2} - j\left(\frac{1}{\omega L_1} - \omega C\right) \tag{3a}$$

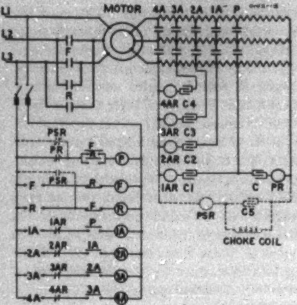


Figure 12. Frequency method. Plugging, reversing controller

This admittance will be a minimum (or its reciprocal, the impedance a maximum) when $\omega^2 = 1/L_1 C$ and for this to occur at frequency f_0 we have

$$\omega_0^2 = \frac{1}{L_1 C} \tag{5}$$

Using the approximation $3a$ the impedance of the total circuit may be written

$$Z \approx \frac{\dfrac{R_1}{\omega^2 L_1^2} + j\left(\dfrac{1}{\omega L_1} - \omega C\right)}{\dfrac{R_1^2}{\omega^4 L_1^4} + \left(\dfrac{1}{\omega L_1} - \omega C\right)^2} + R + j\omega L$$

Assuming $\dfrac{R_1^2}{\omega^4 L_1^4} \ll \left(\dfrac{1}{\omega L_1} - \omega C\right)^2$ in the region where $\omega = 2\pi f_1$, an assumption which can be verified for any particular case, the total impedance reduces to

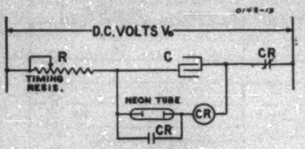


Figure 13. Neon tube timing circuit

$$Z \approx R + j\omega L + \frac{\dfrac{R_1}{\omega^2 L_1^2}}{\left(\dfrac{1}{\omega L_1} - \omega C\right)^2} + j\left(\frac{1}{\dfrac{1}{\omega L_1} - \omega C}\right)$$

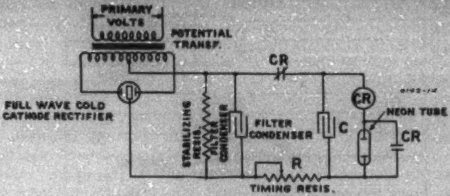


Figure 14. Constant-time timer

but C must be chosen to meet condition 5 hence

$$Z \approx R + j\omega L + \frac{\dfrac{R_1}{\omega^2 L_1^2}}{\left(\dfrac{1}{\omega L_1} - \dfrac{\omega}{\omega_0^2 L_1}\right)^2} + j\left(\frac{1}{\dfrac{1}{\omega L_1} - \dfrac{\omega}{\omega_0^2 L_1}}\right)$$

and for this to be a minimum at frequency f_1 we have

$$\omega_1 L + \frac{1}{\left(\dfrac{1}{\omega_1 L_1} - \dfrac{\omega_1}{\omega_0^2 L_1}\right)} = 0$$

or

$$\frac{\omega_1^2}{\omega_0^2} = 1 + \frac{L_1}{L} \tag{6}$$

Equations 5 and 6 give the relationships that must exist between L, L_1, C, f_0, and f_1 so that the circuit will have a much higher impedance at frequency f_0 than at f_1. Assuming $f_1 = 1.1 f_0$ and using the same relay as before where $L = 1.0$ henry and $R = 25$ ohms, then $L_1 = 0.2$ henry. For $f_0 = 30$ cycles, C takes the value 140 microfarads and for $f_0 = 60$ cycles, $C = 36$ microfarads.

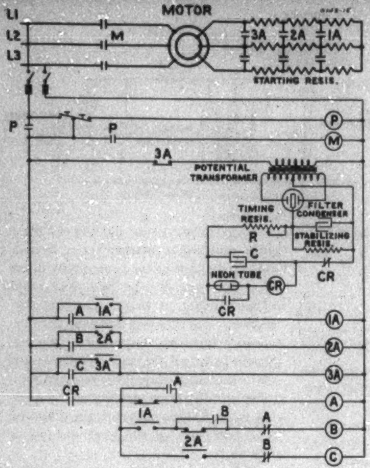
Since no difficulty is presented in designing a choke coil having an inductance of 0.2 henry and a resistance as low as 2.0 ohms, the two assumptions $R_1 \ll \omega_0 L_1$ and $\dfrac{R_1^2}{\omega_1^4 L_1^4} \ll \left(\dfrac{1}{\omega_1 L_1} - \omega_1 C\right)^2$ are now justified for as low as 30 cycles per second. Using all of these constants in equation 4 to determine the variation in impedance of the total circuit with frequency and assuming a voltage equal to 1.5 times its frequency applied to the circuit, the relay current was calculated and plotted in figure 11.

An experimental relay requiring a pick-up current approximately twice its drop-out value was designed along with a choke coil in accordance with the above constants. With the relay adjusted to drop out at 60 cycles, it picked up at 61.5 cycles thus providing the desired operation. Because of the steepness of the current curve (figure 11) between 60 and 66 cycles the operation of the relay was not affected appreciably by a ten per cent variation in line voltage.

The complete wiring diagram of a plugging, reversing controller for a wound-rotor induction motor is shown in figure 12. The controller provides four steps of acceleration as well as plugging protection. Alternatively the controller becomes a four-step starter with plugging-to-rest feature if the series resonance circuit containing relay PR is replaced by a series-parallel resonance circuit containing relay PSR as shown in the diagram. The operation of the controller is as follows.

With motor at rest, suppose the master switch is moved to the point $4A$ in the forward direction. Contactor F closes to apply "forward" power to the motor primary and complete a circuit through its normally open auxiliary contacts to the plugging contactor coil P. Contactor P closes since the plugging relay PR does not open its normally closed contacts under these conditions. The normally open auxiliary contacts on P close to prepare a circuit for the coil of acceleration contactor $1A$ which, however, is not energized until relay $1AR$ opens, closing its normally closed contacts. Similarly, when the acceleration contactor $1A$ closes it prepares a circuit for the $2A$ contactor, which closes after the $2AR$ relay opens. The operation is repeated automatically as the motor accelerates until finally the last acceleration contactor closes, short-circuiting the entire secondary resistance.

Suppose the motor is running in the forward direction and the master switch is removed quickly to the last point reverse. The F contactor will open and R will close, applying reverse power to the motor primary. The frequency in the secondary will now be greater than line frequency and the plugging relay PR will close, opening its normally closed contacts and preventing the closure of contactor P. Near the instant at which the motor comes to rest, relay PR will open and contactor P will close to remove the plugging resistance from the motor secondary. The motor will then accelerate automatically in the reverse direction in a manner similar to that described above.


L1
L2
L3
MOTOR
M
3A
2A
1A
STARTING RESIS.
P
P
3A
POTENTIAL TRANSFORMER
TIMING RESIS.
R
FILTER CONDENSER
STABILIZING RESIS.
C
NEON TUBE
CR
CR
CR
A
1A
B
2A
C
3A
CR
A
1A
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B
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M
1A
2A
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A
B
C

Figure 15. Neon tube method. Multistep a-c starter

Referring to the alternative connections for a starter, suppose the motor is running in the forward direction and it is desired to bring it to rest quickly. All that is necessary is to move the master switch to any of the reverse points, returning it to the "off" position immediately after the *R* contactor closes. Closure of *R* increases the secondary frequency, causing the plugging-to-stop relay *PSR* to close, closing its normally open contacts to provide a holding circuit for the *R* contactor against return of the master to the "off" position and, at the same instant, opening its normally closed contacts to prevent the closure of the plugging contactor *P*. When the motor comes to rest relay *PSR* opens and the *R* contactor drops out, removing power from the primary.

All the relay means described in the foregoing have been made to operate satisfactorily in close agreement with calculated results. Failure to provide forced acceleration by this method of control is not a disadvantage in the case of a wound-rotor induction motor, since it is possible to adjust resistance values in the secondary so that maximum torque is applied at every step of acceleration.

Method II—"Constant-Time" and "Time-Current" Acceleration of A-C and D-C Motors

The frequency relay method just described bears the same relationship to a-c motors as the counter electromotive force method does to the acceleration of d-c motors in that the acceleration contactors close at intervals determined by the speed of the motor. In fact this property is common to all methods based on the current-limit principle as distinguished from those which operate on the time-limit principle, where the contactors close at intervals independent of the motor speed.

While current-limit methods have many important applications, it is generally considered that time-limit acceleration has the wider use. The most frequently used timing devices are dashpot relays (one for each step of acceleration) and motor-driven cam-operated switches. Dashpot relays require careful maintenance and, because of their limited range of adjustment, are not suited to the acceleration of heavy inertial loads since they necessitate an excessive number of acceleration steps. Motor-driven timing devices, on the other hand, do not lend themselves to rapid adjustment in the field and are relatively expensive. In view of these disadvantages it is desirable to use a timing device which operates on electrical principles only and is readily adjustable over a wide range. Such a method will now be described.

When a constant d-c voltage V_0 is applied to a circuit consisting of a resistance R in series with a condenser of capacity C, the voltage V across the condenser at any time t after closing the circuit is given by

$$V = V_0(1-e^{-t/RC}) \qquad (7)$$

If a neon glow-discharge tube is placed across the condenser it will "fire" at the instant the voltage on the condenser reaches the ignition potential of the tube. The time required for the voltage V to reach this potential, V_1 say, is propor-

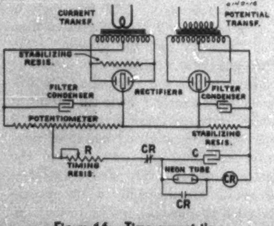
CURRENT TRANSF.
POTENTIAL TRANSF.
STABILIZING RESIS.
FILTER CONDENSER
RECTIFIERS
FILTER CONDENSER
POTENTIOMETER
STABILIZING RESIS.
R
TIMING RESIS.
CR
C
NEON TUBE
CR
CR

Figure 16. Time-current timer

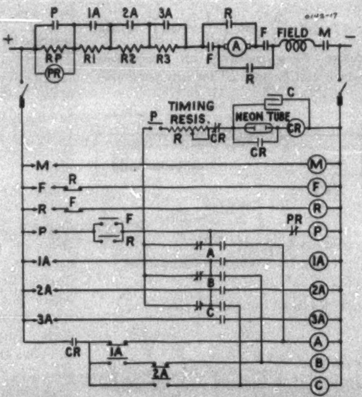
P
1A
2A
3A
R
+
RP
PR
R1
R2
R3
F
A
F
R
F
FIELD
M
−
C
TIMING RESIS.
P
R
CR
NEON TUBE
CR
CR
M
F
R
R
F
P
F
R
PR
1A
A
2A
B
3A
C
CR
1A
2A
M
F
R
P
1A
2A
3A
A
B
C

Figure 17. Neon tube method. Plugging, reversing d-c controller

tional to the product RC and, for a fixed condenser, to R only. Thus a simple and accurate method of timing is provided in which the time interval may be adjusted over any desired range. This method has been used successfully and extensively during the past seven or eight years in timing resistance welders.

Figure 13 shows a control relay CR connected in series with the neon tube in the above circuit. The relay has normally open and normally closed contacts. At the instant the tube fires, the relay closes opening its normally closed contacts to disconnect the timing circuit from the charging voltage V_0. The normally open contacts close to short-circuit the neon tube and provide a holding circuit for the relay against the extinction of the tube. By proper choice of the relay coil it is possible to assure complete, or practically complete, discharge of the condenser, a condition essential to the consistent operation of the timer.

Assuming the extinction potential of the tube to be V_2, the amount of energy given up by the condenser in discharging from V_1 to V_2 is $\frac{1}{2}C(V_1^2-V_2^2)$. This energy must be greater than that required to close the relay CR; otherwise, positive operation will not be obtained. With a 20-microfarad condenser, a tube having an ignition potential of 160 volts and an extinction potential of 80 volts permits the use of a relay sufficiently rugged to be mounted on the same panel as an 800-ampere a-c contactor.

It is always important in automatic motor control work to investigate the effect of variation in line voltage on the acceleration time intervals. If V_0, the voltage applied to the timing circuit, is proportional to the line voltage, it is obvious from equation 7 that the time intervals will decrease or increase as the line voltage is higher or lower. This compensation is in the desired direction.

Application to A-C Motors

In applying the above timing scheme to a-c motors, a source of d-c potential is necessary. This may be obtained by using an ordinary radio rectifying system as shown in figure 14. The transformer has a high-voltage winding, giving 700 volts, while low-voltage windings are available for the range 110 to 550 volts. A cold cathode rectifier is used in preference to the usual '80 tube, since it is free from the time-lag inherent in hot cathode tubes and eliminates the need of control wiring and relays for the filament. The 50,000-ohm permanent load resistor holds the d-c voltage within close limits when the impedance of the timing circuit is varied.

The wiring diagram for a multistep starter is shown in figure 15; its operation is as follows. When the push button is closed, relay P closes to form its own holding circuit against the opening of the button and to complete a circuit to the M contactor coil. The primary of the timing transformer is energized through the normally closed auxiliary contacts on the last acceleration contactor. After a definite time the CR relay operates, closing relay A which, in turn, closes the $1A$ contactor. Relay A forms its own holding circuit against the opening of the normally closed contacts on contactor $1A$, so that A will remain closed as long as CR is closed. Contactor $1A$ forms its own holding circuit against the opening of relay A; therefore it is necessary only that CR remain closed long enough to let contactor $1A$ close. Closure of A opens its normally closed contacts in series with relay B, so that relay B cannot close until A opens and, since A opens only after CR has opened, the $2A$ contactor cannot close until the second time interval has been measured by the timer. The operation is repeated for each step of acceleration until finally the last acceleration contactor closes, disconnecting the timer from the line.

While a wound-rotor induction motor was chosen in the above illustration, any type of a-c motor would have served equally well. One timer only was used for all steps of acceleration, and although not shown in the diagram and usually unnecessary in practice, each step could have been provided with its own timing resistance.

The foregoing acceleration method provides time intervals which are constant and independent of the load on the motor. Time-current acceleration may be provided by the addition of a current-transformer and a second rectifying system connected as shown in figure 16. The rectified voltage from the current transformer is connected in opposition to that from the potential transformer, so that the heavier the load on the motor the lower the voltage applied to the timing circuit and the longer the time intervals. The potentiometer shown across the d-c output from the current transformer provides a means of adjusting the time-current effect to suit load conditions.

Application to D-C Motors

A constant-time, plugging, reversing controller for a series-wound d-c motor is shown in figure 17. When the plugging contactor P closes, its normally open auxiliary contacts close completing a circuit to the timer. After a definite time interval, relay CR closes energizing relay A. Relay A forms its own holding circuit and completes another circuit to contactor $1A$. It is essential that relay CR opens before contactor $1A$ closes, otherwise relay B and contactor $2A$ would close before the second time interval had elapsed. In other regards the controller operates in much the same way as those described elsewhere.

Time-current acceleration may be obtained by placing a potentiometer across the starting resistance and connecting its central tap to the timing circuit. In adjusting the position of the central tap, care must be taken that the voltage applied to the timing circuit is always well above the ignition potential of the neon tube, a condition which limits the amount of time-current effect which can be obtained in this way.

While no practical limitation is inherent in the "neon time" method of acceleration which would prohibit its use for lightly loaded motors, it is finding its widest application in the acceleration of heavily loaded motors requiring long-time intervals.

The Use of Auxiliary Impedances in the Single-Phase Operation of Polyphase Induction Motors

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RECENT literature indicates a considerable interest in the subject of single-phase induction motors which use split-phase starting. In the available motors condensers are often used as auxiliary impedances in the starting circuit and may also be used in the running circuit. Much of the existing literature on these machines deals with the condenser required and the torque developed. For the purposes of this analysis ideal conditions of balancing will be assumed. If the auxiliary impedances are properly chosen the motor windings will be subjected to balanced polyphase voltages. Under these conditions the motor will operate as a polyphase motor and the analysis may be carried out using standard polyphase motor theory and a-c circuit theory. The first machines analyzed will be of symmetrical two- and three-phase design. Following this, motors having unsymmetrical windings will be considered. Some observations on the related subjects of motor size and motor rating will also be included.

Motor Characteristics

In order that the analysis might be made throughout the entire speed range of the motors, the complete motor characteristics were determined from assumed circle diagrams. First a low resistance motor having the characteristics shown in figure 1 was assumed. By doubling the rotor resistance of this motor the characteristics of a high-resistance motor were obtained and are shown in figure 2. The same characteristics have been assumed for two-phase and three-phase machines although this results in slightly different horsepower ratings. In the final tabulations where comparisons were desired the values have been corrected for this difference. In calculating the impedances and the condenser capacitances a line voltage of 100 has been assumed.

Method of Analysis

If a polyphase motor is operated from a single-phase line, and auxiliary impedances are used to shift the phase currents and the motor terminal voltages so that the currents and voltages of the motor are properly balanced both as to phase and magnitude, the motor will operate as though it were supplied from its normal polyphase line. Due to the fact that the motor current and power factor both change as the speed changes, the impedances required also must change with the motor speed. If these changes in the balancing impedances are small so that average values may be chosen which will reasonably approximate the ideal values over a range of speed, then, by using these average values, approximate polyphase performance may be obtained over this speed range with the motor operated from single-phase lines. If the auxiliary impedances required should change radically in value or change in character, then the choice of auxiliary impedances must rest upon other considerations and fixed values of these impedances cannot be expected to give single-phase performance equivalent to the polyphase performance over the speed range, although, it may be possible to have the two performances equal at some speed within the range. This study of the impedance variations was made with the idea that general conclusions regarding these impedances might be drawn. The number of auxiliary impedances considered has been limited to two for each connection, with one of these two assumed a perfect condenser.

Three-Phase Motors

The first motors considered will be of three-phase design. These may be balanced by two different methods. The first method is by series impedances and has been called type *A*. The second method is by shunt impedances and has been called type *B*. Type *A* connection is shown in figure 3, which also shows the vector diagram. From the geometry of the vector diagram it may be seen that Z_A is a perfect condenser, while Z_B is a resistance. The numerical values of these impedances may be found from the following formulas.

$$Z_A = -j\frac{E(1.732 \sin \theta - \cos \theta)}{I}$$

$$Z_B = \frac{E(\sin \theta - 1.732 \cos \theta)}{I}$$

Z_A is condensive when the motor power factor is less than 86.6 per cent, and inductive for greater power factors. Z_B is zero for a motor factor of 50 per cent. For power factors below this value it is a positive resistance. For higher values this impedance becomes a negative resistance. The analysis using this type of connection must be limited to those speed ranges where the power factor is 50 per cent or less. The curves of figure 4 show how these two impedances must vary with the speed if the ideally balanced conditions assumed are to be obtained. In the curves shown in this paper the unprimed values are those calculated for the low-resistance motor, while the primed values are for the high-resistance motor. Considering the speed range between 0 and 50 per cent synchronous speed it may be seen that Z_A is reasonably constant, and that Z_B is quite small actually passing through zero at about 25 per cent speed. This would indicate that for the low-resistance motor Z_A might be assigned an average value and Z_B made equal to zero. For the high resistance motor Z_B' is a negative resistance in this speed range and the connection is not suitable for this motor. Above 50 per cent speed both the low- and high-resistance machines have rapidly changing impedances. No suitable average values are indicated by these curves for the higher speeds.

The type *B* connection is shown in figure 5. The two auxiliary impedances may be determined from the following formulas.

$$Z_1 = -j\frac{E}{I(\cos \theta + 0.577 \sin \theta)}$$

$$Z_2 = j\frac{E}{I(\cos \theta - 0.577 \sin \theta)}$$

Z_1 is a condensive reactance under all conditions. Z_2 is a condensive reactance for power factors less than 50 per cent, and

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1. For all numbered references, see list at end of paper.

inductive for power factors above this value. For a power factor equal to 50 per cent, Z_2 becomes infinite. For this one condition where the power factor equals 50 per cent the type *B* connection becomes identical to type *A*. Curves showing the variations of these impedances are given in figure 6. Referring to these curves it may be seen that during the starting period, considered as between 0 and 50 per cent speed, the impedance Z_2 changes sign. It is, however, a relatively high impedance throughout this range and may perhaps be omitted. In starting the high-resistance motor about the same capacitive reactance is needed for Z_1' as was required for Z_1 for the low resistance motor. In addition an inductive reactance is needed in position Z_2'. Z_2' varies from 57.7 to 24 ohms in the assumed example. An average value of approximately 40 ohms might be used. Experimental checks on a motor having approximately the characteristics of the high-resistance motor showed that the addition of the inductive reactance Z_2' increased the starting torque. Again no impedances can be chosen which will balance the motors, even approximately, for more than one point in the higher speed range.

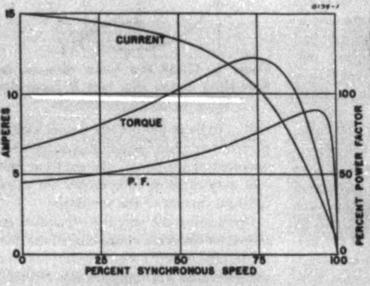

Figure 1 (left). Performance curves for motor with low-resistance rotor

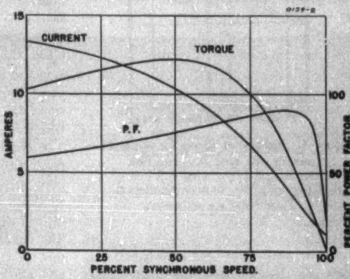

Figure 2 (right). Performance curves for motor with high-resistance rotor

Two-Phase Motors

The type *C* connection for the two-phase motors shown in figure 7 corresponds to the type *A* connection for the three-phase motors. The impedance Z_A is a perfect condenser. Z_B is not a simple impedance. For motor power factors less than 0.707, Z_B is a condensive impedance having a power factor of 0.707. When the motor power factor becomes equal to 0.707 this impedance is reduced to zero, and for higher power factors the impedance becomes an inductance with negative resistance. This type of compensation is of value only where the motor power factor is in the neighborhood of 0.707 as then this second impedance may be omitted. For a high-resistance motor where the starting power factor is 0.60 or higher this impedance may be omitted without too great an error. The values of these impedances are given in the following equations.

$$Z_A = -j\frac{2E \sin\theta}{I}$$

$$Z_B = (j-1)\frac{E(\cos\theta - \sin\theta)}{I}$$

Curves showing the variations of these impedances are given in figure 8. Again no possibility of compensation is indicated by these curves for the higher speeds.

The type *D* connection for two-phase motors corresponds to the type *B* for the three-phase machines. The wiring and vector diagrams of this connection are shown in figure 9. With a condenser used for Z_1, two types of impedance for Z_2 may be used. In the vector diagram the solid lines indicate the values and positions of I_1 and I_2 when a resistance is used for Z_2. The dotted lines show their positions for Z_2 a reactance. In either case I_2 becomes zero for a motor power factor equal to 0.707. The reactance method is the better of the two since where resistances are used negative resistances are required for power factors greater than 0.707. Also for lower power factors the total capacitance required is the same for either method. The impedance curves shown in figure 10 are for the case where Z_2 is a reactive impedance. The equations for Z_1 and Z_2 are as follows. For Z_2 a resistance,

$$Z_1 = -j\frac{E}{I \sin\theta}$$

$$Z_2 = \frac{E}{I(\sin\theta - \cos\theta)}$$

If Z_2 is a reactance these equations become,

$$Z_1 = -j\frac{E}{I \cos\theta}$$

$$Z_2 = j\frac{E}{I(\cos\theta - \sin\theta)}$$

As seen from the curves the low-resistance motor needs two impedances during the starting period. While the ideal impedances vary considerably in the starting range approximate balance may be obtained by choosing average values. Due to the higher power factor of the high-resistance motor the impedance Z_2' is a very high impedance throughout most of the speed range from 0 to 50 per cent speed. This impedance may be left out of the circuit during starting on these high-resistance motors. When the motor power factor is equal to 0.707, Z_2 becomes infinite and the compensation is identical to that of the type *C* connection.

Figure 11 shows still another type of connection in which a two-phase motor

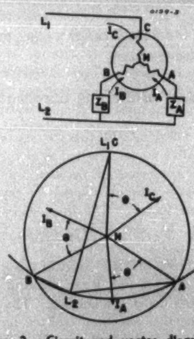

Figure 3. Circuit and vector diagrams for three-phase motors with type A connection

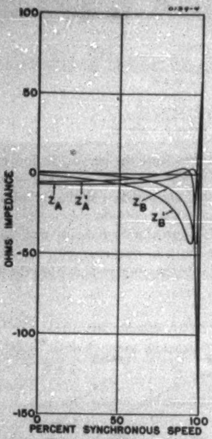

Figure 4. Impedance curves for three-phase motors with type A connection

Z_A and Z_A' are reactive impedances
Z_B and Z_B' are resistances

may be used. This has been called the type *E*. In this case the single-phase voltage is applied to the outside motor terminals and the auxiliary impedances are connected between the neutral point and the lines. Z_1 is a condensive impedance under all conditions. Z_2 is a condensive impedance for power factors less than 0.707 and inductive for greater power factors. At a power factor of 0.707 Z_2 becomes infinite. The variations of these impedances are shown in figure 12. The numerical values are given by the following equations.

$$Z_1 = -j\frac{0.707E}{I(\cos\theta + \sin\theta)}$$

$$Z_2 = j\frac{0.707E}{I(\cos\theta - \sin\theta)}$$

Summary of Starting Impedances

The following table lists average or representative values, selected as giving reasonable compensation between 0 and 50 per cent synchronous speed for the two assumed motors. The impedances calculated for the low-resistance, three-phase motor correspond to a motor power factor of 0.50. Since at this power factor types *A* and *B* are identical they are listed under one heading. The type *C* connection has been omitted for the reasons mentioned above. The values given for the high-resistance motor in the two-phase types have been calculated for a power factor of 0.707 under which one impedance may be omitted. Types *C′* and *D′* being identical have been combined under one heading and type *A′* has been omitted since it would require negative resistance.

In calculating the volt-ampere ratings of the motors the full-load current of the motors originally assumed has been taken as 2.5 amperes.

The second group of values is derived from the first group and takes into account the relative polyphase ratings of the different types of machines. The volt-ampere rating of a three-phase machine is $1.732EI$, while that of a two-phase machine is $2.00EI$. Thus the three-phase machines considered have only had 0.866 the rating of the two-phase machines in types *C* and *D*. The type *E* two-phase motors have phase voltages 0.707 times those of types *C* and *D* and are therefore smaller machines in that ratio. The condenser volt-ampere and microfarad ratings in this part of the table have been increased so that they correspond to those of machines having the same ratings as the two-phase types *C* and *D*. The values of relative weight of the motors are included to point out an advantage which the three-phase motors have over the two-phase motors due to the higher distribution factors of their stator windings.

Since some motors use condensers during the starting period only, the tabulation of per cent of the winding used and the volt-ampere rating of the motor when operated single-phase without auxiliary impedances is included. For motors so operated these values will serve to show the advantages which the three-phase and the two-phase type *E* have over the normal method of connection. The advantages indicated here become of especial importance for motors having a high ratio of exciting current to rated full-load current. In those motors where only a small part of the stator winding is used when operated single-phase, the exciting current may equal or even exceed the rated full-load current of the winding.

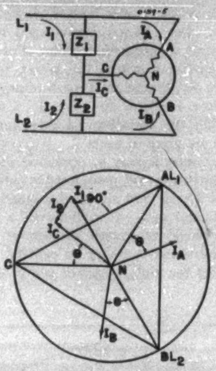

Figure 5. Circuit and vector diagrams for three-phase motors with type B connection

Considering the starting capacitors required for the proper balancing of the low-resistance motors it will be noted that the smallest total volt-amperes are required by the three-phase motor. Also the three-phase motor requires only one capacitor while the total capacitance must be split in the case of the two-phase designs. Although the volt-ampere capacitance is less for the three-phase motor the number of microfarads required is higher than the type *D* since the larger part of those used in the latter connection are operated at a higher voltage. Transformers are sometimes used to get higher condenser voltages. Where these are used the main consideration is the volt-ampere rating of the capacitor.

For motors having higher starting power factors there seems to be little

Table I

	Low-Resistance Motor			High-Resistance Motor		
	A and B	D	E	B′	C′ and D′	E′
Z_A or Z_1 ohms	$-j6.90$	$-j14.0$	$-j3.60$	$-j7.30$	$-j12.35$	$-j4.36$
Z_B or Z_2 ohms	None	$-j20.0$	$-j12.5$	$+j40$	None	None
Z_A or Z_1 microfarads	384	189	736	363	215	608
Z_B or Z_2 microfarads	0	132	212	Inductive	0	0
Total microfarads	384	321	948	363	215	608
Condenser volt-amperes	1,450	1,930	1,790	1,370	1,620	1,146
The following values have been corrected to take account of the relative polyphase ratings of the motors:						
Condenser volt-amperes	1,670	1,930	2,530	1,580	1,620	1,620
Condenser microfarads	443	321	1,340	418	215	860
Condenser voltage	100	141 and 100	70.7	100	141	70.7
Relative weight of motor	95	100	100	95	100	100
If the machines are operated single-phase without auxiliary impedances for full speed running the following factors should be noted:						
Per cent of winding used	67	50	100	67	50	100
Volt-ampere motor rating	289	250	354	289	250	354

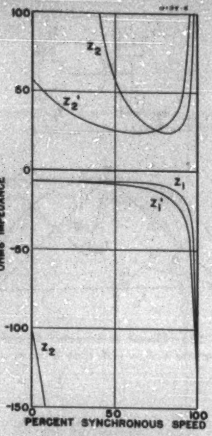

Figure 6. Impedance curves for three-phase motors with type B connection

All impedances are reactive

choice between the different types of connection on the basis of condenser kilovolt-amperes required. The three-phase motor, however, requires a reactance coil as a second impedance which the two-phase designs do not require. In comparing the types D' and E' it should be pointed out that the condenser kilovolt-amperes required is the same in each case and the main difference is the voltage applied to the condenser. Since the voltage across the condenser in the type E' connection is one-half that used in the type D' the capacitance used must be four times as great. If a one-to-two transformer were used with the type E', doubling the condenser voltage, the starting condensers used in the two types would be identical except for modifications needed to make allowances for the transformer exciting current and losses. The main question which arises then is as to the relative advantage of the omission of the transformer versus the use of the entire winding for running.

Unsymmetrical Motors

A disadvantage possessed by unsymmetrical machines is that coils of different wire sizes and number of turns are needed. While not a major item it would be advantageous to use the same sized windings throughout. Three-phase machines with unsymmetrical windings cannot have balanced flux conditions except under special circumstances. Because of this and because they are not used by the industry they have not been considered. Two-phase unsymmetrical machines may readily have balanced flux conditions and are extensively used by industry.

Balanced flux conditions, as used above, are obtained when each phase of the stator supplies the same power and magnetizing force to the rotor, and these are supplied at times corresponding to the mechanical displacement of the phases. In a symmetrical two-phase machine this condition exists when the currents in the two phases are equal and displaced 90 degrees in time. Where the two phases have different voltage ratings, the currents must be different in inverse ratio so that the power and kilovolt-amperes remain the same. The currents in the two phases must still be displaced 90 degrees in time if the coils are displaced 90 electrical degrees on the motor frame.

In figure 13 is shown the vector diagram of the usual type of connection which corresponds to the type D except that phase A is not wound for rated line voltage. In the condenser motor this starting winding is usually wound for a higher voltage than the line voltage. By changing the voltage for which this phase is wound three main effects are produced. First the voltages across the impedances are changed, second the current requirement in that phase is changed, and, third the phase of the voltage across Z_1 is changed. As results of these changes we may find changes in the following: the kilovolt-amperes of the condensers required, the size of the condensers, the arrangement of these condensers, and the power factor at which compensation may be accomplished using a single impedance Z_1. A single condenser in position Z_1 will give perfect compensation when $\phi=\theta$, where tangent ϕ is the starting winding voltage to the running winding voltage. The kilovolt-amperes in the main condenser Z_1 is a minimum for the ratio tangent $\phi=1$, and increases as we depart either way from this value. When ϕ is smaller than θ, Z_2 is also a condenser. Including Z_2 the minimum total kilovolt-amperes of condenser capacity required occurs when $\phi=\theta$. The capacitances of both condensers decrease as the starting winding voltage is increased. The optimum voltage ratio to use would seem to be the highest value possible without overstressing the condensers. For values of ϕ greater than θ, the impedance Z_2 becomes inductive.

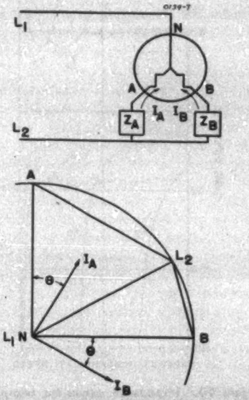

Figure 7. Circuit and vector diagrams for two-phase motors with type C connection

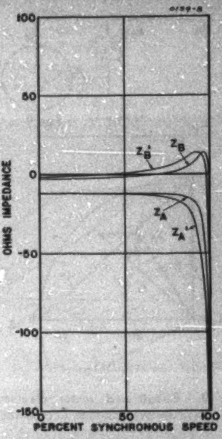

Figure 8. Impedance curves for two-phase motors with type C connection

Z_A and Z_A' are reactive impedances
Z_B and Z_B' values are to be multiplied by $(j-1)$

The two-phase type D with unsymmetrical windings may be compared with the type B. If the two-phase machine were wound with a ratio tangent $\phi=1.732$ and were compensated for a power factor of 0.50, only one impedance Z_1 would be needed. Compensating the three-phase motor for this power factor also would require only one impedance. Correcting for the relative polyphase motor ratings the following comparisons may be made. The condenser voltage for

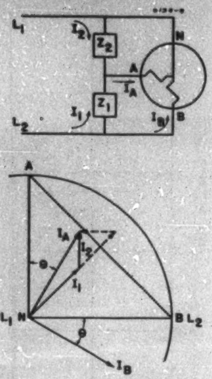

Figure 9. Circuit and vector diagrams for two-phase motors with type D connection

the two-phase motor would be twice that for the three-phase motor. The condenser capacitance for the two-phase motor would be one-fourth that for the three-phase motor. The condenser kilovolt-amperes would be the same for either motor.

The following formulas apply to the unbalanced two-phase type *D* connection.

$$Z_1=-j\frac{E\sin\phi}{I\cos\phi\cos\theta}$$

$$E_1I_1=\frac{EI\cos\theta}{\sin\phi\cos\phi}$$

$$Z_2=-j\frac{E\sin^2\phi}{I\cos\phi\sin(\theta-\phi)}$$

$$E_2I_2=\frac{EI\sin(\theta-\phi)}{\cos\phi}$$

In these equations I is the phase B current.

Figure 14 shows the vector diagram for the unsymmetrical type *E* connection. This modification of the type *E* connection corresponds to a motor in which the two phases have been wound for different voltages, or to one in which the stator has been uniformly wound but the neutral is not exactly the midpoint of the winding.

A single condenser may be used for balancing this type when $\cos\theta=\sin\phi$. As the voltage ratio of the two windings is changed the capacitance required for compensation is also changed. The minimum total capacitance is required for the symmetrical condition where the two phases are identical. This condition does not result in the minimum total kilovolt-amperes which occurs for smaller values of ϕ. When ϕ is less than (90 degrees $-\theta$), Z_2 is inductive. It is questionable if operation in this region is desirable so that minimum kilovolt-ampere condenser rating would, for practical purposes, occur where $\cos\theta=\sin\phi$. For single-phase running without auxiliary impedances the current rating of the higher voltage phase would have to be used in determining the volt-ampere rating of the motor.

The equations covering the unsymmetrical type *E* connection are given below.

$$Z_1=-j\frac{E\sin^2\phi\cos\phi}{I\sin(\phi+\theta)}$$

$$E_1I_1=\frac{EI\sin(\phi+\theta)}{\cos\phi}$$

$$Z_2=+j\frac{E\sin\phi\cos^2\phi}{I\cos(\phi+\theta)}$$

$$E_2I_2=\frac{EI\cos(\phi+\theta)}{\sin\phi}$$

Where $I=I_A\cos\phi=I_B\sin\phi$, and E is the voltage of the single-phase line. Since I_A and I_B both change as ϕ changes a general value of I was desired in the above equations. This particular value was chosen since line voltage times this current was equal to the volt-amperes of either phase.

Conclusions

The results of this analysis show that no appropriate values of impedance can be found for the full-speed running conditions which will give polyphase performance for more than a single point. This statement applies equally well to all the connections considered. It does not mean that the use of condensers in the running circuit cannot be justified. Considerable benefit may be derived from the use of condensers, but these condensers do not make the single-phase performance of the motor equivalent to the polyphase throughout the range of normal operation. A single condenser of proper size may be used to balance the motor for some definite load. This load corresponds to a motor power factor of 0.50 for the three-phase connections and of 0.707 for the symmetrical two-phase connections. For the unsymmetrical two-phase connections the power factor of compensation will depend upon the voltage ratio of the two phases. The ability to balance certain motors at the higher power factors, or higher loads, may be an advantage in some cases.

The best connection to use depends upon the relative weights given the factors involved. For low-resistance motors having starting power factors in the neighborhood of 0.50 the three-phase and the unsymmetrical two-phase type *D* connections require the lowest kilovolt-amperes of condenser capacitance during the starting period, and all this may be in one unit. These two-phase machines should have a voltage ratio of the starting to running winding of about 1.7 to 1. Where the motors have higher resistance, resulting in starting power factors of perhaps 0.60 and above, the symmetrical two-phase types or the unsymmetrical type *D* having a voltage ratio of about 1.25 to 1 are indicated as requiring the

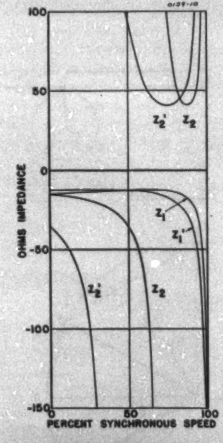

Figure 10. Impedance curves for two-phase motors with type D connection

All impedances are reactive

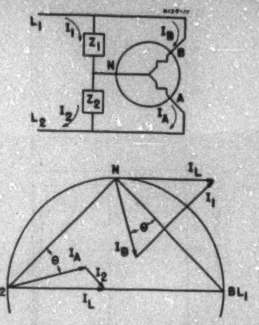

Figure 11. Circuit and vector diagrams for two-phase motors with type E connection

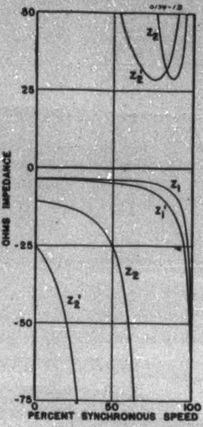

Figure 12. Impedance curves for two-phase motors with type E connection

All impedances are reactive

minimum kilovolt-amperes of capacitance. Where the deciding factor is the capacity of the condenser the connections which place the highest voltages on the condensers must be used, and the unsymmetrical type *D* is the obvious choice unless boosting transformers are to be used. If the deciding factor were the full-speed volt-ampere rating of the motor then the two-phase type *E* would be indicated.

The kilovolt-amperes of condenser capacitance required at starting by the high-resistance motors is not greatly different from that required by the low-resistance motors, the connection requiring minimum kilovolt-amperes being assumed used in each case. This is true in spite of the reduction in starting current caused by the increased rotor resistance. Thus increasing the rotor resistance will have little effect upon the condenser kilovolt-amperes required for complete balancing. The starting torques of these higher resistance motors are higher, so that the starting torque per kilovolt-ampere of condenser will be higher in these motors.

While a small matter in most instances, simplicity in connections for reversing service might be desired. When a single impedance can be used in the three-phase type *B* and the symmetrical type *E* the only change necessary for reversing is the changing of the line side of the condenser from one line to the other.

In speculating upon the indications which this analysis has yielded it has seemed that an excellent motor would be one having a high-power factor and low current at starting. This current and power factor should remain relatively constant during the starting period. For full-speed running the resistance of the rotor circuit should be low and a large part of the stator winding should be active even though running condensers were not used. This combination may be best approximated by a properly designed double-squirrel cage machine. Where the motors are too small to make the double-cage construction economical, deep narrow slots should be of benefit. With the high-starting power factor the type *E* connection could be used with a single impedance at starting. The type *E* connection would also permit the use of the entire stator winding in normal running. The deeply imbedded rotor winding would supply the low-resistance circuit for carrying the main field into the quadrature position. The use of a boosting transformer on the condenser would probably be desired if the motor were wound for low voltage.

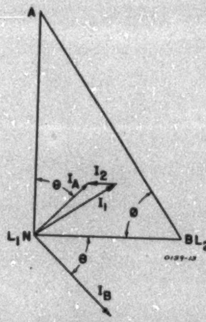

Figure 13. Vector diagram for the unsymmetrical type D connection

This paper has been prepared with the thought in mind that complete balance of the motors was desired. Many applications would not require the high torques which this work has supposed to be needed. Reduced capacitance in the starting circuit will result in reduced starting torque, and this reduction in the capacitance may be demanded by the economics of the case. An item effecting the balance which it is possible to obtain is the saturation of the iron parts of the magnetic circuit. With the higher saturations found in some machines the inductances of the windings are not constant, and the magnitudes and the phase angles assumed for the currents in this work may not be attainable. If this is the case some modifications of the calculated impedances may be desired.

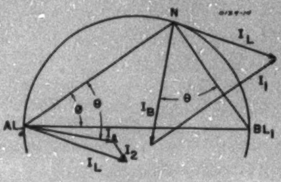

Figure 14. Vector diagram for the unsymmetrical type E connection

Bibliography

1. Notes on Single-Phase Induction Motors, C. P. Steinmetz. AIEE Transactions, volume 17, January 1900, pages 25–61.

2. The Fundamental Theory of the Capacitor Motor, H. C. Specht. AIEE Transactions, volume 48, April 1929, pages 607–13.

3. The Revolving-Field Theory of the Capacitor Motor, W. J. Morrill. AIEE Transactions, volume 48, April 1929, pages 614–29.

4. The Design of Capacitor Motors for Best Starting Performance, B. F. Bailey. *Engineering Research Bulletin* No. 19, Michigan Univ., 1931.

5. Performance Calculations on Induction Motors, C. G. Veinott. AIEE Transactions, volume 51, September 1932, pages 743–54.

6. Segregation of Losses in Single-Phase Induction Motors, C. G. Veinott. AIEE Transactions, volume 54, 1935, pages 1302–06.

7. Capacitor Motor With Double Cage Rotor, A. F. Puchstein. AIEE Transactions, volume 54, 1935, page 1018.

8. Split Phase Starting of Three-Phase Motors, G. F. Tracy and W. E. Wyss. AIEE Transactions, volume 54, 1935, pages 1068–72.

9. Analysis of Unsymmetrical Machines, W. V. Lyon and Chas. Kingsley, Jr. AIEE Transactions, volume 55, 1936, pages 471–6.

10. Induction Motors on Unbalanced Voltages, H. R. Reed and R. J. W. Koopman. AIEE Transactions, volume 55, 1936, pages 1206–13.

11. Induction Motors, Part I, W. J. Morrill. AIEE Transactions, volume 56, 1937, pages 333–8.

12. Single-Phase Induction-Motor Performance, A. F. Puchstein and T. C. Lloyd. AIEE Transactions, volume 56, 1937, pages 1277–84.

Design Factors Involved in the Design of Domestic Motored Appliances

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Synopsis: In all domestic motored appliances, there are certain general design factors to which more detailed design factors are related, depending upon the appliance involved. In this paper the various design factors are enumerated and explained in a way which it is hoped will be of assistance to the many thousands of engineers engaged in the design of motored appliances. Curves show typical speed-torque characteristics of motors used to drive various types of appliances. Tables show the types of motors used with different types of appliances and the principal factors causing noise. General design factors involved in motored appliances are discussed as a group.

Introduction

THE designer of domestic motored appliances is today accepting as his limitations the real values of function, utility, and cost. With these basic elements to start with, motored appliances are now approaching and in many instances, have surpassed all expectations of those who felt the need of a product that functioned both in the home and salesroom.

The consumer is the judge at all times. When he purchases a product and uses it in his home, there must evolve out of this ownership a feeling of satisfaction as to the usefulness of the product. He has every right to expect the utmost in the product. Domestic motored appliances of today are noteworthy examples of what can be done when every consideration has been given to the demands of the consumer. These appliances have higher efficiencies, better quality and appearance. They are more compact, more quiet, and lower in cost.

It is the designer's duty to analyze the factors involved in domestic motored appliances and from this analysis produce the finished product. The author attempts to show in this paper the design factors involved and their application to domestic motored appliances. However, the electrical equipment will be given greater prominence because it is common to all motored appliances, whereas a design treatise on each type of motored appliance would require an individual paper.

Domestic motored appliances cover a wide range of uses, types, and sizes. There are various classifications, such as large and small (or counter) appliances, portable and stationary, or plug-in and permanently wired appliances. However, the author is classifying them according to their use in the various parts of the home as this is often the classification used for marketing purposes.

Kitchen

Refrigerator
Food preparing machines
Dishwasher
Garbage disposal machines

Laundry

Washing machines
Ironing machines

Floor cleaning and finishing

Vacuum cleaners
Floor polishers

Heating

Oil burners
Coal stokers
Unit heaters
Portable room heaters

Air conditioning

Air-conditioning units
Fans
Humidifiers

Bathroom

Electric razors
Hair dryers

Sewing room

Sewing machines

Lawns and garden

Lawn mower
Hedge trimmer

Entertainment

Phonographs
Motion picture projectors

Home work shop and garage

Portable electric drills
Portable electric saws
Small drill presses
Small lathes
Small grinders
Small polishing equipment

Farm house and rural dwellings

Pumps
Churns
Feed cutters

Before starting on a particular design it is necessary for the designers to become familiar with the following subjects and keep them in mind during the design stages.

1. Public demand.
2. Price at which it must be sold.
3. Manufacturing facilities.
4. Marketing.

The following general factors must then be analyzed by the design engineers, and a complete study made of each one.

A. The use for which the appliance is intended.

B. Operation.

C. Frequency and length of time used.

D. Abuses to which appliance is subjected.

E. Quietness.

A. *Use*. The use for which an appliance is intended is the important thing, and requires a thorough study of it. Efforts should be directed toward designing the appliance for a particular use only and not try to make it a jack-of-all-trades. If a design tries to cover too broad a field it soon loses its identity and the efficiency and utility are lost to the use for which it was originally intended.

B. *Operation*. The principles of every phase of operation and the load should be firmly established by thorough study and research before an appliance is ready for the final design stages. Further studies and tests of samples of the final design should then be made under actual operating conditions to determine whether or not changes are necessary prior to the production of the final product.

C. *Frequency and length of time used*. This establishes the temperature rating and very often the type of motor, structural details, and materials to be used.

Some appliances may operate for only a few minutes a day, whereas others may operate for a total of six or eight hours every day of the week. Assume, for instance, a total of 5,000 hours' life for an appliance. A vacuum cleaner is used for an average of about one hour per week. Five thousand hours of operation would represent about 50 years in service. Some machines are so built that they would give that service if given proper attention by the user.

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[1] For all numbered references, see list at end of paper.

For refrigerators and air-conditioning units, for instance, 5,000 hours of operation represents about two years or less service life.

The heating season, in New England and in some other sections, is from October to late in April or May. In many cases the burner may run throughout the summer for heating water. The running time of the burner may be 2,000 hours for the season. On the basis of 5,000 hours' life, the burner would have a service life of only about $2^{1}/_{2}$ years.

The service life for which an appliance is to be designed has never been definitely established because of the many factors involved. The initial investment in a given appliance may mean a great deal of money to some people; therefore, it is necessary that they keep the appliance in service as long as possible. They are willing to forego the advantages that may be obtained in later models. To other people the initial cost does not represent a large investment. They are more anxious in many cases to replace their appliance more frequently to gain the advantages of later models. Therefore, the designer must choose the design and materials that will satisfy the application and give a sufficiently long service life to justify the initial cost to the consumer. The average consumer may then be better able to replace the old model with a new one having the many advantages gained by aggressive research and constant improvements in design and manufacturing methods.

D. Abuses. The abuses to which an appliance may be subjected should be analyzed, as often the success of the appliance is jeopardized because not enough thought was given to them. Considerable impetus has been given to the market for motored appliances by the gradual standardization of voltages and frequencies. However, considerable abuses are experienced by trying to use an appliance on voltages or frequencies, or both, outside the limits for which it was designed. This results in overheating on overvoltage or low frequencies and decrease in torque on low voltages.

Overloading an appliance beyond its capacity is a common fault. Therefore, proper protection should be provided. Failure to lubricate an appliance that requires replenishing the lubrication periodically is another source of trouble. To prevent this trouble a lubrication system should be provided that does not require any additional lubricant or one in which it is a simple matter to add more lubricant. However, the addition of too much oil or grease will be another source of trouble. For instance, in a food mixer excess oil will get into the food. In a sewing machine, the excess oil will get onto the cloth. There are many similar cases that could be mentioned.

Often the bag and rotary brush of a vacuum cleaner are not cleaned frequently enough and this results in lowering the efficiency of the machine. Neglect in defrosting a refrigerator or overloading it reduces its efficiency considerably.

Improper care of the appliance should be anticipated and corrective measures taken in the design to eliminate as many abuses as possible and put out intelligent instructions where the abuses cannot be met by design. There are also many abuses due to handling in service and in transportation which may be anticipated and at least partial protection provided.

E. Quietness. It would be desirable if all motored appliances were noiseless, but that is impossible with rotating apparatus. It is possible, however, to reduce the noise to a degree where it is acceptable to the average user. There are limitations depending upon the type of the apparatus, speeds, use, location of the use, mechanical and electrical limitations, and cost.

The housewife has learned to expect a vacuum cleaner, food mixer, or sewing machine to be noisy, but since she uses it infrequently she will endure a certain amount of noise although she would naturally prefer it to be quiet. However, any great amount of noise in a refrigerator, fan, air-conditioning unit, or oil burner will be annoying because it is running for a portion of the day every day of the year or season. Some of the noise is carried through the walls and floors to other parts of the house, or apartment. Infrequent noises or continuous noises produced by bumping, rattles, or other foreign noise causes apprehension in the mind of the user as to whether or not the appliance is operating properly.

There are high-frequency noises in the motor which may be more annoying to some people than to others. The 120-cycle noise for a 60-cycle motor cannot be eliminated but its intensity can be reduced. Many of the higher frequency noises caused by harmonics can be eliminated through proper design of the motor.

The noises prevalent in an appliance can be divided into four divisions:

1. Motor.
2. Intermediate drive, if any.
3. Functional parts.
4. Resonance in complete unit.

1. Most portable motored appliances are driven by high-speed Universal type a-c-d-c motors in order to reduce the weight of the motor. They are inherently noisy because of the high speed of the motor and driven elements that make up the appliance.

The appliances using constant speed motors do not operate at speeds above 3,600 rpm; in fact, most of them operate at about 1,700 rpm. Therefore, the noise due to the speed is less. There may be other noises, however, that may be just as annoying if one must listen to them all day.

2. The driving unit between the motor and the device, such as gear reductions, couplings, and belts, may be a source of a large per cent of the total noise if not properly designed and manufactured.

3. The noise of the functional parts of the appliance, such as fan blades, pumps, and mechanisms, form a part of the total noise. Any one of the three main sources of noise mentioned above may be responsible for the predominating and most annoying noise.

4. In some cases it has been found that the predominating noise was not due to any one of the above units independently, but the unit was operated at a speed where some frequency component was close to the natural period of some part or parts of the complete unit.

Principal Design Factors

The principal design factors that determine and make up the design of the electrical equipment for motored appliances will be explained in general. It might be well to mention here that the motored appliance design engineer must of necessity be well versed in human engineering, manufacturing, and costs, in addition to engineering fundamentals, as he is placed in contact with people in all walks of life in order to gather information and experience. Engineers will no doubt agree that the engineer who gathers the most valuable information so necessary in design work, gains it by tact and diplomacy whether he is dealing with an editor of a woman's magazine, housewives, the workman at the bench or machine, or cost clerk. The engineer must always keep in mind that only through the combined efforts of the many people with whom he comes in contact and his ability to accept constructive criticism will a design be successful.

Motor

The application determines the horsepower, temperature rating, speed, type, and size of motor. Fractional horsepower motors are used in most applications with few exceptions. They are produced by mass production at minimum costs, and include standardized ratings, sizes, and types so as to assure the manufacturer of motored appliances, etc., the proper motor at low prices.

It is therefore wise to first survey the standardized motors and if possible use them to get the benefit of low costs. If using them sacrifices some mechanical feature a slight modification of a stand-

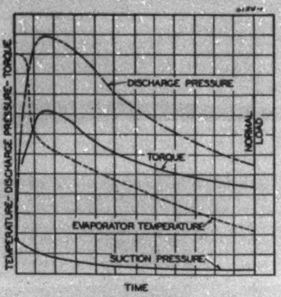

Figure 1. Typical load cycle of a refrigerator at pull down

ard motor may solve the problem. If the use of either of the above motors means a sacrifice in the design of the appliance from a functional or appearance standpoint, then a "custom built" or "built in" motor may meet the requirements.

Rating—Performance

The normal load at which the motor will operate for the greatest length of the time generally establishes the horsepower, speed, and temperature rating.

The speed depends upon the type of load and appliance. For instance, all portable appliances such as food mixers, vacuum cleaners, etc., must be light in weight. The Universal a-c-d-c motor is very desirable for these appliances because the horsepower per pound weight is very high, depending upon the speed.

Other appliances use motor speeds from approximately 870 to 3,450 rpm.

The motor must have sufficient torque to start the load from rest and take it up to speed. It must also have sufficient pull-out torque to prevent the motor from stalling due to a reasonable overload at minimum voltage conditions.

The efficiency should be as high as possible. The limiting factors are the type of motor, first cost, and economy of operation.

The starting current must not exceed the maximum values established by NEMA, National Electric Code, and power companies.

The voltage and frequency range over which the motor must operate has considerable bearing upon the design. For domestic service the voltage will range to a great extent from 115 to 120 with about five per cent of the total house voltages below 115 and above 120. This also includes odd frequencies from direct current through 50 cycles. In countries outside the United States, voltages may be encountered ranging from 90 to 250 in combination with frequencies ranging from direct current to 60 cycles.

The designer should not try to cover too wide a range of voltages and frequencies as it may lead to making sacrifices in the appliance and increase the cost. He should also keep in mind that the torque of the motor decreases with a decrease in voltage from that for which the motor is designed and the temperature of the motor winding will be increased with an increase in voltage.

The following examples of performance are given to illustrate the duty cycle of a few appliances such as refrigerators and air-conditioning units, washing machines, and vacuum cleaners. More detailed explanations are given for refrigerators and air-conditioning units than the others because there is less generally known about them.

Refrigerator and Air-Conditioning Unit Performance

In general the two types of appliances are similar in performance and in some cases the compressor units are similar. Although the method of loading is different, the same type of "pull down" load cycle applies to both units but to a different degree as the temperature gradient for an air-conditioning unit will be less than for a refrigerator.

There are different types of compressors. In most domestic refrigerators a single-phase motor is built into and hermetically sealed with the compressor. In some cases this method of construction also applies to air-conditioning units. For this type, a relay is mounted outside of the compressor unit to drop out or pick up the starting winding as required. There are various types of relays used for this purpose, classified as either thermal or magnetic types. There are variations also in each type, all of which perform their duty in an acceptable manner. For the performance explained below, the magnetic type is assumed. The coil of the relay is in series with the main phase or running winding and the contacts are in series with the starting winding.

Since the type of "pull down" load cycle is the same for both types of units, the refrigerator performance is assumed here. Assume a refrigerator taken out of stock and set up in a warm kitchen. The evaporator of the unit is at the same temperature as the kitchen. The box is filled with food and the door is closed. The circuit is closed and the motor starts the compressor. Figure 1 shows the performance of the unit from where it starts at a stabilized condition and passes through a load cycle until the evaporator reaches a predetermined temperature. At the instant of starting the suction and discharge pressures are the same.

It is to be noted in this case that the load reaches a maximum in about eight per cent of the time that it takes to reach the normal load point and is at the point of greatest differential between the suction and discharge pressures. The average running load may be considered the normal load. However if the box is overloaded the suction pressure will increase, thus increasing the load.

Figure 2 shows the performance of the motor from start throughout the "pull down" cycle. Starting at point *A* the motor pulls up to point *B* on the curve, at which time the relay opens the contacts and drops out the starting winding. The motor then continues from point *C* to run on the main winding. On the "pull down" the speed increases from *C* up to *D*, then back to *E* at maximum load. The load then decreases from there to normal load causing the motor speed to increase from *E* to normal load point *F*. If the load at any time increases too much during its operation, the speed may drop down to point *C*, where the relay closes the contacts, cuts in the starting winding at *B* and tries to get up to speed again. If the motor goes through several of these cycles the motor winding may heat excessively and cause the winding protective device to break the circuit. Possibly by the time the protective device closes again the compressor has equalized or if an unloader is used, the motor will take the compressor up to speed. Often the load may be at a point around *D* or any other load considered as an overload, for a sufficient time for the protective device to break the circuit to protect the

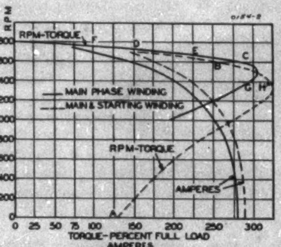

Figure 2. Speed-torque-ampere characteristics of a split-phase-start single-phase induction run motor for refrigerator

winding. Overloading the box, increased ambient temperature or both, will cause an overload on the motor and increase its running time.

The normal horsepower rating will be at the normal load established by the temperature that the evaporator in the cabinet is to maintain, and is an average load. For instance, the control may be so set that when the compressor is running it will bring the evaporator temperature below normal before it cuts out. The temperature will slowly increase until it may be an equal amount above normal at which time the control closes the circuit and the compressor starts. The compressor will continue to cycle over this range or possibly a greater range, depending upon the load or room temperature. The motor should have its maximum efficiency at normal load conditions and if possible over a range near normal load.

Because there are overload conditions that must be considered, it is necessary that the winding does not exceed a safe temperature, for the type of insulation used, over the period of time that the unit may be called upon to operate within the range of normal overloads and extreme cases of operating at normal loads.

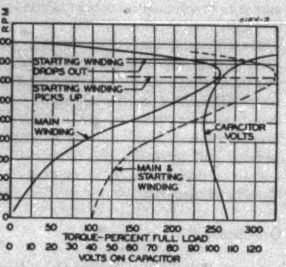

Figure 3. Speed-torque characteristics of a capacitor-start single-phase induction run motor for a refrigerator or air-conditioning unit

Washing-Machine Performance

In a washing machine the tub is filled to the proper level with hot water, soap is added and then the clothes are put in. The motor is then started. The time of operation may be from 10 to 30 minutes depending upon the weight of clothes. The dirty water is removed. Sometimes a pump is attached which is operated by the same motor to pump the water to a drain. Cold water is put into the tub for rinsing the clothes. This water is drained off and clean water added to the tub. Bluing is added to the water and the clothes rinsed or left standing in it for a while and then rinsed. Then the clothes are put through the wringer or spinner, driven by the washing machine motor, to remove the excess water. This operation may take 10 or 15 minutes.

This application has an intermittent load cycle of short duration. It has overloads but for short periods, such as trying to put too many clothes in, or putting too much through the wringer. However, experience has shown that a "short time" rating is satisfactory.

Vacuum-Cleaner Performance

Vacuum cleaners operate over a range of load and with appreciable speed variation. Take, for instance, a motor-driven brush type vacuum cleaner. The brush will take a small part of the total load but the air drawn through the nozzle and fan case will require most of the output of the motor. If the height of the brush is not properly adjusted the load will increase somewhat, but the greatest variation in load is caused by the height of the nozzle above the floor, and whether the cleaner is moving over a bare floor or on different kinds and thicknesses of rugs. The total cleaning time may be less than 30 minutes for any day. Therefore, the motor does not run long enough to get up to maximum temperature. A short-time or intermittent rating should be satisfactory but from a fire hazard standpoint, the Underwriters' Laboratories feel that continuous operation should be expected with the dust bag in place, until the temperature becomes constant with an average load. There are no protective devices on vacuum cleaners; therefore, if the housewife should leave the motor running the above condition would be obtained. It usually takes approximately one hour for the temperature to become constant.

Temperatures are discussed in these examples to illustrate the importance of the proper rating and maximum temperatures on specific appliances. Temperatures are discussed more generally under that subject.

Temperature

The temperature rating is normally set at the normal load and based on the maximum operating time. The motor should also withstand temperatures due to reasonable overloads encountered in the operation of the appliance and at the maximum voltage of the guaranteed voltage range and frequency range. While it may be desirable from a selling and stock standpoint it may be that it is not economical from a cost standpoint.

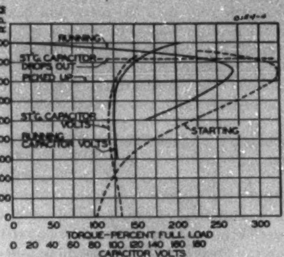

Figure 4. Speed-torque characteristics of a capacitor-start-and-run motor for a refrigerator or air-conditioning unit

There are various limiting hot-spot temperatures established for various classes of insulations by AIEE. Therefore, it is possible to set a rating based on operating time and load. It may be for continuous, short-time or intermittent service. Ambient temperatures to be encountered have considerable bearing upon the temperature rise. The method of measuring temperatures such as by thermometer, resistance and/or thermocouple must be considered in establishing the rating. Proper consideration of these factors often results in considerable reduction in the cost of the motor and appliance, although a smaller motor may be more noisy magnetically.

Consideration must be given to the effect of temperature upon the mechanical parts of the motor, control and appliance. Where there is a combination of copper and steel parts the copper should have only two-thirds the temperature of the steel parts to have the linear expansion of the two metals equal, because the linear expansion of steel is only two-thirds that of copper. In commutator type motors, neglect in considering temperature may result in uneven expansions and warping of the commutator, and breaking of leads to the commutator, high bars and poor commutation.

Clearances between the shaft and sleeve bearings may be affected so as to cause loss of oil film, excess friction or binding. The clearances in ball bearings may be reduced and cause excess friction and noise, and even binding. End play may be reduced to a dangerous point. Temperatures should be considered relative to the stator, rotor, shaft fits and balance. The stator may become distorted at the teeth, and expansion of the rotor diameter may cause the air gap to be reduced so that it causes an unbalanced magnetic pull upon the rotor, and bend the shaft, thus causing the rotor to rub or cause a slip-frequency noise. The rotor may even drag on the stator and further increase the temperature, reduce the torque at the shaft and increase the watts. Reduction of the air gap may also increase the magnetic noise and affect the torque.

High temperatures may also cause excess friction between cylinders and pistons of compressors, also in gears of various appliances causing excess friction and noise. Press fits of parts should be suitable where high temperatures are involved as standard press fits will probably not be sufficient.

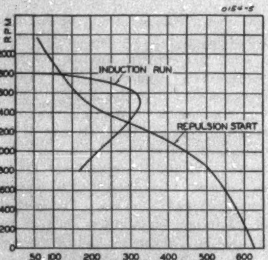

Figure 5. Speed-torque characteristics of a repulsion-start induction-run motor for a refrigerator or air-conditioning unit

Type of Motor

The type of motor to be used depends upon the characteristics required. It is advantageous to use standard motors where possible. For instance, standard $^1/_6$- or $^1/_4$-horsepower, 60-cycle, 1,725-rpm split phase start induction run single-phase high torque short-hour motors are made in large quantities by many motor manufacturers for washing machine drive, and offer many advantages in cost, delivery and service. However, it should be kept in mind that these motors are designed for a specific purpose where there is infrequent starting and a limited time of operation. One of the most serious problems in the application of split-phase high-torque motors is the starting current, which may not in some cases meet the requirements of the power companies. However, due to the infrequent number of starts in washing machine service, they are generally accepted. Other disadvantages of these motors for general application are the greater tendency toward noise and lower efficiency and power factor.

Fractional-horsepower motors are also made in standard ratings and types, open, semienclosed, enclosed, solid or resilient mounting, sleeve bearing and ball bearing; therefore, the appliance manufacturer has a large and varied field from which to choose a satisfactory motor.

Very often "custom built" or "built in" motors are required. The user will gain much if he will use as many of the motor manufacturer's standard parts as possible as new tools are expensive and special parts made in small quantities outside the mass production lines cost more than standardized parts.

The curves showing the different motor characteristics represent motors for specific applications and starting torques, pull-in or pull-out torques are out of line in some cases with standardized NEMA ratings.

For applications requiring high starting and pull-in torques at low starting current, the capacitor-start single-phase induction run motor may be used. Electrolytic capacitors are usually used because the operating time is short and cost of them is low. It is desirable to keep the voltage across the capacitor as low as possible. This type of motor also has fairly high efficiency. Figure 3 shows the characteristics of a capacitor-start motor for a compressor of same size and type as figure 2.

Figure 4 shows the characteristics of a capacitor-start-and-run motor for same application as figure 2. This type of motor is used where higher efficiencies and power factors are desirable.

Figure 5 is a repulsion-start induction-run motor. This type is desirable where unusually high starting torques for a given current are required.

Figure 6 shows a shade-pole type, commonly used for driving fans or other applications requiring low starting and pull-out torques and low starting current.

Figure 7 shows the characteristics of a Universal-type motor used on high-speed applications where weight is a factor.

Insulation

The insulation in most motors for appliances is of the class *A* type. Special insulation is used in hermetically sealed refrigerator and air-conditioning compressor motors. The life of the motor depends upon several factors, one of which is the insulation. The insulation life is also dependent upon several factors, such as temperature, expansion and contraction during load and off periods, electrical stresses, vibration, mechanical stresses, exposure to air subjecting it to oxidation, and moisture.

To withstand the various strains and deterioration due to load and temperature a thorough study must be given to the possible effect of each factor and safeguard against harm to the insulation. Proper impregnation treatments of the winding will minimize the deterioration due to oxidation caused by exposure to the air and moisture.

Quietness

In order to eliminate noise or to reduce it, an analysis must be made as to the

Table I

	Types of Motors					
Appliance	Split Phase	Capacitor Start	Capacitor Start and Run	Repulsion-Start Induction Run	Shade Pole	Universal
Domestic refrigerators	x	x	x	x	–	–
Air conditioning	x	x	x	x	–	–
Pumps	x	x	x	x	–	–
Compressors	x	x	x	x	–	–
Unit heaters	x	–	x	–	x	–
Fans (circulators)	x	x	x	–	–	–
Fans (exhaust, desk, pedestal)	–	–	x	–	x	–
Food mixers	–	–	–	–	–	x
Vacuum cleaners	–	–	–	–	–	x
Sewing machines	–	–	–	–	–	x
Portable tools	–	–	–	–	–	x
Home work shop machinery	x	–	–	–	–	–
Grinders and polishers	x	–	–	–	–	–
Bathroom heaters	–	–	–	–	x	x
Hair dryers	–	–	–	–	x	x
Electric razors	–	–	–	–	–	x
Humidifiers	x	–	–	–	x	–
Dishwashers	x	–	–	–	–	–
Washing machines	x	–	–	–	–	–
Floor polishers	x	–	–	–	–	x
Lawn mowers	–	–	–	–	–	x
Hedge trimmers	–	–	–	–	–	x
Coal stokers	–	x	–	–	–	–
Oil burners	x	x	–	–	–	–
Attic fans (belt drive)	x	–	–	–	–	–
Furnace blowers	x	–	–	–	x	–

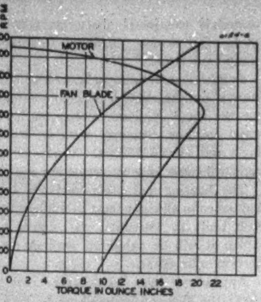

Figure 6. Speed-torque characteristics of a shade-pole motor for a desk fan

cause of the noise. Table II shows from experience the principal sources of noise for the various types of appliances. The application of a noise analyzer will show the frequency and amplitude of the noise of each component part. By examining table II showing the principal sources of noise in various appliances it is noticed that the motor and unbalance are common to all types of motored appliances. This does not mean that they are responsible for all of the noise. In the case of vacuum cleaners, if the motor noise was entirely eliminated the total noise would not be reduced very much. First analyze the noises produced by the motor and unbalanced conditions in a vacuum cleaner and other appliances.

The Universal concentrated pole-type a-c-d-c motor is used for many portable appliances such as vacuum cleaners, food mixers, sewing machines, and portable tools. The speeds vary from 3,000 rpm for sewing machine motors, approximately 10,000 rpm for vacuum cleaners and food mixers, up to about 20,000 rpm for portable tools. The rotational frequencies cause a great deal of the noise. This noise can be kept to a minimum by the choice of the proper number of slots in the armature, and design of the pole face. The surface speed of the commutator is often as high as 3,000 feet per minute. For the best commutation and brush life the commutator mica bars should be as thin as practical and should be undercut. With the commutator running at this speed with undercut mica slots under the brushes it is absolutely necessary that the commutator surface be machined as nearly true as possible with respect to the bearings; otherwise, while some noise will still be present, a poor or untrue surface will result in excessive noise as well as poor commutation.

The grade of material in the motor brushes also reflects upon the noise; therefore, the choice of brushes is important. The clearance around the brushes in the brush holder is important and should be such that chattering of the brushes will not result, but not too close as the brushes may not move freely.

These high-speed armatures should be dynamically balanced to keep noise to a minimum, which increases the life of the bearings.

The noise in single-phase motors may be attributed to several factors all of which must be anticipated and corrected. Inferior shaft and sleeve bearing surfaces, excessive clearances, improper lubrication, dirt and excessive ball-bearing fits, and bump and unbalance will cause noise.

Slip frequency noise is caused by an uneven air gap, which may be caused by improper assembly, bending of the shaft due to unbalance, or excessive bearing clearance.

Another type of noise is caused by magnetic forces in the air gap setting up vibration in the motor frame or stator yoke. The cause may be due to improper choice of rotor slots, and to a weakness in the frame or stator yoke.

Ventilating fans in the motor may cause a siren effect due to close clearance between the end of blade and ventilating holes or noise may result from a high velocity of the air passing through the ventilating holes.

All materials should be analyzed from a noise producing and carrying standpoint.

The appliance to which the motor is mounted may act as a sounding board and magnify the noise. A resilient type of mounting is then desirable. This will prevent transmission of the noise through the appliance and also prevent the motor noise from being transmitted through the floors and walls of the house.

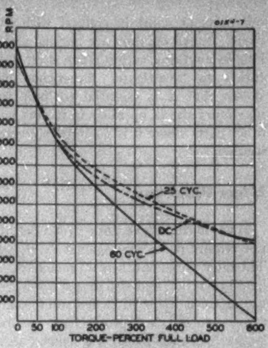

Figure 7. Speed-torque characteristics of a high-speed universal-type motor

Safety

The motor and appliance must be so protected that the user cannot receive injuries from either the electrical or mechanical parts. This means covering electrical circuits and contacts, properly strain-relieving cords, provide proper insulation and sufficient clearance between the frame and live parts, as well as safety releases, such as now provided on motor-driven clothes wringers. Motors open to view and accessible should have the ventilating holes or other openings in the motor of such size that a small child cannot stick its finger into the hole and have it injured by the ventilating fan or mov-

Table II. Examples of Principal Sources of Noise

	Motor and Unbalance	Gears Belt Coupling	Pump	Fan Blades Blowers	Fan Case and Nozzle	Agitator	Flame	Radio Interference	Air
Clothes washers	x	x	–	–	–	x	–	–	–
Dish washers	x	x	x	–	–	x	–	–	–
Fans	x	–	–	x	–	–	–	–	x
Food mixers	x	x	–	–	–	–	–	x	–
Ironers	x	x	–	–	–	–	–	–	–
House pumps	x	x	x	–	–	–	–	–	–
Air conditioning	x	x	x	x	–	–	–	–	x
Refrigerators	x	x	x	x	–	–	–	–	–
Unit heaters	x	–	–	x	–	–	–	–	x
Humidifiers	x	–	–	x	–	–	–	–	x
Floor polishers	x	x	–	–	–	–	–	x	–
Coal stokers	x	x	–	–	–	–	–	–	–
Oil burners	x	x	x	x	–	–	x	x	–
Hair dryers	x	–	–	–	–	–	–	x	x
Room heaters	x	–	–	–	–	–	–	x	–
Vacuum cleaners	x	–	–	x	x	–	–	x	x
Portable tools	x	x	–	–	–	–	–	–	–
Home work shop machinery	x	x	–	–	–	–	–	–	–
Grinders and polishers	x	–	–	–	–	–	–	–	–
Electric razors	x	–	–	–	–	–	–	x	–
Sewing machines	x	Mechanism	–	–	–	–	–	x	–

ing parts or come in contact with live parts. Small children are curious and a hole in anything within reach is an invitation to investigate. Experience has shown that it is essential to limit the maximum size of openings in motors and other parts where it is possible to receive injuries.

Motor Protection

Some motors have duty cycles that do not require protection against overloads. The duty cycles of others do require overload protection and also protection where the motor is liable to locked rotor conditions. Adequate protection assures longer life to the motor and reduces service calls to a minimum. Where overloads are infrequent, overload protection will make it unnecessary to design the motor to withstand the temperature encountered at these overloads and will have considerable bearing upon the temperature rating. In most cases "automatic reset" protection devices are used. However, there are some applications and localities that require "manual reset" protective devices.

Appearance

Good appearance is essential to the popular acceptance of any appliance. It is therefore necessary that the motor and appliance design be so co-ordinated that it will be acceptable as a finished product and will blend into the surrounding of the kitchen or any other part of the home in which the appliance will be used.

About 1929, considerable impetus was given to industrial art, which was no doubt due to the interest, of such artists as Maxfield Parrish, in the streamlining of airplanes, high-speed trains, and automobiles. However, it took several years before some of these ideas really approached streamlining. It was found that this idea not only increased the efficiency by cutting down air friction but greatly improved the appearance. Industrial artists have applied art to motored and other appliances, and have contributed much to the improved appearance and popular acceptance of these appliances.

Such appliances as fans, food mixers, and vacuum cleaners are often given a streamlined appearance. This not only helps the air stream some, in the case of the complete fan, to have the motor streamlined, but streamlining the base as well gives the whole fan the appearance of smoothness in operation. There is no functional improvement to be gained in streamlining a vacuum cleaner or food mixer but it does help the appearance considerably. Incidentally, it helps cost in some cases by giving more draft to drawn or cast parts.

The design of modernized kitchens has caused a further improvement in appearance of refrigerators, washing machines, ironing machines and dish washers, so that they will blend into the kitchen as though they were built into it with the cupboards, electric range, etc. Food mixer finishes have been changed to line up with the finishes of the above appliances.

Air-conditioning equipment, unit heaters, fans, etc., are designed to blend with the woodwork and furnishings of most homes.

In many cases, such as in washing machines, dish washers, open-type refrigerators and air-conditioning units, the artist can design good appearing covers or aprons for the complete equipment, or the open-type unit is assembled within the cabinet of a refrigerator. This makes it possible to use standard type motors.

It is apparent that to obtain the maximum in utility and appearance, the design engineer and artist must co-ordinate their efforts so that the proper functioning of the appliance is not jeopardized. However, it may mean ingenious design on the part of the design engineer to preserve a very desirable art design.

Weight, Size, and Material

Weight, size and materials should be considered together. The performance expected and temperature, open or enclosed, determine the size and weight of the stator and rotor. General practice with standard motors is to use a steel frame and cast-iron brackets. If the application warrants the expense of new tools, the weight of these parts may be reduced by die casting them with zinc or aluminum. Magnesium is gaining in popularity but there are certain precautions necessary in machining. Care must be exercised in the decision to die-cast these parts as there may be a resulting increase in noise.

For some built-in motors, die castings may be adaptable with less possibilities of noise because of the additional material in the appliance. At the same time there is the possibility of creating a sounding board effect. Hermetically sealed compressor motors as a rule use cast-iron housing as that material appears to be best suited to the application.

In portable appliances it is desirable to use die castings of any one of the three materials mentioned above, or pressed parts using steel or aluminum, to reduce weight and costs.

Synthetic materials can be molded into frames and brackets or in combinations for the small appliance motors, such as hand vacuum cleaners, portable drills, fans and electric razors. These materials will cause some reduction in noise.

The housing and other parts of the appliance in all cases must be sufficiently well built to withstand the abuses to which it will be subjected without distortion or breakage.

Appearance may have some bearing upon the type of material to be used. For instance, a smooth surface is not obtainable with cast iron except by expensive polishing methods, whereas die casting and pressed metal parts are smooth and can be given a good finish by painting, plating or by polishing in the case of aluminum. Synthetic materials can be dyed; therefore, requiring no extra finish.

Cost

Before any appliance is marketable, the cost must be at a satisfactory figure and at the same time make a profit. Some appliances may be designed for cost at the expense of quality and reputation. More brains and proper manufacturing facilities can co-operate to produce low-cost high-quality appliances.

Serviceability

The functional service and life of the motor and appliance will depend upon the thought given to the design factors mentioned and the application of them. The expected life of the appliance will more or less establish the degree of quality in many items. However, some elements, if they are to function properly at all, will insure long life. All possible effort should be made to eliminate service troubles, but make servicing easy to do, if it is necessary.

Standards

Recognition and application of standards established by American Standards Association, AIEE, NEMA, Underwriters' Laboratories, trade organizations, power companies and the various recognized testing agencies is necessary to the successful and accepted design.

Conclusions

An outline of the principal design factors involved in domestic motored appli-

Single-Phase Motor Theory—a Correlation of the Cross-Field and Revolving-Field Concepts

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Introduction

THE revolving field theory of single phase induction motor operation resolves the total flux crossing the air gap into two components rotating in opposite directions but constant in magnitude. The cross field theory resolves the total flux into two components of varying magnitude but constant direction with reference to the stator.

The cross field and revolving field theories have been developed by H. R. West[1] and Wayne J. Morrill,[2] respectively, in a complete and analytical manner. From the standpoint of the concepts involved and their final utility, each of these presents certain difficulties. The objective of this paper is to utilize and coordinate both fundamental methods of analysis. A new treatment is used, however, which does not follow the detailed methods or type of development previously used for either theory.

While it is necessary to treat certain factors in an idealized manner, the effort is made to consider a single phase induction motor as it actually is, rather than to say arbitrarily that it is equivalent to some fictitious circuit or other mechanical arrangement as is done with some other theories.

The discussion herein will start with the general expression for voltage induced in a coil rotating in a single phase field (periodically reversing but not rotating).

A two pole motor will be considered throughout for simplicity.

A list of symbols used will be found at the end of the paper.

Rotor Voltage

In a single phase induction motor, there is in practice generally a rough approximation to sinusoidal distribution of flux around the air gap at any instant. In figure 1, it is assumed that flux density is uniform along the X–X axis. The coil has no magnetic core and is simply turning in the field shown. Along the circle traced by the ends of the coil, the flux distribution will then be sinusoidal at any instant.

The flux through the coil will be

$$\phi_c=\phi_M \sin \omega t \cos \theta \tag{1}$$

$$=\phi_M \sin \omega t \cos (p\omega t+\alpha) \tag{2}$$

$$=\phi_M \sin \omega t (\cos p\omega t \cos \alpha - \sin p\omega t \sin \alpha) \tag{3}$$

$$=\frac{\phi_M}{2}\{\cos \alpha[\sin (1+p)\omega t+\sin (1-p)\omega t]+ \sin \alpha[\cos(1+p)\omega t-\cos (1-p)\omega t]\} \tag{4}$$

$$=\frac{\phi_M}{2}\{\sin [\alpha+(1+p)\omega t]-\sin [\alpha-(1-p)\omega t]\} \tag{5}$$

$$=\frac{\phi_M}{2}\{\sin [(2-s)\omega t+\alpha]+\sin (s\omega t-\alpha)\} \tag{6}$$

Voltage in coil $= \dfrac{d\phi_c}{dt}$ (times a constant which is here neglected)

$$=\frac{\phi_M}{2}\{(2-s)\omega \cos [(2-s)\omega t+\alpha]+ s\omega \cos (s\omega t-\alpha)\} \quad (7$$

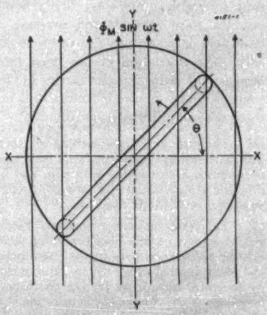

Figure 1. Coil rotating in periodically reversing unidirectional field

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[1] For all numbered references, see list at end of paper.

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ances has been given with the hope that it will help to organize the thinking of the design engineer and assist him in the procedure of a design.

It would be desirable to give a detailed treatise on the design of each appliance but the subject is of such importance that each appliance would require an individual paper to do it justice. Each factor that has been mentioned and discussed involves years of experiences, and has been the subject of many individual papers. It is well worth while for both the motor and appliance design engineer to review the papers presented at the AIEE winter convention in New York, January 1939, the summer convention in June 1940 at Swampscott, Mass., and others (1 to 10 inclusive, Bibliography). Also AIEE and Underwriters' Laboratories' Standards are of interest (11–12 Bibliography).

Bibliography

1. RATING OF GENERAL-PURPOSE INDUCTION MOTORS, **P. L. Alger** and **T. C. Johnson**. AIEE TRANSACTIONS, volume 58, 1939 (September section), pages 445–59.

2. RATING OF ELECTRICAL MACHINERY AND APPARATUS, **R. E. Hellmund**. AIEE TRANSACTIONS, volume 58, 1939 (October section), pages 499–503.

3. DUTY CYCLES AND MOTOR RATING, **L. E. Hildebrand**. AIEE TRANSACTIONS, volume 58, 1939 (September section), pages 478–83.

4. MEASUREMENT OF TEMPERATURE IN GENERAL-PURPOSE SQUIRREL-CAGE INDUCTION MOTORS, **C. P. Potter**. AIEE TRANSACTIONS, volume 58, 1939 (September section), pages 468–77.

5. RATING AND APPLICATION OF MOTORS FOR REFRIGERATION AND AIR CONDITIONING, **P. H. Rutherford**. AIEE TRANSACTIONS, volume 58, 1939 (October section), pages 519–27.

6. DETERMINATION OF TEMPERATURE RISE OF INDUCTION MOTORS, **E. R. Summers**. AIEE TRANSACTIONS, volume 58, 1939 (September section), pages 459–67.

7. HARMONIC THEORY OF NOISE IN INDUCTION MOTORS, **Wayne J. Morril**. AIEE TRANSACTIONS, volume 59, 1940 (August section), pages 474–80.

8. UNIVERSAL MOTORS, **L. C. Packer**. AIEE TRANSACTIONS, volume 44, 1925, pages 587–91.

9. A STUDY OF SHORT-TIME RATINGS AND THEIR APPLICATION TO INTERMITTENT DUTY CYCLES, **R. E. Hellmund** and **P. H. McAuley**. AIEE TRANSACTIONS, volume 59, 1940, pages 1050–5.

10. SOME PROBLEMS IN THE STANDARDIZATION OF TEMPERATURE RATING OF FRACTIONAL HORSEPOWER MOTORS, **C. G. Vienott**. AIEE TRANSACTIONS, volume 59, 1940, pages 1055–61.

11. GENERAL PRINCIPLES UPON WHICH TEMPERATURE LIMITS ARE BASED IN THE RATING OF ELECTRICAL MACHINERY AND APPARATUS. AIEE Standards No. 1, June 1940.

12. STANDARDS FOR MOTOR-OPERATED APPLIANCES, FANS, REFRIGERATION, AND AIR CONDITIONING, issued by Underwriters' Laboratories.

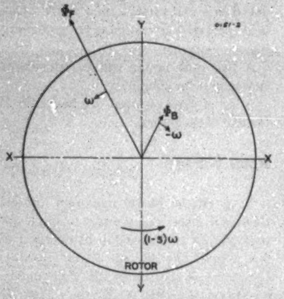

Figure 2. Axes X–X and Y–Y fixed with reference to stator

It is seen that this expression for voltage is precisely what we would have if the single phase field were replaced by two rotating fields of constant magnitude $\phi_M/2 = 1/2$ the maximum value of the single phase field, and each rotating in opposite directions with a velocity of ω radians per second. It can be seen by inspection that these two hypothetical flux vectors are together the vectorial equivalent of the single phase field.

It should be kept clearly in mind just what expression (7) represents. It is the voltage in any rotor turn or single conductor coil due solely to the primary flux (not including voltage due to flux which may exist in the axis in quadrature with the stator poles, and not including that due to leakage flux or resistance drop). In other words, this would be the voltage in a rotor turn consisting of two diametrically opposite rotor conductors, if this turn were open circuited at some point, and no current flowed in this turn nor in any other rotor conductors.

Now with the normal closed circuit rotor squirrel cage, and rotor current flowing, the rotor currents will produce a cross field or quadrature flux in the *X–X* axis. The evaluation of this cross flux becomes the really complex matter in the analysis. The following factors will be listed to show just why it is so involved:

(1). The rotor conductor exciting voltage: This is shown by equation 7. It has two frequencies, each varying with slip, and is different at any instant for each conductor occupying a different angle α.

(2). The rotor conductor current: There will be a rotor current component for each existing voltage component, the magnitude and phase angle of which will be determined by the rotor impedance for the particular frequency of the component in question.

(3). Angular velocity of rotor: The rotor is a moving object; and the angular position of a given conductor will be considerably different at the instant when one voltage component is maximum and the instant when the current due to that voltage is maximum. This fact is completely forgotten in the usual simple textbook "explanation" of the single phase induction motor.

(4). Effect of rotor current on total flux: The existence of the rotor current results in a complex magnetomotive force producing flux in the cross field axis. This flux may be regarded as unbalancing the relative values of the forward and backward rotating field components, tending toward the production of a pure constant forward rotating field as will be discussed in detail later.

It is then evident that we have a mechanism involving a number of interdependent variables; and if an attempt is made to be scientifically precise and to neglect no detail affecting motor performance, and to proceed to set forth expressions for complete rotor currents and voltages, all flux values, and final characteristics such as torque at any given slip in terms of motor constants, the task becomes extremely formidable even to those familiar with motor theory in general.

Referring again to expression 7, it is evident that the first component of voltage may be regarded as that due to the backward rotating flux vector (of the revolving field theory), and the other (slip frequency) component is due to the forward rotating field. Only under open circuit rotor conditions or at standstill, in general, are these two flux components equal ($\phi_M/2$). Attention is here called to figure 2.

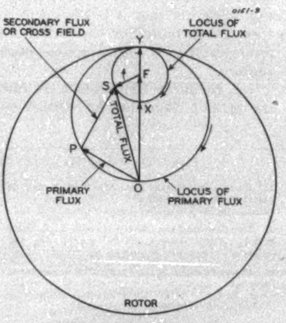

Figure 3. Condition at synchronous speed. Axes fixed with reference to rotor

$$OY=\phi_M=\phi_Y \text{ max}$$
$$OP=\phi_Y$$
$$OX=\phi_{X'}=\phi_X \text{ max}$$
$$PS=\phi_X$$
$$OF=\phi_F$$
$$FS=\phi_B$$
$$\overrightarrow{OS}=\overrightarrow{OF}+\overrightarrow{FS}=\overrightarrow{OP}+\overrightarrow{PS}$$

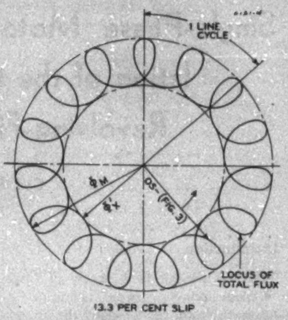

Figure 4. Idealized flux locus of single-phase induction motor under load. Axes fixed with reference to rotor

There are then actually two voltage components in the conductors of the practical rotor, as follows:

$$e_F=\phi_F s\omega \cos (s\omega t-\alpha) \text{ (constant omitted)} \quad (8)$$

$$e_B=\phi_B(2-s)\omega \cos [(2-s)\omega t+\alpha] \quad (9)$$

Now at synchronous speed $s=0$ and $e_F=0$. It is therefore certain that the cross field which exists at synchronous speed is produced by the current resulting from e_B.

Magnetizing Current in Rotor Producing Cross Field

The rotor current due to e_B will be e_B/Z_B, and lagging behind the voltage e_B by an angle whose cosine is R/Z_B. The in-phase component of this current is

$$\frac{\phi_B R}{Z_B^2}(2-s)\omega \cos [(2-s)\omega t+\alpha] \quad (10)$$

The "wattless" component of the current due to e_B will be

$$\frac{\phi_B X_B}{Z_B^2}(2-s)\omega \sin [(2-s)\omega t+\alpha] \quad (11)$$

The flux per square inch in the *X–X* axis is the product of the magnetomotive force and the permeance per square inch. Now if the permeance per square inch is constant, each increment of magnetomotive force multiplied by the area it encloses projected in a plane perpendicular to the *X–X* axis would produce a proportional increment of flux in that axis. For convenience we may then refer to the product of magnetomotive force and its enclosed area thus projected as a "magnetizing effect."

In developing the equations for rotor voltage and current a coil was considered;

this coil lying in a plane which rotated about a line within the plane and perpendicular to the main field flux. In a two pole motor, two rotor bars diametrically opposite with the connecting end rings may be considered as a single turn coil. If the total number of bars or conductors in the rotor is N, there will be a total of $N/2$ turns (or single turn coils) in the rotor. Since the angular conductor density is $N/2\pi$, if equation 11 is multiplied by $\sin\theta(N/2\pi)d\alpha$ and integrated from 0 to π, this will include the projected magnetomotive force of every turn in the rotor. The cross field magnetizing effect of the current component given by equation 11 is thus determined as follows:

$$\int_0^\pi \frac{\phi_B X_B}{Z_B^2}(2-s)\omega \sin[(2-s)\omega t+\alpha]\times \sin\theta\frac{N}{2\pi}d\alpha \qquad (12)$$

$$=\int_0^\pi \frac{\phi_B X_B N(2-s)\omega}{2\pi Z_B^2}\sin[(2-s)\omega t+\alpha]\times \sin[(1-s)\omega t+\alpha]d\alpha \qquad (13)$$

$$=\int_0^\pi \frac{\phi_B X_B N(2-s)\omega}{4\pi Z_B^2}\{\cos\omega t- \cos[(3-2s)\omega t+2\alpha]\}d\alpha \qquad (14)$$

For the integral, with α varying from 0 to π, the second factor or negative quantity in (14) becomes zero, and the magnetizing effect in the X–X axis due to the rotor current of equation 11 is

$$\frac{\phi_B X_B N(2-s)\omega}{4Z_B^2}\cos\omega t \qquad (15)$$

The corresponding integrated magnetizing effect due to the in-phase current component of expression 10 is

$$-\frac{\phi_B N\omega(2-s)R}{4Z_B^2}\sin\omega t \qquad (16)$$

The cross-magnetizing effect of the slip-frequency rotor current is similarly determined as follows, considering first the reactive component:

$$\int_0^\pi \frac{\phi_F X_F}{Z_F^2}s\omega\sin(s\omega t-\alpha)\sin[(1-s)\omega t+ \alpha]\frac{N}{2\pi}d\alpha \qquad (17)$$

$$=\int_0^\pi \frac{\phi_F N\omega s X_F}{4\pi Z_F^2}\{\cos[(2s-1)\omega t-2\alpha]- \cos\omega t\}d\alpha \qquad (18)$$

$$=-\frac{\phi_F N\omega s X_F}{4Z_F^2}\cos\omega t \qquad (19)$$

Similarly, the magnetizing effect of the slip-frequency rotor current in phase with the forward flux is

$$\frac{\phi_F N\omega s R}{4Z_F^2}\sin\omega t \qquad (20)$$

The total magnetizing effect or effective rotor magnetomotive force in the X–X axis is then

$$\phi_B(F_1'-jF_1)-\phi_F(F_2'-jF_2) \qquad (21)$$

where

$$F_1=\frac{N\omega(2-s)^2X}{4[R^2+(2-s)^2X^2]10^8}$$

$$F_1'=\frac{N\omega(2-s)R}{4[R^2+(2-s)^2X^2]10^8}$$

$$F_2=\frac{N\omega s^2X}{4(R^2+s^2X^2)10^8}$$

$$F_2'=\frac{N\omega sR}{4(R^2+s^2X^2)10^8}$$

(The signs in equation 21 should be changed throughout to be mathematically correct as may be observed. This may be done if it is borne in mind that the j factors represent magnetomotive force components in strict time quadrature with the main flux and really determine the relative direction of the cross field. However, equation 21 is written as given to emphasize that the rotor current due to ϕ_B is primarily responsible for positive cross field, and the rotor current due to ϕ_F subtracts from it. No damage is done in subsequent developments by these changes in sign.)

The voltage constant 10^{-8} has been introduced, and the substitutions of $R+j(2-s)X$ for Z_B, and $R+jsX$ for Z_F have been made. Also the time phase factors $\sin\omega t$ and $\cos\omega t$ are dropped, for the sake of brevity.

The revolving field concept visualizes two rotating flux vectors as shown by figure 2. They add in the Y–Y axis to become identical with the mutual flux ϕ_M; and they subtract in the X–X axis to determine the net cross field flux ϕ_X. The cross field by this assumption is maximum at the instant ϕ_Y is zero. The flux in the two axes are thus in both space and time quadrature. Unquestionably in the practical case there are irregularities or harmonics; and also undoubtedly the flux in the X–X axis is not precisely zero at the instant that perpendicular to it is maximum. In any theory however ideal concepts are stated in order that practical mathematics may apply in as simple a manner as possible. The practical problem is to so evaluate the ideal quantities that the calculated results on that basis are within usual limits of experimental accuracy.

The rotor may be regarded as an inductive cylinder which tends to maintain constant flux through itself. As it rotates the rotor flux is forced by the primary magnetomotive force to be maximum at the instant ϕ_Y is maximum. The inductive effect of the rotor prevents this flux from dying to zero when the primary flux is zero, except at standstill. The minimum rotor flux is in the cross-field axis. (If its magnitude were greater than ϕ_M, the backward flux would be negative and no net magnetomotive force would be available to produce positive cross flux.)

The condition at synchronous speed for the flux variation through the rotor is shown by figure 3. Vector FS is the backward rotating flux, and rotates at twice synchronous speed (with respect to the rotor). Vector OF is the forward rotating component, and of course at synchronous speed is fixed and constant with respect to the rotor. OX is the maximum value of cross field flux.

Figure 4 shows the total flux locus with respect to the rotor at a given slip. The locus of total flux traversing the rotor describes a hypotrochoidal curve with respect to it.

In the Y–Y axis the effective rotor ampere turns due to forward and backward rotating currents add instead of subtract as in equation 21; and are balanced by component parts of the primary current. In the X–X axis, however, they must mutually approximately cancel except for that residual magnetomotive force necessary for the cross field.

As the forward flux progresses from the Y–Y axis to the X–X axis, the slip-frequency rotor current at the Y–Y axis changes from the in-phase value through the maximum to the reactive value. Thus the angle of the flux vector ϕ_F at the instant the forward current is maximum at the Y–Y axis depends on the impedance angle of slip-frequency current. The cross-field integrated magnetomotive force of the slip-frequency current is a periodic reversing quantity; and it is out of phase with that due to the "backward" current since the impedance angles of the two are only equal at zero speed.

Figure 5 shows schematically the flux vectors ϕ_F and ϕ_B at an instant just before the flux through the stator winding is maximum. Conductors (1) form a turn in which the rotor current due to ϕ_F is maximum, and conductors (2) carry maximum rotor current due to ϕ_B. Angle (2) will be greater than angle (1), as the frequency of the current due to ϕ_B is greater than that due to ϕ_F.

The magnetizing effect in the X–X axis in time phase with the primary flux is given by adding expressions 16 and 20. The basic hypothesis used herein assumes that this cross field in phase with the main field is negligible.

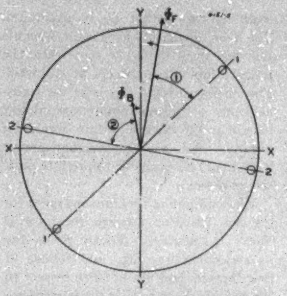

Figure 5. Location of maximum forward and backward rotor current. Axes fixed with reference to stator

The magnetizing effect in the X–X axis in phase quadrature with the primary flux is given by adding expressions 15 and 19.

A hypothetical rotor with zero leakage reactance would be ideal for producing torque in a polyphase motor. In a single-phase stator, such a rotor would not produce a cross field in time quadrature with the main field but would merely in effect shift the axis of the main field. Hence no motor action would result.

A hypothetical rotor with zero resistance and 90° rotor impedance angle would be ideal for setting up a cross field in time quadrature, but would produce no torque.

As a general statement then it may be said that the reactive component of rotor current establishes the rotating field and the in-phase component produces torque. The in-phase component will, however, have an influence on the total flux, which is taken into account in a manner which is sufficiently accurate for practical purposes by the sine factor in equation 23 below.

Determination of Forward and Backward Rotating Flux Magnitudes

The respective values of the forward flux ϕ_F and the backward flux ϕ_B are determined as follows:

(1). The sum of the voltages they induce in the stator winding is equal to the line voltage minus primary impedance drop. (See equation 22.)

(2). The difference between these two fluxes is the cross field. (See equation 23.)

$$\sqrt{2}(E-i_1Z_1)=(\phi_F+\phi_B)T_{\text{eff}}10^{-8} \quad (22)$$

where T_{eff} is the total primary turns (for a two-pole motor) multiplied by the distribution factor. The latter is $\pi/4$ for the ideal case of a winding with sinusoidal distribution. (This corresponds to $N/4$ or 1/2 the total rotor turns, which are linearly distributed.)

$$(\phi_F-\phi_B)\sin\left[\frac{\tan^{-1}\frac{sX}{R}+\tan^{-1}\frac{(2-s)X}{R}}{2}\right]=$$

$$\{\phi_B(F_1'-jF_1)-\phi_F(F_2'-jF_2)\}\frac{\phi_{M0}}{\sqrt{2}i_mT_{\text{eff}}} \quad (23)$$

Let

$$\gamma=\frac{\phi_B}{\phi_F}$$

From (23)

$$\gamma=\frac{\phi_{M0}\sqrt{F_2'^2+F_2^2}+\sqrt{2}i_mT_{\text{eff}}\sin\epsilon}{\phi_{M0}\sqrt{F_1'^2+F_1^2}+\sqrt{2}i_mT_{\text{eff}}\sin\epsilon} \quad (24)$$

The primary current i_1 of (22) flowing through the primary turns (effective value) overcomes the rotor magnetomotive force by an amount necessary to produce the existing mutual flux.

$$i_1=i_m\frac{\phi_F+\phi_B}{\phi_{M0}}+[\phi_B(F_1'-jF_1)+$$

$$\phi_F(F_2'-jF_2)]\frac{1}{\sqrt{2}T_{\text{eff}}} \quad (25)$$

Equation (22) may be modified to the following:

$$\frac{E-i_1Z_1}{E-i_mZ_1}=\frac{\phi_F+\phi_B}{\phi_{M0}} \quad (26)$$

From (25) and (26)

$$\phi_F=\frac{\phi_{M0}}{1+\gamma+\frac{Z_1\phi_{M0}}{\sqrt{2}ET_{\text{eff}}}\times\sqrt{(F_1'\gamma+F_2')^2+(F_1\gamma+F_2)^2}} \quad (27)$$

The treatment of these last equations in algebraic rather than vectorial fashion is an approximation justified for most practical purposes.

Motor Torque

For any slip s, we now have the forward rotating flux vector ϕ_F and the backward vector ϕ_B. Assuming that one is not interested in the instantaneous torque (or the pulsating component), the total or average torque is

$$T_{\phi F}+T_{\phi B}$$

The second factor, torque due to backward rotating flux, is inherently negative, and hence total average torque is the difference of these two component parts. Each torque component may be calculated as for a polyphase motor where given the flux, slip, and rotor constants—as follows.

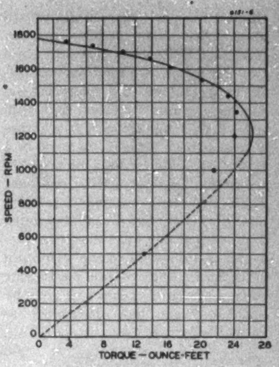

Figure 6 (Curve 1). Speed versus torque

Solid curve—test
Dashed curve—test extrapolated
Circles—calculated

Note: No allowance in calculated values for friction and windage

We have the rotor voltages in equations 8 and 9, whence the in-phase rotor current will be

$$\phi_FG_Fs\omega\cos(s\omega t-\alpha)+\phi_BG_B(2-s)\omega\times\cos[(2-s)\omega t+\alpha] \quad (28)$$

where the conductance $G_F=\frac{R}{R^2+s^2X^2}$

and $G_B=\frac{R}{R^2+(2-s)^2X^2}$

The in-phase component of slip-frequency current is maximum at any instant in the conductor which is at the point of the air gap carrying maximum (forward) flux. Both the forward flux density and the in-phase slip frequency current magnitude will vary around the air gap at any instant as $\cos\delta$; δ being zero where ϕ_F is maximum.

The forward flux per unit length of air gap (=density × length of rotor) at the point where ϕ_F is maximum is ϕ_F/d—where d is the diameter of the rotor. The torque being a tangential force (which is the product of flux density and current) multiplied by the lever arm $d/2$; then the torque due to two rotor conductors forming a turn at the diametrically opposite points of maximum flux density will be found by multiplying the in-phase current by the total forward flux ϕ_F. In symbols, the above may be stated in abbreviated form

$$T=\frac{\phi_F}{d}\times I\times\frac{d}{2}\times 2=\phi_F\times I$$

(for the two conductors or one turn).

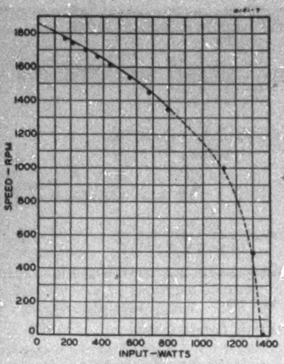

Figure 7 (Curve 2). Speed versus watts

Solid curve—test
Dashed curve—test extrapolated
Circles—calculated

The quantity I is the in-phase "forward" current in the rotor bar in which it is maximum at any instant.

The torque due to ϕ_F for one rotor turn then is

$$\phi_F^2 G_F s\omega \cos^2\delta = \frac{\phi_F^2 G_F s\omega}{2}(1+\cos 2\delta) \qquad (29)$$

Multiplying by $\dfrac{N10^{-8}}{2\pi}\,d\delta$, and integrating from $\delta=0$ to $\delta=\pi$,

$$T\phi_F = \frac{\phi_F^2 N s\omega R}{4(R^2+s^2X^2)10^8} = F_2'\phi_F^2 \qquad (30)$$

Similarly, the torque due to ϕ_B is

$$T\phi_B = F_1'\phi_B^2 \qquad (31)$$

Total average torque is

$$K(F_2'\phi_F^2 - F_1'\phi_B^2) \qquad (32)$$

K is a constant which has the value 0.7376×10^{-8} to reduce to pound-feet.

For a motor having more than two poles, expression 32 will be modified as indicated in Appendix I.

Conclusions

Present methods of calculating the performance of single-phase induction motors are quite tedious, and probably do not convey to the mind of the practical engineer a logical concept of the operation of the motor such as is the case with generally used methods of polyphase motor design calculation. It seems desirable to have a unified and co-ordinated theory for single phase induction motors including formulas which are simplified as much as possible (with justifiable approximations) and which convey unabstruse concepts. The formulas developed may be used as the basis of a routine calculation procedure.

An advantage of formulas of this type is that they deal with real values—actual rotor current in a rotor bar in amperes, and so on. The value of rotor impedance does not change in using the same rotor with two different stator windings for example, as it does when equivalent primary values are used. Flux densities in the cross field axis and other similar values are explicitly evident.

In applying the equations developed, it is necessary first to determine motor constants by established methods (primary impedance, magnetizing current and flux value with open-circuited rotor, and effective rotor resistance and reactance for a turn consisting of two rotor bars 180° apart with connecting end rings common to all turns).

Symbols and Conventions Followed in Text

Primary flux $=\phi_Y=\phi_M \sin\omega t$

This is the total flux component in the Y–Y axis of figure 1, common to stator and rotor. It may also be called the mutual flux.

"Total flux" is regarded as $\overrightarrow{\phi_Y}+\overrightarrow{\phi_X}$ where ϕ_X is the flux in the X–X axis and is referred to as the cross field. ϕ_X' is further taken as that magnitude of flux in the X–X axis, which exists at the instant ϕ_Y is zero. ϕ_X is therefore at any instant $\phi_X' \cos\omega t$.

p = per unit synchronous speed = rpm ÷ synchronous speed $=1-s$ where s is per unit slip
$\omega=2\pi f$ where f is line frequency
R = rotor resistance
X_F = rotor reactance at frequency sf $=sX$
$X_B=(2-s)X$ = rotor reactance at frequency $(2-s)f$
$Z_F=R+jsX$ = rotor impedance at frequency sf
$Z_B=R+j(2-s)X$ = rotor impedance at frequency $(2-s)f$
Z_1 = primary or stator impedance*
ϕ_c = flux through coil of figure 1
θ = angular position of coil of figure 1 at any instant $=p\omega t+\alpha$
α = initial angle of coil when flux ϕ_Y is zero
ϕ_F = forward rotating flux component
ϕ_B = backward rotating flux component
ϕ_{M0} = maximum value of primary flux ($Y-Y$ axis) with rotor open-circuited (standstill with zero rotor current)
T = total primary (stator) turns
T_{eff} = total primary turns multiplied by distribution factor*
N = total secondary (rotor) bars or conductors*
E = line volts, rms*
e_F and e_B = rotor voltages due to forward and backward flux components (respectively) cutting rotor conductors
I_1 = primary-current (rms amperes) at any slip s
i_m = primary magnetizing current (rms) with open circuit rotor $=i_c-ji_m'$
δ = angle measured around air gap from point where revolving flux is maximum at any instant (stator angle measured from traveling reference)
ϵ = angle which is one-half the sum of the two rotor impedance angles at frequencies of sf and $(2-s)f$ respectively

*Multiply by $2/P$ where P is number of poles. See appendix I.

References

1. The Cross Field Theory of Alternating Current Machines, H. R. West. AIEE Transactions, February 1926, pages 466–74.

Table I

Rpm	1770	1740	1700	1660	1615	1535	1445	1345	1000	0
s	0.0167	0.0333	0.0555	0.0778	0.103	0.147	0.197	0.253	0.445	1
$2-s$	1.983	1.967	1.945	1.922	1.897	1.853	1.803	1.747	1.555	1
s^2	2.79×10^{-4}	1.11×10^{-3}	3.08×10^{-3}	6.05×10^{-3}	0.0106	0.0216	0.389	0.064	0.198	1
$(2-s)^2$	3.94	3.86	3.78	3.7	3.6	3.44	3.26	3.05	2.42	1
F_1	0.01	0.00982	0.00968	0.00940	0.00925	0.00892	0.00852	0.00804	0.00605	0.00288
F_1'	0.0234	0.02315	0.02305	0.0228	0.0226	0.0223	0.0219	0.0213	0.01803	0.01335
F_2	0.841×10^{-6}	3.34×10^{-6}	9.28×10^{-6}	1.82×10^{-5}	3.19×10^{-5}	6.5×10^{-5}	1.17×10^{-4}	1.92×10^{-4}	5.9×10^{-4}	0.00288
F_2'	0.233×10^{-3}	0.465×10^{-3}	0.775×10^{-3}	1.082×10^{-3}	1.435×10^{-3}	2.04×10^{-3}	2.75×10^{-3}	3.515×10^{-3}	6.14×10^{-3}	0.01335
$\sin\epsilon$	0.203	0.203	0.203	0.204	0.204	0.205	0.206	0.206	0.208	
γ	0.0241	0.0335	0.0458	0.0586	0.0738	0.100	0.13	0.169	0.337	1
$\phi_F/10^8$	1.865	1.815	1.755	1.70	1.63	1.53	1.44	1.33	1.05	0.618
$\phi_B/10^8$	0.045	0.0608	0.0805	0.0995	0.12	0.153	0.187	0.225	0.354	0.618
T	0.224	0.426	0.66	0.864	1.025	1.26	1.45	1.52	1.335	0
I	$1.426-j2.93$	$1.902-j2.98$	$2.59-j3.07$	$3.22-j3.15$	$3.9-j3.22$	$4.953-j3.34$	$6.0-j3.48$	$7.0-j3.6$	$9.93-j3.9$	$12.04-j4.24$

Note: It would be convenient to have a column for v and w.

2. THE REVOLVING FIELD THEORY OF THE CAPACITOR MOTOR, Wayne J. Morrill. A paper presented at the AIEE winter convention, New York City, January 28–February 1, 1929.

3. A SUGGESTED ROTOR FLUX LOCUS CONCEPT OF SINGLE-PHASE INDUCTION-MOTOR OPERATION, C. T. Button. ELECTRICAL ENGINEERING, March 1937, pages 331–2.

4. SINGLE-PHASE MOTOR-TORQUE PULSATIONS, A. L. Kimball, Jr., and P. L. Alger. AIEE TRANSACTIONS, June 1924, pages 730–9.

Appendix I (Notes)

Calculation methods now in general use refer rotor impedance values to the primary. The values of R and X used herein may be obtained from such primary equivalents by multiplying by $N/(4T_{eff}^2)$. (Multiplying equivalent primary impedance of rotor by $1/T_t^2$ would give total rotor impedance considered as a single turn of all rotor copper in parallel. Multiplying this by $N/4$ would give the effective impedance of a single rotor turn as considered herein.) Rotor impedance values may also be obtained from standstill test. Rotor effective ampere turns = primary effective ampere turns less magnetomotive force for standstill flux. This determines maximum rotor current I. Rotor loss $= I^2RN/4$ and thus R is determined from standstill watts less primary and core loss at standstill. X may generally be assumed about equal to the primary reactance, multiplied by $N/4\ T_{eff}^2$. The standstill flux produces a voltage IZ in one turn of the rotor completely linking it, where $Z=R+jX$.

For a series-connected motor with four poles, the formulas may be applied using half the total primary turns, half the rotor bars, half the primary impedance, and 50 per cent line voltage. The final torque as determined by (32) will then be multiplied by four, since half the motor has been considered and also the rotor radius is twice that of a two-pole motor for which the above half-values would represent the complete motor. Or in general terms, in applying the formulas, E, N, T, and Z, may be multiplied by $2/P$ where P is the number of poles in the motor; and the torque will be found by multiplying the expression for torque by $(P/2)^2$.

Appendix II

(1) SUMMARY OF CALCULATION FORMULAS

$$F_1=\frac{N\omega X}{4\left[\frac{R^2}{(2-s)^2}+X^2\right]10^8}$$

$$F_1'=\frac{N\omega(2-s)R}{4[R^2+(2-s)^2X^2]10^8}=F_1\frac{R}{(2-s)X}$$

$$F_2=\frac{N\omega X}{4\left[\frac{R^2}{s^2}+X^2\right]10^8}$$

$$F_2'=\frac{N\omega sR}{4(R^2+s^2X^2)10^8}=F_2\frac{R}{sX}$$

$$\phi_{M_0}=\frac{\sqrt{2}(E-i_mZ_1)10^8}{\omega T_{eff}}=\text{flux per pole with open circuit rotor}$$

$$\frac{\phi_B}{\phi_F}=\gamma=\frac{\sqrt{F_2^2+F_2'^2}+\frac{\sqrt{2}i_mT_{eff}\sin\epsilon}{\phi_{M_0}}}{\sqrt{F_1^2+F_1'^2}+\frac{\sqrt{2}i_mT_{eff}\sin\epsilon}{\phi_{M_0}}}$$

$$\epsilon=\frac{\tan^{-1}\frac{sX}{R}+\tan^{-1}\frac{(2-s)X}{R}}{2}$$

$$\phi_F=\frac{\phi_{M_0}}{1+\gamma+\frac{Z_1\phi_{M_0}}{\sqrt{2}T_{eff}E}\sqrt{(F_1\gamma+F_2)^2+(F_1'\gamma+F_2')^2}}$$

$$\text{Torque}=0.7376(F_2'\phi_F^2-F_1'\phi_B^2)\left(\frac{P}{2}\right)^2$$

$$i_1=(i_c-ji_m')\frac{\phi_F+\phi_B}{\phi_{M_0}}+\frac{[F_1'\gamma+F_2'-j(F_1\gamma+F_2)]\phi_F}{\sqrt{2}T_{eff}}$$

(2) CONSTANT FACTORS IN FORMULAS

(To be filled in on calculation sheet after determining motor constants)

(a) $\frac{N\omega X}{4}=$ $Z_1=$

(b) $\phi_{M_0}=$ $R=$

(c) $\frac{\sqrt{2}i_mT_{eff}}{\phi_{M_0}}=$ $X=$

(d) $\frac{Z_1\phi_{M_0}}{\sqrt{2}T_{eff}E}=$ $E=$

(g) $\frac{i_c-ji_m'}{\phi_{M_0}}=$ $T_{eff}=$

(h) $\frac{R}{X}=$ $i_m=$

(k) $\frac{0.7376\,P^2}{4\cdot10^8}=$

(m) $10^8R^2=$

(n) $10^8X^2=$

(3) ABBREVIATED FORMULAS USING CONSTANT FACTORS LISTED UNDER (2)

$$F_1=\frac{a}{\frac{m}{(2-s)^2}+n}\qquad F_1'=F_1\frac{h}{2-s}$$

$$F_2=\frac{a}{\frac{m}{s^2}+n}\qquad F_2'=F_2\frac{h}{s}$$

$$\gamma=\frac{\sqrt{F_2^2+F_2'^2}+c\sin\epsilon}{\sqrt{F_1^2+F_1'^2}+c\sin\epsilon}$$

$$\sin\epsilon=\frac{X}{\sqrt{Z_FZ_B}}\ (\text{approx.})=\sqrt{\frac{1}{2}-\frac{Z_F}{4Z_B}-\frac{Z_B}{4Z_F}+\frac{X^2}{Z_FZ_B}}\ (\text{exact})$$

$$\phi_F=\frac{b}{1+\gamma+d\sqrt{v^2+w^2}}$$

where $v=F_1'\gamma+F_2'$ and $w=F_1\gamma+F_2$

$$\phi_B=\gamma\phi_F$$

$$T=(k)(F_2'\phi_F^2-F_1'\phi_B^2)$$

$$i_1=g(\phi_F+\phi_B)+\frac{\phi_F}{\sqrt{2}T_{eff}}(v-jw)$$

(4) SAMPLE CALCULATION

The calculation outlined below was made for a motor having high resistance rotor with brass bars and end rings. (See table I.)

Four pole motor (line volts = 113)

$E=56.5$

$Z_1=1.75=0.7+j1.6$

$T_{eff}=98.5$

$i_m=0.3-j2.7$

$R=12.5\cdot10^{-4}$

$X=2.7\cdot10^{-4}$

$$(a)=\frac{N\omega X}{4}=\frac{18.5\cdot377\cdot2.7\cdot10^{-4}}{4}=0.47$$

$$(b)=\phi_{M_0}=\frac{(56.5-1.75\cdot2.7)\sqrt{2}\cdot10^8}{377.98\cdot5}=1.98\cdot10^5$$

$$(c)=\frac{\sqrt{2}i_mT_{eff}}{\phi_{M_0}}=0.0019$$

$$(d)=\frac{Z_1\phi_{M_0}}{\sqrt{2}T_{eff}E}=44$$

$$(g)=\frac{i_c-ji_m'}{\phi_{M_0}}=(0.15-j1.36)10^{-5}$$

$$(h)=\frac{R}{X}=4.63$$

$$(k)=\frac{0.7376P^2}{4\cdot10^8}=2.95\cdot10^{-8}$$

$(m)=10^8R^2=156$

$(n)=10^8X^2=7.3$

$(v)=F_1'\gamma+F_2'$

$(w)=F_1\gamma+F_2$

An Electrical Engine Indicator for Measuring Static and Dynamic Pressures

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Introduction

THIS paper presents the design of a condenser type indicator and associated electrical equipment, together with the technique of measuring pressures in internal-combustion engines.

The term, pressure indicator, in this paper will be used to refer to the pressure-sensitive element, and not to the type of amplifying and recording systems used in conjunction with the pressure elements.

The selection of an electrical pressure indicator for engine work may involve a choice of several kinds of pressure-sensitive devices, as well as many types of electrical circuits and recording methods. However, it is beyond doubt that no single available combination of pressure indicator and associated electrical equipment meets all the demands of complete engine-pressure analysis. Nevertheless, with a fairly complete knowledge of the defects and limitations of the various kinds of indicators available, the instrument and engine engineer may successfully cope with most of the problems of pressure measurements.

Before discussing the characteristics of the various types of pressure elements, it might be well to consider the important requirements of an ideal engine indicator and its associated amplifying and recording equipment.

1. The ideal indicator must give a linear and undistorted response to any instantaneous pressure encountered in the engine to be studied.
2. The natural frequency of the indicator must be well above any frequencies to be measured.
3. It must have an accurate predetermined calibration that will not be affected by temperature, time, fatigue, or external forces such as vibration or shock.
4. It must be insensitive to all forces except the pressures applied to the diaphragm, that is, it must not pick up engine vibrations.
5. It must be capable of indicating an atmospheric or any other desired pressure datum line.
6. It must be small in physical structure in order that it may be easily installed in modern engines.
7. It must be reasonably simple to operate.
8. It must be capable of recording pressures during a single engine explosion.

At the present time, no known pressure-indicating equipment meets all of these requirements. Fortunately, however, the engine development engineer can compromise on some of these requirements and still obtain instrumentation which is dependable and useful.

For choice of an electrical pressure indicator capable of measuring pressures for a single engine cycle, the following types have been developed:

1. Resistance-type indicators
   - (a). Carbon stack
   - (b). Carbon granule
   - (c). Solid carbon resistor
2. Piezoelectric crystal-type indicators
3. Magnetic-type indicators
   - (a). Moving diaphragm
   - (b). Moving coil
4. Condenser type indicators

All types of pressure-sensitive units have characteristics which make them useful for certain applications. Likewise, most of these indicators possess characteristics which make their use unsatisfactory for some work. The principal advantages and disadvantages of the various types of indicators and pressure-sensitive devices may be briefly outlined to present their possibilities and limitations for any given application.

1. Resistance-type indicator

Advantages:

No electrical amplifiers are needed to operate a magnetic type galvanometer oscillograph. This is especially true of the carbon stack indicator which may be constructed to have a high-current output.

Disadvantages:

(a). Instability of calibration (particularly under severe conditions)

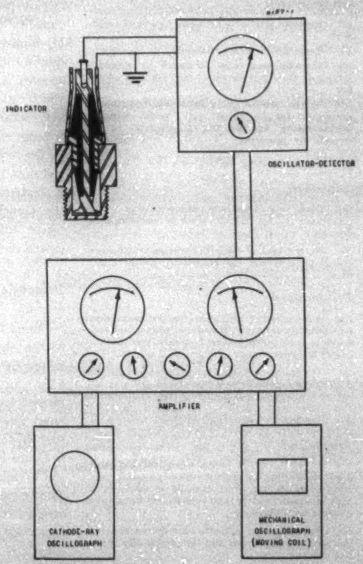

Figure 1. Condenser-plate indicator and electrical equipment

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The authors wish to express appreciation to G. M. Rassweiler of the laboratories for his helpful suggestions in the preparation of this paper.

1. For all numbered references, see list at end of paper.

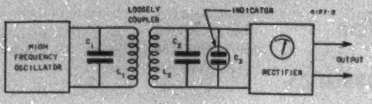


Figure 2

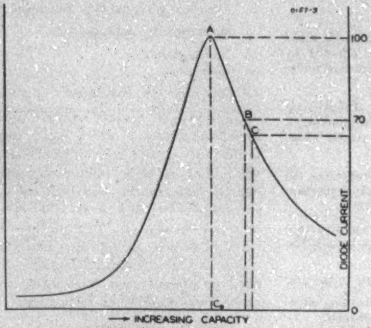


Figure 3

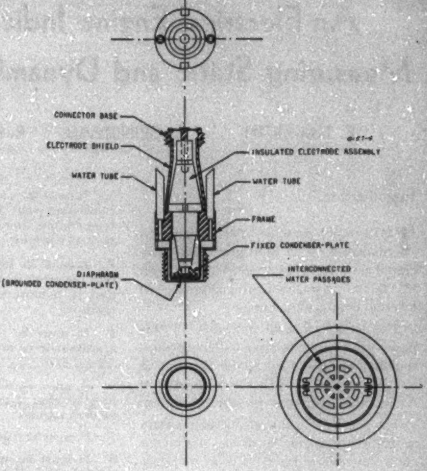

CONNECTOR BASE
ELECTRODE SHIELD
WATER TUBE
0-57-4
INSULATED ELECTRODE ASSEMBLY
WATER TUBE
FRAME
FIXED CONDENSER-PLATE
DIAPHRAGM (GROUNDED CONDENSER-PLATE)
INTERCONNECTED WATER PASSAGES

Figure 4. Water-cooled condenser-plate indicator

(*b*). **Responds to severe vibration**

2. Piezoelectric crystal-type indicator

Advantages:

(*a*). **Highly linear response**
(*b*). **Constancy of calibration**

Disadvantages:

(*a*). **Requires extremely high-gain electrical amplifiers**
(*b*). **Responds to severe vibration**
(*c*). **Extreme care must be taken to minimize leakage in the input circuit of the high-gain amplifier**
(*d*). **In the present state of development, it is unsuitable for low-speed work, static pressure measurements, and for the measurement of extremely low pressures**

3. Magnetic-type indicator

Advantages:

(*a*). **Gives rate of pressure change directly. (A-c impedance bridge circuit gives a pressure-time diagram)**
(*b*). **No mechanical contacts are required between the diaphragm and the electrically sensitive element**

Disadvantages:

(*a*). **A complicated high-gain amplifier is required to accurately integrate the rate of pressure-change response into pressure-time response over the required frequency range**
(*b*). **The present a-c impedance electric circuits for use with magnetic-type indicators do not give accurate reproduction of high-frequency components**

4. Condenser-type indicator

Advantages:

(*a*). **As will be shown in this paper, this indicator lends itself to an exceptionally compact and sturdy construction which is easily serviced, has a high natural frequency, and is relatively insensitive to shock and vibration**
(*b*). **No mechanical contacts are required between the pressure diaphragm and the electrically sensitive element**

Mechanically this indicator offers no disadvantages that are not present in the other type indicators.

The associated electrical equipment for measuring the change in capacity of the indicator may offer certain limitations depending upon the type of circuit used. For example, the electrostatic type of circuit for measuring small changes in capacity has the following limitations: (1) high-gain electrical amplifiers are required, (2) limited low-frequency response, and (3) high-input impedance which makes the electrical amplifiers very sensitive to ignition interference.

Some of these limitations are overcome by the resonance impedance circuit which will now be described in connection with a condenser-type indicator developed in the General Motors Research Laboratories. Limitations introduced are discussed in this paper.

Principle of Operation

Figure 1 shows the general arrangement of the equipment. The indicator illustrated has been drawn in its elementary form for simplicity. Its capacity is formed by the diaphragm and the insulated electrode. This capacity C_3 is a part of a tuned resonant circuit as shown in figure 2 in which L_1C_1 form part of a stable high-frequency oscillator circuit. The tuned circuit, $L_2C_2C_3$ is loosely coupled to a circuit L_1C_1. The high-frequency potential induced in circuit $L_2C_2C_3$ is rectified, and the d-c output current of the rectifier is indicated on a microammeter.

The response of the tuned circuit L_2-C_2C_3 to a constant frequency is shown in figure 3 as a function of the capacity (C_2 plus C_3) of this circuit. This curve is typical of a parallel resonant circuit. Peak output occurs at point *A* when the circuit is tuned to resonate at the oscillator frequency. This circuit is tuned to its operating point *B* by increasing capacitor C_2 until the rectifier meter reads approximately 70 per cent of maximum. Any small change in indicator capacity C_3, due to pressure on the diaphragm, affects the response of the circuit according to the curve of figure 3.

Since the range of capacity change of the condenser-plate indicator is very small, the response of the tuned circuit is limited to a small section such as *B-C* of the response curve. While this section of the curve is not necessarily a straight line, it will be shown later that the output of the detector is approximately linear. The output is amplified before passing to the viewing and recording oscillographs, see figure 1.

In order to accurately reproduce the pressure encountered in engine indicator work, the indicator and its associated electrical equipment should give linear response to frequencies as high as 10,000 cycles per second. This requires an oscillator frequency of 100 kilocycles per

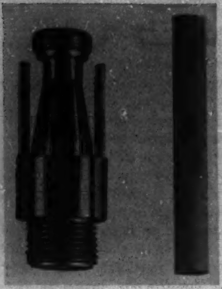


Figure 5 (left)

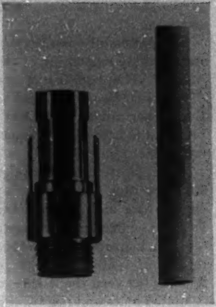


Figure 6 (right)

second or higher to prevent distortion of the high-frequency components. An oscillator frequency of approximately 400 kilocycles per second was selected because reliable circuit components are commercially available in the frequency range of 400 to 500 kilocycles per second.

Mechanical Design of the Indicator

The following mechanical design features are desirable, considering the characteristics of the electrical circuit as well as the indicator applications.

1. The physical dimensions of the indicator must be small enough to facilitate installation on modern engines.
2. The construction must be compact and rigid in order to minimize the effects of severe vibration, that is, the inertia forces must be small.
3. The natural frequency response of the elastic system should be well above any frequency encountered in engine indicator applications. Pressure changes as high as 6,000 cycles per second are common in the combustion chamber of some engines under detonating conditions.
4. The design of the indicator should permit an easy choice of diaphragm thickness to enable measurements over a wide pressure range. To cover the varied requirements of present-day engine research, the range should extend from a few ounces to several thousand pounds per square inch. If the diaphragm thickness is chosen so that its maximum deflection is approximately constant for the maximum pressure change to be measured, in a given application, then the requirements of the associated electrical equipment are much more readily met.
5. The indicator should be constructed so that it can be installed with the diaphragm flush with the walls of the pressure chamber. This type of installation eliminates the possibility of surge effects in connecting passages.
6. Temperature effects should be made as low as possible. Temperature differences, due to changes of engine load and speed, produce a shift in the indicator response. Until a suitable metal is found that will not change its dimensions or elastic properties with temperature, it has been found necessary to provide efficient water cooling.
7. The indicator should be constructed so that it can be readily serviced and assembled.
8. The design should provide adequate electrical shielding combined with a good method of cable connection.

Figure 4 is a cutaway view of the latest type of condenser indicator developed at General Motors Research Laboratories.

The assembly consists of three principal parts: (1) a water-cooled frame and diaphragm, (2) an insulated fixed plate and electrode assembly, and (3) an electrode shield and cable connector.

The frame and diaphragm form an integral unit which is machined from a solid piece of Invar metal. Invar is used to minimize the effect of temperature changes in the diaphragm. Water cooling keeps the temperature of the indicator within the range where the low thermal coefficient of Invar steel is realized.

An important design feature is the water cooling of the diaphragm. After passing through the frame the water flows through a series of interconnected concentric grooves machined on the bottom side of the heavy diaphragm and sealed by a thin disk which is copper-brazed in place, see figure 4. This construction makes the thin sealing disk an integral part of the heavy diaphragm, and thus eliminates any mechanical joints or linkages. The diaphragm is also extremely sturdy.

The fixed-plate and insulated-electrode assembly is made from the mica-insulated center electrode of an aircraft spark plug. This electrode is sturdy and well insulated and has small electrical capacity to the frame of the indicator. Complete electrical shielding is provided by a metal jacket which serves as a unit shield and cable connector as shown in figure 4. In order to minimize extraneous capacity changes such as produced by cable vibration and vibration of parts in the oscillator detector unit, it is desirable to have a large capacity change in the indicator. Since the capacity between parallel plates varies as an inverse function, it is necessary to select a diaphragm thickness so that its movement is limited to approximately 15 per cent of the initial air gap in order to obtain approximately linear response.

Considering the natural frequency of the diaphragm, its elastic properties and limited diameter, a maximum safe dia-

Figure 7

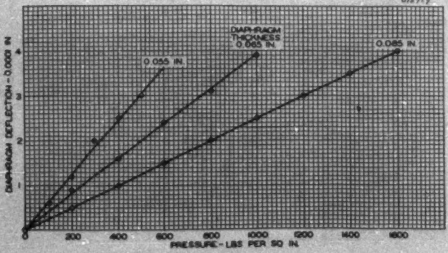
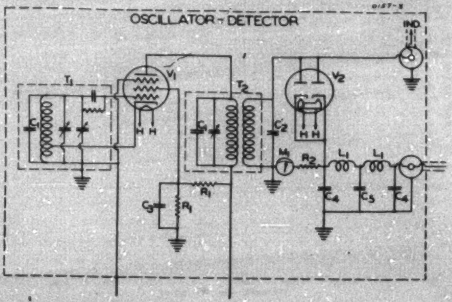


Figure 8 (right)



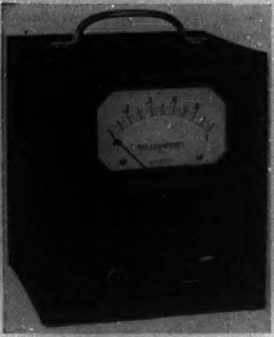


Figure 9

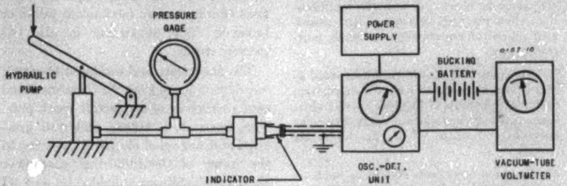


Figure 10

phragm movement of 0.0003 inch was chosen. In order to obtain adequate initial capacity, sensitivity, and linearity with this small movement, an air gap of approximately 0.002 inch was chosen. The initial capacity and sensitivity of the indicator is further improved by the use of a 0.001-mica separator between the plates. This mica also prevents shorting of the plates by minute dust or metallic particles.

For successful operation of the indicator, accurate machining of all parts is essential. The surfaces of the condenser plates must be parallel and separated by the proper amount. It has been found practical to hold the plates parallel within 0.0001 inch.

The indicator shown in figure 4 has a standard 18-millimeter spark-plug thread on the base and a bore diameter of $^7/_{16}$ inch. The fixed plate diameter is held to 0.415-inch in order to clear the root diameter of the $^7/_{16}$-60 tapped hole in the frame. Spacing between the 0.415-inch diameter insulated plate and the $^7/_{16}$-inch bore provides sufficient clearance to assure small capacity between the periphery of fixed plate and the frame. Small capacity in all parts except between the parallel surfaces of the condenser plates is desirable in order to obtain maximum sensitivity. Figure 5 is a picture of an assembled 18-millimeter indicator.

A 14-millimeter indicator, shown in figure 6, has been developed for use in applications where compactness is necessary.

Since it is desirable to limit the movement of the diaphragm to approximately 0.0003 inch, its thickness is determined by the peak pressure to be measured. The curves in figure 7 have been obtained by plotting diaphragm deflection of the water-cooled 18-millimeter indicator versus pressure in pounds per square inch applied to the indicator diaphragm. A suitable diaphragm thickness for any given pressure measurement may be selected by reference to this chart. For example, with peak pressures of 800 pounds per square inch, a diaphragm thickness of 0.065 inch should be used.

Any indicator can be used over a wide range of pressures by changing the gain of the electrical amplifiers. The sturdy diaphragm construction makes it possible for the sensitive indicators to withstand high pressures.

An indicator used in an exhibit at the 1939 New York World's Fair furnishes an example of the ruggedness of this type of construction. Failure occurred after approximately 1,500 hours of operation (about 54,000,000 power strokes) in the combustion chamber of a Cadillac engine. Failure was due to corrosion of the sealing disk.

Electrical Equipment

Oscillator-Detector Unit

A complete schematic diagram of the oscillator-detector unit is shown in figure 8. This circuit corresponds to the skeleton diagram of figure 2. A photograph of the entire unit appears in figure 9.

Tube V_1, in figure 8, and its associated circuits form an electron-coupled high-frequency oscillator. This type of oscillator is used because it provides relatively high output and is inherently stable. The rectifier or diode-detector tube V_2, which is connected as a half-wave rectifier, is coupled to the oscillator through transformer T_2. A microammeter M_1 reads the rectified current flowing through the diode load resistor R_2; therefore, the microammeter reading is an indication of

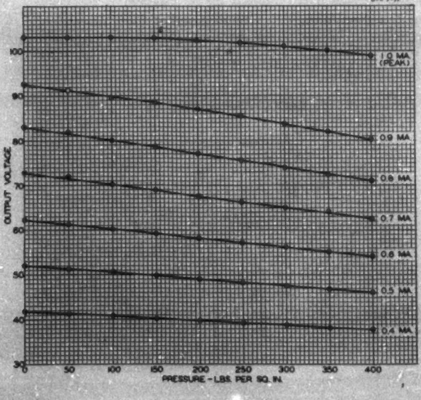
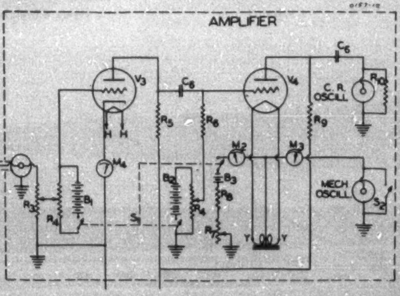


Figure 11 (left)

Figure 12 (below)



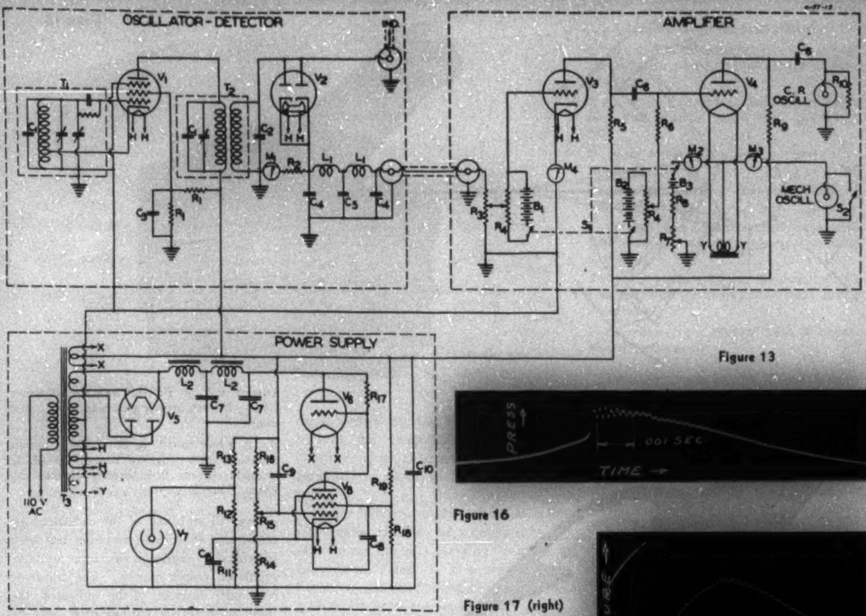

OSCILLATOR-DETECTOR
AMPLIFIER
POWER SUPPLY
IND.
T1
V1
T2
V2
C1
C2
C3
C4
C5
R1
R2
L1
M1
H H
V3
V4
C6
C8
R5
R6
R9
R10
C.R. OSCILL
M2
M3
M4
B1
B2
B3
R3
R4
R7
R8
S1
S2
MECH OSCILL
Y
X
L2
C7
V5
V6
V7
V8
R11
R12
R13
R14
R15
R16
R17
R18
R19
C9
C10
H
110 V AC
T3

Figure 13

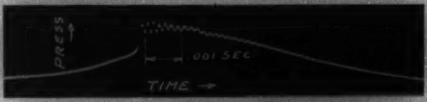


Figure 16

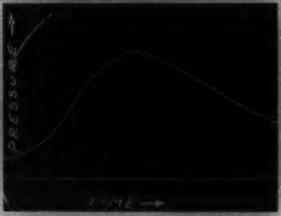


Figure 17 (right)

the output voltage across R_3. A low-pass filter $L_1C_4C_5$, designed to cut off all frequencies above 63,500 cycles per second, removes the oscillator-frequency component from the rectified output voltage.

A concentric cable connects the condenser indicator to the oscillator-detector unit. Since the electrical capacity of the cable forms a part of the tuned circuit in which the indicator is connected, it is apparent that a small change in its capacity will affect the output in the same manner as pressure applied to the indicator diaphragm. Severe vibration of the cable causes small changes in capacity due to the movement of the inner conductor in relation to the outer shield. In order to minimize this effect, the oscillator-detector unit is mounted near the condenser indicator with a short length of low-capacity cable. The diaphragm plate of the indicator is grounded to the engine and fixes the ground point of the oscillator-detector unit. The output voltage developed across R_3 and M_1 in the circuit of figure 8 is at a positive potential when referred to ground.

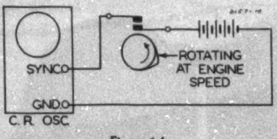

SYNC
GND.
C.R. OSC
ROTATING AT ENGINE SPEED

Figure 14

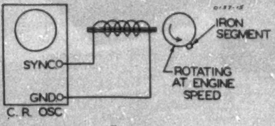

SYNC
GND
C.R. OSC
IRON SEGMENT
ROTATING AT ENGINE SPEED

Figure 15

Static calibration tests were made with a condenser-plate indicator and an oscillator-detector unit to determine their overall sensitivity as well as their linearity of response to pressure. Figure 10 shows the test equipment. A water-cooled indicator of the type shown in figure 4, with a diaphragm thickness of 0.055 inch, was used for this test. At zero-gauge pressure on the diaphragm, the power-supply voltage was adjusted to give a peak oscillator-detector reading of 1.0 milliampere. The detector was then detuned to 0.4 milliampere in successive steps of 0.1 milliampere. At each detector setting, pressure was varied from 0 to 400 pounds per square inch, and the corresponding output voltage measured.

The data plotted in figure 11 show that the output is approximately linear when the detector is operated at initial meter settings of 0.8, 0.7, 0.6, 0.5, and 0.4 milliampere. When it is operated with an initial meter setting of 0.7 milliampere, on the side of the response curve that gives decreasing output voltage with increasing capacity, a pressure change of 400 pounds per square inch gives a 10.5-volt change in the output voltage.

These data show that best linearity of response (over a wide range) is obtained by operating about a point that is approximately 70 per cent of the maximum response as illustrated in figure 3. By operating on the side of the response curve that gives decreasing voltage output with increasing capacity, the nonlinearity of the resonant curve approximately compensates for the nonlinearity of the capacity changes in the indicator.

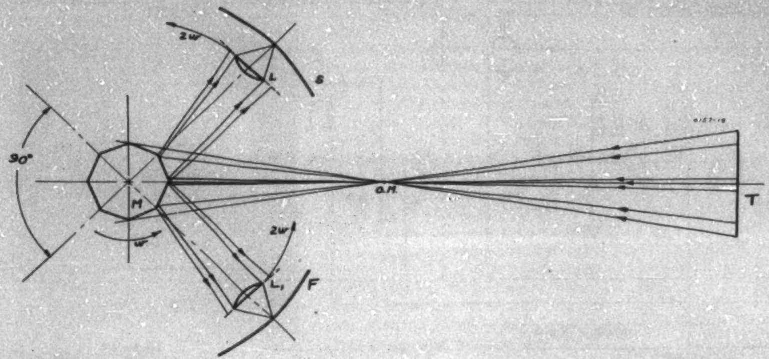


Figure 18

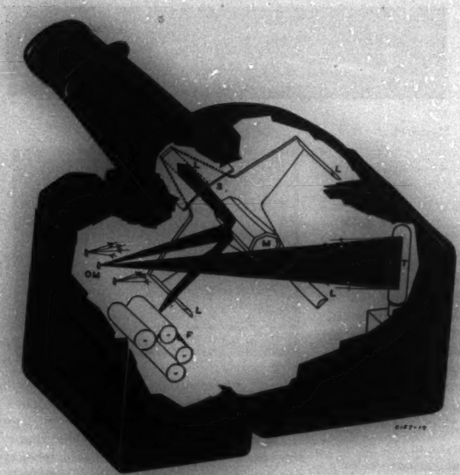


Figure 19

Oscillograph Amplifier

In order to observe or record the output of the oscillator-detector unit under dynamic conditions, the output must be amplified to operate a cathode-ray or a mechanical oscillograph. The schematic diagram of an amplifier built to operate both a cathode-ray and a mechanical oscillograph is shown in figure 12.

The amplifier possesses the following characteristics:

1. Flat frequency response from 4 to 10,000 cycles per second. The low frequency response has been extended to four cycles per second by providing interstage coupling circuits C_5R_8 and C_6R_{10} having high-time constants.

2. A voltage supply in the grid input circuit of the first stage to cancel the initial d-c output voltage of the oscillator-detector and to furnish proper grid bias for the first stage.

3. A power source in the cathode circuit of the second stage to cancel the initial cathode current flowing through the mechanical oscillograph.

Over-all stability of the oscillator-detector and associated amplifier has been improved by using an electronic voltage-regulated power supply. The complete schematic diagram of the indicator, the oscillator-detector, the amplifier, and the regulated power supply appears in figure 13.

For static measurements it is necessary to use a stable d-c amplifier of relatively low gain.

Recording Equipment

For viewing and recording of pressure phenomena in high-speed internal combustion engines, equipment must be used that will respond to the highest frequen-

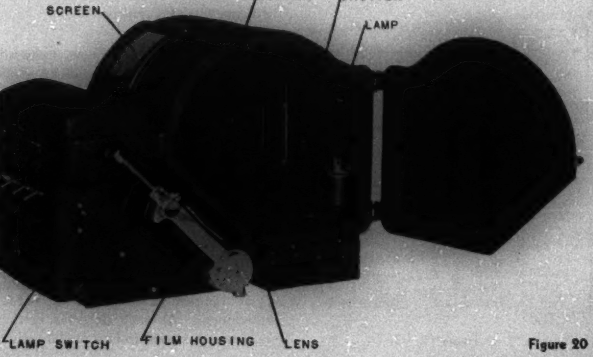
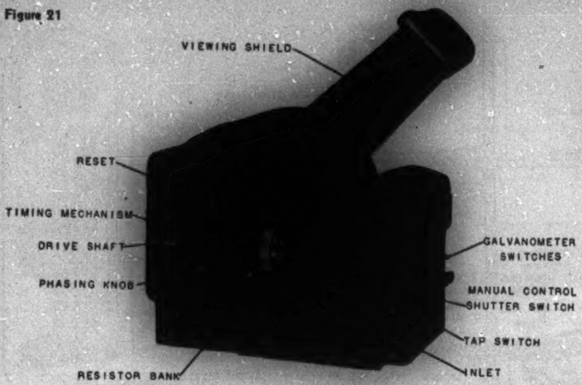


Figure 20

Figure 21



cies to be measured. In general, there are two types of equipment that are practical, namely, cathode-ray and galvanometer oscillographs.

The cathode-ray oscillograph is essentially a voltage-operated device which may have practically unlimited frequency response, whereas, the galvanometer oscillograph is essentially a current-operated device and has a definite limited response. With practical electrical circuits, the frequency response of the galvanometer oscillograph is sufficient for most engine indicator instrumentation. However, for some phenomena such as detonation in a combustion chamber, the use of a galvanometer oscillograph is impractical.

Two methods are available for recording with a cathode-ray oscillograph: (1) by using a moving film to provide for a time axis, and (2) by using stationary film with an electrical time axis.

When an electrical time axis is used, the oscillograph sweep circuit may be synchronized at engine speed by connecting the external synchronizing circuit of the oscillograph to a voltage-impulse arrangement driven by the engine. Either a battery and contactor arrangement as shown in figure 14 or a magnetic impulse generator as shown in figure 15 may be used.

Figure 16 is a pressure card taken in the combustion chamber of a detonating engine by the use of a rotating drum for a time axis. Figure 17 is a similar card taken by stationary film in conjunction with an electrical time axis.

While the cathode-ray oscillograph can be used for recording, it is particularly well adapted for viewing. On the other hand, the mechanical or galvanometer oscillograph is a relatively poor viewing mechanism, but is well adapted for recording.

A multielement mechanical oscillograph has been developed especially for use with engine indicators. The design of this oscillograph includes the following features:

1. Convenience and simplicity of operation.
2. A tungsten-filament light source.
3. An optical system that gives a uniform time axis and permits simultaneous viewing and recording.
4. Revolving short focal length lenses give high photographic speed with stationary film.
5. Four galvanometers may be used simultaneously.
6. Daylight loading and unloading of standard roll film.
7. A rapid film advancing mechanism with automatic spacing and exposure counting device.
8. A device for phasing the oscillograph time axis with the engine crankshaft during operation.
9. A barrel-type shutter with a simple timing mechanism.

Two sketches in figures 18 and 19 show the optical system of the oscillograph. To simplify the diagram in figure 18, the galvanometer mirror, *OM*, is shown acting as a pinhole instead of as a small mirror. The rotating mirror is octagonal and rotates at one-half the angular velocity of the lenses which are four in number. This mirror splits the image of the tungsten filament, *T*, reflecting part of this light to the cylindrical lens *L*, and part to L_1. Each of these two $^{11}/_{16}$-inch focal length lenses receives light from a portion of the filament and forms an image of this portion of the filament on the viewing screen or film. Obviously the length of the image of this portion of the filament depends upon the relative distance of image and object from the lens. A unique and important feature of this oscillograph is the rotating lens arrangement which makes it possible to use short focal length lenses close to the viewing screen and film, resulting in exceedingly fine traces. Since the width of the trace is partly determined by the diameter of the filament, a very fine straight tungsten strip is necessary. Any sagging of the

Figure 22. Phase-changing device

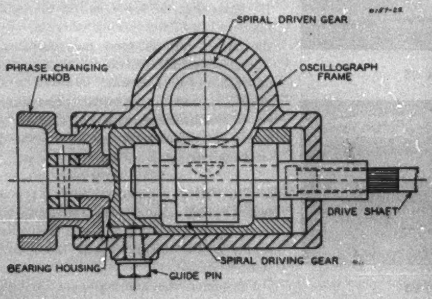
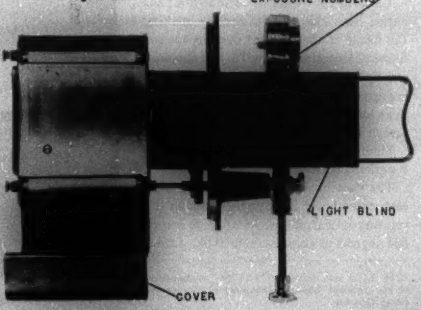


Figure 23



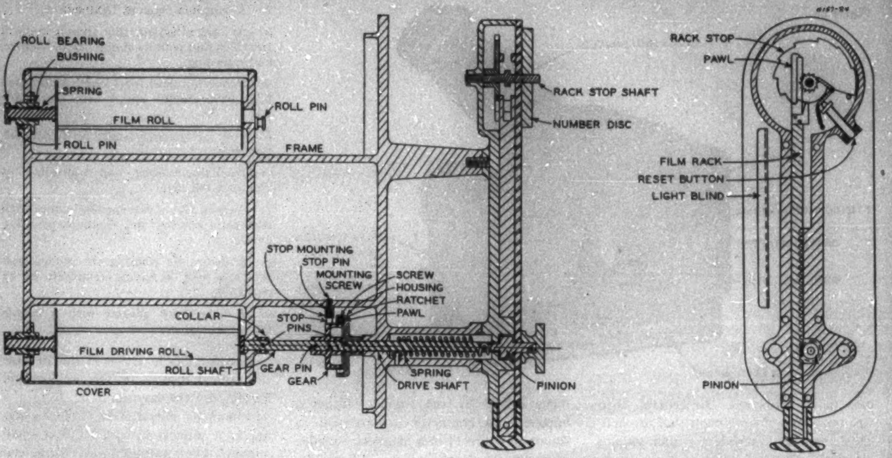


Figure 24

filament in a direction perpendicular to the plane of figure 18 increases the width of the resulting image. Several lamps were tried in an effort to find a suitable filament but since none of the available lamps proved satisfactory, a lamp was designed with a light spring mounted at one end of the tungsten strip.* This type mounting has been successful, as the constant spring tension tends to remove the slack due to expansion when the filament is heated. With the use of this long tungsten filament in place of the carbon arc light employed in former engine oscillographs, the necessity of constantly adjusting the light source is eliminated. The large amount of light, condensed to a very small spot, results in high photographic speed.

It has been shown experimentally that the optical system, as sketched in figure 18 does not introduce an error in the time axis.

Uniform light intensity throughout the entire time axis is obtained by making the image of the filament overlap the cylindrical lenses, see figure 18. In this way the light overlaps the face of the lenses enough to take care of the shifting of the light beam as it leaves the reflecting surface of the rotating octagonal mirror.

Figures 20 and 21 are pictures of the completed oscillograph.

The viewing screen and recording film each extend over 30 degrees of arc in the rotating optical system. In order to record 360 degrees of engine-crankshaft rotation on the 30-degree arc, it is necessary to use a 12:1 reduction gear between the engine crankshaft and the oscillograph. Any desired angle of crankshaft revolution can be viewed or recorded by a suitable gear reduction.

Very often in engine indicator work, it is desirable to study different parts of the engine cycle. To obtain the desired phase of the oscillograph with respect to the phase of the engine crankshaft, some provision must be made so that the phase of the oscillograph may be changed while the engine is in operation. The phase-changing device shown in figure 22 consists of two spiral gears in mesh with their shaft angles at 90 degrees and a method of sliding one of these gears perpendicular to the axis of the other gear. The driving spiral gear is mounted on the oscillograph drive shaft in such a manner that it may be moved back and forth across the mating spiral gear. This sliding movement causes the driven gear which is attached to the oscillograph mirror to revolve, thus changing the phase of the oscillograph with respect to that of the engine. The phase of the oscillograph may be changed with or without the engine running.

Figure 25

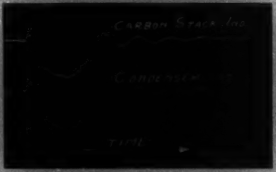


Figure 26

In order to permit continuous viewing, it is necessary that the shutter mechanism does not interrupt the light falling on the viewing screen. An inspection of figure 18 shows that the lower half of the light filament is used for viewing, whereas, only the upper half is used for recording. Therefore a shutter is needed only on the upper half of the filament. A barrel-type shutter is shown in figure 20. The shutter is synchronized with the engine by means of a suitable timing device located on the mirror shaft.

The light intensity is varied to accommodate various engine speeds by a tapped switch which changes the resistance in series with the lamp. A 110-volt d-c supply is used to operate the lamp and shutter circuits.

The camera film housing figure 23, is an

***This lamp was developed with the co-operation of the General Electric Company, Incandescent Lamp Division.**

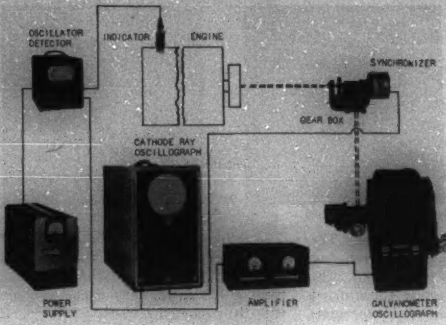

OSCILLATOR DETECTOR
INDICATOR
ENGINE
SYNCHRONIZER
GEAR BOX
CATHODE RAY OSCILLOGRAPH
POWER SUPPLY
AMPLIFIER
GALVANOMETER OSCILLOGRAPH

Figure 27

assembly of a light-tight film holder, film indexing mechanism, and light blind.

The film indexing mechanism shown in figure 24 is a rack and pinion drive which advances the film $3^3/_4$ inches for each index, thus giving ten single exposures on a standard Kodak daylight loading No. 122 roll film. The disk carrying the exposure numbers is set to number 1 at the beginning of each new film roll by pressing the reset button.

To index the film, the rack is pushed down against the rack stop. This forward motion of the rack turns the drive pinion and thus turns the film driving roll. The ratchet mechanism on the pinion drive shaft prevents the film from unwinding on the return stroke of the indexing rack. As the film is wound on the film driving roll, the diameter of the roll naturally increases and therefore requires fewer revolutions with each successive index. In order to advance the film the $3^3/_4$ inches required for the 3-inch oscillograms, and proper spacing, the number of revolutions is controlled by the length of rack travel for each index. This rack travel is regulated by the stepped rack stop. This rack stop is moved one step per index by the pawl on the end of the film rack.

Typical cards taken with the multi-element oscillograph are shown in figures 25 and 26.

Figure 25 is an oscillogram of three pressure records taken simultaneously on a four-cycle gasoline engine. From top to bottom, the records show pressures measured in the exhaust pipe, intake manifold, and combustion chamber respectively. The combustion chamber-pressure record was taken with a low-pressure indicator developed for use in the combustion chamber.

A comparison between the cards taken with a General Motors Research Laboratory condenser-type indicator and a General Motors Research carbon-stack indicator is shown in figure 26. These cards were taken simultaneously at approximately the same position in the intake manifold of a four-cycle single-cylinder engine.

A pictorial diagram of a complete indicator installation is shown in figure 27.

Indicator Calibration

Various methods have been developed for calibrating a single-card indicator. These methods, in general, can be classified as follows:

1. Static calibration, which consists in applying a known steady pressure to the diaphragm of the indicator and recording the oscillograph deflection.
2. Dynamic calibration, which consists of applying a known dynamic pressure to the indicator diaphragm with a compressor or rotating valve, and recording the oscillograph deflection.
3. Calibration in the engine under firing conditions by a supplementary instrument to indicate a point in the pressure cycle corresponding to a known balancing pressure.

The disadvantage of the first two methods of calibrating is chiefly their failure to simulate actual engine conditions in regard to temperature. After calibrating in a suitable device, the indicator must be

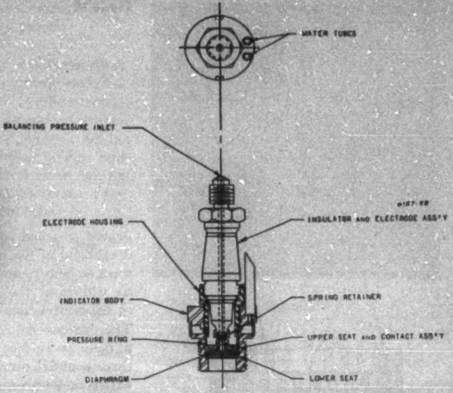

WATER TUBES
BALANCING PRESSURE INLET
ELECTRODE HOUSING
INSULATOR AND ELECTRODE ASS'Y
INDICATOR BODY
SPRING RETAINER
PRESSURE RING
UPPER SEAT AND CONTACT ASS'Y
DIAPHRAGM
LOWER SEAT

Figure 28. Water-cooled high-pressure balanced-diaphragm indicator

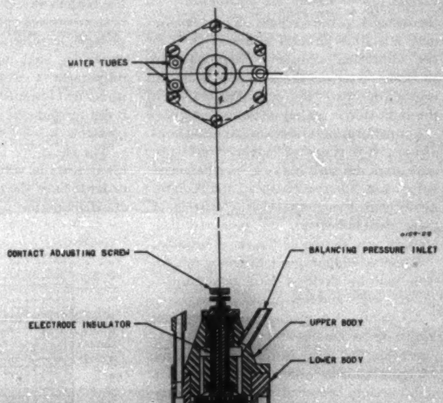

WATER TUBES
CONTACT ADJUSTING SCREW
BALANCING PRESSURE INLET
ELECTRODE INSULATOR
UPPER BODY
LOWER BODY
DIAPHRAGM AND SEAT ASS'Y

Figure 29. Low-pressure balanced-diaphragm indicator

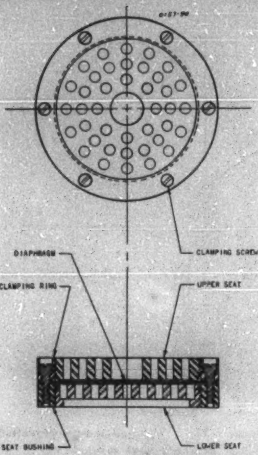


Figure 30. Seat and diaphragm assembly, low-pressure indicator

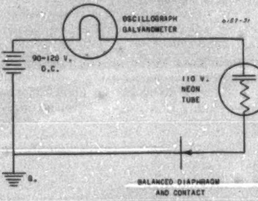


Figure 31

Figure 32

transferred to the engine. This transfer may introduce another source of error in the calibration because of the probability that the forces on the indicator frame in the calibrator will not be duplicated when the indicator is sealed in the engine.

For calibrating under engine firing conditions, two types of balanced pressure indicators are suitable: (1) the balanced valve and (2) the balanced diaphragm. After considering the characteristics of these two indicators, the balanced diaphragm was chosen as the most suitable.

Balanced diaphragm indicators, in general, can be divided into two groups, namely, high- and low-pressure types.

All balanced diaphragm indicators are similar to, or modifications of, the indicator developed by the Bureau of Standards.

Figure 28 is an assembly cutaway of a high-pressure balanced-diaphragm indicator designed and built by General Motors Research Laboratories to include water cooling, small turning radius and improved seat and contact structure. The diaphragm of the high-pressure indicator must be heavy enough (0.002–0.007 inch) to withstand the pressure encountered in the combustion chamber of an engine. The pressure necessary to move the diaphragm away from the contact (one to three pounds) usually is small compared with cylinder pressures. For use in manifolds of engines where maximum pressures are one to ten pounds, it has been necessary to build indicators with extremely thin diaphragms (0.0005 inch) in order to decrease the differential pressure necessary to open the contact. A specially designed indicator had to be built for use with these thin diaphragms to prevent wrinkling, buckling, twisting, sticking, and diaphragm rupture, see figure 29.

The seat and diaphragm are assembled as a separate unit before installation in the indicator frame, see figure 29. The thin diaphragm 0.0005 inch is placed between the two clamping rings and held in place by six counter-sunk screws as shown in figure 30. This method of clamping prevents twisting of the diaphragm. The diaphragm is then stretched by screwing the bottom seat in place. The upper seat is then screwed against the top of the diaphragm, forming a seal where the bottom stretching seat makes contact with the diaphragm (see figure 30). Both the upper and lower seats contain many small holes for passage of the gas to the diaphragm.

The clearance between the upper and lower seats is very small (0.002 inch) in order to limit the maximum movement of the diaphragm.

In figure 29, the seat assembly is shown mounted in the indicator frame. After assembly, the insulated adjustable electrode is lowered until it makes contact with the diaphragm. Caution should be exercised when making this adjustment to avoid rupture of the thin diaphragm. The recommended method of adjustment is to close a high-impedance electrical circuit (such as a neon light and resistor) when the insulated electrode contacts the diaphragm. If this adjustment is properly made, the pressure necessary to break the electrical circuit should not exceed two inches of water.

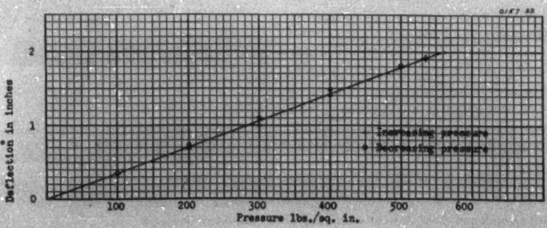
The multielement oscillograph can be used for recording individually and simultaneously the single-card indicator diagrams as well as breaks produced by the balanced-diaphragm indicators. This method eliminates the distortion produced on the single-card indicator diagram by the superimposed calibration marks.

The electrical circuit used to indicate the "make" and "break" of the balanced diaphragm is shown in figure 31.

The neon light is used to give a visual indication of the balanced-diaphragm operation. The adjustable resistor (*R*) is used to limit the current in the circuit and may be adjusted to any desired value.

Figure 32 shows two of a series of calibrated indicator cards taken with the multielement oscillograph. The method of calibration is as follows: referring to card number 6, in figure 32, the balanced diaphragm makes contact at (*A*) when

Figure 33



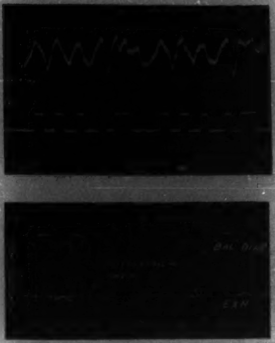


Figure 34

the engine pressure exceeds the known balancing pressure (200 pounds per square inch) above the diaphragm. This contact remains closed until the engine pressure drops below the balancing pressure. This is shown at point *B* on card number 6.

Since the time axes of both diagrams are identical, points *A* and *B* may be perpendicularly projected to the points A_1 and B_1 on the indicator diagrams.

The vertical displacement of points A_1 and B_1 is measured from the atmospheric datum line. This displacement is produced by a known balancing pressure and thus gives two points on a calibration curve. From a series of cards it is possible to obtain a complete calibration curve, see figure 33. This calibration may be superimposed on a single card (card number 7, figure 32).

Some apparent hysteresis between the increasing and decreasing pressure calibration can be seen in figures 32 and 33. The cause of this apparent hysteresis is not definitely known. Studies are being made on this problem.

The peak pressure of 535 pounds per square inch shown on card number 7, figure 32, is obtained from the calibration curve, figure 33.

Figure 34 shows two low-pressure calibrated cards taken in the manifold of an engine. These cards clearly show the ability of the balanced diaphragm to follow low-pressure waves.

Figure 35 is a calibrated high-pressure combustion-chamber record taken on an Allison aircraft engine running at 2,600 rpm.

Other Applications of Condenser-Plate Indicator Equipment

The principle of the condenser-plate indicator and its associated electrical equipment is not limited to pressure measurements. The technique and equipment developed for measuring pressure can be applied to studying the amplitude and wave form of small deflections of moving bodies. A few of the applications in the automotive field are listed below:

Measurement of instantaneous air flow
Measurement of stress and strain
Measurement of flywheel deflection
Measurement of cylinder head and block deflection
Obtaining valve-lift diagrams
Use in propeller-shaft balancing machines

This type of instrumentation might have the following possible uses in other engineering fields:

(1). Measurement or indication of unbalance in high-speed rotating machinery

(2). Vibration measurement

(3). Pressure surges in oil-circuit breakers

(4). Measurement of water hammer and other hydraulic surges

(5). Gun pressures

Bibliography

1. A High-Speed Engine Pressure Indicator of the Balanced Diaphragm Type, **H. C. Dickinson** and **F. B. Newell.** Bureau of Standards, N.A.C.A. Technical Report No. 107, 1921.

2. An Electrical Indicator for High-Speed Internal-Combustion Engines, **J. Obata** and **Y. Yosida.** Aeronautical Research Institute, Report, volume 2, number 28, December 1927, pages 397–405.

3. Electrical Method of Obtaining Indicator Card of a Compressor, **M. E. Fiene.** *Refrigerating Engineering*, volume 15, March 1928, pages 77–80.

4. An Electrical Engine Indicator, **E. J. Martin** and **D. F. Caris.** *Electric Journal*, volume 27, February 1930, pages 87–91, also March 1930, pages 168–72.

5. Piezo-Electric Method of Measuring Pressure Variations in Internal Combustion Engines, **H. G. I. Watson** and **D. A. Keys.** *Canadian Journal Research*, volume 6, number 3, March 1932, pages 322–31.

6. Electrical Indicator With No Inertia Using Semi-Conductor Pressure Element, **W. Glamann** and **H. Triebnigg.** *Forshung*, volume 4, number 3, May-June 1933, pages 137–46.

7. The Engine Indicator, **K. J. DeJuhasz.** New York Instruments Publishing Company.

8. A New High-Speed Engine Indicator, **E. S. Taylor** and **C. S. Draper.** *Mechanical Engineering*, March 1933, pages 169–71.

9. A New Instrument Devised for the Study of Combustion, **C. F. Taylor, C. S. Draper, E. S. Taylor,** and **G. L. Williams.** *S.A.E. Journal* (*Transactions*) volume 34, number 2, February 1934, pages 59–62.

Figure 35

10. Development and Applications of the Cathode-Ray Engine Indicator, **E. M. Dodds.** *S.A.E. Journal*, December 1936, pages 487–95.

11. High-Speed Engine Indicators, **E. S. L. Beale** and **R. Stansfield.** *Engineer*, volume 163, numbers 4233 and 4234, February 26, 1937, pages 240–1 and March 5, pages 270–3.

12. Cathode-Ray Engine-Pressure Measuring Equipment, **H. J. Schrader.** *R.C.A. Review*, volume 2, number 2, October 1937, pages 202–12.

13. Recent Developments in Engine Indicators, **E. M. Dodds.** *Journal I.A.E.*, November 1937, volume 6, number 2, pages 41–67.

14. Motion Pictures of Engine Flames Correlated With Pressure Cards, **G. M. Rassweiler** and **L. Withrow.** *S.A.E. Journal* (*Transactions*) volume 42, number 5, May 1938, pages 185–204.

15. The Cathode Ray Oscillograph and Its Application to Engine Indicators, **A. Towle.** *Journal I.A.E.*, August-September 1938, volume 6, number 10, pages 16–37.

16. Piezo-Electric Indicator. *Engineer*, volume 167, number 4334, February 3, 1939, pages 152–3.

17. Sound and Pressure Waves in Detonation, **C. E. Grinstead.** *Journal of the Aeronautical Sciences*, volume 6, number 10, August 1939, pages 412–17.

18. Erfahrungen mit Elektrischer Druckaufnahme bei Warmekraftmaschinen Von Adolf Watzinger, **Rolf Schnell Larsen.** *V.D.I.—Zeitschrift* August 1939, pages 899–901.

19. The Measurement of Pressure Movement, Acceleration and Other Mechanical Quantities by Electrostatic Systems, **C. H. W. Brookes-Smith** and **J. A. Colls.** *Journal of Scientific Instruments*, volume 16, number 12, December 1939, pages 361–6.

20. Basic Principles in the Design of Cathode-Ray Oscillograph Engine Indicators, **F. D. Smith.** Institute of Mechanical Engineers, *Journal and Proceedings*, volume 143, number 1, April 1940, pages 48–56.

21. Scientific Instruments and Apparatus. *The Engineer*, May 24, 1940, pages 471–2.

22. Condenser Type High-Speed Engine Indicator, **L. C. Roess.** *The Review of Scientific Instruments*, volume 11, number 6, June 1940, pages 183–95.

23. Patent number 537,322 (German), Siemen's Halske Oscillograph.

24. The Admiralty Cathode-Ray Oscillograph Engine Indicator, **F. D. Smith, E. H. Lakey,** and **H. Morgan.** Institute of Mechanical Engineers, *Journal and Proceedings*, volume 143, number 1, April 1940, pages 39–47.

The Geometry of Arc Interruption

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MEMBER AIEE

Synopsis: It is the purpose of this paper to present a rigorous analysis and description of the phenomenon which takes place in an interrupting device during the interval between the parting of the contacts and the current zero at which interruption takes place. It is to be shown that the events which take place during the arcing period prior to interruption play an important part in setting the stage for the drama of circuit interruption. This analysis has a counterpart in the phenomenon which takes place immediately following interruption; namely, circuit recovery characteristics. The knowledge of this latter phenomenon has played a dynamic role during the past ten years. With its aid, oil circuit breaker designs have advanced to the point where their excellent performance has set the high present-day standards of reliable circuit interruption.[2,3] The knowledge of recovery rates[4,5] laid the foundation for the displacement principle;[6] namely, oil velocity multiplied by the oil dielectric (impulse) strength is equal to the maximum permissible rate of rise of recovery voltage.

The stage today, however, is being set for interruptions with devices which employ air as the medium of interruption and insulation.[7,8] New roles are being created. Recovery voltage, although just as important, is being assigned an easier part. It is to be shown by means of a straightforward application of the principles of circuit theory, presented in simple graphical form, that the phenomenon which takes place during the arcing period can be so controlled as to reduce the initial recovery voltage, and its rate of rise, to a comparatively small value. It will be demonstrated that the arc in a particular type of air circuit breaker can be put to work to change the phase angle of the circuit from the 90-degree lagging current, associated with normal short circuits, to a leading phase angle at the current zero of arc interruption. The interval necessary to accomplish this conversion is less than one cycle. Exact arc energy relations are presented which define the watt-hours liberated in terms of the kilovolt-amperes interrupted. Although subject to a strict mathematical proof, the graphical counterpart has been chosen for presentation in which an area method of arc current determination is developed. The graphical method, with its various area relationships and energy criteria, offers opportunities of developing methods of rapid visual oscillograph interpretation. The treatment should prove of interest not only to the user of interrupting devices, but also to students of applied circuit theory.

Mental Springboards

No. 1. The Rheostat Breaker

As an entirely different approach to the problem of circuit interruption, consider a single-phase a-c breaker replaced by a rheostat, figure 1. To represent the closed breaker, all the resistance is turned out. With the development of a short circuit, let it be assumed that the rheostat handle is turned to insert more and more resistance until ultimately the resistance becomes so high as to meet the definition of an insulator. It is agreed that the current will have been interrupted. The voltage across the rheostat will have changed from zero to the full circuit voltage. The evolution of the changes which take place throughout this circuit as resistance is gradually inserted are shown in the circle diagram of figure 1. The understanding of these relationships forms a stepping stone toward the appreciation of the events which take place as resistance is inserted *rapidly* into an otherwise inductive circuit.

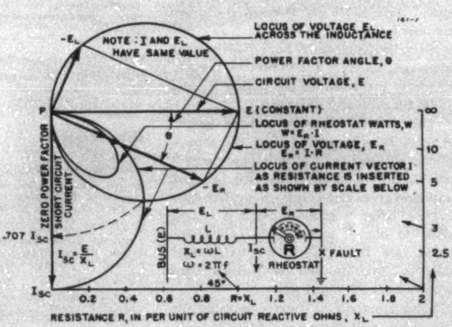
-E$_L$
NOTE: I AND E$_L$ HAVE SAME VALUE
LOCUS OF VOLTAGE E$_L$ ACROSS THE INDUCTANCE
POWER FACTOR ANGLE, θ
CIRCUIT VOLTAGE, E
P
E (CONSTANT)
LOCUS OF RHEOSTAT WATTS, W W = E$_R$·I
LOCUS OF VOLTAGE, E$_R$ E$_R$ = I·R
LOCUS OF CURRENT VECTOR I AS RESISTANCE IS INSERTED AS SHOWN BY SCALE BELOW
θ
-E$_R$
ZERO POWER FACTOR SHORT CIRCUIT CURRENT
.707 I$_{SC}$
I$_{SC}$ = E/X$_L$
I$_{SC}$
BUS (E)
E$_L$
E$_R$
L
X$_L$ = ωL
ω = 2πf
I$_{SC}$
R
RHEOSTAT
X FAULT
45°
∞
10
5
3
2.5
0.2
0.4
0.6
0.8
R=X$_L$
1.2
1.4
1.6
1.8
2
RESISTANCE R, IN PER UNIT OF CIRCUIT REACTIVE OHMS, X$_L$

Figure 1. Circle-diagram analysis of the changes which take place as resistance is inserted gradually in an otherwise inductive circuit

Note that wattage reaches its maximum value when the resistance is equal to the ohmic reactance of the circuit. At this point the current is reduced to only 70.7 per cent of its maximum value

The salient features of figure 1 are: (1) The short-circuit current, I_{sc}, prior to rheostat operation lags the generated voltage, E, by 90 degrees. This is the normal zero power-factor relation encountered in short-circuit studies. (2) The rheostat resistance, shown in the linear scale, is expressed in per unit of the inductive ohms, x_L, which in turn limited the short-circuit current, I_{sc}. (3) A straight line drawn from the point P to the desired rheostat resistance determines the direction or phase angle of the resulting current. (4) The current magnitude is defined by the semicircle designated as "the locus of the current vector, as resistance is inserted." (5) The product of this current and the corresponding resistance defines the voltage, E_R, which appears across the rheostat. This voltage is always in phase with the current and its magnitude is defined by the semicircle designated as "the locus of rheostat voltage." (6) The product of rheostat current, I, and rheostat voltage, E_R, is the rheostat wattage, W. This product is shown as the curve designated as "the locus of rheostat watts." It is of particular significance that this wattage reaches its maximum value when the rheostat resistance in ohms is equal to the ohmic value of the external reactance. (7) The reactance voltage is always at right angles to the rheostat voltage. The locus of the reactance voltage, E_L, is the upper semicircle. Note that the vector sum of the reactance and resistance voltage is at all times equal to the generated voltage, E. (8) Time, and hence energy relations, do not enter this diagram. All values presented are rms or as would be indicated by the voltmeter and ammeter.

The rheostat, like a breaker, can only introduce a resistance in the circuit.

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1. For all numbered references, see list at end of paper.

Hence, many of the above relations have an exact counterpart in the interrupting device. The above explains, for example, why the arc current and arc voltage of a breaker are always in phase, requiring the arc voltage to reverse sign with the arc current. It is of particular importance to note that as resistance is added to the zero power-factor circuit, the current is reduced, and at the same time, its angle with respect to the voltage is decreased. Moreover, as these changes take place, the wattage increases and becomes a maximum at the 45-degree angle, as mentioned above. Note that at this point the current is reduced to only 70.7 per cent of its maximum short-circuit value. As the resistance is increased above the value of x_L, the current, phase angle, reactance voltage, and wattage all decrease. These changes all favor more desirable interrupting conditions. Figure 1 teaches that as resistance is added gradually to the inductive circuit the current and voltage approach unity power factor or zero phase-angle relations only as the resistance approaches infinity and the current approaches zero. It will be demonstrated later that desirable leading phase-angle conditions are created when resistance is added rapidly.

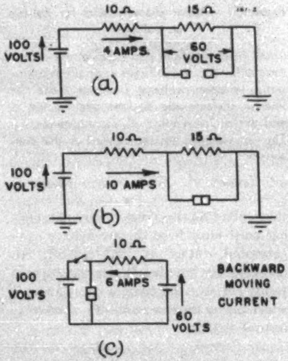

Figure 2. Simple battery circuits which demonstrate that the voltage drop across any element may be used to produce a backward-moving current (figure 2c), which when subtracted from the forward-moving current (figure 2b), produces the net current (figure 2a)

No. 2. Superposition

Consider the simple battery circuit of figure 2*a*. Here a 100-volt battery delivers 4 amperes to a 10-ohm and 15-ohm resistor in series. The voltage drop across the 10-ohm resistor is minus 40 volts, while the voltage drop across the 15-ohm resistor is minus 60 volts. It is of value to compute the current of this circuit in a slightly different manner. For example, let the 15-ohm resistor be shorted out by the primary contacts as shown in figure 2*b*. The resulting current is then ten amperes. Now short the battery out and compute the back moving current produced by the above 60-volt drop, shown in figure 2*c*, as a 60-volt battery. This produces a backward moving current of six amperes. The net current flowing is hence ten minus six, or four amperes.

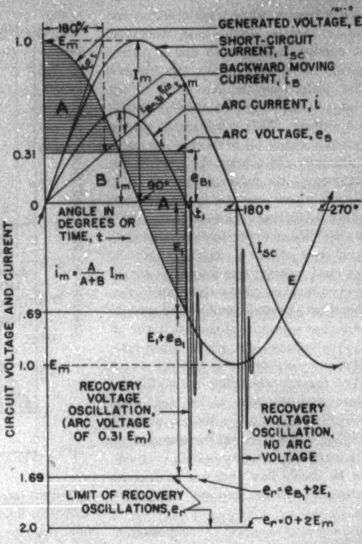

Figure 3. Keynote diagram showing the geometry of arc-current determination

This figure presents a comparison between the associated arc currents, *i*, and the initial recovery voltage, e_r, resulting from two arc voltages; (1) zero value and (2) a flat-top arc voltage of 31 per cent of the generated voltage crest. The entire diagram can be constructed from the generated voltage with the aid of a straight edge and planimeter

This simple procedure embodies perfectly sound principles of superposition. Too little use is made of the advantages it offers. In addition to being particularly useful in facilitating the computation of more difficult circuits than the above, its application will be found of value as an aid in visualizing the accompanying phenomena. It is of particular value when the voltage being studied appears across a circuit parameter which is nonlinear such as the arc of a tube or circuit breaker. As applied to circuit interrupting devices, it states that the voltage produced by the arc of an interrupting device can be used to compute a backward-moving current, which, when subtracted from the short-circuit current that would have existed had not the breaker been tripped, will result in the true arc or circuit current. The voltage across the breaker may vary in any manner whatsoever without altering the validity of the solution. The parameters of the circuit external to the breaker, however, must remain linear. The only parameter external to the breaker for the majority of heavy fault currents is the inductive reactance of the circuit as shown in figure 1. When the arc voltage, and hence the arc current are available, their product determines arc wattage and their ratio arc resistance. Hence, from a design standpoint it becomes possible to investigate various types or families of arc voltage characteristics with the aim of determining which characteristic is most desirable in an interrupting device. Such an examination will now be made.

Determination of Arc Current, Arc Energy, and Maximum Initial Recovery Voltage

Case I. Rectangular or Flat-Top Arc Voltages

With the primary contacts of the breaker closed, the symmetrical short-circuit current, I_{sc}, lags the generated voltage, *E*, by 90 degrees (figure 3). The current, I_{sc}, reaches its peak as the voltage, *E*, passes through the zero value of breaker voltage. This same current would continue to flow in the arc of an opened breaker which developed no arc voltage. Hence, in this case, when the arc current reaches zero and is interrupted, the voltage across the breaker must spring from the zero value of arc voltage to the then crest value of generated voltage. This condition is closely approximated in oil breakers. In negotiating this sudden jump the inherent capacitance of the external circuit comes into action and gives rise to the recovery voltage oscillation. This oscillation permits the voltage to rise to double the generated crest voltage as shown in figure 3. This crest voltage, which appears across the open breaker, will be designated as e_r throughout this study and will be used in addition as a measure of the rate of rise of recovery voltage, over which it has control in any given circuit.

Any breaker which has the ability to reduce the value of e_r to less than this maximum value of $2E_m$ will enjoy a corresponding increase in interrupting ability or in its factors of safety or both. It will now be shown how this can be accomplished through the medium of arc voltage.

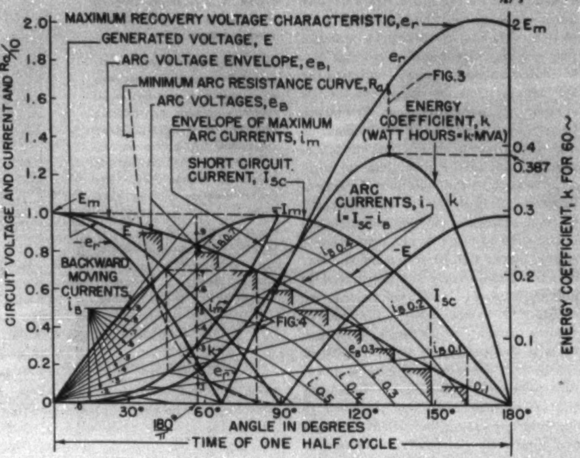

Figure 5. Master characteristics for flat-top arc voltages

These relations show the effect of the magnitude of flat-top arc voltages upon the resulting initial recovery voltage peak, e_r; the arc energy characteristic k; the arc current i; and the minimum value of arc resistance R_a. The arc voltages are introduced as the symmetrical short-circuit current passes through zero

Consider the contacts of the breaker of figure 3 to part as the short-circuit current passes through zero as indicated. Assume that the arc voltage, e_B, which develops throughout the passage of the next loop of current is 31 per cent of the crest of the generated voltage. As previously pointed out this voltage acts to produce a backward-moving current, i_B. For the case of a constant or flat-top arc voltage of $0.31E_m$, acting alone on an inductance L, the current produced rises linearly and at a rate of $0.31E_mt/L$, which is 31 per cent of the initial rate of rise of the short-circuit current, I_{sc}, following a current zero. These relations are all shown in figure 3. Since this current is opposing the original short-circuit current, the true net arc current is expressed in figure 3 as the difference between I_{sc} and i_B. This new current is plotted as i. This current reaches its crest when the arc voltage e_B is equal to the generated voltage. This follows from Kirchhoff's law since at this moment the voltage across the inductance L must be zero. The arc current i arrives at a current zero when the backward-moving current i_B is equal to the forward-moving short-circuit current I_{sc}. This current zero occurs about 45 degrees earlier because of the action of the arc voltage. If interruption takes place at this current zero the voltage must jump from the arc voltage e_B to generated voltage at that instant. In making this transfer it has the opportunity of overshooting a like amount which places the new value of e_r at $1.69E_m$ instead of $2E_m$.

Arc Energy. Arc wattage is the product of arc current i and the arc voltage e_B. The latter, in this case, being a constant makes the true wattage proportional to the arc current. Arc energy in watt-seconds is equal to the area under the wattage-time curve. In this case the energy in watt-seconds is equal to the area under the current curve multiplied by $0.31E_m$. This case gives the maximum energy for the flat-top arc voltages under the conditions of figure 3. Had the arc voltage been higher or lower than $0.31E_m$ the energy would have been less. In this case the arc energy converted to watt-hours gives:

watt-hours = 0.387 MVA (60~)

where MVA = megavolt-amperes interrupted.* Hence the interruption of

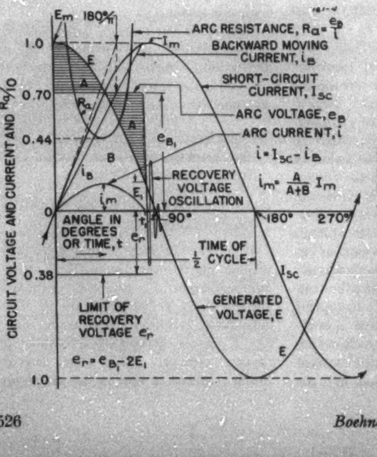

Figure 4. Leading current interruption produced by a flat-top arc voltage of 70 per cent of the generated voltage crest

An oscillogram of this desirable type of interruption is exhibited in figure 12a. The cathode-ray oscillogram of figure 12b shows that the type of recovery oscillation sketched here is so highly damped that the voltage limit, e_r, was never obtained

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*100,000 kva = 100 MVA.

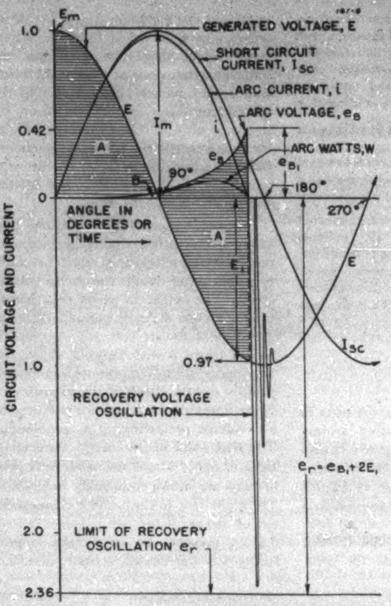

Figure 6. An arc voltage characteristic, e_B, which produces the same arc energy as the case of figure 4

Note the high initial recovery voltage, e_r, of $2.36E_m$ as compared with the corresponding value of $0.38\,E_m$ for figure 4

25,000 amperes at 4,000 volts single phase would release only 39 watt-hours at this maximum energy value. In general, the watt-hours released during any interval will be expressed by the relation:

$$\text{watt-hours} = k\cdot \text{MVA} \tag{1}$$

The relationship of k for the above family of flat-top arc voltages is shown in figure 5.

Graphical Solution. The entire solution of figure 3 can be made with a planimeter and slide rule once the arc voltage characteristic is established. The maximum value of the arc current i is given by the ratio of areas:

$$i_m = \frac{A}{A+B} I_m \tag{2}$$

This relation follows from the fact that the area represented by

$$\int_0^{t}(E-e_B)dt \text{ must equal } L\cdot i=\psi \tag{3}$$

The current at any time t' can hence be determined from the above by measuring that portion of the area A up to t'; namely A'. The current at that instant is then:

$$i' = \frac{A'}{A+B} I_m \tag{4}$$

It follows that when the area between E and e_B is again A, following arc current peak, the current must again arrive at zero. This satisfies the flux linkages of the inductance L, the momentum of the magnetic circuit.

The planimeter solution is particularly suitable when the arc voltage varies in any irregular manner. After computing the wattage curve the planimeter becomes useful again in computing the energies. The majority of the relations developed for this presentation were computed graphically. It is interesting to note that in the case of the flat-top arc voltage of figure 3, it can be proved that maximum energy occurred when the area under the backward-moving current triangle omt_1 is half the area under the short-circuit current I_{sc} up to the instant of arc current zero at t_1.

Leading Phase-Angle Interruptions. It has been shown that the flat-top arc voltage of $0.31E_m$ reduces the initial peak of recovery voltage from $2E_m$ to $1.69E_m$. As higher flat-top arc voltages are assumed this initial recovery peak shrinks rapidly. Figure 4 shows a graphical solution of the case where the arc voltage is $0.70E_m$. Here the backward-moving current i_B intersects the forward-moving short-circuit current I_{sc} at such an early point that the difference gives rise to an arc current i which arrives at an interrupting current zero nine degrees ahead

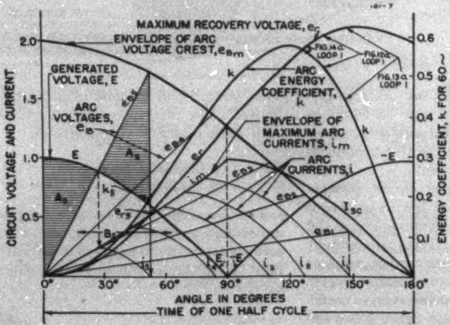

Figure 7. Master characteristics showing the development of arc current, i; initial recovery-voltage peak, e_r; and arc-energy characteristic, k; as a result of linearly rising arc voltages

The arc voltages are introduced as the symmetrical short-circuit current passes through zero

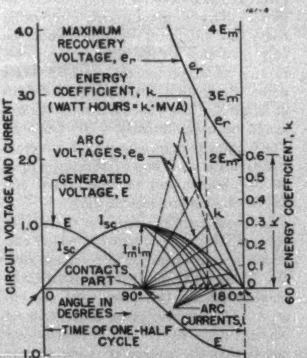

Figure 8. Master characteristics produced by linearly rising arc voltages introduced as the symmetrical zero-power-factor short-circuit current passes through its maximum value. Compare with figure 7

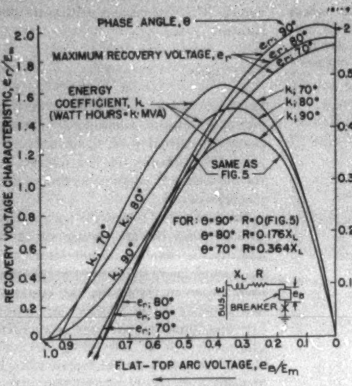

Figure 9. Characteristics showing the effect of external circuit resistance upon the maximum initial recovery-voltage characteristic, e_r, and upon the energy coefficient, k

Note that as the phase angle is decreased due to external resistance the resulting value of e_r is reduced, while the value of k is increased

These relations were obtained with flat-top arc voltages applied as the symmetrical short-circuit current passes through zero. Direct comparison should be made with figure 5

of the voltage zero. Hence at interruption the voltage drops from $0.70E_m$ arc voltage to the generated voltage which, at that instant, is $0.16E_m$. This gives the voltage oscillation an opportunity to reach an initial peak, e_r, of only $0.38E_m$. The damping of the voltage oscillations, however, in modern interrupting devices which are capable of accomplishing this phase-angle change, is so great that the recovery from arc voltage to generator voltage takes place with little or no oscillation. Figure 12*b* shows a cathode-ray oscillogram of the nature of such a transition following the leading phase-angle interruption presented in figure 12*a*.

Arc resistance, the ratio of e_B/i, is plotted on figure 4. The minimum resistance reached was only 4.4 times the external reactive ohms which limited the initial short-circuit current. In actual ohms this minimum resistance would be only 0.70 ohm in the case of the interruption of 25,000 amperes at 4,000 volts under the leading phase-angle conditions of figure 4. The rms current in the arc would be approximately 4,000 amperes. The k value to determine watt-hours from equation 1 is 0.147 hence liberating a total of 14.7 watt-hours for the above 100 MVA, single-phase interruption.

It is interesting to contrast the case of figure 4 with that of figure 6 which embodies a case where the arc voltage rises late in the cycle as shown. Both cases liberate exactly the same total arc energy. The leading phase-angle case of figure 4 gives rise to the extremely low initial recovery voltage of $0.38E_m$ while the corresponding value for the almost zero power-factor interruption of figure 6 is $2.36E_m$. These relations correspondingly define the relative recovery rates associated with the two interruptions. The low arc current of figure 4 results in less metal vaporization during arcing. The shorter arcing time of figure 4 is also in favor of less wear and deterioration.

Characteristic Curves. Just as figures 3 and 4 show the resulting arc currents and recovery voltages as a result of flat-top arc voltages of $0.31E_m$ and $0.70E_m$ respectively, so figure 5 shows the complete story for all flat-top arc voltages. The lines of figure 4 can be found in figure 5 since the latter is constructed by assuming arc voltages of 10 per cent, 20 per cent, ... 70 per cent . . . . . 90 per cent, of the maximum circuit voltage. The corresponding arc currents are shown plotted only for that portion following the arc current peaks. This arrangement permits the locus of arc current peaks to be drawn as shown. The maximum recovery voltage e_r and the energy coefficient k of figure 3 and figure 4 become points on the complete e_r and k characteristic curve shown in figure 5. These latter values should be read from the horizontal time scale at the instant which they occur; namely, coincident with the arc current zero. The minimum arc resistance curve is shown with respect to the time at which the particular minimum value occurred; namely, at the arc current crest corresponding to the flat-top voltage under consideration.

An examination of figure 5 shows that as arc voltage is introduced the maximum recovery characteristic e_r starts to climb to values above double the circuit voltage crest. The height to which this characteristic climbs depends particularly upon the shape of the arc voltage. A comparison of figure 6 with figure 5 shows the flat-top arc voltage to produce a minimum peak of e_r. A study of the geometry of figure 6 reveals the reason for this type of arc voltage producing an e_r of $2.36E_m$. The peak value of the energy characteristic k of figure 5 and the associated phenomena are shown completely in figure 3. The rapid drop in the e_r and k characteristic of figure 5 for arc voltages above $0.31E_m$ presents an attractive opportunity for the circuit breaker designer. The oscillographic exhibits in this paper present a new interrupter which functions with these desirable characteristics.

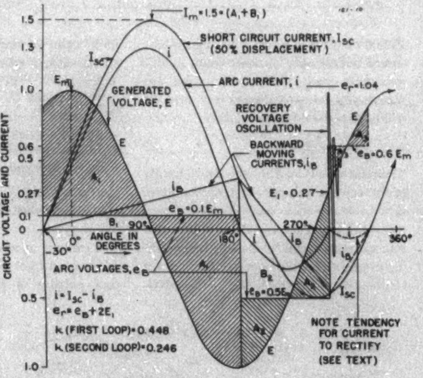

Figure 10. Graphical solution of a multi-half-cycle interruption for a zero-power-factor short-circuit current having 50 per cent displacement

Flat-top arc voltages of $0.1E_m$ and $-0.5E_m$ are applied respectively for the first and second loops. A third current loop is made impossible by the current rectification produced by any flat-top arc voltage greater than plus $0.27E_m$

CASE II. LINEARLY RISING ARC-VOLTAGE CHARACTERISTICS

The foregoing treatment of the flat-top arc voltage as presented in figures 3, 4, and 5 is applicable to any type of arc voltage. When the arc voltage rises linearly, as

$$e_B = K_e t \qquad (5)$$

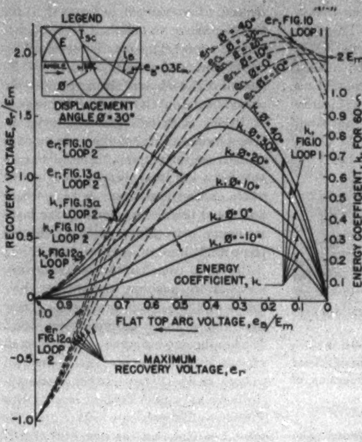

Figure 11. Characteristics of maximum initial recovery voltage, e_r, and energy coefficient, k, for flat-top arc voltages introduced at current zeros which occur at the indicated angles ahead of the circuit voltage crests

These characteristics are particularly useful in compounding the analysis of multi-half-cycle arcs

the backward-moving current is more complex, following the equation

$$i_B=\frac{K_0}{2\cdot L}t^2 \tag{6}$$

For this reason the graphical solution (area method) is often more rapid. The wattage curve is more complex inasmuch as neither the arc current nor the arc voltage is constant. The energy computation, however, is just the same, namely

$$W=\int_0^t e_B\cdot i\cdot dt \tag{7}$$

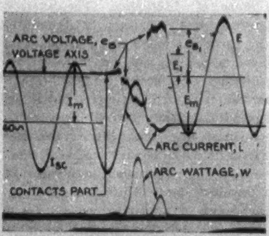

Figure 12a. Oscillogram showing how the Magne-Blast breaker shifts the current zero from 90 degrees behind the voltage to 21 degrees ahead of the voltage in a time of less than one cycle

Current: 13,800 amperes. Voltage: 3,920. The analysis of the first loop can be obtained as indicated on figure 7. This produces a phase displacement of 41 degrees. The second loop can then be analyzed as shown on figure 11. The bottom trace shows arc watts. Measured energies for the two loops give 29.3 watt-hours and 9.8 watt-hours, respectively. Calculated values give 28.4 and 10.2 watt-hours, respectively

The graphical computation of energies is recommended for speed.

Figure 7 presents the complete evolution of the linearly rising arc voltage starting from current zero of a symmetrical short circuit. This case shows slightly different k and e_r characteristics than the flat-top wave. Inspection will show the flat-top wave to be superior to the linearly rising arc voltage.

Case III. Contact Parting at Other Than Current Zero

Figure 8 presents the characteristics which arise as a result of linearly increasing arc voltages which develop as the contacts are parted at the current crest. The graphical solution was made, as previously outlined, keeping in mind that the areas must be balanced at the end of each current zero, as shown in figure 6. This case is important as it is useful in compounding the complete story of the multi-half-cycle interruptions to be studied in the oscillographic cases to be presented.

Case IV. Effects of External Resistance

Short-circuit current studies are usually made neglecting the effects of circuit resistance. For this reason the values of current calculated are usually higher than normally experienced. Under actual conditions, however, fault currents can occur which instead of being 90 degrees behind the generated voltage are only 80 degrees or 70 degrees behind due to the effects of the circuit resistance and the effective resistance of the fault itself. Figure 9 presents the k and e_r characteristics resulting from flat-top arc voltages produced as the interrupting device parts its contacts at a current zero. In this case, the arc voltage produces a backward-moving current, i_B, which is limited by a resistance R in addition to the inductance L. It will be observed that there is beneficial decrease in the values of e_r over those shown in figure 5, the corresponding case without external resistance. This decrease in e_r has been proved oscillographically and shows the presence of external resistance to be helpful. This has been found to be true in spite of the corresponding increase of the energy coefficient k as shown in figure 9.

The values of figure 9 are plotted with respect to the magnitude of the flat-top arc voltage and in this respect differs from figures 5 and 7. The e_r and k characteristics of figure 5 are replotted on figure 9 to the new scale. These are shown as the 90-degree phase-angle curves and are hence available for direct comparison with the corresponding characteristics for the 80-degree and 70-degree phase-angle curves. For all curves the current has been kept at 100 per cent; that is, in spite of the addition of resistance the current has not been reduced. In order to execute the area method in this case it will be necessary to add to the area A the corresponding area under the voltage which appears across the external resistance. For this reason this case is often more readily solved by computing the backward-moving current and subtracting same from the short-circuit current to obtain the true arc current.

Case V. Arc Currents of More Than One-Half Cycle

The principles outlined above are applicable to arcs of more than one-half cycle. Following the current zero of one loop of arc current, the arc voltage reverses sign with the current. The new

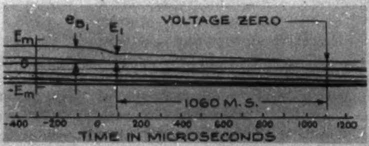

Figure 12b. Cathode-ray oscillogram showing the transition from arc voltage to circuit voltage accompanying the interruption shown in figure 12a. Note the absence of voltage oscillation

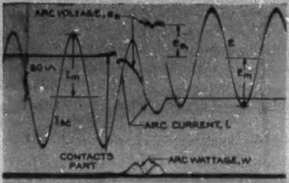

Figure 13a. Oscillogram showing how the Magne-Blast breaker shifts the interrupting current zero 90 degrees to coincide with the generated voltage zero

Current: 5,270 amperes. Voltage: 3,900. The analysis of this oscillogram is indicated on figure 7 (first loop) and figure 11 (second loop). The bottom trace shows arc watts. Measured energies for the two loops give 9.25 and 8.22 watt-hours respectively, while calculated values give 9.45 and 7.95 watt-hours respectively

arc voltage then creates a new backward-moving current which acts as outlined above. Figure 10 shows a graphical solution, which in addition assumes a short-circuit current which is not symmetrical. Unsymmetrical currents are produced in totally reactive circuits when the short-circuit current is initiated at any instant at which the voltage is not at its maximum value. The case of figure 10 shows a 50% displacement at contact parting. The initial arc voltage is $0.1E_m$ of the flat-top variety which reduces the arc current crest from $1.5I_m$ to $1.30I_m$. When the areas balance the current arrives at zero producing an e_r of almost $2.1E_m$; $k=0.448$. The arc current continues in a new environment where the arc voltage is now $-0.5E_m$. This produces a backward-moving current whose slope is now

$$i_B = \frac{-0.5E_m}{L}t \qquad (8)$$

This current is shown in figure 10 and starts where the old backward-moving current of the previous loop left off. The resulting second loop of current peaks at $0.3\ I_m$ and when the net voltage areas balance again, a new current zero arrives. Here the new e_r is only $1.04E_m$ following a k of only 0.246. Here interruption takes place. It is interesting to note that if the current attempted to reverse and produce a flat-top arc voltage above $0.27E_m$, then the resulting current would not have reversed, but would have rectified as shown in figure 10 for an arc voltage of $0.60E_m$. However, if the current cannot reverse, neither can the arc voltage. These conditions, hence, define an impossible state of affairs. Tests show that when this condition tends to exist that interruption invariably occurs. This leads to a new and interesting criterion for interruption which for complete analysis must not neglect the inherent capacitance of the external circuit. The graphical analysis presented in figure 10 reveals a technique which may be applied to any multi-half-cycle interruption of zero power-factor short-circuit current. The oscillograms presented in figures 12, 13, and 14 may be studied with the above in mind.

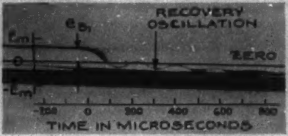

Figure 13b. Cathode-ray oscillogram showing the transition from arc voltage to generated voltage associated with the interruption of figure 13a

Multi-half-cycle arcs are also associated with the interruption of symmetrical short-circuit current. Their analysis follows the pattern as developed in figure 10. The initial loop of arc current, which has been modified by its arc voltage, will have shifted the phase angle of the circuit at current zero an amount depending upon the magnitude and shape of the arc voltage. See figures 5, 7, and 10. This shift sets up the phase angle at which the second loop of current must start. An inspection of the area method of arc current determination shows that it is of advantage to keep the initial loop of arc voltage low if a second loop is to be experienced. For example, in figure 10, if the first loop of arc voltage had been $0.05E_m$, instead of $0.1E_m$, then the second loop would have ended with an e_r of $0.78E_m$ instead of $1.04E_m$ and the k value for the second loop would have been 0.144 instead of 0.246. This fact points directly to design trends which have been mastered in modern air interrupters,[8] namely to provide for a low resistance arcing chamber where an arc in air may develop length and speed prior to its entering the high-resistance path where zero and leading phase-angle interruptions are created.

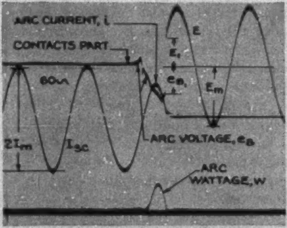

Figure 14a. Oscillogram showing how the Magne-Blast breaker interrupts 20,600 amperes at 3,820 volts, single phase, with a single loop of arc current

The analysis of this interruption is indicated on figure 7. The bottom trace of arc watts measures 44.6 watt-hours while the calculated value is 43.3 watt-hours

Hence, to complete the data necessary for accurate analysis, the characteristic curves of figure 11 are presented. These show the e_r and k characteristics which develop for flat-top waves which start at various angles ahead of the generated voltage peaks. These displacement angles may have been created in a variety of ways by the previous loop of arc voltage. The fact that the new loop of current is dependent only upon the phase angle and not upon how that phase angle was obtained, makes the data of figure 11 extremely useful in compiling the analysis of design or oscillographic data. This is particularly true for the oscillographic cases to be considered here since the second loop of arc voltage is almost invariably of the desirable flat-top variety. A comparison of the data in figure 11 with the equivalent phase-angle changes brought about by the presence of external resistance, figure 9, will be found of interest.

Scope of Application. The analysis presented in this paper is not limited to any particular interrupter. It has been found useful not only in analyzing the performance and design features of certain arc chutes of air interrupters, but it has found its application to oil interrupters, contactors, fuses, and lightning arresters. The examples to be presented here, however, are unique inasmuch as they exhibit a degree of performance unequaled by any device of similar rating.

The method also applies to other studies which do not involve the problem of interruption. For example, the decre-

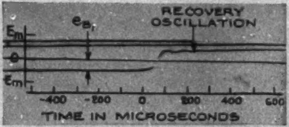

Figure 14b. Cathode-ray oscillogram showing the transition from arc voltage to circuit voltage accompanying the interruption shown in figure 14a

ment factors for the attenuation of unsymmetrical short-circuit currents due to the resistance of an arcing ground may be developed with considerable speed by the above application of multi-half-cycle arc phenomena. Also the problem of the magnitude and shape combinations of the voltage across an electric arc furnace may be studied with the aim of determining which combination gives the maximum heat output in an electric furnace. The above method is particularly adaptable to this problem during the burning-in period where steady-state conditions are not maintained.

Application to Oil Circuit Breakers. It is beyond the scope of this paper to discuss thoroughly the application of this treatment to oil circuit-breaker performance. A review of the above, however, with respect to the typical oscillograms of oil circuit-breaker performance shows that inherently this well-established interrupter does not and should not develop a high arc voltage. The oil device functions best when its arc gap, following interruption, is insulated with clean, cool oil. The development of arc voltage, as demonstrated by old plain break breakers, produced energy (k) and recovery voltage (e_r) characteristics approaching those shown to the right of figures 7 and 8 for the first half cycle of arc. This energy unquestionably blew the oil away from the arc and did not permit the oil, unaided, to reinsulate the arc gap rapidly enough to cope with the arrival of the first peak of recovery voltage (e_r). The arc, on continuing, produced more energy on the next half cycle due to the phase-angle shift created on the first half cycle as indicated by figure 11. This repeated the blowing of oil away from the contacts at the same time heating the oil and the pocketed gas. The result was that interruption could not take place until the contacts became so far apart that re-establishment was impossible. This gave rise to the old long arcing breakers.

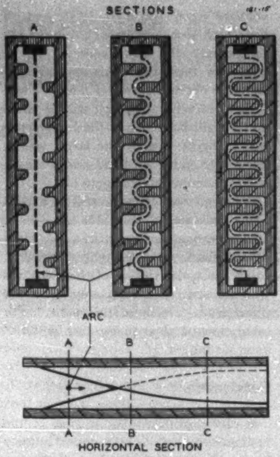

Figure 15. Arc-chute construction of the Magne-Blast air circuit breaker

This figure is repeated here for the convenience of the reader in appreciating how the high arc voltages of figures 12, 13, and 14 were obtained

With the introduction of the oil blast principle[9] the little energy developed during the first half cycle of arc was put to work to blast clean fresh oil across the other contact. These conditions lead to an early interruption with the minimum of developed energy. Arc lengths and arc voltages are hence held to a minimum. The impulse breaker[10], which creates its oil blast by means of an auxiliary piston, creates interruptions at the first current zero following contact separation with extremely low arc voltages. This breaker[4] follows the displacement principle as mentioned in the synopsis of this paper.

Magne-Blast Performance. The Magne-Blast air circuit breaker was described before the Institute in 1939[8] and its extension to larger interrupting capacities is announced with this paper. The 1939 paper described a 150,000-kva, 5,000-volt breaker which produced phase-angle changes from 40 to 50 degrees with accompanying reductions in rates of rise of recovery voltage. The present announcement of a 7,500-volt, 250,000-kva breaker has been made possible by close study of test and design criteria. The new device produces phase-angle changes which exceed 90 degrees and hence gives rise to leading phase-angle interruptions with recovery voltage characteristics of such a nature as to require that new definitions be established to describe them. The operation of the new and larger breaker is the same as previously described[8] with the addition of an improved means of extinguishing the light currents rapidly. Table I presents a typical series of three-phase tests, at various voltages, showing interrupting times well below an over-all time of eight cycles for all currents from transforme magnetizing value to well above the interrupting rating. Particular attention should be paid to the short arcing times for the higher current interruptions. The advantages of this feature are obvious from the text of this paper.

The oscillograms presented exhibit the single-phase performance at approximately 4,000 volts. In addition to the normal oscillographic records of arc voltage and arc current, a new oscillograph record is shown; namely, arc watts. This trace makes available on inspection the relative energies in adjacent loops of arc current. The oscillograms presented were chosen more to present the characteristics of performance rather than demonstrate high interrupting capacity. The latter is adequately presented in table I.

Figure 12*a* shows the interruption of 13,800 amperes, single phase, at 3,920 volts. Neglecting the minor positive kick of arc voltage, the first major arc voltage can be approximated from the triangular

Table I. Interrupting Tests on 1,200-Ampere Indoor-Type Magne-Blast Air Circuit Breaker

Interrupting Rating—250,000 Kva, Three Phase

	Current Amperes in Max. Phase				
Test Volts	Max. Loop Peak Value	Initial in Arc RMS	Kva Three Phase	Arcing Time—Cycles	Cycles From Trip Impulse to Interruption
7,250	12.0	8.4	Transformer	0.6	4.7
7,250	45	31	magnetizing	1.2	5.3
7,250	175	121	currents	1.9	6.0
7,250	870	330		1.9	6.0
7,500	1,690	830	10,800	2.1	6.5
7,500	4,200	2,000	26,000	1.5	6.0
7,500	12,500	5,300	71,000	0.9	5.8
7,500	23,000	9,500	124,000	0.9	5.8
7,500	37,000	14,500	188,000	0.8	5.0
7,500	45,000	21,000	270,000	0.8	5.3
6,600	1,260	640	7,300	2.5	6.7
6,600	2,700	1,400	16,000	1.9	6.3
6,600	12,000	5,200	59,000	1.1	5.3
6,600	34,000	13,600	155,000	0.8	5.3
6,600	58,000	23,000	260,000	0.5	4.8
6,600	72,000	26,000	300,000	0.5	4.8
4,200	70,000	34,000	250,000	0.9	4.4
4,200	97,000	38,000	280,000	0.6	4.0
2,300	77,000	45,000	180,000	0.6	4.9

case of figure 7. This point of figure 12*a* is indicated on figure 7 and gives an e_r voltage of 11,380, a k of 0.528, and produces a phase displacement of 41 degrees. Using this latter displacement as a starting point for the second loop of current we turn to figure 11. Approximating the arc voltage of the second loop as $0.84E_m$, the final loop ends with an e_r of 555, a k of 0.190, and a phase displacement of 21 degrees. This shows a multi-half-cycle case which results in a leading phase-angle interruption as shown. A comparison with figure 4 is instructive. The measured area under the wattage loops shown on figure 12*a* produce 29.3 and 9.8 watt-hours, while the insertion of the above estimated k values in equation 1 produce 28.4 and 10.2 watt-hours respectively.

The maximum e_r value of 555 volts at interruption never occurred due to the high damping of the recovery oscillation, a condition created by the breaker.[8] Figure 12*b* presents the cathode-ray oscillogram of the transition from arc voltage to generated voltage at interruption. The transition is accompanied by no oscillation. The generated voltage then floats through zero some 1,060 microseconds later as shown. Since not even the recovery voltage changed sign at current zero, it follows by the area method that current continuation is impossible. The inherent recovery rate of the circuit under the conditions of figure 12 is 1,100 volts per microsecond. The measured recovery rise for figure 12*b* is less than 1.0 volt per microsecond. The ability of the Magne-Blast breaker to create and maintain arc voltages of this type lies in the novel construction[8] of the new interleaving arc chute, shown in figure 15. This arc chute is designed with one particular objective in mind; namely, to create and maintain high arc voltages.

The analysis of the oscillograms presented in figures 13 and 14 may be carried out as in the case of figure 12. The captions describe the measured watt-hours and other necessary information for the recalculation of the oscillograms from the curves presented in this paper. It is of particular interest to study the wattage trace of figure 13 with respect to the nature of the arc currents and arc voltages upon which it is dependent. Here both loops of current produced approximately the same energy. The k points for these two loops are indicated on the k characteristics of figures 7 and 11, respectively. The first will be found on the right-hand side of the k characteristic of figure 7 while the latter will be found over on the left-hand side of the 30-degree characteristic of figure 11.

Although the cases presented exhibit single-phase performance, the three-phase analysis is just as straightforward. It is beyond the scope of this paper, however, to present a description of their analysis which is merely a compounding of the principles set forth in this paper. For tabulation of three-phase test performance, see table I.

Conclusions

A straightforward application of the fundamentals of electric circuit theory has been made to the analysis of the pre-interrupting phenomena of circuit interrupters. It has been shown that the arc voltage developed by any interrupting device may be considered, quantitatively as well as visually, to produce a backward-moving component of current. This current when subtracted from the initial forward-moving current produces the true resulting arc current. Arc wattages, arc energies, arc resistances, and phase angles are then immediately available. The dependence of these latter quantities in any given circuit upon the arc voltage has made possible the presentation of these relations for various families of arc voltage characteristics. Other curves are presented which enable the fundamental criteria of circuit interruption to be obtained for a wide variety of performance characteristics. Consideration has been given to symmetrical and unsymmetrical short circuits, the effects of external resistance, arcs of more than one half cycle, and other practical cases.

Specific application is made to the unique performance characteristics of the Magne-Blast air circuit breaker.[8] It is shown how this device, through the medium of high arc voltage, transforms a 90-degree lagging short-circuit current to a leading current at interruption with less than one cycle of arcing. Initial recovery voltages are expressed showing enormous reductions. Exact energy relations are presented which express the watt-hours liberated in terms of the kilovolt-amperes of the interruption. The study of the unique Magne-Blast breaker performance characteristics first aided the development of this treatment and in turn has been aided in its extension to higher interrupting ratings by the use of this treatment as a design tool.

Although applied specifically to the Magne-Blast breaker, the principles presented are applicable to oil breakers, contactors, fuses, and other interrupters, both a-c and d-c. It also finds its application to problems apart from interrupting devices, particularly when the voltage across a nonlinear parameter is to be studied. The treatment should prove of interest not only to the user of interrupting devices, but also to students of applied circuit theory.

Bibliography

1. An Analysis of Alternating Circuit Interruptions With Particular Reference to the Interruptions of Heavy Currents in Air, **E. W. Boehne.** Professional Thesis, Agricultural and Mechanical College of Texas, April 1940.

2. Ultra High-Speed Reclosing of Transmission Lines, **D. C. Prince** and **Philip Sporn.** AIEE Transactions, volume 56, 1937 (January section), pages 81–90.

3. A New Multi-Break Interrupter for Fast-Clearing Circuit Breakers, **R. M. Spurck** and **H. E. Strang.** AIEE Transactions, volume 57, 1938 (December section), pages 705–10.

4. Circuit Breaker Recovery Voltages, **R. H. Park** and **W. F. Skeats.** AIEE Transactions, volume 50, 1931, pages 204–38.

5. Determination of Circuit Recovery Rates, **E. W. Boehne.** AIEE Transactions, volume 54, 1935 (May section), pages 530–9.

6. The Theory of Oil-Blast Circuit Breakers, **D. C. Prince.** AIEE Transactions, volume 51, 1932, pages 506–12.

7. Design and Construction of High-Capacity Air-Blast Circuit Breakers, **H. E. Strang** and **A. C. Boisseau.** AIEE Transactions, volume 59, 1940 (September section), pages 522–7.

8. Magne-Blast Air Circuit Breaker for 5,000-Volt Service, **E. W. Boehne** and **L. J. Linde.** AIEE Transactions, volume 59, 1940 (April section), pages 202–12.

9. Oil-Blast Breaker Proved Experimentally, **D. C. Prince** and **E. J. Poitras.** *Electrical World*, volume 97, February 28, 1931.

10. The Impulse High-Speed Circuit Breaker, **W. K. Rankin.** *General Electric Review*, October 1931, page 553.

An Experimental Investigation of Subharmonic Currents

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THIS investigation of the series circuit shown in figure 1, consisting of resistance, capacitance, and nonlinear inductance, was undertaken with a view of determining the behavior of the subharmonic current which may result when a sine-wave voltage of fundamental frequency is applied. The magnetization characteristic of the inductor is given in figure 2, and all results, therefore, are associated with this one particular inductor. All values of resistance are ohmic values.

The behavior of this circuit in a state of subharmonic oscillation is not well known. The increasing use of series capacitors to overcome the inductive reactance drop of transmission systems has brought attention to the inadequate knowledge of the occasionally abnormal behavior of these networks. This application of capacitors usually brings about improved and automatic voltage regulation; however, unusual transient and sustained overcurrent phenomena often result in the system for certain operating conditions. Instead of normal sinusoidal currents there will be currents greatly increased in magnitude, and having a variety of wave shapes of most unusual form. Since series capacitors are nearly always associated with one or more transformers, this type of network may be reduced to a simple series arrangement of capacitance, resistance, and an iron-core inductor whose inductance may not be considered constant. It is the simplest circuit known to exhibit the peculiarities under investigation.

The difficulties encountered in any attempt to analyze mathematically the behavior of this circuit arise from the fact that the inductance is a function of the current. This results in a differential equation which is nonlinear, and therefore not solvable by ordinary methods. It is also necessary to make certain assumptions which might not be justified were all the basic facts known.

Previous investigators resorted to the use of a mechanical differential analyzer to solve certain of these equations.[1,2,3] Idealizing assumptions were made neglecting resistance or any form of dissipation, and neglecting hysteresis and eddy current effects. Fallou[4] has suggested the use of this circuit as a frequency divider. Investigation by others[5–15] was restricted to what has been commonly called "ferroresonance" with only a brief mention of a possible subharmonic response. The location on the voltage-current diagram of subharmonic currents, particularly with reference to the two states of ferroresonance, has been uncertain. The voltage-current diagram is the plot of the rms value of impressed voltage versus the rms value of the current. There is also a confused difference of opinion regarding the effect of initial conditions on the ultimate behavior of the response. Travis and Weygandt[1] give analytical substantiation to the hypothesis that initial conditions, such as switching angle (the point on the voltage wave at which the switch is closed) are important in determining the subharmonic response of the circuit at a time considerably after any switching transient would have died out. Weber's discussion of this paper concludes that switching angle has nothing to do with the ultimate response of current and states further that, "Otherwise any practical use of the relay circuits based upon the critical features of the ferroresonant element would be incomprehensible." Obviously, confusion exists, since the state of subharmonic oscillation, and the state of ferroresonance are entirely different phenomena. To clear up these misunderstandings, and to establish a more substantial basis for further investigation has been an aim of this paper.

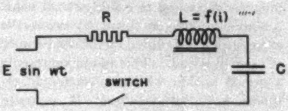

Figure 1. Circuit investigated

The experimental work described includes the following: an account of the regions on the voltage-current diagram capable of supporting subharmonic oscillations, the relation of these regions to the phenomenon of ferroresonance, oscillographic records of wave shape, and the effect of initial conditions on subharmonic response.

The Region of Subharmonic Response on the Voltage-Current Diagram

The definition of a subharmonic response is appropriate. When a sine wave of voltage (unvarying amplitude and frequency) is impressed on this series circuit there can exist currents in which the lowest frequency component is a submultiple of the applied frequency. These currents are called *subharmonics* and the circuit is said to be in a state of subharmonic oscillation. A state of oscillation of order s exists when the resulting current has a period s times as long as the period of the impressed voltage; that is, the wave form repeats every s cycles of the impressed voltage. The component of frequency $1/s$ times fundamental frequency is not necessarily, however, the largest component.

With proper values of circuit parameters, impressed voltage, and initial conditions, a subharmonic current of the most common type, which is of order three, can be obtained by the simple process of closing the switch. This third order current may fall into one of three classifications: sustained and stable; sustained but unstable, that is, erratic or

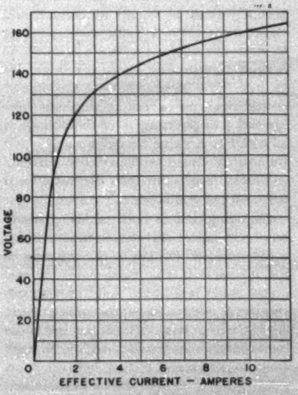

Figure 2. Inductor characteristic—magnetization curve

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1. For all numbered references, see list at end of paper.

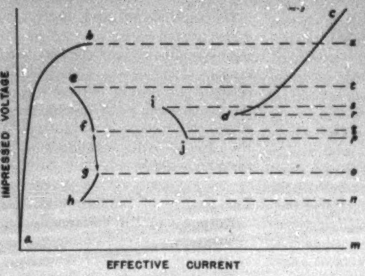


Figure 3. Possible current responses of the series circuit

Section *a-b*, low-current curve

Sections *e-f* and *g-h*, subharmonic of order three

Section *i-j*, subharmonic of order two

Section *c-d*, high current of ferroresonance

"jittery" and having no definite order; and transient. Once the oscillations are initiated, the variation of the rms value of current as the rms value of the applied voltage is changed is described by reference to the curves *e-f* and *g-h* of figure 3. Subharmonic oscillations cannot be obtained if the voltage is greater than at point *e* or lower than at point *h*. Suppose the voltage to be in the range *q-t*. This makes possible the sustained response on curve *e-f*. Having initiated the subharmonic oscillations, increasing the applied voltage within this range so as to approach the point *e* causes the current to decrease, and at point *e*, the oscillations suddenly cease and the current falls back on curve *a-b*, the low-current curve. (This low-current curve is essentially a portion of the magnetizing curve of the inductor.) If, on the other hand, the voltage be decreased so as to approach point *f* the subharmonic current instead of remaining stable until *f* is reached becomes erratic upon nearing this point. When *f* is reached, the oscillations abruptly cease as occurs at point *e*, and the current takes a value corresponding to the curve *a-b*.

Operation on the curve *g-h* is similar to that on *f-e*, except that the unstable or erratic response occurs at the high voltage end *g* of the range. If the voltage is adjusted to a value between *q* and *o* and the switch closed, subharmonic currents will often result, but they die out after a few cycles of oscillation.

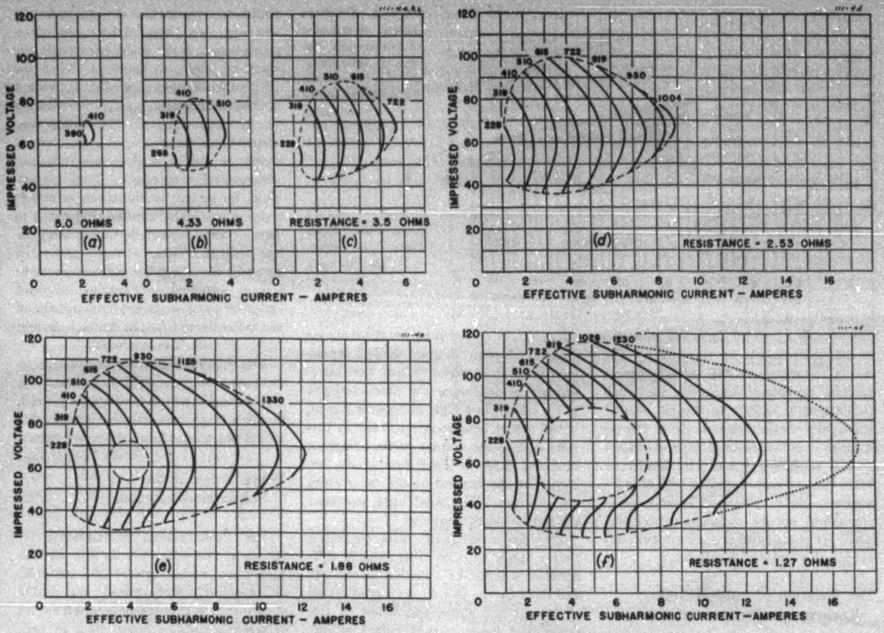
Changing one or both of the parameters, resistance and capacitance brings about marked changes in the response of the circuit. If the capacitance be reduced but resistance left unchanged, then for a given voltage, the subharmonic current will be less. The upper and lower limits of response (*e* and *h*) will be brought closer together, and the gap between *f* and *g* will tend to close. For a capacitance sufficiently small, the two sections *e-f* and *g-h* merge forming a continuous curve. There is a certain lower critical value of capacitance below which, no subharmonic currents can exist. Increasing the capacitance produces much the same effect except that the resulting current is larger. The gap *f-g* will close, and there is a certain upper critical value of capacitance above which no subharmonic currents can exist.

Let us now determine the effect of varying the resistance while keeping the other parameters unchanged. Starting with a high value of resistance it is impossible to obtain any subharmonic response whatsoever. As far as subharmonics go, the region of the voltage-current diagram presents an unbroken expanse of what will be likened to an open sea. The resistance must be decreased to a fairly small value (about five ohms in this case) before the expanse of sea is ruffled by the appearance of a small island. This island represents a region within which subharmonic response may be obtained. It is quite small because of the limited range of capacitance which will support subharmonic currents for this value of resistance, but for any capacitance permitting these oscillations, the resulting curve of current versus impressed voltage is continuous. This small island is shown in figure 4*a*. Continuing the reduction of resistance causes the island to grow larger and the range of capacitance which will support the oscillations to increase. Figures 4*b*, 4*c*, and 4*d* show this clearly.

From figures 4*a* to 4*d* the resistance has been decreased a definite amount. Further reduction brings about an important modification within the island region. No longer is each curve continuous for every value of capacitance. A lake now appears in the island, and the region within this lake is incapable of supporting sustained subharmonic currents; any subharmonic response which can be initiated corresponding to this region rapidly dies out. It is, therefore, a region of transient response. This lake may be fairly small as shown in figure 4*e*, but reducing the resistance enlarges the size of this inner region as well as the island itself, until at a comparatively low value, both the island and the region of transient response are very large. The dotted boundary at the right of figures 4*f* and 4*g* was extrapolated and is only approximate. Lack of sufficient capacitance prevented carrying the experiments above those values indicated.

The boundaries of this lake are precisely determined at the left or lower capacitance side. The upper and lower boundaries are reasonably well defined; the right side boundary is rather indefinite. A region of erratic operation borders this region of transient response in much the same way as a marsh may exist around the shores of a lake; this is shown in figure 5. This erratic current, though unstable in behavior, possesses some sort of stabilizing factor, as it will usually continue to exist with its typical though unpredictable wave form for an indefinite period. The right side of the lake of transient response merges gradually into the marsh of erratic response, however, and a sizable region exists here where the current is more than merely "jittery." Its oscillations are violent and fitful, some cycles having peaks four or five times greater than corresponding peaks of other cycles. Subharmonic oscillations may exist in this state for several minutes and then after a violent oscillation, die out; or, they may go on indefinitely.

Extremely valuable would be information enabling one to adjust the parameters in such a way as to assure a sustained subharmonic response, or to definitely prevent it. The curves of figure 6 show the range of the two parameters, resistance and capacitance, for which subharmonic oscillations may exist either in sustained form, or in transient form, provided that the impressed voltage is correct and the initial conditions favorable. Above, or outside of curve *A*, there can be no subharmonic response, for this region corresponds to operation on the open sea. The region contained between the two curves *A* and *B* is synonymous to operation within the island but outside of any lake of transient behavior that might exist, and therefore to sustained response. Any point below curve *B* corresponds to operation within the region of transient response. The final trace of these curves as the resistance approaches zero is not known, but they probably intersect the abscissa and do not become asymptotic to it. If this is not the case then the results of any analytical investigation which neglects the effect of resist-



ance will prove difficult to substantiate experimentally, and at best would be liable to considerable question. It might be well to mention that many peculiarities not recorded herein were experimentally observed when the resistance was very low.

The foregoing discussion has been concerned with the behavior of the most common type of response, which is of order three. Occasionally a subharmonic current will obtain which is not of this order. Those of order nine, six, and two have been observed experimentally, and others no doubt exist.

Subharmonic currents which repeat every six, or nine, cycles of the impressed voltage, and most probably all others of higher order, exist only within the marshy region of erratic operation. The response in this region is not always, then, what might be termed erratic, and if the rigid requirements of the initial conditions are met the response may be of some definite order higher than three. Oscillograms of such currents are shown in figures 12*a* and 12*b*. It should be stated that the relation of voltage versus current for this oscillation is narrowly confined within the erratic region and is essentially the same as would result if the order were three. Unless an oscillograph were available there would be no way of knowing that the response was not of third order. The conditions permitting such a response are extremely limited and usually a slight variation of impressed voltage will cause the order to change or make the current erratic thus having no definite order.

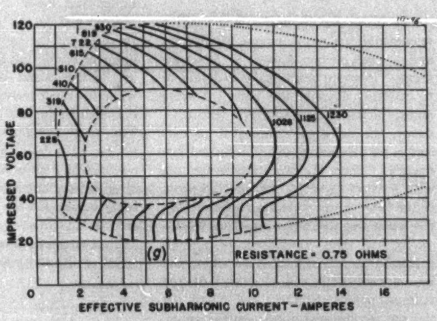


Figure 4. Regions of subharmonic currents of order three

Figures of individual curves refer to capacitance in microfarads. The dotted boundary of *f* and *g* was extrapolated

Subharmonic oscillations of order two behave in much the same way as those of order three. However, the oscillations of order two result in currents roughly twice as great in rms value as corresponding currents of third order, and when plotted on the voltage-current diagram reveal a region quite apart from any regions previously mentioned. Referring to figure 3, a typical characteristic curve is shown as *i-j*. An actual plot of effective current of order two against impressed voltage is disclosed in figure 7. At best these second order oscillations are fickle; requirements of voltage, switching angle, etc, to initiate them are very rigid, as well as the requirements of the circuit parameters in order that their existence be maintained. This response may be abtained only for a very limited range of resistance and capacitance

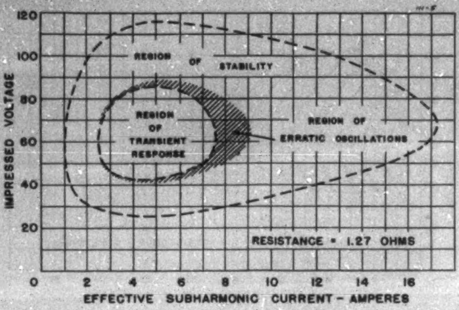


Figure 5. The three regions of response of subharmonic currents of order three, showing in particular the region of erratic oscillation

(about from 1.5 to 2.0 ohms and from 400 to 530 microfarads in this case). The size of the region on the voltage-current diagram capable of supporting sustained oscillations of order two become smaller and finally diminishes entirely as the resistance is increased or decreased from a certain optimum value. It should be noted that no corresponding lake of transient response is evidenced, as such operation could not be obtained experimentally. It is doubtful if such a region exists for this order of oscillation.

The Relation of Subharmonic Response to Ferroresonance

The series circuit under investigation may exhibit both subharmonic oscillations and a phenomenon known as *ferroresonance.* It would be advisable to define, or to carefully describe the latter. During an exhibition of the response of ferroresonance, the principal component of current is very definitely of fundamental frequency. There are two states associated with ferroresonance. The first is known as the low-current state, and is essentially the lower magnetizing current of the inductor. If the impressed voltage is gradually increased from a low value, the low-current curve will be traced on the voltage-current diagram, and when the voltage reaches the upper critical value, the current will suddenly increase. The circuit is now operating in the second, or high-current state and a further increase of voltage merely increases the current correspondingly. If now, the voltage be decreased to somewhat less than the first critical value, the lower critical value of voltage will be reached and the current will collapse suddenly to a value synonymous with the low-current state. In figure 3, the curve *a-b* is the low-current curve and the high-current curve is given by *d-c.*

For a given inductor, the effect of resistance and capacitance on these curves is easily shown. A great deal of experimental investigation[5–15] has been performed to determine the characteristics of the circuit in a state of ferroresonance; it will suffice here merely to refer to this work and to the voltage-current diagram of figure 8 for an exhibition of the behavior of the current as the parameters are changed. The solid lines represent the ferroresonant response.

Previous investigators have suggested that a region capable of supporting subharmonics would be bounded on the left and right by the two ferroresonant states of current and above and below by the upper and lower critical values of impressed voltage. The actual condition is revealed on the plots of figure 8. The outer heavy dotted line forms the boundary of the island of stable oscillation, while the inner heavy dotted line, if present, bounds the lake of transient response. The nonlinear characteristic of the inductor is the essential feature of the series circuit which gives rise to both conditions of operation, but beyond that, the underlying causes for the behavior of the resulting current are probably as different as the two phenomena themselves.

All of the foregoing discussion reveals that for a certain value of impressed voltage, one, two, three, or four current responses may result if the parameters are suitable. Refer to figure 3. When the voltage is adjusted to a value between *n* and *o*, two response possibilities exist for the given parameters. The current may respond on curve *a-b*, the low-current curve, or may be subharmonic in nature and respond on curve *g-h*. Thus it is possible, for certain combinations of circuit parameters including the inductor, to find a condition such that when the voltage is set to a value between *r* and *s*, the response may be a low-current on curve *a-b*, a sustained subharmonic of third order on *e-f*, a sustained subharmonic of second order on *i-j*, or be the high-current of ferroresonance on curve *c-d*.

Although no response other than those indicated was obtained experimentally, Travis[3] indicates that others may exist. In such case, there may be innumerable modes of response for given circuit values.

Wave Form in the Subharmonic Region

The instantaneous subharmonic response of the series circuit may be sustained and stable, erratic and unstable, or transient. A sustained and stable steady state is that response which has an apparently definite order, such as two, three, six, or nine. An erratic response is usually sustained, although it may suddenly die out for no apparent reason, and has no definite order. A transient response is that which occurs immediately after closing the switch, persists for a few cycles, and then dies out. Oscillograms of the wave form of the sustained and stable steady state, and the erratic response are shown.

The change in wave form of one cycle of a sustained third order current as an excursion is made along a characteristic curve on the voltage-current diagram is

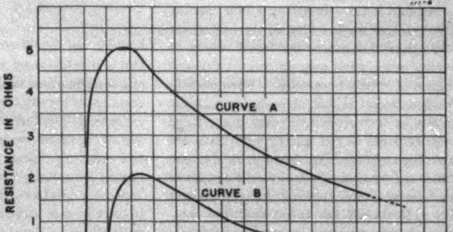


Figure 6. Limiting values of resistance and capacitance for third-order oscillations

Above and to the left of curve *A*, no response; between *A* and *B*, sustained and stable oscillations; below curve *B*, region of transient response shown by figure 9. These were taken on a cathode-ray oscillograph with constant amplification. Note how greatly the wave form changes in the vicinity of the lake region of transient response, figures *c* and *d*, and *e* and *f*. The last record, 9*i*, was taken after the upper limit of the curve had been passed and the current had collapsed to three cycles of what is essentially the magnetizing current of the inductor.

A somewhat similar excursion is given in figure 10,* and figure 11.

Current wave forms of order nine and six are shown in figure 12. In the first record note that the current wave repeats every nine cycles of the impressed voltage, and that while nine fundamental cycles are required for one period of subharmonic current, each third of the period is similar to one typical current cycle for order three. Thus, three similar but different third order cycles can be considered grouped to form one cycle, or period, of order nine. Similarly for order six.

An oscillation of order two is shown by figures 13 and 15*c*. This response is of unusual interest: it is difficult to initiate and maintain, and is quite different from the response of any other order. A resistance as high as 6,000 ohms when connected across the capacitor materially changes the wave form of this oscillation, or may cause it to die out entirely, indicating how sensitive this response is to circuit changes.

The long oscillogram of figure 14 reveals a current response which is erratic. An oscillation such as this may persist indefinitely and never "settle down" to any regular predictable form or it may last for only a short period, say ten seconds or even a minute or two, and then for no obvious reason suddenly snuff out.

There are four possible sustained current conditions which may be obtained for certain restricted but fixed values of the parameters and the impressed voltage. Oscillograms of these four responses for essentially the same impressed voltage and circuit parameters are exhibited in figure 15. Note particularly that the high current of ferroresonance is not subharmonic in nature.

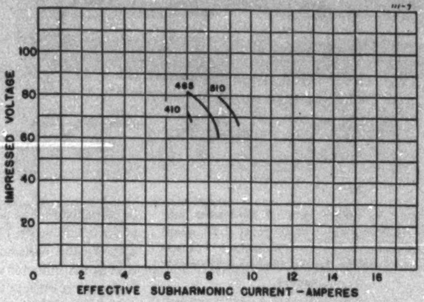


Figure 7. Subharmonic currents of order two; resistance of 1.88 ohms

An examination of the wave forms exhibited leads to two comments. First: if the form of one cycle of a sustained current of third order be taken as a basis, then the form of the currents of orders other than three is comparable and similar to three. Second: at any instant in our circuit maintaining a subharmonic oscillation, there are certain conditions of voltage, charge, and flux linkages, in sign, magnitude, and time rate of change. It would appear then that if all of these conditions identically match at every three cycles of impressed voltage, the current response should then repeat identically thus leading to a sustained third order oscillation. If the matching conditions occur every two cycles, a second order wave results; if it requires nine cycles, then ninth order and so on. If these conditions never match or repeat then a wave form of erratic nature is the result. Thus it seems entirely possible that what might appear to be an erratic response on an ordinary oscillogram is actually a portion of a response of very high order, and an oscillogram of great length would be necessary to detect the order.

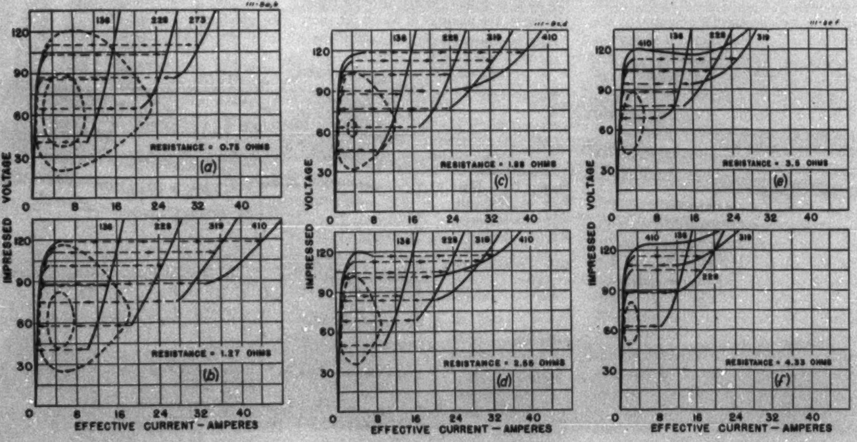


Figure 8. Relative location of the region of third order subharmonic currents to the low-current and high-current states of ferroresonance

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*Note that the capacitor voltage curve is upside down compared to current, since $i = dq/dt$.

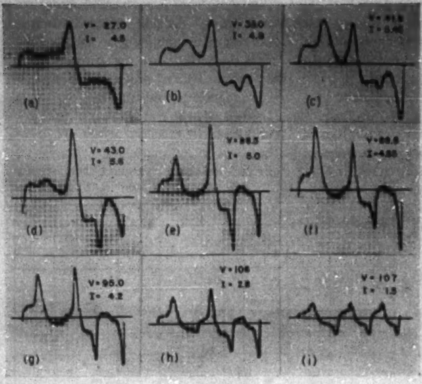


Figure 9. Wave form of third-order subharmonic current, resistance of 1.27 ohms, capacitance of 615 microfarads. V is impressed voltage, I is the effective subharmonic current

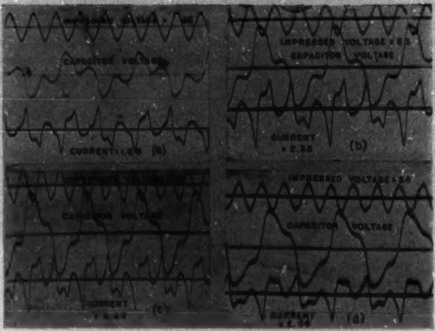


Figure 10. Wave form of third-order subharmonic current, resistance 1.27 ohms, capacitance 319 microfarads

Initiation of Subharmonic Oscillations

There are three factors which act on the circuit initially. These are:

(*a*). Initial flux linkages of the inductor.

(*b*). Initial charge on the capacitor.

(*c*). Manner of applying the impressed voltage.

Assume for the moment that the voltage is applied by suddenly closing the switch. The inductance of the iron-cored reactor is essentially constant until some degree of saturation occurs. Since subharmonics depend for existence on the nonlinearity of the inductor, it is obvious that for any given switching angle, the initial flux linkages and initial charge must be such as to permit the necessary degree of saturation to occur. If the first few cycles of response after closing the switch do not result in the necessary degree of saturation, no subharmonic response will be obtained.

The manner of applying the voltage to the circuit determines whether any subharmonic response at all will result. Experiment proves that in no case is it possible to induce a subharmonic response by impressing the voltage at an unsuitably high or low value, and then gradually bringing the voltage into the range capable of supporting oscillations for the given parameters. This merely results in a high or low current of ferroresonance. To initiate subharmonic oscillations, a shock excitation is required. This must be administered at suitable initial values of switching angle, charge, and flux linkages for the necessary degree of saturation in the inductor which is caused by a high peak of current can be brought about only by a judicious combination of these initial conditions.

A much greater "shock" is necessary to produce oscillations of order two than for the more common type of order three. Occasionally, by repeatedly closing the switch, the circuit may be made to give subharmonics of second order, perhaps once out of fifty closings. However, if the switch is closed and then the inductor momentarily short-circuited, a very high current will result during the short circuit; this operation will very often initiate subharmonics of order two, and can nearly always be counted on to produce the more common variety of order three. Of course, the moment at which the short circuit is removed is certainly comparable in importance to a switching angle, but the high current produced before removing the short circuit allows a wide range of angle consistent to producing the required oscillations. The rarity with which oscillations of order two occur after merely closing the main switch indicates that a rather special combination of initial conditions must exist before such oscillations can be expected.

Subharmonics of order six and nine are indeed rare in occurrence, which must be attributed to the rigid requirements of initial conditions to produce them as well as the requirements for their continued existence.

The effect of initial charge, flux linkage, and switching angle on the wave shape of any third order subharmonic oscillation that may result depends largely on the region of the voltage-current diagram in which the response falls. If the ultimate response is sustained and stable then

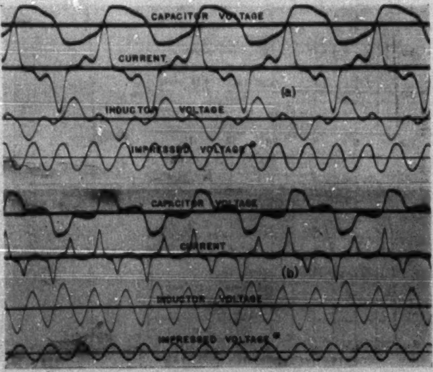


Figure 11. Wave form of third-order subharmonic current, $R=1.27$ ohms

(a)—$C=510$ microfarads, $V=90$ volts, $I=3.6$ amperes

(b)—$C=1{,}026$ microfarads, $V=38$ volts, $I=8.7$ amperes

*Impressed voltage has been sketched in correct phase position

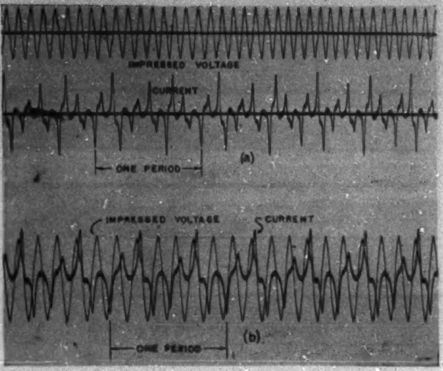


Figure 12

(*a*) — Subharmonic current of order nine. $R=1.88$ ohms, $C=410$ microfarads, $V=73$ volts, $I=3.2$ amperes

(*b*) — Subharmonic current of order six. $R=1.27$ ohms, $C=319$ microfarads, $V=58$ volts, $I=2.45$ amperes

the total response from the instant the switch is closed may be likened to the ordinary linear circuit response which consists of a "switching transient" which shortly dies out leaving the "steady-state" component the wave form of which is independent of initial conditions. If the circuit is adjusted to an oscillation within the lake region, then only a transient subharmonic current will result. However, if the response is erratic a more interesting possibility exits. Travis[3] points out that initial conditions are determining factors in the response for the first excursion of the whole saturation curve, and his region-by-region analysis predicts wave form on what has happened just previously. Thus the instantaneous value of current an hour after closing the switch could be determined by starting with the given initial conditions, and projecting forward to the point desired. That switching angle, for example, could be a factor determining the wave form hours after the switch is closed is incomprehensible to the usual way of thinking, but an examination of the erratic wave of figure 13 certainly suggests this possibility. Certainly, if all initial conditions for this particular oscillogram could be matched, the response for the first few cycles should be repeated and match identically with this one. If at first, why not forever after? In such a case it might be logical to consider the whole response as a transient, even though undamped and lasting for an indefinitely long time, and a steady-state condition, by all the usual definitions, might never be reached.

General Conclusions

Conclusions derived from this investigation may be stated as follows:

1. Subharmonic oscillations are restricted to definite regions on the voltage-current diagram.

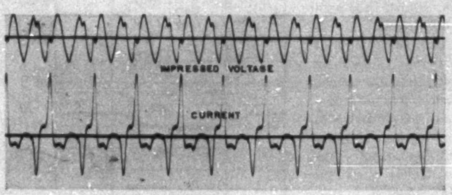


Figure 13. Wave form of subharmonic currents of order two

$R=1.88$ ohms, $C=410$ microfarads, $V=72$ volts, $I=7.1$ amperes

2. The states of ferroresonance and subharmonic oscillation are totally different phenomena unrelated in behavior.

3. For certain suitable but fixed circuit parameters, there may be as many as four different sustained current conditions for the same impressed voltage. They are: low current of ferroresonance, subharmonic current of order three, subharmonic current of order two, and high current of ferroresonance.

4. A "shock" excitation of some form is always necessary in order to initiate subharmonic oscillations.

5. Initial conditions: (1) are important in determining the wave form when the subharmonic response is in the erratic region, (2) are unimportant in determining the wave form of the steady-state sustained response in the region of stable operation; for given parameters, the wave form of current in this region depends only upon the value of impressed voltage.

6. Overcurrent conditions in series capacitor installations may be due to:

(1). Subharmonic oscillations, in which case they may be completely avoided or made to come within the region of transient response by proper selection of circuit parameters.

(2). High current of ferroresonance, in which case the current may be much greater in magnitude than that due to subharmonics, and while it may be reduced by favorable circuit parameters, this phenomenon cannot be wholely avoided except by keeping the voltage down.

References

1. Subharmonics in Circuits Containing Iron-Cored Reactors, **I. A. Travis** and **C. N. Weygandt.** AIEE Transactions, volume 57, 1938 (August section), pages 423–31.

2. Subharmonics in Circuits Containing Iron-Cored Inductors—II, **I. A. Travis.** AIEE Transactions, volume 58, 1939, pages 735–46.

3. Analysis of Series Capacitor Application Problems, **C. Concordia** and **J. W. Butler.** AIEE Transactions, volume 56, 1937, pages 975–88.

4. Sur un demultiplicateur de frequence statique, **J. Fallou.** *Rev. gen. de l'Elec.*, volume 19, 1926, pages 987–91.

5. A propos d'essais effectues sur des interrupteurs a resistance de choc, **A. Maudit.** *Rev. gen. de l'Elec.*, volume 19, 1926, pages 937–50.

6. Existence de deux regimes en Ferro-resonance, **P. Boucherot.** *Rev. gen. de l'Elec.*, volume 13, 1920, pages 827–8.

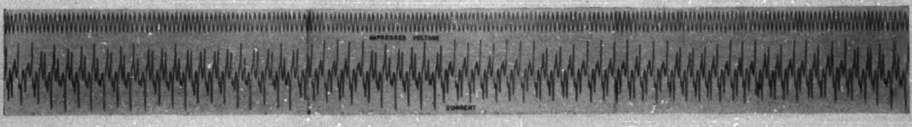
7. Au sujet de l'existence deux regimes en Ferro-resonance, **F. Margand.** *Rev. gen. de l'Elec.*, volume 14, 1921, pages 635–7.

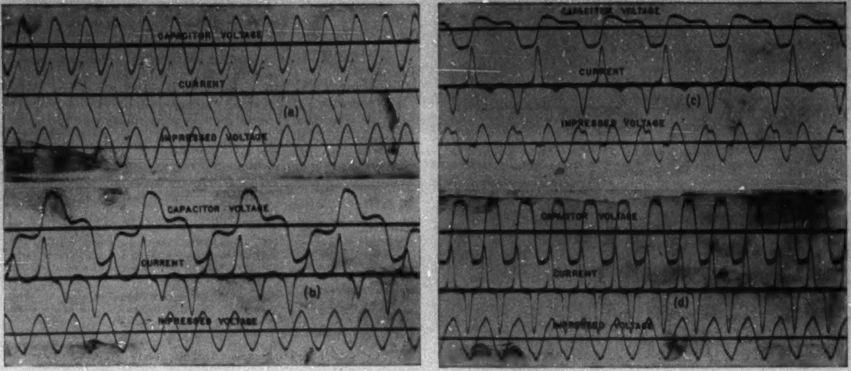
8. Über Schwingungskreise mit Eisenkernspulen, **H. Schunk** and **J. Zenneck.** *Jahrbuch der Drahlos., T.u.T.*, volume 19, 1922, pages 170–94.

9. Experimentelle Untersuchungen über Schwingungskreise mit Eisenkernspulen, **L. Casper, K. Hubmann,** and **J. Zenneck.** *Jahrbuch der Drahlos., T.u.T.*, volume 23, 1924, pages 63–77.

10. A propos de certaines particularities de la ferro-resonance d'un circuit, **F. Margand.**

Figure 14. Wave form of current in the region of erratic operation. R = 1.27 ohms, C = 365 microfarads, V = 46.5 volts, I = 2.9 amperes (this was the average of instrument reading)





(*a*)—Low-current state, $I=0.75$ ampere
(*b*)—Subharmonic of order three, $I=3.4$ amperes

Figure 15. Wave form of the four current possibilities for the same value of impressed voltage and circuit parameters. R = 1.88 ohms, C = 461 microfarads, V = 80 volts

(c)—Subharmonic of order two, $I=7.2$ amperes
(*d*)—High current of ferroresonance, $I=$ 25 amperes (approximate)

Rev. gen. de l'Elec., volume 21, 1928, pages 760–71.

11. Zur qualitativen Theorie Gesattigter Eisendrosseln, **F. Ollendorf.** *Archiv f. Electrotechnic*, volume 21, 1928, pages 6–24.

12. Contribution a l'etude experimentele de la Ferro-resonance, **E. Roulle.** *Rev. gen de l'Elec.*, November 24, 1934, pages 715–38; December 1, 1934, pages 763–80; December 8, 1934, pages 795–819; December 15, 1934, pages 841–58.

13. Critical Conditions in Ferroresonance, **P. H. Odyssey and E. Weber.** AIEE Transactions, volume 57, 1938 (August section), pages 444–52.

14. Resonant Nonlinear Control Circuits, **W. T. Thomson.** AIEE Transactions, volume 57, 1938, pages 469–74.

15. Similitude of Critical Conditions in Ferroresonant Circuits, **W. T. Thomson.** AIEE Transactions, volume 58, 1939 (March section), pages 128–30.

System Load Swings

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Synopsis: Operating records of the Consolidated Edison Company of New York, Inc., revealed the existence of irregular fluctuations in the loads on generating stations and tie feeders. Tests were conducted to determine the response of parts of the system to changes in load, voltage, and frequency. By analyzing the test results to show the relation between these three quantities under various operating conditions, it was possible to establish the origin of the load swings, as well as to determine their magnitude, duration, and recurrence.

Three general types of load swings were observed: small synchronizing power oscillations, short-time swings caused by fluctuating railroad loads, and long-time swings produced by the unco-ordinated adjustments of station loads. These swings are not initiated or sustained by the turbine governors. The synchronizing power oscillations and the short-time swings do not affect system operation, but the long-time swings do affect the loading of the tie feeders and the operation of boiler equipment. It is expected, however, that the greater appreciation of the problem derived from the tests, plus the installation of a common frequency indicator in all stations, will enable the operators to reduce the long-time load swings.

Introduction

The object of the authors whose test results and conclusions are reported in this paper was to secure information on system characteristics which would assist the engineering and operating personnel to meet service requirements adequately and economically.

The tests were conducted at the Hudson Avenue station which carries representative load and is located electrically at the end of the system where it can be isolated easily. Although the steam and electrical parts of a generating station are closely related, the various tests were designed primarily to disclose the electrical response of the system.

Load swings may be defined as deviations from the normal load trend. These have been characterized for this system as:

1. Power oscillations of three to five megawatts with a period of about two seconds in the normal generated load when the station is tied to the system.

2. Short-time load swings with a period of 0.5 to 2.0 minutes and usually not exceeding ten megawatts.

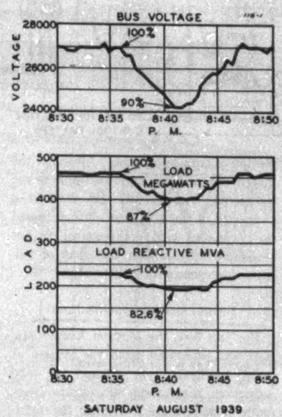

Figure 1. Effect of a reduction in bus voltage on the distributed load

3. Long-time load swings with a period of five minutes or longer and of considerable magnitude.

Importance of Load Swings

On this system the power oscillations and short-time swings do not present any serious problem but the long-time load swings should be kept to a minimum because of the adverse effect upon system economy and operation. Long-time load swings disrupt loading schedules, decrease boiler economy, and add to maintenance costs.

The economy of stoker-fired boilers is reduced by frequent changes in combustion rates which cause irregular fires and reduced furnace efficiency. Thin spots are formed in the fuel bed which reduce combustion efficiency and damage stoker castings with a consequent increase in furnace maintenance. Thicker fuel beds must be maintained to withstand the higher air pressures necessary to pick up sudden load swings. This tends to increase superheater maintenance because of increased secondary combustion.

The rapid variations in steam output decrease momentarily the furnace efficiency of pulverized fuel boilers because of changes in fuel-air ratio.

In boilers having limited steam liberating surface sudden changes in load with attendant change in evaporation rate may cause carry-over. The efficiency of cinder catchers is adversely affected by changes in gas velocities resulting from load swings.

Deviation from economical loading schedules and unnecessary operation over uneconomical valve points decrease turbine efficiency.

Long-time load swings prevent close adherence to system incremental loading schedules and lead the system operator to incorrect anticipation of system load changes. It should be noted that short-time load swings, on the contrary, do not directly affect station economy unless they couple with automatic control equipment in a manner to cause hunting of combustion control apparatus.

Station long-time load swings which are reflected in the tie feeders decrease the effective carrying capacity of these facilities. Additional thermal cycles following these swings accelerate the failure rate of tie feeder cables, reduce tie feeder availability, and increase maintenance costs.

Preliminary Tests

To determine the correct relationship of load to voltage and load to frequency,

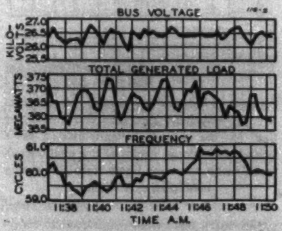

Figure 2. Effect of a change in frequency on the distributed load

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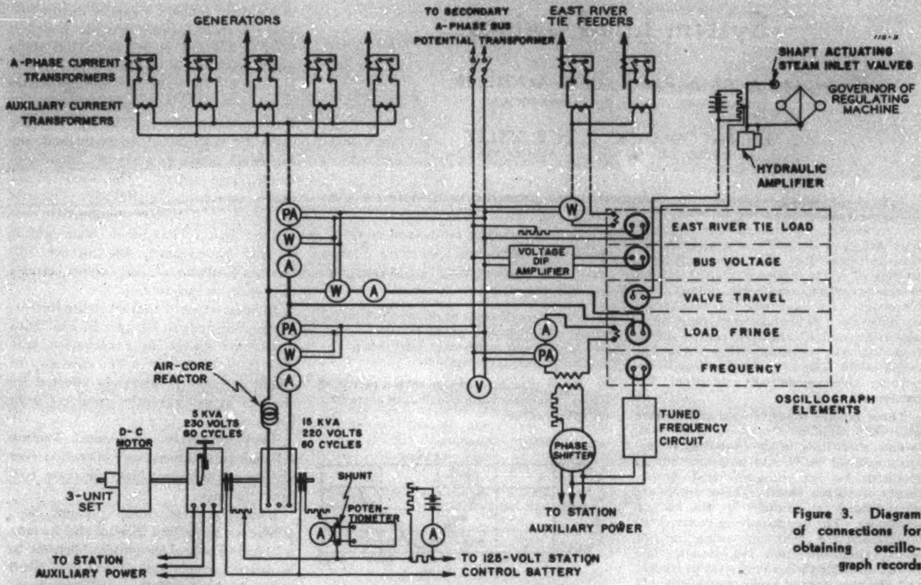

GENERATORS
A-PHASE CURRENT TRANSFORMERS
AUXILIARY CURRENT TRANSFORMERS
TO SECONDARY A-PHASE BUS POTENTIAL TRANSFORMER
EAST RIVER TIE FEEDERS
116-3
SHAFT ACTUATING STEAM INLET VALVES
GOVERNOR OF REGULATING MACHINE
HYDRAULIC AMPLIFIER
PA
W
A
W
A
PA
W
A
V
W
VOLTAGE DIP AMPLIFIER
A
PA
EAST RIVER TIE LOAD
BUS VOLTAGE
VALVE TRAVEL
LOAD FRINGE
FREQUENCY
OSCILLOGRAPH ELEMENTS
AIR-CORE REACTOR
D-C MOTOR
5 KVA 230 VOLTS 60 CYCLES
15 KVA 220 VOLTS 60 CYCLES
3-UNIT SET
SHUNT
POTENTIOMETER
A
A
PHASE SHIFTER
TUNED FREQUENCY CIRCUIT
TO STATION AUXILIARY POWER
TO STATION AUXILIARY POWER
TO 125-VOLT STATION CONTROL BATTERY

Figure 3. Diagram of connections for obtaining oscillograph records

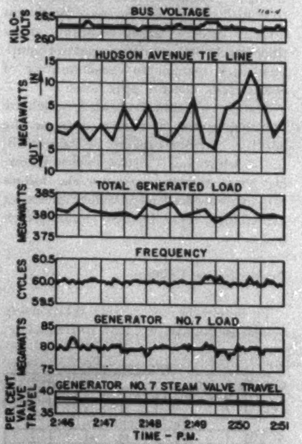


Figure 4. Hudson Avenue load characteristics with the governors automatic and the station tied to the system. Generator 7 is a typical generator. Replot of the oscillograph records with condensed time scale eliminating small power oscillations

preliminary tests were conducted at Hudson Avenue where the information could be obtained on a representative section of connected load. Although the metropolitan New York load reflects the industrial, commercial, and residential character of the area, calculations based on estimated load components do not satisfactorily establish the effect of voltage or frequency on load.

(a). Effect of Voltage on Load

For this test, the Hudson Avenue station was separated from the system at points indicated in figure 14, and induction regulators substations were made nonautomatic. During the lighting peak, the bus voltage was lowered ten per cent in two per cent steps of two minutes each and then gradually increased to normal while the frequency was maintained as nearly constant as possible.

Figure 1 shows that a 10 per cent decrease in voltage resulted in a reduction of 13 per cent in station megawatt demand and 17.4 per cent in reactive megavolt-amperes. The load at the beginning and end of the test was the same. In an emergency requiring temporary reduction of load, the voltage could be lowered within reasonable limits with the consequent rejection of a substantial amount of load without seriously disturbing consumers or endangering stable operation.

(b). Effect of Frequency on Load

The relation of frequency to load within a practical operating range and with voltage held approximately constant is

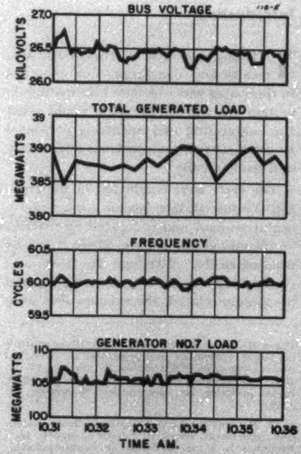


Figure 5. Hudson Avenue load characteristic with the governors automatic and station separated from the system

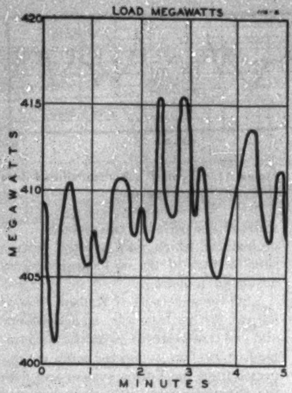


Figure 6. Hudson Avenue load characteristic showing in detail typical short-time load swings with the governors automatic and the station separated from the system

shown in figure 2. A change in frequency of approximately three per cent did not cause any substantial change in load. The load change usually noted when the frequency is altered is the result of the accompanying voltage variation and not the change in frequency. A reduction of frequency in itself does not materially decrease the system load.

Methods of Measurements

To obtain further data on the composite action of the load, generators, governors, automatic frequency control, and tie feeders and their individual contribution to load swings, a group of tests was conducted at the Hudson Avenue generating station.

This required the development of a test procedure to secure continuous and simultaneous records of the following quantities for periods of from 5 to 15 minutes:

1. Total generator load (load fringe)
2. Station frequency
3. Station voltage
4. Load on ties to other stations
5. Steam valve travel of the frequency-regulating turbogenerator

The station recording meters at Hudson Avenue included a totalizing wattmeter, voltmeter, and frequency meter, but these instruments because of their slow chart speed and slow response were considered inadequate for the test and therefore metering circuits which are shown in figure 3, were developed to utilize a nine-element oscillograph with a magazine containing 200 feet of sensitized paper.

(a). Load Fringe Measurement

The important requirement was to record with reasonable precision the fluctuations of about 0.25 to 10 per cent in the total generated load. This was accomplished by a differential current method in which the totalized A-phase current was opposed by a current of constant magnitude and fixed phase displacement. The resultant current was passed through the vibrator circuit of a fin-type single-phase wattmeter element on the oscillograph. The field circuit of this element was energized by a current held at constant magnitude and in phase with the station bus voltage, so that for constant bus voltage the variations in the differential current, and therefore the deflections of the wattmeter element were proportional to the load fringe. The load fringe record was corrected when the bus voltage variations were appreciable. Auxiliary current transformers provided an over-all ratio of 6,000 to 5 amperes for each generator.

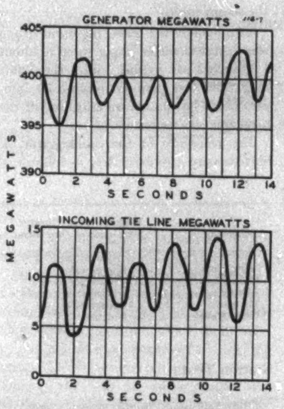


Figure 7. Characteristic power oscillation that exists when Hudson Avenue is tied to the system. Replot of the oscillograph records

The bucking current used to oppose the load current was obtained from a synchronous-synchronous frame shifting motor generator set supplied from the station bus. To hold the magnitude of the bucking current as constant as possible, the fields of the two machines were excited by a station control battery which was isolated for the tests.

To aid in securing stability of the bucking current, an air-core reactor was inserted in the bucking circuit so that the generator could be operated at a voltage near its rated value and still provide the small voltage drop required across the differential circuit whose resistance was of the order of one ohm. Since the inductance of the differential circuit was practically negligible, its impedance drop combined at almost 90 degrees with the much larger drop across the air-core reactor. The use of the air-core reactor had the additional advantage of tending to make the magnitude of the bucking current independent of changes in station frequency. Any changes in the speed and voltage of the bucking generator produced by changes in station frequency were almost entirely offset by proportional changes in the impedance of the air-core reactor.

Before each test run, the proper phase displacement of the bucking current with respect to the load current was secured by a frame adjustment of the motor generator set and the magnitude of the bucking current was adjusted for minimum differential current by varying the field excitation. Following these adjustments the current coils of the indicating instruments in the differential circuit were short-circuited for the duration of the run.

(b). Supplementary Measurements

The frequency record was obtained by passing through an oscillograph vibrator the current in a series resonant circuit energized from the same voltage source as

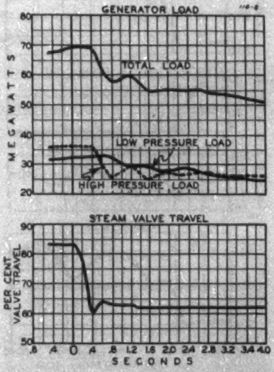


Figure 8. Effect of a sudden change in synchronizing-spring tension on the valve travel and the output of an 80-megawatt cross-compound turbogenerator

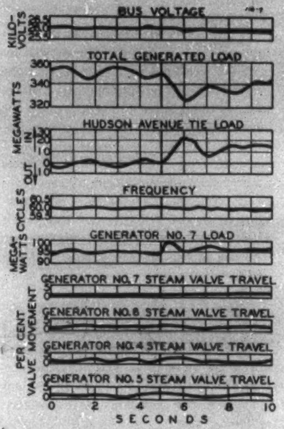


Figure 9. Effect on Hudson Avenue generator and tie-line loads of tripping a generator carrying 20 megawatts. Generator tripped at five seconds on chart. Generator number 7 regulating

the field for the differential wattmeter element used to record load fringe. The circuit was designed for resonance at 62 cycles per second in order to give a practically linear deflection.

A record of the *A* phase-to-neutral bus voltage was obtained by applying the output of a voltage dip amplifier (class *C* amplifier, using a type 53 duotriode) to a sensitive oscillograph vibrator.

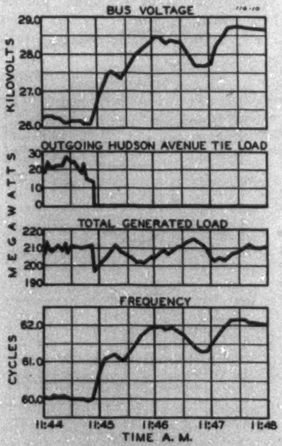


Figure 10. Test results with all governors blocked when an outfeeding tie was tripped. No adjustments made of the field and valve positions

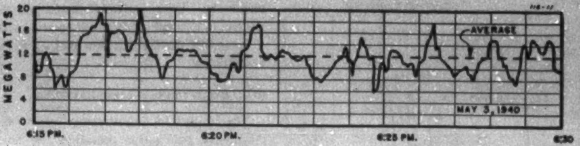


Figure 11. Typical suburban-railroad load demand

The loads on the ties between the Hudson Avenue and East River stations were totalized and recorded on the oscillograph.

To record valve travel, a slide-wire potentiometer was mounted on the turbine frame, with the slider attached to the rod which transmits the movement of the governor's hydraulic amplifier to the steam valves.

Throughout the tests, the oscillograph was operated with a paper speed of about 1.25 inches per second. With the 200-ft magazine, test periods totaling 30 minutes could be run before reloading of the magazine was necessary.

The responses of the oscillograph vibrators were approximately as follows:

Quantity	Magnitude for Vibrator Deflection of One Inch
Load fringe	6 megawatts
Frequency	1 cycle per second
Voltage	2 kilovolts
Tie load	15 megawatts
Valve travel	3 inches

Station Tests

The following tests were made at Hudson Avenue during the flat load period of a weekday except the block governor test which was conducted on Sunday. The weekday demand was about 380 megawatts and the Sunday demand 200 megawatts, and the corresponding spinning capacities were 560 and 430 megawatts respectively. The station voltage was about 26.5 kv. The records of these tests are shown in figures 4 and 5.

The generator load, frequency, and voltage were irregular during both tests but the variations were more pronounced with the station isolated. Short-time load swings were observed with a magnitude of five to ten megawatts and a period of 0.5 to 2.0 minutes. Figure 6 shows in detail the nature of these load swings. When the station was tied to the system there was superimposed upon the generator and tie-line loads an almost continuous power oscillation of about five megawatts with a period of two seconds. The oscillations which were plotted from a short section of the oscillograph record are shown in figure 7.

With the governors at Hudson Avenue normal, further tests were made with one and two ties in service, with the Niagara-Hudson system connected and disconnected from the Edison system and with the automatic frequency control in and out of service at Hudson Avenue. There was no apparent change in the characteristics of the short-time load swings or power oscillations during these tests, thus indicating that these operating arrangements had slight influence on the swings or oscillations.

In order to check whether the governors initiate or sustain these swings or oscillations, a series of tests was conducted in which an exaggerated impulse was introduced at the station in an attempt to start swings or oscillations. The first of these tests was to change suddenly the synchronizing spring tension of the governing mechanism of an 80-megawatt cross-compound turbogenerator. To accomplish this, an auxiliary spring was installed in parallel with the synchronizing spring and arranged with a fusible link so that the tension of the auxiliary spring could be released instan-

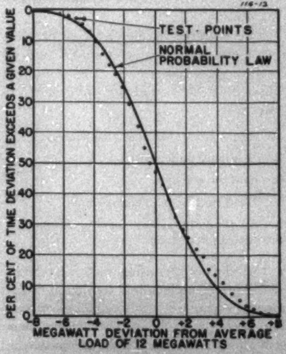


Figure 12. Probability of deviation from average load for typical suburban railroad load

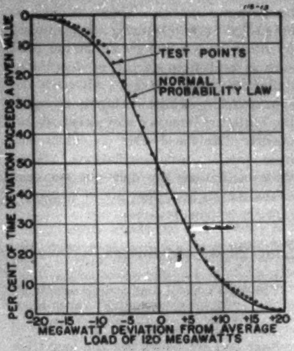
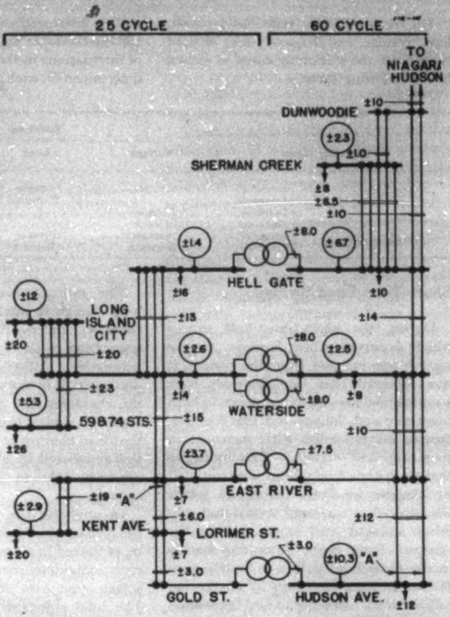


Figure 13. Probability of deviation from average load for typical transit-subway load

taneously. The generator was placed in operation on the bus in parallel with the system and by means of the synchronizing spring the output was adjusted to 56 megawatts. Then by the auxiliary spring the load was raised 12 megawatts to a total of 68 megawatts. The fusible link was blown while the valve travel and the generator load were recorded. Figure 8 shows the results of this test and indicates that the valve traveled to its new position in about 0.75 second. There was slight overshoot, but there was no tendency to continue oscillating. After the steam valve had reached a steady state a highly damped oscillation of small magnitude existed on the high- and low-pressure units of the generator.

To study governor action further, a group of tests was made in which a generator carrying a unity power-factor load of about 20 megawatts was tripped off the line when the station was tied to the system by one and by two tie feeders and with the Niagara-Hudson system alternately synchronized and separated from the Edison system. The period of the resulting transient oscillations corresponded with the natural period of the system and in each case the amplitude of the swing decreased rapidly. Figure 9 shows typical results of these tests. The oscillations were not sustained or amplified by the action of the governors. Either the governor characteristics or the generator and load characteristics had a very marked damping effect on such transient oscillations. This was further substantiated by a test in which one of two parallel Hudson Avenue tie feeders was opened and closed under load to initiate oscillations. The resultant swing died rapidly in less than two cycles of oscillation indicating that the system has a high degree of damping.

Figure 14. Schematic diagram of the Consolidated Edison system showing fluctuations in megawatts on tie feeders, frequency changers, and generating stations caused by variations in railroad load demands. Hudson Avenue was separated from system during tests by opening switches at A



Additional information was obtained on the contribution of governors to load swings by means of a test conducted with all governors blocked and one tie in service. This tie was opened when carrying about 15 megawatts out of the station. During the test no adjustments were made to the steam valves or generator fields. The station frequency and voltage were allowed to adjust themselves to the load. As shown in figure 10 short-time load swings appeared although the governors were blocked. It is of interest to note that because of the initial 7.5 per cent unbalance in load the station frequency increased 3.8 per cent and the voltage increased 10.1 per cent.

Relation Between System Variables

In these Hudson Avenue tests, it was observed that the generator output, voltage, and frequency fluctuated continuously, varying in period from several seconds to a minute or longer. These fluctuations may be initiated in the following ways:

1. A change in load such as the starting or stopping of a train or a large motor.
2. A change in voltage caused by either manual or automatic adjustments of generator field rheostats.
3. A change in frequency by manual control or by automatic operation of governing equipment.

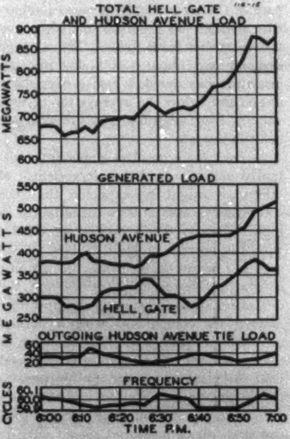


Figure 15. Typical long-time load swings between Hell Gate and Hudson Avenue generating stations

The instantaneous relations between load, voltage, and frequency may be used to indicate the initiating cause as shown in the following table:

Initiating Cause		Effect on		
		Voltage	Load	Frequency
Voltage	Increase		Increase	Decrease
	Decrease		Decrease	Increase
Load	Increase	Decrease		Decrease
	Decrease	Increase		Increase
Frequency	Increase	Increase	Increase	
	Decrease	Decrease	Decrease	

Short-Time Load Swings

During the short-time load swings which occurred when Hudson Avenue station was isolated from the system, it was observed that load increases were accompanied usually by decreases in both frequency and voltage and that load decreases corresponded with increases in frequency and voltage. This indicated that short-time load swings were initiated by changes in distribution load which were traced to the rapid transit and suburban railroad load supplied from the station. Each major generating station in the Consolidated Edison system supplies a substantial amount of fluctuating railroad load. In order to find the magnitude and characteristics of railroad load fluctuations, graphic wattmeter records with a speed of about $1^1/_2$ inches per minute were obtained on typical blocks of suburban railroad load and transit subway load. Figure 11 shows a small part of the record for a section of suburban railroad load during the afternoon transportation peak hour. The average demand was about 12,000 kw but the instantaneous demand varied irregularly above and below the average for periods varying from several seconds to as much as two minutes.

As each train requires power intermittently and is operated independently of others, it might be expected that the composite instantaneous demand of a group of trains would vary in accordance with the normal probability law. Load readings were taken from the graphic charts at five-second intervals and the deviations from the average demand were plotted for comparison with a normal probability curve. The results shown in figures 12 and 13 indicate that railroad load fluctuations appear to conform to the normal probability law and therefore the magnitude of swings for blocks of load similar to those measured should vary as the square root of the average load. For example, if the load is increased nine times the magnitude of the fluctuation will be increased only three times. By this method the approximate magnitude of fluctuations in the railroad load can be determined for each station of this system.

The railroad load fluctuations exist long enough to affect system speed and, therefore, the distribution should be in accordance with turbine governor characteristics. Although individual governors vary, it is probable that the composite characteristic for each generating station is similar to that of the others and that load changes at any point in the system are shared by all generating stations approximately in proportion to their capacities.

In applying the foregoing principles, the magnitude of load swings which would be produced in all parts of the system by each individual portion of railroad load acting separately was first determined. The total swings at any particular part of the systems was obtained by taking the square root of the sum of the squares of the individual swings. Figure 14 shows the calculated maximum load swings which would be expected in each part of the system under normal operating conditions at the time of the maximum railroad load. The calculated values approximate the maximum swings recorded by graphic wattmeters in a number of these circuits.

The sharing by all generating stations of short-time load swings occurring in all parts of the system is a desirable characteristic because the swings on each individual station are less than the fluctuations in the load distributed from the station bus. Although the resulting short-time load swings produce additional heating in tie feeders and frequency changers, calculations show that, in general, this reduces the effective capacity of equipment less than one per cent except in the ties to railroad generating stations where the reduction is three per cent with all ties in service.

Power Oscillations

The Hudson Avenue tests showed that the small two-second oscillations in the generator and tie-feeder loads which appeared only when the station was tied to the system were not caused or maintained by governor action. These oscillations are probably synchronizing swings caused by small sudden changes in the bus load such as result from starting or stopping of individual subway trains. Although the synchronizing swings are highly damped, the initiating impulses may occur so frequently that the system is seldom free from small power oscillations. Since these oscillations are only one per cent of the station load, they are of no importance in system operation.

Long-Time Load Swings

The only type of load swings which affect the operation of boilers and tie feeders and result in undesirable variations in voltage and frequency are the long-time swings lasting five minutes or longer and having a magnitude of five to ten per cent of the station load. These swings can be traced to manual adjustments of the governor valves during periods when the system load is changing rapidly. Examples of such swings are shown in figure 15.

At present the individual station operators have no accurate indication of the operations being performed in other stations so that load is occasionally adjusted manually at two or more stations in a manner to cause heavy tie-line load swings. In order to co-ordinate system operation, a common frequency indicator is being installed in each of the major stations consisting essentially of a synchroscope between a crystal-controlled 60-cycle timing wave common to all stations and the station frequency. The operators at each station with the aid of these instruments will pick up assigned blocks of load in small steps only when the frequency is low and conversely will drop load in small steps only when the frequency is high. By closer control of frequency during the periods of the day when system load is changing rapidly the stations will not endeavor to pick up or drop generator load faster than the system load is changing. Counter adjustment which often results from one station adjusting load so rapidly as to affect the system frequency will be prevented. This will reduce the magnitude of the load swings during the period of rapid load change.

Conclusions

From the results available at the present time, the following general conclusions have been drawn:

1. A reduction in bus voltage will result in the rejection of real and reactive load, and

Power System Governing—the Problem

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Synopsis: Considerable attention has been focused on the subject of load swings, turbine generator governing, and stability not only when operating a system alone but when interconnected with other systems. At a subcommittee meeting of the power generation committee, two assignments were made: One to crystallize the problem and the other to outline the desirable functional characteristics of a governing system. This memorandum, dealing with the former, has been prepared based on answers to a questionnaire submitted to a representative group of operating companies. The response to the questionnaire was excellent, practically all of the operating companies to whom it was sent having made replies.

The report is divided into two parts. Part I presents the questionnaire and the replies. Part II gives a discussion of the elements of the problem of governing on power systems.

Conclusions:

1. The general conclusions seem to indicate that there are no outstanding deficiencies in the modern governing systems.

2. It seems likely that full advantage is not being utilized of the inherent capabilities of many existing governors.

3. The full potentialities of governors often are not utilized until some fluctuating load or unusual incident arises which requires that action be taken.

4. A test program is required to disclose maintenance defects, wrong adjustments, and improper operation.

5. The limitations of the speed governor for interconnected operation must be recognized, and means provided to supplement the governor when close control of frequency or tie-line load is required.

Part I

THE questionnaire submitted to the operating companies is reproduced in the first column on chart I. In carrying out the assignment of the subcommittee, the questions were directed at obtaining a cross section of opinion as revealed by present operating practices. The first group of questions serve to define the operating requirements in so far as they are affected by the size of the system, proportion of steam and hydro, type of present governors, roughness of the load, interconnections, etc. A second group of questions serves the purpose of telling how these requirements are currently being met by showing the amount of droop used and the procedure in regulating frequency and tie-line loads. A third group of questions concerns tests made on governors. It was felt the amount of governor testing would furnish a good index of how serious the governor problem has been in the past. There has been some evidence that ordinary measuring devices, such as graphic frequency meters, tachometers, etc., are not fully adequate for checking governor performance, and a question was included to obtain opinions on improved instruments for this purpose. Question 18 was intended to bring out opinions on the adequacy of present governors, and any forward thinking as to the lines along which improvement should be directed.

Chart I tabulates the individual replies while chart II summarizes these answers in a form that the trend can be recognized. A study of these charts is interesting, but does not reveal a well-crystallized statement of the governor control problem. A surprisingly large number report satisfaction with present equipment. On the other hand, most of those companies reporting present governors as essentially satisfactory have fairly steady loads or have made few if any tests. This may indicate that further study and testing would lead to a more uniform and precise expression of governor requirements. It would also appear that more refined testing instruments are desirable.

Part II

The answers to the questionnaire show that, in general, there is no great concern about the operation of the prime mover governors. It is of interest to note, however, that the systems reporting particular interest in governors are those which serve the heavy industrial or electrified railway loads. One such system is interested in the effect of governors on tie-line loads. As the questionnaire has not disclosed any major difficulties, the following discussion of governing problems has been prepared, based on information derived from published articles, informal discussions of operating problems, and information secured from governor specialists. This discussion has been prepared in four sections:

1. Description of the simple speed governor.

Paper 41-68, recommended by the AIEE committee on power generation, and presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941; prepared through the co-operation of the operating companies listed in the appendix. Manuscript submitted November 13, 1940; made available for preprinting December 14, 1940.

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conversely, an increase in voltage will increase both real and reactive load. A decrease of 10 per cent in bus voltage with the frequency approximately constant resulted in a reduction of 13 per cent in real load and 17.4 per cent in reactive load.

2. Changes in frequency within reasonable limits will not result in appreciable changes in load if the bus voltage is maintained constant. A three per cent change in frequency resulted in no noticeable load change.

3. Continuous small power oscillations of three to five megawatts are present in the tie feeders and generators when tied to the rest of the system. These oscillations have a period of about two seconds and are believed to be synchronizing power swings initiated by sudden changes in distributed load. These oscillations are so small as to be of no consequence.

4. Short-time load swings which have a period of 0.5 to 2.0 minutes appear in the generator and tie-feeder loads and are caused by railroad and transit load fluctuations. Regardless of their origin they are distributed among all generating stations in proportion to the station spinning capacities and amount to less than three per cent of those capacities. These swings have no appreciable effect on the effective capacity of tie lines, generators, boilers, or other equipment.

5. The governors do not contribute to any of the load swings but are operating continuously to maintain the instantaneous frequency within a band of 0.20 of a cycle.

6. The automatic frequency-regulating equipment supplementing the normal governor action has no effect on short-time load swings and synchronizing power oscillations.

7. Transient oscillations on the system such as those caused by tripping a generator or tie feeder are rapidly damped.

8. Long-time load swings are caused by manual operation of governor valves in one or more stations during periods of load change. These are the swings which affect the operation of tie feeders, generators, boilers, and other equipment. By giving the operators in all stations a better understanding of the problem and by providing them with common frequency indicators, it is expected that the long-time swings will be reduced.

Chart I—Answers to Questionnaire on Prime-Mover Governors—

		1	2	3
1	Kw of capacity normally in service for maximum loads?	325,000	511,400	132,500
	(a) Kw of this capacity—steam?	25,000	511,400	132,500
	—hydro?	300,000	None	None
	—Diesel?	None	None	None
2	Kw of steam prime movers having fly-ball governors directly operating the inlet valves?	None	None	None
	(a) Kw of steam prime movers having fly-ball governors with intermediate operating mechanisms (servomotors) between the governor and inlet valves?	25,000	316,500	132,500
	(b) Kw of steam prime movers having fluid governors?	None	222,600	None
3	The no-load, full-load speed regulation varies from a maximum of—%	13.5	5.77	5.0
	(a) To a minimum of—%	1.5	2.25	5.0
	(b) With an average speed-regulation of—%	3.0	3.93	5.0
4	At full load periods what percent of the capacity is operated with blocked governors?	Hydro—100% during high flow; steam—never.	None	11.3
	(a) At light load periods what percent of the capacity is operated with blocked governors?	Hydro—100% during high flow; steam—never.	None	16.6
5	What percent of the operating capacity at full load has automatic combustion control in service on the boilers?	100%	55-full auto. 25-full auto. on air supply only	None
	(a) Percent of the operating capacity at light load?	100%	100	None
6	Are system loads steady?	....		Steady
	weaving?	....	....	....
	rough?	Extremely rough.	Rough	....
	(a) What is the approximate normal load variation during a 15-minute period in percent (plus or minus) of average load?	15.0	6.0	2.0
	(b) If a steel mill, large electric furnace, heavy railroad electrification or other fluctuating load is served and if it affects the load characteristics, describe the type of load cycle in detail.	May get 15% or 75,000 Kw during 2 min. due to steel and railway	During load change may get 11% due to electric furnaces, railway, and rolling mill load	....
7	Have the speed governors been modified or any particular units adjusted to assist or improve the system speed regulation beyond that obtainable with the original governor equipment?	No major modifications. Tests only.	No—Special electric load regulator for 60,000 Kw unit on order.	No
8	Do you operate in parallel with adjacent systems?	Yes	Yes	Yes
	(a) Occasionally?	....	....	....
	(b) Usually?	Always	Yes	Yes
	(c) What is the approximate Kw capacity of the interconnection?	2,500,000	8,500,000	188,000
9	When operating isolated, how is system frequency regulated?	See 8(b)	See remarks	Manually
	, how is tie-line load regulated?		....	
	, how is electric time regulated?		....	
	(a) When operating interconnected, how is system frequency regulated?	By dispatcher of interconnection.	By one of systems in interconnection	Manually
	tie-line load?		Leeds-Northrup	
	electric time?		See remarks	
10	What is the basis and method of allocating load on the various operating units?	Definite blocks according to economy of stations, etc.	Incremental heat consumption, corrected.	Economy
	(a) Does the regulation of frequency or tie-line load conflict with the ideal load distribution?	Yes, almost invariably	No	No
11	Does interconnection with neighboring properties affect your governor practice with respect to number of machines governing on speed?	Yes, dependent upon interconnection.	Not yet	No
	(a) The amount of speed droop?		Not yet	....
	(b) Any other effect?	....	Not yet	....
12	Have you had major difficulties which might be associated with the governing function? For example, pull-out of generating units or generating stations.	No	No but have trouble when isolated due to strip mill.	No
13	Have you had disturbances, not necessarily involving interruption to service, which have caused you to impose limitations on some phase of your system operation and which you feel were associated with governor performance? For example, oscillations hunting any others (a) If such disturbances are traceable to other causes, such as loss of field or low excitation, please distinguish.	No	No	No
14	Do you make any routine tests to determine the speed regulation and speed of response of the governors?	Not reduced to routine matter.	See remarks	Regulation—Yes Response—No
15	Have you made any other tests to determine individual governor performance as a result of hunting or other abnormal operation?	Yes, as required.	Yes, in 1929	No
16	Have you any information derived from governor tests pertaining to governor performance or the variation of the system load with speed and voltage that you can disclose at this time?	Yes, considered of secondary importance on this system.	No	No
17	What type of test instruments have been used for governor tests? (a) How does their speed of response compare with the governor characteristics?	Oscillograph and high speed graphic meters.	Varying from mechanical recorders to magnetic oscillographs (a)—	None made but would use oscillograph if tests were made
18	Are you satisfied with your present governing system?	No	Yes, fundamentally	Yes
	(a) If you are not satisfied, then along what lines should improvements be made?	Frequency regulation	See remarks	....
19	Remarks.		9 Leeds-Northrup frequency regulator 9(a) electric time—By one of system in interconnection 14 Yes, for speed regulation. Tests for speed of response under way. 18(a) To secure same speed of response on various units & adherence to overall speed regulation over load range.	None

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4	5	6	7	8	9	10
578,000	180,000 to 200,000	104,900	See Remarks 1,020,675	1,000,000	27,000	109,250
None	200,000	40,500	1,010,000	747,000	27,000	109,250
578,000	None	64,400	10,675	253,000	None	None
None	None	None	None	None	None	None
Does not apply	None	49,000	None	None	None	4700
Does not apply	227,000	None	1,010,000	562,000	27,000	None
Does not apply	10,000	None	None	185,000	None	109,250
5.0	105.0	0.02	7.6	8.0	10.0	Only with frequency of system
2.0	88.0	−.02	3.6	4.0	7.0	
3.5	5.0	0.0	4.0	6.0	7.0	
90.	None	60	None	By units, determined by boiler capacity, independent of load conditions	0.0	None
0.	None	15	None		0.0	
Does not apply	50	0	None	42	100	None
	90	0	None	90	100	None
....	Steady	Steady	....	....	Steady	Steady
Weaving	....	....	....	....	....	....
....	....	....	Moderately rough	Rough	....	....
3.0	2.0	6.0	140,000 Kw in 15 min. Steel mill and furnace loads cause changes of 40,000 Kw.	5.0 to 20.0	5.0	No appreciable variation from scheduled load for short periods because of interconnection
See remarks	....	....			Fluctuations absorbed by interconnection	
Not materially	Yes, on two units	No	No	Yes, on one later unit	No	No
Yes, see remarks	No	Yes	Yes	Yes	Yes	Yes
....	....	....	....	....	....	....
Always	....	Yes	Always	Always	Yes	Normally
1,378,000	....		1,100,000	2,700,000	500,000	216,750
Whole question now under consideration Electric Time.	Manual	Automatic Manual Manual	See 8(b)	See 8(b)	By turbo-governors	See 8(b)
To limits of ±5 seconds monthly.	....	Auto. or Manual Manual Manual	Manual Interconnecting Co. Manual	See Remarks See Remarks	By interconnected system	By large station of interconnected system (See Remarks)
River flow	Water rate	Base load on some units others take swing	Increment cost curves	See Remarks	Manually	Incremental loading method
Not seriously	....	Occasionally	Yes	Yes	....	Not normally
No	....	No	No	Yes	Yes	No
....	....	....	....	Yes	Yes	....
....	....	....	....	....	....	....
No	No	Yes, on some types of interconnection	No	No	No	No
No	Cannot operate one 10,000 unit as a single unit on system—governor too slow for light swinging load.	Have had cases of instability on one interconnection	No	No	No	No
	....			One (any others)		
	....			No		
Just initiating practice	No	Yes	Overspeed tests only	No routine tests. Speed tests only as required	Speed regulation tests at annual overhaul	No routine tests
Occasionally	Yes, on one unit, See 13	No	No	None in particular	No	No
No	Yes, see 13	No	No	None other than already published	No	No
Have tried recorder frequency meter & mechanical motion recorder synchronized.	Tachometers only.	Portable speed indicators	None	Tachometers for governor regulation	Hand tachometer. Unknown (a)	Hand tachometers ....
Fairly so	Yes, but for one unit.	Not entirely	Yes	Refinements desired	Yes	Yes
Frequency and load regulation	Prevent swings on light loads	Tie-line load control	....	See Remarks	Improve steam admission valves.	....
6(b) Load controlled paper mill load; electric boiler & electric furnace loads. Load variation small. 8 Additional major interconnection now under construction	None	None	Answers do not include data on small or miscellaneous steam or hydro units	9(a) By one company of interconnection 9 tie regulated — same 10 Load increased on station at lowest cost per Kw hr. 18(a) Desire increased sensitivity; more uniform instantaneous speed regulation over entire load range; reduce average speed regulation. Ref. to *Transactions* ASME, April 1940	None	9(a) Tie line regulation accomplished by load scheduling

Chart I (Continued)—Answers to Questionnaire on Prime Mover Governors—

		11	12	13
1	Kw of capacity normally in service for maximum loads?	323,160	550,000	800,000 See Remarks
	(a) Kw of this capacity—steam?	322,860	350,000	275,000
	—hydro?	300	200,000	445,000
	—Diesel?	None	None	80,000
2	Kw of steam prime movers having fly-ball governors directly operating the inlet valves?	None	None	None
	(a) Kw of steam prime movers having fly-ball governors with intermediate operating mechanisms (Servo-motors) between the governor and inlet valves?	358,860	350,000	275,000
	(b) Kw of steam prime movers having fluid governors?	30,000	None	None
3	The no-load, full-load speed regulation varies from a maximum of—%	....	18.0	7.0 All approximate
	(a) To a minimum of—%	....	3.0	2.0
	(b) With an average speed-regulation of—%	....	7.5	3.4
4	At full load periods what percent of the capacity is operated with blocked governors?	None	Steam units occasionally. Hydro units when wasting water	None
	(a) At light load periods what percent of the capacity is operated with blocked governors?	None		None—Load limiting devices sometimes used
5	What percent of the operating capacity at full load has automatic combustion control in service on the boilers?	70	16	60
	(a) Percent of the operating capacity at light load?	None	30	90
6	Are system loads steady?	Steady	Steady	....
	weaving?	....	....	Weaving
	rough?	....	....	....
	(a) What is the approximate normal load variation during a 15-minute period in percent (plus or minus) of average load?	1.5	1.0	2% over 60 minute period
	(b) If a steel mill, large electric furnace, heavy railroad electrification or other fluctuating load is served and if it affects the load characteristics, describe the type of load cycle in detail.	One furnace swings from 0 to 3000 Kw	....	....
7	Have the speed governors been modified or any particular units adjusted to assist or improve the system speed regulation beyond that obtainable with the original governor equipment?	No	One hydro replaced. Several adjusted.	No
8	Do you operate in parallel with adjacent systems?	Yes	No	Yes
	(a) Occasionally?	....	....	....
	(b) Usually?	Usually	....	Yes
	(c) What is the approximate Kw capacity of the interconnection?	428,560	....	950,000
9	When operating isolated, how is system frequency regulated?	Manual	Manual	Automatic frequency control with or without tie-line blocking
	, how is tie-line load regulated?		....	
	, how is electric time regulated?		Manual	
	(a) When operating interconnected, how is system frequency regulated?	Automatic frequency regulator on interconnection	....	
	tie-line load?		....	
	electric time?		....	
10	What is the basis and method of allocating load on the various operating units?	See Remarks	See Remarks	Incremental production costs
	(a) Does the regulation of frequency or tie-line load conflict with the ideal load distribution?	No	No	Insignificantly
11	Does interconnection with neighboring properties affect your governor practice with respect to number of machines governing on speed? (a) The amount of speed droop? (b) Any other effect?	No	See 8	No
12	Have you had major difficulties which might be associated with the governing function? For example, pull-out of generating units or generating stations.	No	No	Occasional mechanical troubles only
13	Have you had disturbances, not necessarily involving interruption to service, which have caused you to impose limitations on some phase of your system operation and which you feel were associated with governor performance? For example, oscillations hunting any others (a) If such disturbances are traceable to other causes, such as loss of field or low excitation, please distinguish.	No	Yes, See Remarks	No
14	Do you make any routine tests to determine the speed regulation and speed of response of the governors?	No	Fairly complete but not routine	Occasional only
15	Have you made any other tests to determine individual governor performance as a result of hunting or other abnormal operation?	No	No	See 14
16	Have you any information derived from governor tests pertaining to governor performance or the variation of the system load with speed and voltage that you can disclose at this time?	No	Published data See Remarks	No
17	What type of test instruments have been used for governor tests?		See Remarks	Mechanical tachometer
	(a) How does their speed of response compare with the governor characteristics?	No tests made	Very satisfactory	
18	Are you satisfied with your present governing system?	Yes	No	See Remarks
	(a) If you are not satisfied, then along what lines should improvements be made?		Reduce frequency swings.	See Remarks
19	Remarks.	10. To secure maximum efficiency of all generating units, including units of interconnection.	10. On hydro—by water available. On steam—economy. 13. Charts showing fluctuations included with answers, not caused by (a). 16. Refer A.S.M.E. Tr. Vol. 62 No. 3 page 207 17. C.E. photocell. Steam engine indicators.	18. Modern governors satisfactory. 18(a) Modernize old governors to improve sensitivity. 1. 80,000 Kw shown is freq. changers.

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14	15	16	17	18	19	20
2,000,000	900,000 to 1,000,000	120,000	600,000	1,866,000	300,000	737,700
2,000,000	See Remarks	70,000	300,000	1,866,000	200,000	737,700
None		30,000	300,000	None	100,000	....
None	None	None	None	None	None	....
None	None	None	None	None	None	None
1,947,000	224,000	104,500	300,000	1,766,000	200,000	647,700
53,000	None	25,000	None	100,000	None	90,000
Not yet available	10.0	8.0	3% slope, approximately, older and smaller units slightly steeper	....	4.8	4.0
	2.5	4.0		....	2.7	3.0
	5.	5.0		3%	4.0	2.0
None. One 53,000 unit on load limit when on one boiler.	100	0	Do not so operate. Use same load limit stops.	None	None on steam, light or heavy. All hydro except during high water when holding freq.	None
	100	0		None		None
60	1/2 of primary steam capacity	31.	20	25	None	23
60		80.	40	50	None	90
Steady	Fairly steady	Steady	Between weaving and steady	....	Steady	Steady
....	....	....		Weaving		....
....	....	....		....		....
Normal, 1–5%; Morning pickup. 12%	≈5.	1.5	3%	7½%, with 2½% minute to minute	3%	2.0
....	Steel mill causes rough load when working	....	....	....	Paper mill on interconnection causes abrupt 2 to 3% change	....
Yes, on one unit.	No	No	None in particular	No	No	No
Yes	Yes	Yes	Yes	No answer given	Yes	Yes
....	....	....	....		....	
Always	Always	Yes	Usual practice		Usually	Always
2,200,000	1,165,000	180,000	100,000's		322,000	25,000,000
See 8(b)	See 8(b)		See 8(b)	L. & N. frequency Controller	Manual Other company Manual	See 8(b)
See Remarks	By one or two stations	Manually	Automatic 30% Manual Automatic 30%	No answer given	Manual Other company Manual	Manually by central dispatching office
Incremental heat rate	Stream flow & economy on steam stations	Efficiency & reliability of lines	Over-all economy	See Remarks	See Remarks	Incremental cost
Yes	At times	No	Frequency only to minor extent		Minor extent when unit is controlling freq.	No
No	Not materially	No	No	No	No, see 6(b)	No
No	No	No	No	No	No	No
No	No	No	No	No	No	No
No	No	During routine overhaul	At periodic overhaul	No	Speed only at overhaul	No
Yes	No	No	Yes with studies of auto. control equipment.	No	No	No
Tests now in progress	No	No	No	No	No	No
Long film oscillograph	None on steam governors	....	High speed oscillograph & frequency meter	No tests made	None	....
Faster			Equally fast			
Yes	In general, yes	Yes	Reasonably so	Fairly so	Yes	Yes
....	....	....	....	Greater sensitivity of response on individual units.	....	....
9. System A regulates by Leeds-Northrup & System B controls tie load. Time—by Leeds-Northrup and crystal oscillator. When B regulates frequency A on selective freq. at one station	1. Kw of steam and hydro varies widely depending on season, amount of rainfall, etc. In general, answers are for steam only	None	None	10. Percentage of total load, one or two units on freq. or tie line control	10. Equivalent increment cost on steam; hydro to reduce steam peaks	

2. Limitations of the simple speed governor.

3. Auxiliary equipment required to assist the speed governor in controlling load.

4. Difficulties encountered in the operation of speed governors.

1. DESCRIPTION

The speed governor is used to control the input to a prime mover in such a manner that the speed will be held within definite predetermined limits. The simple governor, therefore, is fundamentally a device functioning directly from speed, and is not responsive to changes in the prime mover output except in so far as the output affects the speed. The governor, being a mechanical device, is subject to all the variations that result from friction and lost motion in the moving parts, making it impossible from a practical standpoint to produce two similar governors with an exact duplication of characteristics. The governors must, therefore, be provided with some means whereby several governors, presumably set for the same speed, can operate in parallel and share the load. The required characteristics are usually inherent in any governor system which is spring loaded, as at any particular speed, the governor will assume a definite position, usually compressing the spring as the speed increases, and permitting the spring to expand as the speed decreases.

The governor is connected by means of links and power cylinders to the inlet valves in such a manner that the inlet valves take a definite position for each position of the governor. In this manner, the governor can be adjusted to hold the inlet valves closed at normal speed and open them as the speed decreases. The amount of speed droop required to move the inlet valves from the no-load to the full-load position is called the speed droop or speed regulation, and is expressed as the per cent speed change from normal speed required to change the output from no load to full load. The per cent regulation is controlled by the design of the governor springs and the ratio of movement produced by the valve linkage.

It is desirable to be able to shift the no-load speed of the governor both to permit synchronizing and to control the output. This is usually accomplished by an auxiliary spring operating in parallel with the main governor spring, the tension of the spring being varied by mechanically moving one end of the spring. The governor does not directly measure the output of the unit, but sets the inlet valves for a certain output which must be attained if the steam pressure and vacuum are normal. The changes in output which result from inlet or exhaust pressure variations will not affect the governor unless these changes are reflected in the system speed.

2. LIMITATIONS

The following discussion has been written from the viewpoint of governor application on a power system and does not attempt to include a discussion of special applications such as exist in many industrial plants. When two or more prime movers, having speed governors with satisfactory speed droop characteristics and ideal performance for isolated operation are paralleled, certain limitations in the operation of the speed governor immediately become evident. Certain of the prime movers, having good thermal characteristics are expected to operate on a base load schedule. The speed governors, however, are continually shifting the load, and in order to adhere to a constant load, it becomes necessary to either block the operation of the governor by load limit devices or partially close the throttle valve. This procedure transfers the load variations to the less efficient units, and also results in a somewhat greater speed change for a given load variation due to the reduction of the governing capacity. It is common practice to use a broader speed regulation on base load units than on the less efficient units. The usual result of those expedients is the restriction of the operation of the governors.

Another limitation of the speed governor appears when a power system forms an interconnection with another power system of much larger size. The speed variations, in general, are a function of the load changes of the larger section of the interconnection as the load variations of the small system are not usually of sufficient magnitude to materially affect the interconnected system speed. The governors on the small system, no matter how excellent their performance, cannot function to follow the true load on their own system. They will, however, attempt to assist the larger system in regulating the frequency, with the result that the load variations in the tie line may be excessive. Again, the usual solution to these difficulties is broadening the speed regulation of the governors on the small system. These governor limitations would not be so apparent if the interconnected system frequency variation were held within narrower limits.

3. AUXILIARIES

There are several main types of auxiliaries that are used either to assist the speed governors, or to counteract the action of the speed governors when their action interferes with proper load distribution.

The speed changer mechanism is an auxiliary device that is necessary on every governor in order to change the speed calibration. The shifting of the no-load speed by means of the speed changer results in changing the valve opening for any given speed, and, therefore, the unit output is changed. From a practical standpoint, the adjustment of the speed changer of any one turbine on a power system, results in a change in output of that unit. The normal load control procedure on a power system is to first observe the system speed; when the speed varies beyond the normal operating band, the speed changers are adjusted so that the load changes are in agreement with a prearranged loading schedule and this manual manipulation tends to correct any drift of frequency from the normal base. Most governors have an overall speed regulation in the order of three to four per cent and if the speed control were left entirely to the governor there would result a speed droop of one to perhaps $2^1/_2$ cycles on a 60-cycle base, depending on the amount of load change. It is, therefore, necessary that during load change periods, the speed changers be adjusted quite frequently in order to hold the system speed within reasonable limits.

Within the last ten years, many systems have installed automatic equipment for the control of frequency and tie-line interchange. The commercially available equipment consists, in general, of contact-making wattmeters or frequency meters connected to the speed changer drive of the governor. This equipment recalibrates certain governors at periodic intervals, provided the quantity being controlled has departed beyond the control point. There are several practical limitations to this procedure, particularly when a system becomes quite large. The operating range of the regulating units is usually not sufficient to take care of the short-time load changes, and the use of any one particular station for regulation may result in abnormal loads on main tie lines. For these reasons, it may be necessary to provide some regulating capacity in each load area. One school of thought suggests that if each load area must provide assistance for automatic regulation that the speed governor detecting the change in speed can best provide this assistance. However, several systems have found by experience that assistance is best determined by the load indication in tie lines

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rather than by frequency. The simple speed governor is unable to identify any such indication, and must be recalibrated as indicated by tie-line interchange to hold the proper load.

There are cases where the conventional speed changer is not adequate for auxiliary control, and special devices have been installed between the speed governor and the valve control mechanism to operate the inlet valves directly from load. The use of topping turbines has introduced the back pressure regulator on main unit prime movers, displacing the function of the speed governor. These special applications will not be discussed as they do not directly pertain to speed regulation.

Experience has shown that where automatic control equipment has been installed in conjunction with the speed governors, and has been able to hold the system speed within reasonable limits, that many of the limitations of the speed governor are eliminated. Base load operation is satisfactory, and yet, the governors are immediately available for any unusual contingency that may arise. Cases where excessive load variations or swings have been attributed to governors under constant speed operation are usually due to one or two governors functioning improperly.

4. DIFFICULTIES

In discussing difficulties that have existed, it should be borne in mind that this discussion covers a summary of the more common difficulties that are likely to exist. However, no one governor will probably have all of these troubles, and no one system will probably have more than a few of the troubles described. There are certain fundamental factors which must be considered when applying speed governors on the turbine. There are definite limitations to the speed of response and sensitivity which are a function of the prime movers and system load characteristics. These factors are well understood by the manufacturers, and in general, it is believed that few governors have been installed which do not meet the minimum requirements.

Some difficulties are now arising in the application of governors where the newer high-speed turbogenerators, equipped with fast governors are operated in parallel with older units with slow governors. Equipment is available for some types of these fast governors to increase their regulation under transient conditions, thereby providing a means of sharing the load swings without interfering with the speed regulation for steady-state conditions.

Another application trouble results from inadequacies of the oil supply for the operating cylinders, making it impossible to provide the required force to move the inlet valves in step with the governor. This condition may exist on some of the older governors, and can be corrected by the addition of oil accumulators. The steady-state operation of the governor will probably not be affected. Many of the older type governors require the governor to furnish the actuating force for the rocker arms and links between the governor and the inlet valves. The forces required to rapidly move these masses may not be available from the governor for load swings or overspeed. It is necessary to add an intermediate relay and operating cylinder to translate the motion of the governor.

The operating difficulties to which governors are subjected are considered to be these troubles that can be remedied by proper maintenance and adjustment. It has been found that the majority of governor troubles fall into these classifications. In many cases, the blame cannot be placed directly on the governor, because after overhaul periods or other work which has required the tearing down and assembling of the governor, no test procedure has been set up to determine that the operating of the governor is entirely satisfactory. The operation of the governors is usually observed for a period after overhaul, but this type of check is mainly of value in detecting hunting.

The most obvious difficulty and the one that is most commonly observed is governor instability or hunting. This phenomenon can best be described as a tendency of the governor to continually maneuver, often without any direct relation to the speed change. This hunting may take place continually or be observed immediately following a speed or load change, after which it is slowly dampened out. Hunting may be due to several factors, such as an extremely fast governor, very sluggish governor, or improper adjustment of the control mechanism. There may also be indications of hunting when fast governors are operating in parallel with slow governors. An analysis of several of these cases has shown that what appeared to be hunting was nothing other than the fast governors accurately following the speed changes by regulating more than their share of the load until such times as the slow machines became active. Some apparent cases of governor hunting are caused by excessive variations in steam pressure which directly affect the output of the turbogenerators. Another example of generator load swings erroneously identified with governor hunting are those caused by electromechanical oscillations between the various generating units, usually induced by load shifts on the transmission system.

The adjustment of the control mechanism can cause hunting by having the speed of response too fast for the inertia characteristics of the generating unit or by overcompensating a sluggish governor. Another trouble that is frequently found, but can be determined usually only by special tests, is an erratic speed droop. Very few steam turbines have a uniform load response for a given speed variation over the entire load range. This is due to fundamental characteristics of the governor operated by centrifugal force, variation in the angularity of the links and rocker arms, and the output characteristics of the inlet valves. Some types of governors are very sensitive to the variations in forces existing on the inlet valves at various openings. On multivalve turbines, the incremental speed regulation usually varies considerably just preceding and after the opening of an inlet valve. In some cases, valve overlaps, which have been determined for purposes of economy, have been reduced to such a degree that dead spots exist in the governor. Another cause of excessive speed droop, which cannot be overlooked, is the change in steam pressure with the change in load.

The speed droop is usually determined for steady-state conditions. However, actual speed droop will change with the rate of load change. It is not unusual to find a governor which shows an over-all speed regulation of four per cent actually operating at a speed droop of 10 to 20 per cent, for load swings such as those produced by a steel mill.

Another major trouble is lack of sensitivity of the governing system either to speed changes or to load adjustments by means of the speed changers. This difficulty is usually the result of excessive friction. A governor with excessive friction may operate satisfactorily when the speed is steady, being erratic in operation only when adjusted by means of the speed changer. This same governor, under a less steady system speed, may show a tendency to creep.

The speed changer mechanism is frequently a source of trouble. In some cases, the relation between revolutions of the speed changer motor and the response of the governor is such that the speed changer motor must operate at high speed for several seconds in order to initiate a load change. The speed changer then coasts for several more seconds, causing overtravel, and making it difficult

Chart II—Prime-Mover-Governor Questionnaire—Summary of Answers of 20 Companies

1 Kw of capacity normally in service for maximum loads?	4 Co's report 1,000,000 or over; 6 Co's 500,000–1,000,000; 9 Co's 100,000–500,000; 1 Co under 100,000.
(a) Kw of this capacity—steam?	6 Co's report over 500,000; 8 Co's report 100,000–500,000; 4 Co's under 100,000; 1 Co "none"; 1 Co wide variations.
—hydro?	7 Co's report "none"; 5 Co's under 100,000; 7 Co's report 200,000–578,000; 1 Co reports wide variation.
—Diesel?	20 Co's report "none"; 1 Co reports 80,000 in frequency changers.
2 Kw of steam prime movers having fly-ball governors directly operating the inlet valves?	18 Co's report "none"; 1 Co reports 40,000; 1 Co reports 4,000.
(a) Kw of steam prime movers having fly-ball governors with intermediate operating mechanisms (servomotors) between the governor and inlet valves?	3 Co's report "none"; 2 Co's report 2,500; 7 Co's 100,000–300,000; 8 Co's in excess of 300,000.
(b) Kw of steam prime movers having fluid governors?	10 Co's report "none"; 5 Co's report 10,000–100,000; 4 Co's report 100,000–220,000. Note—No. 4 Company is 100 per cent hydro.
3 The no-load, full-load speed regulation varies from a maximum of—%	Max. reported of 19 Co's—18%; 1 Co reports 105% (probably 5%).
(a) To a minimum of—%	Lowest min. of 19 Co's is—.02%; 1 Co reports 88% (probably 12%).
(b) With an average speed-regulation of—%	Average ranges from 0% to 7.5%.
4 At full load periods what per cent of the capacity is operated with blocked governors?	13 Co's report "none"; 3 Co's report over 50%; 4 Co's report a low percent, load limits, or special units.
(a) At light load periods what percent of the capacity is operated with blocked governors?	15 Co's report "none"; 2 Co's report app. 15%; 3 Co's report load limits, or special units only. Hydro units: During high flow periods.
5 What percent of the operating capacity at full load has automatic combustion control in service on the boilers?	6 Co's report "none"; 2 Co's 100%; 11 Co's range from 16% to 70%.
(a) Percent of the operating capacity at light load?	7 Co's report "none"; 3 Co's 100%; 3 Co's report 90%; 6 Co's 30% to 80%. Note—No. 4 Co is 100% hydro.
6 Are system loads steady?	12 Co's report "steady".
weaving?	3 Co's report "weaving".
rough?	2 Co's report "rough"; 2 Co's "moderately or extremely rough"; 1 Co reports between "weaving" & "steady".
(a) What is the approximate normal load variation during a 15-minute period in percent (plus or minus) of average load?	10 Co's report 3% or under; 4 Co's report 5% to 7 1/2%; 4 Co's report 15%–20% including morning pick-ups; 1 Co slight.
(b) If a steel mill, large electric furnace, heavy railroad electrification or other fluctuating load is served and if it affects the load characteristics, describe the type of load cycle in detail.	The 4 Co's reporting "rough", or "extremely rough" report { 75,000 in 2 min. or 15%. 11% in 15 min.—total load 511,400 Kw. 40,000 Kwchange—noperiod given. 20% no period given. 1 Co reporting "steady" has abrupt 2–3% change on interconnection. 1 Co reporting "weaving" has 2 1/2% minute to minute variation.
7 Have the speed governors been modified or any particular units adjusted to assist or improve the system speed regulation beyond that obtainable with the original governor equipment?	15 Co's report "No"; 5 Co's report modifications on specific units; 1 Co reports load regulator on order.
8 Do you operate in parallel with adjacent systems?	17 Co's report "Yes"; 2 Co's report "No"; 1 Co does not answer question.
(a) Occasionally?	No company reports "occasionally".
(b) Usually?	10 Co's report "Usually"; 1 Co—"normally"; 7 Co's report "always".
(c) What is the approximate Kw capacity of the interconnection?	Of 10 Co's who interpreted this question correctly, all reported 1,000,000 to 25,000,000; the 7 others operating interconnected, apparently reported tie-line capacity only.

for the operator to properly estimate the load change. This condition can be improved by reducing the load on the speed changer motor and then changing the speed of the motor so that the motor inertia is not a controlling factor. These difficulties usually are not serious when operating with hand control, but are very evident when automatic frequency or tie-line control is applied.

There appear at present to be four factors which need additional consideration from an application standpoint:

1. Means of adjusting the speed regulation both for steady-state and transient conditions to permit co-ordination of the newer governors with the older governors.
2. Further reduction in the incremental regulation from minimum to maximum load and the elimination of dead spots.
3. Improvements in the speed changer mechanism to provide the required sensitivity and uniform response.
4. Provision for decreasing the speed regulation of the governor within the continuous maneuvering range.

This last suggestion may be subject to some difference of opinion, but is based on the following thought. The system load can be broken down into three components. The fundamental component represents the short-time load variation that has a period lasting from several minutes to an hour or more. This component is easily distinguished by observing the weaving line on the frequency recorder. The second component is the short-time variation which is observed on the frequency recorders as the frequency band. This may vary from a few hundredths of a cycle to several tenths of a cycle, depending on the size of the system and the character of the load. These variations occur every few seconds and many systems, using automatic control, consider it best to have the control inoperative in this range and rely on the system inertia and governors to provide the regulating effort. Applying the same reasoning, the speed governor would have normal sensitivity in this range, but the control mechanism would be damped to prevent excessive operation of the inlet valves until either the speed error exceeded this limit, or errors within the limit existed for an excessive length of time. The third component is due to very short duration load changes due to line switching, motor starting, etc., and these are rarely reflected in the system frequency.

Several systems, which have investigated governor performance at various times, have been convinced that many of the troubles can be laid to inadequate maintenance of the governors. On the other hand, there is evidence that something less than perfect governor performance may still be satisfactory where the steadiness of the load does not demand exceptionally close governing. Governor tests have shown that many of the gover-

Chart II (continued)—Prime-Mover-Governor Questionnaire—Summary of Answers of 20 Companies

9	When operating isolated, how is system freq. regulated?	5 Co's report "manual"; all others report "manual", "automatic" or special instrument.
	, how is tie-line load regulated?	Of those answering directly, 4 Co's report "manual"; 1—by Dispatcher; 2—Instrument; 2—by Governor.
	, how is electric time regulated?	Similar to frequency in most cases.
	(a) When operating interconnected, how is system frequency regulated?	6 Co's report "manual"; 3 Co's report "special instrument"; 2 by large station; 5 by system dispatcher or by one system.
	Tie-line load?	4 Co's report "manual"; 2 Co's report "special instrument"; 2 by large station; 1 by Blocking; 6 by Dispatcher or one system.
	Electric time?	6 Co's report "manual"; 3 Co's report "special instrument"; 1 by large station; 5 by Dispatcher or one system. Note—One Co reports whole question under consideration, another Co gave no answers.
10	What is the basis and method of allocating load on the various operating units?	8 Co's report "station economy"; 9 Co's report some form of incremental loading; 1 base load on some, others swing; 1 water rate; 1 simply by percentage of total load.
	(a) Does the regulation of frequency or tie-line load conflict with the ideal load distribution?	4 Co's report "Yes"; 8 report "No"; 3 report—occasionally; 2 report—to minor extent; 1—no answer.
11	Does interconnection with neighboring properties affect your governor practice with respect to number of machines governing on speed?	15 Co's report "No"; 3 Co's report "Yes".
	(a) The amount of speed droop?	15 Co's report "No"; 3 Co's report "Yes".
	(b) Any other effect?	20 Co's report "No". Note—2 Co's do not operate interconnected.
12	Have you had major difficulties which might be associated with the governing function? For example, pull-out of generating units or generating stations.	17 Co's report "No"; 3 Co's report occasional trouble due to mills or interconnection.
13	Have you had disturbances, not necessarily involving interruption to service, which have caused you to impose limitations on some phase of your system operation and which you feel were associated with governor performance? For example, oscillations	17 Co's report "No"; 1 Co reports a single slow governor; 1—interconnection instability;
	hunting	1 Co sent charts showing weaving frequency.
	any others	1 Co reports one improper governor action—picked up full load.
	(a) If such disturbances are traceable to other causes, such as loss of field or low excitation, please distinguish.	
14	Do you make any routine tests to determine the speed regulation and speed of response of the governors?	9 Co's report "No" to both parts; 6 Co's report routine speed tests, generally with overhaul of unit; 5 Co's report speed tests "as required". Few companies report initiation of speed of response tests.
15	Have you made any other tests to determine individual governor performance as a result of hunting or other abnormal operation?	12 Co's report "No"; 7 others report "as required" or "occasionally"; 1 Co reports tests in connection with automatic control equipment.
16	Have you any information derived from governor tests pertaining to governor performance or the variation of the system load with speed and voltage that you can disclose at this time?	15 Co's report "No"; 2 Co's report published data; 1 reports tests in progress; 2 for particular units.
17	What type of test instruments have been used for governor tests?	7 Co's report no instruments. 6 Co's report tachometers or 1 speed indicators 4 Co's report oscillographs and/or graphic meters 1 Co reports photocell and steam indicators
	(a) How does their speed of response compare with the governor characteristics?	Part (a)—Only 4 Co's answer as follows: 1—equally fast (oscillograph & frequency meter) 1—very satisfactory (photocell & indicators) 1—faster (long film oscillograph) 1—unknown (hand tachometer)
18	Are you satisfied with your present governing system?	12 Co's report simply "Yes"; 2 Co's report "No"; 6 others qualify answer such as "fairly so", "reasonably so".
	(a) If you are not satisfied, then along what lines should improvements be made?	Frequency regulation; similar response and speed regulation over load range; load regulation; prevent swings; increased sensitivity; reduce average speed regulation.

nors which appeared to be satisfactory were not functioning properly. The main difficulties were found to be excessive friction, improper adjustment of intermediate relays, and improper setting of valve cams and compensators.

A few governors are operating today which are entirely inadequate for the duties required of them. These governors can be modified or rebuilt to meet the required standards if and when it can be proved that the expense is justified. In some cases, as long as the governor protects the turbogenerator, it has not been deemed advisable to expend the funds required to accomplish the niceties of speed regulation.

There has been very little said in the above discussions of how well the present governors are doing their job. Many of the faults of governors have been mentioned for two reasons: to summarize the various types of troubles that have occurred in order to provide a basis of attack for analysis of trouble or conduct of test, and in order to form the basis of a discussion. It is not believed at this time that there are any major deficiencies in the governing system. A few systems have recently experienced some large industrial loads, which have made the existing governing system seem inadequate. Other systems have experienced difficulties in tie-line interchanges and the correlation of steam and hydro governors. These problems require much thought and effort for their solution, but this is gradually being accomplished. A few years ago, much reliance was placed upon the use of automatic frequency control, and no doubt the interest in this type of equipment has caused some relaxation of attention to the speed governors. Lately, many operators are returning to the thought that the speed governors are the fundamental speed regulating equipment, and the automatic frequency control is only a vernier control. It is obvious that very few of the automatic frequency control installations are able to cope with any sudden or large load changes without the assistance both of the governors and the operators.

A discussion of load swings has been included in the program. This particular load characteristic has become of more importance in the last few years due to the increasing use of utility service for the supply of the high-speed continuous strip mills, heavy railway electrification, and large electric furnaces. The utilization of power for many of these loads might be impractical without the interconnected capacity made available by the utilities.

The continuous strip mill has a periodic cycle, more or less regular in form, which has the characteristic of increasing from a minimum to a maximum in a relatively few seconds, then holding the load for 10

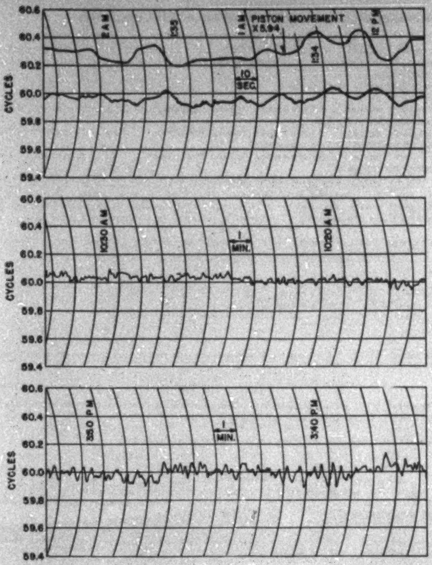

Chart III

or 20 seconds, then rapidly dropping the load. This cyclical type of load is particularly hard to handle because the rates of change are very fast, and the load pickup does not exist for any length of time. The railroad loads, are, at times, as severe as the strip mill loads in so far as rate of pickup is concerned, but do not occur at periodic intervals; therefore, in general, they are somewhat easier to control. The electric furnace load, while not reaching the magnitude of either the railroad or strip mill load, is particularly vicious due to its instantaneous rate of change.

The effect of these loads on system performance varies according to the system setup, and the location of the swinging load relative to interconnections and power stations. An isolated system will experience speed variations, which, at times, are too fast for the governors. Voltage variations often accompany the speed variations and high-speed voltage regulators cannot completely eliminate the voltage variations. These voltage variations result in great activity of the induction voltage regulators. When the system operates interconnected, the speed variation is not so pronounced, and the voltage variations caused by the change in kilowatt load are reduced. However, tie-line control is then a problem. The load swings on the tie line may considerably restrict the safe operating capacity of such an interconnection due to the reduction in average load to hold the maximum load swings within the safe operating limit. The transfer of these load swings through adjacent systems may also introduce operating difficulties. These load swings, as they pass from one interconnection to another, sometimes tend to amplify as the distance away from the origin increases. The presence of these swings in the interconnection is due to the inability to rapidly readjust the generator outputs to compensate for the load swings, and their relative magnitudes are due to the electromechanical characteristic of the power systems. For instance, if a sudden increase of load occurs on system *A*, the initial load swing is partly supplied from the stored energy in system *A*, while its phase displacement is retarded in respect to system *B*, the rest of the load swing is supplied through the interconnection from system *B*. The amount of stored energy released is a function of the load-angle characteristic of the transmission system, including the tie line.

It has been observed that if the governors are immediately readjusted to pick up exactly the same amount of power as is flowing through the tie line, that an appreciable time lag will occur before the tie-line load has returned to normal. This lag may be as great as 15 seconds. A similar phenomenon has been observed when actuating the load shift by means of a phase-angle regulator. This characteristic ordinarily prevents the use of fast response load regulators due to the tendency to hunt. This energy must be returned to the system to advance its phase displacement before the tie-line load can be returned to normal.

Fast tie-line load regulators of the impulse type may be more harmful than advantageous due to the time lag required to readjust the governors. Often the load swings are amplified to twice the original amplitude by action of the load control.

One solution that is being tried is based on utilizing the diversity in load swings between two adjacent systems. The fringe of the adjacent system load is transmitted by telemetering and superposed on the regulated tie line. This procedure permits load variations to exist on the tie line between the two systems which tend to counteract load swings on the adjacent system, and also tends to accelerate the action of the load control when the load swing appears on both interconnections. It has been found desirable to utilize system inertia as much as possible to provide the regulation for the cyclical and other rapid load variations, using automatic control to center or average the speed or tie-line interchange.

Appendix

Excerpts of Complete Answers*

Company 1

Answer 3. Figures of speed droop have a varied and uncertain basis. It is, therefore necessary to be very specific when dealing with this subject. Our steam units have a special device whereby the amount of speed droop can be varied by remote control while the machine is on the line. According to the manufacturer's data, this speed droop at full load can be varied from 1.5 per cent to 7.5 per cent. The present setting is such that the speed droop at full load based on the manufacturer's data showed about 4.5 per cent. These foregoing figures we would, for want of a better term, call the "steady-state speed droop" to distinguish them from the following: by actual test on the machines while running on the system with the above speed droop setting, we find that we have an actual speed droop of about 13.5 per cent which we might call the "dynamic" speed droop.

Answer 6. The system loads are extremely rough. The normal load variation during a 15-minute period is no worse than it might be in a 2-minute period and is approximately plus or minus 15 per cent of the average load. There are large steel mills and heavy railroad electrification on the system; the steel mill cycle is such that we get a peak of from 20,000–25,000 kw once a minute, and the railroad variations are extremely indeterminate both as to time and amount, i.e., they may occur almost any time and the variation may be 50,000 kw in two minutes. The combined *variation* may be occasionally 75,000 kw.

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*Note: These are excerpts from original answers as abstracted for tabulation.

Answer 8. (*a*) and (*b*)—Our system always operates in parallel with two adjacent systems. These three systems are operated as one group as far as economic generation is concerned; this group in turn has interconnections with systems to the north and east.

(*c*)—The total capacity in normal operation of all the groups mentioned is about 2,500,000 kw which includes about 250,000 kw in hydro capacity, which is in addition to our hydro previously mentioned.

Company 2

Answer 6. The system loads are rough. The normal load variation during a 15-minute period is approximately 6 per cent of the average load. However, during load change periods, the 15-minute variation may be as high as 11 per cent of the average peak load. This system supplies many electric furnaces and a large traction load. There is no evidence that these particular loads have introduced any governing problems. There is also a steel mill supply for an 80-inch continuous strip mill which has noticeably affected the governing requirements of the system. A typical load cycle on this mill will vary between the following limits:

(a)		(b)	
Load—Megawatts	Time—Seconds	Load—Megawatts	Time—Seconds
0	0	0	0
20	20	20	10
20	60	20	20
0	80	0	30

These cycles are repeated at periodic intervals averaging between 8 to 12 cycles each 15 minutes.

Answer 17. What type of test instruments have been used for governor tests?

Various types of instruments have been used for governor tests from straight mechanical recorders to magnetic oscillographs.

At present, governor investigations are being conducted with a standard strip chart wattmeter with minute chart speeds and a photoelectric frequency recorder with a similar chart drive. Each of these instruments is equipped with two auxiliary motion recorder pens. The damping of the wattmeter is adjusted to cause the meter to settle at the correct load without overshooting or hunting within one second following a sudden load change. Although this meter is not particularly fast, the damping effect of the inertia and phase-angle characteristics of the turbogenerator make this response adequate. The frequency recorder has a time lag not in excess of 0.2 second for a full-scale change in deflection.

Great care must be taken when using oil pressure recorders to insure that the natural period and damping of these devices is properly chosen to suit the pressure fluctuations to be recorded.

Answer 18. Are you satisfied with your present governing system?

The present governing systems fundamentally meet the requirements of a speed controlling device. Many of the older governors, however, are not capable of meeting the changing requirements as the systems have grown. Certain types of governors are inherently sluggish and do not assist in system regulation. On the other hand, those governors which have been active and sensitive have often been deliberately adjusted to destroy their sensitivity in order to reduce the burden on the boiler rooms.

Major improvements which appear to be desirable in the governing system are means to provide the same speed of response on the various units to give accurate load distribution during transient speed variations, and improvements in the control mechanisms and inlet valves which will result in closer adherence to the over-all speed regulation along the entire load range of the units.

Company 5

Answer 13. Have you had disturbances, not necessarily involving interruptions to service, which have caused you to impose limitations on some phase of your system operation and which you feel were associated with governor performance? Yes. Cannot operate number 5 unit as a single unit on the system.

Answer 16. Have you any information derived from governor tests pertaining to governor performance or the variation of the system load with speed and voltage that you can disclose at this time? See description of trouble.

Answer 18. Are you satisfied with your present governing system? If not, along what lines should improvements be made? All except number 5. This governor is too slow for light swinging load when operating with one condensing unit and steam heat load. Governor permits wide speed swings.

13, 16, and 18—Description of Trouble. Unit 5, a 10,000-kw, 25-cycle, 600-pound turbine, is noncondensing, exhausting into a 185-pound steam header which supplies steam heat systems; 2–10,000-kw, 25-cycle units; or 1–26,000-kw, 60-cycle unit. The 2–10,000-kw, 25-cycle units, also the 26,000-kw, 60-cycle unit can be operated from the 600-pound header through a reducing valve. The 25-cycle system load varies over the 24-hour period from 2,000-kw night load to a maximum of 28,000 kw. The difficulties arise on the light load periods. The governor of unit 5 is too slow for a light swinging load when operating with one condensing unit and steam heat load. Ordinarily the 25-cycle system is connected to the 60-cycle system through 2–5,000-kw frequency changers. At all times when the changers are in operation, no difficulty is encountered due to load swings or hunting between units. On the 25-cycle system at light load periods, the normal swing in load is from 1,000 to 1,500 kw.

With unit 5 operating in parallel with one of the 10,000-kw, 25-cycle units and without the frequency changers the load swings start a frequency variation ranging from 24 to 26 cycles. It is accumulative and has tripped unit 5 on overspeed. The governor regulation of unit 5 has been changed to 8 per cent no load and $4^{1}/_{2}$ per cent full load without satisfactory results. The governor oil pressure has been increased and decreased above and below normal without satisfactory results. To date we have found no correction which is satisfactory.

Company 10

Answer 9. The system frequency and electric time regulation is combined as one operation and is usually regulated by one of the larger generating stations of the interconnected system, with the understanding that on a predetermined low- or high-frequency value all generating plants that are operating interconnected will make adjustments to the frequency to bring the frequency to a normal value as quickly as possible. Tie-line regulation is accomplished by the load schedule set up for the interchange of power in the desired direction in accordance with the economical costs.

Company 12

Answer 12. Unsatisfactory governors cause machine loads to fluctuate unnecessarily resulting in loss of economy on steam equipment, excessive manipulation by the operators, and giving a speed curve which swings as much as 0.15 cycle with a period of 15 to 20 seconds. Chart 3 shows a period of typical operation and a much better period when one 50,000-kw and 2–75,000-kw steam units were operating against stops, and one of the two hydro plants was regulating frequency. The third graph shows the operation of the governor on one of the 75,000-kw units that was causing swings of system speed. This governor had a time lag on the reversals of about four seconds.

Company 14

Answer 9. Either system *A* or system *B* regulates frequency depending on conditions:

1. When system *B* is regulating, the governor of one of the turbines so equipped is controlled by a Leeds and Northrup instrument operating on flat frequency. System *A* maintains a predetermined load transfer on the ties by load regulation. Time is maintained by a device used in conjunction with the Leeds and Northrup controller which changed the neutral point of the controller in proportion to the cycles deviation from perfect time obtained from a crystal oscillator.

2. When system *A* is maintaining frequency they operate on flat frequency and system *B* maintains tie-line load transfer by operating on selective frequency control.

Answer 17. A long film oscillograph is used for measurements, about seven inches of film covering one second. The instrument response is evidently quicker than the response of the things we are measuring since, when changes are made, a gradual change is shown by the photographic record. As an example: the film of the test mentioned in question 15 showed the valve travel on the generator gradually decreasing to a point shortly below its steady state and then increasing a small amount to its final steady value. The total time from the instant of fusing the link to a steady state of valve travel was $^{1}/_{2}$ second and generator load was steady in $2^{1}/_{4}$ seconds.

Company 15

Answer 1. This group of systems is served by a combination of hydro and fuel plants and capacity received from foreign systems on either purchase or exchange bases. The amount of capacity normally in service, therefore, varies within wide limits with seasonal stream flow conditions. Energy requirements, and not short-time peaks, set the goal for capacity. In other words, if the required amount of energy over periods of 16 hours a day for 5 days a week can be met there is little likelihood of any shortage of capacity to carry hourly peaks. When there is plenty of water hydro plants are loaded to capacity with the exception of those assigned to frequency or tie-line control and steam plants and capacity received from other systems, is at a minimum. During dry seasons greater capacities in steam plants are operated to conserve water, and since practically all the hydro capacity is spinning, though not loaded, the total capacity available to carry peaks is much greater than during wet seasons. In transition periods from seasons of plentiful water supply to those when water must be conserved steam and hydro capacity operated varies greatly and the amounts are difficult to express specifically.

The following statement is an attempt to express the conditions prevailing during 1939.

January was unusually dry, and considerable steam was operated, but from February through June stream flows varied from medium to high, which is representative for this season. Monthly peak loads were of the order of 725,000 kw and the capacity in service, some 800,000 kw, of which 725,000 kw was in hydro, 50,000 kw in steam, and 25,000 kw in receipts from other systems. The amounts, expressed in per cent of total, were hydro 91 per cent, steam 6 per cent, and receipts 3 per cent.

During the transition period from July to September loads were some 800,000 kw and the capacity in service, 900,000 kw, the relative amounts being 640,000 or 71 per cent hydro, 200,000 or 22 per cent steam, and 60,000 or 7 per cent receipts from other systems. The dry season lasted from September through December and, in fact, extended into 1940. It was remarkable for its duration, rather than its severity, two other years (1925 and 1931) having had much lower flows. Loads were of the order of 850,000 kw, the peak for the year being 880,000 kw in December. Capacity in service totaled approximately 1,000,000 kw, the relative amounts being 600,000 or 60 per cent hydro, 300,000 or 30 per cent steam, and 100,000 or 10 per cent receipts from other systems.

Answer 4. At full load periods 100 per cent of the steam capacity is operated with blocked governors. At light loads 100 per cent is operated with blocked governors. All of the hydro plants, except the one designated to control frequency, either have their governors blocked so as to prevent the units from increasing their load, or have the port in the governor dash pot reduced to such a small opening that the response is very sluggish to a change in frequency.

Answer 10. Load is allocated to hydro or steam plants in accordance with stream flow conditions primarily. Steam plants are put into operation in ascending order of cost, and those operated are usually fully loaded throughout the peak period of 80 hours per week. Load is assigned to hydro plants on the basis of availability of water, stream flow plants being based during wet seasons and peaked in dry seasons, the operation of storage fed plants being the opposite.

Company 20

Answer 12. System instability has been experienced a number of times in recent years. Every case has been attributed to sustained low voltage, either produced by a sustained system fault or inability of the system to pick up the required quantity of inductive reactive load following the loss of generation. Such periods of instability are characterized by very violent hunting and violent pumping of the governors. In some cases the oscillograph records show traveling phantom faults, indicating that generators or generating stations are out of step with the rest of the system. In every case the system has pulled out of its troubles without losing load other than that lost with the fault by the process of applying maximum field strength to the generators and simultaneously running the governor motors to reduce load on all machines on which all instruments were vibrating over the full scale. Stability has generally been recovered without dropping additional load in some two to five minutes. It is important to note that the turbine operator, during periods of instability and violently pumping governors, seizes the governor rods by hand so as to prevent, or at least greatly reduce, the violent pumping.

Answer 13. On several machines which were found to be unstable, governor nuts were turned down to prevent the machines from taking maximum load. Also, orders were given to the operating department that these unstable machines were to be operated always at full field. The other machines on the system are operated with field currents never less than sufficient to give them a pull-out torque of 150 per cent of maximum load, that is, the pull-out torque against an infinite bus never comes to less than 150 per cent of the load the machine can carry with governor wide open and with normal steam conditions.

Field adjusters are being installed on all generators following a successful trial installation and it is expected that these field adjusters will automatically apply and maintain the maximum possible generator field strength during all periods of instability.

Where it is desired to carry high power factor on a machine having low "steady-state" stability, it is proposed to supply an autotransformer to raise the maximum voltage so that it may continue to operate at a strong field.

Answer 18. Three new 50,000-kw generators are being equipped with resynchronizing governors; two of these machines are being installed in a station some 80 miles from the main group of our generating stations and while these two machines have very good "steady-state" stability, it is believed that they will be liable to fall out of step with the distant system since the local load they carry is relatively small with respect to the load transmitted to the system. It is hoped that the resynchronizing governors will bring these machines into synchronism quickly following an out-of-step condition and that it will never be required that they be disconnected and resynchronized. This same out-of-step governor is being installed in one 50,000-kw topping unit now being installed in one of our generating stations, that is, this machine is being installed on a bus of such low reactance that it may be considered for all practical purposes as connected to an infinite bus.

Legend of Companies Reporting

Boston Edison Company
Buffalo Niagara Electric Corp.
Commonwealth & Southern Corp.
Commonwealth Edison Company
Consolidated Edison Company of New York, Inc.
Detroit Edison Company
Duquesne Light Company
Kansas City Power & Light Company
Louisville Gas & Electric Company
New England Power Company
New York, New Haven & Hartford Railroad Company
Northern States Power Company
Northwestern Electric Company
Oklahoma Gas & Electric Company
Pennsylvania Water & Power Company
Philadelphia Electric Company
Public Service Electric & Gas Company
Scranton Electric Company
Shawinigan Water & Power Company
Union Electric Company of Missouri

Note: Experience indicates that, following a sudden drop in frequency, the governor of a machine having low "steady-state" stability will open up faster than a field adjuster will strengthen the field so that the pull-out torque of the machine may be exceeded.

Effect of Prime-Mover Speed Governor Characteristics on Power-System Frequency Variations and Tie-Line Power Swings

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Introduction

IN the normal steady-state operation of a power system, proper load division, frequency, and time are maintained by manual or automatic system control, while the transient swings occasioned by load changes are absorbed by the inertias and speed governors of the individual generating units. With improvements in reliability and performance of systems and their interconnections, it is a natural development that further careful attention and study be given to the basic speed governors and their effects on system operation.[1,3,4] It is desirable to determine the requirements which a good governing mechanism should fulfill, and the effects of specific governor and system characteristics on over-all system performance. Moreover, under certain conditions some governors may accentuate rather than reduce the magnitudes of the frequency swings, and it is desirable to know the conditions for which this may occur.

The purpose of the present paper is to present the results of an analytical investigation of some of the effects of governing system characteristics on power system performance, and to relate the conclusions drawn from this work to those presented in the companion paper.[4]

Conclusions

The following general conclusions have been drawn.

1. Reduction of dead band, avoidance of too small a regulation, either transient or steady-state, and proper proportioning of the response of the various elements of the governing mechanism tend to improve stability, so that a system will have a maximum tendency for a rapid settling down of frequency and load after a load change and a minimum tendency for the support of a sustained frequency variation. The over-all dead band of the governing mechanism of a prime mover including governor head, hydraulic relays, and valve gear, rather than that of any one particular element of the governing mechanism, is of importance in determining the governor stability characteristics.

2. System characteristics have an important influence on the frequency stability. Load damping (net change of system electrical load and input torque with frequency) and the electrical damping between generators and parts of the system contribute very greatly to stable operation. (The amount of load damping required to assure stable operation regardless of the rate of governor response is inversely proportional to the speed regulation.)

3. The frequency stability of either of two systems may be either improved or lessened by an electrical interconnection, depending upon the magnitude of the tie-line electrical damping.

4. Interconnection of two power systems with good frequency stability may introduce some instability if there is little damping in the tie line. For small tie-line and small load damping, sustained self-excited oscillations can be set up in a system by dead band in the governing system for a considerable range of system parameters. Frequency oscillations at the tie-line natural frequency (producing a tie-line power swing) and at a lower frequency determined by the governor characteristics may be simultaneously excited by dead band in the governing system. On the other hand, considerable damping of larger tie-line power swings may be produced by governor action when these swings cause frequency variations outside the governor dead bands.

5. For machines in service unnecessary system frequency movement or oscillation caused by governing can be most easily and effectively reduced by reduction of dead band and broadening of the regulation of the governor mechanisms on the units tending to cause the unnecessary movement.

6. Steam turbine generators, because of the smaller time lags associated with governing steam flow as compared with water, are inherently capable of producing a greater stabilizing influence on system frequency than are hydro generators.

Discussion of Results

STABILITY

One of the desirable characteristics of a governing system is that it should respond to load changes and settle down to its new operating position with the least possible disturbance. This means that it should be as stable as possible. Therefore the first aspect of governor performance studied was the effect of the various parameters on system frequency stability. The problem of the stability of governing

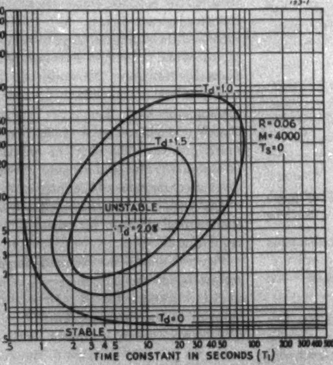

Figure 1. System or single machine

Effect of governor time lags and load damping on stability

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[1] For all numbered references, see list at end of paper.

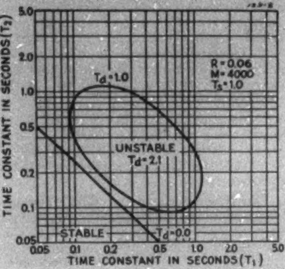


Figure 2. Machine tied to large system

Effect of governor time lags and load or tie-line damping on stability

systems is about as old as governing itself, and, at least in the case of prime-mover governors, has been studied for many years, the first analytical investigation having been reported in 1868 by J. C. Maxwell.[3] The particular contribution of the present work is in showing the effects of the power-system characteristics and of governor dead band.

SINGLE MACHINE

As the simplest approximation to a speed-governed power system the system generating capacity was lumped as a single equivalent machine, having an inertia and a load damping characteristic, and governed by the idealized type of speed governor described in the appendix. Such a mechanism represents the behavior of a steam governor somewhat more exactly than that of a hydro governor having a droop compensation mechanism. Two first-order time lags in the governing mechanism were used because two is the smallest number which will result in the proper qualitative behavior. That is, a governing system whose lags are all lumped into one time constant does not have the possibility of instability exhibited by an actual governing system, while one with two time lags does. For example, a governing system with no lag between speed change and inlet valve position would operate stably even with a large lag due to stored steam or water. As far as stability is concerned, it is because of the inherent physical limitations in the possible speed of response of mechanisms, that a slow rather than a fast response governing system must be used in some cases. Moreover, in the case of water turbines the valve response must be slow because of the water conditions which might otherwise arise during large and rapid gate movements.

Figure 1 shows the results of stability calculations made for this simple system. It is seen that if one of the governor time constants T_1 is made sufficiently small, the system is stable regardless of the value of the other time constant T_2. For a given degree of stability, the allowable total time constant T_1+T_2 is smallest when the two time constants are equal. This result is consistent with the statement that there could be no instability with only one time constant. In general, the fewer the number of stages (and consequently the simpler the mechanism) for a given total time lag, the more stable is the system.

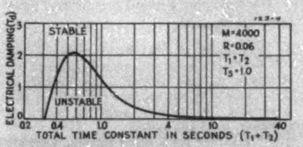


Figure 4. Machine tied to large system

Effect of governor time lag on load or tie-line damping required for stable operation

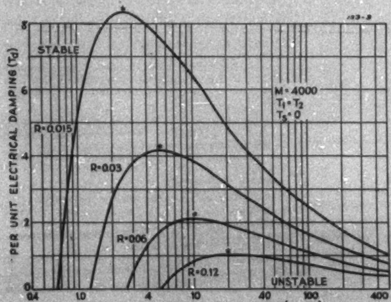


Figure 3. System or single machine

Effect of governor time lag and regulation on load damping required for stable operation

NOTE: Maximum points occur at time (in radians) $T_1=T_2=8RM$, and at damping $T_d=1/(8R)$

Figure 1 further shows that when there is load damping, the system may be made stable by either increasing or decreasing one or both time lags, and that if the load damping is sufficiently great $(T_d \geqq 1 \div (8R))$ stability is obtained regardless of the governor lag. The condition of synchronizing a machine corresponds closely to that of zero damping and no electrical tie, $T_d=0$ and $T_s=0$, and is the least stable condition shown in figure 1.

Figure 2 shows the effect of a synchronous tie between the machine of figure 1 and an infinitely large system, when the tie-line damping is small or zero. Much faster response is required for stability, but the amount of damping required to prevent any instability is not affected appreciably.

It is interesting to note that if there exists small damping either electrical or mechanical, as for example in a salient-pole generator with no amortisseur winding connected to a system at light load, the unit may be more unstable than under the condition of synchronizing (figure 1, $T_s=0$, $T_d=0$). This has been observed in the field and indicates the desirability of amortisseur windings for salient-pole machines. Usually the electrical rotor damping is sufficiently large so that the operation of a single machine connected to a large system is more stable than that of the same machine isolated, or of the entire system operating against the load damping.

The results obtained with two time lags tend to be somewhat more stable than those which would be found for an actual governing system having more than two lags. Therefore, for further study the least stable case, equal time lags, was selected. Figure 3 shows for this case and for the system of figure 1 the effect of regulation on the load damping required to produce stable operation as the time lags are varied. As regulation is increased the allowable range of time lags increases and the required amount of load damping decreases. Moreover, as regulation increases the effect of moderate amounts of load damping $(T_d \leqq 2)$ in allowing longer response times becomes much greater. The effect of regulation on stability is also seen in figure 14, which gives a comparison of the rates of decay of the frequency oscillations caused by a sudden load change, for three different regulations.

Figure 4 shows the required damping torque T_d for the case of $T_1=T_2$ when a machine is connected to a large system through a synchronous tie $(T_s=1.0)$. This curve, when compared with the curve for $R=0.06$ of figure 3, shows the

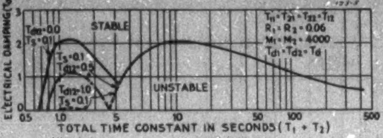


Figure 5. Two interconnected systems or machines

Effect of governor time lag and tie-line damping on load damping required for stable operation

effect of the synchronous tie in changing the range of required time lags for stability. These results show the influence of governor regulation, electrical damping, and synchronizing torques on the stability of the system. Since the electrical damping available is usually limited and determined by system conditions, the results show the desirability of reasonably broad transient regulation.

Just as figures 1–4 show that system electrical damping is required to compensate for the tendency toward instability caused by large time lags in the governing system, calculations have similarly shown that with small time lags the governing system may contribute considerable damping. It may be shown that the possible contribution of the governing system becomes greater as the time lags are reduced, and that for any given speed of response there is an optimum range of governor regulation (which is not however very critical), for maximum damping effort. Because optimum regulation is more or less proportional to the time lag, steam units are inherently capable of a greater system frequency stabilizing influence than hydro units. Field experience seems to bear out this conclusion, and to indicate that for reasonably fast response governors the optimum regulation may be of the order of six per cent.

Interconnected Systems

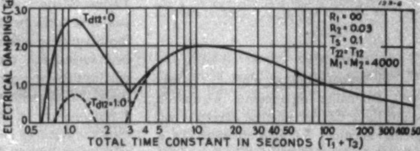
If two similar systems are connected together through a tie line it is evident that they can vary in speed together as one machine or that the two systems can oscillate against each other about a point in the middle of the tie or that both modes of oscillation can occur simultaneously. Thus we may expect to find an unstable region corresponding to that of figure 3 and also one corresponding to that of figure 4 with half the tie-line reactance (twice the synchronizing torque coefficient). Figure 5 bears out this conclusion and shows how a tie between two previously separated systems may introduce a new region of instability unless the tie line has considerable damping.

Figure 5 is for two identical systems with identical governing mechanisms. Figure 6 is included to show the effect of having all the governing effort on one of the systems, with, however, the same effective speed regulation on the combined systems. It is seen that the system is slightly less stable when the governing is not uniformly distributed.

When the connected systems are not identical, physical considerations, as well as the equations given in the appendix, show that the normal modes of oscillation involve both the oscillation of one system against the other and also the oscillation of the combined system at frequencies determined by the system, tie-line, and governor characteristics. Thus when the system becomes unstable, both a tie-line power swing and also a swing of the average system frequency will in general occur simultaneously. The principal component of the tie-line swings usually has a period of from 0.5 to 5 seconds while the swing of average frequency may have a period of from 5 to 60 seconds.

Figure 6. Two interconnected systems

Effect of distribution of governing effort on stability. (Compare with figure 5)



Dead Band

In addition to the instability or hunting which may arise because of the characteristics of the governing mechanism as discussed in the previous sections, another source of hunting or undue frequency variations lies in mechanical imperfections and friction in the mechanism. These effects manifest themselves as a "dead band," or a frequency band within which the governing system will not respond.* There are at least three aspects of system performance which the dead band may affect:

(*a*). Accuracy of load division among paralleled machines,

(*b*). Response to the small frequency variations encountered on large systems,

(*c*). Possible excitation of sustained frequency or tie-line power oscillations.

(*a*). It is shown in the appendix that the possible inaccuracy of load division between two machines for a given total load is approximately equal to the ratio of the sum of the dead bands of the governing mechanisms of the two machines to the sum of their regulations. ($\Delta T = (e_1+e_2)/(R_1+R_2)$.) Since the kilowatt load error is equal and opposite on the two machines, the maximum percentage error evidently occurs on the smaller machine and may in the limit equal the ratio of the sum of the dead bands to the per cent regulation (on its own base) of the small machine. On the other hand, if frequency, rather than total load is maintained constant, the maximum load variation of any machine is equal to the ratio of its dead band to its regulation.

(*b*). Maximum frequency variations on large systems may be of the order of only 0.1 cycle per second, or about 0.2 per cent. It is thus evident that a governing mechanism having a dead band of 0.1 per cent might give a much smaller response to a frequency change than would be computed on the basis of its incremental regulation neglecting dead band. It is further evident that a reduction in dead band may well lead to a greater gain in the magnitude of the response than a reduc-

Table I. Effect of Location and Distribution of Dead Band on Sustained Speed Oscillation

Run Number	e_1 (Per Cent)	e_2 (Per Cent)	e_3 (Per Cent)	Sustained Oscillation (Per Cent)
$T_1=T_2=300$				
69	0.1	0	0	0.28
70	0	0.1	0	0.30
84	0.05	0.05	0	0.41
71	0.033	0.033	0.033	0.51
72	0	0.05	0.05	0.54
$T_1=100$, $T_2=800$				
80	0.1	0	0	0.23
81	0	0.1	0	0.24
82	0.05	0.05	0	0.24
$T_1=800$, $T_2=100$				
86	0.1	0	0	0.26
85	0	0.1	0	0.4

Notes: $R=0.06$, $M=4{,}000$.

e_1 = dead band between speed $p\delta$ and speed indicator $p\delta'$.

e_2 = dead band between intermediate displacement Q and Q'.

e_3 = dead band between control output T_g and turbine input T_g'.

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*See appendix for description of dead band.

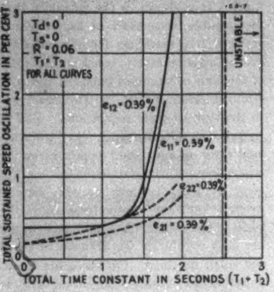

Figure 7. Self-excited speed oscillations

Comparison of various kinds of governing-system dead band

NOTE: The first subscript on e indicates the kind, and the second subscript indicates the location of the dead band (subscript 1 indicates a dead band at the speed point, subscript 2 indicates a dead band at an intermediate point in the mechanism O. (See appendix)

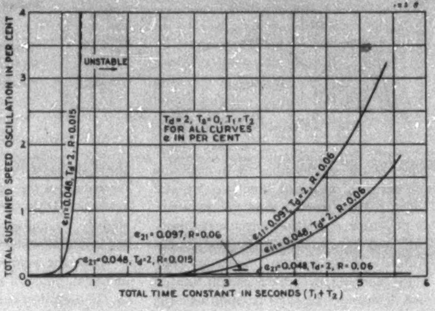

Figure 8. Self-excited speed oscillations produced by dead band

Effect of regulation and time constants of governing system—with system damping. (See figure 7 for definitions of e)

tion in incremental regulation. Moreover, a smaller incremental regulation would have the detrimental effects of reducing accuracy of load division for a given dead band and possibly increasing the frequency variations excited by dead band, while a smaller dead band would have just the opposite effects.

(*c*). The excitation of frequency variations by governor dead band has been studied by setting up the relations derived in the appendix on the differential analyzer of the Moore School of Electrical Engineering, University of Pennsylvania. The results were obtained as curves of frequency, tie-line power, etc., as functions of time, and of frequency versus turbine power input, following a sudden load torque change. The curves were continued long enough to observe the rate and manner of decay of the consequent oscillations, and to determine the magnitude of any sustained oscillation. Figure 13 shows a typical frequency versus time and frequency versus turbine input curve. A few of these curves have been included in the paper, and the principal results are summarized in figures 7–9 and 11–17 inclusive and in the tables.

SINGLE MACHINE

Figures 7–9 show the magnitudes of sustained frequency oscillations excited by governor dead band for the simple system considered in the first section of this discussion. Figure 7 includes the effects of dead bands of the first and second kinds described in the appendix, and shows that (*a*) for a governor with fast response the magnitude of the self-excited frequency oscillation is practically equal to the frequency change within which the governor may be insensitive, (*b*) the magnitude increases sharply as the border line of instability (see figure 3) is approached, and (*c*) the location of the dead band within the governing mechanism does not make very much difference.

Table I also gives information on the effect of location of dead band (first kind), and indicates some tendency for the effects to become greater as the location moves away from the governor head end of the mechanism and as the dead band is more evenly distributed. Of course, dead band cannot actually appear between the control output T_g and the turbine input T_g', since this is beyond the water or steam storage lag.

Figure 8, as compared with figure 7, shows the effect of load damping in suppressing the oscillations due to dead band of the first kind for sufficiently fast response. Figure 8 also shows the effect of a smaller regulation in requiring a much faster response for the same oscillation magnitude. For small time lag, dead band of the second kind is more severe, while for larger time lags dead band of the first kind is more severe.

Figure 9 shows the effect of an un-

Table II. Effect of Location, Response, and Regulation of Governor on Speed and Tie-Line Power Swings

						Tie-Line Power					Per Cent Speed						
								At Five Cycles			First Swing		Maximum		At Five Cycles		
Run Number		R_1	R_2	T_1	T_2	First Swing	Max. Swing	Oscillation	Average	At $t=\infty$ Average	$p\delta_1$	$p\delta_2$	$p\delta_1$	$p\delta_2$	Oscillation	Average	At $t=\infty$ Average
13	$\Delta T_1=0.078$	0.06	0.06	50	50	0.056		0.026	0.039	0.039	0.295	0.310			0.093	0.214	0.209
30		0.06	0.06	800	50	0.077		0.011	0.038	0.039	0.385*	0.439			0.038	0.200	0.209
38		0.06	0.06	50	800	0.055*	(0.056)	0.024	0.041	0.039	0.290*	0.437	0.461		0.087	0.200	0.209
33		∞	0.03		50	0.088		0.003	0.074	0.074	0.390*	0.317			0.012	0.244	0.209
36		0.03	∞	50		0.044		0.026	0.004	0.0042	0.249	0.315	0.329		0.090	0.214	0.209
21		∞	0.06		100	0.075	(0.095)	0.037	0.071	0.071	0.393	0.501	0.501		0.139	0.381	0.378
41		∞	∞			0.735		?	?		0.391	0.707	?	?	?	?	1.95
15	$\Delta T_1=0.0195$	0.06	0.06	50	50	0.016	(0.017)	0.0095	0.0112	0.0098	0.088*	0.110	0.103		0.034	0.032	0.052
40		0.06	0.06	800	50	0.019	(0.021)	0.0093	0.0090	0.0098	0.100*	0.135	0.134		0.029	0.022	0.052
37		0.06	0.06	50	800	0.016*		0.0095	0.0105	0.0098	0.088*	0.139	0.151		0.029	0.027	0.052
23		∞	0.06		100	0.018	(0.025)	0.0126	0.0175	0.0176	0.098	0.142	0.159	0.147	0.044	0.088	0.094
42		∞	∞			0.018		0.0107	0.0098	0.0195	0.100	0.177	?	?	0.037	0.433	0.488

*Indicates presence of appreciable low-frequency oscillation.

$T_{d1}=T_{d2}=2$, $T_s=0.1$, $M_1=M_2=4{,}000$, $e_1=e_2=0.1$ per cent, $T_{d12}=0$. (See figure 11 for definitions of *e*.)

Table III. Effect of Location, Response, and Regulation of Governor on Decay of Tie-Line Load Swings

Run Number	R_1*	T_1	T_{d1}*	T_{d2}	T_{d12}	e_1, e_2 (Per Cent)	R_2	Time Constant (Cycles) $T(\sim)$	(Equiv. T_{d12})		Changes in T_{d12} From Run Indicated at Right	
									**	†	†	
58	0.06	.25	2	2				1.15	3.1	12.4		
63	0.06	.25	2	2		0.1		1.67‡	2.1	8.4	−4.0	(58)
65	0.06	.25	2	2	0.5	0.1		1.35‡	2.6	10.4	+2.0	(63)
61	0.06	.25	2	2			0.24	1.10	3.2	12.8	+0.4	(58)
56	0.06	.50	2	2				2.52	1.4	5.6	−6.8	(58)
57	0.06	.50	6	2				1.50	2.4	9.6	+4.0	(56)
60	∞		2	2				10.6	0.34	1.36	0	
66	∞		2	2	0.5			4.37	0.81	3.24	+1.88	(60)
62	∞		10	10				1.57	2.3	9.2	+7.8	(60)

*On smaller machine base.

**On larger machine base.

†On "effective" machine base—equiv. $T_{d12}=\dfrac{\sqrt{M'T_s}}{\pi T(\sim)}$, where $M'=\dfrac{M_1M_2}{M_1+M_2}$.

‡Until about two cycles, after which rate of decay was much slower, as oscillations were within dead band.

$M_1=4{,}000$, $M_2=1{,}333$, $T_s=0.125$, $R_2=0.12$, $T_2=100$, $T_{d2}=2$, $T_{d12}=0$ except as marked. (See figure 11 for definitions of e.)

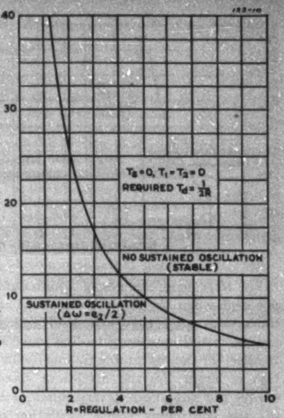

Figure 10. Required value of machine damping T_d to prevent self-excited oscillations

No governor time lags—dead band of second kind, e_2

damped synchronous tie between the machine of figure 7 and an infinitely large system.

Since hunting caused by dead band of the second kind was not suppressed by the load damping considered in figure 8, the amount of damping required to suppress such hunting was computed. Figure 10 shows the results of a calculation made for the most stable case of no time lag in the governing system. The required load damping coefficients are seen to be very large, so that one may expect that dead band of the second kind will always excite frequency oscillations.

Tests have indicated that dead band of the first kind is much more usual than dead band of the second kind, although perhaps neither kind may occur in a pure form. Therefore only the first kind will be considered in the discussion of interconnected systems.

INTERCONNECTED SYSTEMS

Figures 11 and 12 and tables II and III show some of the results obtained in the study of the performance of interconnected systems. The figures show the magnitudes of the sustained self-excited speed oscillations with different governor distributions. Note that a speed oscillation of 0.1 per cent with a period of about 2.1 seconds corresponds to a tie-line power swing of about 2.5 per cent or 25 per cent of the tie-line synchronizing power coefficient. The long period speed swings, however, have no corresponding power exchange, as the systems are then swinging more or less together. The principal self-excited speed oscillation occurred at the tie-line natural frequency in most of the cases studied, although in some cases the principal oscillation had a long period. Both the high- (tie-line) and low-frequency self-excited frequency oscillations may exist simultaneously, although as noted above there is a tendency for the high-frequency oscillations to predominate.

Figure 11 shows that for the case under consideration, if the governors on both systems have dead bands of 0.1 per cent, frequency and tie-line oscillations are excited over a definite range of time lags, but if the governor on one system has no dead band, and the governor on the other system has a dead band of 0.2 per cent, sustained oscillations are not excited.

Comparison of figures 11 and 12 shows that, as noted in figures 5 and 6, with all the governing on one of the systems, the combined system is slightly less stable than with a uniform distribution of governing.

In addition to stability, another aspect of system performance which might be affected by the distribution of governing effort over the system, is the transient swing following a sudden load change on one system. Figures 15 and 16 give information on this point. Figure 15 shows the frequency swings of two interconnected systems, following a load change of two per cent on one system. The upper set of curves is for similar governors of reasonably fast response, the middle set for a slow response governor on the system subjected to the load change, the lower set for a slow response governor on the other system. Figure 16 shows a similar set of frequency curves for a load change of eight per cent. Measurements taken from these curves and from other similar curves are summarized in table II.

It is seen that for load changes of about two per cent the relative speeds of response of the governors on two interconnected systems make very little difference in the maximum swing or continuing oscillations of the system speeds or tie-line load.

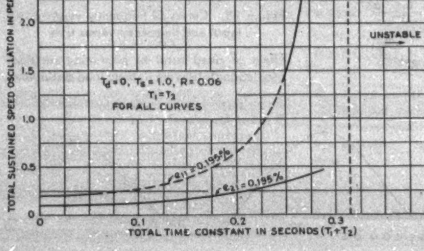

Figure 9. Self-excited speed oscillations produced by dead band

Effect of time lags of governing system—with system tie and no damping. (See figure 7 for definitions of e)

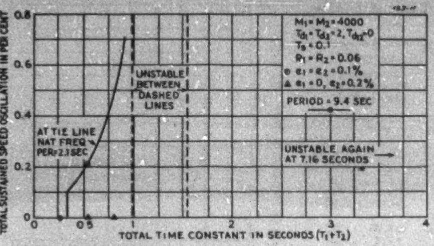

Figure 11. Self-excited system frequency oscillations in two interconnected systems

Effect of distribution of dead band and of time lags of governing system. e_1 and e_2 are dead bands (of first kind) on systems 1 and 2, respectively

For large load changes of about eight per cent when the faster governor is on the system subjected to the load change (*a*) the initial maximum tie-line swing is smaller, (*b*) the continuing tie-line oscillation is larger, (*c*) the maximum speed swings are not affected very much, and (*d*) the continuing speed oscillation is larger. With all the governing on the system subjected to the load change (*a*) the maximum tie-line swing is smaller, (*b*) the continuing tie-line load and speed oscillations are larger, and (*c*) the maximum speed swings are not appreciably different from the case of all governing on the other system. The maximum speed swings are in fact about the same as with the governing distributed over both systems.

Figure 17 shows the tie-line power swings following a load change on one system of an interconnection. The system parameters are different from those of figures 15 and 16. Table III is included to show for the case of figure 17 the effects on the rate of decay of tie-line oscillations, of governor response, dead band, and system and tie-line damping. It is seen from the figure and table that the effect of dead band is slightly to increase the effective time constant of the decay of the oscillations. That is, the component of oscillation which exceeds the dead band decays at about the same rate, but the rate of decay immediately changes to a very small value as soon as the oscillations become less than the dead band, so the result is a slightly smaller effective rate of decay. Doubling the regulation of the large system improves stability, i.e., increases the rate of decay (table III, run 61), while leaving the small system completely ungoverned except for its inherent self-regulation results in a slow decay (run 59). It is further shown that the initially assumed system damping must be increased by a factor of five to create the amount of damping contributed by the governors (run 62).

Summary of Results

Additional results not covered by the conclusions given at the beginning of the paper are as follows:

1. The stability of a system decreases if the speed regulation is made too small, except when the system consists of only one generator having a governor with droop compensation mechanism.

2. The magnitude of the sustained speed oscillations, if they exist, are at least equal to the width of the dead band of the unit having the smallest dead band, and become greater as the system characteristics approach the border line of stable and unstable operation. For dead band of the first or more usual kind, the speed oscillations will be damped out by load damping when the governing mechanisms are sufficiently stable (regulation sufficiently large or response sufficiently fast). Thus considerable information regarding probable system performance can be obtained from stability calculations made neglecting dead band. For units already installed the most effective way to improve undesirable system performance is to reduce the dead band or increase the speed regulation of the units known to be contributing to frequency instability.

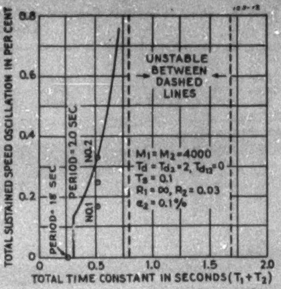

Figure 12. Self-excited system frequency oscillations in two interconnected systems

(Compare with figure 11 for effect of distribution of regulation and for definition of e_2)

3. The sustained frequency oscillations produced by dead band become somewhat greater as the location of the dead band moves from the input (governor head) toward the output (valve lift) end of the governing system. (See table I.) Distribution of a given total amount of dead band over the various stages of the governing system as compared to the same amount of dead band localized at one place in the mechanism increases the magnitude of the sustained oscillations. (See table I.) In spite of these variations, the over-all dead band of the governing mechanism of each prime mover, rather than that of any particular portion of the governing mechanism, is the principal significant factor in determining the magnitude of the frequency oscillations.

4. In analyzing a system, it is not proper to use a single effective dead band which is equal to some kind of average of the dead bands of the various governors on the systems. Instead, as the disturbance following a load change dies down, the governor having the smallest dead band tends to take

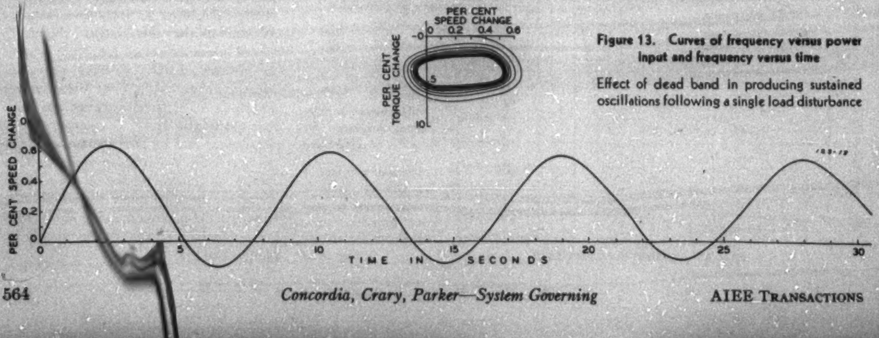

Figure 13. Curves of frequency versus power input and frequency versus time

Effect of dead band in producing sustained oscillations following a single load disturbance

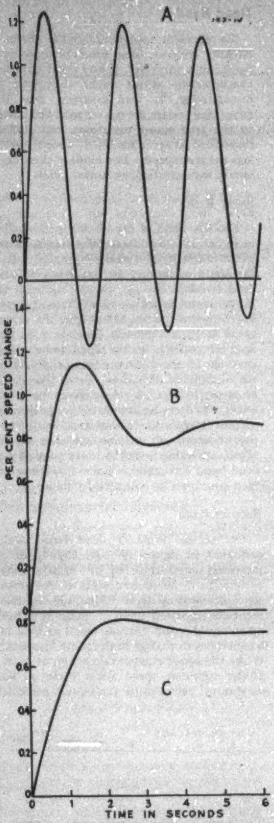

Figure 14. Rates of decay of frequency disturbance

Effect of regulation

Curve *A*—regulation = 1.5 per cent, $\Delta T = 0.32$
Curve *B*—regulation = 6 per cent, $\Delta T = 0.16$
Curve *C*—regulation = 12 per cent, $\Delta T = 0.08$
$T_1 = T_2 = 0.33$ second
$T_d = 2$
$T_2 = 0$
$M = 4{,}000$

control and so produce a more stable condition. (See figure 11.) On the other hand, the average system regulation appears to be a reasonably good indication of the degree of stability, a uniform distribution of regulation being only slightly most stable. (See figures 5 and 6.)

5. For load changes of about two per cent or less, the relative speeds of response of the governing mechanism on two interconnected systems make very little difference in the maximum swing or continuing oscillations of the system speeds or tie-line load. (See figure 15.) Some differences appear when the load change is much larger, but it does not seem that similarity of rates of response is particularly significant for average load changes as long as no response is made so slow as to affect stability.

6. In case of two paralleled machines, dead band in the governors may cause a maximum possible error in load division which varies directly with the ratio of the sum of the dead bands to the sum of the speed regulations of the two machines.

References

1. SUPERPOSED-TURBINE REGULATION PROBLEM, A. F. Schwendner and A. A. Luoma. ASME *Transactions*, volume 58, page 615-FSP-58-8.

2. ADVANCED RIGID DYNAMICS, E. J. Routh. (Fourth edition, 1884, Macmillan and Company), Theory of Watts Governor, page 57, Conditions of Stability, page 170.

3. ON GOVERNORS, J. C. Maxwell. *Philosophical Magazine*, volume 35, series 4, May 1868, page 385.

4. PRIME MOVER SPEED GOVERNORS FOR INTERCONNECTED SYSTEMS, J. B. McClure and R. J. Caughey. AIEE TRANSACTIONS, volume 60, 1941 (April section), pages 147–51.

5. THE FIELD OF SYSTEM GOVERNING, Albion Davis. ASME *Transactions*, volume 62, number 3, April 1940, page 207.

6. REGULATION OF SYSTEM LOAD AND FREQUENCY, Herbert Estrada and H. A. Dryar. ASME *Transactions*, volume 62, number 3, April 1940, page 221.

Appendix. Explanation of System Quantities Used

Single System

The frequency variations of an electric power system may be studied by means of the following approximate torque equation.

$$(Mp+T_d)p\delta+T_g'=\Delta T \tag{1}$$

where

M is the effective inertia of the system, and is numerically equal to the starting time, i.e., to the time in radians required to come to full speed from standstill with rated torque applied. In terms of the conventional inertia constant *H* used in system stability studies, $M=4\pi fH$, where f is normal system frequency in cycles per second

T_d is the load damping characteristic, or slope of the combined prime-mover and load torque versus speed curves. That is, with fixed valve, the turbine torque T_t decreases with increasing speed, therefore $\partial T_t/\partial\omega=-1$ at full load while the electrical load torque T_e may either increase or decrease depending on the character of the load and on the degree of voltage regulation during the speed swings. Then $T_d=\partial T_e/\partial\omega-\partial T_t/\partial\omega$, where $\omega=p\delta$

For example, if the system kilowatt load is directly proportional to frequency $\partial T_e/\partial\omega=0$ which results in a value of $T_d=1$.

T_g' is the change in prime-mover output torque caused by motion of the control valves

ΔT represents the load changes to which the system is subjected. Only a unit function load torque change was used in the present study

$p\delta$ is the system speed or frequency, so δ is the effective system angle

$p=d/dt$

t = time in radians; 1 second = 377 radians for a 60-cycle system

All system torques and speed are expressed in per cent of rated torque and speed respectively throughout the paper.

The governing system, including the lags due to steam or water storage, was repre-

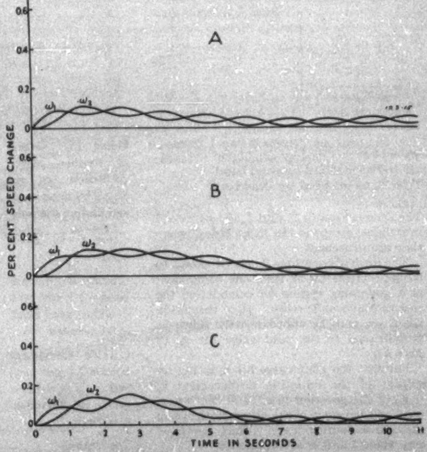

Figure 15. Effect on speed oscillations of relative speeds of response of governors on two interconnected systems

Curve *A*—$T_1 = T_2 = 0.13$ second

Curve *B*—$T_1 = 2.1$ seconds, $T_2 = 0.13$ second

Curve *C*—$T_1 = 0.13$ second, $T_2 = 2.1$ seconds

$R_1 = R_2 = 6$ per cent, $\Delta T_1 = 2$ per cent
$M_1 = M_2 = 4{,}000$
$e_1 = e_2 = 0.1$ per cent

See table II

See figure 11 for definitions of e

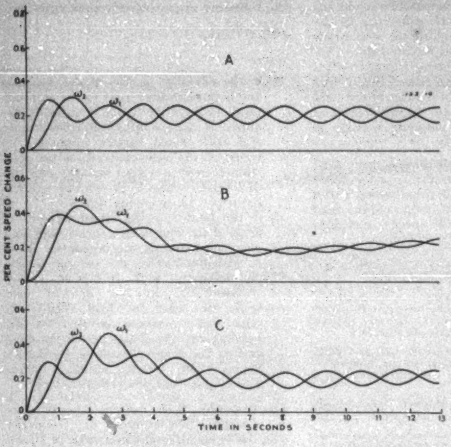

Figure 16. Effect on speed oscillation of relative speeds of response of governors on two interconnected systems

Curve A—$T_1=T_2=$ 0.13 second

Curve B—$T_1=2.1$ seconds, $T_2=0.13$ second

Curve C—$T_1=0.13$ second, $T_2=2.1$ seconds

$R_1=R_2=6$ per cent, $\Delta T_1=8$ per cent
$M_1=M_2=4{,}000$
$e_1=e_2=0.1$ per cent

See table II

See figure 11 for definitions of e

sented by a two-stage amplifier, each stage having a simple first-order time delay. That is, it was assumed that the speed change actuated some intermediate member Q, so that if there was an abrupt (unit-function) change of speed, Q would respond by approaching its final position along an exponential curve. Similarly, if Q changed abruptly, the prime-mover input T_g would respond by approaching its final position along an exponential curve. Such a governing system can be completely specified by two time constants T_1 and T_2, and an effective speed regulation R, where R is the steady-state incremental regulation defined as the ratio of per cent speed change to per cent power input change for a small change from 100 per cent speed. Thus the governor equations are:

$$\left.\begin{aligned} Q &= \frac{1}{T_1p+1}(p\delta)' \\ T_g &= \frac{1}{R(T_2p+1)}Q' \end{aligned}\right\} \qquad (2)$$

where

$p\delta'=p\delta$ as modified by dead band
$Q'=Q$ as modified by dead band
$T_g'=T_g$ as modified by dead band

The time constants T_1 and T_2 are expressed in radians, except in the figure titles, where they are in seconds.

The form of equations 2 may be shown to be appropriate for at least one component of a governing system by considering the case of a hydraulic relay. Here the piston speed px_3 may be approximately taken as proportional to the pilot valve lift x_2, or $px_3=kx_2$.

Further, the pilot valve lift x_2 is the resultant of the initiating displacement x_1 (e.g., of the governor head) and the restoring displacement x_3 of the piston, both acting through a floating lever. Thus, $x_2=lx_1-mx_3$ where l and m are constants.

Combining these two equations, the result is

$$px_3=klx_1-kmx_3$$

or,

$$px_3+kmx_3=klx_1$$

or,

$$x_3=\frac{(l/m)}{\left(\dfrac{p}{km}+1\right)}\,x_1$$

which is of the same form as equations 2.

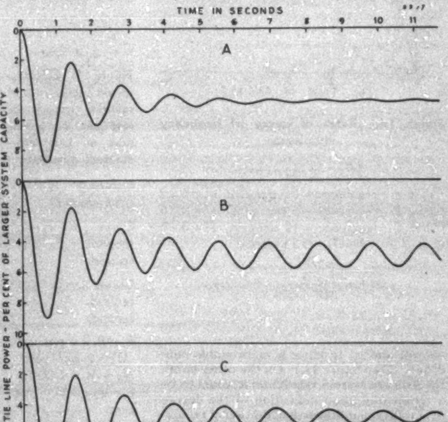

Figure 17. Decay of tie-line power oscillation as affected by dead band and tie-line damping

Curve A—No dead band

Curve B—Dead band=0.1 per cent—within dead band at 11 seconds

Curve C—Dead band=0.1 per cent and $T_{d12}=0.5$—within dead band at 6 seconds

See table III

Dead Band

To represent approximately the effects of sticking friction, loose mechanical connections, pilot valve lap, or any other governor characteristic which might contribute to insensitivity, it was assumed that the governing system did not respond accurately to the true speed variations, but instead responded to some modified "speed" which lags the true speed. Two kinds of these dead bands were studied, as shown below:

First Kind

The first kind of dead band is illustrated in figure 18. As the speed ω increases, no effect is felt by the governing system until the speed arrives at the upper edge of the dead band (point a). From this point on the apparent speed ω', to which the governing system responds, is less than the actual speed ω by an amount depending on the original position in the dead band. For example, if the governor was originally in the middle of the dead band the speed error would be $e_{11}/2$. As soon as the speed ω starts to decrease at point b, the governing system is again in its dead band so the apparent speed ω' remains constant at its maximum value until the lower edge of the dead band is reached at point c. Operation then continues as indicated by figure 18.

Second Kind

The second kind of dead band is illustrated in figure 19. As the speed ω increases no effect is felt by the governing system until the speed arrives at the upper edge of the dead band (point a). At this point the apparent speed ω' suddenly jumps up to equal ω and remains equal as long as ω continues to change in the same direction. When the speed starts to decrease, at point b, the apparent speed again sticks at its maximum value until the speed error is

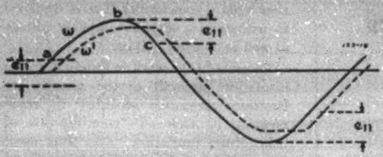

Figure 18 (left). Dead band of first kind

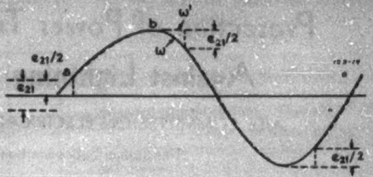

Figure 19. Dead band of second kind

half the defined dead band e_{11}, at point *c*. At this point the apparent speed again jumps down to equal ω, and operation continues as shown by the figure.

The first kind of dead band may represent for example a slack in certain parts of the mechanism, or a constant sticking friction force. The second kind might represent a sticking friction which breaks loose and becomes zero when the applied force (or speed error) is sufficiently large. It is evident that both of these kinds of dead band may also be applied at the intermediate point Q of the governing system, or at the prime-mover input T_g. These locations may be assumed to correspond roughly to particular locations in the actual governing system, from the input (governor head) to the output (control valve).

Two Interconnected Systems

For the case of two power systems connected by a tie line, if the inertia and governor characteristics of each system are considered as those of a single machine, the following approximate torque equations may be written.

$$\left.\begin{aligned}&(M_1p^2+(T_{d12}+T_{d1})p+T_s)\delta_1-\\&\qquad(T_{d12}p+T_s)\delta_2+T_{g1}=\Delta T_1\\&(M_2p^2+(T_{d12}+T_{d2})p+T_s)\delta_2-\\&\qquad(T_{d12}p+T_s)\delta_1+T_{g2}=\Delta T_2\end{aligned}\right\}(3)$$

where

T_{d12} is the tie-line damping torque coefficient, or change in the tie-line torque per unit change in relative system speeds, which depends on the strength of the tie and on the rotor electrical damping of the machines

T_s is the synchronizing torque coefficient, or change in torque per radian change in relative angle, between the two systems connected by the tie line

Subscripts 1 and 2 refer to the two systems 1 and 2 respectively.

The response characteristics of the governing systems are represented, as before, by two time delays between the initiating speed change and the resulting change in input torque.

$$\left.\begin{aligned}Q_1&=\frac{1}{(T_{11}p+1)}(p\delta_1)'\\T_{g1}&=\frac{1}{R_1(T_{21}p+1)}Q_1'\end{aligned}\right\}\qquad(4)$$

$$\left.\begin{aligned}Q_2&=\frac{1}{(T_{12}p+1)}(p\delta_2)'\\T_{g2}&=\frac{1}{R_2(T_{22}p+1)}Q_2'\end{aligned}\right\}\qquad(5)$$

where

T_{11}, T_{21}, T_{12}, T_{22} are the time constants of the two governing systems

R_1, R_2 are the speed regulations

Note that R_1, R_2 must be specified on the same kilovolt-ampere base, and so are not always equal to the conventional per cent regulations.

Dead bands of the first kind, of magnitude e_1 and e_2, were used between $p\delta_1$ and $p\delta_1'$ and between $p\delta_2$ and $p\delta_2'$ respectively, in the study of the two interconnected systems. To study the effects of dead band and the transient response of the system, equations 1 and 2, and 3, 4, and 5 were set up on the differential analyzer of the University of Pennsylvania, and the transient performance for a sudden load change ΔT_1 was determined for the cases discussed in the body of the paper.

Dead band of the first kind was simply represented by introducing a definite amount of backlash in the shaft connecting $(p\delta)'$ to $(p\delta)$, Q' to Q, or T_g' to T_g. Dead band of the second kind was represented by disconnecting $p\delta'$ from $p\delta$ until $p\delta$ reached its prescribed value, then (*a*) stopping the time shaft, (*b*) reconnecting $p\delta'$ to $p\delta$, and (*c*) setting the integrand $p\delta'$ to equal $p\delta$. The time shaft was then operated until $p\delta$ reached a maximum or minimum as explained in figure 19, when $p\delta_1'$ was again disconnected. Q' and T_g' were obtained in a similar manner.

To determine the limits of stable operation without dead band, the characteristic determinants of the coefficients of δ_1 and δ_2 in the equations were tested by Routh's stability criterion[9] over the range of parameters shown in the curves, an unstable system being defined as one in which transient oscillations will not die out.

Accuracy of Load Division

In equations 3, 4, 5 in the steady-state, $p\delta_1=p\delta_2=\omega$ and $\delta_1-\delta_2=\delta_{12}$. If dead bands e_1 and e_2 (of the first kind, on systems 1 and 2 respectively) are included, these equations become:

$$\left.\begin{aligned}T_{d1}\omega+\frac{\omega\pm e_1}{R_1}+T_s\delta_{12}&=\Delta T_1\\T_{d2}\omega+\frac{\omega\pm e_2}{R_2}-T_s\delta_{12}&=\Delta T_2\end{aligned}\right\}\qquad(6)$$

Adding the two equations 6 and solving for system frequency ω for the case of steady load, $\Delta T_1+\Delta T_2=0$, we obtain as the possible speed error,

$$\Delta\omega=\frac{\mp\frac{e_1}{R_1}\mp\frac{e_2}{R_2}}{T_{d1}+T_{d2}+\frac{1}{R_1}+\frac{1}{R_2}}\leqq$$

$$\frac{\frac{e_1}{R_1}+\frac{e_2}{R_2}}{T_{d1}+T_{d2}+\frac{1}{R_1}+\frac{1}{R_2}}\cong\frac{R_2e_1+R_1e_2}{R_1+R_2}\qquad(7)$$

and the possible torque variation of either machine is approximately (neglecting the small effect of T_{d1} and T_{d2}):

$$\Delta(\Delta T)=\frac{\Delta\omega\pm e_1}{R_1}=-\frac{\Delta\omega\pm e_2}{R_2}\leqq\frac{e_1+e_2}{R_1+R_2}\qquad(8)$$

If R_1, R_2 are taken as the per cent regulations on the bases of the individual machines, and if the fractions of the total capacity in machines 1 and 2 are K_1, K_2, then the per cent error of machine 1 is:

$$\Delta(\Delta T)\leqq\left(\frac{e_1+e_2}{\frac{R_1}{K_1}+\frac{R_2}{K_2}}\right)\frac{1}{K_1}\qquad(9)$$

Protection of Power Transformers Against Lightning Surges

AIEE COMMITTEE ON ELECTRICAL MACHINERY

Transformer Subcommittee*

Synopsis: This report presents a brief history of the subject of protection of power transformers against lightning overvoltages. The nature and magnitude of these overvoltages are described pointing out which ones are the most dangerous to apparatus.

The impulse voltage characteristics of transformers are given including volt-time curves, effects of (*a*) steepness of applied wave, (*b*) polarity of wave, and (*c*) mechanical deterioration of insulation in old transformers.

Similarly, impulse voltage characteristics of lightning arresters, rod gaps, and protector tubes are shown, thereby enabling the user to intelligently apply these devices for the protection of transformers.

The question of how to obtain various degrees of protection is fully discussed, ranging from the most effective protection to partial protection by using one or more of the following schemes:

(*a*). Direct-stroke protection and lightning arresters installed.

(*b*). Lightning arresters installed but direct-stroke protection omitted.

(*c*). Direct-stroke protection and station gaps installed, but lightning arresters omitted.

(*d*). Station gaps installed with neither direct-stroke protection nor lightning arresters.

Finally the proper selection of surge protective devices is discussed for the purpose of enabling the user to select the protective scheme that best suits his requirements.

A. General

As a result of the large amount of interest which has developed concerning the problem of protecting power transformers from overvoltages, the transformer subcommittee is presenting here a discussion of the problem in the hope that it will greatly clarify many points which have been given serious consideration in the past, and on which it is believed there have been, and still are, some different viewpoints. By discussing the fundamentals of transformer protection against overvoltage and some of the general controlling factors, a much clearer picture may be obtained of the degree of protection obtainable for transformers including both present-day designs and those built in the not too distant past.

Prior to some 15 years ago, insulation was built into a transformer largely on a 60-cycle basis. In general, sufficient was known at that time about 60-cycle characteristics of the insulation in a transformer to design it to withstand the double rating plus 1,000 specified test voltage, and to give satisfactory operation in service when subject to overvoltages such as normally encountered under operating conditions, exclusive of lightning. When, however, a large number of transformers put into use were subject to lightning conditions about which little was then known either as to magnitude of the lightning or the ability of the transformer insulation to withstand it, it was found that fundamental research on both of these problems was required before real progress could be made.

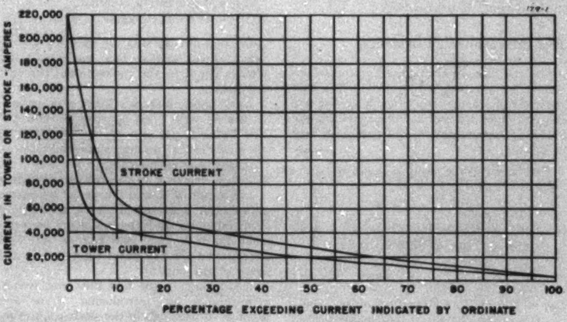

Figure 1. Lightning stroke and tower currents taken from reference 3

Without going into the details of the work done, it is a fact today that results of this research work have made available much data on the magnitude of lightning voltages to which transformers may be subjected in service and also correlated data on lightning or impulse strength of transformer insulation. As now built a transformer thus is designed to withstand a specified impulse voltage as well as 60-cycle voltage.

Progress of the work in building up the insulation strength of the transformer has been previously recorded[1] in which mention is made of "co-ordination of transformer insulation." As thus used, the word "co-ordination" has been used to mean protection of insulation rather than according to its more rigid definition of "having the same or equal" insulation as compared with some other device.

By co-ordination of transformer insulation, what is ordinarily meant is to build into the transformer, both on a 60-cycle and impulse basis, sufficient insulation so that the transformer will not be overstressed due to lightning when protected by readily available, protective devices, suitable for use with such transformers. As a step in this work of insulation co-ordination, there have been set up voltage test values or standards in the recognized preferred voltage ratings to which transformers in particular must conform as a matter of standardization. Standard basic insulation levels have been approved and submitted to the industry January 1941, by the joint committee on co-ordination of insulation of the AIEE, EEI, and NEMA. These levels are in reasonable conformity with the AIEE recommended impulse test values presented in June 1937, and have been recommended for use in place of those now appearing in the Proposed ASA Transformer Standards of 1940.

The problem of protecting transformers against overvoltages requires consideration of several factors. Among these are the following:

1. Magnitude and nature of system overvoltages.
2. Impulse or lightning strength of new and old transformers.

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[1] For all numbered references, see list at end of paper.

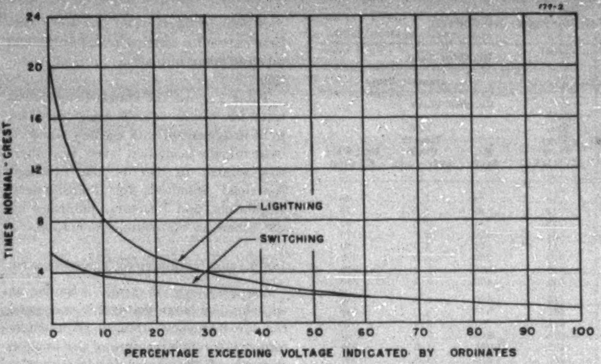


Figure 2. Lightning and switching surges taken from reference 3

3. Voltage characteristics of protective devices.

4. Lightning voltages which may reach the transformer.

These are discussed in some detail under *B*.

B. Factors to Be Considered in Protecting Transformer Insulation

1. Magnitude and Nature of System Overvoltages

In analyzing the voltage to which a transformer may be subjected, there are several types of overvoltages to be considered resulting from the following causes:

(*a*). Generator overspeed (hydro).

(*b*). System switching.

(*c*). Arcing ground faults.

(*d*). Circuit harmonics.

(*e*). Lightning.

(*a*)—*Generator Overspeed Voltage* (*Hydro*). So far as "runaway voltage" is concerned, it appears that the system overvoltage will rarely, if ever, exceed twice normal before governor action takes effect to promptly bring the voltage back to normal limits. Where governor action is more sensitive, the double voltage value may not even be reached. Under these conditions, the resultant overvoltage is one of normal frequency and as a matter of both theory and practice, it appears that the usual applied or induced voltage tests are adequate to insure satisfactory insulation to take care of this situation.

(*b*)—*System Switching Surges.* The overvoltages which may appear on the transformer as a result of system switching surges have been quite thoroughly investigated in the past, particularly by field tests. These rarely exceed 5.5 times normal line-to-ground voltage, and only a very small per cent (2.5 per cent or so) exceed 4.5 times normal. These higher switching surges are of short-time duration and of a high frequency and therefore less severe than normal frequency voltages of the same magnitude. It appears as a practical matter that transformers built to withstand regular 60-cycle double voltage tests and the proposed impulse tests can safely withstand switching surges of the order mentioned above. The relative frequency of occurrence of such switching surges as shown[3] by extensive field measurements is given in figure 2.

(*c*)—*Arcing Ground Fault Voltages.* There are overvoltages[5,6,7] of a character simulating normal frequency voltages that may reach the transformer as a result of fault conditions. These range in the general order of 2.5 times the normal line-to-neutral voltage of the system. These voltages are practically nonexistent and of little or no concern in a solidly grounded-neutral system. They also appear to be adequately taken care of on an ungrounded system by equipment built with insulation designed for the 2×line-to-line voltage+1,000 volts standard as has been used for some time.

(*d*)—*Circuit Harmonics.* With certain relations between direct and quadrature reactances, particularly with salient-pole generators, single-phase faults create large distortions in voltage wave form. In conjunction with line capacitance, crest voltages several times normal are possible. Other sources of harmonic overvoltages[25] may also exist.

(*e*)—*Lightning Overvoltages.* The principal overvoltages which cannot be predicted with engineering accuracy as to magnitude or characteristics if no protection or limitations are applied, are the lightning voltages. The magnitude of such voltages is indicated in times line-to neutral voltage of the system in the upper curve of figure 2. The ability of transformer insulation in oil to withstand lightning surges is essentially independent of the polarity of the voltage, that is, positive or negative, but is known to depend on the rate of rise of the lightning voltage, its magnitude, and duration. Further, transformers may be subjected to overvoltages several times within one second or so as a result of a multiple or repetitive lightning stroke. The tower and stroke currents are indicated in figure 1.

From a consideration of overvoltages,

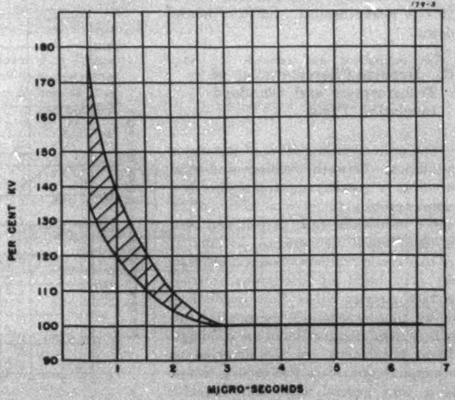


Figure 3. Volt-time curve of typical major insulation in transformers

Based on data in references 12 and 13

Table I. Standard Insulation Tests for Power Transformers

	Rated Voltage Between Terminals			Dielectric Tests			
	Single-Phase				Impulse Tests		
					Chopped Wave		
Insulation Class Kv	For Y Connected on 3ϕ System Kv RMS	For Δ Connected on 3ϕ System Kv RMS	Three-Phase Δ or Y Connected Kv RMS	Low-Frequency Tests Kv RMS	Kv Crest	Min Time to Flash-over—µsec	Full Wave Kv Crest
1.2	0.69	0.69	1.2	10	54	1.5	45
5.0	2.89	2.89	5.0	19	88	1.6	75
8.66	5.0	5.0	8.66	26	110	1.8	95
15.0	8.66	15.0	15.0	34	130	2.0	110
23.0	14.4	25.0	25.0	51	175	3.0	150
34.5	19.9	34.5	34.5	70	230	3.0	200
46.0	26.6	46.0	46.0	93	290	3.0	250
69.0	39.8	69.0	69.0	139	400	3.0	350
92.0	53.1	92.0	92.0	185	520	3.0	450
115.0	66.4	115.0	115.0	231	630	3.0	550
138.0	79.7	138.0	138.0	277	750	3.0	650
161.0	93.0	161.0	161.0	323	865	3.0	750
196.0	113.0	196.0	196.0	393	1,035	3.0	900
230.0	133.0	230.0	230.0	461	1,210	3.0	1,050
287.0	166.0	287.0	287.0	576	1,405	3.0	1,300
345.0	199.0	345.0	345.0	691	1,785	3.0	1,550

it should be apparent that transformers built in recent years apparently have sufficient insulation to withstand system runaway voltages, switching surges, harmonic voltages, and arcing ground voltages, without extensive and perhaps without any voltage limiting devices, but in the case of lightning voltages some protection must be provided to the transformer insulation if destructive impulse voltages and consequent failures are to be avoided to the greatest possible degree.

Knowing the relative magnitude and characteristics of the overvoltage conditions to which transformer insulation may be subjected, the next step is to discuss the characteristics of the insulation strength of the transformer itself and of the available protective devices, and finally the methods of limiting these voltages and providing reasonable margins to insure against insulation breakdown.

C. Impulse Characteristics of Transformers and Standard Insulation Tests

The protection in service of the major insulations (between windings and from windings to ground) in a transformer is a simpler problem than the protection of the minor insulations (coil to coil and turn to turn) which are subjected to internal voltage concentrations. Some transformers may build up dangerous internal voltage stresses either from oscillations or by concentration of the applied impulse in a limited portion of the windings. These phenomena are too involved to attempt treatment in this report, except in a very general way. The impulse voltage to which the major insulation is subjected can be fairly well controlled by protective devices external to the transformer. On the other hand, for a given level and type of protection the internal stresses set up in minor insulation depend on the steepness of the front and length of the incoming waves, the constants of the transformer circuit, and the type of construction used in the transformer.

It is assumed here that both major and minor insulations in modern transformers are well balanced under any condition. But for older transformers an internal (minor) insulation failure may be caused by a lower applied voltage than required to cause failure of the major insulation, especially for steep front waves which overstress the insulation between the line end coils and turns. It is also assumed that manufacturers when providing volt-time curves for existing transformers will take the minor insulations into consideration.

Table I shows the standard insulation tests for power transformers, modified to be in agreement with the new standard basic impulse insulation levels, but otherwise similar to that in the Proposed American Standards for Transformers, Regulators, and Reactors, published by the American Standards Association.

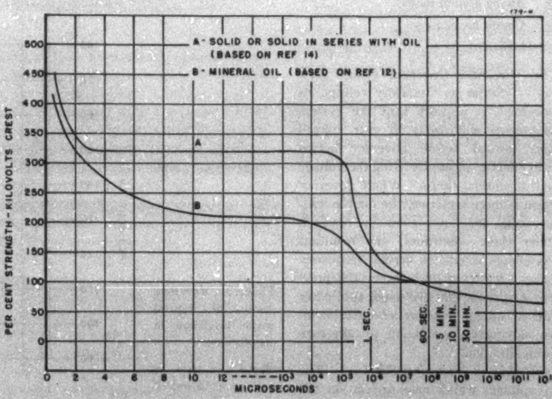
INSULATION STRENGTH OF TRANSFORMERS

Low-Frequency Strength. So far as withstanding surge voltages is concerned the low-frequency strength of a transformer is a rough measure of the impulse strength of the major insulation only. Induced voltage tests demonstrate to some extent the impulse strength between coils (but not between turns) for the slower waves, but do not indicate the strength for steep front waves which impose high stresses between line end coils. The principal value of standard low-frequency tests is in detecting faults due to possible errors in design, material, and workmanship.

VOLT-TIME CURVE OF MAJOR INSULATION

Laboratory tests[4,10,11,12,13] have indicated that the impulse strength, as shown on a volt-time curve of typical major insulation for transformers, decreases with time to approximately three microseconds and remains approximately constant for times longer than this as shown in figure

Figure 4. Typical volt-time curves of insulation



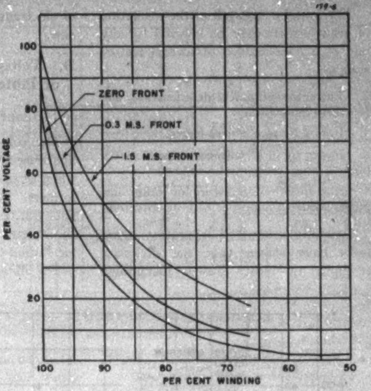
3. In other words, the failure voltage decreases with the "time" from zero to approximately three microseconds and then remains constant up to 50,000 to 75,000 microseconds as shown in figure 4. It then decreases again (during various times of exposure to normal or low-frequency voltage) and tends to flatten out for long periods of time.

The shape of the volt-time curve for oil alone, figure 4, is somewhat different from the curve for insulation mentioned above, the breakdown voltage decreasing from near zero to eight to ten microseconds, remaining approximately constant down to about 1,000 microseconds, then decreasing again and flattening out at approximately one minute time.

The volt-time curve of the breakdown of solid insulation and oil in series is essentially of the same shape as that for solid insulation alone. Where the failure of major insulation in the vital parts of a transformer, having some solid insulation in series with oil, would take place by puncture, the volt-time curve of "solid insulation" can be used for co-ordination purposes. It is recognized, however, that where failure would be by creepage, the volt-time curve may approach that for oil alone.

The "over-all" volt-time curve of a transformer takes into account the volt-time curve of its bushings, as well as the windings. If the long wave flashover of the bushing is higher than the impulse level (demonstrated or estimated strength) of the windings, the volt-time curve to be used for co-ordination purposes can be that of the windings. On the other hand, if the long wave flashover of the bushing is less, then in order to prevent bushing flashover (and consequent service interruption) on long waves, the volt-time curve to be used for protection purposes should be a composite curve of bushing and winding as shown in figure 5.

Figure 6. Effect of wave front on initial voltage distribution in some types of windings



Initial Impulse Voltage Distribution as Affected by Steepness of Applied Wave

The effect of impulse waves applied to transformers which are not designed to produce an approximate uniform voltage distribution differs considerably with the type of winding and the method of insulation of the transformer winding. Concentration of the stresses within the windings varies depending (*a*) on the construction, and (*b*) on the steepness of the applied wave. The effect of the steepness of the applied wave on the initial voltage distribution on some transformer constructions is illustrated in figure 6.

Effect of Polarity of Wave on Transformer Strength

So far as the winding in oil is concerned, the impulse strength is the same for both positive and negative waves. The only way in which the effect of polarity must be taken into consideration in operation is in its effect on the level of the protective device used. Lightning arresters, however, are not affected by polarity but most air gaps, such as rod gaps, bushings, and insulators have higher critical (1.5x 40) flashover for negative waves although the effect of polarity tends to disappear for steep front waves. Therefore, when depending on air gaps alone for partial protection, as discussed later, the effect of polarity should be considered, remembering that the majority of lightning voltages recorded in service are of negative polarity.

Effect of Age on the Impulse Strength of Transformers

The question of the effect of age on the impulse strength of transformers has often been raised.

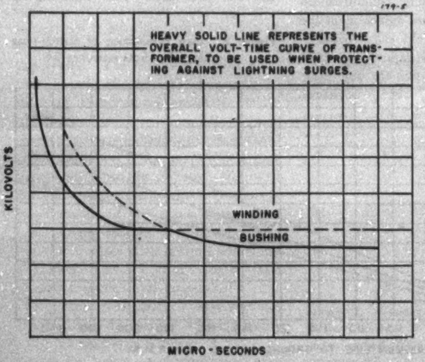
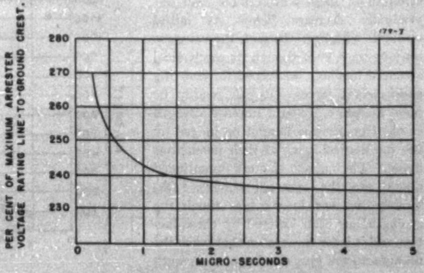


Figure 5. Typical volt-time curve of transformer winding and bushings

Figure 7. Arrester gap breakdown, tolerance ±20 per cent

Taken from reference 24



The impulse strength of the insulation of transformers may be lowered by any one or a combination of several factors such as:

1. Extreme mechanical deterioration, due to excess temperature.
2. Injury by mechanical forces.
3. Injury by surge voltages not sufficiently high to cause complete breakdown.
4. Absorption of moisture or other impurities due to improper care in operation.

Carefully conducted laboratory aging tests have shown that the dielectric strength of most fibrous insulations showed a marked increase in dielectric strength at the start, owing to removal of the last traces of moisture. As the test continued the dielectric strength slowly decreased but did not fall below its initial strength until practically all its mechanical strength was gone; that is, until it became charred and cracked.

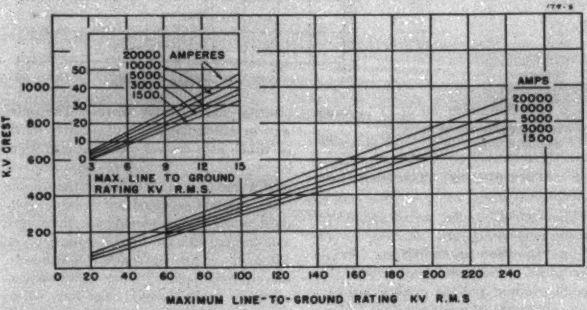


Figure 8. IR discharge voltage, 10–20 microsecond wave of station-type arrester

Tolerance ±15 per cent. Taken from reference 24

It is for this reason that, considered only from the standpoint of the mechanical aging of the insulation, the impulse strength of most transformers does not gradually decrease below its initial strength with age. In fact, the converse may be true, even though its mechanical strength has been weakened for resisting short-circuit forces. It can readily be seen, therefore, that the impulse strength of old transformers cannot be judged by the mechanical aging of their insulations alone. The other factors mentioned above are the controlling ones. Obviously, no one knows how much the strength has been lowered by these factors and it is for this reason that when the manufacturers give the impulse strength of old transformers it is based on "when new."

D. Voltage Characteristics of Protective Devices

(a). Lightning Arresters

The volt-time curve characteristics of lightning arresters should be considered when arresters are used to protect transformers. All arresters have relatively flat volt-time curves. Figure 7 shows the general shape of the volt-time curve for station-type arresters.

The impulse gap breakdown voltages and *IR* drop voltages of lightning arresters based on AIEE test conditions are given in figures 7 to 14 inclusive. The arrester performance is the same for either positive or negative impulses.

(b). Rod Gaps

Tentative volt-time curves for standard rod gaps for 1.5x40 negative waves are shown in figure 15. These curves, especially at short times, are subject to change when the subcommittee of the AIEE-EEI-NEMA joint committee on insulation co-ordination agrees upon the values to be used.

The curves given in figure 15 are the average of test data obtained up to the present time by the Ohio Brass, General Electric, and Westinghouse laboratories.

The 60-cycle and critical (50-50) impulse spark-over voltage of standard rod gaps are shown in figure 16. These curves (except the 60-cycle wet) are based on data agreed upon by the EEI-NEMA laboratory group.

(c). Protector Tubes

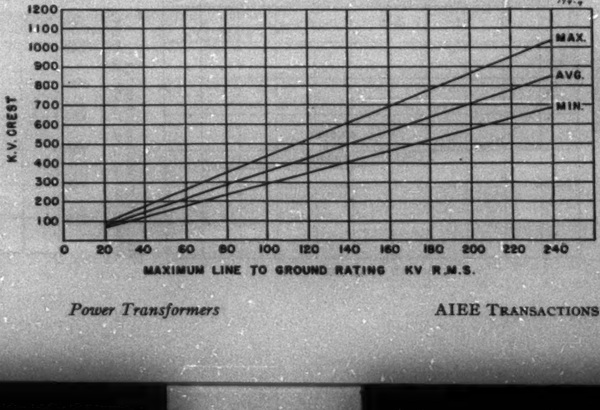
Typical approximate volt-time curves of protector tubes are shown in figure 17. These curves are shown in crosshatched section because the AIEE lightning arrester subcommittee has not yet published final values.

E. Methods of Protection Available

1. Interception of Direct Strokes

With the present knowledge of the characteristics of natural lightning it is possible to provide a means of intercepting direct strokes thereby preventing them from coming in contact with the phase conductors except in rare cases. This type of protection takes the form of shielding. The popular method of intercepting the lightning stroke is by means of a ground wire. The ground wire is placed above the object to be protected and the grounding resistance of this wire is kept low so that the potential of the ground wire or the supporting structure does not become high enough to cause a flashback from the steel to the phase conductor. The ground wire is placed over the substation or over the transmission line or both depending upon the degree of protection desired. Another method of intercepting the direct stroke is to erect a mast or a number of masts on the top of the substation steel to adequately shield the bus structures. This shielding by

Figure 9. Gap breakdown of station-type arresters, taken from reference 24



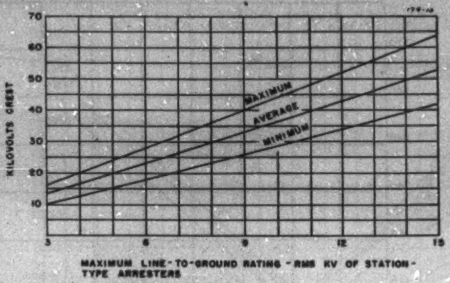


Figure 10. Gap breakdown based on AIEE wave, taken from reference 24

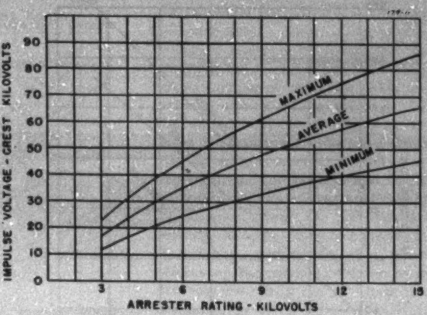


Figure 11. Gap breakdown voltages, line-type arresters, taken from reference 22

masts has in some cases been carried further and the masts have been erected entirely separate from the substation steel. This form of protection has been used where direct stroke protection is desired in connection with old substations or where equipment such as disconnects prevents erecting the masts on the top of the substation steel. Again the mast must be high enough to adequately shield the apparatus under consideration.

In applying any of the above arrangements the angle or area of shielding should be given careful consideration. Initially such shielding was based on a ratio of horizontal radius of area protected to vertical height of mast of two to one (64 degrees to vertical) or greater. The ratio has gradually decreased and recent studies have indicated that one to two may be desirable.

2. Lightning Discharge Devices

A. Lightning Arresters. The lightning arrester is the most generally used protective device as it not only discharges the surge current to offer apparatus protection but it also interrupts the flow of power follow current thereby protecting the service. The lightning arrester volt-time characteristics, as compared to those of transformers, make it ideally suited for this application. Its voltage characteristics are made lower than those of the insulation in the apparatus at long times and due to the arrester's relatively flat volt-time characteristic it provides a larger margin of protection at the short times. This margin is necessary in a number of cases when effects which are explained later of lead lengths and ground resistance are considered.

B. Plain Rod Gaps. Rod gaps are sometimes used in parallel with apparatus to act as discharge devices. A comparison of the volt-time characteristic of a gap compared to the volt-time characteristic of a lightning arrester will readily show that the rod gap is not fundamentally a good protective device because if it is selected to protect the apparatus at the long times it will not provide protection at the short times. On the other hand, if it is set to provide adequate protection at the short times the gap will be so short it will be susceptible to frequent flashovers due to switching transients and minor lightning surges. The practicable gap setting selected, therefore, is a compromise. The plain rod gap has been used in some instances as a primary protective device but it has been more commonly used as a supplemental device.

C. Fuse Gaps. Fuse gaps have the same characteristics as plain rod gaps except a fuse is placed in series to provide a means of interrupting the power follow current caused by flashover. This arrangement therefore gives the added feature of protection to service when compared to the plain rod gap. The maintenance and replacement of fuses are items for consideration in this form of protection.

D. Protector Tubes. The volt-time characteristics of protector tubes offer better protection than those of plain rod gaps but the impulse voltage does increase at the shorter times more rapidly than on a lightning arrester. The protector tube is designed to interrupt power follow current, and this therefore dictates the length of tube for a given voltage which in turn determines the impulse breakdown of the device. Where the system is solidly grounded the tube breakdown can be lower than on a free neutral system so that in the former case it serves as a very good secondary class protective device. It has been quite extensively used on a few towers close to a substation serving as a parallel discharge path to the lightning arrester discharge path.

E. Control Gap. The control gap has been used as a secondary protective device with and without fuses. Its volt-time characteristic is somewhat better than that of the equivalent rod gap but it still has some of the factors of the plain rod gap and is, therefore, classed with it when selecting protective devices.

3. Capacitance

When lightning strikes a line the substation voltage will rise at a rate determined not only by the original voltage on the line but also by the magnitude of capacitance in the substation. The effect

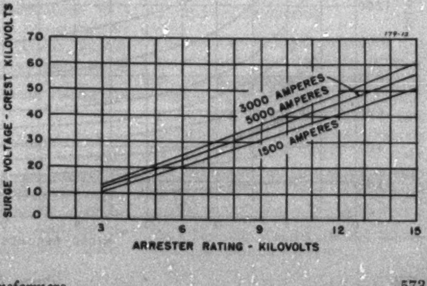


Figure 12. Average IR drop, line-type arresters, taken from reference 22

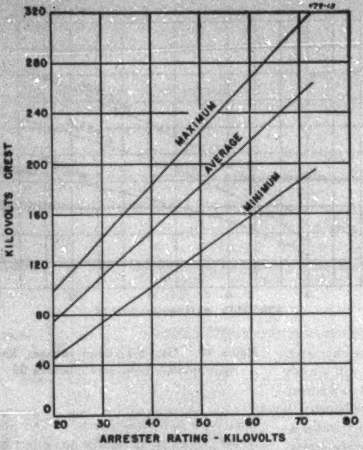


Figure 13. Gap breakdown voltages, line-type arrester

Taken from reference 23

of the station capacitance is to decrease the rate of voltage rise at the transformer and in some cases to reduce the voltage at the transformer. This inherent characteristic of substation equipment is more effective the further the stroke occurs from the substation as the rate of increase of voltage in the substation will be controlled to a greater extent if the capacitance is charged through the surge impedance of the line. If protective devices such as protector tubes are placed on the line the station capacitance will be more effective. However, if station capacitance were depended upon as a protective scheme, there would be no reliable protection. This would be particularly true for strokes occurring close to or directly on the substation.

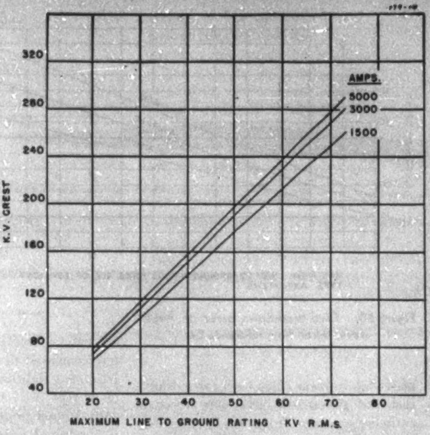


Figure 14. Average IR discharge voltage, line-type arresters, 10–20-microsecond wave

Taken from reference 23

F. Effect of Terminal Conditions

(a). Ground Resistance

The magnitude of ground resistance is very important in securing good protection, particularly where the protective device is not connected to a ground in com-

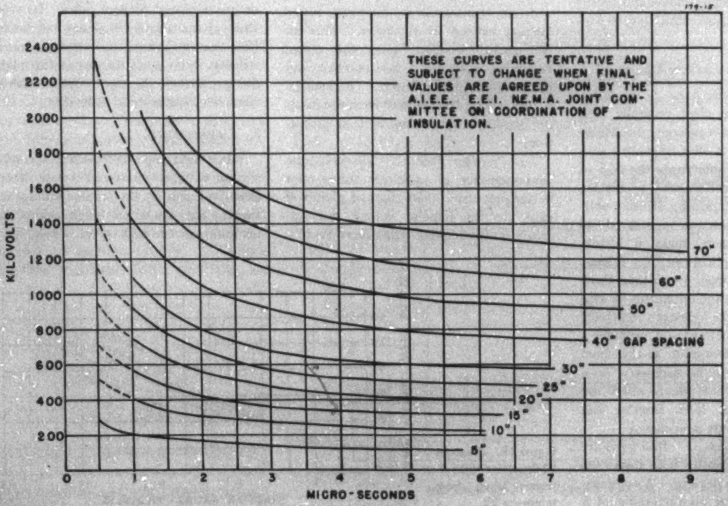


Figure 15. Volt-time curves of rod gaps under standard atmospheric conditions, 1.5x40 negative waves

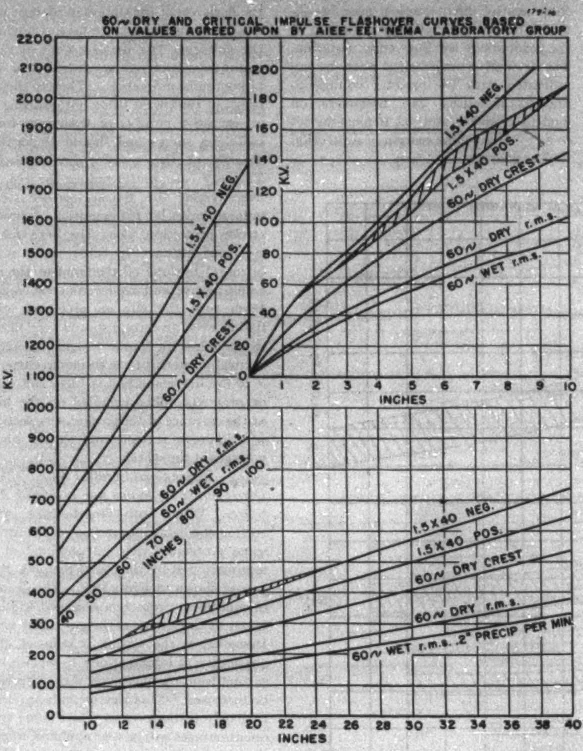


Figure 16. Spark-over voltage of standard rod gaps at AIEE standard atmospheric conditions

mon with the apparatus ground. In this case the resistance drop adds directly to the lightning arrester discharge voltage and this addition may be sufficient to cause flashover or failure of the apparatus to be protected. If the lightning arrester ground is connected to the apparatus ground then the apparatus tanks will rise in potential the same as the arrester ground so only the lightning arrester voltage will appear across the apparatus. This means, however, that although the apparatus so connected will be protected, the bus is subjected to higher voltages, the degree depending upon the distance from the lightning arrester installation. For this and other reasons it is considered good practice to keep the ground resistance of large stations below one ohm. In the outlying regions where a small transformer is the only equipment to be protected it may be economically necessary to let the ground resistance exceed one ohm. Soil conditions will determine the ground grid necessary to achieve the desired results.

(b). Capacitance

As previously discussed, the larger the inherent capacitance of the substation the more it becomes effective as a protective measure. The limitations to a scheme depending upon capacitance are such as to make its application limited. It should be considered as a reserve factor.

(c). Circuit Feet From Protective Device to Apparatus

The distance from the protective device to the apparatus to be protected is a very important factor, particularly for direct strokes and becomes of less importance the further the stroke originates from the substation. Considering the worst case of stroke originating close to the substation the rate of rise of voltage will then have a direct bearing on the voltage appearing at the equipment. In general it can be stated that the voltage at the equipment can reach twice the lightning arrester voltage as a limit. As the distance between the lightning arrester and apparatus is reduced, or as the rise of voltage is reduced, the voltage at the apparatus will approach that of the lightning arrester. It is essential to keep the distance between the protective device and the apparatus to be protected to a minimum, particularly when direct strokes are the criterion of design as in this case the rate of rise of voltage is a maximum.

G. Degrees of Protection

1. Direct-Stroke Protection and Lightning Arresters Installed

Based on present-day knowledge, the most effective protective setup consists, as illustrated by figures 18*A* and 18*B*, of lightning arresters supplemented by proper direct-stroke shielding of the station and overhead-ground wire protection extending approximately 2,500 feet or more from the station. Very careful design of overhead-ground wire arrangement is necessary if complete protection is desired, although it is well to realize that direct-stroke protection cannot necessarily be relied upon 100 per cent. If direct strokes are prevented in the space within the station and for a distance of approximately 2,500 feet from the station, the surge impedance of the line between the arrester and the nearest possible direct stroke will limit the discharge current at the arrester. Under these conditions, modern arresters having suitable performance characteristics and properly located will ordinarily limit the voltage to a value well below the insulation level of the modern transformer. For lines 46 kv and below, the installation of ground wires may not be economically justified.

2. Partial Protection

There may be installations such that the expected revenue and the nature of the load do not appear to justify the installation of the most effective protection. In this case, the user may prefer to take an additional risk of service interruption or apparatus failure, rather than make the expenditure necessary to install maximum protection. Three varieties of partial protection may be classified as:

(*a*). Lightning arresters installed but direct-stroke protection omitted.

(*b*). Direct-stroke protection and station gaps installed, but lightning arresters omitted.

(*c*). Station gaps installed with both direct-stroke protection and lightning arresters omitted.

(*a*)—*Lightning Arresters Installed but Direct-Stroke Protection Omitted.* For years it has been common practice to use lightning arresters without direct-stroke protection to protect power transformers and station apparatus on 15, 23, 34.5, and 46-kv systems. Modern arresters, if the line is not too highly insulated, should afford protection for modern transformers against waves of any rate of rise, originating more than about one-half mile from the station. This practice cannot be relied upon to give protection against direct strokes at or near the station. As shown in figure 18*C*, some protection from nearby direct strokes may be obtained by the installation of a rod gap at the point where the lines enter a station. The setting of such a gap must be coordinated with the arrester voltage to prevent undesired gap flashovers on moderate strokes and yet it must be set so as to afford some margin of protection to the apparatus.

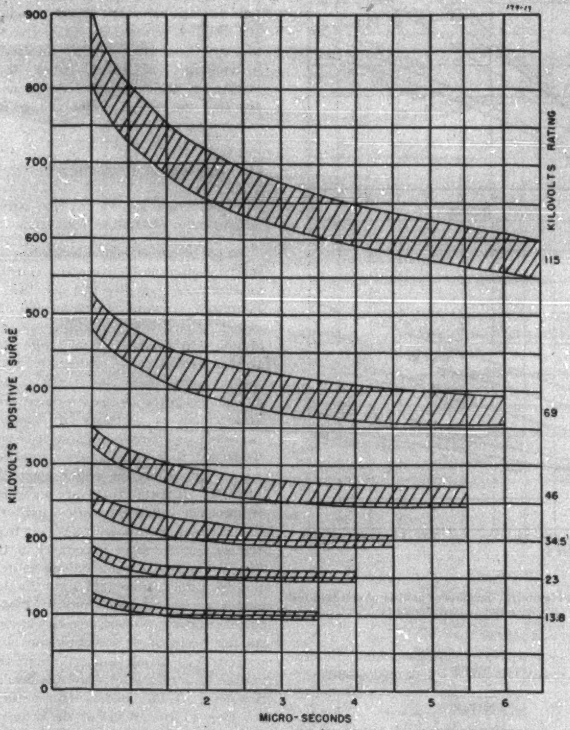


Figure 17. Approximate impulse characteristics of protector tubes

Some degree of direct-stroke protection will result if protector tubes or other protective devices are used for a distance of 2,500 feet out from the station, particularly if line insulation is high. Low-ground resistance for the tubes or interconnection to station ground is desirable for this application to prevent unpredictable voltages from reaching the station.

(*b*)—*Direct-Stroke Protection and Station Gaps Installed, but Lightning Arresters Omitted.* When direct-stroke protection is installed, but lightning arresters are omitted as shown in figure 18*d*, the station equipment may be stressed up to the flashover of the station insulation, unless some other means are taken to reduce the voltage. The use of plain rod gaps might be considered as a method of obtaining some protection. The gap should be set at the lowest value not resulting in an excessive number of flashovers from switching surges and should be located directly at the transformer terminals. The user can greatly reduce the risk of overstressing the transformer by steep voltage waves by the extension of direct-stroke protection along the lines for a sufficient distance (possibly three miles or more) to slope off the front of the incoming wave and reduce its crest value. Each time the gap or protector tube flashes over, an outage should very likely result unless the gap is designed to automatically interrupt the dynamic current.

If the line insulation is high, rod gaps or protector tubes installed on the line at the entrance of the ground-wire section will improve service and reduce surge voltages at the station.

(*c*)—*Station Gaps Installed With Neither Direct-Stroke Protection nor Lightning Arresters.* When both direct-stroke protection and lightning arresters are omitted some protection may be obtained at a minimum cost by the use of the plain rod gap as shown in figure 18*E*. The degree of equipment protection obtained will increase as the setting of the gap is reduced. However, as the setting is reduced, the possibility of interruptions to service resulting from the flashover of the gap will be increased. This scheme finds application only in those cases where more frequent outages will be tolerated and where the user is willing to accept increased risk of transformer failure.

3. SELECTION OF SURGE-PROTECTIVE-DEVICE RATING

The protective device should be selected and the scheme of protection devised so that under the conditions presumed to exist the protection level obtained will be below the insulation level of the transformer.

Protective devices have ratings based on the voltage at which they are designed to operate. In the case of lightning arresters, the rating is the maximum voltage at which power follow current will be interrupted. Protector tubes are rated both on voltage and power follow current magnitude. Rod gaps do not have any established voltage ratings.

The protected level provided by a lightning arrester is generally in direct proportion to its voltage rating, and it is therefore desirable to use as low a rating as practical. In order to do this it is necessary to know something of the magnitude of the dynamic voltages which may be impressed on the arrester under all conditions, including faults on the system. The magnitude of line-to-ground voltages under fault conditions depends largely on the effectiveness with which the neutral of the system is grounded and may be determined by the ratio of zero sequence impedance to positive sequence impedance at the point of application.

The practice has been to manufacture lightning arresters in ratings such that "full voltage" arresters with ratings slightly above line-to-line voltage are available for isolated or "not effectively grounded" neutral systems, and arresters of reduced voltage rating are available for systems with an "effectively grounded" neutral. This has been discussed in many Institute papers.

Reduced rated arresters may be used when applicable to give a greater margin of protection, or in the higher voltages may permit the use of the next lower insulation level on apparatus than would otherwise be used.

The protective devices such as arresters or protector tubes will be most effective when mounted directly on or adjacent to the transformer tank and with line and ground connections between protective device and transformer as short as possible. In large stations more than one set of protective devices may be necessary to keep the circuit-feet between protected apparatus and protective devices within acceptable limits.

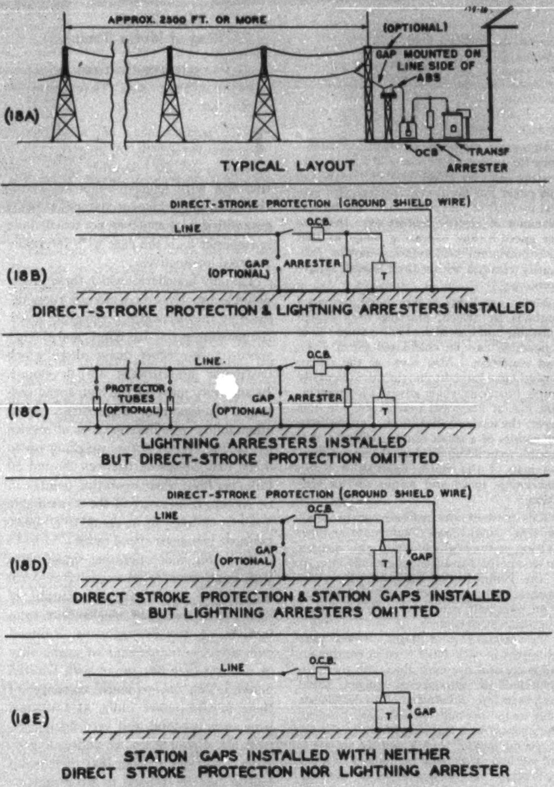

APPROX. 2300 FT. OR MORE
(OPTIONAL)
GAP MOUNTED ON LINE SIDE OF ABS
(18A)
OCB
TRANSF
ARRESTER
TYPICAL LAYOUT
DIRECT-STROKE PROTECTION (GROUND SHIELD WIRE)
LINE
O.C.B.
GAP (OPTIONAL)
ARRESTER
T
(18B)
DIRECT-STROKE PROTECTION & LIGHTNING ARRESTERS INSTALLED
LINE
O.C.B.
PROTECTOR TUBES (OPTIONAL)
GAP (OPTIONAL)
ARRESTER
T
(18C)
LIGHTNING ARRESTERS INSTALLED
BUT DIRECT-STROKE PROTECTION OMITTED
DIRECT-STROKE PROTECTION (GROUND SHIELD WIRE)
LINE
O.C.B.
GAP (OPTIONAL)
T
GAP
(18D)
DIRECT STROKE PROTECTION & STATION GAPS INSTALLED
BUT LIGHTNING ARRESTERS OMITTED
LINE
O.C.B.
T
GAP
(18E)
STATION GAPS INSTALLED WITH NEITHER
DIRECT STROKE PROTECTION NOR LIGHTNING ARRESTER

Figure 18

Bibliography

1. Insulation Strength of Transformers, Committee Report. AIEE Transactions, volume 56, 1937 (June section), pages 749–54.

2. Basic Impulse Insulation Levels, **Philip Sporn** and **C. A. Powel.** AIEE Transactions, volume 59, 1940 (October section), pages 596–8.

3. Lightning Investigation on Transmission Lines—II, **W. W. Lewis** and **C. M. Foust.** AIEE Transactions, volume 50, 1931, pages 1139–46.

4. Insulation Co-ordination and Protection of Power Transformers, **V. M. Montsinger.** *General Electric Review*, October 1937, pages 454–62.

5. Power System Transients Caused by Switching and Faults, **R. D. Evans, A. C. Monteith,** and **R. L. Witzke.** AIEE Transactions, volume 58, 1939 (August section), pages 386–94.

6. Arcing Faults in Power Systems, **C. Concordia** and **H. A. Peterson.** AIEE Transactions, volume 60, 1941.

7. Arcing Grounds and Effect of Neutral Grounding Impedance, **J. E. Clem.** AIEE Transactions, volume 49, 1930, pages 970–88.

8. Protection of Stations Against Lightning, **L. V. Bewley** and **W. J. Rudge.** *General Electric Review*, August and September 1937.

9. The Control Gap for Lightning Protection, **Ralph Higgins** and **H. L. Rorden.** AIEE Transactions, volume 55, 1936, pages 1029–34.

10. Factors Influencing the Insulation Co-ordination of Transformers, **F. J. Vogel.** AIEE Transactions, volume 52, 1933, pages 411–16.

11. Factors Influencing the Insulation Co-ordination of Transformers—Part II, **P. L. Bellaschi** and **F. J. Vogel.** AIEE Transactions, volume 53, 1934, pages 870–6.

12. Dielectric Strength of Transformer Insulation, **P. L. Bellaschi** and **W. L. Teague.** AIEE Transactions, volume 56, 1937, pages 163–71.

13. Co-ordination of Power Transformers for Steep Front Impulse Waves, **V. M. Montsinger.** AIEE Transactions, volume 58, 1939 (April section), pages 183–9.

14. Breakdown Curve for Solid Insulation, **V. M. Montsinger.** AIEE Transactions, volume 54, 1935 (December section), pages 1300–01.

15. Application of Arresters and the Selection of Insulation Levels, **J. H. Foote** and **J. R. North.** AIEE Transactions, volume 56, 1937 (June section), pages 677–82.

16. Application of Spill Gaps and Selection of Insulation Levels, **H. L. Melvin** and **R. E. Pierce.** AIEE Transactions, volume 56, 1937 (June section), pages 689–94.

17. Basic Impulse Insulation Levels, EEI-NEMA Committee Report. AIEE Transactions, volume 56, 1937 (June section), pages 711–12.

18. Flashover Characteristics of Rod Gaps and Insulators, EEI-NEMA Committee Report. AIEE Transactions, volume 56, 1937 (June section), pages 712–14.

19. Insulation Co-ordination, **Philip Sporn** and **I. W. Gross.** AIEE Transactions, volume 56, 1937 (June section), pages 715–20.

20. Lightning Protection of Power Transformers Connected to Overhead Circuits, **K. B. McEachron.** *General Electric Review*, August 1934, page 364.

21. Insulation Co-ordination for System Protection, **C. A. Powel.** *Electric Light and Power*, June 1938.

22. Distribution Lightning Arrester Performance Data, AIEE Lightning Arrester Subcommittee. AIEE Transactions, volume 56, 1937 (May section), pages 576–7.

23. Line-Type Lightning Arrester Performance Data, AIEE Protective Devices Committee Report. AIEE Transactions, volume 57, 1938 (November section), pages 661–2.

24. Station-Type Lightning Arrester Performance Characteristics, AIEE Lightning Arrester Subcommittee. AIEE Transactions, volume 59, 1940 (June section), pages 347–8.

25. Unsymmetrical Short Circuits on Water Wheel Generators Under Capacitive Loading, **C. F. Wagner.** AIEE Transactions, volume 56, 1937 (November section), pages 1385–95.

Complete Analysis of Motor Temperature Rise

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MEMBER AIEE

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Synopsis: The present-day demand and imperative need for higher speeds and increased outputs in every phase of industry and transportation have emphasized the tremendous importance of realizing the maximum utilization of the service capacities built into electrically-driven equipments.

In a great majority of cases the predominant limitation in this direction lies in the heating capacity of the electric motors and this paper has been written to show a general method of "mapping" the heating performance of electric motors over their entire speed-torque operating range so that their maximum performance may be judiciously realized within their heating limitations.

Many excellent papers have described methods by which, starting from design constants and losses, the temperature rise of a machine can be calculated for a given load condition. Also some of the earlier papers[1] have brought out the fundamentally simple functions which govern the temperature rise of electrical machinery. In this paper, the authors show how, starting from the results of a small number of tests, it is possible to build up a complete temperature-rise map of the motor applicable to any conceivable speed and torque service condition.

This method was worked out in detail for the single-phase, commutator-type, traction motors which equip the modern a-c locomotives and the enlightened interest of the Pennsylvania Railroad led to the preparation by two electrical manufacturers of an unusually complete analysis of traction motor performance.

The original calculations, which were submitted in 1935 have been in continuous and successful use ever since, for the locomotives of the type shown in figure 1 and have been fully substantiated by later tests made on motors built on subsequent orders. The authors feel, therefore, fully confident, in recommending that a similar approach and method of plotting be applied to other applications in transportation or industry where widely varying load conditions are encountered and where it is important to secure the utmost utilization of a large number of similar motors.

Part I. Wide-Range Charting of Motor Temperatures

Performance Possibilities of Commutator-Type A-C Traction Motors

The performance of single-phase, commutator-type, locomotives is not limited to definite operating speeds as is the case with induction-motor locomotives, nor to fixed motor voltages as is the case with d-c locomotives, nor to a definite horsepower as is the case with internally powered locomotives.

The only boundaries which these single-phase locomotives recognize are those defined by the maximum permissible speed, the maximum tractive effort, and the top speed-tractive effort curve along which maximum horsepowers are developed. In addition, many of the high-speed passenger locomotives demonstrate their adaptability to different types of service by pulling freight trains weighing up to 5,500 tons at speeds between 30 and 50 miles per hour where operating points can be chosen, from many of the 60 available notches, as freely as in the 60-to-80 miles per hour passenger speed range.

With this wide operating range, with their continuous rated capacity of 4,620 horsepower, and their peak output of 8,500 horsepower, the single-phase locomotives of the Pennsylvania Railroad offer a concentration and an availability of power which has never been realized before. The performance capacity of these motive power units, at the same time very powerful and very finely controllable, should be known under all operating conditions in order that their possibilities may be realized to the utmost.

Traction-Motor Temperature Limitations

Although there are several links in the transformation of electrical energy into mechanical power to drive the wheels of the locomotive, the fact that the space limitations inherent in locomotive construction affect the traction motors more stringently than any other apparatus, and also the fact that the traction motor output is delivered over such a wide speed range, combine in restricting the input of electrical energy to the locomotive to the amount permitted by the commutating and heating capacity of the traction motors. Within the limits of commutation the traction motor heating capacity is the factor which determines the performance of the locomotive as a whole.

This paper explains the analysis by which the temperature rise is determined for any operating cycle, the criterion of a correct application being that the temperature of the windings as measured by their resistance must not exceed the permissible values set by the American standards for railway motors with class *B* insulation. The "normal" temperatures may be considered as the superposition of 120-degrees-centigrade rise for the armature or 130-degrees rise for the field above a 25-degrees-centigrade ambient. The "peak" temperatures may be considered as the same rises (120 degrees centigrade and 130 degrees centigrade) superimposed over a 40-degrees-centigrade ambient.

Temperatures	Normal (Degrees Centigrade)	Peak (Degrees Centigrade)
Armature	145	160
Field	155	170

Complete Mapping of Temperatures

The wide diversity of questions to be answered about the effect on temperature of time, speed, and tractive effort made it quite apparent that a very comprehensive and complete "map" of these relations had to be worked out. At first this appeared very difficult and complicated. However, the wide-range method to be described proved to give a most complete picture of the motor performance in a far shorter time than the partial methods previously used.

Charts of Temperature Rise

Figures 3*a* and *b* show, side by side, the two final charts from which the temperature rise is obtained for the armatures of the twin-armature traction motor shown in figure 2. Six of these twin-armature traction motors, that is, 12 armatures in all, equip the locomotives of the type shown in figure 1, of which more than 100 are in service on the electrified territory of the Pennsylvania Railroad.

Any chosen cycle of operation, including accelerations, overloads, high-speed running, coasting, or layover, can be accurately analyzed from the standpoint of the temperature rise of the armature. Similarly the charts on figure 4*a* and *b*

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[1] For all numbered references, see list at end of paper.

Figure 1. 2-C-C-2, 11,000-volt, 25-cycle, 4,620-horsepower, single-phase locomotive in passenger service

Figure 2 (right). 25-cycle, 770-horsepower, twin-armature, single-phase traction motor

will give the corresponding information in regard to the temperature rise of the field.

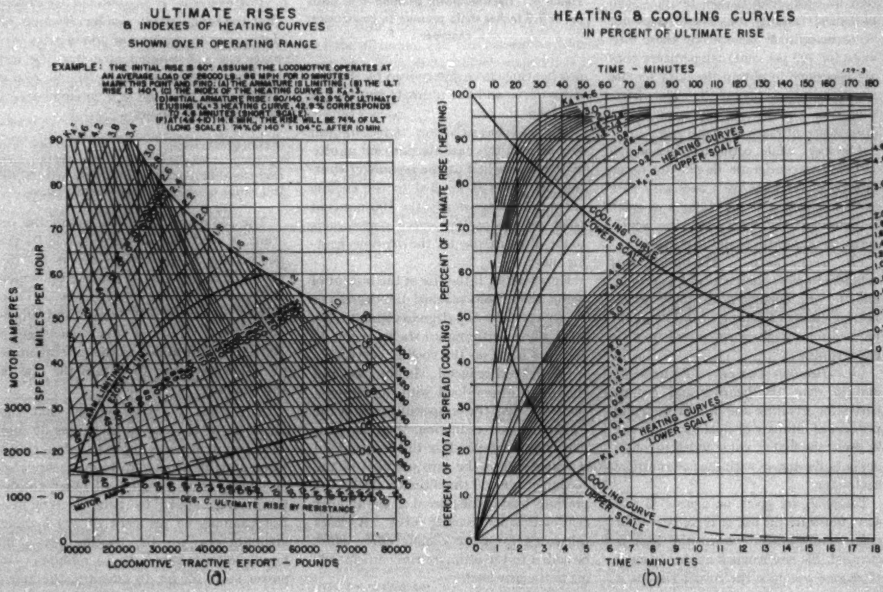
For low-speed operation, where the exciting field is shunted, the effect of speed on temperature rise is very small and, in this short range only, it proved practical to simplify the required charts by preparing curves 5*a* and *b* which are used as follows.

The abscissa of the initial rise (at the time the accelerating tractive effort is applied) gives the equivalent time which it would have taken to reach this temperature starting cold. Adding to this time the minutes of operation at the accelerating tractive effort and reading the corresponding ordinate gives the rise at the end of this period of acceleration.

The use of the charts on figures 3, 4, 5, and 6 will give in a few minutes the temperature data which would otherwise require hours of locomotive operation on test. This is specially suited for determining the tonnage which a locomotive can haul over any portion of the electrified profile.

(*1*). *Ultimate Rises.* Figures 3*a* and 4*a* show the final or ultimate rises which will obtain if any speed and tractive effort are maintained for a long pe-

Figure 3. Traction motor armature heating at 5.8 inches static pressure in commutator chamber



(a) (b)

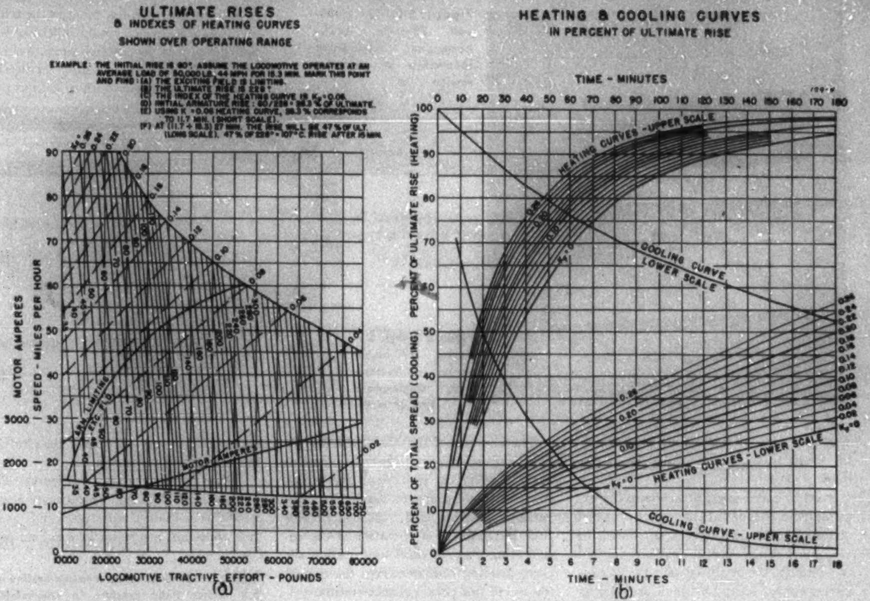


Figure 4. Traction-motor exciting-field heating at 5.8 inches static pressure in commutator chamber

riod of time, four to five hours. Such prolonged running, of course, does not occur in railroad service but these ultimate (or 100 per cent) temperature rises are necessary to obtain the data for the usual short-time operation. The short-time rises can then be expressed in per cent of the ultimate (or final) rise.

Likewise, while ultimate rises above 150 degrees centigrade should not occur in service, yet they are a necessary part of these charts and calculations in order to determine the short-time rises which occur on acceleration and overloads.

(2). *Index Lines.* The broken lines extending across the curves of figures 3*a* and 4*a* are numbered to show which heating curves of figures 3*b* and 4*b* are to be used for any given speed and tractive effort point.

(3). *Heating Curves.* Figures 3*b* and 4*b* show the heating and cooling curves for armature and field expressed in per cent of ultimate or final rise. As will be seen, the higher value index numbers apply to the curves with the faster rate of heating up.

Examples are given on figures 3*a* and 4*a* to illustrate the use of these curves for rising temperatures; when the load is changed, the rise reached at the moment of change becomes the initial rise on a new heating curve determined by the new index curve and the ultimate rise for the new load.

(4). *Cooling Curves.* (*A*). Power shut off: In this case the per cent of ultimate rise reached at the instant of shutting off power is applied to the cooling curve. In other words, the 100 per cent point on the cooling curve corresponds to the ultimate rise for the previous heating curve.

For example, if the rise at the instant of power shutoff has reached 135 degrees or 45 per cent of a 300-degrees ultimate rise, the cooling performance starts at the 45 per cent point on the cooling curve.

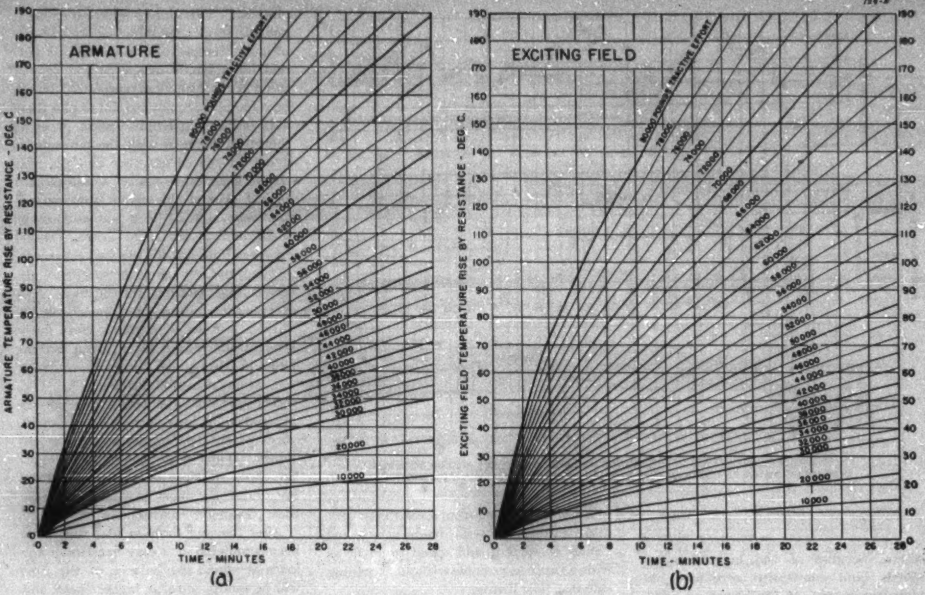
(*B*). Load reduced: In the case of a change to a new load with an ultimate rise lower than the temperature rise at the instant of load reduction, then the ultimate rise corresponding to the new load is considered as the zero per cent of the cooling curve and the ultimate rise of the previous load is considered as the 100 per cent of this new cooling curve. All rises calculated from the cooling curve must be added to the ultimate rise corresponding to the new load.

For example, it is assumed, as before, that the temperature rise has reached 135 degrees or 45 per cent of 300-degrees ultimate rise. Assuming now that the new load has a corresponding ultimate rise of 110 degrees, then the new cooling curve is determined as follows:

300-degrees rise is............100 per cent
110-degrees rise is............ 0 per cent
and the difference or 190-degrees rise is the full range or spread of the cooling curve.

The 135-degrees rise reached at the instant of load change corresponds to the percentage of 135 degrees minus 110 degrees (or 25 degrees) to 190 degrees which is 13 per cent on the cooling curve. Then this 13 per cent of 190 degrees and subsequent lower values of this percentage of 190 degrees are added to the new ultimate of 110-degrees rise to obtain the rise of the winding.

(5). *Time.* It is evident that the abscissae of the curves in figures 3*b* and 4*b* shown in minutes apply to actual times when starting from zero-temperature rise or cooling and that for all other conditions they apply to time increments. For example 45 per cent of 300-degrees ultimate rise on the cooling curve in figure 3*b* corresponds to 15 minutes. If power stays off for 10 minutes, the tem-



perature rise at the end of this period of cooling will correspond to that shown for 15 minutes plus 10 minutes or 25 minutes which reads 31 per cent. This, in turn is 93-degrees rise for the winding.

(6). *Effect of Increased or Reduced Ventilation.* The heating capacity of the traction motors is definitely affected by the amount of ventilating air. Figure 6 shows the relation between the temperature rise and the static pressure in the commutator chamber which in this case is 5.8 inches under normal rating conditions. The per cent shown in ordinates indicates the per cent increase or decrease of the ultimate temperature rise for any ventilating condition between four and seven inches of water static pressure corresponding to approximately 80 per cent to 110 per cent of normal air.

However, the heating curves are also affected by changes in ventilation in such a manner that the time to reach a given per cent of ultimate is shortened in about the same ratio as the ultimate rise is reduced (if the ventilation is increased). This has the effect of partly offsetting the gain obtained by reducing the ultimate rise in degrees. There is still an appreciable gain in temperature if the temperatures are from 80 per cent to 100 per cent of the ultimate. However, increased ventilation has very little effect on very short-time rises, say from 0 to 40 per cent of the ultimate, and this should be taken into account when higher ventilation is proposed to increase the locomotive short-time capacity.

Application to Temperature Study for a Complete Run

Figure 7 shows the performance of a 230-ton locomotive (of the type shown in figure 1), hauling a passenger train of 1,285 tons over a portion of the electrified territory.

Figure 5. Traction-motor temperature rise for slow-speed operation in weak field connection—5.8 inches static pressure

In addition to the usual profile, tractive effort, and speed charts, this figure includes a complete plot of armature and field temperature rises prepared by the use of the method which is the subject of this paper.

It will be noted that in this particular run the armature temperature rise is always higher than the field rise. This is characteristic of passenger train operation

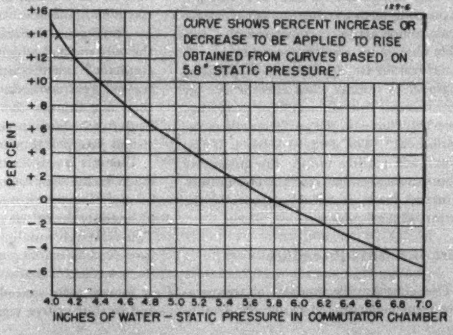
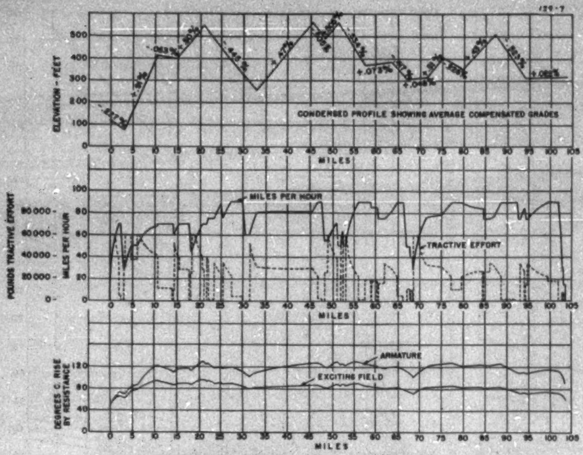


Figure 6. Effect of static pressure in commutator chamber of traction motors on rise by resistance



APPLICATION OF METHOD OF DETERMINING MOTOR TEMPERATURES IN CONNECTION WITH STUDY OF LOCOMOTIVE PERFORMANCE OVER PROFILE — 1515 TONS TOTAL WEIGHT LOCOMOTIVE & TRAIN

Figure 7

where, because of the high armature speeds (and consequent core loss) the temperature rise of the armature windings is higher than that of the field. The corresponding curves for a freight train run show field temperature rises in excess of those for the armature because of low armature speeds and relatively heavy currents.

This shifting of the seat of the limiting temperature rise from the armature to the field according to operating conditions is a further proof of the practical value of a method of analysis which leaves no gap in the coverage of temperature-rise data.

The actual process of applying the described method of temperature charting to a contemplated or actual train run quickly brings out the ease with which it is possible to determine the effect on the peak temperatures encountered of a possible change in operating conditions. It is natural for the application or operating engineer to wonder what effect a five per cent increase in train weight or in average speed will have on the performance of the equipment. The method subject of this paper will readily supply the answer to many such questions and thereby permit, in many cases, material increases in the output of the equipment.

Part II. Test Procedure

The considerable amount of temperature data incorporated in figures 3, 4, 5, and 6 was obtained from a relatively very small number of tests, made at selected values of current and speed to cover as wide a range as possible without exceeding safe test-floor limits.

(1). Ultimate Rises

The ultimate rises of the armature are shown on figure 8. Tests of four or five hours duration were held within the following limits:

(*a*). 1,900 rpm as a speed limitation.

(*b*). 130-degrees-centigrade rise from 1,900 rpm to 1,200 rpm.

(*c*). Below 1,200 rpm, the field heating limited the current to around 1,700 amperes.

Within this continuous capacity operating range which is shown by a dotted line, it was possible to actually determine the ultimate temperatures of the windings by measuring their resistance. This is the only method which gives accurate and consistent results since, by its use, a trained test crew can check within less than five degrees the results of any one load run.[3] Such accuracy permits consistent interpolation and extrapolation.

Ten to fifteen continuous runs were made within this range at the loads shown by circles on figure 8 maintained until the ultimate temperature rises were obtained. This relatively small number of temperature runs is made possible by confining the selection of loads, as shown, to three or four equally-spaced values of current and to four or five equally-spaced values of speed. This limited selection makes it possible to plot, as soon as obtained, the ultimate temperature rises against current for a family of speeds and against speed for a family of currents. It has been found especially helpful to plot these data on semilogarithmic curve paper as shown on figure 8 with the currents as uniformly-spaced abscissas and the rises on the logarithmic scale. When plotted against current the curves obtained are relatively straight-lined for a constant speed and a family of nearly parallel curves is obtained for the various speeds. In the same way using the speeds for abscissas and plotting the ultimate rises in ordinates on the logarithmic scale for various families of currents gives fairly straight-line curves for a given current and a nearly parallel family of curves for the various constant currents.

The fact that the test points line up along straight lines permits a very quick check of the test results as soon as obtained. Thus, if any results look doubtful, check runs can be made without delay under the most favorable and the most conclusive conditions.

After the first three or four runs have been completed, it is easy to estimate the loads at which the balance of the runs will be made, some at higher speed and lower current, others at lower speed and higher current.

In this way a rather complete picture of ultimate rises over the range of speed and current is very quickly obtained as shown in figure 8 and these parallel families of curves lend themselves to, and practically invite, very accurate interpolation. To a certain extent, these curves also make possible extrapolation in both directions, especially toward temperature rises in excess of the values which can be safely run on test. Beyond these limits of continuous load runs, further ultimate rise data are obtained by the use of short-time runs in conjunction with heating shape curves.

(2). Heating Curves

In connection with each ultimate rise or continuous load run, it is possible to obtain test points of the heating curve with very little additional testing effort by stopping the temperature load run 30 or 45 minutes after starting cold (i.e., zero initial rise). After the temperature rise of the motor winding has been determined at the moment of interruption of the load run, the run is resumed and carried to completion.

Dividing the rise obtained after 30 or 45 minutes by the ultimate rise gives the per cent of ultimate reached during the short-time run. This is plotted against

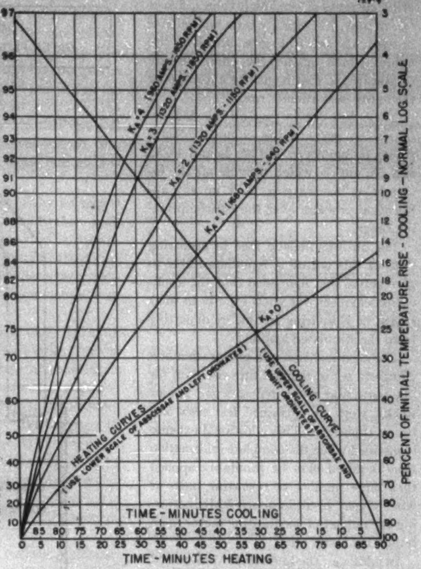


Figure 9 (right). Traction-motor armature heating-up and cooling curves at 5.8 inches static pressure in commutator chamber

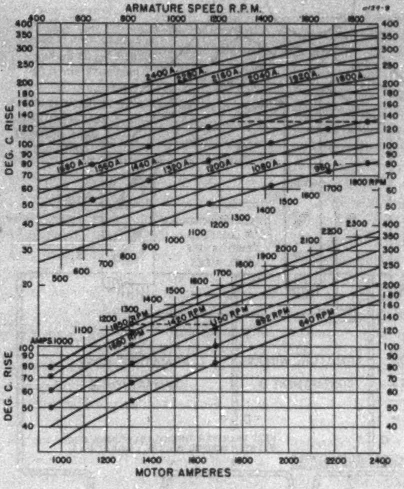


Figure 8. Traction-motor armature ultimate rises by resistance fo 5.8 inches of water static pressure in commutator chamber

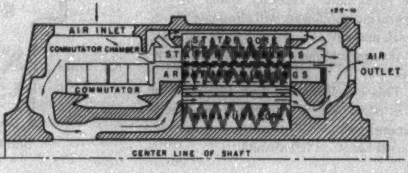


Figure 10. Motor ventilation and air passages

time on a semilogarithmic curve paper using an antilog scale as can be seen from figure 9. Several points are necessary to establish the general heating-up shape for a given load run. In addition to the 30-minute point obtained in conjunction with the continuous load run, other points at 10 or 15 minutes, 45 or 60 minutes are taken to establish the general shape.

Past experience and theoretical investigations have shown that these curves are of a logarithmic nature and, if one of the curves is accurately determined throughout its extent, other curves will be easily established from a small number of test points as follows and as shown on figure 9.

If the known curve (1,320 amperes 1,850 rpm) reaches 70 per cent of ultimate in 12.5 minutes and the test point on a new curve reaches 70 per cent of ultimate in 25 minutes, then we know that for any other per cents of ultimate, the time abscissa for the new curve will be 25/12.5 = 2 times the abscissa for the known curve for the same per cent of ultimate.

When enough of these shapes are drawn up and the time-abscissa ratios are established for various current-speed loads, then it is possible to find a definite relation between the time-abscissa ratio and the load conditions, which in turn permits the plotting of additional heating shapes at any other load. The curves shown for the armature on figure 3*b* and on figure 6 verify a time-abscissa ratio equal to 1+2.2 rpm/amperes or 1+47 miles per hour/amperes for this particular application. For convenience the index number K_A on the heating shapes was taken as 47 miles per hour/amperes and it will be noted that these index lines are numbered from 0 to 4.6.

The curves shown for the field on figure 4*b* verify a similar time-abscissa ratio this time expressed as 1+0.55 rpm/amperes or 1+11.6 miles per hour/amperes for this particular gear ratio and wheel diameter. For convenience the index number K_f was shown as one-fourth of 11.6 miles per hour/amperes so that the index number is 2.9 miles per hour/amperes giving a range of 0 to 0.26.

It is obvious that the ventilation of the motor as illustrated in figure 10 has a considerable effect on this spread of heating curves as affected by speed. If the ventilation of the field is considerably increased by design changes, this spread will be reduced because the field rise will be more nearly independent of the armature heating, in other words, less affected by speed.

(3). Complete Sets of Temperature-Rise Data

Thus as tests proceed, a complete set of data becomes rapidly accumulated, with a minimum amount of cutting and trying, since any test result, provided it is accurate, finds its place in the range which is being mapped out.

The ultimate rises plotted for values in excess of 140 degrees are obtained by a judicious use of the already determined heating curves in conjunction with short-time temperature runs at overloads which if maintained for a considerable period of time would produce temperature rises well above safe limits. Even at accelerating currents of very high values, five or ten-minute temperature tests and reliable heating curves will give ultimate rise values which extend the chart already established so that it may be applied over the entire performance range.

These test curves give all the data required to prepare the complete charts shown on figures 3*a* and 3*b*, 4*a* and 4*b*, 5*a* and 5*b*, and 6, which provide all the information necessary to work up any application problem over the entire operating range.

The authors believe that in preparing data of this nature it is indispensable to give actual test figures so that the application engineer will know exactly to what extent he is inviting trouble by departing from the stipulated loading.

The accumulation and preparation of a complete set of temperature-rise data for one particular motor requires two to four weeks' work by a trained test crew and by an engineer. This additional expense is justified when the complete information thus required may be profitably applied to a large number of operating units or when it may result in appreciably increasing the output of the limiting motor in an important installation.

In addition, the knowledge thus gained of performance capacity under diverse operating conditions will eliminate guesswork and will encourage definite planning in the application of motor-driven apparatus.

This concludes the first two parts of this paper which completely describe a general method of approach and investigation based on test results.

Part III. Empirical Analysis and Hydraulic Analogy

Empirical Equation of Ultimate Temperature Rise

It is now desirable to take advantage of the accumulated heating and cooling data to obtain an empirical formula expressing the temperature rise in terms of speed, current, and flux.

By extrapolating some of the data to zero current or to zero speed and by selecting other data at constant values of current and speed, it was found that the ultimate rise of the armature can be expressed for this particular motor as follows:

$$\text{Rise}=\frac{9\left(\frac{\text{rpm}}{1,000}\right)^{3/2}+18\left(\frac{\text{amperes}}{1,000}\right)^{3/2}+4.15\Phi^{2}\left(\frac{\ }{1,000}\right)^{1/2}\left(\frac{\text{rpm}}{1,000}\right)^{3/2}}{1-0.108\left(\frac{I}{1,000}\right)^{3/2}}.$$

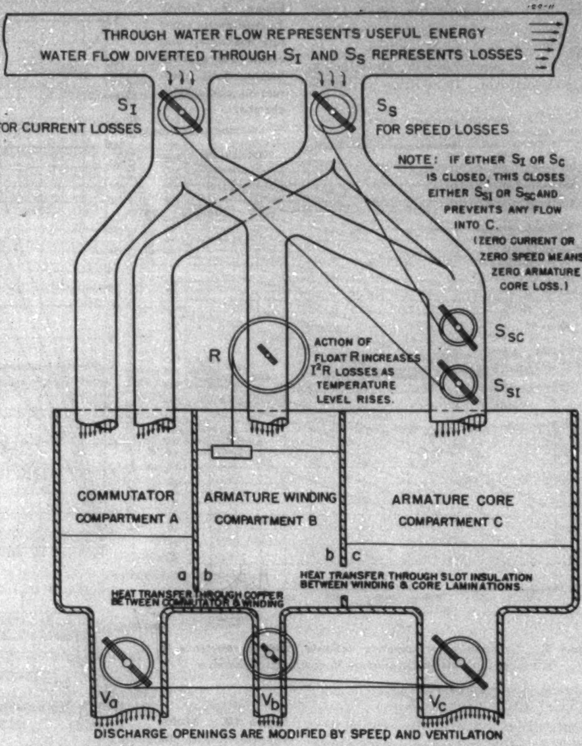

THROUGH WATER FLOW REPRESENTS USEFUL ENERGY
WATER FLOW DIVERTED THROUGH S_I AND S_S REPRESENTS LOSSES
S_I
FOR CURRENT LOSSES
S_S
FOR SPEED LOSSES
NOTE: IF EITHER S_I OR S_C IS CLOSED, THIS CLOSES EITHER S_{SI} OR S_{SC} AND PREVENTS ANY FLOW INTO C. (ZERO CURRENT OR ZERO SPEED MEANS ZERO ARMATURE CORE LOSS.)
R
ACTION OF FLOAT R INCREASES I^2R LOSSES AS TEMPERATURE LEVEL RISES.
S_{SC}
S_{SI}
COMMUTATOR COMPARTMENT A
ARMATURE WINDING COMPARTMENT B
ARMATURE CORE COMPARTMENT C
a
b
b
c
HEAT TRANSFER THROUGH COPPER BETWEEN COMMUTATOR & WINDING
HEAT TRANSFER THROUGH SLOT INSULATION BETWEEN WINDING & CORE LAMINATIONS
V_a
V_b
V_c
DISCHARGE OPENINGS ARE MODIFIED BY SPEED AND VENTILATION

Figure 11. Hydraulic analogy of armature temperature rise

This is an empirical relation which closely approximates the test results and may be considered as giving a fairly correct picture of the effect of speed, current, and flux.

The (3/2) power has been used because it expresses in one term a relation which normally would require several terms. The denominator represents the consequent effect on the temperature rise of the increase of the resistance of the copper with temperature.

Among many interesting applications this equation now makes it possible to analyze an important condition associated with heavy current overloads on acceleration. There is a critical value of current such that, above it, the I^2R loss is so predominant that the cumulative effect of increased I^2R losses with increased temperature brings the ultimate rise to infinity resulting in destruction of the winding at this load even in a very short time. This critical value obtains when the denominator is equal to zero which makes the ultimate rise infinity.

In the particular case which is being covered, this critical value of current is 4,400 amperes which can be compared to a normal continuous current of 1,560 amperes and a locomotive overload setting of 2,750 amperes. This critical value of current gives a rather definite idea of the safe range of overloads and is largely determined by the amount of copper in the winding.

Hydraulic Analogy

Many complicated physical relationships have been clarified by the use of a hydraulic analogy[3] which has proved particularly enlightening in explaining the temperature rise of electrical machinery and which has suggested the arrangement shown in figure 11.

A large tank with three compartments represents the entire armature. The water which is poured into these compartments represents the heat losses developed in the three main parts of the armature: commutator, winding, and core shown in figure 10.

These three compartments communicate with one another to indicate the possibility of heat flowing from one part to the other. Heat dissipation is represented by the flow of water through three discharge openings, one for each compartment and these openings are controlled in accordance with the rate of dissipation (armature speed and amount of ventilating air).

Compartment *A* represents the commutator. The commutator receives heat from brush friction and brush contact loss, both sources of heat being represented by two pipes supplying respectively losses which are a function of speed and losses which are a function of current. Some of this heat can be exchanged with the adjacent compartment through the path *ab* in either direction. Most of the commutator heat is taken away directly by the ventilating air and the surrounding air in which it rotates (see figure 10), and is shown as coming out of V_a.

Compartment *B* represents the armature winding, seat of the I^2R and eddy current loss. Since the I^2R loss is the predominant factor and since resistance increases with temperature rise, a float and valve *R* are shown which increase the amount of heat loss as the temperature rise goes up.

This compartment communicates with the two adjacent compartments *A* and *C* through the openings *ab* and *bc*. This opening *bc* serves to transfer a large amount of the armature copper heat through the slot insulation into the armature core shown in compartment *C*. The rest of the heat generated in the copper is taken away through surface ventilation on the end windings (see figure 10), as shown through V_b which opening, like V_a, is affected to a large extent by speed.

Compartment *C* represents the armature core. All of the core and part of the load loss are functions of speed and flux. These heat losses in addition to that which flows from the armature copper are discharged through the opening V_c which represents the ventilation through the longitudinal ducts in the core (see figure 10).

The temperature rise in each of these three parts of the armature can be compared to the rise in the level of the respective compartments where water is poured at a rate corresponding to the losses and where there is an opening at the bottom which is determined by the rate of dissipation. For the sake of simplicity, it is assumed that the flow through the orifice is proportional to the head and not to the square root of the head as is actually the case. A stable level or ultimate rise will be obtained when the level is such that water flows out at the same rate as it is poured in.

The rate of dissipation is by no means constant, but varies considerably with the operating conditions; in other words, the size of the openings should be regarded as determined by each operating condition, particularly if the amount of ventilating air is varied.

Let it be assumed now that the motor is operating normally with losses from all three sources. All three compartments are being filled at the same time and after two or three or four hours, stable levels will be reached in all containers. Of special interest, of course, is the rise of the armature winding which is measured by the level in the compartment *B* and which is the final limitation in the application of the motor and the loading of the locomotive.

The critical value of current previously referred to can be explained in the analogy by the additional cumulative action of the float mechanism *R* in increasing the flow of water in compartment *B* as the level rises. It will be easily realized that if the level rises high enough in *B* tank and if the water pours in fast enough (as in the case of extreme overload), no stable level is possible and the action of the float will cause a cumulative rise which will lead to faster and faster rises and finally to destruction.

An attempt will now be made to arrive at an equally general interpretation of the heating curves. Assumed first that S_S is closed (zero speed) and that S_I is wide open at the maximum opening (corresponding to the highest possible overload current on the locomotive motors), as the level rises rapidly, the float intervenes to open the valve still more and this somewhat delays the time at which stable temperature levels are obtained, since, so to speak, every time the temperature level approaches a stable point the float action tends to postpone the ultimate stabilization. However, since the value of current used is very far from the critical value of current, the level finally settles at what corresponds to the ultimate rise.

Under these "delayed" conditions (due to the increase in resistance as the temperature rises) the ultimate level or rise will be higher than the constant resistance and it will take a longer time to reach stability. It will also take a longer time to reach any given fraction or per cent of the ultimate rise than with constant resistance. For this reason, it can be said that the corresponding heating curve, expressed in per cent of the ultimate, is slower than would obtain with constant resistance and becomes the slowest of all heating curves within the operating range. This is the curve which is shown in the chart 3*b* by the index $K_A = 0$, and corresponds to maximum current and zero speed.

Assume that the motor remaining stationary, the valve S_S remaining closed, the valve S_I is partially closed to permit only a reduced flow. The level rises less rapidly at first but toward a lower ultimate which is reached more quickly (relatively speaking) because the float control modifies the spigot opening to a smaller extent. In this way there are obtained somewhat faster heating shapes shown in the figure 3*b* by indexes $K_A = 0.2$ or $K_A = 0.4$.

Let it be assumed now that, at a still lower value of current, the motor is brought up to speed. This amounts to closing valve S_I still further and to opening valve S_S. This also has the effect of opening V_a, V_b, and V_c wider because of the more favorable conditions for the dissipation of heat.

The initial rate of rise of the level in compartment *B* is faster because there are additional amounts of water pouring into *A* and *C* which have the effect of partly restricting the flow from *B* out of *ba* and *bc*.

This means that the rate now obtained corresponds to one of the faster heating-up curves shown in figure 3*b* by indexes $K_A = 0.6$ up to 4.6, the higher indexes and faster curves corresponding to the higher speeds, where a greater total amount of losses is dissipated more rapidly by a more complete utilization of the heat-dissipating ability of the motor.

COOLING CURVES

The cooling curves offer a quite different problem. In the heating-up process the rate of rise was determined by the opposition of two conflicting phenomena: (1) generation of losses, and (2) dissipation of losses, or, in other words, water pouring in and water coming out.

In considering the cooling conditions, there is only one of these phenomena, the dissipation of the losses, or, in the analogy used, the rate at which the tank will be emptied if S_I and S_S are simultaneously shut off.

Obviously this cooling or emptying

process is determined by only one set of conditions, the level in the three tanks or the temperature rises in the various parts of the motor and, if the difference between the levels in the three tanks is not excessive there shall obtain to all intents and purposes only one average cooling curve, which has been confirmed by tests. In fact, if there were a very pronounced difference in the three heads, it would tend to level itself out fairly rapidly giving a different rate of cooling for the first few minutes of cooling in the highest tank until levels have equalized.

Exciting Field Data

Figure 4*a* and *b* shows the set of charts prepared for the exciting field which give the same complete information as was given for the armature and which completes the coverage of temperature performance data. A hydraulic analogy similar to that for the armature could be used for the field (but with differently proportioned compartments and openings).

Summary and Conclusions

In the first part of this paper the authors have presented the complete set of charts necessary to work out for one particular motor the temperature rise for any service cycle. In the second part, they have shown in detail how to prepare these charts from a minimum amount of test data.

In the third part of this paper they have shown an empirical analysis of the relationship between temperature rise and the motor current and speed. Further, they have explained by a hydraulic analogy some of the combined phenomena on which this study is based.

Having completed these charts and analysis and having used them with satisfactory results for several years, the authors fully believe that similar charts can profitably be worked out for other types of motors and other complicated service applications and they hope that this paper will facilitate further practical interpretation of the already developed theoretical investigations on the heating and cooling of electrical machinery.

References

1. Thermal Capacity of Electrical Machines, A. E. Kennelly. AIEE Transactions, volume 45, 1926, pages 1290–7.

2. Temperature Testing of Motors, E. I. Staples. *Instruments*, September 1938, pages 219–21.

3. Duty Cycles and Motor Rating, L. E. Hildebrand. AIEE Transactions, volume 58, 1939 (September section), pages 478–83.

Inductive Co-ordination of REA Distribution Systems and Telephone Systems

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Synopsis: The Rural Electrification Administration and Bell System have been carrying on co-operative studies of the inductive co-ordination of REA distribution systems and rural telephone circuits since 1939. The results of these studies have indicated that the two systems can be satisfactorily co-ordinated when (*a*) the influence of the power system is controlled, (*b*) the separation in unavoidable exposures is adequate and reasonably uniform, and (*c*) the susceptiveness of the exposed telephone circuits is limited by care in design and maintenance. The more important factors controlling the influence of REA distribution systems and the susceptiveness of rural telephone circuits are reviewed and certain remedial measures applicable to the power system are discussed, with illustrations from the recent joint field work. Recent trends in the design of rural telephone circuits which result in improved susceptiveness are mentioned.

Introduction

THE inductive co-ordination of power and telephone distribution systems has been co-operatively studied for many years both by operating company engineers and by such groups as the joint subcommittee on development and research of the Edison Electric Institute and Bell Telephone System. The results of these studies have been published in the engi-

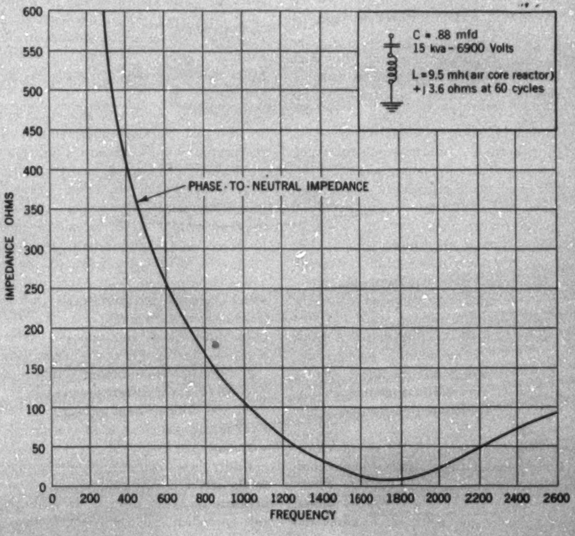

Figure 1. Impedance characteristics of one phase of 1,740-cycle, three-phase resonant shunt

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The authors acknowledge with thanks the co-operation of their associates in the REA and Bell System in the preparation of this paper. The field tests on which the results depend were carried out with the assistance of several of the REA projects and operating telephone companies.

1. For all numbered references, see list at end of paper.

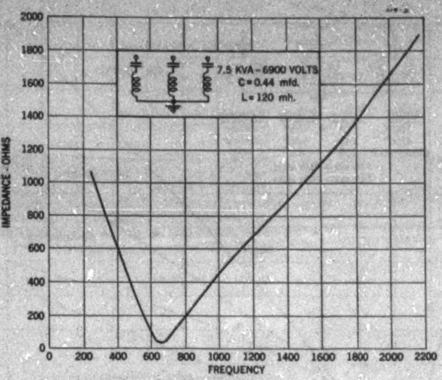
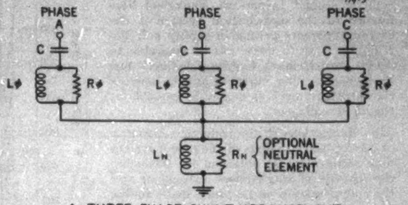
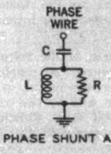


Figure 2. Phase-to-neutral† impedance of 660-cycle resonant shunts

†Impedance to residuals is one-third of phase-to-neutral impedance



A- THREE PHASE SHUNT ARRANGEMENT



B- SINGLE PHASE SHUNT ARRANGEMENT

Figure 3. Schematic diagrams of shunts consisting of capacitance, inductance and resistance for use on distribution circuits

neering reports of the joint subcommittee and in a number of papers before the Institute. These studies were first largely concerned with co-ordination of urban and suburban distribution systems,[1] and only within the past few years has detailed consideration been given to the problems encountered in connection with distribution systems in rural areas. These latter problems were considered in a paper presented before the Institute in 1938,[2] which discussed the theoretical and practical aspects of the co-ordination of rural power and telephone systems and described certain remedial measures which were available at that time. That paper also explained many of the special terms employed in discussions of inductive co-ordination. The present paper makes no attempt to cover the fundamental factors but merely discusses their application to problems encountered recently in connection with the co-ordination of REA type distribution systems and rural telephone circuits, and describes the application of certain remedial measures which have become available since 1938.

Soon after the formation of the REA in 1935, representatives of that organization and of the Bell System discussed methods for handling the co-ordination problems associated with rural power line construction and it was agreed that these problems should be studied co-operatively and co-ordinative measures applied where necessary on the basis of the best over-all engineering solution. Accordingly, in July 1939 a group of engineers from these organizations met and agreed on a program to develop general methods to reduce as far as practicable the number of cases where detailed studies in specific exposures would be necessary. This was deemed important since the REA estimates that its distribution systems will ultimately total about a million miles of line throughout the United States. Arrangements were made to carry on co-operative field investigations in situations which offered an opportunity to obtain development information which would be useful in this connection as well as to assist in the solution of particular problems.

These studies have now been under way for more than a year and the results to date indicate that multigrounded rural distribution systems of the REA type and metallic telephone circuits can be co-ordinated satisfactorily, where the problem is handled co-operatively by the organizations involved. In order to accomplish this, consideration of the following factors is important:

(*a*). Unnecessary exposures between power and telephone systems should be avoided. Where it is necessary to create an exposure, the separation between the two systems should be maintained as nearly uniform as possible in order that transpositions in the telephone circuits will have maximum effectiveness.

(*b*). Perhaps the most important factor in limiting the inductive influence of the distribution system is the restriction of the mag-

Table I. Effectiveness of Experimental 780-Cycle Resonant Shunt on Power-System Influence and Noise on Exposed Telephone Circuit

	Generator Open Circuit (No Devices)	Generator on Bus (No Devices)	Generator on Bus 780-Cycle Shunt on
Measurements on 2,300-volt bus			
Voltage TIF* phase-to-phase	650 (650)	160 (160)	30 (7)
Measurements on 7,200-volt bus			
Voltage TIF* phase *A-G*		42 (39)	7.2 (1)
Voltage TIF* phase *B-G*		31 (25)	15 (2)
Voltage TIF* phase *C-G*		30 (30)	7 (2)
Residual kv·*T**		252 (183)	180 (23)
I·*T** phase *A*		1,800 (1,750)	320 (74)
I·*T** phase *B*		1,800 (1,800)	510 (135)
I·*T** phase *C*		1,800 (1,800)	360 (110)
I·*T** residual		400 (290)	280 (33)
Noise on toll circuit			
Metallic (noise units)		335 (335)	90
Noise to ground (noise units)		605 (480)	180

*Telephone influence factor (TIF) is a measure of the harmonics present in a voltage or current wave, the individual frequencies being weighted in proportion to their inductive influence on exposed telephone circuits. The influence of a voltage is usually expressed as the product of rms kilovolts and TIF (abbreviated kv·*T*). Similarly the influence of a current is expressed as the product of rms amperes and TIF (abbreviated *I*·*T*). These terms are more fully discussed in reference 3.

Note: Numbers in parentheses represent 780-cycle contribution to total TIF or *I*·*T*.

nitudes of the harmonic components present in the voltage at the source of supply to the distribution system. This may involve limitation of harmonics originating in generators or in the power system from which the distribution system is supplied. The importance of harmonics present in the source of supply is often emphasized by the effects of resonance when the line is connected, as discussed later.

(*c*). A second important source of harmonics in the distribution system is the exciting currents arising in supply and distribution transformers. It is desirable to select transformers having relatively low harmonic exciting currents and to operate these transformers as nearly as possible at the rated voltages.

(*d*). Reductions in the inductive influence of three-phase portions of the power system may sometimes be obtained by balancing the lengths of single-phase extensions and the magnitudes of single-phase loads among the various phases. However, where triple harmonic transformer exciting currents are important contributors to the influence, little reduction will be effected by improving the balance.

(*e*). The susceptiveness of the telephone system may be minimized by employing transposed metallic circuits in the case of open-wire construction, and by the control of series unbalance and shunt unbalances (to ground) in both open-wire and cable circuits.

(*f*). So far as practicable in new construction and rehabilitation work or in converting from magneto to common battery operation, it will be desirable to take advantage of the improved susceptiveness of the newer types of subscriber station equipment.

(*g*). Unbalances in telephone central office circuits to which the rural telephone lines may be connected have been found to be of importance in several situations. Improvements in balance may be expected to result in lower noise levels.

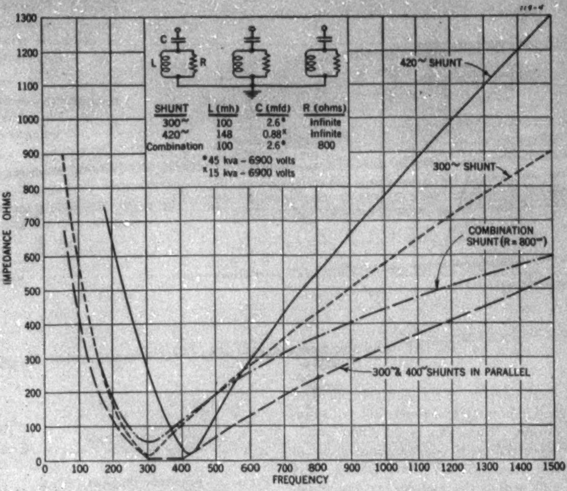


Figure 4. Phase-to-neutral impedances of various shunts for reducing 300- and 420-cycle components

In the usual case attention to the factors mentioned above in planning new construction will result in conditions which will make unnecessary the application of co-ordinative measures on a specific exposure basis. Some of the measures which have been used to bring about the general limitation of influence and susceptiveness on existing systems are:

(*a*). Improvement of the voltage wave shape of the particular generator or motor by means of resonant shunts or other devices at the machine terminals.

(*b*). Use of shunt capacitors, with or without associated reactors and resistors, on the distribution system side of the supply bank to improve wave shape or to minimize conditions of resonance.

(*c*). Use of wave trap in neutral of substation transformer bank to increase the impedance to residual currents at a particular harmonic frequency.

(*d*). Use of auxiliary device (such as autotransformer or regulator) to limit voltage on distribution system to values close to rated values for transformers employed.

(*e*). Rearrangement of distribution system to limit lengths of circuit supplied through a section of line involved in exposures with telephone plant. This may reduce the harmonic currents in the exposure.

(*f*). Improvement in balance of telephone circuits in central offices.

In other cases it has been found necessary to apply measures to telephone circuits in specific exposures, such as:

(*a*). Improvement in transposition arrangements of open-wire circuits.

(*b*). Improvement in series or shunt un-

Table II. Division of Phase and Residual I·T Product Among Three Feeders

	Substation Transformer	A Feeder	B Feeder	C Feeder (1φ)
		Power system normal		
I·T phase *A*	1,800 (1,750)	1,200 (1,135)	1,000 (1,000)	1,200 (1,200)
I·T phase *B*	1,800 (1,800)	230 (229)	1,800 (1,800)	
I·T phase *C*	1,800 (1,800)	1,600 (1,580)	2,000 (2,000)	
Residual *I·T*	400 (290)	2,640 (2,640)	1,100 (1,100)	1,200 (1,200)
		780-cycle shunt on generator bus		
I·T phase *A*	320 (74)	275 (49)	100 (33)	100 (49)
I·T phase *B*	510 (135)	51 (38)	510 (114)	
I·T phase *C*	360 (110)	190 (74)	280 (114)	
Residual *I.T*	280 (33)	360 (118)	480 (49)	100 (49)

NOTE: Numbers in parentheses represent 780-cycle contribution to total TIF or *I·T*.

Table III. Secondary Harmonic Exciting Currents of Rural Distribution Transformers at Rated Voltage

Transformer Kva	A 3	B 3	C 3	D 5	E 5
		Frequency—secondary harmonic exciting currents in mils			
rss	1,180	2,200	1,300	1,380	930
60	1,075	1,910	1,185	1,290	670
180	402	1,170	558	480	444
300	100	530	208	84	185
420	11	228	78	17	86
540	1	100	33	7	38
		Total measured secondary exciting I·T product			
I·T	24	380	125	55	140

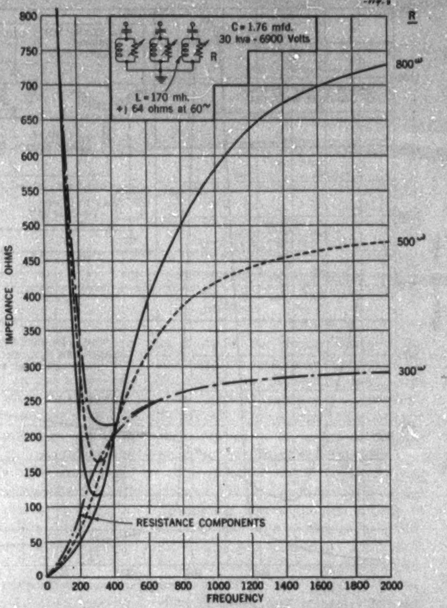
balance conditions of open-wire circuits, and associated equipment.

(*c*). Application of longitudinal impedance or drainage or a combination of these measures to particular circuits to minimize effects of series or shunt unbalances.

(*d*). Replacement of older types of subscriber equipment employing grounded lower impedance ringers for party line service by equipment having lower susceptiveness. This involves the use of higher impedance ringers at magneto stations. Several arrangements of varying degrees of effectiveness are available for common battery stations, including the use of auxiliary cold cathode tubes or relays in the ringer circuit, connection of an additional condenser in series with the ringer, or the use of higher impedance ringers with or without the double condenser.

Specific situations where certain of these measures were tested are described later in this paper. Due to the limitations of space, the results of tests of certain of the co-ordination measures in the telephone plant mentioned above, which were included in the various co-operative investigations, have not been presented in this paper, but are adequately described in the publications listed in the bibliography.

Figure 5. Phase-to-neutral impedance characteristics of various shunting arrangements having resistance component above 300 cycles



While most of the information in this paper is based on the co-operative studies by the REA and Bell System, the REA engineers have also made a number of studies co-operatively with other telephone organizations. Some of these studies have involved telephone circuits utilizing one wire with ground return. This type of circuit will have about 100 times as much noise as a metallic circuit for comparable exposure and influence conditions.[2] The results have shown that it is usually not practicable to limit the inductive influence of rural power systems to values which will permit the operation of ground return telephone circuits under noise conditions comparable with those existing before the power system was energized. However, in some cases it has been possible to reduce the inductive influence so that circuits having the shorter exposures could be retained on a ground return basis while the longer exposed circuits were converted to metallic operation.

The REA has also been carrying on work with the electrical manufacturers looking toward the limitation of harmonics arising in generators, transformers, and other types of power equipment. This work has already brought about worth-while improvements in generator wave shape.

Table IV. Effect of Impressed Voltage on Harmonic Exciting Currents of Rural Distribution Transformers

Transformer Kva	D 5		E 5	
Frequency	Rated Voltage	10 Per Cent Over Rated Voltage	Rated Voltage	10 Per Cent Over Rated Voltage
	Secondary harmonic currents in mils			
rss	1,380	1,730	930	1,550
60	1,290	1,575	670	1,150
180	480	650	444	855
300	84	107	185	403
420	17	20	86	205
540	7	12	38	90
	Total measured secondary exciting I·T product			
I·T	55	75	140	300

Table V. Residual Currents in Supply Bank of 400-Mile System

Impressed Voltage Frequency	7,150 Volts Residual Current		7,520 Volts Residual Current	
	Mils	I·T	Mils	I·T
180	5,880	88	8,130	122
300	1,400	287	1,440	295
420	205	121	320	189
540	54	68	100	125
Total (180–540 cycles)		330		390

Description of REA Distribution Systems

The three-phase four-wire multigrounded neutral distribution system operating at a phase-to-ground voltage ranging from 6,900 to 7,600 volts is generally employed in REA distribution systems. Extensions consisting of one- or two-phase conductors and multigrounded

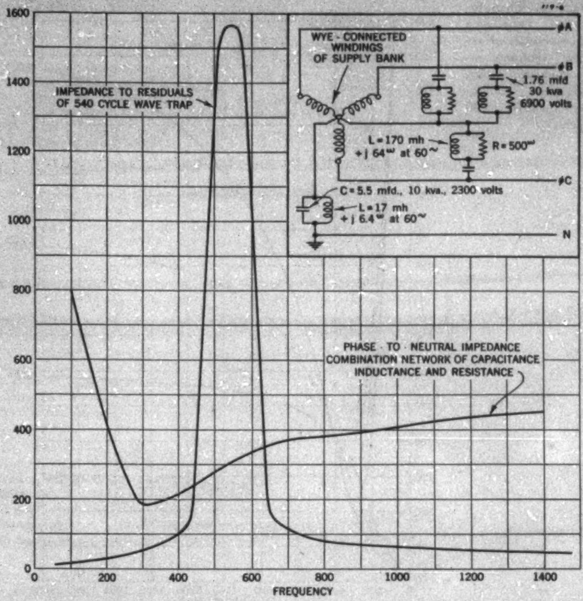


Figure 6. Characteristics and constants of combination network and 540-cycle wave trap

neutral, such extensions being metallically connected to the three-phase circuits, make up a substantial portion of each system. The systems are usually supplied from delta-Y transformer banks ranging from 75 to 2,000 kva (300 kva being the average size). The delta-connected windings on the supply side of the bank range in voltage from 2,300 to 110,000 volts. Power is usually purchased from existing systems, but is sometimes generated in Diesel or steam plants constructed and operated by the REA projects.

The largest proportion of the load on the REA systems is at present supplied single phase from transformers connected between phase wire and neutral and ranging in size from $1^1/_2$ to 5 kva. It is anticipated that the proportion of three-phase load will increase with the increasing use of power in rural areas. The size of the projects varies from small single-phase systems to extensive systems totaling over 1,000 miles and made up of substantial lengths of three-phase feeder with extensions. The average size project consists of approximately 350 miles of line of which about 90 miles is three-phase.

The connected kilovolt-amperes of distribution transformers varies greatly, depending on the type of area to be served. The average is around 5 kva per mile with peak densities up to 20 kva. It is anticipated that the connected kilovolt-amperes of transformers will increase with increasing load.

Description of Rural Telephone Plant

Until quite recently, most of the rural telephone circuits were operated on a magneto basis, but there is a definite trend in the Bell System toward the use of circuits employing common battery signaling, with local or common transmitter battery supply. These circuits are often connected to community dial offices (machine switching equipments of limited capacity). The conversion from magneto to dial operation is generally accompanied by improvements in outside plant conditions, replacement of open wire by cable, and frequently by the substitution of more efficient transmitters and receivers in an antisidetone circuit. These improved sets include higher impedance ringers for party line service. The common battery offices employ d-c signaling circuits which are grounded, in contrast with the magneto circuit, which is isolated from ground at the central office. This is the chief difference in the two types of office circuits from the inductive co-ordination standpoint, the central office ground often tending to increase the importance of series unbalances while at the same time reducing the effect of shunt unbalances.[2] Recent development work has indicated the practicability of improvements in the balance of the dial central office circuits, which are expected to minimize the necessity for special installations of longitudinal chokes or drainage circuits.

Table VI. Effect of Shunt Capacitors[ϕ] on Wave Shape of Power System

(3–150-Kva Transformers 2,300 to 12,500 Volts Supplied From a 500-Kva 2,300-Volt Generator)

	Average Phase to Neutral* (Volts)		Residual (Volts)		Phase (Mils)		Residual (Mils)	
Frequency	Normal	30-Kva Capacitor Per Phase	Normal	30-Kva Capacitor Per Phase	Normal	30-Kva Capacitor Per Phase	Normal	30-Kva Capacitor Per Phase
60	7,500	7,620			1,470	1,340	930	840
660	66	14	96	5	230	60	480	12
780	81	14	78	6	380	66	350	27
1,020	5	1	4.5	1	30	7	10	7
Voltage TIF	48	10						
Kv·T product			350	49				
I·T product					1,710	330	1,900	235

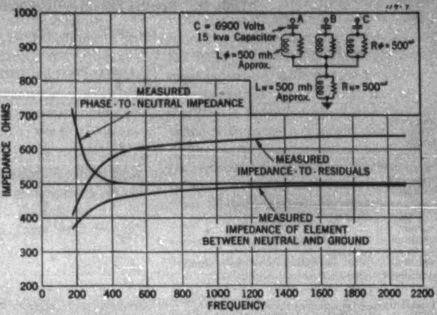
*Currents and voltages measured on distribution system side of supply bank.

ϕ Capacitors connected on distribution system side of supply bank.

Generator Wave Shape

Control of the wave shape of generators purchased by the REA projects for installation in their own plants has been accomplished by including limits on the telephone influence factor (TIF)[3] of the generator voltage in the machine specifications. The open circuit TIF is a measure of the harmonics present in the voltage wave of a machine, the individual frequencies being weighted in proportion to their inductive influence on exposed telephone circuits. Most of these generators are rated below 500 kva and little attempt had been made in designing earlier machines of these sizes to limit the generator harmonics to values which would be considered satisfactory where the machine is to be connected to an extensive rural system of the REA type. As a result of co-operation between the REA and the manufacturers, the wave shape of these machines has been improved, without appreciable increase in cost to the REA projects on impairments in operating characteristics. Out of 32 generators installed and tested up to September 1, 1940, 26 machines had values of open circuit TIF of 25 or less, while the TIF of 15 of these was less than 10. These values compare favorably with those measured on the newer machines of larger size, which for some time have been designed to have relatively low values of TIF. No serious co-ordination problems attributed to harmonics in the impressed voltage have arisen in any situation where generators meeting the TIF limits have been installed.

Figure 7. Arrangement and impedances of experimental three-phase far-end termination



In a particular situation where no TIF limits had been imposed on the generator wave shape, the characteristics of the machine were such as to increase the inductive influence of the REA distribution system and to create a noise problem. In this case, a three-phase series resonant shunt applied to the generator bus was found to be a solution of the problem.

The generator was a 2,300-volt machine of 187-kva capacity supplying the distribution system through a bank of three 50-kva 2,300-volt delta to 7,200/12,450-volt wye-connected transformers. Measurements of the open-circuit phase-to-phase voltage wave shape of the machine indicated that approximately 400 volts of 13th harmonic (780-cycle) voltage was generated in the machine and the open-circuit TIF of the machine was 650. Each phase of the resonant shunt consisted of an air-core inductance of approximately 16 millihenries tuned to resonance at 780 cycles with a 5-kva 2,300-volt capacitor. The effective resistance of the shunts at the tuned frequency was approximately two ohms. Table I shows the effectiveness of the shunts in reducing the influence of the distribution system and the noise on a particular exposed telephone circuit.

The voltage TIF of the generator bus before the resonant shunts were con-

Table VII. Effectiveness of 1,740-Cycle Resonant Shunt on Power-System Influence and Telephone Circuit Noise

(3–400-Kva Transformers 33,000 to 11,000 Volts)

Range of Voltage TIF	Normal		1,740-Cycle Resonant Shunt*	
Phase *A-G*	18–32	(13–22)ϕ	9–10	(1)
Phase *B-G*	26–36	(19–28)	8–12	(2)
Phase *C-G*	16–23	(10–17)	8–10	(8)
Residual kv · *T*	114–147	(66–110)	70–75	(6)
Range of I · T products (supply transformer)				
Phase *A*	5,700–7,100	(3,350–4,700)	900–1,300	(290)
Phase *B*	6,400–8,000	(4,000–6,000)	1,050–1,600	(455)
Phase *C*	3,200–4,500	(2,700–4,000)	800–1,050	(300)
Residual	1,350–1,400	(540–700)	1,000–1,100	(87)
Ground return *I · T* of one three-phase feeder	530–720	(490)	135–153	(18)
Noise metallic on toll circuit (noise units)	450	(350)	200	(90)

*Constants shown in figure 1.

ϕFigures in parentheses are contributions of 1,740-cycle components to over-all influence or noise.

Table VIII. Effectiveness of 660-Cycle Resonant Shunt on Power-System Influence and Noise

(3–100-Kva Transformers 33,000 to 12,500 Volts)

	Phase to Neutral (Volts)ϕ		Residual (Volts)		Phase Current (Mils)ϕ		Residual Current (Mils)		Noise to Ground (Noise Units)	
Frequency	Normal	660-Cycle Shunts*	Normal	660-Cycle Shunts*	Normal	660-Cycle Shunts*	Normal	660-Cycle Shunts*	Normal	660-Cycle Shunts*
42	150	264	20	108	410	550	86	356	33	30
540	35	19	38	72		41	135	29	98	7
660	210	13	144	7	1,020	56	415	106	280	32
Voltage TIF or residual kv · *T*	68	32	400	170						
Phase or residual *I · T*					2,400	710	1,100	675		
Meas. over-all noise (noise units)									400	210

*Constants shown in figure 2.

ϕCurrents and voltages measured on distribution system side of supply bank.

Table IX. Relative Effectiveness of Various Devices on Power-System Influence

(3–200-Kva Transformers 23,000 to 12,500 Volts)

Condition*	Average Phase-to-Ground Volts					Average Phase Current (Mils)				
	A	B	C	D	E	A	B	C	D	E
Frequency										
60	6,750	6,960	7,320	7,130	7,040		8,100	7,100	7,600	7,200
300	350	650	86	50	262		7,500	270	290	2,190
420	100	89	23	71	91		1,430	280	1,370	1,550
Voltage TIF	14	27	6	11	12					
Phase $I \cdot T$							2,400	475	1,180	960
Residual $I \cdot T$							950	380	480	415
Noise on subscriber line (noise units)		1,120	80	112	100					

Legend:
Condition *A*—distribution line disconnected
Condition *B*—normal
Condition *C*—300- and 420-cycle resonant shunts
Condition *D*—300-cycle shunt alone
Condition *E*—combination shunt

*Constants and impedances shown in figure 4.

Table X. Effectiveness of Combination of Capacitance, Inductance, and Resistance in Reducing Power-System Influence

(3–75-Kva Transformers 6,900 to 12,500 Volts)

Condition*	Average Phase-to-Ground Volts						Average Phase Current (Mils)					
	A	B	C	D	E	F	A	B	C	D	E	F
Frequency												
60	6,850	6,850	7,150		7,000	7,100		3,610			3,360	
300	180	680	252		144	47		3,430	955		700	744
420	42	54	30		24	33		157	47		84	202
Voltage TIF	10	22	10	9	6	4						
Phase $I \cdot T$								800	280	245	220	245

A—REA line disconnected
B—REA normal
C—resistance combination on ($R = 300$ ohms)
D—resistance combination on ($R = 500$ ohms)
E—resistance combination on ($R = 800$ ohms)
F—300-cycle resonant shunt on ($R = $ infinity)

*Constants of devices shown in figure 5.

nected was substantially less than the open circuit TIF of the generator, indicating that the impedance of the system as viewed from the generator terminals is low compared with the internal impedance of the machine at 780 cycles. The impedance of the shunts was low compared with either the impedance of the generator or the system, and the 780-cycle component no longer controlled the bus voltage TIF with the shunts in service.

The effect of the shunts on the influence of the phase-to-ground and residual voltages of the 7,200/12,450-volt system and of the currents on the high-voltage side of the 150-kva bank is also shown in table I. The percentage reduction in the phase-to-ground voltage TIF and in the phase $I \cdot T$ products* due to the shunts is about the same as the reduction in TIF on the 2,300-volt generator bus. However, the reduction in the residual kv·T product* and $I \cdot T$ product was less, since frequencies other than 780 cycles, largely from transformer exciting currents, were important contributors to the residual influence.

It is of interest to compare the $I \cdot T$ products of the supply transformer with the corresponding values measured on the three feeders supplied from this bank. As indicated in table II, the arrangement of these feeders was such that there was a substantial unbalance between the currents in the various phases and, due to resonance effects, the harmonic residual currents on the feeders actually exceeded the corresponding current in the substation transformer. It is evident that the application of the resonant shunt substantially reduced the influence of each of the feeders, and in this case, it was not found necessary to consider improvement in the balance of the system.

Motors and Rectifiers

Resonant shunts of design similar to that used in the situation just mentioned present a comparatively simple method of improving the wave-shape characteristics of small motors as well as generators. Special care is necessary in the design of shunts for induction motors which may produce harmonics of varying frequency in order to obtain maximum effectiveness.

Rectifiers have not so far been an important source of harmonic distortion on REA distribution systems, but it is anticipated that this problem will arise as this type of system load increases. It is probable that cases of this sort can be handled by the application of selective devices similar to those employed in other situations.[4,5]

Transformer Exciting Currents

Another source of harmonic components on distribution circuits is the exciting current of distribution transformers. On REA systems, the ratio between the connected kilovolt-amperes of transformer capacity and the system load is relatively large. Consequently transformer exciting currents may control the inductive influence, particularly when the harmonics impressed on the system at the source or arising in loads are relatively small. The importance of transformer exciting currents has geen emphasized by recent changes in the design of certain transformers which have resulted in increases in the harmonic exciting currents. This is illustrated by the values in table III. Considering the three three-kva transformers, of which *B* and *C* are of the newer designs, there is a range of less than 2 to 1 in the rss* exciting current, while the exciting $I \cdot T$ products show a range of over 10 to 1, and the 540-cycle components show a range of 100 to 1. In the case of the five-kilovolt-ampere units, the more recent design (*E*) actually has a lower rss exciting current, but the exciting current harmonics and $I \cdot T$ product are larger. The 540-cycle component may be of particular importance in situations where there are exposures to three-phase portions of the power system, since this triple harmonic component appears as a residual on the three-phase circuits. As indicated later, in several cases it has been found necessary to consider means of reducing this component in order to make any material improvement in the over-all influence of the system.

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*Explained in table 1.

It is well known that the harmonic exciting currents of distribution transformers increase rapidly with increases in the applied fundamental frequency voltages. This is particularly true of some of the more recent transformer designs, as indicated by the comparison of table IV.

In one situation, involving about 400 miles of line with about 1,300 kva of connected transformers, measurements of the lower frequency residual currents in the supply bank were made under two conditions of impressed voltage. As indicated in table V, an increase of five per cent in the voltage resulted in an increase of about 2 to 1 in the 540-cycle residual current. The 300-cycle component of residual current was largely produced by the 300-cycle component in the impressed voltage wave shape and was not appreciably affected by the increase in applied voltage.

In view of the field experience indicating the importance of this source in contributing to the over-all power system influence, consideration is being given by the REA to limits on the harmonic exciting currents of the transformers installed in the future since their experience indicates that the various designs have about the same characteristics from the standpoint of power system operation and cost.

Devices on Distribution-System Side of Supply Bank to Improve Wave Shape or Limit Effects of Resonance Between Line and Supply Transformer

Since REA distribution systems are usually supplied through transformer banks of relatively small kilovolt-ampere capacity, it is often practicable to improve the wave shape of the currents and voltages on the system by the use of shunt elements on the wye-connected side of the supply bank, utilizing the leakage reactance of the bank as a series element. The use of shunt capacitors for this purpose has been covered in a previous paper.[3] Capacitors are used, primarily in cases where the higher frequencies control the inductive influence, to lower the frequency of resonance between the reactances of the supply bank and of the distribution system. At the same time, the capacitor forms the shunt element of a low-pass filter to reduce the higher frequencies. Thus, in the absence of important lower frequencies in the influence, the over-all influence of the system is reduced. Data for one such situation are presented in table VI.

Resonant Shunts

Where the supply bank is relatively large, a considerable amount of capacitance would usually be required to provide adequate reductions in the impressed harmonics by the method described above. Also, there may be adverse resonance effects between the transformer reactance and the impedance of the capacitors at important lower frequencies where sufficient capacitance is used to provide the higher frequency reductions. In such cases, it may be more practicable to use a resonant shunt tuned to the particular frequency which is to be reduced. Such an arrangement can be made to have an effective resistance from 10 to 50 times lower than the reactance of the capacitors at the tuned frequency. The results obtained in a situation where a three-phase resonant shunt for 1,740 cycles was applied to the distribution system side of a 1,200-kva supply bank are indicated in table VII. Each phase of the wye-connected shunt consisted of a 15-kva 6,900-volt capacitor and an air-core reactor, with the impedance characteristics shown in figure 1. As indicated in table VII, the 1,740-cycle components of current and voltage were reduced in the order of 10 to 1 by the shunt, and substantial improvement was effected in the over-all influence and the noise on exposed telephone circuits. While the residual $I \cdot T$ product of the substation transformer was controlled by components other than 1,740 cycles, the results of measurements of the ground return $I \cdot T$ of one of the principal three-phase feeders showed that 1,740 cycles controlled before the application of the shunts.

Resonant shunts have a capacitive reactance at frequencies below the tuned frequency and an inductive reactance above that frequency. Care must be taken in applying such shunts to avoid adverse resonance effects with either the supply bank impedance or the impedance of the distribution system at any frequency of importance in the wave shape. In the case just mentioned, series resonance between the supply bank and shunt reactances occurred at 1,140 cycles, which

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* rss—root sum square.

Table XI. Measured Effectiveness of Combination Resistance Shunt and 540-Cycle Wave Trap in Reducing Inductive Influence of Power System and Noise on Telephone Circuits

(3-100-Kva Transformers 33,000 to 12,500 Volts)

Voltage TIF	Normal (No Devices)	Combination Shunt* Only	540-Cycle Wave Trap* Only	540-Cycle Wave Trap and Combination Shunt
Phase *A-G*	33 (15)(23)(5)ϕ	18	35 (16)(28)(4)	16 (8)(8)(3)
Phase *B-G*	45 (19)(25)(5)	24	31 (18)(14)(2)	19 (11)(9)(3)
Phase *C-G*	35 (11)(25)(6)	21	48 (8)(40)(5)	16 (7)(8)(3)
Residual kv·*T*	335 (24)(213)(75)	200	200 (75)(126)(52)	84 (13)(24)(58)
Ground return I·T				
East feeder	700	300	450	200
West feeder	230	220	230	125
Residual $I \cdot T$ of substation	1,500 (175)(1,120)(380)		570 (290)(248)(20)	190 (72)(52)(15)
Noise longitudinal (noise units)	125 (63)(63)(63)	115 (35)(11)(110)	105 (70)(70)(10)	63 (25)(28)(7)

ϕNumbers in parentheses represent 300-, 420-, and 540-cycle contribution respectively to TIF, $I \cdot T$, or noise.

*Constants of devices shown in figure 6.

happened to be unimportant in the wave shape.

In one case, the reactance of the shunt was utilized as a capacitive element in parallel with the inductive reactance of the supply bank to offer a relatively high impedance to residual currents originating as transformer exciting currents or from unbalanced lengths of single-phase extensions connected to the various phase wires. In this case, the shunt constants were chosen to offer a low impedance at 660 cycles, a frequency prominent in the impressed voltage wave, and to resonate with the 300-kva supply bank reactance at 540 cycles, which was prominent in the load transformer exciting currents and important in the inductive influence of the system. Each of these shunts consisted of a 7.5-kva 6,900-volt capacitor and an air-core reactor of approximately 120 millihenries inductance, as shown in figure 2. The effectiveness of the arrangement is indicated in table VIII. The increases observed in the 420-cycle component with the shunts in service were due to the effect of a 420-cycle voltage in the impressed wave shape on the circuit consisting of the transformer inductive reactance and capacitive reactance of the shunt and line in parallel. However, the substantial reductions in the 540- and 660-cycle components were reflected in reductions in the over-all inductive influence and in the noise.

Combinations of Capacitors, Reactors, and Resistors

The arrangements so far considered are applicable where only the higher frequencies are important in the inductive influence, and where adverse resonance effects are not important. An arrangement which can be designed to offer an impedance which is substantially pure resistance over the voice frequency range has recently been developed by H. W. Wahlquist, and has been tried out on an experimental basis in several of the co-operative studies of the REA and Bell System. Figure 3 shows schematic diagrams of these arrangements as applied to three-phase or single-phase circuits. By properly proportioning the values of resistance, inductance, and capacitance, it is practicable to obtain a wide range of shunt impedance characteristics over the voice frequency range. Some of these are shown in connection with the discussion of specific cases.

Such an arrangement may be applied either at the supply point in the same manner as a shunt capacitor or resonant shunt, or as a resistance termination at the end of the distribution circuit. In the former case it may be used to reduce the effects of resonance between the substation transformer and system and, in addition, to limit the harmonic voltages impressed on the system from the source of supply. Where this is necessary, the shunt constants are chosen so that the impedance at a particular frequency is low compared with that of the supply bank. Where the arrangement is used to terminate a line in order to minimize the effects of line resonance, the constants are usually chosen to offer a resistance of the order of 200 to 500 ohms above about 300 cycles.

In two cases these combinations were applied to reduce the 300- and 420-cycle components present on the distribution system. In the first case (table IX) these components were relatively large in the voltage of the supply system and the 300-cycle component was increased 2 to 1 when the REA system was connected, due to resonance between the supply bank and system. It is of interest to note that the 300-cycle phase current was nearly as large as the 60-cycle current under this condition. Comparative tests were made of the effectiveness of (*a*) resonant shunts tuned for 300 and 420 cycles, (*b*) the 300-cycle shunt alone, and (*c*) a combination of capacitance, reactance, and resistance having the impedance characteristics shown in figure 4. As indicated in table IX, each of these arrangements effected substantial reductions in the influence of the system and in the resulting noise. The resistance arrangement has the advantage that it is not apt to introduce adverse resonance effects at higher frequencies which may be generated in loads connected to the REA system in the future.

The value of 800 ohms shunt resistance in this case was chosen so that the impedance of the combination at 300 cycles would be small relative to that of the supply bank. This was necessary since it was desirable to make considerable reduction in the 300-cycle voltage. In another case, where the 300- and 420-cycle components were not so prominent in the impressed voltage, and where the supply bank had a higher impedance, it was practicable to use lower values of resistance in parallel with the inductance, as shown in figure 5. Table X presents the more important results of these tests and indicates the comparative effectiveness of the various arrangements in reducing the resonance between the supply bank and system at 300 cycles. It is evident that the value of shunt resistance is not particularly critical. The 300-cycle resonant shunt was more effective than any of the various resistance combinations in reducing the 300-cycle component but less effective at 420 cycles. With either type of shunt in service, the 300-cycle line current was largely contributed by the exciting currents of the load transformers.

One phase of this system extended over 50 miles from the supply point. A 300-cycle voltage rise of about 2 to 1 was observed between the substation and the

Table XII. Effectiveness of Far-End Termination on Power-System Influence and Telephone-Circuit Noise

(3–100-Kva Transformers 33,000 to 12,500 Volts)

	Phase-to-Neutral Volts at Substation		Phase-to-Neutral Volts at Point Where Device Was Located		Phase Current at Substation (Mils)		Residual Current at Substation (Mils)	
Frequency	Normal	Combination* on	Normal	Combination on	Normal	Combination on	Normal	Combination on
60	7,500	7,650	7,410	7,550				
420	192	100	231	120	590	350	86	38
540	35	32	35	51		100	135	176
660	102	50	204	83	620	328	415	97
Voltage TIF	43	24	75	34				
$I \cdot T$ product					1,600	900	1,100	360
Noise metallic (noise units)	1,175	295						
Noise to ground (noise units)	1,600	800						

*Constants and impedances shown on figure 7.

distant point. With the resistance combination in service, the small magnitude of 300 cycles at the supply point makes this rise unimportant.

Combination of Shunts and Wave Trap in Transformer Neutral

Another situation somewhat similar to those described involved exposures of a telephone toll line to three-phase feeders supplied in two directions from a supply point. The influence of the feeders and the noise on the exposed circuits was controlled by 300, 420, and 540 cycles. The first two frequencies resulted largely from the effects of impressed voltage and were reduced by resistance shunts of the type already described. However, these shunts offered a low impedance to the 540-cycle residual currents, which originated in the exciting currents of the load transformers, and the over-all ground return $I{\cdot}T$ of one of the feeders was not appreciably reduced. In order to effect reductions in the 540-cycle component and to maintain the reductions at 300 and 420 cycles, a wave trap tuned for parallel resonance at 540 cycles was inserted in the neutral of the supply bank and the shunts were connected as shown in figure 6. The comparative results with these various arrangements in service are shown in table XI.

Resistance Termination at End of Distribution System

Where the inductive influence is largely the result of a condition of resonance, rather than abnormally large harmonics in the supply system voltage, it may be practicable to minimize the resonance effect by using a resistance termination at the far end of the three-phase circuit. This arrangement was tested in one case where resonance was encountered between the supply transformer and system at 660 cycles. The elements and impedances of the device employed are indicated in figure 7. This arrangement offered a resistance in the phase-to-neutral and residual circuits above 300 cycles, and was connected to the main three-phase feeder about 34 miles from the supply point. The results are shown in table XII and indicate about a 2 to 1 reduction in the 660-cycle voltages at the supply point and at the point where the device was applied. The phase current at the supply point was likewise reduced about 2 to 1 but the residual current was reduced over 4 to 1, due to the "balancing" action of the device on the three phases. The transformer exciting currents were not of controlling importance without the device. However, they became more important with the device in service because of the increase in fundamental frequency voltage resulting from the leading current taken by the capacitors.

The application of this type of termination appears limited to cases where resonance conditions are fairly acute. Where the system is extensive, it would probably be necessary to install terminations at the end of each main feeder.

There is always the possibility that an acute resonance condition of this sort may be set up at the time a system is initially energized, when the lines are lightly loaded. As the size of the system increases and the loads build up, the resonance point shifts to a lower frequency and the loads tend to damp out the resonance effects, although they may also contribute sources of harmonics. In several cases, acute noise conditions have occurred when the system was initially energized, and the growth of the system has tended to reduce the influence and noise. It is under such initial conditions that the termination appears most attractive as a remedial device.

Co-ordinative Measures in Telephone System

Co-ordinative measures which are available for improving the susceptiveness of exposed metallic telephone circuits were discussed in a previous paper,[2] and mentioned above. The joint work of the REA and Bell System has yielded no new co-ordinative measures but has emphasized the importance of continuing efforts to limit the susceptiveness of the outside plant and equipment associated with rural subscriber lines. Improvements in outside plant construction, such as extensions of cable on the more important rural routes, better transposition arrangements for open-wire exchange circuits, and the use of improved types of conductors employing rolled sleeve joints, are expected to result in lower susceptiveness of the rural plant. The development of a better balanced battery-supply relay with a fairly low longitudinal impedance has recently made available a new method of minimizing unbalances in dial central offices. This relay is to be employed in new offices in rural areas and will also be available for use in existing offices, as an alternative to the use of longitudinal chokes or drainage circuits,[2] in cases where unbalances are found to be important in contributing to the noise problem. The low longitudinal impedance of the balanced relays has the advantage that electrically induced voltages are drained from the circuit, and their effects on any shunt unbalances present in the line or station equipment are consequently minimized. The improvements in the susceptiveness of subscriber equipment mentioned earlier will also contribute toward better noise conditions in rural areas.

Conclusion

It is evident from the foregoing discussion that there is no universal solution to inductive co-ordination problems. However the experience already secured indicates that, in general, it is practicable to control the influence of the power system and the susceptiveness of the telephone plant so that adequate results can be secured at reasonable cost. The degree of effectiveness of the power system remedial measures has varied with the conditions of each individual case, the greater effectiveness being secured in cases where the influence was relatively high before the application of the devices. It is also apparent that in some cases several alternative arrangements of varying degrees of effectiveness may be applicable. In each case so far investigated at least one arrangement was found which reduced the influence to a point where satisfactory co-ordination with well-balanced telephone systems could be secured.

The success of the joint REA and Bell System inductive co-ordination work to date is a further clear indication that the best results not only from the standpoint of the organizations involved but also from the standpoint of the public can only be secured by close co-operation and joint study. The mutual understanding and background already created through these studies should form a firm basis for continued progress in this work.

References

1. Inductive Co-ordination of Common Neutral Power Distribution Systems and Telephone Circuits, **Coleman** and **Davis.** AIEE Transactions, volume 56, 1937 (January section), pages 17–26.

2. Noise Co-ordination of Rural Power and Telephone Systems, **Wahlquist** and **Taylor.** AIEE Transactions, volume 57, 1938 (November section), pages 613–21.

3. Measurement of Telephone Noise and Power Wave Shape, **Barstow, Blye,** and **Kent,** AIEE Transactions, volume 54, 1935, page 1307.

4. Effects of Rectifiers on Power System Wave Shape, **Blye** and **Kent.** AIEE Transactions, volume 53, 1934, page 54.

5. Engineering report No. 34 of the joint subcommittee on development and research, EEI and Bell System

Power Arc-Over on Overhead Distribution Lines and Newly Developed Equipment for Protection Against Conductor Burndown From That Cause

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Synopsis: With conventional types of switchgear and customary methods of circuit operation, power arc-overs resulting from transient faults on overhead primary lines cause unnecessary burndown of conductors and undesired operation of fuses. The majority of faults on overhead distribution circuits are of transient nature.

Factual data concerning arcing characteristics of covered and bare conductors were established by tests conducted under field conditions. They show that to prevent burndown of covered wire and to hold the arc damage within admissible limits, power arcs must be limited to one-half- to two-cycle (sixty-cycle basis) over-all time, depending upon size of conductor and arc current. Bare conductors are less liable to serious arc damage owing to the phenomenon of arc travel and to the self-extinguishing characteristics of arcs on such conductors. However, the duration of traveling arcs is sufficient to cause undesired operation of sectionalizing fuses unless the line is rapidly de-energized. Speeds of operation required to protect covered wire are also ample to assure against operation of fuses on transient faults.

New switching devices capable of meeting the required one-half- to two-cycle initial circuit clearance have been developed as part of this project. They also provide for automatic reclosure and a sequence of time-delay openings and reclosures before lock-out. Simplicity of construction, ease of installation, and self-actuating principles which dispense with auxiliary power requirements, make investment sufficiently low to bring these devices within the realm of practicability for low revenue distribution circuits as well as the more important distribution circuits. They are adaptable to existing and new circuits alike.

General

FAILURE of overhead primary distribution lines has long been a problem of electric utility operators, despite the fact that the lines possess adequate mechanical and electrical strength to withstand normal interferences. Many of these failures are the result of conductor burndown caused by power arc-over. Table I lists all cases of trouble during 1938 and 1939 on the Detroit Edison distribution system in which primary wires were down. Of the totals shown, investigation indicates that at least 60 to 70 per cent of these cases were initially of the transient fault class in which the wires would not have burned down if the arc had been interrupted soon enough.

Table I. Cases of Trouble on the Detroit Edison Distribution System Involving Primary Wires Down

Month	1938 Total Number of Cases	1938 Number of Electrical Storms	1939 Total Number of Cases	1939 Number of Electrical Storms
January	15	0	18	0
February	37	4	23	1
March	40	4	328*	0
April	56	2	22	1
May	40	6	48	4
June	109	6	277	6
July	165	7	98	7
August	155	5	45	7
September	21	1	88	5
October	26	3	26	2
November	16	0	12	0
December	23	1	8	0
	703		993	

*Sleet storm.

It is evident that burndowns resulting from power arc-overs have their effect on service continuity, although in the opinion of many utility engineers the service now rendered to the various classes of customers is sufficiently satisfactory and in keeping with the character of the loads served. However, the public and maintenance employee hazard as well as costs associated with wires down is, or should be, a matter of serious concern to all operators, and it is this phase of the situation that was chiefly responsible for the studies and developments dealt with in this paper.

From the standpoint of service continuity, the problem of power arc-over becomes more serious as our systems expand and our distribution circuits reach further and further into the outlying and rural districts with proportionate increases in exposure of the lines. Accompanying this, we have the increased difficulty of patrol due to the greater distances and often the lack of good or even passable roads. The customary use of bare wire for rural extensions further complicates the problem because the performance of bare conductors under short-circuit conditions is different from that of covered conductors.

Weatherproof covering, when aged or wet, becomes less effective in preventing short circuits when conductors swing together or are bridged by conducting objects. When breakdown of the covering occurs, the resulting short circuit is of the arcing type because the covering serves as a mechanical spacer preventing metal-to-metal contact. Faults between bare conductors may be of the arcing type or they may be of the metal-to-metal type which cannot be burned clear.

The use of ring primary construction has been found to be an effective means of reducing service outage caused by overhead primary failures, but these measures do not remove or reduce the possibilities of wires down with the accompanying hazards. Furthermore, this scheme is applicable only in the high load density areas where short distances and concentration of customers make the arrangement physically and economically possible. Automatic breakers, sectionalizing fuses, and various types of protective devices have found their places in distribution circuits, although we still experience cases of trouble in which lines are burned down unnecessarily and sectionalizing fuses are needlessly blown.

Let us look at the distribution circuit as a whole and review the usual method of operation. We find the distribution transformers connected to the line through primary fuses to protect the main circuit from troubles that may develop in the transformer or its secondary circuit. We may find sectionalizing fuses protecting the feeders from troubles on the branches. These devices require energy for their operation—energy in the form of a flow of current in excess of that which the fuses are designed to carry under normal conditions. Obviously, if they are to function when a fault occurs, the flow of fault current must be con-

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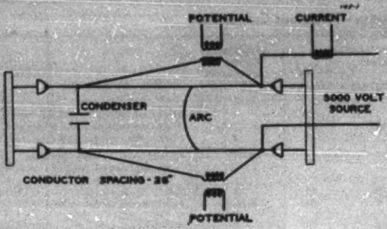

Figure 1. Diagram of experimental line showing method of connecting instrument transformers used in oscillograph circuit

tinued for a long enough time to supply the amount of energy required for that operation. Again, if a fault on the line is not cleared through fuses, continued flow of fault energy is desired to burn the fault clear, if possible. If, in this attempt, the line burns down at the point of fault, the resulting hazard there, although undesirable, is thought to be less serious than hazards that might accompany the total shutdown of the circuit for a prolonged period.

To provide the necessary energy to accomplish these things, the opening of the station breaker is delayed for an appropriate period after fault inception. Moreover, if the fault has not cleared when the breaker recloses, the delayed opening cycle is repeated one or more times in a further effort to clear the trouble before the breaker locks out. On circuits not equipped with breakers it is the general practice to use fuses in an attempt to clear the faulted section without wire burndown.

With this system of operation, a faulted transformer or a fused section of the line which has become short-circuited will be promptly cut off from the rest of the circuit; or a fault on an unfused section may be promptly burned clear so that service, even though it may reach only part of the customers, can be restored at once. But what of the case where lightning paves the way for power arc-over on the line? The power arc may continue as long as energy flows into the line or until fuses are blown or the line is burned down. Here the condition that causes the short circuit or initiates the fault is present for only an instant; yet, intentional delay in interrupting the circuit deliberately provides energy to support the power arc with all its destructive forces. The same sort of thing happens when wires momentarily swing together, and, in fact, when any type of transient fault occurs, if we define such faults as those in which the initiating cause is of momentary nature—lightning, wires swinging together, small conducting objects such as baling wire and radio antenna wire across the line, are all examples of this type. It is apparent then that the delayed initial opening works to advantage on faults of the persistent type but may cause needless damage when faults of the transient type are encountered.

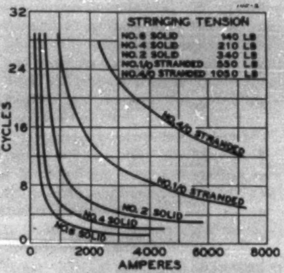

Figure 2. Burndown characteristics of covered copper wire at 5,000 volts; conductor spacing 28 inches

These considerations suggest the desirability of using equipment of a type such that arcs resulting from transient causes will not persist long enough to cause line burndown or fuse operation, but in the cases of persistent faults the fuses will blow or the wires burn down to clear the circuit. We cannot expect the station breaker to differentiate between types of faults on its initial operation, for it has no way of knowing the nature of the trouble with which it is dealing, but we could initially assume every fault to be transient and treat it accordingly. In those cases where the assumption is incorrect and the fault proves to be persistent, we could subsequently treat it as such. Thus, unnecessary burndown and arc damage, and undesired fuse operations, could most likely be avoided when faults are initiated by transient causes. The ratio between transient and persistent faults on overhead distribution systems is difficult to determine accurately from operating records because adequate evidence is usually lacking. Estimates, made from time to time by various individuals and for various types of systems, indicate figures ranging from about 50 to 90 per cent as the relation of transient faults to the total number of faults. It is the author's opinion that of all the faults occurring on the Detroit Edison distribution system about 60 to 70 per cent are initially of transient nature. The Detroit Edison distribution system is operated at 4,800 volts ungrounded and often combines in the same circuit both covered and bare wire.

It will be recognized that the scheme of breaker operation in which "instantaneous" tripping is used for initial breaker opening, succeeded by reclosure and a series of time-delay operations, follows the same patttern as the operating sequence suggested in the previous paragraph. The so-called instantaneous tripping, however, does not accomplish instantaneous circuit interruption because the relays require a certain amount of time for their operation and a relatively much longer time is required for the breaker contacts to part and the arcing at the contacts to cease. Although instantaneous tripping of conventional breakers is successful in many cases in preventing fuse operation on transient faults, the breakers are in general too slow to save covered wire from burndown. It is natural then to inquire as to the necessary speed of interruption if burndown or serious arc damage is to be avoided in case of transient faults, and as to how these requirements vary with different sizes of wire, different spacing and configuration of conductors, and with different values of current in the arc.

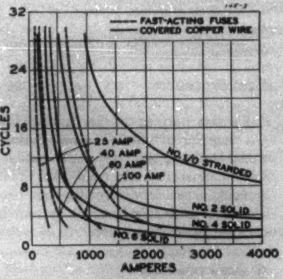

Figure 3. Comparison of the characteristics of fast-acting fuses with burndown characteristics of covered copper wire

Since no data were available with which these questions could be answered, a test project was undertaken to produce factual information upon which an intelligent at-

Table II. Results of Arcing Tests on Bare Conductors—Horizontal Configuration

Kind of Conductor	Size and Type	Spacing (Inches)	Arc Current (Amperes)	Duration of Arcing (Cycles)			Approximate Extent of Arc Travel (Feet)	Remarks
				Maximum	Minimum	Average		
ACSR	No. 2	88	80	26	$1^1/_2$	15	1	Slight surface pits and blisters
ACSR	No. 2	88	190	28	18	23	1.5	Slight surface pits and blisters
ACSR	No. 2	88	500			1	2	Slight surface pits and blisters
ACSR	No. 2	58	200	74	11	35	1	Moderately severe burning; one strand burned open in two of nine tests. Pits fairly dense
ACSR	No. 2	58	500	77	3	32	4	Fairly dense surface pits and blisters
ACSR	No. 2	58	900	78	9	40	6 to 10	Fairly dense surface pits and blisters
ACSR	No. 4	58	200	31	27	29	5	Fairly dense surface pits and blisters
ACSR	No. 4	58	500	43	36	40	6	Fairly dense surface pits and blisters
ACSR	No. 4	58	900	42	34	38	6	Fairly dense surface pits and blisters
Copperweld	$^3/_{16}$ inch—7 strands	58	200	36	21	29	2	Wind against direction of motor action. Pits and blisters quite severe—steel core exposed in spots
Copperweld	$^3/_{16}$ inch—7 strands	58	500	42	39	40	3	Pitting quite severe to depth of copper coating; pits and blisters fairly dense
Copperweld	$^3/_{16}$ inch—7 strands	58	900	35	34	35	5	Pitting quite severe to depth of copper coating; pits and blisters fairly dense
Copperweld	No. 8—solid	58	200	30	24	27	2	Surface pits and blisters quite dense
Copperweld	No. 8—solid	58	500	33	26	30	2	Slight surface pits and blisters
Copperweld	No. 8—solid	58	900	33	30	32	3	Slight surface pits and blisters
Copper	No. 2—solid	58	900	49	27	36	4 to 12	Slight surface pits and blisters
Copper	No. 6—solid	58	200	35	29	32	1	Surface pits $^1/_{32}$ inch deep; some blisters
Copper	No. 6—solid	58	500	37	32	35	2 to 3	Surface pits $^1/_{32}$ inch deep; some blisters
Copper	No. 6—solid	58	900	46	42	44	3 to 10	Slight surface pits and blisters; quite dense where travel was short

tack on this whole problem could be based.

Test Procedure

For this purpose an experimental line of characteristic design using conventional hardware was set up in an outdoor laboratory so that test conditions would simulate actual service conditions. Provision was made for drawing the tension in each size of conductor tested to the value used in regular construction practice. Both bare and covered conductors of the sizes in common use in the company were tested in turn. The covered wire was copper with the standard triple braid weatherproof covering and the bare wire included the ACSR type, bare copper, and Copperweld wire. The covered wire was tested at a horizontal spacing of 28 inches which is the closest horizontal spacing used in the company's distribution system. The bare wire was tested at horizontal spacing of 58 and 88 inches, and at vertical spacings of 12, 18, 24, 30, and 43 inches.

All tests were made single phase, ungrounded, at 5,000 volts, 60 cycles. The fault current was controlled by series reactors and was varied between 80 and 7,000 amperes. The test circuit was arranged so that the flow of fault current could either be sustained until the arc cleared itself or the line burned down, or it could be limited in time duration to $^1/_2$ cycle on a 60-cycle basis ($^1/_{120}$ second).

The electrical performance of the test circuit was recorded with a magnetic oscillograph energized by instrument transformers connected as shown in figure 1. By this scheme of connections the exact instant of the parting of the conductors could be determined.

Short circuits between conductors were initiated in various ways. To simulate

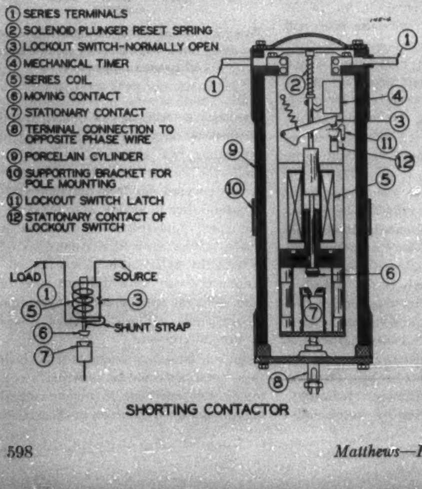
① SERIES TERMINALS
② SOLENOID PLUNGER RESET SPRING
③ LOCKOUT SWITCH-NORMALLY OPEN
④ MECHANICAL TIMER
⑤ SERIES COIL
⑥ MOVING CONTACT
⑦ STATIONARY CONTACT
⑧ TERMINAL CONNECTION TO OPPOSITE PHASE WIRE
⑨ PORCELAIN CYLINDER
⑩ SUPPORTING BRACKET FOR POLE MOUNTING
⑪ LOCKOUT SWITCH LATCH
⑫ STATIONARY CONTACT OF LOCKOUT SWITCH
LOAD
SOURCE
SHUNT STRAP
SHORTING CONTACTOR
①
②
③
④
⑤
⑥
⑦
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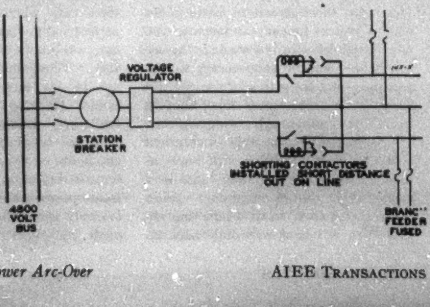
Figure 4. Cross-sectional view of short-circuiting contactor

Figure 5 (below). Application of shorting contactors to ungrounded circuit equipped with station switchgear


VOLTAGE REGULATOR
STATION BREAKER
4800 VOLT BUS
SHORTING CONTACTORS INSTALLED SHORT DISTANCE OUT ON LINE
BRANCH FEEDER FUSED

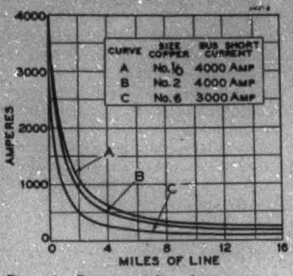


Figure 6. Decrease in line short-circuit current with distance from bus

power arc-over resulting from small conducting objects across the line, a single strand of No. 40 copper magnet wire was drawn across the conductors. The magnet wire was found to fuse in an average of about $^1/_{40}$ cycle, thus establishing the arc-over. To simulate arcing resulting from conductors swinging together, the conductors at mid-span were held in close proximity to one another by tying them with the fine magnet wire. When the line was energized, the fine wire fused allowing the conductors to swing apart. Since no practical difference could be found in the effects of arcs initiated by these two methods, the data from the two types of tests were averaged.

In other tests tree limbs of various sizes and of different kinds of wood, both green and dry, were laid across the conductors to simulate the condition where tree branches fall or are blown into the line.

In initiating the arcs on bare conductors it was only necessary to place the short-circuiting medium in contact with the wire. In the case of covered conductors the covering was in some instances removed all around over a length of two inches to expose the conductor; in other cases contact with the conductors was established by common pins forced through the covering. There was no distinguishable difference in the results of these two methods, and again the data were averaged as a whole.

Results of Tests on Bare Conductors

The motorizing action of the current flux of an arc between line conductors always tends to propel the arc along the line in the direction away from the power source. Under this force, an arc between smooth bare conductors will travel freely along the line. As it moves, the length of the arc "loop," or the ionized stream which carries the arc current, becomes greater and greater until finally it is broken and the arc is extinguished. If, however, the arc in its travel encounters an object, such as a splice in the wire, insulator tie wires, or any similar object that will serve as a point electrode, the arc will become anchored at that point and usually will continue until the conductors burn down or the fault current is interrupted in some other manner. Likewise, tree limbs and similar faults that cause arc-over will prevent the arc between bare conductors from traveling normally. In such cases the arc may periodically leave the location of fault and travel far enough to extinguish itself; but it will generally restrike at the original location and continue this performance until the line burns down or the limb, if it is sufficiently small, burns clear of the line.

The wind also has a marked effect upon the performance of traveling arcs. During certain tests, when the wind happened to be blowing in the same general direction as that of the force propelling the arc, it was observed that speed and extent of travel were increased. When the wind blew in the opposite direction, arc travel was retarded and in some cases the direction of travel was even reversed; that is, the arc was forced to travel toward the power source.

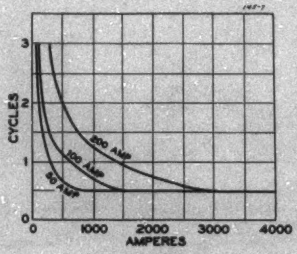


Figure 7. Switching speed of shorting contactor

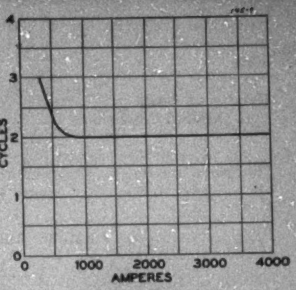


Figure 9. Switching speed of fast-opening and reclosing breaker

These outside influences served to accentuate the normally erratic performance associated with electric discharge so that it was necessary to repeat each test, particularly those of major interest, several times to obtain dependable averages.

Table II gives the results of the tests on bare conductors in which the arcs were initiated with the fine magnet wire and in which the conductors were spaced horizontally at the distances indicated. Table III gives the results of similar tests with the conductors spaced vertically. In all these cases the arcs had a clean sweep along the line and voltage was maintained until the arc blew out. Under "Duration of Arcing" in these tables, the figure shown as maximum in each case is that for the test in which the arcing continued the longest, the minimum figure is that for the test in which the arcing was the shortest, and the average figure represents the average for all tests under the indicated conditions. The extent of travel is given as an average and general

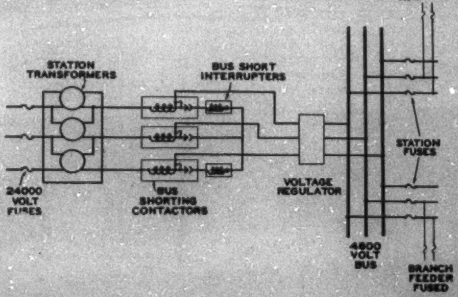


Figure 8. Application of bus-shorting contactors with interrupters to suburban stations with fused feeder circuits

comments concerning the amount of conductor burning are given under "Remarks."

Results of Tests on Covered Conductors

Arcs on covered conductors are prevented from traveling by the presence of the covering. If the arc is allowed to continue, it concentrates its heat at the points where the covering is punctured or the wires are exposed with the result that the line is quickly burned down.

Table III. Results of Arcing Tests on No. 2 ASCR Conductors—Vertical Configuration

Spacing (Inches)	Arc Current (Amperes)	Duration of Arcing (Cycles)			Approximate Extent of Arc Travel (Feet)	Remarks
		Maximum	Minimum	Average		
43	500	56	27	41	5 to 8	Slight surface pits and blisters
43	900			40	6	Slight surface pits and blisters
30	200	144	73	109	2	In one test one strand burned open on top conductor
30	500	306	103	205	15 to 20	Slight surface pits and blisters
30	900			40	6	Slight surface pits and blisters
24	200			83	3	Pits about $^1/_{32}$ inch deep, quite dense
24	500			114	4	Pits about $^1/_{32}$ inch deep, quite dense; one strand burned open
24	900			104	10	Pits about $^1/_{32}$ inch deep, quite dense; one strand burned open
18	140	40	20	30	2	Slight surface pitting
18	200			490	16	Top conductor badly pitted and blistered
18	500	430	57	186	5 to 20	Slight surface pitting
18	900			267	6	Slight surface pitting
12	200	783	56	473	Max. 50	Bad pitting in one case where arc traveled but short distance. Minor pitting in other cases where arc traveled 30 to 50 feet

In the first series of tests on covered wire, voltage was maintained on the line until after conductors burned down so that critical values of arc current and duration of arcing could be determined. Arcs were initiated by the fine magnet wire and the instant of failure was determined from the oscillograms as previously described. The results of these tests are shown graphically in figure 2. The curves represent the locus of combinations of arc current and duration of arcing which are just sufficient to cause fusion of the different wires. Thus, any combination of current and time represented by a point on the curve, or to the right of the curve, will cause burndown of that particular size of wire; combinations represented by points to the left of the curve will not cause actual failure of the conductors but will result in varying degrees of damage, depending upon the proximity of the points to the critical curve.

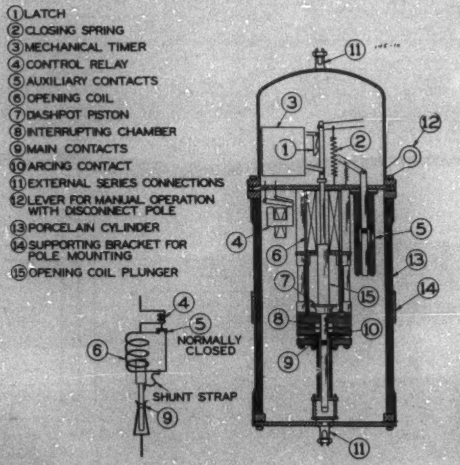


Figure 10. Cross-sectional view of fast-opening and reclosing breaker

The average distribution system has a wide range of fault currents for the different fault and circuit conditions, the maximum possible values obviously being dependent upon the impedance of the circuit up to the point of fault. Since these values are fixed by the characteristics of the circuit and the power available at the source, it is evident that the only function subject to control is the duration of arcing. If the duration could be made sufficiently small, it can be seen from figure 2 that, even though the fault current were high, the conductors could be saved from burndown. This, however, is not enough for practical purposes, for the damage would have to be limited to such a degree that the reduction in cross section of the wire at the point of arc burn would not seriously reduce the physical strength of the wire.

To explore the possibilities of limiting arc damage to admissible amounts, a second series of tests was run in which arcs of various current magnitudes were limited in duration to one-half cycle. The arc-damaged wires were then tested in tension and the reduction of strength determined from comparison with the tensile strength of undamaged wire. The results of these tests are shown in table IV.

Results of Tests With Tree Limbs Across the Line

The following general observations were made during these tests:

1. Tree limbs across the line may cause burndown of any kind of conductor.
2. Limbs of soft or pithy wood are more likely to burn clear of the line than those of hardwood.
3. Limbs of any kind of wood, if over one-half inch in diameter, will generally cause line failure if power arc-over is established.

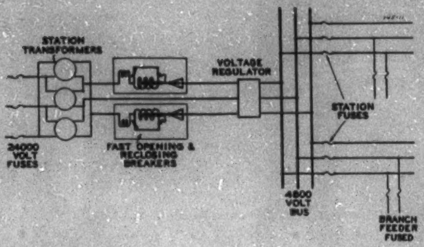


Figure 11. Application of fast-opening and reclosing breakers to light-capacity feeder circuits

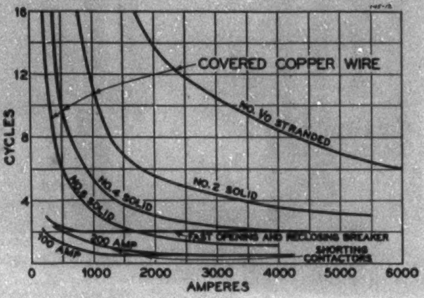


Figure 12. Burndown characteristics of covered copper wire compared with switching speeds of new devices

Analysis of Results

Referring to the results of tests on bare conductors (table II) and taking the data for tests at 58-inch horizontal spacing as a whole, it is seen that, although the arcs were self-extinguishing and did not cause burndown of the line, they persisted up to a maximum of 78 cycles.

Because of the susceptibility of bare conductors to metal-to-metal faults which cannot be burned clear by arcing, sectionalizing fuses are often used in circuits containing bare wire. It is evident from the tests that, with time-delay initial opening, practically any transient fault on the bare sections of such lines will cause the blowing of sectionalizing fuses, although the line itself may remain in perfectly good condition. It might be pointed out that less arc damage to conductors than that reported in the results would be expected in actual service since the duration of arcing would be held to the blowing time of the fuses. Nevertheless, for transient faults the unnecessary operation of fuses causes interruption to a portion of the circuit and long outage may result. The use of repeating type fuses to overcome this difficulty naturally suggests itself, but it must be remembered that the effectiveness of these devices entails frequent inspection and patrolling of the line.

Comparison of the results of tests at 58- and 88-inch horizontal spacing (table II) shows the influence of the wider spacing in reducing the duration of arcing. It can be seen from table III that for the similar wire spacings arcs persist longer on the vertical than on the horizontal configuration—damage to the conductors is correspondingly greater.

The results of the tests on covered wire (figure 2) serve to explain the inadequacy of conventional protective devices to save such wires from burndown or serious arc damage under arcing fault conditions. This is particularly true of the smaller sizes of wire which constitute, by far, the largest percentage of conductor mileage of the whole distribution system. The inability of representative fast-acting fuses to protect covered wire is shown in figure 3, where burndown characteristics of the wire and the blowing characteristics of the fuses are compared directly.

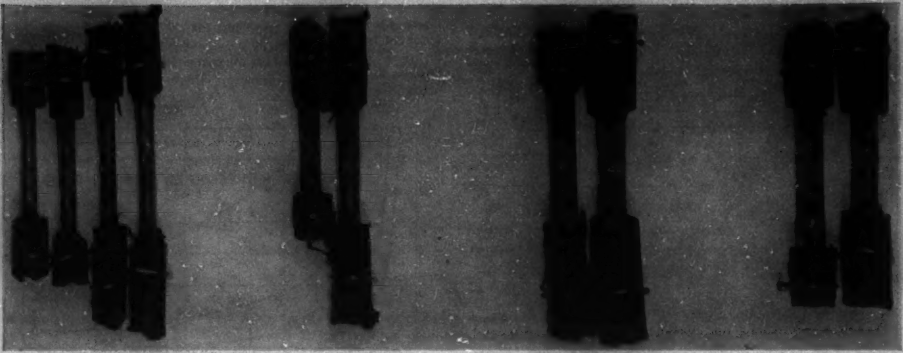
On the basis of the figures of table IV it was possible to reach conclusions with respect to the limiting duration of arcing on various sizes of covered wire if the conductors are to remain in suitable condition for continued use. These values for the range of fault current usually associated with the different sizes of conductors are given in table V.

Having thus established the speed of interruption necessary to avoid needless burndown and serious damage to covered conductors in case of transient faults, it was apparent that speeds adequate for the wire would also protect sectionalizing fuses from operation on transient faults and, therefore, the use of the fuses would be enhanced.

It must be remembered, however, that in addition to transient faults there still remains the persistent type for which energy must be supplied for the intended operation of fuses, or where there are no fuses, to burn the fault clear to avoid total outage of the circuit. The conventional reclosing breaker with time-delay relaying is entirely satisfactory for that operation but, due to the inertia of its parts and its own inherent arcing time, which is about two cycles giving an over-all time of six to eight cycles, it does not lend itself to speeds of operation anything like the required one-half- to two-cycle over-all time, which must be obtained if our objective is to be realized. A survey of commercially available equipment failed to disclose any devices suitable for the purpose and no new commercial developments were in sight. The Detroit Edison Company, therefore, developed three types of equipment which are described in the following sections of the paper.

Fast-Switching Equipment for Circuits Having Reclosing Station Breakers

The device developed for use in combination with station breakers is called the shorting contactor. Its function in case of fault is to metallically short-circuit the line, phase-to-phase, on the source side of the section of the line to be protected. This action collapses the line voltage with such speed that any arc accompanying the fault will be extinguished before objectionable damage occurs to the conductors and before operation or damage to any fuse can take place. After this operation the relatively slow-acting station breaker functions to open the circuit in the usual manner and it is intended that this initial opening will be accomplished by "instantaneous" tripping. The opening of the breaker clears the metallic short circuit applied by the shorting contactor, whereupon contacts of the shorting contactor separate under no-voltage conditions and the device locks in the open position for a predetermined time. If the fault is of transient nature, the entire circuit is left intact so that when the station breaker recloses, service is restored to the whole line. It is also intended that the breakers be arranged for "immediate" first closing which, with conventional breakers, can usually be accomplished in about 30 cycles. This time added to ten cycles, which is approximately the fastest speed at which conventional breakers can be made to open the circuit with "instan-


(*a*)—No. 6 wire one-half-cycle protected by shorting contactor

(*b*)—No. 4 wire one-half-cycle protected by shorting contactor

(c)—No. 2 wire one-half cycle protected by shorting contactor

(*d*)—No. 2 wire two-cycle pro tected by fast-opening breaker

Figure 13. 1,500-ampere arc burns on copper wire protected by the newly developed devices

taneous" relaying, gives a total interruption of about 40 cycles.

If we assume now that the fault instead of being transient is in reality persistent, no harm has been done in the initial attempt to clear the trouble, and little time has been lost. We are then at liberty to apply the usual reclosing cycles with time-delay relaying, the breaker being free of further operations of the shorting contactor during this interval since the latter has automatically locked out. After a sufficient time is allowed for the breaker to have completed its reclosing cycles, the shorting contactor automatically resets and is ready for the next fault.

A diagrammatic view of the shorting contactor is shown in figure 4. It is series operated, self-contained, and fully automatic, requiring no auxiliary circuits or power supply. It is designed for pole or station mounting and can be either air or oil insulated. Being series operated, it is connected in one side of the circuit by means of terminals (1). A lead from the opposite side of the circuit is connected to bottom terminal (8). Figure 5 shows a typical application to a three-phase, ungrounded circuit. It will be noted that the shorting contactors are located a short distance out on the line from the station. If they were located directly at the station, the short circuit caused by the fault would be transformed into the equivalent of a metallic bus short which, on stations of comparatively large size, would cause undesirably heavy shock to the station equipment. The effectiveness of the impedance of the line near the station in cushioning the shock is shown in figure 6 where the steepness of impedance gradient near the station is set forth for typical circuits having different values of bus short-circuit currents.

Table IV. Decrease in Tensile Strength of Insulated Wire Due to Arc-Current Damage

Arc-Current (Amperes)	Duration of Arc (Cycles)	Breaking Load of Arc-Burned Copper (Pounds)	Reduction of Strength	
			Pounds	Per Cent
No. 6 covered copper wire—initial breaking load, 1,165 pounds(average of six specimens)				
400		1,020	145	12.4
700	1	990	175	15.0
1,000	3/4	970	195	16.8
1,100	3/4	940	225	19.3
1,500	5/8	830	335	28.8
1,900	1/2	795	370	31.8
No. 4 covered copper wire—initial breaking load, 1,680 pounds (average of three specimens)				
2,500	3/4	1,265	415	24.7
4,300	5/8	1,225	455	27.0
No. 2 covered copper wire—initial breaking load, 2,820 pounds (average of three specimens)				
4,300	5/8	2,315	505	17.9
7,000	1/2	2,225	595	21.0

In general, No. 6 wire, when built into lines, is applied so that the tensile stress will not exceed 560 pounds. For No. 4 and No. 2 wires these figures are 920 and 1,200 pounds, respectively, under the most severe loadings anticipated.

The shorting contactor is designed to operate at current values 50 per cent over the normal load-current rating of the series coil. Rated load currents can be carried continuously without exceeding normal temperature rise limits, and loads up to 150 per cent of rated load have been carried for several hours without damage to the equipment.

The curves of figure 7 show the switching speed of the shorting contactor with coils of three different load-current ratings. It will be noted that all curves become flat at one-half-cycle over-all time. The different coils are interchangeable in the same housing.

It might be mentioned that the principle of de-energizing a circuit by applying an artificial short circuit at the source for the purpose of extinguishing line arcs is not new. It was suggested many years ago but, as originally proposed, there were no means by which operation within one-half cycle could be accomplished and no provision was made for holding the short-circuiting element inoperative after the initial operation. In the shorting contactor the control mechanism functions to lock the contacts in the open position within four cycles after the plunger resets and, as now built, the lockout period can be adjusted for any length of time up to four minutes.

Application of Shorting Contactor to Circuits Fused at the Station

For the lower capacity circuits that do not warrant station breakers, another form of the shorting contactor has been developed. In this form the contactor has its own interrupter to break the artificial short circuit which it applies between phases. Figure 8 shows an installation of this equipment from which it can be seen that the interrupter is connected in the short-circuit lead from the contactor and that it carries no load current. The design principle of the interrupter was developed by The Detroit Edison Company a number of years ago. In normal operation in this application the interrupter is in the closed position. When a fault occurs, the contactor short-circuits the line through the interrupter, thus collapsing the voltage. Current through the interrupter causes it to open and lock out for a predetermined time. This removes the metallic short circuit and allows recovery of line voltage. If the fault is transient, service is restored but, if the fault persists, the shorting contactor is prevented from de-energizing the line again, since the interrupter is locked in the open position. Fuses are then brought into play and serve to clear the fault. Although the interrupter is capable of opening the artificial short circuit within one cycle, it is purposely delayed in this operation for six cycles to avoid the possibility of re-establishing an arc at the initial location of fault. After a prescribed time, the interrupter automatically resets.

Table V. Approximate Limits of Duration of Arcing on Covered Copper Wire if Serious Loss of Mechanical Strength Is to Be Avoided

Size of Conductors	Current Range (Amperes)	Limiting Duration of Arcing (Cycles)
No. 6	Up to 500	3 to 2
	500 to 1,000	2 to 1
	1,000 to 1,500	1 to 1/2
No. 4	Up to 1,000	3 to 2
	1,000 to 2,000	2 to 1
	2,000 to 4,000	1 to 1/2
No. 2	Up to 2,000	3 to 2
	2,000 to 5,000	2 to 1
	5,000 to 7,000	1 to 1/2
No. 1/0*	5,000 to 10,000	2 to 1/2
No. 4/0*	8,000 to 15,000	2 to 1/2

*Values shown for these sizes are estimated.

This type of installation is particularly suited to stations where a large number of small conductors radiate from the station bus as indicated in figure 8. Such stations are usually of sufficiently small capacity so that application of the short circuit directly at the station is acceptable. Since the total interruption for each operation is only six cycles, there is no particular disadvantage in de-energizing the bus to protect one line from damage; a six-cycle interruption is hardly perceptible to the customer.

Two trial installations of this type, protecting two circuits each, have been in service on the Detroit Edison system for over a year and have given a good account of themselves.

Fast-Opening and Reclosing Breaker

The third device developed is intended for use in suburban and rural circuits of fairly large capacity where the advantages of fully automatic reclosing operation are desirable. In the design of this device advantage was taken of the principle of operation incorporated in the interrupter previously described as developed by the company some years ago. This interrupter had proved itself capable of interrupting fault current consistently with not over one-half-cycle arcing at the contacts and with exceptionally low arc energy, qualities essential in a breaker having over-all interrupting time (two cycles) of the order here required.

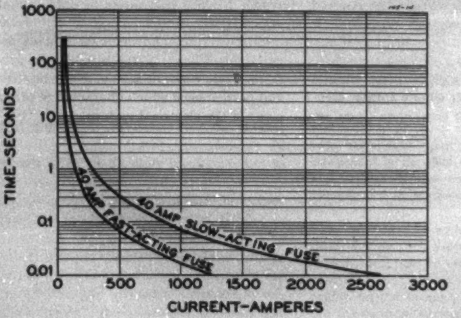


Figure 14. Characteristics of 40-ampere slow-acting and fast-acting fuses

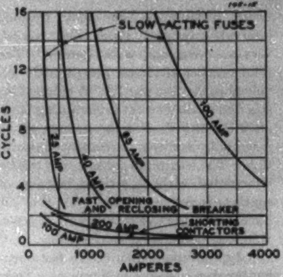


Figure 15. Characteristics of slow-acting fuses compared with switching speeds of new devices

The equipment takes the form of a single-pole, oil circuit breaker, light in weight, self-contained, and fully automatic, having its own mechanical timing mechanism. No auxiliary circuits or equipment are required for its operation. The control mechanism is arranged to give a fast initial opening followed by reclosure and a series of time-delay openings and reclosures before lockout. The mechanism resets automatically from any point in the sequence of operation, short of lockout position, upon restoration of normal circuit conditions. Units, having load-current ratings of 100 and 200 amperes, have been built for extensive trial on the Detroit Edison system. The speed of initial opening for the 100-ampere unit is shown in figure 9. It will be noted that the curve becomes flat at two-cycle over-all time and 700 amperes. These units are set to trip at 200 per cent of continuous rating. Currents up to the minimum current to operate can be carried for prolonged periods without dangerous heating.

In the latest designs the breaker will be built for load-current ratings of 100, 200, and 300 amperes with initial trip settings of 125 per cent of load-current rating.

A diagrammatic view of the device is shown in figure 10. The coil of the control relay (4) is in series with the line. In normal operation the solenoid coil (6) is shunted through switch (5). When a fault occurs, switch (5) opens and fault current flows through coil (6) which causes main contacts (9) to open. Energy stored in a spring during the opening stroke is used to reclose the breaker. In a test of one of the early experimental models of the breaker it was put through a series of complete operating cycles to lock out, at 5,000 volts, on short circuits ranging from 1,000 to 4,000 amperes. The operating cycles included initial opening followed by three time-delay operations with three-fourths second between reclosures. Time-delay tripping intervals averaged about one second, varying inversely with current. The total number of circuit interruptions involved in the test was 72. A detailed inspection of the current-interrupting parts after the test showed the breaker to be in suitable condition for continued operation. The maximum current that can be interrupted without serious burning of contacts is not known but tests thus far have shown that 7,000 amperes at 5,000 volts can be handled successfully.

A typical application of the fast-opening breaker is shown in figure 11. Only two units are required in an ungrounded system for a three-phase circuit and one unit is sufficient for a single-phase circuit. Where the advantage of one-half-cycle switching is desired in circuits equipped with the fast-opening breakers to assure against burndown of No. 6 covered wire, the shorting contactor may be added to the circuit. In an arrangement of this sort an arc will be extinguished in one-half cycle, and the artificial short circuit applied by the contactor will be removed from the system in two cycles by the breaker.

A combined plot of the burndown characteristics of covered wire and the switching speeds of the newly developed devices, all of which have been shown in earlier figures, is given in figure 12. In comparing these two sets of curves it should be remembered that the curves relating to the wires give critical values which will result in actual burndown. To hold arc damage within safe limits, the duration of arcing for given currents must be less than the values given by these curves. For rough approximation it may be said that, if arc duration is limited to about a quarter of the time shown by the burndown curve, arc damage will not be serious and the wire will retain ample mechanical strength for satisfactory continued use.

Figure 13 shows the appearance of 1,500-ampere arc burns on No. 6, No. 4, and No. 2 covered wire (*a*, *b*, and *c*) which were protected by the shorting contactor. The No. 6 wire retains 80 per cent of its original strength. The No. 2 conductor (*d*) shown in the figure was subjected to a 1,500-ampere arc which was cleared in two cycles by the fast-opening breaker. It retains 75 per cent of its original strength.

Fuses

Since the new high-speed switching devices assure the protection of fuses against operation on transient faults, the use of sectionalizing fuses may be extended without fear of creating more outages than may be avoided by their use. This new condition limits the function of fuses to clearing faulted sections or equipment in case of persistent faults only when fault current is purposely held on for a long enough period to cause them to blow. It is obvious then that fuses of the slow-acting type are more suitable with the fast switching methods than the fast-acting type. Progress made in the development of fuses with slow-acting characteristics is shown in figure 14 where the curves for 40-ampere fuses of both the fast-acting and slow-acting types are compared. Similar differences exist in other ratings. It can be seen that, for a given time to blow, the current carried by the slow-acting fuse is much greater than that for the fast-acting fuse. This means that the slow-acting type can be installed at a point in the circuit much closer to the station, where fault currents are higher, than is possible with the fast-acting type. In other words, the restrictions previously imposed upon the location of fuses are now greatly reduced. The coordination between the characteristics of slow-acting fuses and fast switching speeds of the newly developed equipment is shown in figure 15.

Conclusions

Complete interruption of distribution circuits within one-half to two cycles after fault inception will protect covered wire from burndown and unacceptable arc damage and will save fuses from operation in those cases of trouble in which the cause of fault is of transient nature. Since the line is left wholly intact, service is restored to all customers upon reclosure. Where faults prove to be persistent, the immediately following time-delay operations will blow fuses or burn the fault clear, if possible, the same as in present practice. Indications are that well over half the faults on distribution lines are of the transient type.

Several years' experience on a number of systems with so-called "instantaneous" tripping and "immediate" reclosure have disclosed no serious operating or customer problems with this pattern of operation.

Assurance against fuse operation on transient faults makes possible full realization of the potential benefits of line sectionalization.

Another advantage of high-speed initial operation is that a transient fault on one phase of a multicircuit line will not involve other phases or circuits. With prolonged arcing accompanying delayed initial operation, the rapidly increasing size of the hot gas envelope may quickly encompass the other conductors inducing further arc-overs and damage.

The addition of shorting contactors to established circuits having automatic switchgear will provide these circuits with the advantages of one-half-cycle initial interruption without major changes or expense. The addition of quick trip and immediate reclosure to the station breaker is desirable and inexpensive. The shorting contactor with its own interrupter offers a simple means for protecting a group of low-capacity circuits radiating from the station bus against failures on transient faults. The use of fast-opening and reclosing breakers on circuits that would normally be fused at the station will add to such circuits the advantages of fully automatic switchgear in addition to the benefits of two-cycle initial interruption. Single-phase switching will also reduce the number of customer interruptions.

Reducing the cases of wires down will reduce hazards and promote public good will. It will reduce outage and thereby improve service and decrease cost of repairs. It will bring the primary circuits more in line with the safety, dependability and permanence which characterize other parts of the electrical system.

Field Power-Factor Testing of Transformer Insulation and Operating Experience

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ONE requirement of a field test on transformers is that weaknesses which develop in bushing or winding insulation be readily located and failures prevented. This has been effectively accomplished in the American Gas and Electric Company system by a power-factor testing program of transformer insulation which was initiated in the fall of 1933. For some 18 months prior to this time, a field testing program for oil circuit-breaker insulation had been in progress. As pointed out in previous publications,[1,4] these tests had proved satisfactory particularly in locating faulty bushings and bad oil circuit-breaker insulation. It is the purpose of this paper to discuss the tests we have made on transformer winding insulation, the methods used in locating faulty conditions, and the service record of some 5,000 transformers of which 4,000 or so have been included in the test program. The tests have been useful in effectively classifying the condition of insulation thus establishing a guide for the necessary maintenance to minimize service outages. Transformer winding insulation classed unsuitable for further service has been successfully reconditioned. In a number of cases the cause of progressive faults has been detected and eliminated without need of major repairs. This procedure has helped materially in keeping equipment in first-class operating condition, in preventing failures, and in realizing substantial monetary savings.

Figure 1. Light panel delivery test car shown with back door open and power-factor test equipment in its usual position but with the high-voltage cable unreeled, ground wire connected, and in readiness for field test work

Test transformer is on floor while instrument case, writing pad, thermometers, file for test records, etc., can be seen on the shelf. The container for the high-voltage cable, shown resting on the bumper, slides into the spare tire compartment beneath the test transformer

Field Development of Tests on Transformers

Our first tests of transformer insulation were made on 200 or so 44-kv units of 500 kva or less in 1931–32. In these tests the primary object was to locate faulty bushings which at that time were disconnected from their windings and tested individually. Tests in 1933 were extended to instrument and control transformers rated 22 kv and above. In the 1933 tests bushings and windings were tested in parallel and disconnections inside the transformers were in general unnecessary particularly after improved testing methods were developed.

When the routine tests were extended to large power transformers in January 1934, it was thought, in order to locate faulty bushings, they would have to be disconnected from their windings and tested separately. This procedure was started but found unnecessary as well as slow and costly. Losses in the winding insulation were low enough so that a defective bushing might show up to some extent in most units by simply making over-all tests on the entire transformer without any internal disconnections. This procedure, of course, greatly simplified and speeded up the test work on power and distribution transformers.

At first there was considerable speculation as to whether or not faulty bushings could be consistently located by the over-all test on large transformers. During April 1934, however, some faulty bushings were located in medium size power transformers by over-all tests. Shortly after this time the manner of making the tests was changed to a method which, except for refinements, is the same as that used today. With the present high degree of development in the testing of insulation in the field, it is now seldom necessary to make any disconnections inside of transformers. It has been possible on a practical basis to individually classify the quality of the insulation of each bushing as well as that of the windings from one set of simple over-all measurements. The test methods and procedure by which this is done will be described later.

Test Equipment

The test equipment used consists of a transformer for supplying test potential up to ten kilovolt, a wattmeter, a current meter, and a voltmeter, all arranged in suitable cases and equipped with appropriate attachments. As pointed out in previous publications,[1,4] this equipment is manufactured by the Doble Engineering Company and has proved very suitable for field test work.

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[1] For all numbered references, see list at end of paper.

Figure 2 (left). Field power-factor tests being made on a set of 33-kv bus potential transformers

Figure 3. Power-factor test car

Test equipment set up and ready to begin tests on a group of large power transformers at a major station

Three test outfits, each operated by an experienced test man, are now continuously used in making field power-factor tests. Figure 1 shows one of these outfits with the test equipment in the rear of a light-weight panel delivery car. Figure 2 shows tests in progress on a set of 33-kv, bus potential transformers. Figure 3 shows a test car set up and ready to start tests on a group of large power transformers at a major station. Test outfits of this kind have proved to be practical in carrying out the field work.

Test Routine

At the present time we are testing transformers, breakers, etc., about once in every $2^1/_2$ years. It normally requires approximately an hour to test a medium size bank of three or four single-phase, two-winding transformers. If faulty conditions are indicated by the normal over-all tests it occasionally becomes necessary to take readings on individual bushings or other parts, and this, of course, increases the time required to make the tests.

Equipment Tested

When routine power-factor tests were started on transformers, it was planned to test all equipment in stations operating 22 kv and above. At the present time tests are being made on practically all oil-filled transformers located in outdoor stations and operating 11 kv or above with the exception of some small isolated stations. The number of transformers tested or retested each year is approximately as follows:

1931–32	200
1933	200
1934	1,100
1935	1,000
1936	1,150
1937	1,150
1938	1,260
1939	1,400
1940 (7 months)	750

About half of these are instrument and control transformers while the others are large distribution or power units. At the time transformer tests are made, other equipment is also tested such as oil circuit breakers, underground cables, carrier-current coupling capacitors, switch sticks, sticks for hot line work, and other miscellaneous insulating parts. In general, the equipment tested is located outdoors although some indoor equipment is occasionally tested.

Table I. Typical Case of Faulty Bushing Located in a Bank of Power Transformers by Over-all Power-Factor Tests

	Over-all Tests Including Bushings and Windings			
	Other Windings Grounded		Other Windings to Shield	
Transformer	Per Cent Power Factor	Microamps*	Per Cent Power Factor	Microamps*
A—High side	1.5	18,000	1.2	4,500
B—High side	1.3	17,800	1.0	4,400
C—High side	1.6	18,000	2.4	4,400
A—Low side	1.2	27,500	1.0	13,850
B—Low side	1.2	27,100	1.0	13,300
C—Low side	1.3	26,400	1.0	12,800

		Collar Test on Bushings in Trans.		Individual Test Bushing Out of Trans.	
Transformer	Bushing	Watts	Microamps	Per Cent Power Factor	Microamps
A	H_1	0.05	106		
A	H_2	0.04	101		
B	H_1	0.03	101		
B	H_2	0.20	122	1.9	475
C	H_1	0.31	137	19.3	555
C	H_2	0.18	111	1.5	505

*Ten-kv values are shown although low side actually tested at 2.5 kv.

Test Methods Used

In testing a power transformer, one winding at a time is short-circuited, test voltage applied with the other winding or windings grounded. A second test is made in which the winding or windings not under test are connected to the shield circuit of the test set. These two over-all tests for each winding of large transformers have been used for the past six years although some improvements in testing procedure as well as in the test equipment have been accomplished during this period. The over-all tests for instrument or other small transformers are slightly different and were described in a previous[4] publication.

In addition to the over-all tests which are normally made on transformers, the solid types of high-voltage bushings are given a collar test. This is done by simply wrapping a suitable metal band around the porcelain near the top of the bushing and applying ten kilovolt from this band to the bushing conductor. Figure 4 shows a collar test being made on a 22-kv O.C.B. bushing. A flat spiral spring has been used to wrap around the porcelain and is simply contacted by a special fitting on the ten-kilovolt test terminal.

Interpretation of Test Results

Locating Faulty Bushings in Small Transformers

In most instrument or other small transformers, the bushings have capacitances only slightly lower than those of the windings. In making over-all tests with bushings and windings in parallel, any faulty bushings which may be present will therefore show up to about the same extent as a faulty winding. It therefore is not often difficult to determine what parts are faulty from the over-all tests. The following is a typical example of a faulty bushing located in a 33-kv instrument potential transformer.

	Microamps	Watts	Power Factor (Per Cent)
Test on bushing *A* plus bushing *B* plus winding in parallel	2,250	1.41	6.3
Test on bushing *A* with bushing *B* connected to shield	1,125	0.26	2.3
Test on bushing *B* with bushing *A* connected to shield	1,125	1.15	10.3
Test on bushing *A* removed from transformer	445	0.09	2.1
Test on bushing *B* removed from transformer	505	1.21	24.0

It will be noticed from this illustration that bushing *B* is shown to be defective by over-all test and also by those on the individual parts. When making power-factor tests on small transformers one is not likely to overlook a faulty bushing of the above nature.

Figure 4. Method of making collar test

A flat-coiled spring is wrapped around the porcelain of a 22-kv O.C.B. bushing and held in place by a special attachment on the ten-kilovolt test terminal

Locating Faulty Bushings in Large Transformers

In power and large distribution transformers, the capacitance of the windings may be many times that of a single bushing but, by careful analysis, it has been possible to determine from the over-all tests just what part if any is defective in transformers of all sizes. A system of comparisons of test values is used which, after some experience, enables the test man to determine the location of faulty parts. Table I shows a typical example of a faulty bushing located in a bank of three 2,000-kva, 22 to 2.3-kv, single-phase transformers. The over-all tests, of bushings and windings in parallel, point to a faulty high-voltage bushing in transformer *C*. This is indicated by (1) the power factor with the secondary windings connected to ground being slightly higher than that of the other two transformers, and (2) with the secondary winding connected to shield, the power factor is roughly double that of the other two transformers, while (3) the power factors on the low-voltage side of all three units are practically the same. These over-all tests therefore point to uniformity of the winding insulation and a fault in one of the high-voltage bushings of transformer *C*.

Locating Faulty Bushings by Collar Tests

The collar tests in the lower part of table I are very indicative of a fault in bushing H_1 of transformer *C* since the watts loss is considerably higher than that of the other bushings and the current value is also appreciably higher. In addition to this fault bushing *H*2 in transformer *C* as well as bushing *H*2 in transformer *B* showed slightly high collar test. An internal investigation was made which showed that bushing *H*1 in transformer *C* had a power factor of 19.3 per cent. This bushing of course was removed from service. The other two bushings having slightly high collar tests were found to have low power factors and therefore were left in service. In some of the larger power transformers the winding capacitances may be some two or three times that of units shown in table I. This of course makes it slightly more difficult to locate faulty bushings. Collar tests which were first made in our company during 1936 have proved to be very useful in helping to locate faulty bushings in transformers. By observing the over-all tests as well as the collar test, it has therefore been possible to locate bushing faults in all sizes of transformers without making any internal disconnections except in a few unusual instances.

Faults in Transformer Winding Insulation

The nature of transformer winding faults can be determined by an analysis of the test readings. In possibly 90 per cent of the transformers found to have high power-factor winding insulation, the cause has been traced to the presence of

Table II. The Effect of Different Types of Faults in Transformers Upon Power Factor and Charging Current

	Test Results				
	Before Maintenance Operation		After Maintenance Operation		
Item	Per Cent Power Factor 20 Deg C	Milliamps at 10 Kv	Per Cent Power Factor 20 Deg C	Milliamps at 10 Kv	Remarks
1	19.4	10.4	3.4	8.8	Winding dried out
2	6.8	30.6	1.6	24.5	Winding dried out
3	9.3	2.1	3.3	2.2	12.7% power factor oil replaced
4	5.5	10.6	4.2	10.5	19.2% power factor oil replaced
5	4.0	11.8	1.8	11.3	34.1% power factor bushing replaced
6	7.1	25.3	1.6	25.9	59.0% power factor bushing replaced

moisture throughout the insulation. The other ten per cent or so of transformers having high-test readings have been caused mostly by the presence of high power-factor oil. Table II shows typical test results for different types of faults in transformers. Items 1 and 2 of this table show the effect of moisture in the winding insulation. The removal of this moisture by drying not only caused a decided decrease in power factor but also an appreciable decrease in capacitance. The decreased capacitance is shown in item 1 by a decrease of from 10.4 milliamperes at 10 kv before drying to 8.8 after drying. In item 2 the decrease is from 30.6 milliamperes to 24.5 as a result of drying the winding insulation. Items 3 and 4 of this same table show the effect of replacing high power-factor oil in two transformers. It will be noticed that while there was an appreciable decrease in power factor as a result of the oil change, there is very little change in capacitance or measured current values. The presence of high power-factor oil can therefore be distinguished from a fault caused by moisture in that moisture causes an increase in capacitance and high power-factor oil does not. Separate power-factor tests can also be made on oil to determine the effect on over-all test readings.

Test experience has shown that concentrated insulation faults in the main body of transformer windings are exceedingly rare. Moisture which reaches the winding usually seems to become distributed throughout the insulation within a few months, thus causing a general rather than a concentrated fault. Bad bushings or terminal boards may, of course, be considered as concentrated faults. Referring again to table II, items 5 and 6 show the effect of bushing faults upon the over-all test readings of two typical transformers. It will be noticed that the removal of the faulty parts caused an appreciable decrease in power factor but very little difference in current or capacitance values. From such data it is apparent that any concentrated fault, while resulting in high power-factor readings, will not cause abnormally high capacitance.

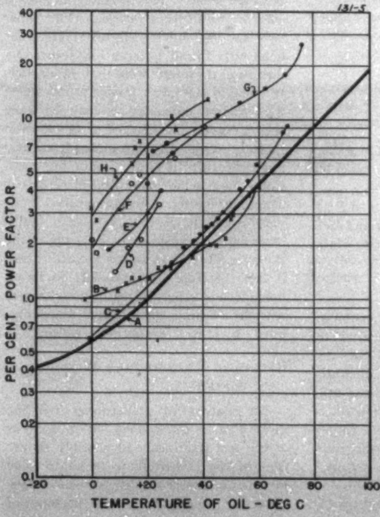

Figure 5. Temperature correction curves.

Curve *A* used in correcting power-factor readings on transformer winding insulation

Curves *B* to *H* are temperature-power factor curves run on transformers in the field and are typical of the ones from which *A* was derived

Information Obtained From Transformer Tests

From the above discussion, it is apparent that from a set of over-all readings on a transformer, collar test on the bushings, and power-factor test of its oil, one can determine:

1. The condition of each bushing to the extent of classifying its insulation.
2. The condition of the oil as far as power factor is concerned.
3. The condition of the winding insulation.
4. The general nature of faults, if any, which may exist in the winding insulation.

This affords a rather complete check of the entire insulation of the transformer.

Power-Factor Limits

Table III shows the power-factor values used in classifying transformer winding insulation. In general if the insulation is considered to be in good or satisfactory condition no maintenance recommendations are made regarding the transformer. When the winding insulation is between 3.5 and 5 per cent power factor (6 per cent for C.T. windings), the insulation is rated deteriorated. This classification is used to indicate that some fault has developed in the winding insulation but the condition is not serious enough to warrant extensive reconditioning such as drying. Transformers in this class however, are often looked over carefully in order to determine the cause of the fault which has developed in the winding insulation. Quite often faulty gasket seals are located or else the breathers may be found to be improperly located. After such minor items are taken care of, most of these transformers have given satisfac-

Table III. Power-Factor Values Used in Classifying the Condition of Transformer Winding Insulation

Classification of Winding Insulation	Power Factor in Per Cent at 20 Deg C
"G" = Good	0—3.5
"D" = Deteriorated	3.5—5*
"R" = Remove from service	Above 5*

*Six per cent power factor for current transformer windings.

tory service performance without any noticeable increase in the winding power factor for several years. Transformer windings having power factors above five per cent (six per cent for C.T.'s) are classified "remove." The cause of high power factor is usually assigned to moisture in the winding insulation. The transformers in this group are therefore generally removed from service and dried out. This treatment has in almost all instances resulted in correcting the faults as evidenced by subsequent tests.

TEMPERATURE CORRECTIONS

Since field tests on transformers are made at all times of the year and following either light or heavy loaded periods, it is necessary to apply temperature corrections to the power-factor test results for analysis purposes.

Curve *A* in figure 5 is used in converting power factor of transformer winding insulation to a base temperature of 20 degrees centigrade. Actual temperature power-factor curves run on transformers which have had considerable service in the field are shown in *B* to *H* of figure 5. These curves are typical of those from which the correction curve *A* was derived. Temperature corrections have proved practical and very useful in classifying and comparing transformer insulation by power-factor tests.

Table IV. Successive Tests on One Obsolete Type of Oil-Filled Current Transformers a Large Part of Which Were Found to Have Moisture in the Winding Insulation

	Fault Rating at Time of Test			Above Six Per Cent Power Factor at 20 Deg C (Some—D, Some—R)	Not Tested	Windings Dried Out
	G = Good	D = Deteriorated	R = Remove			
1934	11	25	16	41	0	17
1935	18	23	11	33	0	8
1938	26	17	5	22	4	8
1940	22	12	10	17	8	

TYPICAL TEST EXPERIENCE

The first transformers dried out and found to be greatly improved in power factor were in a group of some 52, 22-kv current transformers first tested in 1934. A summary of the tests made on this group of transformers from year to year is shown in table IV. The transformer cases and bushings were of an obsolete design and very difficult to make watertight, which accounts for the high percentage of winding faults. The experience with these transformers also illustrates clearly the ability of the power-factor test to indicate typical design faults in equipment. Before power-factor tests were made on these transformers, considerable trouble had been experienced, and it was thought necessary to change the oil frequently in an attempt to prevent winding failures. The first test showed some of the windings to have power factors as high as 15 to 17 per cent. Approximately one-third of the group (those with highest power factors) were removed from service and the windings dried out. Most of these windings returned to about two or three per cent power factor, which was a decided improvement. This group of current transformers (table IV) has been included in the regular testing program for some six years, and only one failure was experienced during this time. This shows that equipment found faulty by the power-factor test can be satisfactorily reconditioned.

That moisture is in many instances the cause of high power factor has been determined by field experience, but a dielectric test on oil drawn from the bottom of the case is in most instances not a reliable criterion. In fact, the dielectric tests on the oil at time of power-factor test or during previous years of service gave no noticeable indication of faulty conditions which existed in the majority of those transformers which were found to have winding insulation requiring drying. These windings when dried out returned to normal good condition as indicated by low power-factor values thus indicating that moisture was present. On the other hand, transformers in which appreciable quantities of water have been found in the oil drawn from the bottom of the tanks quite often have high power-factor winding insulation. This relation, however, is not very consistent, and obviously the best indication of moisture in the winding insulation is the magnitude of its power factor. As contrasted to the lack of correlation between dielectric of the oil and power factor of the winding, our experienced transformer men, when making winding inspections, consistently report findings of rusted spots on iron and other signs of moisture which compare very reasonably with power-factor test. Furthermore, in transformers which have high power-factor windings, a thorough inspection almost always reveals a point of very probable moisture entrance.

Operating Experience

Shown in table V are the operating and test records of some 5,000 transformers. In the table these transformers have been arranged in groups as indicated by columns 1 and 2. The service record of

Table V. Service Record of Transformers January 1, 1934 to August 1, 1940 From Available Records October 15, 1940

(1) Test Rating of Winding Insulation	(2) No. of Transformers	(3) No. Cases of Winding Trouble	(4) Transformer Years of Service = No. Transformer × Years Service	(5) No. Cases of Winding Trouble Per 1,000 Transformer Years Service	(6) Weather Conditions at Time of Trouble	(7)	(8)
					Lightning	No Lightning	Not Recorded
Good*	3,150	22	15,000	1.5	8	14	0
Deteriorated*	840	7	4,000	1.7	1	4	2
Remove*	220	13	500	26.0	2	7	4
Reconditioned and test good*	250	1	400	2.5			1
Not tested until after January 1, 1934**		29	4,500	6.5	13	12	4
Never tested	1,100	88	6,000	14.7	43	34	11
Total	5,310	159	30,000	5.3	67	71	21

*Service record after first power-factor test.

**Service record from January 1, 1934 to time of first test.

each group is shown in columns 3 to 8. A direct comparison of the service performance is shown by column 5 in which is listed the number of cases of winding trouble per 1,000 transformer years of service.

This column shows that there was an average of only 1.5 cases of trouble per thousand transformer years in those windings having good insulation as judged by power-factor testing. The deteriorated transformers have almost as good a record showing only 1.7 cases of trouble. The transformers in the remove group, of course, were in most instances removed from service and reconditioned shortly after the tests were made. However, during the early stages of the test work some of these were left in service for some time, and from the table, it will be noticed that the failure record was much higher than in any other group as there was an average of 26 failures per thousand transformer years of service. In the reconditioned transformers where windings had been dried out and the units placed back in service, there has been one failure experienced or an average of 2.5 failures per thousand transformer years of service. This one failure happened to be in an obsolete type of current transformer in which difficulty was encountered in making tight moisture seals. In the service record of transformers between January 1934, and the time they were first power-factor tested, there was an average of 6.5 cases of winding trouble per thousand transformer years which was considerably higher than was experienced in transformers which tested good. In the group of transformers which has never been power-factor tested, there is an even higher rate of winding troubles as the table shows 14.7 per thousand years of service. This higher figure can be partially credited to the fact that many of these transformers are located in outlying stations and have not received as high a degree of maintenance as those in the more important stations. It so happens that a good many of these transformers are located in fairly severe lightning territory, which also probably had some effect on the troubles experienced.

The last three columns in the table show weather conditions at the time winding trouble was experienced. It will be noticed that in the group which was never tested, there was a comparatively large percentage of troubles which occurred during lightning storms. From the totals shown on the bottom line of table V it would appear, however, that more than half of the cases of trouble took place when there was no lightning.

From table V, it is apparent that transformers with high power-factor winding insulation are much more likely to fail in service than those which have low power-factor values. It is also apparent that the failure rate is appreciably higher in untested than in tested transformers, provided, of course, the transformers which test bad are removed from service or reconditioned before they have time to fail. In general, the data in table V point toward the idea that an appreciable number of transformer winding failures are the result of gradual development in the insulation of weaknesses which can be detected by periodic tests and remedied by simple reconditioning.

Table VI. Successive Tests on Two Groups of Transformers Showing the Effect of Maintenance Operations

104 Power and Distribution Transformers

	Test Rating of Winding Insulation						
	G = Good		D = Deteriorated		R = Remove		
Date Tested	No.	Per Cent	No.	Per Cent	No.	Per Cent	Remarks
1933–34	88	84.6	13	12.5	3	2.9	
1935	83	79.8	16	15.4	5	4.8	
1937	80	77.0	20	19.2	4	3.8	22 transformers overhauled and 16 windings dried
1939	95	91.3	9	8.7	0	0	

163 Instrument and Control Transformers

	Test Rating of Winding Insulation						
	G = Good		D = Deteriorated		R = Remove		
Date Tested	No.	Per Cent	No.	Per Cent	No.	Per Cent	Remarks
1934	111	68	40	24.5	12	7.4	Oil changed in one transformer
1935	102	62.5	46	28.2	15	9.2	Oil changed in 6 transformers and 17 windings dried
1937	112	69	45	27.6	6	3.7	Oil changed in 3 transformers and 7 windings dried
1939	118	72.5	45	27.6	0	0	

Improvements in Maintenance and Operating Conditions

It is apparent that a number of failures has been prevented as a result of the power-factor testing program. While it is, of course, difficult to estimate just how many failures were prevented, it seems conservative, in the light of data such as that in table V, to say that some 100 additional cases of transformer trouble could have been expected if testing of the windings had not been undertaken some six years ago. Placing the cost of a case of winding trouble at $500, this would amount to a saving of some $50,000. Besides this, there has been an appreciable number of faulty bushings located by the test and removed from service before failure took place. By the detection and elimination of these winding and bushing faults, an appreciable number of costly failures has, no doubt, been prevented, thus minimizing interruptions to customers service.

The power-factor testing program has also created a keener interest in maintenance problems throughout the properties. So far as the insulation in bushings and transformer windings was concerned, there was no qualitative test on which the maintenance man could rely prior to the initiation of power-factor testing. For this reason, the maintenance man used to have to work in the dark so far as any accurate classification of insulation quality was concerned. The power-factor test has therefore been useful as a guide to detect the pieces of insulation requiring attention and to indicate the effectiveness of reconditioning or drying operations.

Table VI shows what was done on one property in improving the condition of transformer winding insulation. The first part of this table shows the test records on 104 power and distribution transformers, while the latter half shows the same data for 163 instrument and control transformers. It will be noticed that there is a slight increase in the number of faulty windings as indicated by the 1935 tests. There was, however, some improvement shown by the 1937 tests and a decided improvement in 1939. The increase in the number of faulty windings during the first three years or so can, no doubt, be credited to the usual rate of deterioration of winding insulation due to service conditions. The improvement shown by the 1937 and 1939 tests can be credited to the fact, that a number of transformers were overhauled and windings dried out. This improvement is especially apparent in the transformers which were classified for removal from service, since the 1939 tests did not show any windings which required servicing such as drying the insulation. These data in table VI are typical examples of what has been accomplished due to keener interest and conscientious efforts in the maintenance of insulation quality in equipment in service.

It has been possible to not only determine the nature of faults in equipment in service, but to find the cause so that recurrence of such deterioration can be prevented in the future. Likewise, the cause of minor faults has been determined and steps taken to prevent them from developing into anything of a serious nature. In this way a great deal of major maintenance such as the drying of transformer windings will, no doubt, be eliminated. It has been found that moisture frequently gets into transformers through defective gaskets under manhole or other small cover plates on the top covers of transformers. Under such plates, it has been found to be of considerable advantage to eliminate felt gaskets. The location and types of breathers on transformers have also been studied. A high percentage of faulty windings has been located in transformers where the breathers have their open ends only a few inches above the cover of transformers so that they might at times be completely covered by snow or ice. For this reason, some breathers have been relocated so as to have their open ends extend out over the edge of the top cover of the transformer. By making a few mechanical changes such as this and by eliminating defective moisture seals, we have every reason to believe that much of the equipment is more dependable and that the rate of deterioration of the insulation due to occasional entrance of moisture has been greatly decreased. The maintenance of transformer winding and bushing insulation has therefore been accomplished with a high degree of refinement which would not have been possible without a searching test like the power-factor method to indicate insulation quality and the conscientious efforts in the field to eliminate the causes of insulation deterioration.

Summary and Conclusions

From our operating and test experience with 5,000 or more transformers, and in the light of the foregoing discussion, the following conclusions can be drawn:

1. The power-factor test has proved to be very useful in classifying the quality of insulation in transformer windings as well as that of bushings and oil circuit breakers in service in the field.

2. From a simple set of over-all readings on a transformer, it is possible to determine the location of faults whether they be in the winding insulation or any one of the bushings.

3. In addition to finding the location of faults in transformer insulation it is possible to evaluate the nature or severity of deterioration that has taken place and classify the insulation as to its electrical condition for future service.

4. The power-factor test program as now being carried out on the American Gas and Electric Company system has created a keener interest in maintenance problems and therefore helped appreciably in establishing the high degree of care given to electrical equipment in the field.

5. It has been possible to locate the cause of faults indicated by power-factor test and by simple methods prevent or greatly retard the deterioration of insulation.

6. As a result of field power-factor test work, it is roughly estimated that at least 100 cases of trouble in transformer windings have been prevented at a direct saving of $50,000. By locating and correcting faulty conditions in the insulation of transformer windings, bushings, and other apparatus costly and embarrassing interruptions to customers' service have been prevented.

Bibliography

1. Testing Bushings and Insulation by Power Factor Method, I. W. Gross and H. E. Turner. *Electrical World*, January 13, 1934, pages 68–73; January 20, 1934, pages 111–16.

2. Portable Schering Bridge for Field Tests, C. F. Hill, T. R. Watts, and G. A. Burr. AIEE Transactions, 1934, pages 176–82.

3. Power Factor Testing of Transformer Insulation, J. B. Hodtum. AIEE Pacific Coast convention, August 27–30, 1940.

4. Field Testing of Bushings and Transformer Insulation by the Power Factor Method, I. W. Gross. Electrical Engineering, October 1938, pages 589–96.

5. Power Factor Testing of Transformer Insulation, Frank C. Doble and G. H. Browning. *Electrical World*, December 2, 1939, pages 1583–4 and 1662–3.

Shielding of Transmission Lines

Discussion and authors' closure of paper 40-107 by C. F. Wagner, G. D. McCann, and G. L. MacLane, Jr., presented at the AIEE summer convention, Swampscott, Mass., June 24–28, 1940, and published in AIEE TRANSACTIONS, 1941, pages 313–28.

J. H. Hagenguth (General Electric Company, Pittsfield, Mass.): The authors have presented their subject in such a thorough manner, and have made such exhaustive tests, that their major conclusions, with respect to the protective angle of ground wires, probably are quite correct. Their experiments and subsequent tolerances appear to check the results obtained by Matthias (references 8 and 9 of the paper).

Like all model tests of course the method adopted by the authors is based on certain assumptions. The authors prefer positive polarity because the streamers from the ground electrode have a minimum length under this condition, although it is practically certain that a majority of lightning strokes are of negative polarity. Their choice is principally based on the fact that, in photographs of natural lightning, streamers from the ground are very rarely visible, and, if they are, they are of extremely short length. In the laboratory, arcs with negative point streamers from the positive ground may reach lengths of 50 per cent or even greater of the total gap length.

Now, if the observations of the overwhelming number of negative strokes are correct, and since the photographs of natural lightning rarely show long streamers from the positive ground, quite in contrast with the laboratory experiment, then there must exist profound differences between the lightning stroke and the laboratory spark.

In the laboratory tests, we tacitly assume that not only the dimensions of the model such as cloud height, conductor height, etc., are to scale, but also the voltages. Thus, if a 40-inch gap were used with an approximate critical (minimum) sparkover voltage of 650 kv, then the voltage of a cloud at 1,000 feet is assumed to be $1{,}000 \times 12/40 \times 650$ kv or 300×650 kv $=$ 195,000 kv. We have, however, no proof that this is true, or perhaps the difference in ground streamers observed in the laboratory and from natural lightning proves that this is not true.

If the breakdown gradient required to begin streamer formation at the cloud were the same as that at the end of the rod electrode representing the cloud, then voltages of such high order could be accounted for. However, atmospheric conditions near the base of the cloud, and in the cloud, are substantially different from those near the earth's surface. Just what the gradients are at which leader formation starts is not known. However, it is easily conceivable that they are substantially lower than at the earth's surface. The leader would start then at the cloud when the gradient at the earth is still too low to cause partial breakdown of the air. As the leader progresses, the gradient at the earth's surface increases, and streamers are formed only when the leader approaches the earth electrode close enough, so that with the potential difference available between the tip of the leader and the ground electrode, phenomena as observed in the laboratory can occur. This process of reasoning is of course similar to the one used by Matthias, by which he arrived at the conclusion that the stroke path above a certain height over the ground is immaterial, and the progression of the stroke is not directed toward a definite goal until this goal is approached quite closely.

It seems that this point of view has considerable merit. It will lead probably to the most pessimistic test results if adopted for laboratory tests.

There are of course a number of other dissimilarities between natural lightning and artificial, such as availability of charges within the cloud and in the earth, and the resistance or impedance of the path through which the charges have to be conducted. None of these, however, appears to be of such importance as the actual potential scale adopted for laboratory tests and its implications.

It might be impossible to reproduce actual lightning conditions in the laboratory on a true scale, and therefore compromise tests as explained in this and other papers will probably have to be satisfactory, and the operating engineer has to choose from such tests, the operating experience and the economics of the problem how to provide the most economic shielding of the system.

H. J. Mason (General Electric Company, Pittsfield, Mass.): The question of what "cloud" polarity to use in model tests is a difficult one to answer. Two factors must be considered—first, what actually occurs in nature, and, second, the best means of simulating nature in the laboratory.

Photographs of natural lightning reveal that strokes to the open country are initiated almost entirely by downward leaders, while strokes to the tall Empire State Building are generally initiated by upward leaders. It seems reasonable to expect that there is a transition between these two extreme conditions and that upward and downward leaders of equal length might occur for structures lower than the Empire State Building. It would be very difficult to obtain a high-speed photograph of a stroke to a tall transmission line, but it might well show an upward leader of considerable length.

For this reason, then, in model tests it may not be desirable to reduce intentionally the length of upward leaders by using positive cloud polarity. Moreover, if the theory is held that the upward leader should be of negligible length, even positive polarity is not the solution for model tests, as upward leaders of one-fifth the gap length may be obtained when it is employed. For a cloud height of 1,000 feet, this would mean a leader of 200 feet.

Nor do the facts that the nature of the voltage, the weather conditions, and the scale have less effect on the distribution curves for positive than negative polarity justify the exclusive use of positive polarity in model tests. It is certainly desirable to utilize these phenomena in facilitating experimentation, but a further study of the effect of negative polarity is desirable. With negative cloud polarity a projection upon the ground plane is more attractive and is hence more difficult to protect. In the interest of conservatism this fact should not be neglected.

How to properly simulate ground resistance is also a problem in model tests. On the basis that ground resistance lowers the earth plane below the surface of the earth, Messrs. Wagner, McCann, and MacLane simulated earth resistance by an additional layer of air between the model wires and ground, presumably retaining their solid grounding. Less protection resulted.

It is well, however, to look at the problem from the standpoint of leader phenomena. When an upward leader leaves a projection on the surface of the earth, the amount of current flowing into it, and its length, may be limited by the resistance of the earth. Hence a projection on the surface of the earth where the soil resistivity is high may be less attractive than one where it is low. Tests made in the high-voltage engineering laboratory at Pittsfield support this point of view as do the tests of Messrs. Wagner, McCann, and MacLane in which they insulated the conductor instead of grounding it solidly and obtained more protection. An entirely satisfactory method of simulating ground resistance is not easy to suggest, but a layer of air does not appear to be the best solution.

E. J. Allen (General Electric Company, Pittsfield, Mass.): In this interesting paper the authors have investigated transmission-line shielding in the laboratory through the use of models, and by comparison with operating experience. Adequate shielding is obtained when the overhead ground wires are sufficiently high and properly placed so as to intercept the lightning stroke and prevent its termination on the line conductors.

Shielding is one of the basic principles of overhead ground wire protection which can be incorporated into mechanical design of a line. However, lightning flashover of shielded lines will still occur when the product of the structure footing resistance and the impulse current passing to ground exceeds the line flashover value. Because low-structure footing resistance is as powerful a factor as shielding in providing a high degree of lightning protection with overhead ground wires, its attainment in practice is essential even in the case of the higher voltage steel tower lines. For example, the exceedingly small number of insulator flashovers on the high-voltage lines considered in table II of the paper have been due not only to shielding, but to the adoption of radial or continuous counterpoises as a means of reducing tower footing resistance.[1,2]

Where the attainment of low-structure ground resistance is prevented by high-resistivity soil or rock, or by the economic dictates, protector tubes have been successfully installed to improve line performance under conditions of lightning. Several complete installations of protector tubes have been made on lines already equipped with an overhead ground wire, where the shielding angle in many cases was from 37 degrees to 55 degrees. Although these exceeded the 30-degree shielding angle suggested by the authors, protector tubes were primarily installed for a high degree of lightning protection on these lines where the existing high-tower footing resistance had prevented adequate protection by the ground wires. The protector tubes, being located in direct shunt relation to the insulators of each phase will provide lightning protection independently of the structure footing resistance.

Figure 25 of the paper shows a correlation between tower footing resistance and flashovers per 100 line miles per year for a 132-kv system. A similar correlation between protector tube operations per 100 miles per year versus tower footing resistance was prepared by Wade[3] in 1938 covering 133 miles of 132-kv steel tower line of the same system. This line has a single overhead ground wire with a shielding angle of 37 degrees to 42 degrees depending on the type of tower. The data was supported by approximately 7,700 tube-years of service experience during which records of tube operations on each phase and tower were carefully kept. It was represented by a straight line passing through 50 operations of tubes per 100 miles per year at zero tower footing resistance and 170 operations of tubes per 100 miles per year at 50 ohms tower footing resistance. The intercept at zero resistance, being at 50 operations per 100 miles per year, represents approximately four times the number of flashovers per 100 line miles per year at zero resistance shown for a similar 132-kv line in figure 25 of the paper. Whether or not this represents any significant difference depends upon whether the "flashovers" on the ordinate of figure 25 of the paper are classified as tower flashovers, circuit flashovers, or single-phase flashovers. The last should be compared with the number of tube operations.

It would be expected, as this comparison shows, that there would be more tube operations than insulator flashovers (prior to tube installations), other things being equal, because the impulse discharge voltage of the protector tubes is necessarily below the impulse flashover of the line insulators. Considering this, and since the shielding angles of the 132-kv lines being compared are practically the same, indications are that the effect of tower impedance drop and ground wire impedance for strokes at midspan is appreciable. The effect of tower surge impedance and ground wire impedance for strokes at midspan will, of course, increase with higher structures and for longer spans, as for example at river crossings.

REFERENCES

1. Discussion by **S. K. Waldorf.** AIEE TRANSACTIONS, volume 58, 1939 (June section), pages 305–06.

2. SAFE HARBOR-WESTPORT 230 KV TRANSMISSION LINE, **Edwin Hanson.** ELECTRICAL ENGINEERING, December 1932, pages 834–8.

3. Discussion by **E. J. Wade.** AIEE TRANSACTIONS, volume 57, September 1938, pages 528–9.

K. B. McEachron (General Electric Company, Schenectady, N. Y.): I believe that this investigation represents a valuable contribution to our knowledge. It must be remembered, however, that these tests are subject to all of the questions which arise in connection with model tests, but because of the increased length of the discharge path compared to that used by previous investigators, the multiplying factor is reduced, and therefore one would expect the results to be more directly applicable to outdoor lightning discharges.

I would like to draw attention to a conclusion from my own work with reference to effect of polarity on the direction of propagation. I think in connection with an investigation of this sort it is essential that the conditions be kept as close as possible to those which occur in nature. Sufficient data are now available to indicate very clearly that polarity does not influence the direction of propagation to any great degree, but that the configuration of the electrodes is the determining factor. With a needle point against a plane, propagation begins at the needle point, although streamers may rise from the plane when the leader approaches close to the plane.

Our studies of lightning to the Empire State Building, together with other information which has been accumulated, indicate quite clearly that occasionally the protected zone is violated. It would appear that approximately two per cent of the strokes make contact part way down from the top on substantially vertical surfaces. I suspect that, as the height of the conducting object above the earth's surface decreases, the chance of violating the protected zone becomes less.

We have been conducting tests in Pittsfield for almost a year, following along somewhat similar lines to those reported in this paper. However, in these tests we have felt that it was essential that the polarity be kept negative, although some tests have been made with positive polarity for the purpose of exploring the field more completely. These laboratory tests, together with the photographs which we have obtained of the ground end of strokes out-of-doors, indicate quite clearly that streamers do occur from the relatively flat surface of the earth, and these streamers may be of some significance in connection with the protection offered to transmission lines by overhead ground wires.

I am rather inclined to feel that 1,000 feet does not represent an average height of storm cloud, although it may represent the lowest or nearly the lowest cloud level from which strokes occur. This is a factor on which we have some information but it is not yet in suitable form for presentation.

The authors refer to the current of the return stroke as being of the order of from 10,000 to 200,000 amperes. Data which I have indicate that the order of magnitude of the return stroke can be very much smaller—perhaps of the order of 1,000 amperes or even less.

The picture of a stroke of lightning, based on available data, seems to be one in which a current flows continuously from the time the discharge starts until it is completed, including all of the successive current peaks, which in the past may have been thought were separate identities although they did follow in the same path. It is quite certain now that a continuing current ties all of these successive discharges together. This continuing current may have values apparently as small as ten amperes or less, and the complete phenomenon of the stroke has had a measured duration as great as $1^1/_3$ seconds, although the average duration is of the order of 0.4 second.

Some idea of the protective cone as it occurs in nature may be obtained from the operation of lightning rods on protected buildings. In general, on dwelling houses or farm buildings, the vertical angle will be much greater than 60 degrees, and yet the experience where data are available indicates that such lightning rod installations are about 95 per cent effective in preventing damage to the protected buildings. As the structure becomes higher, it is necessary of course to decrease this vertical angle. It seems doubtful to me, however, if vertical angles of less than 45 degrees will in general be justified on transmission lines from an economical point of view.

S. K. Waldorf (Pennsylvania Water and Power Company, Baltimore, Md.): This paper represents the result of a careful and thorough investigation of the differences between natural and laboratory conditions of shielding. As the authors have stated, there have been inconsistencies between operating experience and the behavior predicted from model tests which have seriously reduced the usefulness of the latter. This paper does much to explain the causes of the discrepancies and points the way for making laboratory conditions better simulate nature.

It has been fortunate that at least some designs have performed better in service than could be predicted from laboratory tests. An instance of this is shown by comparison of data given in table II and figure 16 of the paper, where the performance of a 69-kv line is about six times as good as predicted. And in this connection, it is well to remember the design of a line is not based on the most severe conditions along its length, but rather on an average dictated by the designer's judgment.

There are so many factors which apparently affect the efficacy of shielding that the uninitiated may well think that finding the proper location for overhead ground wires is an almost hopeless task. However, experience shows that the situation is not nearly as bad as it seems, because operating records show that some conventional arrangements of ground wires are far more effective than the laboratory results given in this paper seem to indicate. This circumstance in no way detracts from the value of the paper. The authors' efforts help to advance the problem from the state of an art to that of a science.

W. W. Lewis (General Electric Company, Schenectady, N. Y.): The authors quote Peek's statement from his 1926 paper to the effect that for a single wire, the ground is never hit nearer the projection of the wire than about four times its height above ground.

Lewis and Foust in a paper[1] gave laboratory data which showed that with positive clouds, rods protected an area two to three times the rod height, and with negative clouds 10 to 15 times the cloud height. Hence, we have been using the figure 2 times the rod height for the radius of the protected area or two times the ground wire height for the limit of protection of the ground wire.

The authors find a protective angle of 30 degrees, after allowing proper tolerances, necessary to limit the strokes to the conductor to one per cent of the strokes to the line.

Based on operating experience on a large number of systems over a long period of years,[2] we have developed the following rule for adequate protection: For double-circuit vertically arranged lines two ground wires should be used above the conductors, one on each side of the tower and vertically above the outermost conductors. The ground wires should be at a height above the conductors approximately equal to the spacing of the conductors. For single-circuit lines horizontally arranged, two ground wires should be used, placed one on each side of the center above the conductors and on vertical planes about midway between the conductors or possibly somewhat nearer the outside conductors. The ground wires should be at a height above the conductors equal to about two-thirds of the spacing of the conductors. Thus, for the vertical arrangement the protective angle is approximately zero and for the horizontal arrangement a maximum of about 37 degrees.

On page 324 of the paper it is stated "there should be little difference between the total strokes to a line with a single or double ground wire or between a single- or double-circuit line." If I understand what this means, this is contrary to our experience. We have found double-circuit lines with one ground wire to have about 70 to 80 per cent shielding, as compared with 100 per cent shielding for double-circuit lines with two ground wires arranged in accordance with the rule stated above.[2]

References

1. Direct Strokes to Transmission Lines, W. W. Lewis and C. M. Foust. *General Electric Review*, August 1931, pages 452–8.

2. Protection of Transmission Lines Against Lightning, W. W. Lewis. *General Electric Review*, June and July 1937, pages 276–84 and 314–18.

C. F. Wagner, G. D. McCann, and G. L. MacLane: Most of the discussions are concerned with an analysis of the extent to which laboratory tests simulate natural lightning. Much can be written on this point without arriving at a definite conclusion. In the final analysis, the best procedure is that which was attempted in the paper which consists of comparing laboratory tests with the results of operating experiences upon actual transmission systems.

It is recognized that the similitude relations do not hold in their entirety, either with regard to the geometry of the configuration or to electrical considerations such as electric gradient or actual potentials required to produce breakdown. Mr. Hagenguth raises the question of electric similitude and in his discussion of laboratory representation of natural lightning he follows the philosophy of Matthias. Matthias arrives at the conclusion that beyond a certain cloud height or electrode height, the course of the discharge is independent of height. The laboratory tests, however, show that the distribution curves are not independent of electrode height beyond the ratio of H/h that Matthias assumes for this to be true.

Mr. Mason questions the exclusive use of positive polarity for laboratory tests on the grounds that it may not represent the true leader development of natural lightning. Even if the leader from a transmission line extends to as far as one-fifth the cloud height, which is highly improbable, the laboratory experiments with positive polarity indicate that the results are independent of the leader length, so long as this length is not exceeded. It is true that in laboratory work projections from the ground plane attract strokes from a larger area when the cloud is negative than when it is positive. This does not imply that the same condition is true for natural lightning. The upward leaders from such projections are very much greater in laboratory-produced strokes than is the case for natural lightning. The result of this condition is that the projection attracts strokes from a larger area for negative polarity than for positive polarity, but on the other hand, the result of this effect is to produce greater protection for a ground wire. Thus in terms of protection, positive polarity gives more pessimistic results.

The problem of proper representation of ground resistance as pointed out by Mr. Mason is rather complicated. Aside from the dielectric constant of the earth and the distributed voltage drops therein, this condition can be simulated by a lowering of the effective ground plane and placing a concentrated resistance as the footing resistance of the tower. In the paper the lowering of the effective ground plane was represented by an additional layer of air between the transmission conductor and true ground. Grounding both ground wire and conductor is more pessimistic than grounding only the ground wire through a resistance and permitting the conductor to be free.

Mr. Allen refers to a curve plotted by Mr. Wade which gives the number of protector tube operations as a function of tower footing resistance for the same system for which figure 25 was plotted. The data in figure 25 were obtained before the installation of protector tubes and give a flashover figure of 12 per 100 miles of line per year at zero resistance. The curve by Mr. Wade indicates 50 tube operations per 100 miles of line at zero resistance. This discrepancy may be due to the difference in definitions of flashovers and tube operations. The data obtained by Messrs. Sporn and Gross, for which Mr. Wade's curve is drawn, list the discharges of the tubes for each component of a multiple stroke as a separate tube operation, whereas only one would be indicated on the line on the basis of flashovers. It is also probable that because of the reduced insulation, more flashovers would occur on the line equipped with tubes.

Mr. Lewis recommends for double-circuit vertically arranged lines that two ground wires should be used above the conductors, one on each side of the tower and vertically above the outermost conductor. As stated in the paper, the criterion for determining ground wire location is the protective angle shaded to some extent by variations due to different values of y/h. Whether one or two ground wires be used is dependent entirely upon the most economical manner of obtaining the proper protective angle. If a protective angle of 37 degrees is satisfactory for a single-circuit line, then it should also be satisfactory for a double-circuit line. Of course, if a smaller protective angle can be obtained with very little increase in cost, which is sometimes the case, then it might be better to use the smaller angle.

There is some question in Mr. Lewis' mind regarding the statement in the paper that "there should be little difference between the total strokes to a line with a single or double ground wire, or between a single- or double-circuit line." Perhaps the paper was not sufficiently clear on this point, but the expression refers to the total strokes to the line, including those that strike the ground wire without producing an outage. In other words, the cloud area from which strokes are attracted should be about the same for a single- and a double-circuit line. This is evident from the nature of the distribution curves and has been verified by test.

The authors cannot agree entirely with Mr. McEachron's statement "that the configuration of the electrodes is the determining factor" in the direction of propagation. The laboratory tests have shown that when the point is positive with respect to a flat ground plane, the discharge propagates from the point, but when the point is negative, the discharge is initiated simultaneously from both point and plane. The authors agree with Mr. McEachron as they have stated in the paper that data relating to protective zones around tall buildings cannot be applied to smaller objects.

Since the presentation of the paper, the authors, upon further examination of Bell's* data, have discovered a misinterpretation regarding the ratio of the number of strokes that produced outages and those which produced flashovers. This ratio should be 1.5 instead of three as given in the paper. However, this does not influence the rounded value of 100 used in figure 16 for the number of strokes to lines per 100 miles of line per year.

Arcing Faults in Power Systems

Discussion and authors' closure of paper 40-106 by C. Concordia and H. A. Peterson, presented at the AIEE summer convention, Swampscott, Mass., June 24–28, 1940, and published in AIEE TRANSACTIONS, 1941, pages 340–6.

S. B. Crary: See discussion, page 618.

P. A. Jeanne (Bell Telephone Laboratories, New York, N. Y.): The paper by Messrs. Concordia and Peterson is a useful and interesting contribution to the studies of overvoltages on power systems which have been carried on in recent years with the aid of miniature circuits in the laboratory. The usefulness of this method of analysis, as contrasted with purely mathematical analysis, is evinced by the large number of cases which it was possible to study and discuss in the present paper whereas the labor required for comparable results by mathematical analysis would have been almost prohibitive.

The uncertainty which still exists regarding the characteristics of arcs accompanying faults on operating power systems emphasizes the need for more information on this subject. While I have referred to this matter briefly in a previous discussion before the Institute, I believe it would be interest-

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*See reference 21 of paper.

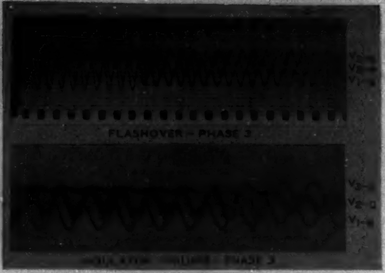


Figure 1. Oscillograms of line-to-ground voltages

ing to show a few oscillograms secured during overvoltage observations on a 550-mile, 44-kv system, which was grounded at one point through a 136-ohm neutral resistor. These data are from overvoltage studies made by the joint subcommittee on development and research, Edison Electric Institute and Bell System, with the co-operation of the American Gas and Electric Company. The first of these, figure 1 of this discussion, shows two oscillograms obtained during troubles on the power line which were correlated as being due to flashovers. The oscillograms show portions of the record before and following circuit-breaker operation. Portions of the record cut away were similar in appearance to the sections shown. The upper oscillogram was obtained with a magnetic oscillograph, and the lower one with a cathode-ray oscillograph. The important point which I wish to emphasize with these two oscillograms is the absence of any serious distortion in the sound-phase voltage waves, and the apparent absence of restriking in the faulted-phase voltage, except at the point where the breaker clears the fault. Here, in the case of the lower oscillogram, there are evidences of two restrikes in the faulted-phase voltage, presumably associated with breaker operation. The transient overvoltage peak on phase 1 resulting therefrom is clearly evident. These characteristics should be contrasted with those in the following slides. Figure 2 shows sections of two oscillograms during faults in which the arc was probably more or less restricted. The upper one, a cathode-ray oscillogram was obtained during a fault caused by cable failure. Here it will be seen that the sound-phase voltages are distorted to a considerable degree. The faulted-phase voltage waves show clearly the fact that the arc tended to extinguish and restrike every half cycle although the voltage reached before restriking was relatively low. The lower oscillogram obtained with a magnetic oscillograph is a record of fault resulting from bushing failure. Here the distortion in the sound-phase voltages is much more pronounced, and the faulted-phase voltage shows that there was restriking, sometimes only on the positive half of the cycle, sometimes only on the negative half of the cycle, the restriking voltage in practically all cases approaching approximately normal line-to-ground voltage. Figure 3 shows the neutral current of the system and the residual current on certain lines during a fault which resulted from bushing failure. The point to which I wish to draw particular attention here is the distortion in the upper or neutral current-wave trace, which indicates that there was extinction and re-establishment of the fault current. Examination of oscillograms of line-to-ground voltage and neutral current obtained during the same fault show that, as would be expected, distortions in sound-phase voltage waves are accompanied by distortion in the neutral current.

Based on these distortion criteria, an examination of the overvoltage oscillograms obtained over two lightning seasons during faults on this 44-kv system showed that most of the ones having pronounced intermediate peaks, as distinguished from initial and breaker clearing transients, were associated with bushing, transformer, or cable failure. Also, 735 neutral current oscillograms obtained over a period of about four years from this same system were examined for evidences of repeated extinction and restriking of the arc. Only 45 such oscillograms were found, and of these, 26 were definitely associated with bushing, transformer, carrier current condenser, and cable failure, while the nature of 13 of them was not definitely ascertained. Similar information on another neutral resistance-grounded system and on one extensive isolated system, seems to indicate that arcs in air seldom clear and restrike in the sense that there is an appreciable interval when the current is zero or an appreciable increase in the voltage on the faulted phase before the arc restrikes, the conditions which are requisite for the higher overvoltages in intermediate portions of a fault. The initiating or clearing transients, of course, are dependent on other factors. These conclusions are based on results from only a few systems and it may be that on some other type of system, for example, short isolated-neutral systems, a different characteristic of arcs in air might be found and it would contribute a good deal to the study of overvoltage if more data on the characteristics of arcs during faults could be obtained from operating systems.

Figure 3. Oscillogram of neutral and residual currents

I_N = neutral current
I_{R1} = residual current lines west
I_{R2} = residual current lines east

As a further tie between the miniature system tests described in the paper and observations on actual systems, the sequence constants of the 44-kv system referred to above which are required for entering the curves of figures 4 and 6 of the paper, were determined and the per-unit line-to-ground voltage which might be expected for this system, as indicated by the curves, was found to be in the order of 2.2 to 2.4 times normal, depending on the fault location, etc. Figure 4 shows the results of observations near one end of this system with both the magnetic and cathode-ray oscillograph. It is seen that a small percentage of the recorded overvoltage reached the values deduced from the curve in the Concordia-Peterson paper. Thus, a reasonably good check is shown between the maximum recorded overvoltage on this system and those indicated by the miniature system.

Additional clarification is needed on how to determine the proper constants with which to enter the curves when the system is complicated. The authors discuss this to some extent but it is not apparent just what should be done in a specific situation. For example, if, as in the case of the system I have been discussing, two main portions of the system radiate from a central large generating source, should the total positive-sequence capacitance be used, or only that up to the generating station on the side of

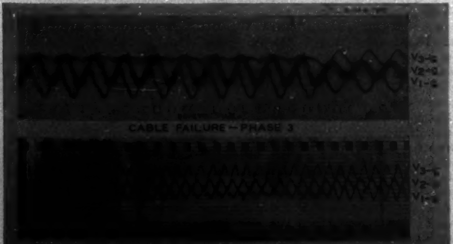


Figure 2. Oscillograms of line-to-ground voltages

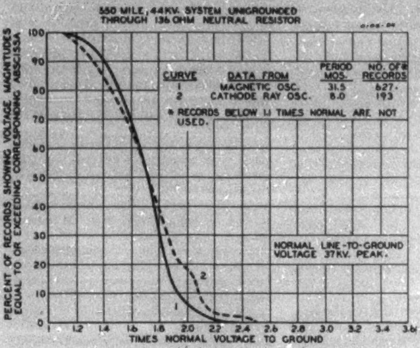


Figure 4. Cumulative percentage curves showing magnitude distribution of overvoltage

the station toward the fault, it being assumed that the capacitance beyond is shorted out by the supply source impedance? Fortunately, for resistance-grounded systems, the curves shown in the paper are relatively flat and this question is not so important. However, it would assume much more importance in the case of isolated or reactance-grounded systems. This phase of the matter may require more study.

There is one minor question as to notation. Is there any distinction intended in the use of the subscript l and L, as used in designating positive-sequence reactance on the co-ordinates of the curves and in the explanatory material, both in the paper and accompanying curves?

A. C. Monteith (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): This paper is of particular interest to me because it gives results of investigations paralleling work we have been conducting since 1937 when the method of analysis, utilizing miniature systems was introduced at the Milwaukee convention. During the intervening time some general investigations have been carried on and a number of specific systems have been studied to determine their characteristics for arcing grounds and switching operations. As a result of this work we agree with the authors that it is better to analyze the particular system rather than depend on too great a generalization of results. This conclusion is due to the large number of variables entering into the problem. It is the complexity of the problem that has made it practically impossible to solve when approached by pure mathematical treatment.

In the paper, reference 2, which outlines some of the work done by Evans, Witzke, and myself, we chose at the time to study a particular system in order to develop a technique of handling arcing grounds and switching transients at the same time getting some ideas of the factors involved. In that particular study high values of voltage were found for reactance-grounded systems when the neutral reactance was one-third that for a ground-fault neutralizer. A review of our subsequent studies on specific systems shows a range of from $^1/_{40}$ to $^1/_8$ which agrees with the findings of the authors.

It is further noted in the conclusion of the paper that a value of five has been selected for the ratio of X_{L0}/X_{L1} as a criterion for the necessity of a study. We have suggested a ratio of 1.5 to 3 for the particular case of grounding a generator through reactance, based on limiting the voltage to 2.0 to 2.5 times line-to-neutral voltage as dictated by apparatus considerations. The limits will have a large influence on the ratio selected.

The fact that we can begin to discuss factors such as these indicates that we now have a method of attacking problems which heretofore have been too involved to attempt. This is emphasized by a case of a general study made with the a-c network calculator method involving 30 man-days work. A conservative estimate of the time to make a similar study by longhand methods indicated 25,000 man-days. With the work now in progress, in co-operation with the operating companies, it is believed much of the veiled mystery of arcing grounds, switching transients, and the like will be better understood. This should result in a better application of apparatus with better resultant system operation.

Philip Sporn (American Gas and Electric Service Corporation, New York, N. Y.): The authors have extended the method of study begun some two years ago to try to evaluate the effect of arcing faults on power systems, and have added valuable additional information on the effects of arcing faults of different types and under different conditions on power systems of various designs.

Of particular interest is the authors' conclusion that, from the standpoint of surges and arcing faults, the solidly grounded system is definitely superior, not only to the isolated-neutral system but also to systems grounded by any other method. This conclusion is not only in accordance with the work of previous analysts but is also checked by the results of our operating experience in grounding scores of high-voltage transformers and distribution networks in the last 20 years.

Messrs. Concordia and Peterson have sounded a note of caution in connection with the use of reactors for system-neutral grounding. This confirms previous analyses and conclusions such as those contained in the reference paper of Concordia and Skeats, and is also borne out by operating experiences as described by R. A. Hentz and J. A. Adams in discussing the same paper. In this connection, as pointed out by Mr. Hentz, the use of relatively small grounding transformers for grounding systems supplied by delta-connected power transformers may in some cases produce an equivalent neutral grounding reactance well above the critical values indicated by the authors in this paper and by the curves of figure 5 in the Concordia and Skeats paper.

Our own operating experience with a few reactor-grounded systems and with a great many grounded transformer installations has so far failed to show any serious overvoltage troubles and a remarkable decrease in difficulty from what had been experienced prior to grounding. However, it is quite likely that this possibility has not been given in all cases the detailed attention it deserves when determining the sizes of grounding transformers to be used in specific applications, and in this regard the work of the authors will be most helpful.

R. L. Witzke (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): In carrying out investigations of arcing faults in the laboratory by means of miniature systems it is necessary to consider a given number of restrikes of the arc. The number of restrikes required to produce a given overvoltage is of importance because as the required number of restrikes is increased, the probability of the voltage being produced is decreased. In the investigation reported by Messrs. Evans, Monteith, and Witzke in the August 1939 issue of ELECTRICAL ENGINEERING, the cases of one and two restrikes were considered. It would be of interest to know how many restrikes were required to produce the higher voltages reported in the paper under discussion.

In carrying out the investigation referred to above, we found that the method of restriking greatly influenced the transient voltages. However, until more data had been accumulated on arcing, we chose to report only restriking which resulted in the highest voltages. This gave a consistent method of attack and gave results which were valuable in comparing methods of system grounding.

An examination of figures 10 and 11 of the paper under discussion would indicate that when switching out a faulted-line section, the voltages produced are low when a ground-fault neutralizer is used. However, these voltages were obtained for the case of "interruption of a faulted conductor by a circuit-interrupting device." When the arcing occurs in series with a sound phase, the voltages produced may be higher and do not decrease when a ground-fault neutralizer is used.

Eric T. B. Gross (College of the City of New York, N. Y.): Because of my contact with the study of earth-fault problems, and especially those involved in the application of earth fault coils (ground-fault neutralizers and similar reactors, more or less exactly tuned to the capacitances-to-earth of the high-tension systems), I have found this paper very interesting, both from a theoretical and practical view. However, though everything is certainly correct from an academic point of view, in drawing conclusions, we have to find out first where the boundary lies for systems with earth-fault coils and other neutral-grounding reactors.

If we assume that the various taps of the earth-fault coils are arranged at a distance of ten per cent from each other (as is amply sufficient), we will be able to tune the coils to the capacitance-to-earth of the system with an accuracy of more than five per cent (over- or undercompensation). A change in the degree of unbalance[1] will occur only when there is a change in the capacitance-to-earth, due to the switching off of some line sections, which is not balanced by a change in tuning. It is self-evident that the installation of the coil is at such a point of the network that it will not become disconnected from the network at any time. Considering the above, we see that the degree of unbalance will never become more than five per cent undercompensation. However, it may go down in the region of overcompensation, when line sections are switched off. This means in other words that, when using an earth-fault coil, no condition will ever exist where the grounding reactance ratio X_{L0}/X_{L1} is greater than about 1.05 times the ratio for an exactly tuned earth-fault coil. We see from all the figures of the paper that such a limitation of the possible reactance of an earth-fault coil assures a reduction of possible overvoltages to the same range as obtained for "solidly" grounded neutrals. Furthermore, in the solidly grounded case, we have to consider that there will always be a certain amount of zero-sequence reactance of the grounding transformer, which acts as a neutral reactance and causes a variation of the possible overvoltages similar to that due to the various degrees of unbalance with earth-fault coil. For this reason, I would say that there is practically no physical difference between the conclusion's points 8*a* and 8*d*, and the investigation shows further that the results with the neutral grounded according to 8*c* are not only satisfactory, but practically the same as 8*a*. It should be mentioned that all this has been proved satisfactorily by experience, not only abroad but also with the earth-fault coils in this country, as is shown in the various American papers recently published on this subject.

However, there remains one fault to be mentioned,[2] in which the results of the investigations with miniature circuits[2] are only academic, as for many years the practice had followed ways to avoid any possible overvoltages when earth-fault coils were used. Earth-fault coils are installed for fundamental reasons at such points of the transmission or distribution system where no disconnection is being brought by the switching off of one or two sections. Even in the case of one double-circuit line with the coil at the energizing end of the transmission, when one circuit is switched off and de-energized, the connection between the second circuit and the coil remains, so that the remaining capacitance is by far large enough to suppress dangerous overvoltages under switching surges. Of course, it has been well known in Europe for many years that overvoltages of considerable amount could be produced when disconnecting earth-fault coil and transmission line by de-energizing the (single) line; however, in such cases, and before the modern-type overvoltage arrester had been developed, an installation was made to de-energize the line by opening the circuit breaker on the low-voltage side of the transformer. After 1929, when the modern-type overvoltage arresters (lightning arrester) with nonlinear damping resistance have been introduced in high-tension networks, such an arrester has always been used as a shunt to the earth-fault coil at the end of a single line to limit the value of the dangerous overvoltages which are otherwise possible. The arresters were of the normal type, and of the type used for systems with a line-to-line voltage equal to the star voltage of the transmission system considered. We see from this that, as far as voltage surges are concerned, there is very little difference between the performance of direct or earth-fault coil grounding, and we cannot say that one way is better than the other.

It seems to me, however, that the use of an earth-fault coil is advantageous from other points of view, one of which I would like to mention. In solidly grounding the neutral we eliminate the disastrous consequences of keeping the neutrals without any connection to earth, and especially the consequences of the capacitive earth-fault currents of arcing grounds, by introducing single-phase short circuits, i.e., we replace one very troublesome disturbance by another, less dangerous one. In both cases the number of short circuits and interruptions would be the same. On the other hand, if earth-fault coils were used, in general a great number of all ground faults would be quenched without any disturbing effect on the whole system whatsoever; in today's American practice in the use of earth-fault coils and their design, this applies only to all arcing grounds. This comparison shows undoubtedly that there are advantages in favor of the use of tuned earth-fault coils. Considering the quenching effect of the earth-fault coil, we might say with regard to this action that it is equivalent to the ultra rapidly reclosing device used in a few solidly grounded systems.

REFERENCES

1. THE DEGREE OF UNBALANCE WITH INDUCTIVE EARTHING AND ITS MEASUREMENT, Eric Gross. Report 326, International Conference on Large High-Voltage Systems, volume 3, Paris, 1937.

2. POWER SYSTEM TRANSIENTS CAUSED BY SWITCHING AND FAULTS, R. D. Evans, A. C. Monteith, and R. L. Witzke. AIEE TRANSACTIONS, volume 58, 1939, page 393.

C. Concordia and **H. A. Peterson:** In regard to the discussion by Mr. Witzke, we have several comments to make. It is true that consideration of a fixed number of restrikes affords a consistent basis for carrying out an investigation. It is true also that the number of restrikes required to produce a given overvoltage is important. However, we believe that the introduction of such consistency may be undesirable since, obviously, no interrupting device is designed to produce an arbitrary number of restrikes under any and all system and fault conditions. Tests have indicated, that an interrupter may have a tendency to restrike several times for one system condition, and not at all under another.

For these reasons, we have not attempted to cling to a fixed procedure as far as the number of restrikes is concerned. Rather, the investigation was carried out with the intention of determining the maximum voltages obtainable for a variety of system conditions when restriking of a particular type occurs. In some cases it was found that a large number of restrikes would cause no serious overvoltages. In others, a few restrikes, one or two, could give rise to serious overvoltages.

The authors appreciate the comments of Dr. Gross, based upon his knowledge of system-grounding practice in Europe. It appears that the results of our investigation, and our experience in methods of grounding agree very well with modern European practice where ground-fault neutralizer coils are concerned.

As Mr. Jeanne has pointed out in his discussion, much remains to be learned about arc characteristics under actual operating conditions. It is extremely interesting to compare the limited number of available oscillograms of actual arcing phenomena with the oscillograms of similar phenomena in miniature systems. In particular, figure 2 of his discussion should be compared with figure 1 of the paper. The arcing phase voltages are quite similar in both cases.

His data summarizing overvoltage magnitudes is most helpful also in interpreting our results in terms of actual system performance. These cross checks are extremely helpful as benchmarks from which work may be continued with confidence.

As systems become more complicated, it becomes increasingly difficult to define those systems in terms of the simple impedance ratios used. As mentioned in the paper, this is particularly true of the isolated-neutral system. However, in the specific case cited by Mr. Jeanne, it would be necessary to consider the capacitance of all of the lines radiating from the bus at the large central generating source, in order to use the curves shown in the paper. In preparing these curves, there was no distinction intended in the use of the subscript *l* and *L*.

It is of interest to note from Mr. Monteith's discussion that their work paralleling ours substantiates our conclusion that the value of neutral grounding reactance giving rise to highest arcing-fault overvoltage possibilities varies through a considerable range. Our figures were from $^1/_3$ to $^1/_{30}$ the value of ground-fault neutralizer reactance while Mr. Monteith gives a range of $^1/_3$ to $^1/_{40}$.

Mr. Sporn, in summarizing the operating experience on the extensive American Gas and Electric Service Corporation systems, has further substantiated the conclusions of the authors regarding methods of system grounding. Consideration of the possibilities of overvoltages should be of particular importance in determining the size of grounding transformers for specific applications.

Mr. Crary has summarized very well the significance of the results of the numerous investigations which have been made to determine several of the fundamental sources of overvoltages in power systems. The availability of such results should be of considerable aid in system planning and designing for better performance.

Abnormal Voltage Conditions in Three-Phase Systems Produced by Single-Phase Switching

Discussion and authors' closure of paper 40-105 by Edith Clarke, H. A. Peterson, and P. H. Light, presented at the AIEE summer convention, Swampscott, Mass., June 24–28, 1940, and published in AIEE TRANSACTIONS, 1941, pages 329–39.

E. A. Williams, Jr. (General Electric Company, Philadelphia, Pa.): The authors have presented a very able analysis of the voltage trouble which may be produced by the operation of single-pole switches and fuses on three-phase systems. The application of the outdoor, triple pole, group-operated horn-gap switch has become general practice, but the single-pole fuse is the accepted standard. Literally hundreds of thousands of single-pole fuses are giving excellent service without causing trouble due to excess system voltages. Some of the reasons for this fortunate situation may be readily deduced from the data given in the paper.

Grounded-Neutral Systems

The great majority of fuses are applied at or near the high-voltage terminals of step-down transformers and in this location are not subject to the production of abnormal voltage conditions. The relatively small percentage of fuses used for line sectionalizing and branch-line fusing rarely cause overvoltages as the existence of one or more of the following conditions results in a safe installation:

(1). The transformer neutrals are grounded.

(2). The transformers are loaded when a fuse blows.

(3). The connected transformer kilovolt-amperes exceeds the minimum required to maintain a safe ratio of x_{c1} to x_m.

An interesting analysis of conditions (3) is indicated in figure 12 of the paper. Assume a branch line, with fuses at the tap off, feeding transformers operated with ungrounded neutrals. The blowing of one or two fuses will not cause excess voltage, even under no-load conditions, provided the connected transformer kilovolt-amperes per mile exceeded that shown in the tabulation below (taken from figure 12 of the paper):

System Voltage (Kv)	Minimum Safe Kilovolt-Amperes Per Mile	
	Two Blown Fuses	One Blown Fuse
2.3	6	3
6.9	50	25
13.8	200	100
23.0	550	275
34.5	1,100	550
46.0	2,000	1,000
69.0	4,500	2,250
115.0	11,000	5,500
138.0	17,000	8,500

An inspection of this tabulation indicates that it would probably be uneconomical to build a 2.3-kv or a 6.9-kv line which would be subject to overvoltage due to single-phase switching. However, the use of branch-line fuses or sectionalizing fuses in lines above 6.9 kv, with transformer neutrals ungrounded, should be carefully checked. With the exception of this one condition, single-pole fuses may, in general, be freely and safely applied in grounded-neutral systems.

Ungrounded-Neutral Systems

The application of single-pole fuses in ungrounded-neutral systems is seldom absolutely safe from the standpoint of excess voltage. However, as pointed out by the authors in conclusion 7, the most probable cause of overvoltage is the breaking and grounding of a line conductor. This condition is entirely independent of fuse operation. The probability of the occurrence of any practical condition which would result in the blowing of only one fuse during a single line-to-ground fault seems rather remote. Therefore, it would appear that the length of line on the system side of the fuse (figure 14 of the paper) may have little bearing upon fuse applications.

Since the ratio of x_{c1} to x_m must exceed 25 to avoid excessive voltages, it may be concluded that branch-line fuses and sectionalizing fuses should not be used generally in isolated-neutral systems above 2,300 volts. With this one exception, the application of single-pole fuses in isolated-neutral systems appears to be reasonably safe from the standpoint of excess voltages.

R. D. Evans (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The paper by Clarke, Peterson, and Light will undoubtedly raise two questions in the minds of many engineers. These questions are:

1. Do the conditions which give rise to the abnormal voltages reported in the paper also apply to the systems for which single-phase switching may be considered for improving system stability?

2. Are these abnormal voltages of frequent occurrence and of much importance from the practical standpoint?

In connection with these questions it is to be pointed out that the abnormal voltages produced by the mechanism described in the paper do not apply to three-phase systems generally but only to a very limited set of conditions. The paper may be described more precisely as dealing with abnormal voltages produced in an ungrounded section of a three-phase system which is partially isolated from a power source by means of single-phase switching. The conditions necessary for producing these abnormal voltages will not be obtained if (1) there are power sources connected to both sides of the single-phase switching equipment, or (2) if the system is solidly grounded on both sides, or (3) if the line under consideration is paralleled by another circuit. It will at once be realized that most of these exceptions will be obtained in any system for which single-phase switching would be considered for the purpose of improving system stability. More definitely, such a system would have power sources on both sides of the single-phase switching equipment, and probably also solidly grounded transformers on both sides. Thus it is clear that the conditions essential for producing abnormal voltages in the manner described in the paper are not to be expected for the ordinary layouts for which single-phase switching might be considered for improving stability. In other situations, single-phase switching may be used but if the load side is effectively grounded, then as pointed out in the paper, abnormal voltage will also not be produced.

Abnormal voltages produced by the phenomena described in the paper are, in our experience, of relatively infrequent occurrence. They do occur however for a wide range of system conditions as brought out by figures 12 and 14. The paper is of value in clarifying the conditions under which these phenomena take place and figures 12 and 14, particularly, will be of use in investigating specific cases.

S. B. Crary (General Electric Company, Schenectady, N. Y.): During the past few years, the subject of power system overvoltages due to causes other than lightning, heretofore quite neglected, has been subjected to intensive study and analysis. This work has been greatly facilitated by the use of miniature networks. This gap in the knowledge of system design and operation became increasingly apparent as the study of lightning phenomena progressed and a correspondingly increased proportion of the failures could be definitely attributed to these "other causes." From the work that has been done, exemplified by the papers of Clarke, Peterson, and Light, and Concordia and Peterson, some general remarks may be in order.

This work shows clearly that the tendency for or probability of a system having equipment damaged or service impaired by overvoltages is determined to a considerable extent by system design and operation, as well as the characteristics and design of the individual pieces of equipment. These system difficulties can be controlled to a very practical and worth-while degree in proportion to the attention given to:

(*a*). The method and details of grounding

(*b*). The proper application of lightning arresters

(*c*). The use of properly designed interrupting devices having a minimum tendency for producing overvoltages

(*d*). The correct application of single-phase switching devices

(*e*). The use of quick-acting voltage regulators for water-wheel generators in case of loss of load

(*f*). The use of properly designed amortisseur windings in water-wheel generators

One of the most outstanding results of these studies is that the highest overvoltages are almost consistently characteristic of the isolated-neutral system. This has provided much sounder reasons than previously for not operating in this manner, except possibly as a means of connecting apparatus together in a small local area which has very little possibility for faults or switching operations.

The seemingly inconsistent experience with reactor grounding has been explained and put on a more rational basis. It appears now that if reactance is to be used for grounding its value should either be very low or have a value near the ground-fault neutralizer value.

Damaging overvoltages need no longer be put aside as unexplainable or due to "surges" but should be brought out for explanation by the results obtained so far or by a transient network-analyzer study, in order that they can be better avoided in the future.

C. T. Weller (General Electric Company, Schenectady, N. Y.): Abnormal voltages unrelated to switching may be obtained with transformers on single-phase circuits under some conditions. For example, in 1915 we were testing one potential element of a 44-kv, 3-phase, 3-wire metering outfit for ratio and phase angle in the laboratory. In such single-phase tests, one primary terminal is grounded and in this case it was the outside terminal or bushing of the number 1 potential element; the "common" terminal was "high." At approximately 40 kv, an arc occurred from the outside terminal of the number 2 "unexcited" element to the nearby "high" lead. This led to an investigation and it was found that a condition of partial resonance existed between the inductance of the primary winding and its distributed capacitance to the grounded secondary winding, core, and tank. As a result, it was recommended that the installation be arranged so that the metering outfit would be permanently connected to the three-phase system.

The investigation was extended to potential transformers of various voltage ratings. Rated voltage was applied from one primary terminal to ground and it was found that considerable voltage existed across the windings in some cases; the voltage was read on the secondary. As a result, the inductance of the primary (and secondary) windings of some potential transformers was reduced, although for the usual application to three-phase systems, the line-to-ground voltage is but 58 per cent of rated voltage.

The abnormal voltage conditions described are analogous to some of those mentioned by the authors. However, I recall but one instance of such abnormal voltages on a three-phase system in practice. This occurred on a 38-kv Canadian system, when a 40-kv potential transformer was temporarily excited on one terminal; it was estimated that approximately double voltage was applied to the bus structure. While the occurrence of these abnormal voltages, both in the laboratory and in the field, fortunately appears to be infrequent, the conditions which may produce them have now been outlined. When such conditions are encountered, suitable precautions should be taken to prevent possible damage to equipment.

Edith Clarke, H. A. Peterson, and P. H. Light: The discussion by Mr. Weller indicates that the basic phenomenon giving rise to overvoltages of the type described in the paper has appeared in a variety of forms and places. Since the mechanism whereby such overvoltages may arise is basically the same for so many different circuit conditions, the quantitative data presented by the authors should be helpful in evaluating possible magnitudes of overvoltages for many conditions other than those mentioned specifically in the paper. The authors are pleased to have Mr. Weller's comments regarding his experience with overvoltage conditions of this general nature.

Mr. Williams has given an interesting and a significant interpretation of the data presented in the paper. We believe, however, that a word of caution is desirable in connection with his discussion of the grounded-neutral system. In some of the low-voltage circuits, short lengths of cable may be used. These may have a capacitance equivalent of 50 or more times the same length of overhead line (see figure 13 of the paper). It is obviously necessary to consider such cable lengths, even though they may be short, in evaluating the possible magnitudes of overvoltages.

In the ungrounded-neutral system, the probability factor, as mentioned by Mr. Williams, must be evaluated in accordance with circumstances attending a particular case. In that connection it may be desirable to consider combinations of fault and blown fuse conditions other than the one of a single line-to-ground fault and one blown fuse which was analyzed in detail in the paper. For instance, two fuses blown and a solid fault on one of the phases gives almost the same basic circuit condition. A double line-to-ground fault with two fuses blown would give a somewhat similar circuit condition. The probability of occurrence of either of these conditions may be greater than for the one analyzed. Only experience can indicate the practical significance of the analysis in this respect.

We agree with Mr. Evans' interpretation of the results of our paper. Undoubtedly single-phase switching can be used in a relatively large number of solidly grounded systems without encountering dangerous overvoltages of the type under discussion. It would seem that in such cases, the only practical limitation would be disclosed by a complete understanding of the effects of line lengths and system voltage levels from the standpoint of arc extinction time. In other words, it is likely that for a given system voltage, a practical maximum length of line exists beyond which length the arc extinction time may be prohibitively great. Even though one phase of an overhead line is open at both ends, the capacitive coupling with the other phases tends to maintain a once-established arc on that phase. This tendency is likely a function of line length, system voltage, atmospheric conditions, and possibly other factors. The practical maximum length of line probably increases as some power of the system voltage, other conditions being equal.

Linear Transformations in Three-Phase Circuits

Discussion and author's closure of paper 40-109 by Louis A. Pipes, presented at the AIEE summer convention, Swampscott, Mass., June 24–28, 1940, and published in AIEE TRANSACTIONS, 1941, pages 351–6.

S. Austen Stigant (Messrs. Johnson and Phillips, Ltd., London, England): Dr. Pipes' application of certain established principles of matrix algebra, whereby the general nonsingular matrix may be diagonalized, to the simplification of the basic equations of the electric circuit is both useful and stimulating. The study might, I think, be extended to a formulation of rules by which the $[A]$, $[B]$, and $[Z]_t$ matrices could be written down at once for any given manner of employing the principles in question, whether in simple or in complicated cases.

An investigation of the particular equations displayed by the author discloses a somewhat simplified manner in which they may be presented, employing minors instead of the n-elements themselves.

Consider, for example, equation 68 of the paper. The principal minors and the various associated second minors of the matrix n-elements are given in table I, and it will be observed that certain of the principal minors therein appear in the various equations 69 to 82. Where indicated therefore, we may make the following substitutions.

$$= Z_{11}\left\|\begin{matrix}1, & 0, & 0\\ 0, & M_{22}, & M_{23}\\ 0, & M_{23}, & M_{33}\end{matrix}\right\| \tag{70a}$$

$$[Z]_t = Z_{11}\left\|\begin{matrix}1, & 0, & 0\\ 0, & M_{33}, & 0\\ 0, & 0, & \dfrac{M_{22}M_{33}-M_{23}^2}{M_{33}}\end{matrix}\right\| \tag{71a}$$

$$= Z_{11}\left\|\begin{matrix}1, & 0, & 0\\ 0, & M_{33}, & 0\\ 0, & 0, & \dfrac{M_1'}{M_{33}}\end{matrix}\right\| \text{or, } \frac{Z_{11}}{M_{33}}\left\|\begin{matrix}M_{33}, & 0, & 0\\ 0, & M_{33}^2, & 0\\ 0, & 0, & M_1'\end{matrix}\right\|$$

where M_1' is clearly the principal minor of the leading element in (70*a*), i.e.,

$$M_1' = M_{22}M_{33} - M_{23}^2$$

$$[A] = \frac{1}{M_{33}}\left\|\begin{matrix}M_{33}, & 0, & 0\\ -{}_pM_{33}\,{}_sM_{21}, & M_{33}, & 0\\ M_{13}, & -M_{23}, & M_{33}\end{matrix}\right\| \tag{72a}$$

$$[B] = \frac{1}{M_{33}}\left\|\begin{matrix}M_{33}, & -{}_pM_{33}\,{}_sM_{21}, & M_{13}\\ 0, & M_{33}, & -M_{23}\\ 0, & 0, & M_{33}\end{matrix}\right\| \tag{73a}$$

$$k = -\frac{M_{23}}{M_{33}} \tag{74a}$$

$$[A]^{-1} = \frac{1}{M_{33}}\left\|\begin{matrix}M_{33}, & 0, & 0\\ {}_pM_{33}\,{}_sM_{21}, & M_{33}, & 0\\ {}_sM_{21}M_{23}-M_{13}, & M_{23}, & M_{33}\end{matrix}\right\| \tag{75a}$$

$$[B]^{-1} = \frac{1}{M_{33}}\left\|\begin{matrix}M_{33}, & {}_pM_{33}\,{}_sM_{21}, & {}_sM_{21}M_{23}-M_{13}\\ 0, & M_{33}, & M_{23}\\ 0, & 0, & M_{33}\end{matrix}\right\| \tag{76a}$$

It may be noted that in (72*a*) etc., the method of expressing the n-elements in the form $-({}_pM_{33}\,{}_sM_{21})/M_{33}$ serves two purposes. The second minor ${}_sM_{21}$ (for instance) is the one giving the required element $(-n_{12})$ in the transformation matrix, but the associated principal minor ${}_pM_{33}$ shows exactly the particular second matrix from which the second minor is derived; having served that purpose ${}_pM_{33}$ is subsequently eliminated by division by M_{33}.

For the case of the general unsymmetric matrix the alternative forms are similar to the foregoing, becoming as follows:

$$[A] = \frac{1}{M_{33}}\left\|\begin{matrix}M_{33}, & 0, & 0\\ -{}_pM_{33}\,{}_sM_{12}, & M_{33}, & 0\\ M_{13}, & -M_{23}, & M_{33}\end{matrix}\right\| \tag{78a}$$

Table I

Matrix Element	Minors: Principal	Minors: Associated Seconds	Matrix Element	Minors: Principal	Minors: Associated Seconds	Matrix Element	Minors: Principal	Minors: Associated Seconds
I	${}_pM_1=n_{22}n_{33}-n_{23}n_{32}$	${}_sM_{22}=n_{33}$ ${}_sM_{23}=n_{32}$ ${}_sM_{32}=n_{23}$ ${}_sM_{33}=n_{22}$	n_{12}	${}_pM_{12}=n_{21}n_{33}-n_{23}n_{31}$	${}_sM_{21}=n_{33}$ ${}_sM_{23}=n_{31}$ ${}_sM_{31}=n_{23}$ ${}_sM_{33}=n_{21}$	n_{13}	${}_pM_{13}=n_{21}n_{32}-n_{22}n_{31}$	${}_sM_{21}=n_{32}$ ${}_sM_{22}=n_{31}$ ${}_sM_{31}=n_{22}$ ${}_sM_{32}=n_{21}$
n_{21}	${}_pM_{21}=n_{12}n_{33}-n_{13}n_{32}$	${}_sM_{12}=n_{33}$ ${}_sM_{13}=n_{32}$ ${}_sM_{32}=n_{13}$ ${}_sM_{33}=n_{12}$	n_{22}	${}_pM_{22}=I\cdot n_{33}-n_{13}n_{31}$	${}_sM_1=n_{33}$ ${}_sM_{13}=n_{31}$ ${}_sM_{31}=n_{13}$ ${}_sM_{33}=I$	n_{23}	${}_pM_{23}=I\cdot n_{32}-n_{12}n_{31}$	${}_sM_1=n_{32}$ ${}_sM_{12}=n_{31}$ ${}_sM_{31}=n_{12}$ ${}_sM_{32}=I$
n_{31}	${}_pM_{31}=n_{12}n_{23}-n_{13}n_{22}$	${}_sM_{12}=n_{23}$ ${}_sM_{13}=n_{22}$ ${}_sM_{22}=n_{13}$ ${}_sM_{23}=n_{12}$	n_{32}	${}_pM_{32}=I\cdot n_{23}-n_{13}n_{21}$	${}_sM_1=n_{23}$ ${}_sM_{13}=n_{21}$ ${}_sM_{21}=n_{13}$ ${}_sM_{23}=I$	n_{33}	${}_pM_{33}=I\cdot n_{22}-n_{12}n_{21}$	${}_sM_1=n_{22}$ ${}_sM_{12}=n_{21}$ ${}_sM_{21}=n_{12}$ ${}_sM_{22}=I$

$$[B]=\frac{1}{M_{33}}\begin{Vmatrix} M_{33}, & -{}_pM_{32}{}_sM_{21}, & M_{31} \\ 0, & M_{33}, & -M_{32} \\ 0, & 0, & M_{33} \end{Vmatrix} \tag{79a}$$

$$k_1=-\frac{M_{23}}{M_{33}} \tag{80a}$$

$$k_2=-\frac{M_{32}}{M_{33}} \tag{81a}$$

$$[Z]_t=Z_{11}\begin{Vmatrix} 1, & 0, & 0 \\ 0, & M_{33}, & 0 \\ 0, & 0, & \dfrac{M_{22}M_{33}-M_{23}M_{32}}{M_{33}} \end{Vmatrix} \tag{82a}$$

$$=Z_{11}\begin{Vmatrix} 1, & 0, & 0 \\ 0, & M_{33}, & 0 \\ 0, & 0, & \dfrac{M_1''}{M_{33}} \end{Vmatrix} \text{ or,}$$

$$\frac{Z_{11}}{M_{33}}\begin{Vmatrix} M_{33}, & 0, & 0 \\ 0, & M_{33}^2, & 0 \\ 0, & 0, & M_1'' \end{Vmatrix}$$

where

$$M_1''=M_{22}M_{33}-M_{23}M_{32}$$

This method of stating the matrix elements as minors of the impedance matrix (68) may lead to a more direct way of reaching the desired results. It certainly makes for co-ordination of expression.

Edward W. Kimbark (Northwestern University, Evanston, Ill.): The material in Pipes' interesting paper may be divided into two parts. One part (sections I, II, III, IV, V, and IX) treats, by matrix notation, the theory of the general linear transformation of three-phase currents, voltages, and impedances, and also treats as special cases several particular transformations, such as symmetrical components. This part is excellently done. The second part (sections VI, VII, VIII, and X) treats the diagonalization of any impedance matrix. This part is disappointing because the theory is not carried to a point where it is of value in the practical analysis of three-phase circuits.

The special transformations which have proved of value are not obtained by following the methods presented in the paper. For example, it is known that the impedance matrix of a symmetrical rotating machine (equation 6 of the paper) is diagonalized by the symmetrical component transformation (equation 22 of the paper). But the application of the method of diagonalization set forth in section VIII of the paper does not lead to symmetrical components nor to anything resembling symmetrical components.

The author seems to have overlooked the fact that a three-phase network usually contains many branches, and that a transformation which eliminates the coupling between substitutive networks in one branch only is of little or no value. What is required is a transformation that eliminates the coupling in as many branches as possible and at the same time reduces the coupling in the remaining branches to as simple a form as possible. Most branches of most three-phase networks are symmetrical and bilateral, or at least nearly enough so that they may be considered so with negligible error; that is, most branches have the type of impedance matrix given by equation 5 of the paper. It is, therefore, reasonable to demand that any transformation which is to be applied to a three-phase network should have the property of diagonalizing the impedance matrices of all symmetrical bilateral three-phase branches. Both symmetrical components and two-phase components have this property. They are not, however, the only transformations having it. Concordia[1] has given four transformations, all of which have this property.

The purpose of this discussion is to present a general transformation which does have the property of diagonalizing the impedance matrix of any symmetrical bilateral three-phase branch and which does reduce to symmetrical components and to the other transformations given by Concordia as special cases. The general transformation is as follows:

$$\left.\begin{aligned} I_{1t}&=\frac{\underline{/\theta_1}}{\sqrt{3}}(I_1+I_2+I_3)\\ I_{2t}&=\frac{\underline{/\theta_2}}{\sqrt{3}}[(\sin\phi\underline{/\theta}+\cos\phi\underline{/-\theta})I_1+\\ &\quad(a^2\sin\phi\underline{/\theta}+a\cos\phi\underline{/-\theta})I_2+\\ &\quad(a\sin\phi\underline{/\theta}+a^2\cos\phi\underline{/-\theta})I_3]\\ I_{3t}&=\frac{\underline{/\theta_3}}{\sqrt{3}}[(\cos\phi\underline{/\theta}-\sin\phi\underline{/-\theta})I_1+\\ &\quad(a^2\cos\phi\underline{/\theta}-a\sin\phi\underline{/-\theta})I_2+\\ &\quad(a\cos\phi\underline{/\theta}-a^2\sin\phi\underline{/-\theta})I_3] \end{aligned}\right\}. \tag{1}$$

This transformation is given in such form that the power is invariant; and the same transformation is to be used both for current and for voltage; consequently, the inverse transformation is the conjugate transpose of it, as follows:

$$\left.\begin{aligned} I_1&=\frac{1}{\sqrt{3}}[I_{1t}\underline{/-\theta_1}+(\sin\phi\underline{/-\theta}+\\ &\quad\cos\phi\underline{/\theta})\underline{/-\theta_2}\cdot I_{2t}+(\cos\phi\underline{/-\theta}-\\ &\quad\sin\phi\underline{/\theta})\underline{/-\theta_3}\cdot I_{3t}\\ I_2&=\frac{1}{\sqrt{3}}[I_{1t}\underline{/-\theta_1}+(a\sin\phi\underline{/-\theta}+\\ &\quad a^2\cos\phi\underline{/\theta})\underline{/-\theta_2}\cdot I_{2t}+(a\cos\phi\underline{/-\theta}-\\ &\quad a^2\sin\phi\underline{/\theta})\underline{/-\theta_3}\cdot I_{3t}\\ I_3&=\frac{1}{\sqrt{3}}[I_{1t}\underline{/-\theta_1}+(a^2\sin\phi\underline{/-\theta}+\\ &\quad a\cos\phi\underline{/\theta})\underline{/-\theta_2}\cdot I_{2t}+(a^2\cos\phi\underline{/-\theta}-\\ &\quad a\sin\phi\underline{/\theta})\underline{/-\theta_3}\cdot I_{3t} \end{aligned}\right\} \tag{2}$$

It will now be shown that the general transformation reduces to certain known special transformations. First, put $\theta_1=\theta_2=\theta_3=0$, and $\phi=0$; whence $\underline{/\theta_1}=\underline{/\theta_2}=\underline{/\theta_3}=1$, $\cos\phi=1$, and $\sin\phi=0$. The general transformation becomes:

$$\left.\begin{aligned} I_{1t}&=\frac{1}{\sqrt{3}}(I_1+I_2+I_3)=I_0\\ I_{2t}&=\frac{\underline{/-\theta}}{\sqrt{3}}(I_1+aI_2+a^2I_3)=I_+\underline{/-\theta}\\ I_{3t}&=\frac{\underline{/\theta}}{\sqrt{3}}(I_1+a^2I_2+aI_3)=I_-\underline{/\theta} \end{aligned}\right\} \tag{3}$$

which is a generalized type of symmetrical components. Putting $\theta=0$ gives the ordinary type of symmetrical components in their normalized form (power invariant). The parameter θ serves to shift the phase of reference.

Next, put $\theta_1=\theta_2=0$, $\theta_3=-90$ degrees, and $\phi=45$ degrees; whence $\underline{/\theta_1}=\underline{/\theta_2}=1$, $\underline{/\theta_3}=-j$, and $\cos\phi=\sin\phi=1/\sqrt{2}$. The result is that I_{1t} still equals I_0, while I_{2t} and I_{3t} become the direct and quadrature axis currents in normalized form:

$$\left.\begin{aligned} I_{1t}&=\frac{1}{\sqrt{3}}(I_1+I_2+I_3)=I_0\\ I_{2t}&=\sqrt{\frac{2}{3}}[\cos\theta\cdot I_1+\cos(\theta-120°)\cdot\\ &\quad I_2+\cos(\theta+120°)\cdot I_3]=I_d\\ I_{3t}&=\sqrt{\frac{2}{3}}[\sin\theta\cdot I_1+\sin(\theta-120°)\cdot\\ &\quad I_2+\sin(\theta+120°)\cdot I_3]=-I_q \end{aligned}\right\} \tag{4}$$

Again putting $\theta=0$, we get normalized two-phase components:

$$\left.\begin{aligned} I_{2t}&=\sqrt{\frac{2}{3}}\left[I_1-\frac{1}{2}(I_2+I_3)\right] \\ I_{3t}&=\frac{1}{\sqrt{2}}(-I_2+I_3) \end{aligned}\right\} \quad (5)$$

It may be shown that the general transformation (1), as well as the four special ones, diagonalizes the impedance matrix of any symmetrical bilateral branch. This general transformation may not be the *most* general one which meets the prescribed conditions, but at least it has several parameters which may be selected so as to simplify the coupling between substitutive networks. The parameters θ_1, θ_2, and θ_3 have no effect in making the transformed mutual impedances vanish; therefore, they are of little value in this study. Parameters ϕ and θ, however, may be used for this purpose.

As an example of the selection of parameters in a particular case, consider a three-phase bilateral network, all the branches of which are symmetrical except one. The one unsymmetrical branch consists of three unequal load resistances ($Z_{11}=3$, $Z_{22}=5$, $Z_{33}=10$) connected in Y with isolated neutral. Let the special transformation (4) be used. This transformation eliminates the coupling between substitutive networks of all symmetrical branches, regardless of the value of θ chosen. On account of the isolated neutral no zero-sequence current (I_{1t}) can flow in the load; and since the remainder of the three-phase network is symmetrical, there will be no current anywhere in substitute network number 1. A value of θ should be selected which eliminates the coupling between networks numbers 2 and 3 in the load branch.

By using equation 18 of the paper we find the mutual impedance between substitutive networks 2 and 3 to be:

$$Z_{23t}=\frac{\underline{/\theta_2-\theta_3}}{6}\left[Z_{11}(\underline{/-2\theta}-\underline{/2\theta})+Z_{22}\times\right.$$
$$\left.(a^2\underline{/-2\theta}-a\underline{/2\theta})+Z_{33}(a\underline{/-2\theta}-a^2\underline{/2\theta})\right] \quad (6)$$

Setting this expression equal to zero and solving for θ,

$$\underline{/4\theta}=\frac{Z_{11}+a^2Z_{22}+aZ_{33}}{Z_{11}+aZ_{22}+a^2Z_{33}} \quad (7)$$

and substituting numerical values

$$\underline{/4\theta}=\frac{3+a^25+a10}{3+a5+a^210}=\frac{-4.5+j4.3}{-4.5-j4.3}=\underline{/272.2^\circ}$$

$$4\theta=272.2^\circ\pm n360^\circ,\text{ where } n=0,\ 1,\ 2,\ 3$$
$$\theta=68^\circ,\ 158^\circ,\ 248^\circ,\text{ or }338^\circ \quad (8)$$

Putting any one of these four values of θ into equations 4 gives a transformation which completely separates networks 2 and 3. Assuming balanced electromotive forces in the three-phase network, the electromotive forces of networks 2 and 3 are in quadrature. The two networks may be solved as separate single-phase circuits, and the currents so found are combined by means of the inverse transformation to obtain the actual currents.

A general three-phase transformation has been presented, which though not the last word on the subject, appears to have more immediate usefulness than the method of diagonalization presented by Pipes. The next step should probably be a study of the number of degrees of freedom (number of independent real parameters) in the most general three-phase transformation which diagonalizes the bilateral symmetrical impedance matrix and which also keeps the power invariant.

REFERENCE

1. RELATIONS AMONG TRANSFORMATIONS USED IN ELECTRICAL ENGINEERING PROBLEMS. *General Electric Review*, volume 41, July 1938, pages 323–5.

L. A. Pipes: Professor Kimbark, in his discussion, points out that the usual three-phase network contains many branches and that transformation matrices $[A]$ and $[B]$, which diagonalize *one* individual impedance matrix, $[Z]_t$ of a branch of the network by a transformation of the form:

$$[Z]_{tt}=[A][Z]_t[B] \quad (1)$$

are of little or no practical value. He points out that most branches of such networks have impedance matrices of the completely symmetrical form:

$$[Z]_s=\begin{bmatrix} Z_0 & Z & Z \\ Z & Z_0 & Z \\ Z & Z & Z_0 \end{bmatrix} \quad (2)$$

and that therefore, matrices $[A]$ and $[B]$ must be capable of diagonalizing matrices of type (2) and at the same time reduce the other $[Z]_t$ matrices of the various branches of the network as nearly as possible to the diagonal form.

The method presented in VI of the paper, while able to diagonalize one general matrix is not capable of diagonalizing several other matrices simultaneously. In this connection, Professor Kimbark's point is well taken and he gives a special matrix that diagonalizes matrices of the type (2) and has parameters that may be adjusted to diagonalize other matrices having a limited amount of dissymmetry.

If the three-phase system has n branches of impedance matrices $[Z]_1$, $[Z]_2 \ldots [Z]_n$, the entire impedance matrix of the system is given by $[Z]$, where:

$$[Z]=\sum_{i=1}^{n}[Z]_i \quad (3)$$

The method of the paper as presented in VI, enables the general matrix $[Z]$ to be diagonalized by the transformation:

$$[Z]_t=[A][Z][B]=[A]\left(\sum_{i=1}^{n}[Z]_i\right)[B] \quad (4)$$

without diagonalizing the individual branch matrices $[Z]_i$ by transformations of the type (1).

In order to do this when the matrices $[Z]_i$ are general matrices, further research is necessary. Since the matrices $[A]$ and $[B]$ may, in the general case, contain a total of 18 *arbitrary* elements and the condition that each matrix $[Z]_i$ be diagonalized by a transformation of the form (1) leads to six equations to determine the elements of $[A]$ and $[B]$ it would appear that at most, only three matrices $[Z]_i$ of general form may be diagonalized simultaneously by the two matrices $[A]$ and $[B]$.

The matrices suggested by Professor Kimbark should prove of value in the analysis of systems having slight dissymmetry. Further research will no doubt lead to the discovery of many more matrices having utility in circuits of certain types of symmetry.

Transient Analysis of Completely Transposed Multiconductor Transmission Lines

Discussion and author's closure of paper 40-108 by Louis A. Pipes, presented at the AIEE summer convention, Swampscott, Mass., June 24–28, 1940, and published in AIEE TRANSACTIONS, 1941, pages 346–51.

E. Weber (Polytechnic Institute of Brooklyn, N. Y.): The claim of the author that the solutions are simple is correct as far as they are presented in this paper. However, the examples given for the dissipationless as well as the general case are not of practical value on account of the special boundary conditions imposed which permit an applied voltage of arbitrary value at a certain point along the line but which assume zero current at the same point for all time. This procedure disregards entirely the boundary conditions arising from the given terminations of lines; the simple expressions obtained are, therefore, of academic value only and cannot be used in the solution of practical problems in which the terminal conditions are of utmost importance. Contrary to the author's claim, the method presented in the paper cannot in any way circumvent the well-known mathematical difficulties of transmission-line problems which are encountered in satisfying specific terminal conditions in a practical case. Furthermore, the assumption that the "entering" current i_0 be zero at all times precludes any transmission of power along the line.

It might also be of interest to consult the various papers by John R. Carson, Bell Telephone Laboratories, on the foundations of wire transmission theory.

Louis A. Pipes: Dr. Weber's contention that the boundary conditions are disregarded is well taken. However it must be emphasized that what is presented in the paper under consideration is an extension of the simple engineering formulation of the n-line problem.

This formulation of the n-line problem is based on the assumptions discussed in section II of the paper. Under these assumptions the long-line problem is regarded as a quasi-stationary field phenomenon. An admirable exposition of the present state of the theory is given by J. R. Carson's paper, "The Present Status of Wire Transmission Theory and Some of Its Outstanding Problems," *B.S.T.J.* 7, 1928, pages 268–80. What is attempted in the paper under consideration is an extension of the simplified engineering theory to the case of perfect transposition.

The use of the transformation matrix [A], separates the equations into independent sets and makes the solution formally possible. The extension of the theory to the case of "entering currents" not zero and to more general engineering boundary conditions is very simple and proceeds directly from the equations 44 and 45.

Some Characteristics and Applications of Negative-Glow Lamps

Discussion and author's closure of paper 40-110 by H. M. Ferree, presented at the AIEE summer convention, Swampscott, Mass., June 24–28, 1940, and published in AIEE TRANSACTIONS, 1941 (January section) pages 8–12.

E. M. Strong (Cornell University, Ithaca, N. Y.): Mr. Ferree presents a timely review of the characteristics and accomplishments of the neon glow lamp family since its introduction in this country nearly a decade and a half ago.

Possibly it should be stressed that the characteristic data are average values which admit of considerable variation in individual lamps. This is of no consequence for most purposes but may be serious if one attempts to use them as oscillators or other more exacting applications. I recall making measurements when they first appeared, and considerable difficulty in finding two alike. Even for a given lamp the extinction voltage in particular is uncertain not only because of photoelectric and temperature effects but also because the discharge does not involve the entire electrode surface at near-extinction values of current, and the particular parts of the electrodes to which the discharge becomes localized vary in uncertain manner.

It seems to me that credit should be given to the neon glow lamp for bringing into existence several offspring designed for special services.

One of the most recent of these is the neon glow starting switch for fluorescent lamps. Probably the first and most important is the 885 type sweep circuit oscillator tube for the conventional cathode-ray oscillograph. Most of us readily recall the time when cathode-ray oscillographs were associated only with Lissajous figures instead of the now usual XY co-ordinates. I was studying neon lamp characteristics with Dr. Bedell at Cornell when he conceived application of the Pearson and Anson oscillator (reference 2) to provide a linear time axis for the cathode-ray oscillograph (AIEE TRANSACTIONS, volume 46, May 27, pages 546–50). I believe this was the first all-electric "sweep" provided for these oscillographs. They were marketed for some time by the Burt Laboratories in California and pioneered until a neon triode was expressly designed for the purpose. While it is impossible to detail the numerous applications which have been found for neon lamps, Doctor Bedell's contribution merits high place on the record. Doctor Reich now at the University of Illinois worked with Doctor Bedell at that time and contributed notably to the literature on neon lamp application in his own right as well as in assisting Doctor Bedell.

My first application of the lamp was for the observation of induction motor slip and synchronous motor hunting and torque angle in lecture demonstration work (*The Sibley Journal of Engineering*, December 1927, page 254). At that time only the one-fourth watt lamp had appeared. By mounting the lamp on the periphery of a small pulley attached to the machine shaft and applying terminal voltage to it, a number of traces or arcs of light equal to the poles were created, which rotated according to the slip or other deviation from synchronous speed. I still use the demonstration with refinements of bias voltage to eliminate alternate traces of light, and a peaking transformer when actual measurements are desired. Elimination of alternate traces permits its use to synchronize an alternator for connection to the station bus.

This application led to a "vector lamp" which comprises a small tabular neon lamp, at one time known to the radio trade as a "tune-a-lite," mounted as the radius of a disk rotated by a small induction type synchronous motor at 3,600 rpm. The lamp has one long and one short electrode so that the length of electrode which appears to glow is (within limits) proportional to the lamp current. When operated on 60 cycles with a peaking transformer a single line or "vector" of light appears during rotation. The length of the line is proportional to the voltage, and the angular position depends upon the phase of the voltages used to light the lamp and to drive the motor. If, for example, a transformer is operated on the same voltage as the motor, and the transformer secondary voltage is applied to the lamp, phase and magnitude changes in secondary voltage are indicated by the "light vector" as various loadings are applied to the transformer. Such demonstrations are of great help in developing early interest and confidence in vector diagrams as practical tools of the profession without demanding too much dependence on mathematical integrity.

One of the simplest applications which exploits neon lamp assets as fully as any I have devised is again a lecture demonstration device. Let us connect a resistor R and an inductance L, of suitable magnitudes in series, so that when 115 volts direct current is applied, time of the order of a second is required for the transient. If two neon lamps be connected, one across R and one across L, some acquaintance with the nature of the circuit is readily obtained. Immediately on applying voltage one electrode of the L-lamp lights; it then dims and extinguishes, and one electrode of the R-lamp lights and stays so in accord with $E = Ri + L\dfrac{di}{dt}$. If the applied electromotive force is now removed, the R-lamp promptly extinguishes and the *other* electrode of the L-lamp lights brightly and then extinguishes. Clearly the polarity of the inductive electromotive force is *reversed* during decay of current and higher magnitude is attained. Details of the entire demonstration need not be enumerated here. Similar demonstrations for simple combinations of R and C and R, L, and C are equally useful. These in no way supplant oscillographic demonstrations of greater detail but have proved of real value in introductory work.

The combined ability of the neon lamp to indicate polarity and approximate magnitude (by brightness as well as ignition and extinction voltages) without serious time lag or power absorption makes possible many applications such as the above which give it distinction as an economy instrument of considerable utility outside of its homely purpose as a low purpose illuminant.

H. M. Ferree: I feel that we are deeply indebted to Professor Strong for his enlightening discussion of our subject. The information which he has given us, and the possible new applications for negative glow lamps which he has described, has certainly done much to make this paper more interesting and profitable to all of us.

Harmonic-Current-Restrained Relays for Transformer Differential Protection

Discussion and author's closure of paper 40-87 by C. D. Hayward, presented at the AIEE summer convention, Swampscott, Mass., June 24–28, 1940, and published in AIEE TRANSACTIONS, 1941, pages 377–82.

L. F. Kennedy (General Electric Company, Schenectady, N. Y.): Mr. Hayward has, I believe, understated the need for an adequate relay for transformer protection. He has pointed out the solution, sometimes used with induction-type differential relays, involving high current and time settings but he has not made clear that, in many such applications, the protection afforded is little better than could be obtained with plain overcurrent relays, thus losing the advantages of sensitive and prompt action normally thought of as inherent to differential protection. Neither has he stressed the fact that the use of the various desensitizing arrangements have been accepted by most relay engineers only because there was no really adequate solution available.

The principle of harmonic restraint has already been described before the Institute. The characteristics of the magnetizing inrush have been described by many writers, so that it is a well-known fact that this phenomenon results in a current wave which is completely offset with respect to the zero axis.[1] From time to time various relay systems have been proposed which would not operate on this type of current wave, such as polarized devices requiring alternate positive and negative impulses and different filter arrangements. The harmonic-restrained relay is, however, the first device, which combines relative simplicity and a high torque, positive acting relay element, to give adequate protection against short circuits and at the same time freedom from operation on magnetizing-inrush currents.

There can be little doubt that the harmonic-restrained relay can distinguish between magnetizing-inrush currents and fault currents. There may be, to some, a question as to whether the relay will be capable of providing sufficient restraint to prevent incorrect operation on an external fault because of current-transformer errors. During the past two years considerable study has been given to current-transformer performance and as a result of these studies, the inadequacies of current transformers have been frequently discussed before the Institute. Engineers are now more conscious than ever before, of the current-transformer side of the story and, naturally, there is a tendency to consider all current transformers as of questionable performance. When we consider practical cases, however, we find that in general the current transformers used with large power transformers are capable of giving a good account of themselves under the most severe conditions to which they can be subjected. I feel, therefore, that despite the caution suggested by Mr. Hayward in the choice of current transformers, application engineers will find that the types of current transformers normally available for such applications are entirely adequate.

As a matter of record there are in service many transformer differential relays of the high-speed type having only through current restraint and operating from conventional bushing-type current transformers. Most of these use some form of desensitizing to prevent operation on magnetizing inrush, but have no special provision other than through current restraint to prevent operation on through faults. The operating record of these relays has been very good and certainly this new relay combining harmonic with through restraint should give sufficient added safety to prevent any danger of incorrect operations for most normal applications.

REFERENCE

1. TRANSFORMER ENGINEERING (a book), L. F. Blume. John Wiley and Sons, 1938.

W. K. Sonnemann and **R. M. Smith** (Westinghouse Electric and Manufacturing Company, Newark, N. J.): At first glance this paper seems to offer a solution to a problem that has long been troublesome. This problem is the one of educating a relay to distinguish between magnetizing-inrush current and short-circuit current of nearly equal magnitudes. Since the former obviously does contain harmonic components, their use in restraining the relay when operation should not occur appears logical. Unfortunately, harmonics also appear in the operating current when tripping should occur and, therefore, the relay must respond to the ratio of fundamental current to harmonic content. This would still be a satisfactory discriminating function if the ratio were always above a definite maximum value for all internal faults, and always below a definite minimum value for magnetizing inrush and external faults. It would also be desirable to have these limiting ratios subject to analysis and calculation. It has not been possible to do this because of the large number of variables involved, including the type of current-transformer design, the amount of iron, the secondary burden, the amount of current, the number of secondary turns, and the magnetic characteristics of the iron used. Possibly, however, the author has found some method of analyzing the problem which has not been included in the paper. This assumption arises from the statement that, "The harmonic-restraint curve *B* lies between the harmonic percentages obtained for the internal fault and the through fault." This would indicate an exact solution has been found. The paper would have been of greater value to the industry if this solution had been given—if such is available.

We note that the third harmonic is not available for star-delta transformers due to circulation of the third harmonic in the delta-connected current transformers. However, in reference to figure 9*D*, which is a test on a single-phase transformer, credit is given to the presence of the third-harmonic current to help desensitize the relay. From the above, we assume that the relay might well have tripped had the same test been made on a star-delta bank.

We note that the relay is restricted in its application to locations where the time constant is not greater than 0.05 second, or 3 cycles on a 60-cycle basis. We would like to inquire if this is an exact solution or if it has been arrived at from test. It would seem that other factors would also have to be specified; for example, the cross section of the iron in the current transformers would be equally important.

J. A. Elzi and **Wm. R. Brownlee** (Commonwealth and Southern Corporation, Jackson, Mich.): The desirability of a relaying system which would provide fast and sensitive protection to all types of transformer banks and connections without danger of incorrect operation on either through short circuits or magnetizing transients has long been recognized. The requirement of eliminating false tripping is particularly acute in the case of transformers since the damage which might be done to this equipment if energized after a fault is so severe that it is customary to make a thorough examination and sometimes detailed electrical tests before attempting to energize it after a differential-relay operation. Particularly in the case of transformers serving loads the operator must choose between possible extensive interruptions, overloading other equipment, or the possibility of severe damage to the transformer if it is re-energized.

The author is to be congratulated on his ingenuity in designing a device which follows so closely the characteristic which he feels is the most desirable, particularly since it is difficult to avoid transients in the various elements of devices containing tuned circuits when the external circuit is suddenly energized or changed.

In applying new principles to old problems it is frequently easier to design a device which will respond to certain quantities than it is to determine definitely the operating margins available in the conflicting characteristics of the circuit to be protected. We would like to ask if the author has made sufficient tests on other transformers to assure himself that the wave form of magnetizing inrush given in table I is closely representative.

Even if the wave form given in table I is representative, the restraint margin may not be adequate for a three-phase bank with customary connections. The author has pointed out that the third-harmonic components of through short-circuit current which are produced by the current transformers themselves may be eliminated when the current transformers are connected in delta for protecting a wye-delta transformer bank. In the same manner the third harmonics of magnetizing-inrush current in two adjacent single-phase transformers of a delta-delta bank will disappear in the common lead to the current transformers whenever the magnitude of the initial peak of two adjacent transformers are approximately equal, this being true since the fundamental components of current in the two adjacent transformers will add at 60 degrees and the third harmonics at 180 degrees. Similarly, the second harmonics of equal current peaks will be reduced with respect to the fundamental component in the ratio of 1.73 since the fundamental currents add at 60 degrees and the second harmonics at 120 degrees. The effective value of the second harmonic in the table would reduce to 36 per cent instead of 63 per cent, the third harmonic would disappear and the rms value of the second, fifth, and other harmonics would be some 50 per cent whereas the curve in figure 5 would indicate an average value of 35 per cent required for restraining operation of the relay, which may not give sufficient margin.

Reasonably good protection has been afforded by conventional ratio-differential relays to large numbers of transformers where an operating speed of four to six cycles is adequate and where a sensitivity of 50 per cent to 75 per cent of full-load current is satisfactory. Experience with over 100 such installations on the various properties of The Commonwealth and Southern Corporation has shown only four cases of difficulty due to magnetizing-inrush current, and in each of these the only difference in application has been that the power transformers themselves had normal magnetizing currents of four to six per cent of full-load current whereas many modern transformers have magnetizing currents less than $1^1/_2$ per cent. It seems likely that difficulties brought about by high normal magnetizing currents might be accentuated in the case of harmonic-restrained relays, since early saturation of the iron could account for improved wave form on the high peaks of the transient wave. Of course this assumes that customary line-current transformers would be utilized so that each relay would receive the sum at 60 degrees of the current in two adjacent single-phase transformers.

Our experience does not confirm the author's statement that it is more difficult to relay a large transformer bank. At a recent plant three transformer banks were equipped with conventional ratio-differential relays with approximately the same sensitivity and speed, one set on a 750-kva, 3-phase transformer, another on a bank of three 1,500-kva units, and a third on a bank of three 15,000-kva units. Serious difficulties with magnetizing inrush occurred on the two smaller transformers and the relay pickup had to be raised to twice full-load current to prevent frequent false operations, whereas no difficulty at all was experienced on the large bank of three 15,000-kva units with a relay pickup of 55 per cent of full load. The small transformers have magnetizing currents of four to five per cent compared with 1.14 per cent for the large bank. The

15,000-kva and the 1,500-kva transformers are energized from busses at which the system short-circuit capacities are in proportion to the transformer ratings. The respective manufacturers of these transformers have advised that the designs of the units represent their standard practice, and the relay manufacturer has stated that the relay connections, applications, etc., are also standard. Many transformers purchased during the past few years have magnetizing currents similar to the bank of three 15,000-kva units although there appears to be considerable variation between units of different manufacture purchased under the same specifications and with competitive efficiencies but without any specified limits of normal or transient magnetizing current. Consideration might be given to placing more emphasis on magnetizing-current characteristics in the purchase of major equipment and to requesting the maximum value of transient magnetizing current for comparison.

A conventional ratio-differential relay could be made to operate at relatively high speed with freedom from false operation on magnetizing transients if its sensitivity could be decreased so that it would operate only on short circuits at the power transformer terminals which would practically always be 6 to 15 times full-load current. It is well then to examine the conditions which require sensitive operation. Since plain ground relays can provide adequate sensitivity for protecting ungrounded-transformer windings where the ground current is much less than the short-circuit current, the most difficult condition normally expected is the limitation of fault current due to the nature of an internal fault in the transformer. Probably the most common occurrence is a failure on a part of the transformer winding not adjacent to the terminals so that the fault is limited by the relatively high reactance between different portions of the winding, this reactance being of such nature as to include considerable iron magnetization effect. Such a fault, both through the action of the iron involved and on account of the low current and high voltage in the resulting arc, will tend to produce harmonics of appreciable magnitude in the current wave tending to prevent correct operation. It seems that an exhaustive investigation including short-circuit tests is in order before it can be definitely determined whether or not the types of faults which require sensitive operation may defeat the apparent sensitivity of the harmonic-restrained relay. On account of the infrequency of transformer failures, particularly those other than bushing flashovers, it is doubtful if operating experience alone could provide the answer to this question except over a period of many years.

It is hoped that the sponsors of the harmonic-restraint relay will continue their study and supplement it with adequate tests to determine the performance of this type of relay under all conditions which may be encountered in the field.

C. D. Hayward: Sonnemann and Smith, in their discussion, have pointed out that a great deal of test work and analysis necessarily lies behind the selection of the amount of harmonic restraint used in the relay to insure proper restraint on magnetizing inrush and through fault-current waves, while allowing high-speed operation on internal fault-current waves. A great many tests of this kind have been made, some on actual transformer installations on operating company properties, others in the laboratories and short-circuit test stations of the General Electric Company. Analytical methods have been used to interpret, co-ordinate, and extend the results. The harmonic-restraint relay has been designed to have the characteristics dictated by these findings. Unfortunately, it is not possible to give all the details of the tests within the limited confines of a single Institute paper. Some of the tests are classed more properly under the subject of a current-transformer investigation rather than a relay investigation and will be included in a future paper under that title. In the present paper sufficient data have been given to explain the operation of the relay and to enable the relay engineer to make all ordinary applications.

Sonnemann and Smith seem to be a little confused as to the purpose of the test illustrated by the oscillogram figure 9*D*. As explained in the text of the paper, this test was made with the through-current-restraint tap plug in the minimum 15 per cent slope position for the purpose of illustrating the aiding effect of the harmonic restraint in preventing false operation on through faults. Normally, for an application where through faults of this severity are met, the tap plug would be placed in the 25 per cent or 40 per cent slope position. As Mr. Kennedy has pointed out in his discussion, there are in service today many high-speed transformer differential relays which rely on through current restraint alone to prevent operation on through faults. In the harmonic-restraint relay the double safeguard of harmonic restraint plus through current restraint is offered. True enough, as stated in the paper, where delta-connected current transformers are used, the harmonic part of the restraint will be lost on balanced three-phase through faults. But these faults are of rare occurrence and, in any case, the through current restraint is always active to prevent false tripping. Sonnemann and Smith also seem to have misread the section of the paper dealing with the selection of current transformers. The use of the harmonic-restraint relay is by no means limited to applications in which the d-c time constant does not exceed 0.05 second. The paper simply states that, as a general rule, ordinary current transformers may be used without special consideration or checking of characteristics up to a fault current value of 20 times rating if the d-c time constant does not exceed 0.05 second. The relay may be used on circuits of considerably longer time constant if the current transformers are adequate.

Elzi and Brownlee have discussed the case of magnetizing-inrush current to a delta-connected transformer bank in which, due to closing at a particular switching angle, approximately equal inrush currents are obtained in two of the three phase windings. It is quite true that, as their analysis shows, the harmonic percentages in the resultant line-current wave will be different from those shown in table I for the winding currents. In the worst case the third harmonic will be canceled; the second, fourth, and seventh reduced to 57.7 per cent of their winding values while the fifth is unchanged and the sixth is increased to 115.3 per cent. The resulting harmonic percentages in the line current then are: 36.6 per cent second, 0 per cent third, 2.9 per cent fourth, 4.1 per cent fifth, 4.3 per cent sixth, and 1.4 per cent seventh. As shown by figure 5, the harmonic percentage required to restrain the relay with normal setting is 35 per cent for the second ranging down to 25 per cent for the higher harmonics. The second harmonic would barely be able to restrain operation alone but a comfortable margin of restraint is supplied by the aid of the higher harmonics. The harmonic percentages for inrush waves taken under differing conditions will vary somewhat from those given for the typical wave in table I but the ranges are not great under conditions encountered in practice and comprehensive tests indicate that adequate restraint is always obtained with the normal harmonic-restraint setting of the relay. If a condition ever were found where the restraint was inadequate it could be readily taken care of by a slight change in the setting.

With reference to the discussion with regard to the relative severity of the magnetizing-inrush current on large transformers versus small transformers, there can be no doubt that the time duration of the magnetizing-inrush transient will be longer for large transformers than for small transformers of similar design when both are connected to power sources of sufficient capacity so that the inrush current is governed by the characteristics of the transformers rather than by those of the source. This is because the time constant of the transformer, which determines the decay rate of the inrush current, is a function of the physical size of the transformer. The magnitude of the first cycle of inrush current, measured in terms of multiples of full-load current, would be the same if the two transformers operated at the same core-flux density and were identical in other ways except physical size. It has been the general practice in years past, however, to design large power transformers to operate at higher core-flux densities than small ones, particularly where the transformer is expected to operate continuously at near full load, because of the higher full-load efficiency obtained thereby. This would mean that the large transformer, because of its higher flux density, would have a greater relative inrush current magnitude. Many factors may upset these broad conclusions, however. The impedances of the source and intervening transmission lines may limit the magnitude of the inrush current and change its rate of decay. Considerations of light-load efficiency or of limiting the magnetizing-inrush current magnitude may necessitate lower flux density. The low value of 1.14 per cent normal magnetizing current quoted for the large transformer described indicates that perhaps these latter considerations have influenced its design. Recently, some transformer purchasers, partly because of past relaying difficulties, have begun to demand low values of transient magnetizing-inrush current in their specifications. Where the relaying problem is the only one involved, it can now be solved by use of the harmonic-restraint relay and the advantages of smaller transformer size and higher full-load efficiency obtainable with the higher core-flux density design can be gained.

A High-Power Oilless Circuit Interrupter Using Water

Discussion and author's closure of paper 40-85 by W. M. Leeds, presented at the AIEE summer convention, Swampscott, Mass., June 24–28, 1940, and published in AIEE TRANSACTIONS, 1941 (February section) pages 85–8.

Philip Sporn (American Gas and Electric Service Corporation, New York, N. Y.): This paper describes an interesting adaptation of an idea worked on extensively by German engineers for using water as a medium of interruption in a circuit breaker. It is true that the particular model which has been constructed and which is tested and described by the author is somewhat clumsy from the standpoint of a finished design but undoubtedly a satisfactory design could, and would, be worked out if serious applications developed.

On this question, it may be of interest to go into the background leading up to the development of water breakers in Europe. There are two outstanding reasons: first, experiences abroad with the use of oil have been disappointing, if not disastrous, in some instances. This has produced a strong incentive to eliminate oil in so far as possible; second, oil itself has been a rather scarce and expensive commodity, its use being discouraged and restricted by governmental authorities.

These factors, for the most part, have not existed in this country. Further, as a result of the extensive research and development work carried out by our American manufacturers and engineers, work to which associates of the author of the paper have made significant contributions, we have had a situation where, in general, oil circuit breakers which have not been called upon to perform the impossible, always gave an excellent account of themselves.

On this question of water breakers, there is need also to point out a number of the weaknesses that these breakers may develop and some of the practical disadvantages in connection with their use. Mr. Leeds recognizes some of these:

1. There is the necessity of replacing the water in the breakers from time to time due to contamination and loss in operation.

2. The corrosive action of water on breaker parts must be taken into account and may develop unforeseen difficulties in operation.

3. The speed of these breakers does not so far compare any too favorably with that of the best oil or oil-poor circuit breaker designs. The interrupting time of from four to five cycles obtained with the experimental model as shown in table I must be compared with a speed of three cycles and under, which has been obtained readily with oil-poor and even with conventional oil circuit breakers.

I note that the author has referred to European claims that the cooling effect accompanying the sudden expansion of the steam from the operation of this breaker brings about an acceleration of the deionization in the arc space. This is a point which I discussed particularly with certain engineers in Europe some years ago, but on no occasion was I successful in discovering any definite experimental data which would substantiate this claim. I am wondering, therefore, if the author of this paper has in his own tests produced any evidence of this effect or whether he has discovered any test data from other sources which would bear out such a claim.

W. F. Skeats (General Electric Company, Philadelphia, Pa.): Mr. Leeds is to be congratulated upon the development of an experimental water circuit breaker of high interrupting capacity at 13,200 volts. The co-ordination of his contact motion is apparently well worked out and it would be interesting to have a general idea of the mechanism which has been used to accomplish this.

The water circuit breaker built by the General Electric Company was described at last summer's convention. Since that time units have been installed of both the 1,000,000-kva and 1,500,000-kva rating. The accompanying illustration shows the 1,500,000-kva 3,000-ampere breaker with cell doors and sides removed. It will be noted that there is very little external difference between this breaker and the corresponding oil circuit breaker. These breakers have closed currents with peak values reaching 118,000 amperes.

As with Mr. Leeds's breaker, distilled water is recommended, but with this breaker extreme purity is not essential, for tap water has been used for most of the tests and no trouble has been experienced either from its relatively low initial resistivity or from such further reduction as takes place as a result of circuit interruptions.

The past two years have seen the advent of water as an interrupting medium in this country and an important expansion in the application of air in this field. Both offer freedom from the fire hazard. They are competitive and one naturally wonders which will survive. The water breaker occasionally derives an advantage from better adaptation to the modernization of breakers now using oil, but for new installations, the air circuit breaker has the advantages of mechanical simplicity due to the flexibility of air operation; accessibility of the contacts for inspection—the moving contact is open to view upon opening of the cell door; and even higher current closing ability—the air circuit breaker has closed 190,000 peak amperes without any sign of distress and without visible flash.

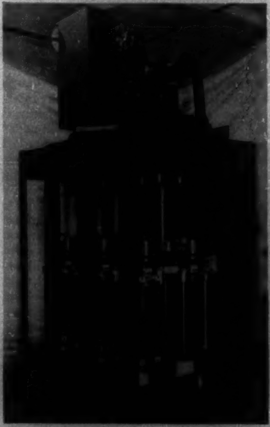


Figure 1. A 1,500,000-kva 3,000-ampere water circuit breaker

H. A. P. Langstaff (West Penn Power Company, Pittsburgh, Pa.): The design of the high-power oilless breaker as described by W. M. Leeds is very interesting especially to central station engineers. He frankly points out factors which give rise to questions of a practical nature. He states that the device does not fulfill the requirements of an ideal breaker, being subject to contamination and loss of water. There also is mentioned the possibility of corrosion of metal parts and of deterioration of the insulation exposed to water or steam. Mr. Leeds hopefully expresses the opinion that these problems are not insuperable, but to me it seems quite evident that considerable work and design will be required to make this type of breaker acceptable for American operating practices.

The paper gives no information as to the means of adding water or of finding out when water is needed. Mr. Leeds states that from four to six short circuits in the range of from 50 to 60,000 amperes can be interrupted without changing the water and that 100 or more low- or medium-current interruptions can be made before the resistivity of the water is such as to require water replacement. Mention is made that the distilled water put into the interrupter usually measures about 100,000 ohms per centimeter. No data is given as to the allowable range of water resistivity to qualify the water for use in the interrupter.

It would be interesting to know how soon the breaker may be reclosed upon a fault after interrupting a fault. Figures 6 and 7 of the paper show oscillographs taken when closing the breaker upon a fault of comparatively low ampere value. It would be interesting to see the same curves for the breaker operating with the 60,000-ampere current range such as illustrated in figures 8 and 9 for the opening operation only. In testing such a breaker to destruction, the question naturally arises as to how it would compare with an oil circuit breaker of the same rating.

The paper states that the closing of high currents on the air-disconnect-switch contacts is accomplished with very little arcing due to the high-speed snap action. It would also be interesting to know the speed of all three closing actions; namely, arcing contacts in water, series disconnect switch, and the main closing switch. I assume it would also be necessary periodically to determine the resistance of the water in the breaker somewhat the same as the making of oil tests. It is noted that practically all of the parts to which the water is exposed are of nonrusting materials but these may affect the water and I assume the manufacturer has some information already available on this as a result of the tests.

The comparative costs of this type of

breaker as compared with one of present-day design I assume to be identical for the same respective ratings in view of the fact that it seems that present day prices are based on rupturing capacity.

In general it may be stated that a high-capacity breaker without oil is a very attractive piece of apparatus. This development can be classified as a step in the direction of that attainment.

R. C. Van Sickle (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The development reported by Mr. Leeds has an interesting relation to the development of earlier water breakers.

The European water breakers were introduced almost ten years ago for services of 6 kv and 10 kv and for interrupting capacities up to 400,000 kva. Since then, the range of interrupting capacity has been raised gradually to 600,000 kva and 800,000 kva at 10 kv.

During this period, our demands for oilless breakers at corresponding American voltages were met by the "de-ion" air circuit breakers of comparable interrupting capacity.

To meet demands for higher interrupting capacity, the investigations reported by Mr. Leeds were made. Consequently this development instead of starting at moderate capacities and expanding into the higher and lower ranges, took advantage of the earlier work of the German designers of "expansion" water breakers and applied it to a breaker for approximately double the interrupting capacity of other competitive types of oilless breakers. This emphasizes the magnitude of the problems and the significance of the work described.

D. C. Prince (General Electric Company, Schenectady, N. Y.): In this very interesting paper, the author expresses the view that the "expansion theory" proposed for the water circuit breaker on the continent has not been sufficiently supported by critical evidence. Instead of that theory, he suggests that the arc is subjected to a "turbulent blast tending to sweep the arc products downward and away from the contact tip". It will be noted that this represents an agreement with our often repeated contention that displacement of arc products by an unimpaired dielectric is the predominant instrumentality in the operation of many circuit breakers.

The author mentions "heavy, local pressures". It would be interesting to know just how heavy these local pressures are and whether the safety valve mechanism employed to control them is effective in keeping the pressures uniform over a wide range of interrupting currents. It is interesting to compare this method of controlling the pressure with that employed by W. F. Skeats in his hydro-blast breaker described before the summer convention of the American Institute a year ago (High-Capacity Hydro-Blast Circuit Breaker for Central Station Service, AIEE TRANSACTIONS, 1940, pages 111–16). Skeats employs a very ingenious arc control device to determine pressure generated in contradistinction to the device described by the author, which merely attempts to prevent the increased evolution at high currents from having destructive effects. A side light on this question is that the Leeds breaker is provided with a steam condensing chamber from which some steam escapes under heavy load (as high as 10 per cent according to the author), whereas the Skeats breaker requires no vent.

Hydro-blast breakers up to 1,500,000 kva are now in service. It would be interesting to know whether a service record is also being obtained for the circuit breaker described in this paper.

J. B. MacNeill (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): In 1918 the writer participated in interrupting tests at 11,000 volts using water in metal tanks that were insulated from ground. At that period, however, the problem before switchgear designers was to secure adequacy of interruption regardless of the medium, and since oil was definitely superior from an insulating view the water development was not pushed.

About 1928 water was taken up seriously in Germany as a substitute for oil in circuit interrupters and a great commercial activity has been developed in that country, particularly by Siemens-Schuckert, for voltages up to 60 kv indoor. A limitation in rupturing capacities at the lower voltages has made the European device unattractive in this country. About 600,000 kva at 15 kv has been used commercially and we understand later developments are good for 800,000 kva at the same voltage. Higher voltages permit some increase of capacity.

The paper by Mr. Leeds outlines the work done to push the interrupting capacity for powerhouse voltages to the point necessary for American service and we believe there is no doubt that 1,500,000 kva can be handled at from 12 to 15 kv satisfactorily with the type described in the paper.

As is universal with devices generating gas for arc extinction, the arcing time is somewhat longer than if a separate source of pressure were used. However, the overall interrupting times, from 3.2 to 5.1 cycles on a 60-cycle basis, are lower than with conventional apparatus and compare favorably with compressed air.

This type of device must generate enough gas pressure at low currents to secure interruption and the physical limit therefore is the pressure which can be withstood under high-current conditions. The materials available for the pressure chamber are fibrous in character and their insulating quality under water must be carefully watched.

An interesting feature is the lack of external ventilating pipes and expansion chambers since the gases formed from water are not permanent and the necessary expansion effect can be taken care of in the mechanism box shown in figure 5 of the paper.

For the usual switch service maintenance of the water is not difficult and various antifreeze compounds have been developed permitting use in low temperatures. These detract slightly from the interrupting ability.

The principal advantage of the water interrupter, in addition to the fact that the liquid is noninflammable, is that it is self-contained and does not require auxiliary equipment, therefore, it can be located conveniently in small groups such as required in station auxiliary layouts and small substations.

R. A. Hentz (Philadelphia Electric Company, Philadelphia, Pa.): The many advantages of the oilless type breaker over the oil type, in spite of a reasonably good performance on the part of the latter, well justify the thought, research, and development that has been given to the various oilless types by the several manufacturers. The trend was started over ten years ago with the development of the air deion circuit breaker, but unfortunately on account of the depression further progress in the general direction of doing away with oil was very slow until the last two or three years.

The type breaker as represented by the air deion has advantages over those requiring the use of compressed air or water, but unfortunately they cannot be developed in the higher interrupting ratings and consequently circuit breaker engineers have had to resort to the use of these other schemes.

Both compressed air and water have been used in Europe and it is unproved which of these two will ultimately be shown to be the most desirable for indoor service in this country. The trend appears to be definitely toward the airblast type, but sufficient operating experience is lacking, though I believe that such difficulties that are met with can be solved satisfactorily.

On the other hand, experience with fifteen breakers in service, the first ones for nearly two years, leads me to conclude that there is no fundamental drawback in the water breaker. Based on the successful experience of the first six that were installed on the Philadelphia Electric Company system experimentally, that company has placed orders for nineteen additional breakers ranging in interrupting rating from 500,000 to 1,500,000 kva and of ampere carrying capacities up as high as 3,000 amperes.

The water breaker covered by Mr. Leeds's paper appears to be well designed and the tests shown in his table II, as well as the oscillographs, show that the breaker can adequately interrupt both heavy and light short-circuit currents. However, like any new development, the day-in and day-out experiences with it under all kinds of operating conditions are necessary before the breaker can have general acceptance. As pointed out above the experience with a water breaker of a different manufacture than that of the company with which Mr. Leeds is connected has shown no fundamental difficulty and it is unfortunate that similar experience is not available on the water breaker that Mr. Leeds described.

In closing I wish to express my concurrence with Mr. Leeds's conclusions.

W. M. Leeds: Mr. Langstaff's doubt that water breaker designs are acceptable for American operating practice because of possible corrosion or insulation difficulties is apparently not shared by Mr. Hentz who seems to have obtained satisfactory service from one type of water breaker. That is not to say, however, that there may not be valid reasons for believing other types of interrupters to have preferable characteristics, but the many years of successful operation in Europe of water breakers in low and medium interrupting ratings as well as the more recent demonstrations of ability to meet heavy duty service requirements seem at least to merit serious consideration for this type of oil less circuit breaker.

A glass water-level gauge, not shown in figure 2 of the paper, is used to determine when water should be added, such water being poured in through the opening under the hinged cover *E*, figure 5. Small drain holes allow this water to reach the interrupting chamber in the same way that condensed steam and water vapor is returned.

By using a series disconnecting switch to remove potential from the water chamber, and by completing the circuit in air during closing operations, the most serious hazards of low dielectric strength water have been avoided. Successful interruption can be obtained with a water resistivity as low as 1,000 ohm-cm, but it has been our practice to use pure distilled water, and keep the resistivity between 10,000 and 100,000 ohm-cm.

Mr. Langstaff also asks how soon the breaker may be reclosed on a fault after interrupting a fault. We have found nothing in our testing which would prevent the use of the water breaker on rapid reclosing duty cycles, applying the same derating factors now standard for oil circuit breakers.

It should be pointed out that the oscillogram of figure 7 of the paper shows a close-open operation on the same circuit that gave approximately 60,000 amperes on the opening tests of figures 8 and 9. The first cycle rms current for figure 7 was 55,000 amperes, the current wave not being completely displaced. The low value of 29,000 amperes for the interrupted current is due, of course, to the decrement during six cycles of short circuit.

When the water breaker is tested to the point of failure, a blowout patch on the vent channel to the mechanism box comes off, allowing steam and water to blow out; and also an open arc is drawn by the disconnect switch. This all subsides at once as soon as the short circuit is opened by the back-up breaker.

The question of the mechanical closing speed has been raised. The arcing contacts in the water chamber close at approximately 10 feet per second, the series disconnect or closing switch at 20 feet per second, and the main paralleling switch at about 5 feet per second.

In investigating possible corrosive effects of distilled water on the various metallic and fibrous materials used in the water breaker no evidence of water contamination was noted, although this might not be the case with certain kinds of tap water.

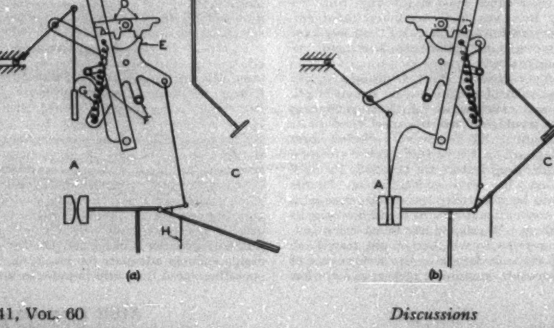
It is of particular interest to note from Mr. Hentz's discussion that after two years of experience with water breakers, he is led to conclude that "there is no fundamental drawback in the water breaker". Many hundreds of water breakers have been put in service abroad, principally by the Siemens-Schuckert Company. We have had the opportunity to follow this field experience, and have reached somewhat the same conclusion as Mr. Hentz. Additional service experience in this country is not contemplated, however, unless present conditions regarding the demand for water breakers should change materially.

In answer to Mr. Skeats's request for more complete information on the breaker operating mechanism, the accompanying illustration is shown, giving the schematic arrangement. The air disconnecting switch is operated by an over-center toggle spring action controlled by latches that are released just before the end of the operating arm motion on both the closing and opening sequences. Knocking rollers strike the switch arm to make the action positive.

With regard to Mr. Prince's interpretation of the arc interruption in the water breaker in terms of his "displacement theory", we should like to point out that even without the water flow arrangement provided in the final design, practically the full rating of the breaker has been successfully interrupted with only turbulence and diffusion effects present. However, the directive effect of this longitudinal flow which moves liquid up to the moving arcing contact tip was found to increase the effectiveness of the diffusion process so that the arcing time could be appreciably shortened.

Figure 2. Water breaker operating mechanism; left, open; right, closed

A. Arcing break in water chamber
B. Series disconnect and closing break
C. Main paralleling switch
D. Holding latches
E. Latch release
F. Knocking rollers
G. Contact operating tension spring
H. Operating rod to solenoid mechanism



While it was not found practical to locate pressure recording devices in the central arcing chamber, certain calculations based on the resistance of the rubber retaining ring and the yield point of certain clamping bolts which were later replaced by a stronger design indicate pressures of approximately 50 pounds per square inch for low current interruptions up to instantaneous peak values approaching 1,000 pounds per square inch at the breaker rating.

Whether or not a specific vent is provided on a water breaker, such permanent gases as hydrogen and carbon monoxide which are mixed with the steam and water vapor must eventually find their way out. The design of water described in the paper has a large condensation chamber and a screened and baffled passage to a vent, so that pressure is rapidly lowered to be ready for the next operation, and yet no connection to an exhaust header is necessary.

Mr. Sporn asks whether we have discovered any experimental evidence to substantiate the European claims that arc extinction in water breakers is due to the cooling effect accompanying sudden expansion of steam. It is evident that in the German "expansion breaker" that expansion does take place and that the arc is extinguished, but the importance of the contribution to de-ionization of the cooling effect does not appear to be quantitatively proved. It is interesting to note in the article by F. Kesselring, "The Expansion Breaker" (*V. D. I.*, volume 78, March 1934, pages 293–7) that emphasis is given to deionization by diffusion in a hydrogen atmosphere as one of the most important factors in the interrupting process.

Medium-Capacity Air-Blast Circuit Breakers for Metal-Clad Switchgear

Discussion and authors' closure of paper 40-125 by R. M. Bennett and B. W. Wyman, presented at the AIEE summer convention, Swampscott, Mass., June 24–28, 1940, and published in AIEE TRANSACTIONS, 1941, pages 383–8.

Joseph Slepian (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The theory of arc extinction in the cross-blast breaker given by the present authors is substantially that advocated by Prince, Henley, and Rankin in their paper presented at the winter convention. This theory is beautifully simple, and leads to a beautifully simple equation.

According to this simple theory, at current zero, the arc column just naturally of itself falls apart at one or more points, thus forming several disconnected pieces. These disconnected pieces then separate from each other with a speed given by the gas velocity. The simple theory does not go into any detail as to the mechanism whereby the arc column is broken into two or more disconnected pieces. Apparently, this mechanism does not require any great gas velocity, since Prince, Henley, and Rankin only require that the gas velocity be great enough to interpose fresh gas between the already separated arc column fragments fast enough so that the rising circuit voltage may be withheld.

The present authors modify the Prince, Henley, and Rankin theory, and make it less simple. The falling apart of the arc column at current zero into distinct pieces no longer just naturally takes place of itself, but must be preceded by the formation of a loop, with considerable gas volume required. Just why the forming of a loop is necessary to make the arc column fall apart is not made very clear. No loop forms in the longitudinal oil-blast and gas-blast breakers, and yet Prince and Skeats, and Prince, Henley, and Rankin do not hesitate to apply their simple theory to these cases.

For a given air blast, to obtain interruption according to their theory, the authors state that the current must not be so large as to prevent loop formation. However, in figure 6, currents as low as 1,200 and 2,100 amperes also gave poorer operation than their theory predicted, so that the annoying question comes up as to whether loop formation failed to occur even for a current as small as 1,200 amperes.

The picture of the arc column blown into loops prior to current zero, shown in figure 4, raises some strong questions as to the possibility of other factors being responsible for the arc extinction than the physical cutting of the arc required by the authors' theory. Figure 4 shows considerable lengths of the arc pushed into small cross section up against the barrier walls. Will not these lengths of arc column have very considerable circuit-interrupting capacity in themselves, particularly since they lie in a region of high turbulence occasioned by the mixing with the main blast of the gas emitted from the barriers?

I would like to ask the authors whether they believe that the translatory velocity next to the barrier surfaces, where they place the arc column, is small or large compared to the main-blast velocity. If it is small, what has become of their theory? Aerodynamists agree in permitting only a small translatory velocity next to a fixed bounding surface.

I would like to ask the authors whether they believe that the gas evolved by the barriers, plays no part at all in the circuit interruption. What success have they had with nongas-forming barriers, or barriers which give less gas than the fiber barriers actually used?

Lastly, in all other breakers, magnetic effects quickly blow large current arcs out to much greater lengths than the actual contact separation. Does not this take place in this breaker? If so, how do they then account for their figure 5, showing contact separation, and not arc length as independent of current for a given circuit-recovery rate?

J. R. North (The Commonwealth and Southern Corporation, Jackson, Mich.): Our companies, along with many others, are keenly interested in the various forms of new oilless circuit breakers and some of our recent 2,400-volt installations are composed of air circuit breakers. Air breakers, both the self-contained units and the blast type of air breaker appear to be particularly promising from the standpoint of

(*a*). Ease of inspection and maintenance,

(*b*). Probable greater inherent interrupting ability on repetitive reclosing duty cycles,

(*c*). Compactness,

(*d*). Elimination of oil in indoor units (despite our reasonably satisfactory operating experience with oil breakers in general).

The papers presented at this and at other Institute meetings are very interesting and have shown some features of these new developments of circuit-interrupting devices, the results of interrupting tests, and structural arrangements. However, judging from experience in applying oilless breakers, it appears that there is a dearth of information concerning the durability of some parts of the arc-interrupting devices and the operating performance which may be expected in service. We should therefore urge the manufacturers to give more thought to the following matters in connection with all new designs.

(*a*). *Operating Performance Over a Range of Service Voltages.* AIEE Standard No. 29 states in part ". . . At the end of any performance at or within its interrupting rating, the circuit breaker shall be in the following mechanical condition . . . the breaker shall be substantially in the same mechanical condition as at the beginning. . . . After a performance at or near its interrupting rating, it is not to be inferred that the breaker can meet its interrupting rating without inspecting and, if necessary, making repairs. . . ." It is no longer considered satisfactory for a circuit breaker to interrupt the circuit and merely not blow up. Modern interrupting devices are generally expected to interrupt their rating without requiring any great amount of maintenance. These operating requirements necessitate the use of durable and simple parts. The arcing contacts in particular should consist of material which will effectively resist the maximum rated interrupting amperes at low-service voltage.

(*b*). *Interrupting Ability at Reduced Service Voltage.* From a practical application standpoint, the interrupting ability of the breaker in amperes at service voltages below the name-plate rating (for example, with a 15-kv breaker, service voltages of 5.0 and 7.5 kv) is vitally important. Some difficulties have been experienced with recent installations of modern 15-kv oil circuit breakers when called upon to interrupt considerably less than their maximum rated amperes (or kilovolt-amperes) at the lower service voltages due to excessive burning of the arcing contacts.

(*c*). *Reclosing Duty Derating Factors.* The theory of operation of air circuit breakers and examination of their component parts would lead one to expect them to be well suited for repetitive reclosing duty cycles, and on this basis it seems doubtful if any derating factors are necessary for such service. In any event, these factors should be materially less severe than with the conventional type of modern oil circuit breakers. We should like to ask the manufacturers as to whether or not tests have been made to determine the performance of air breakers on multiple reclosures and what the effect might be of the number of shots and the time interval between them on the interrupting ability of the breakers and the subsequent maintenance required.

(*d*). *Insulation Levels.* The name-plate voltage rating of a circuit breaker is oftentimes not a correct index of the insulation level for which it was designed. Definite values of insulation levels should be given on circuit-breaker name plates in terms of both low frequency and impulse-withstand test voltages.

(*e*). *Structural Arrangement and Space Requirement.* It is quite likely that as these developments proceed there will be numerous installations of air circuit breakers as replacement of oil circuit breakers and wherever possible the designs of the new air breakers should be such as to permit their installation in conventional-type cells used with oil breakers. This has been given consideration by some of the manufacturers and certain of the designs now on the market can be readily installed in place of oil breakers. Some of the other designs, however, are not so readily adaptable.

J. B. MacNeill (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): This paper portrays an interesting development of a compressed air breaker in space factors suitable for modern metal-clad switchgear service. This breaker is doubtless adequate for the service intended since the current and voltage ranges involved are well within the limits of the cross-blast principle used. The flexibility of the compressed air type is well demonstrated in this design where the air leaves the tank at the top, passes down through an interrupter, and then reverses to reach the atmospheric vent again at the top of the device.

It seems to me, however, that the use of compressed air for this moderate capacity represents a marginal application since self-contained dry air breakers have been produced which do not require air equipment and this must mitigate against the use of the device in station auxiliaries and other small groupings. It is interesting because of its small size. We had not thought to use compressed air in this low capacity.

The vertical-lift design used originated some years ago for oil breakers which naturally have vertically arranged terminals and where the vertical lift is desirable because of the oil content. It does not appear, however, that vertical lift conveys any definite advantage on dry-type switchgear and it is the writer's opinion that horizontal drawout arrangement is definitely better suited for the newer type of equipment.

In this connection not all customers will accept the metal-clad arrangement and some will desire to arrange breakers in concrete or steel cells and for such purposes a horizontal arrangement of the main electric circuit is frequently desirable.

The accessibility of the design given by the authors may be questioned by operating people. The breaker unit is completely boxed in except for the terminals and even when withdrawn from the metal enclosure very few parts of the breaker are visible for inspection or accessibility for repair. The test data given in tables I and II show the design entirely adequate for its rating, the operating speed from trip impulse to inter-

Figure 1

ruption is fast enough for all commercial use, and the arcing times are short. The debatable points about the design seem to be the use of compressed air for this limited capacity; the general arrangement as it affects station layout; and cell structure design and accessibility of parts as affecting inspection and maintenance.

STATIONARY CONTACT
BLAST VALVE
AIR TANK

Figure 2

Figure 1 of this discussion shows a horizontal drawout self-contained dry breaker for 250,000-kva capacity 15,000 volts and below, which we feel would be favored over a device requiring compressed air.

Figure 2 of this discussion shows an arrangement of compressed air breaker which we feel has advantages as to efficiency and inspection; in it grounded metal parts, including tank valves, and lever mechanism, are all arranged at the bottom, the interrupters are mounted on the tank, and mufflers or other means for discharging used gas are arranged overhead away from operators and high-tension electrical circuits. This seems to us the most desirable arrangement of compressed air apparatus for circuit interruption.

D. C. Prince (General Electric Company, Philadelphia, Pa.): In this very interesting paper, the authors described what are believed to be the first oilless circuit breakers sufficiently small and compact to be competitive with oil circuit breakers in metal-clad gear. The paper also contains valuable new material of a theoretical nature bearing upon the mode of operation of the device. Figure 5 shows clearly the greater importance of the regions where the arc is cut, as compared with those where it is supposed to be most exposed to cooling.

Dr. Slepian in discussing the paper, "The Cross Air-Blast Circuit Breaker," by Prince, Rankin, and Henley, presented at the last winter convention, attributed the operation to cooling along the arc and between the barriers. It is believed that the evidence in figure 5 is fairly clear to the contrary.

It may be questioned why the matter of circuit-breaker theory continues to absorb so much attention in the minds of circuit-breaker designers. The reason is a very practical one. One of the heavy charges incurred in the development of high-power circuit breakers is that due to large-scale testing. The cost of this testing goes up rapidly as the size of the circuit breakers to be tested goes up. Many years ago, the full-power test of steam turbines was discontinued because of the prohibitive cost of both steam generation and power absorption facilities. It is now recognized that because of the very comprehensive knowledge of the behavior of the steam turbine, such full-scale testing is not necessary. The substantiation of the displacement theory of breaker operation places similar knowledge in the hands of designers of such type of circuit breakers as employ this principle, and it is then possible to predict with considerable accuracy the behavior of such circuit breakers, far beyond the capacity of existing testing plants. This represents a very large potential saving to purchasers of switchgear.

Philip Sporn (American Gas and Electric Service Corporation, New York, N. Y.): The authors of this paper deserve credit for having given a clear description of the theory and construction details of the air-blast circuit breaker, particularly as applied to metal-clad construction.

It may be of interest that the job here described was installed in the new Central City substation of The Scranton Electric Company in Scranton, Pennsylvania, and that a similar job is being installed in the Third Street substation of The Ohio Power Company at Canton, Ohio.

It may also be of interest to discuss why we are co-operating in this development. Some of the reasons for this are as follows:

1. Almost all of our small or medium-capacity breakers are used in automatic unattended stations, generally indoors. While experience with oil circuit breakers has been quite good, at the same time there is always the possibility of an oil fire in case of a serious breaker failure. One solution for eliminating the bad effects of such a contingency is the installation of CO_2 fire protection but this, of course, is quite expensive. The air-blast breaker eliminates this possibility entirely.

2. In some applications, such as in power station auxiliaries, it is frequently necessary to install breakers in crowded locations where also the temperature conditions may be quite high. Obviously, the elimination of oil in these cases is highly desirable. A specific instance of this sort I previously discussed in connection with the paper presented at the summer convention in 1938 entitled " 'De-ion' Air Circuit Breakers for A-C Feeder, Motor Starting, and Station Auxiliary Service" by R. C. Dickinson, and published in Electrical Engineering under date of November 1938.

3. Inherently, this equipment is lighter in weight than corresponding oil circuit breakers and perhaps ultimately should give us lower-cost switchgear.

While recognizing these actual and possible advantages of the air-blast circuit breaker, at the same time we must not lose sight of the fact that this equipment is radically new in principle and design as compared with the years of background enjoyed by the oil circuit breaker and that many details of design and construction will have to be watched rather closely, at least for some little time. For example, it is certainly true that the breaker in its present design is considerably more complex than the corresponding oil circuit breaker and, if it is found that the breaker has a tendency to get out of adjustment easily, the cost of maintenance may go up. This does not mean that there is any fundamental reason why such difficulties cannot be ironed out eventually, but all of our experience indicates we may expect to go through the inevitable period of childhood diseases before we finally get a satisfactory mechanical job. In this respect I am not only inclined to agree with the authors that one of the outstanding and immediately available advantages of this type of breaker is a sure reduction in maintenance cost, but on the contrary, I am fearful that high maintenance costs may hinder the rapid expansion of application of the breaker.

I feel confident, however, that while such difficulties are to be expected, nevertheless they can and will be overcome and that we have reason to look forward to the time when the complexities of the present design will be appreciably simplified. It is because of this that we are co-operating in the development: We believe that such a course will enable us to more easily gain the material advantages which this breaker should inherently be able to offer.

R. M. Bennett and **B. W. Wyman:** Dr. Slepian's discussion raises interesting questions concerning the mechanism of arc extinction. He asks what we believe to be the translatory velocity of the gas in the region where the arc lies. Tests have been made to determine this gas velocity for use in the displacement formula. These tests were made with a single-barrier arc chute with a transparent side. The speed at which the arc was blasted out by the high-velocity air after it had struck across the opening is clearly shown in the photograph, figure 3 of this discussion. In this photograph the arc forms two loops in the openings of the arc chamber. The left-hand loop had previously been short and portions of it are being rapidly driven down the air passage by the air.

These two photographs were taken $16\frac{2}{3}$ microseconds apart and are representative of a large number of tests. It will be noted that the vertical translatory portion of the arc along the barrier travels at a higher velocity than the middle of the loop as indicated by the points marked on the photograph. The velocity in the middle of the loop is 1,600 feet per second while the vertical translatory portion is traveling at the rate of 2,800 feet per second. The increased velocity in the vertical translatory portion of the arc is undoubtedly due to the evolution of gases from the barrier surface by the intense heat of the arc. These gases evolved by the fiber being chiefly hydrogen have very low density and viscosity and are therefore capable of attaining extremely high speeds. These gases force the arc away from the barrier into a region of high velocity favorable to rapid insertion of a wedge of dielectric.

A wide variety of barrier materials have been tested in the design of air-blast breakers. In general the nongas-forming materials have proved inferior for the construction of arc chutes. Whether or not gas formation is necessary to hold the arc in the region of high velocity, is not proved by these tests for the inferiority of the nongas-forming materials tested can be accounted for by the fact that the surface of the barrier material becomes conducting when subjected to the intense heat of the arc. The extent to which these nongas-forming materials become conducting in the presence of the arc, and the time which they require to regain insulation strength after the arc has been removed have been studied in the laboratory. These insulation-recovery times are much too long for use in air-blast circuit breakers.

Gas evolution from the fiber then performs three functions: first, to provide thermal protection for the insulating surfaces of the arc chute; second, to force the arc away from the barrier into a region of high velocity; and third, to act as a low-viscosity low-density lubricant making possible rapid insertion of a wedge of dielectric at the current zero.

Plotting arc length against interrupting ability yields little information in the case of the air-blast breaker where interrupting ability depends upon the number of breaks in the arc stream. The tests made at reduced air pressure and plotted in figure 5 of the original paper show that interruption of a circuit having a given rate of rise of recovery voltage requires a definite number of breaks in the arc stream regardless of the current magnitude.

The development of the high-voltage air-blast circuit breaker introduced the problem of physical application of the breaker to mounting structures. It was highly desirable to include in the design of this breaker the necessary features to make it suitable for the vertical-lift metal-clad type of switchgear structure in order to retain the advantages already gained through the long use of vertical-lift metal-clad switchgear with oil breakers.

It was early recognized that the contour of the air breaker should conform as closely as possible to that of the oil breaker in order to conserve space in the usual switching structure. It has been possible to retain the vertical lift and easily removable circuit-breaker feature which has been found so useful in the oil-breaker metal-clad switchgear. The advantage is the continuance of a design which has proved its utility in the field. Therefore, no new station designs are necessary nor need any conceptions of this type of equipment be changed. The configuration of the elements of metal-clad switchgear of the vertical-lift type using either oil or oilless breakers is the most economical in floor space and the most convenient in adaptability to the various circuit arrangements where double busses or by-pass disconnecting switches have to be used.

Circuit-breaker configuration is really dictated by the design of an economical and suitable mounting structure for location of control panel, main busses, instrument transformers, and space for cable potheads which can leave interchangeably from below or above. It is much easier to accommodate these features in a structure using vertical-lift breakers. The location of these elements when used with a horizontal draw-out type of breaker is a problem, for if the main bus is connected to the upper studs of the breaker, then there usually is not enough distance between the lower set of studs and the floor to mount one or two sets of current transformers per phase and have enough space left for cable potheads and room for the make-up of cable joints to these potheads. If the busses are connected to the lower studs they come close to the floor and

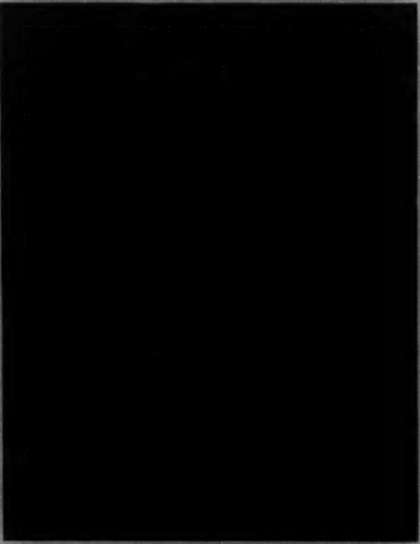


Figure 3

then the problem of cables leaving from below is encountered.

A circuit breaker designed for vertical-lift mounting can be handled by a maintenance man much more easily and quickly because of its decreased weight as compared to one which is mounted in a heavy frame truck.

It is important to have a physical separation of primary disconnecting-device contacts when the breaker is moved from the connected to the test positions. This has been satisfactorily accomplished in the vertical-lift design.

The installation costs are higher with the horizontal drawout type because more care must be taken to insure level station floors and a level entrance surface for each unit. It is extremely fortunate that a type of interrupting unit has been selected that will adapt itself to any breaker configuration, thus allowing the desirable features of the vertical-lift arrangement to be maintained.

Mr. North brings up the question of the application of these breakers at reduced service voltages. The test data at 4.2 kv show the ample margin of safety in this direction. As to reclosing duty derating factors, oil circuit-breaker derating factors are the result of years of field experience combined with laboratory test data obtained by different manufacturers on their various designs of breakers. These derating factors set up for oil breakers are satisfactory for air breakers and are probably conservative as Mr. North points out.

Insulation levels are being worked on by various committees for the co-ordination of insulation levels. As we have always done in oil circuit-breaker design, our insulation level is set up to meet these requirements.

Dielectric Strength of Water in Relation to Use in Circuit Interrupters

Discussion and authors' closure of paper 40-86 by Joseph Slepian, C. L. Denault, and A. P. Strom, presented at the AIEE summer convention, Swampscott, Mass., June 24–28, 1940, and published in AIEE TRANSACTIONS, 1941, pages 389–95.

D. C. Prince (General Electric Company, Philadelphia, Pa.): This very interesting paper seems to substantiate almost the opposite conclusion to that premised by the authors. Although changes in water resistivity would appear to alter the heating of the water by a factor of 200 billion, yet this extraordinary range does not produce any important effects for the short time the voltage is impressed upon the water in circuit-breaker operation. This would appear rather conclusive proof that heating is not an important factor.

Practical experience substantiates this conclusion. Water circuit breakers seem to operate as well on tap water as on perfectly pure distilled water and their operation does not appear to suffer appreciably from contamination of the water in service.

It may be wondered whether conclusions may not legitimately be drawn in another direction from these observations. If the heating of the water has so little effect that a variation of 200 billion is unimportant, perhaps it might be reasoned conversely that cooling of the arc by the water is also an unimportant phenomenon. The authors show that the dielectric strength of the water is a considerable fraction of that of oil, and it is known that this fraction is not changed markedly over a wide range of water resistivity. In the displacement theory, it is the dielectric strength and not the cooling which is important, and the data in this paper would seem to offer evidence that it is the dielectric strength and not the cooling which is the important property in water circuit breakers.

W. F. Skeats (General Electric Company, Philadelphia, Pa.): This paper is an interesting contribution to our knowledge and understanding of the phenomena of nature. The thesis that a long-time breakdown of a water gap is a heating phenomenon seems to be well substantiated.

A study of those tests in which the dielectric stress was high enough to cause breakdown before the boiling point was reached shows that the ratio of water dielectric strength to oil dielectric strength even for these tests is considerably less than the 70 per cent indicated by the surge voltage tests, a strength of 14,000 volts for a one-eighth-inch gap in water corresponding to a value of 40,000 to 50,000 volts for a one-tenth-inch gap in oil. Does this, in the opinion of the authors, represent a reduction in the dielectric strength of water with increase in temperature even before the boiling point is reached?

About a year ago some impulse-breakdown tests were made on tap water with a resistivity of the order of 5,000 ohm-centimeters. The results of these tests are shown on the slide. These show the dielectric strength of this water to be approximately 60 to 65 per cent that of oil as compared with the 70 per cent which the authors have obtained for distilled water of about ten times the resistivity. It thus appears that the dielectric strength of water may be substantially independent of the resistivity.

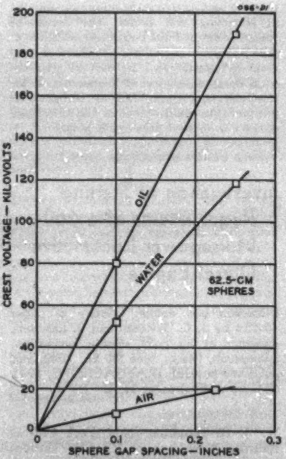


Figure 1. Impulse breakdown strength of insulating oil and tap water

Andrew Gemant (The Detroit Edison Company, Detroit, Mich.): It is known, especially from the work of M. Wien, that water is able to maintain high voltages for a short time, but the present paper seems to be the first investigation on the actual breakdown strength of water. It finally clears away the old distinction between conductors and insulators. Such difference does not exist, at least not for impulse voltages, since the conductivity does not matter in this case. Any material becomes an insulator against surge voltages, if placed between electrodes of a resistivity a few orders lower than that of the material in question, that is, if it is possible at all to apply an impulse voltage to the material. Carrying this idea further, one could say that, if it were possible to construct electrodes of a material having a lower resistivity than that of ordinary metals, for example of a metal in the superconducting state, then the ordinary metal would act as an insulator with a finite and measurable breakdown strength.

J. Slepian, C. L. Denault, and A. P. Strom: We wish to thank Dr. Gemant for his very interesting remarks. As he so well states, there is no absolute distinction between insulators and conductors; electrical breakdown of any material may be said to occur if application of sufficient voltage for sufficient time will cause its conductivity to increase many fold. In the case of water, the arc which forms as the boiling point is reached has a conductivity many times greater than the water from which it is formed.

Concerning the impulse-breakdown strength of water, Mr. Skeats' results are perhaps not too far from our own, namely 60 to 65 per cent of that of oil, compared with our value of 70 per cent. We do not believe that this impulse strength is strongly temperature dependent, although we have no data on this point. We base our belief on the fact that the impulse breakdown is insensitive to conductivity of the water, which is the only property which small temperature changes will alter considerably.

Mr. Skeats refers to our tests with short gaps where high gradients caused breakdown apparently before boiling occurred, at gradients much less than those corresponding to our impulse tests, and asks whether these tests indicate a reduction in the intrinsic dielectric strength of water as the temperature is raised. We believe the answer is in the negative. In some of the tests of this type, voltage was sometimes removed before breakdown occurred. Frequently, under such conditions we observed that the interelectrode space was filled with a cloud of small gas bubbles. We conclude that under high stress the bubbles of dissolved gas set free below the boiling point may cause breakdown.

We are unable to understand Mr. Prince's remark that changes in water resistivity do not produce any important effects for the short time the voltage is impressed upon the water in circuit-breaker operation. Nearly 100 tests are described in our paper which cry out eloquently that the breakdown time for a given voltage varies proportionally with the resistivity for breakdown times ranging from a fraction of a cycle to many cycles.

The Application of Electricity for the Auxiliaries of Railroad Trains

Discussion of paper 40-123 by J. E. Gardner, presented at the AIEE summer convention, Swampscott, Mass., June 24–28, 1940, and published in AIEE TRANSACTIONS, 1941 (January section) pages 34–6.

W. S. H. Hamilton (New York Central Railroad, New York, N. Y.): Several points are brought out by Mr. Gardner on which I wish to comment.

Mention is made of the use of Diesel and propane gas engines applied to each car for operation of air conditioning and furnishing of other auxiliary power. This opens up some very interesting possibilities and is deserving of careful consideration. As this arrangement is just in the initial development stages it is too early to say just how far it will occupy the field in the future. There are many reasons why a Diesel engine set is highly advantageous for the purpose, the main reasons being continuity of power supply independent of whether the train is in motion or not; the ability to dispense with wayside facilities at outlying points; and the possibility of elimination of large sized storage batteries with their renewal expense.

Against these advantages must be balanced the unknown cost of maintenance of the Diesel engine-generator set required. However, with the developments that are proceeding rapidly in the light weight high-speed Diesel engine field [illegible] believe that the advantages will be found to outweigh the disadvantages and that this method of power supply will become an accepted one on air conditioned cars in the future.

Fluorescent lighting is also mentioned. There is no question but that this type of lighting is most advantageous for car lighting. At present it is necessary on cars equipped with a 32 volt d-c lighting system to use some method of inversion to secure a-c power for operation of the tubes. Several types of means for this inversion are available and probably all will be used more or less in the future. While there does not seem to be any possibility at the present time of securing a fluorescent tube that will operate directly on 32 volts direct current, I am confident that one will be developed in the next few years and that the problem while difficult is not insurmountable.

Those railroads which with the advent of electro-mechanical air conditioning adopted 64 volts direct current, now have an additional reason for congratulating themselves on their choice. Fluorescent lamps are now available for use directly on 64 volts d-c supply. These lamps are 15 inches in length, instead of 18 inches as used commonly for lamps operating on alternating current. Stroboscopic effect is of course eliminated and the auxiliaries are very simple consisting of a magnetic switch and a small ballast lamp. They are rather sensitive to voltage variations however and require the use of a lamp regulator but the size necessary for the same illumination is considerably reduced since a fluorescent lamp taking approximately 22 watts total input gives about the same light as a 40 watt incandescent lamp. The efficiency including losses in the lamp regulator is about the same as lamps operated from alternating current through inversion equipment, but the simplicity and ease of control is greater.

The New York Central has had one coach number 2562, with these lamps in operation since October 1939 with entire success and three additional cars are being equipped at the present time.

On cars having 32 volt d-c lighting it is possible also to use a small d-c booster set and secure 64 or higher d-c voltages for fluorescent tube operation. Such an arrangement has the advantage that stroboscopic effect is eliminated and by making the output voltage about 75, standard 18 inch a-c tubes may be used.

It is to be hoped that there also will be developed in the future fluorescent lamps of a more concentrated type that might be used in fixtures such as on platforms and in passageways where the long tubular type of lamp is not so desirable. Such lamps of the cold cathode type have been made but require high a-c voltages for their operation. No doubt the time will come when they will be available for operation on lower a-c voltages and perhaps directly on direct current.

In closing I wish to point out that if the full benefit of fluorescent tubes is to be secured, the fixtures in which they are used must be such as to use the minimum amount of screening and permit the maximum transmission of light consistent with a reasonably good appearance. There is too great a tendency on the part of architectural designers to use arrangements of fixtures that absorb too much of the light output and thus nullify much of the advantage of the use of these fluorescent lamps.

Investigation of Starting Requirements of a 660-Horsepower Locomotive Diesel Engine

Discussion and authors' closure of paper 40-124 by J. C. Davidson and R. Lamborn, presented at the AIEE summer convention, Swampscott, Mass., June 24–28, 1940, and published in AIEE TRANSACTIONS, 1941 (January section) pages 31–4.

W. S. H. Hamilton (New York Central Railroad, New York, N. Y.): The authors are to be congratulated on having set forth clearly and concisely many of the problems and phenomena that take place in the starting of a Diesel engine and for pointing out facts which have not been clearly understood in the past.

It is to be hoped that this paper will stimulate other manufacturers to present similar papers on other engines so that a more thorough study of the whole problem may be made.

Data such as this is very important in determining the number of cells required in the starting battery. Recently there has been a tendency to standardize on 32 cells of lead-acid battery for the larger sizes of engines, say 300–1,000 horsepower, and 16 cells for the smaller ones. These follow the accepted standards in the car lighting field and it will be interesting to see how large an engine it will be found possible to start from a 32 cell lead-acid battery. From the user's standpoint the fewer the cells of battery, the better on account of maintenance.

With the smaller Diesel engines, starting motors are frequently used and in connection with them I believe some erroneous impressions have arisen probably carried over from experience with gasoline engines. Gasoline engines require a spark for ignition which naturally cannot be secured if the battery voltage is allowed to go too low during cranking. The usual limit is about 1.0 volt per cell, I believe. Due to this requirement apparently, the tendency has been to select starting batteries with relatively few cells and a large number of plates per cell. While such a battery gives ample breakaway torque, the cranking speed is limited after the engine turns over and there is a tendency for the time required to fire to be lengthened out.

A Diesel engine on the other hand (except those of the Hesselman type) requires no ignition and the battery voltage can be allowed to go much lower, 0.5 volt per cell being the usual limit. Of course sufficient current to provide breakaway torque must be provided, but on the other hand it is desirable to bring the Diesel engine quickly up to as high a cranking speed as possible in order to heat the combustion air and cause ignition promptly. The combination of these requirements points to the use of a starting battery having "steep" voltage characteristics, i.e., one whose voltage increases rapidly once the engine has started to turn over. This battery characteristic would be obtained by a larger number of cells with fewer plates per cell as compared to the battery that might be selected for starting a gasoline engine. Shortening the time between starting and commencement of firing means that less total kilowatt hours will be required from the battery per start.

This characteristic has been fully recognized in the case of the larger size engines and is shown by this paper, but I do not think it has been fully appreciated by those concerned with the smaller Diesel engines, especially those using starting motors.

I wish to add a word also on the problems of cold engine starting. The relatively large engines used on locomotives do not cool down as rapidly nor are they in general exposed to the outside air as much as are engines in automobiles. Hence it is questionable whether even in severe winter weather the crankcase oil gets anywhere near as stiff as is commonly supposed. We have found by tests on a 300 horsepower engine that the oil hardly gets down to 40 degrees Fahrenheit even in ten degrees Fahrenheit outside air overnight and with doors and windows opened and crankcase covers removed. This feature should be kept in mind in connection with the starting problem particularly in connection with the larger sized engines.

D. P. Orcutt (The Electric Storage Battery Company, New York, N. Y.): This paper is very interesting to me as it gives definite information about the starting characteristics of an engine that I have followed in service from the starting battery side for approximately ten years.

The excellent service record of the electrical cranking equipment used on these engines has been a marvel to me when I consider how little was known of the actual requirements at the time the equipment was designed. It is possible now to consider some of the factors of safety that were in the original design and how they have contributed to this service record.

Test 6 showing how the maximum torque required to start may increase in a hot engine indicates that if initial battery voltage instead of the more conservative value of average battery voltage had been used in the original design, there would have been many failures in service that, with the limited knowledge of the starting characteristics of the engine that were available, would have been hard to explain.

Tests 5 and 9 are especially interesting as representing about the minimum battery voltage that will start the engine. The margin between the voltages in these tests and tests one and seven where the full number of cells were in circuit is considerable, but this margin is no greater than is needed to meet the condition of a hot engine as shown in test 6.

One might gain the impression from comparing tests 7 and 9 that a considerable saving in battery cost might be made by reducing the number of cells from 56 to 40 since the slower cranking in test 9 uses about the same amperes at a lower voltage and therefore less watts. However, test 6 shows that had this been attempted, the engine could not have been started when hot. It would also be true that it could not have been started when the battery was excessively cold due to battery voltage being less under this condition.

There is also a third reason a 40 cell battery of the size used would not be sound engineering. If the watt hours required to make the engine fire in the two tests are calculated, it will be seen that in spite of the fact that the watt input is less with 40 cells, the watt hours are actually about 30 per cent more due to the longer time required to make the engine fire. If the 40 cell battery were engineered with the same factor of safety in number of starts that could be made from the battery, its ampere-hour capacity should be increased so as to make its watt hour capacity approximately 30 per cent greater than that of the 56 cell battery. This, of course, is on the assumption that the same generator windings are used with both batteries.

The tests presented so carefully in this paper confirm my belief that the most satisfactory and the most economical cranking of Diesel engines is obtained by a design which provides rapid acceleration and high speed in cranking. This is even more important under cold conditions when radiation losses are greater or when wear increases the "blowby" of the pistons.

Some of us who have had to live with the results of the early engineering applied to starting of Diesel electric locomotives have been and are deeply grateful for the "leeway provided by the manufacturers" as referred to by the authors under the heading "Conclusions."

R. Lamborn: The discussions by Messrs. Orcutt and Hamilton emphasize the value of definite testing information in connection with selecting starting equipment for Diesel-electric locomotives. Mr. Orcutt has pointed out the necessity for adequate battery equipment whether starting cold or hot. In this connection it should be pointed out that the results shown by the authors apply to a certain specific engine, and as Mr. Hamilton has brought out in his discussion additional information on other engines should be obtained for the benefit of equipment designers and locomotive builders.

An Improved Type of D-C Wattmeter of the Shunted Type

Discussion and author's closure of paper 40-88 by Paul MacGahan, presented at the AIEE summer convention, Swampscott, Mass., June 24–28, 1940 and published in AIEE TRANSACTIONS, 1941 (January section) pages 4–7.

Perry A. Borden: See discussion page 634.

H. N. Walker (New York University, New York, N. Y.): It is remarkable that after 50 years of progress in instrument design, we are still using ammeters and voltmeters for measuring d-c power, thus involving the necessity of two independent readings and a multiplication. Admittedly on most d-c circuits the voltage is reasonably constant, so that current as indicated by switchboard ammeters is usually a satisfactory guide of load.

The great advantage of the so-called d'Arsonval shunted ammeter, and the inconvenience of electrodynamic wattmeters which could not be operated with equal accuracy from ordinary ammeter shunts, has tended to suppress advancement along the lines followed by the author. A reading of d-c power is often preferred over separate readings of amperes and volts, particularly with the varying voltages.

There is nothing new in any of the components of the d-c wattmeter as developed by Mr. MacGahan. However, the resultant assembly itself is new. This paper is an interesting example of how an entirely new and useful device can be produced from existing components. Mr. MacGahan's wattmeter is evidently a hybrid of the d'Arsonval structure and the electrodynamic principle of operation. Operation from a shunt is made possible by retaining the low energy input level of the permanent-magnet instrument. The use of Hipernik for the core material of the electromagnet minimizes residual errors. I believe that a contribution to the art of instrumentation has been made in that, and so far as I know, these things have never been done satisfactorily before.

It is interesting to me to note that progress in the art of instrumentation does not necessarily involve the discovery of new principles, so much as it does the exercise of skill in combining known principles and existing facilities so as to meet a demand.

The question of contact resistance which Mr. MacGahan touches upon is one which seems to me to be too little appreciated by most electrical engineers. It may be interesting to point out that the contact resistance of even copper to copper can be appreciable and prolonged. It has been observed that such contact resistance steadily decreases over periods as long as two months and more. I should like to point out that, up to this time, there has been no adequate explanation given for this phenomenon.

Paul MacGahan: As pointed out by Mr. Borden this instrument was only made possible by the development of new materials. This condition is true in all new developments. Thus the steam engine had to wait many years for the advent of steel before it could be built and thereby bring on the industrial age. As exemplified by Mr. Miller's wattmeter no "principle of operation" can ever become "obsolete." It can only become temporarily set aside until new conditions, or new materials bring it forward again to new uses.

One of the remarks made during the discussion was that such instruments have been made before. It sometimes seems to me that everything new has been made before. Erikson discovered America before Columbus. Many steamboats were sailing ahead of Fulton. Edison discovered the Edison effect in the vacuum of a lamp bulb, Hertzian waves were known and studied, and the spoken word was carried on the wave before Conrad—whose broadcasting achieved revolutionary changes in human affairs. So let us agree that achievement is not invention—but consists rather in "putting it across" at the proper time.

Development in instruments as in other fields has reached a new phase. No one achievement can stand out in advance of the general front of progress. Progress on any section must necessarily depend upon that of other sections.

A New High-Speed Thermal Wattmeter

Discussion and author's closure of paper 40-97 by John H. Miller, presented at the AIEE summer convention, Swampscott, Mass., June 24–28, 1940, and published in AIEE TRANSACTIONS, 1941 (January section) pages 37–40.

P. M. Lincoln (Therm-Electric Meters Company, Inc., Ithaca, N. Y.): John H. Miller's paper describing "A New High-Speed

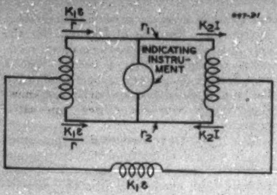


Figure 1

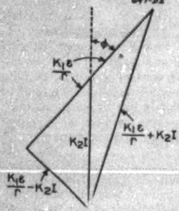


Figure 2

Thermal Wattmeter" is an exceedingly interesting one. It describes a method of measuring watts with which I have been experimenting for over 30 years. It is a method of measuring, which in my opinion, is sure to see much more use as time goes on.

Mr. Miller has described a very ingenious method of securing rapid response. My own experiments have been directed toward physical construction of such a nature that the response would be inherently slow so that the device could be used for the measurement of "maximum demand."

Let us carry Mr. Miller's analysis a little further. Figure 1 of this discussion is based on Mr. Miller's figure 2. Let us assume that the resistance of the hotter of the two rows of thermocouples is r_1 and of the colder is r_2. If $r_1=r_2$, it is obvious that the current through them due to the voltage of the circuit under measurement is $2K_1E/r$, where K_1 is the ratio of the potential transformer and r is the resistance of each of the two rows of thermocouples.

The current of the circuit under measurement is caused to superpose upon the potential current in the manner shown in Mr. Miller's figure 2. The value of this current is obviously K_2I where K_2 is the ratio of the current transformers. The total current in r_1 thus becomes $K_1E/r+K_2I$ and that in r_2 becomes $K_1E/r-K_2I$. The heat generated in resistances r_1 and r_2 is, of course, proportional to the squares of the currents therein. Therefore, the heat generated in resistance r_1 is $(K_1E/r+K_2I)^2r_1$ and that in resistance r_2 is $(K_1E/r-K_2I)^2r_2$. Mr. Miller's setup is such that his measuring instrument gives the difference between these two values of heat. This obviously reduces to $4K_1K_2EI$.

However, the above analysis takes no cognizance of power factor. If the power factor is other than unity, the currents in our resistances due to potential and to the current itself are no longer directly additive and subtractive. The phase angle between the two must be taken into consideration. This is done in figure 2 of this discussion. From trigonometry it is obvious that the current values in resistances r_1 and r_2 are those given in figure 2. Taking power factor into consideration, it is obvious that the voltage applied to Mr. Miller's measuring instrument is $4K_1K_2EI \cos \phi$.

However, it should be borne in mind that the integrity of the above analysis is based on the assumption that $r_1=r_2=r$. If Mr. Miller's rows of thermocouples are composed of materials of zero temperature coefficient, the above analysis will hold true. If, however, there is an appreciable temperature coefficient in the thermocouples, the currents therein due to the voltage E no longer divide equally between the two rows of couples and there will be an error of measurement. This is particularly true on low power factors where the currents flowing through the thermocouples are much increased. It would be of interest if Mr. Miller would give us some information on the temperature coefficient of his thermocouples.

G. S. Lunge (General Electric Company, Schenectady, N. Y.): This new high-speed thermal converter, more familiarly known as the "thermoverter," is especially welcome in the field of carrier telemetering.

With it, it is now possible to totalize economically a number of readings at the transmitting end of the telemeter channel, and even to add in additional readings at intermediate points along the route, all without appreciably impairing the over-all speed of response of the impulse-telemeter equipment. Only impulse telemeters have found general application to carrier channels, because of the simplicity and closer spacing possible of channels so used. Until the advent of the thermoverter, the excessive time lag inherent in thermal converters nullified the reasonably rapid response of the impulse-telemeter equipment and usually necessitated recourse to other methods of totalizing.

Now the sum of the thermoverter-output electromotive forces can be measured immediately (rather than only at regular intervals) by a self-balancing potentiometer, the slide-wire rheostat-arm position of which is "scanned" by an impulse-duration telemeter ("metameter") cam, thereby converting the d-c electromotive force into on-off carrier keying impulses as a single operation, with minimum delay.

The same type of potentiometer device can be used to advantage when a carrier telemeter reading is to be retransmitted to its final destination after adding to it one or more readings that can be obtained by means of thermoverters located at or near the retransmission point. The carrier telemeter receiver at the retransmission point in this case is arranged to provide a suitable (and preferably ripple-free) electromotive force proportional to the quantity measured, for addition in series with the thermoverter electromotive force.

Care must be taken not to destroy the advantage of the rapid response of the thermoverter to changes in load by inappropriate choice of the carrier telemeter system used to transmit the reading or retransmit the totalized value. The impulse-duration telemeter (metameter) is particularly well suited to take full advantage of the high speed of the thermoverter, as, different from the impulse-frequency telemeter, the metameter does not require the thermoverter electromotive force to be converted (by necessarily indirect means) into proportional mechanical rotational speed before carrier keying impulses can be initiated. The impulse-frequency telemeter can, of course, only transmit readings of such quantities as can be arranged to produce continuous mechanical rotation.

Perry A. Borden (The Bristol Company, Waterbury, Conn.): Though it might not so appear on the surface, there exists between the papers of Mr. MacGahan and Mr. Miller a kinship which makes it desirable that they be discussed simultaneously. Both the d-c wattmeter announced by Mr. MacGahan, and the thermal wattmeter described by Mr. Miller, point to the outstanding advances in the art of measurement, as well as in all other fields of electrical activity, which are made possible by use of the new magnetic alloys. In these cases two quite different characteristics of substantially the same material have been advantageously utilized.

The industry has long demanded a d-c wattmeter which could be operated from a conventional shunt with degree of simplicity and efficiency to compare with that of the d-c ammeter; and the instrument described by Mr. MacGahan appears to fill the need.

The rapid response and relatively great power output characterizing the thermal wattmeter described by Mr. Miller have vastly broadened the field of measurement to which this principle may be applied. The fact that it may be used with a rugged millivoltmeter type of indicating instrument over a considerable length of line greatly simplifies power measurement, especially where totalizing is involved, even though the distance is not sufficient to justify application of the term "telemetering". When used with a self-balancing potentiometer, the practical speed of response is limited only by that of the recording instrument, and its performance is eminently suited to automatic load control. The low internal resistance of the output circuit greatly increases the flexibility of use, and makes possible the simultaneous determination of subtotals and a grand total where several groups of feeders are involved.

Now, with the advent of the instrument described by Mr. MacGahan it becomes equally easy to obtain a measurement of d-c power in a form suited to telemetering and totalization, thus providing a practical solution to the long-standing problem of totalizing a-c and d-c power loads on a common instrument.

John H. Miller: Mr. Lincoln asks for the temperature coefficient of the thermocouples. The mean temperature coefficient of the two materials used is 0.00117. However the change in resistance in r_1 and r_2 referred to in the discussion by Mr. Lincoln does not completely determine the distribution of the current obtained from the potential transformer. Since this current flows through several turns in opposite directions on the current transformer, if it tends to separate in other than equal amounts, a differential reactance of a rather high value appears which forces the even distribution of this current in the two parallel circuits.

Actual tests show that if one resistance increases as much as ten per cent above the other, feeding these currents through the split winding on the current transformers will result in less than one per cent difference in the two current values.

The values of r again come in as a function of the generated heat, but if they increase as a function of the temperature, they simply increase the slope of each curve and do not change the form so that such differences can be calibrated into the device through a single spot calibration by means of the adjustment of resistance in the primary of the potential transformer.

Thus we see that actual resistance variations in the thermocouples give rise only to second-order errors and as a practical matter the output curves are quite straight and the compensation for the temperature coefficient of resistance is inherent at least as to first-degree errors.

The Vertical-Flow Interrupter and Its Application to "Oil-Poor" Circuit Breakers

Discussion and author's closure of paper 40-114 by B. P. Baker, presented at the AIEE Pacific Coast convention, Los Angeles, Calif., August 27–30, 1940, and published in AIEE TRANSACTIONS, 1941, pages 440–5.

J. B. MacNeill (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The development outlined by Mr. Baker was initiated in this country to protect the switchgear industry in case the high-voltage oil-poor interrupters used in Europe should demonstrate adequacy for our conditions. The improvements made in dead tank breakers have so far proved satisfactory to operating people and there has been no insistent demand for the oil-poor device. On the other hand the improvement in circuit rupture made possible by this development has been of decided interest in certain European countries and this design is in extensive manufacture in Italy and it is being taken up in a preliminary way in England and Japan.

The results given in the paper, both on factory and field tests, show that interruption occurs in the minimum of time and with great arc efficiency. This makes possible the handling of heavy powers in the limited space and strength of structure provided by a porcelain enclosure.

For use out of doors the device is admirable, in that there is no exposed disconnect switch or operating linkage. The remote control rod passes through the current-transformer chamber, is immersed in oil at all times, and its sole action provides the interrupting break and the disconnect function as well.

Both the interrupting chamber and the transformer chamber are lined with Micarta tubes which preserve the assembly of the breaker in case of porcelain failure so that the porcelain serves only as a weather shield.

The paper describes in detail a 138-kv size but the same structure with dimensional modifications is good for 115 kv and 161 kv. In Italy at the present time a similar structure using two columns is in production for 230 kv.

The Southern California Edison Company is to be commended for its interest in this new development, and for the valuable tests made on it at the Laguna Bell station. While the particular unit tested in the field was designed for 138 kv it handled efficiently short circuits to ground on the Southern California 220-kv system up to approximately 2,250,000 kva on a 3-phase basis.

The transportation of this porcelain structure across the country by railroad and back again, and its erection in California without incident is reassuring to those who may question the adequacy of such a structure for practical switchgear requirements.

J. D. Laughlin (Bureau of Power and Light, City of Los Angeles, Calif.): The paper presented by Mr. Baker emphasizes two important phases in the present trend in the development of high-voltage circuit breakers, namely the reduction of oil requirements and the reduction in interrupting time. Development along these two lines should be very welcome to purchasers of this type of equipment.

The radical departure of this circuit breaker from the conventional oil circuit-breaker design illustrates a phase of the work being done to produce circuit breakers that will not require the large quantities of oil used in past designs. The reduction in oil requirements from past circuit-breaker design should prove of great benefit in the reduced costs of handling oil during construction and in the reduced requirements for oil maintenance and reduction of fire hazard during operation of the equipment.

The data from the tests made on this breaker indicate that it will interrupt currents in times comparable with the fastest conventional type circuit breakers and its interrupting time is approximately constant throughout its capacity range. This fast interrupting time should prove valuable in increasing the transient stability of systems with long transmission lines.

In the construction of the Boulder transmission system, the Bureau of Power and Light of the City of Los Angeles has taken advantage of the use of high-speed low oil content breakers. By using circuit breakers with three-cycle interrupting time, the transient stability of this system was increased over what could be obtained using breakers of conventional design. At each of the two intermediate switching stations, six low oil content 287,500-volt oil circuit breakers are installed. The total oil requirement for the six breakers at each station is 14,000 gallons as compared with a requirement of 140,000 gallons for conventional tank-type breakers of equivalent rating. The greatest quantity of oil in any one compartment in the breakers used is 150 gallons as compared with approximately 8,000 gallons in the tank-type breaker. Due to the low oil requirements of these breakers, no oil handling equipment is installed at these stations. All oil at these stations is filtered and handled with a portable outfit consisting of a filter press and tank mounted on a truck. This piece of equipment is normally used for maintaining equipment at other scattered installations. If tank-type breakers had been used at these stations, it would have been necessary to install special oil storage tanks and oil handling equipment at each station.

In the design of the circuit breaker described in this paper, porcelain and fibrous insulating tubes have been used to provide insulation between live parts and ground, and also to support the interrupting element function performed by the oil and steel tank in conventional design. These departures from conventional designs introduce new problems that will require research and investigation to develop these parts to a point where they will withstand the combined electrical and mechanical stresses due to repeated operations as well as is being done by the conventional design. I would like to ask what assurance the author has of the structural reliability of this breaker.

The Southern California Edison Company should be commended for making the series of circuit-breaker tests on their 220,000-volt system of which this paper reports one test. Such tests which more closely duplicate service conditions than laboratory tests provide a definite check of the ability of circuit breakers to perform under actual system conditions and furnish both manufacturers and operating companies with valuable information regarding circuit-breaker operations.

H. E. Strang (General Electric Company, Philadelphia, Pa.): Mr. Baker has described an oil-poor circuit breaker, which on the strength of the very satisfactory field tests, merits consideration and study even at this time when the trend of new developments in the industry seems to be away from oil breakers, including the low volume variety, toward the oilless types.

Unfortunately it may be confusing to the engineer who is attempting to correlate practical applications and physical construction of interrupting devices, having various theories of operation, to learn that Mr. Baker states that interrupters used in dead tank breakers are not adaptable to use in porcelain structures.

The original high-voltage low oil content breakers in the United States were placed in service on the 287-kv Boulder Dam line, and on the 138-kv receiving station end in 1936. Requirements for the third line in 1939 placed similar breakers at the Boulder end as well as duplicates of both other locations. These breakers are of the impulse type[1] in which a mechanically operated piston forces a stream of oil *across* the arc paths between each of a series of breaks. This type of operation results in minimum arcing time, since oil pressure is always available at the instant the contacts separate. Application of this same type of interrupter has been made to tank-type breakers for 115 kv to 230 kv[2] in which the flow of oil *across* contacts in series is produced by the gas created by other pressure-generating breaks, instead of a spring-actuated piston. This construction results in slightly longer arcing times, but still short enough to demonstrate over-all breaker operating times of 1.96 to 4.22 cycles on 230-kv field tests on the Southern California Edison Company system in 1937.

The device described by Mr. Baker in which oil moves longitudinally with the arc, appears to be related to the well-known explosion chamber, in which the throat provided the "small hole or chimney in the oil." After having served the industry well for more than a quarter of a century, this type of interrupter was superseded in 1930 by the oil-blast explosion chamber[3] following laboratory and field tests which proved that the cross-blast action was about twice as effective as the longitudinal blast.

Mr. Baker also mentions a flow of oil created by the different displacement of two volumes attached to the moving contact assembly, but neglected to make clear whether this differential volume was of sufficient magnitude to have an appreciable influence. If a relatively large volume is displaced and forced along the arc, the piston of course must be actuated by relatively heavy springs to assure a continuance of the flow against the gas pressure created by the arc, and to avoid contact reversal at such a critical time.

REFERENCES

1. Circuit Breakers for Boulder Dam Line, D. C. Prince. AIEE Transactions, volume 54, 1935, page 366.

2. A New Multi Break Interrupter, R. M. Spurck and H. E. Strang. AIEE Transactions, volume 57, 1938 (December section), page 705.

3. The Oil Blast Circuit Breaker, D. C. Prince and W. F. Skeats. AIEE Transactions, volume 50, 1931, page 506.

R. A. Hentz (Philadelphia Electric Company, Philadelphia, Pa.): The breaker described by Mr. Baker is a desirable contribution to the high-voltage circuit-breaker art on two scores.

First, it constitutes one more step toward the reduction of the oil fire hazard. The article states that the interrupting compartment contains 60 gallons and, assuming (from figure 4) that the current-transformer compartment contains about twice that amount, we have approximately 200 gallons of oil per single-pole 138-kv breaker. A corresponding dead tank breaker contains about 1,250 gallons. While 200 gallons of oil (600 for 3 poles) can cause a bad fire, the relative hazard as compared to the dead tank breaker means that this new breaker is much to be desired over the old, especially when conditions require a circuit breaker to be installed in a location where fires would have serious consequences. However, the desirability of eliminating all oil from circuit breakers of all voltages still remains the goal toward which the industry should strive.

Second, it seems that this design should result in a marked reduction in first cost, as well as operating expenses. Figure 3 vividly brings out great differences in physical dimensions which should not only make the breaker itself much cheaper, but the cost of such items as handling, foundations, permanent oil piping (probably not needed at all in the case of the oil-poor breaker). It would be interesting to learn from Mr. Baker if there is a reduction in control current, an important item along with permanent oil piping in the case of a widespread substation designed with substantial separation between its component parts.

The breaker appears to have been designed for ready dismantling for inspection which with the smaller quantity of oil is of interest to the maintenance forces.

In testing circuit breakers it is desirable, of course, to perform as large a number of tests throughout a wide range of short-circuit currents as is possible, for the greater the number of tests, the more reliance can be placed on the breaker. But laboratory limitations usually mean that the higher kilovolt-amperes cannot be imposed upon the breakers and operating limitations mean that only a limited number of tests can be made on a utility's system. By subjecting a circuit breaker to both factory and system tests, a satisfactory over-all testing program can often be obtained. This appears to have been the case in the test described in the paper, about 115 factory tests (figures 7 and 8) being made at the factory with an upper limit of about 3,000 amperes at 132 kv and 4,300 amperes at 88 kv and 4 tests at the Laguna Bell substation at higher short-circuit currents (as high as 5,900 amperes at 124 kv).

H. P. St. Clair (American Gas and Electric Service Corporation, New York, N. Y.): Regardless of the ultimate validity of the author's theory as to the actual mechanics of arc interruption, it appears that the breaker described does produce an effective relative motion of oil and arc which results in consistent interrupting performance, at least throughout the range of recovery voltage conditions applying to the tests reported in the paper.

The voltage recovery characteristics obtained on the Laguna Bell tests have been indicated by the author but no data are given for the laboratory tests. As to be expected from the nature of the system connections supplying the fault at Laguna Bell, these characteristics were distinctly low in severity, particularly on the maximum short-circuit values. Presumably, laboratory tests were made at greater severity. It would be interesting to know what these conditions were. It would also be interesting to check the field performance of this breaker on circuits with higher natural frequencies and correspondingly greater rates of rise of recovery voltage.

The mechanical design of the breaker appears to include a number of good points such as (1) the built-in disconnecting contact with full interior creepage distance, and (2) internal support and operating stresses carried by micarta-tube housing, independent of porcelain shells. On the other hand, possible sources of difficulty in this design might include the stuffing gland where the torsion operating shaft goes through the base compartment of the breaker, this compartment as shown in figure 4 being subjected to the full head of oil equal to almost the full height of the breaker. This head appears to be somewhere on the order of ten feet. While this might not create a serious problem, it certainly would necessitate a completely oil-tight construction and the maintenance of an oil-tight stuffing box operating under considerable pressure.

Current transformers continue to present a problem in breakers of this general type and the solution here, while it is a solution, certainly is far from ideal. Access to the current transformers would appear to require complete dismantling of the breaker.

On the whole this breaker represents an interesting development and Mr. Baker is to be congratulated, both upon the developmental work accomplished and the clear and concise description which has been given in this paper.

Incidentally, the industry is indebted to the Southern California Edison Company for another contribution to the art in lending the facilities of their system for high-power short-circuit tests. We in the American Gas and Electric Company system know that to subject a high-voltage system to repeated full capacity short circuits takes a certain amount of courage and a high degree of confidence in the protective equipment on the system. It has always been our conviction, however, that, since such protective devices have been designed and applied to take care of system faults, there is no better time to find out whether or not they are capable of doing so than under staged short-circuit tests where every opportunity is available for careful observance of system performance and weaknesses. Furthermore, we can truthfully say that, in all of our indulgence in short-circuit testing, the benefits from establishing greater confidence in the reliability of our system protective features and in weeding out such weaknesses or deficiencies as have been occasionally disclosed thereby have far outweighed any harmful results in the way of damage to equipment or disturbance to customers resulting from such tests. It is our understanding that the same conclusion has been reached by the Southern California Edison Company in connection with their testing experience, and we are confident that power systems, in general, would reach the same conclusions after going into more field testing. It is our belief, also, that, while the bulk of the research and developmental testing in the design of circuit breakers must necessarily be made in the manufacturers' testing laboratories, at the same time, as a final proof of a design, a full capacity field test is practically indispensable; and that power companies, in general, would contribute to the development of the art and ultimately to an over-all saving in cost by allowing a greater use of their systems for such tests.

A. C. Schwager (Pacific Electric Manufacturing Corporation, San Francisco, Calif.): In 1938 I had the opportunity to visit a large number of circuit-breaker manufacturers and breaker installations in Europe. At that time the grounded-tank circuit breaker had been replaced mainly by oil-poor circuit breakers. These breakers proved satisfactory for the relatively small interrupting capacities encountered there. Mr. Baker deserves credit for having shown that with proper modifications the oil-poor breaker has possibilities for voltages and interrupting capacities as met in this country. In some European designs most closely approximating Mr. Baker's development, oil is pressed vertically through the interrupting chamber. This oil-blast action was found necessary in order to obtain interruption at low currents; without it failure would occur. It would be interesting to know if Mr. Baker's tests confirm these findings.

Today, breakers are more and more called upon for fast reclosing duty and I believe that considerable hazard would be invited in placing such breakers into service without verification of the interrupting ability during such operation. Mr. Baker's test results seem to show that this breaker loses some of its effectiveness under conditions as they must exist during two successive opening operations. In table X, the arcing time for 60 shots is consistently longer than for 0 shots. This might be due to the fact that a vacuum is produced within the chamber during the closing stroke which will annul some of the desired oil-blast action in the subsequent opening operation. When two or more interruptions take place within a short interval, a similar, except more serious phenomenon is likely to occur. After the first interruption, the arcing chamber is filled with gases at considerable pressure and it is difficult to imagine that upon immediate reclosure any amount of oil can be accumulated and be made useful for the second opening. It is more likely that due to the gas pressure additional oil will be expelled from the chamber during the closing stroke. Has Mr. Baker additional test data to clear up this point?

L. B. Chubbuck (Canadian Westinghouse Company, Hamilton, Canada): While porcelain outdoor oil-poor breakers originated in Europe, it is interesting to note the outstanding improvements provided in Mr. Baker's design. Simple efficient interrupting grids are used, also a compression assembly of all porcelain, with no cemented or clamping rings. No cumbersome external air-isolating switch is required, as a simple oil-disconnecting gap is provided directly in the interrupting compartment. This latter arrangement is particularly suitable for high-speed reclosure.

The extended tests carried out in a well-known high power laboratory, and the very successful field tests in California, give confidence in what is, to most American engineers, a new type of breaker.

Where the current transformer can be taken care of separately, the large porcelain current-transformer assembly can be omitted from the breaker. The interrupting chamber can be mounted on three standard post insulators, and the operating rod from the oil-disconnect gap carried down in oil in a skirted porcelain tube to operating mechanism in the base. Particularly for the higher voltages this would provide a very simple and economical design.

R. C. Van Sickle (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The interrupter described by Mr. Baker is remarkable for the relatively small amount of electrical energy which it dissipates as energy of the arc. The amount of this energy is significant as it controls the depreciation of the oil and the pressures within the structure.

The improvements in interrupting devices made about 10 or 12 years ago were sufficient to permit reductions in oil volumes, and breakers in porcelain using small quantities of oil for interrupting purposes and air gaps for insulation appeared in Europe. The smaller oil volumes naturally increased the importance of the features which control the amount of arc energy liberated, namely, arc voltage, arcing time, and current.

The vertical-flow interrupter liberates very little arc energy. This can be seen from the oscillograms used as illustrations in Mr. Baker's paper. In figure 12, he shows three cathode-ray oscillograms and indicates by arrows the arc voltage. Actually these were so low that they are not distinguishable from the zero line. In the original oscillograms which are 18 inches long, they amount to little more than a thickening of the zero line immediately before interruption. The magnetic oscillograms shown in figure 11 do not have the voltage traces superimposed and here too the arc voltage is not discernible. In figure 6 which contains high power laboratory tests to a larger scale, the arc voltage can be seen but is extremely low.

These low arc voltages and the short arcing times produce very little arc energy. This is an outstanding characteristic of the vertical-flow interrupter and makes it particularly suitable for use in high-voltage breakers employing small quantities of oil and enclosed in porcelain.

H. A. Lott (Southern California Edison Company, Ltd., Los Angeles, Calif.): In October 1936 the first of a series of field short-circuit tests at 220 kv were made on an oil circuit breaker at Saugus substation of the Southern California Edison Company. Since that time 136 test short circuits have been made on 11 different types of interrupters. Each short circuit was single conductor to ground, the intensity varying from $^{1}/_{4}$ million to $2^{1}/_{4}$ million equivalent three-phase kilovolt-amperes.

As a supplement to the short-circuit tests, 47 tests were made wherein the three-phase charging current of 220-kv transmission lines varying in length from 22 miles to 246 miles was ruptured by the switch under test.

In each series of tests an oscillogram similar to the one reproduced in figure 11 of the paper under discussion was obtained for each individual short circuit or line drop test. The oscillograms together with tabulations of results and photographs were obtained and are used by Edison engineers as a basis for their recommendations for modernization of 220-kv oil circuit breakers.

The Edison 220-kv transmission system consists of 1,085 circuit miles of 220-kv line which interconnects the three principal sources of generation with the load. Two sources of generation, Boulder and Big Creek are approximately 250 miles from the load center while the third source, Long Beach steam generating station, is about 25 miles from the load center. On such a system where large blocks of generation are remote from the load, high-speed clearance of short circuits is imperative in order to prevent instability following the clearing of multiphase faults.

Because of the necessity for high-speed modern interrupters, Edison engineers are keenly interested in new developments in the circuit-breaker art, particularly when such developments give promise of being useful in modernizing existing breakers. Accordingly the Edison company was willing to extend the use of its system on the several occasions when it appeared desirable to test the performance of newly developed interrupters which might be used on the Southern California Edison system.

The amount of preliminary work and planning necessary to stage a short-circuit switch test is almost inconceivable to one who has not participated in such a program. Edison engineers have developed special relays for starting the oscillograph and closing test switches, they have developed a travel recorder which records the motion of switch contacts and thus indicates the position of switch contacts at any time during the opening or closing stroke of the switch under test, in addition to several other devices which facilitate making the tests.

When short-circuit tests are made at Laguna Bell substation five values of current are used. These values are obtained by switching of lines and busses at adjacent stations in order to place the test breaker at Laguna Bell at the terminal of transmission lines of various lengths which interposes sufficient reactance to give short-circuit currents varying from 600 to 6,000 amperes. The highest value of 6,000 amperes corresponding to an equivalent three-phase short circuit of $2^{1}/_{4}$ million kva is obtained from the bus at Laguna Bell. The bus sectionalizing switch at Laguna Bell is used as a back-up breaker and is equipped with special relays which are timed for selectivity between the test breaker and the back-up breaker. A special program is prepared outlining all switching, relay changes, and other steps in the procedure which is necessary for dispatchers and station operators to know in order to expedite the test. As a result all tests have proceeded without delay and according to program.

As a result of the several tests the following conclusions have been reached:

(*A*). While the major development and research leading to provision of better circuit breakers must of necessity be provided by the manufacturer, the utility has a distinct place in the development of the art through the staging of field tests.

(*B*). While laboratory tests tend to indicate the success of a circuit-breaker design, experience indicates that the final proof of circuit-breaker performance will only come from system tests wherein the switch is required to interrupt its rated current while connected to an operating system.

(*C*). In so far as 220-kv switches are concerned line dropping tests are a necessary supplement to the short-circuit test since some designs which interrupt short circuits successfully are inadequate in some respects when required to interrupt charging currents.

(*D*). While there is some hazard to service incident to the staging of full capacity short-circuit tests it appears that the benefits occurring are of sufficient value to discount the hazard.

(*E*). When phase-to-ground short circuits are cleared in eight cycles or less the effect on consumers service is negligible.

The tests on the vertical-flow interrupter described in Mr. Baker's paper were made at Laguna Bell on October 9 and 16, 1938. The results from the standpoint of system operation were excellent. There was no dip in system speed and no consumer complaints as a result of the tests. Most consumers were not aware that tests were in progress as the flicker of lights which indicated a short circuit was barely perceptible.

A study of the design and results suggests that the vertical-flow principle might well be applied in modernizing existing conventional breakers.

The author is to be congratulated for an interesting paper and another step in the development of high-speed high-capacity circuit breakers.

B. P. Baker: The author is grateful for the interest shown in this paper by the large number of discussions offered. In addition to the many valuable contributions they have made to the circuit-breaker art, some very pertinent questions were asked which I will attempt to answer.

The breaker described in this paper was operated by a standard solenoid mechanism. The effort required to operate it is substantially the same as that required by other high-speed oil circuit breakers. For applications requiring a reduction in control current a pneumatic or motor-operated mechanism may be used.

The energy required to supply the oil flow which is necessary for efficient uniform operation at all values of current, is obtained partly from the operating mechanism which compresses the accelerating springs during the closing stroke. However, at high currents when the pressure in the exhaust chamber rises, this pressure assists to some extent in supplying the energy for the oil pumping action and thus avoids contact stalling or reversal. Actually the amount of oil which has to be pumped is comparatively small since the passage through which it flows is very small. This is not only desirable to conserve pumping energy, but is a natural result of getting the properly controlled relation between the arc and the surrounding oil. This controlled flow of oil in a direction opposite to contact motion, along the entire length of the arc, is one of the factors which make it superior to other longitudinal blast interrupters. It is entirely different in structure and performance from the earlier type of interrupter to which one discusser referred.

From the instant the contacts part, the venting, rate of gas generation, and deionizing processes are controlled rather than waiting for a certain contact separation to release a random amount of stored energy. The motion of the contacts is always against the flow of, and into fresh oil, and not with the blast into regions already occupied by ionized gas. The arc is held in close contact with the oil and gases over its entire length instead of at a relatively short throat. The long narrow arc passage also prevents sections of the arc from burning in relatively quiet gas bubbles. The passage vents into a chamber in which the pressure is controlled, rather than into an open body of oil. Extinguishing action is not dependent on a self-generated gas blast which varies with current.

Careful measurements on the high-speed closing-opening tests indicate that the breaker was probably tripped out before the contacts had reached their full $1^{1}/_{2}$-inch engagement. This would have resulted in a loss of some of the opening acceleration and thus prolonged the arcing time. I have no data which would allow me to conclude that the opening which immediately follows a closing is not affected by it, however, the effect must be small even after a period of four cycles. Certainly after eight or ten cycles the gas from one test should be well away from the upper end of the passage through which the arc is drawn. At one time in the development a check valve was employed at the lower end of the well through which the space under the moving contact was recharged with clean oil during the closing stroke. This was found unnecessary and was abandoned. If Mr. Schwager's fears are justifiable, this solution would undoubtedly solve the problem.

While there were only 115 factory tests represented in figures 7 and 8, actually there were many times that number made on this interrupter and slight variations of it. At 132 kv the recovery rate of the circuit varied from 2,000 volts per microsecond to almost 4,000 volts per microsecond at the higher values of current. At 88 kv it varied from about 1,600 to 3,200 volts per microsecond.

The current transformer is mounted on the floor of the lower porcelain-clad compartment. The low tension leads are brought out through the grounded metal frame by means of small insulators and are very accessible. Access to the transformer itself requires removal of the superstructure, an operation comparable to removing the bushings of a dead tank breaker. However, Mr. Chubbuck in his discussion has made a unique suggestion for removing it entirely from the breaker and thus making the transformer an auxiliary piece of equipment.

It is regrettable that a most valuable but informal discussion by Lloyd Hunt of the Southern California Edison Company was not written for publication. He discussed the field tests in a very detailed fashion, bringing out some interesting points not covered in the paper.

An Improved A-C Pilot-Wire Relay

Discussion and authors' closure of paper 40-127 by J. H. Neher and A. J. McConnell, presented at the AIEE Pacific Coast convention, Los Angeles, Calif., August 27–30, 1940, and published in AIEE TRANSACTIONS, 1941 (January section) pages 12–17.

H. P. Sleeper (Public Service Electric and Gas Company, Newark, N. J.): This writer made a prediction a few years ago which appears to have been proved incorrect. That fallacy has been further amplified in the paper "An Improved A-C Pilot Wire Relay" by my friends Neher and McConnell. The incorrect prophecy referred to was given as conclusion number two in my paper "Relaying of High Voltage Interconnection Transmission Lines," AIEE TRANSACTIONS, 1933, volume 52. In that paper I predicted that pilot wire relaying would not enjoy much future popularity. However, during the last few years, with the introduction of the Westinghouse type HCB pilot wire relay, and now by the introduction of the General Electric type CPD relay, the popularity of this type of relay has very greatly increased. Furthermore, the increase has proved to be justified due to the very satisfactory operation which has been reported by users of these devices.

The Westinghouse equipment, and the one described in these authors' paper, are sufficiently similar so that the relative merits and differences are minor and do not seem important. Both employ the fundamental improvement over previous pilot-wire systems of using low energy pilot channels so that commercial telephone circuits may be employed. This is of material importance to the operating utilities since the communication facilities are available without capital investment by the operating company and can be abandoned on short notice without withdrawal of the pilot circuit. Furthermore, such channels are usually available at almost an instant's notice, and this is sometimes of importance to the utility.

It is this writer's opinion, based on a very complete laboratory examination and upon a small amount of operating experience, that this type of relay system is fundamentally correct and has been satisfactorily developed by the manufacturers. It should have a very satisfactory operating record and this has been confirmed by the experience to date of a rather large number of operating companies on a considerable number of short transmission line applications.

There still remains the field of longer transmission lines than those at present protected by relays of this type. Present applications are limited by both manufacturers to pilot wires that do not exceed 2,000 ohms or a distance of about 15 to 20 miles between stations. However, there are longer lines which are in the market for protection of this type if the relay scheme can be made applicable. This means that the pilot circuits will be of considerably higher resistance, possibly of the order of 4,000 to 5,000 ohms. Furthermore such circuits, if leased from commercial telephone companies, are usually "loaded," that is, they have a considerable number of loading (inductance) coils in the circuits to improve communication facilities. This increases the a-c impedance of the circuit so that the relay no longer works on a practically noninductive circuit but on one with an inductance of possibly four henries. This of course changes the power factor of the transmitted a-c signal and the relay currents at the two ends of the pilot channel may be considerably out-of-phase. This may be the limiting factor in such an application, as these relays are ordinarily designed to operate on pilot circuits of practically zero inductance. Also, because of the longer pilot circuits, the capacity between wires and to ground is proportionately increased.

There appears to be no fundamental reason why the type of relay described in this paper cannot be made applicable to the longer lines. The relay voltage applied to the pilot wire will have to be increased but the allowable potentials as dictated by the commercial telephone companies appear to be adequate. The effect of the high inductances must be compensated so that the relay currents at the two ends of the pilot circuit are kept approximately in phase. This can be done by means of proper filter equipment which consists merely of series capacitors to compensate the inductance, and parallel inductances to compensate the shunt capacitance. It has been done experimentally by the writer working with the Westinghouse engineers on the type HCB relays. Several applications to lines up to

22 miles are planned by the company with which the writer is connected.

The operating record of these relays will follow directly the service record of the pilot circuits, in this writer's opinion. If the telephone circuit remains continuous and free of faults; if induced potentials are kept below values which will flash protector blocks, which should be capable of accomplishment by means of applying the proper number of neutralizing transformers in the pilot wire circuits; and if the department or the company charged with the maintenance of these telephone circuits provides the proper protection against ambitious or careless personnel who may interfere with these circuits, it is believed that the operating record of these relay schemes will be excellent.

It is especially important that the telephone company leasing these circuits be impressed with the fact that the introduction of foreign potential on these circuits is bound to have serious consequences and cannot be dismissed by a mere "Excuse it, please," which apology is always so gratifying when one has been needlessly wakened by the telephone bell at 2:00 a.m. Education of employees and the co-operation of the supervisors and executives of the telephone company seem to be the answer to this phase of the matter which will so importantly and so directly affect the operating record of these relays. It has been this writer's experience that the telephone companies are, in general, very co-operative and the special telephone rates which are quoted for this service are considered satisfactory.

If the installations of these relays are properly engineered; if the pilot circuits can be controlled directly by the operating utility, or if not, if the full co-operation of the telephone company and its operating personnel can be assured, there appears to be no reason why the operating record of these relays should not equal that of standard relays now in common use on similar applications.

It is the writer's hope that it will not be necessary to reverse this new prophecy.

W. R. Brownlee (The Commonwealth and Southern Corporation, Jackson, Mich.): The use of "standard energy" relay elements with relatively small pilot wire currents instead of the "supersensitive" variety will be welcomed, particularly by those who have experienced trouble with other forms of relays having extremely low energy requirements. Extra sensitive elements are subject to interference from microscopic condensations of matter volatilized from coil impregnating compounds, difficulties with switchboard vibrations, etc. The use of a "current product" element with most of the energy taken from the local source where ample power is available is a fortunate choice.

Some form of backup protection (although probably too slow and insufficiently selective for normal operation) is required with any pilot wire scheme for the following purposes listed in the order of their importance:

1. To clear a fault in an adjacent protected zone when the breaker sectionalizing that zone fails to trip (due to a defective mechanism, open trip coil circuit, etc.).
2. To clear bus trouble from adjacent stations when the bus itself is not separately protected.
3. To provide line protection when the pilot wire equipment is out of service.

The second reason may be removed by installing bus protection and the third may be reduced to a very small value, but the first reason will practically always be sufficient to require backup protection. Such protection will usually involve directional relays, the phase elements requiring a potential supply (inexpensive source such as low tension bus) which will also be adequate for a directional comparison type pilot wire scheme. This situation tends to eliminate one of the three advantages listed by the authors of a differential pilot wire scheme over the directional comparison type.

In some instances the complete immunity of the authors' scheme to an out-of-step condition may be advantageous, although economics will usually limit the length of a pilot wire protected circuit to such a short section that it is difficult for the electrical center of out-of-step condition to pass through the section in a manner to produce false tripping. The use of specially insulated relays is not necessary or desirable for a directional comparison scheme and may be avoided altogether by the use of three-winding neutralizing transformers.[1]

The speed and sensitivity of differential type and directional comparison type of pilot wire schemes are comparable and adequate. When all conditions such as backup requirements are taken into account there is practically no difference in the cost of the two schemes and the choice should then depend primarily upon the over-all reliability to be expected from each. The relay communication system is the heart of the pilot wire protective scheme[2] and the chief engineering disadvantage of the scheme is the hazard of the exposure of the pilot wires themselves.[3] A probability approach to the reliability question indicates, first, that pilot wire circuit trouble should be made practically independent of troubles on the transmission section protected. The use of leased facilities with proper protective equipment does much to accomplish this but brings about an increasing exposure to extraneous influences, both electrical and those brought about by maintenance personnel. The probability approach further shows that troubles due to extraneous sources which are not coincident with power system trouble on the protected section are of little concern provided that (*a*) the relay system is such that extraneous disturbances such as momentary short circuits, open circuits, connection to "test" or "ringing" circuits, will not result in tripping of circuit breakers (in the presence of merely load current) and (*b*) the pilot wire circuit is so supervised that disturbances will be detected immediately and the condition corrected before the relay system is called upon to operate. Over 30 circuit-years' experience with leased pilot wire systems under close surveillance[4] has demonstrated that the surest means of securing adequate over-all reliability is the use of continuous audible and visual supervision of the pilot wire circuit integrity, using current of approximately the same value as is used for operation. This system is even more advantageous where the terminal relay equipment is composed of positive directional elements and direct current relays with wide operating margins found in remote control systems instead of balanced measuring devices of a remote metering system. It is urged that the authors consider a radical increase in the supervisory current of their scheme and that the circuit of figure 8 be modified to include an indication of a grounded telephone protective carbon, permitting such ground to be cleared before a second ground develops, making the circuit inoperative.

I would like to ask the authors the following specific questions:

1. An advantage is claimed for phase angle characteristic of the relay which makes it less responsive to interference from capacitance current. What is the recommended maximum allowable circuit capacitance?
2. Figure 5 shows a maximum pilot wire loop resistance of 2,000 ohms. What is the recommended maximum allowable circuit resistance?
3. In the authors' opinion could the "blind spot" which may occur on rare occasions be avoided by the use of a phase balanced scheme instead of a mixing auto transformer with unequal phase ratios?
4. The authors mention the provision of an adjustable resistor to maintain the ratio differential characteristic of the relay with different pilot wire impedances. Do the authors consider temporary readjustments necessary if emergency rerouting of the pilot wire circuit should increase its resistance by 1,000 ohms or more?
5. It is indicated that as little as five volts loop induction may cause false relay operation. This is a distinct disadvantage compared with direct current schemes in which induced (loop) voltage must be high enough to flash across open contacts to cause trouble. Can this value be raised? What value of d-c voltage across the circuit will cause trouble? What value of standard telephone "ringing" voltage will cause false operation?

REFERENCES

1. NEUTRALIZING TRANSFORMER TO PROTECT POWER STATION COMMUNICATION, George, Honaman, Lockrow, and Schwartz. AIEE TRANSACTIONS, volume 55, 1936, pages 524–9.

2. Reference 1 of authors' paper—conclusion number 6.

3. Reference 7 of authors' paper—conclusion number 2.

4. Reference 6 of authors' paper.

G. B. Dodds (Duquesne Light Company, Pittsburgh, Pa.): The Duquesne Light Company of Pittsburgh, Pa., has been interested in, and has used pilot wire systems for the protection of transmission circuits since 1927. Needless to say, we consider the type of protection described in this paper and the type described in a previous article in the *Electric Journal*, November 1935, by Harder and Bostwick, a great improvement over the earlier types, although some of the earlier types are still in service. The Duquesne Light Company has been using since 1934 the same method described in the present paper of utilizing an autotransformer so that one relay and one pair of pilot circuits would give complete protection on a three phase transmission circuit. The relays used, however, were entirely different, and the currents in the pilot circuit were too large to utilize telephone pairs.

We have been using the HCB relay equipment since it became available in 1938, and have been well satisfied with its performance. We were somewhat concerned about the reliability of the telephone pairs, but to date have not had any interruptions to the telephone pairs used for protection, without having a warning beforehand that they would be taken out of service. There have been no failures of the pairs used for protection to date. The telephone pairs used are owned and maintained by our own company. It is recognized, however, that the telephone pairs are the weakest part of this protective system, and some form of backup protection is always provided.

While the high speed operation of the newer type relay systems is usually an advantage, there are some cases where it is a disadvantage. One of these is where it is desired to apply this type of protection to a circuit on which there is tapped load. In some of our cases, the tapped transformer capacity has been sufficiently large that the pilot relays would operate for low tension faults. In several of these instances, we have added overcurrent relays with very inverse time current characteristics. The contacts of these relays are in series with the contacts of the high speed pilot wire relays. By proper choice of current tap and time lever setting of the overcurrent relay, sufficient time can be obtained for faults on the low voltage side of tapped transformers to permit low tension breakers to open, and yet for high tension faults permit clearing in a relatively short time.

In some applications of pilot protection, the minimum fault current may be lower than the maximum load current. This creates a condition whereby an open pilot circuit will cause tripping from load. To prevent this, we have added in some cases undervoltage relays, the contacts of which are in series with the contacts of the pilot wire relays.

Another condition that is sometimes encountered is where the fault current from one end of a circuit protected by pilot wire is relatively small, while from the other end it is relatively large. This means that there is some possibility, with the breaker open at the end from which the large value of fault current is fed, that a fault might not be cleared by the small amount coming from the other end. To overcome this difficulty, we have in some instances installed an auxiliary relay at the substation supplying the large amount of fault current. This auxiliary relay is operated from auxiliary switches in the oil circuit breaker and is closed when the breaker is closed. The contacts of this relay are in the pilot wire circuit between the insulating transformer and the relay. With the breaker open, the minimum fault current that the pilot wire relay at the far end will recognize is cut in half.

T. G. LeClair (Commonwealth Edison Company, Chicago, Ill.): The authors of this paper have described a new pilot wire relay scheme which has the merit of simplicity and quick response. This paper is an indication of an increasing interest in the use of pilot wire relaying in general.

It has been a puzzle to me why a general acceptance of pilot wire relaying schemes has been so slow in this country. Carrier relaying has become quite popular for high speed relaying and yet carrier relaying inherently is more expensive for short transmission lines and is more complicated than pilot wire relaying for the given results.

In Chicago, pilot-wire relaying has been in common use for many years. One of the first references to pilot relaying on the Chicago system was given in a paper by Mr. R. F. Schuchardt in 1917. The diagram of this scheme was published on page 383 of the 1917 TRANSACTIONS, volume 36.

The major advantage of pilot wire relaying is that selectivity is maintained for practically any line arrangement whether one or more lines may be out of service or whether line connections are transferred from one bus to another. A further advantage of pilot wire relaying over carrier current relaying is the direct comparison of relay currents at the two ends of the line. This avoids the necessity for any complicated "out of step protection" to prevent faulty operation due to system instability.

The standard scheme of pilot wire relaying on the high voltage lines in Chicago is illustrated in figure 919 of the Supplement to the NELA Relay Handbook. This relay scheme, which has been in use for about 15 years, is so sensitive that it will operate on a short circuit current materially less than the normal full load current carried on a line. The only limit to the sensitivity of this scheme is the fact that the charging current of the line appears as a fault current to the relays. Therefore, the relay settings must be above the line charging current. The operating time of the relay at both ends of the line is identical regardless of whether the fault current is supplied from one end or from both ends.

The operating record for pilot wire relays on the Chicago system has been surprisingly near perfect. Pilot wire relays are very popular with the men who determine relay settings because they involve no complicated calculations of short circuit currents.

The method described by the authors for obtaining protection on between-phase and phase-to-ground faults with only two pilot wires has certain advantages. On a power system where an impedance is used in the neutral, this scheme makes it possible to have more sensitive protection for ground faults. A very similar method for obtaining this universal protection on two pilot wires is used in Chicago for those lines where more than two pilot conductors are not available. This scheme is also described in the Relay Handbook and illustrated as figure 920.

It is expected that there should be an increasing trend toward pilot wire relaying because this scheme is ideally suited to operation with modern high speed circuit breakers to insure stable operation with heavy transmission line loading.

L. F. Kennedy (General Electric Company, Schenectady, N. Y.): Messrs. Neher and McConnell should be congratulated for the development of this improved relay system. One outstanding feature, I believe, is deserving of more emphasis. I have reference to the fact that they have adapted a standard relay element, which has been proved through several years service to be highly reliable and stable in its operating characteristics, to a service which limits severely the amount of energy which may be applied to the relay circuit. Through the use of the product principle of operation they have made possible the use of such a proved element. While the proof of reliability of any relay system must finally come from the operating records, it does seem reasonable to expect that the results are bound to be successful when the preliminary design effort is directed toward overcoming the limitation, apparently imposed by pilot wires, of using extremely small operating forces. The possibility of pilot wire currents, due to external causes, producing unwanted relay operation seems to be decreased.

Lest someone might think from the mathematical appendices that settling the pilot relay is difficult, it is pertinent to outline how simple the actual process is. It is merely necessary to calculate the minimum ground fault current for a fault at each end with the adjacent breaker open and the positive and zero sequence currents at each end for a ground fault at the far end with both breakers closed. From these current values a few simple calculations give the items needed to use two curves that determine the tap settings required.

To the application engineer there is a lesson contained in the title of this paper announcing "An Improved Pilot Wire Relay" since it implies that pilot wire relaying is an accepted fact. Despite all that has been published in the past twelve years since Fitzgerald[1] announced the first carrier pilot system, it is surprising to find many who still feel that pilot protection is an ideal system but not a practical one. It is not my purpose to discuss the relative merits of carrier versus physical pilot wire systems, nor more specifically the merits of the various systems using physical pilot wires. I prefer rather to call attention to the fact that this is one more paper in a long series covering various methods whereby the equivalent of differential protection can be applied to lines. There is in this literature ample evidence that the methods of protection once considered ideal have for a long time been practical as well. The Relay Sub-Committee in a report prepared two years[2] ago cited operating experience showing the generally good record achieved by all forms of pilot relaying. Since the evidence shows that pilot relaying is past the experimental stage, then the main remaining reason for not using it despite its obvious operating advantages, is one of cost. In this connection it seems to be in order to point out the permanent value of this form of protection. Pilot protection of all kinds may be installed today not only with the assurance that system changes will not require relay system replacement, but also with the assurance that new developments will not make equipment already in service obsolete. This should not be construed as meaning that we have reached the ultimate goal in relay development. The presentation of this paper by Messrs. Neher and McConnell is evidence that development is still going on. What is meant is that new and improved systems of relaying as they are developed will not make necessary the replacement of equipment already in service and giving satisfactory performance. This feature of permanence has been an outstanding feature of the pilot systems of all types introduced since the relatively fast carrier systems of 1932.[3,4]

REFERENCES

1. A CARRIER CURRENT PILOT SYSTEM OF TRANSMISSION LINE PROTECTION, A. S. Fitzgerald. AIEE TRANSACTIONS, volume 147, January 1928, pages 22–9.

2. HIGH-SPEED RELAYING EXPERIENCE AND PRACTICE, AIEE Protective Devices Committee. AIEE TRANSACTIONS, volume 58, 1939 (November section) pages 588–91.

3. Carrier Current Relaying Proves Its Effectiveness, Philip Sporn and C. A. Muller. *Electrical World*, September 10, 1932.

4. Pilot Protection by Power Directional Relays Using Carrier Current, O. C. Traver, John Auchincloss and E. H. Bancker. *General Electric Review*, November 1932.

R. A. Larner (Texas Electric Service Company, Eastland, Tex.): The development of an improved pilot wire relay should be welcomed by operating relay engineers. Where the expense can be justified, a differential scheme of protection for transmission lines is highly desirable.

The later developments, including this one, have tended toward simplification of the problem of applying pilot wire protection. For example, it is now possible to use a single standard relay element operating only on current transformers at each terminal of the line section to protect against the ten different types of faults, usually considered, on a grounded neutral system. The single telephone pair used as pilot wires are operated at relatively low energy levels and provided with supervision equipment.

This development should increase the use of pilot wire relaying. In some instances it may be installed where carrier current protection would have been installed. Under ideal conditions, its performance should practically equal that of carrier current relaying. Carrier current protection is, of course, expected to be more universally accepted in a larger field.

The equipment described appears to have a minimum of contacts, auxiliary relays, interlocks, sealing contacts, etc. Operating experience soon teaches one to appreciate such simplifications, as well as the statement "It is often true that the simplest device also provides the most satisfactory performance." The mixing transformer which is used for reducing to a single electrical quantity from the three phase current transformer circuit, is a good example. It is simply a tapped autotransformer. While it does not provide exactly the same amount of current for similar faults on each phase, it does apparently furnish ample current for use with a pilot wire scheme. At the other extreme, as in the case of a fault near one end, which may have heavy generating capacity, protection to the relay is afforded by a saturating transformer.

It might be of interest to carry out the solution to show the effect of considerable load being carried within the protected section, which in turn might be in part of a loop fed by several plants. Taking as an example such a section on which is located a comparatively large grounding bank near one end of the line: The ground bank would furnish a major portion of the zero phase sequence current at or near its location. The result would be a very meager residual current from the current transformer for internal ground faults, and conversely, a very large residual current for external faults near by.

The authors are to be congratulated on the development of this relay, which evidently constitutes a step in the direction of the ideal relay scheme, including the possibility of separation at preferred points only in case of loss of synchronism. This paper should also be remembered as a valuable reference in a form which makes pilot wire relay information clear and easily found.

R. M. Smith (Westinghouse Electric and Manufacturing Company, Newark, N. J.): A relay of the type described in this paper is indeed an improvement over the older forms of a-c pilot protection using standard overcurrent, differential, or current balance relays. It is a fact that protection engineers throughout the world have recognized the merits of the a-c pilot scheme which compares a single-phase quantity of limited magnitude using only two wires. This statement is borne out by the fact that since the introduction of the type HCB relay in 1937, hundreds of installations have been made both in this country and in many foreign countries. It is quite apparent now that the industry needed this type of relay for the high-speed protection of relatively short lines and that no real solution had existed prior to that time.

We note from figure 6 that the variable-ratio operating characteristic has been incorporated in the design of the relay. This type of characteristic, rather than the constant ratio characteristic of the older forms of differential relays, appears to have some advantages. We found this to be the case in the type *CA*-6 relay for bus differential protection and the type *HA* generator differential relay, both of which were described before the Institute in recent years.

The authors are to be congratulated on their paper, particularly the mathematical treatise for the relay torque equations. However, it is hoped that a thorough understanding of these equations is not essential for the proper understanding and application of the relay.

C. L. Smith (Rochester Gas and Electric Corporation, Rochester, N. Y.): The relay system described by the authors would seem to have at least two advantages over any comparable system with which I am familiar.

1. It employs a single standard element which has given proof of its sensitivity, reliability, and rugged construction in many other applications.

2. The cost should be somewhat less than for previous systems which involved the co-ordination of several standard units or the design of special elements which were not readily applicable to other schemes of protection.

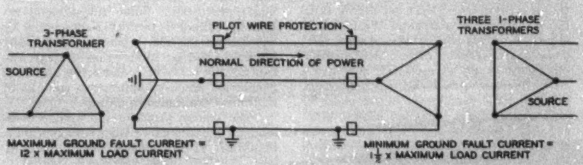
It would seem that this relay system is well adapted to the protection of cable systems where large short circuit currents are common. Assuming that this relay element and auto-transformer have thermal characteristics comparable to those of the power directional induction cylinder relay, we would expect rapid, correct operation over a wide range of fault currents. Our experience with relays utilizing cylinder elements has been quite satisfactory.

If time permits, I would like to ask Mr. McConnell to comment on the application of this relay system to a typical circuit such as shown by figure 1.

Given a maximum to minimum load current ratio of 4/1 and a variation in load power factor from 85 per cent lagging to 85 per cent leading, due to secondary capacitor installations, could tripping of the O.C.B. at the delta-delta bank be accomplished for both phase to phase and phase to ground faults?

The fourth paragraph, page 4 of this paper would seem to indicate that the O.C.B. at the delta-delta bank would not be tripped.

Figure 1


3-PHASE TRANSFORMER
SOURCE
PILOT WIRE PROTECTION
NORMAL DIRECTION OF POWER
THREE 1-PHASE TRANSFORMERS
SOURCE
MAXIMUM GROUND FAULT CURRENT = 12 X MAXIMUM LOAD CURRENT
MINIMUM GROUND FAULT CURRENT = 1½ X MAXIMUM LOAD CURRENT

M. A. Bostwick (Westinghouse Relay Engineer, Newark, N. J.): The authors, Messrs. Neher and McConnell, are to be complimented for their novel adaptation of standard elements to a relatively new problem. It appears that the new device should fill the need of a competitive unit for use in the recently revived field of alternating current pilot wire relaying.

There are three principal points that I believe should be commented upon to avoid any misunderstanding:

1. The author's statement, "Yet few applications of relay systems of this type have been made until recently" can be stated in more definite form. Actually, hundreds of type HCB alternating current pilot wire relays have been installed during the past three years. These are giving excellent service on pilot wires up to 30 miles in length.

2. The author's discussion of "blind spots" is quite thorough; however, it may be profitably reviewed. Why limit the definition of a "blind spot" to relay operating failures "at both ends of the line"? Is not high-speed clearing of faults simultaneously at both ends of a line the preferred object of high-speed relaying? Since the watt-meter element described in the paper requires local polarizing, current in each line terminal, it appears that a number of operating failures at one end of the line, or at best a number of relatively slow-speed, sequential operations may be expected. This condition is anticipated when the distribution factors are such that fault current is largely fed in through one line terminal or a low autotransformer output is obtained. The use of a permanently polarized relay element, as in the type HCB relay, would eliminate all of these slow-speed sequential relay operations and actual operating failures that result from a lack of polarizing current at one of the line terminals. Relays of this type (HCB) are capable of tripping both line terminals even though there is no current in one end of the line.

3. The "low autotransformer output," which causes operating failures or slow-speed tripping, may result from an actual lack of current or the peculiar manner in which the various components of current are added in the autotransformer. This is illustrated in the phase *C* to ground diagram of the author's figure 3. The sum of the positive and negative phase sequence current components is shown 180 degrees from the KI_0 vector; tending to cancel the transformer output. This condition may prove troublesome as the division of the various components of current between the line terminals is changed by switching operations on the transmission system. The magnitude of the single phase autotransformer output for phase to ground faults would be considerably improved if the negative sequence current component were omitted and the positive sequence phase "a" component of current were added in its true position, to the product of the ground

current and its multiplier K. By repeating the vector addition, illustrated in figure 3, with these modifications and using a K factor of at least four and with a maximum value of 16, the action of a positive plus zero phase sequence filter may be observed. Such a study will reveal a freedom from the tendency to cancel the single phase pilot wire signal under any conditions, when the multiplier K is set at 16. A tendency toward cancellation will be observed when a "*b*"–"*c*" to ground fault is encountered and K is reduced to 3.9 (the minimum multiplier used in the type HCB relay). With a positive plus zero sequence filter of this type it appears that the single phase output is much less subject to cancellation than the output of the autotransformer described. This is doubly apparent when the frequency with which phase to ground faults is encountered is compared to that of double phase to ground faults.

Both relay systems appear to have adequate zero sequence compensation to avoid these troubles when properly applied. However, use of the positive plus zero sequence filter simplifies the application as only two quantities need to be considered. It also serves to reduce the danger of encountering "blind spots" when applications are made without a thorough study of the system. Furthermore, use of permanent polarization of the relay element assures simultaneous circuit breaker operation on all faults, above minimum trip value, even though the relay at one end of the line is entirely de-energized.

E. E. George (Ebasco Services, Inc., New York, N. Y.): Messrs. Neher and McConnell present an interesting scheme of protection with some new and desirable features. Among these are the use of a polarizing coil to increase sensitivity, a resonant circuit for reducing the effect of extraneous or undesired current, and the use of an a-c relay to take better advantage of phase angle in reducing interference from cable charging current, induced current, etc.

The authors' comment on the effect of the open circuited or short circuited pilot wires may be misleading. The statement about an open circuited pilot wiring preventing tripping and a short circuited pilot wire causing tripping applies chiefly to the scheme developed by the authors. Another a-c scheme which circulates restraining current instead of operating current has the reverse difficulty in case of pilot wire failure. The direct current scheme operates still differently in case of pilot circuit trouble. In some schemes discrimination is lost if the pilot wire is out of service, and it is therefore probable that half of the operations would be incorrect.

This scheme is the second alternating current pilot wire scheme to be presented to the AIEE within the last year or two. Operating experience with a-c pilot wire seems to be generally satisfactory and most users report plans to proceed with additional installations. However, it should not be inferred that the d-c scheme has been superseded. In any a-c scheme one limitation should not be lost sight of. The use of a single relay for phase and ground protection is a compromise—not necessarily satisfactory for either purpose. With a thoroughly grounded transmission system, with a simple transmission tie line, with continuous power supply at each end, and with a circuit breaker at each end, any a-c scheme involves little sacrifice in either phase or ground protection. The a-c scheme is inherently difficult to apply if ground current is limited. In such cases independent phase and ground relays and the use of the direct current scheme may be preferable.

The directional comparison scheme is less likely to trip on load current in case of pilot wire failure, since in any direct current scheme the pilot wire current has no reaction on relay pickup.

Operating experience with a-c pilot wire protection seems to be much like experience with d-c schemes. Troubles experienced have been in the pilot wire circuit or its protection rather than in the relays. Many of us believe that many of the troubles with pilot wire protection could be avoided by close adherence to the principles and practices of circuit protection used by the Bell System and by other communication services. The use of *three winding* neutralizing transformers and of continuous supervision is frequently essential. The use of insulating transformers generally has been much less satisfactory in power company communication service than the use of neutralizing transformers. Satisfactory explanation of this experience may be difficult. Part of the trouble with insulating transformers has been due to the practice of grounding the primary winding. Such ground connections are preferably omitted and if made at all should be connected to earth remote from the power station. On open wire circuits the elimination of grounding is practically essential in preventing frequent blowing of fuses. It is still believed that most of the troubles on pilot wire circuits are due to rise of ground potential at the terminals rather than to induction or other interference along the circuits themselves.

This relay paper is well prepared from the standpoint of presenting to the prospective user information which he needs in considering the application to his own system. Theoretical discussions of relay torque, etc., are necessary but until recently many relay papers omitted other equally important information. No matter how sound may be the electrical design of the relay the prospective user is not likely to be greatly interested unless he finds that the relay has practical features that are satisfactory for some specific application he has in mind. Such features include operating time, burden, minimum pickup current, range of setting, general wiring diagram and simple description of method of operation.

The manufacturers have spent much time and money trying to get a lot of relay equipment into one small case. Pandora had some sad experience with too many things in one little box. Switchboard space is still cheap—and plenty of it should be provided on relay panels.

Most everyone agrees that relay engineers have done excellent work in protecting transmission lines, even where the protection is extremely difficult, but relay engineers have not had enough influence in preventing the design of transmission and distribution systems that are inherently incapable of adequate protection. Some transmission extensions cannot be protected adequately by any known method, regardless of the cost involved, and on some other systems the expenditure for relay protection is all out of proportion and even approaches the cost of additional transmission mileage.

Some system planners and budget committees are allergic to high voltage circuit breakers. It is difficult to understand why high voltage circuit breakers are sometimes not provided on long transmission lines costing $5,000 to $10,000 per mile and where the cost of breakers is relatively small compared to the over-all cost and to the increase in firm power supply which they provide.

There is an increasing need of information on relay protection that would outline the fundamental principles and limitations of relay protection in a manner *understandable and convincing* to electrical engineers who do not specialize on protection but who are responsible for the planning of most of the facilities a relay engineer has to protect.

L. L. Draper (Bureau of Power and Light, Los Angeles, California): There are two questions that I would like to ask the authors regarding the scheme of pilot wire protection outlined in this paper:

First, is there any objection to connecting the center point of the pilot wire winding on the insulating transformers to ground other than the effect this may have on the pilot wire supervisory schemes? And second, can this type of protection be successfully supplied to a three-terminal line in which the through short-circuit current may be equal in two of the terminals with their sum flowing in the third terminal?

In regard to the first question, if grounds can be applied at both ends of the pilot circuit then any induced potential in the pilot wires or difference in ground potential at the two ends will cause equal currents to flow in the same direction in the two pilot wires and return through the earth. These currents should not affect the relays but would cause the potential to be consumed in impedance drop around the loop instead of concentrated between the pilot wires and the cable sheath at the two ends of the pilot cable.

It is believed that if this plan of grounding the pilot circuit is feasible a great many existing communication cables could be used for pilot wires that would otherwise be unsuitable because the potential difference between the conductors and sheath would be sufficient to flash over the protective gaps and short-circuit the pilot wires.

E. L. Harder (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): In 1937 there was introduced into the relay art, "A Single Element High Speed Pilot Wire Relay," called the HCB, reference number 2 of the current paper. While a distinct field was envisioned for such a device at the time, it must be admitted that its widespread acceptance and application are beyond the most optimistic predictions. It is believed that the large number of installations of pilot wire relaying since the introduction of the HCB indicates not only a particularly successful design but also a distinct trend in the industry toward this form of protection. There evidently was a pent-up need for such an addition to the relay family. It is not surprising, therefore, to read of this paralleling development as described by Messrs. Neher and McConnell. It is encouraging to know that they feel, as we do, that high speed pilot wire relaying has come to stay.

Looking back over the course of this development it is difficult to mention any one application factor which more than others has led to the increased use of pilot wire protection. Some of the outstanding factors are as follows:

(a). Fast simple protection of the short interconnecting lines in metropolitan areas of utilities and power lines of industrials has made possible fast clearing, and clean-cut selectivity, and has avoided accumulating of relay times.

(b). The scheme is universal and does not require individual study of each particular line section, or accurate predetermination of currents or voltages.

(c). The economic upper limit of pilot wire protection, originally considered to be 10 to 15 miles, is found in many cases to be 40 to 50 miles, depending upon the circumstances. This is due partially to the existence of privately owned telephone lines for which the charge to relaying services is comparatively small and partly to the existence of certain high voltage cable circuits which do not lend themselves to the application of carrier current relaying. In other cases where a cable must be installed for communication purposes, the incremental cost of pilot wires for relaying is quite small.

As to the particular form of relay, the circulating current arrangement was adopted for the HCB after consideration of the various possibilities, including the balanced-voltage arrangement utilized by the present authors. ("Circulating Current" refers to the circulation of restraining current over the wires during through-fault conditions as compared with "Balanced Voltage" in which voltages at the two ends are balanced against each other over the pilot wires for the through-fault condition.) The "circulating current" arrangement used in the HCB is entirely free from the effects of switching transient which the authors indicate they have had to reduce by going to a resonant circuit.

Operation based on total fault current with the HCB is considered to be a desirable operating feature since it assures simultaneous operation at the two ends of the line and makes it possible to supply intermediate tapped loads right up nearly to the value of total internal fault current setting.

J. H. Neher and **A. J. McConnell:** The authors are somewhat overwhelmed by the extent of the discussion which this paper has evoked. In the limited space available, therefore, we must restrict our concluding remarks to certain general points which have elicited the most discussion and forego the answering of many of the specific questions which have been asked.

We are gratified to know that many of the discussers concur with us in the belief that pilot wire protection, and particularly that of the a-c type, now represents an accepted reliable method of transmission line protection. The hundreds of such relays which have been mentioned as having been placed in service during the past few years are mute evidence of this point.

As one discusser has pointed out, the procedure for setting the relay is as simple as the relay itself. It is only the equations describing the performance which appear to be complicated.

The application of any pilot wire system to a three-terminal line, to a line from which a considerable load is tapped, or to a line in which there is a large grounding bank, often presents insurmountable difficulties and this relay is not a panacea in this respect. Within limits, however, the relay may be applied in such applications. In regard to applications on lines terminated in a delta-connected bank, it will be evident that the relay at that end cannot respond to certain types of single-phase-to-ground faults, though the fault will be cleared by the operation of the relay at the other end.

As has been pointed out, there is no fundamental reason why relays of this type cannot be used with pilot wire distances greater than the figure of 2,000 ohms, which was given in the paper merely to represent the upper limit of normal short line application. For longer pilot wires the pick-up current of the relays will be increased roughly in proportion to the square root of the pilot wire resistance. There will also be an inherent phase angle shift due to either or both the pilot wire capacity and inductance if "loading" is employed. This may be compensated for by the use of a suitable network or, in some cases, by suitable changes of the relay constants or adjustments.

The stated value of 5 volt loop induction as affecting the relay performance is again a nominal value, and in most instances 15 volts will not cause false operation. However, even 5 volts is very high in telephone practice and no difficulty should be experienced in keeping below the figure. The magnitude of the induced voltage necessary to cause trouble can, of course, be raised at the expense of desensitizing the relay. D-c voltage in the pilot wire will not cause trouble, though it must be realized that there must not be a resultant current flow of sufficient magnitude to affect the operation of the insulating transformers. This may be prevented by introducing capacities into the pilot wire circuit. The introduction of "ringing" voltages will not affect the operation of the relays unless the frequency of the ringing voltage coincides with the power system frequency.

The matter of grounding of the pilot wire circuit depends largely upon the particular conditions at hand. In some instances, the midpoints of the pilot wire windings of the insulating transformers can be grounded. In general, however, such grounds are preferably omitted. If it is essential to reduce the voltages between the pilot wires and ground, the use of neutralizing transformers is believed to be the most satisfactory method.

In regard to pilot wire supervision, we feel that the monitoring system should be designed to best fit the particular conditions at hand and considerable latitude in respect to the circuit of figure 8 is permissible. The relay described in the paper lends itself readily to the use of neutralizing transformers. The d-c monitoring system would be applied in shunt at both ends through suitable filter reactors and the flow of d-c monitoring current is blocked out of the relays by the capacitors already incorporated in the relays.

The correlation of back-up protection with the fundamental protective scheme has been rightly stressed. We agree that in cases where a high speed directional-controlled back-up system is deemed to be necessary, the directional comparison system of pilot wire protection may be preferable despite the attendant complexity of auxiliary relays and contacts. We are more inclined to the belief, however, that where a simple a-c type of pilot wire protection can be expected to give reliable service, the complexity of the back-up protection can be correspondingly reduced, and often the directional feature can be omitted altogether.

Several discussers have indicated that they have obtained satisfactory results from the use of an autotransformer or equivalent circuits in order to obtain a simple electrical quantity for any type of fault. This and the method of vector rotation and addition (or phase-sequence components) are apparently the only two methods which have been used to accomplish this result. Admittedly, both systems leave something to be desired. One system is weakest against single-phase-to-ground faults and the other against two-phase-to-ground faults. Despite the greater frequency of occurrence of the former type of fault, which under certain conditions may cause sequential operation of the relays utilizing the autotransformer method, we feel that this is more than offset by the fact that under certain two-phase-to-ground fault conditions, neither of the relays may operate where the other method is employed. Also, the autotransformer method is considerably simpler and does not require the use of a supersensitive relay element.

It also seems to us that the questionable advantage of "permanent polarization," in order to trip the circuit breaker at the distant end of the line under exceptional conditions, is more than offset by the disadvantages entailed of incorrect operation by surges introduced into the pilot wire circuit, such as have been caused by the discharge of the capacitor associated with the monitoring circuit through accidental short-circuiting of the pilot wires. Polarization at power system frequency will prevent operation from such causes as well as from frequencies other than that of the power system.

The relative merits of the opposed voltage and circulating current systems have been discussed. This is a question long unsettled, and which goes back to the early days of pilot wire protection before these modern systems operative over telephone-type pilot wires were developed. Both systems have their advantages, and a choice between them is largely a matter of preference. Possibly future experience with the modern systems will determine the answer.

Displacement and Diffusion in Fluid-Flow Arc Extinction

Discussion and author's closure of paper 40-115 by Joseph Slepian, presented at the AIEE Pacific Coast convention, Los Angeles, Calif., August 27–30, 1940, and published in AIEE TRANSACTIONS, 1941 (April section) pages 162–7.

D. C. Prince (General Electric Company, Schenectady, N. Y.): In discussing this very interesting paper, agreement should be noted at the outset with the statement that after circuit-breaker performance has been calculated, such calculations should be supported by thorough-going tests. That is undoubtedly the basis upon which all good engineering rests, and the air-blast breaker is no exception. The remainder of the paper does not altogether follow this well-established engineering principle. The rest of the Introduction, section I, is devoted to the varied opinions of European investigators.

Section II is devoted to the discussion of an "ideal" fluid-flow nozzle. To the extent that such a nozzle can be made "ideal," one would not expect the circuit to be interrupted at all since there would be a thread along axis Z in the figure not influenced by the fluid flow. The fact that such breakers do operate is evidence that in practice these "ideal" conditions do not exist, and such a departure from the "ideal" can be easily verified by suspending the thread along the axis Z. During the discharge of air, the thread will be found to be in violent agitation, thus bearing conclusive evidence that the "ideal" is not being reached or even approached. There is no evidence that if the "ideal" were realized, it would bear any given relation to the interrupting properties of the design.

In section III, Dr. Slepian goes on with his discussion but, as before, they are not calculations leading to a numerical result or reports of tests indicating that the calculations were correct.

In section IV, second paragraph, we find the statement that, "Photographic records show that the thickness of this column is too great for simple diffusion or thermal conductivity to account for the high interrupting capacity actually found." Apparently, in following his diffusion theory, a dilemma has developed which has to be explained away. The author goes on to indicate that the boundary layer between the splitters and the air flow would be expected by theory to be of the order of "a few thousandths or a hundredth of a centimeter thick." Such dimensions would be far too small to be observable in the photographs, and, further, if the boundary layer is of such dimensions, it would in no way interfere with the operation of the circuit breaker on the displacement theory.

Later on, in this same section, Dr. Slepian speculates that gas projected from a gas-forming wall will widen the boundary layer enormously. Even if this should happen, and there is no evidence of its happening in the present paper, it would not appear to have any special bearing on the conclusions as to whether the arc is interrupted by displacement or diffusion.

In section V, Dr. Slepian calls attention to the fact that there is a trail of luminous gas instead of a clean break shown in the Biermanns' photographs. This trail is doubtless evidence that with the symmetry which is shown in figure 1, there is a tendency for hot gases to linger along the axis of the nozzles, but again this has no special bearing on the point at issue.

Later, in referring to figure 6, the author attributes to a certain 2-inch length of arc responsibility for the interruption. He holds this belief in spite of the photographic record. The first break in the arc in the photographs referred to occurred at the edges of the splitters. It would not seem to be important which parts of the luminous gases were first blown away thereafter.

Section VI appears to deal entirely with an endeavor to show that the limitations of the simple diffusion theory recognized in section IV might be removed by assumptions allowing a hundredfold increase in the thermal effects. The author has, therefore, overcome the rather serious shortcoming that he recognizes in his own theory, but even that does not appear to be evidence why his theory should be accepted in place of the displacement theory against which he has shown no evidence.

It is not as if there were no straightforward criteria by which it is possible to determine whether interruption is by displacement or diffusion. The process of diffusion is one in which the arc products are cooled and deionized. Such a cooling and deionizing operation can be measured by its effect on arc drop, that is, anything that is done to intensify cooling and diffusion will also increase the amount of arc drop, and that can be observed on the oscillograph. The displacement, on the other hand, does not operate until current zero, and therefore has no inherent relation to arc drop. Careful analyses show that air-blast circuit-breaker performance is substantially independent of arc-drop properties.

Another method of observing the relative importance of diffusion versus displacement, would be to make a structure which would promote diffusion. Such a structure might consist of a large number of barriers transverse to the arc, which would tend to break up the arc, produce eddies in the air blast, and markedly increase diffusion effects. It may be observed that although such structures are in use in low voltage, magnetic blowout circuit breakers, they are not present in the transverse air-blast circuit breakers of the companies producing such breakers. Diffusion effects would be accentuated were the splitters indicated in figures 2 and 3 full of holes. It will be observed that this is not done in the air-blast breakers now produced.

On the whole, it appears that the weight of evidence favors the displacement theory as developed by calculations and supported where necessary by critical tests. Under the circumstances, the fact that the displacement theory is also simple should not be held against it.

W. F. Skeats (General Electric Company, Philadelphia, Pa.): Dr. Slepian is to be commended for attempting to set on paper a critical comparison of the displacement and diffusion theories of arc extinction. He has shown how such considerations as (1) the difference in density between the hot gases of the arc and the relatively cool gas surrounding it, and (2) the emission of gas from the barriers enhance the diffusive effects in the region of the arc to a considerable extent. These same considerations, however, have aspects in which they strongly reinforce the displacement theory, and in a critical comparison these aspects should be given their full weight.

One of these aspects is well brought out in connection with the flow in the double nozzle of figure 1 of the paper. Dr. Slepian quite correctly points out elsewhere in the paper that the gases of the arc proper will have a density less than one twentieth that of the surrounding cool gas. This is a greater discrepancy than exists between water and mercury. In this situation, the heavier fluid, entering the nozzle with radially inward flow, must obviously be expected, on account of its greater inertia, to tend to maintain this direction of flow. The lighter fluid would experience the fast changes in velocity, moving very rapidly in an axial direction away from the central point O so as to make room for the influx of the heavier fluid. Thus a break in the continuity of the lighter fluid may very well be expected notwithstanding the contrary indications of calculations based on the assumption of equal densities.

A similar squeezing out process is to be expected in the region of the barrier tips when the cross blast is used. It is in this connection that Dr. Slepian points out the wide disparity in velocities, but he does not continue to the logical consequence that the fast-flowing hot gas will be rapidly scavenged from the region of the barrier tip. The evolution of gas from the barrier itself also plays an important part in interruption by cross blast. It seems manifest that this gas must push the arc away from immediate contact with the barrier and interpose a layer of gas between. This gas is largely hydrogen and therefore has an extremely low viscosity. Thus it will tend to form the thin boundary layer which, in Dr. Slepian's own words, ". . . would not seriously change the conclusions of the displacement theory."

In section V, Dr. Slepian refers to photographs taken from a high-speed moving picture record presented by Prince, Henley, and Rankin, stating that he believes these include the moment of circuit interruption on the test in question. In connection with this record, however, the desirability of having a definite indication was appreciated, and this was provided by placing in the field of the camera an auxiliary spark gap which was broken down by the recovery voltage. A record of its breakdown was carried on the cathode-ray oscillograph along with a record of the recovery voltage itself, so that the interval between current zero and gap breakdown could be accurately determined. Counting back the proper number of frames from gap breakdown then gives the frame at which current zero occurred. Unfortunately this was four frames after the last of those selected by Dr. Slepian. Thus conclusions based on Dr. Slepian's frames are open to considerable revision.

These considerations, perhaps, do not prove the displacement theory, but they do effectively refute those conclusions of the paper which state that the displacement theory is impossible or is contradicted by motion pictures.

Hugh L. Dryden (National Bureau of Standards, Washington, D. C.): Under the conditions prevailing in gas-blast circuit breakers, there is no doubt that the transport of matter, heat, and ions by turbulent diffusion is much greater than by molecular diffusion. The writer has given elsewhere[1] a review of recent developments in the experimental and theoretical aspects of turbulence in relation to the process of diffusion, which contains a more complete discussion of some of the points mentioned below.

No experimental data are available as to the values of intensity u' and scale L to be expected in the presence of the high speeds and large temperature differences occurring in the arc. The values assumed by Dr. Slepian are not unreasonable. For isotropic turbulence decaying in an air stream of constant speed the changes of L and u'

with time are given approximately by the relations

$$L\frac{d\left(\frac{1}{u'}\right)}{dt}=0.2$$

and

$$\frac{1}{u'}\frac{dL}{dt}=0.2$$

For $L=0.1$ centimeter and $u'=3\times10^4$ centimeters per second, the decrease in u' and increase in L are about six per cent in the first microsecond. Hence the initial large values would remain fairly large for an appreciable time even in the absence of further generation of turbulence.

In any quantitative estimate of diffusion, the "mean free time," which is of the order of magnitude of L/u', plays an important role. For $L=0.1$ centimeter and $u'=3\times10^4$ centimeters per second, the mean free time is of the order of three microseconds. For time intervals small in comparison with the mean free time, the diffusing quantity spreads at a uniform speed proportional to the intensity of the turbulence and the diffusion is not dependent on the scale. For times large in comparison with the mean free time, the diffusing quantity spreads in accordance with the usual diffusion equation with a coefficient of diffusion equal to l_1u', where l_1 is a length defined by $\int_0^\infty R_t dt$, R_t being the correlation coefficient between the speed of any particle at any instant and the speed of the same particle after a time interval t. For turbulence behind a grid, G. I. Taylor found that l_1 was approximately one-half L.

If the intensity of the turbulence in existing commercial devices is actually of the order of the acoustic velocity, there is not much hope for improvement of performance by increasing the turbulence. It might be expected that the device shown in figure 4 would be considerably improved by admitting the air through a series of jets or through an annular nozzle directed from the side toward a point just above the lower electrode. The impingement of the streams of air from different directions should produce an intensity of turbulence much greater than that produced by the arrangement illustrated in figure 4.

REFERENCE

1. TURBULENCE AND DIFFUSION, *Industrial and Engineering Chemistry*, volume 31, 1939, page 416.

J. Slepian: The discussion from so eminent an authority on aerodynamics as Dr. Dryden is certainly most welcome, since the questions at issue in my paper are essentially aerodynamical. Dr. Dryden's published papers have been wonderfully helpful to me in trying to understand in more detail how rapid flow in the gas may aid in arc extinction, and I must express here my appreciation of them. It is most reassuring to me that Dr. Dryden finds nothing obviously wrong in my paper, and believes that the numerical values I used are not unreasonable.

I am highly gratified that Mr. Prince agrees with the analysis in section II of my paper. This is really sensational news! Pure streaming in a frictionless fluid *cannot* interrupt the continuity of an arc! We are now all agreed on this. This is because (second paragraph of Mr. Prince's discussion) "there would be a thread along axis Z in the figure not influenced by the fluid flow." The arc interruption does take place in an actual fluid having friction because, "the thread will be found to be in violent agitation." But this is only another way of describing the turbulence which I also emphasize.

Now that we are in agreement on this very fundamental point, no doubt Mr. Prince will want to modify his simple equation for a-c arc extinction in a flowing fluid, since that equation used only the velocity of pure streaming, which we now agree can play only an indirect part in interrupting the arc. No doubt, Mr. Prince will want to use the intensity of the "violent agitation" of the arc column in his modified equation. Since this intensity is not a simple function of the streaming velocity, but depends on the viscosity of the fluid, and the shape and character of the walls of the channel in which the fluid is accelerated to its high streaming velocity, I am afraid that the modified equation will no longer be simple. I am very sorry for that. Mr. Skeats even suggests that the ratio of the densities of the arc gas and the surrounding fluid will need to be considered, which means that the properties of the arc itself will have to enter into the modified equation. Again that is too bad. The simple equation needed to know nothing of the nature of the arc. "The arc with its varying gas composition, varying pressure, uncertain temperature, varying conductivity, unknown state of ionization, etc., is merely swept away!"

Mr. Prince again confirms his basic agreement with me in the sixth paragraph of his discussion where he states that "with the symmetry which is shown in figure 2, there is a tendency for hot gases to linger along the axis of the nozzles." No doubt, then, Mr. Prince will now want to modify figure 2*a* in "Circuit Breakers for Boulder Dam Lines," ELECTRICAL ENGINEERING, April 1935, page 368, and figure 13 "The Oil-Blast Circuit Breaker," AIEE TRANSACTIONS volume 50, 1931, page 511, to take account of this tendency for hot gases to linger along an axis of symmetry. Also, I am sure he will want to introduce this lingering tendency into his modified simple equation.

The harmony of the agreement between Messrs. Prince, Skeats, and myself is, however, broken when we try to pick out the frames including the moment of arc extinction in those most remarkable high-speed pictures shown in "The Cross-Air-Blast Circuit Breaker," D. C. Prince, J. A. Henley, and W. K. Rankin, AIEE TRANSACTIONS, volume 59, 1940, page 510. In paragraph seven of his discussion, Mr. Prince seems to choose the frame which first shows a break in the arc at the edge of a splitter. This would be frame 6. In my discussion of these pictures, in the present paper I have chosen frames 17, 18, and 19. Skeats, on the basis of oscillographic evidence not given in the paper, selects frame 23.

I was much puzzled by the break in the arc continuity first shown in frame 6, because it occurred while the arc was quite intense, and because it did not grow in length in the succeeding frames, and because it closed again in frame 11, and reopened in frame 13, again not changing in length in succeeding frames. I suspected that this break was not real, but only apparent, and perhaps due to parallax of the various pinholes of the camera with respect to the barrier edge. I wrote to Mr. Prince then, asking about this, and received a reply confirming my interpretation. I therefore chose frame 18 as the first which showed any authentic break in the uniformity of the arc.

Mr. Skeats, apparently, has a cathode-ray oscillogram which he has not submitted along with his discussion, in which one point is correlated with frame 30. By counting back in time on the oscillogram, Mr. Skeats picks frame 23 as showing the moment of arc extinction. However, the oscillogram is undoubtedly one showing the voltage across the arc space, and not the current through the arc. There is a matter of interpretation, however, in picking the moment of substantially zero arc current from the voltage oscillogram. If this was done as is suggested in figure 5 of "The Cross-Air-Blast Breaker," I would not agree with it, but would choose a somewhat earlier time. It is very probable, then, that if I had Skeats' cathode-ray oscillogram before me I might still choose frame 18 as the one showing arc extinction.

Possibilities and Methods of Extending a Carrier Current Relay Channel to Other Uses

Discussion of paper 40-129 by R. M. Smith and S. L. Goldsborough, presented at the AIEE Pacific Coast convention, Los Angeles, Calif., August 27–30, 1940, and published in AIEE TRANSACTIONS, 1941 (January section) pages 23–7.

F. M. Rives (nonmember; General Electric Company, Philadelphia, Pa.): This paper is devoted entirely to a subject which has often been mentioned in the past, but which has not been given the publicity that it perhaps deserved. The fact has long been known that some pilot relay channels, and in particular, carrier relay chanels, are standing idle all but a small fraction of the total time, and that there is, therefore, a possibility of obtaining multiple service from the channel.[1] The fact that carrier channels are particularly easy to adapt to multiple uses has been previously mentioned both in American and European papers.[2,3,4] Operating practice has made general use of this possibility, to signal for checking purposes and for communication when making routine tests. More limited use has been made of carrier channels for the purposes described in detail by Messrs. Smith and Goldsborough.

It is gratifying, therefore, to see a paper dealing exclusively with the possible extensions of the relay channel, as in this way more direct attention is brought to this possibility. The economic advantage of being able to provide several functions for little more expense than would be required for any one alone certainly would seem to justify the authors' suggestion that the expense be evenly distributed between the several services involved.

In order to obtain the maximum dependability and at the same time a reasonable cost, carrier-current equipment designed principally for relaying must necessarily be fairly simple and therefore limited in many respects. Although, as indicated by the authors, there are a number of additional functions that can be supplied by a simple keyed carrier-relay channel, additional functions or channels obtained by tone modulating the carrier may involve requirements and complications that cannot be met with the conventional relay-type equipment.

One tone-modulated channel may possibly be provided with only the addition of a tone-generating source at the transmitting end and a suitable demodulator, filter, and relay arrangement at the receiving end. When more than one tone-modulated channel is desired, or when tone-modulated channels are desired in addition to a voice-communication channel, it is rather questionable whether conventional relay-type carrier equipment can be used. Complex arrangements for shifting the voice frequency channel, or "cutting holes" in the voice channel would appear to depart considerably from the limited facilities provided in the relay-type equipment.

One of the principal limitations in all relay equipment is in the small modulation range provided. Using control-grid modulation, high-level modulation can only be obtained by a large reduction in normal or unmodulated carrier output. Some improvement in this respect may be obtained by using screen or plate modulation, but for all practical purposes the relay equipment must be considered essentially as designed for low-level modulation. While fairly satisfactory emergency-type telephone communication can be provided with low-level or negative modulation, tone-modulation channels require good modulation characteristics, and if elaborate filtering is to be avoided, each tone channel has to be set up to modulate the carrier at a minimum of 15 or 20 per cent. For several simultaneous tone channels, or for a combination of tone and voice channels, a modulation-percentage range must be provided for the sum of the peak modulation percentages used on each channel.

Another important limitation in the conventional relay equipment is in the lack of an adequate automatic volume control in the receiver which is required in multi-channel tone-modulated systems to deliver to the de-modulating and filtering equipment essentially constant signal levels with varying high tension line attenuation.

The most practical means for overcoming these limitations and providing highly reliable channels by tone modulation essentially requires high-grade communication-type carrier equipment. As an example of what can be done in this line, a recent application that is being installed on a new mid-western 220-kv circuit provides a separate carrier channel in each direction with eight tone channels on one carrier and six on the other.

One important method of providing an additional channel with conventional relay-type carrier equipment was not mentioned by the authors. This is the carrier-shift scheme in which the transmitter frequency at one end is shifted a small amount to actuate a relay associated with a special heterodyne receiver at the other station. A very successful installation of this nature has been in operation in Canada for some time in which the shifted carrier signal is used to provide remote tripping of breakers. This feature is provided in addition to the usual pilot relaying and telephone communication. An interesting feature of this arrangement is that the remote tripping has complete preference over all other functions and the tripping signal can be originated or received at any station regardless of whether or not the channel is being used at that point for telephone or telemetering.

The authors have indicated that in some cases, due to system conditions, the relays may be unable to take control of the channel because at the point under consideration no relay action would result. This writer is familiar with an installation of this type which has now been operated successfully for several years with joint relaying and "point to point" telephone where the carrier transmitter was under the joint control of the main carrier relays and additional simple, undervoltage relays. Certainly if there is no possible infeed to a fault under certain conditions the fact that a fault does exist can always be indicated by the voltage failure.

From the foregoing points, it may be concluded that when it is desirable to perform several simultaneous functions or set up several channels over a single carrier circuit it is usually more practical to use communication-type equipment than to load too much complication on conventional relay equipment that is fundamentally designed for much simpler, but still exacting applications.

References

1. The Use of Communication Facilities in Transmission Line Relaying, J. H. Neher. Electrical Engineering, June 1933, pages 595–606.

2. Schnellabschaltung Beim Selektivschutz, H. Neugebauer. *E.T.Z.*, volume 55, February 1934, pages 181–4.

3. Fernmessanlagen in ausgedehnten Landesnetzen, K. Riedel. *VDE Fachberichte*, 1935, pages 160–3.

4. A Faster Carrier-Pilot Relay System, O. C. Travers and E. H. Bancker. Electrical Engineering, June 1936, pages 688–96.

Capacitors and Circuit Performance

Discussion of paper 40-126 by J. W. Butler, presented at the AIEE Pacific Coast convention, Los Angeles, Calif., August 27–30, 1940, and published in AIEE TRANSACTIONS, 1941, pages 408–15.

C. E. Arvidson (The Commonwealth and Southern Corporation, Jackson, Mich.): The timeliness of this paper is evident from the claims made by some engineers as to how they have improved the voltage regulation on circuits by the installation of uncontrolled shunt capacitors. It is encouraging to note that Mr. Butler brought out the fact that such installations do not reduce voltage regulation—they merely raise the voltage level. This review of the fundamental principles involved in the application of shunt and series capacitors should be helpful in bringing about a better understanding of these problems.

W. K. Boice (General Electric Company, Schenectady, N. Y.): This paper should be of particular interest to those engineers who wish to acquire a good understanding of the application of capacitors. The several accompanying figures illustrate many important points. For actual application work, engineers will find the fundamental diagrams of figures 5 and 11 most useful. This fundamental approach is applicable to a variety of conditions which could not be covered by any of the numerous graphs and tables which have been prepared on the subject of capacitor application.

Engineers interested in capacitor application may desire to refer to the following papers which have not been included in the author's bibliography:

1. Analysis of Series Capacitor Application Problems, J. W. Butler and C. Concordia. AIEE Transactions, volume 56, 1937, pages 975–88.

2. Shunt Capacitor Application Problems, J. W. Butler. *General Electric Review*, May 1940.

3. Analysis of Factors Which Influence the Application, Operation, and Design of Shunt-Capacitor Equipment Switched in Large Banks, J. W. Butler. AIEE Transactions, volume 59, 1940, pages 795–800.

4. Why Not Switch Capacitors With the Load? J. W. Butler and E. B. Pope. *General Electric Review*, August 1938.

A Double-Break Oil- or Gas-Blast Power Fuse

Discussion and author's closure of paper 40-134 by G. Leslie Hill, presented at the AIEE Pacific Coast convention, Los Angeles, Calif., August 27–30, 1940, and published in AIEE TRANSACTIONS, 1941, pages 402–07.

A. C. Schwager (Pacific Electric Manufacturing Corporation, San Francisco, Calif.): Mr. Hill has to be congratulated for the successful development of a novel type fuse. Arc interruption in oil circuit breakers and fuses follow identical laws and since the series-break oil-blast breaker has shown an improvement over the conventional explosion chamber breaker, it is but natural to expect the series-break fuse to have advantages over the conventional single-break oil—or expulsion fuse.

Mr. Hill's design has the advantage that the interrupting arc is initiated only after the first arc has accumulated a sufficient volume of gas, independent of the current flowing through the fuse. Uniform interrupting performance is thereby guaranteed over the entire current range, including the smallest currents.

With the present trend in indoor equipment away from oil, the double-break oil-immersed fuse described might be considered a special application. The dry-type fuse however appears to have definite possibilities. Its success will of course depend on commercial considerations. The design seems to be considerably more expensive than that of the conventional expulsion fuse with universal fuse link which is capable of interrupting currents of several thousand amperes.

If the range of the fuse described by Mr. Hill could be extended to include currents in the order of 10,000 or 20,000 amperes, it might replace some of the present designs of powerhouse fuses.

G. Leslie Hill: A. C. Schwager brings out that arc interruption in oil circuit breakers and fuses follow identical laws. The author agrees that there is a corollary existing between the two types of circuit-interrupting equipment. This is especially true when one considers a comparison between the modern breakers where a tank of oil or air is maintained under pressure for utilization in circuit interruption, and the double-break fuse. In the case of the circuit breakers, the oil or air to be used is stored under pressure for use when needed. The energy used for building up the pressure in the tanks is derived from any convenient source, whereas the energy for storing up the pressure in the chamber of the double-break fuse is derived from the fault current in the circuit to be interrupted.

In regard to the use of the dry-type fuse indoors, it must be remembered that the use of a power fuse indoors (either dry or liquid type) calls for some means of cooling or deionizing the expelled gases during circuit interruption. A condenser was developed for use with the double-break fuse during the original experimental work on this device. It was applied to the fuse when filled with a liquid or a dry arc-extinguishing material. It was also applied to the plain expulsion fuse. This condenser, with modifications in the design is also used on the spring-actuated, glass-tube, liquid-filled fuses and the dry-type boric acid, spring-actuated, fuse devices, manufactured by two prominent fuse manufacturers in the United States.

It was pointed out in the text that the double-break fuse was in an intermediate class as to costs, circuit-interrupting duty, etc. It is only obvious that the design is more expensive than the conventional plain expulsion fuse with a universal fuse link, but there are certain disadvantages such as the hot expelled parts and the long arcs, inherent in the plain expulsion fuse, that some operating men do not desire, therefore it is necessary to pay a small additional amount for a higher class device. I wish to point out very specifically that the universal fuse link cannot be used successfully in a *glass expulsion tube.*

The author is positive the range of current interruption of the double-break dry-type fuse may be extended to equal the oil-filled fuse or even extended to a higher value by changing the mechanical structure of the cartridge and the fuse tube. These changes would increase the cost and when it is considered that the present interrupting capacity meets approximately 90 to 95 per cent of our requirements, it is very questionable that any changes increasing the cost would be warranted. Since the double-break fuse was developed, there has been placed on the market, a spring-actuated, cartridge-refill, dry-type fuse that is built especially to cover the higher bracket of power requirement such as station service. These devices have an organic fuse tube, are larger and stronger and are built for heavy duty service and obviously cost considerably more than the fuses described in this paper.

In regard to a verbal question about what arc-extinguishing substance is used in the dry-type fuse: At the present time we are using granular dolomite (marble) which is obtained at a convenient ore mill. Its chemical analysis is approximately $CaCO_3$ 60 per cent and $MgCO_3$ 40 per cent. When the double-break fuse was first developed, various dry-type arc-extinguishing substances were tried out such as baking soda, baking powder, magnesium carbonate, boric acid, and marble, with chlorate of potassium and gun powder mixed, at certain times, in the above substances.

In the liquid-filled fuse, oil and carbon tetrachloride were used and more recently water has been experimented with. Water has been substituted, for the dry granular filler, in the 7-inch and 14-inch refill cartridges, and very interesting results have been obtained. The following table gives some comparative arc times obtained during tests using the 14-inch, 60-kv, refill cartridge.

	Dry Granular Filler			Water Filled
Current Interrupted RMS Amps.	Total Arc Time (Cycles)	Primary Arc Time (Cycles)	Secondary Arc Time (Cycle)	Total Arc Time* (Cycle)
1,700	2.0	1.10	0.9	0.55
3,300	1.4	0.80	0.60	0.50
4,300	1.0	0.50	0.50	0.40

*The primary and secondary arc times could not be distinguished on the oscillograms.

Power-Factor Testing of Transformer Insulation

Discussion of paper 40-130 by J. B. Hodtum, presented at the AIEE Pacific Coast convention, Los Angeles, Calif., August 27–30, 1940, and published in AIEE TRANSACTIONS, 1941 (January section) pages 28–30.

H. S. Endicott (General Electric Company, Pittsfield, Mass.): Although much time has been spent on the problem no single test has yet been devised which, when applied to the terminals of a transformer, will indicate unmistakably the condition of the insulation. The correct analysis of test data still requires a large amount of personal judgment and experience with insulation both in laboratory samples and in factory apparatus. Any new methods of analysis, therefore, warrant serious consideration.

Capacity or charging current readings sometimes have been used as an indication of the condition of insulation. They have the disadvantage that comparison with previous readings is necessary before any conclusions may be drawn. A single power-factor reading on the other hand has considerable meaning (at least in the light of experience). When a series of readings is taken, as in drying out a transformer by short-circuit heat run, capacity may give exactly the same indication as power factor. In the case of a drying run recently made by the writer the curve of capacity as a function of time of drying was the same shape as the power factor curve.

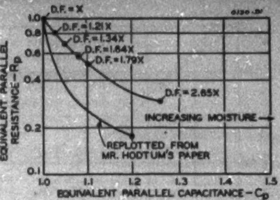


Figure 1. Relation between R_p and C_p for constant temperature and increasing moisture content

Curves for this transformer, in both the wet and dry condition, have been plotted in the same manner as those presented by Mr. Hodtum, except that the equivalent parallel resistance, R_p, is plotted on a logarithmic scale. Mr. Hodtum's curve is re-drawn for comparison. This shows that the sharp break in the curve of R_p against C_p is due partly to the use of a linear scale for R_p in Mr. Hodtum's paper. Some flattening of the curve is to be expected since it has been found that the first small amount of water taken up by dry cellulose is held so tightly that its dielectric constant may be as low as 20. Otherwise the curves are quite similar to Mr. Hodtum's although the moisture content was only in the range of 10 to 15 per cent. It is remarkable that Mr. Hodtum obtained such a small change in dissipation factor when his insulation had taken up 40 per cent water.

It would be desirable to have more data covering transformers with various types of

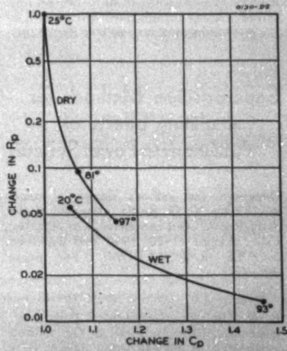


Figure 2. Relation between R_p and C_p for constant moisture content and increasing temperature

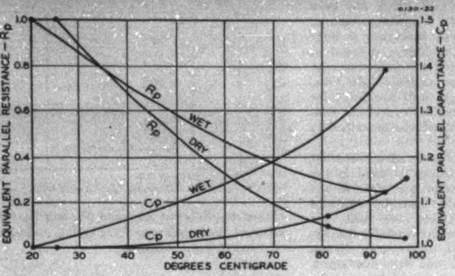


Figure 3. Change of R_p and C_p with temperature

insulation and with known moisture contents. Such data probably already exist in the files of some of the operating companies and we may look forward to having them presented.

A. H. Foley (General Electric Company, Pittsfield, Mass.): It is of interest to note that Mr. Hodtum makes frequent reference to the equivalent shunt values of capacitance and resistance in dielectrics. While our experience has shown equivalent shunt values to be of no great significance at low dissipation factors, they are more representative of the dielectric at high dissipation factors.

As pointed out by Mr. Hodtum, capacitance bridges, in general, express dielectric characteristics in terms of equivalent series capacitances and resistances. In line with this it might be of interest to some to know that the General Electric Company developed a capacitance bridge in 1934 which expresses dielectric characteristics in terms of equivalent shunt values directly. Several such bridges have been in constant use in our factory since that time and have proved the superiority of the bridge method for measurement of dielectric characteristics. For the benefit of those interested in the subject an article is being prepared which describes the network in these bridges and shows how the arrangement can be used in the measurement of transformer insulations.

Superposition Methods for Calculating Effects of Additions to Power Systems

Discussion and author's closure of paper 40-133 by V. G. Rettig, presented at the AIEE Pacific Coast convention, Los Angeles, Calif., August 27–30, 1940, and published in AIEE TRANSACTIONS, 1941, pages 396–401.

E. W. Kimbark (Northwestern University, Evanston, Ill.): In view of the limited availability of a-c calculating boards it is not surprising that attention should be given to the improvement of "longhand" methods of solving networks. Although such methods can never compete economically against calculating boards for carrying out comprehensive studies on complicated networks, it is equally true that the calculating board can never supplant longhand methods for simple problems or simple networks. The present paper helps to extend the field of useful application of longhand solutions to more complicated networks than heretofore.

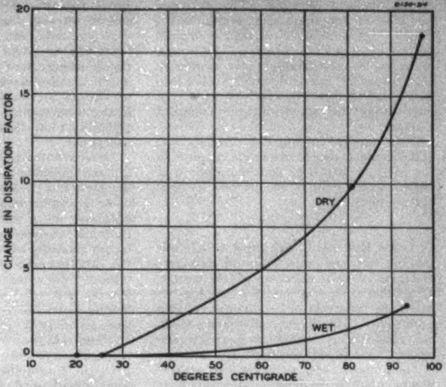


Figure 4. Change of dissipation factor with temperature

The principal features of the method of network solution advocated in the paper may be summarized as follows:

1. The assumption of constant-current loads in preference to constant-impedance loads.

(*a*). Because it more correctly represents power-system loads, and

(*b*). Because it is easier to calculate.

2. The superposition of currents and voltages of

(*a*). An original condition, as determined from readings of station instruments, and

(*b*). Corrections obtained by an algebraic solution of the network of lines.

The present paper by Rettig and the paper on successive approximations by Smith (listed in the bibliography[3]) when taken together make available a practical system for the longhand solution of almost any power network. Smith assumes constant-impedance loads, and in this respect his method is inferior to that of the present paper. But I wish to point out that Smith's method of successive approximations can supplement the method of the present paper in three ways: (1) in determining the currents and voltages of the original operating condition, if data on line impedances and loads are available, even though the necessary instrument readings cannot be obtained; (2) in calculating the correction currents and voltages if the network of lines is at all complicated; (3) in accurately representing any load characteristic, whether constant kilovolt-amperes or otherwise, if the assumption of constant-current loads is not accurate enough, by successive approximations in which constant-load currents are assumed during each step.

Edith Clarke (General Electric Company, Schenectady, N. Y.): It is generally recognized that the calculation of current and voltage distribution in power-system networks is a laborious task. For cases where new loads or circuits are added to an existing network, the superposition methods given by Mr. Rettig simplify the problem, in that calculations are made of *changes* in values of currents and voltages from known initial values and not of the total currents and voltages. The assumptions made, the degree of precision obtainable, and the limitations of the method are clearly stated, thus making the method readily available for problems in which the degree of precision obtainable is adequate.

However, solution of problems of the type given in the paper can be obtained to a high degree of precision on an a-c network analyzer, without the necessity of approximations or simplifying assumptions. For this reason the a-c network analyzer when available will usually be preferred. To those who do not have access to an a-c network analyzer, the methods given in the paper will be of especial interest and value; moreover, they can also be used to advantage for estimating purposes by those who do have such access.

A. A. Kroneberg (Southern California Edison Company, Ltd., Los Angeles, Calif.): Problems of voltage regulation on complex electrical networks have always troubled engineers confronted by them. There is no lack of theoretical material in textbooks and periodicals on network solutions. However, the scarcity of articles such as V. G. Rettig has presented may lead one to believe that there is too big a gap between pages of textbooks and the notebooks of practicing engineers confronted with practical voltage problems.

We appreciate the author's proof that assumed constant currents more nearly represent conditions on an actual system rather than assumed constant impedances. Our comments are chiefly directed to the author's statement that "any method which would permit the elimination of some of the impedances from consideration would greatly simplify any solution." It is our opinion that the simplification lies in the

$$Z = \begin{array}{ccccccccccccccc} \text{NEWMARK} & \text{EATON} & \text{ANITA} & \text{DALTON} & \text{PUENTE} & \text{SANTA FE SPGS} & \text{SAN DIMAS} & \text{GANESHA} & \text{SAN ANTONIO} & \text{CHINO} & \text{UPLAND} & \text{DECLEZ} & \text{COLTON} & \text{PEDLEY} & \text{CARDIFF} \\ \hline 1.85 & 1.50 & 1.34 & .89 & .42 & .05 & .73 & .41 & .54 & .40 & .52 & .46 & .43 & .41 & .43 \\ 1.50 & 6.10 & 3.66 & 2.42 & 1.15 & .15 & 1.97 & 1.11 & 1.46 & 1.09 & 1.40 & 1.24 & 1.16 & 1.12 & 1.16 \\ 1.34 & 3.66 & 4.65 & 3.07 & 1.46 & .18 & 2.50 & 1.41 & 1.85 & 1.39 & 1.78 & 1.57 & 1.47 & 1.42 & 1.47 \\ .89 & 2.42 & 3.07 & 5.01 & 2.38 & .30 & 4.08 & 2.30 & 3.02 & 2.26 & 2.91 & 2.57 & 2.40 & 2.32 & 2.40 \\ .42 & 1.15 & 1.46 & 2.38 & 3.72 & .33 & 2.41 & 2.91 & 2.44 & 2.46 & 2.44 & 2.45 & 2.45 & 2.46 & 2.45 \\ .05 & .15 & .18 & .30 & .33 & 2.50 & .42 & .54 & .56 & .65 & .57 & .61 & .63 & .64 & .63 \\ .73 & 1.97 & 2.50 & 4.08 & 2.41 & .42 & 7.31 & 2.88 & 4.86 & 3.14 & 4.61 & 3.83 & 3.45 & 3.26 & 3.45 \\ .41 & 1.11 & 1.41 & 2.30 & 2.91 & .54 & 2.88 & 6.50 & 3.55 & 4.02 & 3.62 & 3.83 & 3.94 & 3.99 & 3.94 \\ .54 & 1.46 & 1.85 & 3.02 & 2.44 & .56 & 4.86 & 3.55 & 7.00 & 4.16 & 6.59 & 5.31 & 4.68 & 4.36 & 4.68 \\ .40 & 1.09 & 1.39 & 2.26 & 2.46 & .65 & 3.14 & 4.02 & 4.16 & 4.87 & 4.26 & 4.58 & 4.74 & 4.82 & 4.74 \\ .52 & 1.40 & 1.78 & 2.91 & 2.44 & .57 & 4.61 & 3.62 & 6.59 & 4.26 & 8.26 & 6.14 & 5.10 & 4.58 & 5.10 \\ .46 & 1.24 & 1.57 & 2.57 & 2.45 & .61 & 3.83 & 3.83 & 5.31 & 4.58 & 6.14 & 8.45 & 6.33 & 5.26 & 6.33 \\ .43 & 1.16 & 1.47 & 2.40 & 2.45 & .63 & 3.45 & 3.94 & 4.68 & 4.74 & 5.10 & 6.33 & 9.00 & 6.40 & 9.00 \\ .41 & 1.12 & 1.42 & 2.32 & 2.46 & .64 & 3.26 & 3.99 & 4.36 & 4.82 & 4.58 & 5.26 & 6.40 & 10.96 & 6.40 \\ .43 & 1.16 & 1.47 & 2.40 & 2.45 & .63 & 3.45 & 3.94 & 4.68 & 4.74 & 5.10 & 6.33 & 9.00 & 6.40 & 10.25 \end{array} \angle 66.8^{\circ}$$

Figure 1. Impedance matrix of 22–66-kv transmission lines interconnecting 15 substations referred to load currents

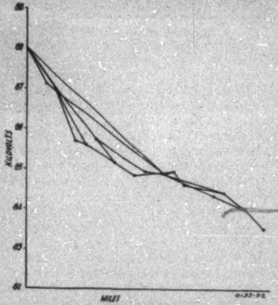


Figure 2. Voltage gradient from supply station under load conditions

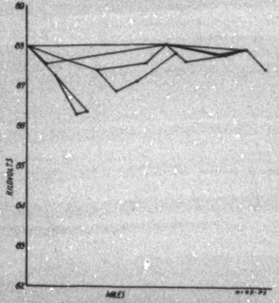


Figure 3. Voltage gradient from supply station under load conditions with a 50,000-kva synchronous condenser added at Chino

elimination of components of load currents in various lines rather than the reduction of number of impedances. Thus in voltage calculations on the system of the Southern California Edison Company, the methods of matrix algebra were used in preference to the method described by the author. In the matrix method, the line currents are not considered and line impedances are transformed into a set of self and mutual impedances of the network-to-load currents.

Figure 1 shows an impedance matrix of 15 substations interconnected by 22–66-kv transmission lines with a supply station (Laguna Bell) where the voltage is regulated. Any current vector may be assumed for load taken from the network at each substation. The voltage drop from the supply station to each substation is then a summation of *IZ* drops in the corresponding row after all columns have been multiplied by the assumed load current vectors. It has been found to be helpful to plot voltage gradients for each set of assumed load currents as shown in figures 2 and 3. Figure 2 shows the voltages at various substations of figure 1 under actual load conditions. Figure 3 illustrates the addition of a 50,000-kva synchronous condenser at Chino. It is merely necessary to multiply the column marked Chino by the additional vector current of the synchronous condenser and add vectorially the resultant impedance drops to the vectors of voltage drops computed for figure 2. Added loads can be computed in a similar manner.

There is little labor saving between this method and the one described by the author when a relatively simple network is studied under a single set of assumed currents. When a more complicated network is analyzed under any number of load conditions, the matrix method results in a very appreciable saving of time.

Reference

Tensor Analysis of Networks (a book), Gabriel Kron. Also Bibliography appended to this book.

V. G. Rettig: The discussions by Professor Kimbark and by Miss Clarke are very much appreciated since they help to more accurately define the field of application of the methods described in the paper. Frequent use for these methods has been found, however, particularly on what might be classed as the medium capacity circuits with voltages of 13.2 to 66 kv on the Cincinnati Gas and Electric Company system. As pointed out by Professor Kimbark, successive approximations based on the paper by Mr. Smith can be applied to the final results obtained by superposition methods if superposition methods are not considered to be sufficiently accurate.

The author's statement, referred to by Mr. Kroneberg in his discussion, that the elimination of some of the impedances from consideration greatly simplifies a solution refers to line or load impedances or both. The author had in mind in particular the elimination of all load impedances leaving only line impedances to be dealt with. This also is the method followed by Mr. Kroneberg in his example since he calculates the various mutual and self impedances from line impedances only and sets them down in matrix form.

Mr. Kroneberg apparently makes one approximation in setting up the matrix that the author has not used and that is—all mutual and self impedances are represented at the same phase angle. For the general power network made up of cables, transformers, and overhead lines, this assumption may lead to large errors. To avoid this, the network could be represented by two matrices, one for the resistances and one for the reactances, but the work of setting up the original matrices would be more complicated as would the solution for final voltage drops.

It should be noted also that Mr. Kroneberg has considered the calculation of voltage drops only and has not indicated how current division in the various lines that make up the network would be found. The example given applies to the calculation of the effects of changing loads on a given network of lines, but if any line additions or changes are made a complete new matrix must be set up which may add considerably to the work to be done.

The use of matrices will, in certain cases however, provide a good method of obtaining complete solutions for a variety of load conditions. Case I described in the paper makes use of only the self impedances represented in the matrix and gives the change in voltage directly only at the point where the load is added or changed.

Some Acceleration Tests on Large Synchronous Pump Motors

Discussion and author's closure of paper 40-120 by Raymond W. Ager, presented at the AIEE Pacific Coast convention, Los Angeles, Calif., August 27–30, 1940, and published in AIEE TRANSACTIONS, 1941, pages 416–22.

A. H. Lauder and C. E. Kilbourne (General Electric Company, Schenectady, N. Y.): Professor Ager has presented a very interesting account of acceleration tests made on laboratory machines and on installed large synchronous pump motors. While it has long been possible to test laboratory machines and to make factory tests on large motors, field tests on machines of this type have been infrequent, though not impossible. Professor Ager's apparatus and method are therefore a distinct advance in the art of field testing of acceleration characteristics.

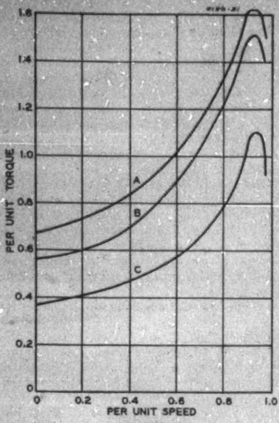


Figure 1

A—Accelerating speed-torque characteristic, 100 per cent voltage at motor terminals

B—Installed accelerating torque characteristic curve *C*, figure 14

C—Calculated accelerating speed-torque characteristic, 100 per cent voltage at switching station

This is quite desirable, as accurate checking of the performance of electromechanical drives can now be accomplished on the assembled unit. Previously, motor characteristics have been matched with the expected characteristics of the mechanical drives and checks on the combined units have seldom been made, particularly if the operation of the unit was generally satisfactory.

Professor Ager's method will be particularly valuable when used in combination with existing methods for determining the desired torque characteristics of motors. This can be done by co-ordinating factory tests and field tests. Factory tests are generally made by the measured power input method, usually under reduced voltage, and data corrected to full voltage. While this method admittedly contains some errors in making corrections, the largest error to be expected, that of saturation, is usually quite small because even under full voltage the internal flux of a motor during its acceleration period is less than that same flux during normal operation. If the data obtained (either by present factory methods or by Professor Ager's method, which also contains an assumption, namely, that rate of acceleration does not affect torque) on the machine without connected load, is combined with data obtained after installation, so that the acceleration characteristics of various types of loads can be better known, then Professor Ager's method will perform its greatest service.

In order to co-ordinate data gathered at different places and under different conditions, the terminal voltage must be known, and when an external impedance comparable to the motor impedance exists between the motor terminals and a point of fixed voltage, either that impedance, or the current drawn by the motor, must be known. Therefore, the usefulness of the tests obtained can be considerably increased if simultaneous oscillograms of voltage and current are taken with the acceleration film.

It is interesting to compare the results obtained by Professor Ager with factory tests and with calculated performance data on one of the large synchronous motors. In figure 1 curve *A* is the 100 per cent motor terminal voltage-torque characteristic, determined by factory test, on the motor whose "installed" test characteristic is given as curve *C* of figure 14 of the paper. The system on which the motor operates has considerable impedance between the motor terminals and the generated voltage. Due to the arrangement of the system it is not desirable to regulate the generated voltage to hold 100 per cent voltage at the motor terminals in any individual pumping plant, but rather to regulate voltage at a central switching station. Curve *C* of figure 1 shows the motor torque characteristic calculated from the 100 per cent voltage characteristic, curve *A*, assuming that 100 per cent voltage is held at the switching station and including the impedance between the motor terminals and this station. It is observed that the "installed" characteristic, curve *B*, falls between curves *A* and *C*. Obviously, if the motor is operated on reduced voltage, curve *B* should fall below curve *A*. Its divergence from curve *C* can be explained by several hypotheses. The system was very lightly loaded when these tests were made and therefore the voltage at the switching station may have been in excess of 100 per cent. The actual impedance between the switching station and the motor terminals may be slightly less than the calculated impedance on which curve *C* is based. Probably a combination of these two reasons is more nearly true.

Professor Ager's tests also give interesting indications of the manner in which the large motors synchronize. If the number of swings made just prior to synchronism are counted, it will be found that figure 7 indicates zero acceleration four different times, figure 10 three different times, and figure 13, neglecting the final swing caused by switching interruption, three times. Figure 10 indicates an earlier attempt of the motor to synchronize than was the case in figures 7 or 13. Possibly field application took place at a lower speed in this instance than in either of the other two. Therefore, since the torque characteristics of all motors are similar, more nearly identical performance of unit number 2 would be expected if the field application relay were set to apply field at a higher speed.

J. V. Kresser (Westinghouse Electric and Manufacturing Company, San Francisco, Calif.): Professor Ager has presented an interesting method of obtaining speed-torque curves of motors under actual load conditions. Obviously, this method is applicable to any type of motor but should be particularly useful in obtaining points between start and pull-out for induction motors, and between start and pull-in for synchronous motors.

Figure 2 of Professor Ager's paper is of particular interest since it is the first time to the writer's knowledge that a measured speed-torque curve of a three-phase motor with single-phase secondary has been published, although Wagner and Evans[1] published one which was calculated and Lamme[2] published one which was obtained by qualitative analysis. Some very interesting results may be obtained from operation with single-phase rotor. In a case investigated by the writer, the motor would not accelerate beyond half speed when started with resistance in only one phase of the rotor (a frequent procedure for medium size motors), even at light load, and vibrated excessively. Analysis of symmetrical components revealed that the torque curve had a cusp which became negative at a slip of about 0.5 and the magnitude of the double-slip frequency-torque pulsation was about 50 per cent greater than the average torque.

This is but one case of the many where heretofore a large part of the speed-torque curve has not been obtainable by tests but has to be calculated, involving complex and tedious calculations, especially for unbalanced or unsymmetrical circuits. In these cases Professor Ager's method will be of great advantage, consisting as it does of a comparatively simple test followed by simple computations.

One disadvantage of the method is the scattering of points obtained as synchronous speed is approached. This may often necessitate drawing the acceleration-time curve to an extended scale in order to obtain average acceleration and torque. However, there is also a benefit resulting from this scattering of points and that is the clear picture obtained of the character and magnitude of the torque pulsations.

It is quite evident that the method can be made quite accurate with care by using a long, well-centered scale, large timing disk, and a constant frequency source for the stroboscope and timing disk. It constitutes a valuable addition to the several existing methods of obtaining speed-torque curves with the great advantage that comparatively simple tests can be made in the field under actual load conditions which cannot very often be duplicated in the field.

References

1. The Method of Symmetrical Components (a book), **Wagner** and **Evans.** McGraw-Hill.

2. Electrical Engineering Papers (a book), **B. G. Lamme.** Westinghouse Electric and Manufacturing Company.

Raymond W. Ager: The torque pulsations recorded in the tests perhaps need some further comment. The complications which they add to the calculations are slight once the causes and effects are understood. All pulsations recorded had frequencies equal to twice the slip frequency. Therefore at 50 per cent speed the time required to complete one torque cycle was equal to $^1/_{60}$ second or the time which elapsed between pictures. At 75 per cent speed the torque cycle was completed in $^1/_{30}$ second while at $83^1/_3$ per cent speed it required $^1/_{20}$ second or 3 cycles of the 60-cycle supply. We may use these facts in locating the curve. At 50 per cent speed each reading is the average of one complete torque cycle. At 75 per cent speed we must average two successive readings. At $83^{1}/_{3}$ per cent speed we must use the average of three readings, etc. Where the reading interval is not every film but, to use examples from the paper, every fourth film as was done for figures 10 and 13 or every fifth as for figure 7, then the method of calculation automatically averages these pulsations at $87^{1}/_{2}$ per cent and 90 per cent speed respectively, and at all multiples of the corresponding slips. A partial averaging is obtained at all other speeds except synchronous speed. Only as synchronous speed is approached is the full value of the pulsation indicated.

The process of pulling into step of the synchronous machines is indicated in the curves. Other curves showing this process in greater detail may be drawn from the data obtained in the tests. This operation at Eagle Mountain was relatively simple due to the high ratio of pull-in torque to pulsating torque. This machine nearly locked into step at 14.3 seconds, it then slipped a pole and locked into step smoothly with the application of the field at 14.8 seconds. The last small negative swing represents hunting after locking into step or it may have been introduced by timing errors of the equipment.

The motor at Iron Mountain had relatively a higher value of pulsating torque. This motor pulled into step, actually going a small fraction of a per cent above synchronous speed, at 6.3 seconds on what appears to have been a combination of reluctance torque and the effect of induced rotor currents. The polarity being reversed the field breaker did not close. Upon the decay of these rotor currents the reluctance torque was not sufficient to hold the motor in step and it slipped a pole. If this interpretation is correct it would substantiate the contention of Messrs. Lauder and Kilbourne that the rate of acceleration may effect the torque. The exact time of field application is not definitely shown by the curves but is believed to have been at 6.5 seconds.

The conditions at Gene were slightly more complicated. The motor was within one per cent of synchronous speed at 7.9 seconds but this motor like the other two happened to come up to speed on this start with reversed polarity. It therefore had to slip a pole, which it did between 8.0 and 8.5 seconds. It then started to slip a second pole, but at 8.7 seconds the field breaker closed pulling the motor back into step.

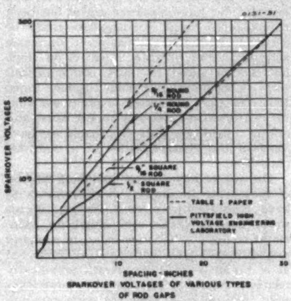
Behavior of Point Gaps at 60 Cycles

Discussion and author's closure of paper 40-131 by J. V. Lebacqz, presented at the AIEE Pacific Coast convention, Los Angeles, Calif., August 27–30, 1940, and published in AIEE TRANSACTIONS, 1941 (February section) pages 44–8.

J. H. Hagenguth (General Electric Company, Pittsfield, Mass.): The paper is an interesting contribution to the subject of point-gap sparkover. The author's curves of figure 2 show very consistent readings of gaps 1, 3, and 7 for all spacings up to about the standard humidity of 15.4 millimeters. I would like to ask what deviations from the average curves were observed for separate test points?

The author's results for gap number 5 ($^{9}/_{16}$-inch square rod ended by a section perpendicular to its axis, vertical gap) check the results obtained in the laboratory with which I am connected with $^{1}/_{2}$-inch square rods mounted horizontally at spacings of 17 inches and higher. At lower spacings the author's values are higher as shown in figure 1 by the lower dotted line, for instance at 8 inches by 18 per cent. The difference at lower spacings probably is due to the difference in mounting of the gap. In my tests the gap was mounted with the rods at a specified height, H, above ground where $H=(1.3S+4 \text{ inches})\pm 10$ per cent with definite specifications for the overhang of the rods beyond the supports ($S=$ spacing). It should be noted that during calibration tests no attempt was made to keep the electrodes clean. Lower values than presented in the paper were obtained contrary to the author's results whose tests indicated higher values when the gaps were not cleaned.

The same figure confirms that a round rod has a higher sparkover than a square rod as shown by the author's results for gap number 2 ($^{9}/_{16}$-inch round rod ended by a section perpendicular to its axis), upper dotted curve, and by the curve for a $^{1}/_{4}$-inch round rod, upper solid curve. From these differences it is clear that very definite regulations have to be followed when a rod gap is used for voltage measurements.



SPARKOVER VOLTAGES OF VARIOUS TYPES OF ROD GAPS (CUT PERPENDICULAR TO THE AXIS)

Figure 1

Mr. Lebacqz obtained a straight-line change of sparkover voltage with humidity for all gaps and all spacings. He observed also a definite change of relative influence of humidity with gap spacing. This latter change checks results obtained in my laboratory although the straight-line variation at least for the larger spacings does not.

The following discussion applies only to square-cut rod gaps mounted according to AIEE Standards. Figure 2 shows the correction factor obtained for different gap spacings as function of absolute humidity. Because these curves were taken several years ago, humidity is expressed in grains per cubic foot. However, a scale is added expressing humidity in millimeters Hg. These curves are a result of a study of humidity effect carried out under normal conditions of humidity and air density during the course of about a year. The deviation of single-test points from the average was about equal for all gap spacings and was of the order of plus or minus three per cent at standard humidity and as high as plus or minus seven per cent at humidity of 20 millimeters Hg.

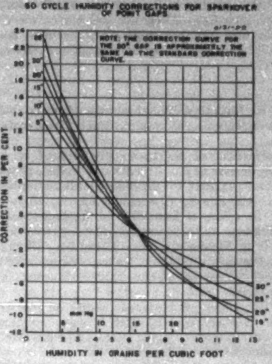


Figure 2

On the basis of the correction factors thus obtained, a comparison of change of sparkover voltages with humidity is as follows for the 30-inch gap (gap number 5):

Table I. Kilovolts Sparkover

Humidity Millimeters	From Paper Figure 2e	This Discussion
10	282	276
15	296	292
20	312	298

It is probably worth-while to point out that humidity corrections at humidity higher than standard (6.5 grains per cubic foot, 0.608 inch Hg or 15.4 millimeters Hg) are usually very unreliable as has also been observed during the author's tests at least at the large spacings. In closing, I would like to ask the author whether, on the basis of his research, he considers some forms of point gap usable as a reliable instrument for measuring voltages? As he states in the introduction, "Arc-over voltages of point gaps in air have been considered rather inconsistent and difficult to reproduce with reasonable accuracy." I am still of that opinion and would much prefer the sphere gap for voltage measurements. Other factors, of course, such as impulse sparkover and time lag enter into the picture to a large extent, when a gap is considered as a measuring instrument, and the sphere gap, of course, is much more reliable all around, especially since its sparkover voltage does not change with humidity. As a matter of fact, rod gaps were used for several years as the yardstick for testing transformers with impulses. Their use now is as a wave-chopping device, while the cathode-ray oscillograph is used as the voltage-measuring device.

P. H. McAuley (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The author has made a useful investigation of sparkover phenomena in a very fertile field. Attention is called, however, to a paper not listed in the bibliography which is as near to a classic on the subject of the effect of humidity on flashover and sparkover as has ever come to our attention. This is Fielder's paper "Humidity Effects on Flashover of High Voltage Apparatus" which appeared in the *Electric Journal* in December 1935.

The effects of electrode shape on sparkover voltage was forcibly brought to our attention about twelve years ago when Mr. Hodnette of our Sharon works and I were obtaining widely different results in the calibration of rod gaps. The standard co-ordinating gap had not appeared at that time and it developed that one of us was using a half-inch square rod, and the other a half-inch round rod, both square cut. When the situation was discovered, the square rod was adopted and further investigation dropped. We were dealing with spacings up to ten inches and it was not appreciated that electrode effects with the round rod extended to the large spacings indicated in the paper.

The influence of electrodes is indicated by the shape of the sparkover curves published by different authors as well as those standardized by the AIEE-NEMA committee on insulation co-ordination. The rate of change of slope at short spacings is neither uniform nor consistent. Similarly, the erratic behavior of the standard rod gap is shown by the bands which it was found necessary to introduce to cover the data from different laboratories for impulse tests. For positive waves the band extends from five to nine inches spacing. For negative waves a band appears between 10 and 23 inches spacing. There is no thoroughly satisfactory explanation of why these bands occur or of why they occur at these particular ranges.

It is our opinion that the author may be giving a little too much weight to electrode condition. After observing the vagaries of high-voltage sparkover for many years, it is felt that many of the phenomena are due to unaccountable conditions in the dielectric rather than to the electrodes. Similar occurrences to those reported in the paper have been observed on apparatus such as bushings where the arc does not necessarily occur from the same point on successive applications. Also, it has been noticed many times in impulse tests where the voltage is raised in small increments that when the first flashover occurs, the voltage must be lowered several per cent to stop flashover. In other words, after the first flashover a somewhat lower voltage will continue to produce flashover, but previous to the first flashover, this same lower voltage can be applied indefinitely without flashover. A flashover apparently leaves a semipermanent effect either in the dielectric or on the electrodes.

On the other hand, with the sphere gap it is often the practice to disregard the first 60-cycle sparkover in any series of tests. It is often much lower in voltage than succeeding applications.

Similarly, the day-to-day changes would seem to indicate the effect of some unknown conditions in the dielectric rather than on the electrodes. The tests in the paper were made in a confined space and it is possible that some of these phenomena may be more pronounced, on this account, than if the tests were made in an open laboratory or outdoors.

Concerning the effects of humidity, the curves in the paper do not extend below ten millimeters of mercury vapor pressure. Bellaschi and McAuley (AIEE technical paper, 40-92) have shown that the absolute humidity for indoor and outdoor laboratories is below ten millimeters (0.4 inch) for all but about four months of the year. Consequently, a great deal of laboratory testing is done at humidities lower than the range covered in the paper. Fielder also observed both that the effect of humidity is lower at short gap spacings and that the effect decreases at high humidities. Because the paper covers only the shorter gap spacings and the higher humidities, the author's results should tend to be lower than if the more practical ranges of humidity and spacings were investigated. Also, the tests reported were made in a chamber with a conducting floor and ceiling. It is not stated whether the test gap was halfway between floor and ceiling. We have observed and it has been reported by other investigators that the effect of humidity on flashover decreases and sometimes becomes negative in slope for certain dielectric field configurations such as bushings and point-to-plane gaps. It is, therefore, desirable to point out that the arrangement with a metal floor and ceiling might possibly approach or have some of the characteristics of a point-to-plane gap.

The author has used an ingenious method of measuring humidity, which appears to have considerable merit. Fielder, in his work, used a Leeds and Northrup automatic recorder. This consisted of wet and dry resistance-type thermometers connected in a bridge circuit to record graphically relative humidity. Absolute humidities were then obtained with the corresponding dry bulb readings.

In the concluding paragraph of section 4, some hints are given which suggest further investigations of a very promising nature. The observation that ether vapor increased sparkover voltage by 17 per cent seems to support our suggestion that unknown conditions in the dielectric cause many of the inconsistent results. An isolated observation of this nature often points the way to a broad fundamental investigation which results in great progress in a given field.

J. V. Lebacqz: In his discussion, Mr. McAuley questions the exact position of the gap in the test chamber. The author regrets that in the description of the equipment he did not make it clear that the gap was halfway (within plus or minus three inches) between floor and ceiling of the test chamber. Since the length of electrode protruding from either floor or ceiling was at least 4$^1/_2$ feet, it seems difficult to assimilate such a setup to a point-to-plane gap.

Concerning the range of humidity and spacing studied, the author observed, during the whole period of the study, only two or three days when the humidity dropped below 10 millimeters Hg pressure, the minimum observed being about 7.5 millimeters Hg. Decreasing the humidity below the ambient value was not considered practical, and the author had to satisfy himself with considering only the range reported in the paper. As to the spacing, it was considered desirable that the dimensions of the test chamber be at least three times the length of gap under test in order to avoid external influences on the field shape between the points. This means that the volume of the test chamber increases as the cube of the maximum gap length considered, and had the gap length been doubled it was feared that difficulties would be encountered in obtaining a homogeneous mixture of the water vapor in the air.

It is quite obvious from the results obtained in this study that changes in the dielectric, as well as the shape and the surface condition of the electrodes, will affect the breakdown voltage and the behavior of point or rod gaps. The author did not wish to give the impression that he underestimates the influence of the dielectric but rather that he believes that both surface and dielectric condition will affect the field shape, upon which, in the final analysis, the breakdown voltage will depend.

The author sincerely appreciates the interesting discussion sent in by Mr. J. H. Hagenguth, containing a comparison with the results obtained by the General Electric high-voltage laboratories. Answering the question raised in the first paragraph, the author wishes to state that the deviation from the average curve for separate test points for clean electrodes (1, 3, and 7), on the same day, did not exceed one per cent, and for at least seventy-five per cent of the points, 0.5 per cent (refer to figure 1 of the paper). However, average curves taken on different days might have differed by as much as three per cent (refer to paragraph "Other factors affecting arc-over voltage" of the paper).

Referring to the end of the second paragraph of the discussion, the author wishes to point out that his tests "indicated higher values when the gaps were not cleaned" for the special case of electrodes ending in one well-defined point (1, 3, and 7) and at small spacings. Because of burning no effect was noticed for gap number 5.

Referring to figure 2 of Mr. Hagenguth's discussion, the author wishes to point out that he does not see any reason for the sudden change in slope in most of the curves at the standard humidity of 15.4 millimeters Hg. No such change was observed in this work, especially not for the square-cut square rod gaps, where all the test points fall within ±1.5 per cent of the average curve for spacings of 18, 24, and 30 inches, at all humidities up to 25 millimeters Hg. Unreliable results at large spacings and high humidities were obtained for all electrodes except the square-cut $^1/_2$-inch square rod.

The author wishes to close these remarks by considering the suitability of some forms of point gap as a reliable instrument for measuring voltages under steady-state conditions (i.e., 60 cycles or direct current).

It appears to him that the main qualities of a voltage-measuring gap should be reliability, insensitivity to surface conditions, and facility of duplication. These requisites are not, in his opinion, found in the sphere gap. Many spurious arc-overs, especially at large spacings, make the results less dependable than could be desired. The sphere-gap arc-over voltage is also highly dependent upon surface conditions; any scratches, rough spots, or pitting due to previous arcs will materially decrease the values obtained, and any deviation from the ideal sphericity will alter the results between different sets of spheres. On the other hand, the tests reported in the paper seem to indicate that the square-cut square rods give results practically independent of surface condition and show no tendency to extremely low or extremely high values of arc-over. The author cannot agree with Mr. Hagenguth's statement that "the sphere gap, of course, is much more reliable all around, especially since its sparkover voltage does not change with humidity." He does not see how this independence from humidity would make one gap more or less reliable than the other. It simply means, to him, that, if rod gaps are used, two correction factors ought to be introduced (air density and humidity), while for sphere gaps only the air density has to be considered. The author thinks, however, that the addition of a second correction factor would be a small price to pay to avoid the use of a voltage-measuring instrument which is not only very expensive to acquire but also very difficult to maintain in perfect shape. Of course, more work covering a wider range of voltages would have to be done before a definite answer could be given to this question, but the author feels that there is a possibility that, provided reliable correction factors were determined, some form of point gap might prove a more accurate method of measuring voltages than the sphere gap.

The author is in complete agreement with Mr. Hagenguth that other factors such as impulse sparkover and time lag enter the picture. He wishes to state again that all the preceding discussion does not apply to impulse phenomena, where the cathode-ray oscillograph used in conjunction with a good voltage divider is probably the most accurate and reliable method of measurement.

Braking Magnets for Watt-Hour Meters

Discussion and authors' closure of paper 40-128 by I. F. Kinnard and J. H. Goss, presented at the AIEE Pacific Coast convention, Los Angeles, Calif., August 27–30, 1940, and published in AIEE TRANSACTIONS, 1941, pages 431–9.

G. J. Wey and V. S. Thomander (Westinghouse Electric and Manufacturing Company, Newark, N. J.): This paper makes quite clear the influence of metallurgical and magnetic aging in connection with the stability of permanent magnets. The reported findings agree very closely with data obtained by the Westinghouse company particularly in connection with the chromium steels.

Granting a suitable treatment for obtaining metallurgical stability, it is interesting to note the influence of demagnetizing methods upon the magnetic aging. The difference in magnetic stability for the two positions can, no doubt, be accounted for by the degree of uniform demagnetization obtained by either the parallel or normal field. In addition to the influence of demagnetizing methods on magnetic stability under conditions of heat cycles, we have found that the methods in like manner have considerable influence upon stability under conditions of external field surges. The surge stability obtained by either of the two methods of coil demagnetization discussed is highly sensitive to the direction of the external field surge applied. However, demagnetization obtained by passing current directly through the center of a magnet largely eliminates the directional effect of the applied external field surge and at the same time reduces the magnitude of the resulting change in damping due to the surge.

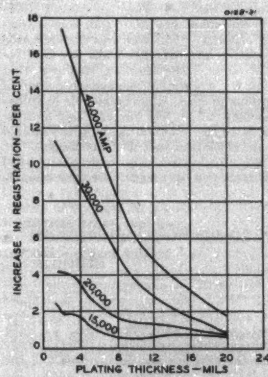


Figure 1. CS—Permanent magnet effect of plating thickness on resistance to current surges

Considering the influence of demagnetizing methods on both magnetic aging and stability under surge conditions, we have found it highly desirable to avoid coil demagnetization wherever possible.

Referring to figure 11, the authors' tests on the various magnet material were obtained on simple bar magnets. While this method gives valuable data for drawing curves of demagnetization versus coercive force, the results cannot be applied directly to show the merits of practical designs since the demagnetization, particularly of chrome magnets, is also a function of the shape of the test magnet. In all of our work we have used standard magnets.

We are surprised to note that the authors obtain such good results from a 0.002-inch thick coating of copper on chrome steel. Curve *A* on figure 11 shows a performance practically as good as Alnico II.

In connection with investigations of surge effects, which led to our recently adopted practice of electroplating magnets, tests were made to determine the effectiveness of varying thicknesses of copper deposition.

The appended curve shows the shielding effect for various thicknesses of copper plating from a minimum of two mils to a maximum of 20 mils. A thickness of about eight times that indicated by Goss and Kinnard (0.015–0.018) was adopted for the standard product, thus retaining the use of chrome steel on which there exists a wealth of experience with regard to permanency in actual practice.

The authors state that the effectiveness of the copper plating is reduced by the inability to build up a coating in the exposed air gaps. Our tests have shown that as long as the external surfaces of the poles are shielded it is not necessary to build up a coating on the working air gap surfaces.

Paul MacGahan (Westinghouse Electric and Manufacturing Company, Newark, N. J.): After reading the excellent paper on "Braking Magnets for Watt-Hour Meters," which was presented by Kinnard and Goss, it seems to me that much of the interesting bibliography record of previous investigations has been omitted. I have reference especially to the following:

THE MAGNETISM OF PERMANENT MAGNETS, **S. P. Thompson.** *Journal* Inst. of Elec. Eng. (British), February 1913.

This carries a very extensive bibliography.

COBALT MAGNET STEEL, **P. H. Brace.** *Electric Journal*, March 1929.

PRINCIPLES GOVERNING CHOICE AND UTILIZATION OF PERMANENT MAGNET STEELS, **R. L. Sanford.** Bureau of Standards Paper No. 567, December 1927.

MAGNETIC PROPERTIES OF STEELS, **Madam Sklowdowska Curie.** *Bulletin* de la Societe d'Encouragement Year 1898, reprinted in *London Electrician*, page 28, year 1898.

This remains as probably the most important contribution to the science of permanent magnets ever made, and is certainly the groundwork of all the modern progress which has been made.

Before Madam Curie published her epochal paper, there were numerous experimenters in the field, attracted to the problem even as now, for producing instruments or meters. Among these the most notable ones in America were Edward Weston, pioneering in this field previous to 1887, Elihu Thomson in 1889, and O. B. Shallenberger in 1888. These early experimenters preferred to keep their findings to themselves as "secret processes" and thus did not publish any of their works for the benefit of the industry. This tradition of secrecy in magnet making is now happily broken down, as exemplified by the Kinnard-Goss paper.

F. A. Redding (Southern California Edison Company, Ltd., Los Angeles, Calif.): The contributions to watt-hour meter engineering knowledge by the authors of this paper in the past have been numerous and have exerted an important influence in this field of endeavor.

Their latest effort, "Braking Magnets for Watt-Hour Meters" is typical of the thorough manner with which they cover their selected subject.

Operating engineers are always interested when a complete change is suggested for one of the vital component parts of our watt-hour meters. While we all realize that progress necessitates changes, we are all concerned that such changes will not gain certain desirable characteristics at the sacrifice of some equally important properties of the unit which is to be replaced.

We have felt that the chrome steel magnet during the past decade has performed in an admirable manner, however with modern installations, certain shortcomings have presented themselves in this type of magnet, where the installation has been subjected to lightning surges or heavy short circuits.

A review of the subject matter and data presented in the paper by Kinnard and Goss definitely gives comparative data, obtained from quite elaborate tests to indicate that Alnico II when properly treated and used for watt-hour meter damping has all the desirable characteristics of chrome steel and in addition, will resist demagnetization from fields caused by heavy direct currents and lightning surges.

The method and equipment used for measuring the permanency of breaking magnets, whereby the personal error was eliminated through the use of photoelectric control gives very definite accuracy of results.

These results indicate a high degree of permanency, following stabilization treatment by aging as outlined, which of course is fundamental and a paramount characteristic in material used for this purpose.

The effective control of temperature errors is shown as even superior to that possible with chrome steel, this is most desirable because more and more of our meters are being mounted in the open without protection of any nature, and the temperature differential during any 24 hours often is a matter of wide range even in our more temperate areas.

The statement is made that "if desirable, this compensation may be increased to allow for errors in other parts of the meter." It is not clear to what other errors reference is made, but it would surely seem desirable to obtain increased accuracy whereever possible.

The data submitted from the varied tests conducted gives the operating engineer a feeling of confidence that the conclusions reached are well supported by technical proof and the Alnico II when used as a braking magnet for our watt-hour meters will prove very satisfactory.

I. F. Kinnard and J. H. Goss: Mr. Redding raises the question regarding just what is meant by adjusting the temperature compensation for the magnet to take into account other errors in the meter. As an example, there is a slight temperature change in permeability of the silicon steel of the electromagnetic driving element of the watt-hour meter and this affects the torque of the meter and is a class I error. This change is small but by adjusting the magnet compensation this error together with several other small errors may be corrected. This gives better over-all performance than if the magnet were compensated independently of the watt-hour meter. For a more complete discussion of this subject, see "Temperature Errors in Induction Watt-Hour Meters", I. F. Kinnard and H. T. Faus, AIEE TRANSACTIONS, volume 44, 1925.

Mr. Redding expresses the interest of operating engineers in the performance of such an important part of the watt-hour meter as the braking magnet. It was the belief of the authors that this interest existed and it was to fulfill this need that the paper was prepared.

G. J. Wey and V. S. Thomander state in their discussion that they have found that the methods of applying the demagnetizing surge have a marked effect on the ability of the magnet to resist the surge.

While they have not given sufficient data or detail to allow an evaluation of their method we would like to point out that in making surge investigations of this type the capacity of the surge generators must be kept constantly in mind. If the path of the conductor carrying the surge current is changed to increase its impedance, the shape of the wave may be changed. It has been pointed out that the demagnetizing effect depends both on the magnitude of the surge current and on the wave shape.

While it is one thing to set up testing methods that give easily repeatable results and minimum demagnetizing effect on a given design of braking magnet, it must be kept in mind that the testing of a specific braking magnet must be done under equivalent service conditions if the evaluation is to be valid.

All of the original work done by the authors on the surge demagnetization of magnets was carried out on General Electric chromium steel watt-hour meter braking magnets. The work was then repeated using bar magnets to obtain the data for this paper. The authors reasoned that data on a specific commercial design of braking magnet, while invaluable in studying that design, had limited value as compared to fundamental data on a standard shape of magnet such as a bar magnet.

We wish further to reiterate that we feel the results on bar magnets are very valuable to the designer from a qualitative point of view. They cannot be interpreted quantitatively but good qualitative agreement was obtained between the tests made on chromium steel braking magnets and bar magnets.

As regards the effectiveness of the 0.002-inch thick copper plate on the bar samples and the 0.015-inch to 0.018-inch thickness used by Wey and Thomander, we must again emphasize, as pointed out in the paper, that the two are not comparable. If the plating thickness on the samples was further increased, further improvement would result whereas there is little or no improvement on a chromium steel magnet with unplated air-gap pole faces beyond 0.010 inch for surges up to 20,000 amperes.

It should also be pointed out that the comparison of copper plating on chromium steel and a high coercive force material is the most favorable for a single surge demagnetization. For multiple surges, the superiority of the high coercive force material is even greater.

A very practical consideration is that should service tests show that further improvement in surge resistance is desirable, an Alnico magnet could be shielded by copper plating to give a magnet with still greater surge resistance whereas the chromium steel magnet with its copper plate cannot be further improved. Performance under service conditions will be needed to find out if the degree of improvement is sufficient.

The authors appreciate Mr. MacGahan's comments and are pleased to have the additional bibliography in his discussion.

Fault Voltage Drop and Impedance at Short-Circuit Currents in Low-Voltage Circuits

Discussion of paper 40-148 by O. R. Schurig, presented at the AIEE Middle Eastern District meeting, Cincinnati, Ohio, October 9–11, 1940, and published in AIEE TRANSACTIONS, 1941, pages 479–87.

A. G. Darling (General Electric Company, Schenectady, N. Y.): The factual data presented by Mr. Schurig is considered a most important contribution to the design of low-voltage systems. The data was obtained as a result of considerable dissatisfaction with arbitrary methods having little or no scientific background. Some disastrous failures have occurred at infrequent intervals to equipment thought to be adequate as determined by rule-of-thumb predicated fault currents. The tests reported by Mr. Schurig show that heavy short-circuit currents can occur in typical modern constructions of low-voltage equipment.

Not a little thought and time were expended in the selection of the equipment to be tested for impedance and arc drop in order that it be representative of equipment in ordinary use without repeating reliable previous investigations. Obviously, not all types of construction could be investigated, nor all types of faults. The majority of typical cases are believed now to have been covered. Certain experiments by other investigators covered a portion of the field but left the impression that severe faults are unlikely. Quite contrary to these erroneous conclusions, Mr. Schurig's data show not only that maximum values of current can be obtained in actual cases but also show that, given the correct constants of the circuit, the expected currents can be calculated mathematically and identify the circumstances under which currents are obtained.

Having available these new and extremely important facts, no undue chances need now be taken in the specification of adequate equipment, nor should there be any serious discrepancy between the results obtained by independent investigators.

As a result of the investigations reported by Mr. Schurig, the general theorem of short-circuit calculations has again been proved to hold true for low-voltage circuits; i.e., the magnitude of short-circuit current is equal to the voltage divided by the impedance. No claim is made that this is a novel or new phenomenon because it has been one which has been known for a long time. The tests merely show that if one wishes to discard guesswork he can be fairly accurate in the calculation of short-circuit currents by using the proper values of impedance. One of the chief reasons for conjectures has been the lack of a fairly accurate description of the circuit, together with a lack of readily available impedance data. Some of the important missing information has been supplied by Mr. Schurig.

Investigation of short-circuit intensities in low-voltage systems did not receive the attention it merits until the past few years when the growth of low-voltage systems in industrial plants and in city lighting voltage distribution brought this subject to the foreground. It is only necessary to realize that $^1/_{25}$ of the impedance of a 2,200-volt system will limit the current in a 440-volt system to the same number of amperes as will occur in the 2,200-volt system in order to appreciate that impedances of low order normally neglected in higher voltage circuits, have a material effect on currents in low-voltage circuits.

Short-circuit currents can be classified into at least four groups of current intensity. In analyzing the possibilities of short-circuit currents, it is well to keep these four groups in mind.

1. The bolted short circuit of minimum contact resistance will produce the highest short-circuit current.
2. An arc fault in a closely congested, enclosed area produces the next highest amount of short-circuit current.
3. An arc fault in assemblies not as closely congested and open to the atmosphere produces the third classification of currents.
4. Arc faults produced by foreign objects of such weight and mechanical resistance to movement as may be readily thrown off from the circuit give the lowest values of short-circuit current and the most rapid current-limiting effect.

Conjectures as to the intensities of short-circuit currents can be based somewhat on the law of probabilities, and with this conclusion there appears to be no serious divergence of opinion. For every source of power there are a number (n) opportunities for failures between source and secondary bus. An additional number (L) chances occur between the secondary bus and the ultimate use of power. The result is that, for one possibility of trouble at the source, there are $L \times n$ chances of trouble in the distribution system. As the impedance increases from source to the point of short circuit, the current decreases. Consequently, in point of number, most fault currents are low compared to the maximum possible currents. This conclusion should, however, not be amplified to the extent that, since the greatest number of faults are of low order of current, there is no need to provide against the maximum fault current. Further, the conclusion should not tend to minimize the importance of adequately protecting against a fault at the point of maximum current as a power failure at that point may cause outages to all loads, whereas a failure at a point of power usage is not likely to affect more than that one usage for a long period of time.

The question then naturally arises as to whether protection should be installed at all places in the system which is adequate against the maximum faults at those locations. Tactics in the design of the entire system can be used to justify the cost of insurance to service. If there is but one line of defense, that line should hold against all expected assaults. If on the other hand there are two lines of defense, each subject to the same intensity of assault but one line backing up the other, the importance of maintaining service through all parts of the outer line should be reviewed before deciding to have maximum required strength in both lines. If a weaker outer line is used, the amount of destruction that can take place is a factor not to be overlooked. Equipment that will not, upon failure, spread damage beyond itself is much more safe than a type that will affect its neighbors. For low-voltage circuits, air-insulated, metal-enclosed circuit-interrupting devices are available and should give a minimum of contingent damage. The ultimate user must take into account the chances for failures of circuit-interrupting devices in comparison with the chances of obtaining short-circuit currents in excess of their rated interrupting ability and arrive at a cost of insurance which he can justify in his business.

Certain types of interrupting devices can be used in series and obtain that degree of co-ordination and clearing time which insures relief from a failure in the outer one. This can be done by adjusting the tripping current of the inner or first defense device at or somewhat below the expected failure current of the outer or second defense line. In this way there is obtained a clearing of the failure before it has a chance to spread beyond its point of initiation.

The data given by Mr. Schurig constitutes a material addition to the factors known to govern short-circuit currents. Heretofore, arcs in general were presumed to materially limit short-circuit current and possibly act to interrupt the circuit before devices could act to do so. In the light of the new data, one must consider the type of construction and insulation used and gauge the results accordingly. In particular, guesswork and rules of thumb should definitely be put aside for the more accurate and safer method of including in the calculation the impedances known to exist in the circuit.

The Application of Ignitrons to Resistance-Welding Control

Discussion of paper 40-146 by M. E. Bivens, presented at the AIEE Middle Eastern District meeting, October 9–11, 1940, and published in AIEE TRANSACTIONS, 1941, pages 471–8.

J. H. Hutchings (General Electric Company, Schenectady, N. Y.): Recent developments indicate that ignitrons are destined to continue to make important contributions in the resistance-welding field.

These same tubes are now being used with magnetic energy storage welders to furnish the high direct current which flows in the welding transformer primary during the charging period. Their well-known qualities of small size, high current capacity, and reliable operation have made their application attractive. Capacity energy storage welding installations use these tubes to trigger the energy stored in the condenser bank into the welding transformer. Another ignitron connected across the welding transformer prevents current oscillation.

Ratings of ignitron tubes for standard a-c resistance welding are ably covered in this paper. However, the economies which follow conservative application of ignitrons within their ratings cannot be overemphasized. These tubes depend on maintenance of good vacuum for their control properties. Inadequate water cooling or overload, even for very short periods, can do irreparable damage and short life can result. Conversely, if proper operating conditions are maintained, extremely long life can easily be obtained. This is proved by the fact that many of the first tubes installed four and five years ago throughout the country are still functioning perfectly.

Proper application of tubes reduces the need for maintenance to a practical minimum. The maintenance problem can be further simplified because ignitron tubes are sturdy, simple devices which are easily tested for most important characteristics. Quite a satisfactory job can be done in checking the ignitron tubes in a welding control panel with only an ohmmeter for measuring ignitor resistance and a hand spark coil for checking vacuum. Tube manufacturers supply complete instructions for performing these simple tests. An example is *GEI*-13627, a two-page publication supplied by the General Electric Company.

As users become more and more familiar with the characteristics and possibilities of ignitron tubes, misuse will tend to disappear and the great economies which tubes provide will become even more apparent.

Two New Methods of Accelerating Electric Motors Automatically

Discussion and author's closure of paper 40-142 by John D. Leitch, presented at the AIEE Middle Eastern District meeting, Cincinnati, Ohio, October 9–11, 1940, and published in AIEE TRANSACTIONS, 1941, pages 487–93.

B. W. Jones (General Electric Company, Schenectady, N. Y.): The author reviews the several methods that have been used in the past for accelerating d-c and a-c motors and refers to them as:

1. Counter electromotive force method
2. Series lock-out method
3. Constant-time method
4. Time-current method

He points out the disadvantage of the several methods and says of "constant-time": "One decided disadvantage of the 'constant-time' method becomes apparent when very heavy loads are encountered, in that the acceleration contactors will close before the motor attains any appreciable speed and the resulting excessive currents may cause considerable damage."

I wish to call attention to some tests that were made a few years ago to show just what does happen under this condition. Some records can be found in the April 1925 issue of the *General Electric Review* and oscillographic records reproduced here show what takes place when a series motor is reversed from full speed in one direction and brought up to full speed in the opposite direction under three degrees of load; namely, no load, rated load, and twice rated load. The type of control used is that which the author calls the constant-time method. The sys-

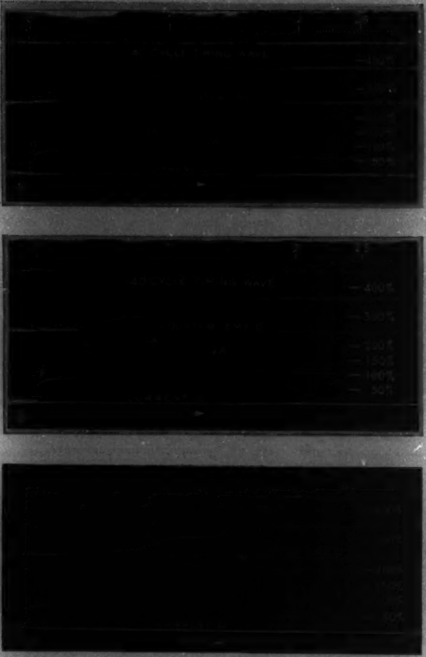


Figure 1. Effects of reversing a series motor under zero load (A), full load (B), and 200 per cent load (C)

tem employed for these records consists of a counter electromotive force effect for controlling the plugging contactor for stopping the motor but the two accelerating contactors were controlled by two relays which interposed a fixed time preceding the closing of each of the two accelerating contactors. These records show that the resulting accelerating current peaks labeled *F*, *P*, 1*A*, and 2*A* were exactly what should have resulted. They increased as the load increased and decreased as the load decreased, but at no time were the current peaks more than proportional to the load being accelerated.

The author states that the constant-time system is very generally used and this record clearly illustrates why the industry is favorable toward this principle. A second reason why constant-time is popular is that it is simple.

It is on the assumption that the current peaks become dangerously excessive that the author bases the need for the "time-current method," but the above records do not substantiate that there is any such need.

John D. Leitch: Mr. Jones seems to have misunderstood my statement in the introduction under the heading "Constant-Time Method." Actually the comparison is being made with the two methods which precede it. The four methods described in the introduction are examples of two entirely different principles—one in which the acceleration contactors close when the motor reaches a definite speed and the other in which the contactors close after a time interval independent of motor speed. The "counter electromotive force" and "series lock-out" methods are applications of the former of these two principles while the "constant-time" and "time-current" methods are applications of the latter.

Mr. Jones' criticism is directed against the "time-current" method in comparison with the "constant-time" method and he has submitted oscillograms to prove or disprove his statements. The oscillograms show the motor current and counter electromotive force for 50 per cent, 100 per cent, and 200 per cent full load on the motor. The peak values of the current at 100 per cent load are quite uniform indicating a good adjustment of the constant-time intervals. No advantage for time-current acceleration can be shown at 100 per cent full load since that is the condition for which the relays are adjusted to give uniform peaks just as they have been for the case shown on the oscillogram.

At 50 per cent full load, the 1*A* and 2*A* current peaks are successively less than that at which the *P* contactor closed whereas with time-current acceleration in which the time interval is automatically decreased with decrease in load, the 1*A* and 2*A* contactors would close earlier and give more uniform peaks and more rapid acceleration of the motor.

At 200 per cent full load, the 1*A* contactor closes before the motor has gained any appreciable speed as shown by the counter electromotive force trace on the oscillogram. With time-current acceleration, more time would be provided automatically before 1*A* closes and the peak current would be lower. Similarly with the 2*A* contactor. Thus, obviously, there is no reason why the peak currents need be proportional to the load if some means is provided for increasing the time intervals with increase in load.

In other words, time-current acceleration provides slower acceleration on heavy loads and more rapid acceleration on light loads with greater protection for the motor, which, after all, is the purpose of any acceleration method. The two new methods described in the paper are also examples of the two fundamental schemes of acceleration. The frequency acceleration relays described in part 1 close at definite motor speeds while, in part 2, "time-limit" acceleration both "constant-time" and "time-current" is provided with a neon tube timing circuit.

The Effect of Overexciting Transformers on System Voltage Wave Shapes and Power Factor

Discussion of paper 40-137 by J. W. Butler and E. B. Pope, presented at the AIEE Middle Eastern District meeting, Cincinnati, Ohio, October 9–11, 1940, and published in AIEE TRANSACTIONS, 1941 (February section) pages 49–53.

Eric T. B. Gross (College of the City of New York, N. Y.): About ten years ago, a few papers were published in Germany concerning higher harmonics due to transformer saturation[1]; but these papers were chiefly of academic interest, as serious difficulties had never arisen because of higher harmonics. Furthermore, investigations on higher harmonics in various transmission and distribution systems of different voltages and character had shown that, in general, higher harmonics decreased with increasing load; thus, it can be understood easily why the effect of higher harmonics due to transformer saturation under "no load" conditions of the network is limited and unimportant.

The problem got a new impulse, however, when war preparations in Germany made it advisable to reduce the demand on iron. In this country, T. C. Lennox had found[2] that the fifth and seventh harmonics in the magnetizing currents of star and meshed connected three-phase transformers are of opposite phase and equalize each other if used in the same or neighboring stations. On basis of this fact, J. Biermanns (Berlin) and R. Willheim (Newcastle-upon-Tyne) suggested[3] the connection of the magnetic flux of one transformer in star and mesh, thus suppressing higher harmonics in the external magnetizing current of the transformer and making it nearly sinusoidal. To adapt the normal construction of transformers to this principle, it is only necessary to split the yokes and to use additional low voltage windings on yokes and cores; these additional windings are to be designed for the magnetizing currents of the higher harmonics and need only to be relatively small. Higher saturation is thus permitted, as far as the harmonics are concerned, so that iron can be saved. However, due to higher saturation the losses are increased and the transformer less efficient than otherwise. Under the present economic conditions in Germany, this is apparently less important than the reduction of the iron volume. It seemed to me that this solution of the problem was of general interest as it might be used with advantage in the construction of transformers with usual flux density and economically designed losses.

1. Distortion of Network Voltage Curve by the Transformers, E. Friedlaender. *Wiss. Veroef. Siemens Konzern*, volume 7, part 2, 1929, pages 1–30.

Choking Coils With Saturated Iron Cores, F. Ollendorff. *Archiv f. Elektrot.*, volume 23, 1929, pages 162–80.

Harmonics in the Magnetizing Current of Three-Phase Transformers, G. Stein. *ETZ*, volume 51, 1930, pages 1061–7.

Harmonics in Transmission Systems With Long Lines, R. Willheim. VDE *Fachberichte*, 1931, pages 82–4.

2. Circulation of Harmonics in Transformer Circuits, *Journal* AIEE, volume 45, 1926, pages 755–7.

3. Magnetic Origin of Harmonics and Their Suppression, W. Kraemer. Int. Conference on Large High-Voltage Systems 1937, paper 116, Paris 1937.

Some Insulator Designs Require Special Features to Insure Radio Quietness

Discussion and author's closure of paper 40-144 by Charles J. Miller, Jr., presented at the AIEE Middle Eastern District meeting, Cincinnati, Ohio, October 9–11, 1940, and published in AIEE TRANSACTIONS, 1941 (February section) pages 62–6.

P. L. Bellaschi (Westinghouse Electric and Manufacturing Company, Sharon, Pa.): While the author deals specifically with line and station porcelain insulators, certain principles on the radio-noise problem which he enumerates have been recognized and established quite generally and therefore they apply equally to other high-voltage apparatus and equipment. For instance, I have reference to the methods of measurement which through the efforts of many (in particular the pioneering work of Aggers and Pakala—*Electric Journal*, December 1933) have been standardized along practical and fundamental lines ("Methods of Measuring Radio Noise"—EEI Publication No. G9—NEMA Publication No. 107—RMA Engineering Bulletin No. 32). Naturally one of the benefits accruing from this systematic standardization process is that a wide range of experience and comparable test data have been accumulated in the past five years to the extent that test codes covering most apparatus and equipment practically are available at this stage.

As these test data became available and experience crystallized, an important principle became apparent. It became clear that the permissible radio-noise voltage levels for high-voltage apparatus and equipment fundamentally should follow upward steps graduated with increase in voltage rating and for two reasons in particular. In the first place, possible sources of increasingly high-radio disturbance inherently are removed from the radio receiver a greater amount (coupling factor) for the higher voltage than for the lower voltage circuits. Second, it is recognized that the problem of designing apparatus for a minimum radio noise is both technical and economical—the higher the voltage class of the apparatus the higher the permissible radio noise-influence voltage level. This does not mean that the same levels are necessary or justified for all apparatus. In fact there are good reasons why higher levels are permissible for station than for line apparatus—a point that the author also has touched upon.

Another consideration of importance has been the test voltage at which the radio noise-influence voltage shall be measured. There has been a good deal of confusion on this point for the voltage at which apparatus may operate is not always defined. In the case of transformer or other potential apparatus, the problem is simplified in that the induced voltage applied to the winding shall not exceed ten per cent above the winding voltage. The desirable test voltage is at 110 per cent of the maximum rated line-to-neutral voltage. For apparatus for which induced tests cannot be made, such as series transformers, circuit breakers, insulators, etc., a test voltage based on 110 per cent of the insulation-class voltage divided by the square root of three appears proper and desirable.

There has also been some confusion as to whether the per cent overvoltage in testing apparatus for the radio noise-influence voltage should be 5, 10, or 15 per cent or some other figure. It now appears that ten per cent is an adequate amount, and a logical figure for this will work toward uniformity in that it makes it possible to test all types of apparatus, either induction or series, on the same percentage overvoltage basis. This test voltage should provide adequate assurance that the radio-noise level of the apparatus is not exceeded even for a voltage somewhat above the normal rating and it also should give the further assurance that the radio-noise characteristic exhibited by the apparatus is of a stable character within the possible operating range.

Charles J. Miller, Jr.: The comments by Mr. Bellaschi are very interesting as they indicate that electrical manufacturers in general desire a standard means of measuring radio noise-influence voltages in the factory so that they may manufacture devices which will not cause radio interference when placed in service.

The instrument and circuits devised by the joint co-ordination committee cover all types and classes of electrical equipment from 110-volt motors to the highest voltage insulators. The circuit for high-voltage devices is but one of several circuits devised, and the testing of insulators is but one application of the high-voltage circuit, as Mr. Bellaschi has pointed out in his comments. However, having an instrument and a coupling circuit, it still remains for each industry to establish its own test code, consisting of test voltages and permissible microvolt levels. Since permissible microvolt levels may vary with the type of device and its application, it is probable that no general levels can be established which apply to all types of equipment.

It is the desire of progressive electrical manufacturers to produce radio-free devices and the advent of a standard method for measuring radio interference in the factory has been welcomed with open arms by a majority of the electrical industry.

A New Current-Limiting Fuse

Discussion and authors' closure of paper 40-158 by H. L. Rawlins, A. P. Strom, and H. W. Graybill, presented at the AIEE Middle Eastern District meeting, Cincinnati, Ohio, October 9–11, 1940, and published in AIEE TRANSACTIONS, 1941 (February section) pages 77–80.

E. A. Williams, Jr. (General Electric Company, Philadelphia, Pa.): The introduction of the paper, "A New Current-Limiting Fuse," seems to imply that the reduction of electromagnetic forces, by a substantial limitation of peak short-circuit current, is only of incidental interest. There may be some logical reasons for this implication when applied to the fuse described by the authors as it is apparently limited to application with potential transformers and small control transformers. However, the real value of "limiting" the short-circuit current has been spectacularly demonstrated by testing 2,500-volt oil immersed contactors in series with conventional fuses and again in series with current-limiting fuses. With an available short-circuit current of 14,000 amperes, the conventional fuse interrupted the 2,500-volt circuit in one-half cycle, but the tank of the contactor ripped open. In contrast, the 200-ampere current-limiting fuse interrupted a 2,500-volt circuit having well over 150,000 kva available with absolutely no damage to the contactor.

Since the authors found it desirable to construct the "new current-limiting fuse" principally from fiber, they must necessarily take precautions to protect this organic material from excessively high temperature. The silver wire, wound directly on the fiber, has a melting temperature of approximately 960 degrees centigrade, and this temperature will destroy fiber in a very short time. This fact has been recognized in the design, and the main or "clean up" fuse element has been "calibrated to melt at a lower current than the auxiliary fuse element." With this limitation in mind, it is rather difficult to see how this main element "can be designed to give any desired time-current characteristics."

The theory and calculations presented by the authors appear to be based on certain assumptions which apply only in a very limited range of short-circuit currents and fuse sizes. The current rise through a fuse of low-current rating does not follow the law, $i = 377 I_m t$ because the resistance of the fuse varies with temperature and is usually high relative to the impedance of the remainder of the circuit. For this reason, during the melting period, the low-current fuse sees a continuously decreasing available short-circuit current which is appreciably lower than that actually available in the circuit in the absence of the fuse.

The current rise through a fuse of high-current rating also fails to follow the law, $i=377I_mt$, but for an entirely different reason. Such a fuse requires so long to melt that even a relatively high short-circuit current will deviate sufficiently from a linear rise to invalidate the assumption. The inaccuracy of the authors' assumptions are well illustrated by reference to the family of current-limiting curves given in figure 1 of this discussion. These curves are based on the results of actual short-circuit tests and show the instantaneous current at the instant the fuse melts, when subjected to a symmetrical current, as a function of the available asymmetrical current.

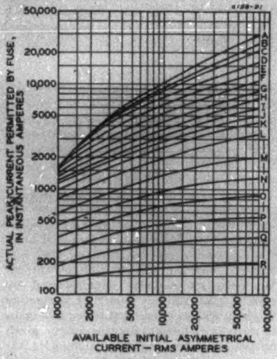


Figure 1.

A—300/200*N* amperes
B—250/150*N* amperes
C—200/125*N* amperes
D—150/100*N* amperes
E—125/80*N* amperes
F—100/65*N* amperes
G—80/50*N* amperes
H—60/40*N* amperes
I—50/30*N* amperes
J—40/25*N* amperes
K—30/20*N* amperes
L—25/15*N* amperes
M—15/10*N* amperes
N—10/7*N* amperes
O—7/5*N* amperes
P—3/3*N* amperes
O—2/2*N* amperes
R—1/1*N* ampere and lower

Note that the 1/1*N* ampere fuse will pass a peak current of 180 instantaneous amperes with 10,000 rms amperes available and that this peak current increases to only 198 instantaneous amperes with 80,000 rms amperes available. According to the authors' predictions, the peak current should have increased to 360 instantaneous amperes. At the other extreme, we find that the peak current permitted by the 300/200*N* ampere fuse increases from 10,000 instantaneous amperes with 10,000 rms amperes available to 29,500 instantaneous amperes with 80,000 rms amperes available whereas the authors' theory would predict this latter peak current to be only 20,000 instantaneous amperes. However, it is appreciated that the authors are dealing with fuses of low-current ratings and that for these particular fuses, the errors in the assumptions are on the safe side.

The statement that the commercial development of the current-limiting fuse for use in this country was abandoned is evidently not intended by the authors to apply generally since this development has been carried forward with very practical results.

A. D. Fishel (A. D. Fishel Company, Cleveland, Ohio): The authors of this paper are to be complimented on their novel approach to designs of current-limiting fuses.

In the calculations under "Theory and Calculations" it is believed that the energy required to supply the latent heat of fusion, is a factor which should also be given consideration in the computation of the maximum value of initial current rush, as a function of fuse wire mass and the prospective fault current.

An ingenious means is employed to suppress abnormal voltages caused by the rapid suppression of current during a short circuit. The voltage surge which occurs during the operation of a current-limiting fuse, is a function of the current magnitude reached prior to the instigation of the current-suppressing process and of the speed with which the current suppression takes place. This means that the problem becomes more difficult as the current-carrying ability of the fuse or the time-lag characteristics of the current-responsive element, are increased.

During the past four years, Schweitzer and Conrad, Inc., have manufactured current-limiting fuses for installations with capacitors and have also a considerable number applied in potential transformer installations on industrial systems, as well as in other specific applications where these fuses are installed to obtain operating experience.

While operating results have been satisfactory, it is felt that the current-limiting fuse is not a substitute for the resistor and fuse combinations, which have found wide application in the past. This is due to these facts: the flashover value of a current-limiting fuse following low-current operation, is less than that of the conventional fuse; the current-limiting type does not possess the same flexibility in time-current characteristics; voltage surges occur during the operation of a current-limiting fuse which do not occur when the conventional fuse and resistor combinations are applied.

The authors' design embodies a general arrangement and function of elements similar to those which Schweitzer and Conrad applied in current-limiting fuses, that is, the spring-actuated, current-responsive element performs the dual function of protecting the current-limiting element during low current, to clear the circuit and obtain a suitable gap between the severed ends. In the authors' design, the clean-up element performs the added function of preventing leakage through the current-limiting portion of the fuse, after operation on heavy prospective fault currents. In the Schweitzer and Conrad design, this duty is not required, because during short circuit, the current-limiting section of the fuse changes from low-resistance conductor to an insulating oxide.

It is the opinion that current-limiting fuses will distinctly improve fuse installations which now contain powder-filled fuse assemblies of insufficient interrupting capacity, and where the available space does not permit the conversion to other modern types of protective equipment. It is felt that for installations in which absolute dependability is required, the current-limiting fuse is not the operating equivalent of either fuse and resistor combinations or modern power fuses.

H. L. Rawlins, A. P. Strom, H. W. Graybill: The discussions presented emphasize both the scope of application and the limitations of the current-limiting fuse. Mr. Fishel points out that the current-limiting fuse has the disadvantage of producing voltage surges during operation. This has been one of the major limitations of current-limiting fuses, and was one of the major factors involved in the design of the new fuse. As stressed in the paper, the construction and characteristics of the fuse are such that overvoltages are definitely limited.

It is further stated that current-limiting fuses have low flashover values after low current operations. To assure high flashover values in this new fuse, the main element is spring operated. A positive gap is, therefore, introduced, even when the fuse operates on very low currents.

Mr. Fishel points out that current-limiting fuses do not have the same desirable time-current characteristics as conventional fuses, and Mr. Williams wonders how suitable time-current characteristics can be achieved in these fuses when the main element must blow before the parallel current-limiting element.

In the new fuse, the main element is essentially a conventional fuse. In order to maintain the parallel current-limiting element—operating in series with the main element before fusing—at a low temperature, it is essential that the minimum or long-time melting current of the main element be only a small fraction of that of the parallel current-limiting element. This allows a wide range in the short-time melting current of the main fuse element without exceeding safe temperatures in the current-limiting element. This range is sufficient to cover any desired time-current characteristics.

The question is raised concerning the effect of the latent heat of fusion on the fusing time. With the current increasing at a constant rate as occurs in this fuse, at the time of fusion, the increase in the calculated fusing time of a silver wire is only five per cent or six per cent when the latent heat of fusion is included. Photographic and experimental evidence proves that only a small part of the heat of fusion has been supplied when arcing begins. These two factors show that the effect of the latent heat of fusion can safely be neglected in calculating the current at the time arcing starts.

As explained in our paper, the equations give the maximum possible value of fusing current as a function of the symmetrical available short-circuit current. Under conditions that give the maximum possible fusing current, symmetrical currents are essentially linear with time up to the fusing point, and follow the law, $i = 377 I_m t$. Practically all currents that are large enough so that the current-limiting function of the fuse comes into play lie in this range.

It should be emphasized that even on asymmetrical currents, fusing currents will never rise higher than calculated by these equations since the average slope of the current curve is then always less than $377 I_m$. On fusing currents that depart from linear due to fusing times longer than about one-sixth half cycle, the actual fusing currents must always be smaller than calculated by the equation. Likewise, on short circuits in which the current is limited by the resistance of the fuse element itself, the maximum fusing current will obviously be smaller than the calculated value.

In deriving the equation, it was assumed that the fuse wire itself, before melting, does not limit the current. This is the worst condition that can be imposed on a current-limiting fuse, and hence always gives conservative results. The departure from the results computed by the equation will depend on the relative increase in impedance introduced by adding the fuse to the circuit. Fuses having low-ampere ratings have high impedance and, therefore, depart farther from calculated results than larger ampere rating fuses. Fuses of the same ampere ratings depart more from the calculated values on large short circuits than on smaller short circuits, and more on low-voltage circuits than on high-voltage circuits.

Curves such as given in Mr. Williams' discussion can, therefore, apply only for one particular design of fuse operated at one particular voltage. Had these data been obtained at, say, ten times the voltage, these curves would undoubtedly approach the limiting conditions as expressed by our equations.

In the second example given in Mr. Williams' discussion where the instantaneous current reaches 10,000 amperes on a 10,000-ampere asymmetric circuit, the fusing current is actually higher than the symmetrical peak. In this case, no current limiting occurs on symmetrical currents, and only slight limiting on full asymmetrical currents. The observed fusing current must, therefore, be lower than calculated by our equation, and cannot be compared to the 80,000-ampere case.

However, if we avoid currents so low that fusing requires most of the half cycle, or so high that the fuse element comprises an appreciable part of the circuit impedance, the data presented by Mr. Williams shows that fusing currents double when the available current increases by eight times, as predicted by the equation. This is shown in the accompanying tabulation of data from Mr. Williams' curves (table I).

Table I

Fuse Rating	Fusing Currents (Amperes) 1,000 Available	8,000 Available	2,000	16,000	5,000	40,000	Ratio of Fusing Currents
3/3 *N*	253	445					1.76
7/5 *N*	350	670					1.92
10/7 *N*	455	905					1.99
15/10 *N*			780	1,530			1.96
25/15 *N*			1,080	2,130			1.97
30/20 *N*					1,900	3,750	1.98
40/25 *N*					2,200	4,300	1.95
50/30 *N*					2,450	4,850	1.98

The equations given in this paper are particularly useful in understanding the operation of these fuses, and in making preliminary design calculations.

This new fuse has characteristics that make it particularly desirable for the protection of potential and small power transformers. The fuse shown in the paper was for such service. However, the design is particularly applicable for higher current ratings where the voltage-limiting characteristic becomes of utmost importance.

Higher voltage surges will occur in a current-limiting fuse where the resistance of the element does not limit the current than where the resistance of the element limits the current. Therefore, the higher ratings of fuses tend to produce higher voltages during operation.

Standard power fuses have been used for years to protect low-rated equipment. Except in the case of extremely low-rated equipment, as mentioned by Mr. Williams, the equipment has been adequately protected. With current-limiting fuses operating in series, the voltages produced during operation add. In the case of conventional fuses, the voltages do not add. The field for current-limiting fuses, therefore, will probably be limited to those applications where their special characteristics are required.

The "Black Band" Method of Commutation Observation

Discussion and authors' closure of paper 40-138 by T. W. Schroeder and J. C. Aydelott, presented at the AIEE Middle Eastern District meeting, October 9–11, 1940, and published in AIEE TRANSACTIONS, 1941, pages 446–51.

T. M. Linville (General Electric Company, Schenectady, N. Y.): This paper ably describes a valuable method. However, it does not point out certain implications of the data presented which are important.

First, a viewpoint of "differential reactance voltage" is frequently used which differs from the concept explained under "Differential Reactance Voltage" (part 4). A coil short-circuited by the brushes (that is, undergoing commutation) is considered to have three voltages, a "reactance voltage" (due to self and mutual induction), a generated voltage (due to the commutating field), and a brush voltage. In an ideal machine, when the reactance voltage and generated voltage are equal there is no brush voltage (for the coil) and no differential voltage. However, if by misadjustment the generated voltage is too big or too little the difference between it and the ideal reactance voltage will not appear as a brush voltage entirely, as suggested by the paper, but will be absorbed in a change in the reactance voltage occurring simultaneously with the appearance of brush voltage.

In a practical machine it is never considered possible to balance the two voltages bringing the brush voltage to zero because the wave shapes cannot be made the same. I believe this difference in wave shape or in instantaneous values is the commonly accepted definition of "differential reactance voltage." This difference can and does exist even when the machine is correctly adjusted in accordance with the black band. It is partly the cause of black-band closure and is one source of the evil when no black band can be obtained for any commutating field adjustment.

It is interesting to note that the majority of the black bands illustrated are substantially uniform in width independent of load. This would indicate that either the reactance and generated voltages do nearly balance each other at all instants, or else the brush voltage is able to take care of this difference without affecting the amount of buck and boost that can be taken care of.

These black bands suggest that there is no limit to permissable reactance voltage since this voltage increases with load while the bands maintain uniform width without tendency to close. In their terms commutation is no better at half load for example than at full load. However, on variable-speed motors the bands invariably are narrower at the top speed, where for equal loading the reactance voltage is higher in proportion to the speed.

The black-band method is not an inflexible measure of commutation performance. For example, a machine having very little commutating pole air gap and little nonmagnetic shims will require less commutating pole ampere-turns to establish the commutating field than a similar machine having twice as much total commutating pole air gap. In the second case the buck and boost applied to the commutating field will be about twice that of the first case for the same effect on commutation. In other words the width of the black band in terms of amperes and in terms of per cent of rated current will be twice as great. And yet the commutation will be no different in the two cases and the margin for changes in the commutator film, brush riding conditions, etc., will be alike. However, the commutating pole shims or the shunted amperes will not have to be adjusted so precisely in the second case.

Mention is made of the effect of commutating pole tip width on the commutation and in agreement with the paper d-c designers seek to determine the optimum width. In this connection figure 11 is interesting showing the effect when the brushes are shifted so that part of the main-pole flux joins the commutating field flux. Varying the main-pole flux from zero to full voltage has substantially no effect on the width of the black band, even though it is equivalent to extraordinary distortion of the commutating field flux. Apparently the distribution is not important in this case and the commutating coils transfer the generated voltage by induction. The paper suggests this in stating that the "trailing edge spark does not appear to be critical of tip width unless the rotation of the machine is such that the last conductor to commutate in a slot is a top conductor." It should be pointed out that many reversible motors commutate equally well in both directions of rotation, although the last conductor is a top one with one rotation and a bottom one when running oppositely.

It is believed that the table under "Black Band Width and Closure" must mean that the narrow black bands with 5,000 feet per minute commutator speed is caused by higher reactance voltage in proportion to the speed and not by the commutator speed itself.

Black-band tests are sometimes made after a fashion without a source of buck and boost power. This is done by replacing all nonmagnetic shims with steel to produce a strong commutating field. The strength of the field may then be varied by shunting current to determine the range of adjustment and the amount of shims to be used or current to be shunted to center the band.

T. W. Schroeder and **J. C. Aydelott:** We appreciate the interest which Mr. Linville has shown in commenting on our paper. His remarks are in large part a valuable addition to the material in the paper. Toward the end of his discussion, however, Mr. Linville makes the statement that "many reversible motors commutate equally well in both directions of rotation." We can agree with Mr. Linville if we can interpret this statement to mean that many reversible motors commutate successfully in both directions of rotation. The fact, then, that one rotation was better than the other would naturally not be observed and would be of no particular consequence, since the commutation in the poorer rotation was entirely acceptable. We wish to repeat, however, that many machines show a distinct difference in commutation in the two rotations. This is sometimes most evident before the final adjustment of commutating pole-face width is selected, and may be minimized if equalizing the commutation in the two rotations is one of the governing considerations in the choice of commutating pole-face width. As stated in the paper, we trace this difference in behavior in the two rotations to the fact that in one rotation the last conductor in the slot to be commutated is in the top polycoil in the slot, whereas in the other rotation, the last conductor to be commutated in the slot is in the bottom polycoil.

In his next paragraph, Mr. Linville comments upon the effect of commutator speed on the closure of "black bands." While we agree that increased reactance voltage proportional to increased speed does tend to cause a narrowing of the "black band" and eventual closure, we also believe that it is a matter of common observation that particularly in the high-speed range, commutator speed itself can be a cause of sparking. This is sometimes called mechanical sparking. Practically, this means that a designer must use lower values of reactance voltage to insure the successful operation of machines with high commutator speeds, as compared with the relatively high values of reactance voltage which can be operated successfully with low-speed designs.

A Comparison of the Relative Efficiency of the Schafer and Pole-Top Methods of Artificial Respiration

Discussion and authors' closure of paper 40-150 by W. B. Kouwenhoven, D. R. Hooker, and O. R. Langworthy, presented at the AIEE Middle Eastern District meeting, Cincinnati, Ohio, October 9–11, 1940, and published in AIEE TRANSACTIONS, 1941 (February section) pages 41–4.

H. B. Williams (nonmember; Columbia University, New York, N. Y.): It seems likely that in any case requiring resuscitation the chance that the effort will be successful diminishes in porportion to the time, perhaps more rapidly than a simple proportion. Mr. Oesterreich's work in the devising of the method of pole-top resuscitation has provided means for commencing this procedure with a minimum of delay. Dean Kouwenhoven and his associates have performed a most commendable service in showing its actual efficiency as a means of ventilating the lungs. I suspect that it does more than this.

Ventilation of the lungs would be futile if movement of blood did not carry the oxygen to the tissues and carbon dioxide away from them. There is good reason to believe that following release from a power line a shocked person will suffer a considerable fall in blood pressure. It seems likely that the pole-top method of resuscitation is also effective in promoting movement of blood. It was shown some years ago at the Physiological Laboratory of Columbia University that even in sheep with ventricular fibrillation it is possible by vigorous manual effort to maintain some circulation. The effect on circulation may be as important an element in the success of the new method in the field as is the promptness with which it can be applied.

This all being admitted, it must still be borne in mind that the pole top is a dangerous place. The position of the rescuer below the victim tends to lessen the danger to the former, though sometimes the accident may be due to electrification of parts of the structure other than the normal conductors and this may increase the danger to the rescuer. It not infrequently happens that the beginning of recovery is attended with convulsions. This was recognized by Mr. Oesterreich in his original communication. That this involves an element of danger is shown by a case brought to my attention last week. An injured lineman was being resuscitated, apparently with success. A convulsion occurred and his hand came in contact with a live wire so that he received a second shock from which it was impossible to resuscitate him. The danger to a rescuer under such circumstances is evident. Such an occurrence might have involved a fatal shock to the rescuer also. It may be possible to devise some simple means of confining the injured person's arms. I believe Mr. Oesterreich recommended removal of the spurs to prevent injury to the rescuer by convulsive movement of the legs.

While I am loath to deny to any injured person the benefit of any method which may offer a better chance of survival I think these considerations should be weighed and that crews should be trained to use judgment as to when the danger of attempting the pole-top method is too great. Any company which has not as yet used the method will do well to secure the services of someone who has had experience to assist in training its crews.

The above represents the personal views of the speaker only as the Resuscitation Review Board of the Edison Electric Institute has not yet formulated an opinion on this matter.

W. B. Kouwenhoven: Dr. Williams' discussion is most gratifying and we welcome his suggestions. This paper presents the report of the first stage of our investigation. The work is being continued. We are now studying the effect of the time elapsing between the shock and the starting of artificial respiration on the chances of recovery. In addition, we plan to determine the influence of artificial respiration upon the circulation of the blood.

The Use of Auxiliary Impedances in the Single-Phase Operation of Polyphase Induction Motors

Discussion of paper 40-139 by Raymond W. Ager, presented at the AIEE Middle Eastern District meeting, October 9–11, 1940, and published in AIEE TRANSACTIONS, 1941, pages 494–9.

Wayne J. Morrill: See discussion, page 669.

R. J. W. Koopman (University of Kansas, Lawrence, Kans.): Professor Ager has taken a problem and has made interesting analyses of many of its ramifications. A study of this paper should be valuable to students trying to gain an understanding of capacitor and split-phase motors.

An interesting point comes up on further consideration of the operation of the motor shown in figure 9 of the paper at less than 70.7 per cent power factor.

The accompanying figure is figure 9 redrawn for a power factor of less than 70 per cent. The dotted lines represent I_1 and I_2 when Z_2 is a resistance and Z_1 is a perfect condenser. The solid lines represent the currents when Z_2 is still a resistance, but disconnected from I_1, and connected in parallel with Z_1. It is clear from the figure that less condenser capacity is required in the latter case than in the former.

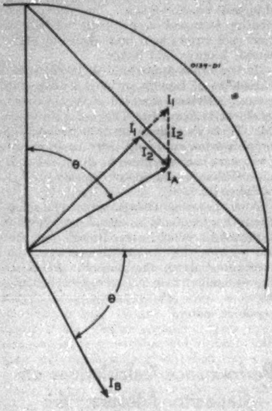


Figure 1

In other words, we have an instance where a leaky or imperfect condenser is superior to a perfect one.

The same connection could be used to give balanced motor voltages for the three-phase motor of figure 5 if it were operating below 50 per cent power factor.

P. L. Alger (General Electric Company, Schenectady, N. Y.): The thought developed by Professor Ager that single-phase capacitor motors should be designed for operation with nearly balanced currents, approaching polyphase conditions, is logical. Many engineers, including especially Biermanns in Germany, have proposed means to this end, but the narrow load range over which any particular set of impedance values gives approximate phase balance makes such balanced operation of only secondary importance.

Under present conditions capacitor motors are usually operated at speed as plain single-phase motors, and capacitors are only used in the running connections when unusually high starting torque per ampere is required. In these cases, the running capacitor is made just large enough to give the desired starting performance, and the resulting characteristics at full speed are accepted without further consideration.

Professor Ager has also mentioned the possibility of using double squirrel-cage rotors in capacitor motors as a means of obtaining better starting conditions and lower full-load slip. These benefits are secured if the motor is operated at a load close to the point of phase balance. However, the large increase in the resistance of a double squirrel cage for double frequency currents makes the extra losses due to the backward revolving field very large, cutting down the motor torque and raising its heating. For most practical motor applications this disadvantage has been found to more than offset the possible gain. For a particular application where the operating load is fixed and known in advance, however, a double squirrel-cage rotor in conjunction with a capacitor adjusted to give balanced operation should give very desirable characteristics.

Performance Calculations on Repulsion Motors

Discussion of paper 40-152 by P. H. Trickey, presented at the AIEE Middle Eastern District meeting, Cincinnati, Ohio, October 9-11, 1940, and published in AIEE TRANSACTIONS, 1941 (February section) pages 67-73.

Wayne J. Morrill: See discussion, page 669.

T. C. Lloyd (Robbins and Myers, Inc., Springfield, Ohio): Mr. Trickey's paper presents a useful extension of the West analysis, by a tabulation simplifying the calculation processes. It is true that such calculations are made at comparatively rare intervals but a much more common problem is that of the starting performance. Inherent with this are the effects of saturation and brush drop. On the latter point Mr. Trickey has indicated a process for obtaining r_3 with no mention of comparative accuracy for starting torque calculations against tests, nor the upsetting effects of brush pressure and temperature.

Mr. Trickey has indicated in the discussion that the accuracy to be expected is not greater than about 25 per cent in calculating starting torque and amperes. This is about the range of accuracy which we have obtained in our own experience. Yet the fault is not due so much to the analysis as to the inherent nature of the problem. Calculations made with older methods of analysis neglecting the effect of r_3 have been off as high as 300 per cent when compared to tests.

Because of this lack of agreement between tests and calculation I question the practicality of following through the involved calculations for starting torque and current. Thinking that some approximations might be justified the following analyses were made.

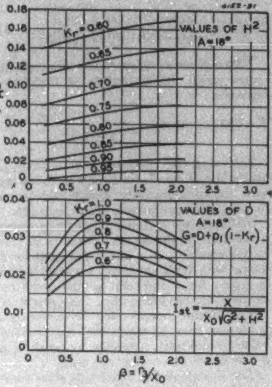


Figure 1

The final torque equation eliminates the effect of r_3 but in light of Mr. Trickey's curves of figure 5 there is some question as to whether such neglect is justified. But by using ratios of constants, curves could be drawn as shown, greatly simplifying the work.

At standstill, West's equation for current becomes:

$$V=I_1\left[jX_m+X_m{}^2\left(\frac{\cos^2 A}{r_3+jX_0}+\frac{\sin^2 A}{r_3+jX_0''}\right)+r_1+jX_1\right] \quad (1)$$

Table I. Calculated and Test Data

Horsepower	3	2	1½	1	¾
Poles	4	4	4	4	4
Com. bars	79	79		79	79
Cond. (primary)	432	576	616	792	1,008
Turns per coil	2(2#12)	2(2#13)	(3#16)	3(2#16)	4(2#17)
Shift angle	18°	18°	18°	18°	18°
Coils shorted	2	2	2	2	2
Length conductor	11.18"	9.62"	8.0"	7.25"	6.76"
Winding factor	0.78	0.78	0.78	0.78	0.78
K_r	0.938	0.926	0.925	0.933	0.932
Ratio a	42.2	56.25	40.2	51.6	49.2
r_1/X	0.166	0.20	0.259	0.263	0.279
r_{coil}	0.0068	0.00745	0.0186	0.0169	0.0264
r_{brush}	0.0115	0.014	0.014	0.0174	0.0232
r_3	32.5	67.7	52.5	91.0	120
X_0	40.0	56.2	64.5	93.7	118
β_2	0.810	1.2	0.811	0.975	1.02
G_2	0.00196	0.00243	0.00274	0.00276	0.0029
H	0.008	0.014	0.011	0.011	0.012
I_1 (calc.)	55.1	30.5	28.2	20.1	15.25
I_1 (test)	61	32.0	32.2	19.0	14
T syn. watts	12,000	7,650	5,350	4,550	3,490
T oz-ft (calc.)	750	479	335	285	217.5
T oz-ft (test)	680	472	370	270	201

This reduces to:

$$I_1=\frac{V}{X_0\sqrt{G^2+H^2}} \qquad (2)$$

Where G and H are derived from curves as shown, plotted for a definite brush shift (18 degrees).

The starting torque equation is:

$$T=I_1^2X_m^2\frac{(r_3^2-r_2^2)X_0 \sin A \cos A}{(r_2r_3-X_0^2)^2+(r_2+r_3)^2X_0^2} \qquad (3)$$

This reduces as shown below by dropping terms which appear to be of little significance:

$$T=I_1^2K_rX_0 \sin A \cos A \frac{\beta^2}{\beta^2+1} \qquad (4)$$

Following this procedure one obtains I_1 by the use of curves for G and H, following with the comparatively simple calculation for torque in synchronous watts from (4).

Although one hesitates to confess to such unscientific methods, one advantage of this torque equation rests in the fact that for a motor with tests available, equation 4 can be solved for β and then for the brush resistance to be used with that grade of brush. Future designs using the same brush grade can then be made with the verified value of resistance, corrected by brush area if new brush dimensions are used.

The accompanying tabulation gives comparative calculated and test results for a line of motors from three-fourths to three horsepower. Although the accuracy is fair, such consistent results are not to be expected, and we are not using the method as a standard part of design procedure.

It is my belief that the main source of error, outside of unpredictable temperature effects, is the saturation of the magnetic circuit. We were recently called upon to design a repulsion-start induction motor crowded down to about half of its usual volume. Fairly satisfactory running performance was obtained, but whereas we had every reason to expect a good starting torque by comparison with previous designs, the starting torque per starting ampere was only half of that obtained in the more generously proportioned motors. No design changes could be found to improve this condition, which could be attributed only to the high degree of saturation of the magnetic circuit. Doubtless this exists to a lesser extent as an unpredictable source or error in all of these calculations.

Our experience with equations 2 and 4 indicates the chief source of error is in the current calculation. That is, when calculated values are finally compared to test results, both current and torque may be off. But when starting amperes by test are substituted in the torque equation, close agreement is brought about. This would indicate a fairly inherent accuracy in the approximations of the torque equations but that current predictions are effected by physical conditions not yet taken into account.

SYMBOLS. (DIFFERING FROM WEST)

$$F=\sqrt{K_r}-K_r\cos^2 A-K_r\frac{\sin^2 A}{\beta^2+1}$$

$H=F+0.5(1-K_r)$

$D=(K_r\beta \sin^2 A)+(\beta^2+1)$

$G=D+p_1(1-K_r)$

$p_1=r_1/X$

$p=r_3/X$

$\beta=r_3/X_0$

C = series conductors on stator

k_w = winding factor, stator

l_c = length of stator conductor in inches

a = resistance of wire in short-circuited coils

r_b = resistance of brush contact in short-circuit path

$$a=\frac{0.5\,Ck_w}{\text{series turns per coil}\times\text{coils shorted under one brush}}$$

$r_3=(r_{\text{coil}}+r_{\text{brush contact}})xa^2$

F. J. Johns (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The resistance of the short-circuited coil circuit is the most difficult of the constants to calculate. Mr. Trickey has called this resistance r_3.

Usually the brush width is slightly less than two commutator bars. In this case several conditions can exist at starting. One or more brushes may be on only two bars in which case only one coil will be shorted. One or more brushes may be on three bars in which case several coils will be shorted. From the number of bars in the commutator and brush width the condition of maximum number of coils should be calculated as this will give the minimum torque at the instant of starting.

A quite satisfactory method of calculating r_3 is to calculate the voltage across the shorted coils. This can be done from primary flux, turns in shorted coil, Kw_3 of shorted coil, and position of coil determined by brush shift. From this voltage subtract the brush drop which is about 1.5 volts for normal brushes at about 200 amperes density or use a curve for the brush in question and at the pressure used on the motor. This voltage divided by shorted coil impedance will give the current. It is usually assumed that the reactance is small so the resistance alone is used. Dividing the total voltage of coil by current will give the value of r_3 to be used. The transformation ratio must be applied to reduce to primary circuit.

As a check the short-circuited current plus the load current can be added to get brush density. If considerably different from assumed density of 200 amperes per square inch, a new calculation of r_3 can be made.

A very important point which should not be overlooked is the change in total flux in the motor. At start this flux is a maximum and as rotor speeds up the flux is reduced. However, this may not be as important on a small motor as it is on a larger one. The magnetizing voltage may be reduced as much as 20 per cent from start to full load on a repulsion-start and run motor.

C. G. Veinott (Westinghouse Electric and Manufacturing Company, Lima, Ohio): In figure 10 of this paper, Mr. Trickey shows the variation of contact drop and contact resisitvity in terms of the amperes per square inch of contact area. This curve shows a wide variation in contact resistance with brush density and might, therefore, throw suspicion on the correctness of assuming a constant-contact resistance in the performance calculations. Apparently the brush curves shown by the author were taken under steady conditions, that is with the particular current density maintained until the voltage drop had become constant. This is the usual manner of taking brush-drop curves. What really should be used in this paper is the dynamic characteristic curve, that is the relation between brush drop and brush resistance when the current density is varied very rapidly; I believe that such curves have been shown by Hellmund, Ludwig, and Baker.

If the brush-drop curve is taken under conditions of rapidly varying density, the contact resistance does not change as much as shown in figure 10. This being the case, Mr. Trickey's assumption that the contact drop does not vary with the current, therefore, more nearly represents the actual conditions than would appear from inspection of figure 10.

A word of commendation should be given the author for giving his calculation schedule in figure 12, which makes it very easy for anyone to use his method without fear of an error in applying the method. Also, his curves figures 2 to 9, inclusive, are enlightening as to some of the characteristics of the repulsion motor.

Performance Calculations on Capacitor Motors—the Revolving-Field Theory

Discussion and author's closure of paper 40-153 by P. H. Trickey, presented at the AIEE Middle Eastern District meeting, Cincinnati, Ohio, October 9–11, 1940, and published in AIEE TRANSACTIONS, 1941 (February section) pages 73–6.

Wayne J. Morrill: See discussion, page 669.

P. L. Alger (General Electric Company, Schenectady, N. Y.): All of the papers on single-phase motors presented today recognize the need for more exact and complete performance calculations; but each author has a different procedure, with its own particular advantages.

One method of attack which has always been attractive to me is that of developing exact formulas or charts for the principal characteristic quantities, such as the maximum torque, on the basis of simplified conditions, and then using approximate formulas to calculate the correction necessary to allow for variations from the assumed conditions. This method was followed in my 1930 AIEE paper on "Induction Motor Performance Calculations." It seems to me that a similar method of attack on the capacitor-motor problem would enable some useful formulas to be derived particularly for the calculation of the breakdown torque.

Exact formulas for the breakdown torque in the case of a plain single-phase motor are available, and it should be a simple matter to prepare a tabular form or a chart for the calculating of this quantity, as a function of the four variables, r_1, r_2, x_1+x_2, and X_m. Core loss may be neglected in this case.

Then, since the employment of a normal capacitor in the auxiliary winding only slightly increases the maximum torque, it should be possible to develop a second formula or chart for the approximate calculation of this increment in the breakdown-torque value. Such formulas would facilitate quick calculations and would also enable the relative effects of various changes in the design constants to be estimated.

L. W. Buchanan (Westinghouse Electric and Manufacturing Company, Lima, Ohio): The apportioning of iron loss and the inphase magnetizing current has caused quite a bit of trouble in the calculation of capacitor motors by the double revolving-field method. In a number of calculations based on constants derived from test values, I have found, as did Mr. Trickey, that the calculated values agreed with test values more closely if the inphase current component of 60-cycle core loss were added to the main phase-calculated values of current for purposes of figuring the input power and current. These values were calculated on the form originated by Mr. Veinott and described in item number 2 in the bibliography of Mr. Trickey's paper. These calculations gave me values which check the main and auxiliary phase watts and amperes within ten per cent, with the main phase watts always having a tendency to be lower than tested values even after adding the 60-cycle iron losses. Does the author in his calculations, obtain the same tendency?

To follow the calculation as outlined in this paper, it is necessary to separate the 60-cycle iron losses from the high-frequency iron losses. I should like to inquire, if the author uses tested values of reference motors, how he separates these losses.

The single-value capacitor motor is often used on variable-speed applications which makes it necessary to calculate the complete speed-torque curve. It seems to me that it might be more accurate to calculate the output torque and then subtract a torque equivalent to the high-frequency loss and the friction and windage loss. The torque corresponding to both of these losses, I believe, will decrease with a decrease in speed of the motor. If one subtracts the high-frequency iron loss and the friction and windage loss from the motor output in watts, he is reducing the output torque by a greater amount for low speeds than he is for high speeds.

For those of us who prefer to do our vector division in polar form the equation taken from Morrill's paper and translated into the terms of this paper of

$$T=R_f(I_m^2+K^2I_a^2+2KI_mI_a\sin\phi)+R_b[2KI_mI_a\sin\phi-(I_m^2+K^2T_a^2)]$$

where ϕ is the angle between the two currents is very convenient for calculating.

A. F. Puchstein (Robbins and Myers, Inc., Springfield, Ohio): Mr. Trickey states that it is not possible to include the stator iron loss in the primary part of the revolving-field equations without making them too complicated, and that good results may be obtained by assuming all of this loss to take place in the main phase external to the machine itself.

In regard to the first of these the performance may be calculated with theoretical accuracy and minimum labor (when low- and high-frequency iron losses are included, with upper harmonics neglected) by the following procedure.

To obtain r_0, select values for low-frequency core loss F_{e1}, and cemf E_{m1} equal to say, $0.85V$, then $r_0 \cong \frac{E_{m1}^2}{Fe_1}$. Next arrange in parallel columns,

Forward field	Backward field
$\frac{1}{r_0} \quad =$	$\frac{1}{r_0} \quad =$
$\frac{-j}{X_m} \quad =$	$\frac{-j}{X_m} \quad =$
$\dfrac{\left(\frac{R_2}{s}\right)-jX_2}{\left(\frac{R_2}{s}\right)^2+X_2^2}=$	$\dfrac{\left(\frac{R_2}{2-s}\right)-jX_2}{\left(\frac{R_2}{2-s}\right)^2+X_2^2}=$
$\text{Sum}=g_f-jb_f$	$\text{Sum}=g_b-jb_b$
$Z_f=\frac{g_f+jb_f}{g_f^2+b_f^2}=$	$Z_b=\frac{g_b+jb_b}{g_b^2+b_b^2}$
$=R_f+jX_f$	$=R_b+jX_b$
$R_{11}=R_f-R_b$	$X_{11}=X_f-X_b$

$$\begin{array}{rr}
r_{1M}=(\quad) & X_{1M}=(\quad)\\
R_f=(\quad) & X_f=(\quad)\\
R_b=(\quad) & X_b=(\quad)\\
\hline
R_{1M}=(\quad) & X_{1M}=(\quad)\\
-X_{11}=-(\quad) & +aR_{11}=(\quad)\\
\hline
(\quad) & +j\quad(\quad)
\end{array}$$

= numerator for I_M

$$\begin{array}{rr}
r_{1S}=(\quad) & X_{1S}=(\quad)\\
a^2R_f=(\quad) & a^2X_f=(\quad)\\
a^2R_b=(\quad) & a^2X_b=(\quad)\\
R_c=(\quad) & X_c=-(\quad)\\
\hline
R_{1S}=(\quad) & X_{1S}=(\quad)\\
+X_{11}=(\quad) & -aR_{11}=-(\quad)\\
\hline
(\quad) & +\ j\quad(\quad)
\end{array}$$

= numerator for I_S

$$\begin{array}{rr}
+R_{1M}R_{1S}=(\quad) & \\
+a^2X_{11}^2=(\quad) & +R_{1S}X_{1M}=(\quad)\\
-X_{1M}X_{1S}=(\quad) & +R_{1M}X_{1S}=(\quad)\\
-a^2R_{11}^2=(\quad) & -2a^2R_{11}X_{11}=(\quad)\\
\hline
(\quad)\quad + & j\quad(\quad)
\end{array}$$

= denominator for I_M and I_S

Currents:

$$I_M=V\cdot\frac{\text{numerator for } I_M}{\text{denominator}}=A+jB \quad (B \text{ is negative})$$

$$I_S=V\cdot\frac{\text{numerator for } I_S}{\text{denominator}}=g+jh$$

Symmetrical components:

Positive sequence

Current, $I_{m1}=\frac{I_M-jaI_S}{2}$ (vectors)

Cemf, $E_{m1}=2I_{m1}Z_f$ (absolute)

Negative sequence

$$I_{m2}=\frac{I_M+jaI_S}{2}$$

$$E_{m2}=2I_{m2}Z_b$$

Low-frequency core loss (theoretically accurate components),

$$Fe_1=\frac{E_{m1}^2}{r_0}\text{ watts}$$

$$Fe_2=\frac{E_{m2}^2}{r_0}\text{ watts}$$

Internal torques:

$$\begin{array}{lr}
G=2a(Ah-Bg)= & (\quad)\\
I_M^2= & (\quad)\\
a^2I_S^2= & (\quad)\\
\hline
\text{Sum}_f= & (\quad)
\end{array}$$

$$\begin{array}{lr}
-G= & -(\quad)\\
I_M^2= & (\quad)\\
a^2I_S^2= & (\quad)\\
\hline
\text{Sum}_b= & (\quad)
\end{array}$$

$$\begin{array}{lr}
T_f=\text{Sum}_f\times R_f & =(\quad)\\
T_b=\text{Sum}_b\times R_b & =(\quad)\\
\hline
T=T_f-T_b & =(\quad)
\end{array}$$

$$T_{F\&W}=\frac{F\&W}{1-S}$$

$$T_{Fe}=\frac{Fe\text{ high frequency}}{1-S}$$

Shaft torque:

$T_0=T-T_{F\&W}-T_{Fe}$ synchronous watts

$$\text{oz-ft}=\frac{112.6\times T_0}{\text{syn.}}$$

Shaft output:

$=W_0=(1-s)T_0=(\quad)$ watts

Input:

$W_t=V(A+g) \quad =(\quad)$ watts

Losses:

$W_L=W_t-W_0$ watts

For checking,

Pri. copper loss, M	$=I_M^2r_{1M}$	$=(\quad)$	watts
Pri. copper loss, S	$=I_S^2r_{1S}$	$=(\quad)$	watts
Capacitor loss	$=I_S^2R_C$	$=(\quad)$	watts
Sec. forward loss	$=sT_f$	$=(\quad)$	watts
Sec. backward loss	$=(2-s)T_b$	$=(\quad)$	watts
F & W loss	$=$estimate	$=(\quad)$	watts
Fe forward loss	$=\frac{E_{M1}^2}{r_0}$	$=(\quad)$	watts
Fe backward loss	$=\frac{E_{M2}^2}{r_0}$	$=(\quad)$	watts
Fe high frequency loss	$=$estimate	$=(\quad)$	watts
Total loss	$=W_L$	$=(\quad)$	watts

As pointed out in Trickey's paper, it is difficult to make both sets of figures agree in slide-rule calculations.

When the symmetrical components I_{m1} and I_{m2} are known, an alternate way to find the internal torques is,

$$\frac{1}{r_0}=g$$

$$\frac{-j}{X_m}=-jb$$

$$\underline{\qquad}=$$

$$\text{Sum}=g-jb$$

$$Z_\varphi=\frac{g+jb}{g^2+b^2}=R_\varphi+jX_\varphi$$

$$\begin{array}{l} Z_{r1}=\qquad\frac{r_2}{s}+jx_2 \\ \hline Z_\varphi+Z_{r1}=\text{sum}=(\qquad) \end{array}$$

$$r_2=2\times\text{Morrill's } R_2$$
$$x_2=2\times\text{Morrill's } X_2$$
$$X_m=2\times\text{Morrill's } X_m$$
$$Z_\varphi=R_\varphi+jX_\varphi$$

$$\begin{array}{l} Z_{r2}=\frac{r_2}{2-s}+jx_2 \\ \hline Z_\varphi+Z_{r2}=\text{sum}=(\qquad) \end{array}$$

The calculation is then completed in the same manner as before.

$$T_f=2\times\frac{|Z_\varphi|^2}{|Z_\varphi+Z_{r1}|^2}\times|I_{m1}|^2\times\frac{r_2}{s}\quad\text{synchronous watts}$$

$$=2\times\frac{|Z_\varphi|^2}{|Z_\varphi+Z_{r2}|^2}\times|I_{m2}|^2\times\frac{r_2}{2-s}\quad\text{synchronous watts}$$

P. H. Trickey: In reply to Mr. Alger, I might say that we do have a means of obtaining breakdown torque which has been developed much in the way he has suggested.

We have a formula for single-phase motors which I believe was largely due to Mr. C. G. Veinott with a correction which I added later for cases of high rotor resistance or high magnetizing current.

$$\underset{\text{(single phase)}}{T_m}=\left(\frac{E}{110}\right)^2\left(\frac{p}{4}\right)\left(\frac{60}{f}\right)\times\frac{326-8.6\left(\frac{r_1+r_2}{X}\right)^2}{r_1+r_2+X}\;\frac{K_r}{1+\frac{r_2^2}{X_0}}\text{ oz-ft}$$

where

E = line volts
P = poles
f = frequency
r_1 = primary resistance
r_2 = secondary resistance referred to primary
X = total leakage reactance $=x_1+x_2K_p$
X_0 = open circuit resistance $=X_m+x_2$
X_m = mutual or magnetizing reactance
$K_r=K_p^2=\frac{X_0-X}{X_0}$

This formula compares with tested values very well for motors of normal proportions.

It is a characteristic of capacitor motors that as the load becomes greater, the capacitor phase shirks its share which is carried by the main phase. With this characteristic in mind, we have developed the approximate formula below.

$$\underset{\text{(capacitor)}}{T_m}=\underset{\text{(single phase)}}{T_m}+\ {}^1/_2FLT\ \frac{\text{rpm at pull out}}{\text{rpm at full load}}$$

If the motor is "balanced" at some other torque than full load, that torque is used in the formula.

Although this formula is very useful and is being used daily in regular design work, its main difficulty is that we do not have good data as to when it does not apply. We do know that when r_2/X_0 is greater than unity, it is obviously in error because the single-phase breakdown torque is zero which would give less than half full-load torque for capacitor pull-out torque, yet the motor would be balanced at full load and operating satisfactorily with additional torque to spare.

In reply to Mr. Buchanan, I have not noticed particularly that one phase tested further from calculation than the other.

We divide the 60-cycle and high-frequency losses arbitrarily except in those cases where we use the more elaborate method of iron loss calculation by Mr. Spooner.

We decrease the sum of high frequency iron loss plus friction and windage as the square of the speed.

Single-Phase Motor Theory—a Correlation of the Cross-Field and Revolving-Field Concepts

Discussion and author's closure of paper 40-151 by C. T. Button, presented at the AIEE Middle Eastern District meeting, Cincinnati, Ohio, October 9–11, 1940, and published in AIEE TRANSACTIONS, 1941, pages 507–12.

Wayne J. Morrill: See discussion, page 669.

C. G. Veinott (Westinghouse Electric and Manufacturing Company, Lima, Ohio): It has been observed before that it is indeed strange that the polyphase induction motor is simpler in theory than the single-phase induction motor, while the latter is simpler in construction, exclusive of the starting mechanism. For polyphase motors, one theory suffices, but there are two accepted theories for single-phase motors, based upon revolving-field and cross-field concepts, respectively. Both theories are useful and a knowledge of both points of view is essential. Arnold, it is said, gave a rigorous proof of the equivalence of these two theories in 1909 in a book of his. Kimball and Alger, in their paper referred to by the author, show too that identical results are obtained by the two theories.

Mr. Button's approach is interesting and helpful. He starts out by setting up the equation for the voltage induced in a coil rotating in a stationary pulsating field, somewhat of a cross-field approach. By deft trigonometric manipulations he shows the existence of the same two frequencies found in the revolving-field method of analysis. As the analysis unfolds, the reader discovers that neither concept is used exclusively, but he does get a helpful picture of just what is happening in the rotor bars. The picture is undoubtedly clearer than that obtained from the usual presentation of the cross-field theory in which consideration is given, not to the actual currents and voltages in the individual rotor bars, but to those same currents and voltages *referred to the stator*, wherein the only frequency is line frequency. And for the latter reason, the cross-field theory is often maligned.

Mr. Button makes the interesting comment, obviously true, that the cross field at synchronous speed is produced by the rotor currents resulting from the voltage induced by the backward field.

The "magnetizing effect" to which Mr. Button refers is the number of ampere turns that would be required to set up the fundamental flux if all those ampere turns were concentrated in one full-pitch coil per pair of poles.

While the assumption of parallel lines of flux, as in figure 1, may be convenient for purposes of analysis, and while the radial component of flux must then necessarily be sinusoidal in distribution, this assumption is fraught with pitfalls when one attempts to consider a practical motor in which all the lines of force are substantially radial and there is no important tangential component as shown in figure 1. The danger is rather simply explained: If the available ampere turns in a full-pitch coil are IN, one might easily say, referring to figure 1, that at the point of maximum flux density in a pole pitch, the ampere turns available to produce flux is IN. However, if radial flux is assumed, the *radial* component of mmf throughout the entire pole face is IN, but the sine-wave equivalent mmf at the center of the pole face is $4/\pi\times IN$.

C. T. Button: In the last two paragraphs of his discussion Mr. Veinott has raised some points which apparently need further explanation.

Suppose we have a single concentrated full-pitch coil in the stator, the plane of which lies in the $X-X$ axis. The cross field or field in the $X-X$ axis may be defined as the flux common to stator and rotor which has no net linkage with this stator coil. If we consider the flux crossing the air gap produced by a pair of rotor conductors diametrically opposite and at a given instant lying in the $Y-Y$ axis, then this rotor turn will be at right angles to the above full-pitch stator coil, there will be no mutual inductance, and the rotor turn is said to produce cross flux in the $X-X$ axis. A rotor turn in the same plane as the stator coil however has maximum mutual inductance with the stator coil and can produce no cross field at that particular instant.

If the angle between the rotor turn and the $X-X$ axis at any instant is θ, then the current in the rotor turn multiplied by $\sin\theta$ is a measure of the amount of cross flux produced by that turn. The integration of all such rotor magnetomotive force components producing flux in the cross-field axis is referred to for convenience as the "magnetizing effect in the $X-X$ axis."

If the total integrated rotor magnetizing effect in the $X-X$ axis at any instant should be called $\text{mmf}_{X\text{eff}}$, then we may state the relation

$$\frac{\text{mmf}_{X\text{eff}}}{\phi_X}=\frac{\sqrt{2}i_mT_{\text{eff}}}{\phi_{M_0}}$$

This is the method of the paper and is the basis of equation 23.

The artifice of assuming parallel lines of flux in figure 1 is felt to hold no danger for the purpose of determining at any instant how much of the total flux links a rotating coil where the flux is sinusoidally distributed around the air gap. The flux could be drawn radially where it crosses the gap (which is about two times 0.015 inch out of a total mean path several inches long in the usual case) but that is of insignificant importance—the basic assumption is that there is sinusoidally distributed flux density around the gap.

The stator winding has in general the same current in each conductor and produces substantially sinusoidally distributed flux density by virtue of roughly sinusoidal distribution of conductor density. The rotor conductors being linearly distributed however can only produce sinusoidal flux distribution at the instant when the primary flux is zero by virtue of roughly sinusoidal distribution of current in the individual rotor bars. When the total cross flux is maximum, the maximum flux density in the gap is at the X–X axis, and the density at the Y–Y axis is zero.

Naturally a single full-pitch coil with ampere turns IN would not produce sinusoidal flux distribution, and the parallel flux lines of figure 1 would not in any way represent that situation. Thus there should be no danger of the possible pitfall suggested by Mr. Veinott. Figure 1 of course is not to be taken to represent a motor but merely a coil rotating in a uniform field periodically varying in density; and it is stated that the resulting voltage in the coil is obviously the same as that which would result in two opposite rotor conductors rotating in an alternating (but not rotating) primary field with sinusoidally distributed density and with cross field zero at all times.

Design Factors Involved in the Design of Domestic Motored Appliances

Discussion and author's closure of paper 40-154 by L. C. Packer, and presented at the AIEE Middle Eastern District meeting, October 9–11, 1940, and published in AIEE TRANSACTIONS, 1941, pages 500–07.

Wayne J. Morrill: See discussion, page 669.

R. E. Hellmund (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The author gives in figure 1 a rather interesting curve of torque-time relations for a refrigerator, which is also typical of conditions in connection with air conditioning. The time schedule is probably omitted intentionally, because under different conditions of service and size the total time varies considerably, thus making the time value for an individual case, such as given in the figure, of secondary interest. It should be noted, however, that the maximum torque in the particular case is 185 per cent of the normal torque, and also that torques exceeding 150 per cent prevail for more than one-fourth of the time. These are overloads appreciably in excess of those usually encountered with motors of this type, and it is therefore likely that in the designing of a motor this pull-down condition is of greater importance than the regular load from a heating point of view. It would therefore seem very desirable for designers of such equipment to determine the practical limits between which the pull-down time varies for various sizes of motors and also how often this condition may be encountered in a given period of time. Without a knowledge of at least the limiting conditions, especially the maximum, it will be difficult to work out safe and economic applications of motors for this service.

E. M. Carville and E. P. Codling (Westinghouse Electric and Manufacturing Company, Lima, Ohio): As motor design engineers, working in a strictly motor manufacturing plant, we wish to add a few thoughts as we look at the problem from a slightly different viewpoint, that of a manufacturer of a wide variety of motors. Our problem is to design motors to be manufactured for use on a number of customer appliances, all of which are different, yet we must use a minimum number of parts in order to facilitate manufacture and keep the cost and development expense to a minimum. In selecting the motor for an appliance, the designer should give due consideration to the quantity which will be produced. When this is small he should make every possible effort to use a standard motor. This will not only aid in keeping the motor cost low, but will facilitate the manufacture of the product and be more satisfactory to the consumer should any replacement be necessary. When the appliance designer finds it impossible to use a standard motor, he should consult with the motor designer in the early stages of laying out his design. Only with the assistance of the motor designer will it be possible for him to lay out a design which will make it possible for him to use the most economical special motor. Many special features may seem simple to the appliance designer, yet require major modifications in motor manufacture and conversely it may be possible with ease to obtain what appears to be very expensive. A slight change in the appliance design may make it possible to use an entirely different type of motor, possibly not normally manufactured in the rating involved, but entirely suitable and more economical for the particular appliance involved. All of this will be determined by the parts available and the details of the manufacture and tooling which will be familiar only to the motor designer.

It is not necessary on some appliances to apply a motor which will have sufficient starting torque to start the maximum possible load. If the appliance is controlled automatically, and the motor is provided with suitable automatic protection, the protection can be relied upon to disconnect the motor in case it is unable to start the load. A motor so applied may be satisfactory if the load will be reduced to a suitable value for starting in a short time. Such is the case with many domestic refrigerators today. If the appliance is manually controlled and proper instructions are given the same may be true. For example, the motor need not be able to start an ironer when the shoe is against the roll. In this position, it would quickly scorch the roll or the material to be ironed, during the time the switch is being closed and the motor is accelerating. A washing machine motor need not be able to start the machine with the wringer jammed. In case of jamming, during operation, instructions are issued to stop the motor, disengage the wringer and agitator, and restart the motor with only the gear load. The cycle of operation for a washing machine as given by Mr. Packer is somewhat different from that recommended by some washing machine manufacturers in that the suds are formed before adding the clothes, and the motor starts the gear load only.

The refrigerator pull-down load-cycle curve shows a maximum load approximately twice as great as the normal load. This as well as the time to reach this maximum load, and the time to level off, varies considerably with the design of the evaporator, the refrigerant, the cabinet, and the refrigerated temperature. On air-conditioning applications, the maximum load and the normal load may be almost the same. On very low-temperature applications the ratio of the maximum load to the normal load will ordinarily be the greatest. The motor must have sufficient pull-out torque to handle this maximum load under the lowest voltage conditions which will be encountered. This affects the motor design and if the ratio is high makes it impossible to design the motor with the maximum efficiency at the normal load point. High overload capacity of necessity results in poorer performance at light loads. Consideration should always be given by the appliance designer to all possible ways of reducing the maximum load if he wishes to obtain the maximum efficiency of the motor at light loads with the minimum cost.

From Mr. Packer's statement regarding "short hour" rating, one not familiar with these terms might receive the impression that these motors are not suitable for continuous operation. To avoid this, recently these motors are being designated as "high torque" motors. These motors are different from the general-purpose split-phase motors in that they have higher torques and lower performance with a resulting lower cost. They are designed for use on infrequently started appliances where the total cost of operation per hour is of less importance than the first cost.

Mr. Packer gives the impression that reduced voltage will not cause excessive temperature rise. When the motor is operating under a heavy load, reduced voltage may have more effect on motor temperature than increased voltage. It should also be pointed out that the torques of the motor vary as the square of the voltage.

Figures 3 and 4 show curves of motors of different types than figure 2, for the same application. These motors would have higher efficiency and lower starting current, but would be more expensive. If this expense is justified, the use of these types of motor is sound. More generally these two types of motors are used on applications requiring higher starting torque and are designed with 200–500 per cent starting torque.

Motor protection may involve other factors than stopping the operation of the motor, such as (1) operation of a visible or audible alarm while leaving the motor connected to the line, (2) operating as a control circuit to reduce or remove part of the motor load without stopping the motor, or (3) both disconnecting the motor and operating the alarm system.

Manual reset protection is used on those applications where there is some hazard involved in having the device reset and restart the motor automatically. On some applications, such as oil burners, this hazard involves the functioning of other controls after stoppage of the motor and before restarting. A still safer procedure than using manual resetting is to establish a sufficiently long time of resetting that these controls will definitely operate first. This can be accomplished by means of a special device involving storage of heat to prevent rapid cooling of the protector. This has the further advantage of preventing complete stoppage of operation should no one be present to manually reset the protector.

The design of domestic motored appliances is quite an involved problem requiring the careful consideration of many details. This makes it necessary to make many contacts with the manufacturing department, the customer, and the supplier, of which one of the most important is the electrical manufacturer.

L. C. Packer: The author is gratified by the reception given this paper at the meeting and grateful for the discussions which have added valuable information to the paper.

Mr. Hellmund's discussion emphasizes the study necessary to the successful application of motors to refrigeration equipment. There are many different types and sizes of this type of equipment and each has its own load cycle; therefore requiring the designer to make a thorough analysis of the load cycle versus time.

The discussion of Carville and Codling has added considerable detailed information to the paper. In commenting upon some of the items I only wish to add that almost every appliance manufacturer has his own ideas about the instructions put out with his device. Many are not clear and do not anticipate many of the things that the user will sometimes do in the operation of the device. Such a case is the washing machine. While the manufacturer puts out what he thinks are proper instructions, it has been the author's experience that many users perform the sequence of operations substantially as described in the paper because they seem more natural. While I did not so state it, when the soap is added, you will find that a woman almost always forms the soap suds immediately. The users sometimes do not even read instructions and often when they do read them, they do not always understand them; therefore, the appliance manufacturer tries to make his appliance as near foolproof as possible against conditions which can and do exist.

The discussion relative to the range of load on refrigerators covers cases that can exist but the ultimate user pays the bill for poor efficiencies. However, there are cases where high efficiencies exist over a wide range of torque, and are accomplished economically by clever designing.

Carville and Codling state that the author gives the impression that reduced voltage will not cause excessive temperature rise. It was not intended to give this impression because conditions can exist where low voltage will cause overheating. While the author cannot speak for all appliance manufacturers it is known that some of them design their devices to operate over the load range under the minimum and maximum voltage conditions. The motor manufacturer can be of considerable help to appliance designers by pointing out to them conditions such as the one just mentioned.

The motor performance shown by figures 3 and 4 are for refrigerator or air-conditioning applications but of different types than the usual domestic type refrigerator. In some cases low starting current is more important than high efficiency while in others both low starting and high efficiency are important. There are cases also where single-value capacitor motors are used.

In the figures 2, 3, and 4, the starting torques are low because the suction and discharge pressures are permitted to become equal at starting, thus requiring relatively low starting torque as there is no pressure differential to start against. Protection for the equipment is quite important and on the whole, a very good job is being done. However, the motor and appliance manufacturers must of necessity co-operate very closely on the design in order to meet all conditions of manufacture, operation, cost, method of selling and servicing the appliance. The author has had many years' experience in the design of both motors and motored appliances and recommends very strongly that the appliance manufacturer co-operate with the motor manufacturer during the preliminary stages of the design rather than after the design is nearly ready for production. The benefits to be derived are well worth while to the appliance manufacturer.

Single-Phase Induction-Motor Performance Calculation

Discussion and author's closure of paper 40-141 by F. S. Himebrook, presented at the AIEE Middle Eastern District meeting, Cincinnati, Ohio, October 9–11, 1940, and published in AIEE TRANSACTIONS, 1941 (February section) pages 55–8.

Wayne J. Morrill: See discussion, page 669.

A. F. Puchstein (Robbins and Myers, Inc., Springfield, Ohio): This paper gives curves for reading with less labor, the quantities from the circle diagram and thence to obtain performance in the usual way for any point on the circle. His independent variable is *MN*, figure 1, and the dependent variables *OT*, *TR*, *TH*, in figure 2, and *MT*, *NT*, *OE/OZ*, in figure 3, all for $K_r = 0.93$. A different set would however be required for different values of K_r; $K_r = 1$ less Blondel's leakage factor $= 1 - M^2/L_1L_2$. A related method that requires a different set of curves for each value of slip was given by Puchstein and Lloyd in ELECTRICAL ENGINEERING, October 1937, pages 1277–84. Other available methods give performance in terms of slip, rotor input, or shaft output. In figure 2, line *TR* apparently should be a curve. For everyday use, the curves would need to be extended and drawn to a larger scale.

At pull out, the complete performance and speed can be obtained. Ordinarily only the pull-out torque or S_{po} are wanted. The latter is convenient if formulas based on slip are to be used. In figures 4–6, the independent variable is K_r, and the dependent variables *OT*, *TR*, *MT*, *NT*, *MN*, *TH*, *OE/OZ*. To obtain the speed at pull out we have available,

1. Sumec's old value $S_{po} \cong 0.576 \cong \frac{1}{\sqrt{3}} \times \sqrt{2 - \frac{1}{\sin^2 \eta}}$ (1903)

2. Branson, T bisects arc VL in figure 1, AIEE (1912)

3. Veinott, equations and curve based on (2) AIEE (1932)

4. Analytical values based on $\frac{\delta T}{\delta S} = 0$ which are not available, but an approximation based on the cross-field theory when primary local impedance and cross-field losses are neglected is,

$$T \cong \frac{E^2 r_2 (S - S^2)}{r_2^2 + x_2^2 (1 - S^2)^2} \quad \text{syn. watts}$$

where

E = cemf
r_2 = secondary resistance, cross-field value
x_2 = secondary leakage reactance, cross-field value
S = speed with synchronism as unity

Placing the derivative of T with respect to S equal to zero gives

$$S^6 - S^4 - S^2\left(1 + 3\frac{r_2^2}{x_2^2}\right) + \left(1 + \frac{r_2^2}{x_2^2}\right) = 0$$

The following table gives the speeds at pull out obtained from this equation.

$\frac{r_2}{x_2}$	S	$\frac{r_2}{x_2}$	S
0.0	100	1.2	66.4
0.1	95	1.3	65.2
0.2	90.5	1.4	64.5
0.3	86.6	1.5	63.9
0.4	82.8	1.6	63.2
0.5	79.4	1.7	62.6
0.6	76.5	1.8	62.1
0.7	74.2	1.9	61.8
0.8	72.0	2.0	61.4
0.9	70.3	2.5	60.2
1.0	68.5	3.0	59.5
1.1	67.3	∞	57.7

F. S. Himebrook: Mr. Puchstein's reminder that additional sets of curves for figures 2 and 3 are required for each different value of K_r, indicates that probably not enough stress was given the total number of curves required. When the set of curves was prepared it was found that in some cases ($K_r \leqq 0.90$) it was necessary to use values differing from one another by one per cent. However, for other values ($K_r < 0.90$) it was necessary to prepare sets of curves in which the value of K_r was changed by only one-half of one per cent. Such a procedure was necessary to maintain as high a degree of accuracy as possible.

In figure 2, the line *TR* is actually a curve as Mr. Puchstein points out. However, the deviation from linearity in the range shown in figure 2 is slight (a maximum of approximately 1.5 per cent). This is true because *TR* is a function of the sine of angle *NHM* (figure 1) and since this angle, for the range shown, does not exceed 11 degrees there is present only a small deviation because in this range the angle and its sine are approximately equal. If the range were extended the departure from linearity would become apparent.

The curves were originally plotted on standard $8^1/_2$ by 11-inch co-ordinate paper and in such work the accuracy of plotting was as great as the accuracy of the values read from the circle diagram.

The range of the curves for the motor whose constants are given in table I is from no-load to about 13 per cent slip. This value is well beyond the normal operating range because most single-phase induction motors are designed to operate at a relatively small slip.

The motor whose constants are shown in table I showed a pull-out speed of approximately 1,450 rpm. The speed calculated by the method described in this paper was found to be 1,415 rpm. A calculation using the method described by Mr. Puchstein yielded a somewhat lower figure.

The writer has recently learned that W. J. Branson (in some unpublished work) has succeeded in locating the center of arc *VLH* (figure 1) mathematically. This would permit an entire set of performance data to be calculated without constructing an actual circle diagram although the method uses the circle diagram as its basis.

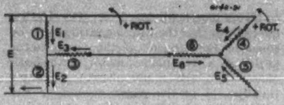


Figure 1

(1)—$E_1 = I_1(Z_1+Z_f+Z_b)+I_2(Z_f+Z_b+jX_1)+aI_3(-jZ_f+jZ_b)$

(2)—$E_2 = I_2(Z_1+Z_f+Z_b)+I_1(Z_f+Z_b+jX_1)+aI_3(-jZ_f+jZ_b)$

(3)—$E_3 = I_3(Z_2+a^2Z_f+a^2Z_b)+a(I_1+I_2)(jZ_f-jZ_b)$

(4)—$E_4 = I_4(Z+Z_f+Z_b)-j\frac{X}{2}(I_5-I_3)+I_5(Z_f\epsilon^{j\beta}+Z_b\epsilon^{-j\beta})-I_3(Z_f\epsilon^{-j\beta}+Z_b\epsilon^{j\beta})$

(5)—$E_5 = I_5(Z+Z_f+Z_b)-j\frac{X}{2}(I_4-I_3)+I_4(Z_f\epsilon^{-j\beta}+Z_b\epsilon^{j\beta})-I_3(Z_f\epsilon^{j\beta}+Z_b\epsilon^{-j\beta})$

(6)—$E_6 = -I_3(Z+Z_f+Z_b)-j\frac{X}{2}(I_4+I_5)+I_5(Z_f\epsilon^{-j\beta}+Z_b\epsilon^{j\beta})+I_4(Z_f\epsilon^{j\beta}+Z_b\epsilon^{-j\beta})$

(7)—$E = E_4 - E_5$

(8)—$E = E_1 + E_2$

(9)—$E = E_3 + E_4 - E_6 + E_2$

(10)—$I = I_1 + I_4$

(11)—$I = I_2 - I_5$

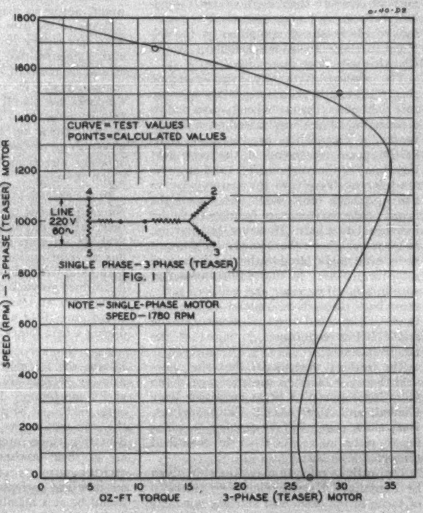


Figure 2. Operating characteristics of three-phase motor operating as teaser motor; 220-volt lines connected to terminals 4–5, figure 1

The Cross-Field Theory of the Capacitor Motor

Discussion and authors' closure of paper 40-140 by A. F. Puchstein and T. C. Lloyd, presented at the AIEE Middle Eastern District meeting, Cincinnati, Ohio, October 9–11, 1940, and published in AIEE TRANSACTIONS, 1941 (February section) pages 58–62.

Wayne J. Morrill: See discussion, page 669.

E. J. Schaefer (General Electric Company, Fort Wayne, Ind.): In this paper, the authors have made certain comparisons between the cross-field theory, the method used by them, and the revolving-field theory of the capacitor motor which was presented some years ago by W. J. Morrill.

With identical assumptions, of course, the final results must be the same, regardless of the method of analysis used. Whether there is any advantage of one method over the other down to the point where they converge to a common result seems to me dependent almost entirely on individual preferences and relative familiarity. There are certain advantages stated by the authors to be inherent in the cross-field theory, however which are based on assumptions that are not rigid and therefore do not actually provide the advantages mentioned.

I believe, for instance that there is too much of a tendency to regard the two axes into which the machine is broken down as actual physical fact rather than analytical artifice. I believe that this tendency is evidenced by the statement that it is easier to visualize differences in saturation and core losses in the two axes by the cross-field theory.

It must not be forgotten that the fluxes in the two axes in fact do *not* exist separately and that in the presence of unequal saturation the fluxes in the two axes cannot be assumed to remain independent of each other which is a basic assumption on which this method of analysis depends. To take into account different saturations in the two axes in making calculations is an attempt to take out of the formulas more than was included in the original assumptions and therefore cannot give correct results.

In visualizing these component fields as separate and physical entities there is also the danger of considering that one or the other represents the maximum flux axis. This, of course, is not the case, for, at some angle between the main and start axis there will be found, by properly adding the two components, an axis of maximum flux, and, 90 degrees displaced from this on axis of minimum flux.

$$\Phi_{max} = \Phi_M \cos\phi + \Phi_S \sin\phi$$

when

$$\tan 2\phi = \frac{2(\Phi_M \cdot \Phi_S)}{|\Phi_M|^2 - |\Phi_S|^2}$$

where Φ_M and Φ_S = vector, main, and cross-axis flux respectively.

In the case of the revolving field point of view Φ_{max} is obtained merely by the addition of the scalar values of forward and backward components.

$$\phi = {}^1/_2\phi'$$

where ϕ' = phase angle between Φ_f and Φ_b, the forward and backward rotating components of flux.

As for more exact methods of allowing for core loss, if it is desired to make allowance by means of an equivalent shunt resistance this resistance may be placed in parallel with X_M with equal facility by either method. There is very seldom a case where the gain in accuracy by this method is worth the extra effort.

As far as the final results at least are concerned, the capacitor-motor analysis has now become quite a familiar story. However, we have the opportunity to test the utility of the analytical tools made available by these studies on different and more complicated problems. One of these, which we recently had occasion to solve and which yielded nicely to the revolving field method of attack was that of a T-connected single-phase teaser motor furnishing unbalanced three-phase power to a conventional polyphase motor. The problem was to calculate the torque characteristics of the T-connected machine with three-phase motor standing still and with one line open; then with line closed to calculate the torque characteristics of the three-phase motor with the T-connected machine running. figure 1 shows schematically the arrangement with the equations which must be solved.

Figure 2 shows comparison of test and calculated torque values for the three-phase motor with the single-phase teaser motor running at a high speed. For each speed of the teaser motor there would, of course, result a different speed-torque curve for the three-phase motor.

C. G. Veinott (Westinghouse Electric and Manufacturing Company, Lima, Ohio): One cannot help agreeing with the conclusions of the authors regarding the inadequacy and inconvenience of the existing design tools for capacitor motors. For this reason this paper is particularly noteworthy. The cross-field theory has already been applied to single-phase induction motors by a number of different authors, and also to repulsion motors. Therefore, it seems only logical to develop it for capacitor motors as well, since the cross-field theory does have certain advantages. This paper is also particularly noteworthy and helpful in that the authors compare their methods with previous methods and give numerical comparisons of the results as computed by the different methods, their own included. Would that this were done more often!

The authors state that "the cross-field theory is blind to the existence of two separate skin effects in rotor conductors and to a pulsating torque superposed on a steady torque." The authors are correct in that, in the usual treatment of the cross-field theory, all of the rotor voltages and currents are referred directly to the stator, wherein line frequency only exists and thus sight is lost of the two separate frequencies in the individual rotor bars. However, Mr. Button, in his paper, shows that the cross field necessarily need not be blind to the two individual rotor frequencies; he shows this, by dealing with individual currents and voltages in the rotor bars themselves, instead of referring these voltages and currents directly to the stator before cognizance is taken of the two frequencies.

The authors' statements that the cross-field theory is blind to the existence of the pulsating torque is a little surprising since Kimball and Alger (AIEE Transactions, June 1924, pages 730–9) develop formulas for the pulsating torque from the cross-field theory.

The authors refer to a relationship given by Punga and Krondl relative to the ratio of turns in the main and starting windings to give a true revolving field. The relation quoted was that the winding ratio should be equal to the tangent of the power-factor angle of a two-phase motor with both phases like the main phase. Trickey's equation 11 in the paper quoted by the authors gives the same identical relationship if the losses in the capacitor are negligible, also if there are equal weights of copper in the main and starting windings. Anyone using Trickey's formula will see that this relationship is approximately true, even if the weight of copper in the two windings is not the same; as a matter of fact, I often make this as the first assumption when making a design calculation on a capacitor motor.

Edward Bretch (The Advance Electric Company, St. Louis, Mo.): In the theoretical consideration of the single-phase squirrel-cage motor it is rather difficult to reconcile some of the assumptions with practical conditions.

In the two-field theory it is assumed that two rotary fields of half the amplitude of the single-phase field, rotating at equal speeds in opposite directions is the equivalent of a single-phase field.

In a single-phase field the flux goes through zero twice during each cycle. As the flux is confined to one path only, all components must go through zero when the total flux does as all flux disappears at the moment of reversal. Thus all components of a single-phase flux must also go through zero and since all components go through zero there could be no rotary components as a rotary field does not go through zero but shifts its position. It is generally admitted that a polyphase system cannot be transformed directly into a single phase, or the reverse, since the polyphase represents a continuous flow while the single phase is intermittent.

It would seem that the same reasoning would apply to magnetic fluxes and it is difficult to see how a single-phase field can have any rotary components. Experiment also indicates that the single-phase field cannot be resolved into oppositely rotating components. If two similar polyphase windings are placed in the same slots of a field structure and an attempt is made to connect them to a polyphase source of power so as to cause the fields to rotate in opposite directions, a short-circuited condition develops.

In the cross-field theory it is assumed that voltages induced in the rotor conductors as they cut across the primary or transformer field set up currents that produce a cross flux at right angles to the transformer flux both as to time and space. This speed field is assumed to vary directly with the speed, being zero at standstill and equal to the transformer field at synchronism thus producing a true rotary field at synchronism.

Assuming synchronous conditions, at the instant the transformer flux as well as the primary current are zero the speed field is at its maximum. It is evident that this cross field must be due to rotor currents, as the primary current is zero. It is also evident that these rotor currents producing it are at their maximum since flux and the exciting current coincide. It is difficult to see how the current due to a speed action would be at a maximum when the flux was zero. This condition indicates that the cross or speed flux is due to a transformer action. Also, if the speed action of the rotor conductors is required to produce the cross field, what produces the torque?

While the theory of the single-phase squirrel-cage motor is difficult to master the practical operation is not difficult to understand.

In synchronous motors and generators it is a common practice to use amortisseur windings. These are simply low-resistance grids or squirrel cages embedded in the pole faces. They are used for two general purposes: (1) to prevent sudden shifting of the flux across the pole faces as for example to prevent "hunting" and (2) to dampen out field-flux pulsations as for example to stabilize the field of a synchronous generator against pulsating single-phase armature reactions.

In a single-phase squirrel-cage motor at synchronism the primary-field excitation sets up a flux in the rotor as well as in the stator. The squirrel cage which is moving in synchronism with the field excitation forms a most effective amortisseur winding by which induced currents sustain the rotor flux substantially constant during the time the primary excitation goes through zero and the rotor moves from pole to pole. Thus these amortisseur currents sustain the rotor flux, and the mechanical rotation of the rotor carries this sustained flux around with it producing the typical rotary field of the single-phase motor. Under running conditions the rotor runs slightly below synchronism. Each time a rotor pole approaches a stator pole the primary magnetizing force pulls the rotor field into synchronism and due to the same action as a polyphase motor tends to pull the rotor along with it, producing the torque. Thus the torque is a series of impulses one for each primary current impulse or of double frequency.

These observations indicate the following:

(1). The cross field is produced by the mechanical rotation of a substantially constant rotor flux.

(2). The rotor flux is sustained while the primary excitation goes through zero by the amortisseur action of the squirrel cage.

(3). These flux-sustaining amortisseur currents are of double frequency at synchronism as they go through a complete cycle while the rotor passes from pole to pole. At speeds below synchronism they are double frequency less slip.

(4). The torque is a series of impulses of double frequency in step with the primary magnetizing impulses.

A. F. Puchstein: We are in agreement with the points raised by Mr. Veinott. With reference to the pulsating torque, in the preparation of this paper we had in mind the fact that the two rotor frequencies have disappeared from the analysis when the equations are put in the form of Steinmetz or West. The cross-field theory then is blind to their existence. Of course these two frequencies can be preserved by some such method as those of Biermanns, Kimball and Alger, or Button, in which instantaneous values are used as pointed out in the paper.

On the matter of balanced operation, we have hitherto missed the equivalent of the relation $a = \tan \phi$ in Trickey's 1932 paper. It is not immediately apparent from his equation 11 but when reduced, with $R_s=0$, it does give

$$a=\frac{V \sin \phi - I_M r_{1M} K_s}{V \cos \phi - I_M r_{1M}}$$

For equal copper this is approximately equal to the tangent, as Mr. Veinott's statements indicate.

Mr. Schaefer points out the danger of visualizing the main and cross fields as separate entities which do not react upon each other. Actually, saturation effects cause them to interfere somewhat with each other. However, his expression for tan $2\phi_1$ makes ϕ_1 a complex quantity whereas it actually is real. The following would seem to be a true analysis.

If ϕ_1 indicates any point on the yoke counted from the center of the S-axis then the yoke flux at this point for any instant t is

$$\Phi = -0.5\Phi_M \sin \omega t \cos \phi_1 + 0.5\Phi_S \times \sin(\omega t+\phi') \sin \phi_1 \qquad (1)$$

The time at which the maximum or minimum flux takes place and the location in the yoke is found by taking the derivatives

$$\frac{\delta\Phi}{\delta(\omega t)} \text{ and } \frac{\delta\Phi}{\delta\phi_1}$$

and equating them to zero. The angle ϕ' used here is the time angle between the main and cross-axis fluxes. ϕ_1 is Mr. Schaefer's ϕ.

The maximum or minimum of Φ occurs at ϕ_1 when ωt has the values given by

$$\tan \phi_1 = \frac{-\Phi_M \cos \omega t}{\Phi_S \cos(\omega t+\phi')} \qquad (2)$$

Also, at time t the flux Φ is a maximum or minimum when ϕ_1 has the values given by

$$\tan \phi_1 = \frac{\Phi_S \sin(\omega t+\phi')}{\Phi_M \sin \omega t} \qquad (3)$$

The values of ωt which satisfy both of these relations are given by equating, so that

$$-\frac{\Phi_M}{\Phi_S}\cdot\frac{\cos \omega t}{\cos(\omega t+\phi')} = \frac{\Phi_S}{\Phi_M}\cdot\frac{\sin(\omega t+\phi')}{\sin \omega t} \qquad (4)$$

From this, the flux Φ is a maximum or minimum at the times ωt given by

$$\frac{\sin 2(\omega t+\phi')}{\sin 2\omega t} = -\left(\frac{\Phi_M}{\Phi_S}\right)^2 \qquad (5)$$

and these maxima or minima exist at the locations given by (3).

Mr. Schaefer also comments on the core loss, suggesting that the accuracy gained by its consideration is not usually worth the effort involved. It has been our experience that the neglect of such loss on small motors seriously interferes with the accuracy to be expected between calculated and test values. It is our belief that the iron loss can be included with theoretical accuracy and little increase in labor, as illustrated in my discussion of Trickey's current paper.

Wayne J. Morrill (General Electric Company, Fort Wayne, Ind.): The papers of this session seem to center so much about the thought of comparing the cross-field theory with the revolving-field theory that I should like to discuss at one time and in a general way the papers by Ager, Puchstein and Lloyd, Trickey, Button, Packer, and Himebrook. I should like to direct my discussion toward the central theme and consider the general merits of the two theories under consideration.

It has been proved and for a long time accepted that the current voltage equations obtained by means of the revolving-field theory are equivalent to those obtained from the cross-field theory. In other words the results arrived at by both lines of reasoning are identical and the end equations obtained through the use of either method can be expressed in the language of the other by mathematical transformation.

It might be questioned why two approaches should be used to attain the same end and it might further be questioned as to whether one of these theories does not have important advantages over the other. In view of these natural questions I should like to consider what a good theory should be expected to do.

In my opinion some of the advantages which we should expect of a good motor theory are:

1. A good theory should present an *accurate* picture by means of which mathematical expressions for the current torque, etc., may be obtained.

2. It should furnish the *simplest possible* picture concerning the important phenomena consistent with the accuracy desired.

3. The theory should furnish *means for readily considering the effects of minor phenomena* when such consideration becomes necessary or desirable.

4. It should, if possible, present a picture *readily susceptible to high-handed simplification* for the determination of limiting conditions such as no load, standstill, balanced operation, etc.

5. The theory *should preferably not change form* or require a new mathematical analysis *when slight normal modifications* of structure are made.

6. Finally, if with no extra complication, the theory can *show the relation between the motor in hand and motors of other types* this is desirable, particularly in an operating design department where motors of many types and connections must be handled simultaneously.

Undoubtedly there are other considerations which should be given to a good theory but these are some that are believed important.

Having proposed a set of considerations by which to compare the merits of alternative motor theories I should now like to consider the relative usefulness of the cross- and revolving-field theories. Throughout the discussion I shall make liberal use of equivalent circuits because through their use information can be set down more compactly and in a form subject to visualization.

1. In the matter of *accuracy* the revolving-field theory and the cross-field theory are equivalent because both give correct expressions for current, torque, etc. On the basis of this consideration the theories are of equal value.

2. The *simplicity* of the one theory as compared with the other is perhaps a matter of personal opinion, though from my own familiarity with both theories I would say that for plain a-c commutator motors the cross-field theory seems the simpler; where both a commutator and a squirrel cage exist, as in a repulsion-induction motor there is probably little to choose between the two basic theories; in the case of induction motors, however, it would appear that the revolving-field theory is simpler. Subsequent comments are directed particularly to consideration of the theories in connection with induction motors.

3. When it is desired to consider minor phenomena such as fundamental stator iron loss or the effect of stator harmonics, the revolving-field theory appears to have some real advantages.

Figure 1*a* shows the complete equivalent circuit for a single-phase motor if the fundamental iron loss is neglected. By the use of "apparent impedances" figure 1*a* may be reduced to the single branch circuit figure 1*b*.

A very close approximation to the effect of stator fundamental iron loss can be made if a suitable resistance is bridged in parallel with each of the forward and backward apparent impedances as shown in figure 1*c*. As in the case of figure 1*a*, figure 1*c* can also be reduced to a single branch circuit by using a new set of apparent impedances as in figure 1*d*. When the apparent impedances, corrected for iron loss, have been obtained the operations required to calculate the current are no more complicated than before. Since it is obvious that stator iron loss does not represent torque, the fundamental stator iron loss of the forward and backward fields must be calculated as E_f^2/R_t and E_b^2/R_t and subtracted from the total forward and backward field losses before using these losses to determine torque.

The effect of stator harmonics is readily determined with the revolving-field theory by first calculating for each harmonic its magnetizing reactance, secondary reactance, and secondary resistance. Using these harmonic constants a single-phase equivalent circuit may be constructed for each harmonic and properly connected in series with the fundamental equivalent circuit as shown in figure 2. (It should be noted that when the harmonic equivalent circuits are employed, the fundamental primary leakage reactance must be reduced by an amount equal to the sum of the forward and backward harmonic magnetizing reactances.) Using the combined fundamental and harmonic equivalent circuit the complete characteristics, including harmonic effects may be determined.

It is difficult to see how the cross-field theory could so simply yield results in connection with the minor phenomena.

4. When it is desired to make high-handed simplifications to quickly determine by inspection what will happen under certain limiting conditions the revolving-field theory really demonstrates its advantage.

At standstill the forward and backward impedances become equal and no current will flow through the central connection and the equivalent circuit figure 1*a* reduces to figure 3*a*. If X_m is very large it may without serious error be assumed infinite and figure 3*b* will apply.

At forward synchronism the slip becomes zero, the forward-field secondary impedance becomes infinite, and the circuit takes the form of figure 4*a* and figure 4*b*. Since the backward slip is 2, the backward secondary impedance is usually very low with respect to X_m and without serious error the circuit may be represented as in figure 4*b*. This circuit and the preceding one are useful in obtaining constants from test.

If it is desired to learn the no-load speed (rotational iron and mechanical losses neglected) it is merely necessary to equate the forward and backward field apparent resist-

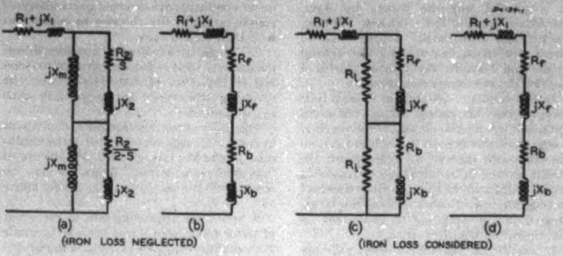


Figure 1. Equivalent circuits for single-phase motors

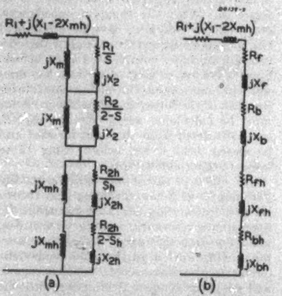


Figure 2. Equivalent circuit for single-phase motor including a stator harmonic

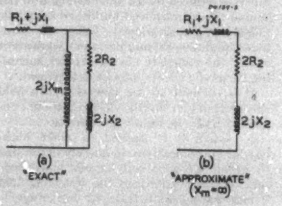


Figure 3. Single-phase motor standstill equivalent circuits

ances and solve for S. A very easy and very accurate approximate method is to assume that at no load the voltages impressed on the forward and backward fields will be the same as at synchronism, figure 4*b*. On this basis the no-load slip may be expressed as:

$$S_{NL}\frac{E_{f0}^2}{R_2}=I_0^2R_{b0} \qquad (1)$$

$$S_{NL}=\left(\frac{I_0}{E_{f0}}\right)^2R_{b0}R_2 \qquad (2)$$

I_0 = input current when $S=0$
E_{f0} = forward field voltage when $S=0$

If it is desired to determine the no-load speed including the effect of rotational losses it is merely necessary to add the rotational losses to the right number of (1).

In a similar way, such questions as, what would happen if the secondary resistance or reactance were zero and what would happen if X_m were made very low, can be readily answered.

5. A modification of induction motor design often resorted to is the multiple cage rotor. The equivalent circuit for a single-phase double cage motor is as shown in figure 5*a*. Again, the multiple branch circuits may be replaced by a single branch circuit through the use of "apparent impedances" as in figure 5*b*. Once these "apparent impedances" have been determined the calculation of performance of a double or triple squirrel cage motor by means of the revolving-field theory is exactly as easy as that of a single cage motor. The same calculation form will handle both calculations. In the case of the cross-field theory, the introduction of a double cage rotor requires a new theory development.

6. As shown in my original paper, when the revolving-field theory is used the current and torque equations for a balanced polyphase motor appear as a special case of the unbalanced motor and the constants and symbols are very closely related. This is advantageous in an operating department in that it permits of handling all induction motors, regardless of type, by means of the same theory. The cross-field theory, on the other hand, bears little resemblance to usual polyphase motor theory.

While the above remarks have been illustrated by the simple case of the single-phase motor they apply in the same way and with relatively the same increase in complication to the split phase motor, the capacitor motor, the unbalanced two-phase motor and, as Mr. Schaefer shows, to even more complicated arrangements such as the "teaser" motor. In all, it can be seen that the revolving-field theory when applied to induction motors is a simple, powerful, and flexible tool.

In conclusion I should like to point out that this very well-attended session indicates a great interest in small motors, which interest is in line with their importance. When we realize that the annual sales of fractional horsepower motors of all types probably amount to about $80,000,000 and that the rated horsepower represented is probably of the order of 3,000,000 horsepower it can be seen that small motors are important. Not only are they important to the motor manufacturers but to the appliance or device manufacturers, the merchandisers, the users, and the power companies. These motors and their related problems deserve a place on AIEE programs.

There are many problems connected with the design and use of small motors which are as yet unsolved. That these problems have a real commercial significance can be appreciated when it is realized that with an annual production of 500,000 motors of one type (there are a number of applications, the motors for which reach these annual quantities) it would be worth while for a good engineer to spend a whole year in order to find a way to save one cent per motor. If possible the reward for good engineering is even greater in connection with mass production small motors than on large apparatus. It is to be hoped that in the future more sessions of this type will be scheduled and, if so, I think we can expect equally great interest.

Figure 4 (below). Single-phase motor equivalent circuit for synchronism

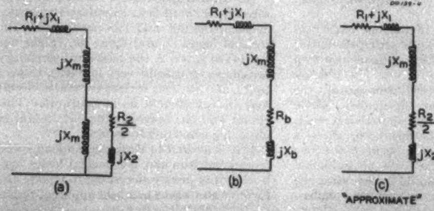
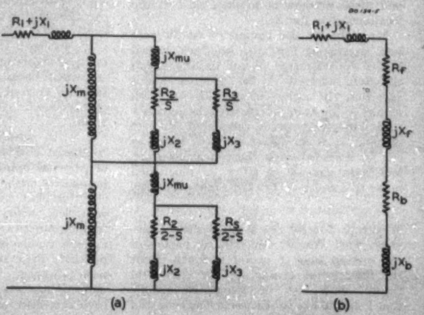


Figure 5. Single-phase double-squirrel cage motor equivalent circuits



A New Air Circuit Breaker With 250,000-Kva Interrupting Capacity

Discussion and author's closure of paper 41-32 by R. C. Dickinson and R. H. Nau, presented at the AIEE winter convention, Philadelphia, Pa., January 27-31, 1941, and published in AIEE TRANSACTIONS, 1941 (May section) pages 197-201.

D. C. Prince (General Electric Company, Schenectady, N. Y.): I am repeatedly amazed to find how much of our present art was originally conceived by such grand old masters as Professor Elihu Thomson. The circuit breakers described by Dickinson and Nau and by Boehne appear to have been inspired by a patent granted to Professor Thomson in 1926, the essence of which is shown in figures 1, 2, and 3.

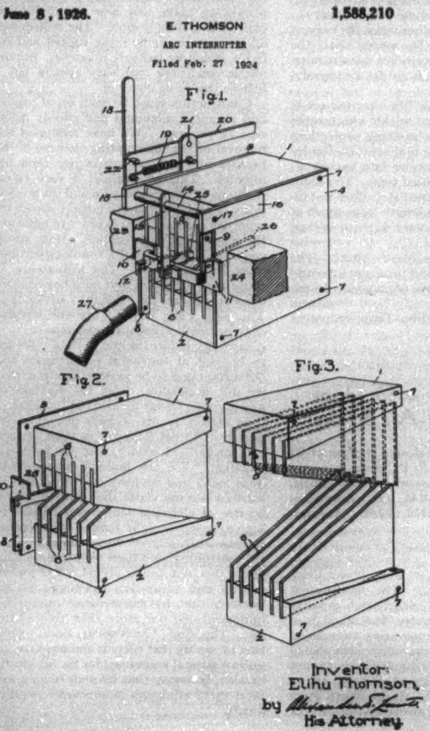
Reading from the patent specification: "Figure 1 shows a combined switch and arc extinguisher. The arc extinguisher is preferably constructed throughout of insulating material and comprises members 1 and 2, which together with plates or slabs 3 and 4 form a passageway open at its two ends. Extending through this passageway in the direction approximately parallel to the sides 3 and 4 is a group of plates 5 supported in any suitable manner by the member 1. A like group of plates 6 partially overlapping or interleaved with but spaced from the plates 5 is supported by the member 2." Later, it is mentioned that these plates may be made of highly compressed asbestos sheets, and still later, "It will be readily understood that an arc between the terminals 10 and 11 will pass the edges of the plates 5 and 6 in a transverse direction, and that if it is forced into the passageway of the extinguisher its length will be greatly increased both due to the fact that it is bowed or bent and because it is forced to follow a zigzag path around the inner edges of the plates. The length of this zigzag path of course increases as the arc is forced farther and farther into the passageway."

These two papers are, therefore, another tribute to the vision of this pioneer.

In examining and comparing the two papers, Mr. Boehne states that in his design the circuit is interrupted by introducing resistance, which improves the power factor, bringing the current more or less into phase with the voltage at final interruption. On the other hand, in the Dickinson-Nau paper, the statement is made that the current is allowed to get to its normal zero in the cycle, whereat intensive deionization is provided to establish dielectric strength. These two statements would appear to be at variance, but the oscillograms produced by the two sets of authors are almost identical, as shown by figure 4, which compares arc drops from figure 7 of the Dickinson-Nau paper with figure 14*a* of the Boehne paper.

In contrast with figure 4, the operation of the circuit breaker described in the paper, "A Conserved-Pressure Air-Blast Circuit Breaker for High-Voltage Service," by Rankin and Bennett, shows an entirely different behavior. Figure 5 is an enlarged portion of figure 8 from that paper, showing the moment of current zero. In this breaker the arc drop is practically negligible so that there is no tendency to bring the current to zero along with the voltage zero as in the case of the Magne-Blast breaker.

Instead of introducing resistance, the operating of this breaker consists of introducing unimpaired dielectric between the contacts at and after the current zero. I am not suggesting that either of these methods of arc interruption is better than the other but indicating that they are very different phenomena, each of which may be the better for some particular application. More directly, the resistance method is quite immune from recovery rate, that is, it operates with equal ease regardless of the recovery rate characteristics of the circuit. On the other hand, it will be noted from the oscillograms that a considerable amount of arc energy is involved. This, however, is of little moment, for the circuit breakers under discussion are relatively small. The air-blast type of circuit breaker is particularly adapted for large interrupting capacity and the fact that it has almost no losses stands it in good stead. For all such applications, the recovery rates are easily predicted so that ample recovery rate capacity can be incorporated in the designs.



Figures 1-3

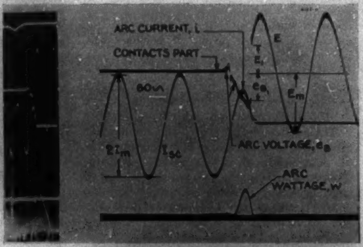


Figure 4 (right)

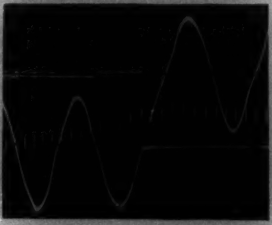


Figure 5 (right)

E. W. Boehne (General Electric Company, Philadelphia, Pa.): The authors have presented an interesting paper. One feature of particular interest is the magnitude of the arc voltage and the corresponding shift in phase angle. The part played by the slight, but costly, staggering of the slots in the chute structure, as shown in figure 2, would be of interest. These phase-angle changes exhibited in the oscillograms, some of which exceeded the angle which produces maximum wattage, give rise to earlier current zeros than would have normally occurred. These facts are not in agreement with the statement appearing on page 197, column 3, namely, "It has been found that definite advantages exist in utilizing a relatively short arc and permitting the current of the arc in interruption to go to its normal zero in cycle. . . ." The authors' conception of "normal current zero in cycle" should be defined. The shift in phase angle, away from the natural cyclic current zero, accompanied by energy increases up to about 45 degrees, has brought about features of the new design expressed by the authors on page 197 in the sentence "The added arc length and distance of arc travel required greater venting to exhaust and deionize the hot gases." This greater venting, in spite of twice the number of insulating plates, is confirmed by the energies available from the oscillograms presented. It is agreed that energies are not the criterion of performance. In fact, the authors have apparently sacrificed energy to achieve the beneficial "low recovery voltage following current zero" as mentioned on page 198, column 2. The answer may lie in the fact that we must first ionize before we can deionize.

A study of figure 11 of the paper "The Geometry of Arc Interruption" shows that rapid reductions of recovery voltage (e_r) do not occur until after the phase-angle changes of more than 45 degrees have been realized. A study of these relations shows the great advantage in having a chute geometry capable of producing phase-angle changes which exceed 90 degrees. As this unity power-factor phase angle is reached both recovery voltage and energy are decreasing at their maximum rates. It becomes apparent that phase-angle changes up to 30 or 40 degrees leading are desirable. Recent random single-phase tests have produced leading phase-angle interruption in excess of 60 degrees with the Magne-Blast breaker.

The authors' reference to the small wear on the arc chamber parts is in confirmation of data collected in thousands of tests. This statement is a measure of the negligible wear experienced when an arc of smaller energy is distributed over a much larger surface, particularly when the surface presents no stopping point for the arc.

R. H. Nau: One might infer from the discussion given by Mr. Boehne and Mr. Prince, of the General Electric Company, that they do not appreciate the advance that has been made in the design and operation of the new magnetic De-ion breaker. To elucidate briefly, the statement, "the period of relative low recovery voltage following current zero" has reference to the transient recovery voltage in its first one-eighth to one-fourth cycle. In practical circuits this time is 2.5 to several thousand microseconds. Obviously the transient recovery voltage is low during this initial period regardless of the location of the point on the 60-cycle voltage wave about which the transient recovery voltage begins to oscillate. The magnetic De-ion breaker will withstand peak recovery voltages which might occur in zero power-factor circuits following interruption.

Mr. Boehne relates at length in his paper, "The Geometry of Arc Interruption," the advantages of low arc energies and high arc voltages, yet in his discussion he says, "It is agreed that energies are not the criterion of performance." The high arc energy of "loop 1" of figure 12*a* in Mr. Boehne's paper shows that continued arcing may be caused by higher energies. Undoubtedly, it is not agreed that breaker performance is independent of the arc energy involved.

Several dominant features set the magnetic De-ion and "Magne-Blast" breakers far apart. The magnetic field as used in the "Magne-Blast" functions primarily to lengthen the arc. Further, this field is produced by a multiplicity of coils, each having many turns. In the magnetic De-ion breaker an intense transverse field, which is produced by one coil advantageously located, is used to produce deionization at current zero and establish high dielectric strength. The high dielectric strength enables the magnetic De-ion interrupter to withstand recovery voltages which may occur following interruption when the current is not suppressed. It seems that the "Magne-Blast" breaker relies upon a near-to-unity power-factor circuit for a successful interruption.

In the "Magne-Blast" breaker the arc is lengthened to many feet within a serpentine path produced by interleaving projections or slabs; while in the magnetic De-ion interrupter the arc is swung into narrow V-shaped slots having closed tops. The slight stagger of these V-shaped slots does not increase the possible arc length by as much as six per cent, and this total length is less than one-fourth that incurred by the serpentine path of the "Magne-Blast" chute. The magnetic De-ion breaker produces interruption by the true principle of magnetic deionization, which has already been described. No inference to the Elihu Thomson patent can be obtained.

A Vertical-Flow Outdoor Compressed-Air Breaker

Discussion and author's closure of paper 41-54 by L. R. Ludwig and B. P. Baker, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (May section) pages 217–22.

H. A. P. Langstaff (West Penn Power Company, Pittsburgh, Pa.): The development of a dependable high-voltage, high-capacity circuit-interrupting device, free of oil is of great interest to all operating companies. Full co-operation and encouragement should be given to research manufacturers toward the practical achievement of such equipment. The engineers of the West Penn Power Company have already witnessed the operation of this breaker and have constructively analyzed and criticized its design and assembly. We believe its advantages have great opportunities for future applications. We have analyzed and made suggestions for location and ratios of protective current transformers and believe that the characteristics of the wound-type current transformers have considerable advantage over the former bushing type available in oil circuit breakers. Their location on one side of the breaker only seems satisfactory. The use of present-day standard capacitor potential devices mounted on an extended steel bracket on the line side of this air-break switch is a suitable substitute for the bushing potential device. For carrier-current relaying application, line traps or carrier choke coils are necessary and we have suggested that they be supported on the top of the coupling capacitor and current transformer provided they are erected on suitable center-lines. We then have a basic device to which is added present-day standard auxiliary equipment which should allow for suitable standardization and aid in reducing costs.

For the control of this new development we deviate from standard and proved electromagnetic facilities such as closing and trip coils, and turn to a pneumatic operational system. The manufacturers know they must and I believe they are doing their utmost to make this new air application entirely free from trouble. The system and all its parts must be quite rugged and reliable, which we feel sure it can be. Atmospheric conditions will vary greatly but I believe this is no obstacle.

Construction must be such as to require and permit a minimum and speedy maintenance program. We have witnessed the removal of the interrupting element of the breaker under discussion in less than ten minutes. This we consider a very essential item. The necessity for and the accessibility of all parts requiring paint should receive due consideration in order to minimize the amount of maintenance. State safety codes for clearance and air-tank inspections must be considered. The availability of single-pole operation and automatic immediate reclosing has definite advantages. The West Penn Power Company has one of these breakers on order and hopes to have it in service before the end of 1941. We are so hopeful of successful performance and the advantages to be gained therefrom that we will, if possible, delay the purchase of any more oil circuit breakers of this class.

In answer to Mr. Edsel's one question, namely—"Is the industry in need of such equipment?"—I wish to cite the following situation. We are building transmission lines today and we have operating data to substantiate our claim that these lines will be free of electrical faults over periods as high as seven or eight years. This reduces the required automatic functioning of breakers controlling such lines to not more than one or two in a similar period. The station control and protective equipment should be such that its maintenance outage requirements be not more than that of the line. Therefore, I answer Mr. Edsel's question by saying that there is and has been for at least several years need for an air circuit breaker, believing that its performance will meet these minimum maintenance requirements.

J. B. MacNeill (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The writer has discussed features of outdoor compressed air circuit breaker design in connection with the Rankin and Bennett paper and most of these points apply equally to the Ludwig and Baker paper.

The outstanding feature of the Ludwig and Baker paper is the impressive test data taken under severe laboratory restored voltage conditions, which demonstrated the high interrupting capacities obtainable with the particular interrupter design presented by the paper. The provisions for easy inspection of the interrupter are noteworthy and a complete dismantling of the interrupter unit takes only six to eight minutes.

J. R. North (The Commonwealth and Southern Corporation, Jackson, Mich.): These two papers present interesting information concerning designs of outdoor high-voltage air breakers. In this class of service the conventional type of oil breaker has given a very good account of itself and in most installations the matter of oil fire hazard has not been considered to be a major factor in the application and selection of equipment. Consequently it is our rather definite opinion that the need for outdoor air breakers should be strongly evidenced before expensive commercial developments are embarked upon. It would also seem that the relative cost of outdoor air or oilless versus outdoor oil breakers would be an important factor and we would like to inquire of the authors as to the estimated relative cost in the different voltage classes.

H. P. St. Clair (American Gas and Electric Service Corporation, New York, N. Y.): After studying these papers and listening to the presentation, I find it difficult to discuss them separately since the devices described have important features in common. While it might appear that two successful high-voltage compressed air interrupters have been developed along radically different lines, at the same time both use actual flow of compressed air at fairly high pressures and both take advantage of the volatilizing characteristics of fibrous insulating material to produce gas which assists in the interruption of the arc.

The breaker described by Messrs. Ludwig and Baker appears to be an ingenious and effective device which really works judging by the impressive performance figures of tables I and II and the accompanying oscillograms. In that connection I should like to ask one or two questions of the authors:

1. In table I, there are shown a number of interruptions, with currents varying from 3,300 to 8,250 amperes and with arcing time varying from 1.4 to 2.1 cycles. I should like to ask if these results indicate that on these tests the arcing extended to the lower half of the interrupter. If so, is there any explanation why in some cases the arc interrupted in the top half only, and, in others, with no apparent correlation as to the current interrupted and the recovery rates involved, the arc extended to the lower half?

2. In test 18 a high value of current (8,900 amperes) was obtained partly by reason of a large offset or d-c component and this offset was still quite a large percentage at the final interruption. With such a large degree of asymmetry would it not be expected that the reduction or difference between the actual tested recovery rate and the calculated circuit recovery rate would be considerably more than that apparently indicated by the cathode-ray oscillograms taken on these tests? Such a reduction does not seem to have occurred on any of the tests since the ratio between test and circuit values is fairly constant throughout.

3. It would also be of interest to know the over-all time, that is, from trip impulse to interruption, obtained with this breaker.

The interrupter described by Messrs. Rankin and Bennett makes use of what would seem to be a sound principle in taking advantage of the increased dielectric strength of air under pressure. While it is not possible to compare the results as to arcing time between this and the vertical flow interrupter, the over-all time indicates a fast breaker.

Both types of breakers seem to offer advantages in the application of ultrahigh-speed reclosing which is now far beyond the experimental stage and a highly successful tool in system operation. The comparative lightness of moving parts and shorter distance of travel both lend themselves to this purpose.

Speaking in a general way of both of these developments, while it is probably true that many years of research and experience in the design of oil circuit breakers have provided something to start on, nevertheless it is significant that in a relatively short time and in virtually a first attempt two separate groups of engineers have produced results which one might say took a lifetime to obtain with oil. We have yet to see whether the radical and apparently successful new devices can bring about any material improvement in the economic picture. If that can be done, and we wishfully hope that it may be possible, then these developments mark an important milestone in the advance of the circuit breaker art.

B. P. Baker: The authors are grateful to the discussers of this paper for their valuable contributions and pertinent questions. Table I of this paper shows the results of 18 interrupting tests at 132 kv on the single pole of a 138-kv compressed air circuit breaker. The current range was varied from 270 amperes to 8,900 amperes. The arcing time varied from 0.6 to 2.1 cycles and shows no tendency to become long at any particular value of current. There is however, as pointed out by Mr. St. Clair, a considerable variation in arcing time for any given range of current. At this particular value of voltage, that is 132 kv on a single pole of a 138-kv breaker, one-half of the interrupting element is not always able to interrupt line-to-line voltage, and thus the arc has to be drawn into the second orifice or other half of the interrupter. If the arcing time exceeds 1.5 cycles, it may be assumed that the second orifice is being called upon to render some assistance to the first one. It may be noted that at 132 kv approximately 70 per cent of the extinctions require only one orifice, while at 66 kv as shown in table II, 100 per cent of the extinctions were obtained by the upper orifice only. This, we believe, explains how at the higher voltages a single orifice may be expected to show limitation and eventually fail if the voltage is carried too high.

The modification of the circuit recovery rate by the asymmetry of the current was very small. This was due to the low power factors of the circuit and to the range of the asymmetry. The test 18 having 8,900 amperes at 123 kv and shown in figures 3 and 4, may be taken as an example. The asymmetry was about 35 per cent at the time of arc extinction, which occurred at the end of a large half-cycle. On a zero power-factor circuit 35 per cent asymmetry reduces the recovery rate only about 6 per cent due to the shape of the sine waves which change rapidly near zero and slowly near their crests. On a laboratory circuit having about ten per cent power factor the current zero normally occurs before the crest of the voltage wave. An asymmetrical wave shifts the zeros and the delaying of the zero after the large half cycle causes first an increase and then a decrease in the transient recovery voltage. In this particular test the 35 per cent asymmetry produced a reduction of only two per cent in the transient recovery voltage. Even if it had extinguished at the end of a small half cycle, the variation still would have been negligible—less than ten per cent. As this example is typical of the largest amounts of asymmetry obtained on tests reported in this paper, it shows why the cathode-ray oscillograms did not show appreciable differences between the circuit recovery rate and the recovery rate obtained on test.

Figure 8 is an oscillogram of the interruption of low current at 132 kv by 138-kv single-pole breaker. In this case the point of energizing the trip coil is indicated and shows a breaker time of $6^{3}/_{4}$ cycles from rise of trip coil current to parting of contacts. 1.2 cycles of this time is consumed by $1^{1}/_{2}$ inches of sliding motion in the finger contact before parting. A large portion of the remaining $5^{1}/_{2}$ cycles is used by the electropneumatic pilot valve. This valve is a standard railway control valve which was not designed for high speed. It is thus quite apparent that a large portion of the $5^{1}/_{2}$ cycles can be eliminated.

The relative cost of outdoor compressed air breakers and outdoor oil breakers is difficult to compare because of the air equipment required with the air breaker. However, comparing four compressed air breakers with three current transformers each and including one air system for the four breakers, we find that this combination is at least 25 to 35 per cent higher in cost than four equivalent oil breakers each with three current transformers.

A Conserved-Pressure Air-Blast Circuit Breaker for High-Voltage Service

Discussion and authors' closure of paper 41-58 by W. R. Rankin and R. M. Bennett, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (May section) pages 193–7.

J. B. MacNeill (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): This paper together with that by Messrs. Ludwig and Baker present two quite different arrangements of outdoor compressed air circuit breakers using modified nozzle-type interrupters and for similar voltage and service conditions. There is a value in having these two developments, which have been carried on in parallel, discussed at the same time before the Institute, since both will soon be placed in service on important utility systems, and the reaction of operating people to their design features will help determine a final type.

The writer wishes to suggest a trial period on a limited number of installations for the accumulation of operating data and experience. After a reasonable period a further discussion before the Institute should be valuable.

In securing such experience, the following points should be observed as they bear on the selection of the most suitable type for American service.

1. Arrangement of disconnecting switch. Some form of disconnecting switch is necessary on all types of compressed air outdoor breakers suggested to the present time. The form showed by Rankin and Bennett is possibly better adapted for weather protection than some others. The form showed by Ludwig and Baker affords a definite mechanical interlock between the interrupter and the disconnecting switch unit. Other forms of disconnect are being studied and it is quite possible an alternate form will be found most serviceable.

2. Insulation characteristics. These breakers rely on air columns between line potential and ground continuously which have internal surfaces that are not corrugated and not glazed. The integrity of this internal surface under such conditions of temperature and moisture as will obtain with outdoor compressed air breakers is of vital importance. This has reference to 60-cycle test values and impulse characteristics. The impulse characteristic of the interrupter unit itself during the short period of time to which it is subjected to line voltage after interruption must be carefully considered.

3. The usability of insulating materials for outdoor interrupters when subjected to moisture and arc products requires further confirmation. This appears a fertile field for research on the special types of insulation materials required.

4. The conditioning of the air used both for circuit breaker operation and for interruption is a matter of some concern. Will it be necessary to bleed a trickle of warm air continuously through the interrupter or will properly dried and cleaned air from the compressor system be adequate when used in an interrupter subjected to the normal variations of surface moisture and temperature.

5. The most economical pressure for storage and breaker operation requires careful study. For indoor equipment storage at 250 pounds with breaker operation at 150 pounds appears adequate. Higher pressures are indicated for outdoor work, possibly 1,000 pounds storage and 300 pounds utilization.

Present oil circuit breakers of all large manufacturers are now of generally the same type, the principal differences are in the interrupter units. This condition has come about through a process of selection of desirable features over a period of years. A similar selection will take place with respect to these new compressed air devices, but time will be necessary to assist a proper selection.

It is gratifying that the large utilities as represented by their joint committee members are willing to proceed on a reasonable schedule of ratings for an experimental period. This method will result in rational development expenses to manufacturers and the accumulation of necessary field experience with a minimum of trouble to operators.

Because of overoptimistic statements regarding possibilities of cost reduction for air circuit breakers over conventional oil types, the writer wishes to caution that the perfection of new devices frequently adds features not thought of at first. Even in oil breakers many of the elements of costs have been introduced in the past few years due to the demand for perfection in service and reduced maintenance. In my opinion a perfected air breaker design will cost as much or more than a corresponding oil breaker up to 115 kv and possibly up to 138 kv, even when both types are produced in equal quantities. For voltages above 138 kv the newer form should have definite cost advantages.

The outdoor compressed air breaker has certain inherent advantages which make it desirable to a system operator. These include: High speed, both during opening and closing; short interrupting time; and probable reduced maintenance time. Present oil breaker apparatus, however, is entirely satisfactory and American system operators will not accept new equipment unless it possesses all the advantages of present products. This makes it vital that compressed air breakers for outdoor service be perfected to a high degree before an effort is made to secure a broad market for them.

J. Slepian (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): I wish to congratulate the authors for the simple method they have found for increasing the voltage interrupting rating of air-blast circuit interrupters. I wish to congratulate them also for not attempting to give any simple detailed quantitative theory of their breaker. There probably is no detailed quantitative theory that is correct and also simple.

Particularly I wish to congratulate the authors for completely omitting any reference to the so-called displacement theory as applicable to their breaker. Even more, I must congratulate the authors for their obvious courageous disbelief in this theory even if unexpressed in their paper, since the devices which they used and found successful are such for which the displacement theory would certainly predict failure.

In late years, my role as a discusser at these switch-gear sessions has frequently been as an exposer of the unwarranted pretensions of this displacement theory. Perhaps then I may be permitted to explain in more detail how the improved devices described by the authors completely contradict the displacement theory.

According to the displacement theory, at the final current zero a thin layer of dielectrically sound fluid interposes itself across the arc column and grows in thickness at a rate determined by the fluid velocity. As it increases in thickness, its total dielectric strength also increases, being given by the product of its thickness and its intrinsic dielectric strength per unit length. If its total dielectric strength at each moment exceeds the rapidly rising voltage impressed on it by the circuit, the arc cannot reignite.

In this displacement theory, the thin growing layer bears the whole circuit voltage. The rest of the arc plays no part in the interruption, and its loss of conductivity during the interruption process is negligible. It is meaningless to talk of its breakdown during the interruption process, since it is broken down and stays broken down for a period of time which is long compared to the actual time of arc interruption.

In the particular case discussed here, the fluid velocity is not changed by the devices added by the authors. Neither, according to the authors, is the density of the fluid at the place where the interrupting layer would form. Hence the dielectric strength of the interrupting layer and its rate of growth must be unchanged, and therefore no improvement in interrupting capacity can result from the devices added by the authors. There is no point in increasing the normal dielectric strength of the arc space outside the interrupting layer, since, according to the displacement theory, this space is already as completely broken down as it can be at the moment of arc interruption, and its normal dielectric strength is of no consequence.

I take it then that the authors must believe that a considerable length of the arc must take part in the actual interruption process, since they themselves say that the modifications which they have made do not affect the conditions at the nozzle where the displacing layer would form, but only the remote parts near the moving electrode. How this longer length of arc is made to recover its dielectric strength so quickly by the gas blast is an interesting theoretical problem. Perhaps my suggestion of an enhanced diffusion by turbulence may be helpful.

J. R. North: For discussion, see page 673.

H. P. St. Clair: For discussion, see page 673.

D. C. Prince: For discussion, see page 671.

W. K. Rankin and **R. M. Bennett:** As Mr. North points out the elimination of fire hazard was a major objective in the development of oilless interrupters for indoor service but is of less consequence where outdoor equipment is concerned.

Experience that has already been obtained by the application of the air-blast principle to medium voltage indoor breakers, indicates that there are certain other inherent advantages in the air-blast interrupter that can be made available in outdoor breakers.

As indicated by Mr. MacNeill the air-blast circuit breaker suffers no deterioration of the insulating and interrupting medium so that maintenance is required only by contacts and interrupter parts. Both field and test experience shows the deterioration of these parts to be remarkably light even at the highest currents.

The authors believe that these other advantages alone justify the development of high-voltage air-blast circuit breakers.

Dr. Slepian's conclusion regarding the effect of the conserved pressure chamber on the rate of growth of the insulating medium after interruption is correct. In short, the addition of a conserved pressure chamber to a short nozzle interrupter did not affect the ability of the breaker to interrupt high rates of rise of recovery voltage. However, as has been shown, the short nozzle failed at high voltages, and its failure was not related to recovery rate.

As the blast pressure of the short nozzle is reduced, the breaker continues to interrupt the circuit momentarily, but progressive dielectric breakdown results in restriking of the current as much as a half cycle or more after the initial interruption. Now deionization and displacement are completed in a few milliseconds at most so that the addition of the conserved pressure chamber had nothing to do with displacement, nor deionization but rather was for the purpose of preventing this progressive breakdown of the contact path after interruption.

The conserved pressure chamber described in the paper uses a spring-loaded valve to control the pressure. At heavy currents where larger volumes of ionized gas are produced this valve opens wider than at light currents. However, provided the maximum current to be interrupted by such a circuit breaker produces volumes of ionized gas that are small as compared with the volume of the main air blast, a simple restriction in the exhaust passage is sufficient to hold the pressure within reasonably close limits and the spring-loaded valve may be eliminated.

Editor's Note—Correction: The untimely death of Mr. Rankin prevented the making of desirable corrections in the Rankin-Bennett paper prior to its publication on TRANSACTIONS pages 193–7 (May section). Mr. Bennett subsequently made available the following corrections:

1. Page 194, column 1, line 5—The statement ". . . . at 38-kv . . . ." should read ". . . . at 88-kv . . . .".
2. Page 194, figure 4—The ordinate scale now reads "Kva = EI/1000" should read "Kva/1000".
3. Page 194, column 3, line 22—word "minimum" should be "maximum."

The Geometry of Arc Interruption

Discussion of paper 41-61 by E. W. Boehne, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941, pages 524–32.

J. Slepian (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The author has done an interesting piece of work in calculating the course of the current in a circuit involving an arc when the arc voltage is known. There are, however, a number of implications given with which issue might be taken.

First is implied that there is some advantage in a high arc voltage, so that the designers of the switch under discussion actually designed to obtain a high arc voltage. A more rational objective to my mind is to design to obtain an interruption as quickly as possible, and with as low an arc voltage as possible. It so happens that by means of the magnetic blast it is as yet not possible to interrupt the circuit with a low arc voltage. Dickinson and Nau in their paper describing a similar breaker, show quite as high arc voltages at corresponding instantaneous currents as Boehne. These gentlemen, however, do not make a virtue of this, but are quite content that the circuit is quickly interrupted.

It is true that with a high arc voltage the magnitude of the oscillating transient at current zero may be reduced, because the current zero does not occur at the moment of peak open circuit voltage. However, with this type of switch the leakage at current zero is so great that the oscillatory transient hardly develops at all, as Boehne points out in his cathode-ray oscillograms—figures 12, 13, and 14. One can hardly take credit for moving away from voltage peak a transient oscillation which does not take place at all.

But is this oscillatory transient which would occur at final current zero with a low arc voltage breaker so much to be feared as to make it preferable to dissipate quite large amounts of energy in the breaker to prevent it? Of course not. First of all, if we could make a low arc voltage breaker of similar size and cost, we would much prefer to use it and take care of the transient if we cared to by other means such as a shunt resistor with trifling energy dissipation. But actually we would do nothing about the transient because all apparatus in our actual system is built to withstand it. When the switch closes upon a piece of apparatus at peak voltage, this same transient appears and must be withstood. Why worry about it, then, if this transient occurs when the switch opens?

The fact is that at the moment we are not able to make a low arc voltage switch of comparable rating size and cost, and that is really why the arc voltage is high.

A superficial reading of the paper by Boehne might leave one with the impression that the course of the current in a circuit using the switch in question can be calculated in quite complete detail. This, however, is not at all the case. The arc voltages are not determined beforehand, but need to be taken from an oscillogram. Since that oscillogram also gives the current, voltage, and watts directly, the calculation serves to give one the satisfaction of verifying that linear circuit theory is valid for the rest of the circuit.

The implication is given in the paper that the particular arc chute pictured in figure 15 has some special virtue in producing a high arc voltage. It is true that for a given position of the arc its sinuous length is much greater than the straight distance between the horns. However, the voltage of an arc depends on many factors besides its actual length. In the switch here considered there is a strong transverse magnetic field blowing the arc up into the chute. If it were not for this magnetic field the arc would rise very little into the chute, and only a low voltage would be produced in spite of the sinuosity. It is because this transverse field produces mechanical forces upon the gas in which the arc is playing, and moves this gas upward that the arc moves upward into the chute. The arc moves up into the chute because if it stood still its arc voltage under the gas blast through it produced by the magnetic field would be higher than its arc voltage when moving with the blast into the chute.

However, the mechanical force of the field upon the arc is given by the *straight* length between horns, and not the sinuous length. One would expect then, that about the same arc voltage will be obtained for a given magnetic field and given distance between horns if similar sufficient impediment is placed in the chute to retard the upward motion of the arc in similar degree, regardless of whether this impediment involves impressing a sinuosity on the arc or not. This thought is supported by the fact that Dickinson and Nau show comparable arc voltages in their switch where the arc is driven by a magnetic field into a narrow slot with no sinuosities.

R. H. Nau (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): In the paper on "The Geometry of Arc Interruption," as well as at two previous Institute meetings, considerable attention has been given to the virtues of current suppression as a function of breaker performance. For the second time the circle diagram and lemon-shaped wattage trace have been given to explain the mechanism of current suppression as advantageously used in arc extinction. During arc interruption the electric circuit is transiently disturbed, and the conditions cannot be represented by the elementary steady-state relations of an a-c circuit as expressed by the circle diagram.

Mr. Boehne attempts to demonstrate that the "Magne-Blast" breaker relieves the circuit of overvoltages, and produces relatively low arc energies. He, however, avoids discussing certain recovery voltages and energies which are found in the oscillograms of figures 12, 13, and 14. At the end of the so-called "loop 1," of figures 12*a* and 13*a*, actually second current loop following the parting of the contacts, recovery voltages (e_r) of more than twice the maximum cir-

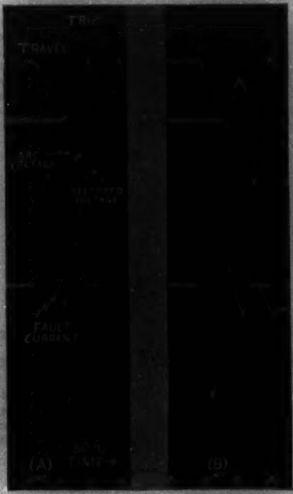


Figure 1. Current interrupted, 40,000 amperes; restored voltage, 4,020 volts

cuit voltage occurred. Note figure 7. It is evident that a short time was required for the arc to move from the parting contacts and into the arc chamber; however, during the so-called "loop 1" the arc had risen high into the arc chamber, dissipating considerable heat energy. At the end of "loop 1" of figures 12*a* and 13*a* interruption should have occurred, but the recovery voltages rose to values approaching twice the circuit voltage and caused the arc to re-establish. The only apparent advantage that exists in a low recovery voltage, as at the end of "loop 2," is the protection given to the "Magne-Blast" breaker itself during short-circuit operation. Magnetic oscillograms taken on the "Magne-Blast" breaker clearly show that there is no tendency to reduce the recovery voltages following interruption of currents below several hundred amperes.

The total energy dissipated in the arc chamber of the "Magne-Blast" breaker seems to be considerably greater than that expected of modern interrupters. From the data given by Mr. Boehne, the heat energy evolved is 0.73 watt-hour per megavolt-ampere for figure 12*a;* 0.85 watt-hour per megavolt-ampere for figure 13*a;* and that for figure 14*a* is 0.57 watt-hour per megavolt-ampere.

An inspection of oscillograms taken with a 5-kv 250,000-kva magnetic "De-ion" breaker reveals that less than two actual current loops are encountered during high-current interruption. The arc energies of these two representative oscillograms are 0.21 and 0.15 watt-hour per megavolt-ampere, respectively. These data definitely indicate that the total heat energy liberated in the "Magne-Blast" breaker is 2.7 to 5.7 times that encountered in the magnetic "De-ion" breakers. It seems very likely that this increased arc energy will hinder the "Magne-Blast" breaker if it is operated on severe duty cycles.

The "Magne-Blast" breaker was first publicized as having an interrupting rating of 100,000 kva at 2,500 and 5,000 volts. In Mr. Boehne's paper he now states that by taking advantage of principles whereby arc resistance is increased to suppress the current from its normal value an interrupting rating of 250,000 kva is obtainable. However, he does not make clear what design features have made this improvement possible.

D. C. Prince: See discussion, page 671.

A Construction Theorem for Evaluating Operational Expressions Having a Finite Number of Different Roots

Discussion of paper 41-37 by P. C. Cromwell, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (June section) pages 273–6.

E. Weber (Polytechnic Institute of Brooklyn, Brooklyn, N. Y.): The so-called "construction theorem" of the author represents a valuable simple method to extend the direct operational method to asymptotic solutions which heretofore could be obtained in complete form only by using the theory of functions of a complex variable. One of the most impressive examples is the application of a unit voltage to an infinite cable with terminal inductance. Using the infinite integral form of solution, J. R. Carson[1] also demonstrated how the infinite series part of equation 31 can be obtained by direct asymptotic expansion, but the method fails to give the first two terms of 31 which are quite essential. Professor Cromwell has shown that it is only by using the differential operator p consistently for all the factors arising in the denominator that one can get the complete solution. His formulation of a principle comparative to the "expansion theorem" but based entirely upon the process of differentiation eliminates an obvious weakness of the elementary approach to operational methods.

Reference

1. Electric Circuit Theory and the Operational Calculus, J. R. Carson. McGraw-Hill, 1926, page 78.

D-C Breakdown Strength of Air and of Freon in a Uniform Field at High Pressures

Discussion and author's closure of paper 41-9 by J. G. Trump, F. J. Safford, and R. W. Cloud, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (March section) pages 132–5.

Wm. C. Brenner (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): I was very much interested to see the results published in this excellent paper because I have recently been connected with the determination of the breakdown strength of air between spheres over a range of pressures greater than those given in this paper and for much smaller spacings.

The interesting point that I wish to bring out is the fact that an empirical equation determined from the work of H. H. Skilling and myself fits the experimental values presented in both papers with equal precision. The equation and its range of application may be seen in the discussion of the paper: The Electric Strength of Air at High Pressure—II, H. H. Skilling and Wm. C. Brenner [AIEE Transactions, volume 60, 1941 (March Section), pages 112–14].

J. G. Trump: Compressed gases are now being used with advantage for the insulation of high-voltage X-ray generators, high-frequency condensers, and to some extent in switches and in cables. It seems reasonable that gases at high pressure will become of increasing importance in other types of high-voltage power apparatus. Of the four media available for the insulation of high voltages—solids, liquids, gases, and vacuum—the latter two, gases and vacuum, have been least investigated and appear to offer the greatest possibilities for the future.

The use as insulators of condensable gases such as Freon introduced some special problems. Since Freon saturates at 85 pounds per square inch absolute and 70 degrees Fahrenheit, the temperature of this gas must be raised in order to prevent condensation at higher pressures. While it is evident that the density of the gas does not increase directly with the pressure when the temperature is caused to increase, the actual departure from a linear relation between pressure and density was only about two per cent and was least at the high-pressure end of the investigated range. Since engineers are concerned with actual pressure rather than calculated density of the gas, the breakdown curves for Freon were drawn with absolute pressure as the independent variable.

I was pleased to note Mr. Brenner's statement that the electric strength of air, as determined in his investigation at voltages up to 60 kv, checks closely with that which we have observed at still higher voltages. Such agreement is usually difficult to obtain, since the breakdown strength is strongly dependent upon factors such as the electrode surface condition, area, and geometry, which are apt to vary in different investigations. The nature of the metal itself is not as important as the surface smoothness and the extent of the surface contamination which is inevitable in a gas-filled gap. We have found, as did Professor Skilling and Mr. Brenner, that repeated sparkover is the most effective manner of "conditioning" the electrodes. Sparkover with excessive energy, however, will roughen the electrode surface and may reduce the voltage which can be insulated. We have observed that the maximum voltage which can be readily obtained under given conditions of pressure and field geometry is less when the area of the electrode is large. This is probably caused by the increased difficulty of approaching ideal conditions over large areas. It may be estimated that the effective area of our interelectrode gap was about a hundred times that of Professor Skilling. Considerations such as these must be taken into account by engineers interested in applying high-voltage data taken on small electrodes to large apparatus.

While the physical understanding of the breakdown mechanism in gases at high pressure is closer to the truth now than it ever was, it seems to me that attempts to describe accurately this complicated breakdown phenomenon in terms of the physical constants of the gas and electrode system must of necessity neglect many of the influential breakdown factors. The situation in a gas-filled gap at the moment when conditions are ripe for a cumulative discharge is remarkable for its complexity. There are present a statistical number of molecules in various states of excitation and ionization with many ion-producing mechanisms contributing to this condition. The gas itself has many physical properties which affect the process, and indeed the gas is usually a complex mixture of different molecules. Recombination is constantly taking place with low probability in the gas, with enhanced probability at the electrode surfaces. The electrodes are contributing to the discharge by photoelectric emission, by secondary emission due to positive-ion bombardment, and by high field emission. These mechanisms have been quantitatively examined separately by careful physical research in high vacuum with thoroughly outgassed electrodes. In a gas-filled gap these mechanisms are taking place together. The electrode surface is covered by many molecular layers of contaminating material which obscures the emission characteristics of the metal itself. The electric field, which may be uniform or of some other geometry because of the shape of the electrodes, is profoundly affected by both the general space charge in the gap as a whole and by the dynamic localized space-charge concentration in the region of electron avalanches. The gap is being irradiated by radiations from naturally radioactive materials and cosmic rays. In spite of this complexity, it is possible and desirable, to understand the role which the several processes play in the general picture of breakdown and many valuable contributions to this phase of the insulation problem have been and are being made.

The Electric Strength of Air at High Pressure—II

Discussion and authors' closure of paper 41-21 by H. H. Skilling and W. C. Brenner, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (March section) pages 112–15.

G. C. Nonken (General Electric Company, Pittsfield, Mass.): The work of Dr. Skilling and Mr. Brenner is a distinct contribution to the field in presenting fundamental data on the strength of air at increased pressure.

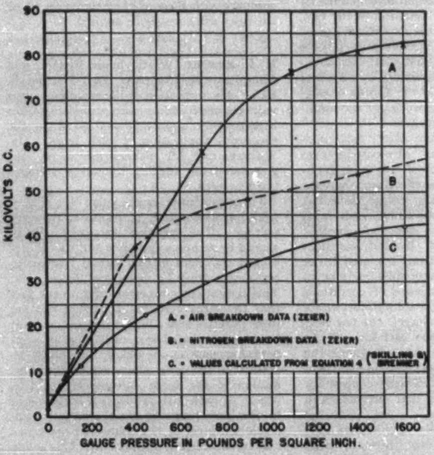
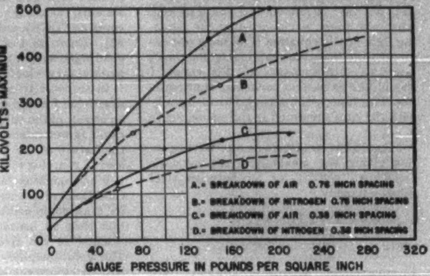


Figure 1. D-c breakdown of 15-millimeter spheres spaced at 0.4 millimeter

Considering the wide spread of the data published by various authors on the dielectric strength of high-pressure gases, it is remarkable the close check between the authors' data and that of A. H. Howell.

The authors do not try to explain the phenomena characterized by the sparking voltage of the first few sparks which are lower than the final stabilized sparking voltage. Maybe this behavior could be accounted for by the possible presence of dust or foreign particles settling on the spheres and lowering the spark-over voltage. A few sparks would burn or force the particles away. H. H. Race[1] showed how small insulating or conducting projections on sphere gaps appreciably reduced the spark-over voltage. His data showed that a piece of insulation, projecting from the sphere $2^1/_2$ per cent of the sphere spacing, decreased the 60-cycle flashover to one-half its normal value and that a very small hemispherical bump on a plane electrode will theoretically increase the gradient at that point to about three times the average gradient between the plane electrodes.

Figure 2. 50-cycle breakdown of concentric cylinders (Finkelmann)



The authors' equation 4 matches Mr. Howell's data remarkably close. It is not quite clear to the writer why the authors chose to describe their data of figure 4, which are plotted as straight lines, with equation 4, which is the equation for a curved line.

If electron emission is responsible for the characteristic drooping curve of sparkover with pressure, the shape of the spark-over curves for different gases should be the same if the same electrodes are used. Figure 1 shows spark-over data by O. Zeier.[2] The tests were made with 15-millimeter diameter steel spheres with a d-c voltage. These data are for smaller spacings and include much higher pressures than those of the authors. The curves for air corresponding to the authors' equation 4 are considerably lower in voltage than Zeier's curve for air but they have somewhat similar shapes. The data for the sparkover in nitrogen show a rapid drooping off in voltage with increase in pressure; in fact the curve calculated from curve 4 checks much closer to the nitrogen curve than it does to the air curve.

Figure 2 shows somewhat similar data by Finkelmann[3] except that the sparkover is between concentric cylinders with a 0.76-inch spacing and 0.38-inch spacing between cylinders and the tests were made with a 50-cycle per second a-c voltage. These data also show the spark-over voltage with nitrogen is about the same as that of air at atmospheric pressure but that the sparkover in nitrogen increases less rapidly than in air with increased pressure.

It may be drawn from these data that some factor other than electron emission causes the nitrogen curves to droop more than the air curves. These comparisons do not prove the absence of an electron emission effect, but they do convey that if there is an electron emission effect due to different electrode materials, it is secondary to the effects due to different gases.

REFERENCES

1. EFFECT OF SMALL PROJECTIONS ON THE BREAKDOWN IN AIR, H. H. Race. *General Electric Review*, volume 43, 1940, page 365.

2. DURCHSCHLAGUNTERSUCHUNGEN IN KOMPRIMIERTEN GASEN UND IN FLUSSIGER KOHLENSAURE, Von O. Zeier. *Annalen der Physik*, Band 14, 1932, page 415.

3. DER ELEKTRISCHE DURCHSCHLAG VERSCHIEDENER GASE UNTER HOHEM DRUCK, E. Finkelmann. *Archiv für Elektrotechnik*, Band 31, 1937, page 282.

Wm. C. Brenner: In response to G. C. Nonken's printed discussion, I should like to say that the choice of an equation giving a curved line to represent the linear data presented for steel electrodes in figure 4 resulted from the assumption made in writing equation 2. Though some had claimed at the winter convention at Philadelphia that this assumption was oversimplifying breakdown theory, the fact remains that the resulting equations (modifications of Paschen's law) agree reasonably well not only with our own data but that of references 5 and 6 as well, whence it might be claimed that the resulting form of the equation is perhaps one degree better than that expressing Paschen's law. In substantiation of this claim, I should like to note in particular that equation 3 for brass electrodes checks identically with the data presented in the curve, figure 3.

I should like to include an alternate equation giving the maximum crest kilovolts breakdown voltage for air between steel spheres, namely

$$V_m = 53.6 \times (P+1.36) \times S + 0.5$$

where

V_m = maximum attainable kilovolts (a-c crest or d-c)
P = absolute pressure in atmospheres
S = gap spacing in inches

This linear equation is purely empirical and was made to fit (even more closely than equation 4) the data collected for the breakdown of air between steel spheres for the range of pressure and spacing as shown in figure 4. If one is willing to accept the equivalence of a-c crest kilovolts to d-c kilovolts, it is interesting to note that when using the above equation (and it is to be noted that there is only very little difference to be found between the use of the above equation and equation 4 of the paper) the predicted breakdown voltages for gaps up to 1.2 inches agree very closely with the results given in the paper "D-C Breakdown Strength of Air and of Freon in a Uniform Field at High Pressures" by J. G. Trump, F. J. Safford and R. W. Cloud [AIEE Transactions, volume 60, 1941 (March section), pages 132–4].

In light of the very close agreement of the above equation with the experimental data for the maximum crest kilovolts breakdown strength of air between steel spheres for both experimental ranges considered, I should be willing to use the equation (with appropriate safety factors* depending on the application) for various design purposes. Although the above equation has only been checked over a range in which the value of V_m was below 1,000 kv, P between 1 and 21 atmospheres, and S between 10 mils and 1.2 inches, it is not my wish to imply that it holds only within these limits, but it is hoped that further checking of experimental data may prove the usefulness of the equation for even wider ranges of pressure and spacing.

H. H. Skilling: The data presented by G. C. Nonken from work by Zeier and Finkelmann are quite interesting, particularly in comparison with the results of Buehl (reference 6 of paper) and of Trump, Safford, and Cloud, respectively, with which they do not agree very closely. In so far as the results are comparable with the results of the present authors, our work tends to agree with Buehl and with Trump, Safford, and Cloud, as well as with Howell (reference 5 of paper) and with Goldman (Breakdown of Compressed Nitrogen in Small Gaps, Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS, volume 18, 1938, pages 89–94). Equation 4 of the present paper gives good agreement on the whole with the experimental work of these latter investigators.

In response to another paragraph of the discussion I would like to attempt to clarify an aspect of the subject that seems likely to result in misunderstanding. This relates to the nature of the ionization processes that lead to sparking in a gas.

When sparking takes place under normal conditions of pressure and temperature, as for example in ordinary air, it is an indication that an unstable condition has been reached in the space between electrodes. The great majority of ions and electrons that carry current in the interelectrode gap at the moment preceding sparking are produced by impact ionization by electrons. These electrons are themselves largely the product of earlier impacts. When the electric field is sufficiently intense the rate of production of electrons becomes almost unlimited and a spark results. This is the condition that Townsend assumed to correspond to an infinite production of electrons.

Although the production of free electrons is largely due to impact ionization, this process alone could not be self-sustaining. A few electrons must be produced at or near the cathode by some other cause. It is probable that at atmospheric pressure this other cause is either irradiation of the cathode by ultraviolet light that results from electrical action taking place in the interelectrode gap, or bombardment of the cathode surface by positive ions, or a combination of the two, possibly complicated by ultraviolet ionization of the gas itself. Whatever the process at or near the cathode that makes the ionizing mechanism self-sustaining, it is evident that any disturbance of the supply of electrons from the cathode will alter the conditions for sparking in the gas.

It is in this connection that "field emission" or "cold emission" of electrons from the cathode as a result of extremely high-voltage gradients can be effective. One cannot expect that a gradient will be reached at which sparking will suddenly result from field current, whereas it previously resulted from impact ionization. Instead, the effect of a relatively few electrons liberated at the cathode by field emission will merely be to disturb the delicate balance of conditions that exist in the gap at a voltage just under the normal sparking voltage; the few electrons released from the cathode will be vastly multiplied by impact ionization in the gas, and the result of field emission will be a slight lowering of the voltage required for that unstable condition that culminates in sparking.

Consequently it is only natural that a reduction of sparking voltage below the value predicted by Paschen's law should be dependent upon the gas as well as upon the electrode material, even though it be assumed that field emission from the electrode is the cause of this lowering of voltage.

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* The safety factor is assumed to apply in every case to the threshold value V_t below which sparking should not occur irrespective of time, where the value of V_t is given by the relation

$$V_t = 0.90 \times V_m$$

Control of the Switching Surge Voltages Produced by the Current-Limiting Power Fuse

Discussion and authors' closure of paper 41-60 by E. A. Williams, Jr., and C. L. Schuck, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (May section) pages 214–17.

J. A. Elzi (The Commonwealth and Southern Corporation, Jackson, Mich.): High-speed circuit-interrupting devices of all types introduce new problems in switching surge voltages. If these surge voltages cannot be limited, it will of course be necessary to take them into consideration in station and equipment designs. The most desirable procedure is that of limiting the surge voltages in the design of the interrupting devices itself as has been done in this case.

In setting up the conditions which determine the allowable magnitude of surge voltage, the authors have expressed the limiting conditions in terms of 60-cycle phenomenon. The oscillograms presented in figure 4 indicate that the arc voltage does not have a particularly steep front, but is of course of a transient nature. This would seem to indicate that the limiting conditions as set up are of the right order and should prove to be satisfactory if the characteristics given in figure 4 are typical.

As shown by equation 1, the transient voltage is effected by the inductance of the external circuit. It would seem important to know how the characteristics of the test circuit used in obtaining the oscillograms compare with those of the circuits on which the fuses will actually be applied.

H. L. Rawlins (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The limitation of the switching surge voltages produced in current-limiting fuses is extremely important and vital to the successful application of such fuses. Mr. Williams and Mr. Schuck are to be congratulated for limiting the surge voltages in their fuses.

I question, however, the value of $\sqrt{2}(2E+1{,}000)$ as a safe limit of transient recovery voltage. The transient recovery voltage should be as low as possible and should not exceed the peak of the normal frequency-recovery voltage by more than 25 per cent. This limitation is necessary due to the fact that the surge voltages of current-limiting fuses in series add. With a line-to-line fault on an ungrounded system having an accidental ground on one phase, the voltage to ground on the other phase will be the summation of the surge voltages of the two fuses. If each fuse produces a surge equal to the limit proposed by the authors, the insulation level of the connected machines may be exceeded by 100 per cent. This is not true in the case of circuit breakers or other types of fuses in which circuit interruption occurs at normal current zero, and the transient recovery voltage is a function of the circuit rather than the interrupter.

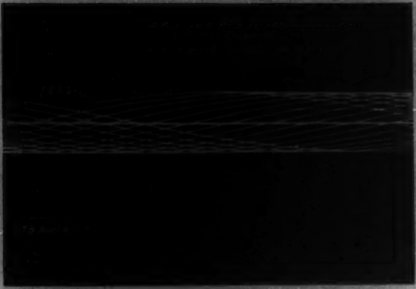

Figure 1

Figure 2

It is possible to make a current-limiting fuse with a very low value of transient recovery voltage. At the Cincinnati District meeting in October, we announced a new type of current-limiting fuse having a positive voltage-limiting action [AIEE TRANSACTIONS, volume 60, 1941 (February section), pages 77–9]. At that time, tests had been limited to 20,000 amperes at 13.2 kv. Since then, tests have been extended to over 100,000 amperes at 13.2 kv and 70,000 amperes at 22 kv. In no case has the transient recovery voltage exceeded the peak of the restored voltage by as much as 25 per cent.

Figure 1 shows the interruption of 86,500 amperes at 13.2 kv (by a 15-kv fuse of the new type). The current was limited to 470 amperes with a follow current as shown. The voltage surge was limited to 16 kv as shown on the cathode-ray oscillogram at the top.

The top cathode-ray oscillogram of figure 2 shows a 15-kv conventional current-limiting fuse interrupting 45,000 amperes at 13.2 kv. The voltage surge rose to a very high value of 58.5 kv.

The lower cathode-ray oscillogram of figure 2 shows the interruption of 69,000 amperes at 22 kv by a 23-kv fuse of the new type. The voltage surge was limited by the voltage-limiting element to only 32.6 kv.

C. L. Schuck and E. A. Williams, Jr.: The authors greatly appreciate the discussions presented and feel that they make for a more thorough understanding of the factors contributing to both the design and application of the current-limiting power fuse.

Mr. Elzi asks a question which has probably occurred to many operating engineers, regarding the test circuit used, and how its characteristics compare with those of circuits on which the fuse is to be applied. A complete description of the test circuit and measuring equipment is given in the article "High-Power Interrupting Capacity Testing Station of the General Electric Company," by G. F. Davis, appearing in the *General Electric Review* for February 1932, pages 99 to 106 inclusive. The generator is rated 100,000 kva and can deliver over 1,000,000 kva, including the d-c component, on three-phase short circuit.

In making interrupting tests on fuses, the most severe testing conditions were deliberately sought, especially in accumulating the data on voltage surges. During interrupting tests, through the complete range of currents between the minimum melting current of the fuse and station capacity, the current was limited by air-core reactors and generator impedance. Most of the short-circuit tests were made without any impedance external to the generator, with only short lengths of bus and cable for making the necessary connections. It is therefore felt that the test conditions were somewhat more severe than those usually encountered in the field, since there is usually line impedance between the generator and the fuse.

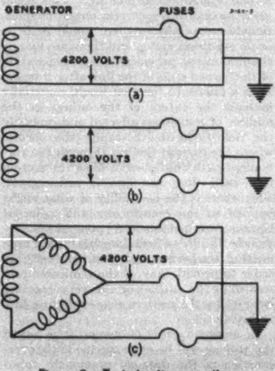

Figure 3. Test circuit connections

Table I. Switching Surge Voltages (Crest), Test Results

Voltages Measured Across Fuses

Fuse Ampere Rating	1-Phase 1-Fuse Figure 3a	1-Phase 2-Fuses Figure 3b	3-Phase 3-Fuses Figure 3c
200 *N*...	8,100	7,800	9,800
		8,000	7,800
			5,200
	12,400	6,600	7,300
		10,100	5,900
			9,600
	10,400	10,000	
		8,400	
150 *N*...	10,500	8,000	4,600
		7,400	10,000
			7,200
	10,900	8,600	9,200
		8,800	5,300
			7,200
	11,700		9,900
			8,200
			6,600
100 *N*...	12,500	8,400	
		8,400	
	12,500	7,300	
		10,100	

Mr. Rawlins' suggestion of the possibility of a ground developing in an ungrounded system, followed by a pure line-to-line fault on the far side of the fuses from the ground appears rather remote. In practice, grounds develop on one line of ungrounded systems quite frequently, but a subsequent fault on one of the other lines is usually also to ground. Table I gives the results of tests made single phase and three phase at 4,200 volts, using fuses rated 5,000 volts. The available short-circuit current in these tests was 86,000 amperes. It will be noted that the transient recovery voltage for tests involving only one fuse is in general higher than any obtained in the tests involving more than one fuse. The recommended limit for fuses of this voltage rating, based on motors rated 4,000 volts, is

$$v = \sqrt{2}[2(4{,}000)+1{,}000] = 12{,}700 \text{ volts}$$

The Protective Link

Discussion and authors' closure of paper 41-59 by J. K. Hodnette and M. G. Leonard, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941 (May section) pages 226–9.

G. F. Lincks (General Electric Company, Pittsfield, Mass.): When a transformer with a protective link is connected onto a distribution circuit it becomes a unitary part of a whole co-ordinated overcurrent protective system from the service entrance fuse on the consumer's meter board to the relays at the generating station. This is true whether such co-ordination is predetermined or just the result of "happenstance." Service on a major part of the system may be disrupted for a long time due to a misapplication or any false operation of the protective link just as it would with any other associated apparatus. Extensive co-operation with operating engineers in all parts of the country on studies for improving overcurrent protection and service continuity has indicated that proper application demands the ready availability of factual data on all protective apparatus especially as to:

1. Melting time-current characteristics, preferably plotted in accordance with EEI-NEMA approved practices.[1]

2. Total clearing time-current characteristics.

3. How accurately these published characteristics can be maintained in commercial production and in service.

4. The reliability of operation in clearing fault currents over the full range from the lowest to the highest.

5. A definite current interrupting rating which can be met with any size of protective element made available for use in the device so rated. (According to AIEE practice this is expressed in rms amperes initial in arc for specified voltages.)

Renewable fuse links for expulsion fuse cutouts have been developed with an accuracy approaching the high standard set by the overcurrent protective relay with which they are associated. This has made possible the many benefits from the co-ordination of externally mounted fuse cutouts and fuse links connected in series with other fuse links and with relays on distribution systems. In the application of the protective link it is imperative to know whether the same high standard of accuracy and reliability of the associated external fuse links and relays is being provided and maintained. This is especially true since the protective link and its associated internal breaker by their very nature take up an unusually large share of the range available for all overcurrent protection on distribution systems.[2] Also field experience has shown that due to inadequate data as to what size of a link is in what transformer it has been necessary in co-ordinating such protective links with other apparatus to consider them on the basis of the worst conditions, namely, the longest time-current characteristics that have ever been applied. This is true even though some of the more modern protective links have been decreased in size so as to present much more of a hazard as to being blown by lightning.

In the paper the authors describe a method for changing the normally faster time-current characteristics of a fuse link when submerged in oil. This steeper characteristic is due to the convection currents in the oil carrying heat away from the fusible wires. At the lower currents where the melting time is sufficient to make this effective in increasing the heat dissipation, it tends to increase the currents at which a certain size of fusible wire melts in a given time. At the higher currents the effect decreases until the heat dissipation from this fusible wire is negligible during the very short melting time. Then the characteristics of the fuse in oil approaches that of the same fuse in air. Shielding the fuse by a tube retards or prevents the convection currents in the oil surrounding the link and thus makes its melting time-current characteristics approach that of a fuse in air. The authors claim to go one step further, combining the 1,083-degree centigrade melting temperature of copper with the blanketing of a surrounding fiber tube to distill gases from the oil. By so doing they claim they secure the slower time-current characteristics of their curve figure 3*c*. Similar characteristics in fuse links for mounting in the oil of distribution transformers are available using tin fusible wires with a 232-degree centigrade melting temperature which is well below the 275-degrees centigrade charring point of fiber. These desired time-current characteristics are provided by using the same design methods that have made possible the high accuracy of the external renewable fuse link. They can be secured with or without shielding the link by a surrounding tube. The authors' method introduces variables such as the distillation temperature, the energy required for distillation, and the possibility of trapped air above the oil in the tube which would totally void such predetermination as described and shown in figure 3. It would appear that these variables may make it impossible to meet the demands of modern overcurrent protection practices for accuracy and reliability.

The authors' discussion of interrupting requirements indicates little theoretical need for high-current interrupting ability but recommends applying the protective link on the basis of maximum available line short circuit. Their field experience corroborates their recommendations evidently due to internal line-to-line or line-to-ground breakdown caused by lightning surges or inadequate overvoltage protection. A large portion of the paper deals with the high kilovolt-ampere interrupting abilities without definitely tying them up to any of the specific types shown in figures 2, 6, and 7 and without reference to rms amperes initial in arc at specific voltages which is the approved AIEE, EEI, and NEMA method for rating fuses. The interrupting function of fuses at different voltages is not always directly proportional to such a kilovolt-ampere rating as is used in the paper.

While the paper deals largely with the maximum interrupting ability it clearly points out that if the link is permitted to do what it is supposed to do, namely, disconnect only winding failures, the majority of fault currents will be very low. Successful low-current operation is accredited to the use of a small auxiliary fiber tube. Such a tube would be precarbonized by the close proximity to the white hot copper link for a second or more before melting. This would make successful low-current operation questionable especially as there would be little or no expulsion action and there are no mechanical means provided to cause separation of the burned ends of the link after it melts. Such a failure to function would materially increase the extent of the outage in the number of consumers affected and probably the time to locate the faulty transformer where the current flowing through the carbonized path continues to blow the section fuse each time the line is re-energized. Then there is the possibility of what might happen to the transformer and adjacent equipment or persons. As pointed out previously, the use of fusible metals having high melting temperatures is not necessary in order to provide any of the requisites prescribed by the authors or the very exacting ones set by the modern renewable fuse link and relay.

In answering this discussion it is hoped the authors will be more specific in the presentation of the data as previously tabulated, tying it up definitely with the voltage and kilovolt-ampere ratings and the types of the transformers in which the different types of protective links are actually employed.

REFERENCES

1. Both EEI and NEMA have standardized on the use of one size curve sheet for plotting the time-current characteristic curves of fuses. This is carried by Kueffel and Esser, New York., under No. 28962.

2. TRENDS IN DISTRIBUTION OVERCURRENT PROTECTION. G. F. Lincks and P. E. Benner, AIEE TRANSACTIONS, volume 56, 1937 (January section), pages 138–52.

S. R. Smith, Jr., and C. R. Craig (Distribution fuse cutout section, General Electric Company, Pittsfield, Mass.): The authors have contributed a very interesting paper on a type of protective apparatus which, due to its important position in the overcurrent protective scheme on the distribution circuit, deserves more than passing study. There are several points not elaborated on sufficiently in the paper to permit a clear understanding of the operation of these devices. It is hoped that a discussion of some of these will lead to a better understanding of the protective link and its functioning.

These devices operate in transformer oil, a relatively noncompressible medium, with more or less violent "explosive action." Transformer tanks are not normally made to withstand high internal pressure. Small unit pressures can produce high mechanical stresses when the tank surface is large. It would appear of real value if data could be included as to the tank pressures developed.

The authors' explanation of the blanketing effect on the fusible element to produce curve *c* of figure 3 is of necessity based on the premise that the element is entirely submerged in the oil. In figure 2 no vent is shown at the top of the tube to allow trapped air to escape and vents were omitted in devices that have come to our attention. While a good part of the air could be absorbed by the oil in time, it would seem that a portion of the fusible element would be in the trapped air above the oil, which portion would determine the melting curve thus making the blanketing effect of the tube of no value in producing the results as described. Variations in the amount of air trapped would greatly affect the accuracy of the melting time-current characteristics.

Referring to figure 5, the authors state—

1. "The electrode travels approximately six inches during the arcing period."

4. "The electrode travels approximately eight inches through oil in the half cycle following the extinction of the arc."

With the method of mounting in actual use as shown in figure 1, the motion of the electrodes would seem to be considerably restricted by the lead insulation and adjacent parts. Are the above data intended to apply in such a case?

The paper states that with the present knowledge, major insulation failures from lightning have been practically eliminated and therefore, line-to-line or line-to-ground failures of transformers are uncommon. The usual fault condition described is of a progressive nature with sufficient winding impedance in series with the fault so that the current does not approach system short-circuit proportions. However, under "Field

Experience" it is brought out that in most cases the lower electrode was violently driven from the end of the tube and that this action is some indication of the magnitude of the current interrupted.

This would indicate that in most cases, the fault current was probably of line-to-line or line-to-ground magnitude since such violent operation would not be attendant upon the progressive low-current failure of the winding for which the link was installed. From this one might conclude that the transformer faults initiated by lightning surge, mentioned in the report on field experience, are the principal cause of failure. There appears to be no other general cause of direct line-to-line or line-to-ground breakdown in the transformers. We would appreciate an explanation by the authors of what seems to be an inconsistency between the field experience and the supporting theoretical discussions in their paper.

E. A. Williams, Jr. (General Electric Company, Philadelphia, Pa.): The practice of placing fuses within transformer tanks has certain advantages over the use of external fuses as pointed out by the authors. To obtain these advantages, it is necessary to sacrifice some of the benefits obtainable with external fuses. Three outstanding sacrifices are:

1. The external fuse is easy to inspect and renew; the internal fuse necessitates dismantling the transformer.

2. The external fuse protects the system against bushing failures and flashovers; the internal fuse does not.

3. The external fuse does not create pressure within the transformer tank; the internal fuse does produce tank pressure.

The authors call attention to the force and power involved in an interruption by the protective link but omit data on tank pressure. What are the magnitudes of tank pressures?

J. K. Hodnette and M. G. Leonard: Some confusion evidently exists as to the distinction between an ordinary fuse and a protective link. We refer to a fuse as an integral device for circuit protection designed so that it can be readily replaced or reloaded after it has functioned to clear the circuit. A protective link is considered an integral part of a transformer designed to clear only faults which result from internal trouble, and therefore normally replaced only when the entire unit is to be repaired or replaced.

Most of the points raised in the discussion have been in regard to the comparative merits and performance of the two devices. These comparisons we are glad to make, particularly for applications in which the two devices have similar functions. Since the discussions overlap in many points, we will combine them all and deal with them in this form.

Co-ordination is one of the questions raised, both as to accuracy and width of band occupied. The band occupied by the protective link and the associated breaker is, of course, wider than would be occupied by a single device, but it is not wider than the comparable combination of a breaker with a fuse. So far as accuracy is concerned, the protective link is more accurate in performance than the fuse, because the fusible element of the link is of noncorrosive material; it is not subject to weathering; it is more uniform in size and resistance; it is many times stronger mechanically than commonly used fuse links. Moreover, it is protected against mechanical strain when used in small sizes, and finally its high melting point protects it from damage by momentary overcurrent.

Uniformity of characteristics, or accuracy, is only one of the requisites of current interrupting devices necessary for satisfactory co-ordination with other devices. The relative shapes of the current-time characteristics are of equal importance. The protective link offers a wide choice in shape, as shown in figure 3 in the paper, which makes it quite simple to fit it to the characteristics of either sectionalizing fuses or feeder breaker relays.

Charring of fiber tubes is suggested as a possible result of using a high-fusing temperature element. Actually, this does not occur, for two reasons: first, because the wire is not in contact with the fiber, and second, because the wire never reaches a high temperature in normal operation, i.e., in operation below the breaker trip characteristic. Operation above this curve occurs only on internal fault, hence for only a short time.

The rating of protective links on short-circuit test is in accordance with recommended AIEE practice. In regard to fusing time characteristic curves, catalogue data are available showing the performance of both the links and the associated breakers as used in distribution transformers.

The electrode travel required during operation has been questioned. The apparatus described in the paper was made to determine fundamental data on performance and was tested over an unusually large range of clearance in order to obtain as much information as possible from the test. Such a long travel is not necessarily required even in power transformers, and in distribution transformers only a relatively short travel is adequate.

The internal pressures produced by link operation have not been measured, for the reason that no difficulty has been experienced in this regard. It should be emphasized that one of the chief advantages of this new design over conventional construction is that its fast and effective quenching action causes a minimum of disturbance, with little burning of the electrodes and no serious pressure rise in the tank.

An Improved Polyphase Directional Relay

Discussion and author's closure of paper 41-62 by Bert V. Hoard, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (May section) pages 246–8.

Wm. R. Brownlee (Commonwealth and Southern Corporation, Jackson, Mich.): The application of the induction-loop principle to polyphase elements multiplies its usefulness. The use of a vertical shaft is a fortunate choice in producing a balanced element with more consistent operation than certain other types of relay elements. The advantages of the new device should be particularly great over the beam type of construction for use as a distance relay element. The inherent inefficiency of the beam has required very small and delicately adjusted air gaps (especially on the voltage element), which change in magnitude when the beam operates, thus depending on exact resetting against a fixed stop for accurate performance. In the writer's experience, minute particles of dirt (almost invisible on a cleaning paper) have resulted in calibration errors of over 30 per cent on beam elements of distance relays. In the new relay form, the air gaps are large and are not affected appreciably by the position of the moving element, and greatly improved operation is expected. The use of the new type element for distance relays should obviate the necessity of using a "negatively restrained" element (which is overbalanced in the tripping direction in the absence of both current and potential) and may permit the protection of moderately short transmission lines for which the beam-type relays had previously been considered inadequate. It is hoped that distance relays using this element may be available soon.

Standard or experimental applications of the induction-loop principle have been made by two prominent relay manufacturers, and both of them have standard applications of the induction-cylinder principle.[1,2] From a comparison of characteristics, it appears that the induction loop has a faster operating time, being on the order of one-third cycle minimum compared with about two-thirds cycle minimum for the corresponding induction-cylinder element. However, according to the author, it has not been possible to overcome the vibrating torque of the induction-loop element on unbalanced faults which is accomplished by the slight damping action of induction-cylinder elements. It would be interesting to know for what range of operation the action of the moving contact spring of the new relay is effective in following pulsations so as to maintain continuous contact.

The chief use mentioned by the author for the new relay is for directional-comparison type pilot-wire installations. Unquestionably the sensitivity at low voltages and currents, freedom from bouncing, etc., provide material advantages over the type CHV3 induction disk relay which was previously recommended for this purpose. However, the operating time appears to be too fast and it is doubtful if contact travel could be increased sufficiently without moving the loop too far away from the magnet poles. Pilot-wire schemes which provide complete coverage (including power supply from one end only) require a plunger relay to operate definitely ahead of the directional fault detector relay. Typical high-speed plunger relays part contacts in slightly less than one-fourth cycle, which is probably too close to the minimum operating time of the directional relay (about one-third cycle) for adequate margin. The pulsating torque characteristic on unbalanced faults is a serious disadvantage in pilot-wire application where it would not be at all serious in a distance element with local instantaneous seal-in auxiliaries.

It is also important that a directional relay with voltage restraint for pilot-wire service should cover adequately the extreme range of phase angles encountered in practice. For near-by faults, the phase angle of the line and fault impedance may approach zero degrees while faults near the distant terminal will be influenced principally by the transmission line which may have an impedance angle of perhaps 80 degrees. With standard watt elements it appears that the relay described by the author must cover an additional 30 degrees, depending upon whether the fault is between phases or involves all three phases. The difficulty of covering adequately a 110-degree range is all out of proportion to the extra 30 degrees, and it is suggested that the new relay be used with delta current-transformer connections, or that the equivalent of this be provided by winding each current coil in two sections, or that some other method[2] be provided to reduce the required phase-angle spread.

Values of sensitivity and operating time apparently apply to a single-element device with directional characteristics. What are the sensitivity and timing values with a two-element unit containing voltage restraint?

References

1. New Network Relays, J. M. Parsons and M. A. Bostwick, *Electrical Journal*, July 1937, page 288.

2. A Single Element Polyphase Directional Relay, A. J. McConnell, AIEE Transactions, volume 56, 1937 (January section), pages 77–80.

A. J. McConnell (General Electric Company, Philadelphia, Pa.): The author's statement that the trend toward high-speed relaying developed a need for a high-speed polyphase directional relay is quite true. In fact, this was recognized several years ago when the first induction-cylinder polyphase directional relay went into service.[1]

I cannot agree with the statement that the inductor-loop relay has all of the desirable characteristics inherent in the induction-disk type relay. Because of reciprocal action between the poles, the torque of the induction-disk relay (and of the induction-cylinder type) has no objectionable net double-frequency component, even when the relay is supplied with single-phase quantities. This is proved by figure 1 which shows two oscillograms of the instantaneous torque of a single-phase induction-cylinder directional relay. The lower film was taken at the maximum torque phase angle while the upper film was taken at only five degrees from the zero torque phase angle. Trace 1 in each instance is the instantaneous torque, trace 2 being the zero torque line.

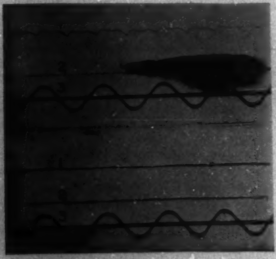

Figure 1

Of course, it is true that under balanced polyphase conditions, the relay described will have a steady net torque. However, during phase-to-phase faults, the pulsating torque may far outweigh any balanced torque which may be present, and tend to cause objectionable vibration. On the basis that the relay is capable of operating in one-fifth cycle (figure 6), one's first thought is that it could operate incorrectly if a pulsating torque were present having a negative loop, as may be the case with a phase-to-phase fault when the current is not at the maximum torque phase angle. However, it is doubtful that such high speed can be obtained in practice, since the time curves of figure 6 appear to be plotted to much higher input levels than would be encountered normally. For example, how would 50 amperes and rated voltage be obtained in an actual application? Furthermore, upon replotting the curves of figure 6, it is found that this relay is no faster than the corresponding induction-cylinder relay.

Although there may be no net pulsating torque under balanced polyphase conditions, there are certain to be pulsating translational forces because of continuous change in the point of application of the force. For example, as drawn in figure 1, the lower magnet will apply, at some instant, its maximum force to the left (assuming clockwise net torque). At some other instant, the upper left-hand magnet will apply its maximum force upward and to the right, and at still another instant, the upper right-hand magnet will apply its maximum force downward and to the right. Thus, there might be continuous sliding of the bearing pin on the bearing, and if so, it would appear that excessive bearing wear might eventually ensue.

With single-phase relays of the loop type it was the practice, at one time, to reduce vibration by means of early saturation of an auxiliary current transformer, with a consequent drastic reduction of torque. Does the relatively small cross section of the current iron (figure 1) signify that the same result is obtained by early saturation of the magnetic circuit of the current coil? What is the torque developed by this relay at rated voltage and current, and at the maximum torque phase angle?

The phase-angle curve at two volts shown in figure 5 is very good. Can all production relays be expected to have a similar characteristic? Ordinarily, current bias torque is caused by manufacturing variations. The relay described appears to have an additional factor affecting current bias, namely, position of each loop relative to its current pole. Is the adjustment of the position of the loops in the air gaps the method of eliminating current bias? If so, is it not necessary to readjust for the elimination of current bias if the contact adjustment is disturbed? If the necessity for this readjustment were not realized, a change in contact setting might cause incorrect directional action at low voltage. Also, if the current bias is dependent upon loop position, will not the low-voltage phase-angle characteristic be different, depending upon which contact is closed when the fault is applied?

Regarding the voltage restraining element of the relay illustrated in figure 7, a nonpulsating net torque with balanced voltages is obtained by using all three combinations of delta voltages. However, during an unbalanced fault, a pulsating net torque appears. In the case when one delta voltage becomes zero, there may be zero average torque, but the magnet supplied with the uncollapsed voltages will have a pulsating torque of considerable magnitude.

Reference

1. A Single-Element Polyphase Directional Relay, A. J. McConnell. AIEE Transactions, volume 56, 1937 (January section), pages 77–80.

Bert V. Hoard: With reference to the oscillograms on torque submitted by Mr. McConnell, it would be of interest for him to describe the method of obtaining them. It is well known that any single-phase source of power produces vibrating torques similar to those shown in the paper, the vibration being largest near the phase angle of zero torque. If these oscillograms indicate the vibration of the induction cylinder after the single-phase torque pulsations have been smoothed out by the relatively high inertia of the moving element, we agree with him, but we doubt very much if the actual torques produced on the moving element by single-phase power are as shown. A similar smoothing effect, but to a lesser extent, is produced by the inertia of the inductor-loop moving element.

We realize it will be an exceptional case if a directional relay has to operate on a three-phase fault of 100 amperes and sustained voltage, which indicates an infinite bus at the relay location. However, if such an operation is possible on a certain system, there is a probability of an unbalanced line-to-line fault occurring at the same fault location, in which case the maximum power supplied to the relay would be half or less than half of the three-phase power which would be supplied during the three-phase fault. Thus, during the unbalanced fault, while pulsating torques are present, the total torque magnitude is materially less than when the equivalent three-phase fault occurs, so that the smoothing out effect of the relay inertia is greater relatively than would be expected. Also, the time to close contacts is correspondingly increased. For these reasons, the possibility of reverse operation due to a negative loop of torque on one phase of the relay has been taken care of in the design. The 90-degree connection gives a torque on the relay equal to the arithmetical sum of the positive and negative sequence components of power. The 60-degree and 30-degree connections give vector summations of these two components of power at 60 degrees and 120 degrees, respectively. Hence, the 90-degree connection gives a greater torque for a line-to-line fault than the other two connections, and, therefore, the timing obtained is nearer to the timing obtained for the three-phase fault.

We agree with the theory that there are pulsating translational forces which in combination with the moving element stop may produce sliding of the bearing pin on the bearing. There is no relay made which utilizes either the principle of the rotating field or the principle of the inductor loop that in theory may not have such sliding within the bearing. Practically, however, there are many relays in which such sliding is negligible and this relay is among this group.

The narrow iron section used for the center leg of the electromagnet fulfills three objectives.

1. For high values of current it saturates, and in this manner prevents any unnecessary increase in relay torque. Thus, the torque increases only by a ratio of 7 to 1 while the current is increasing by a ratio of 20 to 1. At 5 amperes and rated voltage the torque is approximately 30 centimeter grams. The wide air gap allows considerable flux fringing, hence the saturation in the center leg takes place slowly and progressively with increase in current above 30 amperes, and there are no sudden bends in the phase-angle curves.

2. The narrow thickness in the center leg gives a very small amount of current centering torque, so small in fact that at 100 amperes less than 1 volt will operate the relay.

3. The narrow iron section combined with the shape of the section threading the loop provides a small amount of voltage decentering torque. This torque is present on one side of the neutral point and by a simple contact adjustment can be made to act as a small amount of voltage restraint. This restraint, of course, completely disappears at low voltage. The relay is normally adjusted so that the decentering torque will always return the tripping contact to the open position when no current is flowing and this adjustment also automatically locates the loops in the proper position to minimize the effects of the current centering torque. This simple contact adjustment with voltage alone applied can easily be made in the field.

With regard to the voltage restraint element, during a phase-to-phase fault in which one voltage is zero, the torque of two units is reduced to zero because the collapsed voltage is applied to one circuit of each unit; the torque of the remaining unit has 86 per cent of normal voltage applied to each circuit or 75 per cent torque for the unit, but at the phase angle of zero torque for the unit. Thus, the pulsation cannot be greater than one-third times 75 per cent of normal restraint or the equivalent of 2.5 amperes, since normal restraint is 10 amperes. This small value is not objectionable.

The contacts of the relay are effective in following pulsations so as to maintain continuous contact during high-current unbalanced faults except for a range of five degrees away from the zero torque characteristic. For balanced faults this range is approximately two degrees.

We can see no material advantage of the 60-degree connection or its equivalent plus a phase shifter, over the 90-degree connection with a phase shifter both to obtain substantially a 45-degree maximum torque characteristic for three-phase faults. Practically equal performance of the two connections is obtained for unbalanced faults as well as for balanced faults. Mr. Brownlee's reference to standard watt elements and 110-degree range is not clear, unless he feels that the phasing loops used on the current circuit to obtain the 45-degree characteristic are perhaps more complicated than the relay without phasing loops but with additional current transformers to obtain the 60-degree connection.

Values of sensitivity and operating time of the double-element relay which has voltage restraint are available in the standard catalogue section published by the manufacturer.

Modern Relaying for A-C Secondary Network Systems

Discussion and author's closure of paper 41-81 by D. L. Beeman, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (May section) pages 237–41.

R. D. Wilder (Public Service Electric and Gas Company, Newark, N. J.): This single-element network relay is particularly advantageous where secondary networks are supplied by wye-connected transformers such as the 4,150-volt, three-phase, four-wire system utilized by Public Service Electric and Gas Company for their network supply. It is too bad that Messrs. Beeman and McConnell did not have this inspiration a number of years ago, as we have had numerous locations where the *L-N* type relay would not receive its necessary quota of watts on some types of faults.

However, the older types of relay can be improved considerably by a few simple tricks that may not be generally known. If the potential coil-neutral ground connection is opened, the relay neutral will float to a more advantageous point on single and two-line-to-ground faults. Referring to table I in this paper, the per cent of normal torque may be increased from 1.7 to 6.9 per cent for 10 per cent volts, from 11 to 21.5 per cent for 25 per cent volts, and from 81 to 92 per cent for the 70 per cent voltage fault. The double line-to-ground fault, table II shows a corresponding increase from 4.2 to 5.1 per cent. Line-to-line faults are not affected.

Relay operating time is an important item where fault currents are large and selectivity with substation breakers must be considered. The new relay described in this paper shows a notable improvement in operating time. Here too, the older network relays can be improved. Most of these are provided with various leaf springs for obtaining the required reverse-current settings. By using the stiffest spring practicable for getting the desired setting, the relay operating time may be reduced by 40 to 50 per cent. This is presumably due to the greater proportion of free contact travel before the stop hits the restraining spring.

The new relay described by Mr. Beeman does have the improved characteristics which are particularly suited to the smaller capacity medium-voltage networks such as the Public Service has been constructing recently in New Jersey.

D. L. Beeman: Mr. Wilder's suggestion of disconnecting the potential coil neutral, from ground, of conventional network relays with line-to-neutral connected potential coils increases the relay torque for ground faults only. The increase in torque, however, is about one-half as great as if line-to-line connected potential coils are used. Disconnecting the potential coil neutral from ground of line-to-ground connected potential coil relays is of no benefit for unbalanced faults not involving ground. Consequently, it is of no benefit on systems using network transformers with delta-connected high-voltage windings.

Lead Storage Batteries in the Transportation Field

Discussion of paper 41-34 by Roland Whitehurst, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (June section) pages 303–07.

W. S. H. Hamilton (New York Central System, New York, N. Y.): Once in a while it pays us to pause for a moment to take stock of our progress in a given field, see what our wants are at present, and try to picture our requirements for the future. Mr. Whitehurst has covered the first two items well in his paper and some of the rest of us should state what we think the future ought to bring, even if we ourselves are unable to indicate how those results can be accomplished.

Before commenting on the possibilities of the future, there is one point to which I wish to take exception in Mr. Whitehurst's paper. Under section *B*1 he gives the intimation that when the railroads did away with the Planté batteries, they adopted a single-life battery, the intimation being that at the end of the useful life of the elements the entire battery was scrapped. Such has not been the practice on our system and I know it is not on a number of other railroads. We have, ever since the introduction of the pasted-plate batteries, made a practice of renewing elements in existing jars and trays in the case of the wooden-tray batteries and this practice is more desirable than ever in connection with the rubber Monobloc containers. We have in service some jars that are approximately 25 years old and while the life of wood trays of course is nowhere near this long, there is no reason why under reasonably good conditions the life of two sets of elements cannot be secured from even wood trays. Therefore, we do not agree with the policy of discouraging the rebuilding of batteries in the railroad company's shops, and have secured very successful results in recent years from the purchase of wet-charged elements which require the minimum of working to place them in condition for use.

Table III gives a very good idea of the progress which has been made over the years since 1920 in reducing space and weight for the same rating of battery. In between the 1920 and 1940 figures, there could be inserted the figures for a typical pasted-plate battery of 1930 which had a weight of about 1,850 pounds.

While this very great improvement in weight and space has been made possible primarily by two rather radical improvements, namely the substitution of the pasted plate for the Planté, and the rubber Monobloc container for the wood tray with jars, there have also been a great many other gradual improvements over the years which have materially helped in accomplishing the over-all result.

As far as the future is concerned there is still the opportunity for someone to make a radical discovery or improvement in batteries that would change the whole picture completely. Such a battery might even be based on a new concept of storing energy and the change from those of our present knowledge would have to be as distinct in concept as the difference between the reciprocating steam engine and the gas turbine.

The researches in plastic materials indicated by Mr. Whitehurst will be most helpful if a reasonably transparent container results, which will enable inspections to be made without removing the elements, or probing for sediment as is now necessary. As an alternate arrangement there is the possibility of providing a transparent window only in the container.

Looking now toward specific applications in the future, in the Diesel locomotive field there is need for standardization of the number of cells in batteries much the same as has been accomplished on passenger cars by the adoption of 16- and 32-cell batteries. This work has already been undertaken by a committee of the AAR. There are many reasons why 32 cells is a good choice for the larger sizes of locomotives and should conditions ever require it, there is the possibility of series paralleling two groups of 32 cells each for cranking purposes in the manner as described by Mr. Whitehurst for 12/24 volts for cranking Diesel engines in busses.

In the car lighting and air-conditioning field there will undoubtedly be an extension of the use of internal combustion engines on passenger equipment cars in the future and the advent of the d-c fluorescent lamp will probably swing the trend toward the use of the 64-volt system with 32-cell batteries. Incidentally this lamp can be operated on both alternating current and direct current by selecting the proper a-c voltage which adds to its versatility.

For cars with electromechanical air conditioning there should be a decided trend toward providing means on the cars themselves for converting from 220-volt, three-phase, 60-cycle alternating current to direct current for charging the battery. Only those who have been intimately connected with the problem of providing d-c yard battery charging facilities realize the cost and complications involved. The general provision of 220-volt, three-phase, 60-cycle a-c power for stand-by operation of air-conditioning equipment on cars has made available a much less expensive, more efficient, and more readily handled source of power for this purpose. Several means are available for converting this power to direct current: rectifiers, genemotors, and a-c–d-c compressor motors being the ones commonly used, with the genemotor apparently offering the greatest advantages at the present time.

While it may sound fantastic today, I believe that a battery which can be charged directly from alternating current is not beyond the realms of possibility within the next 20 years. Reference was made to this in Mr. Noertker's paper on "Unsolved Problems of Electrical Engineering in the Field of Transportation" (AIEE TRANSACTIONS, volume 59, 1940, pages 992–6) presented at our last winter convention.

Incidentally I have always been a firm advocate of the 64-volt system with 32 cell batteries for electromechanical air-conditioned cars and the New York Central system has 367 of such cars now in service, all with 600 ampere-hour batteries. The individual trays are lighter and more easily handled, the cables, generator, and motor commutators are smaller and one does not have to be continually on the lookout for the least voltage drop at every point in the circuit as with the 32 volt system.

The New York Central system was the first railroad to adopt the wrapped Monobloc container for air-conditioning batteries and we now have approximately 320 such 32-cell batteries in service using 5,120 containers.

In connection with air-conditioning batteries there is need for further investigation of the relative capacities at high and low rates of discharge, especially as the batteries become older. We are much interested in the three-hour rate at the present time and are basing the transfer from heavy duty (electromechanical) cars to light duty (mechanical) cars on tests made at approximately this rate.

We find in these tests that sometimes batteries will more readily meet the high-rate test than they will the low-rate test, and that different makes of batteries act differently in this respect. Further study along these lines is indicated.

In closing I hope I will be here in 1961 and hear what progress has been made in the next 20 years in storage batteries and also see how closely the developments have followed such predictions as are here given.

Complete Analysis of Motor Temperature Rise

Discussion and author's closure of paper 41-29 by Fremont Felix and H. G. Jungk, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941, pages 578–86.

P. L. Alger (General Electric Company, Schenectady, N. Y.): This paper is especially interesting to me because it brings out so clearly the different temperature limitations of electric motors. Most electrical apparatus, however, has other limitations besides thermal ones, and I should like the authors to say what these are and what their relations are to the thermal limits for the railway motor case. I suppose that the slipping point of the wheels establishes a limiting torque for each speed, and that the commutation limit is set by design to be somewhat above this torque limit at all points. It will be helpful if these torque and commutation limits could be indicated on the performance curves.

H. G. Jungk: As Mr. Alger states generally about electrical apparatus, all a-c, single-phase commutator railway motors have definite limits in design and service. The slipping point of the wheels defines the number of poles, field strength, and armature ampere turns required. The maximum current density of the brush contact surface and volts between adjacent bars determines the size and number of brushes and commutator bars.

Maximum accelerating tractive effort per axle and speed to which this maximum accelerating tractive effort is maintained defines the copper size for maximum accelerating currents and the commutation requirements. The range from this speed, to which maximum tractive effort is maintained, up to maximum sustained service speed determines commutation requirements and transformer capacity to supply the power. The maximum sustained speed is limited by the mechanical design of the armature and commutator, gear ratio, and wheel size.

These relations then give the "envelope" or curve from zero to maximum service speed showing all the limits of operation. Then time-temperature curves within this envelope define all the limits due to temperature rise.

The time-temperature curves, as appear in our paper, are limited by such an envelope. The maximum starting and accelerating tractive efforts are limited at the maximum setting of overload switches, the range from accelerating tractive effort to maximum sustained service speed is limited by the highest voltage speed curve available on the locomotive, and the maximum speed is the maximum sustained service speed.

Power Arc-Over on Overhead Distribution Lines and Newly Developed Equipment for Protection Against Conductor Burndown From That Cause

Discussion and author's closure of paper 41-45 by G. A. Matthews, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941, pages 596–604.

J. A. Elzi (The Commonwealth and Southern Corporation, Jackson, Mich.): The comprehensive series of tests made in connection with this investigation are not only indicative of the fact that the equipment developed is effective in protecting covered wire from burndown and avoiding the blowing of fuses on transient faults, but also provides very valuable factual data regarding the behavior of arcs under various conditions. While the author stresses the above two features of this equipment, it appears that the devices which have been developed, particularly the reclosing breaker, possess other desirable features thus providing a welcome addition to available circuit interrupting devices.

Since the reclosing breaker must be selective with fuses or relays after the initial instantaneous operation, it would be interesting to have a little more information regarding the time delay tripping device and whether or not it can be set to be selective with relays and fuses.

The two-cycle switching speed of the

reclosing breaker no doubt produces voltage surges considerably higher than have been experienced on distribution circuits in the past with conventional circuit breakers. This may introduce additional problems, and it would be interesting to know if the author has had occasion to obtain any data on this point.

E. V. Sayles (The Commonwealth and Southern Corporation, Jackson, Mich.): The thoroughgoing analysis by Mr. Matthews of the behavior of transient faults on overhead distribution lines and the results of his work leading to the development of high-speed switching devices are a valuable contribution to the industry in its effort to improve service.

One of the most valuable results of these studies is the development of the short-circuiting device designed to prevent the burndown of even the smallest sizes of weatherproof conductors commonly used for overhead distribution systems. Because of this device, transient faults can be automatically cleared so quickly that branch fuses protecting the circuit will not operate.

It is interesting to consider the use of the fast reclosing breaker as a low tension switching device at small distribution substations where the step-down transformer capacity does not exceed 2,500 to 3,000 kva. The simplicity of the reclosing breaker makes possible its mounting directly on brackets attached to the tank of transformers or regulators, and should simplify distribution substation design where the requirements of the distribution system permit the use of unit type equipment. Substantial savings, subject only to the limitation in capacity of the new device, should result from its substitution for conventional equipment. While the tests described were all made on a 4,800-volt ungrounded system, it is reasonable to assume that the advantages of this new equipment could be realized on the conventional multi-grounded four-wire distribution system as well.

To one who has witnessed many of the tests of this equipment, the greatest single feature of the fast reclosing breaker is the high speed of circuit opening and closing on the initial operation. This over-all time of six cycles is so short that the customer's reaction to the loss of voltage for this duration can be placed in the "insensible zone."

The possibilities of improved distribution system design which these devices offer should serve to stimulate the interest of distribution engineers. Although field experience may lead to improvements of the devices, the fundamental principles on which they are based and the protective scheme which they embody are essentially sound.

A. H. Kidder (Philadelphia Electric Company, Philadelphia, Pa.): Mr. Matthews is to be commended especially for his development of the fast-opening circuit breaker for distribution circuit protection. It should prove to be extremely useful for preventing unnecessary distribution branch-line fuse operations. The breaker is small and well adapted to pole-top installation. It has a load rating and interrupting capacity that is adequate for almost any distribution circuit. It will open quickly enough to protect suitable fuses against operation before nature has been given some opportunity to clear the circuit of the trouble. Distribution branch-line fault sectionalizing is almost indispensable with bare wire construction. Each branch-line fuse operation prevented, however, saves about one hour of outage for the customers on the branch. This, I think, is a very fine opportunity for the Matthews' breaker.

I am not by any means disinterested in the problem of burndown. My own experience indicates that well over 90 per cent of all line faults in four-kilovolt construction is due to other causes than power arcs following lightning flashovers. Mr. Matthews' motion pictures left no doubt that the covered wires will burn down, if a tree limb or other similar construction holds the wires close enough together so that the arc will restrike when the Matthews' breaker recloses. Covered wire burnoffs, figure 2 in the paper, and fuse operations are very nearly alike in their current time characteristics, however, so that a system which will prevent significant wire injury before the first opening of the breaker, can also be used to prevent fuse operation before the first opening of the breaker.

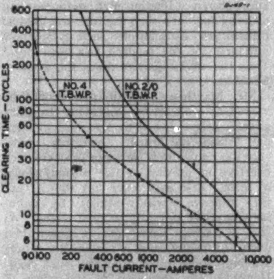

Figure 1. Total self-clearing time for 2,500-volt faults between covered wires

The interval between the initial opening of the breaker and its first reclosure deserves more consideration than has been given to it, I think, because the wires will burn down or the fuses will blow unnecessarily, if the breaker recloses prematurely. My own experience suggests that the lightning flashover of line insulators is almost innocuous at four kilovolts. I attribute this to the likelihood that power arcs will not be ignited by a lightning flashover path whose length is greater than five inches per kilovolt[1] of the operating potential between conductors. The tree branches which cause the preponderance of our distribution circuit troubles may move slowly enough to make a 5- to 15-second delay desirable before undertaking the first reclosure of the circuit breaker. I understand that the Matthews breaker can introduce some such interval, if it should be found to be a sufficiently important factor. For the present, however, it seems quite probable that fast reclosing, such as is characteristic of repeater fuse installations, may not be nearly so effective at four to five kilovolts as it is at the higher voltages.

Mr. Matthews' figure 2 gives only the time required to sever the conductors. For those who may be interested, I am including a figure which shows for two wire sizes of covered wire, the total fault clearing time required to sever the conductor and to extinguish the accompanying power arc at 2,500 volts. Our tests of total fault clearing time were made with slack wires, about four years ago, in an attempt to appraise the maximum probable severity of covered wire faults.

REFERENCE

1. LIGHTNING PROTECTION OF MEDIUM-VOLTAGE LINES, H. N. Ekvall. AIEE TRANSACTIONS, volume 60, 1941 (March section), page 130, figure 2.

D. K. Blake (General Electric Company, Schenectady, N. Y.): One is impressed with the thoroughness and the magnitude of Mr. Matthews' investigation. His data on the burndown characteristics of wire and determination of amount of burning permitted without serious reduction of mechanical strength ought to be very useful to the industry.

Service outage data from other companies, as well as his own, point to the primary system as the place to spend money to realize a substantial reduction in outages to the consumer.

G. F. Lincks (General Electric Company, Pittsfield, Mass.): Mr. Matthews has made a noteworthy contribution to the study and improvement of overcurrent protection on distribution systems not only in providing operating statistics, test data, and new equipment but in helping us think more clearly on this vital subject. Distribution primary lines in general present the greatest exposure, the greatest potential hazard due to location on the city streets and along the traveled way, and by actual record, cause the greatest percentage of consumer minutes outage. Line sectionalizing and branch-line fusing have proved a very effective remedy, employed by an increasing number of utilities across the country during the past few years. Wherever Mr. Matthews' new developments can be included they should prove complimentary to this whole program affording the increased advantages he describes.

It would appear that the data in table I could be of greater value if we could know what percentage of the number of wires burned down in 1938 and in 1939 were of the total outages in the respective years. In other words, is this problem 10 per cent or 90 per cent of the total overcurrent protection problem?

Studies on a large number of systems have shown that a high percentage of them have very largely outgrown the protective orbit of the station breaker and its overcurrent protective relay. Unless some additional protective device, such as a sectionalizing fuse, is provided for this outlaying area, faults must either burn clear or burn the line down with the attendant hazard to life and property. If Mr. Matthews could break down the data in table I to show the percentage within and without the protective orbit of the station breaker and with or without fuse or other protection, it would be helpful in determining the relative values to be derived from protection and by the various methods available. Additional test data would be valuable as to the annealing effect on hard drawn wires from the heating of sustained fault currents which would cause sagging and thus a greater tendency for lines to swing together and cause more faults.

There is one cautionary note that should be sounded. In these days of increased use of automatic open and reclosing and now short-circuiting equipments we should not overlook the fact that practically every consumer is advised of every interruption in voltage supply, no matter how short it may be. The telltale on the electric clock flashes red. The consumer has no way of telling whether the outage was for a fraction of a cycle or for several minutes except by checking some mechanical clock or an outside source. If momentary outages become too frequent, service may be impaired psychologically even in cases when actually it is being improved. The results can show up adversely in the commercial sales from which all our revenue is derived. Every scheme requesting *only a momentary dip in voltage* should be weighed carefully in this respect. Mr. Matthews' methods probably will find a justifiably wide usage whereas other schemes will and should be discarded.

There are two probable reactions to Mr. Matthews' discussion of the problem as it affects future trends in distribution fuse link design. One is to provide a line of slow fuse links having time-current characteristics that line up with those suggested in figure 15 of the paper. The names or ratings may differ somewhat. The other is to make a line of fuse links fast enough to provide equal protection to that of the shorting contactors, even though they do cause a permanent outage to a few consumers and demand manual replacement after each outage.

The requirements for slow fuse links for use with the shorting contactors is reasonably within the possibility of practical design. In fact, a comparison shows a close similarity of these requirements with those for co-ordination with the small low-voltage service entrance breakers[1] for which fuse links are now available. The actual difference between existing fast and these slow links is slight. For example, there is a trend to standardize[2] on a one-tenth second melting current range for fast links, of 10 to 20 times the rating (5 to 10 times the 300-second melting current for any basis of rating). These values are inclusive of the plus 20 per cent allowable variations.[2] A line of the desired slow links would fall within a range including variations of from 17 to 25 times the rating ($8^1/_2$ to $12^1/_2$ times the 300-second melting current). The overlapping of the fast and slow ranges includes only the allowable plus 20 per cent variation of the fast fuse links. To avoid this an over-all range for all fuse links might be made from 10 to 25 times the rating (5 to $12^1/_2$ times the 300-second melting current) with a dividing line at 17 times the rating ($8^1/_2$ times the 300-second melting current). Then fuse links with melting time-current characteristics including variations which fall to the left of this dividing line will meet the requirements of fast links for the best co-ordination with relays. Those falling in the range from 17 to 25 times the rating including variations will meet the requirements of slow links for co-ordination with the shorting contactors and with low-voltage service entrance breakers.

According to the curves figure 3 of the paper, it is not practical to make all ratings of fast fuse links protect all wire sizes from number 6 to 1/0. However, a more careful study of the data for the curves in figure 2 of the paper will probably show that the majority of fuse links normally called upon to protect number 6 conductors in service will provide ample protection and a similar condition will exist with other fuse links and conductor sizes. Doubtless there will be cases where such protection is not possible and this should be taken into consideration. Making fuse links melt faster, while a practical design job, would adversely affect the co-ordination with motor starting demand currents when the fuses are used at transformer installations such as are now provided by the modern fast fuse link.[1] Thus the advisability of doing this is questionable.

Mr. Matthews' contribution should be viewed as a means for improving the gains already accomplished by line sectionalizing and branch-line fusing. Operating experience has shown that such practices are very effective in preventing the burning down of lines. Also experience has shown that they improve service continuity by confining the effect of faults to small areas. As Mr. Matthews points out, a blown fuse makes all faults cause a permanent outage although some might otherwise burn clear. However, the reduction of the effect of permanent outages which otherwise would take out the whole line tends to more than offset the increases in consumer minutes outage from the permanent interruptions to a few customers which otherwise would have cleared without an outage. Mr. Matthews' shunting contactors working in close co-ordination with fuse links make possible a further reduction in consumer minutes outage and the elimination of more of the hazards attendant upon the burning down of lines.

References

1. Discussion by L. M. Olmsted of G. F. Lincks and P. E. Brenner's paper, TRENDS IN DISTRIBUTION OVERCURRENT PROTECTION. AIEE TRANSACTIONS, volume 56, 1937 (July section), pages 897–8.

2. Being considered by the joint EEI-NEMA technical committee.

H. L. Wallau (Cleveland Electric Illuminating Company, Cleveland, Ohio): Distribution engineers should welcome Mr. Matthews' contribution to their art. The data assembled are extremely interesting and valuable. The author states that the percentages of transient to total faults have been variously estimated at from 50 to 90 per cent; he sets the Detroit ratio as of 60 to 70 per cent. I believe his estimate is conservative. Statistics for 1938 and 1939 for five systems with which I am familiar indicate a ratio of 80 per cent for that period. It is probable that some of the nonself-clearing faults included in the 20 per cent of total faults were initially transient (self-clearing) faults and would not have developed into permanent faults if the voltage had been collapsed in a time of the order of $^1/_2$ cycle.

But whether 50 per cent or 90 per cent, if the transient faults developing on a system can be snuffed out before material damage is done to distribution facilities and with scarcely perceptible service interruption, a tangible economic and an intangible psychological gain have been achieved. The net economic gain is the less the greater the cost of the installed facilities required to accomplish the desired results.

The author modestly attributes the development of the devices described to the Detroit Edison Company. I trust you can all read between the lines. He deserves credit for having studied the mechanism of flashover between conductors, for having made quantitative determinations of the effect of arcs of various magnitudes and durations upon them. With these data he developed the simple self-contained and self-operating devices necessary to furnish the requisite protection against conductor burndown, which devices he indicates are sufficiently low in installed cost to justify their use in many localities.

Some of us who have been fortunate enough to see moving pictures of a number of the author's tests can testify as to the effectiveness of the devices.

H. P. St. Clair (American Gas and Electric Service Corporation, New York, N. Y.): It seems to be quite a unanimous verdict, in which I wish also to concur, that the industry is indebted to Mr. Matthews for valuable data on arcing and burning characteristics of distribution lines under fault conditions and for the development of very effective and, we take it, not too expensive equipment for rapidly getting rid of the arcing for transient faults and quickly restoring the lines to service.

It might be of interest to speculate on the extent to which the problem has been affected by the nature of the system involved, that is, the isolated delta 4,800-volt, three-wire system. Certainly, the hazards to the public and to maintenance personnel are affected by the inherent possibilities of having lines on the ground without the flow of fault current for prompt relay action, although this would not apply to conductors placed on the ground as a result of phase-to-phase arcing. Neither does a fallen conductor upon a grounded system of this voltage class insure sufficient fault current for relay action as has been demonstrated many times.

I should also like to ask how much the actual fault current is increased when a phase-to-phase arc is replaced by a dead short circuit by the short-circuiting contactor. In other words, while the contactor kills the arc and prevents the burning of the wire, does it also serve any purpose in providing greater fault current for positive relay action?

Herman Halperin (Commonwealth Edison Company, Chicago, Ill.): Several years ago a number of large utilities exchanged operating records for a few years for distribution systems operating at about four kilovolts. Studies in Chicago in connection with this activity have shown that the average customer has an interruption about once every $2^{1}/_{2}$ years and that the prolonged outages, such as occur incidental to conductor burndowns, occur about once every $4^{1}/_{2}$ years. This seems to be excellent service and results in a brake on any enthusiasm that may develop for additional expenditures to use the ingenious equipment developed by Mr. Matthews. Troubles on primary wires in Chicago are cleared by the functioning of relays and breakers at the substation.

In the Chicago system, which is essentially urban in character, there were 345 cases of trouble involving primary wires burned down in the years 1938 and 1939 as compared with 1,696 on the Detroit system according to table I of the article. The Chicago system had about 1,500 circuit miles of four-kilovolt, four-wire, grounded-neutral overhead primaries, which I suspect was somewhat less than on the Detroit system, which included territory outside of the city of Detroit. It would be of interest to know the approximate number of troubles and miles of circuit for the city of Detroit only.

P. A. Jeanne (Bell Telephone Laboratories, Inc., New York, N. Y.): Mr. Matthews' paper gives, among other things, information which is of special interest in the consideration of joint use of poles by power distribution feeders and telephone facilities. It has been previously determined that in the order of 60 per cent of all contacts between power conductors and telephone plant in the voltage range from 5 to 15 kv are due to power supply conductor failure. Thus, any device or improvement in operating methods which will reduce the probability of conductor burndown is beneficial to joint use.

While the author confines his description largely to the clearing of phase-to-phase faults, it is presumed that the devices he describes could be made equally applicable to phase-to-ground faults on a grounded-neutral system. Perhaps the author would like to comment on this phase of the matter.

The joint subcommittee on development and research, Edison Electric Institute and Bell Telephone System, have recently completed an extensive oscillographic study on a number of grounded-neutral power distribution feeders principally to determine the operating performance of feeders equipped with various protective devices and schemes. The observations were conducted largely on circuits provided with ground relaying. In some instances the feeders were also equipped with repeater-type fuses and pole-top oil-circuit reclosers. The study covered periods from 4 to 55 months of observation on 20 selected distribution feeders having a total mileage of approximately 1,000 miles and operating at voltages ranging from 4 to 13.8 kv. Records from approximately 4,000 accidental disturbances were obtained and as far as possible each disturbance was correlated with the trouble on the feeder.

A few of the pertinent data obtained are listed below:

1. RELIABILITY

It was found that relays, fuses, and circuit interrupters operated consistently and reliably on currents at or above which they were set or designed to operate.

2. CAUSES OF FAULTS

Approximately 70 per cent of all faults involving ground were attributed to lightning and approximately 90 per cent of all faults caused no reported damage and may be classified as transient. Faults, as used in this classification, include disturbances having durations in excess of about one cycle.

3. BURNED-DOWN OR BROKEN WIRES

The percentage of total faults involving wire failures ranged from about 0.5 to 46 for the various feeders. Their frequency of occurrence ranged from about 1 to 23 per year per 100 miles of line. Lightning was responsible for about 19 per cent of the wire failures, the remainder being associated with objects getting into or hitting the line.

4. SPEED OF FAULT CLEARANCE AND RECLOSURE

On feeders equipped with instantaneous relays and for immediate automatic reclosures, average time from initiation of fault to first breaker opening ranged from 7 to 15 cycles for the various feeders; average time from initiation of fault to first reclosure ranged from 34 to 57 cycles.

5. MANNER OF FAULT CLEARANCE

The number of faults which were cleared by circuit breaker operation ranged from a very few on extensively fused systems to 76 per cent on systems with limited fusing. On systems equipped with instantaneous relaying and rapid reclosure, considering only those cases in which the fault current persisted until the breaker first opened, the feeder was clear of fault upon first breaker reclosure in from 50 to 100 per cent of the cases for the various feeders, the average for all feeders being approximately 75 per cent.

J. B. MacNeill (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): Mr. Matthews reviews an intensive investigation of means for improving radial distribution line service. He is correct in saying that commercial apparatus does not give the speeds he finds desirable. The fastest small indoor conventional circuit breaker equipment operated with reclosing relays set for instantaneous first reclosure gives approximately 20 cycles from moment of short circuit until contacts retouch. While this performance seems generally satisfactory, we must doubt its complete adequacy in view of the data presented by Mr. Matthews.

This paper gives a review of service requirements and, in addition, describes three devices:

1. A high-speed reclosing circuit breaker
2. A shorting contactor
3. A shorting contactor with circuit-opening devices.

Each of these units is self-contained and operated by series high-tension auxiliaries.

Because of general interest in this subject, which is being worked on by several groups, a clear understanding of industry requirements at this time is highly desirable.

Over 20 years ago high-speed reclosing was used on feeders that gave three complete operations in a total elapsed time of 1.5 seconds. Also, on the same system at about that time arc suppressors were used on 26-kv lines to short-circuit flashover. However, the general use of reclosing equipment has in later years made the arc suppressor for that voltage obsolete.

Again, great improvements have been made in reclosing equipment especially for high-voltage service. High-speed breakers in themselves have reduced the time necessary to deionize the original fault and have made fast reclosing speed usable. Thus, we have 138-kv apparatus on major transmission lines giving over-all reclosing speed of 18 to 20 cycles.

Comparable improvements can be made in low-voltage equipment if sufficient demand exists. Mr. Matthews' paper strongly indicates that such a demand may be growing. The type of device to service such a demand should be carefully considered. The type described in this paper doubtless has a field; however, the general results of reclosing service reported by the EEI shows that over 80 per cent of all overhead line faults are satisfactorily cleared on a first reclosure and thus present standard equipment is doing reasonably well. If required, considerable improvement can be made along present conventional lines by the use of high-speed interrupters, fast reclosing mechanisms, and proper relays.

I suggest a symposium on this subject, possibly at the summer convention, at which the operators could state their requirements for the future and the interested manufacturers indicate their ability to service such requirements.

Mr. Matthews' paper gives so concretely the advantages to be gained step by step with increased service restoration speed, that a reaffirmation of existing standards or a realignment of ideas on this important subject is required.

Philip Sporn (American Gas and Electric Service Corporation, New York, N. Y.): I was particularly interested in the work of Mr. Matthews and his associates as described in his paper on arc-over on overhead distribution lines from the standpoint of reducing to a minimum the arcing time of a circuit in order to reduce effect on service and damage to the distribution conductors. This is carrying out on a distribution system the same kind of work that we have carried out for the last five years on our high-voltage transmission circuits by the use of our ultrahigh-speed reclosing equipment. The order of time developed by Mr. Matthews of possibly one-half to two or three cycles is given as the maximum fault time now conceded to be that within which damage can be kept to a negligible figure. It is interesting to recall that when the first lightning committee of the AIEE was formed in 1926 that the writer, who was then secretary of the committee, consisting at that time of Messrs. H. H. Dewey, C. L. Fortescue, and M. L. Sindeband, was present at its first meeting, and heard the committee's agreement to the effect that provided relay time on a high-voltage circuit could be kept to a maximum of three to four seconds, no concern need be felt for trouble with conductor burning.

We of course know today that that time has been cut to the order of three or four cycles instead of seconds for high-voltage lines, and Mr. Matthews' work shows that the same order of time is necessary in a distribution system.

Mr. Matthews and his associates are to be complimented upon the equipment that they have helped to develop to effectuate these results. I think, however, it might be pointed out here that other means for accomplishing the same result undoubtedly can and will be developed as times goes on. That, however, is not intended to detract in any manner from the credit that Mr. Matthews and his associates deserve.

G. A. Matthews: The author appreciates the wide interest shown in the paper. Many of the discussions submitted require no comments but the nature of some leads to the belief that there may be confusion with respect to the primary aim that lay behind the research and development reported. This belief is based upon references to operating performances and to statistical data intended, no doubt, to be pertinent to the problem but which, in reality, are not correct measures of the facts with which the project is chiefly concerned.

To clarify this apparent confusion the author wishes first to emphasize that the primary object of this work was not to improve quality of service but to decrease hazards associated with distribution wire burndowns. The word, decrease, is used advisedly, for it was not the intention to eliminate all burndowns. As pointed out in the paper, the expedient of burning down a line to clear a fault is considered valuable and justified in those cases of trouble where otherwise it would be necessary to hold the entire circuit out of service until the fault could be located and removed by other means. The purpose, therefore, was to prevent hazards associated with needless burndowns, or in other words, the burndowns that result from faults defined as transient. It is true that many other advantages accompany the solution of the main problem but in the present case these are inherent in the scheme of operation adopted and in the equipment developed; they work automatically to improve the service rendered to customers.

To review briefly, the power arc is the medium by which conductors are burned down. The destructive forces of the arc are dependent upon the magnitude of arc current and the duration of arcing. The current magnitude is fixed by the characteristics of the circuit and the location of the fault. The duration of arcing, however, is subject to control. Our research studies established the limits of the control required with covered wires to prevent burndown and to keep arc damage within admissible amounts. The solution of the problem, then, lay in providing means for initially interrupting the circuit in case of fault within the intervals of time established. As stated in the paper, these intervals range from about one-half to two cycles depending upon the size of wire and the arc current involved at the point of fault. One of the accompanying advantages referred to above is that this speed of initial interruption also prevents operation and damage to line fuses. In addition, it permits the location of fuses much closer to the power source without danger of unintended operation on transient faults.

It will be noted that all of this deals entirely with the initial interrupting time and is in no way involved with the reclosing practice. That is another matter. The one-half- to two-cycle initial interruption is designed to apply a safeguard against serious arc damage at the instant a fault is detected, and to keep the entire line intact whether covered or bare wire, sectionalizing fuses included, on the supposition that the fault may be transient. If the supposition proves correct, and indications are that a high percentage of faults are of this type, the hazard of a possible burndown will have been avoided, and in addition, the entire circuit will be returned to service upon reclosure. If, however, the fault proves to be persistent, the time-delay control brought into action upon reclosure will serve to burn the fault clear, if possible, as in present practice.

Turning now to Mr. MacNeill's opening remark, the author repeats that the chief aim of the investigation was not to improve radial distribution line service but to reduce wire burndowns and arc damage that would lead to mechanical failure of conductors. It naturally follows that if the number of wires down is reduced, service is automatically improved. With reference to the inadequacy of the speeds of conventional equipment, it should be made clear that reference to performance in terms of reclosing speed, or to use Mr. MacNeill's expression, in terms of total elapsed time from, "moment of short circuit until contacts retouch," is not indicative of ability or inability to prevent wire burndown. The breakers to which Mr. MacNeill refers have an over-all fault clearing time of the order of about ten cycles, two cycles of this time representing arcing within the breaker. This performance, as has been shown, is inadequate for the purpose under consideration.

In reviewing the paper, Mr. MacNeill points out that three devices were developed. This is true, but it is felt that his statement, that the units are operated by high-tension auxiliaries will be confusing to those not so familiar with the equipment inasmuch as the parts referred to as auxiliaries are integral parts of a self-contained mechanism.

With respect to the high-speed reclosing equipment and arc suppressers used 20 years ago, these could not be actuated to clear an arc in one-half cycle which has been found to be necessary if we are to prevent objectionable damage to the smaller sizes of conductors.

The author is familiar with many of the developments in high-speed reclosing equipment but points out that most of the attention has been given to speed of reclosing and not to the speed of opening the circuit to clear arcing faults. In this connection it might be mentioned that the shorting contactor with its own circuit-opening device is capable of not only one-half-cycle interruption but of restoring voltage in a total time as short as four cycles if desired in 7,500-volt breakers.

The statement that 80 per cent of all overhead line faults are satisfactorily cleared on a first reclosure and that thus present standard equipment is doing reasonably well, is not a measure of the number of cases of wires down. The 80 per cent figure checks the experience on the Detroit Edison system, but reference to table I in the paper will show that on that system there was still an average of 845 cases of wires down per year for the period 1938 through 1939. Our studies indicate that improvements cannot be made economically along conventional lines, using high-speed interrupters and available relays and trip coils. However, the shorting contactor used in combination with existing conventional switchgear, as described in the paper, will provide for the quick clearance of faults to reduce arc damage and the number of falling wires in an economical manner.

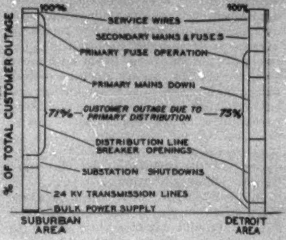

Figure 2

Mr. MacNeill refers in his closing remarks to advantages to be gained with increased restoration speed, a point he has made paramount throughout his discussion. It is the author's opinion that reclosing speeds that have been available in conventional equipment for some time (15 to 30 cycles) are quite satisfactory so that there is no problem along this line. The real need is for a speed of interruption which will save the overhead conductors from needless damage.

The author is enthusiastic over the suggestion of a symposium on this whole subject to bring out the operating requirements. This should prove helpful to all concerned. However, in order that a common understanding of all factors involved be made possible, it is recommended that care be exercised in the choice of terms and expressions used to bring out the picture. Terminology in this field is poorly defined and lacking in many respects. Effort directed toward standardization of methods for expressing the facts would, in the author's opinion, be well worth while.

Mr. Jeanne's comments concerning the advantageous effects on joint use of poles of reducing the number of distribution line failures is pertinent. The situation should be materially improved by eliminating the needless burndowns.

The data abstracted from the studies of the joint subcommittee on development and research are interesting indications of faults and their causes but, as given, the information is not sufficiently complete to permit a comprehensive analysis. Faults are defined, for that study, as disturbances having durations over about one cycle. This prompts one to question whether the results include, as faults, surges that under normal operating conditions might go unnoticed. The point the author wishes to make is that if our definitions of a fault do not agree, then any percentages based upon the number of faults will not be comparable and confusion and misunderstanding may result.

On the whole, Mr. Jeanne's remarks indicate that faster clearance of faults is desirable from the telephone engineer's point of view to reduce the number of interferences between telephone and power lines.

Mr. Jeanne and Mr. Sayles both inquire concerning the applicability of the devices developed to grounded-neutral systems. Used on such systems these devices will be as effective in interrupting arcs to ground as they are in interrupting phase-to-phase

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arcs. There is no difference in the method of application except that in the case of the grounded system there must be one shorting contactor in each phase and the external short-circuiting connection between the devices must be grounded in order to shunt a phase-to-ground short circuit. Likewise, where the fast opening and reclosing breaker is used there must be a unit in each phase whereas in the ungrounded system two units suffice.

The results reported by Mr. Halperin of studies conducted several years ago in Chicago show that from the customer outage point of view service is already excellent. Similar figures were obtained in Detroit and if this were the only consideration involved we would not be justified in spending more money. However, as pointed out above, there are still too many cases of wires down.

Mr. Halperin reports 345 cases of trouble in the Chicago urban area during 1938 and 1939 involving primary wires burned down. On the basis of the reported 1,500 circuit miles, this amounts to 8.7 cases of burndowns per year per circuit mile. In the city of Detroit during the same period there were 703 cases of trouble involving primary wires down in 1,830 miles of circuit. Of these, 147 cases were attributed to an extraordinarily severe sleet storm. Eliminating these from the total of 703 we get 556 cases. If we neglect that the cause of some of these cases was mechanical interference, and assume that all 556 cases were the result of burndown, we obtain 6.6 cases of burndown per year per circuit mile for Detroit as against 8.7 for Chicago.

Since Mr. Halperin's comments refer only to urban territory, for which only one of the devices developed, namely the shorting contactor, would be economically applicable, it is assumed that his stated lack of enthusiasm does not extend to the field for which the fast opening and reclosing breaker is intended which is chiefly suburban and rural.

Mr. Lincks asks the relationship between the number of cases of wires down, as shown in table I in the paper, and the total number of outages occurring on the system during the same period. Outage data for the years 1938 and 1939 do not happen to be readily available in convenient form for a direct comparison but the accompanying chart, figure 2, shows for the period 1933 through 1936 the types of relationship in which Mr. Lincks is interested, but we do not believe it to be different from the period. These data have been divided between the Detroit area and the suburban area and in each case the percentage of the total outage chargeable to the different parts of the system is indicated. As noted, the data include all outages over five minutes in duration. It can be seen that for the Detroit area 75 per cent of the outage was due to primary distribution troubles, wires down accounting for something close to one-half of the 75 per cent. The corresponding figure for the suburban area is 71 per cent, wires down again accounting for about a half. This type of classification of outages, as has been pointed out frequently in the past, shows that the distribution system is the weakest link in the conducting path between power plant and customer.

With reference to the growth of distribution circuits beyond the orbit of the station breaker and its overcurrent protective relay, it may be mentioned that the shorting contactor installed out on the line can be set low enough to trip on faults at the end of the line and still be within the range of load currents at its point of installation. When the shorting contactor operates it will increase the fault current sufficiently to cause operation of the station breaker. This explanation also answers one of the points raised by Mr. St. Clair.

Mr. Lincks states that in these days of increased use of automatic opening and reclosing, and now, short-circuiting equipments, we should not overlook the fact that interruptions, no matter how short, may cause unfavorable psychological reactions on the part of the customer because of the telltale on the electric clock. It is assumed that this refers to the probable greater number of openings to be expected with instantaneous trip than with time-delay initial opening.

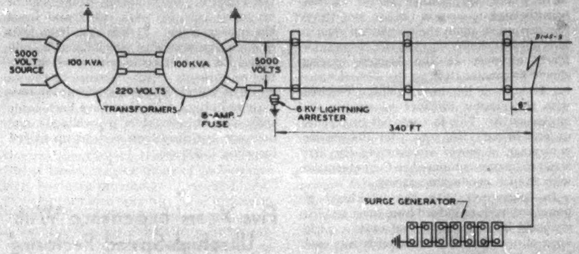

Figure 3. Arrangement used for surge flashover tests on a 5,000-volt line to determine the presence of 60-cycle power-follow current

The presence of 60-cycle current following a surge flashover on the line was ascertained by the color in the arc and the blowing of the eight-ampere fuse

Several years' experience on the Detroit Edison system with some 25 circuits equipped with instantaneous relays, and two years' experience with two circuits equipped with shorting contactors, show that the number of customer interruptions with this system of operation is not objectionable. It is the author's opinion that in going from time-delay to instantaneous tripping on fused circuits equipped with conventional switchgear there is not the increase in interruptions that might be expected because of the fact that the fuses blow on transient faults. When such circuits are equipped with the newly developed high-speed interrupting devices the decrease in customer outage due to protection fuses and covered wire against transient faults will greatly outweigh any disadvantages due to increased momentary interruptions. As a matter of fact, the author believes that the psychological reaction of the customer will be favorable rather than unfavorable when he sees the rapid restoration of service notwithstanding a greater number of interruptions.

Mr. Kidder states that his experience suggests that lightning flashover of line insulators is almost innocuous at four kilovolts and he attributes this to the likelihood that power arcs will not be ignited by a flashover where the path is greater than five inches per kilovolt of operating potential. He gives a reference to the paper by H. N. Ekvall, entitled "Lightning Protection of Medium-Voltage Lines," from which it is understood that the figure of five inches per kilovolt is based on the length of the arc-over path measured along the surface of the supporting structure rather than between the phase wires in air. On this basis the corresponding figure for The Detroit Edison Company's 5,000-volt distribution system, which uses a minimum 28-inch conductor spacing, is about eight inches per kilovolt. In view of this fact and because the majority of wire burndowns on this system occur during electrical storms it was thought advisable to make tests in our surge laboratory to verify the applicability of Mr. Kidder's experience to the Detroit conditions. A description and the results of the tests follow.

A diagram of the test setup is shown in the accompanying figure 3. Standard crossarms with wood pins and porcelain insulators were used. The conductors were number 6 covered wire. As indicated, the lead from the surge generator was connected about six inches from one of the crossarms. The generator rating is 700 kv, 12,000 amperes, and the rate of voltage rise used was 1,600 kv per microsecond. The estimated time required to dissipate the 12,000 amperes was 30 to 50 microseconds. The eight-ampere fuse was connected in the circuit on the source side of the lightning arrester to clear the circuit of fault current in the event of power arc-over. Maximum fault current, at the point of flashover was measured and found to be 240 amperes. In order to simulate conditions during an electrical storm the crossarm near the surge generator connection was sprinkled with water before each test.

In the first tests the conductor spacing was 14 inches in air which gave an arc-over distance over the wood insulation equivalent to five inches per kilovolt at the 5,000-volt phase-to-phase potential used. In nine out of ten tests power-follow current caused operation of the fuse to clear the circuit. The blowing time of the eight-ampere fuse with 240 amperes is six cycles. In spite of the proximity of the discharge to the wet crossarm, flashover in all tests took place in the air space between conductors without touching the crossarm. It appears therefore that the method of calculating inches per kilovolt from the length of the flashover path along the crossarm might be open to question.

In the next series of tests the conductor spacing in air was increased to 28 inches which gave an arc-over distance over wood equivalent to eight inches per kilovolt. In this series 60-cycle power-follow current caused fuse operation in 10 out of 38 tests. In 20 of the 38 tests it was noted that flashover took place directly between the phase conductors and in the remaining 18 tests the flashover followed a path along the crossarm.

In these tests no attempt was made to co-ordinate the application of the surge impulse with the 60-cycle wave. The percentage of cases in which the impulse was superimposed at a point on the 60-cycle wave where the voltage would be sufficiently high to sustain the arc was therefore dependent upon the element of chance. It is obvious, however, that the chances for power arc-over at the 14-inch spacing should be greater than at the 28-inch spacing because of the lower voltage required with the shorter distance to sustain the current path. This factor is felt to explain, at least in part, the reason for the greater percentage of power arc-overs in the first series of tests as compared to the percentage with 28-inch conductor spacing.

In connection with the likelihood of power arc-over, as judged from these results, it should be understood that only a single wave of not over 50 microseconds was used in each test, whereas, in actual service the lines are frequently exposed to lightning waves having durations of the order of 100 to 200 microseconds which greatly increase the probability of power arc-over.

It is indicated by the results of these tests that the figure of five inches per kilovolt or even eight inches per kilovolt is not sufficient to be effective at an operating potential of 5,000 volts. Our operating experience at 5,000 volts also shows that eight inches per kilovolt does not give freedom from power arc-over due to lightning.

The curve data presented by Mr. Kidder pertaining to the total time required to sever covered wire and for the accompanying arc to be extinguished at 2,500 volts is of interest in connection with considerations of the time required to clear persistent faults.

Mr. Sporn's remarks are illustrative of the way in which our conceptions change as time goes on and more factual data and experience are accumulated. The author agrees with the statement that other means can be developed for accomplishing the main objective under discussion, namely, reduction of wire burndowns. It might be added that many important factors enter into the choice of means for the solution of this problem. One of these is the desirability of making the most of investments we already have in lines and equipments. Another is the desirability of taking advantage of any opportunities to help other situations while dealing with the chief point of issue. In the last analysis the best solution is that which gives the greatest over-all benefits consistent with a cost that is economically justified. The solution adopted in this case was chosen after carefully weighing consideration of this character.

Mr. St. Clair questions the extent to which the problem under discussion has been affected by the nature of the system involved, pointing to the fact that the Detroit system is of the ungrounded type. Since both grounded and ungrounded systems are subject to the same exposures it is felt that Mr. St. Clair has answered his own question by stating that a fallen conductor on a grounded system does not insure sufficient flow of fault current for relay action as has been demonstrated many times.

Mr. Blake's remarks that the primary system is the place to spend money to realize a substantial reduction in outage to the consumer is well borne out in the chart previously referred to as figure 2.

To Mr. Elzi's question as to whether the time-delay tripping device of the high-speed opening and reclosing breaker can be set to be selective with relays and fuses, the answer is yes. Means have been built into the equipment by which it can be calibrated for co-ordination with other protective equipment. With reference to the increase in voltage surges accompanying two-cycle switching, tests that have been made show no objectionable overvoltages and none are anticipated on systems up to and including 7,500 volts.

Five Years' Experience With Ultrahigh-Speed Reclosing of High-Voltage Transmission Lines

Discussion and authors' closure of paper 41-52 by Philip Sporn and C. A. Muller, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (May section) pages 241–6.

Eric T. B. Gross (College of the City of New York, N. Y.): In order to appreciate the advantages brought about by the application of ultrahigh-speed reclosing, we should compare the results obtained by the use of this device with the results obtained using similar remedial measures which are also designed to minimize the effects of faults. It seems to me that the earth-fault coil is the only device which can properly be compared with ultrahigh-speed reclosing. The term "earth-fault coil" is used to designate any single- or three-phase reactor which is designed and installed in such a way as to balance the charging current to ground of the network. These coils have been sometimes called by the names of their inventors, namely: Petersen, Jonas, Bauch, Reithoffer.[1] Of course, earth-fault coils are applicable in transmission and distribution systems only where the insulation level of the transformer star points is equal to or greater than phase voltage.

By ultrahigh-speed reclosing is meant the interruption of the arc current by opening the switches and keeping them open as long as necessary for the deionization of the arc. Earth-fault coils accomplish the same result, in the case of single line-to-ground faults, without the opening of any switch whatsoever. The arc current is quenched by the coil current and the very slow recovery of the voltage at the point of the fault allows the ionized air to disperse and thereby prevents restriking.

We may, therefore, compare the results of operation of the two methods of protection under similar fault conditions, namely single line-to-ground faults. Of the 71 flashovers mentioned in the paper, 55 or 77.5 per cent were single line-to-ground faults, which certainly would not have introduced disturbances if earth-fault coils for permanent operation had been used, provided the coils were properly designed and tuned. On the other hand, in 91 per cent of all cases, the ultrarapid reclosing was successful, so that the practical gain of this device is 13.5 per cent = 91 per cent − 77.5 per cent. The question remains as to whether it pays to introduce ultrahigh-speed reclosing to obtain this additional 13.5 per cent. Of course, in networks with solidly grounded neutrals, earth-fault coils cannot be used, and there remains only ultrahigh-speed reclosing. Furthermore, ultrahigh-speed reclosing can, in addition to earth-fault coils, be used to advantage in systems with resonant neutral grounding, to take care of the few shorts which occur between two and three phases.

It seems to me that in the medium voltage systems, such as the 22,000- or 33,000-volt systems, mentioned by the authors, earth-fault coils can be used even more advantageously. Particularly in radial systems, or systems having a small number of points of supply, the earth-fault coils can almost always be added without extensive system changes. I do know from practical experience with radial and network systems in Great Britain,[2] and Continental Europe that in all the faults that occur, the percentage of faults extinguished by the action of the coil is, on the average, between 85 per cent and 90 per cent. For example, in the case of a 290-mile, 23,000-volt mesh-type system in Czechoslovakia, in which most of the lines are without ground wires, the number of interruptions during a period preceding the installation of earth-fault coils (1923) was 183. Though the mileage had increased, in the corresponding period of the following year, after the coils had been put into operation, the number of outages was only 22. This means a reduction in interruptions of 88 per cent. It seems to me that the earth-fault coil is the perfect solution to the problem for subtransmission and distribution networks. I doubt very much that the cost of the high-speed relaying and reclosing equipment for such systems can be made competitive with the cost of earth-fault coils, except perhaps in systems where locating and isolating the few permanent faults introduces great operating difficulties.

REFERENCES

1*a*. PREVENTION OF LINE DISTURBANCES BY SUPPRESSION OF ARCING GROUNDS, W. Petersen. *Elektrotechnik und Maschinenbau*, volume 36, 1918, pages 297–303. *Science Abstracts*, section *B*, volume 21, 1918, page 348.

1*b*. PROTECTION OF HIGH TENSION NETWORKS IF THE ADMITTANCES FROM PHASE TO GROUND ARE UNBALANCED, J Jonas. *Elektrotechnik und Maschinenbau*, volume 38, 1920, pages 453–9.

1*c*. PROTECTION AGAINST EARTHS, A. Roth. *The Electrician*, volume 87, 1921, pages 101–03.

1*d*. Line Grounding Through Earth Fault Coils as Protection Against Ground Faults, R. Bauch. *Elektrotechnische Zeitschrift*, volume 42, 1921, pages 588–91 and 616–22. *Science Abstracts*, section *B*, volume 25, 1922, pages 60–2.

1*e*. New Connections for Earth Fault Coils, M. Reithoffer. *Elektrotechnik und Maschinenbau*, volume 39, 1921, pages 245–9.

2*a*. Line Protection by Petersen Coils, With Special Reference to Conditions Prevailing in Great Britain, H. W. Taylor and P. F. Stritzl. *Journal Inst. Electr. Eng.* (London), volume 82, 1938, page 387, and volume 83, 1938, page 708.

2*b*. Earth Leakage Relays, Eric T. B. Gross. *The Electrician*, volume 123, number 3191, 1939, pages 93–5.

J. W. Jones (Philadelphia Electric Company, Philadelphia, Pa.): The five-year record with perfect mechanical performance and 91 per cent successful reclosure, including some double-line faults, is a remarkably fine showing indicating establishment of the reclosing time on a satisfactory basis and perfection of breaker operating mechanism.

It is interesting to note that the proportion of one-, two-, and three-phase faults is practically the same as experienced on the Pennsylvania-New Jersey 220-kv system which in nine years' experience involving 194 faults, had 82 per cent one-phase, 11 per cent two-phase, and 7 per cent three-phase compared with 81, 11, and 8 per cent, respectively, in the paper.

Performance of the high-speed reclosing might not have been so satisfactory if used on lines operating closer to the stability limit, or on systems in which the power sources and load were not so well distributed.

There was apparently little need for high-speed reclosing in many of the operations. The oscillograms show no sign of instability or serious voltage disturbance after the initial trip-out and in many cases the lines could probably be kept out for the short time required to permit reclosing by the operator. However, there are a few cases of double-line faults, as illustrated by oscillogram 4, in which the high-speed reclosing was undoubtedly a distinct advantage.

S. B. Crary (General Electric Company, Schenectady, N. Y.): The outstanding record for high-speed reclosing as summarized in the paper by Messrs. Sporn and Muller should lend heartening encouragement to those concerned with obtaining the most from their transmission systems. The successful performance is undoubtedly due to the emphasis that has been put on speed of clearing and reclosing.

A major part of the system described in the paper, consists of double-circuit lines bussed together at comparatively short transmission distances which lend themselves particularly well to quick relaying, switching, and reclosing, as a minimum of shock to the system results from the quick switching out of any one of the parallel sections in the case of a single-circuit fault. However, successful reclosures even for single-circuit faults of double-circuit lines allow for a further increase in the line loading for a given number of intermediate stations if the operation is sufficiently quick, approaching as a limit the steady-state stability limit of two circuits.

The data given indicate that the probability for double-circuit faults is small with quick clearing. The double-circuit faults are likely to be most severe from a stability standpoint and also from the reclosing standpoint if the circuits are on the same tower. It is of considerable importance, therefore, to note that out of eight double-circuit faults, six reclosed successfully. It would be of interest to know if the two double-circuit faults which restruck, restruck on both circuits. If reclosure in such a case is successful even on only one circuit, it may be possible to retain synchronism between the two temporarily separated parts of the system. The chances for success would be influenced, among other factors, by the separation between the parallel circuits and the distance between intermediate switching stations which determine the length of single-circuit line left holding the system together.

A goodly percentage of successful double-circuit reclosures indicates even more definitely, than do the single-circuit reclosures of double-circuit lines, that quick reclosure allows an increase in what can be considered the reliable loading of the system. It would be of interest if the authors could give data to indicate the loads being carried at the time of reclosure.

There are factors which make quick reclosure desirable even if synchronism is lost. One of these, the possibility of pulling into step, has been previously mentioned. Another factor which may become of increasing importance, is that it reduces the possibility of separation except at points previously selected in case a sustained system out-of-step condition exists. This makes possible a more flexible generating schedule, in that generating capacity may be more accurately and easily allocated with load in case of a separation.

Rapid reclosure on the single-circuit lines is of particular importance because there exists no parallel circuit to keep the separated parts of the system in synchronism. From a stability point of view they might well be classified with the double-circuit faults of a double-circuit line. It would be of interest to know if there is any distinction between the experience obtained on double-circuit lines as compared with single-circuit lines.

David C. Prince (General Electric Company, Philadelphia, Pa.): One undertakes a great many projects which seem very important at the time but which after five years are so completely forgotten that one has to search the files to find out what the excitement was all about.

The record of ultrahigh-speed reclosing is a conspicuous exception to such experiences, and I regard with a great deal of satisfaction the fact that I was privileged to be in on the ground floor of the experiment, the success of which is recorded in this paper. Those who have been regular attendants at AIEE conventions know that no subject has received more study in more different ways than that of maintaining continuity of service over long-distance transmission lines. Safeguarding continuity has been studied from a number of different angles, including the use of ground-fault neutralizers, expulsion gaps, grounding systems, and overhead ground wires, parallel lines, etc., in addition to the ultrahigh-speed reclosing covered by this paper.

Ultrahigh-speed reclosing presents one important difference compared with any of these other methods of attack. It is always necessary to have circuit breakers and relaying equipment regardless of what other means may be employed in addition to reduce lightning outages. High-speed reclosing does not inherently require additional apparatus but only that the apparatus which is there in any case be designed for this exacting service.

The largest unknown in connection with ultrahigh-speed reclosure had to do with what actual experience would show. It was possible to determine by laboratory methods as reported in the earlier paper referred to by the authors that there was a certain probability of restrike, and what it depended upon. The unknowns had to do with the occurrence in nature of such restrikes. It was not possible to predict whether multiple lightning strokes, for instance, would actually cause many more restrikes than anticipated from laboratory tests or whether, on the other hand, the presence of wind might give a very much smaller number of restrikes.

Now we have the record before us, and as the authors have stated, the operating results have been superior to expectations. This opens the way to increased continuity of service by relatively simple and inexpensive means in a short time. This may be very valuable in the light of our present national emergency, during which it is even more important than in normal times to keep the wheels of industry turning by continuous electrical service.

E. E. George (Ebasco Services, Inc., New York, N. Y.): The paper by Messrs. Sporn and Muller presents the experience record of a very important contribution they have made to the operation of high-voltage interconnected systems. This work has been greatly facilitated by the efforts of the manufacturers in speeding up the operating time of relays, tripping mechanisms, and closing mechanisms.

It is interesting to note that two obstacles originally suggested as limiting the field of high-speed reclosing have not been found in practice to be as serious as was first suggested. A few years ago it was suggested that multiple lightning strokes occurred frequently enough and with proper timing to prevent successful high-speed reclosure in an important percentage of cases. The experience of the American Gas and Electric Company on 132-kv ultrahigh-speed reclosing and of many other transmission companies with immediate reclosing indicates little difficulty from multiple lightning strokes.

Upon the advent of immediate reclosing the manufacturers prepared a derating table for use in connection with immediate reclosing cycles. This table without substantial change is still included in the manufacturers' latest catalogues for use in selecting new breakers as well as in determining the capability of existing breakers. For the instantaneous automatic reclosing cycle a "nonoil-tight" breaker is derated to approximately 30 to 50 per cent of the normal two-minute rating and an oil-tight breaker to 85 per cent or less of the 15-second rating. This derating table has resulted in many companies being slow to take advantage of immediate reclosing for fear of blowing up a large number of their existing circuit breakers, or in lieu of this, being forced to replace them with much larger and more expensive breakers. There is sufficient operating experience from transmission systems by this time to show that this derating table must have some elements of only academic or commercial interest. The table has not been substantiated by operating experience with immediate reclosing. It is interesting to note that Messrs. Sporn and Muller make no mention of breaker derating as a deterrent factor in their plans for ultrahigh-speed reclosing.

Perhaps one of the joint committees might look into the matter of deratings—from the standpoint of operating experience.

L. R. Ludwig (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The problem of reducing transmission line outages, particularly due to lightning, has been tackled by at least four different methods; namely—

1. Overhead ground wires.
2. Installation of protector tubes.
3. Petersen coils.
4. High-speed breaker reclosing.

The authors of this paper have tried several of the above means, and their further comments regarding the relative merits of the various ones would have considerable value.

The paper does not specifically set forth a recommended time for circuit reclosure after completion of the interruption, and their further comments would be appreciated. In this connection, does the interrupting time seem to have any effect on the subsequent reclosing time, other than in possibly preventing a two-line flashover?

There is a growing interest in clearing single-line transient faults by opening and reclosing one pole of the breaker only. Do the authors have any data or recommendations as to this procedure?

The successful use of high-speed reclosing breakers is of great interest to the manufacturers, and the Westinghouse company has done considerable work with various forms of mechanisms and breakers for this service. Motor-operated reclosing mechanisms for 20-cycle over-all operating time have been built and placed in service. A compressed-air mechanism for oil breakers has also been developed and will shortly be in service. This device can be made very fast, it is simple, and particularly attractive because high operating forces are so easily obtained. An air cylinder and piston are used to close the breaker and the opening is by means of a spring in order to make interruption independent of failure of the air system.

Percy H. Thomas (Federal Power Commission, Atlanta, Ga.): The concise and complete statement of the facts offered and conclusions drawn makes questions put to the authors largely superfluous. On the other hand, the very concreteness of the results may make practicable the drawing of conclusions upon which important action can be based. For example, if in an exceptionally exacting situation like that of the operations described, 90 per cent of the lightning and homologous flashovers can be safely handled by quick-acting, self-closing breakers, does it follow that the extreme precautions now often taken to make lines lightning-proof may be relaxed, and materially cheaper construction used? If so, the advance achieved is of momentous importance and a notable event.

There have been many improvements in engineering designs and apparatus looking to more continuous and effective service, but very seldom does any new safety device seem to result in elimination of other and prior safeties with the lowering of cost and decrease of complexity.

The author has before pointed out the great saving in cost and relief from complication that would result in utility operations if each link in the system had an alternate, automatically cut in by trouble on the link, provided such relief, by shift to the alternate, be frankly relied upon and no extraordinary construction be attempted to make any link invulnerable. This high-speed, quick-acting breaker serves much the same purpose to the extent that it renders alternative protective expense unnecessary. It goes without saying, however, that no immediate general dropping of safeguards would be appropriate just because of a new protective means. The suggestion is that this idea of simplification be used as a guide with the purpose that a gradual evolution would result in ultimate economic gain.

Apparently there is a distinct difference in arc interrupting conditions between one case in which all phases of all transmission circuits connecting two centers are involved in a short, and a second case in which only a portion of the circuit phases connecting the centers are involved in the arc, or in which only grounds from some phases are involved. I would like to ask the authors whether they can trace a difference in kind in the operations of the quick-acting breakers between these two cases. Does the existence of some alternative paths for the flow of some energy in parallel to the arc constitute a necessary basis of success?

No statement is made in reporting these results whether any difficulty was met in any of the cases from the loss of synchronous apparatus.

Has any effort been made in these reported cases to connect the degree of success in arc suppression with the magnitude of the short-circuit current, as determined by total synchronous capacity and line impedance?

V. M. Marquis (American Gas and Electric Service Corporation, New York, N. Y.): In addition to the several reclosures discussed by the authors and illustrated by oscillograms, there have been reclosures at other points on the system which are of considerable interest and I should like to discuss four of these.

Referring to figure 1 of the authors' paper, the location and relation to the rest of the central system of the lines which I shall discuss can best be seen from figure 1 of the authors' paper. These are:

1. The Philo-Howard double-circuit 132-kv line which forms an important tie line between the central and western section of the central system.
2. The Saltville-North Bristol line which is a part of the single-circuit line which interconnects the central system with a portion of the system of the Carolina Power and Light Company.
3. The Roanoke-Fieldale line which is part of the line interconnecting the central system with a portion of the Carolina Power and Light Company.
4. The Roanoke-Reusens double-circuit line.

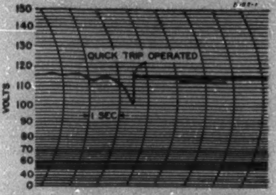

Figure 1. Voltage at Kingsport during ultrahigh-speed operation on the Saltville-North Bristol line

PHILO-HOWARD LINE

Over the past few years there have been a number of double-circuit outages on the Philo-Howard line. These invariably caused the Canton-Ashland-Howard line to trip, with the result that the central and western portions of the central system were split apart. While this did not result in any service interruptions to any part of the system, it did usually upset loading schedules to a considerable extent. In order to improve operating conditions, ultrahigh-speed reclosing was installed on these circuits in 1940. Since the installation date, there have been no cases of double-circuit trouble. However, there has been one case where number 1 circuit was out of service and lightning struck the number 2 circuit during this period. At that time approximately 70,000 kilowatts was being delivered from Philo to Howard over this single circuit. The line tripped and reclosed successfully. It is therefore felt that even with heavier loads on the line, in case of double-circuit outage reclosure will be entirely satisfactory.

SALTVILLE-NORTH BRISTOL LINE

Figure 1 shows the voltage record at Kingsport during a successful reclosure on the Saltville-North Bristol line. This indicates that the voltage was restored in approximately one-half second. This circuit is a section of the 150-mile line extending between Switchback and Waterville, 64 miles of which operate at 110,000 volts and the remainder at 132,000 volts with a 55,000-kva 132/110-kv autotransformer at Kingsport. There is a 7,500-kva condenser at Kingsport, a 9,000-kva condenser at Saltville, and an interconnection at North Bristol with the East Tennessee Light and Power Company. This latter company's system normally has 15,000 to 20,000 kw of generating capacity, most of which is hydro. During this case of trouble there were three 45,000-kva units in operation at Waterville. These units were carrying some 40,000 kw of local load and 55,000 kilowatts was being transmitted to Kingsport, about 48,000 kw of which found its way over the North Bristol-Saltville section of the line.

The point of interest in connection with this is that calculations indicated that during periods when Waterville hydroelectric plant was connected only to a small

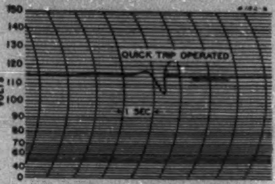

Figure 2. Voltage at Roanoke during ultrahigh-speed operation on the Roanoke-Fieldale line

local load and the central system it was doubtful if reclosing would be successful due to the line capacity and the relatively low inertia of the hydro units at Waterville as compared to that of the large predominantly steam system to the north. Regardless of the paper study, it was decided to install ultrarapid reclosing on the Saltville-North Bristol line and the North Bristol-Kingsport line. In the case just cited there was considerable local load at Waterville, nevertheless, experience indicates that reclosure would be successful even if this local load were small.

This line has also reclosed successfully with the Waterville line open, with the result that no load was lost at Kingsport, North Bristol, or Saltville and that the 7,500-kva condenser at Kingsport pulled in satisfactorily. The operation of both these sections of line indicates that ultrahigh-speed reclosing can be successfully applied to a setup of this character.

ROANOKE-FIELDALE LINE

Figure 2 shows a voltage record during a successful reclosure on the Roanoke-Fieldale line. Here again, the record indicates that voltage was restored to normal in about one-half second. The load being transferred across the line at the time was 25,000 kw. This line section is 37.5 miles in length. The distance from Roanoke to the nearest generation on the Carolina Power and Light Company system is 193 miles. It will be noted from table I of the authors' paper that the ultrahigh-speed reclosing equipment was installed in April 1940, and during the remainder of the year there were 12 reclosures, all of which were successful.

It is realized that the question still remains as to whether the arc will extinguish sufficiently fast to allow reclosing if synchronous equipment is connected at some point on the line. Our only experience has been in connection with the Roanoke-Reusens number 2 circuit. This line is tapped at about the mid-point and connects to a small system having a hydro plant of 1,200-kva capacity and with considerable synchronous load. It will be noted from table I of the paper that this line did not fail to reclose in the three cases experienced since the installation of ultrahigh-speed reclosing.

The application of ultrahigh-speed reclosing on the 132-kv lines of the central system of American Gas and Electric Company has made possible a higher degree of service continuity, the optimum use of generating facilities, both from the standpoint of bulk power transmission and from the standpoint of most economical loading of plants. It has also made possible more efficient use of interconnections with foreign companies.

Philip Sporn and Charles A. Muller: Mr. Thomas has raised a point which is sometimes neglected as advances are made in engineering practice; namely, that when an improvement is made to overcome a specific problem other improvements previously made for the same purpose should then be reconsidered in the light of the new development to see if their continued application is justified. It is the authors' belief that the advances which have been made in lightning protection of lines cannot be generally discontinued as a result of the use of ultrahigh-speed reclosing but that it may be possible to drop a few specific practices with a consequent reduction in costs. The points which must be kept in mind are that any such omissions must not increase the total number of permanent faults and that the increase in cases of lightning flash-over and resultant voltage dips must not affect the general system operation adversely.

The only basic difference between a fault involving all phases of all circuits connecting between two centers and one in which only grounds from some phases are involved, is that in the latter case some energy can still be transferred over the sound conductors, helping to retain synchronism. In this case, of course, the theoretical maximum power which may be delivered without loss of synchronism is increased. The gain, however, is small, since the power flows only until the circuit breakers have opened, which occurs in a maximum of nine cycles after the inception of the fault, and there is actually no practical difference in the success of reclosures involving trouble on one, two, or three phases. In the case of a fault of any kind on a single circuit of a double-circuit line there is, of course, a definite gain in the maximum power which can be transmitted, since in this case the sound line provides a path for continuous transmission of synchronizing power.

Aside from the question of maximum power which may be transmitted, the experience so far indicates that there is more chance of a fault restriking when more than one phase is involved. It is hoped that the development of breakers having faster clearing time will help to eliminate this difficulty by permitting a longer de-energized time with the same over-all reclosing time.

No difficulty has been experienced at any time with the loss of synchronous apparatus during reclosing operations.

Mr. George has brought up the question of derating breaker interrupting capacity when used for immediate reclosing and has pointed out that the manufacturers' latest catalogues still derate a breaker to 30 to 50 per cent of its normal 2-minute rating and to 85 per cent or less of its 15-second rating. It must be remembered that the two-minute rating applies only to old non-modernized breakers and a few low-capacity low-voltage breakers currently listed. Because of the duty imposed on breakers used for ultrahigh-speed reclosing, these breakers are always modern or modernized up-to-date types and are rated on a 15-second basis. For the 138-kv 1,500,000-kva breaker which has been used in most cases, the rating factor for immediate reclosing is 85 per cent of the 15-second rating. So far this has not represented any serious handicap, since interrupting duty has in all cases been well below this value. However, the accumulation of field experience over a period of time with this new method of operation may quite possibly furnish a basis for liberalizing even present derating factors.

Mr. Crary has brought out the important question of line loading at the time cases of double-circuit trouble occurred on lines connecting two centers or single-circuit trouble in cases where only one line existed. Most of the operations have occurred when these loadings were relatively light and there was no evidence of instability. However, Mr. Marquis' discussion has covered a few cases where these loadings were heavier, and it will answer Mr. Crary's question. In both cases listed in the paper where double-circuit faults restruck, the restrike occurred on both circuits.

Mr. Marquis has described the system loading conditions which represent the most severe requirements under which the ultrahigh-speed reclosing has so far been called upon to operate. They show very clearly the valuable contribution which this development has already made to reliable system operation and indicate that considerably more severe conditions may exist before the limits of successful operation are reached.

We believe that the operations Mr. Marquis has described for the Saltville-North Bristol line illustrate the question Mr. Jones has raised regarding successful operation when power sources and load are not well distributed. This was regarded from the beginning as one of the locations where there was most likelihood of unsuccessful performance, and it has been very gratifying to note the results that have been so far obtained.

With reference to Mr. Ludwig's question as to recommended time for circuit reclosure, the time which has been used has been 18 cycles from energization of the trip coil to re-energization of the line. This means that for 8-cycle breakers the line is de-energized 10 cycles and for 5-cycle breakers the line is de-energized 13 cycles. At the time the first ultrahigh-speed reclosing installation was put in service on the Ft. Wayne-Deer Creek line, tests were made to obtain data on minimum de-energized time permissible. These indicated that restrikes occur for one-phase-to-ground faults when the de-energized time is reduced to 6 cycles, and that they occur for phase-to-phase faults after as much as 12 cycles de-energized time. There has been no indication from operating experience as to whether interrupting time has any effect on the subsequent de-energized time required. However, there is some indication from laboratory tests as reported in the paper by Sporn and Prince published in the AIEE TRANSACTIONS, volume 56, pages 81–90, that a definite relation exists between these factors.

The authors have had no experience with clearing and reclosing line transient faults by single-pole breaker operations but will be very much interested in data on results which are obtained with this method of operation.

The question that Mr. Ludwig raises as to the relative merits of various methods of reducing transmission line outages is, of course, one of the important questions in transmission system design and operation. It is obvious that of the four methods of combating the effects of lightning, one is of a preventive nature and the other three are all remedial. As regards the preventive measures, the use of the overhead ground wire has become so well established from a technical standpoint that there doesn't seem to be any question that any important transmission line of even moderately high voltage cannot be considered to be properly designed unless provisions have been made for its proper use. This means not only a design of tower structures to give proper arrangement of conductors and of ground conductors, but also the concomitant devices such as proper grounding and/or counterpoise facilities. The authors are definitely in favor of carrying out on every project that much preventive work but all within economic limits.

Of the three remedial devices, the protective tubes covered a period in the study of the lightning problem and the working out of remedial devices in which they seemed to offer one of the few practical means of combating the effects of lightning flashover. Experience with protective tubes, particularly on high-voltage lines, over the last half a dozen years on the systems with which the authors are associated, has thrown serious doubt, not so much on the adequacy of the device as on the economic soundness of it. This is particularly so in the light of the generally unsatisfactory experience from a life standpoint with protective tubes of various kinds, including straight fiber, fiber-Herkolite, and fiber-Micarta-Herkolite combinations.

The Petersen coil has found considerable application on transmission systems in Europe but in analyzing the reason for its general nonacceptance in the United States, one needs to keep in mind first, the historic development of American systems which have always tended from the very early days to solid-grounded operation, and second, the difficulty that the Petersen coil offers to system operation, particularly in a situation where growth and development is experienced and where system switching may change from day to day and from month to month. This was not found to be a great handicap in European practice but in general it would appear as if the electric systems on which the Petersen coil was applied had more of a static character as regards growth and expansion than most American systems. Further, from a cost standpoint the Petersen coil undoubtedly involves greater expenditures, both in initial cost and in maintenance than the solidly grounded system utilizing high-speed reclosing. The latter obviously requires comparatively little change in the normal switching facilities and as those facilities become more modern, requires almost no change at all since standard modern switching equipment with standard modern relaying provide between them all the necessary facilities to make ultrahigh-speed reclosing possible. It has been the authors' experience that the latter, that is, ultrahigh-speed reclosing, has proved to be the one remedial device superior to all others on high-tension transmission circuits.

The points that Dr. Gross raises in connection with the Petersen coil are covered in the discussion given above on the points raised by Mr. Ludwig.

The authors share with Mr. Prince his feeling of gratification at the development of such satisfactory experience with ultrahigh-speed reclosing. They want to take this opportunity of acknowledging Mr. Prince's contribution to the early development of ultrahigh-speed reclosing.

Surges on Chicago 12-Kv System

Discussion and author's closure of paper 41-51 by F. O. Wollaston and H. J. Plumley, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (June section) pages 260–5.

R. C. Van Sickle (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The authors have made a very valuable contribution to our data on system overvoltages. That these data have been collected systematically over a period of years is especially noteworthy. It practically eliminated the question of whether chance produced the worst possible conditions, a question which is left open by field tests due to the necessarily limited amount of data obtained on them.

The surge voltages which were measured were certainly lower than the worst possible combination of events might have produced. They were explained by the phenomena of the application of faults and the simple opening of circuit breakers without restriking. More complicated phenomena surely occurred also and, although capable of producing higher voltages, evidently did not. As pointed out by the authors, the overvoltages on the Chicago system did not exceed those which could be explained without assuming restriking. These data, therefore, support a paper and another discussion presented by me at the winter convention in 1939. At that time I showed oscillograms which illustrated how circuit breakers by their dielectric characteristics tended to limit the magnitude of the switching surges. One oscillogram of a 220-kv charging current interruption showed seven restrikes, with the maximum voltage reaching less than was theoretically possible with two restrikes.

The large capacitances of the cables caused the natural frequencies to be low for the single line-to-ground faults which were studied. They may account also for the apparent failure of the surges to pass through the transformers.

C. Concordia and H. A. Peterson (General Electric Company, Schenectady, N. Y.): In the past few years several papers discussing system overvoltages have been presented before the AIEE. These have led to an increased understanding of the phenomena involved, and have in several cases explained specific instances of damage to equipment. These papers have been based principally on calculations, although as far as possible they have been confirmed by tests and field experience. Such field experience is extremely valuable, especially in evaluating the relative importance and probability of occurrence of the various theoretically possible forms of overvoltage, and thus also helping to guide future investigations. We are therefore very glad to see this summary by Messrs. Wollaston and Plumley of overvoltage records of the past 20 years on a large system. It is particularly gratifying that the conclusion derived from theoretical investigations[1] (that for systems grounded through resistance or reactance, the maximum transient voltage normally associated with fault application may be taken as a good measure of the maximum switching surge voltages to be expected) is substantiated by this record of field experience.

Moreover, tests on miniature systems corresponding closely with the one under discussion have indicated that under controlled conditions, the maximum voltages to ground that can be obtained as a result of routine switching and ordinary fault conditions is on the order of four times normal line-to-neutral crest voltage. This is in very good agreement with the upper limit of 3.5 indicated in the paper, since the probability of encountering the very worst combination of circumstances is very small.

The relative importance of the various peculiar and special phenomena which may result in high voltages on some systems is not merely proportional to their relative frequency of occurrence. On the other hand, study of cases which have resulted in damage has convinced us first, that it is more often than not those situations that are unusual and special which result in equipment failures, and second, that these dangerous situations are not always easily recognizable without careful consideration and analysis.

There is one question that we would like to ask in connection with this paper. Since presumably sphere gaps and insulation failures were generally at different locations, is it possible that voltages which caused failures at a given location were higher than those indicated by sphere gaps at another location, and perhaps somewhat higher than 3.5 times normal?

REFERENCES

1. OVERVOLTAGES DURING POWER SYSTEM FAULTS, Edith Clarke, S. B. Crary, and H. A. Peterson. AIEE TRANSACTIONS, volume 58, 1939 (August section), pages 377–85.

ARCING FAULTS IN POWER SYSTEMS, C. Concordia and H. A. Peterson. AIEE TRANSACTIONS, volume 60, 1941, pages 340–6.

Herman Halperin (Commonwealth Edison Company, Chicago, Ill.): In connection with conclusion 7 of this article, we know definitely that at least secondary failures are due to overvoltages. Examination of cable and joints that have been secondary failures in each case shows that the insulation was in poor condition, that is, for example, the sheath had been cracked at the duct and exposed the cable insulation. The point I want to make is that cable and joints of present-day design and material that are properly installed are not appreciably affected by transient overvoltages on underground systems. This point, so far as surges are concerned, is substantiated by the fact that the impulse strength of impregnated paper insulation is, according to tests reported by others, equal to at least 1,400 volts per mil of total thickness.

J. M. Dunham (Bell Telephone Laboratories, Inc., New York, N. Y.): In connection with the work of the joint subcommittee on development and research, Edison Electric Institute and Bell Telephone system, on fault current limitation, investigations of overvoltages and fault resistance have been carried on over a number of years. Overvoltage observations on systems grounded through resistance or reactance have been supplemented in some cases by staged tests. These tests included both solid and arcing line-to-ground faults, and measurements of overvoltage. Although the neutral impedance has in most cases been higher than that on the systems described by the authors, there is a similarity between their data and that obtained by the joint development and research subcommittee. In general, our observations have shown maximum overvoltages occurring at or near the initiation of the fault and of essentially a low-frequency nature. Also, about the same discrepancy between observed overvoltages during accidental faults and those obtained during staged tests has been noted in some cases. I would like to ask the authors if they have an explanation for this discrepancy.

Routine observations have shown that faults in enclosed insulation such as cable failures or bushing failures may lead to extinction and restriking of the arc at fundamental frequency. Overvoltages for this type of fault have been higher than for flashovers in air. I wonder if this type of fault has been observed by the authors or is the fault current too large to allow extinction of arcs initiated in the cables.

In regard to fault resistance, the joint subcommittee has co-operated with certain power companies in observing, by means of oscillographs, neutral currents during line-to-ground faults. By correlating these currents with particular fault locations, calculations have been made to determine the fault resistance necessary to yield estimates of neutral current equal to observed values. Most of these data have been obtained on overhead lines and very little data on all-cable systems. While one would expect the fault resistance to be low for cable failures, I should like to ask the authors if their observations have indicated any data on fault resistance.

H. J. Plumley: Concordia and Peterson have brought up the question of the possibility of differences in the peak transient overvoltages at various points in one 12-kv zone. Such differences undoubtedly exist but it is our opinion that they are not large. On figure 1 it will be noticed that in one zone there are two widely separated spark-gap installations. The settings are maintained at the same value at both places. During the sphere-gap period 1932–38, nine line failures initiated transients capable of causing gap operations. In seven of these cases surges were recorded at both installations.

Mr. Dunham inquires about a possible explanation of the discrepancy between observed overvoltages during accidental faults and those obtained during staged tests, the latter not being equal to some of the former in magnitude. We feel that there was no discrepancy in our tests. The production of overvoltages was not the primary objective; hence no effort was made to time the application of the faults to obtain maximum surges. Since only five per cent of all service failures will cause surges exceeding $3N$ (see figure 6 of the paper), the 15 random test shots were insufficient to ensure the production of a maximum surge. We think it likely that maximum surges could be produced almost every shot, if desired, by proper location and timing of the artificial fault.

With reference to intermittent arcing in enclosed insulation, automatic oscillograph records obtained on the Chicago 12-kv system show this phenomenon to be very rare in the case of cable faults but rather common in the case of faults in cable joints. The intermittency is usually of longer period than fundamental frequency, restriking once or twice a second being common. In regard to Mr. Dunham's last question, no effort has ever been made to determine the resistance of a fault on the Chicago 12-kv system. The fault resistance in a 12-kv lead-sheathed cable is probably so low that its exact evaluation would be very difficult, because the resistance of the system neutral resistor is continually changing during a fault due to its resistance-temperature characteristic.

Report on Apparatus Bushings

Discussion of paper 41-76 by the AIEE joint committee on bushings, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (June section) page 266.

J. R. North (The Commonwealth and Southern Corporation, Jackson, Mich.): This report represents the results of a concerted detailed analysis of the electrical characteristics of present-day bushings and their operating performance requirements. The recommendations of the committee are based on providing a reasonable margin of insulation strength above the voltages which may be expected in service, including both low-frequency surges and lightning surges as limited by modern protective devices. A brief discussion of some of the values given in table I may be of interest.

All values of test voltages are expressed in terms of "withstand voltages" rather than as flashover voltages and there are no negative tolerance allowances. The "insulation classification" voltages may be considered as general indices, but bushings may be used for apparatus of a higher voltage rating when adequate for a particular application.

For outdoor bushings the low-frequency wet-test voltages were first determined and after a consideration of surge voltages which might be experienced in practice, these were taken as equal to approximately $3.2E_n+15$ kv. Similarly for the low-frequency dry tests, the values were chosen as equal to approximately $4E_n+15$ kv to provide a suitable manufacturing ratio between dry and wet values and an operating margin above the wet-test values. The outdoor bushings for small apparatus, insulation classifications 15 kv and below, were taken the same as the values for large apparatus but one step lower.

The indoor bushings were considered to require only dry low-frequency tests and the numerical values were taken the same as for the wet tests for large apparatus outdoor bushings.

The impulse test values for both indoor and outdoor bushings were selected from the series of standard basic insulation levels.

It is to be hoped that these bushing standards will receive immediate general acceptance throughout the industry, both by manufacturers and users alike, as this will very definitely result in economic savings of considerable value through the simplification of design and application schedules.

I. W. Gross (American Gas and Electric Service Corporation, New York, N. Y.): The question may well be asked: "How do the proposed withstand values compare with the flashover values for bushings which are now being supplied for standard equipment?" To throw some light on this subject, the committee collected considerable authentic data on the average flashover of bushings in the voltage classification of 8.7 up to and including 230 kv in the course of their study on the bushing insulation problem. These data, together with the committee's recommended withstand values (impulse, wet, and dry withstand) for outdoor bushings, are shown graphically in figure 1.

It will be noted that there has been considerable variation in the average flashover of bushings in the general range from the 15-kv class up to and including the 69-kv class. It will also be noted that the withstand values proposed by the committee are in practically all cases slightly less than the average flashover values reported. This, of course, is to be expected and is necessary to put the values on a withstand basis. A study of these data, however, seems to indicate that the values proposed by the committee are comparable to the general average of bushings which have been supplied to the industry in the past. It is true that some users of equipment specify bushings in a class higher than the nominal classification of equipment. It therefore follows that the average flashover values given in figure 1 do not necessarily mean that these values represent present-day operating practice; on the contrary, they represent present-day manufacturers' rating.

The manufacturers' practice in supplying equipment, the users' practice in its application, and fundamental overvoltages appearing on systems, all have been studied and evaluated in arriving at the committee's recommended values as shown in table I.

In presenting this report, it is particularly gratifying to find that the committee has set up proposed standard values for bushing insulation characteristics without specifically assigning these bushings to any particular piece of equipment. Footnote *a* of the table clearly indicates that the bushings to be applied having the recommended electrical characteristics depend upon the particular application of the bushings to apparatus. From this it also follows that the particular bushing selected for any piece of apparatus may also be influenced by the application of the apparatus itself.

In our own company in the past we have made frequent applications of bushings on large equipment of relatively major importance where so-called oversized bushings have been employed in medium and low-voltage insulation classes. We are particularly desirous in low-voltage class bushings on major equipment of maintaining suitable, mechanical clearances which in turn usually means a bushing with higher electric characteristics. In the higher voltage classes of equipment, we have, however, during recent years been using 115-kv class bushings on 138-kv transformers and experiencing no insulation troubles from this practice.

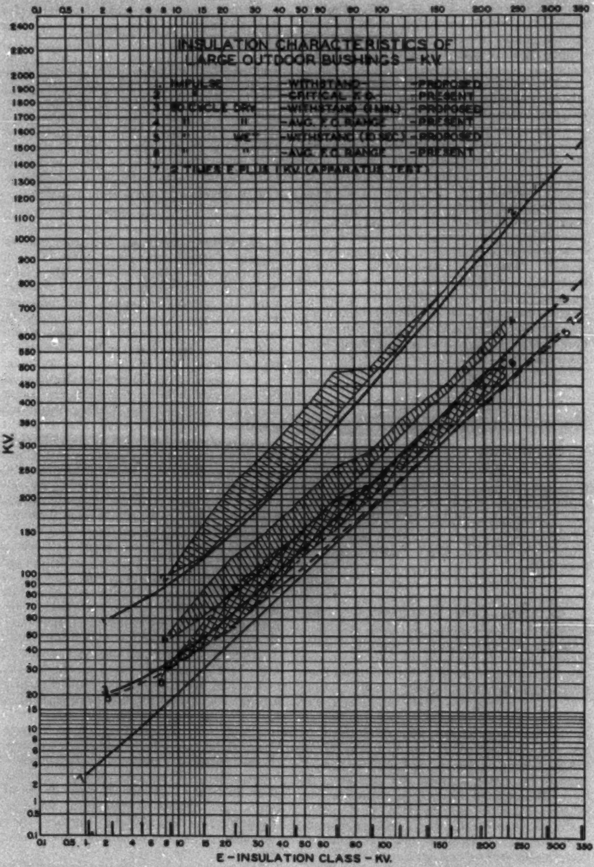

Figure 1

It therefore follows that the insulation classification given in the first column of the table does not in any way require that bushings of a given class must be used on equipment carrying the same system voltage rating. This is a most important point to keep in mind when assigning standard bushings in table I to various classes of equipment. It is hoped that this point will not be lost sight of by the various committees charged with the selection of bushings for different pieces of equipment in the various circuit voltage classifications.

High-Voltage Bushings Designed to Meet Modern Service

Discussion and authors' closure of paper 41-66 by T. F. Brandt and H. L. Rorden, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (June section) pages 255–60.

M. C. Westrate (The Commonwealth and Southern Corporation, Jackson, Mich): This paper has been reviewed with a great deal of interest. The information presented in this paper is both educational and practical.

Of particular interest is the change in the volt-time characteristic of the new type of bushing which, as the authors mention, approaches that of a sphere gap rather than a rod gap. This also makes the bushing volt-time characteristic approach that of transformer insulation and should lend itself to a better co-ordinated over-all design of transformers. The question, then, is what determines the rating of the bushing inasmuch as both the 60-cycle and impulse full-wave flashovers have been materially increased for a given size of bushing.

This change of the bushing volt-time characteristics also presents another problem in that they are not inherently co-ordinated with rod gaps and, therefore, would not be protected by them unless set for considerably lower full-wave impulse flashover, such as now encountered with protection of transformer windings.

A rather interesting observation is made from table I in that for the specified level of radio interference the corona and radio interference points occur at the same voltage with the new type of design whereas older types showed a decided difference.

F. O. Wollaston (Commonwealth Edison Company, Chicago, Ill.): Mechanical troubles with high-voltage bushings have been much more prevalent than the manufacturers would have us believe. For example, more than 70 bushings were repaired or replaced in Chicago because of leaks in the last three years. In some other cases, moisture entered bushings of the oil-filled, free-breathing type, necessitating maintenance of the oil.

Three main requirements for bushings are evident which have not been adequately taken care of in any designs for which service experience has been had in Chicago:

1. Electrical contacts: Concealed contacts should be positive, not depending on wiping pressures, screw-thread pressures, or spring pressures.
2. Entrance of moisture should be made impossible; that is, a completely closed system is essential. There is no objection to including a closed auxiliary oil-reservoir as part of such a system.
3. Internal pressure in the bushing should be controlled and held to a small range of variation, preferably not going below atmospheric.

It has been found in Chicago that gaskets are prone to give way in compound-filled bushings, which are typically subject to high ranges of internal pressure. In oil-filled breather-type bushings it has been found that moisture eventually collects in the bushing, sometimes in considerable quantity. For completely-sealed bushings, it is regarded as essential that the top seal be simple and positive, preferably a soldered seal. It also seems very desirable to keep the internal pressure range low in such bushings, to reduce the duty on gasketed joints to a trouble-free level.

Referring to figure 11, it would be preferable to have a soldered instead of bolted seal on the cover plate of the oil reservoir and to have the magnetic gauge screwed into the side of the reservoir and soldered.

I. W. Gross (American Gas and Electric Service Corporation, New York, N. Y.): An eminent scientist is reported to have said, in effect: "You cannot begin to talk about a thing until you can define it." If the authors had believed this, and accepted the definition of a bushing given by the AIEE sectional committee on electrical definitions *C*-42, they probably would not have presented this paper. The definition referred to reads in principal part: "A bushing is a lining for a hole."

They are, however, to be complimented on visualizing a bushing from a more practical viewpoint, and have discussed modern bushing design accordingly. Two design considerations they have mentioned deserve comment. *First:* That the electrical characteristics should be stable in so far as they affect operating performance. And *second:* A design which is subject to change under operating conditions is undesirable.

The first implies that the materials used in the bushing should be so chosen that effects injurious to any part will not be produced by some other part, such as the mixing of oil with compound, dissolving of varnishes in insulating liquids, and deterioration of sealing gaskets because they are in contact with liquids, or in some cases because they are not in contact with saturating liquids. This brings up the second consideration that changes in the electrical characteristics under operating conditions are undesirable.

As the result of some ten years of systematic and continuous field power-factor testing of thousands of bushings on oil circuit breakers and transformers, we have been convinced that one of the major troubles experienced with bushings in service is the presence in the insulation of moisture usually admitted through faulty gaskets. Various methods have been tried, with more or less success, in an attempt to perfect this gasket sealing problem, such as the use of better gasket material, insulating liquids heavier than water, multiple gasket seals, and even arrangements to restrict the circulation of moisture when it once enters the bushing.

Recognizing, however, that a bushing like any other piece of equipment, must be maintained, and may occasionally require gasket replacement, means should be provided to readily make this change. Concealed gaskets placed behind inaccessible barriers of cement are sources of potential trouble and grief. If found defective before bushing failure occurs, it is often necessary to scrap the bushing, or incur an excessive labor cost in machining out the cement and replacing such gaskets. The type of gasket design described by the authors where constant pressure is maintained and replacement easily made, is a distinct improvement which it is hoped will be eventually extended to bushing designs in all voltage classes.

I would like to ask the authors whether or not the resistance grading obtained by the conducting glaze is going to give a bushing with a stabilized power factor over a period of time, and also whether commercial bushings of this type have practically the same or widely different power factors.

L. Wetherill (General Electric Company, Pittsfield, Mass.): There is a striking relationship between figures 6 and 9 of the paper. In each case the flashover strength is increased by a conducting surface of smooth contour located so as to reduce the voltage gradient in air at the region where flashover would start. Creation of the conducting shielding surface under the lowest petticoat as shown in figure 6 is a natural phenomenon resulting from the application of overvoltage. It is not an uncommon occurrence in dielectric structures that the initial discharge under increasing voltage is such as to increase the dielectric strength and such as to suggest the nature of a modification in design which will give increased strength.

Influenced by the observation that a porcelain petticoat will serve to limit the progress of streamers created by overvoltage and retard the development of a complete flashover, practically all large bushings are now provided with a porcelain petticoat located fairly close to the ground metal of the support flange of the bushing, in order to limit such streamers as much as possible. The older design in which the lowest petticoat is elevated a considerable distance above the ground metal is now rarely manufactured.

Frequently when a useful natural phenomenon is noted, it is possible to improve upon it by proper designing. The shielding shown in figure 9, which utilizes a conducting surface of smooth contour located inside of the bushing shell, offers greater effectiveness in preventing flashover at moderate overvoltage.

Metallic shields similar in location and configuration have been used for this purpose in some types of oil-filled bushings for many years. The original adoption of this construction was for the purpose of suppressing corona and increasing the flashover voltage. A secondary result which has since become important is the almost complete suppression of radio frequency voltage.

The statement is made by the authors that most gasket materials kept under constant pressure between mating surfaces remain oil-tight indefinitely. This is in accord with usual experience. The principal gaskets in oil-filled bushings are kept tight by heavy spring washers mounted on the clamping bolts. For example, the large gasket on a 138-kv bushing is subjected to a spring follow-up force of about 10,000 pounds. Due to the local clamping structure, all of the resilience of the extra-heavy spring washers is available for taking up any set that may occur in the gasket material. None of the resilience need be reserved for thermal expansion of the conductor which in very large bushings may amount to more than $^1/_8$ inch.

There are conditions in which follow-up pressure is not needed. There are a number of gasket materials available which are readily deformed, but are volumetrically incompressible. Many gaskets of this nature have been successfully used in constructions providing complete enclosure of the gasket on all surfaces. Such a gasket cannot flow or take a permanent set. Over 100,000 gaskets of this type have been used in the last ten years with entirely satisfactory results. There have been no cases of leakage reported on volumetrically controlled gaskets of this construction used on liquid-filled bushings when the gaskets were properly assembled in the bushings.

The volumetrically incompressible gasket material is also used against flat surfaces with only the edges exposed. In such a case, spring follow-up pressure is provided to compensate for any slight flow which may occur. Altogether there are some half million gaskets of this material in service and there have been no cases of leakage reported which reflect upon the property of the gasket material.

Flat gaskets provided with spring follow-up pressure have been very satisfactory against oil pressure. However, there is considerable experience indicating that even with spring follow-up pressure, the usual type of composition cork gasket, while satisfactory against oil pressure, is not an adequate seal against gas pressure in a small hermetically sealed structure such as a bushing. For sealing against gas pressure, a less porous gasket material is essential, as indicated by years' experience with manufacture of hermetically sealed liquid-filled bushings.

T. F. Brandt and H. L. Rorden: It may be inferred from the various discussions that the question of greatest concern in the performance of high-voltage bushings in general is that of leakage. A perfect performance record is, of course, difficult to achieve, but bushings of the compression type, in which the gaskets have been kept under constant pressure at all times have an excellent service record. Reaction of the industry to this feature has been very encouraging and we are further gratified at Messrs. Wetherill and Gross' concurrence in this opinion.

In the type of bushing described in the paper, all gaskets which come in contact with the bushing oil are maintained under constant spring pressure. This usually involves three or more gasketed joints and experience has proved these joints to be one of the lesser leakage offenders. All other joints which may involve oil leakage are hermetically sealed.

We believe Mr. Wetherill has confused hermetic sealing with bushings that are oil and air tight. A hermetically sealed bushing would indicate that all mating parts are sealed by fusion and there are no gasketed joints. Obviously, bushing gaskets may be exposed to the atmosphere or other gas pressures. Bushings are then not hermetically sealed and the gaskets are subjected to a difference of gas pressure if the pressure within the bushing differs from that of the atmosphere. The usual composition cork gasket properly treated is an excellent seal against both oil and gas pressure.

While hermetic sealing of the bushing reservoir against possible breathing as suggested by Mr. Wollaston is undoubtedly very satisfactory, we believe the desired result can also be accomplished with properly made gasket seals. In general, bushings with solder-sealed reservoirs still contain gasket joints elsewhere, which are equally vulnerable to his objection. Also we think that gaskets in general should not be condemned based on their performance in older design compound-filled bushings. Present bushings of the type described otherwise satisfy Mr. Wollaston's opinion that bushings should not breathe and the internal pressure range should be restricted.

Mr. Wollaston leaves us in some doubt regarding his statement that electrical contacts have not been adequately taken care of in any bushing designs involved in his experience. It is obviously necessary in bushings of high-current ratings to exercise considerable care with the electric contacts, but in bushings of the most common amperage, wiping, screw thread, and spring pressures have been in common use for many years and have proved generally satisfactory.

We are glad to agree with Mr. Wetherill that designs are influenced by observation of natural phenomena. Undoubtedly by far the greatest part of our information is obtained empirically rather than by rational analysis. The general principle of the shielding referred to by Mr. Wetherill is common knowledge and has been in use for many years. In extending this principle by the application of conduction glazes, we have leaned heavily upon the results of tests rather than mathematical analyses. Factors of major importance in the potential distribution could not be included in the calculation of stresses and so these results could not be given full confidence.

The method of internal shielding which we have described eliminates the tendency to overstress the field at the flange external to the bushing as discussed by Mr. Wetherill. With this internal shielding, bushings having high petticoats have some points in their favor. The fact that they are now rarely manufactured is due to manufacturing rather than electrical stress reasons.

Mr. Westrate raises some interesting questions relative to system co-ordination. The volt-time characteristics illustrated for the new design bushings are between the extremes of the rod gap and the sphere gap, and this raises the critical impulse and 60-cycle flashovers for a given size of bushing. To compensate for this it has been necessary to revise the bushing sizes so that their flashover voltages are consistent with their ratings, although co-ordination is not necessarily expected to be obtained with the bushing, as such, a factor in the flashover levels.

The change in the bushing volt-time characteristics should be an advantage, since the ultimate end is protection of the transformer. These bushing curves more nearly parallel those of transformer insulation, and with an appropriate margin between them should offer ideal co-ordination.

Mr. Westrate's reference to radio levels is of interest since bushings with adequate internal shielding in general have unusually good radio interference characteristics.

In reply to Mr. Gross' question regarding power factor, we wish to state that the power factor of otherwise identical bushings varies between the limits of accuracy obtainable in vitrifying these glazes to within a narrow range of conductivity, which seems difficult to achieve at present. The glaze used on bushings of the type described has widely different characteristics from that used in the treatment of insulators against radio interference. The variation in resistance results in somewhat different power factors, but the power factor of each bushing is stable. The permanence of this glaze has been well established by years of service in other forms of insulators. This power factor is due to the surface conductivity on the dielectric material and is not conductivity through the insulation barriers. Therefore, it is not indicative of the condition of the dielectric material.

Secondary Networks to Serve Industrial Plants

Discussion and authors' closure of paper 41-48 by C. A. Powel and H. G. Barnett, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (April section) pages 154–6.

D. L. Beeman (General Electric Company, Schenectady, N. Y.): The merits of the secondary network system as outlined by the authors should be given full consideration when considering distribution systems for factories in which reliability of service is of particular importance. The basic idea presented in the paper, that is, using high-voltage distribution to small step-down stations located throughout the buildings affords, in many cases, substantial savings in the investment in the electrical system.

Network distribution systems have been used in factories for several years. Noninflammable liquid-filled transformers have been used in such installations and have reduced the over-all costs because they can be located in the buildings at the load centers, thereby saving expensive long runs of heavy copper that may be required if the transformers had to be located away from the load center outside the buildings.

The comparison of initial costs of the radial and network systems in the authors' paper is based on a network system using high-voltage distribution to load centers, and a radial system using low-voltage distribution throughout the plant. They have selected a large enough installation to show that distribution at high voltage, even with the duplication of equipment required for the network, is less expensive than the low-voltage radial distribution system.

It seems reasonable that we should also compare the network system with a radial system using high-voltage feeders supplying small distribution units located at load centers throughout the factory.

Such a comparison will verify the basic advantages of the network given in the authors' paper, i.e., the network is the lowest cost scheme to provide the greatest service reliability. If, however, service reliability is not of paramount importance, this comparison will show that the initial investment may be reduced by using the radial system with high-voltage distribution to load centers. This is because such a radial system requires less equipment as outlined in the following:

1. A lower investment may be required in secondary cable because of the absence of secondary ties.

2. When all transformer secondaries are banked, as they are in a network system, short-circuit currents are increased over what they would be if the same size transformers were not banked. That necessitates larger branch feeder air circuit breakers for the network system or conversely smaller ones in the radial system.

3. Although the authors show limiters or fuses being used in the secondary ties of the network, it is questionable whether or not some form of disconnecting means would be required in the ties to permit greater flexibility in operation: for example, to permit de-energizing a certain area readily in case of an emergency. Such disconnecting means are not required in the radial system.

There are two basic methods of factory distribution; one at utilization voltage, and the other at high voltage. The high-voltage method is applicable to either the network system as outlined by the authors, or at a radial system using distribution stations located through the factory at load centers.

P. W. Crosby (Philadelphia Electric Company, Philadelphia, Pa.): The paper presented by Messrs. Powel and Barnett brings out a very interesting point; that is, the same principle applied so successfully to the distribution systems in high-load density areas of large cities is likewise applicable to distribution systems for industrial plants. That principle is to provide an interconnected distribution system with a protective scheme so designed that in the event of a fault, only the faulty part of the system will be isolated and this, due to the interconnections, without interruption of service to any part of the load.

Two types of low-voltage a-c networks, embodying two different methods of the application of this principle, have successfully met the exacting requirements in high-load density city areas where service continuity is so important. It is therefore axiomatic that these systems should prove advantageous in providing highly reliable service in industrial plants.

The authors have described in their paper only one of these two types of a-c networks; the one in which two or more *radial* primary feeders supply a low-voltage grid through step-down transformers and automatic network protectors. In this system the entire primary feeder and all of its associated transformers are controlled as a unit in the protective scheme. The entire feeder and all of its associated transformers are removed from service in the event of a primary feeder fault.

The other type of a-c network system with several years of successful operation in high-load density city areas is the one in which sectionalized *loop* primary feeders are used. This system has several features which make it applicable for distribution in industrial plants.

The sectionalized loop feeder method of supply to low-voltage networks is, in many respects, fundamentally different from the radial feeder method. The loop feeders are sectionalized by means of automatic primary circuit breakers. The protective scheme consists of a simple direct acting pilot wire system which trips the breakers at each end of the faulty section of feeder. Only one section of primary feeder and its one transformer bank are isolated in the event of a primary feeder or transformer failure. All of the other transformers connected to the feeder remain in service, being supplied from the two parts of the loop feeder which have been separated by isolation of the faulted section. The faulted feeder section and its transformer are disconnected from the low-voltage grid by simple copper link fuses in the secondary leads of the transformer. Since only one transformer is removed from service, it is not necessary to use automatic network protectors. Also due to this feature, the occurrence of a fault causes a minimum of disturbance to the remainder of the system and makes possible a greater utilization of installed transformer capacity.

The sectionalized loop feeder system is particularly well adapted to the supply of both networked and nonnetworked loads from the same feeder, due to the feeder section method of control. This means that large primary voltage motors, frequently encountered in industrial plants, could be supplied from a section of the same feeder as used to supply low-voltage network load. Such nonnetworked loads would be subjected to interruption only in the event of a fault in their own section of feeder.

For systems having extremely high fault duty, control schemes have recently been developed whereby breakers of relatively low interrupting capacity may be used to sectionalize the primary loop feeders. A fusing scheme has also been developed for assuring the blowing of transformer fuses on networks of low available fault current and the selective blowing of fuses on spot networks having only two transformer banks.

It may be interesting to note that a sectionalized primary loop feeder system is now being installed in a large midwest industrial plant. It is believed that thorough consideration should be given to both types of systems to determine the one most suitable for any particular plant.

A. G. Darling (General Electric Company, Schenectady, N. Y.): A considerable number of network systems have been installed in industrial plants over a period of years, some serving power and some serving lighting uses and all seem to satisfy their owners. One large manufacturing plant has in operation over 50 transformers of 500 kva each, supplying several 120/208 lighting networks, and plans to extend these networks as occasion demands. Another plant has connected about six 1,500- to 2,000-kva transformers on a 575-volt power supply ring bus interconnecting several buildings and has plans to extend the network to include ultimately 12 2,000-kva transformers. From each transformer station, radial feeders supply the loads in these two instances, no loads being taken from the tie circuits between transformers.

Still another variety of network utilizes the tie circuits between transformers as a means of supplying the loads through tap-off circuit breakers to local busses from which radial breaker-protected feeders are connected. The transformers are so located as to maintain the voltage on the ring bus at a level to allow for the very small drop in the short radial feeders. Since the ring bus is geographically placed around the building interior at a distance from all outside walls to give short runs from the bus to the walls or to the center of the building, no load need be more than one-fourth of the building width away from the bus. This plan has the distinct advantage of making the most expensive part of the installation the most permanent while the less permanent feeder runs become the less expensive part of the installation. It retains the advantage of other networks in that the transformer capacity may be increased, decreased, or moved to another point on the ring bus. Two kinds of equipment are available for busses—the rigid metal enclosed structural shape conductors or insulated, nonmagnetically metal sheathed cable. Both types can be tapped at any desired points, the bare conductor rigid bus, metal enclosed, being the more accessible of the two types, but the cable being less expensive, giving better voltage regulation and being more susceptible to adding or taking away parallel cables as the loads change.

The authors point to many advantages of the low-voltage network which supplies loads from transformer busses, over the low-voltage radial system of distribution. They mention reliability, good voltage regulation, high economy, and flexibility in adding or taking out transformer capacity and transformer busses from which radial loads are supplied. These advantages likewise apply to the ring bus from which loads are fed but which gives the added advantage of ability to supply radial loads from any point on the ring with minimum radial feed distance and minimum disturbance to transformers and busses. Incidentally a straight line bus may be used if the occasion does not demand a ring bus. Certain experience with layouts of radial and network systems indicates that the several important advantages of the network including automatic tie breakers can be secured for about ten per cent more than the cost of the radial system.

No doubt many designers are giving special thought to reliability in these days. Alternate sources of supply, segregation of loads, quick replacement of unusable equipment as well as flexibility in utilization became relatively more important under uncertain circumstances. Certain forms of network systems offer these advantages and all forms give good regulation and alternate sources of supply.

William W. Edson (Boston Edison Company, Boston, Mass.): A similar application of a secondary network to industrial plants is the supply for turbine and boiler auxiliaries. In both cases consideration of some of the problems which have arisen in the operation of metropolitan networks should be given.

If two transformers are too close together, especially if they are fed by separate high tension feeders, they may not divide the load equally. At light loads one transformer may pump, thereby increasing the wear on the secondary protector. Extreme cases may require balancing transformers to equalize the load.

Large regenerative loads such as elevators might open the protector unless special desensitizing relays are added.

Double-throw disconnects are shown to transfer from a dead high tension feeder to one alive. In order to get back to the original schedule of transformers and feeders it may be necessary to have an interruption of service. Opening the secondary protector removes the necessity of the high tension switch opening load current but the switch would still have to break magnetizing current which is often hazardous. In fact metropolitan units are provided with electrical interlocks to prevent breaking magnetizing currents.

The combination air-cooled transformer and switching equipment offers many advantages but due to the high operating temperatures permitted the design should be such as to encourage frequent cleaning. Access parts, etc., may need to be interlocked with the high tension disconnect. Suitable barriers should be provided between the transformer and the high tension disconnect and the protector.

Several conductors per phase in the secondary bus runs permit use of sectionalizing limiters and is more economical in the use of copper than one large wire, however, the total reactance is less which might be a disadvantage. A fault or starting a large motor will cause a serious voltage dip throughout the plant if the impedances in the network are too small, thus other motors or controls may drop out.

Essential motors might require a throwover switch to another part of the system to prevent an outage if its circuit or part of the net is segregated by fuses.

Special connections will be required to isolate a building or part of a plant out of use or in case of fire.

The following modification of the usual secondary network is offered for consideration:

The secondary network units are located at the various floors or load centers with motors and associated circuits connected to the unit busses somewhat like the usual radial system. Each bus has a reactor which connects to a common reactor bus thus tying the units together.

A fault on one section will have less effect on the rest of the system. The short-circuit duty required for the limiters, circuit breakers, and motor starters is much less. Any section can be shut down or isolated from the rest of the system by opening the reactor tie. In case of a transformer or high tension feeder fault, service is maintained from the remaining transformers through the reactor bus. Additions or changes to any bus section can be made easier. The reactors required are small and in some cases may be eliminated if the impedance of the reactor bus is high. The reactor bus may serve as a duplicate source for essential motors. It may also be used as a starting bus for exceptionally large motors.

A. P. Schnyder (George F. Hardy, Consulting Engineer, New York, N. Y.): The authors express awareness that industrial plants as to system of power distribution defy too broad a generalization. The electrical tools vary in kind, type, and size from group to group to such an extent as to render the task of selecting an all inclusively appropriate pattern even for distribution most difficult.

Commenting on the paper as to possible adaptation of networks in paper mills, for reasons explained in the following, there seems justified doubt that they offer advantages over the present practice of radial feeder distributions.

1. Load densities per 1,000 square feet ground area in mill and yard are extremely small in the wood yard, reach 8 kw in shops, 700 kw in stock preparing room, and several fold the latter amount in grinder rooms.

Feeders supply singly motors driving grinders or paper machine motor generator set varying in size from 1,000–4,000 horsepower, in groups motors between 50 and 500 horsepower in a particular department, all above at 2,200 volts. These larger motors represent roughly two-thirds of total mill load.

2. Mill managements are cost minded and require accurate departmental power load readings which can be readily made available with radial feeders.

3. A mill is constituted by a chain of process equipments from wood yard to shipping platform. A number of equipments are single links and are driven by individual motors. Mechanical troubles and process stock difficulties are incomparably more bothersome than electrical troubles. The latter again originate predominantly in the motors and controls rather than in the distribution system. The greater reliability of network at the comparatively higher cost of installation would merely increase the safety factor which in the radial feeder system is already a very conservative one.

4. Last but not least is the human element. The radial system is more direct and easier understood by the average operator. The lot of a mill electrician is not uniformly enviable. He requires absolute familiarity with his system which is naturally more definitely assured the simpler such system is. To use a somewhat disrespectful term the less "gadgets" the better.

C. A. Powel and H. G. Barnett: The discussers have all added useful thoughts to the subject matter of the paper. The general tenor of the questions raised deals with the economics of the situation rather than the engineering.

It is true that if reliability is sacrificed certain radial type systems may show a saving in cost over network systems. However, industry today is so dependent on electricity and the general understanding by factory operators of electric supply problems is so much higher than it was a few years ago, that we should not be satisfied with anything less than maximum reliability, if this can be obtained without unreasonable increase in cost. A radial system designed to provide reliability approaching that of the network system will certainly cost more than the network scheme.

The loop system described by Mr. Crosby is applicable to factory supply, but we question if it would show any saving over the conventional network system, because of the necessary primary breakers and pilot wire control.

Mr. Edson suggests the necessity of having to interrupt service in order to get back to the original schedule of transformers and feeders after a feeder disturbance. The transfer of a transformer from one feeder to another can always be done without interruption of service, the selector switch being designed to break magnetizing current.

The loss of motors due to trouble in the supply can only originate with trouble in the junction boxes to which they are connected. The probability of loss of an essential motor due to this contingency is no greater than that due to trouble in the motor or its control equipment. Therefore, essential motors should be installed in duplicate and supplied from neighboring junction boxes rather than from the same junction box.

Enclosed Bus-Bar Electrical Distribution Systems for Industrial Plants

Discussion and author's closure of paper 41-64 by E. T. Carlson, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (June section) pages 297–302.

Herbert B. Dwight (Massachusetts Institute of Technology, Cambridge, Mass.): The account of the development of enclosed bus-bar systems given by Mr. Carlson is interesting, as is also the summary of the calculations needed in their design.

The reactance drop of a system of rectangular conductors connected $ABCABC\ldots\ldots\ldots$ as in figure 5 can be calculated by methods available at present, if the currents are known.

If ammeter readings show that for the arrangement of figure 5 the currents in all the conductors are practically equal in magnitude, which is to be expected in ordinary cases, then currents of phases A, B, and C may be taken to be 120 degrees apart in phase. Since the vector sum of all the currents is zero, I_a may be put equal to $-I_b-I_c-I_d\ldots$ where the sub letters represent the different conductors (not the phases). Then the method described by the writer in example II of the paper "Reactance Values for Rectangular Conductors" (reference 9) can be followed, and the drop in any conductor can be expressed in terms of the sum of a number of reactance drops in two-conductor, single-phase circuits. For this the curves of reference 9 will be found accurate and convenient.

An alternative method is to find the drop in any conductor by computing self-inductances and mutual inductances, counting flux up to a certain large distance u, and using self and mutual geometric mean distances as given for rectangular conductors by F. W. Grover in *Proceedings* Institute of Radio Engineers, volume 17, part 2, 1929, pages 2056–7, mentioned in Mr. Carlson's paper, following equation 6.

Thus,

$$L=2\log_n\frac{u}{R}\text{ abhenries per centimeter}$$

and

$$M=2\log_n\frac{u}{D}\text{ abhenries per centimeter}$$

The quantity u will cancel out, in the complete sum. See the first part of example II, reference 9.

The determination of current capacity and temperature rise of enclosed systems such as figure 5 will probably require full-load tests for almost every different size, in the same way that temperature tests are needed for transformers and rotating machines.

O. R. Schurig (General Electric Company, Schenectady, N. Y.): In his paper on "Enclosed Bus-Bar Electrical Distribution Systems for Industrial Plants," Mr. Carlson points out the desirability of having further knowledge relative to a-c–d-c resistance ratio and reactance applicable to commercial bus structures, particularly for closely spaced conductors as employed in commercial busways. The author has also indicated some of the difficulties connected with the analytical method of approach to the problem, the most important unknown factor being the distribution of current in the conductors themselves as determined by skin effect, proximity effect, and circulating currents in the metallic housing surrounding the bars.

In view of these difficulties and in order to obtain practical data on a-c–d-c resistance ratio, and on reactance as well as on impedance unbalance for typical busway assemblies, a series of tests was made at Schenectady.

The busway structures tested included those described in table I. This table also shows the phase current used and the following test data:

In column 6—the rms line-to-line drop V_{AC} measured for 30 feet of busway, across phases AC.

In column 7—the measured three-phase loss W for 30 feet of busway by two-wattmeter method.

In column 8—the impedance per phase to neutral in ohms per thousand feet (derived from V_{AC}) for a going and return circuit having a conductor spacing of 5.25 inches. The values were obtained from the relation:

$$Z=\frac{1{,}000V_{AC}}{30\times 1.73\times I}$$

where I is the line current from column 5. This relation holds for balanced line currents only.

In column 9—the average effective resistance per phase in ohms per 1,000 feet derived from W by the following relation:

$$R'=\frac{1{,}000W}{90I^2}$$

In column 10—the reactance per phase to neutral in ohms per 1,000 feet (derived from Z_{AC} and R') for a going and return circuit having a conductor spacing of 5.25 inches.

In column 11—the calculated value of X_{AC} determined by conventional method neglecting skin effect and circulating currents.

In column 12—d-c resistance.

In column 13—average a-c–d-c resistance ratio derived from columns 9 and 12.

Some of the significant points indicated by the results are the following:

1. The average a-c–d-c resistance ratio for the construction with one 4 by $^1/_4$-inch copper bar per phase at 60 cycles is of the order of 1.5 (see item *a*). This is materially in excess of the value of the isolated-bar skin effect resistance ratio, which is of the order of 1.12 (see Mr. Carlson's references 10 and 15). Obviously a substantial resistance increase due to narrow spacing and due to the metallic enclosure would be expected.

2. For the two-bar-per-phase construction with 4 by $^1/_4$-inch bars, the over-all a-c–d-c resistance ratio was found to be within the range from 1.75 to 1.83 (see item *b*). An isolated two-bar conductor with 4 by $^1/_4$-inch copper bars in the absence of a metallic housing, would have a 60-cycle–d-c resistance ratio of roughly 1.3, not shown in the table.

Table I. Results of Impedance and Loss Measurements for Busway Assemblies

Center Spacing Between Poles $2^5/_8$ Inches; Copper Bars. All Data at 60 Cycles, Except D-C Values. Base Plate Material 12-Gauge Steel, Except for 6-Pole Construction, in Which a 10-Gauge Steel Base Was Used. Cover Material, 12-Gauge Al. In All Tests the Bus-Ways Were Short-Circuited by Copper Straps at One End

								Ohms Per 1,000 Feet Per Phase					
										Reactance Per Phase to Neutral			
Item No.	1 No. and Size Conductors Per Pole	2 No. of Poles	3 Three-Phase Bar Assembly (Base and Housing Not Shown)	4 Clearance Between Bars Within Each Pole	5 RMS Amps Per Phase, Balanced Three-Phase Current	6 RMS Volts Measured Line-to-Line Drop for 30-Foot Bus V_{AC}	7 Measured Three-Phase Loss for 30 Feet (Watts)	8 Impedance Per Phase to Neutral By Test Z_{AC}	9 Resistance Per Phase Average Effective By Test R^1	10 By Test X_{AC^1}	11 By Calc. X_{AC}	12 D-C Resistance	13 Average AC-DC Resistance Ratio
(a)	1 bar $4x^1/_4$	3*	A B C		1,000	2.27	1,412	0.044	0.016	0.041	0.040	0.010	1.6
	1 bar $4x^1/_4$	3*	A B C		790	1.84	895	0.044	0.016	0.041	0.040	0.0106	1.5
	1 bar $4x^1/_4$	3*	A B C		1,250	2.84	2,265	0.044	0.016	0.041	0.040	0.0107	1.5
(b)	2 bars $4x^1/_4$	3*	A B C	$^1/_4$	2,000	3.47	3,520	0.033	0.0098	0.032	0.037	0.0056	1.75
	2 bars $4x^1/_4$	3*	A B C	$^1/_4$	1,597	2.76	2,160	0.033	0.0094	0.032	0.037	0.0052	1.8
	2 bars $4x^1/_4$	3*	A B C	$^1/_4$	1,280	2.19	1,330	0.033	0.0090	0.032	0.037	0.0049	1.83
(c)	2 bars $3x^1/_4$	3*	A B C	$^1/_4$	1,497	3.10	2,358	0.040	0.0117	0.038	0.045	0.0071	1.65
	2 bars $3x^1/_4$	3*	A B C	$^1/_4$	1,200	2.47	1,446	0.040	0.0111	0.038	0.045	0.0066	1.7
	2 bars $3x^1/_4$	3*	A B C	$^1/_4$	961	1.97	908	0.039	0.0109	0.038	0.045	0.0063	1.7
(d)	2 bars $4x^1/_4$	6	A B C A B C Transposed	$^1/_4$	2,400	1.55	2,230	0.0124	0.0043	0.0111		0.0025	1.7
	2 bars $4x^1/_4$	6	A B C A B C Transposed	$^1/_4$	3,200	2.10	4,138	0.0126	0.0045	0.0118		0.0027	1.97

*Not transposed.

3. The two-bar-per-phase construction with 3 by $^1/_4$-inch bars (see item *c*) had an a-c–d-c resistance ratio within the range from 1.65 to 1.7.

4. The six-pole busway assembly with two 4 by $^1/_4$-inch copper bars per pole was found to have an over-all a-c–d-c resistance ratio of 1.7, or slightly less than the corresponding three-pole busway.

5. The reactance values, by test, are also of interest in that they are somewhat lower than the values calculated by the conventional method, at least for the two-bar-per-pole arrangements. This would be accounted for by the current distribution resulting from skin effect and proximity effect.

6. The impedance unbalance for the untransposed arrangements, indicated by the ratio of maximum-to-minimum line-to-line voltage drops was of the order of 1.32 to 1.45 (not shown in the table). This is roughly in accordance with the ratio of the reactance for $5^1/_4$-inch spacing to that for $2^5/_8$-inch spacing.

E. T. Carlson: I wish to thank Professor H. B. Dwight and Dr. O. R. Schurig for their excellent discussions which have added to the fund of information available on the subject of "Enclosed Bus-Bar Distribution Feeders."

Professor H. B. Dwight has referred in his discussion to the paper "Reactance Values for Rectangular Conductors," which describes in detail the determination of the 60-cycle reactance of parallel strap conductors.

He points out the necessity of knowing the current division in a multiple conductor system in order to ascertain the reactance drop by calculation methods now available.

The parallel Professor Dwight has drawn between the methods of determining the temperature rise in other electrical equipment and bus bars is appropriate. Actual tests of each size and each configuration are necessary.

The empirical data given by Dr. O. R. Schurig is a most valuable contribution. For the bus-bar configurations shown under column 3, the reactance values of rectangular conductors can be calculated within an accuracy of ten per cent.

The ratio of a-c to d-c resistance indicates the result of proximity and skin effects in increasing the a-c resistance. Doctor Schurig's data show that on close conductor spacings where the currents in adjacent bars are flowing in opposite directions, the a-c resistance approaches the d-c resistance. (See also reference 15.)

This is a fortunate situation and has given encouragement to the manufacturers of busway systems in their efforts to produce acceptable designs having low reactance which in turn has meant facing the problem of placing bars on very close spacings.

Experience With Preventive Lightning Protection on Transmission Lines

Discussion and author's closure of paper 41-36 by S. K. Waldorf, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (June section) pages 249–54.

F. Von Voigtlander (The Commonwealth and Southern Corporation, Jackson, Mich.): The general conclusions reached by the author regarding distribution of current in tower members, the high degree of effectiveness of single ground wires on towers of vertical configuration, and the doubtful benefits of reducing tower footing resistances below seven ohms have been substantiated by the experiences of companies of the Commonwealth and Southern system.

Similarly, wide discrepancies have also been noted between measured surge voltages and those calculated from surge currents and footing resistances. For ground resistances obtained with the tower footings by themselves, with deep driven ground rods, or with counterpoises, experience consistently indicates that the surge resistances are considerably lower than resistances measured by conventional low-voltage methods. Recent investigations such as those reported by Bellaschi, further confirm the conclusion that surge resistances are lower than the low-voltage resistances. These results would give even lower ratios of tower potentials (based on *IR* drop) to flashover levels than those cited in table III of the paper and would tend to emphasize the importance of reactive components in calculations of surge impedances.

The magnitudes of tower footing resistances regarded as desirable have been developed by making them small compared to the surge impedances of the other components of the electric circuit involved. It is quite probable that with low tower footing resistances, the reactance factors are of greater relative importance than previously considered.

The writer would like to ask if any oscillographic data have been obtained which would indicate whether these apparent reactive components are functions of large surge currents, steep wave fronts, or some combination of both.

C. F. Wagner and **G. D. McCann** (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): This paper presents valuable information relating not only to the performance of actual lines but to the fundamental properties of strokes to transmission lines. Dr. Waldorf has allowed us to examine the large amount of data which he has obtained and from this examination several interesting facts have developed.

Upon breaking up the data into time of occurrence of the stroke, it was found that seasonal effects are important. Figure 1 shows that the stroke current (obtained by adding the current in all the towers thought to have been involved in the stroke) is much larger during the summer than during the spring or fall.

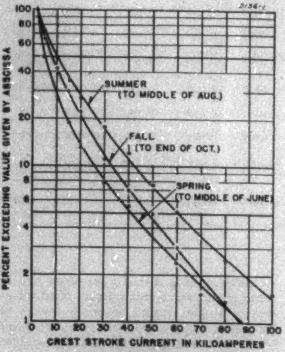

Figure 1. Effect of seasons on crest magnitude of stroke current as shown by Waldorf's data

In figure 2 are plotted distribution curves for both tower and stroke currents segregated into the two common ranges of span length used on the lines described. These curves indicate that a variance of span length of almost two to one has little effect upon the stroke current but, as might be expected, does influence the tower currents, that is, the longer the span the greater the concentration in a small number of towers. Also, the total stroke current curves show little variance between the different individual lines.

For the low range of tower footing resistances obtaining in these lines, very few being in excess of 25 ohms, the data indicate no effect upon either the stroke or tower currents.

A weighted mean of the number of strokes to the lines from table II gives a value of 120 strokes for 100 miles per year. This is the best datum of this nature available and when used in connection with per cent distribution curves permits of converting such curves into working curves of number of strokes per 100 miles per year as a function of the stroke current. In view of other data on this point which give a lower estimate, coupled with the fact that perhaps an appreciable proportion of Waldorf's current indications may be due to induction, a rounded figure of 100 appears reasonable. On this basis the data of both Waldorf and Lewis and Foust[4] have been plotted in figure 3. The dotted curve of Lewis and Foust was plotted directly from their data and the full line curve has been adjusted to correspond to the same minimum current as considered in the curve of Waldorf. Data of stroke currents rather than tower currents are valuable in that they may be used for estimating purposes on lines of different construction and should not vary with span length as do tower currents.

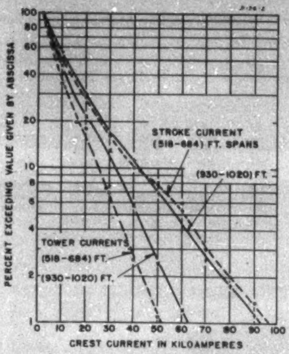

Figure 2. Effect of span length on tower and total stroke currents as shown by Waldorf's data

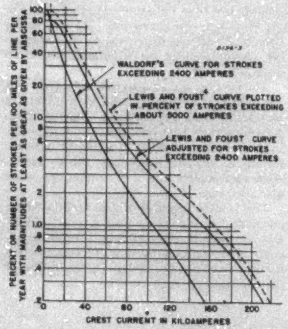

Figure 3. Probability curves of crest currents in lightning strokes to transmission lines

Based on 100 strokes to line per 100 miles of line per year

In view of the many variables involved in the shielding problem, it is gratifying that the results of operating experience agree as closely as they do to the results of investigations on laboratory models. The more optimistic results reported by Dr. Waldorf over that obtained by Wagner, McCann, and MacLane[6] may be due to the low pessimistic assumption of cloud height made by the latter.

While Dr. Waldorf recognized the uncertainty of his conclusions with regard to the possible effect of the inductive drop of the tower in producing insulator flashovers, it might be well to enumerate the important factors which enter into the problem. These are

1. Errors in total tower current arising from measurements of only one leg in the presence of unequal distribution between legs.
2. Contribution of cross braces to total tower current which may introduce correction factors of two or greater.
3. Seasonal variation of soil resistivity.
4. Momentary initially high impedance of footing resistance or counterpoise over the measured resistance.
5. Inductive drop or surge impedance of the tower.
6. Coupling between ground wire and conductor.

The first five factors may account for higher than calculated tower top potentials and the last item effectively decreases the potential across the insulator string below that of the tower top.

E. W. Beck (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): Mr. Waldorf's paper adds considerably to the fund of practical field data on lightning duty. We are particularly interested in the records of discharges through lightning arresters, and we hope that Mr. Waldorf will publish more complete data on his measurements, and more closely define his "small" currents. It is to be expected that the average currents through arresters will be less than the average currents discharged to a line, and that the discharge duty on arresters connected to lines with adequate ground wire protection will be less than on arresters connected to unshielded or poorly shielded lines. In a paper written several years ago, Mr. Opsahl[1] calculated that the usual current discharged through arresters would not exceed a few thousand amperes unless strokes occurred close to the arrester. Mr. Waldorf's comments regarding arrester records indicate that this is what he found, and field studies with which we have been associated show the same result. In fact, it has been recognized that complete over-all protection for apparatus can be more readily realized if the lightning arrester is assisted by shielding of some form. On highly insulated and improved lines not equipped with ground wires, the use of De-ion protector tubes ahead of the arresters has been recommended and used for this purpose. We have had an opportunity to measure discharge currents through an arrester by means of oscillograph and fulchronograph with and without tubes on the line, and have found the discharge currents through the arresters to be increased many fold when the tubes were disconnected.

It must be borne in mind that surges which discharge moderate currents through arresters would, if no arresters were present, produce sufficient voltage at the arrester

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4, 6. See references 4 and 6 of paper.

location to endanger apparatus. For example, speaking in round figures, in the 69-kv class, 300 amperes through an arrester indicates the appearance of a surge voltage, if no arrester were present, of 350-kv the BIL for this voltage class. Mr. Waldorf has recorded currents of 2,600 amperes on shielded lines, if our interpretation of the data is correct.

Arresters were originally designed for, and standard tests were based on, discharge currents rather limited compared to present requirements. As time went on, and more data became available, it was recognized that high currents might occasionally occur. Arrester characteristics and capacities were improved to keep pace with the increased knowledge, and to take care not only of the usual cases but also of the infrequent ones.

The useful part that arresters play in the complete protection of stations and station equipment is well illustrated by Mr. Waldorf's comments regarding arrester discharges involving currents up to 2,600 amperes on shielded lines, and 7,400 amperes, a considerable current as arrester currents go, on unshielded lines.

REFERENCE

1. SURGE CURRENTS IN PROTECTIVE DEVICES, A. M. Opsahl. AIEE TRANSACTIONS, volume 54, 1935 (February section), pages 200–04.

G. D. Floyd (Hydro-Electric Power Commission of Ontario, Toronto, Canada): It is interesting to note (subsection VII) that the author has found that surge current times tower footing resistance may not be a measure of the potential at which flashover will occur. He suggests that there is a reactance drop present which increases the tower potential faster than would be indicated by the product of surge current and tower footing resistance. It is well known that there is a time factor in the strength of the insulation. There may be as great a ratio as two to one in the insulation strength of the insulator string, and this might account in great part for the fact that currents less than those required to produce flashover on the basis of current times tower footing resistance, actually did produce flashover.

Referring to subsection VIII of the paper defining the efficiency of protection, I believe this definition of efficiency could be improved. As it stands, it is likely to be misleading because even on a line where no special measures have been taken to prevent interruption, the efficiency as defined would be quite high. In order to show the benefit of special measures the datum should be higher. It does not seem reasonable to take the total number of strokes as a base when possibly 50 per cent of the total would not produce flashover in any case. This is especially true in the case of higher voltage lines because of the higher insulation level of these lines.

The data presented in table V are interpreted by the author as indicating that the tower design shown in figure 2 of the paper results in very efficient lightning operation. Double circuit construction similar in design to that shown in figure 2 but with the ground wire elevated 12 feet above the conductor in the suspension position, has been operated by the Hydro-Electric Power Commission for a number of years. The operating record of these circuits, however, is not considered to meet modern standards, in fact, in recent construction the design of the tower head has been modified so as to increase the ground wire separation to 17 feet at the tower. It is expected that this change, which brings the design for shielding into accord with recommendations based on recent tests on model lines, will result in noticeably improved performance from this type of tower.

It may be of interest in connection with the problem of shielding, to record less than one year's experience in a single-circuit, 110-kv line, 103 miles in length, on which stringing of the overhead ground wire has been postponed in order to obtain operating experience of the performance of a line with practically no shielding.

The towers on this line are 85 feet in height, being spaced approximately 1,100 feet. Power conductors are spaced 15 feet horizontally. A single ground wire is to be located at the tower head, 25 feet above the center conductor in the suspension position. Prevention of outages from sleet forming on the wires as well as from lightning were first considerations in the design of the towers. The measured average tower footing resistance is approximately five ohms. One individual tower had a resistance of 22 ohms and one of 14 ohms. These were the highest resistances measured at individual towers.

The line went into operation on July 4, 1940, and there have been 20 lightning outages to date. Six of these outages have been located. Of those located, all but one were at towers having footing resistance less than five ohms. This one had passed from the phase wire to the overhead ground wire of another 110-kv line passing underneath the line struck. The distance between the phase wire and ground wire was approximately 12 feet.

Although it would be unwise to draw any conclusions, using less than one year's lightning experience on the line as a basis, it is interesting to note that in at least 20 instances strokes to the line were of sufficient intensity to cause flashover of line insulation. It has been assumed that all strokes were to the phase wire initially, rather than to the tower, because there seems little probability that any of the flashovers were due to rise in tower potential, even if it is assumed that the tower impedance is somewhat higher than the measured resistance.

Data given in subsection XI are interesting. In one of the 220-kv transformer stations of the Hydro-Electric Power Commission there are installed six 220-kv lightning arresters. These arresters have been equipped since they were placed in operation with klydonograph surge recorders placed so as to record any discharge of lightning current through the arresters. In about 60 lightning arrester years of operation, not a single instance of recorded lightning discharge through these arresters has occurred. It should be pointed out that this station has what is considered better than average protection from direct hits in the vicinity of the station. Assuming that the author's data on lightning stroke frequency can be applied, the shielding network over the station and for one mile out on the line should have been hit from 20 to 30 times in the ten years of operation. The absence of a recorded lightning arrester discharge is a measure of the efficiency of shielding.

E. H. Grosser, Jr. (nonmember; Commonwealth Edison Company, Chicago, Ill.): This paper contributes data on some of the factors that enter into the engineering of lightning protective measures for overhead transmission lines. Some of the measurements made, as of tower currents, merely add to data previously obtained along similar lines. Little information, however, has been available heretofore on probable number of strokes to lines. As far as we know, the previous literature on this subject consisted of 1937 Paris conference paper 338, giving early surge current records for the Pennsylvania Water and Power Company lines and the Lewis and Foust data in AIEE TRANSACTIONS, volume 56, 1937 (January section), pages 101–06. These earlier figures indicated a probable average of about 85 strokes per 100 miles per year. The average reported in the present paper is almost 50 per cent higher, and should have some effect on line design.

With reference to Waldorf's comparison of calculated probable outages and actual outages, these can hardly be said to indicate wide discrepancies. Actually the performance records presented appear to show a fairly good agreement with calculated performance, when we consider the approximate nature of the prediction and the many assumptions made in the calculations. Among these may be mentioned: uncertain surge impedance of the lightning stroke; uncertain coupling factors due to corona effect and depth of current ground plane; unpredictable stroke characteristics which involve effect of tower height on tower-top voltages and have an important bearing on flashover voltages of line insulators and mid-span insulation; the consideration of counterpoises as pure leakage resistances; estimation of probable number of strokes to the line, etc. Even the disagreement between the author's data and the 1939 data of Lewis and Foust on distribution of tower lightning currents may introduce a factor of two or more. This particular factor becomes more uncertain for the higher currents, which would indicate that there is probably more chance of making an accurate prediction of performance for a medium-voltage line than for a high-voltage line designed for few lightning outages.

It would be of interest to know whether the calculated figures shown in Waldorf's table VI were complete analyses, or if they included only the effects of strokes to towers. We found, in design calculations for the Powerton-Crawford 220-kv line, that mid-span strokes assumed importance as the protective level of the line was increased, e.g., by added insulation or lowered footing resistance. In other words, for extra high-voltage lines, it seems necessary to consider the effect of strokes to mid-span even to the extent of providing, as we did, increased separation at mid-span between phase wires and ground wires. It is possible that this may account for the differences between the calculated and observed line operation reported in this paper.

It seems desirable to clarify one point regarding the shielding effect of overhead ground wires. Where two ground wires are used, for example, to protect three conductors in horizontal configuration, the center phase conductor should be almost perfectly shielded with nearly any practical placement of the ground wires. This readily follows from a consideration of the geometric arrangement of ground wires and phase wires with respect to the discharge center of the cloud. This effect is indicated to a certain degree in Waldorf's figure 3, but it is probable that a safe protective angle for the middle conductor can be even greater than the 41-degree angle shown.

One conclusion of particular interest may be drawn: that a virtually lightning-proof line is now a practical reality. For economic reasons, this end is normally attainable only for extra high-voltage lines. Where service continuity demands, however, such performance may be obtained on lower-voltage lines, by adequate design.

A. O. Austin (Manufacturing Engineer, Barberton, Ohio): The paper presented by Mr. Waldorf covering lightning performance on the transmission lines of the Pennsylvania Water and Power Company provides information of great value not only for improving existing lines, but as a guide in constructing new transmission systems.

The investigation carried out on the lines of the Pennsylvania Water and Power Company over a period of years has been exceptionally complete, and the electrical profession is indebted to the members of the organization and the company for making this information available. Information such as that presented by Mr. Waldorf, and by Messrs. Sporn and Gross covering the operation of the American Gas and Electric Company lines, can be used very effectively in analyzing different types of transmission line construction and determining the relative as well as probable performance of same.

The paper covers the method of improving the performance of existing lines by lowering the ground resistance or impedance, the other factors such as electrical exposure and length of span being fixed.

If the performance of transmission lines is analyzed, it will be seen that length of span and exposure as well as arcing voltage are very important factors in providing reliability. These factors can be used very effectively for constructing a new line, but are not easily applied to an existing system.

The studies made on the Pennsylvania Water and Power Company's system show the value of reducing the ground resistance or impedance by the use of a counterpoise, which is the easiest method of improving many existing lines.

The discussion and the tables call attention to the value of insulation or arcing voltage in preventing tripouts, as the highly insulated lines are practically free from tripouts even though these lines have a higher exposure and greater length of span.

With most types of transmission line construction an increase in the arcing voltage requires a material increase in the size and cost of the structure, and, furthermore, increases the exposure. For this reason much attention has been given to improvements in transmission line design, which will make it possible to increase the arcing voltage without an adverse effect upon the other factors.

The number of tripouts on some systems have been reduced to a very small percentage by using wood arms to increase the arcing voltage. Where both increased arcing voltage and lower ground resistance are provided, it is evident that the reliability can be greatly increased, and in many cases a higher operating voltage can be used to advantage.

It is interesting to note that Mr. Waldorf has estimated and compared the probable number of flashovers to the service record. The close agreement shows that it is possible to predict the probable performance with a fair degree of accuracy.

Developments in insulation during the last few years have made it possible to develop a higher arcing voltage for many of the two circuit structures, so that not only can tripouts be reduced, but higher operating voltages can be used without causing radio interference.

The relative performance of different types of construction can be readily obtained by taking into account the electrical exposure, ground resistance, length of span, and arcing voltage. To obtain the probable number of interruptions it is necessary to take the length of the line into consideration and the probable number of hits to the system. In other words, the probable number of tripouts can be represented by the following equation, where K depends upon the general storm conditions, L the length of line, E the exposure, G the ground resistance, and FO the flashover:

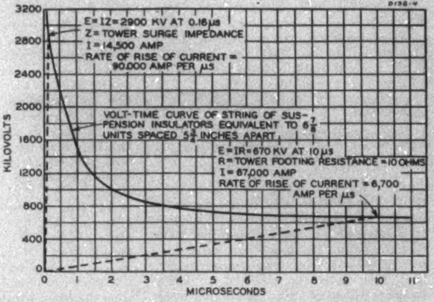
$$K\frac{L\times E\times G}{FO}=\text{probable number of interruptions}$$

A study of the various factors shows that it is possible to modify the type of structure and line to advantage in many cases.

An increase in the arcing voltage requires an increase in the size and cost of the structure. This in turn increases the electrical exposure; therefore much attention has been given to types of construction and insulation which will make it possible to obtain increased arcing voltages to ground without increasing the size of structure and exposure.

While the applications are limited, operating experience shows that much can be expected where all the various factors are given consideration, and it is hoped that more data along the lines given in the paper by Mr. Waldorf will be available.

Figure 4



K. B. McEachron (General Electric Company, Pittsfield, Mass.): In section VII under the heading "Flashovers," Mr. Waldorf suggests the possibility that, for currents in excess of 60,000 crest amperes, the rate of change of current is sufficient to produce an inductive drop great enough to cause flashover of the insulators on 66-kv lines.

It is recognized, of course, that the magnetic link measures crest current values, and does not indicate rates of rise. To assume that currents in excess of 60,000 amperes cause flashover as the result of inductive drop, it is necessary to assume that the higher rates of current rise accompany higher crest values of current. However, I know of no published data which show this to be a valid assumption, although it may be regarded as being reasonable in view of lack of any data to the contrary.

In view of this situation, it seemed worth while to me to examine some of the conditions which might cause flashover. In the figure is given a volt-time curve equivalent to eight disks spaced $4^{3}/_{4}$ inches. This curve is based on data supplied by the high voltage laboratory at Pittsfield, and for times shorter than one microsecond no great accuracy is claimed. Using a tower height of 80 feet, a time of 0.16 microsecond will elapse before a negative reflection from the base of the tower could reach the top of the tower, assuming that a direct stroke made contact with the top. On the curve a time of 0.16 microsecond corresponds to a voltage of 2,900 kv. If the tower surge impedance is taken as 200 ohms, the current at flashover would be but 14,500 amperes, although the rate of rise is 90,000 amperes per microsecond. This rate is much beyond any value yet published, and since the indicated current is much below the 60,000 value indicated by Mr. Waldorf, it does not seem likely that the flashover is due to rates of rise of this order.

If slower rates of rise are examined, it is found that the inductive drop is not sufficient and flashover will not occur, remembering that the first negative reflection reaches the tower top in 0.16 microsecond. However, if a tower footing resistance of 10 ohms is assumed, then any current in excess of 67,000 amperes would cause flashover, while current values appreciably less than 67,000 would not in general cause flashover. One would expect, therefore, in view of the foregoing discussion, that the flashovers which Mr. Waldorf mentioned are more likely due to *IR* drop than to impedance drop.

S. K. Waldorf: Messrs. Austin, Floyd, Grosser, Wagner, and McCann enumerate the several factors which affect the lightning performance of a transmission line. The exact effect of each factor is not known in many cases, but it is significant and fortunate that existing knowledge of shielding, insulation level, tower footing resistance, and perhaps maximum safe tower current allows line performance to be predicted quite well. The several other factors which the discussers have mentioned apparently have little effect individually or as a group.

Mr. Beck requests more complete data on the currents which have been recorded in lightning arresters. In the table which follows are given the complete data on the discharges which have been recorded on lightning arresters connected to circuits having preventive protection. The current of 2,600 amperes recorded in the arresters of the Holtwood-Baltimore lines in 1937 occurred before counterpoise had been provided.

With regard to the accompanying table the four 132-kv circuits have operated without difficulty for the indicated periods without any lightning arresters. Three years ago arresters were provided on *P*5 and *P*6 circuits because two new circuits were connected to them which were not so well protected as are the original ones. While it is recognized that discharges can occur through lightning arresters connected to lines having preventive protection when flashovers occur in the vicinity of the arresters, it is believed that the currents recorded at Highlandtown substation may be due to the arresters there being of the electrolytic type and the magnetic links may have been slightly magnetized during some of the daily power charging cycles. The records on the arresters connected to 13 and 14 circuits, which comprise the Holtwood-Coatesville line, were obtained in a year when expulsion gaps and no counterpoise were installed on three towers near the Holtwood station. The complete record, therefore, justifies the opinion expressed in section 10 of the summary of the paper.

Mr. Floyd suggests another basis for calculating the efficiency of protection provided for a transmission line. Even though no special measures have been provided to prevent lightning interruptions on a given line and some strokes do not cause flashover, it is apparent that protection is being provided even though the design did not include features for that specific purpose. For this reason, and in the interest of simplicity, it is believed that the method suggested in the paper for computing efficiency of protection will prove satisfactory. The other data presented by Mr. Floyd are valuable because they give additional information about the effects of the individual components of preventive protection and about the reduced duty on station lightning arresters when preventive protection is provided on the line.

Mr. Grosser points out that the uncertainty in the measurement of tower lightning currents affects the accurate prediction of performance for a high-voltage line more than for a low-voltage one. In addition to this uncertainty, it must be recognized that the probable outages on a high-voltage line are usually so few that it may take many years before the accuracy of the estimate can be properly compared with operating experience. Mr. Grosser also inquires whether all of the strokes to a line were included when the calculations for probable outages were made. All recorded strokes to the lines were used in the calculations. Just as the Powerton-Crawford line was provided with increased separation between phase wires and ground wires in mid-span, the lines described in the paper similarly are constructed. Mr. Grosser is quite justified in believing that the safe protective angle for a middle conductor in a horizontal configuration can be appreciably greater than shown in figure 3 of the paper. The angle of 41 degrees given in figure 3 is merely the minimum required to shield the inner conductor on the tower illustrated and, if model tests have any significance, probably can safely be made greater.

Dr. McEachron suggests that resistance drop more likely accounts for all of the flashovers listed in table III of the paper than does impedance drop. This may possibly be correct, but the difficulty arises in determining what is the resistance that is to be used in performance calculations. It has been the practice of the Pennsylvania Water and Power Company to include in its ground resistance measurements all auxiliary grounding, such as counterpoise, within one span length of any tower foundation to be measured. When this is done, the resulting resistance does not account for the flashovers in about 50 per cent of the cases which have been noted up to and including 1940.

Messrs. Wagner and McCann have submitted additional interesting analyses of the data obtained on the Pennsylvania Water and Power Company system. The third item which they have listed among the factors affecting lightning protection has been found to be of little importance. It is perhaps surprising to note that records obtained over a period of some years indicate that the ground resistance of counterpoise in the territory studied varies not more than about 10 or 15 per cent from season to season.

In answer to the question of Mr. Von Voigtlander, no oscillographic data have been obtained which would give information about the source of the apparent reactive effects which cause tower flashovers.

Discharges Through Lightning Arresters Connected to Circuits Having Preventive Protection (Including 1940)

Circuit	Arrester Location	Years of Record	Recorded Discharges	
			Year	Current in Amperes
2,302—220 kv	Safe Harbor	4		None
P5—132 kv	No arresters	3		
P6—132 kv	No arresters	3		
P7—132 kv	No arresters	6		
P8—132 kv	No arresters	6		
1— 66 kv	Holtwood	2		None
	Highlandtown	2	1939	− 120
	Highlandtown		1939	+ 630
2— 66 kv	Holtwood	2		None
	Highlandtown	2	1940	+ 120
5— 66 kv	Holtwood	2		None
	Highlandtown	2	1939	− 120
			1940	+ 120
6— 66 kv	Holtwood	2		None
	Highlandtown	2	1939	+ 120
			1939	+ 625
			1940	+ 60
			1940	+ 60
			1940	+ 480
12— 66 kv	Holtwood	4		None
13— 66 kv	Holtwood	3	1938	− 600
14— 66 kv	Holtwood	3	1938	−1,050
			1938	− 200
			1938	− 600

A New Principle of Cable Design

Discussion and author's closure of paper 41-71 by Wm. A. Del Mar, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS (May section) 1941, pages 206–10.

L. F. Hickernell (Anaconda Wire and Cable Company, Hastings-on-Hudson, N. Y.):

EVOLUTION OF GRADED INSULATION

The range in characteristics available in modern cable insulating paper is indicated by the typical data in table I (of this discussion).

From these data, it is evident that as the specific gravity (density) of paper is increased, the physical properties change over a range from 25 per cent (modulus) to 8,700 per cent (air resistance) and the electrical characteristics, from 25 per cent (SIC) to 29 per cent (dielectric strength).

These interrelated physical and electrical properties of paper have created a fertile field for the development of design theories and for the exercise of ingenuity in the patent office. An indication of this productivity is shown in table II (of this discussion).

From this outline, it would seem that practically every combination of high- and low-density paper has been covered by one or more theories, electrically and mechanically. Attention is directed to one vacant space on the chart; namely, the justifica-

Table I. Typical Characteristics of Cable Paper

5-Mil Wood Pulp

Grade	Physical[a]							Electrical[b]	
	Specific Gravity (Density)	Air Resistance (Gurley) Secs/Mil	Oil Penetrativity[c] (Cm-Secs)10^{-3}	Thickness Reduction[d] (Per Cent)	Tensile Strength With Mach. Psi	Elongation at Break (Per Cent)	Modulus Young's $\times 10^3$	SIC 60 CPS 40 Deg C	Dielectric Strength 25 Deg C (VPM)
A	0.739	4	8.0	17	12,600	2.7	405	3.80	1,640
B	0.918	80	5.8	25	16,500	3.3	500	4.30	1,800
C	1.135	352	3.4	40	20,400	3.5	583	4.75	2,120
A to *C* (per cent)	54	8,700	58	135	62	30	25	25	29

Notes: (*a*). Dry paper.

(*b*). Impregnated with unblended mineral oil.

(*c*). For oil with viscosity of 150 SUS at 100 degrees centigrade; surface tension of 30 dynes per centimeter at 100 degrees centigrade.

(*d*). From 100 per cent relative humidity to vacuum.

tion of "sandwich" grading on a mechanical basis. Thus, opportunities for imagination in the field of grading still exist, unless our researchers have missed a reference. Consequently, we will await with interest a future paper covering this missing link.

Analysis of Data

So much for historical background. The remainder of this discussion is devoted to an analysis of the data under the same headings and in the order established by the author.

Relative Imperviousness of Impregnated Paper to Oil Flow

In analyzing figure 1 (of the paper), it should be kept in mind that although the viscosity is shown in terms of Saybolt universal seconds, the actual values above 3,500 seconds were *not* measured on a viscosimeter. In other words, over five-sixths of the curve was obtained by *converting* to Saybolt units, measurements which were obtained under a different procedure. This conversion may be entirely justified. However, from the shape of the viscosity-temperature characteristics so obtained on Habirshaw compound number 7, the procedure would seem to be somewhat questionable. (This point will be discussed in greater detail by Greenfield.)

Elastic Properties of Cable Paper

The author states that different papers have different stress-strain characteristics or moduli. In figure 3 and figure 4 (of the paper), data are shown on type *N* (normal) paper. Since the proposed theory is based upon the *difference* in physical properties between *two* types of paper, it would seem desirable to include data on type *S* (special) paper also. Fortunately, such data are available in the author's patent (U. S. patent number 2,190,017) so that we may have some idea of the quantitative values involved. These data are reproduced in figure 1 (of this discussion).

Reference to figure 1 indicates that over the operating range, say 0 to 5,000 pounds per square inch, the difference in elongations between type *N* and *S* papers is from 0 to 0.1 per cent. In view of the disclosed permanent set in *all* types of paper, this would seem a very small quantity upon which to rely for continued benefit during the life of the cable. Creep under repeated stress also should be considered.

In evaluating this construction, the factor of unequal thickness contractions of two different types of paper should be taken into account. Before drying, the inherent moisture content of the two papers of different densities (or moduli) will be different. Hence, when the cable is dried prior to impregnation, the thicknesses will shrink in different proportions (refer to table I of this discussion). This effect may well mask the small difference in moduli.

Accelerated Aging Tests

Equivalence of Tests Based on Equal Maximum Stress. The author states that "To ensure that our tests should be equally *correlated with service conditions,* we decided to make our tests at the *same maximum stress* as in the Halperin-Betzer tests; namely, 208 volts per mil. . ." This statement is based upon two assumptions; namely that:

1. **The Roper-Halperin-Betzer aging tests, which bore a correlation with service conditions, were conducted at *208 volts per mil* maximum stress.**

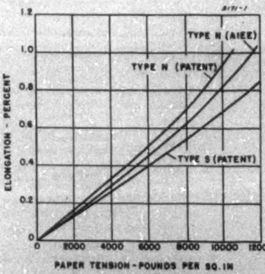


Figure 1. Elongation of impregnated papers—types N and S

Table II. The Evolution of Graded Paper Insulation

Type	High-Density Paper Adjacent to	Design Based on	
		Electrical Theory	Mechanical Theory
Graded	Conductor	1901—O'Gorman—paper[1] 1904—Jona—paper[2] 1921—Emanueil—patent[3] 1925—Meurer—patent[4] 1925—Meurer—paper[5] 1925—Zapf—patent[6] 1934—Vogel—paper[10]	1930—Felten and Guilleaume—patent[7]
Inverse graded	Sheath	1901—O'Gorman—paper[1]	1931—Scott—patent[9] 1938—Del Mar—patent[11]
Sandwich graded	Conductor and sheath	1930—Pirelli—patent[8]	

1. Insulation on Cables, **M. O'Gorman.** *IEE* (British), volume 30, 1901, page 608.

2. Insulating Materials in High-Tension Cables, **E. Jona.** *International Electrotechnical Congress,* St. Louis, Mo., volume 2, 1904, page 552.

3. **L. Emanueil,** U. S. Patent number 1,527,250, filed November 22, 1921, issued February 24, 1925.

4. **H. Meurer,** U. S. Patent number 1,802,030, filed (in Germany) January 27, 1925, issued April 21, 1931.

5. Cable Insulation With Graded Paper, **H. Meurer.** *Sonderheft Vereinigang der Elektrizitätswerke,* Munich, Germany, June 1925 [abstracted by: **M. Hochstadter,** High-Tension Cable Manufacture, *Electrician* (Brit.), volume 97, 1926, page 447].

6. **G. Zapf,** U. S. Patent number 1,771,946, filed (in Germany) May 8, 1925, issued July 29, 1930.

7. **Felten and Guilleaume,** French Patent number 715,645/31, filed (in Germany) June 30, 1930, issued September 28, 1931.

8. **Pirelli,** British Patent number 367/744/32, filed (in Italy) December 10, 1930, issued February 25, 1932.

9. **T. R. Scott,** British Patent number 391,222/33, filed October 23, 1931, issued April 24, 1933.

10. A Few Basic Principles of High-Voltage Cable Engineering, **W. Vogel.** *Archiv Für Elektrotechnik,* volume 28, number 7, 1934, page 391.

11. **W. A. Del Mar,** U. S. Patent number 2,190,017, filed June 16, 1938, issued February 13, 1940.

Table III. Test Voltage Stresses for Cyclic Aging

1-Conductor 750-Mcm 66-Kv Cable, Commonwealth Edison Company

Group	Year	Author	Insulation Thickness (Mils)	Test Voltage: Times Operating	Test Voltage: Conductor Sheath (Kv)	Voltage Stress: Max. (VPM)	Voltage Stress: Avg. (VPM)	Copper Temp. (Deg C)	Remarks
A	1933	Roper	750	1¹/₂	57	124	76	60	Too slow
				2	76	165	101	{50 to 85}	Correlation with service
				3	114	248	152		No correlation with service
B	1936	Halperin and Betzer	750	1³/₄	67	145	89	R.T.+40	Best—too slow
				2¹/₄	86	187	115		
				2¹/₂	95	$\boxed{208}$	127	65	Eliminates poor in 4 weeks
C	1936	Halperin and Betzer	688	1³/₄	67	154	97	R.T.+40	Best—too slow
				2¹/₄	86	198	125		
				2¹/₂	95	218	138	65	Eliminates poor in 4 weeks
D	1936	Halperin and Betzer	625	1³/₄	67	165	107	R.T.+40	Best—too slow
				2¹/₄	86	212	138		
				2¹/₂	95	234	152	65	Eliminates poor in 4 weeks

2. If the author's test were conducted at the *same maximum stress*, the same correlation with service conditions would hold.

The calculated voltage stresses used in the Commonwealth Edison tests are shown in table III (of this discussion).

From this table, it will be noted that the maximum stresses varied from 124 to 248 volts per mil. Only one of the series was conducted at 208 volts per mil; namely, group *B*. This specific value was obtained when 2¹/₂ times operating voltage or 95 kv as applied to 750 mils of insulation. In view of these data, it would be interesting to know why the author happened to select the *lowest* maximum stress corresponding to 2¹/₂ times operating voltage.

Incidentally, it is our understanding that the Commonwealth Edison Company have selected 2¹/₂ times operating voltage or 95 kv as the proper test voltage for 66-kv cable regardless of what insulation thickness is offered. Hence, the bench mark is actual test voltage, not maximum stress.

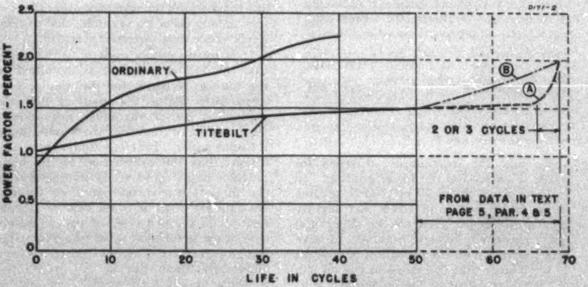
The maximum stress of 208 volts per mil in the Chicago tests, resulted from applying 2¹/₂ times operating voltage or 95 kv to a 750,000-circular-mil, 66-kv cable having *750 mils* of insulation. The selection of 208 (corrected) volts per mil maximum stress by the author to apply on his number 4/0 AWG, 17-kv cable having *220 mils* of insulation resulted in a test voltage of 32.5 kv or approximately three times the operating voltage (17/1.73 = 10 kv) of his cable. As the author points out, Roper found that for 66-kv cable, tests at three times operating voltage bore no relation to service records.

This seemingly paradoxical situation arises from the attempt to test a number 4/0 AWG (212,000-circular-mil), 17-kv cable with 220 mils of insulation at the same maximum stress used on a 750,000-circular-mil, 66-kv cable with 750 mils of insulation; the conductor diameters being 528 and 998 mils, the insulation thicknesses 220 and 750 mils and the geometric factors ($\log_e \times D/d$), 0.603 and 0.916 respectively. Due to this difficulty and because operators usually feel that a simulated service test should bear some relation to operating voltage, the equivalence of tests based on equal maximum stresses have not been accepted generally by the industry. For these reasons, it would seem to be on safer ground to base this type of test on average stress or operating voltage.

Stability Limits. The voltage stress at which the power-factor limit of 2.00 per cent at 85 degrees centigrade is obtained is not stated. Reference to figure 6 (of the paper) would indicate that this measurement is at rated voltage or 10 kv, corresponding to an average stress of 45.5 volts per mil.

Stability of Set Number 2. For set number 2, the tabulated data show that the Titebilt construction reached the power-factor limit (2 per cent at 10 kv and 85 degrees centigrade) in *69 cycles.* These data are shown graphically in figure 6 (of the paper). Unfortunately, the curve for the Titebilt cable is not shown above *50 cycles.* Thus the reader is forced to speculate on the portion of the curve between 50 and 69 cycles, as shown in figure 2 (of this discussion).

Figure 2. Life-cycle tests (compound 7)



The paper states that "the Titebilt cable shows a *smoothly rising* curve which contrasts remarkably with the erratic curve of the ordinary cable. The power-factor curve of the Titebilt cable did not show any final upturn until *two or three cycles before 'failure.'* "

If the term "failure" means reaching the limit of two per cent power factor in the final two or three cycles, presumably the slope would be similar to curve *A*. If the term "failure" applied to final breakdown, then the slope might approach that shown in curve *B*. In any event, the power factor rose from 1.5 to 2 per cent or 0.5 per cent in 19 cycles whereas it took the initial 50 cycles to rise the same amount (1.0 to 1.5 per cent). This would hardly indicate a "smoothly rising" curve, but, on the contrary, instability.

It is probable that, if the tests were conducted for a longer period of time, the results would confirm those obtained on a similar design of inverse-graded cable by Scott of England ("Long Period Aging Tests on Solid-Type Cables," by T. R. Scott, *Journal* IEE (British), volume 85, number 511, July 1939, page 67—Cable EC58/1a).

Compound Number 8

Although no viscosity characteristics are given, presumably compound number 8 with 10 per cent of hydrogenated wood rosin is a less viscous impregnant than compound number 7 with 15 per cent of natural wood rosin. If so, this factor may have some influence on the behavior of the Titebilt cable.

It is stated that there was no indication the Titebilt cable with compound number 8 would attain the power-factor limit in 200 cycles. It would be interesting to know if ordinary cables with compound number 8 showed comparable improvement.

Commonwealth Edison Test Procedure—1939. Again, it is not our understanding that the Commonwealth Edison Company tests reported by Halperin in 1939 ["Load Ratings of Cable," by H. Halperin, AIEE Transactions, volume 58, 1939 (October section), pages 535–53] are based on any specific voltage stress such as 114 volts per mil, average and 195 volts per mil, maximum as stated in the paper. The 1933–36 tests, discussed above, were on 66-kv cable and the test voltage was $2^{1}/_{2}$ times operating voltage or 95 kv. The 1939 tests were on 12-kv cable and the test voltage was two times operating voltage or 24 kv. Accordingly, in both cases the criterion is an actual test voltage related to operating voltage and not average nor maximum stress.

On this basis, the 114 volts per mil average stress, mentioned in the paper, would apply only to a shielded cable having 122 mils insulation. Shielded cables with as low as 100 mils have been subjected to the same test voltage or 24 kv with the resulting average stress of 139 volts per mil.

Shielded Three-Conductor Cable

On the basis of being offered for the same operating voltage (12 kv), it is, of course, interesting to compare the test results obtained on Titebilt shielded cable having 122 mils of insulation with belted cable having $9\times{}^{5}/_{64}$ inches (141×78 mils) of insulation reported by Halperin in 1939. Since 1939, tests on the same procedure have been conducted on two new designs of shielded cable as reported in the technical press ((1) AIEE Transactions, volume 59, 1940, page 1041 and (2) *Electric Light and Power*, July 1940, page 34). It would be interesting to compare the results obtained on the three different designs of shielded cable tested under the same procedure. Unfortunately, it is impossible to do this owing to the omission, in table I (of the paper), of any of the power-factor values above 60 degrees centigrade (i.e., for 80, 100, and 115 degrees centigrade). The absence of other data usually obtained on this type of test, such as pressures, vacua, and sheath expansion, are noted. In addition, the dielectric strength obtained on the AEIC specification sample tests (high-voltage time and cold bend) would be of interest.

To assist the industry in evaluating this construction, it is suggested that the author include in his written closure, the aging data mentioned above. Preferably, this should be in the form established by Halperin in figure 1 of his 1939 paper, and followed by others, in order that the data may be directly compared.

Robert J. Wiseman (The Okonite-Callender Cable Company, Inc., Passaic, N. J.): In the reading of Mr. Del Mar's paper, several questions arise which may help us to appreciate the design of cable which he proposes. It is true that the viscosity of the oil used in solid-type cables is high enough to cause high local internal pressures and these pressures may result in stretching of the paper, enough to take up the increase in volume of the oil. This stretching is more effective on sector and oval-shaped conductors than for round conductors. In the latter case the oil also flows more easily between the tapes to the gaps and out to the sheath. The idea of the oval-shaped conductor for both single and multiconductor cables is based on the theory that the tapes along the minor axis are stretched as the temperature increases and when the cable cools down they return to place, forcing the oil into the inner portions of the insulation. We have tested single oval-shaped conductors and have obtained pressures of 20 pounds per square inch at the sheath when for a round conductor we obtained as high as 85 pounds per square inch. Therefore, any means we have to allow the paper to stretch during the heating of the cable is good for the cable. Has Mr. Del Mar compared tests with round and sector or oval conductors?

I am curious about figure 1. I would like to know more about the method of test and also how Mr. Del Mar obtained such high viscosities. Oil will start to flow very shortly after load is applied to a cable. We have measured 100 pounds per square inch pressure at the conductor one-half hour after load is applied and the pressure at the sheath had not increased. Two hours later the sheath pressure was 85 pounds per square inch. In this case, the conductor temperature had not increased very much in one-half hour.

I would like to see comparative physical properties of the normal and special papers Mr. Del Mar uses in order to appreciate their relative moduli. At the time of the issuance of his patent he showed curves of stress and strain. A comparison with superdense paper and normal paper we are using showed very close similarity. I also would like to know the relative oil absorption of both papers and their specific inductive capacities. I believe he has a reverse gradient cable with its accompanying distribution of electric stress. The difference in comparison to usual gradient will not be very great because of the thin walls involved. In addition, we must consider the electric stress in the oil gaps for the two papers because of the relative specific inductive capacities. If we take as an example the three-conductor, 500,000-circular-mil cable described, and round conductors, an SIC of 4.2 for impregnated special paper, 3.7 for normal paper, and 2.2 for oil, we get the stresses at various positions as shown in the following table:

	Stress at Conductor Surface (Volts Per Mil)		Stress at Beginning of Outer Zone (Volts Per Mil)	
	Paper	Oil	Paper	Oil
Type N inner zone	136	220		
Type S outer zone			101	193
Type S inner zone	129	246		
Type N outer zone			122	204

We find the distribution of stress for paper is better for the usual grading. In the case of oil, the usual method of grading shows higher values of stress and slightly greater difference between the inner and outer zones, not enough, however, to have much influence on the operation of a cable. If we assume that the stress in the oil gap is the major factor in the breakdown voltage of a cable, then we should expect Mr. Del Mar's cable to give higher values than the usual grading. Has Mr. Del Mar made comparative tests?

This cable appears to me to be similar to the "SR" cable put out by the Standard Telephones and Cables Company of England. This cable functions on the assumption that if we have papers with different degrees of oil absorption, with the highest absorption next to the conductor, as the cable cools down, the oil is drawn by capillary attraction to this zone, thereby maintaining better saturation of the cables for all temperatures and better operation.

I would like to see the power factor-temperature curve for the cable referred to in table I of the paper and also the power-factor time plot for all temperatures up to 115 degrees centigrade. The cable operates at higher temperatures than 60 degrees centigrade and, therefore, should show stability at the higher temperatures. How much negative ionization did Mr. Del Mar get at the various temperatures? What pressures and vacuum did Mr. Del Mar get on this cable? If I understand his design correctly, there should be very little, if any, pressure between the cable and sheath when the cable is heated because the oil is not permitted to leave the insulation. Therefore, there should be no lead sheath expansion and when the cable cools down, no vacuum created.

J. B. Whitehead (The Johns Hopkins University, Baltimore, Md.): The results reported by the author for the performance of the Titebilt cable are very impressive. This is especially true of the test of sets 1, 2, 3, and 4. Moreover, the results of examinations after the tests and the performance reported for compound 8 are also very striking.

In view of the superiority claimed for the Titebilt principle, it becomes important to examine the basis upon which claim is made. It is also of great interest to examine the principle which the author invokes to explain the results of his tests. Unfortunately the paper is not completely satisfactory in either of these respects. First as to the test themselves, one would like to know the spread of the results as observed among all the cables tested, the range of initial power factor, and the density and dielectric constant of the several papers studied. Moreover, the program of the test itself is of highly special character, and one would like to have seen the results of other types of test, among them for example, that of relatively short-time dielectric strength.

It is by no means evident in the paper that the Titebilt principle as described by the author accounts for the test results. Apparently the author assumes that the permanent set of the inner low-modulus paper is greater than that of the outer high-modulus paper. It is only under these conditions that the contraction of the high-modulus paper could act in compression on the inner low-modulus paper on cooling. No data are given as to the relative permanent set of the two types of paper. Moreover, if the permanent set of the inner paper constitutes the trouble to be corrected (and apparently there can be no other) this would be absent in the samples constructed entirely of type *S* paper and it should show an equal, if not better, performance than the Titebilt sample.

It appears, therefore, that some other elements than that of the elasticity of the paper are entering to account for the superior results shown from the Titebilt samples. Here again, however, we are in difficulty because of insufficient information furnished by the author. Most cable engineers now agree that initial instability and ultimate failure is caused by the presence or liberation of gas in the oil channels and films. It has recently been shown that in thoroughly impregnated specimens the stress in the oil channels, the stability, and the breakdown strength are greatly influenced by the density of the paper. Breakdown strength and life in accelerated tests are both reduced by increasing paper density. These results are clearly explained by the difference in the dielectric constants of the impregnated paper and the adjacent oil channel. The author of the present paper has not given sufficient data as to either density or dielectric constant to make possible a computation of the relative stresses in the oil channels of his various specimens. One would also like to see points indicating experimental observations on the curves of figures 1, 3, 4, and 6.

With so many variables present in a completed cable, it appears possible that there may be others than those mentioned which also have a bearing on the results found. For example, may it not be that the Titebilt samples were actually more tightly wound than the others?

It is to be hoped that the author will supplement the present interesting paper with further tests and further analyses of the results.

T. F. Peterson (American Steel and Wire Company, Worcester, Mass.): The new design of paper-insulated cable appears to be predicated on the existence of two phenomena which are very difficult for me to accept, namely:

1. In solid-type cable there is substantially no outward flow of oil or compound at ordinary operating temperatures.

2. High modulus of elasticity paper-belt insulation holds inner layers in compression and, after heating, restores inner layers to original condition.

Certainly, figure 1 of the paper does not prove the absence of radial flow. It shows that the flow is slow. Is not this particularly conducive to "ratcheting" out of oil? If there is no radial flow of compound, what has caused lead sheath stretching which has been so prevalent in recent years, and has occupied the attention of operating engineers because it appeared to be so serious?

Now, as to the operation of high-modulus paper in restoring inner paper volume: I contend that the value of modulus has little to do with the degree of restoration unless the outer belt is also "truly elastic" in its stress-strain diagram, that is, unless this is a straight line through the origin both for rising and falling stress. "Creep" or "permanent set" when stress is removed would preclude usefulness of high-modulus paper in restoring inner paper volume. The author indicates that "all papers tested showed more or less permanent set." It seems unfortunate that the "modulus" and "permanent set" curves were given only for so-called normal paper and not for the paper responsible for the remarkable characteristics of the newly designed cable. If the subject were treated mathematically, I think it would be found that even a "low modulus" binder or belt would serve the intended purposes, provided it had negligible permanent set after removal of stress.

Finally, if compound does not move radially, paper does not stretch enough to create oil passages between layers, and high-modulus paper does develop a "permanent set," why not use a truly elastic binder in the form of a hard drawn copper tape or synthetic rubber-like sheath? Quite obviously, some improved characteristics of single-conductor cable have been observed on accelerated life tests. If high-modulus characteristics of paper do not fully explain the improved results, could they be attributed to "inverse grading" with high SIC paper in the outer region contributing to reduced stresses in oil of gaps in paper nearer the conductor?

E. W. Davis (Simplex Wire and Cable Company, Cambridge, Mass.): We have read with considerable interest the paper by Mr. W. A. Del Mar—"A New Principle of Cable Design." The test data submitted would indicate a definite improvement in cable quality, although the discussion of the theories involved in this improvement may be questioned by some engineers and physicists.

We do not completely understand the significance of figure 1. Apparently the rate of pressure transfer is shown for only one constant pressure on one side of the wall of paper insulation. Would not this rate of pressure transfer be a function of both the paper thickness and the value of the constant pressure applied? Being a function of two variables, might not there be a decided change in the shape of the pressure transfer curve?

The meaning of reference to paper stretch is not clear. If the elongation of the paper is not enough to leave oil passages between layers as stated by the author, how can the permanent set be enough to leave voids in cooling if as indicated by figure 3 and figure 4 the set is only one-tenth of the elongation? A space ten times the thickness of the voids should allow the passage of oil or inversely one-tenth of a space too small for passage of oil must be rather small even for a void.

The outer layers of paper are 20 per cent higher in modulus of elasticity than the inner layers. This modulus is stress/strain or the inverse of the slope of the curve of figure 3. Figure 3 shows an elastic limit at about 3,000 pounds per square inch, the modulus being constant below this value and decreasing above it. The higher modulus of the outer paper means that a greater stress is necessary for the same deformation, or less deformation of the outer paper than that of the inner paper for any given stress. We follow the author's explanation this far with no trouble but we find no reference to a difference between the papers that seems important to us. How do they compare as to permanent set? Do papers of higher modulus have correspondingly less permanent set? How does the permanent set-elongation relation of figure 4 vary among papers of different modulus? The squeezing action of the outer papers, especially after several cycles, may be limited more by the permanent set than by the modulus. One is tempted to wonder about the permanence of the improvement if it is due to higher modulus. Presumably, this higher modulus would be called into play at each load cycle. What might happen after 2,000 load cycles? Would the paper still retain its high modulus or would it have become permanently set? Stress-strain data on this paper would be interesting and also any data on fatigue. There is no mention of the well-known fact that it is more difficult to close a void with expansion than to form it initially. Often such a void will withstand appreciable positive pressure before disappearing. Instead of helping "to restore the inner layers to their original condition" the effectiveness of the outer layers may be due to this extra mechanical pressure.

Another explanation may be that high-modulus paper merely reinforces the inner layers preventing them from stretching. Therefore, the oil is forced through the channels between the tapes instead of stretching the paper.

We would like to ask the author if the two papers have differences other than the modulus. Are they appreciably different in air resistance, dielectric constant, or dielectric strength for instance? It is not impossible that some such difference may be an unexpectedly large factor in the cable behavior.

The work with a new impregnating compound is interesting and illustrates the possibilities of newer materials. However, the implications that this compound makes better performance because of its higher resistance to bombardment may or may not be correct. There are two schools of thought as to start of cable failure—

1. That failure is due to cold glow discharge in gaseous voids,

2. That failure is due to ionic bombardment from the conductor when the cable is hot.

From these points of view the conclusion is justifiable, but past laboratory experience has shown equally surprising improvements from changes to compounds not of superior resistance to bombardment providing such compounds are more suited to the particular paper used. We feel there is much to learn yet about combinations of paper and oil under various conditions. Is it not possible that the new compound fitted the particular paper porosity and cable structure better irrespective of its resistance to bombardment?

If this is not so, then we are forced to conclude that even the present improved cable has gaseous ionization or ionic bombardment present, and while life tests show it to be superior, will this superiority be attained in service?

Life tests have distinct value as have similar accelerated aging tests on other insulations. However, experience shows such accelerated tests must be carefully examined and interpreted in the light of service conditions to be sure such tests contain no elements leading to erroneous conclusions. This is particularly true of life tests on paper cable where overvoltages and artificial load cycles can make many changes in such a complicated structure as paper and oil.

E. W. Greenfield (Anaconda Wire and Cable Company, Hastings-on-Hudson, N. Y.):

Mechanical Stresses Within the Cable

This paper is based upon the conception that solid-type paper cable deterioration is fundamentally due to vacuous gaps formed in the cooling cable as a result of excessive pressure on the paper insulation occasioned by the expanding but viscous oil during heating periods. The author arrives at this conclusion from observations of stress-strain and permanent set characteristics of impregnated paper in which he shows for one type of paper the permanent set to be about one-tenth the elongation for any tensile stress (figures 3 and 4). The paper states that "from calculations of the paper tension arising from oil expansion," it is found that permanent set of the paper is sufficient to leave voids on cooling. This point is considered of utmost importance in the mechanisms of the author's design and hence is worthy of examination.

Since neither sample calculation nor method of evaluating the factors is given, it may be pertinent to inquire into the rigorousness of the author's conclusions. The theoretical problem of internally developed tensions is exceedingly difficult and it would seem obvious that an experimental solution would be more practical as well as desirable. The pressure drop across the insulation is a function of paper permeability and oil viscosity and both of these factors are functions of time. In addition to permanent set of the paper, the question of creep is involved and this again brings in the factors of pressure and time in complex relationship. The time factor apparently is neglected since no data nor statements concerning this are given by the author.

As an example of the confusion possible in the calculation of this problem, let us assume we know the form of the pressure distribution through the cable at any instant of time after start of cable heating. On this basis, the relation may be expressed by:

$$P_x, t = Kf(r, t)$$

where

P_x, t is the pressure at a point x in the insulation at a time t

r is the radius of point x from center of conductor

t is the time

The general formula for hoop stress or tension is:

$$\text{Hoop stress} = \frac{(D-T)}{2T} P$$

where

D is the outside diameter

T is the thickness of the serving bearing the pressure

P is the internal pressure

If we wish to calculate the hoop stress at a diameter D_x in the insulation, we must decide first whether a single tape is to carry the full stress, or all the insulation between the diameters D_x and D actually carries the stress. This makes a very large difference in the result. Calculating hoop stresses for both conceptions, we have:

For single-tape carrying stress:

$$S_{x_1} = \frac{D_x - T}{2T}(P_x, t) \qquad (1)$$

where S_{x_1} is the stress in a single tape.

If the remainder of the outer insulation carries the stress:

$$S_{x_2} = \frac{D + D_x}{2(D - D_x)}(P_x, t) \qquad (2)$$

where S_{x_2} is the stress in the rest of the insulation from point x to outer radius.

The ratio,

$$\frac{S_{x_1}}{S_{x_2}} = \frac{(D_x - T)(D - D_x)}{T(D + D_x)} \qquad (3)$$

That the ratio given by equation 3 can have large values is shown by the following example.

Consider a cable of 2.85 inches core diameter, made with 5-mil paper tapes and calculate the hoop stress existing at a point near the conductor, say at a diameter of 1.42 inches. Substituting these values in equation 3, we find the ratio of the stresses for the limiting conditions defined above to be approximately 100. Near the middle of the insulation, the ratio is approximately 80 to 1. Hence, in this case, the stresses borne by the tapes may vary by a factor of 100 to 1 or 80 to 1, depending upon the conception of how the insulation behaves and what point in the insulation we are considering. For normal, well-made insulation it is felt that the tapes act together in transmitting and resisting pressure and hence, the author's theories are not sustained. In 1927, Hochstadter and Barrat,[1] came to this same conclusion.

Development of Internal Pressure

The author states that the source of the excessive internal pressure in solid cable is the "oil viscosity which is so great that very high pressures are created by the expanding oil." In figure 1 of the paper is shown a curve expressing the rate of pressure transmission through 220 mils of insulation as a function of oil viscosity under a driving pressure of about 14 pounds per square inch. The temperature range covered by the curve is only from 60 degrees centigrade to 100 degrees centigrade, yet over this range, the viscosity of the oil is shown to vary from 25,000 seconds to about 200 seconds. We are asked to believe that at 60 degrees centigrade the rate of transmission of pressure is so low through the 220 mils of insulation that equilibrium conditions are reached only after 48 hours and at 100 degrees centigrade after about $1^{1}/_{2}$ hours. This is an enormous range of viscosity and indicates that the saturant is below its flow point (ASTM) at temperatures lower than approximately 65 degrees centigrade.

The author has indicated in his verbal presentation of the paper that the abscissa of his figure 1 is not Saybolt seconds viscosity as marked but rather consists of two types viscosity parameters. The first one-sixth of the abscissa is indicated to be Saybolt universal viscosity and the remaining five-sixths to be "seconds" obtained by penetration through precalibrated paper sheets. It is apparent that these two measures of viscosity are not consistent and that the viscosity function of figure 1 is discontinuous in the region of 3,500 seconds. Hence all the factual data of figure 1 are open to question.

It should also be pointed out that the rate of pressure transmission is a function of the driving pressure in addition to the oil viscosity.

Flow of oil through paper has been investigated extensively by many people, and in particular as to insulating paper, by Whitehead and Greenfield,[2] Race,[3] and Bull.[4] Regardless of whether the D'Arcy or Washburn equations of flow are used, it is known that under a given driving pressure, the rate of oil flow varies directly as the permeability of the paper and inversely as the viscosity of the oil. Since the pressure is transmitted by the oil, the rate of oil pressure is proportional to the rate of oil flow. Hence, conclusions concerning oil flow also define rate of pressure transmission.

The easier the oil can pass through the paper structure, the less pressure on that structure. Therefore, the simplest solution to the problem is to use tightly wrapped, high-permeable paper and low-viscosity oil. The author proposes and shows high-density, high-modulus papers near the sheath and a saturant of abnormally high viscosity (over cable operating range of temperature). These features only aggravate the pressure and insulation distortion possibilities in the cable. On this basis, there is considerable doubt that the improved results the author ascribes to his "Titebilt" construction are due to the mechanism he propounds.

It may be that the improved results shown to have been obtained in the few cycles of aging are due to the electrical effects of reversed grading rather than to any mechanical effect. By placing the lower density paper next to the conductor, the stress in the oil channels in that region is lower than if high-density papers were used. It cannot be emphasized too strongly that the oil channels are probably the weakest part of the insulation. There is now ample experimental evidence and more steadily accumulating to substantiate this point.

Compound 8

Although no viscosity-temperature curve is shown for compound number 8, it may be presumed that, having less rosin, it is probably less viscous than number 7. We wonder if the improved results reported for number 8 compound are not in part due to its lower viscosity as well as to better discharge characteristics. Since the author's tests are not influenced by compound migration, the lower viscosity oil will produce less pressures and permit more rapid and effective self-healing action in the oil gaps. It would be helpful if the paper included comparative viscosity-temperature curves for compounds 7 and 8.

References

1. Sur La Stabilité en Service des Cables Sauterrains Pour Transmission d'Energie, Hochstadter and Barrat. *Revue Générale d'Electricité*, volume 22, 1927, page 1175.

2. Capillary Action in Impregnated Paper Insulation, Whitehead and Greenfield. *Physics*, volume 3, 1932, page 324.

3. Tests on Oil-Impregnated Paper—III,

Fluid Flow, H. H. Race. AIEE Transactions, volume 57, 1938 (October section), pages 578–8.

4. W. Bull. *Journal of Physical Chemistry*, volume 41, 1937, page 463.

John H. Palmer (Phelps Dodge Copper Products Corporation, Yonkers, N. Y.): Mr. Del Mar's paper has dealt rather briefly with belted cables of Titebilt construction, and states that the benefits derived were smaller than those obtained in the single-conductor and three-conductor shielded types of cable.

This is true, if judged by the electrical measurements made. Comparison of Titebilt and old type cables of the belted construction by visual examination, however, after load-cycle tests, gave a very striking example of the effectiveness of Titebilt construction. This was shown by the relative conditions of the filler spaces and of the tapes adjacent to them, both in the belt and in the single conductors.

Results of examination are as follows:

The Titebilt cable, *A* in the photograph, showed only the initial stages of wax formation in the filler spaces. There were only one or two slightly charred spots in the fillers and surrounding insulation of the section examined, and this area remained unusually well saturated considering the treatment which the cable had received.

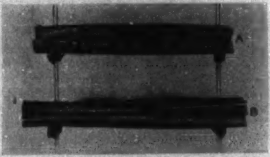


Figure 3. Singles and fillers of belted Titebilt cable (top) compared with belted graded cable (bottom) after 116 cycles of load-cycle test at temperatures of 85 to 100 degrees centigrade and 24,000 volts

Both cables impregnated with compound 7

Note good impregnation of Titebilt cable and dry burned waxy appearance of graded cable

The condition of the old type cable, *B* in the photograph, was about as would be expected. The filler spaces and several layers of the surrounding insulation both in the belt and on the singles contained a great deal of cable wax, charred spots, and dendrites. This area was also quite dry as is usually the case in the presence of much wax.

All other parts of both cables were in excellent condition.

The difference in these cables was so great, that it can easily be distinguished in the accompanying photograph showing the surfaces of the singles which were adjacent to the fillers. The high lights visible on the Titebilt conductors at the top of the picture clearly show the surplus of impregnating compound remaining in this cable.

These cables were of identical construction except for the types and distribution of the paper tapes used. They were impregnated at the same time and in the same tank, using compound 7 and were both subjected to 116 load cycles, giving copper temperatures of 85 degrees centigrade to 100 degrees centigrade.

The filler spaces have long been the weak point in belted cable, where deterioration normally starts. The Titebilt principle of construction has evidently improved conditions in this danger zone.

R. W. Atkinson (General Cable Corporation, Bayonne, N. J.): Del Mar's test results speak for themselves, and show for the experimental conditions a definite superiority for his new construction over the constructions with which it has been compared. His tests, when considered together with the tests made at Chicago which he quotes, seem to be very strong evidence of the value of cable which he describes. However, Del Mar's theory of the results, which seems correctly to explain the *test* results, seems to me to discount the possibility of any permanent benefit from this construction, in terms of *life of cable in operation*.

Del Mar's explanation of the "Titebilt" cable is essentially that the outer layers of paper act effectively as an impervious elastic covering over the remainder of the insulation. Such a theory can readily explain successful results of short-time tests or even tests of the longest duration that Del Mar describes and yet be completely inadequate to correlate these tests with service conditions. Thus, a covering that is nearly tight but not completely so will function, for a time, in a manner substantially the same as a perfectly tight covering, but very slow oil flow through the covering will be cumulative over the period of months and years of the cable in operation and thus eliminate any permanent gain.

In order to permit judgment as to the significance of his tests with respect to operating characteristics, it is necessary to study quantitatively the actual flow of oil in a cable of the "Titebilt" type. The data given in figure 1 are not readily adaptable for such a quantitative study and accordingly we have, ourselves, made such measurements on a carefully prepared cable model.

The model was made on a smooth cylindrical mandrel and tightly hand-wound in a lathe. The tightness of wrapping was at least as great as is permissible on a cable that must be bent, and thus the resistance to oil flow at least as great as for a cable.

The wall thicknesses and types of paper used are those used by Del Mar on his shielded 3-conductor cable as described—90 mils of low-density paper and 30 mils of superdense paper. The oil used was also similar to one of those described by Del Mar, namely 85 per cent paraffin base cylinder oil of 150 seconds viscosity at 210 degrees Fahrenheit and 15 per cent rosin. The diameter of the insulated mandrel was approximately two inches.

After saturation, the radial oil flow through this sample was measured when internal hydrostatic pressure was applied. Measurements were made at temperatures ranging from 116 degrees to 43 degrees centigrade and at pressures of 100 pounds per square inch and 20 pounds per square inch. The values were found to be substantially proportional to pressure and inversely proportional to viscosity. Of 13 measurements made 10 were within 25 per cent of such proportionality while the remaining 3 were only 20 to 35 per cent of the proportional value. The values at different temperatures, as taken from the curve were as follows:

Temperature 0 Deg C	Oil Flow in Grams Per Pound Per Square Inch Per Hour Per Foot of Sample	Kinematic Viscosity (Centistokes)
116	1.35	25.5
76	0.27	122
58	0.093	340
43	0.033	1,000

One may apply the above measured values to the practical case of a cable heated so that the temperature of the outer layer of paper is 76 degrees centigrade. Assume a hydrostatic pressure of 75 pounds per square inch to be produced as a result of the expansion of the oil from heating. The rate of loss of compound per foot length on a cable one inch in diameter (one-half that of the above handmade sample) is 9.5 grams per hour, or about 0.17 cubic centimeter per minute. The total volume increase of oil due to temperature variation is less than three cubic centimeters per foot (two per cent of total volume). Of course the hydrostatic pressure in the cable will fall off as the oil moves out through the paper wall, and the rate of movement will decrease, but clearly a volume of oil equal to a large part of the expansion will go through the wall in a relatively few minutes under the conditions above described. At lower temperatures the rate of movement will be slower due to the higher viscosity of the oil. But, at any temperature at which there is an internal hydrostatic pressure and at which the oil behaves as a viscous liquid, there will be a continuous bleeding that will, during a reasonably short cable life, result in substantial cancellation of the special property claimed for the "Titebilt" construction.

Note that when the load changes from low to high there is relatively high pressure inside the insulation and relatively high temperature, while when the load changes from high to low the pressure and temperature are relatively low. Assuming that for a given temperature and a given pressure difference the rate of flow is the same from the conductor out as from the sheath in (which is substantially true), the actual rate of flow on cooling will be less than on heating, so that apparently some of the compound which gets through the insulation to the sheath will never get back into the cable. Moreover, the greater the absolute resistance to flow of the nonimpervious belt the greater will be this accumulation of compound on the outside.

Let us hope therefore for a new theory that will give some quantitative explanation of the test results found without, at the same time, indicating that these test results are of no practical significance.

Herman Halperin (Commonwealth Edison Company, Chicago, Ill.): Mr. Del Mar's paper is a stimulating contribution to the problem of solid-type high-voltage cable. While the good test results reported in the paper, and confirmed by tests on one sample in Chicago, seem to support the theory underlying this new construction, some of the theoretical considerations seem to be in conflict with observations we have made on cable after operation under test or in service.

Mr. Del Mar's theory assumes that for the heavy compounds used in solid cables practically no radial migration of compound occurs, but the expansion of the compound produces a permanent set of the paper. This theory is in contrast with the finding that frequently the gap under the sheath produced by expansion of the lead is well filled with compound while dryness is found in the body of the insulation. This finding indicates that radial migration of compound must have occurred. Also, if in solid cable the expansion of the sheath were produced only by expansion of the paper insulation rather than by radial migration of the compound, then on aged cable the diameter over the insulation should be found to have increased as much as the diameter over the lead. Actually we have not found this to be the case in the many measurements we have made on cables having appreciable expansion. For cables having large sheath expansion, we have found no appreciable change in the diameter over the insulation, although there have been some indications of slight changes in some cases.

The question as to whether oil can flow along the path between tapes is certainly influenced by the tightness of the insulation as well as the viscosity of the compound. The author points out that, while radial flow does occur with the thin oils used in oil-filled cables, practically no such flow occurs with heavy compounds used in solid cable. It should be noted, however, that the viscosity of solid compound at 80 degrees centigrade is about the same as that of oil-filled cable oil at 30 degrees centigrade. If radial flow of oil can occur in oil-filled cables at low temperatures, the same should be assumed for solid cable compound at high temperatures. Actually, as figure 1 of the paper shows, the rate of flow at 80 degrees centigrade is about one-eighth of the rate of flow at impregnating temperature.

At impregnating temperature the radial flow of compound is obviously large. This temperature is probably in the neighborhood of 100 degrees centigrade. As table I of the paper shows, the test cable was subjected to four cycles with a maximum copper temperature of 115 degrees. It is estimated that the corresponding sheath temperatures were 93 degrees and the average temperature of the compound 104 degrees. These temperatures are of the order of impregnating temperatures, and, therefore, it should be expected that radial flow could have occurred at least in these high-temperature cycles.

We have found occasionally some looseness of the insulation and signs of deterioration in regions where the direction of taping was reversed and we have not found these phenomena particularly associated with regions of change in density and kind of paper tapes.

The main point to me as a user of cable is, however, that the recent samples of cable so far made under Mr. Del Mar's technical direction have excellent characteristics in accelerated aging tests. Probably other factors besides the Titebilt principle help to produce this result.

K. S. Wyatt (Phelps Dodge Copper Products Company, Yonkers, N. Y.): In the novel construction described, improved load-cycle stability has been obtained by taking advantage of the elastic properties of cable paper. Careful study of this paper, however, leads to the conclusion that other factors have contributed to the high degree of stability attained.

In the first place there can be no doubt that the "Titebilt" construction does minimize ionization effects for at post-mortem examinations of samples of Titebilt and normal cable after severe load-cycle aging it is clearly evident that while dendrites and wax were present in considerable amount throughout the inner two-thirds of the insulation wall of the normal cable, there was a complete absence of ionization phenomena in the Titebilt samples except in the outer layers.

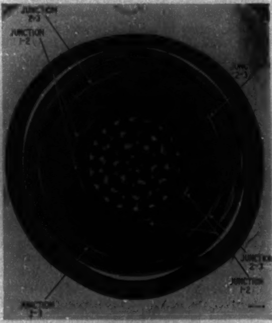


Figure 4. Wafer of new European cable employing dense paper at conductor, less dense in the middle, and still less dense at the outside

In the drying of this cable the different papers shrank away from each other, leaving annular gaps in which ionization may take place

However, I believe some of the improvement is due to another factor. Much cable today is graded with dense insulation next the conductor, less dense paper in the middle and still less dense at the outside. Among such papers of different density there is a fractional variation in SIC, so that when applied with the highest SIC (and highest density) paper next the conductor, the curve of potential gradient was supposed to be smoothed out somewhat, resulting in lower maximum stress at the conductor. With theoretical ideas such as these as a basis, the principle was employed in actual cable construction in Germany, from where it spread to other countries.

A surprising characteristic of cable graded in this manner came to light when it was "waferized" [AIEE Transactions, volume 57, 1938 (March section), pages 141–54]. It was found that at the junction of two different grades of paper, instead of there being close contact between the papers there was an annular gap (figures 1, 2, 3). The radial thickness of these gaps was far more than was necessary to ensure ionization in an operating cable when the compound drained from the gap (as it does in practice because no cable route is perfectly level). The presence of annular gaps and the obvious deduction that serious ionization must occur in them are supported by the common observation among cable engineers, that when a graded cable, after a period of service or accelerated aging is dissected layer by layer, wax in considerable abundance is frequently found at the junctions of the different types of paper. These annular gaps result chiefly from the fact that most dense papers shrink more longitudinally than lighter papers, so that upon drying a cable of composite insulation wall (prior to impregnation), the inner, more dense paper shrinks away from the outer, less dense paper. Thus the widespread practice of grading, based on theoretical considerations of small variations of SIC, neglected to take into account physical

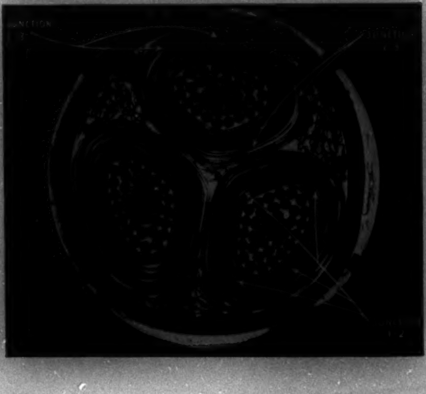


Figure 5. Wafer of 3-conductor 24-kv graded cable showing annular gap resulting from differential shrinkage of three types of paper employed

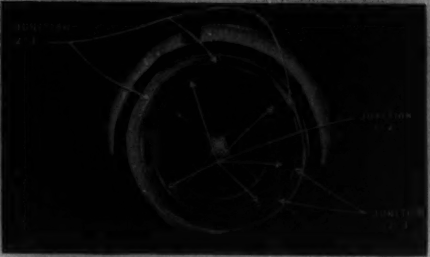


Figure 6. Wafer showing marked development of annular gaps resulting from paper shrinkage in a new cable with thick insulation wall

properties of major importance, and in consequence resulted, instead of in an improved construction, in a cable into which annular gaps or holes had actually been built.

The troubles encountered with grading seem to me to explain why Mr. Del Mar's cable has attained such a high measure of stability. Instead of placing the dense paper next the conductor, he builds his main insulation wall of normal cable paper and outside this he applies dense (high-modulus) paper. Thus when his cable is dried the outer layers of dense paper shrink more than the inner papers, thereby eliminating any gap at the junction of the papers and in fact, probably tightening and supporting the whole insulation wall. What Mr. Del Mar seems to have done, therefore is to follow the practice of a good blacksmith who shrinks the steel tire on to a wooden wheel, not to reverse, with the result that he obtains a tight, sound, and serviceable product.

It should not be inferred from this discussion that all graded cables contain annular hollow spaces within the insulation wall, because the shrinkage properties of paper could be controlled, and it should be possible, therefore, to make a tight construction even though normal grading is employed. There are many difficulties to overcome in practice, however.

In the above considerations differential shrinkage of cable papers, a subject which has occupied my attention for the past four years, is important. This is a broad subject which will be discussed elsewhere.

Mr. Del Mar's construction should result in a cable having unchanged electrical characteristics even after severe bending such as might be encountered under the worst conditions of installation. For example, consider a cable whose insulation wall is made up of two types of paper. If the more easily deformed paper is placed on the outside (where the most severe mechanical strains occur during flexing), when the cable is bent back and forth the outer zone of paper will pull away from the inner zone, and an annular gap will develop at the junction of the two types of paper. Obviously if the stronger paper is placed in the outside, there will be no tendency of the two structures to pull apart after severe bending. Mr. Del Mar's construction places the stronger paper on the outside.

The author's contention that the paper tapes of a normal cable are stretched and given a permanent set when the cable is loaded heavily receives support from figure 4. Here is shown the radial compactness of a new solid cable as measured by a Cenco Compactness Tester. The lower curve shows the greatly increased looseness of the insulation near the conductor after load-cycle aging (85 degrees centigrade copper, 30 load cycles). Thus in normal cable severe loading does mechanically stretch the insulation wall, the effect being more severe when heavy load is thrown suddenly on a cold cable.

Out of the phenomenon exhibited by figure 4 comes an interesting and important deduction. If a void is present within the zone of insulation near the conductor where load-cycle distortion takes place, which in this case would be within 40 per cent of the thickness of the insulation wall or about 25 tapes, the paper tapes adjacent to the void will receive no support when the stretching takes place. In consequence the void will be transferred, after a series of severe load cycles, from the middle of the insulation wall to the vicinity of the conductor. In this position the void is far more dangerous to cable life. While it is difficult to measure *void transfer* specifically it is a matter of common observation, especially in the older loosely constructed cables, that after a period of severe service a gap developed between conductor and insulation as indicated by wafer analysis. The remedy, of course, is to build an insulation wall that is mechanically uniform to start with and to support it externally as Mr. Del Mar has done in his new construction.

The author conceives that the insulation of a solid-type cable is reversible up to some limiting temperature where permanent "set" takes place. This conception is supported by the work of Hochstadter and Barrat (1927).[1] It is also supported by wafer examination of cable aged by load cycling at maximum copper temperatures of 45 degrees centigrade, 55 degrees centigrade, 65 degrees centigrade, and 85 degrees centigrade. No change in mechanical structure could be observed in the 45-degree centigrade and 55-degree centigrade sample whereas slight changes were noticeable in the 65-degree centigrade samples and the 85-degree centigrade samples showed very marked mechanical distortion of the insulation wall. [AIEE TRANSACTIONS, volume 57, 1938 (March section), pages 149–50].

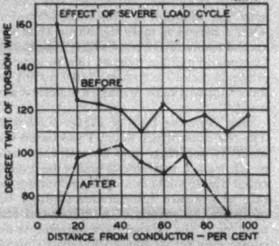


Figure 7

It is my experience that in normal cable of a few years ago, oil and paper expand and contract as one, up to a critical temperature at the conductor of 50 to 60 degrees centigrade ("High Voltage Cable: European and American Practice," minutes, Association of Edison Illuminating Companies, 1936, page 9). This is probably due partly to the elasticity of the paper and partly to the considerable void volume which is present in even the best solid-type cable. (Range of compressible volume at 200 pounds per square inch, probably one-half to three per cent.) It must not be forgotten that very high pressures obtain at the conductor when heavy load is thrown suddenly on cold cable; pressures of 120 pounds per square inch have been measured ("Study of Mechanism of Cable Failure," minutes, Association of Edison Illuminating Companies, 1927) and tapes at the conductor have been reported ruptured as a result.

From the preceding discussion it is evident that high-voltage paper cable insulation requires some external support to prevent the permanent stretching and distorting of the paper layers and the consequent creation of voids of serious radial dimension. This distortion often results from, first, severe bending of the cable during reeling and installation; second, load cycles in excess of some critical temperature, believed to be about 55 or 60 degrees centigrade measured at the conductor; third, the throwing of unusually heavy loads suddenly on a cold cable. In the Titebilt construction the main body of oil-paper insulation is supported by a girdle of high-modulus paper, which serves the double function of mechanical support to the inner layers and of electrical insulation. It would appear, however, that much of the improvement in stability achieved in this manner is due to the elimination of gaps which are prevalent in much "graded" cable, and to the use of the shrinkage properties of the paper to obtain a tight instead of a loose construction by putting the paper with the highest longitudinal shrinkage on the outside instead of on the inside.

REFERENCE

1. EFFECT OF OVERLOAD ON UNDERGROUND CABLES FOR POWER TRANSMISSION, Martin Hochstadter and R. Barrat. *Révue Générale de l'Electricité*, volume 22 (1927), pages 1175–80.

W. A. Del Mar: The main discussion may be divided into two parts, namely:

1. Attempts to explain why the Titebilt principle can't work
2. Attempts to explain why the Titebilt cable does work

In addition to this there were requests for information which I shall endeavor to satisfy before answering these two main subjects of debate.

DATA REQUESTED

Spread of Results in Test Series 1 to 4. The spread of power-factor readings is shown by a typical run of four cables of each of two kinds. In a run of ordinary graded cables, the original power factors had a spread of 0.14 per cent, the average power factor being about 0.9 per cent at 85 degrees centigrade and this spread gradually increased to about 0.24 per cent at the 30th load cycle. Thereafter the spread increased somewhat more rapidly to failure. A typical run of four Titebilt cables showed a spread of original power factors of 0.18, the average power factor being again about 0.9 at 85 degrees centigrade, and gradually increased to 0.32 per cent at 30th load cycle. The subsequent increase was much slower than in the case of the graded cable, until near failure.

Properties of Types S and N Papers

	Type S	Type N
Gurley density, seconds per mil	300	45
SIC impregnated (compound number 7)	3.96	3.60
Dielectric strength, volts per mil	1,800	1,600

Permanent Set. The permanent set of type *S* paper is as follows compared with that of type *N* paper.

Elongation, Per Cent	Per Cent of Permanent Set of Type N
0.9	35
0.8	50
0.7	60
0.6	70
0.5	84

Initial Tightness of Titebilt Taping. Tapes of both types are applied with the same machine tension in all cables reported upon. This tension gives a compactness reading, on a Cenco tester, of about 80 degrees. In this instrument a flat needle is pushed between tapes at a cross section of cable and turned through an angle of 30 degrees by means of a torsion wire. The number of degrees through which the torsion wire must be turned to turn the needle through 30 degrees measures the tightness of taping. The early cables were not taped in an air-conditioned room so that equal machine tensions gave some variability of tightness in the finished cables as shown in figure 9. The later cables, such as that of table I, were taped in an air-conditioned room and show correspondingly uniform tightness in the finished cable.

Negative Ionization in Tests of Table I. The negative ionizations were of the order of half the positive values and occurred when the pressure in the cable was substantially atmospheric.

Pressures, Vacua, and Sheath Expansion in Tests of Table I. Pressures and vacua were measured at the potheads and may not be representative of the cable. The maximum pressure so measured was a little over 100 pounds per square inch and the highest vacuum about 0.6 atmosphere. The lead expansion was measured but the measurements are equally meaningless as the expansion was very little, the measurements of doubtful accuracy by their very nature, and varied along the cable.

Voltage Used for Power-Factor Measurements in Sets 1 to 4. The voltage at which the power-factor measurements were made in connection with the two per cent limit was the AEIC rated voltage of $17{,}000/\sqrt{3}$.

Relative Effects of Titebilt Construction and New Impregnant. The question has been raised, to what extent the stability of the Titebilt cable of table I is due to the Titebilt construction and new impregnant respectively. Tests are under way to estimate this and present indications point to the impregnant as an important element in the long range flatness of the power-factor time curve and the Titebilt construction as the predominating element of its regularity. These tests have now gone over 150 load cycles.

Comparison of Three New Types of Cable. Additional data were requested by Mr. Hickernell to enable him to compare the tests given in AIEE TRANSACTIONS 1940, page 1041 and *Electric Light and Power*, July 1940, page 14, with the Titebilt cable test data. He should have no difficulty in doing so with the data given. The main criterion of stability is the power factor at 60 degrees centigrade which is given at the beginning of the test and once a week thereafter to the end. The power factors at high temperatures are not repeated far enough apart in the standard test to be used for stability evaluation. However, considering the supplemental test, the power factor, at the highest temperature, 115 degrees centigrade, was slightly less after four load cycles at that temperature than it was on the first load cycle due, probably, to a temperature difference that was not observable. The only difficulty Mr. Hickernell will encounter in making a full comparison is the fact that the Titebilt cable was subjected to an additional week of particularly severe load cycles to which the cables described in the publications cited, were not subjected.

Comparative Viscosities of Compounds 7 and 8. From 120 degrees centigrade down to 65 degrees centigrade the viscosities of the two compounds are indistinguishable. Below 65 degrees centigrade they diverge and at 60 degrees centigrade the viscosity of number 8 is about 85 per cent of that of number 7.

Short-Time Dielectric Strength. The plant routine (AEIC) short-time dielectric strength tests have not shown any change since the adoption of Titebilt construction. This, I believe is more significant than any special comparative tests which I might cite. Compound number 8 seems to give some increase in dielectric strength but I do not yet have sufficient data to make a definite statement.

How Were the Data of Figure 1 Derived? The test upon which figure 1 was based was made by applying, from an external source of oil, such a pressure as might be experienced due to voids in a cable and noting the rate of pressure transference through the insulation by means of a differential manometer. The Saybolt viscosimeter is accurate only up to about 700 seconds above which the oil ceases to flow in a stream. Therefore, some other measure of viscosity is required for higher values. Perhaps the Saybolt test might be used, with oil dripping up to 3,500 seconds but the significance of the results would be in doubt. As we were more concerned with the behavior of oil in contact with paper than in contact with metal, we decided to measure the higher viscosities by the time of penetration of oil through paper. The time thus measured was fairly proportional to the Saybolt viscosity over the narrow range that could be measured by both methods. What the relationship may be at higher viscosities we do not know, and, in fact, the question has no meaning. The main point is that the viscosities above 700 seconds were measured by the time of penetration in seconds, multiplying this time by the factor found at viscosities where the methods ran parallel. Theoretically, I could have used the penetration time for the whole scale, and thereby avoided much discussion, but at very low viscosities the time was too short to be measured accurately due both to its dependence on visual observation and the unevenness of the penetration.

The important point about the curve was the existence of a pronounced knee and this was confirmed by tests made on a short length of 500,000-circular-mil cable with 500 mils of insulation sealed at one end and fed with oil from a gravity feed reservoir at the other end. The loss of oil from the reservoir was noted at various oil temperatures, using compound 8 which has about the same viscosity as number 7 from 60 or 65 degrees centigrade up, although less viscous at lower temperatures. These tests showed the same kind of knee at the same viscosity as the earlier tests. The significance of the knee is that when a cable is heated the oil mobility starts low at the bottom right-hand end of the curve exerting great pressure on the inner paper and stretching it. As the knee is passed, the pressure on the paper is released by the oil expanding outwardly by virtue of its increased mobility. The elasticity of the outer paper then comes into play, keeping the inner paper in compression from then on. In cooling this restorative compression continues until the original condition is reached.

Were Comparisons Made Between Round, Sector, and Oval Cables? In reply to Dr. Wiseman's question, no direct comparison has been made between round and sector or oval conductors, as all comparisons have been made between cables of identical geometric design and differing only in the disposition of paper and the kind of impregnant. However, the series 1 to 4 were on round conductors, and a comparative run was made with the sector cable test of table I against an old type cable, i.e., one with ordinary graded insulation and compound 7. It was found that the old type cable started with a 60-degree centigrade power factor of 0.40 per cent, reached 1.37 in four weeks and ended the five weeks test at 1.48 per cent, the intervening values being quite irregular. The test was continued for three weeks more and indicated a very gradual increase which may be compared with the intermediate part of the curve for "ordinary cable" in figure 6. The four weeks rise of 240 per cent (from 0.4 to 1.37) is consistent with the values obtained by Commonwealth Edison Company for standard

12,000-volt cable of 1937 and 1938 vintage, as cited in my paper.

Power-Factor Temperature Data for Cable of Table I. These are as follows:

Temperature, Deg C	Power Factor, Per Cent
20	0.35
40	0.35
60	0.40
70	0.45
80	0.58
100	1.10

Points Raised in Criticism of the Titebilt Principle of Cable

1. *Argument That Load Cycle Tests (Sets 1 to 4) Are Not Indicative of Cable Life.* Mr. Hickernell misquotes me as saying the 1939 Commonwealth Edison Company tests are based on some specific stress. Reference to my paper shows that I said the tests were conducted at "$2^{1}/_{2}$ times operating voltage, corresponding to an average stress of 208 volts per mil," etc. He then goes on to say, in effect, that Halperin and Betzer based their tests on a fixed relation of $2^{1}/_{2}$ to 1 between test voltage and operating voltage regardless of the insulation thickness and therefore had different maximum stresses in different cables. Del Mar, he continues, with a thinner wall of insulation, instead of using the same relation between test and operating voltage, is guided to his test voltage by the maximum stress attained in one out of several Halperin and Betzer cable groups. Why does this particular procedure lead to correlation with the Halperin-Betzer tests? Should not a $2^{1}/_{2}$ ratio have been used, instead?

In my tests, the point at issue was the relative stabilities of cables of equal sizes and insulation thicknesses and in order to tie up with Halperin and Betzer with their invaluable correlation to service conditions, it was necessary to establish a test voltage which would create deterioration at about the same rate, on a smaller cable of lower voltage rating. Deterioration of power factor was found, by our radial power-factor tests, to originate in the regions of calculated maximum stress. It, therefore, seemed most reasonable to adopt a voltage which would give the same calculated maximum stress.

At the time our tests were started, which was long before the publication of the 1936 Halperin and Betzer paper, the only Commonwealth Edison accelerated aging data available were on cables with 750 mils of insulation and the highest maximum stress used which gave satisfactory results was 208 volts per mil. We found to our gratification that our ordinary type cables behaved at 65 degrees centigrade about the same as the best cables shown in Halperin and Betzer's figure 1 for the same temperature and that even at 85 degrees centigrade, the temperature we adopted for our tests, their life was still up to the four-week period regarded as necessary to eliminate questionable cables.

I do not see how we could have used better judgment in formulating our tests. If we had followed Mr. Hickernell's suggestion to use $2^{1}/_{2}$ times the working voltage, the test voltage would have been 24,500 instead of 32,500 and each test, instead of lasting a minimum of 1 month would have lasted a minimum of $5^{1}/_{2}$ months, if the sixth power law is any guide. Such a test would have borne no practical relation to the Halperin and Betzer data.

2. *Argument That There Is Nothing New About Titebilt Cable and That It Has Been Tried Before With Unsatisfactory Results.* Mr. Hickernell gives an interesting list of patents on grading insulation and the impression given is that they have all failed to achieve their purpose and that the idea embodied in Titebilt cable was anticipated by both O'Gorman and T. R. Scott. I took the trouble to look up O'Gorman and found that he mentioned reverse grading only to say that it may occur accidentally due to filtrations of impregnant by cable paper and is to be condemned as being the opposite of his proposed grading system.

T. R. Scott, on the other hand, did make an inverse graded cable that showed promise in the early stages of its development but which ultimately failed due to emphasis being placed on differential oil absorption rather than elastic properties of the paper.

Scott's tests, quoted by Messrs. Hickernell and Wiseman gave the following results.

	Ordinary Grading	Inverse Grading
Max. stress in load cycles calculated on assumption of uniform paper, volts per mil	176	176
Average stress	92	92
No. of cycles to failure—		
Sample 1	36	94
Sample 2	109	96
Power factors at room temperature	Erratic	Consistent
Appearance of insulation after test	Good	Poor

Scott was not seeking power-factor stability, as shown by the absence of power-factor data at operating temperatures. It is of interest, however, to note that his room temperature power factors were very erratic in his graded cable whereas his inverse graded cable except for one point, was quite uniform. One of his graded cables failed at 36 cycles and the other at 109, another sign of nonuniformity. His inverse graded cable was more uniform, failures occurring at 94 and 96 load cycles. Scott's inverse grading was based on the oil capacity of his papers and he paid no particular attention to their mechanical characteristics. Due, however, to a tendency for both oil absorptivity and elastic properties to be related to the degree of hydration of the cellulose, he obtained some slight measure of the characteristic "Titebilt" uniformity, in his inverse graded cable.

Mr. Hickernell seems to have missed the difference between my objective and that of the earlier inventors of graded cable. My aim was power-factor stability; theirs, increased dielectric strength. Whether they attained their objective is not germane to this discussion. The test data presented showed that I attained mine.

3. *If Titebilt Cable Operates on the Principle Claimed, the Action Will Not Be Permanent.* Several speakers, particularly Mr. Atkinson, have suggested that type *S* paper may suffer fatigue in the course of years and lose its modulus and low permanent set, and so the Titebilt action would cease.

It is possible that type *S* paper may suffer fatigue in the course of years, but if so, the much more loosely knit type *N* paper will be even more fatigued and stretched, so that the type *S* paper will not only continue to exert its beneficial influence, but will do so to an increasing extent. That is to say, while under these assumed conditions, the elastic effect might be less than in a new Titebilt cable, it would be much greater than in an aged cable made either with one kind of paper or of two kinds applied as in ordinary grading. For, if the type *S* paper were inside the type *N* paper, the greater fatigue of the type *N* paper would cause it to stretch away from the more stable type *S* paper under it leaving a void space between the two types of paper. That this seems to occur, is shown by the wafers in Mr. Wyatt's discussion.

However, if such creep were to be anticipated, as an important factor, it should be greatest at the start and then fall off, as the paper would be subjected to less stress as it acquired more and more permanent set. Load-cycle tests lasting over a period of six months and including 54 load cycles at 85 degrees centigrade and 62 at 100 degrees centigrade show that no creep occurs even at such high temperatures. Figure 9 shows Cenco tape tightness diagrams for graded and Titebilt cables, otherwise identical in construction, after subjection to the above test at twice operating voltage. It will be noted that the graded cable suffers a loss in tightness throughout whereas in the Titebilt cable the inner layers remain substantially unchanged in tightness, only a few outer layers having suffered. After all, it is the ratio of the moduli that counts, not their absolute values.

Instability of Titebilt Cable Thought to Be Indicated by Imaginary Continuation of the Curve of Figure 6. The imaginary endings *A* and *B* given by Mr. Hickernell to my curve, in his figure 2, have no significance because my curve represents the average of four curves which run together very smoothly for about 50 or 55 load cycles and then diverge, crossing the two per cent line at different points. It is therefore impossible to make a significant average curve above 50 or 55 load cycles and for that reason I stopped the curve at that point. As, however, all the Titebilt cables passed at least 67 load cycles, with an average of 69 before reaching the power-factor limit of two per cent and the graded cables averaged only 30 load cycles, I fail to follow Mr. Hickernell's reasoning that this shows instability of the Titebilt cable.

It Is Questioned How the Permanent Set Can Be Enough to Leave Ionizable Voids and Yet the Stretch, Which Is Several Times as Great, Be Insufficient to Permit Free Flow of Oil. I think that in asking this question Mr. Davis was visualizing a single microscopic tape space in a cable that had expanded with perfect geometric uniformity. That is not what happens, as may be seen by an inspection of cable wafers. Expansion and contraction are no respecters of geometry. A group of tapes will be forced tightly together and their residual gaps, created by permanent set, will be pooled in one layer in which ionization will have full play. Next load cycle this will occur in another place. It is the group closing that virtually shuts off the oil flow and the group openings that permit ionization.

Calculations of Mechanical Stresses in Cables Do Not Support Titebilt Idea. It is claimed by Mr. Greenfield that calculations of mechanical stresses due to oil expansion do not support the Titebilt theory. His own discussion, however, shows how unreliable are such calculations. Moreover, he forgot to include the very important item of residual gas which is by no means negligible in the best of solid-type cables and affords a very considerable cushioning for the expanding oil. My own calculations served as guides for the laboratory work but I lack confidence in their accuracy as representative of the actual complicated conditions in a cable.

All Type S Versus Titebilt. It was suggested that, according to the theory of elastic moduli a cable made of all type *S* paper might be better than one made of two types.

The fact that this is not so is proved by such tests as those described below under "The Oil-Stress Theory." The reason is that the modulus effect cannot come into play unless there is, initially, close and uniform contact of inner layers. This condition cannot be attained with hard, springy, type *S* paper but is ensured by the use of a soft inner paper with low drying shrinkage surrounded by a paper of much higher shrinkage. This ensures that the greater modulus of the outer paper will come into play at an early stage of the oil expansion.

Oil Flow Versus Paper Stretch. The theory upon which the Titebilt cable was designed is not, as several discussers seem to assume that the oil is completely "frozen" in the cable and that all the movement that occurs in load cycles is paper stretching; but that the *deteriorating occurrence* in an ordinary cable is the stretching of the paper and not the outward flow of the oil as hitherto assumed and still maintained by several of the speakers.

Mr. Atkinson made a test to prove that some oil is lost from the insulation when expansion occurs but it was not necessary to make special tests to prove this as we all know it by observation, as pointed out by Mr. Halperin.

The loss, however, is not very great in practice as we find in comparing a new cable with one which has gone through months of severe load cycles. In such cases there is little difference in the amounts of oil between insulation and sheath before and after test.

I am not so much concerned with oil lost by the outer layers of insulation as to make sure that the inner layers are kept free of voids. The oil will contract into its original space, as mercury contracts into a thermometer bulb, if the original space remains unchanged. If, however, the space has been increased by stretching of the paper, the oil cannot fill it by contraction but must be moved in bodily with no force to propel it but the less than one atmosphere due to the partial vacuum formed.

Referring to Mr. Halperin's comparative viscosities, my data indicate that oil-filled cable oil at 30 degrees centigrade has the same viscosity as compound 7 at 120 degrees centigrade, not 80 degrees centigrade.

Mr. Halperin's tests of a Titebilt cable included putting it through three 7-hour heating period load cycles, and one 15-hour heating period load cycle all at a copper temperature of 115 degrees centigrade, and he found the power factor to remain unchanged. As this temperature was near that of impregnation, he concludes that the stability shown by his tests could not have been due to the oil staying in place. True, the oil does not stay in place after it has passed the bend of the viscosity curve and has stretched the paper sufficiently to enable it to flow at its lowered viscosity. My point, is that when the cable cools, the oil will return, provided that the inner layers have not stretched permanently. If they have acquired a permanent set, voids will form and ionization, with its attendant deterioration, will occur. In the Titebilt cable, we have provided means to keep our thermometer bulb, so to speak, to its original dimensions.

That the inner oil in a Titebilt cable contracts into its original habitat after load cycles up to 115 degrees centigrade is shown by the following observation made by Mr. Halperin himself and noted in his report on a test duplicating that of my table I.

> "The impregnation of the fillers and of the conductor insulation was good. There was no sign of deterioration."

If the oil had not stayed in place, would the impregnation have been so good?

EXPLANATIONS SUGGESTED FOR STABILITY OF TITEBILT CABLE

1. *The Oil-Stress Theory.* Messrs. Greenfield, Peterson, Whitehead, and Wiseman suggest that Titebilt cables may owe their quality to the fact that the internal (type *N*) papers are of lower SIC and therefore throw less stress on the oil in the regions of maximum stress. The stability of Titebilt cable is, no doubt, enhanced, due to this feature but if it were the basic explanation of their stability, cables made entirely of type *N* paper should show the best stability and those made entirely of type *S* paper, the least stability. Such is not the case, these two extreme cases showing no difference, whereas the greatest difference is between the ordinary graded and Titebilt which have much less difference in oil stresses. Typical test data are as follows:

Paper	Calculated Max. Stress in Oil	Load Cycles to Reach Two Per Cent (Average of 4)
All type *N*	383	43
All type *S*	348	42
Titebilt	355	69
Ordinary graded	360	30

These figures are based on the following data:

Construction of cable as in sets 1 to 4

Test voltage—32,500 volts

SIC values—type *S* paper—3.96; type *N* paper—3.60 (both impregnated); oil, 2.10

Stresses are figured without increase due to stranding

2. *Shrinkage in Drying.* Mr. Wyatt suggests that Titebilt cable owes its stability in some measure to being built on the principle of a car wheel having a cast-iron core with a heat expanded and shrunk-on steel tire. Type *S* shrinks 20 per cent more than type *N* paper in drying and this shrinkage occurs before impregnation.

I believe that this is a contributory factor in the operation of Titebilt cable, as it is found from inspection of styrene wafers that the strands of such cable are forced tightly together and never show the mild birdgaging, frequently found in other cables. However, the effect of drying a cable is to loosen it and subsequent impregnation does not restore the tightness. The effect of the Titebilt construction is, therefore, not so much to tighten the insulation as to reduce such initial loosening as would normally occur.

Incidentally, the Titebilt principle could not come into full play in a cable having taping applied with irregular stagger and erratic tensions. It is a child of modern taping practice.

3. Titebilt cable owes its stability to the softness of the inner paper which permits it to follow the contour of the conductor more closely than the hard paper ordinarily used at the conductor surface. This is perhaps a contributory factor, but the inferiority of a cable made with all type *N* shows that it is not vital.

4. Titebilt cable owes its stability to the comparatively low oil absorption of its outer paper, thus causing a lessening of the elastic collapse of the outer layers and so keeping the oil passages comparatively open, thereby preventing voids due to stoppage of oil flow.

This is the principle of the Scott cable mentioned in the discussion and may be a contributory factor in the stability of Titebilt cable, but if it were an essential one, a cable made entirely of type *S* paper should be even better, as all the oil passages would suffer less collapse. Tests, however, show that a cable made of all type *S* paper is far inferior in stability to a Titebilt cable.

5. Titebilt cable owes its stability to

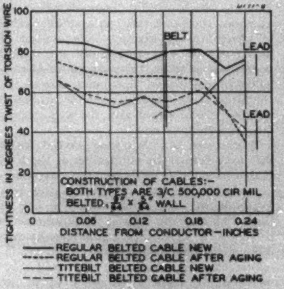


Figure 8. Curves showing radial variation in compactness across insulation wall of cables

Measured across sector base

These cables were aged 54 cycles at 85-degrees-centigrade conductor temperature followed by 62 cycles at 100 degrees centigrade

The rather irregular initial tightness is due to the fact that these cables were made several years ago, before the taping was performed in an air-conditioned atmosphere low permanent set of the outer paper rather than to its high modulus.

The permanent set of type *S* paper is about the same as that of type *N* paper for low elongations, but whereas the permanent set of type *N* increases faster than the elongation, that of type *S* increases less rapidly and, in fact, becomes substantially constant until near failure. This is, undoubtedly a contributory factor in the operation of Titebilt cable but if it were the main factor, an all type *S* cable should be superior, which is not the case.

6. Titebilt cable owes its stability, not to closing of voids, but to pressure put on voids by the outer papers. This suggestion by Mr. Davis seems to imply that the oil in the interior of a Titebilt cable is under sufficient pressure from the belt of relatively impervious paper of higher modulus, as to press into the voids and convert them into regions of high pressure.

If this were an important factor, it should be evident in greater oil pressures at the terminals especially in the vacuum period of the load cycle. We have never seen any evidence of this. However, there may be some local action of this kind which would tend to promote stability.

7. Titebilt cable owes its stability to outer paper preventing stretch of inner paper by virtue of its higher modulus, rather than restoring it to its original condition.

It would be difficult, if not impossible, to differentiate between these two effects. Probably, the higher modulus of the outer paper acts both to restrain stretching of the inner paper and to restore the paper after whatever stretching may occur.

8. Compound 8 owes its stability not so much to its ability to withstand electronic discharges as to either its lower viscosity or that it "fitted the particular paper porosity and cable structure better, irrespective of its resistance to bombardment."

The viscosity of number 8 is somewhat lower than that of number 7 below 60 degrees centigrade, but from there up, there is substantially no difference. Tests were made up to 115 degrees centigrade.

As for compound 8 fitting a particular paper, we find that its superiority holds with papers of such widely different characteristics as type *N* and type *S* used either separately or in combination.

9. There is one factor which is yet under study and which has not yet emerged clearly from a mass of seemingly contradictory data, so that we are not prepared to report it. It ties up with the interesting facts that high-voltage test failures of Titebilt cable frequently originate at the shield rather than at the conductor, as almost invariably happens in graded cables and that in spite of this, the breakdown voltage of Titebilt cable tests, is indistinguishable from that of graded cable.

In conclusion I cannot refrain from noting that there was so little discussion about the new impregnant and of the hitherto unknown relation found between the vacuum cell test of oil stability and the power-factor stability of cable. This surprised me in view of the great interest shown by utility engineers in the stability of cable oils, the subject of an important joint research project which they have sponsored. This research, however, has put its emphasis on oxidation rather than electrical deterioration.

The work on Titebilt cable was performed under the direction of Mr. J. H. Palmer, the details of the load-cycle tests carried out by Mr. Laird McKean, and the work on oil stability by Mr. E. J. Merrell, to all of whom my thanks are due. I also wish to thank the several discussers for their keen analyses of my paper.

Impulse and 60-Cycle Characteristics of Driven Grounds

Discussion and author's closure of paper 41-5 by P. L. Bellaschi, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (March section) pages 123–8.

M. C. Westrate (The Commonwealth and Southern Corporation, Jackson, Mich.): This paper is a contribution on a subject of great interest to those concerned with lightning protection. The inclusion of resistance values measured with a Megger is a valuable addition because of its practical aspect.

Whether the grounds tested are typical is open to question, as it is believed that most companies try to obtain a lower resistance than that obtained with the rods tested. Experience indicates that a resistance of ten ohms or less is desirable for tower footings. The length of the rod of course varies with soil conditions and rods well over 100 feet in length are not uncommon in some areas. Similar tests on longer rods than the ones reported in this paper would also be very interesting.

The data presented by Mr. Bellaschi and plotted in figure 4 of the paper show a decrease in ratio of impulse to 60-cycle resistance with increased current. This agrees with some experience with actual measurements of voltage and current on lightning strokes having much higher current than used in the tests and which showed still lower ratios.

J. Thos. Howard (Tennessee Valley Authority, Chattanooga, Tenn.): Mr. Bellaschi has presented some very interesting data on the resistance characteristics of individual electrodes and a small group of driven grounds with integral values of resistance. It would be of great value to the subject of grounding if the same data were obtained on extensive grounding systems having fractional values of 60-cycle resistance (less than one ohm).

In tables I and III Mr. Bellaschi presents the results of tests at 60 cycles by the voltmeter-ammeter method and by the Megger method. It is to be noted that, in table I, the resistance of both single rods and groups of rods show, with two exceptions, a decrease in the resistance value for increased applied potentials. Table III shows an exceptionally good correlation between measurements obtained by the voltmeter-ammeter method and the Megger method.

During the past several years, the writer has secured data on extensive generating plant and substation grounding systems by the fall-of-potential method, a variation of the voltmeter-ammeter method. Data have been obtained on 11 grounding systems, of which only one showed an integral (more than one ohm) value of resistance. Independent data on the majority of these same grounds have been obtained by the Megger, with this departure from the results shown in table III: that no close correlation exists between the two sets of data, deviations up to 600 per cent being found. In fact, the only case showing any close degree of correlation was noted for the grounding system with the integral value of resistance.

Some of the data have been checked at different values of energizing current into the grounding system. The results bear out the data shown in table I, with one exception. When underground metallic structures connected to the grounding system being investigated were in proximity to the electrodes to which the value of the grounding system was being referred, the resistance value showed an increase for a decreased value of energizing current.

Mr. Bellaschi attributes the decrease in resistance with increase in current to the fact that the increased voltage required to flow more current into a ground electrode breaks down the high-resistance contacts between the more conducting particles of soil. This has not been the case for the extensive grounding systems investigated. These investigations have been made with a potential in the order of either 2,300 or 6,900 volts applied at one end of a transmission line connecting the grounding systems under test, with the other end of the line solidly grounded. The data on the grounding systems at each end of the line have been checked by reversing the source of potential and have been found to check within all practical limits, or approximately ten per cent. From these results it appears that, for extensive grounding systems, at least, the criterion is not the applied voltage but the voltage gradient that individual components of the grounding system can develop.

J. H. Hagenguth (General Electric Company, Pittsfield, Mass.): The paper gives another proof that the impulse resistance of driven grounds tends to be appreciably lower than the resistance measured by means of Megger and 60-cycle methods. It is shown that in the particular locality tested a very definite trend is indicated in that a decrease of resistance results from an increase in current. However, in the author's tests as well as the other tests mentioned,[1] current waves with relatively slow fronts were used. For instance, from author's figure 3, oscillogram *AX*, the maximum ratio of change of current is approximately 700 amperes per microsecond. I do not have any data on driven grounds with higher rate of current rise, but I believe that the author is somewhat optimistic when he states "that the curve (of figure 4) applies for impulses over a wider range of waves as encountered in the field" and "therefore, the range of currents in figure 4 covers field conditions from a traveling surge to a direct stroke of lightning." It seems to me that in a field where so few test data are available and continuity of service depends so much on low ground resistances, a more pessimistic outlook should prevail. Rates of rise of lightning currents have been measured up to 30,000 amperes per microsecond and lightning current amplitudes up to 200,000 amperes on transmission lines[2] with 130,000 amperes through a single tower, and 25,000 amperes[3] through arresters on distribution circuits. Field experience shows that even higher lightning currents occur on distribution circuits.

For these reasons, it is of greatest importance to know the impedance of grounds for the steep fronts and high rates of current rise actually encountered in the field. Results of tests made on counterpoises[4,5,6] will supply a partial answer to this problem. These tests show:

1. (For a rectangular current wave) that a buried wire has a surge impedance of zero, rises to a high value of the order of 100 to 200 ohms in a fraction of a microsecond after the current wave reaches the counterpoise, and then decreases below the value of resistance measured by alternating voltage means at a rate determined principally by the length of the counterpoise.
2. As the rate of current rise is decreased, the maximum impedance of the counterpoise is decreased.

These results were obtained with currents ranging between 150 and 1,100 amperes amplitude and 750 to 2,500 amperes per microsecond. An essential change in surge impedance variations could not be detected within this range of current variations.

There should exist a marked similarity between a driven ground and a counterpoise, at least for the first half microsecond after the wave reaches the ground rod. Both initially constitute a network of capacitances and inductances and the resistance components come into play only gradually. Therefore, the impulse impedance of a ground rod or system of rods should initially reach a high value to be reduced to the a-c resistance or less. Now, for one short rod, this transition period should, of course, be short, of the order of 0.1 microsecond. However, as the length of the rods increases, and, furthermore, if several rods are connected in parallel by a network of wires of considerable extent, the decrease in impedance will occur at a much slower rate. While this short-time high-impedance value probably is of little importance as far as the sparkover of insulators or other line insulation of high-voltage lines is concerned, it is of considerable importance for the low-voltage lines and distribution circuits. High-voltage insulation requires relatively higher voltages to sparkover at short times than low-voltage insulation (for instance, ratio of sparkover at 0.3 microsecond to critical sparkover for the ten-inch gap is 3.3; for a one-inch gap is 1.64). The approximate spark-over voltages at 0.1 microsecond for insulation for the 1.2, 5, and 8.66 voltage class are 70, 180, and 300 kv respectively. On the basis of a 10,000 amperes impulse, 10,000 amperes per microsecond, surge impedance of 60 ohms, the voltage across the ground system would be 600,000 volts, although the long front resistance of the ground may be of the order of ten ohms or less. Therefore, the insulation spark-over voltages of the lower voltage classes would be definitely exceeded. Tests, made by Baatz,[7] on ground rods of 100 feet length show surge impedance values of 100 to 170 ohms at 0.2 microsecond. This paper also shows the reduction of impedance as the rate of current rise is reduced.

The lowering of ground resistance for impulse currents should, therefore, at the best be considered as an additional safety factor and should not be depended upon until the whole volt-time range of ground rods has been investigated to a much greater extent.

REFERENCES

1. References 7, 8, 9 of paper.

2. LIGHTNING SURGES ON TRANSMISSION LINES—NATURAL LIGHTNING, W. W. **Lewis** and C. M. **Foust**. *General Electric Review*, November 1936, pages 543–55.

3. DISCHARGE CURRENTS IN DISTRIBUTION ARRESTERS, II, **K. B. McEachron** and **W. A. McMorris**. AIEE TRANSACTIONS, volume 57, 1938 (June section), pages 307–12.

4. THEORY AND TESTS OF THE COUNTERPOISE, AIEE TRANSACTIONS, volume 53, 1934 (August section), pages 1163–72.

5. Discussion of Reference 4, **J. H. Hagenguth**. AIEE TRANSACTIONS, volume 54, 1935 (February section), pages 220–3.

6. SURGE CHARACTERISTICS OF A BURIED BARE WIRE, **E. D. Sunde**. AIEE TRANSACTIONS, volume 59, 1940, pages 987–91.

7. UBER DEN WIRKSAMEN WIDERSTAND VON ERDERN BEI STOSSBEANSPRUCHUNG, **H. Baatz**. *Elektrotechnische Zeitschrift*, volume 59, number 47, pages 1263–7.

H. M. Towne (General Electric Company, Pittsfield, Mass.): The test results reported in Mr. Bellaschi's paper are a valuable addition to previous work on the behavior of ground resistance to impulse currents. The author's curve, figure 4, showing the ratio of impulse resistance to 60-cycle resistance attending various impulse currents, may be misleading if taken to represent the impulse performance of driven grounds in general. It must be recognized that the range of ground resistance studied with nonoscillatory impulse wave was narrow, from about 22 to 45 ohms at 60-cycle measurement, and hence, while the test points permit one general curve (figure 4) for this range of ground resistance, widely different ratios of impulse resistance to 60-cycle resistance could be expected for the same impulse currents with grounds of say 5 ohms or 100 ohms.

It is interesting that Mr. Bellaschi concurs in the opinion which the writer expressed in 1928, "that the decrease in resistance is probably due to high-resistance contacts between the more conducting particles of soil at, or near, the surface of the electrode, which are bridged over by arcs or sparks when sufficient voltage is applied." If we pursue this theory, then the decrease in ground resistance to impulse current is more basically a function of the soil structure and homogeneity of its conducting elements, and of the actual voltage gradient on the soil, than of the impulse current applied. In other words, it is voltage alone that can cause the arcs or sparks and bridging between conducting particles, and hence for a given character of soil it is expected that a definite applied impulse voltage to the soil close to the ground electrode will produce about the same per cent decrease in resistance of a low-resistance or a high-resistance ground. Hence, current would be significant only in so far as required to produce voltage gradient across the soil.

This can be readily seen from plotting Mr. Bellaschi's data to impulse volt-ampere curves and comparing these to straight-line volt-ampere curves based on the 60-cycle resistance. The writer did this for the author's table IV impulse values of grounds "B and D," "D alone," and "B alone" which had 60-cycle resistances of about 22 ohms, 38 ohms, and 45 ohms respectively. Then taking a given voltage applied, such as 80 kv, the resistance decrease of the 22-ohm ground "BD" was to 0.79, of 38-ohm ground "D alone" was to 0.78, and of 45-ohm ground "B alone" was to 0.80. Thus the decrease of resistance to impulses is practically identical for a given applied voltage to either the 22-, 38-, or 45-ohm grounds.

Using the same volt-ampere curves, and taking a given impulse current through the grounds—such as 3,000 amperes, the reduction ratio for the 22-, 38-, and 45-ohm grounds are 0.84, 0.76, and 0.71 respectively, and are therefore not a direct function of current as might be construed from the author's figure 4.

From this and earlier work, it is believed that the reducing ratio of impulse resistance to 60-cycle resistance of grounds will vary widely, depending on the soil structure and the voltage applied to the ground.

It appears that for the same range of impulse currents used by Mr. Bellaschi, a five-ohm ground would show a resistance reduction to impulses of probably not more than ten per cent (ratio 0.90), whereas a 200-ohm ground might show a resistance reduction to impulses of 70 per cent (ratio 0.30).

Obviously, in conservative grounding practice the indeterminate decrease of resistance to impulse currents should not be depended upon, and grounds should be made sufficiently reliable as determined by the ordinary resistance measurements.

Herbert B. Dwight (Massachusetts Institute of Technology, Cambridge, Mass.): Since ground rods are installed to a large extent with regard to their operation in the case of lightning or other transients, direct measurements of their resistance to transient currents is undoubtedly of much practical value.

Low-frequency measurements of resistance, complete with all dimensions of the ground rods, are not often published and the measurements in this paper offer an opportunity to check up some of the ground rod formulas, as published, among other places, by the writer in reference 3 (AIEE TRANSACTIONS, volume 55, 1936, page 1325).

If the resistance to ground of rod A alone, given in table III, column 1, is substituted in equation 13, reference 3, the resistivity of the ground is $\rho = 14{,}000$ ohms per centimeter cube. From rod B alone, $\rho = 13{,}200$. Substituting the measured resistance to ground of A and B in parallel, 28.0 ohms, in equation 21, reference 3, the value obtained for ρ is 13,300 ohms, which is three per cent less than the average value 13,600 from the two previous measurements. Conversely, using the average value of ρ, 13,600, from the single-rod measurements, the calculated value for A and B in parallel is 28.8 ohms.

Rods C and D have practically the same resistance singly as when in parallel. This fact does not detract from the value of the low-frequency and transient measurements made on them when they are in parallel. Using their measured parallel resistance to ground in equation 21, reference 3, the value obtained for ρ is 15,300 ohms per centimeter cube.

The average of the measured values 13,300 and 15,300 for ground resistivity is 14,300 ohms per centimeter cube. Using this value of ρ with equations 15, 16, 17, and 18 in the manner described for groups of ground rods in the paragraphs following equation 20a, reference 3, the resistance to ground of the group of four rods is found to be 19.6 ohms which is ten per cent greater than the measured value 17.8 ohms given in table III.

In view of the variations in ground resistivity which are known to exist over short distances in almost any location, the above comparisons between calculated and measured values can be considered to be good checks. They are in substantial agreement with those given by Mr. Bellaschi.

If there is a large number of rods in a group, the loss in conductivity per rod given by the formulas and curves of reference 3 should be allowed for in designing an installation of ground rods. Unless wide spacings between the rods are used, the loss in low-frequency conductivity per rod may be surprisingly great.

When a comparatively large number of ground rods are installed, such as 25 or more, a small 60-cycle resistance is obtained by using wide spacing such as 25 feet between the rods, making the extent of the group 100 feet or more. This large distance may decrease the value of the protection as regards transient currents. The distance is particularly large if a single row of ground rods is used instead of a group. At some time in the future, an investigation of the appropriate spacing in large groups of ground rods, made with an impulse generator, showing comparisons between the results with wide spacings and close spacings, should be of value.

P. L. Bellaschi: First I want to thank all who have discussed the paper for their contributions and suggestions. May I state again that one of the objectives of the paper is to fill in the present existing gap in our fundamental knowledge on grounds. Another aim is to stir up interest and develop concerted activity in this important field of work.

In reply to Mr. Westrate, we have made tests on a $^5/_8$-inch steel rod driven 29 feet into the ground (largely clay). The Megger resistance is 12 ohms. At an impulse current of 15,500 amperes, the ratio of impulse to 60-cycle resistance is 0.50. Judging from field observations and discussions, the individual driven rods tested are rather typical, particularly in distribution practice. Should a ten-ohm ground installation be required however, and provided this can be justified economically, the individual rods naturally would be combined in multiple or driven deeper or treated. Mr. Westrate mentions field experience with higher lightning currents than used in the tests. These showed still lower ratios. This is expected. For instance, from a consideration of the gradients developed in the soil, the curve in figure 4 extrapolated would indicate a ratio of 0.30 at 50,000 amperes. A major breakdown in the earth may occur before this current is reached, resulting in an even lower ratio. From tests made, the same rod driven in sand would have a ratio of 0.20 to 10,000 amperes.

The good agreement between the voltmeter-ammeter and the Megger methods in table III was more or less anticipated. Considering the principles involved and the problem, the voltmeter-ammeter method or the equivalent method Mr. Howard refers to, is preferred and in fact, necessary in the case of extensive systems having fractional values of 60-cycle resistance. Potentials of 2,300 and 6,900 volts, particularly when applied to extensive grounding systems, are not likely to develop gradient stresses in the soil sufficient to cause breakdown. On the other hand, potentials in the order of 20 kv and less (table IV) when applied to a single driven ground develop gradients sufficient to cause sparking between particles in the soil, with the result that a lowering of the resistance of the driven ground takes place.

Mr. Howard and Professor Dwight suggest impulse tests on extensive grounding systems and on large groups of ground rods. Such systems essentially are networks of individual grounds connected together. In such a case, the effect of leads is an important factor. I fully recognize the value of extending the work reported in the paper.

As Mr. Towne states in his discussion, the gradient stress is a determining factor. In fact, I have shown in the presentation of the paper, that on the basis of a critical gradient (approximately one to two kilovolts per centimeter), the curve form of figure 4 can be determined by calculation with a good degree of approximation. The determining factors can be stated as the product of the current density in the soil at or near the electrode and the resistivity of the soil. Obviously then, the characteristics in figure 4 will vary with the different electrodes and the various kinds of soil. This point is amply emphasized and developed in the paper. Yet from the practical end, it is convenient to report data in such form as given in figure 4.

The paper attacks the problem from the research approach. Until such time as the impulse characteristics of grounds are more comprehensively established, I agree with Mr. Towne and with Mr. Hagenguth that conservative grounding practices should be advocated.

As to the extent lightning currents are simulated in the tests reported in the paper, I believe Mr. Hagenguth takes a somewhat pessimistic view. It would take this discussion too far afield to review the characteristics of lightning currents, established from the various field investigations to this day. It will suffice to mention that direct measurements (cathode-ray oscillograms) of lightning strokes reported to the CIGRE in 1937 by Messrs. Stekolnikov and Valeev and similar records by Mr. McEachron (*Electric World*, February 10, 1940, page 58) indicate fronts ranging from two to ten microseconds with an average of six microseconds for the eight records. The crest values range from 14,000 to 31,000 amperes. Similar front values were obtained by Professor Norinder. The maximum lightning stroke current in a single tower reported so far is at 135,000 amperes. Considering that the current of the stroke frequently reached ground through several paths—in the case of a tower through the four legs—it is apparent that the tests in figure 4 cover field conditions from a traveling wave to a direct stroke of lightning. There is no sweeping statement made that the more rare but possible severe conditions are necessarily approached. This is a matter for further future studies.

Theoretically it would be desirable to investigate the characteristics of grounds with a rectangular wave front. In most cases, the time constant due to the connecting leads and other inductive effects is such that from an impulse standpoint, it is hardly necessary, even theoretically, to go to such extremes. I appreciate the value of a broad approach in this work. Yet at the same time practical and fundamental considerations indicate that a very fruitful and practical field remains to be explored first with the impulse currents employed in the tests. In some of the tests, the analysis of the oscillograms was extended down to two microseconds.

The charts and formulas Professor Dwight has presented in his analytical treatment on grounds are helpful for estimating purposes, particularly in determining the reduction in resistance expected for the various combinations of the grounds in parallel. However, when it comes to calculating generally the resistance of grounds directly from formulas, I am inclined to counsel caution at this stage, for the reason that our understanding of the resistivity of soils, the factors which affect the resistivity, and the various heterogeneity of conditions in the soil, etc., are still matters that lie at the threshold of a more complete knowledge on grounds.

Minimum Insulation Level for Lightning Protection of Medium-Voltage Lines

Discussion and author's closure of paper 41-14 by H. N. Ekvall, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (March section) pages 128–32.

Eric T. B. Gross (College of the City of New York, N. Y.): There is no doubt that the work in renovating overhead lines has brought about considerable success. After improving the lines over a period of six years, the number of troubles has been reduced by 75 per cent and 62 per cent for the 13-kv and 33-kv systems respectively. It is always good practice to take precautionary measures against any possible trouble that may arise. Dr. Waldorf has very clearly pointed out[1] that there is a fundamental difference between (1) preventive lightning protection and (2) remedial measures, designed to minimize the effect of power arcs following flashovers. As engineers, we should never forget to consider both aspects of the problem and apply all possible means for improvement. I certainly recommend the application of preventive lightning protection, and the work described by Mr. Ekvall belongs in this group. However, it seems to me that the application of resonant neutral grounding (earth-fault coils) would have, in this case, brought at least the same, and very likely a considerably higher, percentage in the reduction of troubles. The reduction in troubles would have come immediately after the installation of earth-fault coils. The cost of these coils would have been comparatively small since the coils must carry only about 10 amperes and 90 amperes for the 13-kv and 33-kv networks respectively. This opinion is based on 15 years practical experience (in Continental Europe and England) in the development and application of resonant grounding.

I have good reason to believe that most of the three wire systems in this country (both overhead line and underground cable systems) with voltages up to 66,000 volts, can be operated with resonant grounding. The only restrictions are that the transformers are sufficiently insulated and that autotransformers are not used for connection to a system with a considerably different voltage. Other apparent difficulties, for example unbalanced charging currents to ground, can in most cases be easily overcome, and also the ground-fault relay problem has its simple solutions.

I want to stress that both preventive and remedial measures should be used, but it is sometimes advantageous to begin with the remedial measures, as they may be more economical and are much quicker to achieve. After all, we are trying to reduce the number of outages and to improve the security of the supply of electric power.

REFERENCE

1. EXPERIENCE WITH PREVENTIVE LIGHTNING PROTECTION ON TRANSMISSION LINES, S. K. Waldorf. AIEE TRANSACTIONS, volume 60, 1941 (June section), pages 249–54.

E. W. Beck (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): 1. The workmanlike survey carried out by Mr. Ekvall and the careful application of the data found have resulted in much improved performance of his lines. This illustrates the practical benefits to be secured from the combination of experience and theory, and the strengthening of the weaker insulations in a system. This paper should be a valuable guide in the design of new lines or the rehabilitation of old ones.

2. In general terms it might be said that good results have been achieved by raising the insulation level of the 33-kv lines to the equivalent of a normal 110-kv circuit.

3. This increase in insulation and removal of low flashover points puts more severe duty on the protective devices in the stations. It is of interest to note that Mr. Ekvall has found no increase in lightning troubles in the stations in spite of this.

4. Additional data on the locations and causes of those interruptions that still occur would be of interest. Perhaps application of protective devices at vulnerable points would improve the performance still more. For instance, the use of protector tubes on pole-top switches has avoided flashovers and interruptions without necessitating increase in switch insulation. Switches are generally vulnerable because when open they are surge reflection points.

5. Experience indicates that even better performance of lines can be obtained by the use of protector tubes. Nevertheless, the improvement in operation obtained by Mr. Ekvall by merely increasing the insulation level is considerable, and whether or not his method or the use of tubes is preferable is largely a question of economics.

F. E. Andrews (Public Service Company of Northern Illinois, Chicago, Ill.): Mr. Ekvall's paper is a distinct contribution to the knowledge and technique of the subject of preventing power-line interruptions caused by lightning.

This discussion will be based on experience with five lines of the Public Service Company of Northern Illinois operating at approximately 34 kv, with grounded neutral, over periods from one to three years.

One point about the method which we question, is the use of wet arc-over distance in determining the length insulation path over porcelain. It would seem that the length of arc path would correspond more nearly to the dry arc-over distance, and we have used this in our data. This difference however does not affect the results materially.

A limited amount of data, from which values similar to those given by Mr. Ekvall's curves can be determined, are available from five single-circuit lines totaling in length about 32 miles, covering 68 mile-years of operation. These data cover construction of both high- and low-ridge pin types of about 3.1 and 1.6 inches per kilovolt phase to phase respectively. The faults have in general been phase to phase or three phase, but in some of the cases the arc scars also indicated a coincident phase-to-ground path, of the same or greater kilovolts per inch. The data, together with values from Mr. Ekvall's curves for comparison follows:

	High-Ridge Pin Construction	Low-Ridge Pin Construction
Miles of line	22.5	9.7
Mile-years of operation	39.2	29.2
Single-circuit structure years of operation	1,093	1,338
Inches per kv phase to phase	3.1	1.6
Power arc faults per 100 structures per year	1.2	0.45
From Ekvall 33-kv curve, faults per 100 structures per year for 3.1 and 1.6 inches per kv, respectively	0.4	0.8
Faults per 100 miles per year	33.2	20.6
1940 average results of all lines in operating division from which above data were obtained, miles of line	44.3	30.7
Faults per 100 miles per year	18.0	19.5

It will be noted that the rate of faults varies widely and inconsistently from the author's curve. In one case our fault rate is three times as high as in Philadelphia, and in the other case about half as great. The lack of agreement of results should not be taken as discrediting Mr. Ekvall's work in any way. This lack of agreement should be considered as an indication of the complexity of the problem and of the need for methods of taking account of such other factors as recovery voltage and those mentioned in the paper, which undoubtedly have great influence on the formation and maintenance of power follow arcs. It also indicates the impracticability of drawing conclusions on this problem from a small number of lines and small amount of data.

One factor which might be responsible for some variation in results in terms of faults per 100 structures, is difference in span lengths. It would seem that the number of ligntning faults would be more directly a function of length of line than number of structures. If this is the case a higher rate per 100 structures would be expected for a line with fewer structures per mile.

It would be interesting to know if Mr. Ekvall in his work found wide divergence from the averages, in the case of individual lines, which might be comparable to the seemingly inconsistent results which we have obtained.

On account of mixtures of old, new, and revised construction in our lines, it has been very difficult to obtain satisfactory comparative data on a year mileage basis. Nevertheless, major reductions in line outage rates are known to have occurred in comparison with the former construction having values of 1.2 inches per kilovolt phase to phase and 0.9 inch per kilovolt phase to ground. Also paralleling Mr. Ekvall's experience, poles having the improved construction have been found splintered by lightning, without any evidence of power arc faults.

It is hoped that others may also contribute data from their systems which in the aggregate may provide a basis for determination of insulation strength values correlated with power arc formation, for general application.

Hollis K. Sels (Public Service Electric and Gas Company, Newark, N. J.): At the winter convention in New York last year, Mr. Ekvall, in discussing a paper on lightning protection of wood-pole lines by Mr. Gothberg and myself, stated "It appears, therefore, that much of the benefit of line performance probably comes from the increased insulation and that a small improvement may come by addition of the protector tubes." I am pleased to have the opportunity of discussing Mr. Ekvall's interesting contribution to the art of improving the protection to wood-pole lines and I hope that my comments may give further enlightenment. Last year we replied to Mr. Ekvall's comments as follows, "Generally speaking, we think that Mr. Ekvall is getting his protection in the deionizing action of the wood as indicated by the splintering so that the economics of his applying tubes depends upon the damage he is experiencing" and I might now add, the further reduction in line interruptions which are desired. In the light of the data which Mr. Ekvall now submits and the additional experience which we have had I see no reason to change our conclusions.

It is appreciated that Mr. Ekvall's data are empirical and that investigations under controlled laboratory conditions would be of considerable value. In fact, such data were presented by A. S. Brookes, R. N. Southgate, and E. R. Whitehead in a paper entitled, "Impulse and Dynamic Flashover Studies of 26 Kv Wood Pole Transmission Constructions" in January 1933 (AIEE TRANSACTIONS, volume 52, 1933, pages 494–501). The paper presented data that closely check Mr. Ekvall's empirical data, that is, at least two inches per kilovolt of insulation are required as a minimum and that I quote from Mr. Ekvall's paper "For the 33-kv system there appears to be no practical value of insulation which will entirely prevent power arc trouble."

Let us look at Philadelphia's performance on 33-kv lines compared to our 26 kv. In 1935, with 36 lightning storm days, Philadelphia had 45 tripouts per 100 circuit miles to our 32 on unprotected lines. In 1940, with 38 lightning storm days, Philadelphia had 17 tripouts per 100 circuit miles to our 1.9 on 345 and 213 miles of protected lines, respectively. This is a 62 per cent reduction compared to a 94 per cent reduction. However, as Mr. Ekvall says, variations in lightning severity in different seasons affect such comparisons and it would be fairer to compare unprotected and protected lines for the same season. Our figures for 1940 per 100 circuit miles are 23.4 for unprotected lines and 1.9 for protected lines, or a reduction of 92 per cent. It would be interesting to have Mr. Ekvall's figures for the remaining 25 per cent or 115 miles of their lines which were unprotected in 1940. I suspect that they have accomplished only about one-half of the benefits which would have been accomplished with the use of protector tubes on the top phase wire. Judging from lines of similar construction, we would expect from 15 to 18 tripouts per 100 miles without protector tubes and we feel that the installation of around 15 tubes per circuit mile is amply justified by the results of 1.9 tripouts per 100 miles or a reduction of approximately 90 per cent.

I wish particularly to call attention to the appreciable increase in cases of pole splintering reported in the paper. We suggest that Philadelphia has raised the insulation level of their line so that certain poles are acting as relief points protecting intermediate poles so that they are experiencing a marked reduction in power arc faults per circuit structure as shown in figure 3, and that the pole splintering in these relief points is principally responsible for the reduction in line interruptions through the deionizing action of the wood. Our suggestion would be that Philadelphia should complete the job by installing protector tubes and realize, first, a further marked reduction in line interruptions and, second, less line maintenance and line outage for that purpose.

H. M. Towne (General Electric Company, Pittsfield, Mass.): Mr. Ekvall's paper reviewing decreased lightning outages of 13.2-kv and 33-kv power lines by appreciable increase of line insulation level is of real interest. While the amount of insulation increase over previous construction is not stated in actual values of 60-cycle or impulse flashover voltage, it would appear that the adopted minimum insulation line to ground is the 13-kv pin insulator plus approximately 23 inches of wood on 13-kv system, and 33-kv insulation plus approximately 44 inches of wood on 33-kv system. Based on published data on impulse flashover of pin insulators in series with wood insulation, the lightning dry flashover level (average of positive and negative $1^1/_2$ by 40 full wave) appears to be of the order of 310-kv and 630-kv crest for the 13-kv and 33-kv lines respectively.

Presumably, the original lines had impulse flashover levels of one-quarter to one-half of these "renovized" flashover levels. Hence, the increase in impulse voltage required to flashover should in itself result in appreciably reduced number of lightning outages. Most authorities believe that lightning voltages from cloud induction are not likely to exceed the order of 500 kv and, therefore, as the impulse flashover level of an exposed line approaches or exceeds the order of 500 kv, there will be a substantial reduction or final elimination of induced lightning flashovers, and the much less probable direct strokes to the line will become the only cause of lightning flashover. It would therefore appear that two factors are present in the substantial reduction of "power arc trouble" and interruptions, which Mr. Ekvall has realized; first, that the "renovized" lines are decidedly less susceptible to lightning voltage sparkover and second, that if and when lightning sparkover does occur the increased arcing distance in "inches per kv" of power voltage tends to prevent development or sustaining of a power arc. Which of these factors is predominant in the over-all reduction of outages may require vigilant observation with particular effort to determine the number of lightning sparkovers of insulation after "renovizing" as compared to before "renovizing." Also, by obtaining the number of lightning sparkovers plus power current which cause interruptions after "renovizing," the relative influence of these factors might be segregated.

It may be found that the "inches per kilovolt" to prevent development or sustaining of a power arc on either 13-kv or 33-kv circuits are influenced by variables at different parts of the system, such as magnitude of available short-circuit currents, system recovery voltage, neutral connections, load current, and, of course, the point on the 60-cycle voltage wave at which the lightning voltage is superimposed.

The notable reductions in system outages already observed from Mr. Ekvall's improved line construction should be of great interest to many utilities dealing with these medium voltage class systems.

J. T. Lusignan, Jr. (Ohio Brass Company, Mansfield, Ohio): In handling the lightning problem on his company's 13- and 33-kv system, the author describes a method of attack which I feel is most clever. The idea of taking field data already available from damaged structures and out of them deriving bench marks for design revisions is obviously a most efficient procedure. I have only a few comments and questions regarding some of the factors he discusses.

He mentions that ". . . . . the arc-over path was considered as being along the surface of the pole or crossarm rather than directly between wires in air." In checking some of the paths between conductors on the structures he illustrates, and comparing them with alternate paths through air, some of our laboratory results described last year in a paper by C. J. Miller and myself ["What Wood May Add to Primary Insulation for Withstanding Lightning" by Joseph T. Lusignan, Jr., and Charles J. Miller, Jr., AIEE Transactions, volume 59, 1940 (September section), page 534] indicate that the surface paths may not always be followed by the flashovers. I am wondering whether the author considered some of the alternate air distances on that basis. I would particularly like to know whether he found any conductor marks in mid-span where some of the diagonal air paths might be unwittingly short.

The author mentions using the "wet arc-over distance" of insulators. I wonder why that was chosen rather than the "dry arc-over distance," or, using the newer phrase, the "striking distance." It seems that the latter would represent the lengths of the flashover and the power follow paths a little better.

The curves plotted in figure 3 are very interesting. I would like to ask whether the grouping of the data points was such that a band could not have been drawn, including within it both the 13-kv and 33-kv sets of data, thereby allowing a possible common "inches per kilovolt" picture. The non-parallelism and crossing of the lines on figure 3 puzzles me.

The author mentions using a relation of two-to-one for the comparative strengths of air and wood insulation paths. Our laboratory results alluded to above, indicate that under average favorable conditions air may only be 20 or 30 per cent better than wood.

The author states that "approximately the same percentage reduction in power arc faults as in line interruptions were obtained." Does he mean "faults to be the detected damaged points on a system and the "interruptions" the switch operations on his log sheets? If so, it is apparent that his data and his conclusions are well founded.

In lengthening insulation paths between phases and to ground, reductions in lightning trouble of course may be the result of reduced initial lightning flashovers due to the greater insulation strengths as well as reduced power follow arcs.

H. N. Ekvall: F. E. Andrews questions the use of wet arc-cover distance in determining the length of arc path over porcelain. Wet arc-over distance was taken because it was thought that under rain conditions the effective arc-over distance would be shortened and therefore the insulators would be somewhat less effective in preventing arc formation than under dry conditions. This was thought to apply particularly in the initial stages of arc formation. However, as indicated by Andrews the difference between wet and dry arc-over distance for the porcelain is small compared to the total length of porcelain and wood in the arc-over path, so that selection of either the dry or wet value would not materially affect the results.

Concerning the inconsistencies and wide variations which are found in comparing Andrews' experience on five lines with data from the curves in figure 3, we agree that it is impractical to draw conclusions from a small number of lines and limited experience. We have found wide divergence from the average values shown on these curves when the number of structures studied was comparatively small. It was only possible to get a consistent trend in results by analyzing the performance of thousands of structures throughout the entire system. In this way we believe we were able to offset to a large extent the localized effect of lightning storms as well as the variable effects of other factors, such as system recovery voltage, magnitude of short-circuit current, circuit tripping time, wind, rain, and others.

It is probably true that the number of lightning faults is more directly a function of length of line than number of structures per mile. However, in our studies span lengths were fairly uniform and therefore comparisons could be made on the basis of faults per 100 structures without introducing any appreciable error. For comparisons with systems having appreciably greater or lesser span lengths than our 33-kv average of 125 feet, it would probably be more accurate to convert our data to faults per mile.

Use of protector tubes on switches as proposed by E. W. Beck has been tried successfully on 13-kv lines, but with some unfavorable results on the 33-kv lines due mainly to the inability of the tubes to clear the heavy follow currents at a number of locations. At present we are attempting to minimize lightning trouble by use of the wood-insulated 33-kv vertical switch shown in figure 5. On the 13-kv lines gang-operated conventional switch units are now being mounted in high-ridge pin configuration and are provided with wood bars in their operating mechanism between phases and between phase and ground.

Eric T. B. Gross has suggested that we install earth-fault coils as a means of reducing line interruptions due to ground faults. We have already investigated use of these coils and have found from experience of others that they are quite effective. However, the method of protection which we have employed minimizes the chances for ground faults and thereby reduces the utility of the earth-fault coils. Our program of renovizing the lines to obtain increased insulation strength has progressed so far now that it appears more economical to complete it than to install coils.

In reply to questions raised by J. T. Lusignan, Jr., our field experience seems to indicate that lightning flashovers generally occur along the wood structure rather than directly between wires in air. A study of twin circuit vertical structures for example showed that even though the air distance between phases was considerably shorter than the path along the arms and pole, there was definite evidence in the form of splintering that the flashover had gone over the wood. It was also found that in going from six-to-eight- and ten-foot arms the rate of power arc trouble decreased, even though the vertical wire spacing remained the same. While burn marks have been observed on the wires near mid-span, there generally has been evidence that the arc traveled along the wires from the structure where it was initiated by lightning flashover. The question regarding use of wet arc-over distance versus dry arc-over distance has been discussed above.

While the data from which the curves in figure 3 were developed could have been plotted to form a band of common inches per kilovolt for the 13- and 33-kv systems, this was not done because the two systems were in different localities and operating under different conditions. Use of the two-to-one relationship for comparative strengths of air and wood insulation paths was considered to be a good approximation based on the meager flashover data available at the start of this investigation in 1935. It is still probably accurate enough for use by the construction forces in relocation of guys. In our analyses "faults" were considered to be the detected damage points on the system, whereas interruptions were the switch operations taken from the log sheets.

We concur with H. K. Sels that use of protector tubes on our ridge pin wire should further reduce line interruptions due to lightning, but are still not certain that sufficient reduction would be obtained to justify their added cost and also whether in later years failure of the tubes themselves may not offset some of the initial gains. Since our 33-kv system neutral is solidly grounded, we would require higher capacity and higher cost tubes than now used on the Public Service Gas and Electric Company 26-kv system having a 75-ohm neutral ground resistance. Also, as mentioned previously, our experience with tubes on our 33-kv system has not been altogether satisfactory. Sels' comparison of the performance of their 26-kv system with our 33-kv system appears to be based on a misinterpretation of our data. Our reported 62 per cent reduction in line interruptions from 1937 to 1940 is for our 33-kv system as a whole and does not apply solely to that part of the system (75 per cent) which has been improved. If the entire system had been improved, then by simple ratio, the reduction in system interruptions would have been 82 per cent. This figure is the one to be compared with the 94 per cent reduction for improved lines with tubes on the 26-kv system. We regret being unable to make comparison by individual lines, since all of our improvement work has been scattered over the entire system. This is the reason for giving our interruption data for the system as a whole and also for making other comparisons on a structure basis rather than by lines.

H. M. Towne indicates that the improvements which we made were such as to decrease the susceptibility to lightning flashover, as well as to reduce the chances for power arcs if and when such flashovers did occur. Actually in renovizing the lines no attempt was made to reduce lightning flashovers, as it was considered impractical to build sufficient insulation into the structures to prevent them. Accordingly, in some instances, the impulse flashover strength to ground was reduced in order to more economically gain greater phase-to-phase arc-over distance. This situation occurred quite frequently where crossarms were lowered two to four feet on the pole to change from flat-top to high-ridge pin configuration. In these cases the reduction in power arc trouble appears to have been due to the greater phase-to-phase arc-over distance minimizing the chances for power arc formation than to any reduction in lightning flashovers. Of course, to prove this would require an accurate knowledge of the actual number of lightning flashovers before and after the improvement. To obtain this information would require use of lightning recording devices on each pole, which for economic reasons was not attempted.

An Experimental Investigation of Subharmonic Currents

Discussion and author's closure of paper 41-11 by J. D. McCrumm, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941, pages 533–40.

C. Concordia (General Electric Company, Schenectady, N. Y.): In regard to two of the conclusions drawn in this paper we should like to make the following comments.

Conclusion 2

We believe the statement that "ferroresonance" and "subharmonic oscillation" are totally different and unrelated phenomena is somewhat misleading, since they are really rather closely related. Both depend in more or less the same way on saturation of inductance and on capacitance, and in general the same sort of circuit modifications may be used to guard against either.

Conclusion 6 (2)

While in laboratory tests over a rather large range of circuit parameters and voltage excitation, ferroresonance as defined by the author may give rise to overcurrent conditions in series capacitor circuits; on the other hand, in actual series capacitor installations only the subharmonic phenomena have been encountered. This is presumably because the system voltage is never high enough to produce ferroresonance.

Stephen J. Angello (University of Pennsylvania, Philadelphia, Pa.): Mr. McCrumm is to be congratulated for his careful, extensive experimental work on this nonlinear circuit. I am very much impressed by the range of parameters that Mr. McCrumm's results cover. His diagrams represent many months of labor and they should serve as a valuable guide for theoretical investigators.

During the progress of these experiments work was done at the University of Pennsylvania on the effect of initial conditions on the response in this circuit.

My qualitative observations of the behavior of this circuit substantiate the descriptions of wave form. The term "jittery" subharmonic is very descriptive. These currents seem to have no definite repetitive form. They have been observed to jitter along for ten minutes, and suddenly die out for no apparent reason.

The existence of a "lake" in the "island" is quite certain and seems to me to be a challenge to the theorist.

I will agree with Mr. McCrumm's conclusions in general with one exception, namely, the statement that the wave form of stable subharmonics is independent of boundary conditions. I have found that initial flux linkages in the inductor have an effect on the wave form. A paper on the results of this work is in the process of preparation for publication.

J. D. McCrumm: In reply to Mr. Concordia's comment on conclusion 2, note that this conclusion states that "ferroresonance" and "subharmonic oscillation" are (*a*) totally different phenomena, (*b*) unrelated in behavior. It was not the intention of the author to imply that the two phenomena are totally unrelated, but merely that the response and certain of the underlying causes are unrelated. Indeed, the statement is made in the paper that "the nonlinear characteristic of the inductor is the essential feature of the series circuit that gives rise to both conditions of operation, but beyond that, the underlying causes for the behavior of the resulting current are probably as different as the two phenomena (meaning response) themselves."

The difficulty lies in the attempt to systematize the observed behaviors of this series circuit. While my observations lead to the statement that a large difference exists among these phenomena and the causes, it is a matter of personal feeling as to the degree of this difference, and Mr. Concordia is no doubt justified in feeling that the degree of difference is, perhaps, slight.

I shall be very interested in seeing the results of Mr. Angello's investigation of subharmonic currents. However, my observations still lead to the conclusion that initial conditions are not factors determining the *steady-state* wave form of current in the case of a sustained and stable response. However, it is perfectly true that a so-called switching transient exists before steady state is reached, and that the wave form during this period depends upon the initial conditions.

Development of the Glow-Switch

Discussion of paper 41-70 by R. F. Hays, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (May section) pages 223–6.

Samuel G. Hibben (Director of Applied Lighting, Westinghouse Lamp Division, Bloomfield, N. J.): It becomes increasingly evident that the fluorescent principle will become a permanent and a vital factor in the business of producing light for an indefinite number of years to come. It becomes likewise evident that the business of producing radiant energy constantly grows more complicated.

Even though the development of the incandescent filament lamp has not been simple, yet comparatively the control of many of the factors was rather definite and the engineering was not too involved. Size of source was largely a mechanical problem of bulk and mass. Life was fundamentally a factor of temperature, and efficiency was basically a problem of conserving radiant heat. True, other refinements were important but so long as the applied voltage was reasonably accurate and uniform, the incandescent lamp makers' concern in the normal functioning of their product did not extend back of the lamp socket. The Mazda tungsten filament lamp was generally independent of frequencies, wave forms, position of burning, temperature of surrounding air; was not concerned with noise nor radio interference, and involved to only minor degree the problems of color.

Now almost without warning comes the fluorescent illuminant, requiring the closest co-operation of almost all of the divisions of the electrical fraternity. The proper performance of this new source calls for proper design of several vital accessories. Not only must the lamp tube itself be carefully proportioned and carefully fabricated but of almost equal importance is the character of the ballast, of the starting switch, and the quality of the power that is supplied to the assembly.

D-c usage becomes a difficult problem. Supply voltages higher than the nominal 120-volt range are much easier to adapt than in the case of tungsten filament lamps. Since but one center design of voltage is standardized for each group of ballasts (for example, 118 volts on all nominal 120-volt circuits) it is now increasingly important to guide all medium voltage supply toward the actual 120-volt figure. Wave form and frequency must be considered. Temperatures are matters to be investigated critically. Life is greatly affected by the number of times the fluorescent lamp is turned on and off. The "burned-out" lamp must be promptly replaced.

All in all the fluorescent lamp, like a highly bred animal, is capable of doing a superb job if superbly engineered and intelligently handled. Nevertheless it is not foolproof! It is resourceful, but temperamental. It is not likely to be limited for many years in its variety of shapes, styles, colors, and performances. It is a device that more than ordinarily calls for the closest co-operation between distribution and power supply engineers, transformer designers, contractors, fixture makers—yesand chemists, glass technicians, and many other specialists in addition to those who fabricate the fluorescent lamp itself. This increased complication in the production of light is a challenge to the co-ordinated working of the electrical engineer in all branches. It challenges us to all work closer together and to think further ahead!

Inductive Co-ordination of REA Distribution Systems and Telephone Systems

Discussion and authors' closure of paper 41-19 by K. J. Plucknett, W. T. Smith, and T. A. Taylor, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941, pages 586–95.

Ward F. Davidson (Consolidated Edison Company of New York, Inc., New York, N. Y.): The authors are to be commended for giving a clear and informative discussion of a very timely topic. They give further proof, if any were needed, of the value of the co-operative approach to problems of mutual concern to supply and communication systems.

Those familiar with the work of the joint committee on plant co-ordination—Edison Electric Institute and Bell Telephone System, will note certain differences in emphasis and approach from those discussed in the present paper. Under the heading, choice between specific methods in the principles and practices for the inductive co-ordination of supply and signal systems issued by the National Electric Light Association and the Bell Telephone System, December 9, 1922, we read:

> "When specific co-ordinated methods are necessary and there is a choice between specific methods, those which provide the best engineering solution should be adopted."

That has been understood to mean that there is no basis of any preconceived determination as to whether remedial or corrective measures should be applied to the power system or to the communication system or some to each system. Consequently the many reports of the Joint Subcommittee on Development and Research deal with corrective measures in both power systems and communication systems.

The present authors, although recognizing in their introduction, and at the end of the paper, that co-ordinative measures can be applied to the telephone system, do not cover such measures in any detail, and do not give any results of their investigation of the application of such measures. This treatment may lead some readers to the erroneous conclusion that there is little to be done to the rural telephone system in connection with co-ordination. I would like to point out that our experience has shown that there are many co-ordinative measures which are most effective in simplifying the over-all problems which are applicable to the telephone system. These measures are referred to in the references given in this paper and these references should be consulted in the study of any particular problem.

C. W. Frick (General Electric Company, Schenectady, N. Y.): The paper states that rural distribution systems of the type described and metallic telephone circuits can be co-ordinated satisfactorily where several things are done, one of which is restricting the magnitudes of harmonics present in the voltage wave shape of generators. I would like to know what value of TIF the authors have actually found necessary. The only information found in the paper is that in certain individual cases where the machines had a voltage TIF of 25 or less the results were satisfactory, while in a certain case where the TIF was 650 a noise problem was encountered.

About ten years ago co-operative studies were made by the power companies and the manufacturers in order to determine the most reasonable values for the TIF of larger machines. These studies covered the experience in the field with respect to co-ordination, the reduction of harmonics in the wave shape of machines by design, and the taking care of cases of trouble encountered by specific remedial measures. As a result of the studies it appeared that from the over-all standpoint the most economical results for the power and electrical industries would be obtained by a limiting TIF of 50 for machines 10,000 kva and above. For similar reasons the limits set for machines under this rating were 60 for 2,500- to 10,000-kva ratings and 125 for 1,000- to 2,500-kva ratings. These values are given in NELA publication 239 entitled "Generator Wave Shape." No limits were set for machines under 1,000 kva.

The construction of rural systems fed wholly from small machines may require different standards from those adopted for small machines for other purposes. However, it would seem reasonable to expect that if limiting values of TIF of 50 and higher in the case of large machines give satisfactory performance with respect to co-ordination on large systems, they would also give similar results with small machines on small systems provided the same standards are maintained for the power system and the paralleling telephone system as regards co-ordination.

It must be remembered that it was shown in the previous study that the cost of these small machines would be increased in order to meet a limit TIF of 50. The authors' statement that they did not meet such an increase may be attributed to the fact that the situation occurred in only a small number of cases. These machines are widely used for industrial purposes and are required in a variety of sizes, voltages, and speeds. This necessitates different designs requiring many slot combinations which affect the magnitudes of the harmonics and thus the TIF. For example, a machine of a certain kilovolt-ampere and voltage rating and speed might have a much higher TIF than another machine of the same kilovolt-ampere and voltage rating but of a different speed. This is so because the possible slot combinations change with speed kilovolt-amperes and voltage. To meet the requirements which the authors consider as necessary for rural systems would mean the avoidance of certain slot combinations and would require special machines. However, a limit of 50 could probably be met in many but not all cases without too radical changes of design.

While the authors have indicated that they were unable to obtain co-ordination unless the TIF of generators was quite low, the experience of one who has had contact with co-ordination over the past several years seems to show that there are other equally satisfactory co-ordinative measures which can be applied both on the power system and to the telephone system if the values of TIF specified for generators are of the order of the values previously considered for the large generators by the power industry.

C. F. Harding (Purdue University, Lafayette, Ind.): It is considered most timely and appropriate that the earlier studies on such co-ordination should be extended to the rural power lines of which such extensive installations have been made during the past five years. This problem is far more difficult, however, and less likely to have equally satisfactory solution for the following reasons:

1. The types of power line and rural telephone construction are basically more susceptible to creative and receptive disturbances respectively.
2. The apparent necessity for the installation of small or at least cheap generators and distribution transformers introduces harmonic induction not so frequently present in the urban utility systems. However, in many REA systems energy is purchased from the larger public utilities and the advantages of the better generator wave forms are retained.
3. Long distribution lines of the REA, with many single-phase, ground-return feeders and widely varying load factors and power factors, frequently supplying grain elevator motors are not conducive to the satisfactory solution of a single corrective application to meet all needs.

With the low load factor on the individual farm transformer, which emphasizes the importance of low core losses, and the less regulation required for such a supply, it would seem that a special design of transformer with the usual close regulation sacrificed in favor of low core loss might be made available at the low cost necessary to "prove in" on such service. The reduction of the higher harmonics, which are causing most of the difficulty on the rural telephone systems, might also be secured in such designs.

Although the resonant capacitor circuits proposed in this paper may correct inductive interference with telephone lines resulting from higher harmonics for a particular line construction, the addition of branch lines, the introduction of induction regulators, and particularly the more recent use of series capacitors for improved regulation on the very long REA lines are tending to disrupt the improvements of such corrective networks.

The paper seems to stress the possibilities of correction which the REA may undertake rather than those available to the telephone company. From the legal standpoint it is probable that the metallizing of the telephone circuits will be the first important requirement of co-ordination as was the case in the earlier correction of interference between the lines of the large urban power utilities and the telephone systems.

K. J. Plucknett (USDA, Rural Electrification Administration, Washington, D. C.): The installation of many small generators in REA-financed generating plants to supply rural distribution systems has made it necessary to specify low TIF limitations for all generators installed in such plants.

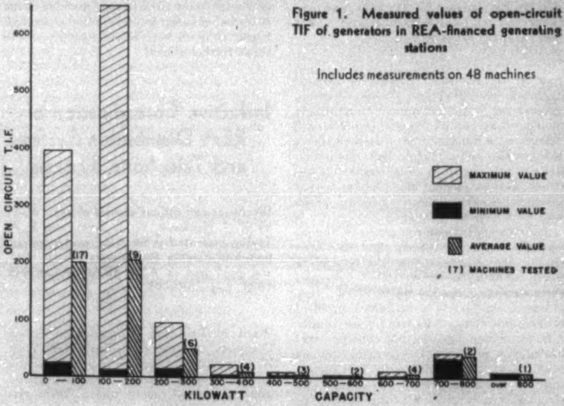


Figure 1. Measured values of open-circuit TIF of generators in REA-financed generating stations

Includes measurements on 48 machines

Figure 1 presents measurements made on 48 machines. It shows that the majority of the machines measured are under 300-kw capacity, and that high values of open-circuit TIF were measured on these machines. In general this chart shows that the TIF decreases as the size of the machines increases.

It is recognized that there are many installations of small generators made in isolated industrial plants, etc., where high TIF is of no consequence, but careful consideration must be given to installations on REA-financed systems to minimize the inductive interference problems resulting from wave shape of generators.

R. D. Evans (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The authors have presented an interesting review of the inductive co-ordination situations involving REA distribution systems. These systems differ from those of other utilities from the co-ordination point of view principally in the greater length of feeders and in the greater use of single-phase extensions of four-wire circuits.

The REA has specified wave-shape requirements for their generators which are relatively severe for machines of their relatively small size. This requirement of low no-load voltage TIF is, no doubt, a logical conclusion of the co-ordination studies involving the REA systems. However, it is not to be inferred that these same requirements should be specified for other generators which are not involved in similar co-ordination problems. It should be appreciated that no substantial improvement in wave shape beyond that of normal commercial machines can be obtained without adverse effects on machine cost or performance. For many years, the co-ordination situation with respect to the smaller synchronous machines has been satisfactory, and in fact machines below 1,000 kva were not even considered in the standardization of TIF levels. Hence for the small machines, TIF levels below those available with normal commercial machines should not be specified unless justified by a particular prospective co-ordination situation. It may be preferable to use filters or other specific measures in the few cases needing attention than to require very low levels on all machines.

In the vast majority of installations the exciting current of distribution transformers has been a negligible factor in inductive co-ordination. In only one co-ordination case that has come to our attention has the exciting current of distribution transformers been the controlling factor and in only a few cases has it been a contributing factor of large importance not considering 8*A*-ringers which are being superseded rapidly. Solution of these cases could be obtained by the application of other measures such as the filters and shunts described in the paper.

In view of the relatively small amount of trouble that has been experienced from the exciting current of distribution transformers it seems to us to be premature to consider setting up a requirement for the exciting-current wave shape. The determination of the wave-shape characteristics of a distribution transformer is a radically different problem from that of determining the wave shape of a synchronous generator. In the case of the generator the wave shape is determined from no-load voltage TIF measurements with the machine disconnected from the system. The measurements are therefore characteristic of the machine. In the case of the distribution-transformer exciting current, however, the measurement of current TIF or the $I{\cdot}T$ product depends not only on the characteristics of the transformer but also on the characteristics of the source as to its harmonic voltages and its impedance at harmonic frequencies. The impedance of the measuring circuit may even be an important factor in the results. We have found that the $I{\cdot}T$ product for one transformer varied over a range of 5.8 to 1 when tested at rated voltage from two different sources, whereas for another unit of about the same rating the $I{\cdot}T$ product varied over a range of only 1.08 to 1 when tested at rated voltage from the same two sources. Consequently, the measurement of the $I{\cdot}T$ product is quite indefinite unless the characteristics of the sources are standardized.

The wave-shape survey conducted by the NELA and Bell system showed that both the balanced and residual $I{\cdot}T$ products for distribution circuits were greater under heavy load than for light load. This indicates that the exciting current is not the controlling factor under heavy load conditions. If provision is made for prospective future loads, the exciting-current problem under initial conditions may be relatively unimportant.

The remedial measures which the authors have applied to solve the field problems encountered with REA systems deserve particular attention. Capacitors, resonant shunts, and filters especially proportioned for the specific case have been used ingeniously in these solutions. This procedure of using inexpensive specific remedial measures appears to provide a better balance in the choice of solutions than would be obtained by an attempt to use general measures alone.

K. J. Plucknett, W. T. Smith, and T. A. Taylor: The discussions offered on this paper have pointed out a number of important considerations in the inductive co-ordination of rural distribution systems and telephone systems. The authors agree with Dr. Davidson that co-ordinative measures in the telephone plant are just as important a consideration in inductive co-ordination with REA lines as with other types of power systems. As pointed out in the paper, no detailed treatment of telephone system measures was included on account of the limitations of space and because experience with these co-ordinative measures had been adequately covered in papers previously presented before the Institute. The co-ordinative measures described in the paper as applicable to rural power systems were tested on an experimental basis. In adopting a final solution for a particular problem all practicable measures were considered, including those available for both the power and telephone systems. Those which represented the best engineering solution were selected. It is interesting to note that, in the cases studied, the final solution usually involved measures in both the power and telephone systems. Thus, it is seen that the co-operative work of the REA and Bell system in fact follows closely the same approach which has been used in the joint work of the private power companies and Bell system for many years.

The discussions of Messrs. Evans and Frick are chiefly concerned with the merits of methods of controlling the inductive influence of rural distribution systems by limitations on the harmonics originating in generators as compared with the treatment of specific cases which may arise where no particular consideration has been given to the limitation of harmonics in their design. There seems to be agreement that these sources may be more important in co-ordination problems involving extensive distribution circuits of the REA type than in problems encountered in the past in connection with urban distribution sytems, or even the less extensive rural systems. Likewise, the number and extent of the REA systems makes it desirable to consider all practicable general approaches to the problem of limiting inductive influence and telephone system susceptiveness in order to minimize the number of specific cases which must be handled in the field. It is with this object in view that the REA has set up wave-shape limitations on generators and is now giving consideration to similar limitations on transformer harmonic exciting currents. The final limits which will ultimately be arrived at by the REA and the manufacturers will depend on the relative costs and the reactions on operating characteristics of incorporating low influence in the fundamental design of generators and transformers as contrasted with the specific treatment of individual situations.

Mr. Frick and Mr. Evans are, of course, correct in saying that limitation of harmonics in generators is much less important where the machines are used in small industrial plants than when they may be employed as the sole source of supply to an extensive rural distribution sytem. However, there have been cases where small rotating machines producing relatively large harmonics have affected the voltage wave shape of power systems from which REA distribution systems were supplied, necessitating the application of remedial measures after detailed field studies.

Mr. Evans points out that few inductive co-ordination problems have been encountered in the past due to transformer exciting currents except in cases where lower-impedance ringer, single-condenser telephone sets were involved. This is undoubtedly true for exposures to urban distribution systems. In such situations the inductive influence is apt to be controlled by harmonics originating in load equipment and the wave-shape survey conducted by the NELA and Bell system and referred to by Mr. Evans was predominantly concerned with urban distribution systems. However, as discussed in detail in reference 2 of the present paper, there are basic differences between urban and extensive rural systems as a result of which harmonic charging and exciting currents tend to control the influence of rural systems. The exciting currents are apt to control the influence where transformers of some of the recent designs which have high harmonic exciting currents are used, and where the harmonics in the voltage of the supply system are small. In the more recent tests it has usually been found that the inductive influence of such rural systems tends to be higher under light load conditions than under heavy load conditions. This is usually due to the combined effects of resonance in the distribution system and the effects of higher exciting currents in the load transformers under light load conditions. As mentioned in the paper the inductive influence of several REA systems has been found to be a maximum when the system was first energized and to decrease as the system loads increased. Another reason for the greater importance of the lower frequency transformer harmonic exciting currents is their tendency to be increased by the application of certain remedial measures which may be used to reduce the effects of harmonic charging currents, as pointed out in the paper.

It is recognized that much remains to be done in working out a satisfactory method of measuring the harmonic exciting currents of distribution transformers. It is expected, however, that agreement on the necessary testing technique can be reached between the REA and the various interested manufacturers.

Professor Harding has mentioned a number of significant points in connection with the co-ordination of rural systems. It is evident from the paper and discussion, however, that the REA is already giving careful consideration to most of the factors mentioned, including limitation of harmonics on their circuits. So far there has been no co-operative experience with the effects of induction regulators and series capacitors but it is felt that their effects will usually not be serious. So far harmonics originating in rotating load equipment connected to the REA systems have not been an important factor in the inductive co-ordination problem. Of course, as the load grows, such problems may arise.

The paper deals primarily with the problem of co-ordination of REA systems and Bell system telephone plant. The legal question of metallizing ground return telephone circuits has never been raised in connection with these studies.

An Electrical Engine Indicator for Measuring Static and Dynamic Pressures

Discussion and authors' closure of paper 40-157 by E. J. Martin, C. E. Grinstead, and R. N. Frawley, presented at the AIEE Middle Eastern District meeting, Cincinnati, Ohio, October 9–11, 1940, and published in AIEE TRANSACTIONS, 1941, pages 513–23.

J. E. Hancock (General Electric Company, Schenectady, N. Y.): In the enumeration of the different types of electrical pressure indicators one of the more important types was omitted under the heading "magnetic type indicators." That is, the variable reluctance type in which the position of an armature controls the a-c impedance of an iron core inductance. At first glance this might seem to be the same as the "moving diaphragm" type which indicates rate of change of pressure instead of instantaneous pressure. The use of the variable reluctance type in an a-c bridge circuit will, however, permit the indication of a pressure-time curve directly without the use of complicated high gain amplifiers and integrating circuits such as are required for the moving diaphragm type. Pressure gauges of this type have been built and used in a variety of applications. The frequency response of such a circuit is constant from zero frequency, or static conditions, up to about one-fourth of the carrier frequency used. In many cases the upper limit of the frequencies which can be recorded is determined by the natural frequency of the elastic system used.

The requirement of a high natural frequency of the elastic system is mentioned in the article together with the fact that frequencies up to 6,000 cycles per second may be encountered, but no mention is made of the natural periods obtained in the elastic systems used. Approximate calculations tend to show, moreover, that the natural frequency of the system is of the order of 6,000–8,000 cycles per second.

It is to be noted that figure 15 shows ripples at a frequency of 6,000 cycles per second which might possibly be due to oscillation at the natural frequency of the diaphragm.

The magnetic oscillograph used in these tests shows the possibilities of providing special equipment to meet particular requirements. The instrument described has an optical system of rotating lenses and mirrors which is well suited to the purposes of this investigation. This special optical system, however, prevents the more general use of the oscillograph where it is often desirable to make slower, long strip records.

The need for a frequency response up to about 10,000 cycles per second is mentioned and considered in connection with the indicator and associated electrical equipment. This requirement is, of course, just as important in the recording element as in the indicating element. Thus an oscillograph galvanometer having a natural undamped frequency of 20,000 cycles per second or higher would be required. It would be interesting to know whether this limit was attained, and if so, by what method.

E. J. Martin, C. E. Grinstead, R. N. Frawley: The natural frequency of the high pressure condenser indicator elastic system is well above the frequency suggested by Mr. Hancock. The natural frequency of the diaphragm is 30,000–100,000 cycles per second dependent on the thickness of the diaphragm.

The theory given in our paper concerning the 6,000-cycle wave shown in figure 16 is not speculative. In the last few years, considerable research has been made on the physical nature of detonation in internal combustion engines, as well as the necessary characteristics of instruments for measuring pressures. (See references.)

The frequency response of the galvanometer oscillograph is limited to approximately 1,000 cycles per second. The galvanometer oscillograph is not used where important higher frequency components are encountered.

REFERENCES

ENGINE KNOCK, Lloyd Withrow and Gerald M. Rassweiler. *Automobile Engineer*, volume 24, August 1934, pages 281–4.

THE PHYSICAL EFFECTS OF DETONATION IN A CLOSED CYLINDER CHAMBER, C. S. Draper. N.A.C.A. report number 493.

SOUND AND PRESSURE WAVES IN DETONATION, Carl E. Grinstead. *Journal of the Aeronautical Sciences*, volume 6, number 10, August 1939, pages 412–17.

The Service Factor Rating of Arc-Welding Generators and Transformers

Discussion and authors' closure of paper 41-33 by R. C. Freeman and A. U. Welch, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (April section) pages 137–41.

J. E. Waugh (General Electric Company, Schenectady, N. Y.): From the point of view of a user of arc-welding equipment, this paper is an excellent step in the right direction toward a more logical method of testing arc-welding generators and transformers.

As indicated by the authors, the present short-time tests of one-half or one hour mean little toward determining what will happen to the equipment on all-day operation. The result has been a general use of equipment of a higher rating than that actually required for a given application.

The choice of a 60 per cent duty cycle is reasonable since probably less than 5 per cent of the present manual welding applications will exceed this figure. It might well be argued that a 60 per cent duty cycle at the rated capacity, under actual average welding conditions of say 1.8 minutes on and 1.2 minutes off, be imposed on the generator or transformer until it has attained a constant temperature. However, if comparable tests indicate that fairly close agreement can be obtained between this method and the proposed testing at 75 per cent of rated current, I am sure that we would all agree to the latter for the sake of greater convenience, testing accuracy, and the closer adherence to existing standards for general purpose motors.

P. L. Alger (General Electric Company, Schenectady, N. Y.): The proposal made in this paper, for an 0.75 service factor rating of arc-welding equipment, seems to me a very desirable step forward. The name-plate amperes will give a single well-understood value of the output delivered during welding operations, while the service factor will give a definite value of continuous heat dissipating capacity that can be accurately checked by test.

In the November 1940 issue of the *BEAMA Journal*, page 76, there is an article entitled "The Rating of Single-Operator Arc-Welding Equipment" by J. W. Bayles, which gives almost identical conclusions, only using a different terminology. In the Bayles paper, the normal ratio of operating to total time, called the "normal intermittency ratio" is suggested to be 0.4, the same as in the French Technical Rules, instead of the 0.6 value recommended by Messrs. Freeman and Welch. It is further suggested that the temperature tests on welding apparatus consist in a continuous run at an output value equal to the square root of the intermittency ratio, which in the British case would be 63.2 per cent, instead of the 75 per cent value here recommended.

In applying this method commercially, it is a question whether to use round numbers for the intermittency ratios or for the service factors. I prefer the latter, since the service factor will appear as a definite part of the rating, while the intermittency ratio will vary for each particular job. For this reason, I suggest that an 0.8 service factor be considered standard, while an 0.7 factor may be used for small equipment or special cases.

K. L. Hansen (Harnischfeger Corporation, Milwaukee, Wis.): As chairman of subcommittee on electric arc-welding apparatus of ASA sectional committee *C*52, I welcome papers of the type presented by Messrs. Freeman and Welch. It has long been recognized that present standards of arc-welding equipment are woefully obsolete and inadequate. This is true of standards pertaining to capacity or temperature rating, and perhaps in greater degree, of standards pertaining to the transient performance, such as current and voltage recovery, etc.

In the early days when arc welding was a repair tool the one-hour rating against a resistance load may have answered the purpose. Now, however, when arc welding has become one of the most important production tools of modern times, it is no longer applicable.

As stated by the authors, production welding is a continuous operation where the loading of the equipment undergoes a more or less regular cycle. Furthermore, it is asserted that a well-ventilated machine with good heat dissipating characteristics may operate continuously at a definite welding load with lower temperature rise than a heavier machine with poor ventilation, although the latter may show lower temperature rise on a one-hour run against a resistance load. This contention is well recognized.

It may be of interest to compare the load cycles given in the paper with those encountered in our own experience. Welding operations in the plant of the Harnischfeger Corporation range from long straight seams on crane girders to welding of small subassemblies requiring considerable positioning between relatively short welds. The average ratio of arc time to total time on the long seams is 73 per cent, and the average ratio for the entire shop is 53 per cent. The suggestion in the paper of considering 60 per cent on and 40 per cent off as the average load cycle for manual welding appears, therefore, to agree very well with our experience, permitting, as it does, a small margin for intermittent overloads.

The recommendation of substituting a

continuous rms current rating based on a 60 per cent load cycle for the present one hour against a resistance load rating is, therefore, a rational one. It constitutes a definite forward step.

While the proposed change is in the right direction, it is a question whether or not it goes far enough. During actual welding, the current drawn from the generator is not a steady one, but fluctuates continuously as the drops of metal pass across the arc. Furthermore, the inrush current, when the arc is struck, may be several times the average welding current. This becomes an important factor when tacking or making a great number of short welds.

Experience has definitely shown that a machine with good transient characteristics will, other things being equal, show less heating on a given job than a machine with poor transient characteristics. I have in mind an installation where a 400-ampere machine outlasted several times to one a 600-ampere machine with considerably larger current fluctuations. In this instance the inferior ability of the 600-ampere machine to stand up was accentuated by the fact that the application was one in which a great number of short welds were made, thus making the inrush current a big factor. The question is: Should the transient performance also enter into the service factor?

The temperature rise by thermometer on the outside of a coil has always been considered a doubtful criterion of the life expectancy of the coil. Coils differ widely in their ability to conduct heat from the inside to the outside, and therefore in thermal gradients. Temperature rise by resistance is more reliable, but neither tells the whole story. A combination of the two furnishes a better picture of the true conditions. It seems rather obvious that a lower hot-spot correction should be applied when the difference between rise by resistance and rise by thermometer is small than when it is large.

In the welding generators we manufacture are coils which show substantially the same rise by thermometer and by resistance. Even with a temperature of 65 degrees (by thermometer and resistance) one of these coils could reasonably be expected to stand up better than a coil with 50 degrees rise by thermometer and 60 degrees by resistance, in spite of the fact that the latter would meet specifications while the former would not.

R. C. Freeman and A. U. Welch: Mr. Hansen's experience with regard to the percentage of welding duty cycles checks well with the values which we have recorded.

The point regarding the effect of transient characteristics upon welder heating is very interesting. In the case cited, it is pointed out that the special nature of the welding duty may have been, to a considerable extent, responsible for the unusual heating of one unit as compared with the other. It is true that such factors as transient characteristics of generator or transformer, the number of short circuits per second in the welding arc, the steadiness with which the arc is held, and the number of starts will influence heating under actual welding conditions.

Thus, although it is realized that other factors are involved to some extent, the fact that experience has shown that welder heating on an average welding load is not greatly different from that on a resistance load, coupled with the need for a simple standard and a test which may be easily performed and consistently checked at various times and locations, has led to the recommendation that heat runs be made on resistance loads.

Among the various accepted methods of temperature rise measurement, no doubt it might be agreed that the resistance method is the most desirable single method for arc-welder windings. Possibly a combination of resistance and surface measurement might be desirable although even in such a case the rise by resistance will be the more reliable inasmuch as it is independent of the location of the thermometer or the thermocouple.

Variations of Atmospheric Temperature With Altitude in the United States

Discussion and author's closure of paper 41-78 by H. W. Tenney, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (May section) pages 230–2.

R. E. Hellmund (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): In his paper Mr. Tenney has supplied data which should prove very valuable in the application of electrical apparatus at high altitudes. In order to avoid any more calculation than necessary, he has confined his study to average temperatures prevailing at various altitudes. Unfortunately, however, it is not the average temperature value which determines the life of the insulation; nevertheless, it is believed that a knowledge of it will be of assistance. It is now generally recognized that the life of insulation is dependent upon time and temperature, and an approximate quantitative valuation can be made by applying the so-called "eight-degree rule," which states that the life of insulation will be decreased by one-half for every eight-degree temperature increase.

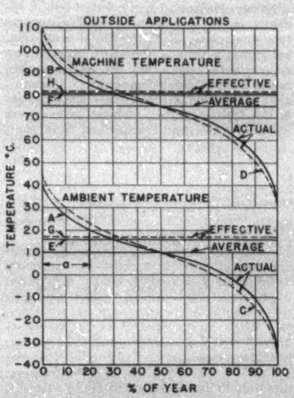


Figure 1

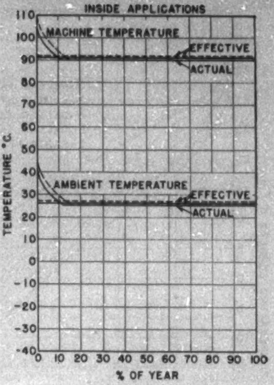


Figure 2

Curve *A* in figure 1 of this discussion is a typical climatic curve showing for a certain locality the number of hours during which a given temperature value is exceeded, the value (*a*) representing the number of hours the temperature exceeds 20 degrees centigrade. By adding to this curve the hot-spot temperature rise of 65 degrees centigrade, curve *B* is obtained. By methods outlined by V. M. Montsinger,[1] it is possible to calculate from curve *B* the constant temperature value (*F*), which in its effect on insulation deterioration is equivalent to that of *B*. By subtracting from the straight line (*F*) the temperature rise of 65 degrees centigrade we obtain line (*E*), which may be considered as giving an effective ambient temperature equivalent to *A*.

Curve *C* represents another climatic time-temperature curve having the same average as *A*, but showing shorter periods for the extreme temperature values. Curve *D* and straight lines *H* and *G* have been derived in the manner previously outlined, and by comparing *E* and *G* it will be seen that the effective or equivalent temperature values vary even though the average temperature is the same in both cases. Complete temperature data such as given in figure 1 are available for but a few localities, and an unreasonable amount of work would of course be required to carry out the calculation indicated in figure 1 for many localities. However, it would seem worth while to make a check of a number of typical localities to make sure that the difference in effective temperatures for different altitudes is at least as great as that between average temperatures. If this fact can be established with reasonable certainty the average values given by Mr. Tenney can be used with safety in applying apparatus at high altitudes, standards for which have been established within the customary low alti-

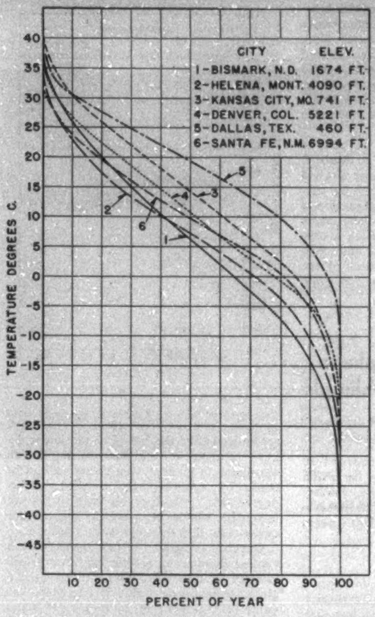


Figure 3. Per cent of year atmospheric temperature is at or above different values for six cities

Average of data for 1932–37 inclusive

tude range between sea level and 3,300 feet elevation.

Figure 1 applies of course only to outdoor installations. In figure 2 it has been assumed that indoor temperatures are about the same as outdoor temperatures whenever they are in excess of 25 degrees centigrade; however, they are assumed to be 25 degrees centigrade during the remaining period. This condition can be safely assumed to exist where there is indoor heating for human comfort.

Reference

1. Temperature Limits for Short-Time Overloads for Oil-Insulated Neutral Grounding Devices. V. M. Montsinger. AIEE Transactions, volume 57, 1938 (January section), pages 39–44.

H. W. Tenney: It must be borne clearly in mind that average temperature data for the entire country, or for any major division of the country, cannot necessarily be applied to a particular locality. Climatic conditions vary considerably with the watershed, latitude, season, and peculiar local conditions and only a general study giving average conditions can be presented in a paper of this type.

As Mr. Hellmund has pointed out in his discussion, it is not the average temperature which determines the life of insulation, but rather a time-temperature function, an approximate evaluation of which is obtained by the "eight-degree rule." Figure 3 gives time-temperature curves for six cities in the United States. These curves were prepared from data transcribed from thermograph charts in the office of the United States Weather Bureau. The number of hours per day the temperature was at or above 105, 95, 85, 70, 50, 32, 10, and −10 degrees Fahrenheit were recorded for the years 1932–37, inclusive. From these data the percentage of the time the temperature was at or above these temperatures was calculated and the curves were drawn. The effective temperatures were calculated both for outdoor and indoor conditions and are given in table I. For the indoor calculations it was assumed that the temperature never falls below 25 degrees centigrade.

While the outdoor effective temperatures show a decrease with altitude in the general order of a decrease in average temperature with altitude, for localities of the same general latitude, there is insufficient data from which to establish any definite relationships.

Table I

Latitude (Deg. N.)	Weather Bureau Station	Altitude (Ft.)	Avg. Temp. 1932–37, Inclusive (Degrees Centigrade)	Decr. in Avg. Temperature With Altitude (Degrees Centigrade)	Effective Temperature (Degrees Centigrade) Outdoor	Decr. With Altitude	Indoor*	Decr. With Altitude
47	Bismarck, N. D.	1,674	6.5	−0.7	14.7	1.8	26.1	0.7
47	Helena, Mont	4,090	7.2		12.9		25.4	
39	Kansas City, Mo.	741	14.1	3.6	20.1	5.9	26.8	1.3
39	Denver, Colo.	5,221	10.5		16.2		25.5	
33	Dallas, Tex.	460	19.2	9.2	22	8.0	26.8	1.6
35+	Santa Fe, N. M.	6,994	10		14		25.2	

*Based on 25 degrees centigrade minimum indoor temperature.

Prime-Mover Speed Governors for Interconnected Systems

Discussion and authors' closure of paper 41-24 by R. J. Caughey and J. B. McClure, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (April section) pages 147–51.

H. K. Sels (Public Service Electric and Gas Company, Newark, N. J.): Remarks by two of the previous speakers, one by one of the authors and the other by a discusser, challenges my understanding of this problem of governor operation during system short circuits and out-of-step conditions. Mr. McClure dismisses these subjects rather lightly when he says that high-speed relays and circuit breakers do not require improved governor performance. Mr. Marquis also remarks that in a properly designed system there is no problem. I should like to ask what is proper. We in America have some rather large and extensive transmission systems, some of which are modern and some of which are not so modern and I should like to speak a word of caution about dismissing so lightly improved governor performance to handle conditions of electric system stability.

Being somewhat familiar with this end of the problem, I would like to call to the attention of governor designers some of the shortcomings in the protective end of the electrical system. In the first place, as I said before, some of the systems are not so modern and still have a number of relatively low-speed relays and circuit breakers which cannot be economically replaced or rebuilt and at the same time keep our operating expenses to a minimum commensurate with low rates. Much as we like to think of our systems as being ideally designed there are several blank spots in our protective system which are not always economic to provide protection for because faults are considered to be so infrequent. However, faults do occur in these blank spots occasionally and they have to be cleared by slower speed backup relays.

Another limitation to the application of high-speed protection is that while we have several excellent schemes of high-speed protection for phase-to-phase faults no one yet has developed an equivalent scheme except those involving pilot wire or carrier for clearing single-phase faults to ground where neutrals are grounded through some form of impedance. Therefore it is possible for a single phase-to-ground fault to develop into a second phase-to-ground fault at some other point in the system so that a two- or three-phase fault is obtained which appears to individual relays as single-phase faults and result in a rather complicated system disturbance and protective problem.

Finally we also have the possibility of failure of high-speed relays to operate and we are forced to fall back on a second line of defense of slower speed operation.

To sum up this problem, the high-speed protection costs a lot of money both on new installations and particularly on revamping of our older systems and I do not think that improvements in governor performance during system short circuits and out-of-step conditions can be dismissed so lightly. In fact, the designer who brings out a governor having these improvements which might eliminate the necessity for such expensive investments in high-speed relays is going to have a better mousetrap and many people will beat a path to his door.

A. F. Schwendner (Westinghouse Elec. & Mfg. Co., Philadelphia, Pa.): The authors describe and define the necessary, desirable, and nonessential governor characteristics required to operate successfully on an interconnected system. The conclusions reached cover the required governor characteristics in the majority of cases. Only on systems which have an oscillating load where the load changes are a high percentage of the total system load do we find a need for governor characteristics not recommended as necessary in the conclusions. The fast load changes call for governors with a not too high transient incremental regulation and reasonably similar rate of response to minimize load exchange between units or stations. The tie-line load regulator has to be made fast in line with the load changes and the governors will have to follow about the same rate.

The large interconnected systems can operate reasonably well with relatively poor governor performance characteristics. Small interconnected systems, independent generating systems, and systems which have to operate occasionally independent from the interconnection require better governor performance characteristics and can use to advantage additional characteristics which would have very little effect on the large interconnected systems. The authors' conclusions should not be used to cover needed governor performance characteristics on all large generating systems. They describe and define the minimum required governor characteristics for large interconnected systems.

V. M. Marquis (American Gas and Electric Service Corporation, New York, N. Y.): Our experience on a large interconnected system has, in general, led us to agree with the conclusions reached by Messrs. Caughey and McClure. This is especially true as to the statement in their summary that governing systems on large interconnected networks should have uniform incremental regulation, accurate response for supplementary control, and small dead band. Experience quite definitely shows that less dead band would materially increase the accuracy of response for supplementary control. As to assistance by operating groups in determining improvements or changes necessary in governing systems, we are anxious to co-operate on any tests that would help to determine what the ultimate sensitivity of the governor should be and believe that such tests should be carried out on some of the older vintage machines as well as the newer units. It also seems apparent that, once these factors are determined, more attention should be given to the maintenance of governors and making available a relatively simple means of checking governor performance.

Since the advent of high-speed clearing of faults, we have had no stability troubles. Prior to that time, however, there were cases of trouble and in some cases we have a suspicion that the governor was aggravating the trouble. One particular case was on The Scranton Electric Company system where instability troubles were experienced at the Stanton plant.[1] Occasionally, governor "pumping" resulted and it was believed that the governor contributed to the unstable condition. A series of tests was conducted at Stanton and these led to the conclusion that a properly designed and adjusted governor did not contribute to any system oscillations so long as synchronism is maintained but, when the unit is out of synchronism, the governor attempts to follow the speed changes as the generator slips poles, with the result that the so-called pumping occurs. However, it was definitely shown that this pumping action is a result rather than the cause of a sustained oscillation and may well be expected when the generator and the system are out of step at a certain slip frequency; furthermore, that it will disappear if the generator will pull back into synchronism. It is interesting to note that in this particular case the trouble was solved by the installation of high-speed switching. It is therefore our belief that governors properly adjusted can be eliminated as having any significant effect on system stability on a power system that is properly designed.

Reference

1. Governor Performance During System Disturbances, R. C. Buell, R. J. Caughey, E. M. Hunter, and V. M. Marquis. AIEE Transactions, volume 50, 1931, pages 354–69.

J. E. Allen (Pennsylvania Water and Power Company, Baltimore, Md.): The authors of this paper refer to a method of simultaneously recording speed on one axis and governor mechanism movement on the other axis. This discussion is intended to show the manner in which it is done and to point out the possibilities in this method for studying governor performance under normal operating conditions. A new frequency recorder is largely responsible for this accomplishment.

Instead of employing an oscillograph as the reference 8 suggests, a continuous recording frequency meter has been developed having the following characteristics: (1) rectangular co-ordinates, (2) high sensitivity, (3) fast speed of response, and (4) large force available in the pen arm. On account of the last feature it has been possible to add a second pen to the recording arm without impairing the performance of the instrument. This second pen is used in combination with a drum which is rotated by the movement of the operating piston or servomotor. The record obtained from the drum represents a simultaneous plot of frequency versus piston travel and the resulting curve serves as a measure of the governor regulation.

The photograph, figure 1, shows the drum and its relation to the second pen of the frequency recorder. It will be noted that another pen is provided to record piston travel on the same strip chart as the frequency meter. Both the drum and the third pen are actuated from a common string or wire attached to the operating piston. Thus two kinds of records are obtained simultaneously while the unit is regulating on the system: (1) a strip chart showing the relation between frequency and piston travel with respect to time thereby permitting an analysis to be made of dead band and rate of response, and (2) a curve of piston travel versus frequency from which the regulation is obtained.

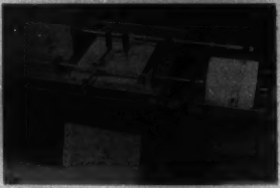
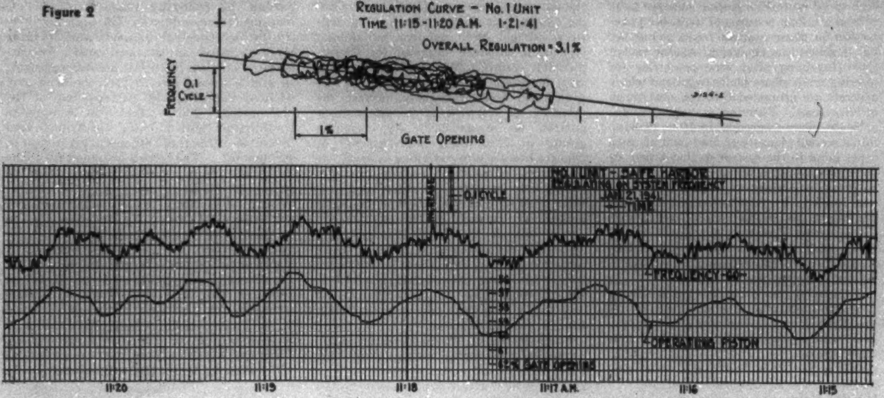


Figure 1. Governor-performance testing equipment

Inherent-speed-droop and servomotor-motion recorders

The important new feature of the equipment as shown in figure 1 is the record on the drum and the method of obtaining the regulation from it. The record consists of characteristic figures resembling an ellipse. When the piston travel is multiplied as shown the figures are greatly elongated and if, after the chart is detached, a line is drawn through the major axis by visual inspection, the slope of the line becomes a measure of the regulation. It is obvious that the regulation thus obtained would be based on the travel of the operating piston or servomotor. If desired, the regulation may be expressed on a kilowatt basis by referring to a test curve of piston travel versus kilowatts output. Since the regulation is obtained while the unit is regulating on the system, the record on the drum consists of a multiplicity of figures superimposed on a common axis. The time required to obtain a sufficient number of figures to establish the slope is a function of the load changes on the system and ordinarily only five to ten minutes are required. The calculations involved in determination of the

Figure 2



slope or regulation are simple and may be done in the field if desired.

The strip chart type of record showing frequency and piston travel with respect to time has been used in the past for governor analysis with gratifying results. However, the possibilities inherent in this simple record have not been fully realized on account of (1) the curved scale charts, and (2) the relatively low sensitivity of the frequency recorders, particularly when used on large interconnected systems where the variations are only about 0.1 cycle. It has been found that the rectangular co-ordinate scale and the high sensitivity as employed in this instrument (0.33 to 1.65 inches per 0.1 cycle) very greatly enhance the value of the strip chart record, making it possible by quick visual inspection to ascertain dead band or rate of response.

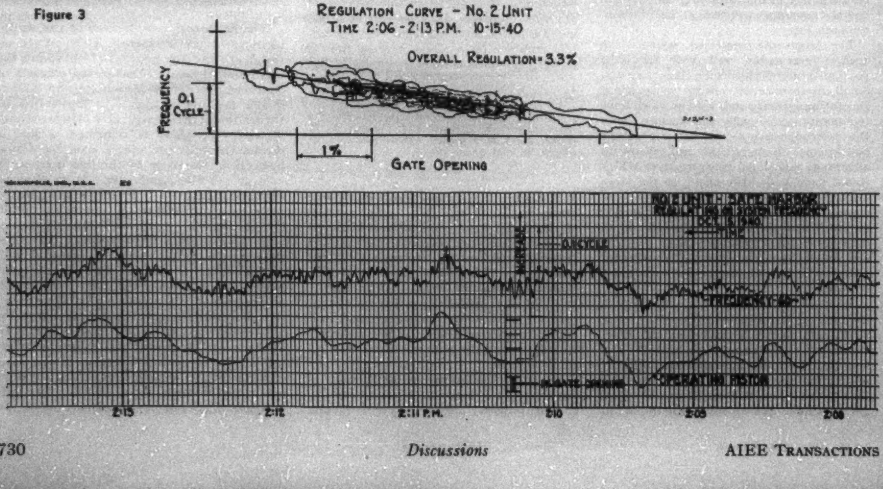
Typical results from this method are given in figures 2 and 3 showing the performance of two hydro governors regulating on the system. The performance of these two units may not at first appear to be very different. However, a closer examination of the strip charts will readily reveal that the governor in figure 2 shows evidence of a larger dead band and a slower rate of response than the one in figure 3. The regulation curves also show that the minor axes of the figures in figure 2 are somewhat greater than those in figure 3. It is believed that the width of the minor axes of the figures on the regulation curve may be used as a relative measure of governor performances. However, caution must be observed in making comparisons since the rate of load or frequency changes may affect the width of the figures due to governor compensation.

The experience acquired from this method of testing has been somewhat limited, but the tests which have been made on both hydro and steam governors have been very encouraging. Since the authors point out that the speed governor is still the backbone of system control, it follows that operating companies should realized the full capabilities of existing governors. Two important needs to this end are: (1) a common basis of analyzing performance, and (2) simple, rugged, and portable testing facilities fully capable of disclosing the high degree of performance inherent in modern governors. It is hoped that the method and equipment briefly described above will prove to be a forward step in meeting both of these needs.

The above method would also appear to be particularly useful as a tool for routine governor maintenance, since the equipment is portable, rugged, and easily operated. The important advantage lies in the ability to determine over-all governor performance while the unit is regulating on the system.

F. Oppenheimer (Pennsylvania Water and Power Company, Baltimore, Md.): I have read all of the papers presented before this group and found in six different places the thought expressed that the per cent speed regulation could or should be increased. In three different places the statement was made that the governor is still the backbone

Figure 3



of system speed regulation. In two places the statement was made that the system frequency should be held within narrow limits to improve certain objectionable conditions.

From these conflicting ideas a complicated conclusion could be drawn as follows: The system frequency should be held within narrower limits, but by increasing the per cent regulation on the governors, as suggested, we prevent them from acting effectively as the backbone of speed control, and we will at the same time have greater frequency variations. A more constructive view of this situation would be to admit that one of the chief purposes of increasing the per cent speed regulation on governors is to hide their shortcomings; therefore correct these, and retain the small per cent regulation or even reduce it, thereby reducing frequency variations, and thus allow the governor to be a real backbone of speed control. In short, I believe that increasing the per cent speed regulation is a step backward.

One of the papers makes the significant statement that "the operation of the governors is usually observed for a period after overhaul." It is probably true that most new governors are observed for a period. A mechanism so complicated requires more than just to be looked at. Suitable instruments should be used to check its performance and when this practice becomes more general, there should be appreciable progress in the speed regulating problem.

R. J. Caughey and **J. B. McClure:** The authors wish to express their appreciation for the discussion which indicates that the subject is a timely one and that many operating groups have contributed to a better appreciation of the problem.

Mr. Marquis, in agreeing with our general conclusions, emphasizes the advantages of small dead band in the governing mechanism in making the response of supplementary control devices more reliable and uniform. Unfortunately we are, as yet, unable to make any concrete suggestions as to what values of dead band should be attained and such values will be arrived at only after more field data have been obtained and analyzed. We would, therefore, be pleased to co-operate with any operating group in making tests on their governors to this end.

Mr. Marquis refers to Scranton tests and these are described in a paper entitled "Governor Performance During System Disturbances" by R. C. Buell, R. J. Caughey, E. M. Hunter, V. M. Marquis, AIEE Transactions, volume 50, 1931, page 354. This reference should appropriately have been included in our bibliography. These tests showed that entrained steam in the machine was sufficient to force the generator out of step regardless of the speed of closing valves and if synchronism were to be maintained the fault duration had to be reduced. Also the pumping phenomenon which is often referred to could be produced only when the machine was running out of step.

Mr. Allen has described a very ingenious method of determining dead band and incremental regulation, and such a method should be useful in making routine tests on the condition of governors during their normal operation.

Mr. Oppenheimer feels that there is some confusion as to what value of regulation is most desirable. In analyzing a record of system frequency there are at least two evident components, first, the slow changing average frequency due to real load changes being accepted by the composite governor characteristics and second, a superimposed fringe which is to a large extent determined by the dead band and regulation. If governors are operated with small regulation, the average frequency will be better and less supplementary correction may be required. However, the frequency fringe will in all probability increase, reflecting itself in more tie-line oscillation and indicating that the margin of stability has been reduced. This suggests that there is an optimum regulation and that variations in frequency may be increased by making the regulation too small as well as by making the regulation too large. Other operating groups have tried various values of regulation and it is interesting to note that they have generally concluded by operating with approximately five per cent over-all regulation, thus obtaining good stability and they have relied on supplementary devices to maintain good average frequency. The suggestion of periodic checks on governing mechanisms is very much in order and will no doubt become a regular routine practice as instruments are made available.

Mr. Sels suggests the desirability of designing a governing mechanism for improved performance during system short circuits and out-of-step conditions. It is generally conceded that the governor performance has very little influence on whether a machine stays in step or loses synchronism during the first swing following a disturbance. This was emphasized by Mr. Marquis in his discussion relative to the Scranton tests. This then brings us to the performance of governors on machines which are operating out of step with the system. Under this condition the machine operates usually at an average overspeed determined by both the regulation of the governor and the induction characteristic of the generator tied to the system. There is also an oscillation in speed at slip frequency and increasing the field has been suggested as a means both of increasing this oscillation, hoping to make it drop down to synchronism, and of increasing the tendency to pull into step when this synchronous speed is reached. Others have found that reducing the speed setting of the governor assists in resynchronizing and some operating companies are now planning to use out-of-step criteria as a means of automatically reducing the speed setting of the governor. It will be interesting to observe the operating experiences of this method. It is common practice to design systems so that the component parts should not lose synchronism during disturbances. Thus it is only when the normal protective equipments fail to function properly that any appreciable benefit from modified governor performance can be realized.

Mr. Schwendner has classified our general conclusions as representing minimum governor requirements for machines connected to a large system. We agree entirely that governor requirements on small systems are different from those on large systems. We believe that the requirements we set forth represent the complete, not minimum, requirements of governing mechanisms for use on the larger systems.

A Power-System Governor Sensitive to Frequency and Load

Discussion of paper 41-15 by Thomas E. Curtis, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (March section) pages 89–92.

V. M. Marquis (American Gas and Electric Service Corporation, New York, N. Y.): The governing system described by Mr. Curtis is most interesting, and actual operating experience with this type of governor will be awaited with interest.

I believe it should be pointed out, however, that on a large system even a governor with the characteristics outlined by Mr. Curtis will still need supplementary control to properly maintain loading on tie lines.

Power System Governing—the Problem

Discussion and author's closure of paper 41-68 by J. J. Dougherty, A. P. Hayward, A. C. Monteith, and S. B. Griscom, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941, pages 547–58.

F. W. Quarles (Consolidated Gas Electric Light and Power Company of Baltimore, Baltimore, Md.): The title of this paper appears to be too inclusive, since it seems to deal almost exclusively with steam turbine governors.

The writer, being of the opinion that the governing of water turbines under transient conditions is a much more difficult task than that of steam turbines, inquires if this condition should not be recognized and a little more consideration be given to this fact in the paper assuming, of course, that the scope of the paper is as wide as the title indicates.

While the statement concerning the impossibility of obtaining two governors having exact duplication of characteristics may be true in the sense that no two devices may be exact duplicates, nevertheless from a practical standpoint the characteristics of two governors of the same design can be made to approach a given standard to a fair degree of satisfaction.

The statement regarding the compression of the governor spring with increase of speed as being usual is considered to be too restricted since there are a large number of governors designed so that an increase of speed extends the governor spring instead of compressing it.

It is suggested that in reference to the complaint about overtravel of speed changer motors because of inertia, this could be limited by use of dynamic braking for d-c motors and solenoid brakes for a-c motors.

Attention is drawn to the fact that the short-time variations of frequency shown on a fast acting frequency meter do not necessarily mean that the changes in governor speed have been of an equivalent amount. This comes about since an a-c generator is not rigidly connected to the bus and load. For the sake of mechanical analogy it may be considered that the generator shaft is connected to the bus and driven load by means of a spring. If the frequency fluctuations originate outside the machine, the spring action together with the rotational inertia of the machine may serve to prevent the speed fluctuations of the generator from being so great. Voltage regulators may even be the source of such indications since a change of excitation changes the phase angle between the generator and the load.

If, on the other hand, machine speed fluctuations originate in the generator unit by reason of faulty governor operation or otherwise, while the unit is carrying load, the frequency meter may not show the full extent of these speed fluctuations.

The spring analogy brings to mind another phase of the governing problem, viz., the oscillating period as related to the units rotational inertia and the spring action. This oscillation, together with that of the weight and spring of a centrifugal weight type governor, constitute the two main oscillating features of such a unit which may result in violent governor action if sizable cyclic variations of the load occur in resonance with a natural period for which there is not a sufficient damping force.

It is observed that in connection with a notation cited from report of company 20 there is the statement that "Experience indicates that, following a sudden drop in frequency, the governor of a machine having low 'steady-state' stability will open up faster than a field adjuster will strengthen the field so that the pull-out torque of the machine may be exceeded." Does not this indicate that it is desirable for high-speed low-inertia machines to have high short-circuit ratios and/or greater field saturation in order to minimize this condition?

The writer is in agreement with the statement as to the need for more refined testing instruments, particularly in regard to obtaining the operating characteristics of a governor under transient conditions. It is considered that the correct basis for such should be machine speed rather than frequency because of the effect of phase-angle variation. If possible, the required instruments should be relatively inexpensive.

In connection with the determination of governor characteristics under transient conditions, Mr. J. E. Allen's discussion of a companion paper "Prime Mover Speed Governors for Interconnected Systems," by R. J. Caughey and J. B. McClure,[1] is quite pertinent. An item of particular interest is the automatic recording or ploting of frequency against turbine valve motion. It is assumed that for an ideal governor, without lag or lost motion, the trace would be only one line. However, the actual trace takes on somewhat the character of hysteresis loops. The difference between the trace with an increasing and that with a decreasing frequency would seem to be the result of various time lags such as that of the turbine speed with respect to terminal or bus frequency; that of the governor weights in reaching equilibrium with the centrifugal force; that of the position of power control valves with respect to governor position; that of the frequency measuring instrument with respect to actual frequency and such lost motions as may occur in the governor or its auxiliary parts and the instrument itself. Here again, the writer recommends the use of turbine speed instead of line frequency unless there is a negligible difference in traces taken under the two conditions.

It may be of interest to mention a device which the writer has used for quite a few years in determining governor regulation. This device is known as a strobometer and consists of a tapered steel cylinder turned by a synchronous motor driven from an electrically maintained 60-cycle tuning fork. A steel wheel carrying an interrupter revolves in contact with the tapered cylinder and can be moved along it by means of a screw. The contactor controls the flashing of a neon lamp which illuminates a symmetrical star on a paper disk attached to the shaft of the turbine. The position of the contactor wheel is determined which causes the star to remain stationary, a suitable scale being provided along the tapered cylinder for reading small deviations of speed from standard. This device is quite accurate but is not suitable for obtaining governor characteristics under actual load since correct values would be dependent upon the alacrity of the observer in obtaining readings.

It is suggested that an optical scheme for obtaining governor performance under transient conditions could very well be used in which a fork timed light would illuminate a suitable figure on the turbine shaft. By taking moving pictures or sufficient still photographs of the figure, together with that of a pointer and scale for valve motion and a clock, suitable data could be obtained. This arrangement should eliminate instrument lag from the problem.

The writer also would like to take this opportunity to strongly urge the unification of governor and control terms to a precise standard.

REFERENCE

1. Discussion by J. E. Allen of R. J. Caughey and J. B. McClure's paper, PRIME MOVER SPEED GOVERNORS FOR INTERCONNECTED SYSTEMS. AIEE TRANSACTIONS, volume 60, 1941, pages 729–30.

R. Rüdenberg (Harvard University, Cambridge, Mass.): It may be of some interest that it is possible to solve with good approximation the problem of the transient interactions between the single parts of a power system, as in the accompanying figure 1, consisting of any number of prime movers of different sizes, with different speed-governor mechanisms and driving different synchronous generators which are interconnected to a common system and loaded, in addition to resistance load, with any number of individual induction motors. The solution[1] which is of remarkable mathematical simplicity, shows that a multitude of oscillations can occur in such a system, the frequency spectrum of which, however, can be divided into three groups, as shown in the accompanying figure 2.

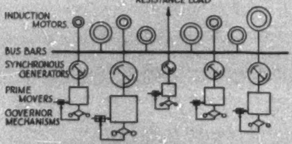


Figure 1. Parts of a power system which suffer transient interactions

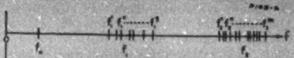


Figure 2. Frequencies occurring during transient exchange of power

In general, there develops a single low frequency f_0 which defines a combined oscillation of all the governors together, comprising the inherent properties of their servomotors as well. Furthermore, the inertias of all the induction motors in addition to those of all the synchronous generators enter into this oscillation, which consists of an in-phase movement of all parts of the electric system. The period of these slow frequency swings of the entire system is ordinarily of the order of 10 to 20 seconds.

The next group of frequencies, f_1, determines the more rapid electromagnetic oscillations which occur between each synchronous generator and the entirety of all the other machines in the system. The number of these frequencies equals the number of different synchronous machines. The values of the frequencies are primarily determined by the data of each synchronous machine but are influenced by the electric load of the machines and also slightly by the data of the governor mechanisms. Since the parts of the electric system oscillate in phase opposition, the frequency swings are moderate. The periods, under perfect conditions of the system, are of the order of 0.5 to 2 seconds; under imperfect conditions one or more frequencies may shift into the neighborhood of f_0.

The third and highest group of frequencies, f_2 in the accompanying figure 2 is caused by the inertia effects of the various induction motors, giving oscillations of a period of 0.1 to 0.2 second. These oscillations are damped, however, very quickly by the effect of the rotor slip of each motor.

All these oscillations are excited by every fluctuation of load or by any impact within the system. The subdivision of such impact on all the machines determines the amplitudes of the various oscillations, constituting subsequently the transient load swings within the system. Full-scale ex-

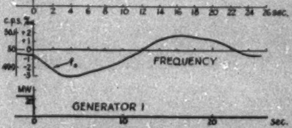


Figure 3. Effect of sudden separation of a generator from an interconnected system

periments with an interconnected system of scores of generators, totaling a power of 800,000 kva and subjected to a sudden impact of 24,000 kw showed initiated load swings and frequency oscillations, as in the accompanying figure 3, which were very close to the computed values.

During such transient oscillations there occur slow and fast exchanges of power between the various parts of the system, temporarily increasing the load on some machines and decreasing it on others, which can be detrimental to good operation. However, tuning conditions can be stated under which all these oscillations reduce to a minimum, thereby yielding the highest possible stability of the entire system. These conditions are: identical natural electromagnetic frequencies of all synchronous machines; identical products of speed droop of the governor with starting time constant of the inertia for all prime movers with their generators; identical closing times of the valve-operating mechanism for every prime mover Although in existing systems it is not always possible to fulfill these conditions completely much can often be done to approach them in order to quiet the operation under adverse circumstances.

Reference

1. Wissensch. Veröffentl. a.d. Siemens-Konzern, 1932, volume 11.1, page 60.

V. M. Marquis (American Gas and Electric Service Corporation, New York, N. Y.): From the answers to the questionnaire in the papers by Messrs. Dougherty, Hayward, Monteith, and Griscom, there does not seem to be any unanimity of opinion as to whether improvements are necessarily advisable on governing systems. This would tend to lead to the conclusion that a great many of us do not know too much about them and points to the desirability of field tests and studies to settle this question. Our experience with the operation of large interconnected systems shows the desirability of proper operation of governors with supplementary control, and that, no matter what was done to improve the governor, in many cases supplementary control in the form of frequency or combined frequency and tie-line control is necessary. Messrs. Dougherty, Hayward, Monteith, and Griscom state that change in steam pressure with change in load is another cause of excessive speed drop Experience certainly bears this out and points to the conclusion that for the ideal system we would really need a boiler that has little or no inertia.

A. P. Hayward: The discussions of this paper indicate a marked interest in the subject of governing, and the committee members responsible for the preparation of this report would like to have these discussions considered as an addition to the report.

One of the discussers has questioned the omission of hydraulic turbine governors from the report. It was realized when this report was being prepared that hydraulic turbine governors have been subjected to rather complete acceptance tests, and in general, have received more attention than steam governors during the initial operation period. The report was limited to a discussion of steam governors as the interest in the governing problem seemed to be based on a desire to learn more about the performance of steam governors.

We would like to summarize some thoughts which have already briefly been mentioned in the report. There is justifiable reason for an investigation of governor performance. The governing system is one of the most important protective devices installed on a prime mover, and probably has not received the attention from the operator that its importance would justify. Methods of determining governor performance and procedure for period tests comparable to similar tests of the electrical protective relays have not been developed on a practical basis.

There are several power systems which have experienced difficulties in maintaining a reasonable speed variation with the existing governors. This condition is the result in general, of supplying steel mill loads. One solution of these difficulties has involved the consideration of governors with low speed regulation and when the speed regulation has approached a value which might indicate the possibility of instability, consideration has been given to the utilization of speed droop compensators. These devices, which provide a means whereby the governor speed regulation can be varied, have considerable merit. They allow units to be operated essentially under base load conditions during full load periods, or when operating with an interconnected system. The governor can then be adjusted for variable load operation during light load periods, or when part of the system may be isolated from the main interconnected system.

A very definite reason for improving governor performance is to assist the electrical system in holding voltage variations to a minimum. The use of shaft exciters with pilot exciters provides an excitation system which produces a voltage variation approaching the third power of the speed change. Large load variations which affect the speed can produce voltage variations which are too fast for satisfactory voltage control. Improvements in the governor performance, which decrease the speed changes, will also decrease the voltage changes and result in more satisfactory overall conditions. Several operators are of the opinion that a lowering of the speed regulation will result in improved speed characteristic without increasing the regulating burden. While it has been shown that there is a limit below which it is safe to reduce the speed regulation of a simple governor, speed droop compensators are available as a means of reducing speed regulation and are operating successfully on the hydraulic units and several large steam units. It seems desirable to further investigate the usefulness of a lower speed regulation before discarding this proposal.

The over-all governor problem involves consideration of the following factors:

The condition of the mechanical speed governor and its ability to respond to speed variations.

The permissible limit of speed variations on the electrical system. This includes a consideration of the frequency variations and its relation to timekeeping, voltage variations resulting from speed changes, and load variations across tie lines.

The relation between governor action and operating conditions in the boiler plant or penstock conditions in the case of hydraulic turbines. The boiler plant should be able to assist system regulation to a considerable degree. Good governor performance does not imply the absence of load variations on the boiler plant.

The degree of perfection required of the governor is an economic balance between governor investment, governor maintenance costs, production costs as affected by governor performance, and expenditures required to realize the operating refinements such as constant frequency, time service, and constant voltage.

Effect of Prime-Mover Speed Governor Characteristics on Power-System Frequency Variations and Tie-Line Power Swings

Discussion and authors' closure of paper 41-23 by C. Concordia, S. B. Crary, and E. E. Parker, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941, pages 559–67.

Wm. A. Wilson (Westinghouse Electric and Manufacturing Company, Philadelphia, Pa.): It is gratifying that a paper has been written which further breaks down the demarcation between mechanical and electrical engineering existing at the turbogenerator coupling. Whereas the static relationship of the unit governor to its connected system is well understood, the dynamic unity of the individual machine and the electrically connected system has previously been given almost no quantitative expression.

Complex phenomena are predicted, even within the limitations of the simplifying assumptions employed. Figures 5 and 6 of the paper are examples of such complexity. The extent to which these theoretical results appear in practice will have to be determined experimentally. They will certainly serve as an excellent guide in the establishment of co-operative programs of investigation by manufacturers and operators.

I would like to ask if the differences in character of the system transients as illustrated by the paper's figures 15 and 16 are not less due to the absolute magnitudes of the disturbances than to the relative magnitudes of the frequency disturbances and the governor dead bands? If the systems were completely linear (without dead band) the shape of the transients would be independent of the magnitude of the disturbance. This means that figure 16 which illustrates the effect of an eight per cent load disturbance is probably more characteristic than is figure 15, corresponding to a two per cent load change. In particular, the differences in the system behaviors for cases *A*, *B*, and *C* during the first five seconds after the upset are almost nil in the case of the slight disturbance. For the larger disturbance these differences are striking and suggest an importance of the other system parameters at least as great as of dead band. The authors have described these differences but have, perhaps, not sufficiently emphasized the effects of the many system variables and the location and character of disturbances on the apparent quality of performance of a particular governor.

The method of attack chosen by the

authors has necessitated employing the simplification of using two equal time constants on each governing unit. They have amply justified this procedure, but the importance of a third or even a fourth constant even of small magnitude should not be belittled. Since virtually all governing systems include at least three time constants, the effect of the third on the stability has been calculated by the writer. Figure 1 of the paper has been extended into a third dimension with the addition of another time constant co-ordinate. (The basis for these calculations was a system identical with that considered by the authors except for the addition of a third time constant and a slightly smaller inertia constant—M = 3,800 instead of 4,000.) Actual space curves were constructed using arithmetic scales for the region of small time constant. These constructions were made for T_0=0 and T_0=1. A photograph of these curves is submitted as figure 1 of this discussion. Figure 2 is a rough logarithmic plot in two dimensions with T_0=1 and T_3 as parameter. It is apparent that the addition of a third constant is very significant in the region of small values of T_1 and T_2 and has the effect of largely nullifying the beneficial influence of load damping. In the region of very slow response, there is virtually no effect of a small added constant.

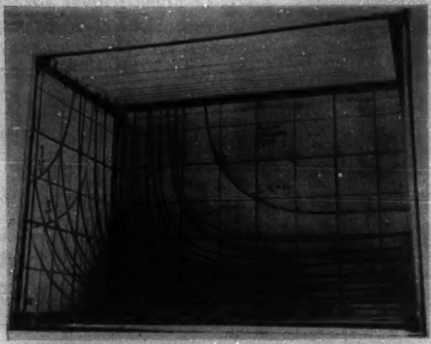


Figure 1. Effect of time lags on governor stability

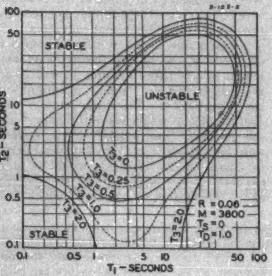


Figure 2. Effect of time lags on governor stability

It may be worth pointing out that the outer region of stability corresponding to large time constants is what makes possible the use of compensators. The minimum time constant permissible for a compensator is dictated by the location of this region on the stability plot for the regulation corresponding to compensation.

I would like to suggest adding to the paper's bibliography the Royal Society paper published August 1937, by Hartree, Porter, Callender, and Stevenson, entitled "Time-Lag in a Control System." This paper includes an exposition of the use of the Differential Analyzer in the solution of differential equations describing control functioning.

H. B. Seeley (Consolidated Edison Company of New York, Inc., New York, N. Y.): Messrs. Concordia, Crary, and Parker have done a very thorough job in exploring the influence of various governor characteristics and system characteristics on frequency stability of systems.

One of the interesting things which their analysis shows is that stable operation is possible for a rather wide range of governor characteristics and system characteristics. It does not appear necessary that each governor in the system should have the same over-all regulation and incremental regulation, the same dead band, and the same speed of response in order to avoid frequency oscillations. This is borne out by the successful operation of many systems having steam and hydro machines of many designs differing considerably in operating characteristics, yet working together in harmony. It would appear that if a majority of governors have small dead band, sufficiently fast response, and large enough incremental regulation, stable operation will result. Even though some units may have poor governor characteristics, the system may be free from sustained frequency oscillations if other governors in the same stations have stable characteristics. Frequency oscillations sometimes appear only when one particular machine in a station is operating by itself.

The authors have pointed out, that, in addition to the governor characteristics, the system characteristics have an important influence on frequency stability. Tests on the Consolidated Edison system illustrate the damping effect produced by load. During recent circuit breaker tests, the test breaker was supplied from a generator which was tied to another station in the system through one high impedance tie circuit. No system load was distributed directly from the generator terminals. The disturbance produced by the breaker test caused this generator to swing with respect to the rest of the system. These swings persisted for 20 to 30 seconds, dying down gradually, which indicated a relatively small damping effect for the generator and tie line alone. On the other hand, in several tests in which system load was distributed directly from the test generator, the swings were very rapidly damped, disappearing almost completely in $1^1/_2$ cycles of swing. This indicates that load distributed at the generator terminals may have a large damping effect on frequency oscillations. It would seem to be a fair conclusion that a system in which a substantial load is distributed directly from each generating station bus would tend to be considerably more stable with respect to frequency oscillations than a system in which some of the stations feed to the load centers through long transmission lines.

Tests made on the Consolidated Edison system did not disclose any swings which could be traced to governor action. Metropolitan systems in general may tend to be unusually stable with respect to frequency oscillations, particularly when:

1. All generating units are steam driven
2. Interstation ties are short, and
3. Load is distributed from each station

When such local systems are tied in with an extended interconnection, they may exert a considerable stabilizing effect on the combined system.

V. M. Marquis (American Gas and Electric Service Corporation, New York, N. Y.): The work carried out by Messrs. Concordia, Crary, and Parker is a valuable contribution to an understanding of the problem of system governing. It is noted that the authors state that the principal component of the tie-line swings usually has a period of from 0.5 to 5 seconds, while the swing of average frequency may have a period of from 5 to 60 seconds. On the central system of the American Gas and Electric Company, the inherent tie-line load swings have a period of about five seconds and we have measured average swings having a period of 120 seconds. The magnitude of the inherent swings varies roughly from 2,000 to 12,000 kw. The period of the measured inherent swings checks very closely with calculated values.

C. Concordia, S. B. Crary, and E. E. Parker: We are very pleased to note that both Mr. Seeley and Mr. Wilson are in general agreement with the results presented in our paper. Mr. Seeley has mentioned confirming system experience and tests, and in connection with these we are presenting figures 3 and 4 of this discussion.

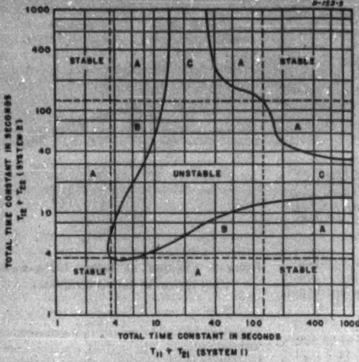


Figure 3. Two machines—effect of solid interconnection with various distributions of governor response

T_s or $T_{d12}=\infty$, $M_1=M_2=4{,}000$, $R_1=R_2=0.06$, $T_{d1}=T_{d2}=1$, $T_{11}=T_{21}$, $T_{12}=T_{22}$

In regions marked stable both machines are stable either separated or tied together

In regions *A* one machine is unstable *if* they are separated

In regions *B* both machines are unstable *if* they are separated

In regions C one machine is unstable if separated and both are unstable if tied together

In region marked unstable both machines are always unstable

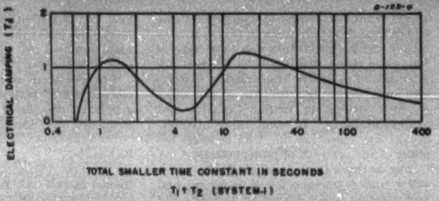


Figure 4. Two interconnected systems with different governor time lags

$M_1=M_2=4{,}000$, $R_1=R_2=0.06$, $T_{12}=T_{22}=16T_{11}=16T_{21}$, $T_S=0.1$
$T_{d12}=0$, $T_{d1}=Td_2=T_d$

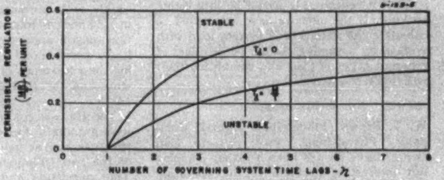


Figure 5. Governing system characteristics—effect of number of time lags on stability

T=total time constant, per unit

Figure 3 of this closure illustrates Mr. Seeley's remarks that even though some units may have poor governor characteristics the whole system may be stable if others have good characteristics, and that good interstation ties are beneficial. This figure shows that a solid tie between two systems one of which is unstable by itself, as in regions (A) of the figure, may be stable as a whole. There is also a small region (B) where the solidly connected system will be stable even if both systems are unstable by themselves.

Figure 4 of this closure is a further illustration of Mr. Seeley's comment that all units need not have identical governor characteristics for successful operation. This figure, with the two parts of the system having widely different speeds of governor response, indicates a somewhat greater degree of stability over the range considered than figure 5 of the paper, in which the governors were alike. However, figure 6 of the paper, for another kind of difference between the governors on the two systems, shows a slight advantage for uniform response. Figure 3 of this closure also indicates some advantage in diversity in governor response. If, for example, the two systems have responses such that they are at the lower right-hand intersection of the dashed lines, the systems are both on the border line of stability when separated but well within the stable region when solidly tied together. On the other hand, if the two systems have equal responses so that they are at the lower left- or upper right-hand intersections of the dashed lines, they are still on the border line of stability when tied together.

Mr. Wilson has mentioned the importance of considering more than two governor time constants, and has made a valuable contribution in this direction. As mentioned in the paper, we may expect the equality of the two time lags to compensate in some measure for their small number. However, it is true that all the results obtained must be considered as only qualitative.

Figure 5 of this discussion is presented in connection with Mr. Wilson's remark that under certain conditions the beneficial effects of load damping may be largely nullified by the addition of a third time constant. It may be seen from the figure that, although the addition of time constants decreases stability, still for any given governing system, load damping is very beneficial. It should be noted that actually a negative regulation is possible with stability with one governor time lag and with load damping, while the figure merely indicates that the regulation may be made as small as desired.

In regard to figures 15 and 16, as Mr. Wilson points out, the relative magnitudes of the frequency disturbance and governor dead band, rather than the absolute magnitude of the disturbance, is the significant factor in determining the character of the system transient. With a given dead band the relative magnitude of the disturbance of course decreases with its absolute magnitude, and we did not think that a 2 per cent load change and a 0.1 per cent dead band were unreasonable values to assume in order to get some idea of the relative performance of the various combinations during small load changes. With either larger load changes or smaller dead bands, the other system characteristics become more important.

We wish to thank both discussers for their contributions.

System Load Swings

Discussion and authors' closure of paper 41-16 by H. A. Bauman, O. W. Manz, Jr., J. E. McCormack, and H. B. Seeley, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941, pages 541–7.

Sherwin H. Wright (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The question, "What causes the observed power fluctuations and can they be controlled to advantage?" has been answered by the authors for their particular metropolitan system, and in so doing they may well inspire others to make related investigations. Most systems have noticeable power fluctuations in at least one section of the system. Sometimes a known widely fluctuating load exists and its effect masks the presence of other power fluctuations which in themselves may be unnecessary and controllable. In other cases a power fluctuation of unknown origin is tolerated unless it becomes unduly large. While widespread investigation, of power fluctuations in interconnections and systems, would not prove valuable in every case, it is none the less likely that in some cases profitable results may be obtained.

As the paper brings out, the expression "load swing" does not imply that all of the swings correspond to actual changes in load consumption or power consumption. Rather, as the paper shows in some detail, some of the swings such as the long-time and synchronizing swings are interchanged between generators or generating stations.

While it was found that the long-time

power swings were important largely because of boiler and fuel economy in this system of steam turbine generators, the same type of long-time swing would also be of great importance in extensive transmission systems and interconnections because of the effect of such swings on tie-line capacity and system stability. In some cases of interconnecting lines, the line load is lower because of the presence of power swings.

The authors have mentioned the possibility under emergencies of reducing real and reactive power output by lowering the voltage slightly and in figure 1 have plotted the measured relation for their system. The actual numerical relation of power output and voltage is of interest because of possible application to other systems, and it is also of importance in stability studies where knowledge of the composite load characteristics is necessary. It would be interesting to know if the authors have attempted to calculate the variation of system load with voltage by assuming certain percentages of different types of load and combining their individual characteristics.

W. T. Brown (Philadelphia Electric Company, Philadelphia, Pa.): System load swings have always been a rather complex but interesting problem to the operating engineer. The authors treat the subject from a practical and test standpoint in a very interesting manner. The observation in conclusion 2 that a three per cent change in frequency resulted in no noticeable load change is worthy of some discussion. For instance, numerous tests[1] on the Philadelphia Electric Company system indicated that one per cent change in frequency resulted in about 2.5 per cent change in power load. Thus test observations on two large electric utility systems have resulted in strikingly different conclusions.

The effect of change of frequency upon load apparently would depend greatly upon the nature of the load. Since induction motors and synchronous motors will reduce their speed with reduction in frequency of the supply system, I believe that the mechanical loads that such motors are driving would, in general, require less power. I would expect that railway load (d-c or a-c series motors) would not be appreciably affected by change in frequency. Lighting load, electric range, electric water heater, and similar load would not be affected appreciably by change in frequency. It would therefore appear that the tests made on the Consolidated Edison Company of New York were made in an area whose loads are primarily made up of lighting, railway, and kindred types of load and that the induction motor and synchronous motor load was probably a much smaller percentage of the total load than in the Philadelphia Electric Company system.

While the mechanical loads supplied by induction motors would decrease with reduction in frequency, I could conceive that that the power input might under certain conditions increase. If the transformers and induction motors are normally operated at a relatively high degree of flux saturation due to the operating voltage, then a reduction in frequency could result in an extremely high exciting current. This would result in an increase in power losses in the induction motors, transformers, transmission and distribution systems which may more than offset the decrease in mechanical load supplied by the induction motors, especially those of the smaller sizes.

I believe more extensive tests on various electric utility systems are required before general conclusions of the effect of change of frequency upon load can be drawn safely.

REFERENCE

1. REGULATION OF SYSTEM LOAD AND FREQUENCY, **Herbert Estrada** and **H. A. Dryar**, Philadelphia Electric Company, presented at the ASME annual meeting, Philadelphia, Pa., December 4–8, 1939, and published in ASME *Transactions*, April 1940.

John G. Noest (Brooklyn Edison Company, Brooklyn, N. Y.): The authors state that: "...... short-time load swings do not directly affect station economy unless they couple with automatic control equipment in a manner as to cause hunting of combustion control apparatus." That such coupling is possible and does occur is illustrated by the experience of the Hudson Avenue station of the Brooklyn Edison Company, Inc.

The Hudson Avenue station is equipped with automatic load-frequency control apparatus which actuates the governors of one of the two 160-megawatt turbogenerators. The magnitude and direction of control impulses to the turbine governor depend upon the frequency and tie-line feeder load deviation from standard. This equipment is designed to regulate for average values rather than for instantaneous ones and is therefore not capable of preventing tie-feeder load swings of as short a period as 0.5 to 2 minutes. It does, however, send impulses to the turbine governor which tend to diminish the tie-feeder swings.

The Hudson Avenue station is also equipped with automatic combustion control which actuates fan vanes and boiler dampers, varying their positions in response to steam pressure variations. The master combustion control regulators, as well as the individual regulators for control of fan vanes and dampers are equipped with fluid dashpot types of damping devices by means of which the time constants of response are adjustable.

These two sets of control devices furnish a coupling between the tie-feeder load variation and the combustion rate in the boiler. The first set, the load-frequency controller, translates tie-feeder load variations into variations of steam pressure by actuating the turbine valves. The second set, the combustion control, translates the variation in steam pressure into variations in boiler combustion rate.

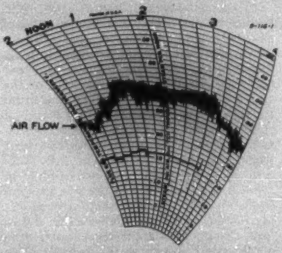


Figure 1. Section of Bailey boiler-meter chart typical of conditions as on September 4, 1940

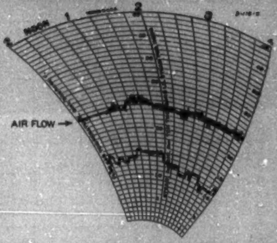


Figure 2. Section of Bailey boiler-meter chart typical of normal conditions

During the period from September 2 to September 10, 1940, increasing evidence of oscillation of the boiler control equipment was noted by the operators. Figure 1 shows a section of a Bailey meter chart of September 4 which is typical of the condition. It shows an oscillation of air flow of approximately 27 per cent and an oscillation in steam flow of approximately 10 per cent, the oscillation being periodic with a period of approximately three minutes. That a coupling existed via the frequency-load controller was established at various times by reducing the governor motor response drastically, which in all cases caused a complete cessation of the boiler oscillations. For comparison with normal conditions, figure 2 is given. The oscillations in boiler output did not appreciably affect the tie-line load. At the time of this trouble it seemed obvious that an increase in the fluid damping, obtainable by restricting the by-pass of the various dashpots in the individual regulators of the combustion control system, should solve the problem. When, however, adjustments were made in this direction, it was found that the oscillations increased in amplitude. Therefore the reverse procedure was tried. When the by-pass valves on these dashpots were opened, thus decreasing the fluid damping and with it the time constant of response, the oscillations ceased. The period during which this hunting of the combustion control equipment occurred coincided with the first cold weather and affected the viscosity of the oil in the various dashpots.

V. M. Marquis (American Gas and Electric Service Corporation, New York, N. Y.): The authors of the paper on "System Load Swings" are to be congratulated on the excellent series of tests which they conducted.

The authors found as outlined in conclusion 2 that changes in frequency did not appreciably change the load provided voltage was maintained constant. On one large interconnected system tests indicated a change of approximately one per cent change in load for a change in frequency of one-tenth cycle. These tests were made by increasing the output of certain plants by the amount necessary to increase the frequency one-tenth cycle, and by decreasing their output by an amount necessary to decrease the frequency by one-tenth cycle. Tests were carried out both during heavy and light load periods with the ratio of load change to total load remaining practically constant.

In connection with conclusion 3 it should be pointed out that the period of the magnitude and period of the inherent and average load swings on tie lines depend on the size and characteristics of the system.

S. B. Crary (General Electric Company, Schenectady, N. Y.): The information given in the papers by Mr. Wild and by Messrs. Bauman, Manz, McCormack, and Seeley regarding the variation of load with sustained changes in voltage and frequency and the damping characteristics of tie lines, is of fundamental importance for the proper study of system performance.

Both of these papers show that the solution of their problems of frequency and tie-line control were obtained chiefly by refinements in the supplementary control, either automatic or manual, which act on the speed governors. Our studies have indicated that the speed governor to fit into this scheme should operate with stability and respond accurately to the frequency and load control mpulses. Close frequency regulation and load allocation for interconnected systems is properly the function of the supplementary control.

Mr. Wild's comment as to what he considers to be satisfactory requirements for governor sensitivity and regulation is appreciated, and we hope that his work will encourage operating engineers of other systems to determine what they believe are rational requirements for their systems.

H. A. Bauman, O. W. Manz, Jr., J. E. McCormack, H. B. Seeley: In answer to Mr. Wright's question, we wish to state that calculations have been made to determine the effect of voltage on system load by assuming certain percentages of types of load and combining their individual characteristics. Since a test to determine the relation between load and voltage can be made very readily with the normal control and meter equipment, we feel that it is not worth while to depend upon elaborate calculations based upon unknown variables.

Messrs. Wild, Brown, and Marquis have presented some interesting data on the relation of frequency and load on other systems. It should be emphasized that the tests on the Consolidated Edison system were made at a station that served a representative block of load and was isolated completely from the system. The frequency was varied by throttle adjustment and the voltage held reasonably constant while the generator outputs were totalized on one sensitive metering device. During the test there was no noticeable change in load when frequency was varied about three per cent.

With the station connected to the rest of the system other tests have been made in which the system frequency was raised by increasing the generator output at one station. These tests indicated that it was necessary to increase the generator output about 20 megawatts in order to raise the system frequency 0.1 cycle. The total load increase appeared on the ties and was about one per cent of the system load. The graphic meters at the other stations showed no apparent load decrease, but we believe that there was a proportionate decrease in load at the other stations because of the drooping characteristics of turbine governors. The sensitivity of the graphic meters is such that this 20-megawatt change in load proportionately distributed would not be readily perceptible against the background of short-time load swings. This latter test is not to be confused with the test reported in the paper where a representative block of load was completely isolated and totalized on one meter.

In addition to the data reported in the paper there were several by-products which proved valuable. A few operating problems developed in connection with consumer utilization equipment and several minor operating weaknesses were revealed on our system and later corrected. Clarification of these difficulties has simplified the task of the operators if the system should separate during trouble. The most valuable by-product developed during the tests was the interest of the station operators and their general appreciation of the problem. The willing application of the knowledge obtained and the use of the newly installed common frequency standard have resulted in a reduction of the load swings so that they are no longer a problem within the Consolidated Edison system. We feel that these by-products in themselves more than paid for the tests.

Methods of System Control in a Large Interconnection

Discussion and author's closure of paper 41-80 by Earle Wild, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (May section) pages 232–6.

E. E. Stevens (Commonwealth Edison Company, Chicago, Ill.): Mr. Wild's paper covers a large field in what has been and what is being done to cope with the interesting problems of frequency, time, and load control among the large interconnected power groups of this country. From the results obtained during the past several years through the conscientious efforts of those responsible for this type of control, I believe that in general a high degree of co-ordination has been attained.

However, as in practically all instances of engineering endeavor, we have not and probably never will arrive at a point where we are willing to let well enough alone.

The ultimate in co-ordination of this type of control, as I now see it, would be the employment of one reference frequency-time standard common to a whole interconnected power system. This standard would probably be in the form of a crystal controlled oscillator.

Another interesting problem is in the automatic control of the output of generating stations in maintaining them on a predetermined load schedule. This form of control should be centralized with respect to the generating stations of an operating company. The logical location would be in the load dispatcher's office.

Another step in the control problem is the automatic control of the loading of individual generators in order to operate them at good efficiencies. This control would naturally be under the supervision of the generating station personnel.

P. B. Juhnke (Commonwealth Edison Company, Chicago, Ill.): With reference to the paper by Mr. Earle Wild, dealing with methods of system control in a large interconnection, these remarks appear warranted.

It is interesting to note that in the particular interconnection in which the methods described in this paper first came to fruition, the basic principle of regulation was agreed upon at a meeting of representatives of the group about 15 years ago. This principle is the one outlined in the paper and specifies that the work of regulating frequency shall be carried on by one given station, while interconnected systems shall merely co-operate, the degree of co-operation depending upon the burden it is expected to place on the interconnected systems. This principle basically is still in vogue. It has been modified by frequency bias and time interlock, but essentially it is the same old principle that is working well, aided materially by the modifications which have been effected.

Next I would like to recall Mr. Sporn's suggestions of some years ago, to install frequency controllers in most of the principal stations of the interconnection and let them all operate in unison. That proposition was different in principle from what we have been conforming to and from what we are doing today, and could be made to work only if we had regulating impulses to guide all the participating stations and, what is equally important, proportionately equal increment or decrement responses. Mr. Sporn's proposal, on account of the difficulties in the way of all conditions for its successful operation, was never put into effect, but the proposal was helpful nevertheless. It was helpful because it resulted in the installation of a number of frequency regulators in various parts of the interconnected system and the co-ordination of these various regulators in the manner described in Mr. Wild's paper. Today this system is equipped to function with a degree of perfection unthought of some years ago, provided all participating parties discharge their full responsibility in the matter of providing necessary reserve capacity required to meet all emergency or ordinary demands of a widely spread out system.

Mr. Wild also devotes considerable space to the relation of controllers to governors. With proper and reliable frequency controllers, if employed adequately, the sensitiveness and regulation of governors become less and less important, except during those brief periods when large variations in the ratio between load and capacity take place. The more important requirement in the regulating turbine responding to such controllers is a more adequate valve characteristic, so that the response of the machine may more closely conform to the dictates of the frequency regulator without excessive reversals of contributions.

In figure 1 the diagram might have been a little clearer if it contained another set of three lines from bottom left to top right, depicting the condition obtaining for a tie line feeding in. The three lines in figure 1 now show this condition only for the tie line feeding out.

H. A. Dryar (Philadelphia Electric Company, Philadelphia, Pa.): Mr. Wild's description of the control of frequency and of tie-line loading of the largest interconnected group of systems in this country is of particular interest as a presentation of the method of regulation found to be satisfactory to the great number of power systems comprising this large group.

The great inertia of these tremendous reservoirs of power to change in speed and the continued presence of the frequency control problem may be said to indicate a failure through the years to recognize the fundamental problem in speed control as that of satisfactory turbine governor operation. Efforts of the past two years have served to crystallize opinion in this respect as former assumptions have not withstood a more thorough investigation.

The great inertia of these tremendous reservoirs of power to change in speed may be reflected in a number of ways:

1. A single regulating station would be subjected to excessive variations in output.

2. Changes in generations to bring tie-line loading to schedule.

(*a*). May affect the speed but slightly if turbine governors are sensitive to such changes in speed.

(*b*). May affect the speed appreciably if turbine governors have too broad a dead band.

(*c*). Subsequent correction of the speed in (*b*) may again find the governors active, upsetting tie-line loading again and resulting in seesawing of frequency and loading of stations.

3. Changes in system load that find turbine governors inactive will cause appreciable changes in speed. Again manual correction may initiate seesawing of load and speed as governors again become active.

4. If turbine governors are sufficiently sensitive, the self-regulating characteristic of interconnected systems should increase with their size as diversity in the load swings are greater.

Consistency of Governor Response

When a speed change occurs and manual changes in the output of a generating station are directed with the frequency meter as a guide, lack of consistency in governor performance creates considerable confusion in the mind of the load dispatcher as to what measures should be taken.

Where governor dead band is small, appreciable changes in the speed will indicate an appreciable change of load. The relatively small change in generation made to bring tie-line loading to schedule would have a negligible effect on the speed. Thus the load dispatcher may proceed with assurance that the measures he directs will be the proper remedy.

Scheduled Tie-Line Loading

The economical operation of a group of systems does not differ from that of an individual system. Any change in the interconnection load is allocated to the marginal system in the same manner that a change in the load of an uninterconnected system is allocated to its marginal station. Thus the loading of the interconnection tie lines may vary in amount and direction throughout the day in the same manner as the loading of the system ties may change.

Frequency Effect on the Load

The effect of change in speed on the system load is not synonymous with the effect of a load change on the system speed as seems to be implied. The general rule to which reference is made may be "a one per cent change in the load will cause a change in speed of one-tenth cycle." This ratio will differ from that of other systems if there is a difference in the response of turbine governors. In the interconnected group of systems of which the Philadelphia Electric Company is one, this ratio, when governors are active, is as follows: A change in load of two per cent will cause a change in speed of one-tenth cycle.

Correction for Droop

As the system load increases, the speed will decrease. If the turbine governors are sensitive to a change in speed, the decrease may be negligible. If the turbine governors do not have a narrow dead band, the change in speed will be appreciable.

Where governors have a narrow dead band, correction for droop as a result of load changes occurring on the plateau of the load curve may be considered unnecessary as speed changes will occur within a narrow frequency band, one counteracting the other, due to the regulating characteristic of responsive governors. During the normal heavy load change periods of short duration, regulation of the load and frequency does not present the same problem as variations are in one direction and changes in loading may be directed without confusion.

Regulating Capacity

In the operation of an interconnected group of systems with approximately 100 generators or more, operating in parallel, the availability of an appreciable number of these units with sensitive governors would find load changes absorbed with negligible effect on the loading of each generating station and relatively little sustained change in the loading of tie lines.

Conclusion

Greater sensitivity of the governors, more uniform response throughout its range, and a reduction of the average speed regulation should provide more consistent control for all changes in speed regardless of degree and promote intelligent regulation of the load.

V. M. Marquis (American Gas and Electric Service Corporation, New York, N. Y.): We are in general agreement with the ideas expressed by Mr. Wild in connection with the speed control of large interconnected systems. The central system of the American Gas and Electric Company operates in parallel continuously with a company which is in turn interconnected with the Chicago system. Frequency is held automatically at our Philo and Windsor stations. Combination automatic frequency and tie-line control is in operation at our Twin Branch, Cabin Creek, and Claytor stations.

As to Mr. Wild's fourth conclusion I am not sure that we know enough about our governors to say that the sensitivity and per cent regulation may vary inversely with the size of the system and nature of the loads. I, of course, note that Mr. Wild does not make a definite commitment on this.

S. B. Crary: For discussion, see page 737.

Earle Wild: The author is pleased with the many favorable comments which he received in the discussion of the paper. In general, there was no disagreement with the principles which were stated in the use and application of the auxiliary speed control devices.

The question was raised by several as to correctness of the figure of a one-tenth-cycle frequency departure causing a one per cent change in system load. In the paper by Messrs. Bauman, Manz, McCormack, and Seeley, the figure indicated no change in load, while in Mr. Dryar's discussion there was an indication that this figure might be even larger than one per cent. It is generally felt that this characteristic is the function of the amount of large industrial load in any system, and it is very likely that the different types of load served in different areas produce the variations in the frequency-load characteristic. The figure quoted was the result of tests which were made at three different times of day, when the frequency was both lowered and raised to produce the load changes.

Mr. Dryar's discussion regarding the relation of governor action to frequency changes is appreciated. In general, there can be no disagreement with the fundamental principles which he states regarding need for sensitive governors. The speed governor is responsible for holding the instantaneous frequency. Now that means are available for testing governors, they should be used and steps taken to correct any excessive insensitivity that might be found. Mr. Dryar, however, probably does not appreciate that the auxiliary speed controllers are essentially load controllers. Probably 95 per cent of the time the frequency is within such close limits that the controllers are responding to real load changes occurring locally and not correcting frequency deviations. The speed governors, if left to themselves, would be allocating this load generally on all turbines of the interconnection. In Mr. Dryar's system, although it is large in capacity, it is compact with large transmission links, and all stations are on the call of one load dispatcher. This is vastly different from the large interconnection covering 16 states, 2 time zones, no plateau load period except for a few hours on Sunday mornings, and a variety of power flow agreements between the many groups. It is under these conditions that the auxiliary speed device has proved so practical on this interconnection, and, as pointed out in Mr. Juhnke's discussion, in the aggregate has given very satisfactory frequency and integrated time. Mr. Dryar's statement that the operation of a group of systems does not differ from that of an individual system may be theoretically true, but in practice each large group in the larger interconnection must handle its own load control problems locally.

In addition to the question of sensitive governors, the other point which has been raised concerns a decrease in the percentage regulation of the governors. This is quite debatable at the present time, especially on systems where the agreements require strict adherence to tie-line load schedules. The author personally feels that in the future it may be practical, after more thought has been given to better maintenance of the existing governors, to consider means for decreasing the per cent regulation.

System Stability

Discussion and author's closure of paper 41-49 by F. W. Gay, presented at the AIEE winter convention, Philadelphia, Pa., January 27-31, 1941, and published in AIEE TRANSACTIONS, 1941 (April section), pages 168-73.

S. B. Crary (General Electric Company, Schenectady, N. Y.): The paper by Mr. Gay raises some interesting questions as to the phenomena associated with the pulling into step of units of a system which have lost synchronism with each other. The associated problem, prevention of loss of synchronism has been given a great deal of attention, resulting in rational system design practices from the instability prevention point of view. Among these has been a more general use of quick relaying and switching, particularly at locations near important generating stations and along the heavily loaded transmission lines. Although intelligent preventive means may greatly reduce the number of cases of instability, it may still be advisable to have a cure for those cases when the relaying and switching do not function properly.

One of the difficulties, pointed out by Mr. Gay is the tripping of overspeed devices following loss of synchronism. When a unit is out of step, it is subjected to a pulsating electrical torque at slip frequency. This torque produces a pulsation in the speed of the unit resulting in an increase in the instantaneous overspeed above that which would exist if this pulsating torque did not exist. The magnitude of the maximum instantaneous speed that may result due to switching off a fault at the most disadvantageous point (180 electrical degrees with respect to the system) may be expressed as,

$$\Delta S=\sqrt{\frac{T_m}{\pi f H}+(\Delta S_0)^2}$$

where

T_m = equivalent maximum of sine wave of pulsating component of electrical torque in times normal

$H=\dfrac{0.231\times WR^2\ (\text{rpm})^2}{\text{kw}\times 10^6}$ inertia constant

ΔS_0 = average per unit overspeed, corresponding to that of no pulsating component

ΔS = maximum instantaneous per unit overspeed

Figure 1 shows the relation between the maximum instantaneous overspeed and the average overspeed as a function of the pulsating component of electrical torque, T_m, and inertia constant, H. On the basis that the speed governor will prevent an overspeed of ten per cent on loss of full load, the average overspeed during a system disturbance will be less than ten per cent because of electrical losses during the fault. However, upon clearing of the fault the unit may be running out of synchronism and subjected to a pulsating electrical torque of appreciable magnitude. Fortunately, however, with the clearing of the fault, the average electrical torque due to the braking and induction generator torques will be appreciable on any unit running above average system speed, so that the average speed of the unit will be rapidly decreased. However, if the fault is cleared when the unit is between 180° and 360° out of phase with the system and during this angular interval the prime mover mechanical torque reaches about zero due to the closing of the steam valves by the governor, the instantaneous overspeed will tend to be a maximum, other conditions remaining the same. If for example, a severe three-phase fault occurs just off the generator bus when the generator is carrying full load such that the average losses during the period the fault is on are comparatively small, the unit may have a maximum average overspeed of six to 8 per cent. Then, if the fault is cleared when the unit is 180 degrees out of phase with the system, the instantaneous overspeed may exceed ten per cent, if the magnitude of the resulting pulsating component of electrical torque is sufficiently large and the inertia of the unit sufficiently small. See figure 1.

It will be noted from figure 1, that if the inertia constant is comparatively small, $H=3$ corresponding to a noncondensing unit and the pulsating torque sufficiently large, $T_m=5$ a maximum instantaneous speed of ten per cent may be obtained with a very small average overspeed.

The conditions which are most apt to lead to tripping of the overspeed device from this cause, with a governor which prevents tripping or complete loss of load are:

1. A severe three-phase fault just off the generator low or high tension bus.
2. Prolonged switching time of the order of 30 to 60 cycles.
3. High kilowatt load on the station, for example carrying a kilowatt load corresponding to full kilovolt-ampere rating.
4. Small turbine-generator inertia constant. For example, 3,600 rpm noncondensing units have a lower inertia constant than 1,800 rpm condensing units.
5. Low overspeed device setting, for example, set below ten per cent.

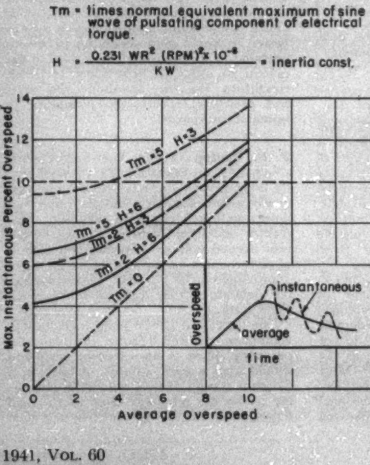


Figure 1 (left). Effect of negative half loop of synchronizing torque during out-of-step condition increasing the instantaneous overspeed above the average overspeed

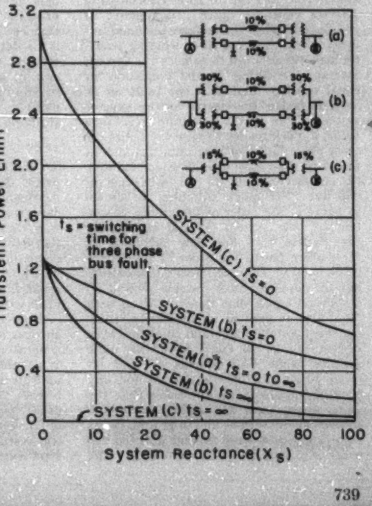


Figure 2. Transient power limits following three-phase faults on system having three-winding transformers compared with more conventional systems

All of these conditions need not exist at once, but they all tend to increase the possibility for tripping. Of considerable interest, is that the quicker the switching time even if it is not sufficiently quick to prevent loss of synchronism, the less chance there will be for high overspeed as the electrical braking torques will become effective sooner. Also, this will reduce the time required for pulling back into synchronism. Accordingly, the quicker the removal of faults, the more quickly the system will regain synchronism and the less chance there will be for tripping of the overspeed devices, even if the switch time is not sufficiently fast to prevent loss of synchronism.

The use of field adjusters to increase the field strength upon low voltage is an effectvie way to increase the steady-state pull-out torque of a generator and may prove particularly so if the unit is operating initially at light load upon the occurrence of a system disturbance, resulting in low frequency and subsequent increase in loading due to the governor opening the steam valves.

If the unit overspeeds when out of synchronism with the system, the more usual case, the field adjusters will increase the generator braking torque which varies as the square of the field current and also increase the synchronizing torque which varies directly with the field current, both effects increasing the possibility for resynchronizing with the system. One of the important considerations in the problem of pulling into step, is the desirability of accomplishing it quickly in order to prevent excessive heating and damage of the rotor structure.

The use of three winding transformers especially designed to reduce the fault severity on a bus section, as discussed in the paper, accomplishes this, but with a reduction in the power limit over the remaining healthy circuit after the faulted line or section has been switched off. Figure 2 of this discussion shows the transient power limits for a three-phase bus fault, as a function of system reactance for the three transmission systems shown in figure 1 (appendix) of the paper. The low-voltage terminal reactance branch of the three winding transformers was assumed to balance the system reactance, resulting in the condition of equal stability limit with the fault on or with the faulted line cleared for this type of system. This is the optimum reactance value, if it is intended to raise the limits for three-phase faults of long duration and at the same time have equally good stability for an insevere fault or just switching off one line. For quick switching as shown in figure 2, the order of preference is system (*c*) over system (*b*) and system (*b*) over system (*a*). For slow switching of three-phase faults, the order of preference is (*a*), (*b*), (*c*). However, with quick switching, the power limits of systems (*b*) and (*c*) are always better than those obtained with (*a*). Also, it may be impossible to transmit rated power for any fault condition with system (*a*), as all faults have the same severity, while with systems (*b*) and (*c*) rated power can be transmitted for any type of fault, with the provision that the more severe ones are quickly switched. Insevere faults such as line-to-ground faults occurring with systems (*b*) and (*c*) need not necessarily require quick switching. The advantages of this method (system *a*) can be compared by analysis against its disadvantages for any given system. However, it appears that the advantages are necessarily limited, as has been previously pointed out, to closely coupled systems having relatively short transmission line distances and where reliance cannot be placed entirely upon the relaying and switching to function properly.

Sherwin H. Wright (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The bulk of the engineering studies and literature on system stability has been devoted to means of maintaining stability and determining stability limits, and while the author has devoted attention to preventive measures, he has also given proper attention and consideration to curative methods to be used in case stability is lost. This is a logical step and that it has entered into system engineering is evidenced by the many applications of out-of-step relays to machines and lines where tripping action is taken after instability is detected.

The efficacy of quick response excitation used as a part of an automatic voltage regulator has been well proved in operation, and since the field adjuster mentioned by the author is a form of quick response, it should give the same benefits as on other systems.

Instead of using three undervoltage relays, one for each phase, the author can simplify the low-voltage relay scheme in cases where only three-phase machines are involved, by utilizing only positive-sequence voltage and a single relay with it. This is because the synchronizing power or pull-out torque of three-phase machines is determined largely by the positive-sequence voltage.

In the closing part of the paper the author suggests the further development and field trial of the several remedies or methods considered. It is believed that most of these controlled methods or remedies have already been developed and applied in one form or another to such a point that the only further development required is to determine just how far each control method need be carried in the particular system involved. This can best be done by the method indicated in the appendix—namely, by making a thorough system study on an a-c network calculator.

J. W. Jones (Philadelphia Electric Company, Philadelphia, Pa.): The paper is particularly timely and interesting in view of the expansion of power systems now under way and the increasing use of pulverized fuel boilers which are so susceptible to shutdown in case of a drop in supply voltage. Many ideas are suggested for improving the reliability of power systems and these ideas will warrant careful consideration.

A reference is made to field adjusters to automatically strengthen the field of generators in case of subnormal voltage. While these are undoubtedly a considerable improvement over no regulation, I do not believe they will be as satisfactory as the usual type of voltage regulator. Most of the generators on the Philadelphia Electric system are equipped with conventional type voltage regulators, in some cases having high-speed features, and no particular trouble has been experienced in their operation or in allocating reactive load as desired by the system load dispatcher. Field current is kept within safe values by means of limit switches on the field rheostats and fixed resistances in the field circuits.

The paper suggests a method of back-up relaying by means of separate low-voltage relays for each individual phase. It might be preferable to have a single low-voltage relay arranged to operate on the average three-phase voltage or the positive sequence voltage which might be obtained by means of a polyphase rectifier or a network. It also seems possible that the complication of a storage battery could be avoided by the use of available types of low-voltage relays.

The three-winding transformers mentioned in the paper which have negative reactance in one winding are a considerable departure from normal design and it would be interesting to know if this involves any construction changes that have an adverse effect on reliability or efficiency.

As pointed out in the appendix of the paper the use of the special three-winding transformers having negative reactance, improves stability while the fault is on the system, but after one part of the system is opened to clear the fault, the stability condition is less satisfactory than with the more usual transformer arrangement. Since most faults are cleared from the system in a few cycles and parts of the system may have to be kept out of service for an extended period it is questionable if the special three-winding transformers are worth while.

In considering voltage at the generator terminals under fault conditions for the two systems shown in figures 2 and 3, it is apparent that while the generator connected to transformer *T*-2 on one side of the system in figure 3, using standard transformers, may have relatively low voltage under fault conditions, there is a correspondingly greater voltage for the generator connected to transformer *T*-1 on the other side of the system, resulting in less possibility of losing both of the generators in case of a single fault. Under some circumstances it may be better, to permit some generators to become unstable or to drop load on a part of the system in order to have more favorable conditions for the remaining machines, and therefore, reduce the possibility of losing all equipment.

F. S. Benson (Pacific Gas and Electric Company, San Francisco, Cal.): We heartily concur with the recommendation that power plant auxiliary motors have sufficient pull-out torque to maintain satisfactory operation down to at least 60 per cent normal voltage. Oscillograph records of disturbances on our system show that at approximately 60 per cent normal voltage our generators often begin to swing against the system.

The suggested power plant loud-speaker equipment controlled from the load dispatcher's office is an excellent idea and should give a good account of itself in a compact system, that is, where the generating plants are relatively close to the load dispatcher's office. However, in a system where generating plants are hundreds of miles away and there are many such plants to be controlled, the loud-speaking equipment does not seem to be the practical solution.

It is difficult to determine how the generator field adjuster mentioned in the paper differs from the high-speed field excitation equipment now furnished with generators; additional information on this device would be appreciated.

The proposal that three-winding transformers having a high equivalent primary negative reactance be used is most interesting; however, the voltage rise to 200 per cent on the unfaulted line seems to preclude this scheme from being generally used as the connected equipment and lines should be constructed to withstand the insulation stresses due to double voltage. It seems, at this time, that high-speed relay and circuit breaker operation is still the better method to obtain the desired system stability.

F. W. Gay: Referring to figure 2 of Mr. Crary's paper, it is important to note that the upper two curves in this paper represent switching operations only. A fault which exists on a system for zero time is no fault at all. Since this paper considers only faults not promptly cleared, and since oscillograph records show that nearly all high-voltage single phase-to-ground faults develop into phase-to-phase ground faults in from 8 to 15 cycles and further develop into three-phase faults in a few more cycles, we must generally consider all sustained faults as three-phase faults, with the exception of pure phase-to-phase faults out on the span caused by "galloping" conductors. With a cross wind of gale proportions, a phase-to-phase fault may remain a phase-to-phase fault until a line burns down.

For sustained faults the upper two curves of figure 2 in Mr. Crary's paper must be thrown out and only the lower three compared. Fortunately, immediately following the clearing of a sustained fault, the system voltage (at least adjacent the fault) instantly rises to a high value and with properly designed field regulators or field adjusters on near-by generators falls slowly to normal value. The power limit varies as the square of the voltage, and in normal operation on a high-capacity system, the transient power limit following a sustained fault should not be much below the steady-state condition shown in figure 5*a*.

Field Power-Factor Testing of Transformer Insulation and Operating Experience

Discussion and author's closure of paper 41-31 by E. W. Whitmer, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941, pages 605–11.

P. L. Bellaschi (Westinghouse Electric and Manufacturing Company, Sharon, Pa.): The author of this paper has presented a fine contribution on the usefulness of the power-factor test as a method for determining the condition of apparatus insulation in service. The power-factor test is a tool which, when properly applied, can give effective results. Paradoxical as it may seem, while the power-factor test serves a useful purpose in the field, its usefulness at the factory appears as yet to be rather limited. I have reference in particular to transformer apparatus. The reason for this becomes apparent from a consideration of the purpose each of the various tests primarily fulfills.

As the author states, in 90 per cent of the cases high power factor of insulation in the field is associated with the presence of moisture diffused in the solid insulation. A capacity higher than normal naturally is also recorded. In the other ten per cent, high power factor is attributed to the oil due to one cause or another. The remedy on the one hand is to dry out the insulation, and on the other to recondition or replace the oil. Both conditions are the result of service.

At the factory, 60-cycle and impulse dielectric tests have a distinct value for these are the tests that verify and check insulation design, workmanship, materials, etc., and that do establish the insulation levels built into the apparatus. A simple illustration is the case of a 115-kv insulation design that was tested to destruction. The power factor before and after the impulse tests, even though the insulation actually was destroyed, was the same, 0.60 per cent in both measurements.

It will be of interest to compare the limits of power-factor values as recorded on some 150 new power and oil-insulated instrument transformers. For the higher voltage classes (46 kv and above), power-factor values within the limits of 1.5 to 0.5 per cent are typical. Higher values in the range of 1.5 to 3.0 per cent are encountered in some of the lower voltage classes. In extreme cases, a value of four per cent may be found. All of these values are for a temperature of 20 degrees centigrade to 25 degrees centigrade.

The correction factors for the power factor versus temperature referred to room temperature are summed up in the following tabulation. The values in column 2 are representative. These approach the correction curve E. W. Whitmer presents in figure 5. Other values encountered are given in columns 4 and 5 and at the bottom of the tabulation. It is clear from this that no single set of correction values is generally applicable, other than as an approximation.

Power Factor of Transformers Versus Temperature (Multiplier)

Temp (Deg C) (1)	Multiplier for 20 Deg C (3,000 Kva, 14.1/2.5 Kv HV to LV and I) (2)	Multiplier for 20 Deg C, (Whitmer, Figure 5) (3)	Multiplier for 20 Deg C (450 Kva, 2.4/0.24 Kv)		Multiplier for 20 Deg C, (Ghen, EEI Journal) (6)
			HV to LV and I (4)	LV to I (5)	
20	1.00	1.00	1.00	1.00	1.00
25	0.76		0.875	0.86	
30	0.59	0.71	0.78	0.77	0.66
35	0.47				
40	0.39	0.48	0.65	0.62	0.42
45	0.32				
50	0.27	0.33	0.58	0.54	0.28
55	0.23				
60	0.194	0.23	0.53	0.48	
65	0.163				
70	0.137	0.16			
75	0.114				
80		0.11			

Other Examples of Multiplier From Experimental Data

Temp (Deg C)	Mult.	Temp. (Deg C)	Mult.	Temp. (Deg C)	Mult.	Temp. (Deg C)	Mult.	Temp. (Deg C)	Mult.	Temp. (Deg C)	Mult.
17	1.0	19	1.0	20	1.0	19	1.0	19	1.0	22	1.0
50	0.5	62	0.41	55	0.55	52	0.62	61	0.36	62	0.41
17	1.0	19	1.0	20	1.0	19	1.0	19	1.0	22	1.0
50	0.35	62	0.33	55	0.51	52	0.61	61	0.49	62	0.51

From the foregoing, the importance in selecting a suitable reference temperature should be noted. In our development tests at the factory, the capacitance was found to be relatively unaffected by the temperature, which fact is a measure of the high degree of drying of the insulation.

V. M. Montsinger (General Electric Company, Pittsfield, Mass.): This paper shows that considerable progress has been made since Mr. Gross published his paper in 1938, in using the power-factor method for the purpose of detecting moisture in transformer windings and bushings under service conditions. I note that the author used the words "insulation deterioration" in several places in the paper. In the usual sense, "deterioration" means that the insulation has been weakened either electrically or mechanically, and cannot be brought back to its original condition. A study of the records in the paper, however, shows that the power-factor tests indicate only the presence of moisture in transformer insulations. I am wondering if a more suitable word or words could not be used to describe the moist insulation.

The author states that faulty bushings can now be detected without disconnecting them from the transformer. In the cases shown where this was done, the power factor of the bushing was quite high, considerably above the minimum value that would be considered safe. I would like to ask Mr. Whitmer if he feels that a bushing, whose power factor is slightly above the safe value, can always be detected without disconnecting it? Whether or not this can be done, the paper shows that operating engineers now have available a promising means of preventing some transformer and bushing failures.

Eric A. Walker (University of Connecticut, Storrs, Conn.): The purpose of field tests on the insulation of any piece of high voltage equipment is to predict the safe operating life of the insulation. In these tests the quantity measured is the power factor and, although there are few, if any, curves relating the breakdown strength of insulation to the power factor, or curves showing a correlation between expected life and the power factor, some such relationship must exist or the method would not have achieved success.

Granting that some relationship does exist which makes a measurement of power factor a useful one, there are two questions which present themselves: First, if a dangerous condition exists in the insulation of a bushing or a transformer, is it usually a localized condition existing only in a limited area or does the condition tend to become general throughout the insulation? Second, is any such fault a linear one which is as prominent at low voltages as it is at high voltages? When these two questions are answered the scope of the tests can be defined.

Let us review the first of these questions in more detail. Suppose a square foot of insulation is being tested and one out of the 144 square inches is defective and all others are in good condition. Then a power-factor test will not reveal the defect. Taking a more specific case, suppose the power factor of the defective square inch is 0.9 and all other areas were perfect or had zero power factor. Then, if one assumes the capacitance per square inch is uniform, the over-all power factor would be about 0.015. This figure is not usually a reason to classify the insulation as defective. However, the test records for bushings and transformers show that few faults have escaped being predicted by a well-interpreted power-factor test. Therefore, the conclusion which must be drawn is that most insulation faults in bushings and power transformers are not of the spot variety but are a result of more general deterioration in the insulation. As the usual cause of deterioration is the entrance of moisture and as the moisture will rapidly permeate cellulose insulation, such results are reasonable.

The second question has to do with the linearity of the fault. It is well known that corona and other forms of gaseous discharges do not occur until a certain critical voltage is reached. It is also conceded that any such discharge in solid insulation is harmful. It is not without reason, then, that some have questioned the adequacy of 10 kv for testing bushings and transformers in the higher voltage classes. Yet again, the record shows that the majority of faults have been successfully diagnosed and a second conclusion must be drawn that the majority of insulation failures in bushings or transformers are not caused by gaseous discharges which are only detectable at operating voltage or, if the faults are of gaseous nature, they do permanent damage to the insulation before failure and this permanent damage can be detected at voltages under the corona voltage.

The next forward step in power-factor testing is the testing of cables and generators. Again, the same two questions must be asked. If faults in cables and generators are caused by over-all, rather than spot deterioration, and, if the deterioration can be found at low voltage, the test will become an important one. Obviously, these questions cannot be answered before a considerable amount of data has been compiled.

H. L. Cole (Westinghouse Electric and Manufacturing Company, Sharon, Pa.): Mr. Whitmer has given us some interesting and valuable data on power-factor testing of oil-insulated transformer windings in the field. The results show that this method of testing, when carried on systematically, will indicate general insulation weakness in time to remove a transformer before failure.

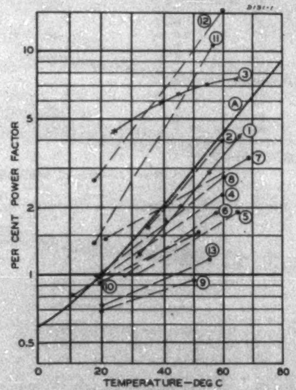


Figure 1. Factory power-factor tests on new transformers

A. Whitmer curve "A"

1. 3,000 kva (22 kv)
2. 65,000 kva (287 kv)
3. 450 kva (2.4 kv)
4. 24,000 kva (161 kv)
5. 10,000 kva (138 kv)
6. 10,000 kva (161 kv)
7. 3,333 kva (115 kv)
8. 667 kva (34.5 kv)
9. 750 kva (40 kv)
10. 333 kva (25 kv)
11. 1,000 kva (13.8 kv) Inerteen transformer
12. 1,000 kva (13.8 kv) Inerteen transformer
13. 500 kva (34.5 kv)

While many power-factor tests have been made at Sharon on new transformers, the power-factor method of insulation testing has been used mainly on solid insulation and oil, separately, rather than on a combination of the two. The power factor of new transformers is usually low (from three-fourths per cent to four per cent at 25 degrees centigrade). Differences in low power factors are difficult to explain. It may be that with further investigation these points will be explained.

It has been shown by test that high power factor which is due to acid oil or the presence of dipoles does not affect the impulse or 60-cycle strength of oil and pressboard structure adversely. Where free moisture is present (as is often the case in the older types of transformers) the high power factor and low strength will go together, but a considerable increase in power factor takes place before the dielectric strength drops—a fortunate thing.

On the other hand, a low power factor may not be an absolute indication of high impulse or 60-cycle strength. Damage to the solid insulation, either electrical or mechanical, can produce a condition which will not affect the power factor because the oil will fill up the damaged insulation. Low power factor is therefore principally an indication that there is low conductivity (due to absence of moisture and acid) and freedom from contamination.

The above points are mentioned here only to indicate that there are limitations in significance of the low-voltage power-factor test which should be recognized, particularly in regard to tests on new transformers.

In connection with figure 5 of Mr. Whitmer's paper, the accompanying curve shows some temperature power-factor data taken at Sharon on transformer windings. The measurements were made on the high-voltage windings. Curve 3 is for a 2,400-volt winding, and the major insulation is of a material having somewhat higher power factor than the other units. Curves 11 and 12 are for Inerteen transformers, which are higher, in general, than oil-insulated units.

It has been found that drying out windings with very low power factor may increase the power factor under certain conditions. This will occur when a transformer is removed from the oil and oven dried at approximately 100 degrees centigrade, and vacuum impregnated again. The heating of the oil-soaked insulation will produce some oxidation of the oil and will increase the power factor. One unit on which such a test was made showed an original measurement of less than two per cent, finally increasing to more than four per cent after drying.

C. F. Hill and **L. J. Berberich** (Westinghouse Research Laboratories, East Pittsburgh, Pa.): Data on insulation in apparatus in service are not only valuable to the operating companies as a maintenance economy and insurance of continuity of service, but also afford information to the manufacturer for the improvement of design and materials of his insulation. The presentation of papers giving such data should, therefore, be of great value to the electrical industry and we hope nonoperating engineers can be encouraged to publish their results.

The papers by Whitmer and others show

definitely that field tests are highly satisfactory on apparatus for which the insulation is subject to certain types of deterioration; namely, to the infiltration or formation of ionizable impurities. Moisture, products of oil oxidation, and even oxidation of organic insulations themselves are readily detectable by power-factor and resistance measurements and such impurities are very definitely a principal cause of insulation failure. This conclusion was more or less anticipated and the design engineer has attempted to protect his insulation from such influences. The data from papers giving the results of field testing should be of help to the design engineer toward further improvement.

I. W. Gross (American Gas and Electric Service Corporation, New York, N. Y.): The field testing of insulation implies three requirements: First, a test method and instrument, that is capable of determining faulty insulation, with acceptable limits of precision under field conditions; second, an effective method of evaluating the seriousness of the faults; and, third, a means of locating the faulty insulation and correcting the condition.

Referring specifically to bushings, transformers, and similar associated equipment, the first requirement has been met with the available power-factor test set which the authors have used on their systems during the past ten years or so. Constant development and improvement in this test equipment and prompt servicing of it by the manufacturer, has kept it available for service at all times.

The second requirement, that the test results should permit a determination of the seriousness of the fault, has been fulfilled by a continuous collection, analysis, and study of field data and the establishment of power-factor rating limits for equipment of different classes, voltage, rating, manufactures, and types of design. This build-up of a sound background for determining the seriousness of insulation faults has resulted from the co-operative efforts of the companies who have been, and are now, carrying on field or service testing. This work has been greatly aided by the manufacturer of the power-factor test set we have been using, as well as by the equipment manufacturers whose apparatus was being tested in the field.

A constant watch is kept on the grading of the faults into good, deteriorated, and remove classifications, and revisions made as field experience justifies. This second phase of the work is important, if the greatest value is to be obtained from the test work.

The third requirement, that means should be available for specifically locating the fault when the over-all bad insulation condition is indicated, cannot be too strongly emphasized. For example, a 25-kv faulty bushing can be promptly replaced without detailed analysis to determine whether the defect is in porous porcelain, split porcelain, voids in the compound, or the like. But the situation is different in a 10,000-kva transformer, for example, where the fault must be specifically located and corrected.

The specific typical cases, in Mr. Whitmer's paper, where detailed analysis has been made of the field data to definitely locate the fault clearly indicates what can be, and has been, done in making the field data give up the secrets of faulty conditions and locations in composite insulation.

It may seem astonishing that relatively small differences of power factors, currents, and watts loss indicated on field tests are capable of telling so much about the quality of the insulation which is considered, and has proved hazardous under operating conditions. Such, however, is a fact, as proved by extensive tests.

Messrs. Baltzly and Schlottere have indicated that the scope of their field test work covers many other items besides bushings (as shown in the paper). Our field test work, likewise, follows very much the same pattern. Their statement that "All equipment rejected by test has, upon investigation, disclosed faulty insulation which warranted its removal from service," also agrees with our experience.

The idea conceived some ten years or more ago that it ought to be possible *and practical* to measure in the field the power factor of insulation and thus determine hazardous insulation conditions has now become an accepted practice in the industry. This has been accomplished, like many other research problems, only through determination that the job could be done and the broad-minded viewpoint of those who cooperated, particularly in its early stages, in carrying on the development. Now that a solution has been found, it is hoped that all concerned will bend their efforts to further perfect the tools and technique for the over-all benefit of the industry.

M. S. Oldacre (Commonwealth Edison Company, Chicago, Ill.): This paper makes out a good case for power-factor testing of transformer and bushing insulation, but it seems to me that the really important facts about the apparatus have been stated in just three or four lines of the paper and these have nothing to do with power-factor testing. In effect, the paper states that moisture is the principal cause of the insulation deterioration, and that if gaskets and other joints and seals are kept tight most of the troubles will be eliminated. Thus, the most important conclusion of all that should be included in the paper is:

Every effort should be directed toward eliminating all openings into parts of apparatus where moisture might have any effect on the insulation. This would mean the elimination of all gasketed joints as well as open vents, and replacement of bolted-on covers, bushings, and other parts by some solid metallic method of jointing such as soldering or welding.

This should be done on all new apparatus and so far as possible on old apparatus, although I realize that in most cases for old apparatus we will have to be content with the best possible type of gasketed joint with the best available gasket materials.

If the above suggestions are carried out and existing apparatus is put in good condition, I feel confident that there will not be enough failures to justify much power-factor testing and the cost of operation can be reduced correspondingly. Objection might be raised that completely sealed apparatus would be difficult to repair. This can be met by building the apparatus carefully enough that repairs will be reduced to a minimum and by designing the sealing arrangement so that when it is necessary to open a unit the expense of doing so will be easily justified. It has been suggested that other methods of sealing might provide easier access and resealing than the existing methods.

It seems to me that the apparatus manufacturers should be quite receptive to sealed-up equipment as a means of reducing complaint calls and costs. So far as we know, no manufacturer has completely sealed apparatus that eliminates all gaskets, although some designs have a materially reduced number of gaskets. I should like to urge that they be reduced to zero.

E. W. Whitmer: Mr. Montsinger has mentioned the possibility of an occasional bushing with a slight fault being overlooked in an analysis of over-all test readings. This of course is possible but as pointed out in the paper, our operating experience has shown that on a practical basis the over-all tests are capable of indicating bushings which are likely to fail before another periodic test is made on the equipment. The collar tests which have been made on bushings during recent years are a great deal of help along this line.

The question of defining the deterioration of insulation in transformers has been brought up by Mr. Montsinger. In the paper the broad interpretation of deterioration has been used in which any decrease in the quality of insulation which makes it less suitable for service operation is considered to be "insulation deterioration." While possibly 90 per cent of such deterioration as indicated by power-factor test is caused by moisture, the remaining 10 per cent, of course, should not be disregarded. High power-factor oil and deposits of carbon or other objectionable materials on the insulation of transformer windings are some of the types of faults which have been located in this group of ten per cent or so of the faulty transformers. Investigations to determine the cause of such conditions have shown a number of transformers to be distinct service failure hazards. Since such faults as these or conditions of excessive moisture may be failure hazards the insulation of the transformer as a whole is considered to be "deteriorated."

Mr. Oldacre brings up the idea which has been prevalent in the industry for some time of welding all transformer tank joints and thus eliminating gaskets. It is pointed out in the preceding paragraph that some serious types of faults are not caused by moisture, and welding the tanks would not necessarily eliminate such troubles. To assume that all troubles in transformer insulation can be eliminated for the normal life of the equipment by simply sealing the tanks without other design improvements seems to be a case of wishful thinking rather than a hard and fast idea which is backed up by operating experience. The elimination of as many gasketed joints as possible from a practical standpoint seems a step in the right direction, but whether or not this should be carried to the complete elimination of all bolted and gasketed joints in all types and sizes of oil-filled transformers, is a subject which is open to a great deal of discussion and investigation. The advantages gained in eliminating gaskets should be weighed against the added inconvenience, expense, and skill of workmanship required in maintaining, breaking, and remarking welded joints as a matter of maintenance after the equipment is put into service. Field experience with some sealed up equipment is certainly necessary before such a practice is put into extensive use. Troubles on the inside of transformers, such as insulation faults, and bushing troubles will be with us no doubt for a great many years regardless of efforts made to retard their development, and a program of periodic checking by a comprehensive test like the power factor seems to be the only way now available in which we can be reasonably certain that the insulation is in good condition either in old or newer transformer.

The question of using one temperature correction curve for tests of all types of transformer windings has been brought up by Messrs. Bellaschi and Cole. It seems quite remarkable that all of the temperature power-factor data on both new and old transformer winding insulation yield correction factors which are so nearly alike. Whether or not the differences encountered are too great for practical use in making corrections from one average curve is the main question and further study may indicate that more than one curve is necessary. The test data presented by Mr. Bellaschi and also that given by Mr. Cole are the type of information needed. It might be expected that continued testing of transformer insulation at the factory will eventually prove useful not only in obtaining temperature corrections but in forming a bench mark from which to evaluate any future tests on the apparatus after it is in service.

The requirements for a field power-factor insulation testing program have been very appropriately outlined by Mr. Gross. Some of the ground work which has been done over the past ten years or so to bring the field testing art up to its present refinement is, of course, one of the things which has been necessary, and future efforts should certainly be made by everyone concerned to improve not only the test procedure and instruments but the equipment which is being tested.

Dr. Walker points out some of the fundamental considerations in locating concentrated faults in a large area of insulation. The location of some faults of this nature is, no doubt, difficult. While some progress has been made in transformer testing, a great deal remains to be done particularly in other types of equipment where the capacitance or area of the insulation may be much larger and the background of testing experience much smaller than for transformers.

Messrs. Hill and Berberich point out some of the fundamental things which influence the power factor of solid and oil insulation. It is gratifying to know that field testing of insulation is being studied by design engineers in an attempt to make further improvements in equipment. While power-factor testing of insulation in service has proved very important from a maintenance standpoint, there is, no doubt, possibility of a great deal of further improvement. The combined efforts of field, factory, and laboratory testing should do much to promote progress along these lines, and it is hoped that much valuable information will be forthcoming from all sources.

Bushing and Associated Insulation Testing by the Power-Factor Method

Discussion of paper 41-72 by C. C. Baltzly and E. L. Schlottere, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (June section) pages 308–12.

E. D. Eby (General Electric Company, Pittsfield, Mass.): The authors have made a helpful contribution to the literature and published data on this subject. The exchange of experience in this field is the strongest factor toward the establishment of a sound practice. The writer offers the following comments upon that part of the paper relating to bushings.

With regard to the adoption of a set of power-factor values, such as those in table *IA*, two things need to be kept in mind if the basis of rating is to be sound. In the first place, some insulating materials undergo an aging process with time which results in an increase in power factor, not dangerous in itself, but trending toward the rated limits. In some types of bushings this aging, which is not due to moisture, may require a graduated scale of limits according to age, or a higher initial limit.

In the second place, classification according to type may need to be refined further than is indicated in table *IA*. Normal power-factor values vary with the kinds and amounts of materials used. As a result, some types have inherently higher values than others. Some common types exceed the "satisfactory" limits in the table when tested new at the factory. The adoption of rating limits must recognize this fact by selecting suitable limits for each type, in order to avoid a false basis of rating and therefore excessive rejections in field tests.

The authors' present practice of basing replacement on the "satisfactory" limits rather than the "follow-up" limits seems a concession to expediency and ease of rebuilding. Unless the foregoing factors are given due weight, this practice may lead to some needless rejections. The "follow-up" testing to determine the trend, whether it be dangerous or normal, has its merits. This feature of the program might bear further study.

The facts that 69 per cent of bushing failures over the past nine years were due to moisture, and that 90 per cent of field rejections for high power factor were also due to moisture, when taken in connection with changed design practice, should be very encouraging for the future. This is reflected in figure 5 which shows the marked decrease in failures since 1935; and in table III which shows the reduced number of rejections in 1940 compared to previous years.

The greatly improved gasket practice on modern bushings naturally diminishes the failures and rejections due to moisture, and therefore reduces the percentage for which this cause is responsible. Reconditioning works to the same end, providing reliable gasket materials and assembly practices have been utilized. Gradually, old bushings with poor gaskets can thus be reclassified as rebuilt bushings with good gaskets. This would result in increasing the proportion of bushings on the system from which trouble due to moisture need not be expected, as was true of some of the older bushings. Eventually, when the older bushings have disappeared or been rebuilt, the moisture problem due to gaskets, as well as other sources of weakness from moisture, should diminish to a negligible minimum. Certainly, modern bushings, both in the matter of gaskets and in other moisture-prevention features, can be expected to give a very superior performance record, in comparison with some of the obsolete types.

In view of conclusion 9, favoring the centralized shop for repair work, the importance of designing bushings for easy and rapid disassembly and reassembly received further emphasis. Obviously, if the operating company is to conduct repairs on an economical and reliable basis, the designer must first provide a structure which lends itself readily to such procedure. In the best designs this is being kept prominently in mind, in order to facilitate such field work as may become desirable as a regular or emergency measure. Designs which require very special or expensive equipment to accomplish their repair, or which take excessive time or labor, are not to be looked upon with as much favor as those which facilitate such operations. This applies even in those designs which commonly give the least trouble.

The authors make no reference to the relative results of power-factor tests made on bushings at different times of year. It has been found by some observers that more failures, particularly those from moisture, occur in spring and fall than in summer and winter, especially in cold climates. It is not inconsistent to expect power-factor values to reflect a similar seasonal variation. On such a basis, it would appear more effective to make the tests during the periods of greatest danger, when the moisture content was greatest and the power-factor values highest. This could be expected to discover some units which might be in dangerous condition at one season, though returning to a passing condition at another. Since the purpose of field testing includes removal of units which constitute a potential hazard to the system, it would appear logical, in so far as such a practice is economically feasible, to select the most dangerous periods for the testing.

C. T. Pearce (Westinghouse Electric and Manufacturing Company, Philadelphia, Pa.): This paper shows the value of field power-factor testing as a guide for doing preventive maintenance of bushings and transformers. Operating experience of this nature has led the manufacturer to provide bushings with long-lived gasket materials that are protected from exposure to the outside air by suitable stops and with provision for taking care of the differential expansion between copper and porcelain. As a further protection against moisture, better waterproof bushing plastics have also been developed. The condensers for both indoor and outdoor types of bushings since 1937 have also been given special moisture resistant treatments prior to varnishing, and better varnishes have been used. With modern condenser bushings we anticipate the test period interval can be extended.

A background of experience for power-factor testing of oil-filled transformers in service is rapidly being built up and this paper is a valuable contribution. The gradual accumulation of moisture in transformers shows up during the tests made from year to year. These tests are particularly valuable for the older transformers that breathe in outside air containing moisture. For many years the larger transformers have been protected by using nitrogen over the oil and excluding outside air. This method has been so successful in keeping out moisture and preventing oil deterioration that our company is now sealing some of the smaller power transformer tanks completely. It is believed that the methods of sealing are sufficient that the moisture problem will become negligible.

Under the heading "Field Testing," an arbitrary value of five per cent power-factor maximum at 40 degrees centigrade is mentioned. This corresponds to only 2.1 per cent at the 20-degree-centigrade base used by the Doble organization and appears to be rather low. We believe that the authors intended this figure to be one of the classification limits instead of a maximum value. This belief is based on the grouping under the heading "Transformers."

During manufacture, transformers are dried out with hot air, followed by the application of a vacuum before the windings are initially immersed in oil. We have found that the leveling off of Megger readings is as reliable an indication of the end of the drying-out period as that given by power-factor readings. When the transformer is dry, it is immersed in oil and power-factor readings have been found to fall within about a seven-tenths to four per cent range at 20 degrees centigrade, depending on the type of construction and voltage class. At the present time, we feel that factory power-factor measurements are of little value but that field tests taken from year to year will indicate by unusual changes the presence of undesirable moisture conditions.

In order to avoid confusion, it is recommended that some reference temperature for power-factor testing of transformers be standardized by the industry. There is a wide variation with temperature and a fixed point of reference is necessary.

I. W. Gross: For discussion see page 743.

M. S. Oldacre: For discussion, see page 743.

The Audio Noise of Transformers

Discussion on paper 41-10 by W. C. Sealey, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (March section) pages 109–12.

J. P. Barton: See discussion, page 749.

Suppression of Magnetic Vibration and Noise of Two-Pole Turbine Generators

Discussion and authors' closure of paper 41-39 by A. L. Penniman, Jr. and H. D. Taylor, presented at the AIEE winter convention, Philadelphia, Pa., and published in AIEE TRANSACTIONS, 1941 (June section) pages 283–8.

E. F. Dissmeyer (The Commonwealth and Southern Corporation, Jackson, Mich.): The authors provide a very interesting discussion of the problem of suppressing the magnetic vibration and noise produced in two-pole turbo generators. Although experience with large 3,600-rpm generators has been comparatively short, operating experience indicates that it is essential that the 120-cycle vibration be limited to minimum possible values. It is not possible to anticipate resonant conditions which may be experienced in the plant itself, relays, disconnects, and other associated equipment. Trouble is usually encountered before a resonant condition is detected.

In most cases after a condition of excessive vibration has been noted, it is possible to apply remedial measures to eliminate the trouble. Cases have been encountered where complete isolation of a device from foundations, etc., will not eliminate vibrations. Evidently the vibration can be air borne and may be of a sound nature. The wave length of this sound is so long that ordinary acoustic materials cannot be used to shield equipment. It is, therefore, essential that all lagging and other flat portions of the machine be so constructed that they do not provide a source for these sound waves. It is recognized that excessive noise in a generating plant is conducive to fatigue of operators which may lead to operating mistakes and accidents.

It appears that the use of a resilient core mounting should reduce the vibration and noise emanating from the generator a considerable amount.

If the rotor in a generator is not located in the magnetic center of the stator, the unbalance magnetic pull may become appreciable. Would it be possible to experience trouble with a resilient core mounting in case the rotor is not properly centered due to poor alignment of bearings, etc.?

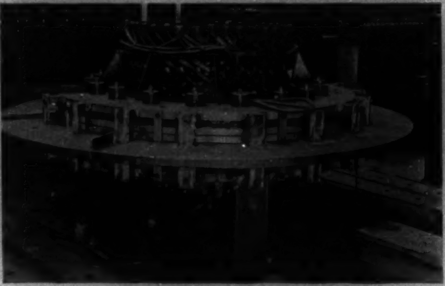


Figure 1. Experimental stator test model

B. A. Rose and M. D. Ross (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The last report of the AIEE power generation committee emphasized the fact that the problem of double-frequency vibration and noise of two-pole turbine generators had not been satisfactorily solved in the machines then in operation and stated that this was the major problem left to be solved in these large high-speed machines. The company with which the writers are associated started up the first of their large 3,600-rpm units in August 1937, and the experience gained at that time indicated that something would have to be done on these large two-pole units to make them comparable in noise and vibration to the 1,800-rpm units which had formerly been used for these generator ratings. The double-frequency stator vibration reacted unfavorably on foundation parts, relays, instruments, and many other parts of the power plant equipment, and special steps had to be taken in certain cases after these first generators were put in operaton to shift parts out of resonance with the 120-cycle frequency or isolate the smaller parts with flexible mountings. While the earlier machines have proved by more than three years' service to have been satisfactory as regards reliability, the development of the flexibly supported stator core has removed one of the major objections to the two-pole, 60-cycle generator.

The authors have presented a very interesting and complete discussion of the phenomenon of double-frequency generator stator vibration. It is interesting to note that they reach the same general conclusions regarding the solution of the problem that C. M. Laffoon and B. A. Rose presented at the AIEE convention at Springfield, Mass., in May 1939. It is also very interesting to note the similarity between the spring suspension members used by the authors and those embodied in the model on which we made our preliminary investigation. The attached figure 1 shows a photograph of the experimental model which was built and tested in 1938 to obtain verification of the calculated performance of the core when vibrating under the influence of the magnetic forces in the air gap. The heavy outer ring represents the generator frame. The core is composed of 12 inches of actual stator punchings with a special winding designed to produce flux variations identical to those in an actual machine.

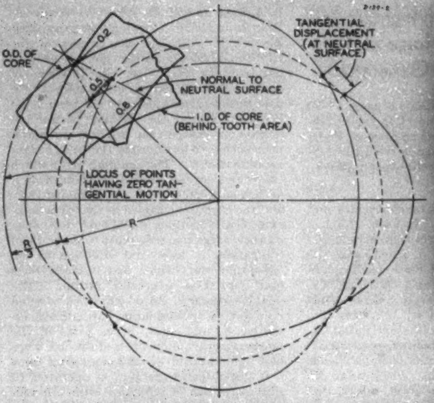

D-39-2
TANGENTIAL DISPLACEMENT (AT NEUTRAL SURFACE)
O.D. OF CORE
0.2
0.5
0.8
NORMAL TO NEUTRAL SURFACE
I.D. OF CORE (BEHIND TOOTH AREA)
LOCUS OF POINTS HAVING ZERO TANGENTIAL MOTION
R
R/3

Figure 2. Sketch illustrating tangential displacements and point of zero tangential motion for a thick ring

Figure 4. 12,500-kva 3,600-rpm generator having flexibly mounted stator core

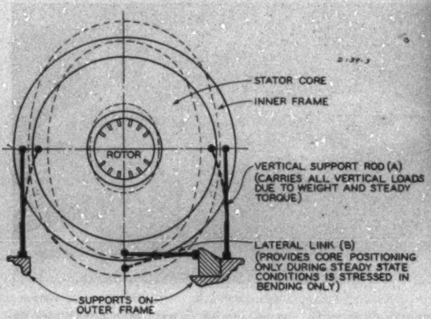

D-39-3
STATOR CORE
INNER FRAME
ROTOR
VERTICAL SUPPORT ROD (A) (CARRIES ALL VERTICAL LOADS DUE TO WEIGHT AND STEADY TORQUE)
LATERAL LINK (B) (PROVIDES CORE POSITIONING ONLY DURING STEADY STATE CONDITIONS IS STRESSED IN BENDING ONLY)
SUPPORTS ON OUTER FRAME

Figure 3. Diagram showing how the stator core of the Westinghouse generator is mounted on resilient supports

Vertical members (*A*) take the static loads of core weight and full load torque reaction; horizontal supports (*B*) hold the core in its proper lateral position

Figure 5. 81,250-kva 3,600-rpm generator having flexibly mounted stator core, built for Consolidated Edison Company of New York

The flexible bars which support the core in the frame are axially situated members which fasten to the outer frame ring at their centers and to the core itself at their ends. The performance of this model was entirely satisfactory and gave the same results which the authors present.

A study of the model in figure 1 will disclose the fact that the flexible tie bars were themselves quite similar to those used in the authors' design except that they were much more rigid tangentially. They were purposely proportioned in this manner to demonstrate whether or not extreme flexibility in the tangential direction was necessary for satisfactory performance of the mounting. As can be noted from the photograph the bars were fastened to the core by spacers which maintained a certain fixed radial position of the flexible bars with respect to the neutral axis of the core cross section. It was found by this test that if the flexible supports were attached to the core at a point beyond the neutral axis equal to one-third the radius to the neutral axis they would have to absorb no tangential motions. This fact was conclusively demonstrated by tests on this model as well as on the several machines which have subsequently been built.

It is very interesting to observe how well the authors' data on the tangential motion of the core agree with the above results. Our figure 2 combines figure 7 and data from figure 8 of the authors' paper to illustrate this point. The outlines of the core are superimposed on the elliptical curve, representing the neutral surface of figure 8. The value of the tangential motion for a ratio of outer diameter to inner diameter of 1.6 are taken from figure 8 of the paper and plotted to scale at their respective positions on the diagram. Lines connecting these points are normal to any circumferential line in the core and if extended a short distance outside the core's periphery intersect at a point of zero tangetial motion. This point as may be seen from a brief study of figure 2, lies at a radius of $1^{1}/_{3}R$ where R is the radius of the neutral surface.

It can be seen from a study of the above information and that contained in the authors' paper that two solutions to the problem are available. One involves the use of supports which are flexible in both the tangential and radial directions and which are clamped to the outer periphery of the stator core. The other involves the use of members having flexibility in only the radial direction which are fastened to points of zero tangential motion just outside the core surface. The Westinghouse company knew of the two solutions and decided to use the second for the following reasons:

1. Its use eliminated the problem connected with the ability of the assembly to withstand the short-circuit torques of the generator. These torques can be as high as 10 to 15 times the full load rating of the machine and anything which can be done to strengthen the supports is well worth while. The supports can be built as rigidly as desired in the tangential direction, thereby allowing an easy and more satisfactory solution to this problem.

2. By the use of an inner frame in which the stator core is securely clamped, the flexible support members could be fastened to the core in the correct radial location to obtain zero tangential motion and afford a construction which would allow easy

Table I. Vibration and Noise Data on Three 3,600-RPM Generators
Vibration Is Given as Double Amplitude

Machine Description	No Field Excitation					Normal Full Load Excitation				
	Average Core Amplitude	Average Frame Foot Amplitude	Average Frame Cover Amplitude	Average Over-all Sound Level (Db)	Average 120-Volt Sound Level (Db)	Average Core (Amp)	Average Frame Foot (Amp)	Average Frame Cover (Amp)	Average Over-all Sound Level (Db)	Average 120-Volt Sound Level (Db)
81,250-Kva hyd. cooled 3,600-rpm generator resilient core mounting	0.0002	0.00015	0.00016	100	91	0.0015	0.0002	0.00029	101	95
31,250-Kva hyd. cooled 3,600-rpm generator resilient core mounting	0.0001	0.00012	0.00013	94.5	75	0.0011		0.00015	95	80
43,750-Kva hyd. cooled turbo of same design as 31,250-Kva generator except solid core fastening to frame	0.0001	0.0001	0.00012	96	76	0.001		0.0009	103	98

Sound data should not be noted for absolute values since the ambient level was rather high and variable from day to day. The decibel values should be used for comparison only.

An interesting although not pertinent observation is that on both of the above-mentioned machines with flexible mounted core, the motion at the base of the spring mounted studs in both the horizontal and vertical direction was so small as to give meaningless values of vibration with a velocity type vibrometer (something less than 0.05 mil).

access to all flexible members for replacement should any fail in service.

3. The stator core is known to be vibrating and from experience with other large machines, it is known that the best construction for these machines is the one which provides a core and support assembly which are tightly clamped together. Unless the supports are so fastened to the core that no slight movement or rocking of one with respect to the other is allowed, trouble may be encountered with wear of the contact points. This is eliminated by the use of the inner frame which is built in two halves and clamped tightly to the core after the core has been built and pressed together.

Figure 3 shows a diagrammatic arrangement of the construction used by the Westinghouse company. Three sets of tangential tie bars support the inner frame and core. These bars are quite rigid in a direction tangential to the core but flexible in the radial direction. The two sets of vertical bars are quite heavy and they act as the major support members taking care of the weight of the core as well as the torques of the generator. The horizontal tie bars act only to center the core in the frame and maintain the air gaps.

As a result of tests on the model of the stator core, the Westinghouse company decided to build and test a 10,000-kw, 3,600-rpm turbine generator incorporating the design as shown in figure 3 and this machine was completed and tested in the early part of 1939. This machine is shown in table I. Extensive tests of noise and vibration with the flexible mounting in operation and with it blocked out by wedges between inner and outer stator frames proved conclusively that the desired results could be obtained in such a design Over 40 short circuits were made on the machine during an investigation of short-circuit torques and the flexible suspension system was therefore given the most severe testing it could possibly get in service. It withstood these tests without failure of any kind.

As a result of the tests on the 10,000-kw generator it was decided to standardize on the flexible mounted stator construction on all the large two-pole, 60-cycle generators and 23 of these units are on order or have been shipped. To date four units have been built and tested with very satisfactory results. Figure 4 gives comparative readings of noise and vibration on three large 3,600-rpm generators. The first two units have the new type core mounting while the third is of the earlier design. It will be noted from a study of these results that the outer frame vibration of the generator with flexible supports increased but a slight amount when excitation was applied whereas that of the conventional machine increased to a large extent. The increase in noise level around the machines is also of interest because both the over-all level as well as the 120-cycle component increased much more with the conventional generator. In line with the experience of Messrs. Penniman and Taylor, it was found that it was impossible to tell by observation when the generator field was applied in the case of the units with the flexible mounting.

The 81,250-kva unit shown on test in figure 5 and the results of which are shown in figure 4 is of interest as representing the largest 3,600-rpm generator yet built. The test results indicate that the noise and vibration of even the largest units can be expected to be excellent.

A. L. Penniman, Jr., and **H. D. Taylor:** Answering Mr. Dissmeyer's question about the possibility of trouble being experienced with a resilient core mounting due to unbalanced magnetic pull in case of the generator field not being properly centered, this might be a real hazard if the core mounting is made as elastic as the conventional type of isolating mounting, which is often rather "floppy." The resilient mounting developed by the General Electric Company and described in the paper is very far from being floppy; for radial displacement of the core, this mounting is actually stiffer than the solid steel rotor. Furthermore, these resilient mountings are applied only to large two-pole machines, in which the air gap is relatively long (about $1^1/_2$ to 2 inches radial length), and in which the gradient of unbalanced pull per unit of displacement off center is inherently low. This is because of the fact that all of the flux entering the rotor through one pole must go out through the other, so that the unbalanced pull is due only to differences in distribution of flux, rather than in amount of flux as in machines with more than two poles. We are confident that there is no danger whatever of trouble from unbalanced magnetic pull with the type of resilient core mounting we have described.

Messrs. Rose and Ross' comments imply that our conclusions regarding the solution of the problem are in general the same as those presented in the Laffoon and Rose paper of 1939. In a very broad sense, this is true, in that we agree that the inherent magnetic vibration of two-pole generator cores cannot be eliminated, and that the most effective way to suppress its effects is to isolate it by means of a resilient core mounting. However, we wish to reiterate that the alternative solution of the deep core-type generator, together with judicious layout of station structures based upon experience, is even more "practical," enjoying the advantage of employing design features proved by long and successful service. Half a dozen or more machines of this type have gone into service within the past year, all of which operate satisfactorily, without objectionable vibration and noise.

Messrs. Rose and Ross made quite a point of the similarity between the spring suspension members used on the Baltimore generator and those employed in the experimental model described by Laffoon and Rose. As a matter of historical interest, the Baltimore spring bars had been designed and some had been made and tested long before the presentation of that paper.

The indirect suggestion that extreme flexibility in the tangential direction is required, if the point of support is not moved out radially to a position well outside the core, is likely to give an erroneous impression. It has been pointed out that the type of resilient mounting used on the Baltimore machine is not of the conventional "floppy" type, but is very stiff for radial displacements of the core. For tangential displacements, it is actually stiffer. The tangential stiffness could be even greater yet, without much change in the effectiveness of the isolation, but this is limited by the requirement that the torsional frequency of the core on its mounting be low relative to 60 cycles, as pointed out in the paper. It may be noted that the spring suspension system used in the Baltimore generator is rigid lengthwise of the machine.

With regard to the relative merits of

the two varieties of spring mounting, the first with both radial and tangential flexibility and the second with extreme radial flexibility but rigid tangentially, there are quite a number of considerations involved, and there is undoubtedly room for differences of opinion. Either type can undoubtedly be built to give creditable results. The reasons for the choice of the first type by the General Electric Company may be summarized as follows:

1. It gave promise of being more effective; that is, of affording a higher ratio of improvement.

2. It avoided the necessity for a double frame, which is not only a complication, but actually seems to work against the effectiveness of the isolation, because the outer frame is necessarily more shallow than it otherwise might be, and this affects its stiffness and resonance properties unfavorably.

3. The manufacture and installation of such a design appeared to be more simple and practicable.

4. While there were other problems involved, particularly with regard to the mechanical effects of rotor unbalance and short-circuit torques, these were by no means insuperable, as was indicated in the paper.

With regard to the question of the ability of the generator to withstand transient short-circuit torques of the order of 10 to 15 times the full load rating of the machine, it should be noted that quantitatively, this is not nearly so drastic as it sounds. This is because, for a speed of 3,600 rpm, rated torque is quite low, and even 10 to 15 times rated torque amounts to only two or three times the gravity load that the spring bars are carrying continuously. The problem is one of resonance, to avoid a condition which would amplify the disturbance. This is accomplished by designing for a moderately low torsional natural frequency of the core as mounted in the frame. Also, the problem is greatly ameliorated by the fact that the disturbing short-circuit torques are transient, and die away so rapidly that there is not time to build up much torsional displacement.

The authors do not know about the trouble with wear of core contact points to which Messrs. Rose and Ross refer. It is clear, however, from the tests described in the paper and illustrated in figures 4 and 5 that, so long as the core itself is tightly compressed between its flanges, it matters little or not at all whether there is a frame around it or not, so far as its magnetic vibration is concerned. The difference in the support of the core between the Baltimore generator and machines with solidly mounted cores is that, whereas in the latter the long dovetail key bars are welded to the frame and keyed to the core, in the Baltimore machine they are keyed to the core and bolted to the spring bars, which in turn are bolted and welded to the frame. In the solid construction, the key bars are an integral part of the frame, and are subject to a microscopic amount of tangential "working" against the core at the dovetails, which has never been known to do any harm. In the spring-mounted construction, the key bars are an integral part of the core and flange structure, and by virtue of the slight flexibility of the spring bars, there is no occasion for any "working," except the intentional slight flexure of the spring bars themselves. In other words, so far as we can see, there should be no wear of contact points whatever with the Baltimore type of spring mounting.

A companion to the 81,250-kva, 3,600-rpm unit mentioned by Messrs. Rose and Ross has been shipped to the same customer. It has the same rating, and is of the deep-core construction. Its operation on test in the factory was excellent. Several other machines of about the same physical size and quite a number of smaller ones are being built with resilient core mountings similar to the one in service at Baltimore.

A Study of Sound Levels of Transformers

Discussion and author's closure of paper 41-53 by H. Fahnoe, presented at the AIEE winter convention, Philadelphia, Pa., January 27-31, 1941, and published in AIEE TRANSACTIONS, 1941 (June section) pages 277-83.

Clifford Mannal (General Electric Company, Pittsfield, Mass.): Mr. Fahnoe's paper is a contribution to the art and undoubtedly represents a summary of a number of years of investigational work. There are four points, however, on which I wish to comment:

1. It is stated that core-type transformers may possess a larger number of natural resonance frequencies than shell-type transformers. This arises from the inference that core-type transformers may vibrate in several planes while the shell-type transformer vibrates only in a single plane. My own studies on core-type transformers indicate that the motion is sensibly in one plane for all practical designs of clamping used on transformers of commercial design.

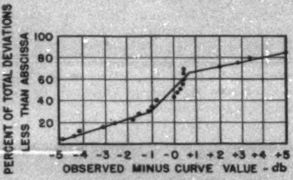


Figure 1. Deviations of observed values from curve values

2. A second point concerns the excitation of the higher resonance frequencies of the core by higher harmonics of the flux frequency. On the basis of a mathematical analysis which makes relatively few assumptions, the higher core vibration frequencies cannot be determined to closer than five per cent. At 600 cycles the uncertainty amounts to 30 cycles. Superimposed on this uncertainty is the shift of the observed from the computed frequencies due to the additional constraints on the core imposed by the oil, tank, and windings. The magnitude of this shift differs from one resonance frequency to the next and can only be estimated very roughly.

Beyond this, there is a basic difficulty which increased computational accuracy cannot remove. The flux in the core may be represented by a fundamental and integral multiples of the fundamental. The resonance frequencies of a rectangular frame cannot be so represented. The natural resonance frequencies of the core are not integral multiples of some fundamental resonance frequency. This is exhibited in curve 6 of the paper. Resonances are indicated at 136 and 292 cycles. The ratio is 2.13. Data given in figure 7 show intervals of 36, 56, 166, and 42 cycles between successive resonance frequencies. A small rectangular core investigated at Pittsfield has resonance frequencies at 696, 756, 1,360, 2,135, and 3,150 cycles.

Since the driving force has components equally spaced along the frequency axis and the core has resonance frequencies unequally spaced on the same axis, any choice of core proportions will locate some of the higher harmonic flux frequencies at or near the core resonance frequencies.

3. It is significant in this connection to examine the practical importance of the higher harmonics of transformer cores. Sound analyses made on a large number of transformers invariably indicate a relatively smooth decrease in the intensity of each succeeding component of the sound. If the higher harmonic resonances were significant from a practical standpoint, a pronounced increase in sound radiation at some particular harmonic frequency should have been observed in at least one of the transformers which have been studied. That this has not been observed seems reasonably conclusive evidence that higher harmonic resonance is not a significant contributor to the sound level of transformers. Resonance at the fundamental or second harmonic frequency would, of course, be a serious matter. Computations and tests have shown, however, that for cores of sizes now used, the fundamental resonance frequency is either well above 120 cycles or is capable of being located at a frequency sufficiently far removed from 120 cycles.

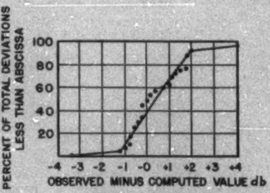


Figure 2. Deviations of observed values from computed values

4. Mr. Fahnoe points out that the sound level in the immediate vicinity of the core face in air may be computed from the dimensions of the core and the characteristics of the steel used. The figure of practical importance is the average sound level radiated by the assembled transformer. A possible method of attack might be to reduce the analysis into four portions:

1. A computation of the amount the core vibration amplitude in the completed transformer is reduced below the amplitude obtained in air.

2. The pressure produced by this reduced vibration amplitude on the tank wall.

3. The tank wall amplitude produced by the pressure as computed under 2.

4. The radiated sound produced by the tank wall vibration.

The subject of sound level in transformers has been under active investigation at the Pittsfield Works of the General Electric Company for the last decade. A large number of transformers have been measured and a quasi-empirical relation between the sound level and the factors of major importance in determining the sound level has been established. Figure 1 shows the deviations of individual transformer sound levels from the best average curve drawn on a decibel versus kilovolt-ampere plot. These transformers ranged in size from 1,000 to 11,000 kva. It can be seen that only some 30 per cent of the values fall within the limits of plus or minus one decibel. The second figure shows a typical distribution of the deviation between observed and computed values. It can be seen that almost twice as many points fall within the limits of plus or minus one decibel as for the previous figure.

Two points of importance are exhibited by these figures: first, that the relation between kilovolt-amperes and sound level cannot be expressed by a simple relation; and second, that the major factors underlying sound level in transformers are known and are subject to engineering control.

J. P. Barton (Carnegie-Illinois Steel Corporation, Pittsburgh, Pa.): I feel that Mr. Sealey and Mr. Fahnoe are to be congratulated in presenting these two pioneering papers (in this country) on the noise generated by large transformers, and some of the engineering aspects of controlling the level of this noise. Of particular interest is the fact that calculations made with the equations shown in Mr. Fahnoe's paper yield values that are quite compatible with those obtained by Mr. Sealey on the finished transformers.

In early 1940, a paper was published in England in the Metropolitan-Vickers Gazette, titled "The Limitation of Transformer Noise," by Mr. Churcher and Mr. King. This paper is of considerable length, covering many aspects of the cause and generation of transformer noise, as well as attempts by soundproofing and other means to prevent the noise from being transmitted at levels obnoxious to human beings in the vicinity of the transformers.

In this paper is given a curve showing the relation between flux density and change in length of silicon steel that may be of interest in consideration of the amplitude of the fundamental and harmonics of the sound generated by this magnetostriction effect in silicon steel. They show that the change in length for increasing values of flux density are not the same as when the flux density is decreased, thus giving a shallow "U" shaped loop for plus or minus values of flux density. This loop would indicate the power used because of the magnetostriction effect, in a manner similar to the power used as illustrated by a hysteresis loop.

The equations 2, 4, 8, and 9 of Mr. Fahnoe's paper, for a core length of 200 centimeters and Δl values in the vicinity of 1.5×10^{-6}, yield decibel values that check very closely with those shown in figure 3 in Mr. Sealey's paper. It should be observed that increasing or decreasing the value of Δl ten times gives a change of 20 decibels, which is a considerable change in sound level.

Of greater interest is a further calculation which appears to tie together quite definitely the equations of Mr. Fahnoe's paper with the observations on finished transformers by Mr. Sealey. From magnetostriction data on transformer grades of silicon steel such as commercially used in large power transformers, average values of the changes in length for various flux densities were substituted in the combined equations 2, 4, 8, and 9 of Mr. Fahnoe's paper using a core length of 200 centimeters and a frequency of 120 cycles. The difference in sound level between 8 and 14 kilogausses from these calculations, was ten decibels, which is the same value for the same range in flux density as Mr. Sealey shows in his figure 2, which is based upon actual tests. It might be added that this good agreement would tend to show that stray sources of sound or noise have been fairly well eliminated, leaving as the primary source of noise the magnetostriction effect in the core.

It is recognized that the type or shape of the core construction will modify the application of these equations in some cases. Also, that other factors will exert their effect in causing the noise of the finished transformer to be higher or lower than calculated. However, such agreements as stated above are indeed gratifying, and illustrate the progress being made in predicting the noise emitted by the finished transformer.

H. Fahnoe: Mr. Mannal's interesting discussion brings out certain facts which were not emphasized in the paper, but which are in line with our experience. In the majority of commercial core-type transformers the core vibrations are principally in a plane parallel with the laminations, but occasionally the designer is forced into abnormal designs in which the core may vibrate in another plane. An interesting case was a core-type transformer for which a very limited floor space was specified. This necessitated a rather tall core structure of relative small core area.

Calculations showed that the fundamental resonance frequency for vibrations in the plane perpendicular to the laminations was approximately 95 cycles and for the "box type" mode, parallel with the plane of the laminations, approximately 145 cycles. The actual measured resonance frequencies were 98 cycles and 150 cycles respectively. At 60-cycle excitation the sound level is approximately what would be expected from the magnetostriction characteristic of the steel, and approximately 15 decibels less than that at the resonance frequencies.

Mr. Mannal made a particular point in his discussion that mechanical resonance of the core at the higher frequencies did not appreciably add to the measured sound level caused by magnetostriction when resonance with the fundamental flux frequency was avoided. I agree, that with present-day transformer steel, it has also been our experience that it is only necessary to design the core of such dimensions that its fundamental resonance frequency is well above or below the fundamental flux frequency.

Protection of Power Transformers Against Lightning Surges

Discussion of paper 41-79 by the AIEE transformer subcommittee, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941, pages 568–77.

V. M. Montsinger (General Electric Company, Schenectady, N. Y.): For the benefit of those using this report, I believe that it will be worth while to work out a couple of cases showing how to use the data when providing protection for transformers.

The two cases selected for illustrative purposes are (1) a transformer having standard insulation, and (2) a transformer having one step below standard insulation. In both cases the transformers are for operation on a 138-kv system.

The figure shows on the left the transformer insulated for 115-kv insulation level for operation on a grounded-neutral system. The winding volt-time curve is based on 625-kv chopped wave at three microseconds and longer, 540-kv full wave. The turnup is based on figure 3 of the paper. The flat part of the bushing curve is based on a withstand test of 550 kv. The solid line shows the level to be protected against winding failure and bushing flashover.

The insulator volt-time curve is shown to indicate the magnitude of impulse voltages that can be imposed on the transformer if no protection is provided. Reflection will, of course, increase the voltage at the transformer.

The negative wave volt-time curve of a 20-inch rod gap is included. It is seen that even a 20-inch gap will not protect the transformer for steep front waves. Moreover by following the dashed (---) line drawn in from the 60-cycle flashover of the 20-inch gap over to the switching surge curve, lower right-hand corner, it will be seen that the gap would be expected to flash over and cause outages on 40 per cent of the switching surges. A 20-inch gap, therefore, is an entirely unreliable means of protection for this setup.

In selecting the proper lightning arrester, the line-to-ground voltage of the 138-kv grounded-neutral system is increased 40 per cent to take care of any possible increase in fundamental frequency voltage. The nearest standard station-type arrester rating above 110 kv $(138/\sqrt{3} \times 1.4)$ is 121 kv rms. The average gap breakdown for 121-kv arrester rating, according to table I, is 430 kv. Since the AIEE standard time of breakdown is based on a voltage rise of 100 kva per microsecond per unit and as there are (10) arrester units required, this fixes the first point on the gap breakdown curve at 430 kv for 0.43 microsecond time. The remainder of the curve is based on curve shown in figure 7.

The *IR* voltage values for 1,500, 3,000, 5,000, 10,000, and 20,000 amperes are taken from table I for 121-kv arrester rating. These voltages are shown at approximately five microseconds time for the reason that five microseconds is the approximate time

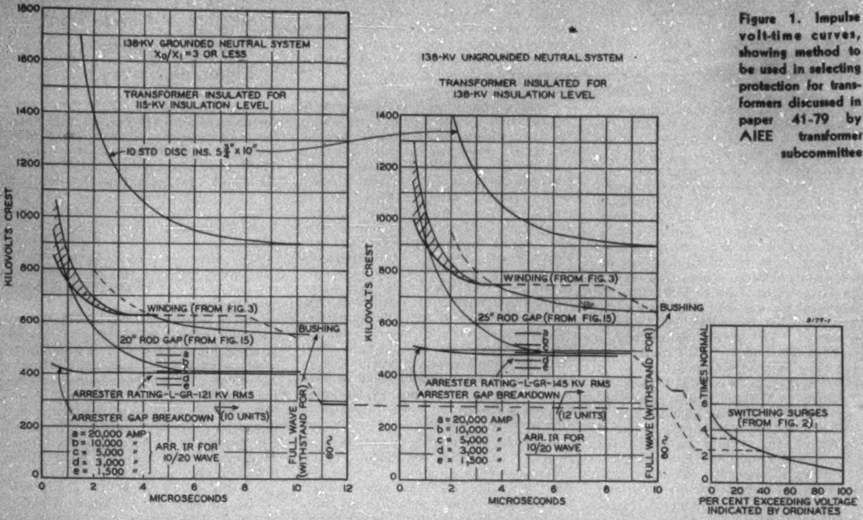


Figure 1. Impulse volt-time curves, showing method to be used in selecting protection for transformers discussed in paper 41-79 by AIEE transformer subcommittee

the voltage reaches crest value for the standard AIEE arrester 10/20 current wave.

The curves to the right of the figure are for a transformer insulated for 138-kv insulation level, for operation on a 138-kv ungrounded neutral system. These curves were worked out in the same manner as just described. In this case the arrester rating is based on line-to-line voltage of 138 kv, for the reason that if one line becomes grounded full line-to-line voltage is imposed on the arrester. The 40 per cent increase is not used here, since it is not likely that the fundamental frequency overvoltage will occur during the time one line is accidentally grounded. The nearest standard arrester rating is 145 kv.

A 25-inch rod gap is shown. Again it is seen that a 25-inch gap will not protect the winding for steep waves. It will be noted that the 25-inch gap would be expected to flash over on 20 per cent of the switching surges.

P. L. Bellaschi (Westinghouse Electric and Manufacturing Company, Sharon, Pa.): This report of the AIEE transformer subcommittee marks another milestone of progress in the protection of power transformer against lightning surges. The comments I have on the report pertain to items that may require further study. I have specific reference at this time to the curve on switching surges in figure 2. It has long appeared to me that the experience from various important sources would justify segregating this general curve into two or more specific curves. The question is more than academic, for switching surges bear a relation to rod gaps and to their settings. As it is evidenced from figure 18 in the report, rod gaps come into the picture of protection and they are likely to remain in the picture to a greater or less extent in future developments.

To illustrate the point, we find the experience on the Chicago 12-kv system, recently reported by Messrs. Wollaston and Plumley [AIEE Transactions, volume 60, 1941 (June section), pages 260–5] sets an

Table I. Arrester Performance Data Taken From References 22, 23, and 24

Arrester Rating (Kv)	Average Gap Breakdown Crest Kv		Average Voltage (IR Drop) Across Arrester							
			Station Type					Line Type		
			Crest Kv When Discharging Currents of:							
	Station Type	Line Type	1,500	3,000	5,000	10,000	20,000	1,500	3,000	5,000
3	13	17	9	10	10	11	12	10.5	12	13.5
6	23	34.5	18	19	20	22	24	20.5	23	25
9	35	49	27	29	30	33	35	30.5	34	37
12	43	58	36	38	40	44	47	40.5	45	48
15	53	66	45	47	50	54	59	50.5	56	60
20	72	75	60	64	67	72	78	69	75	78
25	89	95	74	79	83	90	100	87	94	98
30	106	112	89	95	100	108	118	104	112	117
37	131	135	110	117	124	132	145	129	140	146
40	136	147	119	127	134	144	153	140	151	158
50	178	180	149	158	167	179	191	175	188	198
60	214	217	179	191	200	217	234	210	226	238
73	261	265	217	232	245	262	283	258	278	290
97	345		289	308	323	349	377			
121	430		360	384	403	438	470			
145	515		431	461	487	523	564			
169	602		502	538	566	610	658			
195	691		578	617	647	698	755			
242	860		720	768	806	872	940			

Note: Gap breakdown taken on rate of voltage rise of 100 kv per microsecond per 11.5-kv rating. All test currents 10/20 wave. The 60-cycle spark-over potential of all arresters shown in this tabulation will not be less than 150 per cent of arrester rating.

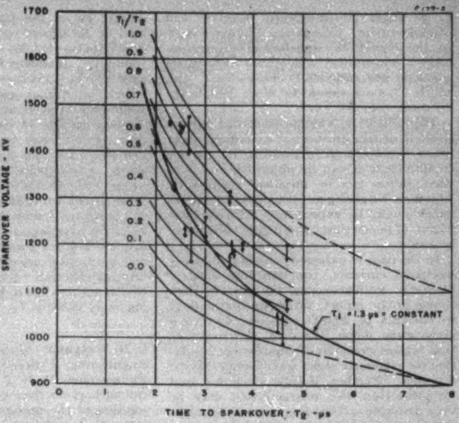
upper limit of 3.5 times the line-to-ground voltage for the highest switching voltages recorded over a period of many years. Rod and sphere gaps were used for the measurements in this investigation. This is a cable system with the neutral in effect solidly grounded. Other important sources of experience and data on this subject matter are references 16, 5, 6, etc., in the report, and in addition the various EEI reports.

In the light of these experiences and data, it would appear that data relating to switching voltages and to similar system overvoltages require some segregating if the curves are to be of more practical use. One curve could probably be drawn for the grounded neutral, in which case from the experience cited, this curve would intersect the zero ordinate at 3.5. One or more curves may be necessary to relate the conditions encountered from the operation of ungrounded neutral system, of systems operating with the neutral grounded through a relatively high impedance, etc., where higher system overvoltages than 3.5 line-to-ground voltages may be developed.

Figure 2. Time to sparkover—T_2 microseconds

Sparkover volt-time area of 50-inch rod gap. Boundaries are $T_1/T_2=0$ and $T_1/T_2=1$. Negative polarity. Standard conditions of humidity and air density

T_1 = effective wave front; T_2 = time to sparkover; points indicate measured values. Arrows point to correct location of test point based on T_1/T_2. T_1 = 1.3-microsecond curve from tests with 1.5x40 wave



W. J. Rudge (General Electric Company, Pittsfield, Mass.): This report is of particular importance and interest because it lists the various recognized factors which must be taken into account when selecting and designing the lightning protective equipment for power transformers.

The report is of greatest value in those items dealing with protection where specific performance values of protective devices available to the industry are given. I refer to the data on lightning arresters, protector tubes and gaps, as these data make possible the evaluation of protective margins.

The report points out that generator overspeed, system switching surges, arcing ground, and circuit harmonic voltages all should be known and evaluated. There have been numerous papers presented before the Institute which resulted from analytical studies, field tests, and transient analyzer studies which constitute ways for evaluating these overvoltages in terms of normal line-to-neutral voltage which is nearly always taken as the reference base. The normal line-to-neutral fundamental frequency voltage is also the base that is usually used for the selection of arrester ratings, and since there are systems having the same nominal circuit rating but which operate at voltages both above and below the nominal rating, it would seem in order to suggest that a survey be made of all systems under the sponsorship of EEI and associated operating companies, listing the actual operating voltages for the various systems, giving both average, minimum, and maximum values. The information so obtained would be to the advantage of both the user and manufacturer of apparatus as it is well recognized that successful standardization of ratings and designs depends upon selection of ratings which will be suitable to the majority of systems.

It is of particular interest to note the tentative volt-time curve shown in figure 15 of the report for plain rod gaps. In connection with this curve I wish to call attention to a paper by J. H. Hagenguth entitled "Volt-Time Areas of Impulse Sparkover" [AIEE TRANSACTIONS, volume 60, 1941 (July section), pages 803–10]. Mr. Hagenguth shows a wide spread in spark-over voltage values occurring on rod gaps, bushings, and similar apparatus, depending on the shape of the applied surge voltage. It is pointed out that these wide areas of sparkover, particularly at short times, show the futility of trying to use a rod gap or bushing as a protective device.

In figures 1 and 2 of the transformer committee report data are given on lightning strokes and tower currents, also switching surge voltages. Probably of greater significance are the currents which have actually been measured through station-type arresters which give both the magnitude and frequency of occurrence. Messrs. Gross and McMorris presented a paper before the Institute last January which gave the results of such a field investigation.

J. H. Hagenguth (General Electric Company, Pittsfield, Mass.): The paper shows the standard insulation tests for power transformers in table I. One of these tests is the impulse test requiring two chopped waves and one full wave at a stated minimum crest voltage. The requirement of impulse strength of the order indicated in the table has improved the design of transformers. However, it should be remembered that the mere fact that a transformer passed a test with impulses of the kind shown in table I will not be complete proof that the transformer winding is immune against surges of different wave shape. Furthermore, even if the transformer passes standard impulse tests, this is not a proof that no failure of any kind did occur. It is at present impossible to detect failures in the winding during impulse test if the percentage of winding involved is smaller than two to three per cent, such as turn-to-turn failure. Of course, from tests made on full-size models, the manufacturer knows the safety factor built into his transformers, but this can only be established by disassembling the winding and thorough investigation of the turn insulation. This procedure, of course, is out of the question for finished transformers receiving commercial tests. The best means available for detecting failure is by the use of a full wave with an amplitude of about 70 per cent or lower of the full-wave amplitude required by table I. If the comparison of the 70 per cent and 100 per cent wave shows any differences, failure of the winding is indicated. This 70 per cent wave should, therefore, be one of the requirements of the standards.

The methods of protection are clearly presented in the order of their effectiveness. One of the protective means mentioned is the rod gap. Tentative volt-time curves of rod gaps are shown in figure 15. As pointed out in the paper, the rod gap is the least satisfactory of the discharge devices. The rod gap has:

1. A marked polarity effect.
2. No sealing characteristics requiring frequent opening of circuit breaker due to switching surges.
3. Steep volt-time characteristic, requiring low spacing to obtain spark-over voltages below the transformer insulation—volt-time characteristic for all wave shapes.
4. A spark-over characteristic markedly dependent on atmospheric conditions.

In a paper which I am presenting at this convention, it is shown that the rod gap exhibits even more undesirable characteristics. The impulse sparkover of the rod gap cannot be represented by a simple volt-time curve, such as shown in figure 15 of the paper, but rather by a volt-time area. A volt-time area means that for a given time to sparkover the corresponding spark-over voltage is not a single value but varies within wide limits, for instance at 2 microseconds to sparkover of a 50-inch rod gap at negative polarity, the spark-over potential varies between 1,130 kv and 1,630 kv as shown by figure 1 (this discussion) or by 44 per cent based on the lower value. This variation of spark-over voltage is not due to erratic behavior of the rod gap but is due to variations in the wave shape of the applied impulse. This spark-over range compared with the curves shown in figure 15 of the paper corresponds to a variation of gap spacing between 43 inches and 65 inches. It is clear that such a device does not provide a very dependable discharge path especially when used in parallel with transformer insulation.

F. J. Vogel (Westinghouse Electric and Manufacturing Company, Sharon, Pa.): In the report on transformer protection, there is one subject which is of importance which is not completely worked out, and which I believe should be more completely determined later.

This subject is, whether increased voltage strength of transformers at extremely short times is necessary, depending on service conditions, if it can be obtained, and if so what values can be obtained reasonably. This includes giving an idea as to gap lengths which might be expected to furnish some degree of protection, where gaps alone may be used.

In the case of extremely steep and heavy discharge currents, the lightning arrester discharge voltages may be greater than shown in the report, which limits the data to currents of 20,000 amperes, and to a given wave shape. In turn, this indicates that higher strengths of transformer insulation for extremely short wave fronts may be desirable. Further, if gaps alone are used for protection, this characteristic may be very desirable. That it can possibly be obtained is indicated in figure 3 of the report. In order to obtain an idea of what might be reasonably done, I would like to make an example of the 115-kv voltage class, having a 3-microsecond strength of 625 kv. If the upper curve of figure 3 were used, a strength of approximately 145 per cent of that at 3 microseconds would give over 900 kv, which would also be equivalent to testing with a 25-inch gap, from curve figure 15, at 1,000 kv per microsecond. If the lower curve figure 3 is used, the voltage value would be 800 kv and the gap length 20 inches. If no additional margin were required, a 20-inch or 25-inch gap might be expected to provide reasonable protection for steep waves, provided it were immediately at the terminals. Also, at 3 microseconds a 25-inch gap would flash at 560 kv, compared to the 625-kv transformer strength, and at about 490 kv for long waves against a transformer strength of 540 kv.

This discussion is not given for the purpose of advocating any definite procedure, but to show how such station gap lengths might be obtained. Probably many operators would feel that these gaps are too small for operation. A further caution should be given, and that is, that this increase in transformer insulation voltage strength at short times is not necessarily obtained, except with great care in design and construction, and if it is to be counted on, a test would be required. This has been under consideration in the transformer subcommittee.

Negative Damping of Electrical Machinery

Discussion and authors' closure of paper 41-12 by C. Concordia and G. K. Carter, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (March section) pages 116–19.

M. M. Liwschitz (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The paper by C. Concordia and G. K. Carter represents an important contribution to the problem of negative damping. It solves the most complicated case in which primary resistance as well as series capacitance and steady-state power limit have influence and gives a new criterion on hunting due to the primary resistance. The authors have found that the hunting criterion $r \leq x_q \tan \delta$, of reference 1 of the paper, is too optimistic for small generator angles and too pessimistic for larger angles. This checks with calculations made by myself. I have found that the values of the negative damping as given by reference 1 are too low for small positive angles δ as well as for any negative angle δ. The new hunting criterion $r \leq x_d' \tan \delta$ as proposed by the authors has to be checked for at least one more value of T_0.

C. N. Weygandt (Moore School of Electrical Engineering, University of Pennsylvania, Philadelphia, Pa.): I wish to congratulate the authors on their complete and rigorous solution of the problem of negative damping. Also, and I speak from experience, I can well appreciate the labor that was involved in testing the roots of the determinant in equation 4.

The only thing that the paper lacks is some experimental checks of the new points of interest disclosed by the theory. Since we have at the Moore School a machine of the type studied by the authors I have made a few tests of the case when the machine is connected to an infinite bus through resistances. Figure 1 of this discussion shows the results of these tests. By comparison with figure 1 of the paper it can be seen that qualitatively the tests check the theory excellently. There is no question about the fact that the curve for higher excitation crosses that for lower excitation as predicted. Also the test curves bear the same relation to the old criterion $r = x_q \tan \delta$ as the theoretical curves of figure 1. It will be interesting to see whether the quantitative check is as good, as soon as time is available to draw theoretical curves for the actual machine tested.

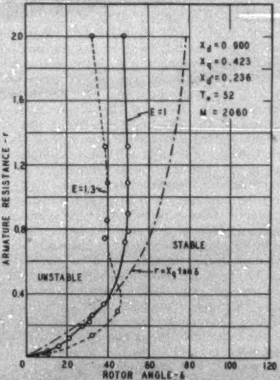


Figure 1. Experimental curves of stability limits, 31.3-kw machine without amortisseur connected to infinite bus

During the test it was observed that the hunting at low values of resistance was much more violent than for high values of resistance, and also that period was shorter. The violent hunting was of course accompanied by rapid swings of armature current, which in turn induced currents in the field circuit. This could easily be observed from the pulsating of the needle of the field ammeter. It seems logical since there must have been an a-c component in the field circuit that $x_d' \tan \delta$ would be a better criterion than $x_q \tan \delta$.

C. Concordia and G. K. Carter: We are very pleased to find that qualitative checks of part of our analysis have been made by both discussers. The test data presented by Mr. Weygandt are especially interesting and are a remarkable confirmation of the rather peculiar contour of the stable region which our calculations showed. Mr. Liwschitz has also found that damping torque calculations made on the basis of the formulas of reference 1 of the paper deviate from his more accurate results. However, as he has shown in his paper,[1] the more commonly used methods of equation 64 of reference 8 of the paper or equation 2 of reference 2 of this closure show no such discrepancies. We agree with Mr. Liwschitz that the hunting criterion should be checked at other values of T_0, and shall do so as soon as time permits.

References

1. Positive and Negative Damping in Synchronous Machines, M. M. Liwschitz. AIEE Transactions, volume 60, 1941 (May section), pages 210–13.

2. Two Reaction Theory of Synchronous Machines—II, R. H. Park. AIEE Transactions, volume 52, 1933, page 352.

Analysis of Short-Circuit Oscillograms

Discussion and author's closure of paper 41-2, by W. W. Kuyper, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (April section) pages 151–3.

C. F. Wagner (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): Mr. Kuyper has raised a very interesting point in interpreting the oscillograms obtained on three-phase short circuits of unsymmetrical machines. In order to form an idea of magnitudes of errors that might result in basing the measured reactance upon the distance between the envelope of the alternating component of short-circuit current, I have made some calculations for short circuits from no load in which the decrements were neglected, that is, instead of projecting the alternating component of current to zero and thus determine the component before it has decreased, it was assumed that there was no decrement. In addition, the assumed envelope was taken as the actual envelope instead of the envelope Mr. Kuyper used as it is a more definite quantity, especially at small values of α. The results of the reactances so obtained expressed as a fraction of the correct value are as follows:

α	0	30	60	90
x_d'' for $\frac{x_q''}{x_d''} = 1.5$	1.0	0.98	0.96	0.95
x_d'' for $\frac{x_q''}{x_d''} = 3.0$	0.98	0.91	0.87	0.86

The departure from correct values is sufficiently great that consideration should be given to this factor. It would be interesting to see a comparison of the theoretical work with results of actual measurements upon machines.

W. W. Kuyper: Mr. Wagner has pointed to the phenomenon of saturation and has suggested that a comparison of results on actual machines with the theory would be of value.

The writer's experience with short-circuit oscillograms of solid-rotor, two-pole synchronous machines indicates that saturation has greater effect on the wave shape than is apparent in the differences between the symmetrical-component and two-reaction theories. Limited studies of oscillograms of short-circuit currents of salient pole machines have indicated that the two-reaction theory should be used in analysis but that saturation disturbs the magnitudes and wave shapes to such a degree that correlation of theory and practice becomes difficult.

Because it did not seem advisable to embark upon what would necessarily have been an indefinite discussion of the complexities introduced by saturation, the subject was avoided. The purpose in presenting the paper was to point out that—based on the two-reaction analysis of the ideal machine—some of the accepted methods of analysis for certain quantities do not appear to have justification.

The Water-Cooled Steel-Tank Rectifier Corrosion Problem

Discussion and author's closure of paper 41-25 by Emil J. Remscheid, presented at the AIEE winter convention, Philadelphia, Pa., January 27-31, 1941, and published in AIEE TRANSACTIONS, 1941 (April section) pages 173-8.

R. O. Nixon (Deceased, formerly of Philadelphia Transportation Company, Philadelphia, Pa.) and **C. T. Pearce** (Westinghouse Electric and Manufacturing Company, Philadelphia, Pa.): In connection with the problem of corrosion of water-cooled steel-tank rectifiers, it is believed that the results of some recently completed tests at the Erie substation of the Philadelphia Transportation Company will be of interest. This station comprises three Westinghouse 1,500-kw, 600-volt rectifier units. Each unit consists of two 750-kw, 6-anode tanks, mounted in a common framework, with one tank above the other.

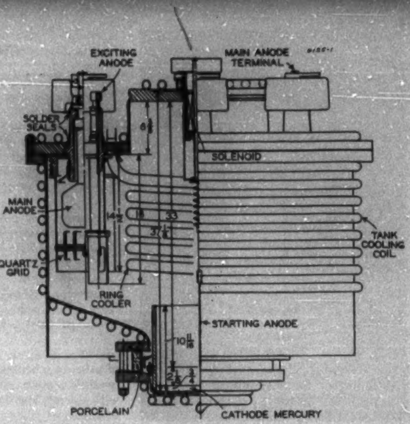


Figure 1. Cross-section view of a rectifier section

A cross-section view of one of the rectifier sections is shown in figure 1. Cooling is obtained by means of a continuous copper pipe welded to the rectifier tank. Each two-section rectifier is equipped with a heat exchanger and a water circulating pump so that the cooling water may be recirculated through a closed system. By-passes with valves are provided so that the heat exchanger can be by-passed and the raw water circulated directly.

In order to ascertain what corrosion and deposition effects would be obtained with raw city water used for cooling, one of the rectifiers, designated as 1, had the heat exchanger cut out by means of the valves and by-passes. The water circulating pump was left in the circuit to provide control and velocity. The performance of this unit was then compared with that of rectifier 2, which was operated with the heat exchanger.

During the tests, hourly readings of d-c kilowatt-hours, cubic feet of cooling water used, and the temperatures of the inlet and outlet water were taken. Water meters were installed in such a location as to measure the total water supplied to the rectifier, including that required by the mercury vapor vacuum pumps.

The results of this test over a year's duration starting October 1, 1939, are given in table I, which is self-explanatory. The variation in water inlet temperatures throughout the year is reflected in the ratio of d-c kilowatt-hours to the cubic feet of cooling water. The outlet temperature of the water from the rectifiers was held substantially constant by means of the temperature regulating valve. The over-all average of raw water discharge temperatures was 52 degrees centigrade for rectifier 1, using city water directly, and 47.5 degrees centigrade for rectifier 2, with a heat exchanger. This average 4.5 degrees centigrade difference in raw water outlet temperatures represents the drop through the heat exchanger.

It is seen that each rectifier operated slightly less than one-half of the total time during the year's period, at an average of slightly over two-thirds of its rating. The load was quite steady, and rarely exceeded full load, except for occasional short peak load periods. This is quite typical for street railway substations.

The flow of water to the mercury vapor vacuum pumps was substantially constant, and was also continuous whether the rectifier was in operation or not. For one complete rectifier, it amounted to some seven or eight cubic feet per hour. In arriving at the data in table I, the cubic feet of water was determined by deducting an arbitrary allowance for the vacuum pumps, from the total volume registered by the water meters. For the year's period, the water furnished to the vacuum pumps figures out to be about two-thirds of that supplied for rectifier cooling.

It may seem surprising that the kilowatt-

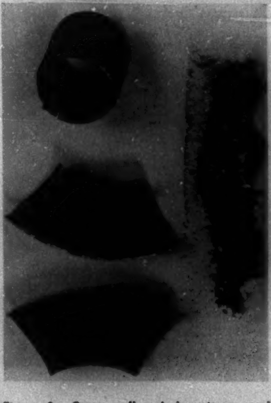


Figure 2. Copper ell and deposit removed from number 1 rectifier after one year's operation with raw city water

Table I

	Number 1 With Raw City Water					Number 2 With Heat Exchanger				
Month	Hrs Operated	Total Kwhr DC	Water (Cu Ft)	Inlet Water (Deg F)	Kwhr (Cu Ft)	Hrs Operated	Total Kwhr DC	Water (Cu Ft)	Inlet Water (Deg F)	Kwhr (Cu Ft)
Oct.	293.0	291,800	7,935	65	36.7	292.0	288,400	7,566	65	38.2
Nov.	319.5	337,000	7,753	52	43.4	296.0	309,000	6,835	52	45.3
Dec.	491.5	490,500	11,141	46	44.0	476.0	495,800	10,232	46	48.4
Jan.	190.5	210,200	4,082	37	51.5	335.0	368,100	6,280	37	58.6
Feb.	341.0	384,300	6,734	38	57.0	309.5	339,800	5,915	38	57.4
Mar.	319.0	361,000	6,823	40	52.9	336.5	355,800	6,932	40	51.2
Apr.	292.5	320,700	7,772	49	41.2	303.5	337,200	7,313	49	46.1
May	279.0	281,000	8,588	61	32.7	323.5	342,900	9,102	61	37.6
June	264.0	268,060	8,212	71	32.6	290.0	300,400	9,503	71	31.6
July	320.0	307,000	11,859	79	25.7	325.0	311,700	11,346	79	27.4
Aug.	255.5	258,000	10,763	79	23.9	291.5	292,900	11,713	79	25.0
Sept.	242.0	241,500	7,972	73	30.2	214.0	216,500	7,486	73	28.9
Total	3,607.5	3,751,000	99,634	57.5	37.6	3,792.5	3,958,500	100,223	57.5	39.4

hours per cubic feet of cooling water are slightly higher for the number 2 rectifier with the heat exchanger. The probable explanation is in unavoidable inaccuracies in the calibrations and readings of the meters, combined with the fact that an arbitrary allowance had to be made for the rather large amount of water taken by the mercury vapor vacuum pumps. Since the water from the unit operating without heat exchanger discharged at an average of 4.5 degrees centigrade higher than that from the other unit, it is obvious that it could not have used quite as much quantity. However, the data indicate that the somewhat greater amount of water required by the use of a heat exchanger is entirely negligible in over-all station consumption, particularly with the low load factor common in railway practice.

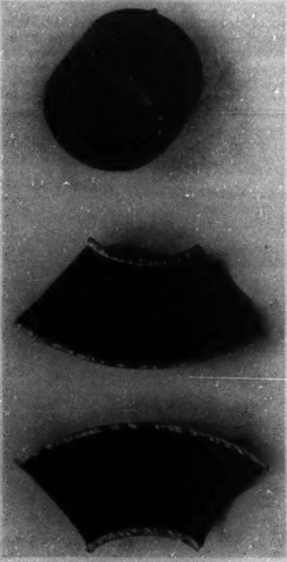


Figure 3. Copper ell removed from number 2 rectifier, after approximately two years' operation with a heat exchanger

At the beginning of the test, the interior of the copper cooling piping at the top of the rectifier section was inspected, by removing the tank thermometer bulb. Both number 1 and number 2 rectifiers had been in operation for about a year at the time this test was started. The interior of the copper piping was found clean with the exception of a light film of a black deposit which could be readily removed with a cloth. An analysis by the city chemist showed that this deposit contained oxides of both copper and iron. A sufficient amount of material was not obtained to permit making a conventional quantitative analysis and determining the relative percentages of these materials present.

In April of 1940, after the test had been in operation for about six months, the tank thermometers were again removed on both rectifiers to examine the interior of the copper piping. On number 1 rectifier, which had been using raw water, the thermometer bulb showed very slight signs of pitting and corrosion, particularly when examined under a microscope. There was a deposit on the thermometer bulbs, and on the interior of the piping, having the appearance and consistency of mud. A chemical analysis of this material indicated the presence of the oxides of iron, aluminum, and copper. On number 2 rectifier, which had been operating conventionally with the heat exchanger, the appearance was the same as on the previous examination at the beginning of the test. This was, of course, to be expected.

At the conclusion of the test, the 90-degree ells in the discharge piping leading from the cathode, were sawed out for inspection and photographing. Figure 2 is a photograph of this ell and the deposit from number 1 rectifier. Figure 3 is a photograph of the ell removed from rectifier 2.

The deposits in the rectifier pipes were similar to those found after six months, except that the amount in number 1 rectifier had increased. The deposit in number 2 rectifier was a thin black film which was very loose and could be easily wiped off. That in number 1 was somewhat harder, but much of it fell off when it became dry. It could all be easily removed, and it should be possible to keep the pipes clean by flushing, at least if some organic solvent is used.

Spectroscopic analysis, which provides approximate quantitative figures was made on the deposit in each rectifier. For number 1 rectifier this indicated the order of 10 per cent of silica, 10 per cent alumina, 22 per cent iron, 22 per cent copper, both iron and copper being in the form of compounds, and 3 per cent organic matter. Number 2 rectifier deposit, much less in quantity, had the same ingredients but proportionately more copper and less than one per cent alumina. The large alumina content in number 1 was probably due to the imperfect performance of the reservoir filter beds, which is mentioned later.

Both the presence of the copper in the deposit, and an examination of the surface of the fitting removed from the number 1 rectifier, indicated some corrosion of the copper. However, the rate indicated was quite slow and there was no evidence of pitting. Practically no corrosion was indicated in number 2 rectifier.

In discussing corrosion and deposition effects, an analysis of the city of Philadelphia water by the Philadelphia city chemist, is of interest. This analysis is given in table II. The recirculated water had a "*p*H" value of 8.3 as compared to a value of 7 for the raw city water used for cooling number 1 rectifier. The cooling water of number 2 rectifier should be slightly less corrosive than that of number 1. This can be explained by the fact that any dissolved gases would be driven off by recirculation, and any inherent acidity of the water would be used up by chemical action. The total suspended matter average of 1.8 ppm is about four to five times what is considered normal and was probably due to constructional work being done at the Queen Lane Reservoir, which prevented maximum efficiency of the filter beds. It would seem, therefore, that the test was conducted under

Table II. Analysis of Erie Substation Water Supply

	September 1939 (PPM)	March 1940 (PPM)
Chlorides	13.6	9.0
Sulphates (SO_3)	97.0	42.0
CO_2	4.0	
Iron	0.07	0.11
Alkalinity ($CaCO_3$)	61.0	28.0
Turbidity	0.5	3.0
Total hardness	161.0	92.0

Total suspended 1.8 ppm average of monthly measurements during year the severest conditions, and the amount of deposit found was probably exaggerated.

It must be remembered that this report covers experience only with Philadelphia city water. Even here, although this experience indicates that obstruction of cooling water passages with deposits could be prevented by periodic flushing and that the rate of corrosion would permit a reasonably long life, greater freedom from maintenance as well as assurance of long life justifies the use of heat exchangers.

C. Van Brunt (General Electric Company, Schenectady, N. Y.): In this paper the subject of the composition of natural waters as affecting the life of a chromate addition is touched upon. I would like to elaborate somewhat on Mr. Remscheid's statements on this point.

There seem to be three main possible sources of consumption of the chromate. The useful consumption, as is stated in the paper, is now believed to consist mainly in a precipitation on the metal of an oxidized iron-chromium complex which constitutes a protective film. This fixation of chromium takes place almost immediately after dosage. We have no data as to how much subsequent destruction and replacement occurs. The main purpose of adding a concentration of chromate which is far in excess of the actual amount fixed upon the iron surfaces, as we do, is to set up a diffusion potential to insure penetration by the solute of crevices and cavities (including preexistent corrosion pits) more or less isolated from stirring influences. Experience has also shown that as is stated in the paper a high content of chloride calls for excess chromate. Since we do not know the exact mechanism of the chloride effect on the protective film we cannot explain this. Our own records do not show any parallelism between chloride content and chromate life. We shall be glad to hear of the experience of others on this point.

The two possible sources of useless consumption of chromate are (1) precipitation as a loose insoluble chromate by metal ions present in the water and (2) reduction of the chromate ion to the chromic ion, which is useless as a corrosion preventive.

Although the first reaction has been observed in other than rectifier installations, a concentration in natural waters of those metals which form insoluble chromates sufficient to cause deposition would be so abnormal as to permit dismissal from consideration here. The second reaction is decidedly more probable. In general it may be said that organic impurity in the water—vegetable matter, bacteria, etc.—or anywhere in the system in the form of contamination on surfaces reached by the solution, is a potential source of loss of chromate. Another possible source of reduction is an imposed cathodic voltage. In both of these the reaction is greatly stimulated by acidity, and, conversely, can be inhibited by a sufficient alkalinity. This is the reason for the advice that sufficient alkali should be added along with the dichromate to convert it to chromate. The limitation to an excess of alkali is fixed in the rectifier application only by the great increase of electrical conductivity of the solution thus produced.

Thus it will be seen that we need not fear the effect of any common mineral impurity in raw waters except possibly chloride on the life of a chromate addition. Organic matter, on the other hand, constitutes a threat which can be minimized by maintaining an alkaline reaction. The observed shorter life of chromate in installations using well water may reasonably be ascribed to its higher content in organic matter combined with an acid reaction, although we have no analytical supporting data in the reported cases.

Mr. Remscheid mentions the dependence of chromate life upon the amount of make-up water added. If this amount measures loss of solution by leakage or spillage the statement is correct. It will be seen from what has been said that loss by evaporation should have no effect unless the added make-up water contributes fresh reducing matter.

The electrolytic test devised and described by Mr. Remscheid for testing adequacy of protection by chromate has given excellent service. But it should be remembered that his observations have been with a limited range of natural waters. It is believed, however, that the test is widely applicable, the only variation in the results with different waters to be looked for is in the visible anode phenomenon he describes. The nature of this will probably vary with the anion composition of the water. The decisive point is the absence of anodic attack.

J. H. Cox (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The sodium dichromate treatment of cooling water, as discussed in Mr. Remscheid's paper, constitutes a completely satisfactory solution of the corrosion problem in the cooling of mercury-arc rectifiers. However, frequently in technical problems, more than one solution is formed, varying in completeness, cost, and complication and the one used depends upon the relationship of these factors to the application.

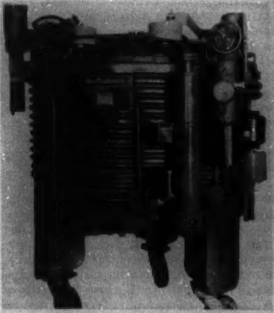


Figure 4. Small multianode rectifier equipped with copper cooling coils welded on

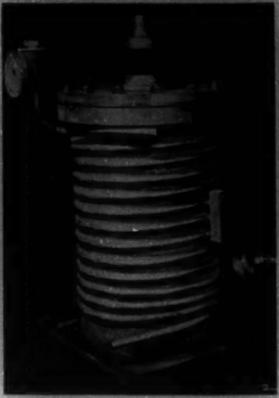


Figure 5. Large ignitron tank with copper cooling coils soldered on

The sodium dichromate treatment of rectifier cooling water involves a number of nuisance factors; namely, the solution is unpleasant to handle, it must be periodically checked and maintained, and it accentuates the problem of electrolysis at the end of the insulating connections to the raw water supply where the rectifier tank is not connected to ground. Obviously, the use of any sort of treated water requires a recirculating system and the use of a heat interchanger. Actually, the use of a closed system with reasonably pure water and without treatment constitutes a practically corrosion-proof system. Oxygen is an essential ingredient in the iron corrosion process. Therefore, if the cooling water is recirculated in a system that is closed to the atmosphere, the only oxygen available is that absorbed in the initial charge and any make-up water. The water jackets of several rectifiers operating with untreated water in a closed system were removed after three

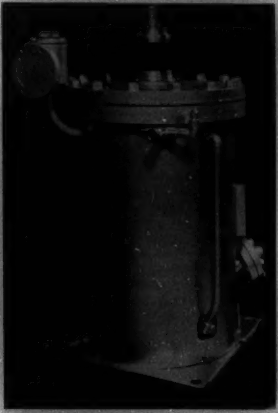


Figure 6. Ignitron tank shown in figure 5. Completed

years of service and the steel surfaces were found to be clean and practically free of corrosion. The rate of corrosion found would indicate a life of at least from 25 to 50 years.

I do not wish to infer that sodium dichromate treatment is not an effective method of corrosion prevention. It is, and I merely want to question whether, for many applications, its nuisance factors outweigh the small degree by which it reduces corrosion over the simple heat exchanger using pure water.

Another complete solution of the corrosion problem is the use of nonferrous cooling provisions. Methods have been worked out for welding, or soldering, copper pipe to the steel walls of a rectifier tank. In designs where the heat density is high, this method has the advantage of superior cooling since the added thermal resistance of the junction of the two metals is more than compensated for by the much greater velocity of the water at the cooling surfaces.

Copper cooling coils can be attached to either large or small tanks, but the difficulties increase with the size of tank. Therefore, the use of copper coils is probably not justified on multianode rectifiers but they work out most attractively on the smaller and simpler tanks inherent to ignitrons.

When equipped with copper cooling coils the only objection to the cooling of rectifiers directly with raw water is the degree to which the water available deposits scale and sediment. Such deposits in rectifier cooling pipes are more than usually objectionable because the pipes are necessarily curved, which precludes mechanical cleaning and deposits must be removed by flushing. Actually the advantage of direct cooling is not great since a recirculating system is necessary in any event to provide proper temperature gradients and control. This recirculating system contains all of the elements of a closed system except the bare heat exchanger, and the inclusion of this item is usually more than justified as insurance from difficulty with scale and sediment.

The following figures show various designs using copper coil cooling jackets.

E. J. Remscheid: In the discussion presented by Mr. Cox he brings to our attention several points in connection with the use of sodium dichromate. These points are of minor importance where the life of apparatus will be actually extended, by the application of an inhibitor. I am in full accord with Mr. Cox in this respect and feel that wherever water is available which will not in a reasonable length of time produce a "corrosion problem," then the application of an inhibitor is unjustified. However, should the "corrosion problem" be a possibility within the expected life of the apparatus, then the addition of a heat exchanger and the application of an inhibitor is low-cost insurance in preventing interruptions in service and expensive repairs.

Mr. Nixon and Mr. Pearce's interesting discussion presents another phase of corrosion in the nonferrous field.

Because of the gradual decay in effectiveness of the sodium dichromate solution as an inhibitor, as explained in Mr. Van Brunt's discussion, periodic examination of the solution is necessary. The frequency of such examination will naturally depend upon the chemical nature of the water, seasonable variation in chemical constituents, and the amount of make-up water added.

It is evident that the use of sodium dichromate is not applicable to all corrosion problems, but it opens up a field of thought whereby all corrosion problems should be studied and a remedy found, so that the expected life of water-cooled apparatus will not be limited by corrosion.

Picture Transmission by Submarine Cable

Discussion and author's closure of paper 41-18 by J. W. Milnor, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (March section) pages 105–08.

W. S. Gorton (Bell Telephone Laboratories Inc., New York, N. Y.): It is gratifying to one who has been connected with the development of submarine cables to learn that submarine cables are now carrying the currents of picture transmission as well as those of printing telegraphy and of the cable code and thus that additional use is being made of the superior stability of the transmission path and the reduced susceptibility to interference offered by the submarine cable. These aspects of the subject have been discussed by the author, and he has given some values of the magnitude of the interference. It would be of much interest if he would give us more detailed information concerning the range between 5 and 500 volts, in view of the connection between cable interference, radio interference and transmission, magnetic storms, and sun spots.[1] It would also be of much interest to know whether the properties of the cable have changed at all in the 14 years since it was laid, and if so in what respect and how much.

J. W. Milnor: The discussion has centered around the stability of the path which is furnished by the submarine cable for picture transmission, and the types of interference to which the path is subject. The static disturbances which find their way into both radio circuits and submarine cables, may be considered to be present at all frequencies between roughly 1 cycle and 10^7 cycles per second. The ocean water tends to shield the cables, so that only the lowest part of the frequency range is of importance. The static currents cause no substantial interference to performance when the working equipment is properly designed to take it into account.

As distinct from the above, the earth potentials which coincide with or shortly follow disturbances in the atmosphere of the sun, are in a frequency range from about one cycle per hour to two cycles per minute, which is well below the operating frequency, but still must be taken into account in the design of equipment. We have learned how to operate cables in telegraph service through the heaviest earth potentials that occur. Picture transmission, however, is impaired when earth potentials are severe.

I know of but one occasion in the past three years when 500 volts was reached, and that continued but a few minutes. Values exceeding 100 volts occurred on 28 different days in that period, the duration of each averaging perhaps 15 minutes. Values of 50 volts occurred more frequently, but they are of less importance.

Since the loaded cable used in this service was laid in 1926 there has been no change in its inductance of amount sufficient to be observed by attendants responsible for equipment or operation of the cable. Measurements of impedance show that the sensitive permalloy loading in some limited sections has suffered changes in inductance, but this has been insufficient to cause noticeable change in the over-all attenuation.

High-Potential Testing Equipment for Quantity Production

Discussion and author's closure of paper 41-55 by C. M. Summers, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (June section) pages 289–92.

J. H. Goss (General Electric Company, West Lynn, Mass.): The author has brought out the major defects of high-voltage short-time testing of low-voltage apparatus that come about in the attempt to speed up the tests.

Many thousands of such test sets have been made and are being used daily, many of which are subject to the criticism of introducing transient surge conditions during the test. While it may not be practical to cover this type of insulation testing in AIEE specifications, it would seem worth while to consider adding a note of warning and to suggest the principles that should be followed in developing such a test. This paper would furnish an excellent basis for such comments or notes.

The use of the cathode-ray oscillograph, as suggested, would be a real addition to many testing sets now in use. Such oscillographs are low in cost, readily available, and give immediate warning of unusual conditions arising during a particular test.

The wave shape in figure 2 is far from a sine wave. While it illustrates the initial surge very well, I should like to ask the author for an explanation of this particular wave shape and just how this oscillograph was obtained.

The point of the effect of conduction current with moist organic dielectric materials in the testing set transformer regulation is well made. This is an excellent example of the additional information that may be obtained from the cathode-ray oscillograph.

J. H. Hagenguth (General Electric Company, Pittsfield, Mass.): Mr. Summers points out the difficulties of high-potential testing in quantity production. The principal source of trouble appears to be the overvoltages produced by connecting the

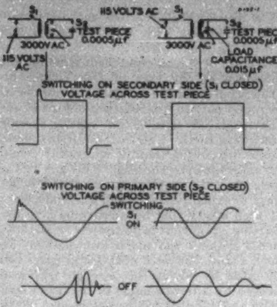

S1
115 VOLTS AC
S2
TEST PIECE 0.0005µf
3000V AC
115 VOLTS AC
S1
S2
TEST PIECE 0.0005µf
3000V AC
LOAD CAPACITANCE 0.015µf
SWITCHING ON SECONDARY SIDE (S1 CLOSED) VOLTAGE ACROSS TEST PIECE
SWITCHING ON PRIMARY SIDE (S2 CLOSED) VOLTAGE ACROSS TEST PIECE
SWITCHING S1 ON
OFF

Figure 1

test piece through the inductance of the testing transformer to the source of power by means of hand electrodes. I agree with him that oscillations and overvoltages should be eliminated since their frequencies and amplitudes will differ widely depending on the type of test equipment used, the capacitance of the test specimen, the point of the 60-cycle wave at which the test piece is connected, the speed at which contact is established, etc. Therefore, different test results will be obtained on test specimens which are identical depending on the factors which are out of the control of the operator. The author has pointed out a solution to this problem which consists essentially in insertion of series resistance in the circuit of sufficient value to damp out these oscillations.

I would like to suggest another method. The details have not been worked out, but the principle is this:

The oscillations in the usual testing circuit are caused because the charging current of the specimen has to be supplied through an inductive circuit. If a capacitor with a capacitance of approximately 10 to 20 times the capacitance of the test specimen is connected permanently in parallel with the high-voltage winding of the testing transformer, the high charge accumulated in this capacitor will be available at the time of spark between the hand electrode and test terminal causing a voltage drop of only a few per cent. Therefore, current does not need to be supplied through the inductive circuit and oscillations and overvoltages are eliminated.

The voltage measured across a capacitor of 0.0005 microfarad as a test piece with and without the large capacitor is shown in figure 1. It is evident that overvoltages do not exist when the capacitor is connected. In this respect the circuit suggested in this discussion is equivalent to that suggested by the author.

I believe that the circuit with capacitance connected has an advantage because the charge stored in this large capacitance will be available to complete incipient breakdown through the insulation of the test specimen when once started by corona or partial breakdown, since series impedance between the test piece and the source of power supply is a minimum. When tests are made with long leads between the transformer and the hand electrodes, some small amount of resistance (of the order of 100 ohms or less) should be connected between the large capacitor and the test specimen to avoid high-frequency oscillations due to the inductance of the leads and the capacitance of the specimen.

C. T. Weller (General Electric Company, Schenectady, N. Y.): The question as to what constitutes suitable equipment for making high-potential tests has been with us for a long time. It came up in connection with the first edition of AIEE Standard 4 on measurement of test voltage in dielectric tests. The differences of opinion that existed in regard to the measurement specifications were negligible compared with the differences in regard to the testing equipment. Even if the desirability of including test-equipment specifications in a measurement standard were conceded, it would still be impossible to give sufficient details to cover all conceivable conditions. Such details are important, since practical testing experience has shown that voltage measurements made when using unsuitable testing equipment may be very misleading. When the subject of the step-up transformer rating is under consideration, the reaction is apt to be that the smallest unit available should be entirely satisfactory because of the light load; frequently this has meant the selection of a potential transformer. Such a conclusion disregards the possibility of surges due to heavy corona streamers, momentary arc-overs, or switching; the effect of switching surges on the ordinary voltage measurements is clearly shown in Mr. Summers' paper for small apparatus. These surges are even more important for large apparatus, for which the rating of the testing equipment becomes a major consideration. I once saw an attempt made to test all three phases of a 25,000-kva generator with high-potential equipment barely large enough to test one phase at a time. It is probable that no other class of equipment is so universally misapplied. In general, it may be concluded that amply-rated, low-reactance equipment is preferable both for routine and for special high-potential testing.

C. M. Summers: The physical size of a high-potential testing transformer could be fundamentally determined if we had accurate information regarding the amount of energy required to produce a failure. How much energy is required to produce an incipient breakdown and how much more energy is required to produce complete failure? In addition to the energy required for breakdown, the testing transformer must be able to supply all of the charging current and leakage current required by the load and at the same time maintain good voltage regulation. Mr. Weller is correct in his statement that testing transformers should be amply rated or co-ordinated with the load requirement. However, a surplus of capacity in the testing transformer will not alleviate the hazards arising from switching surges, and the large transformer may subject the insulation to more severe stress than a small transformer properly controlled.

Mr. Hagenguth has introduced an interesting alternative method for minimizing the effect of switching both in the high-voltage and low-voltage circuits. This method may have merit in the fact that additional energy can be supplied by the capacitor, but this again brings us the question of how much more energy is required to produce a complete failure than incipient breakdown. Incipient failures have been observed on the cathode-ray fault detector an instant before complete breakdown occurred, particularly when the potentiometer control as described in the paper was used in connection with the testing transformer. There is a growing philosophy that nondestructive high-production testing is the ideal test, providing of course the testing equipment will produce a positive indication of impending failure. In addition to the cost of the high-voltage capacitor required by Mr. Hagenguth's arrangement, a control of the magnitude of the test voltages is desirable, and this is inherently obtained with the potentiometer method.

Perhaps the question of wave form mentioned by Mr. Goss needs further explanation. The wave shown in figure 2 of the paper represents the test voltage only while an arc is established during the high-voltage switching period. This is a typical wave form associated with circuits containing an arc. The actual wave form of test voltage when the high-voltage circuit is solidly established is nearly sinusoidal.

Automatic Printing Ammeter

Discussion and author's closure of paper 41-3 by T. G. Le Clair, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (March section) pages 93–9.

M. S. Wilson (General Electric Company, West Lynn, Mass.): The device described by the author is interesting and ingenious. While it does not furnish as complete information as a recording instrument of the conventional type, it does furnish sufficient information for the purpose required in a more compact and convenient form.

It is indicated in the paper that an accuracy of plus or minus one per cent is obtained. From the description it would appear that the current is balanced to within 1 ampere which would be one per cent assuming a full scale of 100 amperes. If, however, the maximum current does not exceed 10 or 20 amperes in some of the circuits, the accuracy would be five per cent or less. This is the penalty paid when a single range instrument is used to measure quantities which may vary widely in magnitude, but it still may be satisfactory for this application.

There is one condition of operation about which we should appreciate further information; in the case of a fluctuating load, what would be the result if the current in a particular circuit were 39 amperes, for example, and the 10's selector switch stops at 3, then the current shifts suddenly to 41 as the units selector switch starts to operate? Would the instrument's switch stop at 1 with 31 as the total, or would the 10's switch move up to 4?

There is one point in the totalized readings which is not clear, and that is that it would appear that the readings are simply added up algebraically instead of vectorially. If this is the case a considerable error might, of course, result. For example, if the current in two circuits were balanced but one operating at 0.50 per cent power factor and the other at unity power factor an error of 13.4 per cent would result.

We would appreciate the author's comments on this point. Further information would also be appreciated on just how much difficulty was encountered due to variable resistance in the contacts and if any unusual precautions were taken to keep the contact resistance at a minimum.

Paul MacGahan (Westinghouse Electric and Manufacturing Company, Newark, N. J.): Looking at it from the point of view of an engineer engaged in meter manufacturing, Mr. Le Clair's automatic printing ammeter is of considerable interest. It fills of course a definite need in powerhouses or substations, as explained so clearly by Mr. Le Clair.

The design and construction of such an instrument seems to me to be quite definitely in the province of the operating engineers rather than that of the manufacturer and is an excellent example of how operating engineers can solve their special problems without calling upon a manufacturer.

For a meter manufacturer such a project would be a special job throughout to suit an individual installation, and every additional outfit would be different, involving as it would, a study of each installation, field development, complete manufacturing, and drawings. The expense of preparation would be prohibitive. It would divert effort which could more profitably be applied in other ways.

Special apparatus can be efficiently developed by a meter manufacturer, and thus become "desirable" when it can pay its own way or else project the maker into future desirable activities. Such special apparatus may turn out to be an important standard product later on, as exemplified by the development of oscillographs and telemeters. It is also desirable as a service to the industry when the meter manufacturer has available talent or tool facilities which do not exist elsewhere.

T. G. Le Clair: The statements in Mr. Wilson's discussion concerning the accuracy of the printing ammeter are substantially correct. If the meter is assumed to have a full scale of 100 amperes, the accuracy of this meter compares favorably with the accuracy of a switchboard-type indicating ammeter with any scale from 25 amperes to 100 amperes. For circuits with a maximum current less than 25 amperes, it might be desirable to use a different multiplier. The meter in its practical application uses a secondary current transformer between the metering element and each of the measured circuits. It is a simple matter to apply the correct ratio of secondary current transformer to obtain any desired multiplier for the various circuits.

When the totalizing attachment is used, it is only necessary that the same multiplier be used for all the group of circuits which are to be totalized together. This is not actually a limitation because if we are going to totalize three circuits, two of which have a maximum load of 100 amperes, and the third a maximum load of 10 amperes, a ten per cent error in the reading of the 10-ampere circuit will not disturb the total reading any more than a one per cent error in a 100-ampere circuit. A practical application of this principle would be in the case of a generating station where three or four large units supply 40 outgoing lines. The meter can then be adjusted for one multiplier for the currents on the outgoing transmission lines, and another multiplier for the currents in the generator leads.

The question which Mr. Wilson brought up concerning the reading of fluctuating load calls attention to a feature not mentioned in the original paper. The meter is provided with auxiliary connections to take care of this particular contingency. Using the example that Mr. Wilson mentioned, if the current in a particular circuit is 39 amperes when the tens selector switch stops at 3 and increases to 41 amperes before the unit selector switch operates, the procedure is as follows: the tens selector switch will print the figure 3 as usual while the current is 39 amperes. Before the unit selector switch operates, the current has increased to the point that nine additional amperes in the units coil will not balance the load current. The units selector switch is equipped with an auxiliary contact which will stop the rotation of the switch in the ninth position and print the figure 9 regardless of whether or not the balancing relay has come to the balance point. This feature is, however, seldom called into play. The time which elapses between the balancing on the tens digit and on the units digit is only a fraction of a second and during this time, the current will not vary much on an ordinary circuit.

Mr. Wilson's statement is correct that the totalizing operation in this meter is algebraic and not vectorial. For the normal application to determine the loading of transmission lines, the algebraic addition is an advantage. The thing which determines capacity needed in transmission lines is not the vector sum of the loads on the various transmission lines, but the loads on each of the lines in a given group which cause the heating of cable in each of these lines. In ordinary cases, where parallel cables serve a common load, it is not possible to have a wide variation in power factor on the parallel cables due to the circuit characteristics. Where these cables are supplied through transformers with varying tap ratios, it is possible to have material differences in power factor and for such cases it is particularly important to determine the algebraic sum of the currents as a measure of the loading of parallel lines.

No difficulty has been experienced due to the variable resistance in the contacts of the metering circuit in two or three years' operation. In fact, one of the instruments was dismantled and put together several times for testing and demonstration purposes. After this operation, no recalibration was found to be necessary. The apparent reason for the stability of calibration is due to the fact that the values of the resistances used in the calibration circuit are high compared to the resistance of the contacts.

Mr. MacGahan's discussion refers primarily to the relationship between the manufacturer and user of metering equipment and is a worth-while contribution to this subject. I believe with Mr. MacGahan that there are times when the design and construction of special equipment is within the province of the user of this equipment. This is particularly true when the equipment is suited only to specialized problems of one user and not suitable for use by a wide variety of users. There are additional times when the development of equipment should be undertaken by the user. This may be the case when the problem is such that it is not easy to explain the needs to a manufacturer in such a way that he can sympathetically develop equipment to accurately fulfill those needs without too much lost motion. In such a case, the most effective progress can be made by having the principle tried and proved by a device made in the user's own shop and adapted to his own needs. From this point on, a manufacturer can adapt equipment for wider commercial applications without losing the fundamental benefits. Frequently the manufacturer can also develop equipment from this starting point which will be better in appearance and in performance from that made by the user's own men.

Such was actually the history of the automatic printing ammeter. The original development work was carried out on the property of the operating company. After the results had proved that the development was justified, it was found possible to have a manufacturer equipped for production of finished apparatus turn out a better and more useful product at no higher cost. By this means, it was also possible to obtain a device which has a broader usefulness and can be applied to a number of problems in addition to the original one which caused the operating company to undertake the problem.

Magnetic Fields in Watt-Hour Meters; Effects of Wave Form on the Registration of Single-Phase Watt-Hour Meters

Discussion and authors' closure of paper 41-43 by C. A. Keener, M. A. Faucett, and M. S. Helm, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (May section) pages 202–05.

M. G. Lloyd (National Bureau of Standards, Washington, D. C.): It would be of some interest to compare the performances of the watt-hour meters of today and those of a previous generation, with respect to the effect of wave form upon their registration. An investigation carried on at the National Bureau of Standards nearly 40 years ago[1] dealt also with single-phase meters of the induction type and applied the third harmonic to produce peaked and flat-topped waves having distortion factors of 10, 24, and 45 per cent. Voltage and current waves had the same form; the load was 100 per cent; and the power factor was kept near unity. Hence no comparisons are available at lower power factors, at different loads, nor with different wave forms for current and voltage. Both investigations applied a third harmonic at different phase angles to distort the wave.

Unfortunately very few comparisons can be made, for although many conditions of the two investigations were identical, the authors of the present paper appear to have given only a few selected results in figures 6 to 10; and for most of these the conditions differ in some items from the former investigation.

In figure 6 with sine voltage and distorted current, the errors are less than 2 per cent up to 40 per cent distortion factor; within 3 per cent up to 65 per cent distortion factor; and become large above 90 per cent.

In figure 7 with sine voltage and 40 per cent distorted current, there is a plus error up to 1.3 per cent for a flat wave. Rosa found no such change with shift in phase when both voltage and current were distorted.

In figure 8 with sine voltage and peaked current, the minus error at unity power factor is changed to a plus error at low power factors.

In figure 9 both voltage and current were distorted (peaked); here the minus error decreased as load increased. At full load, the error is minus 3 per cent for a distortion factor of 50 per cent, compared with only 1 per cent when the current alone is distorted (figure 6). This shows the effect to be multiplied if both waves are distorted, at least for one individual meter (number 1). Comparing this with Rosa's results at 45 per cent distortion factor, meter number 1 is no better than the meters of 1904, and is indeed more affected by wave distortion than three of the 1904 meters.

Figure 10 deals with inductive loads and is not comparable.

Reference

1. **Rosa, Lloyd, and Reid.** *Trans. Intern. Elect. Congress*, St. Louis, 1, (1904) page 555; *Bull. of Bur. Stds.*, 1 (1905) page 421.

Warner T. Smith (USDA, Rural Electrification Administration, Washington, D. C.): The authors of this interesting paper have presented the results of their investigations of important characteristics of watt-hour meters. In extensive systems of the REA type, the effect of harmonics on metering instruments is of unusual interest because resonant conditions on the system, in many cases, accentuate the importance of the harmonic voltages and currents relative to the fundamental.

In a situation recently encountered, in connection with an inductive co-ordination problem, values of currents and voltages were measured at the substation as shown in table I.

Table I

Frequency in Cycles Per Second	Primary (Volts)	Per Cent of Fundamental	Primary Current (Milliamperes)	Per Cent of Fundamental
60	7,270	100	10,133	100
300	827	11.5	11,266	111
420	173	2.4	3,153	31

At the time of the measurements, the power factor of the fundamental voltage and current was approximately 80 per cent leading. The power factor of the harmonics was not determined, nor, was the phase angle between the fundamental and harmonic voltages and current. I would like to ask the authors if their investigations covered situations comparable to that shown by the data, and if so, what effect on the registration could be expected?

L. A. Doggett (The Pennsylvania State College, State College, Pa.): In a long series of three-phase tests of several makes of polyphase watt-hour meters in 1927–29, using impressed voltages consisting of sinusoidal fundamentals with and without fifth harmonics, the following conclusions were reached:

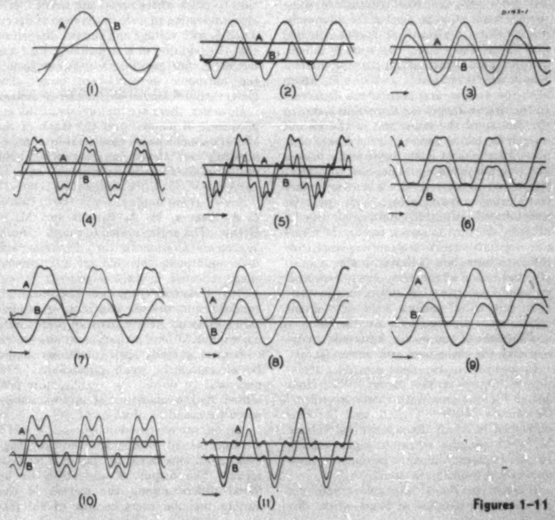
1. For a light-load to full-load range of currents in phase with the fundamental frequency voltages, the greatest error disclosed was less than two per cent; without fifth harmonics;
2. For a similar range of inductive and capacitive loads the greatest error was less than $3^{1}/_{2}$ per cent; without fifth harmonics;
3. When the voltage consisted of a fundamental and a 50 per cent fifth harmonic, the largest error for inductive loads was 9 per cent, whereas for capacitive loads the errors were in all cases very large and of the order of magnitude of 20, 40, 60, and even 100 per cent slow.

In one of our tests harmonic-free voltages were applied to a three-kilovolt-ampere, three-phase transformer, delta-connected primary, wye-connected secondary. A load of six two-kilovolt-ampere capacitors in various series and parallel arrangements was connected to the transformer secondary. Using calibrated indicating wattmeters as standard instruments, these loads were metered by two makes of three-phase watt-hour meters simultaneously. Errors of slow 7 to 8 per cent for one meter and fast 12 to 18 per cent for the other were found.

The results presented in this paper and in the discussion by Mr. W. T. Smith show that the watt-hour meter of today has still smaller errors in the cases they describe. The case of capacitive load, however, does not receive sufficient consideration. It is the capacitive load that magnifies the harmonic effects and exaggerates the fifth harmonic or 300-cycle currents. It is only fair to give the watt-hour meter credit for a high degree of accuracy in metering all ordinary loads and many unusual loads. There is, however, some danger in establishing for the watt-hour meter a reputation for universal accuracy since in the case of capacity loads and distorted voltages the watt-hour meter is definitely not reliable. Whenever a buyer of electrical energy is not sure of the accuracy of his watt-hour meter because his load is predominantly capacitive, he may check his meter very easily by the use of standardized indicating wattmeters and a stop watch.

H. E. Trekell (General Electric Company, Lynn, Mass.) and **G. F. Gardner** (nonmember; General Electric Company, Schenectady, N. Y.): In handling this involved problem of the effect of abnormal wave forms on watt-hour meters, there are many features which may be considered. In their treatment, the authors are to be commended, for many of the limitations have been evaluated and the good performance established without a surplus of less important details.

From our experience in measuring the flux in watt-hour meters and observing the voltages induced in search coils placed in the air gaps, we do not believe that the wave shapes shown in figures 3, 4, and 5 are representative of modern meters. The waves shown by the authors, in their oral presentation, show somewhat less distortion.



Figures 1–11

Table II

Figure	Load Conditions	Per Cent Harmonic Content of Voltage Wave, Harmonic Order				Per Cent Power Factor = 100 × Watts / Volts × Amperes	Per Cent Distortion Factor		Per Cent Effect on Watt-Hour Meters (Referred to Registration With Sine-Wave Conditions)	
		1st	3rd	5th	7th		Voltage Wave	Current Wave	Single-Phase Meter Type I-30	Polyphase Meter Type V-3
1	Half-wave rectifier. Sine-wave voltage					63	0	38	+0.4	
2	Saturated reactor. Sine-wave voltage					50	0	37	+1.2	+1.9
3	Fluorescent lamp installation. Sine-wave voltage					97	0	11	−0.1	0
4	Extreme distortion of voltage wave. Resistance load	100	4	13	4	100	14	14	−0.5	0
5	Same as figure 4 except with resistance and capacitance load	100	4	13	4	73	14	54	−1.1	−0.9
6	Less distortion of voltage wave. Flat-topped wave with resistance load	100	10	6	2	100	12	12	0	+0.1
7	Same as figure 6 except with resistance and inductance load	100	10	6	2	50	12		+0.3	+0.4
8	Same as figure 6 except with peaked wave	100	10	6	2	100	12	12	−0.3	−0.3
9	Same as figure 8 except with resistance and inductance load	100	10	6	2	50	12		−0.4	−0.1
10	Flat-topped wave and 50 per cent distortion with 3rd harmonic. Resistance load	100	58			100	50	50	−1.9	+0.3
11	Peaked wave and 50 per cent distortion with 3rd harmonic. Resistance load	100	58			100	50	50	−2.9	−1.4

DESCRIPTION OF LOAD CONDITIONS

In each of the figures. *A* identifies the voltage wave and *B* identifies the current wave.

For figure 1, the current is that supplied to a step-down transformer and a gas-filled half-wave rectifier tube.

Figures 4 and 5 are examples of extreme distortion in the voltage wave of a power system. It is believed that distortion as severe as this is not often found. However, a recent case with 12 per cent fifth harmonic before correction was the subject of AIEE paper 41-74 entitled "System Lower Harmonic Voltages—Methods of Calculation and Control by Capacitors" by Messrs. Feaster and Harder.

Figures 6, 7, 8, and 9 are examples of less distortion in the voltage wave. The values are based on the maximum harmonics reported in Engineering Report No. 15, "Wave Shape Survey on Operating Power Systems" by National Electric Light Association and Bell Telephone system, February 3, 1932.

Figures 10 and 11 show the extreme distortion required to obtain a 50 per cent distortion factor with the third harmonic.

Although some distortion should be expected because of the use of magnetic materials, it is apparent from the actual performance of modern meters that the effect is secondary.

In considering the significance of this problem, the General Electric Company has from time to time considered some specific loads and the data obtained may be found helpful. This work has been divided into cases where first, the voltage is substantially not distorted but the load draws a nonsinusoidal current; and second, where both the voltage and current are distorted. In the first instance, the distortion is due to the nature of the load, and in the second case, it is due to factors in the power supply. In the tests both single-phase meters and polyphase meters connected as single-phase meters were used. Therefore, when considering polyphase meters, it must be remembered that additional factors may be present, due to the power factors at which the respective electromagnets operate, but the data found are believed to give a valuable indication of the errors to be expected. In table I are given the effects on the watt-hour meters of various load and circuit conditions. Also included are references to oscillograms of the waves, harmonic analyses, and the distortion and power factors.

In addition to the tests described attention is invited to the paper, "Watt-Hour Meter Performance With Power Rectifiers" by Messrs. Weller, Trekell, and Stebbins published in AIEE TRANSACTIONS, volume 59, 1940 (August section), pages 449–56. In this instance, the error in the watt-hour meters was found to be small.

Because of these favorable results on outstanding examples of loads where distortion exists, the conclusions of the authors are confirmed and furthermore, it may be said that considerable distortion must exist before significantly large errors occur. It is believed that very rarely does the distortion factor exceed 40 per cent which limits the practical occurrence of distortion to cases where errors are small. It is also interesting to note in the case of figures 4 and 5, with voltage and current distortion, that the addition of a capacitance load had but little effect on the meters even though the harmonic currents are large. The meters still responded well as energy meters.

However, there are limitations in the applications of meters, and the work of the authors should be of considerable value in pointing out those instances where the errors are likely to be excessive.

C. A. Keener, M. A. Faucett and M. S. Helm: The authors wish to thank Messrs. Trekell and Gardner for their discussion and data confirming the findings and conclusions reported in this paper. It is suggested, however, that since the magnetic conditions in the air gap of a watt-hour meter differ so greatly between points only a few hundredths of an inch apart any search coils used to study such conditions should be of extremely small dimensions. The one used to obtain the oscillograms presented had a diameter of approximately seven thousandths of an inch.

The curves presented in figures 3, 4, and 5 are from actual oscillograms showing magnetic conditions in the air gap of a modern meter. The degree of distortion to be found differs among the meters of the various manufacturers because of the differences in the details of the magnetic circuits, especially those parts that operate at high magnetic density under heavy load.

We agree with the conclusions drawn by Mr. Doggett from his tests. However, in comparing results, care must be taken to keep separate the effects of "leading power factor" and those introduced by the harmonics. No particular study was given cases of fundamental current leading fundamental voltage since it was considered unlikely that such commercial loads would be metered individually. The condition does, however, introduce additional factors of interest and warrants investigation. In polyphase meters such conditions, on a single meter element, will not be uncommon and in the study of such meters must be considered.

It is interesting to note the comparisons offered by Mr. Lloyd. We wish to make clear that figure 7 is for conditions of distorted current only, the potential being sinusoidal. In the discussion of figure 9 mention was made of the absence of appreciable differences in the effects of peaked and flat-topped waves when both potential and current were distorted. Both cases appear to offer reasonable checks with the findings of Rosa. One conclusion that might be drawn from the comparisons given is that the addition of corrective details for overload, temperature change, and other errors has not improved the over-all performance of the meter under every possible condition. This is to be expected.

Mr. Smith has raised a question that is difficult to answer quantitatively for the case cited. As stated in the reply to Mr. Doggett, no particular study was made for capacitive loads giving different degrees of distortion in potential and current. It is believed however that, for a case with a degree of distortion as large as that given, and for a customer whose load was a significant part of the total, the use of indicating instruments of the dynamometer type in the calibration of the meter would be advisable. The registration characteristics of the meters involved being unknown it is difficult to predict the result. If the excessive distortion given occurred only at light load, the error in the total registration would probably be small.

The Detection of Initial Failure in High-Voltage Insulation

Discussion of paper 41-27 by J. B. Whitehead and M. R. Shaw, Jr., presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (June section) pages 267–72.

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Wm. A. Del Mar (Phelps Dodge Copper Products Corporation, Yonkers, N. Y.): The device described by the authors and the results which they have obtained to date, indicate that they have a very useful tool which is likely to elucidate many of the mysteries of cable behavior.

It is well known that a drained cable, that is a cable which has been impregnated with oil and from which the oil has been allowed to drain out, will ionize at a certain critical stress, but if that stress be continued the ionization will stop and will not be resumed until a considerably higher stress is applied.

The reason for this rise in the ionization point is not fully understood. This new instrument might be used to find out something about what is happening inside of the cable while the ionization is dying out.

H. H. Race (General Electric Company, Schenectady, N. Y.): Through operating experience in the field and many types of experiments in the laboratory we are continually learning more as to why liquid impregnated dielectrics behave as they do. In this paper Dr. Whitehead has presented more evidence in support of the conclusions of many investigators that at abnormally high gradients, ionization or localized corona starts in an oil space and if the voltage is maintained, progressive carbonization leads to failure in a few hours.

There are four aspects of this failure mechanism which I should like to emphasize.

First—For any given composite dielectric system there is a voltage threshold value below which ionization will not start. If this voltage is not exceeded and if the physical conditions of the system do not change, the electrical life will be long and final failure will result from slow thermal decomposition or chemical deterioration. This is the desirable service condition and explains the superiority of the oil-filled cable system, which is designed to prevent ionization under operating conditions.

Second—There are conditions under which *failure will not occur even though ionization or localized corona does exist* in portions of the composite dielectric. I believe it is necessary to differentiate between two degrees of ionization: (*a*) one in which the intensity of the corona is too low to cause appreciable thermal decomposition of the surrounding material, and (*b*) spark discharges of sufficient intensity to propagate themselves by carbonization of the surrounding material. The failures in Dr. Whitehead's specimens come in this latter class. On the other hand, it is known that solid-type cables have operated indefinitely with localized corona between layers of paper throughout the insulation wall, as evidenced by wax formation. Apparently the intensity of such discharges is limited, by the amount of energy which can be fed through the capacitance of the portions of dielectric in series with them, to a value too low to cause carbonization and progressive failure.

Third—The greatest danger in abnormal voltage relatively short-time life tests lies in the subconscious tendency to believe that the system which stands up best on such tests will also be best under considerably lower voltage service conditions. In homogeneous dielectrics this may be true but in composite dielectrics it may not, because in long-time service factors leading to chemical change and thermal deterioration may be more important than the physical conditions which determine the ionization starting point.

Fourth—The real contributions of work such as Dr. Whitehead has reported are: (*a*) Improved methods for the study of ionization starting points. (*b*) New knowledge as to the effects of short-time high-voltage applications on oil-impregnated paper together with indications of recovery or self-healing tendencies if the voltage is lowered before material damage has been done.

I hope Dr. Whitehead will greatly extend our knowledge of the latter by a continuation of these studies with corona detection equipment.

Electrolytic Process of Scale Removal From Steel

Discussion and author's closure of paper 41-77 by H. W. Neblett, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (June section) pages 294–6.

F. O. Schnure (Bethlehem Steel Company, Sparrows Point, Maryland): This electrolytic process was added in order to increase the pickling capacity for strip steel of thin gauge, narrow width, and of considerable length, without making a major capital outlay for another line. It appears to be successful.

The first thought in considering this application is that the maintenance required on the additional electrical equipment described, must be considerable because of the bad conditions under which it must operate. Strip steel in a pickle line is in motion only about 50 to 55 per cent of the time, the remainder being employed in threading, welding, tacking, and handling. Consequently, a given increase in the feet per minute speed of the steel while in motion, only results in about half that increase in over-all production. If this electrolytic equipment presents extra maintenance of any consequence, the improvement may not be worth while.

It would seem that the electrolytic features of this addition to the pickling process would always be a nuisance from the maintenance viewpoint and that the increased capacity, although practicable by the method described, might as well be achieved by other means, possibly the addition of another tank in the line.

H. W. Neblett: The comments regarding maintenance are well taken. That part of the equipment that is subjected to sulphuric acid has to operate under severe conditions and the materials that stand up in this service are limited. However, as pointed out in the article, satisfactory materials were found from experimentation. These materials, such as carbon, Pyrex glass, and certain grades of synthetic materials are not affected by acid or electrolytic action and have long life in service free from repair.

The equipment outside the tanks is standard in design and consists of transformers, control, and connections. Therefore, the maintenance on the entire electrolytic equipment as now designed is no greater than any other standard equipment.

The question raised regarding the time consumed in feeding coils to the pickling line including threading, shearing, and welding has to be considered in determining the maximum speed at which any line is to operate. This point is discussed under the heading "Operations."

It is important that a pickling line be so designed that continuous operation of the strip through the pickling tanks can be obtained. This is provided by a looping pit or storage at the feeding end of the pickling tanks, between the tanks and the threading, shearing, and welding equipment. This permits the pickling line to be operated continuously, being supplied with steel from the looping pit during the time a new coil is being handled, sheared, and welded. This permits the strip steel to be in motion continuously through the tanks, which is desirable to prevent the steel becoming overpickled.

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Large Electric-Arc Furnaces—Performance and Power Supply

B. M. JONES
MEMBER AIEE

C. M. STEARNS
NONMEMBER AIEE

ELECTRIC arc furnaces are desirable loads for power systems, and are becoming more and more used for producing high-grade alloy steels, as well as for other types of steel. The authors believe that the data in this article will be of value and assistance to others in the solution of similar problems.

It is necessary in connecting large electric arc furnaces onto power systems that it be done in such a manner that neither the voltage changes, caused by the violent and rapid load swings of these furnaces, will interfere with other load of the furnace customer nor be permitted to reflect back into the system in such a manner as to interfere with service to adjacent customers. Also precautions must be taken so that such widely fluctuating loads will neither interfere with the operations of any of the equipment on the power system nor interfere with any interconnection with adjacent power companies. Further, precautions must be taken so that voltage regulation caused by large short-time loads will not be too great.

Summary

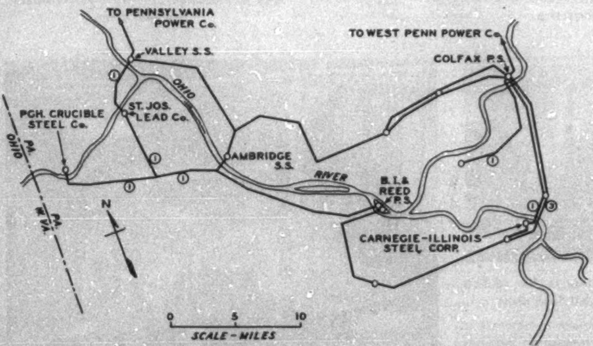
In March 1940, a 25-ton Heroult 3-phase electric arc furnace using a 10,000-kva furnace transformer was connected to the Duquesne Light Company system, and in August 1940, a second furnace duplicating the first was installed at the same location. These installations were made by the Pittsburgh Crucible Steel Company at their Midland, Pa., plant near Pittsburgh.

It was believed that the load swings on the first furnace with the accompanying voltage changes would exceed the standards which the Duquesne Light Company had established for this type of load, viz.

Figure 1. Diagram of power system showing 66-kv lines and power stations

Each line on diagram represents two 50,000-kva lines unless noted. Duquesne Light generating capacity is 504,000 kw with a 60,000-kw unit scheduled for service in July 1941

All lines are double-circuit unless noted by (1) or (3)


TO PENNSYLVANIA POWER Co.
VALLEY S.S.
OHIO
ST. JOS. LEAD Co.
PGH. CRUCIBLE STEEL Co.
OHIO
PA.
AMBRIDGE S.S.
RIVER
TO WEST PENN POWER Co.
COLFAX P.S.
B. I. & REED P.S.
CARNEGIE-ILLINOIS STEEL CORP.
N
PA.
W.VA.
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3
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5
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SCALE-MILES

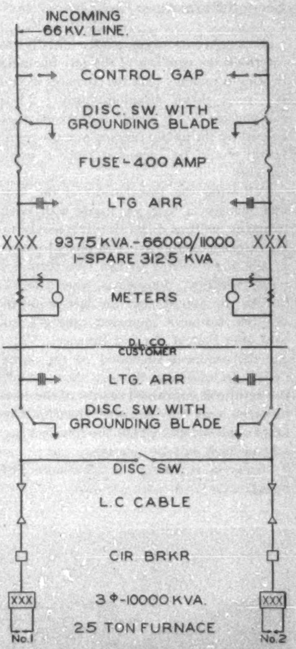
INCOMING 66 KV. LINE.
CONTROL GAP
DISC. SW. WITH GROUNDING BLADE
FUSE-400 AMP
LTG ARR
XXX
9375 KVA.-66000/11000
1-SPARE 3125 KVA
XXX
METERS
D.L.CO.
CUSTOMER
LTG. ARR
DISC. SW. WITH GROUNDING BLADE
DISC SW.
L.C CABLE
CIR BRKR
XXX
3 Φ-10000 KVA.
XXX
25 TON FURNACE
No.1
No.2

Figure 2. Diagram of power-supply substation and furnace substation

The 400-ampere 66-kv fuses provide protection for high-voltage and low-voltage faults

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Paper 41-40, recommended by the AIEE committee on electrochemistry and electrometallurgy, and presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941. Manuscript submitted October 8, 1940; made available for preprinting December 10, 1940.

B. M. JONES is division engineer of system planning and development department, and C. M. STEARNS is subsection engineer of the Duquesne Light Company, Pittsburgh, Pa.

1. For all numbered references, see list at end of paper.

1.5 volts (115-volt base) for the 66-kv system with fluctuations from 3 per hour up to 15 per minute. Precautions were taken in the design of the furnace transformer to include separate reactance to reduce the load swings. Subsequently, it was found that this additional reactance was not eneded and it was removed, thereby improving the operation of the furnace.

The load swings and accompaning voltage changes, after disconnecting the reactance, due to the operation of the first furnace, slightly exceeded the Duquesne Light Company standards. However, no complaints have been received from adjacent customers nor have any difficulties arisen from excessive operation of 4-kv distribution regulators, operation of the power stations, or interconnections.

Second Furnace

Arrangements were made initially to interlock the controls of the two furnaces in such a manner that the furnaces would not melt down simultaneously. Thus the voltage flicker would not exceed that caused by the initial furnace and the regulation would be increased only a small amount.

Following the installation of the second furnace, a test was made with both furnaces melting down simultaneously at the highest secondary voltage. Instead of the load swings of the two furnaces being additive, the swings were diversified to the extent that the net swing of the two furnaces increased only 20 per cent over that of the first furnace. Voltage disturbances increased in the same ratio. In addition, the test showed that the resultant combined swings of the two furnaces were less frequent than for the first furnace alone. Thus the interlocking scheme may not be necessary except as a means of reducing the 15-minute demand.

Method of Approach

This article contains a rather complete treatise of our investigations and presents the industry with these data which the authors hope will be of some use to others in the solution of their furnace problems.

Some data have been published on electric furnace performances and the power system supply (see bibliography), but for the first large installation on any power system the authors strongly recommend getting all data that they can possibly lay their hands on, and making a thorough analysis of them.

Obviously, the correct engineering approach is to determine in advance what the kilovolt-ampere swing and resultant voltage flicker will be, and to provide a suitable system connection with corrective equipment if necessary, to keep the voltage changes within tolerable limits. However, this is not always so easy, for in some cases it is difficult to find out just what the electric performance of the furnace will be.

In addition to the determination of instantaneous load changes, the maximum loads which exist for several seconds at a very poor power factor, and which occur several times during a heat, should be determined. These large loads may result in undesirable voltage regulation on the customer's supply bus or at adjacent customers.

There are many factors involved, and many calculations to be made, not only for normal system conditions but also for the abnormal system condition of reduced capacity, lines out of service, generators off the line, etc. These abnormal conditions may result in voltage disturbances beyond standard voltage limits. However, it will probably not be economical to reduce the voltage changes to standard limitations with lines and equipment out of service because of their infrequent occurrence.

Calculations

Having decided upon the maximum instantaneous furnace load swings, the voltage flicker at the customer's supply bus should be determined from the known or estimated system impedance at that point. The voltage flicker, then, at the nearest customer or substation should also be determined.

The per cent system impedance to the Pittsburgh Crucible Steel Company 66-kv bus in 68-kv terms and on a 10,000-kva base in $R+jX$ terms is:

$(1.1+j4.0)$ per cent

The reactance of each of the 66/11-kv 9,375-kva transformer banks supplying the furnace transformers is 6.0 per cent on a 10,000-kva base. The system impedance is based upon all generating units and all 66-kv lines in service and the West Penn Power tie closed at Colfax.

As the furnace load swings are generally at a very poor power factor, the use of system reactance at the customer's bus will give fairly close results. This system reactance is that ordinarily determined by a short-circuit study. However, the true power does change considerably and if this change and the change in the reactive power are known, then the calculations of the voltage change based upon system impedance rather than reactance will give more accurate results.

The calculations for voltage regulation may be made in substantially the same manner, as described above for voltage flicker.

Application of Corrective Equipment

If the voltage changes calculated as described under the preceding heading

Figure 3. Photograph of power-supply substation—66/11 kv, 18,750 kva—at Pittsburgh Crucible Steel Company, Midland plant

Incoming 66-kv line at left and 11-kv circuits to furnace transformers at right. Lightning arresters mounted at high-voltage transformer bushings

Figure 4. Photograph of charging side of a 25-ton Heroult three-phase electric-arc furnace at Pittsburgh Crucible Steel Company, Midland plant

exceed the allowable values, then corrective equipment should be applied. Discussion here will be limited to correction obtained by synchronous condensers. This as well as other methods of correction have been described by earlier papers, see item 1 in bibliography.

As far as very frequent instantaneous furnace load changes are concerned, the condenser will draw additional leading reactive current to offset some part of the additional reactive furnace load. For a general analysis, it can be considered that the system and the condenser will share the increased furnace reactive load in proportion to the inverse of their respective reactances. Thus the reduction in voltage flicker may be estimated as well as the amount of corrective capacity to obtain the desired reduction.

The condenser can be made to draw more leading reactive current by increasing the system reactance. This may be done by installing a buffer reactor in the furnace supply circuit ahead of the condenser. Actually, this buffer reactor produces increased voltage drop at the terminals of the condenser thus causing it to draw more leading reactive current. The condenser will draw increased leading reactive current in proportion to its subtransient reactance. Thus the speed of condenser field has very little to do with the correction of very rapid furnace reactive changes. Speeding up the field will, however, materially improve regulation for large load changes which last for several seconds.

Duquesne Light Company System and 66-Kv Supply to Furnace

Figure 1 shows the Duquesne Light Company system, including generating stations and the 66-kv transmission ring. The two major sources of interconnections with adjacent utilities are shown, one at Colfax with the West Penn Power Company and the other at Valley substation with the Pennsylvania Power Company. The latter is normally open, but in emergency can be closed and operated in parallel with the other systems, the 40,000-kva regulating transformer between the two systems being able to control both true power flow and reactive flow, the latter providing voltage regulation at this point, see items 6 and 7 in bibliography.

A single 66-kv circuit, ACSR 336,400 circular mils (4/0 equivalent) tapped to the single circuit between Ambridge substation and St. Joseph lead substation, was extended nine miles on wood poles with two ground wires to the Pittsburgh Crucible Steel Company at Midland, Pa. There is an extra degree of insulation and ground protection built into this extension which will make it practically immune from insulation troubles.

Supply Substation

Figure 2 shows the supply substation schematically for the 2 25-ton Heroult furnaces. Figure 3 shows the photograph of the substation with 2 9,375-kva transformer banks and one single-phase spare unit. Normally, each bank supplies one furnace independently and the 11-kv bus tie disconnecting switch is closed only in case one bank is out of service. At this time the furnace must be operated at a reduced secondary voltage to keep the load on the one 9,375-kva bank within its limit.

Figure 5. Photograph showing the pouring of 40 tons of high-grade alloy steel at Pittsburgh Crucible Steel Company, Midland plant

Note the entire furnace tilting to facilitate pouring

Tests on First Furnace

Tests were run during a complete heat on the first 25-ton Heroult furnace shown on figure 4 and figure 5.

Table I. Pittsburgh Crucible Steel Company, Midland, Pa.

Test on 4/10/40

Sec. Voltage	Point No. on Meter Charts	Kw	Reactive Kva	Kva Calculated	Three-Phase Power Factor Calculated*	11-Kv Voltage	Amperes at 11 Kv A	Amperes at 11 Kv B	Amperes at 11 Kv C	Kva Calculated**	Single-Phase Power Factor† (Per Cent) Point	Avg.	Max.	Min.
220	1 High	6,550	6,660	9,300	70.5	10,500	655	750	624	12,800	50	50 Average for 220 volts	98	50
	2 Low	2,280	300	2,300	99.2									
	Inst. change			7,000										
	3 High	7,200	8,700	11,300	64.0	10,000	804	760	792	13,400	50			
	4 Low	2,880	1,680	3,300	87.3									
	Inst. change			8,000										
	5 High	7,250	8,640	11,300	64.2	10,350	866	724	802	14,800	54			
	6 Low	2,400	1,200	2,700	89.0	11,000	192	240	240					
	Inst. change			8,600			674	484	652					
	7 High	6,000	9,840	11,500	52.1	10,000	703	792	888	15,000	60			
	8 Low	3,240	1,320	3,500	92.5	10,850								
	Inst. change			8,000										
240	9 High	8,700	11,520	14,430	60.3	9,400	883	1,065	1,100	16,500	50	85	95	50
	10 High	8,650	11,520	14,400	60.0	10,300	1,060	1,026	1,043	18,600	50			
	Low	8,150	4,800	9,400	86.8		556	605	580					
	Inst. change			5,000		9,770	504	421	463					
220	11 High	4,800	11,500	12,460	38.5	9,250	916	960	760	14,100	50	85	97	50

All measurements are "input to furnace transformers" and adjacent to furnace transformer.

*Calculated on basis of three-phase kilowatts and three-phase reactive kilovolt-amperes input.

**Calculated on basis of average amperes per phase and AB voltage.

†Power factor from charts using A amperes and AB and CB voltage.

General: The average power factor for loads during the refining period varies between 85 and 95 per cent. Inst. change—instantaneous change.

GRAPHIC METER RECORDS

Graphic records were obtained of customer's 11-kv voltage, furnace reactive kilovolt-amperes, kilowatts, amperes in each phase and single-phase power factor, and 66-kv voltages at the nearest attended substations. Sections of these charts during different periods of the heat are shown in figures 6, 8, 10, 12, and 13. These charts were run at approximately six inches per minute speed.

The seven meters at the furnace were driven by a common shaft and a switch was installed to short circuit the current transformers and open the potential leads simultaneously. This switch was operated at 15-minute intervals to aid in synchronizing the charts for analysis purposes.

In order to run the charts at high speed, most of the damping effect is eliminated. This undoubtedly allowed the meters to overswing.

OSCILLOGRAPH RECORDS

An oscillograph record was taken at several points during the heat. Portions of these records are shown in figures 7, 9, 11, and 14. The record of the single-phase wattmeter is intended to show the rapid changes in furnace consumption and not the true single-phase load, in so far as the single-phase wattmeter is 30 degrees out of phase with three-phase balanced loads.

It is almost impossible to obtain oscillograph records of the maximum loads or changes in loads because of their short duration.

Results

Table I shows various values obtained from the graphic charts of kw, reactive kilovolt-amperes, amperes, volts, and power factor. These values are instantaneous ones taken during maximum furnace load swings or furnace loads. Several of the numbered points are shown on the exhibits.

The maximum instantaneous furnace load swing, lasting a fraction of a second, amounted to 8,600 kva, 3 phase with a calculated power factor of 54.5 per cent (change in load). An analysis of the load swings indicated that the maximum occurred only once and only ten exceeded 7,000 kva during the heat.

The maximum instantaneous load of seconds duration, was found to be 14,400 kva or 144 per cent of the furnace transformer rating.

Based upon maximum load swings (8,600 kva) the voltage flicker at the nearest 66-kv customer (St. Joseph Lead Company) amounts to 2.3 volts, at Valley substation, 2.1 volts, and at Ambridge substation, 1.8 volts. See figure

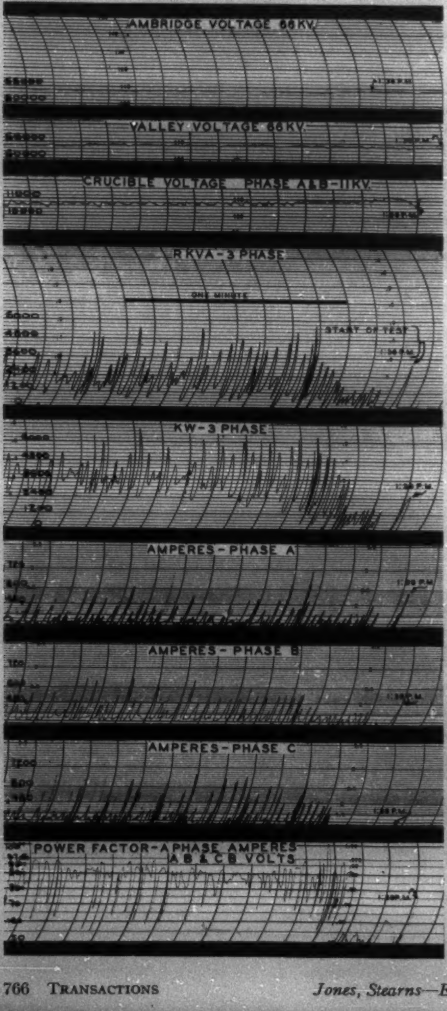
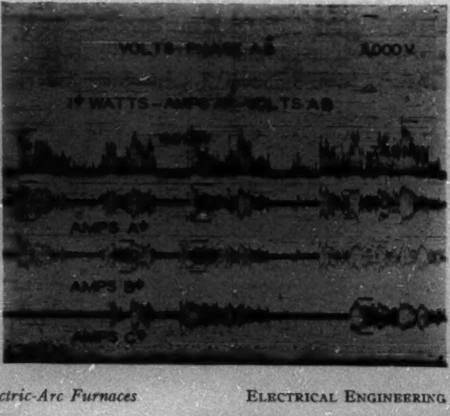


Figure 6. Graphic charts at start of heat at 1:38 p.m.

Furnace swings occur approximately once a second

Figure 7 (below). Oscillograph film at start of heat

A single-phase arc struck and restruck ten times in the space of 15 seconds before all three phases struck. After this initial period, all three phases struck and restruck ten times with currents in all three phases fairly well balanced before the arcs became generally stable. A portion of this performance is shown on this figure



1 for locations. However, due to the fact that the majority of the load swings were considerably less than the maximum, no complaints have been received from adjacent customers and no difficulties have arisen from excessive operation of 4-kv distribution regulators or in the operation of the power station or interconnections.

Thus perhaps corrective equipment should not be installed to correct for the maximum swing but for something less than the maximum, assuming that a few voltage changes in excess of the standard could be allowed.

The location of the furnaces at the end of a long line and with considerable impedance to the supply bus, probably accounts for the fact that the load swings and maximum loads are less than might be expected from a furnace of the size installed.

Due to the fact that no other customers are located near the furnace installation the voltage flicker and regulation at the customer's 66-kv bus are allowed to exceed values that would normally be allowed and the voltage changes at the nearest customer are materially less due to the relatively high impedance between it and the furnace 66-kv bus.

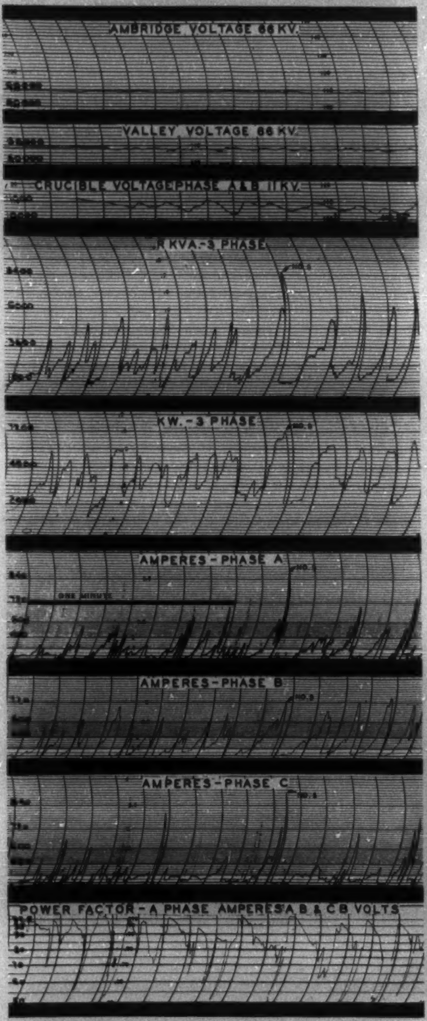


Figure 8. Graphic charts taken 14 minutes after start of heat

The swings have become less frequent, but are greater than shown on figure 6

Table II. Pittsburgh Crucible Steel Company, Midland, Pa.

April 10, 1940

15-Minute Kilowatt Demands

1:30 to 1:45 p.m.	1,800
1:45 to 2:00 p.m.	4,020
2:00 to 2:15 p.m.	3,960
2:15 to 2:30 p.m.	6,720
2:30 to 2:45 p.m.	7,080
2:45 to 3:00 p.m.	8,400
3:00 to 3:15 p.m.	6,480
3:15 to 3:30 p.m.	6,600
3:30 to 3:45 p.m.	3,900
3:45 to 4:00 p.m.	6,960
4:00 to 4:15 p.m.	7,200
4:15 to 4:30 p m.	3,840
4:30 to 4:45 p.m.	1,680
4:45 to 5:00 p.m.	2,640
5:00 to 5:15 p.m.	1,560
5:15 to 5:30 p.m.	0
5:30 to 5:45 p.m.	1,320
5:45 to 6:00 p.m.	840
6:00 to 6:15 p.m.	1,140
6:15 to 6:30 p.m.	1,200
6:30 to 6:45 p.m.	1,320
6:45 to 7:00 p.m.	360

Next heat started at 7:45 p.m.

Complete heating cycle 6 hours and 15 minutes.

Figure 9 (below). Oscillograph film taken 14 minutes after start of heat

The currents have become more stabilized than in figure 7. The arc did not completely go out during the film

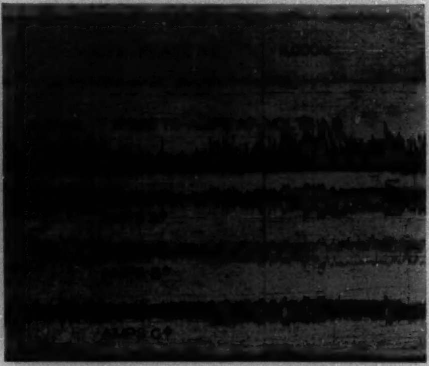


Table II shows the 15-minute kilowatt demand during each 15-minute period of the heat. The maximum demand of 8,400 kw is at approximately 85 per cent power factor or 9,900 kva. Subsequent to this test and prior to the installation of the second furnace the 15-minute demands were somewhat less.

The oscillograms showed maximum amperes of 840 while the graphic charts gave maximum values of 1,100. Probably the true maximum lies between the two, in so far as the graphic meters would probably overswing and the maximum values were probably not recorded on the oscillograph film.

Test on Two Furnaces

Following the installation of the second furnace, test data were obtained of the two furnaces melting simultaneously at the highest secondary voltage. Graphic meter chart records only of voltage, amperes, etc., the same as for the first furnace were obtained. Time has not permitted the preparation of the exhibits for inclusion in this paper but they will be available at a later time for discussion purposes.

During the test, number 2 furnace was melting the initial charge and number 1 furnace was melting the "back charge." Number 2 furnace started the initial melt down 29 minutes prior to number 1 furnace starting the melt down of the "back charge." The test continued from the time number 2 furnace started the melt down of the initial charge until the initial charge was melted, approximately $1^3/_4$ hours. The 11-kv bus tie was closed during the test to facilitate the obtaining of the records.

Results

The maximum combined instantaneous load swing of the two furnaces, lasting a fraction of a second, amounted to 10,580-kva, 3 phase with a calculated power factor of approximately 52.0 per cent (change in load). The next largest swing amounted to 7,500 kva, 3 phase and all other swings were materially less than 7,500 kva.

The maximum instantaneous load of seconds duration was found to be 22,850 kva, and there were several near this value.

Based upon maximum load swings (10,580 kva) the voltage flicker would exceed that of one furnace by about 20 per cent. However, the combined load swings of the two furnaces were much less frequent than for one furnace and actually there seemed to be no flicker

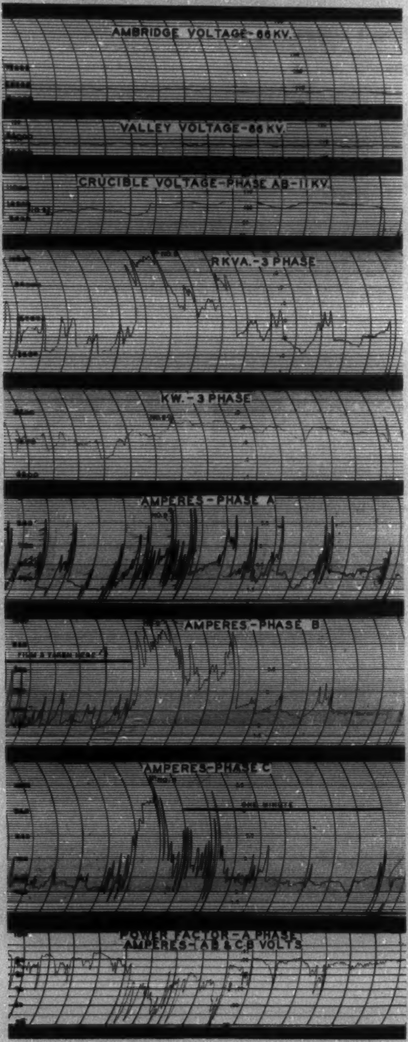
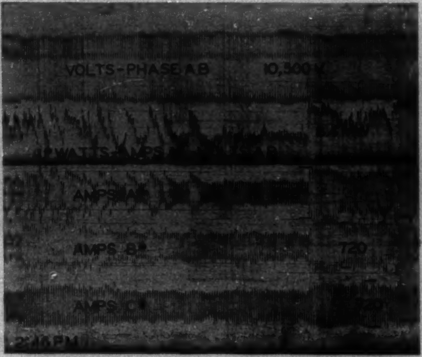


Figure 10. Graphic charts one hour after start of heat

Large furnace swings probably caused by scrap falling against electrodes or falling into molten metal raising its level and short-circuiting the electrodes

Figure 11 (below). Oscillograph film taken during time of figure 10

The exact correlation is not known



problem. Voltage regulation was of course greater but not intolerable.

The maximum 15-minute kilowatt demand was 15,600 at approximately 75 per cent power factor or 20,800 kva.

The graphic charts show maximum amperes of 1,590 per phase in the 11-kv circuit. Due to the larger ammeter constant, overswinging did not seem to be as pronounced as in the initial furnace test.

References

1. Arc-Furnace Loads on Long Transmission Lines, T. G. LeClair. AIEE Transactions, volume 59, 1940 (April section), pages 234–9.

2. Industrial Electric Heating—Transformers and Reactors for Three-Phase Arc Furnaces, N. R. Stansel. *General Electric Review*, May 1937.

3. Power Company Service to Arc Furnaces, L. W. Clark. AIEE Transactions, volume 54, 1935 (November section), pages 1173–8.

4. Three-Phase Electric Arc Furnace, Samuel Arnold. AIEE Transactions, volume 52, 1933, pages 839–43.

5. Modern Electric Furnace Design and Practice, H. F. Walther. Presented at the 36th Annual Convention Iron and Steel Exposition, Association of Iron and Steel Engineers, Chicago, Ill., September 24, 1940.

6. Application of Large Phase-Shifting Transformer on an Interconnected System Loop, W. J. Lyman and J. R. North. AIEE Transactions, volume 57, 1938 (October section), pages 579–87.

7. Controlling the Power Flow in a 300-Mile Transmission Loop, W. J. Lyman and J. T. Mercereau. *Electrical Journal*, October 1938.

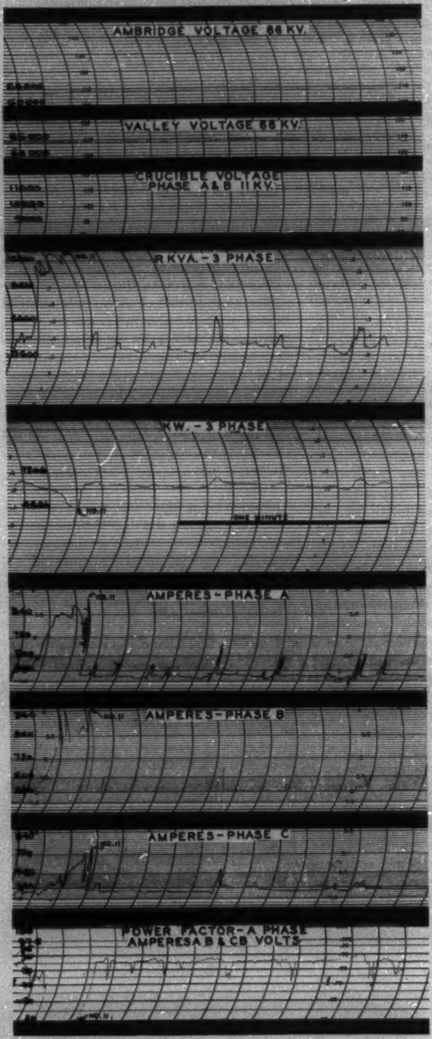


Figure 12. Graphic charts $1^1/_2$ hours after start of heat

The large load, point 11, was probably caused by scrap falling into molten metal and raising its level

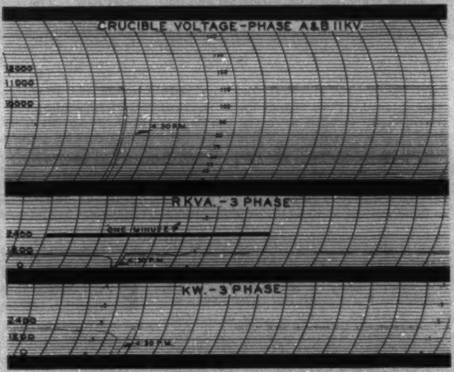


Figure 13. Graphic charts during refining period

A timing device cut off the meters at 4:30 p.m.

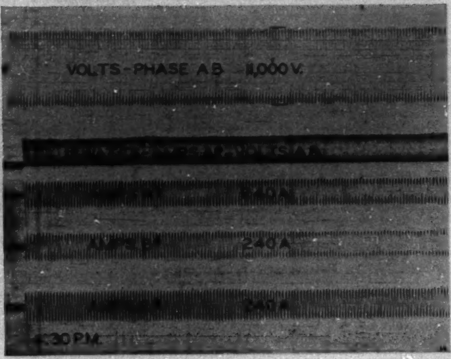


Figure 14. Oscillograph film taken during refining period, corresponding to figure 13

The timing device was operated at the extreme left

The "Electrogear"—A New Electromechanical Vehicle Drive

ERNST WEBER
FELLOW AIEE

Synopsis: Within recent years a new electromechanical transmission known as the "Electrogear" has been developed for use in gas and Diesel engine cars of all types including busses, trucks, rail cars, and track-laying vehicles. The paper describes the principle of its operation which is basically a differential action between direct mechanical power transmission and a generator-motor system, interlinked by means of a permanently engaged planetary differential gear.

At the start, both the electrical and mechanical power flows are in the same direction, giving high torque multiplication. As the vehicle speed increases, the electrical power flow diminishes, goes through zero, and finally reverses its direction, leading to high speed, low torque operation with "overdrive." The torque ratio is thus continuously variable as in the gas-electric drive, and is controlled independent of road conditions by a small auxiliary generator which acts as the "brain" of the system. The design and installation of one specific Bus Electrogear is given in detail in order to illustrate the application to public transportation, though various other models have been built and demonstrated. Extensive test data graphically describe the superior performance of this new vehicle drive which combines the ideal flexibility and low maintenance cost of the gas-electric drive with the economy of the standard gearshift transmission. Actual operation requires only accelerator and brake, both acted on conventionally by means of the conventional pedals, and a selector switch with engine starting, forward and reverse positions. The engine speed varies continuously, and due to the fact that only a comparatively small part of the engine power is circulated through the electrical machines, the weight of the drive is about one-half of that of the gas-electric transmission. Since the Electrogear goes automatically and inherently into "overdrive" at higher car speeds on level road, and can be designed to keep the engine speed in its most economic range during the major part of full throttle acceleration, its fuel economy is at least as good as that of the standard gearshift transmission, and for heavy city schedules becomes even considerably better.

I. Introduction

EVER since internal combustion engines have been used for the propulsion of vehicles, attempts have been made in great number to overcome the fundamental problem of flexible transmission of torque from the gasoline or Diesel engine to the wheels.[1] Yet no practical solution has been found which merely by mechanical devices produces a continuously variable ratio between engine torque and driving torque and is controlled by the action of the driver of the vehicle.

One of the most unsatisfactory elements of the standard gearshift transmission is the mechanical clutch operated by a separate clutch pedal. Practical ways have been found now to replace this clutch by a hydrodynamic coupling or "fluid flywheel"[2,3] which essentially consists of two rotating vane systems; one connected to the driving shaft, acting as centrifugal pump, and frequently called the impeller; the other connected to the driven shaft, acting as a turbine, and frequently called the runner. Torque transmission is obtained by the fluid circulating through both sets of vanes whereby on account of friction losses a certain amount of slip occurs. Any change in impeller speed causes a corresponding but slightly lagging change in runner speed resulting in a smooth operation of the vehicle without jolts. Since this "fluid flywheel" only acts torque transmitting, it must be used in connection with torque multiplying gears, either manually operated as in Chrysler cars, or in connection with automatic gearshifts as in the "hydra-matic" drive of the Oldsmobile[4] or it can be combined with a semiautomatic transmission as the "Fluidgear" developed jointly by the A.C.F. Motors Company and the Spicer Manufacturing Company.[5]

A further step is the hydrodynamic transmission or "torque converter";[2,6] this device adds a stationary vane system to the hydrodynamic coupling as a reaction member changing the tangential velocity of the fluid and thus produces torque multiplication. As the vehicle starts, maximum torque multiplication is obtained with a slip of 100 per cent between impeller and runner, and as the vehicle speed increases, slip and torque multiplication decrease. At a predetermined speed of about 17 miles per hour, direct mechanical drive is established automatically, without action of the driver. Obviously, the efficiency and fuel economy in the hydrodynamic range of driving is considerably lower than in the standard gearshift transmission, though the acceleration can be made to be satisfactory.

In spite of this variety of more or less automatic transmissions now available,[7] none of these can truly lay claim to be the

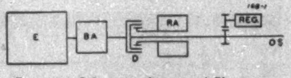


Figure 1. Schematic diagram of Electrogear

BA, RA, REG—Armatures
D—Planetary differential gear
E—Engine
OS—Output shaft

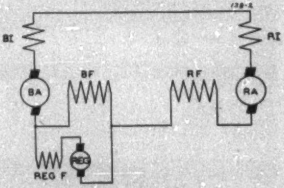


Figure 2. Diagram of Electrogear connections for forward operation

BA, RA, REG—Armatures
BF, RF, REG F—Series field windings
BI, RI—Interpole windings

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Author's acknowledgments: Save for the enthusiastic and freely given co-operation of all the parties mentioned below, it would not have been possible to carry out successfully the very first design and construction of the Bus Electrogear. It is, therefore, a great pleasure to acknowledge the indebtedness to John Whitson of The Power Transmission Company of New York for providing the background, co-ordination and support, and arranging for the co-operation of all concerned; to Colonel H. W. Alden of The Timken-Detroit Axle Company, who was the first to recognize the possibilities of the Electrogear for use in busses and track-laying vehicles, and who initiated the project for designing and building the first units and assigned the design engineering to R. M. Herron, who also supervised the assembly and participated in the installation of the unit; to C. O. Guernsey of the J. G. Brill Company who furnished the bus and provided generous co-operation; to E. R. Martin of the Delco Products Division of General Motors Corporation, who was in charge of the construction of the electrical machines; to G. M. Stapleton of the Ward Leonard Electric Company for the design of the control equipment; to John H. Gethin of the Pittsburgh Testing Laboratory for faithful assistance in the installation and during the tests; to F. W. Kateley and J. F. Chapman of The J. G. Brill Company for their interest, help, and supervision during the tests in Philadelphia. This paper was prepared at the request of the committee on land transportation of which E. B. Fitzgerald of the American Car and Foundry Company is chairman. The writer is indebted to Mr. Fitzgerald for his many helpful suggestions, and to F. W. Kateley of The J. G. Brill Company and Earl E. Kearns of the General Electric Company for the comparative test data on the gearshift and gas-electric drives.

[1] For all numbered references, see list at end of paper.

ultimate solution of the fundamental problem of power transmission in individual vehicles as recently admitted by high authority in the automotive industry.[7a] The only and earliest solution outside the field of strictly mechanical means is the well-known gas-electric transmission in which the engine drives a generator at nearly constant speed and one or two motors are coupled to the driver axle. This gas-electric transmission has recently been considerably improved[8,9] and would appear to be the ideal solution since it gives a continuously variable torque ratio, were it not for the large loss of efficiency and the large size and weight of the electrical machines needed to carry the whole engine power.

The purpose of this paper is to describe the progress in the development of an entirely new principle of power transmission,[10] invented by A. H. Neuland[11] and developed during recent years through the support of The Power Transmission Company of New York City, owners of the patents. Fundamentally, it constitutes an electromechanical drive and is based upon a differential action between directly transmitted mechanical power and d-c electric power furnished by one electrical machine and consumed by another. The driving torque and therewith the torque ratio is controlled by conventional use of the accelerator pedal, and its variation is produced by a small auxiliary generator whose speed is proportional to vehicle speed. Since the transmission embodies electrical machines connected through the elements of a differential gear of the planetary type, the name "Electrogear" has been adopted for it.

The Electrogear has the extreme flexibility of the electric drive and, like this, gives a continuously variable torque ratio. In addition, it has the advantage of high fuel economy, since only a fraction of the mechanical power is transformed into electric power, and since the engine speed though variable can be kept within its most economic range even at the highest vehicle speeds, when overdrive automatically occurs. On account of the partial conversion into electric power, the sizes and weights of the electrical machines are considerably smaller than in the gas-electric drive with the attending advantages of lower currents and smaller commutators.

Designs of the Electrogear have been made and demonstration models built for several 5-passenger cars, for a 36-passenger city bus, and for a 12-ton track-laying vehicle. The Pittsburgh Testing Laboratory made extensive road tests of a five-passenger sedan with the Electrogear drive, and then duplicated the same tests of the same car with its conventional gearshift drive. The comparative results covered in a report[15] should be of interest to the automotive industry as well as to electrical engineers since they show that a transmission using electrical machines in proper combination with a differential gear can be just as efficient as a purely mechanical transmission.

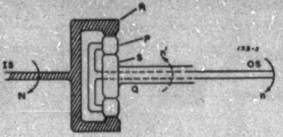

Figure 3. Planetary ring gear differential

R—Ring gear
P—Planets
S—Sun gear
Q—Quill
IS—Engine shaft
OS—Output shaft

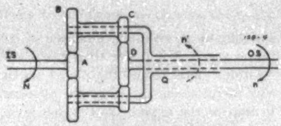

Figure 4. Double-planet differential

A—Input sun gear
D—Output sun gear
IS—Engine shaft
B, C—Planets
Q—Quill
OS—Output shaft

The detail discussions in this paper however pertain to the bus application because the greatest variety of drives has been developed in this field, so that comparisons can readily be drawn from available commercial test and performance data.

II. Principles of Operation and Design

Figure 1 is a schematic sketch of the Electrogear drive with only the armatures of the electrical machines indicated for simplicity, figure 2 shows the connection diagram for forward operation of the vehicle, and figure 3 gives a more detailed sketch of the differential gear. The armature *BA* of the one electrical machine is directly coupled to the engine shaft *IS* and the ring gear *R* of the differential gear; this machine may, for convenience, be called booster. The armature *RA* of the second electrical machine is coupled to the sun gear *S* of the differential gear and usually is mounted on a hollow shaft or quill *Q;* this machine may, for convenience, be called reducer. Booster and reducer are series-wound d-c machines, both connected in series for forward operation of the vehicle as seen in figure 2. The auxiliary generator has its armature *REG* driven by suitable gears or by belt from the output shaft *OS*, which in turn carries the planet spider with the planets *P* of the differential gear. This auxiliary machine, called regulator for convenience, is connected parallel to the booster field; its function is the distinctive feature of the Electrogear.

1. OPERATION OF THE ELECTROGEAR

In forward operation, with the vehicle standing still and the engine idling, the ring gear and booster armature *BA* will turn with engine shaft speed, whereas the planet spider and output shaft are at rest. Thus, the sun gear with the reducer armature *RA* has to turn in a direction opposite to engine shaft rotation. Now, the electrical machines are so connected that under these mechanical conditions the reducer will act as self-excited generator and feed current into the booster, which will act as motor permitting just enough current flow to maintain polarities without moving the vehicle.

Depression of the accelerator pedal increases the engine speed and power and therefore the electric power generated by the reducer which drives the booster as motor in the same direction of rotation as the engine. Thus, the ring gear torque is actually the sum of engine and booster torques. Since both ring gear torque and reactive torque of reducer act in the same direction upon the planet spider, a considerable driving torque multiplication takes place comparable to "first gear" in mechanical transmissions; the planet spider starts revolving and the vehicle starts moving. As the vehicle, and with it the planet spider, speeds up, the reducer slows down so that without any further action the vehicle could reach only a comparatively low speed determined by the electromechanical balance of torques.

However, part of the main current is shunted through the regulator which has its field so connected as to generate a voltage in opposition to the voltage drop across the booster field. Since the regulator speed increases with vehicle speed, the regulator voltage also increases and eventually reverses the current in the booster field. The electrical design must, of course, correlate this period with the period in which the reducer has slowed down enough to be unable to drive the booster as motor, so that the regulator, furnishing from now on the field excitation of the booster in reversed direction, forces it to act as generator. With the main cur-

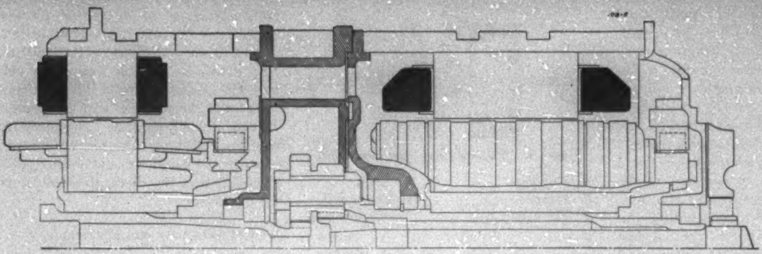

Figure 5. Simplified drawing of Electrogear layout for a city bus

rent continuing to flow in its original direction, the reducer must now reverse its direction of rotation to run as motor. It is quite obvious, at this point, that the regulator is the automatic governor of the booster and determines the over-all action of the electrical power flow which is differentially superimposed upon the mechanical power transfer. Since, in operation, the transitions from motor-to-generator state and vice versa are accomplished smoothly and without any interruption, the resulting variation in driving torque is also continuous and responsive to the degree of depression of the accelerator pedal which, in fact, is an inherent feature of this transmission.

As soon as the reducer reverses its direction of rotation, all three elements of the differential gear, namely ring gear, planet spider, and sun gear are rotating in the same direction. Obviously, at some particular vehicle speed the differential gear will rotate as a solid unit in which case the torque ratio between input and output must be unity, corresponding to "high gear" in the usual mechanical transmission. Advantageously, one can choose to have this occur well below top speed of the engine, preferably close to the most economic speed of the engine with respect to fuel consumption. Further increase of vehicle speed automatically brings about the condition of "overdrive," since the regulator output also increases and forces the booster to generate an increasing voltage at almost constant speed, which in turn speeds up the reducer as motor; thus the ratio of engine to vehicle speed decreases to a value below unity, which is determined primarily by the amount of conversion into electric power at top speed of the vehicle.

Reverse operation of the vehicle is obtained as in any electric drive by reversing the fields of both booster and reducer with the regulator disconnected from the booster field. The booster now acts as self-excited generator driven by the engine and the reducer runs as motor, speeding up in its idling direction of rotation which is opposite to engine shaft rotation. This forces the planet spider also to rotate in the direction opposite to that of the engine shaft which gives backward motion of the vehicle. The maximum reverse speed is of course limited by the electromechanical torque balance under these conditions but is sufficient for all practical purposes, since the maximum tractive effort in reverse operation is as high as 60 to 65 per cent of that in forward operation.

2. Special Features of the Design

The design of this electromechanical transmission requires a careful study of the interrelation of all component elements in order to obtain proper functioning and the economy in weight and cost which is all important for any vehicle drive. The choice of the rear axle ratio (for rear axle drive) and that of the ratio in the planetary differential are interdependent and are determined by the desired rate of initial acceleration and by the top speed of the vehicle. As mentioned above, the fuel economy can be influenced decisively by proper choice of the vehicle speed in full throttle acceleration at which "overdrive" sets in; in this respect particular attention has to be paid to the auxiliary generator or regulator. For best results, individual designs are desirable for different performance requirements, though within any particular class of vehicle a standard design can be used for the average requirements and adjustments made by the use of slightly different rear axle ratios as is customary with the standard mechanical transmissions.

The planetary differential gear of the ring gear type is most commonly used in vehicle drives. Referring to figure 3 and denoting with r the ratio of the number of ring gear teeth to the number of sun gear teeth one finds for the ratio G of the torques on output and input shafts the value

$$T_{OS}=\frac{r+1}{r}T_{IS}=GT_{IS} \tag{1}$$

Obviously, the largest value of G is obtained for the smallest value of r; for mechanical reasons, therefore, $G \leq 1.8$ which restricts this type of planetary differential to use in vehicles with low torque multiplication, as for example in passenger cars.

In order to obtain larger torque multiplication, a differential gear of the type shown in figure 4 and known as "reverted train" has to be used; on account of the two sets of planets it is often called double-planet gear. For reasons of con-

Figure 6. Photographic view of Electrogear on the dynamometer stand

struction the ratio of teeth in *B* to *A* is usually the same as that in *D* to *C*, and denoting it as *q*, one has for the torque ratio *G* of output to input shafts

$$T_{OS}=q^2T_{IS}=GT_{IS} \qquad (2)$$

This type of planet gear can, therefore, be used for any large torque multiplication within practical values, and has the further advantage that the quill *Q* carrying the reducer can be located either to the left or to the right of the planet gear. It has been applied in the bus drive and the drive for the track-laying vehicle.

The main electrical machines have to be designed with utmost economy in the use of materials. In a bus drive with a definite schedule the power requirements can easily be evaluated; the effective mean value of the losses with proper consideration of cooling conditions at the various vehicle speeds will lead to the customary one-hour rating as a basis for the design. In the case of vehicles for general service suitable assumptions must be made with respect to most frequent driving conditions and maximum possible overloads. In the designs built for the bus and track-laying vehicles glass insulation was used throughout to take full advantage of the maximum temperatures permitted by AIEE Standards for traction machines. Of course, only experience can tell what effective cooling the machines may have on account of their particular location on the moving vehicle. The test data below will illustrate these points more fully.

The most important element in the Electrogear transmission is the regulator since it determines and controls the operation of the booster as motor or as generator. Being connected in parallel to the booster field, it carries a large part of the main current in addition to the current which it produces by self-excitation. Furthermore, its magnetic characteristic must be such that at the highest vehicle speeds it will not, or only slightly, saturate in order to provide adequate excitation for the booster. Since it is a very low voltage machine, care has to be taken to obtain good commutation under all driving conditions. All these are requirements that can easily be satisfied by an adequate design, but they are quite different from those met with in the design of general purpose machines.

3. Example of Installation

As an illustration the Electrogear designed for an A.C.F. bus weighing 14,220 pounds empty and 19,420 pounds with seated load will be described. The basic data for the bus were furnished by the engineers of The J. G. Brill Company at Philadelphia. The bus was to be equipped with a Hall-Scott 135 engine mounted under the floor and delivering 116 brake horsepower at 2,000 rpm of which approximately 9 horsepower were used in accessories so that only 107 horsepower were available for the transmission. As a city bus the requirements were assumed to be 10 stops per mile, 11 seconds for each full stop, a scheduled speed of 11 miles per hour and acceleration up to 25 miles per hour between stops. The one-hour ratings of the electrical machines were—

Booster as motor
..........22.5 horsepower at 1,900 rpm
Reducer as generator
....................20 kw at 1,200 rpm

with maximum speeds of the booster about 2,600 rpm, of the reducer about 2,200 rpm; each had four main and four commutating poles, was series wound and designed for optimum voltage from the point of view of economy. Both main machines as well as the regulator were built and individually tested by the Delco Products Division of General Motors Corporation at Dayton, Ohio.

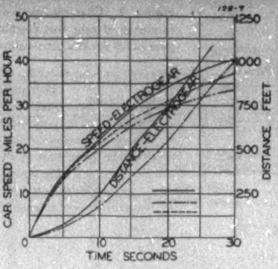

Figure 7. Full throttle acceleration of city bus

——————— Electrogear
— · — · — Gas-electric
- - - - - - - Gearshift

The mechanical layout was primarily determined by the available space between engine shaft flange and universal joint of the propeller shaft in the standard bus chassis. This put a limitation upon the axial extension of the armature end connections, but by placing the planet gear between booster and reducer and shortening the propeller shaft a solution was found satisfactory from all angles. The planet gear is of the double-planet type and uses the smallest feasible sun gear on the input side in order to utilize the space between the two main machines most economically. The complete layout, in simplified lines shown in figure 5, was made by The Timken-Detroit Axle Company, who also designed and built all mechanical parts and assembled the complete unit for installation. As seen, the assembly is extremely compact and utilizes the space within the bell housing, it provides excellent paths for the cooling air which is supplied through air cleaners, and it gives easy access to all brushes for examination and service. A photographic view of the assembly given in figure 6 shows the mounting on the dynamometer stand during preliminary tests. The mechanical construction also involves the brake drum as an integral part of the output shaft flange. Oil seals are incorporated in suitable form and number to insure proper local lubrication without permitting oil to enter into the electrical machines.

The actual weights (in pounds) of the various parts were as follows:

Booster (complete with shaft)	393
Reducer (complete with quill)	534
Regulator (complete with drive)	82
Planet gears	18
Electric control (complete without cover and brake resistor)	56
Air cleaners and miscellaneous parts	29
Total weight	1,112

Inasmuch as this was a sample unit and time was of essence, the material and dimension specifications of the design were not followed closely, so that the above actual weights of the booster and reducer exceeded the computed weights by about 150 pounds. Therefore the given weights do not in any sense constitute the most economic construction.

The electrical control[11] consists principally of a selector switch which connects the electrical machines for the three distinct operations: engine starting, forward, and reverse; and several relays which reduce the effort of the driver to a minimum, namely conventional operation of accelerator and brake pedals, and at the same time provide appropriate interlocks. The control equipment was designed with the co-operation of, and furnished by, the Ward Leonard Electric Company of Mount Vernon, New York.

The "starting" relay is operative only in the "engine starting" position of the selector switch and by interlock only when the accelerator pedal is depressed so as to avoid an accidental short circuit of the battery over the main circuit. Depression of the start button connects the conventional starting motor to the battery to start the engine and also sends current through the booster armature and the booster and reducer main fields in series, establishing the proper polarities for operation in either forward or reverse

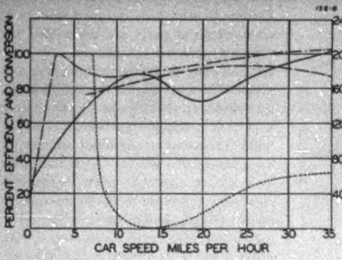

Figure 8. Characteristic performance of city bus Electrogear in full throttle acceleration

—·—·— Gas-electric engine speed
———— Electrogear engine speed
------- Electrogear conversion ratio (per cent of engine power)
— — — — Electrogear efficiency (driver to engine)

direction. This is entirely sufficient for all subsequent operations of the vehicle and no further teasing of the generator is required at any time.

With the selector switch in forward position, depression of the accelerator pedal operates the "accelerator" relay, which permits the reducer to build up self-excitation as generator and to accelerate the vehicle as previously described.

In order to provide the very desirable benefits of electrodynamic braking, the brake relay connects a brake resistor parallel to the two main armatures as soon as the brake pedal is slightly depressed, whereas full depression applies additionally the air brakes. Again, an interlock prevents simultaneous accelerating and braking by de-energizing the accelerator relay when the brake pedal is depressed.

As a protective feature a temperature-operated contact system is inserted close to the reducer main field. Should the temperature of the field coils exceed a critical value, these contacts open and prevent further operation of the accelerator relay and thus of the Electrogear until the electrical machines have cooled down sufficiently to avoid damage. Though this protector never had occasion to act even during the most severe test runs, there might be conditions of getting into deep ruts or similar difficulties when this feature would prove desirable.

III. Comparative Test Data on Bus Transmission

As mentioned above, the bus transmission was chosen as a representative illustration, because in commercial transportation fuel economy, low maintenance cost, durability, acceleration, gradability, ease of operation, and comfort of the driver are of particular importance. Consequently many attempts have been made to develop a bus transmission which would satisfactorily meet all these requirements. Thus, there are at present in actual use, in addition to the standard gearshift transmission, the torque converter[4] developed by the Spicer Manufacturing Company in collaboration with General Motors Corporation, the "Fluidgear" developed jointly by A.C.F. Motors Company and Spicer Manufacturing Company,[5] and the gas or Diesel-electric drives developed by the General Electric Company[8] and the Westinghouse Electric Manufacturing Company.[9] Both the torque converter and the Fluidgear have somewhat lower performance[13] than the standard gearshift transmission, the first with respect to fuel consumption, the second with respect to acceleration, so that comparison of the Electrogear performance will be made with the standard mechanical and the gas-electric drives.

In comparing Electrogear with gearshift performance it should be borne in mind that the test data available on the gearshift transmission are obtained with a skilled driver and constitute optimum results which are seldom attained in actual commercial service, whereas Electrogear as well as gas-electric test performance is reproducible uniformly with any driver, no skill being involved in the operation of these drives, as they require only conventional use of conventional accelerator and brake pedals.

For a fair comparison of these drives it is also necessary to take into consideration the differences in bus weight on account of the drive. It was agreed by the engineers consulted in the preparation of this paper that all test data should be compared on the basis of the following bus weights; with standard gearshift transmission 18,000 pounds; with Electrogear 18,500 pounds; with gas-electric drive 19,200 pounds.

1. Performance Tests With the Electrogear

During the summer of 1939 extensive tests were run with an A.C.F. bus equipped with the Electrogear transmission and a standard $6^1/_4$ ratio rear axle. In all tests the total weight of the bus was about 18,500 pounds. After preliminary tests had demonstrated the satisfactory over-all performance of the transmission and necessary adjustments had been made, final tests were run with calibrated measuring equipment. All the tests were carried on under the supervision of The J. G. Brill Company's (or A.C.F. Motors Company's) engineers and a representative of the Pittsburgh Testing Laboratory.

The acceleration runs were made on a concrete highway west of Philadelphia, and the average taken for several runs in each direction in order to even out possible differences in slight slopes. Speeds were measured by means of a "fifth wheel," mounted on the side of the bus and driving a Hasler-Tel integrating accelerometer, giving mean speed values for each second. The result is shown in figure 7 where speed and distance are plotted against time. According to the records the Electrogear accelerated to 40 miles per hour in 29.2 seconds, and to a distance of 1,000 feet in 26 seconds, which must be considered excellent for any bus of this size. As mentioned before, the Electrogear performance being inherently automatic it can always be reproduced by drivers without skill or familiarity with the principles of operation.

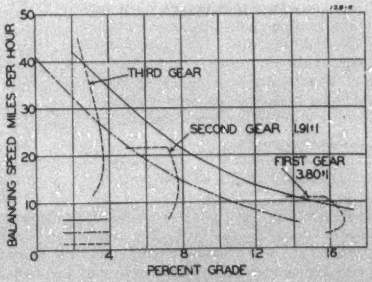

Figure 9. Gradability curves of city bus in terms of balancing speed against per cent grade

———— Electrogear
—·—·— Gas-electric
------- Gearshift

From readings of the currents and voltages during acceleration, the circulating electric power can be computed and compared with engine power. The conversion ratio plotted in figure 8 against car speed graphically shows the percentage of engine power which the Electrogear converts into electric power, and checks very well the theoretical values used in the design. It is quite obvious that during the major part of the acceleration period only a very small fraction of engine power is circulated electrically, so that the losses of the double conversion are very small. This is also expressed in the high values of efficiency, defined here as the ratio of power at the wheels to engine power and shown in figure 8. Finally, in the same figure, the engine speed during full throttle acceleration is also given; the moderate values up to 20 miles per hour vehicle speed, in spite of the high rate of acceleration, are indicative of low fuel consumption for heavy schedules.

In normal driving (at constant speed on level road) the above advantages are, of course, much more marked. Thus, in the present installation the "overdrive" becomes noticeable at about 35 miles per hour, i.e., the engine speed is held below the values which the direct drive of a mechanical transmission would require. Equally, the conversion ratio becomes lower than in acceleration, which improves the over-all efficiency particularly at the higher car speeds.

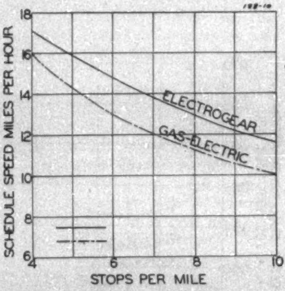

Figure 10. Schedule speed. Stops 11 seconds. Acceleration to about 25 miles per hour

——————— Electrogear
— · — · — Gas-electric

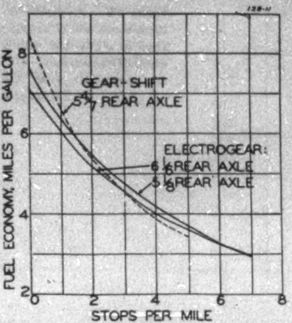

Figure 11. Fuel economy for various stops per mile and different rear axle ratios

——————— Electrogear (18,500 pounds)
- - - - - - - Gearshift (19,980 pounds)

Gradability tests were made on a special test hill near Philadelphia, which is about 700 feet long and has for the major part a grade of 15 per cent, tapering to about 12 per cent toward the top. From a standstill at the foot of the hill the bus was traveling at a speed of 10 miles per hour in less than 15 seconds, and was at the top of the hill at 14.5 miles per hour still accelerating. During this climb the engine speed was very low, with a maximum of 1,650 rpm at the top of the hill. Again, according to the records available to date, this performance was the best obtained on that hill with any bus transmission. On the basis of these and other tests, a gradability curve was computed giving the maximum speed of the bus attainable on various grades; the curve is shown in figure 9.

In order to check continuous performance and temperatures under severe strain the bus was operated for 3 miles with 13 stops per mile, and after a 5-minute interval for $7^1/_2$ miles with 10 stops per mile and 11 seconds per stop, and acceleration up to 25 miles per hour between stops. At the 114th stop the cooling water of the engine boiled over and the brakes were smoking hot. Yet the temperature of the Electrogear machines measured with thermocouples inserted in the field coils was 25 degrees centigrade for the reducer and 43 degrees centigrade for the booster lower than the permissible maximum temperature. The schedule speed during this breakdown test run, as checked repeatedly, was 11.6 miles per hour, taking 5 minutes 11 seconds for every 10 stops. On the basis of this as well as other tests, the schedule speed for various stops per mile was computed and is shown in figure 10. Gasoline consumption was also measured during the runs, giving 2.67 miles per gallon on this severe schedule.

For comparison with other drives, independent fuel consumption tests were carried on at length, for various stops per mile and acceleration up to 30 miles per hour. Figure 11 shows the results for two different rear axle ratios. It is most interesting to observe that the larger ratio $6^1/_8$ which was chosen in the design actually leads to higher fuel economy at heavier schedules than the $5^1/_8$ ratio, which in turn shows up slightly better up to 6 stops per mile. This justifies the selection of the $6^1/_8$ axle for the originally specified schedule of 10 stops per mile, and illustrates the flexibility of the Electrogear showing that it is easily possible to choose the most economical combination for every operating condition. An Electrogear design made for a schedule of 5 stops per mile shows a fuel economy of 8.5 miles per gallon at 30 miles per hour, and of about 4 miles per gallon at 5 stops per mile, which is considerably better than with the design for 10 stops per mile.

2. Comparison With Mechanical Gearshift Transmission

Through the courtesy of The J. G. Brill Company's engineers comparative test data were made available on the standard gearshift transmission in a bus of about the same seated weight and with the same size engine as used for the Electrogear. The ratios in the gearshift transmission were 3.80 for low gear, 1.91 for intermediate, and $5^4/_7$ for the rear axle. The acceleration curves were referred by adjustment of the acceleration rates to the comparative test weight of 18,000 pounds stipulated for the bus with the gearshift drive. The result is plotted in figure 7, demonstrating the superiority of the available continuous driving torque in the Electrogear as compared with the twice interrupted power flow in the gearshift transmission, which in spite of the almost equal initial acceleration rate causes a loss in momentum serious enough to affect the performance. This can be seen very much better in figure 12 in which the available instantaneous acceleration rates are plotted against car speed, and the vertical dotted lines represent the necessary interruptions in acceleration during the shifting of gears. Though the effect is pronounced in acceleration from a standstill, it is much more impressive when accelerating from a given car speed. Thus at 20 miles per hour the mechanical gearshift can produce only 0.76 mile per hour per second acceleration without shifting to second gear, whereas the Electrogear can deliver 1.52 miles per hour per second by mere normal use of the accelerator pedal.

The comparative gradability is plotted in figure 9 in terms of balancing speed, and demonstrates a further advantage of

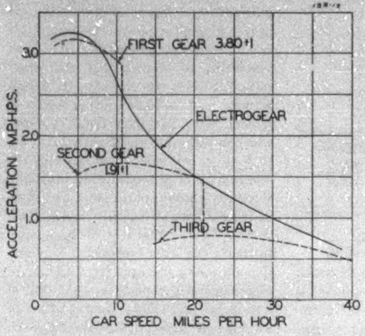

Figure 12. Available instantaneous rate of acceleration of a city bus with a gearshift transmission (----) as compared with the Electrogear

the Electrogear in hilly terrain where the loss of momentum during back shifting of gears is particularly annoying and traffic delaying. Thus, while the standard gearshift bus climbs a grade of only 3.5 per cent at 25 miles per hour, the same bus with an Electrogear will climb a 6.6 per cent grade at 25 miles per hour, which is important with respect to schedule speed on routes with grades. Obviously, this free availability and automatic adjustment of driving torque under all circumstances also indicates a considerable saving in fuel consumption for vehicles traveling in hilly country.

Though no curves are available to show the schedule speed of the gearshift bus directly, reference can be made to an available actual service comparison with the gas-electric drive,[14] which shows that at about the same acceleration rate the gearshift bus schedule is about six to eight per cent slower than that of the gas-electric bus. Since the Electrogear, as shown in figure 7, has better acceleration than the gearshift transmission, the difference in schedule speed in favor of the Electrogear bus would be even greater. At that, it should be kept in mind, in comparing the acceleration curves, that in the case of the gearshift transmission they are obtained with the best driver continuously using extreme care in shifting at the proper moment; such ideal conditions, however, cannot be continuously reproduced in actual service, so that actual schedule speed with gearshift transmissions (manual or automatic) must be lower than theoretically expected, whereas, as pointed out, the Electrogear performance is not dependent on driving skill for uniform efficiency.

In order to compare fuel consumption for various stops per mile and acceleration up to 30 miles per hour, figure 11 shows in dotted line latest data obtained by The J. G. Brill Company on a bus weighing 19,980 pounds and using the same type engine as the Electrogear bus. Since fuel consumption is not proportional to bus weight, reference cannot easily be made to the comparative weight of 18,000 pounds; however, it is quite apparent that the repeated shifting of gears decreases relatively the fuel economy at the heavier schedules, bringing it into the range and below that of the Electrogear beyond about five stops per mile. The influence of shifting gears can best be seen in figure 14 in which the specific fuel consumption for full throttle acceleration is plotted against bus speed, based upon a typical fuel consumption curve of the Hall-Scott 135 engine shown in figure 13 and the engine speed variation shown in figure 8 for the Electrogear, and computed from the gear ratios for the gearshift bus. Over the entire range from 5 to 26 miles per hour (the speed most frequently used in city service) the Electrogear designed for heavy city service keeps the engine in the range of lowest possible fuel consumption. Flexibility of the Electrogear design makes equal or better fuel economy available in busses with lighter city service or for intercity use. In general, then, the Electrogear proves to be at least as efficient as the gearshift drive, and for any particular operating requirements can be made more economical. This clearly shows that the low efficiency of electric drives disappears when the electrical machines are used in proper combination with mechanical elements.

3. Comparison With the Gas-Electric Drive

Through the courtesy of the General Electric Company the performance data of a gas-electric transmission using the same Hall-Scott 135 engine as the Electrogear and the standard gearshift transmission were made available, so that a direct comparison is possible. On account of the greater weight of the gas-electric drive the test weight of the bus in this case was 19,200 pounds. The shunt-wound generator is of the type *GT* 1501 and has a net engine power of 107 horsepower at 2,000 rpm; the series-wound motor is of the type *GE* 1205; the rear axle ratio is $6^1/_8$, comparing with $6^1/_6$ for the Electrogear.

The speed-time curve of figure 7 indicates that the gas-electric acceleration is considerably less than that of the Electrogear, especially at car speeds beyond about eight miles per hour. This is easily accounted for by the fact that for city bus installation the Electrogear is designed to have very little engine power circulating through its electrical machines between 8 and 24 miles per hour car speed, as seen from figure 8, so that there is hardly any conversion loss in that range of car speed. On the other hand in the gas-electric drive the electrical machines continuously carry all the available power so that a considerable conversion loss is sustained. The same explanation holds, of course, for the divergence of the distance-time curves, also shown in figure 7.

For the same reason, the gradability of the Electrogear is considerably higher than that of the gas-electric drive using the same engine, as seen in figure 9, where the balancing speeds are plotted against per cent grades. However, these curves also illustrate particularly well the singular advantage of a continuously variable torque ratio; namely, to permit considerably higher speeds on grades on which the mechanical drive must shift gears and thus lose momentum. Thus at 15 miles per hour car speed the bus with mechanical transmission can climb only a 3.5 per

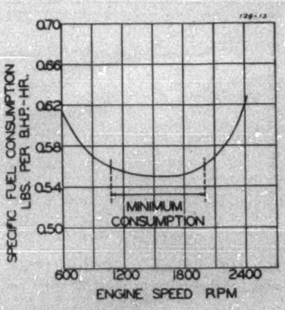

Figure 13. Specific fuel consumption of Hall-Scott 135 engine for full throttle setting

cent grade, the gas-electric bus about a 7.7 per cent grade, and the Electrogear bus even an 11 per cent grade as demonstrated in tests.

Figure 10 shows that the schedule speed of the gas-electric drive is about 1.6 miles per hour lower than that of the Electrogear, assuming 11-second stops in each case. Obviously this difference is due to the considerable difference in acceleration rates already pointed out in connection with figure 7.

Comparison of the fuel consumption per engine horsepower-hour is plotted in figure 14 on the basis of the full throttle specific fuel consumption of the engine given in figure 13 and of the engine speed curves for the various drives. For the gearshift the engine speeds are computed according to the gear ratios with the assumption that shifting is done when the engine speed reaches 2,000 rpm. For the gas-electric drive the engine speed variation was supplied with the performance data as plotted in figure 8 for comparison with the actual engine speed in Electrogear performance. Figure 14 shows the fuel consumption per engine horsepower-hour in full throttle acceleration to be as follows: highest with the gearshift drive (due to the fact that the engine goes through its entire speed range for each gear ratio); up to 12 miles per hour the same with the Electrogear as with the gas-electric drive; lowest with the Electrogear above 12 miles per hour; since the Electrogear uses much less engine power under all conditions it is evident that its total fuel consumption in full throttle acceleration must be the lowest.

Though no curves are available which would permit a direct comparison of the total fuel economy in commercial service of the gas-electric drive with that of the Electrogear, one can judge from comparisons of the gas-electric drive with the equivalent gearshift transmissions obtained in actual operation[14] that the average fuel consumption of the gas-electric drive is about 20 per cent higher if the acceleration is about the same in both cases. The gas-electric total fuel consumption should be even higher in comparison with the Electrogear since it has been established by tests that in a schedule of five stops per mile or more the Electrogear city bus is even more economical than the gearshift bus.

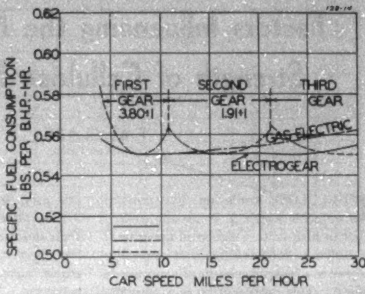

Figure 14. Comparative fuel consumption per engine horsepower-hour of city bus in full throttle acceleration

——— Electrogear
—·—·— Gas-electric
------- Gearshift

IV. Summary

The five principal requirements of an ideal power transmission from the gasoline (or Diesel) engine to the driver axle of a vehicle can be formulated as: first, a continuously variable torque ratio; second, effective loading of the engine under all driving conditions; third, uniformly economic operation at all vehicle speeds; fourth, driving ease; fifth, durability and low maintenance.

The Electrogear has demonstrated that it satisfies all of the above requirements to a higher degree than any known mechanical, hydraulic, or electrical transmission. There is no loss of momentum as in shifting gears, and the engine is never disconnected. Effective engine loading and "overdrive" are inherent features, fully automatic without any facilitating device. The all-around performance of the Electrogear as demonstrated by the Philadelphia tests is superior to that of other known transmissions, and it is significant that the best performance is always automatically produced without calling for skill or judgment. The driver has only the conventional accelerator and brake pedals to use in a conventional way, he has nothing to think about and no controls to manipulate. The durability and low maintenance cost of electrical machines are sufficiently well established by the gas-electric drives still in operation with 400,000 miles or more to their credit. Auxiliary electrodynamic braking (to reduce brake maintenance cost), particularly valuable in busses with heavy city schedules, can easily be made available with the Electrogear.

The design of the Electrogear is simple and flexible, permitting adaptation to any type vehicle. In large vehicles using conventional gas-electric or automatic multishift drives the Electrogear saves space, weight, fuel, and possibly initial cost. Its comparatively low weight opens a wide field of application in medium and light-weight vehicles in which automatically variable infinite ratio transmissions are desirable. To be simple, such transmissions must be electrical.[7a] It is not possible to simulate even by complicated mechanical devices all the desired transmission characteristics which are normally inherent in electric drives. Attempts by mechanical engineers to do so have resulted in increasing the weight and cost of mechanical transmissions to an extent approaching the Electrogear in large production.

Bibliography

1. AUTOMATIC TRANSMISSIONS, **P. M. Heldt.** *Transactions S.A.E.*, volume 32, 1937, page 206.

2. FLUID FLYWHEELS, **P. M. Heldt.** *Automotive Industries*, volume 80, 1939, page 109.

3. HYDRODYNAMIC POWER TRANSMISSION FOR MOTOR CARS, **W. Spannhake.** *Journal S.A.E.*, volume 45, Transactions section, October 1939, page 433. Also *Transactions S.A.E.*, volume 34, 1939, page 433.

4. TRENDS IN DESIGN OF 1940 CARS, **Th. A. Bissell.** *Journal S.A.E.*, volume 45, Transactions section, November 1939, page 461. Also *Transactions S.A.E.*, volume 34, 1939, page 461.

5. FLUIDGEAR, **C. O. Guernsey.** *Bus Transportation*, volume 18, 1939, page 466.

6. NEW HYDRAULIC DRIVE OF YELLOW COACH CORPORATION, **G. A. Green.** *Bus Transportation*, volume 18, 1938, page 448A. Also *Automotive Industries*, volume 79, 1938, page 438.

7. POWER TRANSMISSION FOR BUSSES, **G. A. Green** *Journal S.A.E.*, volume 46, Transactions section, 1940, page 1.

7a. As in 7, page 16.

8. ELECTRICAL DRIVE PROVIDES AUTOMATIC TRANSMISSION FOR BUSSES, **J. C. Aydelott.** *General Electric Review*, volume 40, 1937, page 531.

9. ELECTRIC BUS DRIVES COME BACK, **C. A. Atwell.** *Electric Journal*, volume 35, 1938, page 181.

10. NEW TYPE OF ELECTRO-MECHANICAL TRANSMISSION, **P. M. Heldt.** *Automotive Industries*, volume 76, 1937, page 236. Also in *Transactions S.A.E.*, volume 32, 1937, page 206.

11. **A. H. Neuland:** U. S. Patents No. 2,000,786 of May 7, 1935, No. 2,045,197 of June 23, 1936, and numerous other patents owned by The Power Transmission Company.

12. **E. Weber:** U. S. Patent No. 2,179,364 of November 7, 1939.

13. HYDRAULIC DRIVES, **E. S. Pardoe.** *Bus Transportation*, volume 18, 1939, page 462.

14. OPERATING EXPERIENCES WITH GAS-ELECTRIC-DRIVE MOTOR BUSSES, **R. H. Stier.** AIEE TRANSACTIONS, volume 57, 1938 (June section), page 318. Also in *General Electric Review*, volume 41, 1938, page 229.

15. Report on tests of the Electrogear transmission, Pittsnurgh Testing Laboratory. March 1937.

Factors Influencing the Mechanical Strength of Cellulose Insulation

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CELLULOSE insulation, like mineral oil with which it is usually associated in high-voltage electrical apparatus, is subject to deterioration in an increasing degree as the temperature of exposure is raised. This deterioration is a manifestation of the chemical instability of the cellulose. For a proper understanding of this change, it must be recognized that the fundamental chemical reactions involved are of two types. There are those changes which result from the oxidation of the cellulose and those which arise from the decomposition of its molecule. These changes are invariably intermingled in the normal commercial use of cellulose dielectric materials. To prevent dangerous deterioration and to secure the maximum service life of cellulose insulation, the effect produced by each type of reaction, the probability of its occurrence, and the means for its control must be carefully evaluated.

In a previous article[1] the author has described the mechanical and electrical changes in cellulose promoted by thermal decomposition in contrast to oxidation effects. It has been shown that cellulose will change mechanically and electrically with accompanying increase in color and acidity when heated at temperatures even as low as approximately 130 degrees centigrade under conditions which exclude the possibility of oxidation. These changes have been attributed to a molecular decomposition of the cellulose which culminates in the active evolution of gases, water, and other volatile products at temperatures above 130 degrees centigrade.

One of the objects of the present paper is to evaluate from the standpoint of mechanical strength those deteriorating changes in cellulose insulation which are caused by oxidation in contrast to those which result from the purely thermal decomposition of the cellulose molecule. Linen capacitor paper and Manila paper are used to illustrate the test results. These papers are selected because of their industrial importance. Other cellulose insulations, as for example kraft paper, have been found to be of similar behavior. The tensile strength of the insulation has been accepted as a gauge of its mechanical properties.

The Dormant Period

Cellulose insulation when heated actively exhibits evidence of molecular decomposition only after a "dormant" period, the length of which depends on the conditions of the heat treatment. Foremost among the conditions of importance are the temperature applied, the accumulated thermal history of the cellulose, and the presence or absence of an impregnating liquid such as mineral oil. In general, for unimpregnated insulation, gas evolution and other evidence of serious decomposition are observed after about 12 hours of heating at temperatures higher than 130 degrees centigrade. With oil-treated cellulose insulation, the effect of temperature has been more carefully delineated. Figure 1 illustrates for oil-immersed linen paper the length of the "dormant" period within which no gas is evolved at temperatures from 145 degrees centigrade to 200 degrees centigrade.

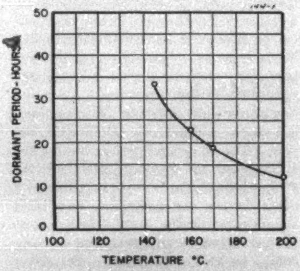

Figure 1. The "dormant" period is the duration of heat treatment which can be applied to cellulose insulation in the absence of oxidation without evolution of gases indicating molecular decomposition

The insulation used is vacuum-dried, oil-impregnated, and oil-immersed linen paper

It is obvious that the single effect of oxidation in promoting cellulose deterioration becomes of major importance only when the effects of molecular decomposition are avoided. It is within this range that the use of an inert gaseous atmosphere such as nitrogen can be expected to have its greatest stabilizing effect. When exposed to more severe conditions of temperature and time so that the "dormant" period is exceeded, molecular decomposition may become the dominant deteriorating factor. Adverse results are produced both directly as a result of pyrochemical change and indirectly because of the corrosive character of the decomposition products formed. It is during this period that the beneficial effects of a nonoxidizing atmosphere such as nitrogen gas are of decreasing, and frequently of negligible, importance.

Conditions Which Affect Aging

When cellulose insulation is exposed to temperatures of normal dielectric use, the rate of deterioration in tensile strength is dependent on the conditions of the heat treatment. Important factors are the temperature applied, the length of the exposure, and the presence or absence of an oxidizing atmosphere. The close intermingling of these factors and the great importance of the first two in producing radical changes in the tensile strength of the cellulose insulation have served to confuse their proper evaluation

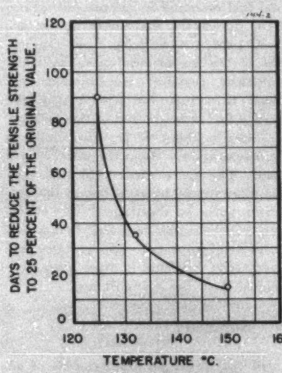

Figure 2. Dry Manila paper will deteriorate mechanically when heated in sealed tubes in contact with nitrogen gas

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[1] For all numbered references, see list at end of paper.

to the extent that oxidation is frequently looked upon as the fundamental cause of all of the deterioration observed. This is not true, for cellulose insulation will deteriorate even in the complete absence of any oxidizing gas or other source of oxygen. This is illustrated in the data of figure 2. In this series of tests, Manila paper, vacuum dried but not oil impregnated, has been heated in sealed bombs at temperatures from 125 degrees centigrade to 150 degrees centigrade under 250 pounds of dry nitrogen gas. The results are expressed in terms of that period of time necessary to reduce the tensile strength to 25 per cent of the original value. The rapid loss in tensile strength with the increasing temperature of the exposure in nitrogen gives warning against too great an emphasis of the oxidation factor as the fundamental cause of cellulose deterioration.

The Stable Period

Available data indicate that, even under the most severe deteriorating conditions, there is an initial period during which the cellulose insulation suffers no loss in tensile strength. During this interval it may even exhibit slight increases. This period has been termed the "stable period." It corresponds to but is not of as great a duration as the "dormant" period discussed in a previous paragraph. After passing through the "stable period" chemical reactions are initiated which affect the mechanical strength of the insulation and which at temperatures above 130 degrees centigrade culminate in the evolution of gases, water, and other volatile products.

The duration of the "stable period" decreases sharply with increased temperature above 100 degrees centigrade. The period of stability is clearly evident in the tests of figures 10 and 11 which describe the behavior of vacuum-dried, oil-treated, and oil-immersed 0.003-inch Manila paper. The "stable period" for each of the relations of figures 10 and 11 is reproduced for purpose of clarity in figures 3 and 4.

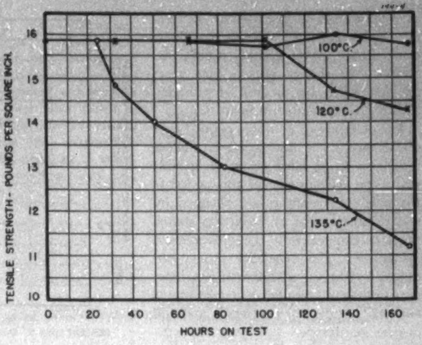


Figure 4. Showing the "stable period" of vacuum-dried, mineral-transformer-oil-treated, and oil-immersed 0.003-inch Manila paper when heated in contact with nitrogen gas

The length of the "stable period" is affected by the presence of an oxidizing atmosphere. This is demonstrated in the following data taken from figures 3 and 4.

Test Temperature (Degrees Centigrade)	Length of "Stable Period" in Hours		Per Cent Decrease in Duration of Stable Period Due to Oxidation
	For Run in Oxygen Gas	For Run in Nitrogen Gas	
100	101	168	40
120	33	101	68
135	16	24	33

The effect of oxidation on the duration of the "stable period" of oil-treated and oil-immersed Manila paper decreases as the temperature is lowered. As illustrated in figure 5, at temperatures as low as 85 degrees centigrade, the accelerating effect of oxidation can be expected to disappear. Tests at temperatures in the range of 85 degrees centigrade are of long duration and subject to wide variation, but in general, have served to substantiate this expectation.

When the cellulose is exposed to higher temperatures, the effect of oxidation on the length of the "stable period" does not increase indefinitely. This results because of the increasing importance of the second major factor, molecular decomposition which also greatly affects the rate of cellulose deterioration. The advantageous effect of the presence of nitrogen gas in prolonging the "stable period" of oil-treated cellulose reaches its maximum value at approximately 120 degrees centigrade. With higher temperatures of aging, the pyrochemical decomposition of

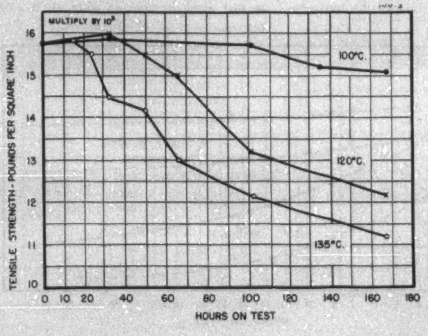


Figure 3. Showing the "stable period" of vacuum-dried, mineral-transformer-oil-treated, and oil-immersed 0.003-inch Manila paper when heated in contact with oxygen gas

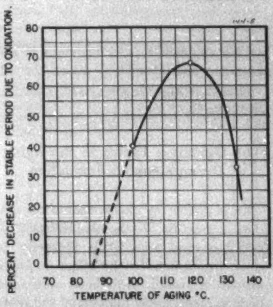


Figure 5. The effect of oxidation on the duration of the "stable period" for vacuum-dried oil-impregnated, and oil-immersed 0.003-inch Manila paper reaches its maximum at approximately 120 degrees centigrade

the oil-treated cellulose assumes increasing importance. As a result, the duration of the "stable period" rapidly decreases even though all oxidizing gases are eliminated by the use of nitrogen. This is also illustrated in figure 5.

The existence of a "stable period" is of importance in the application of cellulose insulation in oil-filled electrical apparatus. The prolongation of this period is the most important result obtained by the elimination of an oxidizing atmosphere in contact with the oil-immersed insulation. The beneficial results obtained by the use of nitrogen gas, however, are largely limited to temperatures between 100 degrees centigrade and 120 degrees centigrade. For lower temperatures, the oxidation effect is of negligible importance. For higher temperatures, the pyrochemical decomposition reaction of the cellulose rapidly increases in importance and ultimately masks the advantages normally associated with the use of nitrogen gas.

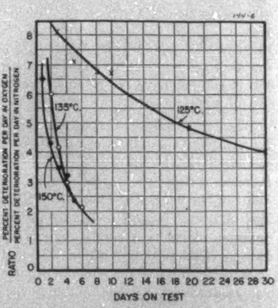


Figure 6. In the heat treatment of vacuum-dried but unimpregnated 0.003-inch Manila paper in sealed bombs under 250 pounds gas pressure, the deterioration in tensile strength is accelerated by the presence of oxygen

The Oxidation Factor in the Aging of Unimpregnated Insulation

In studying the importance of the oxidation reaction in the deterioration of cellulose insulations, it is necessary to differentiate between its effect on those insulations which have been oil-treated and are tested under mineral oil and those insulations which are tested without oil impregnation and in direct contact with an oxidizing atmosphere. In order to illustrate more clearly the effects of oxidation in terms of tensile strength, it has been found desirable to exaggerate the oxidation reaction by the use of oxygen gas in place of air. The effect on unimpregnated insulation is easily demonstrated. Figure 6 illustrates the importance of oxidation as a factor in the mechanical deterioration of dried but unimpregnated 0.003-inch Manila cable paper, aged at 125 degrees, 135 degrees, and 150 degrees centigrade in sealed bombs under 250 pounds of gas pressure. The results are expressed in terms of the accelerated rate of deterioration produced by the presence of oxygen gas as compared to similar aging in the presence of nitrogen. As the deterioration of the cellulose progresses, irrespective of the temperature of test, the accelerating effect of oxygen gas rapidly diminishes. During the initial state of the temperature exposure, however, the increased rate of aging due to the oxidation of the unimpregnated insulation reaches a high value.

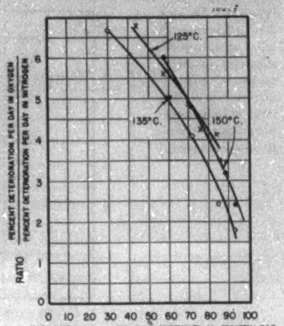


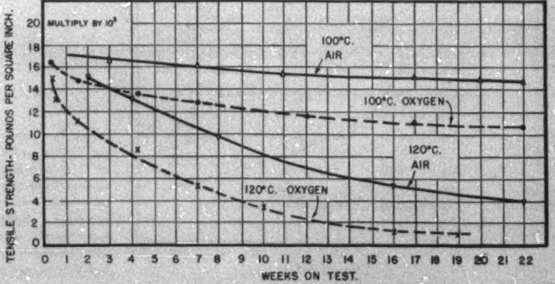
Figure 7. When vacuum-dried but unimpregnated 0.003-inch Manila paper is heated in sealed bombs under 250 pounds of gas pressure, the loss in tensile strength is accelerated by the presence of oxygen gas to approximately the same extent as the temperature is changed within the range from 125 degrees centigrade to 150 degrees centigrade

The marked effect of the oxidation reaction during the initial stages of the high-temperature exposure of dried but unimpregnated cellulose results from the longer duration of the "stable period" when the insulation is in contact with a nonoxidizing atmosphere. When once the "stable period" in nitrogen gas has been passed, however, and the paper begins to deteriorate because of pyrochemical decomposition, the increased rate of deterioration in oxygen gas as compared to that observed in nitrogen rapidly decreases. The decrease is substantially independent of the temperature. This is illustrated in the data of figure 7 which correlate the accelerating effect of oxidation caused by the presence of oxygen gas with the degree of deterioration observed. It is important to note that with dried but unimpregnated Manila paper, even though the effect of oxidation decreases as the deterioration progresses, there is nevertheless a pronounced acceleration of deterioration throughout the whole mechanical life of the insulation.

The Oxidation Factor in the Aging of Oil-Impregnated Insulation

In order to evaluate the importance of oxidation in the deterioration of oil-treated and oil-immersed cellulose insulation, it has been found desirable, as in the study of the unimpregnated insulation, to exaggerate those factors which accelerate oxidation. For this purpose, pure

Figure 8. When vacuum-dried, oil-treated, and oil-immersed 0.003-inch Manila paper is aged in glass tubes sealed at atmospheric pressure, the presence of oxygen gas accelerates the loss in mechanical strength as compared to that which is obtained when air is used



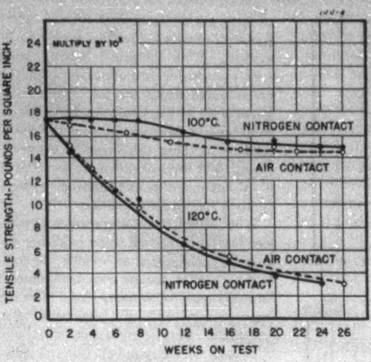


Figure 9. The loss of the tensile strength for vacuum-dried, oil-treated, and oil-immersed 0.003-inch Manila paper when aged in contact with nitrogen gas in glass tubes sealed at atmospheric pressure closely duplicates the deterioration observed when air is substituted for the nitrogen

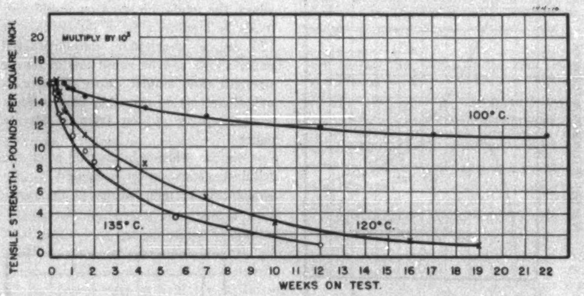


Figure 10. The loss in mechanical strength of vacuum-dried, oil-impregnated, and oil-immersed 0.003-inch Manila paper when heated in sealed tubes in contact with oxygen gas

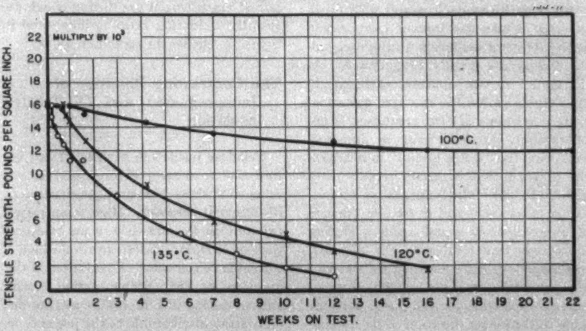
oxygen has been used in place of air. The oxygen gas has been applied both at atmospheric pressure and at 250 pounds per square inch in sealed bombs. Temperatures of test as high as 200 degrees centigrade have been investigated.

The results of accelerated tests in which the possibility of oxidation has been greatly exaggerated must be considered merely as the ultimate limit of oxidation. It must not be concluded from such tests that an equal degree of deterioration will be obtained for the same oil-treated and oil-immersed insulation in contact with the air which is normally present in commercial usage. A typical comparison of the aging of 0.003-inch Manila paper in air and in oxygen is illustrated in figure 8 for tests at 100 degrees and 120 degrees centigrade. The deterioration observed in the insulation aged in contact with air in figure 8 is substantially the same as that observed when the insulation is aged in contact with nitrogen under similar test conditions (figure 9).

Despite the difference in the rate of aging in oxygen and in air, the use of oxygen serves the practical laboratory purpose of bringing out the effects of oxidation under the most favorable, lower temperature conditions at which the effect of nonoxidizing chemical changes are reduced or eliminated. Figures 10 and 11 give detailed illustrations of the aging of 0.003-inch vacuum oil-treated and oil-immersed Manila cable paper sealed in glass tubes at atmospheric pressure, the surface of the oil being in contact with oxygen and with nitrogen gas.

One of the outstanding characteristics which distinguishes the aging of cellulose insulations in contact with oxygen gas from similar aging in contact with nitrogen is the shorter duration of the "stable period" when oxidation occurs. It is obvious from the data presented that any evaluation of the possible acceleration of aging due to oxidation will depend upon that stage of the heat treatment selected for the comparison. If an interval during the "stable period" is selected as the time of comparison, it will be concluded that oxidation is of serious import. If on the other hand, the point of comparison is taken after considerable mechanical deterioration has occurred, entirely different conclusions will be drawn. This is illustrated in figure 12, based on the data of figures 10 and 11. The acceleration of mechanical deterioration due to the presence of oxygen as compared to nitrogen gas is shown as a function of the duration of the aging exposure at 100 degrees centigrade, 120 degrees centigrade, and 135 degrees centigrade. At the lower temperature pronounced oxidation effects are observed during the initial stages of the test. At the end of about four days of aging, the deterioration in oxygen gas at 120 degrees centigrade is approximately $4^1/_2$ times faster than in nitrogen. This results from the longer duration of the "stable period" in the absence of oxi-

Figure 11. The loss in mechanical strength of vacuum-dried, oil-impregnated, and oil-immersed 0.003-inch Manila paper when heated in sealed tubes in contact with nitrogen gas



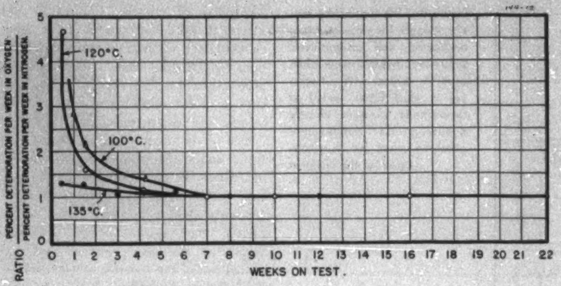


Figure 12. When vacuum-dried, oil-treated, and oil-immersed 0.003-inch Manila paper is heated in glass tubes sealed at atmospheric pressure, the oxidation factor assumes decreasing importance as the aging run progresses

dation effects. At the end of this "stable period" however, rapid changes in the oxidation ratio occur. Deterioration of the insulation in contact with nitrogen gas causes the oxidation ratio to fall rapidly. Within the limits of the available data, it is concluded that the rate of deterioration of the cellulose is progressively less affected by oxidation as the aging continues. When the deterioration has progressed to the extent that approximately 50 per cent of the original strength has been lost, the effects of the oxidation reaction are masked by the increasing importance of pyrochemical deterioration.

A further illustration showing the change from a condition of oxidation type of deterioration to a condition where the pyrochemical type predominates is illustrated in figure 13. In this study the dried, oil-impregnated, and oil-immersed 0.003-inch Manila paper insulation is heat treated at 200 degrees centigrade in contact with oxygen gas and with nitrogen gas. Under this type of deterioration both factors of oxidation and pyrochemical change are greatly exaggerated. As illustrated in the analysis of figure 13 the "stable period" in nitrogen gas under these conditions of heat treatment is approximately one-half hour in duration. Following the "stable period" in nitrogen, the deterioration increases rapidly to a maximum, after which the rate of deterioration in nitrogen and in oxygen appears of approximately equal value. During subsequent heat treatment, the only distinguishable feature between the deterioration resulting from the aging in nitrogen gas as compared to oxygen gas is attributable to the longer duration of the "stable period" in nitrogen gas during which period the oxidation ratio as illustrated in figure 14 reaches its maximum value.

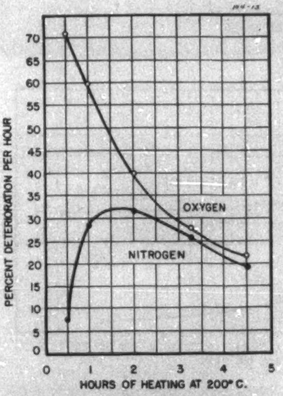


Figure 13. When vacuum-dried, oil-treated, and oil-immersed 0.003-inch Manila paper is aged at 200 degrees centigrade in glass tubes sealed at atmospheric pressure, the effects of oxidation, clearly evident in the initial stages of the heat treatment with oxygen are later masked by the presence of pyrochemical deterioration which occurs even in the presence of nitrogen gas

The Contrasting Behavior of Unimpregnated and Oil-Treated Insulation

Cellulose insulation whether tested before or after mineral oil impregnation and immersion shows the same peculiarities in its chemical behavior. High-temperature exposure in the presence of an oxidizing gas causes an initial deterioration which is traceable to the effect of oxidation on the duration of the "stable period." Examined during this interval, a marked deterioration attributable to the presence of oxygen gas as compared to the corresponding change in nitrogen gas is clearly evident. With continued heating, however, pyrochemical decomposition changes are initiated which become of increasing importance as the insulation is subjected to exhaustive treatment. It is the interrelation of these two types of chemical change and their modification as a result of oil impregnation and immersion which compels special consideration.

When dried but unimpregnated cellulose insulation is aged in an oxidizing atmosphere, the fibers are in intimate contact with what may be considered an unlimited supply of oxygen. Aged after oil impregnation and oil immersion, the amount of immediately available oxygen is limited to that supplied from oil solution. The result is that at temperatures where the oxidation reaction would normally be predominant, as for example at 120 degrees centigrade, unimpregnated paper shows a more rapid loss in tensile strength than similar paper oil impregnated and oil immersed. This is illustrated in figure 15 for tests on Manila paper.

A similar relation holds true when the cellulose deterioration is caused by the application of temperature in the range

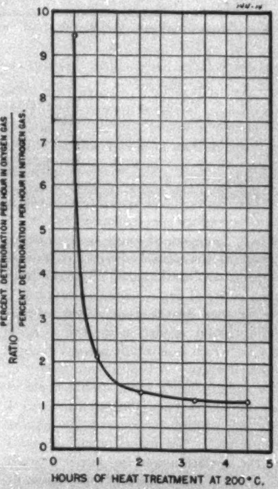


Figure 14. Showing the marked decrease in the oxidation factor after the end of the "stable period" for the aging of vacuum-dried, oil-treated, and oil-immersed 0.003-inch Manila paper in sealed glass tubes at 200 degrees centigrade

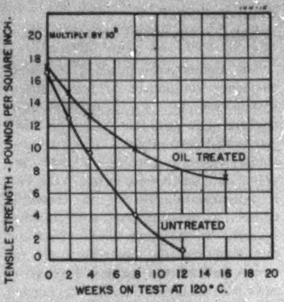


Figure 15. The mechanical aging of vacuum-dried 0.003-inch Manila paper is decreased as the result of impregnation and immersion in mineral transformer oil

The heat treatment in this illustration is carried out in glass tubes sealed at atmospheric pressure, the gas present being dried air

where pyrochemical decomposition is rapid and severe even in the presence of nitrogen. This is not unexpected in view of the fact that, as already described in a previous paragraph, the "dormant" period is increased as a result of oil impregnation and immersion. The longer duration of the dormant period for oil-treated and oil-immersed insulation in the absence of oxidation is undoubtedly related to the slower rate of acid formation in paper when exposed to high temperatures after oil treatment.[1] The result is that at a temperature of 135 degrees centigrade, dried but unimpregnated Manila paper deteriorates in tensile strength at a more rapid rate than does similar paper after oil impregnation and oil immersion. This is illustrated in figure 16.

In comparing the stability of cellulose insulation before and after oil impregnation the conditions of the comparison must be carefully stated. Dried but unimpregnated cellulose may be expected to deteriorate more rapidly than the same material after oil impregnation when the conditions of the temperature treatment are such that severe oxidation or pyrochemical reactions are possible. Heated under conditions such that oxidation and pyrochemical changes are of minor importance, the mechanical deterioration of the cellulose insulation is substantially unaffected by the presence of the impregnating mineral oil in which the material may be immersed.

Summary

1. Cellulose insulation will deteriorate in mechanical strength at a rate which increases with the increasing temperature of exposure. Deterioration will occur in the absence of an oxidizing atmosphere.

2. The rate of cellulose deterioration at a fixed temperature varies as the heat treatment is continued.

3. The mechanical deterioration of cellulose insulation at elevated temperature is the result of oxidation and pyrochemical decomposition.

4. When heated, cellulose passes through a "stable period" during which the mechanical properties are maintained. The presence of oxygen is chiefly effective in reducing the duration of this "stable period."

5. The single effect of oxidation is restricted to the initial periods of treatment at temperatures lower than approximately 120 degrees centigrade. At higher temperatures, pyrochemical decomposition occurs. Even in the most favored temperature range, the effect of cellulose oxidation merges into the effects produced by pyrochemical decomposition as the deterioration progresses.

6. Heated under conditions which favor oxidation or pyrochemical change, the unimpregnated cellulose insulation deteriorates more rapidly than the same insulation after oil treatment and immersion.

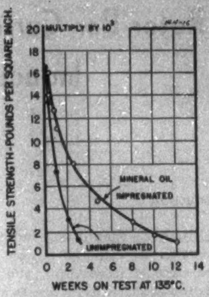


Figure 16. When heated at 135 degrees centigrade in sealed glass containers in contact with nitrogen gas, the rate of loss in tensile strength of dried 0.003-inch Manila paper is substantially decreased as a result of impregnation and immersion in mineral transformer oil

7. The effect of oxidation is accelerated when pure oxygen gas is in contact with the insulation. No substantial difference in the rate of mechanical deterioration is observed when the oil-treated and oil-immersed insulation is aged in contact with nitrogen gas as compared to aging in contact with air.

Bibliography

1. PYROCHEMICAL BEHAVIOR OF CELLULOSE INSULATION, **F. M. Clark.** AIEE TRANSACTIONS, volume 54, 1935, pages 1088–94.

2. CORRELATION OF THE CHEMICAL AND ELECTRICAL CHANGES OBSERVED DURING THE OXIDATION OF MINERAL INSULATING OIL, **F. M. Clark.** ASTM Preprint number 99, 1940.

3. LOADING TRANSFORMERS BY TEMPERATURE, **V. M. Montsinger.** AIEE TRANSACTIONS, volume 49, 1940, pages 776–90.

4. TEMPERATURE AGING CHARACTERISTICS OF CLASS A INSULATION, **J. J. Smith** and **J. A. Scott.** AIEE TRANSACTIONS, volume 58, 1939 (September section), pages 435–42.

5. TEMPERATURE LIMITS SET BY OIL AND CELLULOSE INSULATION, **C. F. Hill.** AIEE TRANSACTIONS, volume 58, 1939 (September section), pages 484–7.

Rotor-Bar Currents in Squirrel-Cage Induction Motors

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MEMBER AIEE

Introduction

THE rotor of a squirrel-cage induction motor is an extremely simple and rugged piece of mechanism; and due largely to this simplicity, it resists accurate analysis of its behavior by ordinary testing technique. Its design has been perfected by cut and try methods, making assumptions with respect to rotor current to simplify the theory; but the actual current in the bars has remained a mystery. The purposes of this paper are: first, to present oscillograms of rotor-bar currents; second, to extend the theory of induction motors to include the computation of rotor-bar currents; and third, to show the effect of the rotor current harmonics upon the efficiency of the motor.

Rotor-Bar Currents

The current in the rotor bars of a squirrel-cage induction motor is a composite wave of complicated form in which, however, three main components are prominent. In addition to the slip-frequency component which increases directly with load, there is a component due to nonuniform reluctance, the tooth-frequency component. The magnitude of this component depends primarily upon flux density but also increases with load; its frequency depends upon the number of stator teeth and the speed. The tooth-frequency component is very pronounced in a machine with open slots and is considerably reduced by using closed slots or nearly closed slots. The third main component of rotor frequency is caused by the nonsinusoidal distribution of stator current, and will be designated the band-frequency component.

A polyphase stator winding excited by current of sinusoidal wave shape gives a stator magnetomotive force with nonsinusoidal distribution. This would produce a rotating field with nonsinusoidal shape, although the flux density at every point along the air gap would vary sinusoidally in time to give a sinusoidal generated voltage in the stator. In a wound-rotor motor, space harmonics in the stator flux wave produce corresponding harmonics in the rotor generated voltage; but in a squirrel-cage motor, irregularities in the flux wave are practically eliminated by currents induced in the short-circuited rotor conductors. The rotor current distribution is such that the total current distribution, rotor plus stator, is sinusoidal. This hypothesis is given in Professor Bailey's book,[1] and is substantiated by analysis of the rotor current oscillograms included in this paper.

Computation of Band-Frequency Component

The method of computing the band-frequency component of rotor current will be shown by a specific example, using a three-phase winding with two-thirds pitch, 12 slots per pole. The curves of stator current distribution over two poles, and the corresponding rotor current distribution curves, are shown in figure 1, at time intervals of 15 electrical degrees. In the stator winding distribution diagram at the top of the figure, the arrows indicate the positive direction of current in each coil side, not the current distribution at a particular time. The cyclic maximum value of stator current per coil side (turns per coil times maximum stator current per conductor) is denoted by I_s. At the time ($\theta = 0$ degrees) the instantaneous values of currents, indicated in the vector diagram at the left, are:

Phase $A = 0$
Phase $B = -0.866 I_s$
Phase $C = 0.866 I_s$

Using these values with the winding distribution diagram, the curve of stator current distribution, S, is obtained for time ($\theta = 0$). The rotor current distribution curve, R, is drawn so that the total current curve, T, is the sinusoidal equiva-

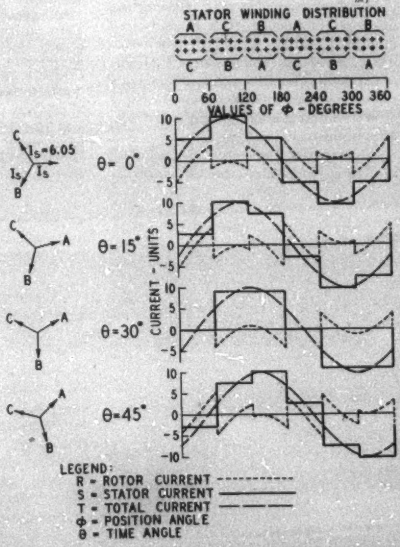

Figure 1. No-load current distribution curves of three-phase winding with 2/3 pitch

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[1] For all numbered references, see list at end of paper.

lent of the stator current curve, that is the fundamental component of the curve S. The amplitude, T_m, may be obtained by use of the formula for the coefficient of the fundamental term of the Fourier series:

$$T_m=\frac{2}{\pi}\int_0^{\pi} S\sin\phi d\phi \qquad (1)$$

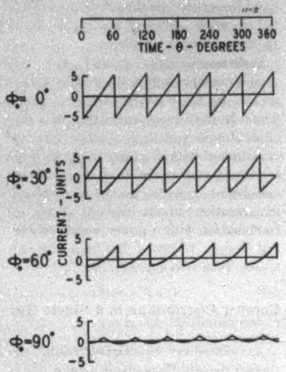

Figure 2. Rotor-bar current at zero slip for three-phase winding with 2/3 pitch

For the time ($\theta=0$) the values of stator current shown give:

$$T_m=\frac{2}{\pi}\left[\int_0^{\pi/3} 0.866I_s \sin\phi d\phi+\int_{\pi/3}^{2\pi/3} 1.732I_s \sin\phi d\phi+\int_{2\pi/3}^{\pi} 0.866I_s \sin\phi d\phi\right]$$

$$=\frac{3\sqrt{3}}{\pi}I_s \qquad (2)$$

and at any other time, the same value will be found for T_m. In figure 1, a value of I_s is chosen arbitrarily to make $T_m=$ ten units, so:

$$I_s=\frac{10\pi}{3\sqrt{3}}=6.05 \text{ units} \qquad (3)$$

and all the curves are drawn to this scale.

The rotor current distribution curves may be used to obtain the time curve of current in a particular bar by following a point on the rotor as it turns with the field. Due to discontinuities in the distribution curves, the time curves of rotor current cannot be obtained by Fourier analysis; however by using the values on the curves in figure 1, the curves of figure 2 may be obtained. The curve ($\phi_0=0$) shows current in the rotor bar which is in position ($\phi=0$) at time ($\theta=0$), so the value 5.24 units is taken at the point ($\phi=0$) on the curve for time ($\theta=0$) in figure 1. At time ($\theta=15$ degrees) the rotor has moved 15 degrees to the right, so the value of rotor current is taken from the ($\theta=15$ degrees) curve at point ($\phi=15$ degrees). The values of rotor current in this bar at $\theta=30$ degrees and 45 degrees are read similarly from the corresponding curves at the 30-degree and 45-degree points respectively. At ($\theta=60$ degrees) the distribution curves are like the ($\theta=0$) curves and the rotor current wave repeats, so the rotor-bar current in this case is a 360-cycle wave of nearly triangular wave shape. The curve ($\phi_0=30$ degrees) in figure 2 is obtained similarly; the first point is taken from the ($\theta=0$) curve at ($\phi=30$ degrees), etc. For different bar positions, the magnitude of this band-frequency component changes as well as the wave shape with a maximum value for the position ($\phi_0=0$):

$$R_b=6.05\sin 60°=5.24 \text{ units} \qquad (4)$$

and only 0.67 unit for the bar in the ($\phi_0=90$ degrees) position.

The curves of figure 2 represent rotor-bar currents in a machine running at synchronous speed. At no load with a small amount of slip, these curves show some of the phases through which the rotor current passes slowly as the bar changes its position with reference to the revolving field.

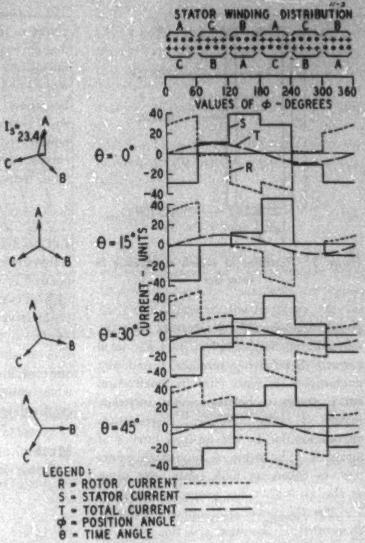

Figure 3. Full-load current distribution curves of three-phase winding with 2/3 pitch

Effect of Load Upon Rotor Current

When the motor is loaded, the stator current is increased by the addition of a

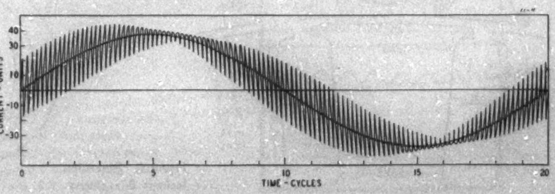

Figure 4. Rotor-bar cunent at full-load with 5 per cent slip, for three-phase winding with 2/3 pitch

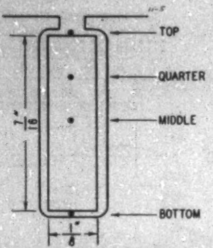

Figure 5. Positions of search conductors in rotor bar

power component in quadrature with the magnetizing component. Since the total current distribution remains practically unchanged, the rotor current distribution curve shows a corresponding increase. Figure 3 shows full-load distribution curves, similar to the no-load curves of figure 1. The value of magnetizing current is taken as 6.05 units, the same as the no-load value given in equation 3. For the power component a value of 22.6 units is assumed, so the value of I_s used in drawing the curves of figure 3 is:

$$I_s = 6.05 + j22.6 = 23.4 \text{ units (at 75 degrees)} \qquad (5)$$

The total current, T_m, equals ten units as before.

The curve of rotor-bar current in figure 4 is obtained from these distribution curves by following a point on the rotor, using a slip of five per cent which corresponds to full load. The rotor current now shows a slip-frequency component with amplitude, R_s, proportional to the power component of stator current:

$$R_s = 22.6 \times \frac{3\sqrt{3}}{\pi} = 37.35 \text{ units} \qquad (6)$$

The band-frequency component increases with the stator current, so the maximum amplitude:

$$R_b = \frac{23.4}{6.05} \times 5.24 = 20.3 \text{ units} \qquad (7)$$

The frequency of this component also changes. At synchronous speed each 60-degree band of the stator winding produces one cycle of the band-frequency wave. As the speed decreases, the band-frequency drops; and for each cycle of slip frequency, the band-frequency component passes through all its phases corresponding to a complete cycle. The band frequency for a value of slip, s, is:

$$f_b = (1-s)6f + sf = (6-5s)f \qquad (8)$$

and at five per cent slip, this frequency is 345 cycles. This band-frequency component is superposed on the sinusoidal (slip-frequency) component; and it passes progressively through the various phases shown in figure 2. In these computed curves the tooth-ripple frequency is omitted, and this should be kept in mind when comparing them with the oscillograms shown later.

Method of Measuring Rotor-Bar Current

In February 1937, an article by Narbutovskih[2] showed a method of obtaining rotor-bar current curves by placing search conductors in the rotor bar. The search conductor is an insulated wire running through the rotor bar under observation, and connected to it at the back end. The front end of the test conductor and the front end of the rotor bar are connected through sliding contacts to the first stage of an amplifier, so no current is drawn from this test circuit. The voltage between the leads is equal to the instantaneous value of iR drop in the bar material adjacent to the test conductor and therefore is proportional to the current density.

The article is most interesting because these oscillograms constitute the first published record of actual rotor-bar currents which has come to the writer's attention. The tests were run on an early type of machine with nearly square rotor bars, and an open slot stator which produced a large tooth ripple in the rotor current. From a study of the oscillograms, it is not apparent that the amplifier used had a frequency range low enough to show slip frequency. The band-frequency component is discernible but is nearly covered up by the large tooth-frequency component.

Preliminary tests showed that this method would permit the study of harmonics in the rotor-bar current, which would be even more interesting in a machine of modern design. An amplifier was constructed with a power stage suitable for direct connection to a magnetic oscillograph; with maximum voltage amplification about 50,000. Used in combination with a power supply of special design, it has a flat-frequency response from 1 to 2,000 cycles.

Current Distribution in a Single Bar

As pointed out by Narbutovskih,[2] the current density throughout a rotor bar is far from uniform, particularly for high-frequency components, due to rotor leakage flux. A search conductor in the rotor bar does not, therefore, measure the current in the bar, but rather the current density adjacent to the test conductor. In a narrow rectangular bar the current density across the bar will be nearly uniform, except for very high-frequency components, but will vary with the depth in the bar. For the tests reported in this paper, four search conductors were used, placed as shown in figure 5. Specifications on the machine tested were:

Rating: 220 volt, 3 phase, 60 cycle, 5 horsepower, 1,800 rpm

Stator: 48 coils, 22 turns No. 15, throw 1-9, connected parallel-wye

Rotor: 45 copper bars, $^1/_8$ by $^7/_{16}$ inch, length 2.38 inches copper end rings, $^1/_4$ by $^{29}/_{32}$ inch, mean radius 2.89 inches

The dimensions of rotor slots and teeth, air gap, and stator slot openings are shown in figure 6, all dimensions in inches.

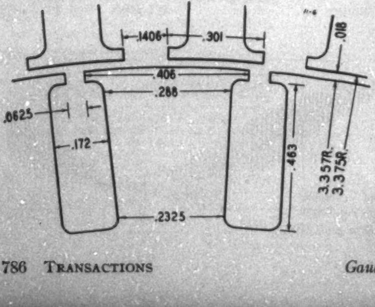

Figure 6 (left). Rotor slot and tooth dimensions

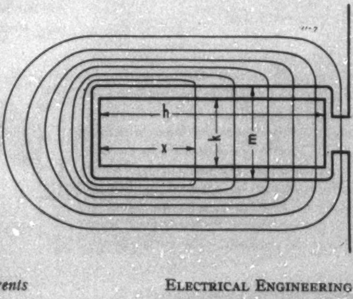

Figure 7. Rotor-bar leakage flux pattern

Figure 7 shows a bar of thickness, k, and depth, h, in a slot of width, m, with all dimensions in inches. Certain simplifying assumptions are made concerning the leakage flux pattern which are quite valid for a narrow rectangular bar nearly filling the slot. Due to the higher reactance of the lower part of the bar, the current density is highest at the top of the bar and diminishes to a minimum at the bottom. Also the current in lower layers is lagging in phase. Both of these effects are most noticeable at high frequency, are relatively small at 60 cycles, and are practically negligible at slip frequency. The current distribution for a particular frequency is given by the formulas:

$$U_x = A\sqrt{2\cosh(2ax) + 2\cos(2ax)} \qquad (9)$$

$$\tan\psi_x = \tanh(ax) + \tan(ax) \qquad (10)$$

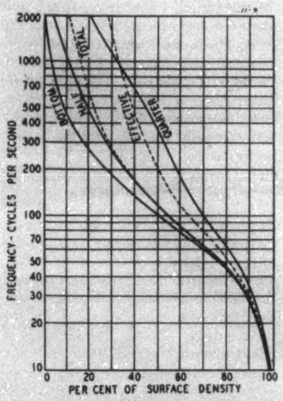

Figure 8. Current density at different levels in per cent of surface density

where U_x is the cyclic maximum current density at a distance, x, from the bottom of the bar.

$$A = \frac{U_h}{\sqrt{2\cosh(2ah) + 2\cos(2ah)}} \qquad (11)$$

where U_h = current density at the top of the bar.

$$a = \sqrt{\frac{1.003kf}{\rho m 10^7}} \qquad (12)$$

ρ = resistivity in ohms per inch cube
f = frequency in cycles per second
ψ_x = phase angle of current at point x with respect to that at the bottom of the bar

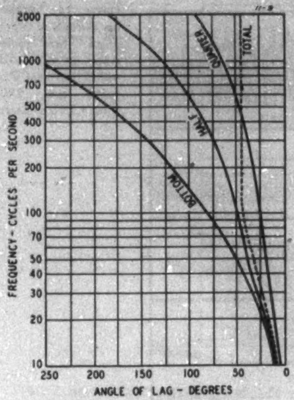

Figure 9. Angle of lag at different levels referred to current at top of bar

In accordance with these formulas, the curves of figure 8 are computed to show the current density at the quarter, middle, and bottom of the bar used, in per cent of the value at the top, for the range of frequencies encountered. Figure 9 shows the angle of lag of the current at the same levels with respect to that at the top of the bar. For the 360-cycle band-frequency component discussed previously, for example, the distribution and phase, taken from these curves, are shown in table I. The current density falls off rapidly with the depth, also the phase shift is large, therefore the average density is only 26 per cent of the density at the surface. The formula for computing this average is:

$$U_{av} = \frac{A}{ah}\sqrt{\cosh(2ah) - \cos(2ah)} \qquad (13)$$

and the phase angle with respect to the current at the bottom of the bar is ψ_{av}, where:

$$\tan\psi_{av} = \frac{\tan(ah) - \tanh(ah)}{\tan(ah) + \tanh(ah)} \qquad (14)$$

It should be noted that the formula is developed in terms of "amperes per bar", that is, the density in amperes per square inch multiplied by the bar area; so the average current density is numerically equal to the total bar current. The formula for effective or rms density, in the same units, is:

$$U_{eff} = A\sqrt{\frac{\sinh(2ah) + \sin(2ah)}{ah}} \qquad (15)$$

These formulas are developed on the basis of sinusoidal wave shape. The band-frequency wave is very nearly triangular so the current density drops a little faster than for a sine wave of the same frequency, due to the higher frequency components of the wave. The wave shape also improves, and becomes nearly sinusoidal near the bottom of the bar. Nevertheless, the curves of figure 8 give a reasonably correct picture of current distribution, especially if a small correction is made for wave shape, and they agree closely with the results obtained by measurements on oscillograms.

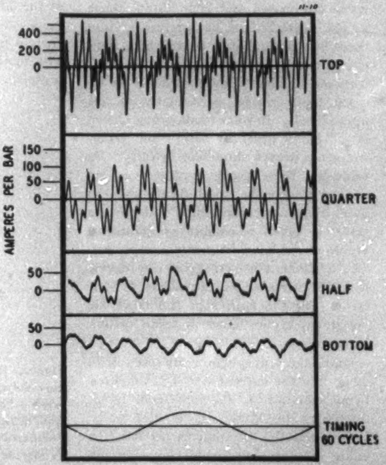

Figure 10. Rotor-bar currents of three-phase four-pole motor at synchronous speed, 1,800 rpm, 220 volts, 3.31 amperes

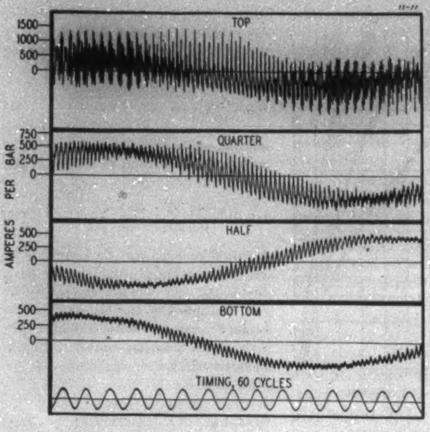

Figure 11. Rotor-bar currents of three-phase four-pole motor at full load with 5 per cent slip 1,710 rpm, 220 volts, 13.15 amperes

Rotor Current Oscillograms

Figure 10 shows rotor current oscillograms for the three-phase motor running at synchronous speed, 1,800 rpm, together with a 60-cycle timing wave. The curves at the different levels in the bar were not actually taken simultaneously, since only one amplifier was used. By means of a stroboscope, the curves were all taken at the same point in the band-frequency cycle; then the waves were lined up on the sheet by means of their separate timing waves, of which only one is shown. By noting the current scales, it is seen how the band-frequency wave diminishes and changes its shape as the depth in the bar increases; also there is a gradual shift to the right, indicating a greater angle of lag. These effects are hidden by the prominent 1,440-cycle tooth ripple, particularly at the upper levels. In some oscillograms shown later, for a different setup, the band-frequency waves show more clearly. In table II, these oscillograph curves are analyzed into the tooth-frequency and band-frequency components. Also in the table is given a column of computed values of the band-frequency component. The agreement is very good considering that the wave shape introduces a little error. All the figures in the table are maximum cyclic values in terms of amperes per bar.

On this test, at synchronous speed, with 220 volts, the current was 3.31 amperes. From equation 4, the maximum band-frequency amplitude, R_b, is 5.24 units. To convert this to amperes per bar, the value of stator maximum amperes per coil side is determined and this is multiplied by the ratio of stator to rotor slots. By equation 3, this is equal to 6.05 units. So:

$$6.05 \text{ units} = \frac{3.31\sqrt{2}}{2} \times 22 \times \frac{48}{45} \text{ amperes}$$

or

$$1 \text{ unit} = 9.09 \text{ amperes per bar} \qquad (16)$$

and

$$R_b = 5.24 \times 9.09 = 47.5 \text{ amperes per bar} \qquad (17)$$

This value appears in table II for "total" current (or average current density). The computed values of current density at the different levels and the "effective" value are figured from this value of total current using the percentages in table I. The tooth-frequency component is more prominent in the top layer, but falls off rapidly. Its effective value is rather hard to determine, but will undoubtedly be lower than that of the band-frequency component in this case.

In figure 11, are shown oscillograms taken under load with readings as follows:

Input—220 volts, 13.15 amperes, 4,570 watts
Output—5.03 horsepower, 1,710 rpm

No attempt has been made to line up this series of oscillograms with the single 60-cycle timing wave shown, and this is included merely to give the time scale which is the same for all curves. A little over one-half cycle of the three-cycle slip-frequency wave is shown at all four levels. As before, the tooth-frequency wave masks the band-frequency wave at the top of the bar. But at the quarter level, and especially at the middle of the bar, the tooth ripple is small and the band-frequency wave is quite prominent; also the similarity between these curves and the computed wave of figure 4 is very marked. The cyclic maximum values of the various components, scaled from these oscillograms, are given in table III.

To check the computation of band-frequency components, from equations 16 and 17 a stator current of 3.31 amperes gives $R_b = 47.5$ amperes per bar. Since this component varies directly with stator current, the value for the "total" under load is:

$$R_b = 47.5 \times \frac{13.15}{3.31} = 189 \text{ amperes} \qquad (18)$$

The maximum values at various levels are computed from this using the percentages of figure 8 for a frequency of 345 cycles. The computed magnitude and distribution of band-frequency current agree very well with the values measured on the oscillograms. Also the effective value of this component is of the same order of magnitude as the slip-frequency component.

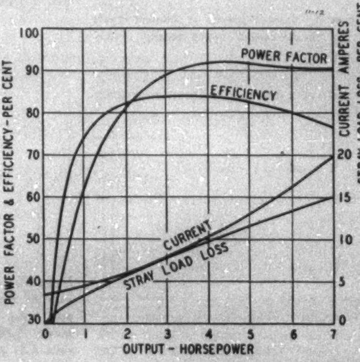

Figure 12. Load-test curves of squirrel-cage motor rated 5 horsepower, 1,800 rpm, three-phase 220 volts. Temperature 55 degrees centigrade

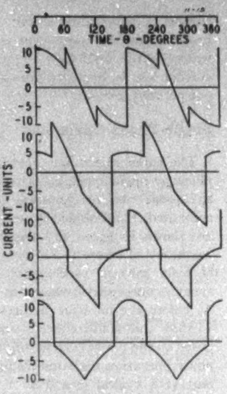

Figure 13. Rotor-bar current at zero slip for single-phase winding with 2/3 pitch

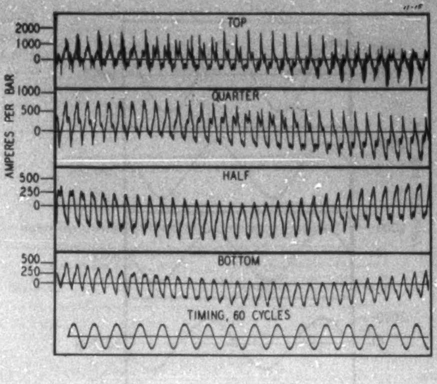

Figure 15. Rotor-bar currents of single-phase four-pole motor with 3.6 per cent slip

The slip frequency is only three cycles, so the value of this component at various levels should be the same as the total. This fact is clearly shown in the oscillograms, although the magnitude does not check so well. The slip-frequency component of rotor current is in space phase with the total magnetomotive-force wave, and is computed from the power component of stator current. The maximum value of this is approximately:

$$\sqrt{2}\times\sqrt{13.15^2-3.31^2}=12.72\sqrt{2}$$
$$=18 \text{ amperes} \qquad (19)$$

The corresponding rotor-bar current, by equations 1 and 2, is:

$$R_s=\frac{3\sqrt{3}}{\pi}\times\frac{18\times 22}{2}\times\frac{48}{45}=350 \text{ amperes} \qquad (20)$$

As a further check upon the magnitude of the slip-frequency component, the slip-frequency loss was computed, also the total resistance per bar. The stator resistance between terminals was 1.139 ohms at 55 degrees centigrade, so the stator copper loss was 295 watts. The fixed loss was 220 watts. Therefore:

$$\text{Rotor input}=4{,}570-295-220=4{,}055 \text{ watts}$$

$$\text{Slip-frequency loss}=\frac{90\times 4{,}055}{1{,}800}=203 \text{ watts} \qquad (21)$$

The total rotor resistance per bar at 55 degrees centigrade is:

$$r=69\times 10^{-6} \text{ ohms} \qquad (22)$$

The amplitude of the slip-frequency component of current, then, computed from the value of loss shown in equation 21 is:

$$R_s=2\times\frac{203\times 10^6}{45\times 69}=359 \text{ amperes} \qquad (23)$$

and this result checks very closely with the value shown in equation 20, but is 13 per cent less than the value scaled from the oscillograms, as shown in table III. This may indicate an error of approximately 15 per cent in the computed scale of the oscillograms, in spite of the care taken to obtain an accurate calibration.

Effect Upon Motor Efficiency, Stray Loss

The total loss determined by the conventional method of losses is:

$$295+220+203=718 \text{ watts} \qquad (24)$$

But the total loss determined by a careful dynamometer test is:

$$4{,}570-3{,}750=820 \text{ watts} \qquad (25)$$

The stray loss at this load therefore is:

$$820-718=102 \text{ watts} \qquad (26)$$

which is over two per cent. Performance curves for this motor are shown in figure 12, including a curve of stray loss in per cent of input. The shape of this curve is in substantial agreement with other published material.

Several writers on the subject of stray-load loss in induction motors agree that the loss is largely due to high-frequency flux pulsations in the stator teeth, plus a relatively small tooth-frequency eddy-

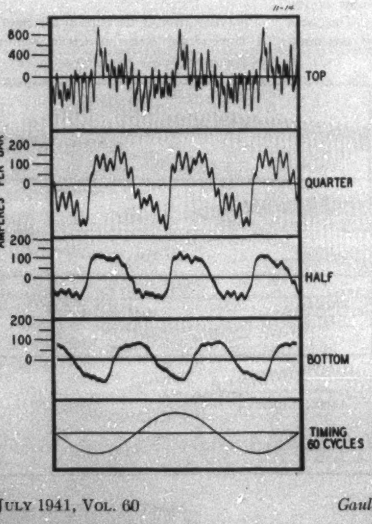

Figure 14. Rotor-bar currents of single-phase four-pole motor at synchronous speed, 1,800 rpm, 220 volts, 5.13 amperes

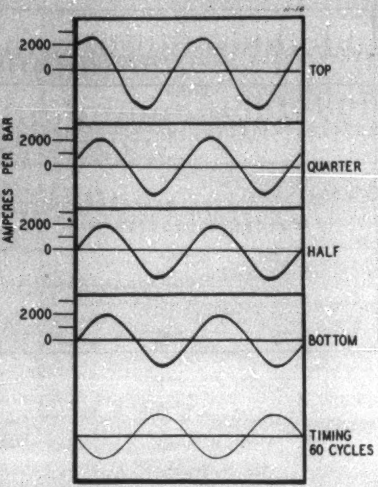

Figure 16. Rotor-bar currents of single-phase four-pole motor with rotor locked, 220 volts, 47.6 amperes

current loss in the rotor bars. In the tests reported herewith, the tooth-frequency eddy-current loss was found to increase with load. The lower frequency band-frequency eddy-current loss was found to be of even greater importance. At full load, this component (see figures 4 and 11) has a frequency of 345 cycles, and its amplitude rises and falls with a beat frequency of $2fs=6$ per second. The rms value of this component per bar is computed to be:

$$I_b=\frac{U_{eff}}{2.38}=\frac{304}{2.38}=128 \text{ amperes} \quad (27)$$

Table I. Current Distribution and Phase for 360-Cycle Component

Level	Current Density (Per Cent)	Angle of Lag (Degrees)
Top	100	0
Quarter	50.1	39.1
Middle	24.2	79.3
Bottom	13.4	155.0
Average (total)	26.0	45.3
Effective	42.6	

Table II. Rotor-Bar Current Distribution in Figure 10

Level	Tooth-Frequency 1,440 Cycles	Band-Frequency = 360	
		Measured	Computed
Top	350	187	183
Quarter	33	55	91
Middle	10	30.5	44
Bottom	3	20	24.5
Total			47.5
Effective			78

Due to the distribution of this component in the end rings, the loss there is very small, and the resistance per bar may be taken as 34×10^{-6} ohms instead of the value given in equation 22. The rotor loss produced by this component therefore is:

$$W_b=45\times(128)^2\times34\times10^{-6}=25 \text{ watts} \quad (28)$$

This is about a quarter of the stray load loss.

In table III is shown a value of 225 amperes for the peak rms value of the tooth-frequency component. This was computed from the curve of measured current density values shown. The loss produced by this component is about 15 watts, so the total high-frequency rotor current loss is 40 watts, which is nearly one per cent of input.

Single-Phase Operation

The same motor was operated single phase by opening one line after it was up to speed. At synchronous speed the computed band-frequency wave shapes are shown in figure 13; the underlying frequency of course is 120 cycles. A set of rotor current oscillograms taken at synchronous speed is shown in figure 14. As before the full-load pictures of figure 15 show the band-frequency wave superposed on the power current, but for single-phase operation the band-frequency current (118 cycles) is a little larger than the slip-frequency current (2.2 cycles).

A locked test was also run on this motor connected single phase, and oscillograms are shown in figure 16. At standstill, all frequency components reduce to 60 cycles, giving a sine wave of current in the rotor bars. The shift in phase is shown by the drift to the right at the lower levels. Values of current density and phase angle with respect to the current at the top are shown in table IV; measured values taken from the film are compared with computed values. The check on phase angle is quite close. There is a satisfactory qualitative check on amplitude also, but the measured values seem to be too high, as noted previously in table III.

Consequent-Pole Winding

By reconnecting the coils of this motor, a three-phase consequent-pole winding

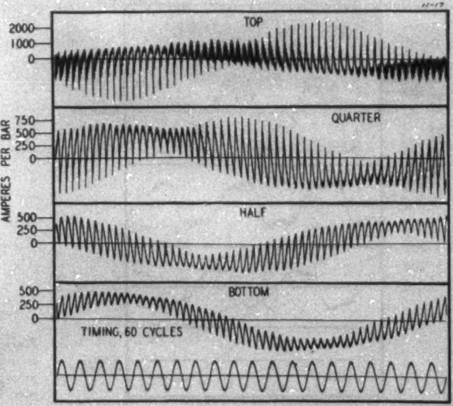

Figure 17. Rotor-bar currents of three-phase four-pole motor, consequent-pole connection 1,710 rpm, 190 volts, 14.4 amperes

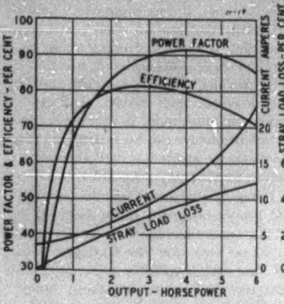

Figure 18. Load-test curves of three-phase four-pole motor, consequent-pole connection, 190 volts. Temperature 55 degrees centigrade

Table III. Rotor-Bar Current Distribution in Figure 11

Level	Tooth-Frequency 1,368 Cycles	Band-Frequency = 345		Slip-Frequency = 3	
		Measured	Computed	Measured	Computed
Top	930	873	707	417	350
Quarter	61	295	357	418	350
Middle	25	137	177	405	350
Bottom	19	86	102	415	350
Total			189	414	350
Effective	225	306	304	414	350

Table IV. Rotor-Bar Current Distribution and Phase in Figure 16

Level	Amperes Per Bar		Phase Angle—Degrees	
	Measured	Computed	Measured	Computed
Top	2,628	2,168	0	0
Quarter	2,153	1,756	28.1	20.4
Middle	1,943	1,578	39.7	40.7
Bottom	1,870	1,530	57.3	57.9
Total		1,578		35
Effective		1,675		

was produced with eight coils per phase group for four-pole speed, that is using a phase band width of 120 degrees instead of 60 degrees and only two polar groups per phase. Upon analysis this gave a band-frequency wave similar to that shown in figure 2, but of half its frequency (180 cycles at synchronous speed) and approximately twice its amplitude. This is noted in the load oscillograms of figure 17. The load-test curves of figure 18 show decreased efficiency, particularly at heavier loads, and a considerable increase in stray-load loss, due almost entirely to the larger band-frequency current in the rotor bars. The test was run on 190 volts to get the same flux density in the motor.

Eight-Pole Operation

The stator winding was reconnected for three-phase eight-pole operation. Using the same coils as before, the coil pitch is four-thirds, which produces a band-frequency wave like that in figure 2; so the load oscillograms shown in figure 19 are quite similar to those in figure 11 except that the tooth ripple is almost entirely gone.

Conclusions

High-frequency components in the rotor-bar current may contribute a considerable portion of the stray-load loss in a squirrel-cage motor. By proper slot design, the tooth-frequency component may be controlled. The band-frequency component, however, depends upon the winding, and may be decreased by co-ordinating pitch and number of phases. If a pitch of five-sixths instead of two-thirds had been used with the three-phase four-pole winding, the band-frequency wave would be as shown in figure 20 instead of that shown in figure 2. An analysis of this wave shows a band-frequency current loss of about one-third as much.

References

1. The Induction Motor (a book), **Benj. F. Bailey.** McGraw-Hill Book Co., pages 102–4, 195–8.

2. Tooth-Frequency Eddy-Current Loss, **Paul Narbutovskih.** AIEE Transactions, volume 56, 1937 (February section), page 253.

3. Measurement of Stray Load Loss in Polyphase Induction Motors, **C. J. Koch.** AIEE Transactions, volume 51, 1932, page 756.

4. Zusätzliche Verluste in Kleinen Drehstrommotoren, **Rogowski and Vieweg.** *Archiv für Elektrotechnik*, volume 14, 1925, page 574.

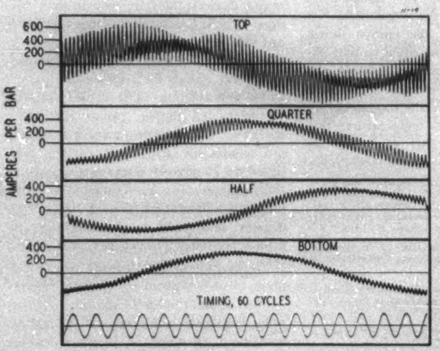

Figure 19. Rotor-bar currents of three-phase eight-pole motor under load with 3.55 per cent slip, 850 rpm, 224 volts, 6.1 amperes

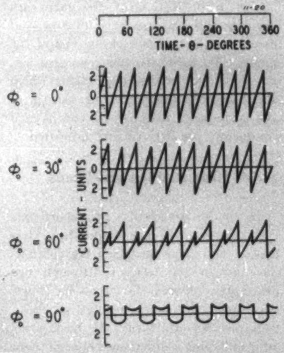

Figure 20. Rotor-bar current at zero slip for three-phase winding with 5/6 pitch

Classification and Co-ordination of Short-Time and Intermittent Ratings and Applications

R. E. HELLMUND
FELLOW AIEE

Synopsis: The paper makes a comparison of permissible temperature rises for short-time and intermittent ratings and permissible rises for continuous ratings. It suggests methods for the application of short-time ratings to various typical classes of intermittent load conditions and presents for each of these classes ways and means for properly co-ordinating the ratings of the major devices (motors and generators) and the ratings of the auxiliary devices and conductors.

Introduction

THE present commercial standards for electrical apparatus and conductors usually provide continuous ratings and short-time ratings for periods of 5, 15, 30, 60, and 120 minutes. The methods used for selecting ratings for various classes of intermittent or varying loads from these available ratings are rather indefinite and at times result in applications which, although generally satisfactory, are often not the most economical. It is shown in this paper that by following essentially the present practices of rating and application but also properly co-ordinating them and supplementing them with additional data, a selection of the most suitable short-time ratings can be made. A very simple procedure applying to the proper co-ordination of the main apparatus and the auxiliary devices and conductors is outlined at the end of the paper. To facilitate this co-ordination, standard temperature-rise values which seem suitable for short-time ratings of the main and auxiliary apparatus and conductors are derived and tabulated.

Short-Time Ratings and Loads

In many standards the short-time ratings are based on temperature rises higher than those on which the continuous ratings are based. Although the differences between the values for short-time and continuous ratings generally were selected arbitrarily, the basic principle of having a difference between them is sound. With short-time loads the maximum temperatures obtained can last for only a relatively small part of the total time. Since the deterioration of insulation depends not only upon temperature but also upon the length of time the higher temperatures prevail, satisfactory life can be obtained if relatively high temperatures prevail for only a small part of the time. Tests have indicated that the life of insulation is halved for each eight- to ten-degree-centigrade increase in temperature, particularly for insulation exposed to oxygen in the air. It is proposed to use this rule here and to calculate for some typical short-time ratings the permissible maximum temperature rises which result in insulation life about equal to that obtained with temperatures specified as standard for continuous operation. (Methods suggested by V. M. Montsinger for transformers[1,2] and also outlined by P. L. Alger and T. C. Johnson[6] will be used for this purpose. These methods and the eight- to ten-degree-centigrade value may not apply to applications where oxygen is excluded, as in oil-insulated transformers with inert air protection or hydrogen-cooled machines.)

Figure 1 shows a number of typical time-temperature curves for short-time ratings of a small induction motor. These curves have been calculated under the assumption that each eight-degree-centigrade increase in temperature halves the life of the insulation. On this basis, each of the curves shown will give insulation life equal to that obtained with a uniform rise of 60 degrees centigrade (by resistance) over the same period of time. Since the cooling curves approach the zero temperature line asymptotically (reaching zero after an infinite time), it was necessary to limit arbitrarily the time for each curve, and the point at which the temperature rise drops to ten degrees centigrade was taken. This provides a margin of safety and results in conservative curves. (All calculations and discussions in this paper deal with temperature rise by resistance; the thermometer method is not very satisfactory for the study proposed.)

It is seen from figure 1 that the permissible temperature rises range from 80 to 93 degrees centigrade, or an excess of 33 to 55 per cent over the 60-degree temperature rise permissible for continuous operation.[10] The use of these higher permissible temperature rises is of importance economically because, according to curve 1, a five-minute rating with a 60-degree limit can be raised to an 11-minute rating for the higher temperature. Similarly, curve 2 shows an increase from a 15-minute to a 46-minute rating; curves 3 and 4 show increases from 30 to 80 minutes and from 38 to 116 minutes respectively. Curve 4 has been chosen as a limiting case with a rate of temperature rise of about one-half degree a minute at 60 degrees centigrade; for major devices such as generators, motors, etc., it is customary to apply continuous ratings rather than short-time ratings if the rate of temperature rise is lower.

The curves of figure 1 are typical of most apparatus, the principal difference being that with larger devices the time values increase. This, however, does not affect the values of the maximum permissible temperatures, which are influenced essentially by the shapes of the curves and which vary to some extent with different types and sizes. Allowing for such variation within usual limits, it seems safe to select for standardization purposes temperature rises 25 to 40 per cent higher for short-time ratings of major devices than for continuous ratings. Table I, column 2, suggests standard values of temperature rise for short-time ratings of major devices corresponding to the values for continuous ratings in the first column. The values are conservative and higher values might be justified. (After testing class *B* insulated d-c motors for short-time ratings now based on 75 degrees temperature rise by thermometer, F. A. Compton recommends a temperature rise of 105 to 120 degrees centigrade by resistance,[8] the table gives 100 or 110 degrees centigrade.)

The considerations thus far can be applied to any type of apparatus or conductors within the limits stated. (Naturally modifications in some of the values mentioned must be made when values

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1. For all numbered references, see list at end of paper.

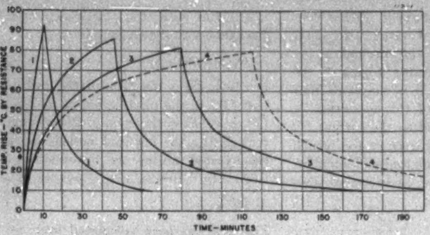


Figure 1. Permissible temperature rises for various short-time loads

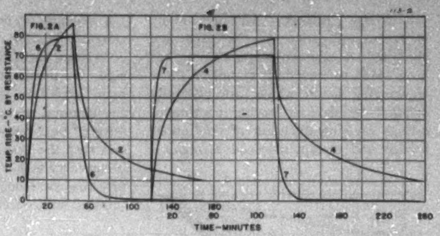


Figure 2. Comparison of permissible temperature rises for short-time loads of apparatus with different time constants

other than 60 degrees permissible temperature rise for continuous load apply.) However, if the main devices are connected in series with various auxiliary devices or conductors having much smaller time constants than the main device, the conclusions derived so far must be modified for the auxiliary devices or conductors. This is demonstrated in figure 2*a*, curve 2 of which corresponds to curve 2 of figure 1 and is considered representative of main devices such as generators, motors, etc. Any smaller device or conductor connected in series with the main device and having a smaller time constant will show more rapid initial temperature rise, and if loaded as long as the main device, will run at temperatures near the maximum for longer periods than the main device. This in turn means that to get equally satisfactory insulation life the maximum temperature-rise values must be kept lower. Curve 6, figure 2*a*, applies to an auxiliary device with a small time constant giving the same insulation life (with the same class of insulation) assumed for the main device. It is seen that 80 degrees centigrade is the maximum permissible value for the auxiliary device instead of 86 degrees centigrade indicated for the main device. Curve 4 in figure 2*b* corresponds to curve 4 in figure 1 and is another typical curve for the main device. Curve 7 has been calculated for an auxiliary device with a smaller time constant to give the same insulation life. Again a smaller maximum temperature rise (71 degrees centigrade) must be chosen for the auxiliary device than the 80 degrees for the main device. Here the auxiliary device reaches constant temperature during the load period.

Table I. Suggested Relation of Temperature Rises (by Resistance) for Short-Time Ratings to Rises for Continuous Ratings

Continuous Ratings (Deg C)	Short-Time Ratings (Deg C) Main Apparatus	 Small Auxiliary Apparatus or Conductors*
20	25	25
25	30 or 35	30
30	40	35
40	50 or 55	45 or 50
50	60 or 70	55 or 60
60 (class *A* insulation)[10]	80	70
80 (class *B* insulation)[10]	100 or 110	90 or 100
100	125	110 or 125

The values in this table are chosen from the preferred values suggested in AIEE Standard number 1.[10] In general they should give satisfactory life of insulation if the rated load is continually reapplied after the apparatus temperature has dropped to about ambient temperature. Practical cases of application of this kind are rare. As a rule, considerable time elapses between the applications of loads, and whenever this is the case the application of apparatus can be made so as to reach even higher temperature-rise values than those suggested for standard rating purposes in the table. Although the value of 80 degrees centigrade suggested in the table to correspond to 60 degrees rise for continuous load does not take full advantage of the permissible rises indicated in figure 1, the increases in time over the 60-degree values are still appreciable, ranging from 65 to 250 per cent.

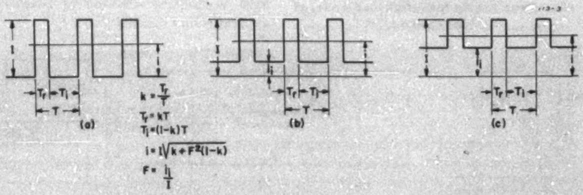
*These values also may be used if the short-time operation of small auxiliary apparatus or conductors results in the same temperature rise obtained with continuous operation at the same load.

In view of the conditions just described, it seems advisable to use lower maximum values of temperature rise for the rating of small auxiliary devices and conductors intended for short-time service. This practice need not be altered by the fact that, as in figure 2*b*, the short-time ratings of the auxiliary devices or conductors give the same temperatures as obtained with continuous operation at the same load. The values in column 3 of table I are suggested for the standardization of small auxiliary devices and conductors used in conjunction with main devices in short-time service. In general, where two values are given, the lower value of one column should be used with the lower value of the other column.

Intermittent and Varying Loads

Typical intermittent and varying load cycles are illustrated in figure 3. During the time (T_r) the apparatus is heavily loaded and carries a current (I), while for the remaining period (T_l) it may carry no load, as in the case of a welding generator[9] (figure 3*a*). Figure 3*b* represents such apparatus as a-c induction motors, which are loaded during the period (T_r) and not loaded during (T_l), but which have to carry a magnetizing current (i_l) during the off periods. There is little change in principle if the motor, instead of carrying a magnetizing current, carries a light load (i_l) as indicated in figure 3*c*. This might occur with an industrial mill which is driven light during (T_l) and loaded during (T_r). In all the applications cited, the rms current (i) is, of course, less than the current (I) during the heavy-load period. (In this discussion, service conditions as illustrated in figures 3 and 10 are of the type which as a rule can be handled by devices having

Figure 3. Typical intermittent and varying (periodic) load cycles



"short-time" ratings. Although many of the considerations in the paper are applicable to other cases, no attempt has been made to include a complete study of varying loads which are generally handled by devices having continuous ratings or a combination of continuous ratings and certain overload ratings.)

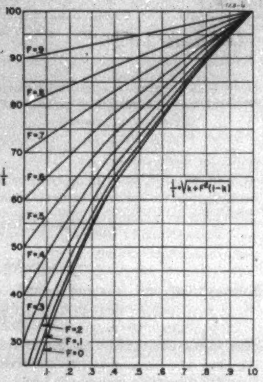


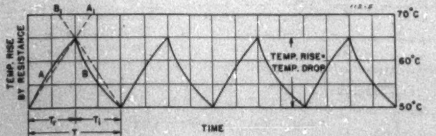
Figure 4. Ratio of rms current to load current for intermittent and varying load cycles

Very often the current (I) during the load period is a significant value in machine design for considerations other than heating. A commutator machine, such as a welding generator[3] for instance, must be able to commutate the current (I) satisfactorily during long periods of machine operation. In an induction motor the value (I) is equivalent to a torque required for driving the load and it is necessary that the pull-out and starting torques of the motor have certain definite relations to the running torque if satisfactory operation and a sufficient margin of safety are to be obtained. Again, the value (I) may be of significance in the regulation of the machine or apparatus. In these examples, current (i) is of importance only in the heating of the machine, while current (I) has considerable bearing on various other characteristics, including the mechanical design. It should therefore appear in the rating structure and it is suggested that the nominal rating of the apparatus be selected to correspond to the current (I) and that the heating current (i_h) which it can carry within safe limits of temperature rise continuously, be indicated by a suitable service factor

$$S_1=\frac{i_h}{I} \tag{1}$$

This has been previously suggested by others (Alger,[4,6] Johnson,[5,6] and Hildebrand[7]) and seems to be a simple method for supplying fairly complete information about heating as well as other characteristics of practical importance. It has also been suggested that no reference whatsoever be made to temperature conditions in connection with the nominal rating corresponding to current (I).[6] However, since practically all nominal ratings of electrical apparatus are now tied in with temperature considerations in some way or other, it seems desirable to retain the temperature rating as the primary basis for all nominal ratings. This can very readily be done because current (I), although it cannot be carried continuously with many devices, can always be carried for at least limited periods and be tied in with one of the standard short-time (5-, 15-, 30-, 60-, or 120-minute) ratings. In this way the length of time the machine can carry the current (I), starting with the machine cold, is indicated. This additional information makes it possible to select for all applications the proper wiring and devices connected in series with the main apparatus, which is very essential for an economical and successful combination of all parts of the circuit. (If the conditions are such that any of the factors, such as commutation, torque, regulation, etc., call for a main device which can carry the current (I) continuously, it can be given a continuous rating (I) and the methods described here will then be of no practical importance with reference to the main apparatus. However, they may be of assistance in the most economical selection of auxiliary apparatus and conductors.)

Figure 5. Temperature variation with intermittent and varying (periodic) load cycles of long duration



The rms current (i), dependent upon the load cycles shown in figure 3, is

$$i=I\sqrt{k+F^2(1-k)} \tag{2}$$

in which (k) is the ratio of the load period (T_r) to the total period of the cycle (T), and (F) is the ratio of the rms current during the low-load periods to the rms current during the heavy-load periods. In figure 3a, with $F=0$, we have

$$i=I\sqrt{k} \tag{3}$$

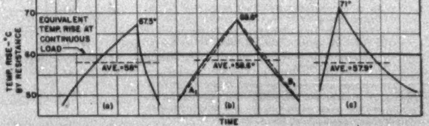
Figure 4 has been calculated from formula 2 and gives the rms current for any value of k and F.

With the regularly repeating (periodic) load cycles shown in figure 3, it is evident that the drop in temperature during each light- or no-load period must at least equal the rise in temperature during each heavy-load period in order to obtain a stable cyclic temperature condition in which permissible temperature values are not exceeded. (See figure 5.) It follows directly that with a heavy-load period (T_r), a time (T_l) must be allowed for the light- or no-load period, resulting in a total time (T) for each cycle. With the time-temperature curves A and B fixed by the loads and apparatus characteristics, the maximum ratio of $T_r/T=k$ is definitely limited. It is dependent upon the ratio (R) of the average rate of drop (B_1) to the average rate of rise (A_1) and follows from

$$k=\frac{R}{1+R} \tag{4}$$

These relations are basic to all regularly repeating (periodic) intermittent or varying loads. A more complete outline of this method of analysis by time-temperature curves is given in a previous paper.[3] In that paper it was generally assumed that the peak temperature rises reached with short-time ratings and during intermittent loads should be the same as the permissible temperature rises for continuous loads. This, however, does not take advantage of the fact that the latter temperature may be safely exceeded for limited periods. An attempt will therefore be made in the following to take advantage of this possibility for

Figure 6. Permissible temperature rises for typical periodic loads



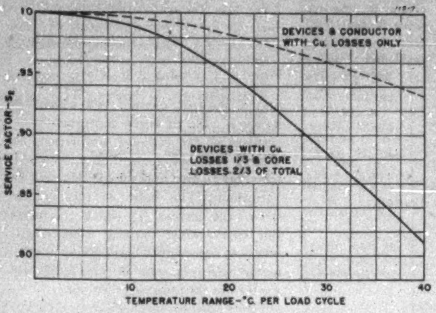


Figure 7. Service factor S_2 takes into account the amplitude of the cyclic temperature variation

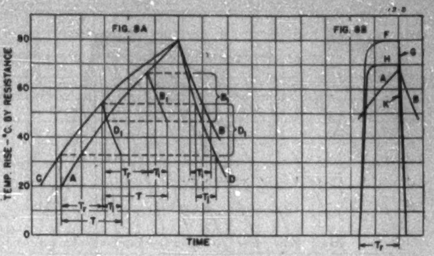


Figure 8. Comparative analysis of temperature conditions for apparatus of different time constants during periodic load cycles

intermittent load applications as in figure 5.

Class 1A Intermittent Service

For this class of service it is assumed that the ventilation conditions are not appreciably changed between the heavy-load and the light- or no-load periods. This applies to most stationary devices or conductors unless they are equipped with adjustable forced ventilation; it also applies in general to constant-speed machinery while running. It is further assumed that the load periods are rather short and the variation in temperature rise will be so small that it can be neglected.

Since with these assumptions the temperature of the machine is about constant, the value (i) should be such as to give a safe temperature rise for continuous operation. For this type of service it is therefore necessary merely to select apparatus having a short-time nominal rating (I) with a service factor at least equal to

$$S_1 = \frac{i}{I} = \frac{i_h}{I} \qquad (5)$$

The method outlined under class 1*A* should be satisfactory if its use is confined to applications where the heavy-load period is limited to ten per cent of the time (T_s) on which the nominal rating is based; that is, in the case of a 30-minute rating, for instance, the load period (T_r) does not exceed three minutes. Then the total temperature variation during the cycle will as a rule be less than ten per cent of the temperature rise for continuous operation and the excess temperature will be less than five per cent.

The wiring and auxiliary devices connected in series with the main machine or apparatus usually have smaller time constants. Therefore, their temperature variations in percentage of the average temperature rise during the time periods just indicated may be larger. This makes it advisable to select them for a continuous rating five to ten per cent larger than the rms value of the load.

Class 2A Intermittent Service

For class 2*A* applications, it is assumed that the cooling conditions are essentially the same during the entire load cycle but that the time for each cycle is increased to the point where the variations in temperature can no longer be neglected. Figure 6 shows three typical time-temperature curves, each for a single cycle with a total temperature change of 20 degrees centigrade. These curves also have been calculated to give a life of insulation equivalent to that obtained with continuous operation at 60 degrees centigrade rise. Figure 6*b* has a curvature in the rising curve about equal to that of the dropping curve. This means that the results are the same as they would be with the triangular time-temperature curve A_1B_1. The maximum permissible temperature rise is 68.6 degrees centigrade. With curves 6*a* having more curvature in the rising curve and thus giving longer periods for the higher temperatures, the maximum must be kept lower, that is to about 67 degrees centigrade. The opposite condition shown in 6*c* led to a higher maximum of 72 degrees centigrade. The average of the temperature rises for these cycles varies within the narrow range of 57.9 to 58.6, that is, 1.4 to 2.1 degrees centigrade, or 2.3 to 3.5 per cent below the temperature-rise value for continuous load. Thus, in order to obtain satisfactory operation the losses must be lowered 2.3 to 3.5 per cent. In devices having only copper losses, this means a reduction of only 1.1 to 1.7 per cent in rms current. In machines designed for intermittent service the core losses are often relatively high, and therefore the copper losses may be as low as one-third of the combined losses. The same reduction in total losses may therefore require a reduction of about 7 to 10.5 per cent in the copper losses, which means 3.5 to 5.5 per cent reduction in the rms current. This is too liberal because the influence of the core losses upon the copper temperature is usually much less than that of the copper losses. It would thus seem that an allowance of five per cent, or a service factor $S_2 = 0.95$, to cover the difference between case 1*A* and 2*A* would be quite safe for a variation of 20 degrees centigrade. The curve for S_2 in figure 7 has been based on this line of reasoning and covers the range of temperature variations usually found in practice.

Determination of the exact temperature variation would require a complete time-temperature curve analysis. It can be safely assumed, however, that it usually does not exceed the value $r(T_r) \div (T_s)$ in which T_r = the time of heavy load per cycle, T_s = the time used as a basis for the short-time rating, and r = the temperature rise upon which this rating is based. It is thus possible to calculate safe maximum limits for the temperature variation and then find the value S_2 for this cycle from figure 7. The service factor for class 2*A* applications is then

$$S = S_1S_2 \qquad (6)$$

The application of smaller auxiliary devices and conductors with main devices in intermittent service of this class cannot be made safely by the method outlined under class 1*A*. Depending upon their time constants, the temperature

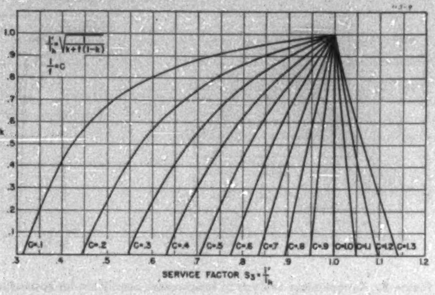


Figure 9. Service factor S_3 takes into account variations in ventilation during the load cycle

rises of the auxiliary devices and conductors may vary over wide ranges and it is hardly feasible to set any safe practical limits for the variation.

Two distinctly different typical conditions are shown in figure 8. In figure 8*a* we have heating curve *A* and cooling curve *B* for a main device. An auxiliary device with smaller time constant and designed for the same short-time temperature rise will have a flatter curve (*C*) for the upper part of the heating curve, and a steeper curve (*D*) for the cooling curve. With a load time (T_r) and idle time (T_i), stable repeating conditions are obtained with a maximum temperature rise of 68 degrees centigrade for the main device and 54 degrees centigrade for the auxiliary device, as found by the methods described in a previous paper.[3] It follows that with conditions shown in figure 8*a*, the auxiliary device will have lower temperature rises than the main device if based on the same temperature rise for the same short-time rating.[9] Entirely different results are obtained if the time constant of the auxiliary device or conductor is so small that maximum temperature is reached during the time (T_r), as shown by curve *F*, figure 8*b*. It is at once evident that with curves *F* and *G* of the auxiliary device or conductor, insulation life will be shorter than with curves *A* and *B* of the main device. This undesirable condition can be largely eliminated by lowering the rating temperature of the auxiliary device or conductor as suggested under short-time ratings in column 3, table I and illustrated by curves *H* and *K* in figure 8*b*. It can therefore be concluded that for intermittent service of this 2*A* class, the auxiliary devices and conductors having the same short-time ratings as the main device can be selected if ratings are based on

(*a*). The same temperature rises (column 2, table I) as the main device, if the auxiliary device does not reach its maximum temperature during the load period (T_r);

(*b*). A somewhat lower temperature rise (column 3, table I) if the auxiliary device reaches its maximum temperature during the load period (T_r).

In general, conditions under (*a*) will apply to all except very small auxiliary devices, while conditions under (*b*) may be found to apply frequently to conductors. If suitable temperature rises, as indicated here, are used in the standardization of devices and conductors for intermittent service, their application becomes exceedingly simple and consists merely in combining apparatus and conductors having the same short-time rating with reference to current and time.

Class 1B Intermittent Service

In some classes of application, ventilation conditions are considerably different during the periods of heavy and light load or no load. This is true in any case where artificial cooling is automatically stopped, except during the heavy-load period. A more common application is represented by motors carrying a current (*I*) while loaded and running and which stand still during idle periods. In these applications the rate of cooling is usually reduced because of the reduced ventilation, although with motors standing still this is at least partially compensated by the absence of those losses (essentially core losses) occurring with the motor running light. In some applications, especially enclosed motors, the elimination of losses may more than counteract the reduced cooling.

If the ratio of the cooling rate for class 1*A* to that for class 1*B* during no-load periods is assumed to be (*f*), it follows that the necessary drop in temperature for each cycle can be obtained for class 1*B* if the no-load time is increased from T_i to $T_i^1=fT_i$. Consequently, we can analyze class 1*B* applications the same as class 1*A* merely by substituting the time (T_i^1) for (T_i). By replacing in formulas 2 and 3 the value *k* by $k^1=\frac{T_r}{T_r+T_i^1}$, we find a reduced current (i^1) which the motor is able to carry continuously regardless of the reduced ventilation, assuming that the total time per cycle is too short to give appreciable temperature variations. The ratio of this current (i^1) to the current (i_h) which the motor can carry while running continuously may be represented by another service factor (S_3). It can be calculated for $F=0$ from the following formula

$$S_3=\frac{i^1}{i_h}=\sqrt{\frac{T_r+T_i}{T_r+fT_i}}=\sqrt{\frac{1}{k+f(1-k)}} \tag{7}$$

Figure 9 gives this service factor for various values of *k* and *f*. With the aid of these curves the current (i_h) can be found directly from the smaller actual rms current (i^1). The total service factor for class 1*B* applications is, therefore, $S=S_1S_3$. Suitable values for (*f*) can be determined by the test method described in a previous paper,[3] taking cooling curves with the motors running and at rest. The method here outlined for class 1*B* motors should be satisfactory if the running time per cycle does not exceed ten per cent of the rating time (T_s).

The application of conductors and auxiliary devices follows the same general principle as suggested for class 1*A*, using the actual rms current as a basis. This, of course, is under the assumption that the cooling conditions of the conductors and auxiliary devices are the same for both the load and the idle period, which is generally true.

Class 2B Intermittent Service

For class 2*B* applications it is assumed that the ventilation is different for the high-load and the low-load or no-load periods, but that the time for each load cycle is so long that temperature variations become of practical importance. Thus, class 2*B* applications have the same relation to class 1*B* as class 2*A* have to class 1*A*. It is therefore necessary to introduce, in addition to the service factor (S_3), the service factor (S_2), which leads to the service factor

$$S=S_1S_2S_3 \tag{8}$$

If auxiliary devices and conductors are combined in this service with the main devices by the methods suggested for class 2*A* applications, they will as a rule have an unnecessarily large margin of safety. The reason for this is that the cooling conditions of the auxiliary devices and conductors usually remain unchanged, while those of the main device are reduced during the ideal periods. However, it is not safe to suggest any other practice which is generally applicable. In specific applications, with known conditions and additional analysis, further economies may be accomplished, but as a rule the gains possible will be of minor importance in the total cost.

General Discussion of Service Factors

S_1 indicates the ratio of the current giving a safe temperature rise with continuous operation to the current corresponding to the nominal (short-time) rating. It is governed by the design of the apparatus and the temperature rises used as a basis for short-time and continuous ratings; it must be determined by test. Minimum values for (S_1) should be known and be standardized for certain types, classes, and ranges of ratings, the same as is done with torque ratios. This is exemplified in table II.

Table II

5-minute ratings	$S_1 = 0.50$
15-minute ratings	$S_1 = 0.60$
30-minute ratings	$S_1 = 0.70$
60-minute ratings	$S_1 = 0\ 80$
120-minute ratings	$S_1 = 0.87$

S_2 makes allowance for temperature variation during long load cycles. It is governed partly by the type of application and partly by the design of the apparatus. However, since it does not vary widely, it can be shown with sufficient accuracy for most practical purposes by curves as shown in figure 7. As pointed out there, higher values than those indicated for S_2 in the full-line curve apply to cases where the copper losses essentially govern the temperature rise of the windings (dotted curve in figure 7).

S_3 makes allowance for differences in ventilation which may exist between the heavy-load and the light- or no-load periods. It is influenced by both the type of application and characteristics of the apparatus. The influence of the former is covered in formula 7 by the factor (k) determined from the service condition. The factor (f), depends upon the apparatus characteristics. It is not recommended here that (f) be made a part of the rating structure but that for various types of motors, values for (f) be established which are safe but which do not result in uneconomic applications.

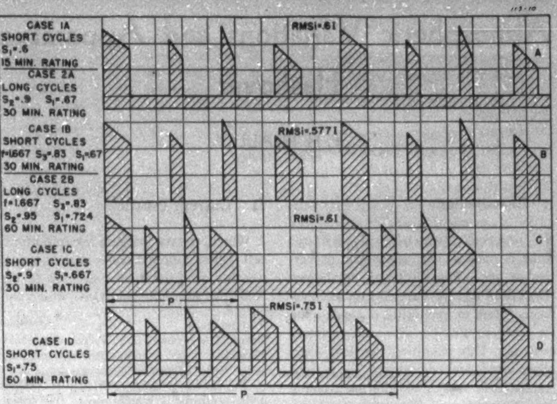


Figure 10. Some typical load cycles analyzed by the methods suggested

Selection of Short-Time Ratings for Intermittent or Varying Loads

(a). Main Apparatus

The variety of conditions met in practice makes it impossible to outline a method of application suitable under all circumstances. However, the data presented here are suitable for the majority of applications and should prove helpful in giving results quickly. Both periodic load cycles as shown in figure 3 and load cycles with irregular intermittent loads as in figure 10 can be calculated as follows.

Divide the entire time into high- and low-load or no-load periods; then calculate the rms current for all the high-load periods. The resultant value is the current (I) of our previous analysis and it is used for the selection of the current value of the nominal short-time rating (or horsepower or kilowatt ratings corresponding to this current value).

The current value (i_r) previously used is found by calculating the rms current for all the low-load periods and it permits the calculation of $F = i_r/I$. The value (k) is obtained by dividing the time of all high-load periods by the total time. The resultant actual rms current (i) can now be found from formula 2 or figure 4. From here on we have to distinguish between different classes of applications.

Class 1A: If with a load as shown in figure 3 or 10A the ventilation is essentially constant and the load periods short, find i/I. Then select from such tables as table II the nominal short-time rating for the current I which has a value S_1 at least equal to i/I.

Class 2A: If, again, with the load as shown in figure 3 or 10A the ventilation is essentially constant but the periods for some of the heavier loads are long, find S_2 from figure 7; determine i/S_2I, and select a rating having a value (S_1) which is at least equal to i/S_2I.

Class 1B: If the ventilation changes between heavy- and light- or no-load periods and the load periods are short (figure 3a or 10B), find S_3 from formula 7 or figure 9, determine i/S_3I, and select a rating at least equal to i/S_3I.

Class 2B: If the ventilation changes between heavy- and light- or no-load periods and some of the periods for the heavier loads are long, find both S_2 and S_3, determine i/S_2S_3I, and select a rating with S_1 at least equal to i/S_2S_3I.

If the load cycles are distributed rather unevenly in time as in figure 10C, it is evident that there will be some fluctuation in the temperature rise even though the individual load cycles are very short. If the periods (P) over which the load cycles are distributed are shorter than the rating time of the device, the use of a service factor of the general order of S_2 will give reasonably correct results. If, on the other hand, the period (P) is rather long as shown in figure 10D, it is safer to calculate results under the assumption that the load conditions prevailing during (P) are continuous. The loads in figures 10A to 10D have all been assumed to have the same value (I) and thus the same nominal current rating. It will be interesting to note the rms values of i/I and S_1 in figure 10 and the time values to be

Atmospheric Variations and Apparatus Flashover

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I. Introduction

IN practically all electrical apparatus use is made of the insulating properties of air. The electrical breakdown strength of air, for the electrode arrangements found in apparatus, depends upon air temperature, pressure and moisture content. This condition has long been appreciated and recognized in various standards for testing and for determining insulation ratings. However, the range and significance of the extreme conditions encountered, is not always appreciated. It is proposed, therefore, to consider the variations found in actual service and determine how well current standards and application practices cover these variations. In addition, the desirability of uniform application practices and reference ranges for different apparatus is suggested.

II. Current Practice

Wherever test margins are important, test values obtained under various actual weather conditions are corrected to the American standard conditions, which are:

Temperature	77 degrees Fahrenheit
Barometric pressure	29.92 inches of mercury
Humidity	0.6085 inches of mercury

According to most apparatus standards, equipment which meets the specified voltage tests, corrected to these conditions, may be used at any of the temperature and humidity conditions encountered in nature and at any barometric pressure encountered at altitudes up to 3,300 feet. The corrections for different air conditions are in many cases quite large and the actual flashover voltage of a piece of equipment in service may differ by as much as 25 per cent from the value for standard test conditions. The internal insulation of apparatus is not affected by the changing dielectric strength of the ambient air and in some cases the internal insulation may not be applied effectively and economically because of this situation. On the other hand, the internal insulation may be overstressed when conditions of unusually high external flashover prevail. Thus the variations in air conditions also are of some concern to the designer in that equipment insulation must be suitable for the most unfavorable conditions encountered in use.

III. Temperature, Pressure, Humidity

(*a*). TEMPERATURE

Flashover (and sparkover) voltages through air vary inversely with the absolute temperature. Figure 1 shows the variation in per cent for temperature ranges encountered in the U.S.A.

(*b*). BAROMETRIC PRESSURE

Flashover voltages vary directly with the barometric pressure. In a given location the pressure ordinarily does not vary through more than a five per cent range. However, barometric pressure and the dielectric strength of air do vary with altitude. Figure 2 indicates the relation.

(*c*). HUMIDITY

Flashover voltages for nonhomogeneous dielectric fields vary with the absolute humidity of the air. The change in these voltages depends upon wave form, polarity, and the kind of apparatus tested. Correction curves, of which figure 3 is typical, must be obtained experimentally.

(*d*). RELATIVE AIR DENSITY

The effect of temperature and of pressure have been combined into a single

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1. For all numbered references, see list at end of paper.

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chosen for the ratings; the latter range from 15 to 60 minutes for the various service conditions.

(b). AUXILIARY DEVICES AND CONDUCTORS

For all classes of applications discussed under "main devices," safe temperature rises are obtained for the auxiliary devices and conductors if they are rated for the same current value (I) and the same time as the main apparatus, assuming that the method of establishing short-time ratings outlined under class $2A$ is followed. For applications $1A$ and $1B$ with short-load periods which are definitely limited by the type of application, further economies can be safely effected by selecting auxiliary devices and conductors having continuous ratings corresponding to the actual rms values, as discussed under $1A$ and $1B$.

Obviously, all the methods outlined here may have to be modified in extreme or unusual cases which depart from the assumptions and limitations indicated in the paper.

Conclusions

The selection of the most suitable and economical short-time ratings of main devices, auxiliary devices, and conductors, all properly co-ordinated, has been reduced to an extremely simple procedure for many of the intermittent and varying loads found in practice. Uncertainties and uneconomical combinations previously encountered can be eliminated by the co-ordinated practices suggested for rating and application.

References

1. EFFECT OF LOAD FACTOR ON OPERATION OF POWER TRANSFORMERS BY TEMPERATURE, **V. M. Montsinger.** AIEE TRANSACTIONS, volume 59, 1940 (November section), pages 632-6.

2. TEMPERATURE LIMITS FOR SHORT-TIME OVERLOADS FOR OIL-INSULATED NEUTRAL GROUNDING REACTORS AND TRANSFORMERS, **V. M. Montsinger.** AIEE TRANSACTIONS, volume 57, 1938 (January section), pages 39-44.

3. A STUDY OF SHORT-TIME RATINGS AND THEIR APPLICATION TO INTERMITTENT DUTY CYCLES, **R. E. Hellmund and P. H. McAuley.** AIEE TRANSACTIONS, volume 59, 1940, pages 1050-5.

4. Discussion of reference 3, **P. L. Alger.** AIEE TRANSACTIONS, volume 59, 1940, pages 1221-2.

5. Discussion of reference 3, **T. C. Johnson.** AIEE TRANSACTIONS, volume 59, 1940, pages 1222-3.

6. RATING OF GENERAL-PURPOSE INDUCTION MOTORS, **P. L. Alger and T. C. Johnson.** AIEE TRANSACTIONS, volume 58, 1939 (September section), pages 445-56.

7. DUTY CYCLES AND MOTOR RATING, **L. E. Hildebrand.** AIEE TRANSACTIONS, volume 58, 1939 (September section), pages 478-83.

8. THE APPLICATION OF CLASS B INSULATION TO AUXILIARY TYPE D-C MOTORS IN SEVERE-DUTY SERVICE, **F. A. Compton, Jr.** AIEE TRANSACTIONS, volume 59, 1940, pages 828-34.

9. Report on Standards Conference at 1940 AIEE Summer Convention, ELECTRICAL ENGINEERING, volume 59, August 1940, pages 337-8.

10. AIEE Standard No. 1, General Principles Upon Which Temperature Limits Are Based in the Rating of Electrical Machinery and Apparatus, June 1, 1940.

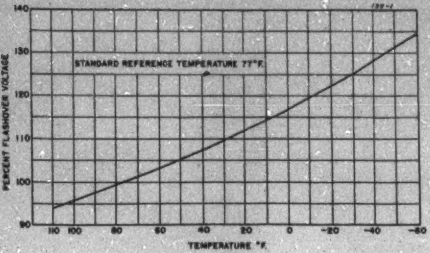


Figure 1. Flashover voltages vary through a range of over 30 per cent due to temperature changes of ambient air

factor, known as relative air density, which is unity for the standard conditions of temperature and pressure. For other conditions the relative air density is

$$\frac{17.95\times\text{bar. press. (inches)}}{460+\text{temp. degrees Fahrenheit}}$$

Flashover voltages actually vary at a rate less than the relative air density, but for practical purposes, the slightly pessimistic assumption that they follow the relative air density directly is satisfactory.[1]

(*e*). Magnitude of Changes

It is seen that temperature alone may produce a change in flashover voltage of 30 per cent. An elevation of 3,300 feet reduces flashover voltage 11 per cent, neglecting daily variations. Annual variations in humidity cause a change in flashover through a 20 per cent range. These effects, therefore are important and their combined results are of interest.

IV. Service Conditions

(*a*). Temperature

Temperatures in the continental U. S. vary from 120 degrees Fahrenheit to −50 degrees Fahrenheit. In terms of the variation from 100 per cent flashover at 77 degrees Fahrenheit, these correspond to −8 per cent and +31 per cent change. Maximum and minimum temperatures for some typical cities, where high voltage apparatus may be used, are given in table I.

(*b*). Barometric Pressure

An altitude map of the U.S.A. is shown in figure 4. Maximum and minimum altitudes by states are given in table II. It follows that a considerable part of the western states is above the 3,300-foot level to which standard apparatus is applicable. From table I also, the elevations of the cities listed range up to 7,000 feet. Consequently, a good deal of equipment is installed at altitudes above the standard range of 3,300 feet.

(*c*). Humidity

In general, the absolute humidity is lowest at low temperatures, where the air becomes saturated at very low moisture contents. For locations near large bodies of water high temperatures and high humidities tend to coincide and these two factors tend to cancel in their effect on flashover. However, table I indicates that, for the high inland locations, absolute humidities are quite low, even at times of maximum temperature, at Denver, for example.

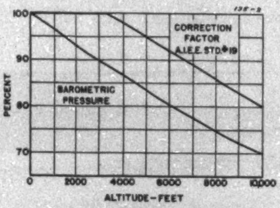


Figure 2. Flashover voltages vary directly with the barometric pressure which is 70 per cent of mean sea level pressure at 10,000 feet altitude

(*d*). Combined Effects

The change in flashover voltages for these extreme conditions in certain cities has been calculated and is shown in table III. The range of the variations likely to be encountered in extreme cases is around 27 per cent. Attention is called, however, to Great Falls, Montana, which is just below the 3,300-foot altitude limit. At this location, standard apparatus would at times suffer a reduction in flashover strength of practically 25 per cent. It should be pointed out that variations in barometric pressure and in humidity from the values assumed or recorded might influence the figures given another two or three per cent in either direction.

V. Rain, Fog, Dirt, Salt Spray

In addition to encountering a reduction in the dielectric strength of air, electrical insulating surfaces, such as porcelain bushings and insulators, frequently are subjected to rain, fog, and foreign deposits. A heavy rain will reduce the flashover voltage of a new clean insulator 30 per cent. Fog or rain on a dirty insulator may reduce the effectiveness of the insulator to the point where operation at rated voltage is impaired. In such cases special types of insulation may be applied or periodic cleaning may be found necessary. Wanger[2] and others have presented quantitative data on this subject, which is outside the scope of this paper.

VI. Apparatus

For a practical illustration of apparatus application, bushings will be considered. The basic insulation levels, based on an impulse voltage-withstand test, for different voltage classes have been suggested for standardization.[3] These levels are shown in figure 5, with the reduced corresponding values for altitudes of 5,000 and 10,000 feet. For the higher voltage classes, the next higher voltage class applies quite well for steps in elevation of the order of 3,000 to 5,000 feet. From 15 to 69 kv the insulation steps do not lend themselves as readily to this practice. Naturally, in any practical application compromises must be made between maintenance of insulation levels and subjecting internal insulation of apparatus to excessive voltages. To avoid an excessive number of designs, it probably is

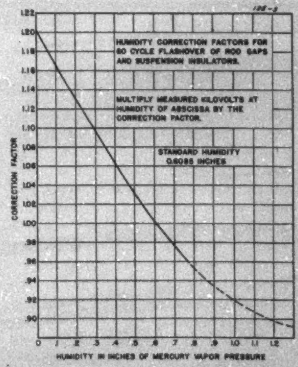


Figure 3. Flashover voltages vary with absolute humidity. The annual range is over 20 per cent for locations at moderate altitudes

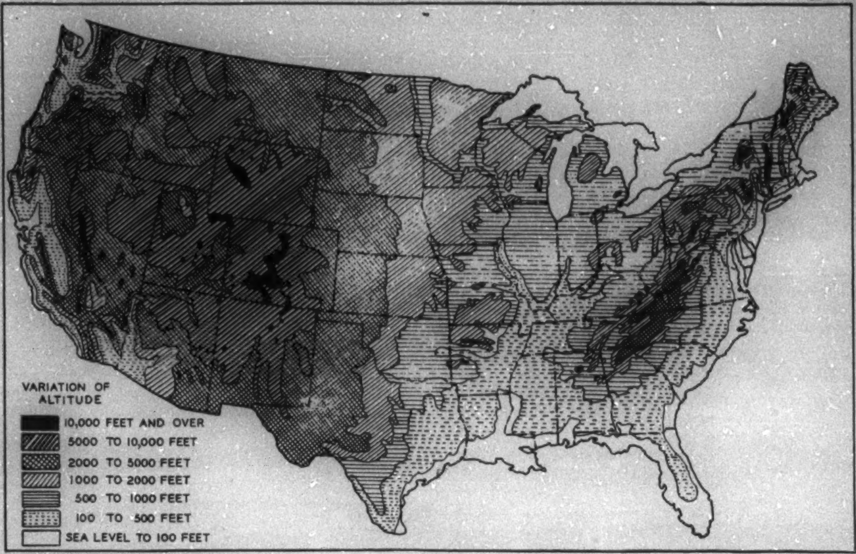


Figure 4. The mean elevation of the United States, including Alaska, is 2,500 feet. About 8.3 per cent of the earth's land lies above 6,000 feet and about 2.5 per cent is above 12,000 feet elevation

most economical to use the next largest bushing with a gap to reduce its flashover at the altitude in question to the basic insulation level of the apparatus. The variations due to temperature and humidity cannot be controlled. Hence, excessive approximations in apparatus application for a given altitude are to be avoided.

Figure 6 shows the effect of altitude on the flashover of 34.5- and 115-kv bushings. The relative flashovers of the next higher voltage steps in 3,300-foot ranges also are indicated. For the 115-kv case, the maximum spread from the basic level using the next higher class of standard bushings is seven per cent. This is likely to be further decreased in practice on account of the lower humidities encountered at higher altitudes. In general, the relative flashover levels lead to a fortunate situation. This is not the case for the 34.5-kv class. The spread between the 46- and 69-kv classes is too great for application in the simple manner found above. For this reason, it has been the practice to use "high-altitude" bushings of appropriate flashover in this range.

VII. Testing Margins

Since apparatus is now tested on a withstand voltage basis, all equipment has a small margin over the basic levels to avoid troubles in acceptance tests. These margins, of course, raise the actual flashover range in terms of voltage. However, although the mean flashover is thereby raised, the width of the band due to atmospheric variations remains the same. The net result is that the internal insulation may be worked a little harder in actual service.

VIII. Upper Air Investigations

A subject of increasing interest is the weather conditions at high altitudes above the ground. Figure 7 gives some early data[7] on this point. Temperature pressure and humidity decrease with altitude, of course. However, the very rapid decrease in absolute humidity and the low moisture content at an altitude of 20,000 feet even with a high value at sea level are very striking.

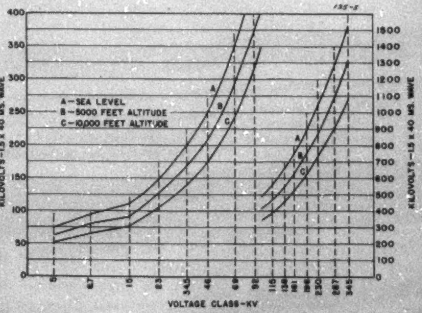


Figure 5. The impulse-withstand voltage ratings of bushings are affected as shown at the 5,000- and 10,000-foot levels

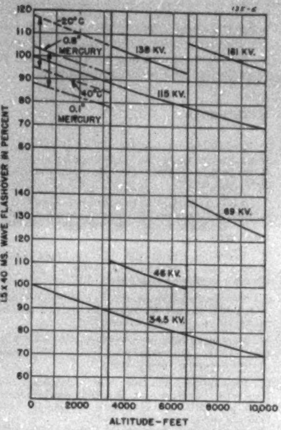
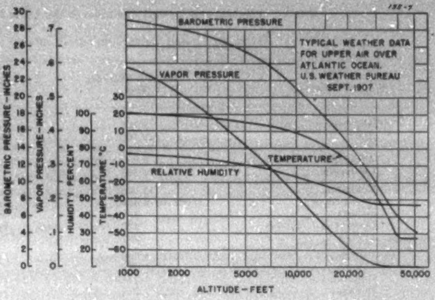


Figure 6. The relation of adjacent voltage classes of apparatus permits applying the next higher classes for different altitude ranges in the higher voltage cases and not in the lower operating voltage classes

Figure 7. Upper air investigations give the variations in weather conditions with altitude. Conditions change of course, with geographical location and with the seasons and time of day



IX. Apparatus Standards

As already stated, apparatus standards permit apparatus of standard voltage rating for elevations up to 3,300 feet. AIEE Standard No. 19 for oil circuit breakers states, "for application at altitudes greater than 3,300 feet, it is recommended that the standard voltage rating of the apparatus should be multiplied by the following factors to obtain the modified voltage rating."

Altitude in Feet	Correction Factor
3,300	1.00
4,000	0.98
5,000	0.95
10,000	0.80

These recommended corrections are shown in figure 2 and their actual relation to the decreasing barometric pressure can be seen. There are a few exceptions to the 3,300-foot standard range, based on thermal rather than flashover limitations. Railway motors, AIEE Standard No. 11, may be used through a range up to 4,000 feet. NEMA Industrial Control Standard gives a range of 6,000 feet and a derating factor for the 6,000–15,000-foot range. These are practical figures based on much experience which help to simplify application problems. However, the adoption of a multitude of ranges as a result of selecting the optimum for each type of apparatus is not to be recommended.

X. Conclusions

1. Temperature changes of the ambient air in the U.S.A. change the flashover voltage of apparatus through a range of 39 per cent.
2. An elevation of 10,000 feet reduces flashover voltages 30 per cent from those at sea level.
3. Variations in absolute humidity result in changes in flashover voltage of nearly 20 per cent.
4. In any one location the combined ef-

Table I. Unusual Air Conditions for Certain Cities of the U.S.A.

Place	Elevation, Feet	Highest-Lowest Temperatures Ever Recorded, Degrees Fahrenheit		Highest Wet Bulb at Noon During July and August for a Six-Year Period			Highest Dry Bulb at Noon During July and August for a Six-Year Period			Mean Relative Humidity at Noon in July
				Dry	Wet	Vapor Pressure*	Dry	Wet	Vapor Pressure*	
Albuquerque, N. M.	4,930	104	−10							
Amarillo, Tex.	3,676			88	74	0.689	100	70	0.402	
Bismarck, N. D.	1,690			90	76	0.747	111	70	0.278	46
Boise, Idaho	2,705	121	−28	93	72	0.55	104	65	0.184	26
Butte, Mont.	5,485	100	−52							
Cheyenne, Wyo.	6,088	100	−38							40
Denver, Col.	5,292	105	−29	92	65	0.319	98	61	0.126	35
El Paso, Tex.	3,778	106	− 5	96	70	0.447	99	67	0.307	33
Flagstaff, Ariz.	6,935	99	−25							
Grand Forks, N. D.	830	108	−43							
Great Falls, Mont.	3,200	107	−49							
Jacksonville, Fla.	100	104	10	98	82	0.921	98	82	0.921	63
Kansas City, Mo.	730	111	−22	98	80	0.829	108	74	0.463	51
Minot, N. D.	1,557	109	−49							
Phoenix, Ariz.	1,075	118	16							30
Pueblo, Col.	4,665	104	−27							33
Reno, Nev.	4,500	106	−19	94	66	0.330	99	65	0.240	18
Salt Lake City, Utah	4,360	105	−20	97	69	0.399	99	63	0.176	29
Sioux Falls, S. D.	1,425	110	−42							
Sante Fe, N. M.	7,013									
Yuma, Ariz.	141	120	22							27

Temperatures from "Air Conditioning Engineer's Atlas," The Industrial Press, New York.

*Vapor pressures correspond to temperatures shown for 30-inch barometric pressure.

Table II. Highest and Lowest Altitudes in the United States

(From the United States Geological Survey)

State or Territory	Highest Altitude, Feet	Lowest Altitude, Feet
Alabama	2,407	Sea level
Alaska	20,500	Sea level
Arizona	12,611	100
Arkansas	2,800	55
California	14,501	276 (below sea)
Colorado	14,402	3,350
Connecticut	2,355	Sea level
Delaware	440	Sea level
District of Columbia	420	Sea level
Florida	301	Sea level
Georgia	4,768	Sea level
Idaho	12,078	720
Illinois	1,241	279
Indiana	1,210	316
Iowa	1,800	477
Kansas	4,135	700
Kentucky	4,100	257
Louisiana	400	Sea level
Maine	5,273	Sea level
Maryland	3,340	Sea level
Massachusetts	3,505	Sea level
Michigan	2,023	573
Minnesota	1,920	602
Mississippi	600	Sea level
Missouri	1,750	210
Montana	12,850	1,800
Nebraska	5,350	825
Nevada	13,058	470
New Hampshire	6,293	Sea level
New Jersey	1,809	Sea level
New Mexico	13,306	2,876
New York	5,344	Sea level
North Carolina	6,711	Sea level
North Dakota	3,500	790
Ohio	1,550	425
Oklahoma	4,750	300
Oregon	11,225	Sea level
Pennsylvania	3,136	Sea level
Rhode Island	805	Sea level
South Carolina	3,548	Sea level
South Dakota	7,242	962
Tennessee	6,636	182
Texas	9,020	Sea level
Utah	13,498	2,000
Vermont	4,364	95
Virginia	5,719	Sea level
Washington	14,408	Sea level
West Virginia	4,860	240
Wisconsin	1,940	582
Wyoming	13,785	3,100

fects of these variables is not likely to exceed a 25 per cent range.

5. Standard apparatus applied at different locations in the range of elevations up to 3,300 feet will vary in flashover over a possible range of 35 per cent.

6. For many applications standard apparatus of the next higher voltage class fits the mean flashover range of the location.

7. For other cases it is necessary to use special apparatus or to gap equipment of excessively high rating to meet the average flashover requirements for the conditions encountered at a given location.

8. The desire to maintain flashover values and to avoid overstressing internal insulation prescribes permitting wide departures from basic insulation levels due to conditions of application.

9. This survey would seem to show greater variations in flashover voltages due to changing weather conditions in service than is generally realized. Although the situation is not to be regarded as alarming, it is well to keep it in mind. A full appreciation of the facts in a particular case should lead to a more effective application of apparatus to meet the conditions. On the other hand, certain applications which may have proved unsatisfactory might be explained by these simple natural phenomena. In addition, standardizing groups often find it necessary to consider information of this nature in formulating and revising standards.

References

1. Effect of Altitude on the Sparkover Voltages of Bushings, Leads, and Insulators, **F. W. Peek, Jr.** AIEE Transactions, volume 33, 1914, page 923.

2. La Tension de Contournement, Sous Ondes de Choc de Differentes Durees, des Isolateurs Salis et Sous Pluie, **W. Wanger.** CIGRE, Paris, 1939.

3. Proposed Standards for Outdoor Bushings, AIEE Transactions, volume 59, 1940 (October section), pages 590–1.

4. Temperature Survey of the United States, **J. J. Smith and H. W. Tenney.** AIEE Transactions, volume 59, 1940, pages 769–75.

5. Recommendations for High-Voltage Testing, AIEE Transactions, volume 59, 1940 (October section), pages 598–602.

6. Reference Values for Temperature, Pressure, and Humidity, **P. L. Bellaschi and P. H. McAuley.** AIEE Transactions, volume 59, 1940 (December section), pages 669–75.

7. The Elements of Hydrology, a book, **Adolph F. Meyer.** John Wiley and Sons, Inc.

Appendix

Altitude Corrections

The effect of altitude on apparatus rating is given in at least four different forms in the AIEE and ASA standards. Altitude affects voltage rating (flashover through air), cooling (air density) and losses (windage). More uniformity of terminology and method of specification appears possible and desirable.

In Standard No. 19 correction factors for voltage rating are given in the form of a table as given in section IX of this paper. The table values correspond exactly to one per cent reduction for each 330 feet above 3,300-feet altitude. In the same standard the correction for temperature rise is given as 0.4 per cent reduction for each 330 feet above 3,300 feet at which the actual installation is to take place.

In ASA Standard *C*-50 for rotating electrical machinery, the probable decrease in

Table III. Effect of Extreme Air Conditions on Flashover Corrections

Location	Boston, Mass.	Bismarck, N. Dak.	Great Falls, Mont.	Denver, Colo.	Cheyenne, Wyo.
Elevation—feet	Sea level	1,690	3,200	5,292	6,088
Barometer—inches	29.92	28.2	26.6	24.6	24.0
Highest temperature recorded	104		107	105	100
Corresponding relative air density	0.952	0.886	0.843	0.782	0.769
Highest dry bulb*	96	111		98	
Corresponding wet bulb	75	70		61	
Corresponding absolute humidity	0.64	0.278	0.2**	0.2	0.2**
Humidity correction	0.993	1.081	1.101	1.101	1.101
Flashover reduced to	0.958	0.819	0.765	0.710	0.698
Altitude rating standards number 19	100	100	100	94	91.6
Lowest temp. recorded	−18	−45**	−49	−29	−38
Corresponding relative air density	1.216	1.22	1.162	1.025	1.021
Humidity correction	1.12**	1.12**	1.12**	1.12**	1.12**
Flashover reduced to	1.085	1.089	1.038	0.915	0.912
Range in per cent	12.7	27.0	27.3	20.5	21.4

*At noon during July and August in a six-year period.

**Estimated.

Volt-Time Areas of Impulse Spark-Over

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Synopsis: On the basis of classification of impulse wave shape, the paper shows that impulse spark-over characteristics of electrodes with nonuniform fields cannot be represented by single volt-time curves but must be represented by volt-time areas of considerable extent. This classification explains the large difference obtained by various laboratories when investigating sparkover between one and five microseconds. The effect of impulse generator discharge circuits in producing these differences is explained. The necessity of insulation co-ordination on the basis of volt-time areas is pointed out.

Introduction

EVER since it was definitely established that lightning strokes contribute the greatest amount of insulator flashovers on transmission lines as well as damage to important equipment such as transformers and circuit breakers, the problem of insulation co-ordination on the basis of impulse sparkover has become of great concern to the operating engineer, not only as a means of eliminating damage, but also of preventing flashovers causing system outages. Many papers[1] have been written on the large problem of co-ordination.

Co-ordination has been defined in various ways. One of the more recent definitions[2] is as follows:

> 'Insulation co-ordination may be defined as the process of setting standards of insulation strength and assigning them to all the items of equipment in the various insulation classes in such a manner as to produce, from an economic and operating standpoint, both a maximum of protection, and, if failure results, a minimum damage to the system."

From this definition it is apparent that the impulse strength of all types of apparatus comprising a transmission system has to be known, within fairly accurate limits, before co-ordination can be applied effectively. Consequent studies of the impulse flashover characteristics of suspension insulators, apparatus insulators, bushings as well as transformer insulation revealed that the impulse flashover in air, oil, or breakdown of solid insulation is not a constant value, but varies with time. Therefore, impulse strength of a given electrode arrangement and dielectric can be represented by a volt-time curve which correlates the spark-over voltage with the time at which the insulator will spark over.

Present Concept of Impulse Spark-Over Characteristics

In figure 1 a series of such volt-time curves are shown for different types of electrodes. The curves are plotted as overvoltage-time curves to show the relative increase of spark-over voltage as affected by the electrode arrangement. Overvoltage is the actual spark-over voltage expressed in per cent of the critical[*] spark-over voltage of the apparatus.

It is evident from a study of these curves that co-ordination on the basis of the critical spark-over voltage of different types of apparatus only cannot be effective, but that co-ordination must be based on the volt-time impulse characteristics of the apparatus. While the various laboratories in the United States of America conducting high-voltage research did agree within fairly close limits on the critical spark-over voltages of rod gaps and insulators,[3] considerable differences in results were obtained for volt-time spark-over curves.[4,5] For instance, the differences in measured spark-over voltages for rod gaps were found to be as great as 20 to 40 per cent at times to sparkover of two microseconds and less. These large differences were at first attributed to measuring methods such as the type of divider—resistance divider, shielded and unshielded, and capacitance divider—as well as insufficient precaution in the oscillograph technique. However, it was soon discovered that the shape of the applied impulse wave before sparkover occurs has a distinct bearing on the spark-over voltage of different types of apparatus.[6] The effect of wave shape,

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1. For all numbered references, see list at end of paper.

[*] The critical spark-over voltage of a test specimen under an impulse of any given wave shape and polarity is the crest value of that impulse when its amplitude is adjusted to cause flashover on 50 per cent of the applications.

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ambient temperature at higher altitudes is recognized and applications at normal rating approved if the operating conditions are within the limits given in the following table.

Altitude In Feet	Altitude In Meters	Maximum Temperature of the Cooling Air in Degrees Centigrade
0– 3,300	0–1,000	40
3,300– 6,600	1,000–2,000	35
6,600– 9,900	2,000–3,000	30
9,900–13,200	3,000–4,000	25

In this case the percentage change varies with the different values of temperature rise since corrections are given in degrees of temperature.

In standard *C*-50 also, a correction of one per cent per 330 feet above 3,300 feet is specified for temperature-rise tests at low altitude on apparatus destined for use at altitudes above 3,300 feet.

ASA standard *C*-57 for transformers, regulators, and reactors gives corrections for temperature rise in per cent per 330 feet above 3,300 feet. Rates from 0.4 to 1 per cent are given depending on whether cooling is largely by radiation or conduction.

AIEE Standard No. 4 for voltage measurement gives a correction for spark-over voltages in air which includes both temperature and pressure (altitude) factors. Measured voltage values are divided by the relative air density to correct to standard conditions.

$$\text{relative air density} = \frac{0.392B}{273+T}$$

where

T = temperature degrees centigrade
B = pressure in millimeters of mercury

ASA standard *C*-50 gives a correction formula for windage losses for different temperature and pressure (altitude) conditions. The losses measured at a given temperature and pressure are multiplied by the factor $690\,\frac{(1+0.004T)}{B}$, in which T = temperature degrees centigrade and B = pressure in millimeters of mercury. Both this and the relative air density formula are unity at 25-degrees-centigrade and 760-millimeter pressure and one is the reciprocal of the other for practical purposes.

The International Electrotechnical Commission proposed specification for a-c circuit breakers (May 1938) states that the limits of temperature rise shall be reduced one per cent for each 300 meters (990 feet) above sea level at which the breaker is intended to work in service. The correction is not applied below 1,000 meters. In this case both the rate and the starting point of the correction are different from American Standards.

however, could not be classified, and, therefore, useful engineering data could not be supplied. Owing to the differences obtained by various laboratories, the volt-time curves were assigned plus and minus corrections of arbitrary value to account for what was thought to be erratic spark-over characteristic of certain types of apparatus.

New Concept of Impulse Spark-Over Characteristics

It will be left to the physicist to translate the following data into electrons, ions, and terms of photo-ionization. This paper will attempt to present the data in terms which the engineer interested in co-ordination and, therefore, in impulse spark-over characteristics of different kinds of apparatus, can use to apply to his problems.

The effect of wave shape is well illustrated by figure 2 which shows a series of volt-time curves and spark-over points for a 20-inch rod gap at negative polarity. Each of these curves was obtained with a constant impulse generator discharge circuit. For the lower curve the impulse wave had a front of 0.5 microseconds, for the middle curve a front of 2.4 microseconds and for the upper curve, 9.6 microseconds at the critical spark-over potential. It can be seen that the spark-over voltages associated with a given time to sparkover vary greatly with the shape of the wave front. For instance, at two microseconds the spark-over voltages for the two extreme wave fronts used are 495 kv and 625 kv respectively with a difference of 25 per cent. If, at the same time to sparkover, widely differing voltage values are obtained, gap spark-over has to be classified as either very erratic or the difference must be due to some physical characteristics of the spark-over phenomenon. If spark-over voltages at two microseconds of 495 kv and 625 kv of a 20-inch rod gap had been obtained in isolated tests and not in the course of obtaining volt-time curves, the spark-over characteristic of the gap would indeed appear erratic.

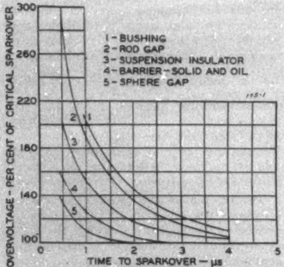

Figure 1. Comparison of overvoltage characteristics of different types of gaps

Negative polarity

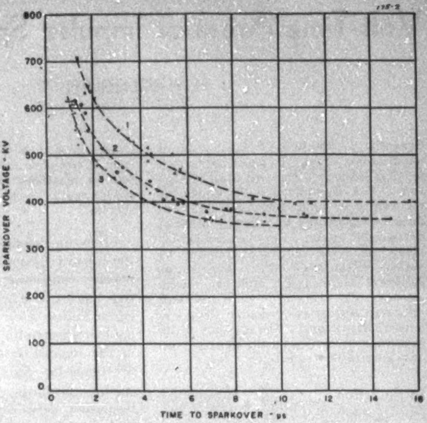

Figure 2. Effect of wave front on spark-over volt-time curves for a 20-inch rod gap

Negative polarity. Vapor pressure 0.26 inch mercury; air density 0.983

Impulse circuit adjusted for: 1—9.6-microsecond wave front, 2—2.4-microsecond wave front, 3—0.5-microsecond wave front

However, since the results shown in figure 2 were obtained with impulse waves of different shapes, it is logical to attribute the difference to wave shape. Since the front of the wave definitely appears to have considerable influence on the variations in spark-over voltage, but, on the other hand, the front of the wave does not stay constant when sparkover occurs between two and ten microseconds, it was decided that the effective front might be used to derive a practical answer for classification of impulse wave shape. The effective front is specified by the slope of the line through the $0.3E$ and $0.9E$ points on the front, where E is the crest of the measured wave. The effective front in microseconds T_1 is the time between the $0.3E$ and $0.9E$ points multiplied by 1.66. For the tests with the slow front, this definition of effective front did not permit sufficient differentiation. Since slow fronts in all cases were of the exponential form, the initial rate of rise was used as the effective front. The time to sparkover T_2 in microseconds is the time from virtual zero time to a point on the wave where voltage is rapidly decreasing toward zero. Virtual zero time is defined as the intersection of the line through the $0.3E$ and $0.9E$ points with the zero line.

The same test points as shown in figure 2 were then replotted as function of kilovolt spark-over and the ratio of T_1/T_2 = (effective front)/(time to sparkover) with time to sparkover T_2 as parameter. Such a plot shows a scattering of points, indicating the trend of lower spark-over voltage for small ratios of T_1/T_2 and higher spark-over voltages for large ratios of T_1/T_2 for equal times to sparkover. In figure 3 the test points were plotted to show the range over which data have been obtained, but the time to

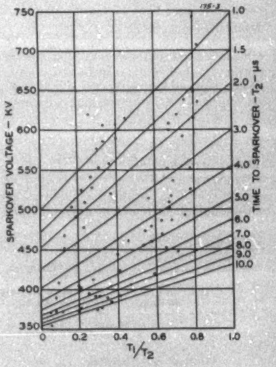

Figure 3. Effect of wave shape on spark-over voltage of 20-inch rod gap

Negative polarity. Vapor pressure 0.26 inch mercury; air density 0.983

T_1 = effective wave front; T_2 = time to sparkover

Test points indicate range investigated

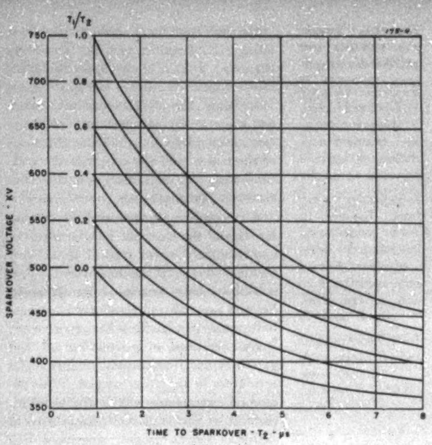

Figure 4. Spark-over volt-time area of 20-inch rod gap

Boundaries are $T_1/T_2=0$ and $T_1/T_2=1$. Negative polarity. Vapor pressure 0.26 inch mercury; air density 0.983

T_1 = effective wave front; T_2 = time to sparkover

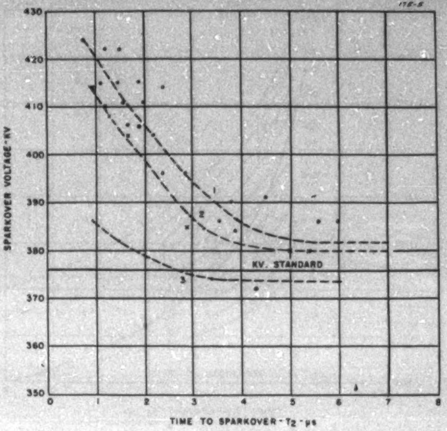

Figure 5. Spark-over volt-time curves of 50-centimeter spheres spaced 15.5 centimeters

Negative polarity. Air density 1

Impulse circuit adjusted for: 1—9.6-microsecond wave front, 2—2.4-microsecond wave front, 3—0.5-microsecond wave front

sparkover associated with each test point was omitted for reasons of reproduction. The network of straight lines, indicating spark-over voltage trends for a given time to sparkover, has been placed such that it most nearly represents the correct interpretation of the test results.

The lower limit $T_1/T_2=0$ and the upper limit $T_1/T_2=1$ have not been reached during these tests. The lower limit, of course, represents a wave with abrupt rising front and practically rectangular tail, while the upper limit represents a front rising in a straight line from the zero point to the spark-over point. Both of these limits are difficult to obtain in practical impulse testing. However, the error in extending the straight lines to both of these limits beyond the available test points should not result in appreciable errors.

To reduce the results to the customary form of the volt-time curves, the straight lines of figure 3 have been used to draw the curves of figure 4, where spark-over voltage in kilovolts is plotted as function of time to sparkover with the ratio T_1/T_2 as parameter.

This figure shows clearly that sparkover of a rod gap for instance cannot be represented by a single "volt-time curve" but rather by a "volt-time area." This volt-time area, in contrast to previous attempts of using correcting factors in connection with volt-time curves, is well defined by recognition of the influence of impulse wave shape on spark-over voltage. It will be shown that the use of the wave shape definition adopted gives results within a few per cent of expected spark-over voltages. Therefore, the use of volt-time areas will eliminate to a large extent the uncertainty heretofore associated with short-time sparkover of insulators, bushings, rod gaps, and other apparatus. It is important to note that this spark-over area, of course, is subject to the variations in spark-over voltage due to humidity and air density. The correct method of correcting for humidity throughout this spark-over area is not known. If the method at present used for volt-time curves is adopted[7] it means that at short times to sparkover the spark-over values will remain practically the same while at the longer times the spark-over voltages will be raised, so that the individual volt-time curves for different T_1/T_2 = constant will have a reduced slope at the longer times to sparkover, and the total spark-over area will be reduced when based on standard conditions of humidity. The air density correction probably will be the same regardless of wave shape.

It has been shown that the impulse spark-over voltage (e) of an electrode arrangement producing a nonuniform field, such as the rod gap, is a function not only of the time at which sparkover occurs, but also of wave shape represented by the ratio of T_1/T_2. Therefore $e=f(T_2, T_1/T_2)$. The physical condition for this dependence of spark-over voltage on wave shape is primarily the fact that corona currents of considerable amplitude have to flow through the dielectric to obtain sufficient ionization before sparkover can be completed.[8] The amplitude of these corona currents in turn depends on the voltage applied to the gap and the length of time voltage has been applied to the electrodes. Obviously wave shape cannot accurately be described by a ratio of T_1/T_2, especially for waves with unusual shapes, but in the range below six microseconds the results obtained indicate a sufficient consistency to justify the use of this approximation until more exact methods are available.

Volt-Time Areas of Different Types of Electrodes

The Quasi-Uniform Field

From the foregoing discussion it should be expected that spark-over voltage of electrode arrangements producing a uniform field should be represented by straight lines parallel to the zero axis (each spacing between electrodes represented by one line) and without the tendency of spark-over areas because heavy corona currents are not required to initiate sparkover. The closest approximation to the uniform field available for testing at higher voltages is the sphere gap at spacings small with respect to the diame-

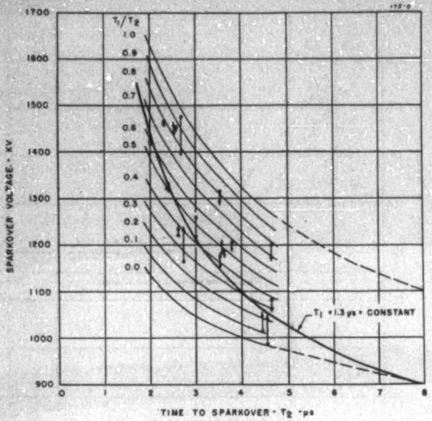

Figure 6. Spark-over volt-time area of 50-inch rod gap

Boundaries are $T_1/T_2=0$ and $T_1/T_2=1$. Negative polarity. Standard conditions of humidity and air density

T_1 = effective wave front; T_2 = time to spark-over

Points indicate measured values. Arrows point to correct location of test point based on T_1/T_2

T_1 = 1.3-microsecond curve from tests with 1.5x40 wave

ter of the sphere. The same types of waves used for the rod gap were applied to a 50-centimeter sphere gap spaced 15.5 centimeters. This spacing was chosen because it required a spark-over voltage approximately equal to the critical spark-over of the 20-inch rod gap.

The results of these tests are shown in figure 5. This figure shows in a greatly enlarged scale the test points and the probable volt-time curves for the three test conditions as outlined for the 20-inch rod gap. Only two test points were obtained for the short front-long tail wave. The differences between the two slow front waves are insignificant. At one microsecond the difference between minimum and maximum measured voltages is only nine per cent. Since the probable measuring error is of the order of three per cent, it can be said that the volt-time area of a sphere gap, representing a nearly uniform field is extremely small as compared to that of a rod gap at times to sparkover of one microsecond and greater. The data do not permit drawing of T_1/T_2=constant curves as shown for the 20-inch rod gap in figure 4.

The Nonuniform Field

In addition to the 20-inch rod gap at negative polarity, volt-time areas have been obtained for the 50-inch rod gap at negative polarity, figure 6, and the 20-inch rod gap at positive polarity, figure 7. In the latter two figures the test points have been superimposed on the plot of the T_1/T_2=constant curves as follows: The spark-over potentials in kilovolts as function of time to sparkover are represented by the solid points. From these test points an arrow is drawn to the expected location of the point according to the ratio T_1/T_2 of the wave. It is apparent that the test points check the expected values very closely. In figure 6 is also shown a conventional volt-time curve for the 50-inch rod gap taken with a 1.5x40 wave. It is evident that such a curve cannot represent the spark-over characteristics of the rod gap. Data of figure 6 and figure 7 have been corrected to standard conditions of humidity and air density according to accepted methods.[7]

Further tests have been made on a string of three suspension insulators (figure 8), a stack of two apparatus insulators (figure 9), and a bushing (figure 10). Only the T_1/T_2=constant curves for these different insulating elements have been plotted to indicate the slope of the curves and the extent of the volt-time areas for comparison with the other electrode arrangements investigated.

A comparison of the volt-time characteristics of the electrode arrangements of approximately equal critical spark-over voltages shows that the slope of the T_1/T_2=constant curves is steepest for the bushing, rod gap, line and apparatus insulators in the order named, while the extent of the spark-over areas is greatest for the rod gap, line insulators, bushing and pedestal insulators in the order named. This sequence corresponds well to the trend indicated by the single volt-time curves of figure 1. The electrode arrangement with the least overvoltage—the sphere gap—has the smallest volt-time area, while the higher overvoltages at a given time to sparkover of the single volt-time curve are indicative of larger volt-time areas.

The effect of wave shape on breakdown characteristics of barriers (representing major insulation of oil-filled transformers)—represented by overvoltage-time curve 4, figure 1—has not been investigated. Judging from this volt-time characteristic, the volt-time breakdown area of such a barrier should be intermediate between the spark-over areas of the sphere

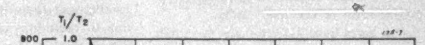

Figure 7. Spark-over volt-time area of 20-inch rod gap

Boundaries are $T_1/T_2=0$ and $T_1/T_2=1$. Positive polarity. Standard conditions of humidity and air density

T_1 = effective wave front; T_2 = time to spark-over

Points indicate measured values. Arrows point to correct location of test point based on T_1/T_2

gap and the line insulators, depending greatly on the type of electrodes and the field between them. Further tests are needed to obtain data comparable to those represented in the paper for air dielectrics.

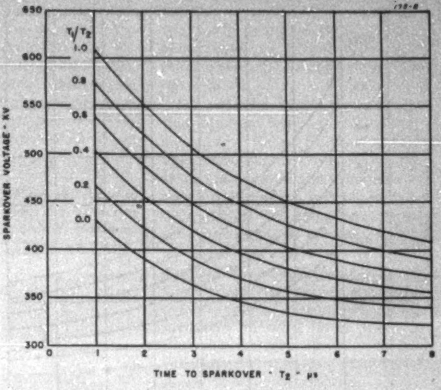

Figure 8. Spark-over volt-time area of a three-unit string of suspension insulators

Boundaries are $T_1/T_2=0$ and $T_1/T_2=1$. Negative polarity. Vapor pressure 0.26 inch mercury; air density 0.983

T_1 = effective wave front; T_2 = time to spark-over

It has been customary[9,10] to represent rod gap spark-over voltages as function of rod gap spacing with the time to sparkover as parameter; similarly, insulator sparkover is represented as function of number of insulators or string arc-over distance. Such curves were based on single volt-time curves taken at various spacings using a standard 1.5x40 wave. On the basis of spark-over volt-time areas, such curves should be drawn for each T_1/T_2 = constant curve, or, since the limiting conditions are of greatest importance, for the two conditions $T_1/T_2=0$ and $T_1/T_2=1$.

The results of the tests presented here permit the plot of such curves for the rod gap between 20-inch and 50-inch spacing at negative polarity (figure 11). To conserve space, curves for the two extremes have been plotted on the same figure and, to avoid overlapping of the extreme ranges, the curves shown range only between two microseconds and four microseconds to sparkover. In addition, typical curves as obtained with the 1.5x40 wave are included.

Effect of Impulse Generator Circuits on Spark-Over Voltage

It is interesting to note that the solid two-microsecond curve as obtained from tests with a 1.5x40 wave (figure 11) is a curve rather than a straight line. This means, of course, that the wave shape at two microseconds to sparkover of the 20-inch rod gap was different from the wave shape during tests on the 50-inch rod gap, although the wave shape at the longer times to sparkover was practically the same in both tests as indicated by the straight lines for three microseconds and four microseconds.

This difference in wave shape at the short times to sparkover is due to the effect of the corona currents required for ionization of the gap to complete spark-over and the resulting voltage drop in the series impedance—inductance and resistance—of the impulse generator discharge circuit. The resulting change of the wave shapes is shown in figure 12. On this figure the wave shape at two microseconds to sparkover is compared with the wave shape at critical sparkover for two types of impulse generator discharge circuits. In figure 12*a* the wave shape is controlled by a series resistance and a small load capacitance which principally consists of the inherent capacitance of the impulse generator plus a small additional load capacitance, a total of approximately 300 micromicrofarads. In order to obtain a 1.5-microsecond front with such a small capacitance, it is necessary to use a series resistance of 1,700 ohms. The corona currents through the gap have an amplitude of the order of hundreds of amperes and, therefore, the voltage drop through the resistor is several hundred thousands of volts. This voltage drop subtracts from the wave as it would have appeared, if the corona currents had not flown through the gap. The effective wave front is, therefore, changed from a 1.2-microsecond front at critical sparkover to 0.6 microsecond at 2 microseconds to sparkover. Therefore, the ratio of T_1/T_2 for this wave at two microseconds is $0.6/2=0.3$ which results in a spark-over voltage of 534 kv from figure 4 when corrected for air conditions. This checks the value of 535 kv obtained from figure 11.

In the other circuit, figure 12*b*, the load capacitance is concentrated close to the rod gap and has a value of 800 micromicrofarads. The series resistance is 800 ohms giving a front of 1.8 microseconds and an effective front of 1.3 microseconds. For equal amounts of corona currents, as for the circuit 12*a*, the voltage drop across the resistor due to the corona is considerably reduced because it can be supplied by the capacitor direct without any appreciable voltage drop. Consequently, the effective front is not changed and at two microseconds to sparkover the ratio $T_1/T_2=1.3/2=0.65$. From figure 4 a spark-over voltage of 587 kv results, which, when corrected to standard conditions of humidity and air density, gives 610 kv as compared to 534 kv obtained with impulse generator circuit of figure 12*a*. This spark-over voltage of 610 kv was actually obtained during tests on the 20-inch rod gap with circuit of figure 12*b* in the process of obtaining a volt-time curve before and independent of this present investigation. Data of figure 4 have been obtained recently. It is, therefore, significant to the value of this proposed classification of impulse wave shapes in terms of T_1/T_2 that test data on the same type of gap taken several years ago should check so well when analyzed on the basis of that ratio.

The solid curves shown in figure 11 were obtained with an impulse generator connected as in figure 12*a* for the 20-inch and 30-inch gap, while the tests for the 50-inch gap were made with circuit of figure 12*b*. If the value of 610 kv for the 20-inch rod gap is plotted on figure 11, a straight line for the two-microsecond test curve results.

These examples show that

1. Differences in spark-over voltages obtained on electrode arrangements with non-uniform field are directly due to differences in impulse generator discharge circuits.

2. Spark-over voltages, when classified according to wave shape as expressed by the ratio of effective front to time to sparkover, are consistent on the basis of spark-over volt-time areas, although they appear erratic on the basis of volt-time curves.

3. The effective wave front produced by some types of impulse generator discharge circuits during investigations of impulse volt-time curves of apparatus with non-uniform field varies considerably throughout the range of times to sparkover from critical to one to two microseconds. Such generators produce volt-time curves where neither T_1 = constant nor T_1/T_2 = constant. The change in effective front depends on the type of apparatus tested, whether large or

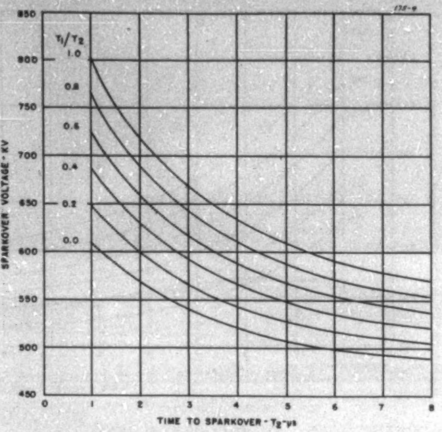

Figure 9. Spark-over volt-time area of a two-unit stack of apparatus insulators

Boundaries are $T_1/T_2=0$ and $T_1/T_2=1$. Negative polarity. Vapor pressure 0.26 inch mercury; air density 0.983

T_1 = effective wave front; T_2 = time to spark-over

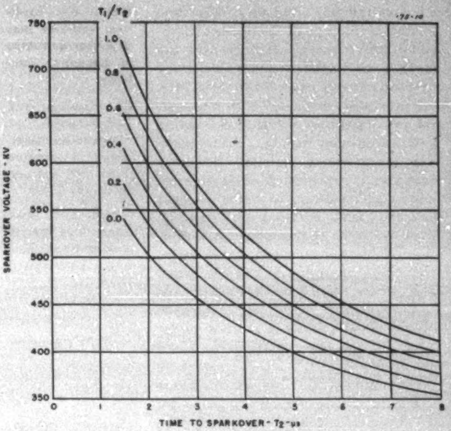

Figure 10. Spark-over volt-time area of a bushing

Boundaries are $T_1/T_2=0$ and $T_1/T_2=1$. Negative polarity. Vapor pressure 0.26 inch mercury; air density 0.983

T_1 = effective wave front; T_2 = time to spark-over

small corona currents are required to complete sparkover.

4. The effective wave front produced by the other type of generator is constant throughout the same range of time to sparkover. Therefore, the volt-time curve produced has a constant T_1. Wave shape at any given time to sparkover is practically independent of the type of apparatus tested. It is, therefore, clear that for purposes of comparison of volt-time curves of apparatus with great differences in electric field distribution, tests made with an impulse generator discharge circuit of the type shown in figure 12*b* are directly comparable. Similar tests made with a generator of the type shown in figure 12*a*, however, are not directly comparable and have to be interpreted with considerable care. Until volt-time areas of all types of insulation structures are available, the effective fronts obtained during tests on volt-time curves should be recorded throughout the range investigated, since they have such a profound effect on the values of spark-over voltages obtained.

Effect of Volt-Time Areas on Co-ordination

The data are obtained on too few types of apparatus to enable complete understanding of all the requirements for effective co-ordination. However, sufficient data are presented to make general recommendations. For this purpose the term co-ordination is used in a restricted sense, such that "co-ordination between two apparatus is obtained if the impulse strength of one of them is below the other, so that sparkover always occurs on the apparatus with lower strength." On the basis of volt-time curves this means that the volt-time curve of apparatus *A* is lower than that of apparatus *B* throughout the expected spark-over time range.

As shown by the examples of the sphere gap and the insulators, quasi-uniform fields between electrodes are associated with small impulse spark-over areas, while the nonuniform field is associated with large impulse spark-over areas. The problems of co-ordinating voltage levels for the large variety of electrode fields encountered in electrical apparatus, therefore, consists in co-ordinating according to spark-over areas rather than according to single volt-time curves. This, of course, complicates the co-ordinating problem to a certain extent.

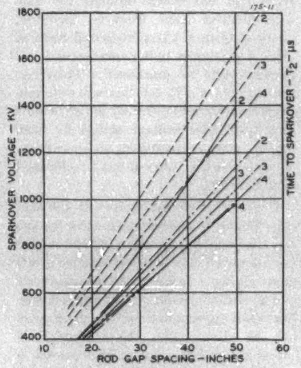

Figure 11. Impulse spark-over characteristics of rod gaps

Negative polarity. Standard conditions of humidity and air density

Solid lines—tests with 1.5x40 wave
Dashed lines—$T_1/T_2=1$ curves
Dot-dash lines—$T_1/T_2=0$ curves

T_1 = effective wave front; T_2 = time to spark-over

However, the problem can be reduced to three fundamental conditions (figure 13):

(*a*). Apparatus with the lower impulse strength (*A*) has a narrow band spark-over area; apparatus with the higher impulse strength (*B*) has a wide band spark-over area (for instance, a sphere gap (*A*) and insulator string (*B*)).

In this case co-ordination can be made effective if the $T_1/T_2=0$ spark-over curve of (*A*) is below the $T_1/T_2=0$ spark-over curve of (*B*) because regardless of wave shape, the impulse strength of (*A*) will be exceeded before that of (*B*) is reached.

(*b*). Apparatus with the lower impulse strength (*A*) has a wide band spark-over area; apparatus with the higher impulse strength (*B*) has a narrow band spark-over area. In this case, co-ordination can be made effective if the $T_1/T_2=1$ spark-over curve of (*A*) is below the $T_1/T_2=1$ spark-over curve of (*B*).

(c). Regardless of the band of the spark-over areas of both apparatus co-ordination is effected if the $T_1/T_2=1$ spark-over curve of (A) is below $T_1/T_2=0$ spark-over curve of (B).

Condition (c), of course, is the most satisfactory method of assuring that apparatus with the lower impulse strength will reduce the impulse voltage before the impulse strength of (B) is reached. However, under conditions where the margin between apparatus (A) and (B) can be kept to a minimum, matching of the spark-over areas as indicated in figure 13, (a) and (b) should give considerable improvement as compared to a consideration of single volt-time curves only. It is obvious that apparatus with nonuniform field and consequent wide band spark-over areas are not suited to limit impulse spark-over voltages in stations where costly equipment may be damaged if the limiting level of the apparatus is set too high. If, on the other hand, the limiting level is set sufficiently low, following condition of figure 13c, the spark-over characteristics will be such that switching surges and other low and medium system disturbances will result in a large number of system outages.

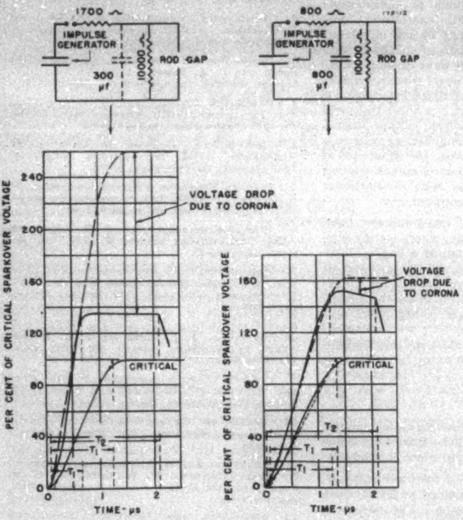

Figure 12. Effect of impulse generator discharge circuits on effective wave front—T_1

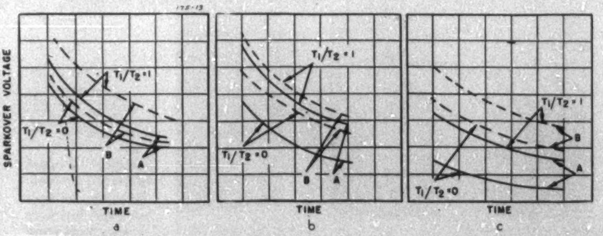

Figure 13. Co-ordination on basis of spark-over volt-time areas

A—Apparatus with lower spark-over strength

B—Apparatus with higher spark-over strength

a—A has narrow band spark-over area, B has wide band spark-over area

b—A has wide band spark-over area, B has narrow band spark-over area

c—Band width of spark-over area irrelevant

Wave Shape Variations on Transmission Lines

The question naturally arises whether apparatus in service will be subjected to waves of such a great variety of shapes as to warrant the investigation of impulse sparkover of areas as has been done in this paper for a few types. The available data are limited, due to the very costly nature of investigations of wave shapes of natural lightning on transmission lines, but approximate limits can be given.

Lewis and Foust[11] give a summary of a study of natural lightning on transmission lines in the United States of America which shows that wave fronts may vary between 1 microsecond and 80 microseconds wave tails between 4 and 160 microseconds. Berger[12] in Switzerland recorded wave fronts up to 80 microseconds causing flashover of insulation at a terminal station. From these limited measurements of traveling waves on actual transmission systems, the possibility of great variations in wave fronts and wave shapes is indicated. Therefore, the large variations in spark-over voltages, as shown to exist for electrode arrangements with nonuniform field, are a distinct possibility and should be taken into account in the co-ordination problem.

Conclusions

1. Impulse spark-over voltages of electrode arrangements with nonuniform field—such as insulators, bushings, rod gaps—cannot be represented with sufficient accuracy by volt-time curves, but must be represented by volt-time areas.

2. The extent of these areas may be considerable. The spark-over voltage of a string of three suspension insulators for instance, ranges between 430 kv and 620 kv with sparkover taking place at one microsecond, a difference of 45 per cent, or at six microseconds to sparkover the spark-over voltage ranges between 330 kv and 430 kv, a difference of 30 per cent.

3. The great differences in voltages at any given time to sparkover are not due to erratic nature of sparkover, but due to differences in the shape of the applied impulse wave before sparkover occurs.

4. The experimental data show that differences in wave shape can be classified by the ratio of the effective front of the wave to the time to sparkover—T_1/T_2. $T_1/T_2=0$ represents a wave with very steep front and practically rectangular tail while $T_1/T_2=1$ represents a wave with a front rising in a straight line from zero time to the time of

sparkover. Wave shapes where the ratio T_1/T_2 is small are associated with the lower spark-over voltages while straight line voltage rise produces the highest spark-over voltage at any given time to sparkover.

This means that the customary volt-time curves give only a very incomplete spark-over characteristic for any given electrode arrangement. The deviations from such curves are summarized in the following table (data from figure 6) for the 50-inch rod gap.

		Time to Spark-Over—Microseconds				
		2	3	4	5	6
Total spread of spark-over voltage in per cent of lower boundary		44	38	33	28	25
Per cent difference from typical volt-time curve with $T_1 = 1.3$ microseconds	+	13	19	21	21	22
	−	28	16	11	6	2

The maximum and minimum differences between the boundaries of the spark-over areas and volt-time curves depend on the impulse generator connection used for obtaining the curves.

5. The dependence of spark-over voltages on wave shape explains the great difference of spark-over voltage for a given electrode arrangement reported from different laboratories. The differences, in the range of one microsecond and more, are principally due to the difference in types of impulse generators used for the investigations. Although two impulse generators may produce the same wave shape at critical sparkover, at shorter times to sparkover (four microseconds and less) the wave shapes become radically different. One type produces a wave with a short effective front. Therefore, T_1/T_2 is small and the resulting spark-over voltage is low; the other produces a wave with a practically constant effective front. Therefore, T_1/T_2 is large and the resulting spark-over voltage is high.

Thus, this classification of wave shape is a satisfactory means for explaining the large voltage differences observed by various laboratories at times to sparkover of one microsecond or more.

6. The same phenomena which cause the upturn of spark-over volt-time curves are responsible for spark-over volt-time areas. Impulse sparkover in the nonuniform field requires corona currents of considerable amplitude to complete sparkover. The amplitude and the length of time of current flow determine the time to sparkover and the crest voltage of the wave causing sparkover. Therefore, for equal times to sparkover, a wave shape for which corona starting voltage is attained in a short time will require a lower spark-over voltage than a wave shape where corona starting voltage is attained later. Likewise, for equal spark-over voltages, early start of corona current will produce a shorter time to sparkover than a late start of corona currents.

7. Therefore, in the quasi-uniform field, such as the sphere gap, where corona currents are of small amplitude or negligible at times to sparkover of one microsecond or more, the effect of wave shape on the spark-over characteristic is very small, the upturn of the volt-time curves is negligible, and the extent of the spark-over area is negligible. At times shorter than one microsecond the effect of corona current becomes considerable and the sphere gap should exhibit increasing extent of spark-over area and upturn of voltage.

8. Methods of co-ordinating system insulation on the basis of impulse spark-over areas rather than volt-time curves are pointed out.

9. The data presented here are only another step to a more complete understanding of spark-over phenomena. The classification of impulse wave shape by means of a simple ratio of effective front to time to sparkover, while not based on a precise scientific interpretation of the phenomenon, nevertheless gives excellent practical results.

References

1*a*. The Co-ordination of Transformer Insulation With Line Insulation, **V. M. Montsinger** and **W. M. Dann.** AIEE Transactions, volume 51, 1932, pages 923–4.

1*b*. Co-ordination of Insulation, **V. M. Montsinger, W. L. Lloyd, Jr., and J. E. Clem.** AIEE Transactions, volume 52, 1933, pages 417–27.

1*c*. Insulation Co-ordination, **Philip Sporn** and **I. W. Gross.** AIEE Transactions, volume 56, 1937 (June section), pages 715–20.

2. Basic Impulse Insulation Levels, **Philip Sporn** and **C. A. Powel.** AIEE Transactions, volume 59, 1940 (October section), pages 596–8.

3. Flashover Characteristics of Rod Gaps and Insulators, AIEE Transactions, volume 56, 1937 (June section), pages 712–14.

4. Short-Time Sparkover of Gaps, **J. H. Hagenguth.** AIEE Transactions, volume 56, 1937 (January section), pages 67–76.

5. Impulse Voltages Chopped on Front, **P. L. Bellaschi.** AIEE Transactions, volume 55, 1936 (September section), pages 985–90.

6. Discussion on reference 4, **J. H. Hagenguth.** AIEE Transactions, volume 56, 1937 (July section), pages 885–7.

7. Recommendations for High-Voltage Testing, **EEI-NEMA Subcommittee Report.** AIEE Transactions, volume 59, 1940 (October section), pages 598–602.

8. Ionization Currents and the Breakdown of Insulation, **J. J. Torok** and **F. K. Fielder.** AIEE Transactions, volume 49, 1930, pages 352–7.

9. Impulse Voltage Strength of Insulators and Materials, **J. C. Dowell** and **C. M. Foust.** *General Electric Review*, March 1937.

10. Flashover Characteristics of Insulation, **P. H. McAuley.** *The Electric Journal*, July 1938.

11. Lightning Surges on Transmission Lines—Natural Lightning, **W. W. Lewis** and **C. M. Foust.** *General Electric Review*, November 1936, pages 543–55.

12. Resultate der Gewitter Messungen Inden Jahren, 1932–1933, **K. Berger.** *Bull. S.E.V*, April 27, 1934.

Effect of Sapphire-Crystal Orientation on the Wear of Watt-Hour-Meter Bearings

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Synopsis: This paper describes the most recent of a series of studies on the life of watt-hour meter bearing materials. It covers an investigation of the effect of jewel crystalline axis orientation upon jewel wear.

Lifetime stability of meter calibration is a function of bearing wear. A study of the wear (after several years' service) of the sapphire jewel cups of meters in relation to crystalline orientation confirms predictions of another investigator; and a corroborative theory, based upon the physics of crystals, is presented.

Introduction

THE importance of the induction type watt-hour meter in the complex structure of the modern distribution system is fully realized. Accurate metering is necessary because the electrical characteristics and the initial and sustained accuracy of the watt-hour meter are directly reflected in terms of revenue produced. Much work has been done over a long period of years to improve the electrical characteristics and to perfect the mechanical parts. The lower bearing which carries the load of the moving system has long been given close study because it plays a real part in sustained meter accuracy.

It is the purpose of this article to describe briefly the results of work that has shown a marked correlation between the wear of the sapphire jewel cups used in the lower bearings and the direction with which the cups are cut with respect to the sapphire crystal structure. The wear of the jewel cups affects the useful life of the lower bearing and the sustained accuracy of the watt-hour meter.

For many years, it has been generally known that certain crystalline substances exhibit different hardnesses on various crystal faces. In 1931, V. Stott[1] working in the National Physical Laboratory experimented with the wear of sapphire watt-hour meter jewel cups cut at various known angles with respect to the major axis of the sapphire crystal.

In 1936 V. Stott released a further account of work done in the National Physical Laboratory in a special report.[2] This report gave the results on 156-watt-hour meter jewel cups cut in definite crystal directions. The report indicated quite conclusively that there was a definite relation between wear and crystal direction. The apparatus used in making these tests was described in the first paper[1] and while it was designed especially for the work, represented a laboratory test which gave different operating conditions than a bearing experiences in a watt-hour meter.

It is desirable at this point, in order to understand properly the meaning of the term, "orientation" which is used to designate the direction with which the sapphire cup is cut with respect to the crystal axes, to consider briefly the subject of crystals and crystal structure.

Crystal Structure

Sapphire is crystalline alumina, Al_2O_3 and is a form of the mineral corundum. It is crystalline in structure and belongs to the hexagonal crystal system.[3] Sapphire is found in nature usually in rough rounded pebble-like masses, though frequently outer crystalline forms are evident. The material is also made synthetically by the Verneuil[3] process.

Figure 1*B* shows a wooden model of a sapphire crystal of hexagonal prism shape. This shape is frequently found in nature. The axis of the long direction of the crystal is defined for the purpose of identification as the *C* axis and is the reference axis of the crystal that will be used for this discussion. This axis will be called the vertical axis and is parallel to the optic axis of the crystal. Figure 1*A* shows a cross section of a watt-hour meter jewel cup cut with the axis of the cup at right angles to the *C* axis. This cup is said to be oriented to an angle of 90 degrees ($\theta = 90$ degrees). Figure 1*C* shows another cross section of a cup where the *C* axis of the crystal and the axis of the

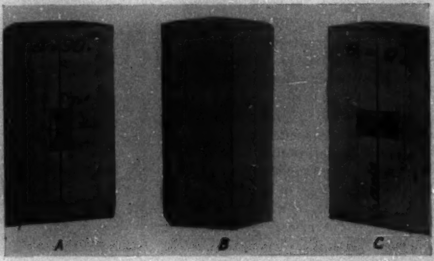

Figure 1. Models of sapphire crystal showing orientation angles of watt-hour-meter jewel cups

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1. For all numbered references, see list at end of paper.

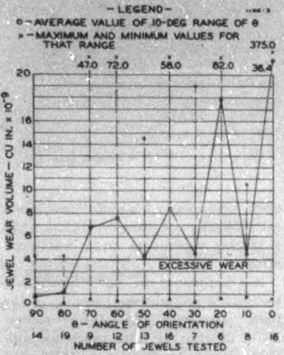


Figure 2. Jewel wear versus orientation

Composite curve of all jewels

jewel cup coincide. The orientation angle is 0 degrees ($\theta=0$ degrees).

If a crystal is considered as being made up of small particles arranged in an orderly fashion, then a material that is noncrystalline, such as glass, may be thought of as being made up of particles with no systematic order of arrangement. A crystal that is allowed to form under ideal conditions will exhibit a definite geometric structure such as has been used in figure 1 to show one crystal form of sapphire.

If a sphere is formed from such a crystal as shown in figure 1, as might happen if it rolled in the bed of a stream for many years, it would be impossible to identify the substance as crystalline from its external shape. This is similar to what happens when synthetic sapphire boules are formed as they are more or less cylindrical in shape and exhibit few flat faces that make possible the identification of the boules as crystalline. Synthetic sapphire boules may give the appearance of globules of glass, yet actually they are identical in crystal structure to a piece of natural sapphire formed in nature which exhibits a perfect crystal shape as shown by figure 1. In the absence of the geometric form either optical methods[4] or X-ray[5] analyses can be used to orient and identify the crystal structure.

The most common method of identifying a sapphire crystal structure is to subject it to an analysis with polarized light and make use of the interference patterns which form definite optical figures.[4] The basal plane of the sapphire crystal may be determined by means of the optical figure. This is the plane in which all axes are equivalent. The axis at right angles to this basal plane is the vertical axis or *C* axis. This is the method of identification and orientation used in obtaining this experimental data.

Experimental Data

Following the publication of the material by V. Stott, it was desirable to check these results as the published data definitely indicated that an improvement in the life of watt-hour meter bearings could be obtained by controlling the orientation of the jewel cup to within the angle $\theta=80$ degrees to 90 degrees. It was impractical to set up life tests with watt-hour meter jewels with known orientations and run them for a period of five to ten years to get this information. There were available in the laboratory several hundred watt-hour meters which had been operated for periods of five to six years under known conditions with jewel cups that had been cut at random with respect to the crystal direction. The logical procedure was to discontinue some of these life tests, examine the bearings for wear, and determine the orientation of each individual jewel cup. A system of measuring the orientation angle was devised and approximately 150 bearings were taken from life tests and examined. The wear was estimated from measurements made by means of a calibrated eyepiece in a 40 diameter magnification microscope. These data are plotted as a composite curve in figure 2. Here the jewels were grouped in accordance with their orientation angle, and the circles represent the average wear value for a particular angular division. The minimum wear and the maximum wear for the angular division are shown by the crosses. It is interesting to note that there is a definite trend of increased wear as the jewels drop from an orientation of 90 degrees to an orientation of 0 degrees. In all cases the jewels oriented within 10 degrees ($\theta=80$ degrees to 90 degrees) showed minimum wear and fortunately 33 jewels were found within this angular range. Figure 2 includes bearings from life test watt-hour meters with both natural and synthetic sapphire jewel cups. A further breakdown of these data is shown by figure 3 which shows the results for the synthetic sapphire cups and figure 4 which shows the results for natural sapphire cups. All of the watt-hour meters from which these bearings were removed were operating to simulate normal load conditions and were equipped with high carbon steel pivots in well-lubricated bearings.

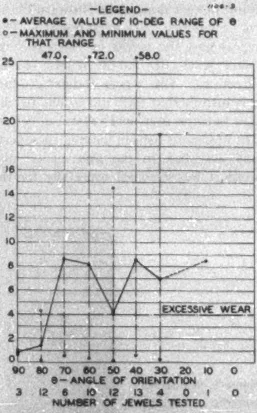


Figure 3. Jewel wear versus orientation

Composite curve of synthetic sapphire jewels

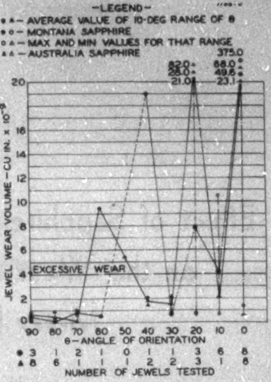


Figure 4. Jewel wear versus orientation

Composite curves of natural sapphire jewels

Natural Versus Synthetic Sapphire

The experimental evidence shows the desirability of controlling the orientation angle as advocated by V. Stott.[2] It is impractical to orient natural sapphire jewel cups due to the fact that natural sapphire is frequently found in small pieces so that it is possible to make but two or three jewel cups from a single piece of raw material. The synthetic sapphire boules are quite large, and it is possible to make many jewel cups from one boule. As the optical procedure for determining the crystal direction of a boule is identi-

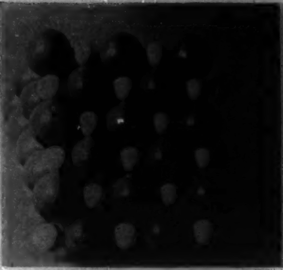


Figure 5. Atomic model of sodium chloride

Scale = two angstroms to one inch

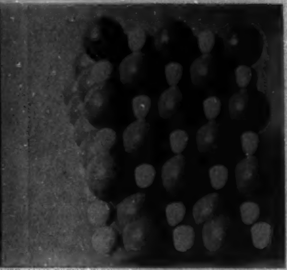


Figure 6. Atomic model of sodium chloride

Scale = two angstroms to one inch

cal with that required for determining the crystal direction of a small fragment of natural sapphire, it is obvious that it is much more expensive to make the determination for natural sapphire on a commercial basis than on synthetic sapphire.

A careful review of extensive life test data on watt-hour meter jewel cups of random orientation, operating at both accelerated and normal speed, had already led to the conclusion that taken as a whole, natural sapphire and synthetic sapphire were giving equivalent performance.

Theory of Orientation

It was suggested that the difference in wear exhibited by the jewel cups cut in the different crystal directions was due to a phenomenon known as translation gliding in crystals. This is demonstrated in certain crystals, as for example rock salt. If the crystal is placed under tension or compression under controlled conditions, one portion of the crystal moves with respect to the other along a plane and rupture does not occur. No fractures are discernible even with polarized light or X-ray diffraction after the crystal has been stressed, even though one portion actually overhangs another portion. A theoretical explanation for this is shown most clearly by figure 5 and figure 6. Figure 5 is an atomic model of salt (sodium chloride, NaCl) actually built to a scale of one inch equals two angstroms (125 million to one). The large balls represent atoms of sodium. An examination of the photograph will show the nesting of the sodium balls between the chlorine balls. The model pictured consists of two parts. The bottom is a single layer, and the top is a triangular pyramid. The model represents but a small part of a crystal and must be thought of as repeating itself in all directions to form a crystal of salt. If the pyramid portion is moved with respect to the bottom layer in any direction except the direction represented by the top and bottom of the page, it will be noted that the chlorine balls tend to ride over and onto the chlorine balls in the bottom layer. As the chlorine particles are negatively charged, we have two negatives coming together with the result that a repelling force is present. If they are forced to move in that direction, the result will be that the negative charges will force the crystal apart. If, however, the pyramid is moved in the direction of the top and bottom of the page, it will be seen, as shown by figure 8, that a new position is assumed which is identical in the relation of the sodium and chlorine balls to that shown by figure 5, except that a portion of the bottom layer has been exposed. This portion of the bottom layer that has been exposed represents, therefore, the portion that is overhanging. This is the crystallographer's explanation of gliding and is substantiated by accepted scientific proof based on X-ray diffraction studies. This is, of course, a very simple example and is not intended to serve as an explanation for the orientation wear effect in sapphire.

Sapphire material has a much more complex crystal structure than salt, which is of the cubic form, the simplest crystalline form known. A model of sapphire was built at the Massachusetts Institute of Technology, under the direction of Professor M. J. Buerger.[10] Figure 7 shows the sapphire model which was made to the same scale as the sodium-chloride model. In figure 7, the reference *C* axis is represented by a vertical line joining the two hands. Figure 8 shows how the model was built to separate. The top half of the model is separate from the bottom half and may be moved with relation to it. While it cannot be seen from the photographs, the model demonstrates that gliding can occur. The gliding is in a plane at right angles to the *C* axis in sapphire.

If the assumption is made that the phenomenon of gliding is an indication of loose atomic bonding on that particular plane, a possible explanation of the results obtained experimentally can be given. That is, particles of the crystal may be more easily removed in this plane of the crystal than in other planes, because the atoms are held more loosely in place than in other directions. Crystals may have more than one plane of gliding, but in sapphire, we are primarily concerned with the plane at right angles to the *C* axis as borne out by the experimental evidence.

Consider a watt-hour meter jewel cup

Figure 7. Atomic model of sapphire

Scale = two angstroms to one inch

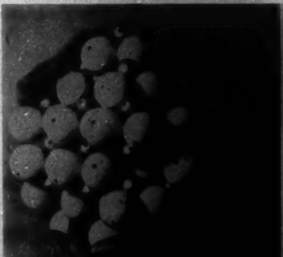
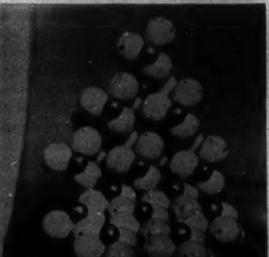


Figure 8. Atomic model of sapphire

View showing internal arrangement of atoms

Scale = two angstroms to one inch



The Dielectric Strength of Glass—an Engineering Viewpoint

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Synopsis: The purpose of this paper is to provide the engineer with co-ordinated information and data regarding the dielectric breakdown of glass which will permit him to apply this insulating material more effectively for high dielectric strength. In the discussion of the subject, particular attention has been paid to "edge effects" and other factors which cause wide variations in breakdown voltages—variations which are responsible for many of the seeming inconsistencies found in published results on the dielectric failure of glass and other materials.

The author has concluded that it is possible to make a reasonable approximation of the breakdown voltage of glass, if the conditions of the test or of the application can be sufficiently well determined.

The characteristic breakdown curves of this paper do not represent the results of any one series of tests, but are rationalized from data selected from many sources, including unpublished tests.

The data curves included refer mainly to Pyrex* glass, but corresponding data have also been included on electrical porcelain because of the interesting comparisons shown.

I. Dielectric Failure of Glass

Within the last few years the physicist has made pronounced progress in the study of dielectric failure, but in spite of this progress, our knowledge of the phenomena involved is still very incomplete. It is generally considered that the dielectric failure of insulating materials, such as glass and porcelain, may occur in one of two ways:

(*a*). In disruptive breakdown
(*b*). In thermal breakdown

Disruptive failure is one which results directly from an electrical overstress of the dielectric material without perceptible internal temperature rise. Recent research indicates that disruptive failure is caused by electronic ionization by collision within the molecular structure of the material.[1] In actual practice it is probable that pure disruptive failure occurs only under somewhat special conditions, and accurate test values of intrinsic dielectric strength are not easily obtained, even with the use of elaborate and careful procedure. The disruptive breakdown voltage is roughly proportional to the thickness, and may be expressed in volts per mil, or kilovolts per centimeter. Table I gives representative data for several materials which have been selected from various sources.

Table I. Disruptive Strength

Material	Disruptive Strength Kv/Cm. (Peak Value)
Lime glass	3,400– 4,500
Lead glass	3,100
Borosilicate glass (Pyrex)	3,900– 4,800
Fused quartz	5,000– 8,000
Electrical porcelain	320– 450
Mica	8,000–10,600

Thermal failure is one which results from the increased temperature of the insulation caused by the applied electrical stress. As the internal temperature rises the losses also increase, which, in turn, raise the temperature still more. Under certain circumstances a cumulative action of this kind may proceed until instability is reached, the current and losses rise suddenly, which represents the thermal failure of the insulation.[2]

Whether the limit of disruptive strength or that of thermal strength is reached first depends upon a number of conditions. In cases of thin glass sections, allowing the rapid dissipation of losses to the electrodes, of short periods of test, and of low ambient temperatures, the failure tends to be of the disruptive type; for the reverse conditions the failure tends to become thermal. When a series of tests is made with samples of successive steps of thickness, or at increasing temperatures, a transition from disruptive to thermal failure is found to

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1. For all numbered references, see list at end of paper.

* Pyrex is a registered trade-mark of the Corning Glass Works and is applied to a group of products rather than to a glass composition. In this paper, glasses identified by Corning as 772, 774, and 776, etc., are referred to.

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with a pivot bearing running in it.[6,7] Then assume a worn point on the pivot which is abrading the jewel surface. If the jewel cup is cut with an orientation $\theta=0$ degrees (figure 1), the pivot is abrading the jewel cup in the most favorable direction for removing material, and it is reasonable to expect that the pivot will find it easier to remove particles of the jewel cup than if the angle $\theta=90$ degrees. When $\theta=90$ degrees the pivot is abrading the crystal right angles to the favorable direction in which material can be removed and the particles of sapphire may be thought of as in a locked position.

Conclusions

The experimental evidence demonstrates that orientation is a definite factor in the wear of watt-hour meter jewel cups, and its control represents another step in insuring longer bearing life.

The experimental data confirm V. Stott's findings that for optimum results, the angle θ should be controlled to within $\theta=80$ degrees to $\theta=90$ degrees.

A possible explanation based on elementary crystal theory confirms the experimental evidence.

References

1. Investigation of Problems Relating to Use of Pivots and Jewels in Intruments and Meters, V. Stott. National Physical Laboratory, volume 24, 1931.

2. Technical Report *T/T25*, British Electrical and Allied Industries Research Association.

3. Gems and Gem Materials, E. H. Kraus and E. F. Holden. McGraw-Hill Book Company, Inc., Copyright, 1925.

4. Elementary Treatise on Physics Experimental and Applied, E. Atkinson. Translated from Ganot's Elements de Physique, William Wood and Company, 1910.

5. Theories and Definitions of Hardness, S. R. Williams. *Instruments*, February 1937.

6. Lubrication Increases Life of Meter Bearings, T. A. Abbott and J. H. Goss. AIEE Transactions, volume 54, 1935 (April section), pages 428–31. Discussion, 1935 (September section), pages 992–3.

7. Watt-hour Meter Bearings, I. F. Kinnard and J. H. Goss. AIEE Transactions, 1937 (January section), pages 129–37.

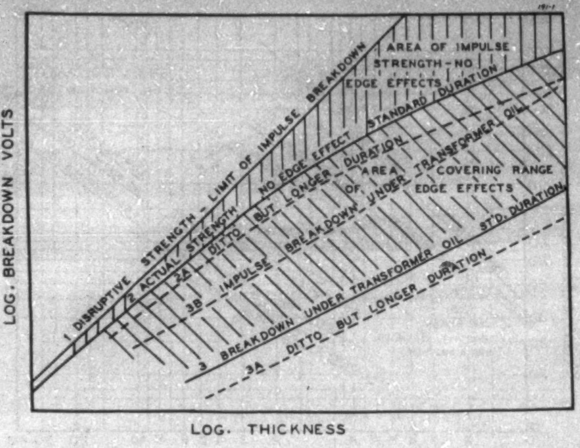
8. Experiments Bearing on the Orientation of Quartz in Deformed Rocks, D. Griggs and J. F. Bell. November 1, 1938. *Bulletin* of the Geological Society of America, volume 49, pages 1723–46.

9. Kunstliche Schiebungen und Translationen in Mineralien, Von Kurt Veit in Kiel, Neuen Jahrbuch für Mineralogie, Geologie und Palaontologie, S. 121–148 und Taf. 1, Stuttgart, 1921, E. Schweizerbart'sche Verlagsbuchhandlung.

10. A Technique for the Construction of Models Illustrating the Arrangement and Packing of Atoms in Crystals, M. J. Buerger and Robert D. Butler. *American Mineralogist*, volume 21, number 3, March 1936.

occur. Certain tests of this sort made on very thin samples of glass have indicated that this transition occurs abruptly. For thicker samples, such as would apply to engineering practice, the indications are that the internal temperature effects within the sample modify the characteristics of disruptive failure very materially before the conditions of thermal failure are reached so that the transition becomes much less apparent.

Curve 1 of figure 1 represents the pure disruptive breakdown voltage of a material such as glass or porcelain as a direct function of thickness. Curve 2 represents the breakdown voltage as modified by thermal effects. For the lower part of the curve the failure is still considered to be substantially disruptive, but for thick sections it will be purely thermal. No definite point of transition is indicated by this curve.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AREA OF IMPULSE STRENGTH - NO EDGE EFFECTS
1 DISRUPTIVE STRENGTH - LIMIT OF IMPULSE BREAKDOWN
2 ACTUAL STRENGTH NO EDGE EFFECT STANDARD DURATION
2A DITTO BUT LONGER DURATION
3B IMPULSE BREAKDOWN UNDER TRANSFORMER OIL
AREA OF COVERING RANGE EDGE EFFECTS
3 BREAKDOWN UNDER TRANSFORMER OIL ST'D. DURATION
3A DITTO BUT LONGER DURATION
LOG. BREAKDOWN VOLTS
LOG. THICKNESS

Figure 1. Graphic representation of breakdown characteristics

II. Factors Governing Breakdown

The principal factors governing breakdown may be listed as follows:

1. Characteristics of the dielectric material
2. Thickness of the section stressed
3. Temperature of the material
4. Duration of application of voltage
5. Characteristics of the applied voltage
6. Electrostatic field distribution in the dielectric and other "edge effects"

1. Wide differences in the characteristics of dielectric materials are indicated by the disruptive strengths listed in table I. Thermal failure is determined largely by the electrical and thermal conductivities of the material, and the variation of these properties with temperature. Glass and porcelain have widely different characteristics in disruptive failure, although in thermal failure these differences are much less pronounced.

2. The rate of change of breakdown voltage with thickness of section is dependent upon a number of conditions. As already stated, the breakdown voltage in pure disruptive failure may be considered to vary directly with the thickness. Mathematical analyses of thermal failure show that for thin sections the breakdown voltage will vary as $\sqrt{\text{thickness}}$, but that as the thickness is increased, this rate of increase is not maintained, and that a limiting condition is reached where a further increase in thickness will not result in any increase in the breakdown voltage. In other words there is a maximum voltage which a single thickness of insulation will withstand, regardless of its thickness. For Pyrex glass this limit has been calculated to be approximately 5×10^6 volts.

When materials such as glass and porcelain are immersed in insulating oil, this rate of change of breakdown voltage with thickness is determined mainly by the geometrical configuration of the insulator and the electrodes. For flat plates with thin electrodes, such as metal foil, the voltage will vary as $\sqrt{\text{thickness}}$ (see figure 1, curve 3). For power insulators of conventional design this ratio becomes approximately $(\text{thickness})^{0.7}$, as is indicated in figure 4.

3. Information regarding the quantitative effect of temperature on dielectric failure is not as complete as would be desired. However, various groups of data for both glass and porcelain indicate that in passing from room temperature (20 degrees centigrade) to 100 degrees centigrade, a reduction in breakdown voltage varying from 20 per cent to 40 per cent may be expected. At temperatures of 350 degrees centigrade or more these materials will have little remaining dielectric strength.

4. The period of time during which the voltage is applied to a dielectric is an important factor in determining the resulting breakdown voltage. Tests made under different procedures in this respect are not directly comparable. In general, it is found that for periods of stress extending from a few microseconds to an appreciable fraction of a second there is little change in the breakdown voltage, but from this point to a period of several minutes the reduction may be of the order of 50 per cent. Beyond this point to indefinite periods of time, further reductions are not great. This time characteristic of breakdown is probably connected with the stabilization of the internal temperature of the dielectric.

In figure 1, the disruptive strength (curve 1) represents the limit of breakdown voltage on impulse tests, curve 2 may be taken to represent the results of tests taken with the voltage raised continuously at the rate of 667 volts, rms per second, which is the standard procedure for insulator testing, while curve 2*A*, may be taken to represent the procedure of raising the voltage in one-minute steps, as is standard for apparatus testing. Curves 3*B*, 3, and 3*A*, are corresponding curves for other test conditions.

The breakdown voltage as measured by the one-minute hold test is usually of the order of 15 per cent to 25 per cent lower than when measured by the standard continuously rising voltage procedure.

5. The question of voltage, whether d-c or a-c, and if the latter, its frequency, must be considered. Evidence indicates that if the failure is purely disruptive, the breakdown is determined by the peak value of the voltage. However, the dielectric losses produced by alternating stresses, even at commercial frequencies, are much greater than the leakage losses caused by direct current, so that in the former case a lower breakdown voltage may be expected, except for very special conditions, such as where very thin sections are involved. For normal sections and commercial frequencies, this reduc-

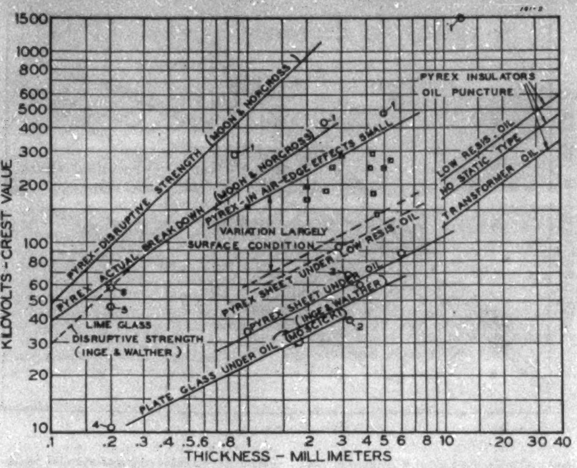


Figure 2. Selected dielectric breakdown data for glass

tion may be of the order of 20 per cent (peak value of the a-c voltage being considered).

As the frequency is increased the dielectric losses increase and the breakdown voltage drops. Some tests have indicated that at a frequency of 10,000 cycles the breakdown voltage may be about 40 per cent of that at 50 or 60 cycles, while for frequencies of the order of 1 megacycle this ratio may drop to less than 10 per cent.

6. In the actual application of insulation, it is not possible to avoid conditions which lead to local field concentrations and similar effects which reduce the breakdown voltage. These are generally referred to as "edge effects," because they were first observed at the edges of the electrodes, although they are not limited to these localities. Even in elaborate tests it is impossible to eliminate edge effects completely, and many discrepancies in the published data on the breakdown of glass result from this cause.

In addition to the edges of electrodes, these effects are found at points of imperfect contact between the electrode and the insulation, scratches in the electrode surface (when metal foil, or a silvered surface is used), voids or lack of homogeneity in the insulation itself, at changing sections of the insulation, and at imperfections of the surface of the dielectric, etc.

One of the principal reasons for the low disruptive strength of porcelain as compared with glass, is believed to be connected with the nonhomogeneous nature of the porcelain body which results in local field concentrations within the material. Glass, on the other hand, is homogeneous.[†] As a consequence of this, porcelain is much less sensitive than glass to certain edge effects of external origin. For instance in tests on glass it was found that with a rough ground surface under the electrodes, the breakdown voltage was only 16 per cent of the value obtained on a corresponding sample with polished surfaces. In the case of porcelain there was little difference in the two tests, and the breakdown voltages obtained on the porcelain samples were of the same order as those obtained on the rough ground glass samples of the same thickness.

Edge effects which result in local concentrations are ordinarily much less serious in the case of thermal failure than of disruptive failure. The temperature conditions resulting from the presence of local areas of high stress concentration will often produce a redistribution and partial equalization of stress, which does not occur when the failure is purely disruptive. Thus, within certain limits, thermal breakdown is in some degree analogous to the mechanical breaking of a ductile material such as soft steel, while the disruptive breakdown is more analogous to the tensile failure of a brittle material such as cast iron. Under certain conditions of test where edge effects are known to be present, a point of failure under the electrode, and away from the edges has been taken as indicative of a thermal failure, while a point of failure at the electrode edge or beyond the edge has been considered indicative of a disruptive failure. This distinction must be used with considerable care, as it is far from being a certain criterion to distinguish these conditions.

In order to avoid these edge effects it it important that the insulation used be without voids or surface imperfections, that the electrode should make a good contact with the insulation at all points, and that the stress at the electrode edges be equalized in some way. One effective method of reducing the edge stress is to thicken the insulation at the point where the electrodes terminate, without sudden changes in section.

Another type of edge effect not yet referred to specifically is one connected with certain phenomena in the ambient medium, such as insulating oil. Dielectric tests are frequently made under oil in order to eliminate flashover of the insulation sample. With the usual procedure for such tests, the electrostatic field at the electrode edges will fringe out into the oil. Insulating oils have a dielectric constant much lower than those of glass and porcelain so that the electrical stress in the oil will be materially higher than otherwise. Under these conditions corona discharges will form in the oil long before the true breakdown strength of glass is approached. As the voltage is raised beyond the corona point of the oil, streamers develop which will extend over the glass or porcelain. At the terminating point of these streamers on the surface of the insulation intense voltage gradients will exist, causing disintegration of the point attacked. If this action is allowed to continue for a sufficient time, it is probable that a hole will be drilled completely through thick sections of glass or porcelain causing dielectric failure.

This type of failure is determined almost entirely by the breakdown of the oil, and does not constitute a valid test of the insulating material, except for those cases where the insulation is to be oil-immersed in actual use.

Corresponding samples of various glasses and porcelain and of mica with wide differences of dielectric characteristics have been found to puncture at essentially the same voltage under this test, which demonstrates that breakdown is not a function of the properties of the insulating material.

[†] This, of course, is relative. Glass bodies may contain "stones, seeds, or blisters," which are not homogeneous with the remainder of the material. Any serious fault of this character, however, can be eliminated readily by visual inspection.

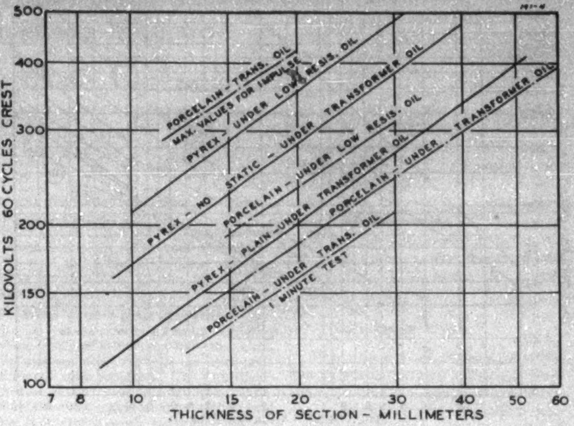


Figure 4. Oil puncture tests on power insulators

60-cycle voltage, raised continuously at 667 volts (rms) per second

Certain investigators state that when streamers occur in the oil the discharges have been found to be highly oscillatory, and that even if the puncture takes place under the area of the electrodes, the measured breakdown voltage may not be a true indication of dielectric strength because of the presence of the high frequency component of voltage.

One of the means used in overcoming this condition, at least to some extent, is by the use of semiconducting liquids in place of regular insulating oil. Oil with a high water content, mixtures of xylol and alcohol, etc., are effective in preventing the formation of corona to a very considerable extent. Semiconducting surfaces on the insulation adjacent the electrodes have also been used to produce the same general effect. In many practical applications the form of the insulation and the electrode can be selected so that the normal voltage stress in the oil is below its corona point. This procedure is commonly employed in the design of oil-immersed transformers. In this case, however, the insulation is usually composed of organic material.

There are still insulation test specifications which call for the oil immersion of the test piece, and many such tests of questionable validity are still made. For this reason an appreciation of the edge effects resulting from the ambient oil is of particular importance. As an instance of this, the oil puncture test is the standard procedure for testing power-line insulators, in spite of the fact that it is generally recognized that the results so obtained are not representative of their performance in service. Figure 4 gives average breakdown data as obtained from a large number of tests made on both Pyrex and porcelain insulators. The curves show that the puncture voltage may be raised either by reducing the resistivity of the oil or by using a semiconducting film on the insulator surface. Tests on Pyrex insulators indicate that, even when low-resistivity oil is used, the puncture results from edge effects, rather than from overstress of the glass.

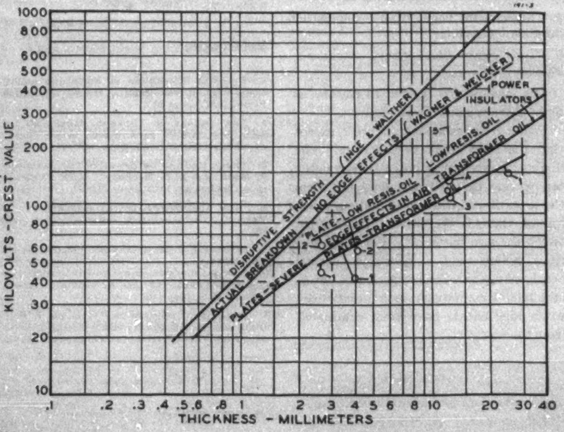
III. Curve Data

Figure 1, representing the general dielectric characteristics of an insulating material such as glass or porcelain has already been discussed. It is to be noted, however, that a single group of such curves will not apply to all conditions. For instance, curve 1, the disruptive strength, may vary in some way with temperature. Curves 2 and 3 will shift to higher or lower voltages with variations of such factors as temperature, frequency, and duration of stress. These two curves represent certain limiting conditions of edge effect, so that for specific cases the relative severity of this factor must be estimated. This is necessarily very much of a guess, unless the engineer has a considerable background of experience to call upon. For actual design purposes there is the additional question of the proper factor of safety to be considered. This will, of course, depend upon the requirements of the type of application in view.

The data presented in the following figures refer to tests made at room temperature, and except when indicated otherwise, the values are for 60-cycle voltage raised continuously at the rate of 667 volts (rms) per second. In the case of some of the published data it was necessary to adjust the breakdown voltages to the condition. It will be noted, furthermore, that all voltages are expressed as peak values, rather than rms values.

Figure 2 includes a few representative

Figure 3. Selected dielectric breakdown data for porcelain



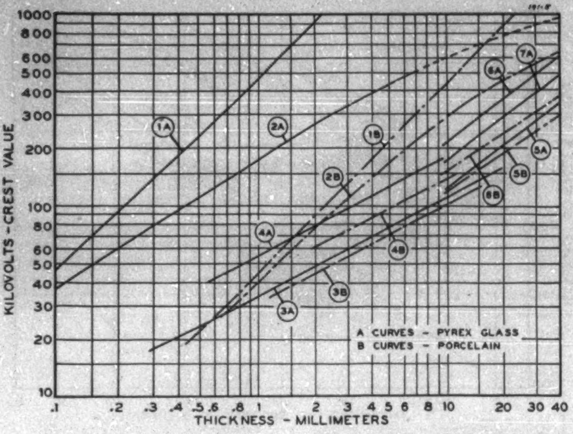


Figure 5. Dielectric breakdown characteristics of Pyrex glass and porcelain

Conditions—Room temperature and for 60-cycle voltage raised continuously at 667 volts (rms) per second

Curve 1—Disruptive strength

Curve 2—No edge effects

Curve 3—Under transformer oil

Curve 4—Under low-resistivity oil

Curve 5—Power insulators under transformer oil

Curve 6—Power insulators under low-resistivity oil

Curve 7A—Pyrex "no static" iridized insulators

data for glass. These are mainly for Pyrex glass, although some values for lime glass are included.

The disruptive strength of Pyrex glass, and also the breakdown values as actually modified by temperature effects are taken from the data of Moon and Norcross.[3] The results indicated by the square points were obtained from tests made by the Corning Research Laboratory on pin-type Pyrex insulators with the heads ground thin and polished to permit their puncture in air without flashover. Voids between the outer electrode and the insulator head, together with other surface irregularities, produced edge effects which were largely responsible for the wide difference in the breakdown values shown. Values indicated[1] represent impulse tests on Pyrex samples. Edge effects were experienced in these tests as well.

Other tests of the Corning Research Laboratory on plate glass under oil (point 2) check closely with the early data of Moscicki.[4] These tests were repeated with the use of semiconducting oil, with the result indicated by point 3. Corresponding tests made on Pyrex plate are seen to give materially higher breakdown voltages. The reasons for this difference are probably the lower dielectric constant of this glass (which will reduce the field strength in the oil), and the higher thermal resistance of Pyrex glass.

Data on lime glass, points 4, 5, and 6, are taken from the many tests of Inge and Walther.[5] They are for the following conditions:

4. Tests made under insulating oil
5. Tests with edge effects eliminated; duration ten seconds
6. Tests with edge effects eliminated; impulse tests

The curves for Pyrex insulators in figure 2 are replotted from figure 4.

Figure 3 corresponds to figure 2, except that the data apply to porcelain. The disruptive strength of porcelain is roughly ten per cent that of Pyrex glass, but when the breakdown is measured under insulating oil, there is very little difference between the two materials. For thick sections, which tend to result in thermal breakdown, the values of glass and porcelain tend to approach each other, even when edge effects have been eliminated from the test.

Points marked 1 and 2, for immersion in low and high conductivity oil respectively, refer to one-minute tests, and are consequently lower in value than the breakdown voltages taken with continuously rising voltages.

Points 3, 4, and 5 represent data published by Bellaschi and Manning[6] on porcelain insulator shells,

3. One-minute hold tests
4. Tests made with continuously rising voltage
5. Represents the limits of breakdown on impulse tests

The curves on power insulators have been replotted from figure 4 which has been discussed already.

Figure 5 combines the dielectric characteristics of Pyrex glass and electrical porcelain on one sheet. These characteristics are indicated in the same manner as figure 1, and the data are taken substantially from figures 2, 3, and 4, with minor modifications.

The information correlated in these curves, and in the data on other specific conditions mentioned above, is admittedly not developed to a point where an accurate prediction of the voltage at which Pyrex glass and porcelain will fail under certain conditions of test or of practical application, but it is believed that rough approximations can be made, and furthermore, that the methods proposed will allow the insulation engineer to obtain a better oriented view of the problems involved, and how the materials discussed might be used to better advantage. It is also hoped that with time, additional test results will allow the approximations to be made with improved accuracy.

References

1. Electric Breakdown of Solid and Liquid Insulators, **A. von Hippel.** *Journal of Applied Physics*, 1937, page 815.

2. The Theory of Thermal Breakdown of Solid Dielectrics, **P. H. Moon.** AIEE Transactions, volume 50, 1931, pages 1008–21.

3. Three Regions of Dielectric Breakdown, **P. Moon and A. S. Norcross.** AIEE Transactions, volume 49, 1930, pages 755–62.

4. Über Hochspannings—Kondensatoren, **J. Moscicki.** *Elektrotechnische Zeitschrift*, 1904, page 527.

5. Durchschlag von festen Isolatoren in homogenen und nicht homogenen elektrischen Feldern bei Beanspruchungen von langer und kurzer Dauer, **L. Inge and A. Walther.** *Archiv. f. Elekt.*, 1929, page 410.

6. Dielectric Strength of Porcelain, **P. Bellaschi and M. Manning.** AIEE Transactions, volume 58, 1939 (December section), pages 651–5.

Overvoltages in Polyphase Induction Motors During Single-Phase Operation

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Introduction

IN the course of the starting, reversal, or normal operation of a polyphase wound-rotor induction motor, it occasionally happens that either the stator or the rotor or both may be allowed to operate single phase. In some cases it has been noted that the voltage of the open terminal or terminals under these conditions has a steady-state peak value well above its normal maximum. While scattered references to this effect may be found in the literature, and though the general subject of single-phase operation of polyphase induction motors has been discussed from time to time, apparently the magnitude of these voltages has not been the subject of a thorough and organized investigation. This paper is based on the results of a study, undertaken to determine analytically the maximum theoretical voltages of a three-phase induction motor under single-phase operating conditions, and, so far as possible, to check these calculated figures against actual test results.

Conclusions

The following conclusions were drawn from this study, for motor operation only; in general, high-voltage conditions would be worse for generator operation:

1. When a three-phase induction motor is operated from single-phase lines, the voltage from the open to either of the active terminals cannot exceed its normal three-phase value.

2. When a three-phase wound-rotor induction motor is operated with one rotor line open-circuited, the voltage from this open terminal to one of the active rotor terminals may reach a theoretical maximum of 2.3 times the rotor open-circuit voltage, assuming zero rotor resistance. This maximum would occur if the stator phase rotation were reversed and the secondary resistance were about 1.3 times the total leakage reactance referred to the rotor. The ratio of 2.3 for the terminal voltages is somewhat reduced in case part of the secondary resistance is internal.

3. When a three-phase wound-rotor induction motor is operated from a single-phase supply with one rotor line open, high voltages can occur on both the rotor and the stator open terminals. The theoretical maxima of these voltages, with resistance and saturation neglected, occur slightly below synchronous speed and recur twice in each slip cycle. The maximum theoretical line-to-line voltage on the rotor is $\sqrt{3}X_m \div 2X_0$ times the normal open-circuit rotor voltage (peak values); that on the stator is $\sqrt{3}(X_m - X_0)/2X_0$ times the normal line-to-line stator voltage (peak values).

4. The rotor voltage is somewhat analogous to the armature voltage of a synchronous machine with a single-phase short circuit.[1] In fact, the expression given here for the rotor voltage agrees with a previously derived expression for the transient voltage of the open terminal of a synchronous machine.[2] The stator open-terminal voltage in the induction motor has no counterpart in the normal synchronous machine. The presence of transient voltages in the induction motor of even higher magnitude than the steady-state voltage seems to correspond to the synchronous machine case when switching occurs at such time as to trap a d-c flux component in the short-circuited winding.

5. The magnitudes of voltages usually experienced are much lower than those calculated, because of various factors such as resistance, saturation, and supply system impedance, which were omitted to make the calculations feasible. In motors of small internal percentage resistance, the calculated values are most closely approached.

6. The conditions of operation and the abnormal voltages discussed in this paper can and do occur with existing systems of control of wound-rotor induction motors. They may be aggravated in cases in which one or the other of the open circuits is of a vibratory, or rapidly recurring, nature. An easy and effective safeguard is to avoid opening completely any one of the rotor lines, as tests show the voltages to be greatly reduced with even a relatively high resistance in the odd rotor phase.

Theoretical Investigations

A. SINGLE-PHASE STATOR

The three-phase induction motor with single-phase excitation has long been studied from several standpoints: as a special case of an unbalanced polyphase machine,[3,4,5] as one type of single-phase motor,[6] and as a single-phase to three-phase phase converter.[7] Operation of this nature may readily be visualized by means of the revolving-field theory of Ferraris and its equivalent expression in terms of symmetrical components or by means of the two-axis method of resolution.[8] Both theoretical considerations and experience show that for this type of operation the open stator terminal will not reach a steady-state voltage greater than normal, so long as the rotor is symmetrical or nearly so.

B. SINGLE-PHASE ROTOR

When a three-phase wound-rotor induction motor is operated with three-phase excitation and a single-phase secondary, the analysis becomes somewhat more difficult than for the case just considered. Nevertheless, adequate solutions have been obtained by the method of symmetrical components[4,5] and by the two-axis method;[9] this is also a special case of "internal cascade" operation,[10] in which tertiary nonsynchronous currents flow through the supply system, and are determined by its total impedance.

For purposes of visualization, with a sinusoidal terminal voltage the stator impressed field is of constant magnitude and rotates at synchronous speed with respect to the stator; thus, neglecting the resistance and leakage reactance drops, a maximum of twice normal voltage could be induced in the rotor if the stator phase rotation were reversed suddenly while the rotor was turning at full speed. A finite impedance connected between two of the rotor terminals allows a single-phase secondary current to flow with no direct mutual reaction on the open rotor phase, which thus keeps substantially twice normal induced voltage. Assuming zero machine resistance and a noninductive secondary resistor, the approximate secondary voltage vector diagram would be as indicated in figure 1, in which point 1 represents the open rotor terminal and points P and Q the rotor terminals shorted through the external resistor, point O being the center of the resistor. As the secondary resistance varies from infinity to zero, P and Q trace out the semicircles indicated. The maximum line-to-line voltage thus is seen to be about 2.3 times normal, for an external resistance of about 1.3 times the total leakage reactance. To account for the rotor resistance, points P and Q should be located on the same line, but a little closer together; they thus trace out

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The authors wish to thank H. Maxwell for providing the opportunity of making this study and arranging for the tests, and J. A. VanLund for assistance in taking tests.

1. For all numbered references, see list at end of paper.

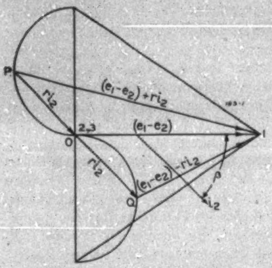

Figure 1. Vector diagram of rotor voltages, single-phase rotor, polyphase stator

curves slightly inside of the semicircles shown.

The conclusions drawn from this visualization are confirmed by an analysis by the two-axis method (appendix *A*).

C. Single-Phase Rotor and Single-Phase Stator

The operation of an induction motor with single-phase stator and rotor has been the subject of study for some time, both qualitatively and quantitatively. Steinmetz[7] discusses this connection under the heading of reaction machines, of which it is one type, and points out a number of interesting characteristics particularly associated with synchronous operation. Lyon[4] makes an analysis by symmetrical components, applying principles found useful in the case of the other types of single-phase operation. This analysis has been extended in a later paper,[11] which gives the equivalent circuit in the form of a ladder network.

Kron[9] has indicated symbolically the analytical solution in terms of a two-phase machine. A similar analysis may be based on the three-phase machine equations of Stanley,[12] also a two-axis resolution. Solutions by both of these methods were carried out for this study, resistance and saturation being neglected. The steady-state solution of Stanley's equations is outlined in appendix *B*. This solution indicates that the maximum voltages occur at a speed slightly below synchronism and are repeated twice in each slip cycle. The shapes of the calculated rotor and stator voltage waves at two points in the slip cycle are shown in figures 2 and 3. The maximum line-to-line voltage for the rotor is $\sqrt{3}X_m/2X_0$ times normal peak line-to-line voltage; for the stator, the ratio is $\sqrt{3}(X_m-X_0)/2X_0$.

In order to visualize the machine conditions corresponding to this state of operation, reference may be made to figures 4 and 5, which represent diagrammatically the physical arrangements of the machine parts and the approximate flux relations.

The voltages include components due to (*a*) transformer effects and (*b*) their modification due to speed of rotation. The first component is most easily recognized in the stationary condition. At any angle θ between the magnetic axes of the single-phase rotor winding and single-phase stator, the former may be assumed replaced by two quadrature coils in series, one of which is in maximum inductive relation to the stator winding, the other in noninductive relation with the excited stator phase, but in maximum inductive relation with the idle stator winding. There is thus a double transformation of stator impressed voltage into the idle stator and rotor windings involving a ratio of transformation dependent upon the angular position of the rotor axis, as illustrated in figure 6. With this variation of transformer ratio there is a corresponding variation of flux density, and of the additional voltage due to speed under running conditions.

When the motor is running, the machine voltages may be thought of as consisting of a speed voltage and also a transformer voltage. Since the curves of induced voltage of figure 6 might be thought of as proportional to the curves of "total flux of induction" for the open coils, the maximum speed voltage at any rotor angle, for constant rotor speed, may be thought of as proportional to the slope of these curves, or to the rate of change of "total flux of induction" with respect to angle (i.e., time). To this speed voltage must be added the transformer voltage itself; and for line-to-line quantities the effects of the excited phases must be considered.

Comparison may now be made with the calculated voltage curves of figures 2 and 3. It is seen that the high-voltage curves fall off from their maxima to zero in about 17 degrees, as shown by the slopes in figure 6. They then acquire smaller negative values, which are appreciably shifted by the transformer and excited-phase voltages.

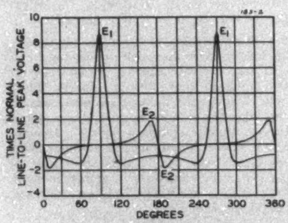

Figure 2. Calculated rotor voltage waves, single-phase rotor single-phase stator, $L_m/L_1 = 10$

E_1—At instant of maximum peak voltage
E_2—At one-quarter slip frequency cycle later

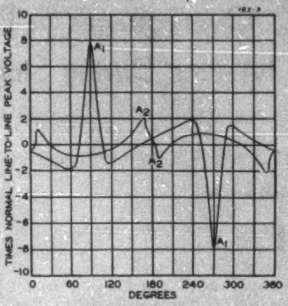

Figure 3. Calculated stator voltage waves, single-phase rotor single-phase stator, $L_m/L_1 = 10$

A_1—At instant of maximum peak voltage
A_2—At one-quarter slip frequency cycle later

It is important to note from this analysis that, although very high voltages may exist, they are not the result of correspondingly high fluxes but of high rates of change of flux. Consequently, the effects of saturation will not be so great as might be supposed. The stator resistance should slightly reduce the magnitude of the currents and fluxes and introduce some phase shift; normal rotor resistance would also be expected to have a relatively small effect, but large values of external resistance would appreciably alter the impedance and result in lower voltages.

Of more importance in many practical cases is the impedance of the power source. This in effect adds to the motor impedances and reduces the current and voltage peaks accordingly. Including transformers as part of the source, the larger the system relative to the motor, the more closely are the theoretical voltage ratios approached.

Experimental Results

Tests were made on a standard three-phase wound-rotor induction motor rated 30 horsepower, 1,200 rpm, 60 cycles, 220/440 volts; the ratio X_m/X_0 for this machine, by test, was 12.8; the ratio of total resistance to total leakage reactance was 0.25. For some of the tests a

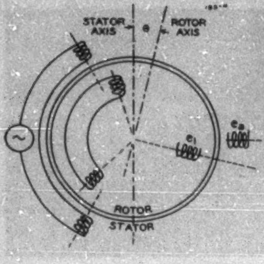

Figure 4. Diagrammatic sketch of three-phase induction motor operating single-phase on both stator and rotor

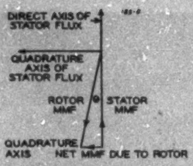

Figure 5 (above). Diagram illustrating vector flux and magnetomotive force relations of figure 4

standard cam-operated controller was used, to simulate operating conditions; in other cases knife switches were used, to give more positive control of the timing.

Prior to the running tests, curves of induced voltage versus rotor position were taken. For these, the stator was excited at a fixed voltage and all line-to-line voltages were measured by voltmeter as a function of rotor position, readings being taken with the rotor blocked. Since in some cases short-circuited rotor phases were involved, only one-fourth rated voltage was applied.

For the running tests, the motor was operated without load, giving a normal speed slightly below synchronism. Observation by means of a cathode-ray oscilloscope gave sufficient confirmation of the analysis for the two cases of three-phase single-phase operation to permit restriction of the further work to the more troublesome case of single-phase single-phase operation. For that case, the magnetic oscillograph was used to furnish permanent records.

Table I shows the comparison of the test and calculated voltage ratios, all on a line-to-line basis, for the single-phase single-phase case. All of these are steady-state voltages; in some of the oscillograms there was evidence of initial transient voltages, higher than the steady-state voltages, which appeared to build up under contactor vibration. Records of this sort of transient condition were obtained only for some of the lower voltage conditions in table I, the maximum observed transient peak being about 30 per cent greater than the corresponding steady-state peak.

Sample wave forms are shown in figure 7, which may be compared with the calculated wave forms of figures 2 and 3.

The tests indicate that resistance, in the normal range, slightly lowers the voltages. The effect of saturation is not indicated for the running condition, though this may be the explanation of the unusual standstill voltages (measured by voltmeter). System impedance, as shown by the effect of removal of the transformer, may modify the peaks to a large degree. Even without the transformer, however, the impedance of the system used was relatively high, resulting in the impressed voltage distortion evident in figure 7.

Table I. Test and Calculated Line-to-Line Voltages for Single-Phase Single-Phase Operation

	Rotor	Stator
Standstill: Ratios of rms voltages		
Calculated	1.61	1.93
Test	1.85	2.15
Running: Ratios of peak voltages		
Calculated (for an infinite bus)	11.1	10.2
Test: Motor connected direct to lines; rings shorted	5.0	4.45
Motor connected through transformer; rings shorted	3.35	3.05
Transformer in; external resistance in rotor circuit four times rotor resistance	3.0	2.55
Transformer in; rings shorted; odd rotor line tied to shorted rings with 300 times rotor phase resistance	1.1	1.0

Well-marked depressions in the supply voltage (wave *B*, figure 7), occurring simultaneously with maximum peak voltages in stator and rotor open phases, indicate the effect of limited supply system capacity in reducing the latter test voltage below calculated values. The wave shape is modified by this and other factors, so that the best comparison with calculated wave shape *A* (figure 3) is not obtained with the maximum induced voltages but at intervals of several impressed voltage cycles earlier or later, where the distortion at the peak has diminished. At these intermediate points in the quarter slip cycle between A_1 and A_2 states (figure 3), the similarity between the oscillogram and the calculated wave shape A_1 may be recognized more clearly despite the relative reduction of the oscillogram peak value.

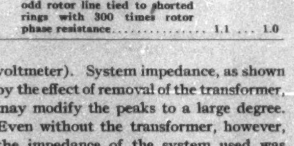

Figure 6. Ratio of induced voltage in open phase to normal phase voltage, as a function of rotor angular position for $L_m/L_l = 10$

Appendix A. Maximum Voltage for Single-Phase Rotor in Three-Phase Stator

The two-axis resolution suggested by Stanley[11] will be applied. Stator resistance is neglected and the rotor resistance is combined with the external resistance, giving a total of r per phase; secondary terminal voltages are taken to the middle of the secondary resistor. With balanced stator terminal voltage (an infinite bus) and with rotor terminal 1 open, the conditions for equations 16 of reference 12 are

$$e_a = V \sin \omega t$$

$$e_b = V \sin (\omega t - 2\pi/3)$$

$$e_c = V \sin (\omega t + 2\pi/3)$$

$$i_a + i_b + i_c = e_2 - e_3 = i_1 = i_2 + i_3 = 0$$

The equations thus become

$$L_0 p i_a - \frac{2\sqrt{3}}{3} M_0 p(i_2 \sin \theta) = V \sin \omega t$$

$$-\frac{\sqrt{3}}{3} L_0 p(i_a + 2i_c) + \frac{2\sqrt{3}}{3} M_0 p(i_2 \cos \theta) = -V \cos \omega t$$

$$M_0 p i_a - \frac{\sqrt{3}}{3} M_0 [p\theta](i_a + 2i_c) - \frac{2\sqrt{3}}{3} l_0 (\sin \theta) p i_2 = \frac{2}{3} e \cos \theta + \frac{2\sqrt{3}}{3} r i_2 \sin \theta$$

$$-M_0 [p\theta] i_a - \frac{\sqrt{3}}{3} M_0 p(i_a + 2i_c) + \frac{2\sqrt{3}}{3} l_0 (\cos \theta) p i_2 = \frac{2}{3} e \sin \theta - \frac{2\sqrt{3}}{3} r i_2 \cos \theta$$

where

$$e = e_1 - e_3$$

These equations may be integrated, giving steady-state solutions for e and i_3. The actual terminal voltages are given by $e_R = (e_1 - e_3) \pm r i_3$. These voltages are shown in the vector diagram, figure 1. The maximum for motor operation is obtained for $[p\theta] = -\omega$, when

$$e_R = \frac{3M_0}{L_0} V \sin 2\omega t \pm \frac{\dfrac{\sqrt{3}M_0}{L_0} V r \sin (2\omega t - \beta)}{\sqrt{4\omega^2 \left(\dfrac{M_0^2}{L_0} - l_0\right)^2 + r^2}}$$

$$\beta = \tan^{-1} \frac{-r}{2\omega\left(\dfrac{M_0^2}{L_0} - l_0\right)}$$

Figure 7. Oscillogram of operating conditions with single-phase stator excitation and with single-phase short circuit on rotor; 30-horsepower, 12,00-rpm, 3-phase, 60-cycle, 220/440-volt induction motor

A—Stator line-to-line voltage
B—Stator line-to-line voltage (impressed)
C—Stator line-to-line voltage
D—Rotor line-to-line voltage
E—Rotor line-to-line voltage

For measured peak values, see table I

The peak value of normal rotor line-to-line voltage, obtained by setting $[p\theta]=0$ and $r=\infty$, is $|e_R|=\sqrt{3}M_0V/L_0$. Also the short-circuit inductance referred to the rotor is $l_l=l_0-M_0^2/L_0$. Thus,

$$\frac{e_R}{|e_R|}=\sqrt{3}\sin 2\omega t\pm\frac{r\sin(2\omega t-\beta)}{\sqrt{r^2+X_l^2}}$$

$$\beta=\tan^{-1}r/X_l$$

As r varies from 0 to ∞, the diagram of figure 1 is traced out.

Maximum voltage occurs for $r/X_l=1.33$; it is $e_R/|e_R|=2.30$. If part of r is internal to the machine, that fraction of the resistance drop is subtracted (vectorially) from the terminal voltage.

Appendix B. Maximum Voltage for Single-Phase Rotor in Single-Phase Stator

Stanley's equations will be applied to this case, as in appendix *A*, assuming sinusoidal applied voltage (an infinite bus) with all resistance neglected. The terminal conditions then are

$$i_1=i_a=i_2+i_3=i_b+i_c=e_2-e_3=0$$

$$e_b-e_c=V\cos\omega t$$

The equations to be solved are as follows:

$$\frac{-2\sqrt{3}}{3}M_0p(i_2\sin\theta)=e_a$$

$$2L_0pi_b+2M_0p(i_2\cos\theta)=V\cos\omega t$$

$$M_0[p\theta]i_b-l_0(\sin\theta)pi_2=(\sqrt{3}/2)e_1\cos\theta$$

$$M_0pi_b+l_0(\cos\theta)pi_2=(\sqrt{3}/2)e_1\sin\theta$$

Finally evaluated, the steady-state phase voltages are the following:

$$e_1=\frac{\dfrac{l_0V}{\sqrt{3}\,M_0}}{\dfrac{L_0l_0}{M_0^2}-\cos^2\theta}\left\{\sin\theta\cos\omega t+\left[\frac{p\theta}{\omega}\right]\cos\theta\sin\omega t-\frac{2\left[\dfrac{p\theta}{\omega}\right]\sin^2\theta\cos\theta\sin\omega t}{\dfrac{L_0l_0}{M_0^2}-\cos^2\theta}\right\}$$

$$e_a=\frac{\dfrac{V}{\sqrt{3}}}{\dfrac{L_0l_0}{M_0^2}-\cos^2\theta}\left\{(1/2)\sin 2\theta\cos\omega t+\left[\frac{p\theta}{\omega}\right]\cos 2\theta\sin\omega t-\frac{(1/2)\left[\dfrac{p\theta}{\omega}\right]\sin^2 2\theta\sin\omega t}{\dfrac{L_0l_0}{M_0^2}-\cos^2\theta}\right\}$$

The line-to-line voltages are

$$e_R=(3/2)e_1$$

$$e_S=-(3/2)e_a\pm(1/2)V\cos\omega t$$

Maximum values for e_R occur when $\cos\theta=\pm 1$, $\sin\omega t=1$; then

$$e_{R\max}=\frac{\dfrac{\sqrt{3}\,l_0}{2M_0}V}{\dfrac{L_0l_0}{M_0^2}-1}$$

Maxima of e_S occur when $\cos 2\theta=\pm 1$, $\sin\omega t=1$; then

$$e_{S_{\max}}=\frac{\dfrac{\sqrt{3}V}{2}}{\dfrac{L_0l_0}{M_0^2}-1}$$

Calling the stator self-inductance L_0 the magnetizing inductance L_m, and the short-circuit inductance $L_0-M_0^2/l_0$ the total leakage inductance L_l, the ratios of the peak voltages as calculated to the normal peak voltages are

$$\frac{e_R}{|e_R|}=\frac{\sqrt{3}}{2}\frac{L_m}{L_l}$$

$$\frac{e_S}{|e_S|}=\frac{\sqrt{3}}{2}\left(\frac{L_m-L_l}{L_l}\right)$$

The former corresponds to equation 24*f* of reference 2.

In practice, the motor may not be connected to a system of low enough impedance to be taken as an infinite bus. In this case, if the generated internal voltage is substantially constant, it may be taken as the source, with the generator reactance transferred to fictitious motor circuits; the generator and line reactances would be expected to have approximately the effect of additions to L_m and L_l in the voltage expressions, giving

$$\frac{e_R}{|e_R|}=\frac{\sqrt{3}}{2}\frac{L_m+L_{ext}}{L_l+L_{ext}}$$

$$\frac{e_S}{|e_S|}=\frac{\sqrt{3}}{2}\frac{L_m-L_l}{L_l+L_{ext}}$$

These voltages would apply to fictitious points within the generator; the actual motor terminal values would differ slightly.

The standstill, or transformer, voltages can also be obtained from the foregoing expressions by setting $[p\theta]=0$. Then the line-to-line voltages to the open terminals are

$$\frac{e_R}{|e_R|}=\frac{\dfrac{\sqrt{3}}{2}\dfrac{L_0l_0}{M_0^2}\sin\theta}{\dfrac{L_0l_0}{M_0^2}-\cos^2\theta}$$

$$\frac{e_S}{|e_S|}=\frac{\dfrac{\sqrt{3}}{4}\sin 2\theta}{\dfrac{L_0l_0}{M_0^2}-\cos^2\theta}\pm\frac{1}{2}$$

The respective maxima are

$$e_R=\frac{\sqrt{3}L_m}{4\sqrt{L_l(L_m-L_l)}}$$

$$e_S=\frac{1}{2}+\frac{\sqrt{3}(L_m-L_l)}{4\sqrt{L_mL_l}}$$

Appendix C. Nomenclature

i_a, i_b, i_c = stator phase currents
i_1, i_2, i_3 = rotor phase currents
e_a, e_b, e_c = stator phase terminal voltages
e_1, e_2, e_3 = rotor phase terminal voltages

Photographic Study of A-C Arcs in Flowing Liquids

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I. Introduction

THE importance of fluid flow in aiding arc extinction in circuit interrupters has long been known. Two general types of explanation of the large effect found have been offered. Slepian[1,2,3] who seems to have been the first to stress the importance of the motion of the gases liberated by the arc from the surrounding oil or solid insulation in oil breakers and expulsion fuses, sought the explanation in reduced arc section at current zero, and increased diffusion rate of ions and molecules arising from the turbulence. Other investigators have expressed similar views, at least in part.[4,5]

Another theory attributes the rapidly rising dielectric strength at current zero to the cutting of the arc path by a film of cool, un-ionized fluid, which grows in thickness at a rate comparable with the fluid velocity, so that a simple relation may be expected between fluid velocity, fluid dielectric strength, and limiting recovery rate of circuit voltage at which extinction takes place. Such a relation is said to have been established experimentally for an oil flow breaker,[6,7,8] but this has not been universally accepted.[9]

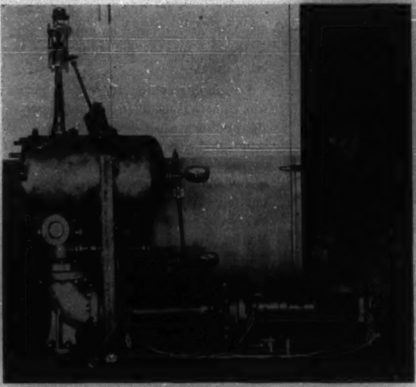

Figure 1. General view of the test structure

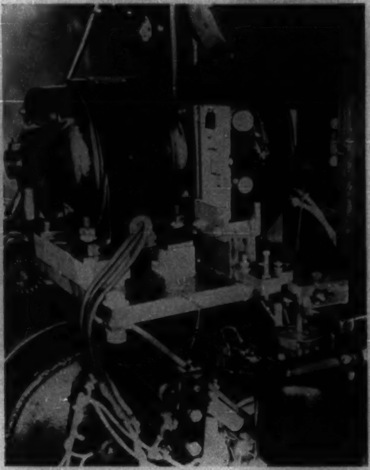

Figure 2. Camera with its synchronous shutter

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1. For all numbered references, see list at end of paper.

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e_S = stator line-to-line voltage
e_R = rotor line-to-line voltage
V = maximum value of impressed voltage
$\omega/2\pi$ = frequency of impressed voltage
θ = rotor position angle
$[p\theta]$ = rotor angular velocity
L_0 = stator inductance per phase
l_0 = rotor inductance per phase
M_0 = mutual inductance between rotor and stator per phase
L_l = leakage inductance referred to the stator
l_l = leakage inductance referred to the rotor
L_m = stator magnetizing inductance
L_{ext} = inductance of supply system
X_l = reactance corresponding to l_l
X_m = reactance corresponding to L_m
X_0 = reactance corresponding to L_l
r = secondary phase resistance

References

1. Discussion by **J. H. Fortenbach** of **Edith Clarke, C. N. Weygandt**, and **C. Concordia**'s paper, OVERVOLTAGES CAUSED BY UNBALANCED SHORT CIRCUITS. AIEE TRANSACTIONS, volume 57, 1938 (August section), pages 453–66.

2. SYNCHRONOUS MACHINES—IV, **R. E. Doherty** and **C. A. Nickle**. AIEE TRANSACTIONS, volume 47, 1928, pages 482–92.

3. INDUCTION MOTORS UNDER UNBALANCED CONDITIONS, **E. O. Lunn**. AIEE TRANSACTIONS, volume 55, 1936 (April section), pages 387–93.

4. APPLICATIONS OF THE METHOD OF SYMMETRICAL COMPONENTS, **W. V. Lyon**. McGraw-Hill Book Company.

5. SYMMETRICAL COMPONENTS, **Wagner** and **Evans**. McGraw-Hill Book Company.

6. THEORETICAL ELEMENTS OF ELECTRICAL ENGINEERING, **C. P. Steinmetz**. McGraw-Hill Book Company.

7. THEORY AND CALCULATION OF ELECTRICAL APPARATUS, **C. P. Steinmetz**. McGraw-Hill Book Company.

8. THE CROSS-FIELD THEORY OF ALTERNATING CURRENT MACHINES, **H. R. West**. AIEE TRANSACTIONS, volume 45, 1926, pages 466–74.

9. APPLICATION OF TENSORS TO ROTATING ELECTRICAL MACHINERY, **G. Kron**. *General Electric Review*.

10. THEORY AND DESIGN OF ELECTRICAL MACHINES, **F. Creedy**. Pitman and Sons.

11. AN EXTENSION OF THE METHOD OF SYMMETRICAL COMPONENTS USING LADDER NETWORKS, **W. V. Lyon**. AIEE TRANSACTIONS, volume 59, 1940 (October section), pages 1025–30.

12. AN ANALYSIS OF THE INDUCTION MACHINE, **H. C. Stanley**. AIEE TRANSACTIONS, volume 57, 1938, pages 751–5.

13. ELECTRICAL ENGINEERING PAPERS, **B. G Lamme**. Westinghouse Electric and Manufacturing Company.

Photograph 272
Taken at current zero
Water velocity = 11 feet per second
Pressure at arc = 1 atmosphere
Circuit voltage = 2,300
Current = 530 amperes

Photograph 273
Taken 0.001 second after last current zero
Water velocity = 15 feet per second
Pressure at arc = 1 atmosphere
Circuit voltage = 2,300
Current = 509 amperes

Photograph 274
Taken one half-cycle after last current zero
Water velocity = 15 feet per second
Pressure at arc = 1 atmosphere
Circuit voltage = 2,300
Current = 515 amperes

Photograph 237
Taken one-tenth half-cycle after last current zero
Water velocity = 40 feet per second
Pressure at arc = 1 atmosphere
Circuit voltage = 2,300
Current = 530 amperes

Figure 3. Arc photographs under water

With this uncertainty as to the nature of the arc extinction process in fluid flow, it is highly desirable that direct experimental investigation be made into the state of the arc near current zero. Short exposure photography near current zero would seem to be a powerful means of attack. High-speed motion pictures, at 5,000 frames per second, have been taken of arcs in a gas blast.[10,11] These show greatly reduced arc sections arising from the blast, but fail to show conclusively any sharp dividing line between a rapidly receding highly ionized luminous region and a growing dark region corresponding to freshly introduced gas in the arc path. Also, a criticism might be made that the location of the current zero is not determined with respect to the pictures. One cannot be certain that the continuous luminous column representing the arc space may not have existed for many microseconds after current zero, since it is quite possible that such a fine column, subjected to the local microturbulence inherent in the gas blast and high-temperature gradients, might have the capacity to withstand considerable voltage without breakdown.

Hoping to shed more light on these questions, particularly for the case where the flowing fluid was liquid, a series of investigations were carried on at the Westinghouse Research Laboratories, of which this is the first report.

The immediate objectives of this research were: (1) to determine the effect on the arc's interrupting ability of independent variations in fluid velocity, static pressure, type of fluid, arc length, electrode shape, baffle arrangements, and circuit natural frequency; and (2) to photograph the arc under these controlled conditions, hoping thus to learn something about the actual mechanism of the interaction between the flowing liquid and the arc in its gas bubble, especially near current zero. In this paper only the photographic results will be described.

II. Apparatus

An arc drawing structure with associated pump was so designed and built that the arc could be subjected momentarily to either longitudinal or transverse oil or water velocities up to 200 feet per second in a 3-inch channel and to independently controlled static pressures up to 15 atmospheres. Figure 1 shows the general arrangement of the structure. The fluid is directed upward from the horizontal pump cylinder through a Micarta and glass liner within the vertical tube and into the discharge tank above. Windows are provided on opposite sides of the arc tube at the point where the arc is drawn.

For photographing the arc, a miniature camera is mounted in front of one of the windows, as shown in figure 2.[12] It is provided with the synchronously driven rotating shutter shown for taking pictures of the arc and its associated gas bubble at any point on the current cycle. Contacts on a rotating drum attached to the shutter disk serve to synchronize the camera shutter with it and to record on an oscillogram exactly when the picture was taken. Because of the importance of that moment, pictures were taken generally at or near to current zero, the exposure time (with a much narrower slit than shown) being about $^{1}/_{8,000}$ second. The arc bubble was shown in silhouette against a very brightly illuminated background. In addition to the magnetic oscillograph, a cathode-ray oscillograph was available for observing the arc reignition or extinction voltage at current zero. A few high-speed moving pictures of the arc at about 2,000 frames per second were also taken.

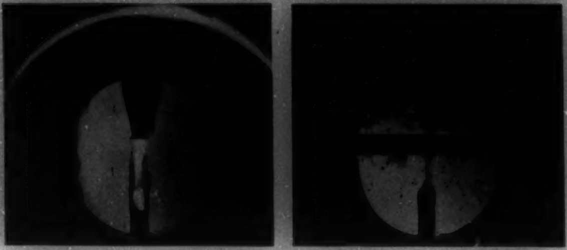
III. Results

Such a large number of arc photographs have been taken that only a summary of the observations illustrated by a few samples can be presented here. Five hundred-ampere arcs, 1 and 2.5 centimeters long between one-fourth-inch (0.635 centimeter) electrodes subjected to parallel flowing oil and water were studied first. The bubble shapes with oil and water were apparently identical, but because of greater transparency and the absence of carbon, the photographs with water were generally clearer than those with oil.

Parallel Water Flow

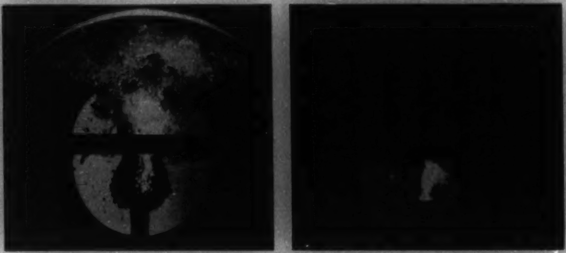
In figures 3 and 4, photographs 272, 273, 274, 237, and 284; are all of tests with parallel water flow. All were taken with the shutter adjusted to open near current zero. In 272, taken at current zero, the velocity (11 feet per second) is so low that droplets of incandescent copper can be seen falling against the flow and the gas bubble, or cloud, is large. In 273 the velocity was slightly greater and the photograph was taken nearly 0.001 second after the arc had been extinguished by opening the "back-up" circuit breaker. Here, streaks of light indicate that the arc terminals were still white hot, and it is evident that the fluid had not yet completely closed in between the electrodes, even though nearly 1,000 microseconds had elapsed since the disappearance of the arc. In 274, taken under similar conditions but one half-cycle after arc extinction, the interelectrode space had been swept clear except for a trail of bubbles still arising from the lower arc terminal. Remnants of the gas bubble were still visible around the upper electrode. The conditions of 237 were similar to those of 273, except that the velocity was considerably greater. Here no incandescence can be seen at the arc terminals, but some of the background light was transmitted through the "neck" of the bubble still existing between the electrodes. Comparison with 273 shows that the increase in velocity resulted in a still smaller tendency for the flow to close in above the lower electrode. This effect of fluid velocity is even more clearly illustrated by 284, in which, at a still higher velocity, the gas bubble at current zero just filled the space in the "wake" of the lower electrode and extended almost unchanged in section all the way to the upper electrode, which was 2.5 centimeters away instead of 1 centimeter, as in the foregoing pictures.

Of especial importance for the theory is the fact that the arc interrupted the 4,600-volt reactive circuit here, showing that this gas bubble of about one-fourth inch diameter was able to withstand $\sqrt{2}\times 4{,}600 = 6{,}500$ volts or a gradient of 2,600 volts per centimeter, shortly (less than 400 microseconds) after this current zero without the aid of any interposed layer of liquid dielectric. Actually, the oscillogram of the test showed that the arc restruck one half-cycle later, presumably by thermal breakdown of the water[13] which must have by then entirely displaced the original gas bubble.



Photograph 284
Taken at last current zero
Water velocity = 87 feet per second
Pressure at arc = 1 atmosphere
Circuit voltage = 4,600
Current = 445 amperes

Photograph 299
Taken one half-cycle after last current zero
Maximum water velocity = 45 feet per second
Discharge pressure = 1 atmosphere
Circuit voltage = 2,300
Current = 442 amperes



Photograph 301
Taken just before current zero
Maximum water velocity = 38 feet per second
Discharge pressure = 1 atmosphere
Circuit voltage = 2,300
Current = 441 amperes

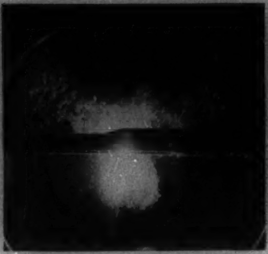
Photograph 302
Taken 1/40 half-cycle before last current zero
Maximum water velocity = 38 feet per second
Discharge pressure = 1 atmosphere
Circuit voltage = 4,600
Current = 474 amperes

Figure 4. Arc photographs under water

Nonparallel Water Flow

In addition to parallel flow, many other more complicated flow arrangements were studied. The first and simplest directing structure was merely a thin one-fourth inch thick) brass diaphragm placed across the flow channel and provided with a central round hole (usually one inch in diameter) through which the arc was drawn by the one-fourth-inch rod electrodes. Its location with respect to the electrodes in early tests is shown in photograph 299 of figure 4, taken one half-cycle after interruption of the circuit by the arc. The flow conditions roughly approximate those with many forms used in practice. Photo. 301, taken just before current zero, is the clearest obtained of a bubble form very often observed under these conditions, pear-shaped with the constricted neck only partially filling the outflow orifice. In contrast is 302, which was taken under apparently almost identical conditions, but which is entirely different. Such random variation in bubble size and shape was found to be characteristic of this type of flow at these moderate velocities. That bubble size was not, as one might expect, a function of instantaneous current is further shown in figure 5, by photograph 356, which was taken at peak current under otherwise similar conditions and shows a bubble of almost the same size and shape as that of 301 where the instantaneous current was near zero.

Associated with the variation in bubble

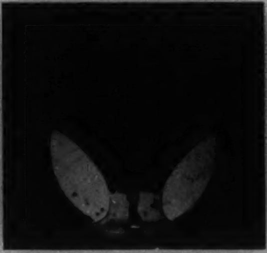


Photograph 356

Taken at peak current
Maximum water velocity = 39 feet per second
Discharge pressure = 1 atmosphere
Circuit voltage = 6,900
Current = 504 amperes

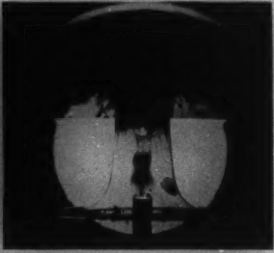
Photograph 805

Taken at last current zero
Maximum oil velocity = 33 feet per second
Discharge pressure = 5 atmospheres
Circuit voltage = 13,800
Current = 554 amperes



Photograph 806

Taken one half-cycle after last current zero
Maximum oil velocity = 23 feet per second
Discharge pressure = 5 atmospheres
Circuit voltage = 13,800
Current = 547 amperes



Photograph 890

Taken 1/20 half-cycle after last current zero
Maximum oil velocity = 22 feet per second
Discharge pressure = 1 atmosphere
Circuit voltage = 13,800
Current = 1,890 amperes

Figure 5. Arc photographs under water and oil

size was the variable interrupting performance of the arc under these conditions from one half-cycle to the next. The correlation between bubble size at current zero and tendency for the arc to reignite is illustrated well by photographs 301 and 302. Even though the circuit voltage in the case of 302 was 4,600 as compared with 2,300 for 301, the arc was extinguished after 302, but restruck with no appreciable pause after 301. Clearly, as one might expect, the small bubble was a very much better arc extinguisher than the large bubble.

FLOW OSCILLATIONS

These wide and apparently random variations in bubble volume and interrupting ability at current zero are apparently due to the characteristically oscillatory nature of the flow of liquid and gas around the arc and through an orifice. These oscillations, which were first revealed by large-amplitude pressure variations in the vicinity of the arc and by corresponding periodic arc voltage peaks, always occurred when the conditions were such as to permit the formation of an appreciably large gas bubble on the high-pressure side of the orifice. The frequency, usually a few hundred cycles per second, depended linearly on the maximum fluid velocity through the orifice and was relatively independent of the orifice size and shape, and also of the arc current and arc length. The relation was $f = 5V$ cycles per second, where V is the fluid velocity in feet per second. There was little, if any, tendency for the oscillation to synchronize itself with the 60-cycle current. Analysis of the oscillographic pressure records and of the mechanics of the flow indicated that periodic growth and collapse of the gas bubble surrounding the arc was an essential part of the oscillation.

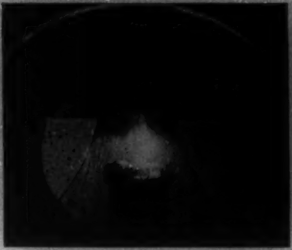
Because of the lack of synchronization of the bubble oscillation with the current zero, the bubble diameter existing at current zero would be expected to vary in a random manner as it did in fact. Variability in interrupting performance was thus also explained. This oscillation could be prevented at moderate currents by so proportioning the flow channel and adjusting the driving pressure that bubble growth in front of the orifice was effectively prevented. Such arrangements gave somewhat better average interrupting performance with water but not with oil.

PHOTOGRAPHS WITH OIL

Photographs 805, 806, 890, 975, 989, 990, 991, and 976 of figures 5, 6, and 7 are samples of a great number obtained while studying the interrupting ability of the arc with oil flow directed by variously shaped orifices. As these orifices were made of a transparent plastic material (Lucite), the effect of a cross-section view was obtained. Photographs 805 and 806 were taken under nearly similar conditions, except that 805 was at the last current zero and 806 followed the last current zero by one half-cycle. In 805, the arc bubble is comparatively small, but still large enough so that the flow was oscillatory. In spite of the design to accentuate radial flow, 806 shows that the flow had not completely closed in over the lower electrode even after one half-cycle, and that the arc terminal there was still luminous. Somewhat in contrast is 890, taken only $^1/_{20}$ half-cycle (400 microseconds) after extinction of a 2,000-ampere arc in an orifice of different shape. The arc terminals were still glowing brightly but the interelectrode space appears to have been swept almost clear of gas even at this comparatively low velocity of 22 feet per second (11 feet per second at the tip of the lower electrode).

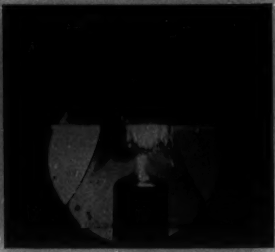
A still different electrode and orifice arrangement is shown in photographs 975 and 976 referred to above. These apparently show successive stages near the beginning of the growth of an arc bubble. Strangely, extinction occurred after 975, but not after 976. All of these pictures illustrate the characteristically very large random variations in appearance of arc bubbles with orifice flow and these last also show the frequent impossibility of distinguishing between bubbles in which the arc will just reignite and those in which the arc will just fail to reignite. This lack of distinction was characteristic of bubbles, whose volumes were always kept small by the flow, such as those in photographs 989, 990, and 991. These were all taken under identical conditions, except for the oil velocities, which were 209, 169, and 127 feet per second, respectively, measured at the constriction, which was here occupied by the arc. At the highest velocity some diminution in the bubble section may be noticed; but at 169 feet per second and 127 feet per second, which values were, respectively, above and below the critical value for ultimate arc extinction (143 feet per second), there was no detectable difference in the arc bubble appearance. However, it should be noted that photograph 990 was taken at a current zero prior to arc extinction.

More recently, sequence pictures have been obtained with a rotating-prism type high-speed motion picture camera[14] which reveal this bubble behavior clearly. The eight-frame sequence of figure 8 shows the gradual growth, collapse, and regrowth of an arc bubble under conditions similar to those of photographs 301 and 302 except that the fluid was oil flowing through the orifice at 68 feet per second, nearly twice as fast as in 301 and 302. The interval between successive pictures was about $^1/_{19}$ of a half-cycle, or 430 microseconds. Examination of the whole film leads to the conclusion that the current passed through a zero value in the vicinity of frames 4 and 5. Figure 9 is a similar sequence of an arc bubble like those of photographs 975 and 976 but also under oil flowing at considerably higher velocity. This sequence does not embrace a current zero. It is now revealed that photographs 975 and 976 represent near-minimum arc bubble sections which approximately coincided with current zeros only by coincidence.



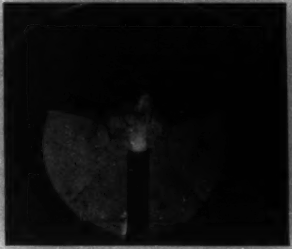
Photograph 975

Taken 1/50 half-cycle before last current zero
Maximum oil velocity = 37 feet per second
Discharge pressure = 1 atmosphere
Circuit voltage = 13,800
Current = 490 amperes



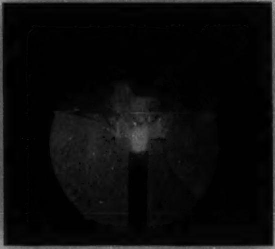
Photograph 976

Taken at current zero
Maximum oil velocity = 34 feet per second
Discharge pressure = 1 atmosphere
Circuit voltage = 13,800
Current = 486 amperes



Photograph 989

Taken 1/20 half-cycle before current zero
Maximum oil velocity = 209 feet per second
Discharge pressure = 1 atmosphere
Circuit voltage = 13,800
Current = 490 amperes



Photograph 990

Taken 1/20 half-cycle before current zero
Maximum oil velocity = 169 feet per second
Discharge pressure = 1 atmosphere
Circuit voltage = 13,800
Current = 490 amperes

Figure 6. Arc photographs under oil

IV. Conclusions

From a study of many hundreds of arc photographs represented here by only a few samples, the following conclusions have been drawn:

1. The gas bubble surrounding an arc under a flowing liquid is generally not displaced by a liquid barrier until some appreciable fraction of a half-cycle after the arc has been extinguished.

2. The existence of a large bubble section at current zero favors reignition of the arc while the existence of a small section favors its extinction.

3. Even under apparently identical conditions, the shape and size of the gas bubble may show very wide variations. These variations are the result of rapid bubble oscillations which are usually not synchronous with the arc current variation.

4. Where the arc bubble is uniformly small, there is no consistent difference between its appearance at a current zero at which the arc is extinguished and one at which it reignites, or between the characteristic bubble appearance at pressures or velocities just above the critical values for extinction and those just below these values.

With regard to the theory, it may be seen that conclusions 1 and 4 are wholly at variance with the idea that arc extinction *results from* the formation between the electrodes at current zero of a growing barrier of dielectric liquid. These and conclusion 2 suggest, rather, that extinction or reignition of the arc is usually determined by conditions existing within the remanent gas bubble at or just before current zero. These conditions probably relate to the character of the gas flow within the bubble, such as can be de-

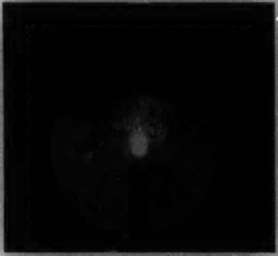


Figure 7. Arc photograph under oil

Photograph 991

Taken 1/20 half-cycle before current zero
Maximum oil velocity = 127 feet per second
Discharge pressure = 1 atmosphere
Circuit voltage = 13,800
Current = 490 amperes

Time
Probable Current Zero

Figure 8 (left). Moving-picture sequence of arc under oil

Sequence taken at 2,300 frames per second

Oil velocity = 68 feet per second
Discharge pressure = 1 atmosphere
Circuit voltage = 13,800
Current = 500 amperes (rms)

scribed in terms of a mean velocity, and an intensity of turbulence. Once the arc has failed to reignite, of course, displacement of the bubble may follow very quickly.

Conclusion 3 seems to offer a plausible explanation for the well-known inconsistency of many types of fluid circuit breakers when tested near their interrupting limit.

References

1. Extinction of a Long A-C Arc, **J. Slepian.** Journal of the AIEE, volume 49, 1930, pages 310–14.

2. The Expulsion Fuse, **J. Slepian** and **C. L. Denault.** AIEE Transactions, volume 51, 1932, pages 157–65.

3. Displacement and Diffusion in Arc Extinction, **Joseph Slepian.** AIEE Transactions, volume 60, 1941 (April section), pages 162–7.

4. **O. Mayr,** *Elektrotechnische Zeitschrift*, volume 55, August 2, 1934, pages 757–60.

5. **F. Kesselring** and **F. Koppelman,** *Arch. f. Elektrot.*, volume 29, 1935, pages 1–33, and volume 30, 1936, pages 71–108.

6. The Oil-Blast Circuit Breaker, **D. C. Prince** and **W. F. Skeats.** AIEE Transactions, volume 50, 1931, pages 506–12.

7. **D. C. Prince** and **E. J. Poitras,** *Electric World*, Volume 97, February 28, 1931, pages 400–04.

8. The Theory of Oil-Blast Circuit Breakers, **D. C. Prince.** AIEE Transactions, volume 51, 1932, pages 166–70.

9. Discussion by **J. Slepian** of **T. E. Browne, Jr.**'s paper, Extinction of A-C Arcs in Turbulent Gases. AIEE Transactions, volume 51, 1932, pages 193–6.

10. **O. Mayr,** *Elektrotechnische Zeitschrift*, volume 53, January 28, pages 75–81; February, 11, 1932, pages 121–3.

11. **J. Biermanns,** *Allgemeine Elektrisitäts-Gesellschaft*, Revised Reprint from *Elektrotechnische Zeitschrift*, volume 59, February 17 and 24, 1938, pages 165–8 and 194–7.

12. **T. E. Browne, Jr.,** *Photo Technique*, February 1941, pages 52–4.

13. Dielectric Strength of Water in Relation to Circuit Interrupters, **J. Slepian, C. L. Denault,** and **A. P. Strom.** AIEE Transactions, volume 60, 1941, pages 389–95.

14. A Non-Intermittent High-Speed 16-Mm. Camera, **F. E. Tuttle.** *Journal of Society of Motion Picture Engineers*, volume 21, December 1933, pages 474–7.

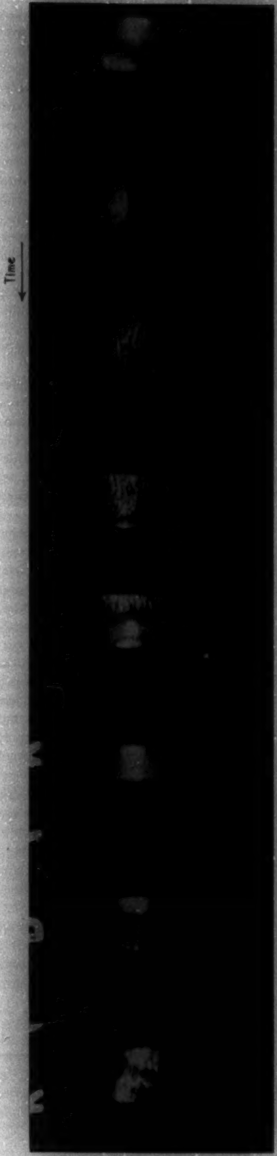


Figure 9 (right). Moving-picture sequence of arc under oil

Sequence taken at 2,300 frames per second

Oil velocity = 56 feet per second
Discharge pressure = 1 atmosphere
Circuit voltage = 13,800
Current = 500 amperes (rms)

Theory of the Brush-Shifting A-C Motor—I

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Synopsis: This is the first of a series of papers explaining the theory and principles of operation of the brush-shifting a-c motor. Operation of the machine, either as a motor or a generator, is explained on the basis of the superposition of currents. The theory of the primary current loci is developed for both high-speed and low-speed adjustments. A method of determining the primary current loci from no-load measurements is presented. Experimental data have shown the theory to be valid.

IN general, a-c polyphase motors are less adaptable to speed adjustment than are d-c motors. However, by use of a combination of regulating winding, a commutator, and brush-shifting facilities, a polyphase induction motor can be so constructed as to provide speed adjustment similar to that obtainable on a d-c shunt motor. These motors are essentially three winding motors. The primary winding placed on the rotor is supplied from the line through slip rings. The stator winding (see figure 1) is a phase wound secondary. The third winding is placed on the rotor and is provided with a commutator. Voltages collected from this commutator are inserted into the secondary circuit. Speed adjustment is obtained by changing the settings of the brushes. These a-c machines have speed-torque characteristics similar to those of the d-c shunt motor. Because of this similarity, they are sometimes called a-c shunt motors.

The authors have presented a simple explanation of the theory of this a-c adjustable speed, commutator type of motor based on ordinary induction motor theory which they have checked and found to be fundamentally sound. This explanation employs an extension of the application of the induction motor circle diagram theory. This new theory, meritorious because of its simplicity, is most helpful in explaining some of the motor characteristics such as power factor correction, design requisites for certain speed ranges, generator action, and its use for regenerative braking. The accuracies of the results obtained by this theory indicate that it may be found useful industrially as well as academically.

The Motor

The speed of an ordinary induction motor can be changed by inserting into the secondary element a voltage of slip frequency, provided this voltage is in such a phase position that it forces a power component of current into the secondary (that is, this current produced in the secondary is not maximum when the flux surrounding the secondary conductors is zero). In the particular motor described here, this current (see figure 1) is obtained from an adjusting winding placed on the rotor. This adjusting winding is provided with a commutator and provision is made for collecting voltage from it by use of adjustable brushes which are connected to the secondary. The secondary is located on the stator and the primary on the rotor. The brushes are mechanically coupled so that they are spaced at the same distance for each secondary phase winding, thereby insuring equal voltages conducted to each secondary phase. When the brushes of each secondary phase are adjusted so that they are in contact with the same commutator segment, the secondary is short-circuited, and no voltage is supplied from the adjusting winding. Under these conditions, the motor behaves like an ordinary induction motor, and the adjusting winding is in no way operative. The speed can be reduced by separating the brushes in a given direction so the voltage collected from the brushes causes a component of secondary current which produces a negative torque. The machine can be operated above synchronism by shifting the brushes, that is, interchanging their positions (*a*) to (*b*), and (*b*) to (*a*)—figure 2, so that the voltage collected by these brushes is in such a direction as to force a current through the secondary which will produce positive torque. The motor can be reversed by reversing two of the leads supplying the primary.

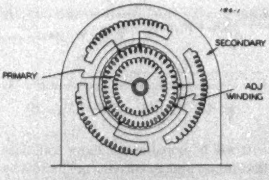

Figure 1. The motor circuit

The Theory of Speed Adjustment

Speed adjustment cannot be obtained merely by inserting a voltage into the secondary from the brushes unless this voltage is in such a direction as to cause a current that is torque producing. A voltage collected by the brushes even though large in magnitude may produce little or no change in speed if the current that it produces in the secondary conductors is in quadrature with the flux surrounding these secondary conductors. Such a condition will materially change the power factor of the motor without appreciable change of speed. Since it is possible to change both the magnitude and direction of this voltage collected from the adjusting winding, there is an infinite number of different brush settings that may give the same speed adjustment. However, there is but one setting of the brushes that will provide a given speed at a given power factor for a given load. Since all of these variables are interdependent, it is desirable that some form of explanation which shows their relation be presented. To show these relations, a specific brush setting will be chosen and the operation of the motor

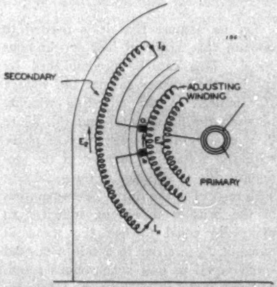

Figure 2. Currents and voltages in the secondaries

Paper 41-86, recommended by the AIEE committee on electrical machinery, and presented at the AIEE North Eastern District meeting, Rochester, N. Y., April 30–May 2, 1941. Manuscript submitted October 18, 1940; made available for preprinting March 7, 1941.

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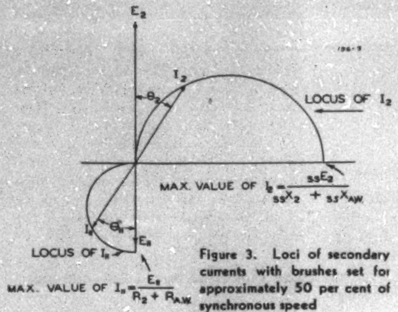

E2
I2
θ2
LOCUS OF I2
196-3
MAX. VALUE OF $I_2 = \frac{{}_{ss}E_2}{{}_{ss}X_2 + {}_{ss}X_{AW}}$
I11
θ11
E11
LOCUS OF I11
MAX. VALUE OF $I_{11} = \frac{E_{11}}{R_2 + R_{AW}}$

Figure 3. Loci of secondary currents with brushes set for approximately 50 per cent of synchronous speed

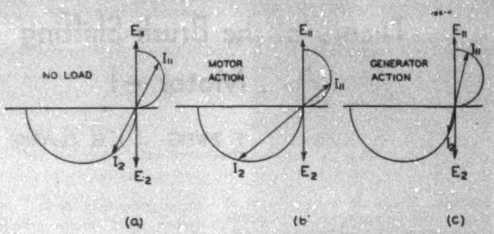


Figure 4. Relations of secondary voltages and currents when brushes are set to make E_2 opposite E_{11} and equal in magnitude at half speed

(*a*)—No load
(*b*)—Machine acting as a motor
(*c*)—Machine acting as a generator

explained for this setting. An understanding of the operation for this one brush setting will be most helpful in determining the conditions for other possible settings.

Operation Below Synchronous Speed

Let us assume that it is desired to operate this motor at half synchronous speed. This can be accomplished by spacing the brushes so that the voltage E_{11} induced in the adjusting winding between the brushes (*a*) and (*b*), figure 2, is exactly 180 degrees out of phase with the voltage E_2 induced in the secondary due to slippage of the secondary with respect to the flux. The separation of the brushes should be sufficient to make E_{11} equal to half the standstill value of E_2 in order to obtain a no-load speed equal to half synchronous speed. In figure 2 and in all following references this induced voltage due to slippage will be designated as E_2 and the current that it forces through the secondary and the adjusting winding included between the brushes will be designated as I_2. The voltage induced in the adjusting winding between the brushes will be referred to as E_{11}, and the current that it causes to flow is designated as I_{11}.

The total current flowing in the secondary for any condition of operation is the sum of the currents I_2 and I_{11}. The magnitude of this total current can be best understood by dealing with these two components separately to determine their nature, and then adding them to obtain the total current.

Assuming that the flux is constant for constant impressed primary voltage, the voltage per turn in the adjusting winding will be constant regardless of the speed. Therefore, for any given brush setting, E_{11} will be constant (independent of slip). The frequency of E_{11} changes with slip and at all times is the slip frequency of the motor. The voltage E_2 induced in the stator by this constant flux is directly proportional to the slip and is of slip frequency. Therefore E_2 and E_{11} are always voltages of the same frequency. The current I_2 which is caused by the voltage E_2 is impeded by the stator phase resistance R_2, the stator phase reactance X_2, the resistance of the adjusting winding R_{AW}, and the reactance of the adjusting winding X_{AW}. The current I_{11} is limited by the same impedance that limits I_2. Expressions for these currents are:

$$I_2 = \frac{E_2}{\sqrt{(R_2+R_{AW})^2+(X_2+X_{AW})^2}} \tag{1}$$

$$I_{11} = \frac{E_{11}}{\sqrt{(R_2+R_{AW})^2+(X_2+X_{AW})^2}} \tag{2}$$

Letting

E_2 at standstill be ${}_{ss}E_2$
X_2 at standstill be ${}_{ss}X_2$
X_{AW} at standstill be ${}_{ss}X_{AW}$
S = per cent slip

Then

$$I_2 = \frac{{}_{ss}E_2S}{\sqrt{(R_2+R_{AW})^2+({}_{ss}X_2S+{}_{ss}X_{AW}S)^2}} \tag{3}$$

or

$$I_2 = \frac{{}_{ss}E_2}{\sqrt{\frac{(R_2+R_{AW})^2}{S^2}+({}_{ss}X_2+{}_{ss}X_{AW})^2}} \tag{4}$$

$$I_{11} = \frac{E_{11}}{\sqrt{(R_2+R_{AW})^2+({}_{ss}X_2S+{}_{ss}X_{AW}S)^2}} \tag{5}$$

It will be noted in the above equations that the secondary currents are limited by an impedance made up among other things of the reactance ${}_{ss}X_{AW}S$. This is the reactance offered to currents of slip frequency, and the voltages that are produced by this reactance and the secondary currents ($I_{2ss}X_{AW}S$ and $I_{11ss}X_{AW}S$) are independent of the primary supply frequency. At synchronous speed these voltages become zero and the secondary current is limited by the resistances only. This is on the basis of course that all the flux cuts all three windings.

On the basis of the equations 4 and 5, the vector diagram of the secondary circuit including its portion of the auxiliary winding can be constructed as in figure 3. From equation 4, it is evident that the extremity of the vector I_2 has a locus following the path of a semicircle as in an ordinary induction motor. Equation 5 shows that the vector I_{11} also has a circle locus. From equations 1 and 2, it is evident that the impedance offered to I_2 and I_{11} are the same, and therefore, these currents must lag their respective voltages by the same angle, therefore the angle $\theta_2 = \theta_{11}$. The maximum value of I_2 (diameter of circle) is equal to $\frac{{}_{ss}E_2}{{}_{ss}X_2+{}_{ss}X_{AW}}$ and the maximum value of I_{11} is obtained at 100 per cent power factor of the secondary or at synchronous speed and is equal to $\frac{E_{11}}{R_2+R_{AW}}$.

The current I_2 produces torque in the direction of rotation (positive) while the current I_{11} produces a negative torque.

The positive torque $= K\phi I_2 \cos\theta_2$
The negative torque $= K\phi I_{11} \cos\theta_{11}$

In these equations, ϕ = flux. It is assumed to be constant in magnitude, and that all of it cuts the primary winding,

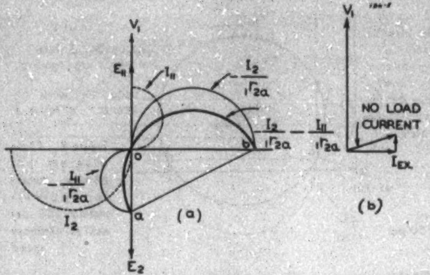


Figure 5. Currents

(*a*)—Of secondary reflected to the primary

(*b*)—Taken by primary at no load

the adjusting winding, and the stator winding. Since $\theta_2=\theta_{11}$ for this brush setting, the total torque is—

$$T=K\phi(I_2-I_{11})\cos\theta_2 \qquad (6)$$

For a condition of no load (zero developed torque) I_2 and I_{11} are equal and opposite. Or in other words, the rotor must slip sufficiently to make I_2 increase to a value equal to and opposite to I_{11}. Since the brushes are so spaced that E_{11} is one-half E_2 at standstill, E_2 and E_{11} will be equal and opposite at 50 per cent slip, and the no-load speed will be one-half synchronous speed. If slippage is increased by further reduction of speed (loading), I_2 becomes larger, I_{11} smaller, and a positive torque will result. This is motor action and it occurs at a speed less than no-load speed. If now the machine is driven at some speed slightly above its no-load speed, I_2 will be reduced, and I_{11} increased. Under these conditions, the torque produced by I_{11} becomes higher than that produced by I_2 and the total torque is negative. This condition results in generator action. Thus with this machine generator action can be obtained at any speed within its speed range. These principles are illustrated further in figure 4.

Primary current for different conditions of load can be determined from the secondary currents and from the no-load exciting current in a manner similar to that used for the ordinary two-element induction motor. However, special consideration must be given to the turn ratios for the particular motor. Any turn ratio involving the adjusting winding is a function of speed adjustment (brush setting).

The effect of the secondary currents on the current taken by the primary can be explained from the diagram of figure 5*a*. This diagram shows the locus of the components of secondary current I_2 and I_{11} when E_2 and E_{11} are 180 degrees out of phase and the machine is adjusted at a no-load speed corresponding to approximately 50 per cent slip or one-half synchronous speed. The current I_2 flowing in the stator and through the adjusting winding will cause a current to flow in the primary of sufficient magnitude and direction so as to neutralize the flux produced by I_2. This component of the primary current is $\frac{-I_2}{{}_1r_{2a}}$ where ${}_1r_{2a}$ is equal to the ratio of the number of turns on the primary to the difference of the number of turns on the stator, and the number of turns in the adjusting winding that are placed in the secondary circuit by this particular brush setting, or ${}_1r_{2a}=\frac{N_1}{N_{2a}}$. The current I_2 passing through both the stator and the adjusting winding will produce fluxes in these two windings that are opposite in direction with respect to the primary circuit. The component of the primary current that neutralizes the magnetizing effect of I_2 is therefore a vector that has its extremity defined by the path of the circle $\frac{-I_2}{{}_1r_{2a}}$ shown in figure 5*a*.

Likewise, the current I_{11} will cause a component in the primary current which by the same reasoning as above must be 180 degrees out of phase with I_{11}. The magnitude of this primary component is $\frac{-I_{11}}{{}_1r_{2a}}$, and is defined by a current locus which is the path of another circle—figure 5*a*. If these two components of the primary current are now added vectorially, their sum would be $\frac{-I_2}{{}_1r_{2a}}-\frac{I_{11}}{{}_1r_{2a}}$ and the locus of this resulting current could be represented by another circle the diameter of which is $ab=\sqrt{\overline{oa}^2+\overline{ob}^2}$. When the machine is running at no load the sum of these currents becomes zero. If the machine is operated at synchronous speed, the sum of these currents becomes *oa*. A resultant current below the line *bo* indicates generator action—a resultant current above this line, which has a component in the direction of the primary impressed voltage V_1, indicates motor action. The locus of motor currents and generator currents is defined by this circle *boa*. This circle however does not show the total primary current because of no-load losses and the requirements of an exciting current. Figure 5*b* shows the no-load loss and exciting components of primary current with respect to the impressed voltage V_1. The total primary current I_1 can now be obtained by adding the current defined by the circle locus *aob* of figure 5*a* to the no-load current shown in figure 5*b*. This construction results in a diagram, figure 6, which shows the relation of the primary current to the primary impressed voltage for different load conditions when brushes are set to reduce speed, to approximately 50 per cent of synchronous speed (${}_{ss}E_2=2E_{11}$).

Operation Above Synchronous Speed

The speed of this motor can be raised to a value above synchronous speed by

Figure 6. Circle diagram of primary current for brush setting corresponding to 50 per cent synchronous speed

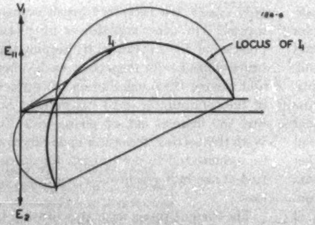
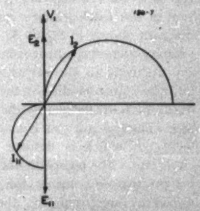


Figure 7. Current loci of secondary currents—brushes set for approximately 150 per cent synchronous speed



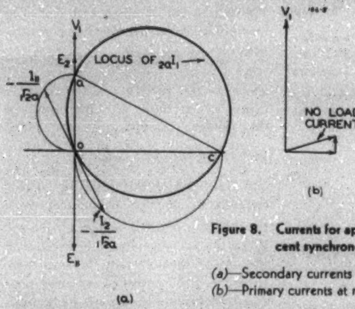


Figure 8. Currents for approximately 150 per cent synchronous speed

(a)—Secondary currents reflected to primary
(b)—Primary currents at no load

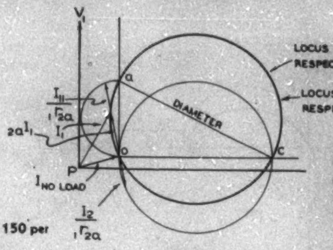

V1
LOCUS OF 2aI1 WITH RESPECT TO POINT O.
LOCUS OF I1 WITH RESPECT TO POINT P.
DIAMETER
a
o
c
P
$\frac{I_{11}}{{}_1r_{2a}}$
2aI1
I1
$I_{NO LOAD}$
$\frac{I_2}{{}_1r_{2a}}$

Figure 9. Circle diagram of primary current for approximately 150 per cent synchronous speed

reversing the potential E_{11} applied to the stator winding. This is done by transposing brush positions, i.e., in figure 2, brush *b* would be moved to the position of brush *a*, and brush *a* to the position of brush *b*. By reversing the direction of the voltage E_{11} applied to the stator, the current I_{11} will have a positive torque and tend to drive the motor at a high speed. For speeds above synchronous E_2 also reverses (since the flux reverses its direction of rotation relative to the stator), and I_2 now produces a negative torque tending to bring the motor back to synchronous speed. The vector diagram for this condition is shown in figure 7. It will be noted that both voltages are reversed with respect to the primary voltage V_1 from their position of figure 5*a*.

The current I_{11} flowing in the adjusting winding and stator will cause a primary current component $\frac{-I_{11}}{{}_1r_{2a}}$ where ${}_1r_{2a}$ is the ratio of the number of turns on the primary to the sum of the number on the stator and the adjusting winding between brushes. For this speed adjustment, the fluxes produced by the stator and adjusting winding are in the same direction with respect to the primary. Likewise, I_2 will cause a primary current component $-\frac{I_2}{{}_1r_{2a}}$.

When these currents of figure 7 are reflected to the primary circuit, with due regard to turn ratios, they provide a portion of the total primary current vector diagram shown in figure 8*a*. The total primary current resulting from the secondary currents in the adjusting winding, and the stator winding is the vector sum of the currents defined by the two current loci. Their vector sum is another current locus ${}_{2a}I_1$ which is defined by another circle *oac*. The complete vector diagram for the primary supply can now be obtained by superimposing the vectors of figure 8*a* on those of the no-load diagram of figure 8*b*. This complete diagram is shown in figure 9.

It will be noted that the resultant locus of the primary current of figure 9 is moved toward the vector V_1 from its position shown in figure 6. With this high-speed adjustment and with brushes set to make E_{11} opposite to E_2, the power factor of the motor can be made quite high at large loads, and the maximum power that the motor can develop is much higher than that of the low-speed adjustment described previously. This is indicated by the maximum power component of the primary current of figure 9 as compared with that of figure 6.

Experimental Check on Theory

In order to more fully verify the theory of the motor advanced thus far, an experimental check was made on one of these motors to see how closely the current taken under load followed the current predicted by the current loci circles. The machine, a General Electric B.T.A., 600–1800 rpm, 6 pole, 60 cycles, 4.1–12.5 horsepower, was first adjusted so that its no-load speed was 600 rpm (one-half synchronous speed). The brushes were set by opening the secondary connections at the brushes and adjusting the brushes so that the voltage across them, E_{11}, was one-half the voltage E_2 across the open-circuited stator coil at standstill. This gives only one of the necessary brush adjustments. The other adjustment for phase position of E_{11} is obtained by keeping the brushes fixed with respect to each other and moving them together on the surface of the commutator until the voltage E_{11} was 180 degrees out of phase with E_2. With this setting, the stator coils can now be reconnected to the brushes for operation at one-half synchronous speed for no load.

The current taken with this brush setting for different values of motor output is indicated by the encircled points in figure 10. The circle which should pass through these points can be located by measuring the no-load primary current (*po*), and the primary blocked rotor current (*pb*).

By erecting a perpendicular to the line *ob* one diameter of the current circle locus is established. If the line *ob* is extended to the point *f* so that $\frac{of}{bf}=\frac{{}_{so}E_2}{E_{11}}$ the point *f* will fall on the locus of $\frac{-I_2}{{}_1r_{2a}}$. From points *o* and *f* the circle $\frac{I_2}{{}_1r_{2a}}$ can be constructed and consequently the point *e* is located. A perpendicular to *oe* will be another diameter to the primary current circle locus. The intersection of the two diameters establishes the center of the circle. The accuracy of this circle diagram is revealed by the proximity of the points obtained experimentally with the current circle locus *cob*.

To check the theory for higher speeds, the brush positions were interchanged so that the voltage E_{11} between them at standstill on open circuit was one-half the voltage E_2 induced in the open-circuited secondary and in phase with it. This adjustment provided a no-load speed of 1,800 rpm (150 per cent synchronous speed).

A current locus circle can be determined in the same manner as for the lower speed adjustment. This current locus along with the experimental points obtained by loading are shown for this speed adjustment (150 per cent synchronous speed) in figure 11. The circle locus predicted on the basis of no-load and short-circuit current provides a fair determination of the current characteristics of the machine. While there appears to be some divergence between this circle and the true current locus indicated by the curve through the encircled points, the actual differences as would be indicated on instruments as to power factor, magnitude of currents, etc., are small. The diagram

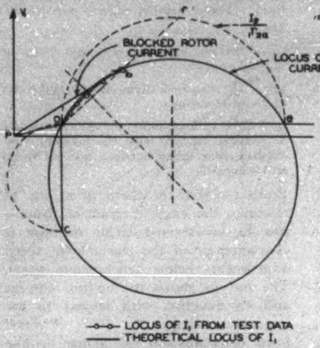
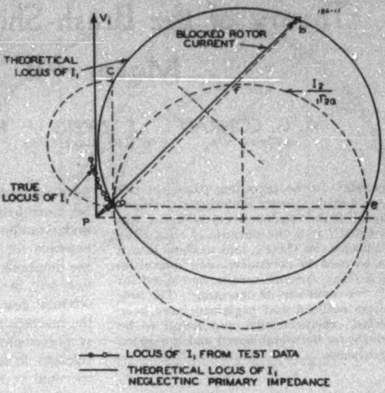

V
r
I_2 / Γ_{2a}
186-10
BLOCKED ROTOR CURRENT
LOCUS OF PRIMARY CURRENT
b
P
o
e
c
LOCUS OF I_1 FROM TEST DATA
THEORETICAL LOCUS OF I_1

Figure 10. Theoretical primary current locus compared with test data—50 per cent synchronous speed

Figure 11. Theoretical primary current locus compared with test data—150 per cent synchronous speed


V_1
186-11
BLOCKED ROTOR CURRENT
b
THEORETICAL LOCUS OF I_1
C
f
I_2 / Γ_{2a}
TRUE LOCUS OF I_1
a
o
e
P
LOCUS OF I_1 FROM TEST DATA
THEORETICAL LOCUS OF I_1 NEGLECTING PRIMARY IMPEDANCE

of figure 11 which displays this difference on a standard machine also, with proper interpretation reveals that the theory thus far developed is sound. However, the differences between the theoretical circle and the experimental current locus can be explained on the basis of the primary leakage reactance.

Effects of Primary Leakage Reactance

In a motor such as described here provision is made for two windings on the rotor and one on the stator. The necessary spacing of the primary winding with respect to the stator introduces primary leakage reactance. This reactance produces a voltage 90 electrical degrees ahead of the current in the primary which subtracts from the applied voltage in such a way that the voltage induced in the primary by the mutual flux (mutual to primary and to the stator) lags the applied voltage. Consequently at speeds above synchronism, this mutual flux must introduce a voltage in the stator winding which is ahead of the voltage that would be present if there were 100 per cent coupling between the two windings. The angle of lead of this voltage in the stator is exactly equal to the angle of lag between the total primary back electromotive force and the back electromotive force produced by the mutual flux. This lead in stator voltage will cause the circle with diameter *oe* of figure 11 to swing about the point *o* in the direction of the rotation of the vectors with increase in load on the motor. Since the leakage between the two rotor windings (primary and adjusting winding) is small, there is little shift of the circle *of* due to primary leakage fluxes.

It has been found experimentally that the true primary current locus represented by the points shown in figure 11 can be predicted from no-load data of three sets of measurements taken on the motor. These determinations are, as follows:

1. The no-load input current and watts.
2. The standstill input current and watts at reduced voltage.
3. The input current and watts on reduced voltage with the machine running.

The first and second determinations above are made in the customary manner. The third determination can be made by applying a reduced voltage to the primary sufficient to rotate the machine at no load at a speed somewhere intermediate between its no-load speed and standstill. A speed of approximately 50 per cent of no-load speed provides fair accuracy in construction of the circle.

From the data of the three above items, it is possible to calculate the current, power factor, etc., for normal voltage. From these three values of primary current and their respective power factors, it is possible to construct the circle locus of the primary current previously developed, and also to locate it in accordance with the shift created by the primary leakage reactance. This more accurate determination of the primary current locus is shown in figure 12. The current *PO* is the no-load current, the current *PS* is the standstill current for normal voltage determined from data taken at reduced voltage, and the current *PR* is a current that the motor would take when running on normal voltage under some load. This current *PR* is determined for a running condition at reduced voltage.

The center of the circle of figure 12 can be found by erecting perpendiculars to the chords *RS* and *OR*. Once this center *C* is located a circle can be drawn through the

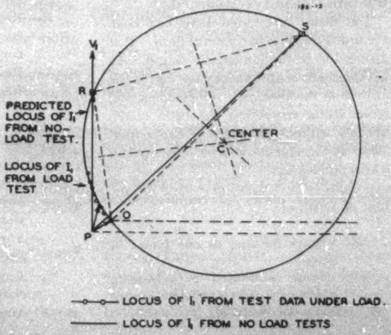

186-12
S
V_1
R
PREDICTED LOCUS OF I_1 FROM NO-LOAD TEST.
CENTER
C
LOCUS OF I_1 FROM LOAD TEST
O
P
LOCUS OF I_1 FROM TEST DATA UNDER LOAD.
LOCUS OF I_1 FROM NO LOAD TESTS

Figure 12. Comparison of primary current locus obtained from no-load tests with the true locus obtained by loading

Theory of the Brush-Shifting A-C Motor—II

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Synopsis: In the preceding paper[1] of this series, a simple theory underlying the circle diagram of the brush-shifting a-c motor was presented. It is the purpose of this paper to advance this theory, and to show how it can be used to determine such quantities as efficiency, current, torque, and speed for different conditions of operation. The procedures outlined and explained have been checked experimentally, and found to be accurate for both high-speed and low-speed adjustments.

THERE is an infinite number of possible brush settings that can be made on the brush-shifting a-c motor. It is possible to move the brushes so as to change the magnitude or phase position, or both magnitude and phase position of the voltage collected from the commutator. Such movements can be used to change speed, or the power factor or both speed and power factor. Any change in brush setting will materially change the circle diagram proportions. Therefore, any circle diagram for this motor is useful only in determining the characteristics of the machine for one brush setting. However, an understanding of the use of the circle diagram for a particular setting of the brushes will be most helpful in obtaining a general understanding of the operation of the machine for other settings. Therefore, the diagrams used for a basis of explanation will be those corresponding to the settings described in Part I, i.e., when the machine is operated at 50 per cent of synchronous speed with the two secondary voltages in direct opposition, and when operated at 150 per cent of synchronous speed with the two secondary voltages in direct opposition. Characteristic circle diagrams for these brush settings are shown in the upper portion of figure 3.

Determination of the Circle Diagram

The circle diagram can be determined from three sets of readings taken at no load. These determinations are as follows:

(*a*). No-load input current and watts at normal voltage

(*b*). The standstill input current and watts at reduced voltage

(*c*). The no-load input current and watts at reduced voltage with the machine running at some speed between no-load speed and standstill

From the data of these three sets of readings, the circle diagram of figure 1 can be constructed. This diagram is characteristic of the machine for speed adjustments below synchronous speed. The line *po* shows the no-load current and its direction with respect to the impressed voltage V_1. It is obtained from determination (*a*) above. The standstill current I_{ss} for normal applied voltage is represented in magnitude and direction by the line *pb*. It is obtained from determination (*b*). A third point, *x*, on the circle is located from determination (*c*). This point is obtained from conversions made on data taken with reduced voltage. Having the points, *o*, *x*, and *b* on the circle, it is possible by erecting perpendiculars to two of its chords to locate the center *r*. Using *or* as a radius, the circle locus of the primary current *oxb* can be located.

Determination of Characteristics From the Circle Diagram

Having established the currents *po* and *pb* in magnitude and direction with respect to the vector V_1, the lines *pe* and

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[1] For numbered reference, see end of paper.

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points *O*, *R*, and *S* which very accurately predicts the primary current locus with respect to the impressed voltage. The accuracy of this method of determining this locus is evident from the proximity of the locus of the points determining the true locus obtained experimentally.

The theory presented here has been verified through experimental data. It is assumed to be fundamentally sound. An advancement of this theory for use in the analysis of the losses, slip, efficiencies, and power factor correction has been made, but falls without the scope of this paper. These advancements will be presented in a later publication.

List of Symbols

E_2—Voltage induced in one phase of stator

E_{11}—Voltage generated in adjusting winding between brushes supplying one phase of stator

I_2—Current flowing in stator and adjusting winding resulting from the voltage E_2

I_{11}—Current flowing in stator and adjusting winding resulting from the voltage E_{11}

I_{ss}—Standstill primary current

R_2—Resistance of stator per phase

R_{AW}—Resistance of adjusting winding per phase

X_2—Reactance of stator per phase

X_{AW}—Reactance of adjusting winding per phase

S—Per cent slip

${}_{ss}E_2$—Stator voltage at standstill (induced)

${}_{ss}X_2$—Stator reactance at standstill

${}_{ss}X_{AW}$—Adjusting winding reactance at standstill

θ_2—Angle of lag between E_2 and I_2 electrical degrees

θ_{11}—Angle of lag between E_{11} and I_{11} electrical degrees

θ—Angle of lag or lead between primary phase voltage and primary phase current

ϕ—Total flux produced by primary windings

K—Torque constant of the motor

T—Torque developed (synchronous watts)

${}_1r_{2a}$—Ratio of the number of turns on the primary to the effective number of turns on the secondary and adjusting winding

N_1—Number of primary turns

N_{2a}—Effective number of secondary turns per phase included in stator and adjusting winding circuit. This quantity is a function of speed adjustment as well as power factor adjustment

V_1—Primary impressed voltage per phase

I_1—Primary current per phase

I_{ex}—Exciting current in primary circuit per phase

${}_{2a}I_1$—Component of primary current flowing as a result of the currents in stator and adjusting winding

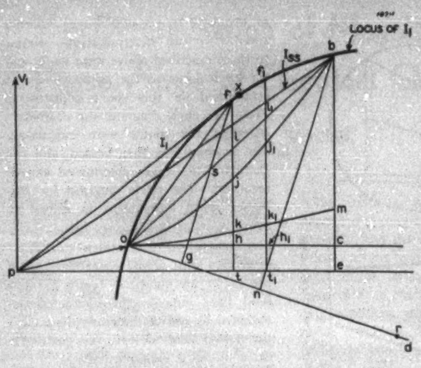

Figure 1. The circle diagram for the low-speed adjustment

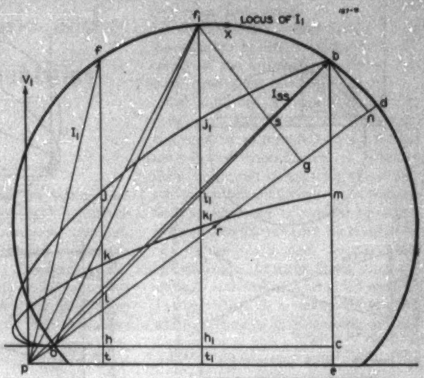

Figure 2. The circle diagram for the high-speed adjustment

oc can now be constructed perpendicular to V_1 and the line *be* perpendicular to *pe*. The power supplied to the motor at standstill per phase is the current *be* multiplied by the voltage V_1. The current *ce* multiplied by the voltage V_1 is the per phase iron loss of the rotor, the friction and windage losses, and a small amount of iron loss in the stator. By measurements of primary resistance, the primary copper losses per phase can be calculated for the normal short-circuit current. This loss is represented by the current *cm*. The loss in the adjusting winding and in the stator per phase for standstill conditions is $(bm)V_1$.

The copper losses can be estimated for any value of primary current in a manner somewhat similiar to that normally used with the ordinary induction motor. Thus for an input current of *pf*, it is assumed that the no-load current is one component and the loss associated with it, $(ce)V_1$, is constant regardless of the magnitude of the load. The other component of the primary current *pf* is *of*, and the losses associated with this component can be determined by the relations—

$$\text{Loss for current } of = \text{loss for } ob\left(\frac{\overline{of}}{\overline{ob}}\right)^2$$

Another method of determining the copper loss for the current *pf* is to erect a perpendicular *bn* from the point *b* to the diameter of the circle, and another perpendicular *fg* to the same diameter. The copper losses associated with the current *of* are expressed as—

$$\text{Loss for current } of = \frac{(bc)(sg)}{bn}V_1 \text{ watts per phase}$$

Having determined the total loss for this current *of* in terms of a current in phase with the voltage, this loss caused by the current *of* can now be laid off on a line *fh* drawn from *f* perpendicular to *oc*. In figure 1, it is labeled *hj*. Similarly, the total copper losses can be determined for other values of input current. Thus for an input current of pf_1, the total copper loss is h_1j_1. By successive determinations of losses, the curve *ojb* can be constructed to show the change in copper losses with different values of input current. If each of the distances *jh*, j_1h_1 are divided so that $\frac{jk}{kh}=\frac{j_1k_1}{k_1h_1}=\frac{bm}{mc}$ a new curve *okm* can be constructed so that the distance h_1k_1 shows the variation in primary copper loss, and k_1j_1 shows the variation in secondary (adjusting winding and stator) copper losses for different values of primary current.

The variables for the motor can now be determined for any value of input current I_1.

(1). Input $=(ft)V_1$ watts per phase
(2). Output $=(jf)V_1$ watts per phase
(3). Iron loss, friction, and windage $=(ht)V_1$ watts per phase
(4). Primary copper loss $=(hk)V_1$ watts per phase
(5). Copper losses of stator and adjusting winding $=(jk)V_1$ watts per phase
(6). Efficiency $=\left(\frac{jf}{ft}\right)$ 100 per cent
(7). Slip $=\left(\frac{jk}{kf}\right)$ 100 per cent of no-load speed
(8). Speed $=\left(1-\frac{jk}{kf}\right)$ times the no-load speed
(9). Torque $=(kf)V_1$ synchronous watts per phase

For speeds above synchronism, a similar treatment can be applied. Figure 2 shows the circle diagram for the same motor as that of figure 1, except the speed is adjusted to approximately 150 per cent of synchronous speed. The standstill current I_{ss} for normal voltage is much larger, and maximum power output much greater. The circle is located from the three sets of no-load determinations described previously. The essential difference of this diagram as compared with that of figure 1 is the shape of the curves showing copper losses for different primary currents. The system of notations on figure 1 has been maintained in figure 2, so that the explanation of the use of figure 1 will suffice for the explanation of the use of figure 2.

Experimental Check on the Theories of the Circle Diagram

In order to check the validity of the theory thus far developed, tests were made on a General Electric B.T.A. 550–1,650-rpm, 4.17–12.5-horsepower, 6-pole, 60-cycle brush-shifting motor. The brushes were set by making the brush voltage E_{11} at standstill equal approximately one-half the voltage ${}_{ss}E_2$ in the stator coil to which they were connected. Care was taken to make these voltages opposite in direction. With this setting, the no-load tests described above under the headings (*a*), (*b*), and (*c*) were made to determine the circle locus of the primary current. The machine was then loaded and readings of speed, torque, current, watts, and volts were obtained for different values of output. The primary resistance was determined by measuring the resistance of the pri-

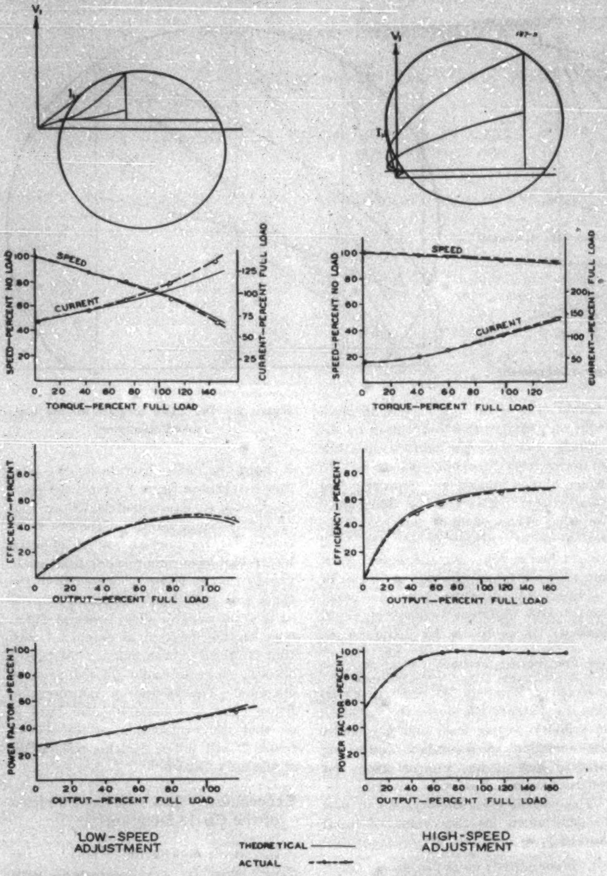

Figure 3. Comparison of the theoretical characteristics taken from the circle diagram with the actual characteristics obtained by tests for no-load speeds, approximately 50 per cent of synchronous speed, and 150 per cent synchronous speed

mary winding with direct current. From the no-load tests the circle diagram of the machine was constructed, and the characteristics determined from its proportions. The theoretical diagram for this machine with this low-speed setting is shown in figure 1.

A second experimental check was made with the brushes set to provide a speed of approximately 150 per cent of the synchronous speed. This can be done by making the voltage E_{11} across the brushes at standstill equal to one-half the stator voltage $_{ss}E_2$ of the stator coil to which they are normally connected. In addition to this, these two voltages should be in time phase with each other at standstill to provide proper running speed. No-load determinations similar to those described above were made and the circle locus of the primary current established for this speed adjustment. From this circle diagram, the characteristics of the motor were calculated. The machine was then loaded, and the characteristics again determined experimentally. The circle diagram for this higher speed adjustment is shown in figure 2.

A comparison of the theoretical characteristics with actual characteristics of the motor are shown in figure 3. The solid lines indicate the characteristics obtained by use of the circle diagram—the dotted lines indicate the characteristics obtained experimentally.

Conclusions

The proximity of the curves obtained from measurements made on the circle diagram to those obtained by the load test, indicates that the circle diagram developed and explained in this series of papers can be used to determine the operating characteristics of the brush-shifting motor accurately. While the circle diagrams of the motor have been shown and checked for only two speed adjustments, the explanation of the theory of the use of these diagrams can be applied to other diagrams constructed for different speed adjustment on the motor. A large portion of the theory thus far developed is directly applicable to the explanation of conditions within the motor when it is adjusted for power factor corrections; however, this is beyond the scope of this paper, and will be presented in another publication.

Symbols

V_1—Primary impressed voltage per phase
I_{ss}—Standstill primary current
I_1—Primary current per phase
$_{ss}E_2$—Voltage induced in stator at standstill
E_{11}—Voltage generated in adjusting winding

Reference

1. **Theory of the Brush-Shifting A-C Motor—I, A. G. Conrad, F. Zweig, and J. G. Clarke.** AIEE Transactions, volume 60, 1941 (August section), pages 829–34.

The Determination of Magnitude and Phase Angle of Electrical Quantities

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MEMBER AIEE

Synopsis: Although the three-voltmeter method for measuring the relative magnitude and phase angle of two voltages or currents is well known, the use of vacuum tube voltmeters, which draw negligible amounts of energy from the circuit, permits the use of the method for purposes which previously have not been considered. A further improvement of the scheme by balancing out one component of either voltage or current adds to the accuracy of the results. The method is considered for two purposes, namely: the measurement of the capacitance and power factor of imperfect condensers, and the measurement of the ratio and phase angle of current transformers.

Purpose

THERE are two well-known methods by which the constants of an unknown impedance may be determined. They are by balancing networks or bridges, of which there are many types,[1] or by the use of a wattmeter, voltmeter, and ammeter. Both methods are used successfully for a wide variety of impedances, even to the determination of the capacitance and power factor of cables and high-voltage bushings.[2,3,4]

There are also two principal methods by which the relative magnitudes of two voltages or two currents and the phase angle between them may be determined. They are by the use of an a-c potentiometer of the Tinsley-Drysdale type,[5] or by use of a modified bridge method.[6]

A third method for determining the magnitude and angle of an impedance, voltage, or current is known as the three-voltage, or three-voltmeter method.[7] The method has been known for a long time but it has been less widely used than the methods mentioned previously because of its supposed inaccuracy. For determining voltages, the method consists of measuring the two unknowns separately and then measuring their sum or difference. The scheme is inaccurate if the circuit on which the voltages are impressed is composed of high-impedance elements, and the instruments draw sufficient current to disturb the vector relations in the circuit. However the development of vacuum tube voltmeters and amplifiers, which draw little or no power from the circuit and thus have negligible effect on circuit conditions, makes a re-examination of the method desirable.

This re-examination is made more important because there are cases when an unavoidable variation in the circuit parameters occurs during the test, making it very difficult to obtain a balance on a bridge. Furthermore there are cases, such as in the testing of current transformers, when it becomes necessary to design circuit elements to carry large currents. Then inductances become undesirable because of the magnetic fields which are produced and capacitors for large currents soon become unwieldy and expensive.

The Modified Three-Voltmeter Method

If one has two alternating voltages of the same frequency it is possible to determine their magnitudes and the phase angle between them by three-voltage measurements and a graphical or trigonometrical analysis. Figure 1*a* shows two voltages across two impedances on which three-voltage measurements can be made because of the common connection. In figure 1*b* E_1 is drawn along the real axis. Then arcs equal in length to E_2 and E_3 are drawn from the ends of E_1 and the phase angle θ is determined. If the measuring instruments do not disturb the distribution of voltage in the circuit the accuracy is limited only by the accuracy of the instruments and the personal error of the operator.

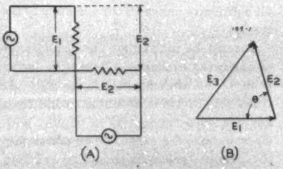

Figure 1. The three-voltage method for determining the magnitude and phase angle of two voltages

One difficulty is present if the two voltages E_1 and E_2 are nearly in phase and equal in magnitude for then a small error in the determination of either of them may result in recording three voltages which will not form a closed triangle. Such an error can be avoided by a procedure which balances out one component of a voltage in a manner which will now be explained. In figure 2*a* the circuit of figure 1*a* is shown with an added element. To the voltage E_2 a voltage E_2' is added which is variable in magnitude and 180 degrees out of phase with E_1. (This requires a transformer with exactly 180 degrees phase shift, a limitation which will be discussed later in this article.) Figure 2*b* shows the voltage relations in this circuit. It is evident that when the voltage E_3 is a minimum the phase angle between E_1 and E_2 is:

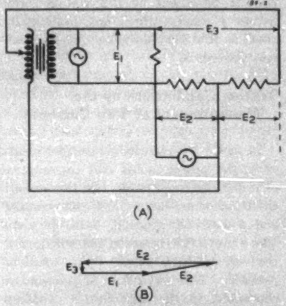

Figure 2. The method of balancing out one component of a voltage vector to determine the phase angle

$$\theta = \sin^{-1}\frac{E_3}{E_2}$$

Therefore it is only necessary to determine these two voltages in order to find the phase angle. For very small angles a further approximation can be made:

$$\theta = \sin^{-1}\frac{E_3}{E_2} \approx \tan^{-1}\frac{E_3}{E_2}$$

$$\theta(\text{radians}) \approx \frac{E_3}{E_2}$$

These formulas are useful in studying the defect angle of condensers or of current transformers.

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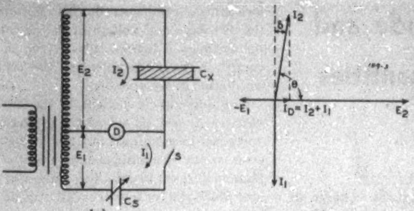
[1] For all numbered references, see list at end of paper.



(A)

(B)

Figure 3. Circuit and vector relations for determining the defect angle of an imperfect condenser

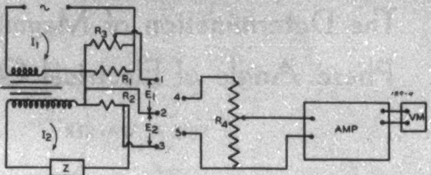


Figure 4. Circuit for determining the phase angle of a current transformer

Although this method has been discussed for voltages it is obvious that the vector relationship is applicable also to currents. Either the currents themselves can be measured or the voltage drop across resistors which are sufficiently small so that they do not appreciably affect the circuit.

Testing Condensers by the Measurement of Two Currents

In figure 3 a circuit is shown for measuring the power factor and capacitance of imperfect condensers. In this circuit the transformer has a 5,000-volt winding and a 100-volt winding, both shielded. The phase angle between the voltages of the two windings should be as small as possible. The imperfect condenser is connected to the high-voltage winding and a leading current flows through the detector *D* as is shown in figure 3*a*. This current leads the voltage by an angle θ and the condenser is said to have a defect angle δ, where $\delta = 90$ degrees $-\theta$. The defect angle is determined by closing switch *S* and varying the air condenser *C* until a minimum current is reached. This is

$$I_D = I_1 + I_2$$

Then

$$\delta = \sin^{-1}\frac{I_D}{I_2} \quad \delta(\text{radians}) \approx \frac{I_D}{I_2}$$

Typical readings made on a high-voltage bushing are as follows:

$I_2 = 1{,}120$ microamperes at 5 kv and 60 cycles per second

$I_D = 60$ microamperes at 5 kv and 60 cycles per second

$$= \frac{60}{1{,}120} = 0.0535 \text{ radian}$$

$$C_x = \frac{1{,}120 \cdot 10^{-6}}{5{,}000 \cdot 377} = 594 \text{ micromicrofarads}$$

(approximately)

These measurements were checked on a Schering bridge. Although power factors which are too low to measure by this method can still be determined by a Schering bridge, this system showed one important advantage in the measurement of the power factor of wet oil. With the bridge the power factor of the oil changed too rapidly for a successful reading to be made, but with this circuit the variation could be followed without much difficulty because as I_d changes there is no appreciable variation in the out-of-phase component of I_2 and the power factor is approximately proportional to I_d.

This circuit is also useful in making tests where corona may be present at higher voltages. The circuit can be balanced at a low voltage and as the voltage is increased the current I_d should increase in direct proportion. Any sudden increase indicates a departure from linearity in the condenser being tested.

It is important to notice that the calibration of the air condenser C_S does not enter into the result. The only requirements are that the voltages E_1 and E_2 must be in phase and that the impedance of the detector be sufficiently low so that the phase angle is not affected. Both of these effects can be checked by placing an air condenser at C_x with a known resistance in series to give a known power factor. For a detector it is possible to use a rectifier type microammeter or, for lower impedance, a noninductive resistance with an amplifier and vacuum tube voltmeter can be used. A filter in the amplifier circuit to suppress all harmonics increases the accuracy of the results.

It must be noted that the capacitance of balancing condenser C_S must be larger than C_x by the ratio of E_1 to E_2. At times this makes it desirable to add fixed low-loss mica condensers in parallel with C_S to extend the range. The power factor of these condensers should be less than one per cent of the condenser being tested.

This circuit has been found very useful for measuring the defect angles of cables, bushings, and other impedances having defect angles from 0.005 to 0.1 radian. It is desirable to shield the device to keep stray currents out of the meter.

Application to Current-Transformer Testing

At present two methods are used for testing current transformers: the mutual inductance bridge method, and the "Silsbee" or comparative method. The first requires one or two calibrated mutual inductances, and the second requires a transformer similar to the one being tested but with a known ratio and phase angle. An arrangement requiring neither of these is shown in figure 4. In this circuit R_1 and R_2 are decade boxes, and *Z* is the burden with which it is desired to test the transformer. The resistance R_3 should be sufficiently large so that it diverts a negligible amount of current from R_1. A ratio of 10,000 to 1 or higher is usually used. The vacuum tube voltmeter consists of a three-stage, resistance-coupled, amplifier with 70 decibels gain and an input resistance of about one megohm. Readings are taken on a rectifier-type microammeter in the output circuit of the amplifier. With the method described here, a calibration of the amplifier-voltmeter combination is not necessary. Greater accuracy can be attained by inserting a 60-cycle band pass filter in the amplifier to eliminate any harmonic voltages from the voltmeter reading.

The method of making the test is as follows: the input terminals of the meter (terminals 4 and 5) are connected to points 2 and 3, and tap of R_4 is adjusted until full scale is indicated on the output meter. This value of R_4 is recorded as R_4'. Next the meter input is connected to points 1 and 2, and with R_4 unchanged the tap on R_3 is adjusted until full-scale deflection is obtained on the voltmeter. Call this setting on R_3, R_3'. Then:

$$E_{23} = E_{12} \qquad E_{12} = I_1 R_1 \frac{R_3'}{R_3} \qquad E_{23} = I_2 R$$

$$\text{Transformer ratio} = \frac{I_1}{I_2} = \frac{R_2}{R_1} \cdot \frac{R_3}{R_3'}$$

The next step is to connect the input terminals of the meter to points 1 and 3. This reading should be much less than the previous ones. If it is not the polarity of the secondary winding must be reversed. The tap on R_4 is now changed until full-scale deflection is obtained once more. This setting on R_4 is recorded as R_4''. One now has:

$$E_{12} \cdot \frac{R_4'}{R_4} = E_{13} \frac{R_4''}{R_4}$$

$$\theta = \sin^{-1} \frac{R_4'}{R_4''}$$

$$\theta(\text{radians}) = \frac{R_4'}{R_4''} \text{ (approximately)}$$

A typical set of readings are:

Rated primary current—50 amperes
Rated secondary current—5.0 amperes (nominal)
$R_1 = 0.01$ ohm $\quad R_2 = 0.10$ ohm
$R_3 = 10{,}000$ ohms $\quad R_4 = 10{,}000$ ohms
$R_3' = 999.0$ ohms $\quad R_4'' = 10{,}000$ ohms
$R_4' = 14.0$ ohms

$$\text{Ratio} = \frac{0.1}{0.01} \times \frac{10{,}000}{999.0} = 1.0010$$

$$\theta(\text{radians}) = \frac{14.0}{10{,}000} = 0.0014 \text{ radian}$$

$$\theta = 28.8 \text{ seconds}$$

It is important to understand that the general method is not limited to the two cases cited. These are two illustrative examples where the method has proved of value and other examples can be found. The measurement of the voltage relations along an artificial transmission line or the measurement of the phase relations in resonant circuits might be mentioned.

References

1. ALTERNATING CURRENT BRIDGE METHODS, **B. Hauge.** Isaac Pitman and Sons, Ltd., London, 1930.

2. A HIGH SENSITIVITY POWER FACTOR BRIDGE, **Kouwenhoven and Banos.** AIEE TRANSACTIONS, volume 51, 1932, page 202.

3. THREE METHODS OF MEASURING DIELECTRIC LOSS, **Doyle and Salter.** AIEE TRANSACTIONS, volume 45, 1926, page 634.

4. THE USE OF A DYNAMOMETER WATTMETER, **E. S. Lee.** AIEE JOURNAL, volume 45, 1926, page 746.

5. A-C POTENTIOMETERS AND THEIR APPLICATIONS, **C. V. Drysdale.** *Journal* Institution of Electrical Engineers, volume 68, 1930, page 339.

6. EQUIPMENT FOR TESTING CURRENT TRANSFORMERS, **F. B. Silsbee, R. L. Smith, N. L. Forman, and J. H. Clark.** *Bureau of Standards Journal of Research*, volume 11, 1933, page 93.

7. ELECTRICAL MEASUREMENTS, **Frank Laws.** McGraw-Hill Book Company, N. Y., 1938, page 328.

Ampere-Squared-Second Recorder

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Synopsis: A device is described which records the ampere-squared seconds in an electrical impulse of less than 0.5-second duration. A number of these have been used in resistance-welding applications to monitor the electrical variables and have contributed greatly to the uniformity of welds. The problem of indicating $^1/_{60}$-second impulses at the rate of 100 a minute with the same device that will correctly integrate a half-second impulse has been solved by the use of a moving element with an infinitely long natural period, combined with a supplementary circuit which returns the element to the starting position in a fraction of a second.

Introduction

IN resistance welding, it is necessary to control many variables to get optimum results. The control of the surface conditions of the material being welded, the size and shape of the electrodes, and the pressure between electrodes are mechanical and metallurgical problems. The control of the energy used in welding is a combined electrical, mechanical, and metallurgical problem. The electrical energy which produces the heat is a function of the square of the current, the time of application, and the resistance. The resistance at the weld will depend upon mechanical and metallurgical conditions which themselves must be subjected to careful control. It is logical then to measure as an electrical entity only those conditions which are responsive to electrical control, namely current and time or $\int i^2dt$. The usual welding circuit is highly reactive and the resistance of the material being welded has but little effect on the current, and therefore the ampere-squared seconds are not greatly influenced by variables which should be controlled by nonelectrical means.

An ampere-squared-second recorder should:

1. Give a record for each weld.
2. Operate over the range of currents and time used in resistance-welding operations, and be adjustable to indicate 100 per cent for any specified value of $\int i^2dt$.
3. Not limit the speed of welding.
4. Give an alarm when specified limits are exceeded.
5. Stand extreme overloads.

With such a device, the supervisor can adjust the sensitivity of the recorder to read 100 per cent for the correct electrical conditions for a given welding job. The operator then works in the usual way, and the device makes a record for each weld. If the electrical conditions change appreciably for any subsequent weld, an alarm is given, and, in some applications, the welding control is automatically locked.

The currents and times used in resistance-welding operations vary over a wide range, but by the use of transformers, any desired fraction of the current and therefore of the ampere-squared seconds in the circuit can be applied to the ampere-squared-second recorder. The most usual range of time is from $^1/_{60}$ to $^1/_2$ second, or from 1 to 30 cycles with a 60-cycle-per-second source. The rate of welding may be as high as 100 per minute on welds of a few cycles duration.

A conventional a-c ammeter can be used to indicate ampere-squared seconds if the duration of the impulse is short in comparison with the natural period of the instrument. If the moving element starts from rest, the maximum throw is directly proportional to ampere-squared seconds. Before the next impulse can be applied, the instrument must have come to rest again. For accurate work, the natural period should be ten or more times as long as the length of the longest impulse to be measured. When the longest impulse is 0.5 second, this would require a natural period of 5 seconds and the rate of welding on the shorter impulses would be less than 12 per minute.

Principles of a Suitable Mechanism

The arrangement finally developed is somewhat similar in operation to the Grassot fluxmeter. Instead of a mechanism for producing kinetic energy from the impulse and then converting to potential energy, as is done in ballistic measurements, a mechanism is used which produces an angular velocity proportional to the current squared which is maintained for the duration of the impulse.

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Figure 1. Moving element for ampere-squared-second recorder

The moving element consists of two separate iron-vane ammeter elements, an eddy-current damping vane, and a pointer, all on the same shaft, as in figure 1. One ammeter element produces clockwise rotation due to the impulse and the other counter-clockwise rotation from a separate source to return the element to the starting position. The deflecting element is at the bottom and the return element at the top in figure 1. The element can also be seen in position in figure 2.

If there were no physical limitations, the element would be made with no inertia, no spring torque, and no friction. Under these idealized conditions and with the element at rest, the torque applied when one ammeter element is energized is proportional to the current squared. The moving element instantly takes on a velocity such that the damping torque is equal to the applied torque. Since the damping torque is directly proportional to the angular velocity, it follows that angular velocity is proportional to the current squared. Under this condition of no inertia, the element will stop instantly when the impulse stops. The angular displacement then is proportional to the integral of i^2dt.

When inertia is present, the element will neither start nor stop instantly. In the appendix, it is shown that the angular displacement from a point of rest to a final rest position is independent of the inertia. In the derivation, friction is neglected because in a practical design the actuating torques will be large in comparison with friction torques.

The equations show that the deflection is proportional to $\int i^2dt$ when: (1) the electrical torque is proportional to the square of the current and independent of the angular position, and (2) the damping is proportional to velocity. In the recorder described, the damping is proportional to velocity, but neither of the other conditions are strictly met. The overall results, however, are such as to obtain substantially linear response with $\int i^2dt$.

Figure 3. Complete ampere-squared-second recorder

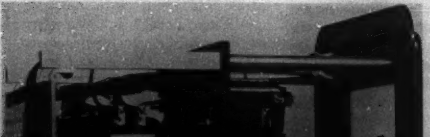

Figure 2. Side view of ampere-squared-second recorder with case off

Auxiliary Devices

The basic element described needs the addition of a recording and alarm mechanism and a device to control the sequence of events to make a complete instrument. (See figure 3.)

A. Method of Recording

A record of the maximum deflection of the pointer for each impulse is recorded on a strip chart. The chart drive is so arranged that the paper moves at a uniform speed as long as the welds are not spaced more than a few seconds apart.

A relay-operated print bar brings the pointer sharply against the strip chart (which is backed up by an inked roller ribbon) after the pointer comes to rest. This makes a short line on the back of the paper. The paper used is thin and the record shows through as a series of dots, one for each weld. The distance of the dot from the starting position is directly proportional to ampere-squared seconds. Once the record is printed, the other ammeter element is energized for a fixed time to return the element to the starting position.

The printing bar is carried by the Z-shaped member in figure 2. The printing bar is of metal except for an insulated

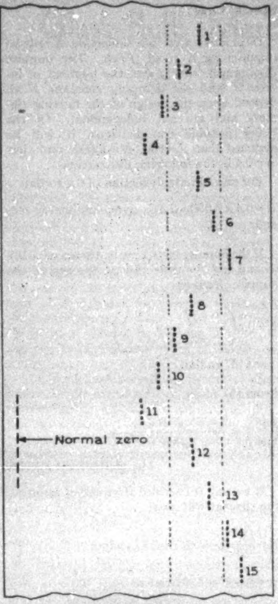

Figure 4. Tracing of an actual record from the weld recorder

Each of the darker dots represents one weld. The allowable limits are represented by the two parallel rows of light dots at the right. Groups 1 through 7 were made with 3 cycles current—the current changed as follows:

1—Current normal
2—Current reduced 5 per cent
3—Current reduced 10 per cent
4—Current reduced 15 per cent
5—Current normal
6—Current increased 5 per cent
7—Current increased 10 per cent

Groups 8 through 21 were made with current and voltage to the welder held constant and timing changed:

8—10 cycles
9— 9 cycles
10— 8 cycles
11— 7 cycles
12—10 cycles
13—11 cycles
14—12 cycles
15—13 cycles

insert at the point corresponding to 100 per cent deflection. An alarm is sounded if the pointer is not beneath the insulated insert when printing occurs. The length of the insert is determined by the tolerance allowed in $\int i^2 dt$. Usual practice permits a variation of ±15 per cent in the ampere-squared seconds. An example of the type of record is shown in figure 4.

B. Sequencing Unit

The "brain" of the device is a unit which knows when to print the maximum deflection, when to energize the return element, and how long it must be energized to come to rest in the starting position.

The sequence of events is:

1. The impulse occurs and the pointer starts to move.
2. There is a waiting period while the pointer comes to rest.
3. The printer operates.
4. The moving element is returned to the starting position.
5. The device is made ready for the next impulse.

The waiting period must start when the impulse ends. The length of the waiting period depends upon the velocity at the end of the impulse, the moment of inertia of the system, the amount of damping, and the accuracy required.

The greatest velocity will be attained for the shortest weld and if the waiting period is sufficient for correct indication then, it will be more than adequate for any longer weld. The relationships used in determining the waiting time are shown in the appendix. On a one-cycle weld, with the recorder in figure 3, the pointer deflects approximately 25 per cent in the sixtieth of a second the impulse lasts and then coasts to nearly 100 per cent in the next tenth of a second. With the same recorder, it will deflect about 95 per cent of the final deflection during a 30-cycle weld, and in the next tenth of a second be substantially 100 per cent of final deflection. With the element used, a tenth-second wait was found adequate.

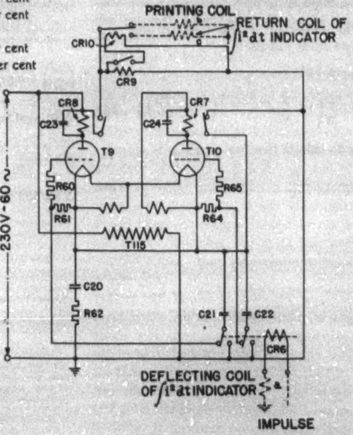

Figure 5 (right). Schematic diagram of sequence unit

Since there is practically no control torque on the moving element, it will remain in any position it is placed. The zero position is determined by a stop, but a slight unbalance or shock might move the element in time. To provide a definite zero position, a small amount of magnetic material is attached to the damping vane. The damping magnets attract this material and hold the pointer against the stop. Only a small deflection is required to carry the magnetic material out of the field of the damping magnets.

Only a short time can be allowed for the return of the pointer, so it is necessary to accelerate it rapidly. On the other hand, if it hits the zero stop with high velocity, it will bounce and the return element will need to be energized until it comes to rest. The return element was designed to have high torque when the pointer was near full scale and to have the torque fall off rapidly as the element approached zero position. The return of the pointer from any scale position to rest at the zero position can be accomplished in 0.15 second.

A schematic diagram of the unit which controls the sequence of events is shown in figure 5. Under stand-by conditions, vacuum tube *T*-10 is conducting and relay *CR*-7 is energized. Tube *T*-9 is blocked by capacitor *C*-20 (which is charged to approximately 300 volts by current passed by *T*-9). The grids of both tubes are at filament potential. When an impulse is received, *CR*-6 is energized which connects the grid bias capacitors *C*-21 and *C*-22 in parallel with *C*-20. The grid bias capacitors then share the charge with the blocking capacitor and some current is passed by *T*-9 to again block itself, but this is not enough to operate *CR*-8. No visible change occurs except that *CR*-6 pulls in and stays in during the impulse. When the impulse ends, *CR*-6 drops out, connecting the charged bias capacitors across each grid. Both tubes are then well beyond cutoff and *CR*-7 drops out. This short-circuits the blocking capacitor *C*-20, but *T*-9 cannot yet pass current because of the high negative grid voltage. The charge on *C*-21 leaks off and 0.1 second after *CR*-6 opened, *T*-9 becomes conducting pulling in *CR*-8, which energizes the printing relay and *CR*-9. When *CR*-9 pulls in, it operates *CR*-10 which transfers power from the print coil to the return coil of the recorder. About 0.15 second after *CR*-8 operated (or 0.25 second after *CR*-6 dropped out) the bias of *T*-10 has leaked off enough to conduct and pull in *CR*-7. This removes the short circuit from the blocking capacitor *C*-20 and it charges up and stops the flow of current through *T*-9 and *CR*-8. Then *CR*-8, *CR*-9, and *CR*-10 drop out and the circuit is again ready for another sequence.

Conclusion

A considerable number of these recorders have been put in service in the last three years. In spite of the apparent complexity, the high-speed operation, and the rough usage they receive, their performance has been satisfactory. While the recorder is particularly designed for resistance-welding applications, the combination utilized represents a new measuring tool that may be valuable in other fields.

Appendix

Assume system starts at rest when $t=0$ (time) and $\theta=0$ (angular deflection). The impulse starts at $t=0$ and lasts for a second (producing a uniform torque) of D dyne-centimeter seconds. The torque is $\left(\frac{D}{a}\right)$ dyne-centimeter. When the system moves there will be a counter torque due to inertia $\left(J\frac{d^2\theta}{dt^2}\right)$ and one due to the damping magnets $\left(N\frac{d\theta}{dt}\right)$.

$$J\frac{d^2\theta}{dt^2}+N\frac{d\theta}{dt}=\frac{D}{a} \tag{1}$$

Let

$$N\frac{d\theta}{dt}-\frac{D}{a}=p$$

$$\frac{J}{N}\frac{dp}{dt}+p=0 \tag{2}$$

$$\frac{J}{N}\ln\frac{pa}{D}+t=0 \tag{3}$$

$$p=\frac{D}{a}e^{\frac{Nt}{J}}=\frac{D}{a}-N\frac{d\theta}{dt} \tag{4}$$

$$-\frac{J}{N}e^{\frac{-Nt}{J}}=t-\frac{Na\theta}{D}+c \tag{5}$$

When

$t=0 \quad \theta=0$

$$\theta=\frac{D}{Na}t-\frac{JD}{N^2a}\left(1-e^{-\frac{Nt}{J}}\right) \tag{6}$$

$$\frac{d\theta}{dt}=\frac{D}{Na}\left(1-e^{-\frac{Nt}{J}}\right) \tag{7}$$

At the end of the impulse $t=a$, $\theta=\theta_1$, $\frac{d\theta}{dt}=\left(\frac{d\theta}{dt}\right)_1$

$$\theta_1=\frac{D}{N}\left(1-\frac{J}{Na}\left(1-e^{-\frac{Na}{J}}\right)\right) \tag{8}$$

$$\left(\frac{d\theta}{dt}\right)_1=\frac{D}{Na}\left(1-e^{-\frac{Na}{J}}\right) \tag{9}$$

The torque due to current ceases at this point and letting

t' = time after impulse stops
θ' = angle traversed after impulse stops

$$J\frac{d^2\theta'}{dt'^2}+N\frac{d\theta'}{dt'}=0 \tag{10}$$

As before let

$$\frac{d\theta'}{dt'}=p'$$

$$\ln\frac{p'}{p_0'}+\frac{N}{J}t'=0 \tag{11}$$

$$\theta'=\frac{J}{N}\left(\frac{d\theta}{dt}\right)_1\left(1-e^{-\frac{Nt'}{J}}\right) \tag{12}$$

The deflection at any time (t) after the *start* of the impulse =

$$\Delta=\theta_1+\theta'$$

$$\Delta=\frac{D}{N}\left(1-\frac{J}{Na}e^{-\frac{N(t-a)}{J}}\left(1-e^{-\frac{Na}{J}}\right)\right) \tag{13}$$

at an infinite time

$$\Delta=\Delta_\infty=\frac{D}{N} \tag{14}$$

Special Case

In practice a certain deflection Δ will be required for a given $\int i^2dt$. The impulse D obtained from $\int i^2dt$, the moment of inertia J, and the damping constant N all depend upon the design of the moving element and are not independent. Of the many possible combinations, it will be assumed that $J=150$, $N=6{,}000$, and that $\Delta=0.4$ in the following illustration.

For an ultimate deflection of 0.4 radian

$D=0.4\times 6{,}000=2{,}400$ dyne-centimeter seconds

If the impulse lasts 1 cycle, then $a=0.0167$ second and the deflection at the end of the impulse (from 8)

$$\theta_1=0.4\left(1-\frac{60}{40}\left(1-e^{-\frac{40}{60}}\right)\right)$$

$$=0.107 \text{ radian}$$

From (9)

$$\left(\frac{d\theta}{dt}\right)_1=\frac{2{,}400\times 60\times 0.487}{6{,}000}=$$

11.7 radians per second

If we wait 0.1 second after end of impulse, the element will coast

$$\theta'=\frac{11.7}{40}(1-e^{-4})=0.287 \text{ radian}$$

$$\Delta=\theta_1+\theta'=0.394 \text{ radian}$$

$$\Delta_\infty=0.400 \text{ radian}$$

For the one-cycle case a change of 25 per cent decrease in the assumed wait of 0.1 second would make 3 per cent difference in overshoot or 2.2 per cent change in indicated value. An increase of 25 per cent will cause less change.

At 30 cycles, $a=0.5$ second

$$\theta_1=0.4\left(1-\frac{2}{40}\left(1-e^{-20}\right)\right)$$

$$=0.380 \text{ radian}$$

$$\left(\frac{d\theta}{dt}\right)_1=0.8 \text{ radian}$$

Waiting 0.1 second, the element will coast

$$\theta'=\frac{0.8\times 0.982}{40}=0.0196 \text{ radian}$$

$$\Delta=0.399 \text{ radian}$$

Glass-Bulb Mercury-Arc Rectifiers for Traction Service

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Synopsis: The paper gives a general survey of the ability of glass bulb mercury-arc rectifiers to satisfactorily meet the stringent requirements of traction service. The subject matter is arranged on the assumption that engineers of the United States and Canada are not familiar with the use of this type of equipment for traction work, and the author has approached the subject from the point of view of an engineer considering the merits of different types of conversion equipment.

Major features pertaining directly to glass bulbs are discussed such as plant capacity, maintenance, efficiency, robustness, etc., but no attempt is made to include wider problems common to mercury-arc rectifiers as a class, namely frequency changing, inversion, harmonics, etc.

Introduction

THE first type of rectifier installed in England (1923) for traction service was a glass bulb mercury-arc rectifier, and installations of this type are widely used for traction service in Great Britain and countries served by British manufacturers. Glass bulb installations exceeding 5,000 kw in capacity are in service operating for the most part within the 500 to 1,600-volt range most commonly employed for D.C. traction. On the London Passenger Transport Board system alone approximately 80,000 kw of glass bulb rectifier plant (distributed among more than 50 substations) is installed for operating trams and trolley-busses. For railway electrification, glass bulb rectifiers are equally serviceable and widely used. Typical examples are London, Midland and Scottish Railway (recently, electrification of the Euston-Watford line required 17 substations each approximately 2,500 kw), Bombay, Baroda and Central India Railway, and the New Zealand Government Railways. In 1938, against world wide competition, one glass bulb rectifier manufacturer received 15,000 kw of a 19,800-kw contract for traction rectifiers in South America. Typical Canadian traction installations are: British Columbia Electric Railways (2–330 kw, 1–660 kw), City of Edmonton 1–700 kw), and Britannia Mining and Smelting Company (1–330 kw).

Approximate over-all dimensions of a 1,000-kw, 550-volt D.C. rectifier, excluding transformer, would be 23 ft long by 4 ft deep by 9 ft high.

The above information indicates that there is nothing new in using glass bulb rectifiers for traction service.

Plant Capacity

The current output of a single glass bulb being limited, engineers unfamiliar with the equipment question the advisability of operating a number of bulbs in parallel. They know that to obtain 2,000 kw at 800 volts D.C., i.e., 2,500 amp, it would neither be economical nor practicable to operate numerous rotary convertors in parallel, and naturally do not immediately appreciate that this arrangement can be utilized with glass bulbs to the greatest advantage. Eight or ten glass bulbs would be paralleled, depending on the size of bulb used.

Arc voltage drop being proportional to the arc current a number of parallel arc paths minimizes the voltage drop and thereby increases plant efficiency. For example, if a single tank rectifier has an arc drop of 20 volts at 500 amp and 25 volts at 3,000 amp, then a 3,000-amp equipment comprising 6–500 amp units would effect a saving of about 15 kw in arc drop. Allowing for extra fan and exciter losses, this may represent an efficiency gain of 1%.[1,4]

It is well known that an arc tends to concentrate within a small space on the anode and steel tank rectifier manufacturers have had to resort to baffles and shields to obtain more even distribution. Arc concentration overheats the anode and gives rise to conditions liable to cause backfire. It has been found that the best way to obtain effective increase in anode area, is not by increasing the size of anodes, but by increasing their number. For the 2,000-kw traction rectifier mentioned above, the steel tank rectifier would probably be equipped with 12 anodes, whereas 8 six-arm glass bulbs would have 48 anodes, thereby having an enormous effective anode area to ensure avoidance of hot spots.[1]

Should inspection or maintenance be necessary on a bulb cubicle, that cubicle can be readily cut out of service without disturbing operation of the remaining cubicles. Thus in the above case, the seven bulbs would share full load between them, slightly overloaded, until the other bulb was restored to service. With other conversion equipment such maintenance would probably entail shutting down the entire unit.

In Great Britain, general development of steel tank rectifiers is in the direction of air-cooled pumpless units whose present outputs are between 250 and 750 amp, and the chief engineer of one manufacturer of both steel tank and glass bulb rectifiers, states that the general trend of application is in the direction of operating convenient-sized units in parallel rather than employing large single units with numerous anodes.[1] The advantages of the multi-unit construction used by glass bulb manufacturers, are now becoming more generally recognized.

Characteristics of Conversion Equipment

The engineer must examine a number of important factors before reaching a decision on conversion equipment. The more important of these factors are discussed below and the degree to which glass bulb rectifiers meet desired characteristics is considered.

1. INITIAL COST

(*a*) *Equipment.* The cost per kw varies considerably, but steel tank, steel bulb, and glass bulb rectifiers are all competitive and show to advantage over rotating equipment.

(*b*) *Buildings.* Glass bulb cubicles can, in general, be arranged in equal or less floor space than single tank rectifiers, and as they have no auxiliaries such as water cooling pipes, pumps, recoolers, blower motors, or special starting equipment, building costs are less than for other conversion equipment.

(*c*) *Installation.* The costly foundations required for rotating equipment are not necessary for rectifiers, and the uniform weight distribution of glass bulb

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1. For all numbered references, see list at end of paper.

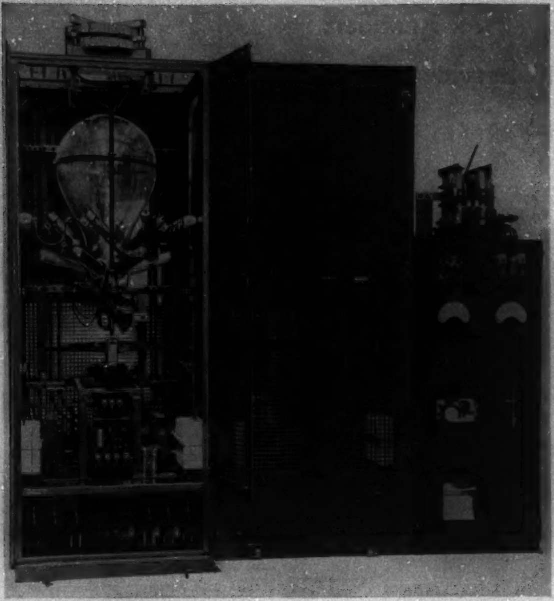

Figure 1. Street railway traction rectifier

330 kw, 550 volts d-c, double bulb unit complete with high speed circuit breaker panel, installed in British Columbia

cubicles can be sustained by a much lighter floor than that required by other types of rectifiers.

Since each cubicle is a self-contained rectifier unit with all auxiliaries (excluding main and auxiliary transformer) the installation, testing, and setting to work is much simpler than that required by more complicated types of conversion apparatus. This represents a considerable saving in initial cost.

2. MAINTENANCE

(*a*) *Attention Required.* In common with other types of conversion equipment, glass bulb substations can be unattended, operating automatically or by remote control. No special attention is required.

(*b*) *Inspection.* Bulb transparency is of considerable practical value to operating engineers since by merely opening the cubicle doors it is immediately apparent whether the rectifier is firing properly. If required, individual bulbs can readily be removed from service and complete inspection carried out without interrupting supply to the remaining bulbs.

(*c*) *Sources of Trouble.* The sources of trouble of rotating equipment are well known and need not be discussed here, but it is advisable to draw some comparison between steel tank and glass bulb rectifiers in this respect.

In most steel tank rectifiers the great majority of breakdowns occur in auxiliary equipment, not in the rectifiers. On one large traction system, the records of 16 rectifiers over a period of 19 months showed maintenance carried out as follows: control gear—45.5%, vacuum pumps—31%, water system—21%, rectifiers—2.5%. Percentages refer to the number of failures. On those occasions when steel tanks must be opened and baked out, the expense involved due to outage of plant and cost of power for bake-out may be considerable.[2]

In glass bulb rectifiers connectors passing through the bulb wall have the same coefficient of expansion as the glass, thereby ensuring no leakage through the seals. The bulb itself is not porous and therefore perfect conditions for a permanent vacuum obtain. No pumping is required. Cooling is provided by a vertical-motor-driven fan, placed directly under the bulb. Ignition may be very simply accomplished with a dipping electrode attracted to a small, external electromagnet operated from the exciter transformer. The only moving parts are the fan and a small contactor which operates when starting. No other auxiliaries are required.

Glass bulb rectifiers, having a minimum of auxiliaries, require negligible maintenance.[6]

(*d*) *Bulb Life.* The life of a glass bulb is indefinite, as no wear occurs. Records show many bulbs in service 15 years without developing trouble. Possibly the best commentary is the fact that glass bulbs of reliable manufacture can be insured at $4^1/_2\%$ per annum to cover a period of 10 years. Experience has shown however, that maintenance and replacement costs are too small to justify even this small premium. For a 2,000-kw traction rectifier of 8 bulbs, insurance at the above rate costs approximately 20 cents per kw per annum.

3. EFFICIENCY

The advantages of lower arc drop and resulting increased efficiency consequent upon multi-unit construction were briefly discussed under "Plant Capacity."

At the load factors encountered in traction service, the all-day efficiency of rectifiers shows to advantage over rotating equipment,[3] due largely to the relatively flat rectifier efficiency curve from $^1/_4$–$^5/_4$ load. Typical efficiencies (in per cent) for glass bulb traction rectifiers are:

D-C Voltage	Load				
	$^1/_4$	$^2/_4$	$^3/_4$	$^4/_4$	$^5/_4$
600	93.75	94.2	94.1	94	93.8
1,200	95.6	96	95.9	95.6	95
1.500	96	96.5	96.5	96.2	95.6

4. POWER FACTOR

The average power factor for glass bulb rectifiers is fairly flat over the normal operating range, varying from 0.90–0.94 approximately between one quarter load and full load.

5. VOLTAGE REGULATION

Glass bulbs, like steel tank rectifiers have a drooping voltage characteristic, the regulation being approximately 5%. Compounding can be effected by grid control, voltage regulator, or by means discussed in other papers.[5]

6. OVERLOADS

(*a*) *Standard Ratings.* For glass bulbs standard overload ratings for traction service are:

25% overload	2 hours
50% overload	$^1/_2$ hour

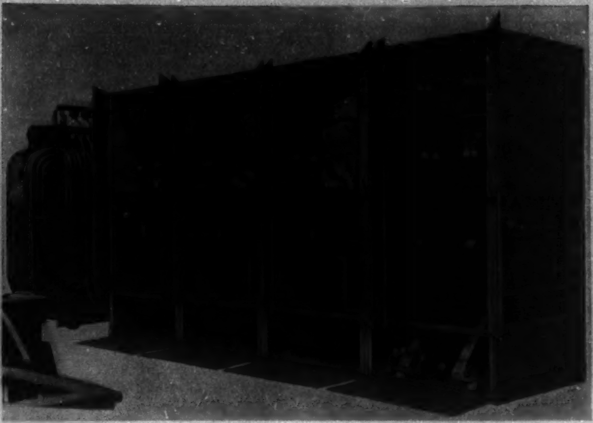

Figure 2. Railway traction rectifier

One of a set of six banks, each 600 kw, 650 volts d-c, with d-c smoothing equipment, England

100% overload	5 minutes
200% overload	momentary peaks

(*b*) *Non-Standard Ratings.* It is a simple matter to supply a glass bulb rectifier with any overload capacity required by simply installing the right number and type of bulb units with a suitable rectifier transformer. Actually each traction installation must be considered on its own merits and equipment supplied to meet particular load requirements.

(*c*) *Short Circuits.* Glass bulbs handle severe track short circuits without distress. Experience indicates that glass bulbs are less affected by short circuits than other types of conversion equipment. One engineer with wide operating experience of both rotating and static equipment confirmed that rectifiers are less sensitive than rotary converters and added that in his experience steel tanks were liable to backfire on track short circuits occurring within a half-mile of the substation but glass bulbs were not affected. The author does not intend this to represent a general condition but merely the experience of a particular engineer. It is of interest, however, that in 1938 another engineer stated "we employ a certain number of these (steel tank rectifiers) for traction purposes, but some of them have not approached the glass bulb type in respect of reliability."[1]

Multi-unit construction with its resultant low current density per anode, large effective anode area, and conditions of permanent vacuum contributes largely to stability under severe overload and short circuit conditions.

7. STABILITY AND ROBUSTNESS

(*a*) *A-C System Disturbances.* It is well known that rectifiers are insensitive to fluctuations of voltage or frequency in the A.C. network and in this respect ensure an absolute minimum interruption of traction supply not obtainable with rotary converters.[2,3]

(*b*) *Starting Time.* Ability to restart instantaneously after shut-down due to system disturbance, or to start up from cold in an emergency and to at once take on full load is of major importance. Glass bulb operation is largely independent of temperature and rectifiers of this type will start up and deliver full load current without delay. Pre-heating is not required except possibly in cold climates where the substation air temperature remains below freezing point for lengthy periods. Room temperatures from 4°C–40°C ensure immediate starting and stable operation.

(*c*) *Robustness.* Objections to glass bulbs are raised on the grounds of fragility and sensitivity to continued vibration or concussion. This has proved to be a fallacy. The bulbs are made of a special quartz type of glass and when handled with reasonable care breakages rarely occur. They are completely enclosed in individual sheet steel or expanded metal cubicles, and if anything, less liable to damage than the exposed commutators, brushes, relays, gauges, etc. of other types of conversion equipment.

The bulbs are secured firmly but flexibly in a suitable cradle and anode and cathode connections made through flexible copper conductors. Free from rigid connections of any type the bulbs are unaffected by continued vibration. Installations operating satisfactorily near gun testing locations at Woolwich Arsenal, underground in mines and subjected to blasting shocks, or in regions subject to earth tremors are ample evidence of their insensitivity to concussion.

Figure 3. Railway traction rectifier

3,600 kw, 1,200 volts d-c, installation comprising three 1,200-kw banks. One of two similar substations, England

8. Paralleling and Extensions

(*a*) *Paralleling Similar Units.* Each bulb unit of a multi-unit installation is adjusted at the factory for correct operation. In the two largest glass bulb substations in the world, 6,500 kw and 7,000 kw at 500 volts D.C. (industrial load) automatic, unattended, and remote-controlled, there are 44 and 40 bulbs respectively operating in parallel.

(*b*) *Paralleling Dissimilar Equipment.* Glass bulb units can and do operate in parallel with rotating equipment or steel tank rectifiers of equal or unequal capacities. Correct load sharing is arranged to suit plant conditions. To avoid considerable complication in grid control gear and high speed relays, it is desirable to arrange the rotating plant with a slight drooping voltage characteristic.

(*c*) *Extensions.* Glass bulb rectifier plant can be extended at any time and to any desired extent by adding the appropriate number of bulb units. This entails a considerable saving in capital cost as extensions to most conversion equipment necessitate installing units of large capacity.

Conclusions

Glass bulb rectifiers are suitable for traction service and in some respects show to advantage over other types of conversion equipment, particularly so in the multi-unit construction, lack of troublesome auxiliaries, high overall efficiency, reliability, negligible maintenance costs, and flexibility.

References

1. Recent Progress in Power Rectifiers and their Applications, **W. G. Thompson.** IEE *Journal*, volume 83, October 1938, and discussions.

2. Equipment and Performance of Steel-Tank Rectifier Traction Substations Operating on the Underground Railways of the London Passenger Transport Board, **A. L. Lunn.** IEE *Journal*, 1935.

3. Rectifier Equipments for Regenerative Working, **J. C. Read.** *B.T.H. Activities*, May–August 1940.

4. Ignitrons for the Transportation Industry, **J. H. Cox** and **G. F. Jones.** AIEE Transactions, volume 58, 1939 (December section), pages 618–22.

5. Voltage Control of Mercury-Arc Rectifiers, **G. R. McDonald.** AIEE Transactions, volume 58, 1939 (November section), pages 563–6.

6. Six Years of Glass-Bulb Mercury Arc Rectifiers, **J. H. Steede.** *Electrical News and Engineering*, June 1, 1938.

7. Frequency Changing With Mercury-Arc Mutators, **R. Feinberg.** IEE *Journal*, volume 85, October 1939.

A New Mercury Rheostatic Element for Regulation and Control

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Introduction

In many regulating and control systems, the primary or actuating stimulus is available only at a low power level and must be amplified to obtain sufficient power output for the various control functions. This requirement, along with the increased use of automatic controls during the past few years has resulted in many industrial applications which require a simple, reliable power amplifier. Electronic tubes and special designs of rotating machines have been used for amplification where a continuous type of control is required, but in most applications it is difficult to justify the use of tubes or rotating amplifiers.

Contact Controls

Contact relays are often used for amplifying power in control circuits of the "on" and "off" type. For continuous control, a single pair of vibrating contacts makes a very simple amplifier. If the contacts are operated in a hovering position, their effective resistance varies with contact pressure and large changes in resistance, which will give large current changes, can be obtained with extremely small variations in pressure. The ratio between the change in resistance and the change in pressure is a measure of the contact amplification. It is possible to obtain very high values of amplification in this manner and to adjust this amplification by changing the amount of contact vibration. A practical application of the contact amplifier to speed regulators was described in a previous paper.[1] The power output of such an amplifier is, of course, limited by the current interrupting capacity of the contacts.

Rheostatic Control

Another simple device for controlling larger amounts of power is the rheostat. By using a large number of steps it is possible to secure practically continuous control. Novel rheostatic control members have been built which require very little force and movement to operate a large number of contacts.[2] In designing a rheostatic control, the following requirements are of importance in most regulating and control systems:

1. The power required to operate the rheostat should be small since the actuating stimulus is usually at a low power level.
2. Hysteresis resulting from frictional and magnetic forces should be small.
3. The inertia of moving parts and their displacement should be low to permit quick response.
4. The contacts should require no maintenance.

The above requirements become more difficult to meet as the output power of the rheostatic control is increased. By making use of the well-known advantages of the mercury switch, a new form of multicontact mercury rheostat has been developed to meet all of the above requirements and at the same time handle relatively large amounts of power. For convenience in describing this new device it has been called a "Mercurystat".

Design of Mercurystat

The contact element of the Mercurystat contains a large number of mercury switches so arranged that they can be operated by a small force and travel. The construction is shown by the schematic drawing, figure 1. A 40-step Mercurystat with its electromagnetic driver is shown in figure 2. Referring to these two figures, small flat terminals

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The author wishes to acknowledge the assistance received from his associates in the laboratories and engineering departments and particularly the many helpful suggestions made by C. R. Hanna. The special speed control application described in this paper was made in co-operation with C. W. Drake and J. Z. Linsenmeyer. The speed matcher application was made in co-operation with C. E. Valentine and R. A. Geiselman.

[1] For all numbered references, see list at end of paper.

each having a central hole, are assembled as shown with an insulating spacer between each pair of terminals. Stainless steel bellows with a plug to reduce the volume of mercury, are attached to the bottom of the terminal assembly. Displacement of the bellows raises and lowers a central mercury column which shorts out, or cuts in, sections of an external resistor connected to the ends of the terminals. A gas space is provided at the top of the terminal assembly to prevent an appreciable change in pressure from the slight changes in volume caused by the bellows displacement. The entire assembly, consisting of bellows, terminals, insulators, and gas space, is hermetically sealed by a special process so that the interior of the Mercurystat can be pumped out and charged with an arc-reducing gas such as hydrogen. It is then sealed off from the atmosphere to eliminate all contact maintenance.

Because of the large ratio between the area of the bellows and the area of the mercury column, only a small displacement of the bellows is required to short out all terminals. In the 40-step Mercurystat shown in figure 2, the terminal assembly is about two inches high but only a few thousandths of an inch displacement of the bellows is required to short out all steps.

Power Capacity

The power capacity of the Mercurystat is fixed by the current interrupting capacity of the contacts and by the internal losses of a given design. The contact duty in a rheostat of this type is much less severe than in the ordinary mercury switch operated at line voltage. The large number of steps reduces the voltage across steps so that relatively high currents can be handled. For example, a small 40-step Mercurystat as shown in figure 2 can easily control six kilowatts. Life tests made with a unit this size operating in a 220-volt d-c circuit and varying the current from 12 amperes minimum to 30 amperes maximum showed practically no sign of contact wear after 5,000,000 cycles of operation.

At higher currents, the internal losses of the above design were greater than could be dissipated with a reasonable temperature rise. These losses result from the resistance of the terminals and the mercury column and from the contact drop between the mercury and the terminals. Greater power capacity can of course be obtained by increasing the cross section of the terminals and the mercury column. This will reduce the resistance of these parts and at the same time give a larger assembly capable of radiating more heat. The contact drop tends to remain constant as in the case of carbon brushes on slip rings and is in the order of about one-half volt for currents up to 100 amperes.

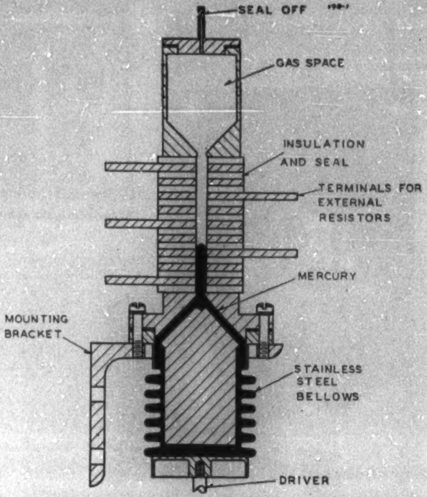


Figure 1. Schematic diagram of Mercurystat

Design of Driver

In some applications the bellows of the Mercurystat can be operated directly by a lever or cam which is a part of the control system. In other cases it is desirable to have an electromagnetic driver so the Mercurystat can respond to electrical stimuli. A driver for this purpose should be free of friction and magnetic hysteresis and should have some reference standard such as a spring force for its calibration. Such a driving element is shown in figure 2. The driving arm is mounted on cross springs to provide a frictionless pivot. An insulating turnbuckle is used to connect the bellows to this arm. A calibrating spring is attached to one end of the driving arm while an iron armature is mounted on the other end. The armature travels in a magnetic circuit having a constant air gap and a laminated iron core. By proper armature shaping the negative magnetic stiffness of the driver can be made to match the positive stiffness of the Mercurystat and calibrating spring so that very small changes in coil current will cause the driver to move through a wide range. Typical response curves of a Mercurystat and driver are shown in figure 3. When working on the curve with a one per cent droop the entire resistance range is covered with only a 0.05 watt change in the driver coil. The change in power output of the Mercurystat may be say five kilowatts so that the power amplification, or ratio of output to input is 100,000. Lower values of amplification can be obtained by merely reducing the spring force. This will result in lower coil currents and a less perfect matching between the negative stiffness of the magnetic circuit and the positive stiffness of the spring. Different amounts of droop secured in this manner are shown by the curves in figure 3. The slight nonlinearity in response can be eliminated by grading the external resistance between steps to give a linear relation between input and output currents.

Changes in temperature and in atmospheric pressure may cause small variations in the calibration of the Mercurystat. The amount of these variations depends upon such factors as the type of drive, the dimensions of the Mercurystat, and its sensitivity when used in a regulating system. In any given application it is usually possible to proportion the design so that these variations are negligible. If the bellows position is positively fixed by the control, changes in atmospheric pressure will have no effect upon the calibration. Temperature changes will cause slight variations in the volume of the mercury which will change the height of the mercury column. These changes can be

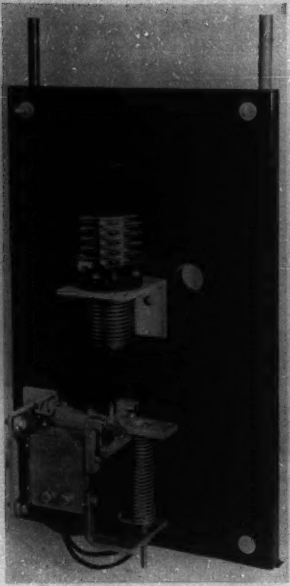
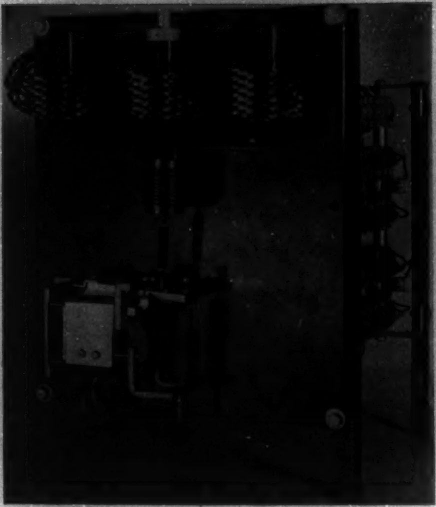


Figure 2. A 40-step Mercurystat and driver

Figure 4. Three-phase Mercurystat



kept small by decreasing the volume of mercury with a plug as shown in figure 1.

If the Mercurystat is used in a regulating system where the bellows are operated by an electromagnetic driver as shown in figure 2, the amount of error caused by variations in temperature and atmospheric pressure will depend upon the droop of the regulator. For example, if the regulator has no droop, any change in the height of the mercury column as a result of changes in temperature and atmospheric pressure will be corrected for with no error. If the regulator has a one per cent droop and changes in temperature and atmospheric pressure should cause the Mercurystat to swing through say 20 per cent of its range, the regulator will correct for this swing with a resulting error equal to one-fifth of one per cent. Errors due to variations in atmospheric pressure can be eliminated by attaching a duplicate sealed bellows to the driving arm in such a position that any change in atmospheric pressure will give equal and opposite moments to the driver arm. By making the volume of this second bellows equal to the gas space at the top of the mercury column, variations resulting from pressure changes of the internal gas with temperature can also be eliminated.

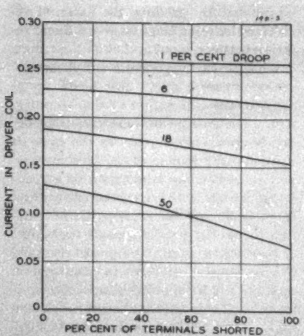


Figure 3. Mercurystat and driver response

Applications

Numerous applications for the Mercurystat will be apparent to those who have worked with regulators and power amplifiers. A few applications will be briefly described to show how it can be adapted to typical control problems.

A Mercurystat designed for controlling the speed of three-phase, wound rotor motors is shown in figure 4. Three sets of terminal assemblies are mounted on a bar which has internal holes connecting the three mercury columns to a common bellows. Movement of the bellows will then give equal changes in external resistors which are connected in each phase of the rotor. If a voltage proportional to speed is applied to the electromagnetic driver of the Mercurystat, it will serve as a stimulus for regulating any speed within the range of the control resistance. When load changes occur which cause the motor speed and the voltage on the Mercurystat driver to change, the Mercurystat will automatically adjust the external resistors to maintain a constant motor speed. The control voltage, proportional to speed, can be obtained from a small electric tachometer driven by the motor. In place of an electric tachometer the voltage across slip rings can be rectified and used to control the Mercurystat. When the motor slip increases, this voltage will increase and the Mercurystat will short out resistance to bring the motor back to its regulated speed.

In another application of speed control, a special speed cycle was made possible by using a cam to drive two Mercurystats. One Mercurystat is used to change the resistance in the rotor of an a-c motor to adjust speed. The other Mercurystat is used to secure rapid changes in speed by controlling the dynamic braking of a d-c generator mounted on the same shaft as the a-c motor. This latter control is obtained by changing the resistance in series with the armature and series field of the d-c generator. The bellows of the two Mercurystats are so coupled that when

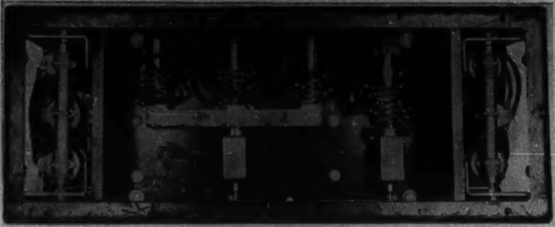


Figure 5. Special speed controller

one is inserting resistance in the rotor of the a-c motor to reduce speed, the other is shorting out resistance in the d-c generator to give increased dynamic braking. This arrangement gives a large number of speed settings which can be reached very quickly by adjusting the main control cam. Figure 5 shows this form of speed controller as built to meet special space limitations. The resistors required by the control are mounted on the sides and in the back of the Mercurystat panel.

In some applications where the operating stimulus is at a high power level, the amplification of the Mercurystat is not important. This was the case in applying the Mercurystat to a speed matching regulator. Here the Mercurystat was used because of its high-current capacity and its sealed-in contact structure. The Mercurystat is shown in figure 6 mounted on a panel which is supported by the synchronous motor and differential housing of an experimental speed matcher. It is a 100-step unit with the terminals connected to control resistors mounted in the rear of the panel. The synchronous motor drives one-half of the differential, and the cone pulley, located just below the Mercurystat resistors, is connected to the other half through the hollow shaft of the synchronous motor. If the speed of the cone pulley differs from the reference speed of the synchronous motor the movement of a traveling nut in the differential is transmitted to a lever on the side of the housing which operates the bellows of the Mercurystat in a direction to change the field excitation of the regulated motor and correct for the error in speed.

It will be apparent to those familiar with the design and application of voltage regulators that the new device can also be applied in this field. If used with an exciter, the Mercurystat has sufficient power capacity to regulate the voltage of medium and large size a-c generators. The circuits used in such regulating systems along with problems of stability and damping, have been described in a previous paper.[2]

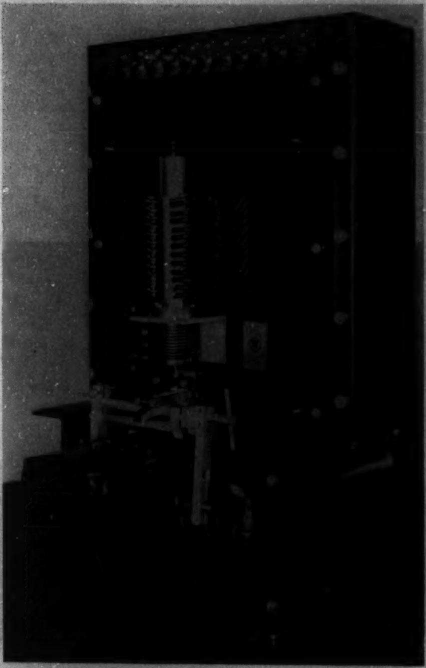


Figure 6. Speed-matcher regulator

Summary

The principal design features, along with several typical applications, were described for a new form of mercury rheostatic element which has been designated as the "Mercurystat". The unique features of the Mercurystat are its large number of hermetically sealed mercury contacts and the small force and travel required for their operation. It was shown that relatively large amounts of power can be controlled by the Mercurystat and because of the small input power required for its operation, power amplification in the order of 100,000 can be obtained. The performance of this new device along with its simplicity and absence of wearing moving parts should find many applications for it in automatic control systems.

References

1. RECENT DEVELOPMENTS IN SPEED REGULATION, C. R. Hanna, K. A. Oplinger, and S. J. Mikina. AIEE TRANSACTIONS, volume 59, 1940, pages 692–700

2. RECENT DEVELOPMENTS IN GENERATOR VOLTAGE REGULATION, C. R. Hanna, K. A. Oplinger, and C. E. Valentine. AIEE TRANSACTIONS, volume 58, 1939, pages 838–44.

Modern Motors Serve City Transit Systems

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Synopsis: City transit motors operated today include many new mechanical and electrical features. Modern vehicles differ greatly from those of a decade ago and propulsion motors contributed substantially to the advances achieved. Meeting present-day needs has required designs for 300 and 600 volts involving extensive use of new materials, application to new types of drive, and suitability for dynamic braking. Mechanical design includes unusual developments in frame construction, method of mounting, lead arrangement, ventilation, housings, and bearings. Compactness has led to ingenuity in brushholder arrangement to secure accessibility. Armatures require judicious selection of materials for shafts, cores, and commutators as well as special seasoning and balancing procedure. Electrical design betterments have been introduced in armature slots, armature coils, field coils, and insulation. Provision for quietness and good commutation are also salient points. The new designs have resulted in high electrical and weight efficiency for series motors used on Diesel-electric busses, trolley coaches, and street cars in comparison with old type machines. Excellent commutation has produced outstanding stability at rapid accelerating and dynamic braking rates. Vehicle performance is exceptional with respect to safety, operating efficiency, available selection of accelerating and braking rates, and the broad speed range of the dynamic brake.

"PEOPLE must be served" has become the success axiom of city transit operators. Creative action has resulted to keep pace with present-day travel demands. The development of modern vehicles is closely associated with progress in propulsion motor construction. Machines in use today include many new mechanical and electrical features. Weight below ten pounds per horsepower, compared with almost three times this figure for old machines, is a measure of what has been achieved with modern designs. These advances have made possible the Diesel-electric bus, trolley coach, and the modern street car.

Performance Requirements

Motors used today differ greatly from older types and are the result of the solution of a number of interesting problems. Machines have been designed for 300 volts and applied two in series to reduce weight in comparison with 600-volt motors. In order to meet the limitations imposed by the space available, new materials have been used extensively. This has assisted in meeting the demand for motors much smaller than older types producing the same horsepower.

The change to different types of drive with spring-supported motors required fundamental departures from previous practices. Gear cases and axle bearings were no longer needed. The new method of mounting protected motors from road shocks and thus permitted lighter construction. An increase in gear ratio was practicable which gave a "step-up" in armature speed and a decrease in over-all weight. Modern vehicles use the motors for stopping as well as starting which introduced the added duty of dynamic braking.

Mechanical Design Features

An evolution has occurred in mechanical design features because of the experience gained during the transition from the axle-hung to spring-supported type machine. The rolled steel frame has replaced the cast type. This new construction has important manufacturing advantages and provides a more effective magnetic circuit. Hidden casting blowholes are eliminated, a condition which often reduced the area of the magnetic section and caused variation in speed between individual motors built in accord with identical designs. There is ample room for original assembly or servicing of the poles field coils, and field wiring between coils The method of motor support is dependent on the vehicles. For example, trolley coach motors are held at the two ends by feet cast on the housings which are suitable for bolting to horizontal cross members. Modern street car motors are supported by clamping them in circular grooves cut in the outside of the frames.

Flexible stranded cable with rubber insulation and heavy rubber jacketing is used for leads. Protection is provided for passage through frames by heavy rubber bushings. Four leads are needed by a series motor, two for the armature with the interpole field, and two for the main field. When field shunting is applied to increase speed, it is obtained by applying a shunt across the main field. The older method of field tapping required an additional lead. A novel terminal construction is used on the modern street car motor which employs four insulated stud type terminals. The construction is illustrated in figure 1. The terminals are made of brass and are tapered at the outer end. A stud bolt between each pair is used for clamping the internally tapered terminals of the car wiring tightly against the motor terminals. This scheme permits easy removal of the car wiring from motors in the restricted mounting space available on trucks.

Commutator and fan end housings are removable castings that fit accurately into the bore of the frame and are bolted to its extremities. Each housing includes its bearing assembly. The commutator end housing supports and provides access to the brush holders as well as being an inlet for the ventilating air. The fan end housing has space for the

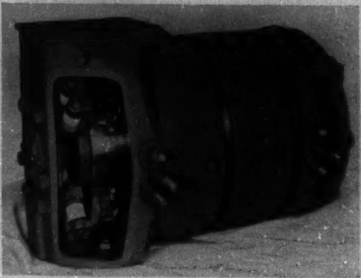

Figure 1. View showing novel lead arrangement of 300-volt, 55-horsepower street-car series motor

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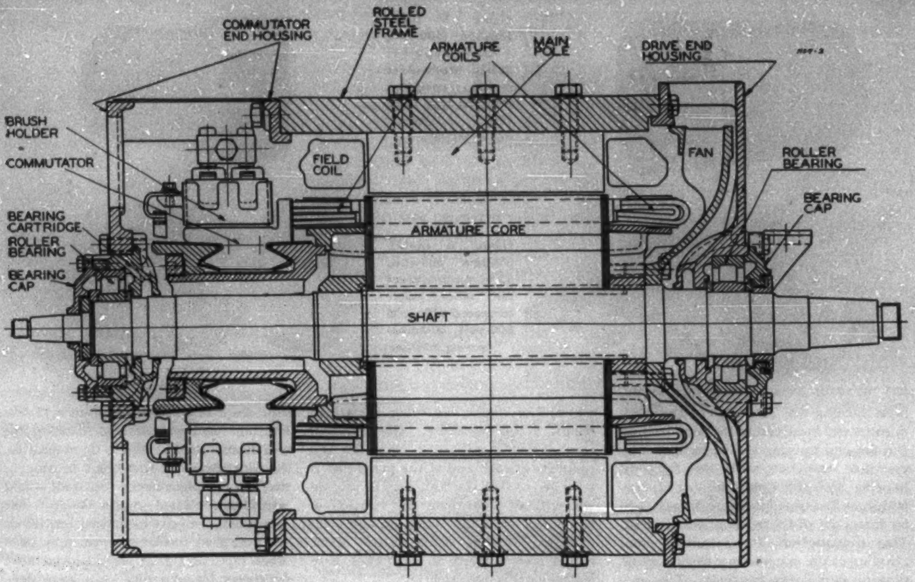

Figure 2. Construction of 600-volt, 140-horsepower trolley coach series motor

fan and is built with air outlets proportioned to give quiet and efficient discharge of the air leaving the fan blades. Adequate ventilation is obtained by the armature shaft fan. It draws air in at the commutator end, through spaces between the field coils and over the surface of the armature, as one path, and through longitudinal ducts in the armature core as a parallel path.

The support of bearings in housings and the method of enclosure have been carefully analyzed in conjunction with recent development work. Roller or ball bearings with grease lubrication are standard. Performance records are exceptional for the designs of the past five or six years in comparison with those of 10 or 12 years ago. This is especially true in the case of roller bearings. It can be attributed to the proper initial internal clearances, the method of mounting and assembly in the housings, and the effective enclosure of the grease space by metal labyrinth seals.

The housing and bearing construction shown on figure 2 is typical. At the fan end, the bearing fits in a bore in the housing and has an outer bolted cap. At the commutator end, the bearing is carried in a separate cartridge having an outer enclosing cap. This cartridge fits in a bore in the commutator end housing and is also bolted to it. The commutator end roller bearing is of the thrust type and locates the armature longitudinally. A "free" type roller bearing is used at the fan end, allowing for expansion and contraction in the length of the shaft with respect to the stationary parts. An important advantage of this type of construction is that the complete armature with bearings may be removed without exposing either of the bearings to dirt. Such a removal is made by taking out the

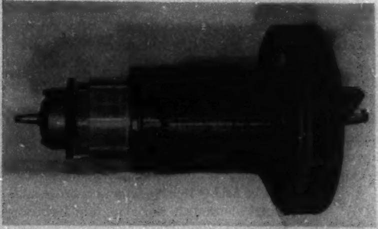

Figure 3. Armature for 600-volt, 140-horsepower trolley coach series motor

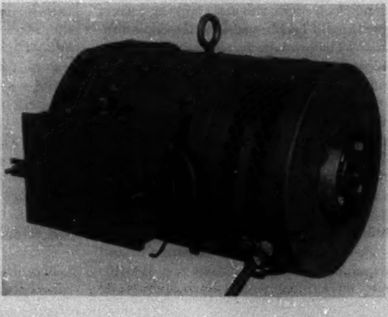

Figure 4 (right). Diesel-electric bus motor for use with 125-horsepower power plant

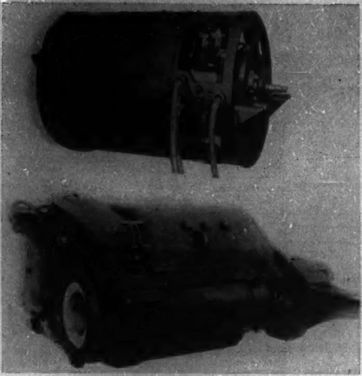

Figure 5 (left). Modern 600-volt, 140-horsepower trolley coach series motor compared with old type 600-volt, 125-horsepower machine

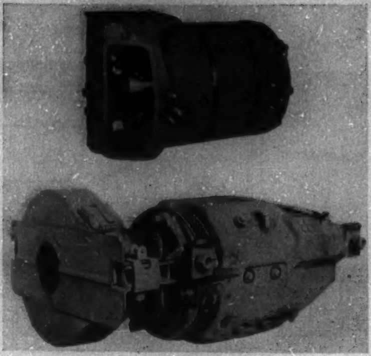

Figure 6 (right). Modern 300-volt, 55-horsepower street car series motor compared with old 600-volt, 50-horsepower machine

bolts holding the cartridge to the commutator end housing and the bolts holding the fan end housing to the frame. The complete armature with the fan end housing and the commutator end cartridge, as illustrated in figure 3, may then be drawn out of the motor at the fan end. This construction also permits a final grinding of the commutator surface while running in its own bearings. The procedure is to support the armature and housing assembly in a grinding fixture and rotate the armature by a small driving motor coupled to the shaft while grinding the commutator surface.

The commutator end housing construction is closely connected with the support and accessibility of the brush holders. Carbon brushes are subject to wear and the brush holders must be available for easy inspection and replacement of carbons. The brush holder used is the bronze box type with two insulated supporting pins. This provides a simple and accurately located mounting with means for radial adjustment as the commutator surface wears to a smaller diameter. Brush holders have adjustable steel clock springs to maintain the correct pressure of brushes against the commutator. The pressure finger and the carbons have flexible copper shunts. Spacing and position are set during manufacture by means of a jig which accurately locates the center line of the brushes relative to the center line of the commutating pole faces. After this setting is made, bolts holding the brush holder support block to the housing are welded in place. Individual brush holders are interchangeable.

Armatures are solidly constructed and capable of operating at high speeds without mechanical distortion. High-grade material in shafts and other parts contribute to this durability. The laminated core assembly with cast armature coil supports at each end of the punchings is arranged so that the shaft may be pressed out without disturbing the relation of punchings, armature windings, and commutator. Broken shafts rarely occur, but in case of injury to shaft ends, easy replacement is essential.

Commutators are seasoned at high temperature while being rotated near the maximum operating speed. This allows the mica between bars to become set to withstand movement at the maximum temperature and speed encountered in service. The seasoning process is a series of heatings and tightenings carried on under these conditions until the commutator runs smoothly either cold or hot at maximum speed. Armature balancing in a modern dynamic balancing machine is essential. The best construction provides for two dynamic balancings: one before the armature is wound, and one afterward. All of the unbalance of the armature except that due to the winding is permanently eliminated and if the armature is rewound, only the unbalance of the armature winding need be provided for. The coil support castings at each end of the punching assembly are provided with balancing pockets into which the required amount of melted lead is poured. This takes care of the unbalance before winding. The small unbalance of the armature winding is compensated for by adding solder to the wire banding on the armature coil ends.

Electrical Design Features

The electrical design of a d-c motor involves the selection of the number, size, and shape of the armature coils and slots and the arrangement and shape of the conductors in the slot. Commutation, flashing characteristics, and heating as well as complete motor size and weight are closely related to slot design. The newly developed insulating materials such as glass insulated conductors, glass fiber tape, and glass cloth backed mica wrappers for armature coils have done much to increase the efficiency of the slot design. The ratio of cross section of copper in the slot to the total cross section of the slot itself has been raised without sacrificing factors affecting commutation, flashing, and heating. Slot wedges for retaining the armature coils in the slots have replaced the use of wire bands on the core. The material used for wedges is usually micarta. The wedge construction is more reliable mechanically than core bands at high peripheral speeds. Their use eliminates eddy current losses produced in bands and the detrimental effect of core bands on commutation.

The small size of modern motors and the use of form wound, single turn, wave type armature coils, makes armature winding a simple procedure. Individual armature coils are varnish treated before winding and the wound armature is given a series of heatings, dippings, and oven bakings using a newly developed insulating varnish. The new varnish has improved characteristics of dielectric strength, more permanent set after baking, and provides a smoother surface that is more repellent to dirt and oil.

The field coil arrangement is made symmetrical around the armature by using the same number of commutating coils and poles as main coils and poles, usually four of each. The coils are either formed of strap copper wound on edge or flat

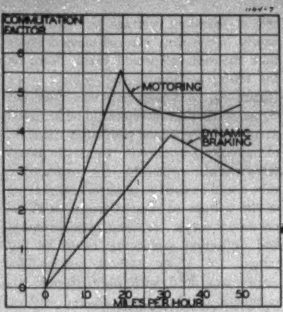

Figure 7. Commutation curve for 600-volt, 140-horsepower trolley coach motor

wound in sections depending on the particular design involved. The field coil construction has also benefited from the available new insulating materials. They occupy a minimum of space and are formed to fit accurately between the pole tips and the frame. The complete frame, pole, and coil assembly are given a dipping and baking treatment with insulating varnish similar to that used for armatures. A smooth hard coating over the insulated surfaces of coils and cable or strap lead connections is obtained. It has high dielectric quality and is resistant to water, dirt, or oil. Modern motors use insulation designated by the AIEE rules as class *B*. The materials consist principally of asbestos, mica, and glass fiber and are little affected by high overload temperatures often encountered in transit service.

Quiet operation is essential for use on vehicles with rubber tires or resilient wheels. Possible sources of motor noise are the brushes, bearings, fan, or armature teeth. All of these noises have been reduced in modern designs. Brush chatter or squeak is prevented by brush holder construction and smooth riding brushes. Bearing noise has been lowered to the point that a noisy bearing indicates that it is defective and must be replaced. Windage noise has been minimized by streamlining fan blades and air outlets. Magnetic noise from the armature is avoided either by skewing the slots or by using slots parallel to the shaft and giving careful attention to shaping of main pole tips and air gap taper. The latter method is more desirable because of greater ease of winding coils in straight slots and because slot skewing often widens the commutating zone under the commutating pole to an undesirable degree.

Commutation and flashing characteristics have been the subject of considerable study to secure long brush and commutator life and to permit the use of service dynamic braking. The higher rotational armature speeds tend to make this problem more difficult. This is largely offset by the use of single turn armature coils which compare with two or even three turn coils for older type machines. Also, the use of two 300-volt motors in series to utilize 600 volts from the trolley, has permitted designs with better commutation and flashing factors. In the case of the single motor for trolley coaches, a 600-volt commutator is required. However, the motor is of sufficient size to permit the use of a single turn armature coil, and other parts affecting commutation and flashing can also be well proportioned. While the commutation and flashing characteristics of a d-c motor are closely related, the design factors favorable to one are somewhat opposed to the other. A satisfactory balance must be obtained that will meet the requirements for both. For example, fewer armature conductors favor low "sparking volts" under the brush, but freedom from flashing calls for sufficient commutator bars, with the required armature conductors, to prevent the maximum volts per bar from becoming excessive at the maximum armature volts encountered in service. A flashover is started by the voltage between adjacent commutator bars becoming so high that arcs between bars are maintained after these bars pass under the brush. One precaution is to avoid main field strength distortion incident to the effect of the armature, thus preventing a peak in the field form. The series motor inherently averts an excessive armature distorting effect because the ratio of armature to field ampere turns is constant and relatively strong in dynamic braking. Under the condition of acceleration with shunted field, some distortion can be allowed in the field form as the terminal voltage is never above the trolley voltage. The field shunt is made with the proper amount of inductance so that sudden changes of load will cause no transient weakening of the field. Good commutation under the condition of maximum accelerating amperes requires that the magnetic circuit of the commutating poles be unsaturated.

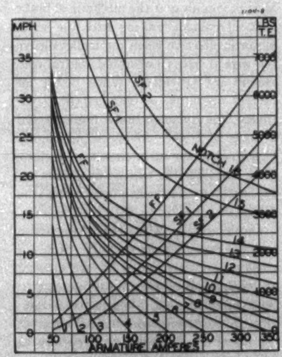

Figure 8. Acceleration curves for 600-volt, 140-horsepower trolley coach motor

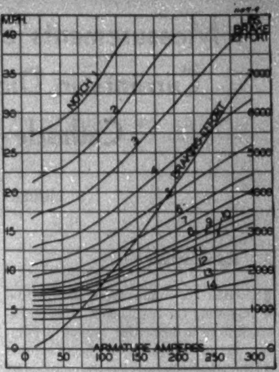

Figure 9. Dynamic braking curves for 600-volt, 140-horsepower trolley coach motor

Typical Modern City Transit Motors

The Diesel-electric bus motor, designated as the 1435 type, is illustrated in figure 4. The machine is capable of handling the output of a 125-horsepower engine-generator power plant. The weight of the motor is 880 pounds which is exceptionally light for the available performance capacity.

The modern trolley coach would be impractical without the 1442-type motor which has been designed for this vehicle. The machine is illustrated in figure 5. It rates 140 horsepower at 600 volts and weighs 1,360 pounds—less than 10 pounds per horsepower. A picture of an old type 125-horsepower motor is shown in the same illustration to give a relative comparison. The latter unit weighs 3,430 pounds excluding gear and gear case—over 27 pounds per horsepower.

The development of the present-day street car is closely associated with the 1432-type, 300-volt motor shown in figure 6. This is a 55-horsepower machine

Measurements of Pre-Breakdown Currents in Dielectrics With a Cathode-Ray Tube

HUBERT H. RACE
FELLOW AIEE

A. Introduction

In order to obtain more information regarding the mechanism of electrical breakdown in insulating liquids, a number of experimenters[1,2,3] have studied current flow as a function of applied voltage, using high-sensitivity d-c amplifiers with indicating instruments having relatively long-time constants. Such instruments will not follow rapid changes in current where the time intervals are of the order of milliseconds or less. Therefore we decided to investigate the possibility of using a cathode-ray tube as a current detector to follow current changes just preceding breakdown. This paper describes current measurements in liquids with 200-microsecond impulse voltage waves and current measurements in liquids and solids with gradually increasing unidirectional voltage applied to the specimens.

B. Measurements on Liquids Using Balanced Impulse Circuits

Breakdown in liquids gives no warning preceding its occurrence which can be used to trigger the sweep of a cathode-ray tube so as to make a breakdown self-indicating. Therefore we applied to the test cell a transient voltage increasing at nearly a constant rate of such a value that the test cell would break down on the rising portion of the voltage wave (figure 1). A sweep trip was built designed so that the sweep could be initiated at any predetermined time after the start of the voltage transient. Since the sweep time was also adjustable, the cathode-ray tube could be used to observe any portion of the voltage or current transient if the necessary single transient writing speed and deflection sensitivity could be obtained. A 15-kv cathode-ray oscilloscope[4] was used with photographic attachment to obtain the desired writing speed.

Because of the high rate of voltage rise necessary to obtain breakdown in one sweep of the cathode beam, the cell capacitance charging current would be much greater than the conductance current. Therefore two balanced, opposing impulse circuits were used, as shown in

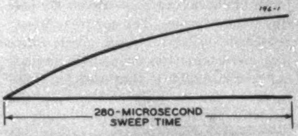

Figure 1. Impulse voltage-time curve

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The author is indebted to R. A. Pfuntner for assistance in making these measurements and to N. Rohats for suggestions and circuit details connected with the cathode-ray equipment.

[1] For all numbered references, see list at end of paper.

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weighing 695 pounds—just over 12 pounds per horsepower. The same picture includes an old type 50-horsepower 600-volt motor and gives a visual indication of the progress made in the new design. The earlier type weighs 2,000 pounds or 40 pounds per horsepower.

Performance Characteristics

Satisfactory commutation requires minimum sparking at the brush faces under the most unfavorable combination of load and speed encountered in service. In figure 7, the "commutation factor" is a measure of the average voltage between bars under the brush. This figure shows how this factor varies throughout the speed range for motoring and dynamic braking for a 600-volt series trolley coach motor. The motor design is such that the maximum value of this "commutation factor" is below that which produces sparking at the brush faces. It is lower for dynamic braking than for motoring.

The series motor has the inherent advantage of a rapidly increasing dynamic braking effort as speed is stepped up. Thus, when a vehicle is gaining speed down grade and the dynamic brake is applied, any desired balanced condition may be quickly obtained. There is no possibility of the motor reaching an unstable condition and permitting the vehicle to run away if the mechanical brake fails. Dynamic braking with the series motor is effective when the trolleys leave the wire if battery operated control contactors are provided. When power for control operation is obtained from the trolley, the dynamic brake is unaffected as long as trolley voltage is sufficiently high to close the main circuit contactors. Thus, considerable voltage drop may occur without adversely influencing the dynamic brake.

Accelerating performance of the series motor is exceptionally smooth. Rates of acceleration, determined by the amount of pedal advance, permit an operator to start a vehicle at a rate selected to conform to traffic requirements. This gives the operating flexibility so necessary in city service. A set of acceleration curves, applicable to the modern trolley coach, is shown in figure 8.

The range of the dynamic brake obtained with the series motor is unusually broad. This is illustrated by the braking curves shown in figure 9. Braking current is effectively regulated to maintain any selected rate of braking up to the maximum capacity of the motor equipment. The uniform motor current which is obtained provides equal tractive effort values on all notches and thus maintains the desired braking rate. The series motor is capable of developing its full braking rate down to three or four miles per hour if these low speeds are desired.

New Motors Extensively Used

The new designs of motors for Diesel-electric busses, trolley coaches, and street cars have been generally applied on city transit systems. The machines available are the product of considerable engineering effort to reduce weight, increase efficiency, and improve output. Results are creditable as a contribution which has assisted in the development of modern transit vehicles.

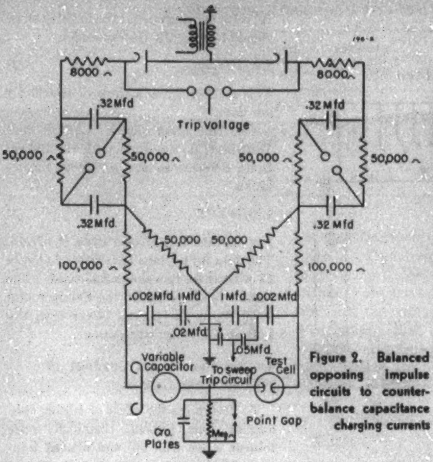


Figure 2. Balanced opposing impulse circuits to counterbalance capacitance charging currents

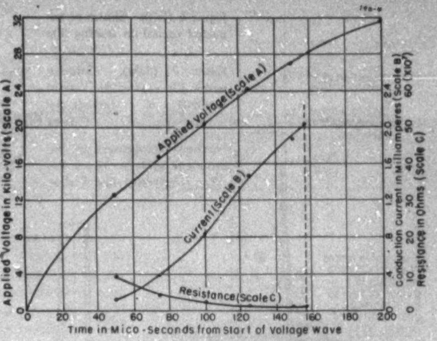


Figure 4. Current-voltage-time curves for a high-conductivity liquid

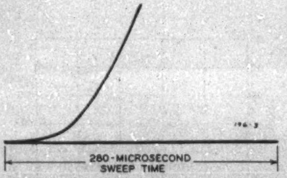


Figure 3. Breakdown current in a high-conductivity liquid

Diphenyl oxide + diphenyl

figure 2.[9] An adjustable air capacitor was used to balance out the capacitance current of the test cell. Under these conditions the voltage between point (*A*) and ground should be zero if the conductance current in the test cell were zero. Also if the cell conductance current were appreciable the voltage drop across the one-megohm resistor could be indicated by the cathode-ray deflection as a measure of this current.

Proper operation of this circuit depends upon equal voltage-time transients of opposite polarity being applied to the two sides of the circuit. The charging currents are then equal and opposite so that only the voltage drop due to the conductance current appears across the measuring resistor. If this resistor was larger than one megohm, the capacitance of the leads and cathode-ray oscillograph deflection plates provided a comparatively low-impedance shunt for the fast transients. Even with one megohm it became necessary to correct the resulting curves, both for the deflection plate capacitance and the circuit unbalance due to the drop in the resistor.

Good results were obtained on a mixture of commercial diphenyl and diphenyl oxide which was continuously circulated through a filtering and degasifying system.[5] Figure 3 is a good example of the type of curve obtained, and figure 4 shows the corrected curve plotted with time, current, voltage, and resistance scales. The pre-breakdown currents in this liquid were very high (one or two milliamperes). However, lower conductivity liquids could not be studied because of insufficient sensitivity. Figure 5 shows a typical shot taken on a low viscosity cable oil (GE number 5314) with a 22-microsecond sweep. After breakdown, the high-frequency damped oscillation is determined by the circuit constants, not the liquid. No pre-breakdown current can be seen, meaning *that the current was less than 10^{-4} amperes up to within a fraction of a microsecond of breakdown.*

C. Measurements on Liquids Using Gradually Increasing Voltage and Photographing Repeating Sweeps on a Cathode-Ray Tube

1. Apparatus

At a low rate of voltage application the capacitance charging current is negligible. Methods were worked out to trip the sweep circuit repeatedly at two-millisecond intervals and to use a continuous feed movie camera without shutter to record the resulting single traces. A 15-kv tungstate screen tube was used to get high photographic intensity. The film moved approximately parallel to the sweep direction at such a speed that successive sweeps uncovered or progressed about one-fifth of the length of a single trace on the film. A resulting hypothetical pre-breakdown current is shown schematically in figure 6. The slowly increasing unidirectional voltage-time curve was measured with a photoelectric recorder.[5] A manually operated selector switch (figure 7) was used so that the operator, while watching the cathode tube deflection, could vary the sensitivity and thus follow a slowly rising current as the voltage was gradually increased. The photographic setup was arranged so that the operator could view the tube screen from the side during the entire run. The 10^5 and 10^6 ∽ resistors were wire wound, the others were electrolytic, using solutions of tetrabutyl ammonium picrate in a mixture of diphenyl and diphenyl oxide. Sealed off glass cells were used to prevent resistance changes with time and the concentration and cell dimensions were varied to give the desired resistances. Since the sweep time used is about 40 microseconds and the sweep interval is 2 milliseconds, we have a 2 per cent prob-

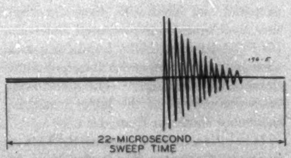


Figure 5. Breakdown in mineral oil

22×10^{-6} seconds sweep time

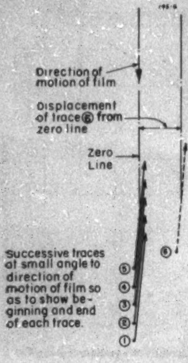
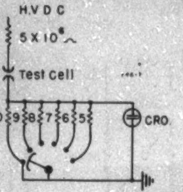

Direction of motion of film
Displacement of trace ⑥ from zero line
Zero Line
Successive traces at small angle to direction of motion of film so as to show beginning and end of each trace.
⑥
⑤
④
③
②
①

Figure 6 (left). Sketch of expected record on moving film

Figure 7 (right). Selector switch for varying the current sensitivity of a cathode-ray tube

Numbers correspond to the exponent to base 10 of the resistance in ohms



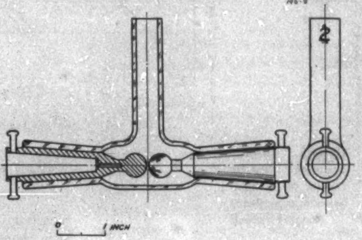


Figure 8 (right). Removable electrode gravity cell

ability of catching a breakdown transient if it is only of the order of microseconds duration.

2. Current-Time Data for a Low-Viscosity Mineral Oil (GE 5314)

For these experiments open cells (figure 8) having spherical electrodes of three different metals were used. The electrodes were polished on a dry clean buffing wheel. Then the electrodes and cells were chemically cleaned, steamed, and dried in a 100-degree centigrade oven as described by Dornte.[6]

Table I shows the last current readings and the breakdown gradients for this series of tests and figures 9 and 10 show the current-time data taken at intervals from the movie film record of the cathode-ray tube deflections. *These data give additional evidence for the conclusion that the conditions of the experiment particularly regarding the electrode surface are the important factors governing pre-breakdown current as well as the breakdown gradient.*[3,5]

3. Pre-Breakdown Current in Heptane

A hemispherical electrode, closed cell prepared and filled with *pure, dry* heptane by Dornte[3] was measured first with his FP54 amplifier below breakdown and then up to breakdown with the equipment described in this paper. The data taken from the records of the latter measurements are shown in figure 11.

It is interesting to examine these data in terms of two possible explanations of their cause. The Wien-Onsager[7] derivation* would predict that if $\log_{10}(I/E)$ is plotted against $\sqrt{\text{volts per centimeter}}$, the asymptotic slope equals

$$\frac{0.434}{T}\sqrt{\frac{8\times 9.64}{\epsilon'}} \tag{1}$$

for this experiment with heptane $\epsilon'=1.91$, $T=300$ degrees Kelvin so that the limiting slope $=0.0092$. Figure 12 shows the data for heptane plotted in this form with the theoretical slope indicated.

Baker and Boltz[1] suggested that the limiting factor in conduction might be a potential barrier at the electrode-liquid interface so that a modified Schottky relation should apply.** Their derivation indicates that if $\log_{10} I$ is plotted against $\sqrt{\text{volts per centimeter}}$, the limiting slope should be given by the expression

$$1.91(\epsilon')^{1/2}/T \tag{2}$$

This relation gives a slope of 0.0089 for our data.

On the other hand, LePage and DuBridge[8] do not agree with the Baker-Boltz derivations and arrive at the relation.

$$1.91/(\epsilon')^{1/2}T \tag{3}$$

For heptane this gives a slope of 0.0045. The data for heptane are plotted in figure 13 with these two slopes indicated. The results for heptane fit the Onsager and Baker-Boltz derivations better than the LePage-DuBridge calculations.

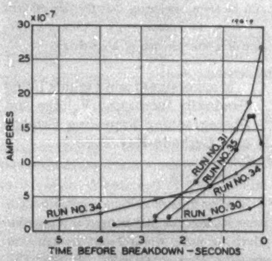


Figure 9. Pre-breakdown current in mineral oil

See table I

4. Pre-Breakdown Current in Conducting Liquids

To determine the effect of adding conducting ions, filtered, chemically pure toluene alone (run 66) and filtered toluene+0.025 per cent tetrabutyl ammonium picrate (run 72) were tested in the quartz cell with stainless steel spherical electrodes. The results are shown in figure 14. Before applying high voltage, the d-c resistance was measured at low voltage in each case as follows, run 66, $R=1.8\times10^{10}$ ohms, run 72, $R=8.8\times10^{8}$ ohms. The breaks in the current curves indicate definite changes in mechanism of conductance as the gradient is increased. Such breaks are commonly observed if the current-measuring equipment has a sufficiently wide range of sensitivity.

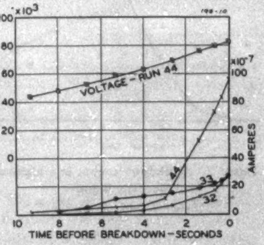


Figure 10. Pre-breakdown current in mineral oil

See table I

Since conducting ions have been added for run 72, the results should be checked against the Wien-Onsager theory. Therefore figure 15 is plotted but, contrary to expectations, the slopes of the conductances versus gradient lines were much

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* This theory is based upon increased dissociation of ion pairs at increased field strengths.

** This theory assumes that the controlling factor in current flow is the ability of electrons to escape into the liquid from the electrode surfaces and therefore is limited by a "work function" similar to that involved in electron emission into gases.

Table I. Pre-breakdown Conditions in 5314 Oil

Run No.	Symbol	Shot	B.D. Gradient (Kv/Cm)	Current before B.D.× 10^{-10} Amperes	Resistance Before B.D.× 10^{-10} Ohms	Electrode Material	Electrode Spacing (Cm)
30		1	600	4.3	8.5	Al	0.061
31	⊙	2	213	27	0.5	Al	0.061
32	×	1	471	27	2.6	Steel	0.15
33	⊗	2	208	27	1.1	Steel	0.15
34	△	1	650	11	4.0	Cu	0.069
35	△	2	265	13	1.4	Cu	0.069
44	ϕ	1	550	96	0.86	Steel	0.15

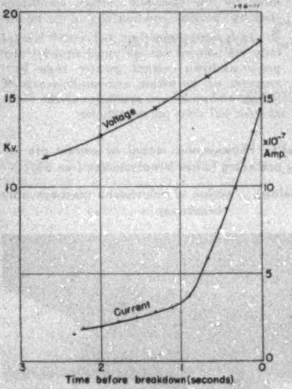


Figure 11. Pre-breakdown current and voltage in dry heptane

lower than would be required by the theory which predicts a slope of 0.008 for toluene. Likewise the same data are plotted in figure 16 to obtain a comparison with the potential barrier theory. The toluene containing conducting ions seems to check the LePage-DuBridge derivation but the toluene alone does not.

Unfortunately this investigation was discontinued before the above calculations were made so that the runs have not been repeated and checks are not possible at present. However, the results seemed worthy of reporting even though they are inconclusive as to the mechanism involved.

5. Discussion of Results on Liquids

In none of the breakdown records on liquids did we catch the breakdown transient. The last trace on each record was substantially like those immediately preceding it, showing the slowly increasing current with increased voltage as shown in figures 9, 10, 11, and 14. It seems reasonable to conclude that the observed current increases are ionic but that the final breakdown is electronic and occurs very fast (fractions of microseconds, see figure 5). Furthermore it seems that the pre-breakdown ionic current may be unrelated to the breakdown electronic current since the results of many investigations indicate that the breakdown gradient appears to be affected more by electrode surface conditions than by the ionic conductivity of the liquid.

D. Pre-Breakdown Currents in Solids

A number of experiments were tried to see whether the methods described in part *C* could be used to measure current just before breakdown in solids. The test pieces were prepared by drilling 3-millimeter coaxial holes from opposite ends of a 1.5×1.5×7-centimeter test piece. The ends of the holes were made hemispherical with a special reamer and the thickness of the test section between the ends of the holes was accurately measured with a micrometer. Very finely divided graphite was then introduced into each hole and brass rod electrodes with rounded ends driven lightly into the graphite. In this manner the field distribution which exists between spheres in a homogeneous dielectric was obtained and corona between the electrode and the dielectric was eliminated.

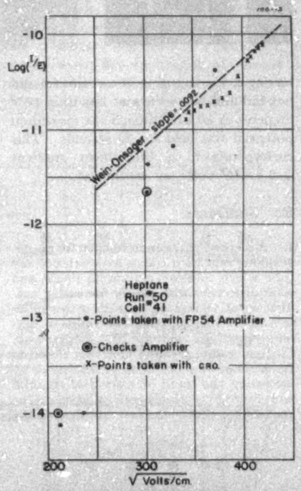


Figure 12. Plot of data for heptane to compare Wein-Onsager calculations

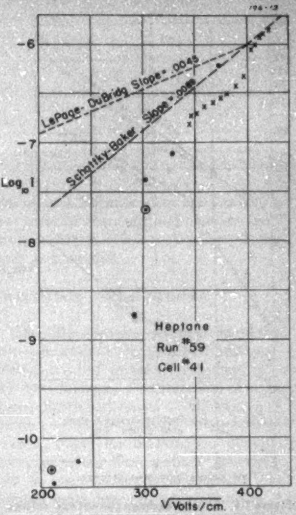


Figure 13. Plot of data for heptane to compare with Shottky-Baker hypothesis

Figure 18 shows a polystyrene specimen (run 53) after breakdown. This specimen had a test thickness of 0.053 centimeter and broke down at 77 kv, giving a breakdown gradient of 1.45×10^6 volts per centimeter. The current in this specimen was less than 1×10^{-8} amperes up to within four milliseconds of failure. The current record just before failure is interesting (figure 17), since the two sweeps before failure show currents of 40 and 106×10^{-8} amperes, respectively. Then failure occurred before the next sweep, which was back on the zero line. Another polystyrene specimen having a spacing of 0.033 centimeter did not fail at 83 kv, when tested at 100 degrees centigrade. This corresponds to a gradient >2.5×10^6 volts per centimeter (6,350 volts per mil). A third specimen at 100 degrees centigrade having a test spacing of 0.013 centimeter failed at 34.2 kv corresponding to a gradient of 2.6×10^6 volts per centimeter (6,600 volts per

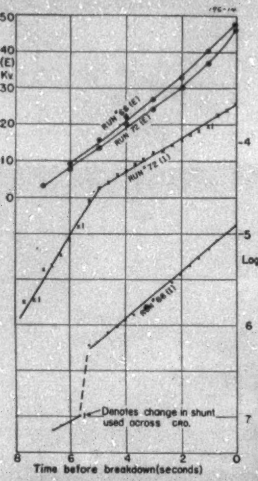


Figure 14. Pre-breakdown current and voltage data for chemically pure toluene (run 66) and toluene + tetrabutylammonium picrate (run 72)

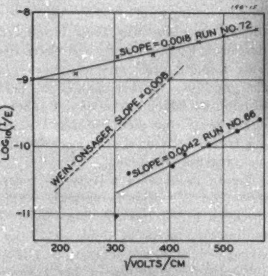


Figure 15. Plot of data for runs 66 and 72 to compare with Wein-Onsager calculations

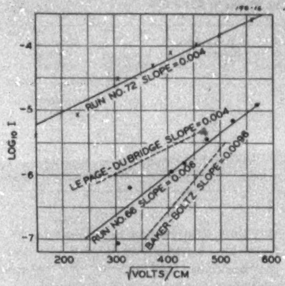


Figure 16. Plot of data for toluene alone (run 66) and toluene + 0.025 per cent tetrabutylammonium picrate (run 72)

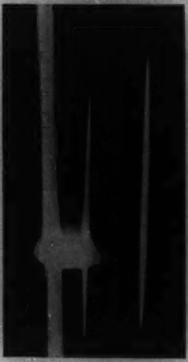
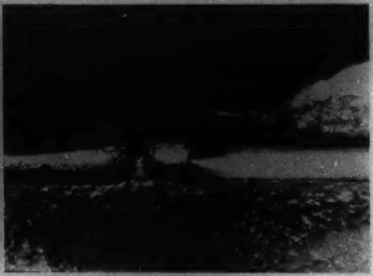


Figure 17 (left). Photographic record of current rise immediately preceding failure in polystyrene (run 53)

Figure 18 (below). Section of polystyrene specimen after breakdown



mil). None of these specimens showed any measurable current before breakdown.

Several specimens of polyvinyl formal resin were tested in the same way. No pre-breakdown current was observed, the failure having occurred between successive sweeps in each case. One specimen having a spacing of 0.038 centimeter did not fail and had a current less than 10^{-8} amperes at 86 kv although the maximum potential was held for 30 seconds. This corresponds to a breakdown gradient $>2.3\times10^6$ volts per centimeter.

E. Summary

1. A method has been developed for photographing repeating traces on a cathode-ray tube which measure current in liquid and solid dielectrics with slowly increasing unidirectional voltage up to within two milliseconds of breakdown. The range of current sensitivity is from 10^{-8} amperes up. Current values at time intervals closer to the breakdown point could be measured by increasing the speed of motion of the film and decreasing the sweep interval. The probability of catching fast transient of the order of one microsecond duration is only 2 per cent (ratio of sweep time to sweep interval).

2. In liquids, the evidence so far obtained leads to the tentative conclusion that the final breakdown current increases from a substantially steady value to failure in fractions of a microsecond. This indicates that the final breakdown mechanism is electronic.

3. In liquids, it appears also that the pre-breakdown ionic current may be unrelated to the breakdown electronic current since the breakdown gradient appears to be affected more by the electrode surface conditions than by the ionic conductivity of the liquid.

4. In polystyrene and polyvinyl formal only one record was obtained showing any pre-breakdown current greater than 10^{-8} amperes up to within two milliseconds of failure at breakdown gradients of the order of 2.5×10^6 volts per centimeter.

References

1. E. B. Baker and H. A. Boltz, *Physical Review* volume 51, page 275, 1937.

2. H. J. Plumley, *Physical Review*, volume 52, page 140, 1937.

3. R. W. Dornte, *Industrial and Engineering Chemistry*, volume 32, page 1529, November 1940.

4. SOME RECENT DEVELOPMENTS IN IMPULSE-VOLTAGE TESTING, C. M. Foust and N. Rohats. AIEE TRANSACTIONS, volume 59, 1940 (May section), pages 257–62.

5. DIELECTRIC STRENGTH OF INSULATING LIQUIDS IN A CONTINUOUSLY CIRCULATING SYSTEM, Hubert H. Race. AIEE TRANSACTIONS, volume 59, 1940, pages 730–7.

6. R. W. Dornte, *Journal of Applied Physics*, volume 10, page 514, 1939.

7. Lars Onsager, *Journal of Chemical Physics*, volume 2, page 599, September 1934 (Equations 37 and 39).

8. W. R. LePage and L. A. DuBridge, *Physical Review*, volume 58, page 61, July 1940.

9. MEASUREMENTS OF SURGE VOLTAGES ON TRANSMISSION LINES, E. S. Lee and C. M. Foust. *General Electric Review*, March 1927, figure 4.

Bus Protection Independent of Current-Transformer Characteristics

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MEMBER AIEE

Synopsis: The paper describes the installation and operating results of a bus protective scheme, which uses a combination of high-speed directional ground relays on all sources of fault current and high-speed plunger type blocking relays on all outgoing circuits. The scheme described is independent of the ratios and characteristics of the current transformers.

Introduction

THE protection of station busses in the past has usually been accomplished by some form of differential relaying or by fault bus relaying.[1]

Differential relaying requires that the current transformers in each circuit on the bus have the same ratio and characteristic.[2] Sometimes auxiliary transformers are used to compensate for differences in the ratios of various current transformers.[3] It has often been necessary to give the differential relays a relatively high pickup and some time delay in order to prevent incorrect operations. The application of the harmonic current restraint relay has overcome some of the difficulties of differential relaying for bus protection.[4]

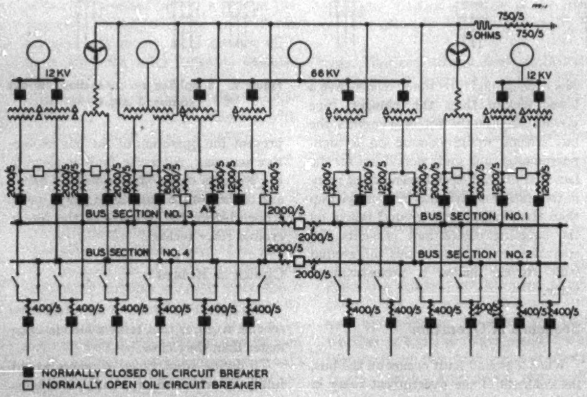

Figure 1. Circuit arrangement of 25-cycle, 22-kv outdoor station

Fault bus relaying requires that all equipment be insulated from ground and connected to a common fault bus, which is grounded through a current transformer which operates some type of relay.[5]

Neither of the above schemes could be applied in a practical way to the bus layout shown in figure 1. Since each feeder has only one breaker, and each pair of feeders has selector disconnects to the busses, the application of current transformers for differential relay protection would not be feasible. This station is a 22-kv outdoor station which was built 20 years ago and does not lend itself readily to the application of the fault bus method.

Description of Bus Scheme

The difficulties in applying either of the two conventional methods of bus protection resulted in the development of a scheme which uses a combination of high-speed directional relays on all sources of fault current and high-speed plunger type current relays for blocking on all outgoing circuits. The method developed can be used for protection against any type of fault, but, after consideration of the speed of operation of the protective scheme and of the fact that the majority of faults in an outdoor station start as one wire to ground or two wires to ground faults, it was decided to install only ground fault protection. The total relay time of the protective scheme is from one to two cycles* and it was felt that this time was sufficiently short to insure relay operation before ground faults have sufficient time to develop into three-phase faults.

Figure 2 shows the simplified a-c connections for bus sections 1 and 2. Figure 3 shows the schematic d-c relay connections for bus sections 1 and 2. The connections for bus sections 3 and 4 are similar to those shown for bus sections 1 and 2.

Each of the high-speed ground directional relays used on circuits which are sources of ground current has one coil connected in the station neutral ground and its other coil in the respective residual current circuit. When de-energized, the contacts of these relays are open, and they close when ground current flows toward the protected bus section. A specially designed *b* switch on the oil circuit breaker in each circuit is connected across the

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[1] For all numbered references, see list at end of paper.

* All times given are on a 25-cycle basis.

normally open contacts of these relays to prevent the relay scheme from being inoperative when a circuit is out of service.

All circuits which are not sources of ground current are provided with high-speed plunger type ground relays, each having two contacts both of which are closed when the relay is de-energized. Since the outgoing circuits can be connected to either one of two busses, it was necessary to use one contact of each of these relays in the protective scheme of each of these bus sections. Thus the operation of these relays, when a fault occurs on a circuit off the bus, blocks the bus scheme of both bus sections to which the circuit can be connected. The contacts of these relays are continuously supervised by indicating lights.

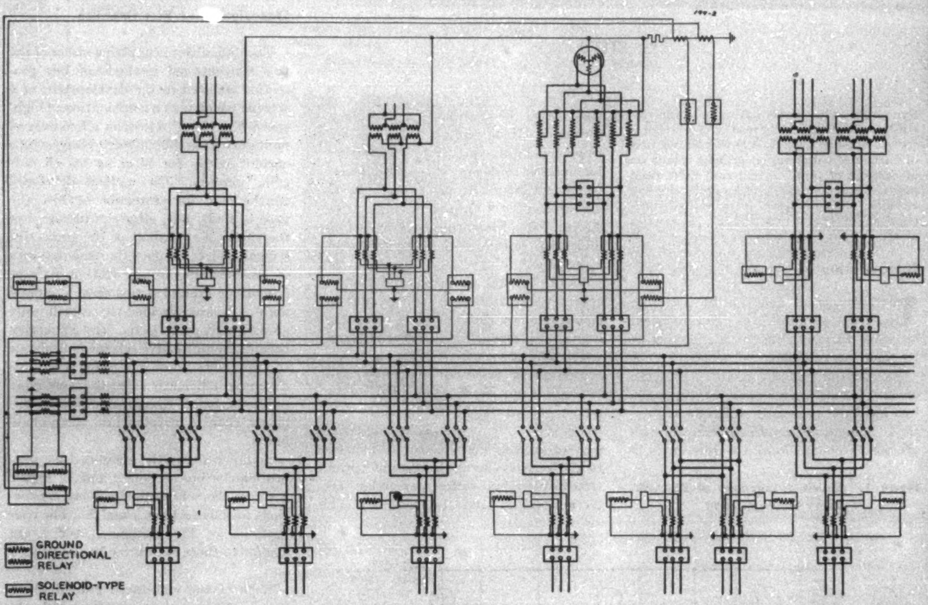

Figure 2. Simplified a-c connections for bus sections 1 and 2

The tie breakers to adjoining bus sections are equipped with both high-speed ground directional relays and high-speed plunger type ground relays with their contacts connected in parallel. Since an adjoining bus section may or may not be a source of ground current, it was necessary to install both types of relays. The normally closed contacts of the plunger type ground relays of the tie breakers also serve as *b* switches when the breakers are open. The tie breakers are normally open at least 90 per cent of the time.

To prevent an accidental operation of the bus scheme and also to prevent its operation on a high-resistance fault on a circuit off the bus, a high-speed plunger type overcurrent relay for each bus section is used in the station neutral ground. The contacts of this relay are connected in series with the operating coil of the gang trip relay, the normally open contacts of the ground directional relays, and the normally closed contacts of the plunger type ground relays for any bus section. Since the ground directional relays have a lower pickup than the plunger type ground relays on the outgoing circuits, the bus scheme would operate on a high-resistance ground fault were it not for the fact that the high-speed overcurrent relay in the station neutral has a higher pickup than the plunger type ground relays on the outgoing cables and prevents the operation of the bus relays on ground fault current below a predetermined value.

Sequence of Operation

When a ground fault occurs on the bus, the contacts of the overcurrent relay in the station neutral and of the ground directional relays in the circuits which are ground sources close, energizing the gang trip relay which trips all the power sources on the bus. Since the outgoing feeder circuits carry no fault current, the contacts of the relays on these circuits remain in the normally closed position.

When a ground fault occurs on a circuit off the bus, the contacts of the plunger type ground relay on the faulted circuit open before the contacts of the overcurrent relay in the station neutral and the ground directional relays close and thus prevent the operation of the bus protective scheme. After the fault is cleared, the contacts of the station neutral relay, and of the ground directional relays, must reopen before the contacts of the feeder ground relay reclose.

Choice of Relays

The correct operation of this protective scheme requires that relay contacts open faster than they close.

To decide what types of relays would fulfill this requirement, a number of tests were made in the laboratory, using an oscillograph, to check the relative opening and closing times of the various relays. It was found that satisfactory tolerance can be obtained between the opening of one contact and the closing of another, even on high-speed plunger type relays which are energized or de-energized simultaneously. The tests also showed that the contacts of five directional relays in series, operating simultaneously, give reliable results especially if the contacts are of the nonbounce type.

The relays that have been selected are all of simple design, and their operating times as recorded by the oscillograph, are described below.

The high-speed plunger type relays in the ungrounded circuits will open their contacts in a quarter of a cycle at all values of current greater than about 30 per cent above their setting, and after the current has been removed, the contacts will reset in one cycle. Of course the setting of these relays should be made as low as possible, because this determines the setting of the relays in the station neutral.

The high-speed plunger type overcurrent relays in the station neutral will close their contacts in a third of a cycle at the maximum obtainable current, which is 300 per cent of their setting. The closing time of the relay contacts increases, as the current decreases, i.e., at about 30 per cent above the setting the time will be one cycle and two cycles at 5 per cent above its setting. The contacts open in a quarter of a cycle after the relay becomes de-energized.

The high-speed ground directional relays have been adjusted to close their contacts in a time, varying from about two cycles at ten per cent above their setting to three-quarters of a cycle at the maximum obtainable current. These contacts will reopen in a quarter of a cycle after the relay becomes de-energized.

Additional tests showed that equally satisfactory results can be obtained with relays used on 60 cycles frequency.

Limitations of the Bus Scheme

1. If the unusual operating condition, should arise of a bus section being fed only through a tie breaker, it would be necessary to make the protective scheme nonautomatic on that section; since the *b* switches on every breaker that normally feed the bus section would be closed. The only contact that would then remain open in the d-c circuit of the bus scheme would be that of the overcurrent relay in the station neutral. Since these contacts close for a fault anywhere on the 22-kv system an incorrect operation of the bus protective scheme could result.

2. The bus protective scheme for a section would be inoperative if a breaker on a ground source circuit were in the closed position while its isolating disconnects were open. This is a situation that might arise while doing maintenance work on a breaker. If a bus ground fault should occur during this period, the ground directional relay on the circuit out of service could not close its contacts. This limitation of the bus scheme can be overcome by by-passing the *b* switch of the breaker on which maintenance work is being done.

3. The bus scheme of any section is inoperative while any of its relays is being tested. This, of course, applies to any bus scheme but with this one there are a greater number of relays to be tested than on a conventional scheme, and therefore more time is required.

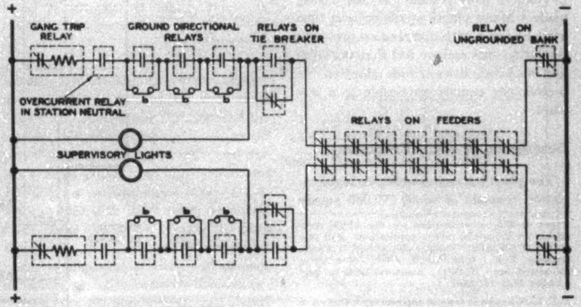

Figure 3. Schematic diagram of d-c relay connections for bus sections 1 and 2

4. This bus scheme does not operate if a bus fault starts as a phase-to-phase or three-phase fault or if a one wire-to-ground or two wire-to-ground bus fault changes to a three-phase fault in less than about one cycle after its start.

Staged Ground-Fault Tests

The various relays of the described bus scheme were installed and put in service in May 1939, but the trip circuits on the gang trip relays were left open temporarily. The purpose of this precaution was to determine, in actual operation, whether any of the relays would operate incorrectly on switching surges or external faults. During this period various disturbances occurred outside the four bus sections during which all the relays of the bus protective scheme performed correctly.

On May 12, 1940 a series of staged ground fault tests were made on each of the four bus sections and on circuits off the various bus sections. In each case the protective scheme operated correctly. Each of the bus faults was cleared in about seven cycles.

Operating Experience

Since May 12, 1940 when the bus scheme was put in service, there have been nine faults on circuits off the bus, and one fault on bus section 4.

The bus scheme was blocked correctly in all cases of faults off the bus.

The fault on the bus, which occurred on the bus side of the breaker bushings marked with the letter *A* in figure 1, was cleared in eight cycles through the operation of the bus protective scheme.

The record of the automatic oscillograph indicates that the disturbance started as a phase 3-to-ground fault. This condition remained for four cycles and after that the fault spread into a two wire-to-ground fault, involving phases 2 and 3.

Conclusion

The bus protective scheme has been applied to an important and rather complicated section of the system, where relatively large generators and power transformers are connected to the busses. Operating experience indicates that the reliability of this bus scheme is not adversely affected by either the magnetizing inrush currents of the power transformers or by the d-c component of the asymmetrical fault currents.

A Distribution System for War-Time Plant Expansion

J. L. McKEEVER
ASSOCIATE AIEE

Introduction

INDUSTRIAL distribution systems to meet the needs of expanding plant facilities for war and defense production are a very live issue today, both in Canada and in the United States. A great deal has already been written on the subject in a general way, as a guide to the proper tailoring of plant distribution to the type of plant and plant production. The purpose of this paper is not to generalize, but to offer a short description of the distribution system installed in the recently expanded facilities of the Peterborough plant of the Canadian General Electric Company, with reasons for the choice made. Many plants at the present time must be faced with distribution problems similar to ours and we feel that our solution has possibilities of wide adoption. It is obviously equally applicable to a new plant.

Description of Plant Extension

The plant extension now nearing completion consists of some 370,000 square feet of floor space in two single story buildings. One building of some 245,000 square feet was constructed to manufacture specialized mechanical equipment while the second building of 125,000 square feet area is to house our wire and cable manufacturing department. They will be designated in this paper as building *A* and building *B* respectively.

The types of load in the two buildings are quite different. In building *A* there will be a connected load of approximately 6,000 hp consisting very largely of small machine tools, plus 900 kva for heating and lighting, while in building *B* with a connected load of approximately 2,500 hp there are an appreciable number of motors in the order of 50 to 200 hp driving rubber mills, Banburys, strainers, etc.

Existing Distribution System

In laying out a distribution system for the new buildings the first thought naturally was to continue with the present system. Power is obtained from four sources, i.e.

Two small hydroelectric stations owned by the company

One steam turbine generator in the company's steam heating plant

Peterborough Utilities Commission

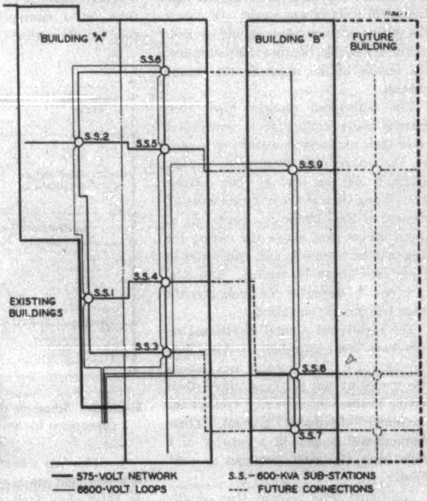

Figure 1. Approximate location of substations in buildings, showing 6,600-volt and 575-volt interconnections

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Grateful acknowledgement is made of the help given in the preparation of this paper by G. R. Langley, chief engineer, and F. A. Athney, switchgear engineering department, Canadian General Electric Company, Peterborough Works.

1. For all numbered references, see list at end of paper.

While the installation requires a greater number of relays than conventional schemes, it avoids the use of additional current transformers and is independent of the current transformer ratios and characteristics, and can therefore be more readily applied to old stations. Economically it compares very favorably with other schemes.

The protective scheme as installed is practically foolproof. Mistakes made while working on the individual relays or on the various current circuits are not likely to operate the bus scheme. Troubles in the secondary wiring or the relays are more likely to prevent the scheme from operating than to cause an incorrect operation, a condition which is desirable from a system operating standpoint.

References

1. BUS PROTECTION, AIEE-EEI committee report. AIEE TRANSACTIONS, volume 58, 1939 (May section), pages 206–11.

2. CONSIDERATIONS IN APPLYING RATIO DIFFERENTIAL RELAYS FOR BUS PROTECTION, R. M. Smith, W. K. Sonnemann, and G. B. Dodds. AIEE TRANSACTIONS, volume 58, 1939 (June section), pages 243–52.

3. RELAY PROTECTION FOR STATION BUSES, W. A. Lewis and R. M. Smith. *The Electrical Journal*, volume 34, November 1937, pages 457–8.

4. HARMONIC-CURRENT-RESTRAINED RELAYS FOR DIFFERENTIAL PROTECTION, L. F. Kennedy and C. D. Hayward. AIEE TRANSACTIONS, volume 57, 1938 (May section), pages 262–71.

5. THE FAULT GROUND BUS. ITS USE AND DESIGN IN BRUNOT ISLAND SWITCH HOUSE OF DUQUESNE LIGHT COMPANY, R. M. Stanley and F. C. Hornibrook. AIEE TRANSACTIONS, volume 49, 1930, pages 201–12.

These sources are paralleled in the switch room of the steam plant and from there power is fed by 6,600-volt underground cables to four strategically located step-down substations. From these stations the existing plant of 790,000 square feet floor area is fed by conventional radial feeders at 575 volts for power and 115/230 volts for lighting.

Distribution Requirements

If the same radial type of distribution were to be used for the new buildings it was obvious that at least two new substations would be required and, considering the large area to be covered, it would be necessary to decentralize the 115/230-volt lighting transformer capacity by placing a number of small single phase units, either Pyranol or air cooled, throughout the buildings. This is of course in line with the best modern practice and the primaries could readily be connected to the 575-volt radial power distribution system, but there were other more serious problems to be solved.

While the load in building *B* both as regards amount and location was fairly well fixed before construction started, the situation with regard to building *A* was quite different. Here, for a variety of reasons, the final total of connected load could not be estimated at all closely and the layout was undergoing continual changes. Furthermore, due to the fact that the work for which this building was being erected was of a temporary nature, the distribution had to be laid out so that the building could be converted readily to a quite different type of manufacturing. In addition all concerned were interested in keeping costs to a minimum consistent with the conditions to be met.

Proposal

Mr. G. R. Langley, the chief engineer of the Peterborough Works, proposed the use of a number of unit substations with the primaries of the transformers connected into a 6,600-volt loop or ring main, and with the 575-volt secondaries interconnected to form a network. The lighting also was to be decentralized with the lighting transformer primaries supplied from the network. This scheme was adopted as a cost analysis showed that it would reduce the first cost and offer other advantages. While the idea of using a low voltage network for industrial distribution is not new, we believe that the scheme proposed has novelty in the use of the primary loop combined with a secondary network with simplified protection. Actually it is an adaptation of the primary network idea as propounded some years ago by Mr. D. K. Blake and others of the General Electric Company.[1] A recent paper[2] pointed out that an industrial network must differ from the well-known 120/208-volt metropolitan type of network in that with the former, secondary cable faults cannot be allowed to burn themselves off. Thus protective means must be employed at each end of each secondary tie. The network diagram accompanying the above-mentioned paper[2] showed the network ties used simply as ties and protected by fuses or current limiters. The power feeders were taken off busses through air circuit breakers and network protectors were used between the busses and the transformers. Our system is admittedly not designed for uninterrupted service due to tie faults and simplifications were made in using the ties themselves as busses protected at each end by air circuit breakers and omitting the network protectors. So far as we are aware no similar system is at present in use in Canada or in the United States.

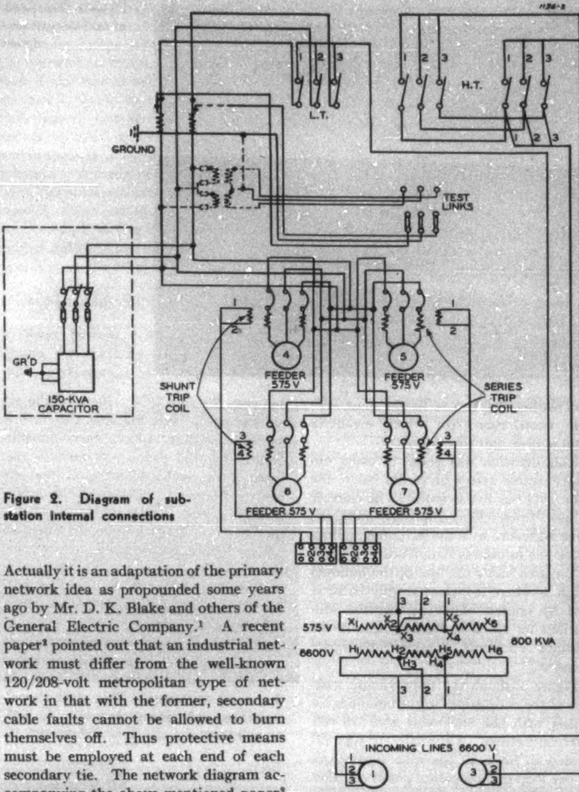

Figure 2. Diagram of substation internal connections

Evolution of Distribution System

Enquiries among other plants and at one plant in particular which had about the same type of load and approximately the same proportion of lighting, electric heating, and power load as we anticipated in building *A*, showed that we might expect a load factor of close to 50% and a power factor of between 70 and 75%. With a radial distribution system, allowance for growth and spare capacity would have necessitated the installation of about 4,500 kva in transformers for building *A* and between 2,000 and 2,500 kva for building *B*. With the network and loop system however, the ease with which a future additional substation can be connected into the system, together with its load dividing properties enables a considerable saving in transformer capacity. Incidentally this feature dispenses with one of the bug-bears of consulting engineers, namely the difficulty in drawing a mean between risk and over-conservatism in deciding upon transformer capacity. The capacity decided upon in our particular case was 3,600 kva for building *A* and 1,800 kva for building *B*. Investiga-

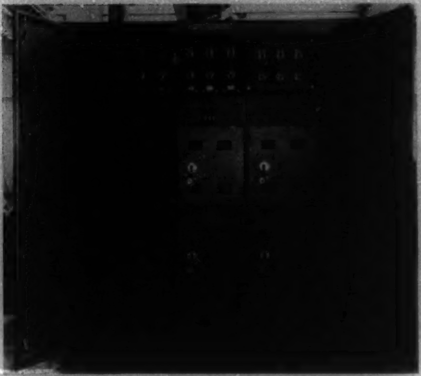

Figure 3. Front view of 600-kva unit substation

tion showed 600 kva to be the most suitable transformer size which meant a total of nine unit substations.

Consideration was given to using oil-cooled transformers in vaults below the floor level but it was found to be cheaper to use Pyranol filled transformers in the same enclosure with the switching. Floor space has minimum manufacturing value in the area along the line of the building columns where it cannot readily be serviced by cranes. Thus placing the substations on the floor backed against the columns represents no appreciable waste of space.

Figure 1 shows the approximate location of the substations in the buildings together with the 6,600-volt and 575-volt interconnections. Originally it was intended to have all the nine substations on one loop but a separate power contract for building *A* made it necessary to have two loops. This works out quite well however, as the wire building loop allows for the addition of three more substations in a future building alongside it so that actually the 6,600-volt cables for the two loops are identical.

The 575-volt switching in each station consists of four 400-ampere air circuit breakers of the dead-front type mounted two high as shown in figure 3. A low tension hook-operated disconnecting switch is provided for disconnecting the transformer from the air breakers, as shown on the top right of figure 3, while the two disconnecting switches appearing at the top left and centre permit the isolation of any section of the loop cable and also serve to disconnect the high voltage side of the transformer from the two ends of the loop.

It was mentioned earlier that the power factor to be expected in building *A* was between 70 and 75%. Each of the six substations in this building therefore is provided with a 150-kva capacitor controlled by the dead-front quick-break, fused knife switch showing at the left centre of figure 3. This is all the equipment provided in the substation except for one current transformer and some test links. Figure 2 shows the connections for the complete substation. The three substations in building *B* are identical with those in building *A* except for the omission of the capacitors.

Various designs of substation structure were considered and the final decision was to enclose transformer and switching in the same enclosure. This arrangement tends towards compactness as is evidenced by the dimensions which are 7 ft. 8 in. width, 8 ft. 6 in. depth, and 8 ft. 4 in. height. All wiring in the building for the 6,600-volt loop and the 575-volt network is overhead and the conduits are brought down the building column and through the roof of the substation.

Figure 1 shows the 575-volt interconnections between stations and the various stub feeders. From junction boxes, taps are taken to 50-kva single-phase Pyranol filled lighting transformers and to GE Trumbull Swing-Wa power panels. In one case in building *B* where a number of controls for large motors are placed in an enclosed force-ventilated control room, two runs of Trumbull Flex-a-power are used between the substations these runs passing through the control room, and connections for the controls are made to them.

Protection

Considerable study was given to the protection of this distribution system against faults in any part of it. Experience with our present system has shown that faults are extremely rare in the 6,600-volt cables and transformers, and this in conjunction with the fact that the rare short interruption of power is not particularly serious with the type of manufacturing involved, permitted simplified protection. The oil circuit breakers at either end of each loop are equipped with inverse-time induction type relays with instantaneous attachments. The loop cables are large enough to feed all stations on the loop from one end, and as already mentioned, disconnecting switches are

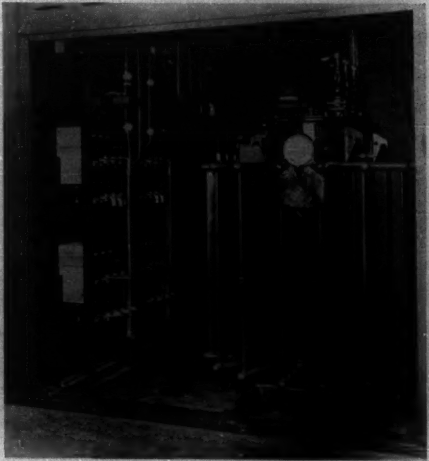

Figure 4. Internal view of 600-kva unit substation showing transformer, capacitor rack, and air circuit breakers

provided at each station so that any section of cable may be isolated. A temperature detector in the transformer winding will ring an alarm upon any serious temperature rise and should it become necessary to remove a transformer for examination or repair before a major fault develops, it can be removed from service without dropping load except on the stub feeders, by the following routine:

1. Trip the four 575-volt air circuit breakers to remove load from the transformer.
2. Open the loop disconnecting switches.
3. Open the 575-volt disconnecting switch and disconnect the transformer high tension leads.
4. Reclose the loop disconnecting switches and the four air circuit breakers.

An actual fault on either transformer or loop cable will open the loop oil circuit breakers and interrupt power for a short period but, as already mentioned, the inconvenience of such a shut-down does not, under our conditions, justify guarding against it.

Figure 5. General view showing small space occupied by 600-kva unit substation

The air circuit breakers in the network are provided with combination inverse-time and instantaneous overload trips which will give adequate selective protection. Calculations of fault currents have been made which indicate the maximum short-circuit current from all six stations at the terminals of any one of the air circuit breakers to be within safe limits. They also show the possible currents at the load side of the Swing-Wa power panels to be within the known tested rating of the standard cartridge fuses used. Actually the calculated fault currents are sufficiently low to allow considerable expansion beyond the six stations and indicate the feasibility of networks of larger capacity with the same standardized design of unit substation.

To provide peak load control, which may or may not be generally required, the network air circuit breakers are provided with shunt trips and a multi-conductor control cable is run from the operating room in the steam plant, allowing the operator to dump any section of the load if the contracted ten minute peak demand is liable to be exceeded. The stub feeders can obviously be disconnected without disturbing the network and in the case of the wire building where mention was made of two 575-volt network ties running through the large motor control room, one of these can be disconnected at each end dumping half the rubber mills but leaving the other half in operation on the other tie.

Direct-Current Supply

Direct current power is usually required for operation of cranes, boring mills, and certain other machines. In our case this power is supplied by motor-generator sets the induction driving motors of which are supplied from the network. With the load dividing feature of the network system such motor-generator sets can be connected into any convenient point close to the machines to be supplied without the prior thought necessary with radial feeders.

Comparison of Costs

In making a cost comparison between the network system described above and the radial system we would otherwise have installed we were fortunate in having access to the actual cost of a radial system recently installed in an Ontario plant of floor plan and area similar to building *A* in our plant. With the radial system the transformer capacity for building *A* alone would have been at least 4,500 kva and the costs from the other plant have been adjusted to this capacity. On this basis the network system showed a saving of $18^1/_2\%$.

With the present radial system in our plant it has been the practice to use reduced voltage starters on all squirrel-cage induction motors of 50 hp and larger. The excellent voltage regulation characteristics of the network system, however, made reduced voltage starters unnecessary and the omission of these brought the saving with the network system to $22^1/_2\%$, representing a figure in excess of $22,000 for building *A* alone.

Conclusion

The saving in capital expenditure realized by the use of the network system just described is sufficiently attractive in itself, but when this saving is coupled to the many other advantages the system offers, there seems to be good reason to expect its wide adoption.

Many plants in these days are faced with expansion problems similar to ours and it is hoped that our experience may be of value to others. The flexibility of the network system for increasing loads was impressed upon us recently by difficulties encountered with one of our existing radial substations in which the power transformer bank was seriously over-loaded. This substation had been built to take care of what at the time was thought to be ample future growth but it is now inadequate both in installed capacity and space for extension. In order to relieve the overloaded transformer bank it was necessary to split the 575-volt bus and feed part of it from a small transformer bank of incorrect characteristics for paralleling located in the test department substation several hundred feet away. Had we had a system similar to the one in our new buildings it would have been an easy matter to build another unit station and to connect it into the 6,600-volt loop and the 575-volt network.

The saving in transformer capacity, the saving in reduced voltage starters due to the good regulation, the high salvage value of equipment, and the simplicity of protection with the low-tension network and high-tension loop system of distribution have already been mentioned. These merits should assure its wide adoption in industrial plants.

References

1. SYMPOSIUM ON THE PRIMARY-NETWORK SYSTEM, D. K. Blake and others. *General Electric Review*, volume 35, number 6, June 1932.

2. SECONDARY NETWORKS TO SERVE INDUSTRIAL PLANTS, C. A. Powel and H. G. Barnett. AIEE TRANSACTIONS, volume 60, 1941 (April section), pages 154–6.

An Improved Frequency Meter for Commercial Power Frequencies

K. J. KNUDSEN
MEMBER AIEE

Synopsis: While frequency meters of prior type admirably serve the purpose for which they are intended, their construction has been such as to necessitate the use of an external reactor. This external reactor is in most cases not objectionable, but for certain applications, where space is at a premium, a self-contained instrument is preferable.

This paper describes a frequency meter in which the external reactor has been eliminated by using an iron-core cross-coil instrument, the iron core serving the purpose of the normally external inductance, at the same time shortening the high reluctance air gap necessary for the moving coils.

Introduction

USING an iron core greatly reduces the ampere turns necessary to establish an adequate field intensity, which is proved by the fact, that while the power necessary to operate frequency meters now being used range from 2.5 to 7.5 watts, depending on the design, this instrument consumes a maximum of 0.8 watt at 115 volts, or one-third the power of the prior instrument using the lowest power.

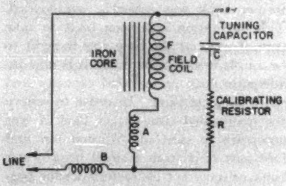

Figure 1. Schematic diagram of frequency meter

While the power consumption in most cases is insignificant with respect to the loss, it is important that a self-contained instrument use no more power than can be readily dissipated within the confinement of the case. Low power loss also simplifies compensation, as there is little change in temperature from the moment the instrument is energized till the temperature of the coils have reached constancy. In a self-contained instrument compensation is further simplified, because all components are mounted within the same container and therefore exposed to the same ambient temperatures, while if an external reactor is used, means must be provided to compensate for a possible difference of the temperature of the instrument proper and that of the reactor.

Theory

Referring to figure 1, the moving coil A is connected in series with the field coils F and therefore tends strongly to align itself with the magnetic flux in the air gap, the current through the coil being in phase with the flux. The capacitor C in series with the compensating resistor R, causes the resultant current in moving coil B to be 90 degrees out of phase with the air gap flux at the frequency corresponding to the center point of the scale. As the frequency varies from the center scale value, the power factor of the current in coil B will change, due to the altered impedance ratio between the field coils and the capacitor, resulting in a clockwise or counterclockwise deflection of the pointer, as the frequency increases or decreases.

As the power factor of the field coils, in series with coil A, is approximately 25 per cent, and the power factor of the current through coil B is about 97 per cent, four times as many turns, of correspondingly smaller size, may be used in coil B than in coil A for an equal heat loss in the two moving coils, resulting in a high order of deflection for a small change in power factor of the current in coil B.

The iron core is made from laminated "47-50" nickel iron due to its high resistance to corrosion and ease of stamping. Damping is provided by an aluminum vane moving through a strong unidirectional magnetic field established by an Alnico magnet.

Comments on Accuracy

The primary function of a frequency meter is, as the name implies, to indicate the frequency of an a-c circuit. Many causes tend to give an indication differing from the actual frequency, the most important of these are: Fluctuation of the line voltage, ambient temperature changes, and distortion of the voltage wave of the circuit.

Compensation for voltage influence has been accomplished by a high ratio of torque caused by the current flowing through the moving coils, to the torque of the ligaments connecting the moving coils to the stationary circuits. Figure 2 is a graph showing the voltage errors from 90 to 140 volts.

While conducting tests for voltage error on a number of instruments, some instruments would indicate erratically when high voltage was applied at certain frequencies. It was found that the two ligaments at one end of the moving coil assembly would short-circuit due to vibration of one of the ligaments, the vibration being set up by interaction between the magnetic flux in the air gap and the current through the ligament. The difficulty was overcome by cutting a turn of the ligament, thereby increasing its natural vibratory frequency above the measuring range of the instrument.

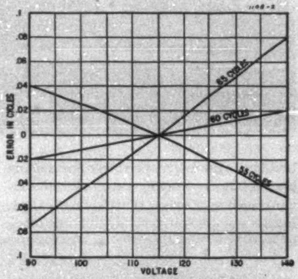

Figure 2. Voltage error curves

There are two main reasons for errors due to ambient temperature changes, they are: Resistance changes of coils and capacity changes of the capacitor used for tuning the field. In order to offset these errors, which have to be compensated simultaneously, a high temperature coefficient resistor is connected in series with the capacitor, thereby maintaining a constant phase relation between the current in the deflecting coil B (figure 1) and the magnetic flux in the air gap for a given frequency, at varying ambient temperatures.

It is important that a high-grade capacitor be used for an instrument, the purpose of which is to indicate small changes of reactance due to frequency changes.

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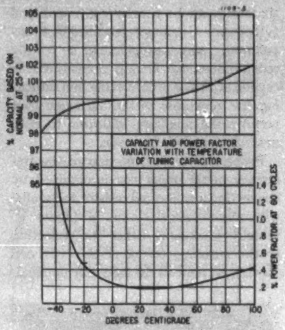

Figure 3. Capacitor-temperature capacity and temperature-power-factor curves

If considerable change of capacity would be caused by normal changes in ambient temperature, the instrument would be worthless as a frequency indicator; for this reason an oil-filled capacitor has been used, which has low and almost uniform temperature coefficient of capacitance within the normal range of temperatures. Figure 3 shows the capacity-temperature curve of the capacitor, and in addition the power factor-temperature curve. It will be noted that the power factor is almost constant within the measuring range, thus aiding the stability of the circuit.

As the temperature coefficient of capacitance is positive, it partly offsets the tendency of the instrument to read high at high temperatures, due to the increased resistance of the copper coils in the field circuit. Satisfactory compensation can be obtained between minus 20 and plus 45 degrees centigrade, by using a resistor in series with the capacitor, the temperature coefficient of the resistor having been adjusted to offset the combined causes tending to give an erroneous reading due to changes in temperature. Figure 4 shows the temperature error before and after compensation. While the capacitor has been under observation for a year, no change of capacity due to aging has been detected; it is however felt that insufficient time has passed to give exact data on aging characteristics.

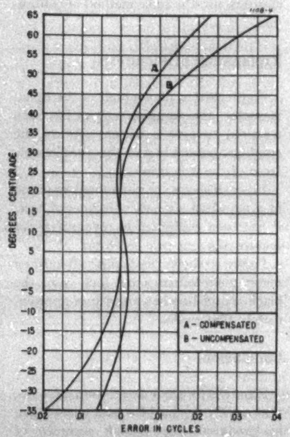

Figure 4. Temperature error of frequency meter

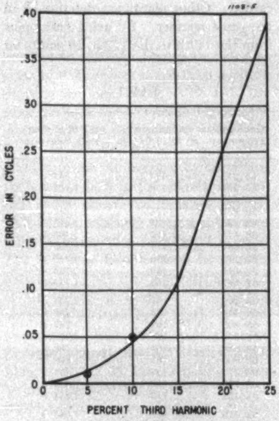

Figure 5. Third harmonic error

As the inductive reactance of the field circuit increases proportional to the frequency, and as the deflection of the pointer is caused by interaction between the moving coils and the magnetic flux of the field circuit, it will be seen, that the lower the harmonic present in the voltage wave, the greater is the tendency to indicate erroneously. Tests have been conducted to determine the effect of the third harmonic on the accuracy of the instrument, because this harmonic is the lowest commonly encountered in commercial supply lines. Figure 5 shows the error caused by varying magnitudes of third harmonic; changing the phase relation between the fundamental frequency and the third harmonic through 120 electrical degrees had no appreciable influence on the error indicated.

While the third harmonic influence seems rather high, it should be realized that the third harmonic, which is generally introduced into the voltage wave by the excitation of iron-core equipment due to the increased current at high saturation, seldom exceeds five per cent.

A four per cent seventeenth harmonic tooth ripple, present in the voltage of the generator used for calibration, had no influence on the accuracy, the indication being the same after eliminating the ripple by means of tuned by-pass filters.

Calibration

In order to obtain accurate calibration of the instrument without waste of time a multivibrator was used, having steps of 87,480, 29,160, 9,720, 1,620, 540, 180, and 60 cycles. The multivibrator is crystal controlled to one cycle in five million, its accuracy ascertained by comparing the time indicated by a synchronous electric clock, connected to the 60-cycles output stage, with radio time signals.

While the method of determining one frequency by its harmonic relation to another is not new, it is felt that it has been limited to a ratio, where it was possible to count the loops on the screen of the oscillograph. In the case where the harmonic ratio is more than 150 to one, counting the loops would be impossible without auxiliary means; a detailed description is therefore given of the method of calibration.

The instrument is calibrated by connecting the horizontal sweep circuit of an oscillograph to the 9,720-cycles stage of the multivibrator, and the vertical sweep to the variable speed generator used for calibration, the vertical sweep being amplified to extend beyond the limit of the screen, so as to show only two or three loops of the horizontal sweep, as indicated in figure 6.

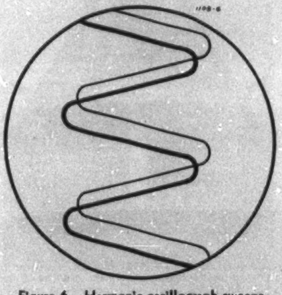

Figure 6. Harmonic oscillograph sweeps

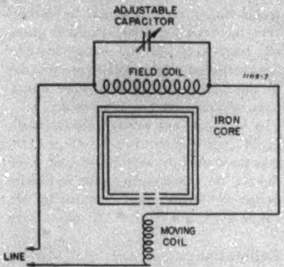

Figure 7. Schematic diagram of auxiliary frequency meter

As the frequency of the multivibrator is the 162nd harmonic of 60 cycles, some means must be employed to determine that the proper harmonic is used, there being only four-tenths cycle approximately between harmonics as compared to 60 cycles. A high-resistance voltmeter is connected in series with the 60-cycles stage of the multivibrator and the variable speed generator. The generator is adjusted until a steady reading equal to the sum of the voltages of the two supplies is obtained on the voltmeter. This reading is only obtainable when the voltages are synchronous and 180 degrees out of phase, the harmonic indicated on the screen must therefore be the 162nd.

In order to keep track of the harmonics used for calibrating the scale, an auxiliary frequency meter is used. The basic circuit of this instrument, shown in figure 7, is the same as that of the instrument under test, except that the restoring torque is provided by springs; the instrument is therefore accurate at only one point of the scale, the deflection at other points being dependent on the line voltage. An adjustable capacitor is used for adjusting the auxiliary instrument to center scale when the proper harmonic is found, by means of the voltmeter, at 60 cycles. As the sensitivity of the instrument is in the order of one-tenth cycle for full-scale deflection, it will be completely off scale if a wrong harmonic is used for calibration, and exact adjustment to center scale is therefore not necessary.

After the auxiliary instrument has been calibrated to 60 cycles, the setting of the adjustable capacitor is recorded and the speed of the generator is then slowly changed until the next harmonic appears on the screen. The auxiliary indicator again adjusted to center scale and the setting of the adjustable condenser recorded. Other points are determined in the same manner. By using harmonics from the 149th to the 177th, 29 points on the scale can be exactly calibrated and actual scale divisions determined by interpolation. Any harmonic can be readily redetermined by setting the adjustable capacitor to the proper value and adjusting the generator to the harmonic that permits the auxiliary indicator to read near the center of the scale.

Table I

Generator Frequency	Subharmonic of 9,720 Cycles	RPM of Four-Pole Generator
54.915	177	1,647.5
55.227	176	1,658.8
55.543	175	1,666.3
55.862	174	1,675.9
56.185	173	1,685.5
56.512	172	1,695.3
56.847	171	1,705.3
57.176	170	1,715.3
57.515	169	1,725.4
57.857	168	1,735.7
58.207	167	1,746.1
58.552	166	1,756.6
58.909	165	1,767.3
59.268	164	1,778.0
59.632	163	1,788.9
60.000	162	1,800.0
60.373	161	1,811.2
60.750	160	1,822.5
61.132	159	1,834.0
61.519	158	1,845.6
61.911	157	1,857.3
62.308	156	1,869.2
62.710	155	1,881.3
63.117	154	1,893.5
63.530	153	1,905.9
63.947	152	1,918.4
64.371	151	1,931.1
64.800	150	1,944.0
65.235	149	1,957.0

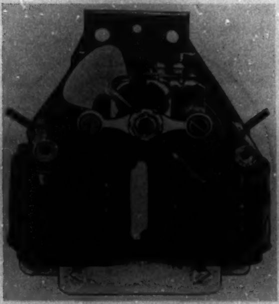

Figure 8. Frequency meter dynamometer

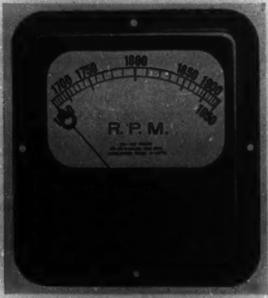

Figure 9. Frequency meter in six-inch case

Proper full-scale deflection of the frequency meter under test is obtained by connecting a calibrated resistor in parallel with either coil *B* or coil *A* (figure 1), thereby weakening either the deflecting or the restoring torque of the cross-coil moving element as the case may demand.

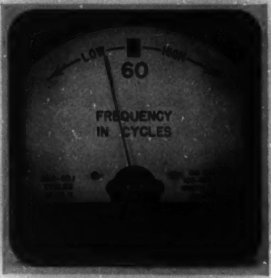

Figure 10. Auxiliary frequency meter in five-inch case

Table I shows the frequency and speed in revolutions per minute of a four-pole generator corresponding to the harmonics being used for calibration, it is included for the convenience of those, who may desire to use the same method of calibration.

Explanation of Pictures

Figure 8 is a photograph of the dynamometer; the damping magnet has been removed to give a better view of the cross-coil assembly and the parallel pole faces of the iron core.

Figure 9 shows the instrument mounted in a standard six-inch switchboard case, it has been calibrated in revolutions per minute of a four-pole generator.

Figure 10 shows the auxiliary frequency meter mounted in a five-inch case, the black center line has a width of approximately 0.01 cycle.

Conclusion

From the foregone description it is seen, that a self-contained frequency meter can be produced, the over-all accuracy of which is better than two-tenths cycle in 60 cycles, permitting a third harmonic

Mechanical Simplicity of Air-Blast Circuit Breakers

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ASSOCIATE AIEE

OTTO JENSEN
ASSOCIATE AIEE

THE papers heretofore presented before the Institute on the subject of air blast circuit breakers have covered primarily their advantages in comparison with oil circuit breakers, the principles involved in current interruption by air blast, descriptions of interrupters developed by several manufacturing companies, the results of tests and related data. It is considered opportune to give an account of several other types of air blast circuit breakers ranging in voltage from 4 kv to 230 kv as developed and built by certain manufacturing companies in Canada and the United States in collaboration with a large power company. The first machine of this general type was a 138-kv air blast breaker placed in service in 1934 at the Beauharnois generating station on lines to Montreal, this breaker having continued in service for six years successfully interrupting all faults; three additional breakers of the same general design with improved details have been placed in service at this station in later years.

Breakers of four types built by three manufacturing companies, all breakers having certain general features in common, will be illustrated by photographs and diagrams. In all of the breakers described, the general design of interrupting contacts is the same. This arrangement of contacts is suitable for the use of arc chutes utilizing any one of the elementary principles of pneumatic arc extinction or combinations of such principles.

Types marked "A" include breakers for 138 kv and 230 kv built by the English Electric Company of Canada.

Types marked "D" are 69-kv breakers built by the same company.

Types marked "B" are breakers of 15 kv to 37 kv built by the I-T-E Circuit Breaker Company, in Philadelphia.

Types marked "C" are breakers of 4 kv to 15 kv built by the Eastern Power Devices Limited, in Toronto.

The diagrams of each type are numbered 1 to 7 to show the sequence of operation.

Type "A"

Photograph *A* shows one of the 138-kv breakers in service. Figure 1*A* is a diagram of this breaker in the open position showing the main parts which are: the tank storing air at 100 lbs. per sq. in., the interrupter, and the isolating switch. In figure 2*A* the closing coil is being energized to close the breaker by admitting air from the tank to the cylinder and piston which closes the isolating switch. This coil is extremely small and becomes fully energized in a fraction of a cycle. The coil only has to move its plunger about one-eighth inch uncovering a port of one-eighth inch diameter, releasing about one-quarter cubic inch of compressed air from under a rubber diaphragm of about two inch diam. which is blown open by the opposing air pressure on the other side, thus opening the valve which admits air from the tank to the isolating switch cylinder and closes the switch. The cylinder of each pole is mounted under the bedplate of the interrupter, at line potential, as close as possible to the switch blade operated by a short link. Electrical, pneumatic, and mechanical inertia are reduced to the practical limit thus minimizing the closing time. A standard auxiliary switch to perform the usual functions, is operated from the isolating switch blade by a 90-degree rotation of the same porcelain tube (of one inch inside diameter) conveying closing air to the switch cylinder. Provision is also made for raising and lowering this porcelain tube one inch, to perform additional functions in connection with quick automatic reclosing.

The jaw contacts of the isolating switch are mounted on the cap of the porcelain cone of the current transformer; this cap is purposely made extra heavy in weight and is supported on rubber bushings to minimize mechanical shock to the porcelain due to fast operation of the switch blade. The jaws are full-floating and swivel to facilitate cracking of ice film in opening; a large sleet hood also covers the contacts.

The interrupter has three identical interrupting elements in series in the 138-kv breaker and six in the 230-kv breaker. Each element has two stationary contacts bridged by one movable contact thus making two breaks in series per element. The movable contact is rigidly secured to a steel piston rod and piston working in a bronze cylinder insulated from adjacent cylinders. Figure 3*A* shows the breaker in the closed position.

As shown in the photograph *A*, the 138-kv breaker has one tank and three blast valves. The 230-kv breaker has three separate tanks to facilitate shipping and erecting the greater length. The 230-kv breaker is built up with the same parts as the 138 kv and almost all of the parts are interchangeable. Due to the greater height, the 230-kv interrupter is brazed with diagonal stacks of standard switch insulators.

The blast valve which is the same for 230 kv and 138 kv, is built into the side of the tank with an inlet of about three square feet section. The outlet of the valve is a funnel-shaped orifice of about 8″ diameter at the valve seat which is normally closed by a thick rubber diaphragm about 20″ diameter, reinforced by a light steel disk vulcanized to the rubber. The diaphragm moves down about one inch to fully open the valve. Air pressure at 100 lbs. is maintained underneath this diaphragm, to hold it closed. The volume of air under the diaphragm is less than one-twelfth of a cubic foot. To open the breaker this small volume of air is exhausted to atmosphere by two or three one-inch diameter orifices each normally held closed by a diaphragm and coil of the same size as

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H. W. HABERL is with the electrical department of the Montreal Light, Heat and Power Consolidated, Montreal, Quebec, Can., and OTTO JENSEN is with the I-T-E Circuit Breaker Company, Philadelphia, Pa.

The authors gratefully acknowledge the assistance of H. Milliken, chief engineer and general superintendent of Montreal Light, Heat and Power Consolidated, in the preparation of this paper.

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distortion of ten per cent, a voltage fluctuation of 90 to 140 volts, and an ambient temperature range of minus 10 to plus 40 degrees centigrade. The power consumed being eight-tenths volt-ampere at 97 per cent power factor.

As the complete instrument can be installed in a $3^{1}/_{2}$-inch case, it is well adapted for use where space is limited. The elimination of an external reactor reduces the work involved in wiring and installation.

Figure A. General view of 138-kv air-blast breaker

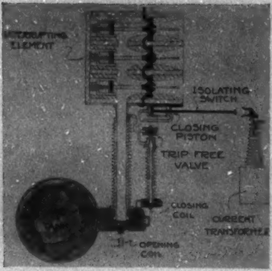
INTERRUPTING ELEMENT
ISOLATING SWITCH
CLOSING PISTON
TRIP FREE VALVE
CLOSING COIL
CURRENT TRANSFORMER
OPENING COIL

Figure 1A. Breaker in the normal open position

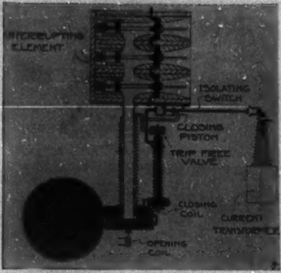
INTERRUPTING ELEMENT
ISOLATING SWITCH
CLOSING PISTON
TRIP FREE VALVE
CLOSING COIL
CURRENT TRANSFORMER
OPENING COIL

Figure 2A. Closing coil energized, isolator moving

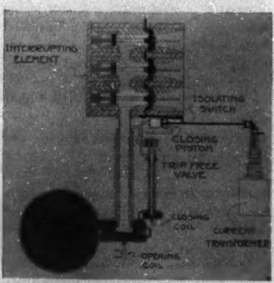
INTERRUPTING ELEMENT
ISOLATING SWITCH
CLOSING PISTON
TRIP FREE VALVE
CLOSING COIL
CURRENT TRANSFORMER
OPENING COIL

Figure 3A. Breaker in normal closed position

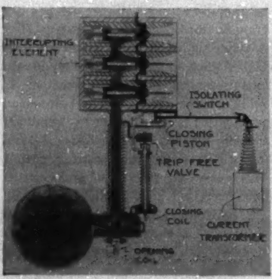
INTERRUPTING ELEMENT
ISOLATING SWITCH
CLOSING PISTON
TRIP FREE VALVE
CLOSING COIL
CURRENT TRANSFORMER
OPENING COIL

Figure 4A. Opening coil energized, air filling blast tube

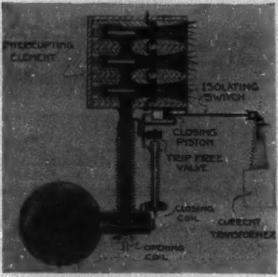
INTERRUPTING ELEMENT
ISOLATING SWITCH
CLOSING PISTON
TRIP FREE VALVE
CLOSING COIL
CURRENT TRANSFORMER
OPENING COIL

Figure 5A. Interrupter pistons open, arc drawn

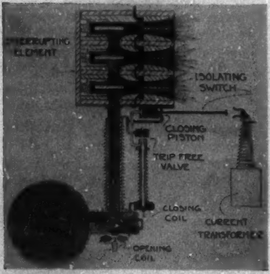
INTERRUPTING ELEMENT
ISOLATING SWITCH
CLOSING PISTON
TRIP FREE VALVE
CLOSING COIL
CURRENT TRANSFORMER
OPENING COIL

Figure 6A. Arc extinguished, isolator moving open

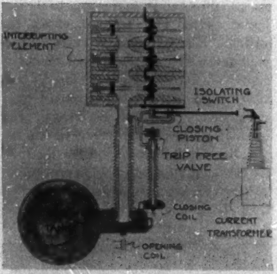
INTERRUPTING ELEMENT
ISOLATING SWITCH
CLOSING PISTON
TRIP FREE VALVE
CLOSING COIL
CURRENT TRANSFORMER
OPENING COIL

Figure 7A. Valves reset, breaker normal open

High-voltage air-blast breaker

For this type of breaker there are three interrupting elements in series, each individually air-operated

that described in connection with the valve used for closing the isolating switch. The weight of the moving parts of the blast valve is small, less than 5 pounds and the accelerating force is about 10,000 pounds, which gives an acceleration of about two thousand times that of gravity. This accounts for the very fast opening of these breakers, about one-sixtieth of a second from energizing the trip coil until parting of contacts, which of course is fast for a breaker of this large size. The interrupting element also contributes to this short opening time, the moving part having a weight of less than ten pounds and an accelerating force of about 2,000 lbs.

Figure 4*A* illustrates the conditions during the opening of the interrupter. The blast valve is open; a wave of air pressure is travelling up the porcelain blast tube, compressing the atmospheric air standing in the tube and header in the interrupter structure, similar to a sound wave, blasting the interrupter pistons to the left, opening the contacts and interrupting the arcs. A pressure wave is also travelling through a half inch pipe to the isolating switch cylinder but it cannot reach the cylinder in less than several cycles, leaving ample time for the interruption of the arcs before the isolating switch parts contact. This is shown in figures 5*A* and 6*A*.

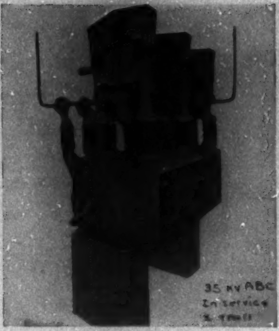
35 KV ABC
In service

Figure B. General view of 35-kv air-blast breaker

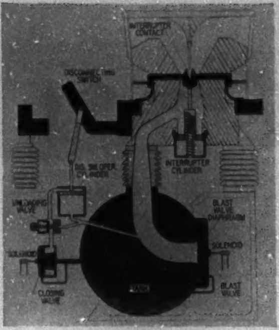
INTERRUPTER CONTACT
DISCONNECTING SWITCH
DIS. SW. OPER. CYLINDER
INTERRUPTER CYLINDER
UNLOADING VALVE
BLAST VALVE DIAPHRAGM
SOLENOID
SOLENOID
TANK
CLOSING VALVE
BLAST VALVE

Figure 1B. Breaker in the normal open position

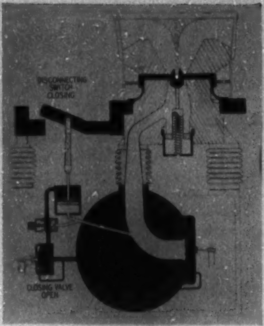
DISCONNECTING SWITCH CLOSING
CLOSING VALVE OPEN

Figure 2B. Closing coil energized, isolator moving

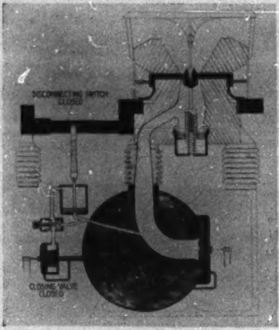
DISCONNECTING SWITCH CLOSED
CLOSING VALVE CLOSED

Figure 3B. Breaker in normal closed position

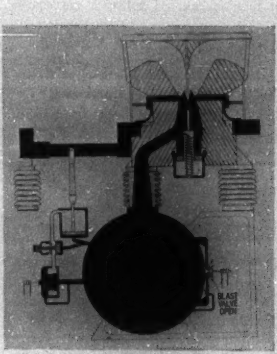
BLAST VALVE OPEN

Figure 4B. Opening coil energized air filling blast tube

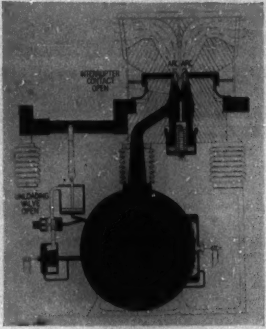
INTERRUPTER CONTACT OPEN
ARC
UNLOADING VALVE OPEN

Figure 5B. Interrupter pistons open, arc drawn

Medium-voltage air-blast breaker

This type of breaker uses only a single blast valve for all three phases and only one interrupter element per phase

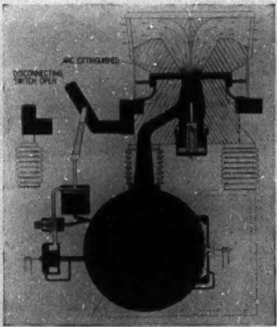
ARC EXTINGUISHED
DISCONNECTING SWITCH OPEN

Figure 6B. Arc extinguished, isolator moving open

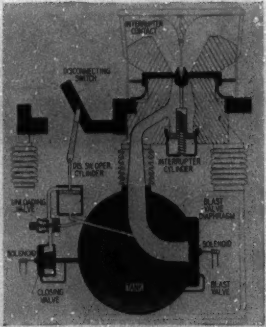
INTERRUPTER CONTACT
DISCONNECTING SWITCH
DIS. SW. OPER. CYLINDER
INTERRUPTER CYLINDER
UNLOADING VALVE
BLAST VALVE DIAPHRAGM
SOLENOID
SOLENOID
TANK
CLOSING VALVE
BLAST VALVE

Figure 7B. Valves reset, breaker normal open

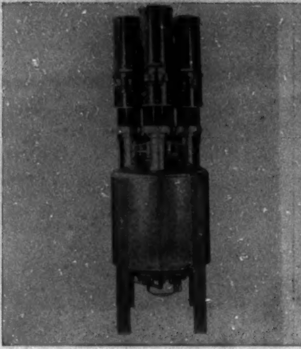

Figure C. General view of 12-kv air-blast breaker

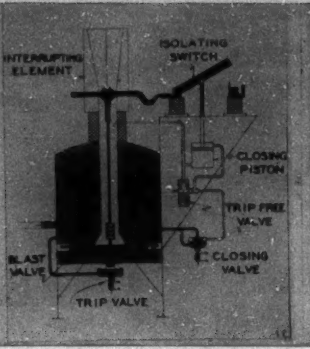
INTERRUPTING ELEMENT
ISOLATING SWITCH
CLOSING PISTON
TRIP FREE VALVE
CLOSING VALVE
BLAST VALVE
TRIP VALVE

Figure 1C. Breaker in the normal open position

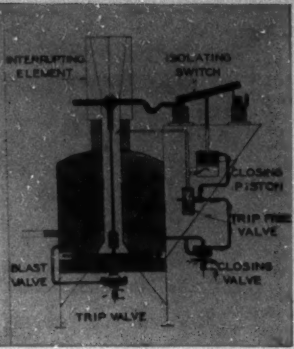
INTERRUPTING ELEMENT
ISOLATING SWITCH
CLOSING PISTON
TRIP FREE VALVE
CLOSING VALVE
BLAST VALVE
TRIP VALVE

Figure 2C. Closing coil energized, isolator moving

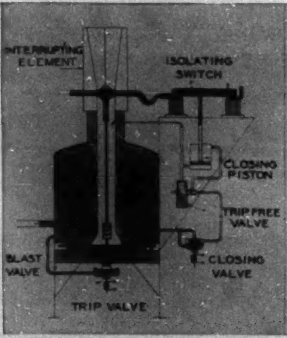
INTERRUPTING ELEMENT
ISOLATING SWITCH
CLOSING PISTON
TRIP FREE VALVE
CLOSING VALVE
BLAST VALVE
TRIP VALVE

Figure 3C. Breaker in normal closed position

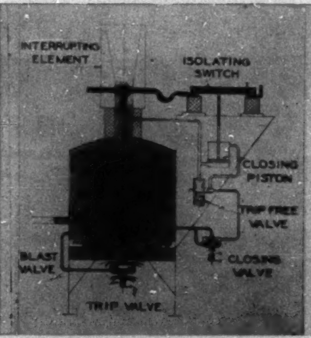
INTERRUPTING ELEMENT
ISOLATING SWITCH
CLOSING PISTON
TRIP FREE VALVE
CLOSING VALVE
BLAST VALVE
TRIP VALVE

Figure 4C. Opening coil energized, air filling blast tube

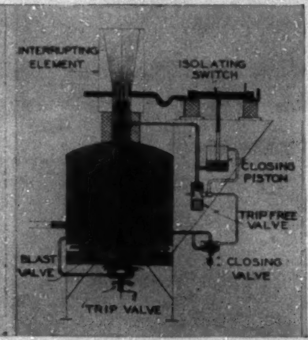
INTERRUPTING ELEMENT
ISOLATING SWITCH
CLOSING PISTON
TRIP FREE VALVE
CLOSING VALVE
BLAST VALVE
TRIP VALVE

Figure 5C. Interrupter open, arc drawn

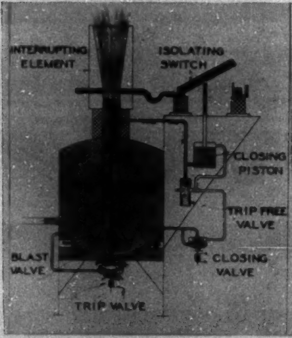
INTERRUPTING ELEMENT
ISOLATING SWITCH
CLOSING PISTON
TRIP FREE VALVE
CLOSING VALVE
BLAST VALVE
TRIP VALVE

Figure 6C. Arc extinguished, isolator moving open

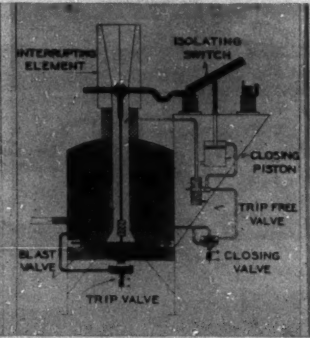
INTERRUPTING ELEMENT
ISOLATING SWITCH
CLOSING PISTON
TRIP FREE VALVE
CLOSING VALVE
BLAST VALVE
TRIP VALVE

Figure 7C. Valves reset, breaker normal open

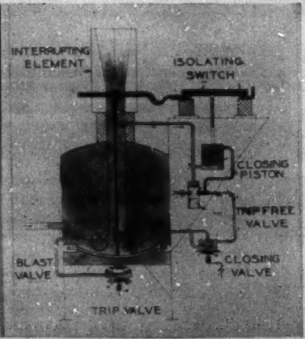
INTERRUPTING ELEMENT
ISOLATING SWITCH
CLOSING PISTON
TRIP FREE VALVE
CLOSING VALVE
BLAST VALVE
TRIP VALVE

Figure 8C. OCO air trip-free valve draining closing air

Low-voltage air-blast breaker

In this type of breaker the main blast diaphragm operates the three phases of the interrupter elements

As the isolating switch is almost fully open, its auxiliary switch opens the trip circuit and the spring in the trip coil closes the trip valve permitting the reestablishing of pressure under the blast valve diaphragm by the flow from the tank. It has been found unnecessary to provide an area under the diaphragm greater than the area on top of it in order to make the diaphragm reseat, because there is an ejector effect which reduces the pressure near the center of the diaphragm due to the high velocity of the discharge and the diaphragm reseats promptly when the trip circuit is opened. Means have been developed for automatically shutting off the infeed of air from the tank to the lower side of the diaphragm while exhausting that space through the trip valve, but in service it has been found that satisfactory timing is obtained without such device. In general, the breaker can be blasted several times in quick succession before the tank pressure is reduced below the safe operating limit. When several breakers are installed in the same station, a common auxiliary storage tank is provided which quickly restores pressure in the breaker tank after which normal pressure is restored in the storage tank by the automatic starting of the compressor.

The 230-kv and 138-kv breakers are well adapted to single pole operation as there is no mechanical connection between poles. The switchboard operator can change the control from three-pole operation to single-pole operation by simply turning a control switch.

Type "B"

Photograph *B* illustrates a 35-kv breaker built by the I-T-E company which has been in successful service outdoors in New England for about two years at a location near the seashore where the salt spray constitutes a test of insulation even more severe than the St. Lawrence River mists on the higher voltage breakers. Figure 1*B* is a diagram of this 37-kv breaker in the open position. The interrupter has only one moving contact per pole; it is rigidly secured to a steel piston-rod and piston moving vertically in a cylinder which is at line potential. There is one blast valve and tank serving the three poles. The blast valve outlet divides three ways inside the tank and the three tubes pass through the tank to their respective poles. The blast valve is of the same general design as that of the higher voltage breakers, with certain modifications in detail, including springs to accelerate reseating of the diaphragm. The three poles of the isolating switch are operated by one cylinder at ground potential.

Figures 1*B* to 7*B* show the sequence of operation of this breaker. All breakers described in this paper are equipped with a pneumatic trip-free device operated by blast pressure to shut off compressed air supply to the closing end of the isolating switch cylinder and exhaust the air trapped in the closing end. The exhausting action is accelerated by the ejector effect of the nozzle in case the isolating switch closes on a short circuit.

Breakers of this same type also have been built by this company for 15 kv and 3,000 amperes. The current-carrying contacts are separate and part slightly before the interrupting contacts, all being rigidly fastened to the same operating piston-rod. In these breakers means are provided for surrounding the stationary contacts by air pressure to prevent premature parting of contacts before the moving contacts have moved away.

Type "C"

Photograph *C* illustrates a type of smaller breaker for 15 kv to 4 kv indoor service, 2,000 amperes, built by the Eastern Power Devices Limited. The main distinguishing feature of this type is the method of operating the interrupter contacts. The blast valve is of the same general design as in the other breakers except that the diaphragm has secured to its center, a steel operating tube which is rigidly connected to the interrupting contacts of the three poles through an insulating rod in each pole. When the valve diaphragm moves down it opens the blast and also mechanically opens the interrupting contacts. In order to insure that the contacts do not part before the blast arrives to interrupt the arc, a slight amount of slack is provided between the valve diaphragm and the operating rod so that the diaphragm moves down opening the valve part way before beginning to move the contacts. To minimize shock on the moving parts in opening, a spring is inserted between the diaphragm and operating tube.

Floor space is minimized by placing the tank vertically on end, the blast valve constituting the lower end of the tank. The blast valve outlet is shown in the diagrams as a single tube but actually it divides three ways passing up through the tank and out the top end of the tank in equilateral triangular formation; each of the three vertically projecting blast tubes is a 4″ standard pipe welded to the top of the tank and acts as the support for one pole of the interrupters. Each interrupter pole consists of a vertical cylinder of insulating material, 9″ inside diameter which provides sufficient space for interrupting elements of ample capacity.

Figure 9C. Opening sequence

Figure 10 C. Oscillogram of interruption

A 2,000-amp. breaker of this type has a width of only 2′6″. This design is well adapted to various indoor layouts of stations and cell arrangements, including a draw-out arrangement of eliminating the usual disconnecting switches. This design can also be turned upside down and adapted to go into cells built for the old style oil breakers with live parts at the bottom.

Type "D"

Photograph *D* shows the general arrangement of 69-kv breakers built by the English Electric Company of Canada. It has one blast valve for the three poles, the valve being of the same design as the other breakers described. The blast tube passes horizontally through the tank to the external header tube parallel to the tank, serving the three poles through vertical stacks of porcelain blast tubes. Inside of each stack is an insulating operating rod opening and closing the interrupting contacts of the same design as in the other breakers. Each of these three operating rods is directly connected to a piston at the grounded end and in addition the three piston-rods are mechanically tied together through short links and a common shaft. The three poles of isolating switch blades are operated by a common cylinder at ground potential. The switch blade is a hollow bronze casting enclosing the hinge of a short toggle blade which makes a swiveling engagement with the stationary full-floating jaw contacts facilitating sleet cracking in opening. The blade is side opening facilitating sleet-hood protection and simplifying insulators, only two stacks per pole being required. A simple arrangement of arcing contacts has been found satisfactory in closing on short circuits with negligible burning; the operation of the isolating switch is very fast.

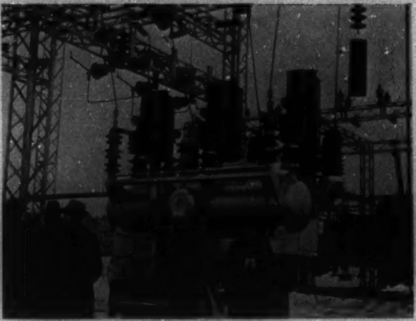

Figure D. General view of 69-kv air-blast breaker

Operating Results

The service record of breakers of this general design dates from 1934 when the first 138-kv breaker was installed. Since that time breakers of the several other voltages have been placed in service and the record is now equivalent to 20 breaker-years of service. The number of breakers recently installed and on order is 30. There have been 15 breaker interruptions reported in service, with no failures.

Interrupting Ratings

The 230-kv breaker is rated 2,500,000 kva; single-pole interrupting tests on sections of this breaker have shown interrupting capacity equivalent to more than 3,000,000 kva on the complete 3-pole breaker. The 138-kv breaker is rated 1,500,000 kva; 69-kv breaker 1,000,000 kva.

Insulation

The standard specifications for breakers of the several voltages are complied with.

Operating Time

The opening time is three cycles or less, from energizing of trip coil to current interruption. Reclosing time is within 20 cycles from energizing trip coil.

Operating Current

Closing coil or opening coil takes ten amperes or less; can be reduced to one ampere.

The Shielding of Permanent Magnets From Transient Magnetic Fields

GEORGE J. WEY
ASSOCIATE AIEE

Introduction

CONSIDERABLE progress has been made toward the development of permanent magnets which are immune to disturbing conditions, such as temperature variation, mechanical vibration, natural aging, and extraneous flux fields. Contributions to this progress include new materials, improved processing techniques, and better manufacturing controls.

Constancy of flux is in most applications quite essential. This is particularly true of electromagnetic damping, in which the damping torque varies as the square of the air gap flux. The permanent magnet of a watthour meter is an example.

Despite the fact that the modern permanent magnet is remarkably stable, there have been cases in which the magnets showed changes in strength, generally becoming somewhat weaker. Isolated instances of such changes in watthour meter magnets have been found by the electric utility companies.

About four years ago, evidence gathered by various utilities pointed to lightning discharges as the probable cause of the demagnetization. After initial laboratory tests failed to demagnetize meter magnets by passing surge currents through the windings alone, it was realized that the demagnetizing field was primarily caused by surge currents in the service wiring itself. Figure 1, curve 1 shows the amount by which forged chrome steel magnets are weakened by a single surge of the value indicated by the abscissa using the test setup shown on the figure 2.

To date, there have been no extensive studies made of the magnitude and frequency of surge currents appearing on the secondaries of distribution systems. Figure 3 shows expectancy curves for lightning discharges in distribution system primary arrester ground leads from the investigations of McEachron and McMorris.[1] These curves show that in rural areas a discharge of 5,000 amperes may be expected at any one arrester location once in 14 years and a 10,000-ampere discharge once in 30 years. In attempting to apply these expectancy curves to the distribution secondaries, however, they should be considered as extremely pessimistic because of the reduced exposure of the secondary circuit.

It is evident from a consideration of the rarity with which high current discharges appear on distribution systems that any program designed to materially reduce the number of weakened magnets found on the system secondary must be applied to a large percentage of the total exposed installations. Thus, it would be desirable to protect not only new meters but also those in service.

Since there are millions of standard forged steel magnets in service, it would be desirable that surge protected magnets should not require new techniques or tools to service them.

High Coercive Alloys

The problem of making permanent magnets impervious to transient magnetic fields has been solved in two ways by the various meter manufacturers. One method has been to make the magnet from one of the new high coercive materials, such as Alnico. The coercive force, H_c, of these alloys ranges from 400 to 600 oersteds which means that when properly applied to a watthour meter such a magnet would withstand surges up to 25,000 amperes without appreciable weakening. Another advantage of these alloys over forged magnet steel is economy of space for a given air gap energy. However, before this advantage can be realized to any appreciable extent, the whole meter must be designed with this object in view. Also, a complete new magnet design must be made and manufacturing processes developed in order to utilize the new alloys.

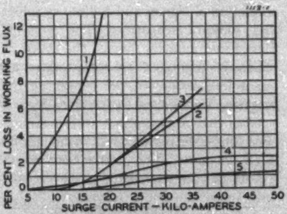

Figure 1. Demagnetization of chrome-steel magnets by transient currents

1. Unshielded
2. Plated with 0.002-inch thick copper
3. Schoop sprayed copper 0.02-inch thick
4. Copper clad (Roth process) 0.02-inch thick
5. High-speed copper plate 0.019 thick

Test waves
Curves 1, 2, 3—24x40 microseconds
Curves 4, 5—36x70 microseconds to 25,000 amperes; 30x50 microseconds 25,000 to 40,000 amperes; 20x35 microseconds over 40,000 amperes

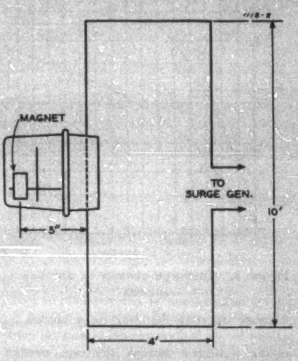

Figure 2. Setup for surge tests on permanent magnets

Electromagnetic Shielding

An alternate solution to the problem of transient fields and the one described in this paper is to use electromagnetic shielding around the existing forged steel magnets. While this principle of shielding is old,[2] it is felt that a description of its application to permanent magnets is new and may be of general interest.

The requirements of an efficient electromagnetic shield are: first, the shield must provide low resistance paths for the flow of eddy currents. Second, the shield must be continuous about the object to be shielded so the eddy currents can flow in any direction.

The above requirements immediately suggest plating the magnets with copper,

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[1] For all numbered references, see list at end of paper.

since copper has low electrical resistance and a continuous sheath of the metal could be easily plated on steel. With the ordinary process of electroplating copper from a copper sulphate solution, however, it is impractical to build up a coat more than 0.002″ thick, and even this thickness requires upward of 24 hours plating time.

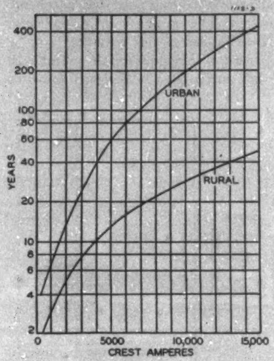

Figure 3. Discharge currents in distribution arresters

Number of years that may be expected to elapse for each discharge of at least the magnitude indicated through any one arrester

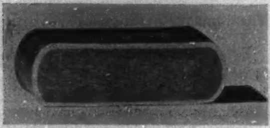

Figure 4. Cross section of plated magnet

Figure 1, curve 2 shows the shielding effect of a 0.002″ thick plated copper sheath using a 24x40 wave (24 microseconds to crest, 40 to half value). By comparing with curve 1 it will be seen that such a shield is quite effective against transients.

Another method of applying a metallic coating is by the Schoop spray process. By this means metallic coatings of any desired thickness can be applied to magnets. Figure 1, curve 3 shows the shielding effect obtained by this method. Due to the porosity and consequent high resistivity of the deposited metal and the difficulty of obtaining a uniform coating, this method was considered inadequate.

Good shielding can be obtained by the use of copper-clad steel made by any of the standard commercial processes. Figure 1, curve 4 shows the shielding obtained from a 0.020″ thick layer of copper formed by the Roth process. This process consists of placing a properly fluxed billet of steel in a vertical mold. After heating the mold and billet to a red heat, molten copper is poured around the billet. The mass is then rolled to the desired bar size. The principal disadvantage of copper-clad steel for magnets is that the conductivity of the copper is lowered during the heat treatment of the magnet steel. Also, the copper coat is not of uniform thickness, which reduces its effectiveness.

High-Speed Copperplating

Early experiments with a new high-speed copper-plating process invented by the E. I. du Pont de Nemours Company gave results worthy of further investigation. The high-speed copper-plating process plates from a hot alkaline bath consisting of copper and sodium cyanides and sodium hydroxide. The work is placed in a rotating barrel, the speed of which is regulated to obtain the best burnishing action on the copper as the magnets tumble over each other.

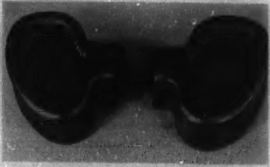

Figure 5. Left, magnet before plating; right, magnet after plating

With this process about 13.1 lb of copper can be deposited in 11 hours on 65 lb of magnets. This corresponds to an average deposition rate of 0.0017″ per hour.

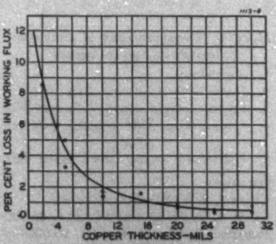

Figure 6. Effect of copper thickness on shielding of chrome-steel magnets

It was found in developing this process for the plating of magnets that the conductivity of the copper plating decreased very rapidly after the solution was used a few times. This condition was traced to an excessive concentration of sodium carbonates in the electrolyte. Excessive amounts of this compound also cause the deposit to be rough and pebbly. It was found necessary to periodically chill the solution and allow the carbonates to crystallize out.

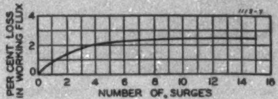

Figure 7. Effect of repeated 30x50 microsecond surges of 30,000 amperes on copper-plated chrome magnets

Figure 4 is an enlarged cross section of a plated magnet.

In order to obtain the maximum calibration adjustment on the assembled magnet, it is necessary to leave the upper pole tip unshielded. This is accomplished by placing a short length of rubber tubing over the upper pole tip before plating. The tubing is thick enough to completely fill the air gap of the magnet, thus reducing the amount of material to be removed in the final grinding. Figure 5 shows a magnet before and after copper plating.

Figure 8. A complete assembly of copper-plated magnets

Since the effectiveness of the shielding depends on the resistance of the plating, this property is used to control the quality of the product. As absolute values are not necessary, the procedure is to measure the total ohmic resistance of the magnet between points just back of the poles using a Kelvin double bridge. The addition of the copper plating reduces the total resistance from 340 microhms for the

System Short-Circuit Currents

Proposed New Calculating Procedure for Application of Interrupting Devices and Relays

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THE current flowing in a short circuit in a simple a-c system can be expressed as a function of time in the form of three exponentially decaying components and a term which is substantially constant. These components are termed the subtransient, the transient, the d-c components, and the steady-state component. In actual systems all these terms are affected to some extent by the number of machines and the phase angles between them and also by their excitations. The phase relations between machines may change during the period of short circuit and thus introduce further complications. Automatic voltage regulators also introduce another factor which tends to increase the short-circuit current, since their action causes increase in machine excitation. Consequently, the accurate determination of short circuit current at any instant may be and usually is quite difficult.

Fortunately such rigorous calculations are not generally required to determine the proper application of interrupting devices and relay settings, for many other factors having greater proportional effect than the variations cited above contribute to the determination of the limits for rational choice of circuit breakers, expulsion protective gaps, fuses, etc. It is, therefore, perfectly proper and logical to make certain approximations in the calculation of short-circuit current to permit simple solutions of electrical systems for the purpose of applications of equipment. Such assumptions have been in general use for a considerable time and recognized by the electrical engineering profession.[1] An early approximation took the form of decrement curves which could be utilized if the reactances of the component branches of the system were known. As the knowledge of machine performance with the resolution of currents into the transient and subtransient components increased and the use of symmetrical components for unbalanced conditions became more generally known and accepted, the original decrement curves were revised[2] and placed on a more substantial basis. These curves, the present decrement curves, included provision for the determination of three-phase, line-to-line and line-to-ground short-circuit currents. At the same time other and more elaborate approximations[3,4] were offered which, while giving more accurate results, involved more extensive and complicated calculations.

The growth and interconnection of systems has resulted in a condition such that if the curves are applied according to the rules, values of impedance outside the range of the curves are often obtained, thus making the curves useless. Recourse was often made to certain expedients, such as the division of the system into several groups, each of which fed short-circuit currents into the short circuit independent of the others. These were so chosen that all the generators in a certain group were approximately the same electrical distance from the fault. In addition, frequent errors have occurred, even among those accustomed to the use of the curves, in confusing the kva of the arbitrarily chosen base on which calculations are made, with the kva of the system. The need undoubtedly exists for

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This paper is the report of a group assigned to investigate the problem under the auspices of the subcommittee on circuit breakers, switches, and fuses of the AIEE committee on protective devices.

[1] For all numbered references, see list at end of paper.

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steel alone to approximately 75 microhms for the combination.

Figure 6 shows the shielding effect of plated copper plotted against copper thickness. No appreciable increase in shielding is obtained after the coat is 0.020″ thick.

Figure 1, curve 5 shows the shielding effect of a 0.019″ thick plated copper shield using the surge values indicated on the figure.

Figure 7 shows the effect of repeated surges of the same magnitude on copper-plated magnets. Since the resistivity of the steel is fairly low (40.5 microhms/cm^3), an appreciable amount of self-shielding is obtained from the steel itself. This means that the demagnetizing effect of the first surge is limited to the outermost layer of magnet steel. After the surge has passed, the inner layers of steel remagnetize the outer layer, thereby weakening the whole magnet. Succeeding surges continue the process of demagnetizing fresh layers of steel until the demagnetized layer is so thick that the eddy currents induced in it are sufficient to prevent the further penetration of the transient flux. This is the same as saying that the demagnetization continues with succeeding surges until the intensity of the transient field minus the opposing intensity of the eddy current field equals the coercive intensity of the magnet material.

A complete copper-plated magnet assembly is shown on figure 8. This magnet is entirely interchangeable with unshielded magnets of similar design. Since the magnet material is the same as that used on unshielded magnets, the air gaps may be cleaned with magnetic cleaning tools without disturbing the calibration.

Summary

It has been shown that the problem of protecting permanent magnets from transient magnetic fields has been solved in a unique manner by the use of a new high-speed copper-plating process to apply a high conductivity copper shield over the conventional forged steel magnets.

Desirable features of this solution are complete interchangeability between unshielded and shielded magnets; forged steel material is used, which is fairly easy to obtain and on which unlimited operating experience is available.

References

1. DISCHARGE CURRENTS IN DISTRIBUTION ARRESTERS—II, K. B. McEachron and W. A. McMorris. AIEE TRANSACTIONS, volume 57, 1938 (June section), pages 307–12.

2. HIGH FREQUENCY MEASUREMENTS, August Hund. McGraw-Hill Book Co., 1933, page 47.

the simplification, if possible, of the present methods of the calculation of system short-circuit currents. Fortunately, due to the increased use of higher speed circuit breakers and faster relaying, this need can be satisfied for an extremely large proportion of the present applications. It is the purpose of this paper to discuss this general problem and recommend simplified methods of calculation for different types of apparatus.

The modern trend of practice is to speed up materially the clearing of short circuits in order to reduce damage and maintain continuity of service. But, in general, the more quickly a short circuit is cleared, the greater is the current to be interrupted. Thus it is quite easy, by speeding up the relay system, to jeopardize the breakers by requiring them to interrupt more current than that for which they were initially chosen. Also quite often there is a tendency to save investment in breakers by the use of slow speed relays to take advantage of the decrease in short-circuit current due to its decrement with time, but even with slow speed relays, there is no guarantee that a short circuit will not change its character and initiate a high current flow in a breaker while it is opening. And there is always the danger of shortening the time setting of slow speed relays, which will tend to increase the interrupting duty on the breaker. Provision of breakers with interrupting capacity adequate for fast relaying is thus at least as essential a part of planning for the future as the usual allowance for increase in generating capacity.

This being the case, the problem of interrupting device application resolves itself into a study of what happens in a system during the first few cycles of short circuit. Even with fast relaying, modern high speed breakers for 60-cycle systems may be expected to part their contacts in times no shorter than given below:

8-cycle breakers—4 cycles (0.0666 second)
5-cycle breakers—3 cycles (0.05 second)
3-cycle breakers—2 cycles (0.033 second)
2-cycle breakers—1 cycle (0.0166 second)

In the above intervals of time, the effects of change in relative phase position and change in excitation from voltage regulators are negligible and permit eliminating their influence in the calculation of short-circuit values. Values so calculated will also be on the conservative side where some types of breakers in the above classes part their contacts in slightly longer times.

Figure 1 gives, in the full-line curves, the magnitude of the three-phase short-circuit current expressed in terms of rated current for various reactances as obtained from the present decrement curves, for the four instants specified; namely, 4, 3, 2, and 1 cycles which are related to the 8, 5, 3, and 2-cycle breakers as discussed above. The rated current and reactances are based on the system kva. Curves are given for both the total rms current and the a-c component only, the difference between the two being due to the d-c component. The larger the d-c time constant, the more closely is the total rms curve approached and the smaller the d-c time constant, the more closely is the a-c component curve approached. The time constant of the d-c component for which these curves were made is 0.15 second or 9 cycles. The d-c time constant may be expressed generally as equal to $\frac{1}{2\pi f}\cdot\frac{X}{R}$ in seconds, or $\frac{1}{2\pi}\cdot\frac{X}{R}$ in cycles, where X is the 60-cycle reactance and R the d-c resistance to the point of fault. Although ratios vary considerably, representative values of the ratio X/R for the major circuit elements for large equipment are given below.

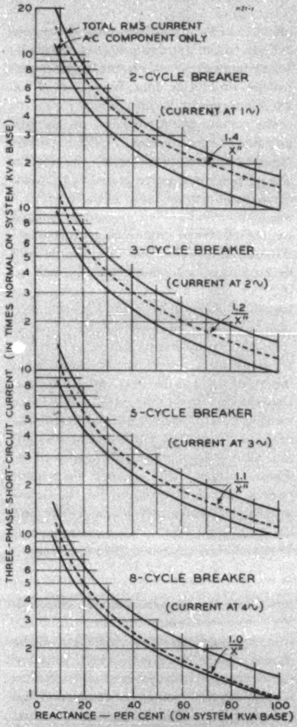

Figure 1. Comparison between decrement-curve calculations and calculations by the proposed method

		Corresponding Time Constant	
	X/R	In Seconds	In Cycles
Generators	80	0.21	13
Current-limiting reactors	50	0.13	8
Transformers	16	0.04	3
Transmission line	4	0.011	0.6
Cable	1.0	0.003	0.2

When any of these elements are connected in series relation the time constants of the combination become much smaller than that of a generator alone. Even a short length of transmission line has a marked effect upon the d-c component of current and a very appreciable effect upon the total short-circuit current. Table I shows the time constant of a particular combination for a fault at the generator, at the transformer terminals, and at two points out on an overhead line.

The Power Circuit Breaker Interrupting Problem

The reactances of figure 1 are based on the kva of the connected generator capacity. When the fault is near a generator the reactance is likely to be small and the d-c time constant large so that for small reactances the actual short-circuit current would approach the values given by the total current curves of figure 1. If, on the other hand, the fault is on the transmission line, the reactance is large and the time constant small. Thus, for large reactances the actual currents approach the values given by the a-c component only. The intermediate curves (dotted) of figure 1 for circuit breaker opening times of 4, 3, 2, and 1 cycles have been computed by the relation $1.0/X''$, $1.1/X''$, $1.2/X''$, and $1.4/X''$ respectively, in which X'' represents the system reactance to the points of fault, subtransient reactances being used for all generators. The factors in the numerators were arbitrarily chosen so that the curves lay between the curves for the total current and a-c component of current, and so the values were numbers that can be remembered easily.

In these curves the effect of the arc drop, which may be quite considerable in decreasing the d-c time constant, has been neglected. For a reactance of 30 per cent an arc drop of only 1.5 per cent in combination with one-half per cent resistance in the system is all that is required to reduce the d-c time constant from the nine cycles used in the present decrement curves to three cycles. On a 440-volt basis this is equivalent to but 6.6 volts, a value that is easily attainable. A reasonably conservative value of arc drop is about 20 volts per inch so that on a 13,800-volt circuit, 1.5 per cent drop requires only about six inches of arc length.

Having shown through the medium of the decrement curves that the relations for 4, 3, 2, and 1-cycle opening times apply quite well for simple systems, the justification has been established for applying the same relations to more complex systems. The use of these simple relations circumvents practically all the disadvantages of the decrement curves enumerated previously. With this method of calculating it is necessary only to obtain the equivalent reactance, expressed in any manner, regardless of the system, and apply the appropriate factor to the reciprocal of the reactance.

The foregoing simplified rules cover practically all applications. For the few remaining, it is sometimes desirable to know the short-circuit current for times longer than four cycles. For these special cases, the present decrement curves[2] or more elaborate methods may be utilized.[3,4,6,7] However, these methods are discouraged in favor of the rules proposed in this paper.

In most cases, faults occur at a point in the system for which a certain amount of other apparatus intervenes between the generators and the point of fault, in which case, the d-c component decays rapidly and the actual current approaches the a-c component. Even if the fault occurs on a generator bus, it is likely that other generators contribute to the short-circuit current through transformers, transmission lines, or cables. Therefore, the calculated values of current will be conservative for most cases.

In extreme cases where all the fault current is produced by large generators located at the point of fault, an accurate calculation with all assumptions on the safe side might show values as much as 20 per cent higher than the values given by this procedure. For the most conservative practice, therefore, when this condition is approached the results may be increased or other methods of calculation used.

In the curves of figure 1 adequate allowance has been made for non-synchronous load. For most central station applications, the additional contribution from synchronous motors will be negligible but where there is an appreciable contribution from them, they should be taken into account. In industrial applications, synchronous motor contribution may be of the same order of magnitude as that from generators and hence must be taken into account.

Synchronous motors are usually operated at about unity power factor; hence the voltage behind their subtransient and transient reactances is but little above terminal voltage. They tend to increase the power factor of the load; hence indirectly they reduce the internal voltage of the system generators. In general, they are much smaller units than generators; hence all of their time constants are faster and the decay of their short-circuit current is more rapid. For all of these reasons, it is justifiable to make somewhat less allowance for the short-circuit current of synchronous motors than for generators. Transient reactance may, therefore, be used in calculating their contribution.

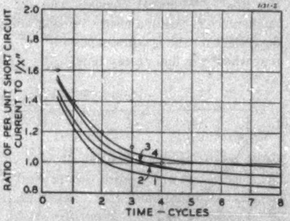

Figure 2. Three-phase short-circuit current from generators fully loaded at rated power factor

Arcing fault at one-fourth mile per kilovolt of transmission line

1. Steam-turbine generator, 0.8 power factor without transformer
2. Steam-turbine generator, 0.8 power factor with transformer
3. Hydroelectric generator, 0.9 power factor without transformer
4. Hydroelectric generator, 0.9 power factor with transformer

Circles show points proposed for calculating duty of equipment

The contribution of induction motors can always be safely neglected at times of two cycles or more.

The Momentary Problem

For many purposes, it is necessary to know the maximum possible rms current which can flow in the circuit. This is given by the current, including both a-c and d-c components, as calculated at one-half cycle. Allowing for overexcitation of generators on account of load, the rms value of current as calculated for zero time is about 1.8 times the value calculated by dividing leg voltage by subtransient reactance. As there will always be some decay even in the first half cycle, however, 1.6 times this value gives a safe figure.

In some instances, as when transmission line or cable reactance is an important factor in the limitation of current, the factor 1.6 may be higher than necessary.

In such applications, at 5,000 volts and below, the ratio of reactance to resistance is sufficiently low so that the d-c component decreases very greatly during the first half cycle. Thus within this voltage range, and except where the bulk of the power is supplied by generators at the point of fault, a standard factor of 1.4 may be used instead of 1.6. A standard factor is particularly desirable in consideration of the difficulties of obtaining actual resistance figures and the distribution of the d-c component among contributing short-circuit sources.[5]

Examples

All of the foregoing has been carefully checked by the calculation of specific examples. Figure 2 has been calculated for four representative simple cases as follows:

1. A representative turbine-generator with both transmission and fault at generator voltage.
2. A representative turbine-generator with a step-up transformer and both transmission and fault at 230 kv.
3. A representative hydro-electric generator with both transmission and fault at generator voltage.
4. A representative hydro-electric generator with a step-up transformer and both transmission and fault at 230 kv.

In each case the transmission line was assumed to have a length of one quarter mile per kilovolt and to have sufficient copper section to carry the entire generator output (either in one line or in several parallel lines) at a current density of 1 ampere per 1,000 circular mils. An arcing fault is assumed with an arc drop of 150 volts at generator voltage and of 2,000 volts at 230 kv.

The points corresponding to the coefficients 1, 1.1, 1.2, 1.4 and 1.6, at 4, 3, 2, 1, and $^1/_2$ cycles respectively are shown on figure 2, and it will be observed that they represent with reasonable accuracy the currents obtained at their respective times.

Figure 2 may be considered representative of the general case in which although some power may be generated near the point of fault, the major part of the short-circuit current comes from some distance away. Figure 3 has been calculated on a similar basis but without any transmission line between generator and fault point. Here it will be observed that the currents are somewhat higher, particularly for times from two to four cycles. It is for this reason that the recommendation is made that some additional allowance be made when the major part of the short-circuit current is obtained from large machines located at the point of fault.

Selection of Apparatus

It is suggested that the following procedure be followed in calculating short-circuit currents for use in applying various kinds of equipment.

CIRCUIT-BREAKER INTERRUPTING CAPACITY

While there is a wide variation in system conditions, the following calculation will usually give a conservative value of required interrupting capacity.

(1). Calculate the highest value of initial rms symmetrical current on the basis of no load, normal voltage, whether for a three-phase, double-line-to-ground, line-to-line, or single-line-to-ground fault, using subtransient reactances for all synchronous generators. Use transient reactance of synchronous motors when their short-circuit contribution must be considered.

(2). Multiply by the appropriate factor for 60-cycle systems from those given below:

8-cycle breakers	1.0
5-cycle breakers	1.1
3-cycle breakers	1.2
2-cycle breakers	1.4

(3). When the condition is approached of a short-circuit fed solely by large machines at the point of fault, these results may be increased up to 20%.

(4). Low voltage (600 volts or less) air circuit breakers are often instantaneous in operation and part contacts during the first half cycle. These breakers, however, are rated on the basis of average current in the three phases, and circuits on which they are installed rarely have X/R ratios exceeding ten. This corresponds to an average rms current during the first cycle equal to 1.23 times the symmetrical current. Breakers may therefore be applied on the basis of $1^{1}/_{4}$ times the three-phase initial symmetrical current using subtransient reactance and including both synchronous and induction motors.[5]

In order to co-ordinate this method of application of low voltage breakers with the conventional method used for other types of breakers, it is interesting to note that an average current for the three phases equal to $1^{1}/_{4}$ times the symmetrical current corresponds to a total rms current in the phase carrying the maximum unsymmetrical current equal to 1.4 times the initial symmetrical component.

In making the calculation for the equivalent system impedances, it should be remembered that at low voltages small impedance values become of importance and all elements of the circuit including current transformers, disconnects, switches, circuit breakers, bus runs, lead wires, et cetera should be taken into consideration.

MECHANICAL STRESSES AND MOMENTARY RATING OF CIRCUIT BREAKERS

The initial mechanical stresses due to the flow of electrical currents and the momentary rating of a circuit breaker are dependent on the maximum possible current which can flow at any time.

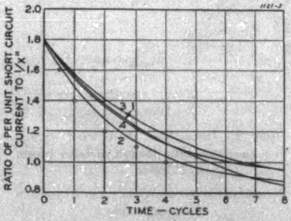

Figure 3. Three-phase short-circuit current from generators fully loaded at rated power factor

Arcing fault at generator or transformer terminal

1. Steam-turbine generator, 0.8 power factor without transformer
2. Steam-turbine generator, 0.8 power factor with transformer
3. Hydroelectric generator, 0.9 power factor without transformer
4. Hydroelectric generator, 0.9 power factor with transformer

Circles show points proposed for calculating duty of equipment

(1). Apart from cases covered by (2) below, the current to be used is 1.6 times the maximum value of rms symmetrical current using subtransient reactance values of both synchronous and induction machines.

(2). At 5,000 volts and below a factor of 1.4, instead of 1.6 should be used except on busses to which the major portion of total fault current is supplied either by directly connected synchronous machines or through reactors.

Directly connected synchronous machines are to be considered as those supplying fault current directly or through lines and transformers which reduce the contribution from those machines by less than 50%.

PROTECTOR TUBES

Inherent limitations of protector tubes require that a maximum and a minimum current rating be assigned to each tube for a given voltage. Because of the speed with which these devices become operative, it has been found possible to incorporate the variability of the d-c component into a rating which involves only the initial rms symmetrical a-c component of short-circuit current.[6]

Tubes should be applied so that the symmetrical short-circuit current at the point of installation of the tube, calculated on the basis of subtransient reactance for all combinations of connected synchronous machines and lines, should fall between the two limits assigned to the tube.

FUSES

Fuses are rated on total current and as the times involved are very short, their required interrupting capacity should be based on the total rms current at one-half cycle.

(1). In the general case, the current to be used is 1.6 times the maximum value of rms symmetrical current, using subtransient reactance for all machines, and including both synchronous and induction motor contributions.

(2). For fuses rated 15,000 V. or below with interrupting ratings not exceeding 3,000 amperes (except when applied on a generator bus) the resistance introduced by line and transformer is so great that the rms current of a maximum displaced wave in the first half cycle will not exceed 1.2 times the calculated symmetrical current, using subtransient reactance, and application on this basis may be considered safe.

PRELIMINARY RELAY SETTING

High speed current actuated relays can be given settings based on, first, the highest initial symmetrical current, using subtransient reactance at maximum system conditions, and second, on the lowest initial symmetrical current using subtransient reactance for minimum system conditions. Time over-current relays are usually set by use of similar values of symmetrical current using transient reactance values.

The settings of distance and pilot wire relays are assumed independent of the magnitude of short-circuit current but their accuracy may be impaired if the currents are outside of the range for which they were designed. This range is determined by the maximum initial symmetrical current for maximum system conditions and the minimum initial symmetrical current for minimum system conditions using sub-transient reactance values if the relays are high speed, or similar values of current calculated with transient reactance values if slower relays are used.

For bus differential relays it may be necessary to make an accurate calculation of both a-c and d-c components for assurance that operation will be satisfactory.

Conclusions

The precise determination of short-circuit currents involves a calculation so laborious as to be wholly impractical. Thus some approximation is required and a degree of judgment is required in the application of any method proposed.

The following rules are proposed for the various applications:

A. Circuit-Breaker Interrupting Capacity

Determine highest value of rms symmetrical current for any type of fault using subtransient reactance for synchronous generators and transient reactance for synchronous motors.

1. *General Case.* Multiply by the appropriate factor for 60-cycle systems from those given below:

8-cycle breakers	1.0
5-cycle breakers	1.1
3-cycle breakers	1.2
2-cycle breakers	1.4

2. *Short-Circuits Fed Predominantly by Large Machines at the Point of Fault.* Increase results of the general case up to 20%.

3. *Air Circuit Breakers Rated 600 Volts or Less.* Apply on basis of average rms current in all three phases, which may be taken as 1.25 times the rms symmetrical current.

B. Mechanical Stresses and Momentary Rating of Circuit Breakers

Determine highest value of rms symmetrical current for any type of fault using subtransient reactance for both synchronous and induction machines.

1. *General Case.* Multiply by 1.6.

2. *At 5,000 Volts and Below Unless Fault Current Is Fed Predominantly by Directly Connected Synchronous Machines or Through Reactors.* Multiply by 1.4.

C. Protector Tubes

1. *Maximum Rating.* Use the highest value of rms symmetrical current determined as in *B* with no multiplying factor.

2. *Minimum Rating.* Use the lowest value of rms symmetrical current determined as in *B* for minimum system conditions, with no multiplying factor.

D. Fuses

Determine highest value of rms symmetrical current as in *B*.

1. *General Case.* Multiply by 1.6.

2. *At 15,000 Volts or Below With Interrupting Ratings Not Exceeding 3,000 Amperes.* Multiply by 1.2.

E. Basis for Preliminary Relay Settings

1. *High Speed Current Actuated*

(*a*). Highest rms symmetrical current based on subtransient reactance at maximum system conditions.

(*b*). Lowest rms symmetrical current based on subtransient reactance for minimum system conditions.

2. *Time Overcurrent.* Make the same calculations as for high speed relays, but use transient reactance.

As compared with the latest decrement curves,[2] this method of calculation of short-circuit currents is somewhat simpler and should reduce considerably the possibilities of misapplication.

This method is in accordance with the present trend toward high speed relaying and eliminates the possibility of exceeding breaker interrupting capacities by modernizing the relay system after installation of the breakers.

It recognizes the acceleration of the decay of the d-c component of short-circuit current caused by the relatively high resistance of circuit elements other than the generator.

Table I. Effect of Location of Fault Upon D-C Time Constant

Location of Fault	Impedance to Point of Fault		$\frac{\text{Total X}}{\text{Total R}}$	Time Constant	
	X	R		In Cycles	In Secs
Generator terminals	20	0.25	$\frac{20}{0.25}$	13	0.21
High-voltage side of transformer	28	0.75	$\frac{28}{0.75}$	5.9	0.10
Inclusion of line having 5 per cent reactance	33	2.0	$\frac{33}{2.0}$	2.6	0.044
Inclusion of line having 10 per cent reactance	38	3.2	$\frac{38}{3.2}$	1.9	0.031

All impedance on same base as generator.

In view of these advantages it is believed that this method will prove more satisfactory in practice than the present standard.

References

1. Rating and Selection of Oil Circuit Breakers, E. M. Hewlett, J. W. Mahoney, and C. A. Burnham. AIEE Transactions, volume 37, 1918, pages 123–38.

2. Standard Decrement Curves, W. C. Hahn and C. F. Wagner. AIEE Transactions, volume 51, 1932, pages 353–61.

3. Calculation of Short Circuits on Power Systems, C. F. Wagner and S. H. Wright. Published in pamphlet form as AIEE paper 32M4, and abstracted in Electrical Engineering, volume 51, 1932, number 2, page 131.

4. Decrement Curves for Specific Systems, W. C. Hahn. Published in pamphlet form as AIEE paper 32M5, and abstracted in Electrical Engineering, volume 51, 1932, number 2, page 131.

5. Short-Circuit Calculating Procedure for Low Voltage A-C Systems, A. G. Darling. AIEE Transactions, volume 60, 1941.

6. Protector Tubes for Power Systems, H. A. Peterson, W. J. Rudge, A. C. Monteith, and L. R. Ludwig. AIEE Transactions, volume 59, 1940 (May section), pages 282–91.

7. Symmetrical Components, Chapter V, by C. F. Wagner and R. D. Evans. McGraw-Hill Company, 1933.

Power-Circuit-Breaker Ratings

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CIRCUIT breaker application is being simplified. A subcommittee of the protective devices committee of the AIEE has studied methods of calculating short circuit currents and has prepared a report which presents a simplified procedure based on standardized contact parting times and typical current decrements.[1] To facilitate the use of this method, circuit breakers should have ratings which conform to the short circuit characteristics assumed in the calculations. At the present time, while the industry is preparing a revised list of short-time current ratings for circuit breakers, an opportunity exists to correlate the short circuit ratings so that a breaker selected for adequate interrupting current by the new method automatically has ratings adequate for the other short circuit currents.

The rating of a power circuit breaker as given by AIEE Standard 19-200 includes the following items:

(*a*). Rated voltage.
(*b*). Rated frequency.
(*c*). Rated continuous current.
(*d*). Rated short-time current.
(*e*). Rated making current.
(*f*). Rated interrupting current.

In addition definitions are given for the rated latching current and two short-time currents (the rated momentary current and the rated five second current).

The first three of these items are independent of each other. Breakers can be designed for any combination of rated voltage, rated frequency, and rated continuous current.

The rated making, momentary, five second, and interrupting currents are interdependent, being values of the maximum short circuit current permissible for the breaker.

The maximum short circuit current is determined by the impedance of the circuit and by the characteristics of the rotating machines supplying power to the short circuit. The initial current is composed of an a-c component determined by the subtransient reactance of the circuit, and a d-c component determined by the point on the wave at which the transient starts and by the circuit inductance and resistance. Both components are a maximum at the beginning and consequently the total rms current also is a maximum, figure 1.

This current during the first cycle is important as it is the maximum current to which the breaker will be subjected and corresponds to the rated momentary current and rated making current. In AIEE standards, "The rated momentary current is the maximum rms total current which the breaker shall be required to carry for any time however small up to one second.—" This current may be initiated either by a short circuiting of the system through some external means or by the breaker closing and completing a short circuit. Consequently, the first cycle current is also the current which the breaker must be capable of closing. By AIEE definition, "The rated making current of an oil circuit breaker is the maximum rms current including the d-c component against which the breaker must be capable of closing, without the welding of or undue damage to the breaker or contacts, with rated control voltage at the closing mechanism." Therefore, the rated momentary current and the rated making current should be at least equal to the maximum current which can be obtained during the first cycle.

Not all breakers are required to close completely and latch when making the rated current. For breakers having relatively low short circuit currents, the electromagnetic and other forces tending to oppose the closing of the circuit breaker may not impose much additional load on the closing mechanism and no difficulty may be encountered in the complete closing and latching of the breaker. Such breakers would have rated latching currents equal to rated making currents.

High interrupting capacity breakers for 15-kv service and below, have to make currents which may impose electromagnetic forces greatly in excess of the other forces opposing the closing, and may require a driving force considerably in excess of that needed for normal service. In this class of breakers, the closing and latching under short circuit conditions is a much more serious problem. The most advantageous position for carrying the current is with the contacts normally closed and the breaker latched, but with high speed tripping of the breaker it may be possible, without completely closing the breaker, to carry the current for the duration of the short circuit. However, the breaker contacts must be closed sufficiently to carry the short circuit current without undue burning or welding until such time as the short circuit is cleared elsewhere or the breaker tripped. In such cases the breaker will perform satisfactorily without latching and a rated latching current equal to the rated making current is not essential. Consequently, the rated latching current has no definite relation to the maximum short circuit current or to the rated interrupting current.

The sustained short circuit current must be carried by a breaker if it is not tripped. From its initial value, the current will decrease with the decreasing d-c and a-c components. The d-c component decreases approximately as a logarithmic function. The amount that the a-c component decreases will depend upon the effect of the short circuit on the generators. This effect is small if the generators do not form the principal limiting impedance and the a-c component may be sustained throughout the duration of the short circuit. Consequently, the maximum current which the breaker will be expected to carry for more than a few cycles while the d-c component is dying out, is the initial value of the a-c component. This value of the current corresponds to the 5 second rating of the breaker. By AIEE definition, "The rated 5 second current is the rms total current including the d-c component which the breaker shall be required to carry for 5 seconds." With increasing speed of breaker tripping this item is losing its significance.

Between the initial and the sustained currents is a current which the breaker must be able to interrupt. It is of great importance as it corresponds to the rated interrupting current which, by AIEE definition, "is the highest rms current at a specified operating voltage which the breaker shall be required to interrupt under the operating duty specified, and with a normal frequency recovery voltage equal to the specified operating voltage. The current shall be the rms value including the d-c component at the instant of contact separation as determined from the envelope of the current wave." The time at which the contacts separate and at which this current is measured depends upon the speed with which the breaker is tripped. It may be in the first cycle in the case of extremely high speed breakers, but is

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1. For all numbered references, see list at end of paper.

generally somewhat later in the short circuit. The interrupting current is between the initial and the sustained values of the current.

The basic rating of a circuit breaker is its interrupting capacity at rated voltage. The interrupting capacity determines the strength of the parts which must carry the short circuit and withstand the forces incident to the interruption of the current. The voltage determines the clearances required for insulation. The normal rated current is a less important factor in determining the size of the breaker, although it does determine to a large extent the cross section of the conducting path, the size of the conductors through the bushings, and the size of the contacts within the breaker. Therefore, the rated interrupting current at rated voltage is the basic rating, and establishes the principal dimensions of a group or class of breakers. Various modifications of this basic design are made to take care of variations in the normal current carrying capacity. For example—a breaker may be developed and tested for an interrupting ability of 500,000 kva at 15 kv and minor modifications will give the various rated normal currents.

Circuit breakers are frequently used at service voltages less than their rated voltage. At these lower voltages, the difficulty of interrupting a given current is less and higher rated interrupting currents can be assigned. Constant kva, or a rated interrupting current varying inversely with service voltage, is customarily assumed. The wider the range of service voltages at which constant kva is maintained, the greater the range of currents which must be interrupted by the breaker. For best designs which have as little compromise as possible, it is desirable to limit the interrupting current, and a limit is established which permits the use of the breaker at full kva down at least to the next lower standard voltage. This maximum permissible current is called the interrupting capacity current limitation and is about 1.5 times the rated interrupting current at rated voltage, but varies, specially for 15-kv breakers which cover a wider voltage range. This rating is one value of the highest short circuit current which the breaker can be required to pass.

The interdependent items of the circuit breaker rating, making current, momentary current, interrupting current, and five second current correspond to currents measured at various points on the short circuit transient current. If definite ratios exist among these values, the ratings of the circuit breaker should correspond to these ratios so that no one of the items becomes the limiting factor and so that if a breaker is selected for adequate interrupting capacity, it will automatically be able to make and to carry the currents associated with that interrupting capacity.

These requirements are related, but in the past variable times allowed for the tripping of the circuit breaker resulted in various durations of the length of the short circuit prior to the parting of the contacts and consequently a variation in the ratio between the initial current and the interrupting current. Consequently, it has not been possible to use definite ratios and any one of the ratings has been able to prevent the application of the breaker on a particular service.

Table 1. Favorable Relations of Interdependent Circuit-Breaker Ratings

Rating Item	Current		
	Measured After Start of Short Circuit at Time Interval of (Cycles)	Ratio of Rating Item to Interrupting Capacity Current Limitation	Ratio of Rating Item to Rated Interrupting Current (Approximate)*
8 Cycle breakers			
Rated making current	0.5	1.6	2.4
Rated momentary current	0.5	1.6	2.4
Rated interrupting current	4	0.67	1
Interrupting capacity current limitation	4	1	1.5
Rated five second current	60	1	1.5
5 Cycle breakers			
Rated making current	0.5	1.45	2.2
Rated momentary current	0.5	1.45	2.2
Rated interrupting current	3	0.67	1
Interrupting capacity current limitation	3	1	1.5
Rated five second current	60	0.9	1.35
3 Cycle breakers			
Rated making current	0.5	1.33	2.0
Rated momentary current	0.5	1.33	2.0
Rated interrupting current	2	0.67	1
Interrupting capacity current limitation	2	1	1.5
Rated five second current	60	0.83	1.25
2 Cycle breakers			
Rated making current	0.5	1.14	1.7
Rated momentary current	0.5	1.14	1.7
Rated interrupting current	1	0.67	1
Interrupting capacity current limitation	1	1	1.5
Rated five second current	60	0.72	1.08

* The ratio beween the rated interrupting current and the interrupting capacity current imitation is a variable depending on the range of service voltages at which full interrupting kva is available. The table uses the ratio 1.5 which is the most common value, but it must not be assumed that this ratio applies to all breakers. Individual cases have conditions which justify using either arger or smaller ratios.

In the report referred to previously on the new method of calculating the current, the variable of the time of parting contacts has been eliminated by the assumption of definite contact parting times dependent only on breaker speed; after 1, 2, 3, and 4 cycles for 2, 3, 5, and 8-cycle breakers respectively. Likewise, the probable ratio between the initial a-c component and the total rms current during the first cycle and at the end of each of these cycles has been established. This new method assumes that the maximum asymmetry which can occur will make the total rms value of the current, at the time of the maximum peak, 1.6 times the a-c component, and that at the end of the first and succeeding cycles, this multiplying factor becomes 1.4, 1.2, 1.1, and 1.0. Consequently, the ratio of the current during the first cycle to the interrupting current for breakers parting contacts after 1, 2, 3, and 4 cycles are (1.6/1.4=)1.14, 1.33, 1.45, and 1.6, respectively. These factors are, therefore, the ratio of the initial current to the interrupted current of 2, 3, 5, and 8-cycle breakers respectively. They represent the minimum ratios between rated momentary current and rated interrupting current which permit utilization of the full rated interrupting current.

The values assigned to the interdependent items of a circuit breaker rating should be based on the interrupting capacity as this is the most important item. The interrupting capacity current limitation is determined from it by the range of voltages at which the breaker is to be used. This value is one measurement of the heaviest short circuit current which will be passed through the breaker. From it, the rated making, momentary, and five second currents can be obtained by use of the suitable factors. Table I shows the relations between these interdependent items as derived from the data in the report on the calculation of the interrupting current.

When low voltage breakers are used at service voltages, much below rated voltages, the making current may be high and require a very powerful closing mechanism. When used near rated voltage, the breakers may require much less energy for closing the corresponding making current. Such breakers could be adjusted to the requirements of the service by limiting the power input to the closing mechanism to that required for a making current corresponding to the rated interrupting current at service voltage. The saving in energy and wear on the breaker may justify this change which amounts of imposing a special rated making current and a special interrupting capacity current limitation which are determined by the reduced closing ability.

With ratings assigned in accordance with short circuit characteristics, circuit

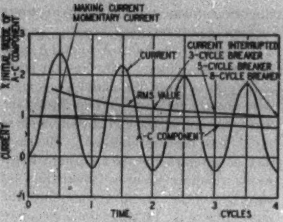

Figure 1. Short-circuit current

breaker application is simplified. Providing the current is calculated by the new method, the breaker can be selected on the basis of its rated interrupting capacity at the system voltage and the ratings of making current, momentary current, and five second current will automatically be adequate.

Since the rated making and momentary currents given in table I correspond to the interrupting capacity current limitations, they usually are higher than are required when a breaker is applied near rated voltage. When interrupting currents, calculated on the assumption of fast tripping, slightly exceed breaker ratings, a more detailed calculation using decrement curves might reveal that the making capacity is adequate and that the interrupting current is down to the breaker rating if a time delay of a few cycles is used.

If the advantages of the new method of short circuit calculation are demonstrated by experience and the method becomes used generally by the industry, another interesting possibility for the further simplification of breaker application should be considered. It gives exactly the same results as those obtained when breakers having ratings assigned in accordance with the assumed current transient are applied by the new method of calculation, but it eliminates from the calculation of the currents the factors dependent on breaker opening time and on special decrements for low voltage circuits.

Breakers could be applied on the basis of the initial a-c component of the short circuit current. The short circuit current calculations could be only the dividing of the voltage by the subtransient reactance to obtain the initial a-c component. The breaker ratings could consist of:

(*a*). Rated voltage.

(*b*). Rated frequency.

(*c*). Rated continuous current.

(*d*). Rated initial a-c component of short circuit current at rated voltage.

(*e*). Rated maximum initial a-c component of short circuit current at reduced voltage.

(*f*). Rated interrupting time.

(*g*). Rated latching current.

Breaker application could consist of selecting a breaker having these items at least as large as the system requirements. Circuit clearing time would depend on the speed of the breaker selected and the current calculation would be the same for 2-, 3-, 5-, and 8-cycle breakers.

Standardization rules could require of the manufacturer that his breakers be capable of closing carrying and interrupting currents having the rated a-c component at rated voltage and the a-c component current limitation at a corresponding lower voltage. Under these suggested rules, breakers would be capable of making a circuit having a total rms current equal to 1.6 times this maximum a-c component. They would be capable of carrying this momentary current. They would be capable of interrupting a total rms current equal to that specified for their contact parting time. They would be capable of carrying the a-c component for five seconds. Demonstrating or proving the performance of the circuit breakers would require these currents to have total rms values bearing the correct relations to the rated a-c component. On classes of breakers where special decrements are justified, for example 5 kv and below, the design of the breaker could be based on the special decrement characteristics and no change would be required in the current calculations. In other words, the requirements and results could be the same whether only the a-c component is specified and the testing requirements define the factors existing between the various currents, or whether as at present the various current values are calculated and checked for each application. Consequently, if the breakers were given rated initial a-c components of short circuit current at rated voltage and rated maximum a-c components at reduced voltage, the making, momentary, interrupting, and five second ratings could be eliminated.

In brief, the requirements would be determined by the initial a-c component of the short circuit current and a breaker selected which would have an adequate rated a-c component and which would be designed for making, carrying, and interrupting the corresponding total rms currents.

Conclusions

The ratings of a circuit breaker can be so correlated that full use can be made of the abilities of the breaker. With the new method of calculating the required interrupting capacity, and with breaker ratings assigned in accordance with the same current decrements, breaker application will be greatly simplified.

If, after trial, the new method of current calculation supplants the present method, a still further simplification in application can be obtained by rating a breaker on the initial value of the a-c component of the short circuit current which it can close, carry, and interrupt.

Reference

1. SYSTEM SHORT-CIRCUIT CURRENTS, W. M. Hanna, H. A. Travers, C. F. Wagner, C. A. Woodrow, and W. F. Skeats. AIEE TRANSACTIONS, volume 60, 1941 (September section) pages 877–81.

Lightning to the Empire State Building

K. B. McEACHRON
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Synopsis: Lightning to the Empire State Building in New York City has been under investigation since 1935. Boys camera photographs of 62 strokes have been taken and 99 oscillograms. Twenty of these were taken simultaneously with photographs of the same stroke. Upward step leaders were discovered and continuing strokes were shown to be in the nature of a direct current arc. A brief résumé of the more important results up to the end of the 1937 season is given.

The paper is chiefly concerned with the oscillographic results of the 1938 and 1940 seasons. The low speed oscillograph in the Empire State Building recorded 17 of the 20 strokes to the building while the high speed oscillograph recorded 41 current peaks in 13 strokes.

1. Of 49 strokes recorded oscillographically during the years 1937–40, no stroke has been entirely positive although 41, or 84 per cent, were entirely negative. The first recorded current peak in stroke 13 was positive and had a crest current of 58,000 amperes.
2. Fifty per cent of the 49 strokes had charge of 25 coulombs or more, the maximum charge measured being 164 coulombs. These values are for the total stroke and not for current peaks.
3. Ninety-one per cent of all strokes recorded to the Empire State Building are classified as continuing, i.e. had a low current component.
4. Fifty per cent of the current peaks were found to have a crest current of 7,000 amp or more, while only 25 per cent of the 33 peaks measured in 11 strokes had a duration exceeding 40 microseconds.
5. The plotted data show that 50 per cent of the current peaks had a charge of 0.13 coulomb or more to half current value, although 5 per cent showed more than 1.5 coulombs.
6. A time of at least 1 microsecond is required to reach crest in 50 per cent of the current peaks, while 65 per cent reach the first crest in a time not greater than 1.5 microseconds—the time to crest of the present standard.
7. The results obtained show that rates of rise of the order of 10 to 20 kiloamperes per microsecond will be encountered quite frequently, and 30 to 40 kiloamperes per microsecond occasionally.
8. Errors as great as ±50 per cent may occur in connection with the highest rate of rise measurements; errors in connection with some of the other measurements may be as great as ±25 per cent, although most of the results will be more accurate than indicated by these figures.
9. It is pointed out that direct stroke data may be applied to transmission and distribution circuits if proper allowances are made for division of currents in the various paths including the effect of grounded neutral transformers. The current to be handled by lightning arresters will in general be much smaller than those measured in the direct stroke.

IN February 1939, the author published in the *Journal* of the Franklin Institute a paper[1] entitled "Lightning to the Empire State Building," and in May 1940 with K. G. Patrick, a book[2] titled "Playing With Lightning." These describe the Empire State investigation in some detail, while the instruments used have been described by Hagenguth,[3] Kettler,[4] and Flowers.[5] Therefore, only a brief mention will be made of the apparatus used, and a summary given of the important results obtained.

Equipment

At the Empire State Building are located two oscillographs—one arranged to record continuing currents up to one second duration, and the other to record the wave shape of individual current peaks. These have been called the low and high speed oscillographs respectively. At present both are of the hot cathode-ray type, although for several years the low speed oscillograph was of the electromagnetic type using a crater lamp for illumination. A schematic diagram of the operation of the high speed oscillograph is shown in figure 1. This equipment successfully[6] recorded in 1938 a multiple stroke consisting of (12) successive current peaks in 0.28 sec. Both of the oscillographs are connected to a non-linear shunt composed of thyrite and a linear shunt in parallel. With these shunts the low speed oscillograph records over a current range of 50 amps. to 24,000 amps., while the high speed range is 1,000 amps. to 200,000 amps.

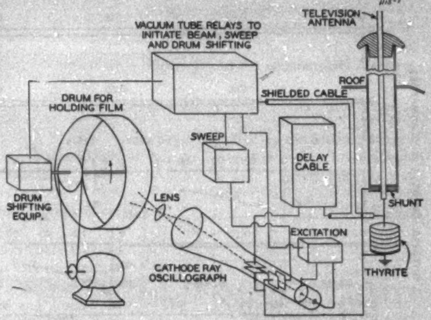

Figure 1. Schematic diagram of high-speed cathode-ray oscillograph equipment used for lightning investigation in the Empire State Building, New York City

Camera equipment is located at 500 Fifth Ave. a distance of 2,550 ft. to the top of the Empire State Building. Slow speed ($^1/_2$ rps) and (2 rps) Boys cameras are installed and also a high speed Boys camera normally having a film surface velocity of 124 ft. per sec. In addition, a moving film camera called a "multiple aperture camera" is used having four matched lenses so arranged that four images appear on the same moving photographic film along a horizontal axis. The lenses are provided with suitable filters and stop openings, so that a range of about one million to one may be obtained in exposure.

In addition to this equipment, another camera referred to as the "oblique camera" was installed on a building at the corner of 34th St. and 8th Ave. The purpose of this camera is to determine the shape of the stroke in a plane at right angles to that of the cameras at 500 Fifth Ave. so that a more accurate determination of the length of the stroke path might be made. This camera, controlled over a telephone circuit by the operator at 500 Fifth Ave., is not a Boys camera; its only purpose being to get the stroke shape, and no attempt is made to record detailed time effects.

A densitometer has been provided in the laboratory at Pittsfield for the study of changes in density found on the photographic records. All of the films used since the study began in 1935 have been provided with a sensitometer strip for calibration of the film against a known light source.

Means were set up in 1940 for the determination of visibility between 500 Fifth Ave. and the Empire State Building. This instrument was provided to make possible a fair determination of current from the Boys camera photographs, and studies are in progress to determine the change in current as one progresses up the

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The author wishes to express his appreciation to the high voltage engineering laboratory staff at Pittsfield, particularly to J. H. Hagenguth, J. W. Flowers, John Russ, and Paul Edwards in connection with designing and operating the equipment, and to Harry Mason for assistance in the preparation of the material used.

[1] For all numbered references, see list at end of paper.

stroke—thus obtaining some data on the space charge around the stroke channel.

Brief Résumé of Results Previously Reported[1,2,6]

Some of the more important results obtained in this study, particularly with the camera equipment, are listed briefly below. Only oscillographic data are discussed in the present paper.

1. Discovery of the initial upward step leader mechanism from the Empire State Building.

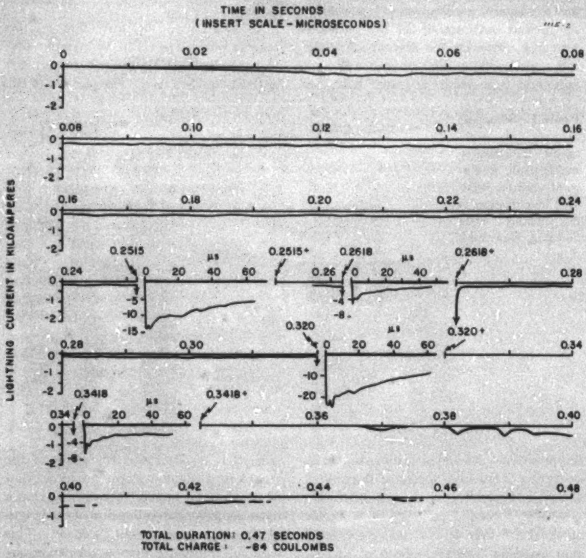

Figure 2. Composite replotted from low-speed and high-speed oscillograms showing our current peaks of stroke 3 to the Empire State Building

June 26, 1940 at 1:29.5 p.m.

2. Proof, both photographic and oscillographic of the existence of continuing current (substantially a direct current arc between line and ground).
3. Direct measure of charge in coulombs in lightning strokes, establishing a curve of frequency of occurrence for a given magnitude.
4. Oscillographic records show that out of 27 records all began with negative polarity, and 3 reversed polarity at end of the record, indicating that the base of the cloud is negative, and a positive region was reached in 3 cases only after a time interval.
5. Lack of so-called return stroke from clouds with upward initial leader mechanism, explained by lack of mobility of charges in clouds compared to the earth, which does produce a return stroke with downward initial leaders.
6. Strokes to ground, whether started by upward or downward initial leader, seem to have same mechanism for successive current peaks without reference to how initiated.
7. The configuration of electrodes probably determines the direction of propagation of initial leader.
8. Thunderless lightning results from relatively slow rates of current rise of upward leader strokes, without successive current peaks.
9. Measurement of leader and stroke propagation velocities and approximate determination of current required by upward step leader.
10. Identification of the upward streamer from earth when the downward initial leader approaches the earth.
11. Determination of wave shape of direct strokes—front, crest, and duration of current peaks of strokes having single peaks and as many as 12 successive current peaks (total of 4 strokes recorded).
12. Strokes to the ordinary countryside, in those cases where the photography was such that good measurements could be made, show the same general photographic characteristics as do those to the Empire State Building, once the stroke is established.

Additional Results

Considerable additional data obtained in 1940 are now available with respect to the wave shape of current peaks. Although many correlating oscillograms and photographs were obtained in previous years, during 1940 most of the strokes were not recorded photographically because they did not occur at night. It is believed, however, based on past experience that most of the strokes recorded were initiated by upward step leaders.

During the 1940 season the low speed oscillograph recorded 17 of the 20 strokes to the building, while the high speed oscillograph recorded 41 current peaks in 13 strokes. The correlation between the low and high speed oscillograms was excellent, there being a detailed record of nearly every current peak of any importance appearing on the low speed record. Such a record for stroke 3 (1940) is shown in figure 2, where the high speed and low speed records have been reproduced together.

Polarity

Of the 49 strokes recorded oscillographically during the years 1937–40, no stroke has been entirely positive although 41, or 84 per cent, were entirely negative. Of the eight strokes having some positive, one is known to have begun with a small positive current (about 200 amps), while four strokes began with negative current. With two other strokes, the current recorded initially appeared to be negative, but had too small a magnitude to be certain. As for the remaining stroke, the first current recorded was positive, although the oscillograph had initiated more than 0.025 sec. before indicating the presence of a current, whose magnitude and polarity is, however, unknown. This stroke (No. 13, 1940) is of considerable interest, as the first recorded current was a positive peak having a crest of 58,000 amps., which is nearly double the highest heretofore recorded oscillographically[6] to the building. This was followed by three negative current peaks, as shown in figure 3. The current peaks appear to be connected together by continuing current, mostly negative.

Stroke Charge

The stroke charge in the entire stroke including current peaks for 49 strokes is given in figure 4, and indicates that at least 50 per cent of strokes have a charge of 25 coulombs or more, although the average charge is 37 coulombs. The maximum measured in New York is 164 coulombs obtained in 1937, while in 1940 the maximum was 150. Three per cent of the total charge measured to date was positive.

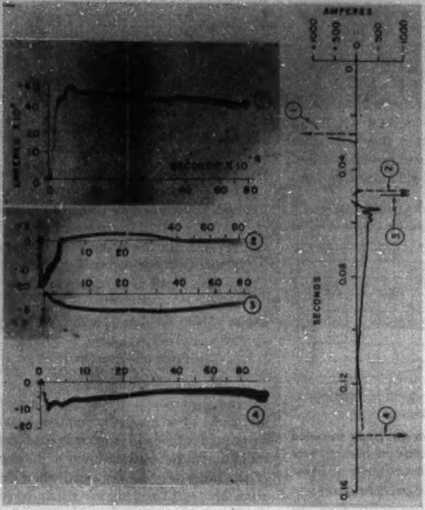

Figure 3. Low-speed and high-speed oscillograms showing four current peaks of stroke 13 to the Empire State Building

June 30, 1940 at 4:34 p.m. Time between peaks 0.021, 0.00092, and 0.087 second

Continuing Strokes

Of the 49 strokes recorded oscillographically, only seven did not show any signs of continuing current, and these due to lack of sensitivity might have recorded current under more favorable recording conditions. Of these oscillographic records 20 correlating Boys camera records were obtained—all but one showing the presence of continuing current. In addition to these photographic records 42 others were obtained—41 of which show the presence of continuing current. Thus 91 per cent of all strokes recorded in New York to the Empire State Building are classified as continuing.

It is of interest to record here that the results obtained thus far with the multiple aperture camera indicate that once a path is found between cloud and ground the current does not cease until the stroke is completed. The current merely increases and decreases in magnitude until the total stroke is completed.

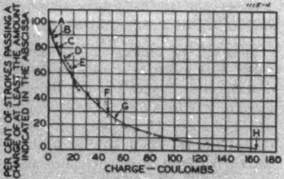

Figure 4. Frequency of occurrence of stroke charge based largely on low-speed oscillographic data

Total charge of lightning strokes recorded by low-speed oscillographs

Empire State Building, New York City
1937, 1938, 1939, 1940

Based on 49 strokes

Coulombs in two-polarity strokes

	A	B	C	D	E	F	G	H
−	1.4	2.0	5.4	12	8.2	30	36	160
+	1.2	1.1	1.8	2.3	9.9	17	17	2.6

Other strokes negative charge only

Stroke Duration

The maximum stroke duration of 1.5 sec. reported in 1937 for a stroke to the building has not as yet been exceeded. Figure 5 shows that half of the 49 strokes recorded have a duration of at least 0.29 sec.

Current Peaks—Crest Current

The data given in figure 6 are plotted in terms of negative current peaks, since all the peaks recorded by the high speed oscillograph were negative but one, which is indicated. When comparing these data with those of other investigators, it is to be remembered that the magnetic link used in measuring currents through transmission towers[7] and distribution lightning arresters[8] indicate only the highest current peak occurring during the time the link is in service. In figure 6 each current peak in a stroke is plotted independently. It is not strange that Lewis and Foust give current values considerably higher than indicated in figure 6. The data given by McEachron and McMorris for distribution circuits are much lower in current magnitude, since the current was measured only at arrester locations, and in most cases is a measure of the traveling wave from some remote point. In many cases, too, the measured surge was the result of the release of a bound charge. The records do indicate arrester duty, but seldom the current magnitude in the stroke itself.

Duration

In terms of time to half value only 25 per cent of the 33 peaks measured in 11 strokes have a duration exceeding 40 microseconds, which is the present standard duration used in impulse testing in the United States. The maximum duration (figure 7) was 115 microseconds.

Charge

Since the tail of the current peak merges into the continuing current it is difficult to determine what is the current peak and what is continuing current. Therefore, when determining the charge in each current peak the area under the current-time oscillogram has been limited to the point where the current has decayed to half value. Figure 8 shows that 50 per cent of the current peaks measured have a charge of 0.13 coulomb or more, although only 5 per cent showed more than 1.5 coulombs.

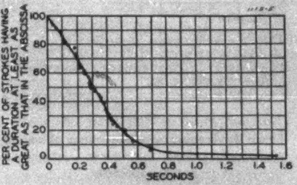

Figure 5. Stroke duration as a function of the frequency of occurrence

Values obtained from oscillographic records

Duration of multiple and continuing lightning strokes recorded by low-speed oscillographs

Empire State Building, New York City
1937, 1938, 1939, 1940

Based on 49 strokes

Since the cloud is not an infinite bus but has limited energy, one might expect that as the current increased the duration might decrease, but figure 9 indicates that the charge is increasing more rapidly than the current, which means of course that the duration is also increasing with the current. If the rate of propagation of the so-called return stroke is independent of the current in the stroke, and if the duration of the tail of the current peak is the time taken to reach the cloud, it would appear then that in general the longer stroke path will contain the greater current. There is yet no real support for either of these assumptions, nor is there any direct measure of the length of the stroke path compared to its current. Furthermore, large and small current peaks are recorded in the same stroke, apparently indicating the presence of other factors beside the length of stroke which may influence the amount of current. While ground resistance may have considerable influence on the amount of current, yet it must be remembered that all of the current peaks recorded in figures 6 or 9, occurred to a structure whose ground resistance was presumably quite constant—certainly it did not change during a stroke.

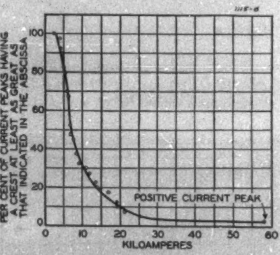

Figure 6. Frequency of occurrence of current peaks of various magnitudes determined oscillographically

Amplitude of current peaks recorded by high-speed cathode-ray oscillograph

Empire State Building, New York City

Based on 13 strokes

Wave Front

The time to the first crest is given in figure 10. These results show that a time of at least one microsecond is required to reach crest in 50 per cent of the peaks. Sixty-five per cent of the current peaks reach the first crest in a time of not more than 1.5 microseconds, the present standard time. It was of interest to plot the time to first crest against the crest value of current, but no trend could be found, except that higher currents usually required longer times to reach crest, but longer times did not necessarily mean a higher current peak.

Rate of Rise

The effective rate of rise in kiloamperes per microsecond, measured through the 10 and 90 per cent points is shown in figure 11. In considering these results it is well to bear in mind the difficulties encountered in recording and measuring accurately when the front becomes almost perpendicular to the time axis. It is clear that as the rate of rise increases the possibility of error increases, and therefore the highest values in figure 11 are to be taken with caution, since an error of the order of ±50 per cent may be present.

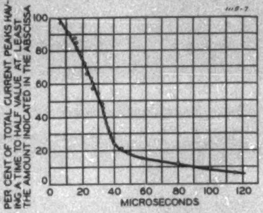

Figure 7. Duration of current peaks measured to half value as a function of frequency of occurrence

Based on 33 oscillographic records

Duration of current peaks recorded by high-speed cathode-ray oscillograph

Empire State Building, New York City 1938 and 1939

Based on 11 strokes

These results do indicate that rates of current rise of the order of 10 to 20 kiloamperes per microsecond will be encountered quite frequently in the stroke, and values as high as 30 to 40 kiloamperes occasionally. These values are to be compared with those of Norinder,[9] which are of the same general order of magnitude although obtained in Sweden by quite a different method.

Accuracy of Measurement

Measurement of the crest of the average current peak (9,000 amp.) is subject to an error of ±20 per cent. The major factors contributing to this error are difficulty in establishing a zero line, uncertainties in the measurement of deflections and in the value used for the oscillograph calibration including the lightning shunt. For smaller deflections, the error tends to be greater and may be less for the larger deflections.

Rate of rise measurements are subject to error in measurement of both current and time. For a crest current of 9,000 amps. and a time of one microsecond, error in the rate of rise measurement may be of the order of ±25 per cent, while for steep fronts (0.1 ms) the error may be as great as ±50 per cent.

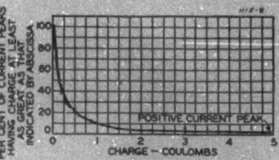

Figure 8. Charge of current peaks as a function of frequency of occurrence

Coulomb values based on time to half value of crest current

Charge in current peaks recorded by high-speed cathode-ray oscillograph

Charge passed until current decays to one-half of crest

Empire State Building, New York City 1938 and 1940

Based on 13 strokes

Measurements of charge are also subject to a possible error of ±25 per cent.

The low speed oscillograms are also subject to a possible ±20 per cent error in current magnitude; while currents smaller than 50 amps. are detectable, the accuracy is very poor, since the trace is often about 150 amps. in width.

It is believed, however, with the exception of the measurements depending upon fractional parts of microseconds, that the average accuracy is good for this type of measurement.

Discussion

Most of the oscillograms of current peaks show evidence of some oscillation near the crest (figures 2 and 3), having a magnitude of from 5 to 20 per cent and a period of about 2.5 microseconds. Since the building is 1,275 ft. high, perhaps this effect is the result of reflection phenomena at or near the base of the building. Of course the size of the building increases near the earth, which confuses the picture somewhat.

In applying these results to the case of lightning striking a transmission or distribution circuit, several assumptions must be made. It seems reasonable to assume that the sum of the currents flowing away from the point of contact with the lightning stroke will equal the current of the stroke, in view of what is now known of the mechanism of the stroke. Thus, if a line is struck at any point except close to

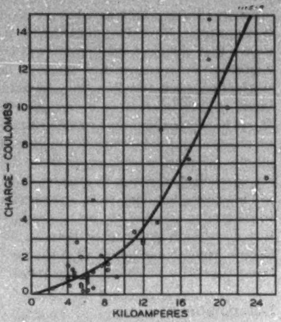

Figure 9. Effect of variation in crest current on the charge contained in each current peak

Relation between charge and crest current of negative current peaks

Charge passed until current decays to one-half of crest

Empire State Building, New York City 1938 and 1940

Based on 13 strokes

Charge of only positive peak recorded was 4.9 coulombs with crest of 58,000 amperes

its end, the current in the stroke will divide equally and will have half of the magnitude of the original measured stroke current. The time to crest and half value will be the same as measured in the stroke itself, thus the rate of rise in terms of amperes per microsecond would be reduced to half that found in this investigation of strokes to a well grounded steel structure.

Current waves traveling away from the point struck are associated with a traveling wave of potential equal to the surge impedance multiplied by the current. Thus stroke 13, figure 3, having a crest current of 58,000 amps., the highest yet measured oscillographically, would have a traveling wave current of 29,000 amps. This corresponds to a traveling wave potential of 14,500,000 volts (using a surge impedance of 500 Ω) which would flashover any insulation now in use, either wood or porcelain. If, however, the stroke made contact with an overhead ground wire at a tower having a ground resistance of 10 ohms, the potential at the tower top would be only 580,000 volts, neglecting the current flowing away from the stricken point which would be only a little over 2,000 amps. until reflections from adjacent towers returned to increase the current.

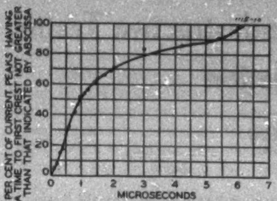

Figure 10. Time to first crest of current peaks as a function of frequency of occurrence

Recorded by high-speed cathode-ray oscillograph

Empire State Building, New York City 1938 and 1940

Based on 13 strokes

Thus the data obtained may be applied to transmission line conditions with reasonable accuracy. If the stroke is considered to have struck a considerable distance away from the ends of the line, then the traveling waves will contain a current half that shown in figure 6 and also a coulomb value of half that given in figure 8. If the stroke is at or very near the end of the line (within a few hundred feet) the traveling wave currents and corresponding voltages will be as originally given.

The wave shapes to reach apparatus will depend upon the point struck with reference to the apparatus, the insulation levels, surge impedance, coupling, and other factors. If overhead ground wires are used and the stroke does not initially make contact with the line conductors, the disturbance will be essentially local, if tower footing resistance is low enough and the line insulation high enough.

The total stroke consists not only of current peaks but also of long time low values of continuing current.

In the case of the transmission line, if the ground wire is struck these continuing currents find their way directly to ground. If the current peak was not of sufficient magnitude to cause flashover, the continuing current will not be of any importance, except for some burning of the ground wire which may result since this current is of the nature of a direct current arc.

If a continuing stroke causes an insulator flashover, the presence of the continuing current following arcover may interfere with the operation of the automatic reclosing equipment, if the latter is sufficiently rapid. Such continuing current in the conductors due to a stroke will be carried to ground through neutrals of transformers if grounded, while if the system neutral is isolated lightning arresters may be forced to carry such currents, which might possibly result in damage to the arrester, although in general the failures of arresters have been very small, indicating that currents of this character do not occur very frequently.

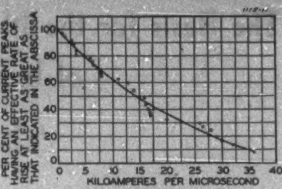

Figure 11. Effective rate of rise* of current peaks of lightning strokes as a function of frequency of occurrence

Recorded by high-speed cathode-ray oscillograph

* Slope of line through points on wave front at 10 per cent and 90 per cent of first crest

Empire State Building, New York City 1938 and 1940

Based on 11 strokes

In the case of distribution circuits, often several paths to ground exist over guy wires, through lightning arresters, and in the case of grounded neutrals continuing currents will find their way through many multiple paths to ground without undue duty on the lightning arresters. No doubt the continuing type of discharge is responsible for some of the fuses blown during lightning storms.

When using direct stroke data, such as given in this paper, to determine what may happen on either distribution or transmission circuits, proper consideration should be given to circuit conditions. Seldom, if ever, can direct stroke values be used directly.

It is also important to point out that there is good reason to believe that ground resistance will have an appreciable effect[1] on the current in a lightning stroke—the lower the resistance the higher the current, other factors being equal. The effect of ground resistance on wave fronts of direct strokes is not known, but one would expect that higher ground resistance would probably effect some increase in the duration of the stroke, although the effect of the cloud distribution of charge is probably controlling.[1]

Conclusion

For the first time, sufficient data relative to wave shapes of direct strokes are available to make possible the drawing of experience curves with respect to front, crest, and tail of current peaks. Similar data with reference to the total stroke are also available.

References

1. Lightning to the Empire State Building, K. B. McEachron. *Journal* of Franklin Institute, volume 227, 1939, pages 149–217.

The Basis for the Nondestructive Testing of Insulation

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MEMBER AIEE

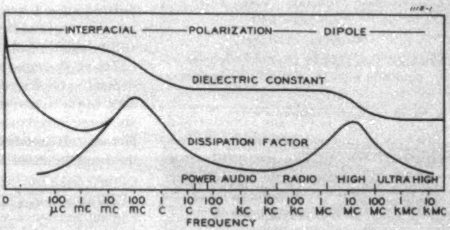
Introduction

DIELECTRIC strength is the property of insulation that is most important to the operating power companies. The factor of safety determined by the ratio of breakdown voltage to operating voltage must be maintained. Yet this breakdown voltage cannot be measured on a finished piece of insulation. Actual breakdown, except in the case of liquids, destroys the insulation. Overvoltage less than that required for breakdown has been found to injure the insulation and leave it with a reduced factor of safety. Non-destructive testing at overvoltage is feasible only if some electrical quantity, such as current, is measured and used as a sensitive indicator of incipient damage. The use of d-c overvoltage from a tube rectifier for cable testing is an example.

Non-destructive testing of insulation, as currently practiced, is the measurement of certain electrical characteristics of the insulation, such as capacitance, dissipation factor,* and leakage resistance, at a voltage not greater than the operating voltage. Ordinarily a relatively low voltage at the operating frequency is used, 100 volts to 10 kilovolts. Of these three characteristics, dissipation factor has come to be considered the most reliable indicator of the condition of insulation. The standard practice is to measure the dissipation factor of insulation at the operating frequency at stated intervals of time after installation. A progressive increase of dissipation factor with time is believed to indicate deterioration of the insulation. Certain limiting values are set depending on the type of insulation. When these limits are exceeded, the insulation is removed from service. Frequently, greater weight is placed upon continual change in the value of dissipation factor than upon its absolute magnitude.

Figure 1. Schematic diagram showing changes in dielectric constant and dissipation factor with frequency (after Murphy and Morgan)

The upper branch of the dissipation factor curve shows the effect of d-c conductivity



Heretofore the assumption has been made that, when insulation deteriorates, a correlation exists between the decrease in dielectric strength and changes in dissipation factor, because the use of nondestructive testing has resulted in a decrease of insulation failures. It is, however, a fact that the greatest success has been attained with the simpler types of bushings and insulators, only fair success with transformers, and little success with generators and cables. This is probably true because the changes in dielectric strength on the one hand and in the three measurable electrical quantities on the other are all effects for which deterioration is the cause. It is also unreasonable to expect that sufficient information will be contained in measurments taken at a single frequency to allow the different kinds of deterioration to be recognized. It will be shown that only by taking measurements over a wide frequency range can all the information obtainable from the electrical characteristics be secured.

These considerations make it desirable to determine whether or not a satisfactory correlation should exist between dielectric strength and one or all of the electrical quantities which can be measured.

Deterioration

The many causes of deterioration occuring in insulation may be grouped under four heads, chemical, mechanical, and electrical changes and the introduction of foreign material. The processing of many commercial dielectrics is incomplete. Either polymerization is not complete or the solvents are not completely volatilized. During the subsequent changes, voids may form throughout the body of the material. Voids are also formed in laminated materials under the vibrational and mechanical forces to which insulation is subjected in rotating machines and in transformers.

Foreign materials, mainly dirt and moisture, may adhere to the surface of insulation or may penetrate throughout the volume, if the body is porous. This is not usually an irreversible change. Cleaning and heating will usually restore the original properties, but, as long as the foreign material is present, dielectric strength may be impaired.

The existence of voids and the presence of foreign material offer the possibility of ionization, even at operating voltage. The surface of the voids will eventually carbonize and add a semi-conducting path. On the outer surface of an insulator this produces tracking. Permanent damage is done in either case.

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1. For all numbered references, see list at end of paper.

* Dissipation factor is the cotangent of the phase angle, while power factor is the cosine.

2. PLAYING WITH LIGHTNING, K. B. McEachron and K. G. Patrick, 1940.

3. LIGHTNING RECORDING INSTRUMENTS, J. H. Hagenguth. *General Electric Review*, volume 43, 1940, numbers 5 and 6, pages 195–201 and 248–55.

4. CAMERAS DESIGNED FOR LIGHTNING STUDIES, C. J. Kettler. *Phototechnique*, May 1940.

5. THE DIRECT MEASUREMENT OF LIGHTNING CURRENT, J. W. Flowers.

6. WAVE SHAPES OF SUCCESSIVE LIGHTNING CURRENT PEAKS, K. B. McEachron. *Electrical World*, volume 113, 1940, number 6, page 428.

7. LIGHTNING INVESTIGATION ON TRANSMISSION LINES—VII, W. W. Lewis and C. M. Foust. AIEE TRANSACTIONS, volume 59, 1940 (April section), pages 227–32.

8. DISCHARGE CURRENTS IN DISTRIBUTION ARRESTERS—II, K. B. McEachron and W. A. McMorris. AIEE TRANSACTIONS, volume 57, 1938 (June section), pages 307–12.

9. LIGHTNING CURRENTS AND THEIR VARIATIONS, H. Norinder. *Journal* of Franklin Institute, volume 220, 1935, number 1, pages 69–92.

Of these various kinds of deterioration the formation of voids and the absorption of moisture are most common.

Mechanism of Breakdown

Breakdown in solids has been studied extensively by von Hippel[1] and others. The application of a voltage produces a motion of free electrons and free ions toward the electrodes. If sufficiently long free paths exist, enough velocity is attained by these carriers of electricity to produce other ions by collision. As long as the motion of the carriers is limited, as by cleavage planes in crystals and by interfaces in mixtures, the insulation is stable. However, when the ionization becomes sufficiently intense, enough carriers are formed to blast an irregular path between the electrodes. This is breakdown.

Whatever increases the supply of free electrons and ions or lengthens their mean free paths lowers the breakdown voltage. This suggests that there may be a good correlation between dielectric strength and d-c conductivity.

Polarization

The values of dielectric constant and dissipation factor obtaining in insulation at any one frequency, or over a range of frequencies, are determined by the existing polarization. A value of dielectric constant greater than unity indicates polarization and an extra contribution to capacitance from polar groups. At optical frequencies a value of the refractive index greater than unity shows that some polarization exists. Any value of dielectric constant greater than the square of the refractive index indicates further polarization. A plot of dielectric constant against frequency has the general shape indicated in figure 1, which is after Murphy and Morgan.[2] The greatest value of dielectric constant occurs at zero frequency.

Two general types of polarization occur over the audio- and radio-frequency range, dipole and interfacial. The former was suggested by Debye and the latter by Maxwell, later amplified by Wagner. Dipole polarization is produced by polar molecules, in which the electrical centers of the positive and negative charges do not coincide. In an alternating electric field, such dipoles attempt to follow the direction of the field. At sufficiently low frequencies there is time enough during each half cycle for nearly all of the dipoles to place themselves in line with the field and that type of polarization is complete. The time required to carry the polarization to $1-1/e$ or 0.632 of completion is called the relaxation time. The corresponding frequency for which the polarization is complete usually appears in the megacycle range for the simpler polar molecules. For the more complex chain molecules, where dipoles occurring along the chain may produce a torsional vibration, this frequency may be found in the audio range.

The rotational motion of the dipoles is opposed by their own heat motion and the motion of the other molecules, with the result that a definite amount of energy is lost at each cycle. The power loss at any frequency is the product of the energy loss per cycle and the frequency. It decreases to zero at both zero and infinite frequencies and hence produces a maximum in dissipation factor at some intermediate frequency.

The d-c conductivity also contributes to the dissipation factor at very low frequencies and causes the numerical value of dissipation factor to become infinite rather than zero at zero frequency. This effect of d-c conductivity at very low frequencies is shown in figure 1, by the upper branch of the dissipation factor curve. For all ordinary dielectrics this contribution to dissipation factor is negligible at audio frequencies.

Interfacial polarization occurs at the interfaces of dissimilar dielectrics. These interfaces can occur as a few large surfaces between two materials or as innumerable small surfaces produced by finely divided particles. The free electrons and ions move throughout the volume of each dielectric and heap up at the interfaces. The charge accumulated at the interfaces during each half cycle contributes to the polarization in exactly the same way as does the lining up of the dipoles in dipole polarization. The resultant changes with frequency in dielectric constant and dissipation factor follow identical laws. In fact, it is not possible to distinguish the two types of polarization from the shapes of their frequency curves.

A condition necessary for the existence of interfacial polarization is an inequality in the product of dielectric constant and specific resistivity for the two dielectrics. This dependence of interfacial polarization on resistivity, and hence on free electrons and ions, suggests that there may be a good correlation between dielectric strength and the dissipation factor resulting from interfacial polarization.

Single Relaxation Time

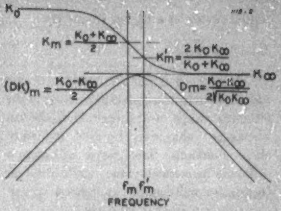

Figure 2. Change of dielectric constant, loss factor, and dissipation factor with frequency for a single-relaxation-time polarization

For a polarization having a single relaxation time the changes of the various electrical characteristics with frequency follow definite laws. The polarization is completely defined[3] by the high-frequency dielectric constant K_∞, the low-frequency dielectric constant K_0 resulting from complete polarization and the relaxation time T. The maximum value of loss factor* is

$$(DK)_m = \frac{K_0-K_\infty}{2} \tag{1}$$

and occurs at a frequency

$$f_m = \frac{1}{2\pi T} \tag{2}$$

which may be called the relaxation frequency. The curve of loss factor is symmetrical about the relaxation frequency and is expressed by

$$\frac{(DK)}{(DK)_m} = 2\frac{f/f_m}{1+f^2/f_m^2} \tag{3}$$

where both loss factor and frequency have been normalized with respect to their values at the relaxation frequency.

The curve of dielectric constant also has a kind of symmetry about the relaxation frequency and is expressed by

$$K = K_\infty + \frac{K_0-K_\infty}{1+f^2/f_m^2} \tag{4}$$

The value of dielectric constant at the relaxation frequency is the average value.

$$K_m = \frac{K_0+K_\infty}{2} \tag{5}$$

The shapes of these curves are shown in figure 2, where frequency and loss factor are plotted logarithmically while dielectric constant is plotted arithmetically. The curve of dissipation factor is also plotted logarithmically in figure 2. In its normalized form the expression for dissipation factor is identical with equation 3 for loss factor, with a maximum value

$$D_m' = \frac{K_0-K_\infty}{2\sqrt{K_0K_\infty}} \tag{6}$$

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*Loss factor is the product of dissipation factor and dielectric constant.

which occurs at a higher frequency

$$f_m' = f_m\sqrt{K_0/K_\infty} \tag{7}$$

Whenever it is inconvenient to obtain values of dielectric constant because of the irregular shape of the dielectric, the capacitance values may be used instead in these equations. Loss factor will have a different numerical value, while dissipation factor will remain unchanged.

Multiple Relaxation Times

The curves obtained for the various dielectrics and for commercial insulation have very much broader maxima than the curve for a single relaxation time shown in figure 2. In addition, the value of maximum loss factor is much less than that calculated from equation 1. The explanation given by Wagner and amplified by Yager[4] is that there exists a group of relaxation times distributed according to some probability law.

The curves shown in figure 3 are given by Yager[5] for a paper base phenolic, curves 15*D* for a dry specimen and curves 15*H* for the same specimen conditioned at 60% relative humidity. The maximum in loss factor at a frequency of ten megacycles is produced by dipole polarization of the cellulose filler and the curve is much broader than that shown in figure 2. The slight rise appearing at audio frequencies in curve 15*D* and caused by interfacial polarization is greatly increased by the moisture added under the conditions of curve 15*H*. The relaxation frequency of this polarization is well below the audio-frequency range. This curve is so broad that loss factor is increased even at ten megacycles.

Circle Diagrams

Cole and Cole[6] have recently offered a simpler and more fruitful explanation for the broad maxima. The properties of a polarization having a single relaxation time are such that a plot of loss factor and dielectric constant, as shown in figure 4*a*, is a semicircle with its center on the dielectric constant axis and with intercepts on this axis equal to the zero and infinite frequency dielectric constants. It becomes apparent immediately that the maximum loss factor must be one-half the change in dielectric constant as stated in equation 1.

The important discovery of the Coles is that when loss factor is plotted against dielectric constant for various insulating materials a circular arc results whose center is depressed below the dielectric constant axis by an angle ϕ as shown in figure 4*b*. The ratio α of this angle to a right angle is taken as the fourth parameter in defining the polarization. It can be considered as a storage coefficient, which determines the ratio of the stored energy to the dissipated energy for whatever mechanism is assumed for the polarization. The data of figure 3 for a paper base phenolic are thus plotted in figure 5. The circular arc is well defined for the dry material and the divergence of the points at the low frequency end shows the existence of a second polarization. For the wet material the effect of the interfacial polarization is very marked but the data were not carried to low enough frequencies to define its circle. Values of the four parameters for the high frequency polarization are given in table I.

Table I. Paper-Base Phenolic

	K_∞	K_0-K_∞	f_m Mc	α
15*D*	3.76	1.08	14	0.52
15*H*	4.01	2.72	20	0.63

The addition of moisture, while slightly increasing the storage coefficient δ, has its greatest effect on the change in dielectric constant K_0-K_∞.

Effect of Temperature on Polarization

The effect of increasing temperature on both types of polarization is to decrease the forces opposing the motion of the carriers and thereby to raise the relaxation frequencies. This moves upward on the frequency scale all the curves of dielectric constant and loss factor without greatly changing their numerical values. At a given frequency dielectric constant always increases with temperature at the lower temperatures and consequently its temperature coefficient is positive. The magnitude of this coefficient is by no means constant but depends both upon the amount of polarization and on the ratio of the measuring frequency to the relaxation frequency. After the polarization is complete at a high temperature, dielectric constant frequently decreases and the coefficient becomes negative.

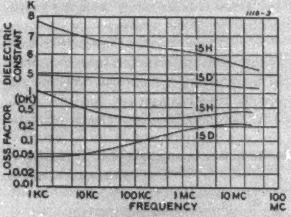

Figure 3. Change of dielectric constant and loss factor with frequency for a paper-base phenolic (after Yager)

15*D* dry, 15*H* conditioned at 60 per cent relative humidity

Loss factor and dissipation factor may increase or decrease with temperature depending on whether the measuring frequency is greater or less than the relaxation frequency. For most commercial insulation showing interfacial polarization the relaxation frequency is well below both audio and power frequencies, and the temperature coefficient is positive. It is always possible, however, for polarization from a single interface or from dipoles to occur at a higher frequency than the measuring frequency and make the temperature coefficient negative.

Effect of Voltage on Polarization

The effect on polarization of increasing voltage is to increase the amount of interfacial polarization because of the increase in the number of free electrons and ions. This will generally increase the value of maximum loss factor and decrease the d-c resistance.

Polarization in Commercial Insulation

Commercial insulation, as used in bushings and insulators, consists of a number of dielectrics in series and parallel. Interfacial polarization, with maximum dissipation factor occurring in the millicycle range,* is usual. Curves of capacitance and dissipation factor for a 25-kv compound-filled bushing are given in figure 6 for four different temperatures. These observations were taken on a Schering bridge operating at 100 volts. The shapes of the curves indicate the existence of both a high-frequency and a low-frequency polarization, but the frequency range is insufficient to allow the construction of circular arcs, even though the lowest frequency used was ten cycles.

The temperature coefficients are all positive and have the values given in table II.

The large variations in temperature coefficient indicate the difficulty of stating a single value applicable under all conditions.

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*From 0.001 to 1 cycle per second. A frequency of one millicycle is 0.001 cycle per second.

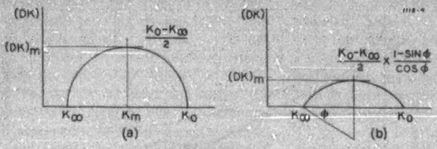


Figure 4. Circle diagrams (after Cole)

(*a*) Center of circle on dielectric constant axis

(*b*) Center of circular arc depressed by angle ϕ

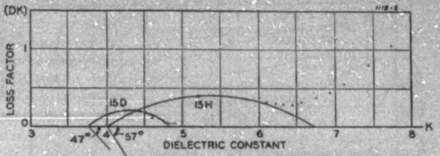


Figure 5. Circle diagrams for paper-base phenolic of figure 3

15*D* dry, 15*H* conditioned at 60 per cent relative humidity

Similar curves for a 66-kv multiple shell porcelain insulator are given in figure 7. Superposed on the low frequency polarization of the porcelain are the polarizations at the interfaces of the porcelain shells and the cement which holds the shells together. The changes in their relaxation frequencies with temperature amount to about 15% per degree centigrade. This is sufficient to cause the dissipation factor curves to cross each other and the capacitance curves to be anomalously spaced. The temperature coefficient of dissipation factor changes sign with both temperature and frequency and cannot be used for correcting to a standard temperature. The average temperature coefficient of capacitance at 60 cycles is 0.3% per degree centigrade.

Table II. Temperature Coefficients for 25-Kv Bushing

f Cycles	C % Per Deg C	D % Per Deg C
10,000	0.24	0.7
1,000	0.25	1.1
120	0.27	3.1
60	0.29	3.8
10	0.45	3.5

The change in the interfacial polarization of porcelain and cement as the moisture content of the cement is changed is startling. In figure 8 is shown the effect of allowing water to stand in contact with the cement for $5^1/_2$ hours. The relaxation frequency is raised to 800 cycles. Further soaking for 4 days raises this frequency to 13 kilocycles. Characteristic changes also occur in the capacitance of the insulator. The existence of such polarizations complicates the determination of the main low- and high-frequency polarizations, but affords added information concerning the structure of composite insulation.

Current-Time Curves

The interfacial polarization occurring in commercial insulation has been shown to be of such a nature that its relaxation frequency is well below the range of bridge measurements and in many instances is not greater than a few millicycles. Curves of charging current against time when a voltage is applied, or of discharge current when the voltage is removed, offer a feasible means of measurement. The existence of this charging current has been a great annoyance in the measurement of d-c resistance, and has led to such arbitrary rules as the specification that d-c resistance shall be calculated from the current flowing one minute after the application of the voltage. It is now obvious that resistance calculated in this manner is only an apparent resistance, which actually is a partial measure of the existing polarization. The complete curve contains the information needed to determine all the parameters which define the polarization. Whitehead and Banos[7] in 1932 analyzed charging current curves for paper taken by means of a cathode-ray oscilloscope and obtained values of dissipation factor which agreed within 2% with measurements taken on a 60-cycle Schering bridge. Murphy and Morgan[8] in 1939 placed this method of analysis on a better theoretical basis. They showed that charge and discharge currents differ by the d-c leakage current only when the polarization during charge has been carried to completion. It is necessary either to subtract the leakage current from the charging current before beginning the analysis or to carry the charging to a time which is long compared with the relaxation time before beginning the discharge. It is very difficult to estimate the correct value of leakage current, and the latter method is preferable in spite of the extra time involved.

The charging current at 500 volts for the 66-kv insulator is given in figure 9. After subtracting the leakage current of 0.62 $m\mu a$, the residual current was analyzed into the five exponentials shown, whose relaxation times vary from 0.4 to 63 seconds. The corresponding curves of dissipation factor are shown in figure 10. Their sum is the total dissipation factor

Figure 6. Effect of temperature on capacitance and dissipation factor of a 25-kv compound-filled bushing

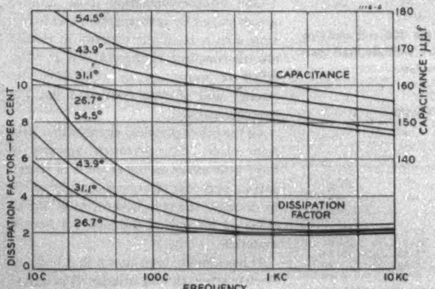
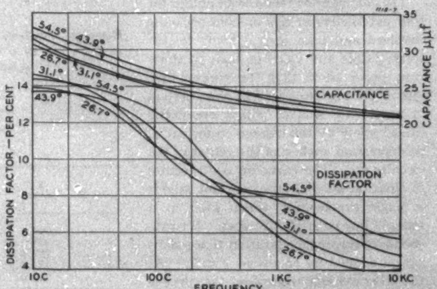


Figure 7. Effect of temperature on capacitance and dissipation factor of a 66-kv multiple-shell, pin-type, porcelain insulator



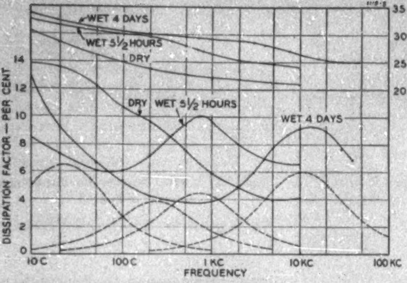


Figure 8. Effect of moisture absorption on capacitance and dissipation factor of a 66-kv multiple-shell porcelain insulator

The contributions to dissipation factor from the polarizations at the cement-porcelain interfaces are shown in dotted lines

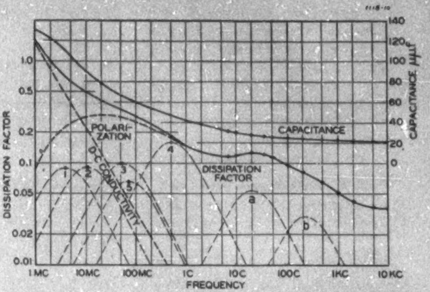


Figure 10. Capacitance and dissipation factor of a 66-kv multiple-shell porcelain insulator as calculated from the exponentials shown in figure 9

produced by the interfacial polarization. The contribution from the leakage current is then added. The corresponding values of capacitance are also given. The resulting circular arc is shown in figure 11. The falling-off of the values of loss factor at the low-frequency end is caused both by the difficulty of estimating the leakage current correctly and by the lack of long time data. The charging current should have been observed for a considerably longer time than 15 minutes. At the high-frequency end the points diverge because of the extra polarizations from the porcelain-cement interfaces. The four parameters derivable from the circular arc are given in the first row of table III.

Table III. 66-Kv Porcelain Insulator

Method	C_∞ μμf	$C_0 - C_\infty$ μμf	f_m Mc	α
Exponential	40.2	122.8	6	0.49
Graphical		115	5	0.4

The calculations required to obtain the four parameters in this manner are quite involved and take several hours to perform. Cole and Cole[9] in their most recent paper show that the current-time curves for any polarization have certain characteristic shapes, which are determined solely by the storage coefficient α, or depression angle φ of the center of the circular arc. The curve for α=0 (center of circle on the dielectric constant axis) is a single exponential, which plots as a straight line on arith-log paper. It corresponds to the single relaxation-time polarization defined by equations 1 to 7. At the other extreme when α=1 the curve is a 45° line when plotted on log-log paper. The family of curves are best plotted logarithmically on both axes. The observed current-time curve, plotted to the same scale, is matched to one of these curves and the values of storage coefficient α, relaxation time *T*, and current at that relaxation time noted. The residual current curve of figure 9 has been analyzed by this method with the results given in the second row of table III. The agreement between the two methods is reasonably good. It is quite necessary in this graphical analysis to use a discharge-current curve which covers at least three time-decades symmetrically disposed around the relaxation time.

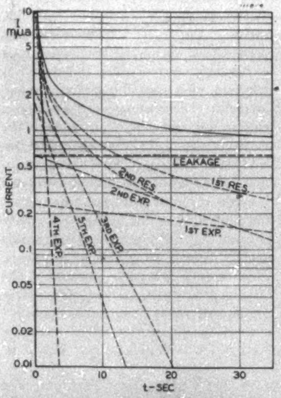


Figure 9. Analysis of the 500-volt charging current curve of a 66-kv multiple-shell porcelain insulator

$E = 500$v
$A = 806 k M\Omega$
$C_A = 21.1$ μμf
$AC_A = 17.0$ sec.

	B kMΩ	C μμf	BC_B sec.
1	2,084	30.1	62.6
2	805	25.9	20.84
3	227.2	16.50	3.74
4	33.0	11.62	.38
5	289.0	8.82	2.55

Effect of Deterioration on the Polarization Parameters

The most common types of deterioration in insulation have been shown to be the absorption of moisture and the formation of voids. In a way they go together, because the existence of voids furnishes an additional porosity which can increase the moisture absorption. This appears to be particularly true of the pasted mica insulation used in large generators. The added moisture increases the dielectric-constant change $K_0 - K_\infty$ both on account of the high dielectric constant of water and also because of the added interfaces and free ions. The relaxation frequency f_m is usually increased. The effect on the storage coefficient α is not known.

The voids formed by chemical or mechanical means provide additional interfaces at which ionization can occur. Under intense ionization these surfaces are damaged by carbonization and oxidation with a resultant increase in the dielectric-constant change $K_0 - K_\infty$: The effect of ionization on relaxation frequency and storage coefficient has not been determined.

Of the other chemical changes that may take place, polymerization appears to have little effect on the amount of polarization, and loss of volatile matter seems to decrease the polarization, as would be expected because of the loss of a foreign material.

It has already been shown that both dielectric strength and interfacial polarization depend on the abundance of free electrons and ions. The changes in the three parameters K_0-K_∞, f_m, and α will not only measure the amount of the deterioration, but will indicate the type of deterioration as well and hence will suggest the probable decrease in dielectric strength. The success of such interpretation will depend greatly on the thoroughness with which interfacial polarization in typical insulating materials is studied by means of current-time curves.

An important by-product of these studies will be numerical values for the temperature and voltage coefficients of the three parameters. At the present time only their signs are known.[10] The dielectric-constant change K_0-K_∞ decreases slowly with temperature, but increases considerably with voltage. Relaxation frequency f_m increases rapidly with temperature, frequently at the rate of a decade in frequency for a rise of ten degrees centigrade. Storage coefficient α often decreases with temperature. The effect of voltage changes on these two parameters is not yet known. The importance of knowing the temperature coefficients is that it will then be possible to reduce data taken at widely differing temperatures to a standard temperature. Difficulties from rapidly varying coefficients such as were illustrated in figure 6 no longer occur for these polarization parameters.

Single-Frequency Measurements

Only when the kind of deterioration and the type of insulation is known will measurements of capacitance and dissipation factor at a single frequency afford a reliable measure of the amount of deterioration. The relaxation frequency is so low that it makes little difference whether an audio frequency or the power frequency is used. There is no virtue in using the operating frequency and there is considerable advantage in choosing some neighboring frequency, because it will then be much easier to eliminate the effect of the intense electric field at the operating frequency in which many measurements must be made.

Measurements spread throughout the audio-frequency range will add little to the information obtained from single frequency data, except as they may disclose unusual polarizations of the type shown in figures 5 and 6, and as they provide values of the infinite frequency capacitance and dielectric constant. These frequencies will be too far removed from the relaxation frequency to be of any help in constructing a circular arc and thereby determining the polarization parameters. For this same reason even wide changes in the applied voltage will have little effect on capacitance and dissipation factor, even though the changes in zero frequency capacitance C_0 and maximum dissipation factor D_m' are appreciable. Measurements made at the power frequency of 60 cycles on both new and deteriorated bushings at voltages from 50 volts to 10 kv bear out this conclusion.

Conclusions

It has been shown that a satisfactory correlation can exist between dielectric strength and low-frequency interfacial polarization in insulation, because of their mutual dependence on the abundance and disposition of free electrons and ions throughout the material. Any decrease in dielectric strength caused by deterioration can therefore be measured by the corresponding changes occurring in this polarization. The polarization is defined by certain parameters and a new method for their measurement has been outlined, involving the graphical analysis of current-time curves.

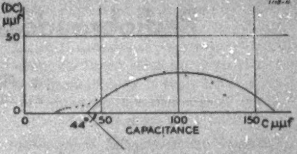


Figure 11. Circle diagram for a 66-kv insulator drawn from the data of figure 10

References

1. Electric Breakdown of Solid and Liquid Insulators, **A. von Hippel.** *Journal of Applied Physics*, volume 8, 1937, pages 815–32.

2. The Dielectric Properties of Insulating Materials, **E. J. Murphy** and **S. O. Morgan.** *Bell System Technical Journal*, volume 16, 1937 pages 493–512.

3. Capacitance and Loss Variations With Frequency and Temperature in Composite Insulation, **H. H. Race.** AIEE Transactions, volume 52, 1933, pages 682–92.

4. The Distribution of Relaxation Times in Typical Dielectrics, **W. A. Yager.** *Physics*, volume 7, 1936, pages 434–50.

5. Dielectric Constant and Loss of Plastics as Related to Their Composition, **W. A. Yager.** *Transactions*, Electrochemical Society, volume 74, 1938, pages 113–34.

6. Discharge and Absorption in Dielectrics, I, A-C Characteristics, **K. S. Cole** and **R. H. Cole.** *Journal of Chemical Physics*, volume 9, 1941, pages 341–51.

7. Predetermination of the A-C Characteristics of Dielectric, **J. B. Whitehead** and **A. Banos.** AIEE Transactions, volume 51, 1932, pages 392–409.

8. The Dielectric Properties of Insulating Materials, III, Alternating and Direct Current Conductivity, **E. J. Murphy** and **S. O. Morgan.** *Bell System Technical Journal*, volume 18, 1939, pages 502–37.

9. Transient Current in Dielectrics, **K. S. Cole** and **R. H. Cole.** Washington Meeting of American Physical Society, May 1–3, 1941.

10. Dielectric Properties of Organic Compounds, **S. O. Morgan** and **W. A. Yager.** *Industrial and Engineering Chemistry*, volume 32, 1941, pages 1519–44.

Radiobroadcasting in Canada

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ASSOCIATE AIEE

Synopsis: The development of radio broadcasting in Canada has been very rapid and there are now 85 broadcast transmitters in daily operation. All broadcasting is under the control of the Canadian Broadcasting Corporation, an independent Government corporation, which in addition to its regulatory functions operates a coast to coast network connecting together, for a nationwide service, its own ten modern stations with almost half of the privately owned transmitters. It cannot therefore be said that radio in Canada is owned and operated by the Government as the CBC is independent and free to apply its revenues derived from licenses and commercial operation, in the best interests of its radio audience. The Canadian system is one of co-operation between a semipublic service and private ownership. Parliamentary enquiries have always found it the best suited to Canada's needs considering its vast area and relatively sparse population.

RADIO broadcasting in Canada began at about the end of the last War when an experimental station originated programs for a few hours a day. Following this the first network broadcast in Canada took place on Christmas Day, 1923, when an experimental station in Ottawa was connected to a station in Montreal. In 1926–27 an Eastern Canada network was organized extending from Moncton, N. B. to Toronto; then in 1928–29 our first coast to coast network was inaugurated. Shortly before this time however, on the occasion of the Diamond Jubilee of Confederation, July 1, 1927, Canada's capital city Ottawa was linked for the first time in a nation-wide broadcast using the combined facilities of the Telephone and Telegraph Companies in Canada.

Thus, in 1932, when the Canadian Radio Broadcasting Commission was created by Parliament, the principal wire companies in Canada were already providing facilities for network broadcast. In 1936, the Commission was superseded by the Canadian Broadcasting Corporation which took over the facilities and personnel of its predecessor.

The present system of radio broadcasting in Canada has, since 1928, been considered the best available by all parliamentary committees of enquiry on this subject. Moreover, the present corporation is different from the Commission, which controlled radio from 1933–36, but the principle of accommodating privately owned stations and public service systems has been recognized as the best for Canada for a number of years. The voting in the House of Parliament and of its special committees treating on radio has always been unanimous.

Of the 85 broadcasting stations at present operating on the standard frequency band in Canada, ten are owned and operated by the CBC; the balance are privately owned. Twenty-five of the latter and the ten CBC stations constitute our national network.

The CBC itself is an independent Government corporation, quite free to run its business as it may please, not using public money in any manner but operating with revenues provided by the receiver license fees contributed to the corporation by every owner of a receiving set; this is supplemented by revenues obtained from the commercial operation of the stations. Outside of permitting the corporation to pay for part of the network cost, the national network, commercially run by the corporation, does not bring any appreciable profit.

The CBC is a non-profit enterprise, operating within its own revenues and serving the interests of the public. There are no shares, there are no bonds. The Board of Governors is entrusted with the administration of the revenues placed at its disposal and is responsible for the honest and efficient conduct of the business to a Minister of the Crown. Any surplus in operation is considered as such and invested in the business. It is not distributed as profits and not returned to the public treasury.

On the other hand, privately owned stations operate under conditions similar to those existing in the United States. They conduct their business as they please, and their operating profit is theirs but, as in the United States, they must, pertaining to program only, submit to certain regulations which in Canada are formulated by the CBC.

The restrictions as to program, self-imposed in the case of CBC stations, are such that restraint of private stations is, in my belief, not serious. For instance, the CBC will not carry commercial spot announcements. It will not carry certain types of commercial programs and generally does not accept a great number of the local commercial programs. These restrictions and many others do not apply to privately owned stations.

To emphasize on this freedom of the commercial side of the business, I may state that there are three privately owned stations in Canada which are outlets of the American networks and there are four CBC stations also having direct network connections with American networks. These facts will immediately prove to you that broadcasting in Canada is neither owned nor operated by the Government.

Our dependence on American networks for a good many of our best programs presents some difficulties at times, but generally speaking, it is considered as a very welcome asset and a great means of fostering closer understanding between our two countries.

From the network operating point of view we have problems in Canada which do not exist in the United States. The CBC network, which is the longest east to west network in the world extending over three thousand air miles, covers five time zones instead of the four south of the border. This great distance separating the two extremes of the network and more particularly the great difference in local time renders the problem of co-ordinating the program service very difficult. In addition we have a bilingual population, as almost one third of all Canadians speak French. In the province of Quebec we have besides our basic trans-Canada network a Regional French network. The French speaking minorities located throughout the other eight provinces are however constantly clamoring for more French programs. This presents a problem requiring a good deal of attention. Of course our greatest difficulty in Canada is the same as that faced by our transportation companies and is due to the wide scattering of a relatively small population across an extremely large area. This means low revenue and yet high cost of service. In the United States there is twelve times the population in five-sixths of the area, while in England the British Broadcasting Corporation serves four times the population concentrated in but one sixty-fifth of the area. In normal times this must, at least from the engineering standpoint, be a broadcaster's paradise, where the whole network is hardly longer than some of our local loops and where lines are only a few

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hundred miles in length as compared to the five or six thousand miles some programs on this continent have to travel before reaching their destination.

The largest studios are located in Montreal and in Toronto where approximately 50% of our programs are produced. For exchange programs from or to American networks we have special switching facilities at Montreal and Toronto for the National Broadcasting Company Red and Blue networks, at Vancouver and Windsor exchanges with the Columbia Broadcasting System are handled, and again at Windsor for exchanges with the Mutual Broadcasting System and the Michigan Radio network. Broadcasts from the British Broadcasting Corporation in England are relayed to our network through our short-wave receiving station in Ottawa. At this station, we have directional aerials and diversity receivers constantly tuned to various points in Europe and South America. Before hostilities began important features or news gathered in this manner were broadcast over the network. From some of these sources activities are now necessarily curtailed. On the other hand, our relations with the British Broadcasting Corporation are even closer than ever. We have at the present time a number of our men in England working in conjunction with the British Broadcasting Corporation personnel.

One very interesting point is that the CBC is in the enviable position of being able to secure for its listeners, at any time, a greater variety of programs than are normally available to any other broadcasting system in the World. They are made up as follows: First it has access to the leading programs of all the principal networks in the United States. Second, broadcasts from Europe for the time being are limited to those from the British Broadcasting Corporation. Third, its own English network and fourth, its own French network.

Some special feature broadcasts have been of considerable credit to the CBC. An example of this was the Christmas Broadcast in 1935. During the two hour feature pick-ups were scheduled from some 35 different and in some cases almost inaccessible spots ranging from a point two miles under the Atlantic ocean to a golf course on an island in the Pacific. It was during this broadcast that eight choirs located at widely separated cities sang in perfect harmony with an orchestra at another point.

A more recent example involving still wider facilities was the Empire Day broadcast of May 24, 1939, during the visit of Their Majesties. The program which lasted an hour began with a rapid visit across Canada with Canadians speaking from Halifax, Montreal, Toronto, Winnipeg, Regina, Edmonton, and Vancouver then it carried the listener to Africa, Rhodesia, Australia, New Zealand, India, Jamaica, Newfoundland, Scotland, Northern Ireland, Wales, and England and then, to climax the broadcast, back to Winnipeg in Canada for the Empire Day message given by the King himself.

The Canadian contributions were brought together by land lines and those from other points of the Empire via short-wave radio relay lines. In Montreal all these elements were blended into a program which was relayed to 54 stations across Canada and 335 transmitters of the NBC, CBS, MBS, MRN in the United States and also to all parts of the British Empire by means of short-wave radio.

In all, we estimate that over 25,000 miles of telephone circuits and 100,000 miles of radio telephone links were used plus the services of some 2,000 technicians simultaneously engaged at the controls, constituting a program which was within the reach of millions of people.

Summing up, the CBC is mainly concerned with national and regional problems of broadcasting. This explains why it maintains a very elaborate network of broadcasting lines of a total length of over 7,000 miles reaching almost every broadcasting station in Canada. As a matter of fact, these lines are owned and operated for the CBC by our two main railway companies: the Canadian Pacific Railway and the Canadian National Railways. Under contract, the telegraph divisions of these two companies must maintain the wire line service between some 70 points across the country, the service being subject to certain specifications concerning the quality of transmission. This network permits the CBC to reach over 90% of the population of Canada within the primary coverage of some broadcasting station, that is within the 5 millivolts per meter contour of some station. If we consider that people living outside of these areas are for the most part isolated from industrial centers and can therefore obtain good reception from much lower field intensities, it may be said that the CBC network reaches practically all the population of Canada.

For commercial purposes, a group of stations has been organized as a network under the name of the National Network. This is composed of five regional networks which may be handled separately from a commercial point of view. Whenever some program of great national importance is handled by the CBC, for instance for all broadcasts of the Royal Visit or when our Prime Minister or your President has a message to send to the public, such broadcasts are offered to all stations in Canada. Those on the National Network carry the program, the others, while they are not obliged to, usually carry it of their own accord. This, I believe gives you a general outline of broadcasting in Canada.

It is difficult to give correct figures on privately owned stations as they are quite independent to run their show as they please and they are not required to submit a report on their financial operations except, of course, in so far as taxes or revenues are concerned, but files on these matters are not open to the public.

As I have said before, the CBC operates under a budget covering each fiscal year. Since its organization on the 1st of November 1936, the corporation has had a total operating surplus of $1,000,000—$600,000 of which has been re-invested in the corporation's plant. Our board is proud to have been able to operate a rather difficult business with a surplus each year. This is not always the case with organizations who may hope to obtain the help of the Government when they get into bad straights.

During less than five years since its inception, the CBC has built four 50-kw transmitters, one of 5 kw, a $7^1/_2$-kw shortwave transmitter, and new studios at two points. Besides this, a 5-kw transmitter was revamped as were two sets of studios. It also had the privilege of leasing from the Manitoba Telephone System the studios built by that organization in Winnipeg.

The Engineering Division is divided into five departments:

The Operation Department,
The Transmission & Development Department,
The Architectural Department,
The Design & Construction Department,
and the Purchasing & Stores Department.

Of a total staff of some 600 we have at present 185 employees in the main divisions of the technical service. Forty of these are graduate engineers and 100 are technicians and operators specialized in radio work.

Had it not been for the war we would probably have two important new buildings, one in Toronto and one in Montreal, to take care of our studio and office requirements at those points. The war has prevented heavy capital expenditures.

Although we ourselves fabricate a num-

Development in Lightning Protection of Stations

E. R. WHITEHEAD
MEMBER AIEE

Introduction

THE AIEE transformer sub-committee has recently sponsored a paper[1] on the protection of power transformers against lightning surges. The most effective method given combines overhead ground wires, line entrance gaps, and transformer mounted lightning arresters. Such a method does give a high degree of protection, but it is primarily applicable to high voltage, high capacity stations where the overhead ground wire system is justified for both line and station.

The conditions noted above are seldom fulfilled on subtransmission systems from 23 kv to 34.5 kv. Effective ground wires and low ground resistances are uneconomical, and the protection provided is usually a more or less modern arrester, not always located and grounded to the best advantage.

On the 23-kv subtransmission system of the Duquesne Light Company, the average transmission distance is approximately two miles. This means that most direct strokes will contact the line not more than one mile from some substation. Much of this transmission is now equipped with wood cross arm braces and guy insulators, and at some stations several spans adjacent thereto are carried on unguyed poles.

Under such conditions, it is possible for very high voltage waves to strike the station and result in arrester discharge currents having high rates of rise.

It is the purpose of this paper to outline briefly some of the field experience pointing to this factor as a cause of transformer and arrester failures, and to present a limited discussion of laboratory tests and calculations relating to a proposed method of achieving a high degree of protection for equipment so exposed.

Field Experience

After several years' experience with improved station insulation and modern types of carborundum block arresters,[2] it has become apparent that arrester failures are confined to a few exposed locations. Furthermore, most cases of insulation failure are associated with definite evidence of direct strokes to the line near the station.

In one station, no equipment failures have occurred, but arrester failures average one per year. In another station there have been no arrester failures, but transformer failures average one every two years. In still another case, the arrester suffered no damage but a transformer bushing flashed over. This particular bushing had an impulse strength over twice the arrester impedance voltage at 5,000 amperes on the standard AIEE test wave.

Such instances have emphasized the necessity of providing some sort of back-up device which would come into play under extreme conditions only, yet not affect service under the great number of minor surges.

In one station, six-inch rod gaps were installed in parallel with the arresters two years ago, yet another transformer failure occurred in 1940.

As a result of this experience, it was decided to make a series of tests in the laboratory in order to determine if some method could be found to afford protection under such conditions.

Performance of Arresters

In analyzing the performance of lightning arresters subjected to high rates of current rise, it becomes apparent that the so-called IR voltage depends both on the rate of current rise as well as on the maximum value of the current. It has been found that a fair approximation to the actual characteristic can be obtained by using an equivalent circuit composed of a constant voltage, a constant resistance, and a constant inductance in series.

Such representation is, of course, entirely empirical, and serves merely to permit approximate calculations. The method of determining the constants is shown in figure 1. The value of the arrester voltage drop at the time of crest current is plotted for two well-separated values of crest current, and the line joining these points extrapolated to current zero to determine the value of E_a. The slope of this line determines the value of R_a. The difference between the crest of the arrester voltage and the value at the instant of crest current is a measure of the fictitious inductive drop at this time. These values are not strictly constant, but

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The author acknowledges the valuable assistance of C. M. Jackson in making the tests, and of B. L. Jarrett in the preparation of this paper.

1. For all numbered references, see list at end of paper.

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ber of the minor parts of our equipment, we purchase the bulk of our plant from the usual equipment manufacturers. Our new 50-kw stations in Toronto and Montreal are Northern Electric transmitters. Those in Sackville and Watrous are RCA. We propose to co-operate with the Canadian Marconi in the operation of an F.M. transmitter on Mount-Royal, Montreal.

All our construction is designed and supervised by our own staff but actual building is assigned by tender to firms who are invited to bid.

Thus, it is seen that radio broadcasting in Canada is conducted under a dual system of individual privately owned stations and of a public service network operated by the CBC. This particular organization, besides having authority to regulate over program contents generally of private stations also owns and operates a number of its own stations and a network composed of both CBC and privately owned stations. Besides, the CBC acts as an agency in booking network programs on all network stations whether they are its own or privately owned. There is also a provision for the setting up of partial networks to meet special demands from sponsors or to take care of the overflow on the national or regional networks. This system has been in operation for more than five years and has proved to meet with the wishes of the very great majority of Canadians across the Country. The public service angle has proved extremely successful in certain instances of national importance and all indications are to the effect that present conditions will be maintained to the best satisfaction of the sponsors, artists, musicians, and the listening public.

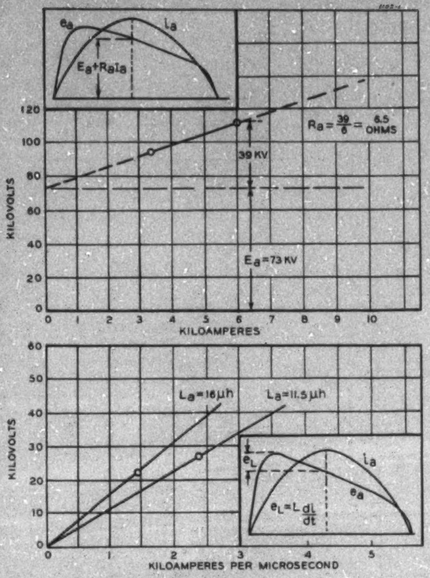


Figure 1. Method of determining constants for equivalent circuit of arrester

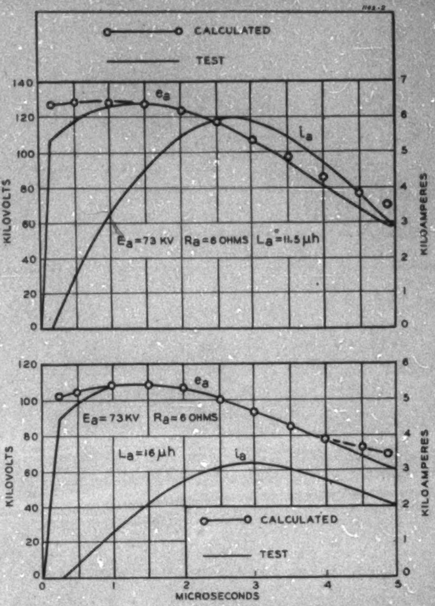


Figure 2. Comparison of test results and calculations from equivalent circuit

decrease with increasing discharge current. Values may be chosen for moderately high current and used with the knowledge that they will be conservative for extreme conditions. Figure 2 shows calculated and measured arrester voltages for the currents and arrester constants shown.

Transformer-Arrester Co-ordination

Because of the relative characteristics of gaps, transformers, and line type arresters, it is not possible to secure complete back-up protection for short times. This situation is illustrated in figure 3 where a modern transformer of the 23-kv class is compared with a modern arrester and a six-inch rod gap.

Gap-Reactor-Arrester Protection

Since the rate of rise characteristic of the arrester is simulated by an inductance, it seemed logical to add an external reactor which would increase the voltage at the gap location for high rates of current rise, and thereby at least partially offset the turn-up in the rod gap characteristic. This scheme is just the reverse of the old arrester-choke coil sequence in that the arrester is transformer-mounted and so limits the voltage at the vital point in the most effective way possible. In the old scheme, the arrester voltages for high rates of current rise were further aggravated by oscillations between arrester and transformer at the natural period of the choke inductance and effective transformer capacitance. In the new scheme, the basic protection is located at the transformer. The inductance and capacitance slope the wave front and lower the arrester breakdown, while the back-up gap is brought into action only when required to keep the arrester impedance voltage at a safe level.

Calculations for Theoretical Scheme

Figure 4 shows such a protection system. Through the substitution of the constant circuit elements to represent the arrester, it is possible to estimate the effect of any assumed terminal current. Although no "turn-up" is shown for the transformer strength, such an increase does exist and would actually provide a greater margin of safety than that indicated. The effect of this protection system at short times is roughly equivalent to the back-up protection of a 3.5-inch gap without the reactor.

Laboratory Tests

Impulse tests were made on the combination protective scheme in order to check the theory within the limitations of the laboratory equipment. Figure 5*a* shows the oscillogram of the voltage across the six-inch gap, while figure 5*b* shows the voltage across the arrester under the same conditions. Figure 5*c* shows the voltage across the arrester for a higher rate of rise of current, and without the reactor.

As a final test of the protection system, it was decided to make a complete set-up including a 100-kva, 3-phase, 22,000/460/230-volt transformer. The particular transformer chosen was an exact duplicate of those mentioned earlier as having flashed over or failed internally three times in six years. The surge generator was adjusted for maximum voltage and all series resistance was short-circuited. Under such conditions, the voltage, if unlimited by gap breakdown, would reach about 650 kv in something less than one-

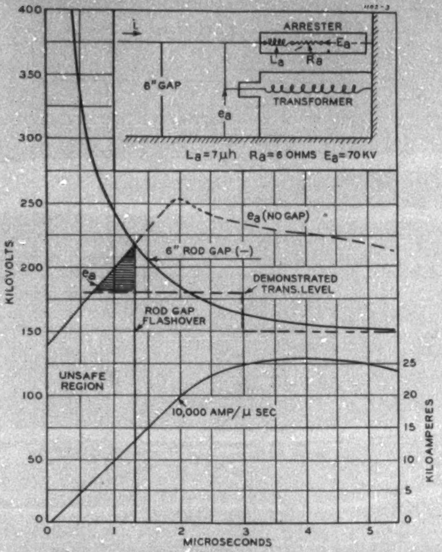


Figure 3. Co-ordination of gap-arrester-transformer combination

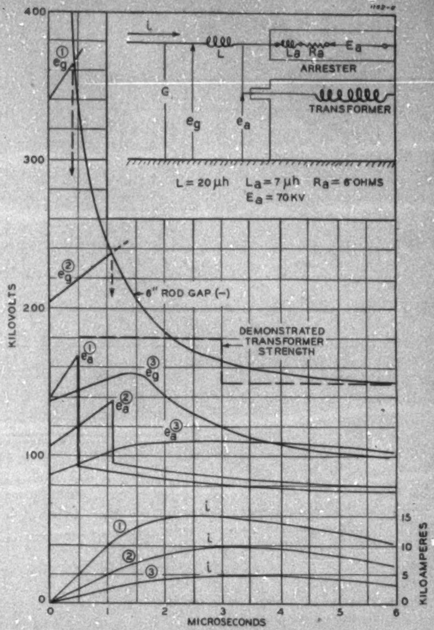


Figure 4. Calculated performance of gap-reactor-arrester protection scheme

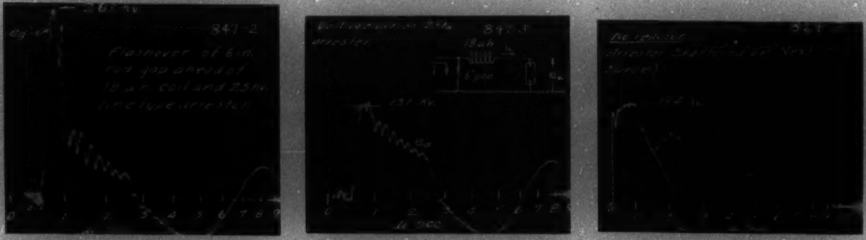
half microsecond. Under these conditions, the generator was discharged into the system several times without damage to the transformer. A maximum voltage of 121 kv was recorded at the transformer bushing. Current measurements under such conditions were impossible, but it is estimated that the rate of current rise approximated 20 ka per microsecond.

Following this test, the transformer was subjected to both full wave and chopped wave tests slightly under the present AIEE standard values, and no evidence of insulation failure was detected.

Conclusion

It appears possible to employ a reactor or "choke coil" to compensate for the steep turn-up of rod gaps when the latter are used as back-up voltage limiting devices in conjunction with transformer-mounted lightning arresters. Moreover, such a combination gives promise of providing a high degree of protection to stations fed at subtransmission voltages over wood pole lines without overhead ground wires.

Figure 5. Laboratory tests on gap-reactor-arrester system



References

1. PROTECTION OF POWER TRANSFORMERS AGAINST LIGHTNING SURGES, AIEE Transformer Subcommittee. AIEE TRANSACTIONS, volume 60, 1941, pages 568–77.

2. LIGHTNING PROTECTION OF 22-KV SUBSTATIONS, E. R. Whitehead. AIEE TRANSACTIONS, volume 57, 1938 (October section), pages 568–71.

Current Rating and Life of Cold-Cathode Tubes

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Introduction

A COLD cathode tube is a gas discharge device capable of serving as a relay, a rectifier, or a voltage regulator.[1,2] It depends for its action upon the properties of gas discharges. The salient property employed is the fact that there is a difference between the breakdown voltage, or voltage necessary to initiate a discharge, and the sustaining voltage, or voltage drop across the tube when conducting. The principles involved are not new but a practical device capable of operating on voltages of 150 volts or less has awaited the relatively recent development of suitable cathode coating materials. The cathode surface employed at present consists of a coating of barium and strontium oxides applied to a nickel base. These oxides are partially broken down to barium and strontium metal during exhaust. During the operation of the tube the barium is continually removed from the cathode by sputtering and is replaced by further reduction of the oxide reservoir. The life of the tube is limited, therefore, by the amount of material present and by the rate of sputtering.

Because the cold cathode tube requires no cathode heating power it is well adapted to those applications in which the service is intermittent. The absence of a hot cathode implies no deterioration during idle periods and the ability of the tube to start instantly upon the application of an input signal. These properties have opened a wide field of use to the cold cathode tube and it is felt that this field may be widened if the tubes are so rated as to take cognizance of the fact that their life is determined by sputtering and the consequent exhaustion of the reservoir of cathode material. Other gas tubes, such as vapor rectifiers and thyratons, have been rated in terms of peak and average current. These ratings have been based on anode power dissipation and electron emission capability of the cathode surface. In the case of the cold cathode tubes it is desirable to have the rating bear a relationship to the rate of sputtering. For a tube of given manufacture, the rate of sputtering is determined solely by the current drawn. The general form of the life rating should be, therefore, a tabular relationship between the average life to be expected and the current. Several values of current and life are needed since the rate of sputtering is not proportional to the current.

If a cold cathode tube is operated until it fails in service certain changes may be observed visually. These are: first, a dark band on the tube walls in the vicinity of the cathode and, second, a change in color and surface appearance of the cathode. These observations may be accounted for by assuming that the cathode material is lost by either evaporation or sputtering. To test the idea of loss of material due to evaporation thermocouple measurements of the cathode were made. These measurements indicate that, in general, the cathode temperature is less than 250 C and invariably less than 375 C. These temperatures are so low as to make the theory of loss of material by evaporation untenable. For example, the cathode temperature at a current of 35 milliamperes was found to be 210 C and the average life 100 hours. Rudberg and Lumpert[3] have measured the vapor pressure of barium at low temperatures and give a formula by means of which their data may be extrapolated. From this knowledge of the vapor pressure of barium at 210 C and the total amount of barium present on the cathode the life may be calculated and it is found to be 45,000 hours compared to the 100 hours actually observed.

Microchemical examination of the tubes indicates that new tubes have considerable metallic barium on the surface of the cathode together with a much larger amount of barium in combination. Similar tests on tubes which have reached the end of their life indicate less metallic barium on the cathode and almost no barium in combination on the electrodes but the total barium which was present when the tube was made is to be found on other parts of the tube, notably the walls.

These facts lead to the conclusion that as the tube is operated barium is continually lost from the cathode surface by sputtering and is continually replaced from the reservoir of barium oxide provided in the cathode coating. When this reservoir is exhausted the tube reaches the end of its life.

Theory of Cathode Sputtering

Most of the literature on cathode sputtering relates to a discharge at relatively low pressures and with high voltage drops across the tube. The only comprehensive survey of sputtering in low voltage tubes is to be found in a paper by C. H. Townes.[3] A review of the conclusions reached by Townes is given below.

Townes divides the rate of sputtering into three factors: (1) the number of ions of a given energy striking the cathode, (2) the amount of material released by each ion, and (3) the fraction of material released from the cathode which diffuses away and ultimately reaches other surfaces in the tube.

There is good evidence that sputtering is due to ionic bombardment of the cathode and that the material sputtered leaves the cathode in all directions and in an uncharged atomic stage. In the low voltage cold cathode tube the cathode dark space, across which appears the cathode fall of potential has a thickness of several electron mean free paths. The mean free path for a positive ion being smaller than that for an electron, it is evident that very few, if any, positive ions reach the cathode with an energy corresponding to that of the cathode fall. The average energy of positive ions impinging on the cathode is found to be a little less than one electron volt. Not only the average energy is necessary but also the probability that an ion has any of the various possible energies. Appreciable variations in the total energy come principally from the variation in the energy acquired at the last free path of the ion. This follows from the fact that collisions between ions and gas atoms are collisions between particles of substantially equal mass so that on the average, an ion will lose half of its energy at each collision. On the average, therefore, the energy acquired by an ion in its last free path is double that which it had at the beginning of this last free path. The final expression for the distribution in energy of the positive ions striking the cathode is an exponential expression of the same form as that describing the distribution of molecular free paths in a gas. This is to

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[1] For all numbered references, see list at end of paper.

be expected since, as pointed out above, the principal variation in the energy comes from that acquired in the last free path, or in other words depends upon the length of the last free path.

Of the mechanisms suggested for the release of material from the cathode under positive ion bombardment the one most successful in explaining the experimental results is the theory of local heating and resultant evaporation of von Hippel.[4] According to this theory the positive ion delivers its kinetic energy to a small number of atoms on the surface which are temporarily given large random velocities and evaporate from the surface just as they would evaporate if the entire cathode were heated so that they had the same thermal energy. The local hot spot of course, cools very rapidly by sharing its energy with the surrounding atoms. Townes has developed an expression for the number of atoms removed from the cathode by each ion impinging with a given energy. For a cathode material such that an atom must have an energy of four electron volts to escape from the surface his expression leads to the following:

Atoms Removed Per Ion Striking Cathode	Energy of Impinging Ions
2×10^{-8}	4 e.v.
0.2	6 e.v.
2	8 e.v.

This demonstrates the rapid variation in the number of atoms removed per impinging ion as the energy of the impinging ions is increased. The expression for the total number of atoms released from the cathode considering the distribution in energy of the positive ions reaching it is complicated and will not be given here.

Since the mean free path of the sputtered particles is much smaller than the distance between the cathode and surrounding surfaces a large fraction of the atoms which leave the cathode will diffuse back to the cathode surface so that they do not represent a net loss of cathode material. Townes shows that for a fixed rate of evaporation of atoms at the cathode the actual rate of loss of material from the cathode is inversely proportional to the gas pressure and depends as well on the distance to the nearest surfaces where deposition may take place. A large number of the evaporated atoms return to the cathode surface. The exact number lost will, of course, depend upon the tube geometry.

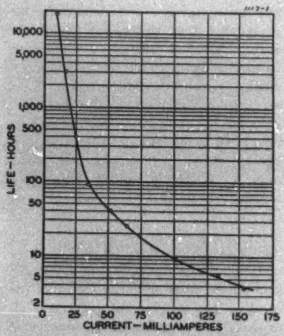

Figure 1. Life of the Western Electric 313C cold-cathode tube as a function of current

The complete expression for the sputtering rate is given by Townes as:

$$I = AF\frac{J}{p}e^{-\left[\frac{V_0}{c}\left(\frac{p^2}{Vi}\right)^{2/3}\right]} \qquad (1)$$

where

I is equal to the mass per second of sputtered material

p is the gas pressure in millimeters of mercury

V is the cathode fall in volts

i is the current density in amperes per square centimeter

V_0 is the energy in electron volts required to remove one atom from the cathode surface

c is a constant characteristic of the gas. For argon $c=20$, for neon $c=33$

J is equal to the total current

F is a function determined by the geometry of cathode and collecting surface. For plain parallel surfaces $F=1/d$ where d is the distance between the surfaces

A is another constant evaluated in terms of the properties of the cathode material in the gas

The constants in this equation may be approximately evaluated on theoretical grounds or they may be determined by experiment.

Experimental Results

A series of tubes have been operated at various currents to determine experimentally the relationship between current drawn and life. The cathodes of these tubes are coated to a nominal thickness of one-half milligram per square centimeter. This is an extremely thin coating and it is difficult to reproduce such coatings exactly from tube to tube. Considerable scattering in the test results must be expected therefore. The points on the curve of figure 1 each represent the average life of three tubes operated under the current conditions noted. The curve represents equation 1 with the constants so selected that it represents the best fit for the data. The agreement between the curve and the experimental data is seen to be quite satisfactory considering the scattering that must be expected in experiments of this type and suggests that the assumption at the outset that the life of the tubes was determined by the loss of cathode material due to sputtering is correct. This being so it seems reasonable to rate the tubes in such a manner as to reflect this fact.

The data presented in figure 1 relate to a Western Electric 313C vacuum tube and form the basis for the current ratings for this tube. The ratings employed are those currents which will give a life of 100, 1,000, and 10,000 hours. Because of the large scattering due to variations in coating thickness and gas pressure in individual tubes it is not expected that the life of a single tube can be predicted with an accuracy better than ±50%. In dealing with large numbers of tubes, however, the average life is predicted with a considerably better accuracy. It is practical, for instance, to use this curve in calculating the number of tube replacements required for a circuit which operates under given conditions.

Application of Current Ratings

Since the life of a cold cathode tube of given manufacture depends solely upon the current drawn a knowledge of the relationship between life and current drain is of vital importance to the circuit designer. In the use of cold cathode tubes it often happens that the duty cycle is so light that the life of the tube may be many times the life of the equipment in which it is used even though the tube, if operated continuously, might last but a few hours. If the circuit designer is furnished with the relationship between life and current drain and knows the duty cycle imposed on the tube he is in a position to determine the average life he may expect from the tube. With this knowledge it is often possible to apply small, inexpensive cold cathode tubes to circuits requiring considerable amounts of current for brief and infrequent intervals rather than to resort to larger tubes requiring hot cathodes and capable of supplying the current continuously. Likewise, it makes it possible for the designer to reach a reasonable compromise in decreasing the cost of associated equipment at the expense of increasing the current drain through the tube and therefore shortening the tube life.

If the current wave form through the tube is irregular the life may still be predicted by dividing the current wave into elements small enough so that the current in each element may be considered constant. Each of these currents may then be considered as lasting for a certain fraction of the cycle. For each of the current values obtained there is a life in hours which may be determined from figure 1. One then has a series of currents $i_1, i_2, \ldots i_n$ each lasting for a fraction $t_1, t_2, \ldots t_n$ of the entire cycle and a tube life corresponding to these currents of $l_1, l_2, \ldots l_n$. The fraction of life used up at each current per hour is, then t/l. The total fraction of life used up per hour is

$$\frac{t_1}{l_1}+\frac{t_2}{l_2}+\ldots\frac{t_n}{l_n}$$

and the socket life of the tube is

$$\text{socket life}=\frac{1}{\frac{t_1}{l_1}+\frac{t_2}{l_2}+\ldots\frac{t_n}{l_n}}$$

Conclusions

For tubes manufactured in any given manner the life will be determined solely by the current drawn. In view of the fact that the tubes do not deteriorate when they are not passing current it seems reasonable to operate them at very large currents if the duty cycle is sufficiently light. It is suggested, therefore, that the most useful type of rating for cold cathode tubes would be to state the current which will give a life of 100, 1,000, and 10,000 hours.

References

1. Ein Neues Relais fur Extrem Schwache Strom, H. Richter and H. Geffeken. *Zeitschrift für technische Physik*, volume 7, 1926, pages 601–06.

2. Cold-Cathode Gas-Filled Tubes as Circuit Elements, S. B. Ingram. AIEE Transactions, volume 58, 1939 (July section), pages 342–6.

3. Theory of Cathode Sputtering in Low Voltage Glow Discharges, C. H. Townes, presented to the American Physical Society, Providence R. I., June 20, 1941.

4. Kathodenzerstaubungsprobleme, A. von Hipple. *Annalen der Physik*, volume 81, 1926, pages 1043–75.

5. The Vapor Pressure of Barium, E. Rudberg and J. Lumpert. *Journal of Chemical Physics*, volume 3, 1935, pages 627–31.

Dielectric Strength of Oil for High-Voltage Testing of Oil Circuit Breakers

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THE purpose of this paper is to report on work undertaken in response to a request, made to the Institute, to determine the minimum dielectric strength of oil for high potential testing of oil circuit breakers.

Many tests were made with 60-cycle and impulse voltages. The tests were made with a rod to plane gap, cone to disc gap, and on oil circuit breakers of commercial types and sizes. The results of these tests have been collected, analyzed, and are included in tabular form in the paper.

The impulse strength of oil circuit breakers was found to be practically unaffected by the 60-cycle breakdown strength of the oil within the range of 16 to 30 kv, as measured in a standard test cup.

The 60-cycle dielectric strength of oil circuit breakers is shown not to be changed in direct proportion to the differences in the dielectric strength of oil as measured in a test cup with 60-cycle voltage.

The effect of low dielectric strength oil on voltage breakdown is discussed and conclusions are drawn which show that it is desirable to test all oil circuit breakers with oil having dielectric strength of 22 kv or higher.

General

The general picture of oil breakdown for some years has involved the lining up of impurities over a period of time ranging from a small fraction of a second to several minutes, with breakdown taking place over the path so formed. From this standpoint it is to be anticipated that long gaps will be relatively less affected by the impurities in the oil than short gaps, and that breakdowns under impulse conditions, where time is not available for lining up of impurities would be less affected by impurities than breakdowns under a one minute test at operating frequency. This is supported by the fact that the impulse ratio of oil, as obtained by comparing Sorensen's[1] data on impulse tests with Miner's[2] data at 60 cycles is considerably greater than the impulse ratio for breakdowns over an air path. Hence, it would be expected that any equipment which will flash over in an air path in preference to an oil path at 60 cycles will have a good margin of safety for the oil under impulse conditions, and it is also to be expected that the dielectric strength of a completed assembly such as an oil circuit breaker would be much less affected by the presence of impurities, in the oil, particularly under impulse conditions, than would be

Table I. Rod-to-Plane Gap Impulse Tests

(See Figure 1)

Gap Spacing (Inches)	Oil	Highest Voltage Held Without Breakdown (Kv)	Lowest Voltage at Which Breakdown Occurred (Kv)
1	New	176	168
1	Old	165	166
2	New	200	186
2	Old	208	190

The tests were made with a $1^1/_2$–40 m.s. positive wave.

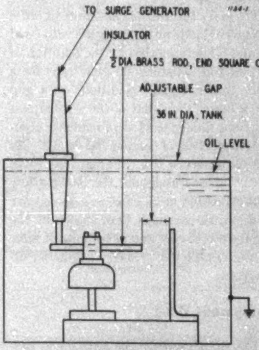

Figure 1. Rod-to-plane gap used in making tests shown in table I

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The authors of this paper are the members of a working group of the circuit breaker switch and fuse subcommittee of the AIEE protective devices committee, who were appointed to investigate and recommend the proper dielectric strength of oil for high-potential testing of circuit breakers.

[1] For all numbered references, see list at end of paper.

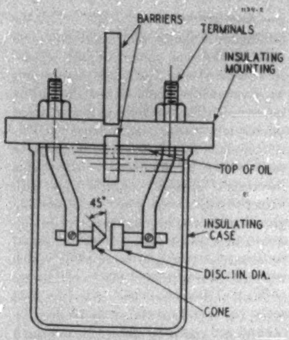

Figure 2. Cone-disc gap used in making tests shown in table II

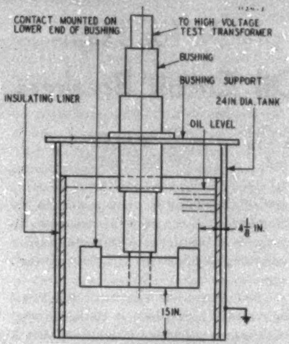

Figure 3. Large gap construction used in making tests in table III

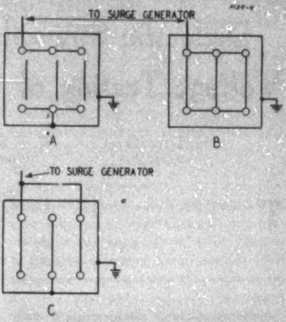

Figure 4. Connections for testing complete circuit breakers 1–10 inclusive, table IV

A. Test on oil across open breaker and to tank

B. Test on all bushings to ground

C. Test phase to phase and to ground

expected from values on oil dielectric strength obtained at 60 cycles in a standard test cup.

To verify this line of reasoning, impulse tests were made on sample gaps with good oil and oil deteriorated by the addition of carbon and moisture, 60-cycle tests were made under similar conditions on a gap much larger than that of the normal test cup, and finally tests were made on completed breakers of commercial design. As will be observed from the data set forth in the tables throughout the paper, these tests indicated in all cases less divergence of dielectric strength than shown in the standard test cup, and in the case of tests on the completed breakers negligible divergence.

Equipment Used

The data used by the authors in preparing this paper were obtained from tests made in the high voltage test laboratories of Westinghouse E.&M. Co. at East Pittsburgh, Pa., and General Electric Company at Schenectady, N. Y., and Philadelphia, Pa.

The oil circuit breakers on which the tests were made were complete commercial designs, sizes, and ratings.

The rod to plane gap was a special gap made by the laboratory and is shown in figure 1. When used for testing, the gap was immersed in a tank of oil.

The cone to disc gap on which tests were made is shown in figure 2.

The wet oil used in making the tests with the cone to disc gap was obtained by forming a saturated solution of water and oil at 80°C. and mixing in measured proportions with oil dried by bubbling dry air through it. The maximum water content was about 250 parts per million.

The used or carbonized oil was obtained from the dirty oil tanks in the laboratory and had previously been used for making oil circuit breaker tests until its dielectric strength had fallen to the point where it was considered necessary to recondition it before further use. In some cases the oil, having been taken from near the bottom of the tank, contained more than the usual amount of foreign material. When let stand without agitation for a short time, these materials tended to settle at the bottom of the tank. This accounts for the fact that in some cases the test value of the oil measured in a standard test cup with 60-cycle voltage varied from 4 to approximately 15 kv.

For the purpose of obtaining data on the dielectric strength of large gaps, using deteriorated oil, a 24-inch circuit breaker tank was filled with oil and a crosshead of corresponding size was supported in the tank in a position corresponding with that which it takes when the breaker is closed. An electrical connection was made to this crosshead through a high voltage bushing, and the tank itself was used as a ground terminal. See figure 3.

Impulse Tests on Rod-to-Plane Gap

Impulse tests were made on a rod to plane gap shown on figure 1. Tests were first made with new oil and then repeated with used oil. The new oil tested about 30 kv in a standard oil test cup. The

Table II. Cone-to-Disk Gap Impulse Tests (See Figure 2)

Ratio of Dielectric Strength Values

	60 Cycles		0.5 M.s.		1.5 M.s.		8 M.s.	
	Std. Gap	Cone Disk Gap	Cone Pos.	Cone Neg.	Cone Pos.	Cone Neg.	Cone Pos.	Cone Neg.
Good oil	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Carbonized oil	0.43	0.48	0.83	0.89	0.72	0.76	0.60	0.62
Wet oil	0.26	0.40	0.89	0.78	0.89	0.80	0.86	0.70

Table III. Low-Frequency Tests on Large Gaps (See Figure 3)

	Oil Dielectric Standard Test Cup Strength		60-Cycle Test With a Large Gap			
	Kv	Ratio	Voltage Held Quietly (Kv)	Noisy (Kv)	Breakdown Voltage (Kv)	Ratio* Max. Quiet
Good oil	30		130	135	150	
Carbonized oil	9	0.30	115	**	118	0.89
Wet oil	14	0.47	112	115	146	0.86

*The values in this column are the ratios of the maximum voltage held quietly by the deteriorated oil, to the maximum voltage held quietly in good oil.

**No noisy region found between voltage held quietly and breakdown.

used oil was taken from a breaker which had undergone severe interrupting duty. The breakdown of the used oil was erratic, varying from 5 to 15 kv.

In these tests, the gap was immersed in the tank filled first with new oil and then with used oil. The tests indicated that the impulse breakdown of the low test oil was as high as that of the new oil. Tests were made with the gap set at one inch and at two inches with results as given in table I.

The small difference in test voltage shown for the two gaps is possibly due to corona forming on the gap electrode at the higher voltage and reducing the voltage required to break down the longer gap.

Impulse Tests Made With a Cone-to-Disk Gap

The tests were made with a 45° cone disc gap, figure 2, with a spacing of 0.025″ using:

(*a*). Good oil
(*b*). Carbonized oil
(*c*). Wet oil

The tests were made with the time to crest at 0.5 m.s. and 8 m.s.

From these values a figure for 1.5 m.s. has been interpolated for use in connection with the standard impulse wave. Table II shows the ratio of dielectric strength values between the good oil taken as 1.00 and the carbonized and wet oils respectively. Each of the impulse values in table II was determined by making approximately 100 tests.

It will be observed that the presence of carbon or water in the oil does not affect the ratio under impulse conditions to the same extent as under steady state voltage conditions.

Tests Made With 60-Cycle Voltage on Large Spacings

Tests were made to determine how the variation in the dielectric strength of oil as measured by the standard gap is reflected in the dielectric strength of a larger assembly such as an oil circuit breaker; the equipment used in making these tests was described previously, and is shown in figure 3. High potential tests were made to determine what 60-cycle voltage could be held for one minute without breakdown or audible internal disturbance. Results are given in table III.

It will be observed that there is a much lower spread in the dielectric strength of oil in a large gap than might be anticipated from the spread in dielectric strength values with the standard gap.

Impulse Tests on Complete Oil Circuit Breakers With New and Old Oil

Impulse tests were made with a $1^1/_2$–40 m.s. positive wave on 12 complete oil circuit breakers to determine if the impulse strength of breakers would be appreciably lowered by carbon or water or both in the oil. The breakers were tested with new oil and then retested with old oil.

The new oil used to test breakers 1–10 inclusive was taken from standard package containers and tested 30 kv in a standard test cup.[3] The used oil with which these breakers were tested was taken from the dirty oil tanks in the high power laboratory and tested from 4 to 16 kv.

Breakers 1–10 inclusive were small three pole breakers with all three poles in a common tank. The results of the tests on these breakers are given in table IV which follows immediately.

Table IV. Tests on Complete Common Tank Breakers

Tests on Completed Circuit Breakers 1 to 10 Inclusive

Breakers Tested		Lowest Flashover (Kv) New Oil Testing 30 Kv	Lowest Flashover (Kv) Old Oil Testing 4–16 Kv	Highest Full Wave Withstood (Kv) New Oil Testing 30 Kv	Highest Full Wave Withstood (Kv) Old Oil Testing 4–16 Kv
Breaker 1		44	44	46	44.5
" 2		40.5	42.5	41.5	40.5
" 3		51.5	51.5	52.1	52.7
" 4		49	52.1	53.5	54
" 5		63.4	65.5	76	78
" 6		80	83.5	81	84
" 7	Closed	97	97	106	108.5
	Open	102	96	104	97.2
" 8	Closed	122	121	133	122
	Open	115	120	115	127
" 9	Closed	119	118	130	121
	Open	96	97	103	101
" 10		127	127	148	148

All flashovers were outside of breaker.

Each of the above values is the lowest flashover or highest full wave of 15 tests.

Tests on Complete Circuit Breaker 11

Breaker 11 was a single pole of a three pole breaker with each pole in a separate tank. The bushings in this breaker had a higher than usual flashover strength in relation to the breakdown strength of the oil between the contacts and the tank. This condition made it possible to obtain breakdowns through the oil in the tank. The tests were made with the breaker closed and both bushings connected to the surge generator. The results of the tests with oils of different dielectric strength are contained in table V. All tests were made with a $1^1/_2$–40 m.s. positive wave.

Test on Complete Circuit Breaker 12

The tests on breaker 12 were made on poles 1 and 2 of a three pole high voltage frame mounted breaker with each pole in a separate tank. Pole 1 was filled with good oil testing between 26 and 27 kv. Pole 2 was filled with oil taken from breakers in service on the shop lines. This oil tested slightly under 19 kv. For convenience in table VI these oils are considered to have test values of 26 kv and 19 kv respectively. The tests made on breaker 12 and shown in table VI show the 26-kv oil to have a slightly higher impulse breakdown value than the

Table V. Tests on Complete Separate Tank Breaker With Oversize Bushings to Secure Breakdown in Oil

	Oil at 30 Kv	Oil 34.5 Kv at Top and 16 Kv at Bottom	Oil at 23 Kv	Oil at 15–17 Kv
Breaker 11	250 kv *B*	226 kv	217 kv	294
	249 *B*	223	2-220	292
	246 *B*	229	223	297
	238 *B*	240 *B*	227	294
	237 *B*	240	228 *B*	302 *B*
	230 *B*	242	2-233	304 *B*
	235 *B*	245	228	2-293 *B*
	232 *B*	248	229	284
	223 *B*	2-256	231	278
	226 *B*	267	2-235	281
	2-226	272 *B*	244 *B*	287
	222	2-272	237	294
	224	278 *B*	2-236	295
	227	2-278	238	308
	226 *B*		239	2-297
	2-223 *B*		240	2-296 *F*
	225 *B*		242	294 *F*
	223		247 *B*	2-289
	226		2-246	294
	214 *B*		244	301
	212 *B*		2-250	303
	221 *B*		253 *B*	309 *F*
	227		2-260	
	228		2-265	
	226 *B*		265 *B*	
Average	228+	254−	241−	295+

B = Broke down through oil to tank.

F = Flashover bushing outside of breaker.

The numeral 2 appearing at the left of a test indicates that two tests were made.

Table VI. Tests on Complete Separate Tank Breaker With Various Striking Distances in Oil

	Full Striking Distance Through Oil		Striking Distance Through Oil Reduced to 35 Per Cent of Full Amount		Striking Distance Through Oil Reduced to 19 Per Cent of Full Amount	
	Oil at 26 Kv	Oil at 19 Kv	Oil at 26 Kv	Oil at 19 Kv	Oil at 26 Kv	Oil at 19 Kv
Breaker 12	3-351	3-351	3-258	2-258	258 *B*	3-192
	3-392	3-392	3-285	2-285	3-192	3-205
	3-432	3-432	3-312	2-312	3-205	2-218
	2-473 *F*	2-473 *F*	3-338	3-338	3-226	2-231 *B*
	5-432	452 *F*	3-372	3-372	245	3-231
	2-450 *F*	452	3-405	2-405	245 *B*	231 *B*
	450 *F*	452 *F*	446	405	245	231
	450	4-432	2-446 *F*	446 *F*	245 *B*	231 *B*
	450 *F*	432 *F*			245	5-244 *B*
	4-540 *F*	432			2-258 *B*	
		3-452			2-258	
		452 *F*			258 *B*	
		452			2-258	
		4-473 *F*			2-258 *B*	
		4-554 *F*			285 *B*	
					285	
					285 *B*	
					285	
					4-298 *B*	
					5-234	
					3-264 *B*	
					264	

The numerals 2, 4, 3, 5, appearing at the left of a number indicate the number of tests made at that particular voltage.

19-kv oil. This is contrary to the results obtained on breakers 1 to 11 inclusive and is probably due to the difference resulting from the two oils being tested in different tanks and on different poles of the breaker. All tests were made with the breaker open, one stud impulsed, frame and all other studs grounded. Approximately 15 seconds elapsed between tests. Both poles were tested and the results given in columns 1 and 2 of table VI. In these tests neither oil could be broken down. All flashovers—marked *F*—occurred in air over the outside end of the bushing. The poles under test were then changed by inserting 2″x$^1/_4$″ copper bar between the contacts and the tank cover to reduce the breakdown distance through the oil to the tank to 35% of the original distance. The tests in columns 3 and 4 were made and again the flashovers occurred through the air over the outer end of the bushing.

The breakdown distance through the oils was further reduced until it was only 19% of the original distance. Tests shown in columns 5 and 6 of table VI were then made. Breakdowns were obtained on pole 1 in good oil at a minimum of 245 kv. In pole 2 breakdowns were obtained through the 19-kv test oil at a minimum of 231 kv as indicated by the letter—*B*. All tests were made with a $1^1/_2$-40 m.s. positive wave.

Discussion of Test Results and Conclusions

The test data indicate that the presence in the oil of moisture, carbon, and other materials, normally tending to lower the 60-cycle dielectric strength of oil, appears not to change appreciably the ability of oil circuit breakers to withstand impulse voltages.

Table I shows that the impulse strength of large gaps in oil is practically constant for oils of different dielectric strength, within the range regularly used.

Table II shows that the impulse strength of small gaps in oil varies, but not directly, with the 60-cycle strength of the oil.

Table III shows that the 60-cycle breakdown strength of the breaker is reduced when the dielectric strength of the oil is lowered, but much less than in proportion to the loss of strength in the oils as measured in a standard oil test cup.

Table IV shows that the [4]impulse strength of ten small oil circuit breakers of different types and ratings was not lowered when they were tested with poor oil.

Data in table V, from tests made on a medium size outdoor breaker, show that the impulse strength of an oil circuit breaker might in some cases be better with poor oil than with good oil.

Data in table VI, from tests made on a large outdoor oil circuit breaker, show the impulse strength of large breakers to be practically unaffected by the 60-cycle strength of the oil within the range regularly used.

Oil for circuit breakers and transformers[4] is purchased with a minimum breakdown value of 22,000 volts. Since oil of this or higher strength is most easily obtained, the authors recommend that the oil used for high potential testing of oil circuit breaker, both 60 cycle and impulse, should have a dielectric breakdown strength of 22 kv or higher, the breakdown strength to be determined as prescribed by the American Society for Testing Materials.[3]

It is to be noted that apart from any reduction in dielectric strength resulting directly from the use of deteriorated oil as an insulating medium, there is a tendency for carbon particles to settle out from the oil upon insulating surfaces. This may in time decrease the creepage strength of these surfaces much more than the breakdown strength through the insulating medium is decreased, and constitutes the greatest danger from allowing poor oil to remain in the breaker. Consequently, the test results reported in this paper are not to be interpreted as suggesting any relaxation of maintenance activities in this respect.

References

1. SURGE-VOLTAGE BREAKDOWN CHARACTERISTICS FOR ELECTRICAL GAPS IN OIL, **Royal W. Sorensen.** AIEE TRANSACTIONS, volume 59, 1940 (February section), pages 78–81.

2. OIL BREAKDOWN AT LARGE SPACINGS, **Douglas F. Miner.** AIEE TRANSACTIONS, volume 46, 1927, page 248.

3. STANDARD METHODS OF TESTING INSULATING OILS. ASTM *Standards* D117-36.

4. ASA PROPOSED AMERICAN STANDARDS FOR TRANSFORMERS, REGULATORS, AND REACTORS. C57. 1-2.050.

Conductor Vibration—Theory of Torsional Dampers

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Synopsis: The paper is a theoretical treatment of the torsional damper which has been applied effectively to the protection of transmission line conductors from fatigue failures due to aeolian vibrations. The formulae of the energy dissipation are derived for three cases, viz., a single rigid damper, two rigid dampers at one end of a span, and a single damper with a resilient joint at the centre of gravity. The results are valuable in understanding the mode of operation of the torsional damper and point to certain optimum features of design. The formulae can be used for numerical calculations when the physical constants are known.

Introduction

FOR many years, electric power companies have been investigating the protection of transmission line conductors from fatigue failures that result from the vibrations excited by steady cross winds of low velocity. The methods of protection that have been proposed are very diversified in character, including the prevention of the vibration at its source with conductors of non-circular cross section, the reduction of concentrated stresses at the clamp by the use of reinforcing cables or rods, and the suppression of the vibrations by the application of damping devices. An energy-absorbing damper has proven to be the most practical solution up to the present time, and has been adopted for a number of lines in North America. An estimate of the efficiency of such devices has been determined, in a few instances, by experimental field studies and, more frequently, by operating experience.

The Hydro-Electric Power Commission of Ontario with power lines of all sizes throughout the province has, of course, been vitally interested in the problem of aeolian conductor vibrations. The need for an effective solution has been most important in recent years since, in modern construction using long spans and high tensions, the conductor is subject to more severe conditions of vibration than in the older construction with short spans and low tensions. To study the effectiveness of various protective schemes, a method was devised for recording the natural vibrations of damped and undamped lines and for analyzing the results statistically. Such experimental data on the frequency characteristics of a type of conductor are essential for the proper design of a protective device for that particular type.

In the year 1935, the theoretical and practical studies indicated that the conductor itself was available to absorb energy, if a suitable restraining force could be applied to the conductor to utilize the interstrand friction and the mechanical hysteresis loss. This force was accomplished with an offset weight attached to the conductor by a horizontal arm, so that a torsional motion was produced between the suspension clamp and point of attachment of the weight when the conductor vibrated. Additional damping was obtained by mounting the offset weights in pairs adjacent to and at different distances from the suspension clamp, when, for most frequencies of vibration, the section of conductor between the two offset weights was also in torsional motion. The torsional device which has become known as a torsional damper has been developed considerably in the last five years and is now practical.

Torsional dampers have been installed for the suppression of aeolian vibrations on the latest ACSR lines built by the Hydro-Electric Power Commission of Ontario. A torsional damper designed for 795,000 cir-mil ACSR is shown in figure 1 and is constructed with a resilient joint of rubber. In California, a rigid type of torsional damper has been adopted for the protection of H-H copper conductor on the exposed sections of the Boulder Dam transmission line.

Theory

The theoretical development of the torsional damper was studied along with the experimental work, and it is the purpose here to derive the theoretical formulae. The formulae were of great assistance in bringing the torsional damper to its present form and are valuable in designing dampers for specific conductors and predicting the approximate performance. The analysis of the torsional damper is limited here to the condition of steady state vibration, and does not include other possible beneficial effects which might exist under initial transient conditions. The reaction of the damper on the motion of the conductor is not taken into account either, since, for the suppression of vibration, the energy dissipative capacity of the damper is of prime importance and is affected only by the amplitude of the motion at the point of attachment, all the other factors being constant. The application of the formulae to actual numerical cases is direct, provided that the requisite physical constants are available.

The mode of vibration at a given point of a line conductor in service usually has a beat characteristic and not a constant maximum amplitude. However, the generality of the analysis is not affected, if it is considered that the conductor vibrates like a perfectly flexible string under tension[1] having the motion

$$y = A_0 \sin (2\pi x/\lambda) \sin \omega t$$
$$= A \sin \omega t \qquad (1)$$

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1. For all numbered references, see list at end of paper.

where

A_0 is the maximum amplitude
λ is the wave length
ω is the angular velocity $2\pi f$
f is the frequency

The differential equation for free torsional motion is

$$I\ddot{\theta}+q\dot{\theta}+\tau\theta=0 \tag{2}$$

where I is the moment of inertia and τ is the angular elastic constant.

With

$$\alpha=q/2I \text{ and } \omega_c^2=\tau/I$$

the solution is

$$\theta=B_e^{-\alpha t}\sin(\omega t+\phi)$$

in which B and ϕ are constants and $\omega^2=\omega_c^2-\alpha^2$. If $\alpha\ll\omega_c$,

$$\omega^2\approx\omega_c^2=\tau/I$$

Therefore, the quantity q which will be defined as the "torsional resistance" is given by

$$q=\delta\tau/\pi\omega \tag{3}$$

because $\alpha=\delta f$ where δ is the logarithmic decrement.[3]

For a rod of circular cross section, the angular elastic constant is inversely proportional to the length of the rod.[2,3] Hence, it is evident from relation 3 that the torsional resistance of a conductor is increased by decreasing the length of the effective part of the conductor.

Single Rigid Damper

The position of a single rigid damper at any moment can be represented as in figure 2. The notation is

M = mass of damper
$I=MK^2/g$ = moment of inertia about the axis of the conductor
$I_G=Mk^2/g$ = moment of inertia about the centre of gravity
l = distance from the centre of conductor to centre of gravity
θ = angular displacement from the horizontal
$y=A\sin\omega t$ = displacement of the point of support
h = distance of centre of gravity from position of equilibrium
$K^2=k^2+l^2$

The velocity of the centre of gravity for small oscillations is $(\dot{y}-l\dot{\theta})$. Considering the transverse energy of the conductor to be independent of θ, the kinetic energy of the damper is

$$S=M(\dot{y}-l\dot{\theta})^2/2g+I_G\dot{\theta}^2/2$$

The potential energy associated with the torsional motion is

$$\tau(\theta+\theta_0)^2/2-\tau\theta_0^2/2=\tau\theta^2/2+Ml\theta \tag{4}$$

since θ_0 is the static rotation and $Ml=\tau\theta_0$. The increase in the gravitational potential energy of the mass is

$$Mh\approx M(y-l\theta) \tag{5}$$

Therefore, the potential energy function is, adding 4 and 5,

$$V=\tau\theta^2/2+My$$

The Lagrangian function is $L=S-V$ and, from Lagrange's equation[4]

$$\frac{d}{dt}\left[\frac{\partial L}{\partial\dot{\theta}}\right]-\frac{\partial L}{\partial\theta}=0 \tag{6}$$

it follows that, adding the dissipative term,

$$\ddot{\theta}+2\alpha\dot{\theta}+\omega_c^2\theta=l\ddot{y}/K^2 \tag{7}$$

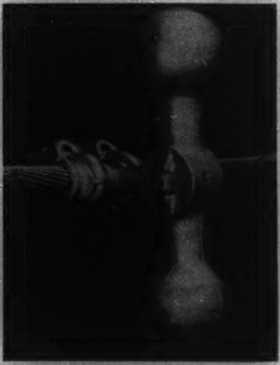

Figure 1. Torsional damper for 795,000-circular-mil steel-reinforced aluminum cable

The steady state solution of differential equation 7 is

$$\theta=-\frac{lA\omega^2}{K^2Z}\sin(\omega t-\beta) \tag{8}$$

with

$$Z^2=(\omega_c^2-\omega^2)^2+4\alpha^2\omega^2$$

and

$$\beta=\tan^{-1}[2\alpha\omega/(\omega_c^2-\omega^2)]$$

The power dissipation is

$$P=\frac{1}{T}\int_0^T q\dot{\theta}^2dt$$
$$=ql^2A^2\omega^6/2K^4Z^2 \tag{9}$$

The graph of the frequency function $f^6/[\pi^2(f_c^2-f^2)^2+\alpha^2f^2]$ contained in equation 9 for constant amplitude is plotted in figure 3 with $f_c=5$ cps. and $\alpha=4$. Actually, most materials dissipate energy by solid damping[3] and then δ, not α, is constant with respect to frequency. From the curve, it is seen that to suppress a range of frequencies the damper has to operate on the region above f_c, i.e.,

$$\omega_c<\omega \tag{10}$$

In determining the damper spacing, the amplitude A which is a function of the frequency is considered in conjunction with the other factors. As a numerical example, let the velocity of propagation (λf) be 400 ft./sec. and the damper be 5 feet from the clamp. For a frequency range 5 to 20 cps.,

$$1\geqq\sin(2\pi x/\lambda)>0.38$$

and the power loss at 20 cps. due to this factor is about seven times that at 5 cps. The magnitude of the divergence is reduced greatly, in practice, because the power loss associated with the transverse vibrations of a conductor excited by the wind increases rapidly with frequency and, consequently, the amplitude A_0 becomes smaller.[5] Thus, the spacing of 5 feet gives a satisfactory response to the frequency variations under these conditions.

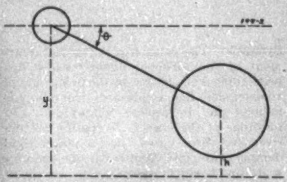

Figure 2. Schematic diagram of a single rigid damper in motion

If the condition $\omega_c\ll\omega$ is maintained, the optimum value of l is given by

$$\frac{d}{dl}(l^2/K^4)=0$$

or,

$$l=k \tag{11}$$

Then, the maximum power is

$$P_M=qA^2\omega^6/8l^2Z^2$$

from which it is deduced that the optimum optimum magnitude of l is the smallest value consistent with conditions 10 and 11.

For a damper with a heavy, concentrated mass and a long arm, $K\to l$; $\beta\to\pi$; $Z\to\omega^2$ and

$$\theta=(A/l)\sin\omega t$$

i.e., the centre of gravity remains stationary in space. This effect is observed in the field for the stated conditions. The power dissipation for this case, substituting the value of q, becomes

$$P=\delta_r f A^2/l^2$$

Two Rigid Dampers at One End of Span

In figure 4, the arrangement of two dampers having the same physical dimensions is shown schematically for the horizontal plane. The notation of the previous section is adopted with the subscripts 1 and 2 referring respectively to dampers 1 and 2. The quantities τ_1, τ_2 and τ_3 represent the angular elastic constants for the three sections of the conductor between the clamps and the two dampers. The displacements at the points of attachment are

$$y_1=A_1\sin\omega t$$

$$y_2=A_2\sin\omega t$$

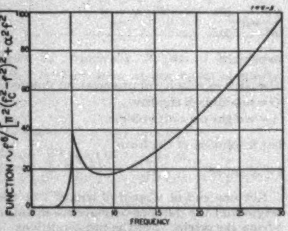

Figure 3. Graph of function for $f_c=5$ cycles per second and $\alpha=4$

The kinetic energy of the system is

$$S=(M/2g)[(\dot{y}_1-l\dot{\theta}_1)^2+(\dot{y}_2-l\dot{\theta}_2)]+I_G(\dot{\theta}_1^2+\dot{\theta}_2^2)/2 \quad (12)$$

Disregarding the gravitational potential energy function which does not contribute to the motion, the potential energy is

$$V=\tau_1\theta_1^2/2+\tau_3(\theta_1+\theta_2)^2/2+\tau_2\theta_2^2/2 \quad (13)$$

If the mutual torsional resistance is designated by q_3, the differential equations of motion with the dissipative terms are

$$\ddot{\theta}_1+2\alpha_1\dot{\theta}_1+\omega_1^2\theta_1+2\alpha_3\dot{\theta}_2+\Omega^2\theta_2=l\ddot{y}_1/K^2$$

$$\ddot{\theta}_2+2\alpha_2\dot{\theta}_2+\omega_2^2\theta_2+2\alpha_3\dot{\theta}_1+\Omega^2\theta_1=l\ddot{y}_2/K^2$$

with

$\omega_1^2=(\tau_1+\tau_3)/I$ and $\alpha_1=q_1/2I$

$\omega_2^2=(\tau_2+\tau_3)/I$ $\qquad \alpha_2=q_2/2I$

$\Omega^2=\tau_3/I$ $\qquad \alpha_3=q_3/2I$

Making the substitutions

$$N_1e^{j\beta}=A_1(\omega_2^2-\omega^2)-A_2\Omega^2+2j\omega(\alpha_2A_1-\alpha_3A_2)$$

$$N_2e^{j\gamma}=A_2(\omega_1^2-\omega^2)-A_1\Omega^2+2j\omega(\alpha_1A_2-\alpha_3A_1)$$

$$De^{j\epsilon}=(\omega_1^2-\omega^2)(\omega_2^2-\omega^2)-\Omega^4+4\alpha_3^2\omega^2-4\alpha_1\alpha_2\omega^2+2j\omega[\alpha_1(\omega_2^2-\omega^2)+\alpha_2(\omega_1^2-\omega^2)-2\alpha_3\Omega^2]$$

it follows that

$$\theta_1=-\frac{l\omega^2N_1}{K^2D}\sin(\omega t+\beta-\epsilon)$$

$$\theta_2=-\frac{l\omega^2N_2}{K^2D}\sin(\omega t+\gamma-\epsilon)$$

The power dissipated by the two dampers is

$$P=\frac{1}{T}\int_0^T[(q_1\dot{\theta}_1+q_3\dot{\theta}_2)\dot{\theta}_1+(q_2\dot{\theta}_2+q_3\dot{\theta}_1)\dot{\theta}_2]dt$$

$$=\frac{l^2\omega^6}{2K^4D^2}[q_1N_1^2+2q_3N_1N_2\cos(\beta-\gamma)+q_2N_2^2] \quad (14)$$

It is interesting to observe that the second term may increase or decrease the energy loss depending on whether N_1 and N_2 have the same phase or opposite.

If the moment of inertia is large and the arm is relatively long, the following approximations hold

$$\left.\begin{array}{l} N_1\rightarrow A_1\omega^2 \\ N_2\rightarrow A_2\omega^2 \\ D\rightarrow\omega^4 \\ K\rightarrow l \\ \beta\rightarrow\gamma\rightarrow\pi \\ \epsilon\rightarrow 0 \end{array}\right\} \quad (15)$$

Therefore, from equation 14,

$$P=\frac{\omega^2}{2l^2}(q_1A_1^2+2q_3A_1A_2+q_2A_2^2) \quad (16)$$

From relation 13 and the fact that the logarithmic decrement is equal to the energy loss per cycle divided by twice the maximum potential energy,[3] the power dissipation for conditions 15 may also be written

$$P=\frac{\delta f}{l^2}[(\tau_1+\tau_3)A_1^2+2\tau_3A_1A_2+(\tau_2+\tau_3)A_2^2] \quad (17)$$

The corresponding coefficients in expressions 16 and 17 are identical, so that the torsional resistances are

$$q_1=\delta(\tau_1+\tau_3)/\pi\omega$$

$$q_2=\delta(\tau_2+\tau_3)/\pi\omega$$

$$q_3=\delta\tau_3/\pi\omega$$

Single Damper With Resilient Joint at Center of Gravity

The inherent damping in some conductors is not sufficient in itself to suppress the vibrations adequately, even with two rigid dampers. Supplementary damping is introduced then by making a resilient joint near the centre of gravity with a material of high damping such as rubber. To facilitate the analysis, the damper is assumed to have the joint at the centre of gravity as in figure 5. In practice, the dampers are installed at both ends of the span and the power dissipation per span is twice the derived formula.

The notation will be the same as before with the subscript r designating the physical constants of the resilient material. The kinetic energy at any moment is

$$S=Mk^2\dot{\Phi}^2/2g+M(\dot{y}-l\dot{\theta})^2/2g$$

The potential energy is

$$V=\tau\theta^2/2+\tau_r(\Phi-\theta)^2/2$$

The damping force of the resilient joint is proportional to the relative angular velocity $(\dot{\Phi}-\dot{\theta})$. Hence, the differential equations of motion including the dissipative terms become

$$\ddot{\Phi}+2\alpha_r(\dot{\Phi}-\dot{\theta})+\omega_r^2(\Phi-\theta)=0 \quad (18)$$

$$\ddot{\theta}+2\alpha_N\dot{\theta}-2\alpha_r\frac{k^2}{l^2}(\dot{\Phi}-\dot{\theta})+\left(\omega_N^2\theta-\omega_r^2\frac{k^2}{l^2}(\Phi-\theta)=\ddot{y}/l\right)$$

where

$$\alpha_N=qg/2Ml^2$$

$$\alpha_r=q_rg/2Mk^2$$

$$\omega_N^2=\tau g/Ml^2$$

$$\omega_r^2=\tau_rg/Mk^2$$

But

$$y=A\sin\omega t$$

Therefore, making the substitutions

$$Q_1e^{j\mu}=\frac{A\omega^2}{l}(\omega_r^2-\omega^2+2j\alpha_r\omega)$$

$$Q_2e^{j\nu}=\frac{A\omega^2}{l}(\omega_r^2+2j\alpha_r\omega)$$

$$\Delta e^{j\zeta}=(\omega_r^2-\omega^2)(\omega_N^2-\omega^2)-\omega^2\omega_r^2k^2/l^2-4\alpha_N\alpha_r\omega^2+2j\omega[\alpha_N(\omega_r^2-\omega^2)+\alpha_r(\omega_N^2-\omega^2-\omega^2k^2/l^2)]$$

it follows that

$$\left.\begin{array}{l} \theta=-\dfrac{Q_1}{\Delta}\sin(\omega t+\mu-\zeta) \\ \Phi=-\dfrac{Q_2}{\Delta}\sin(\omega t+\nu-\zeta) \end{array}\right\} \quad (19)$$

The undamped natural frequencies of the damper are obtained by equating Δ, with the α-terms eliminated, to zero, i.e.,

$$f^4-f^2(f_N^2+f_r^2+f_r^2k^2/l^2)+f_r^2f_N^2=0 \quad (20)$$

The damper is designed so that the lower natural frequency is below the range of frequencies to be suppressed, as discussed in relation to figure 3. When the torsional resistance of the resilient material is greater than that of the conductor, the range of frequencies below the higher natural frequency of equation 20 will not be suppressed adequately if the mode of vibration about the conductor has the lower frequency. In such a case, it is necessary to decrease the frequency f_r of the mode associated with the resilient material.

The rate of energy dissipation is

$$P=\frac{1}{T}\int_0^T [q\dot{\theta}^2+q_r(\dot{\theta}-\dot{\Phi})^2]dt$$

$$=\frac{\omega^2}{2\Delta^2}[qQ_1^2+q_r\{Q_1^2+Q_2^2-2Q_1Q_2\cos(\mu-\nu)\}] \quad (21)$$

The last term approaches a maximum contribution as $(\mu-\nu)\rightarrow\pi$, and its minimum as $(\mu-\nu)\rightarrow 0$. For an undamped condition, equation 18 can be written

$$(\omega_r^2-\omega^2)\Phi=\omega_r^2\theta$$

It is evident that $\theta=-\Phi$ is possible, in this case, only at one frequency, viz., $f=\sqrt{2}f_r$. The relation $\theta=\Phi$ can exist if $f=0$ or ω_r is very large. The first condition is not satisfied; the second is present for a large τ_r and the device would be a rigid damper. Consequently, the resilient joint has a dissipative effect at all operating frequencies, although it reaches a maximum displacement only at a unique frequency.

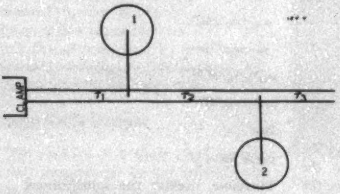

Figure 4. Schematic arrangement of two rigid dampers at one end of a span (horizontal plane)

For a large mass, which is necessary to obtain the low undamped natural frequency, and a high conductor frequency, the following approximations hold:

$$Q_1\rightarrow A\omega^4/l$$
$$Q_2\propto\omega^3$$
$$\Delta\rightarrow\omega^4$$
$$\mu\rightarrow\pi$$
$$\nu\rightarrow\pi/2$$
$$\zeta\rightarrow 0$$

With these values, equations 19 and 21 become

$$\Phi\approx 0$$

$$\theta=(A/l)\sin\omega t$$

$$P=\frac{\omega^2A^2}{2l^2}(q+q_r)$$

Using the mathematical reasoning of the previous section and relation 3, it can be shown that

$$q=\delta\tau/\pi\omega$$

$$q_r=\delta_r\tau_r/\pi\omega$$

Physical Constants

The logarithmic decrement and angular elastic constant are required for numerical computations and can be found experimentally, in one way, from the decrement curves of free torsional motion.

To illustrate the magnitude of these quantities: For a 795,000 cir-mil ACSR, $\delta\approx 0.19$ and τ per foot varies from 230 lb.-ft./rad. at 3,000 pounds tension to 360 lb.-ft./rad. at 5,700 pounds tension. For a 336,000 cir-mil ACSR, $\delta\approx 0.20$ and τ per foot is 46 lb.-ft./rad. at 1,200 pounds tension increasing to a constant value of 78 lb.-ft./rad. at approximately 3,600 pounds tension.

Summary

In the theoretical treatment of the torsional damper, the following conclusions pertaining to the general design of dampers have been deduced:

1. The angular elastic constant and, thus, the torsional resistance are inversely proportional to the length of effective conductor.
2. The undamped natural frequency of the damping system has to be below the lowest prevalent conductor frequency to be suppressed.
3. The damper is placed at the point of highest amplitude, consistent with maintaining a low natural frequency and a high torsional resistance. The effect of amplitude variations over the frequency band on the determination of spacing is not very critical, because the damper has to be effective over such a large range of loop lengths (possibly 5 to 50 feet).
4. For a rigid damper, the distance from the centre of the conductor to the centre of gravity should be as short as possible and equal to the radius of gyration about the centre of gravity. The damper, applied singly, should be placed near the clamp and still satisfy conclusions 2 and 3.
5. With the arrangement of figure 4, the two rigid dampers are mounted in the same loop in order to take advantage of the mutual dissipation. The optimum spacings, which will be short for large torsional resistances, could be determined by calculation for specific cases.
6. The single damper with a resilient joint is required for conductors having a small damping capacity. This type of damper has an advantage over the rigid type in that the damping factor of the resilient material can be varied, whereas that of the conductor is fixed. The spacing for this device is found in the same manner as for the rigid damper. As an example, the torsional damper of figure 1 is installed 6 feet from the clamp on 795,000 cir-mil ACSR.

Appendix

The rigid analysis of the motion of a single rigid damper will be developed to show that the above solutions are approximations strictly valid for short spans.

The torsional wave equation

$$\frac{\partial^2\theta}{\partial t^2}=a^2\frac{\partial^2\theta}{\partial x^2}$$

in which

$a^2=Gg/\gamma=$ velocity of propagation squared
$G=$ modulus of rigidity
$\gamma=$ weight per unit volume

has a solution of the form

$$\theta=X\sin\omega t$$

Let one end of a span of length s be the origin and the damper be placed a distance b from the origin. For the end conditions

$$\theta=0 \text{ at } x=0 \text{ and } x=s,$$

the space function can be written

$$X_1=C\sin\frac{\omega x}{a}: \qquad 0\geq x\geq b$$

$$X_2=D\sin\frac{\omega(x-s)}{a}: \quad b\geq x\geq s$$

The conditions to be satisfied at $x=b$, with I_p representing the moment of inertia of the area of cross section of the conductor, are

$$\left.\begin{aligned}&\theta_1=\theta_2\\&I\frac{\partial^2\theta_1}{\partial t^2}=-GI_p\left[\frac{\partial\theta_1}{\partial x}-\frac{\partial\theta_2}{\partial x}\right]\end{aligned}\right\}\quad (22)$$

Eliminating C and D, the natural frequencies are given by

$$\tan\frac{\omega(b-s)}{a}\left[\tau'\cot\frac{\omega b}{a}-a\omega I\right]=\tau'$$

where

$$\tau'=GI_p$$

For a practical installation, $s\gg b$ and b is small. Then, with $\beta=\omega s/a$,

$$\tan\beta=\frac{\tau'b\beta}{\beta^2a^2bI/s-\tau's} \quad (23)$$

having consecutive roots $\beta_i(i=1, 2, 3 \ldots .)$.

When β has a value such that $\tan\beta \rightarrow \beta$, the natural frequency becomes

$$\omega_c^2 \approx \tau'/bI \tag{24}$$

which agrees with the approximate theory.

For forced oscillations, the solution in generalized co-ordinates is

$$\theta_1 = \sum_{i=1}^{i=\infty} x_i \sin\frac{\beta_i x}{s}; \qquad 0 \geq x \geq b$$

$$\theta_2 = \sum_{i=1}^{i=\infty} x_i \left(\frac{D}{C}\right)_{\substack{\beta=\beta_i\\ x=b}} \sin\frac{\beta_i(x-s)}{s}; b \geq x \geq s$$

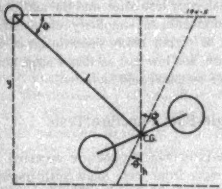

Figure 5. Schematic diagram of a damper with a resilient joint at the center of gravity

The potential energy function is

$$V = \frac{\tau'}{2}\left[\int_0^b \left(\frac{\partial\theta_1}{\partial x}\right)^2 dx + \int_b^s \left(\frac{\partial\theta_2}{\partial x}\right)^2 dx\right]$$

$$= \tau'\beta_i(A_i+B_i)x_i^2/8S^2$$

with

$$A_i = 2\beta_i\left[b-(b-s)\left(\frac{D}{C}\right)^2_{\substack{\beta=\beta_i\\ x=b}}\right]$$

$$B_i = s\left[\sin\frac{2\beta_i b}{s} - \left(\frac{D}{C}\right)^2_{\substack{\beta=\beta_i\\ x=b}} \sin\frac{2\beta_i(b-s)}{s}\right]$$

The kinetic energy function is

$$S = \frac{\gamma I_p}{2g}\left[\int_0^b \left(\frac{\partial\theta_1}{\partial t}\right)^2 dx + \int_b^s \left(\frac{\partial\theta_2}{\partial t}\right)^2 dx\right] + \frac{I_G}{2}\dot{\theta}_{x^2=b} + \frac{M}{2g}(\dot{y}-l\dot{\theta})_{x^2=b}$$

$$= \left[\frac{\gamma I_p(A_i-B_i)}{8g\beta_i} + \frac{I_G}{2}\sin^2\frac{\beta_i b}{s}\right]\dot{x}_i^2 + \frac{M}{2g}\left(\dot{y} - l\dot{x}_i \sin\frac{\beta_i b}{s}\right)^2$$

Applying Lagrange's equation, the differential equation for x_i is:

$$\left[\frac{\gamma I_p(A_i-B_i)}{4g\beta_i I} + \sin^2\frac{\beta_i b}{s}\right]\ddot{x}_i + \frac{\tau'\beta_i(A_i+B_i)}{4I_g^2} x_i = \frac{l\ddot{y}}{K^2}\sin\frac{\beta_i b}{s}$$

For the condition $\tan\beta \rightarrow \beta$

$$A_i \rightarrow B_i \rightarrow 2\beta_c b$$

and the differential equation simplifies to

$$\left(\sin\frac{\beta_c b}{s}\right)(\ddot{x}_c + \omega_c^2 x_c) = l\ddot{y}/K^2$$

Hence, for $K \rightarrow l$ and $\omega \gg \omega_c$

$$\theta_{x=b} = (A/l)\sin\omega t$$

The general solution of torsional dampers with the damping constants is obviously complex. The single natural frequency of the approximate solution is replaced by a series of frequencies in the general solution of a long span, since the distributed mechanical system of the conductor becomes a major part of the vibratory system. However, the lowest natural frequency of equation 23 is less than that of equation 24 for long spans and a damper designed for the approximate conditions will be effective when $\tan\beta \neq \beta$.

References

1. PRINCIPLES OF MATHEMATICAL PHYSICS (a book), **Houston**. McGraw-Hill Publishing Company.

2. GENERAL PHYSICS FOR STUDENTS (a book), **Edser**. Macmillan and Company.

3. VIBRATION PREVENTION IN ENGINEERING (a book), **Kimball**. John Wiley and Sons.

4. VIBRATION PROBLEMS IN ENGINEERING (a book), **Timoshenko**. D. Van Nostrand Company.

5. LABORATORY STUDIES OF CONDUCTOR VIBRATION, **J. S. Carroll**. AIEE TRANSACTIONS, volume 55, 1936 (May section), page 543.

6. DYNAMICS OF A PARTICLE AND OF RIGID BODIES (a book), **Loney**. Cambridge University Press.

7. VIBRATION OF CABLES AND DAMPERS, **R. G. Sturm**. AIEE TRANSACTIONS, volume 55, 1936, part I, (May section), page 455; part II, (June section), page 673.

8. VIBRATION AND FATIGUE IN ELECTRICAL CONDUCTORS, **A. E. Davison, J. A. Ingles, and V. M. Martinoff**. AIEE TRANSACTIONS, volume 51, 1932, page 1047.

9. CONDUCTOR VIBRATION—EXPERIENCE ON OVERHEAD LINES IN NORTH AMERICA, **G. W. Preston**. *The Electrician*, October 15, 1937.

10. PARTIAL DIFFERENTIAL EQUATIONS (a book) **Bateman**. Cambridge University Press.

The Protection of Solid Insulation by Lightning Arresters

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Introduction

THE protection of distribution transformers or other apparatus against lightning involves the basic problem of limiting the voltage applied to the insulation to a value adequately below the insulation strength. The effect on insulation of chopped waves and full waves of standard impulse wave shapes such as 1.5x40 microseconds has been thoroughly investigated in the past. However, there has been a lack of the information necessary to compare the effect of chopped waves with the effect of voltages of other than standard wave shape resulting from arrester breakdown followed by the impedance drop that occurs during the impulse current discharge. Also there have been little data available on the failure of low voltage insulation. For these reasons an investigation has been made using oil immersed solid insulation of a type used in distribution transformers. The puncture voltage and number of shots to failure were determined when the insulation was in parallel with lightning arresters or gaps which controlled the wave shape of the voltage applied to the insulation. These results have been correlated with the single shot puncture voltage determined from tests in which failure occurred on the wave front. It was found that repeated applications of voltages in excess of approximately 70 per cent of the single shot puncture voltage may damage the insulation.

Insulation Samples

The insulation tested consisted of pads of 10- by 10-inch sheets of kraft paper 5 mils in thickness. Thin sheets of paper were used so that the failure voltage could be varied in small steps by changing the number of sheets in series. The line electrodes were pancake coils 2 inches in outer diameter and 1 inch in inner diameter wound from 85- by 155-mil copper, see figure 1. The turns of the line electrodes were wrapped with paper insulation of a type used in transformers. This insulation raised the corona voltage and made the failure voltage of the insulation pads less erratic. The turns of the coils were tied with 15-mil thread since thread and tape are sometimes used on transformer coils. The maximum thickness of the coil insulation at points of contact with the paper was about 20 mils but was variable due to the thread and the paper wrapping. At the ends of the coil, the copper strips were bent perpendicular to the plane of the coil and brought up through a 15-lb cylindrical weight 2 inches in outside diameter used to assure good contact. These leads were connected in parallel to the impulse generator. The pads were immersed in transformer oil in a metal tray that served as the ground electrode. Both the coils and the paper were vacuum treated and oil impregnated before the tests to eliminate moisture and absorbed air. The samples were kept under oil at all times until failure. Although all of the materials in the pads are used for certain types of insulation in transformers, the pads as tested cannot be considered[1] as representing the actual insulation strength of transformers, since in transformers steep impulse waves may result in an unequal distribution of voltage along the turns and the major insulation does not consist of paper sheets alone. In designing the insulation test pieces, it was necessary to consider the requirement of simulating transformer insulation and the additional requirements of simplicity of construction to permit ready observation of the failure, and low cost so that a large number of samples could be tested.

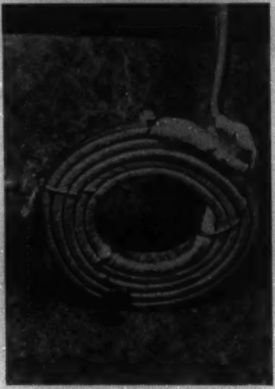

Figure 1. Insulation sample after puncture

A line electrode turned on edge to expose puncture and a small section of the sheet of 10- by 10-inch kraft paper used are shown. The bottom of an oil-filled metal tray formed the ground electrode

Single-Shot Puncture Tests

Tests in which puncture occurred on the wave front were made with impulse voltages of negative polarity that rose at a uniform rate until chopped by insulation puncture. A new line electrode and new sheets of paper were used for each test. The puncture voltage and time were determined, by means of cathode ray oscillograms, see figure 2, for line electrodes alone and line electrodes in series with pads of two, five, and eight sheets of paper. As the thickness of the insulation pads was changed the constants of the impulse generator circuit were adjusted so as to produce puncture at approximately the desired time. In those cases where the wave initially departed from a uniform rate of rise, the virtual zero was determined by a chord through the 30 and 90 per cent points on the voltage oscillogram. The results are plotted in figure 3. The failure in all cases was due to puncture through the paper and tended to occur near the outside of the line electrodes; corona was observed during tests of pads with five and eight sheets of paper.

Since it is desirable to base average curves on all of the available test data, the test results were correlated by means of figure 4 in which the average puncture voltage for failure times of 0.2, 1, and 8 microseconds was plotted against the paper thickness. The solid line curves of figure 3 give the puncture voltage from tests on line electrodes alone and pads with two, five, and eight sheets of paper; all of the original data points were plotted. The broken line curves for three, four, six, and seven sheets were obtained by interpolation from figure 4.

From figure 3 the puncture voltage at

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[1] For all numbered references, see list at end of paper.

0.25 microsecond is about 116 per cent of the value at 1 microsecond. Tests by Montsinger[1] gave a ratio of 160 per cent for the failure voltages at the same value of time. It is suggested that this difference is due to the considerable differences in the structure of the insulation samples tested in the two investigations and to differences in the applied voltage waves. Montsinger's tests were made on insulation barriers 0.64 and 2.5 inches in total thickness with relatively large oil ducts; the samples were designed to simulate the major insulation used in high voltage transformers.

From figure 4, it is evident that the average puncture voltage increased linearly with the total thickness of the paper sheets in the pad. This is evidence that the strength of the individual paper sheets and the distribution of voltage were uniform. Although the line electrode insulation measured 20 mils in thickness including the thread, it had a puncture voltage equivalent to 7.5 mils of the 5-mil paper. Failure tended to occur at the threads presumably due to a concentration of stress.

Puncture of Insulation in Parallel With Arresters or Gaps

A second series of tests was made with pads in parallel with 9-kv distribution arresters or gaps set for values of impulse breakdown within the range permitted by lightning protective devices used on 6,900-volt circuits. Several types of gaps were used among which were $^1/_2$-inch square rod gap set at $^{61}/_{64}$ inch and $2^1/_{16}$ inches; and sphere gaps 6.25 centimeters in diameter set at 16 millimeters. Cathode ray oscillograms were taken to show the failure voltage and time, see figure 5. A new line electrode and new sheets of paper were used for each test but not for each impulse. Although the applied impulse voltage wave was changed during the series of tests, each individual sample was punctured by repeated applications of the same voltage wave except that the tests were stopped in three cases after 100 or more shots. A total of 95 pads was tested and the results are summarized in the tabulation. In all tests the applied voltage rose at an approximate rate of 500 kv per microsecond until reduced by the arrester or gap or by puncture of the insulation. The rate of 500 kv per microsecond exceeds the rate of 100 kv per microsecond recommended in the appendix to "Standards for Testing Lightning Arresters," ASA C-62, and is perhaps representative of the steeper rates to which distribution arresters in service are subjected. The gap and arrester breakdown voltages in the tabulation exceed the values obtained from tests with voltages rising at 100 kv per microsecond. For example the 9-kv valve type arresters have a breakdown voltage of approximately 50 kv on the 100 kv per microsecond tests whereas a breakdown of 69 kv is given in the table for the 500 kv per microsecond wave. The time to breakdown of the arresters or gaps ranged from 0.13 to 0.30 microsecond on the 500 kv per microsecond wave. These values of time to breakdown of the arresters or gaps are small compared to the range of one to four microseconds required for the *IR* voltage to reach crest and are still smaller in comparison with the duration of the *IR* voltages produced by current discharges that decayed to half value in 40 microseconds.

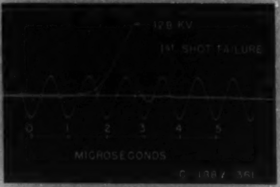

Figure 2. Volt-time oscillogram taken during single-shot puncture tests on eight sheets of 0.005- by 10- by 10-inch paper

When arresters were used, the impulse current had a crest of 5,000 amperes, an initial rate of 2,200 amperes per microsecond, and a time to half value of 40 microseconds. The value of 5,000 amperes crest was chosen for the current since 88 per cent of the discharge currents measured[2] through rural distribution arresters in service did not exceed this value. While comparatively little information is available[3] on rates of rise and duration of currents discharged by arresters in service, it is believed that the impulse discharge used in these tests was of more than average severity.

When gaps were tested, the impulse discharge current crest was 500 amperes or less and the time to half value was 40 microseconds. The value of approximately 500 amperes was selected for the magnitude of the discharge current as a matter of convenience and to give arc voltages and inductive drops that were small. For part of the tests, wire-wound resistors of 120 to 150 ohms were connected in series with the gaps to study the effect of *IR* voltage on insulation failure. Cathode ray oscillograms, see figure 5, showed whether the insulation failure occurred as the voltage was rising to gap breakdown or during the subsequent application of *IR* voltage to the insulation.

Comparison of Test With and Without IR Voltage

By means of the data in the tabulation, it is possible to compare the effect on insulation failure of breakdown voltage without subsequent *IR* voltage, with the effect of breakdown voltage followed by *IR* voltage. The per cent of single shot puncture voltage given in the tabulation is the ratio of the breakdown or *IR* voltage applied to the pad to the average single shot voltage determined from figure 3 for a pad with the same number of sheets. When valve arresters were tested the crest *IR* voltage occurred at one to two microseconds so the single shot puncture voltage curves were read at 1.5 microseconds; the wire-wound resistors gave crest *IR* voltages at two to four microseconds, and the curves were read at 3.0 microseconds. In tests 1 and 2 with valve type arresters puncture always occurred as the voltage was rising to gap breakdown; two-sheet pads failed in 1 to 5 shots and three-sheet pads in 3 to 14 shots. In test 3 with gaps set for

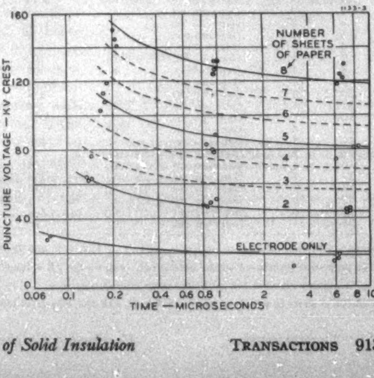

Figure 3. Volt-time curves for single-shot puncture from front-of-wave tests

The solid lines are the average volt-time curves from the original data as correlated by figure 4. All original data are plotted

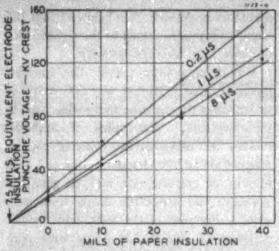

Figure 4. Average single-shot puncture voltage from front-of-wave tests

nearly the same impulse breakdown as in tests 1 and 2 but having no *IR* drop, two-sheet pads failed in one shot. In test 4, six of the three-sheet pads failed in one to nine shots (see footnote in tabulation) and the seventh pad withstood an abnormal number of shots. A comparison of tests 1 and 2 with tests 3 and 4 shows that valve arrester *IR* drops of 74 and 58 per cent of the average single shot puncture voltage did not reduce the number of shots required to fail the insulation.

However, it is evident that the *IR* voltage if sufficiently high will reduce the number of shots the insulation can withstand. This is seen from tests 4, 5, and 6 and tests 7, 8, and 9. The gap breakdown voltage was approximately equal for the three tests 4, 5, and 6. Test 4 was made with no *IR* voltage. In test 5 the *IR* voltage was 84 per cent of the single shot strength and puncture occurred on the *IR* voltage in two out of the seven tests. In test 6 with an *IR* voltage of 95 per cent of the single shot strength, puncture always occurred on the first shot during the *IR* voltage. These tests indicate that repeated shots with *IR* voltages in excess of 84 per cent of the single shot strength will deteriorate the insulation.

The three tests 7, 8, and 9 were all made with the same gap and give additional information on the effect of *IR* voltage. In test 7 there was no *IR* voltage and two pads each withstood 102 shots. In test 8 the *IR* voltage was 69 per cent of the single shot failure voltage and three of the pads tested punctured on shots 25, 34, and 86 while the fourth pad withstood 109 shots without failure. Although it might be inferred from a comparison of tests 7 and 8 that the *IR* voltage reduced the number of shots to fail the insulation, the number of pads are so few in test 7 and the range of shots so large in test 8 that such a deduction is questionable. On test 9 the *IR* voltage was 78 per cent of the single shot puncture voltage and failure occurred in fewer shots than in tests 7 and 8.

The remainder of the tabulation shows the number of shots required to puncture pads with various amounts of insulation when the voltage was chopped by gaps without series resistance. The protective ability of these gaps was approximately equivalent to a rod gap set two inches, since the breakdown voltages ranged from 112 to 125 kv and there was no *IR* voltage. With these gaps in parallel with the pads, it was necessary to have from five to eight sheets in the pads to withstand the same number of shots as were withstood by pads with two or three sheets when tested in parallel with valve arresters that had a breakdown voltage of 69 kv and an *IR* voltage of 34-kv crest.

430 KV/µs
55 KV
0
0.5
1.0
1.5
MICROSECONDS
C-1887-267

Figure 5. Volt-time oscillogram taken on pad with three sheets of paper

The pad was protected by 6.25-centimeter spheres set 16 millimeters with a resistance of 150 ohms connected in series. Puncture occurred during the *IR* voltage at about 1.7 microseconds. This oscillogram was retouched for reproduction

Effect of Repeated Shots on Insulation Puncture

Thus far comparisons have been made between part of the individual tests listed in the tabulation. However, the effect of repeated shots can best be determined by putting the data from all tests on a common basis. This has been done in figure

Tests on Insulation Pads in Parallel With Gaps and Arresters

Test Number	No. of Sheets in Pads	Voltage Limited by	No. of Pads Tested	No. of Shots to Fail Pad	Pad Failed During	Average Voltage Applied—Kv Crest		Average Single Shot Strength, Figure 3—Kv Crest		Ratio of Average Applied Voltage to Average Single Shot Strength—Per Cent	
						Bkd. at 500 Kv/µs Bkd.	40 µs to Half-Current IR	Bkd.	IR	Bkd.	IR
1	2	Valve arr.	7	1–5	Breakdown	69	34	63	46	109	74
2	3	Valve arr.	8	3–14	Breakdown	69	34	80	59	86	58
3	2	Gap—no res.	4	1	Breakdown	69	0	64	0	108	
4	3	Gap—no res.	7	1–78[a]	Breakdown	72	0	78	0	92	
5	3	Gap and res.	7	1–2[b]	Bkd. or *IR*	79	49	77	58	103	84
6	3	Gap and res.	5	1	*IR*	70	55	80	58	88	95
7	4	Gap—no res.	2	102[c]		69	0	99	0	70	
8	4	Gap and res.	4	25–109[d]	Breakdown	69	48	99	70	70	69
9	4	Gap and res.	2	23, 27	Unknown	69	55	99	71	70	78
10	5	Gap—no res.	6	1–4	Breakdown	112	0	105	0	107	
11	6	Gap—no res.	2	3	Breakdown	112	0	120	0	93	
12	7	Gap—no res.	6	5–12	Breakdown	113	0	139	0	81	
13	5	Gap—no res.	6	1	Breakdown	112	0	105	0	107	
14	7	Gap—no res.	6	2–9	Breakdown	113	0	136	0	83	
15	8	Gap—no res.	1	25	Breakdown	115	0	133	0	86	
16	5	Gap—no res.	1	1	Breakdown	117	0	107	0	109	
17	6	Gap—no res.	5	1–2	Breakdown	119	0	119	0	100	
18	8	Gap—no res.	7	4–12	Breakdown	125	0	148	0	84	
19	6	Gap—no res.	4	1	Breakdown	118	0	119	0	99	
20	8	Gap—no res.	4	3–5	Breakdown	124	0	150	0	83	
21	9	Gap—no res.	1	12	Breakdown	120	0	165	0	73	

NOTES: [a] Number of shots to puncture pads: 1, 1, 1, 2, 4, 9, and 78.

[b] Five pads were punctured on the breakdown; two on the *IR* voltage.

[c] Two samples withstood 102 shots and tests were discontinued.

[d] Number of shots to puncture pads: 25, 34, 86, and one withstood 109 shots.

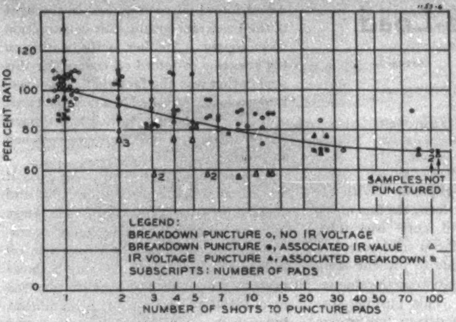

Figure 6. Tests with repeated shots

The applied voltage, in per cent of the single-shot puncture voltage, is plotted against the number of shots to puncture the pads. The insulation pads were in parallel with gaps or arresters

6 where the applied voltage expressed in per cent of the single shot puncture voltage was plotted against the number of shots to rupture the insulation. Data points were plotted from each of the 95 pads tested whereas the tabulation was condensed to include only averages or the ranges of data. A point was plotted for arrester or gap breakdown voltage and another for the *IR* voltage when it existed. The symbols indicate whether the puncture occurred as the voltage rose to gap breakdown or during the subsequent *IR* voltage. Since a large number of insulation punctures occurred on the first few shots, these points have been grouped about the ordinate through the actual shot number. Factors responsible for the scattering of the points include variations in the gap or arrester breakdown voltages, variations in the impulse generator discharge from shot to shot, and variations in the puncture voltage of individual pads.

The curve in figure 6 shows that the number of shots required to fail the insulation increases as the applied voltage expressed in per cent of the single shot puncture voltage is reduced. In all tests in which the insulation was ruptured, either the breakdown or the *IR* voltage was at least 70 per cent of the single shot puncture voltage. It is concluded from this plot that the insulation will withstand a large number of repeated shots provided the applied voltage does not exceed a value of approximately 70 per cent of the single shot puncture strength at any value of time. Within the limits of this investigation, this conclusion holds irrespective of whether the applied voltage is of short or long duration. On the basis that the curves of figure 3 are 100 per cent, it is concluded that curves drawn through 70 per cent of the voltage will approximate the shape of the highest voltage wave that can be repeatedly applied without reducing the number of shots the insulation will withstand. Montsinger[1] found that repeated shot injurious corona appears at voltages of 65 to 90 per cent of the single shot failure voltage and concluded that 80 per cent is a safe average value of voltage for repeated shots. The value of 70 per cent from the present tests is in good agreement with his results notwithstanding the differences in the structure of the insulation tested in the two investigations.

The Effect of Voltage Wave Shape on Insulation Puncture

One object of this investigation was to determine if arrester *IR* voltages of relatively low magnitude might possibly be more destructive to insulation because of long duration than voltages of relatively short duration preceding the breakdown of gaps. This was not found to be the case in these tests. The higher voltages necessary to puncture the insulation samples at short values of time are due to turn up of the insulation volt-time characteristic. Although figure 3 shows that higher voltages are required to cause puncture in short values of time, this might be offset in transformers by nonuniformity in distribution of voltage between the turns of the transformer coils which increases with the steepness of the voltage applied. Although these tests on insulation samples showed no evidence of deterioration of the insulation due to voltages less than 70 per cent of the single shot strength, the conclusion should not be drawn that chopped waves of this magnitude are necessarily harmless to turn insulation in actual transformers.

Conclusions

(1). The single shot puncture of the insulation increased linearly with the insulation thickness over the range tested in this investigation.

(2). The single shot volt-time puncture curves of the insulation tested in this investigation are relatively flat; the average puncture voltage for failure at 0.25 microsecond was found to be only 16 per cent more than the puncture voltage at 1 microsecond.

(3). This investigation shows that valve type arresters, of the type tested and the rating applied on 6,900-volt circuits, are superior to two-inch rod gaps for the protection of insulation when subjected to impulse discharges that are of more than average severity based upon field records.

(4). Tests with repeated applications of impulse voltages showed that as the ratio of the applied voltage to the single shot puncture voltage is reduced, the number of shots necessary to rupture the insulation increases.

(5). The insulation tested withstood a relatively large number of shots provided the applied voltage did not exceed a value of approximately 70 per cent of the single shot puncture voltage at any instant of time during the application.

(6). From life tests during which the insulation was subjected to voltage waves modified by the operation of protective devices, it is concluded that the insulation will withstand a relatively large number of shots provided the instantaneous voltage due to breakdown or *IR* voltages does not exceed approximately 70 per cent of the single shot puncture voltage.

(7). The results of this investigation suggest that in the application of protective devices, the upper limit of the stress that can be safely applied repeatedly to new insulation may be regarded as 70 per cent of the single shot puncture voltage; this is irrespective of whether the highest stress permitted by the protective devices is due to short or long time gap breakdown or short or long duration *IR* voltages.

References

1. Co-ordination of Power Transformers for Steep-Front Impulse Waves, **V. M. Montsinger.** AIEE Transactions, volume 57, 1938 (April section), pages 183–9.

2. Discharge Current in Distribution Arresters—II, **K. B. McEachron and W. A. McMorris.** AIEE Transactions, volume 57, 1938 (June section), pages 307–12.

3. The Lightning Stroke: Mechanism of Discharge, **K. B. McEachron and W. A. McMorris.** *General Electric Review*, October 1936, pages 487–96.

A New Transformer for Base-Load Stations

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Synopsis: This paper describes the application of Hipersil to three 40,000 Kv-a units for the Philo generating station of The Ohio Power Company. Because of Hipersil's ability to carry one-third more magnetic flux, appreciable savings in weight and dimensions are possible—in fact, the transformers will be shipped in oil completely assembled with bushings in place, ready to operate.

Forced-oil and forced-air cooling are provided, with proper relays for protection in the event of excessive copper temperature or failure of auxiliaries. Reliability, economy, ease of installation, and low maintenance have been the objectives sought in the design.

ALTHOUGH technical progress in the art of transmission of electrical energy has been considerable in the last two decades, it is a fact that no single significant contribution looking toward more economical transmission is readily called to mind. Most particularly is this true in the case of the transformer—the essential link in a-c transmission. Thus, major design and construction improvements have been made in impulse strength, in lightning protection, and in impulse testing of transformers. But these developments have only indirectly affected cost. Even more notable is the fact that a design trend of many years background in the direction of continually reducing losses has not been re-examined, nor most certainly altered in the face of decreasing costs of electrical capacity and energy. The development described here, it is believed, marks a definite step in altering that condition.

The essential element in this new design is a new steel, Hipersil.[1] Its outstanding characteristic is its ability to carry approximately one-third more magnetic flux in the transformer core. Figure 3 shows equipment used in a test made on two model cores identical in size—one of silicon steel and one of Hipersil. With the same exciting ampere-turns the flux measured in the Hipersil core was 38% greater than in the silicon steel core. It is hoped at a later date to present some more extensive information on the characteristics of Hipersil. This pushing forward of some of the design limitations which have heretofore held in check transformer designers, even though, for the present at least, the advantages of the new material must be limited to transformers where it can be used with the magnetic flux in line with the direction of rolling, is bound by itself to have a most material effect on transformer design economics and on transformer design art. In the development described, Hipersil has been coupled with forced-oil cooling, and with re-examined criteria for losses in the light of modern power and energy economics. But safety and reliability, as will be evident from the more detailed description to follow, have not been sacrificed.

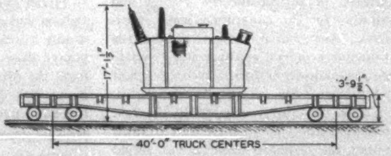

Figure 1. 40,000-kva 138-kv transformer mounted on standard flat car for shipment

In spite of the large size of this transformer, it will be shipped completely assembled with cooling equipment, oil bushings, and lightning arresters in place

Description of Transformer Units

This paper presents specifically the results of an analysis to see what the possibilities of this new material and new ideas would be in the design of large transformers for a base load station. The particular units under consideration are rated 40,000 Kv-a, 138 Kv, with non-solidly grounded neutral. The relatively lower cost of energy at the station bus was fully considered in evaluating the transformer losses and it made possible a closer use to the full of the inherent properties of Hipersil in reducing dimensions and the amount of active material required.

It was found that the smaller weights and dimensions resulting from the greater flux carrying capacity of Hipersil could be reduced still further through the use of forced-oil cooling, and in the light of the basic aim sought, this construction was adopted. Another factor bearing on the selection of forced-oil cooling for this application was the fact that little overload capacity is needed in a base load unit operating on the unit principle with generator and transformer in series switched as a unit.

As a result of these two basic ideas it was found possible to reduce weights and dimensions to a point where these large transformers could be completely assembled in the factory to be shipped in oil with bushings in place. Figure 1 shows an elevation of one of these units mounted on a standard flat car with significant dimensions. The cost of erection and starting in the field will obviously be reduced to a minimum.

Another interesting feature of these transformers is that due to the small size and the relatively small amount of oil involved it was possible to weld the cover on and seal the unit permanently against deterioration of the oil and insulation. There is no provision for breathing.

When operating at full load, these transformers have oil circulating pumps and fans in continuous operation, but much thought has been given to the question of reliability and protective devices. It is believed that with the arrangements described below, a degree of reliability comparable with that of other major apparatus in the generating station has been obtained.

It is interesting to examine the reduction in weights and increase in losses of this transformer compared with a standard self-cooled air-blast unit of the same rating:

Copper windings	reduction in weight	10%
Core	reduction in weight	25%
Losses	increase in weight	5%

Cooling System

The transformer itself is of shell-type construction and is mounted horizontally in the tank. Fin-type coolers employing round tubes of substantial cross-section are mounted on each side of the tank wall and provided with circulating pumps and cooling fans. The two radiator assem-

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[1] A registered trade mark of Westinghouse Electric and Manufacturing Company.

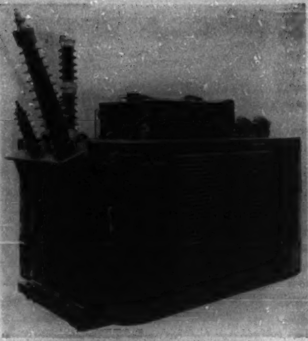

Figure 2. Side elevation of 40,000-kva 138-kv transformer showing principal over-all dimensions

The transformer is completely sealed with cover welded in place. No auxiliary gas equipment is required since adequate gas space is provided in the tank for maximum expansion and contraction due to temperature changes

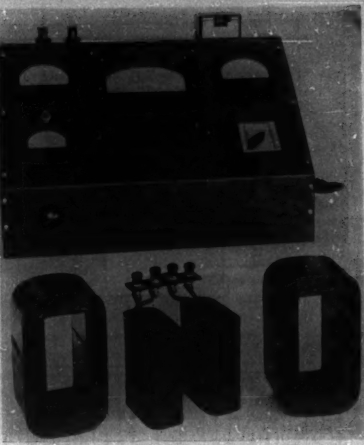

Figure 3 (right). Test to demonstrate flux-carrying capacity of Hipersil

Two cores identical in size—one silicon steel, one Hipersil—were excited with 0.5 ampere through a coil of 154 turns. The voltage across the coil when assembled on the silicon steel core was 92—when on the Hipersil core, 127. Since the flux produced in the core is proportional to the voltage, the Hipersil core carried a flux greater than the silicon steel, in the ratio of 127 to 92, or approximately 38 per cent greater

blies are operated as independent units. Power for the pumps and fans is supplied from two auxiliary transformers mounted inside the main unit—one auxiliary transformer taking care of one radiator bank. However, each of these auxiliary transformers is of sufficient capacity to supply power for the pumps and fans of both cooling units, and the control is so arranged that in the event one auxiliary transformer should fail, its fans and pump can be immediately connected to the other transformer. The control, too, is so arranged that both oil circulating pumps operate at no load but the fans come on as required; that is, when the oil temperature reaches 65°C, one set of fans comes into operation; at 75°C, the second set of fans comes into play.

Differential pressure relays are provided on each circulating pump. With only one pump in operation the capacity of the transformer would be reduced to 80%. Should the oil flow stop through one of the coolers due to failure of the pump or any other cause, the differential pressure relay would sound a warning to the station operator. Should both pumps stop, a signal to clear the unit from the line is given.

Operation by Copper Temperature

Since the thermal capacity of these transformers is small—comparatively little oil being required—excess copper temperatures could be reached in a relatively short time in the event difficulty should develop in the cooling system. For this reason relays are provided in each unit which operate on copper temperature. At 105 degrees centigrade copper temperature (measured by resistance), a warning signal is sounded to the station operator indicating that steps should be taken to reduce the load. It is not likely that the transformers would be actually overloaded due to the limitation of the generating equipment in the station, but it is conceivable that in the event of failure of fans or pumps on one side of the unit, even normal full load would exceed its reduced load carrying capacity. Should the copper temperature continue to increase and reach a dangerous point, the copper temperature relay signals the station operator for immediate shut-down.

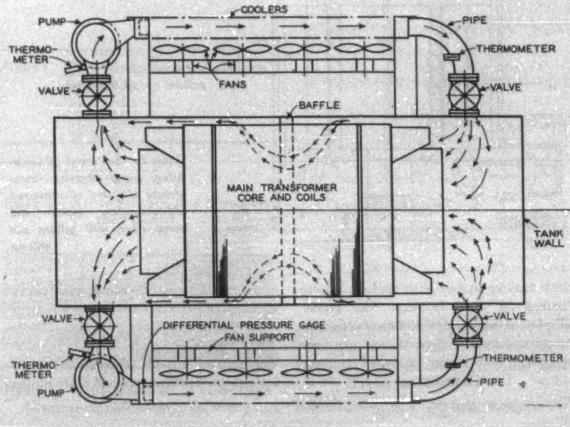

Figure 4. Plan view showing oil circulation system

Pumps and fans are arranged in two independent assemblies each supplied from a separate auxiliary transformer mounted inside the main tank. Protection to the unit is provided in the event of failure of pumps by means of differential pressure relays and in the event of fan failure by copper temperature relays

Fans and Pumps

A number of different blower arrangements were attempted during the course of the design but the use of simple air-

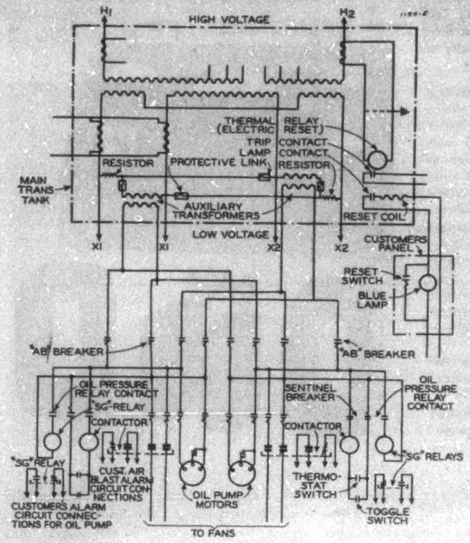
H1
H2
HIGH VOLTAGE
THERMAL RELAY (ELECTRIC RESET)
TRIP CONTACT
LAMP CONTACT
RESISTOR
PROTECTIVE LINK
RESISTOR
MAIN TRANS TANK
AUXILIARY TRANSFORMERS
RESET COIL
LOW VOLTAGE
X1
X1
X2
X2
CUSTOMERS PANEL
RESET SWITCH
BLUE LAMP
"AB" BREAKER
"AB" BREAKER
OIL PRESSURE RELAY CONTACT
"SG" RELAY
CONTACTOR
SENTINEL BREAKER
OIL PRESSURE RELAY CONTACT
CONTACTOR
"SG" RELAY
"SG" RELAYS
CUST. AIR BLAST ALARM CIRCUIT CONNECTIONS
CUSTOMERS ALARM CIRCUIT CONNECTIONS FOR OIL PUMP
OIL PUMP MOTORS
THERMOSTAT SWITCH
TOGGLE SWITCH
TO FANS

Figure 5. Schematic diagram showing arrangement of control apparatus mounted in control cabinets on transformer

Auxiliary transformers mounted inside main unit are provided with protective links to clear circuit in case of winding failures. Either auxiliary transformer will supply all pumps and fans by closing proper *AB* breakers. Fans are controlled from thermostatic relays operating on oil temperature. Pump motors run continuously. Copper-temperature relay protects against excessive copper temperature in the event of overload or failure of auxiliaries

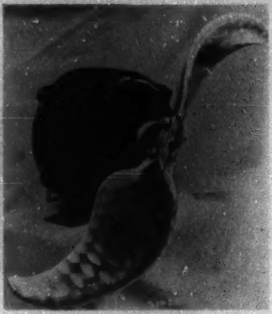

Figure 7. Bird-wing fan mounted in removable assemblies of four units behind radiator coolers

Motors are outdoor capacitor-type equipped with "sealed-for-life" bearings. Capacitors are mounted in control cabinets on the transformer

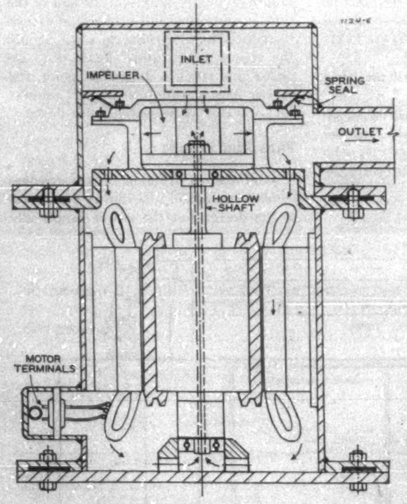
INLET
IMPELLER
SPRING SEAL
OUTLET
HOLLOW SHAFT
MOTOR TERMINALS

Figure 6. Section of oil circulating pump showing completely enclosed oil-immersed self-lubricating motor and pump rated 400 gallons per minute

blast fans equipped with motors having "sealed for life" bearings was finally adopted as the simplest and most economical arrangement. Sixteen fans are employed on each cooler, mounted in racks of four. Consideration has been given to easy replacement of these individual racks should that become necessary as a matter of routine maintenance.

The circulator pumps are of a type previously used for this application. They are equipped with a vertical shaft motor which is completely sealed and mounted in the oil circulation system so that both stator and rotor windings are submerged and bearings continually lubricated at all times by the transformer oil itself. Consideration has also been given to the problem of changing the pump motor and impeller assembly in the shortest possible time should that ever become necessary.

Conclusions

The development of Hipersil has opened up many new avenues for the design of transformers. As shown here, its use, coupled with other design ideas, has made possible production of large units more economically through reduction in size and amount of material required. Weights and dimensions become particularly important in very large units. Using standard magnetic steel these have had to be shipped in nitrogen gas in special containers to get by bridge and tunnel clearances. Upon arrival they required a considerable amount of erection work and expense prior to being placed in service. The use of Hipersil automatically extends the rating size before this becomes necessary. It is apparent, too, that the same development will make possible the extending of the practical size of the portable transformer whether on car or trailer. Finally, the authors believe that this development is but another proof of the vitality of the electric power art and of the ability of modern technology to move rapidly forward even in a well developed field.

The Measurement of Body Currents

ROBERT S. SCHWAB
NONMEMBER AIEE

IN these days of supersensitive amplifiers and recording apparatus, it is very tempting to define life and death in terms of electrical activity. Whether or not this concept is accurate, we can, on present knowledge liken living tissue to a B-battery and dead tissue to a burned out generator. The function of living tissue, however, is so closely allied with its electrical activity that knowledge of the latter has given us better understanding of the working of the human body.

Great names in electrical engineering contributed to our knowledge of the physiology of tissues. Galvani in 1780 made the really startling observation that the muscle of a frog's leg could be made to contract when stimulated with the voltage from the crude Voltaic pile. That the muscle itself, on contracting, gave off a charge of its own was shown in 1870 by D'Arsonval, who noted the deflection of the needle when the muscle contracted across the poles of his galvanometer.

With improvements and greater sensitivity in recording instruments, electrical waves were obtained from the human heart (Waller, 1885; Einthoven, 1903) and from the nerve and muscle (Lucus, 1906; and Piper). In 1929 Hans Berger, with string galvanometer and amplifiers, picked up the first electrical waves from the human brain with electrodes on the scalp.

The first type of body currents to be discussed here is the rather static, direct current potential difference. It is this ever-present E.M.F., found in all living matter, small (few millionths of a volt), but constant as to polarity and relatively unchanging in time or species, that brings up the easy analogy of life to the B-battery. The living cell shows a tiny potential difference between its surface and the centrally located nucleus (Freeman). In plants, we can find a few microvolts of D.C. potential and in one animal, the electric eel, we may find as much as 500 volts. There is strong evidence that these potentials are due to different metabolic rates or energy levels in different parts of the same structure. Thus the relatively inactive, transparent membrane covering the human eye-ball (cornea) is nearly a millivolt different from the highly active, photo-sensitive retina at the back of the eye.

Nearly anywhere in the body, such potential differences may be found and in some cases might be clinically useful. For example, if an electrode is placed in the vagina, and another on the surface of the skin of the abdomen, a sudden increase in the base-line voltage difference is said to indicate that ovulation is occurring in the underlying ovary. These D.C. potentials are always associated with the first and most important character of life, namely: absorbing energy, growth, and change in structure. We call these activities metabolism and catabolism. They are found, for example, in the rose from seed to death, although the flower never moves in the sense that animals swim, fly, walk, etc. The more active part metabolically speaking is negative to the less active part. Thus the former is essentially the −; zinc, or cathode of a battery and the less active tissue the +; carbon, or anode pole.

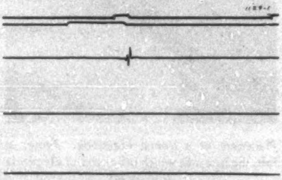

Figure 1. **Electrical response of human biceps muscle to tap on tendon**

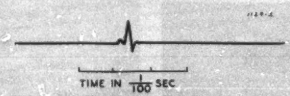

Figure 2. **Action potential in a nerve fiber (from Gasser)**

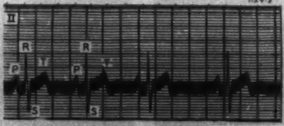

Figure 3. **Action potentials of human heart (electrocardiogram—from P. D. White)**

The second type of body currents is that associated only with the animal kingdom—the electrical activity of locomotion. It is nature's second and perhaps most interesting attribute of life, and is different from any of the D.C. potentials previously described. Contractile tissue is nature's first intrinsic movement in an environment of physical forces all in motion, such as wind, tides, rivers, etc. This tissue, when left alone "lives," i.e., has its D.C. potentials. But, when it is stimulated from without or from within, an astonishing change takes place. It moves actively, withdraws, strikes out. Associated with this defensive or aggressive movement there is an alternating current wave, a diphasic spike, which is lacking in the motionless tissues. An example of such tissue is a muscle fibre. When it is stimulated, it contracts, and a diphasic spike or group of such spikes occurs. This contractile tissue is primitive and found in all organisms that move, and is characterized by no electrical activity when quiet and by burst of such activity when stimulated to action (figure 1).

A similar structure, but more complicated, is nerve tissue. Primarily used to conduct stimuli to the contractile tissue, it acts like muscle in that it has no activity electrically when at rest, but when at work shows the same characteristic diphasic spike. Motion, as an inherent ability, however is not present in nerve tissue. (Figure 2.) In this tissue

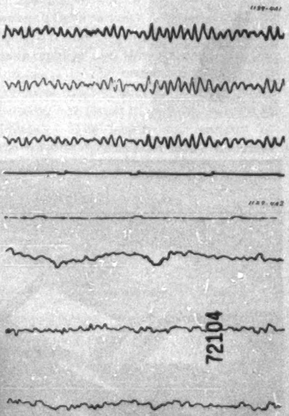

Figure 4A. **Normal patterns of human brain potentials**

Alpha rhythm above and beta rhythm below

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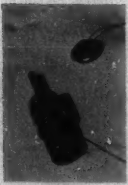
very rapid shifts of metabolic activity occur. For example, A, B, and C are points in line in such a piece of living matter. When at rest they are absorbing energy (being nourished, living) and are negative to their enveloping and supporting structures. Points A, B, and C are in, let us say, contractile tissue or nerve and have a much higher metabolic rate and complexity as far as molecular structure than the simple covering membrane. Hence they are all negative to it. If point A is stimulated its energy is released by a rapid chemical change and A loses its negative electrical charge. This in turn discharges or stimulates point B etc. These rapid changes produce the diphasic wave seen in muscle and nerve potentials. It is to be noted that in 3–4 thousandths of a second the whole process is completed. This nerve conduction proceeds not like electrons in a wire, but slowly as each adjacent point goes through a chemical breakdown and stimulates the next one. The rate of conduction of such an impulse is as far as 100 meters per second in man, but is much slower in lower animals (5 meters per second in reptiles).

A third type of electrical activity is associated with the more highly developed types of contractile tissue. Cilia, tubular digestive tracts, and heart muscle have a "beat" of their own and depend no longer on outside stimuli for their continuous, regular contractions and relaxations. Every "beat" has its accompanying bursts of diphasic waves. (Figure 3.) This shows a normal electrocardiogram. Metal (silver) discs covered with flannel soaked in salt are fastened with elastic bandage to the left wrist and left foot (lead II). Wires from the two discs lead to the poles of a sensitive string galvanometer. Bromide paper moved by clock works at 5 cm. per second receives the light beam from the mirror on the string. Thus electrical changes in the heart show up optically magnified on this recording paper when it is developed. Recording from each arm or right foot and left arm are some of the other leads (I and III). A vast and important field of clinical diagnosis has been developed in the interpretation of these EKG's as they are called. They indicate pathological changes in heart muscle, disturbances in rate, rhythm, and in circulation of blood in the heart muscle itself. No complete examination of the heart can be done without such a measurement. Every hospital now has one and many doctors have them at their disposal. In 1910 they were a curiosity in this continent.

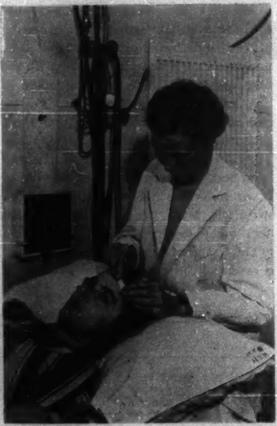
The fourth type of body electrical currents is that associated with the complex tissue that makes up the central nervous system of animals. Here, as the function is continuous during life, the ganglia and brain cell tissues are ever-active electrically and show no periods of rest in the manner of muscle and nerve. (Figure 4.) Each brain cell does not actually beat alone, but by a system of interconnections keep each other stimulated to activity. These "chains" of neurones make up the bulk of the brain and spinal cord of man and animals. According to Herrick, in man, for example there are over $10^{2,783,000}$ possible neuronal combinations—a number far greater than the total sum of all atoms in the entire solar system 10^{76}.

It is this last form of living matter, the cells of the central nervous system that interests most physiologists today. In clinical medicine, the importance of the electrocardiogram in studying the function of the heart is completely established. The newer electroencephalogram or recording of the electrical activity of the

Figure 4B. Obtaining and recording human brain waves in the brain-wave laboratory of the Massachusetts General Hospital and the Harvard Medical School, Boston, Mass.



(Left). Electrodes used. Disc end fastened to scalp



Placement of a frontal electrode. Note, at left, the box into which other end of electrode is plugged

An ink-writing oscillograph recording a brain wave on moving paper

Apparatus room with two sets of amplifiers and two recorders

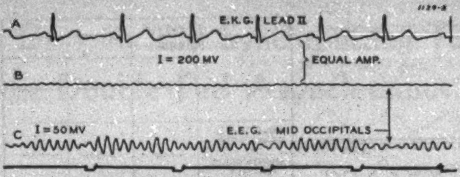


Figure 5. Comparison of human electrocardiogram and human electroencephalogram

In *B* the brain wave is at same amplification as the heart wave in *A*. In C the brain wave is at the standard amplification for recording

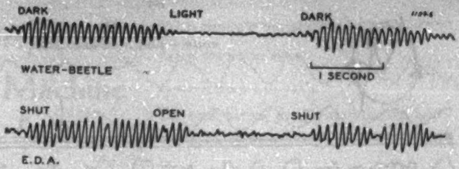


Figure 6. Comparison of the alpha rhythm in the beetle and human (from Adrian)

brain is rapidly becoming just as invaluable in the study of diseases and dysfunctions of the central nervous system. The recording of these brain waves, as the electroencephalograms are called, requires a much more sensitive type of apparatus. The heart currents obtained with surface electrodes on two of the extremities, as previously described, may reach nearly a millivolt and can be recorded directly by a string galvanometer. The brain currents picked up from small (5 mm.) lead discs attached to the scalp by electrode paste and collodion (needles or cutting the hair are not necessary) are only $^{1}/_{20\text{th}}$ of the heart's and require vacuum tube amplification before recording (see figure 4*A*, *B*). Albert Grass of Harvard and Dr. Alfred Loomis of Tuxedo Park both have developed rugged efficient battery and power-operated, push-pull amplifiers that operate ink-writing oscillographs. Offner of Chicago, Garceau of Holliston, Mass., and Rhem of New York, are other engineers who have apparatus in this field. By these brain wave machines the EEG, as the records are called, appear immediately as an ink line on the paper strip. The usual speed of the paper is 3 cm. per second and there are anywhere from 3 to 6 separate "channels" recording simultaneously. Normal human brain waves consist of two sorts. Alpha or 10 per second waves have voltages of 30 to 75 microvolts, are extremely regular, and when dominating the record are associated with easy-going stolid, reliable types (alpha type). The second type of normal activity is beta waves, 20–24 per second of voltages from 10 to 40 microvolts. They are often associated in general with tense, rather high strung people. There are so many exceptions and mixed types that the brain wave pattern is not a real index of personality, intelligence, or state of mind.

(Figure 5.) This shows a brain wave and a heart wave compared. Here is seen the regularity of the 10 per second alpha waves, their continual appearance and the smallness of the voltage as differing from the regularity, discreteness, and high voltage of the heart activity.

(Figure 6.) This shows the phenomenon of the disappearance of the alpha waves when light enters the eye. Professor Adrian of Cambridge University kindly lent me this slide. The lower tracing is from the brain of the famous Nobel Prize winner, the upper from his lowly compatriot, the English Water Beetle. Brain cells, wherever gathered together, act electrically much the same.

(Figure 7.) This shows what the human brain waves are like during sleep. Irregularity and increased voltage are characteristic. Response of the individual to outside stimulus is greatly prolonged. Here the reaction time was over five seconds. (Normal is $^1/_4$ of a second.) In very deep sleep, or when the individual is under anaesthesia, the rhythm becomes slower (3 per second) and the voltages increase. Such waves are called Delta waves. When the brain function is interfered with abnormal waves appear in the electroencephalogram. These seem to result from faster smaller waves being built up into some sort of synchrony. They appear as high-voltage spikes (100 to 200 microvolts) or high-voltage, slow waves (Delta waves). They are something like the sleep waves but usually of more voltage and slower. Periodic disturbances in the normal brain rhythm (cerebral dysrhythmia as it is called by Lennox) are found in patients who have epilepsy. The length and severity of the seizures (seconds to many minutes) are shown by the amount and voltage of these abnormal waves.

(Figure 8.) Record of brain waves of a person who has, clinically, a form of epilepsy. The seizures are short, lasting but a few seconds, and are often periods

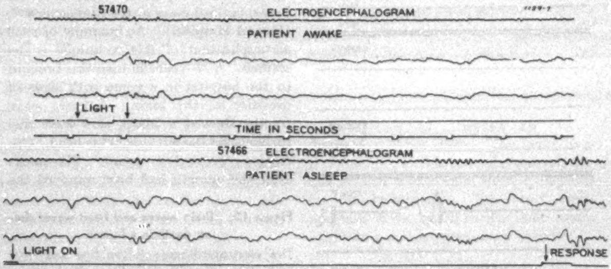


Figure 7 (above). Effect of sleep on brain waves and reaction

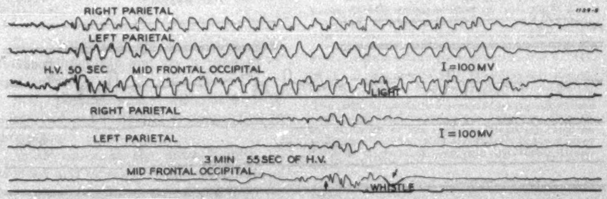


Figure 8 (right). Example of brain waves during a petit mal attack

Upper tracing shows long delay in responding to light. Lower tracing shows beginning of another attack probably stopped by whistle stimulus

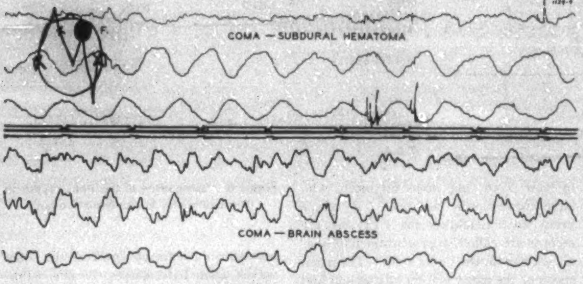
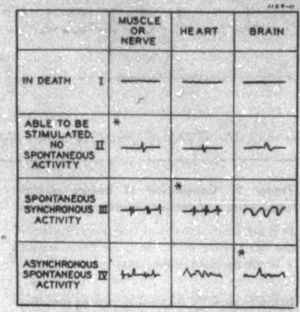


Figure 9. Brain waves from two patients in coma; upper waves from blood clot on brain and lower waves from brain abscess



	MUSCLE OR NERVE	HEART	BRAIN
IN DEATH I			
ABLE TO BE STIMULATED, NO SPONTANEOUS ACTIVITY II	*		
SPONTANEOUS SYNCHRONOUS ACTIVITY III		*	
ASYNCHRONOUS SPONTANEOUS ACTIVITY IV			*

Figure 11. In death I all tissues show no electrical activity; in II only muscle and nerve are normal;* in III heart is normal;* muscle and brain abnormal; in IV only brain is normal*

of unconsciousness. Sometimes they show up as slight twitchings of various face muscles, but do not develop into a real convulsion. These lapses are accompanied by a characteristic wave pattern called *the wave-spike formation*, which is a slow delta followed by a short spike. In this case, the person did not respond to the neon light signal, but did to the extremely unusual stimulus of the sound of a tug-boat whistle. During severe attacks of this kind, many fail to respond even to such a noisy stimulus.

Blood clots and tumors of the brain also cause these high-voltage, abnormal waves. By means of triangulation and phase shifts with the multiple channel sets of apparatus accurate localization of these lesions is possible (84% accuracy in 400 verified cases at the Massachusetts General Hospital). An example of such an application of this technique is described. A 38-year old man was brought to the hospital in a coma with signs of pressure on the brain. A brain wave tracing showed a sharp, slow-wave disturbance on the left side of his head. Operation revealed a tumor. Six weeks after this growth had been removed the

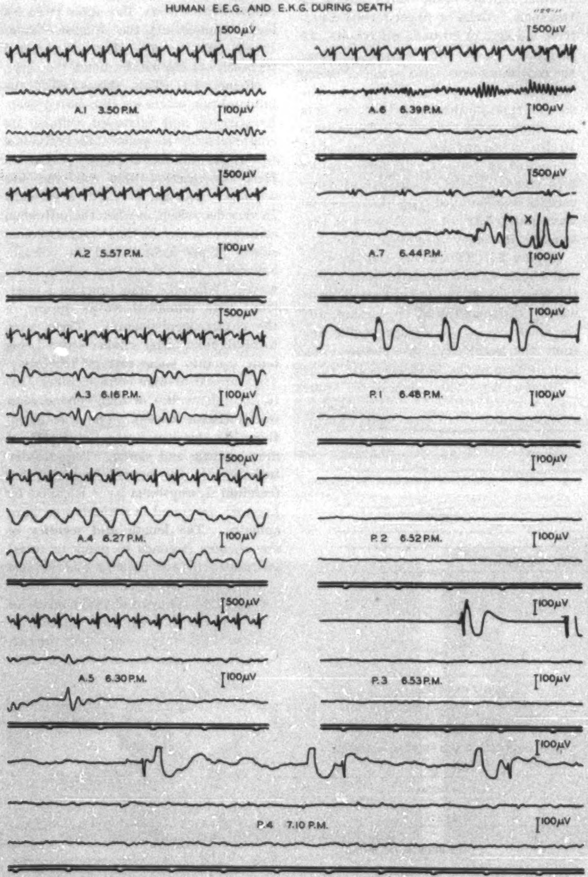


Figure 10. Brain waves and heart waves during death in a human

The electrocardiogram is top line and right and left brain waves are below. Brain waves disappear ten minutes before death. Legal death occurs at the spot marked (x) which indicates cessation of respiration and pulse. (x marks instant when intern placed stethoscope on chest to prove inaudible heart beat. The artifact produced by stethoscope touching electrode wire is shown in the tracing.) Electrocardiogram continues one minute after death, stops and then begins again. Thirty-two minutes after death heart still has three beats from pressure on chest

Damping and Synchronizing Torque of the Double-Fed Asynchronous Machine

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MEMBER AIEE

I. Introduction

THE double-fed asynchronous machine has become important in recent times as fan drive with wide speed range. The system consists of five main machines and the fan (see figure 1). It has many springs, masses, and dampings and therefore many natural frequencies. In order to determine these frequencies, i.e. the dynamic stability of the system, the damping and synchronizing torques of all parts of the system have to be known. Some papers have been published concerning the torques of the synchronous machine. It is easy to determine torques for the D.C. machines of the system. The object of this paper is to derive formulas for the damping and synchronizing torque of the double-fed asynchronous machine. As in the case of the synchronous machine, a complicated derivation and complicated results cannot be avoided.

II. The Differential Equations of the Double-Fed Machine

In order to simplify the mathematical work a two phase machine will be considered, and the usual assumptions, i.e. no saturation and small oscillations, are made. The results will be generalized for the m_1-phase machine. The voltage equations of the two phase machine are (for the symbols see List of Symbols)

$$E_{L1}\sin\omega t = AB_I' + L_1 i_I' + r_1 i_I \quad (1)$$

$$-E_{L1}\cos\omega t = AB_{II}' + L_1 i_{II}' + r_1 i_{II} \quad (2)$$

$$E_{L2}\sin(s\omega t+\delta) = AB_d' + L_2 i_d' + r_2 i_d \quad (3)$$

$$-E_{L2}\cos(s\omega t+\delta) = AB_q' + L_2 i_q' + r_2 i_q \quad (4)$$

where

$$A = \frac{2}{\pi}\tau_p l_i \omega \times 10^{-8} \quad (5)$$

i_I, i_{II}, i_d, and i_q are the currents in the four windings. B_I, B_{II}, B_d, and B_q are the resultant inductions in the axes of the winding produced by all currents. Between the inductions consists the relation[1]

$$\begin{aligned} B_I &= B_d\cos\gamma - B_q\sin\gamma \\ B_{II} &= B_d\sin\gamma + B_q\cos\gamma \end{aligned} \quad (6)$$

where

$$\gamma = \frac{x}{\tau}\pi \quad (7)$$

is the angle between the axes of phase I of the stator and phase I of the rotor. For the inductions B_d and B_q the following equations hold

$$B_d = \frac{x_m}{A\omega}(i_d + i_I\cos\gamma + i_{II}\sin\gamma)$$

$$B_q = \frac{x_m}{A\omega}(i_q + i_{II}\cos\gamma - i_I\sin\gamma) \quad (8)$$

With two new variables

$$\begin{aligned} I_d &= i_I\cos\gamma + i_{II}\sin\gamma \\ I_q &= -i_I\sin\gamma + i_{II}\cos\gamma \end{aligned} \quad (9)$$

the total torque of the machine will be[1,2]

$$T = A\omega(B_q I_d - B_d I_q) \quad (10)$$

With

$$\gamma = \frac{x}{\tau}\pi = \omega_{rav.}t + \epsilon = \omega_{rav.}t + \epsilon_0\sin zt \quad (11)$$

and

$$\alpha = s\omega + z \quad \beta = s\omega - z \quad (12)$$

the following four simultaneous differential equations will be found after some conversions.

$$E_{L1}\left(\cos s\omega t + \frac{\epsilon_0}{2}\cos\beta t - \frac{\epsilon_0}{2}\cos\alpha t\right) = -A(B_d\beta' + B_q') - L_1(I_d\beta' + I_q') - r_1 I_q \quad (1a)$$

$$E_{L1}\left(\sin s\omega t + \frac{\epsilon_0}{2}\sin\beta t - \frac{\epsilon_0}{2}\sin\alpha t\right) = A(B_d' - B_q\beta') + L_1(I_d' - I_q\beta') + r_1 I_d \quad (2a)$$

$$E_{L2}\sin(s\omega t+\delta) = (1+\tau_2)AB_d' + \frac{A\omega}{x_m}r_2 B_d - L_2 I_d' - r_2 I_d \quad (3a)$$

$$E_{L2}\cos(s\omega t+\delta) = (1+\tau_2)AB_q' + \frac{A\omega}{x_m}r_2 B_q - L_2 I_q' - r_2 I_q \quad (4a)$$

III. The Torques

A straightforward solution of the four equations is possible. For the four variables has been found

$$B_d = -E_{L1}\frac{x_m}{A\omega}\left[(a_s\cos s\omega t - b_s\sin s\omega t) - \frac{\epsilon_0}{2}(a_{II}\cos\beta t - b_{II}\sin\beta t) - \frac{\epsilon_0}{2}(a_8\cos\alpha t - b_8\sin\alpha t)\right] \quad (13)$$

$$B_q = -E_{L1}\frac{x_m}{A\omega}\left[(a_s\sin s\omega t + b_s\cos s\omega t) - \frac{\epsilon_0}{2}(a_{II}\sin\beta t + b_{II}\cos\beta t) - \frac{\epsilon_0}{2}(a_8\sin\alpha t + b_8\cos\alpha t)\right] \quad (14)$$

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[1] For all numbered references, see list at end of paper.

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brain wave was normal again in this area.

(Figure 9.) Abnormal, slow-wave pattern obtained from a patient unconscious due to large blood clot on the brain. Abscesses and injuries to the brain can also be localized by this method.

(Figure 10.) Changes in the electrocardiogram and electroencephalogram occurring in a patient dying from cerebral hemorrhage. Electrical activity of the brain in this case ceased before that of the heart.

(Figure 11.) Table of comparison of normal electrical pattern of muscle, heart, and brain. A normal muscle lives at the second level of no electrical activity except on stimulus. When a muscle "beats" rhythmically and continuously, it is a diseased or dying tissue. The heart lives a normal life at this level of rhythmic discharge, and is abnormal when it depends on outside stimulus. The brain exists normally in the fourth level of continuous, arhythmic and asynchronous activity.

Summary

D.C. potentials are associated with static cellular life and are a product of metabolism.

A.C. potentials are associated with many tissues controlling or responding to movement.

Brain tissue is the most highly developed and exists normally in a state of constant, irregular electrical activity.

$$I_d=-E_{L1}\left[(a_{14}\cos s\omega t-b_{14}\sin s\omega t)-\frac{\epsilon_0}{2}(a_{12}\cos\beta t-b_{12}\sin\beta t)-\frac{\epsilon_0}{2}(a_{13}\cos\alpha t-b_{13}\sin\alpha t)\right]\quad(15)$$

$$I_q=-E_{L1}\left[(a_{14}\sin s\omega t+b_{14}\cos s\omega t)-\frac{\epsilon_0}{2}(a_{12}\sin\beta t+b_{12}\cos\beta t)-\frac{\epsilon_0}{2}(a_{13}\sin\alpha t+b_{13}\cos\alpha t)\right]\quad(16)$$

where

$$\lambda_1=\frac{r_2^2+\left(\frac{\alpha}{\omega}\right)^2(1+\tau_2)x_2x_m}{r_2^2+\left(\frac{\alpha}{\omega}\right)^2x_2^2}$$

$$\lambda_2=\frac{\frac{\alpha}{\omega}x_mr_2}{r_2^2+\left(\frac{\alpha}{\omega}\right)^2x_2^2}$$

$$\mu_1=\frac{r_2^2+\left(\frac{\beta}{\omega}\right)^2(1+\tau_2)x_2x_m}{r_2^2+\left(\frac{\beta}{\omega}\right)^2x_2^2}$$

$$\mu_2=\frac{\frac{\beta}{\omega}x_mr_2}{r_2^2+\left(\frac{\beta}{\omega}\right)^2x_2^2}$$

$$\nu_1=\frac{r_2^2+s^2(1+\tau_2)x_2x_m}{r_2^2+s^2x_2^2}$$

$$\nu_2=\frac{sx_mr_2}{r_2^2+s^2x_2^2}$$

$$a=r_2+K(r_1\cos\delta-x_1\sin\delta)$$

$$a_1=\frac{ar_2+bsx_2}{r_2^2+(sx_2)^2}$$

$$a_2=x_m+(r_1\nu_2+x_1\nu_1)$$

$$a_3=\frac{a_1a_2+b_1b_2}{a_2^2+b_2^2}$$

$$a_4=\frac{z}{\omega}(x_m+x_1\nu_1)$$

$$a_5=a_3a_4-b_3b_4$$

$$\alpha'=\omega+z$$

$$a_6=\frac{\alpha'}{\omega}x_m+\left(r_1\lambda_2+x_1\frac{\alpha'}{\omega}\lambda_1\right)$$

$$a_7=a_5+1+K\frac{z}{\omega}x_1\frac{r_2\sin\delta-sx_2\cos\delta}{r_2^2+(sx_2)^2}$$

$$a_8=\frac{a_6a_7+b_6b_7}{a_6^2+b_6^2}$$

$$a_9=\frac{\beta'}{\omega}x_m+\left(r_1\mu_2+x_1\frac{\beta'}{\omega}\mu_1\right)$$

$$a_{10}=a_5-1+K\frac{z}{\omega}x_1\frac{r_2\sin\delta-sx_2\cos\delta}{r_2^2+(sx_2)^2}$$

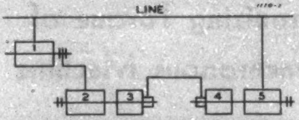

Figure 1. The double-fed asynchronous machine system

1. Double-fed asynchronous machine
2. Synchronous machine
3, 4. D-c machines
5. Synchronous machine

$$a_{11}=\frac{a_9a_{10}+b_9b_{10}}{a_9^2+b_9^2}$$

$$a_{12}=a_{11}\mu_1-b_{11}\mu_2$$

$$a_{13}=a_8\lambda_1-b_8\lambda_2$$

$$a_{14}=(a_3\nu_1-b_3\nu_2)+K\frac{r_2\sin\delta-sx_2\cos\delta}{r_2^2+(sx_2)^2}$$

$$b=sx_2+K(r_1\sin\delta+x_1\cos\delta)$$

$$b_1=\frac{br_2-asx_2}{r_2^2+(sx_2)^2}$$

$$b_2=-(r_1\nu_1-x_1\nu_2)$$

$$b_3=\frac{b_1a_2-a_1b_2}{a_2^2+b_2^2}$$

$$b_4=\frac{z}{\omega}x_1\nu_2$$

$$b_5=a_3b_4+b_3a_4$$

$$\beta'=\omega-z$$

$$b_6=-\left(r_1\lambda_1-x_1\frac{\alpha'}{\omega}\lambda_2\right)$$

$$b_7=b_5-K\frac{z}{\omega}x_1\frac{r_2\cos\delta+sx_2\sin\delta}{r_2^2+(sx_2)^2}$$

$$b_8=\frac{a_6b_7-b_6a_7}{a_6^2+b_6^2}$$

$$b_9=-\left(r_1\mu_1-x_1\frac{\beta'}{\omega}\mu_2\right)$$

$$b_{10}=b_5-K\frac{z}{\omega}x_1\frac{r_2\cos\delta+sx_2\sin\delta}{r_2^2+(sx_2)^2}$$

$$b_{11}=\frac{a_9b_{10}-b_9a_{10}}{a_9^2+b_9^2}$$

$$b_{12}=b_{11}\mu_1+a_{11}\mu_2$$

$$b_{13}=a_8\lambda_2+b_8\lambda_1$$

$$b_{14}=(a_3\nu_2+b_3\nu_1)-K\frac{r_2\cos\delta+sx_2\sin\delta}{r_2^2+(sx_2)^2}$$

From equations 10, and 13 to 16 the torques of a m_1-phase machine are

Constant torque

$$T_c=m_1E_{L1}^2x_m(a_3b_{14}-b_3a_{14})\text{ watts}\quad(17)$$

Damping torque

$$T_d=m_1E_{L1}^2x_m\frac{1}{2z}[a_3(b_{12}+b_{13})-b_3(a_{12}+a_{13})+b_{14}(a_8+a_{11})-a_{14}(b_8+b_{11})]\text{ watt-sec}\quad(18)$$

Synchronizing torque

$$T_s=m_1E_{L1}^2x_m\frac{1}{2}[a_3(a_{12}-a_{13})+b_3(b_{12}-b_{13})+a_{14}(a_8-a_{11})+b_{14}(b_8-b_{11})]\text{ watts}\quad(19)$$

A simple expression for the constant torque has been given in a previous paper.[3]

While the torques of the synchronous machine depend on the angle δ and the excitation, the torques of the double-fed asynchronous machine depend on the angle δ, the secondary voltage, and the slip.

The derivations made above are valid when the synchronous machine (machine 2, figure 1) is larger than the asynchronous machine, for example, when two or more asynchronous machines are connected in parallel to a single synchronous machine and the oscillation of one of the asynchronous machines is considered. In this case the secondary voltage can be assumed independent of the oscillation and proportional to the average slip (equations 3*a* and 4*a*). When the synchronous machine is of the same size as the asynchronous machine the change of the slip has to be taken into acount in the equations 3*a* and 4*a*. The results in this case become much more complicated than in the case considered.

List of Symbols

INDEX 1—STATOR—INDEX 2—ROTOR

E_L terminal voltage

$K=\frac{E_{L2}}{E_{L1}}$

l_i ideal core width

L leakage coefficient

m_1 number of phases of the stator

N number of turns per phase

r resistance

s slip

x leakage reactance

x_m reactance due to the main flux

z angular velocity of the oscillation

δ angle between primary and secondary voltage

$\tau_1=\frac{x_1}{x_m}$, $\tau_2=\frac{x_2}{x_m}$ leakage coefficients

τ_p pole pitch

References

1. DAMPING TORQUE AND DESIGN OF THE DAMPER WINDING OF SYNCHRONOUS MACHINES, M. M. Liwschitz. *Arch. f. Elektrot*, volume 10, 1921.

2. TWO REACTION THEORY OF SYNCHRONOUS MACHINES, I, R. H. Park. AIEE TRANSACTIONS, volume 48, 1929, pages 716–27.

3. POSITIVE AND NEGATIVE DAMPING IN SYNCHRONOUS MACHINES, M. M. Liwschitz. AIEE TRANSACTIONS, volume 60, 1941 (May section), pages 210–13.

Relative Accuracy of Three-Phase Metering Combinations

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Synopsis: This paper grades several three-phase combinations of instrument transformers and watthour meters under various conditions according to their approach to the best standard practice. The combinations are described and sources of uncertainty or of error are discussed. Diagrams are included to show the schematic connections of the component transformers and meters, both separately and in combination and of the power systems to be metered.

Introduction

FROM time to time questions arise in regard to the accuracy of three-phase metering combinations of instrument transformers and watthour meters. A definite estimate of the accuracy to be expected can usually be made for a given installation if sufficient details are available. However, in many cases all that is required is an idea of the relative accuracy obtainable. To meet such situations, relative accuracy under various conditions is here assigned to typical metering combinations now in use and to others sometimes proposed. These combinations are considered principally from the instrument-transformer standpoint. Operating and metering difficulties sometimes encountered with Y-connected potential transformers and the effects of various secondary interconnections on the metering accuracy are discussed. Much of this discussion applies to three-phase indicating wattmeters as well as to watthour meters.

A summary of the paper is given in table I, wherein the relative accuracy is graded from A to E. Grade A is assigned to cases where the metering combination and the conditions of application represent the best standard practice, which ordinarily embodies a close approach to the ideal. Three of the four power systems included are classified as three-phase, three-wire. There are five instrument transformer combinations and three types of watthour meters: 2-element, 2½-element, and 3-element, respectively. In each case, sufficient comments are included to make table I reasonably self-explanatory. Table I is supplemented by figures 1 to 4, which show the schematic connections of the different parts and also of six complete metering applications.

It should be kept in mind that the ideal polyphase metering combination from the standpoint of accuracy is one that can be resolved into two or more component single-phase sections. Each section consists of one potential transformer, one current transformer, and one meter element. The instrument transformers can be checked for accuracy at any time and a portable single-phase indicating wattmeter or a watthour-meter standard can be added to the secondary meter circuits. Thus the accuracy of the polyphase combination can be verified to the satisfaction of any legal authority from the summation of two or more sets of single-phase results obtained simultaneously. For the highest accuracy, complete single-phase sections should be used as standards entirely separate from the polyphase combination. However, the accuracy obtainable with either method will not be uniform over wide ranges of power factor or load.

Power Systems

Three-Phase Three-Wire Systems

These systems may be classified according to the treatment of the system neutral. Systems with isolated neutral (figure 1*a*) or with a ground at one neutral point only (figure 1*b*) are common and require no special comments. Ordinarily it is immaterial from the metering standpoint whether the neutral is isolated or is grounded at one point either solidly or through low or high impedance; a ground-fault neutralizer (Petersen coil) constitutes an example of a high-impedance ground. Separate classifications for the two systems are justified, since single-phase loads could be connected between any line and ground with single-grounded neutral systems, particularly if the neutral is grounded through a low impedance, whereas such loading is not possible with isolated-neutral systems. For systems grounded at but one neutral point, it is assumed that there will be no line-to-ground loads.

Systems grounded at more than one neutral point (multiple grounds—figure 1*c*) in the manner shown probably should be considered as three-phase, four-wire systems, even if not so classified, not only because single-phase loads could be connected between any line and ground as just pointed out, but particularly because such loads can now be applied through the Y-Y-connected power transformers. These transformers often are provided with tertiary windings for closed-delta connection to take care of the triple harmonics in the exciting currents.

Three-Phase, Four-Wire Systems

This classification applies to systems having four metallic conductors (figure 1*d*). Such systems also are common and usually supply single-phase loads connected between any line and the neutral in addition to three-phase, three- or four-wire loads. The system neutral may or may not be grounded at one or more points.

Instrument Transformers

The principal functions of instrument transformers are to insulate low-voltage

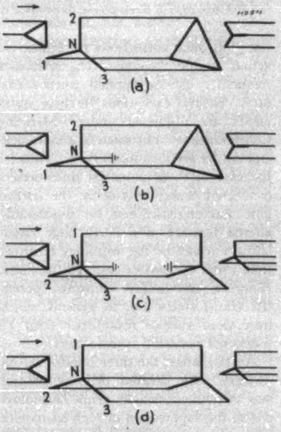

Figure 1. Three-phase power systems

Three-phase three-wire with
(*a*) Isolated neutral
(*b*) One neutral ground
(*c*) Multiple neutral grounds

Three-phase four-wire
(*d*) With or without neutral ground

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1. For all numbered references, see list at end of paper.

measurement devices from dangerous-to-life line voltages and to act as multipliers for them. In the instrument transformer combinations under consideration (figure 2), the current-transformer primaries are connected in the proper lines and their individual secondaries supply individual meter-current circuits. On the other hand, the potential-transformer primaries are connected either line-to-line or line-to-neutral, so their individual secondaries either supply individual meter potential circuits or constitute part of a network.

The currents and voltages obtained from the secondaries of instrument transformers usually represent the corresponding primary quantities with acceptable accuracy without making corrections for the respective ratio and phase angle errors, but such corrections for either the actual or the "average" characteristics frequently can be applied when necessary. Several exceptions to these statements are discussed under "Metering Considerations" but some of a more general nature are mentioned here. Thus for the case of three potential transformers connected closed-delta-delta, the normal unit characteristics may be considerably altered because of a fairly large circulating current in the windings resulting from small differences between the units. However, line-to-line connected potential transformers are, in general, much freer from various disabilities than Y-connected potential transformers.

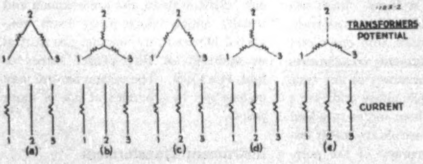

Figure 2. Instrument - transformer combinations for three-phase systems

Primaries of two current transformers and; *(a)* two potential transformers in open delta; *(b)* three potential transformers in wye. Primaries of three current transformers and; *(c)* two potential transformers in open delta; *(d)* three potential transformers in wye; *(e)* two potential transformers in part wye.

(a), (b), (c), (d) For three-phase, three-wire systems; *(d), (e)* for three-phase, four-wire systems

For the latter, the three line-to-neutral voltages with isolated primary neutral may contain appreciable triple harmonics due to the suppression of such harmonics in the respective exciting currents; this distortion actually may cause an erroneous voltage indication, to say nothing about rendering normal transformer corrections of doubtful value. If the primary neutral is grounded, but if the system neutral is ungrounded either permanently or temporarily, the potential transformer line-to-neutral voltages may become very badly distorted and the neutral point, in effect, may even get outside of the supply-voltage triangle due to switching surges.[1,2,3]

Watt-Hour Meters

Two-Element Meters (Figure 3*a*)

These meters have two separate driving elements, which may act either on separate disks mounted on a common shaft or on different segments of the same disk. Such meters are standard for three-phase, three-wire systems.

Two-and-One-Half-Element Meters (Figure 3*b*)

These meters also have two separate elements but each set of current coils is divided into two equal, separate sections. One section and the corresponding potential coil of one element are supplied from one system phase and the other similarly arranged element is supplied from a second system phase somewhat like a straight 2-element meter. Current from the third system phase flows in a reversed direction through the two extra meter current-coil sections in series and thus reacts with the voltages of the other two system phases applied to the two meter potential coils. The vector sum of these two phase voltages usually equals the voltage of the third phase, so it is evident that the "$^{1}/_{2}$-element" then operates correctly but with a synthetic instead of an actual third-phase voltage. However, this vector sum does not always correctly represent the third-phase voltage and under such conditions the "$^{1}/_{2}$-element" registration will be in error; of course, it contributes but one-third the torque and its error but one-third the effect in a balanced system.

These meters also are known as 2-element meters with split current coils. They are standard for three-phase, four-wire systems when the three voltages are approximately balanced and are used to avoid the extra potential transformer, wiring, and expense required for a 3-element meter.

Three-Element Meters (Figure 3*c*)

These meters have three separate elements which are supplied from the three phases of a three-phase, four-wire system for which they are standard under all conditions of load and of voltage balance. The meters are entirely satisfactory for three-phase, three-wire systems having one or more neutral grounds.

Metering Considerations

Single-Phase Metering

The metering of a single-phase system will serve as an introduction to the metering of polyphase systems. A single-phase system requires one potential transformer, one current transformer, and one watthour meter. Certificates for the two instrument transformers will give the respective ratio correction factors to within 0.1 per cent and the phase angles to within three minutes. The watthour meter can be adjusted to be correct within say 0.3 per cent over a medium load range. Thus an overall accuracy within 0.5 per cent over such a load range near unity power-factor could be expected, even if the errors were all in the same direction, which would be most unusual. Better accuracy can be obtained at a definite and fairly steady load point by taking the average of a large number of observations on a high-grade portable indicating wattmeter, which can be checked to within 0.15 per cent over the upper part of its scale. On a similar basis, an overall accuracy within 0.35 per cent could be expected under this condition.

The single-phase metering combination just described represents the simplest case, where the corrections determined for the individual parts can readily be combined algebraically and an estimate made of the best accuracy obtainable. Since a polyphase power system consists in effect, of two or more out-of-phase single-phase systems, it should be obvious that the highest accuracy can be obtained only if single-phase metering conditions are maintained. This accounts for long-standing cautions against some "secondary interconnections" in polyphase metering circuits.

Three-Phase Three-Wire Metering

Applying this idea to a three-phase, three-wire, ungrounded system, two current and two potential transformers and two single-phase watthour meters would

Table I. Relative Accuracy

Case	System	Neutral: Pot. Trans. Pri.	Neutral: Pot. Trans. Sec.	Neutral: Meter	Connections: Pot. Trans.	Connections: Meter	Comments	Figure	Accuracy Grade
3-Phase 3-Wire Systems									
2-element meter—2 current transformers—2 potential transformers									
1	*Isolated				Open Δ	Open Δ	Best standard practice	4a	A
2	**1 ground				Open Δ	Open Δ	Best standard practice	—	A
3	***Mult. gr.				Open Δ	Open Δ	L/N loads not metered correctly	—	D
2-element meter—2 current transformers—3 potential transformers									
4	Isolated	Iso./gr.	Gr.		Y-Y	Open Δ	Indeterminate	—	C
5	1 ground	Iso./gr.	Gr.		Y-Y	Open Δ	Indeterminate	4b	C
6	Mult. gr.	Iso./gr.	Gr.		Y-Y	Open Δ	Indeterminate; related to *case 3*	—	B
3-element meter—3 current transformers—2 potential transformers									
7	Isolated			Iso.	Open Δ	Y	Indeterminate	—	C
8	1 ground			Iso.	Open Δ	Y	Indeterminate	4c	C
9	Mult. gr.			Iso.	Open Δ	Y	Indeterminate	—	C
3-element meter—3 current transformers—3 potential transformers									
10	Isolated	Iso./gr.	Gr.	Iso.	Y-Y	Y	Indeterminate	—	C
11	1 ground	Iso./gr.	Gr.	Iso.	Y-Y	Y	Indeterminate	4d	C
12	Mult. gr.	Iso./gr.	Gr.	Iso.	Y-Y	Y	Indeterminate	—	C
13	Isolated	Gr.	Gr.	Gr.	Y-Y	Y	Neutral instability possible	—	C
14	1 ground	Gr.	Gr.	Gr.	Y-Y	Y	Best standard practice	—	A
15	Mult. gr.	Gr.	Gr.	Gr.	Y-Y	Y	Best standard practice	4e	A
3-Phase 4-Wire Systems									
3-element meter—3 current transformers—3 potential transformers									
16	Iso./gr.	† Iso./gr.	Gr.	Gr.	Y-Y	Y	Best standard practice	—	A
2½-element meter—3 current transformers—2 potential transformers									
17	Iso./gr.	† Iso./gr.	Gr.	Gr.	L/N	L/N	Standard practice	4f	B

NOTES: L/N = line-to-neutral; Iso./gr. = isolated or grounded.

*Figure 1*a*.

**Figure 1*b*—it is assumed that there will be no line-to-ground loads.

***Multiple grounds—figure 1*c*—these 3-phase, 3-wire systems probably should be considered as 3-phase, 4-wire systems and provision made for metering correctly single-phase loads connected between any line and neutral or ground.

†In *cases 16* and *17*, the potential-transformer and system neutrals are tied together; the system neutral may either be isolated from ground (figure 1*d*) or grounded at one or more points. See text for further comments on the 2½-element metering combination.

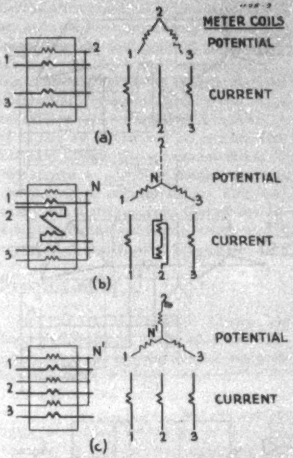

Figure 3. Polyphase watt-hour meters

(a) Two-element for three-phase three-wire systems

(b) Two-and-one-half-element for three-phase four-wire systems

(c) Three-element for three-phase three- or four-wire systems

be required and these would constitute two separate, single-phase metering circuits. Of course, such separate metering circuits are quite unusual, except for special tests. The meters are slightly handicapped by the fact that a phase displacement of 30 degrees exists between the line-to-line voltages and the line currents (at unity power factor).

Thus for *case 1* in table I (figure 4*a*), the first "secondary interconnection" occurs in the substitution of a 2-element polyphase watthour meter for the two single-phase meters, which necessitates an average correction for the instrument-transformer and meter-element errors. Other interconnections occur in the substitution of three secondary leads for four both for the current transformers and for the potential transformers. This tends to complicate the ratio and phase-angle tests, particularly if the secondary leads are long.

For balanced systems, the secondary current in the common lead will equal those in the other two leads, but will be 120 degrees out of phase with them and the resulting additional common burden for the two current transformers must be allowed for in making up the respective test burdens. If this common burden is large due to a long lead or to measurement or protective devices connected in series therewith, it will be necessary to know the phase sequence of the power system, as well as the actual phases in which the current transformers are connected in order to test them properly.

For two equal potential-transformer burdens, the current in the common potential lead will be $\sqrt{3}$ times those in the other two leads and 30 degrees out of phase with them; neglecting the voltage drop in the leads may cause appreciable errors in the ratio and phase-angle results. Further complications occur if a burden is connected across the open delta (third phase without potential transformer); even if its power factor is unity, it imposes its full voltamperes at 0.5 power factor on both potential transformers, leading in one case and lagging in the other, thus again making it necessary to know the phase-sequence of the power system and the connections of the individual transformers thereto: the importance of this proceeding increases with the magnitude of the burden. Somewhat similar considerations apply when double autotransformers are used to shift the phase of the potential transformer voltages to supply var- or voltampere-hour meters, since the net effect is to change the burden distribution. Interconnected, low-power-factor burdens sometimes have surprising effects: when two potential transformers supply such Y-connected burdens, not only does one transformer supply all the energy necessary, but it also tends to excite the other transformer secondary winding! This undesirable effect increases with the magnitude of the burdens and may be overcome by adding suitable non-inductive burdens.

In spite of the various uncertainties just outlined, some of which are inherent in the *case 1* metering combination, satisfactory accuracy is usually obtained. A similar statement applies when the power-system neutral is grounded at one point only as in *case 2*.* Thus the highest relative accuracy—grade A—is assigned to *cases 1* and *2*.

A much greater degree of uncertainty exists when three Y-connected potential

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**Case 3* and others involving grounds at more than one power-system neutral point are discussed under "Three-Phase, Four-Wire Metering."

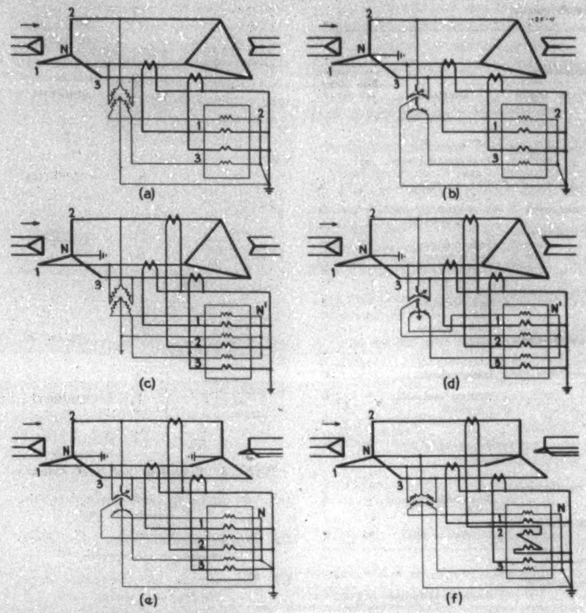

Figure 4. Schematic connections of three-phase metering combinations (cases outlined in table I)

- (a) *Case 1* (figures 1*a*, 2*a*, 3*a*)
- (b) *Case 5* (figures 1*b*, 2*b*, 3*a*)
- (c) *Case 8* (figures 1*b*, 2*c*, 3*c*)
- (d) *Case 11* (figures 1*b*, 2*d*, 3*c*)
- (e) *Case 15* (figures 1*c*, 2*d*, 3*c*)
- (f) *Case 17* (figures 1*d*, 2*e*, 3*b*)

transformers are substituted for two connected open delta to supply the two potential circuits of a 2-element meter as in *cases 4* and *5* (figure 4*b*). Here it is impossible not only to allocate properly the three individual potential transformer corrections, which may differ slightly, but also to resolve the combination into component single-phase metering circuits, which is much more important. The three line-to-neutral transformer voltages will be distorted (peaked) if the primary neutral is isolated and may be distorted if it is grounded with system neutral isolated, as was pointed out under "Instrument Transformers." However, the secondary line-to-line voltages applied to the two meter circuits will not be distorted, assuming sine-wave supply voltages. The uncertainty necessitates assigning a considerably lower relative accuracy—grade C—to *cases 4* and *5*.

A similar indeterminate situation exists when two open-delta connected potential transformers are used to supply the three Y-connected potential circuits of a 3-element meter as in *cases 7* and *8* (figure 4*c*), which is sometimes recommended for low-power-factor loads on three-phase, three-wire systems. Here the three line-to-neutral meter voltages are somewhat distorted due to the suppression of the triple harmonics in the exciting currents. Such distortion will not occur if the inductive meter circuits are replaced by noninductive wattmeters.

The foregoing situation is not improved by the substitution of three Y-connected potential transformers for the two in open delta, if the two secondary neutral points are not connected as in *cases 10* and *11* (figure 4*d*). Here again as in *cases 4* and *5*, the three line-to-neutral transformer voltages will be distorted if the primary neutral is isolated and may be distorted if it is grounded with system neutral isolated, but the secondary line-to-line voltages applied to the Y-connected meter circuits will not be distorted with a sine-wave supply.

The indeterminate features of *cases 10 and 11* are partly overcome by connecting the potential-transformer-secondary and meter-potential-circuit neutral points as in *case 13*. However, the three line-to-neutral voltages may be distorted and potential-transformer-neutral instability may result as with similar neutral conditions in *cases 4* and *10*, due to the attempt to ground the power system through the potential transformers. Because of this, *case 13* is assigned relative-accuracy grade C also.

Grounding the power-system neutral at one point as in *case 14* translates *case 13* into standard practice. *Case 14* closely approaches the ideal of component single-phase metering sections mentioned in the "Introduction." The eight secondary leads (as in figure 4*e*) are practically equivalent to twelve, since the currents in the two common leads normally are negligible. Three single-phase watt-hour meters may readily be added for verification purposes and each meter or meter element operates at the same power factor as that of the load in its respective phase, which is particularly advantageous at low power factors. This favorable condition does not exist with balanced three-phase loads in *cases 1* and *2;* however, these are entitled to share relative-accuracy grade A with *case 14* for most applications.

THREE-PHASE FOUR-WIRE METERING

A 3-element meter with three current and three potential transformers as in *cases 14, 15* (figure 4*e*) and *16* is standard both for three-phase, three-wire systems with at least one neutral point grounded and for three-phase, four-wire systems. Relative-accuracy grade A is assigned to this metering combination. Three-phase, three-wire systems with multiple neutral grounds are equivalent in many respects to three-phase, four-wire systems, even if not so operated with single-phase loads connected between any line and the neutral ground.

A $2^{1}/_{2}$-element meter with three current and two potential transformers as in *case 17* (figure 4*f*) is standard for the usual condition where the three line-to-neutral voltages of the three-phase, four-wire system are fairly well balanced; relative-accuracy grade B is assigned to this metering combination.

Although designed primarily for three-phase, four-wire systems, it should be evident from a consideration of *cases 16* and *17* that the $2^{1}/_{2}$-element metering combination can be substituted for the 3-element metering combination in *cases 14* and *15*. Relative-accuracy grade B would then be assigned unless the system-neutral grounds are effected by means of ground-fault neutralizers; the line-to-neutral voltages may then be consider-

Impulse Strength as a Measure of Cable Quality

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Synopsis: Testing cables by means of the impulse generator, originally developed for use in studying the effect of lightning on cables, is becoming recognized more and more as a useful method for determining cable quality. It is felt that the amount of data accumulated during the past ten years is sufficiently large to be used in formulating a theory concerning the factors governing the impulse strength of cables. It is indicated in this paper that maximum rather than average stress should be taken into account when impulse test results are evaluated, since the maximum stress at breakdown is a measure of the quality of the insulation alone. This view is contrary to that generally accepted in this country although some workers in this field abroad have accepted this theory for some time.

Introduction

THE Detroit Edison Company has in the past sponsored a number of tests to determine the impulse strength of various types of cable in relation to other electrical equipment. Results of some of these tests on "solid-type" impregnated paper (24 and 4.8 kv) and varnished cambric (4.8 kv) cables were published by McCabe[1,2] from time to time. About two years ago the scope of these tests was enlarged to include various types of impregnated paper insulated cables proposed for extra-high voltage service as no impulse test data were then available on such cables. Held and Leichsenring[3] and Foust and Scott[4] have since published results of impulse tests on oilfilled cables.

Oilfilled, oilostatic, compression, and gas-filled (Beaver) cables were in commercial use two years ago at voltages above those recommended for solid type cables. As gas-filled cable was not on hand, oilfilled, oilostatic, and compression cables having equal insulation thicknesses, were tested. The results of these tests indicated that maximum rather than average stress determines the impulse strength of the cable. In order to verify this statement, tests were made on solid type cables of various conductor diameters, conductor surfaces and insulation thicknesses, and on low gas pressure cables. In computing average stresses at breakdown on the solid type cables, no definite correlation could be established between these data and the quality of the insulation as observed by examination of the samples. On the other hand, when maximum stresses at breakdown were computed, values were obtained that showed pronounced increase with improving insulation, reaching a constant maximum value at the best obtainable impregnation. Further tests on oilfilled cables showed that uniformity of taping as determined by the Mildner diagram affects the maximum stress at breakdown in a similar manner as does the degree of impregnation. This correlation is a strong indication of the significance of the maximum stress as the criterion of quantitative breakdown.

Previous Work

No universally accepted theory has been proposed that would allow the computation of the impulse breakdown voltage of a cable of known construction details and known insulation qualities. A study of the published data on impulse tests of cables, however, permits the drawing of conclusions concerning certain factors that influence the impulse strength.

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1. For all numbered references, see list at end of paper.

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ably unbalanced due to unequal capacitances to ground of the three line conductors. Under this condition, the 3-element metering combination should be used. Of course, the straight 2-element metering combination (*case 2*) may be used for all such three-phase, three-wire systems if the single-phase loads to neutral are negligible or if these loads are balanced.

The limitations just placed on the application of 2-element metering combinations are important. In *cases 3* and *6*, for example, a load connected between the line without current transformer and ground would not be metered at all and the meter voltages would not have the proper magnitude or phase for loads connected between the other two lines and ground; this accounts for the low relative-accuracy grades D and E, respectively. The grade C accuracy assigned to the 3-element metering combination in *cases 9* and *12* is due to its being indeterminate and not to the possibility of incorrectly metering single-phase loads to ground.

In the foregoing discussion, the term "indeterminate" has been used to describe conditions where it is impossible to resolve the metering combination into component single-phase sections and thus to allocate instrument-transformer and meter corrections definitely. Although it need not be assumed that "indeterminate" is synonymous with "inaccurate," arguments advanced as to the correctness of such a metering combination may sound unconvincing. It should be emphasized that all important metering installations are at all times subject to verification or checking when questions arise in regard to their correctness.

Conclusions

A general idea of the performance to be expected from several three-phase metering combinations under various conditions is conveyed by the relative accuracy assigned thereto in table I. Among the many cases included, it should be possible to find examples approximating other cases sometimes used or proposed and thus permit grading them also. From the accompanying discussion, it should be evident that in cases where grade A is assigned to the metering combination, very satisfactory accuracy can be obtained if appropriate secondary connections are used and if the instrument-transformer corrections are properly applied.

References

1. Experiences With Grounded-Neutral, Wye-Connected Potential Transformers on Ungrounded Systems, C. T. Weller. AIEE Transactions, volume 50, 1931, pages 299–316.

2. Physical Nature of Neutral Instability, A. Boyajian and O. P. McCarty. AIEE Transactions, volume 50, 1931, pages 317–27.

3. Theory of Abnormal Line-to-Neutral Transformer Voltages, C. W. LaPierre. AIEE Transactions, volume 50, 1931, pages 328–42.

(a). QUALITY OF INSULATION

Held and Leichsenring[3] and Busse and Vogel[5] established that the use of denser papers results in higher impulse strength; and that temperature of the cable during the tests, provided the limits do not exceed those of commercial operation, does not measurably influence the impulse strength.[3] By inference, the latter signifies that the viscosity of the impregnating compound (which decreases as the temperature increases) has also a very limited effect on the impulse strength. Whether "straight" mineral oil is used to impregnate the cable or a mixture of mineral oil and rosin, appears to have no effect on the impulse strength of the cable. The degree of impregnation at a given temperature, however, seems to have a very decided effect.[1] That service aging of solid-type cables, which generally results in a decrease of the degree of impregnation, will also result in a proportional decrease of impulse strength is indicated[2] but this requires further study.

(b). GEOMETRY OF CABLE AND SURFACE OF CONDUCTOR

The effect of conductor diameter (minimum radius) and conductor surface appears to have influenced the results of several workers. For instance when specially constructed copper conductors having the same diameter and smooth outer surface were used, the number of variables was reduced to such an extent that from the results obtained it was possible to disprove the theory still generally accepted in this country, that thinner insulation possesses a comparatively higher impulse strength.[3] Tests on cables with sector shaped conductors disclosed the fact that 80 per cent of the failures were on the "shoulder" (minimum radius of the conductor) of the sector.[1] The same author reports that strand-shielding improves the impulse strength considerably.[2] Davis and Eddy found that the impulse strength of rubber in sheet form is approximately 50 percent higher than that of rubber cable insulation of the same wall thickness.[6]

(c). POLARITY OF THE IMPULSE WAVE

Schneeberger,[7] in discussing the effect of polarity of the impulse wave on the impulse strength of a cable, indicated that when a positive wave was used, the breakdown value was 20 percent lower than with a negative wave. Later reports contradict this finding. One author has found no difference in oilfilled cables but in solid cables the impulse strength, using a negative wave, was from 5 to 13 percent lower than when a positive wave was used.[3] Another paper states that the authors found no difference in impulse strength due to variations in the shape and polarity of the impulse wave.[6]

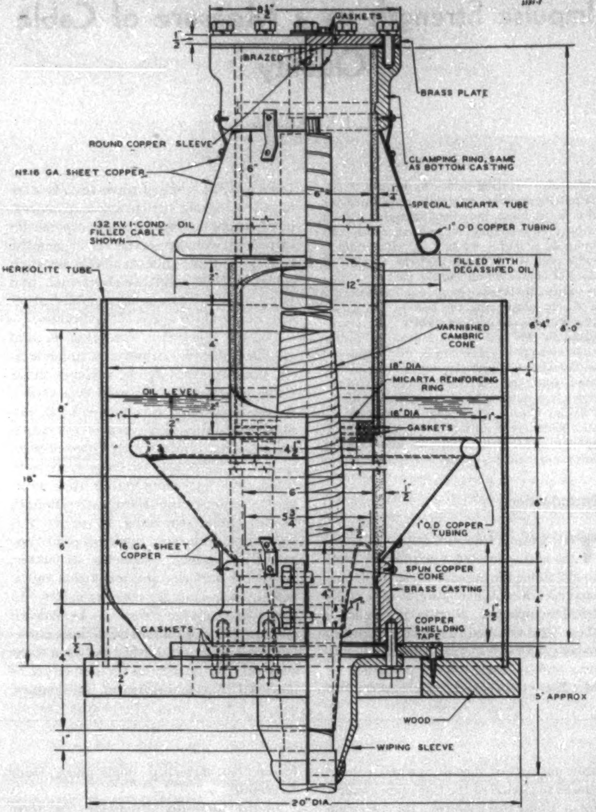
1131-1
8½"
GASKETS
½"
BRAZED
BRASS PLATE
ROUND COPPER SLEEVE
CLAMPING RING, SAME AS BOTTOM CASTING
No.16 GA. SHEET COPPER
6"
6"
¼"
SPECIAL MICARTA TUBE
132 KV. 1-COND. OIL FILLED CABLE SHOWN
1" O D COPPER TUBING
HERKOLITE TUBE
FILLED WITH DEGASSIFED OIL
2"
12"
VARNISHED CAMBRIC CONE
6'-4"
8'-0"
4"
18" DIA
¼"
MICARTA REINFORCING RING
OIL LEVEL
8"
16" DIA
1"
2"
GASKETS
4½"
18"
6"
½"
5¾"
½"
1" O.D COPPER TUBING
6"
#16 GA SHEET COPPER
SPUN COPPER CONE
BRASS CASTING
4"
1"
4"
3½"
COPPER SHIELDING TAPE
GASKETS
½"
4"
2"
5" APPROX
WOOD
1"
WIPING SLEEVE
20" DIA

Figure 1. Impulse terminal assembly for high-voltage surge test

In view of the above data, no definite conclusion can be drawn in regard to the effect of the polarity of the impulse wave on the impulse strength of the cable. For this reason, small variations in absolute values reported by various authors may exist, but the relative rating of the individual samples will probably still remain the same.

Method of Test and Testing Equipment

Engineers of The Detroit Edison Company favor the impulse testing of electrical apparatus with a negative wave because field testing has shown the prevalence and greater importance of negative lightning strokes in connection with transmission line outages. A method of testing with sufficiently high potential surge to obtain breakdown with one surge is also favored for reasons of simulating conditions in the field. All tests described in this paper, therefore, were made with negative wave and attempts were made to obtain breakdown with one surge.

The impulse generator used has been previously described.[1] For these tests additional condensers were used in series and both ends of the cable were encased in terminals built to withstand 1,000 kv and

200 psi pressure. Figure 1 shows the terminal assembly. Figure 2 shows the method used in connecting the cable to the impulse generator. Figure 3 is an oscillogram of the incident wave (approximately 0.05×24 microsecond) with the cable disconnected, and figure 4 is a typical wave recorded during breakdown of sample *J*1. It should be noted that the steepness of the wave front is somewhat lessened when the cable is connected to the generator.

Presentation of Results and Description of Samples

The results of the impulse tests which form the basis of this paper, are graphically illustrated in figure 5, and complete data are supplied in tables I and II.

Table I

Tested in Detroit

No.	Type of Cable	r.	w.	R.	n.	Breakdown Volt. Kv	G. Avg. V/Mil	G. Max. V/Mil	Qual. of Taping	Degree of Impr.
*A*1	Solid	145	156	301	7	230	1,475	2,715	Fair	Fair
*A*2						225	1,440	2,660	Fair	Fair
*B*1	Solid	167	240	407	"Compact" strand	280	1,170	1,880	Very good	Very ltd.
*B*2	Low gas press.	167	240	407	"Compack" strand	300	1,250	2,010	Very good	Very ltd.
*B*3						300	1,250	2,010		
*B*4						315	1,310	2,110		
*B*5						320	1,330	2,150		
*B*6						335	1,400	2,250		
*C*1	Solid	151	281	432	Compack strand	380	1,350	2,390	Very good	Very good
*C*2						390	1,390	2,450		
*C*3						400	1,425	2,520		
*D*1	Solid	166	261	427	Strand shield	480	1,840	3,060	Very good	Very good
*D*2						485	1,860	3,090		
*D*3						500	1,915	3,180		
*E*1	Compr.	290	386	676	18	570	1,475	3,010	Very good	Very good
*E*2	Oilost.					580	1,500	3,060		
*E*3	Compr.					585	1,515	3,080		
*E*4	Compr*					630	1,630	3,320		
*F*1	Oil-filled	542	386	928	37	730	1,890	3,250	Very good	Very good
*F*2						775	2,010	3,450		
*G*1	Oil-filled	514	450	964	25	590	1,310	2,360	Very irr.	Very good
*G*2						680	1,510	2,720		
*H*1						820	1,820	3,280	Very good	Very good
*H*2						825**	1,835	3,300		
*I*1	Solid	340	656	996	18	760	1,160	2,700	Fair	Fair
*J*1	Solid	576	500	1,076	24	830	1,660	2,980	Fair	Fair

*Without pressure.

**No failure.

r. Minimum radius of conductor, mils.

w. Thickness of insulation.

R. Minimum radius over insulation.

n. Number of strands in outer layer.

This chart was compiled to show breakdown voltage and maximum stress at breakdown, for all the samples tested in Detroit as well as those tested elsewhere, when sufficient data were available. It should be noted that the Detroit samples are designated by a letter and a number, each letter representing a cable and each number an individual measurement on a sample of the cable. The data published by Foust and Scott,[4] is represented by the same numbers the authors used in their paper. Number 34 refers to the paper cable sample used by Davis and Eddy[6] and numbers 35 to 40 represent the Held and Leichsenring samples.[8] It should be noted that this latter group represents minimum values of groups of measurements.

Pertinent information regarding the Detroit samples is given in the following:

Samples *A*1, *A*2, *I*1, and *J*1 were solid type, single-conductor cables. The *A* samples had a small conductor and thin insulation. Sample *I*1 had a conductor diameter similar to that of samples *E*1 to *E*4 (compression and oilostatic) but an insulation thickness almost twice as great (170%). Sample *J*1 had a large conductor diameter similar to that of samples *F*1, *F*2, *G*1, *G*2, *H*1, and *H*2 (oilfilled) but greater insulation thickness (130 and 111% respectively).

Samples *B*1, *B*2, *B*3, *B*4, *B*5, and *B*6 were low gas pressure, three-conductor, sector, shielded, 27.6-kv cables. Samples *B*1, *B*2, and *B*3 represent three conductors of one section and samples *B*4, *B*5, and *B*6 the three conductors of another section of the same cable. Ten complete failures were observed in the six samples, seven of which were at the "shoulders" of the sector.

Samples *C*1, *C*2, *C*3, *D*1, *D*2, and *D*3 were solid type, three-conductor, sector, shielded, 24-kv cables. The *C* samples represent the three conductors of one cable, while the *D* samples are the three conductors of another cable. The two cables were identical except for the strand-shielding with semi-conducting tapes employed in the latter group. The manufacturer who made these cables used the same type of paper, same taping machine, impregnated the two cables in the same tank, and lead covered both on the same day. Therefore, except for the strand-shielding, there should be no other variables. Failures in all six samples occurred at or near the "shoulder" of the sector.

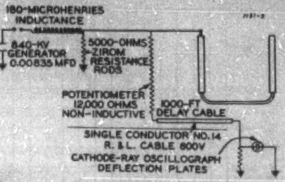
180-MICROHENRIES INDUCTANCE
840-KV GENERATOR 0.00835 MFD
5000-OHMS ZIROM RESISTANCE RODS
POTENTIOMETER 12,000 OHMS NON-INDUCTIVE
1000-FT DELAY CABLE
SINGLE CONDUCTOR NO. 14 R. & L. CABLE 600V
CATHODE-RAY OSCILLOGRAPH DEFLECTION PLATES
1191-2

Figure 2. Impulse-generator wiring diagram

84 0.7-microfarad capacitors, 2 capacitors in series per bank. Capacitors charged to 20 kv per bank

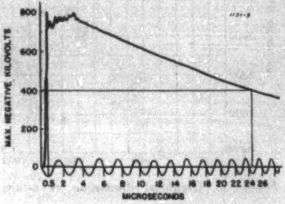

Figure 3. Oscillogram of incident wave

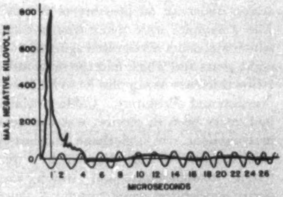

Figure 4. Typical breakdown wave

Samples *E*1, *E*2, *E*3, and *E*4 were taken from the same oval-shaped, single-conductor cable. Samples *E*1 and *E*3 were tested in a steel pipe containing nitrogen under 200 psi (compression cable). *E*2 was tested as an oilostatic cable, that is, the armoring and lead sheath were removed, and the pipe that contained the cable was filled with oil under 200 psi. *E*4 was tested without the application of pressure; in other words, as a solid type cable. All four samples failed near the major axis of the oval conductor. Figure 6 shows the Mildner-diagram of cable *E*.

Samples *F*1, *F*2, *G*1, *G*2, *H*1, and *H*2 were taken from three single-conductor oilfilled cables made by three manufactur-

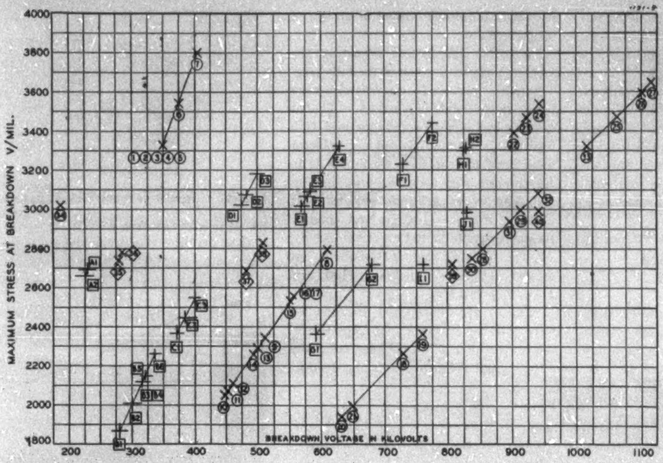

Figure 5. Graphical illustration of results

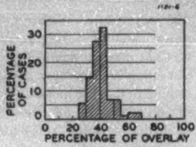

Figure 6. Mildner diagram of cable E

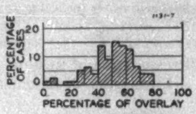

Figure 7. Mildner diagram of cable H

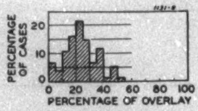

Figure 8. Mildner diagram of cable G

ers, respectively. All six samples were tested under an oil pressure of ten psi. The *F* samples were taken from a cable which was under accelerated aging test for eight years and which had the same insulation thickness as samples *E*1 to *E*4 (compression and oilostatic). Cable *G* which had never been in service, was identical in conductor size and shape, insulation thickness, etc. to cable *H*, made by a different manufacturer and which was under accelerated aging test for a short time. Figure 8 shows the Mildner-diagram of cable *G* and figure 7 of cable *H*.

Discussion of Results

Among the 26 samples tested in Detroit, *A*1, *A*2, *I*1, and *J*1 were single conductor, solid type cables, each having a standard stranded, round, copper conductor. Since the taping and impregnation were approximately the same for the four samples, they offer a basis for comparison of geometrically different constructions. Tabulating the average stresses at breakdown discloses a ratio of 1.43 between maximum and minimum values. Calculating on the basis of maximum stresses after correcting for stranding, yields a ratio of 1.11. This is clearly an indication that maximum rather than average stresses determine the breakdown voltage.

Further indication of the correctness of this theory is obtained in considering another group of three cables of equal insulation characteristics, namely samples *E*4, *F*1, and *F*2. Both cables from which these samples were taken had the same insulation thickness, uniform taping, and high degree of impregnation, but cable *F* had a conductor diameter very much larger than that of cable *E*. The ratio of average stresses of cables *E* and *F* was 1.2 in favor of cable *F*. The ratio of maximum stresses is, on the other hand, 1.01.

Having considered two different groups of cables separately, comparison of the two groups in relation to each other will now be made. The maximum stresses of the first group vary from 2,660 to 2,980 volts per mil, whereas the corresponding values for the second group are 3,250 to 3,450 volts per mil, the averages of the two groups being 1.21. Visual examination of the samples indicated that those in the first group exhibited a lesser uniformity of taping and a lower degree of impregnation than samples in the second group. Thus it can be concluded that the maximum stress, which has been shown to be independent of the geometry of the construction, depends solely upon the quality of the insulation, as measured by the uniformity of taping and the degree of impregnation.

Further and more specific proofs of this statement are supplied by results of tests on other groups of cables. The influence of taping and that of impregnation will be considered in separate paragraphs.

The best evidence in regard to the influence of the quality of taping on the impulse strength of cables was supplied when samples *G*1, *G*2, *H*1, and *H*2 were tested and examined. The cables from which these samples were taken, were identical in dimensions and, both being oil-filled, possessed the same high degree of impregnation. The samples differed from each other in only one respect, i.e. in the degree of uniformity of taping which again was manifested in much lower maximum stress at breakdown (23 percent) for the samples with inferior taping.

The importance of the degree of impregnation without the disturbing influence of variations in taping is demonstrated when the results of tests on the low gas-pressure cable, samples *B*1 to *B*6, are examined and compared with the results of tests on a well-impregnated solid-type cable, samples *C*1 to *C*3. Both types of cables contained sector shaped conductors having nearly equal minimum radii (167 and 151 mils, respectively) at the "shoulders," "compack" stranding was employed, producing approximately equally smooth surfaces; and the taping was equally uniform. The less complete impregnation in the low gas-pressure samples, therefore, appears to be responsible for the 15 percent decrease in maximum stresses.

The smoothness of the electrode surface has for some time been known to influence the impulse flash-over values in gases and liquids. How much practical importance should be attached to smoothing of the conductor surface in cables is indicated by comparing samples *C*1 to *C*3 ("compack" stranded) to samples *D*1 to

Table II

No.	Tested by	Type of Cable	r.	w.	R.	n.	Break-down Volt. Kv	G. Avg. V/Mil	G. Max. V/Mil
1–5	Foust and Scott	Solid	210	187	397	12	350	1,870	3,330
6							375	2,005	3,570
7							400	2,140	3,800
8	Foust and Scott	Oil-filled	945	315	1,260	48	610	1,935	2,895
9							512	1,625	2,430
10							448	1,420	2,120
11							453	1,440	2,150
12							461	1,465	2,180
13							500	1,590	2,370
14							490	1,555	2,320
15							555	1,760	2,630
16							559	1,775	2,650
17							559	1,775	2,650
18	Foust and Scott	Oil-filled	945	500	1,445	48	730	1,460	2,360
19							765	1,530	2,475
20							630	1,260	2,035
21							648	1,295	2,090
22	Foust and Scott	Oil-filled	515	500	1,015	25	902	1,805	3,345
23							928	1,855	3,440
24							940	1,880	3,485
25	Foust and Scott	Oil-filled	515	600	1,115	25	1,067	1,775	3,485
26							1,104	1,840	3,605
27							1,115	1,860	3,640
28							856	1,425	2,795
29							916	1,525	2,990
30							836	1,395	2,730
31							896	1,495	2,925
32							940	1,565	3,070
33							1,014	1,690	3,310
34	Davis and Eddy	Solid	81	94	175	Solid	188	2,000	3,030
35	Held and Leichsenring	Solid	349	118	467	Smooth surf.	280	2,370	2,745
36		Oil-filled	349	118	467		284	2,410	2,785
37		Solid	349	236	585		484	2,050	2,690
38		Oil-filled	340	236	585		510	2,160	2,830
39		Solid	349	472	821		809	1,710	2,715
40		Oil-filled	323	531	854		940	1,770	2,990

r. Minimum radius of conductor, mils.
w. Thickness of insulation.
R. Minimum radius over insulation.
n. Number of strands in outer layer.

*D*3 (strand shielded with semi-conducting tapes). The smoothness of the conductor surface in these samples increased the breakdown voltage by an average of 24 percent. In assessing the significance of this increase, the reduced effective insulation thickness in cable *D*, owing to the substitution of semi-conducting tapes for insulating tapes, near the electrodes, should be taken into account. With an equal thickness of effective insulation, this value would be nearer 30 percent.

The application of 200 psi pressure, gas or oil had a rather curious effect on the maximum stress at breakdown (samples *E*1 to *E*4). It was expected that pressure would increase the impulse strength. On the contrary the results indicated a ten percent decrease due to pressure. If the lower value had been found in only one specimen and the higher value in several specimens, the results might be ascribed to local conditions. However, the single specimen (sample *E*4) tested without pressure was found to have the highest value. Further investigation is recommended before final conclusions are drawn as to the effect of pressure on impulse strength.

Conclusions

The results of the impulse tests conducted by The Detroit Edison Company on various types of impregnated paper-insulated cables seem to indicate that with the type of impulse wave and method of application used in these tests, in a cable having the best obtainable insulation, breakdown occurs at a voltage for which the maximum stress is around 3,300 volts per mil. For a given insulation thickness, therefore, an increase in the minimum radius of the conductor and the use of smoother electrode surface should result in an increase in breakdown voltage. If the quality of the insulation is inferior, owing either to non-uniformity of taping or incomplete impregnation, or both, the maximum stress value can be expected to become less than 3,300 volts per mil. Therefore, the maximum stress obtained from impulse tests, compared with 3,300 volts per mil is a good measure of the uniformity of taping and the degree of impregnation of the insulation. The correlation obtained in these tests between the impulse strength value and the findings of visual examination, such as those shown in Mildner diagrams (also styrene wafers), shows that these visual tests in their turn can justifiably be used as a measure of the impulse strength of the cable insulation.

In view of the above mentioned results, solid, oilfilled, oilostatic, and compression cables should all have the same maximum stress at breakdown. For a given insulation thickness and conductor surface, the oilfilled cable, therefore, having the advantage of an intrinsically large diameter of the conductor will withstand the highest voltage before breakdown occurs.

The tests also indicated that this type of cable will retain this advantage even after aging. The solid, oilostatic, and compression cables, when new, belong to the same group as regards impulse strength. From the decrease of the degree of impregnation in service, it is logical to assume that service aged solid cables have a lower impulse strength than new ones and this assumption is borne out by test. By inference, it may be concluded that the care used in taping solid type cables is more important than the degree of impregnation since age decreases the beneficial effect of the latter, whereas the former will remain practically the same. Oilostatic and compression cables, on the other hand, can be expected to retain their original breakdown values even after aging, although there is as yet no direct experimental verification of this assumption. Low gas-pressure cables, although originally lower than solid-type cables as to impulse strength, are expected to retain this value after service aging and it is possible that they would then show even higher impulse strength than solid-type cables.

References

1. IMPULSE TESTS ON 24–27-KV PAPER AND LEAD CABLE, G. B. McCabe. Transmission and distribution committee, Edison Electric Institute, Chicago, Illinois, May 1938.

2. IMPULSE TESTS ON VARIOUS KINDS OF CABLE, G. B. McCabe. Transmission and distribution committee, Edison Electric Institute, Atlanta, Georgia, October 1940.

3. THE IMPULSE STRENGTH OF HIGH-TENSION CABLE INSTALLATIONS, C. Held and H. W. Leichsenring. International Conference, Paris, June 1939.

4. SOME IMPULSE VOLTAGE BREAKDOWN TESTS ON OIL-TREATED PAPER-INSULATED CABLES, C. M. Foust and J. A. Scott. AIEE TRANSACTIONS, volume 59, 1940 (July section), pages 389–91.

5. Busse and Vogel, V.D.E. Fachbericht, 1935.

6. IMPULSE STRENGTH OF CABLE INSULATION, E. W. Davis and W. N. Eddy. AIEE TRANSACTIONS, volume 59, 1940 (July section), pages 394–9.

7. Schneeberger, International Conference, Paris, 1937.

Appendix

1. In case of a cylindrical concentric cable with smooth surfaces of conductor and sheath, the field stress (*S*) at a point of the dielectric is given by the equation

$$S=\frac{E}{r_1 \log \frac{R}{r}}\ \frac{\text{volts}}{\text{cm}} \qquad (1)$$

where

E = voltage applied between conductor and sheath

The A-C Dielectric-Loss and Power-Factor Method for Field Investigation of Electrical Insulation

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ASSOCIATE AIEE

General

THE use of dielectric loss and power factor for testing the insulation of electrical equipment in service position in the field with one side grounded began with the advent of the apparatus and method described in this paper.

Through its use, tables of standard values have been set up indicating when insulation is "good" for continued operation with an adequate factor of safety or when it is "bad" to a degree that warrants investigation and possible replacement in order to insure continuity of service until the next scheduled test.

Types of insulation that were being discarded as inherently bad, judged by their high failure record, now are being re-established as good insulation after the tests have shown the defects to be largely of a secondary nature and removable or curable.

History

A series of high-voltage bushing failures occurring about 1925 emphasized the need for developing a practical field test for this type of insulation that would indicate the presence of certain types of fault or deterioration which inherently were not detectable by a non-destructive direct-current test.

A considerable amount of research and development work was done and several methods were tried and discarded before the ultimate discovery of the a-c dielectric-loss and power-factor method.

The voltage-distribution method for testing high-voltage live-line insulators which had been in successful use for several years was first tried but this method failed to reveal many types of defective insulation that were responsible for costly service interruptions.

Apparatus was then developed for measuring the charging current of a bushing at operating line voltage by isolating on an insulating support a current instrument arranged for connection in series between the terminal of the bushing and the live line normally leading to it. This test did not reveal certain types of defects in insulation and service interruptions continued to occur.

Finally the method described in this paper was devised in which voltamperes, watts, and the power factor of the insulation were measured in the field with portable instruments. The operation and application of the apparatus during the past ten years have been described in the papers listed in the bibliography. This paper is concerned with the technical electrical features and principles involved. There are different forms in which the apparatus may be constructed. For the purpose of discussion, the details of one form of the apparatus will be used to illustrate the general principles.

Test Apparatus Elements

The elements of apparatus required in an a-c dielectric loss method for field investigation of electrical insulation are shown in figure 1.

A 110-volt supply feeds a step-up transformer, *T*.

From the high-voltage terminal of the high-voltage winding, a cable connection leads to one side of the specimen, C_xR_x, which is grounded.

The measuring means, *M*, is connected between one side of the high-voltage winding of the transformer, *T*, and the grounded specimen under test.

Technical Problems

There are special technical problems involved in apparatus designed to measure losses in insulation as installed in the field with one side permanently grounded and subject to electrostatic induction from adjacent or surrounding circuits and apparatus. Because the ground connection can-

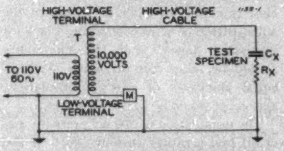

Figure 1. Apparatus to measure a-c dielectric loss in grounded insulation

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Paper 41-132, recommended by the AIEE committee on instruments and measurements, and presented at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941. Manuscript submitted November 22, 1940; made available for preprinting May 21, 1941.

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1. For all numbered references, see list at end of paper.

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r_1 = radial distance of the point from the center of the cable
r = inner radius of insulation
R = outer radius of insulation

Maximum stress for a given cable will be at the surface of the conductor when $r_1 = r$ and the equation will be

$$S = \frac{E}{r \log \dfrac{R}{r}} \tag{2}$$

2. For sector shaped conductors with paralleling ground (shielded cable), and smooth surface of conductor and shielding, the portion of the sector containing the points with the greatest stress, is what is known as the "shoulder" of the sector, having the minimum radius of curvature. In cables with sector-shaped conductors maximum stress was estimated by substituting these minimum radii in equation 2.

3. For elliptical (oval shaped) conductors same consideration as in 2, was applied.

4. To calculate the effect of stranding in cables other than those with compacted stranding, the following formula developed by Serguis Vesselowsky of The Detroit Edison Company was used:

$$S = \frac{0.572nE}{r\left(0.119 + n \log_{10} \dfrac{R}{r}\right)} \tag{3}$$

where

n = number of strands in outer layer
E = voltage applied between conductor and sheath
r = inner radius of insulation
R = outer radius of insulation

Above formula appears to be sufficiently accurate for high values for n (greater than 6).

5. In case of compacted stranding, the maximum value should be somewhat higher than that found in similar cable with smooth conductor surface. However this type of conductor does not lend itself to ready calculations because of differences of smoothness of conductor surface.

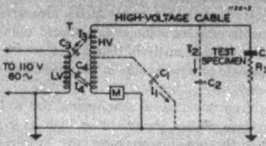

Figure 2. Simplified circuit diagram showing charging currents originating in the test apparatus itself

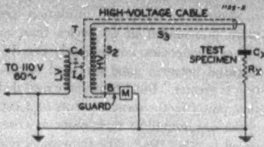

Figure 3. Test circuit diagram with shields to divert I_1, I_2, I_3

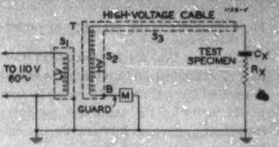

Figure 4. Test circuit diagram with shields to divert I_1, I_2, I_3, I_4

not be removed or even temporarily disconnected, the measuring means cannot be connected between the specimen and ground, and so cannot be connected directly in series with the insulation it is desired to measure. Consequently, the permanent ground limitation may cause the measuring apparatus to be subjected to electrical influences other than the losses in the insulation. For example, there may be:

Charging currents originating in the measuring apparatus itself.

Changes in phase angle and current caused by variations in capacitance.

Effects from extraneous electric fields.

Charging Currents Originating in the Apparatus Itself

In figure 2 the charging currents originating in the apparatus itself are indicated on a simplified diagram of the test equipment without any shielding. The distributed capacitance between the high-voltage winding of the transformer and ground is represented by a single capacitance, C_1, and the resulting charging currents are represented by I_1. The distributed capacitance which extends the entire length of the high-voltage cable is shown conventionally as a single capacitance, C_2, and the charging current as I_2. The distributed capacitance and current from the high-voltage end of the 10,000-volt coil of the transformer are shown as C_3 and I_3. Likewise the distributed capacitance and current between the 110-volt coil and the low-voltage end of the 10,000-volt winding are shown as C_4 and I_4. All of these charging currents, I_1, I_2, I_3, and I_4, flow through the measuring means, *M*, in addition to the current through the specimen which alone it is desired to measure. It is realized that these currents involve both capacitance and loss components but for brevity they are referred to as charging currents.

In figure 3 there have been added to the circuit of figure 2 guard shields S_2 and S_3 around the high-voltage winding of the transformer and the high-voltage lead. Charging currents I_1, I_2, and I_3 are diverted from the measuring means, *M*, by these shields, but I_4 continues to flow through *M*.

In figure 4 there has been added to the circuit of figure 3 a grounded shield S_1 around the low-voltage winding of the test transformer. This shield, S_1, diverts charging current, I_4, from the measuring means, *M*. This is important because the phase displacement of I_4 may be such that it will cause large errors in the watts measurement.

Changes in Phase Angle and Current Caused by Variations in Capacitance and Effects From Extraneous Electric Fields

From a study of figures 3 and 4, it is obvious that the capacitance between ground and the shields S_2 and S_3 is in parallel with the measuring means, *M*. This capacitance shunt diverts some of the test specimen current away from *M*. Correction for this shunting effect can be made by means of a compensating circuit or network. If this shunting capacitance is kept substantially constant, need for adjusting the network is practically eliminated. One method for maintaining constant capacitance is shown in figure 5 where, in addition to the shields S_1, S_2, and S_3 there are grounded shields S_4 and S_5 which surround the guard shields S_3 and S_2 on the high-voltage cable and the apparatus. Means for compensating the shunting effect are provided by the adjustable network *R* and *C*.

An additional function of the grounded shields S_4 and S_5 is to prevent currents induced from extraneous sources from flowing through the measuring means. By reference to figures 3 and 4, it will be seen that currents induced in the shields S_2 and S_3 from extraneous sources, such as adjacent high-voltage circuits, would flow to ground through the measuring means, *M*, thus vitiating the measurement. The shields S_4 and S_5 conduct such currents directly to ground.

This grounded shielding also contributes greatly to the safety of the operator by providing a direct path to ground for stray currents due to accidental breakdown of the insulation of the test circuit or contact with external high-voltage sources. In actual practice, lives of operators have been saved by this ground shielding when defective operating mechanism caused bushings under test to be accidentally alive at line voltages.

Even with the test apparatus and the test cable protected from external fields, the bushing or other test specimen may be so positioned as to have a considerable voltage induced at its high-voltage terminal due to extraneous sources. Voltage so induced in the specimen may in turn produce currents to ground which will flow through the measuring instruments, and it is essential to have means in the set itself for making corrections for these effects. A reversing switch, *SW* (figure 5), performs this function.

In practice, if the test voltage is less than 90 degrees out of phase with the current induced in the test specimen due to external fields, the resultant current and watts, as determined by the instruments, will be too great. If, however, the phase of the induced current is more than 90 degrees out of phase with the test potential, the watts and current indicated on the instruments will be too small. The actual value of watts loss in the insulation under test may be determined by taking two readings, first with the reversing switch in one position and then in the other. The algebraic average of these two readings is the actual watts loss in the specimen, and the algebraic average of the current is substantially the current flowing through the test specimen due to the test voltage. Thus, power factor can be calculated from watts-loss and current values determined under severe conditions of interference.

For example, let W_1 and W_2 be the observed values on the wattmeter, and I_1 and I_2 be the current readings, with the reversing switch first in one position and then in the other. The algebraic average of the watts loss will be $\frac{W_1+W_2}{2}$, and the algebraic average of the current will be $\frac{I_1+I_2}{2}$.* The resulting power factor will be $\frac{W_1+W_2}{E(I_1+I_2)}$. The accuracy of power-factor values thus derived from field readings of watts loss and voltamperes is well illustrated by a comparison with the true power factor of the same test specimen free from induction interference, as follows:

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* In practical application it is found that the current from induced voltages is small in comparison with the current through the specimen due to the test voltage, and hence its effect on the phase of the test current may be disregarded.

Observed values in the field:

$E = 10{,}000$ volts
$W_1 = 1.9$ watts
$W_2 = -1.6$ watts
$I_1 = 895$ microamperes
$I_2 = 685$ microamperes

The formula $\frac{W_1+W_2}{E(I_1+I_2)} =$

$$\frac{1.9+(-1.6)}{10{,}000(895+685)10^{-6}} = 1.9\% \text{ power factor}$$

True power factor, no induction $= 2.0\%$.

In passing it is interesting to note that it is not possible to compute the correct power factor by averaging power factors obtained under conditions of electrostatic interference by direct and reversed connections of the test voltage respectively. The algebraic expression for averaging power factor would be $\frac{\frac{W_1}{EI_1}+\frac{W_2}{EI_2}}{2}$, which equals $\frac{W_1I_2+W_2I_1}{2EI_1I_2}$. Applying this formula to the example cited above gives:

$$\frac{1.9(685)(10^{-6})+(-1.6)(895)(10^{-6})}{20{,}000(895\times10^{-6})(685\times10^{-6})} = -1.1\% \text{ power factor}$$

which negative result obviously is incorrect and in this case meaningless.

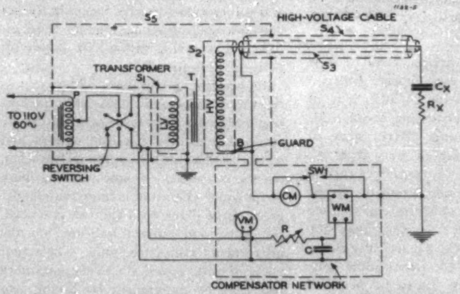


Figure 5. Simplified diagram of the watts-loss and power-factor tester

Test Frequency—60 Cycles

A test frequency of 60 cycles is most desirable because, other things being equal, a test at normal operating frequency reflects actual operating conditions. Moreover, the power factors due to defects in composite insulation are more pronounced at 60 cycles than at higher frequencies, consequently the differential between good and bad insulation is greater. Field trials have revealed that power-factor measurements at 1,000 cycles are limited in their scope as compared to 60 cycles.[8] The thousand-cycle power-factor test proved inadequate to indicate types of serious deterioration which were readily found by the 60-cycle power-factor test.

Test Voltage

A successful field testing apparatus requires a test potential adequate to detect deterioration in insulation of all voltage ratings up to and including the highest kv rating in practical use, which at the present time is 287 kv.

In practice, the operating company is interested principally when insulation has deteriorated to a point where it has become a failure hazard or represents a radio interference problem.

From a design standpoint the manufacturer is interested also to learn of inherent defects or incipient faults which may ultimately cause deterioration even though the resulting destruction of insulation value may not represent an electrical failure hazard until after many years of service operation.

Considering specifically the manufacturers' interest in new design, a test potential equal to or higher than operating voltage might furnish valuable data if it were possible in the field to separate the true loss from abnormal losses due to corona and extraneous interference conditions. However, except for studies of new design the advantages to be derived by the use of over-voltage in routine field testing are heavily outweighed by its obvious disadvantages. Extensive investigation of bushing insulation in the field showed that power-factor measurements at over-voltages introduced complications of analysis that are of no immediate practical value to the power company.

From a technical standpoint, the test voltage chosen should be as high as efficiently possible to meet the requirements of surface contact difficulties, ample unit stress on the insulation under test, and the minimizing of extraneous influences. On the other hand, the difficulties of segregating corona losses from insulation losses and the problems of portability, power supply, and regulation (all of which increase with voltage) compel a compromise in the interest of economy and efficiency. Tests at 10 kv appear to avoid a maximum of the difficulties. Data from ten years' experience with field tests on insulation rated from 10 kv to 287 kv indicate that a 10-kv test potential is adequate to solve the operating maintenance problem and also to make contributions useful to the manufacturer in his design problems.

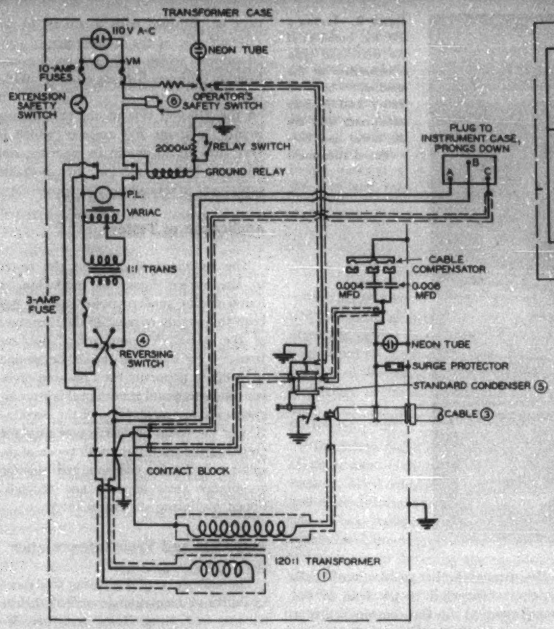
COLLAR TESTS

Need for localized high-voltage stress under special conditions, such as occur in voids in compound-filled bushings, is met by applying a metal band or "collar" to critical parts of the insulation so that the application of 10 kv produces higher unit stress at these local parts than would be produced by a test at full line voltage on the insulation as a whole. For example, tests at 10 kv impressed on metal collars placed under the first or second petticoat of a bushing will produce stresses that will indicate the presence of moisture, voids, and other abnormal and deleterious conditions in their incipient stages. In testing compound-filled bushings, the power-factor measurement is almost universally supplemented by the collar test. The trend of both measurements is carefully tabulated from consecutive tests and both are taken into consideration in the final analysis of the bushing condition.

Test Apparatus Details

In the particular circuit illustrated by figure 5, it will be noted that the measuring means consist of a voltmeter, wattmeter, and current instrument. In the following description (to differentiate this set from other types of measuring equipment which could be used), this equipment will be referred to as the "watts loss and power factor apparatus."

Figure 6 is a wiring diagram of a complete watts loss and power factor insulation test set, shown pictorially in figure 7.


TRANSFORMER CASE
110 V A-C
NEON TUBE
VM
10-AMP FUSES
EXTENSION SAFETY SWITCH
OPERATOR'S SAFETY SWITCH
⑥
RELAY SWITCH
2000ω
GROUND RELAY
PLUG TO INSTRUMENT CASE, PRONGS DOWN
A
B
C
P.L.
VARIAC
1:1 TRANS
3-AMP FUSE
④
REVERSING SWITCH
CABLE COMPENSATOR
0.004 MFD
0.008 MFD
NEON TUBE
SURGE PROTECTOR
STANDARD CONDENSER ⑤
CABLE ③
CONTACT BLOCK
120:1 TRANSFORMER
①

The set comprises:

1. Shielded step-up test transformer as a test voltage supply (120:1 trans.).
2. Shielded set of measuring instruments (voltmeter, ammeter, wattmeter in instrument case).
3. Multi-shielded cable for connection to the specimen under test.
4. Reversing switch.
5. Standard capacitor.
6. Safety relay and switches.

The test voltage supply is a 120:1 ratio, 13,200-volt, 60-cycle potential transformer capable of delivering 150 milliamperes at 10 kv.

The shielded set of measuring instruments includes:

A voltmeter calibrated for a range from 0 to 12.5 kv.

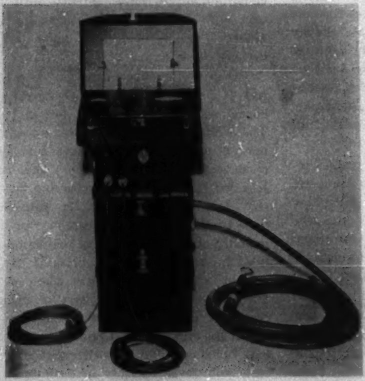


Figure 7. Watts-loss and power-factor tester assembled for use

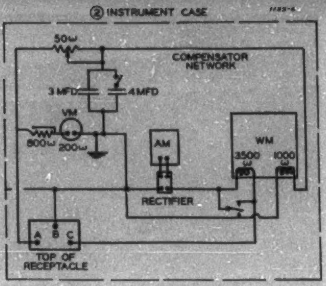

② INSTRUMENT CASE
50ω
COMPENSATOR NETWORK
3 MFD
4 MFD
VM
800ω
200ω
AM
WM
3500 ω
1000 ω
RECTIFIER
A
B
C
TOP OF RECEPTACLE

Figure 6. Watts-loss and power-factor tester

An ammeter having four scales, calibrated to cover a range from 0 to 10,000 microamperes.

A wattmeter of special dynamometer type having two scales with ranges of 0 to 2.4 and 0 to 12.0 watts. (Actual sensitivity of the wattmeter: 0.02 watt full scale.)

The multi-shielded cable has four concentric conductors separated by graded insulations of 500 volts, 10,000 volts, and 500 volts, respectively.

Safety

Important safety features shown in the circuit diagram include a protective relay, series switches, and a safety grounded shielding system surrounding the entire test apparatus including the high-voltage test cable except a hook at the end for contact with the test specimen. Danger from contact with the test voltage is practically eliminated since the hook is the only exposed live part. The arrangement of the shielding is particularly effective in protecting the test operator against trouble arising outside the test apparatus itself, such as the remote but extremely dangerous possibility of accidental direct contact of the hook with the live line, or possible abnormal conditions due to serious voltages induced in the test circuit by short-circuit or ground occurring in neighboring equipment.

Range of Test Apparatus

The testing range of the apparatus described in this paper is from 0 to 2,000 milliamperes, or 2 amperes, at 10 kv.

To meet more special testing conditions, the apparatus has been adapted to convenient ranges of 10, 30, 150, and 2,000 milliamperes. The larger ratings are used for testing power transformers and cables. The design and physical dimensions of the apparatus are substan-

Figure 8. Testing a 220-kv bushing of the draw-lead type independent of the winding in a 220 / 110 / 13-kv transformer with the 30 tester and hot-guard attachment

tially the same up to and including 150 milliamperes, beyond which range additional equipment is required.

Auxiliary Test Equipment

For special tests a shielded current transformer, called "hot guard" transformer, is used in combination with regular testers. For example: in addition to making an overall test on a power transformer, it is advantageous to be able to test the transformer bushings individually without disconnecting the windings at the lower end of the bushing under oil. For this purpose, when the transformer winding terminates in a draw lead through the bushing, a "hot guard" attachment is used to supplement the regular test equipment. Figures 8 and 9 show a set with the "hot guard" attachment arranged for making tests on a transformer bushing in a 23,000-kva transformer. In this apparatus, a shielded current transformer, *CT*, is inserted in series with the high-voltage lead so that transformer bushings of the draw-lead type readily can be measured independently of the transformer winding. The test voltage is applied between the metal tube electrode *A* and ground and only current flowing from this electrode *A*, through the insulation of the bushing, to ground actuates the measuring instruments by flowing through the transformer primary, *W*.

By means of the guard circuit S_6S_7 which is connected to the lead of the transformer at *L*, the charging current flowing into the winding of the transformer through the capacitance *CW* to ground is by-passed from the measuring instruments. The wiring diagram of the "hot guard" test set as adapted to testing transformer bushings is shown in figure 9. It will be noted that the guard circuit, S_6S_7, which by-passes the current transformer winding is at nearly the same potential as that applied to the specimen. Thus, for test purposes, the insulation between the draw lead and the bushing tube need be only sufficient to withstand low voltage. In the test cable already described in connection with figure 6, the concentric (hot guard) shield nearest the center conductor is provided for this special test, and during a "hot-guard" test another shield, called the cold-guard shield, is connected to ground.

For testing generators and cables requiring charging currents up to 2 amperes at 10,000 volts, a shielded resonating choke is connected directly across the high-voltage winding of the test transformer and adjusted until the inductance of the choke and the capacitance of the test specimen are anti-resonant at 60 cycles. Under this condition the test transformer is required to supply only the losses in the choke-specimen resonant system, and 2 amperes at 10,000 volts may be obtained with a supply to the 110-volt side of the transformer of only 20 amperes which easily can be obtained from an ordinary lighting circuit. Without the choke the corresponding current on the 110-volt side of the transformer would be about 200 amperes which is very difficult to obtain in practice, and its complete regulation by portable equipment practically is an impossibility.

Application of Tester

The technique of applying the tester to the varied forms of insulation as found on the modern power system has been thoroughly covered in the literature of the AIEE,[1,2,3,4] and other publications,[5,7,9,10,11] listed in the bibliography.

Designed primarily for a bushing problem, the successful results in this investigation led to an extension of the application of the tester until it is now used for checking the condition of all types of insulation in service position, and also for acceptance tests on new and reconditioned insulating structures.[1,2,3,4]

Test Data and Their Interpretation

The criteria used for rating the serviceability of insulation from field dielectric-loss and power-factor tests are derived from the correlation of thousands of field tests, and many investigations. They are based on certain well known facts about insulation.

The correct interpretation of field dielectric-loss tests requires a knowledge of the detailed construction of the apparatus being tested and the characteristics of the particular types of insulation used. For example, each type of bushing has certain characteristics peculiar to its construction; defective internal insulation of an oil circuit breaker shows up in different ways depending upon the breaker construction.

The dry type porcelain bushing is subject to corona formation which must be considered in analyzing the test data. The condenser type bushing may show a normal power factor but have several condenser layers shorted out; this is revealed by an increase in charging current (capacitance) during the routine dielectric-loss test. A decrease in charging current for some types of compound-filled bushings indicates a break in the ground shield connection. The oil-filled type bushing may have a high power factor as a result of deteriorated oil; if the oil is replaced promptly, there may be no injurious after-effects. Collar tests will reveal defects in compound-embedded bushings that otherwise would be masked.

The condition of the internal insulation of an oil circuit breaker is revealed by a comparison of the sum of the watts loss for the two bushings in a tank during the open-breaker test, with the combined loss of these same two bushings during the closed-breaker test. A study of these data will indicate defective lift rods or flash barriers, defective cross braces, wet or dirty de-ion grids, and many other forms of operating hazards.

No piece of insulation should be condemned until it has been completely isolated, cleaned, and compensated for temperature. Experience has shown that approximately two-thirds of the cases of high over-all dielectric loss initially found are due to bad operating condition such as carbon deposit, etc., that can be removed by ordinary servicing.

Standards for evaluation of test results appear in the literature,[2,3,4,7,9,11] as well as voluminous data concerning the results of their application to the insulation tests of typical operating companies.[1,2,3,4,5,7,9,10,11]

Typical test results[2] have indicated that defective equipment testing high enough to be removed from service is distributed among various types of apparatus in the following order:

	Percentage in "Remove" Class
Oil circuit breaker bushings..	2.1
Oil circuit breaker tank faults	1.6
Transformer bushings.........	3.2 to 3.9
Spare bushings.....................	8.8
Instrument transformer windings....	7.7
Power transformer windings.........	3.1

By the use of the technique and apparatus described, defective insulation can be detected in time for removal from service before it fails.[1,2,3,4,5,7,9,11] Such anticipation of failures not only prevents service interruptions but also makes possible the rehabilitation of defective insulation which otherwise might have been destroyed.

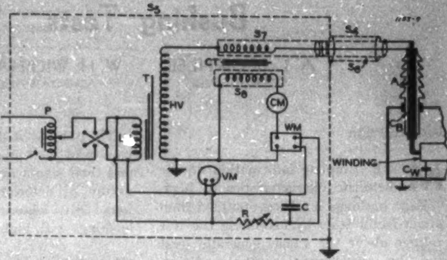


Figure 9. Schematic diagram of the essential portions of the tester circuit when the shielded current transformer "hot guard" is employed for testing transformer bushings of the draw-lead type

Of a large number of bushings tested and indicated defective to a point needing attention, about one-half of the faults were found to be external to the main insulation of the bushing and a considerable percentage of the faults that are in the main insulation can be cured or improved by servicing.[7]

"Insulation faults develop in all types of equipment such as bushings, transformers, wood insulation, and oil regardless of equipment design as it exists today. These faults may develop critically within test period intervals of 18 months to two years, as shown by the experience cited above, and appear, in a few cases, to develop to a lesser degree in a shorter period of time. Periodic testing, therefore, is necessary if a high standard of insulation is to be maintained."[2]

Conclusions

The use of the apparatus and method described in this paper makes it possible successfully to measure the a-c dielectric loss and power factor of all types of high-voltage insulation in service position. Such measurements provide an adequate non-destructive method for economical maintenance of insulation and for detecting deterioration in time to prevent failure.[4] The use of the equipment for tests at regular intervals and proper interpretation of the results will reduce to a minimum service interruptions due to faulty insulation.[2] Experience has proved that the period between tests of equipment in important locations should not exceed one year.[4]

References

1. Discussion of C. F. Hill, T. R. Watts, and G. A. Burr's paper, PORTABLE SCHERING BRIDGE FOR FIELD TESTS. AIEE TRANSACTIONS, volume 53, 1934, pages 618–22, 478–81.

2. FIELD TESTING OF BUSHINGS AND TRANSFORMER INSULATION BY THE POWER-FACTOR METHOD, I. W. Gross. AIEE TRANSACTIONS, volume 57, 1938 (October section), pages 589–99.

3. FIELD POWER-FACTOR TESTING OF TRANSFORMER INSULATION AND OPERATING EXPERIENCE, E. W. Whitmer. AIEE TRANSACTIONS, volume 60, 1941, pages 605–11.

4. BUSHING AND ASSOCIATED INSULATION TESTING BY THE POWER-FACTOR METHOD, C. C. Baltzly and E. L. Schlottere. AIEE TRANSACTIONS, volume 60, 1941 (June section), pages 308–12.

5. DOBLE METHOD OF TESTING BUSHINGS, Frank C. Doble. Presented at meeting of Electrical Apparatus Committee, New England Division, NELA Harvard Faculty Club, April 28, 1931.

6. A PORTABLE POWER-FACTOR APPARATUS FOR STUDYING THE DETERIORATION OF INSULATION, Frank C. Doble. National Research Council, *Report* of Committee on electrical insulation of the division of engineering and industrial research for the year 1932, pages 43–4.

7. TESTING BUSHINGS AND INSULATION BY POWER-FACTOR METHOD, I. W. Gross and H. E. Turner. *Electrical World*, January 13, 1934, page 20.

8. PROGRESSIVE ENGINEERING PAYS, Philip Sporn. *Electrical World*, June 9, 1934, pages 831–5.

9. BUSHING POWER-FACTOR TESTS SHOW FAILURE CAUSE, E. L. Schlottere. *Electrical World*, March 28, 1936, pages 34–7 (894–7).

10. ANALYZE TRANSFORMER INSULATION FAULTS, E. W. Whitmer. *Electrical World*, September 9, 1939, page 71 (747).

11. POWER-FACTOR TESTING OF TRANSFORMER INSULATION, Frank C. Doble and G. H. Browning. *Electrical World*, December 2, 1939, pages 49, 50, 128, 129 (1583–4, 1662–3).

Bushing Tests

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Introduction

MORE complete information on the dielectric characteristics of high-voltage bushings has been obtained from power-factor tests by extending the test voltage above the values generally used and by supplementing the ordinary power-factor tests with laboratory examinations of deteriorated bushings.

Most power-factor tests on high-voltage bushings have been made at a relatively low voltage, about 10 kv, regardless of the operating voltage of the bushing. These tests have been valuable in determining the general condition of the insulation. But by extending the test voltage to higher values, even above the operating voltage, it has been possible to detect defects in new bushings which would shorten their useful life and to detect certain types of deterioration in older bushings, both of such a nature that they are not shown by a low-voltage test.

Laboratory examinations of deteriorated bushings have been valuable in determining the nature, extent, and cause of bushing deterioration. Such information is needed to properly evaluate power-factor test results.

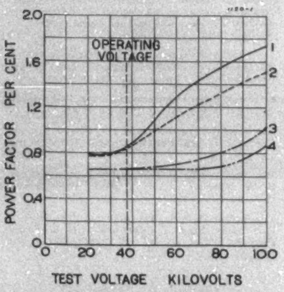

Figure 1. Power factor versus voltage curves, 66-kv condenser-type bushings

The purpose of this paper is to give the results of tests made at higher voltages and their significance and to describe the nature of deterioration in bushings as found by laboratory examinations of bushings with high power factor.

High-Voltage Power-Factor Tests

Power-factor tests on bushings at voltages equal to or above the operating voltage have shown defects in new bushings which would not have been found by a low-voltage test.

Ionization in two new 66-kv condenser-type bushings at a voltage below their operating voltage is shown by the sharply rising power-factor vs. voltage curves No. 1 and 2 in figure 1. Curves No. 3 and 4, taken on similar bushings, are given for comparison and show satisfactory power-factor-voltage characteristics. Curves 1 and 2 show an abnormal increase in power factor with voltage. Corona was present within these bushings at their normal operating voltage, 38 kv, as indicated by the breaks in the curves below that point. The ionization probably occurred in voids in the insulation which may be formed by wrinkles made in wrapping the paper which forms the condenser layers. The photograph of figure 2 shows a condenser-type bushing which was dissected and showed such wrinkles.

Corona has a deleterious effect on the insulation. Discoloration of the insulation around voids has been found in bushings which have been dissected. The action of corona will, in time, produce weak spots in the insulation and might lead to ultimate failure. New bushings in which corona is present at the operating voltage, as shown by high-voltage power-factor tests, are not acceptable.

Power-factor tests at these higher voltages have also shown up deterioration in older bushings which would not have been found by a low-voltage test.

The partial failure of a 44-kv condenser-type bushing is shown by the power-factor and capacitance vs. voltage curves of figure 3. The increasing power factor with increasing voltage, together with the abrupt change in capacitance above 60 kv. indicate that the insulation of one of the condenser layers was deteriorated so that it became partially conducting at the higher voltages. This was confirmed by measurements on the individual layers as given in table I. The outer layer had a very high power factor, 29.6%, compared with an average of 1.65% on the inner seven layers. The high power factor of the outer layer was found to be confined to the area under the ground flange as shown by the curve of figure 4 which gives the power-factor variation longitudinally over the outer layer. The insulation of this outer layer had become contaminated with glycerine from the litharge and glycerine cement used in cementing the ground flange to the condenser.

Partial failure of an oil-filled bushing is shown by the power-factor and capacitance vs. voltage curves of figure 5. While the fault had progressed so far that the power factor was high even at the initial voltage of 10 kv the increasing capacitance with increasing voltage and the changing power factor indicate the nature of the fault—a carbonized path within the bushing. The lower ends of the four inner insulating barriers between the conductor and the equalizing shield were badly burned and heavily carbonized. This conducting path gave rise to the high power factor and to the changing characteristics with increasing voltage.

Laboratory Examinations

Laboratory examinations have been useful in determining the penetration of moisture in bushing insulation and the effectiveness of drying methods. Figure 6 shows a sketch of a 66-kv condenser-type bushing which had a high power factor due to moisture absorption and table II gives the results of the examinations made to determine the distribution of moisture in the insulation and the effectiveness of drying. The power factor of this bushing was not reduced by drying at atmospheric pressure; on the contrary,

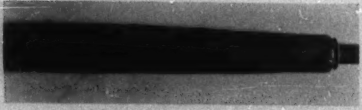

Figure 2. Condenser-type bushing with outer layers of insulation removed

Wrinkled paper at the lower end of the section (left) shows how voids may be formed, in which ionization occurs

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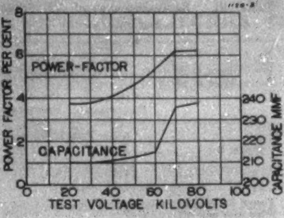

Figure 3. Power factor and capacitance versus voltage curves, 44-kv condenser-type bushing

it increased slightly. Moisture had entered the top of the bushing. This was confirmed by separating the ground wire into two sections, *A* and *B* of figure 6, and measuring the power factor to the upper and lower parts. The power factor was higher under the upper part, *A*. Further drying reduced the power factor somewhat because the separation of the outer layer between *A* and *B* afforded the moisture a means of escape. By measurements on the condenser layers it was established that moisture had penetrated to the second outer layer and that the inner eight layers had not been affected. Also, after prolonged drying at atmospheric pressure the second outer layer still contained a considerable amount of moisture.

Drying under vacuum has been much more effective in reducing high power factors caused by moisture. The curves of figure 7 show the results of drying four condenser-type bushings in vacuum The power factors of all four were reduced to satisfactory limits in reasonable time.

While moisture has generally been considered the chief cause of high power factor in bushings, laboratory examinations have also shown contamination from substances other than moisture.

In the 25-kv solid-type bushing sketched in figure 8*A*, the impregnated paper had been contaminated by acid compounds formed by deterioration of the varnished cloth. The variation of power factor longitudinally is shown in figure 8*B*, curve 1. The power factor was highest in the area under the ground shield. Curve 2 was taken after removing 0.2" of the outer paper and curve 3 after removing 0.1" more. A comparison of these curves shows that the power factor was highest in the outer part of the impregnated paper which was nearest the varnished cloth.

In a 37-kv solid-type bushing shown in the photograph of figure 9, the impregnated paper had been contaminated with a mixture of oil and filling compound. The power factor of this bushing was 9%.

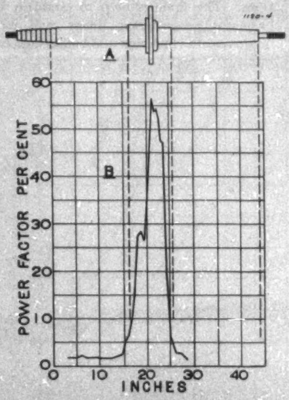

Figure 4. (A) Sketch of 44-kv condenser-type bushing; (B) power factor of outer layer

Table I. 44-Kv Condenser-Type Bushing, Power Factor of Condenser Layers

Layer No.	P.F. %
1	1.83
2	1.81
3	1.77
4	1.61
5	1.44
6	1.21
7	1.88
8	29.6

Note: The layers are numbered from the conductor outward.

Table II. Power-Factor Test Results on 66-Kv Condenser-Type Bushing of Figure 6

1. Bushing as received	Conductor to 10	3.8%
2. After drying in air for 8 days at 90 deg C	Conductor to 10	3.9%
3. After drying in air for 12 days at 110 deg C	Conductor to 10	4.1%
With section of ground wire and outer insulation removed at *C*, figure 6	Conductor to 10*A*	9.4%
	Conductor to 10*B*	3.0%
4. After drying in air for 11 more days at 110 deg C	Conductor to 10*A*	3.4%
	Conductor to 10*B*	2.3%
	Conductor to 1	1.3%
	1 to 2	1.2%
	2 to 3	1.0%
	3 to 4	1.0%
	4 to 5	1.0%
	5 to 6	1.5%
	6 to 7	1.1%
	7 to 8	1.9%
	8 to 9	24.0%
	9 to 10*A*	7.0%
	9 to 10*B*	4.1%
	Conductor to 9	2.2%
	Conductor to 8	0.7%

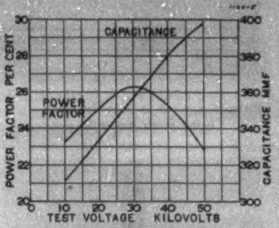

Figure 5. Power factor and capacitance versus voltage curves, oil-filled bushing

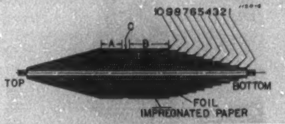

Figure 6. Sketch of 66-kv condenser-type bushing

Power-factor test results are given in table II

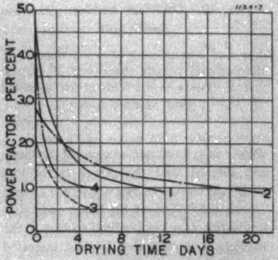

Figure 7. Vacuum drying of condenser-type bushings

Curves showing power factor versus drying time

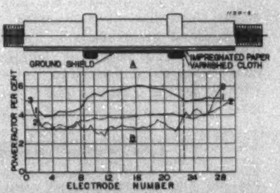

Figure 8. (A) Sketch of 25-kv solid-type bushing; (B) Power factor of impregnated paper

(1) Full thickness, 0.375 inch
(2) Turned down to 0.175 inch
(3) Turned down to 0.075 inch

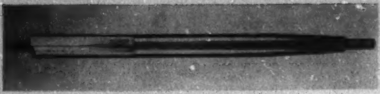

Figure 9. Solid-type bushing with section of insulation removed to show contamination

The path of oil up through the bushing can be seen at the lower end (left). The dark portion in the center shows the penetration of dissolved filling compound from the top

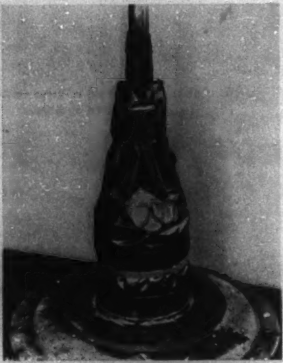

Figure 10. Oil-filled bushing, porcelain-barrier type

View of internal parts showing deposits of disintegrated varnish compounds at the top and of litharge in the folds of the varnished cloth near the center

Power-factor tests on sections of the bushing showed that the power factor was 37.2% at the upper end above the ground shield and 1.78% at the lower end. Examination of a longitudinal section of the insulation showed that oil had migrated from the lower end, through separations in the impregnated paper, to the compound in the top. Compound dissolved in the oil had penetrated the insulation from the upper end. The paths of the oil and compound can be seen in the photograph.

Contamination of the oil has been found in oil-filled bushings of the porcelain-barrier construction. In one instance a 66-kv bushing of this type had a power factor of 26.1%. The power factor of a sample of oil was 84.9% and the dielectric strength of the oil was 14 kv. A photograph of the interior of this bushing is shown in figure 10. Examination showed deposits of disintegrated varnish compounds from the varnished cloth with which the conductor and the ends of the barriers were wrapped. Also, there were heavy deposits of litharge from the litharge and glycerine cement which was used to cement the barriers together.

Deterioration of oil and of impregnated paper under oil has been found by examination of oil-filled bushings of impregnated-paper barrier construction. Table III shows the results of examination and test of the component parts of two 66-kv bushings of this type. On No. 1 the power factors of the core and the barriers were satisfactory. Evidently deterioration of the oil gave rise to the power factor of 2.47% since the power factor was reduced to 0.49% using the same impregnated-paper parts with new oil. On No. 2 the impregnated paper of the core had a high power factor which was not reduced by drying. The insulation had become contaminated by the products of deterioration in the oil. The core was replaced.

Table III. Reconditioning of Oil-Filled Bushings

	Volume, Percentage	Surface, Percentage	
No. 1			
Power factor as received			2.47%
Power factor of parts:			
Core	0.71	0.99	
Cylinder No. 1	1.09	1.06	
2	1.19	1.05	
3	1.23	1.31	
4	1.42	0.94	
Power factor of bushing assembled and filled with new oil			0.49%
No. 2			
Power factor as received			6.60%
Power factor of parts:			
Core	5.52	7.48	Volume power factor not improved by drying. Replaced.
New core	0.97	0.95	
Cylinder	2.02	1.82	
Power factor of bushing assembled and filled with new oil			0.71%

Summary

The value of power-factor tests on bushings at higher voltages has been demonstrated. Tests at low voltages, while valuable, are inadequate. The power-factor and capacitance vs. voltage characteristics at voltages up to or above the operating voltage are important in judging the condition of the insulation.

The nature of bushing deterioration and the significance of high power factor have been shown by laboratory examinations. The findings help to establish a basis for power-factor limits, and for improvements in design and maintenance.

Excitation Circuits for Ignitron Rectifiers

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Introduction

THE development of the ignitron type of mercury arc rectifier, with its outstanding advantage in efficiency, introduced the need of a new type of excitation circuit. For the theory of how the arc is initiated, the reader is referred to the previous papers on ignitrons.[1,2]

Associated with each anode of an ignitron rectifier is an ignitor for initiating the arc. Each ignitor requires a positive impulse of current once each cycle to start the arc at the beginning of the conducting period of the main anode. In order to accomplish control of the output voltage of the rectifier, it must be possible to adjust the phase position of these impulses with respect to the supply voltage, either manually or by automatic provisions. Although the instantaneous current and power of the impulses are quite high, the impulse duration may be such that the average power is low and does not seriously affect the efficiency of rectifiers with the ratings usually encountered.

If an excitation system such as "main anode" is used with which only enough energy is applied to the ignitor each cycle to ignite the arc, the average power is very small. However, with separate excitation systems, every impulse must have the same energy and this must be adequate to ignite the arc under the most adverse conditions encountered.

The wave shape of the impulse is not critical, but the peak value of power required for reliable excitation varies considerably with the time rate of rise of current. With a high rate of rise, the voltage at which a cathode spot is formed is higher than if the voltage is applied more slowly. For a given rate of rise, this voltage varies somewhat from cycle to cycle, depending upon the exact condition of the contact of the mercury and the ignitor. This, of course, influences the time of pick-up of the main anode, so the impulses must be steep enough so that any such variations are negligible.

With the assumption that the impulses have the shape of the positive half of a sine wave, figure 1 has been plotted to show the average power of 60 impulses per second applied to a load with impulses of various peak powers and durations. The duration of the impulses is indicated by the base of the wave in terms of degrees of a 60 cycle wave. The values given by the curves must be increased by the losses in the circuit elements to obtain the total power requirements for excitation. The various systems have varying efficiencies. Circuits with which the authors have had experience have varied in their total power requirements from 75 to 300 watts per ignitor.

There are four types of ignitron excitation circuits that have been used quite extensively in commercial applications which might be termed as follows:

(1). main anode
(2). capacitor-thyratron
(3). rotating impulse generator
(4). saturating reactor

All of the above have been applied with variations, as the art progressed, and depending upon the application. The following is a list of some of the circuits that have been worked on experimentally, but as yet have not proven sufficiently attractive to be applied commercially:

(1). peaked-wave transformers
(2). sine wave transformers phased back with thyratrons
(3). multi-anode control rectifier
(4). rotating commutator or other contact mechanism

A disadvantage of all circuits using thyratrons is that these tubes, having thermionic filaments, have limited life and constitute a replaceable item. An advantage, of course, is that voltage control is extremely flexible, simple, and efficient, since voltage control can be obtained by control of the thyratron grids. This control can consist of a bias voltage on the neutral of a sine wave grid transformer or a phase shifter in the primary of either a sine wave or a peaked wave grid transformer. A suitable phase shifter consists essentially of a wound rotor induction motor with the rotor locked in the desired phase position.

The conventional thermionic cathode of a thyratron is an average rated element; it is not suited to high over currents and thus not best suited to ignitron excitation. Ignitors require relatively high currents for short times. If only average currents were involved, ignitron excitation could be accomplished with very small and inexpensive thyratrons.

For purposes of reliable anode pick up, as well as certain control functions, it is desirable to provide a small excitation current to the main anode shield. Since this is only a nominal amount of power, it can be taken from any of the readily available sources. The figures used to

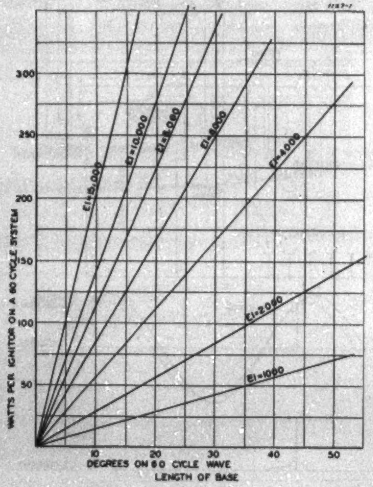

Figure 1. Average power output of a 60-cycle ignitron excitation circuit producing pulses of various durations and peak powers

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1. For all numbered references, see list at end of paper.

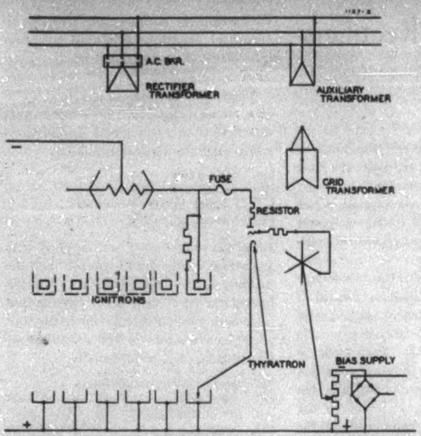

Figure 2. Diagram of a double three-phase ignitron rectifier with main anode excitation and bias control

illustrate the various excitation systems show the shield excitation accomplished in a variety of ways.

Main Anode Excitation

Figure 2 shows a schematic diagram of a simple anode fired ignitron rectifier using a grid bias method of control. A conventional double three-phase rectifier circuit is indicated for the power connections. The excitation circuit is completed for only one phase since the others are duplicates.

In this system, when an anode becomes positive with respect to its cathode, if the thyratron grid is positive, current is passed through the shunting thyratron and ignitor circuit. When this current reaches the value at which the cathode spot is formed by the ignitor, the main anode picks up and the thyratron path is short-circuited by the arc. The rate of application of voltage on the ignitor is a function of the voltage of the rectifier and the amount of grid delay used. The ignitor voltage collapses sharply when the main anode picks up. In this circuit only sufficient energy to ignite the arc is used each cycle and the average excitation power is very small. The major portion of the loss is the power required for heating the thyratron filament. Also, there is ample power available, since it comes from the rectifier transformer.

Although this system is highly efficient and most positive in action, it has certain limitations. Since the ignition voltage must come from the main transformer, arc pick-up does not take place until the anode voltage has reached ignitor voltage. This voltage is such that a 300 volt ignitron rectifier with anode firing operates at a minimum grid delay of approximately 8 degrees. Furthermore, because of the cycle to cycle variation in ignition voltage, this delay angle is not strictly uniform. This tends to cause some unbalance between main anodes, between phase groups, and between more than one rectifier on the same bus, all at low loads. At loads of one-quarter full load and higher, this effect vanishes.

In this circuit the ignition current must also pass through the load. Therefore, at loads below ignition value the entire load current passes through the thyratrons. Actually this is not very important since currents below ignition currents are well within the limits of the thyratrons used, even though maintained for the full load-conducting period.

With main anode excitation, the duty on the thyratron is probably more severe than in separate excitation circuits using thyratrons, since in the former, if an ignitor does not pick up, the current in the thyratron is limited only by the load demands, the resistance in the thyratron ignitor circuit, and the collapse of the interphase transformer voltage.

Capacitor-Thyratron Excitation

Figure 3*a* shows an excitation circuit in which a capacitor is charged through a resistor from a sine wave transformer. At a point near maximum positive charge the capacitor is discharged through the ignitor by triggering the thyratron. The principal voltage and current waves are shown in figure 3*b*. The front of the wave produced is very sharp and the tail is determined by the capacity and resistance of the circuit. The front can be sloped if desired by placing an air core reactor in the discharge circuit.

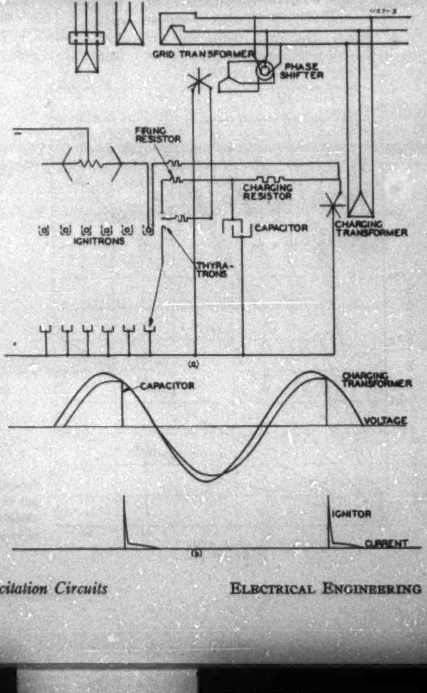

Figure 3. Ignitron rectifier circuit using capacitor - thyratron excitation with resistance charging

(*a*) Circuit diagram

(*b*) Voltage and current waves

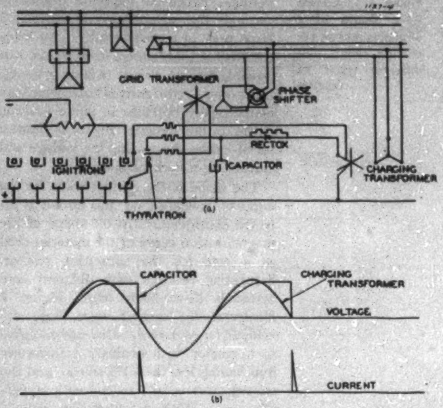

Figure 4. Ignitron rectifier circuit using capacitor - thyratron excitation with Rectox charging

(*a*) Circuit diagram
(*b*) Voltage and current waves

This circuit is simple and direct, and has the advantage of some follow-up current from the charging transformer after the pulse, which maintains an arc for 30 or more degrees and aids pick up of the main anodes at very low loads where the interphase transformer does not function. However, since the capacitor is charged through a resistor and this resistor must limit the follow current, the circuit is limited in the voltage that can be used without prohibitive loss.

The circuit permits a grid delay range of approximately 60 degrees. If a greater delay angle is desired, a transfer switch is necessary to shift connections between the charging transformer phases and the thyratrons. The power consumption can be reduced somewhat by using a reactor in place of the charging resistor. However, with a reactor the range of grid delay is considerably less and the power saved does not justify its use.

A modification of the capacitor-thyratron circuit, in which the capacitor is charged through a Rectox instead of a resistor, is shown in figure 4. This method of charging reduces the power requirements by about 50% and permits a wide range in choice of voltage with a corresponding range in impulse power. The power drawn from the line for either circuit has a power factor of nearly unity.

This circuit permits a wide range of grid delay, without switching terminals. However, if a wide range is used, it cannot be done with a grid bias, but it is necessary to use a phase shifter and a peaking transformer.

Impulse-Generator Excitation

By using concentrated fields and a suitable distribution of armature winding, a rotating synchronous generator can be built which produces impulse voltages. Such a winding arrangement, together with the waves produced and the connections to an ignitron rectifier are shown in figure 5. The impulses produced by units that have been built have a length of about 20 degrees on a 60 cycle base.

Grid delay with this type of excitation is obtained in either of two ways. In one, the generator is driven with a wound rotor induction motor, the necessary d.c. field being provided by d.c. in a pair of the windings. The phase position of the rotor is shifted by shifting the d.c. excitation from one to another of the pairs. Another method is to use a synchronous driving motor with frame shifting equipment.

The impulses provided by rotating impulse generators are highly satisfactory for ignitron excitation and the power requirements compare favorably with other types of efficient separate excitation systems. Furthermore, there are no replaceable parts except field brushes within the normal life of small synchronous M-G. sets.

The impulse generator has the disadvantage of being a rotating device somewhat bulky and noisy. For some applications a more serious objection is the synchronous motor's tendency to hunt and the reflected effect on the output voltage of the ignitron rectifier. By suitable choice of driving motor, this tendency to hunt can be kept to a value having negligible effect on the load where the rectifier capacity is small compared to the supply system. However, where

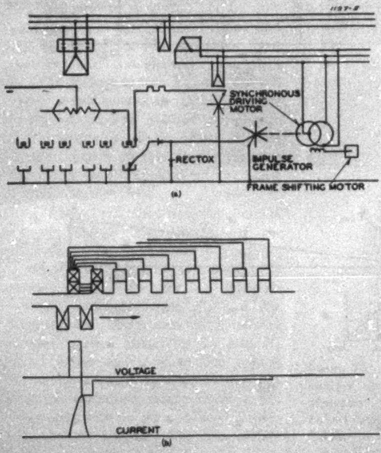

Figure 5. Ignitron rectifier circuit using a rotating impulse generator

(*a*) Circuit diagram

(*b*) Voltage and current waves

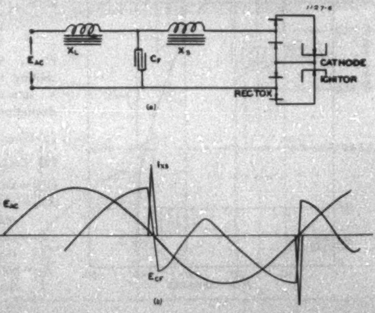

Figure 6 (right). Saturating reactor excitation circuit for ignitron rectifiers

(*a*) Circuit diagram

(*b*) Voltage and current waves

E_{AC} = supply voltage
X_L = linear reactor
C_F = firing capacitor
X_S = saturating reactor

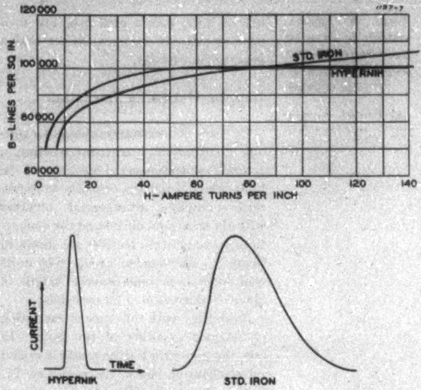

Figure 7. Current wave shapes of a saturating reactor excitation circuit with different types of reactors

the rectifier load is so large that fluctuations in the load affect the frequency of the supply system, it is possible for the natural periods of the impulse generators and the supply system to be such that violent fluctuations build up. Since the size and maintenance features of impulse generators limit their attractiveness to large installations, the danger of hunting practically precludes their use.

Saturating-Reactor Excitation

Figure 6*a* shows a circuit, utilizing reactors and capacitors, which will produce power impulses suitable for ignitron excitation. The linear reactor in this circuit has an iron core with an air gap and is designed to have a constant reactance for various voltages up to the rated voltage and frequency of the circuit. For a 60 cycle circuit, this reactance is approximately 30% of the reactance of the firing capacitor. The saturating reactor is built with a closed iron core and a winding such that at the rated voltage of the circuit, it will draw a large peak of magnetizing current. This peak of magnetizing current occurs when the capacitor, C_F, is charged to its peak value of voltage; hence the capacitor discharges through the saturating reactor and a large peak of current results. By using the Rectox units shown in figure 6*a*, it is possible for one circuit to supply excitation power to two ignitron tanks which are diametrically opposite in phase position in the rectifier circuit. Figure 6*b* shows a set of curves illustrating the voltage and current for a cycle of operation.

The shape of the current wave for the impulse developed by this circuit is affected considerably by the shape of the magnetization curve of the material used as a core for the saturating reactor. Saturating reactors with different core materials have been tried. Figure 7 illustrates the current wave shape developed for each case. One curve applies to a reactor with ordinary transformer iron having less than 1% silicon, and the second curve is for a reactor with Hypernik core. With a saturating reactor using a Hypernik core, it is possible to develop a peak power one hundred times the average power input. Some tests were made using this circuit with a resistor load to show the peak voltage and current it could develop for various values of constant resistance, and the results are plotted in figure 8.

In this circuit, the phase position of the impulses, which control the ignitron voltage, must be controlled by shifting the input voltage to the excitation circuit. This may be accomplished by means of a phase shifter which must shift the entire excitation power. The phase shift-

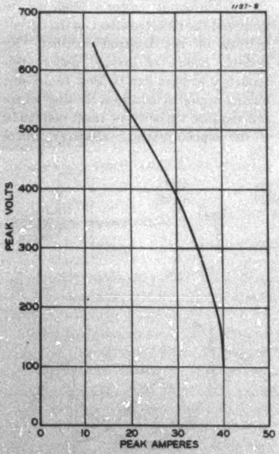

Figure 8 (left). Peak voltage and current developed by the saturating-reactor circuit using a Hypernik reactor

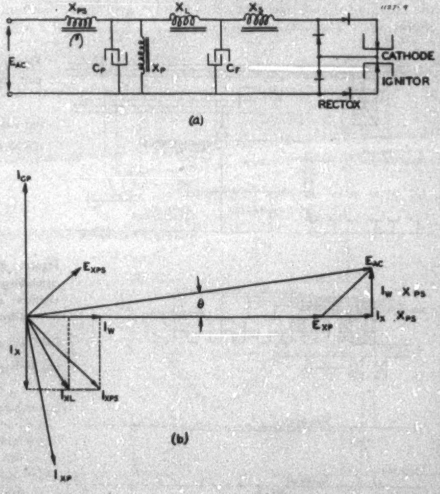

Figure 9. Phase-shifting network for a saturating-reactor excitation circuit

(*a*) Circuit diagram

(*b*) Vector diagram

E_{AC} = supply voltage
X_{PS} = phase-shifting reactor
C_P = parallel capacitor
X_P = parallel reactor

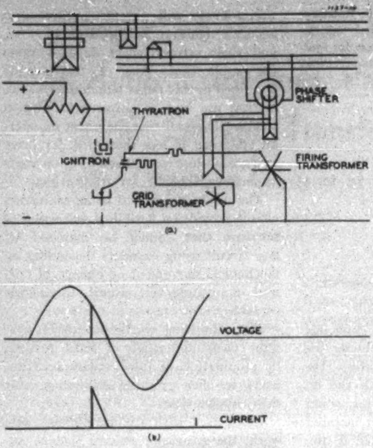

Figure 10. Ignitron rectifier circuit with sine-wave phased-back excitation

(*a*) Circuit diagram

(*b*) Voltage and current waves

ing may also be accomplished by means of a phase shifting network. Figure 9*a* shows a diagram of such a network.

The phase shifting circuit contains all the elements of the circuit shown in figure 6*a* with three additional components. The phase shifting reactor is capable of a gradual change in reactance over a 10 to 1 range by means of d.c. saturation. The parallel reactor and capacitor operate with a circulating current larger than that drawn by the excitation circuit and act as a power source for that circuit. The phase position of the voltage across the parallel combination can be adjusted by controlling the phase shifting reactor.

If the parallel reactor has a closed iron core, and is designed so that normally it operates with considerable saturation, then the voltage across X_P and C_P will remain almost constant with considerable fluctuation in line voltage or change in the reactance of the phase shifting reactor. Figure 9*b* is a vector diagram illustrating the voltage and current phase relations in this circuit.

As shown here, the loss component of current through X_{PS} determines the phase shift for a given value of reactance in X_{PS}. The reactive component of current in X_{PS} operates to maintain constant voltage across C_P and X_P. This reactive component is a balance between current taken by C_P and X_P which varies according to line voltage due to the fact that X_P is saturating. In practical operation this permits a 90° phase shift with no perceptible change in peak voltage output. The line voltage may likewise change 30% with only a 1% change in voltage across C_P and X_P.

Peaked-Wave-Transformer Excitation

The impulse produced by a conventional peaked wave transformer has the shape desired for ignitron excitation. Such a transformer, connected to block out or by-pass the negative peaks, is a satisfactory circuit. However, in a peaked wave transformer the impulse energy is stored periodically in the magnetic circuit and it is found that a transformer to produce impulses of the magnitudes required for ignitron excitation is prohibitively large.

Sine-Wave Phased-Back Excitation

Ignitors can be fired with 60 cycle sine waves. However, if the ignition is permitted to take place on the front of the wave from a circuit having capacity enough to supply the instantaneous power required by ignitors, the power loss is prohibitive. To avoid this high loss the voltage can be phased back or blocked by a thyratron, until a later point in the cycle where the follow current is much less. Such a circuit is shown in figure 10. In any event, the degree to which ignition can be phased back is limited and the minimum follow power attainable results in rather high excitation losses. Furthermore, the power consumption varies greatly with the angle of phase back so the range of grid control is very limited and the highest loss is at the minimum delay angle where rectifiers operate the majority of the time. Practically, control of rectifier voltage with this type of excitation requires that a phase shifter be placed in the power supply to the excitation circuit.

A modification of the circuit shown in figure 10 can be made by placing a single resistor in the neutral of the excitation transformer instead of the individual resistors in the ignitor leads. Such a resistor would cause the circuit to produce 60 degree pulses as in a six phase rectifier.

Multianode-Control Rectifier

The sine wave-phased back circuit can be modified as shown in figure 11, so so as to permit the use of a miniature multianode grid controlled rectifier. Since such a rectifier has a common cathode,

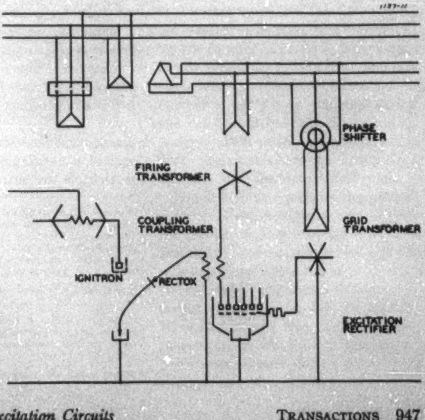

Figure 11. Ignitron rectifier circuit with miniature grid-controlled rectifier excitation

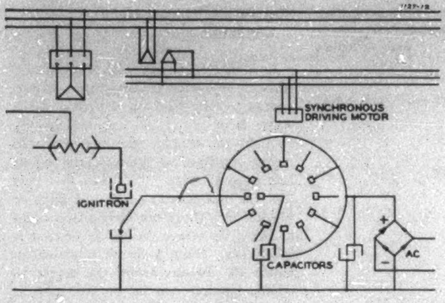

Figure 12. Rotating-commutator excitation system for ignitron rectifiers

coupling transformers are required in order to segregate the individual ignitor circuits.

The advantage of this circuit is that it retains low energy electronic control comparable to thyratron control, but eliminates the replaceable tubes. Its attractiveness is limited to pumped ignitrons where the addition of a small auxiliary pumped device involves little disadvantage. The range of grid control available with this circuit is limited, unless the entire excitation power is shifted, since, in common with the phased-back circuit, the excitation power varies widely with the phase back angle.

Rotating-Commutator Excitation

Figure 12 shows a system of excitation in which a rotating contact member is driven by a synchronous motor. As the moving contact passes under the brushes connected to the d.c. source, the capacitor is charged and when it passes under the brushes connected to the ignitors, the capacitor is discharged through the ignitor. Obviously, the capacitor could be connected to the stationary member and the ignitor contacts rotated if dictated by convenience. Control of voltage can be obtained either by means of rotating the stationary contact members or by any convenient means of shifting the phase position of the driving motor.

This system is most efficient and flexible as regards circuit constants, but it has the disadvantage of including a rotating member and sliding contacts with the resulting maintenance requirements.

Conclusions

In general, the development of the ignitron has progressed to a point where several circuits are available for excitation. These will provide positive and accurate ignition independent of load, and their attractiveness depends upon the application. Any of the circuits described in this paper which are in commercial use have a power consumption sufficiently small that the effect on overall efficiency is less than .1% for large units, and does not exceed .4% on units as small as 200 K.W.

The use of thyratrons in an excitation circuit provides a flexibility and speed of response that cannot be matched by any circuit using elements depending on mechanical movement or change of current in an inductive circuit, to provide control.

The saturating reactor excitation system, consisting entirely of static devices, by elimination of tube replacement appears to offer practical advantages for many applications.

When used with a phase shifting network, the saturating reactor system can be controlled with small power devices or compensating circuits, and has a speed of response adequate for most applications.

References

1. A New Method of Initiating the Cathode of an Arc, J. Slepian and L. R. Ludwig. AIEE Transactions, volume 52, 1933, page 693.

2. Ignitrons for the Transportation Industry, J. H. Cox and G. F. Jones. AIEE Transactions, volume 58, 1939 (December section), page 618.

3. Regulation of Grid-Controlled Mercury-Arc Rectifiers, L. A. Kilgore and J. H. Cox. AIEE Transactions, volume 56, 1937 (September section), page 1134.

4. A New Ignitron-Firing Circuit, H. Klemperer. *Electronics*, December 1939.

A Simple Method for Determination of Ratio Error and Phase Angle in Current Transformers

Correlation of Significant Constants of Current Transformers

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Synopsis: Up to the present time current transformer performance has been described by curves which show ratio error and phase angle over a range of current at various values of burden, (ohms) power factor, and frequency. The number of curves that are required to cover all possible conditions is so great as to render them, by their very bulk, practically useless. Another limitation is that they do not describe performance under transient conditions.

Performance of other types of transformers and certain other apparatus is invariably calculated from a few easily measured constants, according to simple standard formulas. Up to the present time, available methods for calculation of current transformer performance have had the disadvantage of either being difficult to use, or of depending on constants difficult to measure, or else they have not been sufficiently accurate for general use.

A method is described which depends on the following constants:

1. Open-circuit saturation curve measured in the usual way.
2. Secondary coil resistance.
3. Equivalent leakage reactance calculated from the voltage of an exploring coil enclosing the maximum value of leakage flux.

The method includes several important short cuts in the calculation which reduce the numerical work to a practical minimum. Curves are plotted which allow direct reading of the exact components of exciting current which cause ratio error and phase angle.

The objects of the paper are:

1. To present a practical method of calculation.
2. To establish constants (1), (2), and (3) as the fundamental quantities which determine real excellence of the transformer.

I. The Need for a Simple Method for Description of Current-Transformer Performance

THE conventional ratio and phase-angle curves for a current transformer are often inadequate for solution of a particular problem. Furthermore, the tests from which such curves are plotted are expensive, and the data are not available for many of the older transformers now in service.

A simple method for determining current transformer performance from a few easily measured constants (as power transformer performance is determined from copper and iron loss and impedance measurements) has been earnestly sought. This paper will present a method which the author believes to be the best so far proposed.

II. Theory of Current Transformation

The general principle of calculation of current transformer performance is very simple. The ampere turns in the secondary coil are less than those in the primary coil only because of the ampere turns lost in circulating flux in the core. Hence, we need only to calculate the ampere turns or magnetomotive force drop around the magnetic circuit.

However, on account of the leakage flux in the core, this is not as simple as it sounds. This leakage flux is principally the flux which passes between primary and secondary windings, and completes its path in the core outside the primary winding (figure 1). The current transformer is different from other transformers in that this leakage flux is relatively large, often larger than the working flux. This flux has been investigated experimentally[1] and theoretically,[2,3] but no method has been developed by which it can be exactly calculated. Furthermore, it is well known that it enters the core, not at one point, but all over the whole surface, so that it is not the same at any two points in the core

If any one of a number of approximate methods is used to calculate the leakage flux, the ampere turns to circulate it through the core can be calculated by dividing the core into a number of parts, assuming a constant equivalent flux in each part. This is the basis of the method described by Mr. Sinks,[3] and this method, although still not absolutely correct (its limitations are fully described by Mr. Sinks), is the most nearly correct method now available. Its real limitation is the great amount of numerical work involved.

III. Assumptions Which Simplify Calculation

Certain methods have been developed for simplifying this calculation. The most simple method, which does very well for ring type transformers, is to neglect the leakage flux altogether.[4] Another method assumes an experimentally determined flux, approximately related to an average value of leakage flux, to flow in the whole core.[5] These methods are not suitable for general use, because they do not use the real magnitude and distribution of the leakage flux.

IV. Combination of Best Features of All Methods

Giving first consideration to the fundamental soundness of the method, we can say that the method should—

1. Consider division of the core into several parts, as discussed in II with different leakage fluxes in the different parts.
2. Base the distribution of the leakage flux and the division of the core, on values of leakage flux actually existing in the core as measured by exploring coils (figure 2). The exploring coil can be simply ten turns of fine wire.

Giving second consideration to the aspects of economics and convenience, we can say that the method should not divide the core into more parts than is necessary for the required accuracy. Here a good deal of experience is required. Any particular division of the core into any finite number of parts is an empirical device, and can be justified only by general experience which shows that it works. The final result cannot be shown, in any rigorous way, to be the best possible result.

It will help to remember the primary objects of the investigation, and to make the division of the core accordingly. The first object is to calculate an error that is at least as large as the real error, to be on the safe side. The second is to get consistent results, to be able to compare transformers on a true basis. The first object can be obtained if the average flux values in each part of the core are taken to be sufficiently large, even if the

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[1] For all numbered references, see list at end of paper.

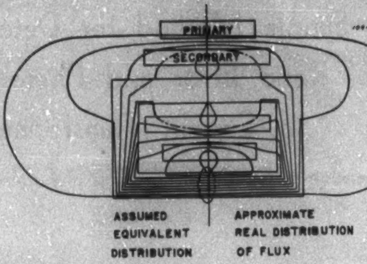
PRIMARY
SECONDARY
ASSUMED EQUIVALENT DISTRIBUTION
APPROXIMATE REAL DISTRIBUTION OF FLUX

Figure 1. The leakage flux passing in the space through and between coils completes its path through the core

The leakage flux is relatively large in a current transformer

core is not too well divided. The second object will be obtained if the flux measurements are made in the same way on all transformers of the same type.

Of course, we do not want to calculate an error too much larger than the true value. This is the disadvantage of not dividing the core at all, but considering the measured value of leakage flux to flow in the whole core; the calculated error will always be too large.

Division of the core into two parts, with the real leakage flux measured by exploring coils, is a compromise which the test results to be cited, and other tests, show to be suitable for general use with dry type transformers. This compromise is, however, definitely not fundamental; the general method to be described can be used with any desired division of core.

When the ratio error exceeds five per cent, the iron is usually working near saturation, and much of the leakage flux which is in the iron at normal flux densities leaves the core at the higher density and completes its path in the air leaving less flux in the core than would be expected. The result is that this method of calculation will give errors which are considerably too large when the error exceeds five per cent. As the method is conservative, it is still useful.

Actually less than the value of flux as measured by the exploring coil shown in figure 2 exists in practically all of the core. It is obviously conservative to assume that it exists in the half of the core which is outside the primary coil. Mr. Sinks has shown that the leakage flux at the center of the core inside the secondary is effectively reversed to the leakage flux in part of the core just outside the secondary. A reasonable assumption, confirmed by tests, is that the average effective value of leakage flux in the inside half of the core is zero, and that the only flux to be considered is the working flux.

V. Simplification of Numerical Work

The proposed method of determining the leakage flux and its distribution does not solve a real difficulty attending the problem—the amount of numerical work for a solution.

Several short cuts have been worked out which do not detract from the accuracy of the calculation. These are:

1. Substitution of equivalent reactance for the leakage flux. It can be shown (appendix I) that the transformer with magnetic circuit divided into two parts, one with, one without, leakage flux, can be represented by the equivalent circuit of figure 4, where the reactance X_L corresponds to the leakage flux and can be determined from the exploring coil voltage at normal current by—(see appendix II)

$$X_L = \sqrt{\left(\frac{N_s}{N_x}\frac{E_x}{I_s}\right)^2 - R_s^2}$$

I_s = secondary current
R_s = secondary coil resistance
N_s, N_x = number of turns in the secondary and exploring coils, respectively
E_x = induced voltage in the exploring coil

(2). When the transformer is considered as the equivalent circuit in figure 4, the problem of calculating primary (input) current from the secondary (output) current is simply vector algebra. The fact that we have limited the method to five per cent ratio error (approximately five per cent total exciting current maximum) justifies the assumption that the voltage drop in X_L is not $(I_s - I_{e1})X_L$ (see figure 4) but $I_s X_L$, and the voltage on the number 2 exciting branch is $I_s(Z_b + R_s + X_L)$ (adding vectorially). The calculation of performance is then exactly as if the two exciting branches were two separate current transformers, one with burden $Z_b + R_s$, the other with burden $Z_b + R_s + X_L$, and the ratio and phase-angle errors will be the sum of the errors of the two.

3. It is well known[7] that the component of exciting current which produces ratio error is:

$F \cos \theta + M \sin \theta$
F, M = watt reactive components of exciting current, respectively
$\cos \theta$ = burden power factor

This value, added to the secondary current, gives the primary current reversed. The formula is:

$$\text{per cent ratio: per cent turns ratio} + \frac{100\,(F \cos \theta + M \sin \theta)}{I_s}$$

I_s = secondary current

Calculating the value of $F \cos \theta + M \sin \theta$ from the ordinary open-circuit secondary saturation and power factor curves is not difficult, but calculation of a number of points is tedious. Now the ordinary saturation curve is plotted between exciting current, which is $\sqrt{F^2 + M^2}$, and secondary voltage. It is not much more difficult to plot $F \cos \theta + M \times \sin \theta$ for a number of values of $\cos \theta$, instead of $\sqrt{F^2 + M^2}$, and the desired value can then be read directly.

$$\text{Phase angle} = \frac{3438}{I_s}\,(M \cos \theta - F \sin \theta)$$

The phase-angle component is $M \cos \theta - F \sin \theta$, and a similar set of curves can be plotted for this value.

Figure 5 is an example of such a set of curves for one transformer and figure 7 is a set of $F \cos \theta + M \sin \theta$ curves for a different transformer. The log-log co-ordinates of figure 5 are more useful if the lower range of the curves is important. This idea was originated by A. M. Wiggins.[6]

The data needed for calculating performance then are:

(1). Saturation curves of the form $F \times \cos \theta + M \sin \theta$ and $M \cos \theta - F \sin \theta$ plotted against secondary voltage, for a number of values of $\cos \theta$ (see figure 5).

(2). Secondary winding resistance.

(3). Leakage reactance X_L measured according to appendix II.

The procedure is:

(1). Given burden and power factor, determine the total burden on the right hand exciting branch of figure 4. (Burden+secondary resistance $= Z_1$.) Also determine the sum of this burden$+X_L$, which is the burden on the left hand exciting branch. $Z_1 + X_L = Z_2$. (Vectorial addition.)

(2). From these two values of burden in ohms and the secondary current, calculate the voltages on the two exciting impedances.

$E_1 = I_s Z_1$
$E_2 = I_s Z_2$

(3). From the $F \cos \theta + M \sin \theta$ curves, read values for E_1 and E_2 and find per cent ratio by—

$$\text{per cent ratio} = \text{per cent turns ratio} + \frac{100}{2} \times \left[\frac{(F \cos \theta + M \sin \theta)_1}{I_s} + \frac{(F \cos \theta + M \sin \theta)_2}{I_s}\right]$$

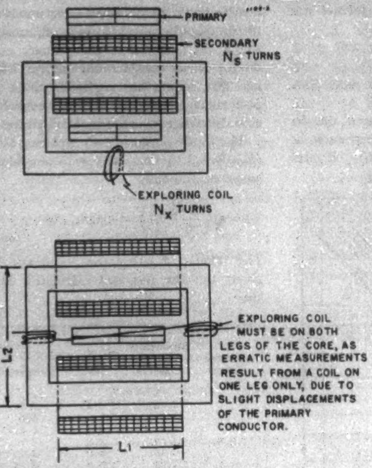
PRIMARY
SECONDARY
N_S TURNS
EXPLORING COIL
N_X TURNS
EXPLORING COIL MUST BE ON BOTH LEGS OF THE CORE, AS ERRATIC MEASUREMENTS RESULT FROM A COIL ON ONE LEG ONLY, DUE TO SLIGHT DISPLACEMENTS OF THE PRIMARY CONDUCTOR.
L2
L1

Figure 2. The most direct and simple as well as the most nearly correct method for determination of leakage flux is measurement by exploring coils

The lengths $L1$ and $L2$ represent relative (not absolute) lengths of the part of the magnetic circuit which can be considered to carry no leakage flux and all the leakage flux, respectively, in a through-type transformer

The subscripts 1, 2, refer to the two parts of the core; the division by 2 takes account of the fact that each component is for only half of the core, whereas the saturation curves are for the whole core. It will be noted that the exact turns ratio must be known. This ratio is often difficult to measure, but the manufacturer can always supply it. It may also be difficult to place the exploring coil on a transformer. This also is easy for the manufacturer, however. General use of this method will probably depend on the manufacturer's measurements of exact turn ratio and leakage flux.

$$\text{Phase-angle minutes}=\frac{3{,}438}{2}\times\left[\frac{(M\sin\theta-F\cos\theta)_1}{I_s}+\frac{(M\cos\theta-F\sin\theta)_2}{I_s}\right]$$

VI. Comparison of Tests and Calculations

The constants which determine the performance of the transformer at a given frequency are:

(1). The secondary coil resistance.

(2). The secondary leakage reactance X_L measured according to appendix II.

(3). The $F\cos\theta+M\sin\theta$ and $M\cos\theta-F\sin\theta$ curves for exciting current plotted against secondary voltage.

(4). The burden.

(5). The secondary current.

The fact that this method gives correct results for ring type transformers, in which there is no leakage flux, has been amply demonstrated.

The method has been demonstrated on several wound type transformers, and the constants for one wound type and one through type transformer will be given, together with calculated and measured results.

(a) Wound-Type Transformer

Secondary coil resistance:

0.3 ohm. (R_s)

Secondary leakage reactance:

Exploring coil voltage E_x 0.148 in 10 turns ($N_x=10$)
At I_s = secondary current = 5 amperes
N_s = secondary coil turns = 239
From equation 7, appendix I

$$X=\sqrt{\left(\frac{0.148\times239^2}{5\times10}\right)-0.31^2}=0.638\text{ ohm}$$

The $F\cos\theta+M\sin\theta$ and $M\cos\theta-F\times\sin\theta$ curves are given in figure 5.

The calculation of a given point is as follows:

Secondary current = 5 amperes
Burden = 0.6 ohm, 90 per cent power factor = $0.54+j\,0.262$
Add R_s (secondary coil = 0.31
(Z_1 = 0.89 ohm, 96 per cent power factor) Z_1 = $0.85+j\,0.262$
Burden voltage $E_1=4.45$
Add X_L = $j\,0.638$
Z_2 = 1.24 ohms 69 per cent power factor $0.85\ \ j\,0.9$
Burden voltage $E_2=6.2$
From figure 5, at $E_1=4.45$, $(F\cos\theta+M\sin\theta)_1=0.0133$
at $E_2=6.2$, $(F\cos\theta+M\sin\theta)_2=0.0229$

$$\text{Per cent ratio}=\left\{\begin{array}{l}\text{per cent turns ratio}=99.584+\\ \dfrac{100}{2}\ \dfrac{(0.0133+0.0229)}{5}=\dfrac{0.362}{99.946}\end{array}\right.$$

Test value = 99.89

The curves of figure 6 were calculated in the same way, and one who desires to may check them very rapidly by the use of tabular calculations.

(b) Through-Type (Two-Coil) Transformer

Good results are generally obtained by dividing the core into two equal parts, in the case of a wound type current transformer, but the division should not be made quite this arbitrarily for a through type current transformer. If the length of magnetic circuit not enclosed by the coils is taken as the part carrying leakage flux, and the axial length of the coil is taken as the part not carrying leakage flux, better results are generally obtained. These two lengths are, more often than not, nearly equal, so that the final division is usually not far from division into two equal parts, but some designs are made with unusually long yokes which carry the leakage flux, and account should be taken of their length. The lengths of these parts of the circuit are usually easily measured, according to figure 2.

The particular design in question was such that the core could be considered to be divided into equal parts.

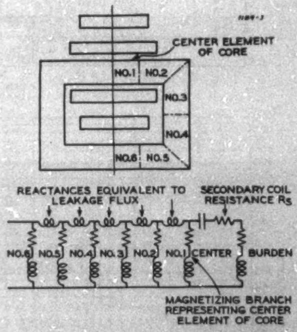
CENTER ELEMENT OF CORE
NO.1
NO.2
NO.3
NO.4
NO.5
NO.6
REACTANCES EQUIVALENT TO LEAKAGE FLUX
SECONDARY COIL RESISTANCE R_S
NO.6
NO.5
NO.4
NO.3
NO.2
NO.1
CENTER
BURDEN
MAGNETIZING BRANCH REPRESENTING CENTER ELEMENT OF CORE

Figure 3. The flux at the center of the core inside the secondary coil is actually less than enough to generate the total burden voltage, because the "leakage" flux passing through the body of the secondary coil actually links some of the secondary turns and induces a part of the secondary voltage

This real reduction of flux in the center element of core can be represented exactly by a negative series reactance or a capacitor C

Each successive element of the core, starting from the center element, carries a greater part of the real leakage flux, represented by added series reactances, and the network is an accurate equivalent

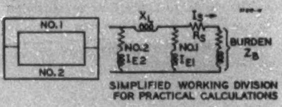

Figure 4. A practical working equivalent for figure 3 is the two-branched network

The curves for this transformer are given in figure 7.

The secondary resistance was 0.345 ohm. (R_s)

The secondary turns (N_s) were 280; ratio 1,400/5 amperes.

Two exploring coils were used, one on each yoke of the core, ten turns each, in series as in figure 2 ($N_x=20$). Erratic results would be obtained from measurements on only one exploring coil, as only slight displacement of the primary conductor causes considerable shifts in leakage flux, which has negligible effect on performance, but which causes considerable difference in individual coil voltages.

The total voltage of two exploring coils (E_x) was 2.15 volts at $I_s=15$ secondary amperes, and thus

$$X_L=\sqrt{\left(\frac{280}{20}\,\frac{2.15}{15}\right)^2-0.345^2}=1.98=2.0 \text{ ohms}$$

The details of calculation need not be given, but the test and calculated data are—

Burden Ohms	Secondary Current Amperes	Per Cent Ratio Error	
		Test	Calculated
1.0	50	0.5	0.59
	75	3.0	4.5
2.0	25	0.1	0.28
	50	3.3	3.1
	55	4.2	4.7
4.0	25	0.7	0.75
	35	5.5	5.4

At larger currents, where errors are much larger, the test values do not agree with calculation.

This design is typical of two-coil through type transformers, and the two-ohm equivalent internal reactance is also typical for this ratio. The burden may often be less than the equivalent internal impedance, so that the performance of the transformer may sometimes not be much affected by burden.

If the ratio is higher than 1,400/5 the internal impedance will be higher, increasing approximately as the square of the ratio, and if the core section is increased the internal impedance will increase almost in proportion. The result is that a two-coil through type current transformer of conventional design will not perform well at primary currents much above 25,000 primary amperes, regardless of the amount of iron in the core.

VII. Other Uses for Saturation Curves and Secondary-Coil Resistance Data

(1) Transient Performance

The use of the secondary coil resistance and saturation curve for determining per-

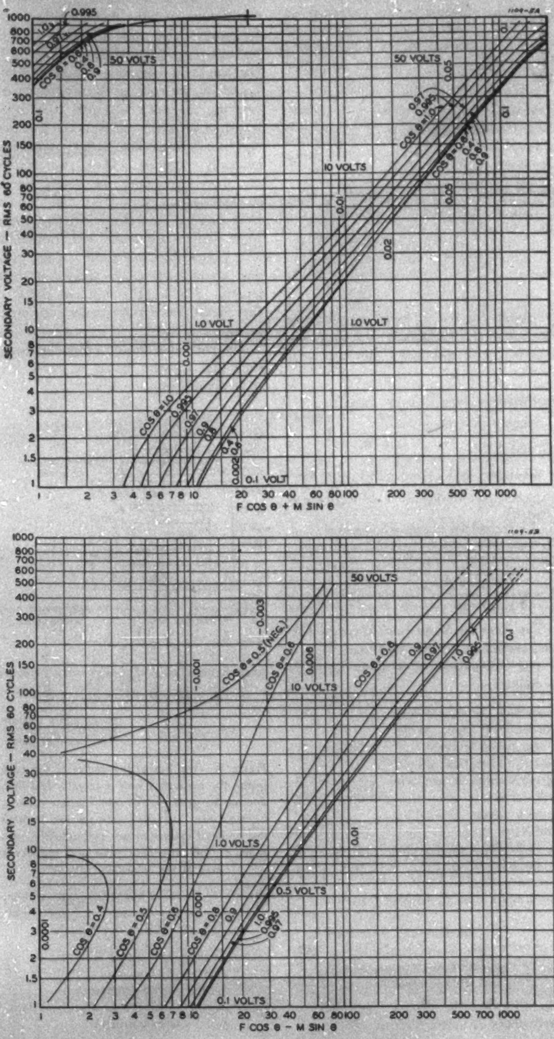

Figure 5. F cos θ+M sin θ and M cos θ − F sin θ curves for a typical transformer

These are calculated from the watt (*F*) and reactance (*M*) components of the open-circuit secondary saturation curve. Cos θ is the burden power factor formance at transient currents has been fully described in previous papers.[8,9]

(2) Parallel Connection

When current transformers are connected in parallel, the primary current of one of the transformers may be very low, or even zero. If it is very low, while the burden voltage is relatively high because of current supplied by the other transformers in parallel, the effective burden impedance (:volts/current) is very high, higher than any usual burden, and the performance of the current transformer at such a burden will seldom be given by the standard performance curves. The performance can then easily be determined by the method given.

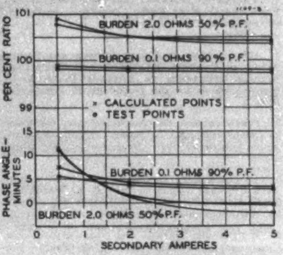

Figure 6. Calculation versus test

Wound-type current transformer calculated from figure 5, and secondary resistance = 0.31, leakage reactance 0.638

If the primary current is zero (primary circuit open) the exciting current drawn by the dead current transformer can be read from the same saturation curve, and its effect on the current supplied to the burden calculated in the same direct way.

VIII. Constants Determining the Excellence of a Current Transformer

The real excellence of a current transformer is determined by examination of ratio and phase-angle data. However, it is not economical, as has been pointed out, to measure ratio and phase angle at every possible condition, and comparison of overcurrent data on two current transformers is usually awkward, and often inconclusive.

When current transformers operate relays, ratio errors of less than five per cent are usually of no consequence. The power factor of the total burden with the usual relays will be in the neighborhood of 60 per cent. If the primary current is the common figure of 10 times normal, or 50 amperes secondary current, $F \cos \theta + M \sin \theta$ must be 2.5 to produce an error of 5 per cent. Then the magnitude of the voltage, according to the $F \cos \theta + M \sin \theta$ curves, at which $F \cos \theta + M \sin \theta$ will be 2.5 is one general measure of the excellence of the current transformer (see figure 5).

The resistance of the secondary winding and the equivalent value of X_L determine how much must be added to the connected burden to determine how much burden the transformer has to carry. Hence these figures also are measures of the excellence of the current transformer.

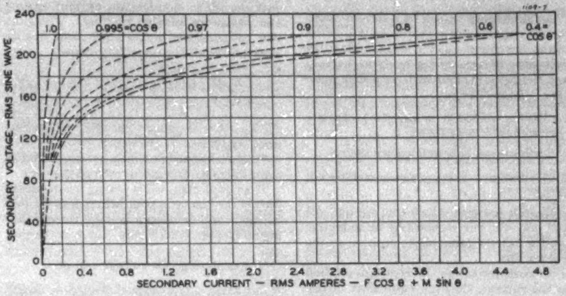

Figure 7. F cos θ+M sin θ curve for a through-type current transformer

$F \cos \theta + M \sin \theta$ is the component of exciting current which must be added to the secondary output current to get primary input current reversed (1/1 ratio basis). Example: burden four ohms at 80 per cent power factor, secondary current 50 amperes, exciting current (from curve) at $4 \times 50 = 200$ volts = 1.8 amperes. Primary current reversed = 51.8, ratio = 100 (51.8/50) = 103.6 per cent

Thus two indices of excellence which can be readily compared are:

(1). Secondary voltage at which $F \cos \theta + M \sin \theta$ for 60 per cent power factor is 2.5.

(2). Internal resistance and reactance.

IX. Conclusions

A simple method for calculation of current transformer performance has been developed. It is not new in fundamental principles, but it is an improvement and simplification of the most accurate existing method.[3]

It is more direct in its use of leakage flux; it uses a real measured value in its calculations; furthermore, the measurement of flux is easily done with exploring coils without other special equipment. The exploring coil can be threaded through the space between core and coil in many transformers, especially older models.

The method is flexible. If further experience shows the necessity for further division of the core, two sets of exploring coils can be used with a division of the core into three parts. It can be made to be as accurate as desired.

It proposes a simple equivalent circuit for calculations which includes an internal reactance. This simplifies the calculation, and isolates and defines a constant of the transformer: the equivalent internal reactance of a current transformer is that reactance which as burden would require an additional flux equal to the maximum measurable value of leakage flux in the core. A formula for the reactance in turns of exploring coil measurements is included.

The errors in the method are on the conservative side. The real exciting current is less than the calculated value, and at the higher values, corresponding to saturation of the core, where the ratio error usually exceeds five per cent, the calculated ratio error is usually much too high. Thus, the method is useful even where it is not too accurate.

Curves are proposed for simplification of the vector algebra. This expedient is of great value when many calculations are to be made. However, this part of the method is not essential. Any correct method of vectorially adding exciting current to secondary current can be used.

The performance under transient conditions, at unusual burdens or currents (including performance of parallel transformers) can all be determined from the characteristics listed above, using available methods[8,9] for transient performance, and the method described above for performance at unusual burdens or currents.

The performance of a current transformer is shown to depend on a few simple constants of the transformer:

(1). Internal resistance

(2). Internal reactance as defined

(3). Open-circuit saturation curve

Comparison of (1), (2), and one point on curve (3) allows formation of judgment about the relative excellence of transformers.

Appendix I. Equivalent Circuit for Current Transformers

An inductance burden on a current transformer requires a voltage 90 degrees ahead of the current, and as the flux to induce the voltage is, in turn, 90 degrees from the voltage, the flux is in phase with the primary current. So we may say:

1. The flux required by an inductive burden is in phase with the primary current.

Furthermore, it is well known that—

2. The leakage flux is also in phase with the primary current.

We can combine statements (1) and (2) and say—the flux required by an inductive burden has the same phase position as the leakage flux.

As both fluxes are directly proportional to the current, they are equivalent fluxes, and the leakage flux can be represented, if desired, by an equivalent inductance in the burden.

As shown by Mr. Sinks,[3] the leakage flux at the center of the core inside the secondary coil has, in effect, a negative value, but has higher values in each succeeding element of the core. For purposes of analysis, we can separate the core elements and represent each one by an equivalent exciting branch. The fact that each element carries a certain leakage flux can be represented by an equivalent inductance in series with the current circuit, as in figure 3. The fact that the leakage flux at the center corresponds to a negative inductive reactance is represented by a capacitance in series with the first branch.

For practical work in which the core is divided into only two parts, one of which is assumed to carry no leakage flux, the circuit takes the simpler form shown in figure 4, in which the series equivalent inductance is determined according to appendix II.

Appendix II. Exploring Coil for Flux Determination

If the secondary winding of a current transformer is short-circuited, the total real linkages of flux with the secondary coil are just enough to circulate the current against the resistance of the winding. This flux can be calculated as an equivalent flux ϕ_s which links all the secondary turns.

$$\phi_s = \frac{I_s R_s 10^8}{4.44 f N_s} \qquad (1)$$

I_s, R_s, N_s = secondary current, coil resistance, and turns, respectively.

This amount of flux may be considered as a working flux flowing all around the core.

If an exploring coil is placed on the outer part of the core at a point where leakage flux exists, it will develop a voltage corresponding to ϕ_s plus (vectorially) the leakage flux ϕ_L.

The exploring coil voltage will be—

$$E_x = 4.44(\phi_s + j\phi_L) f N_x 10^{-8} \qquad (2)$$

j expresses the fact that the vector ϕ_L is at right angles to the vector ϕ_s. In absolute values,

$$\left(\frac{E_x 10^8}{4.44 f N_x}\right)^2 = \sqrt{\phi_s^2 + \phi_L^2} \qquad (3)$$

$$\phi_L = \sqrt{\left(\frac{E_x 10^8}{4.44 f N_x}\right)^2 - \phi_s^2} \qquad (4)$$

$$\phi_L = \sqrt{\left(\frac{E}{N_x}\right)^2 - \left(\frac{I_s R_s}{N_s}\right)^2}\ \frac{10^8}{4.44 f} \qquad (5)$$

The leakage flux ϕ_L can be expressed as an equivalent reactance referred to the secondary winding of N_s turns and peak current $\sqrt{2}\, I_s$

$$X_L = \frac{2\pi f N_s \phi_L 10^{-8}}{\sqrt{2} I_s}$$

$$= \frac{2\pi f}{\sqrt{2}}\ \frac{10^8}{4.44 f} \sqrt{\left(\frac{N_s}{N_x}\ \frac{E_x}{I_s}\right)^2 - R_s^2}\ 10^{-8} \qquad (6)$$

$$X_L = \sqrt{\left(\frac{N_s}{N_x}\ \frac{E_x}{I_s}\right)^2 - R_s^2} \qquad (7)$$

Bibliography

1. Papers on Current Transformers, **H. W. Price** and **C. Kent Duff**. *Bulletin 2*, section 4, University of Toronto school of engineering research, 1921.

2. Calculation of Ratio Errors and Phase Angle in Current Transformers, With Particular Reference to Leakage Flux, **E. C. Wentz**. Thesis for the University of Pittsburgh, 1936.

3. Computation of Accuracy of Current Transformers, **A. T. Sinks**. AIEE Transactions, volume 59, 1940 (December section), pages 663–8.

4. Overcurrent Performance of Bushing-Type Current Transformers, **C. A. Woods** and **S. A. Bottonari**. AIEE Transactions, volume 59, 1940 (September section), pages 554–60.

5. A Proposed Method for Determination of Current-Transformer Errors, **G. Camilli** and **R. L. TenBroek**. AIEE Transactions, volume 59, 1940 (September section), pages 547–50.

6. Ring-Type Current Transformers—Limitations in Design, **A. M. Wiggins**. *Electric Journal*, December 1927, page 621.

7. A Study of the Current Transformer With Particular Reference to Iron Loss, **P. G. Agnew**. Reprint 164, *Bulletin* of the Bureau of Standards, volume 7, number 3.

8. Current Transformers and Relays for High-Speed Differential Protection, With Particular Reference to Offset Transient Currents, **E. C. Wentz** and **W. K. Sonnemann**. AIEE Transactions, volume 59, 1940 (August section), pages 481–8.

9. Current Transformer—Excitation Under Transient Conditions, **D. E. Marshall** and **P. O. Langguth**. AIEE Transactions, volume 48, 1929, pages 1464–74.

Symmetrical Nomenclature of Three-Phase Circuits

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MEMBER AIEE

Synopsis: Two methods of identifying three-phase conductors, voltages, and currents are in general use today. One method, used extensively on power systems and based on 30° voltage relations, is called "standard" nomenclature.[1,2,3] The other method, used frequently for analytical purposes and based on 90° voltage relations, is called "symmetrical" nomenclature.[4]

The following advantages of the symmetrical plan are discussed:

1. It is easier to use.
2. It identifies single-phase currents.
3. It requires fewer angles.
4. It is universal.
5. It is symmetrical.

Introduction

THE transformations most commonly used on three-phase power systems are the delta-Y and Y-delta connections which always introduce voltage phase shifts in the circuit, and the delta-delta and Y-Y connections which sometimes do. It is important to operating and design engineers that the voltage phase position of each terminal and conductor be identified by some arbitrarily adopted system of nomenclature. Such identification becomes more and more necessary as the number of successive transformations is increased.

The two nomenclature methods in general use are contrasted in figure 1 for a delta-Y transformation. Corresponding symbols are assigned to conductor-to-neutral voltages which differ by 90° in the symmetrical plan, and to voltages which differ by 30° in the standard plan. Both methods are expanded in figure 2 to provide a comparison with several variations of delta-Y and Y-delta connections. The two plans are identical with reference to delta-delta and Y-Y transformations, so these are omitted for the sake of brevity. Balanced voltage with counterclockwise (*ABC*) sequence is assumed throughout.

Ease of Application

A simple *ABC* rule facilitates the application of symmetrical nomenclature to all delta-Y and Y-delta transformer connections. For example in figure 2*a*, Y conductor *A* always corresponds with delta conductors *B* and *C* regardless of whether the connection is delta-Y or Y-delta, or whether the polarity is additive or subtractive, or whether the voltage displacement is lagging or leading. There is no exception to this easily remembered rule.

The rule for application of standard nomenclature is more complicated as illustrated in figure 2*b*, for sometimes Y conductor *B* and sometimes Y conductor *C*, corresponds with delta conductors *B* and *C*. This variability not only makes the rule more difficult to remember and use, but it renders standard nomenclature incapable of identifying single-phase fault currents.

Fault-Current Identification

This same *ABC* rule positively identifies the flow of single-phase fault or load current through delta-Y and Y-delta transformations, if symmetrical nomenclature is used. For example in figure 3*a*, conductor *A* always supplies 100% current to a fault between conductors *B* and *C* on the load side of the transformation. Conversely, conductors *B* and *C* always supply 100% current if the fault occurs on conductor *A*. There is no exception to this simple rule.

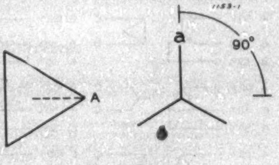
Figure 3*b* illustrates the fact that such identification is impossible without supplementary data if standard nomenclature is used, for 100% current is sometimes supplied by conductor *B*, and sometimes by conductor *C*, through such transformations to single-phase faults between conductors *B* and *C*.

This failure of standard nomenclature to provide definite current identification is a major limitation of the method which cannot be evaded by standardization of connections, for both lagging and leading connections are necessary in a series of transformations to cancel voltage shifts and to obtain zero voltage displacement.

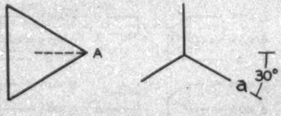
Fewer Angles

If either lagging or leading voltage displacement is allowed to accumulate in a series of transformations, only four kinds of three-phase voltages can result, such that parallel operation between circuits with different kinds is impossible. These four kinds are interpreted quite logically by the symmetrical plan to be the four multiples of 90°, as illustrated in figure 4*a*.

In such a series, the displacement be-



(a) SYMMETRICAL



(b) STANDARD

Figure 1. Symmetrical versus standard plan

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1. For all numbered references, see list at end of paper.

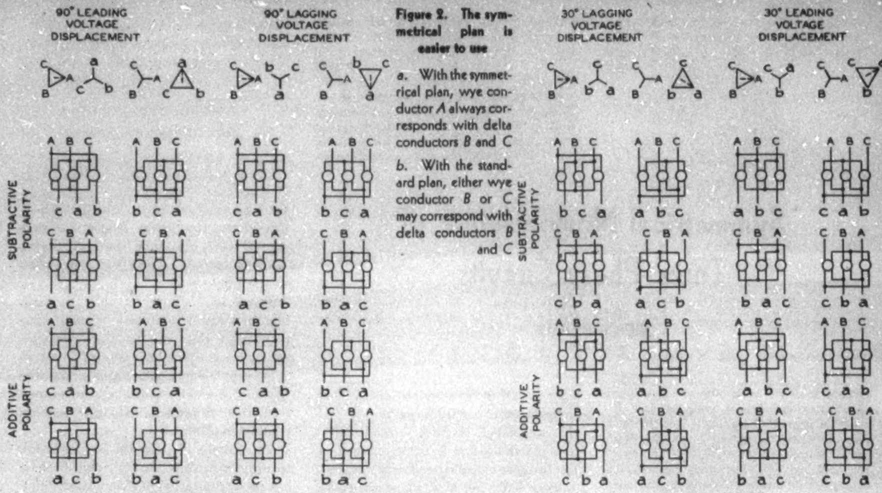

90° LEADING VOLTAGE DISPLACEMENT
90° LAGGING VOLTAGE DISPLACEMENT
30° LAGGING VOLTAGE DISPLACEMENT
30° LEADING VOLTAGE DISPLACEMENT
SUBTRACTIVE POLARITY
ADDITIVE POLARITY
(a) SYMMETRICAL
(b) STANDARD

Figure 2. The symmetrical plan is easier to use

a. With the symmetrical plan, wye conductor *A* always corresponds with delta conductors *B* and *C*

b. With the standard plan, either wye conductor *B* or *C* may correspond with delta conductors *B* and *C*

tween the same four voltage diagrams is interpreted at 30° intervals by the standard plan to present the illusion of twelve phase positions, as illustrated in figure 4*b*. The eight additional angles do not describe new phase positions, but merely represent rearrangements of symbols on the original four voltage diagrams. The use of such electrically duplicate angles, differing by 120° multiples, provides opportunity for confusion and error, and also destroys the ability of standard nomenclature to identify conductors with voltages that are in phase.

Universality

Situations frequently occur in practical applications where standard nomenclature fails to identify the conductors which must be interconnected for parallel operation. The connection diagram in figure 5*b* illustrates such a case where 4 kv conductor *A* originating in one substation must be connected to 4 kv conductor *B* originating in another substation, *B* to *C*, and *C* to *A*. Although all connections in the diagram are uniform and correct, the standard method of nomenclature causes confusion in the identification of 4 kv bus conductors. Circuit connections of this nature can occur in actual practice as a result of placing the 66 kv transformers back to back to reduce substation costs, or as a result of attempting to maintain uniform geographical phase sequence in all substations, such as phase sequence

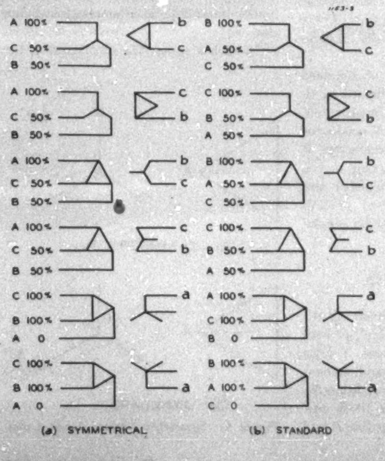

A 100%
C 50%
B 50%
B 100%
A 50%
C 50%
C 100%
B 50%
A 50%
C 100%
B 100%
A 0
A 100%
C 100%
B 0
B 100%
A 100%
C 0
a
b
c
(a) SYMMETRICAL
(b) STANDARD

Figure 3. The symmetrical plan identifies single-phase currents flowing through delta-wye and wye-delta transformations

a. With the symmetrical plan, conductor *A* always supplies 100 per cent current to *BC* fault

b. With the standard plan, either conductor *B* or *C* may supply 100 per cent current to *BC* fault

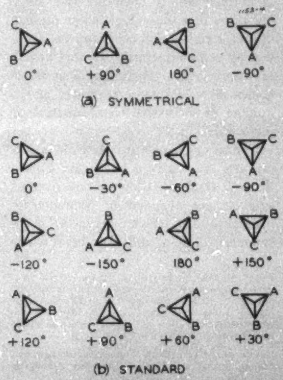
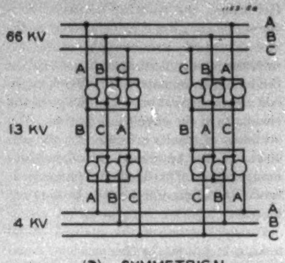
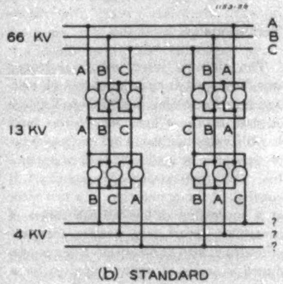

0°
+90°
180°
−90°
(a) SYMMETRICAL
0°
−30°
−60°
−90°
−120°
−150°
180°
+150°
+120°
+90°
+60°
+30°
(b) STANDARD

Figure 4. The symmetrical plan requires fewer angles, contains no duplicates



a. With the symmetrical plan, circuits are always paralleled by connecting conductor *A* to *A*, *B* to *B*, *C* to *C*



b. With the standard plan, it is sometimes necessary to connect conductor *A* to *B*, *B* to *C*, *C* to *A*

Figure 5. The symmetrical plan is universal

ABC from north to south and from east to west. Such situations usually cannot be tolerated for safety, relaying, and other reasons and must be carefully avoided if standard nomenclature is used.

The symmetrical method does not have such limitations for conductors with inphase voltages are always identified by corresponding symbols. For example in figure 5*a*, 4 kv conductor *A* originating in one substation is connected to 4 kv conductor *A* from the other substation, *B* to *B*, and *C* to *C*.

Symmetrical nomenclature is universal to the extent that it can always be applied without confusion to all conductors of a three-phase network, regardless of the sequences of delta and Y connections which may be used, and regardless of the number of interconnections and loops which may be added. It is the only nomenclature plan by which all in-phase delta and Y voltages on a three-phase network of unlimited complexity can be identified by the same symbol.

Symmetry

The symmetrical plan is based on the interpretation that all delta-Y and Y-delta transformations cause either +90° or −90° voltage displacement, and that all delta-delta and Y-Y transformations cause either 0° or 180° displacement. The symmetry of this classification is contrasted in figure 6 with the lack of symmetry of the standard classification.

The mathematical advantage of using such diametrically opposed groups is apparent in the treatment of polarity reversal or the equivalent connection reversal, for according to the symmetrical plan the effect of such reversal is always interpreted to be 180° displacement of voltage. If standard nomenclature is used, it is necessary to resort to the odd interpretation that such reversal causes 180° displacement in case the connection is delta-delta or Y-Y, but only 60° displacement if the connection is delta-Y or Y-delta. This anomaly results from the use of 30° angles to describe voltage phase shifts, and is the cause of many confusing situations in power transmission and distribution work.

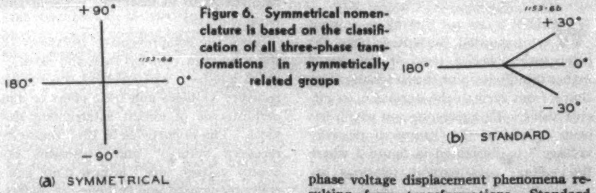


Figure 6. Symmetrical nomenclature is based on the classification of all three-phase transformations in symmetrically related groups

The use of operator *j* is well developed in electrical engineering mathematics and fits perfectly with the symmetrical interpretation that the three-phase voltage displacement resulting from transformations is always a multiple of 90°. Change from standard to symmetrical nomenclature is accomplished by merely interchanging symbols on conductors and vector-diagrams after delta-Y and Y-delta transformations. It involves no change after delta-delta and Y-Y transformations; no change in physical connections; no change in vector diagram procedures; and no change in the use of arrows to identify instantaneous values.

Conclusions

The symmetrical plan of identifying three-phase conductors, voltages, and currents is worthy of more extensive use today and in the future because of the following advantages over the standard plan:

1. It is easier to use because symbols are applied by means of simpler rules.

2. It identifies the conductors which supply current through delta-Y and Y-delta transformations to single-phase faults—an impossibility by means of standard nomenclature.

3. It requires only four angles—the four multiples of 90°—to interpret all three-phase voltage displacement phenomena resulting from transformations. Standard nomenclature requires twelve angles—the twelve multiples of 30°—to accomplish the same purpose.

4. It is universal; it can be applied to transmission-distribution systems with unlimited delta and Y sequences and identify the conductors which must be interconnected for parallel operation. The standard plan sometimes fails to provide correct identification.

5. Mathematical advantages result from its perfect symmetry and the exclusive use of 90° multiples.

References

1. AIEE STANDARDS, TRANSFORMERS, INDUCTION REGULATORS AND REACTORS (Revised), No. 13, May 1930, pages 17–21.

2. NEMA TRANSFORMER STANDARDS, NATIONAL ELECTRICAL MANUFACTURERS ASSOCIATION. Publication number 32-12, seventh edition, 1932, pages 37–42.

3. ASA ROTATION, CONNECTIONS AND TERMINAL MARKINGS FOR ELECTRIC POWER APPARATUS, American Standards Association. C6, September 30, 1938, pages 29–30.

4. SYMMETRICAL COMPONENTS (a book), C. F. Wagner and R. D. Evans. McGraw-Hill Book Company, 1933. Pages 20–1, also 67–8.

The Recovery-Voltage Analyzer for Determination of Circuit Recovery Characteristics

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Synopsis: Widespread interest in transient recovery voltage phenomena has led to the development of a device for determining the recovery characteristics of circuits and apparatus. The problem was solved by making use of the multiple surge testing technique which, in turn, involves the use of a low voltage repeating type surge generator. This paper describes the method and necessary equipment and indicates the range of conditions to which they may be applied. Demonstration tests on miniature and full scale circuits are also shown.

WHEN a current flowing in a circuit is interrupted, the voltage across the interrupting device rises or "recovers" rather rapidly in some characteristic wave shape from zero to the normal open circuit value. This phenomenon which has been defined as the "transient recovery voltage"[7] is illustrated in figure 1 where the voltage and current relations are shown for the application and clearing of a fault on a hypothetical circuit. The characteristic wave shape is dependent on the connected circuit and may consist of an oscillation at one or more frequencies corresponding to natural frequencies of various parts of the circuit.

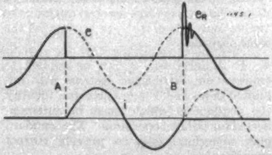

Figure 1. Diagram showing appearance of recovery voltage upon interruption of short-circuit current

e—System voltage
e_R—Recovery voltage
i—Short-circuit current
A—Point of fault application
B—Point of current interruption

The circuit interrupting device in performing its function of opening the circuit must contend with this voltage rise and must introduce insulation strength at a rate adequate to meet the stress of the rising voltage.

Since the comprehensive treatment of the subject in 1931 by Park and Skeats,[1] much has been written[2,3,4,5,6] concerning recovery voltages and their effect on the performance of circuit interrupting devices. The importance of the "transient recovery voltage" and "transient recovery voltage rate"[7] is now quite generally recognized and cathode-ray oscillograph measurements of these quantities are usually considered as part of standard test procedure for circuit breakers and other interrupting devices. Such measurements, though they involve expensive and time-consuming tests, are highly satisfactory and have proven of great value in the development work which has brought power circuit breakers to the present high level of performance.

Frequently, however, it is desirable to know the transient recovery voltage characteristics of a circuit where it is not practical to make interruption tests or cathode-ray oscillograph measurements, or where a breaker is not yet installed. Heretofore, the transient recovery voltage of the circuit was calculated by methods described elsewhere,[1,6,7] or it was sometimes possible to set up and analyze the circuit in miniature.[8,9,10] Such methods are dependent on a knowledge of the constants of the circuit in question. The accuracy of results is limited by the precision of this knowledge which involves many values of bushing and bus capacitance that in general can only be approximated.

By application of the multiple surge testing technique to the problem, a device known as the recovery voltage analyzer was developed. This analyzer permits the direct observation and measurement of the transient recovery voltage without full power interruption tests.

Technique

The multiple surge testing technique was developed as an adjunct to high-voltage impulse testing and the oscillograph electric transient analyzer[11,12] was built to utilize this method in the study of surge performance of a wide variety of apparatus. To briefly review this technique, it consists of the application to a test piece of a succession of low-voltage surges of controlled shape. In precise time coordination with these surges there is produced a series of sweep impulses for a cathode-ray oscillograph. When these voltages are put on the proper deflection plates of the cathode-ray tube, the successive surges superimpose on the tube screen and appear as a standing wave which may be viewed, traced, or photographed. In this way high-speed transient phenomena may be studied with the same facility had in reading steady state conditions with an ordinary voltmeter or ammeter.

Since the transient recovery voltage is also this type of phenomenon, the application of multiple surge testing to the problem was a logical step. This application was developed along two lines. The resulting methods will be distinguished by reference to them as the "substitution method" and the "current injection method" respectively.

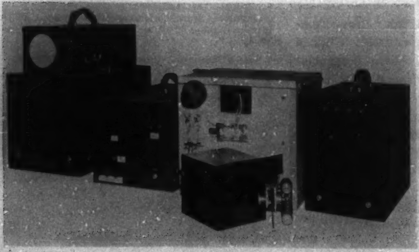

Figure 2. Recovery-voltage analyzer

Left to right, cathode-ray oscillograph, multiple-surge amplifier, multiple-surge transformer, and multiple-surge generator. Foreground, camera and mounting

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1. For all numbered references, see list at end of paper.

a. SUBSTITUTION METHOD

The substitution method was based on the idea of using the oscillograph electric transient analyzer to determine the constants of an unknown circuit or alternatively the constants of a simplified equivalent circuit. From these constants the transient recovery voltage could be calculated or read from curves of the type proposed by Boehne.[6] The justification of the method lies in the similarity of response to an applied voltage surge, of electrically equivalent circuits; and this was demonstrated by showing the transient responses of Boehne's circuits[6] to be identical for a wide range of applied voltage waves. Further demonstration was made by showing for the same wide range of waves that a full size power testing circuit with known constants behaved the same as a simplified equivalent circuit built up in the laboratory from lumped constants.

This method proved to be workable especially where the constants of a circuit are desired as in setting up a miniature representation of a system. However, it was less convenient than the current injection method for general usage.

b. CURRENT-INJECTION METHOD

This method has the advantage that the transient recovery voltage is shown as an oscillogram of the same type as that obtained by high-speed cathode-ray oscillography in connection with interruption tests. The usual measurements of frequency, amplitude, and rate of rise may be made directly from this oscillogram in the accepted manner. In addition the validity of the method is immediately apparent and may be readily demonstrated by mathematical analysis.

The method utilizes the device ordinarily employed in calculations involving current interruption and demonstrated elsewhere[1,6,14]; i.e., a current wave of the same shape as that which is considered as being interrupted is injected into the circuit under study. The resultant voltage is identical with that which would appear on actual interruption, and is therefore the recovery voltage. If interruption of an alternating current at current zero is assumed, the injected current must be of sinusoidal form. However, for the time interval during which the transient recovery voltage is of interest, a sine wave is essentially a straight line so the injected current may be of the form $i=\text{constant}\times\text{time}$. Off-zero interruption or interruption of d-c may also be simulated as will be described later; but the "ideal" interruption case is the most basic condition and the one on which most circuits will be comparable. This case will be considered in some detail.

A current of the desired linear form may be obtained by connecting a source of constant voltage to a linear inductance. Practically this is done by discharging the surge generator capacitance of an oscillograph electric transient analyzer through a suitable inductance. Of course when this arrangement is used to control the current applied to a test circuit the inductance must be sufficiently large in relation to the load so that there will be negligible distortion of the current wave.

Actually the problem was not as simple as it would appear. The inductance called for in the first design turned out to be too large and heavy for a portable equipment. It developed that a coil could not be built with low enough distributed capacitance to prevent the passage of an initial capacitance component of current which would appear as a "kick" at the beginning of the current wave. It was necessary to resort to a transformer in which the inductance could be utilized as before while the series capacitance path was virtually eliminated by electrostatically shielding the primary from the secondary. The same solution to the inductance problem was reached in England by Messrs. Trencham and Wilkinson[13,14] who applied their version of the oscillograph electric transient analyzer to the study of transient recovery voltages.

Further details of technique will become apparent as the various units of the recovery voltage analyzer are described.

Equipment

The present developmental model of the recovery voltage analyzer is shown in figure 2 and the several parts are described below.

a. MULTIPLE-SURGE GENERATOR

This unit is a simplified oscillograph electric transient analyzer[11,12] The actual circuit used is shown inside the dashed lines in figure 3. Briefly its function is the generation of a succession of surges, the shape of which will depend on the elements connected to the terminal marked "surge." Synchronized with the appearance of these surges there are available at the terminals indicated, impulses for initiating the cathode-ray beam and for sweeping the beam across the screen at a predetermined speed.

To give this result the circuit operates as follows: Thyratron T_1 is fired once every cycle of supply voltage by a peaking transformer connected to its grid. This provides a series of positive impulses at the "trip" terminal to initiate the cathode-ray beam and at the grid of thyratron T_3 to trip the surge in synchronism with the sweep voltage produced by the current flowing through the variable resistance R and the capacitor C_2 during the conduction period of T_1. The series

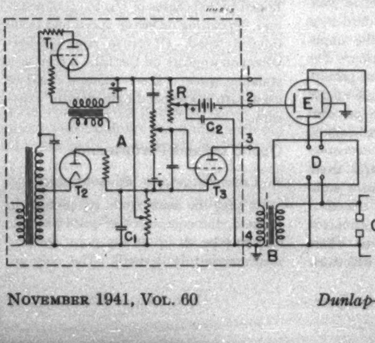
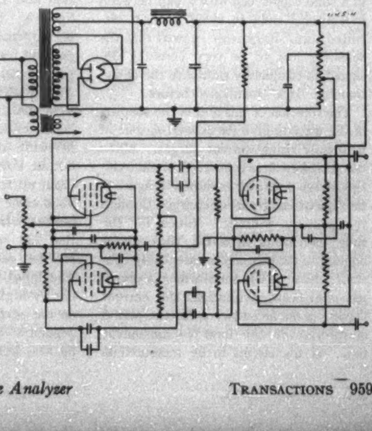


Figure 3 (left). Multiple-surge generator circuit and connections for operation as a recovery-voltage analyzer

A. Multiple-surge generator
1. Trip-impulse terminal
2. Sweep-impulse terminal
3. Surge terminal
4. Ground terminal
B. Multiple-surge transformer
C. Test circuit
D. Multiple-surge amplifier
E. Cathode-ray oscillograph

Figure 4. Multiple-surge amplifier circuit



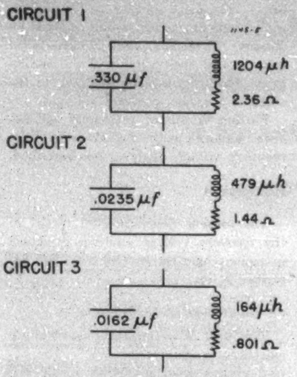


Figure 5. Lumped constant circuits for demonstration of recovery-voltage analyzer performance

Table I. Comparison of Calculated and Measured Recovery Characteristics, Figures 5 and 6

Quantity	Calculated		Measured
Frequencies			
f_1..........	8,000 cycles	...	8,300– 8,400 cycles
f_2..........	47,500 cycles	...	48,000–48,500 cycles
f_3..........	98,000 cycles	...	97,000–98,000 cycles
Amplitude ratios (inductance ratios)			
$\frac{A_1}{A_2}$.........	2.5	...	2.1
$\frac{A_1}{A_3}$.........	7.3	...	5.8
$\frac{A_2}{A_3}$.........	2.9	...	2.8
Damping $\frac{E\phi_2}{E\phi_1}$			
Circuit 1...	0.94	...	0.93
Circuit 2...	0.97	...	0.87
Circuit 3...	0.97	...	0.88

Note: Subscripts refer to the circuit with which the component indicated is associated.

$E\phi_2$ and $E\phi_1$ were measured between envelopes at times corresponding to ϕ_2 and ϕ_1 respectively.

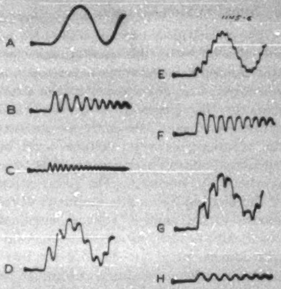


Figure 6. Recovery-voltage analyzer records obtained with the circuits of figure 5

- *A.* Circuit 1
- *B.* Circuit 2
- *C.* Circuit 3
- *D.* Circuits 1 and 2
- *E.* Circuits 1 and 3
- *F.* Circuits 2 and 3
- *G.* Circuits 1, 2, and 3
- *H.* 40-Kc timing wave

of surges is produced by the firing of thyratron T_1 which discharges the main capacitance C_1 through the circuit connected to the "surge" terminal. C_1 is charged through rectifier T_2 on each negative half cycle preceding the firing of T_1. Thus surges with synchronized beam control and sweep impulses are produced sixty times per second. This gives on the screen of the cathode-ray tube a stationary image of sufficient steadiness and brightness for viewing or photographing over the full range of sweep speeds giving sweep times of from 5 to 1,000 microseconds.

b. Multiple-Surge Transformer

The transformer is actually a set of interchangeable iron cored coils which viewed from the load side give a range of inductances from 60 millihenries to 4.5 henries. Each primary coil is electrostatically shielded by complete enclosure in a solid copper can with a gap between overlapping ends to avoid a short circuited turn. Experience showed that the shielding had to be very carefully designed to completely eliminate the objectionable "kick" mentioned before.

The function of this unit is the shaping of the wave to give the desired output of uniformly rising current surges. When the transformer is connected to the multiple surge generator as shown in figure 3, the repeated surges are discharged through the primary winding. Since for the time increments involved, the voltage applied to the primary is essentially constant, and since the transformer is operated well below saturation, the current flowing in the secondary is for the period of interest of the form $i = \text{constant} \times$ time. If the circuit to be measured is connected across the secondary, the current, controlled in form for all practical purposes by the transformer alone, will flow in the circuit and produce the desired response.

Since it is only necessary that the loading effect of the test circuit be negligible, the lower inductance transformer connections are used in conjunction with low impedance circuits in order to obtain a practical maximum of response. On the basis of inductance magnitudes available, circuits of up to 20 ohms impedance may be studied.

c. Multiple-Surge Amplifier

In order to keep the size and required power of the outfit within reasonable limits, it was found necessary to amplify the voltage waves appearing across the circuit to be measured. The amplifier was specially designed for this application and is shown diagrammatically in figure 4. It has an overall amplification of 180 times which falls off less than 10 per cent for frequencies down to 10 cycles per second and up as high as 500,000. Tests with the oscillograph electric transient analyzer showed that standard $1^1/_2$x 40 waves would be passed by the amplifier without measurable distortion. The output voltage is linear with respect to input up to 230 volts crest, while at 300 volts crest the output is only 8 per cent below the linear value. It might be noted that 230 volts gives full scale deflection on the cathode-ray oscillograph used. The input of the amplifier has a sufficiently high impedance so that the effect on the performance of the test circuit is negligible except in rare cases where 50,000 ohms shunt resistance will cause noticeable damping, or where 100 micro-microfarads additional capacitance will change the frequency appreciably. The amplifier contributes to the effective equipment capacitance which is about this value.

d. Cathode-Ray Oscillograph

The oscillograph used has a medium persistence screen 5-inch tube operated at 3,000 volts. All deflection plate leads are brought out directly to terminals and another terminal is provided which allows application of beam initiating impulses to the grid of the tube.

e. Camera

The camera used has an $F2$ lens with a 6.5 inch focus front lens and gives a record approximately $^1/_4$ the size of the actual image of the tube screen. Analysis of records is facilitated by projection of an enlarged image on a surface which may be provided with cross-section lines. With high-speed film good records may be obtained by exposures of $^1/_{10}$ to $^1/_5$ of a second, depending on sweep speed and beam intensity used. Of course, since the phenomenon appears on the tube screen as a standing wave, a slower camera could be used with correspondingly greater exposure.

Operation and Performance

a. Connection and Measurement

As may be seen from its picture in figure 2, the equipment is quite portable and may be set up at any point where a measurement is desired. The only requirements are a source of 110–120 volt 60-cycle power and access to the circuit to be measured, which must be deenergized.

The units are connected together and to the circuits as indicated in figure 3. If the approximate impedance of the circuit is known as is generally the case, the proper combination of coils in the multiple surge transformer can be selected immediately; although if the impedance is not known, the operator is able with a little experience to make the selection by trial. With the equipment in place, measurements may be made directly and rapidly, since with the transient recovery voltage waves standing on the tube screen the effect of any change made in the circuit is immediately apparent.

b. Performance on Lumped-Constant Circuits

As a general demonstration of the analyzer performance, recovery voltage characteristics for the three lumped constant parallel *L-C* circuits of figure 5 singly and in various combinations are shown in the oscillograms of figure 6. It may be seen that the characteristic frequency, amplitude, and damping of each circuit as seen in curves *A*, *B*, and *C* are also present as component parts of the more complex waves of *D*, *E*, *F*, and *G*. Values measured from these records are compared in table I with values calculated from the known circuit values of capacitance, inductance, and resistance.

The comparison based on frequency, amplitude, and damping shows generally good agreement. The maximum deviation in frequency is 5 per cent and this is in the case where the measured value is based on a single cycle. Since the amplitudes should be proportional to circuit inductances, the comparison is made between inductance ratios and ratios based on *y*-axis intercepts of the voltage wave envelopes. Here again the measurement based on the single cycle is subject to some inaccuracy and the greatest deviation is about 20 per cent. For damping the greatest discrepancy of about 10 per cent is not unreasonable since no account was taken of the high frequency losses of the capacitor units.

The oscillograms of figure 7 show the increased damping provided by the connection of resistance in parallel with circuit No. 2. Calculation of damping for these conditions gives the same degree of agreement as found in table I.

Calibration of the time scale for these records is obtained by connecting the analyzer to a circuit with a known natural frequency. A 40 kc timing wave is included in both figure 6 and figure 7. Since it is only necessary that the current be of the desired linearity without restriction on the absolute rate of rise, the value provided by the most appropriate transformer connection is used and a convenient voltage amplitude is obtained by adjusting the amplifier gain. The amplitude of the wave appearing on the screen is therefore arbitrary. In cases where actual values of voltage are desired, a scale may be assigned by taking the axis of the lowest frequency oscillation as equal to the crest value of the rated normal frequency circuit voltage. The determination of this axis may sometimes be facilitated by introducing damping resistance as described in the preceding paragraph.

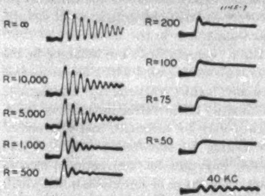


Figure 7. Recovery-voltage analyzer records showing damping effect on circuit 2 of parallel resistance of value indicated on each curve

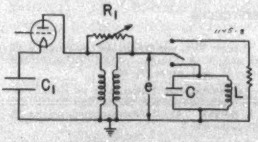


Figure 8. Connections for simulation of off-zero current interruption

C_1—Multiple-surge generator capacitance
R_1—Adjustable resistance 20,000 to ∞ ohms
e—Voltage measured with amplifier and oscillograph
C—0.0018 microfarad
L—1.35 millihenries
R—2 ohms noninductive resistor for current measurement

The range of load conditions under which satisfactory performance of the analyzer may be expected was established by measurements on circuits of the above type with natural frequencies ranging from 1 to 250 kc and with inductances up to 50 mh. While in special cases some lowering of frequency or increase in damping may be caused by the loading of the circuit by the equipment itself, particularly the amplifier as mentioned above, it may be said that the analyzer is adequate for the range of conditions which might be encountered in power circuits.

An interesting sidelight in the study of transient recovery voltage characteristics is the extension of the current injection method to simulate pre-zero interruption. This may be done by adding a d-c component to the normal injected current and is accomplished electrically by providing a resistance path directly between the multiple surge generator and the test circuit as shown in figure 8. Except that this resistance must be high compared to the load impedance, its magnitude may be varied to control the relative magnitude of the d-c component and hence the amount of pre-zero "chopping" simulated. Of course, if the "a-c" component is eliminated by disconnecting the secondary of the multiple surge transformer, interruption of d-c will be simulated.

A demonstration of the effect of pre-zero interruption on the transient recovery voltage is given by the oscillograms of figure 9 which were taken with the figure 8 circuit. The records of current were obtained by switching the analyzer output to a 2-ohm non-inductive resistor. Since the only variable was the resistance *R*, the relative amplitudes of the voltage waves represent over-voltage conditions caused by pre-zero "chopping."

c. Performance on Actual Power Circuits

A rather thorough check of field performance of the equipment was made on the test circuits of a high power switchgear testing station where some sixty different circuit arrangements were studied. Values measured from the analyzer oscillograms showed close agreement with those obtained from high-speed cathode-ray oscillograms taken during actual interruption tests, and in most cases with calculated values. Where there was disagreement between the calculated and analyzer results, the interruption test oscillograms checked with the analyzer records.

Typical transient recovery voltages are shown in the oscillograms of figure 10. Here analyzer records are superimposed on regular cathode-ray oscillograms and good agreement is shown. The small differences in the first part of the wave are due to modifications of the transient recovery by the circuit breaker used in obtaining the oscillograms. Also the method of determining the voltage calibration for the analyzer records is illustrated in this figure. As described above the axis of the lowest frequency oscillation is taken as a voltage equal to the crest

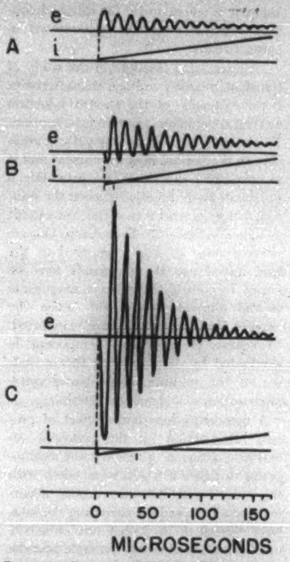


Figure 9. Recovery-voltage analyzer records obtained with figure 8 connections showing effect of off-zero current interruption

A. Voltage and current records, $R = \infty$ interruption at zero

B. Voltage and current records, $R = 200{,}000$ ohms, interruption 9 microseconds before zero

C. Voltage and current records, $R = 20{,}000$ ohms, interruption 34 microseconds before zero

value of the rated voltage of the circuit being measured. For *A* with a 4,200-volt circuit this value is 5.9 kv. For the 8,400-volt circuits of *B* and *C* it is 11.9 kv.

The analyzer has proven useful in determining the characteristics of test circuits, particularly special high recovery rate arrangements where this information was desired in advance of interruption tests. Analyzer records may also be used for comparison with conventional oscillograms to show modifications of the transient recovery voltage caused by the interrupting device itself. In addition, it is frequently desirable to know the characteristics of individual circuit elements and application has been made to transformers and reactors, illustrations of which have been published by Bewley.[15]

Conclusion

A portable device based on the multiple surge testing technique and utilizing the current injection method is now available for the determination of the transient recovery voltage characteristics of circuits and apparatus in the laboratory or field. These characteristics are obtained in the form of oscillograms which are analogous to the regular cathode-ray oscillograms ordinarily taken during interruption tests. They may be measured and interpreted according to standard practice[7] in the same way that the conventional records are. The range of conditions under which satisfactory performance may be expected includes natural frequencies from 1,000 to 250,000 cycles per second and load impedances up to 20 ohms.

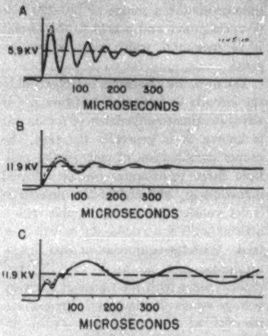


Figure 10. Typical transient-recovery voltages for switchgear testing circuits

Solid lines—cathode-ray oscillograms from interruption tests

Dotted lines—analyzer oscillograms

A. 4,200-volt circuit
B. 8,400-volt circuit
C. 8,400-volt circuit

References

1. Circuit-Breaker Recovery Voltages, **R. H. Park** and **W. F. Skeats.** AIEE Transactions, volume 50, 1931, pages 204–38.

2. Arc-Extinction Phenomena in High-Voltage Circuit Breakers, **R. C. Van Sickle** and **W. E. Berkey.** AIEE Transactions, volume 52, 1933, pages 850–7.

3. Oil Circuit Breaker and Voltage-Recovery Tests, **E. J. Poitras, H. P. Kuehni,** and **W. F. Skeats.** AIEE Transactions, volume 54, 1935 (February section), pages 170–8.

4. Breaker Performance Studied by Cathode-Ray Oscillograms, **R. C. Van Sickle.** AIEE Transactions, volume 54, 1935 (February section), pages 178–84.

5. Circuit Breakers for Boulder Dam Line, **D. C. Prince.** AIEE Transactions, volume 54, 1935 (April section), pages 366–72.

6. Determination of Circuit-Recovery Rates, **E. W. Boehne.** AIEE Transactions, volume 54, 1935 (May section), pages 530–9.

7. Alternating-Current Power Circuit Breakers, Proposed American Standard C37.4 through C37.9, January 1941.

8. System-Recovery Voltage Determination by Analytical and A-C Calculating-Board Methods, **R. D. Evans** and **A. C. Monteith.** AIEE Transactions, volume 56, 1937 (June section), pages 695–705.

9. Power-System Voltage-Recovery Characteristics, **H. A. Peterson.** AIEE Transactions, volume 58, 1939 (August section), pages 405–13.

10. An Electric Circuit Transient Analyzer, **H. A. Peterson.** *General Electric Review*, September 1939, page 394.

11. The Oscillograph Electric Transient Analyzer, **N. Rohats.** *General Electric Review*, March 1936, page 146.

12. Some Recent Developments in Impulse-Voltage Testing, **C. M. Foust** and **N. Rohats.** AIEE Transactions, volume 59, 1940 (May section), pages 257–65.

13. Restriking Voltage and Its Import in Circuit-Breaker Operation, **H. Trencham** and **K. J. R. Wilkinson.** *Journal* Institution of Electrical Engineers, volume 70, 1937.

14. Recurrent Surge Oscillographs and Their Application to Short-Time Transient Phenomena, **K. J. R. Wilkinson.** *Journal* Institution of Electrical Engineers, November 1938.

15. Equivalent Circuits of Transformers and Reactors to Switching Surges, **L. V. Bewley.** AIEE Transactions, volume 58, 1939, pages 797–802.

A Turbine-Governor Performance Analyzer

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Introduction

IN the modern interconnected power system the steam turbine governor plays a very important role. Its performance has an important bearing on system stability, effect of load swings, tie line load control, frequency regulation, and other related phenomena. Consequently, within the last few years there has been an increasing interest in the subject of steam turbine governor performance. Numerous papers[1] dealing with this subject presented at recent AIEE and ASME meetings provide ample evidence of this increased interest. These papers and the discussions which they have elicited reveal a need for improved means of obtaining complete and accurate measurements of turbine governor performance characteristics.

Operating engineers and turbine governor designers alike have felt the need for better measuring instruments. The principal concern of the central station engineer is the maintenance of a high quality of service for his consumers with the minimum operating cost. Accurate measurements of governor characteristics will often reveal imperfect operation and suggest means for improving performance or for increasing operating efficiency. The designer, on the other hand, is concerned with meeting the customer's requirements as to the operating characteristics of the governor. He must also contend with such design problems as the stability of the governor as a speed regulator, and the elimination of spurious effects such as may be caused by friction, mechanical imperfections in fly-ball governors, and by fluctuations in oil supply pressure in hydraulic governors. As a result of the designer's more recent efforts, the modern turbine governor has been developed to a point where existing measuring instruments are no longer adequate for determining its performance. The art of turbine control has, in fact, progressed to a stage where further major improvements are dependent to a considerable extent upon greatly improved instrumentation.

Previous instruments used for checking turbine governor performance have in general possessed limitations which leave their accuracy open to question, or which render them inconvenient to apply and use. Ordinary graphic frequency meters, pressure recorders, and recording tachometers, although entirely satisfactory for the purposes for which they were intended, do not respond rapidly enough to measure accurately the characteristics of the modern turbine governor. In many instances instruments for measuring certain essential quantities, particularly valve travel and hydraulic pressures, have been more or less makeshift, making their application to the turbine and the taking of simultaneous observations a difficult and irksome undertaking.

It is the purpose of this paper to describe a completely coordinated set of recording instruments which have been especially designed for analyzing the performance characteristics of steam turbine governors. These instruments are characterized by their wide range of measurements, their sensitivity, and by their ability to follow changes many times more rapid than can be followed by usual recording instruments capable of measuring similar quantities.[3,4]

General Requirements

The important governor characteristics; namely, sensitivity (or dead-band), speed regulation—both static and dynamic, speed of response, overspeed performance following a load dump, etc., can all be deduced from simultaneous measurements under appropriate conditions of instantaneous values of the following quantities:

1. Electrical output of the generator.
2. The mechanical travel of the turbine steam valve.
3. Various hydraulic pressures within the governor.
4. Angular velocity of the turbine shaft.

In order to overcome the limitations of previous instruments for measuring these quantities, certain features which were deemed essential to satisfactory measurement of the important governor characteristics were incorporated in the new governor performance analyzer. Some of these features which are common to the measurements of all the above quantities are:

> First, unity of design permitting the recording of all quantities, or any combination of them, on a single chart thus allowing visual correlation of the various quantities to each other and to time.
>
> Second, ability to record both direct and alternating components with frequency-response characteristics substantially flat from zero to 50 or 60 cycles per second if feasible.
>
> Third, provision of indicating instruments so that the average value of each quantity can be determined at a glance.
>
> Fourth, complete functional design so that makeshift and "haywire" set-ups can be avoided in the field.
>
> Fifth, complete portability permitting the instruments to be set up and used with equal facility in the laboratory, in the shop, or in the field.

A sixth very desirable feature is the ability to record directly by pen or stylus on a paper chart, thus obviating the inconveniences of photographic recording. However, no multi-element recorder having a sufficiently high frequency response is now available, and due to the urgency of the governor analyzer development, it was considered inadvisable to undertake the development of such a direct-writing recorder for this application. It was decided, therefore, to use the standard oscillographic method of recording at least for the present. If a suitable direct recorder should eventually become available, the instruments to be described can be adapted to it with little difficulty.

Generator Output

For measuring electrical load delivered by the generator, it was thought desirable to provide various full scale ranges conveniently spaced between 1 and 200 megawatts with an accuracy of measurement of the order of ±2 per cent. The decision to use a standard oscillograph made the re-

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The author acknowledges the work of F. T. Hague, manager of steam engineering, and of A. F. Schwendner, W. A. Wilson, and other turbine control engineers of the Westinghouse Electric and Manufacturing Company who proposed this project and who prepared the specifications for the instruments. He is grateful also for the helpful suggestions and encouragement of C. R. Hanna, under whose direction this work was done, and for the help of L. A. Bobula and M. D. Wilson, whose assistance in the design and construction of the instruments was invaluable.

[1] For all numbered references, see list at end of paper.

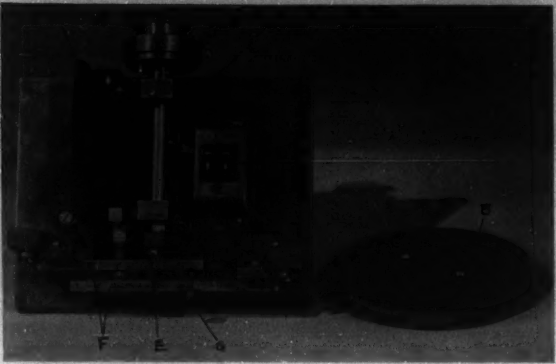


Figure 1. Valve travel recorder

A—Small pulley, *B*—large pulley, *C*—friction clutch, *D*—shaft, *E*—traveling nut, *F*—double-lever system, *G*—ratio-adjusting thumbscrew, *H*—magnetic strain gauge

cording of electrical load a simple matter, since there is already available a polyphase average watt oscillograph galvanometer which measures true watts in three phase circuits. The accuracy of this element is of the order desired, and the frequency response adequately meets the requirements, being flat to about 30 cycles per second, down approximately 10 per cent at 40 cycles per second, and down about 50 per cent at 60 cycles per second. The various full scale ranges can be obtained by using appropriate instrument transformers which are available in all central stations.

Steam-Valve Travel

The device for measuring turbine steam valve travel is illustrated in figure 1. A steel wire with one end attached to the valve operating lever is wrapped around one of the pulleys with the free end of the wire spring-loaded to insure accurate following. As the operating lever moves, the pulley and the shaft rotate proportionately. The traveling nut at the end of the shaft converts the rotary motion into linear motion, which is transmitted to a magnetic strain gauge[2] through the double lever system illustrated. The levers are mounted on crossed spring pivots to minimize friction and to eliminate lost motion.

Two pulleys are provided, one four inches in diameter and the other one inch in diameter. The smaller pulley can be exposed to use only by removing the large pulley, thus insuring the minimum moment of inertia when using either pulley. The ratio of the lever system can be adjusted to any one of four values by moving the thumb-nut to the appropriate tapped hole in the lower lever. The four lever ratios and the two pulley sizes provide a forty-to-one range of full scale motions. The instrument is proportioned so that valve motions of 0.2, 0.5, 1, 2, 4, and 8 inches give full scale deflection of the magnetic strain gauge. Motion of the strain gauge is converted into proportional variations of an electric current which are recorded by one element of the oscillograph. Thus the full travel of the largest steam turbine valve or operating piston, which may occur for example during a load dump test, may be recorded by the instrument as well as the minute motions of the order of $^1/_{16}$ inch or less which occur during normal operation of a highly sensitive governor.

A spring-loaded friction clutch is interposed between the pulleys and the shaft. This clutch, operating in conjunction with a pair of pins in the base and shaft clutch plate respectively, limits the rotation of the shaft to slightly less than one revolution. This feature serves to protect the instrument when the pulley is rotated through more than one revolution as may occur when the speed changer setting of the governor is shifted while the instrument is connected to the valve operating arm.

The frequency response of the valve travel recorder is limited to a value somewhat less than the natural frequency as determined by the inertia of the rotating parts of the recorder and by the stiffness of the wire connecting it to the valve operating lever. In order to obtain as high a natural frequency as possible with a particular wire, the force required to drive the recorder and its moment of inertia have been kept as small as practicable. The conversion of motion by the recorder into a proportional electric current permits mounting the unit very close to the valve lever so that a short and consequently stiff connecting wire may be used. With reasonable care in connecting the unit to the lever no difficulty is experienced in obtaining substantially flat frequency response to 60 cycles per second or higher.

Hydraulic Pressure

The oil pressures of interest in measuring governor performance are (1) the main pump pressure, (2) the speed-sensing impeller pressure (in the oil governor), (3) the transformed pressure, and (4) the pressures acting on the top and bottom of the valve operating piston. In order to cover the range of instantaneous values of these pressures three full-scale ranges were considered necessary, as follows: One pound per square inch variation in any average pressure up to 75 pounds per square inch; five pounds per square inch variation in up to 75 pounds per square inch average; and 50 pounds per square inch variation in up to 150 pounds per square inch average.

The device developed to measure these pressures is illustrated in figure 2. The pressure being measured is applied to the inside of a bellows, one end of which is fixed while the other end is attached to a lever. The motion of the lever is restrained by a stiff calibrating spring and

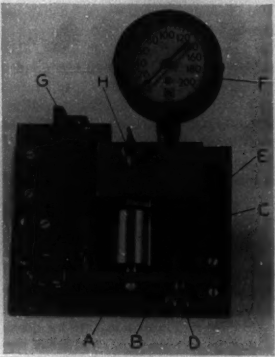


Figure 2. Oil-pressure recorder

A—Bellows, *B*—lever, *C*—calibrating spring, *D*—thumb nut for spring-force adjustments, *E*—needle valve for pressure gauge, *F*—pressure gauge, *G*—magnetic strain gauge, *H*—air drain is measured by means of a magnetic strain gauge. The spring force can be adjusted by means of the thumb-screw to balance the force due to the average pressure within the bellows.

Two different units are provided. One is used for the one-pound and five-pound full scale ranges, the shift from one scale to the other being made in the electric circuit associated with the strain gauge. The other is used for the 50-pound full scale range. The two units are identical except for the size of the bellows and the stiffness of the calibrating spring. Both are provided with gauges to indicate the average pressure. The natural frequencies of the units as determined by the spring stiffnesses and effective inertias of the moving parts are of the order of 140 cycles per second for the low pressure recorder and 170 cycles per second for the high pressure unit. With natural frequencies of this order the frequency responses of the units will be flat to well above 60 cycles per second.

Figure 4. The inductor generators

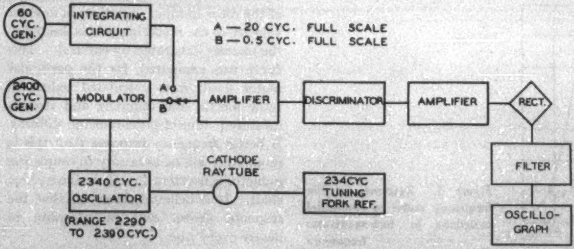
The 2,400-cycle generator is nearer the motor. A brass collar at the end of the rotor corrects for the unbalance caused by truncating the 60-cycle rotor

Angular Velocity

Although angular velocity of the turbine shaft is the quantity measured by the speed-sensing element of the governor, the ends of the shaft in many instances are not available for the attachment of a speed measuring device suitable for governor performance analysis. For this reason a small frequency-measuring synchronous motor driving two inductor generators is used. The motor is connected through a suitable transformer to the generator terminals so that its speed, within certain limitations to be discussed later, is an accurate measure of the generator frequency. Except for a small error due to variations in phase angle between the terminal and generated voltages, the generator frequency is in turn proportional to the speed of the turbine. Whenever it is necessary to avoid the error and limitations just mentioned, provision must be made for driving the inductor generators directly from the turbo-generator shaft. The synchronous motor drive has the advantage of simplicity and also permits the frequency meter described below to be used for the investigation of other problems not specifically related to governors, as for example, the relation between system frequency and tie line load variations.

Two full scale ranges are provided for the frequency meter: a wide range scale extending from 50 to 70 cycles per second to be used, for example during load dumping tests to measure overspeed characteristics; and a sensitive scale giving full scale deflection for ±0.25 cycle variation in a base frequency which may have any value between 59 and 61 cycles per second. In the instrument described below the sensitivity and accuracy are of the order of one per cent of the corresponding full scale ranges for the two scales.

Figure 3. Block diagram of frequency meter



The general scheme of operation of the frequency meter, which is by far the most complicated of the governor analyzer instruments, is illustrated by the block diagram, figure 3. One of the inductor generators produces a nominal frequency of 60 cycles per second for the wide scale range and the other generates a nominal frequency of 2,400 cycles per second for the sensitive range. As the turbo-generator frequency swings throughout the full range of the insensitive scale, the frequency of the 60-cycle generator varies between 50 and 70 cycles per second, and with constant excitation the generated voltage will vary proportionately. As it is desired to use only the frequency variations, an integrating circuit is interposed between the 60-cycle generator and the measuring circuits to eliminate the voltage variations. The output of this integrating circuit is then a true frequency-modulated wave whose amplitude is constant and whose frequency varies throughout the full scale range. This frequency-modulated signal is amplified and applied to a carefully designed discriminator circuit which, for a constant input voltage, gives an output proportional to the difference between the applied frequency and 50 cycles per second. The output of the discriminator is thus zero at 50 cycles per second, has a certain value at 60 cycles per second, and has twice this value at 70 cycles per second. The effect of the discriminator, then, is to convert the frequency-modulated signal applied to it into an amplitude-modulated signal. This signal is further amplified, rectified, filtered, and applied to an oscillograph galvanometer.

When using the sensitive scale the output of the 2,400-cycle generator is applied to a modulator circuit together with the output of a variable frequency oscillator. As the turbo-generator frequency swings throughout the full range of the sensitive scale the frequency of the 2,400-cycle generator will vary from 2,390 to 2,410 cycles per second for a base frequency of 60 cycles per second. If the oscillator is adjusted to produce a frequency of 2,340 cycles per second the modulator output then will be a frequency-modulated wave varying between 50 and 70 cycles per second or throughout the same range as the output of the low-frequency generator during full scale swings on the insensitive range. By this process the turbo-generator frequency variations are increased forty times from ±0.25 cycle per second in 60 to ±10 cycles per second in 60. The output of the modulator is amplified, converted to an amplitude-modulated wave by the discriminator, and recorded similarly to the output of the integrator circuit when using the wide range scale. The voltage of the 2,400-cycle generator also varies with the driving motor speed, but these variations are not multiplied up as are the frequency variations. For this reason the error due to them is insignificant and consequently it is unnecessary to integrate the output of the 2,400-cycle generator.

In order that the base frequency for the sensitive scale be adjustable between 59 and 61 cycles per second, it is necessary that the oscillator frequency be adjustable from 2,300 to 2,380 cycles per second to give the same modulator output wave at all base frequencies. The oscillator control dial is calibrated directly in terms of base frequency.

It is evident from the foregoing discussion that accurate measurement of the average value of frequency when using the sensitive scale depends upon the constancy of the oscillator frequency. Consequently, great care has been taken to provide a highly stable oscillator circuit. In addition, a small cathode ray tube is provided for constantly checking the oscillator frequency, by means of a Lissajous figure, against the frequency of a very stable 234-cycle tuning fork which forms part of the equipment.

As implied above the oscillograph galvanometer will respond to voltage changes as well as to changes in inductor-generator frequency. It is necessary, therefore, to minimize voltage variations in the measuring circuits due to power supply fluctuations and temperature changes. Various expediencies are used to accomplish this end. The amplifier gains are stabilized by the use of large amounts of negative feedback. The discriminator circuit as well as certain other critical circuits are formed of stable elements with low temperature coefficients. The copper-oxide rectifier circuit is designed so that variations in resistance of the rectifier discs are insignificant. Finally, the voltage applied to the field windings of the inductor generators is stabilized by a gaseous voltage regulator tube. As a result of these precautions the oscillograph current varies less than 2 per cent of that corresponding to full scale range of frequency variation for ordinary room temperature changes and for power supply voltage fluctuations of 10 per cent.

Figure 6. The frequency meter

Power-supply circuits are in the lower half and the measuring circuits are in the upper half

The inductor generators have several interesting and noteworthy features. To avoid erroneous readings it is essential that the wave shapes of the generated voltages be very nearly sinusoidal. Consequently, various arrangements have been used expressly to eliminate harmonics. As illustrated in figure 4, the two generators are combined in a single assembly. The rotor of the 60-cycle generator consists merely of a short cylindrical section truncated at one end by a plane at a slight angle to the axis. With this arrangement the length of rotor under any axial element in the pole face varies as the sine of the angle of rotation, and the voltage generated due to a uniform angular velocity is theoretically a sine wave. There is some distortion, however, mostly second harmonic, due to fringing of the flux at the edges of the rotor. This distortion is eliminated by using two diametrically opposed field structures with their windings connected so that the fundamental voltages add while the even harmonics cancel out.

In the 2,400-cycle generator even harmonics due to flux fringing are eliminated by a push-pull field arrangement similar to that used in the 60-cycle generator. The third harmonic and its multiples are eliminated by making the circumferential pole width equal to one third of the rotor tooth pitch. Finally, the fifth harmonic and its multiples are eliminated by a suitably proportioned trapezoidal rotor tooth shape suggested by Mr. C. R. Hanna. Thus the only harmonics not taken care of in the design of the generator are the 7th, 11th, 13th, 17th, etc. These are of such high order and have such small magnitude due to the smoothing effect of flux fringing that they are of no consequence.

The response of the frequency meter to sinusoidal variations in turbo-generator frequency is shown in figure 5. Curve *A* in this figure represents the frequency response of the synchronous motor which drives the inductor generators and illustrates the limitation mentioned earlier in connection with the relationship between motor speed and turbo-generator frequency. Mechanically, the motor consists of a rotating mass coupled to the revolving field produced by the voltage applied to the stator windings. The coupling is provided by the stiffness due to the synchronous torque of the motor and by the damping due to the squirrel cage winding on the rotor. Because it is not feasible to design synchronous motors with nearly perfect coupling, the response of the rotor to rapid variations in angular velocity of the revolving field is limited in the manner illustrated by curve *A*. This curve was computed, for the particular motor used, from calculated values of rotor inertia and damping and from a measured value of synchronizing stiffness. If better frequency response than this is required it will be necessary to couple the inductor generators directly to the turbine shaft. It is believed, however, that the response shown will be adequate in nearly every case.

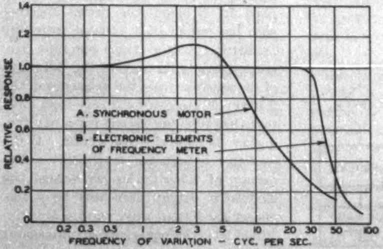


Figure 5. Response of the frequency meter to sinusoidal variations in turbogenerator frequency

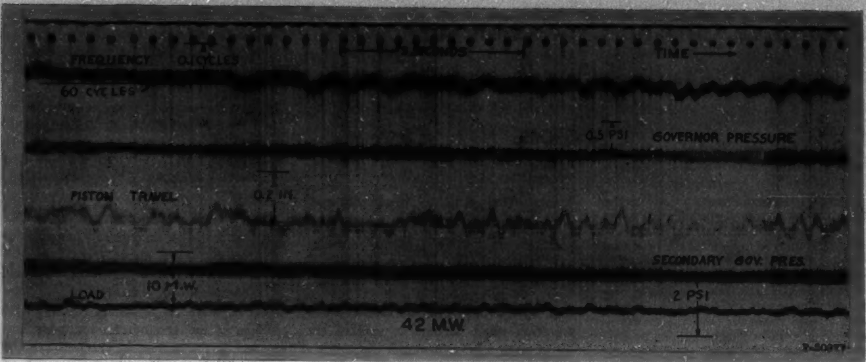


Figure 7. A typical record taken with the turbine-governor analyzer from a modern hydraulic governor controlling a 35,000-kw 3,600-rpm turbogenerator

Curve *B* in figure 5 shows the measured frequency response of the electronic parts of the frequency meter including the oscillograph. This response is very nearly the same as that of the smoothing filter connected between the rectifier and oscillograph, indicating that this filter is the limiting element. The response of the electronic elements of the frequency meter, therefore, can be improved if necessary by using a more elaborate filter.

The assembled frequency meter is shown in figure 6. The meters on the panel indicate generator field current and average turbo-generator frequency, respectively. For simplicity of operation the controls have been kept to a minimum. They include the following: base frequency adjustment, generator field current control, range selector switch, oscillator vernier, and sensitivity control. The cathode ray tube for checking the oscillator frequency is in the center below the base frequency dial. The tuning fork and the inductor generators with their driving motor form separate units.

Conclusion

It has been demonstrated in several applications of the new turbine governor performance analyzer to actual field tests that a minimum of time and effort is expended in both preparation for and during the tests. Furthermore, the information obtained is complete and easily interpreted by anyone familiar with governor characteristics. Figure 7 is a portion of atypical record. A complete test procedure for the use of the instruments was cutlined in a recent paper by A. F. Schwendner.[5]

The instruments themselves are rugged and foolproof and can withstand considerable abuse. When not in use and during transit they are contained in five rugged carrying cases which improve the portability of the equipment.

It is believed that extensive use of the new instruments will add greatly to the operating engineer's knowledge of the characteristics of his governors, and show the governor designer the way to noteworthy improvements in design.

References

1. The following papers are cited as examples:

CONSTANT SYSTEM SPEED AND THE STEAM-TURBINE GOVERNOR, A. F. Schwendner. American Society of Mechanical Engineers *Transactions*, volume 62, 1940, pages 199–206.

STEAM-TURBINE GOVERNORS, Reed J. Caughey. American Society of Mechanical Engineers *Transactions*, volume 62, 1940, pages 191–8.

THE FIELD OF SYSTEM GOVERNING, Albion Davis. American Society of Mechanical Engineers *Transactions*, volume 62, 1940, pages 207–19.

PRIME-MOVER SPEED GOVERNORS FOR INTERCONNECTED SYSTEMS, R. J. Caughey and J. B. McClure. AIEE TRANSACTIONS, volume 60, 1941 (April section), pages 147–50.

POWER-SYSTEM GOVERNING—THE PROBLEM, J. J. Dougherty, A. P. Hayward, A. C. Monteith, and S. B. Griscom. AIEE TRANSACTIONS, volume 60, 1941, pages 547–58.

EFFECT OF PRIME-MOVER SPEED-GOVERNOR CHARACTERISTICS ON POWER-SYSTEM FREQUENCY VARIATIONS AND TIE-LINE POWER SWINGS, C. Concordia, S. B. Crary, Jr., and E. E. Parker. AIEE TRANSACTIONS, volume 60, 1941, pages 559–67.

SYSTEM LOAD SWINGS, H. A. Bauman, O. W. Manz, Jr., J. E. McCormack, and H. B. Seeley. AIEE TRANSACTIONS, volume 60, 1941, pages 541–7.

2. AN INSTRUMENT FOR MEASURING SMALL DISPLACEMENTS, B. F. Langer. *Review of Scientific Instruments*, volume 2, 1941, pages 336–42.

3. OSCILLOGRAPH ANALYZES GOVERNOR PERFORMANCE, J. E. Allen. *Power*, volume 78, November 1934, pages 610–12.

4. APPLICATIONS OF THE PHOTO-ELECTRIC RECORDER, W. L. Carson. *General Electric Review*, volume 39, April 1936, pages 189–93.

5. ANALYZING GOVERNING-SYSTEM PERFORMANCE, A. F. Schwendner. Scheduled for publication in American Society of Mechanical Engineers *Transactions*, January 1942.

Sensitive Ground Protection for Transmission Lines and Distribution Feeders

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Synopsis: If ground fault coils are used in high tension overhead systems and underground cable networks, earth leakage relays may be employed to indicate the location of single ground faults. The influence of the contact resistance at the point of the ground is explained. Earth leakage relays in compensated networks, their operation in radial and meshed networks, and the means for determining their correct operation by tests, are described. The paper is based on the practical experiences gained from such installations in England and in Continental Europe.

1. Introduction

It has always been important to locate the line in a transmission or distribution network which is affected by a ground fault. A general satisfactory solution has not been found to this problem. Therefore, it seems advisable to report on the application of wattmeter-type relays for sensitive ground fault protection in compensated networks. Such relays have been used very successfully in overhead line and underground cable systems in European countries in the past.

The expression "compensated networks" means that earth fault coils (arc suppression coils, grounding arc suppressors, ground fault neutralizers, and resonant grounding reactors) are in use. This means that at the point of the fault the charging current to ground of the system is compensated by the inductive current of the earth fault coil. If the coil is exactly tuned, the fault current at the point of the ground is very small. It consists of the in-phase component of the fundamental wave of the charging current to ground and some higher harmonics. The harmonics shall not be considered in the following, as their influence on earth leakage relays is not of any importance.

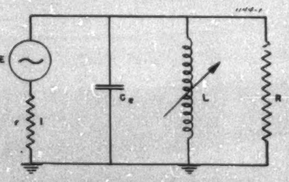

Figure 1. Simplified diagram of a compensated network, one phase of which is grounded through a resistance r

As far as ground faults in high tension systems are concerned there exist two protective schemes which are fundamentally different and which influence nearly every problem in connection with high tension power transmission and distribution. One scheme consists of the direct grounding of the power transformer star points, while the other uses earth fault coils. In the case of solidly grounded neutrals, which shall not be considered in this paper, ground faults are short circuits and the defective line has to be switched off; in the case of compensated networks no dangerous short circuit currents occur in connection with single phase-to-ground faults. It is then possible and permissible to continue operation under steady ground fault conditions.

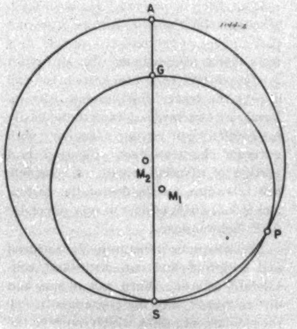

Figure 2. Circle diagrams

Circle M_1: displacement of the neutral corresponding to the simplified diagram, figure 1. Circle M_2: displacement of the neutral when no earth-fault coil is used, and when r varies

The earth leakage relays, which will be described more closely in the following, can be and have been used in radial systems both for signalizing the grounded feeder or switching it off. In interconnected systems, it is not equally easy to use these earth leakage relays for isolating the defective line, and the relays have been in use in the past mostly for signaling; however, for some systems a selective switching off system has been developed and has worked satisfactorily.

In this paper will be considered only how wattmeter-type earth leakage relays, influenced by the neutral-to-ground voltage and the residual current (mutual return of the three current transformers) operate in compensated networks. It is of special interest to investigate first how their sensitivity is influenced by the resistance to ground at the point of the fault. The theory and operation of earth fault coils has been covered in previous publications. We know from these investigations that the simplified diagram of a compensated network[1] is given by figure 1, where E is the negative phase voltage of the grounded conductor, impressed on the system at the point of the fault. The capacitance to ground of the lines is represented by C_e; L is the inductance of the earth fault coils; R is a resistance which represents the line losses and the losses in the coils; r is the contact resistance at the point of the fault.

The fault current I is greatly reduced when C_e and L are in resonance, as R is, on the average, from ten to twenty times greater than ωL. For this trap circuit I becomes $I=\frac{E}{R+r}$, and this is the in-phase component of the fault current. We can see from this immediately that the application of a "grounding transformer," for limiting the single phase short circuit current to about 20–40 amps. is[2] practically the same as a badly tuned earth fault coil, since the capacitances to ground will have an effect.

2. Influence of the Contact Resistance at the Point of the Ground

An exact analysis of the circuit leads to the conclusion that the fault current does not change very much when the tuning of the earth fault coil is not quite correct; as it is very easy to maintain nearly correct tuning, it may be considered in the following that the coil is in tune. The fault current becomes then exactly $I=\frac{E}{R+r}$. R will always be a rather high resistance in comparison with r. If

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[1] For all numbered references, see list at end of paper.

we assume, for example, that the ground fault I_e for an uncompensated 25 kv overhead line system is $I_e = \frac{E}{\omega L} = E\omega C_e = 20$ amps., then the in-phase component will be about 2 amps; hence we get $R = \frac{25{,}000}{2\sqrt{3}} = 7{,}200$ ohms, and we see that I will vary only within narrow limits, if r varies between, say 0 and 1,000 ohms. One of the advantages of the earth fault coil in this connection is, therefore, that the earth leakage relays operate under a rather constant current.

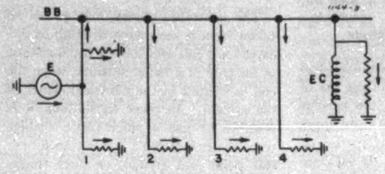


Figure 3. Radial network

Distribution of the in-phase component of the residual current. Ground fault on feeder 1

As r is the contact resistance at the point of the fault, Ir is the voltage-to-ground of the conductor with the fault; the neutral-to-ground voltage then becomes IR. Since $IR + Ir = E$, we see in figure 2 that this vector SA is divided by the ground potential (point G) in the ratio $R{:}r = SG{:}AG$. Since r is much smaller than R, we may express the action of the earth fault coil as reducing the voltage-to-ground of the faulted conductor. Without the application of an earth fault coil the fault current I_u would be

$$I_u = \frac{E}{\sqrt{r^2 + 1/\omega^2 C_e^2}}$$

and for the voltage to ground we get $I_u r$. In this case we have neglected R, as its influence is rather small; however, figure 2 will give an accurate picture.

If we again assume $I_c = 20$ amps. as charging current to ground of a 25 kv network, we find $E = 14{,}400$ volts, $1/\omega C_e = 720$ ohms, and if $r = 720$ ohms, the voltage-to-ground of the faulted conductor becomes $I_u r = E \div \sqrt{2} = 10{,}200$ volts. On the other hand, if a correctly tuned earth fault coil is used, with the same $r = 720$ ohms, and the other data given as above ($R = 7{,}200$ ohms, $I = 2$ amps., $I_c = 20$ amps.), the voltage-to-ground of the faulted conductor becomes only

$$E\frac{r}{R+r} = 1{,}310 \text{ volts}$$

which is considerably smaller.

As already mentioned, the neutral-to-ground voltage is used for the polarizing of the earth leakage relays. Its value is the phase voltage of the system for a direct ground, and nearly as much for an earth fault through a contact resistance. As the voltage-to-ground of the grounded conductor is 1,310 volts in our example, the neutral-to-ground voltage becomes $SG = SA - AG$ or $SG = 14{,}400 - 1{,}310 = 13{,}090$ volts. In general we get $IR = \frac{ER}{R+r} \approx E(1 - r/R)$, as r/R is usually small compared with 1. This means that the earth leakage relays in a compensated network will operate also under a rather constant voltage.

Figure 2 shows the results of a precise analysis of the circuit.[3] The smaller circle, center M_1, is the diagram for the neutral-to-ground and the line-to-ground voltages when an earth fault coil is used and its tuning is changed steadily by varying L while C_e, r, and R are kept constant. The tuning is exact at G, and therefore $AG{:}SG = r{:}R$. The intersection of the two circles, point P, gives the voltages when the earth fault coil is switched off. The bigger circle, center M_2, is the diagram for the same voltages when no earth fault coil is applied; here, r is variable, and R and C_e are constant. The smaller circle is divided by S and G into two parts; at all points of the circle on the right side of SG there is undercompensation of the charging current to ground, i.e., the inductive coil current is smaller than the capacitive ground current. At all points on the left side of SG there is overcompensation. The coil is at resonance at G, at which point the fault current has no reactive component. Point P lies on the right part of the circle, since the coil is switched off at this point.

3. Earth-Leakage Relays

In a network with an insulated neutral and no earth fault coil in use, the point of the ground fault may be considered as that part of all the sections and lines of the whole system where the charging current to ground has its greatest value. We know that the neutral-to-ground voltage should be impressed on the network at this point to give the correct distribution of the charging-to-ground current over the system. This picture of the current distribution changes, as far as the reactive component of the charging current is concerned, when earth fault coils are introduced, since they are additional reactive connections to ground. However, the coils do not influence the dissipation of the in-phase component, since the in-phase component is due to losses, which are not compensated by the coils. This implies that, for getting selectivity, only relays can be used which act upon the in-phase component of the residual current i in each feeder. This means that the relays must react to the formula $ei \times \cos\phi$, where e is the neutral-to-ground voltage and $i\cos\phi$ the in-phase component of the residual current in the feeder. The operation of earth leakage relays in a compensated network will be shown in more detail in the following.

As first example, a radial network is drawn in figure 3, where each line represents a three phase feeder (lines 1, 2, 3, 4); BB are the busbars and EC is the earth fault coil. The resistances represent the losses which are, of course, uniformly distributed over the network. If a ground fault occurs on feeder 1, the distribution of the in-phase components $i\cos\phi$ of the fault current will be as indicated by the arrows. We see that its direction is the same in all the unaffected feeders, away from the busbars and opposite to that in the faulted feeder 1. Hence a sensitive wattmeter-type earth leakage relay will detect the faulted feeder.

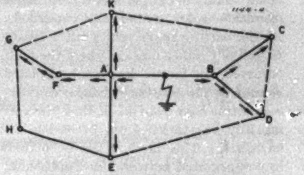


Figure 4. Interconnected network

Distribution of the in-phase component of the residual current. Ground fault on feeder *AB*

Similarly in a mesh or interconnected network, the in-phase component of the residual current is a maximum in the faulted line, and it is very easy to find this section of the transmission or distribution system from the operation of the signalizing earth leakage relays. Figure 4 shows a part of such an interconnected network, the dotted lines indicating that it is really a mesh network. If, for example, the fault occurs at the line AB, the distribution of the active component of the

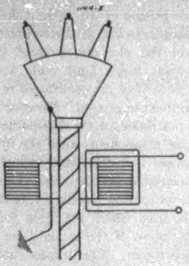


Figure 5. Ring-type current transformer

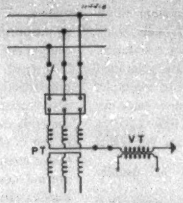


Figure 6. Testing connection when a single-phase voltage transformer is used for supplying the neutral-to-ground voltage

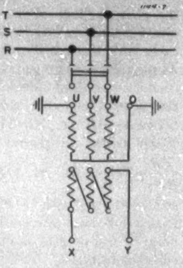


Figure 7. Testing connection when a three-phase voltage transformer is used for supplying the neutral-to-ground voltage

fault current will be as indicated by the arrows. The figure shows clearly that the earth leakage relays will operate on both ends of the faulted line, while on all other lines we find relays operating only on one end. On *BC*, for example, only the relay at *C* will operate thus indicating the direction in which the ground fault is located.

Relays acting on the in-phase component of the residual current could also be used in uncompensated (fully isolated) networks as the distribution of the in-phase component of the residual current is independent of the presence of earth fault coils. However, earth leakage relays have been used mostly in compensated networks since the great reduction of the fault current permitted continuous operation under ground fault conditions. As the earth leakage relays are a means to find out rather quickly where the fault occurred, the faulted line can be isolated as soon as the necessary changes in interconnection have been made, thus preventing any substation from becoming de-energized. This method has been used in compensated networks in Continental Europe for many years and improved by past experience.[4] The method has proved to work very satisfactorily also in Great Britain, where earth fault coils and earth leakage relays have been used only for the last few years,[5] but are now being used in an ever increasing extent.

The earth leakage relays are wattmeter-type relays, polarized by the neutral-to-ground voltage and the residual current. The voltage coils in use are of the normal relay type, but the current coils are usually made from more turns of a smaller wire, in this way increasing the sensitivity of the relay. If accurate current transformers were at hand, they could be used successfully, even with an in-phase component of only 1 amp. or less, and with a current transformer ratio of 100/5 or 200/5 amps. For smaller primary residual currents special current transformers may be advantageous.[6] In Great Britain metal clad switchgear is used almost exclusively; therefore the overhead line sections are usually terminated through a short cable run, and ring-type transformers, figure 5, have been found as a cheap and good solution.

As the ground fault is no longer a single phase short circuit, no overcurrents now appear in connection with that kind of fault in a compensated network; hence it should not be necessary to protect the current winding of the earth leakage relays against damage by heavy excess currents. However, short circuit currents are possible through the earth leakage relay coil in the case of a double ground fault (i.e. two grounds on different line sections in different phases). In networks supplied by stations of very high generating capacity, additional excessive current protective relays in the asymmetry circuit have been used. These overcurrent relays shunt the sensitive earth relay coil and thus protect it against damage by the heavy short circuit currents in case of a double ground fault. Such overcurrent relays are useful too in another respect; they hinder the operation of earth leakage relays under double ground fault conditions. A detailed investigation shows that earth leakage relays might operate incorrectly in interconnected networks under double ground fault conditions.

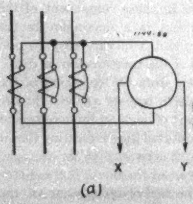
4. Connections and Testing of Earth-Leakage Relays

The neutral-to-ground voltage can be taken from a secondary winding on the earth fault coil, which is the usual method in the case of a radial network, when the coil is located in the electrical center of the system. In other substations, it is possible to use a single-phase voltage transformer between a power-transformer star point and ground, or three single-phase voltage transformers with grounded neutral and the secondary winding in delta for supplying the neutral-to-ground voltage. As has been said, we consider only fundamental questions in this paper, hence we omit some results of theoretical investigations and practical experience by which correct operation of very highly sensitive earth leakage relays was attained.[4]

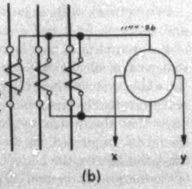
When earth leakage relays have been installed, it is advisable to check their correct operation by tests. There are two different ways of relay testing according to the various methods of getting the neutral-to-ground voltage.

(*a*). If the neutral-to-ground voltage is taken from the secondary winding of the earth fault coil, it is necessary to make actual ground faults, either on the busbars or on one or more feeders. The use of a protective resistance as artificial contact resistance is advisable; its value in ohms should be[3] about $1/\omega C_e$. Then the earth leakage relays should be tested in correct wiring connections and also with reversed voltage. If the ground is made on the busbars, of course, the relays should operate only with reversed voltage, i.e., with the wiring connections reversed.

(*b*). If the potential transformers are used for supplying the voltage coils of the earth leakage relays, actual ground faults for testing the correct operation of the relays are not necessary. According to figure 6, one phase of the power transformer should be disconnected (if the power transformer can be de-energized), provided that only a single-phase potential transformer is used which is connected between the neutral of a power transformer and ground. The potential transformer is then under a voltage-to-ground which is of the same phase angle



(*a*)



(*b*)

Figure 8. Testing connections of the three-current transformers of a line section

(*a*) Incoming power

(*b*) Outgoing power

as if there were a ground in the disconnected phase; the amount of voltage being only half of its value under actual ground fault conditions. If, however, a bank of three potential transformers is used and a similar connection is made (figure 7), the secondary winding will give a voltage of correct phase-angle, but its amount will be only about $^1/_3$ of its maximum value under ground fault conditions; the phase angle will again be that of the opened phase of the potential transformer.

To get a corresponding current in the coil of the earth leakage relay, we now make an auxiliary connection on the secondary side of the current transformers on each outgoing feeder (as shown in figures 8*a* and 8*b*). We use figure 8*b* if power is outgoing and figure 8*a* if power is incoming.

With these auxiliary connections, in both voltage and current circuits, the earth leakage relays are supplied with voltage and current under exactly the same phase-angle as would exist between residual feeder current and ground-to-neutral voltage under ground fault conditions. The earth leakage relays should then operate. If they do not, we must change the connections by reversing either the voltage or the current wires. In similarity with a wattmeter or a watt-hour meter, we are able to find the correct connections only according to the polarized and marked terminals of the potential and current transformers; however, it is always advisable to make sure by actual tests that the relays are put into service with correct connections.

5. Conclusions

(*A*). The application of resonant neutral grounding makes possible the operation of radial and network systems with permanent ground faults. This has been proved by many installations abroad, both for overhead line and underground cable systems, and with line voltages from 3,000 volts up.

(*B*). The installation of wattmeter-type earth leakage relays would give an indication of the defective section of the line. The relays act upon the in-phase component of the residual current and the neutral-to-ground voltage of the system.

(*C*). The in-phase component of the residual current is in most cases large enough to secure correct operation of the relays.

(*D*). The operation of the relays can easily be checked by simple tests.

References

1. The Neutral Grounding Reactor, W. W. Lewis. AIEE Transactions, volume 42, 1923, pages 417–34.

Petersen Coil Tests on 140-Kv Systems, J. R. North and J. R. Eaton. AIEE Transactions, volume 53, 1934 (January section), pages 63–74.

Earth-Fault Coils in Transmission Lines and Distribution Networks, Eric T. B. Gross. *The Cornell Engineer*, volume 6, 1940, page 7.

2. Sensitive Ground Protection for Radial Distribution Feeders, Lloyd F. Hunt and J. M. Vivian. AIEE Transactions, volume 59, 1940 (February section), pages 84–90.

3. Adjustment of Earth-Fault Coils, E. Gross. *Bulletin Schweizerischer Electrotechnischer Verein*, volume 38, 1937, page 165.

4. Earth-Leakage Relays, Eric T. B. Gross. *The Electrician*, London, volume 123, number 3191, 1939, page 93.

Earth-Leakage Relay Sensitivity in Protection Systems, E. Gross and W. Weller. *Elektrotechnik und Maschinenbau*, volume 50, 1932, page 117.

5. Line Protection by Petersen Coils, With Special Reference to Conditions Prevailing in Great Britain, H. W. Taylor and P. F. Stritzl. *Journal* Institution Electrical Engineers (London), volume 82, 1938, page 387 and volume 83, 1938, page 708.

6. Zero-Sequence Current Transformers for Ground Protection, J. W. Graff. *Electrical World*, volume 102, 1933, page 822.

7. Testing of Earth-Leakage Relays, E. Gross. *Electrotechnik und Maschinenbau*, volume 46, 1928, page 1213 and volume 47, 1929, page 372.

8. Ground Faults in High-Voltage Systems (a book), R. Willheim, Berlin, 1936, Verlag von Julius Springer.

The Communication Facilities of the U. S. Forest Service

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FOREST protection in the United States is a job of considerable magnitude. There are, roughly, 550,000,000 acres, which are equal to the area east of the Mississippi River, exclusive of Wisconsin and Illinois, under intensive protection against forest fires. Moreover, there are an additional 138,000,000 acres needing organized forest protection.

Of this vast area, state and private agencies are responsible for about 279,000,000 acres, the U. S. Forest Service for about 208,000,000, and other Federal Government agencies for about 60,000,000 acres.

According to U. S. Forest Service records the annual number of forest fires is increasing materially, though the average size of the fires is becoming progressively smaller each decade. The fact that the average fire is getting smaller, even though there are more of them, is one encouraging evidence of the gradually increasing effectiveness of fire suppression tools and fire control technique.

Dry-battery-operated ultrahigh-frequency automatic-relay station

Upper deck—transmitter; lower deck—receiver

Comparatively recent improvements in communication facilities have undoubtedly played an important part in reducing fire losses. It is generally true that the quicker one can attack a fire the fewer men are needed to put it out, which, of course, points the way toward faster communication and transportation.

One of many examples of the importance which fire fighters place on radio communication is contained in the following quotation from a news story on the recent forest fires in Maryland:*

> "Two-way radio communication proved helpful on a large fire in northern Anne Arundel County which burned more than 3,000 acres. The department of forestry called three forestry radio cars into the area and directed them by radio from the Long Hill lookout tower in shifting fire-fighting crews from point to point.
>
> "Had radio equipment been available on the fire trucks and other vehicles, forestry officials believe the fire could have been held to half its size and much of the forest saved."

Though it is probably true that few, if any, forest protection agencies have sufficient communication facilities, the extent of these systems is surprisingly great. For example, the U. S. Forest Service is

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*"The Washington Post"—April 27, 1941.

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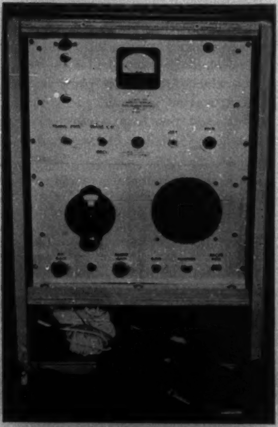


Forest Service type M semiportable radiophone

maintaining over 60,000 miles of telephone line and operates about 3,500 radio stations. State and private forest protection agencies are reported to have about 52,000 miles of telephone line and 1,500 radio stations.

Fundamentally, all forest protection agencies have pretty much the same job to do and go about it in a similar manner, though there are, of course, differences in organization and degrees of protection.

Since I am most familiar with the operations of the U. S. Forest Service, this discussion will deal mainly with that agency.

For administrative purposes, the Forest Service divides the continental United States, Alaska, and Puerto Rico into ten regions. Within each region there is a number of national forest units—a total of more than 150.

National Forest Communication Systems

Since the national forest is the administrative unit, the communication systems are set up on a national forest unit basis.

The backbone of the national forest communication is, of course, wire line—either Forest Service lines or commercial circuits if they are available.

Radio is used where no other satisfactory means of communication are available. Nearly all of the radio communication is of an emergency nature. Ordinarily, it is not considered desirable that both routine administrative and fire traffic flow over the same radio circuits, since in time of emergency the administrative traffic would have to give way to emergency needs which would disrupt the normal forest administration.

The communication system on a forest unit falls roughly into three general classes of use, namely: (1) administrative circuits, as between the forest supervisor and district ranger offices, and between district ranger and CCC camps, ranger stations, and logging camps; (2) fire detection circuits, as between fire lookouts and lookout to central fire dispatcher; (3) fire suppression circuits, as dispatcher to patrolmen, fire crews and fire trucks, communication between main base camp and sector camps on large fires, contact with foot and aerial scouts, freighting planes, communication along the fire line, base camp contact with regional equipment depots, and communication with the mobile fireweather forecast units.

Communication on Large Fires

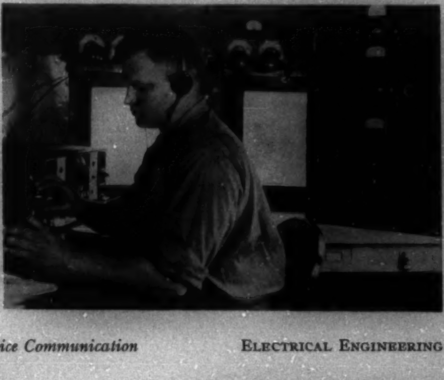
Modern fire fighting, especially on large fires in heavily forested regions, assumes much of the aspect of a military operation with all its complexities of staff organization, service of supply, and front line attack.

Consider, for example, a fire of the "conflagration" class—say 50,000 or 75,000 acres. The perimeter of the fire will likely be 100 or 150 miles. Several thousand fire fighters will be employed. They will be split up in camps of about 200 men each appropriately spaced along the fire line. The fire line will be in the charge of a fire chief. Directly under him will be an assistant chief, chief of service of supply (SOS), transportation chief, equipment chief, head scout, communication chief, paymaster, and sector bosses. Each sector boss is directly responsible to the fire chief for his sector of the fire.

(Lower left) Typical radio installation in Forest Service lookout

Radiophone is designed to fit in standard firefinder pedestal

Interior of Forest Service radio trailer used on large fires



Radio communication on the fire line during "mopping up" phase of fire

Note two hose lines in use

(Right) Forest Service smokechaser "reporting in"

Fire pack includes tools, radio, emergency rations, and blanket

The main communication center will be at staff headquarters in the main base camp. From this point communication circuits will be established with the regional equipment depot, forest supervisor's headquarters, reconnaissance and transport planes, fireweather units, and the sector camps located along the fire line. The sector camps will set up local networks to communicate with the fire line, ground scouts, nearby lookouts, and various fire fighting units within the sector.

Before the advent of radio, communication along the fire line was largely by messenger, which at best is a slow and uncertain method. Consequently, there was too often a conspicuous lack of coordination among the various fire fighting units attacking a single fire. For example, one sector of the fire might be considerably overmanned or overequipped while other crews were losing ground through lack of reinforcements or apparatus. With the development of portable and mobile two-way radio, marked improvements in fire fighting tactics became possible. The entire forces can now be controlled from a single headquarters and men and machines shifted along the line as the vagaries of fire behavior dictate.

Radio Frequencies Used

Originally, forest protection radio communication was entirely in the 2–4 megacycle band. This part of the spectrum was selected because single channel transmitters were used and these frequencies exhibit the least variation in diurnal and seasonal sky wave propagation. The lower limit of the frequency band selected was also influenced by antenna length. In order to preserve portability and obtain reasonable efficiency, horizontal half-wave antennas are used. Experience has shown that the difficulty of finding a clear space in dense forests of sufficient size to stretch an antenna increases rapidly with

(Right) Automatic relay antenna, Mt. Hood, Oregon

(Lower right) Portable radio installation in "sector" fire camp

Communication unit used on large fires

length of the radiator and that 140 feet is a good compromise.

For the last seven or eight years the percentage of ultrahigh frequency stations—30 to 40 megacycle band—has increased rapidly. Frequencies in this portion of the spectrum were selected as representing the best compromise between the desirable characteristics of the ultrahigh frequencies such as minimum static, short antennas, circuit efficiencies, and the better coverage below line of sight obtainable with lower frequencies. Rapid growth in the use of the ultrahigh frequencies has been due both to improvements in apparatus and to increasing experience in the technique. Oddly enough, ultrahigh frequency systems have developed much more rapidly in the mountainous forest of the West than in the flat or rolling terrain of the East. One factor has been the discovery that, in general, the "blind" areas behind sharp mountain ridges are not nearly as sharply defined as they are in areas where the slopes are less precipitous.

Radio Equipment Requirements

Since fire behavior, suppression technique, and transportation facilities vary widely as between forest types, the types of radio apparatus employed also vary. Nevertheless, there are certain general requirements which all types must fulfill.

The equipment must have good reliability and must be consistently capable of withstanding hauling by truck over long distances on rough roads. It must withstand the vicissitudes of packhorse transportation and be workable after parachute delivery. The controls must be foolproof and the manipulation simple enough that untrained laborers may set up and operate the apparatus without difficulty. The component parts must be of standard design and commercially available to facilitate battery replacement and repairs without excessive delay, and because urgent equipment needs greatly exceed available funds costs must be kept to a minimum. In addition, the portable types must be truly portable. In regions where the standard smoke chaser's pack weighs about 35 pounds, 10 or 11 pounds are about the practical limit of additional weight he can carry for communication.

Since the portable radiophones must be low powered, it has been found desirable to restrict the power on even the semiportable units. An insufficient number of frequencies has made it necessary to repeat the same frequency well within interference range so that power must be held to a minimum if the signals from the lowest powered portables are to be protected against interference from the higher powered mobile and semiportable units.

High-Frequency Radiophones

The type *M* is the highest powered fire communication unit used by the Forest Service. It has a nominal output of 25 watts, although the full output is seldom used. The type *M* operates from an a-c power supply or portable generator. The complete unit weighs about 90 pounds, The tube line-up is: 6F6G oscillator, two 6L6G in parallel in radio-frequency amplifier, 6F6G speech amplifier, push-pull 6L6G modulators, and a pair of 83 rectifiers. A low-drift crystal maintains frequency stability. The output stage terminates in a pi matching network to accommodate a wide range of antenna or transmission line impedances.

The receiver section is a conventional superheterodyne having a minimum of operating controls.

The type *M* was designed for use at forest headquarters, central fire dispatchers, central equipment depots, and at communication centers on large fires. Since these radiophones may be expected to create interference over a radius of several hundred miles, lower powered units are substituted wherever possible.

A smaller unit of similar form designed to operate from an ordinary six volt automobile storage battery is also available. This unit has an output of $9^{1}/_{2}$ watts and is a substitute for the type *M* where commercial power is not available and where, because of "standby" requirements or for other reasons, it is not desirable to use a gas engine driven portable generator.

The most widely used three megacycle unit is the type *SPF*. For portable operation the complete unit weighs 21 pounds and has a rated working range of 20 miles. The transmitter section is crystal controlled and, as is the case in all Forest Service portable radiophones, receiving tubes are used throughout in both the transmitter and receiver sections. The nominal power output is 2.25 watts. The receiver section is a five tube superheterodyne having a sensitivity of about three microvolts absolute. Provision is made for continuous wave reception by either headphones or built-in loudspeaker.

Forest Service type SX radiophone with loudspeaker unit as used at emergency lookouts and for semiportable operation

For semiportable service a kitbox is supplied which contains heavy duty batteries and an additional half-wave antenna. This latter form was especially designed for secondary lookouts and road construction crews. The lookout installs the half-wave antenna at his station and operates his radiophone from the heavy duty kitbox batteries. When he goes to a fire he needs only to unplug the battery cable and drop the radiophone in its carrying bag to have a 21 pound portable radiophone complete with light batteries and antenna. Thus, he wastes no time in taking down the antenna and is assured

Portable radiophone used by Forest Service parachutist fire fighters

Forest Service portable type SX ultrahigh-frequency radiophone

of unused batteries for his emergency communication.

Similarly, some road crews use the half-wave antenna and heavy batteries while in camp and each morning take the portable outfit out on the job and set it up for contact with the fire dispatcher.

Though not designed for mobile operation, these units have been operated in planes, boats, and trucks where mobile equipment was not available.

Many of the fire trucks and patrol cars are equipped with two-way three megacycle radiophones. The consistent range of these installations is only about 20 miles, mainly because of antenna limitations.

Where fire trucks are employed intensively, two-way communication adds greatly to their usefulness, since it means the trucks are always "in service," even when making a run. It has enabled the dispatcher to recall trucks from false alarm runs and makes it possible to redispatch them without the necessity of returning to headquarters for orders.

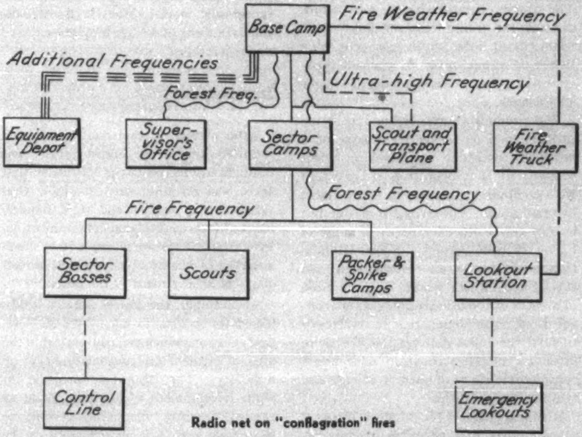
The mobile transmitters are powered by dual vibrators rather than a motor generator in order to eliminate the more difficult servicing of rotating machines and to reduce the drain on the vehicle's battery. The input at the final radio-frequency stage is about 16 watts, but only a fraction is radiated because of antenna inefficiencies. Only a very short antenna can be installed because many of the trucks operate directly on the fire line and are driven through the forests where there are no roads, with a consequent high mortality in antennas, fenders, and running boards.

The shortcomings of the antenna are offset in part by a base loading and tuning unit, but withal the reliable range of this radiophone is only 20 miles, and when possible the ultrahigh frequencies are used, instead, for mobile communication.

Ultrahigh-Frequency Radiophones

In addition to the apparatus already described an increasing proportion of forest protection radio systems utilize the ultrahigh frequency 30–40 megacycle band. Where ultrahigh frequencies are usable they offer obvious advantages over the lower frequencies. Antennas are smaller, and there is substantially no fading or static. The last named feature is especially important since 44 per cent of all fires on the national forests are caused by lightning, and frequently these lightning storms destroy wire or telephones to the point of causing serious delays in the reporting of lightning fires, with consequent heavy fire losses.

Ultrahigh frequency radiophones are used for the same general purposes as the 3,000 kilocycle equipment and are especially effective for interlookout and mobile communication and where the utmost in light-weight compactness is required.



Radio net on "conflagration" fires

The unit designed especially for lookout installation (U. S. Forest Service type *T*) has a crystal controlled transmitter of two watts power.

The receiver contains a 1A7G converter, three stages of 4,050 kilocycle intermediate-frequency amplification using 1N5G tubes, a 1H5G detector and audio amplifier, and a call-buzzer circuit utilizing a 1N5G tube. The intermediate-frequency band width is approximately 50 kilocycles. This comparatively wide band permits reception of modulated oscillators. The buzzer-alarm circuit is useful for night calls and permits installation of extension bells in the towerman's residence.

The low-drain tubes employed in this radiophone give reasonable dry battery economy, even on continuous service.

A new all-purpose portable radiophone known as the type *SX* was developed last year. This complete unit weighs about ten pounds and is applicable to uses of scouts, smokechasers, and others requiring extreme portability. Provision for selecting any of three crystal-controlled transmitter frequencies adapts the unit for operation in fixed frequency networks or ready switching from one network to another. Another useful feature is a panel pushbutton which provides a marker signal and permits setting the receiver on any of the three transmitter frequencies.

For emergency lookout use a kitbox is supplied with the type *SX*, which contains heavy duty batteries and an additional unit containing an audio amplifier and loudspeaker.

Forest Service parachute jumping fire fighters use a condensed version of this radiophone. The parachute jumper's model, which weighs a little less than six pounds, is designed to be carried in a pocket on the leg of the jumper's suit.

General use of ultrahigh frequency in mountainous regions has required the development of an automatic relay station. The transmitter section is conventional design and where—as is usually the case—dry battery is the only source of power, has an output of two watts.

The receiver presents some interesting features. Twelve tubes are used distributed as follows:

Two signal radio-frequency stages
First detector
Crystal oscillator
Frequency multiplier
Three intermediate-frequency stages
Second detector (AVC)
Fifteen kilocycle noise amplifier and rectifier
Relay control tube
Audio amplifier

The system of relay control is based upon the inherent circuit noise and variations of this noise with signal input through operation of the AVC system by the signal. A filter which passes a narrow band centered at 15 kilocycles is branched off the audio input circuit and the resultant 15 kilocycle noise component is amplified and rectified to produce control bias for the relay tube. Fifteen kilocycles is chosen because it is well beyond the range of audio intelligence and, consequently, prevents the communication signal from energizing the relay tube The system operates with the relay, which controls the transmitter, *off* under maximum circuit noise, static crashes or other heavy noise, and relay *on* when an application of a signal causes a reduction in the 15 kilocycle noise component.

The circuit is so arranged and constants proportioned as to keep the noise at a relatively constant top level (in effect limited) so that operation is consistent with combined circuit noise and variable external noise. The circuit is reliable but is only relatively tolerant as to range of battery supply voltage. Some stabilization is effected because of the high gain throughout, which results in production of a small automatic volume control voltage from noise alone; this in turn tends to hold the noise delivered to the noise filter at a relatively constant value over a considerable range of battery voltage and temperature condition.

An inexpensive filter which is incorporated in the unit permits simultaneous transmission and reception on the same antenna.

Satisfactory antennas for high altitude relay stations where icing is especially bad, as well as for lookout house and tower installations, must be extremely rugged mechanically, must provide a heavy d-c path to ground for lightning protection, and must be easy to duplicate without special materials and insulators. An antenna with the best of electrical characteristics would be almost useless unless rugged and sufficiently simple in structure to permit duplication by mechanics in the average forest central garage or machine shop. The antenna structure consists of approximately quarter-wave radiator with an integral coaxial matching section and radial arms of quarter-wave length At one-tenth wave length the coaxial matching section has a resistance of about 65 ohms at the feed point. Line voltage is substantially constant and without standing waves on the transmission line. In field tests this antenna showed a voltage ratio of about 1.8 to that attained on the *J* type antenna.

An additional point of interest in the antenna is the absence of large exposed insulators. The only insulator involved in the antenna proper is the coaxial centering ring, which is placed well inside the coaxial tubing and is thereby not exposed to the weather.

Development

After trying commercially designed radio equipment the Forest Service was reluctantly forced to the conclusion that there was no equipment available that satisfied the peculiar and rigid requirement of forest protection communication. Primarily, the problem was not so much a matter of electrical circuits as of mechanical arrangement of the units.

Accordingly, the Forest Service established its own radio laboratory in Portland, Oregon, which is fairly central to the area of greatest fire and communication difficulty. The laboratory projects include development of new equipment as needed, revision of existing models to keep pace with the current state of the art, and developments in the application and technique of radio communication.

Procurement

The procurement and inspection of all radio equipment used by the entire U. S. Forest Service is centralized at Portland, Oregon. Centralized procurement permits consolidation of orders, helps insure purchase of uniform types of apparatus, and is the most practicable method we have found of filling emergency requisitions for apparatus to be used on going fires with a minimum of delay.

Manufacturing contracts are based on competitive bids Technical specifications are not used; instead, the contractor is required to build exact duplicates of a working model which is supplied him. The whole procurement program is designed to meet unpredictable emergency demands for radio units with a minimum of delay and without carrying large reserve stocks.

Maintenance

Usually Forest Service communication equipment, both telephone and radio, is serviced and maintained by regular personnel. Much of the work is done incidentally to other duties by employees having little or no electrical training other than on forestry radio. This situation requires special consideration in the design of the radio units and in the preparation of servicing instructions.

The variety of uses to which radio can be put in forest protection is increasing as we gain knowledge of its possibilities At present we see a need for a simple, portable direction finder that can be operated by a lookout man in conjunction with his fire finder to guide smokechasers to "sleeper" fires in dense forests. These fires are often most difficult to find and have been known to smolder for weeks and to become a running fire during the next dry windy period.

A portable dry battery operated radio facsimile transmitter and receiver is needed for use by reconnaissance planes and for speeding up message traffic on large fires. Already the traffic load has reached a point where additional circuits must be established or unacceptable delay occurs.

The Modern A-C Network Calculator

W. W. PARKER
MEMBER AIEE

PRODUCTION requires power. The urgent necessity of meeting the immediate demands for electric power—due to national defense and natural growth—has created a challenge to the planning engineers. Local loads are increased and additional loads established, resulting in shifting of load centers. Such changes require increased generating and transmitting facilities. To provide for this expansion in power supply requires accurate planning to avoid waste of time, materials, or effort. The first step is the quick solution of all the electrical problems.

The A-C. network calculator provides accurate means for solving these problems. Therefore, it is appropriate to present the newly perfected features of the latest A.C. network calculators. These provide the means and methods for faster and more accurate analysis of existing or planned power systems and the solutions of electrical design and operating problems. The essential elements of a power system are reproduced in miniature replica, as illustrated pictorially in figure 1 for a simple radial feed from generator to load. The calculator reproduces systems with many generators or generating stations, transformers, lines, and loads. Readings of the miniature network on the calculator are readily converted into actual system quantities.

The electrical performance of the system can be obtained from the calculator studies. The principal studies involve:

1. Voltage regulation and load control studies for normal and emergency operation.
2. Short circuit studies for circuit breaker or protective relay application.
3. Steady state or transient stability studies for determining power limit of transmission systems.

For any of the above investigations, the various steps in the solution are:

1. Assemble and organize the data of the power system.
2. Convert these data to a base or scale suitable for use with the calculator voltages, currents, and impedances.
3. Set up the calculator network and adjust the loads and the generator outputs in accordance with the requirements of the specific problem.
4. Meter the network, obtaining the significant calculator readings such as voltage, current, phase angle, watts, and vars.
5. Reconvert these into system values.

For voltage regulation, load control, short circuit studies, and steady state stability studies, the calculator gives the desired quantities directly, requiring only conversion into system values. Transient stability studies require the same "step-by-step" method used in straight mathematical solutions. The calculator gives the electrical conditions for each step, thus not only saving tedious computations but also permitting the solution of multimachine problems with more generators than can be solved by other methods.

The demand for calculator solutions has been so great that existing facilities have been taxed to full capacity and several new calculators are being constructed. A list of calculators is given in table I.

This paper describes the most recent A-C. network calculator which, although embracing fundamental designs proven over a period of eleven years, has incorporated improved features to facilitate the setting up and metering of the calculator miniature network. A typical calculator of modern design is shown in figure 2.

Fundamental Features of Design

The engineer who uses the calculator is interested primarily in quick, dependable, accurate results. In meeting these requirements, the designer of the calculator considers accuracy, convenience, speed, space, and cost. In these considerations the choice of frequency and power level are predominant factors.

FREQUENCY

The choice of frequency is independent of the frequency of the system to be studied since impedances are used in terms of ohms. The cost and size of reactors and capacitors to cover the range of values needed indicated that a choice of frequency higher than normal would be desirable. The A-C calculator frequency has been established at 440 cycles.[2]

POWER LEVEL

The calculator has rated values for normal operation of 100 volts and 1 ampere, with maximum capacities of 400% for voltage and 300% for current. This power level was arrived at from considera-

Table I. Data on A-C Network Calculators in Use or Under Construction

						Number of Units				
Owner and Location	Date Installed	Frequency Cycles Per Second	Generators	Lines	Loads	Special Reactors	Capacitors	Auto Transformers	Mutual Transformers	Total
1. Massachusetts Institute of Technology, Cambridge, Mass.	1929	60	18	80	40	6	42	12	0	198
2. Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.	1929	440	12	160	40*	20	70	24	30	356
3. Penn. R.R., Pennsylvania Station, New York City	1932	440	6	168	40	40	4	2	16	276
4. Commonwealth Edison Co., Chicago, Ill.	1932	440	6	72	48	12	24	6	6	174
5. General Electric Co., Schenectady, N. Y.	1937	480	12	150	50	0	30	12	15	269
6. Public Service Elec. & Gas Co., Newark, N. J.	1938	480	8	85	30	0	18	10	12	163
7. Tennessee Valley Authority, Chattanooga, Tenn. Initial	1938	440	12	60	20	8**	16	12	10	138
Addnl.	1941	440	6	60	24*	0	18	12	12	132
Total			18	120	44	8	34	24	22	270
8. Bonneville Power Administration, Ampere, Wash.	1939	480	9	50	30	9	30	6	6	140
9 São Paulo Tram. Lt. & Pr. Co., São Paulo, Brazil	1940	440	6	26	14*	14	12	6	12	90
10. Potomac Electric Power Co., Washington, D. C.	1941	440	6	52	18*	18	12	6	8	110
11. Hydro Electric Power Comm. of Ontario, Toronto, Canada	1941	440	9	62	24*	24	18	12	8	157

*Equipped with load adjusters.

**Eight special high resistor-reactor circuits.

Paper 41-143, recommended by the AIEE committee on instruments and measurements, and presented at the AIEE Pacific Coast convention Yellowstone National Park August 27-29, 1941. Manuscript submitted April 14 1941; made available for preprinting July 10, 1941.

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1. For all numbered references see list at end of paper.

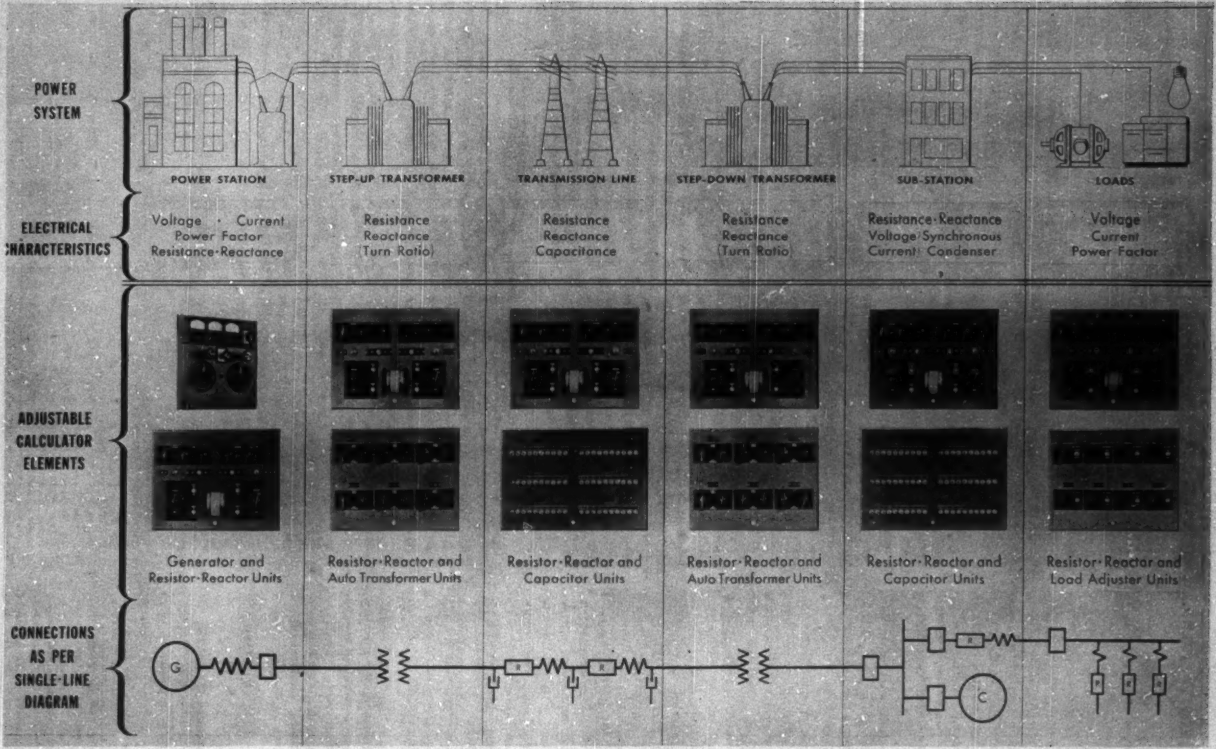
POWER SYSTEM
ELECTRICAL CHARACTERISTICS
ADJUSTABLE CALCULATOR ELEMENTS
CONNECTIONS AS PER SINGLE-LINE DIAGRAM
POWER STATION
STEP-UP TRANSFORMER
TRANSMISSION LINE
STEP-DOWN TRANSFORMER
SUB-STATION
LOADS
Voltage · Current Power Factor Resistance·Reactance
Resistance Reactance (Turn Ratio)
Resistance Reactance Capacitance
Resistance Reactance (Turn Ratio)
Resistance·Reactance Voltage Synchronous Current Condenser
Voltage Current Power Factor
Generator and Resistor·Reactor Units
Resistor·Reactor and Auto Transformer Units
Resistor·Reactor and Capacitor Units
Resistor·Reactor and Auto Transformer Units
Resistor·Reactor and Capacitor Units
Resistor·Reactor and Load Adjuster Units
G
R
R
R
C
R
R
R

Figure 1. **The a-c network calculator electrically reproduces actual power-system conditions**

The a-c network calculator consists of a number of adjustable resistors, reactors, and capacitors, representing the corresponding constants of the actual system, wired to suitable cords so that an actual electrical network can be reproduced in miniature. Sources of voltage whose magnitude and phase position can be varied represent generators and other synchronous machines. Distribution of voltage and current throughout the system is determined by measuring the quantities in the calculator circuits and applying a multiplying factor fixed by the scale to which the network has been set up. The conditions in an existing or proposed system can thus be observed quickly, easily, and accurately

Figure 2. A typical a-c network calculator

The cabinet on the left contains six generator units with their associated instruments for measuring voltages, currents, and power. Below the instruments are the handles for independent adjustment of voltage and phase angle. The bottom drawers contain adjustable capacitor circuits. The next cabinet contains six impedance circuit drawer sections and the master instrument and control desk. (In the calculator described in this paper a separate control desk is used.) The third cabinet from the left is a connection cabinet in which all the circuit terminals are brought out for interconnection. The other cabinets contain load circuits with associated load adjusters, auto transformers, and other line circuits

tion of instrument characteristics and the desirability of avoiding errors due to capacitive or inductive coupling. If a lower voltage base were used, the current base would have to be lowered in proportion to avoid significant inductive coupling. This, in turn, would make the power level so low that, using standard torque instruments, errors would be introduced due to their burden, or high electronic amplification would be required. If the current level were reduced to any considerable extent without reducing the voltage, it would be difficult to avoid errors due to capacitive coupling.

Details of the New Calculator

In addition to these fundamental design features, the later calculators have improved methods of setting up the boundary conditions (i.e. desired loads and generator outputs) and inserting instruments into any element of the calculator network. As a result, accurate determinations of the system conditions are obtained with a minimum of time and effort devoted to manipulation of the calculator. Generally, the solutions are on a phase-to-neutral basis with the use of symmetrical components to solve unbalanced conditions. Complete three phase representation is possible but seldom used.

Generator Units

In order to represent generators or power stations, it is necessary to apply to the calculator impedance network, voltages which are adjustable in magnitude and phase angle. The voltage adjustment corresponds to changes in excitation, and the phase angle adjustment to changes in the governor position. In setting up a specific operating condition, the voltage regulator and phase shifter of the generator units are adjusted until their voltages and currents give the desired inputs. This adjustment requires a minimum of manipulation when there is negligible regulation in the calculator units and the controls of the voltage and phase angle are independent of each other.

Each generator unit has less than 2% regulation from 0 to rated load and has independent adjustment of phase angle and magnitude. This is achieved by means of a novel combination of induction voltage regulators and phase shifter, shown schematically in figure 3. The voltage regulator element consists of two independent, three phase induction regulators, electrically identical, with the rotors on a common shaft. The voltage outputs of the independent stators are equal in magnitude and are varied in phase angle by changing the position of the rotor, as is usual in a 3 phase induction regulator. The two regulator units are mounted mechanically opposite to each other so that turning the rotor shaft in one direction effects a clockwise rotation of the one

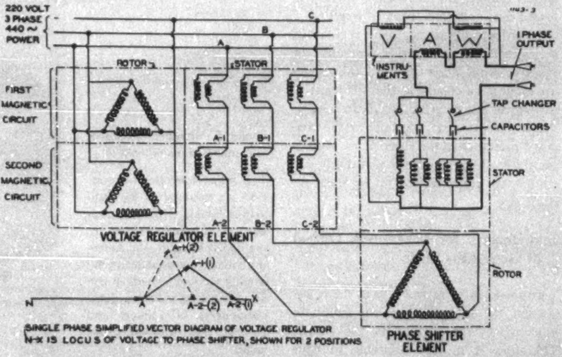

Figure 3. Calculator-generator unit

Actual system sources of synchronous emf are represented by a combination of voltage regulator and phase shifter. Independent control of voltage and phase angle is achieved by a combination of a double-unit three-phase regulator connected to a phase shifter. The voltage regulator has two electrically identical units physically arranged so that adjustment of the rotors does not shift the phase angle of the output to the phase shifter

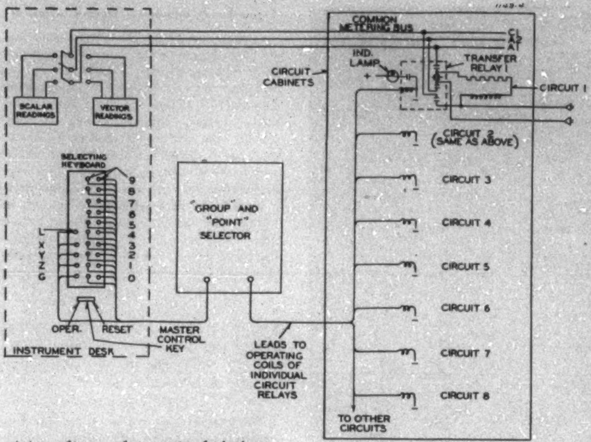

Figure 4. Schematic arrangement of circuit selector for a-c network calculator

The circuit selector consists of the keyboard, a "group" and "point" assembly of telephone-type supervisory relays, and one circuit transfer relay for each circuit. Only 25 keys are required for the selection of any of the present calculator circuits as well as for future additions up to a total of 599 circuits. To select a circuit three keys are pushed, one in each vertical row, and then the control key is thrown to the "operating" position. This closes the selected metering relay located directly behind the respective drawer section in the circuit cabinet. Closing the relay connects the circuit into the metering bus. A guard relay is provided which prevents the selection of any other circuit until the control key is thrown to the "reset" position

stator voltage and a counterclockwise rotation of the other. The two stator windings are connected in series so that the changes in phase angle neutralize each other thus giving a voltage which is always in phase with the supply voltage. This induced voltage is added directly to the supply voltage, thus giving a scalar adjustment of voltage. As shown in the single phase vector diagram of figure 3 the voltage *NA* is the single phase line-to-neutral voltage of the power supply, voltages *A* to *A*-1 and *A*-1 to *A*-2 are the voltages induced in the stators. The output voltage *N* to *A*-2 is adjustable from 0 to twice *NA* in magnitude but always along the phase position as indicated by the dotted line *N-X*. The three phase output of the regulator is then put into the rotor of the phase shifter. A single phase output is taken from the stator for use in the network. Turning the rotor of the phase shifter changes the phase angle of the generator unit voltage as may be required.

In order to obtain negligible regulation, series capacitors are used to neutralize the reactive drop, while the units themselves are made large enough so that the resistive drop gives less than 2% regulation as previously mentioned. Because of the constant reactance of the three phase voltage regulator, the reactance is accurately and completely compensated for by fixed capacitance.

Instruments for Measuring Output of Generator Units

In a calculator network, as in the power system, the adjustment of one generator unit affects the current in the others. Therefore, it is very desirable to have simultaneous indications of the outputs of all the generator units. Since the output can be completely defined in terms of voltage, scalar current, and watts, a voltmeter, and ammeter, and a wattmeter are supplied. Each instrument is equipped with a red marker which can be set to indicate the desired output of the unit. Adjustments of the phase shifters and voltage regulators are made until the indicating pointers line up with the markers. The ammeter is used in short circuit studies and in obtaining equivalent networks. A wattmeter reversing switch is provided to obtain indication of reverse power. A voltage-selecting switch is also provided so that either terminal or internal values of the generator unit may be obtained if the associated impedance unit is set proportional to the internal reactance of the machine being represented.

Circuit Elements

As indicated in figure 1, actual system transformers, transmission and distribution lines, reactors, and capacitors are represented on the calculator by various combinations of resistor-reactor, capacitor, auto-transformer, and mutual reactor circuits. The circuit elements are so designed that for all normal problems there is no necessity for consulting calibration curves to compensate for reactance of resistors, or resistance of reactors.

The value of any resistor or reactor tap is within plus or minus $1^1/_2$% of the setting. The resistors are non-inductive and have negligible (−.00002) temperature coefficient. The iron cores of the reactors have large air gaps so that the permeability of the magnetic circuit is constant

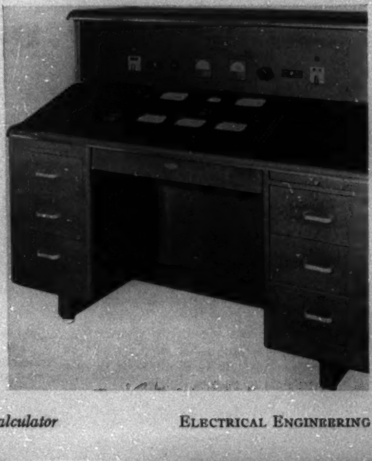

Figure 5. Master instrument and control desk

The motor-generator set controls are mounted on the small vertical panel. Two independent sets of master instruments, the circuit selector, and metering controls are mounted on the sloping sections. Readings can be obtained of scalar or vector components of voltages and currents, watts, vars, and phase angle

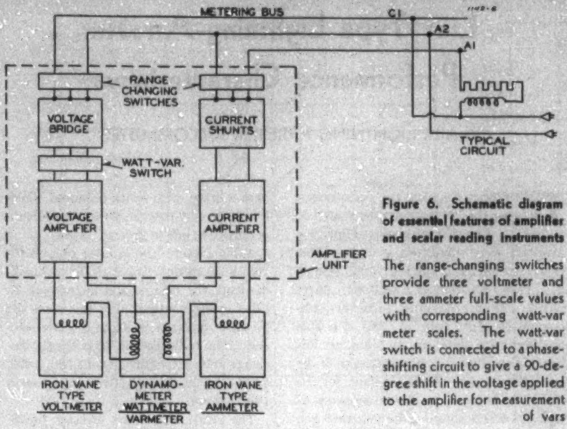

Figure 6. Schematic diagram of essential features of amplifier and scalar reading instruments

The range-changing switches provide three voltmeter and three ammeter full-scale values with corresponding watt-var meter scales. The watt-var switch is connected to a phase-shifting circuit to give a 90-degree shift in the voltage applied to the amplifier for measurement of vars

and hence the reactance is constant throughout the normal range of current. The reactors are designed to keep the resistance to such a low value that its effect can be neglected. This is achieved through the use of "U"-shaped Hypernik punchings (Hypernik having very low core loss at the density used) and by arranging the coils so that the ampere turns are divided equally across the two air gaps. Whenever it is deemed necessary to subtract the resistance of the reactor from the resistance component of the impedance representing an actual circuit element, an average value of 4% of the reactance value can be used.

Ratings

The range of values for the various classes of circuits is shown in the accompanying table.

All circuits are designed for operation at 220 volts, the normal base being 100. Reactors are capable of carrying 3 amperes (300% of normal).

Load and Load Adjuster Units

System loads are simulated on the calculator by means of series or parallel connection of the resistance and reactance elements of the load circuit. The value of this impedance is calculated from the desired kv-a. power factor and voltage at the load bus, using the following formulas:

Series connection

$$R=\frac{K(E')^2}{\text{Kv-a.}}\cos\phi$$

$$X=\frac{K(E')^2}{\text{Kv-a.}}\sin\phi$$

or *parallel connection*

$$R=\frac{K(E')^2}{\text{Kw.}}$$

$$X=\frac{K(E')^2}{\text{R-Kv-a.}}$$

Where K is the power multiplier, or the factor by which the volt-amperes in the calculator circuits are multiplied to give the kilovolt-amperes in the corresponding actual system circuit; E' is the calculator voltage at the load bus; Kv-a. is the 3 phase kilovolt-ampere load desired; and ϕ is the power factor angle.

Range of Circuit Elements

(Since Normal Base is 100 Volts—1 Ampere, 100 Ohms=100%)

Element	Range	
Line impedance (series) $R+JX$	R 0–39 ohms X 0–300 ohms	0 to 100 in 0.2 ohm steps 100 to 399 in 1 ohm steps 1 to 300 in smooth variation*
Load impedance (series or parallel) $R+JX$	R 0–3,990 ohms X 0–2,400 ohms	0 to 1,000 in 2 ohms steps 1,000 to 3,990 in 10 ohm steps 1 to 2,400 in smooth variation*
Capacitance	C 0–4.10 mfd.	In 0.01 mfd. steps
Auto-transformer	%80 to 124.5	In 0.5% steps
Mutual reactors	Ratio 1 to 1†	

*Reactance values are obtained by a combination of fixed air-gap reactors in series with a variable air-gap reactor.

†Value of coupling is determined by impedance circuit connected across either primary or secondary.

Generally, the systems which are set up on the calculator contain several load points and more than one generator; and the voltages at the load busses are not known. For such cases, an assumed value of E' is used in the formula to determine the calculator values of resistance and reactance of the load circuits.

To adjust the values thus obtained to the desired values of load without changing impedances, a simple load adjuster is used. This consists of an auto-transformer with taps corresponding to 1% of the circuit voltage. This auto-transformer is connected ahead of and in series with the load impedance. By means of a reversing switch, the auto-transformer is inserted to increase or reduce the applied voltage. The load adjuster is set by selecting the proper tap and switch position so that the voltage applied to the impedance equals the assumed value. For example, if the assumed value was 100 volts and the voltage read at the load bus was 105, the auto-transformer tap would be set at 5%, and the reversing switch set to lower the voltage. The resistance and reactance do not have to be changed, yet the volt-ampere load taken at the load terminal is equal to the desired value because of the transformer action. The use of these load adjusters is a distinct advance in calculator manipulation.

Circuit Metering

A set of master instruments is provided for determining the electrical conditions of the network. There have been several stages in the development of means for connecting the circuits to the master instruments. The original design utilized manual plugging of an instrument switch associated with each circuit. Later, provision was made for metering a few selected circuits from pushbuttons on the master control desk. These operated transfer relays, one for each circuit. Such an arrangement for all the circuits on a large calculator would require excessive space on the operator's desk and thus would be inconvenient to handle. One of the outstanding features of the new calculator is the use of a 25 pushbutton selector, arranged in a compact assembly like a comptometer keyboard. By pushing the buttons corresponding to the digits of the desired circuit any one of the several hundred circuits is quickly connected to the master instruments. Only one circuit can be connected at a time because of mechanical and electrical interlocks. Figure 4 shows the schematic diagram for this equipment. This simple circuit selector saves valuable space and facilitates circuit connection.

The meter desk shown in figure 5 is the latest development in such equipment, being the result of experience with both horizontally and vertically mounted meters for this application. Having the circuit metering instruments and controls and the writing space on the same sloping plane has been found to eliminate fatigue and to facilitate the recording of readings.

Another distinct advantage of the new desk design is that sufficient space convenient to the operator is made available for using two independent sets of instruments. One set is designed for maximum convenience in obtaining scalar quantities by means of an ammeter, a voltmeter, and a watt-var meter. The second set of master instruments for vector readings, comprises a voltmeter and an ammeter. These, in conjunction with an instrument phase shifter, permit obtaining component values of current and voltage as well as associated phase angle relations. The significant factors in the given problem determine the selection of instruments. For example, the scalar instruments are generally chosen for the ordinary voltage regulation problem; while for stability problems, and certain relay studies where phase-angle and component quantities are desired, the vector instruments are better suited.

All of the master instruments are rugged high torque, instruments with knife edge pointers. The scales are approximately 5 inches long and very uniform so that with a minimum number of full scale values the entire range of electrical quantities can be accurately and quickly obtained. The instruments have a guaranteed accuracy of $^1/_2$ of 1% of full scale for any reading. In order to obtain better results on voltage regulation studies, the voltmeters are calibrated for 100% accuracy at the normal voltage values used in these studies.

Scalar-Type Instruments, Watt-Var Meter, and Thermionic Amplifier

The instruments provided for direct measurement of scalar current, scalar voltage, and watts or vars are connected into the network through a thermionic amplifier which is so designed that the energy taken from the calculator network is negligible. The schematic diagram is shown in figure 6. The amplifier is a negative feed-back type (current feed-back), so that the calibration is independent of temperature changes in the instrument, tube characteristics, and supply voltage variations.[6,8]

Vector Instruments

The vector instruments for measuring component values of current and voltage with respect to any reference or scalar values and relative phase angle are of the dynamometer type. Field coils in these instruments are energized from a phase shifter so that the instrument burden on the calculator network is negligible. The complete description of this scheme has been published previously.[3]

Conclusion

The modern A.-C. network calculator has so improved the technique of operation that little time or effort is required to set up, adjust, and take readings. Errors due to operator's fatigue are practically eliminated. More studies can be made in a given length of time. These benefits are of maximum significance during a period such as the present, when time is the most important factor in power system planning.

References

1. The MIT Network Analyzer, **H. L. Hazen, O. R. Schurig, and M. F. Gardner.** AIEE Transactions, volume 49, 1930, page 1102.

2. An Alternating-Current Network Calculator, **H. A. Travers and W. W. Parker.** *The Electric Journal*, May 1930, pages 266–70.

3. Measuring Phase-Angle Quantities, **W. W. Parker.** *The Electric Journal*, May 1933, page 187.

4. Board Earns Eight Times Its Cost, **T. G. LeClair.** *Electrical World*, November 23, 1935, page 2776.

5. TVA Installs A-C Calculator for Detailed System Studies, **V. J. Cissna.** *Electrical South*, October 1938, page 30.

6. Stabilized Amplifier for Measurement Purposes, **H. A. Thompson.** AIEE Transactions, 1938 (July section), page 379.

7. A New A-C Network Analyzer, **H. P. Kuehni and R. C. Lorraine.** AIEE Transactions, volume 57, 1938 (February section), page 67.

8. An Amplifier-Wattmeter Combination for the Accurate Measurement of Watts and Vars, **G. S. Brown and E. F. Cahoon;** discussion by **W. O. Osbon and W. W. Parker.** AIEE Transactions, volume 59, 1939 (November section), page 598.

Line-Type Lightning-Arrester Performance Characteristics

AIEE LIGHTNING ARRESTER SUBCOMMITTEE

THIS report contains the performance data for the line type arresters rated 20 to 73 kv. The characteristics of line type arresters were submitted as a committee report in AIEE Transactions, volume 57, 1938 (November section), pages 661–2. The lightning arrester subcommittee felt that it was desirable to bring the data up to date at this time; also, that the characteristics of the arresters should be made available for discharge currents up to 20,000 amperes, as well as a curve showing the volt-time gap breakdown characteristics.

The data in this report were obtained from the data submitted by the manufacturers for their respective products. The average values in the report are the average of the manufacturer's average values.

The tolerance from average curve was found to be ±25% for arrester breakdown and ±20% for arrester *IR* discharge voltage.

The arresters covered in this report have a sixty cycle spark potential which is at least 1.5 times the arrester maximum permissible line-to-ground rating.

Table I shows the arrester breakdown values taken on a wave rising to breakdown at the rate of 100 kv/ms per 12 kv of arrester rating. For this wave the breakdown occurs at about 0.4 microsecond. The table also includes the *IR* discharge voltage for currents of 1,500, 3,000, 5,000, 10,000, and 20,000 amperes on a 10×20 wave.

The relation between arrester breakdown voltage for the several voltage ratings is shown in figure 1. The tolerances are also shown.

Paper 41-138, recommended by the AIEE committee on protective devices, and presented at the AIEE Pacific Coast convention, Yellowstone National Park, August 27–29, 1941. Manuscript submitted April 20, 1941; made available for preprinting July 2, 1941.

Personnel of the AIEE lightning arrester subcommittee: H. W. Collins, chairman; H. N. Ekvall, F. R. Ford, I. W. Gross, Herman Halperin, H. G. Lloyd, L. R. Ludwig, W. J. Rudge, J. R. McFarlin, A. H. Schirmer, H. K. Sels, L. G. Smith, H. R. Stewart, E. R. Whitehead.

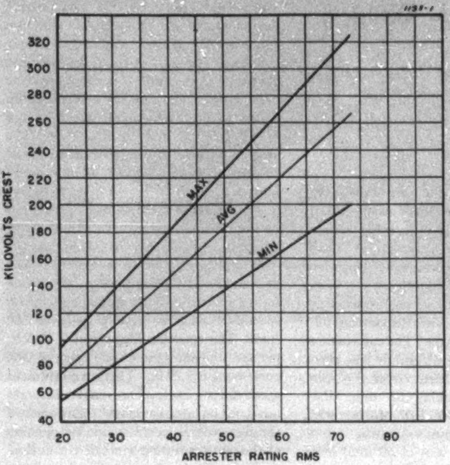

Figure 1. Arrester breakdown voltage at 100 kv per microsecond per 12 kv of rating

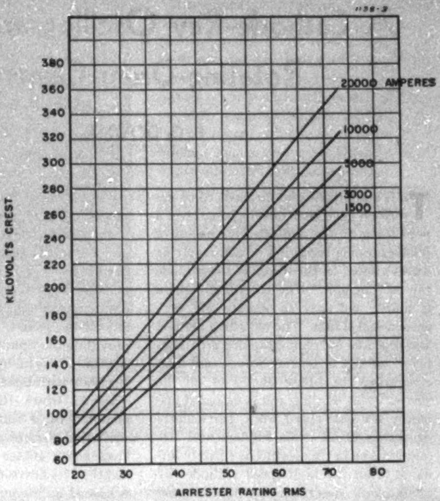

Figure 3 (upper right). Arrester IR discharge

Voltage 10×20
Average values
Tolerance ±20 per cent

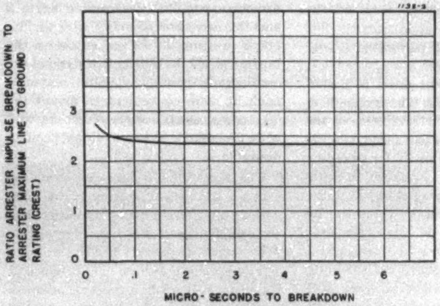

Figure 2. Volt-time impulse-gap breakdown

The arrester volt-time breakdown characteristics are shown in figure 2. The breakdown values are given for times as small as $^1/_4$ microsecond and as great as 6 microseconds.

Arrester average *IR* discharge voltages are shown for all ratings at currents of 1,500, 3,000, 5,000, 10,000, and 20,000 amperes, 10×20 wave.

The data in this report will assist in the evaluation on a volt-time basis of the margin between the arrester performance and the insulation to be protected. Consideration must also be given in such evaluations to other factors including rate of voltage rise, distance (circuit feet) between the arrester and the apparatus, ground resistance, selectivity required.

Performance Characteristics Line-Type Arresters (20–73 Kv)

Arrester Maximum Rating	Arrester Breakdown AIEE Rate			IR Discharge Voltage on 10×20 Current Wave														
				1,500 Amp			3,000 Amp			5,000 Amp			10,000 Amp			20,000 Amp		
	Min	Avg	Max	Min	Avg	Max	Min	Avg	Max	Min	Avg	Max	Min	Avg	Max	Min	Avg	Max
20	56	75	94	57	72	86	62	78	94	66	83	100	73	92	110	81	101	121
25	70	93	116	69	86	103	74	93	111	81	101	121	89	111	133	97	121	145
30	82	110	138	83	103	124	92	115	138	97	121	145	108	135	162	119	149	179
37	102	136	170	102	128	153	111	139	167	119	149	179	131	164	197	145	181	217
40	110	147	181	110	138	165	120	149	183	129	161	193	140	177	213	157	196	235
50	137	183	229	138	172	200	150	187	225	161	202	242	177	222	265	194	243	283
60	165	220	274	168	210	252	181	227	273	193	242	290	217	271	325	238	298	358
73	200	267	333	205	257	308	221	277	333	237	297	355	261	328	392	288	360	432

A Cathode-Ray Oscillograph With Rotating-Drum Camera

E. G. DOWNIE
ASSOCIATE AIEE

THE equipment and methods to be described have been developed to meet a need for a simple and effective means of photographing recurring or non-recurring waves viewed on the screen of the cathode ray oscillograph. The recording camera is simple and easy to operate and very economical of film. The recording method also provides for photographing any number of voltage or current waves, and later comparing the phase angles of all the waves. This oscillograph is designed primarily for slow speed work, particularly on waves of 60 cycle fundamental frequency, and in this feature is entirely different from the high speed cathode ray oscillograph described by Kuehni and Ramo.[1]

The cathode ray tube used is a high vacuum, seven inch screen tube. The screen is of the "fast" type with a highly actinic spot of short persistence. A suitable enclosing sheet steel case and a separately boxed 3,000 volt d-c power supply unit were constructed. When the equipment is used for visual work, sweep voltage is applied to one pair of deflecting plates from an independent sweep circuit.

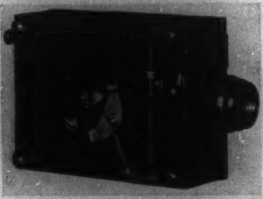

Figure 1. Camera arranged for spring operation of drum

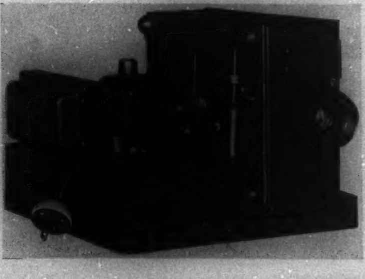

Figure 2. Camera with synchronous motor drive

Permanent records of the screen images are obtained with a revolving drum camera focused on the screen. No sweep voltage is used so that the wave to be photographed appears as a single straight line on the screen; with the moving film providing the time axis. The camera consists of an $8^{1}/_{2}$ in. by $6^{1}/_{2}$ in. by 3 in. sheet steel box with light tight cover, a $5^{1}/_{2}$ in. diameter drum (taking 35 mm. film) mounted on a single ball bearing with shaft and control arm extending out of box; and a Biotar lens of 50 mm. focal length, *F*1.4 aperture. In use, the camera is spaced approximately 15 in. from the tube screen by a light tight tapered sheet steel shield. This spacing is such that the image of the screen approximately fills the usable width of the perforated 35 mm. film.

Rotation of the film drum is accomplished by two methods. The first method, involving only a single revolution of the drum, is used for low film speed, and has been found especially useful for recording 60 cycle wave forms. As shown in figure 1, the drum control lever is locked at the beginning and end of a revolution by spring catches. At the start, the lever is given a short impulse by a spring tensioned lever, when released from the first spring catch. The ball bearing mounted drum has sufficient inertia to make the revolution at practically constant speed, and at the end is stopped by the rubber padded bolt and held by the second spring catch. Drum speed is controlled by changing the impulse lever spring tension and is fairly constant at a fixed setting during successive exposures. The film exposure is made by opening a simple shutter at the start of the revolution and closing it at the end. Transients may be initiated at about one third of the drum revolution by the small toggle switch operated by the drum control lever. The switch is double throw, so it may be used to open or close a circuit, as may be required.

The second method of drum operation is by synchronous motor drive. Continuous rotation is permitted by removing the stop and spring catch assembly. The motor most frequently used was made from a synchronous phonograph motor by special gearing, to give a speed of 550 rpm. The resulting film speed is 13.6 ft. per sec. and the span of one cycle of a 60 cycle wave is 2.72 in. This is a convenient speed for 60 cycle phase angle measurements to be later described. Exposure of the film in this case is made by opening of the shutter during a single revolution, or $^{1}/_{10}$ second. Synchronization of the shutter with the drum is desirable but not absolutely necessary. The motor and camera assembly are shown in figure 2 and the complete assembly with oscillograph in figure 3. In use, of course the tapered shield fits tightly in the end of the oscillograph case. Available materials made it more convenient to mount the $^{1}/_{10}$ second shutter in the shield, instead of on the camera proper, as would be more desirable.

The film drum is removed from the case for convenient loading of the film. The 35 mm. film (either unperforated or the standard perforated motion picture film) is cut to proper length, wrapped tightly around the drum, and fastened at the ends with a small piece of adhesive tape. Loading must of course be done in a dark room. Film used is the most sensitive obtainable, Eastman Super-XX, or Agfa Ultra-Speed Panchromatic; and is developed to high contrast in a "maximum energy" developer such as formula D-82. Such extreme sensitivity is not needed for low frequency waves, but is required when high frequency surges are to be recorded. In practice the intensity of the oscillograph beam is the only factor changed to provide proper exposure at different frequencies. Too high in-

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E. G. Downie is with the General Electric Company, Fort Wayne, Ind.

[1] For all numbered references, see list at end of paper.

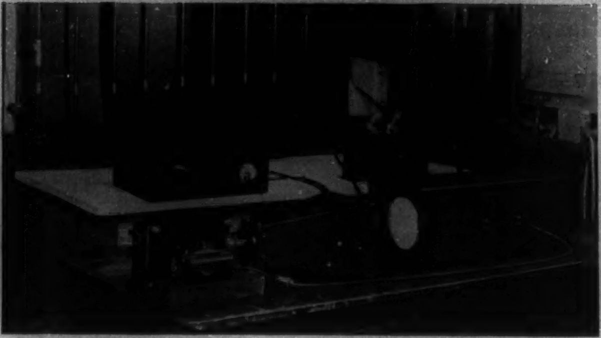


Figure 3. Camera and oscillograph assembly

tensity beam at low frequencies will cause screen retention and blurring on the recording film. The film may conveniently be developed, four lengths at a time, in a small tank such as used for 36 exposure miniature camera film.

The synchronous motor drive provides a uniform time axis, for comparison of successive films. A reference point of phase position on each film then, would also permit comparison of phase angle of waves on successive films. This reference point in each half cycle is obtained by superimposing a small peaked voltage on the wave being studied. This is provided by a coil mounted outside the tube carrying current from a special transformer[8] which produces a small peaked voltage of approximately 5 degrees duration in each half cycle. The magnetic field from the coil deflects the cathode beam in conjunction with the electrostatic field from the internal plates. This transformer must be excited from the same source as the waves being recorded. The voltage shows up on the film as a peak or ripple superimposed on the original wave. It can be easily identified on any except very irregular high frequency waves.

Adjustment of the phase position of the peaked voltage is often desirable to place the peak where it will least interfere and be easiest distinguished on the test wave. Some models of the peaked voltage transformer provide a limited phase shift by a variable resistance in one part of a divided primary winding. A three-phase original voltage source can also be utilized to provide a suitable phase position.

A series of films taken by the above method may be analyzed as shown in figure 4. The films are first arranged in careful register, with respect to the peaked voltage, on a printing frame and a negative film copy made from them. Enlarged prints are then made from this negative, giving black lines on a white background. The vertical line drawn through the peaks is the phase reference, from which measurements may be made on the various waves. Figure 4 shows oscillograms taken of luminous tube transformer secondary voltages, to show the phase relation between no load and various tubing load conditions. This is not conveniently done with a mechanical oscillograph, since only the cathode ray oscillograph, through a capacitance voltage divider, gives a true indication of luminous tube voltages.

In recording an unknown or mixed frequency wave, such as produced by

Figure 4. Phase-angle measurement of luminous tube transformer voltages

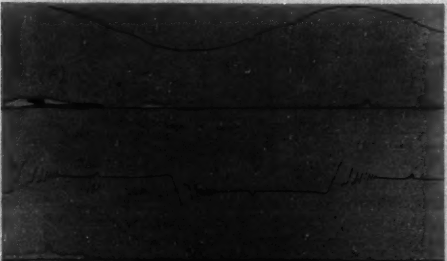
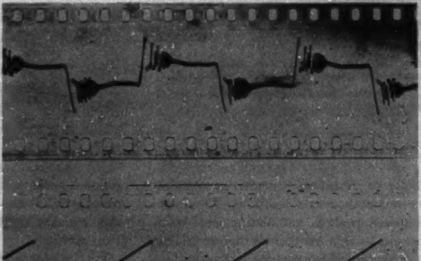


Figure 5. Oscillograms taken with spring operation of drum



High-Voltage D-C Flashover of Solid Insulators in Compressed Nitrogen

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HIGH-voltage apparatus insulated by compressed gas requires the use of solid insulating supports. It is generally recognized that even in a uniform field the flashover voltage of such solid insulation is considerably lower than for the gas-filled gap of equal length. An increase in the allowable gradient along such insulators would in general result in a reduction in the size, weight, and cost of gas-insulated apparatus, and must therefore be considered an important and fundamental objective of high-voltage engineering. It is probable that the flashover strength of solid insulators may be brought closer to that of the gases in which they are immersed, or to their inherent volume breakdown strength, through a better understanding of the influence on insulator flashover of such factors as material, surface condition, corrugations, and gas pressure.

Most of the scant literature on this subject is restricted to relatively low alternating voltages, small samples of solid dielectrics, or to special insulator shapes in non-uniform fields.[1] Reher studied the flashover of plain cylinders of ebonite, paraffin, and porcelain in dry air at pressures up to 15 atmospheres, using alternating voltages up to 140 kv maximum.[2] Goldman and Wul[3] have investigated the flashover of plain ebonite cylinders in nitrogen with alternating voltages up to 125 kv maximum and pressures up to 20 atmospheres. The present paper reports d-c flashover values up to 250 kv across short cylindrical insulators placed in the uniform field between metallic electrodes. These studies were made on both plain and corrugated cylinders of three dissimilar insulating materials immersed in dry nitrogen at gauge pressures up to 400 lbs. per square inch.

Apparatus

The test insulators were mounted between six-inch-diameter steel electrodes supported within a 13-inch-diameter steel pressure chamber. One of these electrodes was fastened to the end of a high-voltage bushing capable of bringing into the tank voltages up to 500 kv; the other, to a movable support which was insulated from ground so that the pre-flashover currents could be measured. The electrode gap was of such shape that a substantially uniform field existed in the central gap area for all spacings up to $1^1/_2$ inches. Windows were provided in the tank so that the appearance of corona and flashover could be observed.

The voltage source was a 500-kv electrostatic generator of the Van de Graaff type. The voltage was measured by a special spiral-line, high-voltage resistor with an estimated error of less than 3 per cent. Dry commercial nitrogen, which is similar in its electrical properties to air and other permanent gases, was used in this work.

Test Pieces

Three types of solid-insulator materials known by the trade names, Lucite, Textolite, and Isolantite, were studied. Lucite is a polymerized methylmethacrylate which has properties of high dielectric strength and resistivity, homogeneity, and machinability, with moderate mechanical strength. Textolite (No. 2029) is a laminated plastic with a paper base bonded with a phenolic compound. This material has a high dielectric strength across laminations and a moderately high resistivity. These two materials have basic differences which influence their performance as insulators, and each may be regarded as representative of a large group of plastics which are useful as solid insulation in high-voltage applications. Lucite is homogeneous in the volume, has the high volume resistivity of 10^{15} ohm cm, and may be given a smooth, nearly optically perfect surface finish; while Textolite is laminated, has moderate sur-

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The Isolantite test pieces were obtained through the courtesy of Isolantite, Inc., of New York, N. Y.

[1] For all numbered references, see list at end of paper.

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noise or vibration, a permanent timing record on the film is desirable. The superimposed peak is not distinguishable on such a wave. Another method of adding the timing record was devised and found to be quite satisfactory. A small hydrogen filled 110 volt incandescent light bulb is enclosed in a shield to emit a small spot of light, and placed at the edge of the tube screen. The filament of the hydrogen filled bulb, operated on 60 cycle alternating current, completely darkens every half cycle and makes a broken trace at the edge of the oscillographic film. A luminous gas bulb, of sufficient intensity to record on the film, could be used as well. A film recording vibration, with timing trace by this method, is shown in figure 5. The other items in figure 5 show a luminous tube voltage wave form and a "saw-tooth" sweep voltage. These were taken with the single revolution method of drum operation.

The oscillograph and camera equipment has been used in regular laboratory testing and developmental work during the past two years. In addition to the above mentioned application, they have been used in analysis of high frequency ripple in output of direct and alternating current generators; measurement of radio frequency surges in both windings of ignition, high potential testing, and luminous tube transformers during high and low side switching; and noise and vibration analysis of motors. Recently the equipment was transported 200 miles by automobile and returned with oscillograms of high potential testing voltages on an assembly line. It is clear that this type of oscillograph does not attempt to replace the mechanical oscillograph, the cathode ray oscillograph with single high speed condenser discharge sweep, or the newly developed multibeam cathode ray tube, but it has been found very useful in a class of work not well covered by any of these.

References

1. A NEW HIGH-SPEED CATHODE-RAY OSCILLOGRAPH, H. P. Kuehni and Simon Ramo. AIEE TRANSACTIONS, volume 56, 1937 (June section), pages 721–8.

2. TRANSFORMERS WITH PEAKED WAVES, O. Kiltie. ELECTRICAL ENGINEERING, volume 51, November 1932, pages 802–04.

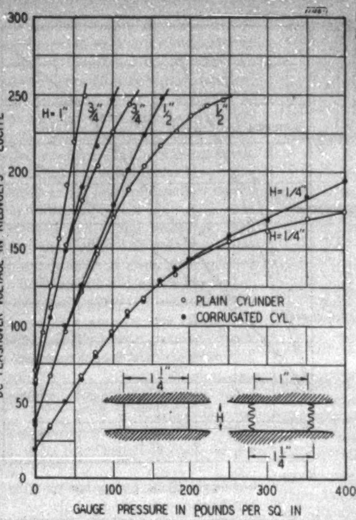

Figure 1 (left). D-c flashover strength of Lucite cylinders immersed in dry nitrogen at high pressures. Uniform field

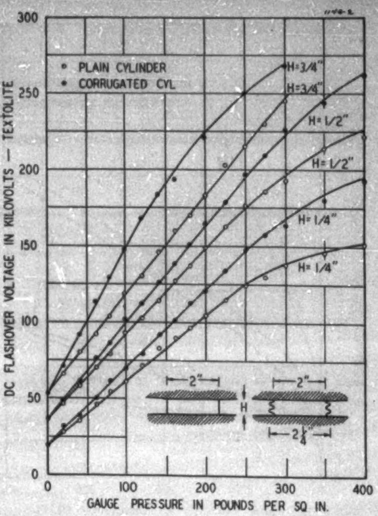

Figure 2(right). D-c flashover strength of Textolite cylinders immersed in dry nitrogen at high pressures. Uniform field

face resistivity and a volume resistivity of about 10^{11} ohm cm, and can readily be given only a moderately smooth surface finish. Isolantite is a porcelain ceramic which has magnesium silicate as its chief ingredient. This material, which may be considered as representative of most electrical porcelains in this application, has high surface resistivity and volume resistivity of about 10^{14} ohm cm, is uniform in the volume, and has the surface smoothness and mechanical properties characteristic of porcelains. The approximate dielectric constant of these materials in the order mentioned above is 4.2, 4.6, and 6.1.

The test insulators were circular in cross section so as to provide axial symmetry, and were $^1/_4$, $^1/_2$, $^3/_4$, and 1 inch in length. The plain cylinders of Textolite and Isolantite were 2 inches in diameter, while those of Lucite were $1^1/_4$ inches in diameter. All of the corrugated pieces had four corrugations per inch, with a 2-inch root diameter for the Textolite and Isolantite, and a 1-inch root diameter for the Lucite. The laminations of the Textolite test pieces were perpendicular to the axis of the cylinders. The cylindrical surfaces of the Isolantite test pieces were glazed.

To ensure good electrical contact between the test insulators and the electrodes, the end surfaces of the former were carefully coated to within $^1/_{32}$ inch of the edges with a conducting graphite mixture. This method was found to be the best of the several means of securing end contact which were tried. To ensure a dry and clean surface condition, the specimens were cleaned with carbon tetrachloride and stored for several days in an atmosphere in which the relative humidity was maintained at less than one percent.

Test Results

The d-c voltage which the insulators could withstand was in every case limited by surface flashover rather than volume breakdown through the material. Figure 1 gives the d-c flashover of plain and corrugated cylinders of Lucite placed in the uniform field between metal electrodes in compressed nitrogen. It is to be noted that with this homogeneous and high-resistivity material the advantage of corrugations becomes considerable only at the higher pressures. The flashover voltages increase linearly with pressure at first, and then more slowly. The flashover values increase nearly directly with the length. Figures 2 and 3 give similar data for Textolite and Isolantite. At the higher pressures Textolite shows a considerably lower flashover voltage than Lucite. For reasons that are discussed below, the influence of corrugations in raising the flashover voltage is pronounced. The flashover values obtained with plain cylinders of Isolantite were normal at atmospheric pressure but increased relatively slowly with pressure. This material was markedly benefited by corrugations.

In figure 4 the flashover strengths of $^1/_2$-inch corrugated cylinders of these three materials between flat electrodes in compressed nitrogen are compared, over the same pressure range, with the breakdown strength of the gap alone. At the lower pressures the flashover strength of corrugated Lucite is nearly the same as that of the gas gap, the departure increasing more rapidly the higher the pressure. At 100 lbs. per square inch the relative flashover strengths of the Lucite, Textolite, and Isolantite cylinders, on the basis of 100 percent strength for the nitrogen gap, were 80 percent, 48 percent, and 40 percent, respectively.

Influence of Surface Irregularities on Flashover

The fact that surface flashover limits the voltage which can be applied to a solid insulator placed between plane parallel electrodes in a compressed gas, may be explained by physical and electrical surface irregularities and by the field-distorting influence of surface-bound space charges. Since solid insulators usually possess dielectric constants in the range from 2 to 6, and are immersed in a compressed gas whose dielectric constant is substantially unity, the effect of physical irregularities—such as a depression—is to distort the otherwise uniform electric field and to increase the electric field intensity on the insulation of the lowest dielectric constant. Figure 5*a*

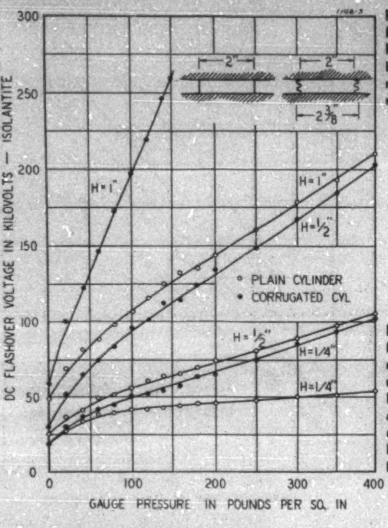

Figure 3 (left). D-c flashover strength of Isolantite cylinders immersed in dry nitrogen at high pressures. Uniform field

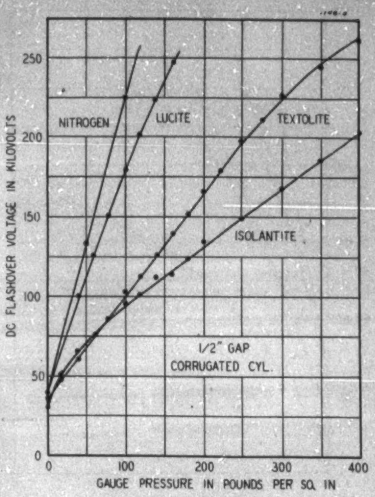

Figure 4 (right). D-c flashover strength of nitrogen and of corrugated cylinders of Lucite, Textolite, and Isolantite immersed in nitrogen at high pressures. Uniform field, 1/2-inch gap

shows the equipotential lines and lines of electric flux in the region of an ideal insulator placed in the uniform electric field between parallel electrodes. Figure 5*b* shows the concentration and distortion of the field caused by a depression in such an insulator. The stress in the region of this depression is nearly *k* times the average stress of the gaseous medium where *k* is the dielectric constant of the solid. Thus, although the average gradient may be substantially less than that required for the breakdown of the gas, the gradient in the region of the irregularity may be sufficient for local ionization. Although the space charge which builds up on the surface of such a defect tends to correct the unfavorable field situation, this local ionized condition of the gas and surface will lead to flashover at a lower voltage than would otherwise be required. This self-healing by ionization in the vicinity of defects in solid insulation is much less effective when alternating potentials are applied, since in this case energy must be expended continuously in the overstressed region. The discussion below is based on applied constant potentials only and should not be assumed to hold in the case of alternating potentials.

Influence of Surface Leakage

Another factor entering into the d-c flashover strength of insulators is the modification of the potential distribution along the insulator surface by leakage. If the surface and volume of the insulator have a uniform leakage resistance, the gradient along the surface is constant and the potential distribution is ideal. Such uniformity of leakage is difficult to obtain, however, particularly when high resistivities such as that of solid insulators are involved. Factors of two in the resistivity of dry, carefully conditioned surfaces of the same insulating material are not uncommon, and the influence of moisture or surface contamination may cause local variations of the resistivity differing from the average by several orders of magnitude. While such nonuniform surface leakage may be modified in some cases by the more uniform leakage through the volume of the insulator, it has the effect of imposing a less favorable distribution of potential along the insulator surface. Local ionization and breakdown of the surrounding gas will originate in the regions of high field intensity, and complete flashover of the insulator will take place at a lower voltage than would be required under uniform leakage conditions.

Effect of Surface-Bound Charges

Again, distortion of the electric field in the vicinity of the surface of an insulator may result from charge accumulations on the surface. This effect should be most pronounced on insulators in which the surface and volume leakage is very small. Such surface-bound charges may be produced by sparking or by corona from an electrode projection. It was observed, in the case of the Isolantite insulators, that at the higher pressures the voltage diminished with repeated flashover to about 50 percent of the normal value, but that the normal voltage was recovered after a lapse of 15 to 30 minutes. This effect was apparently due to the accumulation of surface-bound space charges which distort the electric field. The charge could also be caused to leak off more rapidly by reducing the gas pressure to atmospheric pressure. This effect was not observed with Lucite, although Lucite has even higher resistivity than Isolantite. This is probably because considerable corona preceded flashover in the case of Isolantite, due to the relatively poor end conditions mentioned below. A corresponding space-charge effect has been observed in porcelain high-voltage x-ray tubes. When the inside porcelain wall of such a tube is insufficiently shielded, charge accumulations may occur on the porcelain because of secondary emission. Such an accumulation may displace the electron beam, and this displacement may persist for long periods of time. By introducing air, so as to change from vacuum insulation to a relatively conducting state, this bound charge is quickly removed.

End Effects

A cause of lower flashover voltage can often be found in the improper contact between the solid insulation and the metal electrodes between which it is mounted. It was ascertained that the low flashover voltage of the Isolantite, for example, could be ascribed largely to this condition. As illustrated in figure 5*c*, the ends of these cylinders fitted imperfectly against the flat electrodes, leaving small wedge-shaped gas-filled gaps between electrode and insulator in which ionization readily occurred in the manner described above. This ionization, which may show as a continuous and visible discharge, both lowers the breakdown strength of the gas and distorts the electric field around the insulator by surface-bound charges. In order to confirm this cause of lower flashover, the ends of one of the Isolantite cylinders were reground to secure an improved contact with the electrodes. Though contact was still not ideal, this procedure increased the flashover values by about 20 percent. This difficulty was not observed with either Lucite or Textolite, since these materials could be machined quite accurately, and since their plastic nature insured excellent contact at the electrode-insulator boundaries. The influence of end contact must be regarded as of particular importance because ionization in this region can be supplied with unlimited amounts of power from the electrodes.

Effect of Corrugations on Insulator Flashover

It can be seen from figure 5*d* that the electric field is distorted in the vicinity of corrugations and that the electric field intensity acting on the gas between corrugations is materially higher than average because of the interposition of material of relatively high dielectric constant. It is to be noted, however, that this field distortion is a not-altogether-unfavorable effect, and that the increase in field strength between corrugations is by a factor of about 2 rather than *k*, as in the case of the wall defect of figure 5*b*.*

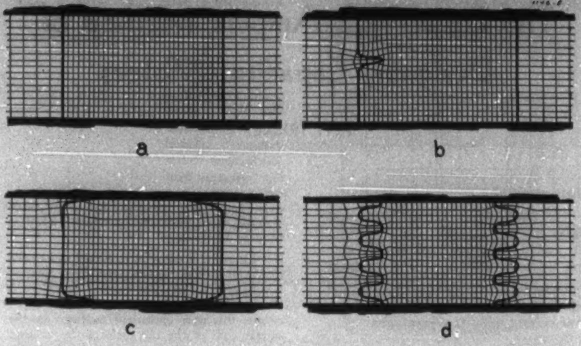

Figure 5. Field pictures of an ideal plain insulator in a uniform field (a), an insulator with a wall defect (b), an insulator with unfavorable end conditions (c), an ideal corrugated insulator (d)

In the case of plain insulators, the imminence of flashover is often revealed by a succession of faint gliding discharges originating on one electrode at the edge of the insulator and proceeding along the insulator surface toward the opposite electrode. On further increase in voltage, these surface gliding discharges increase in brilliance and length, and finally develop into complete and intense flashover. Corrugations, aside from increasing the surface length of the insulator, provide a shielding and localizing effect against such discharges which more than compensates for the field distortion and increased field intensity. The insulator surfaces on the side walls of the corrugations lie in a protected region of low intensity where the disturbing effect of discharges penetrates with difficulty. This is shown by the fact that the final flashover does not in general follow the contour of the corrugated insulator but glides over the tips of the corrugations and leaps across the gas gap between the corrugations.

The net effect of corrugations is to establish longitudinally along the insulator surface regions of alternately high and low field intensity which tend to localize the influence of surface defects, poor end contact, and space charge, and which interrupt the progress of gliding discharges. Corrugations, accordingly, should be expected to play the greatest role in those cases in which most of these conditions exist. The flashover strength of the Isolantite was markedly increased by corrugations, as can be seen from figure 3. In this case, local ionization at the ends caused a surface-bound charge to appear on the nearest corrugation, which reduced the tendency toward further ionization leading to flashover. In the case of Lucite these adverse factors were not present to any considerable extent, so that little advantage from corrugations was obtained until the higher pressures and gradients, at which all influences become more important and critical. It should, however, prove advantageous in general to employ corrugations in all practical applications of solid insulators in compressed gases, since—even with favorable initial conditions—it is probable that imperfections in the insulator surface, non-uniform leakage, or surface-bound charge arising from projections on the electrode may appear in time.

There is considerable tendency in the design of corrugated insulators for use in compressed gas or in high vacuum to have the maximum diameter at the insulator ends. This is intended to avoid the higher gradient which would be found immediately adjacent to the electrode, were the insulator to terminate in the root diameter. It is to be pointed out, however, that very small defects or crevices in the contact will result in local ionizing gradients, and that when this condition exists on the large-diameter ends, the ensuing gliding discharges may proceed from the fault toward the opposite electrode with little opposition. It appears preferable to terminate the insulator in the root diameter, as shown in figure 5*d*, in order that the nearest corrugation may act as a space-charge shield.

Conclusion

The d-c flashover strength of certain solid insulators in compressed gases is less than that of a corresponding gas-

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*If a layer of gas of thickness d_1 and a layer of solid insulation of thickness d_2 and dielectric constant K_2 are arranged in series between parallel electrodes, the electric field intensity acting on the gaseous portion of the gap may be shown to be $G = K_2V + (d_2 + K_2d_1)$ where V is the total applied voltage. If d_1 is small in comparison with d_2, the field intensity acting on the gas approaches K_2 times that which would exist in the gap if it contained gas alone. This approximates the situation in the case of an insulator having a narrow fissure or cleft, as in figure 5*b*, or one which has poor end contact, as in figure 5*c*. When the thickness of the two dissimilar dielectrics are equal, the field intensity on the gaseous gap is $2K_2V/(1+K_2)d$ where $d = d_1 + d_2$ or $2K_2/(1+K_2)$ times that of a completely gas-filled gap. This approximates the situation in the corrugated portion of an insulator, as in figure 5*d*. The field intensity in the gas is in all cases K_2 times greater than in the solid dielectric in series with it. This explanation assumes that the electric field is due to charges on the electrodes alone.

The Equal-Volt-Ampere Method of Designing Capacitor Motors

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Introduction

IN September 1932, the writer presented a method of designing capacitor motors, known as the "balanced" method. Since that time an improvement on that method has been devised which has certain definite advantages.

The Equal-Volt-Ampere Method

The "equal-volt-ampere" method is fundamentally based on the "balanced" method, and consists in departing from the balanced winding in a certain prescribed manner so that the essential features are obtained with a smaller capacitor. The steps are as follows:

1. The motor is calculated as a two phase motor with both phases the same as the main winding of the proposed motor.

2. Using the formulas of the "balanced" method, a balanced winding and capacitor are designed by balancing at the full load point. The performance of this motor is calculated by the balanced method. If the performance is satisfactory, and if the capacitor is not operating up to its rated voltage, an "equal-volt-ampere" winding may be designed.

3. Choosing a value of capacity to be used in the proposed motor (usually the nearest size to give a capacitor voltage slightly less than the rated volts of the capacitor), calculate the new capacitor voltage as the square root of the the ratio of the balance point microfarads to the proposed microfarads.

4. Since the phase angle between the winding voltages is practically 90°, the capacitor winding voltage is obtained as the square root of the difference of the squares of the new capacitor voltage and the line voltage.

5. The winding ratio "K" (which is the ratio of the effective capacitor winding conductors to the effective main winding conductors) is the ratio of the new capacitor winding volts to the line or main winding volts.

6. Calculate the new "K_a" from the ratio $K/K_a = a$ constant, which should be kept the same as in the original balanced motor

$$K_a = \frac{r_{1a}}{K r_{1m}}$$

where

r_{1a} = capacitor winding resistance
r_{1m} = main winding resistance

In other words, the weight of wire in the capacitor phase is kept the same.

7. Choose the capacitor winding turns and wire size to give the new values of K and K_a. The nearest wire size is usually close enough.

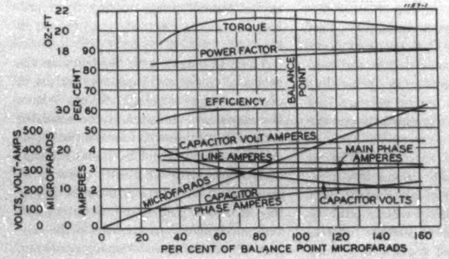

Figure 1. Effect of departure from the balance point while keeping approximately constant capacitor volt-amperes

Results of the Method

The determination of K and K_a completes the design of the "equal-volt-ampere" motor. Its performance will be almost identical to that of the "balanced" motor except for a slight decrease in power factor. This is best illustrated by the example below:

EXAMPLE MOTOR

1/4 hp 1140 rpm 115 volt 60 cycles three speed capacitor motor (speed change obtained by voltage control through regulator)

BALANCE POINT

$K = 1.746 \quad K_a = 2.50 \quad \text{mfd} = 18.85 \quad E_c = 231$

DESIRED CAPACITOR

Mfd = 12.6 (nominal value 12) rated at 330 volts

NEW CAPACITOR VOLTS

$$E_c' = 231\sqrt{\frac{18.85}{12.6}} = 283$$

NEW CAPACITOR WINDING VOLTS

$$E_a' = \sqrt{\overline{283}^2 - \overline{115}^2} = 259$$

NEW WINDING RATIO

$K' = 259/115 = 2.25$

NEW RESISTANCE RATIO

$K_a' = 2.50(2.25/1.746) = 3.22$

where

$$K = \frac{CKw_a}{CKw_m} = \frac{\text{effective series conductors in capacitor phase}}{\text{effective series conductors in main phase}}$$

$K_a = r_{1a}/Kr_{1m}$
r_{1a} = resistance of capacitor winding
r_{1m} = resistance of main winding
E_c = capacitor voltage
E_a = capacitor winding voltage

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[1] For all numbered references, see list at end of paper.

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filled gap or than the puncture strength of the material. It is strongly influenced by surface defects, non-uniform surface leakage, surface-bound space charges, and by end conditions. The flashover strength benefits substantially by increase in the dielectric strength of the gas, and may approach the value of the free-gas gap when favorable field configurations, insulator materials, and proper surface and end conditions are included in the design.[4]

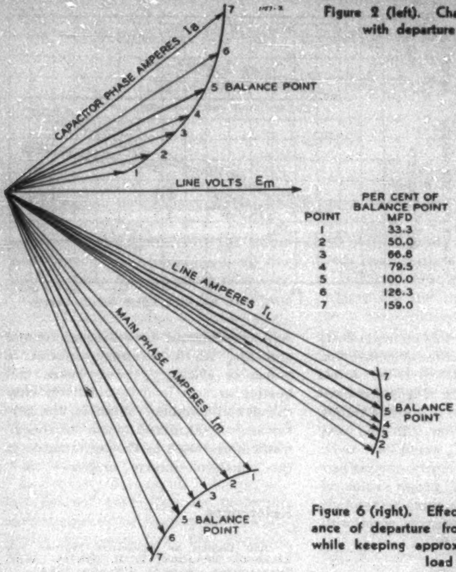
References

1. Durchschlag und Uberschlag in Luft bei Drucken von 1 bis 30 at, C. Reher. *Archiv für Elektrotechnik*, volume 25, 1931, pages 277–98.

2. Die Durchbruchsfeldstärke komprimierter Gase und ihre Verwendung zur Hochspannung-Isolation, A. Palm. *Archiv für Elektrotechnik*, volume 28, 1934, pages 296–302.

3. The Breakdown and Flashover of Solid Dielectrics in Compressed Nitrogen, I. Goldmann and B. Wul. *Technical Physics of the USSR*, volume 3, 1936, pages 519–27.

4. A Compact Pressure-Insulated Electrostatic X-Ray Generator, J. G. Trump and R. J. Van de Graaff. *Physiological Review*, volume 55, 1939, pages 1160–5.


CAPACITOR PHASE AMPERES I_a
7
6
5 BALANCE POINT
4
3
2
1
LINE VOLTS E_m
LINE AMPERES I_L
7
6
5 BALANCE POINT
4
3
2
1
MAIN PHASE AMPERES I_m
1
2
3
4
5 BALANCE POINT
6
7

POINT	PER CENT OF BALANCE POINT MFD
1	33.3
2	50.0
3	66.8
4	79.5
5	100.0
6	126.3
7	159.0

Figure 2 (left). Changes in vector diagram with departure from the balance point

Figure 5 (right). Effect on the speed-torque curve of departure from the balance point while keeping approximately constant full-load capacitor volt-amperes

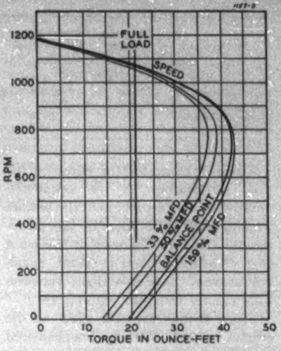
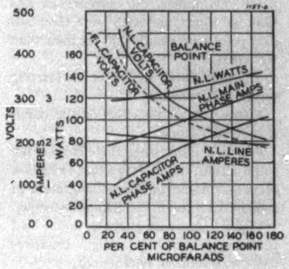


Figure 6 (right). Effect on no-load performance of departure from the balance point while keeping approximately constant full-load capacitor volt-amperes



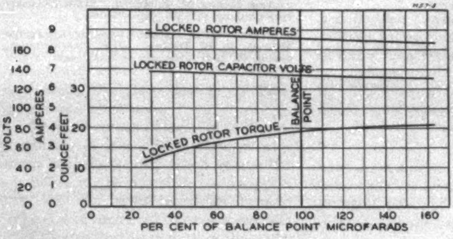
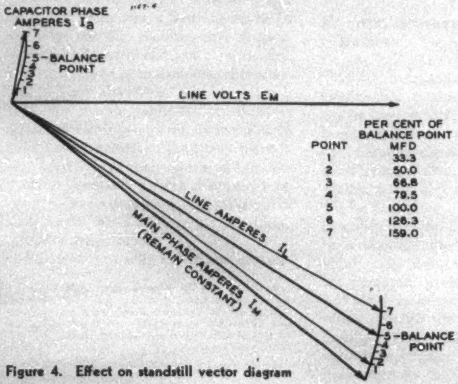
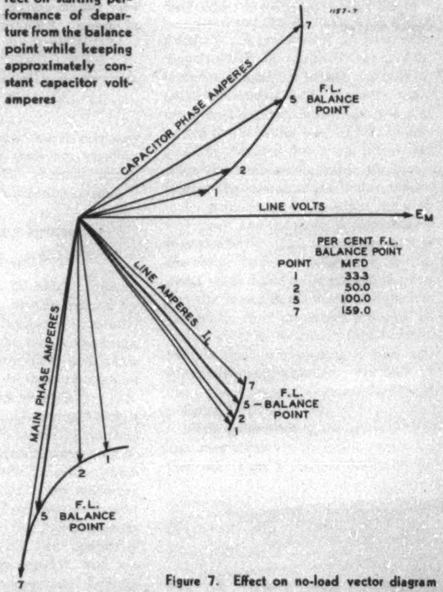


Figure 3 (left). Effect on starting performance of departure from the balance point while keeping approximately constant capacitor volt-amperes


CAPACITOR PHASE AMPERES I_a
7
6
5 - BALANCE POINT
4
3
2
1
LINE VOLTS E_M
LINE AMPERES I_L
MAIN PHASE AMPERES (REMAIN CONSTANT) I_M
7
6
5 - BALANCE POINT
4
3
2
1

POINT	PER CENT OF BALANCE POINT MFD
1	33.3
2	50.0
3	66.8
4	79.5
5	100.0
6	126.3
7	159.0

Figure 4. Effect on standstill vector diagram


CAPACITOR PHASE AMPERES
7
5 F.L. BALANCE POINT
2
1
LINE VOLTS
E_M
LINE AMPERES I_L
7
5 - F.L. BALANCE POINT
2
1
MAIN PHASE AMPERES
1
2
5 F.L. BALANCE POINT
7

POINT	PER CENT F.L. BALANCE POINT MFD
1	33.3
2	50.0
5	100.0
7	159.0

Figure 7. Effect on no-load vector diagram

Performance	Balance Point	Equal-Volt-Ampere Point
Full-load amperes	3.32	3.40
Full-load watts	334	335.2
Full-load rpm	1,070	1,070
Full-load efficiency	60.0	59.7
Full-load power factor	87.5	85.8

Figure 1 illustrates the effect on full load performance as the equal-volt-ampere winding departs from the balance winding with either more or less microfarads. (It should be noted at this point that while the method is called "equal-volt-ampere" there is a slight variation from exactly constant volt-amperes.)

Figure 2 illustrates the effect on the current vectors. With less than balanced point microfarads, the currents are less than 90° apart, and with more than balance point microfarads they are more than 90° apart.

Figure 3 shows the effect on starting torque and current.

Figure 4 shows the effect on the standstill vector diagram.

Figure 5 illustrating the effect on the speed-torque curve, shows that reduction of the microfarads by this method, results in a slight reduction in the maximum torque.

Figure 6 shows the effect on the no load performance which is a slight improvement with reduced microfarads.

Figure 7 shows the effect on the no load vector diagram.

Figure 8. Effect on starting conditions using windings chosen by the equal-volt-ampere method, but using starting microfarads to give maximum starting torque

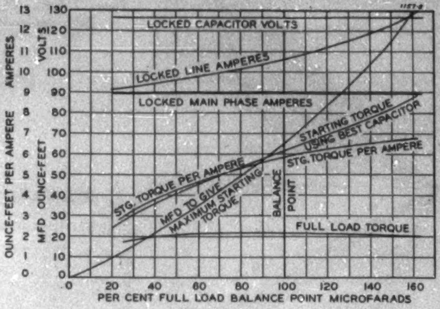


Figure 8 shows how this method affects the possible starting torque of a two value motor. It illustrates definitely that K can not be increased too far if it is necessary to obtain a high value of starting torque on the starting capacitor. In this case, the value of K and K_a would have to be determined by the necessary starting performance. Then the proper value of "equal-volt-ampere" running microfarads would have to be chosen to agree with these values of K and K_a.

Conclusions

The "equal-volt-ampere" method is a very simple and straightforward method of design. A motor is obtained which is well proportioned between performance and cost. All the necessary performance values of efficiency, speed, losses, and heating are obtained by relatively easy calculation formulas. That is, the performance is calculated by the "balanced" method, and used as the performance of the "equal-volt-ampere" motor.

References

1. The Design of Capacitor Motors for Balanced Operation, P. H. Trickey. AIEE Transactions, volume 51, 1932, pages 780–4.

2. The Revolving-Field Theory of the Capacitor Motor, W. J. Morrill. AIEE Transactions, volume 48, 1929, pages 614–29.

3. Performance Calculations on Capacitor Motors, P. H. Trickey. AIEE Transactions, volume 60, 1941 (February section), pages 73–6.

The Inherent Overheating Protection of Single-Phase Motors

C. P. POTTER
FELLOW AIEE

THE problem of controlling and restricting the temperatures of motor windings, is of vital importance to the motor manufacturer, the motor user, and the general public. The motor manufacturer is anxious to have winding temperatures restricted to a safe value, so that his products may have customer acceptance. The motor user is interested in winding temperatures because they affect motor life and maintenance cost. The general public is not conscious of any interest in the matter, but is certainly affected in all cases where excessive temperatures result in fire hazards.

In the early days of the industry, motors did not have adequate protection against overheating, because suitable protective equipment was not available. Motor circuits were fused, but the fuses had to be large enough to accommodate the motor starting current and were, therefore, too large to properly restrict the running current or protect the windings. However, the lack of proper protection was partially offset by the special attention which was given motors by their owners and attendants.

The shortcomings of fuses led to the development of magnetic type relays provided with dash-pots, which retard the operation of the relays to such a point that they can be set to operate close to the full load current of the motor and still have enough time delay to allow the motor to start. These relays were incorporated in motor starting switches and in most cases provide adequate motor protection, but they have one undesirable characteristic. Their operation depends entirely on motor current, while the temperature of motor windings depends upon the room temperature as well as the motor current. A magnetic type relay which will protect a motor located in a room of normal temperature may therefore not protect the same motor when it is operated in a high ambient temperature.

Thermal type relays were the next development and are widely used today in motor starting switches. As the name indicates, these devices are actuated by the heating of a resistor, the temperature of which depends upon both the ambient temperature and the motor current. If the thermal characteristics of the resistor are like those of the motor, and if the relays have the same ambient temperature as the motor, thermal relays installed in starting switches should be entirely adequate for the protection of motor windings. The first of the above conditions is only approximated and the second one cannot always be met, therefore, starting switches provided with thermal relays must also be used with discrimination.

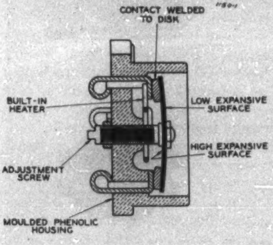


Figure 1. Cross-section of protector with heater coil

Inherent Overheating Protective Devices

Motor overheating is usually due to overloading, but may also be caused by too frequent starting, by foreign material becoming lodged in the motor openings, by installing the motor in a non-ventilated compartment, or in a number of similar ways, and such overheating cannot be prevented by any device which is mounted outside of the motor. Complete protection of motor windings can only be obtained by using thermal devices which are mounted inside the motor enclosure and which make use of resistors or "heater coils" through which the motor current passes. These devices are ordinarily known as "motor protectors" and are designated by the Underwriters' Laboratories as "Inherent Overheating Protective Devices."

Motor protectors have many modifications. When applied on certain types of service they are of the automatic reset type and as the name implies such a protector stops the motor when it reaches a dangerous temperature and automatically starts the motor when its temperature decreases to a safe value. For other types of service, protectors are of the manual reset type which must be reset by an attendant, who presumably inspects the installation to find out what caused the protector to open, before resetting it. Protectors which are used in motors 1 hp and smaller usually break the line current, thus eliminating the necessity of providing the ordinary overcurrent protection required by the Underwriters' Laboratories. Protectors which are used in motors larger than 1 hp do not ordinarily break the line current, but are connected in the pilot circuit of the magnetic switch which starts and stops the motor.

Motor protectors of the automatic reset type contain some sort of a thermostatic device such as a bimetallic disk or strip, which may be mounted adjacent to a heater coil through which the motor current flows. Figure 1 shows a cross-section of a device employing a heater coil, manufactured by the Spencer Thermostat Company. In other types the heater coil is omitted and the bimetallic strip acts as a resistor. A protector of the latter type manufactured by Cutler-Hammer, Inc., is shown in figure 2. In both cases the bimetallic elements carry contacts which are normally closed and which are connected in the motor line. These protectors are mounted inside the motor and when the temperature inside the motor reaches the maximum safe value, the bimetal snaps to its hot position, the contacts open, and the motor is disconnected from the line. When the temperature of the motor decreases sufficiently, the bimetal snaps back to its normal position and the motor is automatically started. Automatic reset protectors are used on motors which drive stokers, fans, blowers, pumps, small refrigerating outfits, etc. Automatic reset protectors are built in many styles, some of which are illustrated in figure 3.

Motor protectors of the manual reset type may employ a bimetallic element or they may have a thermal device of the

Figure 2. Protector without heater coil

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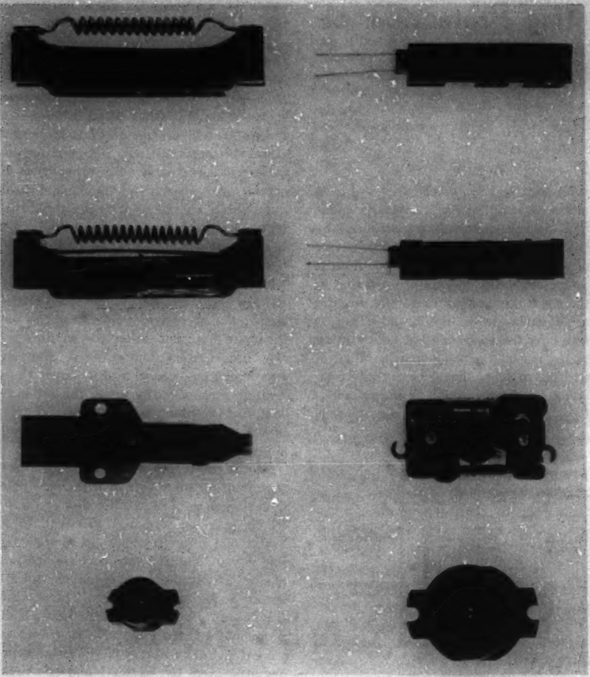

Figure 3. Assortment of automatic reset protectors

"solder pot" variety, mounted adjacent to a heater coil which carries the motor current. Figure 4 shows one of the former manufactured by the Micro Switch Corporation, while figure 5 illustrates a protector of the solder pot type manufactured by Cutler-Hammer, Inc. These protectors are actuated by the temperature inside the motor, but must be reset manually and cannot be permanently reset until the motor has cooled considerably. Manual reset protectors are used principally on motors which operate oil burners and other devices where it is desirable to investigate the cause of motor stoppage before restarting the motor.

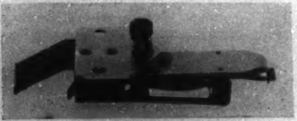

Figure 4. Manual reset protector with bimetallic element

Standard Tests on Motor Protectors

Several temperature tests are required by the Underwriters' Laboratories to determine the suitability of a protector for application on a motor. All temperatures are measured by means of thermocouples which are placed on the surface of the windings and enough thermocouples are used to be sure that the "hottest spot" is located. A series of temperature tests is made with the motor running at about 5% more than normal voltage and with gradually increasing loads, until the protector opens. A single test is made with the rotor stalled at about 5% more than normal voltage, and the protector is allowed to continually trip and reset until the temperature of the windings becomes constant. The object of these tests is to determine the maximum winding temperatures under these abnormal conditions, and restrict them to a safe value. If the temperatures thus taken are found to be reasonable, the application of the protector is considered satisfactory and it is assumed that the motor cannot be damaged no matter how it may be operated. It will be seen that tests have not been mentioned to determine the effect of high ambient temperatures, high or low voltage, high or low frequency, etc., and it would seem to be desirable to make such tests on a representative motor and determine whether motor protectors which have been applied in accordance with accepted rules, actually protect the motors on which they are installed, under all conditions of operation.

Special Tests on Motor Protectors

In order to obtain this information, special tests were made on a $^1/_3$ hp, 1,725 rpm, 110/220 volt, 60 cycle, single phase, repulsion start induction motor equipped with an automatic reset protector, both running and with the rotor locked, at voltages above and below the rated voltage, at frequencies above and below the rated frequency, at various ambient temperatures and with the motor ventilation progressively restricted and finally eliminated. The results of these tests are plotted in figures 6 to 14 inclusive and in order to avoid repetition the term "protector limited temperature" is used to describe the maximum winding temperature which exists when the protector trips. The protector limited temperature

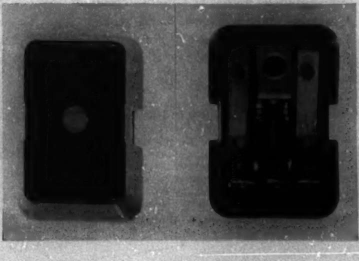

Figure 5. Manual reset protector with "solder-pot"

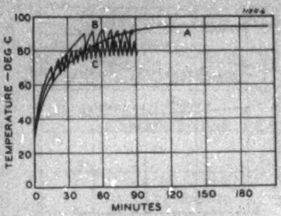

Figure 6. Winding temperature versus time

Normal voltage and frequency. $^{1}/_{3}$-horsepower repulsion-start induction motor with protector

Curve *A*: At maximum current permitted without interruption by protector

Curve *B*: At current 10 per cent greater than that of curve *A*

Curve *C*: At current 20 per cent greater than that of curve *A*

is usually much less than the ultimate temperature which would be reached without a protector.

Curve *A* in figure 6 shows the relation between time and winding temperature for the maximum current which the protector will allow the motor to carry continuously when it is operating at normal voltage and frequency. In this case the protector does not trip but any additional load would cause it to do so. The maximum winding temperature is 94°C, which is much lower than could be allowed. Curve *B* is plotted for a current 10% greater than the maximum current which the protector will carry continuously and curve *C* is plotted for a current 20% greater than the maximum current which the protector will carry continuously. In both these curves the protector trips and resets several times before the winding temperature reaches a constant value, and in both cases the protector limited temperature is less than the ultimate value for curve *A*.

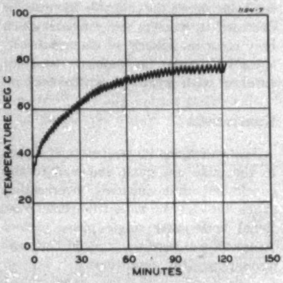

Figure 7. Winding temperature versus time

Rotor locked. Normal voltage and frequency. $^{1}/_{3}$-horsepower repulsion-start induction motor with protector

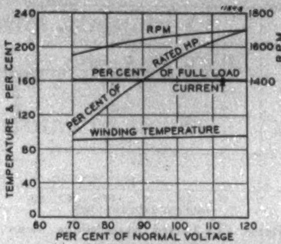

Figure 8. Motor characteristics versus voltage at maximum loads permitted by protector

Normal frequency. 28 degrees ambient. $^{1}/_{3}$-horsepower repulsion-start induction motor

Figure 7 shows the relation between time and protector limited temperature of the $^{1}/_{3}$ hp repulsion start induction motor at normal voltage and frequency with the rotor stalled. This is an abnormal condition, but is one which might exist in case of a mechanical defect in the motor or the machine which it drives. The protector trips and resets many times before the winding temperature reaches a constant value, which, however, is considerably less than for the running condition and is well within safe limits.

Tests were made with the motor operating at voltages above and below normal, to determine the effect of voltage on the winding temperatures permitted by the protector, and the results are shown in figure 8. It is interesting to note that under these conditions the protector is essentially a constant current device and allows the same current to flow at voltages from 70 to 120% of normal voltage. The protector limited temperatures at 70% voltage are slightly less, and at 120% voltage are slightly greater than the values shown in figure 6, but for all practical purposes the protector limited temperatures are not affected by the voltage. Motor loads and speeds are also plotted and as might be expected, they change quite rapidly with the voltage. It will be seen that if this motor is driving a compressor or pump and is considerably overloaded, it will be unable to carry the load at the lower voltages, because the protector will trip and disconnect the motor from the supply circuit.

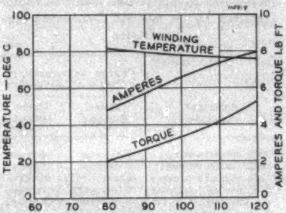

Figure 9. Locked characteristics versus voltage

Normal frequency. 31 degrees ambient. $^{1}/_{3}$-horsepower repulsion-start induction motor with protector

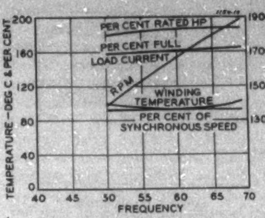

Figure 10. Motor characteristics versus frequency at maximum loads permitted by protector

Normal voltage. 28 degrees ambient. $^{1}/_{3}$-horsepower repulsion-start induction motor

Locked tests were also made at various voltages and the results are plotted in figure 9. In this case the locked current varies at about the same rate as the voltage, but the protector limited temperatures change very little as the voltage varies. This difference in protector operation between running and locked condition is due to the fact that when the motor is locked, the protector is essentially a constant temperature device and its operation depends almost entirely on the temperature of the heater coil which is adjacent to the bi-metal. Since the heater coil temperature and the winding temperature both depend on time and motor current, it logically follows that

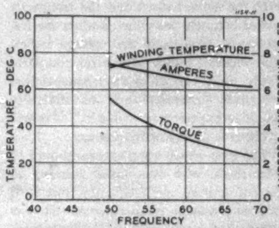

Figure 11. Locked characteristics versus frequency

Normal voltage. 31 degrees ambient. $^{1}/_{3}$-horsepower repulsion-start induction motor with protector

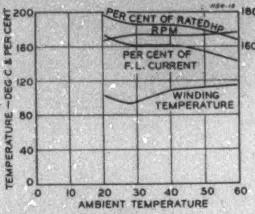

Figure 12. Motor characteristics versus ambient temperature at maximum loads permitted by protector

Normal voltage and frequency. $^1/_3$-horsepower repulsion-start induction motor

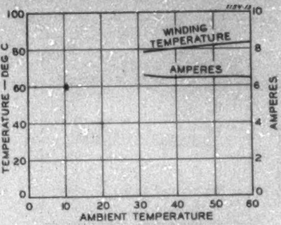

Figure 13. Locked characteristics versus ambient temperature

Normal voltage and frequency. $^1/_3$-horsepower repulsion-start induction motor with protector

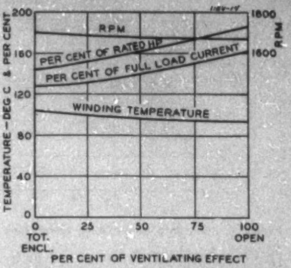

Figure 14. Motor characteristics versus ventilating effect at maximum loads permitted by protector

Normal voltage and frequency. 28 degrees ambient. $^1/_3$-horsepower induction motor

while the time required for the protector to trip a locked motor will vary with the current, the temperatures of the protector and the motor windings when the protector trips are not materially affected by the motor current, and, therefore, do not depend on the voltage applied to the motor terminals.

Figure 10 shows the effect of variations in frequency on the winding temperatures permitted by the protector, while the motor is running. The protector limited temperatures at high and low frequency are only slightly greater than they are at normal frequency and the motor protector takes care of frequency variations while the motor is running, quite satisfactorily. While this motor will carry the same horsepower load at a wide range in frequencies, it obviously cannot operate at frequencies much greater than the normal value because at the higher frequencies, the speed, horsepower, and motor current also increase and the protector will not be able to carry the increased current.

Figure 11 shows the effect of variations in frequency on the protector limited temperatures of the motor with the rotor locked. The operation of the protector with the rotor locked and the frequency varied is much the same as when the frequency is fixed and the voltage varies. The winding temperatures for comparatively wide variations in frequency are practically constant and no difficulty should be experienced with a locked motor at either high or low frequency.

The tests which are plotted in figure 12 were made with the motor running at normal voltage and frequency, but with various ambient temperatures. It was found that with ambient temperatures between 30 and 40°C the protector trips at a fixed value of current and the winding temperature increases at the same rate as the ambient. Above 40°C the protector trips at a smaller value of current and, therefore, the winding temperature does not increase as fast as the ambient. Below 30°C the protector trips at a higher value of current and the winding temperature increases in spite of the decreased ambient. It is evident that the protector limited temperature may increase considerably when the motor is running in a high ambient temperature.

Locked tests at various ambient temperatures were made and the results plotted in figure 13. In this case the winding temperatures do not change materially with the ambient, indicating that the protector trips when the temperature of the bi-metal reaches a definite value, and it makes little difference whether this temperature is produced by the heat of the motor or the surrounding air.

When open type motors are installed in locations where excessive amounts of foreign materials are present, the ventilating openings may become partially or even completely obstructed, and the motor may have an excessive temperature rise. Tests were made to determine whether a motor protector will take care of such a condition and the results are plotted in figure 14. With all the openings in the motor frame enclosed, the protector limited temperature increases 10° and the load which the motor can carry without tripping the protector is reduced 22%. It is unlikely that an open type motor will ever be completely enclosed either accidentally or deliberately; therefore, the probable increase in temperature resulting from foreign materials will be considerably less than 10°.

Variations in Winding Temperatures of a $^1/_3$-Horsepower Repulsion-Start Induction Motor Protected by an Inherent Overheating Protive Device

Abnormal Condition	Change in Temperature (Degrees Centigrade)	
	Motor Running	Motor Locked
120% normal voltage	2 increase	2 decrease
80% normal voltage	2 decrease	4 increase
117% normal frequency	Inoperative	1 decrease
83% normal frequency	6 increase	6 decrease
60° ambient	6 increase	3 increase
All ventilation eliminated	10 increase	No change

Summary

Motor protectors are selected and applied so that they restrict the motor winding temperature to specified values at normal voltage and frequency and a room temperature of 40°C. The accompanying table shows the probable increase (or decrease) in winding temperatures which may occur on account of abnormal voltages, frequencies, or ambient temperatures, or with restricted ventilation.

Conclusions

The variations in temperature shown in the table are small and lead to the conclusion that inherent overheating protective devices mounted inside fractional horsepower single phase motors completely protect them against all abnormal operating conditions.

Criteria for Neutral Stability of Wye-Grounded-Primary Broken-Delta-Secondary Transformer Circuits

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Synopsis: This paper presents the results of an investigation made to determine and evaluate the effect of the factors affecting neutral instability of Y-grounded potential transformer circuits. The method used employs the medium of miniature system representation. The results therefore include the effect of saturation. The effects of the shape of the saturation curve, normal operating voltage, resistance of the winding, system capacitance, and impedance in the broken Δ secondary or across each phase are illustrated. Regions of instability are shown which should be avoided in applying Y-connected potential transformers as ground fault detectors. The analysis shows by means of oscillograms the conditions under which incorrect indications of faults may be obtained.

GROUNDED-NEUTRAL, Y-connected potential transformers with Y- or broken Δ-connected secondaries have been applied to three-phase circuits, the neutrals of which are temporarily or permanently ungrounded. The broken Δ secondary connection affords a means of ground fault detection, the voltage across the break in the Δ under such fault conditions being used to operate relays or otherwise indicate the presence of a ground fault.

In some cases, experience has shown that such a circuit is unstable. The phenomena involved have been described for limited ranges of circuit parameters.[1,2] One oscillographic investigation, however, describes the various conditions of instability in some detail.[3]

More recently other cases of difficulty with this type of circuit have been encountered. While correct indications of true faults were always obtained, indications of faults were sometimes obtained when no faults actually existed. These false indications sometimes resulted in higher than normal fault indicating voltages appearing across the break in the Δ secondary. This phenomenon was traced to the inherent instability of a circuit of this type when the amount of capacitive reactance to ground and the magnetizing reactance of the transformers involved are in certain relative proportions. As a consequence, a comprehensive analysis of the circuit was undertaken, using the miniature system transient analyzer,[4] which disclosed the regions of instability and methods of eliminating such instability. This paper presents the results of that investigation.

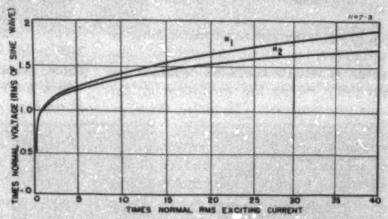

Figure 2. Transformer saturation curves used in the miniature system

1—Typical saturation curve

2—Curve having more saturation than curve 1

Basis of Study

The miniature system shown in figure 1 was selected as a basis for the study. The reactance of the sine wave generator was negligible compared with the external miniature system impedances. Thus balanced sinusoidal line-to-line voltages were applied at the transformer terminals. Consequently the phenomena were independent of positive sequence capacitance so that only zero sequence capacitive reactance, x_{co}, was involved.

Transformers used were single phase and their saturation characteristic is shown in figure 2. Curve #1 is representative of most transformers, and therefore was used throughout the study except for one case in which the #2 curve was used to indicate the effect of more abrupt saturation. The value of x_m is the ratio of normal line-to-neutral rms exciting volts to normal phase rms exciting amperes at normal frequency as defined by the saturation curves. The circuit can be completely defined by the ratio of x_{co}/x_m when the saturation curve used and the normal operating voltage are specified.

Provision was made to control the excitation of the miniature system sine wave set so that system voltage could be varied. It was not deemed practical to operate at voltages higher than normal, as indicated on the saturation curves, since in the system studied, a line-to-ground fault would result in extremely high magnetizing currents in the two unfaulted phases.

The study was made with and without load impedance in the break in the Δ. This is equivalent to putting load impedance across each phase in the case of Y-connected secondaries, the total ohmic resistance being the same in either case. Various impedances were used as burdens to determine the effect of impedance burden magnitude and power factor, both lagging and leading.

In practically all cases studied, the abnormal conditions were obtained by suddenly energizing the miniature system

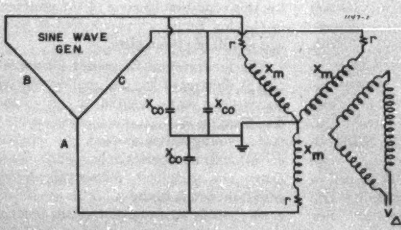

Figure 1. Miniature system studied

x_{co}—Zero sequence capacitive reactance
r—D-c resistance of transformer winding
x_m—Ratio of rms sinusoidal leg volts to rms phase current at a point on the saturation curve corresponding to $E=1.0$ in figure 2
V_Δ Transformer turn ratio 1:1

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1. For all numbered references, see list at end of paper.

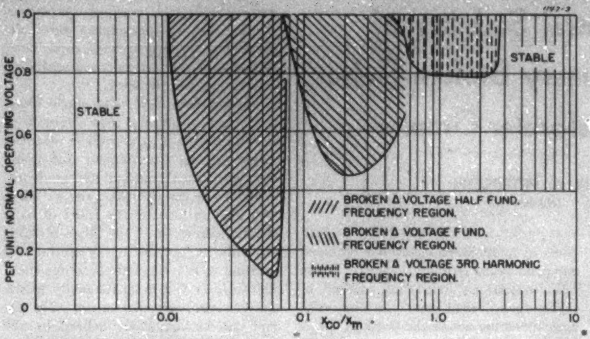


Figure 3. Regions of possible instability of the circuit of figure 1 with transformer saturation corresponding to curve 1 of figure 2

$r/x_m = 0.0003$ for the transformer

r—D-c resistance of the winding

x_m—Ratio of rms sinusoidal leg volts to rms phase current at a point on the saturation curve corresponding to $E = 1.0$ in figure 2

or by applying and removing a fault to ground on one phase, although in some cases, instability could be obtained by gradually raising the excitation of the sine wave generator. However, the manner of obtaining the abnormal circuit conditions, as long as it is reasonable, is of secondary importance relative to the question as to whether abnormal behavior of the circuit is possible or not for given constants. The study is primarily concerned with answering this question.

It is of interest to point out in this connection that regions of possible abnormality may overlap with regions of possible normal circuit behavior. In other words, in regions of abnormal circuit behavior it is sometimes possible for the circuit to behave normally also, but it is never possible in a region designated as stable to have an abnormal circuit condition of the type under discussion in this paper.

It should also be pointed out that the regions of instability are regions where the abnormal conditions persist in the steady-state condition. It is possible for relatively long-time transients to exist near the border line in the stable region. These will die out eventually, however, and normal conditions will always be restored ultimately.

Discussion of Results

For a particular set of transformer characteristics the regions of neutral instability of the system shown in figure 1 are defined in terms of the ratio of system parameters x_{co}/x_m and of the system voltage E where E is the per unit system operating voltage. $E=1$ corresponds to system operating voltage of $E=1$ on the saturation curves of figure 2. In the application of potential transformers as a means of ground fault detection, it is customary to select a transformer rated line-to-line system voltage and operate it at system leg voltage; that is $E=0.58$. In some instances, however, circuits for which $E=1.0$ may be encountered if in a normally grounded system the Y grounded transformer bank is temporarily connected to an otherwise ungrounded circuit, or if single-phase transformers are connected line-to-neutral in a three-phase system which has an open circuit in the neutral at the system grounding point.

The data of figure 3 were obtained using a low-loss transformer with typical saturation characteristics, curve 1, of figure 2. The regions outlined are those in which instability may occur as the result of a shock, such as a line-to-ground fault applied and removed. Stable operation may be obtained within portions of the unstable regions if system voltage is gradually applied or if the Δ secondary is closed until normal steady-state conditions exist.

At each test value of x_{co}/x_m the system voltage was raised to a value corresponding to $E=1.0$. If the application and removal of a line-to-ground fault created an unstable condition as indicated by the presence of an abnormal voltage across the break in the Δ, the system voltage was then reduced to the boundary value at which the unstable Δ voltage collapsed.

As indicated on figure 3 neutral instability for this case may occur in one of three distinct modes, depending on the values of x_{co}/x_m and E. For values of x_{co}/x_m less than 0.01, no instability can occur for values of E less than 1.0. For values of x_{co}/x_m between 0.01 and 0.07, corresponding to relatively large values of zero sequence capacitance, the unstable voltage appearing across the broken delta is approximately of one-half normal frequency. Intermediate values of x_{co}/x_m (between 0.07 and 0.55) give rise to a fundamental frequency Δ voltage, and large values of x_{co}/x_m (between 0.55 and 2.8) give rise to a third harmonic Δ voltage.

The first of these three abnormal conditions is illustrated by the oscillograms in figure 4*a*. For the case $x_{co}/x_m = 0.026$ and $E=0.58$, oscillogram 76-10 shows the broken Δ voltage V_Δ with a single line-to-ground fault on phase *a*. This, of course, is of fundamental frequency and approximately three times normal leg voltage. Oscillogram 77-5 shows the transient V_Δ to the same time and amplitude scale following the removal of the fault on phase *a* at a normal current zero without any subsequent restriking in the fault. It will be observed that this transient voltage is approximately half fundamental frequency. Oscillogram #77-6 shows, with the same calibration, the ultimate steady-state voltage appearing across the break in the Δ. It can be seen that this also is approximately one-half fundamental frequency, of very good wave form, and essentially of the same amplitude as though a line-to-ground fault were on the system.

This phenomenon is referred to as being essentially of $^1/_2$ frequency. Actually it differs from one-half fundamental frequency by a small amount such that line-to-neutral voltages vary with time at a very low beat frequency. Oscillogram 77-7 shows the transient recovery voltage on phase *a*. 77-11, 12, and 13 show the steady-state line-to-neutral voltages on phases *a*, *b*, and *c*, respectively. Oscillograms 77-14, 15, and 16 show the same three respective voltages at a later time to illustrate how slowly the voltages to ground are shifting, due to the "slip" frequency involved giving rise to a beat frequency component of voltage. Phase voltages reach a maximum of two times normal line-to-neutral crest for this case. Transformer currents may be very high for this condition because of the presence of this low frequency component of voltage as will be shown later.

The fundamental frequency phenomenon is illustrated by the oscillograms in figure 4*b* for $x_{co}/x_m = 0.123$ and $E=0.58$. Oscillograms 32-1, 2, and 3, show steady-state voltages to ground on phases *a*, *b*, and *c*, respectively, while 32-8 shows the sustained voltage appearing across the break in the Δ. This condition is very nearly the same as if a fault existed on phase *c* to ground. The voltage on phase *c* is very low—in fact, the fundamental frequency component appears to be practically zero. Voltages to ground on phases *a* and *b* are essentially line-to-line voltages of fundamental frequency, and the broken Δ voltage gives every indication of a fault without there being any fault on the system. The presence of subharmonics of one-half and of one-ninth frequency are evident in the oscillograms.

The third harmonic phenomenon is illustrated in figure 4*c* for $x_{co}/x_m=0.594$ and $E=1.0$. Oscillograms 32-9, 10, and 11 show voltages to ground (one cycle of fundamental frequency) on phases *a*, *b*, and *c*, respectively, while 32-16 shows the voltage appearing across the break in the Δ. The broken Δ voltage is of third harmonic frequency with good wave form and of higher magnitude than that obtained for a fault on the system. Voltages to ground on all three phases are the same in magnitude (about four times normal line-to-neutral crest). There are no subharmonics present in this case.

The normal condition for the same system ($x_{co}/x_m=0.594$) and operating voltage ($E=1.0$) is illustrated by the oscillograms in figure 4*d*. While the system constants are identical with those for the case discussed in the preceding paragraph

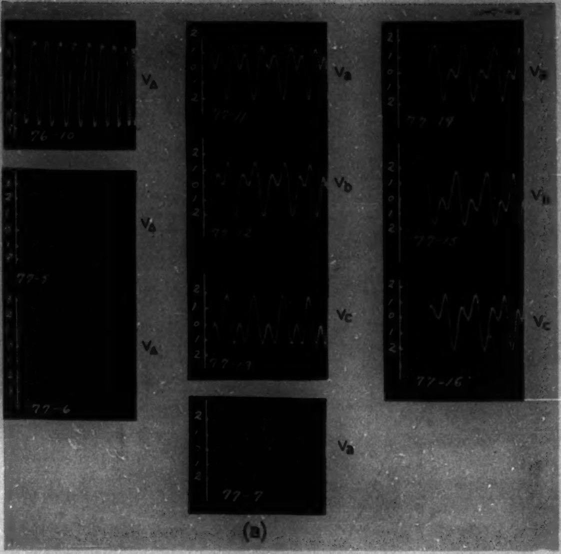
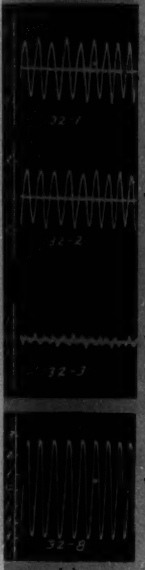


Figure 4a (upper right). Oscillograms illustrating the abnormal condition giving rise to one-half normal frequency broken Δ voltages and balanced phase voltages-to-ground

$x_{co}/x_m=0.026$, $E=0.58$

Note: All voltage calibrations are times normal line-to-neutral crest voltage on the system side of the transformer

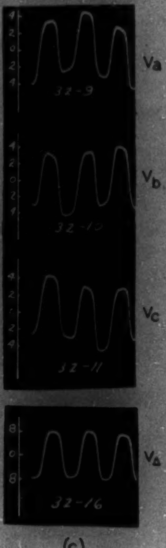


(b)

Figure 4b. Oscillograms illustrating the abnormal condition giving rise to fundamental frequency broken Δ voltages and unbalanced phase voltages-to-ground

$x_{co}/x_m=0.123$, $E=0.58$

Note: All voltage calibrations are times normal line-to-neutral crest voltage on the system side of the transformer

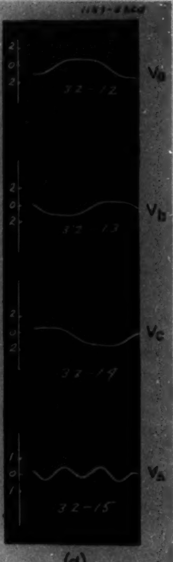


(c)

Figure 4c. Oscillograms illustrating the abnormal condition giving rise to third harmonic broken Δ voltages and balanced phase voltages-to-ground

$x_{co}/x_m=0.594$, $E=1.0$

Note: All voltage calibrations are times normal line-to-neutral crest voltage on the system side of the transformer



(d)

Figure 4d. Oscillograms illustrating the normal condition of some third harmonic voltage across the Δ break with balanced voltages-to-ground

$x_{co}/x_m=0.594$, $E=1.0$

Note: All voltage calibrations are times normal line-to-neutral crest voltage on the system side of the transformer the voltage conditions are quite different, corresponding to "normal" behavior of the circuit. As shown previously, circuits involving saturable impedance elements may give rise to voltage and current conditions which are not single valued functions of the circuit parameters.[5] For this case ($E=1.0$ and $x_{co}/x_m=0.594$), as in many others which could be used as illustrations, either of these conditions may be obtained, the one "abnormal" and the other "normal." Oscillograms 32-12 to -14, inclusive, show the balanced voltages-to-ground on phases a, b, and c, respectively, while 32-15 shows the broken Δ voltage. A third harmonic component is in evidence in this case also, but it is only a very small fraction of that present for the abnormal condition illustrated in oscillograms 32-9 to -11, inclusive, and 32-16. It is also much lower in magnitude than that obtained for the condition of a fault on the system.

Dividing lines between these three distinct modes of instability are not very clear-cut in all cases. For example, in the vicinity of $x_{co}/x_m=0.01$ but in the unstable region, $^1/_4$ frequency components of voltage were in evidence, although the $^1/_2$ frequency components predominated, other subharmonics were also present, sometimes of very low frequency, in this portion of the unstable region.

In the vicinity of $x_{co}/x_m=0.08$, the unstable phenomenon was mainly a combination of fundamental frequency and $^1/_2$ fundamental frequency. The dividing line might be considered as an indication of where for increasing values of x_{co}/x_m the phenomenon changes gradually from something predominantly of $^1/_2$ fre-

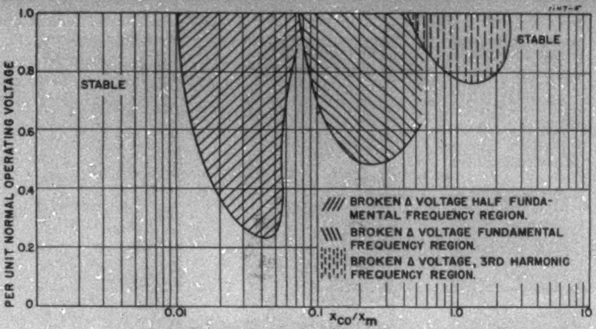


Figure 5. Regions of possible instability of the circuit of figure 1 with transformer saturation corresponding to curve 1 of figure 2

$r/x_m=0.0007$ for the transformer

r—D-c resistance of the winding

x_m—Ratio of rms sinusoidal leg volts to rms phase current at a point on the saturation curve corresponding to $E=1.0$ in figure 2

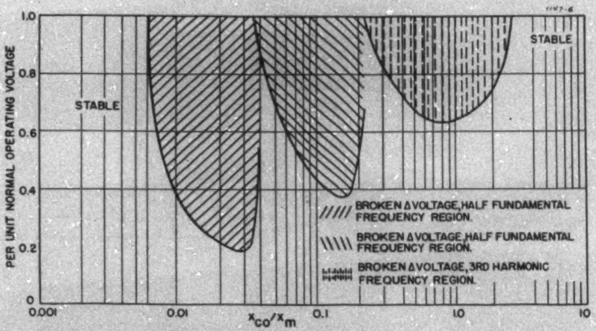


Figure 6. Regions of possible instability of the circuit of figure 1 with transformer saturation corresponding to curve 2 of figure 2

$r/x_m=0.0003$ for the transformer

r—D-c resistance of the winding

x_m—Ratio of rms sinusoidal leg volts to rms phase current at a point on the saturation curve corresponding to $E=1.0$ in figure

Figure 7 (below). Abnormal voltage and current conditions obtained for the circuit of figure 1

Transformer saturation corresponding to curve 1 of figure 2

$E=1/\sqrt{3}=0.58$ $r/x_m=0.0003$

——— Voltage - - - - - - - Transformer current

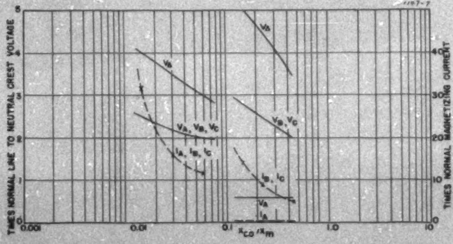
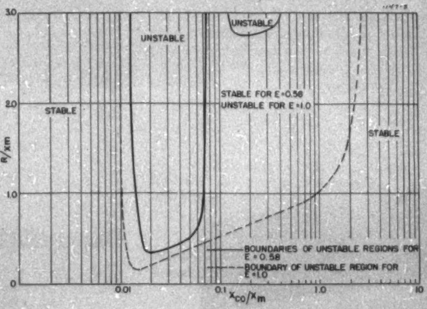


Figure 8. Effect of resistance burden R in the broken Δ on regions of instability

Saturation curve of figure 1 $r/x_m=0.0003$

Note: The total amount of resistance in the Δ is three times the amount of R indicated by the curves since R/x_m is on a per leg basis



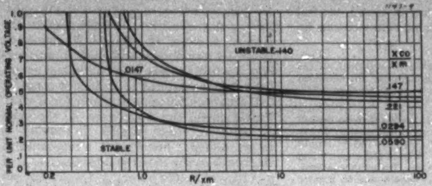


Figure 9. Illustration of the relationship between operating voltage and resistance burden to obtain stability for various values of capacitive reactance

quency to something of predominantly fundamental frequency.

In the vicinity of $x_{co}/x_m=0.5$, the phenomena included a component of approximately twice normal frequency. This was observed only for a very limited range of values of x_{co}/x_m, and the other abnormal conditions occurring over greater ranges of this ratio, were considered of far greater importance.

These facts are pointed out merely to indicate the physical consistency of the information plotted in figure 3. The ratio x_{co}/x_m is proportional to the square of the natural frequency of the circuit studied. For very small values of this ratio, the circuit is always stable. As the ratio is increased beyond a certain value instability is encountered and the unstable phenomena are characterized in succession by voltage components of $^1/_4$, $^1/_2$, 1, 2, and 3 times normal frequency. Thus the predominant frequency of the phenomena is seen to increase consistently with the natural frequency of the circuit.

The effect of increased transformer resistance is shown in figure 5, i.e., greater than that shown in figure 3. It is of importance to note that the regions of instability are only very slightly affected by resistance over a practical range. This fact makes the results of the investigation much more widely applicable.

The results shown in figure 6 were obtained for the saturation curve #2 of figure 2. This curve is somewhat steeper than that likely to be encountered in most actual cases, but the results are of interest in that they show how the unstable regions are broadened and shifted so as to extend the range of instability to lower values of the ratio x_{co}/x_m. This effect is consistent with other phenomena observed in connection with circuit analysis involving saturable impedance elements.[5] The effect of saturation is to broaden the range of abnormality, the broadening effect increasing with increasing abruptness of saturation.

For line-to-line voltage rated potential transformers connected line-to-neutral $E=0.58$ and the regions of instability given in figure 3 are $0.013<x_{co}/x_m<0.075$ and $0.125<x_{co}/x_m<0.45$. Crest values of broken delta voltage, line-to-ground voltage, and of transformer current that may be obtained within these regions are given in figure 7. In the region corresponding to the fundamental frequency broken Δ voltage the voltage-to-ground of one phase is of small magnitude. The voltages-to-ground on the other two phases are approximately equal. Thus line voltages-to-ground as well as the broken Δ voltage are essentially the same as if a fault were on the system.

In the region corresponding to the one-half frequency broken Δ voltage condition, the voltages-to-ground are the same in magnitude on all three phases. However, an interval of several seconds must be considered for this to be true because of the slow "slip" frequency involved as previously described.

The transformer current values shown in figure 7 are of interest also. In the region of one-half frequency phenomena these currents may reach values of 35 times normal magnetizing current in the vicinity of $x_{co}/x_m=0.015$. These currents are the same in magnitude in all three phases. In this range it might be possible to blow potential transformer fuses under such abnormal conditions. In the region of fundamental frequency phenomena the currents are smaller in general, and furthermore, the current in one phase is lower than normal. It is a significant fact that while the voltage conditions in this range of x_{co}/x_m approximate those obtained during a single line-to-ground fault, the currents flowing in the Y winding of the potential transformer may be 10 or 15 times as high as those obtained during a single line-to-ground fault condition, depending upon the ratio of x_{co}/x_m.

Figure 8 shows the results of inserting resistance, R, in the break in the Δ secondary. The curves show that the heaviest burden (lowest resistance) to insure stable operation is required when x_{co}/x_m is about 0.02 or slightly less depending upon the normal system operating voltage. At the lowest point, $R/x_m=0.15$ for operation at $E=1.0$ and $R/x_m=0.34$ for operation at $E=0.58$. These values are on a basis of resistance per leg in per unit of magnetizing impedance per leg for $E=1.0$ on the saturation curves of figure 2. Therefore the curve gives the resistance required per phase for Y-connected secondaries. The value of impedance required across the break in the case of broken Δ secondaries as in figure 1 is three times the quantity R/x_m of figure 8.

The effect of normal operating voltage E on the R/x_m ratio required for stabilization is shown in figure 9. It is of importance to note that as E decreases, the ratio of R/x_m required for stability increases. The values of x_{co}/x_m used in preparing these curves were purposely selected in the range requiring the lowest ratios of R/x_m as indicated in figure 8.

The effect of an inductive reactance burden x_L (power factor=0.2) is indicated in figure 10. A lower ratio of x_L/x_m is required for stabilization than of R/x_m as discussed in the preceding paragraph. The ratio of x_L/x_m at its lowest point is 0.04 for operation at $E=1.0$ and 0.28 for operation at $E=0.58$.

The effect of a leading power factor (capacitive) burden in the break in the Δ was investigated and the results indicated that pure capacitance was not satisfactory. The same conclusion holds for wave traps

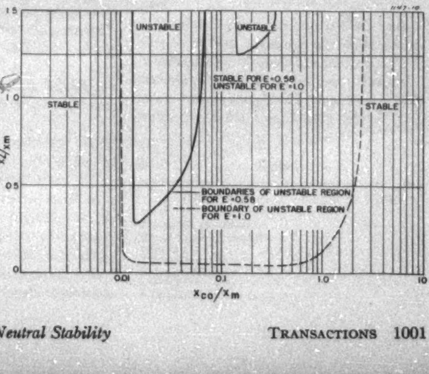


Figure 10. Effect of inductive reactance burden x_L (0.2 power factor) in the broken delta on regions of instability

Saturation curve 1 of figure 2

Note: The total amount of reactance in the delta is three times the amount of x_L indicated by the curves since x_L/x_m is on a per leg basis which were made up of a reactance and capacitance tuned to the third harmonic. These were not effective because other complicating phenomena were encountered which were difficult to account for. Consequently a resistance burden appears to be the simplest and most effective means of insuring stability by means of a secondary burden. Reactance was effective, although a reactance ohmic value would have to be smaller than that of a resistance to insure stability.

Conclusions

1. In using Y grounded primary broken Δ or Y-connected secondary potential transformers for metering or relaying, it is necessary to consider the amount of capacitance to ground that may be permanently or temporarily connected at the terminals of the transformer in order to avoid the possibility of abnormal voltage conditions brought about by circuit instability which might cause incorrect indications of faults. For line-to-line voltage rated potential transformers connected line-to-neutral, corresponding to general practice in the application of such devices ($E=0.58$ in the paper), the range of possible abnormality is defined approximately by $0.012<x_{co}/x_m<0.6$. If a given circuit is found to fall within this range, normal behavior can be insured by:

(*a*). Adding shunt capacitance to ground on the system side of the transformer until the resultant ratio of x_{co}/x_m becomes equal to or less than 0.012, or

(*b*). Connecting load across the break in the broken Δ secondary or in each phase in the case of Y-connected secondary. If resistance is used, it should have a value per phase of 33% of the magnetizing reactance x_m per phase or less; if reactance is used, its ohmic value per phase should not be greater than 28% of x_m per phase. In either case, x_m is referred to the secondary side in accordance with the transformer turn ratio. The impedances required for stabilization are high enough so as not to interfere with the normal functioning of the device.

For these criteria, x_{co} is the zero sequence capacitive reactance to ground on the Y grounded side of the transformer and x_m is the magnetizing reactance per leg corresponding to $E=1.0$ on the saturation curve 1 of figure 2.

2. The regions of abnormality as well as the abnormal phenomena observed are practically unaffected by resistance in the transformer windings or by losses in transformer iron over a practical range.

3. Increasing the steepness of the saturation curve tends to increase the range of possible abnormal circuit behavior. In particular, the region of abnormality is extended to lower values of x_{co}/x_m as the steepness of saturation is increased.

4. For conditions of operation above transformer flux densities corresponding to the normal practice of using *L-N* connected potential transformer with *L-L* voltage rating ($E=0.58$ in the paper), abnormal circuit behavior is extended to include a range of $0.01<x_{co}/x_m<3.0$ approximately for $E=1.0$. Furthermore, the impedance in the open Δ must be made smaller if stability is to be insured. For $E=1.0$, a resistance in the break in the Δ not greater than 45% of x_m (15% of x_m per phase) or a reactance not greater than 12% of x_m (4% of x_m per phase) would be required to avoid abnormal circuit behavior independent of x_{co}/x_m.

References

1. Theory of Abnormal Line-to-Neutral Transformer Voltages, C. W. LaPierre. AIEE Transactions, volume 50, 1931, pages 328–42.

2. Physical Nature of Neutral Instability, A. Boyajian and O. P. McCarty. AIEE Transactions, volume 50, 1931, pages 317–27.

3. Experiences With Grounded-Neutral Wye-Connected Potential Transformers on Ungrounded Systems, C. T. Weller. AIEE Transactions, volume 50, 1931, pages 299–316.

4. An Electric-Circuit Transient Analyzer, H. A. Peterson. *General Electric Review*, volume 42, September 1939, number 9, pages 394–400.

5. Abnormal Voltage Conditions in Three-Phase Systems Produced by Single-Phase Switching, Edith Clarke, H. A. Peterson, P. H. Light. AIEE Transactions, volume 60, 1941, pages 329–39.

Field Testing of Generator Insulation

EEI SUBJECT COMMITTEE ON GENERATOR INSULATION AND TESTING

I—Introduction

NEED for a practical as well as a reliable nondestructive test that will determine the condition of generator insulation has long been recognized by the industry. Simple methods now in general use, the high voltage proof test and the insulation resistance test, do not accurately or positively disclose the overall condition of windings, indicate the degree of deterioration, or necessarily locate weak spots. This lack of suitable insulation tests has been emphasized recently by the return to service of a number of machines either after long shutdown periods during the depression years or from standby service to low pressure elements used in connection with topping units; in these cases knowledge of insulation condition is vital.

As a result of this situation the Edison Electric Institute electrical equipment committee undertook active work on the problem in 1935.[1] In 1937 a special subject committee was appointed on "Generator Insulation and Testing," with the ultimate object of developing a reliable test procedure for determining the condition of generator insulation. The investigation has been carried out by various member companies of the EEI and others in the industry interested in the problem, under the guidance of the subject committee. Close contact has also been maintained with the manufacturers who have made many valuable suggestions, assisted in testing, and given aid in analyzing data.

The study is by no means complete and this paper should not be construed as a final report with definite conclusions, but rather as a progress report on an investigation still under way.

II—Present Status of Insulation Testing

Present testing practice in this country was surveyed in 1938, revealing a wide variation in testing methods and procedure. High voltage proof testing which consists merely of the stressing of the insulation by the application of an alternating potential according to the rules in the AIEE Standards is generally used for acceptance tests for new and repaired machines. There is no general agreement as to advisability of routine proof testing nor magnitude nor frequency when such tests are applied to machines in service. Insulation resistance testing is a common practice though generally a definite routine is not followed. In the past the procedure often has consisted of a "crank and read" type of measurement. Today, increasing recognition is being given the fact that insulation resistance values may be misleading unless careful account is taken of the many factors which may affect the results. No definite practice seems to be established regarding other methods of test, such as high voltage d-c resistance, dielectric loss or power factor tests.

Comparatively few operating companies maintain regular schedules of periodic insulation testing, from which records may be obtained to serve as a guide of the progressive condition of the insulation. Such special tests as are made are carried out usually after a machine winding has been repaired or an inspection indicates it to be in questionable condition.

Foreign practice in this respect also indicates a lack of uniformity. In some countries, notably Germany and Switzerland, experiments have been made with dielectric loss measurements, and it is felt there that this type of measurement has some promise of value as an index of changing insulation condition. A few experiments have made use of a recording instrument giving a curve of dielectric losses versus time during machine operation. It has been suggested that this method may give a definite index of changing conditions within the insulation. The method has not received broad use.

III—Scope of Work

Originally it was the objective of the subject committee to conduct destructive and nondestructive tests on insulation of machines about to be scrapped, rewound, or repaired and to develop from these a workable nondestructive method of determining insulation condition. Valuable information has been obtained from these tests but inasmuch as many of the data consist of "spot" readings taken under a variety of conditions, it has not been possible to correlate all of the data nor to draw definite conclusions. Real need for periodic and standardized measurements has become apparent. The scope of the investigation has been broadened to include and encourage the collection of periodic nondestructive test data on machines in service. By this means information is being accumulated on all classes of insulation, from the old to the new.

Insulation characteristics of both armature and field windings are undergoing investigation. Table I summarizes the machines tested or under test. The machine ratings vary from 1,500 kva, 2,700 volts to 200,000 kva, 16,500 volts.

Table I

Type of Test	Number of Machines
Insulation resistance	106
Power factor of insulation	27
Ionization	8
Voltage strength a-c	21
Voltage strength a-c versus d-c	6*

* Includes 2–2.3 kv motors.

Paper 41-17, recommended by the AIEE standards committee, and presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941. Manuscript submitted November 6, 1940; made available for preprinting December 6, 1940.

Personnel of EEI subject committee on generator insulation and testing: C. M. Gilt and B. Van Ness, Jr., subject sponsors; R. L. Webb, chairman; J. E. Allen, E. F. Dissmeyer, O. E. Fawcett, A. P. Hayward, R. J. Woodrow.

The subject committee wishes to acknowledge the assistance and co-operation rendered in this work by EEI member companies and engineers, by other engineers and utility companies, and by the manufacturers.

[1] For all numbered references, see list at end of paper.

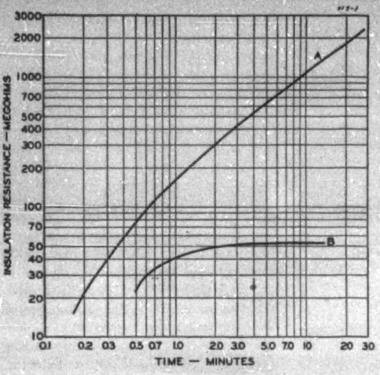


Figure 1. Armature-winding insulation resistance versus time

A—200,000-kva single-barrel generator, 16.5 kv, 60 cycles, 1,800 rpm (insulation dry)

B—18,750-kva single-barrel generator, 13.2 kv, 60 cycles, 1,800 rpm (insulation moist)

The subject committee, recognizing the need for uniform practice in insulation testing, has prepared a guide to be used in making such tests. Copies of this are available to the industry through EEI headquarters.

IV—D-C Insulation Resistance

Although d-c insulation resistance is among the most commonly used methods for testing the insulation of electrical apparatus, records obtained have been baffling of interpretation, and the factors which affect the results are to some extent unknown and often unappreciated. The fundamental that different types of insulating materials may have different volume resistances is obvious; but there are other variables which affect successive tests on a given piece of apparatus and cause widely varying results.

Tests have been directed to establish the manner in which the insulation resistance varies with

(*a*). Differences in insulation materials and structures
(*b*). Duration of voltage application
(*c*). Existing charge in windings
(*d*). Insulation temperature
(*e*). Test voltage magnitude
(*f*). Insulation moisture content
(*g*). Previous operating condition of the generator
(*h*). Various types of winding damage or deterioration

These investigations have confirmed findings obtained in similar experiments made by others and have gone somewhat further in extending the limits to which such tests have been conducted in the field.

INSTRUMENTS

Experiments made so far have indicated that any one of several types of test instruments will give satisfactory insulation resistance results. Commercial forms of insulation resistance testers perform satisfactorily provided they are calibrated periodically and tested to assure their voltage output is proper. The maximum voltage at which measurements have been made with these instruments is 5,000 volts, d-c. Motor driven devices or other constant voltage generators are necessary for taking long time insulation resistance curves. Hand operated units may be used for short time readings.

The voltmeter method may be used with fair accuracy if the insulation resistance is less than 10 megohms. To insure accuracy of readings a high resistance voltmeter should be used.

Considerable success has been attained with electronic rectifier sets having d-c outputs from 100 to 5,000 volts and with dry cell battery sets up to 1,500 volts. These use a voltmeter and a microammeter for measurements which permit calculation of insulation resistance. They must be carefully shielded and guarded.

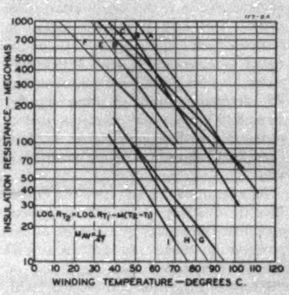
Higher voltages may be obtained from Kenotron test sets. Tests may be carried to any reasonable limit of voltage with this type of equipment, though 5,000 volts appears to be the present limit for sets portable by hand. In all rectifier sets it is essential that a steady supply of voltage be available to prevent fluctuation in charging current to the generator winding capacitance, an automatic voltage regulator being used if necessary.

Where protective resistors are used in test instruments their effect on the voltage magnitude applied to the insulation should be carefully taken into account.

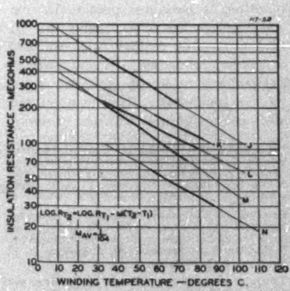
DURATION OF VOLTAGE APPLICATION

While many insulation resistance measurements have been taken by the "crank and read" method, Wieseman[3] and others have shown that the insulation resistance of a winding varies with duration of voltage application. Insulation resistance of a dry winding in good condition may continue to increase for as long as a day or more with the test voltage applied continuously at a steady value. Normally an approximately final value, which is termed "apparent insulation resistance" may be reached after 10 to 15 minutes. Insulation resistance after 30 seconds to 60 seconds may be as little as 10 per cent to 20 per cent of the 15-minute reading.

In figure 1 insulation resistance versus time curves, commonly referred to as dielectric absorption curves, are shown for different conditions of generator insulation. Curve *A* is a good example of the effects of "bone dry" and clean insulation on the final value of insulation resistance and on the shape of the dielectric absorption curve. This curve represents an unusually high insulation resistance for a machine whose rating is 200,000 kva, 16,500 volts, in one stator winding. Curve *B* was taken on an 18,750-kva, 13,200-



Family having more common slope. Data taken at 500 to 20,000 volts after ten minutes



Family having lowest slopes. Data taken at 1,000 to 8,000 volts after one to ten minutes

Figure 2. Insulation resistance versus temperature

volt generator which had been exposed to varying degrees of atmospheric humidity for several years. Considering size alone this smaller unit would be expected to have a higher insulation resistance than the larger unit. However, the insulation conditions are such that the reverse is the case. Leakage resistance effects predominate and the curve flattens rapidly between the 30-second and the 3-minute readings.

Information contained in curves of this nature discloses the reason for at least one difficulty in past attempts to correlate insulation resistance readings, for it is obvious that "spot readings" made without regard to definite time of voltage application can give widely varying results.

Many dielectric absorption curves have been collected. While it is believed that these data give information as to dryness and cleanliness of winding, spot curves have not disclosed a positive relation between insulation resistance and insulation condition. Therefore, periodic curves are being taken and the behavior of the dielectric absorption factor over a long period is being observed and analyzed.

Effect of Existing Charge on Winding

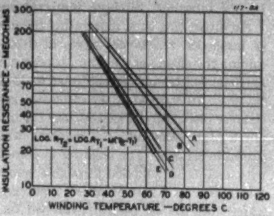
Comparative values of winding insulation resistance are markedly affected by residual charges retained in a winding. For this reason windings should be grounded and discharged for at least ten minutes before applying insulation resistance tests.

Insulation Temperature

Tests have indicated that the insulation resistance of windings is affected greatly by temperature. The most marked effect occurs in class *A* windings but even in class *B* insulation structures, the insulation resistance may vary as much as 10 to 1 for a winding temperature change from approximately 25 degrees centigrade to 75 degrees centigrade, the insulation resistance being lower at the higher temperature.

It is desirable that insulation resistance readings be taken at temperatures as close to 75 degrees centigrade as possible, to secure an indication of resistance at operating temperatures, eliminating the effect of humidity and requiring minimum temperature correction.

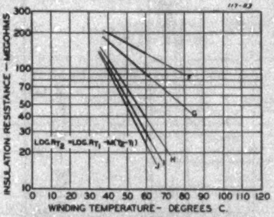
Figure 2 gives the results of field tests undertaken with the objective of establishing factors which will permit readings taken at one temperature to be transferred to a lower or higher temperature. The results so far indicate two families of curves, with similar characteristics of insulation resistance versus temperature in each group. The curves may be represented by the formula:



68,750-kva turbogenerator, 13.2 kv, 60 cycles, 1,800 rpm

Data taken following acceptance test run

A Insulation resistance at 10 min.—$M = {}^1/_{48}$
B Insulation resistance at 4 min.—$M = {}^1/_{60}$
C Insulation resistance at 1 min.—$M = {}^1/_{41}$
D Insulation resistance at 30 sec.—$M = {}^1/_{39}$
E Insulation resistance at 15 sec.—$M = {}^1/_{38}$
Avg.—$M = {}^1/_{44}$



Data taken following about two years of base-load service

F Insulation resistance at 10 min.—$M = {}^1/_{116}$
G Insulation resistance at 4 min.—$M = {}^1/_{76}$
H Insulation resistance at 1 min.—$M = {}^1/_{41}$
I Insulation resistance at 30 sec.—$M = {}^1/_{36}$
J Insulation resistance at 15 sec.—$M = {}^1/_{34}$
Avg.—$M = {}^1/_{60}$

Figure 3. Insulation resistance versus temperature for different periods of voltage application

$$\log R_{t_2} = \log R_{t_1} - M(t_2 - t_1)$$

where R_{t_2} and t_2 are the unknown resistance and temperature at which the resistance is desired, R_{t_1} and t_1 are the measured resistance and the related winding temperature and *M* is a constant equal to the reciprocal of change in temperature for $\log R = 1$. Insulation resistance doubles for each 14 degree centigrade reduction in winding temperature for the curve whose $M = {}^1/_{47}$, and doubles for each 31 degree centigrade reduction for the curve whose $M = {}^1/_{104}$.

Although it might be assumed that the two families of curves bear definite relation to insulation class, this is not the case. Machines with both class *A* and class *B* windings appear in each group. Operating conditions apparently have a large effect on these characteristics, for data indicate that even after years of operation changes in the value of *M* for a given machine may occur from year to year. This is evident from figure 3 in which the curves having an average value for $M = {}^1/_{44}$ were taken in 1938 shortly after a new generator was placed in service. The curves having an average value for $M = {}^1/_{60}$ were taken on this same machine approximately two years later. The machine had been operated about 72 per cent of the elapsed time at an average of 95 per cent of rated capacity and had been run for several days, well loaded, before the last test. It is apparent, therefore that the value of *M* may vary for a given machine but it is not known definitely whether this variation is due to differences in moisture or volatile matter content in the insulation, or whether it is due to some aging or curing action which has been under way during the many continuous hours of high load operation. It is believed that moisture is the most likely cause of the variation.

While extrapolation of the curves of figure 3 appears to meet in a point at low temperatures other data indicate parallelism or a meeting at high temperatures. The reason for and meaning of these variations are not known.

No definite formula applicable to all machines, or even to the same machine under various conditions, can be given at this time for the relation of insulation resistance to temperature. Two readings taken at the same date on a machine will establish a curve for that machine for a definite condition of the insulation. A change in slope of an insulation resistance-temperature curve indicates a change in the insulation condition.

Effect of Moisture Content

Accurate insulation resistance data are very useful in determining the progress of drying machine insulation. It is customary to dry a machine until the insulation resistance readings at constant temperature approach a constant value when plotted with regard to time. When drying out, the attainment of a constant insulation resistance at a constant temperature is a better indication of dryness than is the attainment of some previously selected value.

Figure 4 indicates the variation of insulation resistance of a new turbo-generator whose winding is being dried. Each point of insulation resistance was taken after ten minutes application of test voltage. The insulation resistance dropped rapidly as the winding temperature increased and then recovered as the moisture was driven from the insulation. When the test was terminated, the insulation resistance rose rapidly as the winding temperature decreased.

Figure 5 gives another method of plotting similar data as taken on the initial dry-out run of a 37,500-kva water-wheel generator and illustrates the value of the one- and ten-minute readings in obtaining a clear picture of the conditions during machine dry-out. It will be noted that as the run progressed the separation between the curves increased, indicating an effective drying of the insulation and the resulting increase of absorption effect in the dielectric.

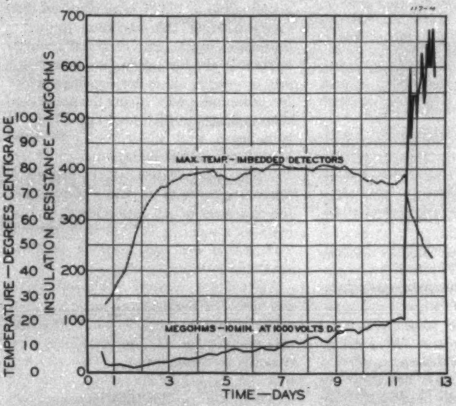


Figure 4. Change in ten-minute insulation resistance with time during generator dry-out

35,300-kva turbogenerator, 13.8 kv, 60 cycles, 1,800 rpm

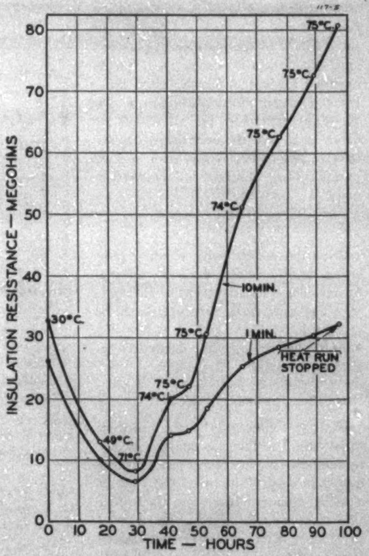


Figure 5 (right). Change in one-minute and ten-minute insulation resistance with time during generator dry-out

37,500-kva hydrogenerator, 13.3 kv, 25 cycles, 100 rpm

The degree of moisture content in the outer layers of generator winding insulation has a very marked effect on the insulation resistance. A machine which has been running regularly will generally have a much higher insulation resistance at room temperature than the same machine after being idle for several weeks in a humid atmosphere. A relatively low insulation resistance resulting from exposure to moist air does not necessarily mean that the insulation is inferior or that the machine is not safe for operation. If the insulation resistance is still reasonably high for the machine in question as shown by reference to periodic test data it may be returned to service and the moisture will be driven from its windings by the increased machine temperature.

An interesting case, illustrating the use of insulation resistance curves in determining the effects of moisture, concerns a machine which had been subjected to flood waters. After dry-out, the insulation resistance increased to a satisfactory though not a high value and the machine was placed in service. The unit was operated with cooling air supplied through a spray system equipped with baffles for removing water from the air before it reached the generator. Later, after fairly continuous service, the generator was found to have low insulation resistance. The cooling air was suspected of carrying moisture into the machine and preventing thorough drying. The unit was operated under test load for two hours using relatively dry turbine room air for generator cooling to learn of its effect on insulation resistance. Curves *A* and *B* of figure 6 show the insulation resistance results before and after this change in cooling air. As a result of these tests the unit, and several others in the same plant were equipped with surface coolers and a closed ventilating system, with improvement in insulation resistance as shown by curve *C* of figure 6.

Spray type air washers have not caused similar trouble at certain other locations where the same type of tests were made on generators whose windings had not been submerged.

Effect of Test-Voltage Magnitude

Insulation resistance measurements at 75 degrees centigrade of dry insulation in good condition will be affected very little by wide variations in test voltage. The effect appears to increase somewhat at lower temperatures as will be noted on figure 7. Windings having high leakage due to moisture or dirt, or other types of deterioration, may show a more pronounced reduction in insulation resistance as the test voltage is increased.

Results of complete tests on two machines, have indicated there may be some merit in comparisons between insulation resistance measurements at two different d-c voltages, at the same winding temperature. A 25,000-kw generator winding section was given a dielectric absorption test at 2.5 kv and at 15 kv d-c. The coils were then punctured on a 60-cycle high potential test and the insulation resistance values at the two voltages were again taken. The first test gave a ratio of approximately 2 to 1 with the 2,500-volt measurement being higher. The second test gave an insulation resistance ratio of approximately 6 to 1 after the insulation had been punctured. This test was repeated on another coil group and the ratio changed from approximately 2 to 1 to a ratio of approximately $4^1/_2$ to 1.

The stator winding of a 30,000-kva synchronous condenser was found in very

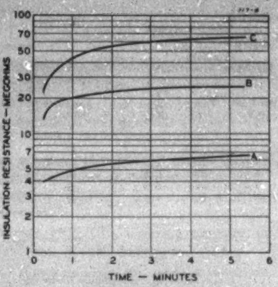


Figure 6. Armature-winding insulation resistance versus time

25,000-kva generator, 13.2 kv, 60 cycles, 1,800 rpm

Tests made at 500 and 1,000 volts d-c. Winding temperature—74 degrees centigrade

A—After using a water-spray air washer. Cooling air humidity—78 per cent

B—After using turbine-room air at approximately 38 per cent humidity during a two-hour load run made immediately after test *A*

C—After using a closed ventilation system and surface coolers for about one month

bad condition on visual inspection in a plant where two machines of the same rating and age were operating. The insulation resistance of the winding which had to be replaced was found to have a ratio of more than 7 to 1 between the 2,500-volt and 20,000-volt tests, respectively. The insulation resistance of the other machine was found to have a relation of approximately 1.25 to 1 for the 500-volt and 10,000-volt tests and 1.2 to 1 for the 2,500-volt and 10,000-volt tests, respectively. The winding of this machine appeared to be in excellent condition.

While sufficient data have not been accumulated to permit final conclusions, they do indicate that a wide difference in insulation values determined by low and high voltage tests should lead one to suspect irregularities in the insulation. Efforts at conducting voltage tests under staged conditions by mechanical damage have met with poor success because of the difficulty of keeping reductions in the dielectric strength of armature bar insulation under control.

V—Insulation Power Factor

Power factor testing of generator windings is not in common use. Experiments have been made, however, to determine whether some test of this type may be useful and practical. The 60-cycle power factor of generator insulation has been studied for different winding temperatures and impressed voltages.

Equipment

Any well constructed and properly shielded power factor measuring equipment is satisfactory. Circuits of the test instruments must be thoroughly shielded and guarded to prevent interference from other energized equipment. Use of the ordinary wattmeter, voltmeter, and ammeter is not likely to give satisfactory results. Suitable commercial measuring equipment is available from several sources. A testing supply of ample capacity to furnish the charging current at the measurement voltage should be used.

Park and Skeats[4] have shown that generator winding capacitance can be determined fairly accurately by use of the formula $C=K_c\dfrac{\text{MVA}}{\sqrt{kv}(1+0.08\ \text{kv})}$ microfarads per phase.

The constant K_e has the following values:

	K_c	Assumed Peripheral Speed Ft/Min
Solid round rotors	0.0187	22,000
Salient-pole generators without amortisseurs	0.0347	10,000
Salient-pole generators with amortisseurs	0.0317	11,000

C is approximately inversely proportional to peripheral speed.

Capacitance of connecting cables must be added if included in the test.

Charging current at high voltage may be 25 per cent to 100 per cent above that determined by the capacitance constant alone, depending upon the extent of the corona losses, which vary with the impressed voltage.

Results

Power factor measurements on machine insulation may be affected by many conditions, most of which are similar to those outlined for d-c insulation resistance. The time of voltage application appears to have little effect on winding power factor unless it causes heating within the insulation. Winding temperature and moisture content will have a pronounced effect, and winding capacitance is, of course, an important factor. Voltage magnitude has an effect on power factor causing it to increase as the voltage is increased over the normal test range.

Insulation power factor should be nearly constant at each voltage below the ionization value.

Figure 8 shows the typical variations of winding power factor with different values of applied alternating voltage at different temperatures.

Power factor measurements have thus far produced little data which may be used to indicate insulation condition, other than moisture content.

No experiments have been conducted on dielectric loss characteristics by the subject committee. European engineers have had some success with this method, particularly as applied to control of manufacturing processes. It has received little attention as a means for supervising the insulation characteristics of an operating machine.

VI—Ionization Measurements

Both qualitative and quantitative ionization measurements[5] have been investigated to determine whether such measurements can be used to detect insulation irregularities in generator insulation. The quantitative tests were designed to obtain a characteristic showing the relative magnitudes of ionization discharge from a generator winding as the applied a-c voltage was varied, and to determine whether this might have any relation to the physical condition of the insulation or to its dielectric strength. Qualitative tests were designed to locate ionization points in the winding on the supposition that ionization would be present at weak points in the insulation.

Apparatus used for the qualitative tests is shown in figure 9. Equipment for the quantitative measurements was connected as indicated in figure 10. Complete shielding of the apparatus is necessary, and even then external disturbances due to radiations from energized equipment nearby are bothersome. For the quantitative tests the radiations in the radio frequency range were amplified, rectified, and measured by a sensitive d-c instrument. The measurements were relative only and not of any definite figure in amperes of ionization current.

Results of Qualitative Measurements

The qualitative ionization tests determined not only the impressed voltage at which ionization began, but were reasonably successful in determining the locations at which more severe ionization was present. On 6.6-kv and 11.4-kv generators having old winding designs varying in insulation structure from 100 per cent class *B* to all class *A* construction in the slot section, the voltage at which ionization was noted varied from about 1,000 volts to approximately 7,000 volts. The

Table II. Summary of Generator-Winding Insulation Tests* 1935–1940

			Armature Insulation		Armature-Insulation Resistance				Armature-Insulation Power Factor			High-Voltage Breakdown (Kv)
Rating (Kva)	Voltage	Speed (RPM)	Age (Yrs.)	Type	Megohms	Time (Min.)	Voltage	Temp. (°C)	Per Cent P.F.	Temp. (°C)	Voltage (Kv)	Armature 60 Cy.
12,000	11,400	750	16	*B*	2,000	6	500	20				35
12,000	11,400	750	15	*B*	1,150	10	500	20	11	20	6.6	45
20,000	6,600	1,500	22	*B*	300	10	500	22	6.7	22	3.8	39
20,000	6,600	1,500	22	*B*	350	10	500	22	6.5	22	3.8	43
14,000	7,800	720	30	*A*	310/185	10	500/5,000	20	11	20	4.5	46
					54/35	10	500/5,000	47				
68,750	13,800	1,800	0	*B*	200	10	8,000	36				
			2	*B*	208	10	8,000	36				
33,000	11,000	360	15	*B*	38/28	10	1,000/5,000	56				
55,000	11,000	360	5	*B*	285	20	5,000	48				
00,000	16,500	1,800	7	*B*	2,200	25	1,000	22				
15,000	13,200	94.7	2		36	10	1,000	22				26 (Before rewinding)
18,750	6,600	112.5	16	*A*	33	10	8,000	32	10	35	2.5	
									13	35	8	8 (Min.) 21.4 (avg. of 18 coils)
30,000	12,000	1,800	10	*B*	32/26	10	5,000/15,000	28				
30,000	9,250	1,500	0	*B*	700/500	10	4,000/9,000	30	7.5/9.0	30	4/10	
3,500	12,000	1,200	27	*A*	16and17	1	1,000	6	22 to 29 (Arm & Fld.)	1	3.5	22.5 Min. (avg.—51.5 kv) (2 units—same rating)
28,600	13,800	1,800	11	*B*	74	10	470					8 to 14 (3 coils tested)
23,750	11,000	1,200	21	*B*	40,000/coil	3	500	11	6.7 (Avg.)	11	6	17 to 35 (1/2 of coils)
23,750	11,000	1,200	12	*B*	460	1	470	21	3.2	35	0.5	
3,500	2,300	1,800	9	*B*	1,000	1	470	21				
4,000	2,300	1,800	25	*B*	200	1	470	21				
8,000	11,000	1,800	23	*B*	340	1	470	21	8.9	31	0.5	
20,000	11,000	1,800	14	*B*	195	1	470	21	6.7	23	0.5	
23,750	11,000	1,200	14	*B*	955	1	470	21	3.5	50	0.5	
23,750	11,000	1,200	14	*B*	345	1	470	21	9.8	46	0.5	
43,750	11,000	1,800	10	*B*	140	1	470	21	5.4	24	0.5	
10,000	11,000	94	26	*B*	475/275	10	1,000	36/60				
10,000	11,000	94	25	*B*	670	10	1,000	63				
13,750	11,000	94	15	*B*	265	10	1,000	69				
10,000	11,000	94	15	*B*	235	10	1,000	60				
10,000	11,000	94	27	*B*	845/157	10	1,000	28/70				
12,000	11,000	116	26	*B*	311	10	1,000	69				
12,000	11,000	116	25	*B*	136	10	1,000	73				
12,000	11,000	94	24	*B*	540	10	1,000	29				
15,000	13,200	94.7	2	*B*	36	10	1,000	22				26 (Before rewinding two years earlier)
15,000	13,200	94.7	14	*B*	35	1	1,000	80				
12,500	13,200	1,800	11	*B*	1,270/200	10	1,000	41/79				
12,500	13,200	1,800	13	*B*	820/164	10	1,000	44/73				
30,000	12,000	1,200	6	*B*	10/7		4,000/12,000	(Room)	10	(Room)	10	14 (Old winding replaced in 1939)
			0	*B*	37/31		4,000/12,000	(Room)	8	(Room)	10	(New winding)
1,500	2,700	1,200	36	*A*	65	10	1,000	20	20	20	3	3.5
2,500	2,300	3,600	13	*B*	150/phase	2	1,000	20	18	20	3	6.5
27,777	13,200	1,800		*B*	1,500/phase	10	2,500	40	4.3	53	10	25 (38°C)
27,777	13,200	1,800		*B*					6.2	53	10	18 (53°C)
30,000	13,600	720	11	*B*	2,200/1,900	10	2,500/10,000	25				
30,000	13,600	720	11	*B*	6,500/2,500 (Ph-1)	10	2,500/10,000	30				26.3 (Avg.)
					5,800/2,700/800 (Ph-2)	10	2,500/10,000/20,000	30				
30,000	13,600	720	0	*B*	400 (1-Ph)	10	2,500	20				
					920 (All Ph)	10	2,500	20				
58,824	13,800	3,600		*B*	950	10	1,000	28				
30,000	13,270	300		*B*	1,000	10	2,500					
14,000	6,000	720	27	*A*	135/105	10	1,000/8,000	28	22/15	28	6	
1,500	11,000	375	0	*B*	1,200	5	25,000	30				
80,000	14,400	1,800	0	*B*	100	1	1,000	70				
					350/200	Final	1,000/10,000	33				
80,000	14,400	1,800	3	*B*	80	10	1,000	43				
80,000	14,400	1,800	2	*B*	19	10	1,000	45				
18,750	13,200	1,800	24	*B*	99/12	10	500	32/63				
18,750	13,200	1,800	24	*B*	125/25	10	500	28/74				
25,000	13,200	1,800	18	*B*	520/50	10	500	37/75				
31,250	13,800	1,800	3	*B*	900/190	10	500	34/72				
30,000	6,600	1,500	22	*B*	170/110	10	1,000/20,000	37	10	52	5	32.7 (Avg.)
24,000	13,800	1,800	21	*B*	100	10	1,000	47				
23,500	13,800	720	3	*B*	950	10	1,000	34				
58,800	13,800	3,600	0	*B*	1,500/2,100	10/35	1,000	27				
188,500	13,800	1,800/1,200	13	*B*	15/19	10	1,000	33				
188,400	13,800	1,800	13	*B*	40/21	10	1,000	25/43				
160,000	11,400	1,500	12	*B*	560 & 660	10	1,000	27	(Machine has two independent windings)			
24,000	13,200	1,800	21	*B*	330	10	1,000	21				
23,500	13,800	720	3	*B*	120	10	1,000	35				
35,000	11,400	1,500	17	*B*	1,000	10	1,000	42				
35,300	13,800	1,800	0	*B*	460	10	500	50				

* Insulation resistance data on 40 machines not listed because of lack of full rating information.

machine on which ionization was detected at 7,000 volts and 21 degrees centigrade, was a 7.8-kv class *A* insulated unit. When this winding temperature was raised to 47 degrees centigrade, ionization could be detected at about 5,000 volts.

Ionization was detected at 1,000 volts on a 6.6-kv machine with mica insulation at a point where the coil passed through an air duct at the center of the stator winding. Though signs of corona pitting were expected in the insulation and heavy paper protective covering actually no evidence of such damage was found.

Where an aerial was used for detection, the only radiations due to ionization which were picked up in the slot sections of the stators were at air ducts and at the outer edges of the stator iron, even though the voltage was raised to more than two times machine rated voltage. This may be explained partially by the shielding effect that the stator conductors receive in the slot sections. Ionization was detected at the end turns of all machines tested by this method, at voltages varying from 25 per cent to 50 per cent of machine rated voltage.

An attempt was made with the search aerial to locate points where intentional damage had been inflicted to the insulation of various generators, by bruising the insulation with hammer blows, by cracking the insulation in bending, and by cutting into the insulation structure with a knife. The qualitative measurements gave no clear indication that ionization had increased or that it developed at a lower voltage.

RESULTS OF QUANTITATIVE MEASUREMENTS

Quantitative ionization tests were made to determine the relation of ionization discharge with change in impressed voltage and to determine whether the rate of change could be used to detect insulation damage or deterioration. Figure 11 shows the curves obtained on a class *A* insulated machine for all three phases, two phases separately, and for a section of another phase winding whose insulation had been intentionally damaged as previously described. It was thought that the curve would show a step type characteristic giving sudden increases of ionization units at increasingly rapid steps as the impressed voltage was increased. This characteristic does seem to be partially realized in the curves for the two separate phase tests, though not to the degree anticipated. The results on the winding section which had been damaged were somewhat surprising since the winding failed to ground as the voltage was approaching 16 kv. This is only 1,000 volts above the highest voltage at which a measurement of ionization units was made. The curve shape gives no indication of approaching failure.

VII—High-Voltage Tests—A-C and D-C

A survey of coil insulation strengths was made on a number of machines where the machine was being removed permanently from service or where the winding was to be replaced. The work consisted of a-c high potential tests, to breakdown, on different phases and different coil sections and, in some cases, was supplemented by tests using d-c voltage from Kenotron testing equipment. These tests were made in conjunction with other measurements on winding sections as found and on coil sections which had been subjected to various types of mechanical damage intentionally inflicted. Special attention was also given to determination of the relative a-c and d-c strength of winding insulation.

EFFECT OF AGE ON DIELECTRIC STRENGTH

Generator armature winding insulation strengths were found remarkably high even after 20 to 30 years of service, as will be noted by reference to table II. There were occasional failures at voltages less than twice normal rated voltage attributable to mechanical injury which had developed in the insulation and which could not be seen until the winding was disassembled. Most of the breakdown voltages were far in excess of the voltage specified for new machines in the AIEE Standards. A 6.6-kv generator with 1915 class *B* insulation failed at the end-turns at approximately $6^1/_2$ times rated terminal voltage on several tests. This is the machine on which ionization was detected in the slot section at the air duct with an impressed potential of only 1,000 volts. It was impossible to obtain a failure on one of the armature bars after the end-turn brackets had been disassembled, though the voltage was raised momentarily to 72 kv and held at 68 kv (over 10 times normal) for a period of 5 minutes. This case is exceptional and is not intended to convey the thought that armature bar insulation has this safety factor as a general thing.

ROTOR DIELECTRIC TESTS

Ten turbogenerator fields were tested to destruction, with breakdown voltages varying from 1 to 5.5 kv. Six of the fields punctured at values between 1 and 2 kv which indicates that, in order to prevent undue stress, routine field testing should preferably be done with a 500-volt source rather than with a 1,000-volt source.

DIELECTRIC STRENGTH—A-C VERSUS D-C

Numerous tests have been made to determine the relative strengths of generator insulation as determined by breakdown voltage using a-c and d-c test equipment. An established ratio between the two types of tests will permit use of low capacity d-c testing equipment on large armature windings.

Such tests were made by selecting a group of four to six coils, each at different locations, around the periphery of the stator for each type of test. Unusually low breakdown strengths were disregarded as being caused by abnormal weaknesses in the coil. Experiments made to date have given the following results:

Table III

	Average Insulation Breakdown Strength		
Machine	A-C Voltage (RMS) Kv	D-C Voltage Kv	Ratio D-C/A-C (RMS)
2.3-kv motor	13.2	20.8	1.6
2.3-kv motor	11.7	21.0	1.8
13.8-kv, 30-mva synchronous condenser	28.3	38.5	1.46
6.6-kv, 30-mva turbogenerator	34.0	46.7	1.38

Similar tests have been performed in the laboratory on a group of spare generator coils, no end-turn supports were used. The slot sections of the coil were grounded carefully by clamping them into metal troughs. The average 60-cycle breakdown was 65 kv. The d-c value varied from 150 to 175 kv. The ratio is approximately 2.5, which approximates values to be expected in cable insulations. It is thought the test set-up, by reducing ionization, may have more nearly represented a cable structure than it did a generator coil structure.

RELATION BETWEEN INSULATION RESISTANCE AND DIELECTRIC STRENGTH

Since the ultimate objective of this investigation has been to determine a test method or a group of tests which could be used to determine the true quality of generator insulation with regard to its ability to withstand high voltage, much time has been spent in attempting to find a relation between dielectric strength as determined by a-c and d-c breakdown and various d-c insulation resistance measure-

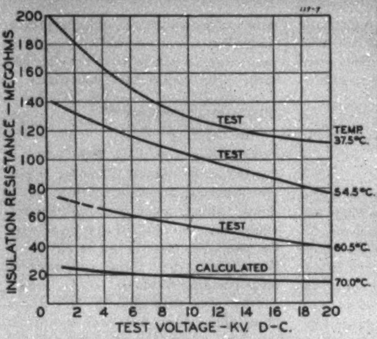


Figure 7. Insulation resistance versus test voltage at different winding temperatures

30,000-kw turbogenerator, 6.6 kv, 25 cycles, 1,500 rpm

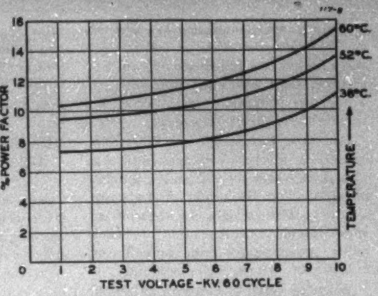


Figure 8 (right). Insulation power factor versus test voltage at different winding temperatures

30,000-kw turbogenerator, 6.6 kv, 25 cycles, 1,500 rpm

ments made at non-destructive voltages. In the early work, insulation resistance magnitudes were compared with 60-cycle breakdown strength on many coils. The investigation has been enlarged to include comparisons between insulation strength determined by a-c breakdown tests with dielectric absorption curve shapes and variations in insulation resistance characteristics at widely varying values of d-c voltage. Comparative insulation resistance and a-c breakdown tests have also been made under many conditions of deliberate mechanical damage to the insulation.

At present, no claim can be made that the ultimate objective has been attained. Comparisons between single insulation resistance measurements and dielectric strength seem to have little or no value and, under certain circumstances, the insulation resistance at moderate d-c voltages has been found to increase after the winding has been punctured on an a-c high potential test.

Location of Dielectric Failures

Stator winding insulation failures on breakdown tests occurred frequently at the end-turns. In some machines several coils were punctured, using both a-c and d-c, without a single failure occurring in the slot section. As indicated in table II the insulation strengths varied widely but invariably if a number of coils were tested, it was found that the weakest portion of the winding structure was at the end-turns.

Most end-turn failures occurred from the winding to the supporting brackets which always involved a flash through a short air gap. The insulation recovery after these failures was such that insulation resistance measurements did not detect any change in the winding characteristics. The failures which occurred in the slot sections appeared at the end of the slot or at the edge of an air duct in the stator iron. It is possible that ionization or swelling of the insulation at these points caused damages which reduced the dielectric strength but we have not been able to confirm the predominating cause. Very few machines have shown severe distress as a result of corona. In the few cases where effects of corona have been noted, the class *A* outer layer only has been affected. There has been no case where direct injury to mica insulation due to corona action has been discovered.

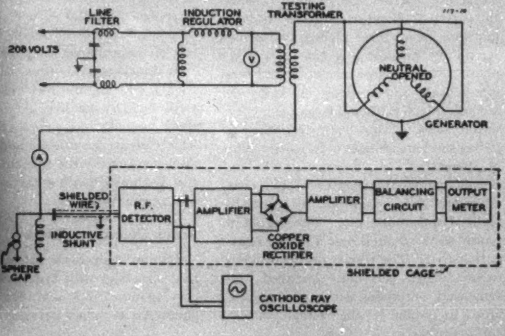
VIII—Conclusions

1. The industry will obtain increased value from its insulation testing if the known causes for variations in the results are carefully taken into account.

2. Insulation resistance varies widely with type and speed of machine, type and condition of winding, degree of cleanliness, temperature, moisture content, and extent of d-c charge on the winding. "Spot" readings, particularly, may be misleading unless carefully corrected for various factors affecting the reading. However, periodic

Figure 9 (left). Ionization detection apparatus ready for use

Figure 10. Connections of apparatus for measurement of ionization intensity in generator windings



readings may indicate a trend of the general condition of the insulation.

3. There is at present insufficient data to permit accurate correction of insulation resistance readings for all factors affecting these readings.

4. Slopes of the resistance-time curves may be of value as an indicator of insulation condition. They are satisfactory indices of excessive moisture content in the insulation.

5. Insulation resistance-time curves are extremely useful to indicate the progress of winding dry-out.

6. Comparisons of insulation resistance-time curves taken at widely different d-c voltages may prove of value in determining insulation condition.

7. The voltage effect on winding insulation resistance appears to be greater as the winding temperature is reduced, or as the moisture content is increased.

8. Power factor tests have given little indication of value in locating faults or incipient faults in generator windings. However, power factor measurements may prove to be an indicator of the trend of the general condition of the insulation including moisture content in a given machine if records are kept of periodic readings.

9. Sixty-cycle ionization tests have given no indication of value in predicting generator insulation condition or locating weak spots in windings.

10. D-c/a-c (rms) relative dielectric strength of generator insulation appears to approach $\sqrt{2}$. Additional tests must be made to establish this more accurately.

11. Dielectric strengths of old armature windings have been found to be generally high.

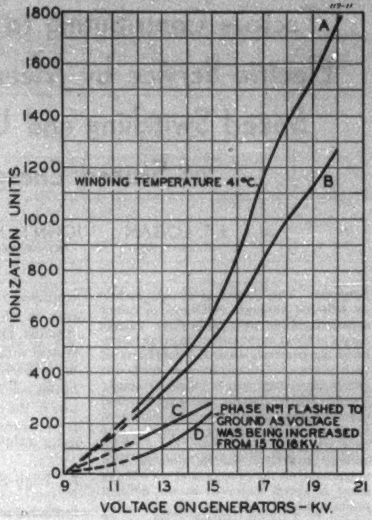
12. The end-turns of old generator windings appear to be weaker dielectrically than the slot sections.

13. Preferably, field windings should not be stressed to more than 500 volts d-c in routine testing.

Figure 11. Ionization intensity measurements on each phase of a generator winding

14,000-kw vertical turbo-generator, 7.8 kv, 60 cycles, 720 rpm

A—Phase 3
B—Phase 2
C—Phases 2 and 3 and section of phase 1
D—Section of phase 1



References

1. Progress Report—Investigation of Test Methods for Determining Operating Condition of Generator Insulation, **H. R. Stewart.** Electrical Equipment Committee, EEI, Eighth Meeting, Chicago, Illinois, October 7 and 8, 1935.

(Editors Note: EEI headquarters at 420 Lexington Avenue, New York, N. Y., advises that the foregoing item is out of print, but that a supplementary mimeographed pamphlet, "Guides for Initial, Periodic, and Experimental Testing of Generator Insulation" is available in a limited quantity.)

2. Insulation Resistance of Armature Windings, **R. W. Wieseman.** AIEE Transactions, volume 53, 1934 (June section), pages 1010–21.

3. The Characteristics of Insulation Resistance, **S. Evershed.** *Journal* of the Institution of Electrical Engineers, volume 52, number 224, December 15, 1913, pages 51–83.

4. Circuit-Breaker Recovery Voltage, **R. H. Park** and **W. F. Skeats.** AIEE Transactions, volume 50, 1931, pages 204–38.

5. University of Illinois engineering experimental station *Bulletins* 259 and 260.

6. Review by **T. Rich** of **G. J. T. Bakker** and **J. C. Van Staversen's** paper, The Insulation of Large High-Tension Synchronous Generators. *Electrician*, volume 102, number 25, June 21, 1939, pages 249–50.

7. Predetermination of the A-C Characteristics of Dielectrics, **A. Banos, Jr.,** and **J. B. Whitehead.** AIEE Transactions, volume 51, 1932, pages 392–403.

8. Insulation Tests of Electrical Machinery Before and After Being Placed in Service, **B. L. Barns** and **C. M. Gilt.** AIEE Transactions, volume 48, 1929, pages 656–65.

9. Insulation for High-Voltage Alternators, **J. F. Calvert** and **C. M. Laffoon.** AIEE Transactions, volume 54, 1935 (June section), pages 624–31.

10. Testing Breaker Insulation by Power-Factor Methods, **F. C. Doble.** *Electrical World*, volume 98, number 9, August 29, 1931, pages 374–5.

11. Testing Electrical Insulation, **E. L. Doty.** *Electric Journal*, volume 35, May 1938, pages 183–5.

12. Insulating Materials—Conditions Which Cause Deterioration, **A. R. Dunton.** *The Electrician*, volume 107, October 9, 1931, pages 483–5.

13. Dielectric Strength Ratio Between Alternating and Direct Voltages, **W. N. Eddy** and **J. L. R. Hayden.** AIEE Transactions, volume 42, 1923, pages 593–9; discussion, pages 612–20.

14. Reconditioning Flood-Damaged Electrical Equipment. Publication number *F13*, Edison Electric Institute, December 1938.

15. The Effect of Heat on the Electric Strength of Some Commercial Insulating Materials, **W. S. Flight.** *Journal* of the Institution of Electrical Engineers, volume 60, February 1922, pages 218–35.

16. Testing Bushings and Insulation by Power-Factor Method, **I. W. Gross** and **H. E. Turner.** *Electrical World*, volume 103, numbers 2 and 3, January 13 and 20, 1934, pages 68–73 and 111–16.

17. The Properties of Dielectrics in Electric Fields Comprising Unidirectional and Alternating Components, **L. Hartshorn** and **E. Rushton.** The British Electrical and Allied Industries Research Association, London, WC2; *Technical Report* L/T 79, July 28, 1936.

18. Improvements in Insulation for High-Voltage A-C Generators, **C. F. Hill.** AIEE Transactions, volume 47, 1928, pages 845–52.

19. High-Voltage Measurements on Cables and Insulators, **C. L. Kasson.** AIEE Transactions, volume 46, 1927, pages 635–47.

20. An Automatic A-C Potentiometer and Its Application to the Nondestruction Testing of Insulating Equipment, **G. Keinath.** AIEE Transactions, volume 58, 1939, pages 887–90.

21. The Influence of Moisture Upon the D-C Conductivity of Impregnated Paper, **G. T. Kohman** and **D. C. McLean.** *Journal* of the Franklin Institute, volume 226, August 1938, pages 203–20.

22. The Predominating Influence of Moisture and Electrolytic Material Upon Textiles as Insulators, **E. J. Murphy** and **R. R. William.** AIEE Transactions, volume 48, 1929, pages, 568–75.

23. Testing of Insulation, **C. R. Reid.** *Electrical News and Engineering*, volume 46, July 1, 1937 page 212.

24. Temperature Limits and Characteristics of Mica as Used in Conjunction With Class "B" Insulation, **Robert H. Spry.** AIEE Transactions, volume 58, 1939 (June section), pages 287–9.

25. The Physics of Insulating Materials, parts I–V, **A. M. Thomas.** *World Power.*

I—Faraday, Kelvin, Maxwell, Mossotti, Debye, Hopkinson, J. J. Thomson, volume 14, December 1930, pages 485–91.

II—Dielectric Absorption and A-C Losses, volume 15, January 1931, pages 23–7.

III—Conductivity of Dielectrics, volume 15, February 1931, pages 113–15.

IV—Failure of Dielectrics Under Electric Stress—Gases and Liquids, volume 15, March 1931, pages 196–8.

V—Failure of Dielectrics Under Electric Stress—Solids, volume 15, April 1931, pages 307–10.

26. Generator-Insulation Tests, **B. Van Ness.** Edison Electric Institute *Bulletin*, volume 6, May 1938, pages 204–06, 231.

27. The Drying of Cellulose Insulation and Electrical Measurements as Measures of Dryness, **Emerson Venable.** Abstracted in Electrical Engineering, volume 58, January 1939, page 41.

28. Gaseous Ionization in Built-up Insulation, **J. B. Whitehead.** AIEE Transactions, volume 42, 1923, pages 921–39. Discussion, pages 953–8.

29. Gaseous Ionization in Built-up Insulation—II, **J. B. Whitehead.** AIEE Transactions, volume 43, 1924, pages 116–26.

Factors Contributing to Improving Electric Service by Means of High-Speed Switching and Utilization of Stored Energy

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Synopsis: The quality of electric service as reflected through variations in motor speeds resulting principally from voltage surges is about as important to many manufacturing processes as is the continuity of such service. This paper sets forth the effects of voltage surges on an extremely speed-sensitive manufacturing process and describes how the effect can be minimized to the extent of becoming harmless. Test data are given which show the effects of voltage surges, of varying duration and magnitude, on the process, and the voltage surge is shown engraved in the finished product as the purchaser sees it. The maximum tolerable driving motor speed variation, without impairment to the product, is established and means are described for preventing a greater variation irrespective of the voltage surge intensity. An alarm system is described which is capable of instantly responding when variations in speed exceed tolerable limits. Use of a synchronous condenser, to supply energy during the interim of severe voltage disturbances or brief power failures, is effective in preventing motor-speed variations from exceeding the tolerable limits. These and other recent improvements will provide highly reliable electric service for the requirements of those manufacturing processes which are very exacting as to speed variations.

ACCORDING to reliable statistics, approximately 85 per cent of automatic interruptions to overhead power circuits result from transient disturbances which are removed within a fraction of a second after the circuit is de-energized, leaving the circuit undamaged. Experience backed by tests conducted in various classes of industrial plants has established the fact that power failures of from one-half to one second duration do not constitute a failure of service, provided the energy stored in current consuming apparatus is utilized during the interim such apparatus is disconnected from the normal power supply. It follows, therefore, that when a power circuit is interrupted from transient causes, if reconnected to the normal power source quickly enough, an interruption to service is averted. Under this practice the circuit breaker employed for controlling the power circuit, in effect, becomes a device which removes disturbances from the power system without impairing service. This paper describes the use of circuit breakers in this manner, together with related factors in the order of their contribution to the improvement in reliability and quality of electric service.

I. High-Speed Circuit Reclosure

A series of tests was made on the Georgia Power Company's system in June 1931 for establishing the feasibility of high-speed circuit reclosure. An automatic circuit breaker reclosing relay, providing for reclosing the breaker without delay after its initial tripping, was applied to a circuit breaker through which some 6,000 kw of industrial load was supplied over a 38-kv overhead circuit. The breaker controlling this circuit was capable of making the complete operation, from closed position to open and closed position again, in about 40 cycles. (All references to frequency are on a 60-cycle basis.)

Time-delay undervoltage-type push-button stations were installed for controlling several motor contactors in one of the textile plants served from this circuit, replacing conventional instantaneous-type push-button stations. The 38-kv circuit was then short circuited several times by drawing pieces of fuse wire across phases. In every test the arc was extinguished by the circuit breaker opening and did not restrike when the circuit was reconnected to the power source, after having been disconnected for about 40 cycles. All motors in the plant equipped with instantaneous-type push-button stations were shut down from the voltage disturbances, but those controlled by the time-delay undervoltage devices continued in operation. The maximum speed drop of these particular motors which were driving twisting frames varied between 10 and 15 per cent during the interim. According to textile experts these speed variations did not appreciably lower the quality of the yarn, and the strain on the motors appeared to be less severe than restarting them from a standstill.

The tests attracted considerable interest and new reclosing relays, or modified conventional reclosing relays, were developed for this new reclosing practice, which was quickly adopted by several power companies in the Southeast.

High-speed circuit reclosing with a duty cycle of 0–15–120 seconds was applied on most of the circuit breakers controlling overhead circuits on Georgia Power Company's system. Certain motor operated breakers necessitated delaying the second reclosure for 45 seconds or more, in order to provide time for the operating mechanism to relatch after tripouts immediately following the first reclosure. After about 15,000 automatic operations of circuit breakers equipped for high-speed circuit reclosure, not one breaker failure has been traced to this reclosing practice.

An early obstacle to the application of high-speed circuit reclosing was the time required for certain types of protective relays to part contact after closing to trip the breaker. The geared-type relays, in particular, then required about three-fourths of a second to part contact and obviously would retrip the breaker upon the initial reclosure. New contact arrangements providing for contact separation in one-fourth second or less were employed.

More than 12,000 interruptions to circuits supplying loads varying from a few hundred kilowatts up to 25,000 kw have been prevented on the Georgia system through the medium of high-speed circuit reclosure during its ten years' use, the effects of circuit tripouts being no more harmful than ordinary voltage surges. Below is a table showing the ten years' performance of certain circuits equipped for high-speed reclosure:

Number of Circuit Tripouts	10,090	100%
Successful Reclosures:		
1st instantaneous	8,400	83.25%
2nd 15–45 seconds	1,084	10.05%
3rd 120 seconds	143	1.42%
Circuit Lockouts	533	5.28%

Actually, the proportion of successful first reclosures including all operations is

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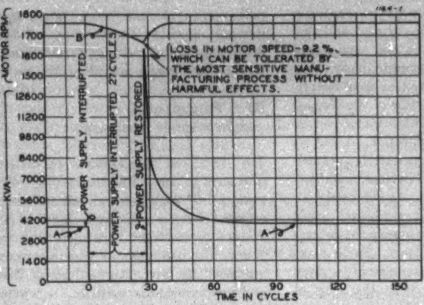


Figure 1. Results from oscillographic analysis of effectiveness of high-speed circuit reclosure in preventing interruptions

Note that load, curve *A*, after circuit tripout and reclosure is practically normal within one-half second after being reconnected to the power supply, and that the speed variation, curve *B*, of a 7.5-horsepower motor is negligible

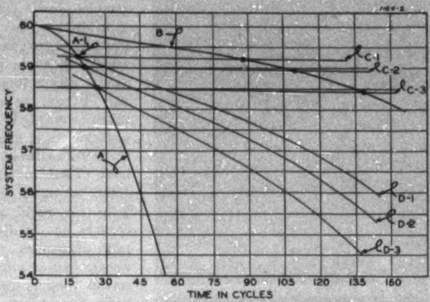


Figure 2. Inertia-relay operating characteristics

Curves *A* and *B* represent, respectively, speed changes for complete and partial loss of power source. Curves C-1, C-2, C-3, and D-1, D-2, D-3 represent, respectively, three of several possible relay-response slopes without and with the inertia element damped. The relay closes contact where slope, say C-1, crosses curve *A* or *B*

above 85 per cent instead of the 83.25 per cent shown, because the tabulation includes 2.3-kv and 4-kv circuits, the performances of which are hampered by tree grounds; and omits some 5,000 operations of 38-kv and 44-kv circuits supplied from automatic or semiautomatic stations where the records might be questioned for lack of devices capable of accurately recording the sequence of breaker operations.

Power consumers, particularly the industrial class, receive the greatest benefits from high-speed circuit-reclosing practices. According to records and observations a power failure, if for only a few seconds, stops completely or seriously curtails the average industrial plant output for five to ten minutes and in many cases as much as one-half hour.[1]

The portion of the load that actually "coasts through" the time required for clearing faults from and reconnecting the circuit to the power source varies, but when motor controls are adapted for this practice, as described under section II following, the service interrupted is negligible, the effects being no more harmful than disturbances that cause moderate light flicker. Figure 1 shows oscillographic results of a test for determining the amount of load actually interrupted during the interim the circuit is disconnected from the power source. The load was disconnected from the normal power source for 27 cycles during which time stored energy supplied the requirements. The fact that the magnitude of the load was practically normal within one-half second after being reconnected to the normal power source is proof enough that for all practical purposes there was no service interruption.

II. Application of Time-Delay Undervoltage Devices

Control apparatus usually supplied with motors incorporates undervoltage protection for the larger or more important motors. Unless specified by the purchaser, this protection is generally of the instantaneous response type and will shut down the motors for voltage dips on the power supply.[2,3] This necessitates restarting the motors after each voltage dip, with resulting loss in production and, in many cases where continuity is essential, of upsetting the entire process for extended periods of time with consequent heavy loss in raw material. Application of suitable time-delay undervoltage protective apparatus will reduce such losses to the practical minimum.

This improved type of undervoltage protection is becoming standard practice, the older instantaneous types having proved entirely unsatisfactory for use on important motors supplied from large interconnected power systems, because of the large number of voltage surges resulting from atmospheric disturbances usually in the form of lightning or sleet. The great majority of these voltage disturbances are cleared from power systems within one second or less, or the fault is so far removed that the surge voltage at the particular location does not drop below 65 per cent of normal. Consequently, the disturbances should be of little concern when the motors are properly protected by time-delay undervoltage devices

Voltage surge tests conducted in many textile plants employing speed-sensitive manufacturing processes indicate that the drop in motor speed in event of a

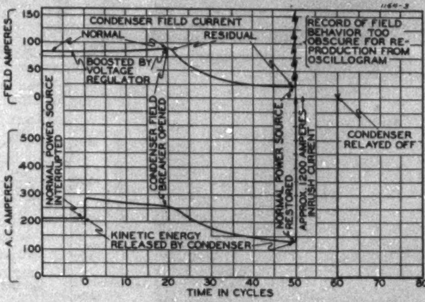


Figure 3. Graphic representation showing effects of high-speed circuit reclosure on synchronous condensers left connected to the circuit with the excitation source removed

Slow decay in field current makes this an unsafe practice. Condenser should be disconnected some 10 to 15 cycles in advance of restoring normal power source

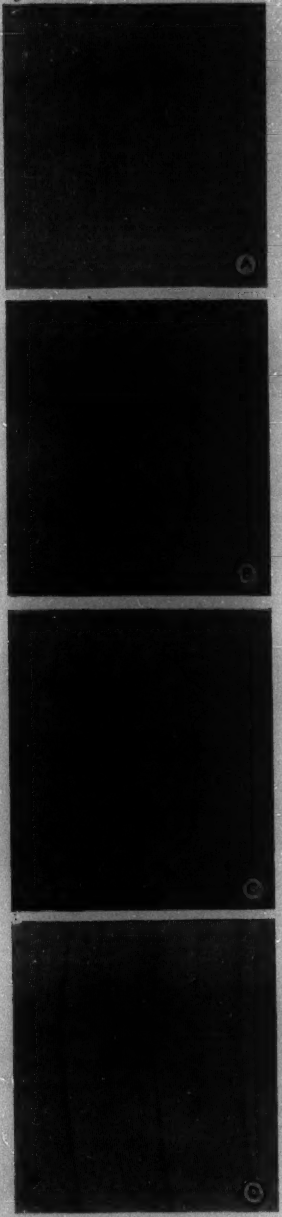


Figure 4. Swatches of hosiery tubing knit from yarn produced under conditions of repeated voltage surges imposed on the power supply

	Surge Voltage (Per Cent)	Surge Duration (Cycles)	Motor-Speed Variation (Per Cent)	Yarn Quality
A	60.0	61	2.2	Good
B	0.0	31	10.2	Good
C	14.7	61	13.7	Defective
D	13.5	122	27.9	Poor

complete power failure for one second is generally less than 20 per cent and that shutting down motors for such speed variations would do more harm than good, even ignoring the trouble of restarting. In fact, the practice of allowing essential motors to be shut down for voltage disturbances of any duration, so long as the motor continues to rotate, is questionable. There are links in many manufacturing processes that are adversely affected by rapid and prolonged speed changes, but usually stopping the process only aggravates matters. Such processes or links in processes should be given special consideration with the view of preventing objectionable voltage surges from being reflected on to the power source driving the particular motors. Means for accomplishing this are discussed under section V.

Continuity of operating through voltage disturbances is not essential, from the standpoint of production or quality of the product, for certain motors in most industrial plants. Whether these motors should be protected by time-delay undervoltage devices is a matter of individual consideration.

Synchronous motors, of course, require special consideration and generally should be disconnected from the line as quickly as possible following an out-of-step condition, unless the motor has excellent pull-in torque characteristics. Even then it would probably be better to provide field removal and restoring features and, possibly, automatic unloading devices.

III. Treatment of Synchronous Apparatus

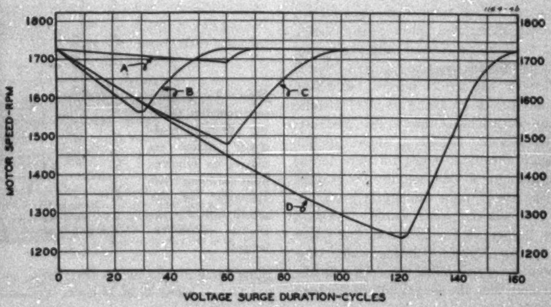
A 3,000-kva synchronous condenser connected to one of the circuits originally equipped for high-speed circuit reclosure required that the initial reclosing time be delayed to allow the fault more time to clear. The condenser in some instances prolonged the arc. An analysis of this problem revealed that suitable relays were not available for the purpose of removing the condenser from the line quickly enough, in order to avoid purposely delaying the initial reclosing time as much as one second or more, thereby defeating the purpose of high-speed circuit reclosure.

Reverse power and frequency responsive relays were tried but when adjusted to operate in about one-fourth of a second, the maximum allowable without purposely delaying the circuit initial reclosure, the condenser would often be unnecessarily disconnected.

In order to solve this problem, the authors developed a relay operating on a new principle, that is, the rate of speed or frequency change. This device, called an inertia relay, combines high speed response with reliability and is ideally suited for not only this application but for other uses discussed later.[4]

The relay differentiates between the relatively gradual rate of speed or frequency change on large power systems where millions of pound-feet[2] of inertia are involved and the rapid and erratic changes that occur on small portions of the system suddenly cut loose from the main power source and left without, or greatly deficient in, power supply.

Referring to figure 2, which shows the inertia-relay operating characteristics,



the relay can be adjusted to operate at any desired point along curve *A*, say at point *A*-1, for disconnecting synchronous condensers or an overloaded generating plant from circuits equipped for high-speed reclosing, this being accomplished in time to allow reconnecting the circuit to the system without delaying the reclosure. The maximum speed change expected on a power system under conditions of ordinary frequency disturbances follows some line usually well above curve *B*, to which the inertia relay will not respond if adjusted to operate on the slope represented by characteristic curves *D*-1, *D*-2, *D*-3 or some other of this family of curves. The relay can be adjusted to operate on the slope represented by curves *C*-1, *C*-2, *C*-3, and so on, if desired by simply lessening the damping effect on the inertia element. Also, the relay can be adjusted to respond within ten cycles or less for curve *A*. However, this would usually be undesirable because of waste in the kinetic energy stored in the rotors of spinning capacity so disconnected.

One of these relays is employed on the Georgia system for sectionalizing a 45,000-kw load in a manner for leaving load within capacity on a local 16,000-kw generating plant when the primary power source, a 110-kv transmission line, is involved by transient disturbances.[4] The relay detects the loss of the 110-kv source and accomplishes sectionalization in time to permit high-speed circuit reclosure to the transmission line without intentional delay, thus avoiding interrupting any portion of the load for transient faults involving the 110-kv source.

Five synchronous condensers and five small automatic hydroelectric plants connected to circuits equipped for high-speed circuit reclosure are also supervised by these relays. Four other installations for similar purposes are now under way.

Incidentally, the relay in addition to sectionalizing the 45,000-kw load area referred to above has proved to be faster and more positive than protective relays in disconnecting the faulted 110-kv line from the hydroelectric plant in time to prevent "swamping" the plant. This could not be done satisfactorily with conventional power directional relays, consequently, two other applications of the inertia relay for similar purposes are being studied.

Whether to disconnect synchronous condensers operating on circuits equipped for high-speed reclosure or to remove their sources of excitation only, during the interim the circuit is disconnected from the power source is a question frequently referred to the authors. Figure 3 shows the results of removing the excitation source only, and indicates that the magnetic energy stored in the machine is not dissipated in the field discharge resistor rapidly enough to make this a safe practice.

IV. Variation Tolerance in Motor Speeds

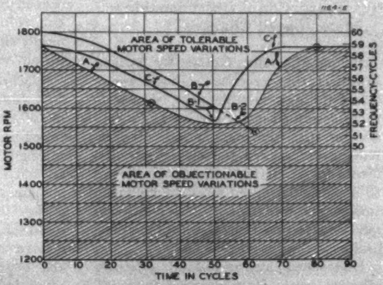
The foregoing sections of this paper deal primarily with improving the quality of electric service by reducing what otherwise would be momentary interruptions to electrical disturbances no more harmful than ordinary voltage surges.

Moderate voltage disturbances can be tolerated by the majority of users of electric service without harmful effects. However, some manufacturing processes are adversely affected from voltage surges to the extent that precautionary measures must be taken for minimizing the effects. In many instances the effects of voltage disturbances have been considered so damaging to the finished product that a local power supply is provided in preference to accepting the consequences of voltage surges that accompany electric service supplied from large transmission systems. A few of the processes which are unusually sensitive and adversely affected by voltage disturbances are employed in the manufacture of rayon and silk; high quality cloth; pulp and paper; broadcasting stations; printing presses.

The damaging effects of voltage disturbances are transmitted to the finished product generally through driving motor speed changes that result from the drop in voltage. Realizing the seriousness of this problem the authors have conducted extensive tests jointly with industries served by Georgia Power Company for the purpose of establishing what might be called a voltage "tolerance factor." The precision with which such a tolerance factor has been established for an unusually delicate or speed-sensitive process should be of interest, particularly to those who question the ability of a power system, subject to frequent lightning storms, to render satisfactory service to industries employing processes which require almost constant speed.

The test procedure provided for spinning yarn while the spinner driving motor power supply was being subjected to repeated artificial voltage disturbances of varying intensity and duration. Oscillographic records of the voltage disturbances and motor speeds were obtained for each series of tests and the yarn marked in a manner for identifying that made under each of four surge test conditions. The quality of the yarn with respect to variations in size and dyeing characteristics, was determined with uncanny accuracy by textile engineers. So accurate were the methods employed, that the yarn size could definitely be tied in with the motor speed, even to the extent of distinguishing that produced during the period of decreasing motor speeds from that produced during the period of accelerating speeds which followed immediately the restoration of normal voltage.

Sections of the four batches of yarn produced under power surge conditions were next knit into hosiery tubing and dyed in a sensitive shade for bringing out dyeing defects. Figure 4 is a reproduction of a photograph of the swatches of hosiery tubing which were knit from yarn spun during the four surge tests. Associated with each swatch is a motor speed curve, corresponding alphabetically with the swatch designation, showing the motor-speed variation for each artificial voltage surge which was repeated every 20 seconds during the test. The effects of the voltage surges as reflected through the driving motor can be seen as bars, faintly in swatch *C* and clearly in



Motor-speed variations in the area above curve *A*, do not damage material produced under extremely speed-sensitive processes. Curve *B* represents the frequency produced by a 2,000-kva synchronous condenser when supplying 2,700 kw of load. Curve *C* represents speed of motors so driven. Load was disconnected from condenser at *B*-1 and reconnected to the normal power source; otherwise, motor speed would have followed trend of *B*-2

Figure 5. Tolerable and objectionable motor-speed variations

swatch *D*. The yarn produced under surge conditions represented by swatches *A* and *B* is classed as normal production by the textile engineers.

A summary of conclusions reached after analyzing the test results follows:

1. The principal interest of the textile company in regard to the effects of electrical disturbances may be measured by the degree to which the spinning machine driving motor speed is affected. Motor-speed variations of not more than 10 per cent in one-half second can be accepted without great concern. Motor-speed variations of 15 per cent during a one-second period lead to variations in yarn size and dyeing disturbances which may be considered objectional. Speed variations on the order of 25 to 30 per cent during a two-second interval lead to pronounced loss of dye quality, due to marked variation in yarn size.

2. A signal system responding to motor-speed changes greater than that tolerable could be employed to advantage for advising the spinning production supervisor that yarn being spun was outside acceptable limits and should be segregated from regular production.

3. Voltage disturbances which do not reduce the driving motor torque below the pullout value are of little concern as the motor slip is increased less than ten per cent. Disturbances that reduce the torque below the pullout value, usually about 65 per cent normal voltage, must be limited to one-half second if the yarn spun is kept within acceptable limits.

4. The effects of voltage disturbances can be reduced to tolerable limits by means of high-speed clearing of faults, or the use of synchronous apparatus for maintaining voltage during system disturbances, to a value above the motor pullout point.

The signal system referred to in (2) above would require an initiating relay of unusual accuracy; otherwise yarn outside acceptable limits might get through unnoticed until too late, thereby causing a large shipment to be rejected as being outside acceptable limits. The inertia relay is ideally suited to initiate a signal for this purpose.

V. Prevention of Objectionable Motor-Speed Changes

Voltage-tolerance factors for various manufacturing processes can be established through suitable tests, similar to those described under section IV in connection with the manufacture of yarn. Having determined the voltage requirements for rendering satisfactory service to a particular speed-sensitive manufacturing process, means for restricting motor-speed variations to within tolerable limits can be considered.

High-speed clearing of faults offers an effective means for preventing damaging motor-speed changes. It was determined from tests described under section IV that zero voltage for one-half second was not objectionable in this particular extremely speed-sensitive process, in that the quality of the yarn was not appreciably lowered by subjecting the spinning motor power source to repeated power failures of one-half second duration. Refer to *B*, figure 4. For reasons more attractive than the prevention of damaging motor-speed variations, high-speed clearing of faults on transmission networks is very desirable and in many instances essential in order to promote system stability and reduce damages from power arcs. Unfortunately, however, this means for preventing motor-speed variations cannot be universally applied, principally because of fuse coordination problems which are encountered on most power circuits supplying industrial plants.

Objectionable motor-speed variations resulting from voltage disturbances reflected on to industrial plant power sources from the transmission system can be prevented where local generation is available and at a very reasonable cost. This may be accomplished by inserting a reactor between the plant bus and the transformer bank connecting the plant to the transmission system. The impedance of the reactor should be such that the local generating source can maintain voltage on the plant bus in excess of motor pullout values, usually about 65 per cent normal. Such an arrangement would limit motor-speed changes during the time of voltage disturbances on the transmission system to less than ten per cent or to within tolerable limits for even the most delicate or speed-sensitive manufacturing processes. Refer to *A*, figure 4. The reactor should preferably be normally short circuited by a high-speed circuit breaker controlled automatically by the transmission system voltage The circuit breaker should be opened only when the transmission system voltage, as measured on the transformer side of the reactor, is below motor pullout values.

Synchronous condensers offer a reliable means for preventing objectionable motor-speed variations. Kinetic energy stored in condenser rotors is generally sufficient for supplying a load equal to the kilovolt-ampere rating of the condenser for one second or more before the frequency drops to a value detrimental to speed-sensitive processes. This would involve high-speed switching, the load and synchronous condenser being disconnected from the main power source the instant the bus voltage dropped below motor pullout values. When voltage returned to normal on the main power source, the condenser would be disconnected from the load a few cycles in advance of transferring the load back to the main power source The condenser would then be returned to normal operation in the usual manner, preferably by automatic means

Figure 5 shows results when supplying 2,700 kw of load from a 2,000-kva synchronous condenser for 50 cycles, curve *B* representing the frequency supplied by the condenser. Motors driven from the synchronous condenser followed curve *C* which remained above curve *A*, the boundary of tolerable motor-speed variations for unusually speed-sensitive manufacturing processes. The condenser was disconnected from the load at *B*-1, a few cycles in advance of reconnecting the load to the main power source. Transfer of the load from the failing power source to the synchronous condenser and back again to the normal power source was accomplished smoothly by automatic control means. The voltage regulator controlling the condenser excitation prevented the bus voltage from dropping below 80 per cent normal which was well above the motor pullout value.

The use of synchronous apparatus connected to circuits equipped for high-speed reclosure for reducing voltage and frequency disturbances during the interim the circuit is disconnected from the normal power source is standard practice on the Georgia Power Company system. This practice is made possible through the inertia relay development which provides for positive isolation of the synchronous apparatus a few cycles in advance of reconnecting the circuit to its normal power source. The result gives the appearance of a short interval of reduced rather than apparent zero voltage. Best results are had by keeping the synchronous apparatus connected to the load until some 10 to 15 cycles prior to the restoration of the normal power source Reference to figure 3 shows that by this practice a great deal of kinetic energy can be fed into the load from synchronous machines thereby reducing the severity of voltage disturbances.

References

1. Faster Reclosing Breakers Needed, **J. T. Logan.** *Electrical World*, April 21, 1934, pages 571–2.

2. Notes on Motor and Circuit Control for Textile Mills, **Marshall E. Lake.** *Textile Bulletin*, July 1, 1940, pages 25–7.

3. Decreasing Lost Machine Time With the Undervoltage Time Delay, **Edwin M. Clapp.** *Cotton*, August 1933.

4. Inertia Relay "Anticipates" to Facilitate Reclosure, **J. T. Logan and J. H. Miles.** *Electrical World*, April 6, 1940, pages 69–72.

High-Pressure Gas as a Dielectric

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Synopsis: Impulse and 60-cycle breakdown strength of sphere gaps, rod gaps, and of samples built up of solid and fluid dielectrics are reported. The tests were made in nitrogen up to 200 pounds per square inch gauge pressure and in Freon up to 70 pounds per square inch gauge pressure. Dielectric strength values were compared with the breakdown of transformer oil between the same electrodes. Gap conditions were found for which the 60 cycle breakdown was twice its impulse breakdown.

RECENT interest in the dielectric strength of gases under pressure has been stimulated by the steadily increasing voltages used in power transmission as well as in X-ray and atomic physics work and by the shortcomings of atmospheric air and of oil insulation. Although scores of articles have been published on high pressure gas dielectrics during the last forty years, they contain practically no information on the impulse dielectric strength of gases at high pressure. This is a serious omission from the point of view of those interested in generating and distributing apparatus since the kind and amount of insulation in these apparatus are influenced greatly by impulse strength. The theoretical background related to the breakdown of gaps in gases is much too incomplete to permit the extrapolation of available data much beyond the exact conditions under which the data were obtained. Therefore, numerous general as well as specific breakdown data must be accumulated in order to make the proper utilization of high pressure gases as dielectrics.

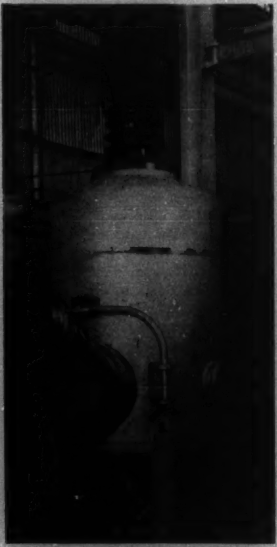

Figure 1. High-pressure electric test tank

This paper will present impulse and 60-cycle breakdown data between gaps in nitrogen up to 200 pounds pressure and in dichlorodifluoromethane (dichlorodifluoromethane, commercially known as "Freon" will be referred to in this paper as CCl_2F_2) up to 70 pounds gauge pressure. In each case, a comparison will be made with the breakdown of transformer oil at atmospheric pressure between the same gaps.

Recent Work

Several valuable contributions have been made to our fund of engineering knowledge of high pressure gas dielectrics during the last decade. Paschen's law, which was looked upon by many as a basic law of the dielectric strength of gaps at different spacing and pressure, has been shown to diverge far from the data at pressures above about six atmospheres. This law states that the sparkover voltage of a uniform field gap in a gas is directly proportional to the product of the gap spacing and the absolute pressure. The breakdown of a fixed gap is lower than the breakdown predicted by Paschen's law at pressures above 100 pounds per square inch. In fact the data[1,2] indicate that for air tested between spheres at a pressure of 600 pounds per square inch the actual sparkover is 55 percent of that predicted from atmospheric pressure data by Paschen's law.

Some gases have been found[3] which have as much as three times the 60 cycle dielectric strength of nitrogen when tested in a uniform field at atmospheric pressure. Notable among them are CCl_3F with a dielectric strength of three times that of nitrogen and CCl_2F_2 the gas commercially known as Freon and in common use in mechanical refrigerators, with a dielectric strength of 2.4 times that of nitrogen at atmospheric pressure.

Most of the earlier dielectric strength tests were made at low pressure or small spacings. Recently d-c breakdown tests[3] on spheres have been made at pressures up to 600 pounds per square inch and 450 kv breakdown. These data show also how slight roughnesses of the electrodes reduce the breakdown by as much as 50 per cent at 600 pounds pressure. Other investigators[4] made test in air and nitrogen up to 1,700 pounds per square inch. They indicate that the rate of increase of dielectric strength of air with increasing pressure continues to decrease at least up to 1,700 pounds per square inch pressure. This and other work[5] indicate that although the dielectric strength of air and nitrogen are practically the same at atmospheric pressure, the dielectric strength of nitrogen increases less rapidly than that of air with increased pressure, particularly above 100 pounds per square inch.

Apparatus

The experimental data presented in this paper were obtained by tests in the special pressure tank shown in figure 1. The pressure tank is four feet in diameter by seven feet high. Although it was tested hydrostatically up to 400 pounds per square inch pressure, it has not been used above 200 pounds gas pressure for safety reasons. It has three five-inch diameter sight windows made of discs of

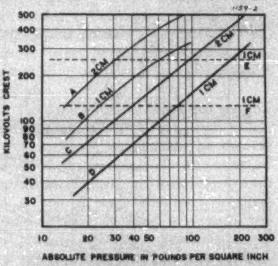

Figure 2. 60-cycle and impulse breakdown between 6.25 centimeter spheres

Curves *A* and *B* = CCl_2F_2, impulse and 60-cycle
Curves *C* and *D* = nitrogen, impulse and 60-cycle
Curve *E* = oil, impulse at atmospheric pressure
Curve *F* = oil, 60-cycle at atmospheric pressure

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1. For all numbered references, see list at end of paper.

methyl-methacrylate and an 18 inch steel door. A single high voltage bushing was used to carry the voltage into the pressure tank. This bushing was of the oil-filled type with a porcelain shell. The bushing was the same size both inside and outside the tank so that it could be used at maximum voltage with atmospheric pressure in the tank.

Two devices were used to adjust or change the electrodes inside the tank without opening the tank. One consisted of a screw which was operated by a shaft passing through the tank wall to a calibrated hand wheel. The gap electrode was mounted on the end of the screw and could be spaced accurately from the outside of the tank. The other consisted of a revolving platform installed in the bottom of the tank. The test samples which were made up of solid and gaseous dielectrics were good for only one breakdown determination. As many as twelve of these solid dielectric samples were placed on the revolving platform. These samples were connected, one at a time, to the voltage source by turning the turn table in the proper position with a driving mechanism which ran through a packing gland in the tank wall. These mechanisms saved a large amount of time by avoiding the necessity of opening the tank after each test.

Gases used were of a commercial grade obtained in high pressure tanks. The nitrogen was dried by passing it through a trap surrounded by liquid air. The CCl_2F_2 with high liquefying temperature was dried in columns of calcium chloride.

The CCl_2F_2 was drawn from the test tank through a compressor which recondensed the gas into its liquid phase and pumped it into a high pressure storage tank for future use.

The 60 cycle test voltage was supplied by a 350 kv, 1,000 kva testing transformer. The voltage was varied by controlling the field on the alternator connected to the testing transformer. The impulse voltage was supplied from a 500 kv impulse generator. The impulse voltage wave shape was checked and the breakdowns were observed with a cathode ray oscillograph connected across the test electrodes through a resistance divider. All impulse data presented herein were obtained with a $1^1/_2$–40 microsecond impulse wave.

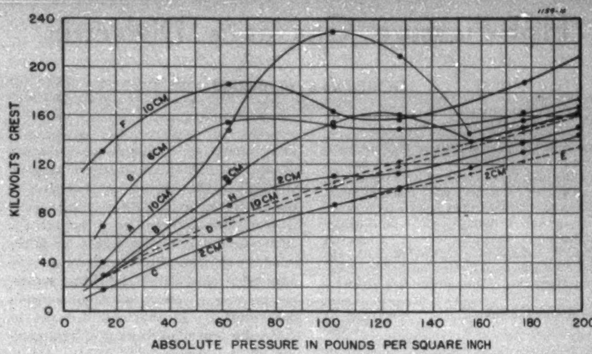


Figure 4. 60-cycle and impulse breakdown between one-half-inch square rod gaps in nitrogen

Curves *A*, *B*, and *C* = 60-cycle breakdown

Curves *D* and *E* = 60-cycle corona starting voltage

Curves *F*, *G*, and *H* = impulse breakdown

Test Results

Breakdown tests were made with the following three types of electrodes:

1. 6.25 centimeter diameter brass spheres.
2. Standard $^1/_2$ inch square brass rod gaps and
3. Electrode arrangements which included solid dielectrics.

The results of experiments made between the 6.25 centimeter spheres are summarized in figure 2. These data show the dielectric strength of nitrogen to increase as the 0.86 power of the pressure instead of increasing as the first power of the pressure in accordance with Paschen's law. The dielectric strength of CCl_2F_2 increases at a slightly smaller power of the absolute pressure than does the dielectric strength of nitrogen.

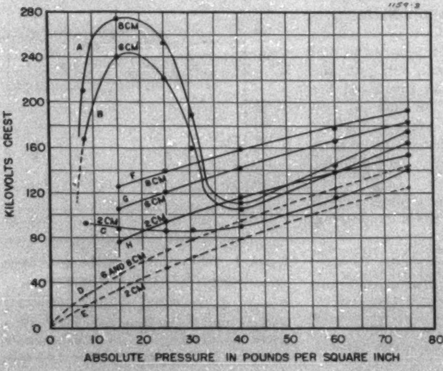


Figure 3. 60-cycle and impulse breakdown between one-half-inch square rod gaps in CCl_2F_2

Curves *A*, *B*, and *C* = 60-cycle breakdown

Curves *D* and *E* = 60-cycle corona starting voltage

Curves *F*, *G*, and *H* = impulse breakdown

These gas breakdown curves represent both the crest value of the 60 cycle breakdown voltage and the crest value of the impulse voltage breakdown with a $1^1/_2$–40 microsecond voltage applied. The 60 cycle tests were made by increasing the voltage at a rate such that the voltage increased from approximately half of breakdown voltage to breakdown in one minute. An elapsed time of one minute was allowed between successive voltage applications. The average of ten breakdown determinations was used to establish a single datum point. The maximum spread of individual values was rarely over five per cent for the 60 cycle tests.

The impulse tests were made by applying five impulses of the same voltage at five or more steps two per cent apart in voltage, above and below the breakdown voltage. The voltage was lowered in two per cent steps until none of the five impulses broke down the gap. The voltage was also raised in two per cent steps until each of the five impulses at a particular voltage broke down the gap. The statistical voltage at which half of the impulses resulted in breakdown was taken as the breakdown value of the gaps. The maximum spread of individual values was with few exceptions not greater than eight per cent of the breakdown value.

Observations showed that the breakdown of a gap was lower immediately after a breakdown. Therefore a procedure was followed such that a five minute interval elapsed between a flashover and the succeeding impulse application. Only one-minute interval was used after a voltage application that did not result in breakdown of the gap before another impulse was applied. This spread in the impulse data was probably increased by the fact that there was no effort to increase the free ion concentration within the tank by radioactive salts or an ultra-violet light source.

Breakdown of 6.25 centimeter spheres spaced one centimeter apart in 10-C transformer oil is plotted here for comparison. The oil breakdown tests were made at atmospheric pressure only. The transformer oil had a 60 cycle breakdown of 33 kv in a standard oil test cup with 0.1 inch gap spacing. The impulse strength of transformer oil with a $1^1/_2$–40 microsecond wave is slightly over two times its 60 cycle dielectric strength, while the impulse and 60 cycle breakdown strength of spheres in the high pressure gases are the same.

Rod-Gap Tests

Requirements of commercial high voltage apparatus other than the electrical insulation strength often dictate that the conductors adjacent to the insulation be made of some other shapes than spheres or curvatures of large radii. The standard $^1/_2$ inch square rod gap was therefore chosen for test electrodes to characterize the more severe conditions of non-uniform field encountered in high voltage apparatus.

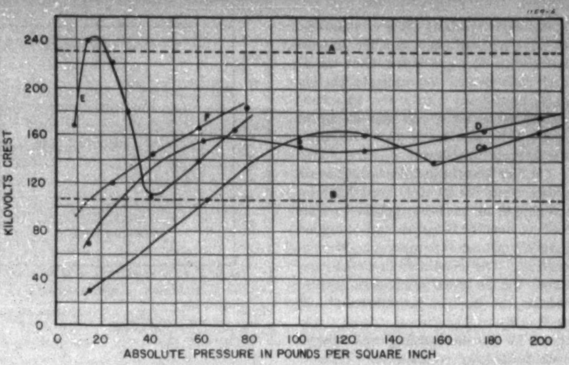


Figure 5. 60-cycle and impulse breakdown between one-half-inch square rod gaps spaced six centimeters in nitrogen, CCl_2F_2, and transformer oil

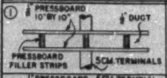
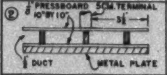
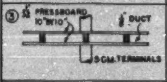
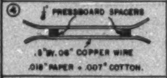
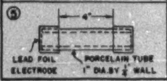
Curve *A* = impulse breakdown in oil
Curve *B* = 60-cycle breakdown in oil
Curve *C* = 60-cycle breakdown in nitrogen
Curve *D* = impulse breakdown in nitrogen
Curve *E* = 60-cycle breakdown in CCl_2F_2
Curve *F* = impulse breakdown in CCl_2F_2

60 cycle and impulse breakdown tests were made on this $^1/_2$ inch rod gap mounted in a vertical plane in the high pressure test tank, figure 1. Tests were made with both 60 cycle and $1^1/_2$–40 microsecond impulse voltages in nitrogen and CCl_2F_2 at various pressures and gap spacings. Figures 3 and 4 give the results of these tests. The corona starting voltage shown on figures 3 and 4 was taken by means of a cathode ray oscillograph which received its voltage from a high frequency amplifier placed across a shunt in series with the test gap

The 60 cycle breakdown of the rod gaps does not steadily increase with pressure. Figures 3 and 4 show how the breakdown voltage increases with pressure up to a critical pressure and then decreases with further increase in pressure until the breakdown of the gap is only slightly above the corona starting voltage of the gap. This critical pressure in the case of nitrogen is about 100 pounds per square inch gauge, while for CCl_2F_2 the critical pressure is only a few pounds above atmospheric pressure. The critical pressures are lower than those found by other investigators[1,4] on tests with point-plain electrodes. Goldman and Wul found the critical pressure for nitrogen to be about 150 pounds per square inch as compared with 100 pounds as shown in this report.

Table I. Relative Dielectric Strength of Solid and Fluid Dielectric Arrangements in Transformer Oil and in Gases Under Pressure

1159-T

INSULATION SET-UP	TYPE OF VOLTAGE	CREST BREAKDOWN IN PERCENT OF N°1 SAMPLE 10-C OIL BREAKDOWN		
		10-C. OIL	NITROGEN 200 LBS./SQ. IN. GAUGE PRESSURE	C Cl_2 F_2 70 LBS./SQ IN. GAUGE PRESSURE
 ① 1/8" PRESSBOARD 10" BY 10" 1/8" DUCT PRESSBOARD FILLER STRIPS 5CM. TERMINALS	60 CYCLE PUNCTURE	100	88	132
 ② 1/8" PRESSBOARD 10" BY 10" 5CM. TERMINAL 3 1/2" 1/8" DUCT METAL PLATE	60 CYCLE CREAPAGE	72	77	120
	1.5/40µS. IMPULSE	142	115	143
 ③ 1/32" PRESSBOARD 10" BY 10" 1/8" DUCT 5CM. TERMINALS	60 CYCLE PUNCTURE	56	37	
 ④ 1/8" PRESSBOARD SPACERS .3" BY .06" COPPER WIRE .018" PAPER + .007" COTTON.	60 CYCLE	41	26	
	1.5/40µS. IMPULSE	48	31	
 ⑤ 4" LEAD FOIL ELECTRODE PORCELAIN TUBE 1" DIA. BY 1/4" WALL	60 CYCLE	102	100	78

The impulse breakdown of rod gaps in nitrogen and CCl_2F_2 under pressure has only a remote resemblance to the 60 cycle breakdown curves. The impulse breakdown for rod gaps in nitrogen spaced at six and ten centimeters has a negative slope between 70 and 120 pounds per square inch, while the impulse breakdown of rod gaps in CCl_2F_2 increases steadily with increasing pressure. Figure 5 gives the impulse and 60 cycle breakdown of rod gaps spaced six centimeters apart in CCl_2F_2, nitrogen, and in transformer oil. The 60 cycle breakdown of CCl_2F_2 is about 2.2 times the impulse breakdown at atmospheric pressure. It is very unusual to have a condition where the impulse breakdown of a gap is less than the 60 cycle breakdown. For example, the impulse breakdown with a $1^1/_2$-40 microsecond wave for a rod gap in air at atmospheric pressure is about 1.8 times the 60 cycle breakdown.

The fact that the impulse breakdown is greater than the 60 cycle breakdown for rod gaps in air has been explained on the basis of time lag before the gap is ionized in such a manner as to make it highly conducting. The condition in CCl_2F_2 gas where the 60 cycle breakdown greatly exceeds the impulse breakdown should be worthy of additional study. The reason for this unusual increase in the 60 cycle breakdown is not clear. There is evidence that the space charge in the gap is responsible for this unusually high breakdown.

Solid-Fluid Dielectric Combinations

Five different arrangements of solid and fluid dielectrics were tested for breakdown when immersed in 10-C oil, nitrogen, or CCl_2F_2. Schematic arrangements of the dielectrics and a comparison of voltage breakdowns are made in table I. Extreme care was taken in preparing the samples for these tests to see that the oil samples were completely impregnated and free from bubbles and that the gas-filled samples were thoroughly vacuum-dried and kept dry until they were tested. All of these dielectric arrangements have very non-homogeneous fields. The dielectric constants of the paper, pressboard, and porcelain used in these tests are all two or more times as large as the dielectric constant of transformer oil. Therefore, where the oil and solid insulation are in series, the oil must stand a disproportionately large part of the voltage stress. Since the dielectric constant of the compressed gases is substantially unity as compared with 2.2 for that of transformer oil, the voltage across the gas dielectric in series with solid insulation is considerably more than the voltage across the oil if it were made to replace the gas. This condition alone puts the gas dielectric to a considerable handicap when used in series with solid dielectrics.

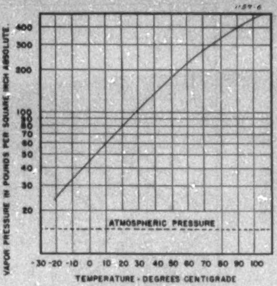


Figure 6. Vapor-pressure temperature curve of CCl_2F_2

It may be seen from table I that in general these combinations of solid and fluid dielectrics had higher dielectric strengths in transformer oil than they did in nitrogen at 200 pounds per square inch pressure or CCl_2F_2 at 70 pounds per square inch pressure.

Temperature Effects

High voltage apparatus filled with compressed gases may be subjected to a wide range of temperature either by virtue of their ambient conditions or by the heat liberated within them.

This change in temperature will change the dielectric strength of the gaseous dielectric in different ways according to the manner in which the gas is supplied to the vessel. The following are generalizations based upon theoretical considerations and checked experimentally for CCl_2F_2 between spheres.

(*a*). If gas were supplied in a manner such that a constant gas pressure was maintained, the dielectric strength of the gas would change inversely as the absolute temperature.

(*b*). If the total quantity of gas in the system is unchanged, the dielectric strength of the gas should remain unchanged with change in temperature. This follows from the fact that the dielectric strength of a gas is proportional to its density.

(*c*). If the gas is in a closed system similar to condition (*b*) but the gas is saturated and an excess of liquefied gas is present in the system, the gas pressure will change with temperature according to the vapor pressure of the gas at the particular temperature. The dielectric strength of the gas will change in such a system nearly the same as the vapor pressure of the gas. Vapor pressure of CCl_2F_2 is shown in figure 6. It is obvious that such a system has much greater change in dielectric strength of the gas with change in temperature than do the systems (*a*) and (*b*).

Summary

1. The impulse and the 60 cycle crest breakdown strength of nitrogen are substantially the same when tested between spheres at less than radius spacing; the impulse and 60 cycle crest breakdown strength of CCl_2F_2 are likewise the same. Nitrogen at 175 pounds per square inch absolute pressure, CCl_2F_2 at 62 pounds pressure and 10-C transformer oil at atmospheric pressure break down at the same impulse voltage when tested between 6.25 centimeter spheres spaced at one centimeter. Under the same conditions the 60 cycle breakdown voltages of the three fluids are the same with nitrogen at 80 pounds absolute pressure, CCl_2F_2 at 25 pounds absolute pressure, and 10-C oil at atmospheric pressure.

2. The fact that there is a condition where the 60 cycle breakdown of a rod gap in CCl_2F_2 is over two times the impulse breakdown of the same gap under the same condition is disturbing to the orthodox theory surrounding the electrical behavior of spark gaps in gases.

The 60 cycle breakdown between rod gaps is substantially higher in nitrogen above 60 pounds per square inch absolute or in CCl_2F_2 above atmospheric pressure than in transformer oil. On the contrary, the impulse breakdown between rod gaps is considerably lower in nitrogen under 215 pounds per square inch absolute pressure and CCl_2F_2 below 85 pounds per square inch absolute pressure than in transformer oil.

3. The breakdown strength of solid insulations tested in nitrogen at 215 pounds per square inch absolute pressure and in CCl_2F_2 at 85 pounds per square inch absolute pressure as compared to its breakdown in transformer oil indicates that with but one exception the tests made in transformer oil showed higher strength than when made in the gases at the above pressures.

References

1. Breakdown of Compressed Nitrogen in a Nonuniform Electric Field—II, I. M. Goldman and B. M. Wul. *Journal of Technical Physics*, USSR, volume 3, 1935, pages 16–27.

2. Breakdown Studies in Compressed Gases A. H. Howell. AIEE Transactions, volume 58 1939 (May section), pages 193–204.

3. Dielectric Strength of Insulating Fluids E. E. Charlton and F. S. Cooper. *General Electric Review*, volume 40, 1937, pages 438–42.

4. Durchschlaguntersuchungen in komprimierten Gasen und in flüssiger Kohlensaure, O. Zeier. *Annalen der Physik*, volume 14, 1932, pages 415–47.

5. Der Elektrische Durchschlag Verschiedener Gase Unter Hohem Druck, E. Finkelmann. *Archiv für Elektrotechnik*, volume 31, 1937, pages 282–6.

An Automatically Reclosing Breaker for Co-ordinating Distribution Systems

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Synopsis: The increasing spread of distribution systems in which fault protection is afforded by fuses has resulted in long outages and high maintenance costs. Excellent coordination is possible with modern fuses, but their inability to differentiate between a permanent and a temporary fault results in frequent outages and costly service trips.

A new automatically reclosing circuit breaker with pronounced inverse time current tripping characteristics makes possible an improved radial distribution system in which outage and maintenance are reduced to a minimum. The effectiveness of coordination is increased to a point where an outage is confined to the immediate neighborhood of a permanent fault. Service trips are no longer necessary unless there is a permanent fault condition to be removed from the system.

Extended Distribution Systems Require New Type of Protective Equipment

IN recent years there has been a large increase in the number of distribution systems having long lines and small loads scattered over considerable territory. Such systems represent an increase in the use and application of electricity as well as affording users the benefits and conveniences of a reliable source of power. The continued extension and successful operation of these systems depends upon a low initial cost and a low maintenance cost.

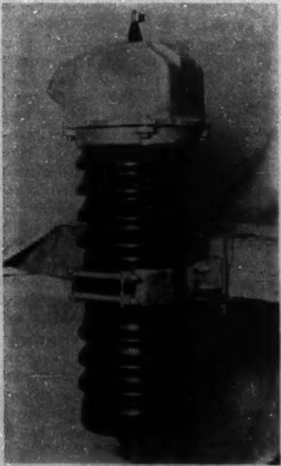

Figure 1. Single-pole automatically reclosing circuit breaker

The use of expensive lines and conventional apparatus for fault protection would result in low maintenance costs, but the installation cost could not be justified. Inexpensive lines with simple fuse protection for faults provide a low installation cost but result in high maintenance cost and unsatisfactory service. A high percentage of the maintenance cost of these extended systems is chargeable to the servicing required by faults. More than 80% of all faults are temporary and will not prevent the immediate restoration of service, so the elimination of service trips on such faults will materially reduce maintenance costs. The successful extension of such systems, therefore, depends upon the low installed cost of a device that will interrupt all faults and then restore service after all temporary faults.

The fault currents on these systems are relatively low and of such a value that distribution cutouts are used for sectionalizing, even to the use of five or more cutouts in series. The excellent coordination possible with modern fuses permits definite restriction of outages to the immediate neighborhood of a fault. On such systems, outages are of long duration and their restriction to limited areas greatly increases the quality of service. Therefore, a further requirement for a device to provide fault protection on such systems is its adaptability to an extensive coordination scheme. This will be accomplished if the device has a time current characteristic approximating that of modern fuses.

The necessarily inexpensive construction used on these systems, together with their length and exposure, has multiplied the number of faults to the point where simple fuses are inadequate for sectionalizing duty. Repeating fuses reduce the necessity for immediate service trips but require a periodic servicing and may not differentiate between a permanent and a temporary fault due to their short reclosing time. Close coordination is not dependable with repeating fuses, for a fuse is a thermal device and the heat stored in a protected fuse may result in its melting upon repeated operations of the fuse protecting it. It is seen, therefore, that an additional requirement is a time current characteristic which is not affected by repeated operations of the preceding protective device.

Since only a limited interrupting capacity is required, it is possible to make an inexpensive automatically reclosing oil circuit breaker to fulfill all of the requirements for operation on extended distribution systems. The requirements are:

1. Adequate interrupting ability.
2. Automatic reclosing to permit restoration of service after a temporary fault.
3. Pronounced inverse time current characteristic to permit close coordination on an extensive system.
4. Consistent time delay on tripping which is not affected by previous circuit conditions.
5. Reliable operation and low oil deterioration to reduce service trips to a minimum.
6. Low cost and a light weight to reduce the total installed cost to a minimum.

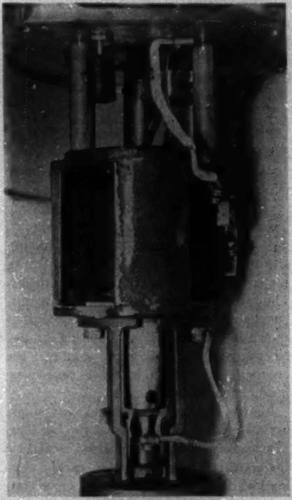

Figure 2. Interrupter and operating mechanism assembly withdrawn from porcelain tank

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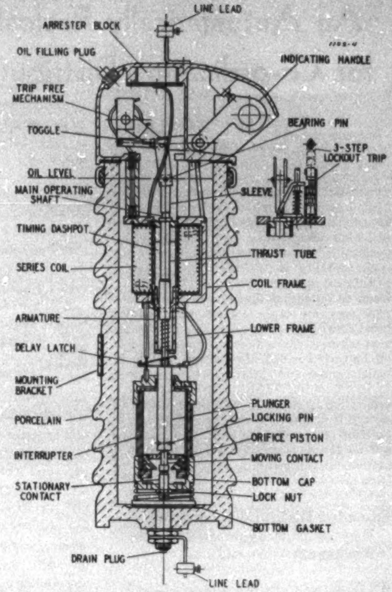
1. For all numbered references, see list at end of paper.

Figure 3. Indicating handle, oil gauge, and operation counter grouped under protecting hood

Figure 4. Sectional view of breaker with side view of lockout trip



7. Complete self protection in that no overload current (within the interrupting rating) or surge voltage shall damage the device.

From the application viewpoint the most important of these requirements is the inverse time current characteristic. The coordination possible with a breaker having the proper time characteristic greatly increases the scope of its application and makes possible the economical extension of low capacity distribution lines.

Construction and Operation of New Breaker

All of the above application requirements are fulfilled by the single pole automatically reclosing circuit breaker shown in figure 1. In addition, it has many other inherent features such as trip free closing; immediate manual reclosing after lockout; lockout on first interruption when manually reclosed against a fault; and multitap coil to permit ready change in rating. The interrupting chamber and all operating parts are easily removed in one unit as shown in figure 2. The indicating handle, oil gauge, and operation counter are grouped under an integral protecting hood, as shown in figure 3.

Figure 4 is a cross sectional view showing all principal elements of the breaker. The breaker is normally biased closed by the contact pressure springs, except during lockout. This differs from more conventional breakers in that, on tripping, the contacts are forced apart against spring pressure instead of being opened by accelerating springs. The energy required to operate the breaker is obtained from a series coil. The small size of coil used to operate the breaker and continuously carry overload currents just below the minimum tripping current is made possible by using an efficient type of interrupter that requires only a short travel. Wide separation of the contacts on lockout is obtained by releasing a spring that opens the breaker in the conventional manner and is reset, on closing, by means of the handle. Additional energy to interrupt heavier currents is obtained from the interrupter itself after the initial separation of the contacts.

Referring to figure 4, the armature when actuated by an overcurrent through the series coil moves upward and forces the entrapped oil from the timing dashpot. This action provides the inverse time characteristic on tripping. The shape of the time current curve is a function of dashpot size and port area together with the magnetic circuit and other factors. After a predetermined travel, the dashpot is opened, permitting the free armature to drive the moving contact open against the contact pressure springs. The moving contact opens rapidly for a short distance and then picks up the orifice piston which forces oil into the arc path resulting in interruption at the end of the first half cycle after the orifice piston starts to travel. On high currents the pressure developed within the interrupter will produce an additional driving force on the plunger thereby forcing oil into the arc path under a pressure proportional to the value of current being interrupted. The moving contact is free to move upward through the armature thus permitting overtravel on high current interruptions. The moving contact is prevented from reclosing at once by the delay latch which is not released until the armature again reaches its starting position. This results in a reclosing time of approximately five seconds. The time interval before reclosing permits the escape of gas from the top of the interrupter and entrance of fresh oil into the bottom. This flushing takes place through washer type valves that close instantly upon the appearance of pressure during interruption.

There is provided a simple, three step lockout trip which is notched up by the travel of the armature during each interruption. On the third interruption it breaks the toggle that holds the trip free mechanism in the closed position, thereby permitting the contacts to be driven to the extreme open position. The manual

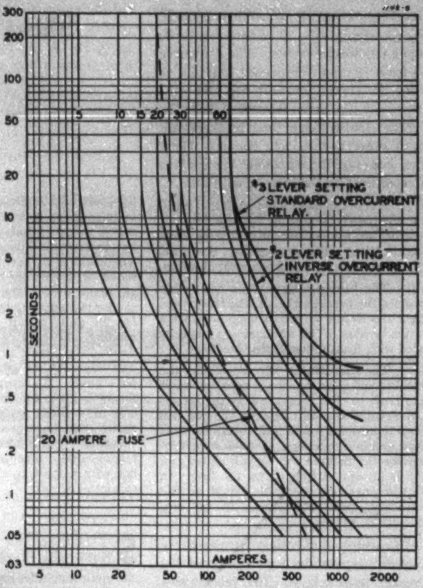
Figure 5. Interruption of 1,120 amperes at 13,800 volts

Note the increased overtravel at high currents

Figure 6. Interruption of 106 amperes at 7,200 volts

Figure 8 (right). Tripping-time current characteristics of new breaker

Note similarity to fuse link and relay curve



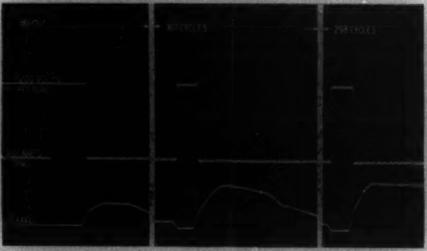


Figure 7. Three interruptions and lockout, 476 amperes at 7,200 volts

Note the travel ofthe contacts to the full open position after final interruption. (Portionsoffilmshowing breaker open between reclosures have been deleted from the original eight-foot film)

return of the indicating handle to the closed position will reset this toggle and the lockout trip and permit the breaker to close after the usual time delay. The lockout trip will be reset only to its second position from which it will lock out the breaker if there is a further interruption. If the fault has been removed from the line, the lockout trip will slowly reset to provide three step lockout on subsequent faults.

The series coil is made up of four separate windings. By connecting these windings in series, series-parallel, and parallel it is possible to obtain the six ratings from two coils, one coil furnishing the 5, 10, and 20 ampere ratings and the other coil furnishing the 15, 30, and 60 ampere ratings. To prevent damage to the coil from transient voltages, a suitable arrester is connected directly across its terminals.

Interruption and Lockout Tests

Repeated interrupting tests at both high and low currents on extremely fast circuits have demonstrated the high efficiency and low oil deterioration of the breaker. Indications are that the breaker can be in service many years before an oil change is required. The interrupting ability of the breaker is determined by the short-time current rating of the series coil rather than by any limitations in the interrupter. Figure 5 shows an oscillogram of a single interruption of 1,120 amperes at 13,800 volts. Figure 6 shows a single interruption of 106 amperes at 7,200 volts. Compare the travel record and note the additional travel of the contacts on the higher current interruption. Figure 7 shows three interruptions and lockout on 476 amperes at 7,200 volts. Note the tripping time, the reclosing time, and the travel of the contacts to the extreme open position after lockout.

Co-ordination Characteristics

The important attribute of this breaker is the time current characteristic. More time delay on tripping allows more time for coordination and therefore permits closer coordination between adjacent ratings. Consequently, as great a time delay as possible has been incorporated in this breaker particularly at the lower values of current just above the minimum tripping current. Since the different ratings are obtained by varying the number of turns on the series coil the curves will be parallel and identical in shape with only a shift in the current axis. Figure 8 shows the time current characteristics for the six ratings of 5, 10, 15, 20, 30, and 60 amperes. The time values have been obtained from oscillographic records of interrupting tests. Sufficient time delay has been provided so that all of these ratings can be coordinated in series. The possibility of coordination between these breakers and fuses is further demonstrated by superimposing a 20 ampere fuse curve upon the breaker curves. The fuse curve is for a 20 ampere universal fuse link as used in distribution cutouts and is a representative curve for such links.

The coordination of these breakers with each other is relatively simple. There are, however, three possible applications in coordinating the breaker with fuses.

(1). A fuse can be selected such that it will be completely protected by the breaker, that is, the breaker will operate three times and lockout without melting the fuse. This would be a rather infrequent application.

(2). The fuse can be used to completely protect the breaker, that is, the fuse would clear the circuit without a single operation of the breaker. This arrangement might be used with transformer protective links.

(3). The fuse can be selected so that the breaker will clear the circuit once to permit the fault to clear, and then on the second operation, if the fault persists, the fuse will blow due to the stored heat remaining in it from the first operation. On the second reclosure the breaker will remain closed and the fault will be isolated by the fuse. In this arrangement the fuse would be the outermost protective device in the sectionalizing scheme.

The shape of the time current curve further suggests the coordination of the breaker with relays. The relay curve shown in figure 7 is for a standard overcurrent relay of the moving disk type. By adjusting it to the proper lever setting and proper current ratio, it is possible to have the relay curve very close to that of the breaker. With the setting shown, the breaker will consistently protect the relay and thus prevent tripping an entire substation. The time delay between operations of the reclosing breaker is sufficient to allow the relay disk to return to its starting point, so the coordination will be the same for each operation. With reclosing fuses, a much larger margin between the fuse curve and the relay curve would be necessary due to the short reclosing time. Here the relay disk would not return to its original position and there would be cumulative turning of the disk and consequent operation of the relay.

A further advantage of this breaker for fault protection on low capacity systems lies in the economies offered by the use of simple gaps for lightning protection. On this type of system a temporary outage is not serious and, with the coordination afforded by these breakers, such an outage due to lightning is confined to a small area and is removed upon the first reclosure.

Conclusions

An automatically reclosing circuit breaker meeting all of the requirements for use on extended distribution systems has been developed and proven by repeated tests. A particularly important aspect of this breaker is the pronounced time delay on tripping that permits coordination of several breakers in series and coordination of the breakers with standard fuses of comparable ratings. The coordination possible with this breaker greatly increases the scope of its application and permits the economical extension of low capacity distribution systems with improved quality of service.

References

1. A NEW SERVICE RESTORER, E. F. Sixtus and W. R. Nodder. AIEE TRANSACTIONS, volume 56, 1937 (January section), pages 180–2, 152.

2. THE OIL CIRCUIT RECLOSER AND ITS OPERATION, L. S. Hobson. *General Electric Review*, November 1935.

Short Circuits in Synchronous-Machine Armatures

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Synopsis: An approximate solution is obtained for the current in a short-circuited coil or group of coils in the armature of a synchronous machine operating under load. The solution is expressed as the sum of a series of harmonics whose terms are in geometric ratio. The results obtained are similar in form to those previously obtained for the short circuit of a complete phase winding, and they reduce to that for the single-phase short circuit in the limiting case when the coil group comprises a complete phase. An experimental check of the theory is given for the case of a short circuit of a single armature coil, and it is shown that close agreement is obtained between theory and test with respect to the fundamental component of current, though not with respect to the small higher harmonics of current.

SOLUTIONS for single-phase and three-phase short circuits of synchronous machines have been obtained by several authors, those of Park[1] and of Doherty and Nickle[4] being notable. These analyses, however, have considered only complete phase windings, and it is the purpose of this paper to extend the theory to treat short circuits of a single armature coil or group of coils. The solution obtained is applicable to machines operating under load, the effect of the load currents being obtained by an approximation that is sufficiently accurate for practical purposes.

As one coil or a group of coils is essentially a single phase, the analysis is similar in form to that of Doherty and Nickle for the single-phase short circuit. Following their procedure, a general expression is obtained for the magnetic flux linkages of armature and field windings. The constant-linkage theorem, which states that the magnetic flux linkages in any closed electric circuit are constant if the resistance is neglected, is applied to determine the initial currents when the short circuit occurs. From physical considerations, these currents are then separated into their sustained and transient components, and the proper decrement factors are applied to the latter.

Assumptions

For simplification, this analysis is applied to an ideal synchronous machine.[2] In such a machine, linear relationships exist between magnetic flux and currents in any part of the machine, so that the principle of superposition applies. The phase windings are identical and symmetrically placed about the armature, and there are only two field circuits, the main field winding in the direct axis and a short-circuited winding in the quadrature axis, both of which are symmetrical about their respective axes. Only fundamental harmonics of magnetomotive force and voltage appear in the phase windings.

In addition to the assumption regarding the ideal machine, other simplifications are made. The resistances of the group of short-circuited armature coils and of the field circuits are neglected in deter-

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The material in this paper is part of a dissertation presented to the graduate school of Yale University in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

[1] For all numbered references, see list at end of paper.

mining initial currents in these windings but are taken into account in obtaining approximate values for the decrement factors. It is also assumed that the effect of the relatively large load resistance is to attenuate the unidirectional and even harmonics of induced current in the load circuit so quickly that these transient elements may be entirely neglected.

The solution is further simplified by making the assumption that, owing to the small mutual inductance between the phases and the coil group, the load currents are unaffected by the short circuit. The value of short-circuit current obtained under this assumption is regarded as the first approximation to the correct result. The effect of this current upon the load currents is then determined, and if the latter are changed an appreciable amount, the analysis is repeated, assuming that the load currents change by this amount. The second approximation is usually sufficiently accurate for all practical purposes, making further refinement unnecessary.

Analytical Development

Since the theorem of superposition is applicable to the ideal synchronous machine, the extension of Blondel's two-reaction method developed by Doherty and Nickle[3] is applied in appendix I to obtain expressions for the inductances of the field and armature circuits. Without loss of generality, the armature winding of which the short-circuited coils are a part is designated as phase *A*, and the angle between the axis of the coil group and that of phase *A* is β. Equations are then developed for the self and mutual inductances for the coil group, and it is found that these inductances may be conveniently expressed as functions of known design constants of the machine and the conventional inductances per phase.

Following the procedure that has been outlined, the inductance equations are used in appendix II to obtain the armature and field linkages of the machine. It is found that the results may be conveniently expressed in terms of the currents i_d and i_q defined by

$$i_d=i_a\cos\theta+i_b\cos(\theta-120°)+ i_c\cos(\theta-240°) \quad (1)$$

$$i_q=i_a\sin\theta+i_b\sin(\theta-120°)+ i_c\sin(\theta-240°) \quad (2)$$

The linkages in the short-circuited coil group are then

$$\psi_s=K_s\{(^3/_2)L_di_d+M_dI_d\}\cos(\theta-\beta)+ K_s\{(^3/_2)L_qi_q+M_qI_q\}\sin(\theta-\beta)+ \{L_{SD}\cos^2(\theta-\beta)+L_{SQ}\sin^2(\theta-\beta)\}\times (i_s-i_a) \quad (3)$$

where i_s is the short-circuit current in the coil group. Expressions for the inductances and for the constant K_s are given in appendix I. The corresponding field linkages are

$$\psi_D=M_di_d+K_sM_d(i_s-i_a)\cos(\theta-\beta)+L_dI_d \quad (4)$$

$$\psi_Q=M_qi_q+K_sM_q(i_s-i_a)\sin(\theta-\beta)+L_qI_q \quad (5)$$

These linkage equations will be used to obtain the solutions for the initial currents after the short circuit.

In conformity with the usual practice in the literature, the equations that follow are expressed in terms of reactances instead of inductances. The reactance corresponding to any inductance is the product of the latter and ω, where $\omega=2\pi f$, f being the rated frequency of the machine. Thus $X_{SD}'=\omega L_{SD}'$.

SOLUTION FOR CURRENTS

When the short circuit occurs, spontaneous unidirectional components of current appear in the coil group and the field circuit tending to sustain the linkages at their initial value. The voltages that support these transient currents are generated through the collapse of the magnetic fields existing at the instant of short circuit.

As the machine rotates, the transient unidirectional current in the field induces a transient fundamental component of current in the coils, and this in turn causes a second harmonic in the field. This "reflection" from field to coils and back again gives rise to a series of even harmonics in the field and a series of odd harmonics in the coils, all of which are proportional to the average induced field current and all of which decay at the rate determined by the field decrement factor σ_f. Similarly the transient unidirectional current in the short-circuited coils causes a series of even harmonics in the coils and a series of odd harmonics in the field that decay to zero according to the decrement factor of the coil group, σ_s.

The rotating magnetic field due to the average field current supported by exciter voltage causes fundamental and higher harmonics of sustained current to appear in the short-circuited coils and also causes a series of even harmonics in the field.

Owing to the relatively large resistance of the load the transient unidirectional and even harmonics of current induced in the phases by the short circuit will be damped out very quickly and may be neglected. Because of the small mutual inductance between the coil group and the phases, only the fundamental of the induced odd harmonics that also appear is of appreciable magnitude. This component, being due to the fundamental of the short-circuit current in the faulty coils and the transient element of the average field current, will have a sustained and a transient value. The latter decays according to the field decrement factor. Neglecting the small harmonics, therefore, the induced load current can be represented by a sustained and a transient fundamental component. These must be added to the load current existing before the short circuit to get the total phase currents. The corresponding currents i_d and i_q, defined by equations 1 and 2, will then have the form

$$i_d=i_{d1}+(i_{d0}-i_{d1})e^{-\sigma_f t} \quad (6)$$

$$i_q=i_{q1}+(i_{q0}-i_{q1})e^{-\sigma_f t} \quad (7)$$

where i_{d0} and i_{q0} are the initial values before the short-circuit and i_{d1} and i_{q1} are the final sustained values. Approximate expressions for i_{d1} and i_{q1} are given in equations 19 and 20.

First Approximation to Short-Circuit Current. Following the procedure previously outlined, the first approximation to the short-circuit current is obtained by assuming that the load currents are unchanged. When the constant-linkage theorem is applied, it is shown in appendix III that if the short circuit occurs at $\theta=\theta_0$, the initial current in the coil group is

$$i_s=i_a+i'+i'' \quad (8)$$

The components i' and i'' are

$$i'=\frac{-2e_{SD}}{X_{SD}'+\sqrt{X_{SD}'X_{SQ}'}}\{\cos(\theta-\beta)+ B\cos 3(\theta-\beta)+B^2\cos 5(\theta-\beta)+\ldots\ldots\} \frac{-2e_{SQ}}{X_{SQ}'+\sqrt{X_{SD}'X_{SQ}'}}\{\sin(\theta-\beta)+ B\sin 3(\theta-\beta)+B^2\sin 5(\theta-\beta)+\ldots\ldots\} \quad (9)$$

$$i''=\frac{e_{SD}\cos(\theta_0-\beta)+e_{SQ}\sin(\theta_0-\beta)}{\sqrt{X_{SD}'X_{SQ}'}}\times \{1+2B\cos 2(\theta-\beta)+2B^2\times \cos 4(\theta-\beta)+\ldots\ldots\} \quad (10)$$

where

$$e_{SD}=K_s\{(^3/_2)X_di_{d0}+X_{md}I_{d0}\} \quad (11)$$

and

$$e_{SQ}=K_s\{(^3/_2)X_qi_{q0}\} \quad (12)$$

are the direct and quadrature components of the voltage in the coil group prior to the short circuit, and

$$B=\frac{\sqrt{X_{SQ}'}-\sqrt{X_{SD}'}}{\sqrt{X_{SQ}'}+\sqrt{X_{SD}'}} \quad (13)$$

is a constant.

By the assumption made, i_a is unchanged, so that the additional current that is due to the short circuit is represented by the components i' and i''. The latter contains the d-c and even harmonics that arise to maintain the linkages at their initial value; therefore it is transient in character and decays to zero at the rate determined by σ_s. The other component, i', contains the odd harmonics and is proportional to the average field current.

As shown in appendix III, the average field current just after the short circuit occurs is

$$I_d(\text{average}) = G_0 I_{d0} \tag{14}$$

where G_0 is a constant defined by equation 60. Since the average sustained field current is I_{d0}, it is $1/G_0$ times as great as the initial value. The sustained value of i', therefore, is $1/G_0$ times as great as the initial value expressed in equation 9. The remaining portion $(1-1/G_0)i'$, is a transient and decays according to σ_f. The first approximation to the short-circuit current in the coil group is therefore

$$i_s = i_a + \frac{1}{G_0} i' + \left(1 - \frac{1}{G_0}\right) i' e^{-\sigma_f t} + i'' e^{-\sigma_s t} \tag{15}$$

The effect of the change in load currents being small, it will be found that this expression gives results that do not differ greatly from those obtained by the more accurate expression, equation 21.

The additional current introduced in the coil group by the short circuit is $(i_s - i_a)$, and for the reasons previously mentioned, its fundamental component is the only one that has appreciable effect upon the load currents. From equations 9 and 15, the sustained value of

$$(i_s - i_a)(\text{fundamental}) = I_1 \cos(\theta - \beta - \alpha_1) \tag{16}$$

where

$$I_1 = \frac{1}{G_0}\left[\left\{\frac{e_{SD}}{X_{SD}' + \sqrt{X_{SD}' X_{SQ}'}}\right\}^2 + \left\{\frac{e_{SQ}}{X_{SQ}' + \sqrt{X_{SD}' X_{SQ}'}}\right\}^2\right]^{1/2} \tag{17}$$

and

$$\alpha_1 = \tan^{-1}\left\{\left(\frac{e_{SQ}}{e_{SD}}\right)\frac{X_{SD}' + \sqrt{X_{SD}' X_{SQ}'}}{X_{SQ}' + \sqrt{X_{SD}' X_{SQ}'}}\right\} \tag{18}$$

Taking approximate account of the effect of this current, it is shown in appendix IV that the sustained values of i_d and i_q corresponding to equations 6 and 7 are

$$I_{d1} = -\frac{3}{2}\,\frac{x_q + X_L}{r^2 + (X_d + X_L)(X_q + X_L)}\left[X_{md} I_{d0} + K_S X_D \frac{I_1}{2}\cos\alpha_1 + \frac{r}{X_q + X_L} K_S X_Q \frac{I_1}{2}\sin\alpha_1\right] \tag{19}$$

$$i_{q1} = \frac{-r}{X_q + X_L}\, i_{d1} - \frac{3}{2(X_q + X_L)} K_S X_Q \frac{I_1}{2}\sin\alpha_1 \tag{20}$$

The corresponding expressions for i_{d0} and i_{q0} are obtained from those for i_{d1} and i_{q1} by putting $I_1 = 0$.

Second Approximation to Short-Circuit Current. The next step in the solution for the short-circuit currents is to assume that i_d and i_q have average values given by equations 6 and 7. Following the same procedure as before, the linkages of the coil group and the field circuits may be obtained and solved for the initial currents in the coil group and the field winding. Separated into its sustained and transient components and with the proper decrements applied, the second approximation to the short-circuit current in the coil group is

$$\begin{aligned}
i = {} & \frac{2}{3}\{i_{d1}\cos\theta + i_{q1}\sin\theta\} - \\
& \frac{1}{G_1}\Bigg[\frac{2\{e_{SD} - K_S(2/3)X_d'(i_{d0} - i_{d1})\}}{X_{SD}' + \sqrt{X_{SD}' X_{SQ}'}} \times \\
& \{\cos(\theta - \beta) + B\cos 3(\theta - \beta) + \ldots\} + \\
& \frac{2\{e_{SQ} - K_S(2/3)X_q'(i_{q0} - i_{q1})\}}{X_{SQ}' + \sqrt{X_{SD}' X_{SQ}'}}\{\sin(\theta - \beta) + \\
& B\sin 3(\theta - \beta) + B^2\sin 5(\theta - \beta) + \ldots\}\Bigg] + \\
& e^{-\sigma_f t}(2/3)\{(i_{d0} - i_{d1})\cos\theta + (i_{q0} - i_{q1})\sin\theta\} - \\
& \frac{1e^{-\sigma_f t}}{G_1}\left[\frac{\{(G_1 - 1)e_{SD} - K_S(2/3)X_d'(i_{d0} - i_{d1})\}\cos(\theta - \beta) + \{(G_1 - 1)e_{SQ} - K_S(2/3)X_q'(i_{q0} - i_{q1})\}\sin(\theta - \beta)}{(1/2)(X_{SD}' + X_{SQ}') + (1/2)(X_{SD}' - X_{SQ}')\cos 2(\theta - \beta)}\right] + \\
& e^{-\sigma_s t}\frac{\{e_{SD}\cos(\theta_0 - \beta) + e_{SQ}\sin(\theta_0 - \beta)\}}{\sqrt{X_{SD}' X_{SQ}'}} \times \\
& \{1 + 2B\cos 2(\theta - \beta) + 2B^2\cos 4(\theta - \beta) + \ldots\}
\end{aligned} \tag{21}$$

where

$$G_1 = \frac{X_{SD} + \sqrt{X_{SD}' X_{SQ}'}}{X_{SD}' + \sqrt{X_{SD}' X_{SQ}'}} \times \left[1 + \frac{2(X_d - X_d')(i_{d0} - i_{d1})}{3X_{md} I_{d0}} + \frac{X_{SD} - X_{SD}'}{X_{SD}' + \sqrt{X_{SD}' X_{SQ}'}} \cdot \frac{2X_d i_{d1}}{3X_{md} I_{d0}}\right] \tag{22}$$

The solution for the coil current for the case of short circuit at no load is obtained by letting $i_{d0} = i_{q0} = i_{d1} = i_{q1} = 0$ in equation 21. This is equivalent to making $e_{SQ} = 0$ and $e_{SD} = K_s X_{md} I_{d0}$.

Because of the relatively small mutual inductance between the coil group and the phase and field circuits $(i_{d0} - i_{d1})$, $(i_{q0} - i_{q1})$, and $(G_1 - 1)$ are ordinarily small. Therefore it is usually possible to neglect the terms involving $\varepsilon^{-\sigma_f t}$ when making calculations using equation 21. For the sake of completeness, however, they have been included.

One component of the sustained short-circuit current is equal to the sustained load current $(2/3)\{i_{d1}\cos\theta + i_{q1}\sin\theta\}$. The other component is the sum of two odd harmonic series whose terms, in ascending order, are in geometric ratio. The ratio, B, is given in equation 13.

The expression for the short circuit current of the group of coils is similar in form to that given by Doherty and Nickle[4] for the single phase short circuit and their discussion of the effects of the machine constants upon the form of the solution applies equally well here. In the limiting case when the coil group comprises a complete phase, the solution for the condition of no load is exactly the same as theirs.

Decrement Factors

Expressions for the decrement factors may be obtained by the approximate method used by Doherty and Nickle,[4] which they have discussed fully. It is then found that the decrement factor for the coil group is

$$\sigma_s = \frac{\omega r_s}{\sqrt{X_{SD}' X_{SQ}'}} \tag{23}$$

where r_s is the resistance of the shorted coils, and that for the field is

$$\sigma_f = \frac{X_{SD} + \sqrt{X_{SD}' X_{SQ}'}}{X_{SD}' + \sqrt{X_{SD}' X_{SQ}'}} \cdot \frac{R_d}{L_d} \tag{24}$$

when R_d is the total resistance of the field circuit.

Experimental Confirmation of Theoretical Results

To verify the theory, experimental measurements were made of the short-circuit current in a single coil of a 125-horsepower, 440-volt, 60-cycle, 900-rpm, three-phase, synchronous motor. This machine had a full-pitch armature winding, no amortisseur on the rotor, and was a specially built laboratory machine with the leads to the armature coils brought out to a terminal board, making each coil accessible for measurements. The measured values of the reactances and resistances of the machine are given in table I.

Table I. Measured Reactances* and Resistances of Test Machine

$X_d =$ 1.520 ohms $X_D =$ 1.050 ohms
$X_d' =$ 0.445 ohm $X_D' =$ 0.266 ohm
$X_q = X_q' =$ 1.205 ohms $X_Q = X_Q' =$ 0.970 ohm

$X_s =$ 0.237 ohm
$X_{md} =$ 37.3 ohms
$r_a =$ 0.048 ohm at 25 C (line to neutral)
$R_d =$ 23.5 ohms at 25 C

* Measured by static test method.

Using the equations that have been developed, it is found that for a single armature coil of the machine,

$K_s = 0.0434$
$X_{SD} = X_{SQ} = X_{SQ}' = 0.0126$ ohm
$X_{SD}' = 0.01109$ ohm

The short-circuit current was measured by taking the voltage drop across a 0.000243-ohm, non-inductive shunt in series with the coil. The wave form of the current was obtained from an oscillogram of this voltage, and the magnitudes of the sustained harmonic components were measured with a vacuum-tube harmonic analyzer.

The coil used in the test was in the center of phase A, so that $\beta = 0$. The current was calculated analytically and checked experimentally for short circuits occurring under two different operating conditions, one when the machine was loaded, and one at no load. Both tests were made at reduced voltage to eliminate the effect of saturation as far as possible.

For the short circuit under load, the machine was operating with a pure resistive load of 2.45 ohms per phase and a field current of 0.992 ampere. The short circuit occurred at $\theta_0 = 215°$. The calculated current for the short-circuited coil was

$$\begin{aligned} i_s = {} & -164.0 \cos(\theta - 17.28°) - 5.13 \times \\ & \cos(3\theta - 23.16°) - 0.161 \times \\ & \cos(5\theta - 23.16°) - \ldots - e^{-0.0133\omega t}[4.80 \times \\ & \cos(\theta + 19.85°) + 0.116 \times \\ & \cos(3\theta + 13.38°) + \ldots] - e^{-0.1067\omega t}[163.3 + \\ & 10.27 \cos 2\theta + 0.324 \cos 4\theta + \ldots] \end{aligned}$$

where $215° < \theta$ and $\omega t = (\theta - \theta_0)$ in radians. In table II the magnitudes of the harmonics of the calculated sustained current are compared with the actual values as measured with a harmonic analyzer.

The short-circuit at no load occurred at $\theta_0 = 180°$ and the field current was 0.988 ampere. The calculated current is

$$\begin{aligned} i_s = {} & -185.5 \cos\theta - 5.85 \cos 3\theta - 0.184 \times \\ & \cos 5\theta - \ldots \\ & -e^{-0.1067\omega t}\{191.5 + 12.06 \cos 2\theta + 0.38 \times \\ & \cos 4\theta + \ldots] \end{aligned}$$

where $180° < \theta$ and $\omega t = (\theta - \theta_0)$ in radians. The magnitude of the sustained harmonics of the calculated and measured currents are compared in table II.

Considering only the fundamental components of sustained current, the data in table II show that the calculated and measured values agree very closely. The analytical and experimental results for the higher harmonics, however, do not agree. This is probably due in large part to the fact that the higher harmonics of mutual linkages between the coil and the other machine circuits were neglected in the analysis. While the voltages due to higher harmonics of flux are ordinarily small in the complete phase due to the effects of fractional pitch and distributed windings, they may be much more important in one or two coils. To neglect these harmonics of flux, therefore, may introduce considerable error in the higher harmonics of current in the short-circuited coils. As they have little effect on the fundamental voltage, however, they may be disregarded in determining the fundamental component of current.

Table II. Comparisons of Peak Values* of Harmonics of Sustained Current in a Single Coil

	Fundamental	Third	Fifth
Machine loaded			
Calculated	164.0	5.13	0.161
Measured	165.5	29.1	6.69
No load			
Calculated	185.5	5.85	0.184
Measured	190.7	30.85	3.49

* All values are given in amperes.

Oscillograms were made of the short-circuit current, and these were compared with a graphical plot of the calculated current. The agreement between the calculated and test results was fairly good, both during the transient period and after the transient components had died out. The effect of the larger values of the harmonics in the actual current was noticeable, however, the actual waves being slightly more peaked than those which were calculated.

Had the test machine been operated at higher flux densities, the effect of saturation would probably have been noticeable. As pointed out by Doherty and Nickle, this would not prevent the use of the results of the theoretical analysis, as the effect of saturation may be taken into account by proper shading of the machine constants.

Appendix I

Inductance Equations

Expressions for the inductances of the field and armature circuits may be obtained by application of the extension of Blondel's method previously mentioned. With the assumptions that have been made, only the fundamental component of magnetomotive force need be considered in obtaining the mutual linkages due to any machine circuit. The procedure is to resolve this space sinusoid into two components, one in the direct axis and one in the quadrature axis, determine the corresponding fundamental linkages due to each component acting separately, and combine these linkages to get the resultant. The fundamental linkages in any other circuit of the machine per unit current in the exciting circuit are equal to the mutual inductance between the two. That portion that is due to the space fundamental of air gap flux is the inductance of armature reaction, and the remainder, which is due to flux in the slots and end windings, is leakage inductance.

SELF-INDUCTANCE OF ARMATURE REACTION OF A COMPLETE PHASE WINDING

Let the flux density per unit magnetomotive force at the armature surface be represented by the even harmonics series[8]

$$P = P_0 + P_2 \cos 2\lambda + P_4 \cos 4\lambda + \ldots \qquad (25)$$

The fundamental component of the linkages with the complete phase A winding of the air-gap flux due to the fundamental component of magnetomotive force of phase A may be expressed as

$$\psi_{aa} = i_a\{L_{AD} \cos^2\theta + L_{AQ} \sin^2\theta\} \qquad (26)$$

where

$$L_{AD} = 3.2Dl(yN_cK_dK_p)^2\{P_0 + (1/2)P_2\} \qquad (27)$$

$$L_{AQ} = 3.2Dl(yN_cK_dK_p)^2\{P_0 - (1/2)P_2\} \qquad (28)$$

are the static inductances of armature reaction for one phase.

MUTUAL INDUCTANCES BETWEEN COIL GROUP AND PHASE AND FIELD WINDINGS

If the space fundamental of magnetomotive force for the coil group is considered, it may easily be shown that the linkages of armature reaction in phase A due to the current in the coil group are

$$\psi_{as} = i_sK_s\{L_{AD} \cos(\theta - \beta) \cos\theta + L_{AQ} \times \sin(\theta - \beta) \sin\theta\} \qquad (29)$$

where

$$K_s = \frac{sK_{ds}}{pyK_d} \qquad (30)$$

and s is the number of coils in the group, y the slots per phase per pole, p the number of poles, K_d the phase belt distribution factor, and K_{ds} the distribution factor for the s coils; that is, sK_{ds} is the ratio of the fundamental of magnetomotive force for the coil group to that for a single coil. The corresponding linkages in phase B are obtained by replacing $\cos\theta$ and $\sin\theta$ by $\cos(\theta - 120°)$ and $\sin(\theta - 120°)$ respectively, and those for phase C may be obtained by using $(\theta - 240°)$ instead of $(\theta - 120°)$. The linkages per unit current in these expressions are equal to the mutual inductances of armature reaction between the phase windings and the coil group. The corresponding mutual leakage linkages will have the same form except that, being independent of rotor position, they will be the same for direct and quadrature axes. These mutual leakage inductances are $K_sL_l \cos\beta$, $K_s2L_{lm} \times \cos(\beta - 120°)$, and $K_s2L_{lm} \cos(\beta - 240°)$ for phases A, B, and C respectively. Adding these components to those due to armature reaction, it is found that the mutual inductance between the coil group and phase A is

$$L_{as} = K_s\{L_D \cos(\theta - \beta) \cos\theta + L_Q \times \sin(\theta - \beta) \sin\theta\} \qquad (31)$$

The corresponding expression for the mutual inductance between the coil group and phase B is

$$L_{bs}=K_s\{(L_D-L_0)\cos(\theta-\beta)\cos(\theta-120°)+(L_Q-L_0)\sin(\theta-\beta)\sin(\theta-120°)\} \quad (32)$$

and that for phase C is similar to equation 32 with $(\theta-240°)$ instead of $(\theta-120°)$.

The procedure followed in the preceding paragraph may be used to obtain the mutual inductances between the coil group and the field windings. These inductances are then found to be

$$M_{sd}=K_sM_d\cos(\theta-\beta) \quad (33)$$

$$M_{sq}=K_sM_q\sin(\theta-\beta) \quad (34)$$

It should be noted that all of the mutual inductances for the coil group are expressed as the product of the constant K_s and the inductances of the machine per phase.

SELF INDUCTANCE OF A COIL GROUP

Although space harmonics of magnetomotive force cannot appear in the complete phase winding of an ideal machine, they must be considered in obtaining an expression for the self inductance of a coil group. In the work that follows, therefore, the complete magnetomotive force of the coil group will be considered.

As the inductance of a coil group is a function of the self and mutual inductances for the individual coils in the group, the first step in this analysis will be to develop an expression for the self inductance of a single coil. The next step will be to obtain a general expression for the mutual inductance between two coils. These self and mutual inductances may then be combined to obtain the total self inductance of the coil group.

Assuming that the current i_s in a single coil of N_c turns is distributed in a line at the armature surface and that the iron has infinite permeability, the magnetomotive force distribution of the coil over the coil span ρ is a square-topped wave of magnitude

$$A_1=(1-\rho/p\pi)(0.4\pi N_ci_s) \quad (35)$$

If the relation between the flux density and magnetomotive force is expressed by equation 25, the linkages, with the coil, of the air-gap flux due to this magnetomotive force may be shown to be

$$\psi_{ss}=[(p-\rho/\pi)\{\pi^2/8(pyK_dK_p)^2\}]\times\{(\rho/2\pi)(L_{AD}+L_{AQ})+(1/\pi)\times(L_{AD}-L_{AQ})\sin\rho\cos 2(\theta-\beta)\} \quad (36)$$

when the small higher harmonics are neglected.

Since the path of the leakage flux in the slots and end windings is practically the same for a single coil and a complete phase and since there are $1/py$ as many turns for the single coil as for the phase winding, the ratio of the leakage linkages for the coil to those for the phase is approximately $1/(py)^2$. As indicated by equations 26 and 36 this is much smaller than the corresponding ratio for the linkages due to air-gap flux. The leakage component is, therefore, only a small percentage of the total linkages of the coil. Since $2L_{lm}$ is also very small in comparison to $(^1/_2)(L_{AD}+L_{AQ})$, the assumption will be made that the total linkages of the coil can be expressed in a form similar to equation 36 with

$$(^1/_2)(L_D+L_Q)-L_0=(^1/_2)(L_{AD}+L_{AQ})+2L_{lm}$$

substituted for $(^1/_2)(L_{AD}+L_{AQ})$. Although this assumption may introduce considerable error in the leakage linkages added, it will make only a small discrepancy in the total linkages, and the equation may be put in a much more convenient form for calculation. The total self inductance of a single coil is then

$$L_{ss}=L_{SD}\cos^2(\theta-\beta)+L_{SQ}\sin^2(\theta-\beta) \quad (37)$$

where

$$L_{SD}=[(p-\rho/\pi)\{\pi^2/8(pyK_dK_p)^2\}]\times[(\rho/\pi)\{(^1/_2)(L_D+L_Q)-L_0\}+(1/\pi)(L_D-L_Q)\sin\rho] \quad (38)$$

$$L_{SQ}=[(p-\rho/\pi)\{\pi^2/8(pyK_dK_p)^2\}]\times[(\rho/\pi)\{(^1/_2)(L_D+L_Q)-L_0\}-(1/\pi)(L_D-L_Q)\sin\rho] \quad (39)$$

The magnetomotive force distribution of the single coil over the armature outside the coil span will be a square-topped wave of magnitude

$$A_2=-(\rho/p\pi)(0.4\pi N_ci_s) \quad (40)$$

Using equations 27 and 28 and making the same assumptions regarding leakage linkages that were made in deriving equation 37, it may be shown that the mutual inductance between two coils separated a small angle Δ is approximately

$$L_{ms}=\{(p-\rho/\pi)(\pi^2/8)/(pyK_dK_p)^2\}\times[\{\rho/\pi-\rho\Delta/(p\pi-\rho)\}\{(^1/_2)(L_D+L_Q)-L_0\}+(1/\pi)\cos\Delta\sin(\rho-\Delta)\times\{(L_D-L_Q)\cos 2(\theta-\beta)\}] \quad (41)$$

The total self inductance of a group of coils may be obtained from the relationship

$$L_{ss}=L_{11}+L_{22}+L_{33}+\ldots+2(L_{12}+L_{13}+L_{23}+\ldots) \quad (42)$$

where L_{11}, L_{22}, . . . are the self inductances of each coil given by equation 37 and L_{12}, L_{13}, . . . are the mutual inductances between coils 1 and 2, 1 and 3, etc., as given by equation 41. It is apparent that the total self inductances of the coil group can be put in the form of equation 37 if L_{SD} and L_{SQ} are given the proper values. Equation 37, therefore, is a general expression for the total self inductance of a group of armature coils whether the group consists of one, two, or more coils.

TRANSIENT INDUCTANCES

The transient inductances L_{SD}' and L_{SQ}' are defined by

$$L_{SD}'=L_{SD}-K_s^2(M_d^2/L_d) \quad (43)$$

$$L_{SQ}'=L_{SQ}-K_s^2(M_q^2/L_q) \quad (44)$$

The inductances L_d', L_q', L_D', and L_Q' correspond to the transient reactances X_d', X_q', X_D', and X_Q' as ordinarily defined in the literature.

Appendix II

Linkage and Voltage Equations

The armature and field linkages for an ideal synchronous machine under normal conditions have been given by Park.[1,2] Following his analysis, it may be shown that if

$$\psi_d=(^3/_2)(L_d+L)i_d+M_dI_d \quad (45)$$

$$\psi_q=(^3/_2)(L_q+L)i_q+M_qi_q \quad (46)$$

then the following voltage equations may be obtained:

$$-\frac{d\psi_d}{dt}-\psi_q\frac{d\theta}{dt}=\frac{2}{3}ri_d \quad (47)$$

$$-\frac{d\psi_q}{dt}+\psi_d\frac{d\theta}{dt}=\frac{2}{3}ri_q \quad (48)$$

where r is the combined resistance of one phase of the machine and load.

When the machine contains another circuit of a group of short-circuited coils, the linkage equations must be modified to include the effect of the additional current in the faulty coils. This may be done by making use of the expressions for the mutual inductances between the coil group and the phase and field windings.

It is convenient to consider the magnetomotive force of the portion of the phase A winding that is outside the faulty coils as the difference between that due to a current i_a in the complete phase winding and that due to the coil group carrying the same current i_a. Similarly, the linkages in this portion of the phase A winding due to other currents may be obtained by subtracting the linkages in the faulty coils from those in the complete phase.

When the inductance equations developed in appendix I are used, the modified forms of equations 45 and 46 become

$$\psi_d=(^3/_2)(L_d+L)i_d+K_sL_D(i_s-i_a)\times\cos(\theta-\beta)+M_dI_d \quad (49)$$

$$\psi_q=(^3/_2)(L_q+L)i_q+K_sL_Q(i_s-i_a)\times\sin(\theta-\beta)+M_qI_q \quad (50)$$

It may then be shown that, to a close approximation, the voltage equations 47 and 48 are applicable to the short-circuited machine when ψ_d and ψ_q have the values given in equations 49 and 50.

Appendix III

Short-Circuit Currents Assuming Load Currents Constant

At the instant of short circuit, $\theta=\theta_0$, $i_d=i_{d0}$, $i_q=i_{q0}$, $I_d=I_{d0}$, and $I_q=0$. Applying the constant linkage theorem, the linkages in the coil group and the field windings are

$$K_s\{(^3/_2)L_di_d+M_dI_d\}\cos(\theta-\beta)+K_s\{(^3/_2)L_qi_q+M_qI_q\}\sin(\theta-\beta)+\{L_{SD}\cos^2(\theta-\beta)+L_{SQ}\sin^2(\theta-\beta)\}(i_s-i_a)=K_s\{(^3/_2)L_di_{d0}+M_dI_{d0}\}\cos(\theta_0-\beta)+K_s\{(^3/_2)L_qi_{q0}\}\sin(\theta_0-\beta) \quad (51)$$

$$M_di_d+K_sM_d(i_s-i_a)\cos(\theta-\beta)+L_dI_d=M_di_{d0}+L_dI_{d0} \quad (52)$$

$$M_qi_q+K_s(i_s-i_a)\sin(\theta-\beta)+L_qI_q=0 \quad (53)$$

In accordance with the method of analysis that has been outlined, the first approximation to the solution is obtained by assuming that $i_d=i_{d0}$ and $i_q=i_{q0}$ remain constant. Then a simultaneous solution of equations 51, 52, and 53 gives

$$(i_s-i_a)=\frac{e_{SD}\{\cos(\theta_0-\beta)-\cos(\theta-\beta)\}+e_{SQ}\{\sin(\theta_0-\beta)-\sin(\theta-\beta)\}}{(^1/_2)(X_{SD}'+X_{SQ}')+(^1/_2)(X_{SD}'-X_{SQ}')\cos 2(\theta-\beta)} \tag{54}$$

$$I_d=I_{d0}-\frac{X_{SD}-X_{SD}'}{K_sX_{md}}(i_s-i_a)\cos(\theta-\beta) \tag{55}$$

where e_{SD} and e_{SQ} are defined by equations 11 and 12. These may also be expressed as

$$(i_s-i_a)=i'+i'' \tag{56}$$

$$I_d=I_d'+I_d'' \tag{57}$$

where i' and i'' are given by equations 9 and 10 and I_d' and I_d'' are

$$I_d'=I_{d0}+\frac{X_{SD}-X_{SD}'}{K_sX_{md}}\,\frac{e_{SD}}{X_{SD}'+\sqrt{X_{SD}'X_{SQ}'}}+\frac{X_{SD}-X_{SD}'}{K_sX_{md}}\left[\frac{e_{SD}(1-B)^2}{2\sqrt{X_{SD}'X_{SQ}'}}\{\cos 2(\theta-\beta)+B\cos 4(\theta-\beta)+\ldots\}+\frac{e_{SQ}(1-B^2)}{2\sqrt{X_{SD}'X_{SQ}'}}\times\{\sin 2(\theta-\beta)+B\sin 4(\theta-\beta)+\ldots\}\right] \tag{58}$$

$$I_d''=\frac{(X_{SD}-X_{SD}')(1+B)}{2K_sX_{md}\sqrt{X_{SD}'X_{SQ}'}}\{e_{SD}\cos(\theta_0-\beta)+e_{SQ}\sin(\theta_0-\beta)\}[\cos(\theta-\beta)+B\cos 3(\theta_0-\beta)+\ldots] \tag{59}$$

The average field current is

$$I_d(\text{average})=I_{d0}+\frac{X_{SD}-X_{SD}'}{K_sX_{md}}\,\frac{e_{SD}}{X_{SD}'+\sqrt{X_{SD}'X_{SQ}'}}=G_0I_{d0} \tag{60}$$

where G_0 is the value of G_1 obtained from equation 22 by setting $i_{d1}=i_{d0}$.

The component of field current, I_d'', contains the odd harmonics and is due to the unidirectional component and even harmonics of the current (i_s-i_a) to which it is directly proportional. It therefore decays to zero according to the decrement factor σ_s.

The current I_d', containing the unidirectional component and even harmonics, must have an average sustained value I_{d0} since it is supported entirely by the exciter voltage. Its sustained value is therefore $(1/G_0)i_d'$. The remainder $(1-1/G_0)I_d'$, is transient in character and, being proportional to the average field current, decays at a rate fixed by σ_f.

Separated into sustained and transient components and with the decrement factors applied, the field current at any instant after the short circuit is

$$I_d=(1/G_0)I_d'+(1-1/G_0)I_d'e^{-\sigma_f t}+I_d''e^{-\sigma_s t} \tag{61}$$

Appendix IV

Short-Circuit Currents Assuming Change in Load Currents

A second approximation to the solution for the short-circuit current is obtained by assuming that i_d and i_q have average values given by equations 6 and 7. It will first be necessary, however, to develop expressions for the currents i_{d1} and i_{q1}.

The currents i_{d1} and i_{q1} may be obtained by use of the voltage equations given in appendix II. Considering only average values, $d\psi_d/dt=d\psi_q/dt=0$. The average value of $(i_s-i_a)\cos(\theta-\beta)$ is $(^1/_2)I_1\cos\alpha_1$ and that of $(i_s-i_a)\sin(\theta-\beta)$ is $(^1/_2)I_1\sin\alpha_1$, where I_1 and α_1 are defined by equations 17 and 18. Likewise, the average value of I_d is I_{d0} and the average of $I_q=0$. Then from equations 47, 48, 49, and 50,

$$(^3/_2)(X_q+X_L)i_{q1}+(^1/_2)K_sX_QI_1\sin\alpha_1+(^3/_2)ri_{d1}=0 \tag{62}$$

$$(^3/_2)(X_d+X_L)i_{d1}+(^1/_2)K_sX_DI_1\cos\alpha_1-(^3/_2)ri_{q1}=-X_{md}I_{d0} \tag{63}$$

These may be solved simultaneously to obtain the currents i_{d1} and i_{q1}. The results are given in equations 19 and 20.

With currents i_d and i_q given by equations 6 and 7, the linkage equations for the short-circuited coils and field windings may be obtained. Following the same procedure as before, these may be solved for the initial currents. When the short-circuit current is separated into its sustained and transient components and the proper decrements are applied, the second approximation to the solution is that given by equation 21.

Nomenclature

i_a, i_b, i_c = currents in phases A, B, and C

i_s = current in short-circuited coil group

I_d = current in main field winding

I_q = current in field winding in quadrature axis

K_d, K_p = distribution and pitch factors for armature phase winding

K_{ds} = distribution factor for the group of s coils as discussed following equation 30

$K_s=(sK_{ds}/pyK_d)$ is a constant

L_D, L_Q = total static inductances, single phase, in the direct and quadrature axes, respectively. These correspond to the commonly used reactances, X_D and X_Q

L_d, L_q = total three phase, line to neutral inductances in the direct and quadrature axes corresponding to the synchronous reactances X_d and X_q

$L_0=(L_l-2L_{lm})$ = zero sequence inductance corresponding to X_0

L_l, L_{lm} = self and mutual leakage inductances per phase

L_{SD}, L_{SQ} = inductances of coil group in the direct and quadrature axes. See appendix I

L = load inductance per phase corresponding to X_L

M_q = maximum mutual inductance per phase between quadrature axis field winding and armature

M_d = maximum mutual inductance per phase between main field and armature

s = number of coils in short-circuited coils

X_d, X_q, X_d', X_q' = usual three phase, line to neutral, synchronous and transient reactances for ,the direct and quadrature axes

$X_{md}=\omega M_d$

X_D, X_Q, X_D' X_Q', = usual static single phase and transient reactances in the direct and quadrature axes

X_{SD}, X_{SQ} = reactances corresponding to L_{SD}, L_{SQ}

$X_L=\omega L$ = reactance of load per phase

β = angular displacement, in electrical radians, of the axis of the group of s short-circuited coils from that of phase A

ρ = coil pitch in electrical radians

θ = angular displacement of direct axis from that of phase A, measured in electrical radians

σ_f = decrement factor of main field, defined by equation 23

σ_s = decrement factor of short-circuited coils, defined by equation 24

References

1. Two-Reaction Theory of Synchronous Machines, Part I, R. H. Park. AIEE Transactions, volume 48, 1929, page 716; Part II, AIEE Transactions, volume 52, 1933, page 352.

2. Definition of Ideal Synchronous Machine and Formula for the Armature-Flux Linkages, R. H. Park. *General Electric Review*, volume 31, 1928, page 332.

3. Synchronous Machines—I, R. E. Doherty and C. A. Nickle. AIEE Transactions, volume 45, 1926, page 912.

4. Synchronous Machines—IV, R. E. Doherty and C. A. Nickle. AIEE Transactions, volume 47, 1928, page 457.

5. Synchronous Machines—V, R. E. Doherty and C. A. Nickle. AIEE Transactions, volume 49, 1930, page 700.

6. A Simplified Method of Analyzing Short-Circuit Problems, R. E. Doherty. AIEE Transactions, volume 42, 1923, page 841.

Transient Overspeeding of Induction Motors

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Synopsis: A transient swing above synchronous speed has been observed when a certain standard squirrel cage induction motor was started without load. This paper presents an explanation of the phenomenon based on the frequency modulation of the power supply caused by the rapid variation of the power taken from the line. Mathematical relations are developed and applied to the operating conditions of this motor. The agreement of the mathematical and test curves indicates that the explanation is basically correct although some secondary effects, not considered, may be present.

A TRUE induction motor cannot, of itself, operate at or above synchronous speed. The torque developed by this type of motor passes through zero at synchronous speed. It should, therefore, never be able to send the motor above that speed. Yet, in making some acceleration tests, a certain motor was consistently found to go above synchronous speed for a few cycles each time it was started.

This motor was from a standard line of squirrel cage induction motors. It had a cast aluminum rotor with deep, narrow slots. There were no salient-pole structures on the rotor, and, further, the rotor bars were skewed so that there was no tendency for the rotor to lock into synchronism. After the motor was up to speed and had settled down, the slip at no load became constant at less than 0.1 per cent.

The tests in which this behavior was observed were similar to those described in a previous paper.[1] A bakelite disc having 500 divisions accurately engraved around the periphery was mounted on the motor shaft. This disc was illuminated by light pulses from a stroboscopic lamp flashing 60 times per second, and the disc's position was photographically recorded at each flash. The motor was reversed on the line and readings were taken as the motor speed dropped from full speed in one direction to zero and then came back up to full speed in the opposite direction. Velocities were calculated by taking position differences, and accelerations were calculated by taking velocity differences. Since the friction forces were very small the entire torque of the motor was used in producing accelerations. The accelerations observed were converted to torques and plotted against speed in figure 1. Figure 2 shows a section of the velocity versus time curve for the same test. In this test the motor was operated at rated voltage. Due to the high torques developed and the low inertia of the rotor, the motor dropped from full speed in one direction to zero and returned to full speed in the opposite direction in a little over one-half second.

The curve of figure 2 shows that the motor passed through synchronous speed about 35.5 cycles after the test was started, and that it rose almost two per cent above synchronous speed on the first over-swing. A second over-swing was recorded and a third one indicated. Reference to the torque curve of figure 1 shows that the torque developed was practically equal to full-load torque as the motor passed through synchronous speed the first time.

The reason for this peculiar behavior was first qualitatively described as follows. The input to the motor had both power and lagging components. The power input to the motor was approximately proportional to the torque developed. The curve of figure 1 would indicate that during the first part of the test the power input was relatively constant. After passing the maximum torque at about 85% of synchronous speed the power taken from the lines dropped very rapidly. The lagging component of the line current, however, certainly did not vary in the same way. A more gradual change of this component would be expected. The power component would reverse if the motor changed from motor to generator action. Such reversal would not occur in the magnetizing component. From full load to no load the lagging component should be nearly constant and therefore cause constant *IR* and *IX* drops. Since it is the rapid variation of these impedance drops which is important and not their magnitudes the effects of the lagging component may be neglected. The power was supplied from a transformer bank connected to a 4,000 volt system. The bank had an appreciable reactance. Neglecting the resistance of the bank and the lagging component of the motor current, the vector diagram would be as indicated in figure 3. In this diagram E_1 represents the transformer primary voltage vector which may be taken as constant in magnitude and rotating at a constant velocity ω_1. E_2 is the motor-voltage vector and *IX* is the reactance drop in the transformer due to the power component of the motor current. As long as the power component is constant the vector E_2 must rotate at the same speed as the vector E_1, although lagging behind it. If the power component decreases the impedance drop must decrease and the vector E_2 must catch up with vector E_1. Therefore during this transient period the vector must rotate at a velocity ω_2 greater than that corresponding to the normal line frequency. With a frequency greater than 60 cycles impressed on the motor terminals, even if only for a few cycles, the motor may attain a speed greater than the 60 cycle synchronous speed if very lightly loaded.

A more quantitative analysis following the above ideas has indicated that this reasoning is correct. The motor on which the tests were made is no longer available and accurate measurements of its constants had not been made. Approximate values of some of these constants had been determined and the others may be estimated accurately enough for present purposes. All values used are thought to be within 3% of the correct values except

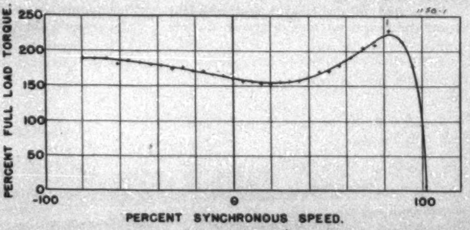

Figure 1. Speed-torque curve for a 15-horsepower four-pole squirrel-cage motor

Curve was determined under transient conditions for rapid acceleration under no load

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The data used in this paper were obtained by C. N. Williamson, a former student of the University of California, now with the U. S. Navy engineering department.

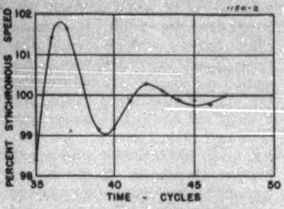

Figure 2. Section of velocity-versus-time curve for same test as figure 1

the full load slip. The full load slip was not determined but the motor manufacturer was consulted before assigning the value used.

Near synchronous speed the torque developed is proportional to the slip, and this torque will all be used in accelerating the rotor if the friction forces are negligible.

$$T = K_1 S = \frac{WR^2}{g} \times \frac{d\omega}{dt} \tag{1}$$

Where S is the slip and ω is the angular velocity of the rotor in radians per second. The full load torque of the motor was 45 pound-feet, and the full load slip was about 2.0 per cent. Therefore the constant K_1 was 45/0.020 or 2,250. The inertia of the rotor, coupling, and disc was 4.0 pound-feet-squared.

From figure 3

$$\omega_2 = \omega_1 - \frac{d\alpha}{dt} \tag{2}$$

The line frequency was 60 cycles per second so that $\omega_1 = 2\pi 60 = 377$. Equation 2 is on a 2-pole basis while equation 1 is for a general p-pole machine. Converting equation 1 to the 2-pole basis it becomes

$$T' = \frac{2K_1}{p} S = \frac{4WR^2}{p^2 g} \frac{d\omega'}{dt} \tag{3}$$

The slip which must be used in these relations depends upon the angular velocity of the rotor and the frequency applied to the motor terminals, and is not the apparent slip based on the nominal line frequency. Therefore

$$S = \frac{\omega_2 - \omega'}{\omega_2} \tag{4}$$

From this

$$\omega_2 = \frac{\omega'}{1-S} \tag{5}$$

Combining (2) and (5)

$$\frac{\omega'}{1-S} = \omega_1 - \frac{d\alpha}{dt} \tag{6}$$

But α is a function of the slip. In the region of interest where the slip is small the lagging component of motor current is nearly constant while the power component is directly proportional to the slip. The angle α therefore varies directly with the slip in this region.

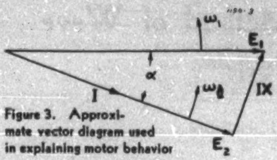

Figure 3. Approximate vector diagram used in explaining motor behavior

$$\alpha = K_2 S \tag{7}$$

The reactance which must be used in computing α is not just the transformer and lead reactance. The motor reactance must be included in this total and will usually be the largest item. For full load the reactance drop was estimated to be 20% of E_1. The angle α was therefore 11.5° or 0.20 radian for 2% slip, and the constant K_2 equal to 10.

Combining equations 6 and 7

$$\omega' = (1-S)\left(\omega_1 - K_2 \frac{dS}{dt}\right) \tag{8}$$

This value of ω' may be substituted in equation 3 giving

$$\frac{2K_1 S}{p} = \frac{4WR^2}{p^2 g} \frac{d}{dt} (1-S)\left(\omega_1 - K_2 \frac{dS}{dt}\right) \tag{9}$$

Differentiating after multiplying by $\frac{p^2 g}{4WR^2}$

$$\frac{K_1 pg}{2WR^2} S = (1-S)\left(-K_2 \frac{d^2S}{dt^2}\right) - \left(\omega_1 - K_2 \frac{dS}{dt}\right)\left(\frac{dS}{dt}\right) \tag{10}$$

Rearranging and dividing by K_2

$$(1-S)\frac{d^2S}{dt^2} + \left(\frac{\omega_1}{K_2} - \frac{dS}{dt}\right)\frac{dS}{dt} + \frac{K_1 pg}{2K_2 WR^2} S = 0 \tag{11}$$

Since the slip is very small in the region of interest the term $(1-S)$ is nearly equal to 1. Also the term $\left(\frac{\omega_1}{K_2} - \frac{dS}{dt}\right)$ is not very different from $\frac{\omega_1}{K_2}$. In figure 2, $\frac{dS}{dt}$ has a value of about 1.8 as the motor passes through synchronism the first time. The tests were made on a 60 cycle supply so that $\frac{\omega_1}{K_2} = 37.7$. Making these approximations, equation 11 becomes

$$\frac{d^2S}{dt^2} + \frac{\omega_1}{K_2}\frac{dS}{dt} + \frac{K_1 pg}{2K_2 WR^2} S = 0 \tag{12}$$

The roots of this equation are

$$D = -\frac{\omega_1}{2K_2} + \sqrt{\frac{\omega_1^2}{4K_2^2} - \frac{K_1 pg}{2K_2 WR^2}} \tag{13}$$

Substituting the numerical values corresponding to the tests in this equation

$$D = -18.8 + \sqrt{355 - 3{,}622}$$
$$= (-18.8 + j57.2)(-18.8 - j57.2) \tag{14}$$

By choosing zero time as the first time the motor passes through synchronous speed or zero slip, and further making the slopes of the two curves the same at this point, the slip equation becomes

$$S = -0.0315\epsilon^{-18.8t} \sin 57.2t \tag{15}$$

This is a damped oscillation having a frequency of 9.1 cycles per second. The frequency of the damped oscillation shown in figure 2 is about 10 cycles per second. The damping is nearly the same in the two cases. A plot of equation 15 is shown in figure 4. The test curve of figure 2 is replotted for comparison. Negative values of slip have been plotted above the axis and the ordinate scaled in terms of synchronous speed so that the curve will appear the same as figure 2. The discrepancies between the test results and equation 15 are due to several causes. First the determination of the constants of the motor were not as accurate as desired. Second the setting up and solution of the differential equation involved approximations. Third the motor was changing speed so rapidly that one might expect that the functions K_1 and K_2, called constants, might not be strictly constant, and their numerical values under these transient conditions might be different from values determined by steady state tests.

Equation 13 indicates that oscillations will be present when the quantity under the radical is negative. This condition exists when

$$\frac{K_1 K_2 p}{WR^2} > \frac{\omega_1^2}{2g} \tag{16}$$

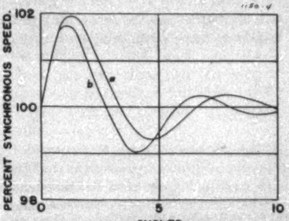

Figure 4

(a) $100(1-S) = 100 + 3.15\epsilon^{-18.8t} \sin 57.2t$

(b) Experimental curve shown in figure 2

A Cathode-Ray Method of Wave Analysis

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Synopsis: Complex periodic functions, when transformed into a corresponding voltage wave, may be analyzed with the use of a cathode ray oscillograph. The complex wave is represented upon the fluorescent screen of the cathode ray tube by a vertical displacement at the same time that a sinusoidal horizontal oscillation exists. If the frequency of the sinusoidal oscillation bears an integral relation to the fundamental frequency of the complex wave, a Lissajous figure is viewed on the screen. The areas of these Lissajous figures are directly related to the coefficients of the Fourier series of the complex wave. The area is determined by measuring a photograph of the figure using a polar planimeter. The method is based on the theory developed by Mr. L. W. Chubb in connection with the Chubb Polar Analyzer.[4] Advantages of the method are that it is quick and convenient, only standard laboratory equipment is necessary, and with special equipment, it promises to make possible the analysis of ultrahigh frequency waves.

PERIODIC waves, such as the vibration of mechanical bodies or the variation of current and voltage in electric circuits, are often complex. In electric circuits, sinusoidally varying functions usually give the most satisfactory operation. Calculations are much easier, values may be expressed graphically in vector diagrams, and vector equations may be used. The same is true of mechanical vibration. Hence, in order to handle complex waves conveniently, it is desirable that they be separated into their sinusoidal components. It is the purpose of this paper to describe and explain a new method of wave analysis using a cathode ray oscillograph. The underlying theory is not new, but the method is thought to be faster and the results more complete than other methods of wave analysis using ordinary laboratory equipment. It is hoped that this method will prove useful in other laboratories.

Application of Lissajous Figures to Wave Analysis

The electron beam in a cathode ray tube is deflected by electrostatic forces. If the beam be deflected in one direction by an alternating voltage, and at the same time by another voltage in a direction at right angles to the first, the resulting position of the beam at any instant of time is a function of both voltages. If one frequency is an even multiple of the other, the resulting pattern upon the screen is a Lissajous figure.

In order to begin a study of Lissajous figures, a simple Lissajous figure is shown in figure 1. The two functions to be compounded are $f(\theta)=A_1 \sin \theta$, and $g(\theta)=A_2 \sin (\theta+\alpha)$. The Lissajous figure is an ellipse with a maximum vertical dimension of $2A_2$ and a maximum horizontal dimension of $2A_1$. If $\alpha=0$, the ellipse becomes a single straight line; if $\alpha=90°$, one axis of the ellipse will be vertical and equal to $2A_2$, and the other axis will be horizontal and equal to $2A_1$; if $\alpha=90°$ and $A_1=A_2$, the Lissajous figure becomes a circle. More complex Lissajous figures are obtained if either or both functions are complex. In this case the fundamental frequencies of the functions must be the same or bear an integral ratio to each other.

The Fourier series for a complex function may be expressed as follows:

$$f(\theta)=A_0+C_1 \sin (\theta+\theta_1)+C_2 \sin 2(\theta+\theta_2)+ \ldots +C_n \sin n(\theta+\theta_n)+ \ldots \quad (1)$$

where θ is a function of time equal to $2\pi ft$. However, due to the difficulty of using the harmonics in their various phase positions, each harmonic is usually resolved into two quadrature components thus:

$$f(\theta)=A_0+A_1 \sin \theta+B_1 \cos \theta+A_2 \sin 2\theta+B_2 \times \cos 2\theta+ \ldots +A_n \sin n\theta+B_n \cos n\theta+ \ldots \quad (2)$$

As a result, wherever complex periodic waves are encountered they are usually resolved into their harmonic components in order to permit a simpler mathematical treatment. The process of harmonic analysis, then, reduces merely to a problem of evaluating the coefficients A_0, A_1, B_1, etc., in the above series. This can be done in a great number of ways.

Experiments have been conducted in the laboratories of the A. & M. College of Texas using a low voltage cathode ray oscillograph for wave analysis. The complex wave is applied to the horizontal plates which give the cathode beam a vertical displacement. This pair of plates will be referred to hereafter as the horizontal plates, and the other pair of plates as the vertical plates. Ordinarily

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1. For all numbered references, see list at end of paper.

For K_1 and K_2 may be substituted their values in terms of full-load torque, slip, and angular phase shift.

$$\frac{T\alpha p}{WR^2S^2} > \frac{\omega_1^2}{2g} \quad (17)$$

This relation indicates that overspeeding may be reduced by increasing the WR^2 of a machine. Large machines have large values of WR^2 but also produce large torques. The inertia constant usually increases more rapidly than the torque, as the machine size is increased, so that these larger motors should show less tendency to over speed. The most important factor is the full-load slip since it occurs in the function as a squared term. Motors with low full-load slip should have much greater tendencies to overspeed than high-slip motors. Operation at reduced voltage would decrease this tendency by changing the relation between the torque, slip, and angle.

Various acceleration methods have been considered as a means of determining the speed-torque curves of induction motors. The above analysis indicates that precautions must be taken to avoid conditions similar to those found in these tests if accurate results are to be obtained. However the errors involved are appreciable only near synchronous speed where the frequency change represents a large percentage of the rotor frequency. Where acceleration methods are used to determine only the low-speed part of the curve, errors due to this frequency modulation should not be important.

Reference

1. SOME ACCELERATION TESTS ON LARGE SYNCHRONOUS PUMP MOTORS, R. W. Ager, AIEE TRANSACTIONS, volume 60, 1941, page 416.

the vertical plates receive a saw-tooth shaped voltage wave, which produces a linear horizontal sweep and allows visual observation of the shape of the complex wave. For wave analysis, the vertical plates are connected to a generator which will produce a nearly pure sine wave.

Since equation 2 is an expression representing a complex wave $f(\theta)$, let it represent the complex wave that is applied to the horizontal plates of the oscillograph. Also let $g(\theta)=-K\cos k(\theta+\alpha_k)$ be the sinusoidal wave applied to the vertical plates. The function $g(\theta)$ has a frequency k times the frequency of the fundamental of $f(\theta)$, so if k is an integer, a Lissajous figure will appear upon the oscillograph screen. The amplitude of $g(\theta)$ is K, and the angle α_k in the expression indicates the phase angle between $g(\theta)$ and the cosine term of the kth harmonic of $f(\theta)$.

Referring again to figure 1 and the discussion of plotting Lissajous figures, it is evident that the shape of the Lissajous figure depends upon the phase angle $k\alpha_k$. There is an infinite number of shapes that may be assumed as $k\alpha_k$ may assume an infinite number of values between 0 and 2π. If the frequency of $g(\theta)$ is not exactly a multiple of the frequency of $f(\theta)$, that is, if the two waves are not synchronized, the figure will gradually change through a complete cycle of shapes as the phase angle changes from 0 to 2π.

The area S_k of one of these figures may be calculated from the expressions of the two functions $f(\theta)$ and $g(\theta)$, and is as follows:

$$S_k=kK\pi(A_k\cos k\alpha_k+B_k\sin k\alpha_k) \qquad (3)$$

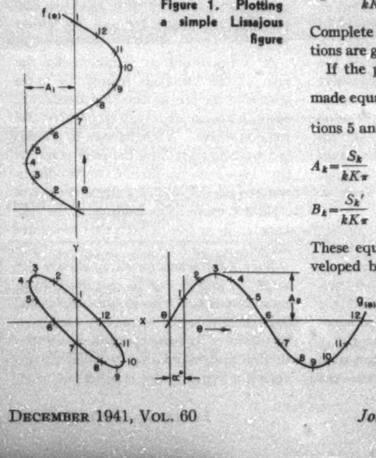

Figure 1. Plotting a simple Lissajous figure

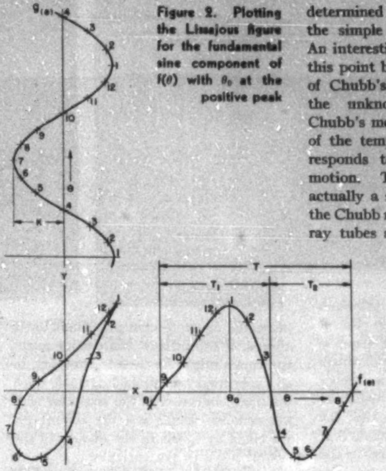

Figure 2. Plotting the Lissajous figure for the fundamental sine component of $f(\theta)$ with θ_0 at the positive peak

It will be noticed that this expression contains only two coefficients of the series for $f(\theta)$. However, it is necessary to be able to solve for one of the coefficients independently. This may be done if another expression can be derived that will contain the same two coefficients. If the phase angle α_k were allowed to change to a new value, say β_k, the area of the figure would change to a new value S_k'. This area may be expressed similarly as

$$S_k'=kK\pi(A_k\cos k\beta_k+B_k\sin k\beta_k) \qquad (4)$$

If equations 3 and 4 are combined, the resulting expressions for A_k and B_k are as follows:

$$A_k=\frac{S_k\sin k\beta_k-S_k'\sin k\alpha_k}{kK\pi\sin k(\beta_k-\alpha_k)} \qquad (5)$$

$$B_k=\frac{S_k'\cos k\alpha_k-S_k\cos k\beta_k}{kK\pi\sin k(\beta_k-\alpha_k)} \qquad (6)$$

Complete derivations of the above equations are given in the appendices.

If the phase angles $k\alpha_k$ and $k\beta_k$ were made equal to 0 and $\frac{\pi}{2}$ respectively, equations 5 and 6 would reduce to

$$A_k=\frac{S_k}{kK\pi} \qquad (7)$$

$$B_k=\frac{S_k'}{kK\pi} \qquad (8)$$

These equations correspond to those developed by L. W. Chubb in connection with the Chubb polar analyzer.[4] However, the fact that it is difficult to accurately adjust the frequency and phase angle of the sine wave generator to a predetermined value precludes the use of the simple expressions in (7) and (8). An interesting analogy may be drawn at this point between this method and that of Chubb's. The vertical deflection of the unknown wave corresponds to Chubb's mechanical deflection by means of the template, and the oscillator corresponds to Chubb's simple harmonic motion. The cathode ray method is actually a simplification in apparatus of the Chubb method, making use of cathode ray tubes and oscillators to replace the Chubb mechanical apparatus. Like the Chubb method, this method will not apply to a determination of the constant term A_0 in a Fourier series. The constant term is relatively unimportant, and if it exists in a circuit, it may be determined by means of a direct current instrument.

It is interesting to note that the expressions 5 and 6 are both in terms of some area divided by π. The graphical method of analysis introduced by C. Runge[2] makes use of a theoretical area obtained by multiplying each ordinate by a function of the corresponding angle. The Lissajous figure automatically produces this same area as an actual figure that may be measured, with the exception that the area is multiplied by K the amplitude of $g(\theta)$, and k the order of the harmonic. Hence both K and k appear in the denominator of expressions 5 and 6.

Discussion of Experimental Procedures

In experimental analysis by means of the cathode ray method, several serious problems have been encountered; (1) the area of the Lissajous figure must be determined easily and accurately, (2) the sine wave source must be of a variable frequency and have very low harmonic content, (3) the frequency must be adjustable to synchronize with the fundamental of the complex wave, (4) the phase angle between the sine wave and the complex wave must be readily determined, and (5) results must be obtained in suitable units.

The Lissajous figure remains stationary on the screen only as long as the two waves are properly synchronized. With a little practice it may be possible to estimate the net area of a figure on the screen. A better practice, however, is to photograph the figure and later measure the area with an ordinary polar planimeter.

Figure 3 (left). Transformer exciting current wave

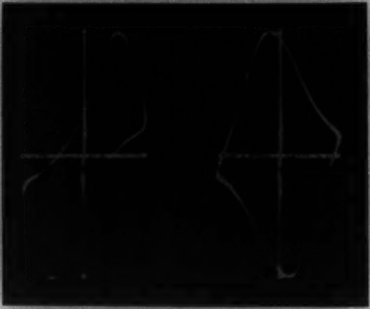

Figure 4 (right). A pair of Lissajous figures for determining the fundamental harmonic

The camera used is important, although an expensive one is not necessary. Better results will be obtained if the room is slightly darkened, but complete darkness is more of a handicap than a help. Since speed is more important than depth of focus, a fast lens is desirable. Also, the size of the image on the film demands that the camera be focused at distances in the vicinity of one or two feet. Once the camera is focused, however, it should not be changed during the process of analyzing a wave, as a change of focus would change the scale of units upon the film.

The sine wave source may be either a mechanical generator or an oscillator. The mechanical generator, of course, would be adaptable only at lower frequencies. In the 60 cycle range, the seventh harmonic is 420 cycles, and a generator producing from 60 to 420 cycles is a rare and expensive piece of equipment and may still develop a high percentage of harmonics. The most desirable characteristic of a mechanical generator would be the accuracy with which synchronism and phase relations could be worked out by mechanical gearing. In general a good quality variable frequency oscillator is more convenient to handle and more easily adjustable.

A number of different types of oscillators may be used. The only requirements are that the oscillator have a nearly constant amplitude over the proper frequency range and that the harmonic content be very small. The power required to maintain voltage across the plates of the tube is very small, so the power rating of the oscillator is relatively unimportant. However, if the output voltage is low, the oscillograph must have an amplifier to produce a figure of the desired size upon the screen.

The frequency of the oscillator must coincide exactly with that of the complex wave if a Lissajous figure is to remain stationary, and frequency drift of either the oscillator or the complex wave will cause the figure to change. There are a number of automatic synchronizing methods which might be used.[5] However, in all cases tried the method depends upon the interconnection of the oscillator with the complex wave, and the advantages are offset by a loss in the quality of the output wave of the oscillator. For this reason manual control of the frequency is desirable and a good frequency control is an important feature of the oscillator.

When the frequencies are synchronized so that the Lissajous figure remains stationary, there are a number of different shapes that the figure can assume. These depend upon the phase angle between the two waves. It is important to be able to determine this phase angle since it enters into the expression for the coefficients of the Fourier series, equations 5 and 6. In figure 2 a complex wave $f(\theta)$ has been plotted on the horizontal axis and a sine wave $g(\theta)$ has been plotted on the vertical axis. The Fourier series for $f(\theta)$ depends upon the location of a reference point where all the sine terms of the series are zero and all the cosine terms have a positive maximum value. Call this point θ_0. In many methods of analysis, the point where the complex wave crosses the axis is chosen for θ_0. For the cathode ray method of analysis, the maximum positive peak of $f(\theta)$ is thought to be a more suitable point for θ_0. The general expression for $g(\theta)$ in figure 2 is $g(\theta)=-K \cos k(\theta+\alpha_k)$. Since θ_0 is a value of θ such that $\sin(\theta_0)=0$ and $\cos(\theta_0)=+1$, θ_0 is some multiple of 2π. Hence, $g(\theta)$ for $\theta=\theta_0$ reduces to

$$g(\theta_0)=-K \cos k\alpha_k \qquad (9)$$

In figure 2, the Lissajous figure has been plotted for the case when $\alpha_k=0$, and $g(\theta_0)=-K$. It will be noted that the Lissajous figure has a peak in the upper right corner a distance $-K$ from the Y axis. If α_k has a value other than zero, $g(\theta_0)$ will be reduced in magnitude and the peak will have moved a distance $(K-K \cos k\alpha_k)$ to the left. The peak can always be located on the oscillogram and $g(\theta_0)$ may be measured directly. Then the angle $k\alpha_k$ is, from equation 11,

$$k\alpha_k = \cos^{-1}(g(\theta_0)/K) \qquad (10)$$

From equation 10, $k\alpha_k$ may be either positive or negative since the cosine is the same for either case, but in writing the sine of $k\alpha_k$, it is important to know whether $k\alpha_k$ is positive or negative. It is positive if $g(\theta)$ has already passed its negative maximum at the time $f(\theta)$ reaches its peak value. The best way to determine this when viewing the figure on the oscillograph screen is to adjust the frequency of the sine wave generator to a value slightly greater than that of the harmonic and stop the figure by synchronizing the two frequencies after the peak has left the upper right corner moving to the left. If the peak is stopped before it reaches the midpoint, the angle will be between 0° and 90°, while if it is stopped between the midpoint and the upper left corner, the angle will be between 90° and 180°. It is good practice to make $k\alpha_k$ about 45° and $k\beta_k$ about 135°.

It is important in planimetering the area of the Lissajous figures, to move always in the proper direction around the loops, for some areas may be positive and some negative. A simple way to do this is always to start from the peak point on the Lissajous figure and move downward to the left, thus tracing through one complete cycle. The reading on the planimeter is or can be corrected to square inches.

Now if the scales for all figures are the same, A_k and B_k can be calculated from equations 5 and 6 in inches. However, it is usually better to convert each area by means of its scale constant to a hypothetical unit, which may be called square volts if a voltage wave is being analyzed,

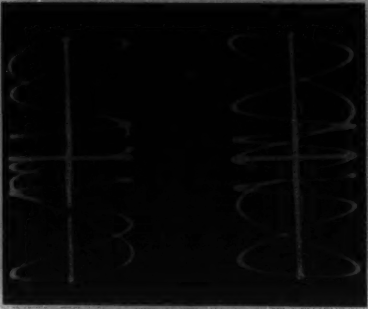

Figure 5. A pair of Lissajous figures for determining the seventh harmonic

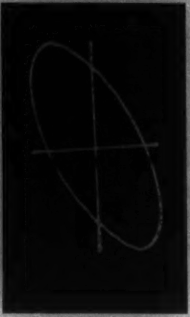

Figure 6. Calibration picture

or square amperes for a current wave. Then K (assumed constant) can be converted to the correct units, and the value of A_k is derived in those units. If K is not constant, equations 5 and 6 must be altered as follows:

$$A_k = \frac{S_k/K \sin k\beta_k - S_k'/K' \sin k\alpha_k}{k\pi \sin k(\beta_k - \alpha_k)} \tag{11}$$

$$B_k = \frac{S_k'/K' \sin k\alpha_k - S_k/K \cos k\beta_k}{k\pi \sin k(\beta_k - \alpha_k)} \tag{12}$$

The scale constant may be determined by taking a picture of a pure sine wave of known amplitude at the same time the other pictures are being taken. In fact, it is a good practice to take two or three calibration pictures during an experiment. Obviously, a change in amplification on the oscillograph or focus of the camera will alter the scale constant. If the scale constant varies slightly for unavoidable reasons, either an average constant must be used or individual constants must be assigned to each picture.

Experimental Results

In order to determine to some extent how practical the cathode ray method of wave analysis is, an exciting current wave for a potential transformer was analyzed. Figure 3 shows an oscillogram of the wave that was analyzed. The results of the cathode ray method were compared with results using another method of analysis. The other method was the graphical method of C. Runge using 72 points.[2] A comparison of results is shown in table I. The analysis was only carried as far as the seventh harmonic as the graphical method becomes very inaccurate for higher harmonics and correspondingly small magnitudes. It is thought that the two methods possess about the same degree of accuracy for low order harmonics up to the third, but for higher harmonics the cathode ray method is thought to be more accurate. The areas of the Lissajous figures for higher harmonics may be rather accurately determined because they are proportional to the order of the harmonic k times the magnitude of the corresponding coefficients.

Table I. Comparison of Results in an Experimental Analysis of a Transformer Exciting Current

	Cathode-Ray Method		Runge Method	
Harmonic	Magnitude Amperes	Phase Angle Degrees	Magnitude Amperes	Phase Angle Degrees
First	0.318	0	0.327	0
Third	0.1120	44.2	0.1135	50.0
Fifth	0.0267	54.5	0.0248	59.0
Seventh	0.00794	7.7	0.00725	12.6

Figures 4 and 5 show examples of some of the Lissajous figures that were used in the experimental analysis. The pictures were taken using a 35 mm. camera with an f-4.5 lens. Exposures were made at $^1/_{25}$ of a second. This was found to give a very good trace on the film when the intensity of the cathode ray was adjusted to a reasonable brightness. Since the light emitted from a cathode ray oscillograph screen is very small, a camera with a fast lens is a distinct advantage. Another desirable feature in this application is a short focus, since the size of the figures on the screen is hardly more than two inches high. The zero axes of the Lissajous figures were obtained by exposing the film once with the exciting current wave shorted out and once with the sine wave shorted out in addition to the exposure for the figure itself. Figure 6 is an example of one of the calibration pictures that were taken. It is a Lissajous figure similar to those in figure 4 except that both waves are sinusoidal.

Figure 7 shows the equipment as it was assembled for the experimental analysis. The cathode ray oscillograph had a 5 inch screen and the size of the figure could be adjusted through step up amplifiers on both the horizontal and vertical plates. The oscillator was a beat frequency type having a frequency range from 20 to 17,000 cycles and a practically constant amplitude over that range. Just in front of the oscillator and to the left is a small tuning condenser which was used in parallel with the main tuning condenser on the oscillator to give a finer adjustment of frequency.

Conclusion

It is important to note that only standard laboratory equipment is necessary for the cathode ray analysis. The method is much faster and less tedious than the graphical method, because most of the calculation is dispensed with. Harmonics that do not exist, or are negligible, are apparent at once from the areas of the Lissajous figures without further calcu-

Figure 7. Apparatus used for the experimental analysis

lation. The other harmonics may be quickly calculated and converted to the correct units.

But much more important than any other advantage is the fact that the cathode ray oscillograph will respond effectively to much higher frequencies than any other type of oscillograph. The vibrator elements in most mechanical oscillographs have a natural frequency of 5,000 cycles per second undamped; so it is evident that any frequency in this range would be somewhat magnified and the results would be erratic. The cathode ray oscillograph may be used in the radio frequency range. Thus, high frequency waves may be analyzed with the same simplicity and completeness as are the power frequency waves.

With the possibility of completely analyzing higher frequency waves, the study of mechanical vibration may be accelerated and improved. A device that would couple the mechanical system to an electrical circuit and reproduce the mechanical oscillations as electrical functions would make possible the analysis of mechanical vibration with a cathode ray oscillograph.

The frequency range of cathode ray oscillographs has been still further increased with the invention of a new microwave oscillograph devised for the centimeter range of wave lengths, where ordinary oscillographs are not adaptable.[8] These high frequency waves are distorted in the ordinary cathode ray oscillograph because the finite time required for an electron to pass across a deflecting field is of the same order of magnitude as the time required for one cycle of the wave. Also, if the two deflection fields are spaced along the direction of the beam, the time lag between the two fields is an appreciable part of a cycle. In the microwave oscillograph, the deflection fields are made small and brought very close together. All dimensions of the tube are likewise reduced, and the image is thrown upon an especially fine grain fluorescent screen. The image itself is so small that it must be observed and photographed by means of a microscope. This new instrument should make ultra-high frequency wave analysis possible, with very little change from the method described in this paper.

Appendix I

Areas of Lissajous Figures

The general expression for the sinusoidal function is

$$g(\theta) = -K \cos k(\theta+\alpha_k) \tag{13}$$

and

$$\begin{aligned} dg(\theta) &= kK \sin k(\theta+\alpha_k)d\theta \\ &= kK(\cos k\alpha_k \sin k\theta + \sin k\alpha_k \cos k\theta)d\theta \end{aligned} \tag{14}$$

The area of the Lissajous figure is

$$S_k = \int_0^{2\pi} f(\theta)dg(\theta) \tag{15}$$

The complex function $f(\theta)$, given in equation 2, may be changed to the following form:

$$f(\theta) = A_0 + \sum_{n=1}^{\infty} A_n \sin n\theta + \sum_{n=1}^{\infty} B_n \cos n\theta \tag{16}$$

The integration may be carried out for each part of $f(\theta)$ independently as follows:

$$\int_0^{2\pi} A_0 kK \sin k(\theta+\alpha_k)d\theta = 0$$

$$\int_0^{2\pi} (A_n \sin n\theta)(kK \cos k\alpha_k \sin k\theta)d\theta = 0 \qquad \text{if } n \neq k$$

$$\int_0^{2\pi} (A_n \sin n\theta)(kK \sin k\alpha_k \cos k\theta)d\theta = 0 \qquad \text{for all } n$$

$$\int_0^{2\pi} (B_n \cos n\theta)(kK \cos k\alpha_k \sin k\theta)d\theta = 0 \qquad \text{for all } n$$

and

$$\int_0^{2\pi} (B_n \cos n\theta)(kK \sin k\alpha_k \cos k\theta)d\theta = 0 \qquad \text{if } n \neq k$$

If $n = k$

$$\begin{aligned} \int_0^{2\pi} (A_n \sin n\theta)(kK \cos k\alpha_k \sin k\theta)d\theta &= \\ A_k kK \cos k\alpha_k \int_0^{2\pi} \sin^2 k\theta d\theta &= \\ (A_k kK \cos k\alpha_k)\pi & \end{aligned}$$

and

$$\begin{aligned} \int_0^{2\pi} (B_n \cos n\theta)(kK \sin k\alpha_k \cos k\theta)d\theta &= \\ B_k kK \sin k\alpha_k \int_0^{2\pi} \cos^2 k\theta d\theta &= \\ (B_k kK \sin k\alpha_k)\pi & \end{aligned}$$

Only two terms remain in the expression for the area S_k,

$$\begin{aligned} S_k &= A_k kK\pi \cos k\alpha_k + B_k kK\pi \sin k\alpha_k \\ &= kK\pi(A_k \cos k\alpha_k + B_k \sin k\alpha_k) \end{aligned} \tag{17}$$

Since it is known that α_k may assume a new value β_k, the expression for S_k' may be written immediately,

$$S_k' = kK\pi(A_k \cos k\beta_k + B_k \sin k\beta_k) \tag{18}$$

Appendix II

Derivation of General Expression for the Coefficients

Equations 17 and 18 may be changed to the following form:

$$A_k \cos k\alpha_k + B_k \sin k\alpha_k = \frac{S_k}{kK\pi} \tag{19}$$

$$A_k \cos k\beta_k + B_k \sin k\beta_k = \frac{S_k'}{kK\pi} \tag{20}$$

These equations may be solved by determinants for the coefficients A_k and B_k.

$$\begin{aligned} A_k &= \frac{\dfrac{1}{kK\pi}\begin{vmatrix} S_k, & \sin k\alpha_k \\ S_k', & \sin k\beta_k \end{vmatrix}}{\begin{vmatrix} \cos k\alpha_k, & \sin k\alpha_k \\ \cos k\beta_k, & \sin k\beta_k \end{vmatrix}} \\ &= \frac{\dfrac{1}{kK\pi}(S_k \sin k\beta_k - S_k' \sin k\alpha_k)}{\cos k\alpha_k \sin k\beta_k - \cos k\beta_k \sin k\alpha_k} \\ &= \frac{S_k \sin k\beta_k - S_k' \sin k\alpha_k}{kK\pi \sin k(\beta_k - \alpha_k)} \end{aligned}$$

$$\begin{aligned} B_k &= \frac{\dfrac{1}{kK\pi}\begin{vmatrix} \cos k\alpha_k, & S_k \\ \cos k\beta_k, & S_k' \end{vmatrix}}{\begin{vmatrix} \cos k\alpha_k, & \sin k\alpha_k \\ \cos k\beta_k, & \sin k\beta_k \end{vmatrix}} \\ &= \frac{\dfrac{1}{kK\pi}(S_k' \cos k\alpha_k - S_k \cos k\beta_k)}{\cos k\alpha_k \sin k\beta_k - \cos k\beta_k \sin k\alpha_k} \\ &= \frac{S_k' \cos k\alpha_k - S_k \cos k\beta_k}{kK\pi \sin k(\beta_k - \alpha_k)} \end{aligned}$$

References

1. Wave Motion and Light (book), E. P. Lewis. Physics for Students of Science and Engineering, 1932.

2. Electric Circuit Analysis (book), M. G. Malti, 1930.

3. The Cathode-Ray Tube at Work (book), John F. Rider, 1935.

4. Analysis of Periodic Waves, L. W. Chubb *Electric Journal*, volume 11, 1914, pages 91–6.

5. Electrostatic Analyzer for Complex Waves of Small Amplitude, J. C. Prescott. *Journal* of the Institution of Electrical Engineers, volume 85, 1939, pages 302–08.

6. A New Frequency Transformer or Frequency Changer, Isaac Koga. *Proceedings* of the Institute of Radio Engineers, volume 15, 1927, page 669.

7. Automatic Synchronization of Triode Oscillators, E. V. Appleton. *Proceedings* of the Cambridge Philosophical Society, volume 21, 1922, page 231.

8. Ultrahigh-Frequency Oscillography, H. E. Hollmann. *Proceedings* of the Institute of Radio Engineers, volume 28, 1940, pages 213–19.

The Apparent-Impedance Method of Calculating Single-Phase-Motor Performance

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Synopsis: In the past several years there have been offered a number of calculation methods to be used in connection with predetermining the performance of single-phase induction motors. While such methods are usually based upon either the cross field or the revolving field theory, they differ from each other according to individual preferences.

In the present paper there is proposed a method which seems to be convenient and simple, and which, because of symmetry, involves a minimum of charts in carrying out the necessary calculations. The method is proposed because it has been used in one design department for a number of years and found to be very satisfactory as a practical method.

In the calculation of performance of single-phase motors by the revolving-field theory, there appear repeatedly certain quantities which have been defined as apparent impedances. The actual determination of an apparent impedance when the characteristic impedances of the motor are known involves a considerable amount of labor and the use of complex quantities. This labor has caused some engineers to avoid a straightforward use of the revolving-field theory and to substitute artificial steps, or actually to use entirely different methods. In a case such as this it is often convenient to consider whether the labor of calculation could be avoided by the use of a suitable chart wherein the calculational work is done once and for all and, for a specific calculation, it is merely necessary to read the values from the chart. It is the principal purpose of this paper to indicate how such charts may be constructed for the determination of apparent impedance and to show how the use of such charts can be made to simplify induction-motor performance calculations.

Per-Unit Apparent-Impedance Charts

The values of the apparent impedances of a single-phase motor were given in equations 54 to 57, inclusive, of my 1929 paper[1] on the revolving-field theory. These values are indicated below in equations 1 to 4.

$$R_f=\frac{X_m^2\dfrac{R_2}{s}}{\left(\dfrac{R_2}{s}\right)^2+(X_2+X_m)^2} \tag{1}$$

$$X_f=\frac{X_m\left[\left(\dfrac{R_2}{s}\right)^2+X_2(X_2+X_m)\right]}{\left(\dfrac{R_2}{s}\right)^2+(X_2+X_m)^2} \tag{2}$$

$$R_b=\frac{X_m^2\left(\dfrac{R_2}{2-s}\right)}{\left(\dfrac{R_2}{2-s}\right)^2+(X_2+X_m)^2} \tag{3}$$

$$X_b=\frac{X_m\left[\left(\dfrac{R_2}{2-s}\right)^2+X_2(X_2+X_m)\right]}{\left(\dfrac{R_2}{2-s}\right)^2+(X_2+X_m)^2} \tag{4}$$

The right-hand members of each of the above four equations contain three independent variables, if R_2/s and $R_2/(2-s)$ be each considered as a single variable. Functions involving three independent variables are usually considered too complicated to chart since they involve families of curves. However, if the apparent impedances and each of the independent variables of the preceding equations be expressed as per unit of one of the independent variables, it is possible to reduce the number of independent variables by one. This can be done if both sides of the four above equations be divided by X_m and both the numerator and denominator of each of the right-hand members be divided by X_m^2. The resulting equations are given in 5 to 8 below:

$$\frac{R_f}{X_m}=\frac{\dfrac{R^2}{X_m s}}{\left(\dfrac{R_2}{X_m s}\right)^2+\left(\dfrac{X_2}{X_m}+1\right)^2}=\text{per unit forward field apparent resistance} \tag{5}$$

$$\frac{X_f}{X_m}=\frac{\left(\dfrac{R_2}{X_m s}\right)^2+\dfrac{X_2}{X_m}\left(\dfrac{X_2}{X_m}+1\right)}{\left(\dfrac{R_2}{X_m s}\right)^2+\left(\dfrac{X_2}{X_m}+1\right)^2}=\text{per unit forward field apparent reactance} \tag{6}$$

$$\frac{R_b}{X_m}=\frac{\dfrac{R_2}{X_m(2-s)}}{\left(\dfrac{R_2}{X_m(2-s)}\right)^2+\left(\dfrac{X_2}{X_m}+1\right)^2}=\text{per unit backward-field apparent resistance} \tag{7}$$

$$\frac{X_b}{X_m}=\frac{\left(\dfrac{R_2}{X_m(2-s)}\right)^2+\dfrac{X_2}{X_m}\left(\dfrac{X_2}{X_m}+1\right)}{\left(\dfrac{R_2}{X_m(2-s)}\right)^2+\left(\dfrac{X_2}{X_m}+1\right)^2}=\text{per unit backward-field apparent reactance} \tag{8}$$

In equations 5 to 8 the independent variables will have the following definitions:

$\dfrac{R_2}{X_m s}$ = per unit secondary resistance at slip s

$\dfrac{R_2}{X_m(2-s)}$ = per unit secondary resistance at slip $2-s$

$\dfrac{X_2}{X_m}$ = per unit secondary reactance

On the basis of the above definitions of the independent variables, there are only two independent variables in the right-hand members of equations 5 to 8. It therefore becomes possible to plot each of equations 5 to 8 in the form of a family of curves. Since the only difference between the expressions for per unit apparent forward field resistance and reactance and the similar expressions for the backward field is the use of R_2/s in place of $R_2/(2-s)$, the curves for the forward and backward field apparent impedances will be identical and only two families of curves need be drawn.

In figures 1 and 2 there are shown curves of per unit apparent resistance and per unit apparent reactance plotted for definite values of per unit secondary reactance and as a function of the reciprocal of the per unit secondary resistance for slips of s or $2-s$. The reciprocal of the per unit secondary resistance has been chosen as the independent variable because the resulting curves spread out in such a fashion as to make them more easily and more accurately read.

In order to make use of the charts and to obtain an actual value of apparent resistance or reactance, it is first necessary to obtain the per unit secondary reactance, X_2/X_m, to learn which of the curves

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1. For all numbered references, see list at end of paper.

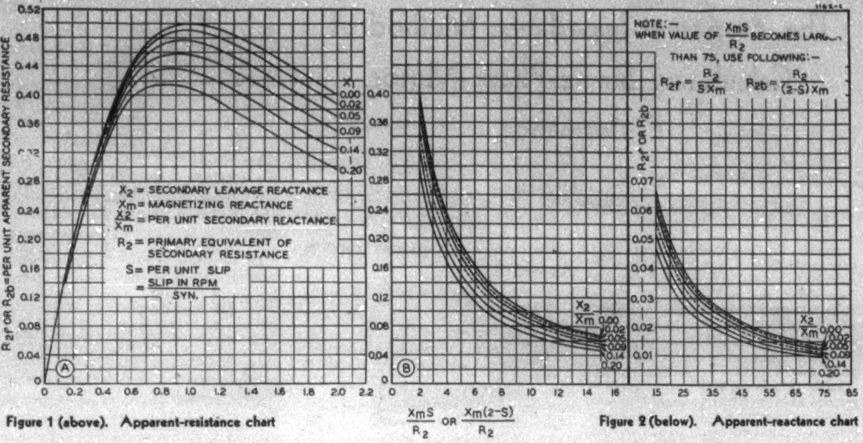
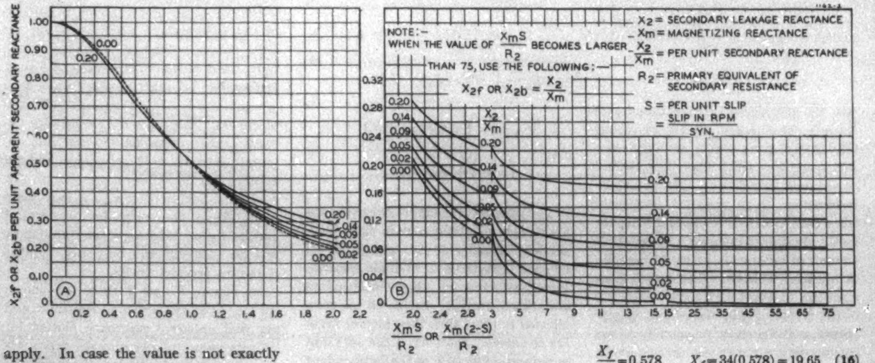


Figure 1 (above). Apparent-resistance chart

Figure 2 (below). Apparent-reactance chart



apply. In case the value is not exactly the same as that of a curve, it is, of course, necessary to interpolate between two adjacent curves.

Once the proper curve has been selected, the corresponding value of per unit apparent resistance or reactance may be read as the ordinate corresponding to the proper value of $X_m s/R_2$ or $X_m(2-s) \div R_2$. The actual apparent resistance or reactance is obtained as the product of the per unit apparent resistance or reactance by X_m, the magnetizing reactance.

As an example of actual use of these curves, consider the case of a single-phase motor having the following constants:

$$R_2 = 2 \tag{9}$$

$$X_2 = 1.7 \tag{10}$$

$$X_m = 34 \tag{11}$$

If this motor is operating at a slip of 0.05, the per unit forward and backward field secondary reactances will be

$$\frac{X_2}{X_m} = \frac{1.7}{34} = 0.05 \tag{12}$$

and the reciprocal of the per unit forward and backward field secondary resistances will be

$$\frac{X_m s}{R_2} = \frac{34(0.05)}{2} = 0.85 \tag{13}$$

$$\frac{X_m(2-s)}{R_2} = \frac{34(1.95)}{2} = 33.2 \tag{14}$$

Using these values and figures 1 and 2, the per unit apparent impedances and the corresponding actual impedance values are given below:

$$\frac{R_f}{X_m} = 0.475 \qquad R_f = 34(0.475) = 16.18 \tag{15}$$

$$\frac{X_f}{X_m} = 0.578 \qquad X_f = 34(0.578) = 19.65 \tag{16}$$

$$\frac{R_b}{X_m} = 0.0273 \qquad R_b = 34(0.0273) = 0.928 \tag{17}$$

$$\frac{X_b}{X_m} = 0.0482 \qquad X_b = 34(0.0482) = 1.638 \tag{18}$$

Calculation of Single-Phase-Motor Performance

In the calculation of motor performance it is usually desirable to proceed from a relatively simple idealized conception and to introduce phenomena of secondary importance by modifying the results obtained from the idealized concept. For example: The equivalent circuit for the single-phase motor which was given in figure 1 of my 1929 paper (figure 3 of the present paper), and which has appeared many times in publications by other authors, makes no provision for repre-

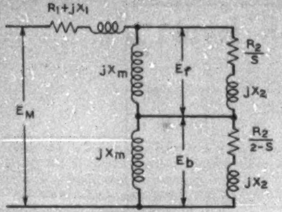
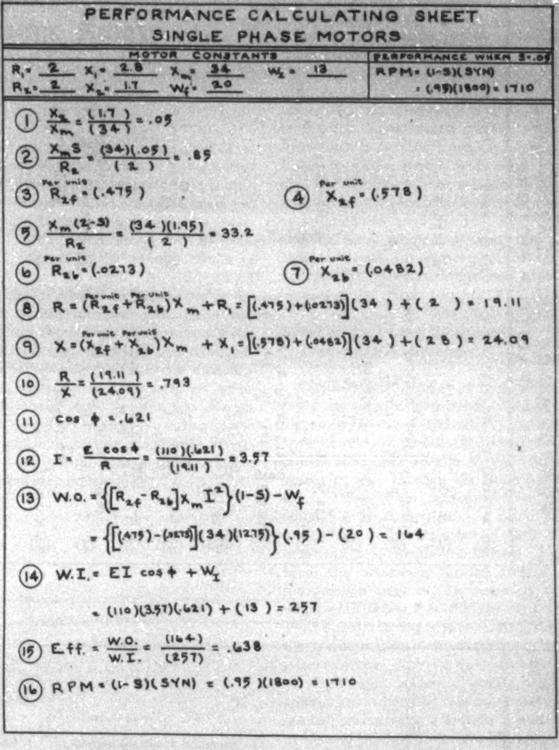


Figure 3. Single-phase-motor equivalent circuit

senting the core-loss current. Such current could be shown as flowing through a high resistance which is connected in parallel with the magnetizing reactance of each forward and backward field. In actual practice it is usually found that the effect of the core-loss current on the motor performance is relatively small since the current itself is usually very small and since its time-phase relation is at 90 degrees with respect to the magnetizing current, which latter is usually relatively large. For this reason, it has usually been found desirable to consider the core loss as if it were the result of a high resistance in parallel with the entire motor and to neglect the effect of the flow of the core-loss current through the primary impedance of the motor. When this approach is employed, the core loss may be introduced as an input which is added to the input determined from the idealized equivalent circuit.

There are two other losses which must be accounted for in making an exact single-phase motor calculation and these are the rotational iron loss and the mechanical friction, sometimes called friction and windage. The rotational iron loss is indistinguishable from friction in its effect upon motor performance since the loss appears as a retarding torque tending mechanically to slow down the motor. The torque of rotational core loss differs from the torque of friction in that, in addition to being a function of speed, it is also a function of the currents in the stator and the rotor and, while an exact expression for the torque would be complicated, it and the corresponding power loss may be approximately represented as proportional to the second power of the stator current.

If it be presumed that the rotational iron loss and the mechanical friction are both known, and that the two rotational losses be classed as friction and represented by the symbol W_f, the effect of this combined rotational loss is properly accounted for if W_f be subtracted from the electrical output as determined from the idealized circuit.



PERFORMANCE CALCULATING SHEET
SINGLE PHASE MOTORS

MOTOR CONSTANTS	PERFORMANCE WHEN S=.05
R_1= 2 X_1= 2.8 X_m= 34 W_I= 13 R_2= 2 X_2= 1.7 W_f= 20	RPM= (1-S)(SYN) =(.95)(1800)=1710

① $\frac{X_2}{X_m}=\frac{(1.7)}{(34)}=.05$

② $\frac{X_mS}{R_2}=\frac{(34)(.05)}{(2)}=.85$

③ $\overset{\text{Per unit}}{R_{2f}}=(.475)$

④ $\overset{\text{Per unit}}{X_{2f}}=(.578)$

⑤ $\frac{X_m(2-S)}{R_2}=\frac{(34)(1.95)}{(2)}=33.2$

⑥ $\overset{\text{Per unit}}{R_{2b}}=(.0273)$

⑦ $\overset{\text{Per unit}}{X_{2b}}=(.0482)$

⑧ $R=(\overset{\text{Per unit}}{R_{2f}}+\overset{\text{Per unit}}{R_{2b}})X_m+R_1=[(.475)+(.0273)](34)+(2)=19.11$

⑨ $X=(\overset{\text{Per unit}}{X_{2f}}+\overset{\text{Per unit}}{X_{2b}})X_m+X_1=[(.578)+(.0482)](34)+(2.8)=24.09$

⑩ $\frac{R}{X}=\frac{(19.11)}{(24.09)}=.793$

⑪ $\cos\phi=.621$

⑫ $I=\frac{E\cos\phi}{R}=\frac{(110)(.621)}{(19.11)}=3.57$

⑬ $W.O.=\left\{[R_{2f}-R_{2b}]X_mI^2\right\}(1-S)-W_f$

$=\left\{[(.475)-(.0273)](34)(12.75)\right\}(.95)-(20)=164$

⑭ $W.I.=EI\cos\phi+W_I$

$=(110)(3.57)(.621)+(13)=257$

⑮ $Eff.=\frac{W.O.}{W.I.}=\frac{(164)}{(257)}=.638$

⑯ $RPM=(1-S)(SYN)=(.95)(1800)=1710$

Figure 4. Single-phase-motor calculation sheet

On the basis of these remarks, the calculation sheet for a single-phase motor may be laid out as shown in figure 4 in which an actual calculation has been carried out using the constants which were employed in explaining the apparent-impedance charts, plus the values for the primary resistance and reactance, the iron loss, and the rotational losses as given below:

$$R_1=2 \quad (19)$$

$$X_1=2.8 \quad (20)$$

$$W_i=13 \quad (21)$$

$$W_f=20 \quad (22)$$

In carrying out the operation of determining the input current, it has been thought desirable to make use of figure 5

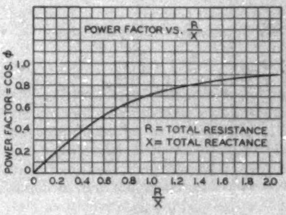
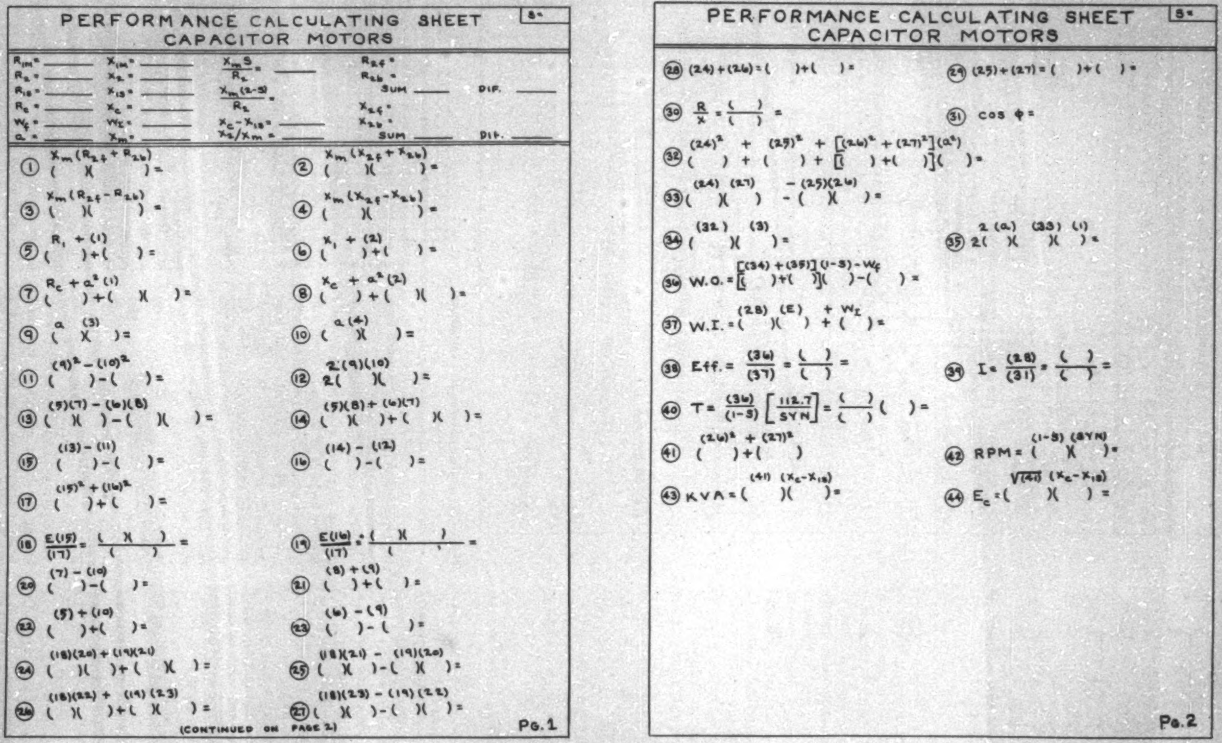


Figure 5. Power-factor versus R/X chart

and the total apparent secondary resistance, thus avoiding the necessity of squaring the impedances and extracting the square root or carrying certain quantities in mind during the calculation, as is necessary with other methods.

It will be seen from the calculation form that the number of steps required for the determination of the performance of a single-phase motor for any given



PERFORMANCE CALCULATING SHEET
CAPACITOR MOTORS
S =

R_{1m} = ______ X_{1m} = ______ $\frac{X_m S}{R_2}$ = ______ R_{2f} =
R_2 = ______ X_2 = ______ R_{2b} =
R_{1s} = ______ X_{1s} = ______ $\frac{X_m(2-S)}{R_2}$ = SUM ______ DIF. ______
R_c = ______ X_c = ______ X_{2f} =
W_f = ______ W_I = ______ $X_c - X_{1s}$ = ______ X_{2b} =
a = ______ X_m = ______ X_2/X_m = ______ SUM ______ DIF. ______

① $X_m(R_{2f}+R_{2b})$ ( )( ) =
② $X_m(X_{2f}+X_{2b})$ ( )( ) =
③ $X_m(R_{2f}-R_{2b})$ ( )( ) =
④ $X_m(X_{2f}-X_{2b})$ ( )( ) =
⑤ $R_1 + (1)$ ( ) + ( ) =
⑥ $X_1 + (2)$ ( ) + ( ) =
⑦ $R_c + a^2(1)$ ( ) + ( )( ) =
⑧ $X_c + a^2(2)$ ( ) + ( )( ) =
⑨ $a\ (3)$ ( )( ) =
⑩ $a(4)$ ( )( ) =
⑪ $(9)^2 - (10)^2$ ( ) − ( ) =
⑫ $2(9)(10)$ 2( )( ) =
⑬ $(5)(7) - (6)(8)$ ( )( ) − ( )( ) =
⑭ $(5)(8) + (6)(7)$ ( )( ) + ( )( ) =
⑮ $(13) - (11)$ ( ) − ( ) =
⑯ $(14) - (12)$ ( ) − ( ) =
⑰ $(15)^2 + (16)^2$ ( ) + ( ) =
⑱ $\frac{E(15)}{(17)} = \frac{(\ )(\ )}{(\ )}$ =
⑲ $\frac{E(16)}{(17)} = \frac{(\ )(\ )}{(\ )}$ =
⑳ $(7) - (10)$ ( ) − ( ) =
㉑ $(8) + (9)$ ( ) + ( ) =
㉒ $(5) + (10)$ ( ) + ( ) =
㉓ $(6) - (9)$ ( ) − ( ) =
㉔ $(18)(20) + (19)(21)$ ( )( ) + ( )( ) =
㉕ $(18)(21) - (19)(20)$ ( )( ) − ( )( ) =
㉖ $(18)(22) + (19)(23)$ ( )( ) + ( )( ) =
㉗ $(18)(23) - (19)(22)$ ( )( ) − ( )( ) =

(CONTINUED ON PAGE 2)

PG. 1

PERFORMANCE CALCULATING SHEET
CAPACITOR MOTORS
S =

㉘ (24) + (26) = ( ) + ( ) =
㉙ (25) + (27) = ( ) + ( ) =
㉚ $\frac{R}{X} = \frac{(\ )}{(\ )}$ =
㉛ $\cos\phi$ =
㉜ $(24)^2 + (25)^2 + [(26)^2 + (27)^2](a^2)$ ( ) + ( ) + [( ) + ( )]( ) =
㉝ $(24)(27) - (25)(26)$ ( )( ) − ( )( ) =
㉞ $(32)(3)$ ( )( ) =
㉟ $2(a)(33)(1)$ 2( )( )( ) =
㊱ W.O. = $[(34) + (35)](1-S) - W_f$ [( ) + ( )]( ) − ( ) =
㊲ W.I. = $(28)(E) + W_I$ ( )( ) + ( ) =
㊳ Eff. = $\frac{(36)}{(37)} = \frac{(\ )}{(\ )}$ =
㊴ $I = \frac{(28)}{(31)} = \frac{(\ )}{(\ )}$ =
㊵ $T = \frac{(36)}{(1-S)}\left[\frac{112.7}{SYN}\right] = \frac{(\ )}{(\ )}(\ )$ =
㊶ $(26)^2 + (27)^2$ ( ) + ( )
㊷ RPM = $(1-S)(SYN)$ ( )( ) =
㊸ KVA = $(41)(X_c - X_{1s})$ ( )( ) =
㊹ E_c = $\sqrt{(41)}\,(X_c - X_{1s})$ ( )( ) =

PG. 2

Figure 6. Capacitor-motor calculation sheet

A Novel Reclosing Relay

P. O. BOBO
MEMBER AIEE

Synopsis: This paper describes a reclosing relay for oil circuit-breaker control which is new in design, new in function, and new in operation. It was designed and built with additional features to meet certain reclosing requirements which cannot be obtained by use of standard reclosing relays. These reclosing requirements, brought about by complete automatization of high-voltage oil circuit-breakers in transmission networks and on lines containing automatic sectionalizing air-break switches, are herein described. Also several schemes of time-sequence-coordination are given for the operation of oil circuit-breakers with automatic line sectionalizing air-break switches. Of special interest is a scheme for materially reducing fuse outages, which utilizes the described reclosing relay for coordinating the operations of a breaker with fuse operations on the line served by the breaker.

BECAUSE of the greater exposure and higher potential stresses of the transmission system over those of the generation and distribution systems, most interruptions to electric service are caused by failures in the transmission process. Although transmission lines can be built almost interruption proof by utilizing modern design, the fact remains that the greater number of lines were built several years past when such quality lines, if at all possible, were so only at a prohibitive cost. Thus the major problem is simply an economical balance between what can be done to reduce the number of interruptions and what can be done to minimize or remove the adverse effects of those which do occur. It is a question whether to rebuild the older lines or accept the interruptions as inevitable and minimize their adverse effects upon service.

It is not the author's purpose to answer this question since each power company must work out its own problems. The author does wish, however, to discuss that side of the question regarding means for minimizing the adverse effects of service interruptions due to faults on the transmission system.

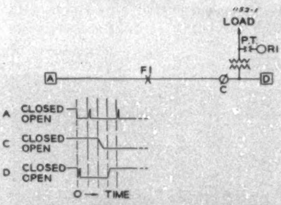


Figure 1. Operation-sequence diagram of line breakers and one automatic sectionalizing air-break switch with low side potential control for opening only

Service interruptions seldom result from failure of the conductor, but are caused by short circuits resulting from lightning. Accompanying voltage disturbances, however, can be practically nullified by the use of high speed relays, a fact which is repeatedly demonstrated on the Oklahoma Gas & Electric Company system. Large industrial motor loads now ride through all disturbances, whereas before installation of such relays these loads were occasionally lost. Moreover, since installing them permanent faults are a rarity and insulator replacements are few because the power is interrupted before burning action effects permanent damage.

Breaker-Reclosing Equipment, Transmission System

Although fast clearing of faults minimizes the voltage disturbance to remaining parts of system and greatly reduces the possibility of permanent line damage, it still interrupts service to consumers served from or through the faulted section. Through the modern practice of equipping breakers with automatic reclosing relays, the duration of such an interruption is greatly minimized. But when the breaker is located within a transmission network, its reclosing requirements necessitate the use of a reclosing relay having greater utility than the standard makes now available. The latter relays are generally designed for use on distribution feeders or radial transmission lines where the only requirements are to perform upon a breaker a certain number of closures at definite time intervals, and to lock out the breaker from further closures if the final closure is unsuccessful because of a permanent fault.

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value of slip is very small. The time required for a single calculation should not exceed five to ten minutes and, where a number of calculations are done in parallel, even this time can usually be bettered.

Apparent impedance charts are not confined to the calculation of single-phase motors, but they may also be applied to polyphase motors, shaded pole motors, shading coils of relays, and other electrical devices. A calculation form for polyphase motors is too obvious and simple to present here, but in a similar way to that which has been used for the straight single-phase motor, it is possible to lay out a calculation form for an unbalanced two-phase motor such as the capacitor motor. An example of a calculation form of this type is shown in figure 6.

Advantages

The advantage of the apparent impedance method as herein outlined for the calculation of induction motor performance lies in the symmetry and naturalness of the approach. This symmetry is particularly indicated by the fact that the curves for the forward and backward field impedances are identical and the further fact that these impedances may not only be used in a simple fashion to obtain the input current, but the same quantities are also used in equally simple fashion in expressions for torque and output.

It is felt that the use of the apparent impedance charts not only eliminates considerable labor, but also a great deal of the possibility of error, for with the use of the charts, the calculational operations are reduced to arithmetic and complex quantities need not be considered.

A further advantage of the method lies in the assistance which it gives to a visualization of how the motor characteristics vary as the constants are changed. The separate points of a calculation are tied together mentally through the continuity of the curves.

In the determination of motor constants from test, the apparent impedance charts are used in reverse because apparent impedances at definite speeds are known and it is desired to find the true impedances.

Reference

REVOLVING-FIELD THEORY OF THE CAPACITOR MOTOR, Wayne J. Morrill. AIEE TRANSACTIONS, volume 48, 1929, pages 614–29.

Such relays cannot be used in transmission networks because they are either unidirectional in operation, which requires extended resetting times, or they cannot be adapted to the conditions desired without use of extra equipment.

Need for a Reclosing Relay for Functioning Under Variable Line Conditions

A reclosing relay for transmission network use must be capable of distinguishing between energized and de-energized line conditions. Also, it must be adaptable for use in conjunction with synchronism-check relays for closing breakers on in-phase conditions. For example, it may be desirable to limit to one terminal the re-energization of a transmission line following tripouts, with closure of the remaining terminal breaker permitted only after successful re-energization of the line. The first requires closures on "de-energized line" only, and the second requires closures on "energized line" only which may also require synchronism-check.

For use in unattended and semi-attended substations a reclosing relay must be capable of one or more subsequent reclosures in addition to the immediate or delayed initial reclosure, since operating experience shows that out of the total tripouts about 85% are successfully re-energized by an immediate breaker reclosure, 10% by a second reclosure, 1% by a third reclosure, and 4% result in permanent faults. The percentage of successful immediate reclosures on circuits equipped with modern high speed fault clearing relays is above average and the percentage of permanent faults is far below average.

Need for a Fast Accurate-Timing Reclosing Relay

Transmission lines are customarily divided into several parts by airbreak switches so that any one section, on becoming faulty, can be removed and service restored to the remaining parts. Most airbreak switches are, as yet, manually operated but with service requirements becoming more exacting, automatization of certain airbreak switches affords an inexpensive means of speeding service restoration to loads tapped on transmission lines.

Airbreak switches are generally incapable of interrupting fault currents. They are designed to be opened under normal service conditions. Therefore, under fault conditions they must be opened after the line is de-energized and the time of opening must be coordinated with the time of re-energization of the circuit by the terminal breakers of the line. To accomplish this time coordination between the automatic sectionalizing switches and line breakers and yet restore service in minimum time, requires the use of a fast accurate breaker-control relay. Short time intervals between breaker reclosures are essential in order to minimize the time gaps between operations of the breakers and sectionalizing switches.

The extent to which an airbreak switch need be automatized depends upon service requirements of the load or loads protected and the quality of the transmission line serving the loads. Several schemes of time-sequence-coordination are given below as examples, in which the operations of airbreak switches are controlled from potential indications only.

Figure 1 shows a single airbreak switch *C* automatized for opening only. No high tension potential device is required for airbreak switch control, the potential indication to control relay *R*1 being received from low side of power transformer as indicated. This scheme is applicable for protecting a load tapped between a long and a short section of line where few permanent faults are expected in the short section and practically all are expected in the long section. Since the airbreak switch is automatized only for opening, it is desirable that opening not take place until after some assurance of a permanent fault. For this reason at least two attempts are made to re-energize the line before the airbreak switch opens.

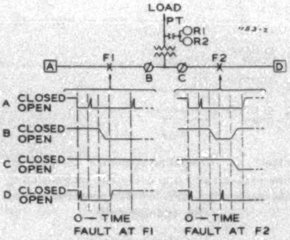
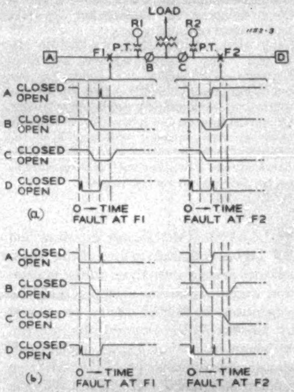


Figure 2. Operation-sequence diagram of line breakers and two automatic sectionalizing air-break switches with low side potential control

Figure 2 shows two airbreak switches controlled by relays *R*1 and *R*2, which also receive potential indications from the low side of power transformers. This scheme assures service to a load should a permanent fault occur on either side of the load.

Figure 3*a* shows two airbreak switches automatized for both opening and closing operations. This requires the use of potential devices connected to the line side of each airbreak switch for control. Since these switches will reclose automatically with line re-energized, less assurance is required that a fault be permanent before the switches automatically open. For this reason only one attempt at re-energizing the line is made before opening takes place. Switches *B* and *C* are identically equipped and controlled. They open simultaneously a definite time after initial attempt to re-energize line. After switches *B* and *C* open, breakers *A* and *D* close simultaneously; breaker *A*, closing on the fault, will reopen, and breaker *D* will remain closed. The line being re-energized to switch *C* will cause this switch to close and re-establish service.



Figures 3a and 3b. Operation-sequence diagram of line breakers and two automatic sectionalizing air-break switches with high side potential control

Figure 3*b* shows two airbreak switches with equipment identical to that of the switches in figure 3*a*, but having different timing control. These switches are automatized so that only one switch is open at any given time. This scheme affords a faster means of service restoration than the scheme of figure 3*a* for faults in the *F*1 section, and slightly longer time for faults in the *F*2 section.

Need for a Quick-Resetting Reclosing Relay

To prevent possible hazards to service within a transmission system, a reclosing relay is needed with quick resetting action following any successful reclosure. However, some time should elapse after a reclosure before permitting the device to reset quickly, as assurance that the reclosure was successful and that no permanent fault exists on the line. This time interval should be greater than the re-opening time of breaker under an existing fault condition, and its length is limited only by the permissible duty on the breaker. In no case, however, should the resetting time be extended until the device has traversed its complete reclosing cycle.

In the case of a breaker equipped with a unidirectional reclosing relay which trips on a transient fault, closes automatically and again trips on a second transient fault, the feeder customers would needlessly be without service until a subsequent reclosure point is reached. Even though this outage is momentary, important motor loads may be dropped, whereas with a quick resetting reclosing relay service would be immediately re-established upon the second tripout. In addition, if a transient tripout occurs during the timing interval between the final reclosure point and the lock-out position of a standard unidirectional reclosing relay, it will lock out without reclosing the breaker. Obviously such hazards can be serious, especially in a transmission system with isolated unattended or semi-attended substations, since it is possible for lines to be out of service or loop circuits to be open for lengthy periods when such unidirectional relays are so employed.

Likewise, on lines containing automatic sectionalizing airbreak switches and whose terminal breakers are controlled by unidirectional relays, subsequent tripouts as described above may cause these sectionalizing switches to open unnecessarily (due to loss of potential) and thereby seriously hazard service. Under these conditions the time coordination between the operations of the breakers and switches becomes disrupted.

Description of the Reclosing Relay

In view of the foregoing and the fact that automatic substations and motor operated sectionalizing switches have increased in number on the Oklahoma Gas and Electric Company system, the author designed an accurate timing, adjustable resetting, automatic reclosing relay for universal breaker application in the company's transmission system. It consists of an assembly of eleven standard telephone type relays mounted in a standard glass covered case as shown pictorially in figure 4 and schematically in figure 5. It contains no fabricated parts other than a terminal board and a mounting plate for the relays. Its principal features are:

Timer

The device contains no motor-driven gears, drum, or camshaft for interval timing. Timing is obtained by a vibrating reed relay acting in conjunction with slow-operate relays to pulse a stepping switch at intervals adjustable between 1 and 15 seconds. The vibrating reed relay *VR*, figures 4 and 5, on energization pulls-up and energizes slow-release relay *SR*, which seals in and de-energizes the *VR* relay; de-energization releases the reed which vibrates to rest, thus energizing (through break contacts) slow-operate relay *SO2*. Relay *SO2* in turn energizes slow-operate relay *SO3* to move the stepping switch one step by energizing the operating coil *SS*. Relay *SO3* de-energizes relay *SR* which causes repetition of the timing cycle for further movements of the stepping unit. The stepping switch has 10 steps which make possible a maximum time duration of $2^{1}/_{2}$ minutes for this reclosing relay. As many as 3 breaker reclosures can be made by it, and they can be accurately spaced from 3 seconds to $1^{1}/_{4}$ minutes apart.

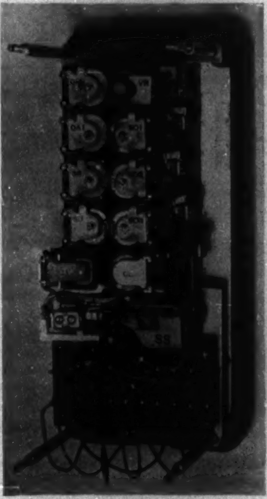


Figure 4. The reclosing relay, an assembly of 11 telephone-type relays

Variable Conditions of Breaker Closure

The recloser contains a built-in "line potential checking relay" (*QAL*) which affords choice of breaker closures on "de-energized line only," "energized line only," or "energized and/or de-energized line." By means of an external terminal connection the device can be interlocked with a synchronism-check relay for closures on "energized line in-phase only" or "de-energized line and/or energized line in-phase."

Variable Conditions of Timer Movement

By this term is meant the conditions under which the stepping switch is advanced following the initial reclosure. For most applications the "conditions of timer movement" and the "conditions of breaker closure" should be the same in order that the timer not move the stepping switch past subsequent reclosure points to the "lockout position" while the conditions of breaker reclosure are incorrect. On the other hand, when the time coordination between operations of breakers and sectionalizing switches is close it is necessary that the reclosing relays serving both terminals of the line advance together regardless of breaker closing conditions.

Quick Reset

The most salient feature of the reclosing relay is its ability to reset following successful reclosures. Reset takes place immediately after the breaker has remained closed a definite time as assurance that the fault was not permanent. Retripping of the breaker before this time expires will cause the relay to progress toward subsequent reclosing points without resetting. This definite time limit is adjustable in impulse intervals of the stepping switch independently after each reclosure. The reset feature may be omitted from any reclosure, as is sometimes desired, to limit the possible rupturing duty that may be imposed upon a breaker with limited rating.

Antipump on All Reclosures

The relay is inherently anti-pump (electrically trip free) on all reclosures, that is, for each reclosure the closing circuit to the breaker is opened immediately upon closure and cannot be re-energized until a subsequent reclosing point is reached, and only then providing the breaker is open and closing conditions satisfy the settings made on the terminal board later described. This feature is obtained by energizing the closing relay *CL* through

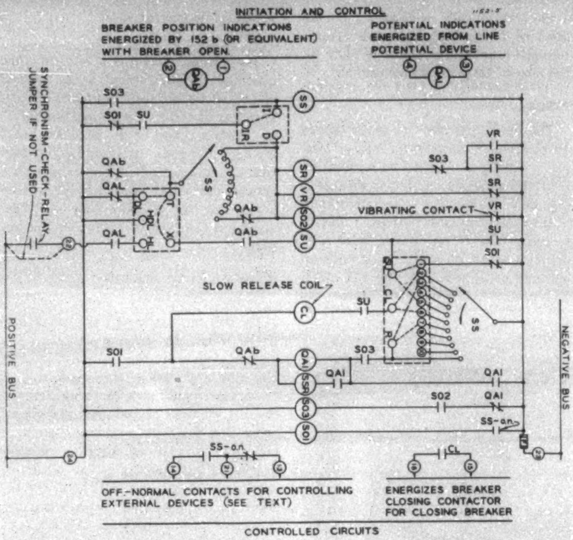

INITIATION AND CONTROL
1152-5
BREAKER POSITION INDICATIONS ENERGIZED BY 152 b (OR EQUIVALENT) WITH BREAKER OPEN.
POTENTIAL INDICATIONS ENERGIZED FROM LINE POTENTIAL DEVICE
QAb
QAL
SYNCHRONISM-CHECK-RELAY JUMPER IF NOT USED
S03
S01
SU
SS
IR
I
D
QAL
DL
HDL
HL
T
X
SR
VR
S02
VR
SR
VIBRATING CONTACT
SLOW RELEASE COIL
CL
QA1
SSR
SO3
SO1
QA1
SO2
SS-on
R
POSITIVE BUS
NEGATIVE BUS
22
24
23
14
21
13
16
15
OFF.-NORMAL CONTACTS FOR CONTROLLING EXTERNAL DEVICES (SEE TEXT)
ENERGIZES BREAKER CLOSING CONTACTOR FOR CLOSING BREAKER
CONTROLLED CIRCUITS

Figure 5. Schematic diagram of internal connections of reclosing relay

contacts of the set-up relay *SU* (figures 4 and 5), which is initiated from points of the stepping switch just preceding the points of closure, and is held energized by its own contacts until dropped by the control relay *QAb* on closure of the breaker.

Setting the Relay

The pulsing intervals of the stepping unit are adjustable between 1 and 15 seconds by screws which vary the reed amplitude of the vibrating reed relay (*VR*). All other settings are made electrically by inserting plug type jumpers in proper jacks on the terminal board, figures 4 and 5. The points of the stepping switch are terminated in a row of jacks numbered 2 to 10 on the board. The number 1 jack represents the zero point or normal position of the stepping switch —being controlled by slow-operate relay *SO*1, which is operated by an off-normal contact "*SS*-on" on the stepping unit.

Directly below this row of jacks is a row of 3 binding posts, *SU*, *CL*, and *R*, from which by use of jumper connections to the row of jacks time settings are made for: (1) initiation of the set-up relay *SU*, (2) energization of the closing relay *CL*, which closes the breaker, (3) resetting of the relay. Reset is accomplished instantaneously by energization of relay *QA*1 which seals in and maintains energized the reset coil *SSR* of the stepping switch until the unit has fully reset and relay *SO*1 is de-energized by the opening of "off-normal" contact "*SS*-on." Note that resetting cannot take place unless relay *QAb* is de-energized as indication that breaker is closed.

The conditions of breaker reclosure are set by placing jumpers from binding post "*X*" to jack "*DL*" for "dead-line" closing conditions, to jack "*HDL*" for "hot and/or dead-line" (non-restricted) conditions, and to jack "*HL*" for "hot-line" conditions. Jacks *DL* and *HL* are controlled by a break and a make contact respectively of line checking relay *QAL*. If synchronism-check is desired, the external terminal stud "22" is connected in series with the contacts of the synchronism-check relay to the positive bus; otherwise this terminal stud is connected directly to the positive bus. These connections to point "*X*" establish the conditions for which the set-up relay *SU* is energized.

The conditions of timer movement are set by placing jumpers from binding post "*T*" to the same jacks as those for the conditions of closure. Such connections to point "*T*" control the conditions for which the timer relays *SR*, *VR*, and *SO*2 are energized.

Immediate or delayed initial reclosure is chosen by inserting jumper from jack "*IR*" to jack "*I*" for "immediate" reclosure, or from jack "*IR*" to jack "*D*" for "delayed" reclosure. Connection to jack "*I*" permits immediate energization of operating coil *SS* of stepping switch, and connection to jack "*D*" interposes one impulse cycle of timing relays before energization of coil *SS*.

Other Features

As can be seen from schematic connection diagram figure 5, only one breaker auxiliary switch (152*b*) is required for operation, or in many cases the reclosing relay may be controlled directly from the breaker alarm or indicating light circuits instead, since the only requirement is energization of relay *QAb* when breaker is open.

The auxiliary relay *CL* which energizes the breaker closing circuit is equipped with a vacuum contact for the high-current capacity required in energizing breaker closing contactors. Its coil is slow releasing in order to permit ample time for the breaker to latch closed without any necessary critical adjustment of the breaker auxiliary switch.

The reclosing relay can be made for operating voltages from 24 to 250 volts d-c, or a-c with a rectifier unit.

A mechanical pointer fastened to the shaft of the stepping switch indicates when the relay is locked out by pointing to line marked "*LO*" on front of stepping unit.

The relay will automatically reset upon manual closing of breaker after lockout, provided breaker remains closed for one impulse interval of timer, otherwise it will remain in the lock-out position.

For remote supervisory indication and for other external functions of control associated with reclosing of breakers, two auxiliary "off-normal" contacts (*SS*-on) are brought out to terminal studs 13, 14, and 21. In attended stations where the relay is located away from control board, remote off-normal supervision indicates relay operations to the attendant. An important control function using an off-normal contact involving fuse operations is described below.

Method of Reducing Fuse Outages

To lessen the outages caused by transient faults on multibranch-fuse circuits served through a common breaker, the Oklahoma Gas and Electric Company is using a unique automatic scheme which permits instantaneous initial tripping of the breaker for a fault at any point on the circuit. Service is immediately restored by the breaker reclosing equipment. The instantaneous opening of the breaker interrupts transient short-circuits before the branch fuses have time to blow. Service can then be restored to the fused branch immediately, instead of waiting until a fuse can be replaced. Of course, service to the entire circuit is momentarily interrupted, but it is doubtful whether such an interruption is more serious than the voltage disturbance caused by a longer clearing time of a fuse.

In order to protect customers on unfaulted branches when a permanent fault occurs, a reclosing relay fulfilling the following requirements is needed. It must remove the instantaneous tripping relays from service prior to the immediate reclosure of the breaker so as to permit coordination between breaker time-delay relays and sectionalizing fuses, and it should hold these instantaneous relays removed from service until it has reset and again is capable of an immediate reclosure, otherwise more than a transient outage may result needlessly to all customers.

The reclosing relay herein described with its auxiliary "off-normal" contact on the stepping switch fulfills all requirements. This "off-normal" contact (figure 5, terminal studs 13 and 21) is connected in series with the instantaneous tripping relay contacts and opens this circuit upon movement of the stepping switch off-normal just before closure of the breaker. The circuit remains open until the relay is fully reset.

The reset feature of the reclosing relay for this service is as useful in preventing fuse outages as the scheme itself.

Operating Experience

The accompanying table is an analysis of operations from January 1939 through June 1941 of the line breakers which are equipped with the reclosing relay.

	Number	Per Cent
I. Breakers serving lines equipped with automatic sectionalizing air-break switches:		
Line terminals equipped	7	
Line faults	123	100
Normal service restored, initial reclosure	101	82
Normal service restored, second reclosure	7	6
Normal service restored, third reclosure	6	5
Relay lockouts	9	7
Reset feature, beneficial	12	10
Permanent faults	12	100
Line correctly sectionalized	8	67
Line sectionalizing failures	4	33
II. Breakers serving 66 kv-loop-circuit lines:		
Line terminals equipped	6	
Line faults	304	100
Normal service restored, initial reclosure	298	98
Normal service restored, second reclosure	3	1
Normal service restored, third reclosure	0	0
Relay lockouts	3	1
Reset feature, beneficial	40	13
III. Breakers serving multi-branch-fuse circuits:		
Line terminals equipped	1	
Line faults	46	100
Normal service restored, immediate reclosure	45	98
Normal service restored, second reclosure	1	2
Normal service restored, third reclosure	0	0
Relay lockouts	0	0
Reset feature, beneficial	3	6

Under (I) above, three of the successful sectionalizing operations were accomplished within 8 seconds by equipment utilizing the scheme of figure 3*b*. The four "line sectionalizing failures" were in no case due to improper functioning of the reclosing relays. One was caused by a blown fuse in an airbreak, switch mechanism; one was caused by the nature of the fault—open conductor, grounded on one end and in clear on the other; one was caused by the failure of a breaker to latch closed; the fourth was caused by manual interference with the automatic reclosing relay.

The most important result shown by the analysis is the fact that the reset feature of the relay was beneficial in restoring service for 55 of the total 471 fault operations, that is, for approximately 12% of the faults, which faults occurred following a successful reclosure and within the total cycle of time for which the relays were set, the quick resetting action of the relay, in addition to speeding restoration of service, prevented the following possible service hazards. (Previously described under "Need for Quick Resetting Reclosing Relay.)

1. Disruption of time-sequence coordination between operations of automatic sectionalizing airbreak switches and line terminal breakers.

2. Longer than transient outages to important loads.

3. Loop circuits remaining open for lengthy periods.

4. Lockouts of reclosing relays without closing the breakers.

Conclusions

The relay meets the following requirements which are essential to successful operation of oil circuit breakers in an extensive transmission network:

1. Variable conditions of closure
2. Variable conditions of timing
3. Fast and accurate timing between closures
4. Anti-pump characteristics
5. Auxiliary off-normal contacts for external control and indications
6. Operation on either alternating or direct current
7. Quick resetting following successful reclosures

It is through the fulfillment of above requirements that the reclosing relay permits a transmission system to render maximum continuity of service. Interruptions caused by transient faults are minimized; hazards caused by untimely tripouts are eliminated; coordination between breakers and automatic sectionalizing switches is made faster and more effective; and fuse outages on certain circuits are greatly reduced. All of this has been definitely proved by 14 installations of the relay in the Oklahoma Gas & Electric Company's system over a period of $2^1/_2$ years beginning January 1939. Experience also has shown that the relay is dependable and virtually trouble free—a tribute to the versatile and highly developed relays originally created for automatic telephone systems.

Formulas for the Inductance of Rectangular Tubular Conductors

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THE formula for the inductance per unit length of two parallel conductors of rectangular cross section, obtained by using the formal expressions for the geometric mean distance of a rectangle to itself and the geometric mean distance between two rectangles with sides parallel, is exceedingly cumbersome. More than a score of logarithms and arc tangents must be evaluated when computing the inductance for a given conductor spacing and cross section. To circumvent such lengthy calculations when computing the reactance of like parallel rectangular strap conductors, Dwight[1, 2] has published curves, each plotted for a certain conductor spacing and cross section, from which the reactance can be obtained. Unless, however, the conductor spacing and the ratio of conductor thickness to breadth coincides with those values for which the curves are plotted, interpolation is necessary. Recently Roth[3] has expressed the inductance of such lines in the form of a rapidly converging series, the parameters of which are the spacing and dimensions of the conductors. Usually, a few terms of this series suffice to yield a value of inductance sufficiently accurate for all design purposes.

As regards rectangular tubular conductors, the only analytical literature is a paper by Dwight and Wang[4] giving formulas for *thin square* tubular conductors. These are derived on the assumption that the conductor walls are so thin that the sides can be considered as but line segments. The geometric mean distances for such segments are then used to calculate the inductance. Such a procedure is equivalent to asserting that the current flows solely on the surface of the conductors. It follows, then, that the values of inductance computed on this basis will be *less* than the actual values. Consequently, while these formulas suffice for many calculations on square tubular conductors, if accuracy is a desideratum of design this minimization of thickness restricts the range of application of these formulas to *very thin square* tubular conductors. Recognizing this restriction and noting, further, that commercial conductors have rounded corners, Dwight and Wang give formulas, empirically deduced, from which corrections for thickness and for rounded corners can be calculated.

The inductance of rectangular tubular conductors of appreciable thickness (and not necessarily square) can be obtained from equations given by the author[5] for the case of a line composed of two parallel conductors, the cross sections of which are geometrically similar but dimensionally different hollow rectangles. However, as these equations are implicit rather than explicit, they are ill suited to numerical computation. Yet, in light of the increasing use of rectangular tubular conductors for main bus bars, electric furnace installations, and other heavy current duties (especially feeders for batteries of welders), it is most desirable to have formulas with which one can calculate quickly the inductance for any conductor spacing or cross section. *Such formulas are derived in this paper*, it being assumed that the conductors are nonmagnetic, square cornered, of such length that end effects are negligible, and carry currents uniformly distributed over the cross sections of the individual conductors.

Although obtained through a procedure involving rather intricate analysis, the formulas themselves are simple in nature. With them a designer can calculate rapidly the inductance of a single-phase circuit composed of solid or tubular rectangular conductors; or of a three, six, or twelve-phase circuit composed of solid or tubular rectangular conductors, the axes of which lie in a plane.

I. The Formal Solution

With reference to figure 1, we seek A_1 which satisfies the differential equations

$$\nabla^2 A_1 = 4\pi w \text{ over } C_1 \qquad (1)$$

$$\nabla^2 A_1 = -4\pi w \text{ over } C_4 \qquad (2)$$

$$\nabla^2 A_1 = 0 \text{ over the remainder of the right half-plane} \qquad (3)$$

and the boundary conditions

$$A_1 = 0 \text{ at infinity} \qquad (4)$$

$$A_1 = 0 \text{ on the line } x = 0 \qquad (5)$$

$$\frac{\partial A_1}{\partial y} = 0 \text{ on the line } y = 0 \qquad (6)$$

The boundary conditions are obtained as follows: (4) from knowledge that the vector potential vanishes at infinity except for an arbitrary constant, taken here as zero; (5) from observation that the line $x = 0$ and the left-hand portion of the circle of infinite radius constitute, in the limit, a line of induction, whence, lines of induction being lines of constant vector potential, the vector potential is zero as on the circle at infinity; (6) from noting that the horizontal component of induction density vanishes on the line $y = 0$, or $B_x = \partial A_1/\partial y = 0$.

For the problem at hand, A_1 is an even function of y, an odd function of x. Accordingly, A_1 is expressed by the double Fourier integral

$$A_1 = (4/\pi^2) \times \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty A_1(X, Y) \sin Mx \times \cos Ny \cos NY \sin MX dX dY dN dM \qquad (7)$$

the same satisfying (5) and (6) unconditionally, and (1), (2), and (3) providing $A_1(X, Y)$ or an integral containing $A_1(X, Y)$ is appropriately determined. Thus, if the double Fourier integral expressing the current density at any point in the conductors,

$$w = (4/\pi^2) \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty w(x, y) \times \sin Mx \cos Ny \cos NY \times \sin MX\, dX dY dN dM$$

$$= (4/\pi^2) \int_0^\infty \int_0^\infty \int_0^h \int_{d-e}^{d+e} w \sin Mx \times \cos Ny \cos NY \sin Mx\, dX dY dN dM$$

$$= (8w/\pi^2) \int_0^\infty \int_0^\infty \frac{1}{MN} \sin Mx \cos Ny \times \sin Nh \sin Md \sin Me\, dN dM \qquad (8)$$

is substituted in the right-hand member of

$$4(M^2+N^2)/\pi^2 \times \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty A_1(X, Y) \sin Mx \times \cos Ny \cos NY \sin Mx\, dX dY dN dM = 4\pi w \qquad (9)$$

obtained by substituting (7) in (1), we

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This paper is based upon a thesis submitted by Professor Higgins to the faculty of Purdue University in June 1941 in partial fulfillment of the requirements for the degree of doctor of philosophy.

1. For all numbered references, see list at end of paper.

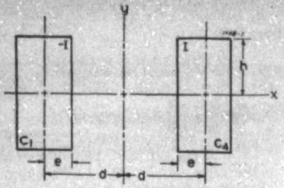

Figure 1

have, on comparing the two members of the resulting equation

$$\int_0^\infty \int_0^\infty A_1(X,Y) \cos NY \sin MX \times dXdY = 8\pi w \frac{\sin Nh \sin Md \sin Me}{MN(M^2+N^2)} \quad (10)$$

Finally, substituting (10) in (7), we have for A_1

$$A_1 = (32w/\pi) \int_0^\infty \frac{\sin Md \sin Me \sin Mx}{M} \times \int_0^\infty \frac{\sin Nh \cos Ny}{N(M^2+N^2)} dNdM \quad (11)$$

For the current distribution of figure 2 we have, analogously,

$$A_2 = (-32w/\pi) \int_0^\infty \frac{\sin Md \sin ME \sin Mx}{M} \times \int_0^\infty \frac{\sin NH \cos Ny}{N(M^2+N^2)} dNdM \quad (12)$$

Superimposing the current distributions of figures 1 and 2 results in that of figure 3. The corresponding vector potential is

$$A = A_1 + A_2 \quad (13)$$

The total electromagnetic field energy W is, by symmetry, four times that associated with the current flowing through the shaded portions of figure 3. Thus, from the usual expression, $W = {}^1/_2 \int AwdS$, we have

$$W = 2\int_0^h \int_{d-e}^{d+e} Awdxdy - 2\int_0^H \int_{d-E}^{d+E} Awdxdy \quad (14)$$

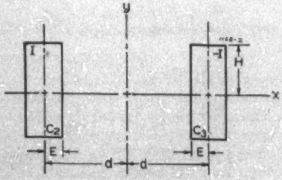

Figure 2

Substituting (11) and (12) in (13), and, in turn, (13) in (14), we have

$$W = (64w^2/\pi) \times \int_0^h \int_{d-e}^{d+e} \int_0^\infty \frac{\sin Md \sin Me \sin Mx}{M} \times \int_0^\infty \frac{\sin Nh \cos Ny}{N(M^2+N^2)} dNdMdxdy -$$
$$(64w^2/\pi) \times \int_0^h \int_{d-e}^{d+e} \int_0^\infty \frac{\sin Md \sin ME \sin Mx}{M} \times \int_0^\infty \frac{\sin NH \cos Ny}{N(M^2+N^2)} dNdMdxdy -$$
$$(64w^2/\pi) \times \int_0^H \int_{d-E}^{d+E} \int_0^\infty \frac{\sin Md \sin Me \sin Mx}{M} \times \int_0^\infty \frac{\sin Nh \cos Ny}{N(M^2+N^2)} dNdMdxdy +$$
$$(64w^2/\pi) \times \int_0^H \int_{d-E}^{d+E} \int_0^\infty \frac{\sin Md \sin ME \sin Mx}{M} \times \int_0^\infty \frac{\sin NH \cos Ny}{N(M^2+N^2)} dNdMdxdy \quad (15)$$

These integrals are such that the order of integration can be varied as desired. Accordingly, integrating first with respect to x and then with respect to y, we have after collecting terms

$$W = (128w^2/\pi) \int_0^\infty \frac{\sin^2 Md \sin^2 Me}{M^2} \times \int_0^\infty \frac{\sin^2 Nh}{N^2(M^2+N^2)} dNdM +$$
$$(128w^2/\pi) \int_0^\infty \frac{\sin^2 Md \sin^2 ME}{M^2} \times \int_0^\infty \frac{\sin^2 NH}{N^2(M^2+N^2)} dNdM -$$
$$(256w^2/\pi) \times \int_0^\infty \frac{\sin^2 Md \sin ME \sin Me}{M^2} \times \int_0^\infty \frac{\sin NH \sin Nh}{N^2(M^2+N^2)} dNdM \quad (16)$$

Now[8]

$$\int_0^\infty \frac{\sin NH \sin Nh}{N^2(M^2+N^2)} dN = \frac{\pi}{2M^3}(MH - \epsilon^{-Mh} \sinh MH) \quad (17)$$

Substituting appropriately in (16), we have

$$W = 64w^2 \int_0^\infty \frac{\sin^2 Md \sin^2 Me}{M^5} \times (Mh - \epsilon^{-Mh} \sinh Mh)dM +$$
$$64w^2 \int_0^\infty \frac{\sin^2 Md \sin^2 ME}{M^5} \times (MH - \epsilon^{-MH} \sinh MH)dM -$$
$$128w^2 \int_0^\infty \frac{\sin^2 Md \sin ME \sin Me}{M^5} \times (MH - \epsilon^{-Mh} \sinh MH)dM \quad (18)$$

The third integral in (18) can be written as the sum of two integrals, such that W is expressed as the sum of four like integrals. Doing so, we have

$$W = 64w^2 \int_0^\infty \frac{\sin^2 Md \sin^2 Me}{M^5} \times (Mh - \epsilon^{-Mh} \sinh Mh)dM +$$
$$64w^2 \int_0^\infty \frac{\sin^2 Md \sin^2 ME}{M^5} \times (MH - \epsilon^{-MH} \sinh MH)dM -$$
$$128w^2 \int_0^\infty \frac{\sin^2 Md \sin ME \sin Me}{M^5} \times \left[\frac{M(h+H)}{2} - \epsilon^{-\frac{M(h+H)}{2}} \sinh \frac{M(h+H)}{2}\right]dM +$$
$$128w^2 \int_0^\infty \frac{\sin^2 Md \sin ME \sin Me}{M^5} \times \left[\frac{M(h-H)}{2} - \epsilon^{-\frac{M(h-H)}{2}} \sinh \frac{M(h-H)}{2}\right]dM \quad (19)$$

Each integral of (19) is of the form

$$\frac{y}{2} f_1(x_1, x_2, x_3, x_4) - {}^1/_2 f_2(x_1, x_2, x_3, x_4, y) \quad (20)$$

As shown in the appendix

$$f_1 = \int_0^\infty \frac{1}{M^4} \sin Mx_1 \sin Mx_2 \sin Mx_3 \times \sin Mx_4 \, dM = \frac{\pi}{3!2^4} \sum_{t=1}^{8} (-1)^p |k_t^3| \quad (21)$$

k_t designating any one of the eight possible terms to be formed from

$$(x_1 \pm x_2 \pm x_3 \pm x_4)$$

by arbitrary selection of the plus and minus signs, while p is the number of minus signs in k_t.

For f_2 we have

$$f_2 = 2\int_0^\infty \frac{1}{M^5} \sin Mx_1 \sin Mx_2 \sin Mx_3 \times \sin Mx_4 \epsilon^{-\frac{My}{2}} \sinh \frac{My}{2} dM = \frac{1}{192} \sum_{t=1}^{8} (-1)^p F(k_t, y) \quad (22)$$

where k_t and p have the same significance as in (21), and

$$F(k_t, y)=(k_t^4-6y^2k_t^2+y^4)\log(k_t^2+y^2)^{1/2}+4yk_t(k_t^2-y^2)\tan^{-1}(k_t/y)-k^4\log|k_t| \quad (23)$$

The fifth column of table I contains the values of k_t used in evaluating the first two integrals in (19), while those for the second two integrals of (19) are in the sixth column.

Defining new variables, $r=2e$, $R=2E$, $s=2h$, $S=2H$, $D=2d$, and $t=e-E=h-H$, evaluating each integral of (19) by means of (21) and (22), and collecting terms, we have for the terms of (19) taken in order

$$W=w^2\{W(r, s)+W(R, S)-2[W(R+t, S+t)-W(t, S+t)]+2[W(R+t, t)-W(t, t)]\} \quad (24)$$

where $W(r, s)$ is defined by

$$W(r, s)={}^1/_6[2F(D, s)-F(D+r, s)-F(D-r, s)+2F(r, s)-2F(0, s)+4\pi r^2s(3D-r)] \quad (25)$$

Collecting the terms of (24), we have

$$W=w^2[W(r, s)+W(R, S)+2W(t, S+t)+2W(R+t, t)-2W(R+t, S+t)-2W(t, t)] \quad (26)$$

The energy W being known in terms of the dimensions and spacings of the conductors, the inductance can be obtained at once from the usual formula, $W={}^1/_2LI^2$. Hence

$L=2W/w^2(rs-RS)^2$ abhenries per centimeter of line length (27)

If $R=S=0$ we have solid conductors, and (27) becomes

$L=2W(r, s)/(wrs)^2$ abhenries per centimeter of line length (28)

The 60-cycle reactance per thousand feet of conductor is obtained by multiplying L by 5.74×10^{-8}.

II. Series Expressions for L

We turn now to finding series expansions which enable one to calculate quickly each of the terms of (26). Expanding (25) in powers of r, we obtain

$$W(r, s)=\frac{1}{6}\left[2F(r, s)-2F(0, s)+4\pi r^2s(3D-r)-r^2\frac{\partial^2F(D, s)}{\partial D^2}-\frac{r^4}{2\cdot 3!}\frac{\partial^4F(D, s)}{\partial D^4}-\frac{r^6}{3\cdot 5!}\frac{\partial^6F(D, s)}{\partial D^6}-\ldots\right] \quad (29)$$

The details of the expansion and expressions for the various derivatives, in terms of the dimensions and spacing of the conductors, are given in the appendix. Substituting appropriately in (29) and collecting terms, we have

$$W(r, s)=2r^2\left\{s^2\left[\frac{7}{12}+\frac{1}{2}\log\frac{D^2+s^2}{r^2+s^2}\right]-\left[\frac{D^2}{2}+\frac{r^2}{12}\right]\log\frac{D^2+s^2}{D^2}+\frac{s^4}{12r^2}\log\frac{s^2+r^2}{s^2}+\frac{r^2}{12}\log\frac{r^2+s^2}{r^2}+2sD\tan^{-1}(s/D)-\frac{2}{3}rs\tan^{-1}(s/r)-\frac{2s^3}{3r}\tan^{-1}(r/s)-\frac{r^4}{180}\left[\frac{1}{D^2}-\frac{D^2-s^2}{(D^2+s^2)^2}\right]-\frac{r^6}{1,680}\times\left[\frac{1}{D^4}-\frac{(D^2-s^2)^2-4D^2s^2}{(D^2+s^2)^4}\right]-\frac{r^8}{7,560}\times\left[\frac{1}{D^6}-\frac{(D^2-s^2)^3-12s^2D^2(D^2-s^2)}{(D^2+s^2)^6}\right]-\ldots\right\} \quad (30)$$

If r$<s$, (30) can be put in a more convenient form. Utilizing the power series for $\log(1+x)$ and $\tan^{-1}x$, we have after expanding appropriately in (30) and collecting terms

$$W(r, s)=2r^2\left\{\frac{s^2}{2}\log\frac{D^2+s^2}{s^2}-\frac{D^2}{2}\log\frac{D^2+s^2}{D^2}+2sD\tan^{-1}(s/D)-\pi rs/3+r^2\left[\frac{25}{75}-\frac{1}{12}\log\frac{r^2(D^2+s^2)}{s^2D^2}\right]-\frac{r^4}{180}\left[\frac{1}{D^2}-\frac{D^2-s^2}{(D^2+s^2)^2}-\frac{1}{s^2}\right]-\frac{r^6}{1,680}\times\left[\frac{1}{D^4}-\frac{(D^2-s^2)^2-4D^2s^2}{(D^2+s^2)^4}+\frac{1}{s^4}\right]-\ldots\right\} \quad (31)$$

If $r<s$ and, in addition, $D>s$, we can expand (31) still further obtaining

$$W(r, s)=2r^2s^2\left\{\log\frac{D}{s}+\frac{r^2}{s^2}\times\left[\frac{25}{72}-\frac{1}{12}\log\frac{r^2(D^2+s^2)}{s^2D^2}\right]+\frac{3}{2}-\frac{\pi r}{3s}+\frac{1}{12}\frac{s^2}{D^2}-\frac{1}{60}\frac{s^4}{D^4}+\frac{1}{168}\frac{s^6}{D^6}-\ldots-\frac{1}{180}\frac{r^4}{s^4}\times\left[\frac{s^2}{D^2}-\frac{s^2D^2-s^4}{(D^2+s^2)^2}-1\right]-\ldots\right\}$$

$$=2r^2s^2\left\{\log\frac{1}{m}+n^2\times\left[\frac{25}{72}-\frac{1}{12}\log n^2(1+m^2)\right]+\frac{3}{2}-\frac{\pi r}{3s}+\frac{m^2}{12}-\frac{m^4}{60}+\frac{m^6}{168}-\ldots-\frac{n^4}{180}\times\left[m^2-\frac{m^2-m^4}{(1+m^2)^2}-1\right[-\ldots\right\} \quad (32)$$

where $m=s/D$ and $n=r/s$.

If, however, $r<s$ and $s>D$, we have from (31)

$$W(r, s)=2r^2s^2\left\{\pi m+\left(\log m-\frac{3}{2}\right)m^2-\frac{m^4}{12}+\frac{m^6}{60}-\frac{m^8}{168}+\ldots-\frac{\pi n}{3}+n^2\times\left[\frac{25}{72}-\frac{1}{12}\log n^2\left(1+\frac{1}{m^2}\right)\right]-\frac{n^4}{180}\times\left[\frac{1}{m^2}-\frac{m^2-1}{(m^2+1)^2}-1\right]-\ldots\right\} \quad (33)$$

where now $m=D/s$ and $n=r/s$.

If $r>s$, we have from (30)

$$W(r, s)=2r^2s^2\left\{\frac{1}{2}\log\frac{D^2+s^2}{r^2}-\frac{D^2}{2s^2}\log\frac{D^2+s^2}{D^2}+\frac{2D}{s}\tan^{-1}(s/D)-\frac{r^2}{12s^2}\log\frac{D^2+s^2}{D^2}-\frac{\pi s}{3r}-\frac{s^2}{6r^2}\left[\log\frac{s}{r}-\frac{25}{12}\right]+\frac{1}{180}\frac{s^4}{r^4}-\frac{1}{1,680}\frac{s^6}{r^6}+\ldots-\frac{1}{180}\frac{r^4}{s^2}\times\left[\frac{1}{D^2}-\frac{D^2-r^2}{(D^2+r^2)^2}\right]-\ldots\right\} \quad (34)$$

If $r>s$ and, in addition, $D>r$, we have from (34)

$$W(r, s)=2r^2s^2\left\{\log\frac{D}{r}-\frac{1}{12n^2}\log(1+m^2)-\frac{n^2}{6}\log n+\frac{3}{2}-\frac{\pi n}{3}+\frac{25n^2}{72}+\frac{n^4}{180}-\frac{n^6}{1,680}+\ldots+\frac{m^2}{12}-\frac{m^4}{60}+\frac{m^6}{168}-\ldots-\frac{1}{180n^4}\left[m^2-\frac{s^2(D^2-r^2)}{(D^2+r^2)^2}\right]-\ldots\right\} \quad (35)$$

where $m=s/D$ and $n=s/r$.

If $r=s$, we obtain from (30)

$$W(r, s)=2r^4\left\{\frac{1}{2}\log\frac{D^2+r^2}{r^2}-\left[\frac{D^2}{2r^2}+\frac{1}{12}\right]\log\frac{D^2+r^2}{D^2}+\frac{2D}{r}\tan^{-1}(r/D)-1.579393-\frac{r^2}{180}\left[\frac{1}{D^2}-\frac{D^2-r^2}{(D^2+r^2)^2}\right]-\frac{r^4}{1,680}\left[\frac{1}{D^4}-\frac{(D^2-r^2)^2-4D^2r^2}{(D^2+r^2)^4}\right]-\ldots\right\} \quad (36)$$

If $r=s$ and, in addition, $D>r$, we have from (36)

$$W(r, s)=2r^4\left\{0.805087-\log m+\frac{m^4}{40}-\frac{11}{504}m^6+\frac{13}{720}m^8-\ldots-\frac{m^2}{180}\left[1-\frac{1-m^2}{(1+m^2)^2}\right]-\frac{m^4}{1,680}\times\left[1-\frac{(1-m^2)^2-4m^2}{(1+m^2)^4}\right]-\ldots\right\} \quad (37)$$

where $m=r/D$.

III. Identification of the Terms of (24)

As (28) can be written in the form

$$L=4\log(D_{14}/D_{11})=2W(r,s)/(wrs)^2 \qquad (38)$$

where D_{14} is the geometric mean distance between C_1 and C_4, while D_{11} is the geometric mean distance of C_1 or C_4 to itself, these latter can be expressed as functions of the terms of (25). Our expressions are somewhat simplified if we introduce the new function

$$G(k_t,y)=F(k_t,y)-2\pi y|k_t|(k_t^2-y^2) \qquad (39)$$

Comparison of $W(r, s)$ with expressions for $\log D_{14}$ and $\log D_{11}$ given elsewhere[6] reveals that

$$12r^2s^2\log D_{14}=2G(D,s)-G(D+r,s)-G(D-r,s)-25r^2s^2 \qquad (40)$$

$$12r^2s^2\log D_{11}=-2G(r,s)+2G(0,s)+4\pi rs^3-25r^2s^2 \qquad (41)$$

Similarly, from $W(R, S)$ we have

$$12R^2S^2\log D_{23}=2G(D,S)-G(D+R,S)-G(D-R,S)-25R^2S^2 \qquad (42)$$

$$12R^2S^2\log D_{22}=-2G(R,S)+2G(0,S)+4\pi RS^3-25R^2S^2 \qquad (43)$$

Like (28) written in the form of (38), we have[5] for (27)

$$L=\frac{4}{(rs-RS)^2}(r^2s^2\log D_{14}+R^2S^2\log D_{23}+2rsRS\log D_{12}-r^2s^2\log D_{11}-R^2S^2\times\log D_{22}-2rsRS\log D_{13}) \qquad (44)$$

Substituting (26) in (27) and comparing the resulting expression with (44), it follows that the as yet unidentified members of (26) must equal

$$-4rsRS\log D_{13}+4rsRS\log D_{12} \qquad (45)$$

Obviously, of the unidentified terms of (26), those containing D comprise the first component of (45), those not containing D comprise the second component of (45). We have, then,

$$12rsRS\log D_{13}=G(D+t,S+t)-25rsRS\; G(D-t,S+t)-G(D+R+t,S+t)-G(D-R-t,S+t)-G(D+t,t)-G(D-t,t)+G(D+R+t,t)+G(D-R-t,t) \qquad (46)$$

$$12rsRS\log D_{12}=2G(t,S+t)-2G(R+t,S+t)+4\pi R(S+t)^2-2G(t,t)+2G(R+t,t)-4\pi Rt^2-25rsRS \qquad (47)$$

where $(D-R-t)>0$.

It is evident that (40) to (47) inclusive are of particular value: with them one can calculate the inductance or reactance of any arrangement of like solid or tubular busbars, the axes of which lie in a plane. For as detailed in most texts on transmission theory, if there are n conductors constituting a polyphase circuit, the reactance of an individual wire (for example, wire a) is given by

$$x_a=0.2794\sum_{j=1}^{n}(I_j/I_a)\log(1/D_{ja}) \text{ ohms per mile at sixty cycles}$$

where

$$\sum_{j=1}^{n}I_j=0$$

Or again, if n conductors constitute a single-phase circuit, the inductance is given by

$$L=\sum_{j=1}^{n}\sum_{k=1}^{n}(-2/I^2)I_jI_k\log D_{jk} \text{ abhenries per centimeter of line}$$

IV. An Illustrative Problem

As an illustration of the rapidity of convergence of the formulas developed in section III, we now calculate the reactance of a line composed of square tubular conductors. We use the dimensional data of a like problem to be found in the previously mentioned paper of Dwight and Wang,[4] and thus simultaneously determine the magnitude of the minimization in the basic formula presented therein, the same being due to neglecting the thickness of the conductors when deriving this formula, and the accuracy of the formula given for correcting for the effect of thickness. The data follow:

$r=s=2.5$ inches; $t=0.5$ inch; $D=10$ inches;
$f=60$ cycles per second;
$X=0.0443$ ohm per thousand feet of conductor;
$L=3.85$ abhenries per centimeter of conductor (calculated from X).

For the data given, we have from (26)

$$W/w^2=W(2.5,2.5)+W(1.5,1.5)+2W(0.5,2)+2W(2,0.5)-2W(2,2)-2W(0.5,0.5) \qquad (48)$$

Thus, to determine W/w^2 each of the terms of (48) must be calculated. From (37) we have for the terms taken in order:

$$W(2.5,2.5)=78.125(0.805087+1.386294+0.0000977-0.0000589)=171.203$$

$$W(2,2)=32(0.805087+1.609438+0.000040-\ldots-0.000025)=77.264$$

$$W(1.5,1.5)=10.125(0.805087+1.897120+0.000012-\ldots-0.000008)=27.359$$

$$W(0.5,0.5)=0.125(0.805087+2.995732+0.000000-\ldots-0.000000)=0.475$$

From (32) we have

$$W(0.5,2)=2(1.609438+0.035875+1.500000-0.261799+0.003333-0.000027+\ldots+0.000022+\ldots)=5.774$$

From (35) we have

$$W(2,0.5)=2(1.609438-0.003329+0.014441+1.500000-0.261800+0.021703+0.000022-\ldots+0.000283-\ldots-0.000026)=5.761$$

The computation necessary to obtain each of these values of $W(x, y)$ was performed on an automatic calculating machine. As a check on both the computation and the series expansions, each value of $W(x, y)$ was then computed independently from (30). Agreement of the first five digits was interpreted as verification of the expansion and the first computations.

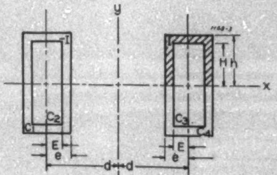

Figure 3

Substituting in (48) as indicated, we obtain

$$W/w^2=171.203+27.359+11.547+11.522-154.528-0.950=66.153$$

From (27) we have for the inductance

$L=66.153/(6.25-2.25)^2=4.135$ abhenries per centimeter of conductor

Multiplying L by $120\pi/10^9$ we obtain the reactance,

$X=496.1\pi/10^9$ ohm per centimeter of conductor

Converting to more suitable units, we have

$X=496.1\pi\times2.54\times12\times10^3/10^9$
$=0.0475$ ohm per thousand feet of conductor

Comparing this with the uncorrected value given by Dwight and Wang—namely, 0.0443 ohm per thousand feet of conductor, uncorrected—we find that our value is 0.0032 ohm per thousand feet larger; or, the difference due to neglecting the thickness of the conductor is, in this case, 6.7 per cent. If the conductor were larger with correspondingly thicker walls, the per cent difference would probably be greater.

For this problem Dwight and Wang obtain from their correction formula 0.0026 ohm per thousand feet increase due to thickness. Accordingly, adding this value to the original value yields 0.0469 ohm per thousand feet of conductor. The difference is now but 0.0006 ohm per thousand feet of conductor. Accordingly, in view of this close agreement one might conjecture that in calculating the reactance of *identical*, *square* tubular conductors for purposes other than wherein extreme accuracy is required, that so far as time of computation is concerned the simpler formulas of Dwight and Wang can be used to the advantage of those given in this paper. If, however, the conductors are other than *both* square and identical the formulas of this paper must be used perforce.

A glance at the individual values of $W(x, y)$ will reveal the extreme rapidity with which they converge. As only three decimal places were used in calculating W/w^2 from (48), it would have been necessary to have computed only the first two terms in each of $W(2.5, 2.5)$, $W(1.5, 1.5)$, $W(2, 2)$, and $W(0.5, 0.5)$. Likewise, several terms could have been omitted in $W(2, 0.5)$ and $W(0.5, 2)$.

Appendix

I. Determination of the Integrals f_1 and f_2

We can transform f_2 as follows:

$$2=\int_0^\infty \frac{\sin Mx_1 \sin Mx_2 \sin Mx_3 \sin Mx_4}{M^5}\times \epsilon^{\frac{-My}{2}} \sinh \frac{My}{2}\, dM$$

$$=\int_0^\infty \frac{\sin Mx_1 \sin Mx_2 \sin Mx_3 \sin Mx_4}{M^5}\times (1-\epsilon^{-My})\, dM$$

$$=\int_0^y \int_0^\infty \frac{\sin Mx_1 \sin Mx_2 \times \sin Mx_3 \sin Mx_4}{M^4}\epsilon^{-My}\, dM dy$$

Substituting the value of the infinite integral,[7] we have

$$f_2=\int_0^y \frac{1}{3!2^3}\sum_{i=1}^{8}(-1)^p\left[(k_i^3-3y^2k_i)\times \tan^{-1}\frac{k_i}{y}-(3yk_i^2-y^3)\log(k_i^2+y^2)^{1/2}\right]dy$$

Table I. Values of K_i for Evaluating Equation 19

x_1	x_2	x_3	x_4	k_i: $x_1=x_2=d$, $x_3=x_4=e$	k_i: $x_1=x_2=d$, $x_3=e$; $x_4=E$
+	+	+	+	$2d+2e$	$2d+e+E$
+	+	−	−	$2d-2e$	$2d-e-E$
+	+	+	−	$2d$	$2d+e-E$
+	+	−	+	$2d$	$2d-e+E$
+	−	+	+	$2e$	$e+E$
+	−	−	−	$-2e$	$-(e+E)$
+	−	+	−	0	$e-E$
+	−	−	+	0	$-(e-E)$

k_i designating any one of the eight possible terms formed from the member $(x_1 \pm x_2 \pm x_3 \pm x_4)$ by arbitrary selection of the plus and minus signs, while p is the number of minus signs in k_i. Integrating and collecting terms, we obtain

$$f_2=\frac{1}{4!2^3}\sum_{i=1}^{8}(-1)^p\left[(k_i^4-6yk_i^2+y^4)\times \log(k_i^2+y^2)^{1/2}+4yk_i(k_i^2-y^2)\tan^{-1}\frac{k_i}{y}-k_i^4\log|k_i|\right]$$

$$=\frac{1}{192}\sum_{i=1}^{8}(-1)^p F(k_i, y)$$

where $F(k_i, y)$ stands for the expression within the brackets.

Noting that

$$f_1=\int_0^\infty \frac{\sin Mx_1 \sin Mx_2 \sin Mx_3 \sin Mx_4}{M^4}dM$$

$$=\lim_{y\to 0_+}\int_0^\infty \epsilon^{-My}\times \frac{\sin Mx_1 \sin Mx_2 \sin Mx_3 \sin Mx_4}{M^4}dM$$

we obtain on substituting for the infinite integral

$$f_1=\lim_{y\to 0_+}\frac{1}{3!2^3}\sum_{i=1}^{8}(-1)^p\left[(k_i^3-3y^2k_i)\times \tan^{-1}\frac{k_i}{y}-(3yk_i^2-y^3)\log(k_i^2+y^2)^{1/2}\right]$$

$$=\frac{1}{3!2^3}\sum_{i=1}^{8}(-1)^p\frac{\pi}{2}|k_i^3|$$

II. Expansion of W(r, s) in a Power Series

By definition

$$W(r, s)=1/8[2F(D, s)-F(D+r, s)-F(D-r, s)+2F(r, s)-2F(0, s)+4\pi r^2 s(3D-r)]$$

Now

$$K(x, s)=2F(D, s)-F(D+r, s)-F(D-r, s)$$

considered as a function of x alone, can be expanded in a power series about the point $x=D$. Accordingly, by Taylor's theorem we have

$$K(x, s)=-r^2\frac{\partial^2(FD, s)}{\partial D^2}-\frac{r^4}{2\cdot 3!}\frac{\partial^4 F(D,s)}{\partial D^4}-\frac{r^6}{3\cdot 5!}\frac{\partial^6 F(D, s)}{\partial D^6}-\dots$$

where

$$F(D, s)=1/2(D^4-6D^2s^2+s^4)\log(D^2+s^2)-D^4\log D+4(sD^3-Ds^3)\tan^{-1}\frac{D}{s}$$

$$\frac{\partial F(D, s)}{\partial D}=(2D^3-6D^2s^2)\log(D^2+s^2)-4D^3\times \log D+4(sD^2-s^3)\tan^{-1}\frac{D}{s}-3Ds^2$$

$$\frac{\partial^2 F(D, s)}{\partial D^2}=6(D^2-s^2)\log(D^2+s^2)-12D^2\times \log D+24Ds\tan^{-1}\frac{D}{s}-7s^2$$

$$\frac{\partial^3 F(D, s)}{\partial D^3}=12D\log(D^2+s^2)-24D\log D+24s\tan^{-1}\frac{D}{s}$$

$$\frac{\partial^4 F(D, s)}{\partial D^4}=12\log(D^2+s^2)-24\log D$$

$$\frac{\partial^5 F(D, s)}{\partial D^5}=-\frac{24s^2}{D(D^2+s^2)}$$

$$\frac{\partial^6 F(D, s)}{\partial D^6}=24\left[\frac{1}{D^2}-\frac{D^2-s^2}{(D^2+s^2)^2}\right]$$

$$\frac{\partial^7 F(D, s)}{\partial D^7}=48\left[-\frac{1}{D^3}-\frac{-D^3+3Ds^2}{(D^2+s^2)^3}\right]$$

$$\frac{\partial^8 F(D, s)}{\partial D^8}=144\left[\frac{1}{D^4}-\frac{(D^2-s^2)^2-4s^2D^2}{(D^2+s^2)^4}\right]$$

$$\frac{\partial^9 F(D, s)}{\partial D^9}=576\left[-\frac{1}{D^5}-\frac{-D^5+10D^3s^2-5Ds^4}{(D^2+s^2)^5}\right]$$

$$\frac{\partial^{10} F(D,s)}{\partial D^{10}}=2{,}880\left[\frac{1}{D^6}-\frac{(D^2-s^2)^3-12s^2D^2(D^2-s^2)}{(D^2+s^2)^6}\right]$$

References

1. Reactance of Strap Conductors, H. B. Dwight. *Electric Review*, volume 70, 1917, pages 1087–8.

2. Reactance Values for Rectangular Conductors, H. B. Dwight. *Electric Journal*, volume 16, 1919, pages 255–6.

3. Champ Magnétique et Inductance d'un Système des Barres Rectangulaires Parallèles, E. Roth. *Revue Générale de l'Electricité*, volume 41, 1938, pages 275–83.

4. Reactance of Square Tubular Busbars, H. B. Dwight and T. K. Wang. AIEE Transactions, volume 57, 1938, pages 762–5.

5. Inductance of Hollow Rectangular Conductors, T. J. Higgins. *Journal* of the Franklin Institute, volume 230, 1940, pages 375–80.

6. On the Geometrical Mean Distances of Rectangular Areas and the Calculation of Self-Inductance, E. Rosa. *Bulletin* of the Bureau of Standards, volume 3, 1907, pages 1–41.

7. Nouvelles Tables d'Intégrales Définies (a book), D. Bierens de Haan. Leyden, 1867. Table 173, number 26, and table 371, number 5.

Stability Limitations of Long-Distance A-C Power-Transmission Systems

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Synopsis: Conventional methods of transmission are analyzed and discussed both for existing transmission distances up to about 300 miles and for greater distances up to a full wave length; also eight different methods that have been proposed for increasing the loading per circuit or distance to which a-c power can be transmitted are discussed. Of these methods, series compensation of the line reactance appears to be the most favorable for straightaway distances up to about 600 to 700 miles at 60 cycles. The problem is analyzed mathematically and with the aid of an a-c network analyzer. Curves are included which give the power limits and reactive kilovolt-ampere requirements for a large number of parameters, including the effect of distance, voltage, conductor size, series and shunt compensation, terminal impedances, and stability margin. This analysis indicates that the limitation of the transmission of a-c power greater straightaway distances than has been accomplished heretofore is primarily that of the cost of the line, the stability limitation can be overcome by a comparatively small increase in cost.

Introduction

TECHNICAL and economic developments in recent years have resulted in an increased and renewed interest in the problems associated with the electrical transmission of large blocks of power over long distances. Although this subject has been treated extensively in the literature, there appears to be a need at the present time for a brief review of the many ideas that have been presented and a further analysis of some of the fundamental considerations of the problem. The extensive use of a-c network analyzers by operating companies for studying their transmission problems and the availability of such means have made it comparatively easy to obtain data for both the existing and theoretical systems. With this information, more general conclusions can be drawn than were previously possible.

This paper is devoted essentially to the technical aspects of the problem of long-distance a-c power transmission. No attempt is made to determine the economic feasibility of such projects, as they necessarily depend upon many widely differing conditions and factors, which can be properly evaluated only for the particular case under consideration.

I. Existing Transmission—100 to 300 Miles

Existing a-c power transmission in the United States and Canada for distances of from 100 to 300 miles is accomplished by overhead transmission lines at 60 cycles with the exception of a few lines which operate at 25 cycles and a few at 50 cycles. All high-voltage overhead lines of conventional design have inductances and capacitances per unit length which fall within a narrow range. Accordingly, the surge impedance and velocity of propagation of these lines are practically the same, i.e. about 400 ohms and almost 186,000 miles per second (appendix *A*). Furthermore most transmission lines of 100 miles or more are operated at full load near unity power factor. Transmission line resistance losses and reactive kilovolt-ampere requirements increase rapidly when the receiving end power factor is changed appreciably from unity. Full load, therefore, in existing lines of 100 miles or more in length corresponds approximately to the surge impedance loading, by which is meant a load of approximately unity power factor of about 400 ohms equivalent impedance to neutral.

If there were no other important limitations such as stability, transmission lines would be expected to have a rated loading of an order of magnitude determined only by their surge impedance and circuit voltage, independent of frequency and length. Furthermore, since all high-voltage overhead transmission lines have practically the same surge impedance, their loading should vary only with, and directly with, the square of the line voltage. A check[1] of the major long-distance transmission lines in the United States and Canada indicates that this is essentially correct and that only five of these lines over 100 miles long are rated as high as 25 per cent above their unity power factor or approximate surge impedance loading. It is of interest to note that these five heaviest loaded lines are all single-circuit lines and are connected to systems which can tolerate the complete loss of the line and its associated generating capacity. Lines operated in parallel are usually more conservatively rated for normal conditions, so that they will attain this heavy loading only under unusual or emergency conditions. Table I, column 2, gives the approximate unity power factor rating, based on a surge impedance of 400 ohms, for different circuit voltages; and column 3 gives, what is termed in this discussion, the maximum practical rating. Column 2 in kilowatts is numerically equal to 2.5 $(kv)^2$ and column 3 is 1.25 times column 2, $(kv)^2$ being the square of the line-to-line voltage in kilovolts. Occasionally a line will be operated above the value given in column 3, under emergency or temporary conditions before additional line capacity is added or other changes are made. Lines operated with these extreme loadings require unusually large reactive kilovolt-ampere capacity, supplied by generators or synchronous condensers.

Table I. Approximate Transmission Loadings Per Circuit of Conventional Overhead A-C Lines of Moderate Length

Column 1 Receiver Voltage—Kv	Column 2 Unity Power Factor Rating* $Kw = 2.5(Kv)^2$	Column 3 Maximum Practical Rating $Kw = 1.25^{**} \times 2.5(Kv)^2$
550	755,000	945,000
495	615,000	765,000
440	485,000	605,000
385	370,000	465,000
330	270,000	340,000
275	190,000	240,000
220	120,000	150,000
154	60,000	75,000
132	44,000	54,000
110	30,000	38,000
66	11,000	14,000

*Based on surge impedance of 400 ohms.

**This factor is indicated by experience of the major existing lines of balanced design (100 to 300 miles).

Since even the longest existing transmission lines of almost 300 miles are being loaded as heavily as lines of 100 miles in length, it is apparent that the reactive kilovolt-ampere and kilowatt losses, and not necessarily stability, have been the important limitation of the loading. If there were no stability limitations,

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[1] For all numbered references, see list at end of paper.

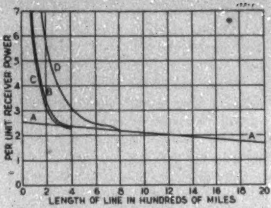


Figure 1. Receiver power with conventional 60-cycle transmission line terminated in dead impedance load

Line constants: $x = 0.8$ ohm per mile
$r = 0.08$ ohm per mile
$b = 5.2 \times 10^{-6}$ mhos per mile

Base kw = $(\text{kv})^2$

Curve *A*—Receiver power with surge impedance load

Curve *B*—Maximum receiver power without overvoltage along the line for impedance loads with impedance angle equal to that of surge impedance of line

Curve *C*—Maximum receiver power without overvoltage along the line for pure resistance loads

Curve *D*—Maximum receiver power without overvoltage along the line for impedance loads resulting in receiving end voltage equal to sending end voltage

economic transmission would dictate heavier loadings per circuit as the distance increases. Up to the present, efforts have been made to bring the transient stability limits due to faults and other disturbances to a point where the loadings of table I (either column) can be obtained with reliability on the circuits remaining in service after the disturbance. This has been accomplished in many systems by means of quick switching, good relaying, intermediate switching stations, almost lightning-proof lines, and good system design. In systems with quick switching and good relaying, where stability limits of the order of column 3 of table I cannot be obtained following a disturbance, experience indicates that the primary reason is the inability of the rotating equipment to provide the emergency reactive kilovolt-amperes required to maintain the bus voltages at the values necessary for the transmission of such loads. This is not necessarily the primary limitation for greater distances.

Conclusions for Existing Lines

1. Existing lines (100 to 300 miles) are operated at maximum loadings as high as 25 per cent above their surge impedance loading. Lines approaching 300 miles in length have loadings equal to or above previously built shorter lines.

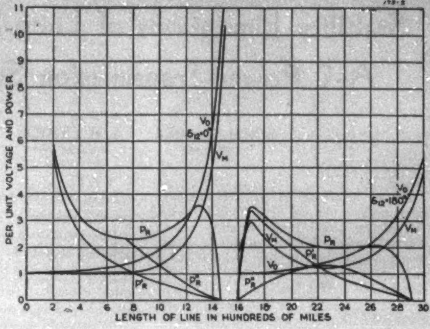


Figure 2. Steady-state stability power limits of conventional 60-cycle transmission lines with zero terminal impedances

Base kw = $(\text{kv})^2$
$E_1 = E_2$
$x = 0.8$ ohm per mile
$r = 0.08$ ohm per mile
$b = 5.2 \times 10^{-6}$ mhos per mile

2. For practical purposes, other conditions remaining the same, the power which can be transmitted over conventional lines varies directly as the square of the circuit voltage.

3. The desirability of using 330 kv as a transmission line voltage for moderate distances will depend to a considerable extent upon whether it is desirable or necessary to transmit 240,000 to 340,000 kw per circuit.

4. With modern equipment and system design practices, line reactive kilovolt-ampere requirements and losses rather than stability have been the important limitations for the transmission of power over conventional 60-cycle lines up to 300 miles in length.

II. Conventional Lines Longer Than 300 Miles

It has been realized for some time that stability will impose more severe limitations as transmission distances are increased. Transmission of a-c power in existing systems with conventional types of load requires that the transmission system develop sufficient synchronizing torque to keep the connected synchronous machines in synchronism. This condition has been termed a live-load requirement in contrast with a dead or impedance load. As transmission distances are increased, the importance of the type of load becomes greater. It is possible to transmit any distance, without overvoltage along the line, a dead load corresponding to the surge impedance load, similar to that used in communication circuits, the limitation of delivered power being determined by the circuit resistance losses. This is not true for live loads, or synchronous-synchronous interconnections, which have a stability limitation.

Curve *A* of figure 1 shows the power that can be delivered over conventional 60-cycle lines of different lengths having the same circuit constants when the receiving end is terminated by an impedance load equal to the surge impedance of the line. The ordinate of figure 1 is plotted in per unit receiver power, with each unit numerically equal to the square of the sending end line-to-line circuit voltage in kilovolts. The abscissa is hundreds of miles of transmission line. The gradual decrease in receiver power with increase in length of line in curve *A* is due to the line resistance losses which

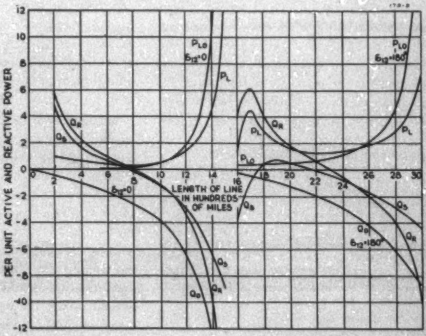


Figure 3. Reactive power and power loss of conventional 60-cycle transmission lines at steady-state stability limit and at no load

Base kva = base kw = $(\text{kv})^2$
$E_1 = E_2 = 1.0$
$x = 0.8$ ohm per mile
$r = 0.08$ ohm per mile
$b = 5.2 \times 10^{-6}$ mhos per mile

could be reduced by using lower resistance conductors. Curve *B* shows the receiver power corresponding to impedance loads which are fractions of the surge impedance. The load power factor is the same as that for the surge impedance load, but the magnitudes of the load impedances are such as to give maximum receiver power at any length of line without overvoltage along the line. For lines longer than approximately 400 miles in length, curves *A* and *B* coincide. Curve *C* is for the optimum pure resistance load without overvoltage along the line. This curve differs slightly from curve *B* because of the small capacitive components in the load impedances of curve *B*. Curve *C* crosses curve *A* between 300 and 400 miles of line. Curve *D* shows maximum receiver power for the case of impedance loads with sufficient capacitance to produce a receiving end voltage equal to the sending end voltage without overvoltage along the line. At slightly less than one-fourth wave length, overvoltages appear along the line at maximum power. (Without resistance, overvoltages under maximum load would not occur for lengths of line smaller than one-fourth wave length.) To reduce these overvoltages to rated voltage, the terminal voltages were reduced, but maintained equal. Curve *D* crosses curve *A* at slightly over one-fourth wave length of line. From these results, it is evident that for distances above approximately one-fourth wave length the maximum power that can be transmitted to an impedance load without overvoltages along the line corresponds to the surge impedance load. Curve *D* of figure 1 and the curves of the other figures in the paper were obtained on the a-c network analyzer.

Figure 2 shows P_R the maximum receiver power that can be transmitted with stability over conventional 60-cycle lines of various lengths between two synchronous systems of zero impedances. The line constants are the same as those of figure 1 and equal voltages are maintained at the sending and receiving ends of the line. Also in figure 2 are shown V_M and V_O the maximum voltages obtained along the line under maximum and no-load conditions, respectively, when the sending and receiving end voltages are equal and of rated magnitude. In the region of multiples of a one-half wave length very high voltages are obtained along the line for the condition of rated sending and receiving end voltages. P_R'' corresponds to the maximum receiver power with the terminal voltages maintained equal, but reduced below rated voltage so that voltages along the line do not exceed rated voltage. P_R' corresponds to the maximum receiver power with terminal voltages maintained equal at a value such that normal voltage is not exceeded at no load.

Figure 3 shows the sending and receiving end reactive kilovolt-ampere requirements corresponding to the power transfer shown in figure 2. Q_S and Q_R are per unit reactive power at the sending and receiving ends of the line, respectively, Q_O is the approximate per unit reactive power at both sending and receiving ends of the line at no load. Reactive power is plotted as positive or negative corresponding to over- or underexcited terminal synchronous systems, respectively. In figure 3 are also plotted P_L and P_{LO} the line losses at maximum load and at no load, respectively.

For distances less than one-half wave length, the stable operating range is in the first quadrant of the angular displacement δ_{12} between the sending and receiving end voltages; while between a half and a full wave length, the stable operating range is in the third quadrant. The no-load conditions were therefore taken as $\delta_{12}=0$ for distances less than one-half wave length, and $\delta_{12}=180$ degrees for distances between one-half and one wave length.

It is of interest to note that no power can be transmitted between two synchronous systems of zero impedances at the terminals of a half wave length line (see appendix *A*), but theoretically power can be transmitted with stability beyond a one-half wave length. It can be shown that for a given distance there is an optimum voltage ratio between the receiving and sending end voltages to obtain maximum power without overvoltage along the line. However, the voltage along the line becomes very sensitive or critical to terminal changes for extreme lengths of line so that operation without overvoltage is difficult.

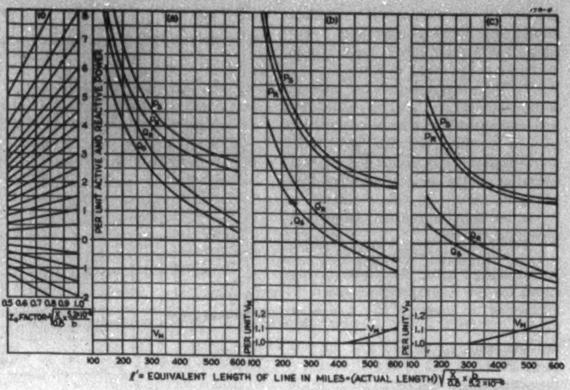


Figure 4. **Steady-state stability curves. $E_1=E_2=1.0$, $r/x=0.1$, base kva = base kw = $(kv)^2$**

(*a*)—Line alone with infinite bus at each end of line

(*b*)—Line with synchronous terminal equipment. Generator plus transformer impedance = 40 per cent on kilovolt-amperes at sending end of line. Receiving end impedance (including transformer) = 25 per cent on kilovolt-amperes at receiving end of line

(*c*)—Same as (*b*) but with 25 per cent stability margin

NOTE: *P* and *Q* are active and reactive power, respectively, in per unit of $(kv)^2$, the subscripts *S* and *R* indicating sending and receiving ends of the line, respectively. V_M is maximum voltage along the line under load

As shown by figure 2, the surge impedance loading can be obtained up to about a quarter wave length (approximately 750 miles at 60 cycles). Beyond this distance the power that can be transmitted falls off rapidly for the condition of no overvoltage along the line. The effect of terminal impedances and the necessity of operating with a stability margin reduce the limits shown on figure 2 still further. These effects are shown in figures 4 and 5.

The curves of figures 4 and 5 summarize the steady-state stability studies made on the a-c network analyzer for conventional lines and compensated conventional lines with compensation uniformly distributed. It can be shown that a transmission line with synchronous terminal equipment is defined by (1) *r*/*x* ratio, (2) equivalent length, (3) Z_0 factor, (4) equivalent steady-state terminal reactances.[2] Figures 4 and 5 differ only in the r/x ratio. In figure 4, $r/x=0.1$; in figure 5, $r/x=0.2$. r is the line resistance per mile and x the inductive reactance per mile. For a compensated line, x is the resultant inductive reactance per mile after series compensation; r is unchanged by compensation if the series capacitors have negligible resistance; b is the resultant capacitive admittance after shunt compensation.

The equivalent length of line and Z_0 factor depend upon the choice of constants for the reference line. A line reactance of 0.8 ohm per mile and a capacity susceptance of 5.2×10^{-6} mhos per mile were selected. For a line with these constants, the equivalent length is the actual length and the Z_0 factor is 1.0.

For a given r/x ratio and given line constants x and b,

Equivalent length of line in miles = (actual length of line in miles) ×

$$\left(\sqrt{\frac{x}{0.8}\times\frac{b}{5.2\times10^{-6}}}\right) \qquad (1)$$

$$Z_0\text{ factor}=\sqrt{\frac{x}{0.8}\times\frac{5.2\times10^{-6}}{b}} \qquad (2)$$

Making use of equivalent length of line and Z_0 factor, with base kva = base kw = $(\text{kv})^2$, and equal voltages at the line terminals, figures 4 and 5 can be applied to conventional lines and compensated conventional lines with compensation uniformly distributed at any voltage and frequency. The equivalent length of line lies between 100 and 600 miles. The r/x ratios are 0.1 and 0.2. The terminal conditions are:

(*a*). Line alone with infinite bus at both ends and equal voltages maintained at the line terminals equal to rated line voltage.

(*b*). Line with synchronous terminal equipment. Generator equivalent steady-state reactance[3] plus transformer reactance = 40 per cent on kilovolt-amperes at sending end of line, receiving end equivalent steady-state reactance (including transformer) = 25 per cent on kilovolt-amperes at receiving end of line, equal voltages maintained at the line terminals equal to rated line voltage. A low value of generator reactance was selected for this study as it will usually be found economical when steady-state stability is a problem to use generators with considerably less than normal reactance.

(*c*). Same as (*b*) but with 25 per cent stability margin. The stability margin is the difference between maximum power and rated power, divided by rated power, where maximum power is the stability limit obtained with the terminal voltages maintained and terminal impedances corresponding to the equipment used for the transmission of rated power.

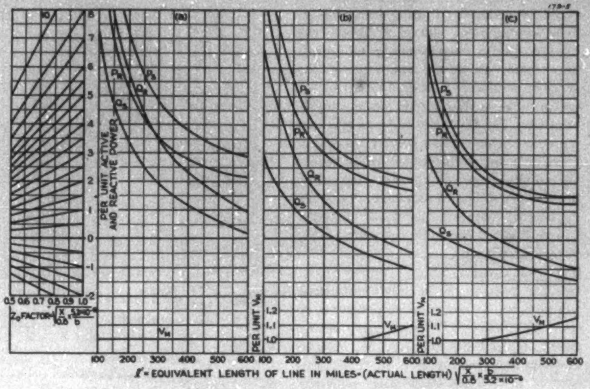


Figure 5. Steady-state stability curves. $E_1=E_2=1.0$, $r/x=0.2$, base kva = base kw = $(\text{kv})^2$

(*a*)—Line alone with infinite bus at each end of line

(*b*)—Line with synchronous terminal equipment. Generator plus transformer impedance = 40 per cent on kilovolt-amperes at sending end of line. Receiving end impedance (including transformer) = 25 per cent on kilovolt-amperes at receiving end of line

(*c*)—Same as (*b*) but with 25 per cent stability margin

NOTE: P and Q are active and reactive power, respectively, in per unit of $(\text{kv})^2$, the subscripts S and R indicating sending and receiving ends of the line, respectively. V_M is maximum voltage along the line under load

In figures 4 and 5 base three-phase power is numerically equal to $(\text{kv})^2$, where kv is the line-to-line rated voltage in kilovolts at the ends of the line. The line terminal voltages E_1 and E_2 are equal in magnitude. P and Q indicate active and reactive power, respectively, in per unit of $(\text{kv})^2$, the subscripts S and R referring to the sending and receiving ends of the line, respectively. Q_S and Q_R are positive when the equivalent synchronous machines at the sending and receiving ends, respectively, of the line are overexcited and negative when underexcited. V_M is the maximum voltage along the line under load.

To apply the curves to a transmission line connecting two synchronous systems with equal voltages held at the line terminals, r/x for the line and the equivalent terminal reactances are calculated or estimated. If $r/x=0.1$ or 0.2, and the terminal conditions correspond to (*a*), (*b*), or (*c*), the equivalent length of line and Z_0 factor are calculated by equations 1 and 2, respectively. With equivalent length of line as abscissa, and using the curves which correspond to the r/x ratio and terminal conditions of the given system, values of P_S, P_R, Q_S, and Q_R are read on the vertical scale corresponding to the Z_0 factor. The vertical lines are labeled for Z_0 factors between 0.5 and 1.0. If the calculated Z_0 factor does not fall within these limits, P_S, P_R, Q_R, and Q_S can be read on the vertical scale corresponding to a Z_0 factor of 1.0 and then divided by the calculated Z_0 factor. The following problems illustrate the use of figures 4 and 5.

EXAMPLE I

Given a 50-cycle, 330-kv 3-phase transmission line 550 miles in length: $r=0.07$ ohm per mile, $x=0.7$ ohm per mile, $g=0$, $b=4.4$ micromhos per mile. Equal voltages are maintained at the ends of the line. How much power can be delivered over the line with (*a*) infinite busses at the sending and receiving ends of the line, (*b*) with synchronous terminal systems at sending and receiving ends of the line having 40 per cent and 25 per cent reactances, respectively, based on sending and receiving end kilovolt-amperes, (*c*) with conditions of (*b*) but with 25 per cent stability margin?

Solution of Example I

$$\frac{r}{x}=\frac{0.07}{0.7}=0.1$$

$$e'=550\times\sqrt{\frac{0.7}{0.8}\times\frac{4.4}{5.2}}=475\text{ miles}$$

$$Z_0\text{ factor}=\sqrt{\frac{0.7}{0.8}\times\frac{5.2}{4.4}}=1.015$$

Entering curves (*a*), (*b*), and (*c*) of figure 4 with $l' = 475$ miles and reading P_R on the vertical scale corresponding to the Z_0 factor of 1.015 (extrapolated), and V_M on its own scale,

(*a*). $P_R = 2.75$ per unit of $(330)^2 = 300{,}000$ kw
$V_M = 1.0$ per unit of 330 kv = not over 330 kv

(*b*). $P_R = 2.2$ per unit of $(330)^2 = 240{,}000$ kw
$V_M = 1.025$ per unit of 330 kv = 2.5 per cent overvoltage along line

(*c*). $P_R = 1.6$ per unit of $(330)^2 = 175{,}000$ kw
$V_M = 1.09$ per unit of 330 kv = 9 per cent overvoltage along line

In cases (*b*) and (*c*) there is overvoltage along the line under load. This suggests the use of shunt reactors to partially compensate for line capacitance.

EXAMPLE II

Given a 60-cycle, 230-kv, three-phase transmission line, 325 miles in length: $r = 0.12$ ohm per mile, $x_2 = 0.8$ ohm per mile, $b = 5.1 \times 10^{-6}$ mhos per mile. The equivalent steady-state reactance[2] of the generators plus transformers based on the kilovolt-amperes at the sending end of the line is 40 per cent, that of the equivalent system at the receiving end of the line, including transformers, is 25 per cent based on receiving end kilovolt-amperes. Equal voltages are maintained at the terminals of the line. How much power can be delivered with stability under steady-state with 25 per cent margin if the reactance of the line is one-fourth compensated? Find also P_S, Q_S, Q_R, and V_M.

Solution of Example II. With line reactance one-fourth compensated, assuming uniformly distributed compensation,

$$x = \frac{3}{4}(0.8) = 0.6$$

$$\frac{r}{x} = \frac{0.12}{0.6} = 0.2$$

$$b = 5.1 \times 10^{-6}$$

$$l' = 325 \times \sqrt{\frac{0.6}{0.8} \times \frac{5.1}{5.2}} = 280 \text{ miles}$$

$$Z_0 \text{ factor} = \sqrt{\frac{0.6}{0.8} \times \frac{5.2}{5.1}} = 0.875$$

Entering curve (*c*) of figure 5 with $l' = 280$ miles and reading P_R, P_S, Q_R, and Q_S on the vertical scale corresponding to a Z_0 factor of 0.875 and V_M on its own scale:

$P_R = 2.55$ in per unit of $(230)^2 = 135{,}000$ kw
$P_S = 2.95$ in per unit of $(230)^2 = 155{,}000$ kw
$Q_R = 0.43$ in per unit of $(230)^2 = 23{,}000$ kvars, overexcited
$Q_S = -0.55$ in per unit of $(230)^2 = 29{,}000$ kvars, underexcited
$V_M = 1.0$ in per unit of 230 kv = not over 230 kv

Figures 4 and 5 show that the surge impedance loading given in column 2 of table I can be realized under favorable conditions of low terminal impedances with lines of conventional design up to about 300 to 350 miles at 60 cycles, and to correspondingly greater distances at lower frequencies. For greater distances, a reduction in loading per $(kv)^2$ would be required with conventional overhead lines. For example, power could be transmitted about 500 to 600 miles at 60 cycles if loadings one-half of those given in column 2, table I, were tolerable. Such a decrease in loading with distance is expensive and undesirable and probably would make it impractical to use conventional a-c transmission for these distances.

CONCLUSIONS FOR CONVENTIONAL LINES LONGER THAN 300 MILES

1. The only limitation to the distance that power can be transmitted to a dead load corresponding to the surge impedance of the line is the line resistance losses, otherwise this amount of power can be transmitted any distance.

2. For transmission of power over lines having synchronous terminal equipment, the limiting distance for a line loading corresponding to unity power factor is, under favorable conditions, about 300 to 350 miles at 60 cycles.

III. Methods for Overcoming Stability and Loading Limitations of Long Lines

As transmission distances increase, the transmission line cost becomes a greater part of the total investment and therefore it becomes of increasing economic importance to maintain or, if possible, to increase the circuit loadings. Under some conditions, studies have indicated that equipment allowing an increase in loading above those shown in table I would be a worth-while investment even for lines of moderate length. Many methods have been proposed to overcome the stability and loading limitations of long lines:

(*a*). It has been suggested that quadrature boost transformers be used to decrease the phase angular displacement between the sending and receiving end systems. The method accomplishes this for a given load, but also decreases the angular displacement at which pull-out occurs, resulting in no gain in stability but a possible decrease for the same line voltage.

(*b*). It has also been suggested that series synchronous condensers be used to overcome the line reactance voltage drop. It can be shown that this method would require extremely large rotating apparatus with a large motor to keep the synchronous condenser in proper phase relation.

(*c*). Series capacitors[3] have been suggested for increasing the power limits. This method appears to be the most promising for the next increase in a-c transmission line distances; for example, from 300–350 to 600–700 miles at 60 cycles. In order to transmit power 600–700 miles by means of a conventional line with a loading equal to the loadings of table I, column 2, it would be necessary to compensate only half of the line reactance. The capacitor kilovolt-amperes required would be about 70 per cent of the kilowatts transmitted. In order to deliver this power at unity power factor, additional reactive kilovolt-amperes either in shunt reactors or in synchronous condenser lagging capacity, also about 70 per cent of the kilowatts transmitted, would be required. Therefore to transmit say 190,000 kw 700 miles at 60 cycles over a single circuit at 275 kv, unity power factor, would require 133,000 kva of series capacitors and 133,000 kva of shunt reactors. A possible doubling in conductor cross section to keep the losses down would also be required (this would be necessary for any electrical transmission method). If it is assumed that capacitors cost \$9.00 per kilovolt-ampere and reactors \$2.00 per kilovolt-ampere, this would amount to \$8.00 per kilowatt transmitted to overcome the stability limitations of transmitting a-c power 700 miles. For shorter distances, the required amount of compensation and therefore the cost per kilowatt to overcome the stability limitations would be correspondingly less.

If series capacitors are used without shunt reactors, the over-all effective surge impedance is reduced, and it is possible to deliver more power at unity power factor per $(kv)^2$ for the same distance. In this

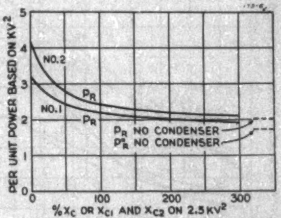
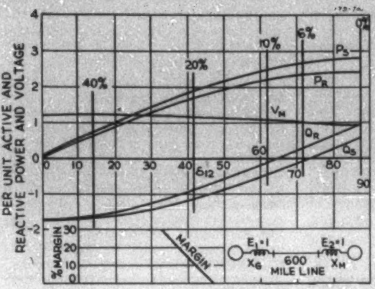


Figure 6. Receiving end power at steady-state stability limit for 600-mile 60-cycle conventional line with one and two intermediate stations as affected by intermediate station reactance

Curve number 1—One intermediate station, $E_1 = E_2 = E_C = 1.0$
Curve number 2—Two intermediate stations, $E_1 = E_2 = E_{C1} = E_{C2} = 1.0$
$x_G = 40$ per cent, $x_M = 25$ per cent $\Big\}$ based on $2.5\,(kv)^2$
$x = 0.8$ ohm per mile
$r = 0.08$ ohm per mile
$b = 5.2 \times 10^{-6}$ mhos per mile



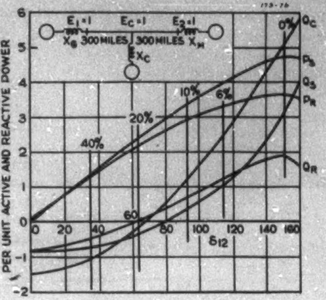


Figure 7. Characteristics of 600-mile conventional 60-cycle transmission line with synchronous terminal equipment, with and without intermediate stations

Base kva = base kw = (kv)^2

$x=0.8$ ohm per mile

$r=0.08$ ohm per mile

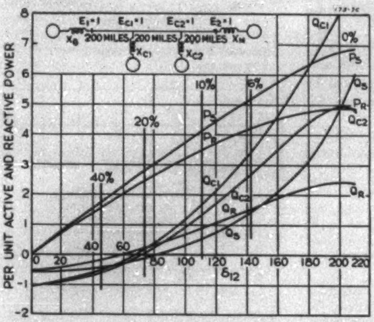
$b=5.2\times10^{-6}$ mhos per mile

Vertical lines give steady-state stability limits with equal terminal and intermediate station reactances indicated based on (kv)^2

(a) (upper left)—No intermediate stations, $E_1=E_2,\ x_G=x_M$

(b) (upper right)—One intermediate station, $E_1=E_2=E_{c1}\ \ x_G=x_M=x_c$

(c) (lower left)—Two intermediate stations, $E_1=E_2=E_{c1}=E_{c2}\ \ x_G=x_M=x_{c1}=x_{c2}$



way an increase in the loading per circuit can be accomplished. Thus series capacitors may be used to increase the loading per circuit or the distance of transmission or both.

In order to distribute the voltage drop across the series capacitors during either steady-state or transient conditions, it would be desirable to locate them along the line at possibly three to six locations for a line that is half compensated. The optimum arrangement can be determined by a study of the particular case under consideration. Protective equipment similar to that used with present-day lower voltage installations could be used to protect them during abnormal conditions. For lines of shorter length, less compensation would be required; while for longer lines, more compensation would be required for a given loading. One advantage in the use of series capacitors is that increased compensation need not be installed until load conditions requiring such compensation are reached.

(*d*). It has been suggested that frequency changers be used to produce lower frequency for the line and thus to extend the distance of transmission or the loading per circuit. Such equipment, in comparison with series capacitors, does not appear to be as economical except for distances appreciably above 700 miles. This method also has the additional disadvantage that the line cannot be as readily tapped and absorbed into the system high-voltage network as a line operated at normal frequency.

(*e*). In this same classification is the use of conversion equipment to impose at the receiving end a load having an impedance equal to the surge impedance of the line, the line being operated with variable voltage. The advantages of this method of overcoming the stability limitation are limited also to very long distances, possibly above 700 miles.

(*f*). It has been suggested that tuned transmission lines be used, possibly operating at higher than normal frequencies so that the line length corresponds to $^3/_4$, $^5/_4$, etc., of a wave length. This method has the disadvantage of requiring conversion equipment and the possibility of dangerous line voltages during fault conditions. Its usefulness therefore appears to be limited to extreme distances.

(*g*). It has been realized that the loading and distance to which power can be transmitted are materially increased by intermediate systems or condenser stations. In order to obtain an appreciable benefit by this method and so make it of practical importance, the intermediate system or station must have a relatively large kilovolt-ampere rating (or low reactance) compared to the power being transmitted. It has not been considered economical to install synchronous condensers for this purpose alone.

The effect of intermediate systems or condensers for the case of a 600-mile line is shown in figure 6. The ordinates are maximum per unit power while the abscissa is reactance of the intermediate system or systems. Curve number 1 of figure 6 is for one intermediate station located at the center of the line and curve number 2 is for two intermediate stations, evenly spaced along the line. It will be noted that the power limit increases rapidly as the reactance of the intermediate station or stations is decreased. If additional intermediate systems are located along the line the power limit is further increased. In appendix *B* is a more complete discussion of the effect of intermediate systems on the power transmitting ability of a line.

With the gradual natural development of a transmission grid over sections of the United States this method of transmitting power has already been accomplished, as many of the systems are normally interconnected, although in many cases the tie-line capacity is small compared to the system capacity.

(*h*). Another method of increasing the limits of a-c transmission is by the use of cable which has inherently a low reactance and would allow a considerable increase in the distance to which the desired circuit loadings could be maintained. Shunt reactors would probably be required to compensate for the excessive line charging capacity. This method depends upon the design of cable which can compete favorably in cost with overhead lines.

(*i*). D-c transmission of power has been much discussed as a method by which the loading per circuit can be materially increased for the same maximum circuit voltage and total effective conductor area. The advantages of direct current over alternating current increase with increase in the straightaway transmission distance, or when the terminal equipment cost becomes small compared with the line cost. On the basis that twice as much power can be transmitted by direct current as by alternating current for the same line cost, it is necessary that the total d-c terminal cost above the total a-c terminal cost (both ends) be less than one-half the a-c line cost if the capital investment is not to exceed that for alternating current. It is apparent therefore that the advantage of d-c over a-c transmission increases with the straightaway transmission line distance and the required cost per mile of line. As the cost of the conversion terminal equipment is reduced, the transmission distances at which d-c transmission compares favorably with a-c transmission become less.

CONCLUSIONS REGARDING METHODS FOR OVERCOMING STABILITY LIMITATIONS OF LONG LINES

1. Of all the methods proposed for transmitting 60-cycle power to greater distances and with loadings exceeding those of existing lines, the series capacitor used in conventional overhead lines appears most favorable at the present time.

2. A-c power may be transmitted across or through interconnected systems for considerable distances, provided the intermediate systems are connected to the transmission system by low-reactance ties and these systems have a kilovolt-ampere rating comparable to the transmitted power.

3. The stability limitation in long-distance a-c transmission is not present in d-c transmission. The advantages of d-c transmission increase with the straightaway distance of transmission and the cost per mile of line.

Nomenclature

e_s, e_r = vector line-to-neutral voltage at sending and receiving ends of line, respectively, in volts

i_s, i_r = vector line currents at sending and receiving ends of line, respectively, in amperes

f = frequency in cycles per second

r, x, z = resistance, reactance, impedance, respectively, in ohms per mile

g, b, y = leakance, capacity susceptance, admittance, respectively, in mhos per mile

α, λ, v = normal attenuation constant per mile, wave length in miles, apparent velocity of propagation in miles per second, respectively

Z_0 = surge impedance to neutral in ohms

θ = hyperbolic angle of the line

A, B, ϕ_a, ϕ_b = scalar value of cosh θ, scalar value of Z_0 sinh $\theta \times 10^{-3}$, impedance angle of cosh θ, impedance angle of Z_0 sinh $\theta \times 10^{-3}$, respectively

l, l' = actual and equivalent length of line in miles, respectively

kv = rated line-to-line voltage in kilovolts

E_1, E_2 = scalar values of line-to-neutral voltage at sending and receiving ends of the line, respectively, in per unit of rated line-to-neutral voltage

V_M, V_O = maximum voltage along the line in per unit of rated voltage under full load and no load, respectively

δ_{12} = electrical angular displacement between e_s and e_r

$\delta_{12(\max)}$, $\delta_{12(\min)}$ = electrical angular displacement between e_s and e_r for maximum and zero power, respectively

P_S, P_R = three-phase active power at sending and receiving ends of the line, respectively, in per unit of base three-phase kilowatts

P_L, P_{LO} = three-phase line losses at maximum load and at no load, respectively, in per unit of three-phase base kilowatts

P_R', P_R'' = three-phase active power at receiving end of line in per unit of base three-phase kilowatts, with equal terminal voltages of such magnitude as to produce no overvoltage along the line at no load and full load, respectively

Q_S, Q_R = three-phase reactive power at sending and receiving ends of the line, respectively, in per unit of base three-phase kilovolt-amperes

Q_O = three-phase reactive power at either end of line at no load in per unit of base three-phase kilovolt-amperes

Q_c, Q_{c1}, Q_{c2} = three-phase reactive power in per unit of base three-phase kilovolt-amperes at intermediate station indicated by subscript

E_c, E_{c1}, E_{c2} = scalar values of line-to-neutral voltages in per unit of base line-to-neutral voltage at intermediate station indicated by subscript

x_c, x_{c1}, x_{c2} = reactance of intermediate station indicated by subscript

x_G = equivalent steady-state reactance[2] of the generators plus transformers (or equivalent system) at the sending end of the line

x_M = equivalent steady-state reactance of the equivalent system (transformers included) at the receiving end of the line

δ_{GM} = electrical angular displacement between the voltages behind x_G and x_M

Appendix A. Characteristics of Symmetrical Three-Phase Transmission Lines With Uniformly Distributed Constants

The following equations[4] apply to transmission lines under balanced operating conditions:

$$e_s = e_r \cosh\theta + i_r Z_0 \sinh\theta \qquad \text{(A-1)}$$

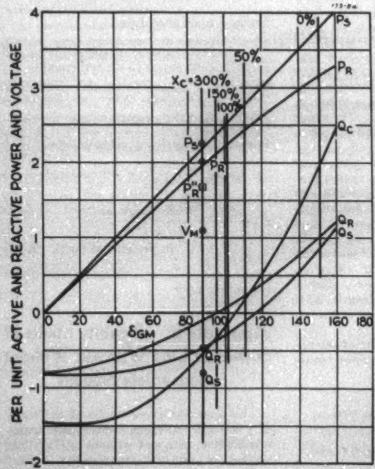
$$e_r = e_s \cosh\theta - i_s Z_0 \sinh\theta \qquad \text{(A-2)}$$

where

$$Z_0 = \sqrt{\frac{z}{y}} = \sqrt{\frac{r+jx}{g+jb}} \qquad z = r+jx$$

Figure 8. Characteristics of 600-mile conventional 60-cycle transmission line with one and two intermediate stations and synchronous terminal equipment

Base kva = base kw = $(kv)^2$
$x = 0.8$ ohm per mile
$r = 0.08$ ohm per mile
$b = 5.2 \times 10^{-6}$ mhos per mile
$x_G = 40$ per cent based on $2.5\ (kv)^2$ [16 per cent based on $(kv)^2$]
$x_M = 25$ per cent based on $2.5\ (kv)^2$ [10 per cent based on $(kv)^2$]



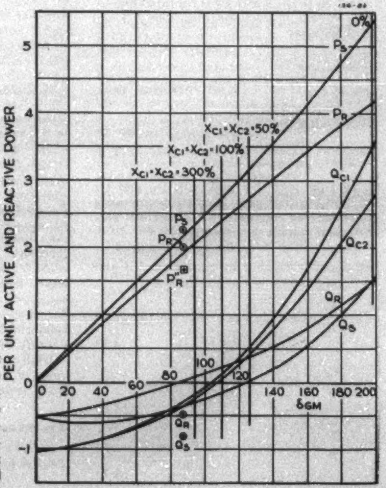
(a) (left)—One intermediate station—see figure 7b for sketch. $E_1 = E_2 = E_c = 1.0$

(b) (right)—Two intermediate stations—see figure 7c for sketch. $E_1 = E_2 = E_{c1} = E_{c2} = 1.0$

NOTE: Vertical lines give steady-state stability limits with intermediate station reactances indicated based on kva = $2.5\ (kv)^2$

⊙ Give P_S, P_R, Q_S, Q_R, and V_M at stability limit without intermediate station

⊡ Indicates P_R'', maximum power with equal terminal voltages and no overvoltage along the line under full load



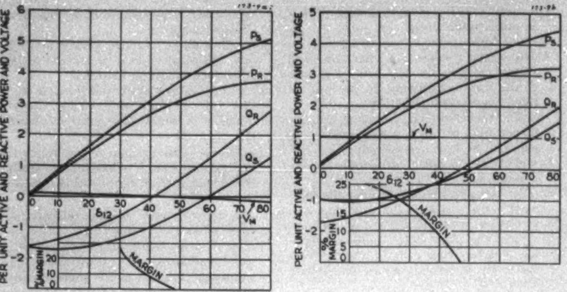


Figure 9. Characteristics of 600-mile conventional 60-cycle transmission line with series compensation infinitely distributed. Series line reactance one-half compensated

Base kva = base kw = (kv)^2
$x=0.8-0.4=0.4$ ohm per mile
$r=0.08$ ohm per mile
$b=5.2\times10^{-6}$ mhos per mile
Reactance of terminal equipment at sending end = 40 per cent on sending end kilovolt-amperes
Reactance of terminal equipment at receiving end = 25 per cent on receiving end kilovolt-amperes
(a) (above)—$E_1=E_2=1.0$
(b) (right)—$E_1=1.0$; $E_2=0.85$

$$\theta=l'\alpha \qquad y=g+jb$$

$$\alpha=\sqrt{zy}=\sqrt{(r+jx)(g+jb)}$$

Expressed in per unit, with base voltage equal to rated line-to-neutral voltage and base three-phase kilovolt-amperes and base three-phase kilowatts both numerically equal to (kv)^2, where kv is the rated line-to-line voltage in kilovolts, equations A-1 and A-2 become

$$E_1\underline{/\delta_{12}}=E_2\cosh\theta+\frac{P_R+jQ_R}{E_2}(Z_0\times10^{-3})\sinh\theta \qquad \textbf{(A-3)}$$

$$E_2\overline{\backslash\delta_{12}}=E_1\cosh\theta-\frac{P_S-jQ_S}{E_1}(Z_0\times10^{-3})\sinh\theta \qquad \textbf{(A-4)}$$

In equations A-3 and A-4, Q_R and Q_S will be positive or negative corresponding to over- or underexcited terminal synchronous machines, respectively.

Substituting $A\underline{/\phi_a}$ for $\cosh\theta$, and $B\underline{/\phi_b}$ for $Z_0\sinh\theta\times10^{-3}$ in equations A-3 and A-4 and solving for P_R, P_S, Q_R, and Q_S:

$$P_R=\frac{E_2}{B}[E_1\cos(\delta_{12}-\phi_b)-E_2A\cos(\phi_a-\phi_b)] \qquad \textbf{(A-5)}$$

$$Q_R=\frac{E_2}{B}[E_1\sin(\delta_{12}-\phi_b)-E_2A\sin(\phi_a-\phi_b)] \qquad \textbf{(A-6)}$$

$$P_S=\frac{E_1}{B}[-E_2\cos(\delta_{12}+\phi_b)+E_1A\cos(\phi_a-\phi_b)] \qquad \textbf{(A-7)}$$

$$Q_S=\frac{E_1}{B}[-E_2A\sin(\delta_{12}+\phi_b)-E_1\sin(\phi_a+\phi_b)] \qquad \textbf{(A-8)}$$

$$\text{Power loss}=P_S-P_R=\frac{1}{B}[(E_1^2+E_2^2)A\times\cos(\phi_a-\phi_b)-2E_1E_2\cos\delta_{12}\cos\phi_b] \qquad \textbf{(A-9)}$$

The voltage along the line at any point F with given values of P_R and Q_R can be obtained from equation A-3 if point F is considered the sending end of the line between F and the receiving end.

With any given voltages E_1 and E_2 maintained at the line terminals and an impedance load, P_R maximum occurs when $\delta_{12}=\phi_b$, P_S maximum occurs when $\delta_{12}=\pi-\phi_b$. With E_1 and E_2 maintained, and synchronous terminal equipment at the sending and receiving ends of the line of zero impedance (i.e., infinite bus at both ends of the line), the criterion of stability[5] is here taken as a positive rate of change of both receiving and sending end power with δ_{12}. For stability,

$$\frac{dP_S}{d\delta_{12}}\geqq0 \qquad \textbf{(A-10)}$$

and

$$\frac{dP_R}{d\delta_{12}}\geqq0 \qquad \textbf{(A-11)}$$

The value of δ_{12} at the steady-state stability limit, $\delta_{12(\max)}$, must satisfy both equations A-10 and A-11, and therefore is either ϕ_b or $\pi-\phi_b$, whichever is smaller. The range of δ_{12}, for stable operation with power transferred, lies between $\delta_{12(\min)}$ and $\delta_{12(\max)}$, where $\delta_{12(\min)}$ may be obtained from equation A-5 with $P_R=0$ and the restriction that $\delta_{12(\min)}$ must lie between $\delta_{12(\max)}-\pi/2$ and $\delta_{12(\max)}$. For a zero resistance line of less than one-half wave length, $\phi_b=\pi/2$, and therefore $\delta_{12(\max)}=\pi/2$ and $\delta_{12(\min)}=0$; while for a line between one-half and a full wave length, $\phi_b=-\pi/2$, and therefore $\delta_{12(\max)}=-\pi/2$ and $\delta_{12(\min)}=-\pi$. For a line of exactly one-half wave length, ϕ_b is indeterminate for a line of zero resistance, since $Z_0\sinh\theta=0$.

With resistance in the line, the operating range of δ_{12} for stability with power transferred is determined from ϕ_b.

If ϕ_b lies between 0° and $\frac{\pi}{2}$, $\delta_{12(\max)}=\phi_b$

If ϕ_b lies between $\frac{\pi}{2}$ and π, $\delta_{12(\max)}=\pi-\phi_b$

If ϕ_b lies between π and $\frac{3}{2}\pi$, $\delta_{12(\max)}=\phi_b$

If ϕ_b lies between $\frac{3}{2}\pi$ and 2π, $\delta_{12(\max)}=\pi-\phi_b$

For lines of low resistance, $\delta_{12(\min)}$ will be approximately zero for ϕ_b between zero and π, and approximately π for ϕ_b between π and 2π. The operating range of δ_{12} decreases as ϕ_b approaches π. For $\phi_b=\pi$, there is no value of δ_{12} which will satisfy both equations A-10 and A-11, and therefore no power can be transferred between two synchronous systems of zero impedance at the terminals of the line. This is true also of lines in which ϕ_b differs but slightly for π, as shown in figure 2 in the region of a half wave length line.

The line constants selected as representative for 60-cycle long-distance power transmission are

$x=2\pi fL=0.8$ ohm per mile
$b=2\pi fC=5.2\times10^{-6}$ mhos per mile
$r=0.08$ ohm per mile
$g=0$

Using these constants,

$$Z_0=\sqrt{\frac{z}{y}}=10^3\sqrt{\frac{0.08+j0.8}{0+j5.2}}=10^3\sqrt{\frac{0.804\underline{/84.29}}{5.2\underline{/90°}}}=393\underline{/-2.86}$$

$$\alpha=\alpha_1+j\alpha_2=\sqrt{zy}=10^{-3}\sqrt{(0.08+j0.8)\times j5.2}=10^{-3}\times2.044\underline{/87.14°}=10^{-3}(0.102+j2.04)$$

$\alpha_1=0.102\times10^{-3}$
$\alpha_2=2.04\times10^{-3}$

$$\lambda=\frac{2\pi}{\alpha_2}=\frac{2\pi\times10^3}{2.04}=3{,}080 \text{ miles}$$

$v=f\times\lambda=60\times3{,}080=185{,}000$ miles per second

Surge Impedance Load

If the line is closed at the receiving end through impedances to neutral equal to the surge impedance, $i_r=e_r/Z_0$; and from equation A-1,

$$e_r=e_s/(\cosh\theta+\sinh\theta)=e_s\epsilon^{-\theta}=e_s\epsilon^{-l(\alpha_1+j\alpha_2)}$$

Power at the receiving end in watts =

$$3e_r\cdot i_r=3e_r\cdot\frac{e_r}{Z_0}=\frac{3|e_s|^2}{|Z_0|}\epsilon^{-2l\alpha_1}\cos(\text{angle of } Z_0)$$

$$P_R=\frac{E_1^2\epsilon^{-2l\alpha_1}\cos(\text{angle of } Z_0)}{|Z_0|\times10^{-3}} \text{ in per unit of (kv)}^2 \qquad \textbf{(A-12)}$$

where bars enclosing a quantity indicate its magnitude only.

Appendix B. Stability Limits of 600-Mile Line With and Without Intermediate Stations

This appendix gives results of the study of a 600-mile, 60-cycle, conventional transmission line, with and without intermediate system or synchronous capacity located along the line. Included also are curves showing the results of series line reactance compensation by means of series capacitors, and line charging compensation by means of shunt reactors, compensation being assumed uniformly distributed.

Figure 7(*a*) shows the sending and receiving end active and reactive power, and the maximum voltage along the line at different angular displacements between the two ends of the line for the conventional line with terminal voltages equal to rated voltage and no intermediate station. Reactive power is positive when the equivalent synchronous machines at the line terminals are overexcited and negative when underexcited. Some overvoltage exists along the line for all angular displacements up to 70 degrees. This overvoltage can be corrected either by shunt reactors along the line, or by the reduction of sending end or receiving end voltage or both. The vertical lines indicate the angles at which maximum receiver power is obtained with stability for different terminal reactances, equal at the two ends of the line and of the magnitudes indicated based on rated voltage and kilovolt-ampere numerically equal to $(\text{kv})^2$. It will be noted that maximum receiver power is 2.4 $(\text{kv})^2$ and occurs at an angular displacement of 87.5 degrees. Steady-state stability at this angular displacement requires zero terminal impedances.

If the line voltage is 230 kv and the sending and receiving end reactances are each 20 per cent based on 100,000 kva and rated voltage, these reactances based on a kva $=(230)^2=52{,}900$ kva are

$$x_G=x_M=\frac{0.20\times 52{,}900}{100{,}000}=0.106 \text{ per unit, or } 10.6 \text{ per cent}$$

The maximum receiver power with 230 kv and these terminal reactances from figure 7*a* is 2.15 $(230)^2=114{,}000$ kw. In a similar manner the reactive power requirements may also be determined. The maximum voltage along the line at this load is about five per cent above normal.

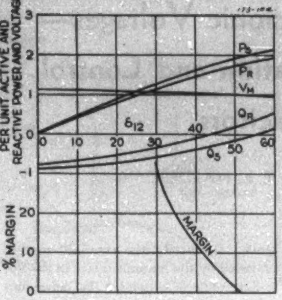
At the bottom of figure 7*a* is a stability margin curve which may be used for determining the line angle and the corresponding line condition of active and reactive power and voltage along the line for a given stability margin. This margin curve corresponds to the condition of $x_G=40$ per cent and $x_M=25$ per cent based on the kilovolt-amperes at the sending and receiving ends of the line, respectively.

Figures 7*b* and 7*c* are similar to figure 7*a* except that they are for one and two intermediate systems, respectively. The reactances of the sending end, receiving end, and intermediate systems are all assumed equal. The curves also give the reactive kilovolt-ampere requirements of the intermediate system or systems for the transmission of power. It will be noted that although with intermediate systems the stability limits are greater, increases in loading above that corresponding to about the unity power factor loading of the line require large amounts of reactive kilovolt-amperes from the intermediate system or systems for the transmission of power. Accordingly, the transmission of power over conventional lines, even with intermediate stations does not necessarily allow for appreciable increases of the line loading above that obtained with existing transmission.

Figures 8*a* and *b* are similar to figures 7*b* and *c*, respectively, except the sending and receiving end reactances are kept constant corresponding to 40 per cent based on 2.5 $(\text{kv})^2$ [16 per cent based on $(\text{kv})^2$] and 25 per cent based on 2.5 $(\text{kv})^2$ (10 per cent based on $(\text{kv})^2$), respectively, while the intermediate system reactance is varied.

For purposes of comparison, figures 9*a* and 9*b* are included to show the effect of series line reactance compensation which may be obtained with series capacitance. Figure 9*a* is for equal sending and receiving end voltages, while figure 9*b* is for a receiving end voltage 85 per cent of the sending end voltage. As will be noted, equal voltage results in a somewhat higher power limit.

Figure 10*a* shows the effect of line charging compensation by the use of shunt reactors; figure 10*b* the effect of both series and shunt compensation. The compensation of the line charging current results in a slight reduction of the power limit. As shown by figures 4 and 5, line charging current is beneficial in increasing the power limit although it would necessarily require compensation at light loads.



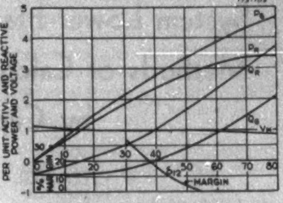


Figure 10. Characteristics of 600-mile conventional 60-cycle transmission line with shunt compensation infinitely distributed

Base kva = base kw = $(\text{kv})^2$

$E_1=E_2=1.0$

Reactance of terminal equipment at sending end = 40 per cent on sending end kilovolt-amperes

Reactance of terminal equipment at receiving end = 25 per cent on receiving end kilovolt-amperes

(*a*) (left)—Line charging capacitance one-half compensated, line reactance uncompensated

$x=0.8$ ohm per mile

$r=0.08$ ohm per mile

$b=(5.2-2.6)10^{-6}=2.6\times10^{-6}$ mhos per mile

(*b*) (right)—Line charging capacitance three-quarters compensated and line reactance one-half compensated

$x=0.8-0.4=0.4$ ohm per mile

$r=0.08$ ohm per mile

$b=(5.2-3.9)10^{-6}=1.3\times10^{-6}$ mhos per mile

References

1. First Report of Power System Stability, AIEE Transactions, volume 56, 1937 (February section), pages 261–82.

2. Equivalent Reactance of Synchronous Machines, S. B. Crary, L. P. Shildneck, and L. A. March. AIEE Transactions, volume 53, 1934 (January section), pages 124–32.

3. U.S.A. Patent No. 1,825,624, issued September 29, 1931 (to C. A. Nickle).

4. The Application of Hyperbolic Functions to Electric Engineering Problems, A. E. Kennelly. McGraw-Hill Book Company.

5. Power Limits of Synchronous Machines, Edith Clarke and R. G. Lorraine. AIEE Transactions, volume 53, 1934 (1933 Supplement), pages 1802–09.

System Lower-Harmonic Voltages—Methods of Calculation and Control by Capacitors

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I. Introduction

A LARGE capacitor bank recently installed at Winchester, Virginia, is supplied with a series reactor to tune it to the fifth harmonic of the power frequency. This is the pioneer installation in the United States of this novel means of reducing the fifth harmonic voltage over a broad expanse of system—33 kv in this case. The paper defines the field of application for this arrangement in comparison with other better known methods of harmonic control.

A technique of system study at harmonic frequency has been developed, based on these findings. Analysis of measurements of lower system harmonics on power systems under varying conditions has shown their chief source to be transformer magnetizing current when power rectifiers are not present. This makes possible analytical determination of the harmonics of interest in the application of shunt capacitors. It involves first, the laboratory determination of the internal voltage and reactance comprising the transformer equivalent circuit, second, the setup of the actual system on the a-c network calculator, using the equivalent circuit, and third, field tests to verify the findings of the network calculator studies, and to prove the adequacy of the tuned capacitor installation. This method of analysis is presented together with confirming tests.

II. Treatment of the Problem

In recent years the price of shunt capacitors has been periodically reduced by the manufacturers. These price reductions have greatly increased the possible savings obtained by the installation of such capacitors by either the power companies or their customers. The result is that the number of capacitor installations is continually increasing, and since the capacitors afford a low impedance path to harmonics present on the power system, due precautions must be taken to prevent their being overloaded from harmonic currents. Ordinarily the standard margin provided by the manufacturers in the up-to-date units is ample for the harmonics encountered. However conditions may arise in which overloading occurs from harmonics, particularly of older units, not provided with present day margins. As the number of capacitors on any system is increased, the general harmonic condition may become worse, and old installations as well as new ones may, under certain conditions, begin to give trouble in the form of fuse blowing, overheating, and excessive noise.

Source of Harmonics

Transformers are the principal source of the lower system harmonics, unless large rectifiers are present. Due to the characteristics of core iron, the exciting current must contain considerable harmonics in order to produce a sine wave voltage. The transformers supply the necessary harmonic voltage to circulate the required harmonic currents. When a low impedance external path is available, the harmonic voltage appearing at the transformer terminals is low. In this respect the transformers act much like constant current harmonic generators when the applied fundamental voltage remains constant. For the purpose of system study they can also be considered as harmonic generators of relatively high voltage and high internal impedance. The harmonic exciting currents decrease in magnitude very rapidly with frequency. The second harmonic and multiples of it are not generated within the transformer, unless d-c saturation is present, while the third harmonic and multiples of it are provided with low impedance circuits by transformers connected in delta. The fifth harmonic is therefore the one most likely to cause trouble to capacitors. Only in rare cases of sharp resonance will the seventh be bothersome.

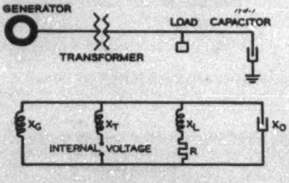

Figure 1. Elementary impedance diagram

Equivalent circuit of transformers connected to system showing possibilities of resonance at harmonic frequencies

The required exciting current of a transformer varies greatly with the applied voltage, a ten per cent overvoltage approximately doubling it. It is therefore important to prevent light load system overvoltage when a harmonic problem is encountered. Considerable improvement may be obtained by a change in the system voltage schedule.

Resonance Conditions

On a normal power system the load and generator impedances are low enough to maintain the harmonic voltage at low level when the system is loaded. At light load periods these impedances become higher and this will result in a considerable change in the harmonic voltage. As capacitors are added to a system a point will be reached where the capacitive reactance will equal the inductive reactance of the generators and load, for the harmonic involved. A condition of parallel resonance[3] then exists on the system and there is no available low impedance circuit for the transformer exciting current of that particular frequency. The result is a high harmonic voltage prevalent over the system.

A further increase of capacitors on the system will place a capacitive load on the transformers. A point may then be reached where the capacitive reactance of the system will equal inductive reactance of the transformer internal impedances. This is a condition of series resonance and the system harmonic voltage may be greater than the transformer internal voltage. In both cases of resonance the system load causes a loss in the tuned circuits and limits the voltage rise. As shown on figure 1, parallel resonance occurs when X_L and X_G in parallel numerically equals X_c. Series resonance exists when X_L,

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The authors wish to acknowledge the assistance of J. E. Barkle in making many of the laboratory tests and calculations.

[1] For all numbered references, see list at end of paper.

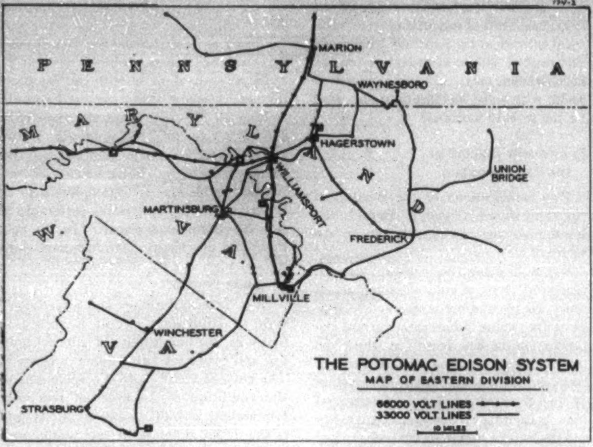

Figure 2. Diagram of Potomac Edison System

X_G, and X_c in parallel are capacitive in nature and equal to X_T.

THE POTOMAC EDISON PROBLEM

On the eastern division of the Potomac Edison System the fifth harmonic voltage was found to reach a peak of 12% of the fundamental; a condition which was general over the entire 33 kv network of approximately 450 circuit miles. Under such a harmonic voltage condition many small capacitors were overloaded with fifth harmonic current. Standard capacitors are designed to stand 135% of rated kva without overheating. Allowing for 10% overvoltage which loads the capacitor to 121% rating, 14% capacity will remain for harmonics. At the fifth harmonic the $\% \text{ kva} = 100\left(\frac{E \text{ harmonic}}{E \text{ rated}}\right)^2 \times 5$ which means the allowable 14% kva will be exceeded when the fifth harmonic voltage exceeds 17%, assuming other harmonics to be negligible. The local rise in voltage due to the leakage reactance of the transformer bank supplying the capacitors frequently increased the 12% harmonic voltage applied to them so that it was far in excess of the safe value. At these locations it was impossible to maintain the capacitors in service due to overheating or fuse blowing.

Table I. Current Wave Analysis Calculated From D-C Saturation Curve for Sine-Wave Flux

Using Fischer-Hinnen Method—All at 104 Per Cent Voltage

Steel No.	Harmonic as Per Cent of RMS					
	1	3	5	7	9	11
1	78.1	55.3	26.2	12.1	3.88	
2	80.0	49.1	29.8	16.1	7.57	3.32
3	82.2	49.2	25.8	12.0	3.58	0.5
4	85.3	45.2	24.6	10.7	4.26	0.935
5	80.2	51.1	27.7	12.6	3.86	

TUNED CAPACITOR

It is possible to place a short circuit to the fifth harmonic at some point on the system by means of a capacitor and series reactor adjusted to resonate at the harmonic frequency. This provides a filter, or circuit of low impedance, and it may be assumed that a large percentage of the system fifth harmonic current will flow through it. These currents can be determined approximately by a very simple method and the proper rating for the capacitor and reactor readily determined.

The maximum possible fifth harmonic current that will flow through the filter is the total fifth harmonic exciting current of the system transformers. Power transformers will on the average require a fifth harmonic exciting current of approximately 0.75% of the full load transformer current when operating at rated voltage. Allowing for some over-excitation, tests have shown that a value of 1% is a close approximation. Thus a system with 100,000 kva of connected transformers would supply $\frac{100{,}000 \times 1{,}000}{2{,}300 \times 1.73 \times 100}$ or 250 amperes of fifth harmonic on a 2,300-volt fundamental frequency base. A capacitor operating with a maximum of 7% overvoltage will be loaded to $(1.07)^2$ or 115% of its rated kva at 60 cycles. Thus 135%–115% or 20% of its rated capacity is available for harmonics. A capacitor's impedance at the fifth harmonic is $^1/_5$ of its impedance at 60 cycles. At fifth harmonic the % kva is $100\left(\frac{I \text{ harmonic}}{I \text{ rated}}\right)^2 \times 0.2$, or is 20% kva when the fifth harmonic current equals the rated current. This means that a capacitor can be operated at 7% overvoltage and still carry 100% of its rated current in fifth harmonic current. A current of 250 amperes at 2,300 volts is the current rating of 1,000 kva capacitor. Thus if the filter is so designed that the fundamental voltage rise through the series reactance does not apply more than 107% voltage to the capacitor, a 1,000 kva capacitor will safely carry all the harmonic current supplied by a system of 100,000 kva transformer capacity. This provides some margin in that not all the harmonic will flow through the filter. The 2,160 kva tuned capacitor described in this paper is applied on the Potomac Edison System having 250,000 kva total connected transformer capacity.

INSTALLATION AT WINCHESTER

Of the various possible remedial measures which were considered for lowering the harmonic voltage, it was decided to use the tuned shunt capacitor as outlined above. Figure 2 shows the extent of the lines comprising the Potomac Edison System and figure 4 shows the fifth harmonic voltage obtained at the Winchester Substation before any remedial measures were applied. Figure 3 illustrates the installation at Winchester. A 2,160 kva capacitor is connected through an 0.81 millihenry reactor to the 4 kv station bus.

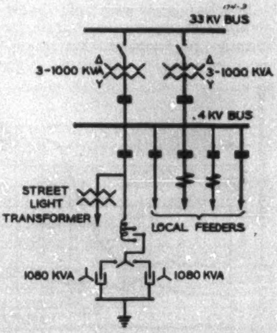

Figure 3. Tuned capacitor installation at Winchester

Table II. Fifth Harmonic Impedances of Loads

Feeder Description	Time	Voltage RMS	Amps. RMS	Fifth Harmonic Impedance, Per Cent on Load Kva Base
Woolen mill-industrial-weaving	Night	2,490	70	53
Commercial-industrial-domestic	Night	2,490	60	68
Commercial-industrial-domestic (unbalanced)	Night	2,490	100	115
Industrial and domestic, 300-hp synchronous motor included	Night	2,490	90	84

Normally, a 0.31 millihenry tap is used, which, in conjunction with the two transformer banks, tunes the complete capacitor to the 33 kv bus. The reduction in the harmonic at Winchester as a result of this measure is indicated in figure 4. It will be noted that the harmonic on the 33 kv system is reduced to approximately 1%. Improvement is also obtained in the harmonic voltage at other points of the system as a result of this installation. Figure 5 shows the harmonic voltage at Frederick, about 50 miles distant, with the filter at Winchester on and off. The results obtained are closely in agreement with the predictions from the analytical study to be described subsequently.

III. Analytical Studies

It was established by field tests on the Potomac Edison System that the fifth and seventh harmonic voltages and currents arose almost entirely from transformer magnetizing current.

In order to study analytically the performance of the system at these frequencies, to determine what remedial measures might be most suitable, it was necessary first of all to represent all the system elements of importance in an electrical network at the harmonic frequency. The method will be described as for the fifth harmonic only. The same procedure could, with some modifications, be used for the seventh harmonic.

Equivalent Circuit of the Transformers

Two modifications of the equivalent circuit are shown in figure 7. In each, the transformer magntizing branch is the source of harmonics.

Figure 7*b* shows the "constant current generator" type of equivalent circuit in which the transformer is assumed to supply a fixed value of harmonic current depending on the flux density at which the transformer is operating. The amount of harmonic current supplied to the system by any given transformer is determined from its exciting current and flux density and the use of a harmonic analysis curve,[7] which shows what proportion of the total exciting current is fifth harmonic. In the absence of data on the specific transformers, the fifth harmonic current from figure 8 may be used. This is expressed as a percentage of the exciting current of the transformer at rated voltage and is based on usual flux densities. This type of equivalent circuit can be used on a network calculator by setting up a voltage source and impedance for the magnetizing branch and adjusting one or the other until the desired current is obtained.

It has been shown,[3] however, that the harmonic current is not a constant for a given density. The curves of figure 9*a* show the effect of introducing reactance in series with the transformer while maintaining the average of applied voltage constant, at the transformer terminals, or while maintaining the fundamental frequency component of exciting current constant.

Figure 9*b* shows the reduction of fifth harmonic current as reactors are introduced, causing an increase in the terminal fifth harmonic voltage. This curve may be used as a criterion for determining whether the "constant current generator" type of equivalent circuit can be used. The curves show that a system harmonic voltage of 5% will cause the current to fall off 15 to 20%. Conversely, it will rise 15 to 20% if the system is capacitive at this frequency. Since in case of harmonic trouble the voltage is likely to be higher than 5%, it appears preferable to use the more exact circuit. In the case described previously, fifth harmonic voltages as high as 12% were encountered on the main system.

The "internal voltage and impedance" type of equivalent circuit is shown in figure 7*c*, the magnitudes of internal voltage and reactance being given in figure 8. The current value plotted corresponds to the condition of no external reactance (equivalent circuit short circuited) and is the upper envelope of test points on a number of transformers in the laboratory and in the field. These constants are dependent solely on the iron. The voltages are expressed in per cent of the normal fundamental frequency voltage corresponding to 100% flux density. The reactances are expressed in per cent on the rated voltage excitation kva base. For example, a 3,000 kva 33,000 volt transformer bank having 4% exciting current, has a rated voltage excitation kva of 120 kva. The equivalent circuit for fifth harmonic has, at normal density an internal voltage of 27% of $33{,}000/\sqrt{3}$ or 5,150 volts, line-to-neutral. The internal reactance is 150% on 120 kva or 13,500 ohms.

$$\text{ohms} = \frac{10\times\%\times\text{kv}^2}{\text{kva}} = \frac{10\times150\times\overline{33}^2}{120} = 13{,}500\text{ ohms}\Big)$$

The transformer rated exciting current is $0.04\times\frac{3{,}000}{33\sqrt{3}} = 2.1$ amps. It will be noted

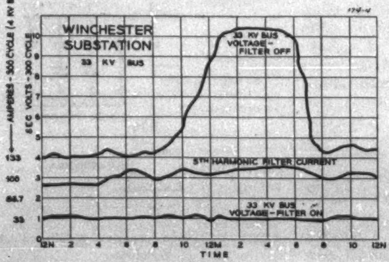

Figure 4 (left). Fifth harmonic voltage and current at Winchester

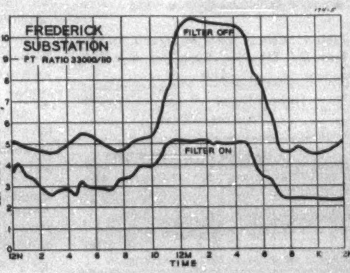

Figure 5 (right). Fifth harmonic voltage at Frederick

(A remote point on the system)

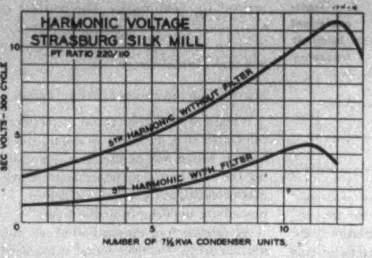

Figure 6. Harmonic voltage at Strasburg Silk Mill

that for zero system impedance the fifth harmonic current is 18% of 2.1=0.38 amp. Or it may be obtained as 5,150 volts÷13,500 ohms=0.38 amp.

At 110% of normal voltage the internal voltage becomes 37% or 7,100 volts, line-to-neutral and the reactance 70% on 120 kva or 6,300 ohms.

Equivalent Circuit by Test

The values of constants for the equivalent circuit, may be determined from test on a sample transformer using the same kind of iron, or may be calculated from saturation curves of the iron.

The circuit for the test method is shown in figure 9. A variable voltage source operating at rated frequency is required together with variable reactance between the source and the transformer under test. By varying the reactance in series with the transformer the internal reactance at a particular frequency can be calculated as the quotient of increase in terminal voltage divided by decrease in current of that frequency. The internal voltage can then be obtained as the terminal voltage plus the drop through the internal reactance just calculated, or may be read as the intercept with the voltage axis of the test curves of figure 9*b*. The reactance is the quotient of voltage and current intercepts.

The flux density should be maintained constant (rather than rms voltage) as the series reactance is varied. This is for the reason that radically different exciting currents may be obtained with the same rms voltage of different wave forms. For the same flux density the crest exciting current is the same, and also it can be shown that the average voltage is the same. Instruments are therefore required to hold crest exciting current or average voltage constant in these tests. The former is preferred because of the rapid variation of exciting current with flux density.

Harmonics Not Independent

Seventh harmonic test points have been plotted also in figure 9*b* and for the higher voltage the function is seen to be double valued. The reduction in seventh harmonic current is obviously not due alone to the increase in terminal harmonic voltage, since it continues to decrease even as the seventh harmonic voltage decreases, but as the fifth harmonic voltage is building up. This illustrates the somewhat expected fact, that the harmonics are not entirely independent as assumed in using the equivalent circuit. For this reason all of the tests were made in a three-phase circuit with the transformer having one delta connected winding, thereby providing a local path for triple harmonics and more closely simulating actual operating conditions.

Analysis From D-C Saturation Curves

The equivalent circuit may also be obtained direct from the saturation curve by analyzing, *first*, the exciting current wave resulting from pure sine wave flux; *second*, the voltage wave resulting from the flux wave which results from pure sine wave exciting current. Obviously, then, irons having the same saturation curves will have identical circuits at harmonic frequencies. Table I shows the harmonic current analysis from the saturation curves of five different irons employed in transformers now or in the past. The conclusion is drawn that for silicon steel the equivalent circuits do not vary widely so that the circuit developed from a sample transformer may be applied quite generally on a system.

Loads

For the fifth harmonic impedance of typical substation loads composed of a mixture of lighting and motor load, analytical estimates were based on 100 kva of load composed of 70 kva of lighting and other static load at a pf. of 95% and 30 kva of motor load consisting of 60 kva of induction motors running at 50% of full load. This combination reduced to its fifth harmonic impedance becomes $(9.3+j66.7)\%$ at 300 cycles on the load kva base. A limited number of field tests have been made the results being recorded in table II. The night time readings are in fair agreement with the theoretical value used in the studies. Caution must be used in the use of average load figures for individual resonance cases. For overall system studies of a network at 33 kv or higher, the resulting harmonic voltages are determined by the average of a large number of loads, and the average values of load impedance may be more safely used.

Generators

The D. and R. subcommittee[6] of EEI and Bell systems have conducted tests on a large number of generators showing their harmonic impedance, referred to fundamental frequency to be approximately 80% of the negative sequence reactance. This report also gives methods

Table III. Self and Mutual Drops for 35 Per Cent Load, Potomac Edison System, Ohms at 300 Cycles, on 33-Kv Base

Drop to	1 Union Bridge	2 Wmspt. 33	3 Millville 33	4 Berryville	5 Warren	6 Winchester	7 Martinsburg	8 Hedgesville
Open Circuit Voltage L–N Kv	2.08-j1.43	1.73-j1.32	2.08-j1.70	2.25-j2.36	2.78-j2.72	2.85-j2.95	2.18-j1.87	2.00-j1.70
For Unit Load at								
1. Union Bridge	85.6+j168.0	23.2+j29.6	28.8+j36.8	39.2+j39.6	42.8+j39.6	47.2+j40.0	30.8+j34.8	26.4+j32.8
2. Wmspt. 33	25.2+j30.0	33.2+j38.8	32.8+j34.8	44.4+j39.6	51.2+j42.4	55.2+j43.6	37.6+j38.8	34.4+j37.6
3. Millville 33	32.8+j34.4	31.6+j34.0	50.8+j57.6	61.6+j59.6	69.2+j60.4	71.6+j58.0	46.4+j43.6	41.6+j41.6
4. Berryville	41.6+j37.6	42.4+j38.8	60.0+j61.6	95.6+j114.8	103.2+j108.8	103.6+j94.0	62.4+j54.4	55.6+j49.2
5. Warren	47.2+j38.0	49.6+j40.8	68.8+j60.4	105.2+j105.2	130.4+j133.6	124.4+j112.8	73.2+j57.6	65.2+j52.0
6. Winchester	46.4+j39.2	51.6+j43.6	68.4+j60.4	98.8+j93.6	116.4+j116.4	138.8+j135.2	76.4+j62.4	68.4+j56.4
7. Martinsburg	32.4+j32.8	36.8+j38.4	46.4+j46.0	64.0+j54.0	73.6+j59.2	80.8+j62.0	57.6+j56.4	49.2+j50.8
8. Hedgesville	28.8+j31.6	31.6+j38.0	39.6+j43.6	54.8+j50.8	63.6+j55.6	69.6+j58.0	46.4+j52.4	

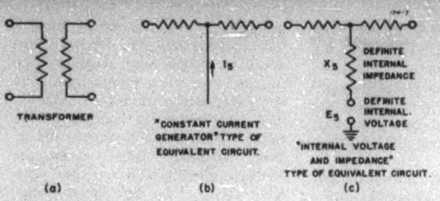
174-7
TRANSFORMER
I_5
"CONSTANT CURRENT GENERATOR" TYPE OF EQUIVALENT CIRCUIT.
X_5
DEFINITE INTERNAL IMPEDANCE
E_5
DEFINITE INTERNAL. VOLTAGE
"INTERNAL VOLTAGE AND IMPEDANCE" TYPE OF EQUIVALENT CIRCUIT.
(a)
(b)
(c)

Figure 7. Transformer equivalent circuits at harmonic frequency

for determining the harmonic impedances of various other system elements.

LINES AND TRANSFORMERS

The methods of determining line and transformer impedances at various frequencies are well known.[5]

NETWORK CALCULATOR STUDIES

Using the methods outlined above, the Potomac Edison System was represented on the a-c network calculator on a fifth harmonic basis.

The "constant current generator" type of equivalent circuit was first used. Although it indicated correctly some of the trends as system changes were made, it proved unsuitable for quantitative results because large system harmonic voltages were developed in the conditions studied. This type of representation is quite inaccurate in such cases. Also, this method was very tedious since trial and error was required in adjusting all the transformer equivalent circuits to the desired current, in phase and magnitude.

The system was later set up using the "internal voltage and impedance" type of equivalent circuit for the transformers. Four conditions were studied corresponding to the light load condition which was taken as a uniform 35%* load on the system. These conditions were:

1. System normal.
2. All capacitors removed except natural capacitance of lines.
3. Normal except 2,160 kva capacitor added at Winchester.
4. Normal except 4,080 kva of capacitors added at three stations.

The results are indicated graphically in figure 10. Some field tests are also plotted on this curve. The light load reading was a single reading and could have been somewhat higher. The network calculator curve was based on all transformers operating at 10% overvoltage except those at generating stations or connected to the 66 kv line. This is somewhat more severe than the actual night-time condition. Since harmonics more than double with a 10% increase in voltage, these two factors appeared to account for the difference between test and calculation. On this basis, the actual characteristic was expected to be a curve as indicated dotted. It is similar in shape to the calculated curve since the shape is determined largely by the system constants.

It had been known previously, from theoretical considerations, that it would be possible for the capacitive paths for exciting current to resonate with inductive paths and result in an increase of harmonic voltage. This study showed quite conclusively that the system was operating in this region, and also provided the valuable information that the addition of an entirely practicable amount of capacity would take the system well "over the hump" and make it immune to further difficulty from fifth harmonic due to further increases in the amount of connected capacity.

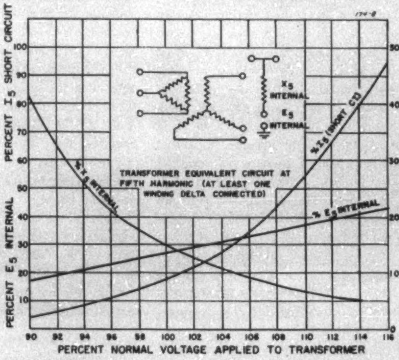

Figure 8. Values of voltage and reactance of equivalent circuit

Based on a normal density of 13,500 gausses

The idea was then conceived of tuning the capacitor to fifth harmonic by means of a reactor in series, and thereby still further lowering the harmonic voltage, a feature which was particularly desirable at neighboring stations where harmonic difficulties were being encountered. The calculating board study showed that the harmonic currents in the capacitor would not be excessive under this condition, a conclusion which has subsequently been verified by field tests. The method of analyzing the effect of the tuned capacitor is given under the heading "Method of Self and Mutual Drops".

METHOD OF SELF AND MUTUAL DROPS

When a system is set up on the network calculator it is desirable to obtain information which will be useful in determining the fifth harmonic performance under other conditions than those which are of immediate importance at the time.

An extension of the method of self and mutual drops[2] has been found convenient for this purpose. In this case the self and mutual drops Z_{11}, Z_{12}, etc. are defined as follows:

Z_{11} is the drop in voltage at point 1 per unit of current drawn from the network at point 1.

Z_{12} is the drop in voltage at point 2 per unit of current drawn from the network at point 1.

For the Potomac Edison network, for example, eight principal load points need to be considered. Nine sets of voltage readings are required. First, the voltages E_1', to E_8' at the 8 load points must be read as a basis for determining drops. Then with load terminal, 1, short circuited the current, I_1, and voltages at the other 7 points E_{12}, E_{13} to E_{17} are read. This is repeated in turn until all 8 points have been short circuited, one at a time. The drops are then computed

$$\left.\begin{aligned} Z_{11} &= \frac{E_1'-E_{11}}{I_1};\ Z_{12}=\frac{E_2'-E_{12}}{I_1};\ \text{etc.} \\ Z_{21} &= \frac{E_1'-E_{21}}{I_2};\ Z_{22}=\frac{E_2'-E_{22}}{I_2};\ \text{etc.} \end{aligned}\right\} \quad (1)$$

As a check

$$Z_{21}=Z_{12},\ \text{etc.} \quad (2)$$

Table III shows the resulting self and

---

* 35% of single step-down transformer capacity.

mutual drops for the Potomac Edison System. It also shows the open circuit voltages E_1' and E_6' for the 35% load condition.

As an example of the use of this method, let the effect of connection of various values of impedance at load point, 6 (Winchester), be determined. In particular what are:

(*a*). The fifth harmonic current and voltage at point 6 (Winchester), and

(*b*). The fifth harmonic voltage at point 1 (Union Bridge), a typical remote point on the system. Union Bridge is near Frederick for which tests of figure 5 apply.

Figure 11 shows the current and voltage at Winchester and voltage at Union Bridge obtained from the following relations:

$$I_6' = \frac{E_6'}{Z_{66}+Z_6} \qquad (3)$$

$$E_6'' = E_6' - I_6'Z_{66} \qquad (4)$$

$$E_1'' = E_1' - I_6'Z_{61} \qquad (5)$$

Z_6 is the reactance X, figure 11.

The following information is directly forthcoming from these curves.

(1). The fifth harmonic current drawn by the capacitor will be less if it is tuned to this frequency than if no reactor is used.

(2). The harmonic current will be within the rating of the capacitor, whether tuned by the reactor or not, provided the entire bank is in service. With only half of the bank in service with one or two transformers a dangerous current may flow, and high harmonic voltages occur.

The voltage reduction at Union Bridge (near Frederick) was verified by later field tests. See figure 5.

Field Tests

Transformers were determined to be the principle source of the fifth harmonic by the following considerations and field tests.

1. There were no large power rectifiers on the system.

2. The generator open circuit harmonic voltages were measured by a harmonic analyzer and the fifth and seventh harmonics found to be very low.

3. A large section of system was supplied over a single transmission line supplied from a single transformer and the transformer taps increased over a 10% range thereby over-exciting all the transformers on the supplied system by 10%. It was found that the fifth harmonic voltages throughout the system approximately doubled for the 10% increase in voltage, measurements being made at various points both during the increase and decrease of taps. Since the exciting currents of transformers are known to approximately double for 10% increase in voltage this gives rather pronounced evidence that fifth harmonic voltages were associated with the exciting currents.

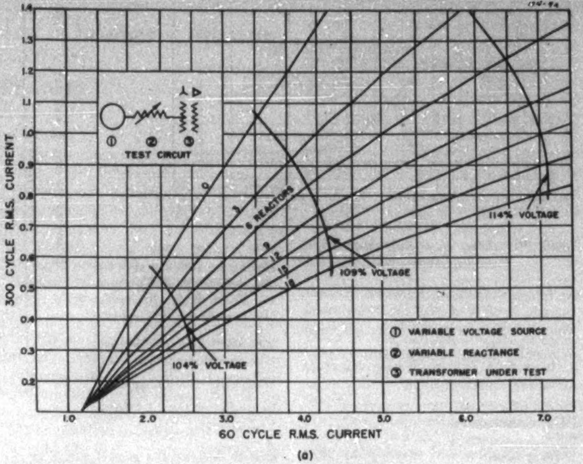

Figure 9a (above). Variation of fifth harmonic current with reactance in series with transformer

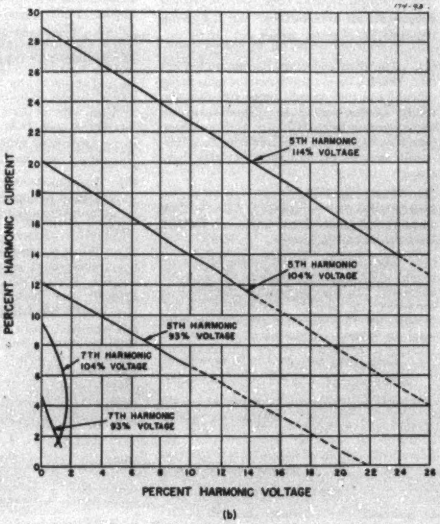

Figure 9b. Reduction of fifth harmonic current with increase in fifth harmonic terminal voltage as reactance in series with the transformer is increased

After the installation had been made further field tests were run to determine the effect on harmonics.

These tests were made at various points on the system and showed that the harmonic voltage was greatly reduced when the Winchester filter was in service. Figures 4 and 5 are the results of these tests. A test was also made at one point of local resonance (Strasburg Silk Mill, figure 6) and it was found that such conditions were also considerably reduced. Harmonic current tests were made on all capacitor banks which had been in trouble and it was found that they were now operating with a safe margin.

Figure 4 shows the current in the capacitor at Winchester over a period of time, including probably the most severe that will be encountered. The maximum

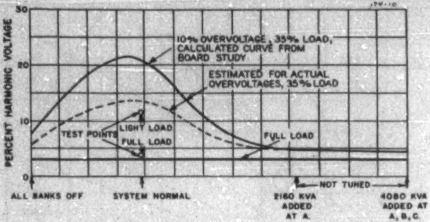

Figure 10. Effect of adding capacitors to Potomac Edison System

current is 40% of the capacitor rating which compares with the estimated value of 47 per cent.

Other field tests have been described previously in the paper.

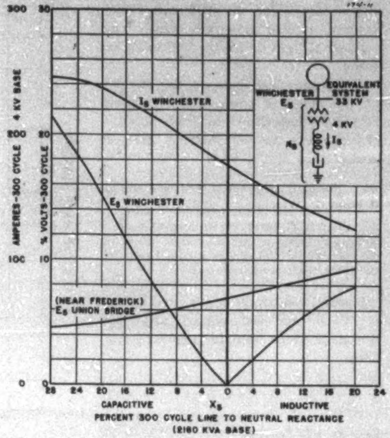

Figure 11. Fifth harmonic current and voltage at Winchester and Union Bridge for various filter impedances at Winchester

Calculated 300-cycle voltages with filter disconnected

Winchester 21.6 per cent

Union Bridge (near Frederick) 13.2 per cent

IV. Conclusions

As a result of this investigation and the subsequent field tests on the tuned capacitor at Winchester, the following conclusions may be drawn:

(1). The fifth and seventh harmonics are due largely to a transformer magnetizing current and may be predetermined for particular system conditions. On this system there were no large six-phase or three-phase rectifiers connected which might be additional sources of fifth and seventh harmonics.

(2). The installation of a large capacitor tuned to the fifth harmonic is a practicable measure for reducing the harmonic voltage level throughout a system.

(3). The capacitor tuned to the fifth harmonic will be net inductive at the seventh harmonic. In the case studied this lowered somewhat the seventh harmonic voltages. In general, the effect on other harmonics than the fifth would not be large.

(4). The analytical methods developed for study of the harmonic problem adequately describe the general nature of the phenomena and give results suitable for making applications and predicting the resulting effects throughout the system. These methods are not limited to the study of this particular type of remedial measure.

(5). Tuning to the fifth harmonic is feasible whenever the capacitor bank represents at least 1% of the total connected transformer capacity. It may also be feasible for considerably smaller amounts, but in this range each case should be studied.

(6). Harmonic voltage reductions as much as 50% may be obtained at parts of the system remote from the tuned capacitor installation.

(7). When a tuned capacitor makes use of the transformers as part of the series reactance, voltage rise at the harmonic frequency may occur at the transformer low-tension bus, depending on what proportion the transformer reactance is of the total reactance. In this instance, the proportions were such that the harmonic voltage on the 4 kv bus was lower during the night with the capacitor in service than out of service; however, the possibility of harmonic voltage increase should be borne in mind in considering other applications.

References

1. HARMONIC ANALYSIS OF OSCILLOGRAMS, **G. H. Cole.** *Electric Journal*, volume 15, number 7, July 1918, page 258.

2. ANALYTICAL SOLUTION OF NETWORKS, **R. D. Evans.** *Electric Journal*, volume 21, number 5, May 1924, page 207.

3. EXPERIMENTAL STUDY OF THE POSSIBILITY OF FIFTH AND SEVENTH HARMONIC RESONANCE IN TRANSFORMERS, **A. Boyajian.** *General Electric Review*, volume 29, number 3, March 1926, page 175.

4. TRANSFORMER ENGINEERING, **Blume.** (A book), Chapter 3.

5. *Engineering Report number 23* of the Joint Subcommittee on Development and Research of the Edison Electric Institute and Bell Telephone System.

6. THE EFFECT OF OVEREXCITATION OF TRANSFORMERS ON THE SYSTEM VOLTAGE WAVE SHAPE AND POWER FACTOR, **J. W. Butler** and **E. B. Pope.** AIEE TRANSACTIONS, volume 60, 1941 (February section), pages 49–53.

A Thyratron Circuit for Theater Lighting

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AT present there is a definite need in small theaters, such as those in colleges and universities, for a flexible, highly advanced system of stage lighting control. During recent years such systems capable of supplying the relatively tremendous loads of the large muscial comedy houses—up to about 25 kilowatts individual circuit capacity—have been developed. However, little attention has been paid to the needs of the small legitimate theater, which requires just as much flexibility of light control and perhaps almost as many circuits, but of much smaller individual capacity—perhaps a kilowatt or two maximum. It is with circuits appropriated for this use that the present paper deals. Experimental circuits employing thyratrons as power rectifiers, supplying uni-directional current directly to a lamp load, were developed as part of the investigation and proved satisfactory.

The fundamental problem in stage lighting is that of supplying voltages of proper magnitude to appropriate lamp circuits in the required sequence. Although it may be difficult to decide which single feature is most desirable in a satisfactory theater lighting control system, a number of characteristics are agreed to be essential, and their comprehension lies in a brief analysis of a few typical examples of the operator's task.

As action on the stage progresses, it becomes necessary to increase the voltage on some circuits, possibly by a different amount on each circuit, to reduce that on some, and to allow that on others to remain constant. Presumably the controls of those circuits requiring changes in the same sense, for example those requiring an increase in voltage, may with more or less difficulty, be ganged mechanically. A moment later the voltages applied to all circuits in use may require other changes, both voltage increases and decreases of various magnitudes, irrespective of either the sense or magnitude of the preceding change. Hence, any grouping of circuits on the basis of changes required at any particular moment must suffer continual revision. With mechanical ganging this is not practicable if the changes are to occur in rapid succession.

The most satisfactory solution to date is embodied in the "preset" type of control, in which predetermined values of lamp voltages for a large number of circuits are selected in sequence. Let us suppose that a given circuit must operate at four different voltages in four successive scenes. The circuit is equipped with as many control handles, called "presets." With some sort of selector in position number one, the setting of preset control handle number one determines the lamp voltage for that circuit; the others may be manipulated without effect. Shifting the selector to position number two transfers control of the lamp circuit to preset control handle number two, effecting a gradual change in voltage from the first to the second of the four values as the setting of the selector is changed. By virtue of its function the selector has been given the name "fader." A large number of lamp circuits equipped with presets are operated from a common fader, thereby allowing simultaneous control of changes in voltage in all these circuits. With such a scheme it is possible to reduce the voltage on some lamp circuits while increasing that on others, since the change in voltage on any one circuit during transition depends only upon the setting of the two successive presets. Circuits to be operated at the same voltage in two successive scenes will have presets for those scenes set at the same value. In actual practice the length of time during which a transition from one set of lighting effects to the next occurs depends only upon how rapidly the operator manipulates the fader.

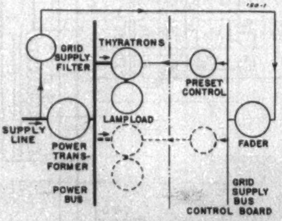

Figure 1. Block diagram showing component employed

From the foregoing, certain characteristics appear to be essential:

(1). A method of presetting in which the requirements of future scenes may be anticipated and the appropriate presets adjusted ahead of time, allowing selection of future lighting effects in rapid sequence if required and with no restriction either as to sense or magnitude of lamp voltage changes required.

(2). Completely centralized control vested in a single operator. (If required, a helper may be employed to adjust presets which are to be used later.)

(3). Transitions between predetermined sets of lighting effects which are smooth and simultaneous in all lamp circuits so controlled.

(4). Constancy during transition from one scene to the next of lamp voltages set for the same value in succeeding scenes.

To those already listed may be added the following requirements:

(5). Location of the control board in the orchestra pit in order that the operator may observe the effects produced. (This requirement places limitations upon the physical dimensions of the board and implies that any associated equipment be placed in some suitable remote location. Furthermore, operation must be noiseless.)

(6). Ease of combination of preset type of control with independent control of individual circuits and with control of a group of such circuits by "masters" where proportionate dimming of the group is required.

(7). Instantaneous response of the lamp voltage to control.

(8). Good voltage regulation for any load within the rated capacity of the circuits and at any voltage setting.

(9). High over-all efficiency.

(10). General simplicity, contributing to dependability of operation and ease of installation and servicing.

The circuit to be described offers all of these desirable features, thereby representing an advancement of the art in at least one important respect. Since its basis is the use of thyratrons as power rectifiers supplying uni-directional current directly to a lamp load, the mathematical analysis of its operation reduces to that of a single-phase full-wave rectifier supplying a non-inductive load.

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The author is indebted to B. R. Teare, formerly of the department of electrical engineering, Yale University, and now of Carnegie Institute of Technology, for his helpful suggestions and encouragement; to H. M. Turner of the department of electrical engineering, Yale University, for his constructive criticisms, to S. R. McCandless of the department of drama, Yale University, for his help in formulating the problem and in criticizing the performance of experimental circuits, and to Eugene Braun and Andrew Herzog of the Radio City Music Hall for their kind cooperation.

1. For all numbered references, see list at end of paper.

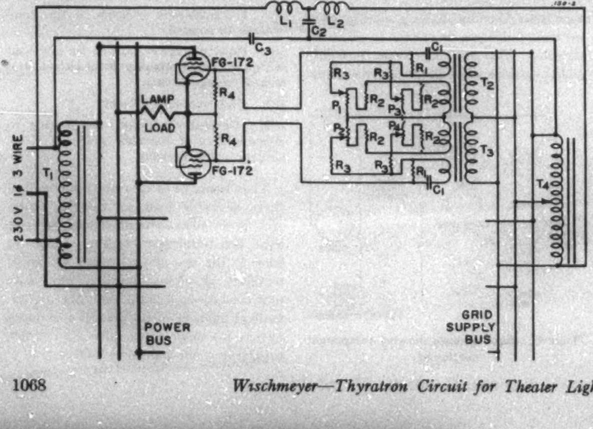
Basic Circuit

The diagram of figure 1 indicates the units required and their functions. The heavy lines represent load power connections and the light lines those of control power. Thyratrons controlled by an alternating grid voltage of variable phase are used in pairs to provide full-wave rectification.

Limitations as to size and power dissipation in control circuit parts seemed to indicate the use of thyratrons,[1,2,3] which inherently permits the control of a large amount of power in the anode circuit by a small amount of grid circuit power, which is independent of load. For any instantaneous value of anode voltage there exists a critical grid potential at which the initiation of arc discharge occurs. When the grid potential is increased from a large negative value to the critical voltage, electrons emitted by the hot cathode acquire sufficient velocity to ionize by collision the mercury vapor within the tube, and the arc is formed. The grid is then unable to stop or modify the discharge, which persists until the anode voltage is decreased below that required to sustain the arc. In the present application the periodic reversals in polarity of the alternating anode voltage continually interrupt the discharge, while the grid controls its restarting, thereby permitting adjustment of the magnitude of anode current.

The effective voltage output of a full-wave rectifier operating from a single-phase three-wire line will be less than the effective line-to-neutral voltage by the tube drop. Hence, the required line-to-line voltage must be slightly greater than twice the desired lamp voltage. Thus, to obtain a lamp voltage of 115 volts from a 230 volt supply it is necessary to provide by means of auto-transformer T_1, figure 2, an additional voltage of approximately 12 volts each side of the neutral. See appendix I for rating of this auto-transformer.

Figure 2. Wiring diagram of one lamp circuit equipped with four-scene preset control



In the interest of simplicity the use of half-wave rectification was considered, but appeared inadvisable because of perceptible flicker with lamps rated at less than about 100 watts.

While the power output of a pair of thyratrons may be increased by increasing the anode voltage, two transformations are necessary, since from practical considerations the lamp voltage must be 115 volts.

Grid-Voltage Phase Control

Grid circuit design comprises two problems: that of producing a control voltage of proper magnitude and phase, and that of incorporating the control elements in a circuit permitting presetting.

Tube characteristics for low anode voltages are shown in figure 3. Figure 4 shows the critical grid potential corresponding to instantaneous values of a sine wave of anode voltage. It should be noted that the angle of lag of grid voltage with respect to anode voltage is not equal to the angle of delay of conduction except at the two points where the critical grid potential curve crosses the axis. Although this difference is small, five degrees or less throughout the entire range of control in the experimental circuits, the analysis to follow is in terms of the angle of lag, account being taken of the increase in the angle of conduction.

Figure 5 shows the variation of effective output voltage with angle of lag for the full-wave grid controlled rectifier. Since at low anode potentials large positive grid voltages are required to produce arc discharge, a range of phase control of grid voltage of approximately 15 to 165 degrees lagging with respect to the anode voltage permits control of lamp voltage from a maximum value to zero. Further it is evident that making the maximum amplitude of the sinusoidal grid voltage large compared to the maximum negative critical grid potential minimizes the effects of changing or slightly dissimilar tube characteristics.

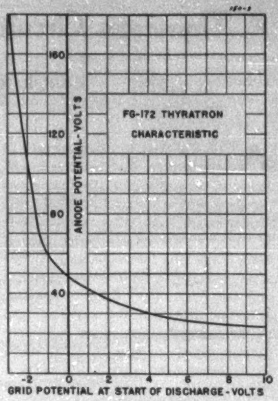


Figure 3. FG-172 thyratron characteristics

Shield grid connected to cathode, ambient air temperature 75 degrees Fahrenheit, heater voltage five volts

A method of obtaining a voltage variable in phase throughout the desired range is indicated in figure 6. The grid voltage E_G is the sum of two components: E_F, the phase and magnitude of which are fixed by the constants of the *R-C* circuit; and E_V, which is in phase with the anode voltage and the magnitude of which is varied by means of the preset potentiometer *P*. The vector diagram of figure 7 shows the phase of the grid voltage for several settings of the preset potentiometer. Although the magnitude of E_G varies with the phase angle, no objectionable effect is encountered provided the minimum peak value is large as compared to the maximum negative critical grid potential.

Presetting

The method of predetermining lamp circuit voltages is based upon the division of the individual preset potentiometers into two groups, energized by a variable

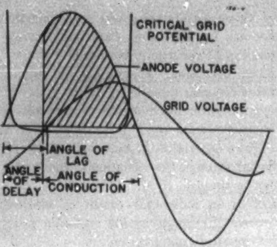


Figure 4. Voltages determining angle of conduction (one tube)

auto-transformer so connected that the sum of the voltages impressed upon the two groups is constant. See figure 8. As the voltage across one group is varied from zero to a maximum that across the other is varied simultaneously from a maximum of equal magnitude to zero, thereby transferring control, or "fading," from a preset potentiometer of one group to one of the other group. Hence, the variable component of grid voltage, E_V, is now the sum of voltages E_{V1} and E_{V2}, shown in the vector diagram, figure 9. The selection of individual preset potentiometers is accomplished by switches, only one potentiometer of a group being in use at a time.

The normal sequence of operation of the four-scene preset control shown is as follows: Suppose preset potentiometers P_1 and P_2 are connected. With the auto-transformer tap at A, figure 8, maximum voltage is applied to P_1 and none to P_2. The magnitude of E_V is then determined by the setting of the tap of P_1, which has been set to give the angle of lag of grid voltage corresponding to the desired lamp voltage in the present scene. The tap of P_2 is set to give the desired lamp voltage for the same circuit in the next scene. When it is desired to make the transition from the first to the second lamp voltage for the circuit, the auto-transformer tap is moved either rapidly or slowly, depending upon the speed with which the transition is to occur, from A to B. Control has now been transferred smoothly to P_2. Preparatory to transferring control or "fading" from P_2 to P_3, P_3 is connected and then P_1 is disconnected. Moving the auto-transformer tap back to its initial position A will then transfer control to P_3. Such a switching sequence is necessary in order not to open the grid circuit, which would result in immediate loss of control of the lamp voltage. The transitions from P_3 to P_4, and from P_4 back to P_1, and so on are made similarly. The capacity for any required number of presettings may be attained simply by providing enough potentiometers. Since only a few milliamperes of grid current are drawn through them, the potentiometers are inexpensive and compact and dissipate little power.

When both P_1 and P_3 are connected while energized, which would result only from a mistake on the part of the operator and which could be prevented by appropriate relays, current in excess of the potentiometer rating tends to flow if the settings of P_1 and P_3 are near opposite ends of the scale. Resistors R_2 and R_3, figure 2, serve to limit this current.

Operating Characteristics

In order to accomplish the subtle blending of the light output of the different lamp circuits, it is essential that all desired intensity changes occur simultaneously and, as a matter of convenience, approximately linearly with auto-transformer fader control, which is possible if the light output of a lamp circuit be made a linear function of the variable component of grid voltage E_V. An analysis of the factors involved shows a satisfactory approximation to be attainable.

The light output for several sizes of lamps is plotted as a function of applied voltage in figure 10, which together with the curve of figure 5, determines light output as a function of the angle of lag of grid voltage, shown in figure 11.

The phase angle of the R-C combination in figure 6 determines the phase of the fixed component of grid voltage E_F, upon which, together with the magnitude of E_F, depends the required range of the variable component of grid voltage E_V to give any resultant grid voltage phase. For the circuits constructed the variation of angle of lag with variable component of grid voltage E_V is shown in figure 12, which together with the curve of figure 11 determines the over-all characteristic of light output versus variable component of grid voltage, figure 13. Between 5 and 90 per cent of rated lumens output the curve approaches a straight line; below 5 per cent the number of lumens output is sufficiently small that the departure from linearity is not objectionable in practice, and above 90 per cent the departure is not noticeable because of the logarithmic response of the human eye.

In order to set the limits between which the light output of a circuit can be operated, means should be provided for adjusting the phase of grid voltage component E_F, which determines the minimum value, and for limiting the maximum value of E_V, which determines the maximum value. The former may be done by adjusting R_1, figure 2, the latter by adjusting the protective resistors R_2. A generally acceptable range of control from 0.03 of one per cent to 99 per cent rated lumen output means limiting control from the equivalent of 28 to 72 per cent E_V, figure 13.

It should be pointed out that dissymmetry of the curve of light output versus variable component of grid voltage E_V about its 50 per cent lumens output point will cause objectionable peaks during transition when the increasing light output from one circuit is added to the decreasing output of another.

As pointed out under essential control characteristics, it is sometimes necessary to maintain constant illumination intensity of one or more lamp circuits while others are varied. The transition from one presetting to another of equal intensity introduces no deviation, because

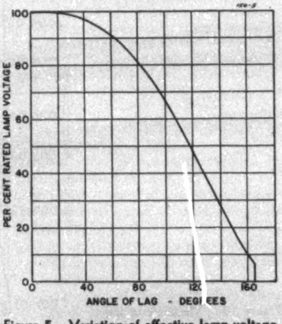


Figure 5. Variation of effective lamp voltage with phase of grid voltage

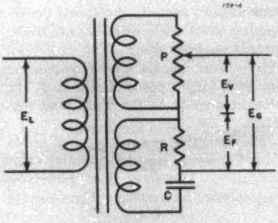


Figure 6. Basic phase-shifting circuit

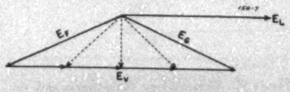


Figure 7. Resultant grid voltage the sum of fixed and variable components

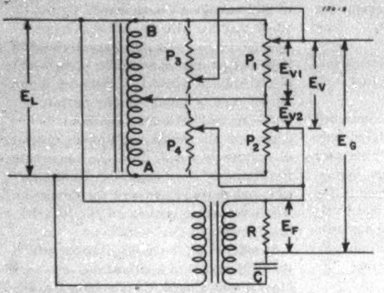


Figure 8. Basic phase-shifting circuit modified to permit presetting

the sum of the voltages applied across the two groups of preset potentiometers is constant. The total light output of two identical lamp circuits initially of illumination intensities *A* and *B* and finally of *B* and *A* respectively remains approximately constant during the transition, as shown in figure 14. The smooth and simultaneous changes in illumination intensity of two lamp circuits for a typical case in which all initial and final intensity settings are different are shown in figure 15.

Comparison With Other Types of Control

At present nearly all small theaters are equipped with the resistance type of dimmer, which has several serious limitations: the lamp voltage for any setting of the dimmer is dependent upon the number and size of the connected lamps; the flexibility with regard to presetting is limited; the size of the control board is necessarily large; and the power dissipated in the resistance dimmers is usually sufficient to produce an objectionably large amount of heat. Replacing the resistance units with variable auto-transformers obviates many of the objectionable features; but the space requirements are still large, power leads must be brought directly to the board, and presetting is limited.

The saturable reactor type of preset control possesses superior operating characteristics. However, since control is accomplished by varying a direct current through an inductive circuit (the d-c winding of the saturable reactor) there is an accompanying time lag, which is definitely objectionable in some instances. Since the reduction in lamp voltage is produced by an impedance drop, some sort of feed-back circuit is required to maintain constant lamp voltage independent of load if calibration of control setting is to be significant.

The preset type of control described in this paper embodies all the characteristics cited as desirable, including rapid response of light output with control, limited only by thermal lag in the lamps. Regulation is approximately that normally encountered with the supply transformers. Tube life of 3,000 hours is guaranteed by the manufacturer, but it is believed that in the present circuit, in which operation is at relatively low anode potentials, considerably better life may be expected. Like the other types of circuits mentioned, the present one may be operated from a three-phase supply. Aside from the division of the circuits into groups for operation from the separate phases, the only change required is the substitution of three auto-transformers mechanically coupled for the fader, each unit having one-third the capacity required for single-phase operation. Essentially the apparatus becomes three independent systems operated from mechanically coupled auto-transformer faders.

Since control of the lamp voltage is effected through the variation of one voltage in the grid circuit, independent control of an individual circuit or master control of several circuits requiring proportionate dimming is easily accomplished in essentially the same way as in resistance dimming, except that with the present thyratron circuit only control voltage need be varied.

When a large load is to be supplied, it is subdivided and connected to pairs of thyratrons the grids of which are operated in parallel for simultaneous control. This method has been found successful and from the user's standpoint is practical, since no individual load unit normally exceeds about one kilowatt.

With a grid controlled rectifier supplying a maximum load of the order of 100 kilowatts, as might be required by a small theater, the problem of telephone interference due to power line harmonics may become serious. For a three-phase installation this would mean a maximum load per phase of 30 to 35 kilowatts. However, a Fourier analysis of line current, the results of which are summarized in appendix II, shows that all higher harmonics present reach maxima for an angle of conduction of approximately 90 degrees, under which conditions a load of 30 or 35 kilowatts per phase at full voltage would be reduced to half. On the basis of the Fourier analysis a telephone influence factor of 1,200 was computed for an angle of conduction of 90 degrees. This

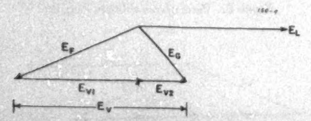


Figure 9. Grid-voltage components and resultant for intermediate setting of auto-transformer tap

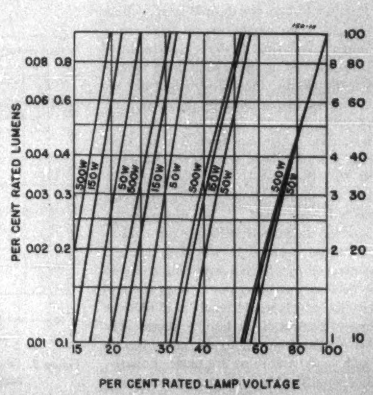


Figure 10. Variation of light output with voltage for several sizes of 115-volt lamps

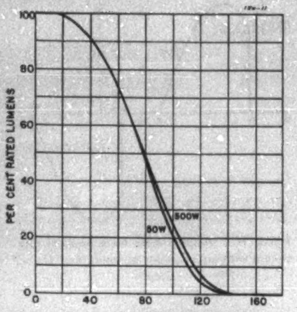


Figure 11. Variation of light output with phase of grid voltage

figure is somewhat high in view of the assumptions upon which the Fourier analysis is based. Furthermore it cannot be taken as representative of actual operation because it implies that all lamp circuits are adjusted to the same voltage. Should the attenuation of line harmonics in a particular installation be necessary, resonant shunts, or a combination of series inductance and shunt capacitance forming a low-pass filter, or series inductance in conjunction with resonant shunt capacitance may be employed.[5]

To prevent distortion of grid voltage wave form by line voltage harmonics a single T-section low-pass filter of cut-off frequency slightly above 60 cycles was interposed between the supply line and the auto-transformer fader. Condenser C_3, figure 2 compensates for the phase shift introduced by the filter.

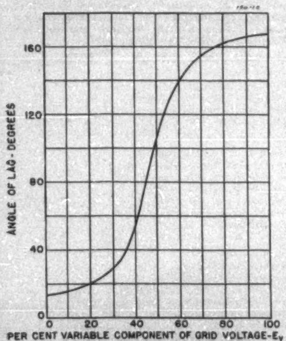


Figure 12. Variation of phase of grid voltage with variable component in experimental circuits

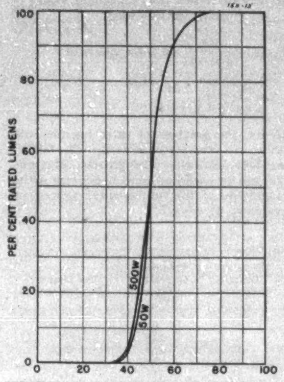


Figure 13. Variation of light output with variable component of grid voltage

At first difficulty was encountered with stray capacitance currents flowing in the grid circuit, the effects of which were minimized by electrostatic shielding of the grid supply and heater transformers, of the grid voltage phase shifting components, and of the grid leads. All transformer and control component shields were connected to the neutral of the supply line and that for each pair of grid leads to the cathodes of its associated pair of tubes. Shielded grid leads 200 feet in length were found to operate satisfactorily.

Appendix I. Anode-Transformer Rating

A transformer with two identical secondaries was connected as an auto-transformer to supply anode power. See T_1, figure 2. The secondaries are, therefore, loaded alternately. If iron losses are assumed constant, a conservative estimate of the permissible rating might be based on that loading which would not exceed in any part of the winding the copper losses in normal use.

Let—

I_p = peak value of primary current in normal use

I_s = peak value of secondary currents in normal use

N_p = number of turns of primary winding

N_s = number of turns of each secondary winding

r_p = copper resistance of primary winding

r_s = copper resistance of each secondary winding

I_p' = peak value of primary current when supplying a full wave rectifier

I_s' = peak value of secondary currents when supplying a full wave rectifier

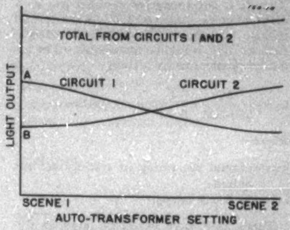


Figure 14. Characteristics of experimental circuits for transition from scene one to scene two

Equating secondary copper losses—

$$\frac{I_s^2}{2} r_s = \left[\frac{I_s'}{2}\right]^2 r_s$$

Equating primary and secondary magnetomotive forces—

$N_p I_p = 2 N_s I_s$ in normal use
$N_p I_p' = N_s I_s'$ in rectifier use

And—

$$I_p' = \frac{\sqrt{2}}{2} I_p = 0.707 I_p$$

Therefore, the rating for rectifier use with assumptions as stated is 70.7% of the normal rating.

If iron losses are assumed constant, an estimate of the permissible rating may be calculated on the basis of the constancy of total copper loss. Such an estimate may not be sufficiently conservative, since the division of losses between primary and secondary windings will be different.

Equating total copper losses—(same notation as before)

$$\frac{I_p^2 r_p}{2} + I_s^2 r_s = \frac{I_p'^2 r_p}{2} + 2\left(\frac{I_s'}{2}\right)^2 r_s$$

Equating primary and total secondary copper losses in normal use—

$$\frac{I_p^2 r_p}{2} = I_s^2 r_s$$

Solution with the magnetomotive force relations given above leads to—

$$I_p' = \sqrt{\frac{2}{3}}\, I_p = 0.817 I_p$$

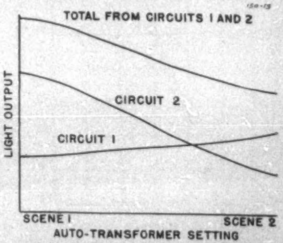


Figure 15. Characteristics of experimental circuits under typical operating conditions

Therefore, the rating for rectifier use with assumptions as stated is 81.7% of the normal rating. A value between those calculated, such as 75%, would be reasonable. The required rated capacity is then—

$$\text{kva} = \frac{23}{230+23} \times \frac{1}{0.75}\ \text{kw} = 0.126\ \text{kw}$$

where—

kva = normal use rating of transformer required

kw = d-c power output

Tubes used were FG-172's rated at 6.4 amperes average anode current, which in the present application means a peak tube current of 20 amperes and hence an effective load current of 14 amperes from two tubes. At 115 volts the individual circuit capacity is 1.6 kilowatts maximum, requiring an anode transformer capacity of 0.2 kilovolt-ampere per circuit.

Appendix II. Fourier Analysis of Line Current

Neglecting the effect of exciting current for transformers, inductance in the supply, change of lamp resistance with current, and arc drop, line current may be represented as follows:

$$\left.\begin{array}{ll} 0<\theta<\alpha, & f(\theta)=0 \\ \alpha<\theta<\pi, & f(\theta)=A\sin\theta \\ \pi<\theta<\pi+\alpha, & f(\theta)=0 \\ \pi+\alpha<\theta<2\pi, & f(\theta)=A\sin\theta \end{array}\right\}\text{See figure 16}$$

The wave contains no even harmonics and no uni-directional component. A general expression for the coefficient of the nth term is given by the following:

$$C_n = \frac{A\sqrt{2}}{\pi(1-n^2)}\left[(3+n^2)+(1-n^2)\cos 2\alpha - 2(1-n)\cos(1+n)\alpha - 2(1+n)\cos(1-n)\alpha\right]^{1/2}$$

where n is the order of the harmonic.

For the first term—

$$C_1 = \frac{A}{\pi}\left[(\pi-\alpha)^2+(\pi-\alpha)\sin 2\alpha - {}^1/_2\times \cos 2\alpha + {}^1/_2\right]^{1/2}$$

It may be shown that maximum values of C_n are given by the following:

$$C_{n\,\max.} = \frac{2A}{\pi(n-1)}\sin\alpha \text{ for values of}$$

$$\alpha = \frac{2\pi}{n+1}, \frac{4\pi}{n+1}, \frac{6\pi}{n+1} \ldots$$

Hence, the greatest of these maxima will occur in the vicinity of $\alpha=\pi/2$. The following table expresses the greatest of these maxima for each harmonic in per cent of the magnitude of the fundamental component evaluated at $\alpha=\pi/2$.

Order of Term n	Frequency Cycles Per Second	Per Cent of Fundamental
1	60	100.0
3	180	53.7
5	300	23.2
7	420	17.9
9	540	12.8
11	660	10.7
13	780	8.7
15	900	7.7
25	1,500	4.5
35	2,100	3.2
45	2,700	2.4
55	3,300	2.0
65	3,900	1.7
75	4,500	1.5

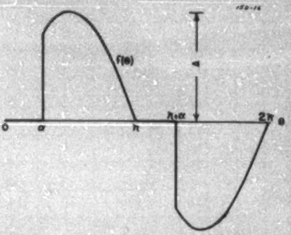


Figure 16. Approximate wave form of line current with equal angles of conduction for all circuits

References

1. HOT CATHODE THYRATRONS, **A. W. Hull.** *General Electric Review*, April 1929, pages 213–23; July 1929, pages 390–9.

2. GRID AND PLATE CURRENTS IN A GRID-CONTROLLED MERCURY VAPOR TUBE, **A. C. Seletzky and S. T. Shevki.** *Journal* of the Franklin Institute, March 1933, pages 299–326.

3. VOLTAGE IMPULSES FOR THYRATON GRID CONTROL, **M. M. Morack.** *General Electric Review*, June 1934, pages 288–95.

4. THE TELEPHONE INFLUENCE FACTOR OF SUPPLY-SYSTEM VOLTAGES AND CURRENTS, engineering report number 33, volume 3, *Engineering Reports* of the Joint Subcommittee on Development and Research, Edison Electric Institute and Bell Telephone System, New York, 1937.

5. SELECTIVE DEVICES FOR REDUCING HARMONICS IN POWER SYSTEMS, Part II, A-C SYSTEMS, engineering report number 34, volume 4, *Engineering Reports* of the Joint Subcommittee on Development and Research, Edison Electric Institute and Bell Telephone System, New York, 1937.

6. MEASUREMENT OF TELEPHONE NOISE AND POWER WAVE SHAPE, **J. M. Barstow, P. W. Blye, and H. E. Kent.** AIEE TRANSACTIONS, volume 54, 1935 (December section), pages 1307–15.

7. EFFECT OF RECTIFIERS ON SYSTEM WAVE SHAPE, **P. W. Blye and H. E. Kent.** AIEE TRANSACTIONS, volume 53, 1934 (January section), pages 54–63.

8. APPLICATION OF ELECTRON TUBES IN INDUSTRY, **D. E. Chambers.** AIEE TRANSACTIONS, volume 54, 1935 (January section), pages 82–92.

9. THÉORIE DES REDRESSEURS À L'ARC À COMMUTATION RETARDÉE (SUITE ET FIN), **M. Marcel Demontvignier.** *Révue Générale de l'Électricité*, tome 32, numéro 20, 19 Novembre 1932.

10. INDUSTRIAL ELECTRONICS, **F. H. Gulliksen and E. H. Vedder.** John Wiley and Sons, New York, 1935.

11. THYRATRON LIGHTING CONTROL, **W. R. King.** *Electrical Engineering Handbook*, volume 5, section 16, pages 22–6.

12. MERCURY ARC-POWER RECTIFIERS, THEORY AND PRACTICE, **O. K. Marti and H. Winograd.** McGraw-Hill Book Company, New York, 1930.

13. ELECTRICAL CONTROL FOR VARYING LIGHTING INTENSITIES, **S. R. McCandless and F. M. Wolff.** Illuminating Engineering Society *Transactions*, volume 31, pages 41–68.

14. THEATER LIGHTING CONTROL, **D. M. Rollins and B. S. Burke.** *Electric Journal*, volume 32, pages 477–81.

15. THYRATRON REACTOR LIGHTING CONTROL, **E. D. Schneider.** AIEE TRANSACTIONS, volume 57, 1938 (June section), pages 328–34.

16. WAVE-SHAPE CHARACTERISTICS OF POWER CIRCUITS SUPPLYING RECTIFIERS, engineering report no. 22, volume 3, *Engineering Reports* of the Joint Subcommittee on Development and Research, Edison Electric Institute and the Bell Telephone System, New York, 1937.

Electric Shock

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Introduction

A REVIEW of the literature has revealed very little information regarding the effects of small 60-cycle electric currents on man. These currents are not sufficient to produce unconsciousness or death but they are sufficient to produce involuntary muscular contractions. The hazard to the public has been increased because of the recent development and application of electrical devices with exposed electrodes, such as the electric fence, the electric fly panel, the electric insect trap, the electric cattle prod, and the a-c welding apparatus. The fatal accidents that have occurred due to these devices have emphasized the importance of obtaining additional knowledge on electric shock. This report is based on experiments just completed on 120 men. A preliminary investigation had been made during 1936–37 on 56 men.[1] Some of the data collected then have been included in the body of this paper, but not in the statistical analysis. It is hoped that the results may help to improve the design and safety of some of the new electrical devices and may be useful in clarifying the widespread misunderstanding of electric shock.

Before presenting the experimental results, a review of the effects of 60-cycle alternating current on man may be instructive. If the electrodes are held in the hands, the threshold of perception is about one milliampere. As the current is gradually increased, the sensation of tingling in the fingers, hands, and wrists is intensified. At the same time the muscles of the fingers and hands, and later also of the arms, tighten involuntarily. At the higher currents (six to eight milliamperes and above) the discomfort from severe muscle contraction is more evident than the sensation of tingling, and may even be painful. For currents of this magnitude the individual may still be able to release his grip of the electrode by a determined muscular effort. As the current increases, the involvement reaches the shoulder girdle and extends to the external muscles of the chest so that respiration becomes somewhat difficult. The higher currents cause a sensation of mental discomfort which may even amount to a nameless fear, although the individual knows that at any indication from him, such as a nod or a cry, the current would be shut off. Figure 12*a* is a picture of one of the subjects at his 60-cycle limit. Figure 12*b* illustrates the reactions at two milliamperes greater current. Figure 12*c* is a photograph of another subject and illustrates the muscular reactions and test procedure used in determining the let-go currents. The subjects were unaware that the pictures were about to be taken and the sittings were not posed.

This investigation was not concerned with currents that produce unconsciousness, accompanied by cessation of either respiratory or heart action, or of both. However, the higher values of the currents investigated, which produce nothing more than severe involuntary muscle contraction, can result in death. The cause of death is not definitely known, but is ascribed to exhaustion with respiratory failure and resultant asphyxia.

Problem

Because of the increasing prevalence of electrical devices with exposed electrodes and the likelihood that individuals may come in contact with them, it was considered desirable to investigate the effects of currents at different frequencies on voluntary muscular control. It is generally conceded that current and not voltage is the proper criterion of shock intensity. This fact was verified by an experiment in which subjects received open-circuit voltages varying from 5 to 9,720 volts. The muscular reactions and sensations, although varying to some degree, were substantially in agreement. Several factors must be considered in deciding what constitutes a maximum safe current. The victim of an accidental shock [illegible] often release himself by using muscles little affected by the current. The[illegible] muscular reactions may tend to break the circuit rather than to improve it, or loss of balance may free the victim. However, it is doubtful if any of these methods would be of avail when contact is established by gripping an electrified wire so that the path of the current extends across the body to the other arm or to the legs. Under these circumstances it is concluded that the safe electric current for man is that value which can be

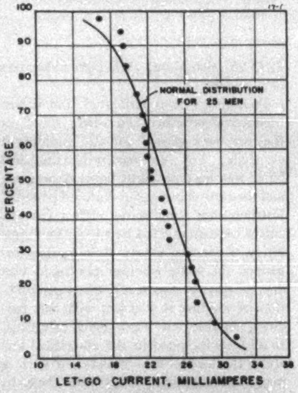

Figure 1. Five-cycle let-go current distribution curve

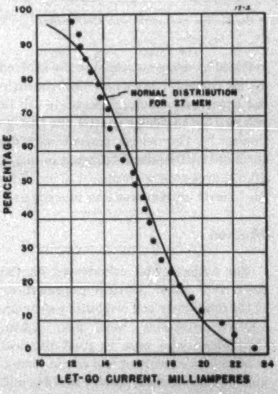

Figure 2. Ten-cycle let-go current distribution curve

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Table I. 60-Cycle Let-Go Tests

Subject Number	Voltage (Volts)	Current (Milli-amperes)	Resistance (Ohms)	Percentile Rank (Per Cent)
73	26	9.7	2,680	99.6
7	45	10.7	4,210	98.3
19	26	10.7	2,430	98.3
62	32	11.1	2,880	97.0
5		11.3		96.1
118		11.5		95.2
104		11.6		94.3
12	43	11.7	3,680	93.0
33		11.7		93.0
82		12.0		91.2
16	53	12.0	4,420	91.2
20	30	12.3	2,440	89.5
75	20	12.3	1,630	89.5
88		12.4		87.7
10		12.4		87.7
2	44	12.5	3,520	85.9
26	27	12.5	2,160	85.9
69	31	12.6	2,460	84.7
53	26	12.7	2,050	83.8
90		12.8		82.9
91		13.0		82.0
11	30	13.1	2,290	80.7
70		13.1		80.7
89		13.3		79.4
83		13.5		77.6
15	43	13.5	3,190	77.6
84		13.5		77.6
38	28	13.6	2,060	75.0
46	32	13.6	2,350	75.0
68	30	13.6	2,200	75.0
13	34	13.7	2,480	71.5
102		13.7		71.5
25	23	13.7	1,680	71.5
113		13.7		71.5
41	22	13.7	1,610	71.5
47		13.9		68.9
27		14.0		68.0
52		14.1		66.7
79		14.1		66.7
1	63	14.2	4,430	64.9
58	31	14.2	2,180	64.9
30	48	14.4	3,330	63.6
45	38	14.7	2,580	62.7
77		14.9		61.8
74	44	15.0	2,930	59.7
72	34	15.0	2,270	59.7
78		15.0		59.7
29	32	15.0	2,130	59.7
59	31	15.1	2,050	57.4
39	29	15.2	1,910	56.1
32	28	15.2	1,840	56.1
116		15.3		53.9
28		15.3		53.9
86		15.3		53.9
43	44	15.4	2,860	52.2
31	33	15.6	2,120	50.0
76		15.6		50.0
4	38	15.6	2,430	50.0
51	30	15.6	1,920	50.0
67	34	15.7	2,170	47.8
100		15.8		46.5
6	37	15.8	2,340	46.5
119		16.0		43.8
42	43	16.0	2,690	43.8
21		16.0		43.8
94		16.0		43.8
8	34	16.1	2,110	42.1
44	31	16.1	1,920	42.1
96		16.3		39.0
48	55	16.3	3,370	39.0
85		16.3		39.0
106		16.4		36.8
36		16.4		36.8
101		16.5		35.5
57	35	16.6	2,110	33.8
87		16.6		33.8
23	36	16.6	2,170	33.8
121		16.7		32.0
37		16.9		31.1
61	48	17.1	2,810	29.8
114		17.1		29.8
60	75	17.3	4,330	28.1
120		17.3		28.1
56		17.4		26.3
55		17.4		26.3
92		17.5		24.1
34	38	17.5	2,170	24.1
93		17.5		21.9
35	36	17.6	2,040	21.9
65	32	17.6	1.820	21.9
117		18.0		19.7
71		18.0		19.7
95		18.0		19.7
49	31	18.1	1,710	18.0
109		18.2		17.1
54	51	18.4	2,770	16.2
108		18.5		14.9
81		18.5		14.9
97		18.6		13.6
111		18.7		12.7
22	30	19.1	1,570	10.5
98		19.1		10.5
110		19.1		10.5
80		19.1		10.5
112		19.2		8.3
64	44	19.5	2,250	7.0
103		19.5		7.0
40	58	19.8	2,930	5.3
99		19.8		5.3
50	38	20.0	1,900	3.9
107		20.3		3.1
14		20.5		2.2
3	53	20.8	2,550	1.3
105		21.6		0.4
115*		13.2		

* Female, not used in the statistical analysis.

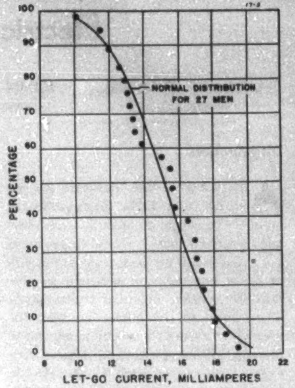

Figure 3. 25-cycle let-go current distribution curve

released by using muscles directly affected by the current. We have called the highest current at which an electrode can be released by muscular control the "let-go current." The let-go current was determined for frequencies ranging from 5 to 10,000 cycles on a group of 120 individuals. Direct current was also investigated.

Method

The subjects who volunteered for the tests were given physical examinations of the circulatory and respiratory systems. Electrocardiograms were also taken. Only those who were in good physical condition, who had not had recent illness, and who had normal blood pressures and electrocardiograms, were admitted. The age of most of the subjects ranged from 21 to 25, with about 15 between the ages of 26 and 46.

Preliminary tests indicated that it was immaterial whether the other hand or the feet were placed on the indifferent electrode. For convenience, the left hand was used in right-handed persons, and the right hand in left-handed persons. Furthermore, they showed that current values were unaffected by the surface condition of the test hand; that is, they remained the same whether the hand was dry, whether it was moist from perspiration, or whether it was wet with salt (sodium chloride) or weak acid solutions. In all tests included in the statistical analysis the hands were moistened with a weak salt solution. This was done in order to secure uniform conditions, to lower contact resistance, and particularly to lessen unpleasant sensations due to high-current densities at localized spots.

The general procedure was as follows: After the subject had placed his hands on two large brass electrodes, the current was turned on gradually. This permitted him to become accustomed to the particular sensations of the frequency used. Then the subject firmly grasped the test electrode, which consisted of a 12-inch length of number 6 copper wire connected to the circuit by a flexible lead. His other hand remained on the indifferent electrode, a brass plate eight inches in diameter. This permitted him to break the circuit should he so desire, even though it was necessary for an observer to hold this hand flat to prevent it from curling due to the muscular contraction.

When the subject was ready, the current was raised at moderate speed to a certain value, at which point the subject was commanded to drop the electrode. If he succeeded, the current values were increased by steps until he was no longer able to let go of the wire; if he failed, the test was repeated at a lower value. The end point was checked by several trials insufficient in number to fatigue the individual but enough to determine the let-go value accurately. In order to minimize the effects of fatigue, the number of tests at different frequencies on any one subject were limited to two or three per day.

Let-Go Currents

The tabulated data show the actual currents and the voltage across the body,

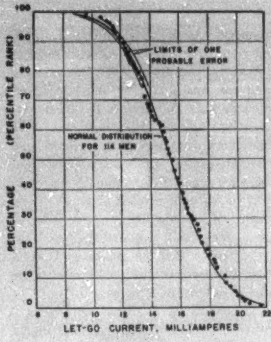

Figure 4a. 60-cycle let-go current distribution curve

corrections having been made for the instruments in the circuit. The 60-cycle let-go currents were determined using the commercial 60-cycle source regulated with a variac. The results are given in figure 4 and table I. Let-go currents for 114 men were: minimum, 9.7; mean 15.5; maximum, 21.6 milliamperes (rms). As previously stated, these results were obtained on the right hand in right-handed persons and on the left hand in left-handed persons. The values averaged about one-half milliampere less in the few cases in which the other hand was tested. Since voltages were considered relatively unimportant, only random measurements were taken in order to expedite the experimental procedure. The range observed was from 20 to 75 volts. The body resistance varied from 1,570 to 4,430 ohms. As the salt solution on the hands dried, the voltage and the resistance increased, although the let-go current remained approximately the same. This confirms the conclusion that current and not voltage is the proper criterion of shock intensity.

All values in table I are the results of the first 60-cycle test, although some subjects were tested first on other frequencies. It may be of passing interest to mention that a let-go current of 13.2 milliamperes was obtained for the one woman (subject 115) who took the test. A few of the subjects refused to take more than about five milliamperes, hence they were not included in the series. The preliminary results previously published gave the average let-go current for 42 men as 12.6 milliamperes.[1] The higher value obtained in the present series is believed to be due to improved technique.

The curve of normal distribution derived from these results approaches the 0 and 100 percentiles asymptotically, as shown in figure 4. Therefore an arbitrary choice of limits was required and the 99.5 percentile was chosen to permit a conservative conclusion. Statistical methods were then applied to examine the reliability of the results. The probable error of the mean was found to be 0.166 milliampere. The probable error of the 99.5 and 0.5 percentiles was found to be

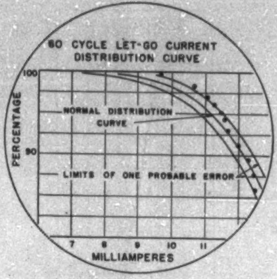

Figure 4b. Magnified section of figure 4a

Table II. D-C Tests

Subject Number	Voltage (Volts)	Current (Milliamperes)	Resistance (Ohms)
1	113	75.0	1,510
8	127	75.5	1,680
10	130	71.0	1,830
12	116	61.0	1,900
21	129	83.0	1,560
22		80.0	
23		71.0	
28	130	75.0	1,740
30	116	80.0	1,450
31	128	73.0	1,750
32	115	72.0	1,600
33	109	65.0	1,680
36	122	70.0	1,740
37	109	75.0	1,450
40	126	71.0	1,780
45		62.0	1,440
47	129	81.0	1,590
54	117	75.0	1,560
55	131	67.0	1,960
57	117	76.0	1,540
60		79.0	
61	130	72.5	1,790
62	123	75.0	1,640
64	128	70.0	1,830
69		80.0	
70	124	75.0	1,650
74	126	75.0	1,680
75	121	76.0	1,590
115*		56.0	

*Female, not used in the statistical analysis.

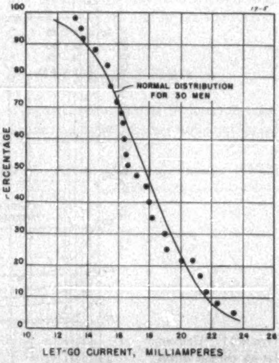

Figure 5. 180-cycle let-go current distribution curve

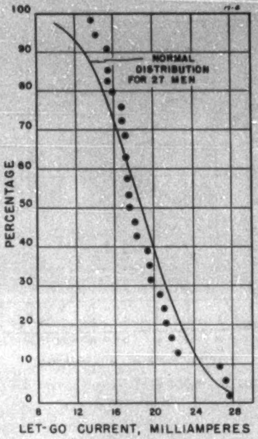

Figure 6. 500-cycle let-go current distribution curve

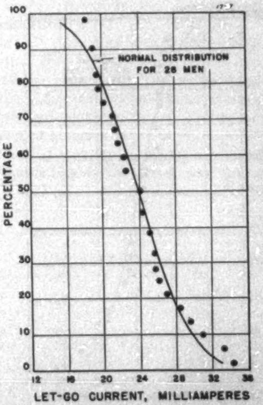

Figure 7. 1,000-cycle let-go current distribution curve

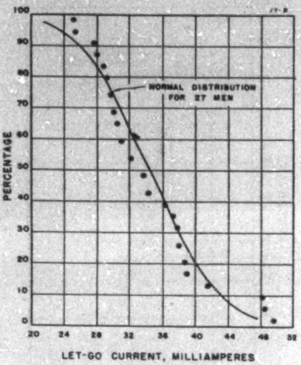

Figure 8. 2,500-cycle let-go current distribution curve

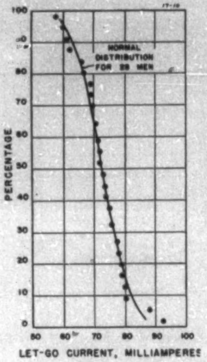

Figure 10. 10,000-cycle let-go current distribution curve

0.80 milliampere. Thus, if it is assumed that the 114 subjects represent an accurate sample of all normal men of the same age range, the following applies: If 1,000 men had been tested, 0.5 per cent or 5 men would have been able to let go of currents as high or higher than 22.3±0.8 milliamperes. Similarly, 99.5 per cent or 995 men would have been able to let go of 8.8±0.8 milliamperes or more, while five men would not have been able to let go of that value. Less conservative values may be taken directly from the curve of figure 4. For example, 97.5 per cent of all men should be able to let go of 10.4±0.3 milliamperes and 2.5 per cent, or 1 man in 40, would not. On basis of these data, the authors are of the opinion that eight to nine milliamperes is a reasonably safe 60-cycle current for normal men to handle. The value may well be less for women and children, and in the interest of safety no definite value should be stated until adequate investigations have been made.

For frequencies less than 60 cycles, two alternators mounted on a common shaft were driven at the proper speeds to generate 5, 10, and 25 cycles. The machines were connected in series and, in order to minimize harmonies in the output, the stator of one was shifted with respect to the stator of the other. The results are given in figures 1 to 3 inclusive. A beat frequency oscillator and a power amplifier were used to produce the higher frequencies Vacuum tube instruments were used for measurements and the let-go currents were determined at various frequencies up to 10,000 cycles. The equipment was adjusted to maintain sine wave shapes for all load conditions and the wave shape was continually observed with a cathode-ray oscillograph. The results are given in figures 5 to 10 inclusive. Statistical predictions for large groups other than on the 60-cycle tests were believed unwarranted because of the limited number of subjects used.

Tests with steady direct current produced a sensation of internal heating of the hands and arms with only slight muscular contraction, though a severe shock was experienced each time the circuit was broken. None of the subjects had any difficulty in releasing the electrode. Many stated that they would almost prefer to maintain contact in spite of the burning sensation, in order to avoid the severe shock that was produced when they released the wire. The values given in table II represent the limit of endurance instead of the let-go value. The prevalent opinion that direct current is safer than 60-cycle alternating current was confirmed. This should be of interest to operators of electrical devices, such as welding equipment, who work in hazardous locations and who may have a choice of power source.

Derived distribution curves of let-go current plotted against frequency are

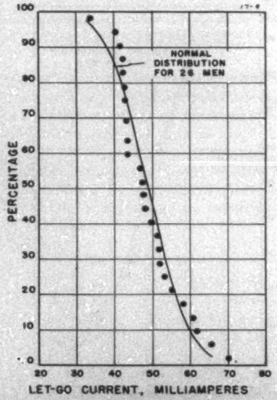

Figure 9. 5,000-cycle let-go current distribution curve

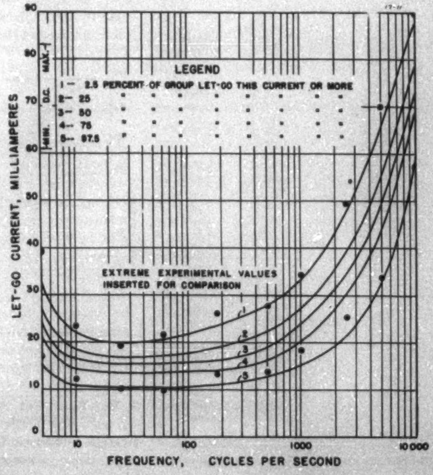

Figure 11. Derived let-go current distribution curves versus frequency

Figure 12a (left). Subject 65—at his 60-cycle let-go current (17.6 milliamperes)

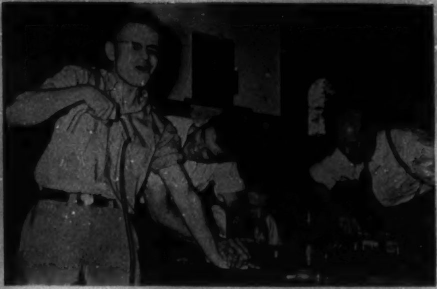

Figure 12b. Subject 65—about two milliamperes in excess of his 60-cycle let-go current

Figure 12c (right). Subject 82—at his 60-cycle let-go current (12.0 milliamperes)

shown in figure 11. The data were obtained from the normal distribution curves shown in figures 1 to 10 inclusive. The extreme points obtained experimentally, as well as values for direct current, have been inserted for comparison. The shape of the curves is not only interesting, but also instructive since it shows the ordinary power frequencies to be the most dangerous.

A summary of the experimental and significant statistical data is given in table III. The current values for 60 cycles agree very well with the values for 50 cycles obtained by Webber[2] and at the Electrical Institute of Zurich.[3] The values are considerably higher than those reported by the Underwriters' Laboratories.[4]

Effect of Electrode Size

Electrodes consisting of the number 6 copper wire and brass rods approximately one-half, three-fourths, and one inch in diameter were investigated by determining the let-go current at 60 cycles. Although variations occurred in individuals, the averages for the group were practically identical. The experimental results with the number 6 wire and the one-inch rod are given in table IV. It was concluded that the let-go current was independent of the size of the conductors used.

Effect of Repeated Trials

An attempt was made to determine the degree to which a person could develop an immunity to electric shock. It was felt that a definite psychological factor was involved which might be overcome by repeated tests at intervals on the same frequency. Naturally the well-being and strength of an individual varies from time to time, depending on sleep, diet, anxiety, etc. These physiological factors could not be controlled. The let-go current test at 60 cycles was conducted at weekly intervals on ten subjects. The data are plotted in figure 13. The results are too much at variance to permit any conclusions although the trend was upward.

Table III. Statistical Data and Measures of Validity

Frequency	Direct current	5	10	25	60	180	500	1,000	2,500	5,000	10,000
Number subjects	28	25	27	27	114	30	27	26	27	26	28
Voltage											
Maximum volts	131	80	68	56	75	60	46	49	61	76	82
Minimum volts	109	29	24	24	20	22	18	18	22	31	45
Impedance											
Maximum ohms	1,960	3,810	3,330	3,910	4,430	3,140	2,150	2,000	1,970	1,310	1,010
Minimum ohms	1,430	1,340	1,610	1,620	1,570	1,380	1,040	923	706	720	634
Current											
Maximum milliamperes	83	39.1	23.2	19.3	21.6	26.1	27.7	34.4	49.6	70.0	92.5
Minimum milliamperes	61	17.0	12.1	10.1	9.7	13.2	13.7	18.2	25.2	33.8	57.6
Mean milliamperes	74	24.05	16.30	15.14	15.55	17.80	18.90	24.20	34.60	49.30	73.00
Probable errors											
Mean		0.604	0.376	0.32	0.166	0.378	0.595	0.588	0.871	1.106	0.911
2.5 and 97.5 percentile		1.61	1.01	0.857	0.443	1.01	1.59	1.57	2.32	2.96	2.43

Table IV. Effect of Electrode Size on 60-Cycle Let-Go Current

Subject Number	Electrode: Wire Number 6	Electrode: Rod One-Inch Diameter
12	13.9	14.9
21	18.0	19.4
22	21.6	20.6
23	14.5	15.7
27	15.6	15.7
30	19.6	19.4
31	15.5	15.6
32	15.7	16.2
33	12.7	12.7
37	16.9	16.6
41	16.7	17.1
43	15.4	16.0
45	12.6	12.8
55	23.0	18.6
61	16.5	16.7
62	17.2	16.8
64	19.4	17.6
65	17.6	16.5
67	14.8	14.0
74	12.8	12.0
76	17.0	17.6
Average	16.5	16.3

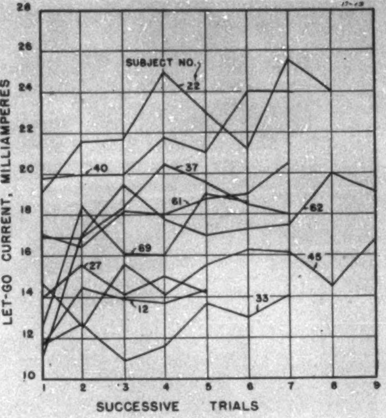

Figure 13. Effect of repeated trials on let-go for 60-cycle current

They demonstrate that variations due to psychological and physiological factors are important in an investigation of this type. These factors cause a distribution of results due to chance that require statistical methods for correct evaluation.

Effects of Wave Form

It was thought desirable to investigate the effect of wave form on let-go currents. In all tests the wave form was observed with a cathode-ray oscillograph, and the equipment was adjusted to maintain sine waves. A bank of three transformers, connected wye on the primary and broken-delta on the secondary, were energized from the 60-cycle commercial source through a three-phase phase-shifting transformer. The third harmonic obtained from the broken-delta was amplified and superimposed on the 60-cycle source to give the wave shapes of figure 14. The combined output was controlled with a potentiometer and metered by means of an iron vane milliammeter and a vacuum tube voltmeter connected to a shunt and calibrated to read peak milliamperes. To obtain comparative results, tests were taken of the three wave shapes on the same afternoon. The sequence of the wave shapes was taken at random in order to minimize the effects due to fatigue. The wire test electrode was used in these tests. The results are given in table V. Inspection of the averages indicates in a striking manner that muscular response depends on the crest value of the current and not on the rms value.

Table V. Effect of Wave Form on 60-Cycle Let-Go Current

Subject Number	Peak Wave: Crest (Milliamperes)	Peak Wave: RMS (Milliamperes)	Flat Wave: Crest (Milliamperes)	Flat Wave: RMS (Milliamperes)	Sine Wave: Crest (Milliamperes)	Sine Wave: RMS (Milliamperes)
1	19.9	11.0	20.4	15.8	21.0	14.9
12	20.2	11.0	20.2	15.7	20.4	14.5
21	21.3	11.5	22.4	18.0	20.5	14.5
23	19.6	10.1	20.2	16.0	20.4	14.5
27	19.3	10.9	20.2	15.6	19.9	14.1
28	20.7	11.2	21.3	16.8	21.5	15.3
30	25.2	13.8	27.5	22.8	28.3	20.0
31	23.5	13.0	24.4	19.2	23.8	16.8
32	21.8	12.0	21.3	15.6	20.7	14.7
33	18.5	10.0	19.6	15.6	19.9	14.1
37	25.5	14.6	26.0	20.9	26.0	18.4
40	26.0	15.0	26.7	22.0	28.3	20.0
45	19.9	10.7	21.0	15.2	19.9	14.1
47	20.2	10.5	20.2	15.8	19.6	13.9
54	23.5	13.2	25.0	20.0	23.5	16.6
55	24.4	14.0	27.5	21.9	29.4	20.8
57	24.7	13.4	23.5	19.0	26.0	18.4
61	25.2	14.6	25.8	20.0	25.8	18.2
62	25.2	14.6	25.0	19.5	25.2	17.8
65	24.7	14.1	22.7	18.2	24.1	17.0
67	19.6	10.8	19.9	15.5	20.7	14.7
69	22.1	12.2	26.0	20.4	22.7	16.0
70	17.9	9.7	19.3	14.6	18.5	13.1
74	17.9	9.7	18.5	14.4	18.7	13.3
76	23.8	12.8	22.7	18.0	23.8	16.8
115*	21.0	11.7	17.4	13.8	18.7	13.2
Average	22.0	12.2	22.5	17.7	22.6	16.0

*Female used in computing averages.

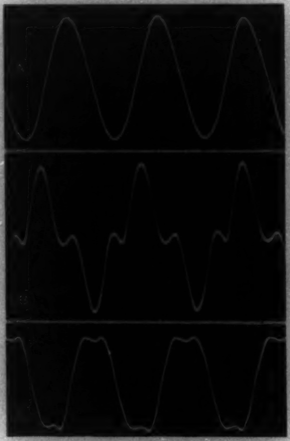

Figure 14. Current wave shapes used in determining 60-cycle let-go currents

(a)—Sine wave
(b)—Peaked wave
(c)—Flat wave

Conclusions

1. The maximum safe noninterrupted electric current is that value which can be released by using the muscles directly affected by it. For man, the safe 60-cycle current is eight to nine milliamperes.

2. Muscular reactions are proportional to the peak value of the wave and *not* to the rms value.

3. The let-go currents are affected by frequency. Unfortunately for man, the minimum values are in the range of commercial power frequencies.

4. The let-go current is unaffected by the surface condition of the hands or the size of the electrodes.

5. The voltages which correspond to the limit of muscular control are relatively low. For a current pathway between the hands, with the hands moist but skin intact, the minimum observed 60-cycle value was only 20 volts.

6. It is possible for man to develop a degree of immunity to electric shock, however the increase is negligible from a practical standpoint.

7. The results of the investigation emphasize the importance of continual vigilance and education of both the public and the electrical workers with regard to hazards and safe procedures in the use of electrical apparatus.

References

1. Danger of Electric Shock, C. F. Dalziel. *Electrical West.*, volume 80, number 4, April 1938, pages 30–1.

2. Wie hoch muss eine Spannung sein um dem Menschen gefährlich zu werden? H. F. Webber. *Elektrotechnischer Verein*, bulletin 21, 1928.

3. Beitrag zur Kenntnis der Vorgänge beim Stromdurchgang durch den menschlichen Körper. Elektrizitätswerken des Kantons Zurich. *Bulletin des Schweizer Elektrotechnischen Vereins*, volume 2, number 13, January 1929, pages 428–41.

4. Some Impromptu Determinations of Bearable Electric Shocks. Underwriters' Laboratories data, volume 11, September 1930, page 203.

Rural Automatic Telephone Networks

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MEMBER AIEE

Introduction

THE communication problems of rural areas offer an unique opportunity for the application of automatic switching methods. In no other situation is the combination of factors so favorable. Automatic exchanges are particularly suited to rural needs, especially when they are installed as unattended units in connection with nearby attended exchanges. The principal benefit springs from the fact that in a small manual exchange the number of operators cannot be adjusted to take care efficiently of the maximum traffic, because too often there is not enough traffic to occupy the time of even one operator.

The low traffic factor in rural areas therefore results in a twofold handicap—a handicap which only automatic equipment has been able to remedy. Experience has shown first that it is the infrequency of calls in an exchange, rather than high traffic, that makes the operator inattentive, and often causes delay in putting through calls. Service being the essence of all telephone facilities, this is a condition which cries aloud for a remedy.

The second phase of this handicap is its economic inefficiency, because of the low number of calls even during the busiest hours. Small exchanges, if they give twenty-four-hour service, have an irreducible minimum of three operators per twenty-four hours. Yet calls come in very infrequently during the night and other off-peak periods. The operator's time is therefore very incompletely utilized.

Where two or three nearby communities each can be served by an automatic exchange, the local calls can be handled by the automatic units while the toll calls for the entire area can be centralized at a nearby attended exchange. In this way, the maximum benefits are derived from automatic service, while the operator is not only kept efficiently occupied, but is concerned solely with duties that call for personal judgment.

The problem of rural exchanges is not new—the first of the so-called community automatic exchanges was installed in Fifield, Wisconsin in 1916—but its solution is not well enough known. This paper will therefore review a number of the factors entering into the operation of a network of rural automatic exchanges.

Technical Considerations

A network of automatic exchanges may take either of two general forms. The first type consists of a number of exchanges arranged in a long line with a trunk line or lines connecting the first to the second, to the third, etc., and finally to a toll center. In general, only the toll center is attended by an operator and it is equipped with control means to enable her to switch calls between the various exchanges of the group, as well as between any of the exchanges and points outside the group. This type of network is usually dictated by geographic conditions and would be used, for example, to serve communities located along a valley.

The second type of network is more common and may in fact include the first. In this type all or most of the small unattended exchanges are connected by trunk lines directly to the toll center. The exchanges not connected directly to the toll center have indirect connections through other exchanges of the group. Figure 1 shows a block diagram of the two types. Each of the lines connecting the various exchanges represents one or more trunk lines. The number to be provided in each case will be determined by the traffic requirements.

The local exchange equipment at the toll center may be of any type either manual or automatic, because all the traffic to and from the outlying exchanges is handled by a toll operator. The type of toll board used is immaterial, although the remote control toll board[1] is being used by several companies with conspicuous success.

The outlying exchange equipment will be determined by the requirements of the community in which it is to be installed. It has been found in practice that they will vary in size from a dozen or so lines up to two or three hundred lines. The smaller sizes usually serve only party lines, whereas the larger ones provide all types of service in the towns and party line service in the country.

The very small automatic exchanges are particularly worthy of note. A typical board is arranged to serve 19 lines and 3 trunks and is equipped with 4 finder-connector link circuits. The

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[1] For all numbered references, see list at end of paper.

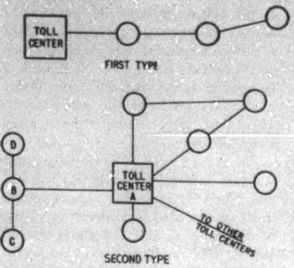

Figure 1. Block diagram of network types

finders and connectors are of the rotary switch type, which reduces the size of the unit. Telephones may be of the following types:

1. Individual line.
2. Two party divided ringing, full selective.
3. Ten party divided code or harmonic ringing.
4. Five party bridged code or harmonic ringing.
5. Post-payment paystation.

These classes of service are provided on a common battery, full metallic basis, but grounded line adapters can be used if required. These boards come fully equipped and self-contained with batteries, charging and switching equipment all in one cabinet.

Two types of cabinet are available, one for indoor mounting and the other which is weatherproof, for outdoor pole mounting. Pole mounting of automatic exchanges is a comparatively new idea and it has met with great success. It is probably not suited to all climates, but where the weather is favorable it is a useful method. It does away with the necessity of finding a suitable room in a satisfactory building as well as the expense of renting it. If a room is not available, it eliminates the expense of erecting a building. This type of exchange is mounted on a platform built between two poles as shown in figure 2. The two poles are located at any convenient place in the pole line where power is available. Since power is usually available anywhere, the switchboard can therefore be located at the most advantageous point from the wire plant standpoint. This may or may not be in the center of the town. In a number of cases it has been a cross road in open country. The simplicity of this type of mounting makes it extremely flexible. If the traffic outgrows the capabilities of the board, the latter can easily be removed and a larger exchange installed in a permanent building. If the expected traffic fails to materialize the board can be removed entirely without any great loss of installation expense.

The switching equipment for the larger exchanges may be of any of the well known automatic types. The type chosen will depend somewhat on the size of the exchange as well as on the preferences of the operating company. In general, relay type equipment is only suitable for exchanges of less than 200 lines ultimate capacity[2,3] but standard Strowger switches used with either plunger lineswitches, rotary lineswitches, or finders are suitable for use in systems up to any ultimate capacity. Systems of less than 100 lines are not ordinarily equipped with selectors but those of more than 100 lines are equipped with selectors.

Figure 3 shows a photograph of a small exchange designed to serve 50 or 90 subscriber lines and 10 trunk lines. This board is equipped with rotary switch 50 point finders and Strowger type two-motion 100 point connectors. The finders are arranged in two groups of 50 and 40 lines, only one of which need be installed originally. The additional group can be added whenever growth requirements justify it. The following classes of service can be supplied.

1. Individual—metallic or ground-return lines.
2. PBX trunk.
3. Two party, full selective—metallic lines only.
4. Four party, semi-selective—metallic lines only.
5. Ten party, semi-selective—metallic lines only.
6. Twenty party, semi-selective—metallic lines only.
7. Ten party bridged—metallic or ground-return lines.
8. Twenty party bridged—metallic or ground-return lines.
9. Post-payment paystations.

Figure 4 shows a trunking diagram of a typical rural exchange of the larger type. This particular exchange is equipped with linefinders and first selectors. It will be noted that both trunk lines and subscriber lines terminate on line equipment. When a subscriber wishes to initiate a call his line is found by the finder in the usual manner. If he wishes

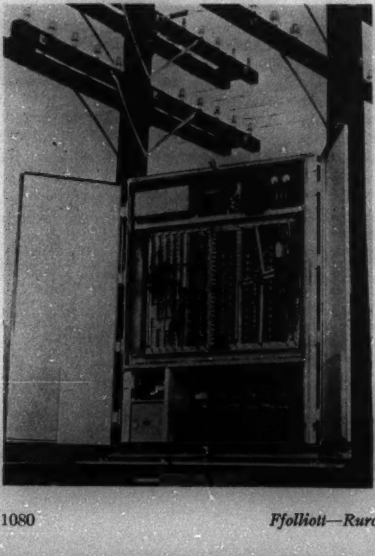

Figure 2. Pole-mounted exchange showing method of mounting and with doors open for inspection

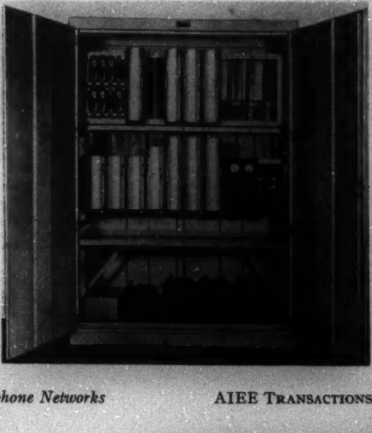

Figure 3. Small automatic exchange to serve 50–90 lines

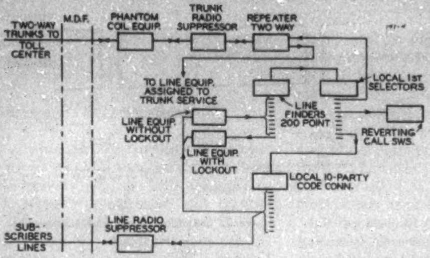

Figure 4. Trunking diagram of a small automatic exchange to serve more than 100 lines

to make a local call he dials two and goes out over the second level of the selector bank to the connector which responds to the last two digits, and completes the call. If he wishes to make a call to an outside point he first dials zero and goes out over the tenth level of the selector, through the repeater over the trunk line to the operator in the toll center. He then gives his instructions and the operator completes the connection.

An incoming call to the exchange comes over the trunk line, through the repeater and the line equipment to mark a contact on the finder bank. The finder finds this contact and connects the toll operator through to the first selector which returns dial tone. Upon hearing the tone she completes the call by dialing the subscriber's number in the usual manner.

It will be observed that the line equipment associated with the first finder level is furnished with "line lockout" but that the remaining line equipments are not. This feature is designed to disconnect automatically from the switchboard a line which becomes "permanent" due to faulty line conditions or failure of the calling party to dial promptly after dial tone is received. Inasmuch as rural lines are more likely to develop faults than are the town lines it is the usual practice to equip only the rural lines with the lockout feature.

Automatic exchanges must provide some means by which a subscriber can call another station on his own line. In the exchange illustrated in figure 4 this is done by means of reverting call switches. To make a call of this type with this equipment the subscriber must dial a special number. The first digit of the special number is that of the selector level to which the reverting call switches are connected (in this case seven) and the second and third digits are the code digits (usually the last) of the called and calling subscribers' numbers. After the number has been dialed, the calling subscriber hangs up to allow ringing to proceed. The called subscriber's code is rung first followed by the calling subscriber's code. These continue alternately until the called subscriber answers. This stops the ringing and so notifies the calling subscriber that his call has been answered and that he should pick up his receiver to complete the talking connection. If the subscribers' bells are on the same side of the line or if bridged ringers are used, only the called party's code is rung which both subscribers hear. If they are on opposite sides of the line each will hear only his own code. In this system the finder, selector, and reverting call switch are held during the conservation.

Small exchanges that do not require selectors sometimes use another type of reverting call service in which the reverting calls are made by directory number. As its name implies in this system it is not necessary to dial a special number to call a party on your own line. The calling subscriber dials the directory number of the wanted subscriber and then hangs up when he gets busy tone. Ringing takes place in the manner previously described, except that the calling subscriber does not hear it unless he is on the same side of the line with the called subscriber or unless bridged ringing is used. When the ringing stops or after a suitable length of time, the calling subscriber lifts his receiver to complete the talking connection. In this system all the switches are released after the talking connection has been established and talking battery is fed from the line relay.

A reverting call tone has recently been used on some exchanges to give the answering subscriber a warning that the call he has answered is from someone on his own line and that he should not hang up until the calling party has a chance to pick up the receiver.

It is the practice of most companies to provide at least one paystation in each community. The line thus equipped usually allows free local service, but provides means to give the operator in the toll center a distinctive signal whenever a toll call is originated from that station.

The exchange illustrated in figure 4 is of the finder type. The use of plunger or rotary lineswitches would change only the first portion of the switch train leaving the part from the selector onward very much as it is shown. Figure 5 shows a typical unheated building used to house an exchange of this type.

With reference again to figure 1, it will be noted that in both types of network some of the exchanges do not have direct trunk lines to the toll center. Some means must therefore be provided to allow the operator to supervise the calls between each pair of tandem exchanges, as well as calls from them to the outside. This is accomplished by the use of revertive call repeaters.[4] These repeaters may take several different forms to suit particular local conditions but they all have the essential feature of allowing the toll operator to dial back over the same trunk on which she was called. By this means she can, for example, set up a call between subscribers in exchanges *B* and *C* or between exchanges *C* and *D* of figure 1. After the connection is completed she retains supervision over the trunk line, and can therefore time the call and make the appropriate charge. The advantages of this system are briefly:

1. The transmission circuit includes only the trunk line between the two subscribers.
2. Only one trunk line to the toll center is held during the conversation.
3. The operator retains full supervision.

A variation of this scheme has also been used in which the operator does not retain full supervision but releases the trunk line as soon as the call is set up. She can check in on the progress of the call by dialing back at some later time over a monitoring circuit. This method of handling revertive dialing allows more efficient use of the trunk lines at the ex-

Figure 5. Type of unheated building used to house unattended exchanges

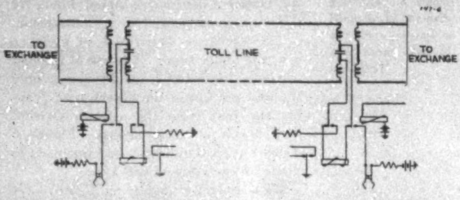

Figure 6. Simplified diagram of loop-dialing circuit

pense of less efficient supervision. There are in general, three methods of dialing between exchanges.[5,6]

1. Loop dialing. (Figure 6)
2. Simplex dialing. (Figure 7)
3. Composite dialing. (Figure 8)

Each of these is designed to suit particular conditions. Loop dialing is the simplest and it is usually used when the trunk lines are not too long. The circuit consisting of the operator's dial, the two wires of the trunk line, and the pulsing relay in the distant exchange, is broken momentarily by the impulse springs of the dial, allowing the pulsing relay to fall back. The details of the general method are too well known to require further description. In long dialing circuits of this type (and the other types as well) the battery supplying the dialing current is always located at the originating office, the reason for this being that if it were reversed the line leakage would tend to make the pulsing relay sluggish in its action. Furthermore in two way circuits, such as shown here, there is no battery drain due to line leakage such as there would be if the connections were reversed.

The simplex method makes use of both line wires in parallel as one leg of a ground return dialing circuit. This method is used on trunk lines which are too long for loop dialing but which are not part of a phantom group.

Simplex dialing is sometimes used on subscriber's lines that are longer than the allowable loop pulsing limit. For this use the dial is connected between the mid-point of the ringer coils and ground, and the line is simplexed at the exchange.

Composite dialing is used wherever the trunk lines are arranged in phantom groups. This method provides an extra leg that can be used as a spare dialing circuit or may be used as a telegraph wire.

In addition to the three methods discussed above there are two alternating current methods that have been used. These are known usually as low frequency a-c dialing and as voice frequency dialing.[7] The first of these uses 60 cycle alternating current and the second uses 1,000 cycle alternating current. The low frequency method has been used quite extensively in Europe and has been tried but not generally adopted in this country. The voice frequency method is in use over long toll circuits in South America, and it has been used to some extent here. They are mentioned only for the sake of completeness.

The resistance, the capacitance, and the leakance of a trunk line all tend to distort the dial impulses traveling over it. For this reason it may be necessary to introduce pulse correcting repeaters at various points of the circuit. It is the usual practice to have incoming and outgoing repeaters on all trunk lines. Tests have been made on circuits of this type where the impulses were repeated 10 times before reaching the final switch. In one particular test the impulsing circuit was about 170 miles long which was beyond the range of good transmission.

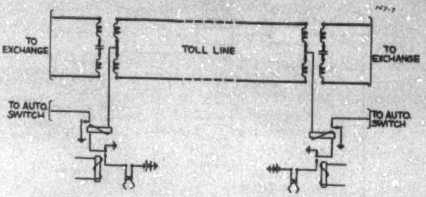

Figure 7. Simplified diagram of simplex-dialing circuit

A rural automatic network offers a unique opportunity for toll dialing. By this term is meant a system of operation in which the originating toll operator dials the subscriber in the distant office directly without the help of another operator. The originating operator reaches the distant exchange by dialing from one exchange to the next until she finally comes to the one she is seeking. She does this by dialing up the incoming selector of each exchange in turn to the correct level to give access to the next office until finally in the last one she dials the subscriber's directory number.

The advantages of this method of operation are:

1. The elimination of the time of one operator on each call.
2. An increase in the capacity of the trunk lines by eliminating any waiting time in dealing with the distant operator.
3. The elimination of errors and wrong numbers due to misunderstandings between operators.

To make this scheme effective complete directories of all exchanges must be maintained in each of the offices doing inter-exchange dialing. In the few cases

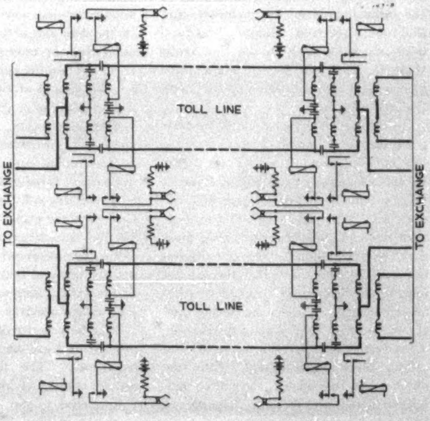

Figure 8. Simplified diagram of composite dialing on a phantom group

where the number cannot be found in the directory the operator obtains it from the distant information clerk and then dials the number herself.

Economic Considerations

It has been the experience of the companies operating rural networks that the subscribers offer very little opposition to conversion to automatic and that all opposition disappears soon after the cut-over. In the smaller communities the residents are usually more concerned over what is to become of the operator after the conversion than they are over any other phase of it and they may insist that she be properly taken care of before they will consent to a change. This concern is undoubtedly present because the operator in a great many cases acts as a sort of community secretary and has therefore a wide circle of friends who are solicitous as to her future.

The usual practice of one company in arranging for a change to automatic is to obtain the cooperation of the subscribers and the town council who petition for the conversion, with an increase in rates if any is required, to bring them into line with similar exchanges in other parts of the network. This method has been found most satisfactory since the subscribers do not feel that something is being forced on them against their will.

The Carolina Telephone and Telegraph Company, which operates one of the largest rural networks in the country from a standpoint of number of exchanges, has found that they can install a pole mounted automatic to serve a minimum of 35 subscribers. It has been their experience that invariably the number of subscribers increases greatly soon after the cut-over and may continue to increase until a larger exchange is required. In one case of phenomenal growth a pole automatic installed for 15 advance subscribers grew within a few months to 93 stations. They have found that in addition to increasing subscribers there is also a marked increase in the amount of toll traffic. This is attributed to the better grade of service available as well as to greater secrecy, because of automatic operation and because of the fact that the toll operator is in another town and is unknown to the subscriber or his neighbors.

It sometimes happens that a small community around a cross road, for instance, may be served by one or two multi-party lines from a large exchange some distance away. Such a location makes a very good nucleus for a pole mounted exchange.

In rural networks, particularly in agricultural districts, the rural lines make the town lines more valuable and a company can afford to break just even on them because of their value to the town.

The method of providing paystation service for the very small requirements of a small exchange is something of a problem. If has been found by experience that paystations will not be used for local calls, but that they are necessary for occasional toll calls made by nonresidents. The method of handling this problem used by a large operating company can be considered as typical. This company provides two general classes of paystations, semi-public and full public. The first of these classes is sub-divided into partial paystations and guaranteed paystations. The partial paystation plan is used mainly in the smallest exchanges. Under its provisions the regular business telephone of one of the subscribers (who pays the regular business rate) is equipped with a coin box for toll service. As an inducement the subscriber gets a single party line for price of a four-party line.

Under the terms of the guaranteed paystation plan a regular three-coin paystation is installed in some business house for their regular use. The owner of the establishment guarantees to pay for his local calls at the rate of five cents each with a minimum number such that the total amount paid is 50 cents less than the regular business rate. The guarantor then gets a 25% commission on excess local calls but gets no commission on toll calls.

Full public paystations are installed on a commission basis only in public places such as hotel lobbies, drug stores, railway stations, and the like.

It is the practice in many places served by manual exchanges and volunteer fire departments for the telephone operator to sound the fire alarm and call the firemen. When such an exchange is converted to automatic, there are a number of ways in which the alarms can be handled. In some places the telephone assigned to fire alarms is located in an all night business and the person there gives the alarm and calls the firemen. In other places an a-c relay is connected in parallel with the fire alarm telephone. The relay locks up when the telephone is called and sounds the alarm. The difficulty with this system is that there is no check on false alarms. One unique system makes use of a 10-party line as the fire telephone. The town in which this system is used is divided into ten districts and subscribers in each district report a fire by dialing the fire number with the last digit corresponding to their own location. When the telephone at the fire station rings it automatically codes the location of the fire.

Conclusion

It has been approximately 25 years since the first unattended community automatic exchange was installed. Since that time the design of boards to serve localities of various sizes has become crystallized to the point where today standard equipment is available to meet the needs of any operating company. On the other hand the basic units are so flexible that they can easily be adapted to any special requirement that may arise.

Especially noteworthy is the attitude of the user after a community automatic exchange is placed in service. Before a conversion, objections to change in the form of service, while seldom serious, do sometimes arise. Once the users experience the benefits of automatic service, however, they invariably become enthusiasts. All things considered there is every reason to believe that the number of automatic networks will continue to increase.

References

1. Remote-Control Toll Board, **Gilbert Sorber and Arthur Bessey Smith.** AIEE Transactions, volume 58, 1939, pages 696–704.

2. Economic Aspects of Various Automatic Switching Systems, **C. F. Ffolliott.** *Strowger Technical Journal,* volume 7, June 1938, pages 12–15.

3. Some Notes on Relay Switching Equipment for Automatic Telephone Systems, **B. B. Johnson.** *Telephone Engineer,* volume 44, September 15, 1940.

4. Remote-Control Toll Dialing for Automatic Exchanges, **C. F. Ffolliott.** *Strowger Technical Journal,* volume 7, December 1939, pages 34–41.

5. Automatic Switching of Toll Traffic, **Arthur Bessey Smith.** *Journal* Western Society of Engineers, volume 29, February 1924, pages 45–64.

6. Impulsing Circuits for Strowger Systems, **O. D. Grandstaff.** *Strowger Technical Journal,* volume 9, June 1940, pages 1–10.

7. Voice Frequency Dialing for Toll Circuits, **O. D. Grandstaff and John Wicks.** *Strowger Technical Journal,* volume 8, December 1939, pages 25–31.

Powerton-Crawford 220-Kv Line—System Operating Features and Terminal Design

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Synopsis: A 220-kv line, 150 miles long has been constructed between Powerton Station near Peoria, Illinois, and Crawford Station in the city of Chicago. This line will probably operate continuously near the static stability limit and will deliver 150,000 kilowatts to the city from Powerton Station near the coal fields of southern Illinois. This line has the highest capacity of any in this area and includes several novel features, such as a unique carrier current system for tripping the breakers at the remote end of the line, for telemetering, load control, and communication.

The 180,000-kva transformer bank at Powerton has two 22,000-volt primary windings connected to different busses. The Crawford transformer bank has a 150,000-kva winding at 220-kv, a 75,000-kva winding at 12-kv, and two 90,000-kva windings at 69-kv for connection to separate 66-kv systems. Regulating transformers are provided at both terminals. A 75,000-kva synchronous condenser at the Crawford terminal is connected to the 12-kv transformer winding for voltage control. No 220-kv breakers are used at either end of the line.

This paper describes only the operating features and the terminal facilities; a companion paper by Messrs. Oldacre and Wollaston describes the construction features of the line.

Introduction

THERE are four generating stations within the Chicago city limits, all depending upon the flow of the Chicago River for condensing water. The flow of the river has been limited to 1,500 cubic feet per second by U. S. Supreme Court order, so that even with their ultimate developments the present generating stations would not be adequate to supply the Chicago load. The lake front would be an ideal location for generating stations, but has been dedicated for park and other recreational purposes and is restricted against the building of industrial plants. Therefore, Chicago must depend for some of its supply on sources outside of its boundaries.

The locations of the four generating stations are shown in figure 1.

It is interesting to note the effect during the past ten years of these limitations on the various generating sites in the development of the Commonwealth Edison Company bulk power supply system.

In the year 1928 less than 5 per cent of the electric energy distributed in the city of Chicago came from sources outside the city limits; in the year 1939 one-half the energy distributed in the city came from this source. This is a remarkable change in the eleven years and is due partly to the fact that the newer stations outside of the city are more efficient, having been built prior to the beginning of the modernization of some of the older stations within the city.

Powerton Station

One of these new efficient generating plants is Powerton Station, 150 miles southwest of Chicago, within fifty miles of a substantial coal field and with ample cooling water for a generating capacity of 800,000 kilowatts.

Powerton Station comes very near to fulfilling the requirements of "mine-mouth generation" which was a popular subject for discussion some years ago. This station is on the edge of an unlimited coal supply and on a river with ample condensing water. It has a further advantage in being at the center of a reasonably good load area which would justify its construction whether or not additional capacity were required to supply Chicago. These various considerations were responsible for the development of Powerton Station as a necessary source and as an adjunct to the sites within the city which are fast reaching their maximum capacity.

Powerton Station was originally connected by 132-kv overhead lines to the system of the Public Service Company of Northern Illinois and other systems in the territory between Powerton and Chicago. After these lines were completed for local transmission, comparatively inexpensive reinforcements and extensions made possible the transmission of 50,000 kilowatts to 100,000 kilowatts over this transmission system to the city. Further increases in the load required by Chicago seemed to justify a direct transmission system connecting the two points. An

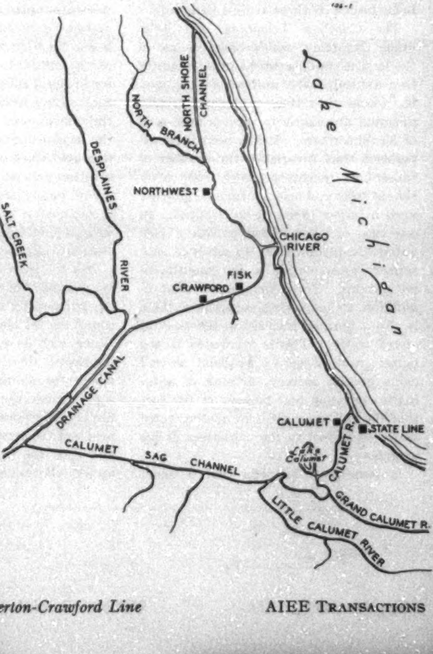

Figure 1. Location of generating station sites in Chicago

The map shows the location of each of the generating stations in Chicago in addition to State Line Station. The map also shows the streams on which the stations are located. The Chicago River, whose flow has been reversed to drain into the Mississippi through the drainage canal is the largest source of cooling water

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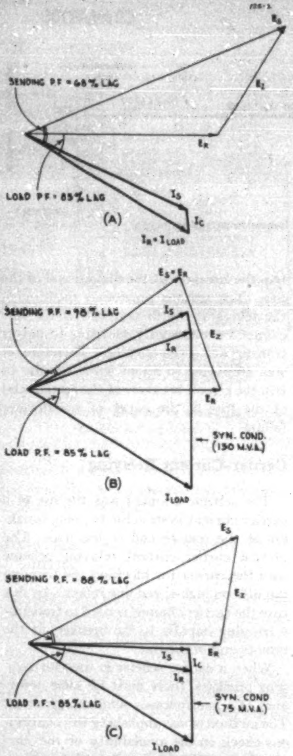


Figure 2. Current and voltage conditions in high-voltage transmission circuit, consisting of 150 miles of 220-kv line and terminal transformers while supplying a load of 150,000 kw at 85 per cent power factor lag

A—Tap changers only, no synchronous condenser

B—Synchronous condenser only, no tap changers

C—Tap changers and synchronous condenser

E_s = sending voltage
E_r = receiving voltage
E_z = voltage drop in line and transformers
I_s = sending current
I_r = receiving current
I_c = line-charging current
I_{load} = load current

Table I. Comparative Data on Crawford and Powerton 220-Kv Terminals

Item	Crawford	Powerton
I. 220-kv yard		
A. Area (occupied)	83,000 sq. ft.	80,000 sq. ft.
B. Distance: 220-kv yard to 22- or 66-kv yard	450′	725′
C. Distance: 220-kv yard to control board	1,000′	1,200′
II. Terminal towers		
A. Number	One	Four
B. Height: ground to conductor	*C*φ—59′ *B*φ—80′ *A*φ—101′	62′
C. Height: ground to ground wire	125′	90′
III. Transformer and condenser house		
A. Dimensions	146′x52′x62′ high	58′x80′x81′ high to low point on roof
B. Doors	Two main roller shutter doors, west and east sides, 25′ wide x34′8″ high	Two main roller shutter doors, north and south, 28′ wide x47′ high
C. Crane capacity	200 tons	125 tons
D. Height: floor to crane hook	53′2″	62′
E. Height required for untanking large transformers	45′1½″	47′10″
IV. Synchronous condenser		
A. Capacity	75,000 kva	
B. Speed	514 rpm	
C. Cooling	Air	
D. Total weight	690,000 pounds	
E. Rotor weight	350,000 pounds	
F. Starting motor	3,750 hp slip ring	
V. Main power transformer		
A. Number of single-phase units	Four	Four
B. Number of windings per transformer and their ratings:	No. Kv Kva	No. Kv Kva
1. High voltage	1 220 50,000	1 230 60,000
2. Low voltage	2 69 30,000	2 22 30,000
3. Tertiary	1 12 25,000	
C. Method of cooling	Self and fan	Self and fan
D. Height over bushing	30′6½″	30′8″
E. Height over tank	19′7½″	20′
F. Floor space	21′5″x29′2″	19′10″x25′4″
G. Net weight with oil	491,000 pounds	410,000 pounds
H. Net weight without oil	310,000 pounds	251,000 pounds
I. Oil capacity of each	24,133 gallons	20,500 gallons
VI. Regulating transformers		
A. Number	Four	Eight
B. Voltage range in per cent	±10%	±10%
C. Kva capacity per transformer	5,000 kva	3,000 kva
D. Exciting voltage	12 kv	22 kv
E. Location of series winding	Ground end of 220-kv winding	Inside 22 kv delta
F. Method of cooling	Self	Self
G. Floor space	14′10½″x18′0¼″	14′x17′
H. Net weight with oil	129,000 pounds	90,000 pounds
I. Net weight without oil	79,200 pounds	50,000 pounds
J. Oil capacity of each	6,640 gallons	5,250 gallons
VII. Control		
Location of equipment	Carrier-current control house contains carrier-current equipment and control for 220-kv disconnect switch. Condenser house contains all relays and recording equipment. Control room (main station) contains all controls and metering equipment	Two control houses contain panels for carrier current, instrument transformer secondary transfer, and relaying, control and indicating equipment
VIII. Fire protection		
A. Type of fire walls	Concrete	Concrete
B. Size of walls between main power transformers	34′ high x31′8″x12″ to 15″ thick	30′6″ high x24′x6″ to 15″ thick
C. Stone fill under transformers	3′5″	24″
D. Size of walls between voltage regulating units	22′ high x25′4″x12″ to 15″ wide	24′6″ high x21′x6″ to 12″ wide
E. Stone fill under regulators	3′5″	18″
F. Fire fighting equipment	Bulk CO_2 system on transformers and synchronous condenser	Water and portable foam equipment

economic analysis indicated that such a tie was advisable.

Such a direct transmission system into a congested territory immediately met with right-of-way limitations. The necessity of transmitting an ultimate of 400,000 kilowatts on one right-of-way practically compelled the use of the highest transmission voltage then available, which was 220,000 volts at the time this project was initially authorized. Calculations showed that it was possible to deliver nearly 400,000 kilowatts with a double-circuit transmission line over the 150-mile route by loading these transmission lines practically to their maximum stability limit.

Electrical Characteristics of the 220-Kv Line

The fundamental design of the transmission system produced some interesting technical problems. The line is intended to deliver 150,000 kw or more per circuit at a delivered power factor of 85 per cent. This is a very high loading for a transmission line of this length and requires special consideration to overcome the voltage drop in the line, which would be nearly 40 per cent including the transformer drop. The use of taps in the transformer windings or separate regulating transformers could be adopted to overcome this voltage drop. However, tap changers set for a voltage boost of 40 per cent or more in the direction of transmission would increase the current excessively in the sending end of the line and would also result in extremely high voltage in case the line opened at the receiving end while carrying full load.

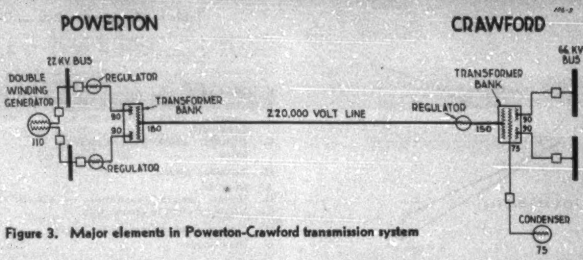


Figure 3. Major elements in Powerton-Crawford transmission system

Another method of controlling voltage regulation would be the use of synchronous condensers at the receiving end to operate the line at approximately constant voltage. In this case the capacity of synchronous condensers required at the receiving end would be approximately the same as the kw of power delivered. The cost of these synchronous condensers would be excessive. A careful study of the problem indicated that the most economical solution to the problem was to obtain half the compensation by the use of a synchronous condenser and the remainder by the use of tap changers. Transformers at both the sending and receiving end of the line have a fixed ratio equivalent to 5 per cent boost from Powerton towards Chicago. A regulating transformer bank is used at each end of the line with a range of plus and minus ten per cent; in addition, a 75,000-kva synchronous condenser is connected to the tertiary winding of the receiving transformer bank. This combination not only results in the minimum total cost for transmission but has certain other advantages illustrated in figure 2.

At full load, the power factor is approximately the same at both the sending and receiving end of the line and is within the normal range for satisfactory operation of the generators. The use of synchronous condensers on a tertiary winding protects the line to a marked degree against overvoltages should the circuit breakers open at the receiving end while carrying full load. The synchronous condenser is also used to energize the line during the starting process before it is synchronized to the system. This eliminates the voltage disturbance which would be created by closing 37,000 kva of line charging current to a generating station bus.

The total range of the tap changing equipment is from a 30 per cent boost in one direction on the line to 10 per cent in the reverse. This is accomplished by fixed taps in addition to the tap changer units. The result is that the line can operate at full load from Powerton to Chicago or at approximately half load from Chicago toward Powerton. There is no indication of any bus loading conditions which would call for more than half load in the reverse direction.

The essential parts of the transmission link, including terminals, are shown in figure 3. This one-line diagram shows the three-winding transformer bank at Powerton which is used to supply the line from two different 22-kv bus sections. At the Crawford end of the line, power is delivered to two separate 66-kv systems. Therefore, the Crawford transformers have two 66-kv windings in addition to the 220-kv winding and the 12-kv winding for the synchronous condenser.

Operating this line so near its stability limit has an interesting effect on the short-circuit currents. For a symmetrical three-phase short-circuit on the 66-kv winding of the Crawford terminal transformers the short-circuit current sent out from Powerton is nearly equal in value to the normal full-load current at Powerton. For a short-circuit on the 220-kv side of the Crawford transformers, the short-circuit current is somewhat higher but the increase in current is not sufficient for relaying purposes.

Relaying of the 220-Kv Line

Relaying for ground faults on the transmission line is comparatively simple. There are no high-voltage circuit breakers and any neutral current indicates a fault on this line. Therefore, a simple neutral relay can be set selective for faults on the full length of the line. The relaying for faults between phases is by means of high-speed impedance directional type relays.

Faults in the transformers or regulating units are detected by transformer differential relays. These relays trip the circuit breakers on the low-tension side at the near end of the line. However, with no high-voltage breakers it is necessary to trip the breakers at the distant end of the line. For reasons previously mentioned, the relays at the distant end of the line cannot be sufficiently sensitive to detect transformer short-circuits. Therefore, it was necessary to adopt some scheme to trip the circuit breakers at the remote end of the line in the event of transformer failure.

Carrier-Current Relaying

The scheme adopted was the use of a carrier current system for tripping breakers at the remote end of the line. The normal carrier current relaying scheme uses the carrier for blocking the tripping impulse on high-speed line relays. In this case the carrier channel is used to transmit a tripping impulse to the breakers at the remote end of the line.

When a carrier scheme is used for tripping breakers, there must be some assurance of a continuously available channel. The method of accomplishing this continuous check on the availability of the carrier channel is quite novel and interesting. One carrier channel is used to transmit carrier frequency in one direction over one pair of conductors of the line and an additional channel transmits a continuous carrier signal in the reverse direction over a separate pair of conductors of the line. Each of these carrier channels is modulated by several modulating frequencies for various purposes to be described later. As a part of the remote tripping scheme, one of the modulating channels is used to transmit a continuous signal from Crawford to Powerton which picks up a relay to return continuous modulating signals over the other carrier channel to Crawford. So long as both carrier sets are in operation, a signal light is lighted on the switchboard. Any trouble in either the carrier or modulating frequencies breaks this circuit and rings an alarm. Separate modulating channels on the same carrier frequency are used for sending the tripping impulses. The tripping and blocking circuits are so interlocked as to maintain

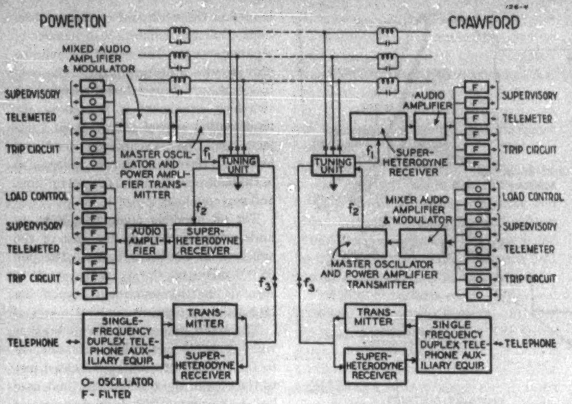

126-4
POWERTON
CRAWFORD
MIXED AUDIO AMPLIFIER & MODULATOR
SUPERVISORY
TELEMETER
TRIP CIRCUIT
MASTER OSCILLATOR AND POWER AMPLIFIER TRANSMITTER
f_1
TUNING UNIT
LOAD CONTROL
SUPERVISORY
TELEMETER
TRIP CIRCUIT
f_2
AUDIO AMPLIFIER
SUPERHETERODYNE RECEIVER
f_3
TELEPHONE
SINGLE-FREQUENCY DUPLEX TELEPHONE AUXILIARY EQUIP.
TRANSMITTER
SUPERHETERODYNE RECEIVER
O- OSCILLATOR
F- FILTER
AUDIO AMPLIFIER
SUPERVISORY
TELEMETER
TRIP CIRCUIT
TUNING UNIT
f_1
SUPERHETERODYNE RECEIVER
f_2
MIXER AUDIO AMPLIFIER & MODULATOR
LOAD CONTROL
SUPERVISORY
TELEMETER
TRIP CIRCUIT
MASTER OSCILLATOR AND POWER AMPLIFIER TRANSMITTER
f_3
TRANSMITTER
SINGLE FREQUENCY DUPLEX TELEPHONE AUXILIARY EQUIP.
TELEPHONE
SUPERHETERODYNE RECEIVER

Figure 4. Block diagram of elements in carrier-current equipment

practically constant supervision over this tripping circuit in a manner not described in detail in this paper.

Additional Uses of the Carrier Current

Carrier equipment for tripping of the breakers at the remote end of the line was a necessity for protection of costly terminal equipment. Having justified the carrier equipment for this purpose, it was relatively inexpensive to extend the carrier scheme for the five following major purposes:

1. Tripping of circuit breakers at the remote end of the line for transformer or terminal equipment failures.
2. Transmitting from Powerton to Chicago, telemetering impulses to indicate at the dispatcher's office the total output of the Powerton Station.
3. Transmitting from Chicago to Powerton telemetering impulses which indicate at Powerton the total output of all generating plants in the Chicago district system.
4. Transmission of impulses from Chicago to Powerton to control the loading of turbines.
5. Two-way telephone communication for dispatching and routine business.

Figure 4 is a block diagram showing the elements involved in this carrier scheme. As a further insurance of the reliability of the carrier channels, each carrier channel is coupled to two phases of the transmission line. The carrier transmitters have sufficient power, should one conductor be open-circuited or grounded, to continue to operate satisfactorily with phase-to-ground coupling on the remaining line conductor.

Referring again to figure 4, the communication unit has a single carrier band to be modulated with voice frequencies. The other two channels each have one carrier frequency modulated with six or eight separate audio frequencies used in connection with the several purposes of the carrier scheme.

As a result of the high-speed relaying and remote trip, it is expected that even without high-tension breakers on the transmission circuit, faults will be cleared from the system in less than one-sixth second.

Line Terminal at Crawford Station

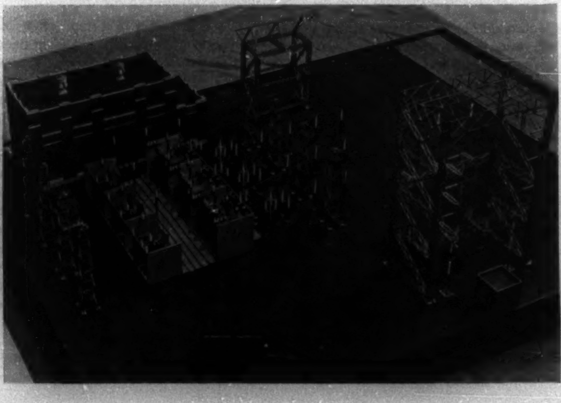
The terminal for the line at Crawford Station presents a number of novel features. In addition to being the first transmission terminal for 220,000 volts between the Alleghenies and the Rockies, this terminal receives an unusually high load per line. The amount of load on the one line is such that for operating safety it is desirable to divide the output of the line between the so-called "red" and "blue" systems in Chicago. These are the two halves of the 66,000-volt transmission system which operate as parallel paths between all stations. To meet this condition a transformer bank was designed with four separate windings; one winding at 220 kv with a capacity of 150,000 kva, two 66-kv windings, each with a capacity of 90,000 kva, and a fourth winding at 12 kv with a capacity of 75,000 kva for direct connection to the synchronous condenser. The division of reactive kva and power flow is such that all four windings may be loaded to rated capacity at the same time.

Figure 5 is a photograph of the Crawford terminal. Actually, the photograph is of a scale model made in advance of construction in order to assist the designers in working out some of the more complicated design details. Because the model is quite complete, it is used to show the complete structure more effectively than a drawing. The three conductors of the 220-kv line are connected to three strain insulators in the foreground on the tall rectangular tower. This tower is constructed so that a second line, when built, may be terminated on the opposite side.

There are no high-voltage circuit breakers, and the 220-kv disconnects are directly beneath the large strain tower. The lightning arresters are adjacent to the transformer compartments in order to have them as near as possible to the transformer bushings for maximum protection.

Figure 5. Crawford terminal

Photograph of scale model used in working details of design



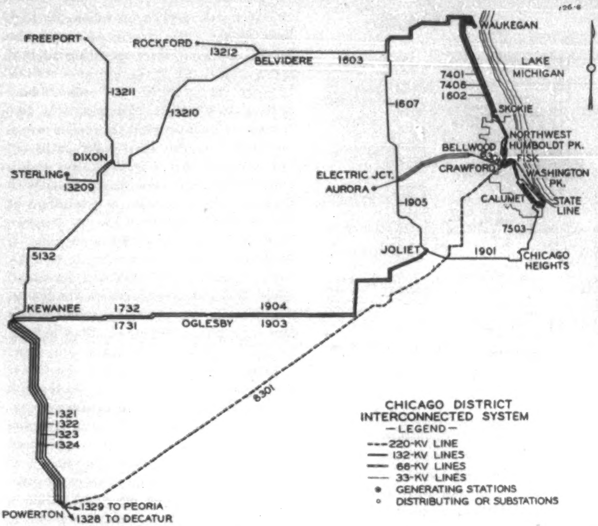


Figure 6. Chicago district interconnected system

This map, approximately to scale, shows the relation of the new 220-kv line to the existing 132-kv system between Powerton and Chicago

The entire structure is covered by a network of overhead ground wires for lightning protection. The line ground wires are brought into the top of the 125-foot rectangular tower in the foreground to complete the lightning shield.

Each of the three single-phase transformers is in a separate compartment, mounted on a movable truck, and located over an oil pit filled with crushed stone. Each regulating transformer is in a separate tank in a separate compartment opposite the main transformer on the corresponding phase. Each of the main transformers covers a ground area of approximately 29 feet by 22 feet, and is nearly 20 feet high. The total height of the bus is slightly more than 30 feet. Each unit contains 24,000 gallons of oil and weighs 491,000 pounds.

The regulating units cover a ground area of 18 feet by 15 feet and are 15 feet high. The regulating transformers contain about 6,600 gallons of oil.

Synchronous Condenser Building

The large building in the background houses the 75,000-kva synchronous condenser in the left end of the building. In the right end of the main portion of the building is the pit which is used for assembling and maintaining the transformer units. There is a 200-ton crane in the building with a maximum lift of 60 feet above the floor of the transformer pit. The purpose of the pit for handling the transformers is to reduce the total height of the building and use the same crane for assembling the synchronous condenser and the transformers. The transformer pit is 31 feet by 36 feet and 18 feet below the main floor level.

Synchronous Condenser

The 75,000-kva 514-rpm condenser weighs 690,000 pounds. The heaviest part is the rotor, which weighs 350,000 pounds. It is started by a separate motor of 3,750 horsepower. This motor is appreciably larger than necessary for starting purposes alone. It is used to excite the transmission line and synchronize it with the system to avoid the voltage disturbance caused by closing the circuit breaker on a long line with a high charging current. The transmission line is connected to the synchronous condenser at rest and the condenser brought up to speed with the starting motor. Then the field is applied to the synchronous condenser to bring the line up to normal voltage for synchronizing with the 66,000-volt system.

The relay boards, the auxiliary power, and miscellaneous electrical equipment are housed in the south end of the synchronous condenser building in a number of separate rooms. This end of the building also includes a heating plant. The control for the synchronous condenser and the switching equipment is located in the main station control gallery about 1,000 feet from the synchronous condenser building. Here it is under the supervision of the same operators who control turbines and outgoing lines.

The oil handling equipment is in an underground section to the east of the synchronous condenser house. The two 25,000-gallon oil storage tanks are in a separate underground room beyond the oil handling equipment.

The 12-kv outdoor oil circuit breaker which connects the synchronous condenser to the 12,000-volt winding is located just to the west of the synchronous condenser house adjacent to the main transformer bank.

Spare Equipment

The terminal is provided with a spare single-phase main transformer unit mounted on a truck. In event of transformer failure, the defective transformer can be rolled out of place and into the building for repairs. The spare unit can then be rolled into the place of the one removed. Such a replacement, including the disconnecting of oil and water pipes, bushings, etc., can be accomplished in a few hours.

There is also a spare regulating transformer unit. This regulating unit can also be wheeled into place as a substitute for any regulator which might fail in service. The use of regulating transformers in separate tanks has been standard practice with this company for many years because regulating transformers have many more parts subject to failure than do the main transformers. Experience has shown that it is economical to separate the tap changing equipment from the main unit so that trouble, if it should occur, would be confined only to the smaller capacity regulating transformer.

The small building directly in front of the tall tower contains the carrier relaying and communication equipment. The photograph of the model shows only two of the carrier coupling units. Carrier coupling units are used on all three phases in order to perform the several functions previously referred to.

Line Terminal at Powerton Station

The Powerton terminal is similar to the Crawford terminal for the same line in some respects. However, the Powerton transformer bank contains three windings in contrast to four at Crawford, and obviously, no synchronous condenser is necessary at the sending end of the line. The transformer handling house is much smaller than the one at Crawford because it contains space for the transformer and oil handling only.

Comparison of Powerton and Crawford Terminals

Tabulation I attached includes a number of detailed figures on the Powerton and Crawford terminals. For comparison the essential features of the two terminals are shown in parallel columns. There is a surprising similarity between the designs in spite of the fact that the two terminals were designed by different engineering organizations, and that one terminal includes four-winding transformers and a synchronous condenser while the other terminal has three-winding transformers and no synchronous condenser.

In both terminals it was found to be necessary to have a building with a crane for assembling and possibly repairing transformers. This was in part dictated by the fact that the transformer units were so large that they could not be shipped completely assembled. The core and coils had to be sent in shipping tanks and assembled on the job in the main transformer tanks with radiators. The transformer handling house is different in the two cases for a very good reason. At Powerton the only large equipment to be handled is the transformer and, therefore, it was economical to build a tall building with the transformer on the ground level and with clearance below the crane rails sufficient to lift the core and coils out of the tank. At Crawford the heaviest single part of the synchronous condenser is near the same size as the heaviest single part of a transformer. However, the synchronous condenser requires much less head room than that needed to remove core and coils from the transformer tank. Therefore, the most economical building design was obtained by having the clearance under the crane sufficient for the synchronous condenser and by digging a pit at one end into which the transformer could be lowered for removing core and coils. With this design, the total volume of building is considerably less than would be the case if the crane rails were high enough for transformer handling.

The local conditions at Powerton and Crawford have a very material effect on the steel structure where the line terminates. At Powerton where ample land is available and where a future outgoing line would be on a separate single-circuit tower line, no structure is provided for any future high-voltage switching equipment. At Crawford a future second circuit will be brought into the station on the opposite side of the same double-circuit tower and brought around the terminal. The terminal tower is, therefore, built to take the strain of the present and future incoming line; also, the structure is complete for a later installation of the line and transformer circuit breaker when they are required in connection with the second parallel circuit. This difference between the two terminals accounts for the larger amount of structure at the Crawford terminal.

Spare Equipment

There is another material difference between the local conditions at Powerton and Crawford. Powerton Station, being farther away from the city, has neither the men nor machinery available for rapid changing of transformers in case of failure. It was, therefore, decided to have the spare transformer and spare regulating units permanently installed. Bus work is provided so that in the event of transformer failure the spare transformer can be connected in the proper phase of the line without any physical moving of the equipment. Such a change can be made with a relatively small force in a few hours. The intention was that the time required to make replacements would be of the same order of magnitude at both ends of the line.

Conclusion

The building of this 150-mile 220-kv transmission line with two terminals results in a material improvement in the reliability of the bulk power supply to metropolitan Chicago. The line and terminals are built to be physically much more reliable than any others in this area. In addition to this, the line traverses a route which is entirely different from that of any existing 132-kv line supplying Chicago. It seems reasonable to assume that no sleet storm or any other contingency would simultaneously strike the routes of all these transmission lines. In the event that some disturbance would take out part or all of the 132-kv lines going north from Powerton, much of the generating capacity can be transferred to the 220-kv line and delivered to Chicago over this route.

Figure 6 is a map of the interconnected system showing all of the 132-kv lines in addition to the new 220-kv line. With this transmission system power from outside sources can enter Chicago through three different gateways. By so diversifying the routes it is believed that the reliability of supply from these stations at some distances from Chicago is satisfactory for the high service standards necessary in a large city.

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Television Transmission Over Wire Lines

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Synopsis: Intercity networks appear vital to the success of television broadcasting. Experiments with wire lines for this purpose and for local transmission of present-day television signals are reported herein. The design and construction of the equipment used are described and its performance characteristics given.

The intercity lines discussed involve carrier transmission over coaxial cable with repeaters which pass a net band of about $2^{3}/_{4}$ megacycles. For local intracity connections video transmission of about a 4 mc band is obtained over existing telephone plant or by means of special low attenuation cable. Various circuit arrangements including the facilities used in bringing scenes from the Republican Convention in Philadelphia to the N.B.C. in New York are shown together with their overall television transmission characteristics.

Introduction

If the development of television broadcasting follows in the footsteps of its predecessor in the sound broadcasting field, networks for interconnecting television stations will be very important. In fact many students[1] of the problem believe that such networks are a virtual necessity because of the expected high cost of programs.

Considerable progress in the development of a wire line technique for this purpose has been made in connection with the Bell System's study of coaxial conductor systems for use in wide band telephony. Data previously published[2,3,4] have been supplemented recently by certain tests and experiments in the transmission of 441-line television images, the results of which are presented in this paper. This will cover the transmission characteristics of facilities both for intercity and local distribution, including the wire lines which were used during the television broadcast in New York of the proceedings of the Republican Convention in Philadelphia during the last week of June 1940. This broadcast was undertaken jointly by N.B.C. and A.T.&T. and its associated companies as an experiment in the furtherance of the television art. A large part of the experimental facilities used were manufactured by the Western Electric Co.

Long Haul Coaxial Systems

For long distance broad-band transmission, coaxial systems have certain natural advantages which have been previously pointed out. In common with all long distance systems for multiplex telephony, the carrier method of transmission is essential and has been found to be relatively straightforward. For long distance television transmission, the carrier method is necessary with the present coaxial lines and coaxial repeaters, due to the fact that satisfactory long distance transmission cannot be obtained at the very low frequencies involved in a video television signal. Hence for television the entire signal must be raised bodily to a higher frequency. The modulating means developed for this purpose will be described in detail later. In this section we will confine discussion to the transmission of a broad-band of frequencies independent of how this band is used.

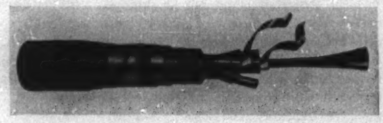
Figure 1. Photograph of coaxial cable

Cable

The transmission characteristics of ideal coaxial cables have long been known. The properties of practical structures so far built including matters of crosstalk or shielding are also now well understood.[5,6] Certain mechanical improvements in construction have been made recently[7] and may be illustrated by a photograph, figure 1, of the recently installed Baltimore-Washington coaxial cable. A similar construction was used in a cable completed last summer between Stevens Point, Wisconsin and Minneapolis, Minnesota.

These cables each contain 4 coaxial units. Two of these are used to provide a normal broad-band system having one pipe for each direction of transmission. The other two provide spare facilities for each direction. The construction of the coaxial unit itself can be seen from the photograph to use a single longitudinal copper tape for the outer conductor. This is formed into a tube which is held to a fixed diameter by the width of the tape and is prevented from collapsing by the interlocking of its saw-toothed edges. Two layers of steel tape provide the needed support against buckling and also give additional shielding. This construction results in somewhat improved transmission characteristics and lower manufacturing costs as compared with other types of construction with which we have experimented. Improvements in transmission include lower attenuation, due to a reduction in the effective resistance of the outer conductor, and a smoother impedance frequency characteristic due to greater mechanical uniformity. In spite of the thinner outer conductor satisfactory crosstalk characteristics are obtained.[8] Typical attenuation, crosstalk, and impedance characteristics of this cable as a function of frequency are shown in figures 2 and 3 for a 5-mile length of installed cable.

Repeaters

The band width of a coaxial system, at least over regions which we have studied, is limited only by the amplifiers with which it is provided. The amplifiers which have been built most recently for use in these systems are known as "3-megacycle amplifiers" and were intended to provide about a 2-megacycle band of suitable characteristics for telephone purposes or about a $2^{3}/_{4}$ megacycle band suitable for television transmission.

Figure 4 shows one of these amplifiers. It is a three stage feedback device using two small pentodes in parallel in each stage. The mathematical design of the circuit is beyond the scope of this paper and has been treated elsewhere.[9] This type of pentode has an initial transconductance of from 2,000 to 2,500 micromhos and an output power of .1 to .2 watt at 130 volts as used in this system. These tubes are in parallel only to give added reliability. The gain of this amplifier is very roughly the complement of the line loss as a function of frequency. With this amplifier and the cable described above, these repeater sections are about $5^{1}/_{4}$ miles in length. As illustrated in figure 5, the difference between the gain

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[1] For all numbered references, see list at end of paper.

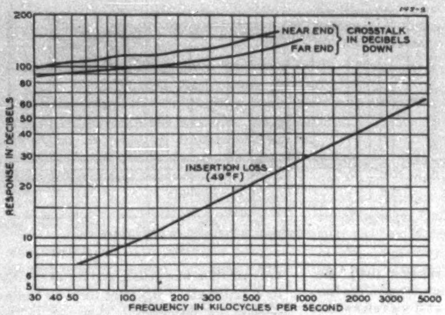


Figure 2. Attenuation, crosstalk

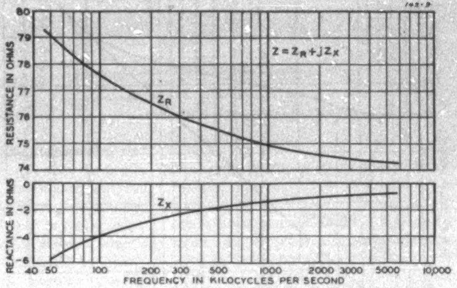


Figure 3. Impedance of five miles

and line loss is made up by a line equalizer so that to a first approximation, zero loss in transmission is obtained at all frequencies within the band over each repeater section. About 30 db of feedback is effective over the telephone frequency band (i.e. up to 2,000 kc) around the entire amplifier with about 10 db additional around the final stage. From 2 mc up to 3 mc the feedback gradually falls off about 10 db. This arrangement gives the high degree of transmission stability and linearity required for long telephone systems with hundreds of amplifiers in tandem and satisfactorily meets present requirements. Limited experience with television transmission so far indicates satisfactory performance. The linearity is illustrated in figure 6 which shows measurements of 2nd and 3rd order modulation products of a 1,000 kc signal in a typical amplifier at various signal levels. As in previous coaxial systems, power for operating the amplifiers is transmitted at 60 cycles over the coaxial cable itself from main stations located at about 50-mile intervals.

REGULATION

In order to compensate for changes in attenuation due to temperature change of the copper conductors, the gain of the amplifier is regulated automatically by a device located at each amplifier point which is operated from a pilot channel. In this system a pilot frequency of 2,064 kc is transmitted along the line with the signal. At the output of each amplifier, a high impedance highly selective crystal filter is bridged on the circuit to select the pilot frequency. This is then amplified, rectified, and used to control the output of an oscillator. The oscillator output in turn is used to control the resistance of one element in the feedback circuit of the amplifier. This variable element is a very tiny thermistor[10] made up of certain oxides which have a very large negative temperature coefficient of resistance. The regulator is "back-acting" and maintains a substantially constant output voltage at the pilot frequency over a range of about 9 db in input voltage. The feedback circuit of the amplifier is so designed that the changes in the resistance of the thermistor produce changes in gain over the entire frequency band in such a way as to compensate for the changes in loss in the coaxial conductors, as illustrated in figure 7. Changes there shown are for ±70°F., which is about the maximum which is expected in a repeater section, even though the cable is of the aerial type.

In a long system, it has not been feasible to make the accuracy of equalization and regulation in each 5-mile section sufficient to give the desired overall uniformity of transmission. Hence, certain supplementary adjustment is required. Devices for such adjustment have been installed at 50-mile points on the Stevens Point-Minneapolis system with satisfactory results. Also, two additional pilot channels have been provided, one at 64 kc and one at 3,096 kc. These serve to indicate the circuit performance and the need for manual adjustment. These pilots could be used to actuate automatic regulators if desired. For longer systems, it is expected that additional, and necessarily more complicated, supplementary devices will be required at intervals of perhaps 200 to 500 miles.

PERFORMANCE

A complete repeater containing amplifiers for each direction of transmission, automatic regulators, equalizers, power supply, and various automatic alarm features is mounted in a box about 2x2x1 ft. as shown in a photograph (figure 8). Measurements on the overall performance of systems with many such repeaters in tandem indicate a high degree of transmission stability and freedom from noise. In the neighborhood of the pilot frequency the transmission variations are in the order of 0.1 db. At other frequencies there are slow drifts due to aging of tubes which, when they reach a few db, will require readjustment. These changes are now effected manually at the attended stations.

Interference from all sources both external and internal is very low in this system. The largest contributions of such interference are from tube noise and from thermal agitation in the conductors and circuit elements. The effect of interference from external sources so far encountered is lower than the above, although the presence of radio broadcasting stations can be detected. Intermodulation of signals traversing the system simultaneously has been very carefully

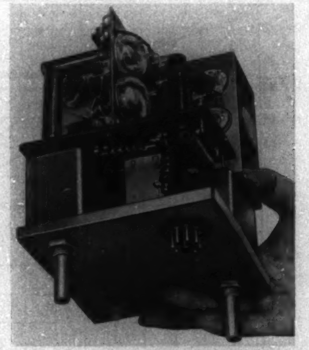
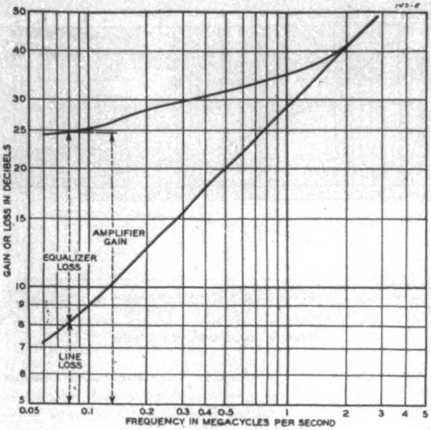


Figure 4. Three-megacycle amplifier in hand



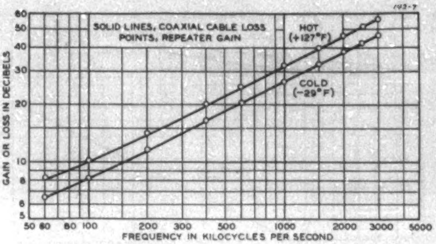


Figure 5 (left). Repeater-gain, line-loss, and equalization characteristics

Figure 7 (above). Regulation hot and cold versus frequency

measured because of its importance for multichannel telephony and television. In telephony, because of the large number of modulation products, principally 2nd and 3rd order, these appear as random interference.

The method of measurement of interference from all sources was to transmit over the system a wide band signal having a continuous spectrum such as thermal noise. At the sending end a narrow band elimination filter was inserted. At the far end the noise was measured within that same band.

The total noise so measured depends upon the signal energy levels at the input and the output of the repeaters, the former controlling the effect of line and resistance noise and the latter controlling the effect of modulation. These levels in turn are a function of repeater spacing. The tests that have been made indicate that it is practicable to keep this type of interference within desirable limits on long telephone or television circuits.

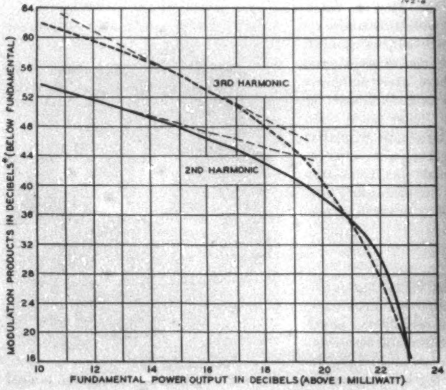
Due to the 60 cycle power supply used on the system, power frequency modulation products require special attention. Sixty cycle sidebands are produced on all signals transmitted due to the traces of nonlinearity in the system. As these are very small in magnitude and result mostly in a 120 cycle component they are unimportant for telephony. However, in the television transmission system used, this component is larger because of the presence of a strong carrier and one or more pilot channels. Also 120 cycle sidebands produce a very disturbing type of horizontal bar pattern across the picture. This type of interference will increase as the circuit length is increased, and may become more visible as receiving tubes are improved. On systems so far available for test, however, it has been possible to hold this type of interference within acceptable limits, on present day television broadcast images.

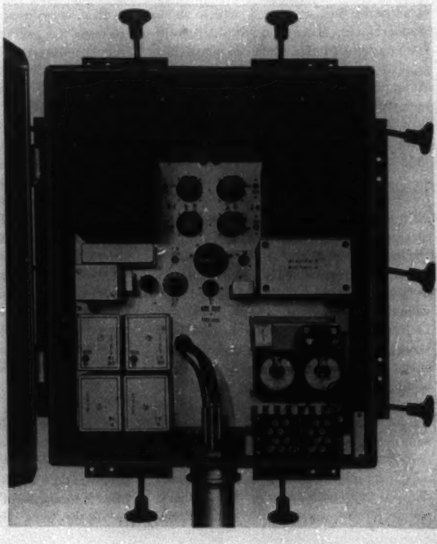
Distortion in Television Images

Departures from ideal transmission in the line, equipment or in a radio path

Figure 6 (below). Amplifier linearity

Figure 8 (right). Complete repeater



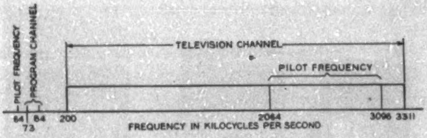


produce distortion in the form of negative or positive fringes or "ghosts." These occur when there is a lack of proportionality between phase shift and frequency through the system. This trouble in television images is perhaps more easily understood if one thinks of it as an actual difference in time of transmission of various parts of the signal. In discussing this matter in this paper, we will use the term "delay" to mean the time of transmission of the envelope of a modulated wave. This quantity is often more accurately referred to as "envelope delay."[11] If this quantity varies too widely there is an actual difference in the time of transmission of various parts of the signal, producing distortion in the form of fringes or "ghosts" which are exhibited by many television images today.

BAND WIDTH

A band width of about 3 mc is required to give equal resolution in the vertical and horizontal directions in a 441-line, 30-frame interlaced image. Recent experiments[12] with out-of-focus moving pictures have shown not only that the eye is quite insensitive in its requirement for equal detail in the two directions but also that the loss of detail due to a narrowing of the frequency band from 4 mc to $2^{3}/_{4}$ mc will pass unnoticed by many careful observers at normal viewing distance.

Figure 11. Frequency allocation on line


PILOT FREQUENCY
PROGRAM CHANNEL
TELEVISION CHANNEL
PILOT FREQUENCY
64
84
73
200
2064
3096
3311
FREQUENCY IN KILOCYCLES PER SECOND

Television on Coaxial Systems

As mentioned above, no practical method has been found for transmitting television over long haul coaxial circuits in the video frequency range. By the carrier method, however, the video frequency band may be raised to a region suitable for transmission. To conserve frequency space, single sideband transmission, of course, is desirable. The actual method chosen involves also a modest vestigial band since it appears impracticable to select a single sideband involving video frequencies as low as 45 cycles in any other way. The present coaxial amplifiers pass a band from about 64 kc to about 3,100 kc. The region useful for television, however, appeared to be somewhat less than three megacycles on account of the difficulty of equalizing the delay distortion near the lower edge of this band. About 100 kc was allotted to obtain proper shaping of the vestigial sideband. The carrier was, therefore, placed at about 300 kc and a net television band of about $2^{3}/_{4}$ mc was obtained. If we attempt to move a 3 mc video band up 300 kc in a single step of modulation, the result is an overlapping of the sidebands which hopelessly distorts the signal. Two steps of modulation are therefore resorted to as shown in figure 9.

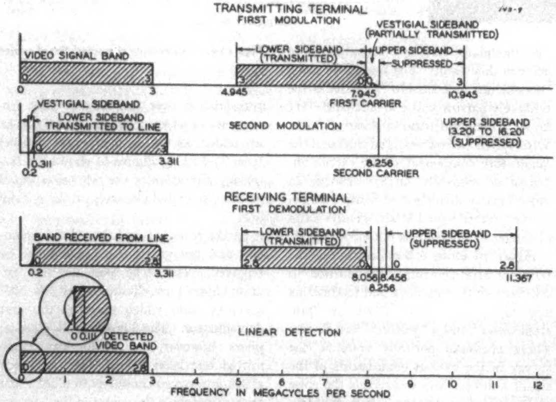

TRANSMITTING TERMINAL
FIRST MODULATION
VIDEO SIGNAL BAND
0
3
LOWER SIDEBAND (TRANSMITTED)
VESTIGIAL SIDEBAND (PARTIALLY TRANSMITTED)
UPPER SIDEBAND
SUPPRESSED
4.945
7.945
FIRST CARRIER
10.945
VESTIGIAL SIDEBAND
LOWER SIDEBAND TRANSMITTED TO LINE
SECOND MODULATION
UPPER SIDEBAND 13.201 TO 16.201 (SUPPRESSED)
0.311
0.2
3.311
8.256
SECOND CARRIER
RECEIVING TERMINAL
FIRST DEMODULATION
BAND RECEIVED FROM LINE
LOWER SIDEBAND (TRANSMITTED)
UPPER SIDEBAND (SUPPRESSED)
0.2
2.8
3.311
8.056
8.456
8.256
11.367
LINEAR DETECTION
0 0.111
DETECTED VIDEO BAND
0 1 2 3 4 5 6 7 8 9 10 11 12
FREQUENCY IN MEGACYCLES PER SECOND

Figure 9. Modulation steps diagram

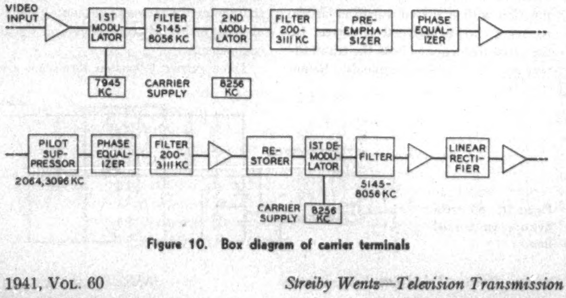

VIDEO INPUT
1ST MODULATOR
FILTER 5145-8056 KC
2ND MODULATOR
FILTER 200-3111 KC
PRE-EMPHASIZER
PHASE EQUALIZER
7945 KC
CARRIER SUPPLY
8256 KC
PILOT SUPPRESSOR
2064,3096 KC
PHASE EQUALIZER
FILTER 200-3111 KC
RESTORER
1ST DEMODULATOR
FILTER
5145-8056 KC
LINEAR RECTIFIER
CARRIER SUPPLY
8256 KC

Figure 10. Box diagram of carrier terminals

The energy of a television system is concentrated in the lower frequencies or, in a carrier system, near the carrier. To take most advantage of the coaxial system, the carrier should be at the low end where the full feedback in the amplifiers is available. The four lines in figure 9 illustrate the four stages of modulation, two at the transmitting terminal and two at the receiving terminal. As can be seen the signal is first modulated with a carrier of about 8 megacycles and the lower sideband, part of the carrier, and a portion of the upper sideband, are selected by a band filter. This signal is then modulated again with a carrier of about 8.3 megacycles and the lower sideband again selected. In this position of the signal, which is the position at which it will be transmitted over the coaxial line, the frequency which corresponds to d-c in the video signal sideband is at 311 kc, the main sideband extends from 311 to 3,111 kc and the vestigial sideband from 311 kc down to 200 kc.

The receiving terminal is in general the inverse of the transmitting terminal and will not be discussed in detail. The sideband shaping[4] is accomplished by the four filters, two at the transmitting terminal and two similar ones at the receiving terminal, acting in conjunction. The re-

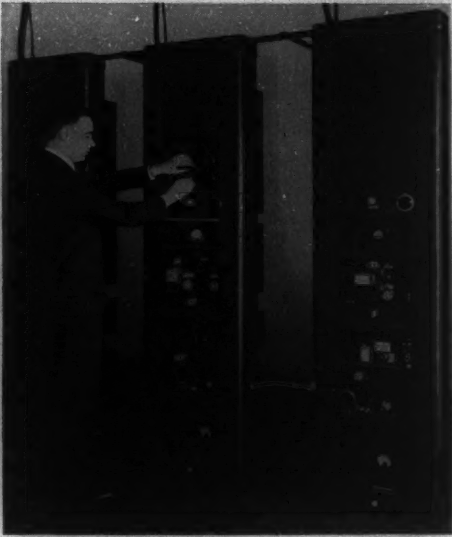


Figure 12. Carrier terminals

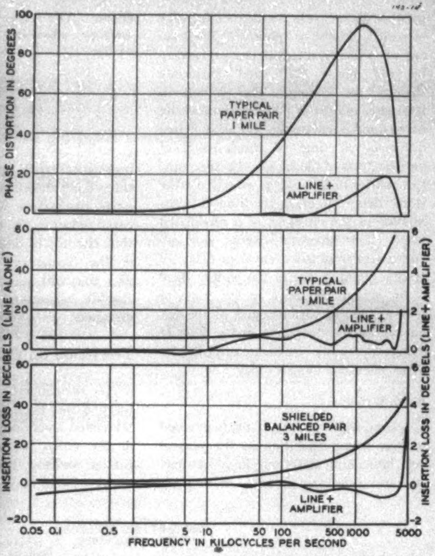


Figure 14. Transmission characteristics of cables

sult is that at the final stage of demodulation the contribution from the vestigial sideband when added to the contribution from the shaped portion of the main sideband gives back very nearly an undistorted video signal. This last stage of demodulation is accomplished in a linear detector. The carrier amplitude at the input terminals of this detector is about six db greater than the amplitude of the video envelope of the modulated signal, the amount of carrier which was mixed with the sidebands at the output of the first modulator having been adjusted to achieve this result. The reason for using this amount of transmitted carrier is the relatively narrow vestigial sideband—111 kc vs. a main sideband of about $2^3/_4$ mc. With such a narrow vestigial sideband the quadrature component of the carrier envelope is relatively large, resulting in objectionable distortion at sharp changes in the picture signal if the greater ratio of carrier to sideband is not employed.[13]

Figure 13. Photograph of video amplifier

Figure 10 shows a box diagram of the terminal arrangements. In addition to ordinary video amplifiers and modulators and filters mentioned above, a "pre-emphasizer" and a "restorer" are shown. These networks partially equalize the energy in the various components of the signal, and thus help to override the noise and spurious modulation products introduced by the line and amplifiers. A phase equalizer is also shown which, in conjunction with a similar equalizer at the receiving end, is designed to correct for the phase distortion in both the transmitting and the receiving terminals. Before transmission over the coaxial, pilot frequencies of 64 kc, 2,064 kc, and 3,096 kc are added, as well as a program channel from 73–84 kc. Figure 11 shows the frequency allocation of the television signal and its associated channels on the coaxial line.

At the receiving end the pilot frequencies and the program channel must be removed. The 64 kc pilot and the program channel are eliminated by the 200–3,111 kc filter which precedes the first demodulator. The 2,064 kc and 3,096 kc pilots, however, are within the transmitted television band. The frequency allocation was so chosen as to place them approximately in the center of the "empty energy regions"[14] of the television spectrum where they can be eliminated by sharp selective networks without appreciably distorting adjacent television signal components.

Three carrier television terminals are

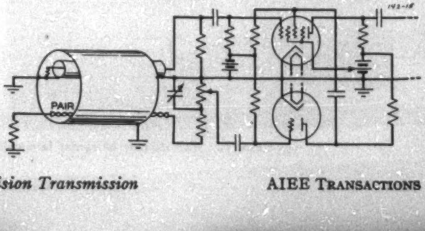


Figure 15. 60-cycle balance on coaxial line

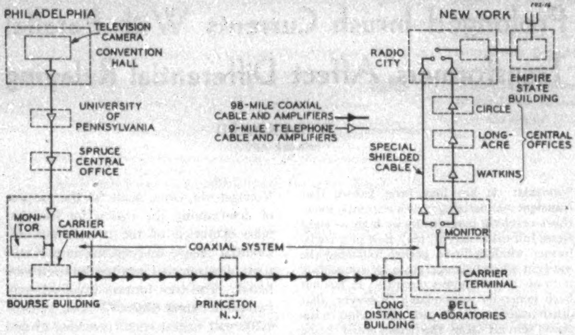


Figure 16. Map of New York-Philadelphia network

shown in the photograph, figure 12. The one on the right is a transmitting terminal, the two on the left receiving terminals. Each terminal occupies one six foot relay rack bay and is complete with power supply and means for adjustment.

Short Haul Lines for Television

For the pickup or transmission of television within cities or metropolitan areas, it appears to be more economical, as would be expected, not to use the carrier method described above but to transmit "video" frequency signals over cable circuits. For this purpose existing telephone cables may be used or special cables may be provided. In either case amplifiers and special equalizers are required which will overcome the attenuation and delay distortion of the cable circuits. Because of high frequency crosstalk usually only a small fraction of the circuits in any existing telephone cable can be used simultaneously.

Video Amplifiers and Equalizers

Television pickup and broadcasting equipment is quite naturally designed on an unbalanced (i.e. one side grounded) basis. Unbalanced amplifiers for the video band have been available for some time.[15] New amplifier designs have been worked out for use with balanced lines. In general, the problem is to provide approximately zero loss and constant delay between unbalanced terminals a mile or more apart. Thus, an unbalanced to balanced amplifier is required at the sending end, the converse at the receiving end. If the circuit is long, balanced amplifiers are most convenient for use at intermediate points. The equalization problem has been successfully met even if ordinary telephone cables are used. A series of variable equalizers have been experimented with which have several degrees of flexibility. A variety of circuits ranging in length up to 9 miles have been equalized with this arrangement with considerable success. A typical amplifier, equalizer, and power supply are shown in the photograph, figure 13.

Telephone Cables

Ordinary fine wire paper insulated cables have very high attenuation at the frequencies required. Typical values for loss and net loss after amplification and equalization are shown in figure 14. Experience has shown that the noise levels in such cables even at the higher video frequencies are rather high so that amplifiers are required at intervals of a mile or even less. Local telephone cables are usually laid out with many branches. At high frequencies these branches introduce irregularities which, similar to those produced by obstacles along a radio path, cause delay distortion. Plant changes are frequently required to obtain a clean circuit free from such bridged taps. When amplifiers and proper equalizers are added, however, substantially flat transmission is obtained as shown in the figure.

Phase distortion characteristics of a typical cable circuit are also shown in figure 14. After the amplifiers and equalizers are added, the phase distortion is made substantially negligible.

Coaxial Cables

Coaxial cables may be used for video transmission for short distances but power or other low frequency interference may introduce serious problems. Coaxial units of the size discussed above have been used in a few cases a mile or so in length. Even for such distances, however, it has been found desirable to reduce the power interference by balancing it out. One method which we have used is shown in figure 15. This has given an improvement at power frequencies of the order of 50 db in certain cases.

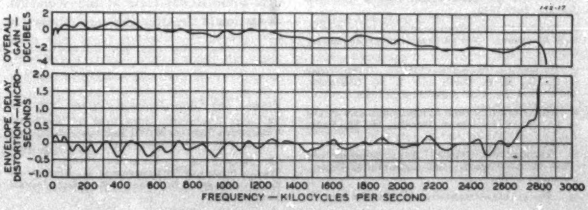
Balanced Shielded Cables

The ideal type of transmission line for video signals combines the balance feature with low attenuation and a high frequency shield. The distance over which such cables could be used appears to depend upon the perfection of balanced video amplifiers and the equalization, although power interference may also present difficulties. Such cables have been built using a pair of wires and a disc type of insulation analogous to the coaxial structure described above. Attenuation measurements on a 3-mile test length installed in New York City are shown in figure 14. This figure also shows the net result after amplifiers and equalizers were added. The attenuation of this special type of cable is such that amplifiers would be required at only about 5-mile intervals. The useful range of such a cable for video transmission has not been determined but in any case it should be considerably greater than that of the paper-insulated telephone cable circuit.

Experiment in Network Broadcasting

During the last week of June 1940, the proceedings of the Republican Convention in Philadelphia were broadcast in New York by television. The facilities

Figure 17. Attenuation and delay—New York-Philadelphia network



used included the 3-megacycle coaxial system plus certain video connections at each end as shown in figure 16.

Because of the interest in this circuit and its good performance in transmitting 441-line television, the overall attenuation and delay characteristics are given on figure 17. It will be noticed that a net band of about $2^{3}/_{4}$ megacycles was transmitted and that over most of that band the delay distortion did not exceed ±0.2 microsecond. The random noise, modulation, and other distortions introduced by the wire line network appeared to be unimportant when viewed on a commercial television receiver.

Conclusion

The experiments so far made in the transmission of present-day television indicate that wire lines can be provided at least for moderate size intercity networks, also, that such lines if properly equalized for delay and attenuation do not materially alter or distort the transmission of present-day 441-line images, even though the frequency band is somewhat narrower than the nominal 4-mc band.

The use of ordinary telephone cables for local television connections also has been found to be feasible for all of the conditions so far tested. The $2^{3}/_{4}$ mc television transmission experiments over wire lines reported herein have proved very successful. Experiments with wider band coaxial systems are being undertaken.

References

1. Television Broadcasting (a book), **L. R. Lohr.** McGraw-Hill Book Company, 1940, chapter 8.

2. Wide-Band Transmission Over Coaxial Lines, **L. Espenschied** and **M. E. Strieby.** AIEE Transactions, volume 53, 1934 (October section), pages 1371–80.

3. Million-Cycle Telephone System, **M. E. Strieby.** *Bell System Technical Journal*, January 1937.

4. Coaxial Cable System for Television Transmission, **M. E. Strieby.** *Bell System Technical Journal*, July 1938.

5. Crosstalk Between Coaxial Transmission Lines, **S. A. Schelkunoff** and **T. M. Odarenko.** *Bell System Technical Journal*, April 1937.

6. Crosstalk in Coaxial Cables—Analysis Based on Short-Circuited and Open Tertiaries, **K. E. Gould.** *Bell System Technical Journal*, July 1940.

7. Stevens Point-Minneapolis Coaxial Cable, **O. S. Markuson.** Bell Laboratories *Record*, January 1940.

8. Crosstalk Between Coaxial Conductors in Cable, **R. P. Booth** and **T. M. Odarenko.** *Bell System Technical Journal*, July 1940.

9. Relations Between Attenuation and Phase in Feedback Amplifier Design, **H. W. Bode.** *Bell System Technical Journal*, July 1940.

10. Thermistors—Their Characteristics and Uses, **G. L. Pearson.** Bell Laboratories *Record*, December 1940.

11. Delay Measurements, **H. Nyquist** and **S. Brand.** *Bell System Technical Journal*, July 1930.

12. The Subjective Sharpness of Simulated Television Images, **M. W. Baldwin.** *Bell System Technical Journal*, October 1940.

13. Effect of Quadrature Component in Single Sideband Transmission, **H. Nyquist** and **K. W. Pfleger.** *Bell System Technical Journal*, January 1940.

14. A Theory of Scanning and Its Relation to the Characteristics of the Transmitted Signal in Telephotography and Television, **P. Mertz** and **F. Gray.** *Bell System Technical Journal*, July 1934, pages 464–515.

15. Analysis and Design of Video Amplifiers, **S. W. Seeley** and **C. N. Kimball.** RCA *Review*, October 1937.

Prolonged Inrush Currents With Parallel Transformers Affect Differential Relaying

C. D. HAYWARD
MEMBER AIEE

Synopsis: It has long been known that transient magnetizing inrush currents, sometimes reaching magnitudes as high as eight times full-load current, may flow in a transformer winding for a period following the moment when it is energized by connecting it to an electric power circuit.[1] It has not been generally appreciated, however, that other transformers already connected to the power circuit near the transformer being switched may also have transient magnetizing currents of considerable magnitude at the same time, although they themselves are not switched but remain continuously connected to the power circuit, carrying load. It has not been appreciated, moreover, that with this arrangement the transient periods of the inrush currents may be very long, the currents dying away at a much slower rate than would the inrush current of the transformer being switched if the others were not connected. This paper discusses the cause of these phenomena and describes tests made to investigate their occurrence. The results of a mathematical analysis for the currents in the circuit under various conditions, supporting and extending the test results, are given in the appendix. The amount of resistance in the transmission line circuit connecting the parallel transformers with the generating source is shown both by tests and mathematics to be a determining factor; the magnitude of the inrush current in the already connected transformers increasing to values of over twice full load current as the line resistance increases.

Magnetizing inrush currents tend to operate the transformer differential relays, causing the circuit breakers to trip and remove the transformer from service when no fault exists within it. Various means have been devised to prevent the operation of the relays on magnetizing inrush current. This paper discusses the effectiveness of these various means when subjected to the inrush currents occurring with parallel-connected transformers, with particular reference to the harmonic restraint relay described in a recent paper.[2]

Ohio Edison Company Tests

In some recent tests at the Wood St. station of the Ohio Edison Company in Youngstown, Ohio, made for the purpose of determining the reason for frequent false operation of the transformer differential relays, magnetizing inrush currents of extremely long time duration were found. The transformers involved were two similar three-phase, 60 cycle, 23,000/2,300 volt banks, each consisting of two 3,333 kva single-phase transformers connected in open-delta. The two banks were connected in parallel to busses on both the high and low voltage sides. Load was fed from the 2,300 volt bus, while the 23 kv bus was supplied through two transmission lines, each of which led back to generation over a network of lines having an equivalent single-line length of about five miles. The oscillogram figure 1 was taken during a light load period with only one of the two lines feeding the station in service. One transformer bank was in use, carrying about 20 per cent normal load. The other transformer bank was switched on the 23 kv bus with its 2,300 volt side open. The three traces on the oscillogram show the three phase currents in the 23 kv line feeding the bus. The currents are magnetizing inrush currents to the two transformer banks superimposed on the 20 per cent normal load current being carried by the first bank. The currents decay at a very slow rate;

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The author wishes to acknowledge with thanks the cooperation and aid extended by F. W. Funk, N. A. Hooper, and A. E. Flink of the Ohio Edison Company in making tests at the Wood Street station. L. F. Blume, H. A. Petersen, G. Camilli, L. F. Kennedy, and C. R. Mason, all of the General Electric Company, also contributed much thought and effort to the investigation described in the paper.

1. For all numbered references, see list at end of paper.

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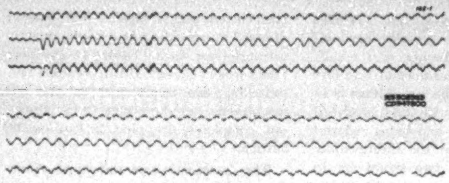

(Lower three traces are a continuation of the upper three)

Figure 1. Long duration magnetizing inrush currents obtained with parallel transformer banks at Wood Street station of the Ohio Edison Company

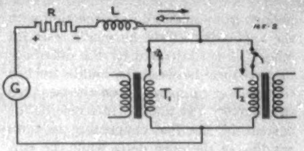

Figure 2. Schematic diagram of circuit used in laboratory tests

the magnetizing inrush components at the end of the oscillogram, 66 cycles after the second bank was switched on, are still of nearly full-load current magnitude. Other oscillograms taken with the first bank disconnected from the bus while the second was switched on showed magnetizing inrush currents of much shorter duration, the rate of decay as measured from the oscillograms being over ten times as fast. The time constant as measured from the oscillograms was 1.68 seconds when the parallel transformer was connected compared with 0.166 second when it was not connected.

Laboratory Tests

Following the Ohio Edison Company tests the occurrence of inrush currents in parallel transformers was extensively investigated in the General Electric Company laboratories. In one series of tests, two identical single-phase, 333 kva, 60 cycle, 13,800/460 volt transformers were used in a circuit shown schematically in figure 2. For convenience, connection was made to the 460 volt windings of the transformers from a 500 volt source. With one transformer already connected to the source, the second was switched in parallel with it by means of a synchronous closing switch set to close at the zero point of the voltage wave to obtain maximum inrush current. In addition, the direction and magnitude of the residual core flux in the transformer being switched was fixed before each test by passing a direct current through the transformer winding for a short period. The oscillogram figure 3*A* is representative of a series taken with no load on either transformer. The trace of vibrator No. 2 shows that an inrush current flows in the transformer being switched as would be expected. The trace of vibrator No. 3 however, shows that an inrush current having a maximum amplitude about one-third as great flows in the transformer which is already connected. The inrush current peaks in the two transformers occur on alternate half-cycles and are oppositely directed as is verified by the wave of the current in the incoming line traced by vibrator No. 4.

The amount of resistance in the incoming line circuit was found to be a great factor in determining the magnitude of the inrush current in the transformer already connected. Curves showing the variation of the maximum amplitude of the currents in the two transformers with change in line-circuit resistance are shown in figure 4. It should be noted that increasing the line resistance rapidly increases the maximum current in the already connected transformer up to a certain point after which it increases at a much slower rate with further increase in resistance. The maximum crest occurs at increasingly early current cycles as the resistance is increased as is indicated by the cycle number marked for each observed point. Other curves drawn in broken lines on the diagram show the crest amplitudes of the current at various numbered cycles after the start of the transient. These show that the two currents have the same amplitude after the first few cycles and die away at the same slow rate. This indicates a current composed of the direct current and even harmonic components circulating in the loop circuit of the two transformer windings without flowing in the incoming line. The slow decay is due to the very low R/L ratio of this circuit owing to the high magnetizing inductance of the transformer. Due to saturation of the core by the direct current the effective inductance is not as high as would be indicated by calculation from exciting current measurements made at normal voltage without the direct current. Calculations made from the rate of decay measured on the oscillograms indicate an effective inductance of only about 2.5 per cent of this value. Even with this great reduction, however, the inductance is still high enough to cause the rate of decay to be very slow. On one test made with a long oscillograph film the inrush current was found still to have a crest value equal to the crest value of a sine wave current of 30 per cent full-load magnitude after 150 cycles.

Increasing the inductance of the incoming line circuit has little effect except to reduce the magnitudes of both the inrush currents.

Increasing the resistance of the transformer loop circuit rapidly reduces the magnitude of the inrush current in the transformer already connected and increases the rate of decay of the circulating current. This represents the case of two parallel transformers separated by an appreciable length of transmission line instead of being connected directly to the same bus.

Loading the transformer already connected to the bus reduces the inrush current to it by only a small amount while affecting current in the transformer being switched even less. The oscillogram figure 3*B* shows a test made under the same conditions as for oscillogram figure 3*A* except that a steady resistance load equal to 87.5 per cent of rating was applied. The trace of vibrator No. 3 shows the inrush current superimposed on the load current. In a differential relay circuit, of course, the load current would be subtracted and the inrush current would appear alone.

A series of three-phase tests was made with small transformers connected in various delta and wye combinations to determine if any new phenomena not shown by the single-phase tests would be found. In particular, it was thought possible that some switching sequence and timing might set up a circulating current in a single delta-connected bank. No such effect was found, however, although extensive tests were made on delta-delta, wye-wye, delta-wye, wye-delta, and open-delta connections both with and without the neutral points grounded. When a three-phase transformer is switched in parallel with another three-phase transformer the long time constant circulating current is set up between corresponding phase windings in the two transformers in the same way as for single-phase transformers. When delta or open-delta primary connections are used the two delta-winding inrush currents combine to form a line current having a different wave form because of cancellation of certain harmonic components. This will be explained in more detail in connection with the application of the harmonic restraint relay. It is illustrated by the difference in wave form between the middle trace in the oscillogram figure 1 and the two outside traces. The middle trace records the current in the line connecting to both winding of the open delta while the outside traces record the single winding currents in the other two lines.

Cause of Phenomenon

The cause of this phenomenon, as first suggested by Mr. L. F. Blume, may be explained by reference to the diagram figure 2. Before the switch is closed the generator G sends a steady-state alternating current through the transmission line and the winding of transformer T_1. When the switch is closed, connecting transformer T_2, a magnetizing inrush current is drawn through the generator, transmission line, and the T_2 winding having a direct current component whose magnitude and direction depend on the point of the voltage wave at which the switch is closed. For the present discussion it may be assumed to be in the direction indicated by the solid arrows on the diagram. This direct current component produces a d-c IR drop voltage across the total resistance R of the transmission line and generator circuit having a polarity indicated by the plus and minus signs. This voltage, being applied to the winding of transformer T_1, causes a direct current to begin flowing through the circuit made up of the generator, transmission line, and the winding of transformer T_1 in series, the direction of which is indicated by the dotted arrows. This current increases at a rate determined by the IR drop voltage, and the R/L ratio of the circuit in which it flows. At the same time, the IR drop voltage, acting in the circuit consisting of transformer T_2, the transmission line, and the generator, causes the current indicated by the solid arrows to decrease at a rate determined by the R/L ratio of this circuit. After a time the two currents become substantially equal, and since they flow in opposite directions in the transmission line, they cancel each other and produce no more drop across the line resistance. The two currents then become substantially a single circulating current, flowing around the loop circuit made up of the two transformer windings in series and slowly dying away at a rate determined by the R/L ratio of the transformer windings.

The flow of direct current in the transformer windings causes the cores to become saturated in one direction. As a result the a-c exciting current becomes unsymmetrical, being greatly increased due to saturation of the iron on the half-cycles in which the transformer core flux required to support the a-c voltage is in the same direction as the flux produced by the direct current and being reduced practically to zero on the intermediate half-cycles when the two fluxes are in opposite directions. Since the direct current flows in opposite direction in the two windings, the cores become saturated in opposite directions and hence the current peaks in the two windings occur on alternate half-cycles.

Relay Application

Induction relays, of either the percentage-differential or overcurrent types, are most commonly used for differential protection of transformer banks, particularly banks of low kva rating. These depend on current setting and their inherent time-delay operation to ride over the magnetizing inrush period without tripping. This is quite satisfactory for general applications where parallel transformers are not involved and where the moderately long time delay operation introduced on internal faults is of no great importance either from the point of view of increased damage to the equipment or of system stability. Where parallel transformers are involved, however, the extremely long-time and high-current settings necessary to prevent operation on the long duration magnetizing inrush currents may reduce the protection afforded so seriously as to make it practically ineffective. In some cases it will be found that the maximum time setting available on standard relays is insufficient to prevent the tripping.

Several methods of preventing operation on magnetizing inrush, applicable to high speed relays, have been devised. These involve the use of auxiliary relays, operating on the transformer voltage, which in effect desensitize the differential relays sufficiently to prevent tripping on the inrush current during a set time delay period following the application of voltage to the transformer. Any of these methods would obviously be ineffective in preventing operation on inrush current flowing in the transformer already connected to the bus when the second transformer is switched on since, with voltage continuously applied to the transformer before and during the inrush period, the voltage relays cannot reset and apply the desensitization. The differential relays, with the inrush-trip-preventing device inoperative, are unable to distinguish between the inrush current and an internal fault current of equal magnitude. As a result, then, in a transformer station having two or more transformers connected to a common bus, switching one transformer on the bus might cause all the others to be tripped out, completely dropping the load on the station.

The harmonic restraint type transformer differential relay, in contrast with the others described, is admirably well suited for the protection of parallel banks. This relay, described in a recent paper,[2] is able to distinguish between the differential current due to an internal fault and that due to magnetizing inrush by their difference in wave form, operating with high speed on the fault current and being reliably restrained from operating by the magnetizing inrush current. The current due to an internal fault is nearly sinusoidal in wave form, containing little or no harmonic components. The magnetizing inrush currents, on the other hand, as may be seen from the oscillogram figure 3A, are nonsinusoidal, containing, in addition to the fundamental, large proportions of harmonics, principally the second and third. The fundamental component tends to operate the relay while the harmonic components tend to restrain it. Operation occurs only when the harmonic percentage is below the minimum amount occurring in magnetizing inrush currents. The harmonic restraint is effective as long as the magnetizing inrush current lasts, hence the relay will not operate on the magnetizing inrush current no matter how long it lasts, yet it is instantly ready at all times to operate on internal fault currents.

In the application of harmonic restraint relays to parallel transformers each transformer bank should, in general, be protected by a separate set of relays. The practice sometimes followed in the past of including two or more parallel transformers within a single differential zone with one set of relays may result in magnetizing inrush tripping due to cancellation of both the second and third harmonics in the differential current. When a delta or open-delta connection of the transformer primary winding or a delta connection of the current transformers is employed cancellation of the third harmonic occurs due to combination of the delta currents in the line. In the worst case, when the switch is closed at such a point on the voltage wave that equal inrush currents of opposite polarity occur in two of the windings, complete cancellation of the third harmonic occurs and the second is reduced to 57.7 per cent of its percentage in the winding. The harmonic

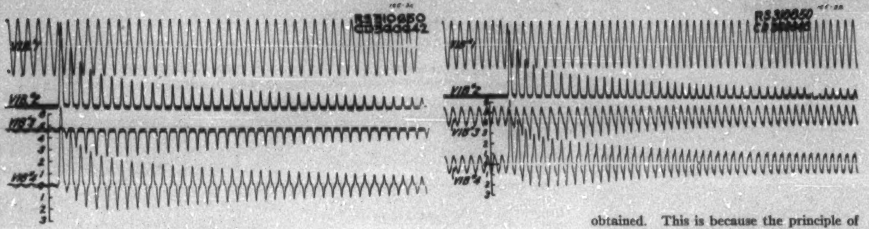

Figure 3. Oscillograms obtained in laboratory tests

Vibrator 1—Transformer bus voltage

Vibrator 2—Primary current in transformer T_2 being switched on bus

Vibrator 3—Primary current in transformer T_1 already connected to bus

Vibrator 4—Current in line. Calibration scale in multiples of one transformer full-load current crest amplitude

For these tests the line resistance was 0.0165 ohm, transformer T_1 resistance, 0.0026 ohm, transformer T_2 and switch resistance, 0.0034 ohm. The line inductance was 0.00005 henry, while the effective inductances of transformers T_1 and T_2 calculated from exciting current at normal voltage were 0.045 and 0.054 henry, respectively

Oscillogram *A* (left)—No load

Oscillogram *B*(right)—Resistance load of 87.5 per cent normal rating on transformer T_1

restraint setting of the relay is such, however, that even with this reduced amount of harmonic current reliable restraint is obtained. If in addition, however, the two transformer banks are included within the same differential zone, cancellation of the second harmonic occurs in the latter part of the transient, due to combination of the inrush currents to the two banks, since these two currents are opposite in polarity and differ 180 degrees in phase position. The center trace on the oscillogram figure 1 shows the nearly sine-wave form assumed by the current when both these effects occur together. If the power transformers are wye-wye connected and the current transformers are also wye-connected, the third harmonic is not lost and may be relied on to restrain relay operation alone if a slightly lower harmonic restraint setting than the normal 25 per cent is used. In this case it would be permissible to include both transformers within the same zone with one set of relays if desired. With only one set of relays, of course, a fault in either transformer will require both to be removed from service.

Conclusions

1. When a transformer is switched on an electric power circuit in parallel with one or more other transformers at a point some distance from the system generators, magnetizing inrush currents of considerable magnitude flow, not only in the transformer being switched but also in the other parallel transformers. (By "parallel" it is meant that the transformers are connected to the same power circuit on the side being switched. The other windings of the transformers may or may not be paralleled.)

2. The magnitude of the inrush current in the parallel transformers not being switched increases as the equivalent resistance of the transmission line circuit connecting the parallel transformers to the generators is increased.

3. Line inductance has little effect except to reduce the magnitudes of all the inrush currents.

4. Load carried by a parallel transformer does not prevent the inrush currents from flowing in its winding although the magnitude is slightly reduced and the rate of decay somewhat increased.

5. Induction type percentage-differential or overcurrent relays commonly used for transformer differential protection may be unsatisfactory when applied to parallel transformers because of the high current and long time settings necessary to prevent tripping on the long-duration magnetizing inrush currents.

6. High speed differential relays using transformer-voltage-operated auxiliary relays to prevent tripping on magnetizing inrush are unsatisfactory for application to parallel transformers under conditions where the inrush currents described occur because the inrush-trip-preventing device is inoperative on the transformers not being switched.

7. The harmonic restraint transformer differential relay is well suited to the protection of parallel transformers giving high speed sensitive operation on internal faults while being reliably restrained from operating on the long-duration magnetizing inrush currents, provided only one transformer is included within the differential zone.

Appendix

Mathematical Analysis

An accurate mathematical solution for the currents flowing in the various circuit branches, including the effect of saturation of the transformer cores, cannot readily be obtained. This is because the principle of superposition on which the methods of the operational calculus,[3] commonly used for the solution of circuit problems, are based does not apply when the circuit inductances vary with changes in current magnitude, as is the case when core saturation occurs. An analysis has been made, however, assuming the inductances to remain constant. The results of this analysis are given below. While admittedly not giving the current magnitudes and wave forms accurately, these results do show in a general way the steady-state and transient current components present in the various branches and indicate the way in which the magnitudes and time-constants of these components depend on the inductances and resistances of the various circuit branches. The analysis is thus an aid in visualizing and understanding the phenomena observed in test as well as in qualitatively predicting the effects of circuit changes not covered by tests.

General Formulas

The current in transformer T_1 already connected to the bus:

$$\begin{aligned} i_1(t) = E\Bigg[& \frac{1}{Z_1}\sin\,(\omega t+\theta-\beta)+ \\ & K_{11}\sin\,(\omega t+\theta-\alpha_1)+K_{12}\sin\,(\omega t+\theta-\alpha_2)- \\ & K_{11}\sin\,(\theta-\alpha_1)\epsilon^{-b_1t}-K_{12}\sin\,(\theta-\alpha_2)\epsilon^{-b_2t}\Bigg] \end{aligned}$$

The current in transformer T_2 being connected:

$$\begin{aligned} i_2(t) = E\Bigg[& -K_{21}\sin(\omega t+\theta-\alpha_1)- \\ & K_{22}\sin\,(\omega t+\theta-\alpha_2)+ \\ & K_{21}\sin\,(\theta-\alpha_1)\epsilon^{-b_1t}+K_{22}\sin\,(\theta-\alpha_2)\epsilon^{-b_2t}\Bigg] \end{aligned}$$

The current in the line:

$$\begin{aligned} i_0(t) &= i_1(t)+i_2(t) \\ &= E\Bigg[\frac{1}{Z_1}\sin\,(\omega t+\theta-\beta)+ \\ & (K_{11}-K_{21})\sin(\omega t+\theta-\alpha_1)+ \\ & (K_{12}-K_{22})\sin\,(\omega t+\theta-\alpha_2)- \\ & (K_{11}-K_{21})\sin\,(\theta-\alpha_1)\epsilon^{-b_1t}- \\ & (K_{12}-K_{22})\sin\,(\theta-\alpha_2)\epsilon^{-b_2t}\Bigg] \end{aligned}$$

These are the currents which flow after the switch is closed, assuming that the voltage on the bus is $E\sin\,(\omega t+\theta)$ previous to the closing of the switch at time $t=0$.

$$Z_1=\sqrt{R_1^2+(\omega L_1)^2}$$

$$\beta=\tan^{-1}\frac{\omega L_1}{R_1}$$

$$K_{11}=\frac{R_0-L_0b_1}{Bb_1-2C}\cos\alpha_1$$

$$K_{11}=\frac{(R_0+R_1)-(L_0+L_1)b_1}{Bb_1-2C}\cos\alpha_1$$

$$K_{12}=\frac{P_0-I_0b_2}{Bb_2-2C}\cos\alpha_2$$

$$K_{22}=\frac{(R_0+R_1)-(L_0+L_1)b_2}{Bb_2-2C}\cos\alpha_2$$

$$\alpha_1=\tan^{-1}\frac{\omega}{b_1}$$

$$b_1=\frac{B}{2A}-\sqrt{\left(\frac{B}{2A}\right)^2-\frac{C}{A}}$$

$$\alpha_2=\tan^{-1}\frac{\omega}{b_2}$$

$$b_2=\frac{B}{2A}+\sqrt{\left(\frac{B}{2A}\right)^2-\frac{C}{A}}$$

$$A=L_0L_1+L_1L_2+L_2L_0$$

$$B=R_0L_1+R_0L_2+R_1L_2+R_1L_0+R_2L_0+R_2L_1$$

$$C=R_0R_1+R_1R_2+R_2R_0$$

L_0 and R_0 are respectively the total equivalent inductance in henries and the total equivalent resistance in ohms of the transmission line and generation circuit supplying the transformer bus. L_1 and R_1 are the magnetizing inductance and resistance of transformer T_1. L_2 and R_2 are the magnetizing inductance and resistance of transformer T_2.

Case of Two Identical Transformers

These general formulas appear rather complicated but if the assumption is made that the two transformers are identical a great simplification results.

Placing $R_2=R_1$ and $L_2=L_1$ we find that the equations for the currents become:

$$i_1(t)=E\left[\frac{1}{2Z_1}\sin(\omega t+\theta-\alpha_1)+\frac{1}{2Z_{01}}\sin(\omega t+\theta-\alpha_2)+\frac{1}{2Z_1}\sin(\theta-\alpha_1)\epsilon^{-\frac{R_1}{L_1}t}-\frac{1}{2Z_{01}}\sin(\theta-\alpha_2)\epsilon^{-\frac{2R_0+R_1}{2L_0+L_1}t}\right]$$

$$i_2(t)=E\left[\frac{1}{2Z_1}\sin(\omega t+\theta-\alpha_1)+\frac{1}{2Z_{01}}\sin(\omega t+\theta-\alpha_2)-\frac{1}{2Z_1}\sin(\theta-\alpha_1)\epsilon^{-\frac{R_1}{L_1}t}-\frac{1}{2Z_{01}}\sin(\theta-\alpha_2)\epsilon^{-\frac{2R_0+R_1}{2L_0+L_1}t}\right]$$

Where:

$$Z_{01}=\sqrt{(2R_0+R_1)^2+\omega^2(2L_0+L_1)^2}$$

$$\alpha_1=\tan^{-1}\frac{\omega L_1}{R_1}=\beta$$

$$\alpha_2=\tan^{-1}\frac{\omega(2L_0+L_1)}{(2R_0+R_1)}$$

Both α_1 and α_2 will be very close to 90 degrees in practical cases.

The transient components of the currents are shown to consist of two parts: one, a current circulating around the loop formed by the two transformer windings in series without flowing in the transmission line as evidenced by the form of the exponent of the first exponential term:

$$\frac{R_1}{L_1}=\frac{R_1+R_2}{L_1+L_2}$$

and the other a current flowing in the circuit consisting of the transmission line and generator in series with the two transformer windings in parallel as shown by the exponent of the second exponential term:

$$\frac{2R_0+R_1}{2L_0+L_1}=\frac{R_0+\dfrac{1}{\dfrac{1}{R_1}+\dfrac{1}{R_2}}}{L_0+\dfrac{1}{\dfrac{1}{L_1}+\dfrac{1}{L_2}}}$$

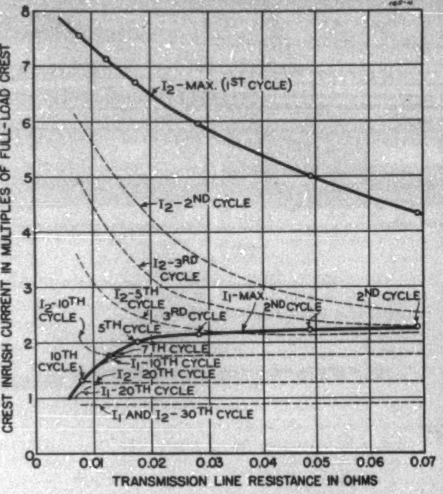

Figure 4. Variation of magnetizing inrush current crest values with line resistance

Measured from oscillograms similar to figure 3*A*. Circuit constants except for line resistance same as for figure 3*A*

I_1—Current in transformer T_1 already connected

I_2—Current in transformer T_2 being switched

This may be proved by solving for the circulating current:

$$\frac{i_1(t)-i_2(t)}{2}=\frac{E}{2Z_1}\sin(\theta-\alpha_1)\epsilon^{-\frac{R_1}{L_1}t}$$

and for the line current:

$$i_1(t)+i_2(t)=E\left[\frac{1}{Z_1}\sin(\omega t+\theta-\alpha_1)+\frac{1}{Z_{01}}\sin(\omega t+\theta-\alpha_2)-\frac{1}{Z_{01}}\sin(\theta-\alpha_2)\,\epsilon^{-\frac{2R_0+R_1}{2L_0+L_1}t}\right]$$

An important difference between this equation for line current and the actual trace of the line current shown by vibrator No. 4 on the oscillogram figure 3*A* should be noted here. The oscillogram shows the effect of the long time constant circulating current, whereas the equation does not contain the circulating current transient term. This difference is due to the assumption made in the analysis that the circuit inductances remain constant. Actually the variation in the transformer inductances caused by the circulating current causes a variation in the magnitudes of the steady-state terms which has the appearance of a transient.

It is interesting to note that the circulating current, both with respect to magnitude and rate of decay, is entirely independent of the characteristics of the transmission line circuit, being determined solely by the inductance and resistance of the transformers themselves in conjunction with the initial bus voltage, the frequency, and the switching angle.

The two transient current components in the expression for $i_1(t)$ have opposite signs whereas in the expression for $i_2(t)$ they have the same sign. In practical cases the amplitudes are nearly equal but the rates of decay may be quite different, depending largely on the magnitude of R_0 in comparison with R_1, since L_0 is small compared with L_1. Curves of the two transient current components are shown in figure 5 together with the curve of their sum which represents the total transient current in the transformer T_2 being switched and the curve of their difference which represents the total transient in the parallel transformer T_1. The latter curve has been drawn in the inverted position above the zero axis rather than in its true position below the zero axis to conserve space. The close correspondence between the forms of these curves and the forms of the direct current components of the actual currents shown by the oscillogram figure 3*A*, or of curves drawn through their maximum values, should be noted.

The reason for the dependence of the current in the two transformers on the resistance in the transmission line is now apparent. If R_0 becomes very large Z_{01} increases so that the amplitude is reduced. At the same time the exponent $(2R_0+R_1)+(2L_0+L_1)$ becomes very large and the rate of decay becomes very fast. In the limit the current in both transformers would approach the value of the circulating current alone, assuming the initial voltage could be applied to the bus through the high resistance transmission line. At the other limit is the case where both R_0 and L_0 approach zero. This is the case of a generator directly connected to the transformer bus with no intervening transmission line. In this case the two transient current components approach equality both in amplitude and rate of decay. The transient current in the transformer being switched approaches double its value at the end of the high resistance line while the transient current in the parallel transformer approaches zero.

CASE OF TRANSFORMERS OF UNEQUAL SIZE OR MORE THAN TWO TRANSFORMERS

The case of transformers of unequal size may be investigated by substituting:

$R_2 = mR_1$
$L_2 = mL_1$

in the general equations, where m is a constant which may take any positive value, either greater or less than unity. This substitution assumes that as the size of the transformer varies the ratio of its resistance to inductance remains constant. This assumption does not hold strictly in practice; in general the ratio of resistance to inductance tends to become smaller as the size of the transformer increases. The great simplification of the general equations obtained by this substitution, however, seems to justify examining the results obtained even though it is understood that they apply in the strict sense only to a special case. The trend of effects to be expected when the R/L ratio varies over the range normally found in practice can be deduced therefrom without too great error. The effects produced when several transformers are already connected to the bus when one is being switched on can be approximated by combining their resistances and inductances to obtain the equivalent resistance and inductance of a single transformer. This approximation should be reasonably close if the transformers do not differ greatly in their R/L ratios. The results of the substitution are as follows:

$$i_1(t) = E\left[\frac{m}{(1+m)Z_1}\sin(\omega t+\theta-\alpha_1) + \frac{m}{(1+m)Z_{0m}}\sin(\omega t+\theta-\alpha_m) + \frac{1}{(1+m)Z_1}\sin(\theta-\alpha_1)\epsilon^{-\frac{R_1}{L_1}t} - \frac{m}{(1+m)Z_{0m}}\sin(\theta-\alpha_m)\epsilon^{-\frac{(1+m)R_0+mR_1}{(1+m)L_0+mL_1}t}\right]$$

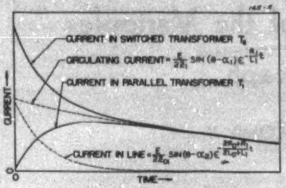

Figure 5. Transient current components as derived by mathematical analysis

$$i_2(t) = E\left[\frac{1}{(1+m)Z_1}\sin(\omega t+\theta-\alpha_1) + \frac{1}{(1+m)Z_{0m}}\sin(\omega t+\theta-\alpha_m) - \frac{1}{(1+m)Z_1}\sin(\theta-\alpha_1)\epsilon^{-\frac{R_1}{L_1}t} - \frac{1}{(1+m)Z_{0m}}\sin(\theta-\alpha_m)\epsilon^{-\frac{(1+m)R_0+mR_1}{(1+m)L_0+mL_1}t}\right]$$

where:

$$\alpha_m = \tan^{-1}\frac{\omega[(1+m)L_0+mL_1]}{(1+m)R_0+mR_1}$$

$$Z_{0m} = \sqrt{[(1+m)R_0+mR_1]^2+\omega^2[(1+m)L_0+mL_1]^2}$$

and the other quantities have the same values as for the case of two equal transformers.

Consider first the case of a small transformer being switched on the bus in parallel with either a single large one or a number of transformers whose aggregate size is relatively large. For this case m has a value greater than unity in the equations. Examining the first transient term it is seen that the circulating current has the same rate of decay as before but its magnitude is greater in the ratio $2m/(1+m)$ than it would be if it were switched on in parallel with another equal small transformer. (The Z_1 used in the equations for two identical transformers becomes mZ_1 in making this comparison, since the impedance Z_2 of the small transformer is equal to mZ_1.) Examining the second transient term, which governs the transient current flowing through the transmission line and dividing between the two transformers in parallel, we see that the division of current between the transformers is in inverse ratio to their impedances as would be expected. The magnitude in each transformer is smaller in the approximate ratio of $2/(1+m)$ than it would be if the transformer were paired with one of equal size. (This assumes Z_{0l} approximately equal to Z_1 and Z_{0m} approximately equal to mZ_1 which is permissible since the magnetizing reactance ωL_1 appearing in the impedance expressions is large compared with the line reactance and the resistances.) The rate of decay is modified, having a value between the values it would have for two equal transformers of the smaller size and that for two equal transformers of the larger size supplied from the same transmission line circuit. The net result is that the maximum amplitude of the total transient current in the small transformer being switched is about the same as it would be if switched in parallel with another of equal size while the amplitude of the current in the large transformer is less in the approximate ratio of $2/(1+m)$ than it would be if another equally large transformer were switched in parallel with it.

Consideration of the case of a large transformer being switched on in parallel with a small one, in which case the value of m is less than unity, leads to the conclusion that the maximum amplitude of the transient current in the small transformer is larger in the approximate ratio of $2/(1+m)$ than it would be if both transformers were small. This ratio approaches the value 2 as m approaches zero, which means that the d-c transient current approaches the magnitude it would have if the small transformer were switched on the line without the other transformer being connected. The maximum amplitude of the total transient current in the large transformer is about the same as it would be if the transformer were switched on in parallel with another of equal size but the rate of decay is faster; approaching the rate it would have if the transformer were switched on the line alone, without the presence of the smaller transformer; when m approaches zero.

References

1. STARTING CURRENTS OF TRANSFORMERS, T. D. Yensen. University of Illinois *Bulletin* number 55, February 19, 1912.

TRANSFORMER ENGINEERING (a book), L. F. Blume. John Wiley and Sons, Inc., 1938, chapter 2, pages 23–36.

2. HARMONIC CURRENT RESTRAINED RELAYS FOR TRANSFORMER DIFFERENTIAL PROTECTION, C. D. Hayward. AIEE TRANSACTIONS, volume 60, 1941, pages 377–82.

3. OPERATIONAL CIRCUIT ANALYSIS (a book), V. Bush. John Wiley and Sons, Inc., 1929.

4. INTRODUCTION TO ELECTRIC TRANSIENTS (a book), E. B. Kurtz and G. F. Corcoran. John Wiley and Sons, Inc., 1935.

Basic Principles of the Varioplex Telegraph

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Synopsis: A new kind of channel-sequencing system allots the full capacity of a telegraph circuit to the busy channels alone. A busy channel cuts itself out of its sequential position by becoming idle but may re-enter the sequence at will by offering one letter for transmission. Entry, mixing, and exit are accomplished by electromechanical control of standard start-stop and multiplex printing telegraph apparatus. Special signal or code combinations are used to denote changes in the pattern of subchannels cut in, and to act as a check on correct distribution. For greater efficiency, ordinary message letters, by a new method, may be interspersed among successive elements of switching combinations, and the cascading of combinations is also employed. Dummy channels, high-speed receivers, and other devices may be used to prevent or utilize unusual channel speeds. Varioplex systems may be repeatered either by subchannel storage or by storage involving several subchannels handled as a unit.

In the time-division multiplex telegraph the channels form fixed component parts of the lane of traffic over which they operate. There is a great convenience in channeling but it is necessary to assign multiplex channels with care because idle channels secure and waste their proportion of lane time, and channels with practical speeds are limited and precious.

The varioplex is a new kind of channel-sequencing system which permits channeling without waste of lane time. It secures this result by allotting the capacity of the lane to working channels only, so long as those channels have anything to send. The application of the new system to the Western Union telegraph plant is described in a companion paper;[1] it is the province of the present treatment to outline the general principles of varioplex theory.

Analogy of Capacity

Since comprehension of the purpose and underlying principles of the varioplex is facilitated by analogy, an examination of the theory of operation of an easily understood form of varioplex will be preceded by reference to two forms of physical transportation.

Figure 1 illustrates analogies for single channel, multichannel, and expanding or variable channel telegraph systems, in the form of paint factories connected to packaging plants by pipe lines. In the first instance it is assumed that the factory manufactures blue paint only. Its single pipe is large enough to carry the entire factory output without congestion. The cross section of the first pipe shown at the right-hand side of figure 1 reveals that the pipe is completely filled, but carries only blue.

Assume that although transmission of one color through one pipe is satisfactory, the factory wishes to diversify the same output among five colors. Under certain conditions the colors might be sequenced through the single pipe, intercolor swabs (corresponding to addresses on telegrams, or machine-switching combinations) being used to remove residues. In the present case, however, assume that ever-open channels are required. Four additional pipes are installed, as shown in the middle figure, each having the same capacity as the original pipe. As the cross sections indicate, the price paid here for the convenience of separate channeling is a tremendous increase in unutilized capacity. Under similar conditions there would be a corresponding increase in unutilized wire capacity if a printer channel capable of handling the traffic of five Chicago branch offices were to be supplemented by four additional similar channels between New York and Chicago in order that the branch offices might be served directly from New York.

The lower part of figure 1 is an analogy for the expanding channel or varioplex system. A single pipe is equipped with five thin flexible partitions which expand as far as they can while in use and collapse when empty. The capacity of the pipe is in this way divided among the channels which are in use, and if we assume that the shell of the pipe is large enough to carry the output of the factory, we automatically rule out all chance of delay at the factory end, or point of dispatch. In similar fashion does the lane of a varioplex operate at full capacity until devoid of load, and if this capacity is not exceeded by the combined load there can be no delay on any channel. This is true even when the traffic is so diverse that all channels are in use at once and the volume handled per channel per unit of time correspondingly low.

Analogy of Sequence

Figure 2 is an analogy for a multiplex in the form of a belt conveyor carrying alphabet cards from three transmitting positions on the right to three receiving positions on the left. The sending dealer (or distributor) works downward, picking up one card from a channel at a time. The receiving dealer distributes the cards as they arrive in a similar manner and in proper sequence. The letters on the belt in figure 2 seem to be badly jumbled but if the reader will substitute himself for the receiving dealer he will find them forming the correct words and sentences as soon as they are put into the proper channels.

If we assume, as in figure 3, that channel 3 has been closed out before its sentence, "PACK MY BOX," had been started through the conveyor, nothing but blank cards would be available at the third channel for the sending dealer to put on the belt. The belt thus carries alphabet cards for channels 1 and 2, and blank cards for channel 3. Numerals have been drawn above the letters in the diagram to indicate their respective channels. The receiving dealer, proceeding in sequence, must lay down blank cards as well as "live" cards in channel order as they arrive. This simple system, corresponding to the multiplex, would at such times waste one-third of the total lane space.

If the belt conveyor were operated on the varioplex principle, as in figure 4, the sending dealer would note the stoppage of transmission on channel 3 and throw a single blank card on the belt to indicate it. Thereafter he would skip channel 3 and would send out cards from channels 1 and 2 exclusively. When the blank card arrives at the receiving end, the dealer there is to be imagined as distributing it in regular sequence to the third channel, as shown in figure 5. The receiving dealer interprets such a card as an indication of the closure of channel 3 and proceeds to skip that channel in the future.

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[1] For all numbered references, see list at end of paper.

The lane now carries the cards of channels 1 and 2 at a more rapid rate because the third channel's position on the lane is no longer claimed and held by a string of blank cards.

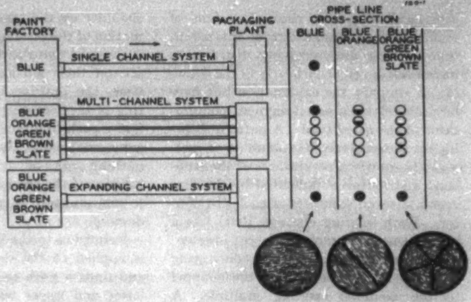


Figure 1. Pipe analogy to compare the efficiencies of single-channel, multichannel, and expanding-channel systems

At this point it is advisable to anticipate the ultimate re-entry of channel 3 because the sending dealer cannot start third channel cards without notice. To this end small numbers may be imagined as painted on the belt below the cards to represent the moments at which channels 1, 2, and 3, can be entered. This is shown in figure 6. These numerals are employed solely for the entry of channels and have no relation to the channel identity of the cards that happen to be above them.

The sending dealer now proceeds to enter channel 3 by putting a card marked "In" on the next space designated by "3" that the belt brings up. The arrival at the receiving station of such a card in such a position is the cue for the receiving dealer to pick up the blank card from the third channel and, having discarded both the blank and "In" cards, include that channel in future distributions.

Channel 3 has been used as an example but obviously the other channels of the varioplex can be entered, mixed, and cut out in similar fashion. The belt numbers in figure 6 can be dispensed with if the rims of the pulleys are numbered correspondingly. The transmission of special signals, corresponding to "In" cards at predetermined times, interspersed among regular signals, is an important feature of the varioplex system because the movement of traffic continues while additional channels are being entered and cut out.

Although the efficiency of a varioplex lane is not impaired by delay in entering it should be understood that entry is not instantaneous. However, on a 68-cycles-per-second varioplex lane 53 characters are transmitted every two seconds. Fifty-three channels could therefore be entered on such a lane in two seconds with an average entry delay of one second.

Methods of Entry and Exit

Another way of entering channels is to show a "pattern" of the channel cut-in situation, i.e., to transmit a group of signals representing the status of a group of channels. "In" channels may be represented by marking pulses and "Out" channels by spacing pulses of the pattern signals. The complete pattern can be shown at a prearranged time, say at the point "50 on the clock," and that point in time always reserved for it at both ends of the lane. This kind of entry will be described in detail when we come to figure 7. Another way of using the pattern is to introduce it into traffic by a special break-in signal, and in this case there is no delay at all in controlling the channels. Patterns may be cascaded by forming them into groups that enter on a time basis; after the group is selected a portion of the pattern is exhibited to reveal the change that is desired in the group. On the same 68-cycle per-second varioplex lane considered above, 55 channels could be entered by cascade in 11 groups with an average entry delay of a quarter of a second. Patterns may be confirmed by repeating them. "Code" entry is effected by following up the special break-in signal with regular Baudot code combinations which indicate the channel being changed. A break-in signal followed by one multiplex letter will immediately control any one of a group of 32 channels. Two kinds of control may be had from the same combination, the first appearance signifying entry, the second exit, the third entry again, and so on.

There are many methods of cutting channels of a varioplex in and out of operation, and sometimes it is expedient to use two methods in the same system. Thus, the periodic transmission of a pattern signal representing the channel situation does not preclude the use of a single exit signal when a particular channel is to be cut out. Double-exit means in such a case would be useful in improving the efficiency of the lane and the second exit would be a check on the first. The signals for altering channel distribution need not be sent on the circuit traversed by the traffic of those channels, in fact added secrecy can be obtained by having such signals sent over another route. The channel distribution can even be altered manually in response to signals or messages sent from one terminal to the other, and in many instances this is an economical and advantageous method of obtaining the benefits of the varioplex.

The selection of combinations for appropriation by the varioplex is a serious problem, because there are only 32 in the Baudot five-unit code and few of them can be spared for the purpose. The place of the one that is appropriated must be filled in such a way that there will be no difficulty when connecting various kinds of keyboards and printers to the varioplex channels. Provision must also be made to keep the traffic channels from sending switching combinations at significant times. A line failure may result in loss of channel synchronism, so the combination should be made difficult to obliterate and easy to check. With these characteristics a good combination will almost always come out of a lane in recognizable form and if it does not meet rigid requirements, alarms are set off or automatic means put into action to resynchronize.

In addition to the safety of special combinations, channels may be installed on a varioplex which have no other function than to open, send test signals, and close again. If such channels open at the proper time and the proper signal is received, it is practically certain that everything is in order. If checking channels are given slightly different periods on different circuits and directions, there will be immediate indication of channel mismating due to crossed or open line wires.

Entry and exit of channels presented difficulties of the first order in the early days of the varioplex, and for this reason the subject has been covered in the foregoing in some detail.

Simple Form of Varioplex

Figure 7 illustrates diagrammatically a simple form of varioplex operating over a seven-unit stop-start channel or lane of traffic. The commutators of the seven-unit sending and receiving distributors have been developed linearly in the drawing to simplify the explanation but may be replaced by a seven-segment commutator of the usual type. A pattern-showing revolution precedes a number of message-transmitting revolutions of the stop-start distributor as indicated by the first group of seven segments of each distributor. Each sending channel (left) has a tape autostop contact arm, a cut-in relay, a tape transmitter, and a sending gang relay for interconnection of the channel to the common sending multiple. A polar relay bank is connected to the sending multiple for storage of characters sent by the various channels. The tongues of the polar sending bank connect to the sending segments of the sending distributor. Pulses for local sending functions are obtained from a local ring on the sending distributor. On the receiving side (right), the pattern channel and message channels correspond to those on the sending side. Message channels on the receiving ring are multiplied to the coils of the receiving polar bank relays, and the tongues of these relays apply battery to the selecting magnets of a particular receiving device when its corresponding gang relay is operated. Both ends of the circuit are equipped with mixers to distribute local ring pulses in the correct order and thereby control the operating channels in rotation, the brush arm of each mixer having three stop positions in the case of three channels, as shown.

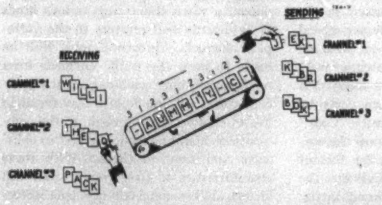


Figure 2. Belt-conveyor analogy for a three-channel time-division multiplex

The sentences under transmission are WILLIAM-JEX-QUICKLY - CAUGHT, THE - QUICK - BROWN-FOX, and PACK-MY-BOX. Dealers move from top to bottom

Let us assume that shortened tape loops have lifted the autostop arms on all sending channels and that the tongues of the sending polar bank are at rest on a certain combination, for example the all positive combination. When the brushes at both terminals are in the positions indicated, a circuit is established between negative and negative via the top sending segment, the line, the top receiving segment, and the receiving start magnet. As the receiving start magnet is not energized under these conditions, its armature is held to the right by its spring and a shoulder on the armature blocks further motion of the receiving brushes. Since it is assumed that the receiving cut-in relays are on their left-hand contacts at this time, the positive battery on the local ring of the receiving distributor does not find a path to ground through any of the gang relay coils, and as a consequence nothing can be transferred from the tongues of the receiving polar relay bank to the magnets of any of the receiving devices.

Returning to the sending end, positive is applied to the sending local segment and finds a path to ground through the lower and upper windings of the three sending cut-in relays. The upper winding of each of these relays is in opposition to its lower winding and overcomes it because of the greater number of turns. The polar relays in the diagram are wound in such a direction that current passing from right to left causes the armature of the relays to move from left to right, and vice versa. When all channels are cut out the tongues of the sending cut-in relays thus assume positions on their left-hand contacts and apply positive battery to the first three sending segments of the pattern channel and positive battery to the three segments of the sending mixer. The arm unit of the mixer tends to rotate at a speed somewhat faster than the speed at the sending distributor but it is frictionally connected to its shaft and can be stopped by the stop armature with the brush engaging any one of the three segments.

Under the assumed condition the sending mixer arm unit rotates freely because whenever the brush touches a segment, positive battery is immediately connected through the brush to the stop magnet and the stop armature is withdrawn from the path of the stop arms. As the sending brush moves downward, a start pulse of positive polarity is sent to the line. This energizes the start magnet at the receiving end, operates the stop armature of the receiving distributor, and permits downward movement of the receiving brushes over the segments of the pattern channel. While the receiving brushes are on their way to the next stopping point, three positive pulses are received and laid down on the three control segments of the pattern channel. This polarity maintains the tongues of the receiving cut-in relays on their left contacts. Positive battery is thus applied to all segments of the receiving mixer and its constantly energized magnet permits its brush arm unit to rotate freely without stopping. The sending brush of the sending distributor continues to advance at a uniform rate, sending rest, start, and all-positive channel pulses throughout its travel, while the receiving brush assembly proceeds through similar stages, laying down positive pulses on each of the channels in turn, which maintain the tongues of the receiving polar bank to the left.

Entry of First Channel

As messages accumulate on the first sending channel, additional tape is punched, and the slackness of the loop permits the autostop arm to fall and ground out the upper coil of the first cut-in relay. When the sending brush of the sending distributor comes again to the pattern channel at the top of the figure, a positive local pulse passes through the lower coil of the first channel cut-in relay, is not neutralized by the upper coil of the relay as before, and now operates the tongues of the relay to the right. This applies negative to the first channel's segment in the pattern channel as an indication to the receiving end that the first channel is prepared for operation. The lower tongue of the cut-in relay removes positive from the first channel's segment on the sending mixer and connects this segment to local operating segments on the sending distributor. These local segments are without battery for the moment, and the unenergized segment on the mixer quickly causes the mixer brush to stop upon it. The negative pattern pulse of the first channel arrives at the receiving end and operates the tongue of the first channel cut-in relay to the right. This removes positive battery from the first channel's segment of the receiving mixer and connects it to local segments of the receiving distributor which are also open at the moment. The brush of the receiving distributor likewise quickly comes to rest on the open first channel's segment. When the brushes of the sending distributor have passed to the next rest and local pulse positions, an operating circuit is completed from the local segment through the magnet of the first channel gang relay, the contacts of the first channel cut-in relay, and the brush and magnet of the sending mixer.

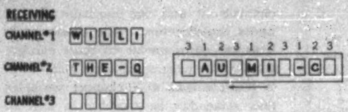


Figure 3. Belt-conveyor analogy for a three-channel time-division multiplex with channel 3 idle and its "turn" wasted

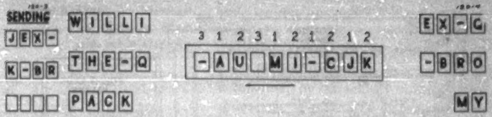


Figure 4. Belt-conveyor-out analogy for a three-channel varioplex in the act of closing out channel 3

Note that distribution changes from three channels to two channels after a blank card denoting exit has been sent by channel 3 on "turn." Established channels can transport their own demolition combinations

This releases the brush of the sending mixer which makes one revolution and comes to rest again in the same spot.

The sending gang relay has six contacts. The lower one opens the magnet circuit of the transmitter, permitting the pins to find their corresponding holes in the tape, and moving the transmitter tongues to negative when they do. The downward operation of contacts 1, 2, 3, 4, and 5 of the gang relay applies the marking and spacing elements of the letter perforated in the tape to the coils of the sending polar bank for transmission to the line by the segments of the sending distributor. At the receiving end, the marking and spacing elements of the received letter cause corresponding movements of the tongues of the receiving polar bank. When this letter has been completely received, a local receiving ring pulse passes through the magnet of the first channel gang relay, the contacts of the first channel cut-in relay, the first channel's segment of the receiving mixer, and through the magnet of the receiving mixer to ground. The operation of the receiving gang relay completes the circuits of the five selecting magnets of the receiving device to the tongues of the receiving polar relay bank. The sixth tongue of the gang relay affords an operating pulse for the receiving device or printer of the first channel. The operation of the receiving mixer stop magnet permits the brush of the receiving mixer to make one revolution and stop again in the same place. At the termination of the local pulse on the sending distributor, the sending gang relay for channel number 1 is de-energized, the tongues of the transmitter disconnect from the sending polar relay bank and the magnet of the transmitter re-energizes. The energization of the magnet of a transmitter of standard design withdraws the pins from the holes and steps the tape. The cycle described above is repeated and the line is completely occupied with transmission from the first channel.

Function of Pattern Channel

At regular intervals, say after 25 letters of traffic, a show of the cut-in situation is made by passage of the brushes over the pattern channel. After transmission from the first channel has continued for a while, assume that additional tape is punched for the third channel, permitting the autostop arm of the channel to descend and bid for entry. When the brushes of the sending distributor next arrive at the position shown in figure 7, local ring positive battery finds its paths to the ground through the lower coil of the first channel cut-in relay, the lower and upper coils of the second channel cut-in relay, and the lower coil of the third channel cut-in relay. This confirms the position of the tongues of the cut-in relays on the first two channels and operates to the right the tongues of the cut-in relay on the third channel. As the sending brush of the sending distributor reaches the pattern channel, it transmits negative from the first control segment to indicate the cut-in condition of the first channel, positive from the second segment to indicate the cut-out condition of the second channel, and negative from the third segment to indicate the cut-in condition of the third channel. At the receiving end the new pattern confirms the operation of the first receiving cut-in relay to the right, the second to the left, and operates the third to the right. The sending and receiving mixers now have two stopping positions without permanent battery, namely, segments for channels 1 and 3. In a manner similar to that explained for channel 1, the gang relay of sending channel 3 now operates in sequence with the gang relay of sending channel 1, and alternate letters from these channels traverse the line and are properly deposited on the printers or receiving devices of channels 1 and 3. In a similar manner, channel 2 can cut in and share proportionately in the capacity of the line. When this occurs, the mixer brushes are stopping at every channel. When a channel cuts out, the local stop arm is lifted, the local sending pulse passes through both coils of the cut-in relay, restoring the tongues to the left and resulting in a skipping of that channel by the sending mixer. The same effect is secured on the receiving mixer by restoring the tongues of the receiving cut-in relays to the left. Thus, it is seen that in accordance with the varioplex principle, the line circuit is loaded to capacity irrespective of the number of operating channels, and the channels are cut in and out automatically as the traffic varies.

Mechanical Type of Mixer

Figure 8 shows a mechanical method of mixing and changing channels, adaptable to the sending and receiving ends of many forms of varioplex systems. It will be

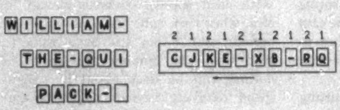


Figure 5. Belt-conveyor analogy of a three-channel varioplex with channel 3 closed out and its "turn" abolished

The lane carries traffic from channels 1 and 2 at a faster rate

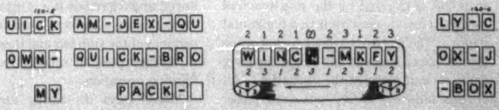


Figure 6. Belt-conveyor analogy of a three-channel varioplex with channel 3 in the act of entering

"Channel time" numerals are painted on the belt and pulleys to add significance to the "In" card. The sending dealer gives channel 3 a "turn" by putting an "In" card on the belt above the number 3. The arrival of this card above the number 3 causes the receiving dealer to include the third channel in future distributions

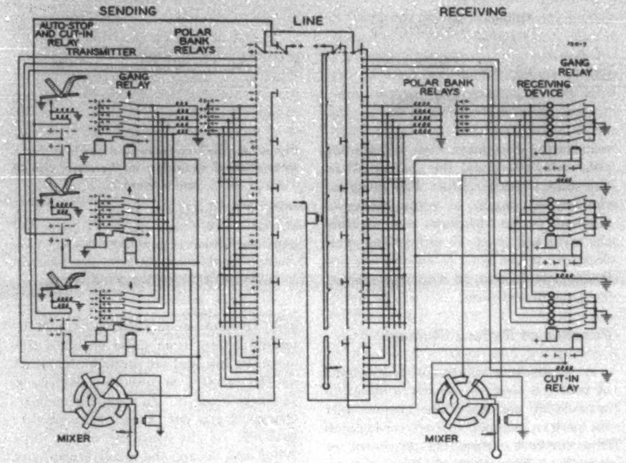


Figure 7. A simple form of varioplex having three subchannels on a start-stop lane of traffic

Entry and exit are accomplished by use of a periodic show of the channel cut-in pattern

fitted into the receiving side of figure 7 to show how it can be substituted for the mixer, the cut-in relays, and the gang relays of that figure.

In figure 8 each varioplex channel is represented on the mechanical unit by a stop bar, a contact lever, and possibly a character counter for telemeter service. The stop bar on the right and the clock above it can be disregarded for the moment.

When all channels are "out" their stop bars are lifted out of the path of the stop arm, which rotates by friction in the space between the stop bars and the contact levers. If channel 1 wishes to enter, a negative pulse from the first segment of the pattern channel energizes the cut-in magnet and lowers the stop bar of that channel into the path of the stop arm. The stop arm quickly comes to rest against it as shown in figure 8. The sixth pulse magnet is soon energized by a pulse, and the stop arm shaft is pulled down against a spring until the stop arm clears the lower edge of the stop bar. The contact lever, which is below and beyond the stop bar, is engaged by the stop arm and rotated a few degrees by it in a horizontal plane. The stop arm comes to rest when the torque of its shaft has closed the gang contacts and metered the character. All of this happens during one-sixth pulse, and at the termination of the pulse the stop arm is lifted by its spring until it again clears the contact lever. It then continues its rotational search for the stop bar of the next cut in channel.

A spring restores the contact lever to its normal position and this removes the ground from the magnets of the receiving device on channel number 1. A simple cut out is employed to keep the stop arm from interfering with the contact levers when all channels are idle.

The varioplex distribution principle is adhered to throughout because the tongue circuits of the polar bank are completed through the receiving devices of operating channels only.

A different method of cutting in and out is applied to the stop bar at the right-hand side of the mechanical unit. The clock assembly is mechanically or electrically geared to the receiving distributor. Since the clock magnet is connected through three condensers to the three pattern segments it is never energized except when a pattern is coming in, and not even then unless changes in the cut-in pattern reverse the charges on the condensers. When such a change does occur the clock arm is so phased that its sword will be above the stop bar of the channel to be changed, and the kick of the sword will result in the stop bar's being raised if it is down, and depressed if it is up. A spring on the sword permits it to follow the cam surfaces of the stop bar and then return to a centered position suitable for the next flip-flop operation.

Many ways of mixing and cutting channels in and out mechanically are under consideration. The design problems are quite similar to those encountered in the telegraph printer field and it is possible that a 50-channel varioplex may some day be mixed in a unit no larger than a printer. By limiting the mixing and entry parts of a subchannel to two small stampings and a contact block, a big step can be taken toward the goal of longer and more lightly loaded subchannels.

Varioplex Speeds and Capacity

Since excessive channel speeds should be utilized or avoided, the reader is referred to figure 7 for a discussion of the subject. It has been assumed heretofore that the whole speed of the lane could be allocated to any of the three channels without undue punishment of the channel apparatus. Transmitters and printers of standard design work very well at 80 words per minute so that the speed of the lane is not restrained by them until that point has been reached. Let it now be assumed that the aggregate load has increased to a point where a lane speed of 150 words per minute is required. Channel speeds in figure 7 are then 0, 50, 75, and 150 words per minute. Since a 150-word speed is impracticably high for standard transmitters and printers, special high-speed apparatus must be employed if it is desired to retain it. For such a speed a special transmitter has been designed, having ten or more pins and stepping two or more letters on every local operation. The receiving unit can be a storing device handling two letters or more on every local operation. The gang contacts shown in figures 7 and 8 is under these conditions also increased in number.

Another way of governing the speed of the figure 7 varioplex is to open additional channels carrying deferred traffic as the regular channels are closed out. Or, dummy channels without transmitters or receivers may be cut in when the speed threatens to be excessive. Such channels may be controlled by increment relays in the autostop multiple connecting to the local segment of the pattern channel on the sending side.

The governor of a varioplex ought not to function until channel printers have attained their maximum practical speeds. With such speeds, channels should be clear when they enter their efficient busy periods, and if they still have traffic to send when the dummies cut in again nothing can be done about it. High-speed receiving devices might be em-

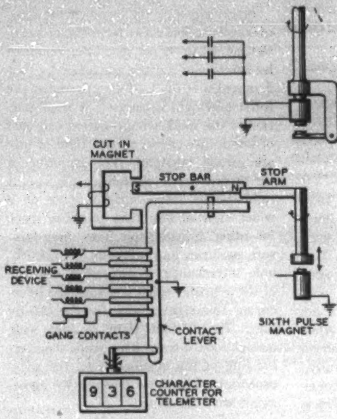

CUT IN MAGNET
STOP BAR
S
N
STOP ARM
CLOCK ARM
RECEIVING DEVICE
GANG CONTACTS
CONTACT LEVER
SIXTH PULSE MAGNET
9 3 6
CHARACTER COUNTER FOR TELEMETER

Figure 8. Mechanical form of mixer with cut-in elements

ployed on loaded submarine cables and at varioplex repeaters, but they would be an unnecessary complication on ordinary circuits unless the periods of overload were prolonged and alternated frequently with periods of underload. This does not happen on commercial varioplexes which are supplied with traffic from many places by operators who send at net speeds of considerably less than 65 words per minute.

Varioplex Subchannels on Multiplex Circuits

A start-stop lane of traffic was chosen for the figure 7 varioplex to provide dead time for local operations. If multiplex distributors are substituted for start-stop distributors time for such operations can be furnished by employing overlap storage. A different way is to separate the lane channels, instead of strapping them, and to allow the working subchannels to attach themselves to the lane channels. If unattached lane channels remain idle, the governor problem is solved along with that of local operating time.

Figure 9 shows a separated channel multiplex lane carrying the traffic of nine subchannels. Sixty-five words per minute is assumed to be the speed of the sending and receiving legs, and the speed of each of the four multiplex channels. The subchannel speed varies from 0 to 65 words per minute. As an example, when sending begins on the first leg at Chicago, the distributor-perforator fabricates tape which feeds into the transmitter of the first varioplex subchannel. A relay at the grid junction 1*A* closes and makes a semipermanent connection between the transmitter and the *A* channel of the multiplex. The corresponding grid relay in the New York receiving chain also closes, and traffic begins to move at 65 words per minute. In similar manner multiplex channels *B*, *C*, and *D* are seized by subchannels 3, 4, and 5 which are assumed to open up. When subchannel 9 enters the picture there is no place for it, so the relay chains begin to step and mix the multiplex channels with the varioplex subchannels. Under this condition a certain subchannel makes contact with varying multiplex channels like a tooth of a five-toothed gear making contact with varying teeth of a four-toothed gear that is driving it. This arrangement is illustrated by the analogy at the bottom of figure 9. If more than five subchannels are cut in, the sending opportunity of each is made less frequent, just as the left gear would revolve more slowly if it were larger and had more than five teeth.

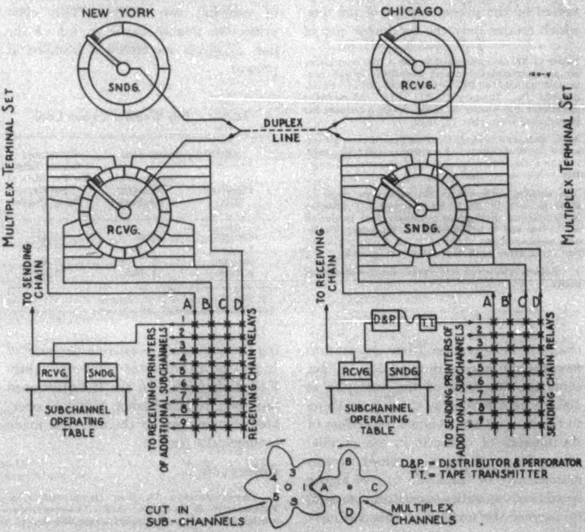
The advantages of the varioplex are most apparent when subchannels are extended beyond the system terminal. Repetition of one channel into another channel of higher speed is always a simple matter, but when the reverse condition occurs it is necessary to provide storage of some description. Individual channel storage is not required when several channels are repeatered from one varioplex lane into another because a single storing device can handle the combined traffic of all such channels. The repetition of varioplex subchannels forms an essential part of through channel networks.

Conclusion

While the efficiency of the multiplex system depends upon the proportion of

Figure 9. Layout of a varioplex with nine subchannels operating over a lane consisting of four separated multiplex channels

Leg equipment, relay chain mixing, and channel mixing order are here shown


NEW YORK
SNDG.
CHICAGO
RCVG.
DUPLEX LINE
MULTIPLEX TERMINAL SET
RCVG.
SNDG.
MULTIPLEX TERMINAL SET
TO SENDING CHAIN
TO RECEIVING CHAIN
A B C D
1 2 3 4 5 6 7 8 9
TO RECEIVING PRINTERS OF ADDITIONAL SUBCHANNELS
RECEIVING CHAIN RELAYS
RCVG
SNDG
SUBCHANNEL OPERATING TABLE
D&P
T.T.
A B C D
1 2 3 4 5 6 7 8 9
TO SENDING PRINTERS OF ADDITIONAL SUBCHANNELS
SENDING CHAIN RELAYS
RCVG.
SNDG.
SUBCHANNEL OPERATING TABLE
D&P = DISTRIBUTOR & PERFORATOR
T.T. = TAPE TRANSMITTER
4 3 0 1 5 9
B A C D
CUT IN SUB-CHANNELS
MULTIPLEX CHANNELS

Powerton-Crawford 220-Kv Line—Design and Construction Features

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THE placing in service of the 220-kv Powerton-Crawford transmission line of the Commonwealth Edison Company marks the completion of a project begun 12 years ago and subsequently dropped during the depression years. When work was suspended in 1932, most of the right-of-way had been secured, tower sites were located, the tower contracts had been let, typical towers had been built and tested, and some of the terminal apparatus was completed.

This line, with a normal rating of 150,000 kva and an emergency rating of approximately 200,000 kva, is to serve as a very important link in the supply to the Chicago area. The plans were carefully reviewed when the project again became active, with the object of incorporating the new ideas that had been developed in the industry for protecting overhead lines from hazards of lightning, sleet, vibration, and galloping conductors. The desired objective was virtually trouble-free operation and this, rather than the lowest cost, was the first consideration. The extent of the advance in the industry is revealed by the present design of the line, which retains only the conductor size of the original designs of 1928 to 1930. Towers, insulators, ground wires, hardware, and all other elements are of present-day design. It is anticipated that the line will have an outstandingly good operating record.

Statistics pertaining to the line are given in the accompanying tables, while the more important elements that affected the design are discussed more generally.

The line is 147 miles long and transverses the rather flat farm region of Illinois between Powerton station near Pekin and Crawford station in Chicago. A right-of-way 250 feet wide was acquired to permit two single-circuit lines for 130 miles from Powerton. The present line is located 75 feet from one edge of the right-of-way to the center of the towers, leaving room for a future circuit 75 feet from the other edge. The remaining 17 miles to Crawford station is through a suburban area dotted with small communities. The only feasible right-of-way was approximately parallel to two main-line railroads in a narrow strip sufficiently wide for only a single tower line capable of carrying two circuits. This comprises the double-circuit section of the line, in which one circuit is installed at present.

Table I. Fair Weather Corona Loss

ACSR Conductor Size		Calculated Corona Loss Kw Per Mile at 240 Kv, Three Phase
Circular Mils	Approximate Diameter (Inches)	
636,000	1.0	0.85
715,000	1.05	0.70
795,000	1.1	0.55
900,000	1.162	0.40

Practically all of the single-circuit right-of-way is an easement from the property owners, about 92 per cent being obtained by agreement and the remainder by condemnation of the land directly under the tower and right of access. A small portion of the single-circuit section and part of the double-circuit section right-of-way are owned outright. The majority of the double-circuit section is on a 25-foot right-of-way. This strip is part of a wider right-of-way that provides space for other transmission lines, high-pressure gas lines, and railroads. A remarkable interchange of ownership and use of the available space was involved in securing this right-of-way for the 220-kv line. The 25-foot strip is just wide enough for the tower base, while the overhanging of the conductors on the adjacent rights-of-way is covered by agreements with the owners.

Basic Considerations of Design

As natural effects of the weather constituted the greatest hazards to trouble-free operation, all the weather information was carefully considered as the basic data in designing the line. Records indicate that an average of about four tornadoes a year may be expected in Illinois. Wind velocities in mild tornadoes are known to be of the order of 100 miles per hour. In the most severe cases the wind may attain a velocity of about 400 miles per hour. It was concluded that the economic limit in the tower design was about reached if the line could withstand wind velocities of 150 miles per hour.

The line route is in the general direction of the Illinois River and DesPlaines River valleys, where frequent and heavy sleet storms occur. Observations in this territory indicate that $^{1}/_{2}$-inch ice may be

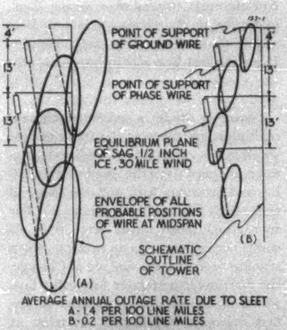
4'
13'
13'
POINT OF SUPPORT OF GROUND WIRE
POINT OF SUPPORT OF PHASE WIRE
EQUILIBRIUM PLANE OF SAG, 1/2 INCH ICE, 30 MILE WIND
ENVELOPE OF ALL PROBABLE POSITIONS OF WIRE AT MIDSPAN
SCHEMATIC OUTLINE OF TOWER
(A)
(B)
4'
13'
13'
AVERAGE ANNUAL OUTAGE RATE DUE TO SLEET
A-1.4 PER 100 LINE MILES
B-0.2 PER 100 LINE MILES

Figure 1. Sleet clearance diagrams at midspan for Chicago 132-kv lines

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1. For all numbered references, see list at end of paper.

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channels in operation from moment to moment and may vary from 0 to 100 per cent, the efficiency of the varioplex is 100 per cent so long as there is anything to transmit on any channel, regardless of the number of channels in use or idle. The gain in efficiency obtained in this manner may be employed to give higher speeds and capacities to existing channels, to increase the number of channels having the same loads as existing channels, or to provide an unlimited number of lightly loaded channels. In the last-mentioned use, long, lightly loaded, privately operated channels are for the first time made economically feasible.

Reference

1. The Varioplex: A New Development in Telegraphy, E. R. Shute. AIEE Transactions, volume 60 (April section), 1941, pages 181–4.

encountered once or twice a year, $^3/_4$-inch ice will occur every few years, and $1^1/_2$-inch ice is not beyond the bounds of possibility. The line was designed for trouble-free operation under such loads.

Lightning storms are as frequent and severe as any place in the country with a few exceptions. Extensive investigations by others in recent years have led to the development of methods of making high-voltage lines practically lightning-proof, and full advantage of this information was taken in designing the line.

Conductor

Economic studies for the conductor indicated lowest annual costs for one in the neighborhood of 500,000 to 600,000-circular-mil copper equivalent, although a fairly wide choice of sizes could be used without appreciable effect on annual charges. Studies of corona loss, following recently developed methods, indicated fair weather losses as shown in table I.

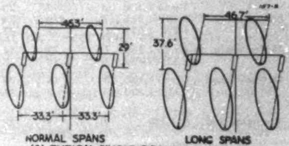
A 900,000-circular-mil ACSR conductor of 1.162-inch diameter and 32,300 pounds guaranteed strength was selected because of over-all economy thereby obtained on the conductor and tower installation. The ACSR conductor also had the advantage of a considerable background of satisfactory service, availability of standard splicing and dead-ending equipment, and freedom from corrosion. Actively corrosive conditions due to smoke and fumes are a minor hazard near the terminals and are nonexistent elsewhere along the line.

A minor factor in the choice of the conductor diameter was that the eventual installation of a Petersen coil was seriously considered, although initially the transformer neutrals will be solidly grounded. It was desired to maintain low corona losses at the occasional higher voltages incidental to Petersen coil operation. Laboratory tests in Chicago showed the fair weather corona starting voltage, with insulators and hardware in place, to be at least 150 kv, or the equivalent of a three-phase voltage of 260 kv. In these tests corona appeared only

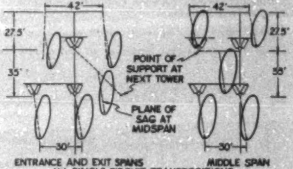
Table II. Statistical Data, Powerton-Crawford 220-Kv Line

Line		
Number of 3-phase 60-cycle circuits:	present	1
	future	22
Line-to-line kv:	sending end (Powerton)	42
	receiving end (Crawford)	198
Nominal line capacity, kw		150,000
Maximum capacity as limited by thermal and stability limits, kw		172,000 to 200,000
Approximate full load loss, including transformers and condenser, per cent		7
Length of single-circuit tower section, miles		130
Length of double-circuit tower section, miles		17
Total length of line, miles		147
Conductor		
Type		ACSR
Circular mils		900,000
Aluminum strands, number and diameter (inches)		54×0.1291
Steel strands, number and diameter (inches)		7×0.1291
Outside diameter, inches		1.162
Weight per foot, pounds		1.158
Guaranteed ultimate strength, pounds		32,300
Single circuit	normal sag, feet (ruling span, 1,200 feet)	38.6
	ruling tension, 60 deg F, no wind, pounds	5,400
	maximum design tension, 0 deg F, 8-pound wind, $1^1/_2$-inch ice, pounds	21,950
Double circuit	normal sag, feet (ruling span, 800 feet)	16.6
	ruling tension, 60 deg F, no wind, pounds	5,600
	maximum design tension, 0 deg F, 8-pound wind, $1^1/_2$-inch ice, pounds	19,500
Total pounds used, approximately		2,750,000
Overhead Ground Wire		
Number		2
Type		Special 30 per cent conductivity, $^1/_2$-inch Copperweld
Stranding		7 #6
Outside diameter, inch		0.486
Weight per foot, pound		0.515
Guaranteed ultimate strength, pounds		22,510
Single circuit	normal sag, feet (ruling span, 1,200 feet)	26.8
	ruling tension, 60 deg F, no wind, pounds	3,470
	maximum design tension, 0 deg F, 8-pound wind, $1^1/_2$-inch ice, pounds	15,480
Double circuit	normal sag, feet (ruling span, 800 feet)	9.2
	normal tension, 60 deg F, no wind, pounds	4,400
	maximum design tension, 0 deg F, 8-pound wind, $1^1/_2$-inch ice, pounds	13,540
Total pounds used, approximately		816,000
Span		
Single circuit	ruling span, feet	1,200
	average span, feet	1,080
	maximum span, feet	1,275
	minimum span, feet	599
Double circuit	ruling span, feet	800
	average span, feet	700
	maximum span, feet	1,095
	minimum span, feet	229
Insulators		
Phase-wire suspension:	size, inches	$5^3/_4$x10
	strength, pounds	15,000
Phase-wire strain:	size, inches	$5^3/_4$x10
	strength, pounds	25,000
Ground-wire suspension:* (one unit)	size, inches	$5^3/_4$x$6^1/_2$
	strength, pounds	25,000
Ground-wire strain:* (one unit)	size, inches	7x12
	strength, pounds	36,000
Standard suspension insulator string		18—($5^3/_4$x10—15,000 lb)
Standard angle insulator string		Double 20—($5^3/_4$x10—15,000 lb)
Standard strain insulator string		Double 20—($5^3/_4$x10—25,000 lb)
Clearances		
Conductor to tower, inches for 8-pound wind at 60 deg C	single circuit	103
	double circuit	108
Dampers		
For conductor; weight, pounds		16
Number used per span, for all lengths of span		2
For ground wire; weight, pounds		6
Number used per span, span lengths up to 1,000 feet		2
Number used per span, span lengths over 1,000 feet		4
Maximum permissible stresses in tower steel		
Tension, pounds per square inch		33,000
Compression, pounds per square inch for:	L/R up to 150	38,000–165 L/R
	L/R between 150 and 200	28,000–100 L/R
Shear on bolts, pounds per square inch		26,000
Bearing on bolts		52,000

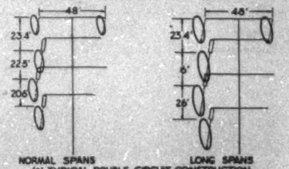
*For insulating one ground wire, on double-circuit towers, for use as supply conductor for obstruction lights.



(a) TYPICAL SINGLE CIRCUIT CONSTRUCTION



(b) SINGLE CIRCUIT TRANSPOSITIONS



(c) TYPICAL DOUBLE CIRCUIT CONSTRUCTION

Figure 2. Sleet clearance diagrams at midspan for Powerton-Crawford 220-kv line

on hardware, and for the conductor alone the corona voltage was more than 300 kv three-phase.

With this conductor, the sag at 60 degrees Fahrenheit is 38.6 feet in the ruling span of 1,200 feet and the tension is 16.7 per cent of ultimate. Under the assumed worst loading of zero degree Fahrenheit, 1½-inch ice, eight-pound wind, the tension reaches 68 per cent of ultimate, so that the conductor should safely withstand these severe conditions. Similar considerations hold for the double-circuit tower section with shorter spans. The data are given in detail in table II.

Protection Against Sleet

Sleet-prevention and sleet-melting procedures are being used very effectively on the 132-kv lines in the Chicago territory. However, the size of the conductor and the electrical characteristics of the 220-kv line are such that it is impracticable to apply these procedures. Hence it was necessary to design the line for trouble-free operation under heavy ice loading, with its accompanying hazard of galloping conductors.

The envelopes of probable positions of the wires during extreme galloping were determined by an empirical rule developed by A. E. Davison[1] from observations of dancing conductors in the field. The validity and practical value of the procedure were first checked by applying it to all important transmission lines in the Chicago district. It was found that the operating records agreed closely with the findings based on the envelopes of galloping, or sleet clearance diagrams. Where inadequate clearances were indicated, those lines had high rates of outage due to sleet storms. Where clearances were ample, the outage rates were zero or negligible. Figure 1 gives typical examples of sleet clearance diagrams and operating data for the 132-kv lines. It was noted that wire spacings commonly accepted as adequate for 132-kv construction were in reality quite inadequate in several cases from the standpoint of maintaining service with ice on the wires, as is essential with this line.

The most difficult problem in providing adequate spacings is encountered where the wires are in vertical configuration. In single-circuit construction this affects the positioning of the ground wires above the conductors. In double-circuit construction it involves also the positioning of the phase wires one above the other. The difficulty resides in the fact that galloping conductors have been observed to soar up at least as high as the level of their supporting clamps and to drop down

Table III. Wire Spacings on Towers

	Single-Circuit Towers						Double-Circuit Towers						
				Wire Spacing at Tower—Feet						Wire Spacing at Tower—Feet			
				Vertical**	Horizontal					Vertical**		Horizontal	
		Towers						Towers					
Class of Tower	Span Length (Feet)	Type	Number Installed	Conductor to Ground Wire	Conductors	Ground Wires	Span Length (Feet)	Type	Number Installed	Conductor to Ground Wire	Conductor to Conductor	Bottom and Top Conductors	Middle Conductor and Ground Wires
Suspension	Up to 1,130	*A*	277	29	33.3	46.3	Up to 800	*I*	68	23.4	22.5, 20.6‡	40	48
	1,131–1,215	*B*	259	34	33.3	46.7	801–925	*J*	12	23.4	26	40	48
	1,216–1,275	*C*	37	37.6	33.3	46.7							
15 deg angle	Up to 1,130	*D*	21	31.6*	30.3, 36.7†	46.3	Up to 940	*K*	4	25.4*	27, 26‡	40	48
	1,131–1,199	*E*	13	35.6*	30.3, 36.7†	46.7	Up to 940	*L*	18	27.9*	27, 26‡	40	48
Dead end and 45 deg angle	Up to 1,130	*F*	16	29	31	42	Up to 800	*M*	8	23	21, 20‡	40	48
	1,131–1,220	*G*	6	34	31	42	801–940	*N*	7	23	26	40	48
							1,095	*P*	2	28.5	32.5	30	48
Transposition	0–1,130	*H*	8	27.5	30	42	Up to 800	*O*	5	23	21, 20‡	40	48
Total			637						124				

*Not corrected for deviation of insulator string from vertical; corrected spacing is about the same as for suspension towers.

**Vertical spacing between conductor and ground wire at midspan is increased approximately ten feet on single-circuit section and approximately seven feet on double-circuit section.

†First figure is spacing from middle conductor to conductor on inside of angle, second figure is spacing to conductor on outside of angle.

‡First figure is spacing between top and middle conductor, second figure between middle and bottom conductor.

Table IV. Design Loads for Towers

	Single-Circuit Towers					Double-Circuit Towers				
		Specified Loads*					Specified Loads*			
Tower Type	Line Angle at Tower	Radial Ice, In.	Wind, Lb. Per Sq. Ft.	Broken Wires	Specified Margin of Safety, Per Cent	Line Angle at Tower	Radial Ice, In.	Wind, Lb. Per Sq. Ft.	Broken Wires	Specified Margin of Safety, Per Cent
Suspension	0 to 2	½	8	One conductor or one ground wire	50	0 to 2	½	8	One conductor or one ground wire	50
		½	8	Any or all	0		½	8	Any two wires	25
		1½	8	All intact	15		½	8	Any three wires	10
							1½	8	All intact	15
Angle	2 to 15	Same as for suspension tower				2 to 15	Same as for suspension towers			
Dead-end	0 or 15 to 45	½	8	Any or all	50	0	½	8	Any or all of 8 wires	50
						45	½	8	Any or all of 4 wires on one side of tower	50
		1½	8	All intact	15	45	½	8	Any or all of 8 wires	25
						0 to 45	1½	8	All intact	15
Transposition	0 to 2	Same as for suspension tower				0 to 2	Same as for dead-end tower			

*Includes dead weight of tower and wind load on tower of 13 pounds per square foot on 1½ times the projected area of one face.

NOTE: Where two or more broken wires are specified, all broken wires are to be in the same span. "One side of tower" refers to location on tower, not to span.

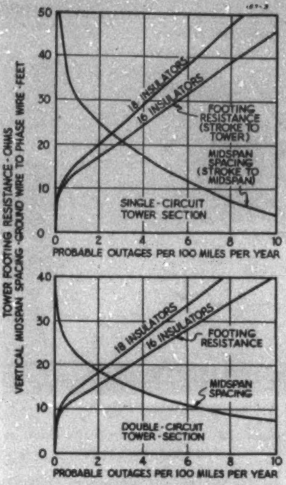


Figure 3. Probability of lightning outages as affected by line insulation and tower footing resistance

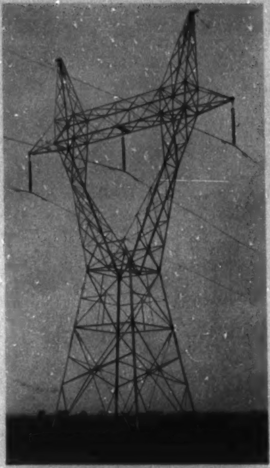


Figure 5a. Typical single-circuit suspension tower

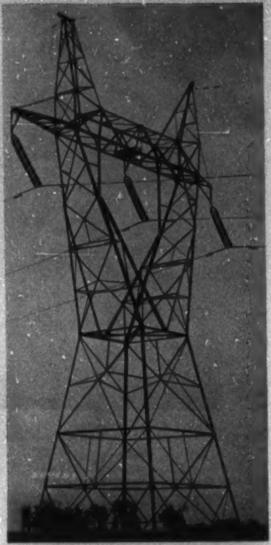


Figure 5b. Typical single-circuit 15-degree-angle tower

somewhat below the normal rest position. This means that the vertical spacing from one wire to the next should be at least as great as the sag in the upper wire. Where the sag is very large, as in unusually long spans, the vertical spacing at the towers should likewise be large and the tower would be correspondingly expensive. For shorter spans the vertical spacings can be smaller and the towers cheaper. In designing towers for adequate wire spacings, it thus becomes necessary to choose one of the following two alternatives: (1) have only one design but the tower must be as large as the longest span requires; (2) have at least two varieties of tower, one for spans of normal length and another for unusually long spans. This point must be studied individually for each class of tower, that is, suspension, angle, transposition, and dead-end.

For the Powerton-Crawford line the second alternative proved to be the most economical and was adopted. Complete data on tower types and wire spacings are given in table III. It should be noted that the tower sites were fixed by prior right-of-way negotiations. If this had not been true, some simplification and economy might have been achieved by shortening some span lengths, thus eliminating some of the types of towers with

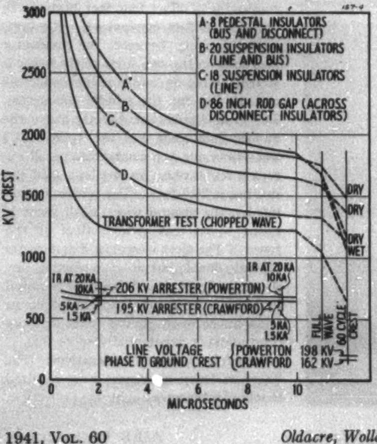


Figure 4. Co-ordination of insulation at terminals of Powerton-Crawford 220-kv line

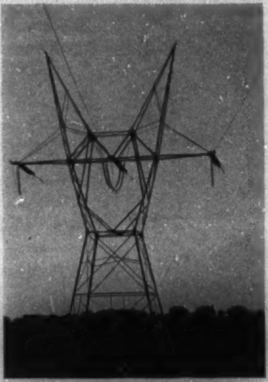


Figure 5c. Typical single-circuit 45-degree-angle and dead-end tower

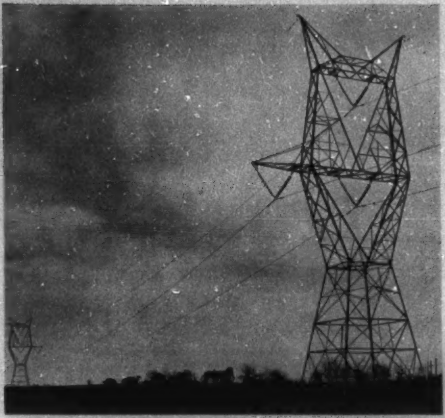


Figure 5d. Typical single-circuit transposition tower

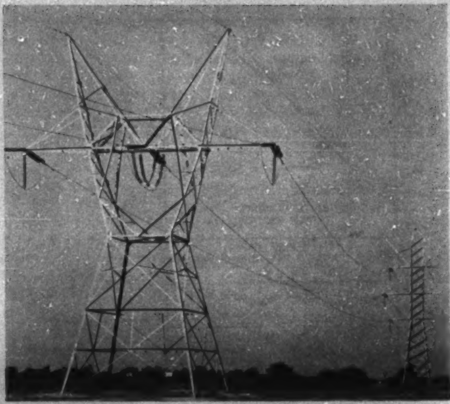


Figure 5e. Transition from single-circuit to double-circuit tower

the larger vertical spacings. On single-circuit towers the ground wires are 29 feet above the phase wires for all spans up to 1,130 feet long, 34 feet for longer spans up to 1,215 feet, and 38.3 feet for the longest spans up to 1,275 feet. These spacings are used not only for the suspension towers but also for the 15-degree-angle towers and dead-end or 45-degree-angle towers. The vertical spacings between the ground and phase wires at midspan are in all cases about ten feet greater than at the tower. This was necessary to provide sufficient clearance and was obtained by stringing the ground wire in a flatter sag than the conductor. The conductors are spaced 33.3 feet apart horizontally, giving ample clearance to tower steel. The two ground wires are inset approximately ten feet from the outer conductors, which helps to maintain galloping clearance. The sleet clearance diagram for typical single-circuit construction is shown in figure 2*a*. Special consideration was given to the transpositions, which are described elsewhere in the paper. The sleet clearance diagram in figure 2*b* shows that ample clearances were provided.

The double-circuit towers have a minimum vertical spacing of 20 feet between conductors for spans up to 800 feet. The longer spans up to 940 feet have 26-foot spacing. One long crossing span of 1,095 feet has $32\frac{1}{2}$-foot spacing. The middle conductor is offset four feet beyond the top and bottom conductors. The horizontal spacing between the conductor positions on the two sides of the tower is 40 feet for the top and bottom conductors and 48 feet for the middle conductors. The ground wires are directly above the middle conductor and are spaced 23.4 feet above the top conductors on all except a few towers; in one long span the spacing is 28.5 feet. The midspan spacing for the ground wire on all spans is about seven feet greater than at the tower. The sleet clearance diagrams for typical double-circuit construction are shown in figure 2*c*. A point to be noted in figure 2*c* is that the vertical spacing at midspan between ground wires and top conductors is substantially more than is required from sleet considerations. The larger midspan clearances were required to secure satisfactory lightning protection,

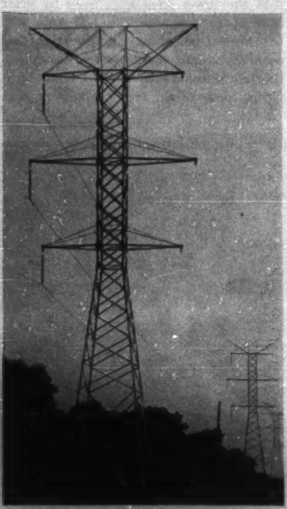


Figure 5f. Typical double-circuit suspension tower

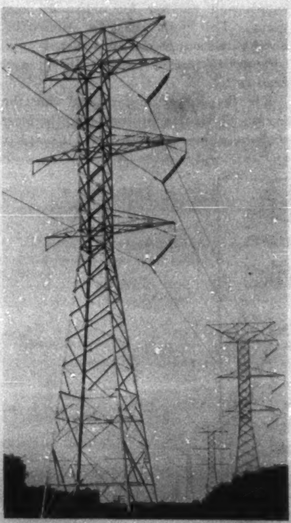
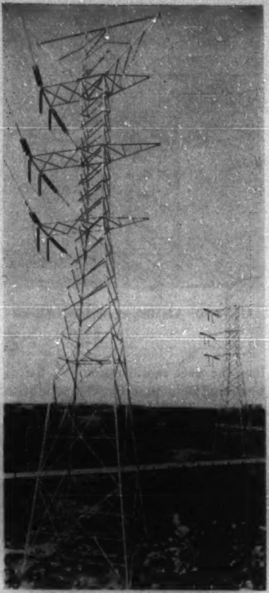
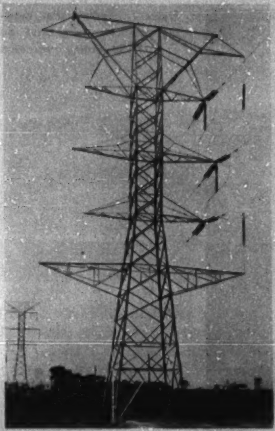


Figure 5g. Typical double-circuit 15-degree-angle tower





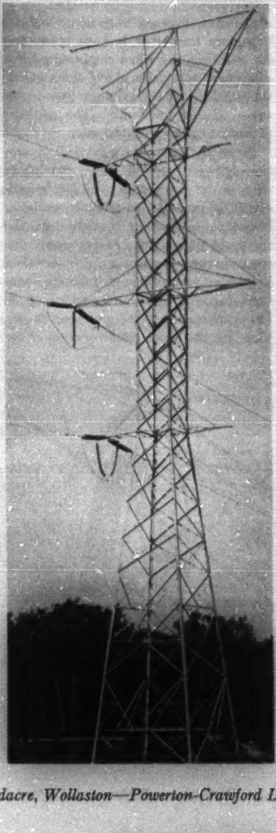


Figure 5h (above). Typical double-circuit 45-degree-angle and dead-end tower

Figure 5i (top center). Typical double-circuit transposition tower

Figure 5j (lower right). Special double-circuit crossing tower

as is brought out in more detail in the section on lightning protection.

Lightning Protection and Insulation

The degree of shielding of the power conductors from lightning strokes that is effected by the ground wires is usually expressed as a function of the protective angle, which is the angular departure of the conductor from a position vertically below the ground wire. For the vertical position, the protective angle is zero and the shielding is the best obtainable. Larger protective angles mean greater departures from the vertical and reduced efficiency. However, it is generally agreed that the reduction in effectiveness of shielding is negligible for protective angles up to 20 degrees. The towers were therefore designed so that the protective angle would not exceed 20 degrees, and two ground wires were installed on all towers.

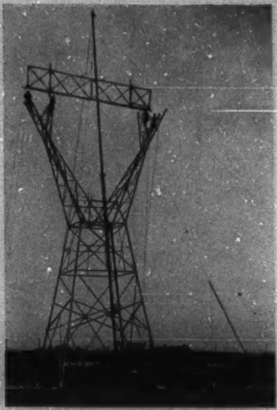


Figure 6. Erecting tower with extensible boom

On the single-circuit section of the line the normal protective angle for tangent and dead-end towers is from 15 to 19 degrees at the tower. At midspan the differential sagging improves the protective angle by about four degrees. The protective angle for transposition towers is about nine degrees at the tower and eight degrees at midspan. On the double-circuit section of the line the protective angle is zero, as the ground wires are directly above the outermost conductors. The location of the ground wires relative to the phase wires on all angle towers, both single- and double-circuit, is the same as on tangent. With these provisions, the chance of a direct stroke hitting a phase wire on any part of the line is exceedingly small.

On the assumption that all direct strokes would terminate on a tower or ground wire, calculations of outage probability were made to determine the effects of line insulation, tower footing resistance, and location of ground wires with respect to the phase wires. The results of these calculations are shown in figure 3.

Based on this study, the line insulation was set at 18 standard 10-inch diameter by $5^3/_4$-inch spacing cap and pin type insulators, having a full-wave flashover voltage of approximately 1,585 kv for a 1.5 by 40-microsecond positive wave. The standard-size insulator was selected following an economic study which showed the cost per kilovolt to be from 10 to 18 per cent less for this insulator than for insulators with 6- or $6^1/_2$-inch spacings, in the string lengths under consideration. For physical reasons double strings

Figure 7. Earth anchor, showing installation of ground rod in footing excavation

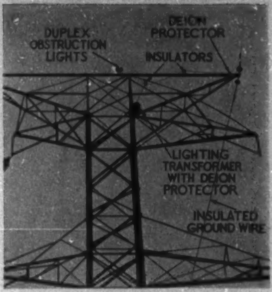

DUPLEX OBSTRUCTION LIGHTS
DEION PROTECTOR
INSULATORS
LIGHTING TRANSFORMER WITH DEION PROTECTOR
INSULATED GROUND WIRE

Figure 10. Lighting equipment on tower

of insulators of standard or high strength were used on special towers and special locations, with the number of disks increased to 20 for dead ends.

In order to keep lightning outages down to the desired low level, that is, an average of substantially less than one outage per 100 line-miles per year, figure 3 indicates that tower footing resistances should be held to values below 10 or 15 ohms. However, to provide for the usual uncertainties in measuring ground resistances, seasonal variations, and the departure from true leakage resistances for counterpoises when conducting lightning currents, a value of five ohms was established as the maximum permissible tower footing resistance. The methods of grounding are described later.

Figure 3 shows that a vertical separation between ground wires and phase wires of the order of 35 to 40 feet is indicated to properly co-ordinate midspan insulation with tower insulation on the single-circuit section. As previously discussed in the section on protection against sleet, the prevention of galloping contacts required midspan spacings of 39 feet or more, depending upon span length. Thus, for the single-circuit section of the line, the sleet consideration was the determining factor in setting this separation.

For the double-circuit section, figure 3 indicates that a vertical midspan spacing between ground wire and phase wire of 25 to 30 feet was required from a lightning protection standpoint. This was more than needed to prevent sleet contacts, and was the controlling consideration in determining the vertical spacings for this part of the line.

In both cases, economic studies showed

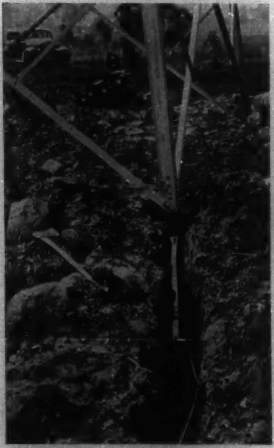


Figure 8. Counterpoise connection to tower leg

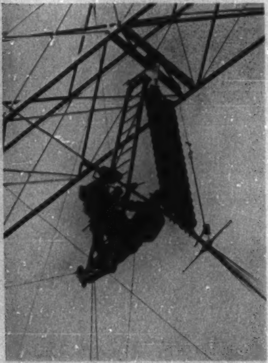


Figure 9. Installing tapered armor rods

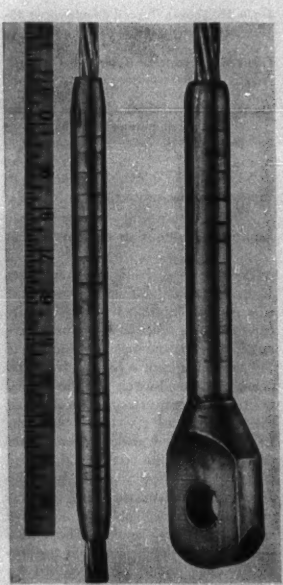


Figure 11. Ground-wire splice and dead end

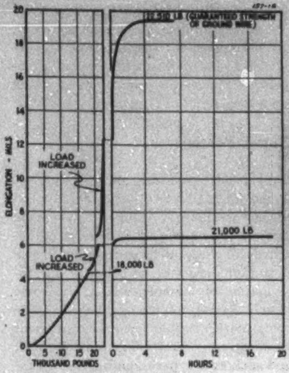


Figure 12. Stress-strain test on dead-end fitting for ground wire

that these large midspan spacings could be most cheaply obtained by employing differential sagging between ground wires and phase wires.

Terminal Insulation

At both the Powerton and Crawford stations the line terminates on steel structures and is conducted over suspended busses, supported from 20-unit insulator strings, to disconnect switches. From the switch to the transformer terminal at Crawford a rigid bus is carried on eight-unit stacks of pedestal-type insulators. These stacks are mechanically graded, with two ten-inch-diameter bolt circle insulators at the bottom, one transition insulator, and five insulators with seven-inch-diameter bolt circle. Eight-unit stacks, rated at 287 kv, were selected to provide additional leakage distance under the atmospheric contamination prevalent at the generating stations.

Similar stacks of insulators are used for the disconnect switches at both terminals. To insure against possible lightning flashover between line side and bus side of the switch when open, the pedestal stacks on each side are provided with rings spaced 86 inches apart. For this spacing the impulse flashover voltage is approximately 20 per cent less than that across the open switch.

At Crawford station, the 230-kv transformer windings are protected by 195-kv station-type arresters installed approximately 50 circuit-feet from the transformer bushings. Because of the higher operating voltage at the generating end of the line, the arresters used at the Powerton end are rated at 206 kv. Figure 4 shows the insulation levels for the various equipment at the terminals.

Figure 13. Stringing wire under tension

To protect the terminals from direct strokes, a network of ground wires is installed above the exposed high- and low-voltage busses. The elevations of these wires above the busses, and their necessary ground resistances, were calculated to give greatest practicable immunity from lightning flashover of the bus insulation.

Ground Wires

The following factors were considered in selecting the ground wire: strength consistent with the flat sagging required, cost, freedom from corrosion and vibration troubles, previous operating record, and availability of splices and hardware. These requirements appeared to be most adequately met by a one-half-inch, 30 per cent conductivity, extra-high-strength Copperweld wire, specially selected for ten per cent increased breaking strength. Complete data on the wire are given in table II. The guaranteed strength is 22,510 pounds. This wire could be sagged approximately ten feet less than the phase wires in the single-circuit section and seven feet less in the double-circuit section with an equally high factor of safety for phase and ground wires. The normal tension at 60 degrees Fahrenheit, no wind, is 15.4 per cent of ultimate for the single-circuit section and 19.6 per cent for the double-circuit section. The maximum design tension at zero degree Fahrenheit, $1^1/_2$-inch ice, eight-pound wind, is 68.7 per cent for the single-circuit section and 60.2 per cent for the double-circuit section. These values are considered to be quite conservative. The permanent set in the unloaded sag will not exceed six feet in any span following this loading, which will occur very rarely if at all.

Tower Design

The lack of means for preventing sleet formation resulted in a rather radical departure from common practice in setting up tower loading schedules. As shown in the loading schedule given in table IV, the towers were designed to withstand $1^1/_2$-inch radial ice of normal density on the conductors combined with the usual transverse wind loading of eight pounds per square foot with all conductors intact. Under this condition the specified margin of safety is 15 per cent. This necessitated unusually heavy construction in certain parts of the towers. In addition, the specification called for the usual 50 per cent safety margin with one ground

Figure 14. Pulling equipment for stringing under tension

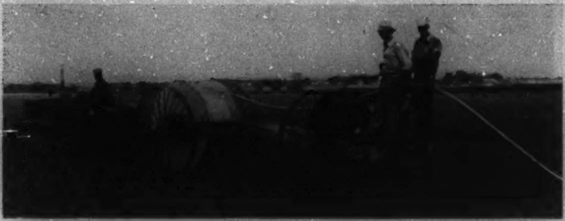


Figure 15. Stringing wire by laying out on ground

wire or one phase wire broken, but in this case the loading was reduced to the customary medium schedule; that is, one-half-inch radial ice and eight-pound wind. The combination of these two requirements resulted in suspension towers and towers for small angles that would almost, but not quite, withstand "any or all" wires broken under the medium loading schedule, instead of the usual "one broken conductor" or "one broken ground wire." The additional steel required to permit these towers to meet the "any or all" condition was so small that it was decided to design the towers to meet it. This results in the unusual situation of having suspension and small-angle towers capable of acting as dead-end towers under all but the most severe sleet and wind loadings. Another consequence is that all towers are heavy enough to withstand very high wind pressures in summer when there is no sleet loading. The calculated maximum steady wind velocity (actual) that can be withstood is about 150 miles per hour, which should enable the towers to withstand all but the most severe tornadoes experienced in this territory. Naturally, the tower strength under storm conditions is expected to be somewhat lower than estimated, because of the effects of dynamic shock produced by gusts of wind or breaking wires.

The double-circuit towers have ten per cent more strength than called for in the specifications. It was decided not to test these towers, due to lack of time and of adequate testing facilities, and so the extra ten per cent strength was included in lieu of testing. The cost of the extra weight of steel required to provide the additional ten per cent margin of strength was about the same as the cost of testing would have been. The same procedure was followed in the case of the single-circuit transposition towers. The single-circuit suspension, 15-degree angle, and dead-end towers were thoroughly tested.

Typical single- and double-circuit towers are shown in figure 5. The towers are of galvanized steel parts bolted together. Heavy lockwashers were used to ensure permanence of the bolted connections, as Chicago tests and experiences indicated heavy lockwashers to be the most reliable device. In addition, the nuts were locked by prick-punching after final tightening.

The erection of most of the single-circuit suspension towers was done very efficiently by using the extensible boom shown in figure 6. The boom had a mobile mounting and was 115 feet long when fully extended. An advance crew assembled the tower into small subsections such as side panels, main section of crossarm, and ground-wire peaks. The assembled sections were laid out on the ground at the tower site in a prearranged pattern. The erecting crew set up the hoisting boom and erected the tower by sections. Suspension towers were often erected in as little as $2^{1}/_{2}$ hours by this method. The erection of the remaining towers was done with a slung gin pole, as has frequently been done before.

Transpositions and Dead Ends

The methods of making transpositions on single- and double-circuit types of construction are thought to have advantages over some other methods that have been commonly used. In the single-circuit transposition, shown in figure 5*d*, there is no crossing of wires in the spans. This materially reduces the chance of wires coming in contact under storm conditions. The double-circuit transposition tower is pictured in figure 5*i*. Only three main crossarms are used, and the tension of each dead-ended conductor is balanced by an equal opposing tension. This makes for a relatively light, neat-looking tower.

The general principle was adopted of having as little dead-end construction on the line as possible. Experience of other utilities indicated that electrical and mechanical troubles were more apt to occur at dead-end towers than elsewhere. Consequently, dead-end towers were installed only where absolutely necessary. In particular, it was considered undesirable to install dead-end towers on tangents for the sole purpose of breaking up a long series of suspension spans.

Tower Footings

Pyramid-type earth anchors were selected for the tower footings. A typical anchor is shown in figure 7. Concrete anchors were not used because of anticipated difficulties with weather and transportation during construction. The installation of anchors was to be done mainly in the winter of 1939–40. The cold weather would have been a great handicap to pouring concrete in the field, and anchors of types requiring that operation were considered unsuitable. This eliminated the Malone type and the grillage-cast-in-concrete type. Precast concrete anchors were ruled out on the grounds of being expensive, difficult to transport from storage yard to tower site, especially over muddy fields, and difficult to set accurately on account of their great weight.

The steel for the earth anchors was designed to have the same safety margins as specified for the towers. The bearing surfaces, however, were designed to have an additional margin of 20 per cent in the case of suspension and 15-degree-angle towers and 33 per cent for dead-end towers. These safety margins apply only to the bearing and uplift pressures in the soil. The maximum allowable

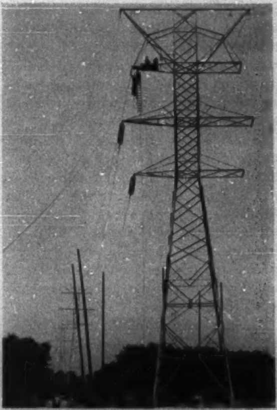


Figure 16. Making compression dead-end on crossarm

Figure 17. Laboratory setup for radio noise and corona tests

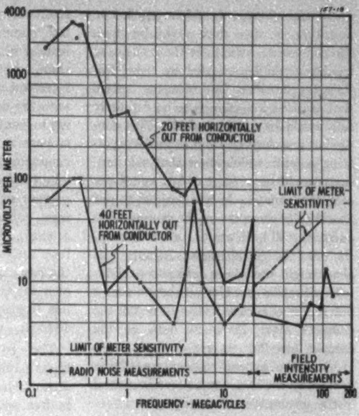


Figure 18. Radio noise under artificial rain—220-kv insulator string and conductor. Conductor energized at 140-kv alternating current to ground

bearing pressure including overloads was specified as 10,000 pounds per square foot. This meant that the maximum actual pressure would be about 5,000 pounds per square foot (10,000/(1.33×1.50)) for the worst condition and the normal pressure would be about 1,200 pounds or less per square foot. Resistance to uplift was figured on the basis of an inverted frustrum of a 30-degree pyramid of earth weighing 100 pounds per cubic foot. A few rock anchors were used, consisting of a post angle carrying crosswise clip angles concreted in rock. The concrete is carried six inches above the ground line and is designed to take the horizontal shear stresses at the ground line.

All anchors are of galvanized steel and are spliced three feet below the ground line to permit replacing any parts that may succumb to ground-line corrosion. As a further precaution, a protective coating of asphalt paint was applied from a point three feet below the ground line to a point six inches above it. Actually no corrosion trouble is anticipated if the galvanizing is intact when the steel is installed, since examinations of some galvanized steel tower footings in this territory which have been in service as long as 33 years show them to be in remarkably good condition.

Tower Grounding

From the standpoint of securing low resistance tower grounds for protection against lightning flashovers, the Powerton-Crawford line is fortunately situated. As shown by Card,[2] Northern and Central Illinois have in general fairly low soil resistivity as compared with most of the United States.

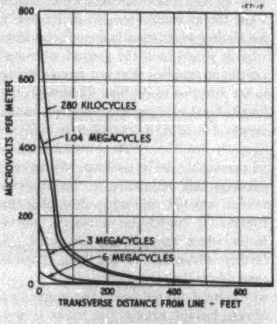


Figure 19. Radio-noise measurements on Powerton-Crawford 220-kv line

The large grillage anchors provide extensive contact area with the soil. At the bottom of each anchor a ten-foot galvanized steel rod was driven and connected to the footing, as shown in figure 7. This procedure resulted in bringing 92 per cent of the towers to the desired resistance of five ohms or less. For 57 towers, some of which were in rocky soil where rods could not be driven, the measured ground resistance averaged 12.2 ohms and had a maximum value of 47 ohms. Footing resistances of these towers were reduced in a few cases by deep ground rods and in the remainder by counterpoises. An average of 1,420 feet of counterpoise per tower resulted in an average final footing resistance for these towers of four ohms. Special precautions were taken to assure permanence of the grounds. Ground rods, counterpoise wire, and connections have extra heavy galvanizing. Galvanized steel was used throughout to avoid having contacts between dissimilar metals that might cause galvanic action.

Counterpoises are of the discontinuous type parallel to the line. Half-inch stranded galvanized steel wire was used, buried 18 inches in the ground and connected to the tower legs with galvanized steel straps as shown in figure 8. Wires 300 feet long were installed successively on the four tower legs until the resistance had reached the specified value. Where necessary, additional wires of the same length were installed parallel to the first wires. Except in a few instances, where interference with the counterpoise wires was encountered, they were spaced 50 feet apart. The maximum amount of counterpoise used on any one tower was 3,600 feet.

When the grounding work was completed, all towers on the line had footing resistances below five ohms. The average for the 761 towers on the line was 1.8 ohms.

Vibration Protection

The available information on vibration troubles on overhead wires covers a wide range of causes of vibration such as tension, material, and construction of the cable. It was concluded after studying this information that any overhead wire might be subject to vibration and that preventive devices should be applied to any type of wire that might be used. There is some indication that vibration trouble could be avoided by using low stringing tensions, but it seems certain that tensions low enough to be definitely safe would be uneconomical. This conclusion was derived from extensive operating records which show that safe tensions would be at least as low as ten per cent of ultimate. It was decided to put Stockbridge dampers on the ground wires and to install both armor rods and dampers on the conductors. Armor rods alone might have been sufficient to eliminate vibration troubles on the conductor but it was considered safer to install the dampers also. Pertinent data on the damper installations are given in table II, and the general appearance of the devices on the line is to be seen in several of the accompanying photographs.

A view of the armor rods being installed is shown in figure 9. Besides vibration protection, they will provide protection for the conductor against flashover burns and mechanical damage in the region of the clamp. Protection against flashover burns could have been obtained by using grading shields, but only at the expense of reduced clearances to tower steel or of larger towers. Grading shields were not used, as they would have no function beyond protecting the insulators against flashover damage. This seemed unnecessary, in view of the general experience that modern insulators do not shatter badly enough to come apart because of flashovers. Any lesser damages such as loss of insulating value would probably be confined to a few units in the string. This would be of little consequence because several units could be short-circuited in the string without affecting normal operation of the line. This results from the fact that the string length was selected on the basis of impulse strength and its 60-cycle strength happens to be considerably higher than necessary.

Insulator and Hardware Assemblies

The selection of the hardware assemblies presented several points of interest. Double strings as illustrated in figure 5*b* were used at all crossings of important highways for conductors in suspension. The double clamp arrangement shown in the photograph was preferred to the more usual yoke-plate and single clamp because the total length worked out about the same as for the single string. Consequently, the clearance to the tower steel was not affected by the inevitable last minute decisions to install double instead of single strings at certain locations. Longer armor rods to match the increased clamping length were installed with the double strings. It was surprising to learn from tests that even the single suspension clamp would not permit the conductor to slip through it under broken-wire tensions. This is desirable as it will prevent spans adjacent to a broken span from coming down all the way to the ground. The slip is not needed to relieve the tension. If a conductor breaks, the suspension insulator strings will pull over far enough in the direction of the line to slack off the tension as much as 40 to 70 per cent.

The ground-wire suspension assemblies, figure 9, were made flexible and about 18 inches long. This puts the wire in free suspension, so that the tension will be equalized between the spans. Thus the maximum tension under heavy ice loading is substantially reduced from what it would be if the ground-wire supporting assemblies were not free to swing in the direction of the line. Likewise, the longitudinal swing of the assembly in the event of a broken ground wire will reduce the loading on the tower by 15 to 50 per cent. As in the case of the conductors, this reduction in broken-wire tension was not utilized in designing the towers. It constitutes a reserve safety factor which was cheaply obtained. A further advantage of the long, flexible assembly is that the chance of the ground wire being damaged due to the supporting clamps locking against the tower is negligible.

The hardware for all suspension assemblies was selected on the basis that the elastic limit of the weakest piece should be at least equal to the maximum design load on the assembly. In general, the ultimate strength of each item making up the assembly is at least $2^1/_2$ times the design load. The insulators were selected so that the maximum continuous loading would not exceed 40 per cent of the M&E rating. However, it was deemed permissible[3] to go to 60 per cent of M&E rating for the maximum design load. The maximum loading is for the condition of very heavy sleet on the wires, and is expected to have a duration of not more than about 48 hours. On the basis of these considerations, the standard ten-inch insulators having an M&E rating of 15,000 pounds were selected for suspension duty.

The insulators in the conductor dead-end assemblies were selected on the same basis as the suspension insulators. Because of the heavier design loadings, double strings of standard ten-inch insulators with an M&E rating of 25,000 pounds were required. The hardware for all dead-end assemblies was selected on the basis that the ultimate strength of the weakest item would be at least equal to the breaking strength of the wire. The splices and dead-end fittings for the ground wire had to meet these requirements and, in addition, it was desired to take no chances on galvanic corrosion at the point where the fittings contacted the copper-covered steel strands. A new splice and dead-end fitting, figure 11, were developed to meet these needs. They are pressed on the wire with the same press and dies used for splicing and dead-ending ACSR wire. The material is a special bronze, to avoid corrosion and season cracking troubles. The weight is extremely low, to avoid vibration troubles. Results of accelerated loading tests with and without severe heat cycles were very satisfactory. Some typical test data are shown in figure 12. The compression sleeve of the dead-end fitting is identical to the splice, which is a bronze sleeve with a sprayed coating of Nichrome on the interior of the bore. When the sleeve is pressed on the wire, the hard particles of Nichrome are embedded in the softer metals of the sleeve and wire coating and form a bond between them.

Wire Installation

The conductor is the largest of the ACSR type that has ever been installed by pulling out under tension. The setup at the feeding end is shown in figure 13 and the pulling rig is shown in figure 14. In figure 13 the conductor is being paid off the three reels in the foreground and the ground wire from the other two reels. The braking drums for applying enough tension to keep the wire from dragging on the ground are directly in front of the cable reels. In part of the work, all five wires were pulled out simultaneously with the two tractor-mounted rigs shown in figure 14. Another method of pulling out under tension which was used extensively was to hitch the pulling lines to three tractors and drive them forward. One tractor pulled the middle conductor; the other two each pulled one outer conductor and ground wire. The tractor crews were directed by radio telephone. The pulls were about 3,600 feet long for both the conductor and ground wire. The pulling lines were attached to the wires with woven-wire basket grips.

Another method of stringing was to lay out the wire on the ground and then raise it into position on the towers. This scheme was used for the outer conductors and ground wires wherever the ground was smooth enough to ensure that the wire would not be damaged. The reels were mounted as shown in figure 15 and the wire was paid out as the tractor moved forward. The middle conductor on the single-circuit towers was always pulled out under tension, since it had to pass through the center of the tower.

It was found necessary to watch closely the surface of the stringing blocks, particularly the steel block used for the ground wire. The steel pulling line tended to wear grooves in the surface which could cut the copper coating of the wire. The ground wire tended to twist as it was being pulled through the blocks under tension, which made the cutting action worse. A little trouble was experienced from this cause but was largely avoided by close inspection.

The wires were temporarily dead-ended during installation at anchors set in the ground near midspan. Permanent dead ends and splices were usually made up on the ground. However, at some crossings it was not feasible to lower the wires again for the compressing of the dead ends after the wires had once been raised into position. In these cases a platform was set up on the crossarm and the dead ends were made on the tower, figure 16. Sagging was done by transit, and occasional checks were made with a dynamometer. The observer at the span being sagged directed the adjustment of the tension over a field telephone set connected to the conductor. All the wires were installed without prestretching.

Tower Lighting

The towers for the first ten miles from Crawford station are located in the general vicinity of the Chicago Municipal Airport and its associated airways and are to be provided with red warning lights in accordance with an agreement with the Civil Aeronautics Board. The usual method of providing a supply line to the towers by a distribution line would have been quite expensive, as an entirely new line would have been required throughout most of the ten miles. Instead, resort has been made to the unusual arrangement of insulating one of the ground wires and using it to supply distribution-type transformers on each tower. The return circuit is through the earth and the other ground wire. The insulation of the energized ground wire, figure 10, is sufficient for its normal potential of 2,160 volts alternating current, 60 cycle, but the impulse strength is only about 80 kv. De-ion protectors are shunted across this insulation so that a lightning stroke will go to ground through the protectors, except possibly for very steep-front strokes. Thus the energized wire will also function as a ground wire.

A distribution-type transformer of the CSP design is installed on each tower and supplies a standard 120-volt, 150-watt lamp. The fixtures are the standard Civil Aeronautics Board duplex obstruction lights, which have a throwover relay to place the second lamp in use after failure of the first one. The lamp sockets are mounted in specially developed spring supports to protect the filaments against vibration. The normal load current in the ground wire has been kept very low, about three amperes average, by using the 2,160-volt feed. The maximum current that would occur due to a fault has been limited to ten amperes by a reactor in the supply circuit. These limitations of current were necessary to secure inductive co-ordination, as discussed in the following section. Protective devices are provided in the circuit to open it in case of a fault anywhere external to the transformers on the towers. If a tower transformer becomes faulted, it will be automatically disconnected without interrupting the circuit.

Inductive Co-ordination

Some difficult problems in inductive co-ordination were encountered in the 17-mile section of double-circuit towers at the Chicago end of the line. The route throughout this section parallels three main trunk telegraph leads operated by two large telegraph companies. The separations originally varied from 50 feet to 3,000 feet. About 57 telegraph circuits are involved. Many of them are varioplex and four-channel multiplex circuits operating at signaling speeds of about 66 words per minute and their sensitivity to disturbing voltages is quite high. In addition, the telephone, telegraph, and signal wires of two important railroads are carried on the same poles with the commercial telegraph circuits. Some carrier telegraph and telephone circuits are now being superposed on the existing wires and there is an increasing trend toward this type of plant.

Co-ordination was secured by a variety of methods. The pole lines for the signal plant were relocated to eliminate physical conflict, but it was not possible to obtain wide separations. About $4^1/_2$ miles of parallel has only about 200-foot spacing. The induction on the sensitive high-speed signal circuits due to normal operation of the power line is nullified by neutralizing amplifiers[4] connected to the telegraph circuits. A pilot wire on the signal lead picks up the same voltage as is induced on the signal wires. This voltage is then applied in opposite phase to each affected signal wire, thus virtually canceling the induction. The signal circuits other than high-speed telegraph, for example telephone and Morse telegraph, are not provided with neutralizing equipment. For the benefit of such circuits, the mitigative measures included the retransposition of telephone circuits where necessary and the provision of resonant reactors for some of the telegraph circuits. A co-ordination measure of general benefit to all signal plant was to lay out the transpositions of the power line in such a way as to minimize the disturbing effect of earth currents induced in the ground wires by balanced current in the phase wires. The line was transposed at five points in the 17-mile section for this purpose.

The induction due to power line abnormals was calculated to be several thousand volts. This is far beyond the capacity of the neutralizing equipment, which has an efficient working limit of about 100 volts. However, it was seen that abnormals would very seldom occur since the construction used on the power line is of very high grade. On this basis, arrester relays to drain the 60-cycle induced current from the signal wires during power line abnormals were deemed adequate.

The supply circuit for the obstruction lights on the towers presented a difficult co-ordination problem, since it is essentially a multigrounded circuit with earth return and parallels the signal leads for ten miles. Co-ordination was accomplished by specially designing the circuit and its feeding equipment to hold the normal load current and the maximum fault current to the very low values previously mentioned. The induced voltages due to these currents are well within the capacity of the neutralizing amplifiers to nullify and in fact would have very little disturbing effect if the neutralizing amplifiers were out of service.

The only co-ordination problem of any consequence encountered on the 130 miles of single-circuit construction was solved by specially locating one of the transposition points. Only four transpositions were made, as the main purpose was merely to balance the impedances of the three phases. The four transpositions gave two complete barrels, one about 80 miles long and the other about 50 miles.

The progress of co-ordination engineering in the last ten years is well illustrated by the work done in connection with this line. When the project was started in 1930, the co-ordination measures then proposed were the best that were available. The present measures, however, are very much more effective and satisfactory to all the interested parties. This improvement in results did not entail increased costs. On the contrary, the present cost is more than 30 per cent less than the cost of the measures proposed in 1930. Another striking feature was the simplicity of the negotiations from an engineering standpoint. Due to the work of various committees in developing and publishing data on co-ordination problems, it was surprisingly simple to arrive at agreements between the many interests involved.

Radio Noise

Because of the proximity of the line to the Chicago Municipal Airport and its relationship to the Chicago radio range beacon, some questions arose as to possible interference with aviation radio communication which might be caused by the line. An investigation was conducted by three methods: (1) measurement of radio noise from a typical insulator string and section of conductor in the laboratory, (2) determination of radio noise at selected locations near the line and airport before and after the line was placed in service, and (3) checks on existing 220-kv lines in Pennsylvania and New Jersey.

In the laboratory investigation, an 18-unit string of insulators, supporting a 50-foot length of 900,000-circular-mil ACSR conductor, including the armor rods and vibration dampers, was arranged for test as shown in figure 17. Voltages up to 185 kv to ground, or equivalent 320 kv three phase, were applied to the conductor and measurements of radio noise-influence voltages made, using the EEI-NEMA-RMA circuit.[5] A frequency range of 150 kilocycles to 20 megacycles was covered.

The laboratory tests also included measurements made with an antenna placed at various distances from the conductor and insulator string. These measurements covered a frequency range of 150 kilocycles to 125 megacycles with special emphasis on the frequencies in common use in aviation radio. Radio noise at the lower frequencies and up to 20 megacycles was explored by means of a meter having the recommended EEI-NEMA-RMA circuit; the higher frequencies with an RCA high-frequency field intensity meter. This latter meter was applicable only for the measurement of radio-frequency field strengths; consequently, the lower and higher frequency measurements were not strictly comparable.

With the insulators dry, radio noise was practically negligible, even directly underneath the conductor. Results for an artificial rain falling on the insulators are shown in figure 18. At the closest point of measurement, an actual distance of 28 feet from the suspension clamp, field strengths as high as about 3,000 rms microvolts per meter (quasi-peak voltage of radio-frequency wave) were measured around 300 kilocycles. There was a marked decrease in radio noise for the higher frequencies, until at 20 megacycles and above no field strengths that exceeded the meter sensitivity were noted. Attenuation normal to the conductor was very rapid; the highest value measured 45 feet from the suspension clamp was only 100 microvolts per meter. As shown in the graph of test results, figure 18, much lower values were measured over most of the frequency range between 300 kilocycles and 125 megacycles. It appeared from these laboratory tests that the interference of the line with radio services would be negligible.

This tentative conclusion was supported by field measurements made during a co-operative investigation this year by several organizations on operating 220-kv lines. Data from this investigation are to be published. It was found that the attenuation of radio noise normal to the lines was quite high. At a few hundred feet the radio noise was negligible even at the lower frequencies, which included the radio range beacon services.

The general conclusions drawn from the foregoing tests were confirmed by measurements on the Powerton-Crawford 220-kv line. Figure 19 shows the characteristic rapid attenuation normal to the line and decrease in noise at the higher frequencies, for fair weather conditions. There has been no opportunity as yet to obtain corresponding data for wet weather conditions. It is anticipated, however, that the line will not be a source of radio interference, during any weather conditions, to receiving stations located a few hundred feet or more from the line. A careful check has shown that the line has had no detectable effect on the radio range beacon.

References

1. Ice-Coated Electrical Conductors, A. E. Davison. *CIGRE* Paper No. 229 (1939).

2. Earth Resistivity and Geological Structure, R. H. Card. AIEE Transactions volume 54, 1935 (November section), pages 1153–61.

3. Duration of Load and Its Influence on the Ultimate Strength of Suspension Insulators, John J. Taylor. *CIGRE* Paper No. 228 (1939).

4. Control of Inductive Interference to Telegraph Systems, J. W. Milnor. AIEE Transactions volume 59, 1940 (August section), pages 469–74.

5. Methods of Measuring Radio Noise, 1940, EEI Publication *G9*.

Short-Circuit Calculating Procedure for Low-Voltage A-C Systems

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Introduction

FAULT currents in electrical systems depend upon the impedance of the circuit for their intensity. Stresses, both thermal and mechanical, occur due to such currents and at various time intervals following initiation of the fault. Such stresses are of primary interest in system and equipment design. The possible amount of short-circuit current in any system equals the quotient of the voltage divided by the impedance. The latter term usually includes certain elements which formerly were indeterminate but now can be estimated with reasonable accuracy. Short-circuit currents measured in present-day (600 volts and less) types of equipment were found to check closely with currents calculated from the impedance of the circuit and the applied voltage.[1] It is, therefore, advisable to discard "rule-of-thumb" or "discounting" procedures for predetermining fault currents.

In the discussion which follows, a method is given whereby the symmetrical current may be calculated from the E/Z relation; the amount of d-c component is shown to decrease from a theoretical maximum at zero time at various rates according to the relation of the circuit reactance to its resistance; a simplified and convenient step-by-step procedure is given for the determination of balanced three-phase short-circuit currents using the foregoing relationships; the time is shown at which currents that determine the interrupting and short-time duties of circuit interrupting devices are identified in order that the corresponding value of reactance and d-c component can be used for calculation of the imposed current.

Impedance values of apparatus, conductors, devices, etc., generally in the metallic circuit, also arc drop and contact resistance have been assembled in sufficient variety for most cases accompanied by explanations, where necessary, of their uses.

The material presented, while not entirely complete, will, it is expected, give the reader a fairly comprehensive view of short-circuit calculations and supply him with a procedure and essential impedance data.

Scope

The procedure suggested is a fairly rigorous method and applies in particular to a-c systems of 600 volts and less. It applies specifically to "bolted faults" in which the values of impedance are known. Data, however, are listed in the appendixes which indicate possible arc drop in certain types of arc faults. The procedure does not attempt to discuss the limitation of currents when affected by interrupter action.

The procedure outlined gives a method of calculation which is of high accuracy, permitting the application of apparatus on a reasonably sound basis. Involving, as it does, a degree of technical knowledge, the need is indicated for a simplified approach which may be agreed upon as being capable of reasonable accuracy and which can be applied by industry at large. Certain efforts are now being made to simplify the determination of short-circuit current during transient conditions, but until such methods have been developed and agreed upon the procedure outlined herein, will, it is expected, be of definite assistance to those interested in predicting short-circuit currents. These methods, however, will not obviate the necessity for taking into account all the impedance elements in low-voltage circuits.

Objective

The object of a short-circuit calculating procedure is to obtain that value of current which occurs at a particular time, as required by the characteristics of the equipment upon which the duty is imposed. The duty imposed by the short-circuit current may result in mechanical or thermal stresses or stresses due to the function of circuit interruption, relay operation, or system disturbance.

Principles Underlying the Calculation of Short-Circuit Currents in Low-Voltage A-C Circuits

1. Short-circuit currents should be determined by calculation from the impedance constants and the voltage of the circuit, that is

$$I = \frac{E}{Z}$$

The accuracy of the calculation depends upon the completeness and the accuracy of the impedance values used.

2. In the calculation, the impedance of all apparatus, devices, busses, and conductors up to the point of fault, should be considered, assigning the proper numerical values available for each item. Although impedance values of individual devices or parts of the circuit may appear negligible, the sum of a number of small impedances may be of appreciable magnitude in the total impedance. Missing circuit impedances should be estimated as well as possible using good judgment based on experience with similar types of circuits.

3. Values of impedance (resistance and reactance) for the principal devices and conductors in distribution circuits have been established by recent studies and added to available information. These data together with the constants of apparatus are given in the appendixes. Data given in appendix A, pertaining to apparatus, is general in nature but may be used in the majority of cases for "estimating" purposes. For specific apparatus, specific data should be obtained from the manufacturer when accuracy of such degree is warranted.

(NOTE: While it is expected that specific information concerning equipment and apparatus given herein will be repre-

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[1] For all numbered references, see list at end of paper.

sentative of that manufactured by other concerns, the data included have been obtained from apparatus and equipment manufactured by the company which the authors represent.)

4. Arbitrary reduction factors should not be applied to calculated short-circuit currents. Likewise, the practice of using a fixed short-circuit current value, arbitrarily established regardless of circuit constants, should be discontinued.

Short-Circuit Characteristics of Systems

GENERAL

Short-circuiting a system at any point permits an abnormal current to flow immediately in that system. The amount and persistence of this current rush depend upon the characteristics of the synchronous and induction motors and generators connected to the system at the time, and upon the impedance in the circuit between the sources and the point of short circuit.

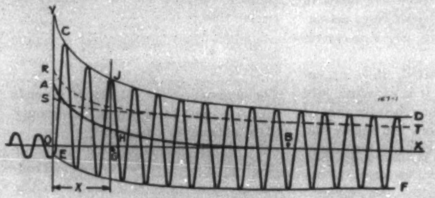

Figure 1. Short-circuit current flowing in one phase of a three-phase system

The greatest transient disturbance of the system that can occur at the point of application of the interrupting device, bus-bar supports, current transformers, etc., when the system is short-circuited governs the selection of those parts. The rms value of current is the unit in which the strength or ability of those parts is defined. A diagram showing the current flowing in one phase of the external circuit, when a system is short-circuited under ordinary conditions, is shown in figure 1. In this diagram, representing a current-time curve, the origin *O* is located at the instant fault occurs. The axis of abscissas, *OX*, represents time and the axis of ordinates, *OY*, represents current. The wave to the left of *O* represents the current during normal operation while that to the right represents the current from the start of the short circuit to the time it has become constant or sustained. It will be noted that the current wave at the instant of short circuit for the condition shown is initially unsymmetrical in respect to the *OX* axis, but that the wave gradually approaches the axis during the increasing increments of time, until it again assumes a position of symmetry. This phenomenon is dependent upon the point of the voltage wave at which the short circuit is established. It is possible by short-circuiting at different points on the normal voltage wave to secure short-circuit current waves ranging anywhere from those symmetrical about the *OX* axis to those totally unsymmetrical. Referring again to figure 1 showing short-circuit current based on an unsymmetrical wave, *CD* is a curve passing through the maxima of the wave of the total current, and *EF* is a curve passing through the minima. *AB* is a curve which cuts the vertical everywhere midway between *CD* and *EF*.

The wave of total current whose crests lie along curves *CD* and *EF* and whose ordinates are measured from the axis *OX* may be regarded as having two components, namely:

1. A direct component.
2. An alternating component.

The direct component is represented at any time by the ordinate to the curve *AB* or at the time *X* by the ordinate *GH*.

The alternating component is a wave whose crest value at any time is the difference between the ordinates to the curves *CD* and *AB*. This difference at the time *X* has the value *HJ*. The rms values of this alternating component are shown on curve *ST*. At any instant, this component is considered to have the same rms value as an alternating wave of constant amplitude whose crest value is represented by one-half the distance between curves *CD* and *EF* at that instant.

The effective total current at any instant is the square root of the sum of the squares of the value of the d-c component and the effective value of the a-c component. The quantity, so obtained, is the rms value of a steady-state current having the same magnitudes of a-c and d-c components as those which existed at the instant under investigation. The rms values of the waves whose peaks are defined by curve *CD* (major peaks) are shown on the curve *R-T*.

The rms current at any point of a system under short-circuit conditions is affected by the following factors:

1. Synchronous and induction rotating machine voltage, reactance, and resistance.
2. Circuit arrangement, voltage, reactance, and resistance.
3. Contact or arc resistance at the short circuit.
4. The nature of the short circuit, whether single-phase, multiphase, or phase-to-neutral.
5. The kilovolt-amperes and power factor of the load being carried at the time of the short circuit.
6. The point of the voltage wave at which the short circuit is established.
7. The effect of automatic voltage regulators is not considered herein as it is usually not noticeable until approximately 0.2 second after the short circuit.

SOURCES OF SHORT-CIRCUIT CURRENT AND THEIR BEHAVIOR

Practically all the rotating machines represent sources of short-circuit current, including induction as well as synchronous machines, whether applied as generators, motors, or converters. The short-circuit characteristics of the principal classes of machines differ in some respects as follows.

Synchronous Machines

A Steady-State Portion (I_s) controlled by the synchronous reactance x_d and field excitation which persists as long as speed and excitation are maintained, providing the circuit remains closed.

Plus a Transient Portion ($I'-I_s$) whose initial magnitude is controlled by the transient reactance (x_d') and which diminishes exponentially with a time constant generally between the limit of one-fourth and eight seconds, depending upon specific machine constants and electrical proximity to the fault.

Plus a Subtransient Portion ($I''-I'$) whose initial magnitude is controlled by the subtransient reactance (x_d'') which also diminishes exponentially but at a much faster rate with the time constant in the order of $^1/_{30}$ second.

It is to be noted that a quantity which is diminishing at an exponential rate is reduced to approximately one-third of its original value in a time interval equal to the time constant.

Induction Machines

The short-circuit current contribution from an induction motor or generator consists essentially of a single a-c component which diminishes exponentially.

No steady-state component is present due to the absence of separate excitation. The time constant of decay is of the same order as the subtransient component in synchronous machines, which permits common treatment of the two.

Synchronous Converters

Considering only the case of synchronous converters, singly or, if in multiple, supplied as a group from a common bus and paralleled on the d-c side, the reactance of the armature circuit and its resistance, together with the impedance of the converter's transformer, will limit the a-c current fed by the converter to a source-side fault.

If separate sources of supply feed converters operated in multiple on the d-c side, the impedance of the converter circuit is materially changed. This case deserves separate consideration.

Large Systems Feeding Small Systems

When giving consideration to a small system, fed in part or in whole by a large system, the large system should be carefully analyzed in order to be fairly accurate. The total impedance of the large system to the small system, incorporating the subtransient reactance, should be calculated from accurate data. It is inadvisable to assume that the voltage of the large system will be maintained during a fault on the smaller system or that the effect of the subtransient reactance of the sources in the large system can be neglected until a careful review of the large system diagram and its constants has been made and compared with the impedance of the transformers connecting the two systems.

If the necessary information required for the foregoing procedure is not available, the most accurate impedance data that are available should be used. Generally, these will be in the form of transient reactance and an incomplete knowledge of the system from which an approximation of the large system subtransient reactance can be obtained.

The effect of motors, generators, and converters on the secondary side of the transformer should also be taken into account.

RESISTANCE

Resistance of the circuit has two effects:

First, it limits the flow of current in the circuit by creating a voltage drop. Except that the drop is at a 90-degree phase angle with respect to the drop due to reactance the two are similar. To obtain the impedance to point of fault, it is necessary to obtain the total resistance and total reactance and combine them in the relation:

$$Z=\sqrt{R^2+X^2}$$

substituting either ohmic or per unit values and obtaining results in the same unit. If resistances of arcs are to be taken into account, they should be added directly to the resistance of the metallic circuit. Appendix *F* suggests values of arc resistance for specific types of faults. The effect of an arc varies from practically no effect to that of a current-limiting effect by which the arc is self-extinguishing.

Second, the d-c component of total short-circuit current decays more rapidly as resistance increases. The effect will be seen in the d-c decrement factor, plate *B*, figure 2.

COMPLETE SHORT-CIRCUIT STUDY

For a three-phase system, a complete short-circuit study involves various types of faults:

1. Three phases
2. Phase-to-phase
3. Phase-to-neutral
4. Multiple phases and neutral

Many low-voltage a-c systems do not use a grounded neutral and, for the remainder, the three-phase fault practically always develops maximum initial current. Where other than three-phase faults are to be considered, it will be advisable for the investigator to use the symmetrical component or other like procedure.

The general procedure suggested herein should be used for three-phase and other types of faults up to the point of calculating the particular fault. From this point on, the procedure pertains only to three-phase faults on three-phase systems. For other types of fault, additional impedance data will be required.

Specific Procedure for Calculating Three-Phase Short-Circuit Currents in a Three-Phase System

Step 1. Prepare a one-line system diagram on which the reactance and resistance of component parts, referred to a common base, will be noted.

Step 2. Determine the resistance and the reactance of all equipments, devices, and conductors and the transient and subtransient reactance of synchronous machines, the reactance of induction machines which is equivalent to the subtransient reactance of synchronous machines, and the resistance of synchronous and induction machines.

Step 3. Express all resistance and reactance values in terms of "per unit" (pu) on a convenient size, common base, and insert these values on the system diagram—step 1.

To express a pu value, referred to one base, such as the rating of a definite piece of apparatus, on a different base use the relation:

$$\text{pu value base}_2=\text{pu value base}_1\times\frac{\text{base}_2}{\text{base}_1}$$

The value of any resistance, reactance, or impedance in ohms may be expressed in pu terms by:

$$\text{pu value on base kva}=\frac{\text{base kva}\times\text{ohms}}{\text{kv}^2\times 1{,}000}$$

Step 4. Calculate, by conventional methods or by the use of a calculating board, the total pu reactance and resistance (expressed on the common base) to the point of fault.

NOTE: A method of combining parallel impedances into an equivalent single circuit is given in appendix J.

Step 5. Calculate the impedances and rms value of symmetrical short-circuit current by the relations:

$$Z_d'=\sqrt{x_d'^2+r^2}$$

corresponding to transient reactance.

$$Z_d''=\sqrt{x_d''^2+r^2}$$

corresponding to subtransient reactance. Z_d', Z_d'', x_d', x_d'', and r should be expressed in pu values.

The total subtransient reactance (x_d'') is obtained by adding the subtransient reactance of the synchronous machines or the reactance of induction machines directly (in series circuits) to the fixed reactance of connections, devices, transformers, induction regulators, etc. Similarly, the total transient reactance is obtained by adding the transient reactance of rotating machines directly (in series circuits) to the aforementioned fixed reactances.

$$\frac{I_B}{Z_d'}=I' \text{ (rms amperes)}$$

corresponding to transient reactance.

$$\frac{I_B}{Z_d''}=I'' \text{ (rms amperes)}$$

corresponding to subtransient reactance. I_B should be expressed in amperes of the base kilovolt-amperes at the voltage of the point for which calculations are being made.

Step 6, Symmetrical Current at Time t_1. The quantitative relation between values of x_d'' and x_d' varies for different sources of short-circuit current. To be exact for a number of different sources would involve considerable investigation.[3] The characteristics of various types of sources have been consolidated into a single set of characteristics and are used in the following procedure which determines the symmetrical current due to the decrease in subtransient reactance effect at time (t_1) by the relation

$$I_{(\text{rms sym.})t_1}=I_B\left[\frac{1}{Z'}+C\left(\frac{1}{Z''}-\frac{1}{Z'}\right)\right]$$

which is the same as

$$I_{(\text{rms sym.})t_1}=I'+C(I''-I')$$

Factor (C) is obtained from plate A, figure 2. It represents the rate at which the subtransient reactance decreases in relation to the transient reactance as time progresses. The time (t_1) at which C is selected may be taken from table A which specifies the time (t_1) needed for general uses. Obtain factor C at time (t_1) and calculate $I_{\text{rms}_{t_1}}$ from formula above.

Step 7, Unsymmetrical Currents. Select step 7*a* or step 7*b* according to results desired; see table A.

Step 7a, Maximum Unsymmetrical Current in One Phase at Time (t_1). Select the curve from the d-c decrement factor, plate B, figure 2 corresponding to $\dfrac{\text{pu } x_d'}{\text{pu } r}$ and obtain the factor K for the time (t_1) at which current is to be determined. The total rms unsymmetrical current at time (t_1) including the effects of subtransient reactance and d-c component is obtained by

$$I_{(\text{total rms unsym.})t_1}=KI_{(\text{rms sym.})t_1}$$

Step 7b, Unsymmetrical Current, Maximum Average of Three-Phase Currents at Time (t_1). Multiply the symmetrical current at time $t_1={}^1/_2$ cycle obtained in step 6 by a factor ($K_{\text{avg 3 ph}}$) selected from figure 3, according to the x''/r ratio of the system to the point at which current is to be obtained.

$$I_{\text{avg 3 ph}}=I_{(\text{rms sym.})t1}\times K_{\text{avg 3 ph}}$$

General Factors to Obtain Total Current From Symmetrical Current

By the foregoing fairly rigorous procedure, the total unsymmetrical current may be calculated for any particular time desired. The particular time for which current is to be calculated depends upon the purpose under consideration. For the practical purpose of determining the duties on devices and equipment in circuit, the standards to which they are built must be consulted. It will be found that there is no general agreement among existing standards and in some instances no standards exist. Suggestions of duties to use pending adoption of standards are offered later in the text.

Certain efforts are now being made to simplify the procedure by nomination of multiplying factors by means of which total current may be obtained from symmetrical current in order to put the calculation of total current on a common basis for specified equipment or devices and to eliminate the necessity for use of the x''/r ratio. Attention is called to a factor for three-phase low-voltage air circuit breakers which is the only one for low-voltage equipment that has come into somewhat general use and is explained in a footnote under table A. As other factors are agreed upon, they may be used within the limitations of their definition.

For determination of total currents under specified conditions of x''/r and elapsed time from initiation of current, table A synopsizes existing standards of current as a variable with time which should be calculated or suggests elapsed times in lieu of such standards.

In lieu of the determination of the specific average of the three unsymmetrical, available currents for a specific x''/r ratio, the factor 1.25 may be used to increase the symmetrical current calculated in step 6 to obtain the average of the three un-

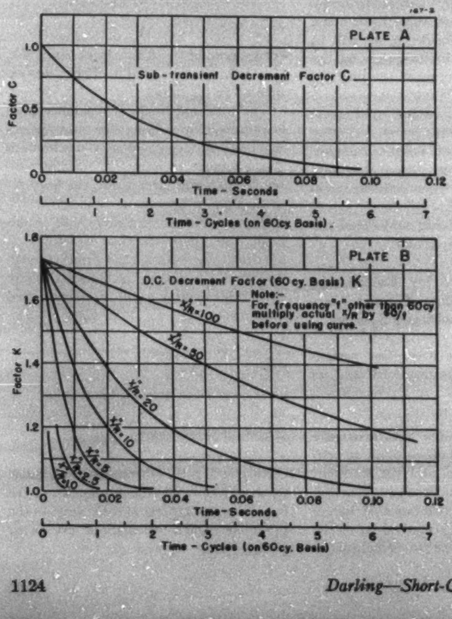

Figure 2. Short-circuit current decrement curves for subtransient and direct currents

I total rms $=K[I'+C(I''-I')]$

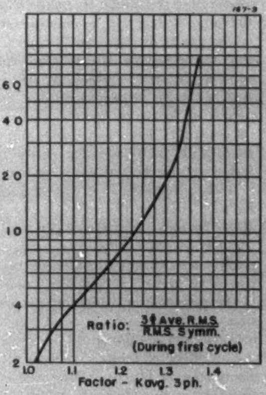

Figure 3. Short-circuit current decrement curve for three-phase average d-c decrement

Ratio of three-phase average rms total current to rms symmetrical during first cycle of fault-current flow

Curve represents maximum ratio for a simultaneous three-phase fault with initial components of:

Phase	A	B	C
A-c sym.	1.0	1.0	1.0
D-c	1.42	0.71	0.71

Table A. Short-Circuit Currents for Establishing Mechanical, Thermal, and Interrupting Duties on Devices and Equipment

Device or Equipment	Mechanical	Thermal	Interrupting
A. Stationary equipment, e.g., current transformers, bus bars, and supports, etc.	Step 7*a* ($t_1 = {}^1/_2$ cycle)	$I_{\text{therm.}}$*	
B. Interrupting devices, e.g.			
1. Air circuit breakers, 3 phase, 600 volts and less**	Available current Step 7*b* ($t_1 = {}^1/_2$ cycle)		Available current Step 7*b* ($t_1 = {}^1/_2$ cycle)
Air circuit breakers, 1 phase, 600 volts and less	Available current Step 6 ($t_1 = {}^1/_2$ cycle) × 1.25		Available current Step 6 ($t_1 = {}^1/_2$ cycle) × 1.25
Cartridge and plug fuses, 600 volts and less, and current-limiting fuses, all voltages			Available current Step 7*a* ($t_1 = {}^1/_2$ cycle)
2. Power circuit breakers	Step 7*a* ($t_1 = {}^1/_2$ cycle)	$I_{\text{therm.}}$*	Step 7*a* (t_1 = minimum relay time + minimum contact separation time)
3. Industrial controllers			
a. Those whose circuit opening and closing ability = 10 × current at hp rating	Available current Step 7*a* ($t_1 = {}^1/_2$ cycle)		Available current Step 7*a* ($t_1 = {}^1/_2$ cycle)
b. Those incorporating in a manufactured assembly, hp rated contactors and short-circuit protective devices such as fuses and instantaneous trip air breakers	Available current Step 7*a* ($t_1 = {}^1/_2$ cycle)		Available current Step 7*a* ($t_1 = {}^1/_2$ cycle)
c. Those which comprise a single switching element having published hp and short-circuit ratings	Available current Step 7*a* ($t_1 = {}^1/_2$ cycle)		Step 7*a* (t_1 = minimum relay time + minimum contact separation time)

*$I_{\text{therm.}}$ In selecting a current for this designation, the object is to choose a current which, if sustained, would produce the same heating as is created by the actual thermal duty cycle under consideration. The times at which currents are to be calculated are partially indicated in existing standards.

**In lieu of the determination of the specific average of the three unsymmetrical, available currents for a specific x''/r ratio, the factor 1.25 may be used to increase the symmetrical current calculated in step 6 to obtain the average of the three unsymmetrical, available currents in a three-phase low-voltage air circuit breaker when the x''/r ratio is less than 12:1.

symmetrical, available currents in a three-phase low-voltage air circuit breaker when the x''/r ratio is less than 12:1.

For a clearer understanding of table A, the comments which follow should be noted.

Mechanical stresses, such as determine momentary ratings of breakers, current transformers, and bus rigidity,[3,4] are calculated from the highest total rms current (including the d-c component) during the first half cycle.

Thermal stresses occur as an integration of current, squared, over a time interval. Present thermal ratings are determined from currents at selected periods or assume that currents are held at a constant value. The standards for the equipment or device under consideration should be used to obtain a guide to calculating the thermal duty.

The ability of interrupting devices is, at the present time, defined by either of two conventional methods. In one instance the duty is expressed as to current (termed "available current") which would flow in the circuit if the interrupting device is considered to have negligible impedance—measured in testing by substituting a short-circuiting bar of negligible impedance for the interrupting device.

NOTE: Where the term "available current" is used in this procedure, it is intended to mean maximum available current. In the other instance the duty is expressed as the current which exists at the moment the contacts of the interrupting device part or arcing begins. Since the construction and adjustment of the devices determine the time element from inception of short circuit to contact separation, the current must be calculated at a suitable increment of time after short circuit. In the interests of simplification, a common method of expressing interrupting device ability would be very helpful.

Short-circuit duty standards have not been adopted for industrial controllers. In general, the varieties of controllers can be classified into three groups. Suggested means to calculate the duties of these groups are as follows:

Group A. Controllers rated only in horsepower. In general, such controllers are capable of switching the current corresponding to ten times the horsepower rating of the controller. Available current might be used to express the duty on this group of controllers.

Group B. Controllers consisting of horsepower rated contactors and short-circuit protective devices, such as fuses and instantaneous trip air circuit breakers, all assembled by the manufacturer in a unified structure in such a manner as to practically preclude the possibility of a fault short-circuiting the impedance of any portion of the combination. Current that is available on the source side of the combination might then be used to express the duty on the controller.

Group C. Controllers comprising a single switching element which have a published horsepower rating as a controller and a published short-circuit rating as an interrupting device. In this case the short-circuit duty might be expressed in terms of the short-circuit rating of the switching element which will generally be found to be in terms of the current existing when the switching element is in circuit and at the time its contacts part.

Interrupting Devices Classified

CLASS I

Class I includes those interrupting devices whose duty is expressed in terms of available current.

Fuses for 600 Volts and Less. Some conventional cartridge fuses will, on heavy currents, open the circuit before the peak of the first half cycle. It is believed that, when a-c ratings are assigned, they will be in terms of available current. Until standards for a-c interrupting ratings are established, it is suggested that the duty be calculated as of the maximum rms current during the first cycle.

Fuses—Current Limiting for All Voltages. Domestic and foreign current-limiting fuses for 2,500 and higher voltage circuits are available and some foreign makes of low-voltage current-limiting fuses are made. Since these operate before the peak of the first half cycle, their duty is expressed in available current and therefore it should be expressed in terms of the maximum rms current during the first cycle.

Circuit Breakers—Air (600 Volts and Less). The interrupting rating of an air circuit breaker of 600 volts or less is expressed in total available rms amperes, which occur during the first cycle following inception of the short-circuit current. (This rating in amperes remains constant for any circuit voltage within the rating of the breaker.) The momentary rating of an instantaneous trip air circuit breaker is numerically equal to its interrupting rating.

In the case of three-phase breakers, the three-phase duty is taken as the average of the three-phase short-circuit currents. A convenient means of determining the average as a function of the ratio of subtransient reactance to resistance in the circuit is given in figure 3. These data are derived from that relation of currents in the three phases which gives the highest average of the three currents. In the case of phase-phase duty followed by three-phase duty, such as might occur with nonsimultaneous short-circuiting of the three phases, it is possible to calculate higher average currents than given in figure 3. This case appears to be sufficiently improbable to allow it to be discarded from normal calculated circuit breaker duty and instead to depend upon the ability of the air breaker to interrupt the slight additional expected current which can occur under these remote circumstances. When the more exact method, indicated above, of obtaining the average of the three phase currents is not required, the factor 1.25 may be used to increase the symmetrical current calculated in step 6 to obtain the average of the three unsymmetrical available currents when the x''/r ratio is less than 12:1.

In the application of breakers, in single-phase circuits, a factor of 1.25 times the symmetrical rms current in the first cycle is suggested as representative of the average total unsymmetrical rms current likely to be encountered in actual cases. The maximum unsymmetrical current can, if desired, be obtained from step 7*a* ($t_1 = 1/2$ cycle).

Class II

Class II includes those devices whose duty is expressed in terms of the current existing at the instant arcing begins in the interrupting device. This definition requires the interrupting device to be in circuit at the time a performance test is made.

Power Circuit Breakers. Beside momentary and five-second current ratings, the duty is calculated for this group of circuit breakers functioning without purposely delayed action at a certain inherent time delay after inception of short circuit, i.e.—time required for breaker, tripping and protective relays—sufficient to cause separation of the contacts. The time taken to cause contact separation is generally expressed in terms of whole cycles on a 60-cycle basis. The current to be calculated is the maximum rms value, one phase, that occurs at the instant of contact separation, regardless of the number of poles for which the breaker is built.

Inverse time overcurrent relays operate on the integration of current over the period of time elapsing during short circuit. For purposes of preliminary setting, it is safe to use transient reactance (x_d') since the first settings are generally modified by trial after installation. Minimum tripping current exists under maximum impedance conditions instead of minimum impedance needed for maximum current.

System arrangement and operating characteristics depend upon momentary current, current after a few cycles, and sometimes current after a longer period of time.

Appendixes

Contents

IMPEDANCE DATA

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    (a) Generators, distributed pole
    (b) Generators, salient pole—without amortisseur
    (c) Generators, salient pole—with amortisseur
    (d) Condensers
    (e) Motors
    (f) Small turbogenerators—distributed pole
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I. Current-limiting characteristics of circuit interrupting devices (title inserted to complete list; material not yet available)

MISCELLANEOUS

J. Combining parallel impedances into a single equivalent circuit.

Appendix A. Rotating Machines

Impedance data for rotating machines is being subjected to analysis from the standpoints of calculating methods, tests, and technique of making tests. For close calculations, specific data for specific apparatus should be obtained until greater refinement in general form can be issued.

Where ranges of data are given, it is expected that the reader will select values within the range which will exemplify the case in hand. Only when very little is known of the particular machine should the mean value be used.

The values given in the following tabulations are only approximate and are intended merely to indicate the order of magnitude of the machine reactance and of the time constant. The designers may deviate from these average values quite appreciably in order to obtain other desired characteristics of the machines.

A-1. Synchronous Machines

See tables I and II.

A-2. Synchronous Converters

See figure 4.

A-3. Induction Generators and Motors

The short-circuit reactance of induction machines (in per unit on its own kilovolt-ampere base) may be taken as the reciprocal of the times normal stalled rotor current under conditions of rated impressed voltage and frequency. Induction generators, generally, will have the same reactance as induction motors of like rating.

Appendix B. Transformers

B-1. Distribution Transformers

See table III.

B-2. Power Transformers

See table IV.

Appendix C. A-C Reactors and Feeder Regulators

C-1. Reactors

The reactance is a portion of the rating.

The voltage drop through the reactor at rated current and frequency divided by the line-to-neutral voltage of the circuit gives the per unit reactance on the current rating of the reactor. (This will also be the per unit reactance on the kilovolt-ampere rating of the circuit, if the rated reactor current is the same as the rated current of the circuit.)

The effective a-c resistance may be obtained from the losses (I^2R) of the reactor. The d-c resistance is about 60 per cent of the effective resistance.

C-2. Feeder Regulators

See table V.

Appendix D. Conductors

D-1. Rectangular Bars and Channels

Resistance of Flat Copper Bars. See table VI.

Resistance of Channels. The resistance of channel conductors will generally be negligible when compared to their reactance.

Reactance of Bars and Channels. All reactance values given apply to 60 cycles. Reactance values at any other frequency f multiply the 60-cycle value by $f/60$.

The reactance of rectangular bars used for busses and connections depends on their length, size, grouping arrangement, and

Table I. Reactances and Time Constants of 60-Cycle Synchronous Generators and Motors

Minimum-Mean-Maximum Per Unit Values

	x_d''	x_d'	T'_{d0}	T_a
Distributed pole generators				
2 pole, 625–9,375 kva	0.06–0.09–0.13	0.10–0.13–0.20		
2 pole, 12,500 kva up	0.06–0.10–0.12	0.11–0.15–0.21		
4 pole, 12,500 kva up	0.10–0.14–0.17	0.20–0.23–0.28		
Salient-pole generators—without amortisseur				
12 or less poles	0.15–0.25–0.35	0.15–0.25–0.35		
14 or more poles	0.25–0.35–0.45	0.25–0.35–0.45		
Salient-pole generators—with amortisseur				
12 or less poles	0.10–0.18–0.25	0.15–0.25–0.35		
14 or more poles	0.18–0.24–0.40	0.25–0.35–0.45		
Synchronous condensers	0.09–0.24–0.38	0.21–0.43–0.73	2.7–6.0–10.5	0.08–0.14–0.28
Synchronous motors				
Motors of M-G sets and general-purpose motors				
6 pole	0.07–0.10–0.16	0.10–0.15–0.22	2.16–2.24–2.45	0.03–0.07–0.08
8–14 pole, incl.	0.11–0.15–0.22	0.17–0.24–0.36	1.24–2.50–3.50	0.026–0.057–0.25

NOTE: Refer to ASA standards, group 10, sec. 35 for definitions of:

x_d'—Direct axis transient reactance.

x_d''—Direct axis subtransient reactance.

T'_{d0}—Direct axis transient open-circuit time constant.

T_a—Short-circuit time constant of armature winding

Table II. Reactance and Time Constants of 60-Cycle Small, Distributed-Pole Turbogenerators, 625–9,375 Kva

		Per Unit Reactances			Time Constants in Seconds at 75 Deg C	
Kva	Volts	x_d''	x_d'	x_d	T_a	T'_{d0}
625	480	0.088	0.120	1.30	0.050	1.25
	600	0.094	0.125	1.25	0.064	1.25
		0.090	0.120	1.30	0.053	1.30
781	480	0.078	0.105	1.17	0.053	1.45
	600	0.079	0.105	1.21	0.058	1.40
	2,400	0.092	0.120	1.26	0.075	1.40
937	480	0.074	0.100	1.15	0.063	1.65
	600	0.071	0.100	1.12	0.075	1.60
	2,400	0.074	0.100	1.19	0.061	1.65
1,250	480	0.097	0.120	1.40	0.041	1.80
	600	0.095	0.120	1.33	0.051	1.60
	2,400	0.090	0.114	1.25	0.044	1.75
1,563	480	0.084	0.110	1.35	0.035	1.70
	600	0.085	0.110	1.28	0.047	1.60
	2,400	0.091	0.125	1.29	0.053	1.60
1,875	480	0.086	0.115	1.38	0.058	2.05
	600	0.085	0.115	1.36	0.042	1.95
	2,400	0.086	0.115	1.41	0.049	2.05
	2,400/4,160	0.093	0.120	1.40	0.055	2.00
2,500	480	0.102	0.130	1.35	0.063	2.20
	600	0.115	0.140	1.39	0.060	2.20
	2,400	0.110	0.135	1.39	0.068	2.30
	2,400/4,160	0.105	0.130	1.33	0.060	2.20
3,125	480	0.092	0.120	1.35	0.050	2.50
	600	0.086	0.120	1.32	0.055	2.60
	2,400	0.085	0.115	1.35	0.063	2.60
	2,400/4,160	0.085	0.115	1.32	0.067	2.55
3,750	600	0.121	0.160	1.37	0.103	2.95
	2,400	0.106	0.140	1.33	0.091	2.95
	2,400/4,160	0.117	0.155	1.37	0.100	2.80
	6,900	0.104	0.140	1.37	0.077	2.90
4,375	600	0.095	0.130	1.35	0.088	3.00
	2,400	0.095	0.130	1.35	0.088	3.00
	2,400/4,160	0.094	0.130	1.37	0.081	3.00
	6,900	0.101	0.135	1.34	0.090	3.00
5,000	2,400	0.089	0.120	1.30	0.090	3.15
	2,400/4,160	0.095	0.130	1.31	0.096	3.20
	6,900	0.084	0.115	1.27	0.81	3.45
6,250	2,400	0.083	0.115	1.35	0.088	3.75
	2,400/4,160	0.080	0.110	1.28	0.084	3.50
	6,900	0.085	0.115	1.34	0.082	3.60
7,500	2,400	0.110	0.150	1.30	0.110	3.95
	2,400/4,160	0.112	0.150	1.30	0.103	3.95
	6,900	0.110	0.150	1.26	0.116	3.90
	13,800	0.113	0.150	1.29	0.109	3.75
9,375	2,400	0.100	0.135	1.34	0.117	4.40
	2,400/4,160	0.105	0.140	1.38	0.122	4.45
	6,900	0.104	0.140	1.37	0.121	4.55
	13,800	0.104	0.140	1.25	0.105	4.30

NOTE: Refer to ASA standards, group 10, sec. 35 for definitions of:

T_a—Short-circuit time constant of armature winding.

T'_{d0}—Direct axis transient open circuit time constant.

spacing. The order of reactance magnitude at 60 cycles is from, say, 0.030 to 0.100 ohm per 1,000 feet of each conductor. While a reactance of 0.00006 ohm representing, say, the reactance of one foot of conductor, will not ordinarily be of serious importance in limiting short-circuit currents, a 50-foot length of conductor in a circuit, that is, 50 conductor feet at, say, 60 microhms per foot will represent 3,000 microhms reactance, or 0.003 ohm. At a current of 40,000 amperes, for example, the voltage drop across this reactance would be 120 volts, which is a sizeable voltage magnitude in a circuit of 600 volts or below.

The reactance at 60 cycles of rectangular bars used for busses and connections depends on their length, size, grouping arrangement, and spacing. Figures 5, 6, 7, and 8 give data for the more usual constructions. These values are primarily for long lengths but will be only slightly less for short lengths. In the case of joints between conductors of different sizes or spacings, the reactances for the various portions of the circuit are calculated separately and then added together. For general purposes of short-circuit calculations, the effect of angles and bends in the bus runs may be neglected and the reactance obtained from the total lineal distance.

D-2. CABLES IN DUCTS

Cables of particular interest in the calculation of short-circuit currents in low-voltage circuits include rubber, varnished cambric, and paper insulated types designed with insulation for 600, 3,000, and 5,000 volts.

Basic data are given in table VII for three-conductor and three single-conductor cables in a duct for the following basic conditions:

Standard-strand conductors.
Nonmagnetic binder construction.
Nonshielded design.
Operation in nonmagnetic ducts.

Note that the lead sheath increases the effective a-c resistance approximately five per cent above the a-c resistance shown. The lead sheath also tends to decrease the reactance from the values shown. The figures given are estimated to be within five to ten per cent. The impedance values shown do not include the increase in resistance and decrease in reactance mentioned.

Tables of resistance include the skin and proximity effects of three-conductor cables at 60 cycles. Tables of reactance for different types of insulation of any one voltage rating have been combined when the calculated values were within five per cent. This has also been done for cable impedances to reduce the number of columns.

Multiplying Factors

For conditions other than those specified, the following multiplying factors may be used to obtain *approximate* values.

(*1*) *Sector Conductor Cable* (*Compact*):

Size in MCM	Ratio = $\frac{\text{Round Cond. Reactance}}{\text{Sector Cond. (Av.) Reactance}}$
300	1.03
400	1.04
500	1.05
600	1.07
700	1.075
850	1.08

Table III. Resistance, Reactance, and Impedance of Single-Phase 60-Cycle Distribution Transformers[†]

Per Unit Values Are Given*

H.V. 2,400/4,160Y L.V. 120/240; 240/480; 600

Data at 2,400 volts, full wdg.

Kva Cont.	Pu Resist.	React.	Imped.
1.5	0.0247	0.0170	0.0300
3	0.0194	0.0216	0.0290
5	0.0192	0.0230	0.0300
7.5	0.0196	0.0186	0.0270
10	0.0188	0.0194	0.0270
15	0.0174	0.0220	0.0280
25	0.0156	0.0232	0.0280
37.5	0.0137	0.0256	0.0290
50	0.0124	0.0262	0.0290
75	0.0124	0.0328	0.0350
100	0.0120	0.0329	0.0350
150	0.0120	0.0329	0.0350
200	0.0111	0.0332	0.0350
250	0.0113	0.0436	0.045
333	0.0106	0.0437	0.045
500	0.00974	0.0439	0.045

H.V.2,400/4,800/8,320Y L.V.120/240; 240/480; 600

Data at 4,800 volts, full wdg.

Kva Cont.	Pu Resist.	React.	Imped.
1.5	0.0254	0.0254	0.0358
3	0.0203	0.0207	0.0290
5	0.0208	0.0202	0.0290
7.5	0.0213	0.0182	0.0280
10	0.0195	0.0157	0.0250
15	0.0190	0.0178	0.0260
25	0.0165	0.0226	0.0280
37.5	0.0151	0.0259	0.0300
50	0.0136	0.0268	0.0300
75	0.0129	0.0336	0.0360
100	0.0125	0.0338	0.0360
150	0.0123	0.0339	0.0360
200	0.0112	0.0342	0.0360
250	0.0113	0.0436	0.0450
333	0.0106	0.0437	0.0450
500	0.00974	0.0439	0.0450

H.V. 6,900/11,500Y L.V. 115/230; 230/460; 575

Data at 6,900 volts, full wdg.

Kva Cont.	Pu Resist.	React.	Imped.
1.5	0.0287	0.0234	0.037
3	0.0247	0.0276	0.037
5	0.0240	0.0240	0.034
7.5	0.0234	0.0218	0.032
10	0.0217	0.0208	0.030
15	0.0187	0.0235	0.030
25	0.0164	0.0251	0.030
37.5	0.0144	0.0264	0.030
50	0.0134	0.0268	0.030
75	0.0120	0.0485	0.050
100	0.0123	0.0484	0.050

L.V. 115/230

Kva Cont.	Pu Resist.	React.	Imped.
150	0.0115	0.0487	0.05
200	0.0117	0.0486	0.05
250	0.0113	0.0487	0.05
333	0.0106	0.0489	0.05
500	0.0097	0.0490	0.05

L.V. 230/460; 575

Kva Cont.	Pu Resist.	React.	Imped.
150	0.0114	0.0487	0.05
200	0.0114	0.0487	0.05
250	0.0108	0.0489	0.05
333	0.0100	0.0490	0.05
500	0.0093	0.0491	0.05

H.V. 13,800 L.V. 115/230; 230/460; 575

Data at 13,200 volts, full wdg.

L.V. 115/230

Kva Cont.	Pu Resist.	React.	Imped.
2.5	0.024	0.0179	0.030
5	0.0224	0.0200	0.030
10	0.0212	0.0197	0.029
15	0.0193	0.0216	0.029
25	0.0154	0.0234	0.028
37.5	0.0141	0.0230	0.027
50	0.0134	0.0234	0.027
75	0.0125	0.0380	0.040
100	0.0123	0.0377	0.041
150	0.012	0.0391	0.041
200	0.012	0.0392	0.041

L.V. 230/460; 575

Kva Cont.	Pu Resist.	React.	Imped.
250	0.0108	0.0488	0.050
333	0.0100	0.0489	0.050
500	0.0093	0.0491	0.050

H.V. 22,000 L.V. 240/480

Data at 22,000 volts, full wdg.

Kva Cont.	Pu Resist.	React.	Imped.
5	0.033	0.0409	0.0525
10	0.0285	0.0442	0.0525
15	0.0237	0.0469	0.0525
25	0.0204	0.0485	0.0525
37.5	0.0183	0.0493	0.0525
50	0.017	0.0497	0.0525
75	0.0151	0.0504	0.0525
100	0.0139	0.0506	0.0525
150	0.0126	0.051	0.0525
200	0.0121	0.0511	0.0525
250	0.0109	0.0514	0.0525
333	0.0101	0.0516	0.0525
500	0.0093	0.0517	0.0525

H.V. 22,000 L.V. 120/240; 600

10–500, kva pu imped. = 0.0525 ≐ 5 per cent

H.V. 33,000 L.V. 120/240; 240/480; 600

15–500 kva pu imped. 0.0525 ≐ 5 per cent

H.V. 44,000 L.V. 120/240; 240/480; 600

25–500 kva pu imped. 0.055 to 0.065

H.V. 66,000 L.V. 240/480

Data at 66,000 volts, full wdg.

Kva Cont.	Pu Resist.	React.	Imped.
25	0.0225	0.0662	6.5 to 7.5
37.5	0.0191	0.0673	6.5 to 7.5
50	0.0175	0.0678	6.5 to 7.5
75	0.0151	0.0684	6.5 to 7.5
100	0.0142	0.0686	6.5 to 7.5
150	0.0123	0.0689	6.5 to 7.5
200	0.0123	0.0689	6.5 to 7.5
250	0.0120	0.0690	6.5 to 7.5
333	0.0113	0.0691	6.5 to 7.5
500	0.0100	0.0694	6.5 to 7.5

H.V. 66,000 L.V. 120/240; 600

50–500 kva pu imped 0.065 to 0.075

[†] For three-phase transformers of 500 kva and less, use the data of single-phase distribution transformers at one-third the kva rating of the three-phase transformer.

Obtain data for three-phase transformers over 500 kva from table IV.

* Data are given for specific designs. To allow for variations in manufacture, it is suggested that the resistance, reactance, and impedance data be rounded off to two significant figures.

D-c resistance may be derived from the 60-cycle resistance values by dividing them by factors varying from 1.0 to 1.2 using the larger factors for the higher normal ampere ratings.

(2) *Three-Conductor Cable With Magnetic Binder.* Multiply X by 1.25.

(3) *Cable in Magnetic Duct:*

Three-conductor cable—multiply X by 1.12
Three single-conductor braid-covered cables { multiply X by 1.25; multiply R by 1.12
Three single-conductor lead-sheath cables { multiply X by 1.12; multiply R by 1.30

(4) *Balanced Three-Phase, Four-Wire Circuits.* Use same values as three-phase, three-wire circuits as the increased spacing is small in comparison to the random arrangement of conductors.

As a result of field test data,[5] the spacing of the three single-conductor, braid-covered cables was increased 45 per cent to allow for the random arrangement of the cables in the duct. The test data also indicate that the reactance of lead-sheath cables for the lower voltage ratings is nearly the same as that for braid-covered cables. The data in table VII have, therefore, been made accordingly.

Since the tables for three single-conductor cables give line-to-neutral values, the resistance, reactance, and impedance of single-conductor cables on isolated single-phase circuits may also be determined by assuming the same spacing and proximity effect. Test data on the spacing of two single-conductor cables in a duct are not available but it is known that there will be some "snaking" of the conductors and the spacing, thereby, increased.

D-3. Stranded Conductors in Air

By means of table VIII, the resistance and diameter of various conductors may be obtained, and by means of the curves, figures 10 and 11, the reactance in ohms per mile at 60 cycles at various spacings of conductors may be obtained.

For frequencies other than 60 cycles, ohms at frequency

$$F=\frac{\text{ohms at 60 cycles}}{60}\times f \text{ cycles per second;}$$

i.e., for 25 cycles, ohms = ohms at 60 cycles × 25/60.

For three-phase circuits in which the conductors are not symmetrically arranged, at equal spacings, the reactive voltage drops in the three conductors are unequal, even for the case of balanced currents.

For approximate calculations in three-phase circuit with balanced currents and with unequal spacings, s_1, s_2 and s_3 between conductors, the mean spacing $s=\sqrt{s_1s_2s_3}$ may be used for calculation of average reactance per phase to neutral.

Appendix E. Current Transformers

The impedance of current transformers, though generally of a low order of magnitude in ohms, should be considered in short-circuit current calculations in low-voltage circuits because it may have an effect on the magnitude of the short-circuit current for heavy current short circuits, that is, of the order of 25,000 amperes or larger.

The impedance of the current transformers depends on the current rating and on their design, and varies over a wide range. Therefore, a single value of impedance applicable to a variety of current transformers, cannot be given.

With Primary Circuits of the Wound Type

Approximate data on resistance and reactance at 60 cycles for current transformers of type *W*, covering current ratings from 100 to 800 amperes, based on tests at short-circuit currents, are given in table IX. The values in table IX apply to transformers with a secondary burden of one volt ampere or less at five amperes or at normal current. For higher burdens, the impedance referred to the primary side will be somewhat increased, but the increase is far less than that occurring at normal currents, on account of the reduced mutual inductance between primary and secondary windings. The impedance values based on low burden are conservative for calculations of maximum short-circuit current.

Having Bar-Type Primary Conductor

For bar-type current transformers with current ratings from 1,000 to 4,000 amperes, such as the following types:

JS2—600 volts
WC15—5,000 volts
KC50—7,500 volts

the resistance and reactance have the following approximate order of magnitude:

Resistance—0.000025 ohm
Reactance—0.000070 ohm

at currents within the range from 10,000 to over 80,000 rms symmetrical amperes, with or without d-c component.

The reactance depends on the spacing between phases, since a considerable amount of air flux links the primary bar conductor. The value given is that for nine-inch phase spacing with the transformers side by side, representing an average value for the three phases for three-phase short circuits. Strictly speaking, the reactance in the three phases will be unequal in a side-by-side assembly of current transformers, but for short-circuit current calculations an average value can ordinarily be used without serious error.

It is a fair approximation to say that the reactance for bar-type transformers is equal to the air reactance of the primary conductor, considering its length, size and shape, and the spacing between phases. (See appendix D on reactance of bar conductors.)

Appendix F. Contact Drop and Fault-Voltage Drop Including Current-Limiting Effects

Contact Drop

Contact resistance drop in permanent joints is generally negligible in comparison with the impedance of the apparatus, conductors and devices in the circuit. In practical calculations, the resistance of permanent joints may be considered equal to the resistance of a length of conductor equal to the length of the joint.

Fault-Voltage Drop Including Current-Limiting Effects

The effect of the impedance at the point of fault on the magnitude of the short-circuit current in low-voltage distribution circuits varies widely, depending on the type of fault, that is, whether of the bolted type as in the case of a faulty connection, or of the arc type in which the member causing the short circuit is quickly repelled. A single value for impedance or resistance or current-limiting effect of the fault itself, can therefore not be given.

The current-limiting effect of an arc fault is due to its resistance. The procedure recommended for allowing for the effect of arc resistance in short-circuit current calculations is to use representative values of arc voltage in the calculating method outlined below. Though the arc voltage for many kinds of faults is relatively large, and may reach, or even exceed 50 per cent of the available voltage in the circuit, a correspondingly large current reduction due to the arc voltage is not obtained because the arc drop is a resistance drop and is therefore in quadrature with the reactance drop of the circuit. This caution is important, since short-cut methods of calculation using large reduction factors to take account of arc resistance have been erroneously used in the past.

Faults may be classified into the following groups from the point of view of their effects upon the magnitude of short-circuit current, as indicated by the magnitude of the arc voltages given in table X, to which the following groups of comments also apply.

1. *Short Circuits of the Bolted Type* are treated in the class with those in which the conductors are held in contact by other means, or in which a highly conducting member is firmly held or wedged against the conductors short-circuited.

The voltage drop at the fault will be that due to the contact resistance (or resistances) and will be less than, say, ten volts in most cases. The current-limiting effect of this type of fault will be negligible.

Should the contact at the fault become slightly loosened so as to produce a small arc, across a small gap of the order of, say,

Table IV. Resistance, Reactance, and Impedance of Single-Phase 60-Cycle Outdoor Self-Cooled Power Transformers†

Per-Unit Values Are Given.* Data at Full Winding

Kva Cont.	Pu Resist.	React.	Imped.
H.V. 2,200/3,810Y to 15,000 L.V. 440 to 600			
667	0.0082	0.0544	0.055
833	0.0074	0.0545	0.055
1,000	0.0073	0.0546	0.055
1,250	0.0075	0.0544	0.055
H.V. 15,001 to 25,000 L.V. 440 to 600			
667	0.0092	0.0594	0.06
833	0.0082	0.0594	0.06
1,000	0.0079	0.0595	0.06
1,250	0.0080	0.0595	0.06
H.V. 25,001 to 34,500 L.V. 440 to 600			
667	0.0082	0.0594	0.06
833	0.0077	0.0595	0.06
1,000	0.0076	0.0596	0.06
1,250	0.0079	0.0595	0.06
H.V. 5,001/8,661Y to 15,000 L.V. 2,200/3,810Y to 5,000/8,660Y			
667	0.0079	0.0544	0.055
833	0.0075	0.0544	0.055
1,000	0.0073	0.0546	0.055
1,250	0.0070	0.0545	0.055
1,667	0.0066	0.0546	0.055
2,000	0.0062	0.0597	0.060
2,500	0.0059	0.0597	0.060
3,333	0.0058	0.0647	0.065
4,000	0.0057	0.0647	0.065
5,000	0.0055	0.0647	0.065
H.V. 15,001/25,000 L.V. 2,200/3,810Y to 5,000/8,660Y			
667	0.0081	0.0594	0.06
833	0.0078	0.0595	0.06
1,000	0.0074	0.0596	0.06
1,250	0.0070	0.0596	0.06
1,667	0.0066	0.0597	0.06
2,000	0.0065	0.0596	0.06
2,500	0.0062	0.0648	0.065
3,333	0.0059	0.0698	0.07
4,000	0.0058	0.0698	0.07
5,000	0.0056	0.0699	0.07
H.V. 25,001 to 34,500 L.V. 2,200/3,810Y to 5,000/8,660Y			
667	0.0078	0.0595	0.06
833	0.0074	0.0596	0.06
1,000	0.0070	0.0596	0.06
1,250	0.0068	0.0596	0.06
1,667	0.0067	0.0597	0.06
2,000	0.0066	0.0598	0.06
2,500	0.0063	0.0649	0.065
3,333	0.0060	0.0698	0.07
4,000	0.0060	0.0698	0.07
5,000	0.0057	0.0698	0.07
H.V. 37,001 to 46,000 L.V. 2,200/3,810Y to 5,000/8,660Y			
667	0.0082	0.0645	0.065
833	0.0075	0.0645	0.065
1,000	0.0071	0.0646	0.065
1,250	0.0069	0.0647	0.065
1,667	0.0064	0.0648	0.065
2,000	0.0062	0.0649	0.065
2,500	0.0061	0.0698	0.070
3,333	0.0059	0.0698	0.070
4,000	0.0056	0.0749	0.075
5,000	0.0054	0.0749	0.075
H.V. 50,001 to 69,000 L.V. 2,200/3,810Y to 5,000/8,660Y			
667	0.0090	0.0744	0.075
833	0.0085	0.0745	0.075
1,000	0.0081	0.0746	0.075
1,250	0.0076	0.0746	0.075
1,667	0.0070	0.0747	0.075
2,000	0.0067	0.0747	0.075
2,500	0.0064	0.0748	0.075
3,333	0.0060	0.0748	0.075
4,000	0.0057	0.0749	0.075
5,000	0.0055	0.0749	0.075
H.V. 25,001 to 34,500 L.V. 5,001/8,661Y to 15,000			
667	0.0080	0.0595	0.06
833	0.0076	0.0596	0.06
1,000	0.0072	0.0596	0.06
1,250	0.0070	0.0596	0.06
1,667	0.0066	0.0598	0.06
2,000	0.0063	0.0598	0.06
2,500	0.0062	0.0598	0.06
3,333	0.0061	0.0698	0.07
4,000	0.0060	0.0698	0.07
5,000	0.0057	0.0749	0.075
H.V. 37,001 to 46,000 L.V. 5,001/866 to 15,000			
667	0.0085	0.0644	0.065
833	0.0077	0.0645	0.065
1,000	0.0073	0.0645	0.065
1,250	0.0072	0.0646	0.065
1,667	0.0065	0.0648	0.065
2,000	0.0064	0.0648	0.065
2,500	0.0063	0.0698	0.070
3,333	0.0060	0.0698	0.070
4,000	0.0056	0.0749	0.075
5,000	0.0054	0.0749	0.075
H.V. 50,001 to 69,000 L.V. 5,001/8,661Y to 15,000			
667	0.0093	0.0744	0.075
833	0.0088	0.0745	0.075
1,000	0.0083	0.0746	0.075
1,250	0.0078	0.0746	0.075
1,667	0.0073	0.0747	0.075
2,000	0.0068	0.0747	0.075
2,500	0.0065	0.0748	0.075
3,333	0.0060	0.0748	0.075
4,000	0.0057	0.0749	0.075
5,000	0.0055	0.0749	0.075

†For three-phase transformers, use the data of single-phase transformers of one-third the kva rating of the three-phase transformer.

*Data are given for specific designs. To allow for variations in manufacture, it is suggested that the resistance, reactance, and impedance data be rounded off to two significant figures.

D-c resistance may be derived from the 60-cycle resistance values by dividing them by factors varying from 1.0 to 1.2 using the larger factors for the higher normal ampere ratings.

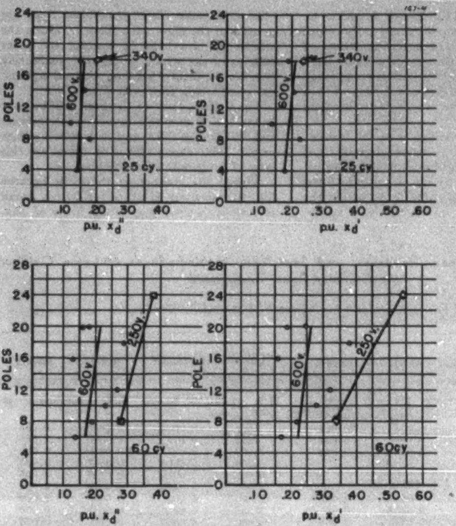


Figure 4. Synchronous converter reactance for use in isolated converter or when converters are paralleled on d-c side and fed from same a-c source

Add transformer reactance:

For compound wound converters 12 pu

For shunt wound converters 0.7 pu

General note: converter resistance 0.003 pu; transformer resistance 0.008 pu for small sizes to 0.005 pu for large sizes

$^1/_8$ inch, arc voltages of the order of 40 volts may be obtained, according to Slepian and Strom, who found arc voltages ranging from 40 to 100 volts at currents from 2,500 to 6,400 amperes between bare copper bars one-fourth inch apart.

Since arc voltages of the order of 40 volts instantaneous will generally not have an appreciable current-limiting effect, and since faults produced by firmly held conducting members may not form arcs at all in the initial stage of a short circuit, the faults of the group classified as "bolted faults or their equivalent" will develop the highest short-circuit current. Their magnitude is to be calculated on the basis of the impedance of the metallic circuit up to the point of fault, neglecting the fault resistance as in the case of a bolted fault resulting from a faulty connection.

2. The Current-Limiting Effect of Accidental Arc Faults formed by objects readily repelled or destroyed by the arc in enclosed, congested structures may be of a minor order. Arc-fault currents as high as 100 per cent (average three-phase value) of the rms symmetrical bolted fault short-circuit current may be expected in circuits up to 600 volts line-to-line open-circuit voltage.

Fault arcs in these structures may not be self-clearing within normal protective device clearing time.

Three-phase short-circuit currents for arc faults of this class as high as 99,000 amperes rms average value for the three phases, have been obtained in tests at 600 volts line-to-line voltage.

The arc voltage drop for faults of this type for conductor clearances of the order of two inches between phases and between circuits (case 2*a* in table X) was found to be as low as 80 volts crest, line-to-neutral value at circuit voltages within the range from 300 to 600 volts line-to-line voltage in three-phase circuits.

Arc faults between bare conductors with two-inch phase spacing in a metallic enclosure in the absence of other circuits within the same enclosure (case 2*b* in table X) gave arc voltages from 150 volts crest, line-to-neutral value.

Faults started as single-phase, line-to-line faults developed into three-phase short circuits within one-half cycle, in both cases 2*a* and 2*b*.

3. Arc Faults Between Isolated Bare Conductors away from enclosing walls will often have substantial current-limiting effects when the member initiating the fault is readily repelled or destroyed by the short-circuit current. In tests of faults of this kind, maximum peak currents have often been limited to values below 25,000 amperes, and the faults have cleared themselves within the first half cycle, but a limitation of current to this order of magnitude cannot be depended on.

4. Arc Faults Observed in Insulated Cables initiated by piercing the insulation with conducting members capable of being repelled gave currents as high as 80 per cent (approximate rms value) of the rms symmetrical bolted fault short-circuit current indicating that large current-limiting effects, in excess of 20 per cent with respect to the rms symmetrical bolted fault current, cannot be depended on.

Short circuit caused by piercing cable insulation with light metallic members such as a $^1/_8$-inch diameter drill, short-circuiting the line conductors, resulted in currents limited to values well below half of the rms symmetrical bolted fault short-circuit current.

Faults of the above character in exposed cables (not enclosed in ducts) were self-clearing within one or two half cycles.

Table V. Short-Circuit Impedance of Feeder Regulators of 60-Cycle Design

Range—Plus 10 Per Cent, Minus 10 Per Cent Volts

Type	No. of Phases	No. of Cores	Circ. Volts	Kva of Regulator	Per Cent Impedance (on Base of Circ. Kva) Min.	Avg.	Max.	Ratio of Reactance to Resistance
Induc.	1 or 3		2,400 to 4,800	12 to 96	0.65	0.85	1.00	10:1
				Amp. of Regulator				
Step	1	1	2,400 to 13,800	All ratings	0.0+		0.6	4:1
Step	3	1		to 160	0.0+		0.7	4:1
Step	3	3		over 160	0.15		1.0	4:1

Note: The impedance of a given step regulator is modified by the position of the tap changer, and becomes a maximum at maximum voltage boost. It is minimum at neutral position, while at the maximum buck the impedance is higher than at neutral.

Table VI. Resistance of Flat Copper Bars

	Ohms Per 1,000 Feet at 77 Deg F At D-C	At 25 Cycles	At 60 Cycles
1 bar 2"x$^1/_4$"	0.0170	0.0173	0.018
2 bars 2"x$^1/_4$" ($^1/_4$" apart)	0.0085	0.0090	0.010
1 bar 3"x$^1/_4$"	0.0113	0.0117	0.0125
2 bars 3"x$^1/_4$"	0.0057	0.0061	0.007
1 bar 4"x$^1/_4$"	0.0085	0.0089	0.0097
2 bars 4"x$^1/_4$"	0.0042	0.0049	0.0058
1 bar 6"x$^1/_4$"	0.0057	0.0061	0.0068
2 bars 6"x$^1/_4$"	0.0028	0.0034	0.0042
4 bars 4"x$^1/_4$", in groups with 2" clearance between groups	0.0021	0.0027	0.0038

Calculating Procedure for Estimating Short-Circuit Currents in Circuits With Arc Faults

The arc voltages for different kinds of faults in low-voltage circuits will vary depending on

The conductor spacing between phases,
The spacing between adjacent circuits,
The type of construction, whether open or enclosed,
The clearances to ground and to enclosing walls,
The type of conductors, whether bare or insulated,
The manner in which the fault is initiated,
The character of the enclosing walls, etc.

It will not be attempted to consider all of these in detail. Some of the fault conditions of particular interest in low-voltage switchgear construction are those caused by conductors dropping across phases in the following types of structures:

(*a*). Heavy current bar-conductor assemblies with low to minimum spacings, metal enclosed, with two or more bar-conductor circuits in close proximity to each other.

(*b*). Bar-conductor circuits, remote from other live conductors, metal enclosed.

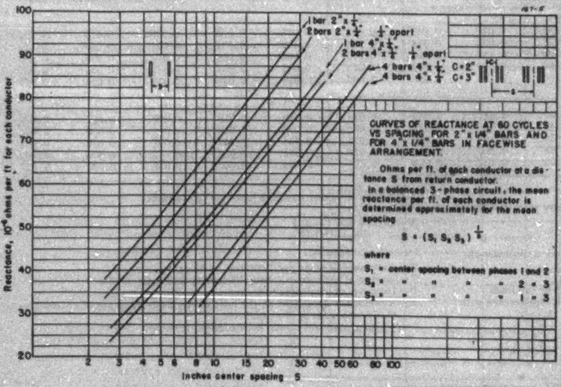


Figure 5

Less severe types of faults than those for (*a*) and (*b*) above may occur in open bar-conductor circuits remote from live conductors, and may give rise to short-circuit currents with more pronounced current-limiting effects.

When low to minimum clearances are used in structures of types (*a*) and (*b*), short-circuit currents due to objects (which are capable of being repelled or destroyed) bridging across phases may be roughly estimated from the values suggested in table X.

Basis of Formulas. As a first approximation, the vector voltage equation 1 for three-phase short-circuit including arc may be written assuming sine-wave voltages and currents:

$$\mathbf{E}-\mathbf{V}-r\mathbf{I}-j\mathbf{I}x=0 \text{ rms volts to neutral} \qquad (1)$$

where **E**, **V**, and **I** are vectors representing impressed voltage, arc voltage (line-to-neutral value) and line current respectively.

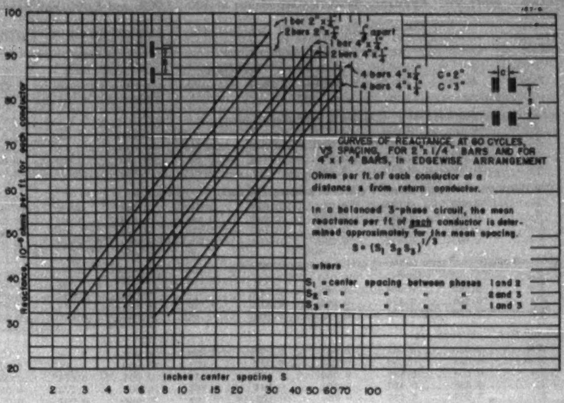


Figure 6

Numerically, the values are defined as follows:

E = scalar value of $\mathbf{E}=0.58x$ (line-to-line voltage of circuit)
V = scalar value of $\mathbf{V}=0.71x$ (crest value of arc voltage to neutral from figure 7 or 8, as required)
i = scalar value of $\mathbf{I}$ = rms symmetrical short-circuit current, amperes
$j=\sqrt{-1}$
r = resistance and reactance, respectively, of metallic circuit, ohms per phase, to the point of fault

The scalar value of current in rms amperes per phase, derived from equation 1 is:

$$i=-\frac{rV}{(r^2+x^2)}\times\left(1-\sqrt{1+\frac{(r^2+x^2)(E^2-V^2)}{r^2V^2}}\right) \qquad (2)$$

Figure 7

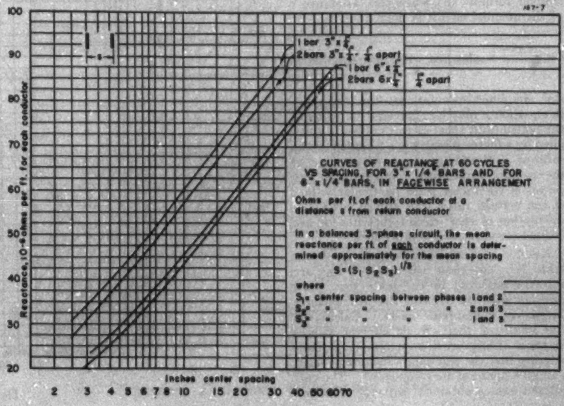


Table VII. Resistance-Reactance-Impedance of 600-, 3,000- and 5,000-Volt Rubber, Varnished Cambric, and Paper Cable

Ohms Per 1,000 Feet at 25 Deg C–77 Deg F; Line to Neutral—60 Cycles; Nonshielded, Nonmagnetic

	B&S Gauge								Thousand CM			
Type*	No. 8	No. 6	No. 4	No. 2	1/0	2/0	3/0	4/0	250	350	500	750
600-volt single-conductor cable—3 cables in nonmagnetic duct												
X-a	0.046	0.045	0.043	0.041	0.040	0.039	0.039	0.038	0.038	0.037	0.037	0.036
X-c	0.049	0.047	0.044	0.042	0.040	0.040	0.039	0.039	0.038	0.037	0.037	0.036
Z-a	0.662	0.414	0.254	0.164	0.112	0.093	0.078	0.065	0.060	0.050	0.044	0.040
R-a	0.662	0.414	0.254	0.157	0.103	0.085	0.067	0.053	0.046	0.034	0.024	0.017
3,000-volt single-conductor cable—3 cables in nonmagnetic duct												
X-a	0.052	0.049	0.047	0.044	0.042	0.041	0.040	0.040	0.039	0.038	0.037	0.037
X-c	0.056	0.053	0.050	0.047	0.045	0.043	0.042	0.041	0.041	0.040	0.038	0.038
Z-a	0.665	0.416	0.250	0.163	0.110	0.093	0.078	0.066	0.061	0.051	0.045	0.041
R-a	0.665	0.416	0.250	0.163	0.101	0.083	0.067	0.052	0.045	0.033	0.024	0.018
5,000-volt single-conductor cable—3 cables in nonmagnetic duct												
X-a	0.056	0.053	0.050	0.047	0.044	0.043	0.042	0.041	0.041	0.039	0.039	0.038
X-c	0.060	0.056	0.053	0.049	0.046	0.045	0.044	0.043	0.042	0.041	0.040	0.039
Z-a	0.656	0.435	0.265	0.173	0.113	0.095	0.079	0.067	0.062	0.052	0.046	0.042
R-a	0.656	0.435	0.265	0.168	0.104	0.084	0.067	0.053	0.045	0.034	0.024	0.017
600-volt three-conductor cable												
X-b	0.035	0.032	0.031	0.030	0.029	0.028	0.028	0.028	0.027	0.027	0.026	0.025
X-d	0.038	0.036	0.033	0.032	0.031	0.030	0.029	0.029	0.029	0.028	0.027	0.026
Z-b	0.658	0.425	0.264	0.167	0.110	0.091	0.074	0.060	0.054	0.044	0.037	0.031
Z-c	0.658	0.425	0.264	0.164	0.108	0.090	0.073	0.059	0.053	0.043	0.035	0.030
R-b	0.658	0.425	0.264	0.163	0.104	0.085	0.067	0.054	0.046	0.033	0.024	0.017
3,000-volt three-conductor cable												
X-b	0.039	0.036	0.033	0.031	0.030	0.029	0.028	0.028	0.028	0.027	0.026	0.025
X-d	0.044	0.042	0.040	0.037	0.035	0.034	0.033	0.032	0.031	0.030	0.029	0.027
Z-b	0.656	0.425	0.270	0.169	0.111	0.091	0.075	0.063	0.056	0.045	0.038	0.033
Z-c	0.656	0.425	0.270	0.166	0.109	0.089	0.073	0.061	0.054	0.043	0.036	0.031
R-b	0.656	0.425	0.270	0.166	0.104	0.085	0.067	0.054	0.046	0.033	0.025	0.017
5,000-volt three-conductor cable												
X-b	0.040	0.038	0.035	0.033	0.031	0.030	0.029	0.029	0.028	0.027	0.027	0.026
X-d	0.050	0.046	0.043	0.040	0.037	0.035	0.034	0.033	0.032	0.031	0.030	0.028
Z-b	0.655	0.425	0.271	0.168	0.109	0.092	0.077	0.064	0.057	0.047	0.039	0.033
Z-c	0.655	0.425	0.271	0.168	0.109	0.091	0.075	0.061	0.054	0.044	0.036	0.031
R-b	0.655	0.425	0.271	0.165	0.104	0.085	0.069	0.054	0.046	0.033	0.025	0.017

*Cable insulation and finish types:

X-a—Reactance of V.C. with braid cover or V.C. or paper with lead sheath.
X-b—Reactance of paper or V.C.
X-c—Reactance of rubber with braid cover or lead sheath.
X-d—Reactance of rubber.
Z-a—Impedance of rubber, V.C. or paper with braid cover or lead sheath.
Z-b—Impedance of rubber.
Z-c—Impedance of paper or varnished cambric.
R-a[6]—Resistance of single-conductor cable with braid cover (no lead).
R-a-c[7]—Resistance of three-conductor cable (no appreciable difference with braid cover or lead sheath).

NOTE: The reactance data are based on the formula:

$$X=0.023\left(\log_e\frac{2S}{D}+K\right)$$

$\log_e=\log_{10}\times 2.3025851$

X = Reactance in ohms per 1,000 feet at 60 cycles.
S = Spacing of conductors (center to center) in inches.
D = Diameter of conductor in inches.
K = A coefficient dependent on the ratio of the inside diameter of a conductor to outside diameter of the conductor.
For cables of standard-strand construction, $K=0.25$.

The general formula (2) reduces to the simpler formula (3) when the resistance r of the metallic circuit is negligible.

$$i=\sqrt{\frac{E^2-V^2}{x}}\text{ amperes, when } r \text{ is negligible} \quad (3)$$

For simplified calculations, the current i in equation 2 may be expressed in terms of the bolted-fault short-circuit current, multiplied by a factor p which is a function of the ratio of V/E and the power factor $\cos\theta$ of the metallic impedance. With the aid of curves of the factor p for particular values of the voltage ratio V/E and of $\cos\theta$, the calculation of short-circuit currents for arc faults becomes a simple procedure.

Results of calculations are given in table XI. In these calculations the arc voltages observed in the tests were used together with the metallic circuit reactance based on bolted-fault tests. Calculated and observed short-circuit current values for the arc faults are in good agreement, considering that the test values include at least a small amount of d-c component. The calculated values obtained with the assumed

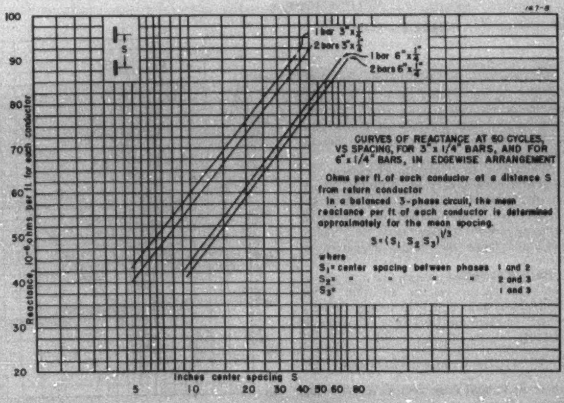


Figure 8

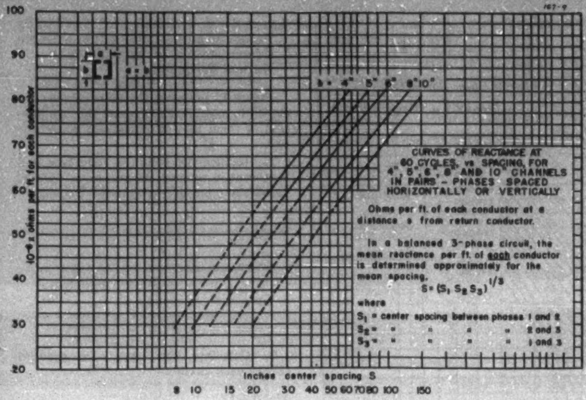


Figure 9

sinusoidal form factor of 0.71 for the arc voltage are probably somewhat high since the arc voltage is generally flat topped. Possibly a form factor of 0.8 or 0.85 for the arc voltages would be more nearly correct. The calculations of the table were made with a form factor of 0.71.

Appendix G. Switching and Control Equipment

G-1. OIL CIRCUIT BREAKERS

The reactance of certain oil circuit breakers with their contacts closed is indicated by the approximate values in table XII.

The data are given in ohms per pole at 60 cycles. The reactance values are based on calculations considering the size of the loop and the spacing between poles, and have been checked by test data for about one-third of the cases. The data do not apply after the contacts have begun to open; i.e., the resistance due to arc voltage during short-circuit interruption is not included. Obviously, the reactance per pole will not be the same for the three phases for three-phase balanced current flow. Approximate average figures have been given, since the unbalance of voltage drops in the different phases will almost always be negligible in the over-all impedance of the circuit.

The order of magnitude of the a-c resistance of oil circuit breakers of the above-mentioned types is from, say 0.00040 ohm per pole for a 200-ampere breaker to 0.000030 ohm for a 2,000-ampere breaker.

G-2. AIR CIRCUIT BREAKERS AND NETWORK PROTECTORS (600 VOLTS AND LESS)

The reactance of air circuit breakers with their contacts closed is indicated by the following:

	AE-1B-600 A. Coil Rating, Amps.		
	15	225	600
Resistance in ohms	0.033	0.00022	0.00010
Reactance in ohms	0.110	0.00085	0.00020

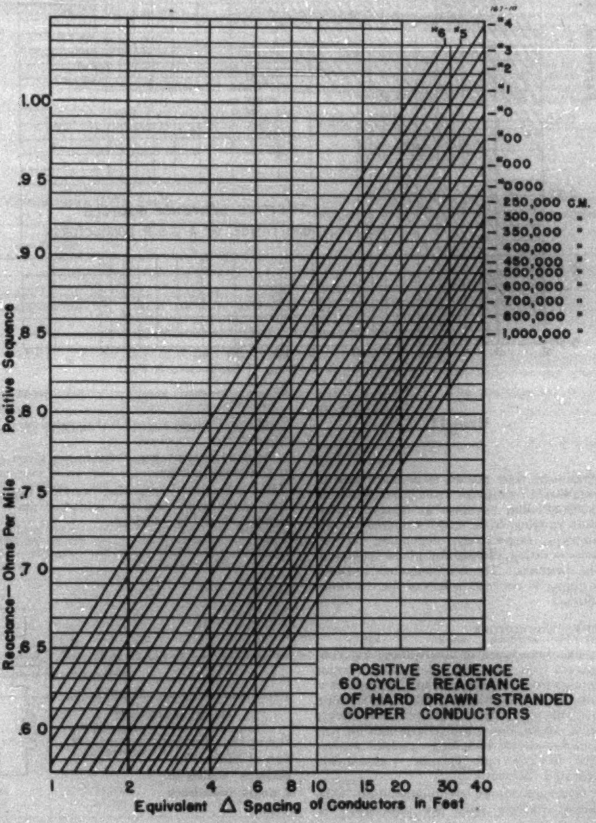
For air circuit breakers and network protectors of representative designs in rating above 200 amperes, the reactance in ohms is approximately 0.2/continuous rating in amperes. The a-c resistance of such breakers and protectors is in the order of 25–50 per cent of the reactance.

This does not include the impedance due to arc drop when a circuit breaker is in the presence of interrupting a short circuit. However, arc drop in interrupting devices need not enter into short circuit current calculations when circuit breakers are rated and selected on the basis of the prospective short circuit current, i.e., the current obtainable in the circuit when the breaker is replaced by a conductor of negligible impedance.

G-3. LEVER SWITCHES AND DISCONNECTING SWITCHES

The reactance of lever switches and disconnecting switches for low-voltage circuits

Figure 10



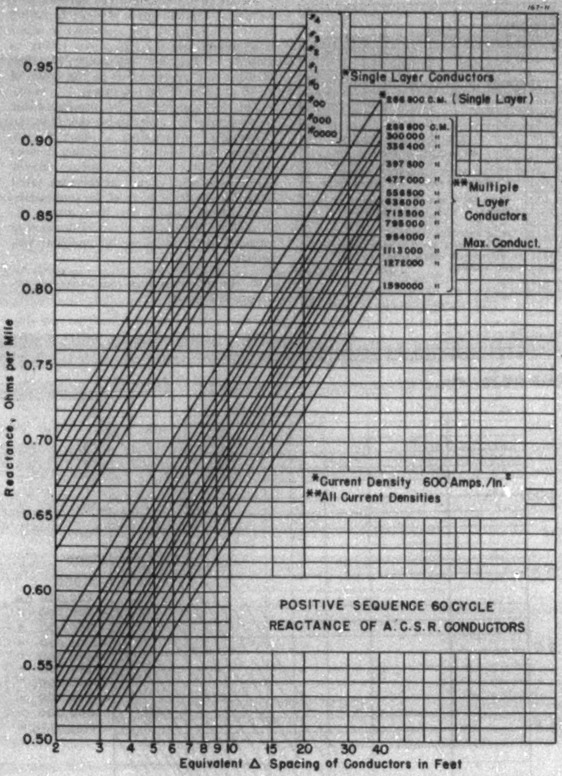


Figure 11

(600 volts and below) is of the order of magnitude ranging from 0.000050 to 0.000080 ohm per pole at 60 cycles, for sizes ranging from 4,000 amperes to 400 amperes, respectively, depending on the ampere rating, design, and phase spacing of the switches. The a-c resistance in ohms is about 30 per cent or less of the reactance ohms.

G-4. Contactors

The impedance of low-voltage a-c contactors (600 volts and below) with current ratings from 600 amperes to 150 amperes is roughly of the order of 0.001 to 0.003 ohm per pole at 60 cycles, the resistance being from about one-third to one-half of the impedance, not including overload protective devices and current transformers operated therewith.

Overload relays commonly used with motor starters have an impedance of roughly $5/I^2$ ohms where I is their rated current.

Data on current transformers are given in a separate section.

For calculations requiring accurate figures, additional data on contactor impedance should be obtained.

Appendix J. Solution of Combining Paralleling Impedances Into a Single Equivalent-Circuit Impedance at Normal Frequency

Figure 12 indicates the nomenclature assigned to multiple circuits and the combination of them into an equivalent single circuit.

1. Reduce the values of resistance and reactance in each of the two given parallel circuits to a per unit (pu) basis. Example:

	r	X
Branch *A*	0.05	0.15
Branch *B*	0.008	0.108

2. Calculate the impedance squared (Z^2) of each circuit

$Z^2 = r^2 + X^2$

Branch *A*: $r_a^2 + X_a^2 = Z_a^2$,
e.g., $-\overline{0.05^2} + \overline{0.15^2} = 0.025$

Branch *B*: $r_b^2 + X_b^2 = Z_b^2$,
e.g., $-\overline{0.008^2} + \overline{0.108^2} = 0.0117$

3. Obtain the ratios of r/Z^2 for each circuit

Branch *A*: r_a/Z_a^2, e.g., $\dfrac{0.05}{0.025} =$ 2.0

Branch *B*: r_a/Z_b^2, e.g., $\dfrac{0.08}{0.0117} =$ 0.683

4. Add the foregoing $r/Z^2 = G_0 =$ $\overline{2.683}$

5. Obtain the ratios of X/Z^2 for each circuit

Branch *A*: X_a/Z_a^2, e.g., $\dfrac{0.15}{0.025} =$ 6

Branch *B*: X_b/Z_b^2, e.g., $\dfrac{0.108}{0.0117} =$ 9.2

6. Add the foregoing $X/Z^2 = B_0 =$ $\overline{15.2}$

7. $Y_0^2 = G_0^2 + B_0^2$, e.g., $= \overline{2.683^2} + \overline{15.2^2} =$ 238.2

8. $r_0 = G_0/Y_0^2$, e.g., $\dfrac{2.683}{238.2} = 0.0112$

9. $X_0 = B_0/Y_0^2$, e. g., $\dfrac{15.2}{238.2} = 0.0642$

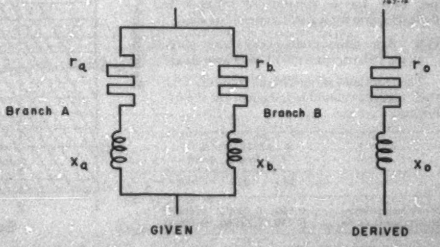


Figure 12. Resolution of multiple impedance circuits to a single impedance circuit

Table VIII. Resistance and Diameter of Stranded Copper and Aluminum Conductors

60 Cycles—Conductor Temp. 25 Deg C (77 Deg F)

Hard Drawn Stranded Copper Conductors: Size in Circ. Mils or B&S	Res. in Ohms Per Mile	Outside Diam. in Inches	Aluminum Cable Steel Reinforced Conductors: Size in Circ. Mils or B&S	Res. in Ohms Per Mile	Outside Diam. in Inches
1,000,000	0.0636	1.152	1,590,000	0.0607	1.544
950,000	0.0664	1.123	1,510,500	0.0637	1.506
900,000	0.0696	1.094	1,431,000	0.0671	1.465
850,000	0.0732	1.062	1,351,500	0.0709	1.424
800,000	0.0773	1.031	1,272,000	0.0752	1.382
750,000	0.0819	0.998	1,192,500	0.0801	1.337
700,000	0.0873	0.964	1,113,000	0.0857	1.292
650,000	0.0934	0.929	1,033,500	0.0922	1.246
600,000	0.101	0.893	954,000	0.0997	1.196
550,000	0.109	0.855	900,000	0.106	1.162
500,000	0.120	0.813	874,500	0.109	1.146
450,000	0.132	0.772	795,000	0.119	1.093
400,000	0.148	0.728	715,500	0.133	1.036
350,000	0.169	0.681	666,600	0.142	1.000
300,000	0.197	0.630	636,000	0.149	0.977
250,000	0.236	0.575	605,000	0.157	0.953
(B&S) 0000	0.278	0.528	556,500	0.168	0.912
000	0.350	0.470	518,000	0.180	1.000
00	0.440	0.419	500,000	0.187	0.904
0	0.555	0.373	477,000	0.196	0.845
1	0.699	0.328	397,500	0.235	0.771
2	0.882	0.292	336,400	0.278	0.710
3	1.11	0.260	300,000	0.311	0.670
4	1.40	0.232	266,800	0.350	0.632
5	1.77	0.206	(B&S) 0000	0.464	0.564
6	2.23	0.184	000	0.579	0.501
7	2.73	0.164	00	0.718	0.447
8	3.44	0.146	0	0.893	0.398
			1	1.12	0.355
			2	1.41	0.316
			3	1.78	0.281

Table X. Arc Voltages for Various Types of Arc Faults

Type of Fault	Arc Voltage Instantaneous Volts	Current-Limiting Effect
1. Short circuits of the bolted type and their equivalent	Less than 50 volts and generally less than 10*	Not appreciable. Current to be calculated from impedance of metallic circuit, neglecting fault resistance
2. Arc faults in enclosed congested structures typical of modern switchgear and station design, formed by objects readily repelled or destroyed by the arc		
a. Highly congested construction with a multiplicity of closely spaced bare conductors	From 80 volts up*	Not of a large order; may be determined by calculation
b. Less congested structures	From 150 volts up*	Higher than 2*a*, may be determined by calculation
3. Arc faults between bare conductors in the absence of nearby live conductors and away from enclosing walls; arc voltages highly variable depending on the circumstances	May be of the order of magnitude of the circuit voltage and higher*	Highly variable, calculation not recommended
4. Arc faults in cables produced by piercing the insulation with conducting members capable of being repelled; arc voltages may vary widely	May be of the order of magnitude of the circuit voltage and higher*	Highly variable; large current-limiting action cannot be depended on; calculation not recommended

*Line-to-neutral values for three-phase short circuits.

Table XI. Comparison of Calculated and Measured Values of Short-Circuit Currents for Arc Faults in Enclosed Double-Bus Assembly

RMS Line Voltage Open Circuit	Arc Voltage Crest Value From Test Line-to-Neutral Volts	Metallic Circuit Reactance X Ohms	Short-Circuit Current RMS Sym. (Amperes): Calculated	Test
300	146	0.00402	35,000	37,500
600	128	0.00352	95,000	99,000

Table IX. Over-All Resistance and Reactance of Type W Current Transformers Referred to Primary Winding*

Current Rating Primary Winding (Amperes)	Resistance (Ohms)	Reactance at 60 Cycles (Ohms)
100	0.0017	0.0035
150	0.0008	0.0017
200	0.00045	0.0010
250	0.00030	0.00066
300	0.00025	0.00050
400	0.00015	0.00032
500	0.00010	0.00022
600	0.00008	0.00019
800	0.00004	0.00012

*Approximate values at short-circuit currents with d-c component; rms symmetrical component ranging from 15,000 to 55,000 amperes.

Notes: These values are also representative of the order of magnitude of the impedance for current transformers of the following types, rated at 5,000 volts: *JW*1, *JW*4, *JW*6, *JW*14, *WC*12, *WF*1, *WF*6, and *WF*12.

Impedances for other designs of current transformers of the wound primary type may be estimated by applying the following approximate factors to the values of table IX:

Type of Current Transformer	Factor to Be Applied to Resistance and Reactance Values in Table IX
*KF*85—7,500 volts	1.8
*JS*1—15,000 volts	0.4

The foregoing may be tabulated for convenience in solving a number of parallel pairs of circuits:

	r	X	$Z^2=r^2+X^2$	r/Z^2	X/Z^2
Branch *A*	( )	( )	( )	( )	( )
Branch *B*	( )	( )	( )	( )	( )
Branch *C*, etc.					

Addition of the last two columns gives $\overline{G_0=(\ \ )\ B_0=(\ \ )}$. The combination of the circuits results in:

$$r_0=\frac{G_0}{Y_0^2}, \qquad x_0=\frac{B_0}{Y_0^2}, \qquad Y_0^2=G_0^2+B_0^2$$

Any number of parallel circuits may be accommodated by additional horizontal columns same as branch *A* and branch *B*,

Table XII

Type	Volts	Amperes	Reactance Per Pole at 60 Cycles (Ohms)
FK-20	2,500	200,300	0.000080
FK-33	5,000	200,400	0.000125
FK-42	5,000	800	0.000150
FK-42	7,500	400–600	0.000160
FK-143	7,500	1,200	0.000180
FK-143	15,000	600	0.000200
FK-45	5,000	2,000	0.000180
FK-46	7,500	1,200	0.000210
	15,000	600	0.000230
FH-123	15,000	2,000	0.000620
FHK-330-20*B*	15,000	2,000	0.000550
FHKO-339-24*A*	23,000	2,000	0.000780

Surge Characteristics of Two-Winding Transformers

REINHOLD RÜDENBERG
MEMBER AIEE

1. Introduction

THE object of this paper is to show the performance of transformers with two windings in the rapid-transient state when surges develop in their conductors. On the one hand, we will determine the characteristic constants for traveling waves, as surge-impedance and wave velocity, from the data of the transformer. On the other hand, we will consider the rapid interaction of the primary and secondary windings, with different numbers of turns, which for travelling waves is very different from the coupling under quasi-steady-state conditions.[1,2,3]

In a previous paper[11] it is shown that the propagation of traveling waves of moderate frequency in coils and windings is similar to the propagation in smooth lines and that the values of surge-impedance and traveling-velocity of such waves can be derived from the total self-inductance and the total ground-capacitance of the windings under steady state conditions. Furthermore, very steep parts of incident waves, corresponding to the highest frequencies, are cut off and converted within the winding into a flattened front, the length of which depends on the internal capacitance between the turns or coils. In this paper these results for moderate frequencies will be used but the influence of internal capacitances for very high frequencies will not be considered, as this would only obscure the principal objective of the paper.

2. Characteristics of Ideal Transformer Coils

Consider a single-layer tubular coil as represented in figure 1. The axial length and the diameter may be large compared with the insulating distance b to the inner core. However, the wall thickness of the coil may be small compared with the distance b, while the outer tank may be at any distance from the coil. The surfaces of the core and the tank are considered as being smooth, and concentric to the winding and it is assumed that both are electrically perfect conductors. Then no rapid-transient flux can penetrate their interior, so that the magnetic field is parallel and the electric field perpendicular to their surfaces. The core and the tank are assumed to be grounded.

First we derive the steady-state values of capacitance and self-inductance of the coil since these are basic for the surge characteristics. We use the electromagnetic system of units and shall add conversion factors for the practical system where it is useful.

A uniform voltage E in the winding produces over the internal breadth b and the external distance d electric gradients which are represented in figure 1 as E_i and E_e. If w is the wire length of one turn, equal to the circumference of the coil, and h is the axial length of the winding, the electric charge of the entire winding under the influence of E is given by the total electric displacement over the areas on both sides of the coil as

$$Q=\frac{wh}{4\pi v_0^2}\left(\epsilon\frac{E}{b}+\epsilon\frac{E}{d}\right)=\frac{\epsilon wh}{4\pi v_0^2}E\left(\frac{1}{b}+\frac{1}{d}\right) \tag{1}$$

where ϵ is the dielectric constant of the insulating spaces on both sides and v_0 is the velocity of light in vacuum. Hence the total ground-capacitance of the coil, as the quotient of charge and voltage, is

$$C=\frac{Q}{E}=\frac{\epsilon wh}{4\pi v_0^2}\,\frac{b+d}{bd}\;(\times 10^9 \text{ in farads}) \tag{2}$$

A steady-state current I in the coil produces a magnetic flux which is entirely in the axial direction, and is equal in magnitude and opposite in direction within the insulating spaces on both sides of the coil. The flux densities B_i and B_e in the internal and external gaps are shown in figure 1. If N denotes the total number of turns of the coil, the flux in the gaps on either side of the coil is

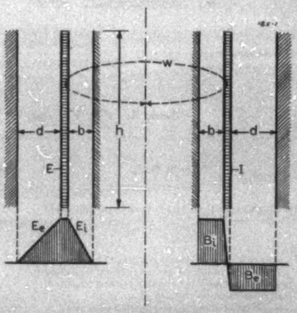


Figure 1. Ideal transformer winding with distribution of radial electric field and axial magnetic flux

$$\Phi=\frac{4\pi\mu wNI}{h/b+h/d} \tag{3}$$

where μ is the magnetic permeability on both sides of the winding. Hence the self-inductance of the entire coil is

$$L=\frac{N\Phi}{I}=4\pi\mu\frac{w}{h}N^2\frac{bd}{b+d}\;(\times 10^{-9} \text{ in henrys}) \tag{4}$$

It contains the physical dimensions h, d, b of the transformer in the same mutual arrangement as in equation 2 for the capacitance.

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[1] For all numbered references, see list at end of paper.

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etc., their resultant (r/Z^2)'s and (X/Z^2)'s being added together to obtain G_0 and B_0.

References

1. FAULT-VOLTAGE DROP AND IMPEDANCE AT SHORT-CIRCUIT CURRENTS IN LOW-VOLTAGE CIRCUITS, O. R. Schurig. AIEE TRANSACTIONS, volume 60, 1941.

2. STANDARD DECREMENT CURVES, W. C. Hahn and C. F. Wagner. AIEE TRANSACTIONS, volume 51, 1932, page 353.

3. PRACTICAL CALCULATION OF SHORT-CIRCUIT STRESSES IN SUPPORTS FOR STRAIGHT PARALLEL BAR CONDUCTORS, Schurig, Frick, and Sayre. *General Electric Review*, August 1936.

4. METHOD OF CALCULATING ELECTROMAGNETIC FORCES ON CONDUCTORS WITH BENDS, SHORT LENGTHS, AND CROSS ARMS, C. W. Frick. *General Electric Review*, May 1933.

5. IMPEDANCE OF THREE-PHASE SECONDARY MAINS IN NONMETALLIC AND IRON CONDUITS, L. Brieger. Edison Electric Institute *Bulletin*, February 1938.

6. LOSSES IN ARMORED SINGLE-CONDUCTOR LEAD-COVERED A-C CABLES, O. R. Schurig, H. P. Kuehni, and F. H. Buller. AIEE TRANSACTIONS, volume 48, 1929, pages 417–35.

7. RESISTANCE AND REACTANCE OF THREE-CONDUCTOR CABLES, E. H. Salter, G. B. Shanklin, and R. J. Wiseman. AIEE TRANSACTIONS, volume 53, 1934 (December section), pages 1581–9; discussion, volume 54, 1935, pages 324, 436.

If we denote the total wire length of the winding by

$$a=wN \tag{5}$$

the velocity of traveling waves within the coil is determined by total self-inductance and total ground-capacitance as

$$v=\frac{1}{\sqrt{lc}}=\frac{a}{\sqrt{LC}} \tag{6}$$

l and c being the respective values per unit length of the wire. If we insert herein the values of C and L from equations 2 and 4, we see that all physical dimensions cancel and there remains the relation

$$v=\frac{v_0}{\sqrt{\epsilon\mu}} \tag{7}$$

This shows that the velocity of waves along the wire of such transformer coils is always equal to the universal velocity of light, 300 m/μsec. divided by the square root of dielectric constant and magnetic permeability.

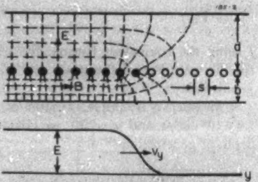


Figure 2. Electromagnetic field of a steep-front wave traveling through a coil

This fundamental law was found empirically[11] as a result of measurements on many actual transformers as a limiting case for small internal capacitance. We see now this law is well founded theoretically and that electromagnetic waves on conductors wound into coils with negligible internal capacitance travel with the same velocity as in straight wires. Hence the increase in self-inductance by winding the wire in a cylindrical coil is exactly compensated by the decrease in ground-capacitance caused by the crowded arrangement of the turns.

The surge impedance of the coil, which determines the ratio of voltage and current of travelling waves, is also determined by equations 2 and 4 as

$$Z=\sqrt{\frac{l}{c}}=\sqrt{\frac{L}{C}}=4\pi v_0\sqrt{\frac{\mu}{\epsilon}}\frac{N}{h}\frac{bd}{b+d}$$
$$(\times 10^{-9}\text{ in ohms}) \tag{8}$$

As the value of v_0 is universally given, we can simplify the numerical magnitudes and obtain, in practical units,

$$Z=120\pi\sqrt{\frac{\mu}{\epsilon}}\frac{N}{h}\frac{b}{1+b/d}\text{ (in ohms)} \tag{9}$$

Thus we see that in addition to ϵ and μ the surge impedance of transformer windings[13] is dependent mainly on the number of turns per unit length N/h and on the internal insulating distance b, the larger external distance d entering only in a correction term.

A model oil transformer[6] with grounded core, the latter consisting of a cylindrical steel sheet, had a single-layer winding of flat copper wire 1.2×7 mm in cross-section and $a=59$ m long. The wire was wound with $N=52$ turns on an insulation tube of 36.3 cm diameter and an axial length $h=65$ cm. The inner distance from coil to core was $b=5.4$ cm and there was no outer metal tank. From the shape of the switching-on voltage curves, measured with a cathode-ray oscillograph, in comparison with those of a known smooth line, the propagation velocity through the turns of the winding was determined as $v=155$ m/μsec, and the surge-impedance as $Z=920$ ohms approximately.

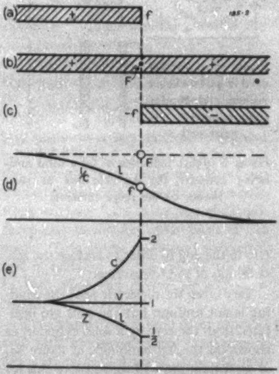


Figure 3. Finite end of a coil and its electromagnetic characteristics

With an average $\epsilon=3.5$ and $\mu=1$, equations 7 and 9 give

$$v=\frac{300}{\sqrt{3.5\cdot 1}}=160m/\mu s$$

$$Z=120\pi\sqrt{\frac{1}{3.5}}\cdot\frac{52}{65}5.4=870\Omega$$

Thus our conceptions have enabled us to derive proper values of the coil characteristics.

Average transformer-windings in oil with $\epsilon=3.5$ and $\mu=1$, having a winding density of $N/h=10$ turns/cm and insulating distances $b=3$ cm to the core and $d=30$ cm to the tank, may have a surge-impedance

$$Z=120\pi\sqrt{\frac{1}{3.5}}\cdot 10\cdot\frac{3}{1+3/30}=5{,}500\Omega$$

This is about ten times the value of the surge-impedance of ordinary overhead lines.

A study of many power transformers of various types shows that

For rated sizes of 5 to 500 to 50,000 kva
The winding density varies from 100 to 10 to 1 turns/cm

These values refer to the high voltage side; for the low voltage side the values of winding density are about $^1/_3$ to $^1/_{10}$ of the foregoing figures. Since the insulating distance b varies only from 1 cm for low voltages to 5 cm for high voltages, while the tank distance d is always relatively large, we can expect that the windings of oil transformers of the sizes just considered will have surge impedances of the order of

Z_{HV} = 20,000 to 5,000 to 1,000 ohms
Z_{LV} = 4,000 to 600 to 100 ohms

for the high voltage and low voltage sides respectively. These figures agree well with sporadic measurements published in the literature.[1]

If a wave of any form travels with high velocity v along the wire of a helical winding, the axial velocity of propagation of the electromagnetic state will be much smaller than v. The shape of the circumferential wave remains preserved in axial direction; however, it is condensed in the ratio of wire length w to pitch s of a turn. Thus the axial velocity of the wave is only

$$v_y=\frac{s}{w}v=\frac{h}{a}v \tag{10}$$

expressed also in terms of axial coil length h and total wire length a.

If an impact wave E of a form shown in figure 2 travels through the coil, the turns ahead of the front are still without voltage and current while the turns behind the front have full voltage and current. The electric and magnetic lines of force therefore follow curves as shown in figure 2. The wave front travels circumferentially along the helical wire with the velocity of light in the dielectric medium. The entire electromagnetic excitation of the coil, however, propagates in the axial direction of the coil with much lower velocity, as given by equation 10. Because of this slow axial propagation, the electromagnetic field in the longitudinal cross-section shown in figure 2, is nearly static, so that it is easy to trace the lines of force. Thus the structure of the electromagnetic field of the transformer coil is wave-like in the circumferential direction but quasi-stationary in the axial direction.

3. Approximation to Actual Windings

Until now we have assumed that the coil is infinitely long. In order to consider the effect of a finite end, as shown in figure 3*a*, the electromagnetic field of this end may be conceived as being composed of the field of an infinite coil, as in figure 3*b*, and the field of a finite coil, as in figure 3*c*, extending to the other side and having the opposite sign from the infinite coil. A superposition of the two coils of figure 3*b* and 3*c* gives exactly the coil of figure 3*a* with a finite end. If F is the well-known electromagnetic field of the infinite coil, and f the unknown field at the end of the finite coil in figure 3*a*, the field at the end of the auxiliary coil in figure 3*c* is $-f$. The superposition gives

$$F-f=f \text{ or } f=F/2 \tag{11}$$

Hence the electromagnetic field and therefore the specific properties, such as electromagnetic and electrostatic inductance, both per unit length, decrease towards the end of a coil to exactly half the value which they have in an infinite coil or at the center parts of a long finite coil. This is represented in figure 3*d*, while figure 3*e* shows that the specific capacitance c, the reciprocal of the static inductance, is doubled at the end.

The velocity of the waves, determined by the product of l and c, as in equation 6, remains spatially constant up to the end of the coil. The surge impedance, however, determined by the quotient of l and c, as in equation 8, decreases towards the end to one-half its original value. This is shown in figure 3*e*. The axial distance up to which these changes in v and Z extend into the coil depends entirely on the breadth b of the dielectric gap and actually is only a small multiple of the distance b. As a whole, a finite end of the coil acts as if it were loaded with some additional capacitance and some inductance were removed. Those properties of the coil which are determined by the wave velocity, as for example the natural frequencies, are not changed by the alterations near a finite end. The surge impedance on the other hand, comes into effect mainly with transmission of the waves, coming from or going to adjacent lines, which for the most part have much smaller surge impedance than the coil. Halving of the surge-impedance near the coil end produces therefore only moderate alterations in the transmitted voltage.

In actual transformers, as in figure 4, neither the coren or the tank have a smooth surface as in our idealized picture, but the insulating distances vary around the circumference. However, since we will not determine the variation of voltage around a single turn, we may replace, without actual loss of accuracy, the irregular surfaces of both core and tank by concentric circles which give an equivalent interior and exterior insulating distance. As the distance d to the tank is comparatively large and its influence, according to equation 9, is small, it does not matter whether or not the tank is eccentric to the coil in order to enclose the second limb of the transformer.

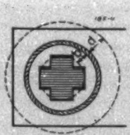


Figure 4. Actual and equivalent ground electrodes surrounding a circular coil

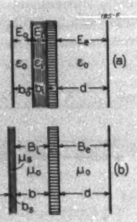


Figure 5 (right). Influence of (a) a solid insulating cylinder, (b) the skin flux in steel sheets, on the surge constants

Very often the dielectric near the winding is not uniform but the coils are insulated from the core by a solid cylinder, as shown on the left-hand side of figure 5*a*. If we denote the wall thickness of this insulating tube by b_i and its dielectric constant by ϵ_i, while the remainder of the distance to the core is b_0 with the constant ϵ_0, we have instead of equation 1 for the electric charge of the coil under the voltage E

$$Q=\frac{wh}{4\pi v_0^2}E\left(\frac{\epsilon_0}{d}+\frac{1}{b_i/\epsilon_i+b_0/\epsilon_0}\right) \tag{12}$$

By comparison with equation 1 we obtain therefore as mean dielectric constant for the whole transformer

$$\epsilon=\epsilon_0\frac{\dfrac{1}{\epsilon_0/\epsilon_i\cdot b_i/b+b_0/b}+\dfrac{b}{d}}{1+b/d} \tag{13}$$

With actual figures this gives for oil transformers resultant values of about $\epsilon=3.5$, which may be used for the wave-velocity in equation 7 as well as for the surge-impedance in equations 8 and 9.

The interior and exterior boundaries of the electromagnetic field of actual transformers do not consist, as in our idealized picture, of material with perfect electric conductivity but are built up of iron with finite electric conductivity and high magnetic permeability. Figure 4 shows that the surface consists partly of rolled steel sheets, partly of magnetic or anti-magnetic constructional steel plates, and partly of the edges of the steel sheets which form the active magnetic core. High frequency magnetic oscillations, of which every travelling wave is composed, penetrate only into a thin layer of these materials, the equivalent depth of which is

$$b_s=\frac{1}{4\pi}\sqrt{\frac{s}{\mu_s f}} \tag{14}$$

f being the equivalent frequency, s the specific resistance of the steel, and μ_s its permeability.

If we put, for example, $f=10^4 c/s$, $s=1.5\times10^4\text{ cm}^2/s$, and $\mu_s=200$, we obtain a depth of penetration

$$b_s=\frac{1}{4\pi}\sqrt{\frac{1.5\cdot10^4}{200\cdot10^4}}=6.9\times10^{-3}\text{ cm}$$

The magnetic flux within this skin supplements the air flux and, since the flux density in steel is μ_s times that in air, we have an additional equivalent air breadth for the internal magnetic flux as $\mu_s\cdot b_s=1.39$ cm in our example.

For several reasons it is difficult to decide without further experiments which value of the permeability actually comes into effect. The edges of the highly magnetic steel sheets are deteriorated magnetically by the cutting process, the magnetization by the travelling waves is superposed on the steady state magnetization, the magnetic circuit for the wave flux is not iron closed, and the hysteresis in steel parts of poorer magnetic quality gives an additional screening effect.

If the effective permeability μ_s of the steel surfaces were known, we could compute the additional effect on the permeability μ_0 of the insulating space as in figure 5*b*. In confining ourselves to the effect of the more important core surface, we can derive the resultant surge permeability for a steel core winding as

$$\mu=\mu_0\frac{1+b/d}{\dfrac{1}{\mu_s/\mu_0\cdot b_s/b+1}+\dfrac{b}{d}} \tag{15}$$

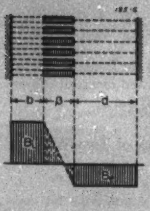


Figure 6. Influence of the magnetic field within the winding space on the surge constants

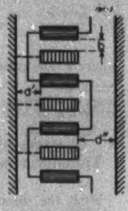


Figure 7. Effective distances for pancake windings with grounded secondary

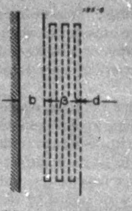


Figure 8. Effective distances for concentric layer windings

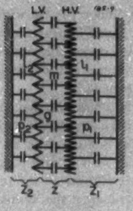


Figure 9. General linkage diagram for two-winding transformers

an expression which in its structure is similar to equation 13. If the value of μ exceeds considerably that of μ_0, it decreases the wave velocity given by equation 7 and increases the surge-impedance given by equations 8 and 9. Since b_s, according to equation 14, is dependent on frequency, we then obtain a variation of the characteristics of the winding with frequency.

With the values of the last example and insulating distances $b=3$ cm and $d=30$ cm, we obtain a resultant permeability

$$\mu=1\frac{1+3/30}{\dfrac{1}{200/1\cdot 6.9\cdot 10^{-2}/3+1}+\dfrac{3}{30}}=1.43$$

For smaller frequencies this magnitude may become considerably larger.

The copper winding also permits some penetration of the magnetic flux since it is not perfectly conducting. As its permeability μ_c is always equal to 1, then according to equation 14 the skin depth becomes with $s=2\times 10^3$ cm$^2/s$ and $f=10^4 c/s$,

$$b_e=\frac{1}{4\pi}\sqrt{\frac{2.10^3}{1.10^4}}=3.56\cdot 10^{-2}\text{ cm}$$

This value is so small compared with the insulating distances that one is almost always justified in neglecting it.

We have assumed that the wall thickness of the coil in figure 1 is very small. Actual transformers, however, have a considerable coil thickness β, as in figure 6, where the variation of the flux density between the interior and exterior insulating spaces is indicated by an inclined dotted line. The intermediate flux within the cross section of the winding is only partially linked with the turns and gives therefore, as is well known, only an additional breadth $\beta/3$. However, as transformer coils are usually wound with solid copper wire with high skin-effect as mentioned above, the transient flux penetrates only the insulation of the wires and not the copper. With β' as the total internal radial insulation between the turns of figure 6, we thus obtain a correction factor

$$\mu'=1+\frac{1}{3}\frac{\beta'}{b} \qquad (16)$$

which may be used as an additional permeability for the electromagnetic space.

As figure 6 indicates, the capacitance of the winding to ground is not noticeably altered by a finite wall-thickness of the coil even if the winding consists of single sections, with small distances between. Since neither the total self-inductance nor the total ground-capacitance is materially influenced by subdivision of the winding into single discs, sectionalized cylindrical coils as in figure 6 can also be treated by the method already outlined. However, the last discs at the ends of the entire coil-stack will have some extra ground-capacitance due to the flat outer faces of the stack. This can be considered as lumped end-capacitance of the coil in addition to the other end effects already mentioned.

If the transformer is built up of pancake coils with interleaved primary and secondary sections as in figure 7, it is still possible to compute the total self-inductance and ground-capacitance in a manner similar to that already described, when the secondary winding is short-circuited and grounded, thus forming a part of the entire ground. We have only to consider, according to familiar rules, the formation of the electric and magnetic fields within all interspaces of the primary and secondary coils with distances b, d', and d'' as in figure 7. Also the multiple concentric layer winding of figure 8 can be treated likewise, except that in this case the internal capacitance between adjacent layers far outweighs the ground capacitance, so that wave velocity and surge-impedance may become dependent on frequency.[11]

4. Interaction of Primary and Secondary Coils

In real transformers there are always two windings acting together and influencing each other.[2,3] Whether or not the windings have different numbers of turns, their interaction can be analyzed by consideration of the voltages induced magnetically and statically by the currents. We again consider surge waves of such lengths that the mutual inductances of neighboring elements, in the winding under consideration as well as in the other winding, can be combined to give equivalent self- and mutual-inductances, as indicated in a previous paper.[11]

Then the spatial variation of voltages in each of the windings, distinguished by the subscripts 1 and 2, is

$$\left.\begin{aligned}-\frac{\partial e_1}{\partial x_1}&=l_1\frac{\partial i_1}{\partial t}+m\frac{\partial i_2}{\partial t}\\ -\frac{\partial e_2}{\partial x_2}&=l_2\frac{\partial i_2}{\partial t}+m\frac{\partial i_1}{\partial t}\end{aligned}\right\} \qquad (17)$$

The effective self- and mutual-inductances l and m per unit length of the conductors are related to the constants L and M of the entire windings under steady state conditions, as

$$l_1=L_1/a_1 \quad l_2=L_2/a_2 \quad m=M/\sqrt{a_1a_2} \qquad (18)$$

a_1 and a_2 being the lengths of the windings.

Since the mutual inductance m is the same in both equations 17, it must be formed symmetrically with respect to both windings. Furthermore, it must be derived from the mutual inductance M between the entire windings by dividing by a length. These dimensional conditions are satisfied by the last expression in equation 18.

The variation of voltage with time for each winding is determined by the spatial variation of both currents

$$\left.\begin{aligned}-\frac{\partial e_1}{\partial t}&=p_1\frac{\partial i_1}{\partial x_1}+g\frac{\partial i_2}{\partial x_1}\\ -\frac{\partial e_2}{\partial t}&=p_2\frac{\partial i_2}{\partial x_2}+g\frac{\partial i_1}{\partial x_2}\end{aligned}\right\} \qquad (19)$$

where p and g are Maxwell's self- and mutual-potential coefficients. These are reciprocal values of capacitances and thus are determined by the potential coefficients P and G of the entire winding as

$$p_1=P_1a_1 \quad p_2=P_2a_2 \quad g=G\sqrt{a_1a_2} \qquad (20)$$

The last expression again satisfies the dimensional considerations.

Figure 9 shows the self-constants l and p and the mutual-constants m and g for the high-voltage and the low-voltage windings of a transformer, both situated between the internal and external ground electrodes. In addition to these parameters actual transformers have internal capacitances between adjacent turns of each winding which produce, however, no coupling interaction between the two windings. Therefore we will not consider them and will thus obtain the interaction of the transformer coils in the purest form.

The differential equations 17 and 19 can be solved by linear dependences between voltage and currents in both windings, as

$$e_1 = Z_1 i_1 + z i_2 \qquad e_2 = Z_2 i_2 + z i_1 \tag{21}$$

Herein Z_1 and Z_2 denote the self-surge impedances of the two coils while z is a mutual surge-impedance between both coils as indicated in figure 9.

In order to determine these constants, we insert equations 21 in the left-hand sides of equations 17 and 19. For their first lines, expressing e_1, we obtain

$$\left.\begin{aligned} -\frac{\partial}{\partial x_1}(Z_1 i_1 + z i_2) &= \frac{\partial}{\partial t}(l_1 i_1 + m i_2) \\ -\frac{\partial}{\partial x_1}(p_1 i_1 + g i_2) &= \frac{\partial}{\partial t}(Z_1 i_1 + z i_2) \end{aligned}\right\} \tag{22}$$

For waves travelling with constant shape, differentiation in time and in space is proportional. We can then divide the first equation 22 by the second and obtain

$$(Z_1 i_1 + z i_2)^2 = (l_1 i_1 + m i_2)\cdot(p_1 i_1 + g i_2) \tag{23}$$

Since this is valid for any currents i_1 and i_2, the coefficients of the respective currents on both sides must be equal. We have, thus, the three conditions

$$Z_1^2 = l_1 p_1 \quad z^2 = mg \quad 2Z_1 z = l_1 g + p_1 m \tag{24}$$

The first two conditions give the surge impedances

$$Z_1 = \sqrt{l_1 p_1} \qquad z = \sqrt{mg} \tag{25}$$

and substituting these surge impedances into the third condition, we find

$$p_1/l_1 = g/m \tag{26}$$

If we insert the last equation of (21) into the last lines of equations 17 and 19 we obtain from the corresponding conditions the surge impedances

$$Z_2 = \sqrt{l_2 p_2} \qquad z = \sqrt{mg} \tag{27}$$

and the additional condition

$$p_2/l_2 = g/m \tag{28}$$

The second equations in (25) and (27) are identical and the coincident equations 26 and 28 indicate certain qualities of symmetry in the windings. Thus we have defined the three surge impedances which determine the interaction of currents and voltages in the coils according to our statement (21).

In order to determine the wave velocities in the two windings, we differentiate the first equation 22 with respect to t and the second with respect to x_1. Subtracting eliminates the surge impedances, and arranging the terms with regard to the currents gives

$$l_1 \frac{\partial^2 i_1}{\partial t^2} - p_1 \frac{\partial^2 i_1}{\partial x_1^2} = -m \frac{\partial^2 i_2}{\partial t_2} + g \frac{\partial^2 i_2}{\partial x_1^2} \tag{29}$$

Since this relation is valid for any value of i_1 and i_2, both the sides must disappear independently of each other. Therefore each side forms the well-known wave equation, and thus the wave velocities along the wire length x_1 of the winding 1 are

$$v_1 = \sqrt{p_1/l_1} \qquad v_1' = \sqrt{g/m} \tag{30}$$

The first expression 30 gives the velocity of the self-induced waves, the second gives that of the mutually-induced waves, and by inserting condition 26 in the second expression, we see that both velocities coincide.

By subjecting the second lines of equations 17, 19, 21 to the same operations as performed in the first lines, we obtain the wave velocities in the wires of winding 2 as

$$v_2 = \sqrt{p_2/l_2} \qquad v_2' = \sqrt{g/m} \tag{31}$$

Consideration of equation 28 shows that also in winding 2 there exists only one well-defined wave velocity along the wires. Furthermore, we see by comparison of v_1' and v_2' in equations 30 and 31 that all the velocities along the wires of both windings are exactly the same.

These conditions show that the primary as well as the secondary winding of any transformer possesses one and only one wave velocity which, furthermore, is the same in both windings, measured along the conductors of each winding. The presence of the other winding and the flow of currents therein does not alter this electromagnetic property of every coil. This does not mean, as we see later, that the windings do not interact with each other. However, it has the result that along the axis of the winding an electromagnetic field will travel only with a velocity corresponding to the pitch of the winding under consideration, independent of the presence of the other winding. Therefore we have to distinguish two different axial velocities, namely those of the H.V. and of the L.V. windings. Corresponding to equation 10 these velocities are

$$v_H = \frac{s_1}{w} v = \frac{h}{a_1} v \qquad v_L = \frac{s_2}{w} v = \frac{h}{a_2} v \tag{32}$$

where v now means the fundamental velocity given by all the identical expressions in equations 30 and 31.

5. Characteristics of Two-Winding Transformers

With the same assumptions for coils and insulating spaces as previously made, figure 10 represents the cross-section of a two-winding transformer. Now b_1 and d_1 are the internal and external distances to ground of the H.V. coil 1, while b_2 and d_2 are those of the L.V. coil 2. The self-potential coefficients are then reciprocal to equation 2

$$\left.\begin{aligned} P_1 &= E_1/Q_1 = (4\pi v_0^2/ewh) b_1 d_1/(b_1 + d_1) \\ P_2 &= E_2/Q_2 = (4\pi v_0^2/ewh) b_2 d_2/(b_2 + d_2) \end{aligned}\right\} \quad (\times 10^{-9} \text{ in } 1/\text{farad}) \tag{33}$$

As shown in figure 10*b*, the potential of winding 1 drops linearly from its own voltage E_1 to zero at the ground electrodes. If we introduce into this field a secondary winding at position 2 in figure 10*a*, it takes up by static induction a voltage

$$E_2 = \frac{b_2}{b_1} E_1 = \frac{4\pi v_0^2}{ewh} \; \frac{b_2/b_1}{1/b_1 + 1/d_1} Q_1 \tag{34}$$

where the value of the voltage E_1 is inserted from equation 33. The mutual potential coefficient between the windings 1 and 2 is, therefore,

$$G = \frac{E_2}{Q_1} = \frac{4\pi v_0^2}{ewh} \; \frac{b_2 d_1}{b_1 + d_1} \quad (\times 10^{-9} \text{ in } 1/\text{farad}) \tag{35}$$

The form of this expression is symmetrical to both coils, as we see from figure 10, since all the denominators of equations 33 and 35 have the common value

$$b_1 + d_1 = b_2 + d_2 = D \tag{36}$$

giving the distance between both ground electrodes, independent of the positions of the coils.

The self-inductance of each of the windings follows from the previous equation 4, as

$$\left.\begin{aligned} L_1 &= 4\pi\mu \frac{w}{h} N_1^2 \frac{b_1 d_1}{b_1 + d_1} \\ L_2 &= 4\pi\mu \frac{w}{h} N_2^2 \frac{b_2 d_2}{b_2 + d_2} \end{aligned}\right\} \quad (\times 10^{-9} \text{ in henrys}) \tag{37}$$

N_1 and N_2 denoting the number of turns of the two windings.

The distribution of the primary magnetic flux over the insulating spaces is shown in figure 10*c*. The secondary coil is linked with only a part of this flux within the inner space, namely

$$\Phi_2 = \frac{b_2}{b_1}\Phi_1 = 4\pi\mu w \, \frac{N_1 I_1}{h} \, \frac{b_2/b_1}{1/b_1+1/d_1} \tag{38}$$

where the value of the flux Φ_1 is inserted from equation 3. Thus the mutual inductance between the coils becomes

$$M = \frac{N_2\Phi_2}{I_1} = 4\pi\mu \, \frac{w}{h} \, N_1 N_2 \, \frac{b_2 d_1}{b_1+d_1} \quad (\times 10^{-9} \text{ in henrys}) \tag{39}$$

This expression is also geometrically symmetrical to both coils, as we see from figure 10.

Now we can determine the specific constants per unit length according to equations 18 and 20, by dividing or multiplying the total values by the wire lengths

$$a_1 = N_1 w \quad a_2 = N_2 w \tag{40}$$

We see immediately that all these constants, inserted in equations 30 and 31, give only a single value for the wave velocity along the wires, namely,

$$v = v_0/\sqrt{\epsilon\mu} \tag{41}$$

which again is identical with the universal velocity of light, adapted to the electromagnetic constants of the transformer space.

On the other hand, insertion of the constants in equations 25 and 27 gives the three surge impedances

$$\left.\begin{aligned} Z_1 &= 4\pi v_0 \cdot \sqrt{\mu/\epsilon} \cdot N_1/h \cdot b_1 d_1/D \\ Z_2 &= 4\pi v_0 \cdot \sqrt{\mu/\epsilon} \cdot N_2/h \cdot b_2 d_2/D \\ z &= 4\pi v_0 \cdot \sqrt{\mu/\epsilon} \cdot \sqrt{N_1 N_2}/h \cdot b_2 d_1/D \end{aligned}\right\} \quad (\times 10^{-9} \text{ in ohms}) \tag{42}$$

dependent on winding density and insulating distances. We derive from the last expressions the ratio of the mutual-surge-impedance to both of the self-surge-impedances, which will often be used, as

$$\frac{z}{Z_1} = \sqrt{N_2/N_1}\,\frac{b_2}{b_1} \qquad \frac{z}{Z_2} = \sqrt{N_1/N_2}\,\frac{d_1}{d_2} \tag{43}$$

If ϵ and μ are heterogeneously distributed with respect to both windings, equivalent distances b and d can be determined easily, with ratios frequently increasing towards unity.

From equation 21 we can derive that the ratio of voltages is determined directly by the ratio of surge impedances. The coupling ratio for travelling waves in a transformer is given therefore, according to equation 43, by the square root of the turn ratio, in contrast to the steady state coupling which is given directly by the ratio of turns. This is advantageous when a wave is transmitted from the L.V. to the H.V. side, but very unfavorable for the transmission of waves from the H.V. to the L.V. side of a transformer.

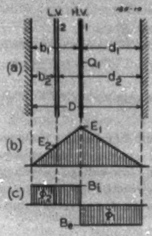


Figure 10 (left). Mutual inductive and capacitive interaction of two concentric windings

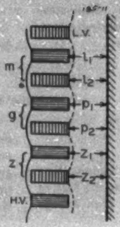


Figure 11 (right). Coupling coefficients for pancake coils

The coupling ratios in equation 43 are determined further by a geometric factor given simply by the distances. The ratio of the inner distances b is always materially smaller than one, while the ratio of the outer distances d for the most part is slightly smaller than one. In order to avoid high transmitted surges from the H.V. to the L.V. winding, it is thus preferable to arrange the L.V. winding as the inner coil and the H.V. winding as the outer coil, which actually is the usual practice with transformers having only two coils.

Pancake windings, as in figure 11, satisfy also the general coupling diagram of figure 9 and can therefore be treated in the same way as concentric windings. However, the mutual interactions appear here as more intensive and the subdivision into sectional coils is more pronounced.[13]

6. Wave Transmission Between Two Windings

The electromagnetic field propagates axially through the H.V. winding with the moderate velocity v_H, and through the L.V. winding with the higher velocity v_L, both determined by equation 32 and being much slower than the velocity of light. We call these the natural velocities of the windings. The complete electromagnetic state of the transformer is composed of both these slow and fast moving fields. Each of them cuts the helical wires of its own winding with a velocity of light. The wires of the other winding, however, are cut with a velocity different from that of light. The problem therefore arises as to the manner in which a winding responds if a wave is impressed on it, the velocity of which does not coincide with its natural velocity.

Let us investigate this performance of windings by means of a method which is easy to visualize. We perceive, as in figure 12, a coil with a definite natural axial wave velocity v_N, and along which an electric charge Q is moved with a given velocity V. Then three different cases are possible:

(a) The Velocity of the Charge Is Equal to the Natural Velocity of the Coil

In this case the electromagnetic induction of the moving charge produces voltage and current in the coil which move in synchronism with the charge. For an observer moving with the charge, the waves of current and voltage represented in figure 12 seem to be at rest. They form electromagnetic energy which travels along the coil always in parallel to the charge. This case is present in transformers with a turn ratio 1:1, which, however, is quite rare. The waves on both windings here interact in the same way as waves on parallel smooth lines, the performance of which is well known.

(b) The Velocity of the Charge Is Higher Than the Natural Velocity of the Coil

In this case, represented by figure 13, the moving charge Q is not able to bind any opposite charge on the coil for such charge can move only with a smaller natural velocity v_N and thus cannot follow the high velocity V of the charge Q. If no charge on the coil can be bound, then no current can flow in its turns, and if no current exists in the wires, the coil performs exactly like an insulator, having no conductivity for waves moving with higher

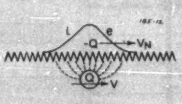


Figure 12 (left). Interaction of a helical coil with a moving charge

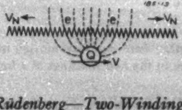


Figure 13 (left). Fast-moving charge influencing a high-voltage winding

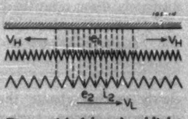


Figure 14 (above). High-voltage winding statically affected by a fast-moving low-voltage wave

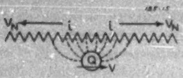


Figure 15 (left). Slow-moving charge influencing a low-voltage winding

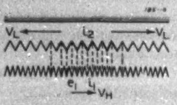


Figure 16 (left). Low-voltage winding screening the field of a slow-moving high-voltage wave

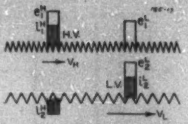


Figure 17 (above). Voltages and currents in high-voltage and low-voltage windings produced by fast-moving and slow-moving waves of natural velocity

velocity than the natural velocity of the coil.

On account of the low natural velocity, any voltage on the coil produced by a faster moving charge is therefore merely a statically induced voltage which cannot flow away from the zone of influence of the charge Q. The coil performs therefore in interaction with a fast-moving charge as if it consisted only of conducting rings not connected with one another, instead of turns of a helical wire. The coil thus appears transparent for the electromagnetic excitation of a fast-moving charge.

These results are transferred to our transformer coils in figure 14. Any wave consisting of voltage e_2 and current i_2 on the L.V. winding and moving with higher velocity v_L than the natural velocity v_H of the H.V. winding can induce within the latter only a static voltage wave e_1 but no current i_1. The H.V. winding is therefore subjected merely to a capacitive voltage division. This winding does not absorb any energy; the entire energy of the wave is rather conducted by the L.V. winding with its high natural velocity. As, under these circumstances, there is no reaction from the H.V. winding back to the L.V. winding, the H.V. winding appears nonexistent from the viewpoint of the L.V. winding and its waves.

(c) THE VELOCITY OF THE CHARGE IS LOWER THAN THE NATURAL VELOCITY OF THE COIL

In this case, as represented by figure 15, the charge Q can bind opposite charges on the coil, as it would do even at rest. However, no voltage can be formed opposite the moving charge since every voltage immediately flows away in both directions with the higher natural velocity v_N of the coil. The winding performs therefore like a metallic layer, conducting in the axial direction. From the viewpoint of the slowly moving charge the coil appears opaque and acts as a perfectly conducting screen. The counter-charges moving with the low velocity V of the charge Q produce currents varying in space and time in the coil. Here exists therefore an intensive magnetic interaction between the slow-moving charge and the coil.

Figure 16 represents this interaction within transformer coils if a low velocity wave propagates with v_H on the H.V. winding. Its voltage e_1 and current i_1 induce only current i_2 at the L.V. winding but not voltage e_2. The L.V. winding appears therefore, from the viewpoint of the H.V. winding and its waves, as perfectly conducting in every direction or as a complete metallic screen. The induced current i_2 is carried along the L.V. winding directly opposite the slow-moving H.V. wave. The entire energy of the electromagnetic field is conducted by the H.V. winding with its low natural velocity.

In order to determine the consequences of this different performance of the windings in transformers, we go back now to our analytical expressions. Figure 17 shows in a comprehensive manner by means of short rectangular waves the voltages and currents which can flow in the windings. Both types of wave, the fast one with the energy concentrated on the L.V. winding and the slow one with the energy concentrated on the H.V. winding, travel independently of each other and form by superposition the entire electromagnetic state of the transformer. We distinguish both wave systems by superscripts L and H on all currents and voltages, and consider first the fast L.V. waves.

With reference to figure 17 we have in the transparent H.V. winding

$$i_1^L = 0 \tag{44}$$

and therefore equation 21 simplifies to

$$e_2^L = Z_2 \cdot i_2^L \qquad e_1^L = z/Z_2 \cdot e_2^L \tag{45}$$

We see that for the L.V. winding there remains effective only the self-surge impedance Z_2, while in the H.V. winding a transmitted voltage appears, the magnitude of which is determined by the ratio of surge impedances given by the second equation 43. The H.V. winding thus has no influence on the performance of the L.V. waves; it suffers only a voltage division according to the coupling factor z/Z_2. From the standpoint of the L.V. winding the H.V. winding appears as air.

For the slow H.V. waves, on the other hand, we have, as in figure 17, within the opaque L.V. winding

$$e_2^H = 0 \tag{46}$$

The second line of equation 21 therefore gives

$$i_2^H / i_1^H = -z/Z_2 \tag{47}$$

by which the current in the secondary winding is determined. If we insert this in the first equation 21, we obtain the primary voltage

$$e_1^H = Z_1 i_1^H - z^2/Z_2 \cdot i_1^H = Z_1^H i_1^H \tag{48}$$

Thus we can define an effective surge impedance of the H.V. winding,

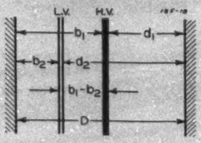
$$Z_1^H = Z_1(1 - z^2/Z_1 Z_2) \tag{49}$$

We see that this is considerably diminished by the reaction of the L.V. winding with respect to the slow-moving H.V. waves.

By introducing into equation 49 the values of equations 42 and 43, we can determine the effective H.V. surge impedance by the dimensions of the windings as

$$Z_1^H = 4\pi v_0 \sqrt{\mu/\epsilon} \cdot N_1/h \cdot d_1(b_1 - b_2)/d_2 \quad (\times 10^{-9} \text{ in ohms}) \tag{50}$$

Figure 18. Distances deciding the effective surge-impedance of the high-voltage winding



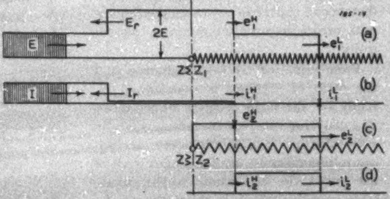


Figure 19 (right). Fast- and slow-moving traveling waves released by an incident high-voltage surge in the two windings of a transformer with open low-voltage terminal

If in figure 18 we examine the distances given in the last fraction of equation 50 and compare them with equation 8 and figure 1, we see that Z_1^H is constituted exactly as if the L.V. winding were replaced by a ground electrode. The L.V. winding therefore reacts intensively on waves in the H.V. winding, in screening their rapid electromagnetic field completely. The effective surge impedance of the H.V. winding is therefore given by the leakage flux of both windings, as indicated already by the form of equation 49. Any influence of the magnetic steel core disappears here completely.

For verification of these results we consider measurements made by H. Norinder[8] with a 450-kva oil transformer for 20 kv/112 v with $b_1-b_2=1.4$ cm insulating distance between the windings. The H. V. winding consisted of 80 sectional coils each with 20 turns, each coil covering an axial length of 0.865 cm. The secondary winding was grounded at the neutral. The dielectric constant can be taken as $\epsilon=3.5$, the permeability as $\mu=1$, and the tank wall was far away. We compute by equation 50 an effective H.V. surge impedance

$$Z_1^H=120\pi/\sqrt{3.5}\cdot 20/0.865\cdot 1.4= 6{,}530 \text{ ohms}$$

This magnitude was measured by cathode-ray oscillograms and was found to be 6,600 ohms, a very close agreement indeed. The wave velocity in the wires was $148m/\mu s$ in accordance with equations 7 and 41. Since the L.V. winding had a distance to the core $b_2=1.4$ cm, the unreduced H.V. surge impedance would be, with $b_1=2.8$ cm, $Z_1=13{,}060$ ohms, which is much higher than the effective as well as the measured value.

The fact that the slow wave within the H.V. winding cannot transmit voltage to the L.V. winding does not mean that no voltage at all appears on the L.V. winding. This would occur only if the L.V. winding were grounded throughout. In many actual cases, however, an incident H.V. surge produces both wave systems at the terminals of the winding, the slow and the fast one, in the primary as well as in the secondary winding and these waves travel further through the windings.

Figure 19 shows the performance of a terminal in a special but important case, to which we will confine ourselves for the sake of simplicity: the surge impedance of the incident H.V. line may be small compared with that of the H.V. winding, while the L.V. winding may be open at the terminal. Then the H.V. terminal voltage, doubled by reflection as in figure 19*a*, is composed of both voltages

$$e_1^L+e_1^H=2E \tag{51}$$

and the L.V. terminal current is, as in figure 19*d*,

$$i_1^L+i_1^H=0 \tag{52}$$

By using equation 45 we can transfer e_1^L into e_2^L and by successive application of equations 48, 47, 52, 45, we can also transfer e_1^H into e_2^L. This gives, with use of equation 49,

$$2E=e_2^L(z/Z_2+Z_1^H/z)=Z_1/z\cdot e_2^L \tag{53}$$

and thus we obtain for the most interesting voltage at the open L.V. terminal

$$e_2^L=z/Z_1\cdot 2E=2\cdot b_2/b_1\cdot\sqrt{N_2/N_1}\cdot E \tag{54}$$

Hence this voltage, transmitted by the travelling wave interaction of the windings, is given by twice the value of the surge coupling ratio and therefore is determined by the square root of the turn ratio and by the ratio of the insulating distances between windings and core. For actual transformers both distances b_1 and b_2 may be corrected with respect to the heterogeneous distribution of ϵ and μ in accordance with the method of section 3.

On account of the importance of the secondary surge voltage, many investigations have been published on transmitted voltages. However, only a few of them are made under such well-determined conditions that they can be compared with equation 54. The table gives results of three experiments[3,5,9] in which a short-front wave impinging on the H.V. winding was transmitted to an open L.V. winding, primary and secondary waves being measured by cathode-ray oscillograms. The characteristics of the transformers are given in the table, so far as they have been published, and estimated values for the distances are added, appropriate to power and voltage. The measured secondary surge voltages are all only slightly smaller than the theoretical values computed by equation 54, and thus the main conceptions underlying our calculations are verified.

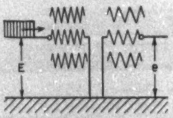
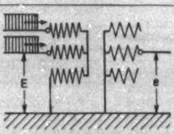
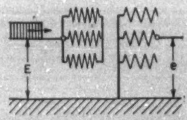
The influence of loading the secondary terminals and the consideration of other terminal conditions, as well as the further behavior of the waves after their production at the terminals, remains to be investigated.

7. Summary and Conclusions

In transformer windings with coils which are large as compared with their insulating distances, the velocity of travelling waves along the conductors of both windings is equal to the velocity of light adapted to the magneto-dielectric constant of the adjacent space. The surge impedance of the windings is determined by the turn density of the coils and the distances of insulation and is in the range of 100 up to 20,000 ohms for power transformers of low voltage and high power up to high voltage and low power.

On the arrival of a surge at the terminals, two different wave fields travel axially through the windings, a slow one coupled with the H.V. coils and a fast one coupled with the L.V. coils, both propagating with light-velocity helically in their respective windings. The H.V. winding appears transparent, or nonexistent, for the axially fast-running

Table I. Comparison of Computed and Measured Secondary Surge Voltages in Transformers

Investigator	Palueff and Hagenguth[3]	Rebhan[5]	Elsner[9]
Connecting diagram at test			
Rated high voltage	220 kv	15 kv	110 kv
Rated 3-phase power	78,000 kva	400 kva	15,000 kva
Turn ratio N_2/N_1	1:7.05	1:37.5	1:5.33
Distances b_2/b_1, estimated	1:2.5	1:2	1:3
Transmitted voltage, e/E, computed	30.1%	16.3%	28.8%
Transmitted voltage, e/E, measured	30%	15%	26%

waves; the L.V. winding appears opaque, or perfectly screening, for the axially slow-running waves.

This mechanism effects a transmission of surge voltages between the two windings entirely different from any steady state transformation. The surge-coupling factors of the primary and secondary windings are determined by the ratio of the insulating distances and by the square root of the turn ratio. For transmission from the H.V. to the L.V. side this coupling factor is larger than the turn ratio.

All the principal results of this paper capable of numerical examination are verified by experiments collected from the previous literature.

Special Symbols

a = wire length of the winding
b = internal insulating distance
d = external insulating distance
f = field at the end of a finite coil
g = mutual potential coefficient per unit length
h = axial length of the entire coil
p = self-potential coefficient per unit length
s = pitch of the helical coil
s = specific resistance
v = velocity of waves
w = wire length of a turn
z = mutual surge impedance
D = distance between core and tank
F = field of an infinite coil
G = mutual potential coefficient
N = number of turns of the winding
P = self-potential coefficient
Z = surge impedance
β = insulating distance across a multi-layer coil

References

1. Contribution Expérimentale à l'Étude des Surtensions dans les Transformateurs, **J. Fallou.** *Bulletin* de la Société Française des Électriciens, 1926, 4e Serie, tome 6, page 237.

2. Transient Oscillations of Mutually Coupled Windings, **L. V. Bewley.** AIEE Transactions, volume 51, 1932, page 299.

3. Effect of Transient Voltages on Power Transformer Design—IV. Transition of Lightning Waves From One Circuit to Another Through Transformers, **K. K. Palueff** and **J. H. Hagenguth.** AIEE Transactions, volume 51, 1932, page 601.

4. Resolution of Surges Into Multivelocity Components, **L. V. Bewley.** AIEE Transactions, volume 54, 1935 (November section), page 1199.

5. Modellversuche zur Wanderwellenübertragung auf die Unterspannungswicklung von Transformatoren, **H. Neuhaus** und **R. Strigel.** *Wissenschaftliche Veröffentlichungen aus den Siemens-Werken*, volume 15, number 1, 1936, page 51.

6. Lightning Protection of Generator Transformer Units, **J. H. Hagenguth.** *Electrical World*, volume 106, 1936, page 327.

7. Neuere Untersuchungen zur Frage der Stossbeanspruchung von Transformatoren, **R. Elsner.** *Archiv für Elektrotechnik*, volume 30, 1936, page 368.

8. Die Transformatoren (a book), **E. Arnold** and **J. L. la Cour.** Third edition, 1936, page 195.

9. Zur Frage der Übertragung von Stossspannungen auf die Unterspannungsseite von Drehstromtransformatoren, **R. Elsner.** *Wissenschaftliche Veröffentlichungen aus aen Siemens-Werken*, volume 16, number 1, 1937, page 1.

10. Die Übertragung von Überspannungen von der Oberspannungs- auf die Unterspannungswicklung von Transformatoren, **M. Wellauer.** *Bulletin des Schweizer Elektrotechnichen Vereins*, volume 30, 1939, page 124.

11. Performance of Travelling Waves in Coils and Windings, **R. Rüdenberg.** AIEE Transactions, volume 59, 1940, page 1031.

12. Electric Oscillations and Surges in Subdivided Windings, **R. Rüdenberg.** *Journal of Applied Physics*, volume 11, 1940, page 665.

13. Electromagnetic Waves in Transformer Coils Treated by Maxwell's Equations, **R. Rüdenberg.** *Journal of Applied Physics*, volume 12, 1941, page 219.

Methods of Determining Natural Frequencies in Coils and Windings

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Synopsis: A knowledge of the natural frequencies of transformer, reactor, and generator windings is essential for the determination of the transient response of such coils to lightning waves and switching surges, as well as for the calculation of the recovery characteristics of circuit breakers and expulsion gaps. Different testing methods have been employed to determine these natural frequencies, and it has usually been taken for granted that each of these different testing procedures gives results in substantial agreement. Recently, however, in a study of transformer transients it was found that several different methods of test failed to agree, and an investigation as to the cause of the discrepancies disclosed that all of these usual methods are not equally valid or reliable.

It is, therefore, the object of this paper to discuss various testing procedures, and to point out their theoretical limitations toward yielding the supposed information.

The voltage distribution along the winding of a grounded neutral transformer is discussed analytically and the basic equations are developed.

Results of voltage distribution tests made by several methods are reported and compared. The various methods of testing are discussed and compared with the conclusion that the method of free oscillation is, in general, preferable.

Introduction

THERE are four general methods for ascertaining the natural frequencies of windings or distributed circuits:

1. Cathode ray oscillograms of free oscillations excited by an impulse generator.
2. Resonance frequency determined by varying the applied frequency so as to cause a maximum or minimum voltage at a selected point of the winding. The applied frequency may be a sustained alternating voltage supplied, for example, from a vacuum tube oscillator, or it may be a damped oscillatory wave furnished by an impulse generator. The voltage may be measured by a vacuum tube voltmeter, a cathode ray oscillograph, or a spark gap.
3. Voltage-current ellipses on the screen of a cathode ray oscillograph as a sinusoidal applied voltage is varied in frequency.
4. Impedance-frequency curves calculated from corresponding voltage and current readings for sustained sinusoidal applied voltages covering a sufficiently wide range of frequencies.

In order that the limitations of each method of test may be examined critically from a theoretical point of view, consider the case of an idealized uniformly distributed circuit having self inductance, capacitance to ground, capacitance between adjacent turns, and energy losses; such that its transient response to an infinite rectangular applied voltage wave (a "unit function") consists of a set of space harmonics, each oscillating at its own natural frequency about a final distribution as an axis of oscillation, and decaying exponentially, as given in equation 1 of appendix I.

It is immaterial whether such an idealized circuit actually exists or not in practice (although it is known to be closely approximated by transformer windings, transmission lines, generator coils, etc.), because if it can be shown that this theoretical circuit cannot yield unquestionable results to a given method of test, then it can hardly be expected that circuits which are not ideal (with respect to pure harmonics) will do so.

Appendix I gives the solution, through an application of Duhamel's theorem, to this same idealized circuit when the applied surge is a damped oscillatory wave, and as a special case thereof for a sustained alternating voltage (zero decrement factor for the applied wave), particularly when in resonance with one of the natural frequencies.

I—Free Oscillations Excited by an Impulse

One method of testing for the transient voltage distribution along the winding of a transformer subjected to an impulse is to apply a surge of the desired shape from an impulse generator and take oscillograms of the voltage to ground at different points along the winding with a cathode ray oscillograph. Figure 1 shows a set of such oscillograms (tracings from the images on the screen of a transient analyzer), taken on a 66 Kv, 3,000 Kva high voltage concentric disk-type coil stack of a transformer from which the low voltage winding had been removed. The applied wave in this case had a very steep front and a long tail (0.2×275).

The voltage from the 50% point shows a predominant fundamental of 41,000 cycles, distorted during the first cycle by higher harmonics. The decrement factor for the fundamental is approximately $\gamma_1 = 0.022$ (where time is measured in microseconds). Assuming that the transient consists of pure sinusoidal space harmonics, the distortion is due to those of odd order—the 3rd, 5th, 7th, etc., and theory shows that their amplitudes rapidly decrease in size as their order increases. The segregation of a higher harmonic, which is not at all pronounced, from a decaying oscillation is not very certain or satisfactory, and considerable error may be involved.

The second space harmonic is a maximum at the 25% and 75% points of the winding, and passes through zero at the midpoint. By connecting the oscillograph between the 25% and 75% points, the second harmonic is doubled in amplitude, the fourth harmonic is zero at both these points, and all odd harmonics cancel out. Thus, this is a strategic connection for the segregation of the second harmonic. The oscillogram of figure 1 shows this harmonic to have a frequency of 105,000 cycles (2.5 times that of the fundamental), and a decrement factor of 0.035. There is very little distortion shown in this oscillogram, but in many transformer tests the 6th harmonic is sufficiently pronounced to be segregated.

If oscillograms are taken from the 25% and 75% points to ground, then the distortion due to the odd harmonics is quite pronounced, as shown in the two bottom oscillograms of figure 1. It is difficult to match these two separate oscillograms with sufficient accuracy to yield, by subtraction, the preceding oscillogram, and, therefore, it is always best to measure between the 25% and 75% points. There are no other strategic points along the winding which will permit a definite segregation of any of the other harmonics. Thus, this method of test does not ordinarily yield good results for other than the fundamental and the second harmonic, but for these two the data are definite and reliable.

The advantages and limitations of this method of test are as follows:

(*a*). *The natural frequencies* of the fundamental and second harmonic are definite and reliable, and their decrement factors can be obtained with reasonable accuracy.

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(*b*). The natural frequencies and decrement factors of some of the remaining harmonics can be roughly obtained, but only in cases where they are pronounced.

(*c*). From oscillograms of the voltages to ground all along the coil stack, the envelope of oscillations, as well as the initial and final distributions, can be obtained as in figure 2. The difference between the initial and final distributions can be analyzed by Fourier's method to determine the amplitudes of all of the space harmonics, and these values may be checked against the amplitudes of the fundamental and second harmonic as determined directly from the oscillograms of figure 1. If an oscillograph is not available these distributions can be obtained by means of a spark gap.

(*d*). The effect of applied wave shape—steepness of front, length of tail, etc.—in reducing or amplifying the oscillations can be studied experimentally, and, therefore, the effects of these factors on voltage stresses throughout the winding can be studied in detail.

II—Resonance With Damped Oscillatory Waves

If a damped oscillatory wave is applied to a winding, and the frequency varied so as to cause the voltage at a selected point of the winding to become a maximum, a condition of resonance is indicated. Figure 3 shows this condition in the neighborhood of the resonance frequencies for the fundamental and second harmonic. The oscillograms for the fundamental and second harmonic at resonance are shown in figure 4. The selected connection for the fundamental was the 50% point to ground, and for the second harmonic between the 25% and 75% points of the winding.

As a check, the envelopes of the oscillations shown in figure 4 can be calculated with fair accuracy from the data of figures 1 and 2. The solution for a damped oscillatory applied wave is given by equation 4 of appendix I. If the applied frequency is equal to one of the natural frequencies, $b=\omega_s$, and, if the decrement factor γ_s is small compared to ω_s, as is the case in practice, then the *dominant* term of the transient is

$$A_s \sin\frac{s\pi x}{l}\,\frac{E}{2}\sqrt{\frac{b^2}{\gamma_s^2}+1}\times\left(\frac{\epsilon^{-at}-\epsilon^{-\gamma_s t}}{a/\gamma_s-1}\right)\sin(bt-\lambda_s)$$

and this has a maximum occurring at

$$t=\frac{\log\ (\gamma_s/a)}{\gamma_s-a}$$

The fundamental has an amplitude, as determined from the oscillogram taken at the 50% point of figure 1, of

$$A_1=y\epsilon^{\gamma_1 t}=0.13\epsilon^{0.022\times 60}=0.49$$

and this agrees reasonably well with the Fourier analysis of figure 2.

The decrement of the applied wave of figure 4 is $a=0.014$. Then

$$\frac{a}{\gamma_s}=\frac{0.014}{0.022}=0.635 \qquad \frac{b}{\gamma_s}=\frac{2\pi\,41{,}000}{22{,}000}=11.7$$

The time at which the envelope of the oscillation reaches maximum is

$$t=\frac{\log 1.575}{0.008}=57$$

and the maximum itself is

$$\frac{0.49}{2}\sqrt{11.7^2+1}\left(\frac{\epsilon^{-0.014\times 57}-\epsilon^{-0.022\times 57}}{0.635-1}\right)=1.27$$

The maximum value of 1.27 at 57 microseconds has been plotted as a "calculated point" on figure 4, and is seen to agree well with the tested value.

The corresponding values for the second harmonic are:

$$A_2=0.22 \qquad \gamma_2=0.035 \qquad a_2=0.035$$

and the calculated maximum is 0.77 occurring at $t=26\mu s$. Between the 25% and 75% points of the winding the maximum is twice as much, or 1.54, which is in fair agreement with the value shown on figure 4.

This method of determining the natural frequency appears to give reasonably accurate results for the first few harmonics. But, as can be seen from equation 4 of appendix I, it is not an exact method because many other terms are present in addition to the "dominant" term. In circuits with high decrement factors the error may be appreciable, particularly for the higher harmonics.

IIb—Resonance With Sustained Oscillations

Instead of applying a damped oscillatory wave, it is usually easier to apply a sustained sinusoidal voltage by means of an oscillator. The frequency can then be varied in the neighborhood of a resonance frequency until a maximum voltage is obtained at some selected point of the winding; or better, complete distributions can be taken along the winding, as in figures 5 and 6, so as to obtain the maximum both in space and in time. Thus, in figure 5 the distribution for an applied frequency of 42 Kc (the frequency giving the maximum envelope), and partial distributions for 40 and 44 Kc are shown. Likewise in figure 6, the distributions for applied frequencies of 100, 112, 117, and 120 Kc are shown.

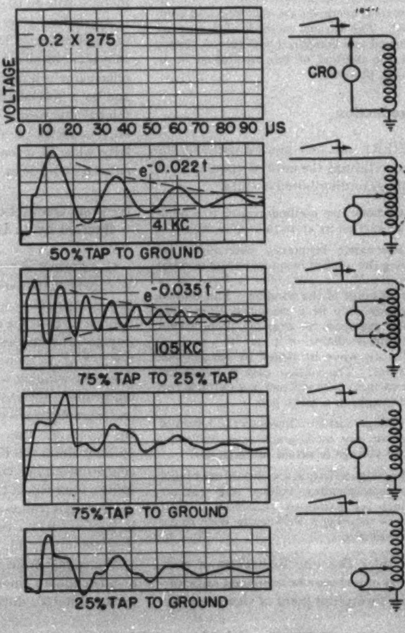


Figure 1. Free oscillations excited by impulse generator

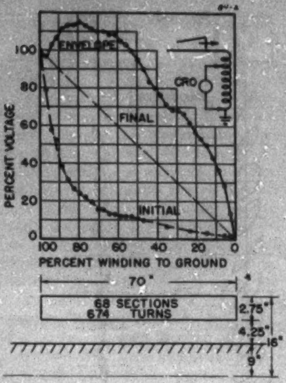


Figure 2. Envelope of oscillations and initial distribution

It is seen that a small departure from resonance causes a very great reduction in the maximum voltage distribution, and the maximum is very sensitive to frequency in the neighborhood of a resonance point.

Equation 8 of appendix I gives the solution for a sustained sinusoidal applied wave, after steady state conditions prevail. Under resonance conditions $b=\omega$, and for the usual conditions of decrement factors small compared to the angular velocities, the solution reduces to equation 10, from which it is seen that the phenomenon consists essentially of an oscillating axis and a dominant harmonic, although all the other space harmonics are present. In transformer windings the contribution of harmonics, other than that under resonance excitation, is practically negligible.

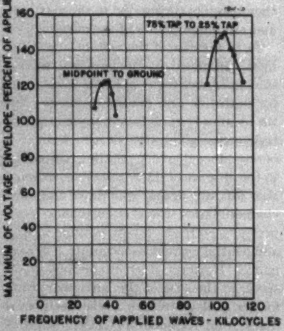


Figure 3. Resonance with damped oscillatory waves

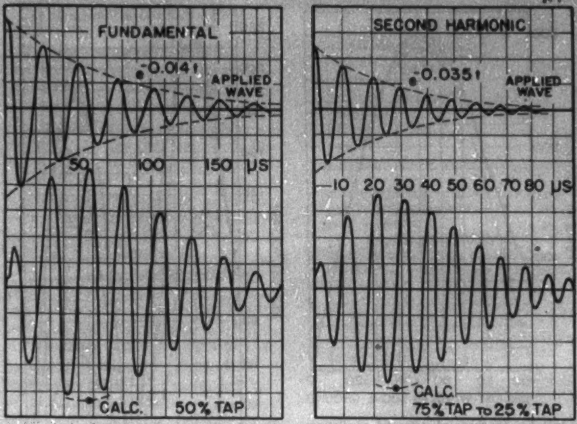


Figure 4. Resonance with damped oscillatory waves

Thus, it is impossible for the circuit to oscillate in a single space harmonic to the exclusion of all other space harmonics, when it is excited in this manner. The other space harmonics contribute to the total voltage and current, and, therefore, it cannot be concluded that a maximum at a point on the winding is an exact condition for true resonance.

It will be seen also from equation 10 that the harmonic oscillations take place about an oscillating axis, and in space this axis is the initial distribution curve. The total maximum voltage to ground at any point, therefore, is composed of two parts: the initial voltage to ground and the harmonic voltage. The amplitude of the latter depends considerably on the decrement factors of the harmonic oscillations. For high decrement factors the amplitude of the harmonic voltage will be low and the initial voltage component will be an appreciable part of the total amplitude.

The equations show that the nodal points of harmonic oscillations are located along a curve represented by the initial voltage distribution. This is indicated in figure 6 where the nodal point for the second harmonic at the midpoint of the winding lies on the initial voltage distribution curve. Also, in figure 5 the envelope of voltages to ground produced by the first harmonic when in resonance is shown by the dashed line. This line is obtained by subtracting the initial voltage distribution curve, figure 1, from the test points obtained for an applied sustained oscillation of 42 Kc.

For the winding here considered:

$$\frac{b_1}{\gamma_1}=\frac{2\pi\,41{,}000}{22{,}000}=11.7$$

$$\frac{b_2}{\gamma_2}=\frac{2\pi\,105{,}000}{35{,}000}=18.9$$

Equation 10 then gives for the amplitudes of the fundamental and second harmonic, respectively:

$$e_1=e(0,x)\cdot\sin b_1t-0.49E\ \sin\frac{\pi x}{l}\times$$
$$(-0.75\sin b_1t+5.85\cos b_1t)+\ldots\ldots$$

$$e_2=e(0,x)\cdot\sin b_2t+0.22E\sin\frac{2\pi x}{l}\times$$
$$(-0.75\sin b_2t+9.45\cos b_2t)+\ldots\ldots$$

The calculated maximum values for these resonant conditions are then

$e_1=2.91$, and $e_2=2.11$

as compared with test values from figures 5 and 6 of 3.00 and 2.20, respectively.

The first term of the equations gives the initial voltage, and as seen in the above equations, this may be an appreciable part of the maximum voltage.

III—Voltage-Current Ellipses

A method sometimes used to determine resonance conditions is to apply a sinusoidal voltage, which is connected to one axis (pair of plates) of a cathode ray oscillograph, and measure the corresponding terminal current on the other axis of the oscillograph. The oscillograph then records the instantaneous voltage-current relationship at the line terminal. According to equations 12 and 13 of appendix I the

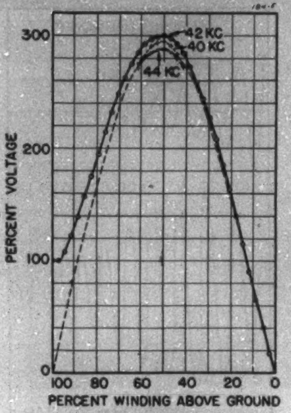


Figure 5. Envelope of sustained oscillations near fundamental resonance

Dashed line = contribution of harmonic oscillations to envelope of maximum voltage at 42 kc

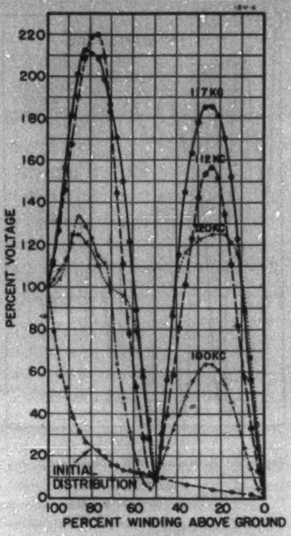


Figure 6. Envelope of sustained oscillations near second harmonic resonance

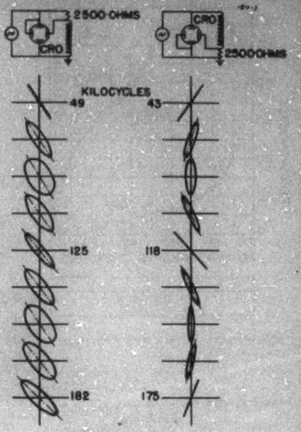


Figure 7. Voltage-current ellipses

steady state voltage and current at the line terminal are of the form:

$$e=E\sin bt,\qquad i=I\sin(bt+\alpha)$$

As shown in appendix II, these are the parametric equations of an ellipse with an inclined major axis. The inclination of the major axis shifts as the applied frequency is varied and the ellipse becomes a straight line for $\alpha=0$ when current and voltage are in phase.

For an applied frequency equal to the harmonic frequency $b=\omega_s$, the dominant terms of the current, according to equation 11 of appendix I are:

$$i\cong bCE\left[\frac{x^2}{2l}\cos bt-\frac{lA_s}{s\pi}\cos\frac{\pi sx}{l}\left(\frac{b}{2\gamma_s}\right)\sin bt\right]$$

The amplitude of the harmonic (A_s) is always less than unity and grows successively smaller with an increase in s (the order of the harmonic). Thus at the line end, $x=l$, the cosine term may be of the same order of importance as the sine term for the fundamental, and of even greater importance for the higher harmonics. However, at the neutral end, $x=0$; the cosine term

$$\frac{x^2}{2l}\cos bt$$

vanishes. This term principally represents the charging current entering the line end, which is dissipated along the winding directly to ground and does not flow through the neutral. The neutral current is

$$i\cong bCE\frac{lA_s}{s\pi}\left(\frac{b}{2\gamma_s}\right)\sin bt$$

Consequently, at resonance, the voltage-current ellipse is practically a straight line for the neutral current, but for the line current resonance occurs under the condition of an open ellipse, although this ellipse may be quite slender.

This conclusion is confirmed by the test results shown in figure 7. As the applied frequency was varied, the inclination of the ellipse shifted back and forth, tending towards a straight line at the points of current resonance and towards an open ellipse at the points of voltage resonance. The swing is between two straight lines on opposite sides of the vertical ellipse for neutral current, because the neutral current for the even harmonics is 180° out of phase with the corresponding part of the line current.

It is difficult to determine, by the slenderness of the ellipse, the exact points of resonance, particularly with line current, since the presence of the cosine terms materially opens up the ellipse. Now there is no assurance that an actual circuit under test will behave in accordance with an idealized circuit, and for an unknown circuit it would be highly unreliable to determine resonance by matching neutral current with line voltage. For these reasons the ellipse method is not recommended as a means for determining resonance, except in those cases where it is known definitely to give a fair indication.

IV—Impedance-Frequency Characteristics

A commonly-accepted method of testing for the natural frequencies of a multi-element circuit is to apply a sustained constant voltage varied over a sufficiently wide range in frequency and measure the current at each frequency. From these data the *impedance* is plotted against the frequency, as in figure 8, and the points of "series" and "parallel" resonance are then taken to correspond to the maximum and minimum points of the curve.

As in the case of the voltage-current ellipses discussed in the previous section, the impedance curve of figure 8 was tested for both line and neutral currents. For the idealized circuit here assumed the neutral current gives more definite indication of resonance than does the line current due to the absence of the x^2 term in equation 9. But for any actual circuit of unknown characteristics, there is no reason to expect any correlation between line voltage and neutral current as a condition of resonance. However, it is interesting to observe, in the case of the particular transformer coil on which these tests were made, that its behavior closely approximates that of the idealized circuit whose analysis is given in appendix I and consequently, for this case, that the neutral current is a more definite indication of resonance than the line current, as seen in figure 8.

If "resonance" is defined as a maximum or minimum terminal impedance, then a curve of this type certainly gives the resonance points. However, if resonance is defined as an oscillation at a natural frequency, then this method does not give an exact result for, referring to equation 9 of appendix I, it is evident that the condition $b=\omega_s$ is not synonymous with a maximum or minimum current except for the case of zero decrement factors. For small decrement factors the approximation may be very good but for normal decrement factors, such as occur in transformer windings, there may be an appreciable shift of the maxima or minima points away from the natural frequencies. To determine the direction of shift, put $b=\omega_s+\Delta$ in equation 9, where Δ is a small deviation from ω_s. Then upon expanding coefficients in series form it becomes apparent that the current is a maximum for positive values of Δ, that is, for applied frequencies somewhat greater than the resonant values; but maximum current is minimum impedance, and, therefore, the minima points of the impedance curve are further to the right than the true resonance points.

In figure 8 the minima points in the vicinity of the fundamental and second harmonics for grounded neutral are at frequencies of 44 and 120 kilocycles, respectively, whereas the true natural frequencies, as given in figure 1, are 41 and 105 kilocycles.

It is also shown in appendix I that the ratio of line terminal voltage to line terminal current under resonant conditions is *not* the surge impedance for the resonant frequency.

Conclusions

Results obtained on a transformer coil by the several methods and their variations are given in the following table.

	Harmonic Frequencies (Kilocycles)		
Method	1st	2nd	3rd
Free oscillations	41	105	Indefinite
Resonance with damped oscillations	40	105	Not tried
Resonance with sustained oscillations	42	112 117 (?)	Not tried
Voltage-current ellipses (neutral current)	43	118	175
Voltage-current ellipses (line current)	49	125	182
Impedance-frequency curve (neutral current)	44	120	180
Impedance-frequency curve (line current)	43	120	180

The agreement is not bad, with the exception of the line-current ellipse method. The ellipse and impedance methods indicate higher values than the oscillation tests and this is in accord with the theoretical reasoning; but the normal testing error by most of these methods is probably five or ten per cent and for the line-current ellipse method perhaps twice as much.

It is believed that the most satisfactory method of test is by free oscillations using an impulse generator and a cathode ray oscillograph, if the frequencies of the fundamental and the second harmonic will suffice. This method gives much more detailed information about the characteristics of the transient than the other methods; however, it fails to give definite information on harmonics higher than the second (unless they are pronounced), and, of course, selected points of the winding must be accessible for connecting to the oscillograph.

The second choice is resonance with sustained oscillations, using an oscillator and a CRO. Here again points of the winding must be accessible, but higher harmonics can be segregated.

If only the terminals of the circuit are accessible, the impedance method using line current gives the most reliable information, although there may be an appreciable deviation of the maxima and minima points from the natural frequencies.

Appendix I

The voltage distribution along the winding of a grounded neutral transformer subjected to an infinite rectangular applied wave ("unit function") is of the form:

$$e(t)=E\left[\frac{x}{l}+\sum_1^\infty A_s\epsilon^{-\gamma_s t}\cos\omega_s t\ \sin\frac{s\pi x}{l}\right] \quad (1)$$

in which

l = total length of winding
x = distance along the winding measured from the neutral
s = order of space harmonic ($s=1, 2, 3, \ldots$)
A_s = amplitude of the s space harmonic
γ_s = decrement factor of the s space harmonic
ω_s = angular velocity of the s space harmonic

According to this equation, the transient voltage distribution consists of an infinite series of space harmonics oscillating at different frequencies, and decaying exponentially, about a fixed linear distribution (x/l).

If, now, the applied surge is a damped sinusoidal wave

$$E(t)=E\epsilon^{-at}\sin bt \quad (2)$$

then the corresponding solution is given by Duhamel's theorem

$$f(t)=E(t)e(o)+\int_0^t E(t-u)\frac{\partial e(u)}{\partial u}\,du \quad (3)$$

Substituting (1) and (2) in (3) and simplifying, there results

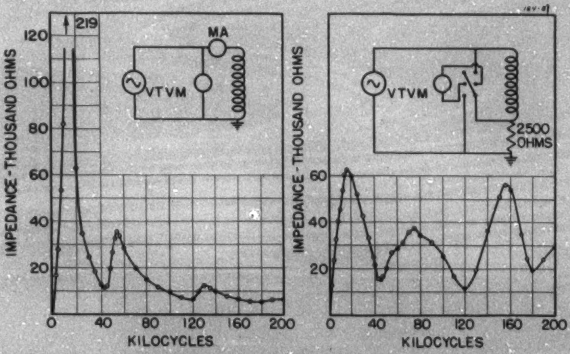
$$\frac{e}{E}=\frac{x}{l}\epsilon^{-at}\sin bt+\sum_1^\infty A_s\sin\frac{s\pi x}{l}\left\{\epsilon^{-at}\times\sin bt+\frac{1}{2}\sqrt{\frac{\gamma_s^2+\omega_s^2}{(a-\gamma_s)^2+(\omega_s+b)^2}}\times\left[\epsilon^{-at}\sin(bt+\psi_s)+\epsilon^{-\gamma_s t}\sin(\omega_s t-\psi_s)\right]+\frac{1}{2}\sqrt{\frac{\gamma_s^2+\omega_s^2}{(a-\gamma_s)^2+(\omega_s-b)^2}}\left[\epsilon^{-at}\sin(bt-\lambda_s)-\epsilon^{-\gamma_s t}\sin(\omega_s t-\lambda_s)\right]\right\} \quad (4)$$

where

$$\psi_s=\tan^{-1}\frac{a\omega_s+b\gamma_s}{\gamma_s(a-\gamma_s)-\omega_s(\omega_s+b)}$$

$$\lambda_s=\tan^{-1}\frac{a\omega_s-b\gamma_s}{\gamma_s(a-\gamma_s)-\omega_s(\omega_s-b)}$$

Figure 8. Impedance-frequency characteristics



It will be noticed from equation 1 that the *initial* voltage, at $t=0$, is

$$e(o)=E\left[\frac{x}{l}+\Sigma A_s \sin\frac{s\pi x}{l}\right] \tag{5}$$

so that the first two terms of (4) may be written

$$\frac{e}{E}=e(o)\epsilon^{-at}\sin bt+\ldots\ldots \tag{6}$$

The corresponding current is

$$i=C\int\frac{\partial e}{\partial t}\,dx=I(t)-CE\frac{x^2}{2l}\sqrt{b^2+a^2}\,\epsilon^{-at}\times$$
$$\sin\left(bt-\tan^{-1}\frac{b}{a}\right)+\sqrt{b^2+a^2}\,CE\sum_1^\infty\frac{lA_s}{s\pi}\times$$
$$\cos\frac{s\pi x}{l}\left\{\epsilon^{-at}\sin\left(bt-\tan^{-1}\frac{b}{a}\right)+\right.$$
$$\frac{1}{2}\sqrt{\frac{\gamma_s^2+\omega_s^2}{(a-\gamma_s)^2+(\omega_s+b)^2}}\left[\epsilon^{-at}\sin\left(bt+\psi_s-\right.\right.$$
$$\left.\tan^{-1}\frac{b}{a}\right)+\sqrt{\frac{\omega_s^2+\gamma_s^2}{b^2+a^2}}\,\epsilon^{-\gamma_s t}\sin\left(\omega_s t-\psi_s-\right.$$
$$\left.\left.\tan^{-1}\frac{\omega_s}{\gamma_s}\right)\right]+\frac{1}{2}\sqrt{\frac{\gamma_s^2+\omega_s^2}{(a-\gamma_s)^2+(\omega_s-b)^2}}\times$$
$$\left[\epsilon^{-at}\sin\left(bt+\lambda_s-\tan^{-1}\frac{b}{a}\right)+\right.$$
$$\left.\left.\sqrt{\frac{\omega_s^2+\gamma_s^2}{b^2+a^2}}\,\epsilon^{-\gamma_s t}\sin\left(\omega_s t-\lambda_s-\tan^{-1}\frac{\omega_s}{\gamma_s}\right)\right]\right\} \tag{7}$$

where $I(t)$ is an integration constant with respect to x (and therefore a possible function of t), whose evaluation depends upon the particular assumptions made regarding the inductance of the winding. It will not be necessary for the present purposes to evaluate $I(t)$.

For a sustained sinusoidal applied wave ($a=0$), and after the transient has died away ($\epsilon^{-\gamma t}\rightarrow 0$), equations 4 and 7 reduce to

$$e(o)\sin bt+\sum_1^\infty A_s\sin\frac{s\pi x}{l}\times$$
$$\left\{\frac{1}{2}\sqrt{\frac{\gamma_s^2+\omega_s^2}{\gamma_s^2+(\omega_s+b)^2}}\sin(bt+\psi_s)+\right.$$
$$\left.\frac{1}{2}\sqrt{\frac{\gamma_s^2+\omega_s^2}{\gamma_s^2+(\omega_s-b)^2}}\sin(bt-\lambda_s)\right\}\ldots\ldots \tag{8}$$

$$i=I(t)+bCE\frac{x^2}{2l}\cos bt-bCE\sum_1^\infty\frac{lA_s}{s\pi}\times$$
$$\cos\frac{sx\pi}{l}\left\{\cos bt+\frac{1}{2}\sqrt{\frac{\gamma_s^2+\omega_s^2}{\gamma_s^2+(\omega_s+b)^2}}\times\right.$$
$$\cos(bt+\psi_s)+\frac{1}{2}\sqrt{\frac{\gamma_s^2+\omega_s^2}{\gamma_s^2+(\omega_s-b)^2}}\times$$
$$\left.\cos(bt-\lambda_s)\right\}\ldots\ldots \tag{9}$$

If the applied frequency is in resonance with some particular harmonic ($b=\omega$), then (8) and (9) give, for γ_s small compared with ω_s,

$$e\cong e(o)\sin bt+EA_s\sin\frac{s\pi x}{l}\times$$
$$\left(-\frac{3}{4}\sin bt+\frac{b}{2\gamma_s}\cos bt\right)+$$
$$\Sigma[\text{remaining harmonic terms of (8)}] \tag{10}$$

$$i=I(t)+bCE\frac{x^2}{2l}\cos bt-bCE\frac{lA_s}{s\pi}\times$$
$$\cos\frac{s\pi x}{l}\left(\frac{1}{4}\cos bt-\frac{b}{2\gamma_s}\sin bt\right)+$$
$$\Sigma[\text{remaining harmonic terms of (9)}] \tag{11}$$

It is clear from these equations that the steady-state of the circuit to a sustained sinusoidal applied wave in resonance with one of the natural frequencies of the circuit is by no means restricted to the space harmonics corresponding to that particular frequency. It is only under the condition of zero losses ($\gamma_s=0$) that the resonant term predominates to such an extent as to create the illusion that it is the only harmonic present.

At the line end ($x=l$), (10) and (11) give

$$e=E\sin bt \tag{12}$$

$$i=I(t)+bCE\frac{l}{2}\cos bt-bCE\frac{lA_s}{s\pi}\times$$
$$\cos s\pi\left(\frac{1}{4}\cos bt-\frac{b}{2\gamma_s}\sin bt\right)+$$
$$\Sigma[\text{remaining harmonic terms of (9)}] \tag{13}$$

Thus at the line end, e and i are substantially in phase (for $b/2\gamma_s$ large) and not 90 degrees out of phase as might be suspected under resonant conditions. It will be observed that

$$\left.\frac{e}{i}\right|_{x=l}\neq\text{(surge impedance for the resonant harmonic)} \tag{14}$$

At the neutral end ($x=0$)

$$i=I(t)-bCE\frac{lA_s}{s\pi}\left(\frac{1}{4}\cos bt-\frac{b}{2\gamma_s}\sin bt\right)+$$
$$\Sigma\text{(harmonic terms)} \tag{15}$$

from which it is clear on comparing (13) and (15) that the current is not completely symmetrical at the two ends of the winding; although in the case of small losses the dominating terms might make it appear to be so.

For an applied frequency which is small compared to that of any of the natural frequencies ($\omega_s\gg b$), so that b may be neglected in comparison with ω_s, there is $\lambda_s\cong 180°$ and $\psi_s\cong 180°$ and (8) then reduces to

$$e=E\frac{x}{l}\sin bt \tag{16}$$

Thus at 60 cycles the distribution is substantially linear.

Appendix II

For a sinusoidal applied terminal voltage

$$e=E\sin\omega t \tag{1}$$

The steady-state terminal current, according to equation 9 of appendix I, is of the form

$$i=I\sin(\omega t-\alpha) \tag{2}$$

Eliminating ωt between (1) and (2)

$$\left(\frac{i}{I}\right)^2+\left(\frac{e}{E}\right)^2-2\cos\alpha\left(\frac{i}{I}\right)\left(\frac{e}{E}\right)=\sin^2\alpha \tag{3}$$

and for a given value of α this can be plotted as a curve having (i) and (e) as coordinates. If the equation is transformed to a new set of axes, i', e' by rotation of axes through an angle β such that

$$i=i'\cos\beta-e'\sin\beta \tag{4}$$

$$e=e'\cos\beta+i'\sin\beta \tag{5}$$

then equation 3 becomes

$$\left[\frac{\cos^2\beta}{I^2}+\frac{\sin^2\beta}{E^2}-\frac{\cos\alpha\sin 2\beta}{EI}\right]i'^2+$$
$$\left[\frac{\sin^2\beta}{I^2}+\frac{\cos^2\beta}{E^2}+\frac{\cos\alpha\sin 2\beta}{EI}\right]e'^2+$$
$$\left[\frac{\sin 2\beta}{E^2}-\frac{\sin 2\beta}{I^2}-\frac{2\cos\alpha\cos 2\beta}{EI}\right]e'i'=\sin^2\alpha \tag{6}$$

This is the equation of an ellipse if the coefficient of the $e'i'$ term can be made to vanish by a suitable choice of β. The condition is met by

$$\tan 2\beta=\frac{2EI}{I^2-E^2}\cos\alpha \tag{7}$$

Thus (1) and (2) define an ellipse, the inclination of whose axes are given by (7).

In particular, if $\alpha=90°$

$$\left(\frac{i}{I}\right)^2+\left(\frac{e}{E}\right)^2=1 \tag{8}$$

and this is an ellipse whose axes coincide with the coordinate axes. If $\alpha=0°$,

$$e=\frac{E}{I}i \tag{9}$$

which is the equation of a straight line with a slope of

$$\Theta=\tan^{-1}\left(\frac{E}{I}\right) \tag{10}$$

Now since the current will either lag or lead the voltage by 90°, the angle is restricted to the range ($-90°\leq\alpha\leq 90°$), for which $\cos\alpha$ is always positive; and therefore by (7) the angle β varies from a negative value (when $I<E$) to a positive value (when $I>E$), and at the two extremes ($\alpha=0$) the ellipse degenerates to a straight line. Thus, as the frequency is varied over a wide range, nodes and anti-nodes are encountered successively (that is, $I<E$ and $I>E$); and correspondingly the axis of the ellipse swings back and forth between the straight lines to which it degenerates. This is indicated by the test results in figure 7.

Protection of Low-Voltage Circuits by Air Circuit Breakers in Cascade Arrangement

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Synopsis: This paper deals with and outlines a new solution to the problem of providing adequate short circuit protection for low-voltage distribution systems and circuits operating at 600 volts or less.

Recent tests and field experience have dispelled the illusion that protective devices of very limited interrupting ability will satisfactorily protect *any* low-voltage circuit. A recently published calculating procedure makes possible the accurate determination of the short circuit currents obtainable in low voltage circuits or systems. Modern types of low-voltage air circuit breakers in a wide range of interrupting ratings are now available. While the best short circuit protection is obtained by selecting those of interrupting ratings at least equal to the calculated short circuit current, such a procedure is not always justifiable.

In many instances a relatively large number of main feeder circuits, emanating from a bus fed through one or more main breakers or incoming line breakers, supply an even larger number of branch feeders. The short circuit currents obtainable at the branch-feeder breaker locations may not be materially less than those obtainable at the main bus. Yet the selection of branch feeder breakers, or possibly even main feeder breakers, with interrupting ratings high enough to match these obtainable currents, may prove completely impractical when viewed with any reasonable regard for space requirements or cost.

Air circuit breakers of varying interrupting ratings, properly coordinated in design may be used in a "cascade" or series connection in such manner that in event of a severe fault, the main breakers "back-up" the smaller main feeder breakers, and these, in turn, "back-up" the branch feeder breakers. Only the main breakers need have interrupting ratings equal to the obtainable short-circuit current, yet the system is adequately protected, within the limits set forth in the paper.

Cascading also makes possible a simple means of selecting air circuit breakers to provide economical yet adequate protection for relatively small installations, where a complete short-circuit study might be difficult to justify and where a poor selection of protective devices would result in either an unnecessarily expensive installation or a serious hazard—generally the latter. An application table for such installations, based upon the principle of cascade operation is included.

Introduction

RECENT papers[1,2] on the subject of short circuits in low-voltage a-c systems (600 volts or less) have dispelled prevalent illusions with respect to faults in low-voltage systems and low-voltage gear. The first reference[1] states that faults in typical low-voltage bus structures are not self-clearing and that fault arcs do not show the drastic current-limiting effect which they were previously assumed to have exerted upon short circuits in low-voltage systems. The magnitude of fault currents obtained in this series of tests showed the fallacy of applying arbitrary and large discount factors to the calculated "bolted fault" short circuit current values in low-voltage systems. This justifies the present practice of determining the short circuit current on the basis of a "bolted fault."

The second reference[2] outlines a procedure for calculating the short circuit current for "bolted faults." The accuracy of the method has been verified by calculation and subsequent measurement of short circuit currents extending to values beyond 100,000 amperes. (Reference 1 and appendix *A* show typical examples.)

When using the calculating procedure of reference 2 for determining the interrupting duty imposed upon low-voltage air circuit breakers, the following specific steps are generally all that need be considered:

Determination of Short-Circuit Duty Placed on Air Circuit Breakers

A.—Low-Voltage Alternating-Current Systems

The maximum interrupting duty is assumed to occur 0.5 cycle from the inception of the fault. During this interval there is a decay of both the a-c and d-c components. For the various kinds of apparatus here used it has been possible to determine a factor *N* which represents the average decay of the sub-transient portion of the a-c component during this interval. Similarly, a factor *D* has been determined to express the average effect of the d-c component. This factor may be used to increase the calculated symmetrical current (a-c component) to obtain the average of the three unsymmetrical, available currents. Thus,

$$i_{0.5\ \text{cycle}} = Di_{ac} \qquad (1)$$

where i_{ac} is the a-c component.

However, the a-c component at any time, *t*, during the first few cycles, may be represented by:

$$i_{ac} = i'' - N(i'' - i') \qquad (2)$$

where,

i'' represents the maximum initial value of the a-c component, which is determined by the sub-transient reactance and other circuit impedance, and

i' represents the current determined by the transient reactance, and other circuit impedance. Hence,

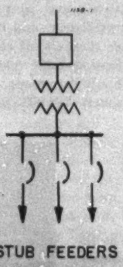

Figure 1. One-line diagram of low-voltage installation of limited capacity

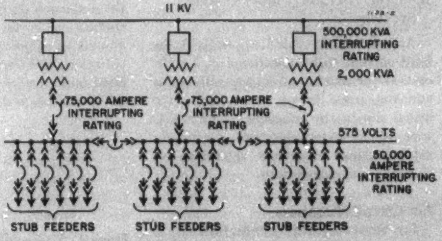
11 KV
500,000 KVA INTERRUPTING RATING
2,000 KVA
75,000 AMPERE INTERRUPTING RATING
75,000 AMPERE INTERRUPTING RATING
575 VOLTS
50,000 AMPERE INTERRUPTING RATING
STUB FEEDERS
STUB FEEDERS
STUB FEEDERS

Figure 2. One-line diagram of extensive low-voltage installation

Calculated short-circuit current (at 575 volts) approximately 100,000 amperes, at 0.5 cycle

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[1] For all numbered references, see list at end of paper.

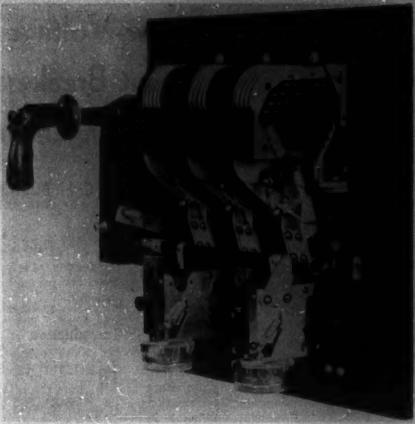


Figure 3. Manually operated air circuit breaker, 3-pole, 15,000-ampere interrupting rating, with arc-quencher side removed to show construction

$(i''-i')$ represents the sub-transient portion of the a-c component which diminishes exponentially.[2]

For most rotating apparatus, the average ratio of $i''/i'=1.5$ while the factor N is approximately 0.2, at 0.5 cycle.[2] The term $N(i''-i')$ at 0.5 cycle, then becomes approximately $0.07i''$.

Since, at 0.5 cycle the increase in short circuit current, obtainable under load conditions (due to increased excitation) is of the order of 5%, the second term in equation 2, indicating a decrease of the order of 7%, is largely offset and can be omitted for all practical purposes, resulting in,

$$i_{0.5\text{ cycle}}=Di'' \tag{3}$$

For air circuit breakers applied to single or three-phase circuits supplied by transformers or rotating apparatus, the factor D at 0.5 cycle, equals 1.25. Accordingly, for rotating apparatus, the total rms current at 0.5 cycle, becomes:

$$i_{0.5\text{ cycle}}=1.25i'' \tag{4}$$

For transformer banks connected to relatively large systems (see appendix *B*), practically the entire limiting impedance is found in the transformer, permitting equation 4 to be written as:

$$i_{0.5\text{ cycle}}=1.25i \tag{5}$$

where i is the a-c component, determined by the transformer impedance.

To the currents determined by equations 4, 5, must be added the short circuit current contributed by induction and synchronous motors. This current (including both a-c and d-c components but allowing for the effect of typical motor circuit impedance) at 0.5 cycle, is approximately 6 times rated current for synchronous motors and 4.5 times rated current for induction motors. (See appendix *B*, tables II and III.)

B.—LOW-VOLTAGE DIRECT-CURRENT SYSTEMS

As indicated in table I, appendix *B*, the fault current to be considered in d-c circuits is the maximum current obtainable and not some lower value during the initial rise in the transient. The resistance of the external circuit, frequently negligible in a-c systems, should be included in the calculations.

Air Circuit Breakers for Short-Circuit Protection

In the past, oil circuit breakers had commonly been used on low-voltage a-c systems. Because of smaller space requirements, lower cost, and faster interruption, the present practice is to apply air circuit breakers to such systems.

When there is a single circuit breaker between the source and load (figure 1) the interrupting rating of the circuit breaker must be at least equal to the fault current obtainable in the circuit. Air circuit breakers of adequate interrupting rating may be selected from table I, appendix *B*.

In low-voltage a-c systems of limited capacity, such as illustrated in figure 1, the circuit breakers are relatively few in number and the calculated short circuit current usually falls within the lower interrupting ratings given in table I. Hence the selection of air circuit breakers for adequate short circuit protection is a simple matter.

As the system capacity increases, the selection of air circuit breakers presents a more difficult problem. Figure 2 illustrates such a system, where, despite the influence of relatively long runs of connections to the transformers and the absence of appreciable short circuit contribution from the motor load, the calculated fault current on a feeder circuit, at 0.5 cycle reached almost 100,000 amperes. Application of air circuit breakers of 100,000 ampere interrupting rating throughout, would result in an equipment of high cost and occupying more space than available or justifiable in the average installation.

An alternative sometimes suggested is to use smaller air circuit breakers, but to render their tripping means inoperative whenever the short circuit current exceeds their interrupting ratings, thus supposedly "locking-in" the smaller breakers and transferring the severe interrupting duty to some larger or "group" circuit breaker. All modern air breakers are inherently and instantaneously trip-free and thus would be difficult to "lock-in," even though their mechanisms and contacts were made much larger, heavier, and more costly to permit them to close and hold their contacts closed against the strong opening forces produced by the excessive current. Many air circuit breakers could not withstand, for any appreciable interval, the heating losses of as much as 40,000 times normal that would result. For these and many other cogent reasons, this scheme may be dismissed as totally impractical.

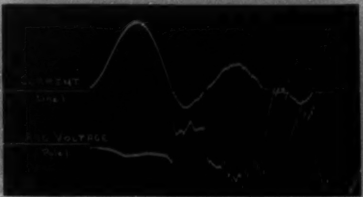


Figure 4. Low-voltage air circuit breaker (15,000-ampere interrupting rating) on verge of failing to interrupt short-circuit current considerably in excess of rating

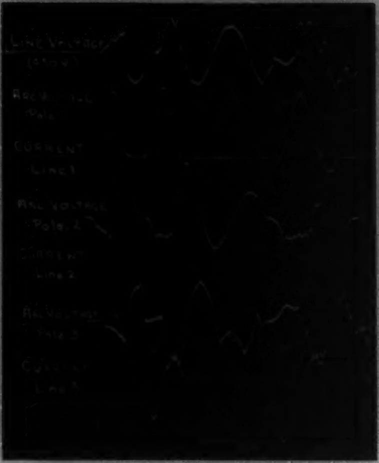


Figure 5. Low-voltage air circuit breaker (25,000-ampere interrupting rating) failing to interrupt satisfactorily short-circuit current exceeding 200 per cent of rating

Limitation of the short circuit current by sectionalizing or by the use of reactors provides a solution which is feasible under some conditions, but consideration must be given to the possible effect on cost, voltage regulation, and space requirements.

A generally more practical solution of the problem is the use of air circuit breakers in "cascade." "Cascading" consists in the use of air circuit breakers of interrupting ratings less than the calculated short circuit current but instead of being "locked-in," these breakers are allowed to trip and are assisted or "backed-up" by one or two larger breakers, the one nearest the source of power having a current interrupting rating at least equal to the calculated short circuit current. In figure 11 circuit breakers (*A*) (*B*) and (*C*) are in cascade. This scheme permits the use of air circuit breakers having lower over-all cost and occupying less space than a layout of breakers having full interrupting rating throughout.

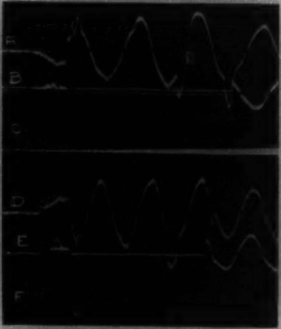


Figure 6. Low-voltage air circuit breakers of 25,000-ampere and 100,000-ampere interrupting ratings operating in series, in circuit calibrated for 89,000 amperes, short-circuit current

Curves *A*, *D*—Arc voltages, number 2 and number 3 poles of 25,000-ampere breaker

Curves *B*, *E*—Arc voltages, number 2 and number 3 poles of 100,000-ampere breaker

Curves C, *F*—Currents, lines 2 and 3

Actual rms values average 62,000 amperes

Performance Required of Breakers to Be Cascaded

It must not be assumed that *any* combination of air circuit breakers will operate effectively in cascade arrangement. They must be carefully and properly selected. For example, tests have revealed that—of the types of breakers with which the authors are familiar—the so-called "thermal" breakers, when subjected to currents considerably beyond their ratings, may clear the circuit and yet, if the current is too great, be destroyed during the first peak of 60-cycle current. Some users have been willing to accept this hazard, using larger breakers to protect the circuit only and recognizing the impracticability of preventing damage to the "thermal" breakers. This, however, is not considered as true "cascading," where one of the principles is that no breaker shall be damaged beyond repair, nor shall any breaker constitute a potential hazard to the safety of personnel or adjacent equipment.

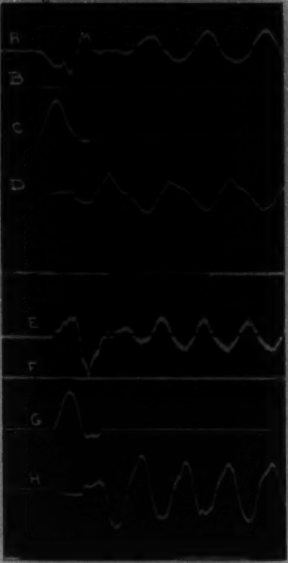


Figure 7. Low-voltage air circuit breakers of 25,000-, 50,000-, and 100,000-ampere interrupting ratings operating in cascade connection, in circuit calibrated for 89,000 amperes, short-circuit current

Curves *A*, *E*—Arc voltages, (*C*) breaker, 25,000-ampere rating

Curves *B*, *F*—Arc voltages, (*A*) breaker, 100,000-ampere rating

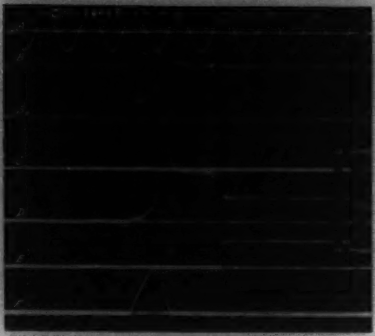
Curves C, G—Currents, lines 2 and 3

Actual rms values average 45,000 amperes

Curves *D*, *H*—Arc voltages, (*B*) breaker, 50,000-ampere rating

The authors, therefore, confine their recommendations for "cascading" to air circuit breakers of the conventional "magnetic" type (figure 3) somewhat larger and of greater thermal capacity than the corresponding "thermal" breakers. Although based upon breakers of the type with which the authors are most familiar[3] the following requirements may serve as a guide for the selection of any breakers having characteristics that permit their use in "cascade."

The smaller breakers (such as *C* in figure 11) must meet the following requirements throughout the range of short-cir-



Available short-circuit current 70,000 amperes

Generators number 5, 6, 9, and 10

Curve *A*—Timing wave, 60 cycles

Curve *B*—Line voltage, 250 volts

Curve *C*—Voltage across station breaker

Curve *D*—Voltage across breaker number 9

Curve *E*—Voltage across breaker number 15

Curve *F*—Short-circuit current, 28,000 amperes actual

Figure 8. D-c interrupting test on low-voltage air circuit breaker number 9 (15,000-ampere) left pole and breaker number 15 (50,000-ampere) left pole

cuit currents for which they are to be recommended:

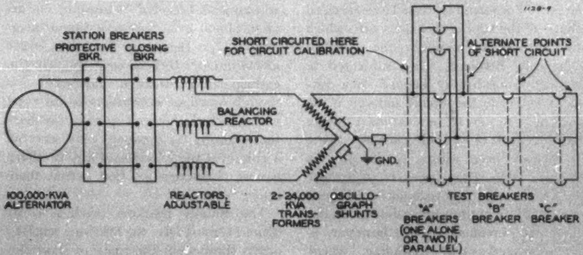
1. They must be capable of mechanically withstanding the "impact" resulting from the electro-magnetic forces set up by the excessive current. This is much more readily accomplished if these breakers have a quick-acting instantaneous tripping means [see reference (*a*) below table I, appendix *B*].

2. Having a quick-acting tripping mechanism, they should not otherwise be restrained from separating their contacts nor from transferring the current from contacts to interrupting unit. Undue delay may cause serious damage, but rapid current transfer, preferably accelerated by the electro-magnetic forces previously referred to, will transfer the burning from the contacts to the arc runners (see figure 3) where even a considerable amount of burning will not impair subsequent current-carrying ability.

3. Their interrupting units must be capable of withstanding arcing for such time as the "back-up" breakers (such as (*B*) in figure 11) may require to interrupt the current.

Representative tests depicted in figures 4 and 5, show that the foregoing requirements can be met. Arc voltage records show that the main contacts are out of the circuit in substantially less than one cycle and—even though the currents were such that these breakers, operating alone, were on the verge of failing to interrupt—the contacts were not seriously damaged. The decided current-limiting tendencies of the arc quenchers can be seen, as can also the temporary interruption of current. Circuit interrupting failures of air circuit breakers are not always—as has been commonly supposed—characterized by prolonged, continuous arcing without appreciable diminution of current.

Figure 9. Diagram of connections for cascade tests on low-voltage air circuit breakers (a-c tests)



Figures 4 and 5 give a picture of what might be called the "mechanism of failure" of the smaller sizes of modern breakers. It can be seen that, while "back-up" breakers (such as (*B*) in figure 11) interrupting during the first cycle of current might share the interrupting duty with the (*C*) breakers, such rapid operation would impose much greater duty on the "back-up" breakers than would operation during the subsequent cycle. In fact, figure 5 indicates a strong probability that the "back-up" breaker might merely serve to remove potential from the protected circuit, without interrupting any current whatever.

Since the "back-up" breakers must protect not only the circuit but also the smaller breakers, the following requirements are indicated for the (*A*) or (*B*)* breakers of figure 11:

1. They must trip whenever the current reaches a value equal to or slightly less than the interrupting rating of the smaller breakers next in line.

2. They must completely interrupt this current (or, in the case of the (*B*) breakers, must at least reduce its magnitude) just rapidly enough to avoid serious damage to the smaller breakers, yet not so rapidly as to participate unnecessarily in the actual interruption of current, thereby requiring unnecessarily increased maintenance. A reasonable and effective solution is that the "back-up" breaker need not actually separate its contacts in less than the normal interrupting time of the smaller breaker next in line, so long as the current does not greatly exceed the rating of the smaller breaker. However, the operating time should become proportionately shorter at higher currents in order to obtain better circuit protection.

Fortunately, a "back-up" breaker of the next larger or even the second larger size (see table I, appendix *B*) generally meets this requirement. A breaker more than two sizes larger would introduce an unnecessarily long time element, as shown in figure 6, where a 100,000 ampere breaker operates in series with the 25,000 ampere breaker of figure 5. Although this large "back-up" breaker did prevent "restriking," it was not rapid enough to assure satisfactory protection had the 25,000 ampere breaker not functioned quite so satisfactorily.

Much better protection is obtained (in the same circuit) when a 50,000 ampere breaker is inserted, as in figure 7. Here, although the "back-up" breaker still performs no appreciable interrupting duty, it now assures complete clearing of the circuit in one cycle instead of 3 cycles. Incidentally, it may be noted that the 100,000 ampere breaker did not receive an impulse of sufficient magnitude or duration to cause it to trip. This was due, at least in part, to some current limiting action resulting from the addition of the 50,000 ampere breaker to the circuit. Although the circuit was the same, the actual currents were reduced from approximately 62,000 amperes (figure 6) to approximately 45,000 amperes (figure 7).

Similar results in 250 volt direct-current circuits are typified by figure 8, where a 50,000 ampere breaker backs up a 15,000 ampere breaker in a 70,000 ampere circuit, limiting the actual current to 28,000 am-

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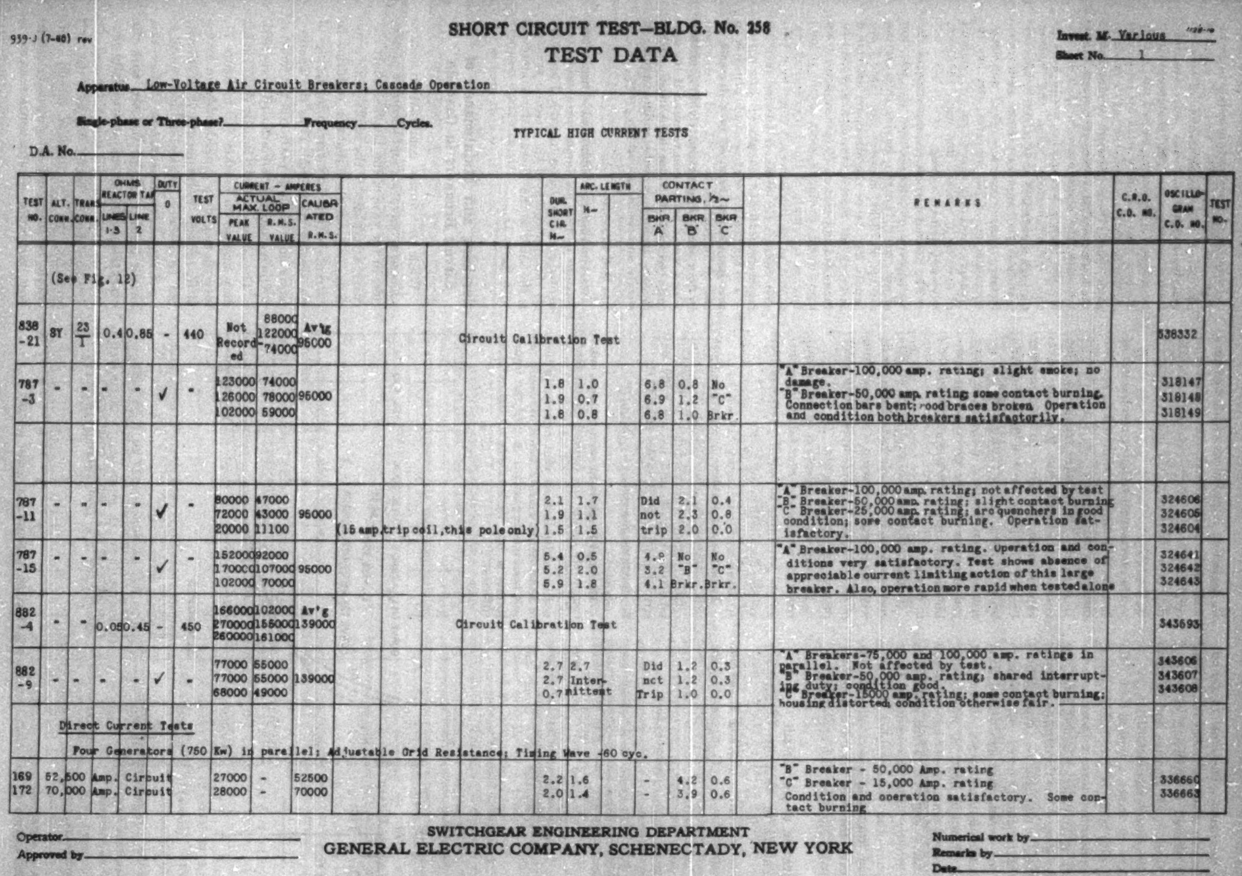
*The (*B*) breakers, when operating at currents beyond their ratings must also meet the requirements previously outlined for the (*C*) breakers.



939-J (7-40) rev

SHORT CIRCUIT TEST—BLDG. No. 258
TEST DATA

Invest. M. Various

Sheet No. 1

Apparatus Low-Voltage Air Circuit Breakers; Cascade Operation

Single-phase or Three-phase? ______ Frequency ______ Cycles.

TYPICAL HIGH CURRENT TESTS

D.A. No. ______

TEST NO.	ALT. CONN.	TRANS. CONN.	OHMS REACTOR TAP LINES 1-3	OHMS REACTOR TAP LINE 2	DUTY 0	TEST VOLTS	CURRENT – AMPERES ACTUAL MAX. LOOP PEAK VALUE	CURRENT – AMPERES ACTUAL MAX. LOOP R.M.S. VALUE	CURRENT – AMPERES CALIBRATED R.M.S.		DUR. SHORT CIR. ~	ARC. LENGTH ~	CONTACT PARTING, ½~ BKR. "A"	CONTACT PARTING, ½~ BKR. "B"	CONTACT PARTING, ½~ BKR. "C"	REMARKS	C.R.O. C.O. NO.	OSCILLOGRAM C.O. NO.	TEST NO.
	(See Fig. 12)																		
838 -21	8Y	23/1	0.4	0.85	-	440	Not Recorded	88000 122000 -74000	Av'g 95000	Circuit Calibration Test								338332	
787 -3	-	-	-	-	✓	-	123000 126000 102000	74000 78000 59000	95000		1.8 1.9 1.8	1.0 0.7 0.8	6.8 6.9 6.8	0.8 1.2 1.0	No "C" Brkr.	"A" Breaker-100,000 amp. rating; slight smoke; no damage. "B" Breaker-50,000 amp. rating; some contact burning. Connection bars bent; wood braces broken Operation and condition both breakers satisfactorily.		318147 318148 318149	
787 -11	-	-	-	-	✓	-	80000 72000 20000	47000 43000 11100	95000	(15 amp trip coil, this pole only)	2.1 1.9 1.5	1.7 1.1 1.5	Did not trip	2.1 2.3 2.0	0.4 0.8 0.0	"A" Breaker-100,000 amp. rating; not affected by test "B" Breaker-50,000 amp. rating; slight contact burning "C" Breaker-25,000 amp. rating; arc quenchers in good condition; some contact burning. Operation satisfactory.		324606 324605 324604	
787 -15	-	-	-	-	✓	-	152000 170000 102000	92000 107000 70000	95000		5.4 5.2 5.9	0.5 2.0 1.8	4.9 3.2 4.1	No "B" Brkr.	No "C" Brkr.	"A" Breaker-100,000 amp. rating. Operation and conditions very satisfactory. Test shows absence of appreciable current limiting action of this large breaker. Also, operation more rapid when tested alone		324641 324642 324643	
882 -4	-	-	0.05	0.45	-	450	166000 270000 260000	102000 155000 161000	Av'g 139000	Circuit Calibration Test								343693	
882 -9	-	-	-	-	✓	-	77000 77000 68000	55000 55000 49000	139000		2.7 2.7 0.7	2.7 Intermittent	Did not Trip	1.2 1.2 1.0	0.3 0.3 0.0	"A" Breakers-75,000 and 100,000 amp. ratings in parallel. Not affected by test. "B" Breaker-50,000 amp. rating; shared interrupting duty; condition good. "C" Breaker-15000 amp. rating; some contact burning; housing distorted; condition otherwise fair.		343606 343607 343608	
	Direct Current Tests Four Generators (750 Kw) in parallel; Adjustable Grid Resistance; Timing Wave -60 cyc.																		
169 172	52,500 Amp. Circuit 70,000 Amp. Circuit						27000 28000	- -	52500 70000		2.2 2.0	1.6 1.4	- -	4.2 3.9	0.6 0.6	"B" Breaker - 50,000 Amp. rating "C" Breaker - 15,000 Amp. rating Condition and operation satisfactory. Some contact burning		336660 336663	

Operator ______

Approved by ______

SWITCHGEAR ENGINEERING DEPARTMENT
GENERAL ELECTRIC COMPANY, SCHENECTADY, NEW YORK

Numerical work by ______

Remarks by ______

Date ______

Figure 10

peres, maximum. The same 15,000 ampere breaker when used alone in the same circuit allowed a maximum current of more than 30,000 amperes and failed to clear the circuit satisfactorily.

Figure 10 shows a typical test data sheet, with some of the higher current tests summarized thereon to indicate the current magnitudes obtained, and the manner of recording the data, as applying to one of the representative combinations of breakers actually tested. Other varieties, not shown, were substituted from time to time during the tests. The a-c test circuit (shown in figure 9) was capable of delivering currents as high as 140,000 total r.m.s. amperes. These data supplemented by visual observation of these and many other tests establish the possibility of "cascading" air circuit breakers and pave the way toward determining reasonable limits for such applications.

Application of Air Circuit Breakers in Cascade Connection

Using the current available at the line terminals of breaker (*A*) (figure 11) (that is, neglecting impedances of all breakers and their connections), the maximum combination of three breakers in cascade may be applied up to the limits indicated in the table.

Breaker	Application Limit
(*A*)	Calculated short circuit duty placed on air circuit breaker nearest source must be within its published interrupting rating
(*B*)	Calculated short circuit duty placed on second air circuit breaker must be within 200% of its published interrupting rating
(*C*)	Calculated short circuit duty placed on third air circuit breaker must be within 300% of its published interrupting rating

The overcurrent tripping mechanism of breaker (*A*) must be adjusted to operate instantaneously when the total current through any (*B*) breaker reaches a value equal to or slightly less than the interrupting rating of (*B*). In this manner breaker (*B*) takes care of fault currents up to its interrupting rating, without assistance from breaker (*A*). The same consideration must be given to the combination of circuit breakers (*B*) and (*C*) in cascade.

Combinations of two and three air circuit breakers in cascade have been used, with successful operation. A maximum of three circuit breakers appears to meet present day requirements.

To assist in the selection of air circuit breakers in cascade, the following comments pertaining to specific types of low voltage installations are given:

1. Systems Supplied by Single Transformer Banks

For these frequently encountered, and relatively simple installations, a practical tabulation is given and its use fully described in appendix *B*, tables II and III.

2. Systems Supplied by Multiple Sources

The larger breakers in such systems must completely isolate the faulted circuit from *all* sources of short circuit current. It cannot be safely assumed that mere reduction of short circuit current (such as would result from opening only 1 or 2 of the transformer or bus tie breakers of figure 2) is sufficient.

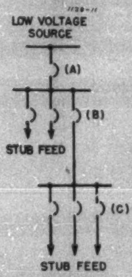


Figure 11. (above) Cascade connections

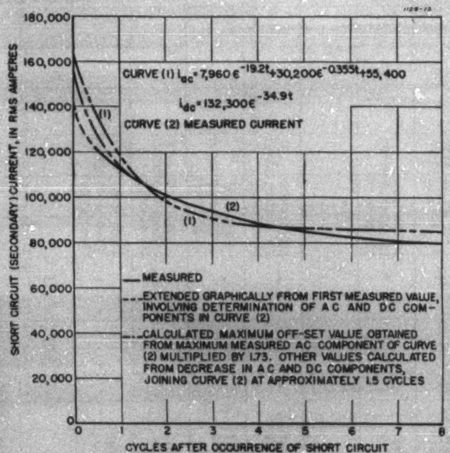


Figure 12. (right) Calculated and measured short-circuit currents obtained in a 440-volt circuit, producing maximum rms values in excess of 100,000 amperes

In applying the cascade principle to the installation illustrated in figure 2, where a calculated short circuit current of approximately 100,000 amperes was available, feeder breakers rated 50,000 amperes could be selected if properly "backed up." It is obvious that complete back-up protection cannot be obtained from the bus tie breakers alone, since these breakers cannot isolate the fault from all sources of current. Consequently the transformer breaker supplying the faulted bus section would have to trip instantaneously when the current through it exceeded $^1/_3$ of 50,000 or approximately 16,700 amperes. While each transformer breaker requires this automatic operation at 16,700 amperes, the bus tie breakers (normally closed) are not required to operate in cascade and may, therefore, have any desired setting.

The normal current requirements of the transformer circuits dictated the selection of air circuit breakers having interrupting ratings of 75,000 amperes (see table I appendix *B*). Since the maximum calculated short circuit current through any one of them did not exceed 70,000 amperes (two transformers feeding a short circuit at low-tension terminals of third transformer, assumed to be disconnected from high tension system) the breakers selected met the requirements for (*A*) breakers.

The application of the cascade principle to this installation, resulted in a decrease in cost of approximately 50% and a reduction in floor area to approximately 50% of that which would have been required if the cascade principle had not been used. Other factors include a substantial saving in weight and resulting building costs.

3. D-C Systems

Except that the calculated short circuit current is the maximum obtainable (see quoted standard following table I appendix *B*) the application of the cascade principle is the same as that discussed in connection with figure 11.

Servicing and Maintenance of Circuit Breakers in Cascade

While the cascade combination assures successful operation and adequate protection in the case of the heaviest short circuits, a majority of the faults occur some distance from the breakers, resulting in currents within the interrupting ratings of the smaller breakers. Thus the maintenance due to a large portion of the faults is not increased over that ordinarily expected.

The occurrence of a nearby short circuit may impose on breaker (*C*) in figure 11, interrupting duty considerably in excess of its rating. Consequently it is reasonable to require inspection and possible servicing on such occasions, before this circuit breaker can be reclosed with the expectation that it can successfully perform its usual duty.

When faults occur beyond the (*C*) breakers, the (*B*) breaker should preferably be inspected after each interruption, but little—if any—maintenance is expected and it should be capable of reclosing and carrying its rated current until inspection and minor maintenance can be provided. When subject to faults occurring on the source side of the (*C*) breakers, the (*B*) breaker then becomes the smallest breaker in the series and will require inspection and possible servicing as outlined in the preceding paragraph.

The main breakers (*A*) will probably operate only in event of very severe faults. Since these breakers are not applied beyond the limits of their interrupting ratings, their interrupting performance should be within the qualifications of the AIEE and NEMA Standards. Inspection and maintenance should be provided after completion of the standard interrupting duty-cycle, i.e., two operations at 100% of interrupting rating.

In the cascade connection, the greater the number of smaller circuit breakers protected by any "back-up" breaker, the greater will be the probability of severe faults requiring the "back-up" breaker to operate and the greater will be the maintenance it will require.

Conclusions

Cascading of suitably co-ordinated air circuit breakers is a practical and economical system of protection for extensive low voltage radial feeder systems. Proper performance of breakers in cascade has been verified by test, and several years' experience with the system has been satisfactory, both from the standpoint of operation and maintenance.

It is true that in the cascade arrangement severe faults will occasionally trip the "back-up" breakers and thus interrupt power supply to some circuits not in trouble. However, this might happen even though circuit breakers with interrupting ratings to take care of the full short circuit current were used in the same connection. But, such faults are not common and most of the troubles will be taken care of by the individual feeder breakers without tripping the "back-up" breakers and without the loss of service to the rest of the system.

Appendix A. Calculated Vs. Measured Short-Circuit Current in 100,000-Ampere Test Circuit

In the tests made to compare actual current obtainable with the calculated current available in the low tension circuit of figure 9, the reactors were set to limit the current to approximately 100,000 amperes, average of three phases, through about 10 ft. of bus between the transformer terminals and the point of connection to the short circuited air circuit breakers. All connections were securely made and the short circuit was applied and removed by closing and opening the station breakers.

Since the rating of the transformer was appreciable compared with that of the alternator, low tension short-circuits had a more pronounced effect on the alternator voltage, than would be found in the usual application, where the ratio of alternator to transformer rating is of the order of 10:1 or greater. Hence, it became necessary to convert the reactance and resistance of the low voltage circuit to equivalent primary circuit values and combine these with the remaining constants of the primary circuit, including those of the alternator. The method used in calculating the short circuit current, including both a-c and d-c components, was based on the per-unit system.[4] It was found that the resistance had an effect of less than 1% on the total impedance, but did reduce the d-c time-constant to approximately one-third that of the alternator.

The equations of a-c and d-c components, in terms of the secondary circuit, are shown in figure 12, and the total rms current, plotted as curve (1).

The actual measured value of the maximum current (of the three phases) is plotted as curve (2) in figure 12. This curve was extended graphically to 0 time, after determining the a-c and d-c components. By multiplying the a-c component by 1.73 and graphically determining the rate of decay of the a-c and d-c components, it was possible to obtain the extension located between curves (1) and (2). This extension indicates the current that might have been obtained if the fault had occurred so as to produce maximum off-set.

The close agreement between curves (1) and (2) of figure 12 is indicative of the soundness of the method. No attempt was made to obtain additional refinement since the purpose of the test was to check the results obtained from calculations based on a generally accepted method and confining the data to that which was readily available. Therefore, it can be said that for all practical purposes, the measured values agreed with those obtained from the calculations.

Appendix B. Air Circuit-Breakers —Ratings and Selection

Table I. Standard Ratings of Low-Voltage Air Circuit Breakers

Interrupting Rating*	Continuous Current Rating
15,000 amperes	(15–225 amps., incl.)
25,000 amperes	(15–600 amps., incl.)
50,000 amperes	(15–1,600 amps., incl.)
75,000 amperes	(2,000–3,000 amps., incl.)
100,000 amperes	(4,000 amps. and above)

* Air circuit breakers are rated in amperes interrupting capacity as follows, in accordance with NEMA Large Air Circuit Breaker Standards SG-7-25:

"A. *Interrupting Rating*

1. The published ampere interrupting rating shall be given in rms total amperes at the rated voltage of the breakers, and based upon the standard operating duty (duty cycle).

2. The rms total amperes shall be determined by measuring the current flow in the test circuit as follows:

(*a*). The circuit breaker shall be short circuited or omitted.

(*b*). In alternating-current circuits, the current measured shall be the total rms current including the d-c. component, if any; it shall be measured at an instant one-half cycle after the short circuit occurs and shall be calculated in accordance with Appendix to AIEE Oil Circuit Breaker Standard No. 19, dated April 19, 1938. (For three-phase test circuit, the rms total current shall be the average of all three phases. For a single-phase test circuit, three successive tests shall be made to determine the average current in that circuit.)

(*c*). In direct-current circuits the current measured shall be the maximum value."

Reference (*a*)—By instantaneous is meant that no delay is purposely introduced in the action of the device. In accordance with present standards, "all air circuit breakers shall be inherently instantaneous at current values in excess of the following:

(*a*) 2,000 amperes and below—15 times rated current of the breaker.

(*b*) Above 2,000 amperes—12 times rated current of the breaker, but not exceeding 75% of the interrupting rating."

Selection of Air Circuit Breakers for Low-Voltage Systems Supplied by Single Transformer Banks

When single transformer banks of various sizes are supplied from relatively complex high tension systems, selection of low-voltage air circuit breakers is based, in part, on the short circuit KVA available at the primary terminals of the transformer. When more definite information is lacking, this KVA value may be deduced from the KVA interrupting rating of the power circuit breaker, connected to the primary or high tension terminals of the transformer, on the assumption that the power circuit breaker has been economically and properly applied. Furthermore, it is assumed that the available primary KVA is a measure of the a-c component obtainable from the primary system, and hence is merely a means for determining the equivalent reactance of the primary circuit from the power transformer to the generated source of voltage.

Referring to the example given in figure 13, it will be seen that the equivalent reactance of the primary system can be combined readily with that of the power transformer, so that the three-phase symmetrical short circuit current can be quickly calculated. (In most cases only the transformer reactance is given, and the omission of the resistance has, as a rule, very little effect on the a-c component.) This method makes possible the plotting of a series of curves, from which the short circuit current, available from a given size of transformer, having a specified reactance, can be determined. Those plotted in figure 13 are on the basis of 5% reactance, although the example readily indicates the procedure to be followed if some other reactance value is used. In order to obtain the average rms values at 0.5 cycle, the calculated symmetrical values shown in figure 13 should be multiplied by 1.25.

The short circuit currents obtained from the transformers listed in tables II and III were determined in accordance with the method shown in figure 13. Thus for a given low tension voltage, and for the transformer

Tables II and III—Selection of Low-Voltage Air Circuit Breakers (Interrupting Ratings)

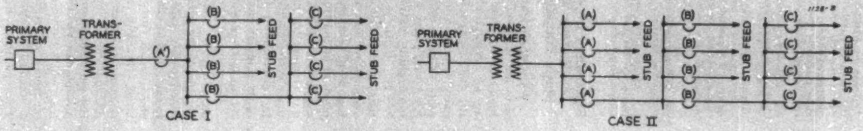


Table II.* 208 and 220 Volts

Transformer Rating 3-Phase Kva (5% Reactance Assumed)	Max. Short-Circuit Kva Available From Primary System	208 Volts								220 Volts							
		Normal Continuous Current (Amps.)	Short-Circuit Current (RMS Amperes) From			Interrupting Rating of Air Circuit Breaker Recommended				Normal Continuous Current (Amps.)	Short-Circuit Current (RMS Amperes) From			Interrupting Rating of Air Circuit Breaker Recommended			
			Transformers	Motors†	Total	(A')‡	(A)	(B)	(C)		Transformers	Motors†	Total	(A')‡	(A)	(B)	(C)
300	Unlimited	834	20,800	2,000	22,800	50,000	25,000	15,000	15,000	790	19,750	1,900	21,650	50,000	25,000	15,000	15,000
450	25,000	1,250	22,900	3,000	25,900	50,000	50,000	15,000	15,000	1,180	21,700	2,900	24,600	50,000	25,000	15,000	15,000
	50,000		26,600		29,600	50,000	50,000	15,000	15,000		25,200		28,100	50,000	50,000	15,000	15,000
	Unlimited		31,200		34,200	50,000	50,000	25,000	15,000		29,500		32,400	50,000	50,000	25,000	15,000
600	250,000	1,665	39,800	4,000	43,800	75,000	50,000	25,000	15,000	1,580	37,600	3,900	41,500	50,000	50,000	25,000	15,000
	Unlimited		41,700		45,700	75,000	50,000	25,000	25,000		39,500		43,400	50,000	50,000	25,000	15,000
750	50,000	2,080	40,000	5,000	45,000	75,000	50,000	25,000	15,000	1,970	37,700	4,800	42,500	75,000	50,000	25,000	15,000
	150,000		47,500		52,500	75,000	75,000	50,000	25,000		44,800		49,600	75,000	50,000	25,000	25,000
	Unlimited		52,000		57,000	75,000	75,000	50,000	25,000		49,500		54,300	75,000	75,000	50,000	25,000
1,000	25,000	2,780	38,800	6,800	45,600	75,000	50,000	25,000	25,000	2,620	36,600	6,400	43,000	75,000	50,000	25,000	15,000
	50,000		49,400		56,200	75,000	75,000	50,000	25,000		46,600		53,000	75,000	75,000	50,000	25,000
	500,000		66,600		73,400	75,000	75,000	50,000	25,000		63,000		69,400	75,000	75,000	50,000	25,000
	Unlimited		69,500		76,300	75,000	100,000	50,000	50,000		65,600		72,000	75,000	75,000	50,000	25,000
1,500	50,000	4,160	65,000	10,000	75,000	100,000	75,000	50,000	25,000	3,940	61,500	9,600	71,100	100,000	75,000	50,000	25,000
	150,000		87,000		97,000	100,000	100,000	50,000	50,000		82,000		91,600	100,000	100,000	50,000	50,000
	250,000		92,500		102,500	100,000	§	75,000	50,000		87,500		97,100	100,000	100,000	50,000	50,000
	500,000		98,500		108,500	100,000	§	75,000	50,000		93,000		102,600	100,000	§	75,000	50,000
	Unlimited		104,000		114,000	§	§	75,000	50,000		98,500		108,100	100,000	§	75,000	50,000

Table III.* 440 and 550 Volts

Transformer Rating 3-Phase Kva (5% Reactance Assumed)	Max. Short-Circuit Kva Available From Primary System	440 Volts								550 Volts							
		Normal Continuous Current (Amps.)	Short-Circuit Current (RMS Amperes) From			Interrupting Rating of Air Circuit Breaker Recommended				Normal Continuous Current (Amps.)	Short-Circuit Current (RMS Amperes) From			Interrupting Rating of Air Circuit Breaker Recommended			
			Transformers	Motors†	Total	(A′)‡	(A)	(B)	(C)		Transformers	Motors†	Total	(A′)‡	(A)	(B)	(C)
300	Unlimited	395	9,870	1,900	11,800	25,000	15,000	15,000	15,000	315	7,880	1,500	9,380	25,000	15,000	15,000	15,000
450	25,000	590	11,000	2,900	13,900	25,000	15,000	15,000	15,000	473	8,800	2,300	10,100	25,000	15,000	15,000	15,000
	Unlimited		14,800		17,700	25,000	25,000	15,000	15,000		11,900		14,200	25,000	15,000	15,000	15,000
600	25,000	790	13,300	3,900	17,200	50,000	25,000	15,000	15,000	630	10,700	3,100	13,800	50,000	15,000	15,000	15,000
	Unlimited		19,800		23,700	50,000	25,000	15,000	15,000		15,750		18,850	50,000	25,000	15,000	15,000
750	50,000	985	18,900	4,800	23,700	50,000	25,000	15,000	15,000	790	15,200	3,900	19,100	50,000	25,000	15,000	15,000
	Unlimited		24,700		29,500	50,000	50,000	15,000	15,000		19,800		23,700	50,000	25,000	15,000	15,000
1,000	25,000	1,310	18,300	6,400	24,700	50,000	25,000	15,000	15,000	1,050	14,600	5,100	19,700	50,000	25,000	15,000	15,000
	50,000		23,400		29,800	50,000	50,000	15,000	15,000		18,600		23,700	50,000	25,000	15,000	15,000
	150,000		29,000		35,400	50,000	50,000	25,000	15,000		23,100		28,200	50,000	50,000	15,000	15,000
	Unlimited		32,800		39,200	50,000	50,000	25,000	15,000		26,300		31,400	50,000	50,000	25,000	15,000
1,500	25,000	1,970	22,400	9,600	32,000	75,000	50,000	25,000	15,000	1,575	18,000	7,700	25,700	50,000	50,000	15,000	15,000
	75,000		35,250		44,850	75,000	50,000	25,000	15,000		28,200		35,900	50,000	50,000	25,000	15,000
	100,000		37,800		47,400	75,000	50,000	25,000	25,000		30,200		37,900	50,000	50,000	25,000	15,000
	500,000		46,500		56,100	75,000	75,000	50,000	25,000		37,000		44,700	50,000	50,000	25,000	15,000
	Unlimited		49,200		58,800	75,000	75,000	50,000	25,000		39,400		47,100	50,000	50,000	25,000	25,000
2,000	25,000	2,625	25,300	12,800	38,100	75,000	50,000	25,000	15,000	2,100	20,400	10,200	30,700	75,000	50,000	25,000	15,000
	50,000		36,300		49,100	75,000	50,000	25,000	25,000		29,100		39,400	75,000	50,000	25,000	15,000
	75,000		43,100		55,900	75,000	75,000	50,000	25,000		34,300		44,600	ϕ	50,000	25,000	15,000
	100,000		47,000		59,800	75,000	75,000	50,000	25,000		37,600		47,900	ϕ	50,000	25,000	25,000
	500,000		60,500		73,300	75,000	75,000	50,000	25,000		48,500		58,800	75,000	75,000	50,000	25,000
	Unlimited		65,600		78,400	75,000	100,000	50,000	50,000		52,500		62,800	75,000	75,000	50,000	25,000

*All selections in tables II and III are based on:

(1) Multiplying factor, $D = 1.25$, giving average rms value of three phases, at 0.5 cycle.

(2) Negligible external resistance and reactance.

(3) Connected motor load as indicated, (†), not more than approximately 25 per cent of which is synchronous.

(4) Breaker selections are based on all breakers having modern arc quenchers, and properly co-ordinated in operation.

†For table II a total connected motor load of 50 per cent, and for table III, of 100 per cent, of the transformer rating is assumed.

‡Breaker (A') selections are in all cases governed by the necessary continuous current ratings of the transformer, and this accounts for the fact that in some cases the interrupting ratings are higher than the ratings given for the corresponding (A) breakers.

(ϕ) Requires special consideration.

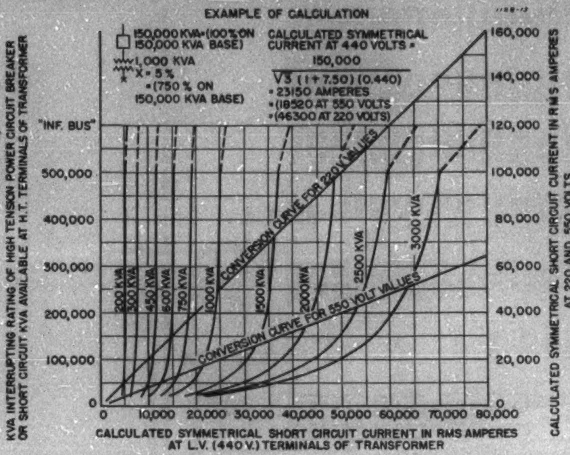


Figure 13. Calculated symmetrical three-phase short-circuit current obtained at low-voltage terminals of transformers (having assumed reactance of 5 per cent) in terms of kva-rating of high-voltage power circuit breaker or short-circuit kva available at high-voltage terminals of transformer

For average rms value, at 0.5 cycle, multiply calculated (symmetrical) value by 1.25

ratings listed in the first column, together with the primary system KVA listed in the second column, the short circuit currents obtainable from the transformer may be found in the fourth column. To these values are added the short circuit current contributed by connected motors as shown in column 5, giving the total short circuit current as shown in column 6.

The remaining four columns give the required interrupting ratings for the circuit breakers in cascade. The values listed are the standard steps of interrupting ratings of air circuit breakers (table I).

As an example of the use of these tables, assume an installation consisting of a single 1,500 KVA transformer supplying a 440 volt system, with a power circuit breaker on the high tension side, having an interrupting rating of 250,000 KVA, from which it may be deduced that the high tension system will deliver not more than 250,000 KVA in the case of a short circuit at the high tension terminals of the transformer. Data pertaining to this installation would be found under table III (440 volts). Since the power circuit breaker rating is 250,000 KVA, and this value does not appear in table III opposite the 1,500 KVA transformer rating, use the next higher value, namely, 500,000 KVA. Although the short circuit current values will increase as the primary system KVA increases between the limits of 100,000 and 500,000 KVA, the increase is not sufficient to affect the selection of low-voltage circuit breakers. Hence, the intermediate primary KVA values are not listed.

Reading across from the 500,000 KVA value in column 2, the transformer has a continuous rating of 1,970 amperes (column 3) and the required rating of a transformer breaker (A') would be 2,000 amperes (next standard size over 1,970 amperes). Such a breaker has an interrupting rating of 75,000 amperes (table I) which is adequate, since the transformer contributes only 46,500 amperes to the short circuit (column 4). The motor load contributes 9,600 amperes (column 5) giving a total of 56,100 amperes (column 6). If there is no transformer breaker (A') circuit breaker (A) would be required to have an interrupting rating of over 56,100 amperes, resulting in the choice of a 75,000 ampere circuit breaker (next higher standard rating). The requirement of breaker (B) is that its rating exceed 56,100/2 which results in a 50,000 ampere circuit breaker at this location, while that of (C) is 56,100/3, resulting in a 25,000 ampere circuit breaker. The continuous ampere ratings of (A), (B), or (C) depend on the rating of the feeder in which each is connected. The range in continuous current values for a given interrupting rating is indicated in table I.

References

1. Fault-Voltage Drop and Impedance at Short-Circuit Currents in Low-Voltage Circuits, O. R. Schurig. AIEE Transactions, volume 60, 1941, pages 479–87.

2. Short-Circuit Calculating Procedure for Low-Voltage A-C Systems, A. G. Darling. AIEE Transactions, volume 60, 1941, pages 1121–36.

3. Modern Trends of Low-Voltage Air Circuit Breakers, J. W. Seaman. AIEE Transactions, volume 59, 1940 (January section), pages 24–9.

4. Calculations of Short-Circuit Currents in A-C Networks, W. M. Hanna. *General Electric Review*, March, April, June, and August 1937.

Diode Rectifying Circuits With Capacitance Filters

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Synopsis: This paper presents the results of an investigation of the half wave and full wave rectifier circuits with a simple capacitance filter and using either high vacuum or mercury vapor tubes. In the fore part of the paper an equivalent circuit is set up such that when the tube is conducting, a mercury vapor tube is represented by a constant voltage drop or a high vacuum tube by a constant resistance. A detailed analysis of this circuit is carried through in the appendix, and it is found that the output characteristics depend upon three parameters.

The results of this analysis are presented in the form of curves giving the angles at which the tubes begin conducting and cease conducting and also the d-c output voltage. For the more special case of no tube drop, curves of ripple voltage, maximum tube current, and tube inverse peak voltage are presented. The comparisons made between the calculated results and test results show very satisfactory agreements. It is shown that the curves given in the paper may be used in the design of rectifier circuits with capacitance filters.

THE need for a high voltage, low current d-c power supply such as those used to supply the plate voltage requirements of the tubes in small radio receivers or of the cathode ray tubes in television receivers, is fulfilled by the half wave or full wave rectifier circuit with a single condenser as the smoothing filter and storage capacity. Up till now accurate knowledge as to the d-c voltage output and the ripple of the output voltage has been incomplete. Glaser and Müller-Lübeck[1] have solved the half wave case with no drop in the rectifier tube when conducting but gave no experimental results confirming their theoretical curves. Roberts[2] has discussed the half wave and full wave circuits using high vacuum rectifier tubes and also gives some excellent experimental curves of the d-c voltage output. Others[3-7] have also treated this subject at some length.

Equivalent Circuit and Wave Form

The half wave rectifier circuit shown in figure 1*a* has R_L as the load resistance and C as the smoothing capacitance. The corresponding full wave circuit is shown in figure 1*b*. While a mercury vapor rectifier tube is conducting, the voltage drop across the tube is almost constant and may be indicated in an equivalent circuit by a battery voltage V. Roberts[2] has shown that during the conduction period a thermionic vacuum tube rectifier may be considered as either a constant resistance R_R or a constant resistance R_R plus a constant voltage drop V. The resulting equivalent circuit is given in figure 1*c*, where $e = E_m \sin \theta$ is the alternating voltage applied. E_m is the maximum value of this voltage, and $\theta = \omega t$, where $\omega/2\pi$ is the frequency of the source and t is the time in seconds. Switch S is closed when the tube is conducting and is open when the tube is not conducting.

The waveform of the half wave circuit is given in figure 2 with 180 degrees of the impressed voltage wave also shown. The tube starts conducting at $\theta = \alpha$ because the potential of the anode becomes sufficiently positive with respect to the cathode. The tube current i_R shown in figure 2 flows until $\theta = \beta$, i.e., when the current becomes zero. It is zero thereafter until the tube begins to conduct again at $\theta = \alpha + 360°$. The condenser C is charged during part of the period that the tube is firing and is discharged during the rest of the cycle. The waveshape of the resulting load current i_L is shown in figure 2, and this is also the shape of the voltage across the load resistance. The waveform of the full wave circuit given in figure 3 is very similar to that of the half wave circuit except that the condenser C is charged twice every cycle instead of once. In the following n is taken as one for the half wave circuit and for the full wave circuit, $n = 2$.

Parameters of the Circuits

If the Fourier series coefficients of the load current i_L are determined, there will be a d-c component I and the a-c components, the most important of which are M_1 and N_1 with a frequency the same as the supply frequency in the half wave case or double the supply frequency in the full wave case.

$$i_L = I + M_1 \sin n\theta + N_1 \cos n\theta + M_2 \sin 2n\theta + N_2 \cos 2n\theta + \dots \quad (1)$$

where

$$I = \frac{n}{2\pi} \int_{\alpha}^{\alpha + \frac{2\pi}{n}} i_L \, d\theta \quad (2)$$

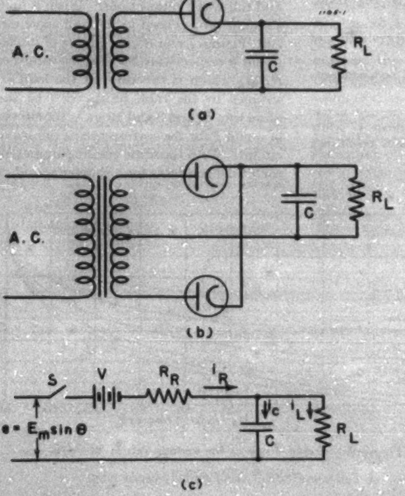

Figure 1. Rectifier circuits

(*a*) Half-wave circuit
(*b*) Full-wave circuit
(*c*) Equivalent circuit

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[1] For all numbered references, see list at end of paper.

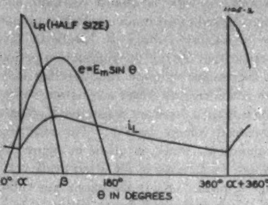


Figure 2. Wave form of half-wave circuit

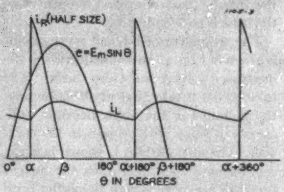


Figure 3. Wave form of full-wave circuit

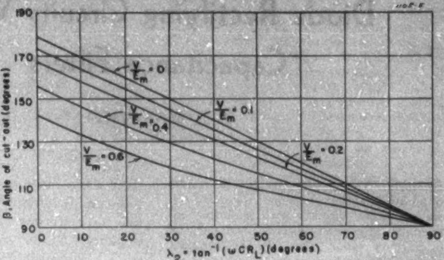
$$M_1=\frac{n}{\pi}\int_{\alpha}^{\alpha+\frac{2\pi}{n}} i_L \sin n\theta d\theta \tag{3}$$

$$N_1=\frac{n}{\pi}\int_{\alpha}^{\alpha+\frac{2\pi}{n}} i_L \cos n\theta d\theta \tag{4}$$

and with similar expressions for the rest of the *M*s and *N*s. In order to study the voltage output of these rectifier circuits it will be advantageous to define two ratios. The first is the ratio of the load d-c voltage IR_L to the maximum value of the a-c supply voltage E_m and will be denoted by IR_L/E_m. Since M_1 and N_1 will be large compared to the coefficients of the higher frequencies in the load ripple voltage, the ripple fraction r will be defined as the ratio of the effective value of the load ripple voltage involving M_1 and N_1 to the d-c voltage across the load.

Figure 5. Cut-out angle β for various values of V/E_m

Half-wave or full-wave circuit using mercury vapor tubes



$$r=\frac{\sqrt{\frac{M_1^2}{2}+\frac{N_1^2}{2}}R_L}{IR_L}=\frac{\sqrt{\left(\frac{M_1R_L}{E_m}\right)^2\left(+\frac{N_1R_L}{E_m}\right)^2}}{\sqrt{2}(IR_L/E_m)} \tag{5}$$

In appendix II it is shown that the quantities α, β, IR_L/E_m, and r depend on three parameters, and for a mercury vapor tube these parameters are (ωCR_L), V/E_m, and n. If the vacuum tube is considered as a resistance R_R alone, the parameters become (ωCR_L), R_L/R_R, and n. The purpose of this paper is to indicate how α, β, IR_L/E_m, and r vary with the above parameters, and to explain the use of the resulting curves in the design of the rectifier circuits. Several assumptions have been made to simplify the results derived in the appendix, and these assumptions are:

1. The secondary voltage of the transformer is sinusoidal;
2. The leakage reactance and resistance of the transformer is zero;
3. The tube resistance when non-conducting is infinite;
4. When conducting, the mercury vapor tube has a constant voltage drop V, while the vacuum tube has a constant resistance R_R; and
5. The filter capacitance C has zero power factor, and the load resistance has no inductance.

Angles of Cut-in and Cut-out

The angle of cut-in α at which the tube begins to conduct and the angle of cut-out β at which the tube ceases to conduct are shown in the first group of curves constructed from calculated results. All of the curves are plotted against the angle λ_2 where $\tan \lambda_2=(\omega CR_L)$. The angle α is always between zero degrees and 90 degrees, while angle β is between 90 and 180 degrees. For the half wave circuit using a mercury vapor tube, figures 4 and 5 show curves of angles α and β respectively for several values of V/E_m, the ratio of tube voltage drop to maximum value of a-c voltage. The corresponding curves for the full wave circuit are those of figures 6 and 5. In this particular case angle β is the same for both the half wave and full wave connections. When a high vacuum tube is used in the half wave circuit, figures 7 and 8 give curves of angles α and β respectively for various values of R_L/R_R which is the ratio of the load resistance to the tube resistance. In the same way figures 9 and 10 show the curves of α and β for the corresponding full wave circuit. With the same circuit parameters each tube conducts for a shorter period of

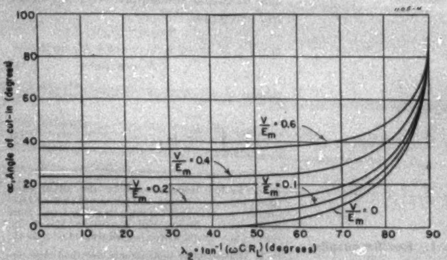


Figure 4. Cut-in angle α for various values of V/E_m

Half-wave circuit using mercury vapor tube

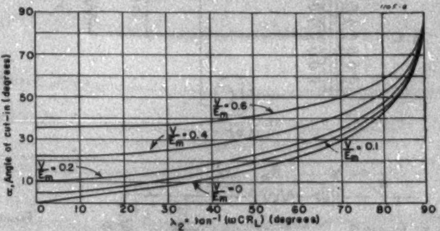


Figure 6. Cut-in angle α for various values of V/E_m

Full-wave circuit using mercury vapor tubes

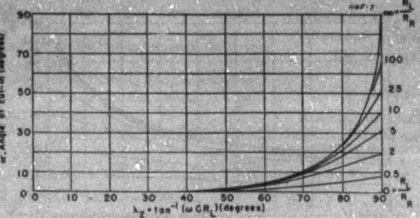


Figure 7. Cut-in angle α for various values of R_L/R_R

Half-wave circuit using high vacuum tube

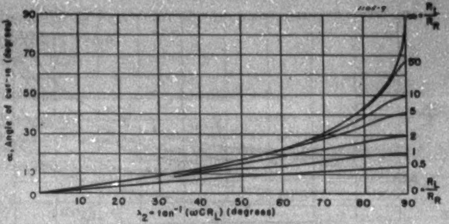


Figure 9. Cut-in angle α for various values of R_L/R_R

Full-wave circuit using high vacuum tubes

time in the full wave circuit than in the half wave circuit.

Output Voltage Characteristics

The ratio of the average d-c voltage across the load resistance to the maximum value of the a-c voltage impressed on the circuit, i.e., IR_L/E_m, is shown in the second group of curves which are also obtained from calculated results. The ratio IR_L/E_m varies between zero and one, but is never more than one since the maximum average output voltage cannot be more than the maximum value of the a-c voltage. Figures 11 and 12 show the curves of IR_L/E_m for the half wave and full wave circuits respectively using mercury vapor tubes. For the corresponding circuits using high vacuum tubes figures 13 and 14 give the curves of IR_L/E_m. The right side of each of these figures corresponds to light loads, while the heavier loads would be on the left side. In general, the output voltage is less for the half wave circuit than for the full wave circuit.

The ripple fraction r, which is the ratio of the rms value of the voltage of lowest frequency in the load to the load d-c voltage, is the third quantity shown in curve form. In most applications of these circuits using mercury vapor tubes V/E_m is very small and may be taken as zero with small loss in accuracy. Similarly for circuits with high vacuum tubes R_L/R_R is large and may be assumed infinite. Under these circumstances the ripple fraction curve for the half wave circuit is given in figure 15 as the solid line curve from calculated results. Figure 16 shows the corresponding curve for the full wave circuit. For the same parameters the full wave circuit will have a ripple fraction of approximately one half that for the half wave circuit. Also the frequency of the ripple voltage for the full wave circuit will be twice that of the half wave case.

Maximum Current and Inverse Peak Voltage of the Tube

When choosing the rectifier tube to be used in a given circuit, it is necessary to know the maximum current i_m that will flow when the tube is conducting and also the inverse peak voltage E_p that the tube must withstand when it is not conducting. Since I/n is the average tube current, the quantity ni_m/I will be the ratio of the maximum tube current to the average tube current. For the case when V/E_m is zero or R_L/R_R infinite, ni_m/I is given in figure 17. The half wave case is indicated by $n=1$ and the full wave by $n=2$. The ratio of the inverse peak voltage to the maximum value of the a-c voltage, i.e., E_p/E_m, is shown in figure 18. These two figures indicate that for the same circuit parameters, the tube in the half wave circuit has the lower maximum current to average current ratio and the lower inverse peak voltage than the tube in the full wave circuit.

Verification of Solution

The calculated results for the case where V/E_m is zero or R_L/R_R is infinite were checked experimentally by setting up the half wave and full wave circuits and measuring the d-c output voltage and current. The output ripple voltage was determined by means of a wave analyzer. To eliminate as many sources of error as possible the following precautions were observed:

1. The transformer used was much larger than necessary so that the leakage reactance and resistance would be small;
2. A mercury vapor tube with a small voltage drop while conducting was used, and also E_m was so large that V/E_m was practically zero; and
3. The condensers had a low power factor, and the voltmeter and wave analyzer a very high input resistance compared to the load resistance.

For the half wave circuit the circles in figure 19 show the experimentally ob-

Figure 8. Cut-out angle β for various values of R_L/R_R

Half-wave circuit using high vacuum tube

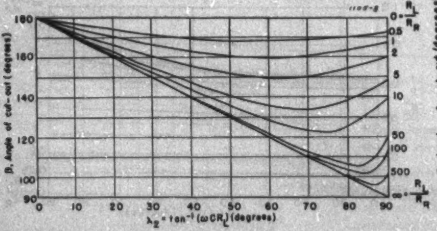
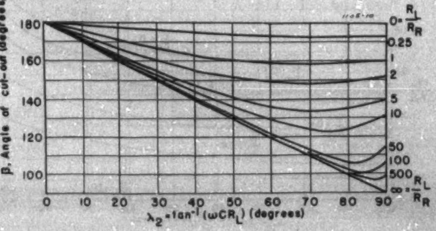


Figure 10. Cut-out angle β for various values of R_L/R_R

Full-wave circuit using high vacuum tubes



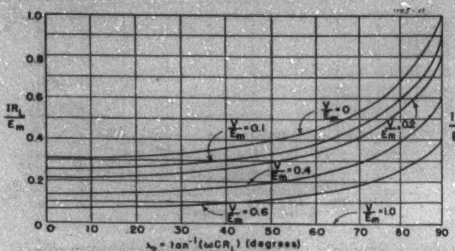


Figure 11. Voltage ratio IR_L/E_m for various values of V/E_m

Half-wave circuit using mercury vapor tube

tained values of IR_L/E_m as compared to the calculated curve which is the solid line curve. Figure 15 shows the results for the ripple fraction r. The same results for the full wave rectifier are shown in figures 19 and 16. The deviation of the observed values from the calculated ones may be explained in view of the sources of error mentioned above.

The calculated values of IR_L/E_m for the circuits using a high vacuum rectifier as given in figures 13 and 14 compare very well with the observed values as given by Roberts.[3] The values of the ripple fraction r as shown in figures 15 and 16 compare fairly well with the approximate expression derived by Reich[6] as long as r is less than one per cent. This approximate expression is

$$r=\frac{1}{n\omega R_L C} \qquad (6)$$

Method of Design

In many practical applications the ratio V/E_m is so small that it may safely be assumed zero. When this is true, figures 15, 16, 19 give the values of r and IR_L/E_m. Usually the d-c output voltage, output current, and maximum allowable output ripple voltage are known, and it is desired to obtain the circuit parameters. In the case of the half wave circuit ($n=1$), figure 15 will give the value of λ_2 corresponding to the maximum ripple voltage allowable. The d-c output voltage and current will give the load resistance R_L, and from the λ_2 obtained above the minimum value of C can be determined. Figure 19 will give the value of IR_L/E_m corresponding to the λ_2 obtained, and from this value of IR_L/E_m may be obtained the voltage that the secondary of the transformer must deliver to the circuit. Also for the above value of λ_2 and from figures 17 and 18, it is possible to predict the necessary maximum current and the required inverse peak voltage that the tubes must withstand. Thus from these data and the average current, a suitable rectifier tube may be selected. The procedure for the full wave case ($n=2$) is exactly similar except that figures 16 and 19 are used for r and IR_L/E_m.

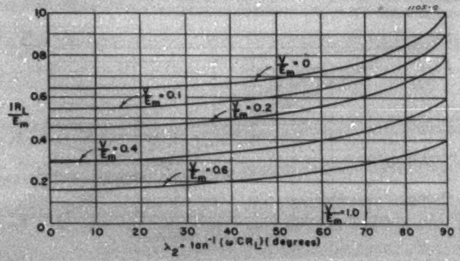
If V/E_m is not small enough to be assumed zero, or R_L/R_R not large enough to be assumed infinite, a tentative value of λ_2 may be obtained as in the preceding paragraph, and this may be used with the value of V/E_m or R_L/R_R to determine α and β from figures 4 to 10. More exact values of r may then be obtained from the equations for r in the appendix and the value of IR_L/E_m from figures 11 to 14.

The choice between the full wave circuit and the half wave circuit depends upon a number of factors, among which are:

1. The half wave circuit needs only one tube and half the transformer secondary required by the full wave case; and
2. The d-c output voltage is higher and the ripple voltage lower for the full wave case. The ripple voltage is also of double frequency in the full wave connection.

Figure 12. Voltage ratio IR_L/E_m for various values of V/E_m

Full-wave circuit using mercury vapor tubes



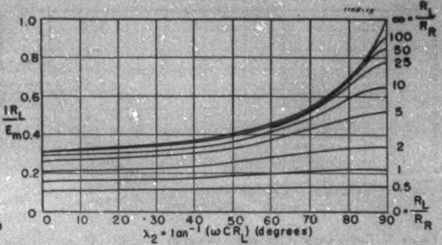


Figure 13. Voltage ratio IR_L/E_m for various values of R_L/R_R

Half-wave circuit using high vacuum tube

Conclusion

The solution of the equivalent circuit equations gives results which agree well with the experimental results, and from these results it is possible to predict the average voltage output and the load ripple voltage for any values of the circuit parameters. In addition these circuits may be designed readily from the curves given in the paper, and both the proper circuit constants and the proper tubes may be chosen.

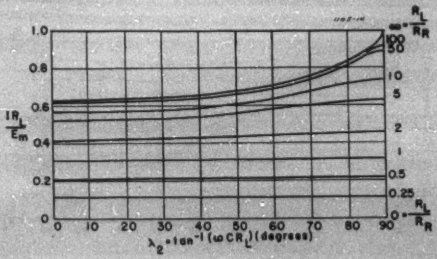
Appendix I—Glossary

$\theta=\omega t$ where the applied frequency is $\frac{\omega}{2\pi}$

t = time in seconds

Figure 14. Voltage ratio IR_L/E_m for various values of R_L/R_R

Full-wave circuit using high vacuum tubes



E_m = the maximum value of the a-c applied voltage

i_R = the current flowing through the rectifier tube

i_C = the current flowing into the condenser

i_L = the current flowing through the load resistance

R_R = the resistance of the rectifier tube in ohms

V = the constant voltage drop of the rectifier tube

R_L = the load resistance in ohms

C = the smoothing capacity in farads

$$n=\begin{cases}1 \text{ for the half wave circuit}\\ 2 \text{ for the full wave circuit}\end{cases}$$

$$Z_1=\sqrt{(\omega CR_LR_R)^2+(R_R+R_L)^2}$$

$$\lambda_1=\tan^{-1}\left(\frac{\omega CR_LR_R}{R_R+R_L}\right),\ 0°\leq\lambda_1\leq 90°$$

$$Z_2=\sqrt{R_L^2+\left(\frac{1}{\omega C}\right)^2}$$

$\lambda_2=\tan^{-1}(\omega CR_L)$, $0°\leq\lambda_2\leq 90°$

α = angle in radians at which the tube starts to conduct current (cut-in angle)

β = angle in radians at which the tube stops conducting (cut-out angle)

i_α = current i_L at angle α

i_β = current i_L at angle β

$\gamma=\beta-\alpha$

I = average direct current through R_L

$$r=\frac{\text{effective value of lowest harmonic voltage across } R_L}{IR_L}$$

λ_3 = an angle such that $\tan\lambda_3=2\tan\lambda_1$, $0°\leq\lambda_3\leq 90°$

λ_4 = an angle such that $\tan\lambda_4=2\tan\lambda_2$, $0°\leq\lambda_4\leq 90°$

q_α = charge on condenser C at $\theta=\alpha$

Appendix II—Calculation of Performance Characteristics From the Equivalent Circuit

(a) Cut-in and Cut-out Angles

Referring to the equivalent circuit of figure 1*c*, when i_R is flowing (S closed) the equations for the currents are:

$$i_R=i_L+i_C$$

$$E_m\sin\theta=V+R_Ri_R+R_Li_L$$

$$\frac{1}{\omega C}\int_\alpha^\theta i_c\,d\theta+\frac{q_\alpha}{C}=R_Li_L$$

These three equations are solved for the three currents i_R, i_C, and i_L, and then the condition, $i_L=i_\alpha$ at $\theta=\alpha$, is substituted to give the following results:

$$i_R=\frac{E_m}{Z_1}\frac{\sin(\theta-\lambda_1+\lambda_2)}{\cos\lambda_2}-\frac{V}{Z_1\cos\lambda_1}+\frac{\sin(\lambda_1-\lambda_2)}{\cos\lambda_2\sin\lambda_1}\left[i_\alpha-\frac{E_m}{Z_1}\sin(\alpha-\lambda_1)+\frac{V}{Z_1\cos\lambda_1}\right]\epsilon^{-(\theta-\alpha)\cot\lambda_1}\quad(7)$$

$$i_C=(\tan\lambda_2)\left\{\frac{E_m}{Z_1}\cos(\theta-\lambda_1)-(\cot\lambda_1)\left[i_\alpha-\frac{E_m}{Z_1}\sin(\alpha-\lambda_1)+\frac{V}{Z_1\cos\lambda_1}\right]\epsilon^{-(\theta-\alpha)\cot\lambda_1}\right\}\quad(8)$$

$$i_L=\frac{E_m}{Z_1}\sin(\theta-\lambda_1)-\frac{V}{Z_1\cos\lambda_1}+\left[i_\alpha-\frac{E_m}{Z_1}\sin(\alpha-\lambda_1)+\frac{V}{Z_1\cos\lambda_1}\right]\epsilon^{-(\theta-\alpha)\cot\lambda_1}\quad(9)$$

Again from figure 1*c* when i_R is not flowing (S open), the equations for the currents are:

$$i_L=-i_C$$

$$\frac{1}{\omega C}\int_\alpha^\theta i_C\,d\theta+\frac{q_\alpha}{C}=R_Li_L$$

These equations are solved for i_L and i_C, and then the condition, $i_L=i_\beta$ at $\theta=\beta$, is substituted to give

$$i_L=-i_C=i_\beta\epsilon^{-(\theta-\beta)\cot\lambda_2}\quad(10)$$

At $\theta=\beta$, (9) becomes:

$$i_\beta=\frac{E_m}{Z_1}\sin(\beta-\lambda_1)-\frac{V}{Z_1\cos\lambda_1}+\left[i_\alpha-\frac{E_m}{Z_1}\sin(\alpha-\lambda_1)+\frac{V}{Z_1\cos\lambda_1}\right]\epsilon^{-(\beta-\alpha)\cot\lambda_1}\quad(11)$$

At $\theta=\alpha+\frac{2\pi}{n}$, (10) becomes:

$$i_\alpha=i_\beta\epsilon^{-\left(\alpha-\beta+\frac{2\pi}{n}\right)\cot\lambda_2}\quad(12)$$

At $\theta=\alpha$:

$$i_\alpha=\frac{E_m\sin\alpha-V}{R_L}\quad(13)$$

At $\theta=\beta$:

$$i_\beta=\frac{E_m\sin\beta-V}{R_L}\quad(14)$$

Substitution of equations 13 and 14 into 11 and 12 gives:

$$\left(\sin\alpha-\frac{V}{E_m}\right)\epsilon^{\left(\alpha+\frac{2\pi}{n}\right)\cot\lambda_2}=\left(\sin\beta-\frac{V}{E_m}\right)\epsilon^{\beta\cot\lambda_2}\quad(15)$$

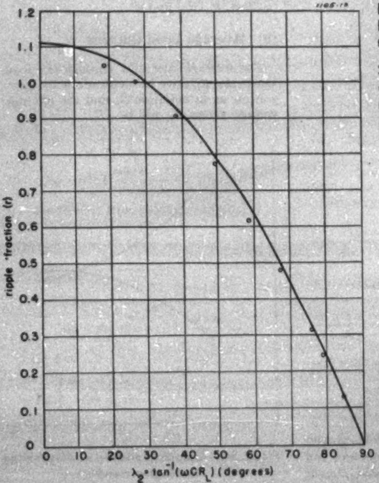


Figure 15 (left). Ripple fraction r for the half-wave circuit

Solid curve is calculated while the measured points are indicated by circles

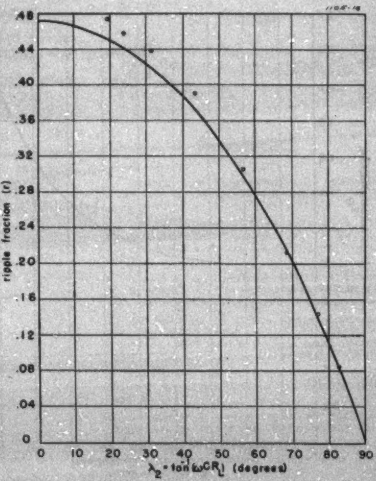


Figure 16 (right). Ripple fraction r for the full-wave circuit

Solid curve is calculated while the measured points are indicated by circles

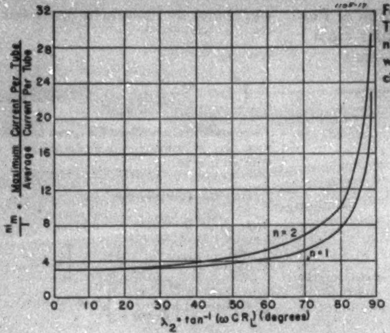
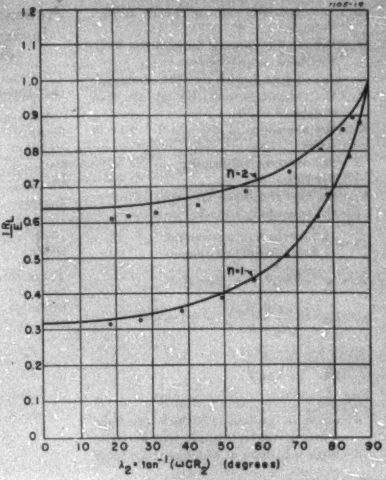


Figure 17 (left). Tube-current ratio ni_m/I for both full-wave and half-wave circuits



$$\left[\sin(\alpha-\lambda_1+\lambda_2)-\frac{V}{E_m}\frac{\cos\lambda_2}{\cos\lambda_1}\right]\epsilon^{\alpha\cot\lambda_1}=$$

$$\left[\sin(\beta-\lambda_1+\lambda_2)-\frac{V}{E_m}\frac{\cos\lambda_2}{\cos\lambda_1}\right]\epsilon^{\beta\cot\lambda_1} \quad (16)$$

These two equations 15 and 16, must be solved simultaneously to determine the angles α and β. To facilitate this let $\gamma=(\beta-\alpha)$. Equations 15 and 16 then become:

$$A_1\sin\alpha+B_1\cos\alpha=C_1 \quad (17)$$

$$A_2\sin\alpha+B_2\cos\alpha=C_2 \quad (18)$$

where:

$$A_1=1-(\cos\gamma)\epsilon^{\left(\gamma-\frac{2\pi}{n}\right)\cot\lambda_2}$$

$$B_1=-(\sin\gamma)\epsilon^{\left(\gamma-\frac{2\pi}{n}\right)\cot\lambda_2}$$

$$C_1=\frac{V}{E_m}\left[1-\epsilon^{\left(\gamma-\frac{2\pi}{n}\right)\cot\lambda_2}\right]$$

$$A_2=\cos(\lambda_1-\lambda_2)-[\cos(\gamma-\lambda_1+\lambda_2)]\epsilon^{\gamma\cot\lambda_1}$$

Figure 19 (right). Voltage ratio IR_L/E_m for both the half-wave and the full-wave circuits

Solid curve is calculated while the measured points are indicated by circles

$$B_2=-\sin(\lambda_1-\lambda_2)-[\sin(\gamma-\lambda_1+\lambda_2)]\epsilon^{\gamma\cot\lambda_1}$$

$$C_2=\frac{V}{E_m}\frac{\cos\lambda_2}{\cos\lambda_1}(1-\epsilon^{\gamma\cot\lambda_1})$$

If (17) and (18) are solved for α:

$$\alpha_1=\sin^{-1}\left(\frac{B_2C_1-B_1C_2}{A_1B_2-A_2B_1}\right) \quad (19)$$

$$\alpha_2=\cos^{-1}\left(\frac{A_1C_2-A_2C_1}{A_1B_2-A_2B_1}\right) \quad (20)$$

A value of γ must be found such that when substituted in equations 19 and 20, α_1 will equal α_2. This may be done graphically as in figure 20 by plotting α_1 and α_2 against γ. The point of intersection of the two curves has the required γ as its abscissa and α as its ordinate. The angle β may be found by adding α and γ. The angle α lies between 0° and 90°, while β lies between 90° and 180°. Angle γ ranges between 0° and 180°.

From equations 15 and 16 it is seen that α and β depend only on the four parameters, λ_1, λ_2, V/E_m, and n. Since λ_1 depends on (ωCR_L) and R_L/R_B, and λ_2 depends on (ωCR_L), the angles α and β depend on the four parameters (ωCR_L), R_L/R_B, V/E_m, and n. For mercury vapor tubes the parameters are (ωCR_L), V/E_m, and n, while for vacuum tubes the parameters are (ωCR_L), R_L/R_B, and n.

(b) Average Load Current

The average current I through the load resistance R_L may be determined by integration as in equation 2, and the average voltage across R_L will be IR_L.

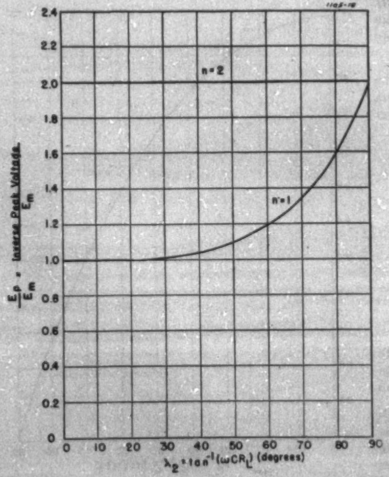


Figure 18. Tube-voltage ratio E_p/E_m for both full-wave and half-wave circuits

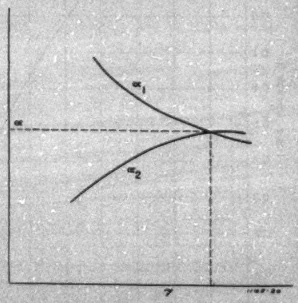


Figure 20. Graphical method of determining the angles α and γ

$$I=\frac{n}{2\pi}\int_{\alpha}^{\alpha+\frac{2\pi}{n}} i_L d\theta=\frac{n}{2\pi}\frac{E_m}{R_L}\left(1-\frac{\tan\lambda_1}{\tan\lambda_2}\right)\times$$
$$\left[\frac{\cos(\alpha+\lambda_2)-\cos(\beta+\lambda_2)}{\cos\lambda_2}-\frac{V}{E_m}(\beta-\alpha)\right]\quad(21)$$

The two expressions for i_L are given in equations 9 and 10, and after the integration is performed, the expression for I may be simplified by the use of equations 11, 12, 13, and 14. IR_L/E_m is found to depend only on (ωCR_L), R_L/R_R, V/E_m, and n.

(c) Ripple Fraction

The ripple fraction r is defined by equation 5 in which M_1 and N_1 are obtained from equations 3 and 4. By the use of equations 9 to 14 inclusive, M_1R_L/E_m and N_1R_L/E_m may be expressed as follows:

For $n=1$:

$$\frac{M_1R_L}{E_m}=\frac{\sin(\lambda_2-\lambda_1)}{2\pi\sin\lambda_2}\left\{(\beta-\alpha)\cos\lambda_1-\right.$$
$$\left.\frac{1}{2}\left[\sin(2\beta+\lambda_1)-\sin(2\alpha+\lambda_1)\right]\right\}+$$
$$\frac{\sin\lambda_2}{\pi}[\sin\beta\sin(\beta+\lambda_2)-\sin\alpha\sin(\alpha+\lambda_2)]-$$
$$\frac{\sin\lambda_1}{\pi}[\sin\beta\sin(\beta+\lambda_1)-\sin\alpha\sin(\alpha+\lambda_1)]+$$
$$\frac{V}{E_m}\frac{\cot\lambda_2}{\pi}\{(\sin\lambda_2)[\cos(\beta+\lambda_2)-$$
$$\cos(\alpha+\lambda_2)]-(\sin\lambda_1)[\cos(\beta+\lambda_1)-$$
$$\cos(\alpha+\lambda_1)]\}\quad(22)$$

$$\frac{N_1R_L}{E_m}=-\frac{\sin(\lambda_2-\lambda_1)}{2\pi\sin\lambda_2}\left\{(\beta-\alpha)\sin\lambda_1+\right.$$
$$\left.\frac{1}{2}[\cos(2\beta+\lambda_1)-\cos(2\alpha+\lambda_1)]\right\}+\frac{\sin\lambda_2}{\pi}$$
$$[\sin\beta\cos(\beta+\lambda_2)-\sin\alpha\cos(\alpha+\lambda_2)]-$$
$$\frac{\sin\lambda_1}{\pi}[\sin\beta\cos(\beta+\lambda_1)-\sin\alpha\cos(\alpha+\lambda_2)]-$$
$$\frac{V}{E_m}\frac{\cot\lambda_2}{\pi}\{(\sin\lambda_2)[\sin(\beta+\lambda_2)-$$
$$\sin(\alpha+\lambda_2)]-(\sin\lambda_1)[\sin(\beta+\lambda_1)-$$
$$\sin(\alpha+\lambda_1)]\}\quad(23)$$

For $n=2$:

$$\frac{M_1R_L}{E_m}=-\frac{\sin(\lambda_2-\lambda_1)}{3\pi\sin\lambda_2}[\sin(3\beta-\lambda_1)-$$
$$\sin(3\alpha-\lambda_1)]+\frac{\sin(\lambda_2-\lambda_1)}{\pi\sin\lambda_2}\left\{\sin(\beta+\lambda_1)-\right.$$
$$\sin(\alpha+\lambda_1)+(\sin\lambda_3)[\sin(2\beta+\lambda_3)\times$$
$$\left.\sin(\beta-\lambda_1)-\sin(2\alpha+\lambda_3)\sin(\alpha-\lambda_1)]\right\}-$$
$$\frac{\sin\lambda_3}{\pi}[\sin\beta\sin(2\beta+\lambda_3)-$$
$$\sin\alpha\sin(2\alpha+\lambda_3)]+\frac{\sin\lambda_4}{\pi}\times$$
$$[\sin\beta\sin(2\beta+\lambda_4)-\sin\alpha\sin(2\alpha+\lambda_4)]+$$
$$\frac{1}{\pi}\frac{V}{E_m}\left\{\frac{\sin(\lambda_2-\lambda_1)\cos\lambda_3}{\cos\lambda_1\sin\lambda_2}[\cos(2\beta+\lambda_3)-\right.$$
$$\left.\cos(2\alpha+\lambda_3)]\right\}+\frac{1}{\pi}\frac{V}{E_m}\{(\sin\lambda_3)\times$$
$$[\sin(2\beta+\lambda_3)-\sin(2\alpha+\lambda_3)]-(\sin\lambda_4)\times$$
$$[\sin(2\beta+\lambda_4)-\sin(2\alpha+\lambda_4)]\}\quad(24)$$

$$\frac{N_1R_L}{E_m}=-\frac{\sin(\lambda_2-\lambda_1)}{3\pi\sin\lambda_2}[\cos(3\beta-\lambda_1)-$$
$$\cos(3\alpha-\lambda_1)]+\frac{\sin(\lambda_2-\lambda_1)}{\pi\sin\lambda_2}\{-\cos(\beta+\lambda_1)+$$
$$\cos(\alpha+\lambda_1)+(\sin\lambda_3)[\cos(2\beta+\lambda_3)\times$$
$$\sin(\beta-\lambda_1)-\cos(2\alpha+\lambda_3)\sin(\alpha-\lambda_1)]\}-$$
$$\frac{\sin\lambda_3}{\pi}[\sin\beta\cos(2\beta+\lambda_3)-$$
$$\sin\alpha\cos(2\alpha+\lambda_3)]+\frac{\sin\lambda_4}{\pi}\times$$
$$[\sin\beta\cos(2\beta+\lambda_4)-\sin\alpha\cos(2\alpha+\lambda_4)]+$$
$$\frac{1}{\pi}\frac{V}{E_m}\left\{-\frac{\sin(\lambda_2-\lambda_1)\cos\lambda_3}{\cos\lambda_1\sin\lambda_2}[\sin(2\beta+\lambda_3)-\right.$$
$$\left.\sin(2\alpha+\lambda_3)]\right\}+\frac{1}{\pi}\frac{V}{E_m}\{(\sin\lambda_3)[\cos(2\beta+\lambda_3)-$$
$$\cos(2\alpha+\lambda_3)]-(\sin\lambda_4)[\cos(2\beta+\lambda_4)-$$
$$\cos(2\alpha+\lambda_4)]\}\quad(25)$$

For the case of no tube drop or tube resistance, equations 22 to 25 inclusive, become:

For $n=1$:

$$\frac{M_1R_L}{E_m}=\frac{1}{2\pi}\{\gamma-\sin\gamma\cos[\gamma+2(\alpha+\lambda_2)]\}\quad(26)$$

$$\frac{N_1R_L}{E_m}=\frac{1}{2\pi}\{\sin\gamma\sin[\gamma+2(\alpha+\lambda_2)]\}\quad(27)$$

For $n=2$:

$$\frac{M_1R_L}{E_m}=\frac{1}{\pi}\left\{\sin\beta-\sin\alpha-\frac{\sin3\beta-\sin3\alpha}{3}+\right.$$
$$(\sin\lambda_4)[\sin\beta\sin(2\beta+\lambda_4)-$$
$$\left.\sin\alpha\sin(2\alpha+\lambda_4)]\right\}\quad(28)$$

$$\frac{N_1R_L}{E_m}=\frac{1}{\pi}\left\{\cos\beta-\cos\alpha-\frac{\cos3\beta-\cos3\alpha}{3}+\right.$$
$$(\sin\lambda_4)[\sin\beta\cos(2\beta+\lambda_4)-$$
$$\left.\sin\alpha\cos(2\alpha+\lambda_4)]\right\}\quad(29)$$

The ripple fraction depends only on (ωCR_L), R_L/R_R, V/E_m, and n.

(d) Maximum Tube Current

To determine the maximum tube current i_m for the case of no tube drop:

From equation 7:

$$i_R=\frac{E_m}{R_L}\frac{\sin(\theta+\lambda_2)}{\cos\lambda_2}$$

Since maximum i_R is i_m:

$$\frac{di_r}{d\theta}=\frac{E_m}{R_L}\frac{\cos(\theta+\lambda_2)}{\cos\lambda_2}=0$$

or $\theta=90°-\lambda_2$

First case when

$90°-\lambda_2\geq\alpha$

or for

$n=1,\quad 0\leq\lambda_2\leq74.3°$

for

$n=2,\quad 0\leq\lambda_2\leq64.65°$

$$i_m=(E_m/R_L)\frac{1}{\cos\lambda_2}$$

$$\frac{i_m}{I/n}=\frac{\text{maximum current per tube}}{\text{average current per tube}}$$
$$=\frac{n}{(IR_L/E_m)\cos\lambda_2}$$

Second case when

$90°-\lambda_2<\alpha$

or for

$n=1,\quad 74.3°\leq\lambda_2\leq90°$

for

$n=2,\quad 64.65\leq\lambda_2\leq90°$

$$i_m=(E_m/R_L)\frac{\sin(\alpha+\lambda_2)}{\cos\lambda_2}$$

$$\frac{i_m}{I/n}=\frac{\text{maximum current per tube}}{\text{average current per tube}}$$
$$=\frac{n\sin(\alpha+\lambda_2)}{(IR_L/E_m)\cos\lambda_2}$$

(e) Inverse Peak Voltage

To determine E_p for the case of no tube drop:

For $n=1$, the inverse voltage on the rectifier tube between $\theta=\beta$ and $\theta=(\alpha+2\pi)$ is

$$-E_m\sin\theta+R_Li_L=-E_m\sin\theta-$$
$$E_m(\sin\beta)\epsilon^{-(\theta-\beta)\cot\lambda_2}\quad(30)$$

The angle at which the maximum inverse voltage occurs is found by putting the derivative of (30) with respect to θ equal to zero and solving the resulting equation for θ.

This is

$$-E_m\cos\theta+E_m\times$$
$$(\sin\beta\cot\lambda_2)\epsilon^{-(\theta-\beta)\cot\lambda_2}=0$$

and if the angle θ thus found, is substituted in (30), the inverse peak voltage E_p will result.

For $n=2$, $E_p=2E_m$.

References

1. Theorie der Stromrichter (a book), A. Glaser and K. Müller-Lübeck, volume 1, 1935, pages 56–61.

2. The Diode as Half-Wave, Full-Wave, and Voltage-Doubling Rectifier, N. H. Roberts. *Wireless Engineer*, volume 13, 1936, pages 351–62, 423–30.

3. Analysis of Rectifier Filter Circuits, M. B. Stout. AIEE Transactions, volume 54, 1935 (September section), pages 977–84.

4. An Analysis of Copper-Oxide Rectifier Circuits, P. O. Huss. AIEE Transactions, volume 56, 1937 (March section), pages 354–60, 366–7.

5. Half-Wave Gas Rectifier Circuits, C. M. Wallis. *Electronics*, volume 11, October 1938, pages 12–14.

6. Theory and Applications of Electron Tubes (a book), H. J. Reich, 1939, pages 539–42.

7. Behavior of Half-Wave Rectifiers, M. B. Stout. *Electronics*, volume 12, September 1939, pages 32–4.

D-C Machine Flashover and Bus Short-Circuit Protection

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Synopsis: The transient induced voltage across the interpole and compensating windings of d-c motors or generators on short circuit or flashover provide a quick and convenient means of actuating circuit breaker or other protective devices. Such means have made possible quicker tripping times than are obtained with the conventional series overload or reverse current protection.

LIKE an Indian Scout with his ear to the ground—quick to forewarn of approaching danger—the interpole transient may be used to instantly and infallibly guard the dynamo against costly outage.

It is generally appreciated that the concentration of power on a-c systems in the last twenty years resulting from growth of generating capacity and inter-connection of systems has been accompanied by a proportionate improvement and development in protective relay systems so that the large investment in apparatus could be protected from damage resulting from short circuit or ground faults. This protection is achieved by relays such as differential, impedance, carrier current, ground relays, and the like, as well as by better insulation, adequate bracing of windings, etc., in the machines themselves.

Such latter means prevent damage to a machine until the relaying equipment can act upon occurrence of a fault. The relaying means themselves are operated by characteristics of the system which become active only under the particular condition to be protected against.

In d-c systems it is generally recognized that the machine commutators are the weakest link in the system under short-circuit conditions. This is so because arc-over occurs above a limiting value of current due to the limiting value of volts per commutator bar and commutating ability consistent with economical design. The machine windings and pole pieces are not subject to like limitations as they may be adequately supported for the maximum stresses, as is also the case with bus bars.

In d-c systems in which individual generating units are relatively separated, such as traction systems, short circuit currents are not usually of such magnitude as to present a serious problem, due to the relatively large feeder resistance between stations. It is unusual for a large amount of power to be concentrated on one bus in a single station.

The application of large blocks of d-c power in the steel industry, however, presents a different condition; particularly illustrative is the continuous hot strip mill, and the recent cold reducing mills, which, with modern demands for high speeds and high tonnage, require large amounts of d-c power. Furthermore, accurate speed control and ease in mill manipulation require that the motors and generators be on a single bus of as low resistance as possible to give good speed regulation. The apparatus connected to a continuous hot mill bus may have as high as 500,000 amperes short circuit capacity.

Until recently, short circuit protection for d-c machines consisted simply of an overload trip coil on the air circuit breaker of either time delay or instantaneous type, and where required, reverse current trip coils. Such devices must be set high enough to prevent tripping on maximum load. In addition, even the instantaneous trip type has an inherent delay, which, on short circuit, necessarily allows the current to rise to a higher value than its setting before the circuit can be interrupted and stresses set up are proportionately high.

Attempts have been made, with partial success, on constant or relatively constant loads, to provide inductively actuated trip coils so that tripping would be accomplished by the rate of increase in current, thus distinguishing between load currents and short circuits.

On a hot strip mill particularly, load is applied suddenly to each motor as a strip enters the mill. Though current may and does increase at different rates on load application as compared to short circuits, yet, due to difficulty in predicting such factors as momentary load fluctuations and total inductance of the system, and due to the existence of system disturbances, the protective device has to be set with a wide margin, so that a large proportion of the quickness of response on short circuit is lost.

A great deal of study has been given to d-c breaker design in an effort to secure interruption in the shortest possible time both by quick breaker opening time and improved means of extinguishing the arc. Very commendable results have been accomplished and proved by thorough tests. The object sought is, of course, to stop and arc before the current has risen to a destructive value, or in as short a time as possible after current has risen above the maximum safe overload value.

To protect the machine commutator, however, it is desirable to *prevent*, if possible, any excessive current. Protection on a large steel mill system is more vital than on other d-c systems, since the loss of a single commutator will result in a costly shutdown, for each machine forms a link in the production process. Unlike a utility, the load cannot be shifted to other units on the system. The loss of a machine means loss of production until repairs can be made or a spare machine set in place. The total apparatus protected may cost from 500,000 dollars to a million dollars, and considering overhead, the outage cost may run 200 dollars per hour and more.

The system to be described has been in use for some time on busses up to 40,000 kw capacity. While no bus to bus short circuits have occurred since its installation, there have been some few commutator flashovers. These resulted in the clearing of the circuits by breaker opening with decidedly less current flow in the cases observed, as judged from arcing at the breaker contacts, under flashover condition than under openings due to motor overload. Effects on the commutator were negligible.

There have been two schools of thought on d-c bus layout and protection:

> Separate positive and negative busses sufficiently spaced so that forces between them can be neglected under any possible condition of current. Busses with extra heavy cross section to give best motor speed regulation, supported with heavy insulators. Breakers of sufficient capacity to open maximum short circuit current, with a breaker in each of the positive and negative leads of each machine giving double break. This construction entails heavy expense.

Or

> Only heavy enough bus to be consistent with good motor speed regulation with nominal bracing. Dependence on a protective scheme to prevent excessive current on commutator flashover and short circuit.

The second scheme is based on the experience that bus to bus short circuits

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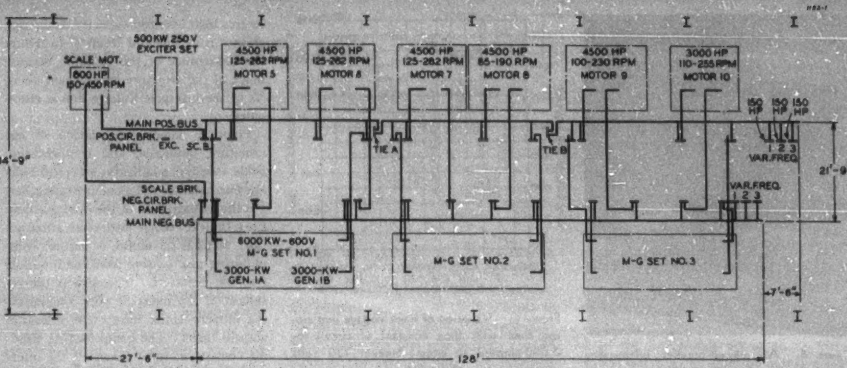

SCALE MOT.
800 HP
150-450 RPM
500 KW 250 V
EXCITER SET
4500 HP
125-282 RPM
MOTOR 5
4500 HP
125-282 RPM
MOTOR 6
4500 HP
125-282 RPM
MOTOR 7
4500 HP
85-190 RPM
MOTOR 8
4500 HP
100-230 RPM
MOTOR 9
3000 HP
110-255 RPM
MOTOR 10
150 HP
150 HP
150 HP
MAIN POS. BUS
POS. CIR. BRK. PANEL
EXC.
SC. B.
TIE A
TIE B
1 2 3
VAR. FREQ.
64'-9"
21'-9"
SCALE BRK.
NEG. CIR. BRK. PANEL
MAIN NEG. BUS
VAR. FREQ.
1 2 3
6000 KW - 600 V
M-G SET NO. 1
3000-KW GEN. 1A
3000-KW GEN. 1B
M-G SET NO. 2
M-G SET NO. 3
7'-6"
27'-6"
128'

Figure 1. Bus circuit layout for modern hot strip mill

seldom ever occur. If one should occur and damage result, the outage would result in less cost than the fixed investment under the first scheme. Both arrangements demand that the machine windings and pole pieces be adequately secured against worst conditions.

The bus shown in figure 1 has the construction of the first scheme and, in addition, the protective relay scheme.

As shown, there is connected to the bus 26,750 horsepower in motors and 18,000 kw in generators, or a total of 38,000 kw connected short circuit capacity, within a space less than 200 feet in length.

Each main bus consists of nine $^1/_4 \times 8$ inch copper bars. Figure 2 shows main positive bus and motor bus for stand number 7 motor. Figure 3 shows an installation with each positive and negative main bus of 2–10 inch aluminum channels and 2–10 inch aluminum channel cross over busses. This view indicates the simpler system of bus supports due to the rigidity of the aluminum channels. With the latter system the bus has a somewhat lower inductance, which has a direct bearing on service requirements of the breakers. (See figure 6.)

Figure 2 (left). Main positive bus and cross-over bus for 4,500 - horsepower mill motor

Figure 3 (below). Main positive bus and crossover bus, channel aluminum construction

To determine how effectively such systems are protected by standard breakers, it is well to examine some standard breaker characteristics.

Figure 4 shows an oscillogram of the interrupting of 41,200 peak amperes d-c overload current in a total time of 0.082 second or 4.92 cycles. Under this test there was no inductance except that inherent in the test generator and connections. Tripping was done by an instantaneous series overload trip coil.

Figure 5 shows test interrupting times of a similar air circuit breaker under both a-c and d-c heavy short circuits. Figure 6 shows variation of system back voltage and arcing time with circuit inductance.

It is to be particularly noted that in each test, although current interruption is relatively fast, the arc energy expended is large. This gives rise to contact deterioration, and the currents involved produce heavy stresses in the connected apparatus.

The effect of these momentary overloads can best be determined by examin-

Figure 4. Air circuit-breaker interrupting test on overload at 42,100 amperes

(No external inductance)

(6) Zero line of back voltage
(7) Zero line of current
(8) D-c overload current
(9) Back voltage across breaker contacts

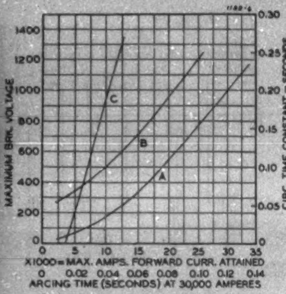


Figure 6. Variation of back voltage and arcing time with time constant of circuit for 6,000-ampere air circuit breaker 750 volts

Overload trip coil shots tested on 275 volt circuit with additional inductance 0.6 mil henry. Overload trip set at minimum approximately 6,450 amperes. The circuit inductance was varied to give time constants as shown in curve B by changing the value of an external inductance coil for the several tests

(*A*) Back voltage per ampere interrupted
(*B*) Variation of time constant with current interrupted
(*C*) Variation of arcing time with time constant

ing some characteristics of the d-c machines under such conditions.

Most of such large concentrations of d-c power are of 600 or 700 volts, have low bus resistance, and are composed of high capacity machines each with a full load *IR* drop in armature and interpole winding of 2 to 4 percent of rated voltage. Thus the short circuit current of a machine may be as high as 25 to 50 times the full load current, which would produce very severe mechanical stresses in the windings and mechanical parts.

If a short circuit occurs on the bus outside the machine, the time for the current to build up to maximum will depend on the resistance and inductance of the complete circuit. Since the bus run must necessarily be short in most large d-c power installations (see figure 1), the resistance and inductance of the lines is a comparatively small part of the whole and will be neglected in the following calculations. The factor of inductance is the only delaying force in the build up of current from the machine, and this has been reduced quite low in modern high capacity machines by the pole face or compensating windings. The compensating winding is spaced, and is of such strength in the face of the main poles as to completely neutralize the cross-magnetizing field of the armature. The purpose of the compensating winding is primarily to limit the maximum voltage between commutator bars to a safe value, and to reduce leakage from the interpoles in order to give high overload capacity to the machine. An incidental result is to reduce the inductance of the machine because much of the armature leakage and about $^1/_3$ of the interpole leakage flux is eliminated.

Figure 5. Air circuit-breaker interrupting time for heavy a-c and d-c short circuits

(*A*) Average time for arc extinction on d-c
(*B*) Maximum time for arc extinction on d-c
(*C*) Average time for arc extinction on a-c
o-points, A.C.B. 8,000 amperes, 600 volts
x-points, A.C.B. 5,000 amperes, 600 volts
Δ-points, A.C.B. 5,000 amperes, 250 volts

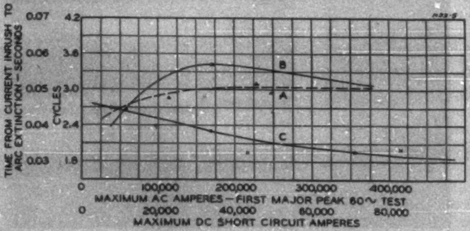


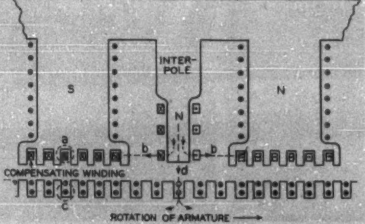
Figure 7 shows the relation of the armature, interpole, and compensating fields, in which *a* indicates a typical leakage flux path around a compensating bar; *b* is the leakage path of the interpole coil; *c* is a typical leakage path of an armature slot; and *d* is the useful, or mutual, commutating flux passing from the interpole into the armature. This flux is the resultant of the interpole plus compensating ampere turns minus the armature ampere turns. The compensating winding consists of turns taken off the interpole coil and disposed in the face of the main pole, resulting usually in the same number of turns in the pole face winding as there are on the interpole.

Figure 8 is a diagram showing the relation between the various magnetomotive forces, and the net resultant magnetomotive forces producing the mutual flux between the armatures and interpoles.

a is the shunt coil magnetomotive force
b is the armature magnetomotive force
c is the interpole and compensating magnetomotive force
d is the net resultant magnetomotive force which is usually from 20 to 40 per cent of the armature magnetomotive force at the center of the interpole

Figures 7 and 8 illustrate the fact that the interpole and armature fields are normally in opposition to each other on the same magnetic circuit, with each field having its own leakage flux. Using this as a basis for determining the action of a d-c machine under transient current loads, we find that the effective inductance is the standard equation of two coils with differential coupling, or $L = L_I + L_A - 2M_{IA}$.

Figure 7. Relationship between armature, interpole, and compensating fields in a d-c machine



L = effective inductance (1)

L_I = self inductance of interpole and compensating fields

L_A = self inductance of armature

M_{IA} = mutual inductance factor, where

$M < \sqrt{L_I L_A}$

The current change after a short circuit, as in figure 9*a*, is:

$$i = \frac{E}{r_1 + r_2}\left[1 - \epsilon^{-\frac{(r_1+r_2)t}{L_1 + L_A - 2M_{1,A}}}\right]. \tag{2}$$

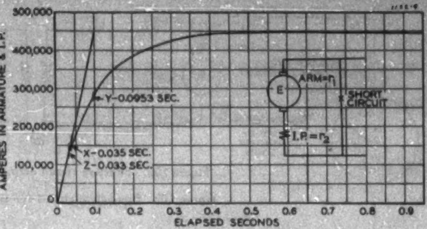


Figure 9a. Effective circuit during bus short circuit

Figure 9b. Current build-up at bus short circuit

Figure 9*b* shows the current build up on external short circuit in a 300 volt, 12,000 ampere generator. Calculations indicate that the effective inductance of a compensated machine is approximately 25 per cent of the unit resistance of the machine (rated volts divided by rated amperes) divided by the rated angular velocity, $\pi Pn/60$ (where P is the number of poles and n is the speed in revolutions per minute). For a non-compensated machine the inductance is about twice this amount. In equation 1 the value of M_{IA} is 60 to 70 per cent of $L_I L_A$ for a compensated machine, and the ratio of L_I to L_A is about 2 to 1.

The initial rate of current build up in figure 9*b* is about 4,500,000 amperes per second, which produces a transient voltage across the interpole and compensating fields of 200 volts, compared to a steady full load voltage across this winding of 3 volts. During normal operation the load builds up on this generator at a maximum rate of about 60,000 amperes per second, giving a normal transient voltage of about 4.8 volts plus *IR* drop. This shows that on external short circuit an interpole voltage of approximately 40 times the magnitude of normal load transient voltage is induced.

It is not uncommon to get a flashover on the commutator of a 600 volt machine because of some transient disturbance. It is worth investigating the difference between transient induced flashovers and short circuits external to the machine. Referring to figure 10*a*, suppose we have a motor with current flowing in at the positive line, through the armature, out the interpole to the negative line. When a flashover occurs, the current will flow in the armature in the direction of the counter electromotive force, while the line will feed in parallel through the arc and into the interpole winding in the normal direction of current. This produces the effect of the armature field and the interpole and compensating fields being additive instead of the normal subtractive condition, and the equivalent circuit is that for two coils in additive coupling connected simultaneously to two separate voltage sources, figure 10*b*. The solution for this case is:

$$i = A\epsilon^{-M_1 t} + B\epsilon^{-M_2 t} + C \tag{3}$$

derived from the fundamental expression,

$$E_1 = i_1 r_1 + L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

which is equation 4 in the appendix.

Figure 11 shows the current flowing into the interpole winding from the line after a flashover on the commutator of a large motor. The maximum rate of current rise is 460,000 amperes per second, and the voltage drop across the interpoles is close to 600 volts as long as the voltage of the line is sustained, since the voltage drop in the arc across the commutator is small. The initial lag in current build up is caused by the mutual induction effect with the armature field, the induced voltage in the interpoles being nearly as high as the line voltage because the turns on the interpole and compensating field are about 30 per cent more than on the armature. This conclusively demonstrates that normal overload protection is subject not only to the handicap of delay due to inaction until the current from the line builds up to an excessive value, but also remains inoperative during the initial lag in line current build up, while the commutator is being subjected to excessive circulative currents.*

Flashover protection should also take care of the third possibility of machine operation—a commutator flashover when the machine is not connected to the line, such as a generator at full voltage on open circuit. The solution remains the same as equation 6 (derived in the appendix), but the result can be simplified because of the open circuit secondary which is the interpole circuit. In this case, $E_1 = L_1\frac{di_1}{dt}$ when $t=0$, which is the time of most rapid current build up in the armature caused by the flashover. Then the interpole voltage will be $E_2 = M\frac{di_1}{dt}$, so that normally the maximum voltage that appears across the interpoles $= E_1 \times \frac{M}{L_1}$. The factor $\frac{M}{L_1}$ is usually above $^3/_4$, so that the initial voltage appearing across the interpoles at the very start of the flashover is far above any normal operating voltage.

Referring to figure 9*b*, in case of a bus short circuit, assuming values of tripping time from figure 5, tripping will occur at point *x*, corresponding to 145,000 amperes, in 0.035 second, when using a standard overload trip coil (neglecting the effect of any external inductance).

If the average tripping time obtained from figures 4 and similar tests, is used, tripping will occur as indicated at point *y* at 275,000 amperes in 0.0953 second. Serious deterioration of the breakers will result and stresses will be very high. These values are conservative, because time constants under test are less than

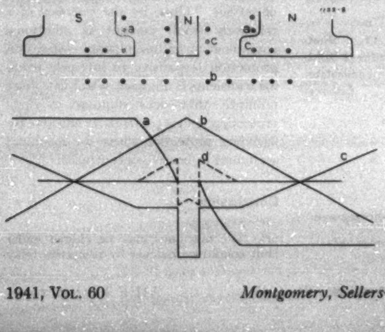


Figure 8. Magnetomotive forces in a d-c machine

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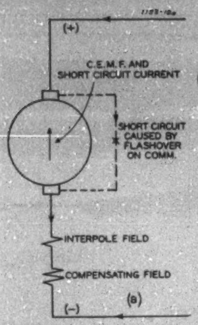
* The circulative current referred to is a current flowing through the arc from brush to brush, while the current actuating the series overload trip must flow from the bus.


(+)
C.E.M.F. AND SHORT CIRCUIT CURRENT
SHORT CIRCUIT CAUSED BY FLASHOVER ON COMM.
INTERPOLE FIELD
COMPENSATING FIELD
(a)
(−)

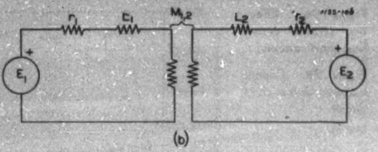


Figure 10a. Effective circuit during commutator flashover

Figure 10b. Equivalent circuit during commutator flashover

those found under actual conditions of installation. Consequently, the tripping time would be more nearly that corresponding to figure 4. However, since latest design of breakers give a shorter tripping time, the average obtained as above is used.

Referring now to a case of commutator flashover, as illustrated in figure 11, and assuming the same values of tripping time, tripping occurs in 0.075 second at 25,000 amperes, as indicated at point *x*, or in 0.135 second at 52,000 amperes, as indicated at *y*. It is highly desirable that circuit interruption be effected before these high values of current are reached.

To accomplish this, advantage may be taken of the machine transient conditions described above, in order to obtain improved protection. Figure 12 shows an effective transient protective scheme when used for two mill motors supplied by two generators in parallel on one bus.

Consider motor number 2. This machine is protected by a circuit breaker *GM*2 in its positive lead. This breaker has two shunt trip coils. One is a standard trip coil (*GM*2—#2 trip) shown in the lower left hand corner of the diagram as operated by a control switch in conventional manner. A special low resistance shunt trip coil (*GM*2—#1 trip) is connected across the motor interpole winding. As previously explained, during normal operations transient voltages of not more than 4.8 volts occur across the interpole, whereas on a bus to bus short circuit the transient voltage produced is 200 volts. By means of resistance R_1 the value of voltage at which this trip coil will operate is set to a safe value above 4.8 volts. Thus it has a small current flowing through it normally. Upon a bus to bus short circuit, however, the induced voltage across the interpole and compensating winding rises and quickly trips the breaker. Since the voltage across the interpole instantly becomes at least 10 times its maximum load voltage, it immediately actuates the special breaker trip coil (*GM*2—#1 trip) before line current can build up to an excessive value.

On a flashover, the circuit becomes as shown by the dotted line around the armature (figure 12), and full bus voltage is instantly projected across the interpole and also the interpole trip coil.

Imposition of full bus voltage on the interpole trip coil, normally set to be actuated at, say 10 volts, causes very rapid actuation of this coil to trip the breaker. Thus it is seen that an interpole trip coil is effective for rapid circuit interruption upon occurrence of a fault, whether that fault be an external short circuit or a commutator flashover. If breaker construction is such that the special interpole shunt trip coil opens the breaker in two cycles, or 0.033 second, referring to figure 9*b*, the circuit will be interrupted at point *z* corresponding to 135,000 amperes on a bus to bus short circuit rather than at 150,000 amperes with conventional breaker overload protection. The advantage is even more pronounced on commutator flashover. Referring to figure 11, the circuit will be opened at point *z* corresponding to 7,000 amperes, if the machine is operating at no load, rather than the 25,000 amperes under conventional protection. With certain machine construction, the current from the bus will even reverse initially. Breaker construction can be made such that tripping from the special interpole trip coil is accomplished in less than the two cycles given above.

While the above discussion relates to motor protection, it is equally applicable to a d-c generator as illustrated in figure 12.

While the interpole trip coil is operative for both types of faults, its operation is even more rapid in the case of commutator flashover, because it is then subjected to full bus voltage.

In a mill installation, however, since a fault on one machine completely interrupts continuity of production, it is desirable to instantaneously trip all breakers, clearing the bus upon occurrence of such fault. Consequently, a blocking rectifier is inserted in the circuit and a common connection is made from the point between R_1 and the interpole trip coil, figure 12, in the special tripping circuit of motor number 2 to the corresponding point in the circuit to each of the other machines on the system. The presence of the rectifiers prevents the transient voltage, due to fault on one machine, from being fed into the interpole circuit of any other machine so as to interfere with its commutation. However, all trip coils are energized from the fault transient in any machine interpole. Also, relays *X*, *X*′, and *X*″ are connected to the common trip circuit. Contacts of these relays energize the standard shunt trip coils from the control voltage source and serve as back up protection. Each machine armature main circuit knife switch is provided with an extra contact blade to isolate the special trip circuit across its interpole when the machine is taken out of service. Thus, by the insertion of a relatively small amount of additional equipment at an insignificant cost circuit protection responsive to interpole winding transients is obtained, which acts more promptly than does standard overload protection, and is particularly effective for flashover protection where d-c machines are found in heavy concentration.

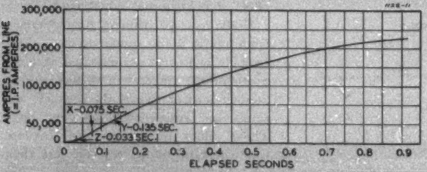


Figure 11. Short-circuit current build-up for commutator flashover

Build-up of current in interpoles from bus after flashover on commutator on a 3,500-horsepower 600-volt $\frac{165}{400}$-rpm d-c motor (Using constant inductance, line voltage, and motor counter emf)

Conclusion

(*a*). D-c machines may be cleared under fault conditions quicker by protective relay

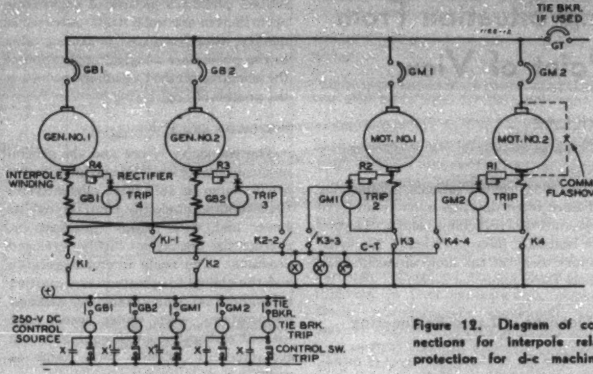


Figure 12. Diagram of connections for interpole relay protection for d-c machines

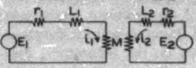
equipment responsive to transient machine characteristics than by conventional means responsive to load current.

(*b*). Such protection eliminates the necessity for bus tie breakers for sectionalizing faults.

(*c*). With adequate back up protection for breaker tripping, bus construction may be safely used with less margin against maximum ultimate short circuit stresses.

Appendix

Derivation of Transient Current for Accumulative Coupling of Two Coils When Connected Simultaneously to Two Different Voltage Sources (D-C)



$(M<\sqrt{L_1L_2})$

$$E_1=i_1r_1+L_1\frac{di_1}{dt}+M\frac{di_2}{dt} \quad (4)$$

$$E_2=i_2r_2+L_2\frac{di_2}{dt}+M\frac{di_1}{dt} \quad (5)$$

Differentiate (2):

$$0=r_2\frac{di_2}{dt}+L_2\frac{d^2i_2}{dt^2}+M\frac{d^2i_1}{dt^2} \quad (6)$$

From (1):

$$\frac{di_2}{dt}=\frac{1}{M}\left(E_1-i_1r_1-L_1\frac{di_1}{dt}\right) \quad (7)$$

Differentiating (7):

$$\frac{d^2i_2}{dt}=-\frac{r_1}{M}\frac{di_1}{dt}-\frac{L_1}{M}\frac{d^2i_1}{dt^2} \quad (8)$$

Substituting (7) and (8) in (6):

$$0=\frac{r_2}{M}\left(E_1-i_1r_1-L_1\frac{di_1}{dt}\right)-\frac{r_1L_2}{M}\frac{di_1}{dt}-\frac{L_1L_2}{M}\frac{d^2i_1}{dt^2}+\frac{M^2}{M}\frac{d^2i_1}{dt^2} \quad (9)$$

Simplifying (9):

$$\frac{d^2i_1}{dt^2}\left(\frac{L_1L_2-M^2}{M}\right)+\frac{di_1}{dt}\left(\frac{r_1L_2+r_2L_1}{M}\right)+i_1\left(\frac{r_1r_2}{M}\right)=\left(\frac{E_1r_2}{M}\right) \quad (10)$$

Simplifying (10):

$$\frac{d^2i_1}{dt^2}+\frac{di_1}{dt}\left(\frac{r_1L_2+r_2L_1}{L_1L_2-M^2}\right)+i_1\left(\frac{r_1r_2}{L_1L_2-M^2}\right)=\left(\frac{E_1r_2}{L_1L_2-M^2}\right) \quad (11)$$

Solution for (11) is:

$$i_1=\frac{E_1}{r_1}+A_1\epsilon^{m_1t}+A_2\epsilon^{m_2t} \quad (12)$$

Similarly:

$$i_2=\frac{E_2}{r_2}+B_1\epsilon^{m_1t}+B_2\epsilon^{m_2t} \quad (13)$$

Auxiliary equation is:

$$m^2+\left(\frac{r_1L_2+r_2L_1}{L_1L_2-M^2}\right)m+\left(\frac{r_1r_2}{L_1L_2-M^2}\right)=0 \quad (14)$$

By binomial equation:

$$m_1=\frac{-(r_1L_2+r_2L_1)-\sqrt{(r_1L_2+r_2L_1)^2-4r_1r_2(L_1L_2-M^2)}}{2(L_1L_2-M^2)} \quad (15)$$

By binomial equation:

$$m_2=\frac{-(r_1L_2+r_2L_1)+\sqrt{(r_1L_2+r_2L_1)^2-4r_1r_2(L_1L_2-M^2)}}{2(L_1L_2-M^2)} \quad (16)$$

When $t=0$, $i_1=0$

$\therefore 0=I_1+A_1+A_2$ from (12)
$$\therefore A_2=-(I_1+A_1) \quad (17)$$

and

$0=I_2+B_1+B_2$ from (13)
$$\therefore B_2=-(I_2+B_1) \quad (18)$$

From (12) and (17):

$$i_1=I_1+A_1\epsilon^{m_1t}-(I_1+A_1)\epsilon^{m_2t} \quad (19)$$

From (13) and (18):

$$i_2=I_2+B_2\epsilon^{m_1t}-(I_2+B_1)\epsilon^{m_2t} \quad (20)$$

Eliminate $\frac{di_2}{dt}$ between (4) and (5):

$$L_2\times(1) \text{ and } -M\times(2) \quad (21)$$

$$L_2E_1=i_1(r_1L_2)+\frac{di_1}{dt}(L_1L_2)+\frac{di_2}{dt}(L_2M) \quad (21a)$$

$$-ME_2=-i_2(r_2M)-\frac{di_2}{dt}(L_2M)-\frac{di_1}{dt}(M^2) \quad (21b)$$

$$L_2E_1-ME_2=i_1(r_1L_2)+\frac{di_1}{dt}(L_1L_2-M^2)-i_2(r_2M)$$

or

$$i_2=\frac{1}{r_2M}\left[i_1(r_1L_2)+\frac{di_1}{dt}(L_1L_2-M^2)-L_2E_1+ME_2\right] \quad (22)$$

When $t=0$, $i_2=0$,

So

$$0=(r_1L_2)(I_1+A_1-I_1-A_1)+(L_1L_2-M^2)\times[A_1M_1-(I_1+A_1)m_2]-L_2E_1+ME_2 \quad (23)$$

Simplifying (23),

$$0=A_1(m_1)(L_1L_2-M^2)-I_1m_2(L_1L_2-M^2)-A_1m_2(L_1L_2-M^2)-L_2E_1+ME_2 \quad (24)$$

Simplifying (24)

$$A_1=\frac{L_2E_1-ME_2+m_2I_1(L_1L_2-M^2)}{(m_1-m_2)(L_1L_2-M^2)} \quad (25)$$

Similarly

$$B_1=\frac{L_1E_2-ME_1+m_2I_2(L_1L_2-M^2)}{(m_1-m_2)(L_1L_2-M^2)} \quad (26)$$

The Incandescent Lamp Situation From the Engineering Point of View

PRESTON S. MILLAR
MEMBER AIEE

Introduction

AT this time, when the incandescent electric lamp is perhaps at the zenith of its career, there is a certain propriety in recording its status, in appraising it as an illuminant, and in reviewing the circumstances surrounding its production and use.

As an instrument in the advance of civilization, the lamp is highly important. In its technical and economic relationships it has been, and is, the subject of intensive engineering study. Thereby has come benefit to the art of illumination, which it serves well, and to the public welfare, to which it has made large contributions. Incidentally, the lamp illustrates the process of transformation of a luxury through intensive and widespread use into a necessity.

Somewhat of an assay of the incandescent lamp being proposed, it has seemed in order for the writer to undertake it because for several decades, though connected with neither a manufacturer of lamps nor an electric light and power company, he has had continuing contacts with both through independent lamp testing services. This circumstance accounts for an unusually comprehensive acquaintance with developments in lamp engineering. In this respect, this paper may be considered a sequel to an Institute paper presented in 1936 entitled, "The Qualities of Incandescent Lamps."[1]

While the approach in this paper is from the engineering point of view, it will be observed that considerable attention is given to the economic and commercial aspects of the lamp situation. Experience teaches that under American conditions engineering and commercial considerations cannot well be divorced. Economics, of course, is an essential element in engineering consideration. Engineering advance often depends upon commercial exploitation of its achievements. The two are mutually interdependent. No satisfactory assay of the lamp situation may ignore the commercial aspects.

Objectives of Lamp Engineering

Among the evident objectives of incandescent electric lamp engineering are:

High efficiency of light production
Low cost
Simplicity and universality of application
Consistency of performance

EFFICIENCY OF LIGHT PRODUCTION

Though low in ratio of transformation of electrical energy into visible light, the record of gain in light produced per watt expended (figure 1) reflects sustained improvement. It is necessary to turn to luminescence to secure higher efficiency in commercial illuminants.

COST OF LAMPS

As efficiencies of light production have advanced and use of lamps has been extended, costs of lamps have been reduced. The record of figure 2 reflects a continuing high degree of industrial progress.

COST OF LIGHT

The combination of increasing efficiency of light production and decreasing prices of lamps affords a record of increasing lumen output for each cent of lamp cost which is shown in figure 3.

This sustained advance through the years has been more or less paralleled by a continuing decrease in the cost of electrical energy supplied to lamps. The public has benefited by an ever diminishing unit cost of light. Merrill[3] has diagrammed this as in figure 4, in which, for lamps of specification qualities, the chart of his devising has been brought up to date. The outer rectangle represents the unit cost of light in 1921. The culmination of twenty years' effort is shown by the inner rectangle. Data are for the best grade of lamps which consistently have met the Federal Specifications.

The charts which have just been described present a picture of engineering operations in one small field. Advance in quality, reduction in cost, increasing value to the public, are a logical application of the scientific method to affairs—which is the process of engineering.

SIMPLICITY AND UNIVERSALITY

The simplicity of use and the general applicability which Edison demonstrated for his lamp in 1879 have characterized the incandescent electric lamp through its career and, with minor qualifications, characterize the present highly developed products. The ready interchangeability and the ability to operate in any position throughout a wide range of ambient temperatures, even under water, and without elaborate control equipment are characteristics offering large value to users.

Of note also is the close-to-unity power factor which assists in avoiding those extravagant expenditures in power generating, distribution and utilization equipment that a low power-factor lamp would involve.

CONSISTENCY OF PERFORMANCE

The degree to which performance of lamps can be made to approach uniformity is a test of manufacturing competency and is one of the measures of service satisfaction. It has been the history of each type of incandescent lamp of specification quality that progress has been made toward performance consistency so long as the demand for the lamp continued to grow. Figure 5 offers in illustration two comparisons of, respectively, early and fully matured lamp products showing, for performance in the laboratory, distribution of hours lives of individual lamps in terms of the average.

A like progress toward uniformity with each largely-used type of lamp has been

Table I. Rating Tolerances for General Purpose Gas-Filled Lamps

(Federal Specifications)

	Lumens Per Watt (Per Cent)	Watts
Single coil tungsten filament lamps		
1921	±12	±18
1923	±12	±14
1925	±12	±12
1927	±11	± 8
1929	± 7	± 5
1932	± 6	± 4
1933	± 5	± 4
1937	± 4	± 3
1941	± 4	± 3
Coiled coil tungsten filament lamps		
1937	± 7	± 5
1940	± 6	± 4
1941	± 5	± 4

Paper 41-123, recommended by the AIEE committee on production and application of light, and presented at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941. Manuscript submitted March 31, 1941; made available for preprinting May 14, 1941.

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Appreciation is expressed to L. J. Lewinson for assistance in collecting material for inclusion in this paper.

[1] For all numbered references see list at end of paper.

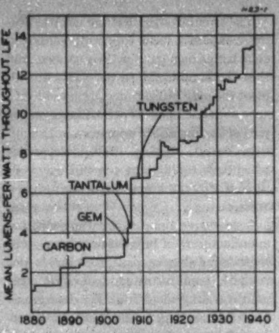


Figure 1. The record of advance in efficiency of light production

(Howell-Schroeder statistics up to 1915)

achieved in the matter of light output per watt for new lamps. Specification quality lamps have been brought within successively narrowed tolerances.

These statements as to attainment of lamp engineering objectives apply to the record of lamp products made by the larger and more competent manufacturers. Their products may be described as specification-quality lamps in that throughout the years they have been made to comply with the requirements of the Federal Specifications for Incandescent Lamps. There are other lamp products that do not attain to the quality required by these Federal Specifications and to which these statements are not applicable.[1] The data apply, however, to the majority of the lamps which have been made and sold in the United States.

The Basis of Lamp Engineering

In the United States it has been the endeavor to conform lamp engineering policies to the best interests of the public. Such an assertion rests upon no claim to especially high ethics or to unusually developed social consciousness on the part of anyone concerned. It rests upon the fact that enlightened selfishness dictates adoption of the best interests of the public as the objective to be sought in engineering procedures.

Of the individual in society Adam Smith in the "Wealth of Nations" wrote:

> "By directing . . . industry in such a manner as its produce may be of the greatest value he intends only his own gain and he is in this, as in many other cases, led by an invisible hand to promote an end which was no part of his intention. Nor is it always the worse for the society that it was no part of it. By pursuing his own interest he frequently promotes that of the society more effectually than when he really intends to promote it."

and on the other side of the picture Smith added:

> "I have never known much good done by those who affected to trade for the public good."

The writer has had the experience of having participated informally in discussions of lamp standardization, at times with responsible representatives of leading lamp manufacturers, at other times with responsible representatives of the largest public utilities, and at still other times with illuminating engineers who are variously connected. He has found these interests, in their capacities as, respectively, manufacturers, purchasers, and users of lamps, to be very generally actuated in their planning and their decisions by a desire to arrive at adjustments and procedures which will best serve the public interest in lamp matters.

Variables Confronting the Engineer

Underlying the standardization of lamps there must be decisions as to selection among available alternatives.

BULB SIZES

For example, it must be decided which of several possible bulb sizes will be used for lamps of a certain watts consumption. This decision is reached with realization that, other things being equal, smaller bulbs make possible more compact luminaires but involve greater blackening and higher temperatures than do larger bulbs. So far as the blackening is concerned, this is illustrated in the case of 25-watt vacuum-type round bulb lamps which are available in both $2^5/_{16}$-inch and $3^2/_{16}$-inch diameter bulbs. Data for the maintenance of light output and efficiency during life are as follows:

Table II. Bulb Size and Output Maintenance

	25-Watt Vacuum Tungsten Filament Lamps	
	G-$18^1/_2$ ($2^5/_{16}$ In.) Bulb (Per Cent)	G-25 ($3^2/_{16}$ In.) Bulb (Per Cent)
Mean throughout life in per cent of initial		
Lumens	85.0	87.7
Lumens per watt	80.2	92.4

TYPE OF TUNGSTEN FILAMENT

Sometimes decision must be reached whether a gain in efficiency of light production is worth more than a greater degree of filament ruggedness. For example, some twenty years ago the manufacturers were using two kinds of filaments offering such a choice.

Table III. Relative Performance of Lamps Having Different Types of Tungsten Wire (1920 Data)

	Type 1 (Per Cent)	Type 2 (Per Cent)
Filament breakage in new lamps after shipment	3.2	1.0
Filaments broken in careful handling during laboratory life tests		
At 300 hours	2.0	1.7
At 590 hours	10.8	3.7
At 880 hours	25.9	4.8
At 1,160 hours	31.5	7.6
Breakage index on a certain shock test*		
New lamps	64	30
After 500 hours	30	16
After 1,000 hours	21	15
Average laboratory life, at common initial efficiency, hours	1,090	2,270
Mean values throughout life, in per cent of initial		
Lumens	93.2	81.6
Watts	97.4	102.7
Lumens per watt	95.7	79.5

* Arbitrary scale.

A selection between these two types of filaments involved the kind of engineering decision which is here referred to. Actually, in the cited case, it was decided to select the first type of filament with its somewhat greater fragility but its better light-producing performance in the hope that gradually the manufacturers would be able to overcome the fragility difficulty. This subsequently proved to be the case.

CONVENIENCE VERSUS COST

Frequently, decisions involve the balancing of convenience against cost and the more or less arbitrary evaluation of each. A recent illustration of this is afforded by the two-filament lamp, used principally in

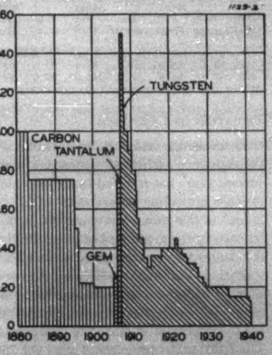


Figure 2. The record of decreasing list prices for the most largely used size of lamps

(Howell-Schroeder statistics up to 1915)

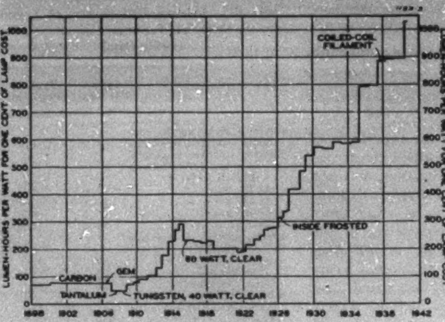


Figure 3. The record of lumen output per watt, per cent of lamp cost

(Howell-Schroeder statistics up to 1915)

Standard central station lamps

portable lighting equipment. To serve convenience, additional switching is provided and more costly lamps are made available in relatively small bulbs, that result in relatively poor light output maintenance through life. It might be difficult to justify such a development on any ground except provision of an appreciated convenience and facility to the public. Certainly it results in an increase in the unit cost of light over that which would be involved in the use of a single filament lamp.

Table IV. Mean Values Throughout Life of Three-Light (Two-Filament) and Single-Filament Lamps in Per Cent of Initial

	Two-Filament Lamps (Bulb Area, 48 Sq. In.)				Single Filament Lamp (Bulb Area, 100 W Lamp 27 Sq. In. 200-Watt Lamp, 56 Sq. In.)	
	Simultaneous Operation		Alternate Operation			
Filament (Watt)	Lumens	Lumens Per Watt	Lumens	Lumens Per Watt	Lumens	Lumens Per Watt
100	80.1	82.0	81.7	83.6	91.5	93.1
200	83.0	84.8	83.3	85.0	89.9	91.9

DURATION OF LAMP LIFE

A decision reached and reaffirmed through considerations extending over several years has resulted in the adoption of the prevailing convention of 1,000 hours' or 750 hours' average life in the laboratory for most sizes of general lighting lamps.[1,3] Setting aside unusual installations that might justify a longer or a shorter lamp life, and dealing with the generality of use, experience appears to have confirmed engineering judgment. The 1,000 hours (or in some cases the 750 hours) represents a balance between two considerations. In the case of 60-watt lamps, selling at 13 cents, and supplied with 4-cent current, the lowest cost of light would be obtained through use of a lamp designed for a life of the order of 330 hours. Thus, considering only the cost of lamps and energy, a shorter than prevailing life for lamps would be indicated. On the other hand, the advantages of convenience and freedom from replacement annoyance dictate a longer life. Here an important consideration is that relief from replacement increases largely with increased lamp life, while the cost of light increases but slightly with increased lamp life, throughout the range of commercial applications.

In all such matters the guiding principle has been the greatest satisfaction to the public upon which the maximum acceptance of lamps depends.

Favorable Aspects and Good Results

Numerous favorable dispositions, some perhaps peculiar to the United States, have affected the development of the lamp and its use with varying results.

EMPHASIS UPON GOOD QUALITY

It is difficult to say whether it was the spirit of the times or a chance disposition of fortune that gave rise to early emphasis upon good quality in incandescent lamps. We do know that Edison stressed this point from the start of commercial production of his lamps (figure 6). Following his example, there arose toward the latter part of the last century a group of men who perceived the essential importance of achieving in lamps the highest quality which it was commercially feasible to accomplish. These men introduced a regard for high quality, demonstrated its desirability, and so entrenched it in practice that to an unusual degree it has influenced the entire history of the incandescent lamp in this country.

The circumstance that the electric utility companies were supplying incandescent lamps as a part of their service, and that the lamp in the earlier years was almost the only utilization equipment led to specialization upon its performance on the part of leading utility operators, with salutary results. Those utility companies which now supply lamps in this way continue to exercise a large influence for good quality.

A predisposition among the principal manufacturers of lamps toward research, engineering development, and highly developed manufacturing technique met with encouragement from the determination of these largest purchasers of incandescent lamps to secure the best lamp performance that was commercially attainable. Manufacturers' expenditure for research and improvement in quality brought reward because principal purchasers were measuring quality and purchased the products of highest attainment. It is a fact that purchasing interest will stimulate producers to provide good quality while purchasing indifference will fail to bring such results. Lord Lyttleton, decades ago, illustrated this principle by

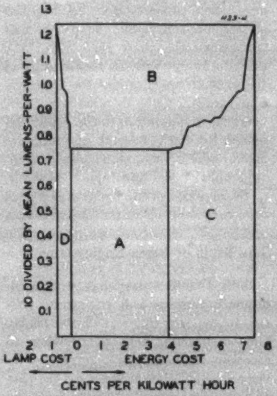


Figure 4. Lamp and energy cost of light (dollars per million lumen-hours) through the last 20 years

A—Cost of light—1941.........($3.04)

B—Reduction in cost since 1921 through increase in lamp efficiency...........($3.38)

C—Reduction in cost of light since 1921 through lowered rates for energy.....($3.25)

D—Reduction in cost of light since 1921 through lowered lamp prices.......($0.54)

A+*B*+*C*+*D*—Cost of light in 1921....... ($10.21) making his beautiful city girl in the country say—

> "Where none admire
> 'Tis useless to excel,
> Where none are beaux,
> 'Tis vain to be a belle."

Today, standards of quality in the incandescent lamp field are high. The Federal Specifications for Incandescent Lamps, revised periodically, call for good performance. The more competent manufacturers of lamps through research, engineering development, and advanced manufacturing technique quite generally equal or exceed these requirements. Their methods for accomplishing this include continual independent checks of performance to insure that it is accomplished, which same checks, if need be, may demonstrate its accomplishment.

A natural effect of critical purchasing methods in the United States has been that the lamp producing business has gravitated toward manufacturers who have been willing and able to produce the best lamps. The insistence by large purchasers that lamp products shall attain to high quality has tended to encourage manufacturing enterprises which were capable of meeting requirements that best serve the public.

The same influences which made for high quality of incandescent lamps tended to bring about mass production. This brought lowered costs and reduced prices for lamps.

Some evidence as to relative prices in several countries has been afforded by comparisons made by Swope[4] on the basis of the length of time an average workman in certain comparable industries in the several countries would have to toil to earn enough to purchase an incandescent lamp. The comparison was as follows:

Table V. Number of Hours' Work Required to Purchase an Incandescent Lamp (1937)

	Hours		Hours
Austria	1.8	Germany	1.4
Belgium	2.5	Holland	2.4
England	1.3	Hungary	3.4
France	1.2	Italy	2.0
United States of America	0.2		

Out of the laboratories and factories of the principal manufacturers of incandescent lamps in the United States have flowed a series of advancements and refinements which little by little have improved the qualities of incandescent lamps. From the same sources have come production economies which little by little have lowered the unit cost of the lamps.[5] In consequence, the American public has enjoyed the highest expression of the incandescent lamp making art at remarkably low prices.

Search for Markets Caters to the Public

It is a characteristic of the free enterprise, profit system that business, to prosper, must seek ever widening circles of purchasing power. Accordingly, manufacturers of lamps and electric light and power companies independently and by such means as seemed appropriate to their circumstances, have sought to further the use of lamps. On the principle that every satisfactory experience encourages other like experiences, and as a matter of enlightened self-interest, they have engaged in the development and promotion of good practice in lamp use. This has aided the public while serving their own commercial interests. The principal manufacturers of lamps through publications, service in technical societies, conduct of lighting schools, assistance with regard to new installations, etc., and the electric utility companies through like measures, including services in assisting with lighting layouts and advice regarding lighting, have participated in the advance of illuminating engineering applications. Thus has been formed a beneficent circle, featured by mass production of lamps, improved quality and lowered prices of lamps, greater use of lamps by the public, and more sale of less costly electrical energy for supply of lamps. Commercial interests and the public have benefited accordingly.

Importance of Long-Term Concern

Generally speaking, production, distribution, and use of the lamp may profit by the guidance of those who have long-term, comprehensive concern in the success of electric lighting. It is believed to have been an advantage that the principal manufacturers of incandescent lamps have also manufactured a wide range of electrical apparatus and equipment. Among their products there is a measure of mutual dependence and mutual stimulation. Deficiencies in any one hamper growth in another. Success of one stimulates others. Such manufacturers therefore unquestionably take an industry-wide view and feel a concern in the continuing success of lamps.

Manufacturers who have a long-term concern in the success of electric lighting and of a wide variety of equipment involved therein have no incentive to produce and sell novelties which offer little of real value but which temporarily might effect a profitable sale. To insure long-term business success, such manufacturers must be careful of their reputations and must seek to avoid advertising misrepresentations or guidance of the public that ultimately may prove misleading. Through years of testing of lamps privately for clients, during which test results obtained for them have been available to them for commercial purposes, a very general desire to represent findings accurately and honestly has been evident to the writer on the part of the more competent manufacturers of incandescent lamps.

Research in the Lamp Industry

The achievements of organized research which have been responsible for the advance in light production during the last 40 years are too well known to need recitation. American conditions in the lamp industry have resulted in the application to the lamp of the highest type of skillful scientific research, which has been facilitated by adequate equipment and resources. But for these, it is doubtful that electric lighting would have advanced and that the public would have benefited thereby to the existing degree.

This comment regarding research achievements applies as well to the niceties of manufacture. New methods, that make for more consistent and better products, are the result of engineering proficiency. Under the conditions that have existed in the United States, they have been developed to an advanced stage.

Black-Body Radiation an Advantage

Environmental influence, great though it be, is subject to the influence of heredity. The sense of sight in human beings has been evolved under the influence of sunlight. When this came to be supplemented at night by firelight glow, the abundance of daylight was replaced by the sparsity of firelight, and the white light of the sun was replaced by the red glow of the fire. The common characteristic of sunlight and firelight was a continuous spectrum throughout the visible range differing in color principally in that firelight was relatively much stronger in the red or longer wave lengths at one end of the visible spectrum and was much weaker in the blue or shorter wave lengths. It may be noted in passing that the redder glow of firelight became associated with warmth, relaxation, and pleasurable sensations in contrast with the white light of the sun which was associated with the intense activities of the day. There is thus evolutionary basis for speaking of the redder tints as warm and for associat-

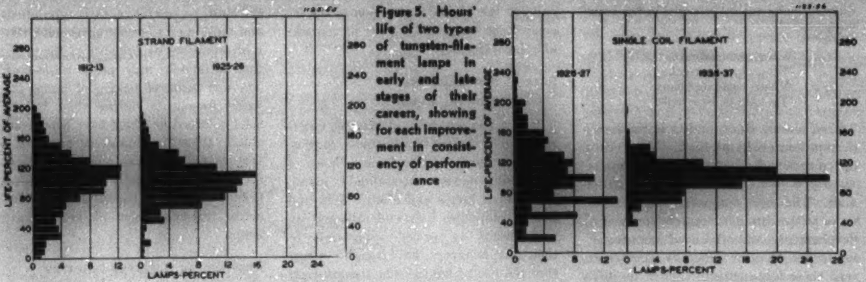


Figure 5. Hours' life of two types of tungsten-filament lamps in early and late stages of their careers, showing for each improvement in consistency of performance

ing them with social functions, while associating closer attainment to white light with the industries.

As artificial illuminants were developed, the incandescence of the flame, whether of fats or oils and whether burned crudely or with controlled drafts, found place along the black-body locus somewhere between the red glow of the fire and the white light of the sun. Not until the incandescent mantle gas lamp made its appearance did there enter into common use an illuminant which departed materially from the characteristic of black-body radiation. When electric lamps made their appearance, the carbon arc with its approach to the light color of the sun and the incandescent lamp with its lower color temperature were also found to lie on the locus of black-body radiation. Only the flame arc lamp with its selectively radiating impregnated electrodes, and, in modern times, mercury, sodium, neon, and the fluorescent lamps with their selective radiation, have departed materially from the black-body radiation characteristics. The incandescent electric lamps have a history of progress along the black-body radiation curve leading gradually away from the fire-glow in the direction of the white light of the sun (figure 7).

Light which does not depart materially in color characteristic from black-body radiation seems to satisfy human visual wants and to be pleasing. It meets with ready acceptance, apparently as a result of association and of developments of the evolutionary process. It is a valuable asset of the incandescent electric lamp that it produces light[6] which is eminently satisfactory in this particular.

Standardization

Lighting practice in the United States has benefited to an unusual degree by the fruits of lamp standardization. The prescience of Edison in selecting operating voltages for multiple circuits and in devising bases and sockets influenced later practice in these matters. The unfortunate inability early in the carbon lamp era to control carbon filaments led to dispersion of operating voltages throughout the 100- to 125-volt range in an endeavor to conform them with the voltages at which lamps as produced would yield stipulated candlepower. With the coming of the controllable tungsten filament, this trend as to operating voltages was reversed, with the result that during the last three decades operating voltages have tended toward standardization with 120 volts as the apparent goal.

Frequencies in the United States, with few exceptions, were standardized at 60 cycles per second—a frequency high enough to avoid flicker difficulties in the case of incandescent lighting.[7]

Lamp bases have long been standardized.[8] Their dimensions are controlled with care to keep them within close tolerances.

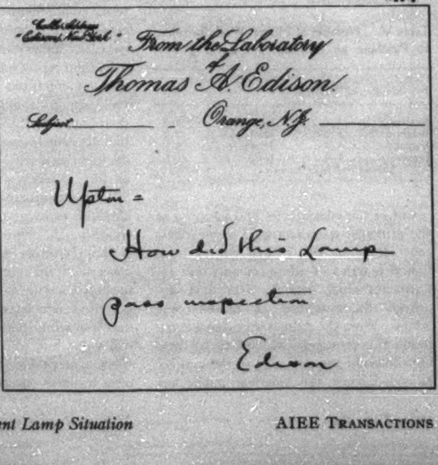
Sizes of lamps (identified for multiple lamps by watts consumed) have been well standardized, though sometimes special requirements have resulted in a larger number of sizes than engineering considerations would sanction.

Bulb dimensions have been arrived at as a compromise, but are well standardized, with consequent advantage to the trade and to the public.

The standardization of lamp efficiencies and lives for specification-quality lamps has resulted in more efficient and somewhat shorter lived lamps in the United States than in some other countries.

Modern lamp standardization has proceeded on the theory that for general lighting purposes the lamp should produce light as efficiently as practicable and should be of standardized construction with a minimum of variants. This theory for general purposes leaves to auxiliaries the task of redirecting, diffusing and, if desired, tinting the light. Through such auxiliaries general purposes may be served and specialized requirements may be met.

There are plenty of exceptions, however, to prove this rule. For specialized



Cable Address
"Edison New York"

From the Laboratory
of
Thomas A. Edison.

Subject ________ Orange, N.J. ________

Upton =

How did this Lamp
pass inspection

Edison

Figure 6. Illustration of Edison's concern for good quality

(Note by courtesy of Mrs. E. P. Lovejoy, nee Jane Howell) applications bulbs are partly silvered, internal reflecting surfaces are provided for certain bulbs to produce the so-called reflector and projector lamps and sealed beam headlamps for motor vehicles. These do not detract, however, from the merit of the principle of standardized light sources for general purposes.

In proportion as standardization succeeds, opportunities are afforded for making something different and for selling it because it is different. The uninformed public may be approached with the argument that something which is different is better. To the degree that such approaches succeed, the purposes of standardization are somewhat frustrated. So in the case of incandescent lamps, bulb shapes and finishes which deviate from the conventional are sometimes adopted as manufacturing variants and attain to a certain sale, irrespective of value to the purchaser. These are perhaps exceptions which prove the rule of the value of standardization.

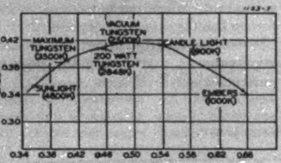


Figure 7. Color temperature of incandescent amps with reference to the locus of black-body radiation

Promotion and Use of More and Larger Lamps

Early incandescent electric lamps were but feeble light sources. So little light was produced that it was necessary to use the light without refinement in its crude state. As light came to be produced more efficiently, as lamps became cheaper, and as the costs of electrical energy were reduced, it became possible to utilize the light more effectively. Instead of employing it in its raw form that produced unsatisfactory illumination, it became possible to some extent to benefit through adherence to the principles which illuminating engineers had come to recognize. Lamps could be shaded. Light could be diffused and redirected with improvement in conditions for vision. The process of furthering conformity with illuminating engineering tenets involved the use of larger sizes of lamps (figure 8) which cost less per unit of light and which produced more light per watt. They were therefore more economical to use, and were better adapted to produce the improved illumination which was sought by all who were concerned in the success of electric lighting.

Table VI. Relative Cost of Light Produced by Lamps of Various Sizes

Lamps (Watts)	Cost Per Hundred Thousand Lumen-Hours*
25	$0.47
60	0.31
100	0.27
200	0.25
300	0.23

* Lamps at list prices, current at 4 cents per kilowatt-hour.

It is interesting to compare the crude lighting of, say, 1911 with that now provided. A 1911 living room with a candelabra fixture and wall brackets equipped with small, bare lamps may be compared with a similar modern room lighted by lamps which conform with IES Recommended Practices. Here are some of the comparative data:

	1911	1941
Total watts of lamps in room	260	400
Lumens produced	1,820	6,400
Maximum brightness to which eyes are exposed, candles per sq. in.	20	3
Average horizontal footcandles	2	8
Cost of light for 75 hours	$1.70	$1.26

It would be a reproach to engineering if, over a period of 30 years, there had been no marked advance in lighting effectiveness and economy. The actual progress is very creditable. The rough comparison indicates that a great deal more illumination is provided with better conditions for visibility and with a reduction in cost as compared with the accomplishments of 30 years ago. It now costs about one-fifth as much to obtain a given amount of lighting as it did in 1911. Actually, more light is now used, refined to greater serviceability, at a cost which is materially less than that incurred for less light more crudely used in 1911.

In proportion as improved results of engineered illumination were achieved, they were so satisfactory and acceptable to the public that they created demands for further applications. There resulted increased sales of lamps, increased sales of electrical energy for supply to lamps, and increased satisfaction to the public from improved illumination. And, due to engineering advance, such improved illumination costs but little more than the crude light formerly provided by less efficient, smaller lamps, supplied with electricity at higher rates. Both business interests and the public profited.

While there has been considerable advance along these lines, much further progress in application of illuminating engineering principles is required to satisfy aspirations for the welfare of the public as affected by artificial lighting. (Though it is not germane to this paper, it is hoped that fluorescent lamps, with their greater production of light per watt, will assist ultimately in achievement of the desired goal of well-engineered illumination.)

Electric Wiring

The advance of electric lighting through use of incandescent lamps has been somewhat hampered by absence from wiring procedure of some of the influences which have been beneficial in the case of the lamp. Following early developments of electric lighting, wiring practice came under the guidance of interests which had neither responsibility nor concern for the ultimate success of electric lighting. It seems to be generally agreed that as a result much electric wiring is inadequate for modern requirements.[9]

Clearly, the lamp cannot render optimum service if its preformance is subject to undue potential drop or to other inadequacies of wiring. As clearly, the achievements of large-scale production in decreasing the cost of light cannot be of maximum advantage to the public if electric wiring is needlessly costly.

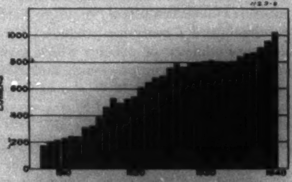


Figure 8. Average initial lumens of large incandescent lamps sold in the United States

Latest Attainments in Incandescent Lamps

The lamp manufacturers advise that the tiniest incandescent electric lamp regularly produced for commercial use is the "grain-of-wheat" lamp for surgical use, consuming about $1^{1}/_{2}$ watts. On the other extreme, the largest incandescent lamp regularly made and sold is intended for flood lighting applications and is rated at 50 kw. Within this range there are thousands of sizes, styles, and types of incandescent electric lamps. There are about 200 which are regularly listed by the manufacturers. The 60-watt, 120-volt lamp is the most generally used size.

It is estimated that during 1940 approximately 575,000,000 "large" incandescent electric lamps, or 4.4 lamps per capita, were sold in the United States. It has been estimated that about one-third of the domestic use of electric energy is for lighting.

So well is the lamp industry being handled that a laboratory procedure, which concerns itself with difficulties encountered in lighting service, finds about as many difficulties arising from the relatively simple auxiliary equipment with which lamps are used as it does from the delicate and complicated lamps themselves. This, in the writer's judgment, is attributable to the fact that, in the case of lamps, manufacturers' quality control methods are highly developed, with results that are constantly being checked by critical purchasers. In the case of sockets, switches, etc., the undoubted competency of many of the manufacturers receives no equivalent stimulus from purchasing interests.

The provisions for determining the performance of the principal makes of incandescent electric lamps and for securing correction where deficiencies are encountered have been described previously[10] and are an outstanding example of measures taken by industry to insure good serviceability of consumer products. Under the influence of policies which give rise to this advanced procedure, and as a result of the procedure itself, standards of lamp quality in the United States are of the highest. It appears that this relatively satisfactory lamp situation is attributable to the fact that, with the public interest as a touchstone, leadership is exercised by manufacturing and purchasing interests that have a long-term concern in the success of electric lighting.

References

1. The Qualities of Incandescent Lamps, Preston S. Millar. AIEE Transactions, volume 55, 1936 (May section), pages 516–23.

2. The Quality of Incandescent Lamps, Howell and Schroeder. Journal AIEE, volume 42, 1923, page 813.

3. More Light for Your Money and Why, George S. Merrill. *Electrical World*, 1937, page 1752.

4. Standards of Living, Gerard Swope. *The Atlantic Monthly*, March 1938.

5. *a*. The Value of Quality in Mazda Lamps, E. W. Beggs. *The Electric Journal*, December 1937, pages 475–7.

b. Precision in Manufacture Improves Lamp Quality, E. W. Beggs. *The Electric Journal*, February 1938, pages 51–4.

6. Search for High-Efficiency Light Sources, Saul Dushman. *Journal*, Optical Society of America, volume 27, 1937, page 1.

7. Lamp Flicker, Utilities Coordinated Research, Inc., 1936.

8. ASME Transaction number 1474. *Transactions*, 1915, pages 25–9, and ASA Standard C44, 1931.

9. The Economics of Light Production With Incandescent Electric Lamps With Particular Reference to Operating Voltage, G. S. Merrill. *Transactions*, Illuminating Engineering Society, 1937, pages 1077–90.

10. Safeguarding the Quality of Incandescent Lamps, Preston S. Millar. *Transactions*, IES, volume 26, 1931, pages 948–58.

New Current Transformer for Bus Differential Protection

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Synopsis: Perfect current transformers would make all differential relaying simple, but such transformers would be of excessive size and cost. A new idea in current transformer design makes it possible to maintain a linear relation between primary and secondary currents in a transformer of reasonable size, even where high current values, completely offset, are encountered. Such a current transformer is particularly valuable for applications where high speed bus differential protection is required. The basic principles of design and test results are reviewed showing that the air gap core current transformer retains a fixed ratio over the full range of current for which it is designed. There is thus made available a current transformer of reasonable size, which may be easily applied and may be used with simple relays.

1. Introduction

SINCE practically all protective relays are energized from the secondaries of current transformers, the overall performance of the relay system is considerably influenced by current transformer action. This is particularly true with the differential forms of protection, where, if the current transformers reproduce accurately the current flowing in each primary circuit, the relay to be used can be of the most simple type. The various features of relays used for differential protection, such as percentage restraint, harmonic restraint, or even time delay, are used, or were developed, to overcome the effects of known or suspected current transformer errors.

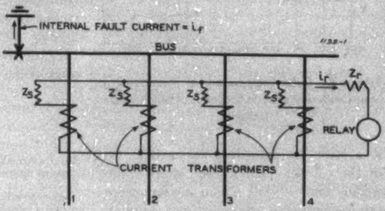

Figure 1. Schematic diagram of current transformers differentially connected to relay

In simple series differential connections, such as are commonly used for generator protection, where each current transformer has the same primary current, it is not essential that the current transformers be capable of reproducing the primary current exactly. If such current transformers are matched (i.e. have identical characteristics) a very simple type of relay is adequate. In practice, matched current transformers are not generally used, preference being given to the percentage restrained relay as a means of overcoming slight differences in current transformer characteristics.

When differential protection is applied to a station bus section, even matched current transformers are not necessarily suitable, because each of the current trans-

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1. For all numbered references, see list at end of paper.

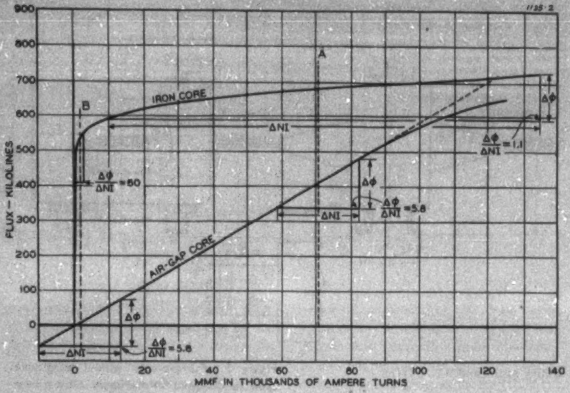


Figure 2. Comparison of magnetization curves for air-gap-core current transformer and conventional current transformer of same dimensions without gaps

formers may carry a different value of current and therefore have a different ratio error (refer to figure 1). In such applications, the vector sum of all the primary currents for an external fault is of course zero, but the vector sum of all the secondary currents will be zero only if each secondary current bears the same relation to its primary current as each other secondary does to its primary. In the application of current transformers to bus protection problems, it has been found that conventional transformers deviate very materially from their nominal ratio at the high current values frequently encountered. The amount of deviation is a function of transformer design, current magnitude, secondary burden, and d.c. time constant. To obtain even reasonably good current transformer performance where high speed bus differential protection is required, as for example, on a 13.8 kv bus section with a short circuit capacity of 1,000,000 kva, requires a transformer of excessive physical size. To overcome the errors introduced by transformer saturation by the use of special relay characteristics, requires a very careful and laborious study of both transformer performance and relay characteristics over a wide range of conditions to insure that the protective equipment will always operate to trip all internal faults and never cause tripping for any external faults.

No amount of relay development seems capable of overcoming these difficulties, but the development of a special type of current transformer in which the secondary current bears a linear relation to the primary current up to values of 25,000, 50,000, or 100,000 amperes would be an ideal solution. This paper describes such a transformer which at all times reproduces the a.c. component of the primary current with the same degree of accuracy. At the same time, in effect, it rejects the d.c. component present in a displaced wave, so that the secondary current is always proportional to only the a.c. primary component. Such transformers when applied to bus problems (or other problems) will produce no relay current for external faults except that due to permissible, and easily predictable manufacturing tolerances.

2. New Principle

As mentioned before, the difficulty with conventional current transformers is not so much that they have errors but that the errors are different with different currents. This variation of error with current is caused by the change of permeability of the iron with flux density. A current transformer having a core material of constant permeability, such as air, would be free from such variations and therefore, it is reasonable to consider the performance of an air-core transformer.

The following mathematical analysis will show that a group of such identical transformers operating together will produce no false differential currents with either symmetrical or assymmetrical current waves.

It is well known that the relation between the primary and secondary currents in a current transformer having constant inductances and resistances can be calculated as follows:

Let

M = Mutual inductance—henrys

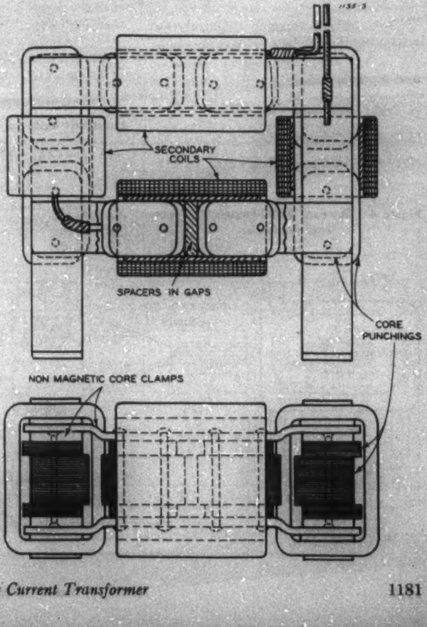


Figure 3. Core and coil construction of air-gap-core current transformer

L_c = Self inductance of the secondary with the primary open—henrys
L_b = Inductance of burden—henrys on individual transformers
$L_s = L_c + L_b$—henrys
R_s = Resistance of individual secondary circuit including secondary coil—ohms
$Z_s = R_s + L_s d/dt$
i_p = Primary current—amperes
i_s = Secondary current—amperes
t = Time in seconds
L_r = Inductance of relay
R_r = Resistance of relay
$Z_r = R_r + L_r d/dt$

The voltage generated in the secondary is, therefore, given by:

$$E_s = -Mdi_p/dt$$

and in the case of one transformer operating a relay E_s also must equal

$$i_sR_s + L_sdi_s/dt + i_sR_r + L_rdi_s/dt$$

Therefore, for one transformer the equation relating the currents is:

$$i_s(R_s+R_r)+(L_s+L_r)di_s/dt = -Mdi_p/dt \quad (1)$$

If now several transformers, say 4, are differentially connected to a relay as in figure 1, and i_f = fault current in the protected area and i_r = the relay current, the equations for the individual current transformers of identical design and identical burdens become for

$$\text{(CT \#1)}\quad -Mdi_{p1}/dt = i_{s1}R_s + L_sdi_{s1}/dt + i_rR_r + L_rdi_r/dt$$

or

$$-Mdi_{p1}/dt = Z_si_{s1} + Z_ri_r \quad (2)$$

and similarly

$$\text{(CT \#2)}\quad -Mdi_{p2}/dt = Z_si_{s2} + Z_ri_r \quad (3)$$

$$\text{(CT \#3)}\quad -Mdi_{p3}/dt = Z_si_{s3} + Z_ri_r \quad (4)$$

$$\text{(CT \#4)}\quad -Mdi_{p4}/dt = Z_si_{s4} + Z_ri_r \quad (5)$$

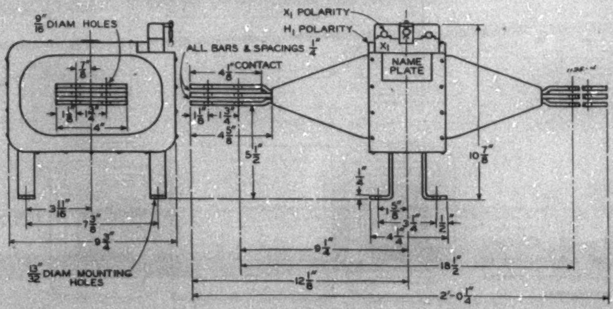


Figure 4. Standard 3,000-ampere JK-9 current transformer

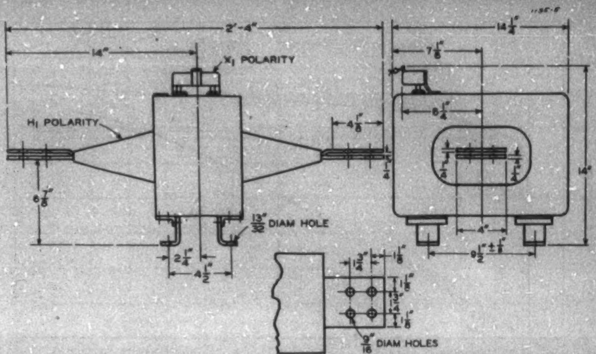


Figure 5. Air-gap-core current transformer type JD designed for maximum rms a-c component of short-circuit current of 50,000 amperes with 100 per cent d-c transient of any time constant, ratio 2,500/1

adding equations 2 to 5 gives:

$$-Mdi_{p1}/dt - Mdi_{p2}/dt - Mdi_{p3}/dt - Mdi_{p4}/dt = Z_si_{s1} + Z_si_{s2} + Z_si_{s3} + Z_si_{s4} + 4Z_ri_r$$

or

$$-Md/dt(i_{p1}+i_{p2}+i_{p3}+i_{p4}) = Z_s(i_{s1}+i_{s2}+i_{s3}+i_{s4}) + 4Z_ri_r$$

but

$$i_{p1}+i_{p2}+i_{p3}+i_{p4} = i_f$$

and

$$i_{s1}+i_{s2}+i_{s3}+i_{s4} = i_r$$

therefore

$$-Mdi_f/dt = Z_si_r + 4Z_ri_r$$

or

$$-Mdi_f/dt = i_r(R_s+4R_r) + (L_s+4L_r)di_r/dt \quad (6)$$

On steady state a.c. this equation becomes

$$-jX_mI_f = I_r(Z_s'+4Z_r') \quad (7)$$

where

$X_m = 2\pi fM$
f = frequency
I_f = effective value of fault current
I_r = effective value of relay current
Z_s' and Z_r' = conventional a.c. impedances corresponding to Z_s and Z_r

It should be noted that equation 6 relating to the fault current in the protected area to the relay current is the same in character as equation 1 relating to the primary current of an individual current transformer to the secondary current. Hence for identical transformers the relay current i_r bears a fixed relation to the total internal fault current i_f regardless of the distribution of this current through the several current transformers, and there can be no relay current unless there is an internal fault. Moreover, this statement holds irrespective of wave form, d.c. transients, or frequency. The principle of the air cored current transformer is thus demonstrated to be sound.

However, for the idea to be practical, the value of i_r corresponding to the minimum internal fault current must be sufficient to operate the relay. It will be noted in equation 7 that the impedance Z_r is multiplied by 4, the number of current transformers. If there were N current transformers, this impedance would be multiplied by N. The relay burden, therefore, acts as if it were N times its actual value. Consequently, assuming a one to one ratio for simplicity, and realizing X_m will always be somewhat smaller than Z_s', the value of X_m must be at least as large as NZ_r' in order that I_r will be a reasonable proportion of I_f. Moreover, any attempt to increase I_r by decreasing secondary turns is ineffective because a decrease in secondary turns decreases X_m.

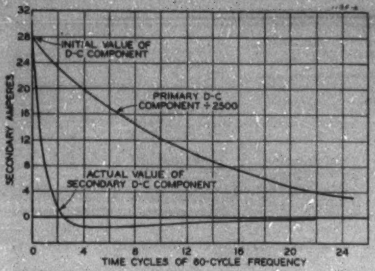


Figure 6. Transient performance of air-gap-core current transformer for 100 per cent offset of primary current wave

Reproduction of d-c transient in current transformer with air-gapped core

Rated 2,500:1 ampere

Therefore, with a core of a given permeability and length there is a minimum value of core cross section which will provide the requisite value of X_m and the longer the core length the greater this cross section. With such low permeability as air, this core cross section has to be quite large even with a normal length of magnetic circuit with the result that the copper of the secondary has to be large to keep down the value of secondary resistance. This in turn requires a larger than normal length of magnetic circuit requiring a still larger cross section so that by the time everything is made large enough to get any desired amount of power out of the secondary of an air core transformer such a device becomes substantially as large as a closed iron core transformer designed for the same service.

3. Practical Use of the Principle

Obviously, therefore, it is desirable to replace the air with a material of higher constant permeability, if satisfactory performance is to be obtained with a transformer of reasonable size. Unfortunately, such a material is not yet available. A reasonable approach to the ideal can be obtained by providing an iron core with gaps in it to maintain a linear characteristic providing the total length of gaps required is not as great as the length of the complete magnetic circuit. It so happens that in the range of short circuit currents usually encountered the total length of gaps required is only about 17% of the necessary length of magnetic circuit. Thus, by using an iron core with gaps, a core which will not saturate but which has about 6 times the effective permeability of air can be constructed. This 6-fold improvement is sufficient to reduce the size of the air core transformer to a reasonable value with the result that a transformer only moderately larger than standard transformers can be built to operate properly at the highest short circuit currents encountered, for which the standard transformer would not be suitable.

A question that is likely to arise at this point is "If a transformer of conventional design requires so much cross-section of iron to reproduce the d.c. transient in the secondary circuit,[2,3] how are air gaps in the magnetic circuit going to make possible a reduction in this iron." The answer is that in the air gap core transformer the d.c. transient can hardly be said to be reproduced at all. A d.c. transient current does occur in the secondary, but its rate of fall is so high and its duration so short that the flux required to generate the voltage corresponding to it is small.

4. Principles of Design

If the equation for the actual primary current (including the transient) be substituted for i_p in equation 1, it can be shown[1] that for a transformer having a constant permeability core the reproduction of the a.c. component of current in the secondary is unaffected by the presence of the d.c. component. This fact, together with the fact that the d.c. in the air gap core current transformer is not reproduced to any extent provides the criterion for the size of the gaps because these two conditions mean that the maximum value of magnetizing ampere turns acting on the core of the transformer will be something slightly less than the sum of the maximum primary d.c. transient ampere turns and the maximum crest value of the a.c. exciting ampere turns that will be required. The latter quantity will ordinarily be a rather small part of the total magnetizing ampere turns and will be determined by the maximum primary current and the maximum percent a.c. error that will be considered permissible. (The limitations of this error will be discussed later.) If the transformer is a bar or window type de-

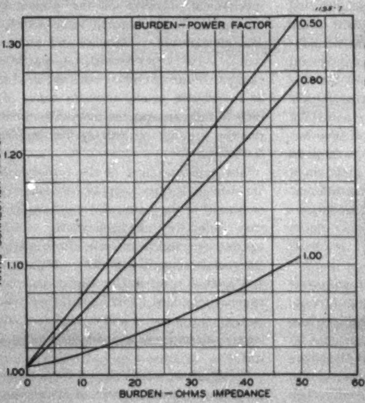


Figure 7 (left). Ratio errors of air-gap-core transformer shown in figure 5, 2,500/1 ratio

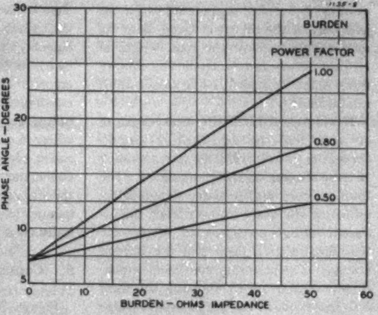


Figure 8 (right). Phase-angle errors of air-gap-core transformer shown in figure 5, 2,500/1 ratio

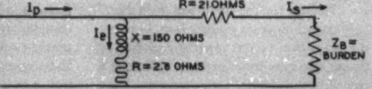


Figure 9 (right). Equivalent circuit air-gap-core transformer shown in figure 5, 2,500/1 ratio

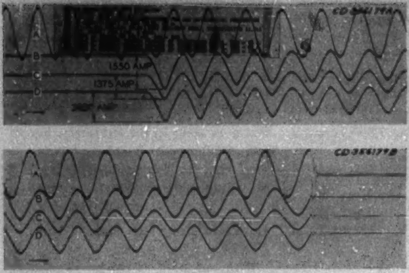


Figure 10. Oscillograms of transformer currents and relay currents for transformers connected as in figure 1

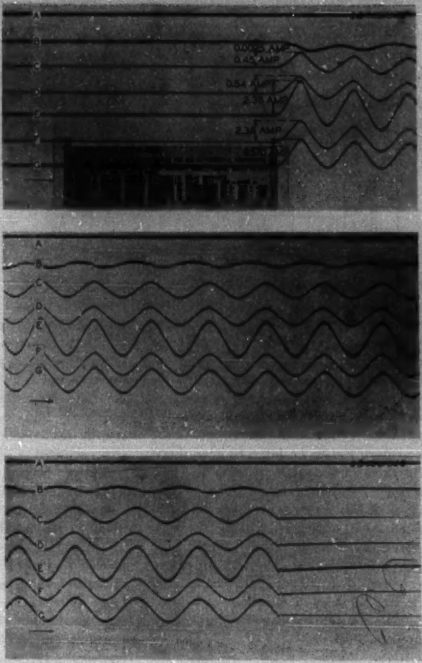
Fault outside protected area

Above:

Curve *A*—generator voltage, 1 mm = 262 v
Curve *B*—current through primary of transformer No. 1
Curve *C*—current through primary of transformer No. 2
Curve *D*—current through primary of transformer No. 3

Right:

Curve *A*—current through differential relay contacts
Curve *B*—current through differential relay coil
Curve *C*—current through secondary of transformer No. 1
Curve *D*—current through secondary of transformer No. 2
Curve *E*—current through secondary of transformer No. 3
Curve *F*—current through secondary of transformer No. 4
Curve *G*—current through primary of transformer No. 4



sign as will usually be the case for bus differential protection, the maximum ampere turns acting on the core are then established. If now the maximum flux density that can be permitted without requiring appreciable ampere turns for the iron part of the circuit is found from the magnetization curve the total length of air gaps necessary to keep the flux density down below this permissible maximum is established.

Having established the total length of air gaps and the a.c. exciting ampere turns, the a.c. flux density corresponding to these ampere turns is determined. The total a.c. core flux necessary is then determined in the usual manner from the induced voltage necessary to supply the secondary coil *IR* drop plus the burden voltage. The necessary core cross-section is thus established by dividing the total core flux required for the induced voltage by the flux density which the permissible a.c. exciting ampere turns will produce.

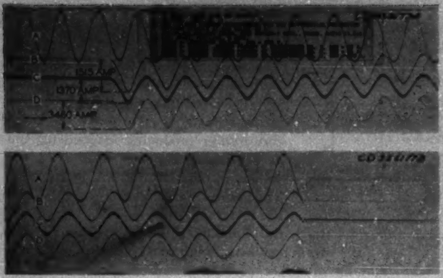
Figure 2 shows a comparison between the operation of the magnetic circuit of the air gap core transformer and that of a conventional current transformer of the same core and coil dimensions without the gaps. Without any d.c. in the magnetizing current, the increment of exciting current required for the a.c. flux is very much smaller in the conventional transformer than in the air gap core transformer, but with much d.c. present in the magnetizing current, the exciting ampere turns in the conventional transformer for the same increment of a.c. flux density is increased enormously, even way above that required for the air gap core transformer whose a.c. exciting current remains the same. It is this constancy of ratio between flux and exciting ampere turns in the air gap transformers which make them balance properly when operating differentially even though their errors are large.

5. Construction

Figure 3 shows the mechanical construction of the air gap core current transformers. The punchings of the core are rounded and stepped back at the gaps to prevent concentration of flux at the edges thereby avoiding local saturation of the iron. Several gaps are employed rather than one and symmetrically disposed in the core in order to reduce interference effects to a minimum as well as to reduce the fringing of flux that would occur with one large gap. Non-metallic spacers of textolite are used in the gaps to insure the mechanical stability of the punchings against the large magnetic forces exerted at high currents. The core and coil assembly is held together with non-magnetic clamps. Four secondary coils are used so as to provide nearly uniform turns per inch of core around the magnetic circuit thereby minimizing leakage flux and interference.

The coils and core of the transformer are surrounded by a copper case soldered or brazed at the joints thereby forming a very effective electromagnetic shield against stray flux interference from nearby current carrying conductors. Figures 4 and 5 show comparative dimensions between a standard 3,000 ampere bar type current transformer and a sample 2,500 ampere air gap core transformer made for test.

This sample was designed for a maximum short circuit current of 50,000 am-



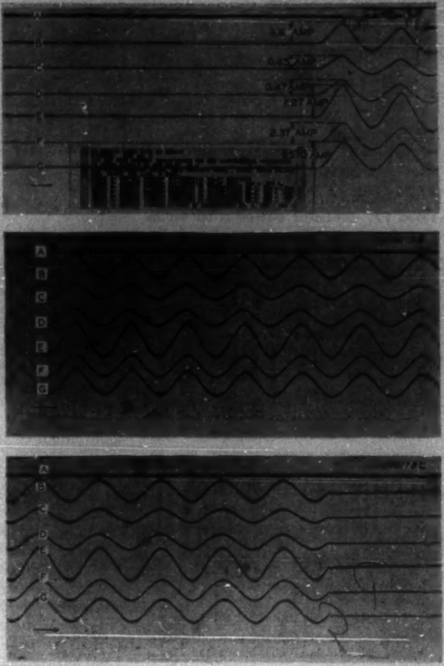


Figure 11. Oscillograms for same conditions as figure 10 except secondary leads of number 4 transformer reversed

Above:

Curve *A*—generator voltage
Curve *B*—current through primary of transformer No. 1
Curve *C*—current through primary of transformer No. 2
Curve *D*—current through primary of transformer No. 3

Right:

Curve *A*—current through differential relay contacts
Curve *B*—current through differential relay coil
Curve *C*—current through secondary of transformer No. 1
Curve *D*—current through secondary of transformer No. 2
Curve *E*—current through secondary of transformer No. 3
Curve *F*—current through secondary of transformer No. 4
Curve *G*—current through primary of transformer No. 4

peres r.m.s. a-c component of current with 100% d-c transient of any time constant. The following tabulation shows a comparison between this sample and a conventional current transformer designed for the same service assuming a 0.25 second time constant for the d-c transient.

Table I

	Air-Gap-Core Transformer	Iron-Core Transformer
Weight	120 lb	500 lb
Size	28"x14"x14¼"	30"x18"x18"

6. Characteristics of the Air-Gap-Core Transformer

Figure 6 shows the transient performance of the 2,500 ampere transformer referred to above with primary current wave 100% offset and a d-c time constant of 0.2 second. Figures 7 and 8 show ratio and phase angle errors of this same transformer plotted against burden. It is not necessary to plot curves of accuracy against current, since the per cent error for a given burden is fixed (within 1% or less) for any current up to the limit of the design.

Figure 9 shows the equivalent circuit of the transformer from which operation under any conditions can be calculated. It is interesting to note that since the d-c in the secondary is almost negligible and since the impedances are constant, any calculation of the performance of the air gap current transformers, either individually or in a group, becomes simply a conventional a-c network problem.

It will be noted from the figures given on the drawings that a one-ampere secondary was used on the transformer referred to. The reason for this as well as the reason for the selection of the impedance values given will be discussed under application.

While theoretically air gap core current transformers of identical design with identical burdens should be perfectly balanced and give no false differential current when operated differentially, in practice some unbalance must be expected not so much on account of any difficulty of matching the transformers and burdens accurately but mostly because of the variation of the secondary resistance with temperature. An analysis of the equivalent circuit, figure 9, will show that the transformer error is affected considerably by variation in the secondary resistance, a 15% variation in the value of 21 ohms producing approximately a 2% vectorial shift of secondary current. A 15% change in resistance will result from about 37°C temperature change, a difference which may readily exist between two transformers on different branches of the same bus, if possible differences in ambient temperature and load current in the two locations are considered. To reduce this effect means an increase in the size of core to increase the magnetizing impedance, or in other words, reduction in the permissible per cent exciting current. This temperature variation factor is, therefore, one of the factors limiting the permissible errors of the transformer.

7. Tests

In order to check the calculations and theory on the air gap core transformer, many tests were made, the more important of which will be described here.

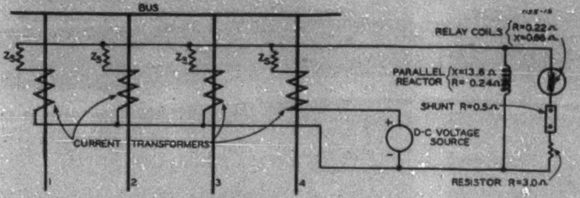


Figure 12. Schematic diagram of test set up for high currents and superimposed d-c in the secondary of number 4 current transformer

First, to be sure that no unforeseen saturation effects existed, an exciting current test on a sample transformer was made in which the crest value of flux was measured by a rectifier voltmeter across the secondary winding and the crest value of current was also measured by a rectifier voltmeter across the secondary of a mutual inductance whose primary was in series with the secondary coils of the transformer. The results of this test are:

Table II

RMS Amperes	Crest Amperes	Crest Amp Turns	RMS Secondary Volts	Average* Secondary Volts ×1.11	Ratio of Average Volts ×1.111 to Crest Amps
12	17.0	42,500	1,832	1,814	106.8
18	25.5	63,700	2,740	2,740	107.2
20.2	28.6	71,500	3,050	3,020	105.5
23.8	33.6	84,000	3,600	3,600	107.2

* Average voltage is measure of crest flux.

It is apparent from the figures that the ratio of flux to exciting ampere turns was linear within the accuracy of the measurements, and that the maximum ampere turns were in excess of 70,000 amperes, the d-c component corresponding to a 50,000 ampere rms a-c component.

Tests were also made for interference effects which were found to be approximately 1% without the copper casing and practically imperceptible with the case.

To find the effect of variation in gaps and iron stacking, impedance measurements were made on four sample transformers and three of the four were then balanced against the remaining transformer one at a time to find the actual differences in ratio and phase angle errors. It was found that although the mutual inductances M and the self inductance L_c of the transformer secondaries varied between one transformer and another by as much as 4% the difference in the errors was less than 1%. The reason for this, is of course, that variations in L_c are always almost exactly the same in percent as the variations in M because practically the same magnetic circuit controls both inductances. Therefore, the accuracy is not greatly affected by small variations of inductances between individual transformers of the same design.

A survey of testing facilities suitable for testing the air gap core current transformers under actual operating conditions showed that about the only way large short circuit currents with large d-c time constants could be obtained was to use a large generating station. As such stations are not readily available for current transformer tests, two sets of tests were devised to duplicate actual conditions as nearly as possible with available equipment.

In the first set of tests, 4 sample 2,500:1 air gap core current transformers of design previously described were connected together essentially as shown in figure 1. Current was fed in through lines 1, 2, and 3 and out through line 4. For the condition of a through fault outside the protected area, the secondaries were connected as in figure 1 and to get the effect of a fault inside the protected area the secondary leads of transformer #4 were reversed. The current was supplied through large reactors in order to get long d-c time constants. Because of saturation, the usefulness of these reactors was limited to a few thousand amperes.

Although the currents were too low to prove the capacity of the current transformer, these tests did demonstrate the correctness of the principles as indicated by the oscillograms, figures 10 and 11. It will be noticed that the primary d-c transient was of long duration whereas the d-c in the secondaries disappeared very quickly. The current in the relay for the through fault test in figure 10 was about 0.6% of the total fault current referred to secondary and the relay did not operate, whereas for the "internal fault" conditions of figure 11 the relay current was twice the fault current referred to the secondary and the relay did operate.

Since long d-c time constants could not be obtained at high currents, in order to produce d-c magnetizing ampere turns on the core equivalent to actual operating conditions a d-c voltage was inserted in the secondary circuit of current transformer #4 and the relay by-passed by a low resistance reactor as shown in figure 12. The d-c voltage produced a d-c circulating current in the secondary of #4 transformer producing magnetizing ampere turns on the core similar to what would be produced by a long d-c transient. The purpose of the reactor in parallel with the relay was to by-pass the d-c around the shunt of the differential circuit without by-passing an appreciable amount of a-c so that the oscillograph sensitivity could be made high enough to measure the unbalance in the transformers without the elements being forced off scale by the d-c, most of which returned through the differential circuit.

The maximum sustained d-c current in the secondary of #4 transformer that could be obtained with the voltage available was 20 amperes making 50,000 ampere turns. The primary a-c which had a short d-c time constant was raised to slightly over a 100,000 amperes rms symmetrical with 50,000 d-c ampere turns superimposed on #4 transformer.

The results of these tests were essentially the same as the tests made at low currents with the large reactors. The maximum unbalance observed was about 2%, some of which was undoubtedly due to the extra burden of the generator armature in #4 transformer secondary. That it was not due to saturation effects was evidenced by the fact that the per cent unbalance was the same for all currents. When the secondary leads of #4 transformer were reversed, the relay current was again approximately equal to twice the total fault current referred to the secondary.

With these tests completed it was concluded that the engineering calculations had been adequately confirmed by test.

8. Choice of Ratings

The selection of transformer sizes and ratings to be available was based primarily upon the maximum rms value of the symmetrical current which might be encountered. Transformer cores suitable for use with primary currents of 25,000, 50,000, and 100,000 amperes, have been designed. These correspond approximately to short circuit capacities of 500,000 kva, 1,000,000 kva, or 2,000,000 kva at 13.8 kv and are intended in fact to be applied on this basis.

Nominal primary ratings which are really continuous current ratings have been established corresponding to the above as 1,000 amperes, 2,500 amperes, and 4,000 amperes. It is possible, however, to wind on these cores suitable coils to permit other nominal ratings if a need for them should arise. It was felt that the ratings selected above were, in general, high enough to carry continuously the maximum single circuit current which might be encountered on busses having the designated short circuit capacity. Since these transformers are intended for differential protection only, there is no need to rate the current transformer down to the circuit rating.

Since these transformers are not suitable for metering service it was felt that the ratio did not have to be such as to give 5 amperes secondary at nominal rating, particularly if there was some good reason for using some other value. As far as the performance of the relay system is concerned, once a selection of relay types and a transformer core size has been made it does not matter what the ratio of transformation may be. Any changes in ratio require a corresponding change in relay setting and hence in relay burden. If we decrease the ratio, using fewer turns, we can increase the relay setting and reduce its burden. But as long as the only load on the transformer is the relay, the overall performance remains unchanged. The reduction in burden is compensated for by the lowered burden capacity of the transformer with fewer turns. Thus, considering only the relays and the transformers it would make no difference what ratio was chosen.

There is, however, the lead burden in addition to the relay burden imposed on the current transformers. Frequently the lead burden may be larger on one transformer than on another. If the accuracy curves figures 7 and 8 were based on 5 ampere secondaries instead of 1 ampere secondary the impedances given would all be divided by 25 and a variation of $^{1}/_{2}$ ohm in lead resistance would be equivalent to a variation of 12.5 ohms on the curves as plotted. The difference in accuracy resulting from such a burden difference is apparent from the curves. A $^{1}/_{2}$ ohm difference in burden on the 1 ampere secondary transformer on the other hand makes practically no difference in accuracy. Since in many applications the secondary lead lengths will vary considerably the selection of a 1 ampere secondary should greatly simplify the application problem by eliminating any necessity of balancing lead resistances.

9. Application

The application problem is made very simple through the use of this air gap core current transformer. Up to the present time the application of high speed bus differential protection has been very largely a matter of judgment. While it is possible to calculate with fair accuracy the performance of any particular installation of conventional design transformers, these calculations are quite laborious and require data on the transformers which have not always been available. With the data from such calculations it has been possible, however, to determine the type of relay necessary to obtain correct performance of the relay system, provided of course that the errors as determined by the calculation are not too great. The amount of time required for such exact determination is so great, however, that most applications have been made based on the individual's judgment of the expected performance and the required relay characteristics to offset the expected error. Despite the generally satisfactory results obtained in service most engineers have felt a considerable degree of dissatisfaction because of the inability to prove the soundness of application made in this way.

The amount of application effort where high speed differential protection is to be applied has proved to be very great. Furthermore, many cases studied showed that standard current transformers were not capable of giving the desired characteristics. When special designs of conventional construction were considered these were found to be either physically too large or too costly to permit installation. Thus, many busses do not have adequate protection.

With the air gap core current transformer, there are no uncertainties regarding performance. The difficulty formerly experienced in finding the required space has been minimized or entirely eliminated since the transformer shown in figure 5 can be made as a bushing type suitable for mounting over either a bushing in station type cubicles or the insulated conductor in metal clad gear. Knowing the maximum symmetrical short circuit current which can be supplied, the current transformers are selected which give a linear performance up to this current value. There is no need to attempt to determine the time constant of the d-c component since the transformer performance is not affected by this factor.

Since for any value of short circuit current within the limit for which the current transformer is designed, a linear relation is maintained between primary and secondary currents, there will be essentially no difference in current for any external fault, and it is thus possible to use a simple overcurrent relay of the instantaneous type. The setting of this relay would be such that it would have a safe operating margin, say 2:1, to insure operation on the minimum internal faults.

With the air gap core current transformer, application difficulty is encountered only where the current to a single phase to ground fault is limited by neutral impedance to a small percentage of the maximum phase fault. While the design and manufacture of the transformers is such that agreement within 3% is expected and that the maximum differential current for any through fault would be generally less than this amount, it is felt that good practice for the moment dictates a minimum relay setting corresponding to not less than 10% of the maximum fault current. More sensitive settings are of course theoretically possible and can be used where necessary but with more careful consideration of all factors.

10. Conclusion

Through the development of the air gap core current transformer there is made available for high speed differential protection a transformer having the following advantages.

It has been designed to operate at currents as high as 100,000 amperes and has a *constant ratio* up to its designed value.

It is of reasonable size and weight.

It is easily applied because it requires only the determination of the maximum symmetrical current to which it may be subjected.

It is not affected by the d-c component of the short circuit current.

It can be used with the simplest type of relay, i.e., instantaneous overcurrent or any other type desired.

It does not produce any appreciable d-c component in its secondary.

Simple and brief calculations establish its performance with certainty for all possible conditions of operation up to the current limit for which it is designed.

References

1. Current-Transformer Excitation Under Transient Conditions, D. E. Marshall and P. O. Langguth. AIEE Transactions, volume 48, 1929, pages 1464–74.

2. Current Transformers and Relays for High-Speed Differential Protection With Particular Reference to Offset Transient Currents, E. C. Wentz and W. K. Sonnemann. AIEE Transactions, volume 59, 1940 (August section), pages 481–8.

3. Harmonic-Current-Restrained Relays for Differential Protection, L. F. Kennedy and C. D. Hayward. AIEE Transactions, volume 57, 1938 (May section), pages 262–6.

4. Considerations in Applying Ratio Differential Relays for Bus Differential Protection, G. B. Dodds, W. K. Sonnemann, and R. M. Smith. AIEE Transactions, volume 58, 1939 (June section), pages 243–9.

Measurement and Control of Conductor Vibration

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ASSOCIATE AIEE

Synopsis: A method is outlined for determining the effectiveness of devices used for prevention of vibration fatigue in transmission conductors. Measuring equipment is described which provides records of conductor vibration in the quantitative form essential for such determinations. Vibration records obtained on typical constructions are analyzed and correlated with fatigue tests and operating experience to determine the probable life of the conductor. Data are given on the performance of test spans of a special loose-core A.C.S.R., as well as A.C.S.R. of standard stranding, for various span lengths, tensions, and equipped with various devices, including a double-suspension clamp, armor rods, Stockbridge dampers and a newly-developed torsional damper. Of the devices tested, dampers are shown to be most effective in reducing vibration stresses.

The Problem

THE occurrence of fatigue failures in conductors, and the more rapid deterioration observed with increase of span length and tension, disclosed a serious hazard in existing transmission lines. Pioneer investigations of Relf and Ower,[1] Varney,[2] and others showed that the fatigue failures were due to vibration of conductors in steady winds of relatively low velocity, and that the vibration was excited by wind eddies which exert alternating vertical forces on the conductor.

Many subsequent studies have been directed toward prevention of vibration fatigue in conductors and in these numerous investigations there naturally were developed many devices for vibration control. Unfortunately no method for comparing their effectiveness has been generally accepted and as a result, such widely different devices as armor rods, festoons, dampers, or special clamps have been extensively applied. Although reports of operating experience indicated satisfactory performance for some of the devices, no definite opinion could be formed of their comparative effectiveness.

Preliminary Studies

The Hydro-Electric Power Commission of Ontario, faced with this difficulty in adopting measures for protection of existing and proposed lines, commenced a program of vibration research. Some phases of this program have been described previously,[3,4,5] in which analyses have been made of the effect of conductor shapes, stress distribution, and energy loss in conductors due to travelling waves, and fatigue characteristics of steel wires.

Preliminary studies indicated the desirability of testing vibration-suppressing devices on full-size wind-excited spans, because no other method took full account of the complex relations between the wind eddies and conductor. In field tests of this nature it was recognized that the vibration records should reliably indicate the effectiveness of the various devices tested, in reducing or eliminating vibration fatigue. A survey was made to determine the limitations of available recording apparatus for this purpose.

Recording Apparatus Available

Clock-driven live-line recorders of the Zenith and Servis types[6] indicate amplitude approximately, but give no indication of frequency. These may be supplemented by cycle counters of the Jaquet type,[7] which register the total number of vibration cycles above a fixed amplitude. From the combined records, it is possible to estimate the approximate amplitude, number of hours vibration per day, and average frequency.

More complex recording mechanisms, using motor driven wax charts[6] or oscillographs[8] have been described. These measure the amplitude, frequency, and wave shape at a point on the conductor two feet or more from the suspension clamp, and thus indicate the effectiveness of dampers in suppressing vibration. They do not, however, provide a measure of the stress-relieving properties of special clamps or conductor reinforcements.

Two methods of recording the actual bending in a conductor adjacent to a suspension clamp, have been described. One[8] uses a mechanical linkage, connected by waxed strings to the conductor on opposite sides of the suspension clamp. Another[9] employs an optical arrangement to record on photographic film, the motion of the vibrating conductor relative to the suspension clamp.

A carbon-pack type of curvature recorder[4] has been found useful for oscillographic recording of travelling waves in conductors, and for recording curvatures due to aeolian vibration. However, its tendency to change calibration when left unattended, made it unsuitable for continuous vibration recording.

Development of Recording Equipment

None of the devices available appeared entirely satisfactory for the purpose of this investigation. In the development of new equipment for recording conductor vibration, the following requirements were recognized:

1. Because all fatigue breaks occur within an inch of suspension or strain clamps, knowledge of stress conditions in that region was considered essential.

2. No strain gauge was available to measure stresses in the individual strands under field conditions. A satisfactory alternative appeared to be the measurement of conductor curvature adjacent to the clamp. This could be accomplished by measuring the deflection of the conductor, relative to the clamp.

3. Deflections of the conductor occurring within an inch of the clamp, even under severe vibration, amount to only a few

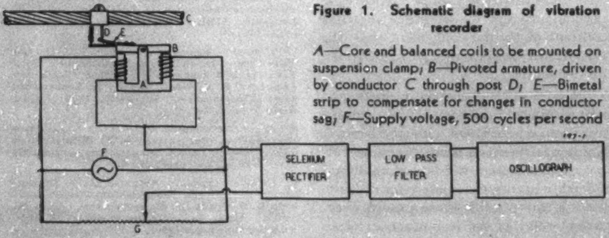

Figure 1. Schematic diagram of vibration recorder

A—Core and balanced coils to be mounted on suspension clamp; *B*—Pivoted armature, driven by conductor *C* through post *D*; *E*—Bimetal strip to compensate for changes in conductor sag; *F*—Supply voltage, 500 cycles per second

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[1] For all numbered references, see list at end of paper.

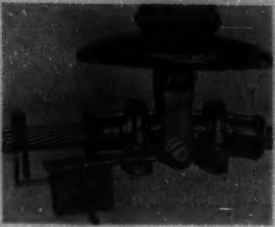

Figure 2. Magnetic device for recording deflections of conductor relative to suspension clamp

thousandths of an inch. One essential of the recording device was therefore a sensitivity of 0.0001 inch.

4. Ability to record vibration on live lines was not considered essential because adequate test spans were available to simulate any existing or proposed construction. Tests could be made on new lines during construction, and system conditions were such that many existing lines could be interrupted for suitable test periods.

5. Other desirable characteristics of the measuring device were the ability to record deflections at all frequencies up to 60 cycles per second, stable calibration, lightness, and ruggedness.

A magnetic method for measurement of small mechanical movements, described by Mershon[10,11] and others, appeared applicable to this problem. This type of measuring device uses an iron magnetic circuit, excited at a relatively high frequency, usually between 500 and 2,000 cycles per second. The length of a small gap in the magnetic circuit is altered by the motion to be recorded. The resulting change in impedance of the exciting coil may be made to produce an electrical output proportional to the motion.

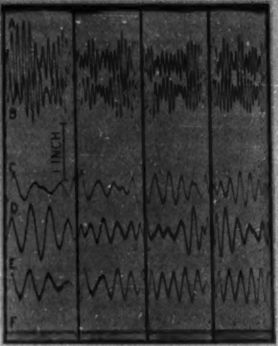

Figure 4. Vibration records on 220-kv line without dampers

A and B—Skywires $^3/_8$ inch steel. One inch = 0.005 inch

C, D, and E—Power conductor 795,000-circular-mil ACSR "Drake." One inch = 0.010 inch

F—Timing wave, 60 cycles per second

Figure 3. Automatic equipment for recording vibration on five conductors

Left—Timing devices and 500-cycle generator

Right—Five-unit control (top) and six-element oscillograph

Figure 1 shows some details of the device developed for recording conductor vibration. Use of the double magnetic circuit shown, resulted in linear calibration over practically the whole travel of the armature. Effect of the non-linear relation of resistance to current in the selenium rectifier at low current was avoided by using a suitable bias current, obtained by initially unbalancing the rheostat (*G*). Sensitivity of the recording elements could be varied to provide ratios of oscillograph deflection to conductor deflection of any value between 10 and 1,000.

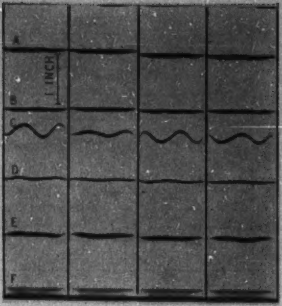

Figure 5. Damped span vibration records

Scales same as figure 4

A and B—Skywires with 25-foot festoons of $^3/_8$-inch steel in five loops

C—North 795,000-circular-mil ACSR with two Stockbridge dampers per span

D and E—Center and south power conductors with two torsional dampers per span

F—Timing wave, 60 cycles per second

Provision was made for an over-all calibration after the magnetic device had been installed on the suspension clamp as shown in figure 2. For this purpose a dial micrometer was temporarily clamped to the device and used to indicate displacement of the armature produced by

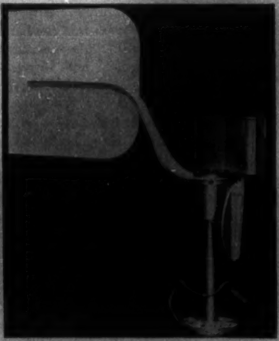

Figure 6. Anemometer for recording instantaneous values of low-velocity winds

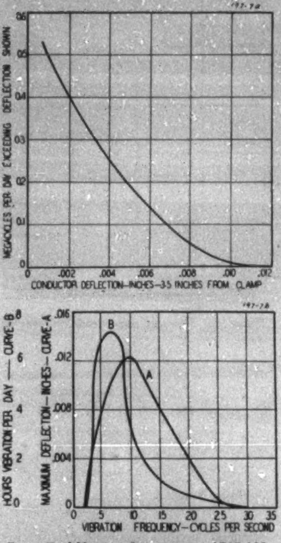

Figure 7. Vibration characteristics of 795,000-circular-mil ACSR "Drake" in 1,150-foot spans at 5,500 pounds tension

No dampers. Suspension clamp as shown in figure 2

lifting or depressing the conductor. Correlation of micrometer readings with oscillograph deflections provided a calibration which was found to be reliable, and accurate to plus or minus five per cent on an one-inch oscillograph deflection. The static calibration values were found to apply, without correction, for the range of frequencies encountered in power conductor vibration. Although measurements could be made of conductor deflections to within 0.1 inch of a clamp, it was customary in field tests to record conductor motion about three inches from suspension clamps.

Five such units, in conjunction with a multiple filter and a six-element oscillograph, provided continuous records of conductor vibration on 5 spans, the sixth oscillograph element being used for a timing wave. Figure 3 shows the recording equipment in portable form as used on high voltage lines during construction.

Provision was first made to run the oscillograph film at a speed of two inches per hour, obtaining a continuous record of amplitude, and once every half hour, to increase film speed momentarily to record amplitude, frequency, and wave shape. The high speed records were found to provide the more useful information, and in later tests, the low speed records were discontinued. Figure 4 shows a record obtained on the two skywires and three power conductors of a 220 Kv. line before dampers had been erected. Figure 5 shows a record after the installation of dampers on the power conductors and festoons on the steel skywires.

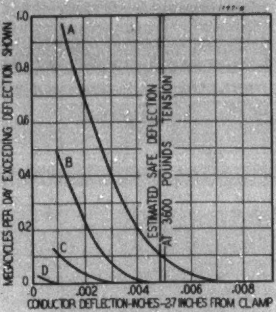

Figure 8. Effect of span length and tension on vibration of 477,000-circular-mil ACSR

A—1,400-foot span at 3,600 pounds tension

B—650-foot span at 3,600 pounds tension

C—650-foot span at 1,800 pounds tension

D—Same as *A* with two dampers per span

Anemometer Records

The linear relation of vibration frequency to wind velocity, determined by Relf and Ower,[1] has been generally accepted. Nevertheless, it was thought desirable to obtain field records of instantaneous wind velocity, since most of the values reported were obtained by rotating-cup anemometers which are unable to record rapid fluctuations in velocity or direction. Following, in some respects, the designs of Sherlock and Stout,[13] an anemometer was developed for recording instantaneous values of wind velocity and direction on the same film as the vibration records. Figure 6 shows the anemometer, which was capable of following rapid fluctuations of wind velocity and direction and had a linear calibration over a range of 3 to 15 miles per hour.

Correlation of the anemometer records with those of conductor vibration have confirmed the generally accepted ideas that most vigorous vibration occurs in steady winds at velocities of two to ten miles per hour directly across the line; that conductors vibrate in variable winds, but at lower amplitudes; and that wind gusts produce no appreciable vertical motion of the conductor. Vibration frequency was occasionally found to deviate rather widely from the accepted relation to wind velocity. However, it was concluded that wind velocity records, even of the instantaneous type, added little to the information available from the vibration records.

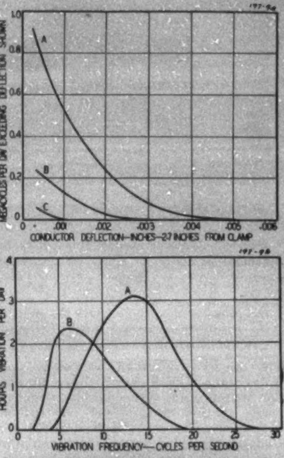

Figure 9. Comparison of standard and loose-core ACSR in 650-foot spans

A—477,000-circular-mil ACSR "Hawk" at 3,600 pounds tension

B—477,000-circular-mil loose-core cable at 2,975 pounds tension

C—Span *A* with two dampers per span

Test Spans

A number of towers, providing a choice of span lengths from 300 to 1,400 feet were available in a level district where aeolian vibration occurs on undamped spans about 60 per cent of the time. By erecting suitable conductors and accessories in these spans, full scale models of existing or proposed line construction could be provided.

Comparison of vibration records obtained on the test spans with those obtained on similar spans of new lines being constructed, showed close agreement in maximum amplitude, range of frequencies, and number of cycles per day. Predictions of the performance of vibration suppression devices based on records in the test spans were confirmed by subsequent tests on actual lines.

Test spans were also provided for

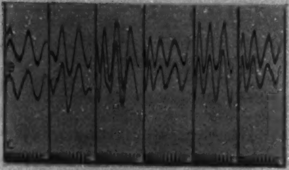

Figure 10. Conductor deflections recorded on opposite sides of rocking-type suspension clamp

A and B—Deflections measured 3.5 inches from suspension clamp

C—Timing wave, 25 cycles per second

making endurance tests on 150-foot lengths of cable, strung at normal tension between concrete piers. At one end the cable was supported in a rigid clamp; at the other end, a motor drive was connected to produce vibration in the span at the desired frequency and amplitude.

Test Procedure

Under normal wind conditions at the test spans, it was found that vibration records covering a period of two weeks were sufficient to establish the performance of any span. During this period, the full range of effective wind velocities, vibration frequencies, and amplitudes were encountered, and additional records did not affect the average results appreciably. In testing damped spans, simultaneous records on undamped spans were usually obtained for comparison. It was found desirable that such undamped conductors be supported on separate towers to prevent excitation of the damped spans by tower vibration.

In analyzing the field data, certain assumptions were made to simplify the calculations and to permit presentation of the results in graphical form. For any one test, at least 500 momentary records of amplitude and frequency, taken one-half hour apart, were analyzed. Each half-hourly record, although it covered a time of less than one second, usually showed cycles of varying amplitude. Only the maximum amplitude was measured, and this was assumed to persist for the entire half-hour period. The frequency indicated was also assumed to persist for the half-hour period. Use of the maximum amplitude shown in each record introduces a conservative factor in the analysis, but this same factor would occur in analyzing any low-speed vibration record, which can register only the maximum values of quickly changing amplitudes.

Analyses of the records may be plotted to show the relations of amplitude, frequency, number of cycles per day, or number of hours vibration per day. The most useful types of graphs were found to be those shown in figure 7. The frequency range determined for any particular construction is essential information for the design of suitable dampers. The number of cycles per day at various deflections may be correlated directly with fatigue tests to determine the probable life of the conductor. It is not possible in fatigue tests to duplicate the varying amplitudes and frequencies which occur on field spans, but by choosing the frequency at which maximum stresses occur in the field, and by measurement of conductor deflection relative to the supporting clamp with the same instrument used to record field vibration, a satisfactory correlation may be obtained.

Effect of Span Length and Tension

In order to determine quantitatively the effect of variations in span length and tension, vibration records were obtained on three spans of 477,000 circular mil A.C.S.R. "Hawk" stranding. These spans were chosen to show the effects of fifty per cent reductions in either span length, tension, or both. Figure 8 shows the results obtained.

Reduction of span length only is shown to considerably reduce both the number of cycles per day and the maximum deflections. Reduction of both span length and tension further reduces the severity of vibration. The most significant fact, however, is that the installation of effective dampers on the longer span at the higher tension, provides much better suppression of vibration than is obtained by decreasing span length and tension.

Comparison of Loose-Core Cable and ACSR of Standard Stranding

Figure 11. Whiffletree suspension clamp on 477,000-circular-mil ACSR with deflection recorder mounted on ring-type suspension clamp

A form of cable described by Preiswerk,[13] is claimed to be vibrationless because of interference between the wind-excited outer aluminum envelope and the loose steel core. A 1,000-foot length of this cable (477,000 circular mil) was erected in suspension clamps on two towers, providing a test span of 650 feet, at a tension of 2,975 pounds. The ends of the cable were fitted with special strain clamps which provided the specified division of tension between the aluminum envelope and the steel core. On the same towers was erected a 477,000 circular mil A.C.S.R. "Hawk" stranding at a tension of 3,600 pounds. Records of vibration were obtained on both cables, with results as shown in figure 9. The loose-core cable shows considerably smaller deflections, due to vibration, than the undamped standard cable and entirely eliminates vibration at frequencies above 18 cycles per second. The suppression obtained on the loose-core conductor is, however, less complete than that provided by effective dampers on the standard cable.

Comparison of Suspension Clamps

Because vibrations in adjacent spans are seldom of the same frequency, conductor motions on the two sides of a suspension clamp continually shift in phase relation. When this relation is such that conductor motion is upward (or downward) on both sides of the clamp simultaneously, it is evident that the clamp,

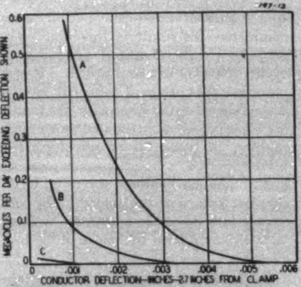

Figure 12. Comparison of single and double suspension clamp on 477,000-circular-mil ACSR Hawk in 650-foot spans at 3,600 pounds tension

A—Single ring-type suspension clamp

B—Two ring-type suspension clamps on 30-inch whiffletree

C—Single clamp on span equipped with dampers no matter how light or well pivoted, will remain stationary, and will react on the conductor as a rigid clamp. Thus an ideal clamp with zero pivot friction and zero inertia could be expected to reduce the number of stress cycles, but not to reduce the maximum stresses. Field measurements show that available types of pivoted suspension clamps fulfil this condition. Figure 10 is a record of deflections on opposite sides of the suspension clamp shown in figure 2, which shows the instantaneous conductor stresses to be equal on opposite sides of the clamp.

An attempt to overcome this limitation of the single pivoted suspension clamp resulted in development of a whiffletree clamp which consists of a bar about three feet long, supported at its centre, and carrying a suspension clamp at each end as shown in figure 11. A comparison of this form of clamp with the single suspension clamp is shown in figure 12. The benefit of the double suspension indicated in these tests has been confirmed by field experience, in which a decided reduction in the rate of fatigue breaks has been obtained over a 10-year period of observation, in comparison with single suspensions.

New Damper Developed

In the course of these investigations, a new principle was employed in the suppression of aeolian vibrations, which resulted in development of an effective and economical damper. It is well known that when a vertical motion is imparted to a 1,000-foot undamped span of power conductor, the resulting travelling wave may be clearly identified after several cycles of travel between towers. It was observed that when a torsional motion was imparted to the same span, the resulting torsional wave attenuated much more rapidly. A converter to transform the wind-imparted vertical oscillations to highly damped torsional waves is, in its simplest form, a mass clamped to the conductor with its centre of gravity horizontally displaced from the centre of the conductor.

On certain conductors, having exceptionally high torsional damping, the rigid eccentric weight is reported[14] to give satisfactory suppression of aeolian vibration. However, on long spans of A.C.S.R. at high mechanical tension, the simple torsional damper did not give the desired degree of suppression. A modification of this damper, employing a resilient joint of rubber at the centre of gravity of the weight, permitted more effective and economical design, and showed the following advantages in operation:

1. **The resilient connection between the weight and conductor lessened the possibility of stress concentration in the conductor at the point of attachment.**

2. **By properly proportioning the damper weight, moment of inertia, arm length, and resilience of the joint, effective suppression of aeolian vibration could be obtained over the entire range of frequencies encountered on any particular conductor.**

The determination of optimum values for these factors was guided by mathematical analysis[15] but in all cases, designs arrived at by theoretical considerations were checked for performance on wind-excited spans. Figure 13 shows a design of torsional damper found effective on 1,150-ft. spans of 795,000 c.m. ACSR "Drake" at 5,500 pounds tension using two dampers per span, one mounted 6 feet from each suspension clamp.

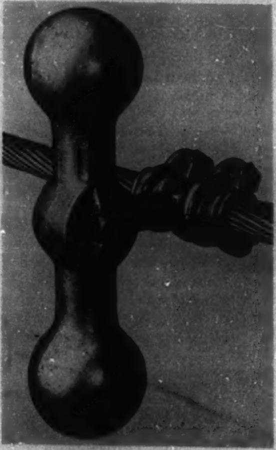

Figure 13. Torsional damper for 795,000-circular-mil ACSR

Comparison of Dampers and Reinforcing Devices

Devices designed to suppress the amplitude of vibration throughout the span to a value which will not produce fatigue failures in the conductor, are properly called "dampers" and in this investigation two forms were tested, the well-known Stockbridge damper, and a newly-developed torsional damper.

Devices which act primarily as reinforcements may effect only a small reduction in vibration amplitude throughout the span, but by stiffening the conductor in the region where failures usually occur, seek to prevent stress concentration. Of this type, two devices were tested: the well-known tapered aluminum armor rods which form a spiral wrapping around the conductor, and a cable absorber consisting of a six-foot length of the power conductor, carried above the main conductor in a double-barreled clamp, its ends fastened to the main conductor with light aluminum clamps.

A comparison of these four devices, installed as shown in figure 14, was made on spans representing a proposed 220 Kv. line. The essential information for comparing their effectiveness is also plotted in figure 14. Examination of the oscillographic records discloses other differences in operation. Spans equipped with reinforcing devices vibrate throughout the same range of frequencies as the undamped span, whereas dampers suppress all but the lowest frequencies. The Stockbridge damped span permitted vibrations at frequencies up to 3 cycles per second and deflections of 0.003 inch, of which a typical record is shown in figure 5. On the torsional damped span, no deflections exceeding 0.001 inch were recorded, and no single frequency could be identified to indicate resonant vibration.

The possibility of extending the effective frequency range of torsional dampers still lower, to suppress galloping of conductors has been considered. Present conceptions of a damper to be effective at 0.5 cycle per second appear unreasonably large for general use.

Estimated Safe Deflection

For any conductor, under normal tension and supported in a specified clamp, there is some value of alternating deflection which the conductor will withstand indefinitely without fatigue failure. One method of determining this value would be to vibrate specimens of the cable at normal tension and at various deflections until a strand failure occurs. By plotting a curve of deflection on number of cycles to failure, the deflection which the cable can withstand indefinitely may be determined. This procedure, especially for large aluminum conductors which vibrate normally at low frequencies, and which may require up to 500 million cycles to produce failure, is not considered justifiable until further investigation has determined the effects of such variables as corrosion, infrequent

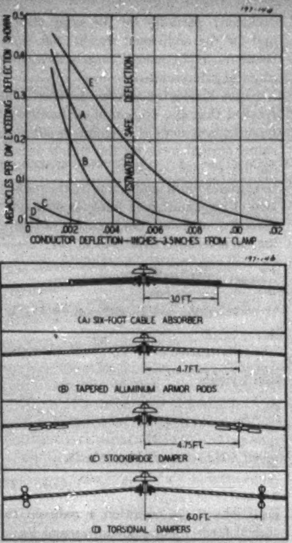

Figure 14. Vibration-control devices and performance curves for 795,000-circular-mil ACSR in 1,150-foot spans at 5,500 pounds tension

A—Six-foot cable absorber
B—Tapered aluminum armor rods
C—Stockbridge dampers, two per span
D—Torsional dampers, two per span
E—Undamped span

overstresses such as those produced by galloping conductors, surface defects, or abrasions. These uncertainties must now be allowed for by an arbitrary safety factor.

The fatigue characteristics of an aluminum strand subjected to bending stress reversals, but without tension load, may be determined readily on a high speed fatigue test machine such as the Haigh-Robertson.[5] As shown in figure 15, the stress-life relation is such that if a stress which causes failure in one million cycles be reduced by 50 per cent, infinite life can be expected. Similarly, a stress reduction of 30 per cent will increase a ten million cycle life to infinity.

Little information is available concerning the fatigue characteristics of aluminum under the combined stresses of tension and reversed bending. It has been shown[16] that for steel, the superposition of tension on reversed bending stresses reduces the allowable stress range, but that the shape of the *S-N* curve is not greatly altered. Assuming that a similar relation exists for aluminum, it should be possible, from endurance tests on a stranded conductor at one measured deflection, to estimate the safe deflection below which fatigue failures would not occur. For example, an endurance test was carried out on a 477,000 circular mil A.C.S.R. at 3,600 pounds tension, set up to approximate the field test conditions of curves *A* or *B*, figure 8. The cable was vibrated to produce a deflection of 0.006 inch at a distance 2.7 inches from the clamp, and the first strand failure occurred at 26 million cycles. From figure 15, it appears that a stress reduction of 20 per cent would have prolonged conductor life to infinity. Referring again to figure 8, the estimated safe deflection for the conditions of curves *A* and *B* would be $0.006 \times 0.80 = 0.0048$ inch.

A third method, which was used in estimating the safe deflection shown in figure 14 is based upon operating experience with the actual conductor. Several hundred miles of 795,000 circular mil A.C.S.R. equipped with cable absorbers as shown in figure 14 have been in operation for 10 years. Periodic inspections have shown no fatigue failures at suspension clamps. Curve *A* of figure 14 shows that this construction has withstood deflections exceeding 0.006 inch totalling 0.030 megacycle per day, or about 100 million cycles in the 10-year period. Figure 15 shows that a stress reduction of 15 per cent would increase conductor life from 100 million cycles to infinity, hence the estimated safe deflection is $0.006 \times 0.85 = 0.0051$ inch.

Conclusions

1. Vibration of large diameter A.C.S.R. conductors in spans exceeding 1,000 feet and at tensions exceeding one-sixth the ultimate strength may be expected to produce fatigue failure unless protective devices are applied.

2. Reduction of such span lengths and tensions by fifty per cent provides less reduction in vibration stresses than is accomplished by the application of suitable dampers.

3. A special stranding "loose core" A.C.S.R. effectively suppresses vibration in the upper range of frequencies. The overall suppression is, however, much less effective than that provided by dampers on A.C.S.R. of standard stranding.

4. Reinforcing devices such as armour rods, are much less effective than dampers in reducing vibration stresses on large A.C.S.R. conductors.

5. Available dampers reduce conductor deflections well below the limit indicated as safe both by operating experience and by fatigue tests on conductors. The factor of safety thus provided is considered adequate to cover all uncertainties in field observations or in estimating fatigue limits for the conductors.

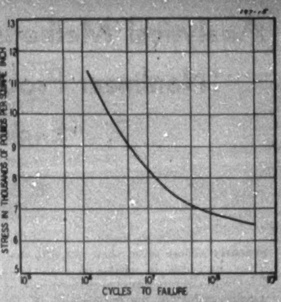

Figure 15. Fatigue curve for 0.1059-inch diameter aluminum wire as determined with Haigh-Robertson machine

References

1. The Singing of Circular and Streamline Wires, E. F. Relf and E. Ower. *Aeronautical Research Committee*, number 825 (Ae. 76), March 1921.

2. Notes on the Vibration of Transmission-Line Conductors, Theodore Varney. AIEE Transactions, volume 45, 1926, pages 791–5.

3. Vibration in Electrical Conductors, A. E. Davison, J. A. Ingles, V. M. Martinoff. Electrical Engineering, volume 51, November 1932, pages 795–8.

4. Vibration Analysis—Transmission-Line Conductors, W. B. Buchanan. AIEE Transactions, volume 53, 1934 (November section), pages 1478–84.

5. Fatigue Tests on Zinc-Coated Steel Wire, D. G. Watt. *Proceedings* ASTM, volume [illegible], 1940, pages 717–32.

6. Field Tests on Conductor Vibration, E. M. Wright, J. Mini. AIEE Transactions, volume 53, 1934 (July section), pages 1123–7.

7. Les Vibrations des lignes électriques aeriennes et les moyens d'amortissement et de controle, E. Jaquet. Association Suisse Des Electriciens, *Bulletin number* 10, 1937.

8. Vibration Studies Made With Oscillograph, C. O. Crane. *Electrical World*, September 17, 1932, pages 378–80.

9. Engineering Features of Boulder Dam-Los Angeles Lines, E. F. Scattergood. AIEE Transactions, volume 54, 1935 (May section), pages 494–512.

10. Vibration Recorder for Electrically Measuring and Recording Small Mechanical Movements, E. V. Mershon. AIEE Transactions, volume 45, 1926, pages 1007–11.

11. Electric Strain Gauge, E. V. Mershon, Rusher. AIEE Transactions, volume 57, 1938 (November section), pages 645–8.

12. An Anemometer for a Study of Wind Gusts, R. H. Sherlock, M. B. Stout. *Engineering Research Bulletin number* 20, University of Michigan, May 1931.

13. Nonvibrating Cables for Overhead Transmission Lines, *Elektrotechnische Zeitschrift*, December 1934, page 1225.

14. Conductor Vibration, G. W. Preston. *The Electrician*, October 15, 1937, pages 431–2.

15. The Theory of Torsional Dampers, J. W. Speight. AIEE Transactions, volume 60, 1941 (October section), pages 907–11.

16. Fatigue Properties of Steel Wire, S. M. Shelton and W. H. Swanger. *Bureau of Standards Research Paper R.P.754*, January 1935, pages 17–32.

Analytical Methods of Solving Discrete Nonlinear Problems in Electrical Engineering

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Synopsis: More often than not, the non-linear problems of electrical engineering arise from discrete physical systems and are usually reducible mathematically to the solution of systems of non-linear total differential equations or to systems of non-linear integral equations. Six independent methods of solving discrete non-linear problems are given in this paper. Each method is illustrated by means of an electrical engineering problem. The illustrative examples employed pertain to a-c and d-c non-linear circuits, reluctance-induction motors, hunting of and dynamic braking of synchronous machines. References to additional methods are given in the bibliography. The 88 given references listed represent approximately ten per cent of the field, but many of the entries contain a bibliography on their respective fields.

The accelerated growth of research in the field of non-linearity is due to different causes. The general advancement of science requires increasingly more precise expressions for the laws of science. Accurate non-linear equations frequently depart from the linearized or postulated linear equations which have been previously used for approximate results. The quest for perfection and generalization and the love of difficult investigations by professional mathematicians play a large part in this growth. A recent incentive is the increasingly exacting requirements of modern manufacturing. These requirements are born of the competitive necessity of producing ever improved machines and equipment in the most economical manner.

Definition; Characteristics of Two General Fields

A NON-LINEAR problem (discrete or continuous) may be defined as one which, when formulated mathematically, reduces to (one or) a system of differential integral, or integro-differential equations such that at least one of the three quantities; a derivative, an integral, or a dependent variable is involved transcendentally or algebraically to a power other than the first in at least one equation of the system or in at least one mathematical boundary condition of the system.

Non-linear problems resolve themselves into two general types; continuous and discrete. The first type deals with the behavior of quantities in a field or in at least one continuous region of space and more often than not reduces mathematically to systems of non-linear partial differential equations. Problems relating primarily to this field have been treated by Dr. Theodore von Kármán in a paper[1] which is both a mile stone and a beacon of progress in that it is an admirable exposition and inventory of the non-linear problems of continuous fields and at the same time an inspiration and invitation both to the engineer and mathematician for further advancement in this difficult field. Discrete non-linear systems are characterized by a finite number of degrees of freedom. They are at least frequently reducible mathematically to systems of non-linear and ordinary (total) differential equations.

Scope and Plan of Paper

The material of the present paper is restricted to the field of discrete non-linearity. Six powerful methods of solving discrete non-linear problems are given but references to as many more are found in the bibliography. The method of presentation is to (*a*) display the fundamental systems of equations and forms of each method, (*b*) illustrate the application of each method by the solution of a non-linear problem in electrical engineering. The methods range over a time span from Euler to Galerkin, but the solutions of the engineering problems are new.

It should perhaps be stated that the elementary methods of variables separable, homogeneous forms, etc., of a first course in differential equations are excluded from this paper. They comprise a field with which everyone is familiar or can become so with little effort. Likewise, the solution of non-linear equations as power series in the independent variables is also excluded; partially for the same reason and for the additional reason that such solutions are, in general, largely devoid of physical interpretation.

Before displaying the first method it is recalled that any system of ordinary differential equations is reducible to the so-called normal form by the simple introduction of new variables. The reduction to normal form is made clear by the reduction of an equation of hunting of a synchronous machine. An equation of hunting,[2] in a certain reduced form, is

$$\frac{d^2\theta}{dt^2}+k(1-b\cos 2\theta)\frac{d\theta}{dt}+c\sin 2\theta+(1-\epsilon^{-at})\sin\theta=T \quad (1)$$

The substitution of $\theta=\theta_1$, $\dfrac{d\theta}{dt}=\theta_2$ in equation 1 yields

$$\frac{d\theta_1}{dt}=\theta_2$$

$$\frac{d\theta_2}{dt}=T-k(1-b\cos 2\theta_1)\theta_2-c\sin 2\theta_1-(1-\epsilon^{-at})\sin\theta_1 \quad (2)$$

Equations 2 are equation 1 reduced to normal form. The general normal form for an *n*th order system of differential equations is

$$\begin{aligned}&\frac{dx_1}{dt}=f_1(x_1, x_2, \dots, x_n;\ t)\\ &\dots\dots\dots\dots\dots\dots \qquad (3)\\ &\frac{dx_n}{dt}=f_n(x_1, x_2, \dots, x_n;\ t)\end{aligned}$$

or

$$x_i'=f_i(x_j;\ t) \qquad (i, j=1, 2, \dots, n) \qquad (4)$$

In equation 1 the terms represent respectively the inertial, amortisseur, reluctance, synchronous, and load torques.

Six Methods of Solution

1. Solution in Power Series in Parameters; Type I

The first method considered, and the second oldest, is due to Euler. If a system of differential equations is, or can be reduced by changes of variables, or by any method whatsoever to the form

$$x_i'=rf_i(x_j, r;\ t) \qquad (i, j=1, 2, \dots, n) \qquad (5)$$
$$x_j(t_0)=a_j$$

where the $f_i(x_j,r;\ t)$ are analytic in x_j and r and continuous in t for

$$|x_j-a_j|<r_j,\ t_0\leqq t\leqq T, \text{ and } |r|<r_0$$

then there exists a solution in the form of the infinite series

$$x_j=x_j^{(0)}(t)+x_j^{(1)}(t)r+x_j^{(2)}(t)r^2+\dots \qquad (j=1, 2, \dots, n) \qquad (6)$$

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[1] For all numbered references, see reference list at the end of the paper and not sections of the bibliography.

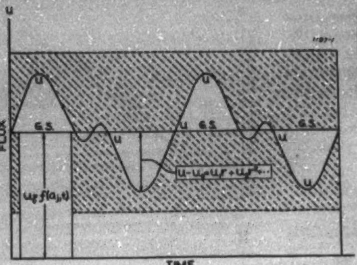

Figure 1 (left). Type I. Solution of a nonlinear problem whose generating solution is a constant

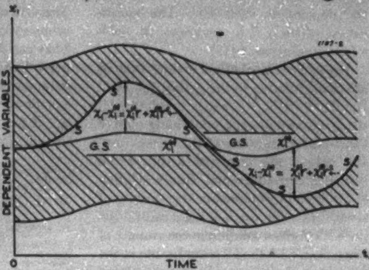

Figure 2 (right). Type II. Solution in the vicinity of a function

which is convergent for some value of $r>0$. The proof, which is by means of dominant functions, of the above statement is found in the literature.[3] The primary objective of this paper lies in the methods of construction of solutions.

To obtain the infinitude of functions $x_j^{(k)}$ in equations 6 expand the $f_i(x_j,r;t)$ in power series in x_j-a_j and r, substitute in these expansions the values of x_j-a_j from equations 6, equate to zero the coefficients of each power of r in the resulting equations thus obtaining the sequence of equations

$$x_i'^{(1)}=f_i(a_j,0;\ t)\quad (i=1,2,\ldots,n) \tag{7}$$

$$x_i'^{(2)}=\sum_{j=1}^{n}\frac{\partial f_i}{\partial x_j}x_j^{(1)}+\frac{\partial f_i}{\partial r} \tag{8}$$

……………………

$$x_i'^{(k)}=\text{polynominals in }(x_j^{(1)},\ldots,x_j^{(k-1)};\ t)\quad (k=3,4,\ldots) \tag{9}$$

Equations 7, 8, 9 are linear equations since the partial derivatives are constants, the x_j having been replaced by a_j. Equations 7 are integrable at once. The values of $x_j^{(1)}$ obtained from (7) are substituted in (8). The values obtained from (8) are substituted in (9). Equations 7, 8, 9 are thus said to be integrated sequentially. The solution of a system of differential equations defining the effect of resistance in non-linear control circuits[4] is an application of the first method. The equations are

$$\begin{aligned}u'&=-r(1+3b_3y_1^2+5b_5y_1^4)u\cos^2(t+v)\\ v'&=+r(1+3b_3y_1^2+5b_5y_1^4)\sin(t+v)\times\cos(t+v)\end{aligned} \tag{10}$$

where the b_n are constants and

$$y_1=u\sin(t+v),\quad y_2=u\cos(t+v)$$

By the theory there exists the solution

$$\begin{aligned}u&=u_0+u_1r+u_2r^2+\ldots\\ v&=v_1r+v_2r^2+\ldots\end{aligned} \tag{11}$$

where the functions $u_0,u_1,\ldots$ and $v_1,v_2,\ldots$ are defined by the equations

$$u_0'=0$$

$$u_1'=f_2\sin 2t+f_4\sin 4t+f_6\sin 6t,$$

……………………

$$\begin{aligned}v_1'&=-(e_0+e_2\cos 2t+e_4\cos 4t+e_6\cos 6t),\\ v_2'&=(g_2\sin 2t+g_4\sin 4t+g_6\sin 6t)u+2v_1(f_2\cos 2t+2f_4\cos 4t+4f_6\cos 6t)\end{aligned}$$

…………………… (12)

Since the right members of equations 12 are functions of the time only, the problem is reduced to quadratures.

The geometrical significance of the solution of equations 5 or 10 is important in view of the methods which follow. In figure 1 it is observed that the solution u is in the vicinity of a constant value labelled u_0. The parameter r measures the departure of u from u_0. Scale values are not shown on the figure because time t is not in seconds and u and v are functional values largely devoid of physical significance. (Reference 4 may be consulted.)

2. Solution in Power Series in Parameters; Type II

This method also had its beginning with Euler, but the analytical perfection of the method (questions of convergence, etc.) is due to Cauchy[5] and Poincaré.[6] The fundamental system of differential equations in the normal form is

$$x_i'=f_i(x_j;\ t)+rg_i(x_j,r;\ t)\quad (i,j=1,2,\ldots,n) \tag{13}$$
$$x_j(t_0)=a_j$$

where the $f_i(x_j;\ t)$ satisfy the same conditions as in method 1. The $g_i(x_j,r;t)$ are analytic in $x_j-x_j^0$ and r uniformly with respect to t and are continuous in t for all x_j, r, and t for which

$$|x_j-x_j^{(0)}|\leqq r_j,\quad |r|\leqq\sigma,\quad t_0\leqq t\leqq T\leqq T^{(0)}$$

The first step in the solution of (13) is to find the solution of the system

$$x_i'=f_i(x_j,t),\ x_i(t_0)=a_i,\ (i,j=1,2,\ldots,n) \tag{14}$$

The solution of this system may possibly be accomplished by the elementary methods of a first course in differential equations. If the solution of equations 14 is difficult, recourse may be had to the methods of this paper. Let us suppose then that the solution of (14) has been accomplished and denote this solution by

$$x_j=x_j^{(0)}(t)\quad (j=1,2,\ldots,n) \tag{15}$$

Solution 15 is called the generating solution of (13). It is provable[7] that there exists a solution of (13) of the form

$$x_j-x_j^{(0)}=x_j^{(1)}(t)r+x_j^{(2)}(t)r^2+\ldots\quad (j=1,2,\ldots,n) \tag{16}$$

which is convergent for r sufficiently small.

To construct the formal[7] solution of (13) expand $f_i(x_j,t)$ and $g_i(x_j,r;t)$ as a power series in $x_j-x_j^{(0)}$ and r, substitute in these expansions and in equation 13 the value of $x_j-x_j^{(0)}$ as given by equations 16, and finally equate to zero the coefficients of each power of r. The resulting differential equations are

$$x_i'^{(0)}=f_i(x_j^{(0)};\ t),\quad x_j^{(0)}(t_0)=a_j$$

$$x_i'^{(1)}-\sum_{j=1}^{n}\frac{\partial f_i}{\partial x_j}x_j^{(1)}=g_i(x_j^{(0)},0;\ t)$$

$$x_i'^{(2)}=\sum_{j=1}^{n}\frac{\partial f_i}{\partial x_j}x_j^{(2)}$$

$$=\tfrac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n}\frac{\partial^2 f_i}{\partial x_j\partial x_k}x_j^{(1)}x_k^{(1)}+\sum_{j=1}^{n}\frac{\partial g_i}{\partial x_j}x_j^{(1)}+\frac{\partial g_i}{\partial r} \tag{17}$$

……………………

There are the following observations relative to equations 17. The solution of the first system of (17) is (15). (If this solution can not be obtained, then the present method is not applicable.) When the values $x_i^{(0)}(t)$ for the $x_j^{(0)}(t)$, determined by the solution of the first of (17) are substituted in the next system of (17) these differential equations are linear in $x_j^{(1)}$. The second system is now solved for $x_i^{(1)}$ and the values obtained are substituted in the third system whose dependent variables are $x_j^{(2)}$. Thus equations 17 are solved sequentially. All of equations 17, the first excepted, are linear in the dependent variables. The substitution of the $x_j^{(k)}$ in equations 16 yields a formal solution of (13).

The gist of this method is that if the rg_i are small relative to the f_i (say $1/_{10}$ of f_i) and if equations 14 are easy to solve and if the g_i are complicated, containing most of the troublesome terms of the system (usually only a few terms of a differential system are troublesome) then a solution of (13) can be obtained in the vicinity of $x_i = x_i^{(0)}(t)$. The method is a powerful one since it affords a solution in the vicinity of a function. Method 2 is applicable in the case of transients in synchronous machines when resistances are taken into account. Again it is applicable in perturbation theory in celestial mechanics,[8] where the method originated. If the perturbative forces of a third or of n bodies are slight, then the generating solution is the elliptical orbit in the problem of two bodies. The engineering problem illustrating this method is combined with that of method 4. See figure 2.

3. METHOD OF VARIATION OF PARAMETERS

Of the methods here presented the method of variation of parameters[9] is the oldest. Its first beginnings, in geometric form, are due to Newton. Let the fundamental differential equations be

$$x_i' = F_i(x_j;\ t) \equiv f_i(x_j;\ t) + g_i(x_j;\ t)$$
$$x_j(t_0) = a_j, \qquad (i, j = 1, 2, \ldots, n) \qquad (18)$$

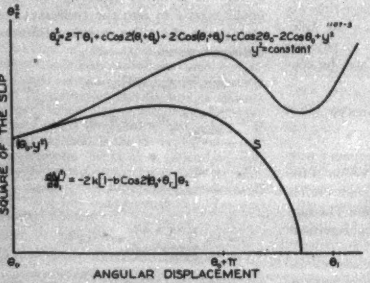

Figure 3. Hyperelliptic method

Synchronous motor operating as an induction-reluctance motor employed as an illustrative example

Generally, the n functions F_i can be resolved into $2n$ functions f_i and g_i in more than one way. This resolution is usually accomplished such that the f_i and g_i possess physical significance.

Let the solution of

$$x_i' = f_i(x_j;\ t) \qquad (i, j = 1, 2, \ldots, n) \qquad (19)$$

be

$$x_i = \phi_i(y_1, y_2, \ldots, y_n;\ t)\ (i = 1, 2, \ldots, n) \qquad (20)$$

The y_k in equations 20 can be viewed as n arbitrary constants of the solution of (19). They can also be viewed as a set of n new dependent variables and (20) are then not only the solution of (19) but are also equations of change of variables from x_i to y_i. If the second point of view is taken, the substitution of (20) in (19) yields

$$\frac{\partial x_1}{\partial y_1}\frac{dy_1}{dt} + \frac{\partial x_1}{\partial y_2}\frac{dy_2}{dt} + \ldots + \frac{\partial x_1}{\partial y_n}\frac{dy_n}{dt} + \underline{\frac{\partial \phi_1}{\partial t}} =$$
$$f_1\underline{[\phi_j(y_1, y_2, \ldots, y_n;\ t);\ t]} + g_1[\phi_i(y_k;\ t);\ t]$$
$$\frac{\partial x_2}{\partial y_1}\frac{dy_1}{dt} + \frac{\partial x_2}{\partial y_2}\frac{\partial y_2}{dt} + \ldots + \frac{\partial x_2}{\partial y_n}\frac{dy_n}{dt} + \underline{\frac{\partial \phi_2}{\partial t}} =$$
$$f_2\underline{[\phi_j(y_1, y_2, \ldots, y_n;\ t);\ t]} + g_2[\phi_j(y_k;\ t);\ t]$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (21)$$
$$\frac{\partial x_n}{\partial y_1}\frac{dy_1}{dt} + \frac{\partial x_n}{\partial y_2}\frac{dy_2}{dt} + \ldots + \frac{\partial x_n}{\partial y_n}\frac{dy_n}{dt} + \underline{\frac{\partial \phi_n}{\partial t}} =$$
$$f_n\underline{[\phi_j(y, y, \ldots, y_n;\ t);\ t]} + g_n[\phi_j(y_k;\ t);\ t]$$

Those terms in (21) which are underlined vanish because equations 20 satisfy (19).

The new system of differential equations to be solved then is (21) with the underlined terms deleted. This new system is solvable by determinants for dy_1/dt, $dy_2/dt, \ldots, dy_m/dt$. When this has been done the resulting system in $y_1, y_2, \ldots, y_n$ is in the normal form This system, in general, is non-linear but it may be more easily solvable than (18). Moreover, the system is of a form to which many devices[10] are applicable. For example, if it is known from physical considerations that the y_k are slowly changing functions of the time, then the y_k in $g_i(y_k;\ t)$ are replaceable by constant values. The accuracy is not seriously impaired. Under this condition the system (21) is linear.

The equation of hunting is used to illustrate method 3. The system is

$$\theta_1' = \theta_2$$
$$\theta_2' = T - k(1 - b\cos 2\theta_1)\theta_2 - c\sin 2\theta_1 - \sin\theta_1 \qquad (22)$$

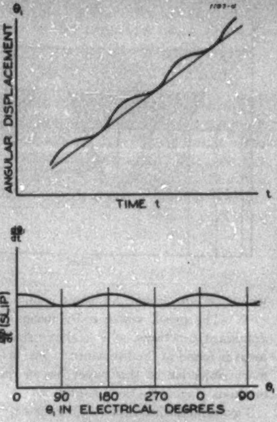

Figure 4. Variation of parameters

Hunting equations of a synchronous machine

$$\theta_1(t_0) = \theta_0,\ \theta_2(t_0) = y$$

Division of the second by the first of (22) yields

$$\frac{d\theta_2}{d\theta_1} = T - c\sin 2\theta_1 - \sin\theta_1 - k(1 - b\cos 2\theta_1) =$$
$$f(\theta_1, \theta_2) + g(\theta_1) \qquad (23)$$

The solution which satisfies the boundary conditions of (22), of

$$\frac{d\theta_2}{d\theta_1} = \frac{T - c\sin 2\theta_1 - \sin\theta_1}{\theta_2} \qquad (24)$$

is

$$\theta_2^2 = 2T\theta_1 + c\cos 2(\theta_1 + \theta_0) + 2\cos(\theta_1 + \theta_0) -$$
$$c\cos 2\theta_0 - 2\cos\theta_0 + y^2 \qquad (25)$$

where y^2 is an arbitrary constant or a new variable. The equation corresponding to (21) is

$$\frac{\partial\theta_2}{\partial(y^2)}\frac{d(y^2)}{d\theta_1} + \frac{\partial\theta_2}{\partial\theta_1} = \frac{T - c\sin 2\theta_1 - \sin\theta_1}{\theta_2} -$$
$$k(1 - k\cos 2\theta_1) \qquad (26)$$

or

$$\frac{d(y^2)}{d\theta_1} = -2k[1 - b\cos 2(\theta_0 + \theta_1)]\theta_2 \qquad (27)$$

Equation 27 is solvable by method 1. It should perhaps be stated that although (22) are not the latest[11] equations of hunting of a synchronous machine they serve the purpose of this paper.

4. HYPERELLIPTIC METHOD

This method is one of great power in the solution of problems involving non-linear springs, non-linear circuits, and in problems where the periods of the motions (currents) are dependent upon some nonlinear function of the displacements (charges). The fundamental system of differential equations and the general procedure are both evident from the solution of an illustrative example.

Let it be required to solve the nonlinear differential equation of motion of the rotor of an induction-reluctance motor. If the last term in the second of (22) is deleted then the resulting equations are the equations of performance of an induction-reluctance motor. If $2\theta_1 = x_1$ the equations become

$$\frac{dx_1}{dt} = x_2$$

$$\frac{dx_2}{dt} = 2T - kx_2 - 2r\sin x_1 + k\mu r x_2 \cos x_1 \qquad (28)$$

where $\mu r = kb$ and where typical numerical constants for machines of present manufacture are

$0 < b < 0.5$, $0.028 < k < 0.11$ (k for electrical degrees)
$0.3 < T < 0.8$, $0.250 < r < 0.50$

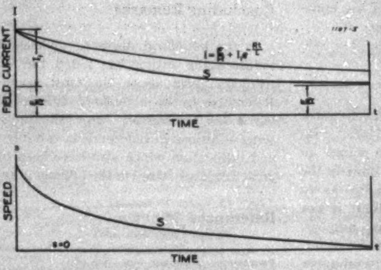

Figure 5. Method of nonlinear integral equations

Dynamic braking of a synchronous motor

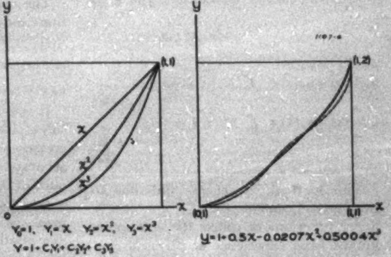

Figure 6. Exact and approximate solution by Galerkin's method

In the construction of a solution of (28) several characteristics of the desired solution must be kept in mind. It is known from experience and physical considerations that the steady-state solution of (28) consists of an oscillatory component superimposed on a constant component of slip. (See figure 4.) The oscillating component of slip is induced by the reluctance and damping torques represented by the last two terms in the second of (28). The oscillating component of the steady-state solution is periodic of unknown period. The period of the oscillating component and the magnitude of the constant component of slip are, from physical considerations, mutually dependent. The purpose of this solution is to determine the position of the rotor at the time of switching on of the field voltage for at this time the motor becomes a synchronous motor. In other words the present solution determines the initial conditions for the solution of (22). If in equation 1 the term representing the synchronizing torque is omitted and the changes of variables $2\theta = x_1$ and $t = (1+\delta)\tau$, where $\delta = \delta_1 r + \delta_2 r^2 + \ldots$, are performed then the resulting differential equation is

$$\frac{d^2x_1}{d\tau^2} = 2T(1+\delta)^2 - k(1+\delta)(1 - b\cos x_1)\frac{dx_1}{d\tau} - 2r(1+\delta)^2 \sin x$$

or, in the normal form

$$x_1' = x_2$$
$$x_2' = 2T(1+\delta)^2 - k(1+\delta)(1 - b\cos x_1)x_2 - 2r(1+\delta)^2 \sin x_1 \qquad (29)$$

where $x' = dx_i/d\tau$. Expand $\cos x_j$ and $\sin x_j$ in powers of $x_j - x_j^{(0)}$, substitute

$$x_j - x_j^{(0)} = x_j^{(1)} r + x_j^{(2)} r^2 + \ldots \quad (j = 1, 2) \qquad (30)$$

in (29) and equate to zero the coefficients of like powers of r. The resulting sequence of equations is

$$x_1^{(0)\prime} = x_2^{(0)} \qquad (31)$$

$$x_2^{(0)\prime} = -kx_2^{(0)} + 2T \qquad (32)$$

$$x_1^{(1)\prime} = x_2^{(1)} \qquad (33)$$

$$x_2^{(1)\prime} = -k(\delta_1 x_2^{(0)} + x_2^{(1)} - \mu x_2^{(0)} \cos x_1^{(0)}) - 2\sin x_1^{(0)} + 4T\delta_1 \qquad (34)$$

$$x_1^{(2)\prime} = x_2^{(2)} \qquad (35)$$

$$x_2^{(2)\prime} = -k\{\delta_2 x_2^{(0)} + \delta_1(x_2^{(1)} - x_2^{(0)}\mu \cos x_1^{(0)}) + x_2^{(2)} + x_1^{(1)} x_2^{(0)} \mu \sin x_1^{(0)} - x_2^{(1)} \mu \cos x_1^{(0)}] - 2x_1^{(1)} \cos x_1^{(0)} - 4\delta_1 \times \sin x_1^{(0)} + 2T(\delta_1^2 + 2\delta_2) \qquad (36)$$

……

The general solution of (31-32) is

$$x_1^{(0)} = -\frac{A_0}{k}\epsilon^{-k\tau} + \frac{2T}{k}\tau + C_0 \equiv -\frac{A_0}{k}\epsilon^{-k\tau} + e\tau + C$$

$$x_2^{(0)} = A_0\epsilon^{-k\tau} + \frac{2T}{k} \equiv A_0\epsilon^{-k\tau} + e$$

Since only a steady-state periodic solution is desired $A_0 = C_0 = 0$ and thus

$$x_1^{(0)}(0) = 0, \qquad x_2^{(0)}(0) = \frac{2T}{k} \qquad (37)$$

$$x_1^{(0)}(\tau) = e\tau, \qquad x_2^{(0)}(\tau) = e \qquad (38)$$

The general solution of (33-34) is

$$x_1^{(1)} = e\delta_1\tau + \left[\frac{e(k^2\mu+2)\sin e\tau - k(\mu e^2 - 2)\cos e\tau}{e(e^2+k^2)}\right] - \frac{A_1}{k}\epsilon^{-k\tau} + C_1 \qquad (39)$$

$$x_2^{(1)} = e\delta_1 + \left[\frac{k(\mu e^2 - 2)\sin e\tau + e(k^2\mu+2)\cos e\tau}{(e^2+k^2)}\right] + A_1\epsilon^{-k\tau} \qquad (40)$$

Evidently, $A_1 = C_1 = 0$. If a term linear in τ remains in (39-40) then the solution of (35-36) will contain terms of the form $\tau \sin e\tau$ and $\tau \cos e\tau$. Periodicity renders this impossible. Consequently $\delta_1 = 0$.

Next (38) and (39-40) are substituted in (35-36). The periodicity condition determines δ_2 which does not vanish. The process of sequential integration can be carried as far as desired.

The final solution in θ_i as far as the terms in r^2 is

$$\theta_1 = \frac{e\tau}{2} + \frac{r}{2} \times \left[\frac{e(k^2\mu+2)\sin e\tau - k(e^2\mu - 2)\cos e\tau}{e(e^2+k_2)}\right] + \frac{r^2}{2}\left[\frac{(k\beta - 2e\alpha)\sin 2e\tau - (k\alpha + 2\beta e)\times \cos 2e\tau}{2e(k^2+4e^2)}\right] + \ldots$$

$$\theta_2 = \frac{e}{2} + \frac{r}{2}\left[\frac{k(\mu e^2 - 2)\sin e\tau + e(k^2\mu+2)\cos e\tau}{(e^2+k^2)}\right] + \frac{r^2}{2}\left[\frac{(k\beta - 2e\alpha)\cos 2e\tau + (k\alpha + 2\beta e)\times \sin 2e\tau}{(k^2+4e^2)}\right] + \ldots$$

where

$$\delta_2 = \frac{2 - \mu e^2}{e^2(e^2+k^2)}, \quad \alpha = \frac{k^2\mu(\mu e^2 - 2) - (k^2\mu + 2)}{(e^2+k^2)},$$

$$\beta = \frac{ke^2\mu(k^2\mu+2) + k(\mu e^2 - 2)}{e(e^2+k^2)}$$

The solution is periodic of period 2π in τ, and of period $2\pi(1+\delta)$ in t.

5. Method of Nonlinear Integral Equations

If a problem is reducible to Lalesco's non-linear integral-equation system

$$u_1(x)=\phi_1(x)+\int_0^x K_1[x,\xi,u_1(\xi),\ldots,u_n(\xi)]d\xi$$
$$u_2(x)=\phi_2(x)+\int_0^x K_2[x,\xi,u_1(\xi),\ldots,u_n(\xi)]d\xi$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$u_n(x)=\phi_n(x)+\int_0^x K_n[x,\xi,u_1(\xi),\ldots,u_n(\xi)]d\xi \qquad (41)$$

then there exists a solution which is the limit of the infinite sequences

$$u_1(x)=\phi_1(x)$$
$$u_1^1(x)=\phi_1(x)+\int_0^x K_1[x,\xi,\phi_1(\xi),\ldots,\phi_n(\xi)]d\xi$$
$$u_1^2(x)=\phi_1(x)+\int_0^x K_1[x,\xi,u_1^{(1)}(\xi),\ldots,u_n^{(1)}(\xi)]d\xi$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$u_2^{(0)}(x)=\phi_2(x)$$
$$u_2^{(1)}(x)=\phi_2(x)+\int_0^x K_2[x,\xi,\phi_1(\xi),\ldots,\phi_n(\xi)]d\xi$$
$$u_2^{(1)}(x)=\phi_2(x)+\int_0^x K_2[x,\xi,u_1^{(1)}(\xi),\ldots,u_n^{(1)}(\xi)]d\xi$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$u_n^{(0)}(x)=\phi_n(x)$$
$$u_n^{(1)}(x)=\phi_n(x)+\int_0^x K_n[x,\xi,\phi_1(\xi),\ldots,\phi_n(\xi)]d\xi$$
$$u_n^{(2)}(x)=\phi_n(x)+\int_0 K_n[x,\xi,u_1^{(1)}(\xi),\ldots,u_n^{(1)}(\xi)]d\xi$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \qquad (42)$$

This method, in general, fails due to complexity, unless the K_i are nearly linear functions or unless the solution is known from experience or physical considerations to be monotone increasing or decreasing.

An application is to dynamic braking of synchronous machines. The differential equations of dynamic braking of a synchronous machine are

$$\frac{dI}{dt}=\frac{(E-IR)}{L}\frac{\left[\left(\frac{rs_0}{s}\right)^2+x_dx_q\right]}{\left[\left(\frac{rs_0}{s}\right)^2+x_d'x_q\right]}-$$
$$\frac{2KPr^3I^3}{JI_0^2s^4}\frac{x_qs_0^2(x_d-x_d')\left[\left(\frac{rs_0}{s}\right)^2+x_q^2\right]}{\left[\left(\frac{rs_0}{s}\right)^2+x_d'x_q\right]\left[\left(\frac{rs_0}{s}\right)^2+x_dx_q\right]^2}$$

$$\frac{ds}{dt}=-\frac{KPrI^2}{JsI_0^2}\frac{\left[\left(\frac{rs_0}{s}\right)^2+x_q^2\right]}{\left[\left(\frac{rs_0}{r}\right)^2+x_dx_q\right]^2}$$

By change of dependent variables $s=s_0\epsilon^{-z}$, $I=E/R+I_1\epsilon^{-y}$ (suggested by an oscillogram of the field current and by a speed curve) the differential equations are reducible to the non-linear integral equations

$$y=\frac{Rt}{L}+\int_0^t\frac{Rdt}{A_0^2+(r\epsilon^z)^2}+\frac{2KPr^3\mu}{Js_0^2I_1I^2}\times$$
$$\int_0^t\frac{[x_q^2+(r\epsilon^z)^2]\left[\frac{E}{R}+I_1\epsilon^{-y}\right]^2\epsilon^ydt}{[A^2+(r\epsilon^z)^2]^2[A_0^2+(r\epsilon^z)^2]\epsilon^{-4z}}$$

$$z=\frac{KPr}{JI_0^2s_0^2}\int_0^t\frac{\left[\frac{E}{R}+I_1\epsilon^y\right]^2}{\epsilon^{-2z}}\frac{[x_q^2+(r\epsilon^z)]^2}{[A^2+(r\epsilon^z)^2]^2}dt$$

where $\mu=x_q(x_d-x_d')$, $A^2=x_dx_q$, $A_0^2=x_d'x_q$, I and s are the instantaneous field current and speed respectively and t is the time in seconds. For the meaning of the remaining symbols (mostly reactances) the reader is referred elsewhere.[12]

Once z has been found the number of revolutions before the rotor of the machine comes to rest is given by

$$N=\frac{s_0}{2\pi}\int_0^\infty\epsilon^{-z}dt$$

The start of the solution of the equations above is the identification

$$u_1(x)=y_1=\phi_1(x)=\frac{R}{L}t,$$
$$u_2(x)=z_1=\phi_2(x)=0, \text{ etc.}$$

It is evident that the complexity of the successive approximations u_1^1, u_1^2, .. increases much too rapidly to afford an accurate solution. An upper limit to the number of revolutions will suffice as an answer provided the upper limit is not much in excess of the actual number. Since both I and s are known to be monotone functions (figure 5) this useful upper limit is easily obtained by replacing functions of the time within the integrands of the successive integrals by appropriately chosen constants.

6. Method of Galerkin

Galerkin's method resembles in certain features the method of Ritz[13] for linear partial differential equations. Essentially, the method consists of setting up a sequence (sometimes series) of functions which satisfy precisely the boundary conditions of a system of differential equations and which satisfy the differential equations themselves very closely. The method is primarily one of solving systems of linear equations, but it will in certain cases attack successfully non-linear problems. In the interest of brevity, we shall examine it relative to a most trivial exercise and refer elsewhere[14] for the theory. However, a gist of the method is evident from the exercise. The equation $di/dt=Ri^2$ can be considered the differential equation of a non-linear d-c circuit with non-linear resistance. Let the initial conditions be $i=1$ for $t=0$. By Galerkin's method choose

$$i=1+c_1t+c_2t^2$$

for the interval (0, 1). Denote the result of substituting i in the differential equation by ϵ. Galerkin imposes the condition

$$\frac{\partial}{\partial c_i}\int_0^a\epsilon^2dt=0$$

which, in the present exercise, yields

$$\int_0^1[(c_1+2c_2t)-R(1+c_1t+c_2t^2)^2]\times[1-2R(1+c_1t+c_2t^2)t]dt=0$$
$$\int_0^1[(c_1+2c_2t)-R(1+c_1t+c_2t^2)^2]\times[2t-2R(1+c_1t+c_2t^2)t^2]dt=0$$

Values of c_1 and c_2, which satisfy the last equations, can be found by means of implicit function theory.[15] The accurate solution of the differential equation is $i=1/(1-Rt)$. Graphs of the accurate and approximate solutions (for $R=1/2$) are shown in figure 6.

Concluding Remarks

In the non-linear discrete problems of industry the author has found the six methods given above the most useful. References to the methods of differential variations, successive approximations, series solutions in independent variables, and collocation which also have been of great value are found in the bibliography.

References

1. The Engineer Grapples With Nonlinear Problems, **Theodore von Kármán.** *Bulletin*, American Mathematical Society, volume 46, 1940. An encyclopedia of nonlinearity in continuous systems with a bibliography of 178 entries, also nonlinearity in certain discrete systems.

2. Pulling-Into-Step of a Salient-Pole Synchronous Motor, **H. E. Edgerton** and **Paul Fourmarier.** AIEE Transactions, volume 50, 1931, pages 769–78.

Pulling-Into-Step Equations Solved (Differential Analyzer Solution), **H. E. Edgerton, G. S. Brown, K. J. Germeshausen, R. W. Hamilton.** Electrical Engineering, volume 52, 1933, pages 707–11.

3. Les Méthodes Nouvelles de la Mécanique Céleste, **H. Poincaré,** volume 1, page 48.

4. Resonance Theory of Series Nonlinear Control Circuits, **E. G. Keller.** *Journal* of the Franklin Institute, volume 225, 1938, pages 561–77.

Beat Theory of Nonlinear Circuits, **E. G. Keller.** *Journal* of the Franklin Institute, volume 228, 1939, pages 319–37.

5. Collected Works, **A. Cauchy,** volume 7, first series.

6. Problem of Three Bodies, **H. Poincaré.** *Acta Mathematica*, volume 13, pages 5 and 266.

7. Differential Equations (book), **F. R. Moulton,** page 47.

8. Periodic Orbits (book), **F. R. Moulton,** Carnegie publication 161.

9. **J. Lagrange's** Oeuvres, 4, pages 9 and 159.

Ordinary Differential Equations (book), **E. L. Ince.**

10. Mathematical Engineering, **E. G. Keller,** volume 2, chapter 3.

11. Pull-in Characteristics of Synchronous Motors, **D. R. Shoults, S. B. Crary, and A. H. Lauder.** AIEE Transactions, volume 54, 1935 (December section), pages 1385–95.

Some Effects of Variable Excitation on Synchronous Motor Oscillations, **J. G. Barry.** *Journal* of the Franklin Institute, April 1940.

12. Mathematics of Modern Engineering, **R. E. Doherty and E. G. Keller,** volume 1, page 185.

13. Uber Variationproblem der Mathematischen Physik, **W. Ritz.** *Journal für Mathematik,* volume 135, 1909.

14. Reference 21 in bibliography.

15. References 22, 23, 24, 25, 26, 27 in bibliography.

Bibliography

Theory

(1). Systems of Differential Equations Solved as Power Series in Parameters

1. Mécanique Céleste, **F. F. Tisserand,** volume 3, chapter 6.

2. Traité d'Analyse, **É. Picard,** volume 2, 1905, pages 255–60.

3. Introduction to Celestial Mechanics, **F. R. Moulton,** pages 264–5.

(2). Variation of Parameters

4. **John Bernoulli,** *Acta Erud.*, 1697, page 113.

5. **J. Lagrange,** *Nouv. Mem. Acad.*, Berlin, volume 5, 1774, volume 6, 1775, page 190.

6. Théorie de la Lune (book), **C. E. Delaunay.**

(3). Differential Variations

7. Periodic Orbits (book), **F. R. Moulton.**

8. New Methods in Exterior Ballistics, **F. R. Moulton,** chapter 4.

9. Les Méthodes Nouvelles de la Mécanique Céleste, **H. Poincaré,** volume 1, chapter 4.

(4). Hyperelliptic

10. Fonctions Elliptiques, **C. Briot and Bouquet,** page 388.

11. **C. Briot and Bouquet,** *Académie des Sciences,* Paris, volume 40, 1885, page 342.

12. **F. R. Moulton,** *American Journal of Mathematics,* volume 34, pages 177–202.

13. **H. Schwarz,** *Crelle Journal,* tome 70.

14. **E. Christoffel,** *Annali di Matematica,* tome 1.

(5). Method of Successive Approximations

15. Differential Equations, **F. R. Moulton,** chapter 11.

16. Traité d'Analyse, **E. Picard,** volume 2, 1905, page 340.

17. **É. Picard,** *Journal de Mathématiques,* fourth series, volume 6, 1890, pages 197–210.

(6). Series Solutions in Independent Variables of Nonlinear Equations

18. A Treatise on Analytical Dynamics, **E. T. Whittaker,** chapter 16.

19. **W. O. Pennell,** *Journal of Mathematics and Physics,* 7, 1927, page 24.

(7). Method of Collocation

20. **R. A. Frazer, W. P. Jones, S. W. Skan,** R. and M. number 1799 (2913). *ARC Technical Report,* 1937, Air Ministry, London. (Method is primarily one for linear equations, but is extensible by means of implicit function theory to non-linear equations.)

(8). Method of Galerkin

21. **W. J. Duncan,** R. and M. 1798 (3287). *ARC Technical Report,* 1937, Air Ministry, London.

(9). Implicit Function Theory

22. Periodic Orbits, **W. D. MacMillan,** Carnegie publication 161, chapter 1.

23. **W. D. MacMillan,** *Bulletin* of the American Mathematical Society, volume 17, 1910, pages 116–20.

24. **W. D. MacMillan,** *Mathematische Annalen,* volume 72, 1912, pages 157–202.

25. **G. A. Bliss,** *Bulletin* of the American Mathematical Society, volume 16, 1910, pages 356–9.

26. Les Méthodes Nouvelles de la Mécanique Céleste, **H. Poincaré,** volume 1, chapter 3.

27. Treatise on Theory of Functions, **Harkness and Morley,** chapter 4.

(10). Existence Theorems

28. Ordinary Differential Equations, **E. L. Ince,** chapter 13. (Theorems on equations of first order, but not of first degree).

29. Ordinary Differential Equations, **E. L. Ince,** chapter 14 (Theorems on equations of second order).

30. Fonctions Elliptiques, **C. Briot and Bouquet,** page 338.

31. **C. Briot and Bouquet** (*Comptes Rendus*) *Académie des Sciences,* Paris, volume 40, 1855, page 342.

32. **Gambier,** *Comptes Rendus Académie des Sciences,* Paris, volume 142, 1906.

33. **P. Painlevé,** *Bulletin* Soc. Math. France, volume 28, 1900, page 252 (This reference and the next two refer to theorems on equations of order three or higher).

34. **Chazey,** *Comptes Rendus Académie des Sciences,* Paris, volume 145, 1907, pages 305 and 1263.

35. **Garnier,** *Comptes Rendus Académie des Sciences,* Paris, volume 145, 1907, page 308.

36. **Garnier,** *Comptes Rendus Académie des Sciences,* Paris, volume 147, 1908, page 915.

37. **P. Painlevé,** *Comptes Rendus Académie des Sciences,* Paris, volume 116, 1893, pages 88, 173, 362, 566.

38. **P. Painlevé,** *Comptes Rendus Académie des Sciences,* Paris, volume 117, 1893.

39. **P. Painlevè,** *Comptes Rendus Académie des Sciences,* Paris, volume 133, 1901.

(11). Nonlinear Integral Equations

40. **T. Lalesco,** *Journal de Mathématiques,* volume 4, 1908, page 165.

41. Volterra, Leçons sur les Équations Intégrales, **T. Lalesco,** page 90.

42. Lecons sur les Équations Intégrales (book), **V. Volterra.**

43. Quasi-nonlinear Differential Equations, **E. Cotton.** *Bulletin,* Soc. Math. France, volume 38, 1910, pages 144–54.

44. **H. Galajikian,** *American Mathematical Society Bulletin,* volume 19, 1913, pages 342–6.

45. **H. Galajikian,** *Annals of Mathematics,* volume 16, 1915, pages 172–92.

46. **E. Schmidt,** *Mathematische Annalen,* volume 65, 1908, pages 370–99.

47. Integralgleichungen und Gleichungen mit Unendlichvielen Unbekannten, **E. Hellinger and O. Toeplitz,** pages 1481–1504.

(12). Integro-Differential Equations

48. Theory of Functionals (book), **V. Volterra.**

49. Sur les singularités des équations différentielles et intégro-différentielles à une ou plusieurs variables, **L. Pomey.** *Proceedings* of the International Mathematical Congress, Toronto, 1924.

50. Intégration d'un système comprenant une . . . d'inconnues, **L. Pomey,** *Comptes Rendus Académie des Sciences,* Paris, 1926, first half.

Applications

(1). Mechanics

51. Forced Vibrations With Nonlinear Spring Constants, **S. J. Mikina.** *Transactions* ASME, 1932.

52. Steady Forced Vibration as Influenced by Damping, **L. S. Jocobsen.** *Transactions* ASME, 1930.

53. Damping Proportional to Square of Velocity, Theory of Sound, **Lord Rayleigh,** volume 1, page 77.

54. Damping Proportional to Square of Velocity, **M. V. Ostrogradsky.** *Memoires* de l'Académie Des Sciences de St. Petersbourg, sixth series, volume 3.

55. Damping Proportional to Square of Velocity, **Burkhard.** *Zeitschrift, für Math. und Physik,* volume 68.

56. Erzwungene Schwingungen bei veranderlicher Eigenfrequenz (book), **G. Duffing.**

57. Stability of Nonlinear Systems, **E. Trefftz.** *Mathematische Annalen,* volume 95, 1925, page 307.

58. Subharmonic Resonance, **J. G. Baker.** *Transactions* ASME, volume 54, 1932, page 162.

59. Forced Nonlinear Vibrations, **J. P. Den Hartog and S. J. Mikina.** *Transactions* ASME, volume 54, 1932, page 153.

60. Method of Collocation in Mechanics (reference 20).

61. Method of Galerkin in Mechanics and Elasticity (reference 21).

62. Motion of a Rotating Projectile, New Methods in Exterior Ballistics (book), **F. R. Moulton,** chapter 6.

63. New Methods in Exterior Ballistics, **F. R. Moulton.**

64. Spherical Pendulum, **F. R. Moulton,** *Carnegie Publication number 161.*

(2). Physics

65. On the Theoretical Determination of Earth Resistance From Surface Potential Measurements, **A. F. Stevenson.** *Physics,* volume 5, 1934.

66. General Theory of the Plasma of an Arc, **L. Tonks and I. Langmuir.** *Physical Review,* volume 6, 1929.

(3). Celestial Mechanics

67. Les Méthodes Nouvelles de la Mécanique Céleste, **H. Poincaré.**

68. Periodic Orbits About an Oblate Spheroid, **W. D. MacMillan.** *Transactions* American Mathematical Society, volume 11, 1910.

69 Oscillating Satellites About the Straight-Line Equilibrium Points, **F. R. Moulton,** *Carnegie Publication number 161.*

70. Oscillating Satellites When the Finite Masses Describe Elliptical Orbits, **F. R. Moulton.** *Mathematische Annalen,* volume 73, 1913.

71. Isoceles-Triangle Solutions of the Problem of Three Bodies, **D. Buchanan,** *Carnegie Publication number 161.*

72. Periodic Orbits of Infinitesimal Satellites and Inferior Planets, **F. R. Moulton.** *Transactions* American Mathematical Society, volume 7, 1906.

73. Bruns' Theorem on Algebraic Integrals, **Bruns.** *Acta Mathematica,* 11, page 25.

74. Non-existence of Integrals of Certain Types, **H. Poincaré.** *Acta Mathematica,* 13, page 259.

75. A Class of Periodic Orbits of a Particle Subject to the Attraction of n Spheres Having Prescribed Motion, **W. R. Longley.** *Transactions* American Mathematical Society, volume 8, 1907.

(4). Nonlinear Circuits

76. On Relaxation Oscillations, **B. van der Pol.** *Phil. Mag.*, volume 2, 1926, page 978.

77. Frequency Demultiplications, **B. van der Pol.** *Nature,* September 1927.

78. **B. van der Pol,** *Phil. Mag.*, volume 3, 1927, page 65.

79. **A. Boyajian,** *General Electric Review,* volume 34, numbers 9 and 12.

80. **O. Martienssen,** *Physikalische Zeitschrift,* volume 11, 1910, page 448.

81. Critical Conditions in Ferroresonance, **P. H. Odessey and E. Weber.** AIEE Transactions, volume 57, page 444.

82. **Margand,** *Révue Générale d'Electricité,* 1921, page 635.

83. Electric Circuit Theory and the Operational Calculus (book), **J. R. Carson,** chapter 10.

84. Theory and Calculation of Variable

Transient Torques in Squirrel-Cage Induction Motors, With Special Reference to Plugging

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WHEN a squirrel-cage motor is switched in any way, whether while running or at rest, transient torques, which are usually several times the steady-state torque, occur and must be considered in design. It was desired to obtain a general semi-quantitative view of these torques by considering a limiting case which is accessible analytically with the expectation that a simple rule would be apparent for estimating the magnitude of these torques from design data. No such rule was found, and we must be content to exhibit limiting transient torque curves for certain types of motors in the commercial range and to provide accurate formulas for calculating such limiting torques for types not considered here.

The limiting case studied involves the following assumptions:

1. The windings of stator and rotor are sinusoidally distributed.
2. The iron is worked on the straight portion of the *B-H* curve (constant reluctance of the magnetic circuit).
3. There are no losses in the iron.
4. The applied voltage is sinusoidal.
5. The moment of inertia of the rotor and attached apparatus is large, so that in the limit there is no change of rotor speed during the interval of time considered.

Differential equations appropriate to this case have been derived by H. C. Stanley.[1] We shall use those for the two-phase motor.

$$\left(R+L\frac{d}{dt}\right)I_\alpha+M\frac{di_\alpha}{dt}=E\sin 2\pi ft$$

$$\left(R+L\frac{d}{dt}\right)I_\beta+M\frac{di_\beta}{dt}=E\cos 2\pi ft$$

$$M\frac{dI_\alpha}{dt}+MnI_\beta+\left(r+l\frac{d}{dt}\right)i_\alpha+lni_\beta=0$$

$$-MnI_\alpha+M\frac{dI_\beta}{dt}-lni_\alpha+\left(r+l\frac{d}{dt}\right)i_\beta=0 \quad (1)$$

I_α and I_β are the stator currents. i_α and i_β are currents flowing in the face of the rotor under the corresponding stator phases. They are not the instantaneous currents flowing through fixed points in the rotor face. R and r are the ohmic resistances of stator and rotor phases. L and l are the corresponding self-inductances and M is the maximum mutual inductance between a rotor and a stator phase. n is the rotor speed in electrical radians per second. In order that the limiting case be the best approximation to an actual case, R, r, L, l, and M should be the effective values at the supply voltage and frequency for the stator windings, and at the frequency actually occurring in the rotor windings.

In terms of these quantities, the torque (in watt-seconds) is

$$T=M(I_\beta i_\alpha-I_\alpha i_\beta)$$

The quantity of interest is the ratio of instantaneous torque to steady-state torque. The form of equation 1 has the disadvantage that many different combinations of $R, r, L, l, M, n, E,$ and f give the same relation between torque-ratio and the time in cycles. A fractional horsepower motor may have just the same transient characteristics as a much larger motor, for which all the above design parameters are different. To clarify this situation it is desirable to introduce new parameters and variables having identical values for motors of identical torque ratio characteristics, as follows:

$$I=\frac{E}{R}$$

$$x_1=\frac{I_\alpha}{I}\quad x_2=\frac{I_\beta}{I}\quad x_3=\frac{i_\alpha}{I}\quad x_4=\frac{i_\beta}{I}$$

$$A=\frac{2\pi fL}{R}\quad B=\frac{2\pi fM}{R}\quad C=\frac{2\pi fl}{R}$$

$D=\frac{r}{R}\quad \rho=-\frac{n}{2\pi f}$[*] $\quad u=2\pi ft$

The number of independent design parameters has been reduced to five, and these and the variables have become dimensionless. Substituting those, and introducing a phase-shifting parameter kappa (κ) which we shall need later, we obtain

$$\left.\begin{aligned}
&\frac{d}{du}(Ax_1+Bx_3)+x_1=\sin(u+\kappa)\\
&\frac{d}{du}(Ax_2+Bx_4)+x_2=\cos(u+\kappa)\\
&\frac{d}{du}(Bx_1+Cx_3)-\rho(Bx_2+Cx_4)+Dx_3=0\\
&\frac{d}{du}(Bx_2+Cx_4)+\rho(Bx_1+Cx_3)+Dx_4=0
\end{aligned}\right\}\quad (2)$$

Further modification of these equations to bring out maximum symmetry is desirable. The necessary transformation, with its inverse, is given below.

$$\left.\begin{aligned}
&Z_1=Ax_1+Bx_3 && Z_3=Bx_1+Cx_3\\
&Z_2=Ax_2+Bx_4 && Z_4=Bx_2+Cx_4\\
&x_1=cZ_1-bZ_3 && x_3=-bZ_1+aZ_3\\
&x_2=cZ_2-bZ_4 && x_4=-bZ_2+aZ_4
\end{aligned}\right\}\quad (2a)$$

$$a=\frac{A}{AC-B^2}\qquad b=\frac{B}{AC-B^2}\qquad c=\frac{C}{AC-B^2}$$

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[1] For all numbered references, see list at end of paper.

[*] Rotation of the rotor from the positive axis of the stator coil of phase β toward the positive axis of α has been taken as making ρ positive, which is the opposite of Stanley's convention for n. Since the voltage $E\sin(u+\kappa)$ applied to stator phase α lags the voltage $E\cos(u+\kappa)$ applied to phase β, the field of the stator windings also rotates from β toward α, and with our convention $\rho=+1$ for synchronous speed.

Electrical Systems, J. R. Carson. *Physical Review*, 1921.

(5). **Rotating Electrical Machines**

85. Starting Performances of Synchronous Motors, T. M. Linville. AIEE Transactions, volume 49, 1930, pages 531–45.

86. Integraph Solutions of Differential Equations, V. Bush and H. Hazen. *Journal* of the Franklin Institute, volume 204, 1927, page 575.

87. Transient Torque-Angle Characteristics of Synchronous Machines, W. V. Lyon and H. E. Edgerton. AIEE Transactions, volume 49, 1930, pages 686–98.

88. Starting Performance of Synchronous Motors, H. V. Putman. AIEE Transactions, volume 46, 1927, pages 39–57.

(6). **Additional References**

89. An Operational Treatment of Nonlinear Dynamical Systems, L. A. Pipes, *Journal* of the Acoustical Society of America, volume 10, 1938, pages 29–31.

90. Mathematical Methods in Engineering, (book), Th. von Kármán and A. Biot, McGraw-Hill Book Company, pages 150–7.

which gives

$$\left.\begin{aligned}
&\left(c+\frac{d}{du}\right)Z_1-bZ_3=\sin(u+\kappa)\\
&\left(c+\frac{d}{du}\right)Z_2-bZ_4=\cos(u+\kappa)\\
&-DbZ_1+\left(Da+\frac{d}{du}\right)Z_3-\rho Z_4=0\\
&-DbZ_2+\rho Z_3+\left(Da+\frac{d}{du}\right)Z_4=0\\
&T=MI^2(ac-b^2)(Z_2Z_3-Z_1Z_4)
\end{aligned}\right\}\qquad(3)$$

Since $L\ l-M^2$ is necessarily positive, $AC-B^2$ is necessarily positive, as are a, b, and c.

Equations 3 can be solved by straightforward methods. The solution is:

$$\left.\begin{aligned}
Z_1={}&(F_1\cos h_1u+F_2\sin h_1u)e^{g_1u}+\\
&(F_3\cos h_2u+F_4\sin h_2u)e^{g_2u}+\\
&\alpha\sin(u+\kappa+\lambda)\\
Z_2={}&(-F_2\cos h_1u+F_1\sin h_1u)e^{g_1u}+\\
&(-F_4\cos h_2u+F_3\sin h_2u)e^{g_2u}+\\
&\alpha\cos(u+\kappa+\lambda)\\
Z_3={}&(F_5\cos h_1u+F_6\sin h_1u)e^{g_1u}+\\
&(F_7\cos h_2u+F_8\sin h_2u)e^{g_2u}+\\
&\beta\sin(u+\kappa+\mu)\\
Z_4={}&(-F_6\cos h_1u+F_5\sin h_1u)e^{g_1u}+\\
&(-F_8\cos h_2u+F_7\sin h_2u)e^{g_2u}+\\
&\beta\cos(u+\kappa+\mu)
\end{aligned}\right\}\qquad(4)$$

where

$$2g_1=-c-aD+\sqrt{\frac{q_1+\sqrt{q_1^2+q_2^2}}{2}}$$

$$2g_2=-c-aD-\sqrt{\frac{q_1+\sqrt{q_1^2+q_2^2}}{2}}$$

$$2h_1=-\rho-\sqrt{\frac{-q_1+\sqrt{q_1^2+q_2^2}}{2}}$$

$$2h_2=-\rho+\sqrt{\frac{-q_1+\sqrt{q_1^2+q_2^2}}{2}}$$

$$q_1=(c-aD)^2-\rho^2+4b^2D$$
$$q_2=2\rho(c-aD)$$

When q_2 is negative interchange h_1 and h_2.

$$\begin{aligned}
(k_3-k_5)F_1&=-k_5\Gamma_1-k_4\Gamma_2-k_7\Gamma_3-k_6\Gamma_4\\
(k_3-k_5)F_2&=-k_4\Gamma_1+k_5\Gamma_2-k_6\Gamma_3+k_7\Gamma_4\\
(k_3-k_5)F_3&=+k_3\Gamma_1+k_4\Gamma_2+k_7\Gamma_3+k_6\Gamma_4\\
(k_3-k_5)F_4&=+k_4\Gamma_1-k_3\Gamma_2+k_6\Gamma_3-k_7\Gamma_4\\
(k_3-k_5)F_5&=-k_2\Gamma_1-k_1\Gamma_2+k_5\Gamma_3+k_4\Gamma_4\\
(k_3-k_5)F_6&=-k_1\Gamma_1+k_2\Gamma_2+k_4\Gamma_3-k_5\Gamma_4\\
(k_3-k_5)F_7&=+k_2\Gamma_1+k_1\Gamma_2-k_3\Gamma_3-k_4\Gamma_4\\
(k_3-k_5)F_8&=+k_1\Gamma_1-k_2\Gamma_2-k_4\Gamma_3+k_3\Gamma_4\\
\Gamma_1&=Z_1^{(0)}-\alpha\sin(\kappa+\lambda)\\
\Gamma_2&=Z_2^{(0)}-\alpha\cos(\kappa+\lambda)\\
\Gamma_3&=Z_3^{(0)}-\beta\sin(\kappa+\mu)\\
\Gamma_4&=Z_4^{(0)}-\beta\cos(\kappa+\mu)
\end{aligned}$$

$Z_i^{(0)}$ is the value of Z_i when $u=0$

The k's are functions of the g's, and h's, and b and c as follows:

$$\begin{aligned}
k_1={}&[-h_2(g_1+c)^2+h_1(g_2+c)^2-\\
&h_1h_2(h_1-h_2)]+b\\
k_2={}&[+(g_1+c)(g_2+c)(g_1-g_2)+h_1^2(g_2+c)-\\
&h_2^2(g_1+c)]+b\\
k_3={}&h_1(h_1-h_2)+(g_1+c)(g_1-g_2)\\
k_4={}&h_1(g_2+c)-h_2(g_1+c)\\
k_5={}&h_2(h_1-h_2)+(g_2+c)(g_1-g_2)\\
k_6={}&-b(h_1-h_2)\\
k_7={}&-b(g_1-g_2)
\end{aligned}$$

The relation

$$k_4(k_3+k_5)+k_1k_7+k_2k_6\equiv0$$

may be used as a check on the numerical work.

$$\alpha=\left(\frac{a^2+\sigma^2}{G^2+H^2}\right)^{1/2}\qquad\beta=\frac{b}{(G^2+H^2)^{1/2}}$$

$$\sin\lambda=\frac{-aG+\sigma H}{(a^2+\sigma^2)^{1/2}(G^2+H^2)^{1/2}}$$

$$\cos\lambda=\frac{aH+\sigma G}{(a^2+\sigma^2)^{1/2}(G^2+H^2)^{1/2}}$$

$$\sin\mu=-\frac{G}{(G^2+H^2)^{1/2}}\qquad\cos\mu=\frac{H}{(G^2+H^2)^{1/2}}$$

$$G=a+c\sigma\quad\sigma=\frac{1-\rho}{D}\quad H=ac-b^2-\sigma$$

It is worthwhile to form the expression for the torque as certain terms cancel. The result is:

$$\begin{aligned}
Z_2Z_3-Z_1Z_4={}&\alpha\beta\sin(\mu-\lambda)+\\
&L_1e^{2g_1u}+L_2e^{2g_2u}+\\
&[L_3\cos(h_1-h_2)u+L_4\sin(h_1-h_2)u]e^{(g_1+g_2)u}+\\
&[L_5\cos(h_1+1)u+L_6\sin(h_1+1)u]e^{g_1u}+\\
&[L_7\cos(h_2+1)u+L_8\sin(h_2+1)u]e^{g_2u}
\end{aligned}\qquad(5)$$

where

$$\begin{aligned}
L_1={}&F_1F_6-F_2F_5\\
L_2={}&F_3F_8-F_4F_7\\
L_3={}&F_3F_6-F_2F_7+F_1F_8-F_4F_5\\
L_4={}&F_1F_7-F_3F_5+F_2F_8-F_4F_6\\
L_5={}&+F_6\alpha\sin(\kappa+\lambda)+F_5\alpha\cos(\kappa+\lambda)-\\
&F_2\beta\sin(\kappa+\mu)-F_1\beta\cos(\kappa+\mu)\\
L_6={}&-F_5\alpha\sin(\kappa+\lambda)+F_6\alpha\cos(\kappa+\lambda)+\\
&F_1\beta\sin(\kappa+\mu)-F_2\beta\cos(\kappa+\mu)\\
L_7={}&+F_8\alpha\sin(\kappa+\lambda)+F_7\alpha\cos(\kappa+\lambda)-\\
&F_4\beta\sin(\kappa+\mu)-F_3\beta\cos(\kappa+\mu)\\
L_8={}&-F_7\alpha\sin(\kappa+\lambda)+F_8\alpha\cos(\kappa+\lambda)+\\
&F_3\beta\sin(\kappa+\mu)-F_4\beta\cos(\kappa+\mu)
\end{aligned}$$

When $Z_i^{(0)}=0$ (no currents flowing in motor at the instant of switching) the torque is independent of the phase κ of the voltage at which the motor is thrown in the line. This was to be expected from the complete symmetry of the squirrel-cage motor and the polyphase field.

When $q_2=2\rho(c-aD)=0$, in which case the burden of computation is considerably reduced, interesting special cases occur.

Case 1:

$c=aD$
$\rho\neq0$
$U^2\equiv\rho^2/_4-b^2D>0$
$g_1=g_2=-c$
$h_1=-\frac{\rho}{2}-U$
$h_2=-\frac{\rho}{2}+U$

This is the case in which rotor and stator windings have the same phase-angle when each is energized at the supply frequency with the other open circuited, and in which the absolute speed of rotation is at or near synchronous speed. In this case the formulae for the k's are simplified as follows:

$k_1=+2bDU$
$k_3=+\rho U+2U^2$
$k_5=+\rho U-2U^2$
$k_6=+2bU$
$k_2=k_4=k_7=0$

Case 2:

$c=aD$
$\rho\neq0$
$V^2\equiv b^2D-\rho^2/_4>0$
$g_1=-c+V$
$g_2=-c-V$
$h_1=h_2=-\frac{\rho}{2}$

This case has the same phase-angle relation as case 1, but the absolute speed of rotation, while not zero, differs considerably from synchronous speed. The k's simplify to

$k_2=-2bDV$
$k_3=2V^2$
$k_4=\rho V$
$k_5=-2V^2$
$k_7=-2bV$
$k_1=k_6=0$

Case 3:

$c=aD$
$\rho\neq0$
$b^2D=\rho^2/_4$
$g_1=g_2=-c$
$h_1=h_2=-\frac{\rho}{2}$

This is the transition case between case 1 and case 2. The Z's are

$$\begin{aligned}
Z_1={}&[(\Gamma_1+\Gamma_5u)\cos b\sqrt{D}u+(-\Gamma_2+\Gamma_6u)\times\\
&\sin b\sqrt{D}u]e^{-cu}+\alpha\sin(u+\kappa+\lambda)\\
Z_2={}&[(\Gamma_2-\Gamma_6u)\cos b\sqrt{D}u+(\Gamma_1+\Gamma_5u)\times\\
&\sin b\sqrt{D}u]e^{-cu}+\alpha\cos(u+\kappa+\lambda)\\
Z_3={}&[(\Gamma_3+\sqrt{D}\Gamma_5u)\cos b\sqrt{D}u+(-\Gamma_4-\\
&\sqrt{D}\Gamma_6u)\sin b\sqrt{D}u]e^{-cu}+\beta\sin(u+\kappa+\mu)\\
Z_4={}&[(\Gamma_4+\sqrt{D}\Gamma_6u)\cos b\sqrt{D}u+(\Gamma_3+\\
&\sqrt{D}\Gamma_5u)\sin b\sqrt{D}u]e^{-cu}+\beta\cos(u+\kappa+\mu)
\end{aligned}$$

where

$$\Gamma_5=b(\sqrt{D}\Gamma_2+\Gamma_3),\quad\Gamma_6=b(\sqrt{D}\Gamma_1-\Gamma_4)$$

$$\begin{aligned}
Z_2Z_3-Z_1Z_4={}&\alpha\beta\sin(\mu-\lambda)+\\
&(N_0+N_1u+N_2u^2)e^{-2cu}+\\
&e^{-cu}[(N_3+N_4u)\cos(b\sqrt{D}+1)u+\\
&(N_5+N_6u)\sin(b\sqrt{D}+1)u]
\end{aligned}$$

where

$$N_0=\Gamma_2\Gamma_3-\Gamma_1\Gamma_4 \quad N_3=-\sqrt{D}(\Gamma_5^2+\Gamma_6^2)$$
$$N_1=-\Gamma_3\Gamma_6-\Gamma_4\Gamma_5+\sqrt{D}\Gamma_2\Gamma_6-\sqrt{D}\Gamma_1\Gamma_5$$
$$N_2=-\Gamma_4\alpha\sin(\kappa+\lambda)+\Gamma_3\alpha\cos(\kappa+\lambda)+\Gamma_2\beta\sin(\kappa+\mu)-\Gamma_1\beta\cos(\kappa+\mu)$$
$$N_4=-\sqrt{D}\Gamma_5\alpha\sin(\kappa+\lambda)+\sqrt{D}\Gamma_6\alpha\cos(\kappa+\lambda)-\Gamma_6\beta\sin(\kappa+\mu)-\Gamma_5\beta\cos(\kappa+\mu)$$
$$N_5=-\Gamma_3\alpha\sin(\kappa+\lambda)-\Gamma_4\alpha\cos(\kappa+\lambda)+\Gamma_1\beta\sin(\kappa+\mu)+\Gamma_2\beta\cos(\kappa+\mu)$$
$$N_6=-\sqrt{D}\Gamma_6\alpha\sin(\kappa+\lambda)-\sqrt{D}\Gamma_5\alpha\cos(\kappa+\lambda)+\Gamma_5\beta\sin(\kappa+\mu)-\Gamma_6\beta\cos(\kappa+\mu)$$

The above formulae are valid when $\rho=-2b\sqrt{D}$. When $\rho=+2b\sqrt{D}$ change the sign of $\sqrt{D}$ wherever it occurs. α, β, λ, μ, and the Γ's have the same values as before.

Case 4: $\rho=0$. Here the equations are as follows:

$$Z_1=J_1\epsilon^{\omega_1 u}+J_2\epsilon^{\omega_2 u}+\alpha\sin(u+\kappa+\lambda)$$
$$Z_2=J_3\epsilon^{\omega_1 u}+J_4\epsilon^{\omega_2 u}+\alpha\cos(u+\kappa+\lambda)$$
$$Z_3=J_5\epsilon^{\omega_1 u}+J_6\epsilon^{\omega_2 u}+\beta\sin(u+\kappa+\mu)$$
$$Z_4=J_7\epsilon^{\omega_1 u}+J_8\epsilon^{\omega_2 u}+\beta\cos(u+\kappa+\mu)$$

where

$$J_1=\frac{(c+\omega_2)\Gamma_1-b\Gamma_3}{\omega_2-\omega_1} \qquad J_2=\frac{-(c+\omega_1)\Gamma_1+b\Gamma_3}{\omega_2-\omega_1}$$

$$J_3=\frac{(c+\omega_2)\Gamma_2-b\Gamma_4}{\omega_2-\omega_1} \qquad J_4=\frac{-(c+\omega_1)\Gamma_2+b\Gamma_4}{\omega_2-\omega_1}$$

$$J_5=\frac{c+\omega_1}{b}J_1 \qquad J_6=\frac{c+\omega_2}{b}J_2$$

$$J_7=\frac{c+\omega_1}{b}J_3 \qquad J_8=\frac{c+\omega_2}{b}J_4$$

α, β, λ, μ, and the Γ's have the same values as before, and ω_1 and ω_2 are the roots of

$$(\omega+c)(\omega+aD)-Db^2=0$$

These roots are always real and unequal.

$$Z_2Z_3-Z_1Z_4=\alpha\beta\sin(\mu-\lambda)+K_0\epsilon^{(\omega_1+\omega_2)u}+\epsilon^{\omega_1 u}(K_1\cos u+K_2\sin u)+\epsilon^{\omega_2 u}(K_3\cos u+K_4\sin u)$$

where

$$K_0=\frac{\omega_2-\omega_1}{b}(J_2J_3-J_1J_4)$$
$$=J_3J_6+J_4J_5-J_1J_8-J_2J_7$$
$$K_1=-J_7\alpha\sin(\kappa+\lambda)+J_5\alpha\cos(\kappa+\lambda)+J_3\beta\sin(\kappa+\mu)-J_1\beta\cos(\kappa+\mu)$$
$$K_2=-J_5\alpha\sin(\kappa+\lambda)-J_7\alpha\cos(\kappa+\lambda)+J_1\beta\sin(\kappa+\mu)+J_3\beta\cos(\kappa+\mu)$$
$$K_3=-J_8\alpha\sin(\kappa+\lambda)+J_6\alpha\cos(\kappa+\lambda)+J_4\beta\sin(\kappa+\mu)-J_2\beta\cos(\kappa+\mu)$$
$$K_4=-J_6\alpha\sin(\kappa+\lambda)-J_8\alpha\cos(\kappa+\lambda)+J_2\beta\sin(\kappa+\mu)+J_4\beta\cos(\kappa+\mu)$$

The foregoing sections apply only to the case where the two phases are switched simultaneously. Larger transient torques are found when the switching (plugging) is done one phase at a time. To compute these it is necessary to solve the equation for the case of only one phase closed and the case of no phase closed.

Single-Phase Case

Here, if we assume that it is the β phase which is opened, the second of equations 3 becomes indeterminate and the condition $x_2\equiv 0$ is substituted for it. The solution is

$$\left.\begin{aligned}Z_1&=E_1\epsilon^{su}+\epsilon^{gu}(E_2\cos hu+E_3\sin hu)+\alpha'\sin(u+\kappa+\lambda')\\Z_2&=b[E_7\epsilon^{su}+\epsilon^{gu}(E_8\cos hu+E_9\sin hu)+\gamma'\cos(u+\kappa+\nu')]\\Z_3&=E_4\epsilon^{su}+\epsilon^{gu}(E_5\cos hu+E_6\sin hu)+\beta'\sin(u+\kappa+\mu')\\Z_4&=c[E_7\epsilon^{su}+\epsilon^{gu}(E_8\cos hu+E_9\sin hu)+\gamma'\cos(u+\kappa+\nu')]\end{aligned}\right\}\quad(6)$$

where s is the real root of the equation

$$cs^3+p_1s^2+p_2s+p_3=0$$

and g and h are coefficients in the form of complex roots $g\pm hi$. Here

$$p_1=D(2ac-b^2)+c^2$$

$$p_2=D(ac-b^2)(Da+2c)+\rho^2c$$

$$p_3=D^2(ac-b^2)^2+\rho^2c^2$$

$$E_1=\frac{[(c+g)^2+h^2+Db^2]\Sigma_1-b(2g+Da+c)\Sigma_3+\rho bc\Sigma_2}{(g-s)^2+h^2}$$

$$E_2=\Sigma_1-E_1$$

$$\left.E_3=\frac{[(g-s)(s+c)(g+c)+Db^2(g-s)-h^2(s+c)]\Sigma_1+Pbc(g-s)\Sigma_2+b[h^2-(g-s)(c+Da+s+g)]\Sigma_3}{h[(g-s)^2+h^2]}\right\}$$

$$E_4=\frac{E_1(c+s)}{b}$$

$$E_5=\frac{E_2(c+g)+E_3h}{b}$$

$$E_6=\frac{-E_2h+E_3(c+g)}{b}$$

$$\rho bcE_7=E_1[(c+s)(Da+s)-Db^2]$$

$$\rho bcE_8=E_2[(c+g)(Da+g)-h^2-Db^2]+E_3h(Da+c+2g)$$

$$\rho bcE_9=-E_2h(Da+c+2g)+E_3[(c+g)(Da+g)-h^2-Db^2]$$

Checks:

$$E_4+E_5=\Sigma_2$$
$$E_7+E_8=\Sigma_3$$
$$DbE_3+hE_5-(Da+g)E_6+\rho cE_9=0$$
$$DbE_2-(Da+g)E_5-hE_6+\rho cE_8=0$$

where

$$\Sigma_1=Z_1^{(0)}-\alpha'\sin(\kappa+\lambda')$$

$$\Sigma_2=Z_3^{(0)}-\beta'\sin(\kappa+\mu')$$

$$\Sigma_3=\frac{-bZ_2^{(0)}+aZ_4^{(0)}}{ac-b^2}-\gamma'\cos(\kappa+\nu')$$

and

$$P_1=c(1-\rho^2)-D^2a(ac-b^2)$$
$$P_2=D^2b(ac-b^2)$$
$$G'=c-p_2$$
$$Q_1=D(2ac-b^2)$$
$$Q_2=Dbc$$
$$H'=p_1-p_3$$

$$\alpha'=\left(\frac{P_1^2+Q_1^2}{G'^2+H'^2}\right)^{1/2}$$

$$\beta'=\left(\frac{P_2^2+Q_2^2}{G'^2+H'^2}\right)^{1/2}$$

$$\gamma'=\frac{Db|\rho|}{(G'^2+H'^2)^{1/2}}$$

$$\sin\lambda'=-\frac{P_1G'+Q_1H'}{(G'^2+H'^2)^{1/2}(P_1^2+Q_1^2)^{1/2}}$$

$$\cos\lambda'=\frac{-Q_1G'+P_1H'}{(G'^2+H'^2)^{1/2}(P_1^2+Q_1^2)^{1/2}}$$

$$\sin\mu'=\frac{P_2G'-Q_2H'}{(G'^2+H'^2)^{1/2}(P_2^2+Q_2^2)^{1/2}}$$

$$\cos\mu'=-\frac{Q_2G'+P_2H'}{(G'^2+H'^2)^{1/2}(P_2^2+Q_2^2)^{1/2}}$$

$$\sin\nu'=\frac{-H'}{(G'^2+H'^2)^{1/2}}$$

$$\cos\nu'=\frac{-G'}{(G'^2+H'^2)^{1/2}}$$

The above formulae for $\sin\nu'$ and $\cos\nu''$ hold when ρ is positive. When $\rho<0$ the signs should be reversed.

For the single phase case

$$Z_2Z_3-Z_1Z_4={}^1/_2c\alpha'\gamma'\sin(\lambda'-\nu')-{}^1/_2b\beta'\gamma'\sin(\mu'-\nu')+{}^1/_2c\alpha'\gamma'\sin(2u+2\kappa+\lambda'+\nu')-{}^1/_2b\beta'\gamma'\sin(2u+2\kappa+\mu'+\nu')$$

in the steady state.

In the case that the roots of the cubic $cs^3+p_1s^2+p_2s+p_3=0$ are all real the formulas are

$$Z_1=\rho bc[f_1\epsilon^{s_1u}+f_2\epsilon^{s_2u}+f_3\epsilon^{s_3u}]+\alpha'\sin(u+\kappa+\lambda')$$

$$Z_3=f_1\rho c(c+s_1)\epsilon^{s_1u}+f_2\rho c(c+s_2)\epsilon^{s_2u}+f_3\rho c(c+s_3)\epsilon^{s_3u}+\beta'\sin(u+\kappa+\mu')$$

$$\frac{Z_2}{b}=\frac{Z_4}{c}=f_1[-Db^2+(c+s_1)(Da+s_1)]\epsilon^{s_1u}+f_2[-Db^2+(c+s_2)(Da+s_2)]\epsilon^{s_2u}+f_3[-Db^2+(c+s_3)(Da+s_3)]\epsilon^{s_3u}+\gamma'\cos(u+\kappa+\nu')$$

where

$$f_1=\frac{+[Db^2+(c+s_2)(c+s_3)]\Sigma_1-b(c+Da+s_2+s_3)\Sigma_3+\rho bc\Sigma_2}{\rho bc(s_1-s_2)(s_1-s_3)}$$

$$f_2=\frac{+[Db^2+(c+s_1)(c+s_3)]\Sigma_1-b(c+Da+s_1+s_3)\Sigma_3+\rho bc\Sigma_2}{\rho bc(s_2-s_1)(s_2-s_3)}$$

$$f_3=\frac{+[Db^2+(c+s_1)(c+s_2)]\Sigma_1-b(c+Da+s_1+s_2)\Sigma_3+\rho bc\Sigma_2}{\rho bc(s_3-s_1)(s_3-s_2)}$$

When $\rho=0$ the formulas are

$$Z_1=m_1be^{s_1u}+m_2be^{s_2u}+\alpha'\sin(u+\kappa+\lambda')$$

$$Z_2=m_1(c+s_1)e^{s_1u}+m_2(c+s_2)e^{s_2u}+\beta'\sin(u+\kappa+\mu')$$

$$\frac{Z_3}{b}=\frac{Z_4}{c}=\Sigma_3e^{s_3u}+\gamma'\cos(u+\kappa+\nu')$$

where

$$m_1=\frac{-(c+s_2)\Sigma_1+b\Sigma_2}{b(s_1-s_2)}$$

$$m_2=\frac{(c+s_1)\Sigma_1-b\Sigma_2}{b(s_1-s_2)}$$

In this case s_1, s_2, s_3 reduce to

$$2s_1,2s_2=-c-Da\pm\sqrt{(c-Da)^2+4Db^2}$$

$$s_3=-\frac{D}{c}(ac-b^2)$$

α', λ', Σ, etc., have the same meanings as in general.

"No-Phase" Case

When both stator phases are open, we have

$$\left.\begin{aligned}&x_1=x_2\equiv0\\&C\frac{dx_3}{du}+Dx_3-C\rho x_4=0\\&C\frac{dx_4}{du}+Dx_4+C\rho x_3=0\\&x_3=\delta e^{-Du/C}\sin(\rho u+\epsilon)\\&x_4=\delta e^{-Du/C}\cos(\rho u+\epsilon)\end{aligned}\right\}\qquad(7)$$

where δ and ϵ are determined by the initial values of x_3 and x_4 when $u=0$. The torque is, of course, identically zero in this case.

Transformation of Formulas

For computational purposes it is generally necessary to be able to modify the preceding formulae by means of the following rules. They should be understood as mere devices to obviate the necessity of writing down new sets of formulae; the *formulae* are to be transformed as described, while the significance of the *letters* remains unchanged. E_α and E_β denote the voltages applied to the stator α and β phases, respectively.

To make the one-phase formulae for stator phase α closed ($E_\alpha=E\sin(u+\kappa)$), β open apply for α open, β closed($E_\beta=E\times\cos(u+\kappa)$), interchange Z_1 and Z_2 (also $Z_1^{(0)}$ and $Z_2^{(0)}$); interchange Z_3 and Z_4 (also $Z_3^{(0)}$ and $Z_4^{(0)}$); change the sign of ρ in the formulae; and increase the phase constants λ', μ', ν', calculated from the transformed formulae, by $\pi/2$. Thus we obtain

$$\Sigma_1=Z_2^{(0)}-\alpha'\sin\left(\kappa+\lambda'+\frac{\pi}{2}\right)$$

and so on. The actual numerical value of ρ to be used in the new formula must be determined by the convention adopted, that ρ is positive when the rotor turns from the positive axis of stator-phase β toward the positive axis of stator-phase α.

To reverse the polarity of stator-phase α, that is, to make the two-phase formulae which are valid when $E_\alpha=E\sin(u+\kappa)$, $E_\beta=E\cos(u+\kappa)$, apply to the case when $E_\alpha=-E\sin(u+\kappa)$, $E_\beta=E\cos(u+\kappa)$, the procedure is exactly the same as in the preceding paragraph, except that here there are only the two phase constants λ, μ to be increased by $\pi/2$. A specimen transformed formula is

$$\Gamma_1=Z_1^{(0)}-\alpha\sin\left(\kappa+\lambda+\frac{\pi}{2}\right)$$

Formulae applicable when E_α, E_β are given by still other expressions are obtainable from the foregoing by assigning a suitable value to κ. It may be mentioned, however, that the polarity of the β phase may also be reversed by means of the procedure above, except that the phase constants λ, μ are to be decreased (not increased) by $\pi/2$.

Sequential Switching

As an example of the use of the above formulae consider the case of a motor which has been running forward long enough to have reached a steady condition. The general method is to switch one phase out at a selected phase of the voltage and consider the motor as running single-phase in the transient condition. At a second selected instant and phase, the second phase is switched out. The stator currents (i.e. x_1 and x_2) are zero during the "no phase" interval but the rotor currents die away slowly in the manner described by equations 7. Before the rotor currents are gone one stator phase is switched in with the polarity which it had originally and the motor runs single phase again in the transient condition. Before the steady state is established the second phase is switched in with reversed polarity and the motor runs "plugged" through the transient condition into the steady state. The computation details are as follows:

The Z's are computed from equations 4 for the particular phase of u at which one phase is to be switched out, bearing in mind that the absolute value of u is so large that the exponential terms drop. From the values of the Z's at the instant at which one phase is switched out the Σ's of equations 6 are computed. These are substituted in equations 6 for the instant the second phase is switched out and the corresponding Z's are computed. x_3 and x_4 are now computed from equations 2*a*. (Note that the values of x_1 and x_2 so computed are not zero in general; in practice the energy accruing from their annihilation is dissipated in arcs at the contacts.) δ and ϵ are computed from these values and substituted in equations 7 and x_3 and x_4 computed for the end of the "no phase" interval. These are translated into Z's via equations 2*a* with x_1 and x_2 zero, new values of the Σ's of equations 6 are worked out and the Z's computed for the instant the second phase is reclosed reversed. Equations 4 are transformed as described above, new Γ's computed for these equations and the transient followed as far as desired. Since the coefficients of explicit formulae for the Z's have been evaluated for each interval, it is possible to compute the instantaneous torque at any time before, during, and after the switching sequence.

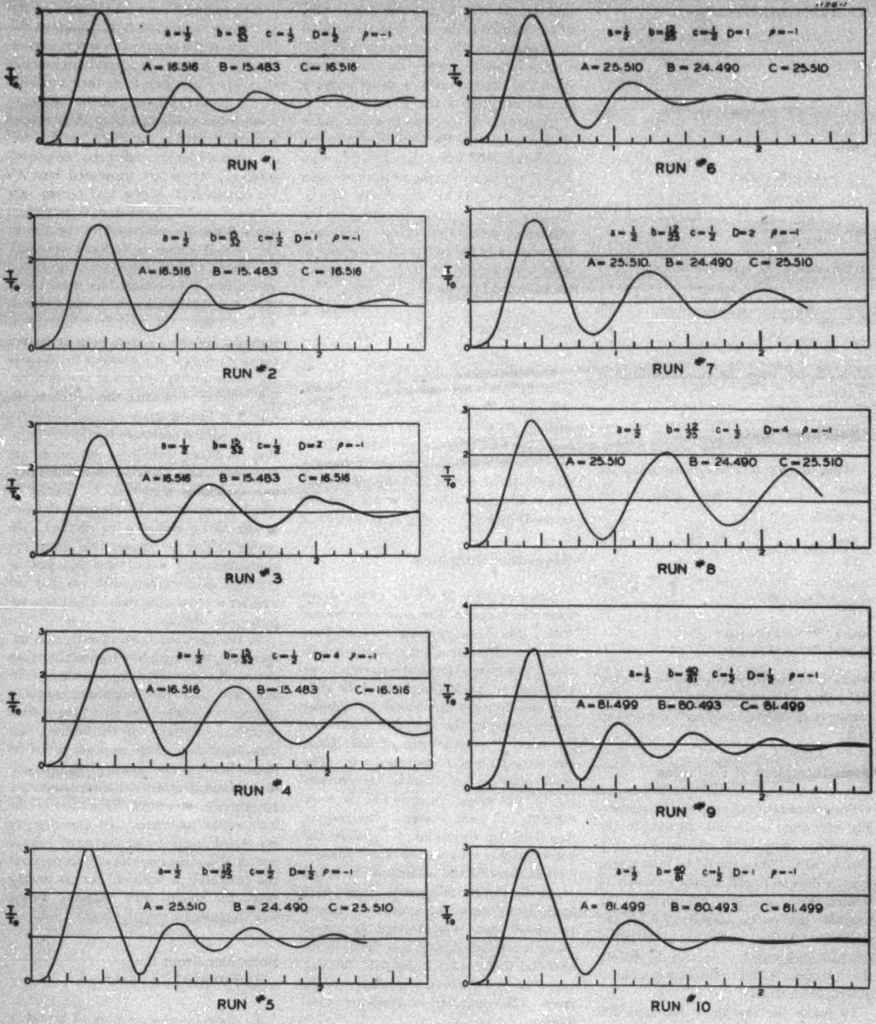
The changes necessary for the computation of other switching sequences are fairly obvious. There are in all ten different switching sequences by means of which a motor can be plugged, the two phases being regarded as distinct. In reality it can be shown that from the computational point of view they may be regarded as interchangeable, so that the number is reduced to five. These five appear to be distinct.

In the cases we have investigated numerically, the maximum transient torques occur when the switching is done at voltage zero and the minimum transient torques occur when the switching is done at voltage maxima. In the limiting case considered here there appears to be no upper limit to the transient torques obtained from arbitrary switching sequences. In practice, of course, saturation of the iron would intervene. In one case we computed opening and closing at voltage zero, and leaving one phase in throughout the switching, a transient torque twenty times the steady state plugging torque was computed.

Solutions From the Differential Analyzer

To obtain a general view of transient plugging torques resulting from simultaneous switching equations 3 were solved on the Massachusetts Institute of Technology differential analyzer.[2]

The use of gears for multiplying constants into the machine restricted us to simple rational fractional values of a, b, and c. The results are shown graphically in the accompanying illustrations. The ordinates are ratios of instantaneous



RUN #1

RUN #2

RUN #3

RUN #4

RUN #5

RUN #6

RUN #7

RUN #8

RUN #9

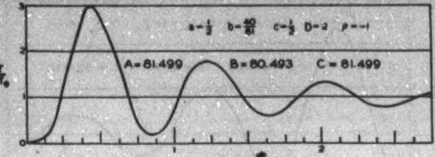
RUN #10

Transient plugging torques of induction motors

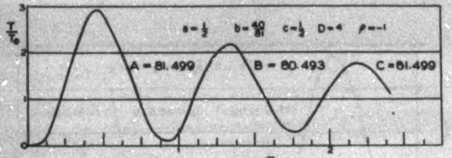
Ordinate: $\frac{T}{T_0}$, ratio of instantaneous torque to steady plugging torque

Abscissa: number of cycles

$$A = \frac{2\pi f L}{R} \quad B = \frac{2\pi f M}{R} \quad C = \frac{2\pi f l}{R} \quad D = \frac{r}{R} \quad \rho = -\frac{n}{2\pi f}$$



RUN #11



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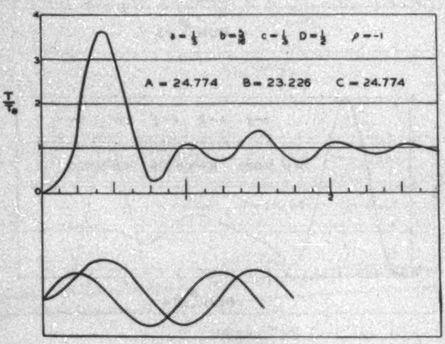

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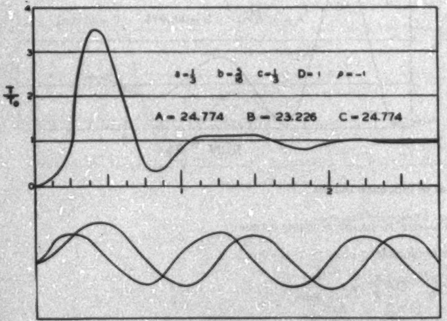

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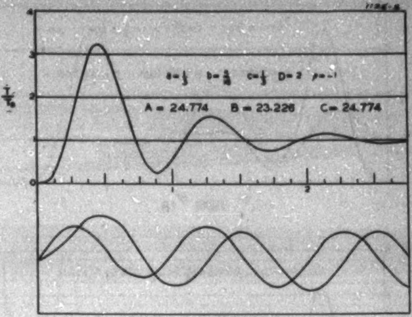

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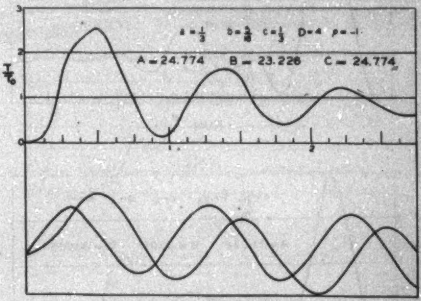

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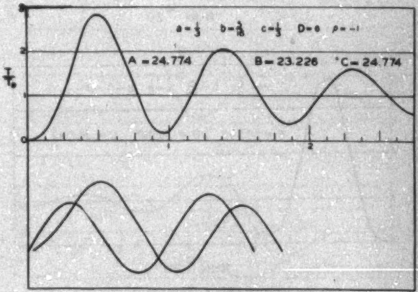


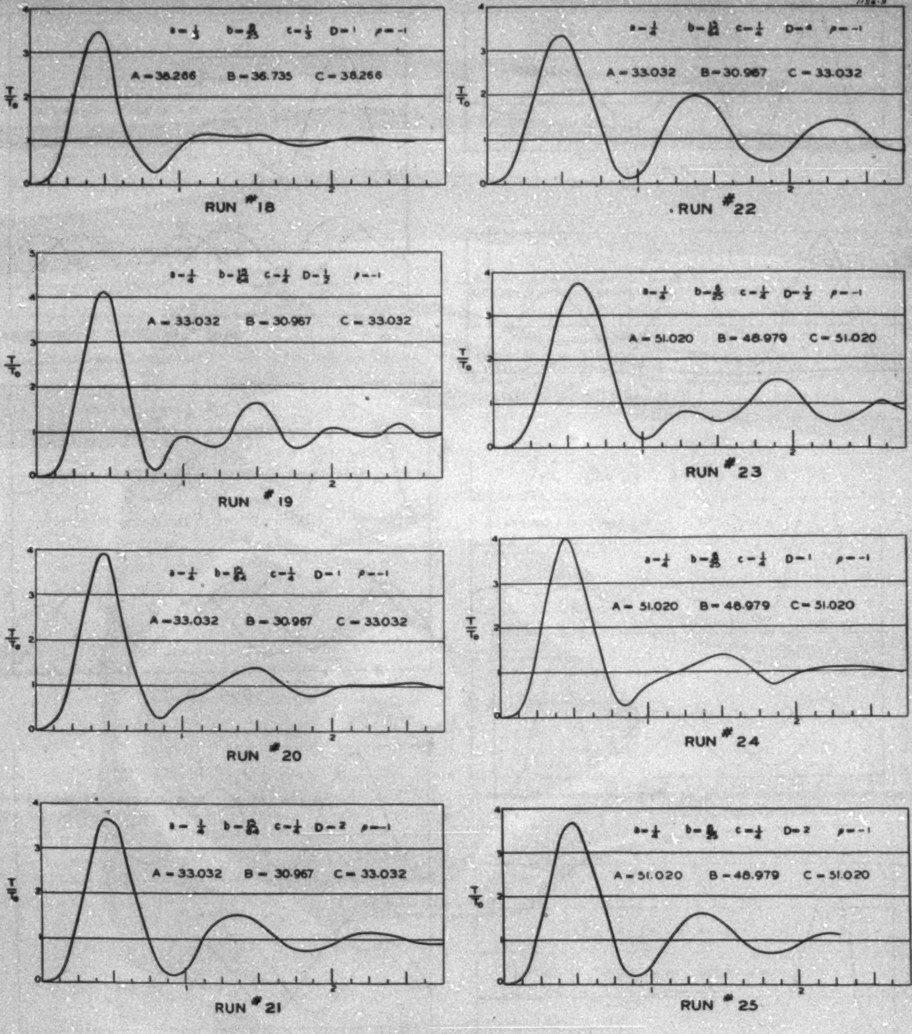
RUN #17

Transient plugging torques of induction motors

Ordinate: $\frac{T}{T_0}$, ratio of instantaneous torque to steady plugging torque

Abscissa: number of cycles

$$A=\frac{2\pi fL}{R} \quad B=\frac{2\pi fM}{R} \quad C=\frac{2\pi fl}{R} \quad D=\frac{r}{R} \quad \rho=-\frac{n}{2\pi f}$$



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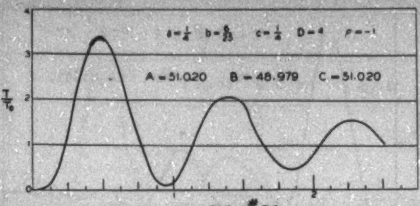

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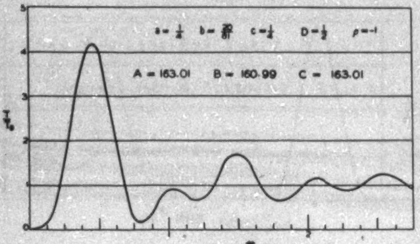

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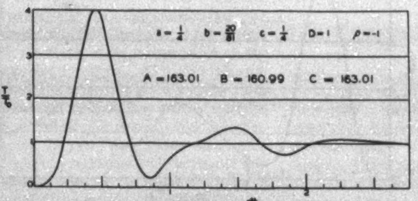

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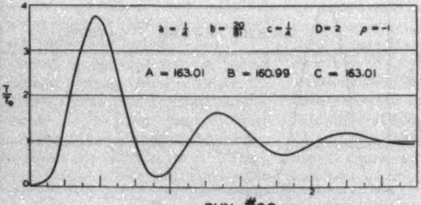

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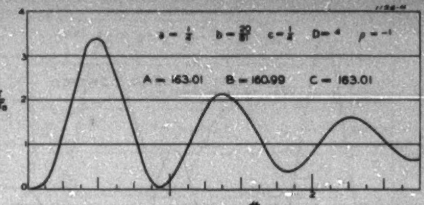

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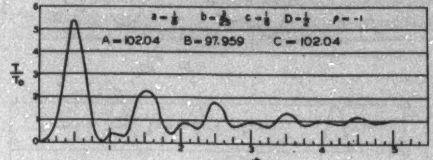

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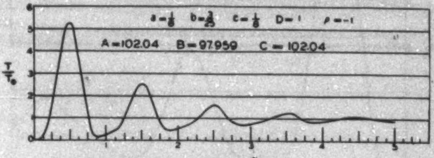

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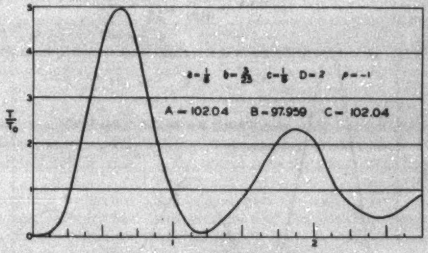

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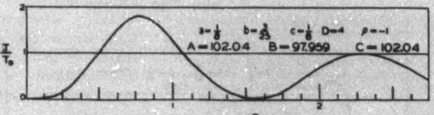


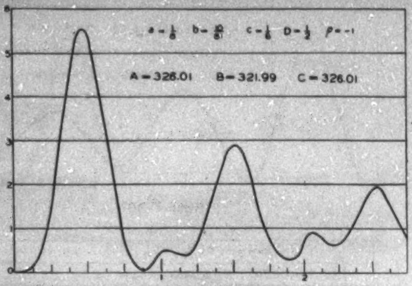
RUN #34

Transient plugging torques of induction motors

Ordinate: $\frac{T}{T_0}$, ratio of instantaneous torque to steady plugging torque

Abscissa: number of cycles

$$A=\frac{2\pi fL}{R} \quad B=\frac{2\pi fM}{R} \quad C=\frac{2\pi fl}{R} \quad D=\frac{r}{R} \quad \rho=-\frac{n}{2\pi f}$$



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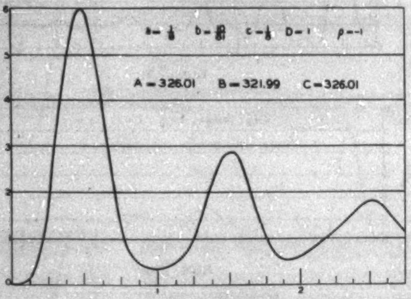

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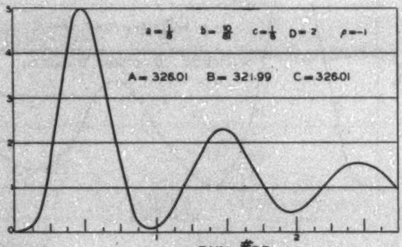

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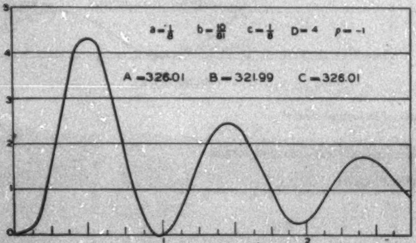

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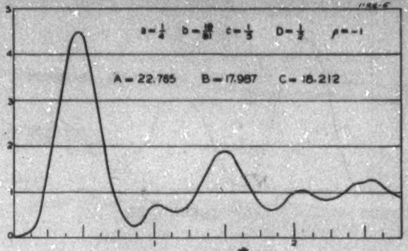

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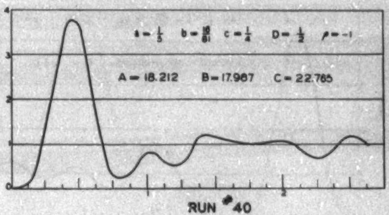

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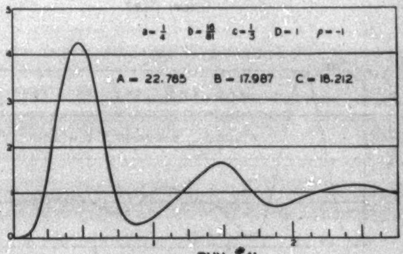

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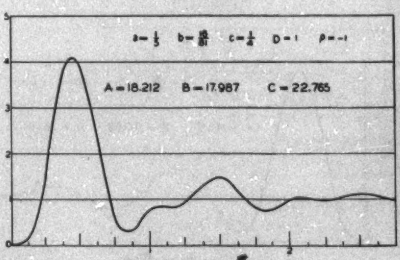


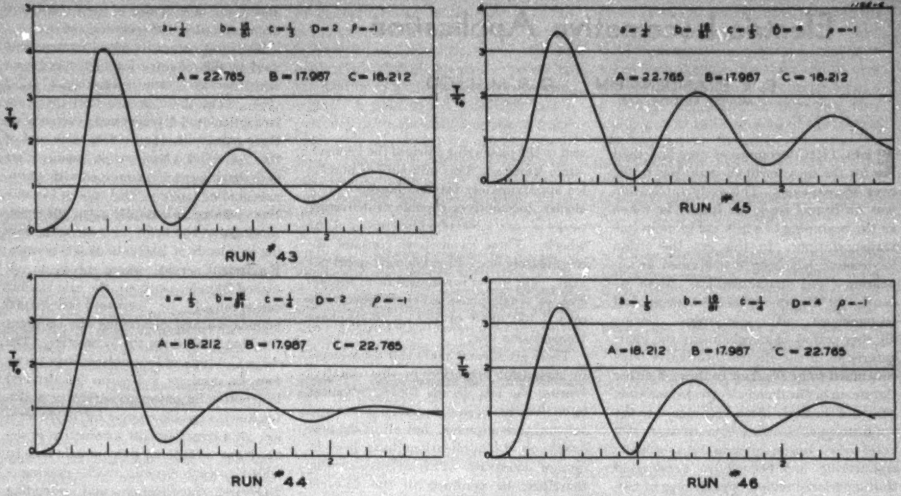
RUN #42

Transient plugging torques of induction motors

Ordinate: $\frac{T}{T_0}$, ratio of instantaneous torque to steady plugging torque

Abscissa: number of cycles

$$A=\frac{2\pi fL}{R}\quad B=\frac{2\pi fM}{R}\quad C=\frac{2\pi fl}{R}\quad D=\frac{r}{R}\quad \rho=-\frac{n}{2\pi f}$$



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Abscissa: number of cycles

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torque to steady plugging torque. The abscissas are cycles.

The accuracy of the instantaneous torques from the differential analyzer was expected to be of the order of two percent of the steady-state torque based on the machine constants. All curves were run until a steady state set in and the value compared with the first term of equation 5. They always agreed within two percent. In many cases the curves were integrated graphically and the results substituted directly in equations 3 for a value of u near the maximum of the instantaneous torque. The discrepancies were never greater than three and usually less than two percent. This agreement between two totally different methods of solving equations 3 amounts to proof that both solutions are correct.

It will be observed that the highest transient plugging torque is 5.9 and the lowest 1.9 times the steady state plugging torque. As the parameters used sweep out the commercial range these numbers represent approximate limits between which maximum theoretical torques will lie for motors of any number of phases provided the rotor currents are initially zero and all phases are switched simultaneously. The best results we have on actual motors lie in this range. In general, the effect of sequential switching will be to spread the torques outside this range.

It is interesting to note that the torques for simultaneous switching never fell below zero by more than the error of the machine, though many of them came very close to zero following the maximum. In sequential switching negative torques are usually found at some points.

It will assist in the interpretation of the curves to remember that large values of A and C correspond to circuits of high inductance-to-resistance ratio, and, in ordinary commercial practice, to large motors. D, it will be remembered, is the ratio of rotor to stator resistance. The difference between B and the mean of A and C is the measure of the leakage reactance of the machine.

For several cases the magnetizing currents x_1+x_3 and x_2+x_4 were computed and plotted with the torque curve. It will be seen that the transient torque maxima are due more to shift of phase than to shift of amplitude.

References

1. An Analysis of the Induction Machine. H. C. Stanley. AIEE Transactions, volume 57, 1938, pages 751–7.

2. The Differential Analyzer. A New Machine for Solving Differential Equations. V. Bush. *Journal* of the Franklin Institute, volume 212, number 4, October 1931.

Electric Locomotive Application

E. W. BRANDENSTEIN
MEMBER AIEE (Deceased)

D. R. MacLEOD
ASSOCIATE AIEE

ELECTRIC locomotives were first used for the movement of main-line traffic over 45 years ago. This early application was for tunnel work but it may be taken as the beginning of a new era in main-line transportation. In this era the steam locomotive has been transformed in appearance and characteristics. More recently the Diesel engine has been applied to passenger and freight trains. During all this period of development the characteristics of the electric locomotive have continued to be superior to those of either the steam or the diesel-electric locomotive. Many of the early ideas with regard to the application of electric locomotives are still valid today, but progress in transportation engineering has become so accelerated that a periodic review of each type of motive power must be made by those who are responsible for their application.

A discussion of the engineering fundamentals which enter into the selection of an electric motive power unit for main-line work will be presented. An understanding of these fundamentals is essential to the specification of a balanced design. A locomotive which is specified on this basis will give all of the advantages of momentum operation augmented by high electrical horsepower at high speed. This is one of the unique advantages of the electric locomotive.

Development

Great as have been the advances in steam and diesel-electric motive power, the developments in the electrification of main-line railroads are still more spectacular. Figure 1 depicts the tremendous improvement which has been made in electric motive power during the last 20 years. This advance is due in no small measure to the pooling of the engineering talent of the two leading electrical manufacturers and to the pioneering spirit of the Pennsylvania Railroad. This railroad has in service approximately 900,000 horsepower of electric locomotives capable of delivering peaks of over 1,500,000 horsepower at the wheels. These locomotives consume approximately one billion kilowatt-hours per year. The power is supplied from 364,000 kva of single-phase 25-cycle generators over 1,100 miles of 132-kv single-phase transmission line.

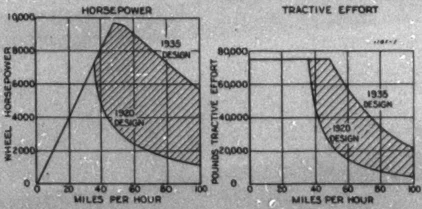

Figure 1. Comparison of 1920 and 1935 designs of electric locomotives

There are several 3,000-volt d-c systems in successful operation in this country. During the last decade steady advances have been made in d-c motive power and substation equipment, but all of the new 3,000-volt d-c applications have been in foreign countries. This discussion will, therefore, be confined to the 25-cycle single-phase system of electrification.

The *Electrical World* records that during 1939 consumption of electric power by the electrified railroads and transit systems was more than four per cent of the total amount of power consumed in the United States. Railroad electrification is, therefore, of great importance to every electrical engineer who has anything to do with power generation and distribution. Many of the problems associated with electrification have to do with getting power to the pantograph. A knowledge of the requirements of the locomotives and an appreciation of what is expected of them are essential for an economic design of that part of the system.

Types of Electric Locomotives

During the last ten years a great deal has been learned about the tracking qualities of high-speed electric locomotives. Extensive tests were made with different wheel arrangements, different weights on drivers, different degrees of restraint, different amounts of lateral in the drivers, and so on. The results of these tests have been printed in the technical press.

The most successful wheel arrangement used in this country for high-speed passenger service is classified as the 2-C+C-2 type. (The C+C means that there are two articulated driving trucks each having three axles. The figure 2 on each end of the 2-C+C-2 classification means that each driving truck is equipped with a two-axle guiding truck.) This type of locomotive operates successfully at speeds somewhat above 100 miles per hour with driving axle loads of 50,000 to 55,000 pounds. For freight service, where speeds do not exceed 70 miles per hour, the axle loading can probably be increased to 70,000 pounds without destroying the excellent tracking qualities of the locomotive. The 2-C+C-2 type of locomotive may, therefore, be used for a 150-ton (on drivers) locomotive for passenger service or a 210-ton (on drivers) locomotive for freight service. If a larger freight locomotive is desired one additional driving axle can be added to each driving truck, making a 2-D+D-2 classification and providing 280 tons on drivers.

Locomotive Characteristics

The speed-tractive effort characteristics of three locomotives of the 2-C+C-2 type are shown in figure 2. All three locomotives have the same wheel arrangement the only differences being in the motor and equipment capacity and the weight on drivers. It should be understood that all horsepower ratings are given in wheel horsepower, which is contrary to the usual practice for steam and diesel locomotives. Steam ratings are often given in terms of indicated horsepower, and diesel locomotive ratings in terms of engine output. Neither the steamer nor the diesel-electric includes transmission losses in its nominal horsepower rating.

Experience has taught us that at least 21 per cent adhesion can ordinarily be maintained on limiting grades with heavy rails and good ballast. The short-time maximum horsepower rating available at this adhesion is, therefore, quite important because it is used to accelerate trains not only from stops but also from every slow down. The overall schedule speed depends to a considerable extent on the

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D. R. MacLeod is with the General Electric Company, Erie, Pa., and E. W. Brandenstein was with the same company at the time of his death, May 1941.

1. For all numbered references, see list at end of paper.

short-time rating. The rating corresponding to 21 per cent adhesion on the 6,000-horsepower locomotive is 69,300 pounds tractive effort at 56 miles per hour representing 10,350 horsepower at the wheels. For the 7,500-horsepower freight locomotive the short-time rating for 21 per cent adhesion is 88,200 pounds at 49 miles per hour representing 11,500 horsepower at the wheels. This is about twice as much power as is obtainable from the largest steam locomotive, and five to six times as much as has been installed in a single-cab diesel-electric locomotive.

The advantage of having locomotives, with characteristics similar to those shown in figure 2*B*, which can be used interchangeably in freight or passenger service is obvious. The peak passenger traffic on most lines does not coincide with the peak freight traffic. The latter usually occurs in October while the peak passenger traffic usually occurs in the vicinity of the Christmas holidays. These peaks are difficult to predict. The passenger peak is particularly variable as was evidenced by the very great increase for 1940 over that for 1939. The greatest flexibility in operation with the smallest number of locomotives is obtained if an appreciable number can be used in either freight or passenger service. The pool of universal locomotives can then be used for freight service in October and for passenger service in December.

The ability of the electric locomotive to utilize its overload capacity, together with its high availability, makes it an ideal type of motive power for handling peak traffic conditions with a minimum interference with schedules and with a minimum number of units. The high horsepower at speed gives it the ability to speed up the freight and get more miles per locomotive. Similarly it is possible to add as much as 50 per cent to the average passenger train weight and make the same or better schedule than was made with steam operation.

A railroad gets the greatest use of its track capacity and other facilities if the daily freight movement avoids the peaks of the passenger movement. If a locomotive can come into a terminal on a freight train and go out on a passenger or vice versa, a much smaller number of locomotives will be required than would otherwise be the case. The number of extra locomotives required as "protect locomotives" is cut in half. Maintenance is simplified, shop facilities reduced, and training of personnel simplified by the universal type of locomotive.

The electric locomotive has a further advantage in that it can be double headed without an extra crew so that excess motive power at one terminal can be moved to another terminal in anticipation of the requirements at that end.

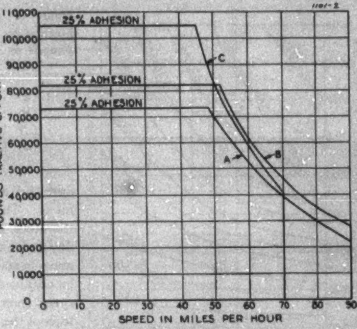
A careful analysis of the traffic must be made when selecting the motive power for the division of a railroad that is to be electrified. A peak day of passenger and a peak day of freight should be selected and the trains rearranged for electric operation. The rearrangement of traffic can only be done intelligently by one who is very familiar with the division. The weight of the locomotive that is selected will depend upon the sizes of trains to be hauled, the schedule speeds that are to be made, and by the characteristics of the limiting grade. The continuous rating of the locomotive should be in the neighborhood of 700 to 1,250 horsepower per driving axle, depending on whether the axle loading is 50,000 pounds or 70,000 pounds or between these limits.

Figure 2. Speed–tractive effort characteristics of three types of 2-C+C-2 electric locomotives

A—4,800-horsepower passenger locomotive

B—6,000-horsepower freight and passenger (universal) locomotive

C—7,500-horsepower freight locomotive



Momentum Operation

One of the outstanding advantages of the modern electric locomotive is its performance in momentum operation on relatively short grades. The flexible control and high tractive effort that are available at high speeds enable the electric locomotive to reap much greater benefit from momentum operation than is possible with other forms of motive power. The principle involved is that of the conversion of the kinetic energy of the train into the potential energy required to lift the train up the grade. The relation is expressed by the formula

$$\text{Lb per ton} \times \text{miles} = \frac{V_1^2 - V_2^2}{72}$$

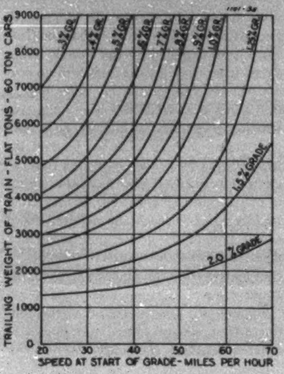
where V_1 and V_2 are the speeds in miles per hour at the beginning and the end, respectively, of the slow down. Thus a train that starts up a grade at 70 miles per hour and slows down to 20 miles per hour gives up energy equivalent to 62.5 pounds per ton-mile. If the train resistance (friction) is all supplied by the electric power taken from the line, the train will go over a 0.625 per cent grade five miles long, a 0.3125 per cent grade ten miles long, etc., starting at 70 miles per hour at the bottom and ending with 20 miles per hour at the top in each case. The advantage that the electric locomotive has over other forms of motive power is its ability to accelerate rapidly so as to reach the required speed at the foot of the grade, and in its high tractive effort at all speeds to supplement that gained from the kinetic energy of the train.

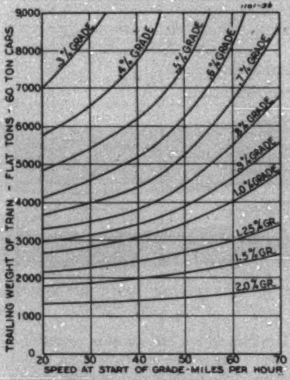
Figures 3*a*, *b*, *c*, and *d* show the tonnages that can be taken over grades of different lengths. The curves were calculated for 60-ton cars as these represent a good average weight of car for a loaded train. The 6,000-horsepower locomotive characteristic of figure 2*b* operating at 21 per cent adhesion up to 56 miles per hour was used for the tractive effort. The train resistance was calculated by the formula

$$TR = 1.5 + \frac{72.5}{W} + 0.015V + \frac{0.055V^2}{W} \quad \text{lb per ton}$$

where W is the weight of the car and V the speed in miles per hour. This formula gives values of train resistance more in line with the performance of modern equipment on a good road bed than does the conventional Davis formula.

An inspection of the curves reveals a useful criterion by means of which a momentum grade for a high-speed locomotive may be recognized. This criterion is that a momentum grade for an electric locomotive operating at high speed is one which has a characteristic of eight per cent grade-miles. Thus a two per cent grade four miles long, a one per cent grade eight miles long, etc., are momentum grades for

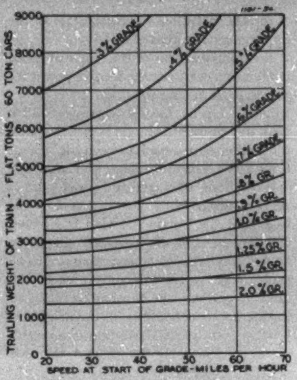




an electric locomotive. This characteristic would be considerably lower for a constant horsepower motive power unit having the same weight on drivers. The high tractive effort that is available at all speeds as the train slows down is a unique characteristic of the electric locomotive.

Heating and Commutation of Motors

Felix and Jungk[1] have presented a method for predetermining the temperature rise of an a-c traction motor. This enables the application engineer to take a train over the profile and determine the tonnage rating of a given locomotive on the basis of the heating of the motors. Ratings established in this way are then checked against the ratings permitted on the basis of momentum operation. In the case of relatively easy grades of the order of 15 miles or longer it may be necessary to provide greater thermal capacity to obtain the full benefit of the high short-time overload capacity of the electric equipment. The ventilation of the motors is, therefore, an important part of the motor design for a given application.[1] The maximum benefit with a given equipment will be obtained if the engine is brought up to speed soon enough to permit the blowers to cool the motors down before attacking the grade. Since the blowers on a locomotive consume an appreciable quantity of power, it can be seen that variable speed blowers have a definite advantage over those that operate at constant speed. Forced ventilation can then be provided when it is needed.



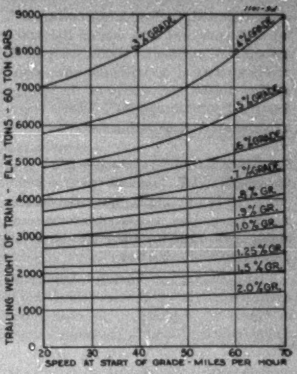


Figure 3. Momentum operation of 6,000-horsepower electric locomotive; 330,000 pounds on drivers; 21 per cent adhesion to 56 miles per hour

(*a*) Length of grade $2^1/_2$ miles
(*b*) Length of grade 5 miles
(*c*) Length of grade $7^1/_2$ miles
(*d*) Length of grade 10 miles

When an engine is limited in its tonnage rating by heating, the capacity can be increased by simply changing the gearing to lower the top speed. Thus if a 70-mile-per-hour engine is found to be limited by heating, consideration should be given to changing the gearing of some of the engines to 70 mile-per-hour gearing and assigning these permanently to freight service. The only difference between a freight locomotive and a passenger is the gearing if this plan is adopted.

On the other hand, it is possible to apply larger motors so that the continuous horsepower will be greater with only a slight increase in weight on drivers. Such a locomotive does not have the heating limitations of the usual 90-mile-per-hour passenger engine and can be used in freight service with no more restrictions than are applied to the 70-mile-per-hour freight engine. For instance, a 6,000-horsepower locomotive of the 2-C+C-2 type with characteristics as shown in figure 2*b* can be built without exceeding 55,000 pounds per driving axle. The continuous rating of this 6,000-horsepower locomotive is approximately 50,000 pounds tractive effort with 90-mile-per-hour gearing. The continuous rating of the 4,800-horsepower locomotive, figure 2*a*, when it is geared for 70 miles per hour is 43,000 pounds tractive effort. It is, therefore, evident that the universal 6,000-horsepower locomotive, which is good for either high-speed passenger service or for freight service, will have a higher tonnage rating than will the 4,800-horsepower locomotive when it is geared for 70 miles per hour. This 6,000-horsepower locomotive represents what is meant by a balanced design.

There are many details of motor design which must be considered for a given application. Such details are beyond the scope of this paper, but the following empirical relation, which has been borne out by operating experience, is of value when a preliminary study is being made.

Number of poles on motors per axle =
weight per driving axle in tons

Control

The adjustments of the main and interpole fields which are necessary for the best performance of an a-c series motor are under the control of relays.[1] Figure 4 shows the essential elements of the circuits by means of which the motors are controlled. There are two small preventive coils X_1X_2 and Y_1Y_2 and one large preventive coil X_3Y_3. This method of tap changing under load permits rapid operation of the control and is the most economical in weight and space of the available tap changing schemes. Since the motors are designed for relatively low voltage a number of armatures are connected in series.

When a wheel slips, the motors that are geared to the slipping axle consume most of the voltage and overspeed. Ogden[8] has proposed an autotransformer connection which equalizes the voltages across the slipping and working motors that are in series. This limits overspeeding and gives substantially the same performance as if the motors are connected in parallel. The practice at the present time is to notch back when slipping occurs rather than to add automatic anti-slip equipment. A slip relay tells the engineer of a locomotive when to notch back and the control is made fast enough to reapply power without complete loss of tractive effort. A slow control would be a disadvantage because there would be some loss of headway which might be sufficient to stall the locomotive when it is operating at very low speeds. Relatively slow controls have been built, but locomotive crews who have operated both types prefer the fast control.

The preventive coil method of tap changing under load has the disadvantage of causing high switching surge currents. The contactors all make and break load current and must be designed to make considerably higher currents than they have to break. When a contactor (figure 4) is opened, the load current passing through one leg of either X_1X_2 or Y_1Y_2 is not balanced by a corresponding current in the other leg so that one-half of the total load current saturates the core of the coil that is being switched. When a contactor is closed to change to the next notch the fluxes in the coils must change suddenly to new values determined by the main transformer tap voltages to which the terminals X_1, X_2, Y_1, and Y_2 are connected. This causes transient currents to flow in the preventive coil circuits and through the tapping contactors. The preventive coils are designed with certain densities and air cored reactances to keep the surge currents to a value which will be within the interrupting rating of the contactors.

The duty on the contactors is greatly reduced if a resistor is connected from X_1 to X_2 before the coil X_1X_2 is switched, or from Y_1 to Y_2 before Y_1Y_2 is switched. The resistor commutates the load current instantly so that the current, which was passing through the contactor before the switch opened, passes through the resistor immediately after the contactor opens. The direction and magnitude of the current in the leg of the preventive coil that is being switched is maintained so that the coil does not saturate. Additional contactors and some elaboration of the control are required to cut the resistor into circuit before the preventive coil contactors are opened and to cut it out after the transition is complete.

When one leg of a preventive coil is open the other half must carry twice the load current and is, therefore, subject to approximately four times the heating. The coils are usually of the forced air cooled type which does not have the thermal storage of an oil-insulated type. It is, therefore, desirable to keep the currents balanced in the legs of the preventive coils on all running notches.

The number of available notches may be multiplied three times by the addition of the small buck-boost transformer marked Z_1Z_2 in the diagram figure 4. The secondary is in series with the motor and the primary is connected in one direction or the other across a pair of taps on the main transformer. A Thyrite resistor is connected across the primary in order to hold down the voltage that would otherwise appear when the primary is open-circuited with the secondary carrying load current. This feature gives a very smooth control and is particularly useful in freight service when working near the limits of adhesion.

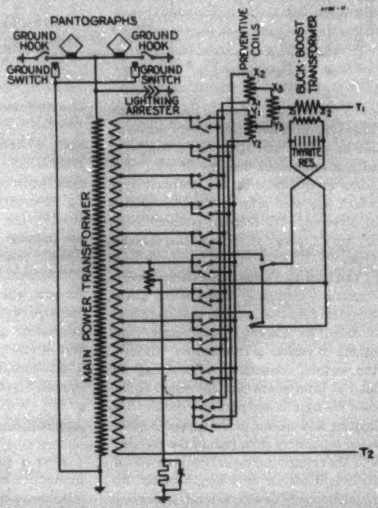


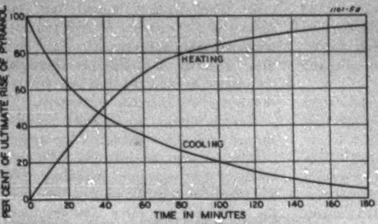
Figure 4. Main power transformer with low-voltage taps and auxiliaries for control of a-c series motors

Transformer Heating

The preventive coils and buck-boost transformers are of the forced air cooled type. They have a thermal time constant (reach 63 per cent of their ultimate temperature) of approximately eight minutes. Their thermal design is, therefore, determined by the short-time performance that is required from the locomotive.

The main power transformer converts the relatively high voltage of the trolley (12,000 volts) to the relatively low motor voltage. The maximum secondary voltage that is used at the present time is of the order of 1,400 volts. The reactance of the transformer should be kept relatively low because it is in reality a part of the distribution system. The per cent reactances used on all locomotives should be approximately the same so that the notching curves will line up for multiple-unit operation. The impedance of the motor circuit adds vectorially to that of the transformer and preventive coils to give tractive effort-speed curves that are relatively steep. Increments of tractive effort equivalent to about one per cent of the weight on drivers are obtainable from zero speed until the top characteristic is reached.

Figures 5*a*, *b*, *c*, and *d* show characteristic heating and cooling curves of a 25-cycle locomotive transformer. The transformer is Pyranol-insulated. A motor-driven pump immersed in the transformer tank circulates the liquid through the coils and the radiator in series. The heat is removed by force ventilating the radiators. The time constant of the transformer is nearly one hour, whereas that of the cop-



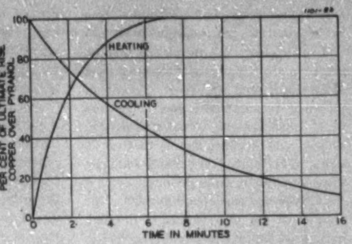
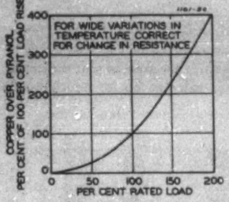


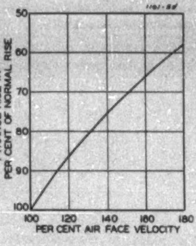
Figure 5. Pyranol insulated forced cooled locomotive transformer

(*a*) Heating up and cooling down curves of Pyranol with pump and blower on

(*b*) Heating up and cooling down curves of windings—rise over Pyranol



(*c*) Winding rise over Pyranol for constant conditions and constant resistance



(*d*) Pyranol rise versus air velocity for constant conditions

per is of the order of two to three minutes. This type of transformer is very economical in weight and space when compared with a self-cooled design. It also compares favorably with the air-blast type which was used prior to the introduction of Pyranol. Its thermal characteristics are very superior to those of the air-blast type.

The load on a locomotive transformer is constantly changing. The heating can be calculated by the methods that have been outlined by Cooney, Montsinger,[4] and others in various AIEE papers. Briefly, the method consists in determining the load at intervals of time for which significant changes in temperature occur. The heating and cooling of the copper is calculated separately from that of the Pyranol, and the resultant temperatures are added. In freight service on a long haul the Pyranol may only be able to reach 50 per cent of its ultimate before it starts to cool down again. The required capacity of the transformer can be determined theoretically by taking a train, which can be handled by the motors and preventive coils, over the profile and calculating the temperature rise for an assumed rating. It is well known that liquid-cooled transformers will stand overheating for short intervals without appreciably affecting their life. It is, therefore, not necessary to make allowances for emergency operating conditions when selecting the rating for a given service.

High-Voltage Insulation

The insulation level of a locomotive is determined by that required for the primary of the transformer and the high-tension cable. The existing a-c electrifications in this country operate on a nominal 12-kv trolley and the prevailing practice is to use three insulators. These have an impulse flashover of approximately 355-kv crest on a 1.5 by 40 positive wave. The insulation level of the Pyranol transformers which have been in service on locomotives for more than five years is equivalent to a full wave AIEE impulse test of 120 kv. They are protected with 15-kv (max.) Thyrite lightning arresters mounted on the transformer.

Application

After a suitable rating and design of locomotive has been selected the next step is to make a more detailed study of the traffic than was made in the preliminary selection. An analysis can only be made after making a prolonged study of the movement of traffic over the divisions that are to be electrified as well as those that feed into the electrified zone but which will remain in steam operation. The delays that occur from day to day in steam operation must be studied to see to what extent these delays may be reduced or eliminated. Such details as track capacity in terminals may affect the possible length of passenger trains. Speed restrictions, cross-overs, passing sidings, and the like will affect the schedule speeds. The possibility of combining freight trains to take advantage of higher speed operation, made possible by the more powerful electric locomotives, will be affected by the facilities that are available in classification yards and by the service that is available on lines that feed into the yard. The combination of sections of passenger trains into larger sections will be affected by the origin of the cars, by the facilities for switching at terminals, and by the inclusion of mail or express cars with passenger cars. Finally, it is essential to examine the performance of all facilities that contribute to the make-up, dispatch, and movement of trains under peak traffic conditions.

When all the fundamental data have been assembled, a train sheet can be prepared for electric operation and compared with steam operation. The number of locomotives that are required for that day can be determined from the electric train sheet. The ratio of the number of steam locomotives that were required for the day that was studied to the maximum that had ever been used gives a factor by which the number of locomotives may have to be increased when a complete change-over has to be made. Electric operation often proves to have many nonpredictable operating advantages so that the number of locomotives actually required is less than the calculated figure. It is, therefore, advisable to make an initial purchase of less than the calculated requirements and buy added units as needed.

Conclusion

Single-cab electric locomotives with high horsepower at speed have been in successful operation for several years. They are maintaining high-speed passenger and freight schedules which would be impossible with any other type of motive power having the same horsepower rating. The universal type of locomotive is conservatively designed on the basis of this experience for use in either freight or passenger service. Its electrical characteristics are co-ordinated with its weight on drivers and it combines high horsepower at speed with high availability. It is, therefore, capable of moving more gross ton miles per train hour with fewer units than any other type of motive power.

References

1. Complete Analysis of Motor Temperature Rise, Fremont Felix and H. G. Jungk. AIEE Transactions, volume 60, 1941, pages 578–86.

2. Protective Relays on Pennsylvania Locomotives, B. Luther. AIEE Transactions, volume 53, 1934 (February section), pages 261–4.

3. H. S. Ogden, U. S. Patent 2,209,826.

4. Temperature Limits for Short-Time Overloads for Oil-Insulated Neutral Grounding Reactors and Transformers, V. M. Montsinger. AIEE Transactions, volume 57, 1938 (January section), pages 39–44.

The 220,000-Volt System of the Hydro-Electric Power Commission of Ontario—II

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Introduction

AT the summer convention of the Institute in 1930, Mr. E. T. J. Brandon presented a paper under this same title,[1] describing the design of the initial components of this system, which had then been in operation approximately one and one-half years. At that time, one and one-half circuits, having a combined length of 350 miles, were in service, transmitting approximately 110,000 kw. to a receiving terminal station in the Toronto area of 180,000 kva. capacity. At the present time, the Commission is operating a total of 1,000 miles of single-circuit 220,000 volt construction, with one receiving terminal of 420,000 kva. rated capacity and is placing into service immediately 45 miles of double-circuit construction and a second receiving terminal of 150,000 kva. capacity.

This paper presents a brief history of the development of the system and places on record the experience gained in 8,400 circuit-mile-years of operation of the transmission circuits. Data are presented regarding lightning outages and the behavior of the circuits under sleet and conductor vibration.

These data are then used to indicate the reasons for certain revisions made in the design of new single-circuit construction carried out during 1940–41, and to indicate the factors that influenced the design of a new 45 mile double-circuit extension. The paper concludes with a discussion of the relay protection system and the improvements now being incorporated therein.

General System Arrangement

The 220,000 volt system under discussion forms part of the Commission's 25 cycle Niagara system, figure 1, which supplies a highly developed area of some 12,000 square miles in the peninsula formed by Lakes Huron, Erie, and Ontario. This system distributes power over approximately 1,350 miles of 110,000 volt lines. The total primary load, which equalled 710,000 kw. in December 1929, reached nearly 1,125,000 kw. in December 1940.

This 220 kv. system has been the channel over which all growth of load in the Niagara system has been supplied since 1928, from generating sources largely in the neighbouring Province of Quebec. An initial 60,000 kw. was transmitted over the first circuit in October 1928, increasing to 515,000 kw. transmitted over three such circuits in December 1940.

Table I. Lightning Outage Record

Classified According to the Type of Fault

Involving one wire and ground	56– 59%
Involving two wires and ground	25– 26%
Involving three wires	14– 15%
Total	95–100%
Involving one circuit only	95– 98%
Involving two circuits simultaneously	1– 1%
Involving three circuits simultaneously	1– 1%
Total outages due to lightning	97–100%

Until this summer, this supply has been delivered at the Leaside receiving terminal, adjacent to the City of Toronto, which it will be noted lies at the easterly extremity of the main 110,000 volt system. This fact has in itself created problems of distribution, the solution of which will be materially aided by the placing in service of the new terminal shown at Burlington.

Development of 220,000-Volt System

The 220,000 volt system was initially conceived as a two-circuit system, with a mid-point interswitching station, transmitting some 200,000 kw. purchased under contract from the Gatineau Power Company, from that Company's Paugan development some 230 miles east of Toronto. Later contractual undertakings and the construction of the Chats Falls development on the Ottawa River, brought the total capacity available from eastern sources to approximately 615,000 kw.

The third 220 kv. circuit was constructed in 1931, when the Chats Falls development was first brought into service. The connection shown from Beauharnois to MacLaren to Chats Falls was built during the depression years, to deliver the power which became available from the former sources to Chats Falls for transmission over the three-circuit system. This was an expedient, adopted as the most economical means of effecting this delivery, following an analysis of the transmission capacity of these three circuits in the light of the then existing knowledge of stability problems. This analysis indicated that, even without a mid-point interswitching station, the dependable capacity of these circuits, given proper fault clearance times, could be increased from an earlier rating of 330,000 kw. to approximately 450,000 kw.

The terminal capacity at Leaside has been increased progressively, by the addition of four 45,000 kva. banks, duplicate of the two banks originally installed, and two 75,000 kva. banks, making a total installed rating of 420,000 kva.

Until the outbreak of hostilities in September 1939, it was planned that this system would need to be extended for service in the fall of 1942. The outbreak of hostilities, however, brought the expectation of rapidly accelerated power demands and the construction of a fourth circuit from the Beauharnois development of the Beauharnois Light, Heat and Power Company, in the Quebec section of

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[1] For all numbered references, see list at end of paper.

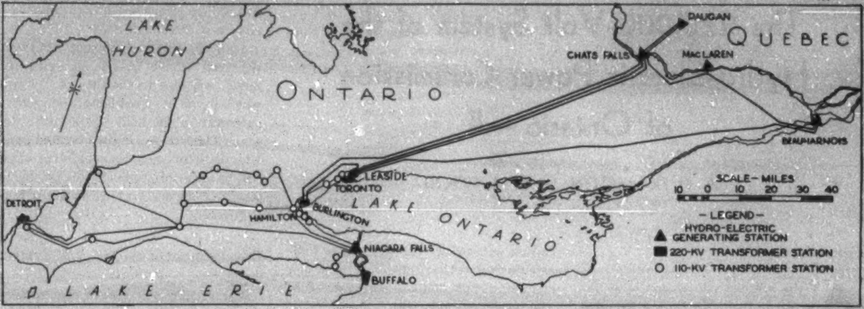


Figure 1. The Niagara system of the Hydro-Electric Power Commission of Ontario, showing major generating and transformer stations and 220-kv and 110-kv lines

the St. Lawrence River, was immediately undertaken. The selection of a westerly terminus for that line presented a problem upon which considerable time and study has been expended.

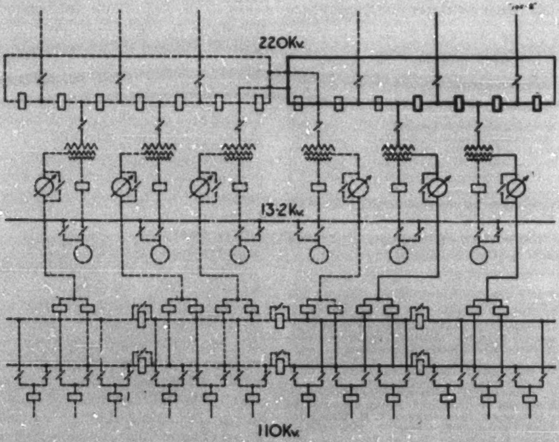
The Leaside terminal, being located in the metropolitan area of the City of Toronto, in which approximately 40 per cent of the total primary demand of the Niagara system occurs, provided a convenient point of distribution for the power delivered during the building-up years. In later years, however, Leaside has been expanded beyond the capacity originally contemplated, so as to make the most efficient use of the transmission system, thereby creating an increasing distribution problem.

Furthermore, as is obviously desirable, the power transmitted over this system was purchased under contracts which require high load-factor deliveries, much higher, in fact, than the load-factor of the demands within the immediate vicinity of the receiving terminal.

For these reasons it has been necessary to distribute from Leaside, to gradually increasing distances, power and energy delivered in excess of the Toronto area demands. This distribution distance increases and decreases daily, with the variations of local demand, and is considerably greater in the summer than in the winter. During recent summer months power generated 250 miles east of Toronto has actually been delivered to the immediate vicinity of Niagara Falls.

Figure 2. Burlington 220-kv receiving terminal

Proposed ultimate diagram for six 220-kv circuits and six 75,000-kva transformer banks. Initial (1941) installation in heavy lines



It had been planned that the second 220 kv. terminal would also be in the Toronto area, but on its westerly outskirts. Further study, in the light of the increased capacity of Leaside, has resulted in the new terminal being located at Burlington, some thirty miles west of Toronto. At that point the new station is adjacent to the rapidly expanding load area of the city of Hamilton and also is situated where a number of existing 110 kv. circuits intersect.

This new terminal is supplied by diverting to it the shortest of the existing 220 kv. circuits, namely, one originating at the Chats Falls development, thus holding the longer Beauharnois circuit to its minimum length by terminating it at Leaside. In addition, a 220 kv. tie-circuit between Burlington and Leaside is provided, interconnecting the 220 kv. lines so that they operate as a four-circuit system. These two circuits are carried around the metropolitan area of Toronto and to Burlington

Table II. Lightning Outage Record

Classified According to Tower-Footing Resistance

Average Tower-Footing Resistance—Ohms	Number of Outages	Circuit-Mile-Years of Operation	Outages Per 100 Circuit-Mile-Years
(a) For 70 located faults on the Leaside-Chats Falls Circuits			
Under 25	3	2,150	0.14
25–50	6	1,950	0.31
50–200	16	2,000	0.8
Above 200	45	1,100	4.1
	70	7,200	0.97
(b) For all outages due to lightning			
Under 25	5	2,600	0.19
25–50	9	2,200	0.41
50–200	27	2,500	1.08
Over 200	56	1,100	5.1
	97	8,400	1.15

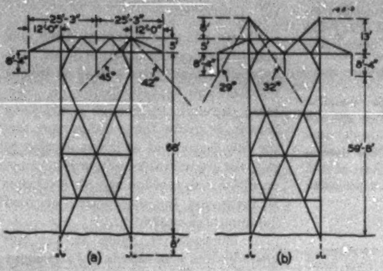


Figure 3. (a) Outline of original single-circuit 220-kv tower, showing major dimensions and shielding angles

(b) Corresponding outline of revised 1940 tower

on double-circuit structures, the first of such construction adopted by the Commission.

The Burlington transformer station is being constructed on a sixty-acre site, designed to accommodate ultimately six banks of three-25,000 kva., single phase, 220/110/13.2 kv., forced-air-cooled transformers. Two banks totalling 150,000 kva. are being installed initially. A schematic diagram of the proposed ultimate station is shown in figure 2, on which the initial installation is indicated in heavy full lines. The addition of a third bank at this station will complete the existing phase of development, providing for the delivery of some 675,000 kw. over the four 220 kv. circuits. Beyond that point further delivery at Burlington will depend upon the development of new power resources, as for example, upon the Ottawa or St. Lawrence rivers.

Summary of Operating Experience

The first of these 220 kv. circuits was placed in service on October 1, 1928, since when a total of 560 circuit-miles has been added, making a total of 790 circuit-miles in service as of March 31, 1941. Of this mileage some 85 circuit-miles are not actually operated by the Commission, being located in the Province of Quebec, but are included in the following record. The whole of this construction in general conforms to the designs described in the earlier paper, the standard suspension tower being shown in figure 3*a*.

In approximately thirteen years, 8,400 circuit-mile-years of operating experience have been secured and a total of 111 faults due to all causes have been experienced. Of these, four were occasioned by various construction hazards in the early years and eight were due to miscellaneous causes, chiefly external interference. Each of these faults involved only one wire and ground and, except in the period when only one circuit was in service, created no serious disturbance. It is of interest to discuss the salient features arising out of the remainder of this extensive operating record.

(a) LIGHTNING

Of the remaining 99 faults experienced in this period, 97 are attributed to lightning, equal to an average of 1.15 lightning outages per 100-circuit-miles per year, in a territory in which the thunderstorm frequency is between thirty-five and forty storms per year. The classification of these lightning faults as to single-wire-to-ground, two-wire-to-ground, and three-wire and as to those involving one, two, or three circuits is given in table I. The frequency of occurrence of such faults has varied tremendously; for example, three outages have been experienced within ten minutes; on another occasion, four within an hour and again five in one day and yet one period of eighteen months passed without a single outage.

These averaged data do not give a complete picture of the performance of these circuits. The three Chats Falls-Leaside circuits are constructed over terrain varying from rich farm land to rocky undeveloped bush country. The latter section, which extends almost continuously for a distance of about 90 miles, is characterized by surface rock formations with pockets of muskeg in the rock depressions. Towers are frequently erected on the rock outcrops. Low footing resistances are, therefore, difficult to obtain. In figure 4 an approximate footing-resistance profile for the Chats Falls-Leaside section is given, as compared to the known location of 70 of a total of 89 outages in this section attributed to lightning. It will be seen that some 90 per cent of the located lightning outages occurred in the section of high footing resistance.

In table II is presented an attempt to co-relate the lightning outage record with footing resistance. The data are presented first for the 70 located faults in the Chats Falls-Leaside section and then for all 97 faults, based on locations for those not traced, as estimated from relay target and oscillograph records. It will be observed that, in the territory where footing resistances are considered to be below 25 ohms, the actual outage record approximates 0.2 outage per 100-circuit-miles per year.

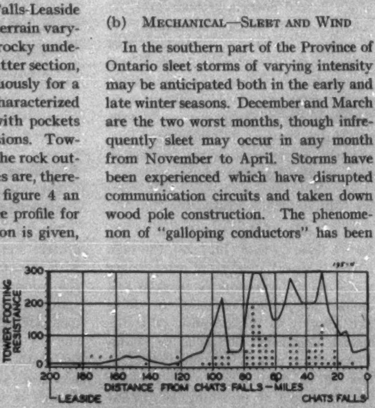
Reference to figure 3*a* shows that the tower design in these lines provides a shielding angle of 42 degrees and a ratio of "height of ground wire above power conductor to total height of ground wire" of 0.182. The experimental results of Wagner, McCann, and MacLane[2] and the data presented by Waldorf[3] would indicate that good lightning performance could be expected and the record proves, that given low footing resistances, such has been the case.

Some crowfoot counterpoise work was done in the rock section along these three circuits during 1934–35. Short sections of highest footing resistance were so treated, with some success in lowering the measured values, but presumably the distances the crowfoot wires were carried to reach good grounds were too great to secure any considerable benefit.

(b) MECHANICAL—SLEET AND WIND

In the southern part of the Province of Ontario sleet storms of varying intensity may be anticipated both in the early and late winter seasons. December and March are the two worst months, though infrequently sleet may occur in any month from November to April. Storms have been experienced which have disrupted communication circuits and taken down wood pole construction. The phenomenon of "galloping conductors" has been

Figure 4. Approximate footing-resistance profile of three 220-kv circuits, Chats Falls to Leaside, showing grouping of 70 located lightning outages



Each dot represents one lightning fault in the respective four-mile section (five towers per circuit per mile). Tower-footing resistance figures are average values for the 60 towers in each four-mile section

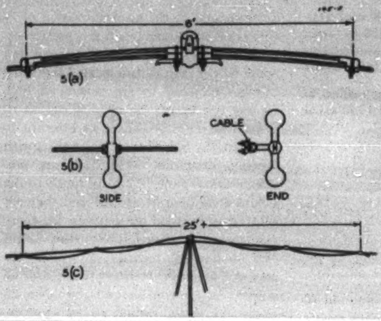


Figure 5. (a) Conductor reinforcement on first (1927–28) construction

(b) Torsional-type vibration absorber provided on one-half 1940–41 single-circuit construction; Stockbridge damper provided on remainder of such construction

(c) Ground cable festoon used on 1940–41 construction

observed, at not infrequent intervals, on various sizes and spans of conductors up to 605,000 c.m., A.c.s.r. at a span of 880 feet. Consideration must therefore be given to sleet conditions for all lines designed or constructed in this territory.

Two outages in the period under review are attributed directly to sleet, and these, incidentally, occurred within a few minutes of one another. Heavy sleet had formed on both power conductors and ground cables on certain hill tops, but only lightly on an intervening long valley span. As a result, the power conductors in the long span were pulled up by the unbalanced loading on the two sides of the adjacent suspension insulation, so that the ground cables, sagged to their normal loaded positions, appeared below the plane of the power conductors at mid-span. Two flashovers occurred, the second of which burned down a ground cable, resulting, on account of inaccessibility, in 27-circuit-hours of outage. In two other similar cases, a condition of unbalanced sleet loading was set-up, greatly increasing the sag of the ground cables without compensating sag of the power conductors, resulting in inadequate clearances. Fortunately, in these cases, the conductors were not distributed by wind.

(c) MECHANICAL—VIBRATION

At the time of the construction of the first of these circuits (1927–28), the then relatively recent adoption of much longer spans and higher conductor tensions had brought the problem of conductor vibration strongly to the attention of transmission engineers. Remedial or palliative measures were as yet under investigation. In the original design this problem was recognized by providing, at suspension points, a reinforcement consisting of a 6-foot length of the conductor fastened at its outer extremities and supported above the main conductor in a double-seated suspension clamp, figure 5*a*. In the second and third circuits this form of reinforcement was replaced by the standard armour rods.

Each of these circuits was designed for a maximum conductor tension, at $^{1}/_{2}$ inch ice, 8 pound wind, and 32 degrees F, of 10,000 pounds, conductors being 795,000 c.m., A.c.s.r having an ultimate strength of 28,500 pounds. This represents a maximum tension of 35 per cent of ultimate and also represents a tension at 60 degrees F of 16 per cent of ultimate. These figures will be recognized as indicating a design in which conductor fatigue due to vibration might be anticipated.

Actually a considerable record of vibration has accumulated in operation, though nothing approaching a serious condition has been observed to date. Quite early a loosening of tower members was experienced, but this was remedied by the use of locknuts or their equivalent at all one and two bolt positions. Careful periodic examination of the conductors has revealed a few broken strands though such damage does not indicate the need for any further palliative measures for some years.

Revisions in Design Incorporated in the 1940–41 Single-Circuit Construction

The recent extensive additions to this system were naturally not undertaken without re-consideration of the various design factors in the light of the experience enumerated above. It is of interest therefore to review such revisions as were made and to discuss the reasons associated therewith. The discussion is perhaps best arranged under the previous headings, though it is difficult to separate those adopted for improved lightning design from those which aimed at improvement in the mechanical performance.

(a) LIGHTNING

The record indicates that good lightning performance can be expected of the original single-circuit tower, so long as footing resistances are kept sufficiently low. Perhaps, therefore, no change would have been made in the tower design were this the only factor, but as it was decided to raise the ground cables primarily for sleet operation, this revision may also be taken as improving the expectation of good lightning performance.

As illustrated in figure 3*b*, the two ground cables were raised eight feet above their original location, to a point of support 21 feet above the point of support of the power conductor in the suspension tower. This has had the result of decreasing the shielding angle to 29 degrees, while increasing the ratio of "height of ground cable above power conductors to total height of ground cable" to 0.263.

Measurements made by the Meg-Earth tester shortly after tower erection indicated that, in the earth sections, after consolidation of the back-fill, tower footing resistances generally would not exceed 15 ohms. In these sections, no treatment beyond the occasional crowfoot is contemplated. In the rock section, it was decided to lay a continuous counterpoise, consisting of $^{5}/_{16}$ inch steel conductors available in salvage stores. These cables are in general buried to an average depth of 18 inches under the outer phase wires. Occasionally, however, they are taken around rock outcrops, when by so doing they could be buried, and, in isolated cases, are actually carried over the top of the rock. The performance of this new circuit will be carefully compared to that of the existing circuits as such performance will largely dictate whether counterpoise should be added to the older construction.

(b) MECHANICAL—SLEET AND WIND

Our experience would seem to indicate that, under the operating conditions existing, sleet is more liable to form on the ground cables than on the power conductors and under such conditions, the vertical separation provided between ground cables and power conductors has proven insufficient. Mr. A. E. Davison, transmission engineer for the Commission, has actively studied this problem of conductor clearances. These studies are based on Lissajous figures[4] and the locus of motion of the conductor under "galloping" conditions is taken as the criterion. The axes of this motion have been determined empirically from recorded field observations and from analyses of motion picture records of a number of actual occurrences. It is possible to at least approximately delineate the various loci. A tower design in which these approximate loci indicate adequate clearance between all conductors is considered much more satisfactory than one in which overlapping loci occur.

However, the extent to which this method should be applied to heavy conductor, long-span construction is still somewhat of an open question, being largely based on the behaviour of smaller conductors and shorter span construction than is involved in this case. No conclusive data are available indicating that 795,000 c.m., A.c.s.r. at 1,056 foot spans will "gallop" at all, and, even if it does, whether it will be in one continuous loop between the points of support or in something of a wave motion raising not more than one-half the span above its normal position at one time (that is, in half-loops). Messrs. Oldacre and Wollaston,[5] in describing the Powerton-Crawford line, illustrated the use of Lissajous figures and assumed the movement of the conductor in one loop.

In our case, however, a substantial mileage of towers was available in stock and much quicker deliveries could be obtained if the basic original design was not changed. This design was based on a longitudinal loading equivalent to $^{1}/_{2}$ inch ice and 8 pound wind, but with the transverse loading increased to $^{3}/_{4}$ inch and 11 pound wind, the latter increase an added factor of safety since considered unnecessary. It was decided that part of this excess strength could be utilized to provide greater vertical separation between ground cables and power conductors, though in designing this improvement it was considered sufficient to assume movement of the conductors in half-loops only. It will be noted in figure 6*a* that the redesign has removed the loci of motion of the ground cables from those of the phase conductors, providing adequate clearances under the assumed conditions. The ice-loaded mid-span position of the ground cable at rest, as compared to the corresponding position of the unloaded phase conductor, is also shown, indicating the maintenance of substantial clearances even with a $^{3}/_{4}$ in. ice differential.

(c) Mechanical—Vibration

The balance of the excess strength in the original tower design has been utilized to increase the ruling span from 1,056 feet to 1,150 feet. In order to maintain the same ground clearance at this longer span, the higher strength 26x7 strand, 795,000 c.m., A.c.s.r. has been used, strung to a maximum designed tension of 12,000 pounds. Based on an ultimate strength of 30,900 pounds, the designed maximum and "60 degree" tensions therefore approximate 39 per cent and 17.6 per cent of ultimate strength respectively.

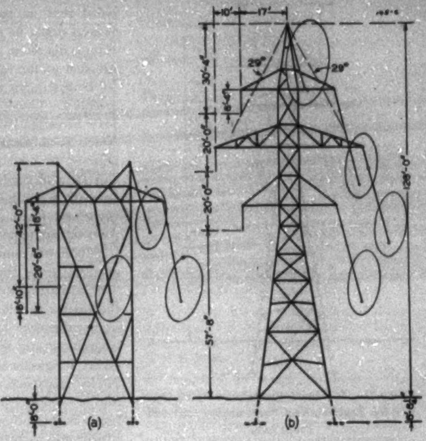


Figure 6. (a) Clearance diagram (Lissajous figures) for 1940–41 single-circuit construction

Loci based on half-loops, i.e., movement of quarter-point in suspension span. Note also mid-span positions of $^{3}/_{4}$-inch ice-loaded ground cable and unloaded phase conductor

(b) Corresponding diagram for 1941 double-circuit construction

Note full loop movement of ground wires assumed at 880-foot span

The lengthening of the span in the new circuit was based partly upon the vibration record of the existing circuits and partly upon the favourable results obtained from an extensive laboratory and field investigation of damper design and performance.[6] It is felt that the development in this field now safely permits securing the economy inherent in longer spans and higher conductor tensions. The full possibilities in this connection were not realized in this case, due to the decision to retain the basic original tower design, but had a completely new design been permitted, spans as great as those encountered in some other recent construction would have been given serious consideration.

Associated with this increased span length the application of vibration absorbers was decided upon, rather than the previously used armour rods. Approximately one-half the line is equipped with the Stockbridge damper and one-half with the torsional damper, figure 5*b*, developed in the Commission's laboratory and described in a companion paper by Mr. G. B. Tebo.[6] Both dampers are used singly, that is, two per span. Vibration of the ground wires is protected against by the use of festoons, figure 5*c*.

Double-Circuit Construction—Leaside to Burlington

In considering the two-circuit extension of these 220 kv. lines, from the existing terminal at Leaside to the new terminal at Burlington, a number of factors were brought into consideration. Single-circuit construction was initially considered, the Commission not having previously operated any double-circuit construction at 220 kv. In fact, there had been evident in the Commission's engineering a tendency to avoid such construction at all voltages, in favour of various single-circuit configurations. However, as some twenty miles of the Burlington extension necessarily encircled the metropolitan area of Toronto, this viewpoint was brought sharply into conflict with the question of right-of-way costs. The final decision was in favour of the double-circuit construction, though it will be noted that a relatively conservative design has been adopted.

The initial decision was to adopt a span of 880 ft., utilizing 795,000 c.m., A.c.s.r. at a maximum designed tension of 10,000 pounds. However, a change in this decision was brought about by the exigencies of the present situation and the line is actually being constructed utilizing the type HH segmental, hollow-core, copper conductors, 500,000 c.m., 1.02 inch outside diameter, seven segments, having an ultimate strength of 21,200 pounds. At the 880 ft. ruling span the maximum designed tension, at $^{1}/_{2}$ in. ice, 8 pound wind, and 32 degrees F, is 9,500 pounds, the 60 degree tension being 4,700 pounds, equivalent to 45 per cent and 22 per cent of ultimate respectively.

The tower design adopted is shown in figure 6*b*. It will be noted that a single ground cable is used, located at the tower peak, 30 ft. above the point of support of the upper phase conductor in the suspen-

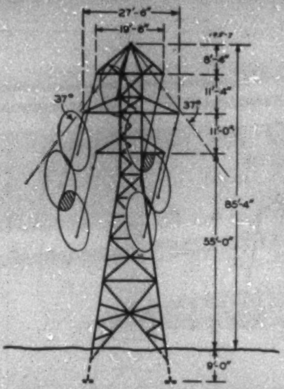


Figure 7. Clearance diagram for 1920-type 110-kv double-circuit construction—one-half loops only

Note overlapping loci indicating anticipation of sleet outages

sion position. A shielding angle of 29 degrees and a ratio of "height of ground wire above power conductors to total height of ground wire" of 0.236 results. Footing resistance data on this construction are not yet available, but the line being all in good agricultural land it is not anticipated any particular treatment of the footings will be found necessary

Again in the new double-circuit construction the same principles of design for sleet operation were used. It has been found more difficult to provide adequate clearances economically in this type of structure than in the single-circuit design. For example, in an earlier 110 kv. design, utilizing 605,000 c.m., A.c.s.r. conductors at 880 ft. spans and providing a 4-foot offset of the centre phase wire, figure 7, actual outages due to "galloping" have been experienced. That these outages might be anticipated, however, is indicated when the design is analyzed by means of Lissajous figures, though it will be seen that rather extensive revisions will be needed to effect full clearances. The clearance diagram for the 220 kv. design finally adopted is shown in figure 6*b*, based on half-loop movement of heavy copper power conductors but full loop movement of the relatively light ground cable.

No special precautions are being taken to protect this double-circuit construction against conductor vibration. The extensive studies reported by other authors are interpreted as indicating that, except perhaps in certain cases, no such precautions are necessary. Furthermore, it was decided that no particularly special provisions would be made in the suspension clamp, figure 8*a*, and that copper compression joints, figure 8*b*, would be used for both straight joints and dead-end assemblies. Time has not permitted complete investigation of this latter practice, which is at variance with the practice adopted in certain other lines utilizing this form of conductor,[7] but, as the full strength of the conductor is developed in these new joints and as the mass of all parts subject to possible vibratory stresses is reduced to a minimum, no objectionable operating experience is anticipated.

A structural revision incorporated in the double-circuit tower consists of designing the lower panel so that all diagonal connections are made above grade. In earlier designs, including the single-circuit 220 kv. construction, the point of connection of the lower diagonals and the main legs is located below ground level. In this climate and particularly in clay soils, frost heaving has been found to occur which reacts against this below-grade diagonal, causing bending and in some cases actual failure.

Relay Protection and System Stability

It is finally of interest to describe briefly the relay protection provided on this 220 kv. system and to discuss the improvements being made, both in the existing protection and in the protection of the newer construction, associating these improvements with the operating record and with the data obtained from network calculator analysis of the system.

For "phase" faults, the earlier relaying consists of directional, two-stage impedance distance type. The instantaneous range of such relays is set to cover 85 to 90 per cent of the line length, the overlapping second range set to cover the remainder of the line and being given a definite time delay of 0.6 to 0.8 second. This protection effects simultaneous clearance of all faults in the mid-section of any line, but results in delayed opening of the distant breaker for end-zone faults.

For "ground" faults, similar protection is used, except that the relays are supplied with line residual current and phase to ground voltage and the instantaneous range is set to cover the whole length of the line, with some margin if the remote end is open. This results in simultaneous clearance of mid-section faults, though it also effects sequential clearance of faults in the end zones, that is, clearance without the delay associated with the timed second range.

Certain of the line sections terminate in breakers of earlier design, which originally gave a clearance time, with instantaneous tripping, as high as 0.5 to 0.6 second. The more modern equipment clears within 0.2 to 0.25 second. Improvements have been made from time to time, in the original relaying and in the older circuit breakers, so that total clearance times now vary from 4.5 to 10 cycles, with an average of about 6 cycles (based on 25 cycles) for all faults except those cleared by the second ranges.

This protection, though admittedly below present-day standards, has nevertheless given adequate service during the building-up period on this system. Fortunately, all but a very few of the 39 multi-

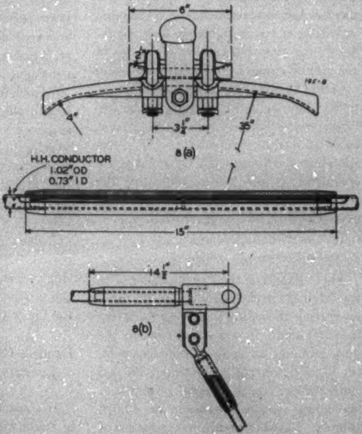


Figure 8. (a) Suspension clamp adopted for type HH segmental copper cable, 500,000 circular mils, 1.02 inches outside diameter

Span 880 feet, maximum designed tension 9,500 pounds, 45 per cent of ultimate strength

(b) Jointing and dead-end assemblies for type HH copper conductors

phase faults have occurred in the high-footing-resistance territory in the mid-section of the Chats Falls-Leaside lines, where this protection effects simultaneous clearance.

Improvement necessary in the protective equipment, when operating at the higher recent loadings, has been the subject of several network calculator studies. These studies bring out quite clearly the inherent stability of a 25-cycle system. Assuming simultaneous clearance times of 0.2 to 0.25 second, as would be obtained with standard modern equipment, it is found that the three-phase fault may be adopted as the stability criterion, rather than the two-wire-to-ground fault usually adopted in 60-cycle systems.

In figure 9 is presented a family of curves obtained in such analysis, which indicate the relative severity of different types and locations of faults. At a loading of approximately 150,000 kw. per circuit, curves *A* and *B* indicate that three-phase faults near the generating sources must be cleared simultaneously in approximately 0.2 second, if stability is to be maintained. If such faults occur away from the generating sources, curve *C* indicates that the increased loading of generators thereby created is sufficient to maintain stability with simultaneous clearance at 0.25 second. Curve *D* shows the relatively lesser severity of the two-wire-to-ground fault, which does not disturb the system stability with sequential clearance as long as 0.2 and 0.5 second. These results are taken to indicate that, with protective equipment effecting simultaneous clearance of all faults in 0.2 to 0.25 second, loss of system stability at loadings of 150,000 kw. to 170,000 kw. per circuit need not be anticipated.

It is proposed to attain this aim by superimposing carrier pilot control on existing and on all new two-stage impedance relaying. With the exception of the new Leaside-Burlington line, where standard "transfer-block" carrier relaying is proposed, a system of transfer-trip carrier control is being developed. The first of this equipment is being installed on the Beauharnois-MacLaren-Chats Falls connection and on the new Beauharnois-Leaside circuit. It consists of 400 watt carrier communication transmitters, single-frequency voice-actuated the speech frequency range being limited to a band of 200 to 2,500 cycles. Tone generators will be used to transmit relay control signals, which operate to remove the time-delay feature of the distant-end second-range relays. Thus simultaneous clearance is obtained over the full line length, the speed of clearance being limited to that of the various terminal breaker equipments. This protection is as yet experimental and its performance in service will form an interesting study.

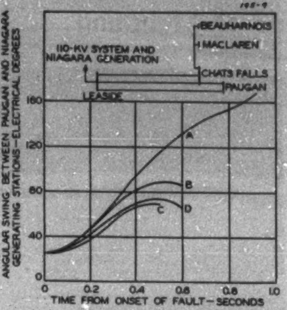


Figure 9. Stability curves for three-circuit 220-kv system, at loadings of approximately 150,000 kw per circuit

A. Three-phase fault at Chats Falls, cleared sequentially in 0.2–0.5 second—unstable

B. Three-phase fault at Chats Falls, cleared simultaneously in 0.2 second—stable

C. Three-phase fault 30 miles west of Chats Falls, cleared simultaneously in 0.25 second—stable

D. Two-wire-to-ground fault at Chats Falls, cleared sequentially in 0.2–0.5 second—stable

Conclusions

1. Operating experience with some 8,400 circuit-mile years of 220 kv. single-circuit overhead line construction is submitted, which to a high degree confirms conclusions which may be drawn from the application to designs of published principles derived from theoretical and experimental analysis.

2. Changes made in the latest designs of towers to provide improved operation under sleet conditions also provide desirable improvements in design against lightning.

3. Counterpoise on towers of high-footing-resistance which has largely been omitted on earlier construction is being installed on 1940–41 lines.

4. Standard two-stage impedance type relay protection has given good satisfaction on these lines. To provide the best operation under heavy line loadings, carrier-current features are being superimposed on both new and old relaying to extend high-speed simultaneous fault clearance to cover the full length of each line

References

1. The 220,000-Volt System of the Hydro-Electric Power Commission of Ontario, E. T. J. Brandon. AIEE Transactions, volume 49, 1930, pages 1400–17.

2. Shielding of Transmission Lines, C. F. Wagner, G. D. McCann, and G. L. MacLane, Jr. AIEE Transactions, volume 60, 1941, pages 313–28.

3. Experience With Prevention Lightning Protection on Transmission Lines, S. K. Waldorf. AIEE Transactions, volume 60, 1941 (June section), pages 249–54.

4. Ice-Coated Electrical Conductors, A. E. Davison. *Bulletin* of the Hydro-Electric Power Commission of Ontario, September 1939, pages 271–80.

5. Powerton-Crawford 220-Kv Line Design and Construction Features, M. S. Oldacre and F. O. Wollaston. AIEE Transactions, volume 60, 1941, pages 1108–20.

6. Measurement and Control of Conductor Vibration, G. B. Tebo. AIEE Transactions, volume 60, 1941, pages 1188–93.

7. Engineering Features of the Boulder Dam–Los Angeles Lines, E. F. Scattergood. AIEE Transactions, volume 54, 1935 (May section), pages 494–512.

Field Investigations of Lightning

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Synopsis: In the investigation described in this paper, measurements have been made and statistical data obtained on the multiple character and wave shape of the currents in direct strokes and the currents discharged by lightning arresters. These data, in conjunction with other data on the field performance of arresters, have revealed definite differences between the nature of direct strokes and the currents they produce in arresters.

1. Arrester discharge currents have fewer components than found in direct strokes. Only about 30 per cent were found to be multiple and not over 8 components were recorded, while from 50 to 70 per cent of direct strokes were multiple with as many as 32 components recorded.

2. Components of arrester discharges are of lower crest magnitude. Only 30 per cent of those measured exceeded 1,000 amperes, while for direct strokes 50 per cent exceeded 5,000 amperes.

3. The initial high current portions of the components of arrester discharges do not vary greatly in wave shape and are similar to those of direct strokes, having for both types of discharges times to half value which, in general, lie between 25 and 100 microseconds and average about 50 microseconds.

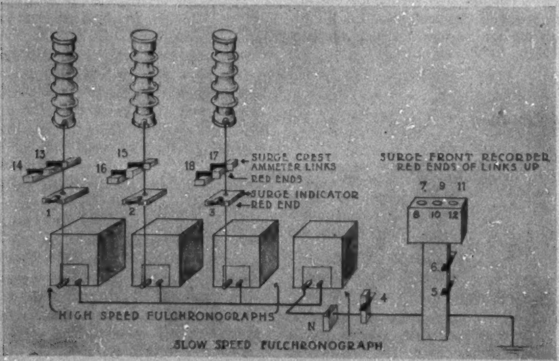
4. The long duration low magnitude portions of direct strokes vary over wide limits, both in magnitude and duration. However, for arrester discharges they are seldom present to any appreciable degree. This portion of the discharge may have current magnitudes as high as 1,000 amperes and measurable durations from 50 to 20,000 microseconds in direct strokes, while lightning discharges through arresters seldom last more than 300 microseconds.

5. There is definite evidence that system neutral ground conditions influence the currents discharged by arresters on systems, and that the most severe duty is imposed on arresters on ungrounded systems, while the least severe service conditions appear on four-wire grounded neutral circuits. This is shown by the fact that no long duration surge components have been recorded on four-wire circuits, while about 50 per cent of the records obtained on ungrounded circuits had components of long duration. It is also shown by the performance record of the new Autovalve lightning arrester. With over 40,000 arrester years service this arrester has experienced only 18 lightning failures or a lightning failure rate of 0.045 of one per cent. Of 27,000 arrester years experience on four-wire systems there have been no failures, while of 4,500 arrester years of service on ungrounded circuits there have been 11 lightning failures.

6. There is indication that one of the factors influencing the nature of a lightning stroke is earth resistivity or geological structure. Observations lead to the inference that lightning discharges of long duration are more prevalent in regions of high earth resistivity and ancient geological structure than elsewhere. Thirteen of the 14 long duration arrester current records and 16 of the 18 arrester failures due to lightning occurred on four systems in two regions of high earth resistivity for which there was only 1,078 arrester years experience.

7. These data indicate that it is only with a combination of high earth resistivity and type of system for which the transformers cannot absorb the long duration portion, that appreciable long duration arrester discharges and accompanying severe duty on arresters is obtained. The data on which this is based are not yet conclusive, but the indications are sufficiently definite that this probability should not be overlooked.

Figure 1. Schematic diagram of typical lightning-recording installation in a three-phase arrester bank


13
14
15
16
17
18
SURGE CREST AMMETER LINKS
RED ENDS
SURGE INDICATOR
RED END
1
2
3
SURGE FRONT RECORDER RED ENDS OF LINKS UP
7 9 11
8 10 12
6
5
4
N
HIGH SPEED FULCHRONOGRAPHS
SLOW SPEED FULCHRONOGRAPH

Introduction

IN spite of the concentrated efforts of many years of lightning investigations, there still remains much to learn regarding the nature of the lightning discharge proper and its effects in power systems. The greatest dearth of information pertains to the wave shape of individual components of discharge currents, the number and time intervals between components, the total duration of the discharge, and the factors which may affect these quantities. The purpose of the investigation in progress is to collect statistical information regarding the foregoing characteristics, not only with regard to the lightning stroke itself, but also with regard to the discharge currents of lightning arresters. The accumulation of statistical information of this nature has been hindered heretofore by the lack of adequate instruments which were sufficiently low in cost and maintenance to permit their distribution in large quantities, yet with sufficient resolving power with respect to time to record the wave shape. The present investigation has been in progress for the past two years with instruments of new types which meet these requirements. Among these are the fulchronograph, the magnetic surge front recorder, and the magnetic surge integrator, which have been described elsewhere.[1]

The investigation has been made possible through the cooperation of a number of utilities and other organizations, reference to which will be found in the paper.

Table I. Direct-Stroke Stations

Location	Height in Feet
1. Anaconda Copper Mining Co., Stack at Anaconda, Montana	585
2. Anaconda Copper Mining Co., Stack at Great Falls, Montana	535
3. Cathedral of Learning, Univ. of Pittsburgh, Pittsburgh, Pa.	530
4. Radio Station WADC, Akron, Ohio	360
5. United Broadcasting Co., Station WHK, Cleveland, Ohio	300
6. United Broadcasting Co., Station WCLE, Cleveland, Ohio	300
7. Philadelphia Electric Co., North Wales Substation, Philadelphia, Pa.	80

Recognizing the probable difference in character of currents in the lightning channel and those produced in power systems, this investigation has been divided into two major parts; first, the measurement of lightning strokes to tall objects and to open ground and, second, the measurement of lightning arrester discharge currents. The performance record of arresters in the field has provided additional information which coordinates with the data obtained by the instruments.

In order to clarify the nomenclature, some of the terms will be defined. The entire phenomenon associated with current flowing in a given lightning channel will be referred to as the lightning stroke

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C. F. WAGNER is consulting transmission engineer, G. D. McCANN transmission engineer, and EDWARD BECK lightning arrester engineer, all with Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.

[1] For all numbered references, see list at end of paper.

Figure 2. Photograph of a typical lightning-recording installation in a three-phase arrester bank

or discharge. A multiple stroke or a multiple discharge is one having more than one component. It is important to distinguish between the individual components of a multiple stroke and the two distinguishing portions of a single component, the high current short duration portion and the low current longer duration portion. These are sometimes referred to as "cold" and "hot" lightning.[2] The high current portion produces high voltage on systems and is important from the insulation standpoint. The low current long duration portion is not important from an insulation standpoint, but can impose severe duty on the arrester. In the case of arrester installations involving more than one arrester pole, the group of records obtained is called a composite record. The data associated with the surge current in each or any arrester pole is called an arrester discharge record. For example, if three arrester phase legs discharge on one stroke, they will be regarded as three arrester discharges. Similarly, the failure of two arresters in a bank will be reckoned as two failures. As in the case of direct strokes, an arrester discharge with more than one component will be classified as one discharge.

Direct-Stroke Stations

The most direct manner in which to measure the current in a lightning channel is to connect the instruments in a circuit which intercepts the stroke itself. This suggests the use of tall objects in order to enhance the probability of obtaining a stroke to the object. On the other hand, it is known, not only from field work, such as the investigation on the Empire State Building,[3] but also from laboratory work,[4] that a large projection from a plane alters the nature of the discharge. The investigator, therefore, faces the dilemma of choosing between a small amount of data of unquestionable character and a larger quantity which may be questionable in regard to being representative of strokes to power systems. In the present investigation objects of different heights have been chosen and it is hoped that with time, sufficient information may be collected to determine the extent to which height affects the results.

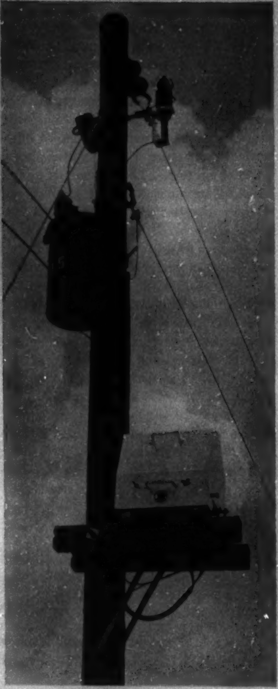


Figure 3. Typical single-phase fulchronograph installation

Table II. Lightning Stations in Power Systems

	Company	Kv of System	Types of Stations: Fulchronograph: Three-Phase	Types of Stations: Fulchronograph: Single-Phase	Types of Stations: Surge Integrator	Types of Stations: Surge-Front Recorders
(1)	American Gas and Electric Service Corporation					
	Appalachian Elec. Power Co.	12 and 132	3			3
	The Ohio Power Co.	33	2			2
(2)	Mississippi Power Co.	110	1			1
(3)	Duquesne Light Co.	22	2*			
(4)	West Penn Power Co.	25	2			2
(5)	Monongahela-West Penn Public Service Co.	22	1			1
(6)	Georgia Power Co.	6.9	2			2
(7)	South Carolina Electric and Gas Co.	4.6		12		
(8)	Rural Electrification Authority	7.2**		10		
(9)	Commonwealth Edison Co.	2.4**			2	
(10)	Wisconsin Public Service Corp.	6.9			5	
(11)	Pennsylvania Water and Power Co. (on legs of two-line towers)	220				2
	Total		13	22	7	13

*In neutral lead only.

**Line-to-neutral voltage.

Seven direct stroke stations have now been established, at each of which a "high speed" and a "low speed" fulchronograph, a surge front recorder, and a number of surge crest ammeter links have been installed. These stations vary in height as enumerated in table I.

Stations in Power Systems

The stations established on power systems are summarized in table II. There are 44; 42 in lightning arrester and protector tube ground leads and two on transmission line towers. Thirteen of the arrester installations are on three-phase arrester banks in stations with circuit voltages varying from 12 to 132 kv. Except for the two installations on the system of the Duquesne Light Company, these are of the general type shown schematically in figure 1 and in the photograph of figure 2. They consist of a high speed fulchronograph in the ground lead of each arrester and a slow speed fulchronograph and surge front recorder in the common lead to ground. In addition, surge crest ammeter links and surge indicators are installed in each phase and the neutral. The surge indicator consists of a surge crest ammeter link in a magnetic iron circuit surrounding the conductor. It is sensitive enough to measure currents as low as 50 amperes and provides a simple means of checking for records.

There are 29 single-phase installations at arresters on systems of voltage ratings from 4 to 7 kv. Twenty-two of these have high speed fulchronographs installed either in the arrester ground lead of a single arrester protecting a transformer connected phase to neutral or in the individual ground lead of one of the two arresters protecting a single-phase transformer connected phase to phase. Figure 3 is a photograph of a typical fulchronograph single-phase installation. Seven of the single-phase stations consist of surge integrators and surge crest ammeter links installed in the arrester ground leads.

Camera Studies

Photographic studies have also been carried on by means of modified motion picture cameras, in which the shutter and claw have been removed, thus permitting the film to move across the open lens at uniform speed. The lightning stroke constitutes its own shutter and the motion of the film imparts the time element. By this means certain properties of multiple strokes can be determined. A number of these cameras have been constructed and interesting records have been contributed by D. D. Clarke of the Kansas City Power and Light Company and C. M. Lear of the Westinghouse Electric and Manufacturing Company.

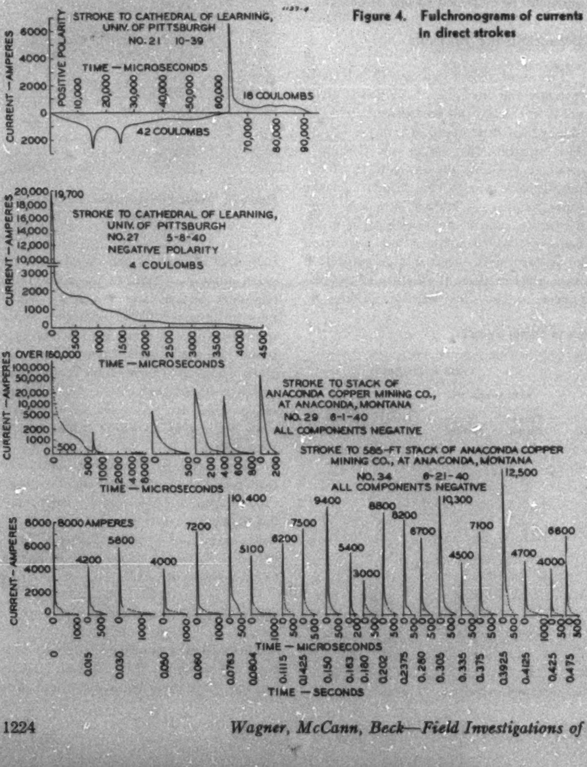


Figure 4. Fulchronograms of currents in direct strokes

Number of Records Obtained

Records have now been obtained of 12 direct strokes having a total of 57 components. Representative fulchronograms of these records are shown in figure 4. Fifty-three composite records have been obtained at the arrester installations. Thirty-two of the composite records were obtained on three-phase installations and comprise 54 arrester discharges having a total of 78 individual components. At the single-phase fulchronograph stations 9 arrester discharges were recorded and at the single-phase surge integrator stations 12 composite records were obtained of 19 arrester discharges. Fulchronograms of several of the arrester currents are shown in figure 5. With the surge front recorder, data were obtained on the character of the fronts of four of the direct strokes, 11 of the arrester surges, and one transmission line tower current. During 1940, the first full season of operation of most of the installations, surges were recorded at 20 of the 44 arrester stations. The number at any one station varied from 1 to 6. Seventy-three of the individual arresters were studied during all of the 1940 lightning season. The duty on these arresters is given in figure 6. As shown by this figure, 46 per cent of the arresters discharged no surges, 54 per cent had at least one surge, 26 per cent had two, and 1.4 per cent had 6, the highest number recorded. This may not be typical of duty on an average group of arresters, since the locations were chosen because experience indicated that they were subject to severe lightning conditions.

Thirty-seven direct strokes to open ground have been photographed with the modified motion picture cameras. Three of these are shown in figure 7 and the statistical data obtained by this means are shown in figures 10, 11, and 12. An analysis of the statistical data on both direct stroke and arrester discharge currents recorded with the fulchronograph is given in figures 9 to 14. The surge front recorder data are summarized in table III and the surge integrator records in table IV.

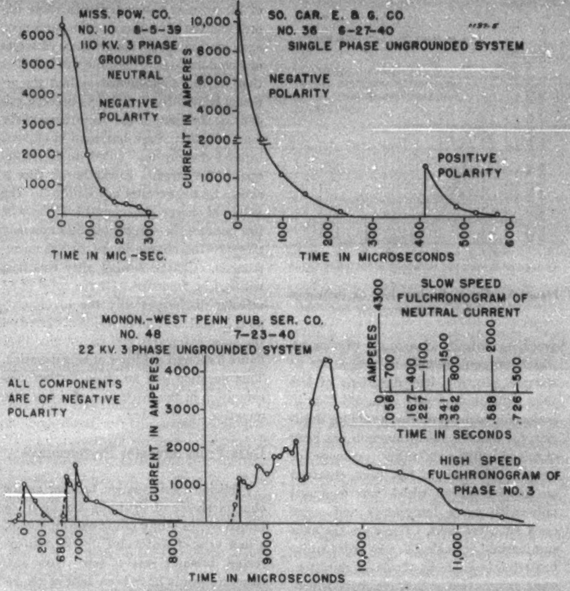
Direct-Stroke Records

Six strokes occurred to the Cathedral of Learning of the University of Pittsburgh during 1939 and 1940, three of which were recorded during operation of the fulchronographs. These records are of two types; first, single-component strokes in which the current can be divided into a high-current short-duration and a low-current long-duration portion of which record No. 27 of figure 4 is typical; and second, one in which the current is more or less continuous with peaks of relatively small magnitudes, such as record No. 21 of figure 4. The latter record, with its relatively low initial current, is similar to those obtained by McEachron in his investigation of the Empire State Building where most of the strokes were initiated at the building and propagated to the cloud. This initially low current represents the streamer formation current. It is believed that at the Cathedral of Learning with a height of 530 feet, strokes can be obtained which are initiated either from the building and propagate skyward or which are initiated from the cloud and propagate earthward. The two records of the first type were of negative polarity with crest magnitudes of the order of 20,000 amperes and long duration tails. They lowered to ground charge of the order of four to six coulombs. The third record (No. 21) consisted of several components having crest values of only a few thousand amperes and discharged 60 coulombs. It had a total duration of 92,000 microseconds, for all of which time considerable current was flowing. To obtain measurements of the thermal effect of the strokes fusible copper wires of varying sizes and a one-inch gap with one blunt electrode and one pointed electrode are installed at the Cathedral station. Information regarding fusion is given in the caption of figure 8. Of the three strokes which occurred while the fulchronographs were not in operation, surge integrator, surge crest ammeter, and fusion data indicate that they probably lie in one of the two groups mentioned above.

An Autovalve distribution arrester connected in the circuit withstood without damage a discharge of the type of record No. 27, but another was damaged by a discharge of the type of record No. 21.

During 1940, the first year that the station on the Anaconda Copper Mining Company's stack at Anaconda, Montana, was in service, four strokes were recorded. Fulchronograms of all of these were obtained, of which two are shown in figure 4. Although enough strokes have not yet been recorded to draw any definite conclusions, there appear to be marked differences between the strokes to this stack and to the Cathedral of Learning. All of the Anaconda strokes were multiple, having from four to 22 components. Only one of all 52 individual components had a measurable duration greater than 1,500 microseconds and only 14 over 500 microseconds. The magnitude of the long duration portion of all these components is considerably smaller than those for strokes to the Cathedral. One of the four components of the first stroke at Anaconda (record No. 29 of figure 4) exceeded 160,000 amperes, the maximum range of the equipment. This represents the highest known magnitude in which the total current was measured at one point, as contrasted with measurements obtained by summing up currents in parallel paths. Record No. 34 of figure 4 is of interest because of the large number of components and its total duration.

All the recorded direct strokes were negative, that is, lowered negative charge to ground, except two to the Cathedral which were oscillatory but initially negative.



PHASES NOS 1 & 2 HAD SINGLE COMPONENT NEGATIVE DISCHARGES OF ABOUT 300 AMPERES AND NEGLIGIBLE DURATION.

Figure 5. Fulchronograms of lightning-arrester discharge currents

Arrester Discharge Currents

Of the 32 composite records on three-phase arrester installations, 25 permitted analysis of current distribution between phases. Of these, nine showed current in only one phase, eight in two phases, and eight in three phases. In only one of the cases in which a surge was recorded in more than one phase was the current predominately greater in one phase than the others. This is record No. 48 of figure 5. In 21 of the 25 cases the maximum current recorded in the neutral was, within the accuracy of the instruments, the sum of the phase currents; in three, it was more; and in one, less. In a few cases of multiple strokes the number of components was not the same in each phase. Twenty-three per cent of the surges were positive polarity and only three surges were oscillatory. Nineteen per cent of the station arrester surges were positive as compared to 29 per cent for the distribution arresters.

The analysis of the individual components of the arrester currents given in figures 9 to 14 show that neglecting record No. 48, the times to half value vary from 25 to 80 microseconds with an average value of 47 microseconds, and none of the measurable durations exceed 300 microseconds. Thirty additional fulchrono-

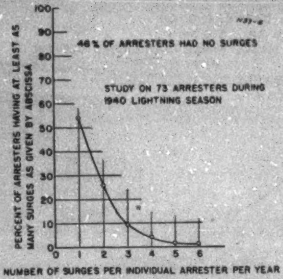


Figure 6. Curve showing duty on individual arresters

graph records of 48 arrester discharges, which were of insufficient magnitude to warrant plotting, had durations of less than 100 microseconds. Record No. 48 is the only arrester discharge of long duration for which a fulchronogram has been obtained, although other evidence of long duration surges has been obtained on another circuit which was equipped only with surge integrators and surge crest ammeter links. These are the data summarized in table IV. In order to interpret this table in terms of duration, some assumption as to wave shape is necessary. For example, if it is assumed that the discharge drops quickly to a constant current of 100 amperes, then a charge of one-half coulomb would correspond to a duration of 5,000 microseconds. While lack of definite data on wave shape precludes the possibility of drawing definite conclusions, it does appear that these charges are considerably greater than those determined elsewhere in this investigation. Further, the failure of the arrester associated with these records and the appearance of the parts upon examination verify the presence of surges of long duration. This table also indicates that on this ungrounded system the arrester failure did not always become evident at the time of the occurrence of the long duration surge but may have been injured thereby and failed in the subsequent discharge. Evidence of this is shown by the arrester at location 5. The values of charge given in table IV may be low because the surge integrator can only measure the charge up to 10,000 microseconds. Charge flowing after this time will not be recorded. Of the total of 82 arrester discharges only the 13 enumerated in table IV, all of which occurred on the same extension, and record No. 48 of figure 5 showed evidence of long duration. This question will be given further consideration in the discussion of the service record of the arresters.

Data From Arrester Performance

Additional information bearing on the characteristics of lightning has been obtained by the study of the performance of a new type of Autovalve distribution arrester, brought out a few years ago. Careful records have been kept of the locations and performance of this arrester. Laboratory tests had indicated that this new arrester has greater ability to handle the severe surges experienced on rural distribution systems than arresters available heretofore. The field experience has borne this out. For some years arresters have been able to withstand discharges of high current and short duration, but the new type has also greater capacity for moderate currents of long duration. It had been concluded from theoretical considerations and certain evidence from the field, that surges of this type were the principal cause of lightning failures of arresters on rural distribution circuits.[5] This has been confirmed by those discharge current records which were obtained at the time of arrester failure. Seven of these recorded by surge integrators are shown in table IV. The eighth such case so far recorded is record No. 48 which was accompanied by the failure of a 25 kv line type arrester of obsolete construction.

One of the factors affecting arrester discharge currents is the nature of system grounding. By providing a path to ground paralleling the arrester the transformers of grounded neutral systems can to varying extents, depending upon the time constant and resistance of the parallel path, absorb the low magnitude long duration portions of surges and thus relieve the duty on the arresters.[6] In other words, the arresters protect the system against the high voltage high current portions of the discharge and the system protects the arresters for the low voltage low current and long duration portions. Ungrounded systems cannot perform this function. Table V analyzes the operating experience with the new arrester with regard to the type of system on which it was installed as well as other factors contributing to arrester damage.

Over 40,000 arrester years of experience have been accumulated with this arrester since the early part of 1938. During this time there has been a total of 28 failures from all causes, representing a failure rate of 0.07 of one per cent. Of these, 10 are attributed to dynamic overvoltage, leaving 18 failures caused by lightning, or a lightning failure rate of 0.045 of one per cent. All of the failures occurred in rural service.

Table V shows the importance of the type of system grounding. Although by far the greatest portion of the service has

Figure 7. Photographs of multiple strokes to open ground

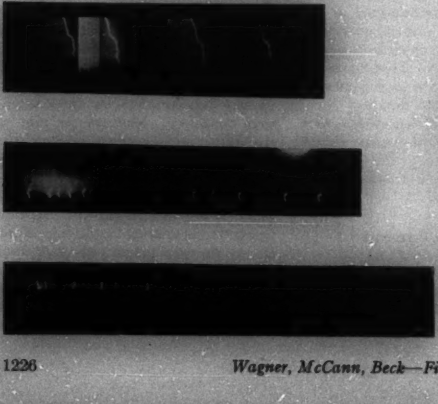
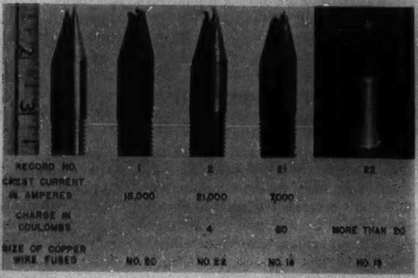


Figure 8. Fusion data on strokes to the Cathedral of Learning of the University of Pittsburgh



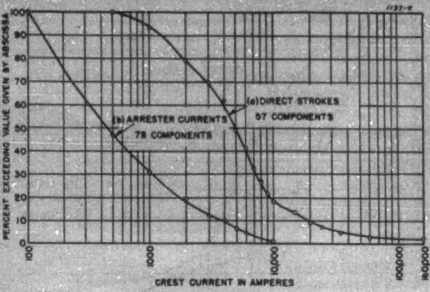


Figure 9. Percentage distribution of crest magnitude of individual components of surge currents

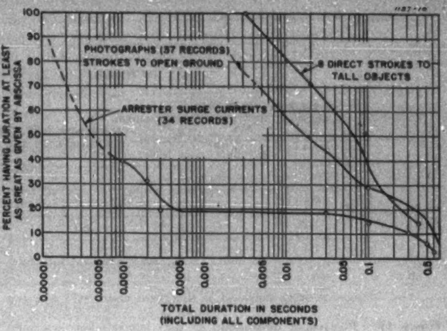


Figure 10. Percentage distribution of total duration of lightning surges

been accumulated on four-wire grounded neutral systems, no arrester failures have been reported on such circuits. This type of system provides the maximum amount of surge current drainage, because both the substation and distribution transformers constitute paths to ground in parallel with the arresters. At the other extreme are the delta connected or ungrounded systems, which provide no parallel drainage path, so that arresters must bear the full brunt of the surge, unless the line flashes to ground. Table V shows that on delta systems, although the service experience was much less, the failure rate was 0.24 of one per cent. Intermediate between these extremes is the type of system fed from a grounded neutral substation transformer but in which no neutral wire is carried out. The distribution transformers being connected phase to phase, the only drainage path to ground is that provided by the substation transformer. The effectiveness of this path decreases with increasing distance between the substation and the arresters involved in the discharge. In 8,500 arrester years of service on such systems, only seven lightning failures have occurred, indicating a rate of 0.08 of one per cent. All of these occurred at points remote from the substation transformers, a fact which corroborates the theory that arresters close to the grounded neutral substation transformers are relieved of discharge current.

This effect of system grounding upon the nature of the arrester discharge is also shown by the current records which have been obtained. Of the 82 arrester discharge currents recorded, 30 were on four-wire circuits, 21 on three-wire circuits, and 31 on ungrounded circuits. However, all 14 of the surges with long duration components were obtained on ungrounded circuits. All of the rest had measurable durations of less than 300 microseconds. It might be argued that, since about 70 per cent of the arrester currents measured had crest magnitudes less than 1,000 amperes, the presence of a long duration component of proportionate magnitude to that appearing in direct stroke currents would be indistinguishable in most of the records. However, of the 24 discharges with crest magnitudes exceeding 1,000 amperes only 12 were recorded on ungrounded circuits and nine of these showed evidence of long duration portions.

Another factor in the performance of arresters appears to be the earth resistivity in the region of the installations. Surveys[6] indicate that in certain regions in which the arresters are installed the earth resistivity is high, 1,000 meter ohms or more. Only two arresters were damaged in regions where the resistivity is thought to be low; both occurred on delta systems; while 16 were damaged by lightning in regions where the resistivity is known to be high. These 16 cases occurred on four systems representing only 1,078 of the 40,000 arrester years of service. The 16 failures have been further broken down in table VI, according to type of system and earth resistivity. Because of difficulty of securing accurate data, the segregation of arrester years of service according to earth resistivity is very approximate. However, a very considerable error in these is permissible without vitiating the qualitative character of the results and conservative figures are used. The 13 long duration discharges recorded with the surge integrator were obtained on one of the four systems in a region of high earth resistivity which gives further evidence of this effect.

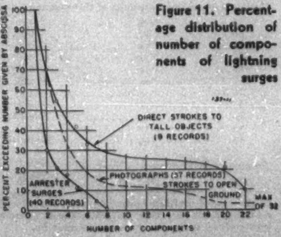


Figure 11. Percentage distribution of number of components of lightning surges

While the data are still meager, it appears from table VI that earth resistivity influences the character of arrester discharge currents and that a combination of high earth resistivity and delta or three-wire grounded system is required to produce appreciable rates of arrester failure, whereas four-wire grounded systems appear to be the most satisfactory from the arrester standpoint regardless of earth resistivity. This is important from the operating standpoint.

Discussion of Results

The foregoing data show that there are marked differences between the currents in direct strokes to tall objects and the currents discharged by protective devices on power systems.

MAGNITUDE

As shown by figure 9, the maximum measured value of crest current discharged by an arrester is about 10,000 amperes (record No. 36 of figure 5), and only 50 per cent of the crest magnitudes exceed 450 amperes, whereas the maximum measured value of crest current obtained in a direct stroke exceeded 160,000 amperes and 50 per cent of the components had crest values exceeding 5,000 amperes. These data differ from those

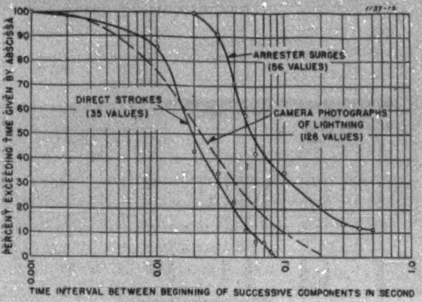


Figure 12. Percentage distribution of time intervals between components of multiple strokes

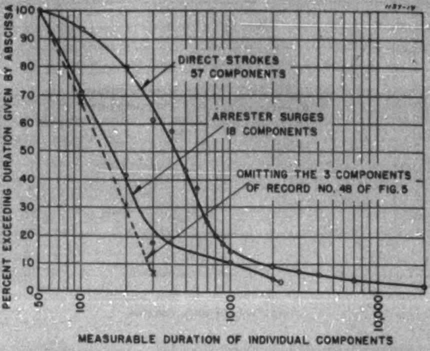


Figure 14. Per cent distribution of duration of individual components of lightning-surge currents

previously published by other investigators in that the quantity plotted is the crest magnitude of the individual components in distinction to the maximum crest magnitude of all the components in a stroke, the value measured by a surge crest ammeter. In common with the results published by others, the arrester currents are less than those in direct strokes. This may be attributed to three primary causes. One, all of the direct stroke current is not likely to pass through an arrester because more than one arrester may operate thus dividing the current, or flashovers may occur on the lines thereby limiting the arrester discharge current. Two, the arrester may be operated by attenuated surges reaching the arrester from strokes to the system at remote points. Three, the arresters may be discharged by induced surges.

POLARITY

Although all of the direct strokes recorded in this investigation, as well as those published by Stekolnikov[7] and McEachron,[8] were either negative or oscillatory, but initially negative, an appreciable number of positive discharges were recorded on arrester installations. It is thought that this is so because most of the positive discharges are produced by induced strokes, which, although relatively low in magnitude may yet be sufficiently high to discharge arresters. This is further substantiated by the fact that the proportion of positive discharges increases with decreasing discharge voltage of the arresters. The station arresters, which varied in ratings from 12 to 109 kv, showed 19 per cent positive discharges, while the distribution arresters, which varied in ratings from 3 to 9 kv, showed 29 per cent positive discharges.

MULTIPLE CHARACTER OF STROKES

In figures 10, 11, and 12 are plotted curves giving statistical data on the total duration, the number of components, and the time intervals between successive components of lightning currents. These curves present data of three general types, arrester discharge currents, strokes to open country, as given by the photographic records, and strokes to tall objects. More components and longer durations were recorded in strokes to tall objects than to open ground. Of practical importance is the indication that arrester discharge currents have fewer components and shorter durations. This is probably caused by the diminution of the magnitude of the components from the original magnitudes in the stroke for the reasons enumerated under "Magnitude." Thus, some of the smaller components may be reduced to such an extent that they are below the recording range of the equipment, or have associated voltages below the operating voltage of the arresters. Consequently, arresters and other

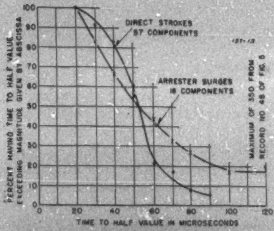


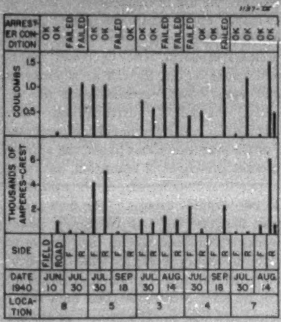
Figure 13. Per cent distribution of times to half value of individual components of lightning-surge currents

Table III. Records Obtained With the Magnetic Surge-Front Recorders

Record No.	Polarity	Crest Current Amperes	Surge Front Micro-sec.	Average Rate of Rise Amps Per Micro-sec.
Currents in direct strokes				
24	Neg.	19,000	2.5	7,600
34	Neg.	Multiple See ful'grams	0.5– 9.9	1,300–6,000
59	Neg.		0.5–19	174–2,000
60	Neg.		0.5– 6	1,140–2,000
Currents in common grounds of three-phase arrester banks				
5	Neg.	100	More 10	Less 10
55	Neg.	1,600	2	800
30	Pos.	500	1	500
41	Neg.	500	1.1	450
31	Pos.	600	More 6	Less 100
42	Pos.	825	More 20	Less 40
38	Pos.	1,100	3.9	280
47	Neg.	2,300	6	380
45	Neg.	200	More 10	Less 20
58	Neg.	5,100	1.6	3,200
48	Neg.	Multiple	Less 0.5 to 20	500–2,600
Transmission-line tower current				
66	Neg.	50,400	1.16	43,600

In the case of multiple strokes, because of interference between the indications upon the magnetic recorder resulting from successive impulses, the surge-front recorder provides only a range within which all of the fronts of the successive components must lie.

Table IV. Summary of Discharge Records Obtained by Surge Integrators on 6,900-Volt Ungrounded Circuit of the Wisconsin Public Service Corporation



The recorded data on coulombs of charge may be less than the actual because the surge integrator does not record charge flowing after 10,000 microseconds.

terminal equipment are exposed to less severe duty than would be indicated by the data on direct strokes to tall objects or to open country. The curves of figure 12 indicate that the time intervals between successive components of strokes to tall objects and open ground are approximately the same. The time intervals between components in arrester discharges, on the other hand, are appreciably greater. This would be expected from the reduction of some of the components as mentioned above. It is also possible that some of the multiple fulchronograph records which were regarded as the result of a single stroke are actually the result of more than one stroke occurring between inspections of the instruments. The data of figure 12 shows that there is seldom more than one component discharged in a period equivalent to one-half cycle of 60 cycles.

Wave Shape

The data obtained on wave fronts by means of the surge front recorder summarized in table III show that of the four direct strokes for which records of the front were obtained only one had a single component. This record (No. 24) had a crest value of 19,000 amperes, a front of 2.5 microseconds, and an average rate of rise of 7,600 amperes per microsecond. The three strokes of multiple character indicated a possible range of fronts between 0.5 and 19 microseconds, and a possible rate of rise of between 174 and 6,000 amperes per microsecond. The highest rate of current rise which has been recorded is that of the one transmission line tower current. This value (43,000 amperes per microsecond for a current with a crest value of 50,400 amperes) is also the highest rate of rise which is known to have been measured in natural lightning surges. However, this value was determined by instruments on only one leg of a tower and the result of this leg multiplied by four.

Of the arrester discharge current records, the times to crest varied from about 1 to 20 microseconds. The highest rate of rise is 3,200 amperes per microsecond obtained, however, for a surge of only 5,100 amperes crest.

Table V. Performance Record of the New Distribution Arrester, 1938–1940

	Number of Failures		Estimated Total Arrester Years of Service	Annual Failure Rate Per Arrester-Year	
	All	Lightning		All	Lightning
Total number	28	18	40,000	0.0007	0.00045
Year					
1938	0	0	250	0	0
1939	5	3	8,000	0.0006	0.0004
1940	23	15	32,000	0.0007	0.0005
	28	18	40,000		
Probable causes					
Lightning		18	40,000		0.00045
Dynamic voltage only	10		40,000	0.00025	
Type of service					
Rural	28	18	30 000	0.0009	0.0006
Urban	0	0	10,000	0	0
	28	18	40,000		
Type of system					
Four-wire grounded neutral	0	0	27,000	0	0
Three-wire grounded neutral	8	7	8,500	0.0010	0.0008
Delta	20	11	4,500	0.0046	0.0024
	28	18	40,000		
Location					
Regions of high soil resistivity					
A	7	7	10	0.70	0.70*
B & C	11	2	68	0.16	0.03*
D	8	7	1,000	0.008	0.007
All others	2	2	39,000	0.00005	0.00005
	28	18	40,000		

*These percentages may be distorted because of the small number of arrester years of service and the possible influence of system voltage conditions.

Table VI. Effect of Earth Resistivity and Type of System on Arrester Performance

Type of System and Earth Resistivity	Arrester Years	Number of Failures	Failure Rate Per Cent
Four-wire, grounded neutral	27,000	0	0
Low earth resistivity	26,000*	0	0
High earth resistivity	More than* 1,000	0	0
Three-wire, grounded neutral	8,500	7	0.08
Low earth resistivity	7,500*	0	0
High earth resistivity	1,000+	7	0.7
Delta	4,500	11	0.24
Low earth resistivity	4,000*	2	0.05
High earth resistivity	500*	9	2

* Approximate estimate.

The times to half value of the high current portions of the direct strokes and arrester discharges, as shown by figure 13, range from 25 to 100 microseconds with an average of about 50 if record No. 48 is excepted. The total duration of the individual components including the long duration portions, however, as shown by figure 14, varied over a wide range. Direct strokes show a range of from 50 to 20,000 microseconds, while arrester discharges varied from less than 50 to 2,300 microseconds.

McEachron in his investigations on the Empire State Building shows that most of the strokes to it are initiated from the building and propagate upward to the cloud. The current measured in the building in such cases may persist at relatively small values between successive peaks. No doubt this tendency increases with the height of the object. On the other hand, the fulchronograms obtained in the present investigation show that such low currents can follow high current peaks even to lower objects. There is nothing in the mechanism of stroke propagation as proposed and developed by Schonland that precludes the possibility of lower currents of long duration following high current peaks in strokes to ground. The only direct evidence that this characteristic is present in strokes to open ground is provided by the fulchronogram (record No. 48 of figure 5) and the magnetic surge integrator records of table V, although there has been indirect evidence to this effect provided by the markings left on arresters returned from the field and by camera studies, notably those of Schonland.

In general, there is considerably less evidence of long duration portions of the current in arrester discharges than in di-

Power-Circuit Instruments for the Higher Range of Audio Frequencies

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Synopsis: The extending use being made of frequencies from 900 to 12,000 for special purposes such as induction furnaces, etc., introduces new problems in instrument design and calibration. Previous practical a-c instruments were either of the electrodynamic or moving iron types designed for low power frequencies or thermal and rectifier types for audio and radio frequencies, these latter usually being confined to measurements of only current and voltage.

The ordinary low frequency electromagnetic instruments have such high coil reactance and losses that they are unsuitable for the higher power frequencies in question without special arrangements, which are described in the paper. Furthermore, the usual calibration methods and standards as previously used for low frequencies are not adequate, and therefore, new methods of calibration were introduced.

Introductory

FOR many years, the frequency range within which measurements were required, was within the scope of ordinary power application requirements. This range was in general below 150 cycles. Then there followed certain higher frequencies for high speed motors, as in metal working and spinning, which involved the use of frequencies up to 600 or 1,000. The higher frequencies gave rise to no new serious instrument problems, except that they disqualified the induction principle of operation which was used in earlier instruments designed for low frequencies.

Higher frequencies than those ordinarily used for operating motors were used largely in communication work and were therefore termed "audio frequencies." This name has prevailed for want of a more exact term. It is, of course, somewhat inexact since it does not distinguish from frequencies such as 60 cycles, which are also audible. Moreover, the higher frequencies in the audio range were also used for long wave radio communication. Likewise, the range of audio frequencies overlaps the range of radio frequencies, which begins at 10 kilocycles and extends in practice to 60,000 kilocycles or higher.

Principles of Operation

Electrical instruments may be classified by applicability in various frequency groups, roughly as follows:

1. Direct current.
2. Low frequency power (up to several hundred cycles).
3. High frequency power, including the complete audio-frequency range.
4. Radio frequencies.

Practical indicating instruments required in the radio and upper audio-band communication activities are well covered by instruments which measure currents and voltages. Requirements for measuring power, energy, or reactive power or power factor are not insistent for these fields.

The extension of industrial power applications to induction electric furnaces requires the use of frequencies within the so-called upper audio-frequency range, involving the usual power supply measurement of watts, reactive power (vars) and power factor, as well as of amperes and volts. Such measurements are required not only for investigation work, but also for control or operating purposes. For the purposes of this paper, the various present principles of operation, as found in practical instruments, can be said to be of two distinct groups. The first group was designed for low frequency power work, and the second, for high audio- or radio-frequency work, for which the ordinary instruments of the first group could not be used because of their high reactances and losses. The second group has other inherent limitations which generally restrict its use to measurement work where the more sturdy, less sensitive types cannot be used.

Until recently, the instrument requirements for higher audio-frequency measurements have been taken care of by the use of thermal instruments for current and voltage measurements in the same manner as for radio frequencies. However, the development of power equipment for heat-treating equipment has

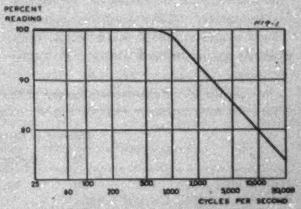

Figure 1. Frequency response of repulsion moving iron ammeter

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rect strokes. This is to be expected from the consideration of the absorption of these portions in varying degrees on grounded neutral systems, and is confirmed by the results of the investigation, which so far has shown evidence of long duration portions only on ungrounded circuits. It is indicated that there is a considerably greater prevalence of the long duration portions in strokes in regions of high earth resistivity, since at least 13 and possibly all of the 14 long duration discharges were obtained in such regions, which comprise only a small part of the scope of the investigation. This is given additional weight by the fact that 16 of the 18 cases of lightning damage to arresters occurred in two regions of high earth resistivity, although they represent only a small part of the total arrester experience.

References

1. New Instruments for Recording Lightning Currents, C. F. Wagner and G. D. McCann. AIEE Transactions, volume 59, 1940, page 1061.

2. Lightning Strokes in Field and Laboratory, P. L. Bellaschi. Electrical Engineering, volume 58, November 1939, page 466.

3. Lightning to the Empire State Building, K. B. McEachron. Electrical Engineering, volume 57, December 1938, page 493.

4. Shielding of Transmission Lines, C. F. Wagner, G. D. McCann and G. L. MacLane, Jr. AIEE Transactions, volume 60, 1941, pages 313–28.

. Lightning and Lightning Protection of Distribution Systems, R. C. Bergvall and Edward Beck. AIEE Transactions, volume 59, 1940 (August section), page 442.

6. Earth Resistivity and Geological Structure, R. H. Card. AIEE Transactions, volume 54, 1935 (November section), page 1153.

7. L'Etude de la Foudre dans un Laboratoire de Campagne, I. Stekolnikov and Ch. Valeev. *CIGRE*, 1937, bulletin number 330.

given rise to the necessity for instruments having heavy overload capacity, quick response, and other characteristics found in industrial power instruments. The necessity for measuring power and power factor also has made it necessary to extend the use of wattmeters and reactive meters to include this audio-frequency band.

The electrical measurements involved include (*a*) current, (*b*) voltage, (*c*) true power, (*d*) vars (reactive power), (*e*) power factor. In any particular installation, one of the last two measurements is usually sufficient in any electrical circuit.

AMMETERS

The magnitude of an alternating current usually is measured by moving iron vane, electrodynamic, or electrothermic type ammeters. The electrothermic type ammeter, which utilizes the thermocouple principle in measuring a current by its heating effect, is entirely free from frequency errors over the audio-frequency range. However, the disadvantages of this type, such as low overload capacity, delicacy of mechanism, and uneven scale distribution, render it undesirable for power work, or in fact, for any source where the measurement can be made accurately by other indicating means. Likewise, the electrodynamic ammeter has relatively low overload capacity, and furthermore, the usual parallel connection of the moving and stationary coils introduces difficulties in matching circuit branch impedances as required in order to minimize frequency errors. This narrows the choice down to the moving iron vane type ammeter. This type is characterized by inherently high overload capacity and simplicity. The frequency influence is relatively small and no particular difficulty is involved in calibration.

The frequency error of the repulsion moving iron ammeter is shown in the curve, figure 1. In the higher audio-frequency range, the eddy current losses in metal framework parts, damping chamber parts, and even the jewel screws and nuts of the conventional low frequency forms, cause excessive temperature rise, making it necessary to fabricate these parts from special low-conductivity materials having suitable physical characteristics. Temperature rises are thus kept within permissible limits. Further points of interest in this connection are the necessity of laminating the iron vanes to sufficiently thin sheets to limit temperature rise caused by eddy currents in the vanes themselves, and the fact that at least in five- or ten-ampere ammeters, there appears to be no necessity for subdividing the conductors used in the coils. Calibration of ammeters is accomplished by using the thermocouple type standard, with current obtained from an oscillator source adjusted to the desired frequency.

VOLTMETERS

The value of an alternating voltage is ordinarily measured by electrodynamic, moving iron, thermocouple, or rectifier type instruments. For high voltage work, electrostatic types have been used. Electrodynamic voltmeters readily may be built for frequencies up to about 3,000 cycles by designing the windings with the requisite low inductance. For higher frequencies, the necessary further decrease in inductance which is accompanied by increase in energy consumption, causes the coils to operate near their safe maximum temperature, thus limiting overload capacity. Moving iron type voltmeters inherently have a high ratio of inductance to resistance which limits their use to commercial power frequencies. For example, a well-designed repulsion iron vane type voltmeter calibrated for 60-cycle service may indicate 12 percent low at 500 cycles. This inductance varies with the angular position of the moving element for different parts of the scale, and therefore the compensating means for wattmeters (which will be described further) cannot be employed readily for voltmeters.

The overload capacity of voltmeters must be greater than is provided by thermocouple type instruments, since induction furnace generators usually have poor regulation. The no-load voltage may be 20 to 30 percent higher than full-load voltage, and this no-load overvoltage condition may exist for prolonged periods under operating conditions. Also, there exists a possibility of surges when the load is applied or removed, which would damage thermocouple type instruments. Therefore, thermocouple voltmeters would seem to be undesirable for this application.

Another inherent disadvantage of the thermocouple type voltmeter, as well as of the ammeter, is that the scale distribution is a function approximately of the square of the applied quantity.

Electrostatic voltmeters would seem to have the ideal electrical characteristics, but are far from satisfactory from the viewpoint of mechanical requirements in practical operating instruments.

The remaining practical possibility for voltmeters is the rectifier type principle. Rectifier voltmeters, as is well known, consist of a permanent magnet moving coil mechanism with a copper-oxide rectifying unit. All rectifier types measure the average value of the a-c wave, but in practice, they are calibrated on a sine wave to read the rms value. Thus, on other wave forms, errors are introduced. It is necessary to provide a filter to eliminate errors which arise from variable harmonics present in the distorted wave forms usually present in high frequency furnace circuits. The importance of a suitable filter is further emphasized by the fact that the wave form changes with various load conditions. Recent improvements in Rectox units used with rectifier voltmeters vir-

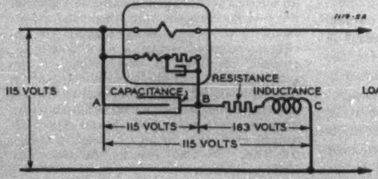

Figure 2a (left). Phase-shifting network for varmeter

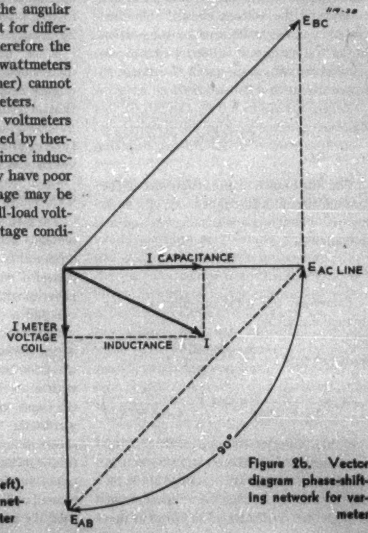

Figure 2b. Vector diagram phase-shifting network for varmeter

tually have eliminated the frequency error within the audio range and reduced the temperature error to commercially allowable limits.

WATTMETERS

The measurement of power is accomplished by a conventional electrodynamic type instrument provided with a simple method of compensating for the voltage circuit inductance. Since the inductance of the voltage circuit causes the current in that circuit to lag the impressed voltage, an uncompensated instrument will read low at high frequencies because the deflection is proportional to the in-phase components of current and voltage. Actually, the inductance of the voltage circuit can be made very low. At power frequencies, the voltage circuit current lags the impressed voltage only 10 to 20 minutes, and this is therefore negligible. At higher audio frequencies, however, this angle of lag assumes very large proportions; for example, if the angle of lag at 60 cycles is 18 minutes, it becomes almost 40 degrees at 9,600 cycles. This would correspond to an error of 23 percent in the reading. It is therefore necessary to provide compensation for the voltage circuit inductance.

Compensation for the inductance of the voltage circuit is accomplished by a simple method which makes the instrument practically free of frequency error. This is done by connecting a capacitor of suitable value in parallel with the non-inductive resistor of the voltage circuit. The impedance of the resistor capacitor portion of the voltage circuit is the impedance of a parallel circuit composed of resistance R_1 in parallel with a capacitor C.

$$Z_{\text{Res.}}=\frac{1}{\frac{1}{R_1}+jWC}=\frac{R_1}{1+jWCR_1}=\frac{R_1-jWCR_1^2}{1+W^2C^2R_1^2}$$

The impedance of the remainder of the voltage circuit is the impedance of a moving coil which has a resistance R_2 and inductance L.

$$Z_{MC}=R_2+jWL$$

Therefore, the total impedance of the voltage circuit is:

$$Z_{\text{total}}=R_2+jWL+\frac{R_1-jWCR_1^2}{1+W^2C^2R_1^2}$$
$$=R_2+\frac{R_1}{1+W^2C^2R_1^2}+jW\left(L-\frac{CR_1^2}{1+W^2C^2R_1^2}\right)$$

At 10,000 cycles, the value of $W^2C^2R_1^2$ for a well-designed wattmeter is $(2\pi 10{,}000)^2 \times (0.002\times 10^{-6})^2(2{,}500)^2 = 9.88\times 10^8\times 4\times 10^{-18}\times 2.5\times 10^6 = 9.85\times 10^{-3}$, which may be neglected with respect to unity so that approximate impedance $Z_{\text{total}}=R_2+R_1+jW(L-CR_1^2)$. If R_1 is adjusted so that $L=CR_1^2$, then $Z_{AC}=R_1+R_2$ which is the same as the measured resistance.

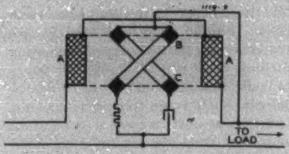

Figure 3. Schematic diagram power-factor meter

VARMETERS

A wattmeter equipped with suitable frequency compensation, as described, may be used for the measurement of vars by providing a phase-shifting network in the voltage circuit to turn the voltage circuit vector 90 degrees in a lagging direction. The instrument will then read zero at unity power factor and full scale at zero power factor of the measured circuit, assuming a sinusoidal wave condition. It is customary to build varmeters with zero center scales to provide indications for both leading and lagging reactive power. The phase-shifting network consists of a circuit with capacitance, inductance, and resistance paths arranged as shown in figure 2*a*. The vector diagram is shown in figure 2*b*. In practice, the 90-degree lag of the current in the moving coil behind the applied voltage is accomplished by adjusting the value of inductance until I_M lags E by exactly 90 degrees.

While thermal wattmeters and varmeters also could be applied to the frequency ranges in question, it is believed that in view of the practicability of the modified conventional electrodynamic type as above described, their use would not be justified.

POWER-FACTOR METERS

The measurement of single-phase power factor is accomplished by a conventional cross-coil type electrodynamic instrument provided with a non-inductive resistance in series with the horizontal cross-moving coil and a capacitance in series with the vertical cross-moving coil. The inductance of the horizontal moving coil is compensated for by a small capacitor in parallel with a portion of the series resistance in the same manner as described for the wattmeter. The circuit arrangement is shown in figure 3. The principle of operation is otherwise the same as for conventional single-phase electrodynamic type crossed coil power factor meters provided with the resistor-reactor network, except that, because of the high frequency, it is more practical to substitute a capacitor for the usual inductive reactance, the corresponding moving coil being reversed since the current in it leads instead of lags the voltage.

The production of resonant type frequency meters for the ranges involved is a much easier problem than that of the reactive meters. However, so far as the present types of induction furnace circuits are concerned, there seems to be no necessity for frequency indication, as the generators are motor-driven at a speed sufficiently constant for good performance. This speed is not ordinarily changed during operation.

If variable frequency systems are introduced later, they would be of the tube circuit type, in which case, frequency meters would be desirable. These might be of the wave meter type as used in radio applications.

Measurements for Induction-Furnace Circuits

Reactive measurements are important in the operation of an induction furnace or high frequency heating load for two reasons:

(1). The power factor of the load is quite low; approximately 10 percent in the case of an induction furnace. It is not practical to design a generator to supply, say, 100 kw of load and 900 kva of reactance. Power factor correction by means of capacitors is, therefore, used to bring the power factor up to unity, and in some cases, even to make it slightly leading.

It is important that the power factor of the circuit being supplied be checked at all times, because the physical conditions, such as permeability and resistivity of the charge, will vary with temperature, and the initial power factor will not be maintained. Adjustments, therefore, have to be made in the amount of corrective capacitance as the heating cycles proceed. In other words, the power factor meter tells the operator when it is necessary for him to make an adjustment in the number of capacitor units connected. This may not be so important in the case of the short time heating cycles because in that case, an average power factor must be used, since the time is too short to make any adjustments. However, the power factor meter is used in this case to predetermine the correct value.

(2). The generator usually is arranged with an automatic voltage regulator to maintain its voltage constant. If, then, the power factor becomes increasingly lagging, the excitation required would be very greatly increased, resulting in overheating of the generator field. If the generator voltage is not automatically regulated, then the kw input to the load would be reduced materially, since the voltage naturally would drop with a fixed excitation on an increasingly inductive load. For these reasons, the power factor is important.

In more recent installations, the use of varmeters has been recommended rather than power factor meters, because it is believed that a closer control of the tuning of the circuit can be obtained by the use of the former.

Electrodynamic wattmeters are a more accurate form of instrument than the corresponding power factor meter, as the accuracy of the latter is influenced by the constant but slight torque of the spiral strips used for conducting the current to the moving coil.

The power factor meter operates by a difference between two electrical torques, and thus the presence of a third torque, such as that of the connecting strips, introduces an error at light loads which cannot be calibrated out. The influence of wave form distortion caused by harmonics is present in both varmeters and power factor meters. However, the oscilloscope shows a somewhat better wave form on the power supply circuit than anticipated, especially when a large capacitance is connected.

A word of caution is necessary in the interpretation of readings of reactive quantities such as vars and power factor. Both varmeters and power factor meters operate on the assumption of a sinusoidal wave form, and only under this condition is power factor equal to the cosine of the angle of lag. Under conditions where harmonics are present, the power factor and reactive component calculated from the wattmeter readings and the voltamperes will not necessarily agree with the readings of the power factor meter or the varmeter. Thus the readings of the reactive instruments should be used for control rather than for exact measurement purposes. Their usefulness is principally for indicating the required resonance condition of the circuit. As between the choice of a power factor meter or a varmeter for this purpose, the authors are strongly in favor of the latter. Figure 4 shows the elementary instrument diagram of connections of a typical installation.

In order to simplify the metering diagram, the 60-cycle auxiliary control circuit for the generator is not shown. The elementary diagrams for measurement and control are virtually the same for frequencies from 960 to 11,520 cycles.

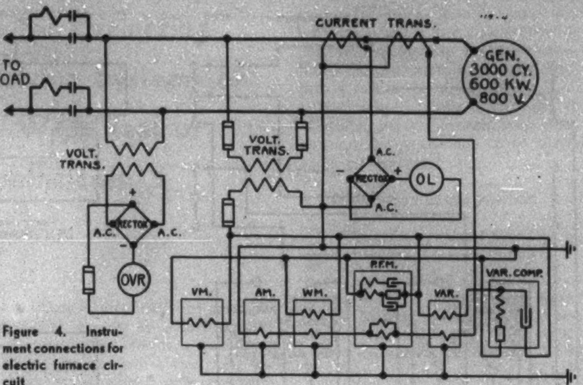

Figure 4. Instrument connections for electric furnace circuit

An important operating precaution in audio-frequency measurements involving instruments having both current and voltage circuits is to have the voltage terminal which is connected to the voltage coil, connected to the low potential or grounded line. This brings the moving and stationary coils to the same level of potential which eliminates errors due to capacitance effects which at the higher audio frequencies are by no means negligible. It is, of course, conventional practice to connect the voltage terminal from the resistor end of the voltage circuit to the opposite line from the current coil, even in power frequency applications. However, at the higher audio frequencies, the effects are so much more pronounced that this point must not be overlooked.

Calibration

The calibration of modified power type instruments for audio-frequency application required new equipment, including the adjustable frequency power sources having adequate capacity and calibration standards.

The schematic wiring diagram of the audio-frequency source is shown in figure 5. Taps are provided for various voltages and load impedances. The beat frequency oscillator is adjustable for frequencies from 20 to 20,000 cycles a second. This equipment provides a suitable power source for the calibration of ammeters, voltmeters, wattmeters, and varmeters. The calibration of power factor meters requires two separate audio-frequency sources (one for voltage and one for current), with provision for varying the phase angle between current and voltage.

Ammeters are calibrated by setting the oscillator for the desired frequency and using a thermocouple type instrument as a standard with the instrument under test. A tap is selected for proper impedance to maintain sine wave output.

The calibration of voltmeters is accomplished similarly, using a thermocouple voltmeter connected in parallel as a standard.

Wattmeters are adjusted and dials marked in the conventional manner on a 60-cycle source. Frequency compensation adjustment is accomplished by passing a current of the order of 12,000 cycles a second through the current coil and short circuiting the voltage circuit terminals. Any movement deflection resulting from this short circuit indicates the necessity of compensation. When the condenser is connected across the proper portion of the resistance, no deflection results from the short circuit, and the instrument is practically independent of frequency error over the complete frequency range to 12,000 cycles.

The varmeter instrument proper is identical to the wattmeters except for dial markings, so calibration adjustments and compensation are the same for both instruments. The varmeter compensator is adjusted to shift the voltage circuit current 90 degrees at the desired frequency by adjusting the value of inductance and resistance until I_M lags E by exactly 90 degrees and is of proper magnitude. The vector relationship of currents and voltage in the circuit are shown in vector diagram figure 2*b*. Phase shifter adjustments for obtaining proper phase angle and magnitude are made quite simply by the test and calibration circuit, figure 6. Since the current in the instrument voltage circuit must be in quadrature with the line voltage, the phase angle adjustment made by adjusting the phase shifter reactor will produce a quadrature shift in current which gives zero deflection on the voltmeter.

The magnitude of the voltage output of the phase shifter is made equal to the input voltage by adjusting resistance R_1. The calibration circuit for power factor meters is shown in figure 7. The voltage applied to the voltage coil of the power

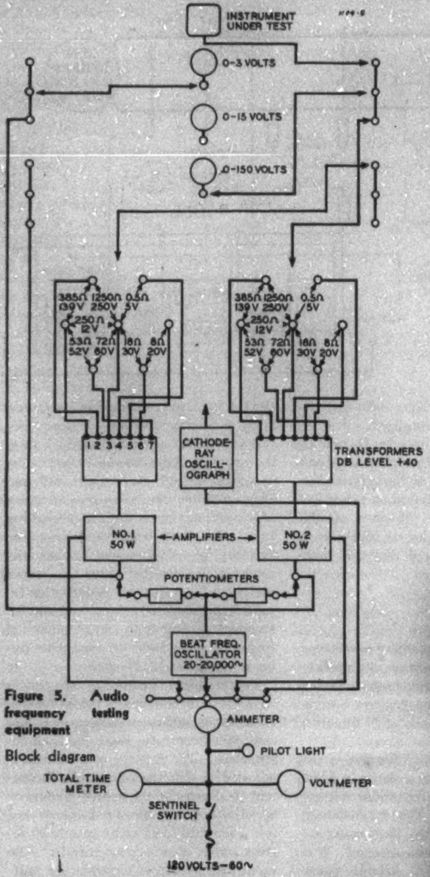
INSTRUMENT UNDER TEST
0-3 VOLTS
0-15 VOLTS
0-150 VOLTS
385Ω 139V
1250Ω 250V
0.5Ω 5V
250Ω 12V
53Ω 52V
72Ω 60V
18Ω 30V
8Ω 20V
1 2 3 4 5 6 7
CATHODE-RAY OSCILLOGRAPH
TRANSFORMERS DB LEVEL +40
NO.1 50 W
AMPLIFIERS
NO.2 50 W
POTENTIOMETERS
BEAT FREQ. OSCILLATOR 20-20,000~
AMMETER
PILOT LIGHT
TOTAL TIME METER
VOLTMETER
SENTINEL SWITCH
120 VOLTS—60~

Figure 5. Audio frequency testing equipment

Block diagram

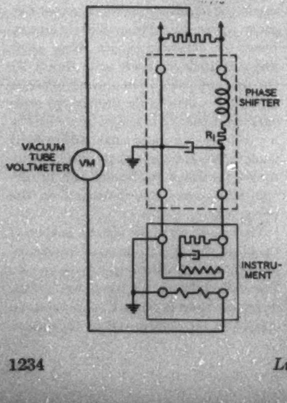
PHASE SHIFTER
R_1
VACUUM TUBE VOLTMETER
VM
INSTRUMENT

Figure 6 (left). Calibration circuit for varmeter

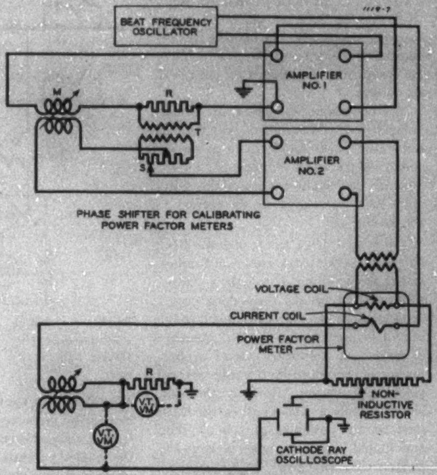
BEAT FREQUENCY OSCILLATOR
AMPLIFIER NO. 1
AMPLIFIER NO. 2
M
R
T
S
PHASE SHIFTER FOR CALIBRATING POWER FACTOR METERS
VOLTAGE COIL
CURRENT COIL
POWER FACTOR METER
R
V.T. VM
V.T. VM
NON-INDUCTIVE RESISTOR
CATHODE RAY OSCILLOSCOPE

Figure 7 (above). Circuit diagram for calibrating power-factor meters at audio frequencies

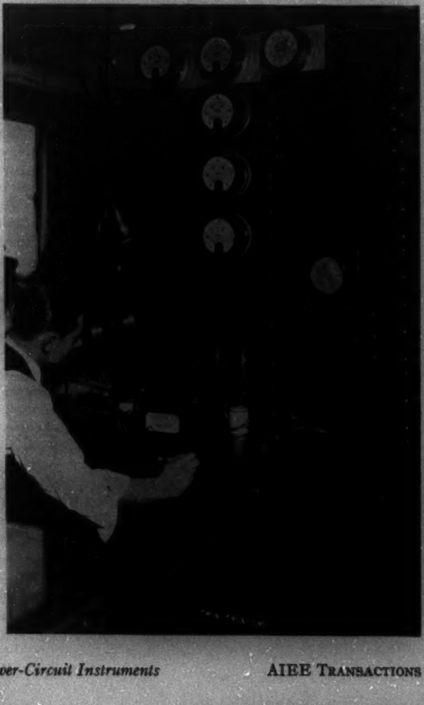

Figure 8 (right). High-frequency instrument-calibrating equipment

A Short Method for Evaluating Determinants and Solving Systems of Linear Equations With Real or Complex Coefficients

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NONMEMBER AIEE

THE purpose of this paper is to describe a short method for solving arbitrary systems of linear algebraic equations, and evaluating determinants, the quantities involved being either real or complex. For proofs and for treatment of complex systems whose symmetrical coefficients are conjugate, see appendix. The cases considered are:

1. Arbitrary systems with real coefficients, which occur in obtaining stresses in structures, in solving systems of linear differential equations with constant coefficients (transient problems), etc.

2. Symmetrical systems with real coefficients, which occur with d-c networks, undamped vibration, deflections in structures, least square processes, Ritz's method, etc.

3. Symmetrical systems with complex coefficients, which occur with a-c networks, and forced vibration with dissipation.

4. Arbitrary systems with complex coefficients, which occur in certain vibration problems involving gyroscopic action.

5. Systems involving two sets of variables, which occur when the currents in a network are to be found for a variety of impressed voltages, also in the approximate solution of integral equations arising in electric field problems.

The work of solving a system of equations (or evaluating a determinant) is largely concentrated in the determination of an "auxiliary matrix," and is roughly half that required by a matrix multiplication. The process is particularly adapted for use with a computing machine, for each element is determined by one continuous machine operation (sum of products with or without a final division). The setting down of this matrix and of the final solution is the only writing required by the process. The work involved is cut almost in half if the given equation (or determinant) is symmetrical, as very often happens. A "check column" can be carried along if desired.

The amount of work required to obtain a solution is considerably less than that required by Gauss's method,[1] even when there is symmetry and the coefficients are real, in which case Gauss's method has been considerably refined by Doolittle.[2] (Gauss's method is much shorter than a solution by determinants.)

The method as given is applicable to m equations in n unknowns, there being no restriction on the rank of the matrix of the coefficients.

2. Description of the Method

Let the given system of equations be specified by its *given matrix*,[2] thus

$$\begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & = \\ 12.1719 & 27.3941 & 1.9827 & 7.3757 & 6.6355 \\ 8.1163 & 23.3385 & 9.8397 & 4.9474 & 6.1304 \\ 3.0706 & 13.5434 & 15.5973 & 7.5172 & 4.6921 \\ 3.0581 & 3.1510 & 6.9841 & 13.1984 & 2.5393 \end{array} \tag{1}$$

the first equation being $12.1719x_1 + 27.3941x_2 + 1.9827x_3 + 7.3757x_4 = 6.6355$. The solution requires the formation of one matrix and a set of final results; thus we have an *auxiliary matrix*

$$\begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & = \\ 12.1719 & 2.2506 & 0.16289 & 0.60596 & 0.54515 \\ 8.1163 & 5.0720 & 1.6793 & 0.0057629 & 0.33632 \\ 3.0706 & 6.6327 & 3.9585 & 1.4193 & 0.19891 \\ 3.0581 & -3.7316 & 12.7526 & -6.7332 & 0.060806 \end{array} \tag{2}$$

and a *final matrix*

$$\begin{array}{l} x_1 = 0.15942 \\ x_2 = 0.14687 \\ x_3 = 0.11261 \\ x_4 = 0.060806 \end{array} \tag{3}$$

The procedure for obtaining the auxiliary matrix from the given matrix is contained in the following rules.

1. The various numbers, or *elements*, are determined in the following order: elements of first column, then elements of first row to right of first column; elements of second column below first row, then elements of second row to right of second column; elements of third column below second row, then elements of third row to right of third

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[1] NUMERISCHES RECHNEN, Runge, page 65.

[2] PRACTICAL LEAST SQUARES, Leland, page 40.

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factor meter is applied to one set of plates of a cathode ray oscilloscope, while a voltage drop in phase with the current in the current coil is applied to the other set of plates. The amplifiers in the cathode ray oscilloscope must have no phase shift. This readily can be checked by applying the same voltage to both plates and noting the image.

It is possible to determine the phase angle by measuring the areas of the elliptical image formed on the screen of the oscilloscope, but precise measurements are not possible, as it is difficult to read exact deflection values. A more accurate method of determining the phase angle is to bring the two voltages applied to the plates exactly in phase, which produces a precise straight-line image on the cathode ray screen. By means of vacuum tube voltmeters, it is then possible to measure the voltage in phase with the non-inductive resistance R, which is in phase with the current in the current circuit of the power factor meter, and to measure the voltage across the mutual inductance which is 90 degrees leading the current in the current circuit of the power factor meter. In bringing the image on the oscilloscope to a straight line, this latter voltage (the out-of-phase component) is adjusted in magnitude so that the vector sum of the two voltages is brought in phase with the voltage across the voltage coil of the power factor meter. The ratio of these two voltmeter readings is therefore a measure of the tangent of the phase angle from which the phase angle and power factor can be calculated accurately.

Figure 8 is a view of the calibrating outfit which is shown diagrammatically in figure 5.

column; and so on until all elements are determined.

2. The first column is identical with the first column of the given matrix. Each element of the first row except the first is obtained by dividing the corresponding element of the given matrix by that first element.

3. Each element on or below the principal diagonal is equal to the corresponding element of the given matrix minus the sum of those products of elements in its row and corresponding elements in its column (in the auxiliary matrix) which involve only previously computed elements.

4. Each element to the right of the principal diagonal is given by a calculation which differs from rule 3 only in that there is a final division by its diagonal element (in the auxiliary matrix) [3]

As examples we have the following typical calculations made in obtaining (2), the letters *R* and *C* representing the words "row" and "column," respectively.[4]

$$\begin{array}{lrl}
R1C3 & 0.16289 & = 1.9827 \div 12.1719 \\
R2C2 & 5.0720 & = 23.3385 - 8.1163\cdot 2.2506 \\
R4C2 & -3.7316 & = 3.1510 - 3.0581\cdot 2.2506 \\
R2C5 & 0.33632 & = (6.1304 - 8.1163\cdot 0.54515)\div 5.0720 \\
R3C3 & 3.9585 & = 15.5973 - 3.0706\cdot 0.16289 - 6.6327\cdot 1.6793 \\
R4C3 & 12.7526 & = 6.9841 - 3.0581\cdot 0.16289 + 3.7316\cdot 1.6793 \\
R3C4 & 1.4193 & = (7.5172 - 3.0706\cdot 0.60596 - 6.6327\cdot 0.0057629)\div 3.9585 \\
R4C4 & -6.7332 & = 13.1984 - 3.0581\cdot 0.60596 + 3.7316\cdot 0.0057629 - 12.7526\cdot 1.4193 \\
R4C5 & 0.060806 & = (2.5393 - 3.0581\cdot 0.54515 + 3.7316\cdot 0.33632 - 12.7526\cdot 0.19891)\div(-6.7332)
\end{array}$$

Since a modern computing machine gives in one continuous operation a sum or difference of products with or without a final division, we see that *each element of the auxiliary matrix is given by a single machine operation.*[5]

The procedure for obtaining the one-columned final matrix from the auxiliary matrix is contained in the following rules.

1. The elements are determined in the following order: last, next to last, second from last, third from last, etc.

2. The last element is equal to the corresponding element in the last column of the auxiliary matrix.

3. Each element is equal to the corresponding element of the last column of the auxiliary matrix minus the sum of those products of elements in its row in the auxiliary matrix and corresponding elements in its column in the final matrix which involve only previously computed elements.

We see that in forming products only those elements of the auxiliary matrix are used which lie to the right of the principal diagonal and to the left of the last column. The calculations made in obtaining (3) are

$$\begin{array}{ll}
R3C1 & 0.11261 = 0.19891 - 1.4193\cdot 0.060806 \\
R2C1 & 0.14687 = 0.33632 - 1.6793\cdot 0.11261 - 0.0057629\cdot 0.060806 \\
R1C1 & 0.15942 = 0.54515 - 2.2506\cdot 0.14678 - 0.16289\cdot 0.11261 - 0.60596\cdot 0.060806
\end{array}$$

It may be noted that *each element of the final matrix is given by a single machine operation.*

It is not necessary but is strongly recommended that the values of the unknowns, which compose the final matrix, be substituted in each of the given equations, the result being a number of checks equal to the number of equations. Since the satisfaction of these checks guarantees the correctness of the solution, *it is not necessary to check the calculations which gave the auxiliary matrix and the final matrix.*[6] The first of the four checks obtained from (1) and (3) is

$$12.1719\cdot 0.15942 + 27.3941\cdot 0.14687 + 1.9827\cdot 0.11261 + 7.3757\cdot 0.060806 = 6.6355$$

Evidently *each check requires but one machine operation.*

The above method is applicable as described to *n* equations in *n* unknowns. The only writing involved is that required in recording the auxiliary matrix and the final matrix.

3. Systems Having Symmetrical Coefficients

If there is symmetry, the work of computing the auxiliary matrix is cut almost in half by the fact that *if the coefficients of the unknowns (or the elements of the given matrix) are symmetrical about the principal diagonal; each element of the auxiliary matrix below the principal diagonal gives, if divided by its diagonal element, the symmetrically opposite element above this diagonal.*[3] Elements below the principal diagonal of the auxiliary matrix are thus obtained as by-products of calculations made in determining elements above this diagonal.

As an example the symmetrical set of equations

$$\begin{array}{rrrrr}
x_1 & x_2 & x_3 & x_4 & = \\
245.95 & 768.49 & 233.08 & 261.13 & 40.384 \\
768.49 & 2665.85 & 880.54 & 915.95 & 90.887 \\
233.08 & 880.54 & 688.91 & 458.17 & 6.5783 \\
261.13 & 915.95 & 458.17 & 652.85 & 24.471
\end{array} \qquad (4)$$

has the auxiliary matrix

$$\begin{array}{rrrrr}
x_1 & x_2 & x_3 & x_4 & = \\
245.95 & 3.1246 & 0.94769 & 1.0617 & 0.16420 \\
768.49 & 264.63 & 0.57536 & 0.37800 & -0.13338 \\
233.08 & 152.25 & 380.42 & 0.40259 & -0.029928 \\
261.13 & 100.03 & 153.15 & 276.13 & -0.0017367
\end{array} \qquad (5)$$

and the final matrix

$$\begin{array}{ll}
x_1 = 0.55591 & x_3 = -0.029229 \\
x_2 = -0.11591 & x_4 = -0.0017367
\end{array} \qquad (6)$$

In the auxiliary matrix the element in row 3 and column 4 is

$$\frac{(458.17 - 233.08\cdot 1.0617 - 152.25\cdot 0.37800)}{380.42} = \frac{153.15}{380.42} = 0.40259$$

---

[3] A *matrix* is a rectangular array of numbers, or *elements*. Those elements which have the same row and column index form the *principal diagonal* which slopes down to the right starting with the element in the upper left corner. The *diagonal element* of any element to the right of the principal diagonal is that element of this diagonal which lies in the same row as the given element. The diagonal element of any element below the principal diagonal is that element of this diagonal which lies in the same column as the given element.

[4] All numerical data (except where exact) were obtained by omitting a certain number of decimal places from the corresponding original data, which were obtained using a ten bank computing machine retaining ten decimal places throughout. This applies to the given equations as well as to other data, for the problems used here as illustrations were, for the most part, previously solved for other purposes.

[5] In carrying out such an operation it is not necessary to foresee whether the result is positive or negative.

[6] Since in solving a given set of equations only a limited number of decimal places can be carried, the question arises as to how much the error accumulated during the course of the calculations affects the final results. An answer to this question cannot be given, since it depends upon whether or not the determinant of the system is on the point of vanishing (or, more generally, upon whether the matrix is on the point of shifting rank). Nevertheless it will be mentioned that in solving each of six systems of four equations in four unknowns using a ten bank computing machine, the final checks showed the given equations to be satisfied except for errors numerically less than three units in the ninth digit. Also it is known (from a separate investigation) that the values (10) of the unknowns of the complex system (8) computed with a ten bank machine contain errors each of which is numerically smaller than four units in the tenth digit. Although no general statement can be made, it is believed that in the usual case the accuracy tends to remain high during the course of the calculations.

the numerator 153.15 being recorded in the symmetrically opposite position before the final division by the diagonal element 380.42 is carried out.[4] The final matrix is obtained in the usual manner.

4. Evaluation of Determinants

It is true that *the value of a determinant is equal to the product of the elements which form the principal diagonal of the auxiliary matrix.* The auxiliary matrix is computed as described in section 2, the elements of the determinant being treated in the same manner as the coefficients of the x's before. The auxiliary matrix is square, since there is no column corresponding to the last column in (2); also, there is no final matrix.

As an example, the determinant composed of the first four columns of (1) has an auxiliary matrix composed of the first four columns of (2); hence the value of the determinant is $(12.1719)(5.0720)\times(3.9585)(-6.7332)=-1645.4$.

If the determinant is symmetrical about the principal diagonal, the work of evaluating it is cut almost in half because of the smaller amount of work required in computing the auxiliary matrix. (See section 3.) For example, the determinant composed of the first four columns of (4) has an auxiliary matrix composed of the first four columns of (5). The value of the determinant is hence $(245.95)(264.63)(380.42)(276.13)=6.8368\cdot10^9$.[4]

5. Continuous Check on Calculations

If desired, a "check column" may be written at the right of the given matrix, each element of this column being the sum of the elements of the corresponding row in the matrix. This column is now treated in exactly the same manner as the last column of the given matrix, the calculations being carried along with those for the other columns, and the result being the addition of corresponding "check columns" to the auxiliary matrix and the final matrix. The check columns thus obtained for (1), (2), and (3) are, respectively,[4]

$$\begin{array}{l} 55.560 \\ 52.372 \\ 44.421 \\ 28.931 \end{array},\quad \begin{array}{l} 4.5646 \\ 3.0214 \\ 2.6182 \\ 1.0608 \end{array},\ \text{and}\ \begin{array}{l} 1.1594 \\ 1.1469 \\ 1.1126 \\ 1.0608 \end{array} \tag{7}$$

These columns provide checks at all stages of the computation, because

1. **In the auxiliary matrix any element in the check column is equal to one plus the sum of the other elements in its row which lie to the right of the principal diagonal.**
2. **In the final matrix any element in the check column is equal to one plus the sum of the other elements in its row.**

For example noting (2), (3), and (7), two of the checks are

$$1+1.6793+0.0057629+0.33632=3.0214$$
$$1+0.11261=1.1126$$

The above statements are true and the procedure the same for any number of equations and unknowns; also, for the evaluation of determinants, in which, however, only statement 1 is applicable.

6. Improvement in Accuracy

Since the number of decimal places in the computations is limited, the values obtained for the unknowns are in general not exact. However, if they are placed in the given equations and the differences between the two sides are obtained, and if these differences are then inserted in place of the right hand sides of the given equations, the resulting equations have as their solution the corrections to the values first obtained. Noting that the above differences are obtained in applying the final checks (see section 2), and that the auxiliary matrix for the modified equations is the same as that for the original equations except for the last column, it follows that *if the column of the differences[7] obtained in applying the final checks be annexed to the given matrix and then treated in the same manner as the last column, the corresponding column obtained in the final matrix is composed of the required corrections.*

Since the problem of solving the modified equations is similar to the original problem, the above process may be repeated; thus the final checks on the corrections give data for another column in the given matrix, which leads to a column in the final matrix composed of corrections to the first corrections, etc. In the usual case each application of this process increases the number of significant figures in the results by approximately the same number obtained with the original solution, the data in the given equations being considered exact.

7. Systems of Equations Having Complex Coefficients

Since the proofs which establish the above method do not require the quantities involved to be real, the method is applicable to complex equations. The only question is that of whether the required calculations can be easily performed on a computing machine.

Since $(a+jb)(c+jd)=(ac-bd)+j(ad+bc)$, we see that the real and imaginary parts of a sum of products of complex numbers are each a sum of products of real numbers, given by one machine operation. It follows that *each element of the final matrix and each element of the auxiliary matrix which lies on or below the principal diagonal can be obtained by two machine operations one of which gives the real part, the other, the imaginary part.* It also follows that *each check run on the final matrix consists of two machine operations.*

Each element of the auxiliary matrix which lies to the right of the principal diagonal is given by a sum of products with a final division. The sum is first computed and recorded, after which the final division by the diagonal element $A=a+jb$ is carried out as a separate *multiplication* by $1/A=a/(a^2+b^2)-jb/(a^2+b^2)$. Since the best form for recording the supplementary data depends upon how much of it there is, we shall consider separately the following two cases.

1. Symmetrical Equations

Noting section 3 we see that *if the coefficients in the given equations are symmetrical about the principal diagonal, the sum required for computing any element of the auxiliary matrix above the principal diagonal is already recorded as the symmetrically opposite element below this diagonal.* It follows that only the last column of the auxiliary matrix requires separately recorded sums, which may be written in a *right supplementary column.* Data on the final multipliers $1/A$ may be placed in *two left supplementary columns.* [See (9).] In determining the auxiliary matrix the elements in any row which lie in the supplementary columns are determined just before those in that row to the right of the principal diagonal.

As an example the given set of four equations

$$\begin{array}{rrrrr} x_1 & x_2 & x_3 & x_4 & = \\ 84.276 & -91.342 & 32.463 & -45.417 & 6558.50 \\ 71.184j & 10.741j & -12.142j & -32.984j & -\ \ 185.00j \\ -91.342 & 45.374 & -21.212 & 80.414 & -8780.75 \\ 10.741j & 52.741j & 74.342j & -20.732j & 670.75j \\ 32.463 & -21.212 & 72.713 & 37.642 & -3463.25 \\ -12.142j & 74.342j & -20.221j & -77.814j & -3535.41j \\ -45.417 & 80.414 & 37.642 & -20.114 & -4268.87 \\ -32.984j & -20.732j & -77.814j & -29.741j & 1449.04j \end{array} \tag{8}$$

---

[7] In recording the computed values of these differences, as many nonvanishing digits should be kept as can be handled in the subsequent machine calculations.

where the imaginary part of each element is written below the real part for compactness, has the auxiliary matrix

$$\begin{array}{cc|ccccc|c}
a^2+b^2 & 1/A & x_1 & x_2 & x_3 & x_4 & = & RS \\
12170. & 0.0069251 & 84.276 & -0.56973 & 0.15379 & -0.50745 & 44.336 & \\
 & -0.0058493j & 71.184j & 0.60867j & -0.27397j & 0.037241j & -39.644j & \\
13101. & -0.000009789 & -91.342 & -0.1282 & 0.41654 & -0.10413 & -29.887 & -5156.8 \\
 & -0.0087368j & 10.741j & 114.458j & 0.087841j & -0.30098j & 45.088j & -3426.6j \\
7119.4 & 0.0111588 & 32.463 & -10.107 & 79.444 & 0.75944 & -29.404 & -2574.2 \\
 & 0.0039928j & -12.142j & 47.665j & -28.426j & -0.77633j & -8.3789j & 170.19j \\
3284.5 & -0.00050088 & -45.417 & 34.462 & 38.265 & -1.6452 & -51.713 & 1369.5 \\
 & -0.0174415j & -32.984j & -11.880j & -83.263j & 57.287j & -22.421j & -2925.6j
\end{array} \qquad (9)$$

where the supplementary columns are partitioned off by vertical lines, and the final matrix[4]

$$\begin{array}{llll}
x_1 = & \begin{array}{l}20.471\\ 10.782j\end{array} & x_2 = & \begin{array}{l}27.275\\ -31.498j\end{array} \\
x_3 = & \begin{array}{l}-42.652\\ 37.913j\end{array} & x_4 = & \begin{array}{l}-51.713\\ -22.421j\end{array}
\end{array} \qquad (10)$$

The following are typical calculations made in obtaining the above solution. In each case the part computed is specified by "real" or "imag.," after which the letters *R*, *C*, *LS*, and *RS* are used in place of "row," "column," "left supplementary," and "right supplementary," respectively. In the auxiliary matrix[4]

Real, *R3C3* $\quad 79.444 = 72.713 - 32.463 \cdot 0.15379 + 12.142 \cdot 0.27397 + 10.107 \cdot 0.41654 + 47.665 \cdot 0.087841$

Imag., *R4C3* $\quad -83.263 = -77.814 - 45.417 \cdot 0.27397 + 32.984 \cdot 0.15379 - 34.462 \cdot 0.087841 + 11.880 \cdot 0.41654$

Real, *R3CRS* $\quad -2574.2 = -3463.25 - 32.463 \cdot 44.336 + 12.142 \cdot 39.644 - 10.107 \cdot 29.887 + 47.665 \cdot 45.088$

R3CLS1 $\quad 7119.4 = 79.444 \cdot 79.444 + 28.426 \cdot 28.426$

Imag., *R3CLS2* $\quad 0.0039928 = 28.426 \div 7119.4$

Real, *R3C4* $\quad 0.75944 = 0.0111588 \cdot 38.265 + 0.0039928 \cdot 83.263$

Imag., *R3C5* $\quad -8.3789 = 0.0111588 \cdot 170.19 - 0.0039928 \cdot 2574.2$

In the final matrix

Real, *R2C1* $\quad -42.652 = -29.887 - 0.41654 \cdot 27.275 - 0.087841 \cdot 31.498 - 0.10413 \cdot 51.713 + 0.30098 \cdot 22.421$

2. NONSYMMETRICAL EQUATIONS

In the general case where there is no symmetry, and a sum must be recorded for every element of the auxiliary matrix to the right of the principal diagonal, the supplementary data may be recorded in a *supplementary matrix* having the same number of rows and columns as the auxiliary matrix. Each element to the right of the principal diagonal is the sum required in computing the corresponding element of the auxiliary matrix; the diagonal elements are the final multipliers $1/A$; and values of a^2+b^2 may be placed in a left supplementary column.

Since to obtain any element of the auxiliary matrix or the final matrix a real and imaginary part must be obtained, and since the sums involved have twice as many terms as they would have in the case of real coefficients, we see that *the work involved in solving a set of equations with complex coefficients is a little more than four times that required in solving a similar set with real coefficients.*

The procedures just described under case 1 and case 2 can be used to obtain the square auxiliary matrix required in evaluating a determinant with complex elements. (See section 4.)

A check column can be carried along if desired, the entries being complex. (See section 5.)

8. Equations Involving Two Sets of Variables

If two sets of variables are involved, the above method may be used to obtain one set in terms of the other. For example the given set of equations[4]

$$\begin{array}{cccccccc}
x_1 & x_2 & x_3 & x_4 = & y_1 & y_2 & y_3 & y_4 \\
12.172 & 27.394 & 1.983 & 7.376 & 6.636 & 0 & 0 & 0 \\
8.116 & 23.339 & 9.840 & 4.947 & 0 & 6.636 & 0 & 0 \\
3.071 & 13.543 & 15.597 & 7.517 & 0 & 0 & 6.636 & 0 \\
3.058 & 3.151 & 6.984 & 13.198 & 0 & 0 & 0 & 6.636
\end{array} \qquad (11)$$

of which the first is $12.172x_1+27.394x_2+1.983x_3+7.376x_4=6.636y_1$, has the auxiliary matrix

$$\begin{array}{ccccccccc}
x_1 & x_2 & x_3 & x_4 & = & y_1 & y_2 & y_3 & y_4 \\
12.172 & 2.251 & 0.1629 & 0.6060 & & 0.5452 & 0 & 0 & 0 \\
8.116 & 5.072 & 1.679 & 0.00576 & & -0.8724 & 1.308 & 0 & 0 \\
3.071 & 6.633 & 3.958 & 1.419 & & 1.039 & -2.192 & 1.676 & 0 \\
3.058 & -3.732 & 12.753 & -6.733 & & 2.699 & -4.877 & 3.175 & -0.9855
\end{array} \qquad (12)$$

and the final matrix

$$\begin{array}{lcccc}
 & y_1 & y_2 & y_3 & y_4 \\
x_1 = & -9.187 & 17.053 & -12.117 & 5.643 \\
x_2 = & 3.800 & -6.607 & 4.734 & -2.343 \\
x_3 = & -2.791 & 4.730 & -2.830 & 1.399 \\
x_4 = & 2.699 & -4.877 & 3.175 & -0.9855
\end{array} \qquad (13)$$

from which we see that $x_1 = -9.187y_1 + 17.053y_2 - 12.117y_3 + 5.643y_4$, etc. Each *y* column of (11) is treated in the same manner as the last column of (1), the result being a corresponding column in the final matrix. Four final checks on each of these is obtained by equating the corresponding *y* to 1 and the other *y*'s to 0, and substituting (13) in (11). If a check column is not used, it is desirable to finish one *y* column and apply the final checks before filling in the other *y* columns. (The calculations for each *y* column are independent of those for the others.) If a check column is desired, the procedure and rules given in section 5 are worded so as to apply to the present case, the *y* columns being included with the others. There is no restriction on the number of equations or the number of *y*'s.

9. m Equations in n Unknowns. Unusual Cases

Let there be given *m* equations in *n* unknowns. These may be arbitrarily (in any order) labeled 1, 2, . . ., *m*, and 1, 2, . . ., *n*, respectively, and the above procedure followed as though the equations and unknowns were actually written in the orders indicated by the labels. We shall consider the rows and columns both numbered in the orders in which they are completed in the auxiliary matrix. In general these orders are those in which the equations and unknowns are actually written, so that the labels are superfluous; however, in determining the auxiliary matrix two unusual cases may arise, as follows:[8]

1. If in completing a column the diagonal element is found to vanish, the (unlabeled) row chosen to be completed and labeled is one which has a nonvanishing element in this column.
2. If in completing a column all of the newly computed elements are found to vanish, that column is left unlabeled; but is (with its zeros) in its final form, and requires no further calculations.

The calculation of the auxiliary matrix is continued until either the rows or the unfinished x columns are exhausted, the total number of labeled rows and columns then being R. (R is the *rank* of the matrix of the coefficients in the given equations.) The m-R unlabeled rows and n-R unlabeled x columns will be called *excess*. Either, both, or neither of the quantities m-R and n-R may vanish, depending upon the given equations.

The one column (or more if there are several y columns as in section 8) arising from the right hand sides of the equations is now completed, the procedure being identical with that used in completing the last labeled column. If in so doing the computed elements (which lie in excess rows) all vanish, the given equations are compatible; otherwise they are incompatible.

If the equations are found to be compatible, we next omit the excess rows (the equations corresponding to which are superfluous), and determine the final matrix from the remaining rows. Each excess column is treated like the last column, and contributes a column to the final matrix, in which it is labeled $-x$ with the appropriate subscript. We thus obtain R unknowns as linear combinations of the others.

As an example, the given equations[4]

$$\begin{array}{rrrrr} x_1 & x_2 & x_3 & x_4 & = \\ 47.126 & -74.150 & 14.222 & -64.372 & 27.304 \\ -29.312 & 54.332 & -37.998 & 70.028 & -80.476 \\ 32.470 & -46.984 & -4.7770 & -29.358 & -12.934 \\ -5.7490 & 17.257 & -30.887 & 37.842 & -66.824 \\ 8.9070 & -9.9090 & -11.888 & 2.8280 & -26.586 \end{array} \tag{14}$$

have the auxiliary matrix

$$\begin{array}{rrrrr} x_1 & x_2 & x_3 & x_4 & = \\ 47.126 & -1.5734 & 0.30179 & -1.3660 & 0.57938 \\ -29.312 & 8.2113 & -3.5502 & 3.6522 & -7.7324 \\ 32.470 & 4.1056 & 0 & 0 & 0 \\ -5.7490 & 8.2113 & 0 & 0 & 0 \\ 8.9070 & 4.1056 & 0 & 0 & 0 \end{array} \tag{15}$$

and the final matrix

$$\begin{array}{lrrr} & -x_3 & -x_4 & \\ x_1= & -5.2843 & 4.3805 & -11.5871 \\ x_2= & -3.5502 & 3.6522 & -7.7324 \end{array} \tag{16}$$

The first unknown is $x_1=5.2843x_3-4.3805x_4-11.5871$. Since the two labeled rows and columns of the auxiliary matrix are in consecutive order, the labels are omitted, the excess rows and columns being clearly indicated by the block of zeros. Here $R=2$.

m final checks can be obtained on each column of the final matrix by substituting the results in the given equations, the other columns in the final matrix and the corresponding columns in the given equations being omitted.

If a check column is desired, the procedure and rules given in section 5 are worded so as to apply to the present case. In addition it is true that in the auxiliary matrix those elements of the check column which lie in excess rows must vanish if the given equations are compatible.

If the given equations are homogeneous (right hand sides all zero), all columns of the final matrix arise from excess columns of the auxiliary matrix. In those systems ordinarily met in stability and vibration problems $m=n=R+l$.

Mathematical Appendix

1. Introduction

The purpose of this appendix is to give the proofs which established the method, but which were omitted in describing the technique. To avoid repetition, familiarity with this technique is assumed. The method was originally obtained by combining the various processes which compose Gauss's method, and adapting them for use with a computing machine; however, direct proofs of the induction type[9] (as opposed to proofs based on the evolution of the method) were found to be the only ones which are short, clear, and capable of disposing of unusual cases.

2. n Equations in n Unknowns

Denoting the given matrix, the auxiliary matrix, and the final matrix by $\|G_{ij}\|$, $\|A_{ij}\|$, and $\|F_{i1}\|$, respectively,[10] the method is contained in the following equations:

$$A_{ii}=G_{ii}-\sum_{k=1}^{i-1}A_{ik}A_{ki}$$

$$A_{ij}=G_{ij}-\sum_{k=1}^{j-1}A_{ik}A_{kj} \qquad \text{if } i>j \tag{17}$$

$$A_{ij}=\left[G_{ij}-\sum_{k=1}^{i-1}A_{ik}A_{kj}\right]\frac{1}{A_{ii}} \qquad \text{if } i<j$$

and

$$F_{i1}=A_{i,n+1}-\sum_{k=i+1}^{n}A_{ik}F_{k1} \tag{18}$$

where any Σ whose lower limit exceeds its upper vanishes. At present we consider the usual case where the diagonal elements $A_{ii}\neq 0$, in which case the final matrix has but one column.

It is true that *if in the auxiliary matrix the elements of the principal diagonal be replaced by 1's, and the elements below this diagonal be replaced by 0's, the resulting matrix is the augmented matrix of a set of equations entirely equivalent to the given set.*[10] More explicitly the set of equations having the augmented matrix

$$\left\|\begin{array}{lll} G_{11} & G_{12}\ldots & G_{1,n+1} \\ G_{21} & G_{22}\ldots & G_{2,n+1} \\ \ldots & \ldots & \ldots \\ G_{n1} & G_{n2}\ldots & G_{n,n+1} \end{array}\right\| \tag{19}$$

and that having the augmented matrix

$$\left\|\begin{array}{llllll} 1 & A_{12} & A_{13} & A_{14}\ldots & A_{1n} & A_{1,n+1} \\ 0 & 1 & A_{23} & A_{24}\ldots & A_{2n} & A_{2,n+1} \\ 0 & 0 & 1 & A_{34}\ldots & A_{3n} & A_{3,n+1} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & 0\ \ldots & 1 & A_{n,n+1} \end{array}\right\| \tag{20}$$

have the same solution. Since the solution of the equations corresponding to an augmented matrix is not altered if the rows are multiplied by any numbers, or if to any row there is added a linear combination of the other rows; the above statement will be proved by induction[9] if we note that the first row of (20) is $1/A_{11}$ times the first row of (19), and if we can show that the [i^{th} row of (20)] $=\frac{1}{A_{ii}}\{$[i^{th} row of (19)] $-A_{i1}$[1st row of (20)] $-A_{i2}$[2nd row of (20)] $-\ldots-A_{i,i-1}$[$(i-1)^{\text{th}}$ row of (20)]$\}$, or more explicitly

$$\left.\begin{array}{l} 1=\left[G_{ii}-\sum\limits_{k=1}^{i-1}A_{ik}A_{ki}\right]\dfrac{1}{A_{ii}} \\ 0=\left[G_{ij}-\sum\limits_{k=1}^{j-1}A_{ik}A_{kj}-A_{ij}\right]\dfrac{1}{A_{ii}}, \\ \qquad j=1,2,3,\ldots,i-1 \\ A_{ij}=\left[G_{ij}-\sum\limits_{k=1}^{i-1}A_{ik}A_{kj}\right]\dfrac{1}{A_{ii}} \\ \qquad j=i+1,i+2,i+3,\ldots,n+1 \end{array}\right\} \tag{21}$$

---

[8] These cases may also arise in evaluating a determinant. Should case 1 occur, the sign of the product of the diagonal elements must be reversed if the number of inversions of the row labels is odd; otherwise, left unaltered. Should case 2 occur, the value of the determinant is zero.

[9] A proof by *induction* establishes a relation for all positive integer values of n by verifying it for $n=1$ and then showing that if it is true for $n=1, 2, \ldots, i-1$, it is also true for $n=i$.

[10] The element in the i^{th} row and j^{th} column of the matrix $\|G_{ij}\|$ is G_{ij}. (See (3).) The *augmented matrix*, or given matrix, differs from the matrix of the coefficients of the unknowns only in that it has an additional column composed of the right hand sides of the equations. The usual matrix notation (double vertical bars) is used here, since we are concerned only with proofs.[9]

where any Σ whose upper limit is zero vanishes. But these equations follow directly from (17); hence the above statement is proved.

In a similar manner we can show that *the equations corresponding to the final matrix, whose augmented matrix is*

$$\left\|\begin{matrix} 1 & 0 & 0 \ldots 0 & F_{11} \\ 0 & 1 & 0 \ldots 0 & F_{21} \\ 0 & 0 & 1 \ldots 0 & F_{31} \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 \ldots 1 & F_{n1} \end{matrix}\right\| \qquad (22)$$

are equivalent to those of (20), for the n^{th} rows of (20) and (22) are identical ($F_{n1}=A_{n,n+1}$), and the [i^{th} row of (22)] = [i^{th} row of (23)] $-$ $A_{i,i+1}$[$(i+1)^{th}$ row of (22)] $-$ $A_{i,i+2}$$(i+2)^{th}$ row of (22)] $- \ldots -$ A_{in} [n^{th} row of (22)], as is expressed by the relation

$$F_{i1}=A_{i,n+1}-\sum_{k=i+1}^{n} A_{ik}F_{k1}$$

which agrees with (18). The equations corresponding to (22), which constitute the result given by the final matrix, are thus equivalent to (20) and hence (19); and therefore express the required solution.

3. Evaluation of Determinants

To show that the value of a determinant is equal to the product of the elements of the principal diagonal of the auxiliary matrix we note that although augmented matrices and determinants are alike in that any row can be increased by a linear combination of the other rows, they differ in that division of a row of an augmented matrix by any number gives an equivalent matrix, whereas division of a row of a determinant by a number divides the value of the determinant by that number. The procedure used in section 2 for augmented matrices can hence be repeated for determinants; however, since the i^{th} row is divided by the diagonal element A_{ii}, we must compensate by retaining A_{ii} as an external factor. Corresponding to (20) we thus have

$$\begin{vmatrix} G_{11} & G_{12} \ldots G_{1n} \\ G_{21} & G_{22} \ldots G_{2n} \\ \ldots & \ldots \\ G_{n1} & G_{n2} \ldots G_{nn} \end{vmatrix} =$$

$$\begin{vmatrix} 1 & A_{12} & A_{13} & A_{14} \ldots A_{1n} \\ 0 & 1 & A_{23} & A_{24} \ldots A_{2n} \\ 0 & 0 & 1 & A_{34} \ldots A_{3n} \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & 0 \ldots 1 \end{vmatrix} A_{11}A_{22}A_{33} \ldots A_{nn}$$

But the determinant on the right hand side is 1, as can be seen by developing it by minors with respect to the first column, second column, and so on; hence the value of the given determinant is $A_{11}A_{22}A_{33} \ldots A_{nn}$.

4. Symmetrical Equations and Determinants

We shall now show that in the symmetrical case where

$$G_{ij}=G_{ji}, \quad i, j \leq n \qquad (23)$$

elements of the auxiliary matrix which lie in symmetrical positions on opposite sides of the principal diagonal are related by the equation

$$A_{ij}=\frac{A_{ji}}{A_{ii}}, \quad i<j\leq n \qquad (24)$$

Using induction[1] we note from (17) and (23) that (24) is true for the first row and column of the auxiliary matrix. Assuming it true for the first $i-1$ rows and columns, it follows from (17), (23), and (24) that

$$A_{ji}=G_{ji}-\sum_{k=1}^{i-1} A_{jk}A_{ki}$$

$$=G_{ij}-\sum_{k=1}^{i-1}(A_{kk}A_{kj})\left(\frac{A_{ik}}{A_{kk}}\right)$$

$$=G_{ij}-\sum_{k=1}^{i-1} A_{ik}A_{kj}=A_{ij}A_{ii}, \; i<j\leq n$$

Hence (24) is also true for the i^{th} row and column.

5. Check Columns

If in a matrix the last column is the sum of the other columns, this fact evidently remains true if a row be multiplied by a constant, or if to any row be added a constant times another row. Since (20) can be obtained from (19), and (22) from (20) by just such operations, we see that if a check column is annexed to (19), the corresponding columns obtained in the auxiliary matrix and the final matrix are the sums of the columns of (20) and of (22), respectively; and hence have the required properties.

6. Complex Equations Whose Symmetrical Coefficients Are Conjugate

It is true that *if in the given equations each coefficient below the principal diagonal is the conjugate of the symmetrically placed coefficient above this diagonal, those on the diagonal being real; then in the auxiliary matrix the elements of the principal diagonal are all real, and every element above this diagonal is obtained by dividing the conjugate of the symmetrically opposite element below this diagonal by its diagonal element.*[2] It follows that complex equations of this type, which occur in certain least square processes, are far easier to solve than any other type of complex equations, because in determining the auxiliary matrix

To prove the above statement we must show that if

$$G_{ij}=\overline{G}_{ji}, \quad i, j\leq n \qquad (25)$$

in which case G_{ii} is real (since $G_{ii}=\overline{G}_{ii}$), bars indicating conjugates; then

$$\begin{aligned} &A_{ii} \text{ is real}, \quad i\leq n \\ &A_{ij}=\frac{\overline{A}_{ji}}{A_{ii}}, \quad i<j\leq n \end{aligned} \qquad (26)$$

Using induction[1] we note from (17) and (25) that (26) is true for the first row and column of the auxiliary matrix. Assuming it true for the first $i-1$ rows and columns, it follows that

$$A_{ii}=G_{ii}-\sum_{k=1}^{i-1} A_{ik}A_{ki}$$

$$A_{ii}=G_{ii}-\sum_{k=1}^{i-1}(A_{kk}\overline{A}_{ki})\left(\frac{\overline{A}_{ik}}{A_{kk}}\right)$$

$$=\overline{G}_{ii}-\sum_{k=1}^{i-1} \overline{A}_{ik}\overline{A}_{ki}=\overline{A}_{ii}, \quad i\leq n$$

since the conjugate of a sum of products is the sum of products of the conjugates. A_{ii} is therefore real, since $A_{ii}=\overline{A}_{ii}$. Again, noting (17), (25), and (26),

$$A_{ij}=\frac{1}{A_{ii}}\left[G_{ij}-\sum_{k=1}^{i-1} A_{ik}A_{kj}\right]$$

$$A_{ij}=\frac{1}{A_{ii}}\left[\overline{G}_{ji}-\sum_{k=1}^{i-1}(A_{kk}\overline{A}_{ki})\left(\frac{\overline{A}_{jk}}{A_{kk}}\right)\right]$$

$$=\frac{1}{A_{ii}}\left[\overline{G}_{ji}-\sum_{k=1}^{i-1} \overline{A}_{jk}\overline{A}_{ki}\right]=\frac{\overline{A}_{ji}}{A_{ii}}, \quad i<j\leq n$$

Hence (26) is also true for the i^{th} row and column.

7. m Equations in n Unknowns

The justification of the procedure described for m equations in n unknowns follows from the statements

A. If one column of a matrix is a linear combination of certain other columns, it remains that same combination if a row be multiplied by a constant, if to any row be added a constant times another row, or if such operations be repeated any number of times.

B. If after determining i rows and i columns of the auxiliary matrix, the remaining elements are determined by the relation

$$A_{pq}'=G_{pq}-\sum_{k=1}^{i} A_{pk}A_{kq}, \quad p, q>i \qquad (27)$$

(ordinarily used only in completing the $(i+1)^{th}$ column) instead of (1); the resulting matrix has the property that if in the first i columns the elements of the principal diagonal be replaced by 1's, and the elements below this diagonal, by 0's, thus

$$\left\|\begin{matrix} 1 & A_{12} & A_{13} \ldots A_{1i} & A_{1,i+1} & A_{1,i+2} & \ldots A_{1,n+1} \\ 0 & 1 & A_{23} \ldots A_{2i} & A_{2,i+1} & A_{2,i+2} & \ldots A_{2,n+1} \\ 0 & 0 & 1 \ldots A_{3i} & A_{3,i+1} & A_{3,i+2} & \ldots A_{3,n+1} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 \ldots 1 & A_{i,i+1} & A_{i,i+2} & \ldots A_{i,n+1} \\ 0 & 0 & 0 \ldots 0 & A_{i+1,i+1}' & A_{i+1,i+2}' & . A_{i+1,n+1}' \\ 0 & 0 & 0 \ldots 0 & A_{i+2,i+1}' & A_{i+2,i+2}' & . A_{i+2,n+1}' \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 \ldots 0 & A_{m,i+1}' & A_{m,i+2}' & . A_{m,n+1}' \end{matrix}\right\| \qquad (28)$$

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[1] Elements below the principal diagonal are obtained as by-products of calculations made in determining elements above this diagonal, the procedure being similar to that used with symmetrical equations.

[2] The divisions by the real diagonal elements can be performed directly as final divisions, no supplementary columns being required.

Self-Excitation of Induction Motors With Series Capacitors

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FELLOW AIEE

INDUCTION motors with series capacitors connected in the supply circuit can under certain conditions become unstable. Upon application of the voltage the rotor will come up to partial speed and continue to rotate at this reduced speed. This phenomenon should not be confused with ferro-resonance which is accompanied by violent wave distortion, abnormally large magnetizing currents, and fluctuations in voltage. Both of these phenomena were analyzed as applied to induction motors by Butler and Concordia.[1] It is the purpose of this paper to present further thoughts regarding the stability problem. A new approach is utilized which is much simpler and at the same time provides a clearer conception of the physical processes involved. Somewhat different conclusions result with the use of this analysis than with the analysis of others. In this analysis conventional induction motor nomenclature and symbols will be used as it is believed that induction motor engineers will better understand the problem by this procedure.

Simple Analysis of Problem

Currents in circuits consisting of a capacitor, inductor, and resistor in series may originate in two ways. First, they may be forced by an externally applied voltage in which case the frequency of the currents corresponds to that of the applied voltage. Second, they may flow in accordance with the natural frequency of the circuit. In the latter case, the current flows in response to some change in the circuit and gradually decays to zero, the rate being dependent to a large extent upon the resistance. The smaller the resistance the longer will the currents persist. If a negative resistance, such as may be obtained by certain combinations of vacuum tubes, is inserted into the circuit and is of such characteristic as to just equal the positive resistance, then currents of the natural frequency of the circuit flow continuously. This is the kind of condition that may exist in an induction motor in which series capacitors are connected in the stator circuit.

The upper circuit of figure 1 shows the conventional schematic diagram of an induction motor in which

x_s = leakage reactance of the stator
x_r = leakage reactance of the rotor
x_m = reactance of the magnetizing branch
r_s = resistance of stator
r_r = resistance of rotor
x_c = capacitive reactance of series capacitor
s = slip, expressed as a fraction of synchronous speed

All impedances are expressed in per unit quantities which are the same as per cent, except that unity is taken as a base instead of 100. The loss in the resistor $\frac{1-s}{s}r_r$ represents the shaft power.

The lower circuit is the same as the upper one, except that it is resonant for a frequency f times the system frequency. It is assumed that the source has zero impedance, and since it has no emf of resonant frequency, this point of the circuit is merely represented by a line of zero impedance. After connection of the motor to the system voltage, the speed increases until it reaches a value equal to that corresponding to the synchronous speed associated with the frequency f. Beyond this speed, its slip, s_1, with respect to the synchronous speed of the resonant circuit becomes negative, and with regard to this circuit the motor begins to function as an induction generator. Since s_1 is negative, then the resistor $(1-s_1)r_r/s_1$ is negative and the shaft power, which is represented by the loss in this resistor, is also negative. Thus, s_1 so adjusts itself that the negative resistor $(1-s_1)r_r/s_1$ just annuls the effect of the two other positive resistors r_s and r_r; the circuit will self excite and the currents of frequency f will flow continuously. The resistor $(1-s_1)r_r/s_1$ corresponds to the vacuum tubes mentioned previously. The only connection between these otherwise independent circuits is

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then each row of (28) differs from the corresponding row of the given matrix by a linear combination of the preceding rows of (28). The equations corresponding to (28) (considered as an augmented matrix) are thus equivalent to the original equations.

A is self-evident. The proof given with (20) and (21) is applicable to (28), and establishes B for the first i rows. For the p^{th} row, $p>i$, it is true that the [p^{th} row of (28)] = [p^{th} row of given matrix] $-A_{p1}$[1st row of (28)] $-A_{p2}$[2nd row of (28)] $-\ldots-$ A_{pi}[i^{th} row of (28)], or

$$0=G_{pq}-\sum_{k=1}^{q-1}A_{pk}A_{kq}-A_{pq}, \quad p>i,\ q\leq i$$

$$A_{pq}'=G_{pq}-\sum_{k=1}^{i}A_{pk}A_{kq}, \quad p>i,\ q>i$$

For here the first equation follows from the second of (17), and the second follows from (27); hence B is proved.

In solving a set of m equations in n unknowns two unusual cases may arise in determining the auxiliary matrix; however, case 1 requires no comment, and we lose nothing by supposing the equations to be written in such an order that this case does not occur. Turning to case 2, suppose that in completing the $(i+1)^{\text{th}}$ column of the auxiliary matrix the computed elements, which are identical with $A_{i+1,i+1}'$, $A_{i+2,i+1}'$, $\ldots$, $A_{m,i+1}'$, all vanish; then this column is evidently a linear combination of the first i columns of (28), whence it follows from A and B that the $(i+1)^{\text{th}}$ column of the *given* matrix is the same linear combination of the first i columns of that matrix. Were this excess $(i+1)^{\text{th}}$ column shifted to any other position to the right in the given matrix, and i or more rows and columns completed as usual in the auxiliary matrix before applying (27) to obtain a matrix similar to (28); then since the first i columns are the same as before, and since the same linear combination is involved, the corresponding excess column obtained is (with its zeros) identical with that obtained before. It is thus true that *each excess column in the auxiliary matrix is obtained in its final form when first calculated.*

Writing the excess columns together and just to the right of the R columns for the other unknowns, the finished auxiliary matrix differs from (28) only in that i is replaced by R. Since the last $m-R$ or more elements in each excess column in this matrix vanish, it follows from B that for compatibility the last $m-R$ elements of the $(n+1)^{\text{th}}$ column, which arises from the right hand sides of the equations, must also vanish; in which case the last $m-R$ elements of the check column (if one is used) must vanish due to A.

In determining the final matrix the excess columns are considered as transposed to the right hand side of the equations, which fact gives rise to minus signs in the column labels.

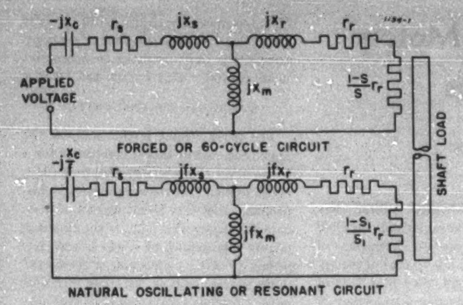


Figure 1. Schematic diagram of induction motor with series capacitor

the shaft. Power is taken by the shaft from the upper circuit and fed back into the lower circuit. The current flow in the lower circuit is limited only by the amount of power that can be drawn from the upper circuit at the particular speed necessary to make the resistance of the lower circuit zero. This power appears as a negative quantity in the branch which represents shaft loading and must equal the I^2R loss in the r_s and r_r branches of the resonating circuit.

The actual speed, p, of the rotor can be expressed in terms of both s and s_1. Thus, the speed is

$$p = 1 - s = (1 - s_1)f \qquad (1)$$

from which

$$s = 1 - (1 - s_1)f \qquad (2)$$

Quantitative Relations

Consider next the conditions which must be satisfied to permit natural oscillations. The resonant circuit of figure 1 must have an impedance equal to zero as viewed from any point in the network. It will be found more convenient for the present purpose to view the circuit from a point in the rotor circuit. For the sake of simplicity in deriving the following expressions f will first be assumed equal to unity. It will be reinserted in the final results by dividing x_c everywhere it appears by f and multiplying the inductive reactances by f. In this paper all reactances, both inductive and capacitive, will be given in terms of their values at system frequency. The two resistances in the rotor can be combined into the single resistor r_r/s_1. Upon paralleling the condensive branch with x_m and adding to the rotor impedances, the impedance as viewed from a point in the rotor becomes

$$\frac{jx_m[r_s + j(x_s - x_c)]}{r_s + j(x_m + x_s - x_c)} + \frac{r_r}{s_1} + jx_r = 0 \qquad (3)$$

or

$$\frac{jx_m[r_s + j(x_s - x_c)][r_s - j(x_m + x_s - x_c)]}{r_s^2 + (x_m + x_s - x_c)^2} + \frac{r_r}{s_1} + jx_r = 0 \qquad (4)$$

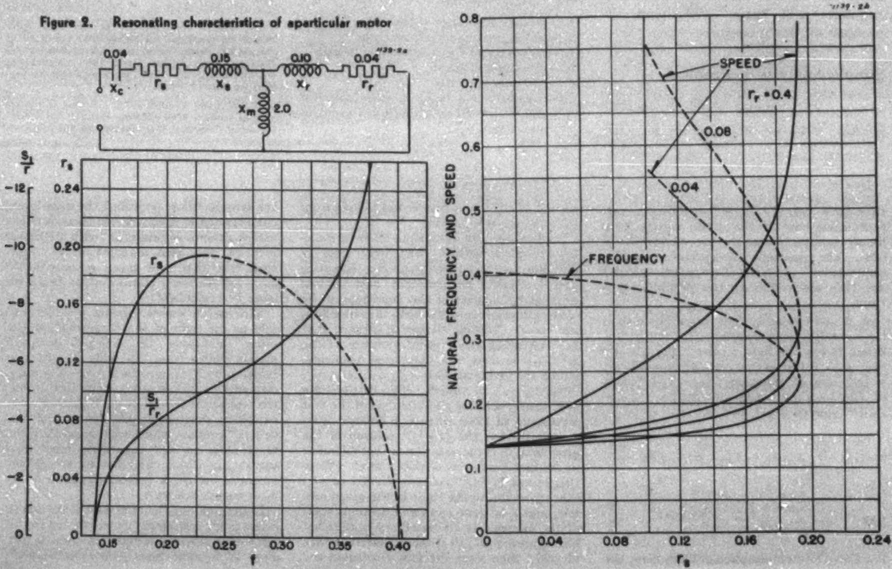
The condition that the circuit be self-exciting is satisfied when both the real and the imaginary components of the above impedance are individually equal to zero. Equating the imaginary component to zero, there is obtained

$$x_m r_s^2 + x_m(x_s - x_c)(x_m + x_s - x_c) + x_r r_s^2 + x_r(x_m + x_s - x_c)^2 = 0$$

from which

$$r_s^2 = -(x_m + x_s - x_c)\left(x_s + \frac{x_r x_m}{x_m + x_r} - x_c\right) \qquad (5)$$

Figure 2. Resonating characteristics of aparticular motor



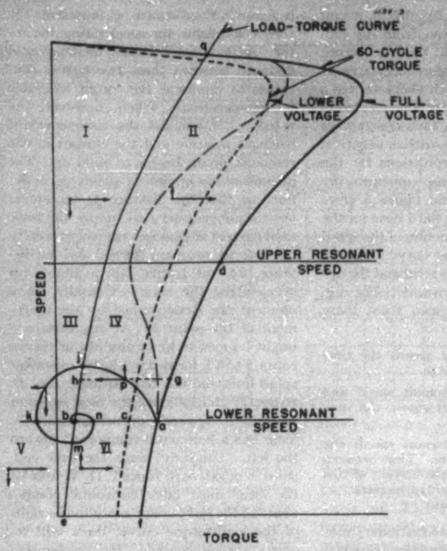


Figure 3. Speed-torque characteristic to show that the lower resonant speed is stable

and upon equating the real components to zero, there is obtained

$$\frac{s_1}{r_r} = -\frac{r_s^2+(x_m+x_s-x_c)^2}{x_m^2 r_s} \tag{6}$$

After squaring (6) and inserting r_s from equation 5, there results

$$\left(\frac{s_1}{r_r}\right)^2 = -\frac{x_m+x_s-x_c}{(x_m+x_r)^2\left(x_s+\dfrac{x_r x_m}{x_m+x_r}-x_c\right)} \tag{7}$$

Upon reinserting the f term, equations 5 and 7 become

$$r_s^2 = -\frac{1}{f^2}[f^2(x_m+x_s)-x_c] \times \left[f^2\left(x_s+\frac{x_r x_m}{x_m+x_r}\right)-x_c\right] \tag{8}$$

$$\left(\frac{s_1}{r_r}\right)^2 = -\frac{f^2(x_m+x_s)-x_c}{f^2(x_m+x_r)^2\left[f^2\left(x_s+\dfrac{x_r x_m}{x_m+x_r}\right)-x_c\right]} \tag{9}$$

The quantity (x_m+x_s) represents the exciting reactance of the motor and will be designated x_M; $x_s+(x_r x_m)/(x_m+x_r)$ is the blocked rotor reactance for r_r equal to zero and will be designated by x_B. Equations 8 and 9 then become

$$r_s^2 = -\frac{1}{f^2}[f^2 x_M-x_c][f^2 x_B-x_c] \tag{10}$$

$$\left(\frac{s_1}{r_r}\right)^2 = -\frac{[f^2 x_M-x_c]}{f^2(x_m+x_r)^2[f^2 x_B-x_c]} \tag{11}$$

Extraneous roots have been introduced in the manipulation of the equations, but consideration of equation 6 shows that s_1 will be negative. These two equations represent the two conditions which must be satisfied to produce self excitation. For a given r_s, the value of f which satisfies equation 10 determines the resonating frequency. Equation 11 determines the slip corresponding thereto and consequently the speed.

Figure 2 shows the general resonating characteristics of an induction motor with a series capacitor for a particular case. The upper curves represent the solutions of equations 10 and 11 plotted as a function of f. The lower curves were obtained from the upper ones and through the use of equation 2. They show how f and s vary with the stator and rotor resistors. For $r_s=0$ two resonant points occur, the lower one representing the frequency at which the capacitor resonates with the magnetizing reactance and the upper one the frequency at which the capacitor resonates with the magnetizing reactance and the upper one the frequency at which the capacitor resonates with the blocked rotor reactance. For other values of r_s two resonating points likewise exist, but, as will be shown presently, only the lower point is a stable operating point. The unusually low magnetizing impedance was chosen to take care of saturation.

Mechanism of Pull-in and Operating Points

As in any resonating circuit, the current increases until the actual losses in the circuit just equal the negative losses produced by the negative resistance. In the present case the losses are supplied through the shaft and thus the current is limited solely by the ability of the 60-cycle circuit to supply power to the resonant circuit at the speed corresponding to the resonant frequency.

In figure 3 are given the shaft torque characteristics of a motor with a series capacitor as determined from the conventional 60-cycle circuit. Likewise is given an assumed load-torque curve and two horizontal lines which indicate the speeds

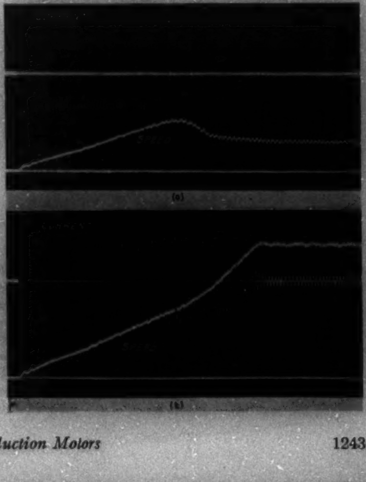


Figure 4. Oscillograms showing rotor speed and stator current as voltage is suddenly applied to a motor with series capacitor

(a) Low applied voltage—rotor stalling at resonant speed

(b) Higher applied voltage—rotor passing through resonant speed corresponding to the upper and lower resonant frequencies. On the lower resonant speed the distance *oa* is the total shaft torque. Of this torque *ob* is required to supply the load requirements. The excess *ab* is the torque which when converted to power at the particular speed represents the power input into the resonant circuit. If the applied voltage were lowered so that the 60-cycle torque curve were reduced to that shown by the dotted line, then the power into the resonant circuit would be reduced to that corresponding to the torque *bc*. Similarly, if the load torque characteristic were increased there would be less power available for circulation of current in the resonant circuit.

The right hand portion of the upper curve of figure 4*a* represents the steady-state stator current, which can be seen to consist of two components. One component has the same frequency as that of the 60-cycle applied voltage and the other a lower frequency corresponding to that of the natural frequency of oscillation.

The point *d* of figure 3, where the speed-torque curve intersects the upper resonant speed line, is the other solution from a curve, such as shown in figure 2. It is the purpose of this discussion to show that the point, *a*, represents a stable operating point and the point, *d*, an unstable point. For any particular speed, an induction motor with a series capacitor has three modes of oscillation. By this is meant that the transient resulting from any sudden change is composed of three terms. Two of these terms always have a negative time constant, which characterizes a condition wherein the components always tend to decrease. The time constant of the third term may be positive, negative, or infinitely large. Expressions for the calculation of these time constants are given in appendix I and figure 5 gives numerical values for the third term for the circuit of figure 2 as a function of the speed of the motor. The lower curve gives the natural frequency of this term and the upper curve the time constant. The significant facts to be drawn from these curves are that:

1. At the two resonant speeds the time constants are infinitely large.
2. Below the lower resonant speed and above the upper resonant speed, the time constants are negative.
3. Between the two resonant speeds the time constants are positive. Thus between the two resonant speeds the current of this term of the transient tends to increase; at all other speeds, to decrease.

If voltage is applied to a stationary motor for which the characteristics are given in figure 3, the distance, *ef*, represents the torque available for accelerating the rotor. As its speed increases, the horizontal distance between these two curves continues to represent the torque available for acceleration. When, however, the point, *a*, is reached the time constant becomes positive and the current in the resonant circuit begins to build up. The instantaneous operating point, *p*, is defined so that the distance *pg* represents the torque necessary to supply the resonant current at that instant (the power input into the resonant circuit) and the distance, *hp*, the torque still available for accelerating the rotor. Consider for a moment the locus formed by the movement of the point *p*. At any instant *p* might happen to lie in any one of the six zones, I to VI, formed by the two resonant speed lines and the load torque curve. In zones III and IV since the time constant is positive, *pg* tends to increase and the locus has a horizontal component toward the left. Similarly, since the time constant is negative in zones I, II, V, and VI the locus must have horizontal components to the right. For zones to the right of the load-torque curve, there still remains torque available for acceleration

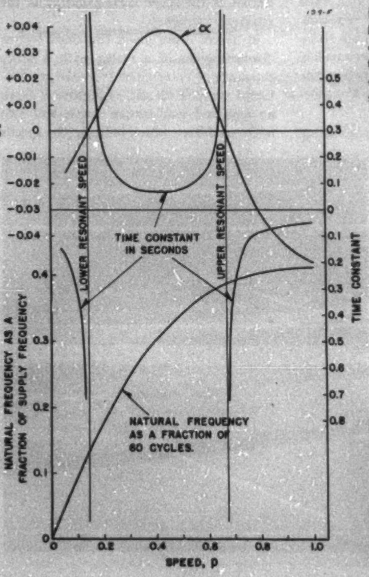


Figure 5. Natural frequency and time constants of induction motor having constants of those shown in figure 2

$r_r = 0.04$

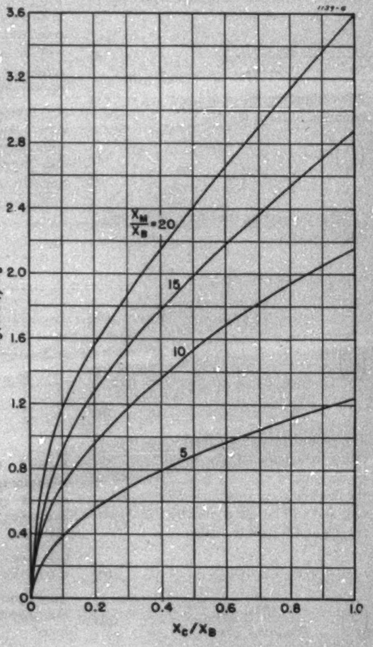


Figure 6. Maximum series resistance in stator, $r_{s(max.)}$, above which circuit will not resonate expressed as a function of locked rotor reactance, x_B, and no-load reactance, x_M

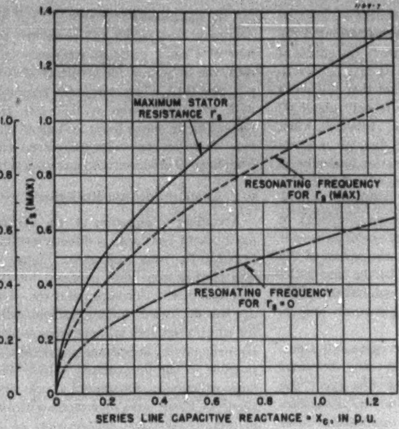


Figure 7. Effect of series capacitance for a specific case

$x_M=3.24$
$x_B=0.40$

of the rotor. Thus zones II, IV, and VI must have upward components of motion of *p*. Similarly, points to the left of the load-torque curve tend to decelerate and have downward vertical components. The point *p* moves from *a* with an upward and leftward motion, the upward component becoming smaller and smaller as the load-torque is approached. To the left of *j* (in zone III) the locus has a downward and leftward component, the leftward component decreasing with increasing time constant as the lower resonant speed line is approached at *k*. Following this reasoning it may be seen that the locus follows the spiral and reaches a stable point at *b*.

The foregoing assumed that the inertia of the rotating part was quite high or that the power available for acceleration was low, so that the current had sufficient time to build up. If on the other hand the above conditions are not fulfilled, the locus may take a path, such as indicated by the dashed line, finally attaining the stable operating point *q*.

Note that in neither case did the motor tend to operate at point *d*, the point for which the curves of figure 2 indicated a solution in the dotted portion of the graphs. If the upper resonant speed is greater than 1.0, the rotor must lock in at the lower speed for all cases.

Verification of the foregoing theory is presented in figure 4 which shows the rotor speed and stator current for the case (*a*) in which the rotor pulls in at the resonant speed and (*b*) in which it passes through this speed and finally attains a speed corresponding to normal operating conditions with a small slip. In both cases the motor was operating at no load, the only difference in the two cases being that the applied voltage for (*b*) was about 3 per cent higher than for (*a*). In the former case it can be seen that the speed overshoots the final value and finally settles down to the resonant speed. It differs from the story depicted in figure 3 in that the resonant current changes so rapidly that only the components in zones IV and III are of importance. The oscillogram also shows that the resonant frequency component of the stator current is a maximum a short time after the speed is a maximum, which conforms to the expectations of the foregoing theory. As the speed in figure 4*b* reaches the resonant speed, the resonant frequency component of the stator current begins to build up and then decreases again as the speed exceeds the upper resonant speed.

General Results

By slight transformation, equation 10 can be rewritten as

$$\frac{r_s}{x_c}=\frac{1}{f}\sqrt{\left[f^2\frac{x_M}{x_c}-1\right]\left[1-f^2\frac{x_B}{x_c}\right]} \qquad (12)$$

For specific values of x_M/x_c and x_B/x_c curves of r_s/x_c similar to that of r_s in figure 2 can be plotted as functions of r. It is impossible for the circuit to resonate for x_s/x_c in excess of the maxima of these curves. These maxima, $r_{s(\text{max.})}/x_B$, are plotted in figure 6 as a function of x_c/x_B and x_M/x_B. Both x_B and x_M include the external series reactance of the motor.

Figure 7 gives the maxima of r_s for typical values of x_M and x_B. In addition, the resonant frequency, f, corresponding to $r_{s(\text{max.})}$ is also given together with f for $r_s=0$. For intermediate values of r_s, f must lie between these two curves.

As pointed out by Butler and Concordia for other cases of instability, this particular type also can be alleviated by paralleling the capacitor with a resistor. The solution for this condition is given in the appendix for the special case in which there is no series resistance in the stator. The results of calculations employing this method are given in figure 8, which gives the minimum resistance necessary to suppress resonance. Of course, as series resistance is added to the circuit, the minimum parallel resistance increases. When the value of series resistance has reached the magnitude given in figure 6, no parallel resistance is necessary. This corrective method is particularly good because at system frequency so little of the current is by-passed but at the lower frequency of resonance sufficient current is by-passed to prevent resonance effects.

Utilizing the same general approach as that given for the development of equations 8 and 9 or that in appendix II, it is possible to calculate the parallel resistor required to suppress resonance for any particular value of series resistance. The same method can be extended further to cover any combination of resistors, reactors, and capacitors.

Test Results

In order to verify still further the correctness of the foregoing theory, tests were made upon a 220 volt, 7½ hp, three-

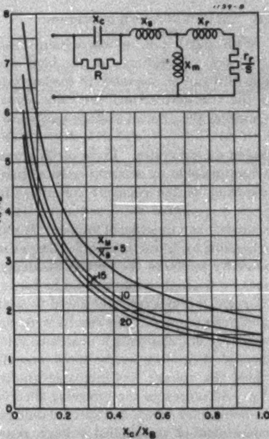


Figure 8. Minimum parallel resistor, $R_{\text{min.}}$, below which an induction motor with no series stator resistance will not resonate

phase, wound rotor induction motor. Power was supplied from a 100 kva, 2,200 volt generator. The generating voltage was stepped down from 2,200 to 220 volts by means of a 15 kw, three-phase transformer. Capacitors were connected in series in the 220 volt side. It was found that conditions of ferro-resonance were easily set up for this condition. Resistance representative of line resistance was then inserted in the 2,200 volt side of the circuit sufficient to suppress ferro-resonance. Additional resistance was at times placed in series in the low voltage side.

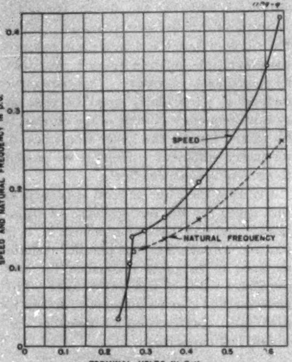


Figure 9. Variation of speed and natural frequency with terminal voltage

(a) Changing Applied Voltage

With 0.021 p.u. of capacitance and 0.071 p.u. of resistance in the high side and 0.16 p.u. of reactance in the low side of the transformer and the rotor short-circuited, the applied voltage was varied with no load on the shaft except an unloaded d-c generator. From oscillograms it was possible to determine the natural frequency of the resonant currents. In figure 9 is plotted the speed and natural frequency as a function of voltage. It was impossible to obtain the data for the entire range by applying the voltage suddenly, since for the higher voltages, the motor accelerated through resonant speed to near synchronous speed. The results were obtained by first applying a moderate voltage and then after steady speed was attained the voltage was increased gradually. According to the foregoing theory, the natural frequency and speed should be independent of the applied voltage, resonant current being limited only by the power input into the shaft for the particular speed and terminal voltage as determined by the system frequency characteristics of the motor. The variations in the curves of figure 9 are due to the effects of saturation. This may at first appear illogical in view of the fact that the maximum terminal voltage is only about 60 per cent of rated voltage. However, when it is recalled that the flux for a given amplitude of voltage varies inversely as the frequency, it may be seen that the influence of the resonant frequency voltage is very great. In addition, the effect of the resonant frequency voltage is not measured by the root-mean-square of the combined voltages but by the sum of the maximum values, the lower frequency being given greater weight inversely proportional to its frequency.

The break in the speed curve at 0.13 p.u. synchronous speed represents the lower resonant speed. Below this point the speed is determined purely by the nature of the shaft load; the oscillograms show no evidence of resonant frequency currents. The minimum frequency at which currents were discernible was 0.12 p.u.

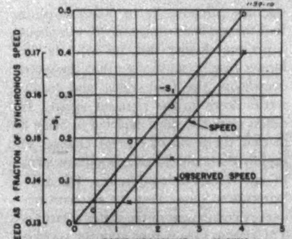


Figure 10. Effect of changing rotor resistance

In figure 9, as the applied voltage is increased the magnetizing impedance becomes lower. Likewise, the ratio x_m+x_B decreases quite rapidly but x_c/x_B increases but slightly. Thus, the limiting values of r_s, as determined from figure 6, decrease. Eventually, therefore, as the applied voltage continues to increase the resistance in the stator finally becomes greater than the limiting value corresponding to the changed constants. At this point the slope of the speed curve becomes infinitely large and the speed increases to the normal load speed near synchronous speed and the resonance current ceases to flow. This corresponds to the observed facts.

For a given terminal voltage, as the shaft load is increased, less power remains available to circulate resonant frequency currents. Thus to check the effect of saturation the circuit was left the same but the shaft load was increased. For the maximum load which still permitted the flow of a small resonant frequency current, the frequency for a voltage of 0.50 p.u. was 0.14. This compares favorably with the above value of 0.12 p.u. frequency at the much lower voltage of 0.34 p.u. Fifty per cent voltage under no load from figure 9 produces a frequency of 0.19.

(b) Changing Rotor Resistance

According to equation 11, the slip for fixed circuit constants is directly proportional to the rotor resistance. Figure 10 was plotted from data obtained for the $7^1/_2$ hp motor. For each point the voltage was decreased to the minimum value for which resonance still persisted so that the magnetizing impedance was as close as possible to the unsaturated value. The data indicates a substantial check with theory. These tests were made with capacitors of 29 ohms and resistors of 44 ohms in the high side of the transformer.

(c) Changing Capacitors

With a 44 ohm resistor in the high side of the transformer the capacitors were changed with the results upon the natural frequency shown in figure 11. Calculations indicated that the frequency should vary approximately as the square root of x_c. These tests verify this relation.

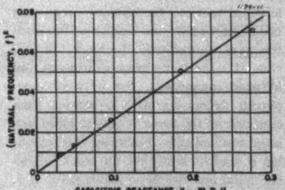


Figure 11. Effect of capacitive reactance upon natural frequency for a particular case

Appendix I. Generalized Angular Velocity of Induction Motor With Series Capacitor

The same general procedure will be followed as that used by Lyon[2] and Rüdenberg.[3] Figure 12 shows the schematic diagram of the induction motor. The symbols are self-explanatory. The instantaneous current, i_s, in the stator is

$$
\begin{aligned}
i_s &= \text{real part } [I_s\epsilon^{mt}] \\
&= \text{real part } [I_s\epsilon^{(\alpha+jf)t}] \qquad \text{(A1)}
\end{aligned}
$$

in which I_s is complex and m is the generalized angular velocity, a quantity whose real component determines the variation in amplitude with time and whose imaginary component determines the angular velocity. In the rotor the current will be designated by

$$i_r = \text{real part } [I_r \epsilon^{(m-jp)t}] \quad \text{(A2)}$$

For the stator the equation can be written

$$(r_s + ml_s)I_s + \frac{1}{mC}I_s + mM(I_s+I_r) = 0 \quad \text{(A3)}$$

and for the rotor

$$[r_r + (m-jp)l_r]I_r + (m-jp)\times M(I_s+I_r) = 0 \quad \text{(A4)}$$

Collecting terms

$$[mCr_s + m^2C(l_s+M)+1]I_s + m^2CMI_r = 0 \quad \text{(A5)}$$

$$(m-jp)MI_s + [r_r + (m-jp)\times(l_r+M)]I_r = 0 \quad \text{(A6)}$$

Solving equations $A3$ and $A4$ for m, there results

$$[(l_s+M)(l_r+M) - M^2]Cm^3 + [r_r(l_s+M) + r_s(l_r+M) - jp\{(l_s+M)(l_r+M) - M^2\}]Cm^2 + [r_r r_s C + (l_r+M) - jpr_sC(l_r+M)]m + [r_r - jp(l_r+M)] = 0 \quad \text{(A7)}$$

Now let ω be equal to 2π times system frequency and

$$l_s = \frac{x_s}{\omega} \quad \text{(A8)}$$

$$l_r = \frac{x_r}{\omega} \quad \text{(A9)}$$

$$M = \frac{x_m}{\omega} \quad \text{(A10)}$$

$$C = \frac{1}{\omega x_c} \quad \text{(A11)}$$

p will be expressed as a fraction of ω. Then Equation $A7$ becomes

$$[(x_s+x_m)(x_r+x_r) - x_m^2]\left(\frac{m}{\omega}\right)^3 + [r_r(x_s+x_m) + r_s(x_r+x_m) - jp\{(x_s+x_m)(x_r+x_m) - x_m^2\}]\times \left(\frac{m}{\omega}\right)^2 + [r_r r_s + (x_r+x_m)x_c - jpr_s(x_r+x_m)]\times \frac{m}{\omega} + [r_r - jp(x_r+x_m)]x_c = 0 \quad \text{(A12)}$$

Figure 5 gives the solution of the significant root of this equation for particular constants as a function of p. The other two roots of this equation are relatively unimportant because their real parts are negative. Letting

$$m = \alpha + jf = \frac{1}{T} + jf \quad \text{(A13)}$$

it can be seen from figure 4 that there are two rotor speeds for which α is zero ($T = \infty$). The natural frequencies, f, corresponding to these two points are the values of f found from a solution of equation 8, and the speeds, p, are the corresponding values calculated from the slip s_1 of equation 9.

Appendix II. Resistance Shunted Capacitor

If the resonant frequency, f, is unity, then considering the diagram from figure 8 the impedance of the resistor, R, and capacitance, x_c, in series with x_s is

$$\frac{Rx_c^2 - jR^2x_c}{R^2+x_c^2} + jx_s$$

Upon paralleling this impedance with jx_m and taking $r_r + jx_r$ in series, there is obtained

$$\frac{jx_m\left\{\frac{Rx_c^2}{R^2+x_c^2} + j\left[x_s - \frac{R^2x_c}{R^2+x_c^2}\right]\right\}}{\frac{Rx_c^2}{R^2+x_c^2} + j\left[x_s + x_m - \frac{R^2x_c}{R^2+x_c^2}\right]} + r_r + jx_r = 0$$

Equating the imaginary component to zero

$$x_m\left\{\left(\frac{Rx_c^2}{R^2+x_c^2}\right)^2 + \left[x_s - \frac{R^2x_c}{R^2+x_c^2}\right]\times\left[x_s + x_m - \frac{R^2x_c}{R^2+x_c^2}\right]\right\} + x_r\left\{\left(\frac{Rx_c^2}{R^2+x_c^2}\right)^2 + \left(x_s + x_m - \frac{R^2x_c}{R^2+x_c^2}\right)^2\right\} = 0 \quad \text{(B1)}$$

This equation reduces to

$$(x_c^2+R^2)\left(\frac{Rx_c}{R^2+x_c^2}\right)^2 - \left[(x_s+x_m) + \left(x_s + \frac{x_rx_m}{x_m+x_r}\right)\right]\frac{R^2x_c}{R^2+x_c^2} + (x_s+x_m)\times\left(x_s + \frac{x_rx_m}{x_m+x_r}\right) = 0 \quad \text{(B2)}$$

Letting

$$x_s + x_m = x_M \quad \text{(B3)}$$

$$x_s + \frac{x_rx_m}{x_m+x_r} = x_B \quad \text{(B4)}$$

then the above equation becomes

$$(x_c^2+R^2)\left[\frac{Rx_c}{R^2+x_c^2}\right]^2 - (x_M+x_B)\frac{R^2x_c}{R^2+x_c^2} + x_Mx_M = 0 \quad \text{(B5)}$$

This can be further reduced to the following

$$\left[1 - \frac{x_M+x_B}{x_c} + \frac{x_Mx_B}{x_c^2}\right]\left(\frac{R}{x_c}\right)^4 + \left[1 - \frac{x_M+x_B}{x_c} + \frac{2x_Mx_B}{x_c^2}\right]\left(\frac{R}{x_c}\right)^2 + \frac{x_Mx_B}{x_c^2} = 0$$

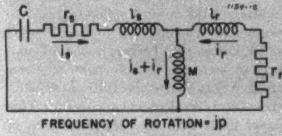


Figure 12. Schematic diagram of induction motor with series capacitor

Upon multiplying all inductive reactances by f and dividing x_c everywhere by f, there results

$$\left[1 - f^2\frac{x_M+x_B}{x_c} + f^4\frac{x_Mx_B}{x_cx_c}\right]\left(\frac{R}{x_c}\right)^4 + \frac{1}{f^2}\times\left[1 - f^2\frac{x_M+x_B}{x_c} + 2f^4\frac{x_M}{x_c}\frac{x_B}{x_c}\right]\left(\frac{R}{x_c}\right)^2 + \frac{x_M}{x_c}\frac{x_B}{x_c} = 0 \quad \text{(B6)}$$

This equation provides the relation from which the results of figure 8 were obtained.

References

1. ANALYSIS OF SERIES CAPACITOR APPLICATION PROBLEMS, J. W. Butler and C. Concordia. AIEE TRANSACTIONS, volume 56, 1937 (August section), page 975.

2. TRANSIENT CONDITIONS IN ELECTRIC MACHINERY, W. V. Lyon. AIEE TRANSACTIONS, volume 42, 1923, pages 157–76.

3. ELECTRISCHE SCHALTVORGANGE (a book), R. Rüdenberg. Julius Springer, Berlin, 1926, page 78.

Lightning Strokes in Field and Laboratory—III

P. L. BELLASCHI
FELLOW AIEE

LIGHTNING research is classified here for convenience into three main divisions. First, there are the meteorological and physical processes which give rise to the lightning stroke proper. Second, we have the characteristics of the lightning stroke as these appear at the point of incidence of the stroke to electric lines, to protective systems, and to earth. And third, comes that branch of the art connected with the development of these characteristics over electric circuits and their effects on electrical apparatus. This paper deals with the second division of lightning research. It is essentially a continuation of a previous report[1] and presents the results of investigations carried on particularly through 1937 to 1940.

The first part of the paper covers data and findings which pertain to the long-duration low-current components of lightning. The relative frequency of occurrence of these long-duration components is discussed. From these studies, we find that the electric charge of lightning discharges ranges from a few to 200 coulombs. Higher values are not entirely precluded but they would be rare. The second part of the paper refers to the type of component characteristics for lightning-stroke discharge prevalent to earth and to electric systems (figure 20).

The method of analysis in these investigations consists in comparing the physical effects on objects tested in the laboratory to the field data on similar objects that have been hit by lightning, such as lightning rods, ground wires, conductors of power and communication lines, coaxial cables, metal sheets, and other objects. The degree, nature, and amount of fusion on the metal is determined by ordinary measurements as well as by means of macrographs and micrographs of the objects affected.

Investigations on Long-Duration Lightning Currents

SIXTY-CYCLE FUSION TESTS

In some of the early laboratory investigations because of convenience in testing, sixty-cycle currents were used to correlate the fusion effects of the long-duration components in lightning discharges. The test arrangement and oscillograms typical of these tests are given in figures 1*A* and 2. Table I summarizes the more important data from these tests and figures 3 and 5 illustrate fusion effects obtained on electrode terminals. The amounts of fused metal in the tabulation correspond to a range which is representative of the fusion observed on similar objects that have been hit by lightning. Analysis of these data and of the physical processes involved will indicate that these fusion effects provide a practical and effective method to study lightning-stroke discharges.

Microscopic examinations of the wires and rods at the fused ends were made. As indicated by changes in the original grain structure of the metal, the depth of heat penetration is surprisingly small in every case. For instance, for the copper rod in figure 4, the portion which melted is shown by coarse columnar grains in contrast to the fine equiaxed original grains. The depth of heat penetration amounts to 0.075 inch (1.9 mm.). Micrographs of brass rods show an extremely small heat penetration due probably to the "boiling off" of the fused metal at the tip, the total depth measuring 0.0125 inch (0.33 mm). The wires and rods in this investigation had equiaxed grains, representative of annealed material. Differences in the original grain structure of wires may bring about different results. Even so, the visible depth of heat effects would probably not be much greater than observed here. For example, in a microscopic examination of cold-drawn copper conductors fused by lightning,[2] a depth of heat penetration less than one mm was found.

The physical factors that determine the amount of the metal fused become apparent from a consideration of the energy balance in the fusion process. Table IV presents data illustrating briefly the results of analyses that were made. In general, the energy absorbed from the arc is consumed largely in fusing the metal. This fact is also borne out by the limited penetration of the heat into the body of the metal. Where the conditions are such that the metal being fused does not progress at a rate sufficiently rapid to reabsorb the heat conducted away, then an appreciable proportion of the arc energy input goes simultaneously into conduction. For instance, in sample *K*, the bulk of the arc energy is absorbed in fusing the metal; whereas in sample *A*, a greater proportion is lost in conduction. Of interest, but of secondary importance here, are the radiation, thermal conduction through the gas, i^2R in metal, etc.

On the supply side, the energy developed from the arc is the factor of primary importance. Physicists have long studied the cathode and anode drop at the root of the arc,[3,4,5] but due to the complex phenomena these quantities are still largely a matter of experimental determination in each case and problem. For the practical purposes of this investigation, for a copper arc an energy input at the electrode of six to eight watt-seconds per coulomb appears reasonably substantiated from the good agreement indicated in the energy balance (table IV). About the same relative amount of energy input holds for other common metals as brass, aluminum, and iron. According to some investigators, the amount of fusion can vary with the electrode polarity, while others[6,7] have established by actual measurements about equal distribution for the two polarities. All considered, the results reported in this paper would indicate about equal energy input at the electrode for either polarity. It should be noted also that the shape of the electrode may influence the amount of melting. For instance, if there is an opportunity for the arc terminal to wander about an electrode, the melting may be reduced.

IMPULSE FUSION TESTS

These tests were made with the lightning-stroke generator[1] which is shown schematically in figure 1*B*. The generator can be operated to develop the component parts of the lightning-stroke discharge, either individually or combined.

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[1] For all numbered references, see list at end of paper.

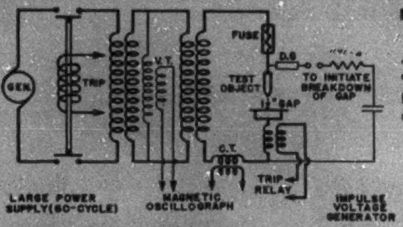
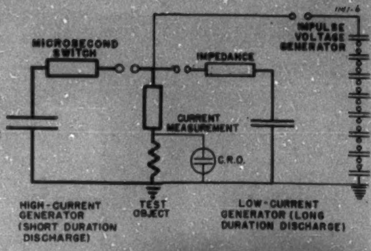

GEN.
TRIP
V.T.
FUSE
D.G.
TEST OBJECT
1" GAP
C.T.
TRIP RELAY
TO INITIATE BREAKDOWN OF GAP
LARGE POWER SUPPLY (60-CYCLE)
MAGNETIC OSCILLOGRAPH
IMPULSE VOLTAGE GENERATOR

Figure 1. Test arrangement

A (left). 60-cycle circuit

B (right). Lightning-stroke generator circuit


MICROSECOND SWITCH
IMPEDANCE
IMPULSE VOLTAGE GENERATOR
CURRENT MEASUREMENT
C.R.O.
HIGH-CURRENT GENERATOR (SHORT DURATION DISCHARGE)
TEST OBJECT
LOW-CURRENT GENERATOR (LONG DURATION DISCHARGE)

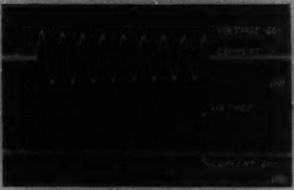


Figure 2. Sixty-cycle oscillograms

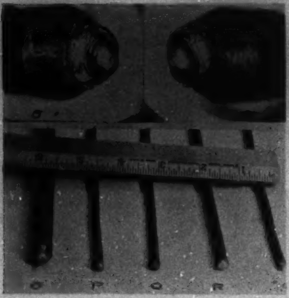


Figure 3. Fusion on copper (O, P, Q, S) and brass (R) rods with sixty-cycle tests

Discharge 165 coulombs (*O, P, Q, R*) and 22 coulombs (*S*). Top insert shows macrograph of rod *O*

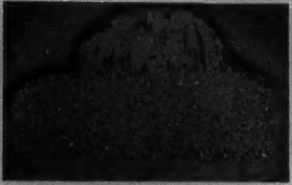


Figure 4. Micrograph of fusion on rod O (20 diameters)

First the results of the tests with the long-duration currents alone are presented. Typical oscillograms of these currents are given in figure 6. In figure 8, we have the discharge of a 2,000 ampere, 0.0023 second duration, unidirectional current between a copper rod and an aluminum plate. Table II summarizes some of the more important data obtained. In addition to the general scope of this investigation, individual samples were tested with the specific purpose of comparing and correlating the fusion effects with identically similar electrode objects hit by lightning.

The two sets of data in tables I and II substantiate each other quite well. It is apparent again that the conduction of heat to the body of the electrode assumes in certain cases some importance. For instance, for the same impulse discharge practically twice the volume of metal is fused from the wire of sample *S*9 than on the sheath of sample *S*7.

As to the probable error of the measurements, the electrical quantities are accurate within about four or five per cent. A greater error naturally is expected in determining the metal fused. This question was looked into from various angles. For example, the metal fused in the six samples, *S*10 to *S*17 in figure 9, was calculated by the usual caliper measurements. In addition to this, each sample was sectionalized and from the micrographs of the sections, the volume of metal fused was determined. The comparison between the two shows an agreement on the average within twenty per cent in spite of the fact that the volume of the metal fused in the last three samples is very small. In general, the probable error in determining the amount of the metal fused is not likely to exceed 20 to 25 per cent for the individual cases. When the data of a group are considered together, the mean error is even much less.

The "burning" effect produced on metals by the high current of lightning strokes is confined to a thin top layer at the surface of the metal. Table III presents typical results on samples that were

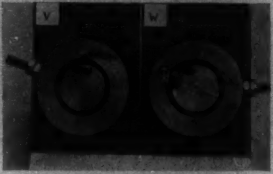


Figure 5. Fusion on tower gaps with sixty-cycle tests

Charge 104 coulombs (*V*) and 147 coulombs (*W*)

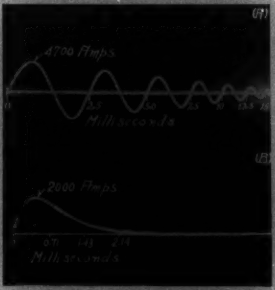


Figure 6. Low-current impulse oscillograms

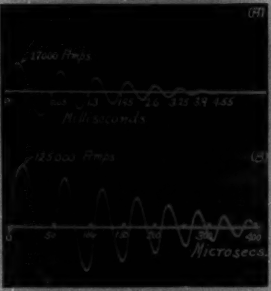


Figure 7. High-current impulse oscillograms

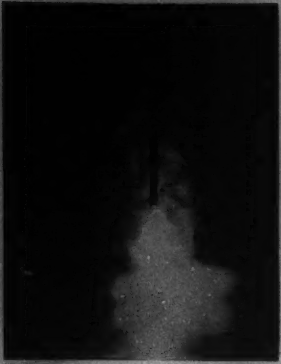


Figure 8. Low-current impulse discharge from rod to plate

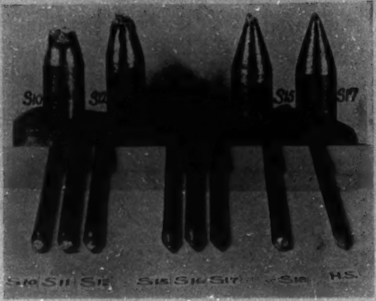


Figure 9. Fusion on copper rods with impulse tests

Top insert shows macrograph of samples

tested with currents as shown in figure 7. The duration of the initial high current in lightning discharges is relatively short, not more than 200 microseconds. For such currents, the average current density at the metal surface in contact with the arc is from 300 to 1,000 amperes depending somewhat on the metal and the polarity. Therefore the surface of metals is merely marked or blistered by the high current.[8] Thus for sample *H.S.*, the current density at the surface of the aluminum sheet is 650 amperes per square mm, so the corresponding discharge is 0.08 coulomb per square millimeter. For a drop of eight to ten volts at the root of the arc, the energy input at the surface of the metal then accounts for some 0.003 inch (0.1 mm) depth of metal fused, which is an amount in the order of the value observed. At these high currents, the ohmic losses would also add to the energy input.

Data and Test Results—Their Application

The fusion data in tables I and II for copper have been plotted in figure 13. An average of 2.0 to 2.2 coulombs per cubic mm of copper fused is representative, though for specific cases the amount may be from 2.5 to 1.8 depending on the electrode terminal, volume fused, and similar factors. Similar curves can be established for other metals, either by test or they may be estimated from the properties of the metal.[9] The physical properties would indicate that for the same discharge and identical electrodes, about three times the volume of fused metal is expected for aluminum (pure) than for copper, about the same amount for iron as for copper, and slightly more for brass than for copper. However, the experimental determination is preferred for dependable results. For instance, in sixty-cycle arcing tests on iron electrodes,[10] the amount of metal fused was found one cubic mm per coulomb.

For the best results, the specific value (coulombs per cubic mm of metal fused) to apply in evaluating field data naturally requires comparing the field specimen with a similar laboratory specimen, such as indicated in the tabulations and in the figures. From this correlation and the curves, the fusion method provides effective and practical means for determining the electric charge, the relative current, and the duration of long-duration compo-

Table I. Sixty-Cycle Current Fusion Tests

Sample	Test Piece	Current: Amps RMS	Current: Time Cycles	Charge Coulombs	Fusion Effects	Metal Fused Cu. Mm.: Cu	Metal Fused Cu. Mm.: Brass	Metal Fused Cu. Mm.: Al	Coulombs Per Cu. Mm. Fused: Cu	Coulombs Per Cu. Mm. Fused: Brass	Coulombs Per Cu. Mm. Fused: Al
A	1/4" dia. Cu rod 0.020" Al sheet	3,400	1/4	12.8	Rod tip fused 0.1" Sheet intact	4.3			2.98		
B	0.20" dia. Cu wire 0.020" Al sheet	3,500	1/2	26.2	Wire tip fused 1/8" 1/4" hole burned in sheet	8.6		16.2	2.93		1.62
C	0.20" dia. Cu wire 0.035" Al sheet	3,500	1/2	26.2	Wire tip fused 1/8" Crater spot in sheet	8.6			2.93		
D	0.20" dia. Cu wire 0.030" Al sheet	500	8 1/2	63.8	Entire wire tip fused 1/2" hole burned in sheet	34.4		97	1.86		0.66
E	0.20" dia. Cu wire 0.035" Al sheet	500	8 1/2	63.8	Entire wire tip fused 1/4" crater in sheet	34.4			1.86		
F	1/4" dia. Cu rod 0.020" Al sheet	1,350	8 1/2	172	Tip plus 1/16" rod fused 1 1/2" hole burned in sheet	118		580	1.46		0.30
G	1/4" dia. brass rod 0.020" Al sheet	1,350	8 1/2	172	Tip plus 1/16" rod fused 1 1/2" hole burned in sheet		93	580		1.85	0.30
H	0.20" dia. Cu wire 0.035" Al sheet	1,350	8	156	Tip plus part of wire fused 1 1/16" hole burned in sheet			510			0.31
I	1/2" dia. Cu rod 0.035 Al sheet	1,200	8	144	Rod tip fused 1/4" 1 1/16" hole burned in sheet	67.6		510	2.13		0.28
J	1/2" dia. brass rod 0.055" Al sheet	1,200	8 1/2	153	Rod tip fused 1/4" 13/16" hole burned in sheet		67.6	467		2.26	0.33
K	1/4" dia. Cu rod 0.095" Al sheet	1,300	8	156	Tip plus 1/16" rod fused 1/2" crater in sheet	93			1.68		
L	1/2" dia. brass rod 0.095" Al sheet	1,200	8 1/2	158	Rod tip fused 1/4" 1/2" crater in sheet		67.6			2.26	
M	1/4" dia. Cu rod 0.020" Al sheet	7,000	17	1,790	Tip plus 1 1/2" rod fused 4 1/4" hole burned in sheet	978		4,660	1.83		0.38

Comments—Refer to figure 1*A* for test set up. For typical oscillograms, see figure 2. End of rods machined to cone form, height equal rod diameter. In all the tests Al sheet is grounded. The polarity of the rod or wire on the first half-cycle is positive for the tests on samples *A*, *B*, *C*, *D*, *E*, *F*, *H*, *J*, *L* and *M* and negative for the other tests.

Table II. Long-Duration Low-Current Impulse Fusion Tests

Sample	Test Piece	Current		Charge Coulombs	Fusion Effects	Metal Fused Cu. Mm.	Coulombs Per Cu. Mm. Cu
		Amps Crest	Time Sec.				
*S*1	1/4" dia. Cu rod 0.035" Al sheet	4,750	0.028	40	Rod tip fused 0.17" 1/4" crater in sheet	16.3	2.45
*S*2	0.20" dia. Cu wire (body) 1/4" dia. Cu rod	4,750	0.028	40	Wire fused on one side Rod tip fused 0.17"	16.3	2.45
*S*4	0.20" dia. Cu wire 1/4" dia. Cu rod	4,750	0.028	40	Wire tip fused Rod tip fused 0.17"	16.3	2.45
*S*5	0.20" dia. Cu wire 1/4" dia. Cu rod	3,050	0.028	25.7	Wire tip fused Rod tip fused 0.13"	8.6	3.0
*S*6	Tower gaps (brass) 1/16" electrode spacing	4,750	0.028	40	Surface fused in semi-circle 0.40" dia.	15 to 25 (estim.)	
*S*7	Coaxial cable (Cu) Conductor 0.127" dia. Sheath 0.039" Air gap 0.15"	4,750	0.028	40	Porcelain spacer flashed over Conductor body partly fused 0.15" Sheath nearly burned through in 0.08" × 0.20" ellipt. area	17.5 (approx.) 13.0 (estim.)	2.3
*S*9	0.093" dia. Cu wire (on end) Cu sheet	4,750	0.028	40	0.20" fused off end of wire	22.2	1.8
*S*10	0.20" dia. Cu wire 1/4" dia. Cu rod	4,750	0.028	40	Wire tip fused Rod tip fused 0.175"	17	2.35
*S*11	0.20" dia. Cu wire 1/4" dia. Cu rod	3,050	0.028	25.7	Wire tip fused Rod tip fused 0.155"	13	2.0
*S*12	0.20" dia. Cu wire 1/4" dia. Cu rod	2,050	0.028	17.3	Wire tip fused Rod tip fused 0.115"	6.5	2.65
*S*13	0.093" dia. Cu wire (body) 0.20" dia. Cu wire	4,750	0.028	40	0.14" of 0.093" wire heavily fused Wire not sundered		
*S*14	0.051" dia. Cu wire (body) 0.093" dia. Cu wire (loop)	4,750	0.028	40	Large segment 0.051" wire fused 0.093" wire fused apart		
*S*15	0.20" dia. Cu wire 1/4" dia. Cu rod	2,000	0.0023	2.40	Rod tip fused 0.055"	1.18	2.0
*S*16	0.20" dia. Cu wire 1/4" dia. Cu rod	1,300	0.0023	1.56	Rod tip fused 0.035"	0.80	2.0
*S*17	0.20" dia. Cu wire 1/4" dia. Cu rod	850	0.0023	1.02	Rod tip fused 0.031"	0.36	2.8

Comments—Refer to figure 1*B* for test set up. For typical oscillograms, see figure 6. Under "Test Piece" the rod-sheet, wire-rod, etc. are the electrode arrangements of vertical gaps, air spacing 2 1/2 to 3" (see figure 8). Bottom electrode is grounded. The polarity of the top electrode on the first half-cycle or for the unidirectional waves is positive for the tests on samples *S*1, *S*2 and negative for the other tests.

Table III. High-Current Impulse Fusion Tests

Sample	Test Piece	Current		Charge Coulombs	Fusion Effects
		Amps Crest	Time Sec.		
*S*18	1/4" dia. Cu rod 0.035" Al sheet	17,000	0.0045	23	Rod fused 0.195". Tip of rod blasted 1/4" crater fused on sheet and splashed out
H.S.	1/4" dia. Cu rod 0.035" Al sheet	125,000	0.0004	15	Rod blistered heavily at tip, no concentration of fusion. Discharge blasted up rod 1.5". Fusion on sheet concentrated in 5/8" dia. (core of current discharge), to estimated depth 0.005 to 0.010". Metal fused approx. 25 to 50 cu. mm. Fused metal in 7/8" cusp form extending from core indicate molten metal was sucked up during discharge and deposited back on sheet. Sheath of discharge blasted out several inches from core

Comments—See footnote to table II. Typical oscillograms given in figure 7. Al sheet is grounded and rod at positive polarity on first half-cycle.

nents, as well as other important characteristics of the lightning-stroke discharge. It is estimated that in most cases the electric charges can be determined to within 20 to 25 per cent of their actual value. For the purpose of this study, where a mass of specific and statistical data are considered, the method enables establishing within a good degree of approximation, the minimum, the average, and the maximum values of the forementioned characteristics of lightning, their probable curve of distribution, and their frequency of occurrence.

Correlation of Field Data and Laboratory Tests

LIGHTNING RODS, WOOD-POLE GROUND WIRES, AND OTHER TERMINALS

The common lightning-rod terminal affords a fruitful source of field data on fusion, for thousands of lightning-rod systems are installed throughout the land and elsewhere. Figure 14 is a set of rod points that have been hit by lightning; these were obtained in Sweden.[11] The ends are of platinum, a metal which requires half the arc energy to fuse as copper.[9] To produce the fusion indicated, we estimate an electric charge in the stroke for each respectively in the order of 2.5, 33, and 100 coulombs.

Substantial field data and experience were kindly made available by the lightning-rod manufacturers for the purpose of this study. In one case, where in the course of time a group of 25 lightning-rod systems were examined, which according to the property owners had been hit, out of the 25 points hit, hardly half were fused. The fusing was about the same as on the samples in figure 15; the electric charge of fusion corresponding to these three samples is estimated at 10, 15, and 25 coulombs. From another source covering extensive field experience, we find that the solid points of copper-bronze terminals usually are not melted down more than approximately one-fourth of an inch. However, greater amounts of fusion have been found on hollow-shell points. As a rule, the hollow-shell point is installed upon the top of an air terminal. Apparently this type of terminal is exposed to the more severe as well as all other lightning discharges. An ample number of cases are on record where these points have been melted down for a distance of one to two inches from the tip. Also many cases are known where the entire point, including the hollow copper tube air terminal, has been crushed and fused into a solid mass. A case of the kind is shown in figure 16. In these instances, the copper fused reaches and even exceeds 100 cubic mm so the electric charge in these severe lightning discharges attains and may exceed 200 coulombs. The crushing effect is clear

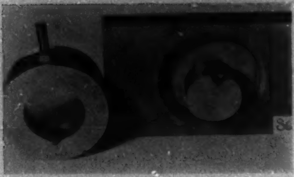


Figure 10. Fusion on tower gap with impulse current tests

Table IV. Energy Balance Estimated (Cu Samples)

Sample	Metal Fused Cu. Mm.	Charge Coulombs (Q)	Energy Absorbed by Metal (Watt-Sec.): Heating & Fusion of Metal (1,060°C)	Heating of Fused Metal (1,600°C)	Conduction to Rod (Estim.)	Total	Energy Input at Electrode (Watt-Sec.) (Q×E)*: (E=6)	(E=8)
A	4.3	12.8	27	12	10–30	60	77	103
B	8.6	26.2	54	24	30–50	120	157	210
D	34.4	63.8	217	96	80	393	383	510
K	93	156	586	260	150	996	935	1,250
F	118	172	747	330	150	1,227	1,032	1,380
M	978	1,790	6,160	2,730	150	9,040	10,700	14,300

**E* = watt-sec./coulomb. *Q* = coulombs (column 3).

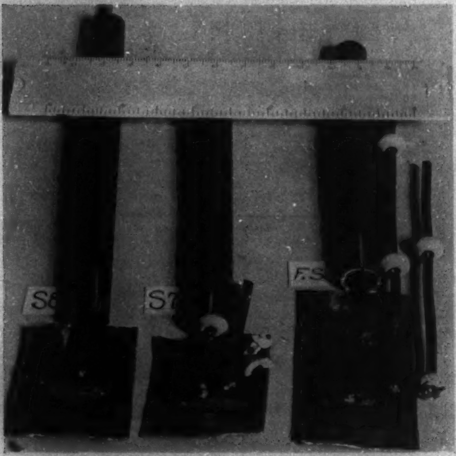


Figure 11. Fusion on coaxial cables

Field specimen (*F.S.*) and samples (*S7* and *S8*) from laboratory impulse tests

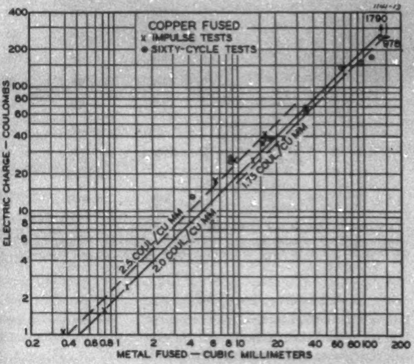


Figure 13. Curve data of electric charge versus volume of copper fused

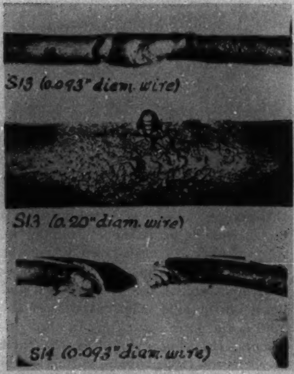


Figure 12. Fusion effects on conductors in laboratory impulse tests

evidence of the presence of a severe initial high current.[15]

The following illustrates the case of the more severe strokes encountered to wire terminals. On a 66 kv wood-pole line protected only with wood-pole ground wires, lightning struck the terminal of a ground wire (0.2 inch diameter copper) fusing the tip to the extent shown in figure 17. From it, 75 to 100 cubic mm of fused metal are estimated. The electric charge accounted by the long-duration component of this discharge is estimated in the order of 150 to 200 coulombs. The stroke was severe also from the standpoint of the high current, for it side-flashed from the wood-pole ground wire to the line conductors over six-foot cross arms and seven-unit suspension insulator strings affecting all three phases. From these data and the ground resistance (95 ohms meggered value), the high current is at least 50,000 amperes and possibly higher. The entire data point clearly to a severe lightning discharge.

The copper terminals in figure 18 are of particular interest, for here the current and the duration of the stroke for terminals 3 and 4 have actually been recorded.[12] For terminal 3, the metal fused is estimated at one cubic mm. The wave of the current is reported elsewhere.[13] The total charge is about four coulombs. A 20,000 ampere crest, short-duration initial current precedes the long-duration component, the entire discharge lasting 0.02 second. The charge associated with the low current, long-duration component is 2.5 to 3 coulombs. In the case of terminal 4, the current may be considered essentially a long-duration component for the entire duration of the discharge which is close to 0.10 second.[14] A total charge of 60 coulombs is associated with this stroke. The fusion of the rod tip is estimated at 25 to 30 cubic mm. In each case, the correlation is confirmation of the fusion method, of the sound principles on which the method is based. In the case of terminal 2, a 16,000 ampere crest was recorded on a magnetic link and a

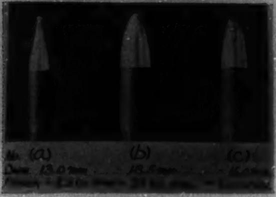


Figure 14. Lightning-rod terminals fused by lightning[11]

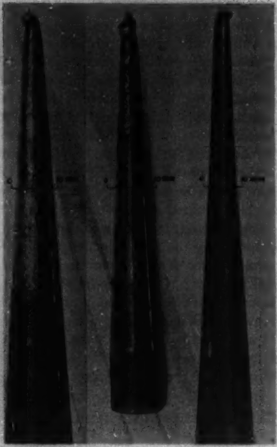


Figure 15. Other specimens of lightning-rod terminals fused by lightning

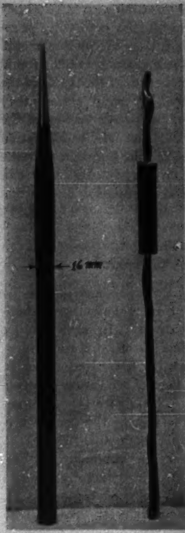


Figure 16. Lightning rod crushed and fused at tip by lightning[23]

Original rod shown

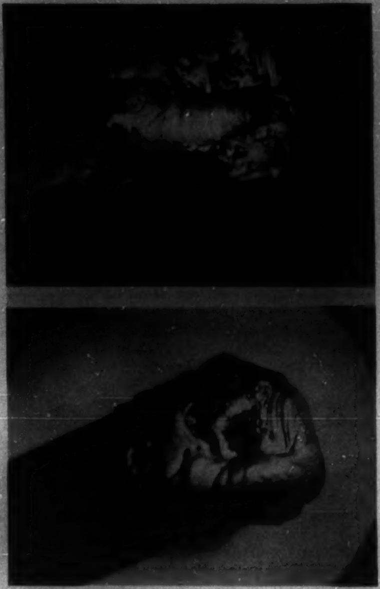


Figure 17. Fusion at end of wood-pole ground wire hit by lightning. Two macrograph views

0.032 inch (0.813 mm) diameter copper link carrying the discharge was fused. From these data the discharge apparently is of the same general form as that for terminal 3, except that judging from the amount of fusion of the rod tip, the long component would be substantially greater.

Conductors on Communication and Power Lines

The experience on open-wire telephone lines is another source of pertinent field data. For 1938, out of 1,435 cases of open-wire toll line troubles attributed to lightning, it is estimated that at least fifty per cent involved the burn-down of one or more conductors from direct strokes. In 1939, there were 1,210 wire troubles attributed to lightning—about half represents wire burn-downs from direct strokes. The average rate of wire troubles per 100 route miles for 1939 is three. The three common sizes of conductors used for toll lines are 0.104 inch (2.74 mm), 0.128 inch (3.38 mm), and 0.165 inch (4.36 mm) diameter copper. In any one instance, the number of conductors parted vary. For example during a storm in a given locality, eight 0.104 inch and two 0.165 inch diameter conductors were affected over a 1.7 mile section of the line; at places the conductors were fused in groups of two or more.

In general, the conductors are fused at a single point. From the fusion effect alone, the tests on samples $S13$ and $S14$ (figure 12) would indicate that a long-duration current of 40 coulombs is required to fuse the smaller conductor (0.104 inch). Actually it is less, for the high short-duration current in the lightning discharge, if sufficiently large, imparts an impact to the conductor over the span (about 50 kg for a 25,000 ampere current) which contributes to rupture the smaller conductors when the section of the wire is partly fused. An example is a three mm (0.12 inch) diameter conductor sundered apart with less than half the section affected by fusion (metal fused five cubic mm).[3] This example and calculations of the impact effect involved indicate that the lighter telephone line

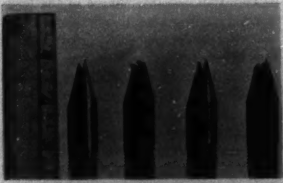


Figure 18. Rod terminals fused by lightning in lightning investigations[14]

Original rod shown

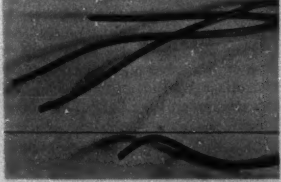


Figure 19. Fusion and sundering of distribution line conductors

conductors may be sundered by lightning discharges from a couple coulombs and up. The fusion of heavier conductors or of conductors in groups is evidence of higher charges. Thus, the field data on open-wire toll lines likewise establish a high frequency of occurrence of the long-duration components in strokes. Estimates of the electric charges from this source range from a few coulombs up into the higher figures.

The larger conductors on power lines rarely are affected; however, in the course of time, a medium-sized conductor would be found burned in two by lightning. In one case a copper conductor about 0.25 inch (0.64 mm) diameter was parted in the span—the evidence bearing the earmarks of fusion from the long-duration current. The charge in this stroke is estimated up in the 100 coulombs. Conductors on distribution lines occasionally are burned apart by lightning. For instance, two 0.128 inch (3.38 mm) diameter copper wires were parted at the point of incidence of the stroke and crushed solidly together over sections of the service span. This and other damage indicate both a severe initial current and a severe sustained current in the stroke discharge. Figure 19 shows a three-strand conductor (each strand 0.108 inch diameter) which was fused and sundered apart by a stroke. From this and other dependable evidence at adjacent electrical apparatus on this distribution line, the charge of the long-duration current of this stroke is estimated at 60 coulombs and greater. This case illustrates furthermore one of many characteristics which distinguishes the long-duration, low currents from the initial, short-duration, high current in the stroke. The latter usually is limited in its effects within a restricted area from the point of incidence of the stroke to the line. On the other hand, the long-duration currents appear to reach out over greater distances of the line—virtually miles in some instances. In the preceding case, the effects of the long-duration currents were clearly in evidence over a five mile section of the line. Similar cases of effects appearing over a large section of the line are on record. The evidence precludes as a general explanation the presence of multiple strokes from a single flash to points over a large section of the line, although simultaneous discharges of this character to the line may occur at times.

Globules and protrusions of fused metal, such as shown in *S*13 of figure 12, which we find on the conductors of communication lines,[16] on overhead ground wires, etc., are other effects associated with the long-duration currents. They are produced by the magnetic force or so-called pinch effect on the molten metal. A number of these globules may be found within a few inches or, as the result of the arc displacement by wind action, they may spread out over 30 to 40 feet of the conductor. They are definite indication of the presence of long-duration currents in the stroke discharge. Also they can give other useful information on the discharge.

Coaxial Cables, Tower Gaps, Metal Sheets, Miscellaneous

At various times coaxial cables at broadcasting stations have been damaged from lightning. In one instance where protection had not been provided, lightning struck the tower and the cable was fused. A view of the fusion effects is shown in item *F.S.* of figure 11. Approximately 30 cubic mm of the copper was melted from the conductor. From the fusion of either the conductor or the sheath, the electric charge estimates about the same—fully 60 coulombs. A separate but lighter fusion spot will be noted; which indicates a second discharge of about ten coulombs. The field and laboratory samples (figure 11) reveal close similarities to each other. However, there are visible indications from the relative blast action on each that the field current must have been of longer duration than the laboratory discharge.

Another group of field data is supplied from tower gaps. These are applied at the footing of transmission-line towers as described elsewhere.[17] Samples of these gaps tested in the laboratory with the long-duration currents appear in figures 5 and 10. The field survey reveals a variety of effects on the gaps, ranging from surface "blistering" to concentrated fusion and combinations of the two. The gaps affected by lightning discharges are located at some 500 towers. Out of the total number affected, five evidenced complete fusing to the extent that some of the metal fused bridged the air gap and has left a filament about 0.12 inch (3.05 mm) diameter frozen across the gap. Comparing the fusion on one of these five field gaps with the laboratory samples, an electric charge well up to 100 coulombs is estimated. Furthermore, from the development of the fusion, the relative blast-action, etc., which appear on the laboratory samples and on the field sample, convincing evidence is present to the effect that the long-duration currents of the lightning discharge in nature must have lasted well up to one second and possibly even longer.

Other field data on metal objects may be mentioned briefly. For instance, from a report on atmospheric discharges to aircraft in flight,[18] evidence of fusion is presented. In a similar survey, out of sixteen cases in nearly half there is substantial evidence of the long-duration currents present in the discharge. In at least three, the charges which are required to account for the fusion are estimated well up in the order of 100 to 200 coulombs. All these discharges are likely to have been between clouds and may have been initiated by the moving plane itself.[19] There is also a mass of miscellaneous data, but much of this requires confirmation and should be used with considerable caution. One source[20] mentions the possibility of one to two thousand cubic mm of fused metal where the discharge enters the metal. While this amount of fusion is intended to relate only to the most severe strokes, even then it should be questioned and corroborated by specific instances before it can be accepted. Terminals on highly elevated objects[21] may indicate profuse fusion, but this would result because these objects frequently receive strokes, some of which must no doubt be quite severe. Other effects of interest associated with the long-duration currents in lightning discharges are the firing of combustible materials, fusion of sand into fulgurites, etc. These have been discussed in the previous report.[1]

Characteristics of Lightning-Stroke Discharge

At this point it is desirable to examine the type of lightning discharge prevalent to electric systems and to earth. Figure 20 shows in schematic form the pattern of the discharge. In this figure are presented the conclusions based on correlation studies of the lightning problem that have extended over a period of years.[1,23] From a recent survey in this same connection, other investigators[24] have come essentially to the same conclusions.

In the first place, the characteristics outlined schematically in the figure conform with the mechanism of the lightning-stroke discharge from cloud to earth.[25,26] The high-current component is associated with the main stroke which blazes its way up the channel upon the instant of contact of the tip of the leader stroke with the earth. The duration of the main stroke is from 10 to 100 microseconds. It determines in a large measure the duration of the high-current component. The low-current long-duration components are related to the discharge processes that follow the main stroke. The source of these components apparently resides largely in the cloud and they are due to a continuation of the lightning discharge in the cloud proper.[27,28] The lightning flash may well be a multiple stroke in which case there is a series of such component groups as indicated in the figure. It should not be construed from the schematic representation that the three components are necessarily uniform and of simple form. Under certain conditions, some may even be reduced and others intensified.

Likewise, the pattern of these characteristics conform with the data from the various field investigations as well as the mass of experience from electrical systems. In general, the high-current component is the one which is better established of the three. It is more or less exponential with possible variations from this form, rising to crest in some 2 to 10 microseconds and receding more slowly on the tail.[29,30] The time required for the current of the main stroke to flow from its source in the charge on the earth into the channel, precludes front durations of the high current component below a certain limit. These considerations and field data[31,32] indicate that the maximum current rate of rise on the front seldom, if ever, attains 50,000 amperes per microsecond, while the average is more in the order of 5,000 amperes per microsecond. The amplitudes of the current and the frequency of occurrence are quite well established.[33,34,35] They range from 5,000 to 200,000 amperes. Currents of 200,000 amperes or more are rare.[15] Actual direct measurements of the voltage component at or near the point of incidence of the stroke to the line likewise are few.[36,37] In this connection, the extensive experience on the performance of electrical apparatus to direct strokes affords a source of important information.[38,39] More data on the wave form of these two components are desirable to better establish the essential details on lightning-stroke discharges. However, it is apparent that the mass of the established data and experience available on lightning phenomena sustain the deductions presented in figure 20 as to the pattern of the characteristics of lightning discharges to objects near to the surface of the earth.

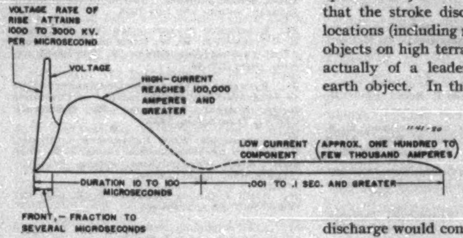

VOLTAGE RATE OF RISE ATTAINS 1000 TO 3000 KV. PER MICROSECOND
VOLTAGE
HIGH-CURRENT REACHES 100,000 AMPERES AND GREATER
11-41-20
LOW CURRENT COMPONENT
(APPROX. ONE HUNDRED TO FEW THOUSAND AMPERES)
DURATION 10 TO 100 MICROSECONDS
.001 TO .1 SEC. AND GREATER
FRONT,— FRACTION TO SEVERAL MICROSECONDS

Figure 20. Schematic representation of lightning-stroke discharge

In regard to the long-duration components, it is only in the recent three or four years that this constituent part of the lightning discharge has received particular attention.[39,40,23,12] It should be pointed out that in this group of investigations, those in which the lightning-stroke phenomena are recorded at the terminal of very high structures, while they no doubt will contribute to the knowledge on lightning, yet the results from them need be interpreted carefully before these can be applied to electric systems and to protection systems located near the surface of the earth. For instance, in the greater part of the cases of strokes to the Empire State Building, the leader is from the structure up into the clouds, a process which is the reverse of the lightning-stroke process from cloud to ground in open country. There is the possibility that the stroke discharge from elevated locations (including mountain tops, ridges, objects on high terrain, etc.) may consist actually of a leader initiated from the earth object. In these special cases, the discharge would consist initially of a low-current, long-duration component possibly followed later by high current spurts as the leader reaches into and taps the cloud.

The following are examples drawn from the paper which illustrate the characteristic components in the lightning discharge. In the case of the stroke to the wood-pole ground wire discussed in connection with figure 17, the high-voltage, the high-current, and the low-current components are clearly in evidence. The crushing of the lightning-rod tubes and the fusion at the terminals, such as in figure 16, present evidence of severe multiple strokes consisting of high currents accompanied by long-duration components. The case of two service conductors crushed solidly together over sections of the span and parted at the point where the stroke entered the conductors is another similar example. As discussed in the paper, the fusion and the sundering of conductors on communication lines result from the combined effect of the two components of the current. The shattering of line and station apparatus as well as other objects in which the concurrence of fusion of metallic parts can only be ascribed to the low-current component is further indication of the presence of the two components in the lightning discharge. The crash of thunder followed by a sustained arc combined with a flicker of the flash are evidence of a multiple stroke with all the components shown in figure 20.

The presence in the lightning discharge of the two components simultaneously can be distinguished quite readily from a number of distinguishing features inherent with one which are absent in the other, with overlapping in part for some of the effects. As discussed at length previously, the earmarks indicating the presence of long-duration currents is found principally in the fusion effect produced at the surface of the conductor where the metal is in contact with the arc of the stroke. In the case of the high-current components, their presence is evidenced by shattering, crushing, bursting, and similar effects as well as "blistering" at the surface of metal, the fusion of conductors through ohmic loss, the development of high voltages, etc. For instance, a thin-wall, hollow tube was set up for test. A moderate high-current discharge would readily crumble the tube, whereas even a heavy long-duration discharge would leave the tube completely intact.

In regard to the electric charge associated with long-duration currents, the fusion data in this investigation are of particular interest and significance. If to this component charge the generally smaller amount associated with the high current is added, we have the total charge of the stroke. In summary, from the combined fusion data, the total charge for strokes to ground is found to range from a few coulombs and possibly less up to 200 coulombs. There is evidence from the high volume of fusion in limited cases that the electric charge of unusually severe and rare strokes may reach 200 to 300 coulombs. A few isolated and unconfirmed cases would suggest even higher values, but in the opinion of the writer, these must be dismissed until future confirmation becomes available. The statistical data in the paper indicate that practically all strokes to earth have long-duration currents of greater or lesser amounts associated with each. From the mass of the data, the mean value of the charge is estimated between 15 to 30 coulombs. That is 50 per cent of the strokes exceed this amount. In a previous estimate determined from a limited amount of data, a somewhat higher average was suggested between 25 to 50 coulombs.

Close similarity is generally found between the fusion of the test samples and that on the specimens from the field. However, in one respect the nature of the fusion on field specimens would point to longer sustained duration and correspondingly lower currents in the field than for the currents used in the laboratory tests. In some of the field specimens, the smooth appearance of the fusion and other evidence connected thereto are indicative of a relatively gradual development of the fusion which in turn suggests current discharges in the field estimated in effect well up to one second. In general, the effective currents and corresponding durations of the long-duration components cover a range as shown in figure 20.

For purpose of general comparison, the following investigations may be cited. In a recent investigation in New Mexico[41,42] the electric charge from cloud to ground was determined from electric field recorders located at the surface of the earth at widely separated stations. Out of more than 200 discharges occurring during thunderstorms, all of the cloud ground strokes measured have transferred negative charge from cloud to ground the quantity of charge ranging from 5 to 200 coulombs with an average of 30. In the Empire State Building investigation,[40] out of 27 strokes the charge of the stroke discharges ranges from a fraction to 164 coulombs, the data indicating an expectancy of 35 coulombs or more on 50 per cent of the cases. Amounts of charge within these limits are found from the field investigation at the Cathedral of Learning and elsewhere.[13,14] We should note again that on account of the inverted mechanism of the lightning discharge on highly elevated structures, the data from these sources may not be altogether comparable to the data for strokes from cloud to earth which normally represent more nearly the conditions encountered in practice. In spite of all considerations on the methods of measurement, the source of the four groups of data, etc., similarity in the results can hardly be considered coincidental.

References

1. Lightning Strokes in Field and Laboratory, **P. L. Bellaschi.** Electrical Engineering, volume 58, November 1939, pages 466–8.

2. Damages Caused by Lightning on Overhead Lines of Communication Circuits, **W. Peters.** *Elektrotechnische Zeitschrift*, April 1, 1937 and April 8, 1937.

3. Elektrische Gasentladungen, **A. v. Engel** and **M. Steenbeck,** volume 2, pages 133–8.

4. The Cathode Fall of An Arc, **R. C. Mason.** *Physical Review*, August 1, 1931.

5. Investigation of the Voltage Distribution in the Welding Arc, **H. v. Conrady.** *Elektroschweissung*, June 1937 and July 1937.

6. Calorimetric Study of the Arc, **P. P. Alexander.** AIEE Transactions, volume 49, 1930, pages 519–22.

7. Cathode Energy of the Iron Arc, **Gilbert E. Doan.** AIEE Transactions, volume 49, 1930, pages 723–31.

8. Lightning Strokes in Field and Laboratory—II, **P. L. Bellaschi.** AIEE Transactions, volume 56, 1937 (October section), pages 1253–60.

9. Handbook of Chemistry and Physics. **Charles D. Hodgman,** page 1241. Chemical Rubber Company.

10. New Type of Switch for Interconnected Networks, **F. Kesselring** and **W. Kaufmann.** *CIGRE Transactions*, 1939, volume 1, page 104.

11. Current Values in Lightning, **H. Norinder.** *Särtryck ur Nordisk Familjeboks Manadskrönika.* nr. 8/1938.

12. New Instruments for Recording Lightning Currents, **C. F. Wagner** and **G. D. McCann.** AIEE Transactions, volume 59, 1940, pages 1061–8.

13. Direct Stroke Proves Length of Lightning Tail, **C. F. Wagner** and **E. Beck.** *Electrical World*, July 20, 1939.

14. Field Investigation of Lightning, **C. F. Wagner, G. D. McCann,** and **E. Beck.** AIEE Transactions, volume 60, 1941, pages 1222–30.

15. Lightning Currents in Field and Laboratory, **P. L. Bellaschi.** AIEE Lightning Reference Book, page 1468.

16. Telecommunication Protection Devices, **B. S. Cohen.** *The Electrician*, November 11, 1938, page 564.

17. Symposium on Operation of the Boulder Dam Transmission Line—Insulation and Lightning Protection, **Bradley Cozzens.** AIEE Transactions, volume 58, 1939 (April section), pages 140–7.

18. Lightning and Aircraft, **G. C. Simpson.** Publication of Meteor Office (British) 1934.

19. Meteorological Conditions Associated With Aircraft Lightning Discharges and Atmospheric, **E. J. Minser.** *Journal of the Aeronautical Sciences*, December 1939.

20. Code for Protection Against Lightning. *Handbook number 21*, U. S. Bureau of Standards, Washington, D. C.

21. Washington Monument Lightning Protection, **M. G. Lloyd.** *Quarterly* of the National Fire Protection Association, July 1935, part 1, page 26.

22. Physics of the Air, **H. J. Humphreys.** McGraw-Hill Book Company.

23. Lightning and Lightning Protection, **P. L. Bellaschi.** AIEE Connecticut Section, meeting December 14, 1938.

24. The Mechanism of the Lightning Discharge and Its Effect on Transmission Lines, **C. E. R. Bruce** and **R. H. Golde.** *Journal* of the Institution Electr. Engrs., 1941.

25. Progressive Lightning, **B. F. J. Schonland** and **H. Collins.** *Proceedings* of the Royal Society, series A, volume 143, 1934, page 654.

26. Progressive Lightning—II, **B. F. J. Schonland, D. J. Malan,** and **H. Collins.** *Proceedings* of the Royal Society, series A, volume 152, 1935, page 595.

27. Progressive Lightning—III, **D. J. Malan** and **H. Collins.** *Proceedings* of the Royal Society, series A, volume 162, 1937, page 175.

28. Progressive Lightning—V, **B. F. J. Schonland, D. B. Hodges,** and **H. Collins.** *Proceedings* of the Royal Society, series A, volume 166, 1938, page 56.

29. L'Etude de la Foudre dans un Laboratoire de Campagne, **I. Stekolnikov** and **Ch. Valeev.** *CIGRE Transactions*, 1937.

30. Wave Shapes of Successive Lightning Current Peaks, **K. B. McEachron.** *Electrical World*, February 10, 1940.

31. Résultats des Mésures Effectuées au Cours des Orages de 1934–1935, **K. Berger.** Association Suisse des Electriciens *Bulletin*, volume 27, number 6, March 20, 1936.

32. Lightning Currents and Their Variations, **Harald Norinder.** *Journal* of the Franklin Institute, volume 220, July 1935.

33. Lightning Investigation on Transmission Lines—VII, **W. W. Lewis** and **C. M. Foust.** AIEE Transactions, volume 59, 1940 (April section), pages 227–34.

34. Research on Operating Disturbances Due to Lightning and on Protection of Overhead Lines Against Lightning, **H. Grünewald.** *CIGRE Transactions*, 1939.

35. Experience With Preventive Lightning Protection on Transmission Lines, **S. K. Waldorf.** AIEE Transactions, volume 60, 1941 (June section), pages 249–54.

36. AIEE Lightning Reference Book, page 697 (**Edgar Bell** and **A. L. Price**).

37. AIEE Lightning Reference Book, page 873 (**R. R. Pittman** and **J. J. Torok**).

38. Dielectric Strength of Porcelain, **P. L. Bellaschi** and **M. L. Manning.** AIEE Transactions, volume 58, 1939 (December section), pages 651–6.

39. Co-ordination and Protection of Station Insulation Against Lightning, **P. L. Bellaschi.** *CIGRE Transactions*, 1939.

40. Lightning to the Empire State Building, **K. B. McEachron.** Electrical Engineering, volume 57, December 1938, pages 493–505; also *Journal* of the Franklin Institute, February 1939.

41. The Electrical and Meteorological Development of Thunderstorms, **E. J. Workman** and **R. E. Holzer.** *International Geophysical Union*, September 1939.

42. The Origin and Types of Discharges in Thunderstorm, **R. E. Holzer** and **E. J. Workman.** *International Geophysical Union*, September 1939.

An Analysis of Transient and Sustained Voltages in Ground-Fault Neutralizer Systems

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Introduction

THE first ground-fault neutralizer (Petersen coil) in the United States was installed on the Alabama Power Company system in 1921. The technical press at that time devoted considerable attention to the substantial improvement in service continuity which was obtained with the neutralizer, and also described the overvoltages which occurred on the system during switching operations.[1] Since this first application, more than fifty ground-fault neutralizers have been installed on systems in the United States in all voltage classes from 22 kv to 230 kv. Considering the total number of neutralizer-years of successful operating experience that have now been accumulated, it can therefore be stated that the ground-fault neutralizer has definitely proved its value as a service protective device, and that practical means have been found to eliminate the overvoltage troubles encountered in the first application. The very favorable operating record obtained to date indicates that ground-fault neutralizers may be expected to find increasingly widespread application in the future.[2,3,4,5]

The question of overvoltages, however, is still raised from time to time with reference to prospective applications of ground-fault neutralizers, presumably because of the attention which the overvoltage problem has received in the past,[6,7,8,9,10] and it is felt that an analysis of the possible causes of and the means used to control overvoltages is in order. It is the specific purpose of this paper, therefore, to present data to indicate possible sources of overvoltages in ground-fault neutralizer systems, to point out the factors affecting the voltage magnitudes, and to show how the overvoltages are limited by present design and application practices.

It is to be noted that a major change in ground-fault neutralizer design occurred after the first application. The first neutralizer was a non-saturable air core reactor. Its reactance remained substantially constant, regardless of the voltage existing between its terminals. All subsequent neutralizers, however, have iron cores, carefully designed to saturate on the occurrence of voltages exceeding normal line-to-neutral voltage of the systems to which they are applied. This characteristic plays an essential part in limiting voltages of the series resonant type to harmless values. Furthermore, all ground-fault neutralizers, except the first one, include Thyrite by-pass arresters connected between their high voltage terminal and ground to limit the magnitude of any abnormal voltages that might appear at the system neutral.

In this analysis particular emphasis is placed on the condition of "series resonance." This condition was long considered to be, although erroneously, a dangerous source of overvoltage. In modern installations, however, it is realized that voltages of the series resonant type, if system conditions permit them to occur at all, are limited by the saturation of the neutralizer iron cores and by the lightning arresters mounted integral with the neutralizers. The quantitative effects of neutralizer saturation and the part played by lightning arresters in limiting overvoltages are illustrated. Out-of-phase switching conditions are also analyzed; and the advantages of short interrupting time in circuit breakers are pointed out.

In most of the previous work of analyzing system performance, the effects of saturation and of lightning arresters have been omitted, since to include them with their non-linear characteristics would be practically impossible by means of longhand calculation. It has been found that the miniature system representation method of attack used in this analysis is eminently suited for including these factors, and thereby readily facilitates the acquisition of much new knowledge concerning power system performance and the part played by protective equipment.

What Produces Series Resonant Overvoltages

In a three-phase system the method of grounding (whether grounded solidly, through resistance or reactance, including the ground-fault neutralizer, or not grounded) has no effect on the normal steady-state flow of balanced three-phase power. Under unbalanced or transient conditions, however, the method of grounding is an important factor in determining the behavior of the system. The condition which permits series resonant overvoltages in a ground-fault neutralizer system occurs only when one or two conductors of a three-phase system become open circuited; when all three phases are intact no series resonant overvoltages can occur.

With an opening in either one or two of the phases in a ground-fault neutralizer system, a flow of current tends to persist between the two parts of the system located on either side of the break, by taking a path from the system to ground through the neutralizer, in parallel with the line-to-ground capacitance on one side of the break, and from ground back into the system by way of the line-to-ground capacitance on the other side of the break. The opening in the circuit, in effect, places the ground-fault neutralizer inductive reactance in series with the system capacitive reactance, and since this constitutes a tuned circuit, the net impedance to the flow of current is very small. If the tuned condition is allowed to exist for all values of load and fault current, overvoltages may occur. With respect to this possibility several practical factors may be evident:

1. The larger the current previously flowing in the one or two opened conductors, the higher will be the resulting voltages. A line-to-line short circuit on the load side of the open point would be an extreme example of this condition.
2. Opening one or two conductors on a line carrying only charging current is not expected to cause serious overstress of the system insulation, particularly if the line is short relative to the extent of the system. However, the unbalance produced by such an opening will cause a sustained current to flow through the neutralizer. This must be taken into consideration in the neutralizer design and application.

Figures 1 and 2 are presented for the benefit of those who are familiar with the

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1. For all numbered references, see list at end of paper.

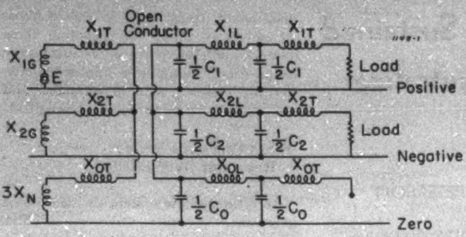


Figure 1. Equivalent circuit for one open conductor

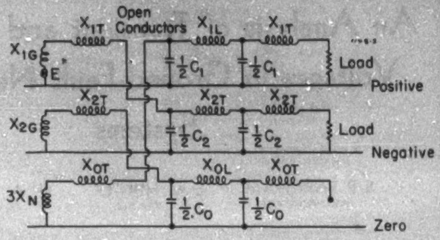


Figure 2. Equivalent circuit for two open conductors

X_{1G}, X_{2G}: Generator positive- and negative-sequence reactance

X_{1T}, X_{2T}, X_{0T}: Transformer positive-, negative-, and zero-sequence reactance

X_{1L}, X_{2L}, X_{0L}: Line positive-, negative-, and zero-sequence reactance

C_1, C_2, C_0: Line positive-, negative-, and zero-sequence capacitance

use of symmetrical components. It has been shown previously[11] that the case of one open conductor may be represented diagrammatically by inserting the negative and zero sequence network diagrams, in parallel, into the open point of the positive sequence network, as shown in figure 1. The effect of the negative sequence network is negligible, since it is effectively "short circuited" by the low impedance series resonant circuit of the zero sequence network. The case of two open conductors has likewise previously been shown[11] to be represented as shown in figure 2. Here the negative and zero sequence networks, in series, are inserted in the opening in the positive sequence network. The presence of the negative sequence network, in this case being in series between the load and the zero sequence network, tends to reduce the magnitude of current that can flow, and to this extent tends to reduce the overvoltages that may occur. However, if saturation in the ground-fault neutralizer has occurred, its beneficial role in destroying resonance may be partially offset by the presence of the additional unsaturated negative sequence reactance in series with it. From figures 1 and 2 it will be apparent that in a single neutralizer system the mechanism which may give rise to overvoltages is independent of the location of the open point except in case the opening is at the terminals of a load transformer.

How Series Resonant Overvoltages Are Limited

Fortunately, series resonant overvoltages can readily be limited. Since they result from a tuned circuit, it follows that any means which are effective in detuning the circuit will necessarily reduce the magnitudes of the overvoltages. This detuning is successfully accomplished by building the ground-fault neutralizer with iron cores which are carefully designed to saturate when the voltage across them exceeds the normal system line-to-neutral voltage. The effect of the neutralizer saturation is, of course, to reduce the neutralizer inductive reactance, and the change in reactance effectively detunes the circuit.

During all the time that ground-fault neutralizers have been applied, it has in general been recognized that an open conductor in either one or two phases of the system could create the condition of series resonance. One of the application rules, therefore, has been that no ground-fault neutralizer system may include line sectionalizing fuses or sectionalizing switches which are not gang-operated.

A broken conductor may, of course, set up the condition for series resonance. However, if a conductor breaks, it will usually fall to earth, thus grounding one side of the opening and destroying the condition of resonance. Broken conductors have occurred on ground-fault neutralizer systems without producing dangerously high voltages. This condition almost invariably increases the neutralizer current sufficiently to cause the by-pass switch[13] to close, thereby solidly grounding the system neutral. Up to this point, the performance is the same as if a sustained line-to-ground fault had occurred on the system. However, when the by-pass switch closes, but no circuit breaker operation follows, the experienced system operator recognizes that an open conductor has occurred, and efforts are made to locate and repair it.

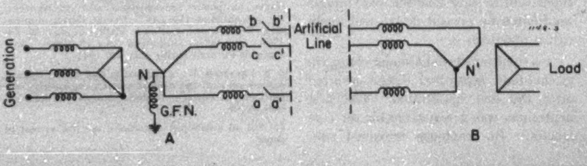
Results of the Miniature System Study of Series Resonance

As a typical illustration of a series resonant condition, the system shown in figure 3 was studied by means of the miniature system method of analysis. In this figure, the ohmic values of source reactances are purposely omitted because the results are essentially unaltered by these values as long as the ohmic impedance of the neutralizer is large by comparison. Therefore, the results of this study apply not only to one specific system operating at a given voltage, but apply in general to all ground-fault neutralizer systems conforming to the circuit arrangement shown in figure 3.

Since the description of the basic equipment and general procedure used in investigations of this type have been disclosed elsewhere in the literature,[7,8,10,12] it will not be repeated here. The miniature representation of ground-fault neutralizers and by-pass or conventional lightning arresters, however, is believed to be unique to this study, and a few words concerning these devices appear desirable.

The miniature ground-fault neutralizers were built for operation at 110 volts rms. They were constructed so as to have saturation characteristics and losses closely approximating those of a large neutralizer as shown in figure 4. For flexibility in tuning and adjusting to a

Figure 3. System used as a basis for study



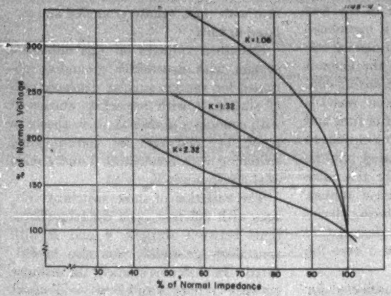


Figure 4. Typical ground-fault-neutralizer saturation curves

K is the ratio of impedance at 100 per cent of normal voltage to the impedance at 200 per cent voltage. Values of *K* in most modern neutralizers lie between 1.06 and 2.32, depending upon the type of neutralizer and the tap setting

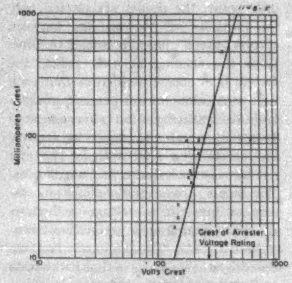


Figure 5. Comparison of miniature system and full size arrester volt-ampere characteristics

——— 240-kv (20-unit) arrester reduced to 190-volt base which was the normal line-to-line voltage in the miniature system

× Test points from oscillograms of instantaneous voltage across the miniature arrester and current passing through the arrester

Note: For representing arresters of different voltage ratings in the miniature system, it was necessary to reduce their volt-ampere characteristic to the 190-volt base and then put in the number of miniature discs to duplicate this characteristic in the miniature system

variety of system sizes, the miniature models were equipped with a total of 25 taps. At 110 volts, the effective impedance of these units covers a range of values of from 1,750 ohms on tap 25 to 460 ohms on tap 1, all at 60 cycles.

The miniature lightning arresters were Thyrite discs or wafers, about one inch in diameter and $^1/_8$" thick. Several of these could be stacked together as desired to obtain a voltampere characteristic duplicating to scale that of a full size arrester as shown in figure 5. The homogeneity and electrically stable characteristics of Thyrite material even in such relatively small pieces permitted this miniature representation.

A means for duplicating the spark-over of an arrester gap is illustrated in figure 6. Use of this device was helpful in determining the effect of an arrester sparking over during transient conditions. By adjusting the grid bias voltage, the spark-over voltage could be controlled as desired. The energy dissipated in an arrester following spark-over could be determined from the oscillograms of voltage and current during the transient period. This energy might be considered a measure of the severity of overvoltage disturbances from the standpoint of duty imposed upon protective equipment.

The first study was to show the effect of the degree of saturation in the neutralizer core. To distinguish between degrees of saturation, a factor *K* was introduced. It is defined as the ratio of the neutralizer effective impedance at normal voltage to impedance at two times normal voltage. The larger the value of *K*, the higher the degree of saturation in the neutralizer.

A curve showing the relationship between overvoltages and the factor *K* for phase *A* opened at the generating end of the system described in figure 3 is shown in figure 7. Similar data are given for phases *B* and *C* open on figure 8. In these and subsequent figures and tables the subscripts indicate the points between which the voltages were measured. In case only one subscript appears, the voltage was measured to ground. These two curve sheets show that neutralizer saturation is an important factor in limiting the magnitude of voltages caused by series resonance.

The second study was to show how the overvoltages vary with the physical location of the break in the system, and was made with a saturable core neutralizer having a *K* factor of 2.2. Crest values of voltage as a function of the location of open phase *A* in the system for no load conditions is shown on figure 9. Similar data for phases *B* and *C* open are given on figure 10. This study was repeated with the system loaded, and the results are given on figures 11 and 12.

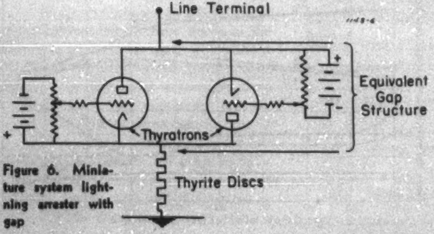
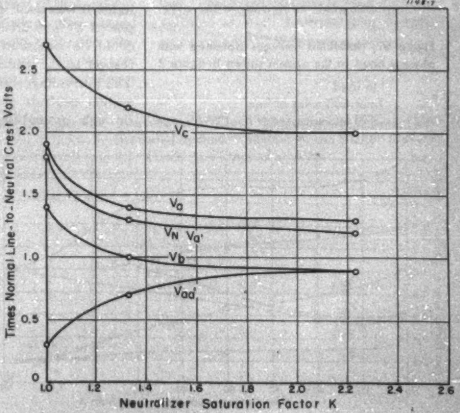


Figure 6. Miniature system lightning arrester with gap

Figure 7 (right). Effect of neutralizer saturation factor on sustained overvoltages for system shown in figure 3

500 ohms resistance load on system
Phase a open at *A* 180 miles of line



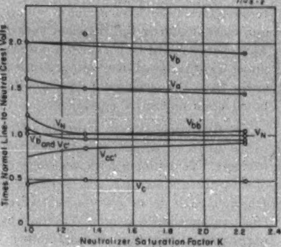


Figure 8. Effect of neutralizer saturation factor on sustained overvoltages for system shown in figure 3

500 ohms resistance load
Phases *b* and *c* open at *A*
180 miles of line

These results indicate that in a given typical system, the highest overvoltages are obtained when all of the capacitance is on the load side of the open conductor or conductors. The overvoltages decrease in magnitude as the break position occurs nearer the load. In a typical network system consisting of many lines, only a small percentage of the total system capacitance could be on the load side of an open conductor, and therefore the overvoltages would be limited.

It is to be pointed out that the series resonant overvoltages shown in these studies are hardly high enough in magnitude to spark over a lightning arrester. Furthermore, the voltages build up gradually requiring 4 to 5 cycles to reach their maximum values, so that they are very unlikely to occur while switching off a line by means of an oil circuit breaker.

After this general study was made, an attempt was made to reproduce the overvoltage conditions obtained in the first neutralizer application. As mentioned before, this neutralizer was a non-saturable core reactor. To reduce the amount of current flowing through it during conditions of normal operation, it was adjusted so as to be slightly out of tune and on a tap such that the charging current in the fault was undercompensated, that is, the neutralizer had a greater reactance than that required for precise tuning. Overvoltages were obtained during oil circuit breaker operations on this installation of sufficient magnitude to flash over the bus insulators.

For the miniature system selected for this study, the non-saturable core neutralizer was in tune with the system at 818 ohms. The voltage across the neutralizer, which is a direct measure of the voltage on the system, for phase *A* open at the source end of the system with a neutralizer in tune, is shown in figure 13, oscillogram 80-2. The voltage across the neutralizer for the same conditions but with a reactance of 854 ohms is shown in oscillogram 80-5, and for 935 ohms is shown in oscillogram 80-6. It is seen from these oscillograms that detuning the non-saturable core reactor in this manner gave rise to very high series resonant overvoltages. The same conditions were set up using a miniature system saturable core neutralizer (set on tap #9) and the voltage across the neutralizer is shown in oscillogram 80-7. The effectiveness of saturation in limiting series resonant overvoltages is at once apparent.

Out-of-Phase Switching

In the miniature system investigation of out-of-phase switching, the system of figure 3 was used with the load at *B* replaced by a source of generation which could be varied in phase position with respect to the source of generation at *A*. The overvoltages were recorded with the system neutrals at *A* and *B* grounded through ground-fault neutralizers, and also with them solidly grounded and isolated in turn.

Sustained voltages on this miniature system with neutralizer grounding obtained for various angular displacements of station *A* with respect to station *B* with conductor *a* open at *A* are shown in figure 14. Voltages for corresponding switching with conductors *b* and *c* open at *A* are shown in figure 15.

The variation of these switching voltages with different open conductor locations between stations *A* and *B* with neutralizer grounding was also investigated. In the cases studied, the distance between stations *A* and *B* was equivalent to 100 miles, although because of the symmetry existing on both sides of the 50 mile point, the results are plotted only for locations of the opening between station *A* and the midpoint of the line. With the opening at this point, the system is of course divided so that each part constitutes an independent ground-fault neutralizer system. For one conductor open refer to figure 16, for two conductors open refer to figure 17.

These curves indicate that angular displacements between two portions of a system can give rise to voltages as high as 3.5 times normal on a ground-fault neutralizer system. The significant fact disclosed by these curves is that if the open conductor location is such that on either side of the break there is a tuned system, no overvoltages of any consequence are obtained for even the worst condition of 180 degrees out-of-phase switching, except across the contacts of the breaker itself, which is to be expected regardless of the method of system grounding.

The significance of this portion of the study has no background of definite field

Figure 9. Sustained voltages obtained with phase a open in the system shown in figure 3

No load
Total 180 miles of line
GFN saturation factor $K=2.2$

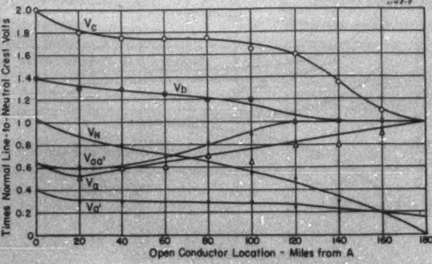
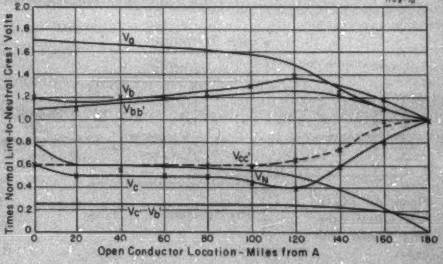


Figure 10. Sustained voltages obtained with phases b and c open in the system shown in figure 3

No load
Total 180 miles of line
GFN saturation factor $K=2.2$



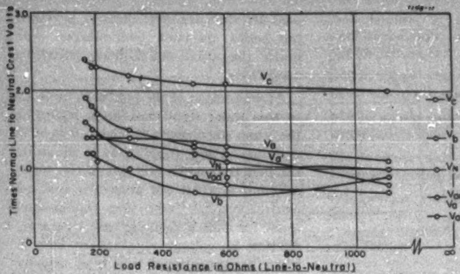


Figure 11. Effect of resistance load on sustained overvoltages for system shown in figure 3

Phase *a* open at *A*
180 miles of line
Saturation factor *K* = 2.2

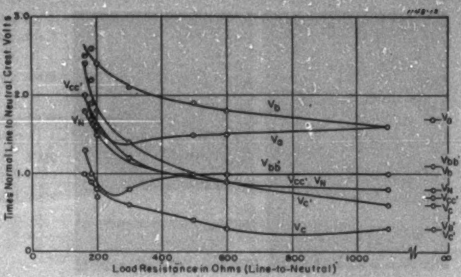


Figure 12. Effect of resistance load on sustained overvoltages for system shown in figure 3

Phases *b* and *c* open at *A*
180 miles of line
Saturation factor *K* = 2.2

experience, since there have been no clear indications that out-of-phase switching on ground-fault neutralizer systems has given rise to troublesome overvoltages. In order to make comparisons of the severity of these overvoltages on ground-fault neutralizer systems with systems using other methods of neutral grounding, the miniature system study was extended to include data for the same system with neutrals grounded solidly and later, ungrounded. The relative severity of out-of-phase switching in solidly grounded, isolated, and ground-fault neutralizer systems is shown in table I.

The results of this miniature system study show that out-of-phase switching in any system regardless of the method of connecting the neutral to ground may present severe breaker interrupting duty requirements. It is probable, however, that the worst conditions of out-of-phase switching as outlined here are seldom realized in practice.

IDEALIZED BREAKER OPERATION

The effect of arcing time in clearing a circuit under out-of-phase switching conditions is illustrated by the results in table II. In obtaining these results, a three-phase synchronous switch was used which interrupted all three phases of the circuit at adjacent current zeros without restriking—an ideal circuit interruption. Arcing time may be of importance since it determines the maximum possible interval of time between the opening of the first phase and the opening of the last phase. In particular, it is believed to be important when recovery voltages differ in severity for the first and last phases to clear. While short arcing time offers a distinct advantage in any system, this advantage is most apparent in the ground-fault neutralizer system, because it prevents occurrence of any voltages of the series resonant type which otherwise might arise from this cause. This follows from the previously mentioned fact that such voltages take several cycles to build up to their full magnitude.

Effect of Lightning Arresters

The lightning arrester consists of two component parts, a gap element and a valve element. For surge voltages below the gap breakdown voltage, the system response to these voltages is as though the arresters were not connected to the system. For surge voltages of sufficient magnitude to cause the gap to become conducting, the transient overvoltages are limited to the *IR* drop through the arrester valve element. It is through this valve element that the energy of the surge is dissipated.

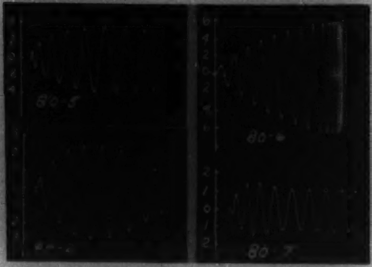
Arresters of the ungrounded neutral class are used on the isolated neutral, and the ground-fault neutralizer system. The minimum gap breakdown of this arrester is approximately 2.6 times the normal line-to-neutral crest voltage for the relatively slow transient switching voltages studied. The frequency of these voltages is for the most part less than ten times normal frequency. Arresters placed in the system neutrals to by-pass the neutralizers have a maximum permissible line-to-ground voltage rating equal to at least the system line-to-neutral voltage, and have a minimum gap breakdown voltage equal to approximately 1.5 times the line-to-neutral crest voltage.

The ability of neutralizer by-pass arresters to limit overvoltages is indicated in table III. This table lists three cases of overvoltages caused by 180° out-of-phase switching in ground-fault neutralizer systems. Case I lists the overvoltages obtained without any arresters on the system. The next case shows how these voltages were reduced at station *B* by a single arrester connected from the system neutral to ground by-passing the neutralizer. The third case shows how the overvoltages at both stations were reduced when each neutralizer was equipped with a by-pass arrester.

From this table it is clear that such by-

Table I. Relative Effects on Sustained Overvoltages of Open Conductors in Ground-Fault Neutralizer, Isolated Neutral, and Solidly Grounded Systems. Sources A and B 180° Out-of-Phase. (See Figure 3.) Conductors Opened at A

Miles of Line Between Sources	Type of System	$V_{N'}$	V_N	V_a	V_b	V_c	V_a'	V_b'	V_c'	$V_{aa'}$	$V_{bb'}$	$V_{cc'}$	Voltage Measured at Station
100	G.F.N. (K = 1.08)		1.8	1.6	2.3	2.0	3.3			3.0			A
		2.5		3.2	2.8	1.5							B
			1.7	1.0	2.5	0.8		2.5	2.6		3.2	3.3	A
		2.7		2.2	2.9	3.5							B
100	Isolated neutrals		1.0	1.9	0.2	0.8	1.0			2.8			A
		0		1.0	0.5	1.2							B
			2.0	1.0	2.7	2.7		1.0	1.0		3.3	3.3	A
		0		1.0	1.0	1.0							B
100	Solidly grounded		0	1.0	0.6	0.2	1.4			2.5			A
		0		1.0	0.7	1.1							B
			0	0.4	1.0	1.0		1.5	1.3		2.4	2.2	A
		0		1.0	1.0	1.0							B



100 miles of artificial line. Calibration: times normal line-to-neutral voltage

80-2 Air-core reactor of 818 ohms—correctly tuned

80-5 Air-core reactor of 874 ohms—undercompensated

80-6 Air-core reactor of 935 ohms-further undercompensated

80-7 Iron-core reactor, tap 9—correctly tuned

Figure 13. Transient voltage across the neutralizer of the system shown in figure 3 following an opening in one phase

Table II. Transient Voltages Produced by Idealized Three-Phase Switching at A (See Figure 3)—Sources A and B 180° Out-of-Phase

Miles of Line Between Sources	Type of System	$V_{N'}$	V_N	V_a	V_b	V_c	$V_{a'}$	$V_{b'}$	$V_{c'}$	$V_{aa'}$	$V_{bb'}$	$V_{cc'}$
100	G.F.N. ($K=1.08$)	1.5	1.7	3.0	2.0	1.5	1.7	2.2	2.2	3.5	4.0	3.2
100	Isolated	0	1.8	2.4	2.8	1.0	1.5	1.2	1.0	3.5	3.9	1.0
100	Grounded	0	0	1.6	1.5	1.6	1.9	1.6	1.6	2.6	2.5	1.9

Table III. Effect of Neutralizer By-pass Arresters in Limiting Overvoltages. See Figure 3. 100 Miles of Line—Sources A and B 180° Out-of-Phase, Phase a Open at A

Type of System	System Voltage	Remarks	$V_{N'}$	V_N	V_a	V_b	V_c	$V_{a'}$	$V_{aa'}$	Voltages at Station	Case
G.F.N.		No arresters		1.7	1.5	2.3	2.0	3.2	3.0	*A*	I
			2.5		3.2	2.8	1.3			*B*	
G.F.N.	230 kv	12 unit arrester at *N′*		1.1	1.9	0.7	1.1	1.5	2.7	*A*	II
			0.9		1.5	1.4	1.0			*B*	
G.F.N.	230 kv	12 unit arresters at *N* and *N′*		0.6	1.5	0.6	0.7	1.7	2.8	*A*	III
			0.7		1.7	1.2	0.5			*B*	

NOTE: In these ground-fault neutralizer systems, $K=1.08$. One coil is in the neutral at each source.

Table IV. Effect of Lightning Arresters in Limiting Overvoltages. See Figure 3. 100 Mile of Line—Sources A and B 180° Out-of-Phase—Phase a Open at A

Type of System	System Voltage	Remarks	$V_{N'}$	V_N	V_a	V_b	V_c	$V_{a'}$	$V_{aa'}$	Voltages at Station	Case
G.F.N.		No arresters		1.7	1.5	2.3	2.0	3.2	3.0	*A*	I
			2.5		3.2	2.8	1.3			*B*	
G.F.N.	230 kv	20 unit arrester on phase *a* at *B*		1.7	2.6	1.2	1.7	1.4	3.0	*A*	IV
			1.2		1.3	1.7	1.7			*B*	
Isolated		No arresters		1.0	2.0	0.2	0.9	1.0	3.0	*A*	V
			0		1.0	0.5	1.2			*B*	
Isolated	230 kv	20 unit arrester on phase *a* at *A*		1.1	1.5	1.1	1.1	2.0	3.2	*A*	VI
			1.2		2.1	1.7	1.0			*B*	

NOTE: In these ground-fault neutralizer systems, $K=1.08$. One coil is in the neutral at each source.

Table V. Effect of Lightning Arresters in Limiting Overvoltages. See Figure 3. 100 Miles of Line—Sources A and B 180° Out-of-Phase—Phase a Open at A

Type of System	System Voltage	Remarks	$V_{N'}$	V_N	V_a	V_b	V_c	$V_{a'}$	$V_{aa'}$	Voltages at Station	Case
G.F.N.		No arresters		1.7	1.5	2.3	2.0	3.2	3.0	*A*	I
			2.5		3.2	2.8	1.3			*B*	
G.F.N.	230 kv	20 unit arrester on phase *a* at *B*		1.7	2.6	1.2	1.7	1.4	3.0	*A*	IV
			1.2		1.3	1.7	1.7			*B*	
G.F.N.	46 kv	4 unit arrester on phase *a* at *B*		1.7	2.7	1.0	1.7	1.0	3.0	*A*	VII
			1.0		1.0	1.5	1.7			*B*	

NOTE: In these ground-fault neutralizer systems, $K=1.08$. One coil is in the neutral at each source.

pass arresters are very effective in reducing series resonant overvoltages even under the worst out-of-phase switching conditions.

An inspection of the results in table IV reveals some significant facts regarding the same phenomena in isolated neutral systems. These cases serve to illustrate the fact that while voltages of the series resonant type are not present in isolated neutral systems (note that voltages are lower for case V than for case I), the reduction of voltage obtained in case V when a conventional line type arrester sparks over (case VI) is relatively slight (from 2.0 to 1.5 at the arrester location). A comparison of the voltages shown for cases I and IV shows that the voltage at the arrester location was reduced from 3.2 to 1.3. In other words, after spark-over, the voltage across the arrester tends to be greater in the isolated neutral system than in the same system equipped with neutralizers. This emphasizes one important characteristic of series resonant voltages, namely, that they are inherently of very poor regulation. That is, as soon as any resistance path for current to flow to ground is introduced at the point of highest voltage (e.g., an arrester), the voltage collapses quickly at this point to a value below the arrester rating. It possibly also explains in part the fact that arrester troubles on some ground-fault neutralizer systems have been observed to be greatly reduced over what they were when the system was operated with neutrals isolated.

These results indicate also that in systems operating with only partial arrester protection, e.g., arresters at only one end of the line, there is a tendency to "shift" the overvoltages from the protected area to the unprotected area. That is, after an arrester gap has sparked over, voltages are reduced in the vicinity of that arrester, but may become higher in remote portions of the system than they were before the arrester gap broke down. This is another good reason for applying arresters to all apparatus on a system. From an inspection of the results in table V, it is apparent that arresters are more effective in lower voltage systems than in higher voltage systems as far as reducing overvoltages of the type described here is concerned. (Compare values of V_a at B, the voltage across the arrester, for the three cases in table V.) This is explained by the fact that an arrester will pass approximately the same follow current at rated voltage regardless of the arrester rating. Series resonant overvoltages are intimately associated with transmission line constants which are relatively independ-

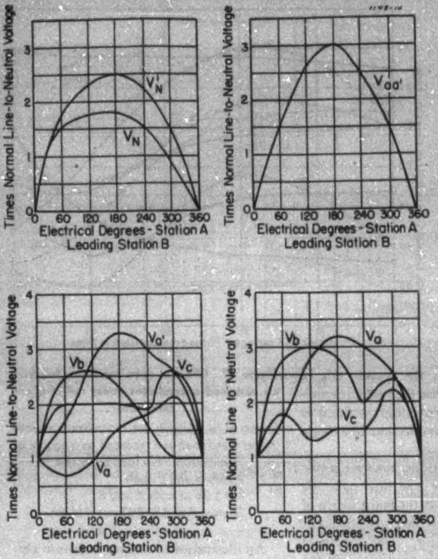


Figure 14. Sustained overvoltages caused by an open conductor during out-of-phase conditions

System as in figure 3, but neutralizers at both *A* and *B* and load at *B* replaced by a source of generation. No load on system. Phase *a* open at *A*.

$K=1.08$ at both *A* and *B*

(*a*) Voltages at the neutrals
(*b*) Voltage across the opening in phase *a*
(*c*) Phase voltages to ground at *A*
(*d*) Phase voltages to ground at *B*

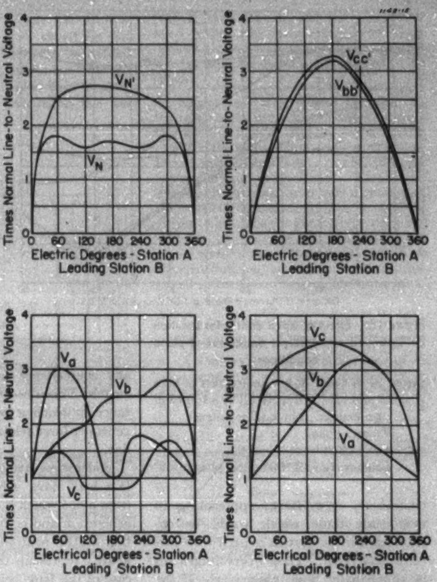


Figure 15. Sustained overvoltages caused by two open conductors during out-of-phase conditions

System as in figure 3, but neutralizers at both *A* and *B* and load at *B* replaced by a source of generation. No load on system. Phases *b* and *c* open at *A*

$K=1.08$ at both *A* and *B*

(*a*) Voltages at the neutrals
(*b*) Voltages across the openings in phases *b* and *c*
(*c*) Phase voltages to ground at *A*
(*d*) Phase voltages to ground at *B*

ent of voltage. Therefore, the effective impedance of an arrester is lower in a low voltage system, and the arrester is thus more effective in reducing such overvoltages. It is important to note that voltages across the valve element are always low after an arrester has sparked over, and therefore regardless of system voltage, the arrester gap spark-over voltage is the factor limiting instantaneous overvoltage magnitude.

In interpreting the results of tables III, IV, and V, it is important to bear in mind the fact that the extremely severe condition of 180° out-of-phase switching has been considered. Furthermore, a relatively low neutralizer saturation factor ($K=1.08$) was used. As a general conclusion following these and numerous other similar tests, it may be said that in most practical open-conductor cases an arrester will not be required to function. If, in extreme cases, such as these under discussion, it should be required to function as a result of overvoltages reaching the gap spark-over value, it is apparent that such overvoltages are immediately and effectively reduced to harmless values.

Since it is evident that series resonant voltages of the type under discussion, if produced at all, must be in evidence at the system neutral (i.e., across a neutralizer) an arrester selected on the basis of system normal line-to-neutral voltage plus whatever factor is deemed necessary to avoid the possibility of overstressing the arresters during a single line-to-ground fault condition, should afford excellent protection against the most severe overvoltages of this type. Such an arrangement in no way affects the normal operation of the neutralizer. Every ground-fault neutralizer in this country, except the first one, has been equipped with a shunting by-pass arrester mounted on the neutralizer tank and it undoubtedly has been a contributing factor to the absence of any series resonant overvoltages in subsequent neutralizer installations.

Switching Off Neutralizers

When a system is to be equipped with more than one neutralizer, it is desirable that the neutralizers be located in the system so that they, and the system sections to which they are tuned, can be switched as units. It is recognized that some switching operations will inevitably detune the neutralizers. Staged tests on ground-fault neutralizer equipped systems have demonstrated that successful arc extinction may be obtained even if the system capacitance is as much as 25% overcompensated and 10% undercompensated.

The undercompensated condition, that is, when the neutralizer impedance is greater than that required for exact tuning, results in making the system line-to-ground impedance capacitive rather than inductive. This is an approach to the condition of isolated neutral operation. If a ground fault should occur under these

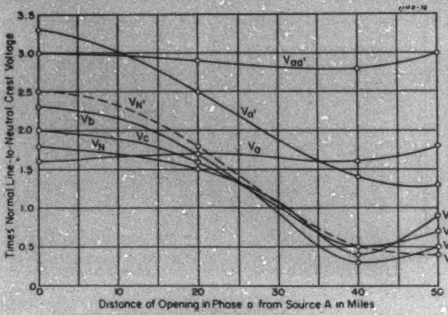


Figure 16. Effect of open conductor location on sustained overvoltages under out-of-phase conditions

System as in figure 3, but neutralizers at both *A* and *B* and load at *B* replaced by a source of generation. No load on system
Phase *a* open
$K=1.08$ at both *A* and *B*
Sources *A* and *B* 180° out of phase

conditions, overvoltages approaching in magnitude those which would be encountered if the system neutral were isolated may occur. These overvoltages can be abnormally high, particularly if the system is large in extent and a large share of the generation is lost during such switching operation.[10] Furthermore, in the event of an open conductor, saturation of the excessive neutralizer reactance, instead of detuning the system would actually tend to bring the system closer to a resonant condition.

It has long been recognized by operating companies using neutralizers that to obtain the full benefit of the neutralizer protection, continued operation in the detuned condition is to be avoided, because in addition to the overvoltages encountered in the above-mentioned undercompensated condition, the arc may not be extinguished. Diligence in always keeping the neutralizers correctly tuned to the connected system capacitance reduces the chances to the minimum that dangerous overvoltages will be encountered, and insures that the maximum benefits in service continuity will be realized. The latest design of neutralizers has a motor mechanism for changing the taps remotely which is an aid in keeping the neutralizers in tune.

Effect of Bus Capacitance

On typical systems there would usually be several lines radiating from a bus. Switching off any one of these lines would not be expected to cause overvoltages of any consequence on the system side of switch. This is because the system capacitance to which a neutralizer is tuned is generally large relative to that of a single line.

Under some conditions, however, a system may have only one line radiating from a bus which is at the main source of generation and is, furthermore, the location of the neutralizer. The system of figure 3 represents this condition. In the case of the fault being cleared on such a line, the circuit would look like that in figure 18 for the last phase to clear, assuming the faulted phase clears last. Under such conditions certain values of circuit constants may give rise to overvoltages provided the interrupting device has characteristics which permit restriking and reclearing of the arc to occur in a certain sequence. Such phenomena have been investigated and reported for a wide range of circuit constants, and criteria have been presented for eliminating these overvoltages.[8,9]

A curve of overvoltages which may be obtained under such conditions is shown in figure 19. In this case the ground-fault neutralizer was tuned to approximately 90 miles of line. Generation was such that the ratio of zero sequence to positive sequence reactance in the source was 30. The capacitance connected to the bus was varied. It is of particular interest to note that for the last line being switched off a bus, the transformer winding capacitance to ground is of the right order of magnitude to give rise to this phenomenon.

While it is not necessarily true that all circuit interrupters will give rise to a restriking sequence, the fact that the constants may be such as to render this circuit susceptible to these phenomena suggest that in switching the last line from a bus, protective equipment may be called into operation.

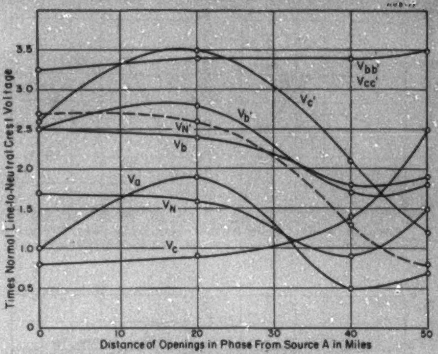


Figure 17. Effect of open conductor location on sustained overvoltages under out-of-phase conditions

System as in figure 3, but neutralizers at both *A* and *B* and load at *B* replaced by a source of generation. No load on system
Phases *b* and *c* open
$K=1.08$ at both *A* and *B*
Sources *A* and *B* 180° out of phase

An illustration of this phenomenon giving rise to arrester spark-over is shown in figure 20. For this case, the bus capacitance was equivalent to that of approximately one mile of overhead line. The arrester gap in phase *A* was set to spark over at about 1.5 times the crest arrester rating. The oscillograms in figure 20 show the gap spark-over and subsequent discharge to ground and arrester discharge voltage.

A detailed analysis of these oscillograms and others of a similar nature indicate that a conventional arrester is very effective in limiting the magnitude of the overvoltages of this type as well as others previously mentioned. Either the conventional line-type arrester or an arrester with a line-to-ground voltage rating connected across each neutralizer should give good protection and experience is substantiating this contention.

It is to be noted that the energy to be dissipated in this type of overvoltage is relatively small for circuit constants likely to give rise to this phenomenon in ground-fault neutralizer systems. Furthermore, the overvoltages are relatively short in time duration with the result that the arrester duty is comparatively light.

Other Factors

The size of system (length of line) is relatively unimportant from the standpoint of possible overvoltages. Line losses, at least up to the point where the

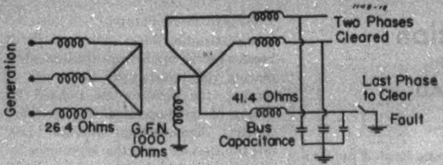


Figure 18. Equivalent circuit for last phase to clear a fault near a bus when no other lines are connected to that bus

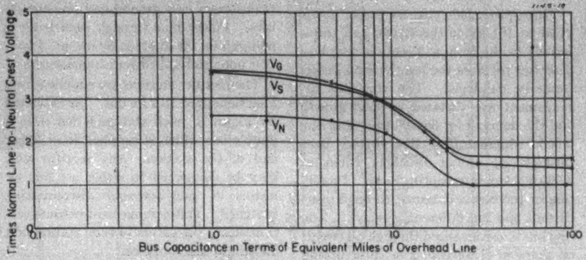


Figure 19. Maximum transient voltages which may be obtained as a result of restriking in the switch for the system shown in figure 18

V_S—Maximum voltage across switch
V_G—Maximum voltage from phase to ground
V_N—Maximum voltage at the neutral

line resistance is equal to the line reactance are also only of minor importance.

Source reactances (positive and negative sequence) have only minor effects on the general phenomenon. This is to be expected because it is mainly the zero sequence circuit that is affected, and practically all of the zero sequence reactance is in the neutral or neutrals.

Waterwheel overspeeding increases the frequency and in ground-fault neutralizer systems tends to make the system zero sequence reactance capacitive viewed from any fault location. If a line-to-ground fault were accompanied by a considerable amount of generator overspeeding, dynamic voltages to ground would be increased and might reach magnitudes sufficiently great to damage protective equipment.

Conclusions and Summary

As a result of this investigation, a number of conclusions have been drawn. In evaluating these, however, it is important to bear in mind that while these and other investigations have shown that it is possible for overvoltages to exist in various types of power systems, it was necessary in many instances to study extreme cases in order to obtain any that were dangerously high. In an actual system, therefore, the probability of the occurrence of dangerous overvoltages is very small. The best evidence of this with respect to the ground-fault neutralizer system is the excellent trouble-free operation to date of more than 50 neutralizers in service in the United States alone to say nothing of the great number operating satisfactorily elsewhere.

1. Saturation in ground-fault neutralizers is generally very effective in preventing the occurrence of dangerously high overvoltages.

2. With a given neutralizer saturation characteristic, the smaller the current interrupted by an opening in one or two conductors of a line, the lower will be the voltages obtained.

3. While tests and operating experience show that successful arc extinction is obtained even when neutralizers are somewhat out of tune, care should be taken to avoid having the neutralizer reactance higher than the "tuned" value.

4. The number and location of ground-fault neutralizers on a system is relatively unimportant as far as resonant overvoltages are concerned. Operating experience indicates, however, that in an extensive system better service continuity may be obtained if more than one neutralizer is used.

5. When one or two conductors are open on a given line, loads on the remainder of the system do not appreciably affect the overvoltages resulting. However, the more lines there are radiating from the source (that is, the more extensive the system) the lower will be the series resonant voltages, because the ratio of current interrupted to rated neutralizer current will, in general, be smaller.

6. Source reactance, which includes the reactance of the grounding transformer or of the power transformer bank in the neutral of which the ground-fault neutralizer is connected usually has only a slight effect on series resonant overvoltages. This reactance normally constitutes only a small part of the total reactance required for neutralization, but since in general, it does not saturate, its tendency is to produce slightly higher overvoltages than would occur if all the neutralizing reactance were concentrated in the saturable ground-fault neutralizer.

7. Line resistance has practically no effect on the magnitude of resonant overvoltages.

8. A fault-to-ground on either side of an open conductor will reduce resonant overvoltages to harmless values. In practice this is the condition which would most probably occur in event of a broken line conductor, since it is improbable that a broken conductor could avoid contact, either directly or indirectly with ground.

9. Unequal pole openings of circuit breakers is undesirable if the interruption in the different phases is more than 3 or 4 cycles apart. Short arcing time and a minimum tendency to restrike are desirable.

10. Fuses or other single pole interrupting devices should not be used on the lines of a ground-fault neutralizer system. Such devices may, however, be used in the terminals of load transformers.

11. Lightning arresters, connected in the system neutral or to the lines, are effective in limiting transient or resonant overvoltages to the gap spark-over levels of the arresters.

A survey of these conclusions, together with the steadily growing number of successful ground-fault neutralizer installations, establishes the fact that practical and effective means have been found to eliminate difficulties with overvoltages. With the modern designs of ground-fault neutralizers, and with proper attention to their application, the record of trouble-free operation and effectiveness in service protection should continue to be excellent. On a ground-fault neutralizer system having only one radial line, the voltages occurring across a breaker may be appreciably higher than if the system

Arrester current
Arrester voltage

Figure 20. Oscillograms of arrester current and voltage following sparkover

Calibrations:
Voltage: times normal line-to-neutral crest voltage
Arrester current: times crest current corresponding to discharge at system normal voltage

Alcoa Rectifier Installation

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IN 1938, 15 mercury arc rectifier units, with a total capacity of 82,500 kw, were placed in service by the Aluminum Company of America to supply d-c power for the electrolytic production of aluminum.[1] In the plant at Alcoa, Tenn., ten of these units were installed to supply two 45,000 ampere, 600 volt, d-c circuits. In the plant at Massena, N. Y., five units were installed to supply one such circuit.

These installations are noteworthy as they marked the first application of power rectifiers in this country for the electrolytic production of metal. At that time the Alcoa installation was the largest concentration of rectifiers in the country, with the highest kilowatt capacity, installed in a single station and operating on a common bus. Since 1938, one unit was added to each station in space reserved for it. At Alcoa twelve additional units were installed in a second station; six of these were placed in service in May 1940, and six in January 1941.

This paper will describe the Alcoa rectifier installations. The Massena station is practically identical with the first Alcoa installation except for the number of units.

Power Circuits

A single-line diagram of the power circuits for the Alcoa rectifier installations is shown in Figure 1. The stations are designated as number 3 and number 4. The buildings are indicated by dotted lines.

The 13.2 kv, 3-phase, 60-cycle, a-c power is supplied to the station busses over four lines from a number of 42,000 kva transformer banks located approximately 100 feet away.

Three-phase, 60-cycle, a-c power is supplied at 161 kv to the 42,000 kva transformer banks from a 4-section ring bus. This bus replaces the longitudinal busses formerly employed. The ring bus arrangement was dictated by load growth and the demand for flexibility. The ring bus is supplied from five 161 kv lines through oil circuit breakers rated at 2,500,000 kva interrupting duty. Protection of the bus is obtained through overcurrent and bus differential relays. The ring bus is sectionalized through oil circuit breakers of 2,500,000 kva interrupting duty and 8-cycle timing. Transformer and 13.2 kv bus protection is provided by bus differential, overcurrent, transformer case ground and pressure relays.

The 42,000 kva wye-delta transformer banks are connected to the ring bus in groups of two to four banks per section. The high-tension sides of the transformers are disconnected from the bus by means of gang-operated, horizontal double-break disconnecting switches. The disconnecting switches are provided with arcing tips to break the magnetizing current. Some of these transformers are fan-cooled for emergency overload operation. The 13.2 kv sides of the transformer banks are provided with oil circuit breakers of 1,500,000 kva interrupting duty. These oil circuit breakers serve to interrupt the load when placing the transformers in or out of service; the breakers are isolated by means of gang-operated, high-pressure contact, disconnecting switches. Low side transformer protection is provided by instantaneous and time overcurrent, transformer case ground and pressure relays. Power flow through each bank is metered on the low side by means of suitable potential and current transformers.

The rectifier stations are supplied from a sectionalized 13.2 kv bus. As indicated by Figure 1, each station is fed from a double bus which is supplied from either end of the station. Any rectifier unit may be connected to either a-c bus by means of gang-operated disconnecting switches. This arrangement permits sectionalizing the halves of each station and facilitates maintenance.

Inasmuch as the rectifiers feed a continuous electro-chemical load, continuity of supply is essential. Consideration of this factor has influenced the arrangement of busses and equipment as described above. Under the existing arrangement, individual items of equipment, and in certain instances complete bus sections, may be isolated as necessary for maintenance or repairs without interruption to the station supply. At the stations a spare transformer is available and each station has sufficient capacity to permit the isolation of any one complete unit. A constant source of d-c power for operation of controls is insured by making the storage battery at each station available to another station or stations.

A comprehensive program of preventive maintenance has proved an important factor in minimizing power interruptions resulting from equipment failures. One feature of this program is the regular testing of the power factor of insulation.

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neutral were solidly grounded, but no more severe than if the system neutral were completely isolated. However, for the network or multi-circuit type of ground-fault neutralizer system, the breaker voltages are correspondingly less severe than for a single radial line.

References

1. OPERATING PERFORMANCE OF A PETERSEN COIL, **J. M. Oliver and W. W. Eberhardt.** AIEE TRANSACTIONS, volume 42, 1923, page 435.

2. TEST AND OPERATION OF PETERSEN COIL ON 100-KV SYSTEM OF PUBLIC SERVICE COMPANY OF COLORADO, **W. D. Hardaway and W. W. Lewis.** AIEE TRANSACTIONS, volume 57, 1938 (June section), page 295.

3. PETERSEN COIL TESTS ON 140-KV SYSTEM, **J. R. North and J. R. Eaton.** AIEE TRANSACTIONS, volume 53, 1934 (January section), page 63.

4. OPERATING EXPERIENCE WITH PETERSEN COILS ON 66 KV-SYSTEM OF METROPOLITAN EDISON COMPANY, **H. M. Rankin and R. E. Neidig.** AIEE TRANSACTIONS, volume 58, 1939 (November section), page 569.

5. SOME ENGINEERING FEATURES OF PETERSEN COILS AND THEIR APPLICATION, **E. M. Hunter.** AIEE TRANSACTIONS, volume 57, 1938 (January section), page 11.

6. POWER-SYSTEM TRANSIENTS CAUSED BY SWITCHING AND FAULTS, **R. D. Evans, A. C. Monteith, and R. L. Witzke.** AIEE TRANSACTIONS, volume 58, 1939 (August section), page 386.

7. ABNORMAL VOLTAGE CONDITIONS IN THREE-PHASE SYSTEMS PRODUCED BY SINGLE-PHASE SWITCHING, **Edith Clarke, H. A. Peterson, and P. H. Light.** AIEE TRANSACTIONS, volume 60, 1941, page 329.

8. ARCING FAULTS IN POWER SYSTEMS, **C. Concordia and H. A. Peterson.** AIEE TRANSACTIONS, volume 60, 1941, page 340.

9. EFFECT OF RESTRIKING ON RECOVERY VOLTAGE, **C. Concordia and W. F. Skeats.** AIEE TRANSACTIONS, volume 58, 1939 (August section), page 371.

10. OVERVOLTAGES DURING POWER-SYSTEM FAULTS, **Edith Clarke, S. B. Crary, H. A. Peterson.** AIEE TRANSACTIONS, volume 58, 1939 (August section), page 377.

11. SIMULTANEOUS FAULTS ON THREE-PHASE SYSTEMS, **Edith Clarke.** AIEE TRANSACTIONS, volume 50, 1931 (September section), page 919.

12. AN ELECTRIC-CIRCUIT TRANSIENT ANALYZER, **H. A. Peterson.** *General Electric Review*, September 1939, page 394.

13. AUXILIARY EQUIPMENT FOR GROUND-FAULT NEUTRALIZERS, **E. M. Hunter and S. B. Farnham.** *General Electric Review*, September 1940, page 352.

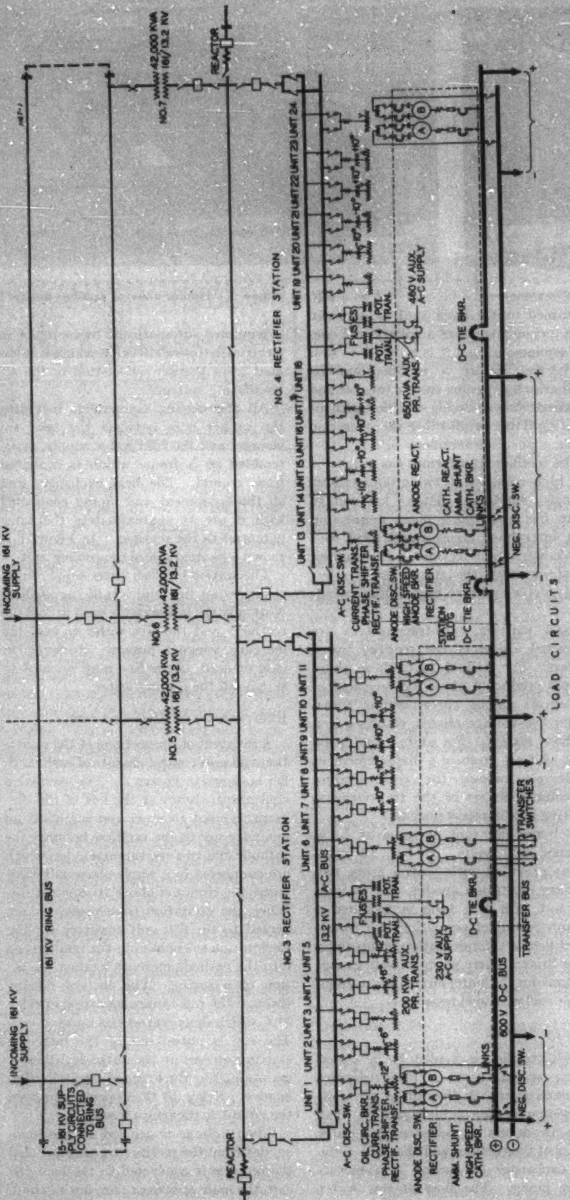

INCOMING 161 KV SUPPLY
161 KV RING BUS
15-161 KV SUPPLY CIRCUITS CONNECTED TO RING BUS
REACTOR
NO.5 42,000 KVA 161/13.2 KV
NO.6 42,000 KVA 161/13.2 KV
NO.7 42,000 KVA 161/13.2 KV
NO. 3 RECTIFIER STATION
NO. 4 RECTIFIER STATION
UNIT 1 UNIT 2 UNIT 3 UNIT 4 UNIT 5 UNIT 6 UNIT 7 UNIT 8 UNIT 9 UNIT 10 UNIT 11
UNIT 13 UNIT 14 UNIT 15 UNIT 16 UNIT 17 UNIT 18 UNIT 19 UNIT 20 UNIT 21 UNIT 22 UNIT 23 UNIT 24
13.2 KV A-C BUS
FUSES
POT. TRAN.
200 KVA AUX. PR. TRANS.
230 V AUX. A-C SUPPLY
650 KVA AUX. PR. TRANS.
480 V AUX. A-C SUPPLY
A-C DISC. SW.
OIL CIRC. BKR.
CURR. TRANS.
PHASE SHIFTER
RECTIF. TRANSF.
ANODE DISC. SW.
RECTIFIER
AMM. SHUNT
HIGH SPEED CATH. BKR.
CURRENT TRANS.
HIGH SPEED ANODE BKR.
ANODE REACT.
CATH. REACT.
CATH. BKR.
LINKS
NEG. DISC. SW.
600 V D-C BUS
TRANSFER BUS
TRANSFER SWITCHES
D-C TIE BKR.
STATION BLDG
LOAD CIRCUITS

Figure 1. Single-line diagram of the rectifier stations and the power-supply system

Each major item of equipment, including transformers, oil circuit breakers and bushings, is tested when installed, or overhauled, and annually thereafter. Any indication of faulty insulation is investigated immediately and corrected. Failures have been averted by correcting faulty conditions in the initial stage. The problem of transformer maintenance has been simplified by close attention given to the oil.[3] Activated alumina bypass treaters are used for the treatment of oil; similarly activated alumina is used for treating bushing oil. Activated alumina breathers have been successful in excluding air-borne moisture from transformers, thereby maintaining a high dielectric property of the oil.

The a-c switching equipment, phase-shifting auto-transformers, rectifier transformers, anode disconnecting switches, negative disconnecting switches, and d-c busses are located outdoors. Stranded all-aluminum cables are used for the connecting lines between the a-c station busses and the main 13.2 kv bus. The a-c busses are made of aluminum channels. The d-c busses are constructed of aluminum bars. Aluminum tubing, bars and channels are used for the power circuit connections between the component parts of the equipment.

The d-c busses are suspended from brackets on one wall of the station building. The d-c load circuits are connected solidly to the positive and negative busses. The negative bus runs the full length of each station with a solid tie between the busses of the two stations. The positive bus is sectionalized in the middle of each station through a bus-tie breaker. The positive busses of the two stations are connected through a tie breaker to permit transfer of power between the stations if desired.

Rectifier Station 3

Equipment and Layout

A view of the outdoor equipment of station 3 is shown in Figure 2. A view of the station interior is shown in Figure 3. All equipment is located on one floor.

The station is divided into two sections, each containing five units, with a spare unit (unit 6) in the center which may be operated on either section. Near the center of the station, outdoors, are located two sets of potential transformers and two 200 kva, 3-phase transformers for supplying 230 volt power to the rectifier auxiliaries and station lights. One auxiliary power transformer and one set of potential transformers are connected to each a-c bus.

Figure 2. View of a-c switching equipment and transformers, rectifier station 3

Figure 3. Interior view of rectifier station 3

At the center of the station inside the building are located the control benchboard, a switching cubicle for the 230 volt auxiliary power supply, and a 600 volt d-c bus-tie switch. On the benchboard are mounted the circuit breaker control switches and meters for the incoming lines, the rectifier units, and the load circuits. Control switches are also provided for regulating the load current of individual rectifiers by grid control.

The connections between the neutral bushings of the rectifier transformers and the negative bus are run through the building below the ceiling. The connections between the cathodes and the d-c breakers are located in floor trenches.

A diagram of connections of one rectifier unit is shown in Figure 4. Each rectifier unit consists of the following equipment: two 3-pole, a-c disconnecting switches; a 3-pole solenoid-operated oil circuit breaker; three current transformers; a phase-shifting auto-transformer, if required; a 3-phase rectifier transformer; two steel-tank, 12-anode, grid-controlled rectifiers; two sets of cooling equipment for the rectifiers; a twin cubicle containing the rectifier excitation and grid-control equipment; a 2-panel switchboard on which are mounted the vacuum meters, the a-c overload relays and the control equipment for the rectifier auxiliaries; two solenoid-operated, 6,000 ampere, high-speed d-c breakers which will clear in approximately 0.75 cycle on a 60-cycle basis; two 6,000 ampere negative disconnecting switches located outdoors and operated by means of a lever inside the station building; and 12 anode disconnecting switches. The cubicles and cooling equipment are located at the rear of the rectifiers. The switchboards are located along the aisle in front of the rectifiers.

The rectifier units are so designed that between the transformer and d-c bus each rectifier may be operated as a self-contained unit. Each tank is provided with a complete set of auxiliary and control equipment, and it is possible to shut down and isolate a rectifier and its auxiliaries by opening the d-c breaker and the anode disconnecting switches, without shutting down or affecting the operation of the companion rectifier.

The rectifier transformer has a delta or wye high-voltage and two groups of low-voltage windings. Each low-voltage group has a double 6-phase connection with interphase transformers, and supplies power to one rectifier. The rectifier transformers in the station have alternately delta and wye high-voltage windings.

Phase shifting auto-transformers are connected ahead of the rectifier transformers. For units 1 to 5, these phase shifters produce a phase displacement of 12 degrees among corresponding transformer low-voltage phases, resulting in a 30-phase system. For units 6 to 11, the phase shifters produce a displacement of 10 degrees among the corresponding low-voltage phases of the transformers, resulting in a 36-phase system.

Each rectifier tank is rated at 4,580 amperes, 600 volts. It has 12 main anodes provided with control grids, two auxiliary excitation anodes, and an ignition rod. On the tank is mounted a vacuum pump set consisting of a water-cooled mercury condensation pump and a motor-driven rotary pump, also an expansion tank for the water circulating system, and a panel with grid resistors.

Cooling System

Each rectifier is cooled by a closed cooling system using a water-to-water heat exchanger. A diagram of the cooling system is shown in Figure 5. The water is circulated between the water jackets of the rectifier and the shell of the heat exchanger by means of a motor-driven pump. The flow of raw water through the tubes of the heat exchanger is regulated automatically by means of an electrically-operated valve which is actuated by a thermostat located in the recirculating system.[3]

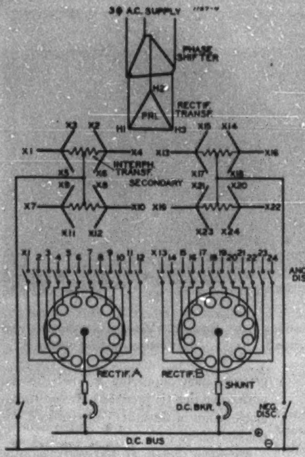
All the cooling equipment, including the rubber hose between the heat exchanger and the cold water supply, is assembled on a frame which is insulated from ground. The heat exchanger and all the equipment and piping connected with it are at approximately the same potential as the rectifier. In Figure 6 is shown a photograph of the cooling unit.

The station has two water supply lines, a city water line and a mill supply line. Normally the mill water is used to cool the rectifiers and the city water to cool the mercury vacuum pumps. However, in case of need, either line may be used to supply all the cooling water.

Excitation and Grid Control

A diagram of connections of the excitation and grid control circuits of each rectifier is shown in Figure 7. The excitation equipment, shown at the left of the diagram, is used to start and maintain an auxiliary arc in the rectifier between the cathode and two excitation anodes, which are connected in a single-phase full-wave rectifying circuit of about 10 amperes d-c. When the excitation transformer is connected to the 230 volt auxiliary supply, the ignition solenoid pulls the ignition rod into the cathode mercury against the action of a spring. The current flowing through the rod causes operation of relay 31*X* which de-energizes the ignition coil. The rod is pulled out of the mercury, starting an arc at the cathode which is maintained by the fixing of the excitation anodes. Relay 53 then operates to open the circuit to the ignition rod.

When voltage is applied to the main anodes from the rectifier transformer and the rectifier is connected to the load circuit, the main anodes can fire during those parts of the voltage cycle when they are


3φ A.C. SUPPLY
PHASE SHIFTER
H2
RECTIF. TRANSF.
PRI.
H1
H3
X3
X2
X15
X14
X1
X4
X13
X16
INTERPH. TRANSF.
X5
X6
X17
X18
SECONDARY
X9
X8
X21
X20
X7
X10
X19
X22
X11
X12
X23
X24
X1 2 3 4 5 6 7 8 9 10 11 12
X13 14 15 16 17 18 19 20 21 22 23 24
ANODE DISC.
RECTIF. A
RECTIF. B
SHUNT
D.C. BKR.
NEG. DISC.
D.C. BUS
⊕
⊖

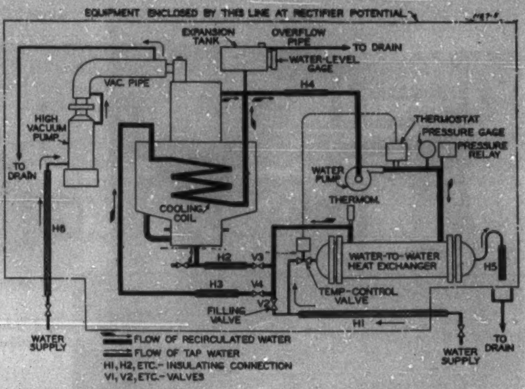

EQUIPMENT ENCLOSED BY THIS LINE AT RECTIFIER POTENTIAL
EXPANSION TANK
OVERFLOW PIPE
TO DRAIN
WATER-LEVEL GAGE
VAC. PIPE
H4
HIGH VACUUM PUMP
TO DRAIN
THERMOSTAT
PRESSURE GAGE
PRESSURE RELAY
WATER PUMP
THERMOM.
COOLING COIL
H6
H2
V3
WATER-TO-WATER HEAT EXCHANGER
H5
H3
V4
V2
TEMP-CONTROL VALVE
FILLING VALVE
H1
WATER SUPPLY
FLOW OF RECIRCULATED WATER
FLOW OF TAP WATER
H1, H2, ETC.- INSULATING CONNECTION
V1, V2, ETC.- VALVES
WATER SUPPLY
TO DRAIN

Figure 4 (left). Diagram of connections of a rectifier unit in station 3

Figure 5. Diagram of cooling system for rectifier in station 3

positive with respect to the cathode. The start of the firing of each anode is controlled by its grid. An anode can start firing only when the grid is at a positive potential to the cathode. During the firing period of the anode the grid has no controlling effect. It is at the potential of the arc, and carries a small current as determined by the voltage and impedance of its external circuit. It can regain control at the conclusion of the anode firing period. This property of the grid is used for controlling the output voltage of each rectifier. The rectifier voltage may be reduced by retarding the start of the anode firing during each cycle. The anodes may be blocked from firing altogether by applying to the grids a negative potential.

In the grid control circuit of Figure 7, provision is made for regulating the rectifier d-c voltage and for applying to the grids a high negative potential to block the anodes upon occurrence of a backfire in the rectifier. The grids are connected through resistors to the 6-phase secondary winding of a grid transformer, the primary winding of which is connected to a 3-phase circuit. The neutral point of the grid transformer secondary is connected to the cathode over a grid bias circuit which is supplied by a 3-phase full-wave selenium-type rectifier. The voltage between each grid and the cathode thus consists of an a-c component and a variable d-c component. The point of the cycle at which the grid-to-cathode voltage changes from negative to positive, to permit the pick-up of its associated main anode, is determined by the relative values of the a-c and d-c components. This point can be retarded by increasing the negative d-c bias voltage, which can be adjusted by means of a motor-operated grid control rheostat.[4]

For the units provided with a phase shifter on the high-voltage side of the rectifier transformer, a small auto-transformer type phase shifter is connected in series with the primary winding of the grid transformer, with the same angle of shift as the main phase shifter, in order to maintain the same phase relation between the grid and anode voltages for all the units. A fixed resistor, *Rm*, connected between the end of the grid control rheostat and the cathode, provides a small negative bias to keep the anodes blocked when the supply to the grid transformer is open.

Relays 77*T* and 77*G* are used for grid blocking of the anodes. Each relay has two coils which are connected to current transformers in two phases of the a-c supply line to the rectifier transformer. Upon occurrence of a backfire, the increase of current in the a-c line causes operation of these relays. Relay 77*T* short-circuits the primary winding of the grid transformer, thus removing the a-c components of the grid voltage. Relay 77*G* connects the neutral of the grid transformer to the negative terminal of the bias supply circuit, increasing the negative bias on the grids. One contact of this relay inserts additional resistance in the grid bias circuit to prevent overloading of the auxiliary rectifier. Short-circuiting of the grid transformer increases the current through the sealing current transformer, which holds the relays in the operated position. It also increases the voltage on the coil of the auxiliary relay 78, causing it to operate. The contacts of this relay take over the function of relays 77*T* and 77*G* and permit them to reset by short-circuiting the sealing current transformer. Relay 78 also trips the oil circuit breaker of the unit. Tripping of the circuit breaker resets relay 78 by short-circuiting its coil through an auxiliary switch.

PROTECTION

The individual rectifier units are protected against overload by induction type overcurrent relays, connected to current transformers on the high-voltage side of the rectifier transformer. These relays are connected to trip the oil circuit breaker of the unit. The rectifiers are protected against backfire by the high-speed cathode breakers which are provided with reverse current trips, and by the grid-blocking relays. Upon occurrence of a backfire, current flows to the faulty anode from the transformer low-voltage winding through other anodes of the rectifier, and from the d-c bus through the cathode. The grid-blocking equipment stops the flow of current from the other anodes, and the tripping of the cathode breaker interrupts the flow of current from the d-c bus.

Operation of the grid-blocking relays or automatic tripping of the cathode breaker also trips the unit oil circuit breaker, in order to insure complete interruption of current flow from the other anodes of the rectifier to the faulty anode. The unit is returned to service immediately after tripping of the breakers. A reverse current relay is connected to the ammeter shunt of each rectifier. These relays are

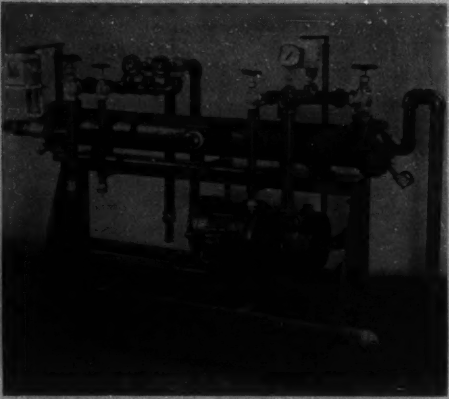


Figure 6 (left). Cooling unit for rectifier in station 3

Figure 8. View of a-c structure and transformers, rectifier station 4

connected to trip the cathode breakers of all the rectifiers operating in parallel on the d-c bus, and are set to operate only in the event that the reverse current flowing from the d-c bus to the backfiring rectifier is not cleared successfully by its cathode breaker. This back-up protection was considered essential in view of the number of rectifiers connected in parallel to the d-c bus.

Rectifier Station 4

Rectifier station 4 differs from station 3 in several respects, particularly in regard to the switching and control equipment and the cooling system of the rectifiers. The changes involved important innovations in the engineering practice relating to the application of rectifiers for electrochemical service.

Equipment and Layout

As may be seen in Figure 1, the electrical arrangement of the a-c and d-c busses and their connections to the supply and load circuits are similar to those in station 3. The width of the station building and the longitudinal space per unit are also the same as in station 3. A view of the outdoor a-c equipment is shown in Figure 8. A view of the station interior is shown in Figure 9. The station building is windowless. Illumination is provided by three rows of fluorescent lighting fixtures.

The rectifier units are divided into two groups, each supplying a load circuit, with a d-c bus tie breaker between them. Each group of six units constitutes a 36-phase system.

Auxiliary power, at 460 volts, is supplied by two 650 kva, 3-phase transformers. This power is distributed through a switching cubicle and an auxiliary bus provided with outlets through thermal-release switches. More auxiliary power is required than in station 3 because of the additional load for the fan motors of the rectifier cooling and station ventilating equipment. This made it advisable to use 460 volt distribution. These motors and the vacuum pump and water pump motors are rated at 460 volts. The other auxiliaries of each rectifier are supplied with 230 volts through a $7^1/_2$ kva, 460/230 volt transformer.

The station building is provided with a basement in which are located the control cables, the cathode bus-bars and reactors, the $7^1/_2$ kva auxiliary power transformers, and the station ventilating units. These units consist of fans and self-cleaning air

Figure 7. Diagram of connections of excitation and grid-control circuits for rectifier in station 3

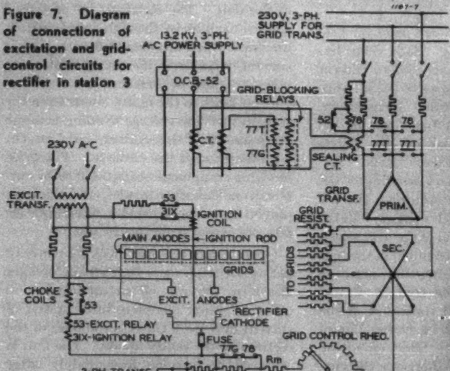

1167-7
230V, 3-PH. SUPPLY FOR GRID TRANS.
13.2 KV, 3-PH. A-C POWER SUPPLY
O.C.B.-52
GRID-BLOCKING RELAYS
52
78
78
78
77T
77T
77T
77G
C.T.
SEALING C.T.
230V A-C
EXCIT. TRANSF.
53
31X
IGNITION COIL
GRID TRANSF.
PRIM.
GRID RESIST.
MAIN ANODES
IGNITION ROD
SEC.
GRIDS
TO GRIDS
CHOKE COILS
53
EXCIT.
ANODES
RECTIFIER CATHODE
53-EXCIT. RELAY
31X-IGNITION RELAY
FUSE
GRID CONTROL RHEO.
77G
78
Rm
3-PH. TRANSF.
230V 3-PH. A-C
SELENIUM RECTIFIER 400V D-C
GRID BIAS CIRCUIT
78
77G

Figure 9 (below). Interior view of rectifier station 4

filters. Air from the outside is drawn through the filters and discharged into the basement. From there it passes through floor gratings into the rectifier room, and is discharged through roof ventilators.

RECTIFIER UNITS

A diagram of connections of the power circuits of one rectifier unit is shown in Figure 10. Each unit is connected to the a-c busses through gang-operated air-break disconnecting switches which are mounted on top of the bus structure. They are capable of interrupting the magnetizing current of the rectifier transformers. There is no circuit breaker on the high-voltage side of the rectifier transformer, as in station 3. Its function is performed by a 6-pole, high-speed, anode breaker between each rectifier and the low-voltage side of the rectifier transformer.

The high-voltage windings of the rectifier transformers are connected in delta or wye. Four units of each group of six are provided with phase-shifting auto-transformers for phase multiplication. Three through-type current transformers are connected on the high-voltage side of each unit. For units having phase shifters, the current transformers are located inside the phase shifter tanks.

Each rectifier transformer has two groups of 6-phase low-voltage windings, with interphase transformers. Each phase winding is connected to two of the rectifier anodes through a paralleling anode reactor.

The cathode of each rectifier is connected to the positive d-c bus through a reactor of approximately 0.1 millihenry, and a semi-high-speed breaker. The purpose of the reactor is to reduce the rate of rise of the current from the d-c bus to the rectifier during a backfire. This was considered desirable in view of the lower impedance of each low voltage phase winding of the rectifier transformer as compared to the transformers of station 3, which have a separate winding for each anode.

PROTECTION

The anode breakers are pneumatically operated. The six poles may be opened and closed simultaneously by means of an electrically operated air valve. Each pole is independently latched, is trip-free, and is provided with a reverse-current trip attachment. The interrupting time is approximately 0.75 cycle. A view of the anode breakers is shown in Figure 11. The anode reactors are located below the breakers. The anode reactors are connected to the anode terminals of the rectifier by means of cables passing underneath the station floor.

The use of anode breakers, instead of a circuit breaker on the high-voltage side of the rectifier transformer, has several advantages. Upon occurrence of a backfire, the circuit of the faulty anode is opened at high speed, thus cutting off the flow of current from the other anodes of the same rectifier as well as the current from the d-c bus. Only the backfiring rectifier is removed from service, without affecting the second rectifier of the same unit. A circuit breaker on the high voltage side of the rectifier transformer would have to be of relatively high interrupting capacity in view of the large power concentration, and would require a more elaborate outdoor installation.

The individual rectifier transformers are protected by induction-type over-

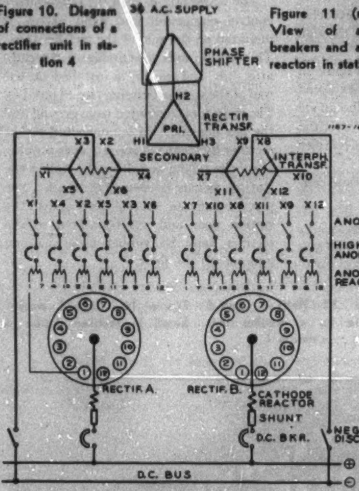

3Φ A.C. SUPPLY
PHASE SHIFTER
H2
PRI.
RECTIF. TRANSF.
H1
H3
1187-10
X3
X2
X9
X8
SECONDARY
INTERPH. TRANSF.
X1
X4
X7
X10
X5
X6
X11
X12
X1 X4 X2 X5 X3 X6
X7 X10 X8 X11 X9 X12
ANODE DISC.
HIGH SPEED 6-P ANODE BKR.
ANODE REACTORS
RECTIF. A.
RECTIF. B.
CATHODE REACTOR
SHUNT
D.C. BKR.
NEG. DISC.
D.C. BUS
⊕
⊖

Figure 10. Diagram of connections of a rectifier unit in station 4

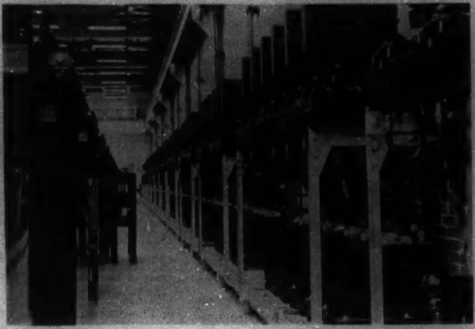


Figure 11 (right). View of anode breakers and anode reactors in station 4

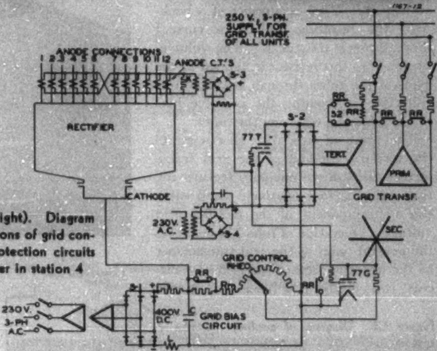

1187-12
250 V. 3-PH. SUPPLY FOR GRID TRANSF. OF ALL UNITS
ANODE CONNECTIONS
1 2 3 4 5 6 7 8 9 10 11 12
ANODE C.T.'S
S-3
RR
52
RR
RR
RR
S-2
RECTIFIER
77T
TERT.
PRIM.
GRID TRANSF.
CATHODE
230V. A.C.
S-4
SEC.
GRID CONTROL RHEO.
RR
Rm
RR
77G
230V. 3-PH A.C.
400V D.C.
GRID BIAS CIRCUIT

Figure 12 (right). Diagram of connections of grid control and protection circuits or rectifier in station 4 current relays connected to the current transformers on the high-voltage side of the unit. These relays are connected to trip the oil circuit breakers supplying the a-c busses.

The semi-high-speed cathode breaker of each rectifier is equipped with reverse-current and overload trip attachments. These breakers provide overload protection for the individual rectifiers and back-up protection for the anode breakers. They are also used for isolating the rectifiers from the positive bus and for switching the load circuit.

Each rectifier is provided with high-speed grid-blocking protection, the purpose of which is to limit to a minimum the number of anodes in a backfiring rectifier participating in the flow of current to the faulty anode.

GRID CONTROL

A diagram of connections of the grid control and blocking circuit is shown in Figure 12. This circuit is similar to that of Figure 7 except that thyratron tubes, 77*T* and 77*G*, are used in place of mechanical relays. The grids of these tubes are actuated by a rectified impulse from differentially-connected anode current transformers. In normal operation, when the currents of parallel anodes are balanced, there is no residual current through resistor *K*. Upon occurrence of a backfire, the resulting unbalance current flows through resistor *K*. The voltage-drop across this resistor is applied to the grids of the tubes through auxiliary rectifier *S*3, overcoming the negative bias of the grids and releasing the tubes.

The firing of tube 77*T* short-circuits the tertiary winding of the grid transformer through the auxiliary rectifier *S*2, thus reducing to a relatively small value the a-c component of the rectifier grid voltage. Operation of tube 77*G* connects the neutral point of the grid transformer to the negative side of the grid bias circuit, thus applying a blocking negative voltage to the grids of the rectifier. Operation of tube 77*T* also causes operation of reset relay 78. The contacts of this relay short-circuit the primary winding of the grid transformer and tube 77*G*, taking over the function of the tubes and permitting their grids to regain control. Opening of the anode breaker resets relay *RR* by short-circuiting its coil through auxiliary switch 52.

Figure 14. View of rectifier cooling equipment on roof of station 4

The grid-blocking circuit of Figure 12 has greater selectivity and higher speed than the circuit of Figure 7. The anode current transformers are clamped over the ends of the anode bus bars of the rectifier (see Figure 11).

The operation of the grid voltage control of the rectifiers in station 4 is similar to that in station 3.

COOLING SYSTEM

A diagram of the rectifier cooling system is shown in Figure 13. Due to the limited supply of raw water at this installation, it was decided to use water-to-air instead of water-to-water heat exchangers.

The cooling system of each rectifier is self-contained. Water is circulated between the rectifier and the radiator of the cooling unit. The radiator is cooled by a fan driven by a two-speed motor, with a rating of 10 and $2^{1}/_{3}$ horsepower at the high and low speeds, respectively. The water temperature is regulated by means of a 3-way valve combination which controls the relative rate of water flow through the radiator and the by-pass circuit at the rectifier.

The water pump, with the temperature control equipment and hand valves, is mounted on the side of the rectifier tank. The cooling unit, consisting of the radiator, fan, and motor, is installed on the roof of the station building (see Figure 14). The radiator is insulated from ground and the fan housing. It is connected to the rectifier by means of copper pipes running along the ceiling and down the wall of the station, as may be seen in Figure 9. The pipes terminate several feet above the station floor and are connected to the rectifier tank by rubber hoses passing underneath the floor.

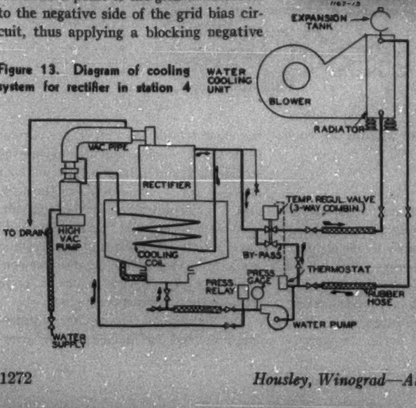


Figure 13. Diagram of cooling system for rectifier in station 4

Figure 15 (left). Control cubicle for a rectifier unit in station 4

Figure 16. Control switchboard in rectifier station 4

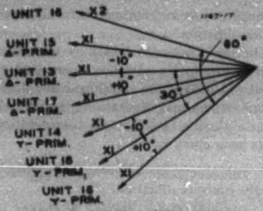


Figure 17. Vector diagram of 36-phase system for units 13 to 18 in rectifier station 4

The cooling units are designed to dissipate the full-load losses in the rectifier at an air temperature of 105 degrees Fahrenheit. During the greater part of the year the fans operate at low speed. An antifreeze solution is added to the cooling system for the winter months.

The high-vacuum pumps are cooled by water from the city supply, as in station 3.

SWITCHBOARDS

The excitation and grid-control equipment and motor starters of the vacuum and water pumps, for the two rectifiers of each unit, are assembled in a cubicle shown in Figure 15. The grid control rheostats are mounted on the swinging doors. The cubicle is located in front of the rectifiers, facing the cathode breakers.

The control switchboard in the center of the station is of the totally-enclosed duplex-type construction. It is provided with a passage-way between the front and rear panels and a door at each end. A front view of the switchboard is shown in Figure 16.

On the front panels are mounted the control switches and indicating instruments of the rectifier units, incoming a-c lines, and load circuits. On the control panels of the rectifier units are mounted the instruments and control switches required for complete supervision and control of each rectifier, including the vacuum meters, drop annunciators, grid control switches, position indicators of the temperature regulating valves, and control switches for the fan motors. On the rear panels of the switchboard are mounted relays, battery switches, and the incoming line watthour meters.

The terminal blocks of the switchboard and cubicles project through the floor into the basement. This arrangement facilitates the connections of the control cables and conserves switchboard space.

LOAD SWITCHING

The cathode breakers and the control grids of the rectifiers are utilized for switching the load circuits. To drop the load on one of the load circuits, the positive bus section to which it is connected is isolated by opening the adjoining bus tie breakers. The cathode breakers of the rectifiers feeding the load circuits are then tripped simultaneously through a multicontact relay by turning the control switch of the load circuit to the "trip" position. This operation also opens a grid contactor which supplies 3-phase voltage to the grid transformers of these rectifiers. With the a-c components of the grid voltage removed, the rectifier anodes are "blocked" by the fixed negative bias applied to the grids.

To pick up the load, the anode and cathode breakers of the rectifiers are closed. The load circuit is then "closed" by turning the control switch to the "close" position. This energizes the grid contactor and applies voltage to the grid transformers, permitting the rectifier anodes to fire in their regular sequence.

The load circuits of station 3 are controlled similarly. Originally, the load circuits in that station were connected to the positive bus through 50,000-ampere breakers, which have since been removed. The omission of these breakers simplified the installation by making it unnecessary to bring the heavy connections between the load circuits and the positive bus into the station building.

PHASE MULTIPLICATION

As was mentioned previously, each group of six rectifier units operates as a 36-phase system which cancels or reduces to a negligible value the harmonic components of the current in the a-c supply line, below the 35th harmonic. This is accomplished by displacing the phases of the units by equal angles from each other, as indicated in Figure 17. This diagram shows the voltage vectors of low-voltage phase $X1$ of the rectifier transformers for units 13 to 18. The phase displacement is effected by delta and wye high-voltage windings of the rectifier transformers and ten-degree phase shifters, as indicated in Figure 1. Units 13 and 18 have no phase shifters. Their phases are displaced by 30 degrees due to the delta and wye connections. Units 14 and 15 are shifted by ten degrees in the lagging direction, and units 16 and 17 by ten degrees in the leading direction.

The a-c line current of each 6-phase unit contains all the odd, nontriple harmonics: 5th, 7th, 11th, 13th, 17th, 19th, etc. In Figure 18 are given the vector diagrams of some of the harmonic components for the six units, showing their relative phase positions in harmonic degrees. It may be seen readily that the sum of the vectors in each of the diagrams shown is equal to zero. They are therefore cancelled from the line current.[5] This applies to all the harmonics below the 35th. The vectors of the 35th and

Figure 18. Vector diagrams for a-c current harmonics of units 13 to 18 in rectifier station 4

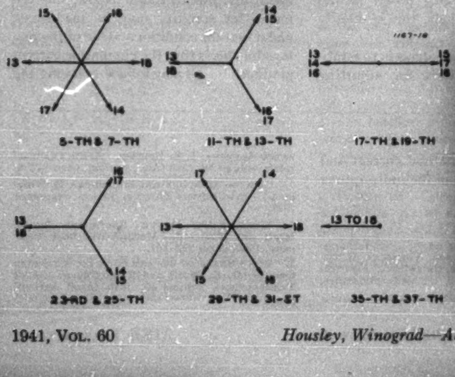
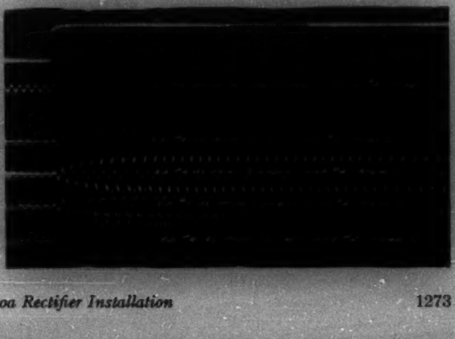


Figure 19 (right). Oscillogram of a-c currents in supply line and rectifier units, connected in a 30-phase system



Oscillogram taken while closing the load circuit

Operation of Load-Ratio Control Transformers Connected in Parallel and in Networks

F. M. STARR
MEMBER AIEE

SUBSTATIONS equipped with automatic load-ratio control equipment are frequently operated with their secondaries in parallel. The units may be physically together, the secondary tie being a bus; or they may be some distance apart connected together by a tie line or group of tie lines as in a primary network. At the present time large numbers of factory-built distribution substations are being sold, and well over half of them are equipped for some form of network operation.

As with most voltage regulators, automatic load-ratio control equipment is usually controlled by means of a line-drop compensator and a contact-making voltmeter.* This automatic control equipment has two principal functions:

1. It serves to maintain some specified voltage regulation at some point on the distribution system.
2. It serves to maintain regulator stability and to keep circulating currents at a minimum.

To be stable a regulator must not "pump" (that is, successively buck and boost unchecked) under any operating condition, and it must be so compensated that there will be no tendency to run away (that is, go to full boost or full buck) under any operating condition except as dictated by regulation requirements. Pumping is prevented by adjustment of the voltmeter band width, and the units operating in parallel are held together by proper adjustment of the reactance compensation as will be explained below. This same compensator adjustment, of course, serves to correct for circulating currents resulting from any cause, such as changes in high-side supply voltages or unbalanced loads.

When a single compensator is used for the dual purpose of providing voltage regulation as well as regulator stability in a network, it naturally will not be adjusted in the conventional manner of adjusting the compensator on a radial feeder regulator. Instead of adjusting the resistance and reactance elements to some value of line impedance, the reactance compensation is greatly reduced (sometimes made negative) to insure stability, and the resistance compensation is substantially increased to obtain the prescribed voltage regulation at peak load. Generally, the compensator and voltmeter can always be adjusted as will be shown, to

1. Insure stability,
2. Correct for circulating currents,
3. Provide the desired voltage at a specified location at peak load.

This may mean some compromise in desired voltage levels at light load periods but, because of changing power factors during light load periods, this is usually not the case.

The purpose of this analysis is to establish quantitative criteria for adjusting compensators and voltmeters for this type of application.

In the discussion which is to follow, three criteria for adjusting compensators and voltmeters are developed and their applications and limitations are stated. In developing these criteria consideration has been given to:

1. Different assumed degrees of accuracy in the control calibration.
2. Different assumed rates of change in voltage.
3. Large and small groups of units.

The case of two similar units operating in parallel has been examined in great detail because this case occurs most frequently in practice, and also, because of its simplicity it is an excellent one from which to develop principles.

Definition of Stability Criteria and Statement of Conditions to Be Studied

CRITERIA FOR STABILITY

Assume a load-ratio-control transformer, operating in parallel with several other units, makes a tap change, $\overline{\Delta E}$ in magnitude. This change will initiate a circulating current which, by virtue of the voltage drop it produces in the transformer and compensator impedances, contributes a voltage change at the voltmeter of $\overline{\Delta V}$. Obviously, to avoid a succession of tap changes in the same direction $\overline{\Delta V}$ must be less than $\overline{\Delta E}$, for otherwise the voltmeter would never receive any indication that the correction it had called for had been consummated.

Thus, one criterion for insuring stability would be:

$\overline{\Delta V}<\overline{\Delta E}$ (criterion A)

This criterion is frequently used in adjusting compensators on induction voltage regulators operating in parallel. However, as will be disclosed in the more detailed analysis to follow below, this criterion for stability may be inadequate under certain conditions. One of the reasons for this is that the circulating current produced by $\overline{\Delta E}$ may induce opposing tap

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* Hereafter to be referred to simply as compensator and voltmeter.

37th harmonics of the six units are in phase, and therefore are present in the line current, in full. The vectors do not indicate the relative magnitudes of the harmonics.

In Figure 19 is shown an oscillogram taken while closing a load circuit supplied by five rectifier units, connected as a 30-phase system. The a-c line current and the a-c current of four of the units appear in the oscillogram.

References

1. RECTIFIERS ADOPTED FOR AMERICAN ALUMINUM, D. I. Bohn and W. E. Gutzwiller. *Electrical World*, December 3, 1938.

2. RECONDITIONING OF INSULATING OILS BY ACTIVATED ALUMINA, J. E. Housley. AIEE TRANSACTIONS, volume 58, 1939 (April section), pages 172–8.

3. COOLING MERCURY-ARC RECTIFIERS, H. Winograd. *Allis-Chalmers Electrical Review*, September 1937.

4. MERCURY-ARC POWER RECTIFIERS (book), Marti and Winograd. McGraw-Hill Book Company, 1930, pages 397–401.

5. WAVE SHAPE OF 30- AND 60-PHASE RECTIFIER GROUPS, O. K. Marti and T. A. Taylor. AIEE TRANSACTIONS, volume 59, 1940 (April section), pages 218–24.

changes in other units resulting in increased circulating currents, and a total change in voltage contributed by circulating currents which is greater than $\overline{\Delta E}$. As will be shown below, to avoid this possibility, further restrictions must be placed on the magnitude of $\overline{\Delta V}$ such that:

$\overline{\Delta V} < \frac{\overline{\Delta E}}{2}$ (criterion *B*)

This criterion will insure that parallel units will never get more than two tap changes apart (except when correcting for circulating currents due to other causes). It is practically impossible to keep all parallel units on the same tap by compensator adjustments, but positive insurance against more than one step apart (again except when correcting for circulating currents due to other causes) can be obtained by further restricting $\overline{\Delta V}$ to a value:

$\overline{\Delta V} < 0$ (criterion *C*)

This criterion states that $\overline{\Delta V}$ is always negative which, of course, is obtained by a relatively large amount of negative reactance compensation.

The three basic criteria defined above have to do with keeping the units in step and correcting circulating currents, and are established by compensator adjustments. The stability criterion to avoid pumping, on the other hand, is determined principally by voltmeter band-width adjustment. Pumping will be avoided if the net change in voltage at the voltmeter, following a tap-change of $\overline{\Delta E}$, is always less than the voltmeter band-width. In combination with criteria *A* or *B* above, this condition is always satisfied if:

$\overline{\Delta E} < \overline{B.W.}$

In combination with criterion *C* above, however, since $\overline{\Delta V}$ is always negative, this condition is satisfied only if:

$\overline{\Delta E} - \overline{\Delta V} < \overline{B.W.}$

Therefore, in the following more detailed examinations which will be made, the following three general criteria will be applied:

Criterion *A* $\overline{\Delta E} < \overline{B.W.}$
$\overline{\Delta V} < \overline{\Delta E}$

Criterion *B* $\overline{\Delta E} < \overline{B.W.}$
$\overline{\Delta V} < \frac{\overline{\Delta E}}{2}$

Criterion *C* $\overline{\Delta E} - \overline{\Delta V} < \overline{B.W.}$
$\overline{\Delta V} < 0$

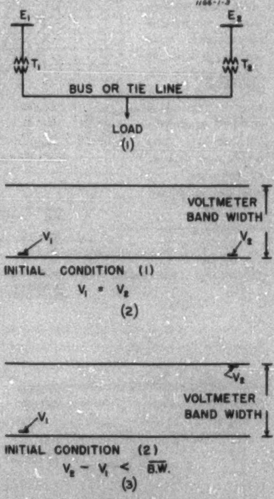
Control-Equipment Calibration

A considerable amount of study on the problem of stability has shown that the adequacy of any particular criterion is greatly influenced by the degree to which the actual calibration of the compensators and voltmeters deviates from the intended values. To establish quantitatively the magnitude of such influences, this analysis has been made on each of two different assumed conditions.

Condition 1.—All voltmeters and compensators have zero errors in their calibration and always function as intended.

Condition 2.—Compensator and voltmeter calibrations are in error to the extent that any two units which should normally respond together for a given set of conditions actually respond as if the delivered voltages to the two voltmeters differed by a value approaching band width in magnitude.

These two assumptions represent two extremes, the actual operating condition probably lying somewhere between them.



Figures 1, 2, and 3. Study of the case of two similar units operating in parallel was based upon two different assumed conditions of instrument calibration

Condition 1. All voltmeters and compensators have zero errors in their calibration and always function as intended. This is illustrated in figure 2 where the voltages V_1 and V_2 as seen by the voltmeters of T_1 and T_2 are equal as intended

Condition 2. Compensator and voltmeter calibrations are in error to the extent that any two units which should normally respond together for a given set of conditions actually respond as if the delivered voltages to the two voltmeters differed by a value approaching band width in magnitude

Sudden Voltage Dips

This analysis has further indicated that the adequacy of any particular stability criterion is influenced by the rate at which the voltage changes. All cases studied were analyzed for two different assumed conditions of rate of voltage change:

1. Voltage rate of change is sufficiently slow that the actual change occurring during the time required to make a tap change is small.
2. Sudden voltage dips occur. The actual magnitudes of voltage dips considered were the critical ones pertinent to each particular case.

Compensator Adjustment for Voltage Regulation

The key to the stability criterion is the value selected for $\overline{\Delta V}$. A large part of this quantity consists of the voltage drop in the compensator produced by circulating currents. As noted above, stability assurance increases as $\overline{\Delta V}$ decreases, and $\overline{\Delta V}$ is reduced by reducing the reactance compensation. The compensator must also provide the desired voltage boost and regulation required at peak load. Since the reactance compensation has been reduced below normal, regulation can be obtained only by increasing the resistance compensation. As might be expected, the more that reactance compensation is reduced to reduce $\overline{\Delta V}$, the more resistance compensation is required to give a specified voltage boost at peak load.

Thus, an appropriate balance between resistance and reactance compensation adjustments is necessary to best meet the stability and regulation requirements.

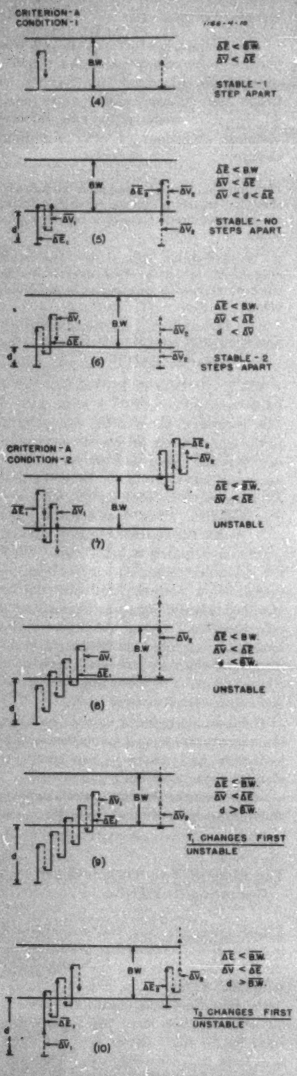
All three of the stability criteria defined above have been studied from this point of view.

The Case of Two Transformers Operating in Parallel

The case of two load-ratio-control transformers operating in parallel is the simplest one. It is also one of the most common. Many radial substations consist of two three-phase transformers serving a distribution bus. Also, many primary networks consist of only two units or at least start with only two. Thus, the case of two units in parallel is a good one from which to develop principles.

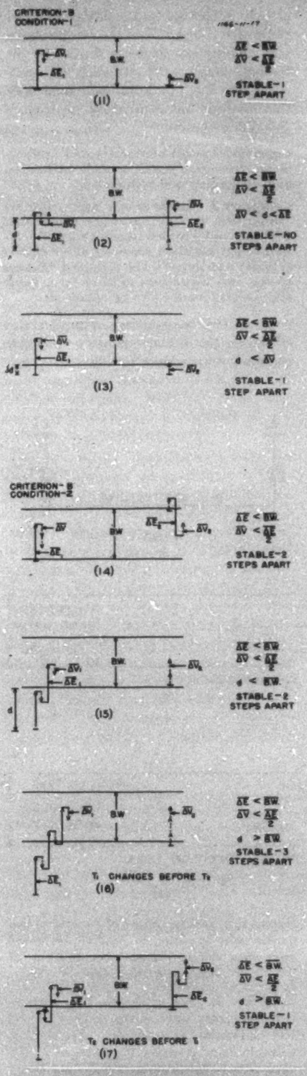
It will be assumed in this analysis that two similar units with the same impedance characteristics and having the same compensator settings are operating in parallel, either on the same bus or over a tie-line.

The case of two load-ratio-control


CRITERION-A
CONDITION-1
1166-4-10
B.W.
$\overline{\Delta E} < \overline{B.W.}$
$\overline{\Delta V} < \overline{\Delta E}$
STABLE-1 STEP APART
(4)
B.W
$\overline{\Delta E}_2$
$\overline{\Delta V}_2$
$\overline{\Delta V}_1$
$\overline{\Delta E}_1$
d
$\overline{\Delta E} < B.W$
$\overline{\Delta V} < \overline{\Delta E}$
$\overline{\Delta V} < d < \overline{\Delta E}$
STABLE - NO STEPS APART
(5)
$\overline{\Delta V}_1$
$\overline{\Delta E}_1$
B.W
$\overline{\Delta V}_2$
d
$\overline{\Delta E} < B.W.$
$\overline{\Delta V} < \overline{\Delta E}$
$d < \overline{\Delta V}$
STABLE - 2 STEPS APART
(6)
CRITERION-A
CONDITION-2
$\overline{\Delta E}_2$
$\overline{\Delta V}_2$
$\overline{\Delta E}_1$
$\overline{\Delta V}_1$
B.W
$\overline{\Delta E} < \overline{B.W.}$
$\overline{\Delta V} < \overline{\Delta E}$
UNSTABLE
(7)
$\overline{\Delta V}_2$
B.W
$\overline{\Delta V}_1$
$\overline{\Delta E}_1$
d
$\overline{\Delta E} < B.W.$
$\overline{\Delta V} < \overline{\Delta E}$
$d < \overline{B.W.}$
UNSTABLE
(8)
B.W
$\overline{\Delta V}_1$
$\overline{\Delta E}_1$
$\overline{\Delta V}_2$
d
$\overline{\Delta E} \leq \overline{B.W.}$
$\overline{\Delta V} < \overline{\Delta E}$
$d > \overline{B.W.}$
T_1 CHANGES FIRST
UNSTABLE
(9)
BW
$\overline{\Delta E}_2$
$\overline{\Delta V}_2$
$\overline{\Delta E}_1$
$\overline{\Delta V}_1$
d
$\overline{\Delta E} < \overline{B.W.}$
$\overline{\Delta V} < \overline{\Delta E}$
$d > \overline{B.W.}$
T_2 CHANGES FIRST
UNSTABLE
(10)

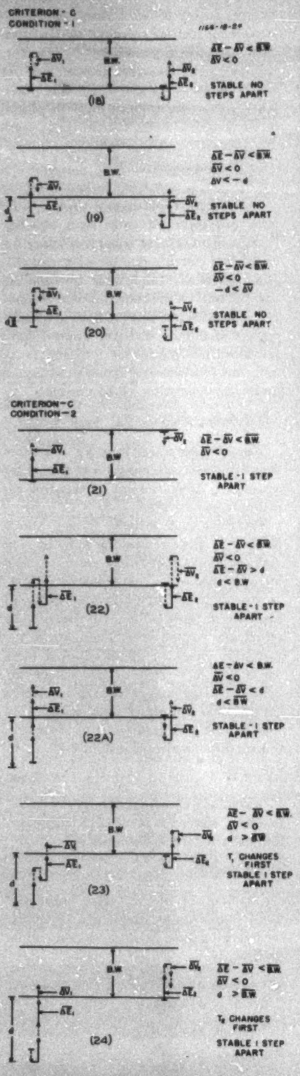
Figures 4–10. Regulator operation with compensator and voltmeter adjusted according to criterion A for seven different assumed initial values of V_1 and V_2

Note that the operation under the conditions shown in Figures 7–10 is unstable. Refer to Table I


CRITERION-B
CONDITION-1
1166-11-17
$\overline{\Delta V}_1$
ΔE_1
B.W.
$\overline{\Delta V}_2$
$\overline{\Delta E} < \overline{B.W.}$
$\overline{\Delta V} < \frac{\overline{\Delta E}}{2}$
STABLE-1 STEP APART
(11)
B.W.
$\overline{\Delta V}_1$
ΔE_1
$\overline{\Delta V}_2$
ΔE_2
d
$\overline{\Delta E} < \overline{B.W.}$
$\overline{\Delta V} < \frac{\overline{\Delta E}}{2}$
$\overline{\Delta V} < d < \overline{\Delta E}$
STABLE-NO STEPS APART
(12)
$\overline{\Delta V}_1$
$\overline{\Delta E}_1$
B.W
$\overline{\Delta V}_2$
d
$\overline{\Delta E} < \overline{B.W.}$
$\overline{\Delta V} < \frac{\overline{\Delta E}}{2}$
$d < \overline{\Delta V}$
STABLE-1 STEP APART
(13)
CRITERION-B
CONDITION-2
$\overline{\Delta V}$
ΔE_1
BW
$\overline{\Delta E}_2$
$\overline{\Delta V}_2$
$\overline{\Delta E} < \overline{B.W.}$
$\overline{\Delta V} < \frac{\overline{\Delta E}}{2}$
STABLE-2 STEPS APART
(14)
$\overline{\Delta V}_1$
$\overline{\Delta E}_1$
BW.
$\overline{\Delta V}_2$
d
$\overline{\Delta E} < \overline{B.W.}$
$\overline{\Delta V} < \frac{\overline{\Delta E}}{2}$
$d < \overline{B.W.}$
STABLE-2 STEPS APART
(15)
$\overline{\Delta V}_1$
BW
$\overline{\Delta V}_2$
ΔE_1
$\overline{\Delta E} < \overline{B.W.}$
$\overline{\Delta V} < \frac{\overline{\Delta E}}{2}$
$d > \overline{B.W.}$
STABLE-3 STEPS APART
T_1 CHANGES BEFORE T_2
(16)
$\overline{\Delta V}_1$
$\overline{\Delta E}_1$
BW
$\overline{\Delta V}_2$
ΔE_2
$\overline{\Delta E} < \overline{B.W.}$
$\overline{\Delta V} < \frac{\overline{\Delta E}}{2}$
$d > \overline{B.W.}$
STABLE-1 STEP APART
T_2 CHANGES BEFORE T_1
(17)

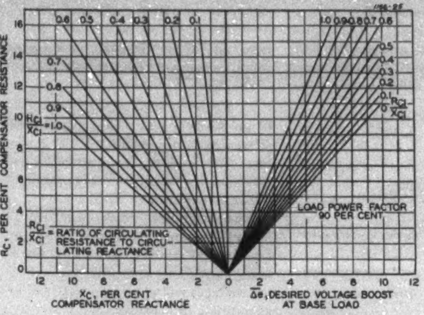
Figures 11–17. Regulator operation with compensator and voltmeter adjusted according to criterion B for the same respective initial values of V_1 and V_2 assumed in Figures 4–10

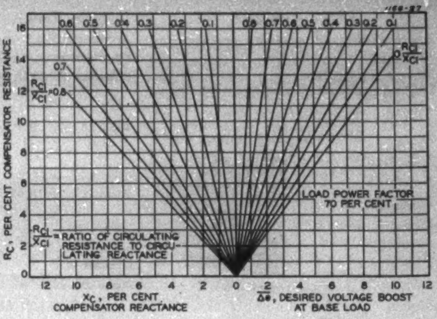
Refer to Table II

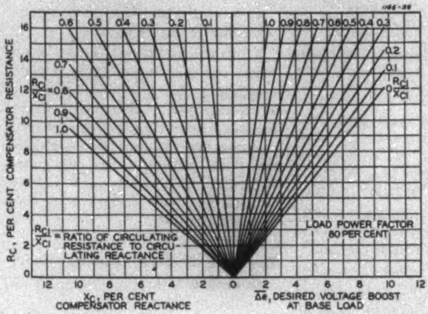

CRITERION-C
CONDITION-1
1166-18-24
$\overline{\Delta V}_1$
$\overline{\Delta E}_1$
B.W.
$\overline{\Delta V}_2$
$\overline{\Delta E}_2$
$\overline{\Delta E} - \overline{\Delta V} < \overline{B.W.}$
$\overline{\Delta V} < 0$
STABLE NO STEPS APART
(18)
B.W.
$\overline{\Delta V}_1$
$\overline{\Delta E}_1$
$\overline{\Delta V}_2$
$\overline{\Delta E}_2$
d
$\overline{\Delta E} - \overline{\Delta V} < \overline{B.W.}$
$\overline{\Delta V} < 0$
$\Delta V < -d$
STABLE NO STEPS APART
(19)
$\overline{\Delta V}_1$
$\overline{\Delta E}_1$
B.W
$\overline{\Delta V}_2$
$\overline{\Delta E}_2$
d
$\overline{\Delta E} - \overline{\Delta V} < \overline{B.W.}$
$\overline{\Delta V} < 0$
$-d < \overline{\Delta V}$
STABLE NO STEPS APART
(20)
CRITERION-C
CONDITION-2
$\overline{\Delta V}_2$
$\overline{\Delta V}_1$
$\overline{\Delta E}_1$
B.W
$\overline{\Delta E} - \overline{\Delta V} < \overline{B.W.}$
$\overline{\Delta V} < 0$
STABLE-1 STEP APART
(21)
B.W
$\overline{\Delta V}_2$
$\overline{\Delta E}_1$
$\overline{\Delta E}_2$
d
$\overline{\Delta E} - \overline{\Delta V} < \overline{B.W.}$
$\overline{\Delta V} < 0$
$\overline{\Delta E} - \overline{\Delta V} > d$
$d < B.W$
STABLE-1 STEP APART
(22)
$\overline{\Delta V}_1$
$\overline{\Delta E}_1$
B.W.
$\overline{\Delta V}_2$
$\overline{\Delta E}_2$
d
$\Delta E - \Delta V < B.W.$
$\overline{\Delta V} < 0$
$\overline{\Delta E} - \overline{\Delta V} < d$
$d < \overline{B.W}$
STABLE-1 STEP APART
(22A)
B.W
$\overline{\Delta V}_1$
$\overline{\Delta E}_1$
$\overline{\Delta V}_2$
$\overline{\Delta E}_2$
d
$\overline{\Delta E} - \overline{\Delta V} < \overline{B.W.}$
$\overline{\Delta V} < 0$
$d > \overline{B.W}$
T_1 CHANGES FIRST
STABLE 1 STEP APART
(23)
B.W
$\overline{\Delta V}_2$
$\overline{\Delta E}_2$
$\overline{\Delta V}_1$
$\overline{\Delta E}_1$
d
$\overline{\Delta E} - \overline{\Delta V} < \overline{B.W.}$
$\overline{\Delta V} < 0$
$d > \overline{B.W.}$
T_2 CHANGES FIRST
STABLE 1 STEP APART
(24)

Figures 18–24. Regulator operation with compensator and voltmeter adjusted according to criterion C for the same respective initial values of V_1 and V_2 assumed in Figures 4–10 and 11–17

Refer to Table III







Figures 25, 26, and 27. These curves give the values of reactance and resistance compensation (R_c and X_c) necessary to insure stability and also to give a specified amount of voltage boost at peak load for the case of two similar units operating in parallel on the same bus

All values of X_c are negative and have a ten per cent marginal factor of safety to insure stability

transformers operating in parallel will be examined:

1. For each of the three stability criteria defined above—that is, *A*, *B*, and *C*.
2. For two assumed conditions of voltmeter and compensator calibration as defined above and illustrated in Figures 2 and 3.
3. For both gradual changes in voltage and sudden voltage dips of critical magnitudes.

CRITERION A

To repeat, this criterion states that when T_1 makes a tap change of a magnitude $\overline{\Delta E_1}$, the voltage drop in the transformer (and system back of it) plus that in the line-drop compensator due to the resulting circulating current must be less than $\overline{\Delta E_1}$. That is:

$$\overline{\Delta V_1} < \overline{\Delta E_1}^{*} \qquad (1)$$

Now consider this criterion for condition 1 above (Figure 2) where both V_1 and V_2 are on the verge of calling for a tap change in the same direction. Assume V_1 and V_2 are dropping slowly and V_1 calls for a tap change. The resulting condition, shown in Figure 4, is stable with the two units one step apart. V_1 has increased by an amount $\overline{\Delta E_1} - \overline{\Delta V_1}$ and $\overline{V_2}$ has increased by an amount $\overline{\Delta V_2}$ which is equal and opposite to $\overline{\Delta V_1}$.

A second case to consider is that when V_1 and V_2 are not changing slowly as above, but drop suddenly to a value $(V-d)$. This case must be studied for two values of *d:*

$$\overline{\Delta V} < d < \overline{\Delta E} \qquad (2)$$

$$d < \overline{\Delta V} < \overline{\Delta E} \qquad (3)$$

A larger value of *d* need not be considered since any larger value would revert to one of the above two values after the necessary changes in voltage.

Consider first the case of $\overline{\Delta V} < d$ and assume that T_1 makes the first tap change. This will be followed immediately by another tap change by T_2, since $\overline{\Delta V_2}$ is not sufficient to put V_2 back in the voltmeter band. This is illustrated in Figure 5. Following the second tap change both V_1 and V_2 are inside the voltmeter band and the two units become stable and are in step.

Now consider the case where $\overline{\Delta V} > d$. If T_1 makes the first tap change, since this puts V_2 back in the voltmeter band, T_1 will make a succession of tap changes until

$$n(\overline{\Delta E} - \overline{\Delta V}) > d \qquad (4)$$

where n = number of tap changes.

Following this T_2 will likewise call for tap changes in the reverse direction resulting in instability unless

$$n\overline{\Delta V} - d < \overline{B.W.} \qquad (5)$$

This is illustrated in Figure 6 where three tap changes of T_1 were necessary to obtain stability and then it was obtained only with large value of $\overline{B.W.}$, a much larger value than is considered desirable for good voltage regulation.

Three cases have been considered for condition 1, that is, when V_1 and V_2 are substantially together initially. Now consider condition 2 with V_1 and V_2 substantially apart by some value approaching $\overline{B.W.}$ Thus, assume the condition in Figure 3 exists initially and that V_1 drops gradually until a tap change is called for. As soon as T_1 makes a tap change, an opposite one is called for in T_2. Both V_1 and V_2 are now outside the band, which calls for two further opposing tap changes, and so on until the regulators are in full opposition. This case, illustrated in Figure 7, is unstable and obviously not satisfactory.

Again, with the initial condition of Figure 3, assume that, instead of a gradual change in V_1 and V_2, they drop suddenly by an amount *d*. Consider first the case where $d < \overline{B.W.}$ and leaves V_2 barely inside the voltmeter band. T_1 will then make a succession of tap changes until—

$$n(\overline{\Delta E_1} - \overline{\Delta V_1}) > d \qquad (6)$$

and stability will result only if

$$n\overline{\Delta V_2} < \overline{B.W.} \qquad (7)$$

This is illustrated in Figure 8. Here again, as in Figure 6 several tap changes and a wide $\overline{B.W.}$ were necessary to secure stability if it can be obtained at all.

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* See appendix for definition of symbols.

Table I. Criterion A

Condition 1.		
Figure 4	$\overline{\Delta E}<\overline{B.W.}$ $\overline{\Delta V}<\overline{\Delta E}$	Stable—one step apart
Figure 5	$\overline{\Delta E}<\overline{B.W.}$ $\overline{\Delta V}<\overline{\Delta E}$ $\overline{\Delta V}<d<\overline{\Delta E}$	Stable—no steps apart
Figure 6	$\overline{\Delta E}<\overline{B.W.}$ $\overline{\Delta V}<\overline{\Delta E}$	$d<n(\overline{\Delta E}-\overline{\Delta V})$ $n\overline{\Delta V}<\overline{B.W.}+d$ Stable—two steps apart
Condition 2.		
Figure 7	$\overline{\Delta E}<\overline{B.W.}$ $\overline{\Delta V}<\overline{\Delta E}$	Unstable
Figure 8	$\overline{\Delta E}<\overline{B.W.}$ $\overline{\Delta V}<\overline{\Delta E}$ $d<\overline{B.W.}$	$n(\overline{\Delta E}-\overline{\Delta V})>d$ $n\overline{\Delta V}<\overline{B.W.}$ Unstable
Figure 9	$\overline{\Delta E}<\overline{B.W.}$ $\overline{\Delta V}<\overline{\Delta E}$ $d>\overline{B.W.}$	$n(\overline{\Delta E}-\overline{\Delta V})>d$ $n\overline{\Delta V}<\overline{B.W.}$ T_1 changes first Unstable
Figure 10	$\overline{\Delta E}<\overline{B.W.}$ $\overline{\Delta V}<\overline{\Delta E}$ $d>\overline{B.W.}$	$n(\overline{\Delta E}-\overline{\Delta V})+\overline{\Delta V}>d$ $n\overline{\Delta V}+\overline{\Delta E}-\overline{\Delta V}<\overline{B.W}$ T_2 changes first Unstable

Again, starting with the initial condition of Figure 3, assume $d>B.W.$ leaving V_2 slightly outside the voltmeter band. The resulting criterion for stability now depends on whether T_1 or T_2 changes taps first. If T_1 changes taps first, V_2 is restored inside the band and relations 6 and 7 above still hold (see Figure 9). If T_2 changes taps first, T_1 will have to follow with n changes until—

$$n(\overline{\Delta E_1}-\overline{\Delta V_1})+\overline{\Delta V_1}>d \qquad (8)$$

and stability will result only if

$$n\overline{\Delta V}<\overline{B.W.} \qquad (9)$$

This case is illustrated in Figure 10. Here again stability is obtained only with the two units several taps apart and using a large value of $\overline{B.W.}$

A summary of conditions for the seven cases studied under criterion A and illustrated in Figures 4 through 10 is tabulated in Table I.

CRITERION B

The second criterion as defined above is:

$$\overline{\Delta V_1}<\frac{\overline{\Delta E_1}}{2}$$

This criterion will be examined for the two initial conditions illustrated in Figures 2 and 3 and for each of the cases analyzed under criterion A. The results of this examination are shown diagrammatically in Figures 11 through 17 which may be compared directly with Figures 7 through 10, respectively, for the corresponding cases under criterion A.

The conditions and results of criterion B for these seven cases are tabulated in Table II.

Table II. Criterion B

Condition 1.		
Figure 11	$\overline{\Delta E}<\overline{B.W.}$ $\overline{\Delta V}<\frac{\overline{\Delta E}}{2}$	Stable—one step apart
Figure 12	$\overline{\Delta E}<\overline{B.W.}$ $\overline{\Delta V}<\frac{\overline{\Delta E}}{2}$ $\overline{\Delta V}<d<\overline{\Delta E}$	Stable—no steps apart
Figure 13	$\overline{\Delta E}<\overline{B.W.}$ $\overline{\Delta V}<\frac{\overline{\Delta E}}{2}$ $d<\overline{\Delta V}$	Stable—one step apart
Condition 2.		
Figure 14	$\overline{\Delta E}<\overline{B.W.}$ $\overline{\Delta V}<\frac{\overline{\Delta E}}{2}$	Stable—two steps apart
Figure 15	$\overline{\Delta E}<\overline{B.W.}$ $\overline{\Delta V}<\frac{\overline{\Delta E}}{2}$ $d<\overline{B.W.}$	$n(\overline{\Delta E}-\overline{\Delta V})>d$ $n\overline{\Delta V}<\overline{B.W.}$ Stable—two steps apart
Figure 16	$\overline{\Delta E}<\overline{B.W.}$ $\overline{\Delta V}<\frac{\overline{\Delta E}}{2}$ $d>\overline{B.W.}$	$n(\overline{\Delta E}-\overline{\Delta V})>d$ $n\overline{\Delta V}<\overline{B.W.}$ T_1 changes first Stable—three steps apart
Figure 17	$\overline{\Delta E}<\overline{B.W.}$ $\overline{\Delta V}<\frac{\overline{\Delta E}}{2}$ $d>\overline{B.W.}$	$n(\overline{\Delta E}-\overline{\Delta V})+\overline{\Delta V}>d$ $n\overline{\Delta V}+\overline{\Delta E}-\overline{\Delta V}<\overline{B.W.}$ T_2 changes first Stable—one step apart

Since any operating condition will revert into one of the above seven assumed possible conditions, and since each of the above seven conditions results in stable operation, criterion B may be considered satisfactory so far as stability is concerned.

CRITERION C

The third criterion has been examined for these same seven cases analyzed under criteria A and B above.

Criterion C states:

$$\overline{\Delta V}<0$$

This criterion states that $\overline{\Delta V}$ must always be negative, requiring a large amount of negative compensation—more than may be obtainable in some compensators.

Figures 18 through 24 show graphically the regulator operation for the seven cases. The conditions for these cases correspond respectively to those in Figures 3 through 10 for criterion A, and to those in Figures 11 through 17 for criterion B.

The conditions and results of criterion C are tabulated in Table III.

CONCLUSIONS FROM TWO-UNIT ANALYSIS

1. Criterion A, $\overline{\Delta V}<\overline{\Delta E}$, should never be used. If the voltmeters and compensators are not closely calibrated, instability of the regulators may result. In any case, an excessively high band width would be required to insure stability most of the time.

2. Criterion B, $\overline{\Delta V}<\overline{\Delta E}/2$, appears to be an excellent one. This criterion will always guarantee stability of the regulators. When the input voltages are equal and when the compensators are set alike, the units will usually remain within one step of each other. Under conditions of bad calibration they may stabilize two taps or possibly three taps apart. Circulating currents set up by differences in input voltages will always be corrected by appropriate tap changes.

Criterion B is not too severe to interfere with effective voltage regulation.

3. Criterion C, $\overline{\Delta V}<0$, is also an excellent one since it will insure stability usually at no steps apart and occasionally at one step apart. Criterion C, however, requires so much negative reactance compensation that voltage regulating requirements will probably be seriously interfered with. It does not appear to have sufficient merit over criterion B to warrant its preference.

The Case of More Than Two Units in Parallel

Again, in the case of more than two units operating in parallel, the problem may be treated generally, whether the units are paralleled on the same bus or whether they are connected through tie lines.

The case of two units was a simple one to the extent that a change in voltage on one unit, $\overline{\Delta E}$, produced a circulating current which flowed through each unit and which produced equal and opposite voltage changes at each voltmeter, that is, $\pm\overline{\Delta V}$.

In the case of more than two units, if there is a change in voltage in T_1, the resulting circulating current divides in the remaining units in inverse proportion to the transfer impedances between unit 1 and the other units. Thus, the stimulus of a change in T_1 to cause secondary changes in the other units becomes increasingly less as the number of units increases, and conversely the influence on T_1 of these secondary effects becomes less as the number of units increases. Thus, the operation of T_1 in a large network or in parallel with a large number of units approaches the operation of T_1 in parallel with an infinite bus. On the other hand, the operation of T_1 in a small network or in parallel with a small number of units

Table III. Criterion C

Condition 1.		
Figure 18	$\overline{\Delta E}-\overline{\Delta V}<\overline{B.W.}$ $\overline{\Delta V}<0$	Stable—no steps apart
Figure 19	$\overline{\Delta E}-\overline{\Delta V}<\overline{B.W.}$ $\overline{\Delta V}<0$ $\overline{\Delta V}<-d$	Stable—no steps apart
Figure 20	$\overline{\Delta E}-\overline{\Delta V}<\overline{B.W.}$ $\overline{\Delta V}<0$ $-d<\overline{\Delta V}$	Stable—no steps apart
Condition 2.		
Figure 21	$\overline{\Delta E}-\overline{\Delta V}<\overline{B.W.}$ $\overline{\Delta V}<0$	Stable—one step apart
Figure 22	$\overline{\Delta E}-\overline{\Delta V}<\overline{B.W.}$ $\overline{\Delta V}<0$ $\overline{\Delta E}-\overline{\Delta V}>d$ $d<\overline{B.W.}$	Stable—one step apart
Figure 22(a)	$\overline{\Delta E}-\overline{\Delta V}<\overline{B.W.}$ $\overline{\Delta V}<0$ $\overline{\Delta E}-\overline{\Delta V}<d$ $d<\overline{B.W.}$	Stable—one step apart
Figure 23	$\overline{\Delta E}-\overline{\Delta V}<\overline{B.W.}$ $\overline{\Delta V}<0$ $d>\overline{B.W.}$	T_1 changes first Stable—one step apart
Figure 24	$\overline{\Delta E}<\overline{\Delta V}<\overline{B.W.}$ $\overline{\Delta V}<0$ $d>\overline{B.W.}$	T_2 changes first Stable—one step apart

approaches the operation of the two-unit network analyzed in detail above.

If T_1 were operating in parallel with an infinite bus, all secondary effects would be zero, and it is obvious that the necessary and sufficient conditions for stability of T_1 are:

$$\overline{\Delta V_1}<\overline{\Delta E_1}<\overline{B.W.}$$

In the two unit analysis it was shown that the minimum criterion for stability was:

$$\overline{\Delta V_1}<\frac{\overline{\Delta E_1}}{2}$$

$$\overline{\Delta E}<\overline{B.W.}$$

Thus, with several units operating in parallel, it appears that the proper criterion for a value of $\overline{\Delta V}$ varies between something less than $\overline{\Delta E}$ and something less than $\overline{\Delta E}/2$. Since most networks are small, the criterion, $\overline{\Delta V}<\overline{\Delta E}/2$ probably should always be used in preference to the less conservative one. The author uses criterion *B* for all networks or groups having less than six units in parallel.

General Conclusions

Load ratio control transformers operating in parallel on the same bus or in a network can be made to operate in a stable manner, be made to correct for circulating currents, and be made to give a specified amount of voltage boost at peak load by proper adjustment of their compensators and voltmeters. Three different criteria may be used in making these adjustments:

Criterion *A*—$\overline{\Delta E}<\overline{B.W.}$
$\overline{\Delta V}<\overline{\Delta E}$

Criterion *B*—$\overline{\Delta E}<\overline{B.W.}$
$\overline{\Delta V}<\dfrac{\overline{\Delta E}}{2}$

Criterion *C*—$\overline{\Delta E}-\overline{\Delta V}<\overline{B.W.}$
$\overline{\Delta V}<0$

The criterion used will depend on:

1. The amount of circulating current permissible.
2. The amount of voltage regulation required at peak load.
3. Whether the number of units in parallel is small or large.

Criteria *A*, *B*, and *C*, in this order, give increasingly more positive assurance of stable operation and increasingly greater limitation on circulating currents. Criteria *A*, *B*, and *C*, in this order, give an increasingly smaller amount of voltage boost possible at peak loads for a specific compensator.

Criterion *A* should preferably not be used for less than six units operating in parallel. It is well suited to large networks. Criterion *B* is the preferred one since it represents the best compromise for stability control and voltage regulation. Criterion *C* would only be used where it is desired to keep all units within one tap of each other (except when correcting for circulating currents), and to insure maximum stability.

Appendix. Compensator Adjustments

Case of Two Units in Parallel

The foregoing analysis has indicated criterion *A* to be unsatisfactory, the choice being between *B* and *C*.

First consider criterion *B* which states that:

$$\overline{\Delta E}<\overline{B.W.} \text{ (to prevent pumping)} \tag{10}$$

$$\overline{\Delta V}<\frac{\overline{\Delta E}}{2} \text{ (for stability)} \tag{11}$$

By definition:

$$\overline{\Delta V}=\frac{\overline{\Delta E}}{Z_{ct}}\left[(R_t+R_c)\cos\theta_{ct}+(X_t+X_c)\sin\theta_{ct}\right]$$

$$\cos\theta_{ct}=\frac{R_{ct}}{Z_c}$$

$$\sin\theta_{ct}=\frac{X_{ct}}{Z_{ct}}$$

$$Z_{ct}=\sqrt{X_{ct}^2+R_{ct}^2} \tag{12}$$

This is simply the value of the voltage consumed in the transformer impedance (and the system back of it) and the compensator impedance by the circulating current due to one tap change.

Combining (11) and (12) we get:

$$R_c\cos\theta_{ct}+X_c\sin\theta_{ct}<\frac{Z_{ct}}{2}-R_t\cos\theta_{ct}-X_t\sin\theta_{ct}<\frac{R_L\cos\theta_{ct}+X_L\sin\theta_{ct}}{2} \tag{13}$$

Since

$$Z_{ct}=(2R_t+R_L)\cos\theta_{ct}+(2X_t+X_L)\sin\theta_{ct} \tag{14}$$

From relation 13

$$X_c<\left(\frac{R_L}{2}-R_c\right)\left(\frac{R_{ct}}{X_{ct}}\right)+\frac{X_L}{2} \tag{15}$$

If the two units are bussed, R_L and X_L become zero and equation 8 becomes:

$$X_c<-R_c\left(\frac{R_{ct}}{X_{ct}}\right) \tag{16}$$

Thus X_c is limited by the value of R_c and R_c is determined by the value of regulation required at normal load. This is:

$$e=(R_t+R_c)\cos\theta_L+(X_c+X_t)\sin\theta_L \tag{17}$$

Substituting the value of X_c (equation 15 in 17) we get:

$$R_c>\frac{e-(R_t\cos\theta_L+X_t\sin\theta_L)}{\sin\theta_L\left(\dfrac{\cos\theta_L}{\sin\theta_L}-\dfrac{R_{ct}}{X_{ct}}\right)}-\frac{\left(\dfrac{X_L}{2}+\dfrac{R_LR_{ct}}{2X_{ct}}\right)}{\left(\dfrac{\cos\theta_L}{\sin\theta_L}-\dfrac{R_{ct}}{X_{ct}}\right)} \tag{18}$$

If the compensator is not required to compensate for transformer regulation at peak load, the term in the numerator, $(R_t\cos\theta_L+X_t\sin\theta_L)$ drops out.

If the two units are bussed and there is no tie line, R_L and X_L are zero and the last term in equation 18 drops out.

If both of these conditions prevail, that is, the units are bussed and transformer compensation is not required, equation 18 becomes:

$$R_c>\frac{e}{\cos\theta_L-\dfrac{R_{ct}}{X_{ct}}\sin\theta_L} \tag{19}$$

Allowing a 10 per cent factor of safety for X_c, relations 16 and 19 become:

$$X_c=1.1R_c\frac{R_{ct}}{X_{ct}} \tag{20}$$

$$R_c=\frac{e}{\cos\theta_L-1.1\dfrac{R_{ct}}{X_{ct}}\sin\theta_L} \tag{21}$$

These have been plotted for several load power factors, values of e and R_{ct}/X_{ct}, in Figures 25, 26, and 27.

Now consider criterion C which states that:

$$\overline{\Delta E}-\overline{\Delta V}<\overline{B.W.}\text{ (to prevent pumping)} \quad \textbf{(22)}$$

$$\overline{\Delta V}<0 \quad \text{(for stability)} \quad \textbf{(23)}$$

Relation 22 states that not only must X_e be always negative, but sufficiently so that $\overline{\Delta V}$ is also always negative. This is a rigid criterion, but will insure the two units against ever getting more than one step apart in any case and it will insure against even one step apart if the two units are on the same bus. This criterion also means that the change in voltage at the voltmeter following a tap change will always exceed the magnitude of the tap change.

Expanding relation 23, we have:

$$\overline{\Delta V}=\frac{\overline{\Delta E}}{Z_{ct}}[(R_t+R_c)\cos\theta_{ct}+(X_t+X_c)\times \sin\theta_{ct}]<0 \quad \textbf{(24)}$$

$$[(R_t+R_c)\cos\theta_{ct}+(X_t+X_c)\sin\theta_{ct}]<0 \quad \textbf{(25)}$$

$$X_c<-\left[(R_t+R_c)\frac{R_{ct}}{X_{ct}}+X_t\right] \quad \textbf{(26)}$$

Combining equations 17 and 26, we obtain a value of R_c to give the desired voltage regulation e at normal load.

$$R_c<\frac{e-(R_t\cos\theta_L+X_t\sin\theta_L)}{\sin\theta_L\left(\frac{\cos\theta_L}{\sin\theta_L}-\frac{R_{ct}}{X_{ct}}\right)}+\frac{\left(\frac{R_tR_{ct}}{X_{ct}}+X_t\right)}{\left(\frac{\cos\theta_L}{\sin\theta_L}+\frac{R_{ct}}{X_{ct}}\right)} \quad \textbf{(27)}$$

If 10 per cent factor is applied in adjusting X_c, relations 26 and 27 become:

$$X_c=1.1\left[(R_t+R_c)\frac{R_{ct}}{X_{ct}}+X_t\right] \quad \textbf{(28)}$$

$$R_c=\frac{e-(R_t\cos\theta_L+X_t\sin\theta_L)}{\sin\theta_L\left(\frac{\cos\theta_L}{\sin\theta_L}-1.1\frac{R_{ct}}{X_{ct}}\right)}+\frac{\left(\frac{R_tR_{ct}}{X_{ct}}+X_t\right)}{\left(\frac{\cos\theta_L}{\sin\theta_L}-\frac{1.1R_{ct}}{X_{ct}}\right)} \quad \textbf{(29)}$$

The term $(R_t\cos\theta_L+X_t\sin\theta_L)$ may be dropped from equations 20 and 22 for the value of R_c if regulation to compensate for transformer impedance is not necessary.

Having determined X_c and R_c, $\overline{\Delta V}$ should be computed from (24) and the value of $\overline{B.W.}$ determined from (22).

In using criterion B, it might be found that R_c is too great to be obtained on standard compensators. If so, it can be reevaluated from (26), and it can be set at its maximum value. In this case, of course, e will have a value less than desired. Its actual magnitude may be determined from (29).

Example

Assume two similar transformers are networked over a tie line. Assume five per cent voltage boost is required at peak load (compensation for transformer regulation not required), and that the units are never to get more than two steps apart except to correct for circulating currents arising from outside influences. Assume the system constants as follows:

$\overline{\Delta E}=1.25\%$ $\quad X_L=6\%$ $\quad e=5\%$

$X_t=8\%$ $\quad R_L=3\%$ $\quad \cos\theta_L=0.80$

$R_t=2\%$ $\quad \sin\theta_L=0.60$

From these we obtain:

$X_{ct}=(2\times8+6)=22$

$R_{ct}=(2\times2+3)=7$

Criterion B will be applied in this case and from (15) and (18) above we get:

$$R_c>\frac{5}{0.6\left(\frac{0.8}{0.6}-\frac{7}{22}\right)}-\frac{\left(\frac{6}{2}+\frac{3\times7}{2\times22}\right)}{\left(\frac{0.8}{0.6}-\frac{7}{22}\right)}>4.7\%=5.3\%\text{ (allowing 10\% margin)}$$

$$X_c<\left(\frac{3}{2}-5.3\right)\frac{7}{22}+\frac{6}{2}<1.792=1.6\text{ (allowing 10\% margin)}$$

In this case,

$$\overline{\Delta V}=\frac{\overline{\Delta E}}{Z_{ct}}[(R_t+R_c)\cos\theta_{ct}+(X_t+X_c)\sin\theta_{ct}]$$

$$=\frac{1.25}{23.0}\left[(2+5.3)\frac{7}{23}+(8+1.6)\frac{22}{23}\right]$$

$$=0.61$$

The band width in this case would probably be set at about two per cent.

Suppose, now, that these two units had been bussed in which case R_L and X_L would be zero. From (16) and (19) above

$$R_c>\frac{5}{0.8+\frac{7}{22}\times0.6}>5.61=6.2\%\text{ (allowing 10\% margin)}$$

$$X_c<-6.2\frac{7}{22}<-1.97=-2.3\%\text{ (allowing 10\% margin)}$$

$$\overline{\Delta V}=\frac{1.25}{23.0}\left[(2+6.2)\frac{7}{23}+(8-2.3)\frac{22}{23}\right]=0.43$$

$$\overline{B.W.}>1.25+0.43>1.68=2\%\text{ or more}$$

Compensator-Adjustment Case of More Than Two Units in Parallel

As was pointed out above, criterion A would be adequate in a large network with a large number of units, but probably would not insure stability in a network of from two to five units. Consider the case where criterion A is adequate.

$$\overline{\Delta E}<\overline{B.W.} \quad \textbf{(30)}$$

$$\overline{\Delta V}<\overline{\Delta E} \quad \textbf{(31)}$$

Combining relations 31 and 12 we get—

$$X_c<\frac{Z_{ct}-(R_t+R_c)\cos\theta_{ct}-X_t\sin\theta_{ct}}{\sin\theta_{ct}} \quad \textbf{(32)}$$

Substituting this value of X_c in equation 17 we get for R_c:

$$R_c>\frac{e-(R_t\cos\theta_L+X_t\sin\theta_L)}{\sin\theta_L(\cot\theta_L-\cot\theta_{ct})}-\frac{Z_{ct}-(R_t\cos\theta_{ct}+X_t\sin\theta_{ct})}{\sin\theta_{ct}(\cot\theta_L-\cot\theta_{ct})} \quad \textbf{(33)}$$

Relations 32 and 33 cannot be further simplified, as in the case of two similar units, because Z_{ct} cannot be expressed in terms of the other quantities. The value of Z_{ct} for any particular unit will be the driving point impedance from the source through that unit and into the network.

The term $(R_t\cos\theta_{ct}+X_t\sin\theta_{ct})$ may be dropped from equation 33 if regulation to compensate for transformer impedance is not necessary.

If the network is small and it is decided to use criterion B instead of A, the proper expressions for R_c and X_c may be determined by replacing the term, Z_{ct}, in equations 32 and 33 with $Z_{ct}/2$.

Definition of Symbols

E_1, E_2—System voltages back of network on T_1 and T_2

V_1, V_2—Voltages impressed on contact-making voltmeter of T_1 and T_2

$\overline{\Delta E_1}$, $\overline{\Delta E_2}$—Change in E_1 and E_2 due to one tap change or regulator step

$\overline{\Delta V_1}$, $\overline{\Delta V_2}$—Change in V_1 and V_2 due to circulating current in transformer and compensator impedances following a tap change

R_{t1}, R_{t2}—Transformer resistances for T_1 and T_2

X_{t1}, X_{t2}—Transformer reactances for T_1 and T_2

R_{c1}, R_{c2}—Compensator resistances for T_1 and T_2

X_{c1}, X_{c2}—Compensator reactances for T_1 and T_2

X_L—Line reactances

R_L—Line resistances

R_{ct}—Circulating resistances

X_{ct}—Circulating reactance

Z_{ct}—Circulating impedance

$\overline{B.W.}$—Band width of voltmeter

e—Voltage boost required by compensator at normal load

θ_{ct}—Power-factor angle of circulating current

θ_L—Power-factor angle of load

d—Critical magnitude of sudden voltage dips

n—Number of tap changes required to reach a stable condition

All of the above voltage and impedance quantities are in per cent on base kilovolt-amperes and voltage. R_t and X_t include system resistance and reactance back of the transformer.

Tennessee Valley Authority Hydroelectric Stations—Electrical Design

RAYMOND A. HOPKINS
MEMBER AIEE

THE Tennessee Valley Authority, since its establishment in 1933, has, among its many other activities, developed an electric generating and transmission system which now has a generating capacity of 1,050,200 kilowatts in 69 generating units, a one-hour peak load of about 907,200 kilowatts, and a monthly output of 520,000,000 kilowatt hours. The original generating facilities taken over by the Authority in the fall of 1933 consisted of the Wilson Hydro and Steam Plants both built at Muscle Shoals, Alabama, by the United States Government during the years 1918 to 1925 and totaling 244,000 kilowatts in generating capacity. During subsequent years the Authority has acquired and leased from private utilities previously operating in the Tennessee Valley area 5 hydro plants and 4 steam plants, totaling 292,300 kw in generating capacity, with transmission lines. Up to the present time the Authority has built and put into operation 6 new hydro plants and extensions to existing hydro plants, totaling 513,900 kilowatts in 15 generating units, with transmission lines. The Authority now has under construction 6 new hydro plants, 1 new steam plant and extensions to 2 existing hydro plants, totaling 969,200 kilowatts in 29 generating units, with transmission lines. The system as now operating and under construction, including the future generating units definitely provided for, will have a total capacity of 2,584,700 kilowatts in 115 generating units.

This paper discusses the principal electrical design features of the ten hydroelectric stations designed and constructed or under construction by the Tennessee Valley Authority. Ocoee No. 3 and Apalachia, authorized in July 1941, are now in the early stages of design and are not described herein.

The Transmission System

Figure 1 presents a map of the Tennessee Valley area showing, as of August 1941, locations of the principal generating stations, transmission lines, substations, and points of interconnection with adjacent systems. Table I lists the principal generating stations as of August 1941, giving the number of units and capacities at each station.

The voltage of the main transmission system, nominally 154-kv, was chosen because study indicated that this voltage would be adequate for transmission purposes within the Tennessee River Basin, and also because 154-kv was already established at Muscle Shoals. The 500-mile transmission line connecting Norris, Watts Bar, Wheeler, Wilson, and Pickwick dams and the Memphis substations has, however, been designed for 220-kv operation to provide the higher transmission voltage if it is required in the future, but is now operated at 154,000 volts. All new generating stations have been connected to the 154-kv transmission system.

The lower voltages, 110-, 66-, and 44-kv, were also existing in the Valley. At Norris, Pickwick, and Guntersville, auto transformers are installed to supply 110-kv transmission. At Chickamauga 3-winding step-up transformers are provided to serve the 44-kv transmission system of the city of Chattanooga, both from the generators and from the 154-kv system. As a rule, step-down points have been located near load centers and not at generating stations, but in every case where the most economical step-down location is in the vicinity of a generating station, the economies of centralized station attendance have been secured by combining the step-down and generating stations.

The Generating Stations

Two general designs of generating station arrangement have been developed to suit the two general classes of sites: (1) the tributary developments, and (2) the main river developments. While these two patterns do not represent all the variations of design employed to meet various specific requirements of each site, they do give a general picture of the two fundamental classes of developments.

The tributary development, illustrated by Figures 2, 3, and 4 is in many respects typical of Norris, Hiwassee, and Cherokee. There is no navigation lock. The dam is high, and the spillway is short. The concrete dam abuts rock on both sides of the river. The water wheels are Francis type, of relatively large capacity and small number. Being usually away from load centers, the plant requires only a minimum number of transmission lines. Sloping, rocky, and often steep terrain usually requires a high, stepped switchyard structure.

The main river development, illustrated by Figures 5, 6, and 7 is in many respects typical of Wheeler, Pickwick, Guntersville, Chickamauga, Watts Bar, Kentucky, and Fort Loudoun. This type of development usually requires a navigation lock, a long adjustable spillway, long earth embankments, propeller-type water wheels of relatively small capacity in large number, and a number of transmission lines. Sufficient level natural terrain or fill is usually available for a low, flat, economical switchyard arrangement.

The electrical bay is generally located downstream from the generator room and extends the full length of the generator room in space available over the draft tubes without adding to the total width of the structure. Within the electrical bay are located the generator-voltage switchgear and connections, the station service transformers and switchgear, the control cable tunnel for serving the generating units, and the generator fire protective equipment.

The service bay is at the shore end of the station. It contains facilities for the handling of incoming equipment, space for assembly of water wheel and generator parts, and auxiliary equipment common to the entire station such as machine shop oil purifying and storage facilities and sump pumps. The control room is located in the upper part of the service bay where it is somewhat central between the generators and switchgear in the station and the transformers and switchgear in the yard. Office space and public facilities are also located in the service bay.

The transformers and switchyard are usually located on shore slightly downstream from the centerline of generating units, although the exact location is largely determined by the topography. At all plants except Pickwick, local topography permits the alternating of generators and lines along the bus.

Main Electrical Connections

The single line wiring diagram of Figure 8 is in many respects typical for the new

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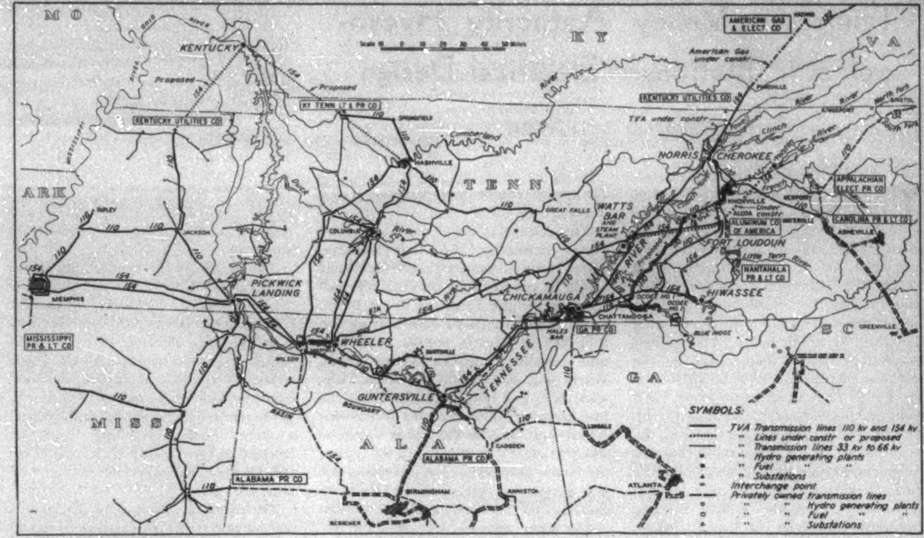
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PICKWICK LANDING
WHEELER
GUNTERSVILLE
CHICKAMAUGA
WATTS BAR
WATTS BAR STEAM PLANT
NORRIS
CHEROKEE
FORT LOUDOUN
HIWASSEE
KENTUCKY UTILITIES CO
KY TENN LT & PR CO
AMERICAN GAS & ELECT CO
American Gas under constr
TVA under constr
APPALACHIAN ELECT PR CO
CAROLINA PR & LT CO
ALUMINUM CO OF AMERICA
NANTAHALA PR & LT CO
GA PR CO
ALABAMA PR CO
MISSISSIPPI PR & LT CO
Proposed
MEMPHIS
JACKSON
RIPLEY
NASHVILLE
SPRINGFIELD
COLUMBIA
GREAT FALLS
KNOXVILLE
CHATTANOOGA
HALES BAR
OCOEE NO 1
OCOEE NO 2
BLUE RIDGE
ASHEVILLE
GREENVILLE
KINGSPORT
BRISTOL
BIRMINGHAM
BESSEMER
GADSDEN
ANNISTON
ATLANTA
HUNTSVILLE
WILSON
Tennessee River
Cumberland River
Duck River
Clinch River
Holston River
French Broad River
Little Tenn River
Ohio River
Mississippi River
110
154
SYMBOLS:
TVA Transmission lines 110 kv and 154 kv
" Lines under constr or proposed
" Transmission lines 33 kv to 66 kv
" Hydro generating plants
" Fuel " "
" Substations
Interchange point
Privately owned transmission lines
" " Hydro generating plants
" " Fuel " "
" " Substations

Figure 1. Map of principal transmission lines within and adjacent to the Tennessee Valley area as of August 1941

hydro stations designed and built by the Authority.

All generation for the new plants is at 13,800 volts, with step-up transformers connecting the generators to the 154-kv transmission system. At Norris, Pickwick, Guntersville, Watts Bar, and Hiwassee, one transformer bank is provided for each generator, with no circuit breakers between the generator and transformer, so that they are operated as a unit with all switching on the high voltage side of the transformers. At Wheeler, Kentucky, Cherokee, and Fort Loudoun, one transformer bank is provided for each pair of generators, and generator circuit breakers are provided in addition to the breakers on the high voltage side of the transformers. At Chickamauga, one bank is provided for each generator, but since three-winding transformers are required to serve the 154-kv and 44-kv systems, generator breakers are provided for switching the generators without interrupting the 154-kv–44-kv connections.

The scheme of bussing and switching of the 154-kv circuits has proved to be simple, economical, adaptable to all system operating requirements, and susceptible to a number of modifications to meet specific requirements. Fundamentally it is the main-and-transfer-bus scheme with each circuit connected to the main bus by one breaker and to the transfer bus by a motor-operated disconnecting switch. The conventional bus tie bay with bus tie oil circuit breaker is not provided but two oil circuit breakers are provided for at least one and usually for each main step-up transformer for connecting to both the main and transfer busses. The saving in eliminating the bus tie bay largely offsets the cost of the additional breakers which are considered as spare breakers and which can be substituted for any of the normal transformer or line breakers for use in case of breaker trouble, for maintenance and inspection of breakers, and for special system operating setups. Each spare breaker is equipped with a set of overcurrent and directional ground relays for emergency use in the event of failure of normal transmission line relays.

The main-and-transfer bus arrangement is also used for the 110-kv switchyards and for some of the lower voltages which are fed from the 154-kv busses through auto-transformers. Following the 154-kv practice, bus tie bays are not provided for the spare breakers but instead the auto-transformer circuits are provided with double low-voltage circuit breakers to connect the auto-transformers to both the main and transfer busses. At Chickamauga, the double-bus double-breaker arrangement for 44-kv circuits is used to provide the most reliable service possible for the important load of the city of Chattanooga.

Modifications of the main-and-transfer-bus scheme can be readily made without any major changes to structures or moving of equipment. During the early stages of development of a station, when not more than two circuits are installed, for instance a transformer with one outgoing line, a single high voltage breaker with bypass disconnect can be used. Later, with three or four circuits, the ring-bus scheme with one breaker per circuit can be used by extending the original structure. With this scheme, each circuit is connected to two breakers and the busses are included in the relay protection of the circuits. With more than four circuits the ring-bus scheme is readily developed into the main-and-transfer-bus scheme by again extending the same structures. At stations where lines and generators can be paired, the adding of sectionalizing disconnects in the transfer bus permits the breaker-and-a-half scheme of operation with one and a half breakers per circuit. Under this method of operation, each circuit is connected to two breakers, the busses are provided with differential relay protection, all breakers are operated closed and all bus sections energized. Considerable operating flexibility is secured to meet emergency or special loading conditions or for operating with the system split. The conventional double-bus double-breaker scheme with two breakers per circuit can be secured, if needed, by merely removing a disconnect motor mechanism and adding a breaker and isolating switch for each line circuit.

Generators

Ratings of turbines at all plants are based on normal heads. Greater capacities, averaging in the neighborhood of 16 percent greater, are inherently available at maximum heads. A normal generator rated at 60-degree rise, allowable with class *A* insulation, has a capacity about 15 percent greater if operated at 80-degree rise, allowable with class *B* insulation. To match the variable turbine outputs due to head variations, the generator is provided with class *B* insulation but is given a normal rating at 60-degree rise about equal to the turbine rating at normal head. It, therefore, operates at conservative temperatures ranging from 60-degree rise to 80-degree rise with full gate, at heads ranging from normal to maximum. It is expected that the total time of operation at the higher temperature under high heads will not be sufficient to reduce materially the normal life of the insulation. Table II illustrates the above coordination of ratings by actual data from four typical installations.

Power factor ratings are all 0.9. Actual power factor under full load generation is usually above 0.95 lagging. At some plants, particularly those near heavy load centers, generators are often floated at zero load, leading power factor to assist power transmission and to stand by as spinning reserve generating capacity. Voltage rating is uniformly the AIEE standard, 3-phase, 60-cycle, 13,800-volt with performance tolerance of 5 percent plus or minus.

The general type of generator at all plants is vertical shaft with bearings below the rotor. The shaft is coupled to the water wheel; the rotor is overhung above the thrust and guide bearings; the ventilating air is re-circulated by means of fans on the rotor within a closed housing and is cooled by water from the scroll case; the main and pilot exciters are on the shaft above the rotor and uniquely telescoped one within the other. Table III presents principal turbine-generator data from four typical installations.

The shaft is a solid forging with a flange at the lower end for coupling to the turbine shaft, and in most cases it is also flanged at the upper end to receive the rotor which is centered by a male-female fit, keyed by a diametrical torque key and secured by bolts.

The rotor is of conventional design with a fabricated or forged hub, fabricated arms, laminated rim, and field poles. It is assembled and stacked on a center pedestal on the erection floor of the powerhouse.

The thrust bearing and the guide bearing are located immediately below the rotor in the same lubricating oil reservoir, and both bearings are self-lubricating without the assistance of oil pumps. The oil is cooled by water coils. Temperatures of the oil and of the stationary shoes are measured by thermometers which indicate and record on the turbine gauge board.

The stator is shop-fabricated in four or more sections, and in most cases completely assembled and wound at the factory with the exception of the joint coils and their connections, which must be assembled and connected at the site. At Chickamauga, all the stator coils were installed at the site after the stator was completely assembled. All stator windings are arranged in two parallel paths per phase, and both ends of each path are brought out to terminals for protective relaying. The neutral ends are bussed within the generator housing and one lead is carried out for grounding.

The neutral is grounded through an individual reactor for each generator. The value of reactance is chosen to limit the phase-to-ground fault current to a value about equal to the phase-to-phase fault current. This amount of neutral reactance is low enough to assure dependable generator relay action and to avoid danger of resonant voltages and is high enough to avoid excessive fault currents.

The excitation system consists of a main exciter and a pilot exciter directly connected to each generator main shaft above the rotor, together with rheostats, switching, and regulation. Spare exciters are not provided since experience records of low-speed, direct-connected exciters indicate generally better reliability than other essential parts of the entire turbine-gener-

Table I. Generating-Station Capacities (as of August 1941)

Station	Number of Units	Kva Per Unit	Power Factor	Station Kilowatts	Group Kilowatts
Taken over in 1933					
Wilson Hydro	4	32,500	0.8	104,000	244,000
	4	25,000	0.8	80,000	
Wilson Steam	3	23,530	0.85	60,000	
Acquired hydroelectric plants					
Great Falls	1	14,300	0.9	12,900	137,270
	1	20,625	0.8	16,500	
Blue Ridge	1	25,000	0.8	20,000	
Ocoee No. 1	5	4,500	0.8	18,000	
Ocoee No. 2	2	11,750	0.8	18,800	
Hales Bar	10	3,750	1.0	37,500	
	4	4,250	0.8	13,600	
Acquired steam plants					
Nashville	1	23,600	0.85	20,000	155,000
	1	15,625	0.8	12,500	
	1	8,125	0.8	6,500	
	3	3,000	1.0	9,000	
Hales Bar	2	25,000	0.8	40,000	
Parksville	2	8,125	0.8	13,000	
Memphis (leased)	4		0.8	54,000	
New stations in operation					
Norris	2	56,000	0.9	100,800	513,900
Wheeler	4	36,000	0.9	129,600	
Pickwick	2	40,000	0.9	72,000	
Guntersville	3	27,000	0.9	72,900	
Chickamauga	3	30,000	0.9	81,000	
Hiwassee	1	64,000	0.9	57,600	
New stations and extensions under construction					
Pickwick	2	40,000	0.9	72,000	969,200
Wilson	6	28,000	0.9	151,200	
Watts Bar	5	33,333	0.9	150,000	
Cherokee	3	33,333	0.9	90,000	
Ft. Loudoun	2	35,500	0.9	64,000	
Kentucky	5	35,500	0.9	160,000	
Watts Bar Steam	3	66,667	0.9	180,000	
Ocoee No. 3	1	30,000	0.9	27,000	
Apalachia	2	41,667	0.9	75,000	
Future units definitely provided for					
Wilson	4	28,000	0.9	100,800	565,300
Wheeler	4	36,000	0.9	129,600	
Pickwick	2	40,000	0.9	72,000	
Guntersville	1	27,000	0.9	24,300	
Chickamauga	1	30,000	0.9	27,000	
Hiwassee	1	64,000	0.9	57,600	
Cherokee	1	33,333	0.9	30,000	
Ft. Loudoun	2	35,500	0.9	64,000	
Watts Bar Steam	1	66,667	0.9	60,000	

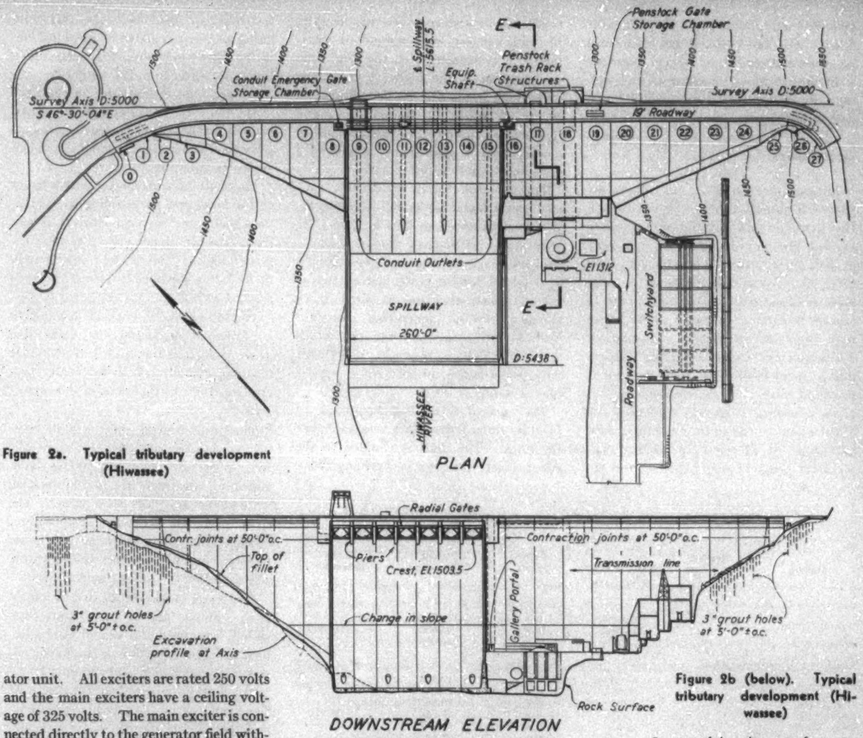
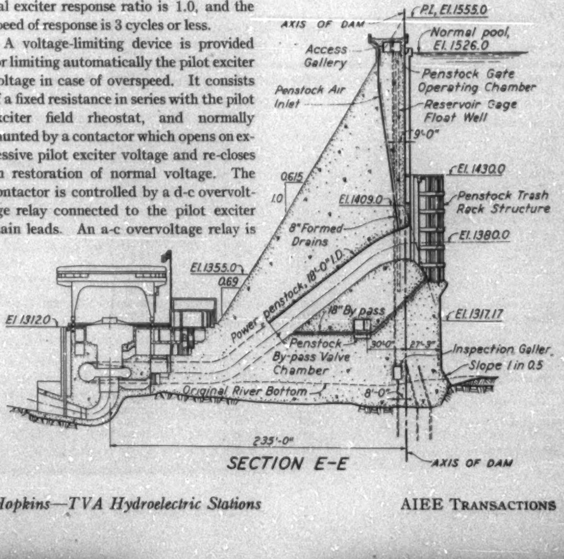

PLAN
Survey Axis D:5000
S 46°-30'-04" E
Conduit Emergency Gate Storage Chamber
℄ Spillway L:5615.5
Equip. Shaft
Penstock Trash Rack Structures
Penstock Gate Storage Chamber
19' Roadway
Conduit Outlets
SPILLWAY
260'-0"
D:5438
El 1312
Switchyard
Roadway
HIWASSEE RIVER
E
DOWNSTREAM ELEVATION
Radial Gates
Contr. joints at 50'-0" o.c.
Top of fillet
Piers
Crest, El. 1503.5
Contraction joints at 50'-0" o.c.
Transmission line
Gallery Portal
Change in slope
3" grout holes at 5'-0" ± o.c.
Excavation profile at Axis
Rock Surface

Figure 2a. Typical tributary development (Hiwassee)

Figure 2b (below). Typical tributary development (Hiwassee)

Section of dam shown in figure 2a


SECTION E-E
AXIS OF DAM
P.I. El. 1555.0
Normal pool, El. 1526.0
Access Gallery
Penstock Gate Operating Chamber
Penstock Air Inlet
Reservoir Gage Float Well
9'-0"
El. 1430.0
0.615
1.0
El. 1409.0
Penstock Trash Rack Structure
8" Formed Drains
El. 1380.0
El. 1355.0
0.69
Power penstock, 18'-0" I.D.
El 1312.0
18" By pass
El. 1317.17
Penstock By-pass Valve Chamber
30'-0"
27'-3"
Inspection Gallery
Slope 1 in 0.5
Original River Bottom
8'-0"
235'-0"

ator unit. All exciters are rated 250 volts and the main exciters have a ceiling voltage of 325 volts. The main exciter is connected directly to the generator field without rheostat or breaker. Voltage control is by electrically operated rheostat and breaker in the main exciter field circuit. The pilot exciter field circuit has no circuit breaker but is provided with a manually operated rheostat which is given a permanent setting during preliminary operation and adjustment of the unit. The rheostats and breaker, together with the voltage regulator contactors, are located in a steel cubicle on the operating floor close to each generator with the result that the entire excitation circuit is short and compact.

Voltage regulation acts upon the field rheostat of the main exciter. The regulator measures voltage through a three-phase torque motor connected to potential transformers on the generator main leads. Cross-current compensation is used where two generators are connected to one transformer bank to assure proper division of wattless current between the two generators. The regulator is the standard face-plate, rheostatic type with high and low contactors for quick response. The nominal exciter response ratio is 1.0, and the speed of response is 3 cycles or less.

A voltage-limiting device is provided for limiting automatically the pilot exciter voltage in case of overspeed. It consists of a fixed resistance in series with the pilot exciter field rheostat, and normally shunted by a contactor which opens on excessive pilot exciter voltage and re-closes on restoration of normal voltage. The contactor is controlled by a d-c overvoltage relay connected to the pilot exciter main leads. An a-c overvoltage relay is also provided and connected to potential transformers on the main generator leads. The relay actuates the quick-lowering contactor on the main exciter field rheostat and is set to operate at about 16,000 volts.

Carbon dioxide fire extinguishing equipment is provided at each station for the protection of the generators. The enclosed generator housing, which is made especially tight for this purpose, renders this type of protection particularly effective. A centralized assembly of standard 50-pound, 850-pounds-per-square-inch, carbon dioxide cylinders is provided for a group of one to five generators. The liquid is automatically discharged and routed through valves and permanent piping to the particular generator indicated, where it is discharged through nozzles into the generator. The initial discharge provides a concentration of not less than 50 percent within the generator, which is sufficient to extinguish a fire and is effective in cooling any hot metal. This is followed by delayed discharges automatically timed to maintain for 30 minutes a concentration of not less than 25 percent, which is sufficient to prevent reignition. The initiation of the discharge is actuated either by thermostats within the generator housing, by the operation of the generator differential relays, by a break-glass push-button station located on the turbine gauge board, or by a manual release at the centralized assembly of carbon dioxide cylinders

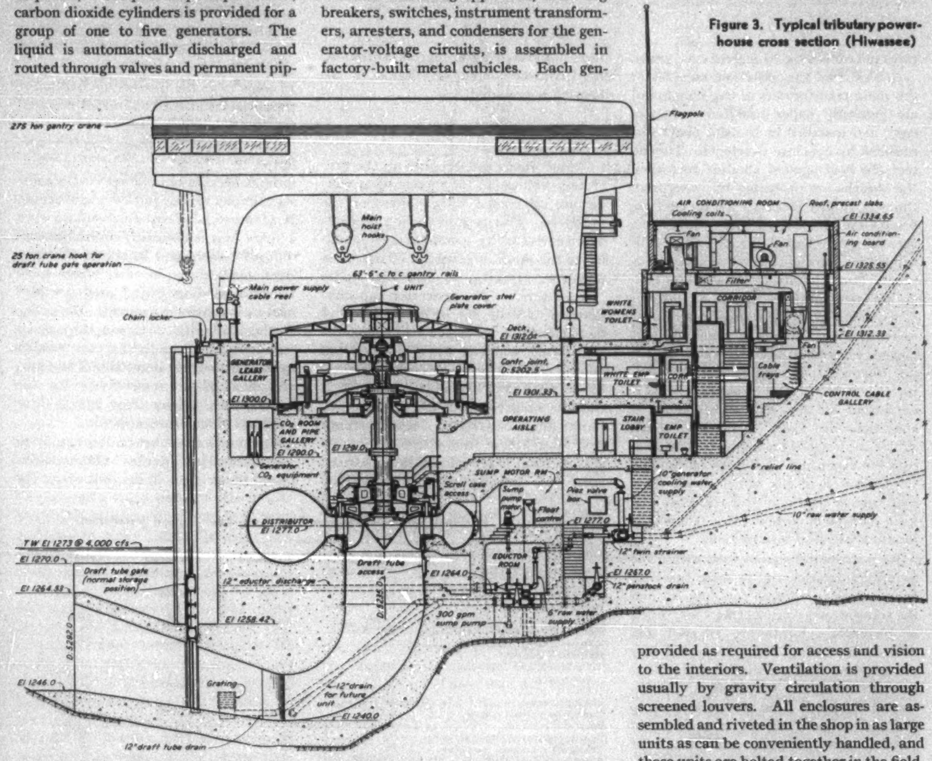

275 ton gantry crane
Flagpole
Main hoist hooks
25 ton crane hook for draft tube gate operation
63'-6" c to c gantry rails
Main power supply cable reel
℄ UNIT
Generator steel plate cover
Chain locker
Deck, El 1312.0±
WHITE WOMENS TOILET
AIR CONDITIONING ROOM
Roof, precast slabs
Cooling coils
El 1334.65
Fan
Fan
Air conditioning board
El 1325.55
Filter
CORRIDOR
El 1312.33
Fan
Cable trays
CONTROL CABLE GALLERY
GENERATOR LEADS GALLERY
Contr joint, D. 5202.5
WHITE EMP TOILET
CORR
El 1300.0
El 1301.33
OPERATING AISLE
STAIR LOBBY
EMP TOILET
CO2 ROOM AND PIPE GALLERY
El 1290.0
El 1291.0
Generator CO2 equipment
SUMP MOTOR RM
6" relief line
10" generator cooling water supply
Scroll case access
Sump pump motor
Float control
Piez valve box
10" raw water supply
℄ DISTRIBUTOR El 1277.0
El 1277.0
12" twin strainer
T W El 1273 @ 4,000 cfs
El 1270.0
Draft tube gate (normal storage position)
El 1264.53
Draft tube access
12" eductor discharge
El 1264.0
EDUCTOR ROOM
El 1267.0
12" penstock drain
D. 5235.0
El 1258.42
300 gpm sump pump
6" raw water supply
D. 5292.0
El 1246.0
Grating
12" drain for future unit
El 1240.0
12" draft tube drain

Figure 3. Typical tributary powerhouse cross section (Hiwassee)

Generator-Voltage Switchgear and Connections

Indoor switching apparatus, including breakers, switches, instrument transformers, arresters, and condensers for the generator-voltage circuits, is assembled in factory-built metal cubicles. Each generator circuit is isolated in a separate room in the electrical bay. Enclosing cubicles are of steel or aluminum, as best adapted to each application. Breaker enclosures requiring considerable structural strength are of steel plate reinforced with structural steel frames and protected by Parkerizing and painting. Barriers between phases are of non-magnetic steel or aluminum. In locations where re-painting is difficult, copper-bearing steel is used. Enclosures for disconnecting switches, instrument transformers and other light equipment and for the busses are generally of aluminum sheet with aluminum barriers. The principal advantages of aluminum for enclosures are ease of fabrication, light weight, and permanence without paint or other protection. Recently, due to greater need of aluminum elsewhere, steel cubicles with copper or transite barriers are being substituted. Hinged doors and glazed windows are provided as required for access and vision to the interiors. Ventilation is provided usually by gravity circulation through screened louvers. All enclosures are assembled and riveted in the shop in as large units as can be conveniently handled, and these units are bolted together in the field.

Cables from the generators to the switching cubicles are insulated with corona-resisting rubber compound, shielded with copper tape, and covered with tape and braid. At each end the braid and copper shield are removed for a sufficient distance back from the lug to provide adequate creepage distance. The shielding tape is belled out at each end to form a stress cone and is grounded. Such circuits are designed for conductor tempera-

Table II. Typical Co-ordination of Water-Wheel, Generator, and Transformer Capacities

	Chickamauga	Cherokee	Pickwick	Hiwassee
Turbine continuous rating (normal head) hp	36,000	41 500	48,000	80,000
Corresponding generator output, kw	25,900	30,000	34,800	58,200
Turbine maximum capacity (max. dead) hp	42,000	50,000	55,000	93,000
Corresponding generator output kw	30,200	36,000	39,800	67,700
Generator continuous rating, kva	30,000	33,333	40,000	64,000
Generator 60° capacity at 0.8 pf, kw	24,000	26,667	32,000	51,200
0.9 pf, kw	27,000	30,000	36,000	57,600
1.0 pf, kw	30,000	33,333	40,000	64,000
Generator 80° capacity at 0.8 pf, kw	27,600	30,600	36,800	58,800
0.9 pf, kw	31,000	34,400	41,400	66,200
1.0 pf, kw	34,500	38,300	46,000	73,600
Transformer continuous rating, kva	27,000	30,000	36,000	56,250
Capacity with fans, kva	36,000	40,000	48,000	75,000

tures not exceeding 70 degrees centigrade.

Cables from the switching cubicles to the main transformers in the switchyard are generally paper-insulated, lead-covered, and installed in transite ducts surrounded by concrete envelopes. To protect the lead against alkaline corrosion, the sheaths are protected by waterproof tapes or rubber jackets. To prevent excessive sheath currents and voltages, the sheaths are grounded at each end and insulated by porcelain sleeves at about midpoint. Such circuits are designed for conductor temperatures up to 82 degrees centigrade. Where a vertical rise of over approximately 30 feet is required, varnished cambric insulation with non-migrating impregnating compound and shielding tape is used. The covering is braid if conditions are dry, and rubber jacket if conditions are likely to be wet. Such cables are designed for conductor temperature not exceeding 78 degrees centigrade. Configuration of the cables in all these long runs is chosen to give the most uniform division of current consistent with a practical physical arrangement.

Bare structural shapes used within the switching cubicles and in some cases for heavy circuits between the cubicles and the transformers consist of flat bars, tubes, or channels. Joints are silvered and bolted where many connections and taps are required and are welded where long uninterrupted runs are involved. At Hiwassee the leads from the generators to the transformers, having a capacity of 3,200 amperes per phase and a length of about 180 feet, consist of two aluminum 6-inch channels per phase, forming a hollow square with ventilation top and bottom and with welded joints. With the absence of low voltage busses, the short circuit stresses at all plants are within the limits of conventional bus supports. Enclosures for these heavy circuits are of non-magnetic material, usually aluminum plate. Such circuits with silvered and bolted joints or with welded joints are designed to operate at conductor temperatures not exceeding 85 degrees centigrade. At Hiwassee forced ventilation of the housing is provided.

Transformers

Transformers for stepping up the generation voltage to the transmission voltage are self-cooled with blowers on the radiators. This type of cooling provides a range of capacity somewhat corresponding to the range of generator 60-degree to 80-degree capacity, as described under "Generators." The generator capacity, however, at 80-degree rise is 115 percent of its capacity at 60-degree rise, while the transformer capacity when blown is 133 percent of its capacity self-cooled. Hence, for a generator having for example a capacity range from 100 to 115, the transformer might be assigned a capacity range from 86 to 115, or from 100 to 133, or any $33^{1}/_{3}$-percent range intermediate between these two extremes. Since experience indicates that the transformer is usually designed on a little closer margin than the generator, and since the blower capacity is secured by forced cooling which results in a greater spread between hot-spot temperature and indicated temperature, the transformer rating is chosen somewhat above the middle of the permissible range, say, 96 to 128. This coordination of transformer and generator ratings is illustrated in Table II by actual data from four typical installations.

Auto-transformers for power transfer between two system voltages at the same generating station are also self-cooled with blowers, but their ratings are determined entirely by power system requirements. In some cases the blowers are omitted in the initial installation and added as power-transfer requirements increase.

The three-phase transformer is preferred to a bank of single-phase transformers because of its lower cost, smaller size, simpler wiring, and less maintenance. It, of course, has a cost disadvantage when a spare unit is required, and it becomes difficult to ship and handle in the large sizes, particularly in cases where more than two windings are required, or where load-ratio control is required. The policy of the Authority is to use three-phase transformers generally for the smaller banks, particularly in multi-unit stations, and single-phase transformers for the larger banks and for those having three windings or load-ratio control.

The transformers are built generally to conventional standards. Oil is maintained by inert gas in the tank above the

Table III. Principal Turbine-Generator Data From Four Typical Installations

	Norris	Hiwassee	Wheeler	Chickamauga
Continuous rating, kva	56,000	64,000	36,000	30,000
Power factor	0.9	0.9	0.9	0.9
Voltage, kv	13.8	13.8	13.8	13.8
Normal speed, rpm	112.5	120	85.7	75
Runaway speed, rpm	215	235	155	207
Efficiency, per cent:				
Turbine	92	93	91	91
Generator	97.9	97.4	97.5	96.6
Unit	89.1	90.6	88.7	87.9
Shaft diameter, in	35	36	33.5	33.5
Condenser capacity, kva	40,000	50,000	20,000	16,500
Short-circuit ratio	1.045	1.1	1.1	1.12
Synchronous reactance, per cent	97	98	102	107
Transient reactance, per cent	34	35	39	46
Fly-wheel effect, WR^2:				
Turbine	2,600,000	2,600,000	4,537,000	6,700,000
Generator	62,740,000	60,000,000	62,260,000	75,000,000
Major crane lifts, lbs:				
Turbine	154,000	142,000	525,000	490,000
Generator	483,000	489,000	380,000	465,000
Weight of rotating parts, lbs:				
Turbine	165,000	142,000	262,000	332,000
Generator	543,000	554,000	523,000	604,000
Hydraulic thrust	385,000	410,000	1,033,000	1,052,000
Total weight on thrust bearing	1,093,000	1,106,000	1,818,000	1,988,000
Rotor diameter, ft, in	27'-8"	26'-2"	29'-3"	31'-8"
Overall diameter, ft, in	32'-6"	37'-10"	35'-0"	46'-3"
Neutral reactor impedance, ohms	0.84	0.97	0.86	0.78
Exciter rating, kw				
Main	275	250	200	300
Pilot	15	10	17	28
Main and pilot exciter voltage	250	250	250	250

oil level. Standard no-load ratio changers are provided in the neutral ends of the high-voltage windings. For the three-winding step-up transformers at Chickamauga and for most of the auto-transformers used for system power transfer, load-ratio changers are provided with remote control and indication at the main switchboard. The autotransformers at Norris and Pickwick and the three-winding transformers at Chickamauga are provided with automatic voltage control equipment to maintain predetermined voltages on the 110-kv and 44-kv busses.

The neutrals of the wye-connected high-voltage windings are grounded through a single neutral reactor for each transformer bank. The value of neutral impedance is chosen between two criteria: high enough to limit phase-to-ground fault currents to values not greater than three-phase fault currents, and low enough to limit the ratio of zero to positive sequence impedance to not greater than two.

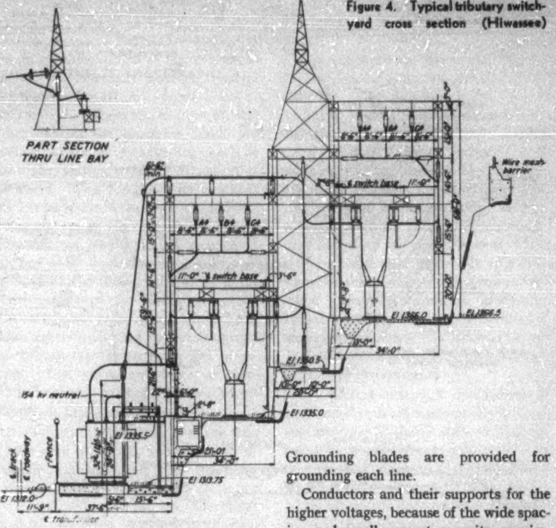


Figure 4. Typical tributary switchyard cross section (Hiwassee)

Transmission-Voltage Switchgear and Connections

Two general designs of switchyard arrangements have been developed to suit the two general classes of plants described under "The Generating Stations." The tributary plants where the terrain is steep are illustrated in Figure 4, and the main river plants where ample level ground area is available are illustrated in Figure 7. Both of these general types are designed to be developed into the ring-bus scheme, the main-and-transfer-bus scheme, the breaker-and-a-half scheme, or the double-bus double-breaker scheme of connections as described under "Main Electrical Connections."

Oil circuit breakers, disconnecting switches, and other equipment are all of conventional outdoor design. The 161-kv breakers are 2,500,000-kva 8-cycle tripping class except Guntersville and Norris which are 1,500,000-kva, 8-cycle tripping class. The 115-kv breakers are 1,500,000-kva, 8-cycle tripping class. Except for the original installations at Norris and Wheeler all line breakers are of the 35-cycle reclosing type. At the present time, however, the breakers are not operated as 35-cycle reclosing breakers but instead the breakers are arranged to reclose only on a de-energized line or bus or on an energized line which is in synchronism with the bus as determined by a synchronism check relay. In the future, after additional tie lines have been installed to give greater assurance that synchronism will be maintained between the two ends of a line which has tripped out, it is expected that the high-voltage line breakers will be provided with relays to reclose the breakers in 35 cycles without checking synchronism or voltage. Higher speed closing and reclosing breakers are now being considered for critical locations on the system. Breakers of each rating are interchangeable with respect to locations of current transformers, mechanisms, oil connections, and other accessories. Oil supply and return lines are installed under the surface of the yard near the breakers with outlets for temporary hose connections for filling, draining, and filtering from an oil storage and treating plant located either in the yard or in the powerhouse.

Disconnecting switches have two fixed insulator stacks and one rotating insulator stack, and are mounted either upright or underhung. Those used for isolating circuit breakers and other devices are 3-pole manually operated. Those used for connecting circuits to the transfer bus and for sectionalizing the busses are 3-pole electrically operated. Interlocking of the electrically operated disconnects with breakers to prevent operating them under load is not found necessary, but as an operating aid, two control switches for each disconnect are used on the control board, and are so wired that both must be turned simultaneously to operate the disconnect. This scheme has proved to be simpler and just as reliable as interlocking the disconnect with the breaker. Grounding blades are provided for grounding each line.

Conductors and their supports for the higher voltages, because of the wide spacing and small current capacity requirements, constitute a design problem which is principally a structural one. Busses are of four-inch galvanized steel pipe with welded connections. The welds are zinc-sprayed outside and asphaltum-painted inside. The busses are supported on pedestal insulator stacks. At the terminals of equipment such as transformers, arresters, and disconnects, flexible connections are provided to prevent unnecessary stresses on the porcelains. Connections from breakers to lines are of stranded copper cables supported on strain insulators.

Structures are of conventional latticed truss, galvanized steel construction, standing on concrete foundations. Trusses and columns are shop-riveted in maximum allowable shipping lengths and field-bolted as erected.

The surface of the switchyard is drained and finished with from 6 to 12 inches of crushed stone which is intended, among other things, to dissipate heat in case of an oil fire. Walkways are provided for the most traveled routes.

Relaying

Each generator is protected against grounds, open circuits, and turn-to-turn faults by induction differential relays connected to give both over-all-differential and split-winding protection. The differential relays are connected in the full winding at the line end and half the wind-

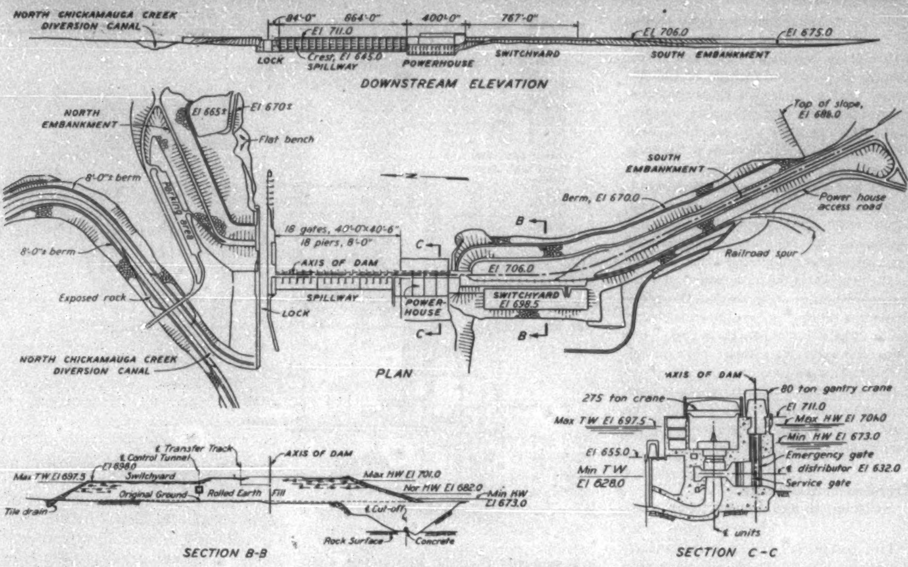

NORTH CHICKAMAUGA CREEK DIVERSION CANAL
84'-0"
664'-0"
400'-0"
767'-0"
El 711.0
Crest, El 645.0
LOCK
SPILLWAY
POWERHOUSE
SWITCHYARD
El 706.0
El 675.0
SOUTH EMBANKMENT
DOWNSTREAM ELEVATION
NORTH EMBANKMENT
El 665±
El 670±
Flat bench
8'-0"± berm
Parking area
8'-0"± berm
Exposed rock
18 gates, 40'-0"x40'-6"
18 piers, 8'-0"
AXIS OF DAM
SPILLWAY
LOCK
C
POWER-HOUSE
C
B
SWITCHYARD El 698.5
B
El 706.0
NORTH CHICKAMAUGA CREEK DIVERSION CANAL
PLAN
SOUTH EMBANKMENT
Berm, El 670.0
Top of slope, El 686.0
Power house access road
Railroad spur
℄ Transfer Track
℄ Control Tunnel
El 698.0
Max TW El 697.5
Switchyard
Original Ground
Rolled Earth
Fill
AXIS OF DAM
Max HW El 701.0
Nor HW El 682.0
Min HW El 673.0
Tile drain
℄ Cut-off
Rock Surface
Concrete
SECTION B-B
AXIS OF DAM
80 ton gantry crane
275 ton crane
El 711.0
Max TW El 697.5
Max HW El 701.0
Min HW El 673.0
El 655.0
Emergency gate
Min TW El 628.0
℄ distributor El 632.0
Service gate
℄ units
SECTION C-C

Figure 5. Typical main river development (Chickamauga)

ing at the neutral end. An overcurrent ground back-up relay with a low current long-time setting is also provided in the generator neutral ground lead to protect against ground faults near the neutral end of the stator windings which would not ordinarily trip the main differential relays and also to serve as a back-up for the main differential relays. The relays are arranged to clear the generator, remove the excitation, shut down the turbine, and initiate the discharge of CO_2 into the generator housing. Where a generator is connected to its step-up transformer or to its auxiliary power transformer without breakers, the transformer is cleared along with the generator.

Each step-up transformer, auto-transformer, and main auxiliary power transformer is protected against internal faults by induction differential relays arranged to clear the transformer. Where a transformer is connected to a generator without breakers, the generator is also shut down but CO_2 is not released.

Main busses are protected by high-speed overcurrent relays which are differentially connected to isolate the bus section at fault. Auxiliary power busses are protected by overcurrent relays on incoming circuits.

The 154-kv transmission lines are provided with carrier current pilot relays, using the center conductor, *B*, of the transmission line flat configuration, and with directional overcurrent ground back-up. The 110-kv lines generally have distance phase relays and directional overcurrent ground relays with provision for future carrier current operation. The 66-kv lines are generally provided with distance relays. The 44-kv and 12-kv lines generally have 3-shot automatically reclosing breakers with induction overcurrent relays and both high-speed and standard-speed overcurrent directional ground relays, the high-speed ground relays being locked out after the first ground fault tripping. The high-speed and standard-speed ground relays are so coordinated with the 44-kv and 12-kv transformer and branch circuit fuses that for momentary ground faults the high-speed relay causes the breaker to clear the fault and reclose without blowing the fuse, and for permanent ground faults the standard-speed relay allows the fuse to clear the fault on reclosure of the breaker. A number of the shorter lines of various voltages are provided with pilot wire relays.

Annunciation

Audible and visual annunciation is provided for excessive temperatures of bearings, windings, air, oil, and water, abnormal liquid levels and flows, and faulty operation of equipment throughout the station. Those deserving the attention of the control room operator are located in the main control room, and those requiring the attention of the turbine room operator are located on the governor actuator boards. The audible signal can be silenced by a push button, but the visible signal persists until normal conditions are restored.

In connection with the annunciator, in the more recent installations, an operation recorder located on the operator's desk in the control room automatically records all annunciations and all normal operations such as starting, stopping, and switching. Each record on the strip chart of the recorder identifies the source of the signal and records the date and time when the signal is received. The time required by the recorder to set up the type wheels and print a signal is about $2^1/_2$ seconds. When signals are received at intervals shorter than $2^1/_2$ seconds, they are stored and subsequently recorded at $2^1/_2$-second intervals.

Lightning Protection

The overhead ground conductors coming in from the transmission lines are carried across the top of the switchyard in the form of a network supported by and grounded to steel peaks rising from the tops of the steel structures. The network is sufficiently extensive to include all equipment and conductors within a protective cone having a base radius equal to its height. The multiple points of support for the overhead network are thoroughly grounded to a ground network below the station which is also connected by underground cables to the footings of the nearest transmission line towers. Low resistance earth connections are secured by connecting the ground network to buried ground mats of copper cable or copper strap, driven ground rods and pipes, abandoned steel sheet piling, buried pipes, and penstocks. Arresters are provided at transformer high-voltage terminals, and in shunt with the transformer neutral grounding reactors. Each arrester is chosen with as little spread as possible between the break-down voltage and the reseal voltage and the latter is chosen with a safe margin above the maximum voltage expected at generator run-away speed. Spill gaps are provided at line entrances wherever a higher level of insulation is used on the line than in the switchyard, and in some cases as back-ups to the arresters. Arresters and condensers are provided at the terminals of generators.

Switchboards and Controls

The main electrical controls, instruments, and relays are centralized in the control room in the service bay at the end of the powerhouse while the turbine controls and instruments are located on the governor actuator cabinets on the operating floor near the turbines. The control room is partially soundproofed by the use of linoleum floor, acoustical tile ceiling, and double-glassed doors and windows. A specially designed asymmetrical indirect lighting system is provided, utilizing an arched ceiling and a deep gutter at the front of the room behind the operator. By this arrangement the front part of the ceiling is brightly and uniformly lighted, thus lighting the fronts of the boards without causing glare in instrument faces, and the rear part of the ceiling is lighted to a much lower intensity, thus avoiding any high lights within the vision of the operator facing the board. The room is air-conditioned, summer and winter. Files, supplies, toilet facilities, and lunching accessories are provided in small rooms adjoining the control room.

Within the control room, the controls and mimic busses are located on a free-standing benchboard. Instruments and relays are on two back-to-back vertical boards at the rear of the benchboard. Recorders and load-frequency controllers are on a vertical board at the left, while battery and standard frequency control and distribution are on a vertical board at the right. On the bench, instrument, and relays boards the generator panels are grouped at the left and the line panels at the right, leaving room for expansion at each end. With the generator panels at the left end toward the recorder and load-frequency board, the controls and indications for the generators are grouped at one end of the control room, and the controls and indications for the lines at the other end thus facilitating operation. The mimic bus presents a single-line diagram substantially like the actual switchyard, both diagrammatically and physically. The operators' desks are centrally located in front of the benchboard.

All switchboards are of steel cubicle construction with black finish. At the later stations all indicating and recording instruments are semi-flush type. Relays are generally of the conventional projection type although some of the later relay boards are being equipped with flush draw-out relays. Control handles are of distinctive shapes and colors for the various functions, uniform at all plants. Mimic busses are of chromium-plated brass, polished brass, and polished copper, representing various voltages.

Wiring of the switchboard panels is usually performed by the Authority's construction forces. It is carried in steel wiring gutters extending from top to bottom behind panel joints. At most of the generating stations the gutters extend down through the floor to terminal cabinets in a terminal room below. At some of the installations now under construction it is planned to locate the terminal blocks on wing panels behind the switchboards. From the terminal blocks, in each case, rubber-insulated, rubber-jacketed, multiconductor control cables are carried away on transite cable trays through a control cable tunnel extending the full length of the powerhouse and in the larger stations extending also out into the switchyard. From the cable trays in the tunnel, the control cables are carried in individual steel conduits to their destinations at equipment. Color coding of wires is not used but all conduits, cables, individual wires and terminal studs are tagged or marked with a uniform designation which is carried consistently throughout all conduit drawings and wiring diagrams. The multiple conductor control cables range in size from 2 conductors to 19 conductors. Each conductor is insulated with 35-percent performance rubber and the cable is covered with a 60-percent tough rubber jacket.

The control storage battery and the two duplicate motor generator sets operate at 250 volts. They are generally located immediately below the battery control and distribution board in the main control room. Smaller 48-volt batteries and charging rectifiers are provided for the annunciators and for the telephones.

A standard frequency source is provided in some of the stations for automatic time control of the power system. The source consists of an accurately adjusted, 60-cycle tuning fork, operating in a shock-proof, constant-temperature enclosure. Its accuracy is within three-fourths of a second in 24 hours. It is checked by Arlington radio time signals every day and corrected when necessary.

A 6-element automatic oscillograph is provided at each of the generating stations and arranged so that it may be plugged into the current and potential circuits of the transmission lines so that a record may be obtained of the current and voltage conditions in the lines under fault conditions. The oscillographs are of the switchboard type and are mounted on the main control switchboards. Darkroom facilities are provided close to the control room.

The governor cabinets on the turbine operating floor contain all essential auxiliary controls and instruments for operation of the turbines. These controls consist principally of bearing oil and cooling water pump controls, thermometers and pressure gauges, air brake controls and gauges, manual CO_2 release for generators and emergency shut-down switches. The generating units are started and brought up to speed by the turbine operator. Synchronizing and loading of the units is done by the main control room operator. The governor drive for most of the plants consists of a permanent magnet generator direct-connected to the generator shaft and a synchronous motor direct-connected to the governor head. At several of the plants, potential transformers connected to the generator main leads are used to drive the governors. Both methods of governor drive have proved satisfactory.

Auxiliary Power

The amount of auxiliary power load varies so greatly with the type of development that the demand can be estimated only by careful analysis of the connected load after the major features of the development and the expected operating procedures have been determined. At

Guntersville, with about 100 motors, 30 heaters, and the lighting system, the connected load amounts to 3,750-kva, and the demand is under 1,000-kva. At Hiwassee, with 66 motors, 35 heaters, and the lighting, the connected load is 1,895-kva, and the demand is under 600-kva. It is generally found that the demand amounts to from 25 to 30 percent of the connected load.

Figure 9 shows a single-line diagram of a typical auxiliary power supply and distribution system, simplified with respect to the distribution circuits for convenience of presentation. Two duplicate auxiliary power transformers constitute the normal supply. Each is connected to the low-voltage leads of a main step-up transformer and hence each has two sources of supply, one from the power system, the other from the generator. Each has a capacity sufficient for the entire auxiliary power demand of the station so that each may be considered a 100-percent spare for the other, although they may be operated separately at about half load. A third source consists of an auxiliary power transformer usually of much smaller capacity, stepping down from an outside source coming eventually from the transmission system. It may be used in case of emergency and for necessary lighting, heating, and pumping during any period when all generators are shut down and the main step-up transformers are de-energized. At Chickamauga, where the three-winding step-up transformers are connected to two high-voltage transmission systems, each of the two duplicate normal auxiliary power sources is always expected to be available and in this particular case the auxiliary power transformer from another outside source is not considered necessary. At some of the dams where it is of vital importance that spillway gates be operable in the extreme emergency of all electric power sources being lost, a small gasoline-engine-driven generator is provided for this purpose.

Voltage of motors and large heaters is 440-volt, 3-phase, 3-wire, ungrounded, and that for lighting and smaller heating units is 220/110-volt, single-phase, 3-wire, grounded. At a few of the larger plants such as Wheeler, a 2,300-volt, 3-phase, 3-wire, ungrounded system is also provided for the larger motors and for longer feeders.

Main feeders are provided with interlocked breakers at source and double-throw switches at destination so that sources of supply cannot be paralleled. Every bus and bus section is provided with a normal and an emergency source of supply. Transfer from normal to emergency is by manual switching. Automatic switching to the emergency source in case of loss of the normal source is not considered warranted, because of the complex relaying required, because such automatic switching always involves risk of spreading the fault, and because momentary interruptions to the auxiliaries do not affect power production. The manual transfer is accomplished by a momentary interruption of voltage which is considered more acceptable than the possibility of a surge resulting from momentary paralleling of sources of supply.

Inspection and maintenance of all parts of the auxiliary power distribution system may be done without seriously curtailing generation. The main auxiliary power bus is in two essentially duplicate sections, either of which, together with its supply and feeder breakers, can be retired leaving the other section and all essential auxiliaries in full operation. The bus sectionalizing breaker can be withdrawn for

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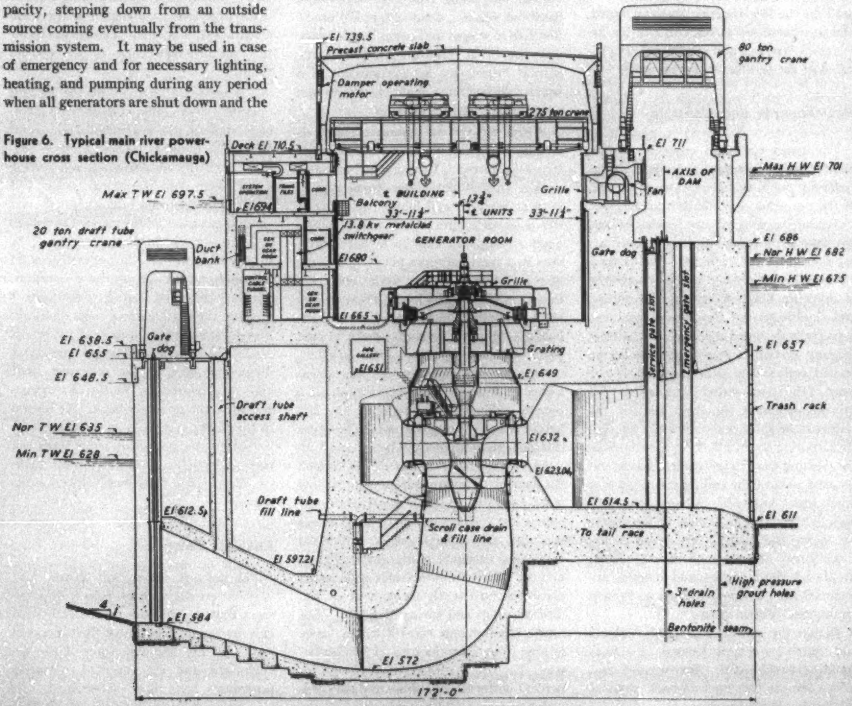

El 739.5
Precast concrete slab
80 ton gantry crane
Damper operating motor
275 ton crane
Deck El 710.5
El 711
Max H W El 701
AXIS OF DAM
SYSTEM OPERATION
TRANS FILES
CORR
Max T W El 697.5
El 694
℄ BUILDING
13½"
Balcony
33'-11⅛"
℄ UNITS
33'-11⅛"
Grille
Fan
13.8 kv metalclad switchgear
GENERATOR ROOM
20 ton draft tube gantry crane
Duct bank
GEN SW GEAR ROOM
El 680
Gate dog
El 686
Nor H W El 682
Min H W El 675
Grille
CONTROL CABLE TUNNEL
El 665
El 658.5
El 655
Gate dog
PIPE GALLERY
El 651
Grating
Service gate slot
Emergency gate slot
El 657
El 648.5
El 649
Draft tube access shaft
Trash rack
Nor T W El 635
El 632
Min T W El 628
El 623.04
El 612.5
Draft tube fill line
El 614.5
El 611
Scroll case drain & fill line
To tail race
El 597.21
3" drain holes
High pressure grout holes
4
1
El 584
Bentonite seam
El 572
172'-0"

Figure 6. Typical main river powerhouse cross section (Chickamauga)

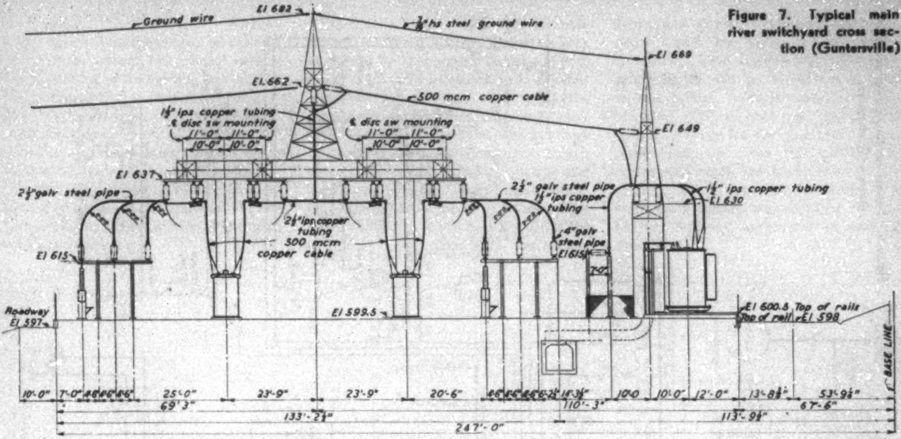

Ground wire
El 682
3/8" hs steel ground wire
El 669
El 662
500 mcm copper cable
1½" ips copper tubing
℄ disc sw mounting
11'-0" 11'-0"
10'-0" 10'-0"
℄ disc sw mounting
11'-0" 11'-0"
10'-0" 10'-0"
El 649
El 637
2½" galv steel pipe
2½" ips copper tubing
500 mcm copper cable
2½" galv steel pipe
1½" ips copper tubing
1½" ips copper tubing
El 630
4" galv steel pipe
El 615
El 615
7'-0"
Roadway El 597
El 599.5
El 600.5 Top of rails
Top of rail El 598
BASE LINE
10'-0" 7'-0" 4'-6" 4'-6" 4'-6" 25'-0" 23'-9" 23'-9" 20'-6" 4'-6" 4'-6" 4'-6" 6'-2½" 18'-3½" 10'-0 10'-0" 12'-0" 13'-8½" 53'-9½"
69'-3"
110'-3"
67'-6"
133'-2½"
113'-9½"
247'-0"

Figure 7. Typical main river switchyard cross section (Guntersville)

maintenance, leaving both bus sections in operation. Each unit bus may be retired whenever its generator is shut down. Either section of the common bus may be retired without shutting down essential auxiliaries. Each circuit from the various distribution centers may be isolated individually by its own disconnecting switch.

Transformers are generally located indoors in order to secure short, well-protected primary and secondary connections, and are therefore of the noninflammable liquid-filled type. They are self-cooled and their rooms are adequately ventilated. Primary and secondary terminals and circuits are totally enclosed.

Protection of auxiliary power transformers is by differential relays and of the main auxiliary power busses by overcurrent relays. Motor and heater circuits are protected against overload by thermal relays on the contactors, and against short circuits by fuses on the disconnecting switches.

Switchboards are of dead-front, steel cubicle construction. The circuit breakers on the main 440-volt auxiliary power board are air-break type, having an interrupting rating of 20,000 rms amperes. The incoming breakers and the bus section breakers on this main board are electrically controlled from the main control room while the feeders are manually controlled at the board. Motor and heater circuits are generally electrically controlled at destination.

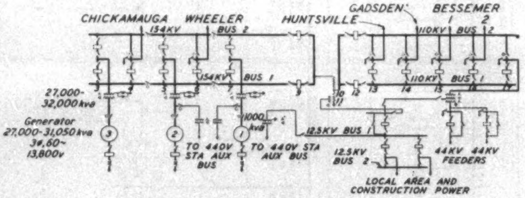
Wiring for auxiliary power circuits consists of single conductor cables insulated with 35-percent performance rubber, and covered with 60-percent tough rubber jacket. It is installed in steel conduits.

Communication

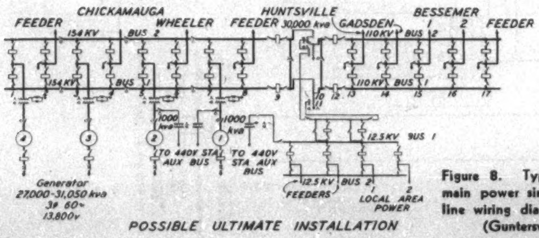
An automatic telephone system is provided in each generating station to provide communication, code call, fire alarm, police call, executive right-of-way, and conference service within the station, the switchyard, the dam, and the village. A manually operated key-type turret is located on the operator's desk in the main control room, through which connections can be made to the carrier current telephone system, establishing communication with other generating stations and with the load dispatcher's central office. The two outside phases, *A* and *C*, of the transmission system are used for the carrier telephone channel.

Lighting and Heating

The lighting and heating system originates at the two lighting and heating transformers, each of which is fed from a separate section of the main auxiliary power bus. Each transformer and its primary feeder has a capacity of about 75 percent of the combined lighting and heat-

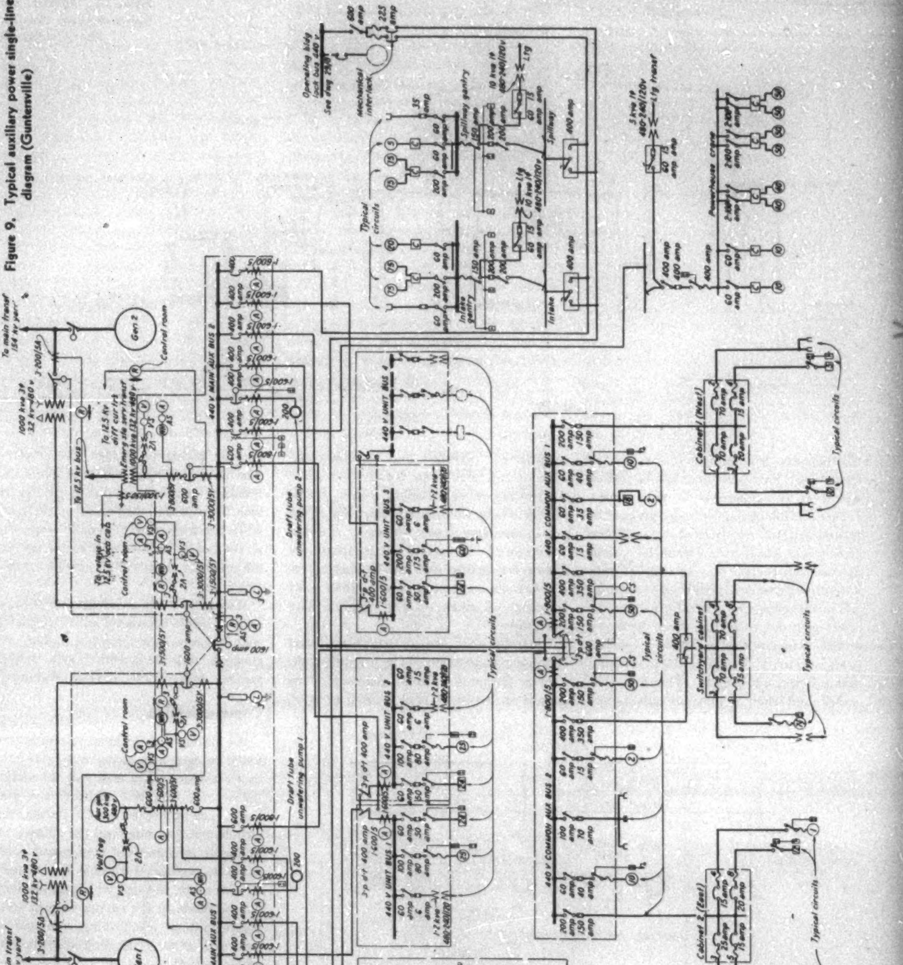

CHICKAMAUGA
WHEELER
154KV
BUS 2
HUNTSVILLE
GADSDEN
BESSEMER
1
2
110KV
BUS
2
154KV
BUS 1
110KV
BUS 1
9
10
11
12
13
14
15
16
17
27,000-32,000 kva
Generator 27,000-31,050 kva 3φ, 60~ 13,800v
3
2
1
1000 kva
12.5KV BUS 1
TO 440V STA AUX BUS
TO 440V STA AUX BUS
12.5KV BUS 2
44KV 44KV FEEDERS
LOCAL AREA AND CONSTRUCTION POWER

INITIAL INSTALLATION


FEEDER
CHICKAMAUGA
154KV
BUS 2
WHEELER
HUNTSVILLE
FEEDER
30,000 kva
GADSDEN
110KV
BUS
BESSEMER
1
2
FEEDER
154KV
BUS 1
110KV
BUS 1
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
4
3
2
1
1000 kva
1000 kva
12.5KV BUS 1
TO 440V STA AUX BUS
TO 440V STA AUX BUS
12.5KV BUS 2
FEEDERS
1
2
LOCAL AREA POWER
Generator 27,000-31,050 kva 3φ 60~ 13,800v

POSSIBLE ULTIMATE INSTALLATION

Figure 8. Typical main power single-line wiring diagram (Guntersville)



To main transf 154 kv yard
Gen 1
Gen 2
1000 kva 3Φ 13.2 kv-480 v
Control room
To 12.5 kv bus
Emerg sta serv transf
440 V MAIN AUX BUS 1
440 V MAIN AUX BUS 2
Draft tube unwatering pump 1
Draft tube unwatering pump 2
440 V UNIT BUS 1
440 V UNIT BUS 2
440 V UNIT BUS 3
440 V UNIT BUS 4
440 V COMMON AUX BUS 1
440 V COMMON AUX BUS 2
Typical circuits
Air conditioning and ventilating distribution board
150 kva 1Φ 480-240/120 v
Ltg & htg bus A
Ltg & htg bus B
Operating bldg lock bus 440 v
Mechanical interlock
Spillway gantry
Intake gantry
Spillway
Intake
Powerhouse crane
Cabinet 1 (West)
Switchyard cabinet
Cabinet 2 (East)

Figure 9. Typical auxiliary power single-line diagram (Guntersville)

ing demand. The transformers are normally operated independently on the two halves of the system, but manual switching is provided on the lighting and heating distribution board so that the entire system may be fed from either transformer. With this arrangement, 75 percent of the entire lighting and heating demand which includes all essential requirements for station operation can be carried indefinitely in case of outage of one transformer, one primary feeder, or one section of the main auxiliary power bus.

Emergency lighting from the 250-volt battery charging motor generator sets supported by the control battery is provided at all vital locations such as main control rooms, governor actuator boards, auxiliary power boards, pump rooms, electrical galleries, stairways, passages, and switchyard. The emergency lights form a part of the normal lighting system and burn along with the normal lights whenever lighting is required. The switching is so arranged as to promote the use of the emergency lights along with the normal lights, and in essential places the emergency lights are given precedence in switching. This assures that the emergency lights will be in service in case of a-c failure, and it stimulates the prompt replacement of burned-out lamps. Energy for the emergency lights comes normally from the motor generator sets which are driven from the a-c system, but in case of a-c outage the emergency lights are supplied by the battery without automatic or manual switching.

Short-Circuit Calculating Procedure for Low-Voltage A-C Systems

Discussion and author's closure of paper 41-67 by A. G. Darling, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941, pages 1121–36.

R. H. Kaufmann (General Electric Company, Schenectady, N. Y.): The present tendency toward the use of circuit interrupting apparatus having shorter operating time, coupled with the more general application limitation by apparatus momentary current ratings, have given impetus to a more accurate method of evaluating maximum short-circuit currents, particularly within an interval of a few cycles following initial short circuit.

One outstanding feature associated with the proposed method of calculation procedure is represented by the ability to recognize and interpret the variable rate of decay of the initial d-c component of short-circuit current.

A brief discussion of the origin of the decrement curves and their significance would seem to be of interest.

In short-circuit analysis procedure there are several firm anchor points or bench marks which include:

1. Initial alternating component of current (I'') represented by E/Z''.
2. Initial d-c component which, at the most unfavorable switching angle may assume a value equal to the crest of the initial a-c component.
3. The initial transient portion of a-c current (I') represented by E/Z', to which value the initial total a-c component rapidly approaches following the short circuit. This quantity would represent the initial a-c current component were there no amortisseur effects in any of the rotating machines.

The d-c component of fault current decays exponentially at a rate controlled by the ratio of inductance to resistance in the a-c network between the internal generated voltages and the point of fault. Increased resistance in a given a-c network produces a correspondingly more rapid decay of this d-c component.

The subtransient component of alternating current ($I''-I'$) also decays exponentially at a rate controlled by the ratio of inductance to resistance in amortisseur effects within the contributing machines. It happens that the effective time constant of this subtransient component varies only over a moderate range for various types of apparatus which will be encountered. The short-circuit subtransient time constant for synchronous apparatus will generally be between the limits of 0.015 and 0.04 for machines having ratings up to 10,000 kva. The entire a-c short-circuit current contribution from induction machines is subject to exponential decay with a time constant generally between the limits of 0.02 and 0.04. The presence of external reactance in the a-c network between contributing machines and the fault tends to increase the effective time constant. However, as the external reactance becomes large, the magnitude of the subtransient current component diminishes to a small value and hence moderate inaccuracy of time constant is no longer significant. As a result of these conditions, it is proposed to include the entire induction motor a-c current contribution in the subtransient component and utilize a common time constant of 0.04 second in evaluating the time rate of decay.

The transient component of a-c current, disregarding subtransient effect ($I'-I_s$) is actually also subject to exponential decay, but at a much slower rate. During the first one-eighth second the reduction in total a-c component due to this effect, would be limited to less than ten per cent in practically all cases. The presence of considerable reactance, external to the a-c machines, and the operation of automatic voltage regulating equipment, if actually considered, would both function to diminish the magnitude of this effect.

In view of the fact that the present calculating procedure is limited to an initial time interval of about one-eighth second with the portion of greatest interest being below one-tenth second, it is proposed to disregard the reduction in current resulting from transient component decay.

In accordance with this premise, the total current consists of a d-c component

$$\sqrt{2}I_{a\text{-}c}''e^{-t/T_a}$$

plus an a-c component whose rms value is

$$[(I''-I')e^{-t/T''}+I']$$

The equivalent rms value of the combined current (representing the rms value of a steady-state current wave having the same d-c and a-c components as exist at the time t) becomes:

$$I_{\text{total}}\text{ rms}=\sqrt{(I_{d\text{-}c})^2+(I_{a\text{-}c})^2}$$

$$=\sqrt{(\sqrt{2}I_{a\text{-}c}''e^{-t/T_a})^2+[(I''-I')e^{-t/T''}+I']^2}$$

Since the effect of the diminishing d-c component is of much greater significance than the reduction in subtransient component, an approximation for simplification may be made as follows:

$$I_{\text{total}}\text{ rms}=K[(I''-I')e^{-t/T''}+I']$$

where

$$K=\sqrt{2e^{-2t/T_a}+1}$$

$$T_a=\frac{La}{Ra}=\frac{1}{\omega}\frac{Xa}{Ra}$$

$$=\frac{1}{377}\frac{Xa}{Ra}\text{ (at 60}r)$$

The factor K becomes a function of time and the ratio of inductance to resistance in the a-c network. The latter may also be expressed in terms of the ratio of reactance to resistance at a specific operating frequency.

In determining the reactance-resistance ratio, it is proposed to use subtransient reactance (X'') for simplification, which is justified by reason of the fact that X'' is substantially the same as X_2 and, furthermore, in the event that X_2 be actually higher, the initial maximum magnitude of d-c component would then be decreased, thus constituting a compensating effect.

Since the subtransient time constant (T'') may reasonably be taken at 0.04 second, the total rms current may be expressed as:

$$I_{\text{total}}\text{ rms}=K[(I''-I')e^{-t/0.04}+I']$$

$$=K[(I''-I')C+I']$$

where

$$C=e^{-t/0.04}$$

The resulting expression for total rms current becomes that which appears in the proposed analysis procedure. Included in the paper are curves giving values of the factor K and C as a function of time for various ratios of 60-cycle reactance to resistance covering the range of values to be encountered in practice.

It is not expected that the reactance-resistance ratio of the a-c network be accurately determined. At generating centers with current-limiting reactors in the major number of interconnected circuits, it would be expected that the 60-cycle reactance to resistance ratio might assume values in the order of 30, while in remote secondary switching centers the 60-cycle reactance to resistance ratio might assume values of three or even less. Primary distribution centers and primary load centers will be found to assume 60-cycle reactance to resistance ratios of intermediate value. The principal objective is to determine whether a given switching center may be reasonably considered to possess an X/R ratio not in excess of say 3/1, 5/1, 10/1, etc., rather than to accurately define the reactance-resistance ratio at a specific value between 9/1 and 10/1, for instance.

The proposed method of analysis requires the solution of the subtransient reactance network (including induction machine reactances), the transient reactance network, and the resistance network between the source or sources of short-circuit current and the point of fault. In making approximations in the solution of the resistance network, it should be the objective to determine a resistance value which is as low as can be obtained under any operating condition.

O. R. Schurig (General Electric Company, Schenectady, N. Y.): The basis of the procedure for calculating the magnitude of a short-circuit current is the well-known relation that the current in a circuit is the voltage applied divided by the impedance of the circuit as pointed out in the paper. The impedance of the circuit includes:

1. Those occurring in the power supply circuit including power sources, line, and step-down transformers, and
2. Those in the distribution circuit including conductors, circuit breakers, control equipment, and other devices.

Much has been written in the past of the impedance and short-circuit characteristics of rotating machines and of other equipment in the power supply circuit; and as further studies of this apparatus are made, their characteristics and impedance values will be even better understood.

On the other hand, the impedance values of the circuit elements in the distribution circuit, though fundamentally less complex than those of the power supply circuit, have not been so well understood. This is indicated by the fact that a number of approximations giving widely different results have been used to estimate their magnitude, or their effects on the short-circuit current. By recent investigations, the necessary impedance constants of the principal circuit elements in the distribution circuit have been established and are included in the paper. Now that the impedance values are available, as given in simple curves or tables (see appendix of the paper), there should remain no mystery as to the magnitude of the impedances such as those of flat bar conductors at various spacings or of channel bars used in busses. Nor should there be any uncertainty as to the order of magnitude of the impedances of current transformers of conventional design, of air circuit breakers, oil circuit breakers, and of other switching and control devices. There is no doubt that with the aid of the new data given, reasonably accurate calculations of short-circuit current in low-voltage circuit can be made, eliminating guesswork.

There have also been widely divergent ideas on the current-limiting effect of the arc at faults of different kinds in low-voltage distribution systems. Recent investigations have shown that the voltage drop at the point of fault may vary considerably, ranging from a negligible value to a magnitude in excess of the crest value of the circuit voltage, with a correspondingly wide range in current-limiting effect. The results of these investigations are summarized in the paper in table X. They have also been reported in full elsewhere.[1]

Suffice it to consider here how the arc voltage magnitude affects the magnitude of short-circuit currents to be used for the following purposes in typical modern switchgear and station design:

(*a*). As a basis for the mechanical design of main busses to resist short-circuit forces, and

(*b*). For the selection of the main-line protective devices.

The following basic considerations are of interest here:

1. Since large current-limiting effects cannot be depended on in the case of arc faults in enclosed congested structures typical of modern switchgear and station design, maximum short-circuit current values closely approaching bolted fault conditions may be expected.

2. While it is true that many arc fault conditions having substantial current-limiting effects may occur in less congested structures, or at points remote from the power supply source, the less severe fault conditions cannot be considered representative of the maximum short-circuit currents which determine the maximum stresses for mechanical design of main busses and which may need to be interrupted by the main-line protective devices in structures typical of modern switchgear and station design.

REFERENCE

1. FAULT-VOLTAGE DROP AND IMPEDANCE AT SHORT-CIRCUIT CURRENTS IN LOW-VOLTAGE CIRCUITS, O. R. Schurig. AIEE TRANSACTIONS, volume 59, 1940, pages 479–87.

Wm. A. Del Mar (Phelps Dodge Copper Products Corporation, Yonkers, N. Y.): It has been brought out in the course of discussion that the actual deterioration of circuit breakers is generally less than would be expected from calculated short-circuit currents. This may be explained in part by the cable data used in making the calculations. I do not know to what extent the impedance of the cables affects the short-circuit current but I do not believe that much reliance can be placed in the cable impedance data given in the paper. Mr. Darling has undoubtedly used the best information available and if I were to try to write up a procedure such as his I would have used the same data. However, I would have added a note saying that the basis of the information presented is exceedingly questionable due to lack of adequate data on the reactance and a-c resistance of large cables in iron conduits. The principal field test data relied upon are those of L. Brieger whose tests were confined to size 4/0 and 500,000 circular mils. The Brieger factors, however, are used in table VII which goes up to 750,000 circular mils and it is in precisely the range between 750,000 circular mils and 1,000,000 circular mils that a rapid rise in impedance occurs.

A. G. Darling: Mr. Del Mar's point that actual deterioration of circuit breakers is generally less than would be expected from calculated short-circuit currents is well taken. The calculating procedure gives an anchor point in establishing the maximum short-circuit duty which the breaker may be called upon to withstand. The factors which will cause less duty are arc drop at the fault and an increase in impedance during the fault such as might be created by spreading of conductors. Since the breaker will usually be selected for a fault location giving minimum impedance, naturally the duty will be decreased as the fault location introduces more circuit impedance. The longer the circuit protected by the breaker, the less the chances are for a high percentage of severe faults, always remembering that there remains at least one chance of a fault of maximum intensity at the terminals of the breaker.

The subject of cable impedances has been given considerable study. Mr. Del Mar's suggestion that further study may reveal more accurate data is recognized as true and any contributions Mr. Del Mar or others can make to the cable and other impedance data will be welcomed by all concerned.

Electric Shock

Discussion and author's closure of paper 41-7 by C. F. Dalziel, J. B. Lagen, and J. L. Thurston, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941, pages 1073–9.

Horatio B. Williams (nonmember; Columbia University, New York, N. Y.): For a long time the spectacular effects of large currents drew attention away from the fact that electricity can kill by stimulation as well as by destruction. Death due to fibrillation of the ventricles of the heart brought on by electrical stimulation is now generally recognized. The authors call attention to another manner in which currents of small size may become dangerous. Current passing from hand to hand, of strength such as to prevent the subject from releasing himself, may kill if the strength is sufficient to cause continuous contraction of the muscles of respiration; or by an effect on the nerves controlling these muscles which renders them incapable of conducting the normal nerve impulses. This effect was described by Einthoven some 25 years ago as an experimental fact, the nature of which is still obscure. Under experimental conditions nerves in this state may recover within a half hour. It is possibly in such conditions that resuscitation by artificial respiration succeeds most easily.

Currents of such magnitude as to "freeze" the subject to the circuit are very painful. An early effect of pain is to raise blood pressure, but if pain is long continued the pressure falls, fainting results, and if the cause is not removed recovery may not take place. The subject who is unable to breathe is also unable to make outcry and so may be unable to secure help even when it is near at hand. The increase of blood pressure due to pain is brought about by constriction of small blood vessels. When this occurs suddenly in a person with a damaged heart that organ, unable to contract against the sudden increase may dilate and this may determine a fatal issue. When the current is such as to produce a muscular contraction as great as or greater than that which the subject can normally produce voluntarily, then if both sides of the circuit are gripped with the palms of the hands he will certainly be unable to let go since the muscles which close the hand are stronger than those which open it.

The stimulation of these muscles will depend not on the current through the entire arm so much as on the current density in the arm. We should expect then that for a given total current the ability to let go would diminish with the size of the forearm. This is of particular importance in connection with the electric fence hazard since very small children are exposed to them and might be expected to be "frozen" by currents from which an adult could let go.

If Professor Dalziel still has access to many of the group who served as subjects for his investigation, it would be of great interest to have measurements of the maximum diameters of the forearms in those who had the greatest and least let-go values as well as in some of the others.

In determining the maximum safe current which may be permitted on an electric fence it would be most helpful to know the let-go limit when the current passes for one-tenth second and is interrupted for nine-tenths second, this being common fence practice. In order to insure effectiveness of these fences in all circumstances the makers desire a rather high current limit. It is assumed that interruption will always make it possible for a person to clear himself. If such experiments are tried I would suggest that current be not passed through the body, but from a hand to upper arm. Any difference might be evaluated by trying both arrangements at low current levels.

The authors' finding that stimulating effect is related to peak rather than to rms value of current is an agreement with current physiological observations as is also the observation that higher frequencies stimulate less effectively than commercial frequencies. There is a definite relationship between strength of current, duration, and stimulating effect and when time is made short enough stimulation will not occur when the current strength becomes so great as to destroy. For this reason physicians employ current of high frequency to produce deep heating of body structures, a procedure known as diathermy just now engaging the attention of engineers because it is a source of radio interference. The relatively smaller effect of direct current is also in agreement with physiological facts which it would take too long to develop, but its safety is only *relative*. The Bureau of Mines has recently reported numerous fatalities on 250 volts direct current in Alabama coal mines.[1]

REFERENCE

1. *Information Circular 7120*, Bureau of Mines, June 1940.

M. G. Lloyd (National Bureau of Standards, Washington, D. C.): This paper contains very valuable data regarding the "let-go" values for men, and it would be very desirable if similar data could be obtained for women and children. The one woman reported in this paper gave results between about 20 per cent of the men on one side and 80 per cent on the other.

Such experiments have value in setting the limits for the amount of current which should be permitted in connection with the regulation of use of electric fences Several such regulations have been set up, one of which constitutes part of the National Electrical Safety Code. In the latter the limit of eight milliamperes is set for currents which can be steadily drawn from the fence without use of an interrupter.

More than two dozen fatalities have, to my knowledge, already occurred to human beings in connection with electric fences, most of which were equipped with home-made controllers, and probably none complied with present standards. One child was killed in Oregon last summer, in which case the child had wet feet and was reported to have been scratched by the barbed wire which penetrated through the skin, resulting in greatly reduced resistance through the child's body. It was found by laboratory measurements that eight milliamperes was delivered by the fence controller with a resistance of 2,000 ohms in circuit. The results in this paper today show that some men have a resistance lower than this when contact is made with wet hands, and if the child's skin was pierced, it is probable that the resistance was lower than this in the Oregon case, and the current consequently larger than eight milliamperes. It is noted that the authors conclude that a safe limit for men is eight to nine milliamperes.

Jellinek, who has written extensively on this subject in Europe, claims that electrocution of a child has occurred with as little as ten milliamperes.

In an experiment made years ago at the National Bureau of Standards by Dr. F. Wenner, the resistance of the body between two hands immersed in salt water approached a lower limit of 500 ohms.

Attention appears to be a defensive factor, so that a larger current can be tolerated when it is expected than when it is not expected, and fatigue may also be an element in this problem.

Walter A. Curry (Columbia University, New York, N. Y.): Several years ago the writer carried out some tests, with himself as subject, which were similar to those described in this paper. In particular, with 60-cycle potential applied between dry hands grasping the electrodes (large metal masses so that the whole palm was practically in contact) and an extreme limit of 50 volts was found, at which voltage the current was 10 milliamperes. With wet soapy hands, the extreme limit was 22 volts, with a current of 11 milliamperes. The conclusion was "a body current of about 10 milliamperes is extremely unpleasant and will paralyze the hands and arm muscles." In these tests, the criterion was not the "let-go" value, but the extreme discomfort which could be endured.

In addition, some 60-cycle tests were carried out with the subject sitting in an ordinary porcelain bath tub, with potential applied between ground (water piping connected to tub) and a tin plate which one or both hands grasped. The water level was about waist high. With one hand grasping the plate, the voltage was raised slowly to 25 volts, and the corresponding current read as 16.9 milliamperes, representing a body-path resistance of some 1,500 ohms. The test was made with clear water and when repeated with soapy water the current at the limit voltage of 25 volts was 18.3 milliamperes, corresponding to a body-path resistance of about 1,400 ohms; soapy water would thus be indicated as slightly increasing the body conduction and therefore slightly more hazardous. A second test was carried out with the soapy water, with both hands grasping the tin plate; in this case the current corresponding to 25 volts was 22.6 milliamperes, giving a body-path resistance of about 1,100 ohms. Under this condition there was a rather violent contraction of the arm and hand muscles, and there was a peculiar vibratory sensation in the chest. These bath-tub tests probably held a certain hazard due to the possibility of inducing ventricular fibrillation of the heart and have never been repeated. With a body resistance of 1,100 ohms, the current flowing with 110 volts will be 100 milliamperes; since many fatal bath-tub cases have occurred at the normal service voltage (110–20 volts) this substantiates the opinion that 100 milliamperes passing through the chest may cause death.

C. F. Harding (Purdue University, Lafayette, Ind.): This paper presents several new data and a unique contribution to the limits of "holding" or "letting go" of electrical circuits by human beings. Fortunately the medical profession is now assisting in the interpretation of many puzzling phenomena in connection with human reactions under electrical exposures. Further study should be undertaken of such conditions as: (1) the reported greater surface heating due to direct current as compared with alternating current of the same effective value and (2) the greater shock as direct current is broken when contrasted with the breaking of an equivalent alternating current. Can it be that there is an inductive reactance in the human being which results in effects comparable to the breaking of the direct current in a magnetized coil?

Although it has been made evident that it is the magnitude of the actual current flow through the body rather than the actual applied potential that causes shock, and no doubt death as well, it seems rather questionable to divert the mind of the layman from the probable danger of high-voltage circuits by the statement that "subjects received open-circuit voltages varying from 5 to 9,720 volts." Furthermore, the frequency of the supply and surface capacitance of the human body must be taken into consideration in determining the current *through the vital organs of the body* which may correspond to an applied high potential. The statement of the importance of current as opposed to potential measurement would seem to be somewhat neutralized by the later reference in the paper to the effects of peak versus effective values of various potential waves.

With respect to the effects of the higher harmonics in distorted wave forms it has often been noted that capacitors which tested satisfactorily on a sine wave soon puncture upon the distorted wave form which may be augmented locally by the connection of the capacitor itself to the circuit. Thus such circuits as electric fences with capacitors involved may prove to be additional risks on the distorted waves frequently found on rural power distribution systems.

Charles F. Dalziel: Most persons are aware of the danger incident to contact with high-tension circuits, and high-voltage warning signs generally command respect. There is no question of the shock hazard from power circuits, and it is regrettable that there is a tendency for this attitude to disappear when handling low-voltage circuits, especially ordinary lighting circuits. It is probable that many accidents with home appliances are due to ignorance, lack of appreciation, or disregard of safe procedure and the inherent dangers associated with electrical apparatus. This may assume importance in explaining accidents involving worn out or defective home appliances used near well-grounded objects, where excessive moisture is present, or when standing in water, such as in wet basements, laundry rooms, bathrooms, etc. There was no intention to divert the mind of the layman from the danger of contact with high-voltage circuits. Experiments in progress indicate that the maximum safe 60-cycle voltage is about 5 volts, and about ten times this value for direct current. Voltage is very important, but due to the variability of contact and skin impedances, voltage in itself may be relatively unimportant in producing or explaining damage. Muscular

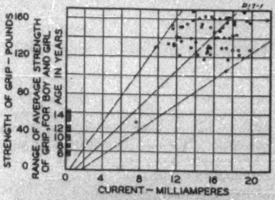


Figure 1. Muscular grip in pounds versus let-go current in milliamperes

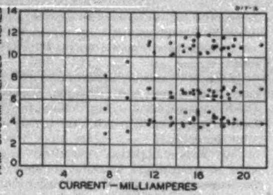


Figure 2. Arm circumference in inches versus let-go current in milliamperes

reactions and sensations are due to the current flowing through the body, and therefore current and *not* voltage is the proper measure of shock intensity. This is of special importance when considering the safety of electric fences, fly panels, insect traps, stock driving canes, etc. In these devices the open-circuit voltage may be several kilovolts; however, the hazard depends upon the maximum current output and has no relation to the voltage output.

The investigators have given thought to the determination of the let-go current for children. The essential contents of this paper were presented before the San Francisco section AIEE technical meeting on December 13, 1940. The question of testing children was also raised in the discussion following the presentation. Several attempts were made to obtain children, but only one was available. The child tested was a well-developed boy, nine years of age, weight 62 pounds. His 60-cycle let-go current was 7.6 milliamperes, and he released direct currents up to 15 milliamperes easily and without comment. Higher direct currents were not tried for reasons of safety. Mr. Royce E. Johnson, director electrical standards laboratory, University of Wisconsin reports that several years ago let-go current tests were made on a few subjects. Although the experimental procedure was slightly different the findings were within the range of our results. He reports that "a 60-cycle current of seven milliamperes was found to paralyze the grip of a five-year old boy who weighed about 45 pounds." These two values for children are valuable in connection with the permissible maximum current output for noninterrupted a-c fences.

An attempt was made to correlate let-go current and the strength of grip, with the hope that a simple and unobjectionable method of estimating probable let-go currents for women and children might result. The results are shown in figure 1. The wide scattering of points and the lack of data at low values preclude drawing definite conclusions. Possibly other investigators may be able to secure additional points for children, thereby settling the issue.

At the suggestion of Professor Williams, the maximum forearm and minimum wrist measurements were made on all available subjects, and are given in figure 2. It was thought that possibly let-go currents might be more closely related to the current density in the conducting tissues rather than to the over-all measurements. An attempt was made to estimate the amount of conducting tissue by subtracting the wrist values from those of the forearm. These values are also given in the figure. Although those having smaller measurements also had smaller let-go currents, the range of the arm measurements was so small that little can be said regarding the probable lower limits.

Dr. Williams and several fence manufacturers have suggested performing let-go studies using interrupted a-c shocks of 0.10 second duration followed by an off period of one second. Some tests were made using currents up to 26 milliamperes, and none of the subjects had difficulty in releasing the test wire electrode during the off period. Tests were not made at currents higher than 26 milliamperes due to the violent muscular reactions and painful sensations produced by currents of this magnitude. The entire subject of electric fencing needs further study and clarification, and our efforts are now being directed to this phase of the subject.

The interest and enthusiastic response accorded this investigation is much appreciated.

Powerton-Crawford 220-Kv Line—Design and Construction Features

Discussion and author's closure of paper 41-57 by M. S. Oldacre and F. O. Wollaston, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941, pages 1108–20.

J. W. Milnor (The Western Union Telegraph Co., New York, N. Y.): The description of the Powerton-Crawford 220-kv line is of much interest as an indication of the variety of problems which must be met in the design of a line for transmission of high power.

It is regarding the problems of inductive co-ordination that I wish particularly to speak. The route of the power line is closely parallel to an important line of Western Union and of the Atchison, Topeka and Santa Fe railway, and exposures to two other telegraph lines occur at somewhat greater separations. Under the circumstances it is inevitable that the magnetic and electric fields of the power system should link with the telegraph circuits.

Fortunately, methods of co-ordination have kept pace with the power system developments. Some years earlier, an attempt of the power and communication interests to share this general route would have resulted in an impairment of most of the telegraph services west from Chicago. As the co-ordination has finally been applied, the interference has been controlled to an extent that the remaining effects into communication circuits thus far have been sufficiently small to be unimportant. These results have been obtained in part through considerations of design of the power system, for the purpose of reducing its inductive influence as far as practicable. What remained of the interference to telegraph is corrected by neutralizing equipment applied in each paralleling system. The co-ordination, of course, was not obtained without cost; but the expense has been small as compared with the total investment in the power and in the communication lines concerned.

The extent of the success in controlling interference will be better appreciated when it is considered that a high-grade telegraph circuit may be harmed by a small fraction of one watt, and that the total energy carried by the power system is six billion times larger.

Edwin Hansson (Pennsylvania Water and Power Company, Baltimore, Md.): The authors are to be congratulated on this very excellent paper. The line stands as a credit to its designers and they should be able to look forward to a very satisfactory performance.

In one respect I should like to differ with the Commonwealth engineers. This difference applies, however, more to execution than design. In order to obtain low tower footing resistances, ground rods were driven in the bottom of the footing excavations. I doubt very much if these rods influenced the final values of ground resistance sufficiently to warrant their cost. The four steel grillages comprising the foundations of the towers form a very large electrode when compared with the ten-foot rods, and overshadow their effect.

As far back as 1923 we found that an eight-foot ground rod driven within a few feet of a six-foot deep concrete footing had no appreciable effect on the combined resistance. In 1931 we tried driving eight-foot rods 50 feet from similar foundations and found that the connecting wire was just about as effective without the ground rod as with it. In 1932 we tried driving rods in the bottom of the excavation up to 16 feet below the concrete and again found them ineffective. After these trials we came to the conclusion that rods in connection with tower foundations are justified only where it is possible to reach soil of much higher conductivity than that directly adjacent to and surrounding the foundation.

J. T. Lusignan, Jr. (Ohio Brass Company, Mansfield, Ohio): This paper gives a splendid review of the various construction features of the newest 220-kv line. It particularly points out some interesting new factors that were considered in this line design and some new ways of treating old problems.

The development of the clearance envelopes to allow for conductor galloping is most interesting. I am wondering what wind velocities the authors counted on for this and how closely the conductors will approach one another under the worst galloping conditions.

It is stated that the flashover voltage between tops of the line and bus switch stacks must be about 20 per cent greater than the line-to-ground flashover voltage. The problem of running the tests to determine this was put up to our laboratory, as was the case for similar switch arrangements for the Boulder Dam lines. Literally the tests cannot be made as it is obviously impossible to put any more voltage, nonetheless 20 per cent more, on the switch than the flashover voltage to ground of each stack. Accordingly a compromise test was necessary which involved determining the flashover voltage to ground of the stack, then adding enough insulators to each stack to gain over 20 per cent more voltage. Following this the horizontal gap was adjusted for a flashover voltage in excess of this.

The authors point out one advantage of the long ground-wire suspension assemblies chosen. Another advantage is that the long suspension arrangement decreases measurably the angle of rocking and thereby the rubbing at the tower attachment. This obviously reduces the wear there appreciably.

In describing their studies of radio noise on their line, the authors, mention tests made on wet suspension strings, the results of which are plotted in figure 18. Our laboratory studies indicate that ordinary standard suspension units, wet or dry, give negligible radio interference. We know, however, that water drops on smooth metal surfaces, in intense electric fields, cause appreciable corona and streamer formations. We therefore wonder whether the small interference values, indicated on figure 18, cannot be attributed to such water drops on the conductor and hardware surfaces.

A. E. Davison (Hydro-Electric Power Commission of Ontario, Toronto, Canada): Mr. Oldacre and associates should be congratulated on supplying such complete and intimate data of a modern (1940) 220,000-volt line.

The transmission line is particularly interesting being located in a highly productive plain or river valley, there being some evidence that, considering the area on a 100-mile square basis, the line is in the most productive area in the world. Evidence of this important consideration is found in the special method of minimizing right-of-way rights and costs. Condemnation is provided only for the area directly under the tower together with the right of access to that area. Easements are provided apparently for the cables.

Special thought and study have been given to design for protection from sleet and following loads. Records are available which are indicative of what may happen to a line when involved in a gusty wind storm for which the recorded velocity was 50 miles per hour. It would seem that very few existing lines could successfully withstand velocities much above the gusts occurring in a 50-mile wind. This design provides for calculated loads for steady winds up to 150 miles per hour.

References made in the paper under "Basic Consideration and Design" to "practically lightning-proof" lines. Lightning proofing of transmission lines is a considerable undertaking. It would appear from the reading of other parts of the paper (for instance see the paragraph, beginning "Desired low level—less than 1 outage per 100 mile-years") that the writer of the article expects outages.

Referring to "Protection Against Sleet" a chart (figure 1) indicates the writer's interpretation of desirable loops (Lissajous figures) for the tower and normal sag which is described in the article. Superimposed are the loops or figures as prepared by the writers of the article (see figure 2c normal span). The point in recording these variations was to demonstrate variations in interpretations. As a matter of fact, the phases are so separated that the loops or figures for both interpretations are reasonably clear of one another.

There is some evidence that a combination of 220,000 volts and the normal electrical currents, found in lines such as those described by the writers of the article, will reject certain sleet loads. The amperes per square inch would appear to be of the order of 700. It is a fact that sleet has been rejected or practically rejected on at least two or three occasions when the amperes per square inch ranged from 700 to 800. Not all of these cases were 220,000 volts.

"Loops" are referred to under "Protection Against Sleet." The proving of any advantage there may be in using these was left to Mr. Oldacre who has done so quite satisfactorily (see figure 1) for 132,000-volt lines. If any reader is interested in securing copies of the article wherein loops are developed (see reference 1) these developments have been published in *The Bulletin* of the Hydro-Electric Power Commission for September 1939. Copies can be secured by writing the undersigned or by writing to *The Bulletin*, Hydro-Electric Power Commission, 620 University Avenue, Toronto, Canada. Some engineers looking at the group of figures (see figures 1 and 2) of the paper may be confused. A review of reference number (1) might assist further studies by interested engineers.

Under "Transpositions and Dead Ends" it is stated that there is no crossing of the wires in the spans of single-circuit transpositions (see figure 5*d*). Looking at this photograph it would appear that a ground wire and the upper phase do not cross at transposition tower. They may cross at some other tower. Heretofore mechanical failures and burning down of sky cables has occurred only at transpositions in one fairly large system having several thousand mile-years' operating experience.

Under "Tower Footings" it is a matter of experience of others as well as of the writer of the paper that hot galvanizing at the ground line lasts for a great many (33 or more) years.

Referring to tripod type of anchor, it is known that the existence of any complicated structure at or near the surface of frozen ground may account for extraordinary stress in the leg members between the frost line and the grillage due to frost heaving. It may be that leg members in this particular design are strong enough to raise the grillage and that it may settle back during the following warmer season. Considerable attention must be given to this feature where footings are set in certain clay soils.

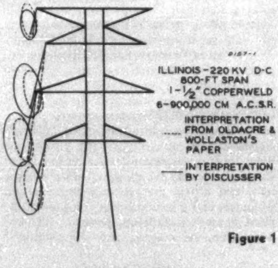

0187-1
ILLINOIS - 220 KV D-C
800-FT SPAN
1-½" COPPERWELD
6-900,000 CM A.C.S.R.
INTERPRETATION FROM OLDACRE & WOLLASTON'S PAPER
INTERPRETATION BY DISCUSSER

Figure 1

Under "Vibration Protection" the absence of grading rings minimizes the anvil which a suspension clamp offers for the hammering action of vibrations carried into it by the cable.

The "Tower Lighting" described in the paper is interesting. The effect of lightning discharges if any, upon the lighting fixtures and equipment will be watched with interest.

The paper as information for use by engineers would be effectively supplemented if the writers could give some attention in their closure to the following.

METHOD OF SETTING OF FOOTINGS

Are footings set with a templet, with part of a tower as a templet, or are they set as four separate footings by instrument? The latter method seems to provide for the minimizing of internal stresses in the completed structure and may be a comparatively economical procedure. Were there any special methods adopted in setting footings in excavations where the sides of the excavations are not stable?

MICROVOLTS PER METER CHART

Referring to figure 19, it is not clear to the reader, from the text, whether these records apply to a single circuit or to one circuit of a double-circuit structure.

OPERATING PROCEDURES BY WHICH SLEET CAN BE KEPT OFF LINES

If there is some specific experience or procedure whereby operating organizations can so interconnect their lines during sleet periods, that sufficient current flows to cause sleet rejection, this data and information would be greatly appreciated by engineers who have to design and operate services like those described by the writers of technical paper 41-57.

F. O. Wollaston: Along most of the route of the line the location of the water is such that a ten-foot ground rod in the bottom of the anchor excavation will reach soil of substantially higher conductivity than would be contacted by the anchor grillage alone. It is felt that the ground rods were definitely worth while, since very low footing resistances were obtained for most of the towers.

The clearance diagrams for galloping were based on 0 degrees Fahrenheit, $^1/_2$-inch ice, and 30-mile wind. The minimum clearances between galloping wires are shown in figure 2 and are a few feet greater than the calculated flashover distance. It is agreed that, as Dr. Lusignan suggests, the radio noise values under wet conditions shown in figure 18 are probably due to water drops on the conductor and hardware.

The calculated outage rate of the line due to lightning is low, but finite, for the conditions assumed in the calculations. The assumptions were very conservatively severe, so that the phrase "practically lightning-proof" seems justified when referring to the line under actual service conditions. Mr. Davison is correct in inferring that one of the phase wires crosses under a ground wire in the single-circuit transposition. This was unavoidable because of the inset of the ground wires with respect to the outer conductors, but very ample clearances were provided.

In reply to Mr. Davison's question, all footings, except a few at the start of construction work, were set by transit. This proved more satisfactory than using templets, which were not rigid enough to hold the heavy anchors in line during installation. Where sandy soil was encountered, the sides of the footing excavations were supported with sheeting. This was either the usual $^1/_4$-inch corrugated steel type braced with steel channel wales, or wood sheeting with wood wales made to fit the job. Sheeting was driven into place with air hammers operated from portable compressors.

The radio-noise measurements of figure 19 of the paper were made on the double-circuit section of the line. Similar measurements have also been made on the single-circuit section. Although weather conditions were not strictly comparable, these measurements showed about the same order of magnitude of radio noise as on the double-circuit section of the line.

A discussion of sleet-prevention and sleet-melting procedures, outlining methods in use in our company, was given in an article, "When Sleet Comes," by W. R. Kunkel, *Electric Light and Power*, October 1940, page 52. Those interested in the prevention of sleet troubles on transmission lines are referred to this article.

Stability Limitations of Long-Distance A-C Power-Transmission Systems

Discussion and authors' closure of paper 41-73 by Edith Clarke and S. B. Crary, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941, pages 1051–9.

C. Concordia (General Electric Company, Schenectady, N. Y.): In this paper it has been concluded that if 60-cycle electric power transmission is to be extended to distances very much greater than about 350 miles, series capacitor compensation of the line, with otherwise conventional apparatus is the most practical and feasible way of accomplishing such an extension. In view of recent work which has been done on possible negative damping produced by too much series capacitor compensation, a natural question is whether or not the amounts of series compensation required for reasonable increases of line length will be sufficiently within the safe operating regions as far as negative damping is concerned. We are therefore presenting approximate calculations on two bases, as shown by figures 1 and 2 attached.

Figure 1 shows the no-load damping torque coefficients[1] for a typical large water wheel generator, with an amortisseur, as affected by the amount of capacitive reactance compensation x_c. It is seen that only with an unusually high-line resistance ($r=40$ per cent on the generator base kilovolt-amperes) and high compensation does increased compensation cause a decrease in the amount of damping, and even then the damping is not less than that with no compensation. A 40 per cent line resistance corresponds to an uncompensated line reactance of about $(40/0.15)=270$ per cent of generator base impedance if the line (r/x) ratio is 15 per cent. The corresponding line length is then nearly as great as the lengths considered in the paper.

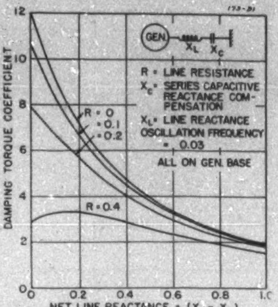


Figure 1. No-load damping-torque coefficients of typical synchronous generator as affected by capacitive-reactance compensation X_c

Figure 2 shows the self-excitation regions with constant generator speed as affected by series capacitor compensation,[2,3] and shows that the values of line resistance required for stability on this basis are not unduly large. From this figure it can be concluded that the per cent of line reactance which can be series capacitor compensated is only slightly affected by the relative length of line but tends to increase with line length. This may be seen by assuming any fixed line (r/x) ratio and determining the allowable compensation from the curves.

The lower limits to line resistance established by figure 2 and the upper limits established by figure 1 are sufficiently far beyond the range of compensation required for moderate increases in transmission line length, and the phenomena involved are sufficiently well understood, that negative damping should not prove to be a serious limitation to the gains in power transmission possible with series capacitors.

REFERENCES

1. TWO-REACTION THEORY OF SYNCHRONOUS MACHINES—II, R. H. Park. AIEE TRANSACTIONS, volume 52, 1933, pages 352–5.

2. TWO-REACTION THEORY OF SYNCHRONOUS MACHINES, S. B. Crary. AIEE TRANSACTIONS, volume 56, 1937, page 27.

3. NEGATIVE DAMPING OF ELECTRICAL MACHINERY, C. Concordia and G. K. Carter. AIEE TRANSACTIONS, volume 60, 1941.

E. E. George (Ebasco Services, Inc., New York, N. Y.): This paper is probably the first to give a quantitative discussion of the electrical performance of transmission lines over 300 miles long. It is also the first to carry out the very effective idea of using the network analyzer to obtain the answers. While the authors' results are largely confined to electrical performance of long circuits, a brief correlated discussion of present economic limitations may help in evaluating the results obtained by the authors and in suggesting a field for future papers.

The authors state that no attempt was made to determine the economic feasibility of long-distance a-c power transmission. Perhaps future Institute papers will cover the economics of typical conditions, as well as technical limitations other than stability.

Although the authors refer to the present-day limit of straightaway high-voltage transmission as being about 300 miles, no particular explanation of this limit is given, and it might be inferred that certain technically feasible changes in design of the line and associated equipment would result in greatly extending this distance. The actual conditions are not this simple.

There is much evidence that most well-designed high-voltage lines reach the economic limits of voltage drop, power loss, and operating stability almost simultaneously at about present-day transmission limits. *To extend these limits, there must be a decrease in the cost of lines and equipment, or technical advances to permit maintaining the present percentages of line loss and regulation and the present stability margin with longer lines. Removing only one of these three limitations is not enough—the other two are close at hand to block further progress.*

The authors show that the stability limit decreases about 40 per cent when a 300-mile line is doubled in length. It must also be kept in mind that doubling the length of line more than doubles the per cent line loss and per cent voltage regulation in terms

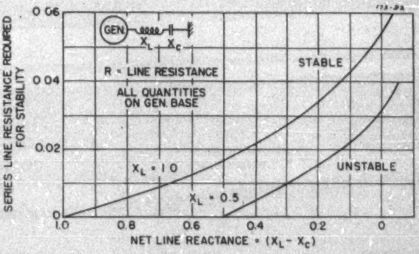


Figure 2. Self-excitation regions of a typical synchronous generator connected through series-capacitor compensated tie line to infinite bus—effect of line resistance

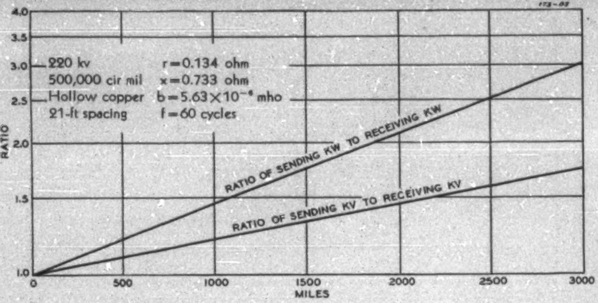


Figure 3. Performance of typical transmission line with load equal to characteristic impedance

To use chart for any conductor size other than 500,000 circular mils equivalent, change mileage in linear proportion to ratio of r to 0.134. Power and voltage ratios are substantially independent of voltage and other characteristics except conductor resistance

of maximum loading—regardless of voltage. To hold the same percentage loss it is necessary to double the conductor area. Therefore, doubling the length of line would probably treble its cost, if the same performance ratios must be obtained.

It is popularly thought that increase in voltage can offset the increase in line length, but this supposition overlooks several factors.

1. The economics of line cost tend to set a definite *per cent* limit of power loss. This is shown by the fact that long lines are designed to work at the maximum possible loading and the resulting *per cent* loss has about the same relation to power cost regardless of whether the line is 100 miles or 300 miles long and regardless of whether the voltage is 110 kv or 287 kv.

2. The economics of terminal equipment and the frequent necessity of tieing between existing facilities generally limit the permissible *per cent* voltage regulation regardless of distance and voltage. Economics likewise limit equipment reactance. Equipment of 25 per cent equivalent reactance on rating *including the transformer* would be prohibitive in cost and size if we may judge by recent discussions of standard short-circuit ratios.

3. The economics of line and terminal costs are such that a 220-kv line hauls energy at about the same cost per kilowatt-hour as for a 110-kv line; more power is hauled further by the higher voltage—that is the major difference.

4. Above about 250 miles distance becomes an increasingly serious obstacle. Even under the most favorable conditions of load and power factor, power loss and voltage drop are logarithmic functions of mileage and exceed a linear relation at a rate increasing with line length.

In view of the foregoing, it is apparent that in designing practical lines, *the cost of power and the cost of lines and equipment are necessarily major factors on one side of any equation for stability, regulation, or power loss.* As the authors show, stability can be increased, although the effect on the cost side of the equation is not clear without detailed study of typical or particular cases.

Concurrently with stability, regulation, and power loss must be studied. The present paper does not attempt to discuss these latter two factors. However the authors have laid the foundation for a quick and effective analysis of the technical relations thus facilitating a determination of the economic factors.

The authors present an equation for power loss for surge impedance loading. This is a form of a well-known communication equation. A similar communication equation provides a simple formula for voltage drop at surge impedance loading. (See analysis at the end of discussion.)

It is also known from communication theory that the loss ratio is a minimum and the voltage regulation is the best at surge impedance loading. The authors have therefore made an important contribution in showing that *most modern lines operate so closely to their surge impedance load that the performance of practical lines may be studied under this load condition.* This simplifies physical performance in every respect except as to stability. For instance, power loss and voltage drop become smooth exponential curves without any relation to wave length. As shown on the accompanying chart (figure 3), power loss and voltage drop may be drawn as straight lines on a semilog chart of distance against ratio of sending to receiving power or voltage. We can now see where *economics has placed a transmission limit much shorter than that set by stability.* This limit is substantially proportional only to length and conductor resistance per mile. The latter is limited by cost of supporting structures as well as by the cost of the conductor itself, and although it can be shown that the percentage power loss and percentage voltage drop can be maintained constant for any length of line by increasing the conductor area linearly with the line, our economic judgment tells us that it is no use to follow this relation very far.

We thus come back to where we started and with little hope of rapid progress unless some one overcomes I^2R loss—because present-day lines at maximum loading approach closely to the I^2R loss calculated on a d-c basis (equal current throughout the line length) and that the cost of that loss is ordinarily one of the three major economic limitations to long-distance transmission.

From equation A-12 in authors' paper:

$$P_R=\frac{1{,}000E_S{}^2\epsilon^{-2H}\cos PF_S}{Z_0}$$

$$P_R=P_S\epsilon^{-2H}$$

$$\frac{P_S}{P_R}=2.72^{2H}$$

$$\frac{E_S}{E_R}=2.72^{H}$$

$$H=\frac{Mr}{2R_0}$$

M=miles of line
r=resistance per mile
R_0= surge resistance=Z_0 times cosine of angle of Z_0
Angle of Z_0=2 to 10 degrees
H=real component of propagation constant times length

H. P. St. Clair (American Gas and Electric Company, New York, N. Y.): The authors of this paper are to be commended for bringing together in compact form a large amount of information, both practical and theoretical, on the performance and possibilities of long transmission lines. The many interesting performance curves based on exhaustive network analyzer studies would seem to answer almost any theoretical question (except economical ones) regarding the possibilities of transmission lines up to any conceivable length.

The authors' discovery or observation that most existing lines, 100 to 300 miles in length, are rated at a load corresponding to an impedance equal to the surge impedance of the line is rather striking; also, the conclusion that such a load, if a dead or a nonsynchronous load, can be carried over any length of line even 3,000 miles without overvoltage along the line. Somewhat more obvious, however, is the observation or discovery that existing long lines carry their rated loads at approximately unity power factor. This is necessarily true since even small components of reactive kilovolt-amperes carried over the high reactance of long transmission lines produce excessive voltage drops.

In connection with the very significant surge impedance loading described by the authors, it might be of interest to point out another characteristic of this same loading, perhaps inferred but not specifically mentioned by the authors, and, that is, that it corresponds to the load at which the I^2X reactive kilovolt-ampere loss in the line just equals or neutralizes the capacitance charging kilovolt-amperes of the line. This, of course, is the reason that a "dead load" of this amount could be carried over any length of line: the distributed shunt capacitance load is completely neutralized by an equal reactive kilovolt-ampere loss.

This value of loading, which I have called for convenience the "critical" loading, for a given voltage line is often very useful as a bench mark for checking purposes. For example, knowing that a 132-kv line has a charging kilovolt-ampere of approximately 9,000 kva per 100 miles, it is also apparent that at the critical loading of 44,000 kva the I^2X reactive kilovolt-ampere loss is the same amount, or 9,000 kva, and that for any other loading the I^2X loss will vary as the square of the loading. Thus, by simple arithmetic it is possible to determine the approximate power-factor requirements at the two ends of a line for any load condition. To illustrate, assuming that a 100-mile 132-kv line delivers 66,000 kw at unity power factor, what power factor or reactive kilovolt-ampere component exists at the sending end? This load is 1.5×

44,000 kva, so that the I^2X loss would be $(1.5)^2$ or $2.25\times9{,}000$, or approximately 20,000 kva. Subtracting the 9,000-kva charging current, there would remain approximately 11,000 lagging reactive kilovolt-amperes at the sending end. This is rather rough, of course, but at the same time quite useful for quick estimating purposes.

This criterion or bench mark is also useful in making network analyzer power flow studies where it is usually necessary to add or subtract corrections for charging kilovolt-amperes for each line entering or leaving a bus. By comparing the actual loading on the line with this criterion of surge impedance or unity power-factor loading, it is possible by inspection to determine whether the change in power factor between the two ends of the line is in the right direction and of the right order of magnitude and thus quickly discover possible errors in the application of the charging kilovolt-ampere corrections.

As mentioned above and as already brought out by other discussers, the economic aspects of extra long lines have scarcely been touched upon by the authors in this paper. Further consideration should be given to this phase of the subject, and in order to go into it more carefully we hope to submit a discussion on this in writing at an early date.

E. C. Starr (Oregon State College, Corvallis, Ore.): Miss Clarke and Mr. Crary have made a useful contribution to the literature on electric power transmission. They have summarized the present and probable future status of a-c and d-c transmission, and have pointed out the important limitations. As time progresses, occasions will doubtless arise for the transmission of increasingly larger blocks of power over distances greater than now practiced or deemed feasible. It is consequently important that we anticipate these requirements and be prepared to meet them adequately when the occasions arise.

A primary objective of all agencies dealing in electric energy is the delivery of that energy to the consumer at the minimum annual cost. This objective can be attained only when the sum of all the individual costs involved in the generation, transmission, and distribution of energy is a minimum.

Mathematical analyses have been made of a number of generating plants and their associated transmission systems. The results have shown that in every case involving lines of the order of 200 miles or more in length, the maximum possible loading of the transmission lines, based on their present accessories and operating practice, is not sufficiently great, by a considerable margin, to develop the minimum annual cost of delivered energy.

The cases examined involved lines operating at 220 kv or higher and employing conductors of the order of 500,000 circular mils, copper or copper equivalent. The equations of maximum economy indicate that the least costly of these lines should be loaded to approximately 600 amperes per conductor. The more costly lines require heavier loading. These values are based on the use of no additional auxiliary equipment over that already installed on the systems studied. Line currents of the order of 600 amperes are considerably below the heating limit of the conductors employed, but are very materially above the currents obtainable in these systems because of reactive kilovolt-amperes and power-limit considerations. Further studies have shown that the use of series capacitors, either in single or multiple circuits, will permit the currents of maximum economy to be obtained at a very moderate increase in total system cost. As a matter of fact, if the cost of necessary series capacitors is included in the solution of the equations for maximum economy, the current of 600 amperes named above is reduced only to approximately 550 amperes.

The line losses will, of course, be very materially increased, but the utilization of the investment is so increased as to result in minimum total annual cost. If the capacity factor of the line is less than unity, as it most always is, and if the costs of fuel or hydraulic storage are appreciable items, the current values named above, and consequently the normal rated power capacity of the lines, would be increased somewhat. The economic power limits named are from approximately $1^1/_2$ to 2 times the limits now obtainable on the systems examined, and the investment per kilowatt transmitted as well as the cost per kilowatt-hour of energy delivered is materially less than at present.

We are looking forward with a great deal of interest to the possible use in the not too distant future of series capacitors on transmission systems already in service, or planned for future construction, for the purpose of improving the economy of delivered energy or making possible the successful economic operation of lines now considered too long to be feasible.

High-voltage direct current also enters the picture at this point. The excessive reactive kilovolt-ampere requirements involved in the operation of 60-cycle lines at high-current densities as well as the stability limitations of synchronous equipment are not present. The transmission circuits required appear to be much less costly on a per kilowatt basis and afford an opportunity to attain the desired currents of maximum economy. On long circuits a very large saving in line costs would be available for added terminal equipment.

J. M. Barry (Pennsylvania Water and Power Company, Baltimore, Md.): It would seem desirable to amplify somewhat the first conclusion drawn by the authors regarding lines longer than 300 miles, namely that the only limitation to the distance that power may be transmitted to a static load corresponding to the surge impedance of the line, is the line resistance losses. This conclusion is based on curve *A* of figure 1 of the paper, which shows a load of approximately 120,000 kw being delivered (less losses) any distance up to 2,000 miles. Inasmuch as practical operating experience has been with lines under 300 miles in length the curves of figure 4 of this discussion were prepared to show some of the operating characteristics of longer circuits. This diagram shows a number of voltage regulation curves for lines varying in length from 100 to 600 miles. In calculating these curves conductor resistance was neglected, the load power factor was taken as 100 per cent and the sending end voltage assumed constant at 220,000 volts. It will be noted that all of these curves intersect at a common point, i. e., the surge impedance load. Since resistance has been neglected, the receiving end voltage is equal to the sending end voltage regardless of the circuit length at this particular load. However, these curves also indicate some of the difficulties which might be expected in the operation of lines of length of the order of 500 to 600 miles. For instance in the case of the 600-mile line voltage regulation would be very poor for loads approximating the surge impedance loading and at no load the receiving end voltage rises to nearly three times that of the sending end.

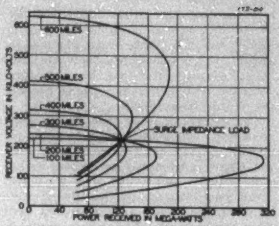


Figure 4. Voltage-regulation curves for 220-kv circuits

The curves of figure 4 are presented as a matter of general interest rather than criticism, as it is realized that the authors are not claiming that lines of such length are practical at present.

R. D. Evans (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The paper by Miss Clarke and Mr. Crary may be considered as two parts:

1. A presentation of steady-state stability limitations of a-c transmission systems as a function of transmission distance, and
2. A review of the present status of this problem.

The stability limitations of various systems are given in terms of the surge-impedance loading. Such a reference is convenient for general studies but not for a study of a particular project. The surge-impedance loading corresponds to the "critical load" used by Percy Thomas and Harold Goodwin in their papers before the Institute in 1909 and 1923, respectively. As would be expected from the scope, the second part of the paper reviewing the present status of the problem does not lead to conclusions presenting new thoughts.

The principal interest of the paper is in the discussion of measures to improve stability. High synchronizing power by low-reactance apparatus and high-speed fault-clearing measures are now accepted as standard for systems with long lines and the stability question narrows down to that of what supplementary measures are to be used. Mention is made of the quadrature-booster transformer scheme which appears to be based on the common but inaccurate statement that the stability limit occurs at an angular displacement of 90 degrees for a simple system without loss. Actually, the limit occurs at a displacement of 90 degrees from the displacement which corresponds to zero-load transfer. On this basis it becomes clear that the quadrature-booster scheme is not operative to produce the results claimed for it.

The authors conclude that the two most promising supplementary measures for improving stability are series capacitors and shunt synchronous condensers in the order named. With this view we agree. Series capacitors are particularly effective for increasing steady-state limits and have other applications. The curves given in the paper for the intermediate condenser would, it seems to us, be more useful (1) if based on condenser internal voltages higher than that of other machines as would be the practical case, and (2) if based on adjusting the machine kilovolt-amperes in accordance with the increased capacity made possible by the increased limits obtainable with the condenser.

This paper is of interest in its reduced emphasis on d-c transmission and its increased attention to methods for increasing stability limits of a-c systems.

R. Rüdenberg (Harvard University, Cambridge, Mass.): I would like to say a word in favor of the much abused inductance of transmission lines. Although I agree completely with the authors in regard to the favorable effect of series capacitors with respect to the stability limits of long lines, we should keep in mind that by the effect of the inductance we have a fortunate opportunity of compensating in a-c transmission systems the detrimental resistance voltage drop between sending and receiving stations. We have only to send over the line, as indicated in the accompanying figure 5, a flow of reactive power Q reverse to that of active power P in order to produce an inductive rise of voltage XI_q able to neutralize completely the ohmic drop RI_p in every section of the line.

Since for high-power overhead lines R is small compared with X, a moderate amount of back-flowing inductive power, easily produced by proper excitation, suffices to arrange for constant voltage in all stations over the entire system. This method[1] is used in a European transmission system, for which I served as consultant, over a total distance of more than 400 miles. In d-c transmission lines there is no such simple possibility of compensating the ohmic drop.

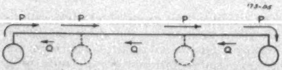


Figure 5. Reverse flow of reactive power for neutralizing ohmic drop of active power

If we neglect for a moment the resistance of the a-c line, we can easily clarify the significance of that special power which the authors call "surge impedance loading" or "unity power factor rating." On every transmission line, current as well as voltage consists of two sets of waves proceeding in opposite direction with velocity of light, as indicated in the accompanying figure 6, and constituting a power wave P proceeding in one direction and a second power wave P' traveling in the reverse direction. If the distant end of the line is open, both waves are equal in magnitude and P' is the reflected wave of P.

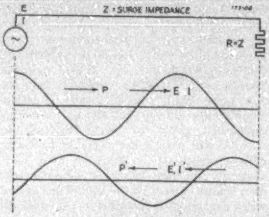


Figure 6. Avoidance of reflected power flow by proper end load

The maximum voltage on the line is given by the sum of both voltage waves and thus the line insulation can be a minimum if the reverse voltage wave disappears. The total current in the line is given by the sum of both current waves and thus the energy losses become a minimum if the reverse current wave disappears. Both conditions can be fulfilled simultaneously by preventing any reflection at the end of the line. This is attained if, and only if, there is an equivalent end resistance R equal to the surge impedance Z of the line, as indicated in the accompanying figure 6. The load is then wholly active and of the magnitude $P=E^2/Z$. The line and the end station are operating with unity power factor and with unidirectionally proceeding waves which have constant voltage throughout. With a surge impedance $Z=400$ ohms, we have for $E=220$ kv directly $P=$ 120 megawatts, an optimum load which I have called[1] for the transmission system mentioned above the "natural power" of the line. This special power can be transmitted over any distance without limitation due to the inductance or the wave length of the line. Such transmission requires relatively inexpensive means for angular stabilization at high loads and for power-factor compensation at low loads.

REFERENCE

1. *ETZ*, 1929, volume 50, page 970.

A. M. Taylor (Universal Power Transmission, Ltd., London, England): Clarke and Crary's paper contains a vast amount of information and withal excellently set out.

But the busy engineer will, the speaker fears, be rather bewildered by the abundance of the information. He wants to get some tangible figures for distance, voltage, and power transmitted, and may undervalue the paper because he does not know which of the many sets of curves represent practical conditions. The following remarks are intended to help.

Table I is exceedingly valuable as showing the effect of voltage in increasing the powers which can be put through—*must* be put through—lines, as they get longer, to make them pay commercially.

The speaker is very pleased to see 550 kv suggested. He is certain we shall have to come to it eventually for great distances.

Figure 1 deals with only "dead" loads and is therefore not suitable for practical conditions. Figure 2 deals with terminal generators and receivers which have no reactance whatever and are therefore not to be obtained practically.

Figure 4 comes to practical realities. Generator reactances of 40 per cent and receiver reactances of 25 per cent are considered. A steady-state stability factor of 25 per cent is introduced.

The curves tell us that, at 300 miles and 230 kv, we can transmit 122,000 kw with stability; and that at 550 miles and 330 kv we can transmit 163,000 kw.

Figure 7*a* gives very valuable hints as to effect of changing reactance values at generator and receiver stations and effect on stability. But on the whole, it makes no advance on figure 4—gives only 141,000 kw at 330 kv and 550 miles.

Figure 8*a* is for 600 miles (only) but with an intermediate synchronous condenser station. At 330 kv the power transmitted is increased to 354,000 kw as against 218,000 kw *without* an intermediate station. But synchronous condensers of four times the base power are required in the case of the intermediate station scheme, which largely discounts the advantage.

Figure 9*a* is for the use of 50 per cent of series capacitors in the line and gives the figure of 228,000 kw transmitted over 600 miles at 330 kv. Figure 10*b* combines 50 per cent of series capacitors with 75 per cent of line capacitance kilovolt-amperes canceled by shunt reactances, and obtains 228,000 kw transmitted over 600 miles at 330 kv with 25 per cent stability margin. The speaker, however, estimates that with quadrature boosters he can transmit 610,000 kw with a 50 per cent stability margin at same voltage and distance.

The speaker would here explain that his calculation for the figure of 610,000 kw mentioned above is arrived at by the use of Miss Clarke's figure 4 given in her Institute paper in February 1926. (This figure omits ohmic resistance.) The reason for his being able to get such an excellent result is that he normally employs the boosters not only to cancel out the reactance of the line, proper; but also that of the step-up and step-down terminal transformers.

If the line plus transformers are cut out in this way, we have:

$$P_{\max}=\frac{(1)^2}{\sqrt{\underset{(x_G)}{0.583}+\underset{(x_R)}{0.406}}}=2.055 \text{ per unit} =1{,}235{,}000 \text{ kw on a 600-megawatt base}$$

If reactance of line and transformers are *included*, the value of $P_{\max}$ immediately drops to 415,000 kw. This result is of the greatest importance.

It will no doubt be objected that, since the quadrature booster gives only a constant secondary voltage (i.e., boost), for the maximum steady-state stability current, a sudden increment in synchronizing current traversing the line would give increased reactive drop in the line and a consequent sudden lowering of voltage at receiver terminals, with a further demand for synchronizing current; and thus a "vicious circle" would begin; unless the ordinary regulating arrangements have had time to operate—say 0.5 second, at least.

Consequently, we must face this fact and assume (which is considered reasonable) that, at the moment prior to the sudden demand, the boosters at a steady fixed line load of 610,000 kw have just exactly canceled out the reactance of line and transformers. Now let a sudden increment of load—at receiver—demand momentarily ten per cent more current in the line (i.e., 61,000 kw); and reconstruct the Clarke diagram to take account of this "equivalent extra reactance" (and *only* this). The speaker finds, on doing this, that the value of P_{max} is now changed from 1,235,000 kw to 1,080,000 kw (i.e., by only 12.5 per cent).

He therefore concludes that, since $P_{max} =$ 1,235,000 kw is given by Clarke's formula, when *ignoring* said deficiency of boosters, it is absolutely safe to work at 610,000 kw which gives a 50 per cent margin. This particular value is taken because it represents the value at which the capacitive kilovar equals the inductive kilovar and suits the quadrature booster system. But a greater power may be taken if desired, under certain modified arrangements; and would undoubtedly be *quite* safe as regards stability.

The speaker has observed that the authors, in section III, subsection (*a*), of the paper state that the quadrature booster method "decreases the angular displacement at which pull-out occurs." Surely however, they must have overlooked that it is *not* the reduction of the angular displacement *in the line itself* that is the matter at issue; since the reduction of this, if any existed, would actually *increase* the stability. For instance, taking the extreme case, viz., that of a system in which the transmission line consisted only of a pair of bus bars, it will not be denied that here we have the conditions of maximum stability. It is not, therefore, the phase angle between the ends of the line that counts, but only that across the generator plus that across the receiver, which the authors would appear to have overlooked.

Consequently (even looking at things in their worst light) in a system which normally approaches the equivalent of a solid bus bar between stations, there will be only a loss of stability to a trifling extent; corresponding, in fact, with only a $12^{1}/_{2}$ per cent diminution (already established earlier) in the maximum pull-out power for a ten per cent increment in line power. Where, however, there is already a great phase angle between the two ends of the line, or parts of the line, as in most systems, the conditions are presumably very much less stable; because other factors than mere line inductance are at issue.

This very interesting paper is so full of extremely valuable matter that the speaker feels that these few remarks do not in the least represent all that he ought to have touched upon; but he ventures to think that they are important and hopes that the summary of the salient features of the several diagrams which he has presented may be acceptable to the authors—and to any engineers who have perhaps not fully assimilated the paper.

Edith Clarke and S. B. Crary: Several of those who discussed the paper have pointed out that the chief limitation imposed upon the transmission of power over long distances is an economic one. With this we agree. As stated in our paper, no attempt was made to present an economic study; it was thought that the technical features should first be thoroughly understood.

Messrs. George and St. Clair have indicated that they intend to present more complete discussion of the economic features at some future time. These discussions should be of considerable interest and value.

Mr. George is correct in stating that in addition to stability, regulation and power loss are important factors. In our paper the curves presented give directly the power loss in the line, as well as the reactive kilovolt-amperes at both the sending and receiving ends of the line required to maintain terminal voltages. The economic studies we have made for given sets of assumed conditions indicate that the cost of transmitting a kilowatt-hour per mile can be kept practically constant for distances from 200 to about 500 miles. As the distance increases, it is necessary that a higher voltage be used and a larger block of power be transmitted in order to keep down the transmission cost per kilowatt-hour per mile. For these longer distances, the series capacitor provides a means of maintaining a more reasonable transmission cost. As Mr. George has pointed out, the cost goes up with conventional transmission as the distance to which power is transmitted is increased. Without series capacitors, the cost per unit of energy per unit distance may be tripled when the distance is doubled. However, with series capacitors, if the distance is doubled, the transmission cost may be only doubled rather than tripled. Mr. George's chart shows clearly the limitations due to losses as the transmission distance is increased and the necessity of using lower resistance conductors for long distances.

Mr. Barry's curve shows the effect on the amount of power that can be transmitted by changing the resistance of a static receiver load. The values from his curve check those of curve *C* of figure 1 of our paper in that they indicate, for distances beyond about 400 miles, no increase in power above the surge impedance load without overvoltages along the line.

Professor Starr points out the interesting result that even for lines of existing lengths, series capacitors appear to be a worthwhile means of increasing power limits. Our economic studies bear out Professor Starr's, and indicate very definitely that series capacitors at present costs should be considered as a means of reducing transmission costs for either existing or new lines, greater in length than about 200 miles.

Mr. St. Clair has given an interesting application of the surge impedance loading concept in making power-flow studies.

Professor Rüdenberg points out the value of some reactance in the line. The considerations which Mr. Concordia has discussed place definite limitations on the amount of compensation that can be used. It is of interest to note that some reactance, as Professor Rüdenberg has pointed out, for control of voltage on a transmission line may be valuable; while as Mr. Concordia points out, with series compensation of line reactance, it is necessary that there exist some resistance between the two parts of the system as well as possibly some uncompensated reactance. The resistance in the line becomes a necessary factor only in the case of series compensation.

We are glad to learn that Mr. Evans is in general agreement with our conclusions. We would like to point out, however, that we believe series capacitors provide a much more effective method for increasing power limits than synchronous condensers, used for that purpose alone. Mr. Evans has suggested that our studies of the intermediate condenser would be more useful ". . . (1) if based on condenser internal voltages higher than that of other machines as would be the practical case, and (2) if based on adjusting the machine kilovolt-amperes in accordance with the increased capacity. . . . . ." The first consideration was taken into account in arriving at the results presented in the paper, for when the line loading exceeded that of the unity power-factor loading, the synchronous condensers were required to furnish the system with overexcited capacity in order to maintain the terminal line voltages; therefore, their internal voltages were appreciably higher than those of other machines of the system. The effect of the second consideration pointed out by Mr. Evans is taken care of on the curves, as figures 7 and 8 show the intermediate station kilovolt-ampere requirements as well as the intermediate station reactances. Thus it is easy to arrive at the required intermediate station kilovolt-amperes and reactance. Mr. Evans has taken this paper as an indication of reduced emphasis on d-c transmission. The intention was to restrict this paper to stability limitations of a-c systems, as it is believed that any rational conclusions relating to power transmission must be based upon a clear and accurate comparison of all available methods. Although series capacitors provide a means of overcoming some of the limitations of a-c transmission, this does not mean that d-c transmission will not find application under conditions favorable to d-c.

In closing we wish to thank all those who discussed the paper for the important contributions they have made to this subject.

Powerton-Crawford 220-Kv Line—System Operating Features and Terminal Design

Discussion and authors' closure of paper 41-26 by H. E. Wulfing and T. G. LeClair, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941, pages 1084–9.

R. A. Hentz (Philadelphia Electric Company, Philadelphia, Pa.): Tabulation shows eight regulating transformers at the Powerton end, six for regular operation and two spares. The cost of one of these 3,000-kva regulating units must be considerable and it would be interesting to know why the second spare was preferred over permanent switching equipment by means of which one spare could be electrically connected in place of any one of the six regular units.

H. E. Wulfing and **T. G. LeClair**: Mr. Hentz is correct in the statement that the cost of the two 3,000-kva spare regulating units at Powerton was considerable. A careful estimate showed, however, that the use of two spare regulating units would cost less than one spare regulating unit and additional switching.

Two regulating units are needed for each single phase main transformer in regular operation because of the two low voltage windings. This is shown in figure 3 of the paper. To provide connections so that only one spare regulating unit is necessary, involves cable and disconnecting switches for the series and exciting winding to be connected in six different places. These connections would be in addition to those for substituting the main transformer spare for the main transformer on each phase.

The spare main transformer has a spare regulating transformer permanently connected to each of the secondary windings. This combination of main and two regulating transformers is treated as a single unit for switching. The number of facilities required for replacing the combination of transformer and regulating units is the same as the number required for a spare transformer alone. The complete unit will be taken out of service for any case of trouble regardless of whether it is in the main transformer or one of the regulating units.

Rural Automatic Telephone Networks

Discussion and author's closure of paper 41-47 by C. F. Ffolliott, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941, pages 1079–83.

Emmett D. Talbot (Consulting Telephone Engineer, Rochester, N. Y.): The application of automatic switching methods to rural communication problems has resulted in increasing demands for service protection facilities. Some of the more important developments in service protection are:

Intercepting Service Facilities

In all telephone networks, it becomes necessary from time to time to change the subscriber's telephone numbers, and there are some subscribers who discontinue their telephone service. Number changes can be corrected and brought up to date with the issue of a new telephone directory, but during the interval between issues of the directory it is desirable to notify subscribers of number changes as economically as possible.

Notification of number changes can be accomplished in several ways, at the present time the preferred methods in use are:

Supplemental equipment is provided so that it is possible to cross-connect a changed or discontinued subscriber's number to this supplemental equipment, whereby, a subscriber calling a changed number will receive a tick tock tone indicating that the number called has been changed or discontinued. Where this method is used subscribers are advised that upon hearing a tick tock tone, they should hang-up and dial the operator, generally zero, the operator will ask the calling subscriber the number being called and will furnish the calling subscriber correct information relative to the status of the called number.

This method of intercepting has been found satisfactory in a number of rural areas and is not expensive to apply.

In many rural dial areas, intercepting service is provided through the use of supplemental equipment, to which changed or discontinued numbers are cross-connected, but instead of notifying the calling subscriber by means of a tone that the number has been changed the apparatus automatically connects the calling subscriber to a trunk to the zero operator. The operator furnishes the calling subscriber with the correct number or other information required to satisfy the calling subscriber. This method is somewhat more costly to apply but has the advantage over the first scheme of giving the subscriber direct connection to the zero operator.

Another important service protection that has been developed is in connection with the various alarms associated with a dial central office equipment. The more important alarms are those indicating a failure of the commercial power supply, blown fuses in battery and generator control circuits, and open fuses or heat coils individual to the subscriber's lines. Alarms are classified as major (power, battery, and generator supply), or minor (subscribers' line protection).

Means are provided so that when an alarm condition occurs in an unattended dial office, this condition is transmitted automatically over a trunk to the parent attended office. When such an alarm is indicated at the attended office, anyone on duty at the attended office can pick-up the trunk carrying the alarm signal and by means of a dial at attended office and special switch mechanism at the unattended office, determine by dialing a prearranged set of digits, what type (major or minor) alarm exists. If a major alarm condition is found, a trouble man can be dispatched to correct the condition, if a minor alarm is indicated, the condition is recorded and reported to the trouble man to be corrected as soon as it is convenient for him to visit the unattended office. The transmittal of the alarm conditions to the attended office insures against service interruptions being left uncorrected for any appreciable length of time.

One other feature that improves the service which is gaining favor is the use of time control equipment in connection with exchanges on a flat rate basis, especially those rural unattended areas having free call privileges into some nearby dial or manual area. The time control feature makes it possible to automatically control the length of line occupancy by a subscriber. If a six-minute use is desired the time control apparatus can be arranged so that at the end of a six-minute period, a connection is automatically cut-off. The control can be applied to all local or free service calls, and not be effective on toll calls. Where time control service is given, there are generally some subscribers who dislike this service, however, the majority of subscribers are pleased with the service. Time control tends to discourage long telephone conversations, which tie up the apparatus.

D. L. Solomon (Consulting Telephone Engineer, Chicago, Ill.): This paper describes one of the lesser known applications of automatic telephone switching equipment to the solution of communication problems in rural networks. While it is widely known that automatic telephone switchboards are available for use in small communities, it is not generally known that these automatic switchboards can be easily interconnected to form a practical and economical network.

Because of the more congested areas and higher density of population in European countries, with resultant problems of furnishing satisfactory and cheap telephone service in rural areas, this problem has been attacked to a rather amazing degree. There, in certain countries, notably Belgium, Holland, and Switzerland, not only have most of the rural exchanges been converted to dial operation, but a general plan has been developed for complete national dialing, where any subscriber can dial direct to any other subscriber in the nation, without the aid of an intervening toll operator. This has been accomplished by the development and use of automatic toll ticketing, a system wherein the necessary commercial details of the toll call, normally recorded by a long distance or toll operator, are stamped automatically on a slip of paper, thus dispensing with any human intervention in the progress of either a local or a long distance telephone call.

These European rural exchanges are quite similar in operation to those described in this paper, with the possible exception that provisions for party-line working are not provided, since party-lines are not in general use outside of the United States and Canada. They are also furnished in compact self-contained units, but the pole mounted type has not yet been used, so far as is known.

As the author has stated, much of the inter-exchange dialing in Europe is performed by the use of alternating currents, and standards for this method of long-distance dialling have been set up by the Comité Consultatif International Téléphonique, which is the governing body of the various telephone administrations in Europe and elsewhere.

F. E. Norris (Associated Telephone Company, Ltd., Santa Monica, Calif.): The economic considerations incident to the establishment of an unattended dial controlled switchboard must, among other things, recognize the possible effects on toll line investment and revenue. It is not unusual to find that the toll lines connecting the manual office under consideration for conversion with the "master" or control exchange are working to capacity or are actually overloaded. Both conditions indicate that all toll facilities are in use frequently. This may present a serious service problem after dial operation is established if the toll lines are not reenforced. The difficulty arises out of the fact that the "storing" possibilities existent with manual operation (and the subsequent release of such calls in the order received) do not exist in the unattended dial exchange. In other words, in a manual office the customer always is able to reach the operator to inquire the status of his toll call when all toll lines are in use, whereas in a dial office he receives an impersonal busy tone upon dialing the toll code. Furthermore, he may dial for the operator repeatedly but not at the particular moment a toll channel is available, and at that moment another customer may, upon the first attempt, seize the channel.

In anticipation of the service difficulties attendant to this situation, the usual practice is to compute the toll line facilities on a more liberal basis. For example, a delayed call probability of 20 in 100 may be adequate when the exchange under consideration is operated manually, but after the conversion of the exchange to dial operation, a delayed call probability of 6 in 100 may be necessary. To reduce the delayed call probability from 20 in 100 to 6 in 100 will indicate in many cases the necessity for increased toll line facilities and the cost of providing them should be reflected in the conversion to dial operation study.

The conversion to dial operation frequently results in an increase in stations served and consequently an increase in toll business. The possibility that the increased toll business may demand an increase in toll line facilities should be considered together with the increased revenue.

In some cases, nearby communities are served only from toll line stations (using toll lines other than those connecting the unattended dial exchange with the master exchange). These frequently present a special problem (and special equipment) as they may have a different toll rate to many points than the unattended dial office has to the same points. Other conditions may make it impractical to operate such stations on a dial (or common battery) basis. Special equipment to connect such lines with the toll trunk lines, when the receiver is removed or the magneto operated, is available. This special equipment involves an appreciable additional cost which should be considered in the conversion study.

In some instances the exchange to be converted may connect to a master or control exchange owned by another company. Further, the toll line facilities connecting the exchange to be converted with the master or control office may be owned by another company. There may be either or both traffic and terminal compensation agreements which will require modification when the conversion from manual to dial operation is made. It seems only reasonable that such agreements be reviewed and discussed with the connecting company and the conversion study made to reflect any revenue gains or losses resulting from possible contract revisions before proceeding to install dial equipment.

The manufacturers have steadily increased the operating range of their equipment as regards subscribers loop resistance. All of the longer loops should be investigated however to determine if they fall within the supervisory limits of the equipment under consideration. If some do not, an improvement in the line or special central office line equipment is required. The investigation of such lines should also be made with the possibility of high resistance joints in mind and such joints corrected.

Drop wires should be considered when a conversion to dial is contemplated as experience has shown in many cases that drops which may work with fair satisfaction in a magneto, and sometimes common battery manual exchange, are not satisfactory for dial operation. It is rather difficult to determine in advance of the conversion, particularly if the exchange considered for conversion is not magneto, which drop wires must be changed. For study purposes, some companies estimate that 10 per cent of all drops will have to be replaced following a conversion from magneto manual to dial operation, but this percentage, of course, may be varied upward or downward, depending upon a knowledge of the conditions within a particular plant. The need for such replacement may not become apparent immediately following the cutover, if the weather is fair, but low insulation may show up with the first rain if the drops are not in good condition.

C. F. Ffolliott: Mr. Talbot's comment is apt and covers services that are available for use on standard boards. Alarm supervision such as he describes is a standard feature of most unattended exchanges. Intercepting service and conversation timing are optional features that can be supplied with most boards if desired by the operating company. There is a considerable difference of opinion among operating men, supported by ample arguments on both sides, as to when or where these services should be used. They are available however for those who want them.

The points in Mr. Norris' comments are well taken and serve to emphasize the importance of planning for and providing adequate facilities in all departments of the telephone plant. As he points out and as I mentioned in my paper, both toll and local traffic invariably increase with the introduction of automatic. This point should always be kept in mind by those companies who are planning to convert existing offices to automatic and adequate provision should be made for outside plant improvement and extension. In passing it might be of interest to mention that as a general rule a rural exchange should never be engineered with less than two circuits to the toll center.

Mr. Solomon's comments are interesting since they call attention to current European practices as regards the rural telephone problem. It might be pointed out by way of extending his remarks that the European and American problems are somewhat different because of essential differences in the way of life in the rural communities of the two continents. In Europe, contrary to the practice in this country, the farmers live in small villages located at frequent intervals throughout the country. This characteristic of their rural life combined with the fact that most European administrations operate on a message rate and not on a flat rate basis tends in some cases to reduce the local telephone traffic to the vanishing point and leaves only toll traffic to be handled by the exchange. The absence of party lines makes line identification simple and allows the extended use of "multiple metering" for automatic toll dialing. Automatic toll ticketing mentioned by Mr. Solomon is a more recent development and as such has not been used in the past to the same large extent as the "multiple time and zone metering" scheme. It is, however an important advance in the art and will undoubtedly see more extended use.

Prolonged Inrush Currents With Parallel Transformers Affect Differential Relaying

Discussion and author's closure of paper 41-65 by C. D. Hayward, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941, pages 1096–1101.

A. E. Flink (The Ohio Edison Company, Youngstown, Ohio): Tests such as those described by the author, both field and laboratory, are highly desirable for the application of relays protecting transformers on account of the infrequent occasions upon which their protective equipment is called upon to operate. No transformer failures requiring differential relay operation at Wood Street substation of the Ohio Edison Company have been reported during the past ten years.

Perhaps more attention should be concentrated from a transformer design standpoint on wide variations in normal magnetizing current of transformers of supposedly similar design. The single phase units at Wood Street are similar and of the same manufacture but were not all purchased together. One of them has an exciting current of greater than six per cent of its full load current, and after preliminary test to determine the cause of false relay operations it was suggested by the manufacturer (who is also the relay manufacturer) that one of the units was defective since the unusually high magnetizing current indicated the probability of a core short. This possibility was abandoned when a similar transformer located at another station showed substantially the same magnetizing current. Difficulties with differential protection on inrush current have occurred on units having unusually high steady state magnetizing currents.

The specific effects of parallel transformers described by the author were not foreseen by the relay manufacturer and were not suspected until after several conclusive false operations. During the few months in which harmonic restraint relays were left in service at Wood Street there were three incorrect operations of the harmonic restraint relays in which the substation load was interrupted with no other relay operations or system switching taking place within hours of the false operation. System conditions were normal, and the most plausible cause of any disturbance (none was noticed by operators or dispatchers) may have been faults or switching in the equipment of some minor industrial customer. With relatively new and untried relaying devices it is possible that there are other unknown special effects such as the one described by the author which should certainly be discovered and corrected before operators can have sufficient confidence in the relays to apply them extensively.

From the standpoint of the protection engineer a relay which may be subject to little known effects is particularly unfortunate when applied to a power transformer. Incorrect operations extending over a few weeks or months may require that the relays should subsequently give perfect performance for over a century in order to compare favorably with general relaying performance on the system. It is hoped, therefore, that the manufacturer will encourage extensive tests both of the relay network elements themselves and of different types of transformers and that this new type of relay may become thoroughly reliable.

Eric T. B. Gross (College of the City of New York, New York, N. Y.): The solution to the problem of prolonged inrush currents with parallel transformers has been known for many years. In 1913, K. Kuhlmann published a paper[1] which contains a very complete treatment of all the many questions concerned. He also reported on his experiments and published some of the oscillograms which proved the results of his circuit analysis. Kuhlmann's investigations were considered important enough by other authors to be included in textbooks on transient phenomena.[2]

In principle, there is no difference between the prolonged switching-in current in a single phase and in a three phase transformer. The magnitude of the greatest amplitude of the inrush current depends on the voltage at the time of closing the switch, and it is a maximum if the closing happens to be made at zero voltage. Therefore, in case of a single phase transformer, the chances for a heavy transient current are not very great. However, as the phases of a three phase transformer are shifted by $^2/_3\pi$, there will always be one phase near its zero voltage and a heavy, unsymmetrical transient current between the transformers will circulate. In 1925, I had the privilege of making investigations of this kind for transformers in actual service in the "Sokolska trida" substation of the Prague Electric Utility Power Company, Czechoslovakia, where several transformers of considerable size are installed. When another transformer was switched in, the prolonged inrush current was so heavy that the overcurrent relays operated and isolated both the transformer just switched in, and most of the other transformers which had been in service before. Fortunately, the relays were installed in two phases only, and disconnection did not always occur, because of the unsymmetry of the current distribution. Sometimes, the inrush current had died out before the very long setting of the relays was reached. However, the measurements of the currents in all three phases showed that nearly the same maximum currents occurred each time the transformer was switched in, which was in good agreement with Kuhlmann's analysis. The high inrush currents were due to the relatively high resistance of a long underground cable connection between the Holesovice Power Station and the substation mentioned. As a remedy, it was recommended that a second transformer be put in service by closing first the resistance breaker connected to the winding on the other side. One of the contacts of each phase of the resistance breaker consists of two parts, a short and a long finger, which are connected through a resistance. During the closing period, the resistance is temporarily inserted in series with the circuit, but the resistance is short circuited as soon as the shorter contacts have been closed. The later connection of the power circuit to the already magnetized transformer does not give rise to inrush currents. For the damping of switching-in currents, resistance breakers were used in Europe quite extensively for transformers, at that time.

REFERENCES

1. REACTION OF TRANSFORMER SWITCHING-IN CURRENTS ON THE SUPPLY SYSTEM, **K. Kuhlmann.** *Bulletin Schweizer. Elektrotechnischer Verein*, volume 4, 1913, pages 141–52 and also *Archiv für Elektrotechnik*, volume 1, 1912–13, pages 527–38.

Science Abstracts, Section B, volume 16, 1913, page 466.

2*a*. MAGNETIC TRANSIENTS IN ELECTRICAL MACHINES (a book), **J. Biermanns,** Berlin, 1919, J. Springer.

b. OVERCURRENTS IN HIGH-TENSION SYSTEMS (a book), **J. Biermanns,** Berlin, 1926, pages 23–30, J. Springer.

Wm. R. Brownlee (Commonwealth and Southern Corporation, Jackson, Mich.): The inherent reliability of power transformers makes the accumulation of field experience data on transformer differential relays extremely difficult. Short circuits within the protected zone are rare, the interval usually being from five to twenty years, and of these faults a large proportion consists of bushing flashovers so that the sensitive action of differential relays is not required. This same inherent reliability makes it mandatory that transformer protective relays be much freer from false operations than transmission line protective relays. Coordinated economical system design based on observed failure rates results in the construction of many transmission substations consisting of only one transformer bank with two to four transmission lines. Therefore, the author is especially to be congratulated on his laboratory tests of model units after field tests became too cumbersome. Since the principle itself of the harmonic restraint relay for transformer protection is a relatively new application, it is earnestly suggested that tests be expanded so as to uncover other special difficulties and also to determine definitely whether the design sensitivity will actually be obtained in transformer faults which are of such a nature as to limit fault current appreciably.

The cancellation of third harmonics in certain instances is not confined to delta-delta or open delta transformer banks. When the power transformer windings are connected in wye it is practically always necessary to connect the current transformers on these windings in delta, either on account of phase relation with current from the other windings or to prevent false operation on external ground faults when a delta tertiary winding is provided but not brought out. Naturally a delta current transformer connection provides the same cancellation of harmonics as a delta power transformer connection. Since the only exception to the delta connection of either transformer windings or current transformers is the relatively rare case of a wye-wye connected (usually three phase) unit with no tertiary winding, it is suggested that no reliance should be placed on triple harmonics for relay restraint.

The author has not cited any supporting data for his statement that current due to an internal fault is nearly sinusoidal, containing little or no harmonic components. It is of little moment whether bushing or terminal flashovers result in a sinusoidal wave since current limited only by the power transformer impedance will practically always be adequate for the operation of even relatively insensitive differential relays. The occurrence of faults which limit the current to a fraction of its normal short circuit value is the chief reason for desiring a sensitive differential relay, and it seems entirely possible that the influence of arcs under oil or the presence of iron in a high reactance path between portions of transformer windings (in case of partial winding faults) might introduce sufficient harmonics to defeat the desired relay sensitivity.

It would be of interest to know details of cases which have come to the author's attention of transformers already energized being tripped falsely by their differential relays when another transformer was energized from the same bus. Aside from this possibility, the desensitizing of transformer differential relays during the inrush period either with time delay overvoltage relays or control switch connections is open to the very serious objection that transformers should be adequately protected at the time they are first energized.

Previous methods of transformer differential protection admittedly fall short of adequate performance. It is therefore hoped that the author will be encouraged to make exhaustive tests on the harmonic restraint relays, both to determine whether there are any other special effects which will naturally arise in connection with a new device or new principle and whether genuine sensitivity is secured on the type of internal low current, transformer short circuits for which sensitive relay operation is essential.

E. L. Michelson (Commonwealth Edison Company, Chicago, Ill.): Mr. Hayward's paper brings to light a very interesting case of magnetizing inrush currents, and his analysis should be of aid to power system engineers in avoiding difficulties due to this situation. In Chicago, there has recently

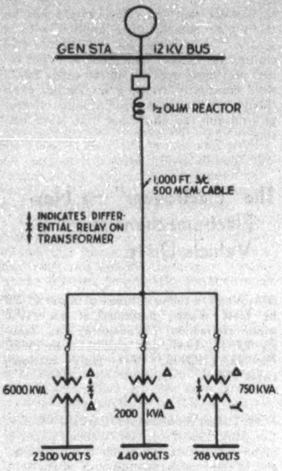


Figure 1

been experienced another unusual type of inrush current problem, which appears to be of a different nature than the one described in this paper. The system involved is shown on the attached diagram.

It was found that the magnitude of inrush currents varied considerably with a different number of transformers connected to the 12 kv line. When the three transformers were energized simultaneously, the differential relays (one set for each transformer) operated in a very large percentage of the trials, while with only one transformer being energized, the differential relays tripped only in a small percentage of the trials.

Oscillographic tests showed that the exciting current inrush was of the usual form regardless of the number of transformers being energized, and only the magnitude varied. Records made of the 12 kv bus pressure showed that there was no appreciable change in this pressure when the transformers were energized.

It appears that some type of mutual effect exists in this system to create the high value of inrush currents. However, it does not appear to be the same type of phenomenon described in this paper inasmuch as the resistance of the supply system in this case is extremely low.

W. K. Sonnemann (Westinghouse Electric and Manufacturing Company, Newark, N. J.): The fact that an inrush current will flow in the power transformer already connected does make a very good case for the use of harmonic components for the prevention of false tripping. The magnetizing inrush does have, at all times, a second harmonic component. From this standpoint, then, the use of the second harmonic in particular does have its advantages for identifying a magnetizing inrush. It should be noted, however, that in the event of a heavy internal fault which has a direct current component, d-c saturation of the current transformers may occur and thus introduce harmonic components of all frequencies at a time when the relay should trip. The point is that while utilization of the harmonics of the magnetizing inrush has its advantages, they must be used with care.

An important point to note is that the inrush in the connected transformer is not too severe when compared with the inrush of the transformer being connected. The paper indicates that a maximum crest of about $^1/_3$ that which occurs in the transformer being connected is experienced. This undoubtedly accounts for the reason that the field has not experienced more trouble with this condition than it has. There are in service any number of installations of more than one transformer bank, protected by differential relays, and connected to the same bus. These relays, more generally of the induction percentage differential type, have not been tripping their banks off of the bus every time a transformer bank has been energized. The evidence seems to indicate that the special conditions described by Mr. Hayward do not reach troublesome proportions except in the rarest of cases.

The mathematical formulas which are given in the appendix are quite long and involved and the derivations of these formulas are not given. It is presumed that the derivation has been omitted for the sake of brevity. The writer believes this is just as well, inasmuch as the formulas certainly should be considered with the utmost caution. The reason for this is that the formulas assume the inductance values to remain constant which is an assumption very far from the truth. Also, the inductance values for the two transformers have been assumed to be their normal exciting inductances, whereas, during inrush, the effective inductance is reduced as indicated by Mr. Hayward's figures, to approximately 2.5% of the normal excitation value. With such a range in values through the inrush period, and with a variation in inductance throughout any one cycle, the formulas certainly cannot be taken too literally.

It is believed that in figure 5, the curve for the "current in the line" is in error in that the ordinates should have approximately twice the value shown on the curve. At time, $t=0$, the d-c current in the line certainly equals the d-c current in the switched transformer, T_2. Further, the formula for the "current in the line" shown on figure 5 has a 2 in the denominator which is not present in the text.

The explanation given for the cause of a direct current component in the transformer already connected is quite valid, although we feel that it is a bit left-handed. It is felt that a more straight-forward explanation is given by the fact that the two transformers are, after all, connected in parallel and the voltage drop across one is necessarily the same as the voltage drop across the other one.

The voltage drop, e_2, across transformer, T_2, caused by the d-c component, i_2, is expressed by the following equation:

$$e_2 = i_2R_2 + L_2\frac{di_2}{dt} \qquad (1)$$

In this case, the inductive component of the voltage drop is both larger than and negative with respect to the resistive component, hence, the total voltage, e_2, is negative. The result is that a d-c current builds up in the transformer, T_1, in the direction shown by the dotted arrow in figure 2 of the paper.

C. D. Hayward: It is very gratifying indeed to have a paper received with so much discussion. Apparently the subject is new and interesting to many people even though to Dr. Gross it is "old stuff." Frankly, I was not aware that the phenomenon described had ever been identified and investigated before. None of a rather large group of engineers with broad experience in the relay and transformer fields with whom I conferred had ever heard of it before. While preparing the paper I checked several of the standard indices covering all the leading technical periodicals, both American and foreign, for twenty years back without finding any reference to the phenomenon whatsoever. Apparently the phenomenon, although described by Kuhlmann twenty-eight years ago, as Dr. Gross tells us, was not recognized in this country until the investigation described in the present paper was made. It is strange indeed that a phenomenon of such magnitude should remain unrecognized so long. Since publication of the paper, several operating engineers have reported cases of long duration inrush currents which have puzzled them in the past but which are now seen to be explained by the effect described. No doubt many other cases have been unreported or perhaps ascribed to other causes because the real cause was not understood. It would certainly seem true that in a great many instances when a transformer is magnetized there is another transformer already connected and carrying the load, whose presence may have had more to do with the inrush current tripping experience than the design of the transformers themselves. It would be interesting if other operating company engineers who have experienced inrush trouble would review the circumstances surrounding each installation and report those instances in which inrush tripping was experienced when other transformers were connected in parallel with the one being switched.

The case described by Mr. E. L. Michelson in his discussion appears to be one in which inrush currents of increased magnitude are set up when parallel transformers are switched on a line whose resistance is quite low compared with the transformer winding resistances. I am inclined to agree that the low line resistance would prevent the phenomenon as described in the paper from being set up in any appreciable magnitude, but to analyze the case further more complete data are needed.

Mr. W. R. Brownlee's discussion relates chiefly to the harmonic restraint relay rather than to the inrush phenomenon which is the main subject of the paper. I heartily concur in his opinion that devices employing new principles of operation should be thoroughly tested before being applied. The harmonic restraint relay has been so tested. It is rarely if ever possible, however, to anticipate completely all the special conditions which a new device will encounter in service. Sometimes it is a long time before these conditions are discovered. The subject of the present paper is a case in point. The percentage-differential induction relay is an old device, yet in all the years it has been applied no one of all the thousands of engineers and testmen intimately concerned with its use, employed in all the operating companies of the United States as well as in the manufacturing companies, ever before found out why it would trip more frequently on inrush current when another transformer was present in parallel with the one being switched.

With regard to the question concerning the possibility of arcs under the oil in the case of a partial winding fault so distorting the wave form as to reduce the sensitivity of the relay, it should be pointed out that the relay has a very nearly constant per cent harmonic slope characteristic right down to pick-up; that is, a current having a little less than 25 per cent third harmonic content (or the equivalent in other harmonics) and of a magnitude just above the pick-up setting of the relay will cause the relay to operate as readily as a current of sine wave form having the same magnitude. In general, partial winding faults introduce so much linear equivalent leakage reactance and linear resistance into the circuit in comparison with the nonlinear resistance of the short arc path that the current wave form is not distorted to the degree required to restrain relay operation.

The reference to possible current wave distortion produced by "the presence of iron in a high reactance path between portions of transformer windings" is difficult to comment on because it is not clear what is meant. If a high *reluctance* path is meant as, for example, the presence of the steel tank of the transformer in the stray flux field of a faulted winding section, it seems obvious that no appreciable distortion would be introduced due to this cause since most of the reluctance of the leakage flux path is through oil or air rather than through the iron. The flux density in the core itself will not increase, of course, due to the fault, since the applied voltage does not increase.

Mr. W. K. Sonneman in his discussion points out what appears to him to be an error in figure 5 of the paper. Actually, no error exists although perhaps the labelling of one of the curves is a bit confusing if one does not read the text of the paper too carefully. The curve labelled: "current in the line," shows the transient component of current which flows in the transmission line and divides between the two transformers, half flowing in one and half in the other if the two are identical. The curve is plotted at the half amplitude value at which the current flows in the transformers so that it can be directly added to the other transient component shown by the curve labelled: "circulating current" to give the current in the switched transformer T_2 as shown by the curve so labelled and subtracted from it to give the current in the parallel transformer T_1 also shown by a curve with the appropriate label. This is all explained in the appendix in the section headed "Case of Two Identical Transformers."

I have no objection to Mr. Sonneman using differential equations rather than a word description to explain the setting up of the transient currents if he prefers. However, if this is done a complete set of simultaneous differential equations, including the resistances and inductances of the line and of transformer T_1 as well as those of T_2, should be used. Otherwise the whole point of the control of the phenomenon exerted by the line resistance is entirely missed.

The derivation of the formulas in the appendix was omitted to conserve space, but I will be glad to send a copy of it to anyone interested. Since the derivation is the result of a rather straightforward application of the usual methods of the operational calculus it was felt that it was of little general interest. The correctness of the final equations may be readily checked by anyone by the simple process of substituting in limiting conditions; that is, by letting some of the resistances or inductances become zero or infinite in various combinations.

Mr. A. E. Flink in his discussion relates that in the tests at the Ohio Edison Co. Wood St. substation one of the transformers was found to have an exciting current in excess of six per cent of its normal full load current. This is explained by the fact that this exciting current was measured with 13 per cent overvoltage on the transformer and that no correction was made in the readings for this overvoltage condition. The overvoltage at which the transformers were being operated also explains the variation in exciting current measured on the different units since the exciting currents were measured at varying amounts of overvoltage. The transformers were being continuously operated with a high side line voltage in the neighborhood of 23 kv applied to winding taps rated at 19.82 kv. It is conceivable that at the time the three relay operations were reported without switching at the station the voltage may have for a short period increased to as much as 25 kv. This is equivalent to 26 per cent overvoltage on the 19.82 kv tap. With this degree of overvoltage the steady-state exciting current would increase to a magnitude sufficient to cause a current greater than the relay pick-up current to flow in the differential circuit. The third harmonic present in the exciting current would be cancelled in one phase due to the open-delta connection of the transformers and the relay in that phase would operate. The remedy for this condition would be to use a less sensitive pick-up setting of the relay or, if possible, to operate the transformer on a tap rated nearer to normal line voltage. Faults in the equipment of some industrial customer causing overcurrent to be carried through the protected transformers is not a plausible cause of the operations. In addition to harmonic restraint the relays are provided with the usual through current restraint of a percentage-differential relay. This adequately prevents operation on through faults as was definitely proved at the Wood St. station itself when a number of through faults occurred while the relays were in service.

It seems scarcely fair or logical to blame a new device for the existence of a phenomenon simply because the phenomenon was discovered by its use. Neither does it seem logical to hesitate to use a new device which overcomes the serious shortcomings of its predecessors on the grounds that it may possibly have shortcomings of its own as yet undiscovered. The conditions with regard to inrush currents at the Wood St. station are admittedly severe but during the course of the tests no evidence was found to indicate that the relays would not be thoroughly reliable when applied with a separate set of relays for each of the two transformer banks. The induction type relays previously applied, on the other hand, were constantly tripping on inrush even though they were set for a high current pick-up and the maximum time lever setting.

The "Electrogear"—a New Electromechanical Vehicle Drive

Discussion and author's closure of paper 41-28 by Ernst Weber, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (July section), pages 770–7.

F. M. Turner (nonmember, General Electric Company, Erie, Pa.): Dr. Weber's paper is highly interesting, inasmuch as it is indicative of the considerable amount of thought and effort that has been devoted during the past decade to the development of a continuous torque, fully automatic power transmission for use on large urban busses and other vehicles. Because we have been actively engaged for a number of years in the development of electric drive for busses, Dr. Weber's comparison of the Electrogear with the conventional electric drive is of the utmost interest to us.

We have accordingly made some preliminary calculations to compare an adequate Electrogear transmission with a conventional electric drive. This comparison indicates that the electrical machines of the Electrogear might weigh as little as 75 to 80 per cent of the combined weight of the motor and generator of a conventional electric drive. This comparison excludes the weight of the planetary gearing, which, of course, must be added to that of the electrical machines to obtain the total weight of the Electrogear. Cost of such an Electrogear, manufactured by comparable methods, might be about equal to that of the electric drive.

At first glance, the Electrogear might be expected to weigh considerably less than the conventional electric transmission. There are several logical reasons, however, why there is so little difference.

It is pointed out that with the Electrogear only part of the power is handled electrically while a large part flows directly through gearing from the engine to the drive shaft. The optimum point is when the reducer is standing still and all the power is being transmitted through the gearing. However, at this point the reducer must provide sufficient reactive torque to hold the planet spider at standstill, and it requires a considerable current flow through the reducer to develop this stall torque. It is well to keep in mind that in electrical apparatus the size of machine is closely related to the torque which it must develop and that heating of the machine will occur whenever it is maintaining torque, even though there is no power output involved, as at standstill.

It will be noted that during the course of an acceleration, the booster torque does pass through zero, but in this case another condition exists which is unfavorable to electrical apparatus. At this point the field of the booster is zero, which accounts for the zero torque, but it is necessary that a large amount of current flow through the booster armature in order to maintain the torque of the reducer, since the booster and reducer are connected in series. Thus, although the torque of the booster passes through zero, heating still continues due to the continuing flow of current through the machine.

Thus, the electrical machines of the Electrogear must not only contribute some share of the total torque multiplication obtained, but must also provide the reactive torque, or the fulcrum, if you will, for the torque multiplication that is obtained through action of the planetary gearing. Therefore, it may be deduced that the Electrogear fundamentally would have to be approximately the same size and weight as a conventional electrical transmission to handle the same torque conversion at the same rotative speeds. The corollary to this is that the principal contribution toward any weight reduction which might be possible with the Electrogear design is due to the higher rotative speeds of the reducer unit driving through the planetary gearing, which permits some reduction in the physical size of this portion of the equipment. The development of standard rear axles having correspondingly higher gear reduction would allow equivalent reductions in the weight of the conventional generator-motor transmission by permitting the electrical apparatus to be designed for operation at higher rotative speeds.

Compared with the conventional electric transmission, we believe that the ultimately successful Electrogear will offer little, if any, advantage in weight, cost, or efficiency. The feature of electric braking can be incorporated in the conventional electric drive, and, in fact, this has been done in a number of recent applications. The Electrogear has the disadvantage of requiring a mechanical connection between the engine and the rear axle, which limits the flexibility of the power plant mounting to isolate vibration and noise.

Dr. Weber has stated well the five principal requirements of an ideal power transmission. However, durability and low maintenance are not demonstrated satisfactorily by a comparatively few test operations of a few experimental vehicles. In these characteristics, no other fully automatic transmission has even remotely approached the record made by the conventional electric drive in approximately one billion miles of revenue operation.

In closing this discussion, we should like to register one small observation relative to the speed-time curves shown in figure 7 of Dr. Weber's paper, particularly to the curve indicating performance of the bus equipped with the manually-shifted, step-gear transmission. This curve indicates no loss of momentum nor time allowed for shifting from one gear to another. It has been repeatedly proven that the performance of a bus with a step-gear transmission is no better, even under optimum testing conditions with a highly skilled driver, than the performance of a similar bus equipped with electric drive; and further, that under ordinary service conditions, the electric drive bus enjoys a margin of superiority over the mechanical drive bus. In fact, this statement is confirmed in the text of Dr. Weber's paper.

Albert A. Nims (Newark College of Engineering, Newark, N. J.): The equipment described in Dr. Weber's paper is of especial interest to the writer because it confirms conclusions reached by him some twenty years or more ago. At that time experience with electric vehicle drives and a consideration of steps to improve them pointed to a combination of a motor, a generator, and a planetary gear as the most hopeful means of providing the desired flexibility, with a minimum of power translated back and forth between electrical and mechanical forms. It is hoped that, in solving so happily the problems of flexibility, power control, weight and fuel economy, the mechanical difficulties usually attendant upon the use of a planetary gear have been as effectively overcome.

It is noteworthy that both the motor and generator involved have series excitation instead of shunt. Series motors, of course, have practically a monopoly in the traction field, but series generators are a bit unusual. When it is realized that the motor and generator functions interchange, a reason for the series excitation of both machines is apparent.

The use of a small generator, driven by the final output shaft, to stabilize and regulate the speed-torque relationship is also worthy of note. The possibility presents itself, however, of separately exciting this regulator from the vehicle battery instead of by a series field; would the effect on the performance be satisfactory, and sufficiently desirable, to warrant the extra drain on the vehicle battery? The low voltage and high current rating of the regulator suggests the speculation as to whether this might be a possible application for a homopolar generator.

It may be of interest to recall one or two other equipments which were developed prior to the gas-electric drives for present day busses. One of these was covered by the Entz patents and featured the Owen magnetic automobile which had a brief vogue some 20 or 25 years ago. As in the subject of today's paper, there were two electric machines, arranged so that only part of the engine output was converted into electrical power. Both armatures were rigidly connected to the propeller shaft. The field structure of the motor unit was mounted upon the car frame. The field structure of the generator served as the engine fly wheel. The generator speed was the slip between the crank and propeller shafts and the motor armature was permanently connected to the generator armature. Speed control was obtained through the engine throttle, and regulation of the field strengths of motor and generator gave the equivalent of a flexible gear ratio. High initial cost and weight, inadequate torque range, and high operating costs militated decisively against a drive which for those days was exceedingly flexible and easy to handle.

Some thirty years ago Messrs. Macfarlane and Burge described before the Institution of Electrical Engineers a drive which was developed by Crompton and Company for battery-driven busses. The heart of this system was the CMB auto-converter, a four-pole d-c machine with a ring-wound armature and four brushes. One pair of field poles was magnetically independent of the other, so that the voltage of one half of the armature could be increased or decreased or even reversed without affecting the voltage of the other half. One half of the armature was connected across the battery, while the other was in series with the driving motor. By a foot controlled rheostat the voltage of the series portion of the armature was made to buck or boost the battery voltage and so control the speed.

The same controller could also be connected to a constant speed engine and the field excitation of the parallel half of the armature so compounded that, in starting, or in slow speed, heavy torque driving, its voltage was lowered, and the battery discharged; then when a higher speed was attained, the voltage of this part of the armature increased and the battery was charged.

It was claimed that, with the frequent and repeated cycles of charge and discharge thus associated with every stop and start or every hollow in the route, a comparatively small battery was of sufficient aid to a comparatively light engine to give very satisfactory and economical performance. In comparison with the Electrogear, this arrangement suffered from the handicap that all the driving power was translated from mechanical power to electrical and back.

From the description in the present paper it appears that the electrical equipment of the vehicle in question consists of the three machines of the Electrogear, the starting and lighting battery, its generator, and the engine starting motor. Quite a multiple unit plant. Since only part of the driving power is handled electrically in the Electrogear, it would seem profitable to consider the possibility of eliminating the charging generator and starting motor and incorporating a somewhat larger battery into the driving unit in such a way as to get frequent charging and discharging cycles. This mav result in some simplification of the complete power plant, a slight increase in the peak output with a given engine, and the automatic assurance of proper polarity of all circuits at starting.

E. Weber: The comments by Mr. Turner on the Electrogear try to prove theoretically that the Electrogear transmission should be essentially as heavy, inefficient, and uneconomical as the gas-electric drive, notwithstanding the fact that the actual performance data presented in the paper, and corroborated during the oral discussion, demonstrate beyond any doubt the outstanding superiority of the Electrogear on all these points, namely, at least 38.5 per cent lighter weight, 15 per cent better efficiency, and 22 per cent better fuel economy. The only conclusion that one can rightfully draw is that Mr. Turner's theoretical discussion completely fails to understand the principle and function of the Electrogear in spite of ample opportunity to inform himself more accurately of actual facts and test results; indeed, the following will serve to bring out the obvious confusion and inconsistency in Mr. Turner's remarks.

Mr. Turner states that the electrical machines of the Electrogear might weigh about 75 to 80 per cent of those used in the conventional electric drive. Against this stands the fact that the comparative weights accepted by mutual agreement between the J. G. Brill Company, the General Electric Company and ourselves were 18,000 pounds for the mechanically, 18,500 pounds for the Electrogear, and 19,200 pounds for the electrically equipped bus. (See the paper.) This, by mutual agreement, defines the Electrogear as 500 pounds heavier than the mechanical transmission, but also defines the gas-electric transmission as 1,200 pounds heavier than the mechanical transmission.

The actual fact is that the complete Electrogear installation in the test bus weighed only 61.5 per cent of the gas-electric drive as admitted by Mr. Turner on another occasion. But even this would not be the most economical weight for the Electrogear since as stated in my paper, the design specifications were not followed closely, and the electrical machines could have been built about 10 per cent lighter, as admitted by Mr. E. R. Martin of the Delco Products Division of General Motors, the company which built the electrical machines for the bus Electrogear. It is, therefore, absurd to suppose that the cost of manufacturing the Electrogear should be equal to that of the gas-electric drive.

The deductions with respect to weights and sizes which Mr. Turner makes based upon torque and heating conditions do not apply to the Electrogear, as design data would clearly show. For the purpose of this discussion, however, it is sufficient to say that electrical machines have an unmistakable way of expressing themselves, if they are not built to stand the loads—they break down. As stated in the paper, under no condition of testing, even under most severe schedules of 13 stops per mile, when the engine overheated and the brakes were smoking hot, did the Electrogear machines ever refuse to give the accelerative torques required, and their temperatures always remained far below permissible values as standardized by AIEE.

Mr. Turner's claim that the lighter weight of the Electrogear machines is due to their running at higher speeds than the gas-electric drive is disproved by the General Electric Company's own data which were plotted in figure 8 and clearly show that the engine speed of the gas-electric drive is continuously higher than the actually observed speed of the Electrogear.

The lower cost and weight of the Electrogear must in themselves offer considerable advantages over the gas-electric drive. As to fuel economy: The paper presented by Mr. R. H. Stier of the Philadelphia Rapid Transit Company and published in AIEE Transactions, volume 57, 1938 (June section), page 318 and in the *General Electric Review*, volume 41, page 229, favors the gas-electric drive and admits that its fuel consumption determined by their tests indicated an increase of 22.9 per cent at four stops per mile and of 23.1 per cent at eight stops per mile over the mechanical drive. Since the Electrogear tests showed better fuel economy than the mechanical drive for four or more stops per mile, its advantage over the gas-electric drive with respect to fuel economy is self-evident. As to overall efficiency: Actual tests show the Electrogear efficiency to be over 90 per cent which is about the same as the mechanical drive efficiency, and certainly much higher than the efficiency of the gas-electric drive could ever be.

The all-around advantages of the Electrogear over the gas-electric drive were brought out during the oral discussion relating the experience with the Electrogear in the ACF bus.

The mechanical connection between the engine and the rear axle is not a disadvantage, in fact, it is desirable as it avoids undesirable free wheeling. The isolation of vibration and noise is taken care of in the same manner as in the gas-electric drive, where a solution had to be found with respect to coupling the generator to the engine. There is no doubt that the gas-electric drive has an impressive record with respect to maintenance and durability, but it is only natural that one should expect the same type of record for the Electrogear, since the same principal elements are involved, namely electrical machines.

Finally, the full throttle acceleration curve for the bus with mechanical transmission as given in figure 8 is the curve submitted by the J. G. Brill Company and represents their actual test results admittedly obtained under optimum conditions. It is not my opinion that one should choose performance curves of a rival device obtained under unfavorable conditions in order to gain additional advantage. That under ordinary service conditions the schedule speed of the gas-electric drive is higher than that of the mechanical transmission has been stated in my paper and is readily admitted by proponents of the mechanical transmission, in fact this is one of the main reasons why the interest has centered upon automatic transmissions in recent years.

Professor Nims' discussion is singularly interesting since on the basis of theoretical reasoning he came to the logical conclusion that the prohibitive weight, high cost, and low efficiency of the gas-electric drive can only be reduced by reducing the amount of circulating electric power through the use of a planetary gear, which is exactly the principle embodied in the Electrogear. The possibility of eliminating the charging generator and starting motor has been considered in application to the Electrogear, in fact in smaller models of the Electrogear the starting motor has actually been eliminated with satisfactory results. This, however, is a matter of economic consideration.

System Lower-Harmonic Voltages—Methods of Calculation and Control by Capacitors

Discussion of paper 41-74 by W. C. Feaster and E. L. Harder, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941, pages 1060–6.

A. C. Monteith (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The authors are to be congratulated on the completeness with which they have analyzed this complicated system problem. The problem is not new, but to the best of my knowledge this is the first attempt at an approach by other than more or less cut and dry methods, rounded out with confirming field tests. There are two points that might bear emphasis, one the use of the a-c network calculator and the other the new data on harmonic characteristics of the magnetizing currents of transformers.

It has been of interest to watch how new uses for the a-c network calculator have been devised beyond the original conception of studies of regulation, system stability, and short circuit currents. It has been adapted for use in transient analysis and now in this paper, for the analysis of fifth harmonic conditions on systems. This method of attack should be useful for the solution of the problem on other systems.

The authors have also presented some new data on the magnetizing characteristics of transformers as they are applied on systems. Considerable data have been published on the magnitude of the different harmonics in exciting current but this is not consistent, there being a range of 10:1 in the data. These inconsistencies may be due to a difference in the way the iron is worked and also to the fact that a single phase test has been made on the actual system, delta's are present that act as a short circuit to the third harmonic component. The authors' data should be of use in connection with this problem as it has been carefully taken in the laboratory for stated conditions that are believed average for actual systems. Their development of equivalent circuits should also be helpful in solving specific cases.

C. T. Pearce (Westinghouse Electric and Manufacturing Company, Philadelphia, Pa.): This study has been of special interest to me in view of participation in one of the cases of local resonance to harmonics coming from the main system. This was one of the instances of capacitor heating which led to the general study. It would be of interest to know whether the authors feel that the methods of calculation developed give equally good results on other systems.

S. C. Miller (The Chesapeake and Potomac Telephone Company of Baltimore City, Baltimore, Md.): The novel arrangement described in the paper by Messrs. Feaster and Harder will naturally be of interest to engineers concerned with the inductive coordination of supply and telephone systems at noise frequencies. It is, therefore, the purpose of this discussion to present the available facts relating to the coordination aspects of the capacitor installation at Winchester, Virginia.

It is evident from the information in the paper under discussion that the capacitor-reactor arrangement is, in reality, a 300-cycle resonant shunt across each phase at the Winchester 33-kv bus and is designed so as not to affect the system impedance to residuals. Such a shunt would, of course, be expected to reduce the fifth harmonic voltage on the 33-kv circuit at Winchester and at other points when applied to the bus as compared to conditions that obtain when the capacitor is not energized. It was to be expected that the application of the capacitor would cause a considerable increase in the fifth harmonic current in one or the other or both of the 33-kv circuits which supply Winchester. In general, the line impedance drop of this current would tend to hold up the 300-cycle voltage somewhat at points remote from Winchester; hence the most pronounced changes in fifth harmonic voltage and current on the 33-kv system would be expected in the neighborhood of the capacitor, except in situations where the effects of phase change are important at 300 cycles.

The 300-cycle voltage on the 4-kv bus at Winchester would, of course, either increase or decrease when the capacitor is applied depending, among other things, upon the relative magnitudes of the reactance of the tuning reactor and the transformer leakage reactance.

With the foregoing considerations in mind, the interested power and telephone engineers decided to make a few observations on a cooperative basis to investigate the effects produced by the installation on telephone noise metallic in some of the exposures which might be affected. It was decided to make these observations during the daylight hours of a normal week day because experience has shown that daytime wave shape conditions on this power system are fairly indicative of effects to be expected during the hours which are most important from the standpoint of telephone traffic.

In order to obtain data on the effects produced in the vicinity of the capacitor noise measurements were made on the open-wire telephone toll circuits in the exposures to the 33-kv circuits between Winchester and Martinsburg, West Virginia, and between the latter point and Hagerstown, Maryland, with the capacitor energized and deenergized. Observations were also made at the Winchester central office on local telephone circuits exposed to the four-kilovolt system with which the capacitor is associated and simultaneous measurements were made of harmonic voltages and currents in various supply circuits at the Winchester substation.

It was found that the 300-cycle voltage at the Winchester 33-kv bus dropped nearly 70 per cent, while the 300-cycle current in the Martinsburg-Winchester 33-kv circuit increased by a ratio of about 3 to 1 when the capacitor was applied. The 300-cycle current in the Stephens City-Winchester 33-kv circuit decreased by a ratio of almost 10 to 1. The phase-to-neutral 300-cycle voltage on the four-kilovolt bus at Winchester increased by about 35 per cent. It should be kept in mind that these data were obtained during the day and are, therefore, somewhat different from comparable measurements referred to in the paper which were made during the night.

Despite these relatively large percentage changes in 300-cycle magnitudes in the power system no important changes were observed in the noise metallic on the telephone circuits tested although a few minor changes were noted in the magnitudes of some of the frequency components which comprise the noise. This phenomenon can be attributed to the fact that the 300-cycle components of the noise were relatively unimportant in the presence of controlling magnitudes of higher frequency components which were not materially affected by the capacitor in this instance under power system conditions which obtained at the time the observations were made. Some effects on telephone noise would have no doubt been observed had the higher frequencies not been present, but the indications are that such effects would not have been important in this case.

It appears that, in future installations of the type described in the paper, consideration should be given to the usual possibilities of undesirable inductive effects presented by any primary capacitor installation, but it should be kept in mind that such devices may be quite helpful in reducing telephone noise over fairly wide areas in situations where induction from fifth or seventh harmonics is important and no serious adverse effects are produced in exposures near the capacitor.

R. D. Evans (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): I wish to comment briefly on the factors which affect the fifth harmonic voltages which may appear on systems. The field form of generators and the operation of six-phase rectifiers may produce fifth harmonic voltages of appreciable magnitude. These sources were unimportant in the case investigated by the authors who found that the cause was transformer exciting current. Yet all systems have transformers but the harmonic voltages vary through quite a range of magnitude. There are a number of factors which may contribute to this. (1) The exciting current magnitudes vary widely, particularly in relation to system constants. (2) On some systems, certain transformers may be operated at considerable overvoltage, particularly under some load conditions. (3) Resonant conditions on the system may increase or decrease the harmonic voltages in the external circuit. (4) The phase positions of the harmonic sources in the transformer may vary considerably in respect to each other since they are controlled by fundamental frequency voltages which are displaced by different amounts in different parts of the circuit and under different load conditions. (5) The actual transformer connections may have an important effect. For example, if a delta-star transformer is replaced by two equal transformers, one connected delta-star and the other star-delta, the combination will substantially suppress all of the fifth and seventh harmonics. While such a method is not advocated, it should be recognized that such connections do affect the magnitudes of harmonics, especially on systems.

The accepted method of determining the distribution of harmonic exciting currents and voltages in the external circuit of a transformer, is by the use of a familiar equivalent circuit of the transformer with the exciting branch replaced by a harmonic source and a harmonic impedance. This arrangement proposed by Dr. H. S. Osborne in 1915 appears to work well for the third harmonic. The present authors have shown a method for accurately determining the fifth harmonic voltages. For appreciable higher harmonics the use of a simple but accurate equivalent circuit does not appear possible, a circumstance which may be due to the interaction of the important lower-harmonic exciting currents.

My last point deals with the effect of capacitors on harmonics. A capacitor does not, of course, generate harmonics but if harmonics are present in the source, the connection of a capacitor to the circuit may decrease or increase them. If a long feeder is supplied from a source with harmonics and if a capacitor is added at the supply end, the harmonic voltages may be decreased by a filtering action, or if the capacitor is added at the far end, the harmonic voltages may be increased by a resonant action. On the system investigated by the authors the addition of capacitor kilovolt amperes was accompanied at first by increases in harmonic voltages. This raised a question as to the effect of adding a large capacitor bank to the system. The analytical studies showed that the effect would be to reduce the harmonic voltages. Confirmation of this result by installation and test is most reassuring in connection with capacitor applications. The method developed by the authors using analytical work, simple field tests, and the a-c network calculator will have application to other systems.

Television Transmission Over Wire Lines

Discussion of paper 41-42 by M. E. Strieby and J. F. Wentz, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941, pages 1090–6.

A. P. Godsho (Bell Telephone Company of Pennsylvania, Philadelphia, Pa.): A striking feature of the high frequency transmission art in the paper by Mr. Strieby and Mr. Wentz is the development of the technique for its application to the common everyday variety of telephone plant—the paper insulated cable. The significance of this feature cannot be overemphasized, for the cable plant reaches everywhere television programs are likely to originate and of equal importance where they may be received. Fundamentally, it would appear, therefore, that the framework of a basic television program pick-up and distribution system is already available.

The past decade has seen cables originally designed to carry voice frequencies adapted to transmit frequencies up to 60,000 cycles and now we have this new art making the transmission of frequencies up to 3,000,000–4,000,000 cycles practical over the same fundamental type of plant. Presumably, given enough time, opportunity, and justification, this band may be further extended.

Just as the analytic approach to developments of this type is taken for granted, the Strieby-Wentz paper indicates that methods of measurement have had to be devised simultaneously with the main objective. To those of us who have had any experience in the conditioning of plant for television transmission, involving such items as the elimination of bridged taps, obtaining proper signal-to-noise ratios at various locations along the transmission path, etc., this is a familiar story.

To introduce a speculative aspect into the Strieby-Wentz presentation, perhaps a day will soon be here where cables to many locations, at least in the metropolitan centers, will be provided with a small number of pairs which will be free of all bridges, with intermediate amplifiers in strategic locations, so that the pick-up of television programs will be the commonplace rather than the unusual thing. Technically, the present paper reveals that much progress has been made towards this objective. Of course, there are many other features, mostly of an economic nature, which still require formulation and development.

Volt-Time Areas of Impulse Spark-Over

Discussion and author's closure of paper 41-75 by J. H. Hagenguth, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (July section), pages 803–10.

V. M. Montsinger (General Electric Company, Pittsfield, Mass.): Mr. Hagenguth's paper contributes one more important step in the coordination of laboratory testing technique. For several years there has been a large discrepancy between the different laboratories on front of wave flashover of a simple rod gap, and we, who build transformers that occasionally have to be tested on the front of wave basis using the rod gap as a voltmeter, were rather discouraged because no satisfactory explanation for the wide variation in the flashover values could be given. Now that we have the explanation, it should be possible to drop the rod gap as a voltage measuring device and make front of wave tests by kv—the same as is being done with long wave tests.

The second point I wish to bring out is the unreliability of the rod gap, as brought out in the data in the paper, for coordination or protective purposes. The rod gap not only has a faster turn up for short times to flashover than insulation but, its wide variation in flashover voltage for any given time less than approximately two microseconds makes it almost useless as a practical protective device unless set so low as to be prohibitive from outage standpoint. For example: the 20-inch and 50-inch rod gaps show from 40 to 45 per cent variation in flashover at two microseconds as compared with 20 per cent variation at 8 microseconds (figure 7). Plus or minus 20 to 22 per cent flashover, coupled with the larger turn up of the gap at two microseconds than for insulation, explains why it is necessary to use such low spacings—as low as 50 per cent of the spacing for long waves—to give any protection to transformers against steep waves.

P. L. Bellaschi (Westinghouse Electric and Manufacturing Company, Sharon, Pa.): This paper calls attention again to the question on wave form and to definitions, to the significance of wave form in impulse testing and the problem on insulation coordination. While the subject at large lends itself to theoretical treatment, the difficulty lies in translating the larger problem into practical and simple form that is both effective and feasible in engineering application.

An item of great significance is the question of definitions. I wish to confine this discussion largely to this question for it provides the constructive approach to the larger problem. The following example will illustrate the point.

Up to the time impulse testing and insulation coordination were confined to critical flashover and relatively long chopped waves (1.5x40 microsecond wave), the front of the wave remained of secondary importance. Therefore the definitions established for the front were quite adequate. As front-of-wave testing made headway, it became clear to the writer and to others that a better and more complete definition of the front was essential, if large apparent variations at the same laboratory and at different laboratories were to be reduced to acceptable limits. Theoretically, the complete shape of the wave defines the wave best, but obviously this method does not lend itself to the necessary simplicity required in engineering.

Two important steps were considered. First the 30 per cent–90 per cent method of defining the front proper was proposed, as inherently by this method the definition assumes greater physical significance and furthermore variations present at the foot of the wave (below 30 per cent) would automatically be excluded. Second, to define the wave beyond the 90 per cent point, it was proposed that the ratio of the virtual front to the effective front (virtual zero to point of breakdown) should not exceed 0.80. Rather infrequently are ratios of 0.90 or 0.95 exceeded in actual testing. Thus a rather simple definition was evolved and formulated which practically is significant in the engineering sense, and which should eliminate much of the difficulties encountered in the past.

Other distinct benefits can acrue from establishing suitable and practical definitions. For instance, in the case cited, large variations in results could be narrowed down to the point where it became clear that measurement was not the decisive factor.

It would take the length of another paper to adequately discuss the pros and cons of the various points to which Mr. Hagenguth calls attention in connection with insulation coordination. Surely the question discussed above on the definition of the wave form is an important related consideration.

P. H. McAuley (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): Mr. Hagenguth has presented a detailed quantitative analysis of the possible variations in volt-time curves due to changes in impulse voltage wave shapes. To this extent it appears that a concept which was much discussed qualitatively in the early days of impulse testing with the oscillograph has been revived. Attention is called to the following articles: "Surge Testing of Suspension Insulators," F. D. Fielder, *Electric Journal*, July 1931; "Time Lag of Breakdown," by Fielder and McAuley, *Electrical World*, November 23, 1929; "Time Lag of Insulators," by Wade and Smith, *Electrical World*, August 18, 1928. Fielder in the 1931 article stated: "Naturally, with variations in both rate of rise and rate of fall, an infinite number of wave combinations are possible, making it necessary to use some arbitrary standard in the laboratory. A review of the other articles will indicate that these authors knew and appreciated the effect of changing wave shapes many years ago.

With full appreciation of the situation, the standardizing groups adopted the 1.5x40 wave, thereby reducing the flashover band to a curve. It has been possible to forget until recently, the changes in effective wave front with change in time to flashover. The change in circuit arrangements of the General Electric surge generators has revived the problem. As long as all laboratories were using similar waves and testing circuits, fairly good agreement in test values were obtained. The change in technique of some laboratories brought the problem forcibly to the attention of the laboratory subcommittee of the EEI-NEMA joint committee on coordination of insulation in 1938 and it has been studied by this subcommittee in connection with the correlation of laboratory data. Mr. Hagenguth advocates investigation of the complete band of impulse sparkover, which greatly complicates testing and application. Undoubtedly, it would be much simpler to standardize testing circuits or to define the test wave more rigidly. This would appear to be a reasonable and practical simplification of the problem.

In the early days of insulation coordination, it was considered desirable and necessary to grade the apparatus insulation in the station in steps of the order of five per cent. In this way, any flashover which would occur would take place at points where the least damage would be caused. It was soon found that due to the many steps and to testing tolerances required, the range in insulation strength from the incoming lines to the internal insulation of apparatus was so great as to be impractical. Insulation coordination now means that all the apparatus of the station is above a certain level of strength which is, of course, above transient values permitted by the protective apparatus. Insulation levels are based on tests with the 1.5x40 wave. Most of these tests involve only the critical flashover or withstand values for equipment. Actually, the critical flashover voltage, as used in the past, might be regarded as a band rather than a point or line, since it is usually obtained by averaging several test points. However, it has not been found necessary to work with the maximum and minimum regions of this band for the purpose of coordinating insulation. In view of the satisfactory experience on this point, Mr. Hagenguth's statement that this type of coordination "cannot be effective" seems very pessimistic. It is always desirable to keep testing procedures as simple and inexpensive as possible and any attempts to explore the complete band of values possible with different types of impulse waves on different apparatus seem somewhat complicated and impractical. Furthermore, it is possible that other factors such as lead lengths under conditions of installation may have a greater influence on the performance of gaps and insulation in parallel for very short times to flashover than moderate changes in the testing wave. This possibility was discussed briefly in the *Electric Journal* for February 1939, page 56.

It seems apparent that difficulties will be encountered in checking values from different laboratories as long as it is possible for different groups to make different interpretations of the definition and procedure for using the 1.5x40 wave. Therefore, either the definition and procedure must be tied down by further standardization or a wave which cannot be varied by different interpretations must be used. This step has been suggested, first, I believe, by Ohio Brass engineers. If a one microsecond front were used instead of a 1.5, there should be less change in front for different parts of the volt-time curve. This would tend to reduce values obtained with surge generators with the shunt capacitor and to raise, slightly values obtained on generators with only series resistance. At least, this is one procedure which shows much promise at present and it seems worthy of a trial. In addition, it has the merit of agreeing more closely with the International standard wave which is 1x50 microseconds.

J. J. Clark (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): As Mr. Hagenguth points out in his paper an accurate knowledge of the impulse strength of all apparatus on a system is necessary for proper coordination. For apparatus involving solid insulation only the withstand strength test is practical, since a breakdown destroys the insulation. Apparatus which uses air as a dielectric, such as bushings, insulators and rod gaps, all have electrode arrangements giving a nonuniform field.

Comparing the limiting values for negative waves as given in figures 4, 8, 9, and 10 of the paper shows the following spreads between the $T_1/T_2=1$ and $T_1/T_2=0$ curves at 2 microseconds. For the 20-inch rod gap the spread is 47 per cent, for three suspension insulators 41 per cent, for two apparatus insulators 28 per cent, and for a bushing 32 per cent. Since these limits are not actual test values but extrapolated from data obtained at other ratios, the agreement is remarkably close. This would indicate that the change in sparkover voltage with change in wave shape is not greatly different for the electrode arrangements in different apparatus. Hence, volt-time curves taken with the same wave are, for practical purposes, directly comparable even though the effective front does not remain constant for all times to sparkover.

Either of the impulse generator circuits of figure 12 of the paper can be used to obtain volt-time curves suitable for coordination, though the results obtained with the two circuits cannot be compared directly. Most of the generators in use at the present time use the circuit of figure 12*a* not only because of the high cost of a load capacitor but the addition of a large capacity load reduces the voltage which can be supplied by a generator of given rating.

To secure agreement between different laboratories either the testing circuit can be standardized and the wave front allowed to shorten as sparkover time decreases or the wave front can be maintained constant. The latter course would necessitate constant readjustment of the surge generator circuit and increase the time and expense of impulse testing without securing greater accuracy of coordination. In this case, it appears technical perfection might be sacrificed for practical reasons.

L. Wetherill (General Electric Company, Pittsfield, Mass.): Mr. Hagenguth's paper brings out clearly the inadequacy of volt-time curves based on the 1.5x40 microsecond wave except in cases where waves of substantially this form are being considered. Most existing impulse volt-time data covering high-voltage bushings probably are based on the 1.5x40 microsecond wave.

The volt-time characteristics of bushings are of interest in connection with the protection of the bushing from flashover, for which case the curve of $T_1/T_2=0$ is useful, or in connection with the use of the bushing as a protective device in which case the curve of $T_1/T_2=1$ is useful. Thus it would appear that future volt-time data covering bushings should be obtained not on the 1.5x40 microsecond wave but rather for the area boundaries represented by the expressions $T_1/T_2=0$ and $T_1/T_2=1$.

One result of Mr. Hagenguth's paper is that the usefulness of a bushing as a protective device can be more accurately evaluated. Another result is to show that furnishing of anything approaching adequate protection by this means is considerably more difficult and more expensive than was previously thought to be the case. This results not only from the increased margins which would be required to allow for the wide range of flashover performance represented by the volt-time areas in comparison with single curves, but also from the need for an increased amount of impulse test data to determine the volt-time performance of various types and sizes of bushings on the more complex basis of volt-time areas.

These considerations indicate that the use of a bushing as a protective device is becoming more difficult than ever to justify. Furthermore, with the bushing removed from the protective device picture, the lower boundary of its volt-time area ($T_1/T_2=0$) assumes the place of chief importance while the upper boundary becomes unimportant from the co-ordination viewpoint. Therefore, we may find that in the future most volt-time data on bushings will be taken for the condition $T_1/T_2=0$, and that data of this type will supplant the former volt-time curves taken with the 1.5x40 microsecond wave.

J. H. Hagenguth: The discussions of the paper cover two principal points:

1. The effect of volt-time areas on the co-ordination problem.
2. The problem of impulse testing.

Mr. Wetherill's conclusion that the most useful volt-time curve for a bushing will be the lower boundary of the volt-time area ($T_1/T_2=0$) is quite correct. Similar considerations apply to apparatus such as transformers where the breakdown phenomenon is initiated by corona streamers only to a minimum extent, and which represent apparatus in the transmission system which are to be protected against breakdown or flashover. On the other hand, apparatus such as lightning arresters which are used to protect costly equipment require tests which will show the upper boundary ($T_1/T_2=1$) of the volt-time area. Finally, there is a group, such as insulators, which requires the knowledge of both boundaries to fully understand the interrelation of different parts in the system as far as sparkover characteristics are concerned. Unless it is questioned that only co-ordination according to figure 13 will give complete protection, it is necessary to investigate lower volt-time area boundaries for apparatus which is being protected and upper boundaries for protective devices. Any volt-time curves which extend within the volt-time areas, such as the volt-time curve of figure 6 of the paper labeled $T_1=1.3$ microseconds = constant, are of no practical significance from the point of view of co-ordination where considerable margins are required between maximum voltage permitted by the protective device and the minimum permissible withstand voltage of the protected apparatus. It is, therefore, rather immaterial what kind of impulse generator is being used to determine the sparkover characteristics of greatest interest from the point of view of co-ordination as long as these most important volt-time curves can be obtained.

Mr. Clark discusses the question of impulse generator and arrives at the conclusion that the best type of generator is one similar to figure 12*a* of the paper for practical reasons, although its use will sacrifice technical perfection. Mr. McAuley believes that a change in wave front of the standard wave from 1.5 microseconds to 1 microsecond would provide better agreement between results obtained in different laboratories. I believe, as pointed out above, that it is much more important to definitely settle on which of the limiting volt-time curves are to be obtained for different types of apparatus and then to obtain these curves than to standardize on impulse generators or wave shapes. Mr. McAuley points out that early experience in impulse testing showed variations in sparkover voltages with wave shape. Many more references can be found showing similar experiences. However, as far as I know, it has not been possible to identify the change in sparkover voltage with simple factors expressing wave shape and, therefore, the solution to use a standard wave was chosen as the best way out. Of course, the choice of a standard wave was made principally because measurement technique in the early days of impulse testing was not so very good and, also, for comparisons between laboratories. Furthermore, the 1.5x40 wave was a good average of all lightning waves measured on transmission lines with cathode-ray oscillograph.

Mr. McAuley quite correctly states that other factors may upset the co-ordination picture such as lead lengths. I fail to see that on account of this we should close our eyes to the wide sparkover variations possible due to difference in wave shape. For best results we must take into account all the factors involved and should gradually modify our standards accordingly. I do not believe that it is pessimistic to say, as Mr. McAuley states, that co-ordination according to customary volt-time curves cannot be effective. For instance, the complete sparkover band for a 20″ rod gap at one microsecond comprises the volt-time curve points of all rod gap spacings between 16″ and 24″. Likewise, the band for a 50″ rod gap at two microseconds corresponds to rod gap spacings from 43″ to 64″. Such large spreads are entirely too great to be ignored. The reason for obtaining sparkover characteristics of apparatus is to obtain a basis for co-ordination of different types of apparatus. The function of the laboratory is to obtain the required data in such a form that they are useful for co-ordination purposes. Therefore, the different laboratories can devote their energies to determine those sparkover characteristics which are most useful for co-ordination purposes, especially since the question of measurement accuracy for times to sparkover of one microsecond and longer has practically been settled.

Mr. Clark contends that, since the volt-time areas of figures 4, 8, 9, and 10 of the paper do not deviate much from each other, single volt-time curves taken with the same wave are directly comparable, especially since apparatus involving solid insulation receives only a withstand test. I would remind Mr. Clark that a withstand test is a test which proves that a given apparatus will not break down under given conditions. The type of generator used for the withstand test, especially if this test involves the use of a rod gap for chopping the wave, is therefore, very important. It can be shown that certain types of transformer insulation would withstand a test with a 20″ rod gap, for instance, using an impulse generator of figure 12*a*, while, when using figure 12*b*, a rod gap of only 17″ spacing can be used to protect the same specimen. This is one of the reasons why Mr. Montsinger's suggestion of dropping the rod gap as a yard stick for front of wave tests on transformers is logical. In this same connection, Mr. Bellaschi's suggestion of using closer tolerances on the wave shape, allowing a minimum ratio of $T_1/T_2=0.8$, is a step in the right direction. However, even with such seemingly close tolerances, differences in sparkover potential of a given rod gap spacing of as much as 10 per cent at 0.5 microsecond and 6 per cent at one microsecond to sparkover will occur. At these short times to sparkover the question of accuracy of voltage divider also becomes more important.

In the paper discussion was limited purposely to sparkover at one microsecond and longer to eliminate the question of divider influence on the measurement.

Messrs. McAuley and Clark are concerned about practical means of impulse testing. From the point of view of practical testing, it is very difficult with present impulse generator designs to obtain either the lower or the upper boundaries of the volt-time areas directly. As shown in the paper, it would not be difficult to obtain these limits for a few voltage ratings of types of apparatus as indicated by figure 11 for the rod gap. Even though these boundaries may not be determined with very great accuracy, they would much more nearly represent the required information for co-ordination than any other single volt-time curves obtained according to present practice.

Whenever single volt-time curves are needed to demonstrate a specified strength of an apparatus in the laboratory, generator of figure 12*b* is preferable because it produces a constant effective front throughout the volt-time curve range. Only such curves are directly comparable for different types of apparatus with different electrostatic fields. Any other impulse generator will be satisfactory, provided that its discharge circuit is changed to maintain a constant effective wave front. With generator figure 12*b* no changes are necessary since the effective wave front is automatically maintained practically constant. It also seems necessary to allow smaller tolerances for the front than have been used when defining the standard wave. The present tolerances of $\pm 1/2$ microsecond for the front will result in sparkover voltage differences of 15 to 20 per cent for sparkover at two microseconds.

In answer to Mr. Clark's objection, a load capacitance which is required for generator figure 12*b* represents only a relatively small expense compared to total cost of impulse generator equipment. The use of a load capacitance keeps the effective test wave front practically constant. While the test voltage at critical sparkover is a few per cent lower for impulse generators of figure 12*b* than for 12*a*, at sparkover at two microseconds and less the available test voltage from generator 12*b* is considerably greater than that of generator 12*a*. A volt-time curve obtained with generator figure 12*b* can be used directly to obtain the upper and lower boundaries of the volt-time area since it almost diagonally cuts across the whole area (see curve T_1, figure 6). By taking a few points on either side of the curve with different wave shapes, the area is established.

Field Testing of Generator Insulation

Discussion and author's closure of paper 41-17 by the EEI subject committee on generator insulation and testing, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (December section), pages 1003–11.

R. E. Hellmund and **P. H. McAuley** (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): It is evident from the data given in the paper, as well as from similar studies, that a great deal more investigation will be required and that considerable time will elapse before it will be possible to evaluate the conditions of the complicated insulation structure of large machines by such factors as ionization, insulation resistance and power factor. Much progress has been made, however, and these tests are valuable in detecting moisture and other defects, where it is possible to detect and analyze insulation deterioration by periodic observations over long periods of time. Continued study of these methods is to be encouraged and they are already of great assistance in maintenance problems for certain types of apparatus.

The general problem is complicated by the fact that one weak spot may make a machine unfit for service but has little or no effect on many tests which must include the whole mass of the insulation of a machine. A typical case is punctured solid insulation through which a few thousand amperes of impulse current have been passed but which tests perfectly good as far as megger and similar low voltage indicators are concerned. For similar reasons, none of the non-destructive methods suggested so far furnishes a dependable indicator in advance of actual breakdown. This in turn means that dielectric overvoltage tests will have to be depended upon for quite some time. It would, therefore, seem well worth while to give considerable attention to improving this method of test.

The conventional one-minute a-c test does not necessarily indicate resistance to surge voltages. Also, on large high voltage apparatus, it may produce corona and other disturbances which result in voltages in different parts of the conductors that differ from the impressed test voltage. Inductance and capacity of the windings and harmonics in the test voltage lead to similar effects. Consequently, the possibilities of d-c testing, which are mentioned briefly in the paper, seem worthy of serious consideration. The use of a d-c test voltage with definitely limited and properly controlled rates of voltage rise and fall would eliminate many of the disadvantages of high voltage a-c testing. For test values below 100 kv the equipment using thermionic rectifiers is not unduly complicated by comparison.

D-c testing has been given a great deal of attention in connection with cables, principally because, with their high electrostatic capacity, tests with a-c voltages were found impractical where portable equipment was required. For large generators the problem is somewhat similar with the additional complications previously mentioned. In view of the greatly varying ratio of d-c to a-c breakdown which have been found to exist for different kinds of insulation structures, there is, of course, a question as to the d-c voltage to be used. This, however, should not be considered a deterrent if it is realized that the widely used a-c values have been selected entirely arbitrarily and without any theoretical background.

In considering suitable tests, especially tests for machines which have been in service, it seems worth while to strive for a type of test which more closely approximates conditions to be met in actual service than the conventional one-minute a-c test with a relatively high overvoltage. In actual practice machines are likely to be operated with a-c voltages moderately in excess of the rated voltage and, therefore, a one-minute a-c overvoltage test with a voltage 125 or possibly as high as 140 per cent of the rated voltage would be justified. In addition to the normal frequency voltage stresses, machines and apparatus not directly connected to the line and, therefore, not subject to lightning surges, are likely to be subjected to switching surges of perhaps two to four times their rated voltage. These switching surges are usually of very short duration, and the first suggestion might, therefore, be to subject the machines to the more or less standardized impulse waves of 40 to 50 microseconds duration. Such tests, however, again have the disadvantage that it will be impossible to determine the oscillations and voltage stresses in different parts of the windings. For this reason it might be of advantage to investigate the possibilities of a d-c test of one or several seconds duration, with properly limited and controlled rates of increase and decrease of the voltage. Such a test, with suitably chosen amplitude, might be effective in detecting mechanical weaknesses in the winding, which are the usual cause of breakdown in service. The reason is that, in the case of mechanical difficulties, there is usually at least one part of the breakdown path through air. Some evidence to this effect is given in the paper. If proper means for reliably indicating a breakdown during the short period of test can be devised, the main object of locating weakness in the insulation has been accomplished without stressing other parts of the winding unnecessarily and without danger of introducing unknown overvoltages in any part of the insulation structure.

All operating engineers are vitally interested in a suitable method for proving the adequacy of the insulation of machines in service, and in connection with insulation to ground or between windings, it is believed that the method suggested here will be very helpful. However, the question may arise as to how many breakdowns in service are caused primarily or initially by a breakdown to ground or between windings. Especially for class *A* insulated conductors, it would not be surprising if a large percentage of failures were originally caused by breakdown between turns resulting from mechanical deterioration of the insulation due to heating. None of the methods of insulation test suggested so far is likely to give any indication in advance of an actual breakdown, except that possibly a surge test may stress the insulation of the end turns sufficiently for this purpose.

F. C. Knowles (Evershed and Vignoles, Ltd., London, England): In attending this convention it was not my intention to take any part in the discussion, but I am pleased to respond to the request of some of your members to give my views on the practice in England.

The splendid work which has been done over here in the matter of the consideration of electrical insulation has been followed with much interest by engineers in England. From time to time we have received reports issued not only by this Institution but by other authorities on this side, dealing particularly with the question of ascertaining the condition of the insulation of electrical apparatus by non-destructive tests.

The particular aspect of the subject in which I am most interested is that which has been termed over here "dielectric absorption tests." These tests are based on the continuous application of a steady voltage over a period of time, and in that respect it reminds me of the tests which telegraph and cable engineers have carried out for over half a century and which are termed "electrification," it being common practice to specify cables for telegraph and submarine work as having a certain insulation after so many minutes' electrification, and in many specifications there is a statement of the decrement of the reading on a galvanometer for successive minutes' electrification.

I have had the opportunity of discussing this question of dielectric absorption as it may apply to the insulation in generators with a number of prominent British engineers, but have not yet found enthusiasm for the application of such a method. It is quite possible that conditions in our country are quite different from yours. For example, we do not have such extremes of temperature and humidity. It is our practice to measure the insulation of a transformer, generator, or other large electrical unit before putting it into service, noting the temperature and weather conditions and then to repeat that test when the apparatus is taken off circuit under hot conditions. The results of these insulation tests are recorded usually on a card and are compared with previous readings. If there is any considerable disparity, the matter of the insulation is then investigated.

After reading the paper which has just been presented and talking to some of the members of the committee on this subject, I shall return to England with a better appreciation of the work that you are doing and will endeavor to present it to engineers on our side in the hope that we may be able to do something to add to the good work already done.

The question of what is meant by dielectric absorption, I feel has been very ably expressed in the contribution to the discussion just made by Mr. R. J. Woodrow. Both the diagram and the formula which he gives seem to me to be a basis for the production of a reasonably simple method for the operating engineer not only to understand the basis of the tests, but also to be able to obtain an empirical formula based on the theoretical formula which should enable him to ascertain quickly whether there is any unreasonable decay or change in the insulation of the apparatus under test.

I would also like to add my appreciation of the bibliography given at the end of the paper, but would suggest that possibly a paper given before the British Institution of Electrical Engineers by J. Lindley Thompson and H. Walmsley on the drying out of transformers might be of some interest in this connection. The paper will be found in the IEE *Proceedings*, volume 64, page 505.

I would also like to add that it is the common practice in England to take spot insulation test readings during the drying-out process of cables, generators, and motors.

M. S. Oldacre (Commonwealth Edison Company, Chicago, Ill.): This paper is a report on a valiant effort that so far has brought mostly negative results. The investigation has eliminated some tests as of little or no value, but a definite indication of a really reliable non-destructive test has not yet been found.

I do not see under the heading of "conclusions" in the paper an item recommending that the work be continued to find a definite test to show the condition of the insulation. I think that it should be continued. It may be that this work could be divided into two parts: first, to determine whether the winding is free enough from moisture to permit safe operation; second, to determine whether the internal insulation is satisfactory. Probably some of the insulation resistance tests with high-voltage d-c supply and sensitive instruments for a suitable length of time will take care of the first part.

Some and most likely a fairly high proportion of failures are initiated by weakening of the internal insulation before the insulation to ground will permit failure at the usual test voltages. It has been suggested that for multi-turn coils a high-frequency voltage applied across the winding would give a better test than any now used. We do not know enough about the insulation effects under such conditions and would like to urge the generator insulation engineers to study the possibilities of such tests and help lay out a test program, particularly on old coils. Such a test might provide for the second item mentioned above.

In either case, some sort of voltage test will still be required to show up the really weak spots due to damage, poor workmanship, and deterioration of the main or ground insulation.

One manufacturer has specified in certain cases that a high-voltage d-c insulation resistance test be made for a fairly long time with the further requirement that the insulation resistance continue to increase for at least ten minutes, this test to be followed by the usual dielectric test. Some of the committee results seem to indicate the value of such tests, and this should be explored still further.

V. J. Egan (Commonwealth and Southern Corporation, Jackson, Mich.): We believe the EEI committee is performing a service of inestimable value to the electrical industry by consolidating the various methods of insulation testing and assisting in interpreting the test results. Practically all companies now make some kind of generator insulation tests but the recorded data usually have not contained enough information to allow comparison with later test results. By following a standard procedure very little additional work is required to obtain results which will be valuable to the company making the test and which will also contribute to the development of the testing routine.

Although the original objective of the committee as stated was to correlate non-destructive test results and the dielectric strength of generator insulation, it seems possible that there is no particular correlation between the dielectric strength of a generator winding and the useful years of life remaining in the winding. However, with the proper cooperation of the operating companies making insulation tests the committee should eventually be able to determine whether it is dielectric strength, d-c resistance, power factor, or some other factor that indicates the expected life of the generator winding.

R. J. Woodrow (Philadelphia Electric Company, Philadelphia, Pa.): Since the data were collected for the paper on "Field Testing of Generator Insulation" some additional tests have been made which expand somewhat the total sum of information as presented in the paper. These tests were made on a 30,000 kva synchronous condenser, 13.6 kv, 60 cycles, 720 rpm with class *B* insulation throughout. The stator of the machine was completely reinsulated and tests were made just after the reinsulation was completed. Approximately a year after the reinsulation the previous tests were repeated and some additional tests were performed, and it is the purpose of this discussion to analyze the difference in results of the two tests and to present some of the interesting features of the recent tests.

Shown in figure 1 of this discussion are curves of insulation resistance versus temperature for various periods of voltage application. The insulation resistance is that measured to ground of all three phases in parallel. The points taken on the original tests just after reinsulation fell exactly on

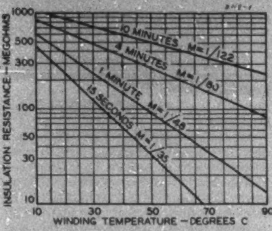


Figure 1. Insulation resistance versus temperature for different periods of voltage application

Data taken at 2,500 volts

the curves plotted from the recent test results showing that the volume insulation resistance to ground had not changed. The values secured for M to be inserted in the equation of insulation resistance agree very closely with those given in figure 3 of the paper for a large turbo-generator after 2 years of service. It might be pointed out that the equation given in the paper is in logarithmic form whereas the exponential form is generally easier to use, where

$$R_{T_2}=R_{T_1}\epsilon^{-2.3M(T_2-T_1)}$$

The values secured from tests all followed very closely the average curves shown in figure 1 except for one set of values taken at 30 degrees centigrade. However, the set of points taken at 30 degrees centigrade which did not conform to the rest of the curve were taken early in the morning after a night of high humidity ($RH=90$ per cent) whereas all the rest of the points were taken at a much lower humidity ($RH=40$ per cent to 60 per cent). Since there was another set of tests made at the same temperature but at the lower humidity it is interesting to compare the two sets to note that a theory can be developed which will explain the difference.

The dielectric absorption phenomenon can be explained by assuming that insulation responds to the application of a constant d-c voltage as if it consisted of a leakage resistance (value of insulation resistance after infinite time of application) in parallel with a complicated system of distributed resistances and capacitances (an internal insulation property producing the response of apparently increasing insulation resistance with time). However it is well known that almost any system of distributed resistances and capacitances can be closely approximated by two or at most three resistance-capacitance branches in parallel. Such a circuit is shown in figure 2 of this discussion, with the equations for current and insulation resistance given.

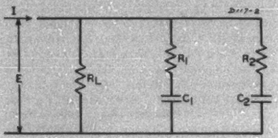


Figure 2. Equivalent circuit for insulation resistance versus time

$$I=E\left[\frac{1}{R_L}+\frac{1}{R_1}\epsilon^{\frac{-t}{R_1C_1}}+\frac{1}{R_2}\epsilon^{\frac{-t}{R_2C_2}}\right]$$

$$\text{Insulation resistance}=\frac{E}{I}=\frac{1}{\frac{1}{R_L}+\frac{1}{R_1}\epsilon^{\frac{-t}{R_1C_1}}+\frac{1}{R_2}\epsilon^{\frac{-t}{R_2C_2}}}$$

By applying the equations for the circuit of the accompanying figure 2 to the test results at low humidity and determining the constants to be inserted, curve A of figure 3 results. It will be noted that the curve follows the test values, which are also plotted, very closely, thus showing that the use of only two resistance-capacitance branches as in figure 2 of the discussion is sufficiently accurate. If the test had been run until constant conditions were reached it is possible that a third resistance-capacitance branch would be necessary to simulate the results, which would have meant one additional term containing R_3 and C_3 in the equation for insulation resistance.

Turning to the test results secured at the same temperature but at much higher humidity, it will be noted from curve B of figure 3 of the discussion that the test points start approximately the same for a short time but level off much more rapidly. It is believed that this effect is due to particles of moisture at the outer surface of the insulation and not due to any infiltration of water vapor into the insulation structure. If this is true then the only term which would change in the equation of insulation resistance is R_L, which represents the final leakage resistance. This final leakage resistance would be reduced due to improved contact between the outer layer of insulation and ground by means of the conducting property of the moisture in contact with it. No change should be noted in the terms R_1, R_2, C_1, or C_2 since these are assumed internal properties of the insulation. In accordance with this explanation a new value of R_L can be calculated for the test results taken at high humidity, and, assuming the other constants remain the same, curve B results. It will be noted that curve B follows the test points as closely as does curve A, showing that the theory given does explain the results.

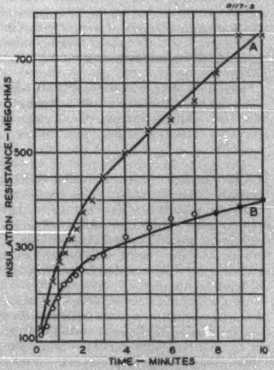


Figure 3. Insulation resistance versus time by theory and test

Data taken at 2,500 volts at winding temperature of 30 degrees centigrade

$$\text{Insulation resistance}=\frac{1}{\frac{1}{R_L}+\frac{1}{R_1}\epsilon^{\frac{-t}{R_1C_1}}+\frac{1}{R_2}\epsilon^{\frac{-t}{R_2C_2}}}$$

$R_1=455$ megohms
$R_2=125$ megohms
$R_1C_1=$ 6 minutes
$R_2C_2=0.48$ minute

A—Insulation resistance at 40 per cent relative humidity; $R_L=1{,}110$ megohms

B—Insulation resistance at 90 per cent relative humidity; $R_L=475$ megohms

In addition to the tests described above on all three phases in parallel to ground, additional tests were made on the same machine for each phase to ground with the other two phases connected to ground. These tests showed little correlation with the values for all three phases in parallel to ground, and were in every case much less than what might be expected i.e. three times the values for all three phases to ground. The reason for this is that the insulation resistance between phases and also additional leakage over the end turns to ground acts in parallel with the volume insulation resistance and reduces to a large extent the overall measured resistance. In order to remove the last two mentioned effects the other two phases were connected to the guard circuit of the test instrument to secure as accurate a measure as possible of the volume insulation resistance of one phase to ground. After this was done the dielectric absorption curves had values for periods of time from 15 seconds up to 10 minutes almost exactly three times the values for three phases in parallel.

In comparing the results on individual phases from the tests just after reinsulation and from those a year later, no change was noted in the volume insulation resistance measured with the other two phases connected to the test instrument guard circuit. This checked the results on three phases in parallel. However a definite increase in the insulation of each phase was noted when the other two phases were connected to the guard circuit. This was believed due to drying out of the varnish on the end turns which reduced the leakage over the end turns.

In conclusion it might be said that further application of the theory which has been used to explain differences in dielectric absorption due to humidity may also be used to determine differences in other insulation properties, which may show up as a change in the initial response instead of as a change in the final value. Measurement of individual phase resistances with the other two phases connected to the test instrument guard circuit can be used to check the phase volume resistance, and with the other two phases connected to ground to check end turn conditions.

Wm. B. Creagmile (James G. Biddle Company, Philadelphia, Pa.): There are two widely separated points in the paper to which I wish to call attention.

Under "Instruments," it is stated that any one of several types of test instruments appear to give satisfactory insulation resistance results and the following types of instruments are mentioned:

> Commercial forms of insulation resistance testers (presumably of the cross-coil ohmmeter type with self-contained generator).
>
> Voltmeter insulation resistance testers.
>
> Electronic rectifier sets using a voltmeter and a microammeter.
>
> Kenotron test sets.

In connection with commercial forms of insulation resistance testers, it is stated they will perform satisfactorily, provided they are calibrated periodically and tested to assure their voltage output is proper. It should be noted that this statement would appear to apply with equal force to the other types of test sets mentioned.

In the cross-coil ohmmeter type of instrument, readings are not affected by considerable deviations from normal test voltage. Of course, it is understood that the voltage must be steady during dielectric absorption tests. Furthermore, we see from figure 7 of the paper that deviations in the voltage of as much as 10 per cent or even more from the normal value, affect the indicated insulation resistance to only a slight extent. Accordingly, considerable departure from the rated voltage and slight lack of calibration will have but little effect on the general form of dielectric absorption curves.

Under "Conclusions," we believe there is general agreement that a single "spot" reading of insulation resistance may be misleading; however, it is sometimes all that can be had. Also, it is a fact that many intelligent conclusions have been arrived at from such readings, augmented no doubt by intimate knowledge of the apparatus under test and good judgment on the part of the man making the test.

The paper under discussion deals particularly with large machines, generating for the most part 11,000 volts or more. There are countless smaller generators as well as motors and other electrical apparatus whose small size and low voltage rating make dielectric absorption tests not worth while, or where, for lack of time or other reasons, the maintenance personnel does not apply such tests.

For many years and on hundreds of machines, periodic "spot" readings of insulation resistance have been taken after a fixed time interval of electrification, such as 30 seconds or 1 minute. These "spot" readings may be taken monthly or at any other convenient interval, and plotted in a curve against time, to give a continuous record of the changes occurring. Such curves appear very definitely to indicate a trend of the general condition of the insulation, when consideration is given to temperature and other common factors.

The EEI subcommittee is to be congratulated on the progress made toward a nondestructive method of testing generator insulation which will give as full information as possible. It seems likely, however, that if resistance-time curves (showing dielectric absorption) are to be of maximum value, they should also be taken periodically over a long period of time, and values of insulation resistance plotted in a curve against time, for subsequent interpretation. This of course is not so easily done as if a single reading is taken at each test period, but possibly, for example, sufficient data may be recorded by plotting three curves in different colors on the same sheet or card, showing insulation resistance values at the end of 1, 3, and 10 minutes.

H. W. Bousman (General Electric Company, Schenectady, N. Y.): The vast amount of work done by the committee in collecting this information is bound to be helpful even though we do not yet know fully how to use the facts brought together.

I should like to suggest an interpretation of the observations on the appearance of dielectric absorption in the insulation resistance measurements as machine insulation dries. This is that the absorption current is present all the time but masked by a larger leakage current in some cases. Consider for example the curves shown in figure 5 of the paper. If the constant temperature parts of these curves, say the final 50-hour portions, are plotted in terms of insulation conductance instead of insulation resistance, the separation between the 1-minute and 10-minute curves is constant, indicating no change in the absorption current. This comment is made merely for what value it may have in showing the physical phenomena underlying the direct experience gotten from the measurements. The practical man with a machine to dry may still prefer a resistance plot such as figure 5 in the paper.

Dielectric strength testing at routine time intervals has not been greatly used because of the likelihood of damaging entirely serviceable insulation. It may be that different sorts of dielectric strength tests cause deterioration at different rates. For this reason, the committee in its future work on the dielectric strength of generator and motor insulation should include consideration of any difference in the deteriorating effect of various sorts of high-potential testing short of breakdown. That is, does a d-c test or an impulse test to 80 per cent of breakdown voltage have less deteriorating effect than a 1-minute a-c test to the same percentage of breakdown? Some work done in England[1] on cable testing rather points this way, even to the extent that routine d-c high-potential testing at several times operating voltage was done at regular intervals on an extensive underground cable system.

Reference

1. Routine Overvoltage Testing of High-Voltage Cables, C. **Kibblewhite**. *Journal* IEE, volume 79, July 1936, pages 82–7.

R. W. Wieseman (General Electric Company, Schenectady, N. Y.): Insulation resistance readings of armature windings were considered by some to be a haphazard compilation of unrelated numbers. One reason for this was that insulation resistance readings were rarely taken under the same conditions. In the past few years, however, an effort was made to measure insulation resistance of armature windings under specified conditions regarding the applied voltage, the time of voltage application, and the winding temperature. This procedure indicated that insulation resistance readings are not simply a set of erratic numbers with the result that more confidence is being given to insulation resistance testing.

This paper is further evidence of the importance of insulation resistance as a winding maintenance index. Unfortunately insulation resistance itself cannot be an absolute criterion of the condition of a winding because it gives only a relative indication of the amount of foreign matter in or around the winding.

One of the interesting features about insulation resistance is the manner in which dielectric absorption aids in evaluating insulation resistance readings. If the absorption effect is small, the insulation contains foreign matter of one kind or another, but if the absorption effect is large, the insulation as a rule does not contain foreign matter. This phenomenon is very clearly shown in figures 1 and 5 of the paper.

If the insulation resistance curve is drawn as in figure 1 of the paper, the winding is usually bone dry when the slope of the curve is about 45 degrees at one minute of voltage application. In fact, a dry winding can be determined by observing the slope of the curve as well as by the insulation resistance itself. Thus the insulation resistance combined with the absorption effect offers a double check on the condition of the insulation. The manner in which insulation resistance varies with the applied voltage, figure 7, is still another check on the winding. It follows that the manner in which insulation resistance varies with both time and voltage aids greatly in interpreting insulation resistance readings.

In conclusion, insulation resistance will not indicate when a winding will fail, and judgment, therefore, must be used in deciding when a winding should be reconditioned or reinsulated. Insulation resistance, however, is an excellent maintenance barometer as shown by figures 4, 5, and 6 of the paper.

H. L. Wallau (Cleveland Electric Illuminating Company, Cleveland, Ohio): Technical paper 41-17 presents in concise form an account of research work done by a group of interested engineers in an endeavor to develop non-destructive electrical tests for application to machine windings for determining the condition of their insulation to forestall the development of faults in service.

Definite tests of such character would be of inestimable value to engineers operating and responsible for the service continuity of rotating electrical machinery. To dismiss the possibility of their development as chimerical, because as yet no conclusive answer has been reached, would be puerile in the light of the data submitted in this paper. Too much praise cannot be given the members of this subject committee for their unflagging efforts in exploring any and all trails which might lead to the coveted goal. The subject is one involving many variables making it most difficult to devise methods by which all variables but one are maintained at constant values while the effect of variations in that other one are studied.

The curves of figures 1 and 6 of the paper show a qualitative indication of relative winding dryness, the steeper the slope of the insulation resistance-time curves during the initial period of electrification the higher the ultimate ten-minute insulation-resistance values become.

In one of the committee's early reports to the electrical equipment committee EEI was a plate showing a series of such curves taken on a 68,750 kva generator after a heat run and at relatively small differences in winding temperature while the machine was cooling off. This family of curves led to speculation as to the possibility of establishing the relation between insulation resistance and winding temperature changes. If this could be established, then the author believed a ten-mile spot check at, say, yearly intervals, could be corrected to some standard reference temperature, average operating temperature of the windings, for example, and any change evaluated. These notions were communicated to Mr. Webb who went into them with the results indicated in his figures 2 and 3. Unfortunately, as figure 3 indicates, the slopes of the curves for the longer electrification periods of four and ten minutes, respectively, may change materially as a result of a generator's service history. That the resistances for all electrification-times and temperatures are higher in the second test is within expectancy; that the slopes for the longer electrification periods have changed so greatly is to be deplored, since what gave possible promise of an easily applied and reasonably reliable test is shown to be vain.

In closing, I wish to congratulate Mr. Webb and his committee for painstaking work over a period of five years, and for an enthusiasm as yet undimmed.

C. F. Hill and L. J. Berberich (Westinghouse Research Laboratories, East Pittsburgh, Pa.): The paper by Messrs. Gilt, Van Ness, Jr., and Webb, on generator insulation tests, shows that the operating engineer has not been very successful in evaluating insulation quality in this apparatus. Here the arrangement of conductors and insulation and the nature of materials limits the type of tests that can be used, but more important is the fact that insulation failures are no doubt due chiefly to mechanical causes and these are not detectable by the usual electrical method.

Surface contaminations can be detected, and, of course, do the insulation no good, but for a machine kept dry by continuous use, it appears a visual inspection is still one of the safest measures of condition, at least when coupled with operating experience. Such results as reported in the paper are not to be taken as negative, however, as they may lead to more emphasis on the mechanical properties of such insulation to withstand thermal cycling, vibration, and other mechanical causes of insulation defects. What we would actually anticipate is that there must be an optimum compromise between electrical and mechanical properties. It is, however, highly desirable to find some electrical method adaptable to the evaluation of such insulations in machines. To date it appears that the dielectric absorption method coupled with a continuous history of a particular insulation to detect changes is the best available. We hope the authors and their co-workers will continue such a search and we assure them of our interest in the problem.

Some of the data on generator insulation are of special interest fundamentally.

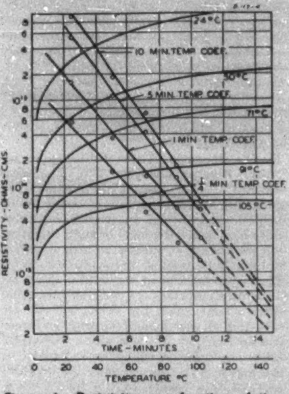


Figure 4. Resistivity as a function of time and temperature for 6,600-volt mica folium measured at 500 volts

Figure 3 of the paper shows the expected increase in resistance as well as an increase in dielectric absorption as the insulation is cured in service. This family of temperature coefficient curves, however, shows abnormal behavior in that if extrapolated the curves appear to intersect in the neighborhood of 25 degrees centigrade. This would indicate that the insulation resistance is independent of the time or that dielectric absorption is absent at this temperature. Such behavior would be inconsistent with dielectric theory. A more normal behavior in this respect is illustrated in figure 4 of this discussion which gives the results of our measurements of insulation resistance against time over a range of temperature on mica folium removed from a machine after eleven years of service. The temperature coefficient curves quite definitely converge at the high temperature end and should become asymptotic at some temperature above 150 degrees centigrade. Most all insulation follows this pattern, that is dielectric absorption decreases as temperature increases until finally ionic conduction becomes so large that the dielectric absorption current is completely masked and the resistance becomes independent of time.

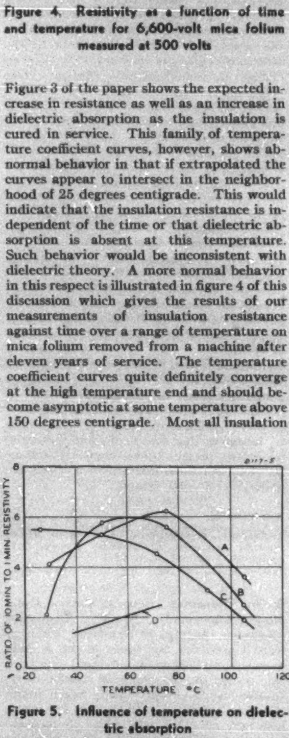


Figure 5. Influence of temperature on dielectric absorption

A—Mica tape—asphaltic-type bond
B—Mica tape—asphaltic-type bond
C—Mica folium—shellac bond
D—Data from figure 3 of paper on generator insulation

We have used the ratio of the ten minute resistivity to one minute resistivity as a crude measure of dielectric absorption. When this ratio approaches unity the dielectric absorption becomes small and the resistance becomes independent of the time. The variation of this ratio with temperature is shown in figure 5 of this discussion for a number of samples of generator insulation. The data for curve *D* were taken from the second family of curves of figure 3 of the paper. Curve *C* is taken from our figure 4. Curves *A* and *B* are for two experimental samples of a newer type of mica tape insulation. Again the normal expected behavior is illustrated by curve *C*. Curves *A* and *B* show the very unexpected abnormal behavior in which the ratio goes over a maximum as the temperature increases. Curve *D* probably also would have shown a maximum had the data covered a wider temperature range. The power factor data did not help to explain these results which, incidentally, show samples *A* and *B* to have a considerably lower power factor than that of sample *C* over a wide range of voltage and temperature. This is introduced merely to show that a careful fundamental study of this complex insulation is necessary before the insulation resistance data can be interpreted more intelligently.

We have also investigated insulation resistance over a range of voltage from 5 to 175 volts per mil at temperatures up to 105 degrees centigrade on a considerable number of samples of both the newer and the older type of generator insulation. The a-c peak operating stress on these types of insulation averages only 60 volts per mil. The power factor data indicated that some of the samples contained internal voids which might influence the high voltage d-c measurements. It was found, however, that the insulation resistance was substantially independent of voltage over the whole range on all samples. The samples with voids gave somewhat unsteady readings. When the charging current under high voltage d-c was amplified and fed into a loud speaker the more or less periodic discharging of the voids could be heard distinctly. It should be remembered that there is an important difference between measurements in a machine and measurements made on samples cut from the slot section and equipped with guard rings. A measurement in a machine on a single phase with the other two phases grounded involves three components: namely, (1) the volume resistance of the slot section, (2) leakage resistance of the end winding, and (3) leakage resistance from the phase group under test to the other two phases. In our work only the first entered, but in a machine all three are in parallel and might result in quite different behavior.

We were much interested to see the results of d-c and a-c breakdown strengths compared on generator insulation. The data show the d-c and a-c peak breakdown strengths to be of the same order of magnitude. This suggests the possibility of substituting d-c for the a-c overvoltage tests now required.

The question of corona in a-c proof testing bringing about voltages much higher than the sphere gap setting through resonance should be considered. A d-c overvoltage test would eliminate this possibility. We wish, therefore, to encourage obtaining more data comparing a-c and d-c breakdown strengths.

R. L. Webb (chairman of committee): We are greatly pleased by the interest shown and by the extent of the discussion aroused by this paper. It is our hope that the paper will help to stir further interest among field maintenance groups when it is printed and made available to the industry.

Suggestions have been received from many interested engineers from time to time and these have assisted to guide this study. The matter of studying temperature effect was suggested by Mr. Wallau more than a year ago. Some of the data collected are presented in the paper. It is obvious that further attention to the characteristics observed will be necessary before all available information can be correlated and fully explained.

Mr. Woodrow's figure 1 shows data similar to those in figure 3 of the paper. Both sets of data were obtained on modern shielded insulation made by the same factory. His theory and formula which conveniently simulated, by equivalent circuits, the different current paths in an insulation structure will prove a valuable tool for analyzing data to determine where changes are occurring in the dielectric.

We agree thoroughly with Messrs. Hellmund and McAuley regarding the advisability of continuing to investigate the possibilities of high voltage d-c testing. This type of test has the advantage of light weight testing apparatus and it shows some promise of giving an index of insulation condition at non-destructive voltage values. We shall continue this phase of the work vigorously.

It was a privilege to have Mr. Knowles present and to hear a few words from him. His review of insulation resistance versus time testing on telegraph cables as long as 25 years ago is interesting and points to the early attention given this type of test at low voltages. Mr. Knowles and his company deserve much praise for the attention given to low voltage d-c testing for the last 30 to 40 years and for the manufacture of fine test instruments which have been so broadly used throughout the world in checking insulation resistance. We appreciate his suggesting the additional paper to supplement the bibliography.

Mr. Creagmile is quite right in stating that it is equally important for the voltage output to be proper in all types of test instruments. However, a continuous voltage indication is provided the operator, so he may easily supervise the voltage during the actual testing, where battery equipments, electronic sets, or Kenotron outfits are used. In the case of the cross-coil ohmmeter the voltage output is not indicated and must be taken for granted. An occasional check on the voltage output of these instruments will suffice since any reduction will likely occur slowly. The insulation resistance of a machine may be sensitive to and may vary inversely with impressed voltage. Readings taken at voltages different from that anticipated will give inaccurate data not comparable with past or subsequent test information if the voltage has changed appreciably. He is also quite right about the need for spot readings in the general run of testing. We are concerned here with improved accuracy for supervision of the insulation on relatively large and important machines.

Mr. Oldacre's suggestion on testing turn to turn insulation is on our calendar for attention as time permits.

Mr. Bousman's observations agree with those of Mr. Woodrow and they appear to be quite consistent with the test data on hand. His recommendation that tests be made to determine the relative effects of a-c and d-c testing as well as impulse testing shall be given early attention.

As stated in the paper, data have been obtained similar to that shown in figure 4 of the discussion by Dr. Hill and Dr. Berberich. The data in figures 2 and 3 of the paper show the convergence of the curves at the low temperature end. The curves of figure 1 by Mr. Woodrow show the same characteristic. We agree this deserves further study and testing to determine the cause of these discrepancies. The data in the paper and in Mr. Woodrow's discussion were taken on modern shielded insulation manufactured by the discusser's company. They may wish to do further testing on some factory samples. We agree fully with the interest expressed in a-c and d-c testing. We shall continue work on this phase of the investigation at every opportunity.

The slope of the dielectric absorption curve at one minute, does as Mr. Wieseman states, give an index of insulation dryness. His other comments apply correctly to low voltage insulation testing. We are hopeful that better and more accurate indices will be obtained using higher voltages.

Mr. Egan has touched on an important point—namely, "Industry Co-operation." The subject committee needs more and more test data taken according to the procedure already set down. Copies of this procedure can be obtained from J. O'R. Coleman at the Edison Electric Institute, 420 Lexington Avenue, New York, N. Y.

Atmospheric Variations and Apparatus Flashover

Discussion and author's closure of paper 41-35 by P. H. McAuley, presented at the AIEE winter convention, Philadelphia, Pa., January 27-31, 1941, and published in AIEE TRANSACTIONS, 1941 (July section), pages 798-803.

L. Wetherill (General Electric Company, Pittsfield, Mass.): The total effect of atmospheric conditions, combined with the presence of dirt, can undoubtedly reduce the flashover strength of a bushing or other similar insulator to some fraction of its normal strength. However, under usual operation conditions, the decrease in strength of a bushing which may result from such factors is not quite so great as it would first appear.

For example, extreme variations in air temperature may account for a 39 per cent variation in the flashover strength of a bushing. However, as compared with the reference temperature of 77 degrees Fahrenheit, the range of variation of flashover of 39 per cent consists of six per cent possible decrease and 35 per cent possible increase. The danger of flashover of a bushing caused by variation in air temperature appears to be slight. Most of the effect of variation in air temperature appears as an increase in strength which to many operators is not undesirable.

While the flashover strengths as affected by air density may decrease 30 per cent at 10,000-feet altitude, we are primarily concerned only with an 11 per cent decrease at 3,300-feet altitude, since at altitudes exceeding 3,300 feet it is the usual practice to furnish bushings with increased external strength. It is not clear whether the curve for effect of altitude refers to 60-cycle flashover or impulse flashover. What little data are available on characteristics at densities encountered at high altitude seem to indicate that the impulse flashover is affected somewhat less than the 60-cycle flashover. The result of such a condition would be that for bushings in service at high altitudes the actual impulse flashover strength would be somewhat in excess of the value which would otherwise be expected.

Fluctuating atmospheric humidity may cause variation of 20 per cent in flashover strength. However, the high atmospheric humidity which usually accompanies lightning conditions is on the favorable side and tends to increase the flashover strength of a bushing with the result that the effect of humidity on flashover strength may, on the whole, be beneficial in preventing flashovers due to lightning which is the chief source of extremely high overvoltages in service.

R. E. Hellmund (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): Although it must be conceded that the change in flashover voltage with weather conditions is troublesome to users and manufacturers alike, a study of Mr. McAuley's paper indicates that in general there is not much that can be done about it. The present standards and practices applying to bushings in particular are reasonable both from an operating and an economic point of view. There is, however, one possible inconsistency in the voltage de-rating curve shown in figure 2 of the paper, which is taken from existing standards. This curve shows how apparatus is de-rated above a certain altitude. It is obviously not very practical for users to change the line voltage in a given transmission system as it goes from one altitude to another. Therefore, since small increments in flashover are only possible when gaps can be used to reduce excess flashover margins, it becomes more practical to make applications in a series of steps at selected levels rather than by means of a smooth curve.

Standards co-ordinating committee 1 (reference values for standardization) is investigating the possibilities of establishing standard limiting ranges of altitude for applying apparatus about as follows:

0– 3,300 feet
3,300– 6,600 feet
6,600– 9,900 feet
9,900–13,200 feet

Assuming that these standard ranges were established, it would seem possible to determine to which ranges each of the standard voltage classes of apparatus apply and where it would be expedient to use special apparatus or bushings, or in some cases, higher voltage apparatus with reduced flashover by means of gaps.

Considering the effect of temperature upon flashover, certain limited risks may be taken because lightning surges, which might overstress internal insulation on account of the higher air densities with low temperatures, occur but rarely at the lower temperatures.

J. H. Hagenguth (General Electric Company, Pittsfield, Mass.): The paper shows clearly the effect of atmospheric variations on the flashover of bushings. As pointed out, similar effects occur on rod gaps, insulators, or other apparatus with non-uniform fields. This effect, of course, has been recognized clearly since many years and correction factors have been obtained experimentally, an example of which is figure 3 of the paper. At low vapor pressures a correction factor of 1.2 is shown if the standard humidity of 0.6045 inch of mercury vapor is used. That means that the rod gap sparkover would be reduced from 100 at standard humidity to $\frac{100}{1.20}=83.5$ at vapor pressure 0. Humidity affects apparatus flashover regardless of what value of standard humidity is chosen. Therefore, the principal use of a standard value of humidity is that of permitting comparison of flashover values of different types of apparatus on a common basis.

The whole range of humidity corrections for different types of apparatus, of course, should be known. However, for laboratory tests, it always is desirable to make tests under as nearly standard conditions as possible. This is desirable, especially where large correction factors are involved and, furthermore, where correction factors are not known within any great degree of accuracy as is the case with humidity corrections. The humidity variations at the Trafford and Sharon laboratories of the Westinghouse Company have been shown in a graph, figures 6 and 7, in a previous paper by Mr. P. L. Bellaschi and the present author. A similar curve is shown in figure 1 of this discussion for the Pittsfield high-voltage engineering laboratory of the General Electric Company. This figure also shows the average vapor pressure for the year to be 0.31 inch, for the period of 7 months (November–May) 0.2 inch mercury, and for the five-month period (June–October) 0.45 inch mercury. Only during two months of the year—July and August—the present standard vapor pressure of 0.6085 inch mercury is exceeded for an appreciable time. Similar conditions exist in the author's laboratories as shown in reference 1.

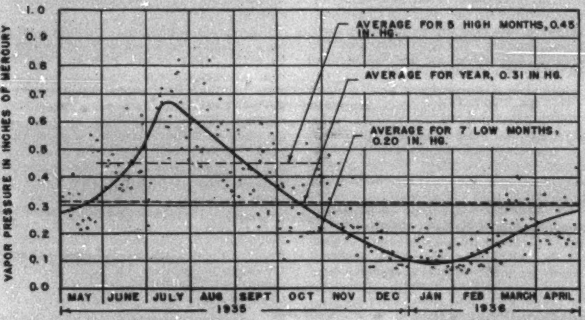


Figure 1 (above). Humidity data obtained in high-voltage engineering laboratory, Pittsfield works, General Electric Company, during the period from May 1, 1935 to April 30, 1936

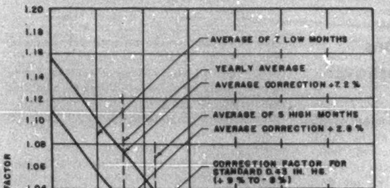


Figure 2 (left). Humidity correction curves for suspension insulators. Positive polarity

In figure 2 (of this discussion) the humidity correction curve for impulse flashover of insulators has been plotted on the basis of standard vapor pressure of 0.6085 inch mercury (curve *A*) and on the basis of 0.43 inch mercury as the standard condition (curve *B*) (ICE standard conditions). Superimposed are the dotted vertical lines indicating the average vapor pressure for the year, seven winter months, and five summer months. By shifting the correction curve—which, of course, is merely a mathematical procedure and the effect of vapor pressure on flashover is not changed by this procedure—three advantages result:

1. The yearly average correction to be applied to test flashover voltages to calculate standard flashover voltages has been reduced from 7.2 per cent at 0.6085 inch mercury to 2.8 per cent at 0.43 inch mercury.

2. Taking the maximum variation of vapor pressure during the year from figure 1 (this discussion) as 0.8 and 0.05 inch mercury, the maximum corrections for a standard vapor pressure of 0.43 inch mercury are +9 per cent and −9 per cent as compared to +14 per cent and −4 per cent for a standard vapor pressure of 0.6085 inch mercury. While the overall range is the same, the maximum correction is reduced by 5 per cent.

3. A humidity of 0.42 inch mercury in the laboratory is not nearly as objectionable from the point of view of physical comfort and the general health condition of the laboratory men as a humidity of 0.6085 which creates the impression of a tropical jungle when artificially created in the laboratory. Therefore, it would become practical to maintain a nearly constant humidity of 0.43 inch mercury for all flashover tests. The high-voltage engineering laboratory of the General Electric Company in Pittsfield is equipped to maintain a constant humidity up to approximately 0.6 inch mercury. Clearly accuracy of measurements on this basis would be greatly increased.

There are two principal objections against a change in standard conditions:

1. As mentioned in reference 2, the flashover voltages established under the 0.6085 inch mercury standard would all be lowered—as shown by figure 2 (this discussion) for impulse flashover voltages of insulators a reduction of 4.5 per cent would result.

2. The humidity during conditions favorable for thunderstorms is mostly at or above the present standard condition of 0.6085 inch mercury, and, therefore, standard values of flashover at 0.6085 inch mercury are more nearly representative of actual service conditions.

However, as pointed out previously, the fixed standard level does not influence the flashover of a given apparatus. Therefore, apparatus flashover of 96.5 kv at 0.43 inch mercury indicates the same performance characteristics as 100 kv at 0.6085 inch mercury for any apparatus with humidity correction as shown on figure 2. As long as the whole range for different apparatus groups is investigated, flashover voltages for all conditions of humidity are known regardless of the standard chosen. However, the lower standard value has the advantage of giving lower corrections during tests and permits testing at more nearly standard conditions throughout the year, and, therefore, results in greater accuracy of measurements.

The paper points out the great variations in flashover voltages caused by conditions of the air. It is also pointed out that "the internal insulations of apparatus is not affected by the changing dielectric strength of the ambient air." The large variations in flashover voltages due to air conditions together with the large variations in flashover voltages due to impulse wave shapes as shown in my paper presented at the convention[2] clearly indicate that apparatus with non-uniform fields are not suitable for coordination purposes of costly station equipment. Since internal insulation is not affected by air conditions, the means of protection employed should also be unaffected and the only device of fulfilling this condition is the modern lightning arrester.

REFERENCES

1. REFERENCE VALUES FOR TEMPERATURE, PRESSURE AND HUMIDITY, P. L. Bellaschi and P. H. McAuley. AIEE TRANSACTIONS, volume 59, 1940 (December section), pages 669–75.

2. VOLT-TIME AREAS OF IMPULSE SPARK-OVER, J. H. Hagenguth. AIEE TRANSACTIONS, volume 60, 1941 (July section), pages 803–10.

P. H. McAuley: The discussions consider and amplify many practical points related to the general subject of the paper and require no further comment for the most part.

The correction curve for altitude is given in Standard 19, apparently with 60-cycle flashover in mind. The derating factors actually are applied to the standard voltage rating of apparatus. However, similar variations seem to occur with critical impulse voltages. This is recognized in the corrections for impulse sparkover of rod gaps and sphere gaps given in Standard 4. It is probable, however, that the phenomenon was not investigated over a wide range of pressures such as encountered up to 10,000-feet altitude. Fielder (*Electric Journal*, December 1935) found that the correction for relative air density decreases with the time to flashover. This applied as well to humidity and, in this case, current laboratory practice makes allowance for the effect.

Mr. Hagenguth's data confirm a situation discussed in the paper on reference values. The question of changing a reference value is a matter for thorough and wide discussion in view of the many standard values, catalogue data, and insulation levels which would require adjustment.

Factors Influencing the Mechanical Strength of Cellulose Insulation

Discussion and author's closure of paper 41-44 by F. M. Clark, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (July section), pages 778–83.

J. G. Ford (nonmember; Westinghouse Electric and Manufacturing Company, Sharon, Pa.), and **C. F. Hill** (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): A stable period, similar to that reported by Clark in which cellulose tends to retain its mechanical strength at the beginning of a heating cycle has been observed in our studies in the presence of oxygen. We have assumed it due to the formation of oxidation products of cellulose, possibly by hardening of residual resins. The existence of a stable period due merely to thermal action is, I believe, not a previously known behavior. I am wondering if the technique used by Clark completely eliminates the action by oxygen. A dormant period as a phenomenon distinct from the stable period is rather surprising as it would indicate two rather independent mechanisms in cellulose due to the application of heat. So far as I know, the cellulose chemist has not anticipated two such independent effects.

In general, the deterioration of cellulose due to thermal action at high temperature and deterioration due to oxidation are well recognized and known phenomena, and so far as I can see, Clark's results are in general agreement with other results. There is one conclusion in Clark's paper, however, which I have been unable to justify from his results. I refer to conclusion 7 in which it is stated that there is no substantial difference in the rate of mechanical deterioration of oil-immersed cellulose in the presence of nitrogen and of air. It would be my interpretation that in Clark's method the oxygen of the air is captured rather quickly by the oil and that in effect the amount of oxygen is too limited to produce sufficient acid concentrations in the oil to affect the cellulose appreciably, so that the tests are both run essentially in a nitrogen atmosphere. At the elevated temperatures of 100 degrees to 130 degrees centigrade or more, the oxygen would disappear from the gas space within a very few hours. It seems to me this conclusion would only be justified if the oxygen supply in the gas were kept constant. Then the effect of oil oxidation products on the cellulose should become the dominant factor and not the direct action of oxygen on the cellulose. Figure 8 of Clark's paper shows an appreciable effect with five times as much oxygen as in his figure 9 and it would appear that the effect due to oxygen is about proportional to the available oxygen. Acidity measurements on the oil itself would have been of value.

A quantitative comparison of our own results with those of Clark may be of interest. We have used the bomb test in the form of a sealed glass tube very similar to that used by Clark. Figure 12 of my paper presented two years ago before this group was obtained on a Manila paper in a fireproof transformer liquid in such a glass bomb. The residual tensile strength after 96 days at 140 degrees centigrade was about 15 per cent of the initial strength. Clark reaches less than six per cent residual in that time in oil. Since publishing my paper, I have had my technique of using the sealed bomb questioned by cellulose chemists in that by-products of the decomposition may react on the cellulose. Previously we had made tests on cotton base materials in oil and a nitrogen atmosphere in which 25 per cent of the initial strength was retained after two years at 140 degrees centigrade, but in that case the by-products could escape at one atmosphere. These results were demonstrated in figure 8 of my paper. I would be interested in Mr. Clark's comments on this effect of residual by-products and particularly to know if he has made any studies in which the action of such by-products and also of oxygen are both eliminated.

An interesting experiment upon condenser paper heated in a vacuum is of interest in this general problem of thermal resistance of cellulose. Samples of the paper were heated one month at 125 degrees centigrade in vacuum and then the temperature was raised to 150 degrees centigrade for one month in vacuum. The initial strength of this paper was 3.91 pounds for a $^{1}/_{2}$-inch strip and the strength after heating was 2.37 pounds. The paper had thus retained 60 per cent of its initial strength. From such results as just quoted and from results of some transformer life tests, we have therefore, concluded that the bomb tests which we made some three years ago are questionable so far as practical application is concerned.

V. M. Montsinger (General Electric Company, Schenectady, N. Y.): I would like to discuss this paper from the standpoint of application of the data on the rate of aging of insulation to the operation of transformers.

In the first place, the data show quite conclusively that even though an oxidizing agent is excluded from contact with either the insulation or the oil in which the insulation is immersed, the tensile strength of the insulations when subjected to temperatures in excess of approximately 100 degrees centigrade decreases rapidly with time and eventually becomes unsuitable as an insulating material. This obviously means that the insulations in oil-immersed transformers having either conservators or cushions of inert gas over the oil cannot carry heavy overloads for indefinite periods of time without their life being seriously shortened.

In the second place, let us see how the aging of the insulations versus temperature bear out the eight-degree rule which was published in my 1930 AIEE paper on "Loading Transformers by Temperature." It is true that all the curves do not follow the eight-degree rule, and it would be rather surprising if they did. Mr. Clark clearly shows that there are two important factors causing the deterioration of fibrous insulation, one of these being oxidation, and the other chemical decomposition. We should not expect to find any logical correlation between these two factors.

Excluding the tests in which the insulation was aged in oxygen, since it is not a practical operating condition, an examination of the curves shows that in general the aging doubles for approximately each 8-degree increase in temperature, the range being from about 7 to about 15 degrees although most of them are quite close to the 8-degree rule. For example:

Figure 2 shows that the relative times to produce the same aging are, 90 days at 125 degrees and 20 days at 142 degrees. The eight-degree rule would show 22.5 days ($^{1}/_{4}$ of 90) at 141 degrees (125+16). Therefore, it is quite close to the eight-degree rule.

Figure 4 shows that the relative aging for 120 degrees and 135 degrees follows the 7 to 7.5-degree rule.

Figure 11 shows that to reach 12 pounds per square inch the 100-degree and 135-degree temperature curves follow the 9-degree rule. Comparing the 100-degree and 120-degree curves, the rule is approximately 7 degrees. Comparing the 120-degree and 135-degree curves, the rule ranges from 15 to 17 degrees. The over-all average rule is approximately 10.5 degrees.

Comparing the curves given in figures 15 and 16, it will be noted that when increasing the temperature 15 degrees, that is, from 120 to 135 degrees, the aging of oil-treated insulation doubles for each 3.5-degree increase in temperature, while the aging of the untreated insulations doubles for each 7.5-degree increase in temperature. This, of course, is comparing the aging in air and in nitrogen. Other tests, however, figure 9, show that the aging is essentially the same in air and in nitrogen.

Taken as a whole, I think it is safe to say that the data given in Mr. Clark's paper come fairly well within the seven- to ten-degree rule. This is fortunate for without some kind of rule it would be practically impossible to determine the effects of either short-time or continuous overloads on the life of transformers.

Wm. A. Del Mar (Phelps Dodge Copper Products Corporation, Yonkers, N. Y.): Mr. Clark states in the introduction to his paper that the tensile strength of the insulation has been accepted as a gauge of its mechanical properties. Some years ago I made tests of somewhat similar character but gauged the mechanical condition of the oil-impregnated paper not only by tensile strength but also by folding endurance and tearing strength. I found that when cable paper was aged in air at high temperatures the tensile strength tended to remain constant, even at temperatures almost up to the charring point of the paper. However, it was noted that long before the charring occurred, both the tearing strength and folding endurance, particularly the latter, fell off quite rapidly.

I therefore question whether the "stable period" described by Mr. Clark deserves to be known by that name and I question its value as an indication that cellulose insulation may be subjected to overloads corresponding to this stable period without sensible deterioration, unless the insulation is not to be bent.

I realize that both the tearing strength and folding endurance are somewhat more erratic than the tensile strength and that it is not so easy to get a good-looking curve of deterioration, but would not such a curve be more useful even though depending upon somewhat more erratic points?

Mr. Clark's oxidation tests were made with oxygen gas and I believe that this is entirely justifiable for evaluating the oxidation of low-voltage insulation. If, however, high-voltage insulation is to be evaluated it would seem that ozone should be used as ozone is the principal deteriorating agent in such cases and produces entirely different chemical effects from ordinary oxygen.

Mr. Clark's data on the relative effects of oxygen and nitrogen are exceedingly interesting. I should like to ask whether he considers that the deterioration in nitrogen is due, in large measure, to oxygen occluded or dissolved in the insulation.

Herman Halperin (Commonwealth Edison Company, Chicago, Ill.): The contrasting data and views presented by different investigators show that there is still need for such carefully controlled studies as reported in Mr. Clark's paper. In figure 9, he shows that there is practically no difference between the effects upon the mechanical properties of paper of heating impregnated paper in the presence and absence of air. This finding is apparently in contrast with the views of several other investigators. The Swiss oil specifications ("Conditions techniques pour huiles isolantes," Publication 124 of the Association Suisse des Electriciens) use in the oil oxidation test as a criterion of the oil deterioration the decrease in tensile strength of cotton yarn kept in contact with the oxidizing oil. C. F. Hill (AIEE TRANSACTIONS, volume 58, page 484) reports that oxidation products of the oil peroxides in particular and perhaps some of the low-molecular weight acids react with cellulose to cause its decomposition. Putman and Dann (AIEE TRANSACTIONS, volume 58, page 505) report that test results showed that insulating materials in oil protected against contact with air will withstand appreciably higher temperatures than in open oil. It is noted that Mr. Clark's tests on impregnated paper were made in sealed tubes, i.e., with a limited supply of air or oxygen. Possibly this fact is the reason for the apparent discrepancy between Mr. Clark's findings and those of other investigators. What was the ratio of gas and insulation volume in these tests in sealed tubes?

The data on the dormant and stable periods are of considerable practical interest. The table shown in the paper shows that for 120 degrees centigrade the stable period is 101 hours. Are there any comparable data available as to the total number of hours of intermittent heating before the end of the stability period is reached? In service, obviously, the hours at high temperature during each emergency will usually be a small fraction of 101 hours.

Aging tests of oil-filled cables in Chicago produced apparently less mechanical deterioration of the paper than indicated by figure 11 of the paper. In the Chicago tests the inner tapes of the insulation were subjected to temperatures in excess of 120 degrees for over 2,000 hours, but not more than 100 hours continuously in most cases. After these tests the mechanical strength of the tapes was found to be decreased to about 50 per cent of the original strength as determined by tearing tests, while according to figure 11, 120 degrees for 2,000 hours would cause a decrease in tensile strength of 81 per cent.

What is the limit of temperature below which thermal decomposition in the absence of oxygen becomes negligible? According to figure 11, thermal decomposition takes place even at 100 degrees centigrade. It is noted, however, that after 16 weeks no further deterioration occurred. Could it be possible that residual acids or other oxygen-carriers in the oil and paper were responsible for the deterioration that stopped after these agents were used up? This opens another question, namely, whether the deterioration is taking place in the cellulose or is due to noncellulose materials in the paper.

The statement is made based on figure 5 that for temperatures below 100 degrees "the oxidation effect is of negligible importance." As at these lower temperatures thermal decomposition is probably not of great importance, does the statement imply that at such lower temperatures deterioration of paper is practically absent? E. J. Murphy presented test results before the 1938 meeting of the Conference on Electrical Insulation of the National Research Council showing that decomposition of dry paper under vacuum is appreciable at 75 degrees centigrade with no electrical stress applied. It is possible that the deterioration found by Mr. Murphy was due to residual moisture being driven out by the combined effect of a very high vacuum, 10^{-4} millimeters of mercury, in combination with an elevated temperature, 75 degrees. However, serious deterioration due to oxidized oil has been found in some transformers that have operated at temperatures below 100 degrees.

F. M. Clark: The deterioration of cellulose insulation resulting in a mechanical weakening is definitely the result of a chemical change and as such presents a difficult problem for its commercial control. To adequately discuss the various phases of the problem raised by the discussion presented would justify another paper of equal length as the original. The following paragraphs attempt to outline explanatory suggestions in answer to the more important points raised:

1. There is not sufficient evidence at present available to determine whether the existence of the commercially important stable period is due to the initiation of a favorable (resin hardening) reaction as suggested by Dr. Hill or to the occurrence of a period of inhibition. We have observed its occurrence in a variety of cellulose insulations of widely different resin content.

2. The dormant period should not be considered as an independent phenomenon but merely as the culmination of a series of preceding pyrochemical reactions which eventuate in a simple decomposition, one of whose products is gaseous.

3. The explanation that the protective action of mineral oil is due to its preferable reaction with the oxygen present does not appear tenable, since reactions under high oxygen pressure where an excess of the oxygen is always present lead to the same general conclusions as are drawn from deterioration studies at lower gas pressure.

4. The deterioration of cellulose is definitely accelerated by the presence of the decomposition products. This has been demonstrated in a previous publication (ELECTRICAL ENGINEERING, volume 54, number 10, pages 1088–94). This effect is present whether the insulation be aged in the presence or absence of oxygen. From studies recently made it would appear that the mechanical deterioration of cellulose proceeds along the same chemical trend whether gaseous oxygen be present or absent, possibly due to the oxidizing action of the products formed as a result of pyrochemical decomposition. I refer Dr. Hill to the article mentioned for a more complete description of the action of heat on capacitor paper under reduced pressure.

5. It is gratifying to note that the rate of cellulose deterioration recorded in this paper agrees with the so-called eight-degree rule already described by Mr. Montsinger. This eight-degree rule may have many important practical applications within the range of commercial practice. Its extension to include a wider range of temperatures than those described by Mr. Montsinger, however, can be expected to be difficult because of the initiation of chemically different types of reactions as the extremes of temperature are encountered.

6. Mr. Del Mar cites an interesting case with reference to cable paper which retained an approximately constant tensile strength almost to the point of charring. This phenomenon has not been observed in the series of researches of which the data of this paper are a part. Serious deterioration in tensile strength has always been observed long before the deterioration has proceeded to the point of charring.

7. The tearing and folding strength of papers have invariably been found to be extremely erratic and do not lend themselves to reliable analysis. Al-

though the deterioration in tensile strength is slower than that for the folding or tearing test, it is more constant in its behavior and has, therefore, been considered as a more reliable test tool although for certain electrical applications information regarding the tearing and folding strengths of the paper may well be demanded.

8. In any study of the mechanical deterioration of cellulose insulation, careful attention must be given to the conditioning of the insulation before the aging test. Many of the discrepancies reported in the literature and referred to by Mr. Halperin can be traced to neglect of this important factor.

9. In a general sense, the intermittent exposure of cellulose insulation to a deteriorating temperature gives an additive effect.

10. The cable experiments cited by Mr. Halperin are of interest, but it must be remembered that the extent of the mechanical deterioration of cellulose insulation caused by pyrochemical effects is not only determined by the temperature and the presence or absence of oxygen or oil, but also by the mechanical conditions of the exposure. Like many chemical reactions of this type, environment has such an important effect, that it is impossible to transfer the exact results obtained in a laboratory setup to the deterioration obtained in commercial practice. The loss or retention of the products of the initial cellulose deterioration, for example, is a factor which greatly influences the type and rate of all subsequent reactions and effects.

11. Paper will deteriorate from many causes and the statement that the effect of oxidation is of negligible importance below 100 degrees centigrade certainly must not be interpreted to mean that paper cannot be made to deteriorate at a lower temperature. The statement merely indicates that at low temperature the presence or absence of oxygen is not of major practical importance in determining the rate of the deterioration observed. The effects of vacuum applications and other factors not included in the paper cannot be anticipated from the data presented.

Classification and Co-ordination of Short-Time and Intermittent Ratings and Applications

Discussion and author's closure of paper 41-13 by R. E. Hellmund, presented at the AIEE winter convention, Philadelphia, Pa., January 27-31, 1941, and published in AIEE TRANSACTIONS, 1941 (July section), pages 792-8.

Wm. A. Del Mar (Phelps Dodge Copper Products Corporation, Yonkers, N. Y.): The case of cables differs from that of other apparatus due to the fact that cables are generally associated in a thermal way, with the structure in which they are carried, such structure being unrelated to the cable itself.

In many cases the structure is so extensive that the element of time for heating it is of prime importance. The heat capacity which comes into play, therefore, depends on the rate of change of load. For instance, cables in underground ducts will have a high thermal capacity for overloads lasting an hour or more, due to the heat capacity of the ducts and ambient earth. In fact a cable in an underground duct will never attain the ultimate temperature rise corresponding to the balance of its steady heat generation and dissipation. On the other hand, for overloads lasting a second or less the available heat capacity will be little more than that due to the copper conductors.

It is advisable to consider the cases of impregnated paper cable and of rubber and varnished cambric cables separately.

The rule that each eight degree centigrade rise in temperature halves the life of insulation does not apply to paper cables, because this rule seems to apply only to insulation exposed to air, whereas the insulation of power cables is generally out of contact with air. Furthermore, the rule is based on the assumption of thermo-chemical deterioration only, whereas in high voltage paper insulated cables, which constitute the bulk of those used for wholesale power work, the effect of high temperature is to promote ionization through cyclic expansion and contraction.

The cases of rubber and of varnished cambric insulations are important in the present connection because such cables are directly connected to rotating machinery, and station apparatus. The voltage, in general, is low and, in most cases, a considerable part of the cable does not have the advantage of thermal association with any structure. In such cases, only the thermal capacity of the copper, insulation, and coverings come into play and, in the absence of metallic sheathing, chemical deterioration of the insulation will be a prime factor in its life, if subjected to high temperatures.

Available data on the connection between life and operating temperature of cable insulation do not fall into any well defined pattern, so that the applicability of the eight degree law is undetermined. An example of such data, carefully determined, will be found in Mr. S. Rosch's paper on "The Current Carrying Capacity of Rubber-Insulated Conductors" [AIEE TRANSACTIONS, volume 57, 1938 (March section), page 157]. Code grade rubber insulation was aged at 50 degrees and 60 degrees centigrade. According to the eight degree rule, a difference of 10 degrees centigrade should multiply the aging by 2.3. The above data however, indicate a four-fold decrease in tensile strength. The data on elongation at break depart even further from the eight degree centigrade rule.

This, of course, raises the question of what is to be the measure of aging and that will depend on the application of the cable. In one case tensile strength may be the criterion and in another, flexibility, or, perhaps, some electrical characteristics.

In general, Mr. Hellmund's proposed rule permits conductor temperatures five degrees centigrade in excess of the normal values. This rule does not seem to be applicable to Code grade rubber insulation with a permissible temperature rise of ten degrees centigrade from an ambient of 40 degrees, as the table is based on the assumption of a 10 degree centigrade rise as base. In other words, the rule would eliminate Code rubber for connection to apparatus with overload rating, unless much heavier copper were used than would be required for normal load. I doubt whether the industry is prepared to endorse this. Of course, the standard carrying capacities of code wire are based on 30 degrees ambient but when co-ordinating the carrying capacities of wires and apparatus connected thereto, one should, obviously, use the same ambient.

V. M. Montsinger (General Electric Company, Pittsfield, Mass.): Mr. Hellmund's paper gives some very useful information on determining under load cycle conditions either the safe continuous rating (service factor rating), or the permissible short-time overloads in terms of a continuous rating.

The paper makes use of both the root-mean-square method and the temperature integration method. The rms method is, of course, easier to handle than the temperature integration method that I used in my papers, references 1 and 2. As pointed out by Mr. Hellmund, the use of the rms method should be limited to conditions where the range in temperature is moderate. The reason for its limitation is that the deterioration of insulation is not proportional to temperature.

As is probably well known, the mechanical deterioration of class *A* insulation doubles for each 7 to 10, or 8 to 12 degrees centigrade increase in temperature. I have used eight degrees in my work.

I wish to show under what load-cycle conditions the root-mean-square method is applicable to transformers, and when the temperature integration method should be used. For purposes of comparing the two methods, I have assumed four different load cycles, starting with 10 per cent overload for one hour and gradually increasing the overloads to 40 per cent and the time to six hours.

Figure 1 shows the calculated oil and hot-spot temperatures for a transformer operating under these four different load cycles. The transformer has the following assumed characteristics at rated load:

1—40 degrees oil rise
2—65 degrees hot-spot
3—3:1 ratio of copper to iron loss
4—time constant of 3
5—30 degree ambient

Table I gives the equivalent rms load for each load-cycle, the load-cycle and continuous rms load hot-spot temperatures, the relative aging of the insulation, and finally the life of the insulation under load-cycle operation in per cent of the life under continuous operation at the rms load. The

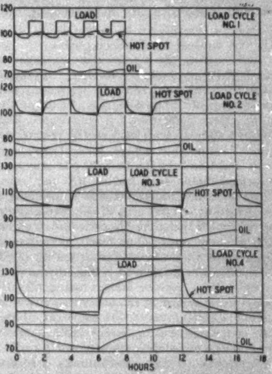


Figure 1. Hot-spot and oil temperatures for a typical power transformer operating under various load cycles—in 30 degrees ambient

Table I

No.	Load Cycle: Percentage Load	Load Cycle: Time in Hours	Equivalent RMS Percentage Load	Ultimate RMS Load Hot-Spot Degrees C	Max. Load Cycle Hot-Spot Degrees C	Aging Units for 1 Cycle Resulting From: Load Cycle	Aging Units for 1 Cycle Resulting From: Cont. RMS Load	Life on Load Cycle in Per Cent of Life on RMS Load
1	110 100 repeat	1 1	105.12	99.0	102.0	10,656	10,280	96.5
2	120 100	2 2	110.5	104.0	110.7	36,920	31,800	86*
3	130 100	4 4	116	109.5	119.5	126,220	101,000	80
4	140 100	6 6	122	115.0	132.0	382,280	245,000	64

* 92% for 12 degree rule.

temperature areas were integrated "by parts" using the formula:

$$A = txe^{kT}$$

where

A = aging units
t = time in hours
k = 0.0865 for 8 degree rule
T = temperature degrees centigrade
e = 2.718

It will be noted that for the first load cycle of 110 per cent load for one hour and 100 per cent load for one hour, the two methods differ by only 3.5 per cent. If used for load-cycle 2—120 per cent load two hours, 100 per cent load two hours, the difference is 14 per cent. If the 12 degree rule had been used for load-cycle 2, the difference in the life expectancy would be only eight per cent. It appears, therefore, that the rms method could be used for load-cycle 1 and possibly for load-cycle 2. Where the overloads (coupled with the times shown) exceed these, the temperature integration method should be used. The rms method can, of course, be used for heavier overloads if the time is shorter and the range in temperature is small. That is, the magnitude of the overload alone is of little importance. In other words, both overload and time should be considered in determining when the rms method is suitable for use.

When using the temperature integration method in my paper, reference 1, I found that the peak load could be expressed as a percentage of the load-factor. Based on the most severe condition, i.e., maximum load on part time and no load on the rest of the time, I found that a peak load of three per cent could be held for each 10 per cent that the load-factor was below 100 per cent. That is, if the load-factor was 50 per cent a peak load of 115 per cent of the rating could be held with the same life expectancy as for 100 per cent continuous load. This would correspond to a service factor rating of 87 per cent of the peak load. The rms method would give a service factor rating of 70.7 per cent, or looked at the other way it would permit a peak load of 141 per cent of the safe continuous rating, which is 23 per cent more than the other method gives.

Referring to Mr. Hellmund's paper—he states that the curves shown in figure 6 have been calculated to give a life of insulation equivalent to that obtained with a continuous operation at 60 degrees rise, but he does not state how the calculations were made. I assume that he used the eight degree rule in integrating the temperature areas.

P. H. McAuley (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The author has classified different types of load conditions and given a procedure for selecting apparatus to fit these conditions. In this connection, it is interesting to observe how different types of motors perform under different periodic operating conditions. The table gives test data on a single frame size motor with different designs of the windings and ventilating systems and different cooling conditions. The weak field motor had a low magnetizing current, the strong field motor a relatively high value, and the no-load losses differ accordingly. Under changing ventilation the motors were stopped during the no-load period of the cycle. With constant ventilation, the motors ran continuously but carried no load for part of the duty cycle, although the no-load losses continued. The relative effects of these losses and of the ventilation are indicated by the rates of drop of temperature (by resistance) of the windings. The significance of the ratio R was discussed in reference 3 of the paper.

7.5 Horsepower 30-Minute Frame Size

	Rate of Temperature Change Near 60 Degrees Centigrade in Degrees Per Minute				
		Changing Ventilation		Constant Ventilation	
	Rise	Drop	R = d/r	Drop	R
Weak field open	1.9	2.5	1.32	4.2	2.2
Strong field open	1.15	1.6	1.38	1.0	0.87
Strong field closed	2.7	0.95	0.35	0	0

For the weak field motor, the magnetizing current is low and the winding cools 1.7 times as fast running light as it does standing still. A similar figure was given for another motor in the closing discussion of reference 3. For the strong field motor, the no-load losses are not compensated by the continued ventilation and the rate of cooling running idle is only 63 per cent of that in the stopped position. When this same motor is enclosed the no-load losses maintain it at the limiting temperature rise and no continuous rms rating is available. This means that the motor cannot be applied advantageously to any service where the motor is connected continuously to the line. It can be used, however, to good advantage for applications where the motor is disconnected during the idle periods. It, therefore, follows that for this enclosed motor case, the simplified methods in the present paper cannot be used. Either slight modifications will be necessary or it is possible to resort to the fundamental methods given in the previous paper of reference 3.

Comparing the open cases, on loads which permit stopping between load cycles, the two motors will perform similar service as indicated by nearly identical values of the ratio R. Where these motors run between load cycles, that is, with constant ventilation, the weak field motor will function satisfactorily on heavier load cycles, as indicated by the larger R constant, which permits larger values of k. Thus, although the 30 minute ratings in all cases are practically identical, there is a wide difference in performance of the motors under the different conditions of periodic service.

P. L. Alger (General Electric Company, Schenectady, N. Y.): Mr. Hellmund has made a considerable contribution to our knowledge of the increasingly important subject of short time rated apparatus. What he has said may be divided into two distinct categories. The first category is that of rating of apparatus. In this connection, he recommends the use of somewhat higher temperatures for the short time rating tests than for continuous apparatus to obtain the same total life, on the basis of our modern knowledge of insulation aging. Also, he recommends the use of service factors to express the ratio of the continuous to the short time ratings. Both of these ideas seem sound and desirable for adoption in various standards.

The second category is that of application. Under this category come all the discussion of intermittent duty cycles, thermal time constants, and their associated effects. These ideas, as well as those expressed by Mr. V. M. Montsinger in his various articles on temperature aging of transformer insulation, are very helpful in the selection of the proper rating of apparatus to operate on a particular duty cycle, but they do not form a part of the apparatus rating itself.

It seems to me that these two questions of rating and application ought to be kept quite distinct. The former requires the setting up of simple methods of rating apparatus that are definite, commercially convenient, and easily proven by simple performance tests. The rating thus establishes the manufacturer's responsibility and permits ready comparison of different makes of apparatus. The latter or application method permits the user to select the proper rating and judge its probable performance under any particular conditions, but forms no part of the guarantees or acceptance tests.

R. E. Hellmund: Since this session was held under the auspices of one of the Standards co-ordinating committees, it is very gratifying to have engineers interested in different types of apparatus and conductors participate in the discussion. Mr. Del Mar's remarks concerning cables are very pertinent; however, conditions in connection with cables are after all not so very different from those encountered in apparatus and machines. Frequently various types of apparatus are associated in a thermal way with their supporting or enclosing structures and with other apparatus; machines and transformers often have their insulation out of contact with air; and many times there are problems arising from expansion, contraction, and ionization. Whenever such modifying conditions are of practical importance, they must of course be taken into account regardless of whether they apply to cables, apparatus, or machines.

I agree with Mr. Del Mar that the methods in my paper relate essentially to applications where cables or conductors are directly connected to machinery. Naturally it is to be expected that the rate of deterioration in material like rubber is somewhat different from that found in class *A* materials used in machinery, but very likely a suitable rule can be determined for rubber insulation and it is hoped that some efforts will be made in this direction.

From the data quoted from Mr. Rosch's paper it would seem that an excess of five degrees for short-time ratings of rubber insulated conductors, i.e., a total temperature of 55 degrees centigrade as compared with 50 degrees centigrade for continuous ratings, is quite conservative. Assuming that these values are reasonably correct, it would simply follow that with a continuous cable rating based on a 10 degree temperature rise and a 40 degree ambient, the temperature rise can be increased to 15 degrees for a short-time rating, and with a continuous rating based on a 20 degree rise and a 30 degree ambient, the temperature rise can be increased to 25 degrees. This is equivalent to an increase in current rating of at least 22.5 and 12 per cent respectively and therefore means worthwhile economies.

I am glad that Mr. Montsinger in his discussion on transformers has emphasized the fact that the ordinary rms method is definitely limited in its application to some load conditions. His load cycles range from two hours to 12 hours, with the load period being one-half of the total time. He finds that for his shortest load cycle the rms method gives reasonably correct results. With the application for which the methods in my paper are most useful, the rms method gives incorrect results for much shorter periods because in machines without the large heat-absorbing capacity of the oil and with other factors reducing the time constant, the temperatures follow load changes much faster. Mr. Montsinger is correct in his assumption that the eight degree rule has been used for the temperature curves in my paper; the method can, however, be applied for any other value.

Mr. McAuley's discussion gives some very valuable test data on motors. He is entirely correct in stating that the method of my present paper cannot be applied to some enclosed motor ratings. His figures further indicate that the ratio between the rate of temperature drop with the motor running and the rate with the motor standing still is influenced appreciably by such minor design changes as variations in the field strength of the motor. I therefore believe that, in general, the method given in a previous paper prepared jointly with Mr. McAuley will, with some slight modifications, be more suitable for all those load cases where the ventilation or core-loss conditions during the idle periods are considerably different from those during the load periods. The method in this paper was worked out chiefly for use by those who prefer a modified rms method for applications where it will give satisfactory results.

Mr. Alger is correct in stating that part of my paper relates to the application of apparatus rather than to methods of rating. However, my main purpose in going into the methods of application was to point the way toward a rating structure which would permit of simple and correct application of apparatus and conductors. In any intermittent service with wide load fluctuations, the heat-absorbing ability of apparatus is a factor of utmost importance, because without it every load change would mean an instantaneous change in temperature. This is the reason why I have suggested a short-time rating giving an indication of the heat-absorbing ability combined with a continuous rating expressed by a load factor, and also why I have subsequently demonstrated methods for applying such a rating structure. A simple rating structure is of little value if it does not permit of simple methods of application. Unfortunately the present method is not as generally applicable as the one given in the previous paper; however, it has the advantage of being very similar to the conventional rms method and of covering a much broader field than the latter.

Large Electric-Arc Furnaces—Performance and Power Supply

Discussion and authors' closure of paper 41-40 by B. M. Jones and C. M. Stearns, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (July section), pages 763–9.

N. R. Stansel (General Electric Company, Schenectady, N. Y.): The term *large* in the title of the paper is relative with reference to power systems. These vary widely in sizes and methods of service. A large furnace for one system may be a small furnace for another system.

The two functions of reactance in an arc-furnace circuit are: (*a*) circuit stability, and (*b*) limitation of current surges. The first relates to furnace performance; the second, to the voltage regulation of the power system.

As regards furnace performance, the desirable value of reactance is that which gives a satisfactory degree of stability after the initial erratic period of operation, that is, after a certain amount of molten metal is obtained, which means temperature and gas conditions in the chamber which are favorable to the maintenance of the arc. This value of reactance usually corresponds to an average power-factor within the range of 80 to 90 per cent, that is, 60 to 45 per cent reactive voltage drop with the normal current of the arc for a given voltage.

The circuit of an arc-furnace is defined by counting all the reactance and resistance in the conductors from the point in the power system at which the voltage is held constant. The maximum power in this type of circuit occurs at 0.707 power factor. The i^2r loss in the circuit for a given current, subtracted from the power in the circuit, gives the value of the power in the arc. The maximum power in the arc is obtained with a value of current less than the current value which gives maximum power in the circuit.

The value of current, for a given applied voltage, for maximum power in the arc is:

$$i_{\text{opt.}}=\frac{e}{\sqrt{2}x}\sqrt{1-\frac{r}{z}}\ \text{line amperes} \tag{1}$$

e, γ, x, and z phase to neutral values.

The corresponding power-factor of the circuit is obtained from the expression,

$$\tan\phi=\frac{x}{z+r} \tag{2}$$

The value of maximum power is,

$$p_{\text{max.}}=\frac{3e^2}{2(z+r)}\ \text{watts} \tag{3}$$

The resistance of the circuit is, or should be, small in comparison with the reactance. Hence, we can write

$$p_{\text{max.}}\approx\frac{1.5e^2}{x}\ \text{watts} \tag{3a}$$

This approximate expression emphasizes the importance of the value of the reactance of the circuit of an arc-furnace installation.

An analysis of the power system at the point of a proposed furnace installation gives the maximum permissible value of current surges at that point. An analysis of the circuit of the proposed furnace, as outlined by equations 1, 2, and 3 shows the performance of the furnace circuit. These two analyses can be correlated to predetermine the operating results from a proposed installation. The exception is the effect of the frequency of current surges—which can not be predicted—with reference to the class of customers affected, as discussed by L. W. Clark in a paper entitled, "Power Company Service to Arc Furnaces," AIEE TRANSACTIONS, volume 54, 1935 (November section) pages 1173–8.

O. B. Falls, Jr. (General Electric Company, Schenectady, N. Y.): This paper is very timely due to the marked increase in the use of arc furnaces during the past two to three years. Messrs. Jones and Stearns present a study which should be carefully reviewed by all utilities interested in this class of service, with a view towards handling such loads with the least possible delay and at a reasonable cost, when called upon to do so.

The authors point out the multiplicity of data and calculations required to determine the effects of arc furnace loads on power systems. Would not the following be a simple and conservative basis on which to start?

Assume the system connections on which the furnace will be operated most of the time. Then, determine the maximum allowable change in current to give a voltage change which is within the specified limits for the system in question. This step should be quite simple since the necessary information has usually already been prepared for system short circuit studies. The total reactance in the circuit of arc furnaces, below approximately 5,000 kva, is of the order of 50 per cent at the nominal current of the furnace. This means that the maximum current under a short circuit (of the furnace electrodes which may occur during the cold-melt period) will be twice the full load current of the furnace. With these two facts, the full load current rating of the furnace is determined. The maximum power input of the furnace circuit corresponding to this current is at 70.7 per cent power factor. Deducting the I^2R loss in the furnace transformer and connections from this value of maximum power gives the maximum rate of heat development in the furnace. Thus, it is possible to determine the maximum individual furnace load which can be supplied at a given point in the power system.

This method, from one viewpoint, will indicate a furnace load smaller and from other viewpoints a furnace load larger than could actually be tolerated.

The results obtained from the use of this method will depend a great deal on the discretion of those using it in properly taking into account the various factors involved. Its use, however, will help to indicate the order of magnitude of furnace load allowable at any one point in a system. If results obtained are favorable, then a more detailed study, as indicated by the authors, may be justified.

With regard to corrective means, the authors have mentioned only synchronous condensers. This is probably due to the fact that the present conditions of operation indicate that no correction will be required, and further detailed study of this problem was not necessary. It would be interesting to know if any other methods of correction, such as the use of series capacitors, has been considered in connection with this problem. A series capacitor will "balance out" a part of the reactance of the circuit, or can be made large enough to maintain practically constant voltage at any given point in the system, such, for example, as the furnace bus. The series capacitor is a *lumped* reactance, and as such, produces all the voltage rise at its location. Thus, the voltage on the *load* side of the series capacitor is all that is effected. Therefore, the series capacitor would necessarily have to be placed far enough back in the circuit to be at a point where the voltage fluctuations without a series capacitor would not be objectionable.

Since these furnaces are fed by a line tapped in on a 66-kv ring, two series capacitors would very likely be necessary to eliminate voltage flicker at both Valley and Ambridge substations. One would be placed at each substation on the side opposite the source of trouble. The economics, and probably the operating results obtained would quite likely not justify the application of series capacitors in this case. However, it would be interesting to know how the calculated results and costs would compare to other methods of correction.

Along this same line of thought, I would like to point out that a series capacitor has been installed in Chile, South America for just this type of application. The installation, which has been in successful operation for approximately three years, consists of three single-phase units, each rated 115 line amperes, 264 kva, 2,300 volts. The equipment is installed at the present time in a 33-kv three-phase circuit which will ultimately be changed over to 66-kv.

The units are installed at the receiving end of a single transmission line just ahead of the step-down transformer bank. This bank furnishes power for normal city distribution, as well as an arc furnace load. This series capacitor has proven itself quite effective in eliminating voltage flicker on the low voltage side of the step-down bank.

I would like to describe briefly a similar problem in connection with arc furnaces which I have had the opportunity of analyzing.

A certain southern utility served power to a three-phase electric arc furnace by means of a separate feeder direct from the generating plant in order to segregate the voltage flicker caused by the arc furnace load. It was the desire to eliminate this special feeder at the same time that certain other changes were being made in the system.

The problem involved was one of determining from tests made with existing system connections, voltage flicker conditions which might result under future system connections.

Using calculated characteristics of the system, as well as test data, it was found that there were four possible system connections which would result in less than 1.35 per cent voltage variation, with a minimum of approximately 0.9 per cent variation. Since the furnace was operated only at times when one of the four allowable system connections would normally be used, the utility proceeded with work on the changes with the assurance that when these changes were made, and the furnace load changed over to the distribution bus, the results would be satisfactory insofar as light flicker was concerned.

One other interesting result obtained from the study conducted by Messrs. Jones and Stearns is that the diversified operation of the two furnaces results in load swings only 20 per cent greater than obtained by one furnace, and that the frequency of fluctuation of load swings was actually reduced when the second furnace was installed. This would suggest the possibility, from an engineering standpoint, of using two or more smaller furnaces, rather than one larger furnace to meet a given production demand.

S. B. Griscom (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The authors have performed an outstanding service for power supply companies and electric furnace operators in presenting the co-ordinated data on furnace power loads. Several problems in connection with electric furnace loads have come to the writer's attention recently, and in those cases where the installation was not already in operation there was much indecision as to the proper loads on which equipment application should be made. The authors by presenting graphic charts and oscillograms at sampling intervals throughout a heat have presented a very clear picture of the nature of an electric arc furnace load in all its phases. In addition figures of 15 minute demands, maximum instantaneous three phase swing with power factor change are established for one furnace installation and corresponding figures for simultaneous operation of two furnaces. These figures, of course, apply definitely only to the particular installation, and in using them on projected jobs, intelligent modifications must be made to allow for different ratings, methods of operation, and type of scrap metal.

The writer's principal contact with power supply to electric furnace has been in connection with synchronous condensers for power factor correction or reduction of voltage flicker. For power factor correction the principle item of interest with the usual form of demand billing is the power factor at the peak interval demand. In one case coming to attention, this was 82 per cent as compared to the 85 per cent given by the authors.

For flicker determinations, strictly speaking, the maximum single phase *change* in load with its attendant power factor is of chief importance, since lighting is done at single phase. The authors evidently base their flicker calculations on the three phase load change. This leads to a lower calculated value of flicker voltage than if based on single phase quantities. For example, points 5 and 6, table I show a three phase change of 8,600 kva or 450 amperes per phase. However, phases *A* and *C* each drop about 660 amperes. Considering that the voltage change is 1.15 times as great for a given current from line to line as for line to neutral, the change in voltage based on single phase quantities would be $\frac{660}{450}\times 1.15$ or 1.42 times that based on a load swing of 8,600 kva three phase.

I would be interested in the authors' reasons for adopting the three phase basis. There would appear to be some justification for this procedure following the reasoning given in paragraph 5 under "Results." That is, if the worst swings are to be eliminated because of their infrequency, the use of three phase quantities may be a satisfactory way of doing this. Another factor is the difficulty in obtaining instantaneous single phase currents and power factors as a watt-element oscillograph is about the only available means, and this would require an excessive amount of film.

B. M. Jones and **C. M. Stearns:** Answering Mr. O. B. Falls, Jr., series capacitors were considered but not to the extent of obtaining cost estimates. Service for each furnace was required in so short a time that very little time was allowed for exhaustive engineering research and estimates. In the case of each furnace certain restrictions were required; viz., separate reactance for the first furnace and staggered operation for the second. Fortunately no corrective equipment was required for the first furnace or for the first two. Had corrective equipment been required, we have two 7,500-kva synchronous condensers which could have been "borrowed."

The amount of furnace swing, that the system can stand at the point of connection, has been increased slightly since the first furnace was placed in operation. It will further increase a small amount this summer (1941) when an additional 60,000-kw unit is placed in operation at the Reed station and will increase quite materially in 1942 when the new 60,000-kw unit, near Ambridge substation, is placed in operation. For this reason, and for others, we wished to avoid the installation of corrective equipment if possible.

As has been mentioned, we are to supply service to two more (a total of four) furnaces at this location in the summer of 1941. Our experience with the first two furnaces leads us to believe that voltage flicker will not be too severe with four furnaces. Voltage regulation of course will be worse and voltage regulation, rather than flicker, may require us to install corrective equipment.

For capacity reasons for two more furnaces we are cutting the tap to Pittsburgh Crucible off from the line between Ambridge and St. Joseph lead substation, extending it back to a point across the river from Ambridge and connecting it to the second line between Brunot Island and Ambridge. (See figure 1 in original paper.) This will connect the furnace load at a point nearer the center of the system and allow a greater furnace swing than at the present.

In 1942 a new power station with a 60,000-kw unit will be located across the river from Ambridge substation and the line to Pittsburgh Crucible Steel will be connected to the station bus through an oil circuit breaker.

Answering Mr. S. B. Griscom; we did not stress the single phase change, particularly because there seemed to be considerable discrepancy between the instantaneous current readings obtained from the graphic ammeters and the current calculated from the 3ϕ-kw and *R*-kva readings. For instance, at point 5 on table I (main paper), the average amperes per phase, calculated from the kw and *R*-kva readings was 630, while the average of the ammeter readings was 824.

The observed values of voltage change at the nearest attended substations are not as great as Mr. Griscom indicated they might be, based upon single phase furnace load swings. This leads us to believe that there might be overswinging or some peculiar reaction of the ammeters to rapidly changing loads which differs from that of the kw and *R*-kva meters.

As pointed out by Mr. Griscom, the oscillograph is the only way to get one-phase amperes and power factors. However, as he agreed it is necessary to use a very large amount of film in order to assure obtaining the maximum values which occur during the heat. In addition, without special connections, the one-phase wattmeter element will ordinarily not record true watts because of the incorrect phase angle relation between line amperes and line voltages.

Mr. L. W. Clark presented a very interesting slide showing the maximum size of a three-phase arc furnace which could be connected to the Detroit Edison Company system at locations with varying amounts of short circuit kva. This slide showed a family of curves for various values of allowable voltage flicker. We do not have such curves, but actually apply the principle of the curves for new furnace applications. The short circuit kva at the point where the tap to the Pittsburgh Crucible Steel connects to the system is approximately 500,000 kva. This value was believed too low for connection of a 10,000 kva furnace. We allowed the furnace to be connected with the restrictions mentioned before. The load swings of a furnace are of varying magnitude, the maximum values occurring only a few times during a heat, and as a result the voltage flicker has in no way been objectionable.

In closing we would like to say that we are by no means afraid of electric furnaces and believe that the power companies should encourage this business, but of course feel that the proper method of connecting furnaces to their system should be determined so as to prevent interference with other customers, making necessary reinforcements as required.

We are in hopes that the data presented in the original article and these additional comments might be of some use to others in their furnace application problems.

Rotor-Bar Currents in Squirrel-Cage Induction Motors

Discussion and author's closure of paper 41-1 by J. S. Gault, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941 (July section), pages 784–91.

M. M. Liwschitz (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The paper by J. S. Gault gives a very interesting and valuable picture about the currents in the rotor bars of a squirrel cage motor; it shows the shape of these currents as well as their distribution. The author tries to draw from his tests conclusions on the stray load losses. Unfortunately, the present state of our knowledge does not permit such conclusions.

The currents and the corresponding losses in the rotor bars are due:

1. To the slot openings of the stator.
2. To the step harmonics of the stator.
3. To the phase belt (= band) harmonics.

While the first losses depend on the flux density, the second losses depend on the current. While the first losses increase with increasing slot opening, the second losses decrease with increasing slot opening. The first losses appear at no load, the second and third at load.

In most modern machines these losses are low. The losses under (1) equal 10 to 20 per cent from the total no-load losses and the losses under (2) and (3) are about the same percentage of the total stray load losses. The main part of the stray load losses consists of:

(*a*). The eddy current and hysteresis losses at the surfaces of the tooth crowns.

(*b*). The eddy current and hysteresis losses in the tooth bodies.

Due to the additional currents on the rotor bars very little losses will be in the bodies of the rotor teeth of the squirrel cage motor, but higher losses than in the wound rotor motor will occur in the tooth bodies of the stator. It is well known that the no-load losses of a squirrel cage motor remain about the same when the motor runs with squirrel cage or without squirrel cage. The high frequency currents on the rotor bars force the superimposed flux wave due to the stator out of the rotor teeth and produce additional iron losses in the stator teeth.

In two-pole motors the losses due to the phase-belt harmonics can become considerably high. The influence of these harmonics on a 20-horsepower two-pole motor I have shown in the discussion of the paper by T. H. Morgan, W. E. Brown, and A. J. Schumer (AIEE TRANSACTIONS, volume 59, 1940, pages 1208–09).

R. E. Hellmund (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): In connection with the problem discussed in Mr. Gault's paper, an article which I wrote in 1906 on the subject, "Magnetizing Currents in Polyphase Induction Motors," (*Electrical World*, volume 48, page 329) may be of interest inasmuch as it outlines some of the basic considerations for determining primary and secondary current wave shapes and their higher harmonics caused by the fact that the stator distribution of magnetomotive force is not sinusoidal under the usual assumption that the primary magnetizing current is sinusoidal. Data relating to current wave shapes and higher harmonics in both stator and rotor of single-phase machines are also given in my paper "No-Load Conditions of Single-Phase Induction Motors and Phase-Converters" (AIEE TRANSACTIONS, volume 37, 1918, pages 539–626). Actual test data given in a discussion of the latter paper by A. M. Dudley and C. A. M. Weber may also be of interest. Unfortunately the analytical methods above referred to, as well as those of Mr. Gault's paper, are too complicated to be used in everyday work for determining corresponding losses; nevertheless they are valuable, as are also the tests, in that they indicate to the designer the most favorable arrangements.

In his paper Mr. Gault emphasizes the practical importance of the band-frequency currents and leaves the impression that the tooth-frequency components are of secondary importance. This is correct for the particular motor tested because its primary chording gives a high value to the band-frequency currents, and its slot arrangements minimize the tooth-frequency currents. However, in actual practice very few three-phase motors are designed with the unfavorable two-third or full pitch chording, the majority of them being built with an intermediate value, which, as mentioned in the paper, very materially reduces the band-frequency currents. As a consequence the losses caused by them are in the majority of motors of less importance than in the particular motor tested. Furthermore, many motors, particularly some of the older designs, differ from the one tested in that they are designed with an appreciably greater number of secondary slots than of primary slots. This, together with the fact that the slot bridges of entirely closed slots or tooth tips often become saturated under load and thus bring about a condition approaching that of open slots, tends to materially increase the variations in air-gap reluctance and to increase the tooth-frequency currents. I am therefore confident that the tooth-frequency currents are of greater importance than the band-frequency losses in many designs. I fully agree with Mr. Gault that under some of the conditions indicated above, appreciable losses are caused by high-frequency currents in the rotor.

P. L. Alger (General Electric Company, Schenectady, N. Y.): The test methods and results presented in this paper must be of great interest to every designer of induction motors, because of the clear picture they give of stray load loss phenomena. Although the conclusions reached have been qualitatively known for some time, the quantitative results appear more precise than those heretofore published.

Modern three-phase induction motors normally have windings with a pitch in the neighborhood of $^5/_6$, selected to minimize the belt leakage (or band) harmonics. As the author points out, in reference to figure 20 of the paper, the band frequency losses in this case are much less than for the $^2/_3$ pitch winding used in the test motor. Considering the fifth and seventh harmonics only, the ratio of flux densities is only 27 per cent as great in the former case, for the same value of fundamental flux wave, giving an expected loss ratio of only seven per cent. The test motor also had a slot ratio of 45/48, unusually close to unity, which greatly reduces the tooth frequency losses (at the expense of increased magnetic noise).

In usual motors, therefore, the load losses are due chiefly to the tooth frequency fluxes and associated rotor currents rather than the band frequency currents. At these higher frequencies, the iron losses are relatively greater as compared to the copper losses. Also, the higher flux leakage between rotor and stator windings makes the assumption of a sinusoidal total flux wave highly inaccurate. Hence, considerably more test information is needed for a thorough analysis of the problem.

An important aid in studying these effects is to take oscillographic records of the flux pulsations in a stator tooth, that are produced by the high-frequency rotor currents. Very useful data can undoubtedly be obtained by the author's methods, if they are applied to motors of more usual winding distribution and slot ratios.

J. S. Gault: The discussions submitted indicate careful study of the paper by recognized authorities in this field. For this and the helpful nature of their criticisms I am deeply grateful.

The primary object of the paper was to present oscillographic records of rotor bar current under various conditions, together with a theoretical study of methods for computing the different components. The division of losses found in the case analyzed showed a band-frequency rotor current loss which was about 25 per cent of the total stray load loss at full load. It was not intended that this would indicate a probable average value for other motors, as Mr. Liwschitz implies. It was stated in the "conclusions" paragraph of the paper that this loss was high due to the two-thirds pitch winding employed; and that a change in pitch to five-sixths would reduce this loss to about one-third. This would be in closer agreement with the figures Mr. Liwschitz presented as representative average values. It should also be noted that, for motors with wider phase bands, this component of the stray load loss becomes more important; for example, two-phase and consequent pole windings and many single phase windings.

Mr. Hellmund's remarks concerning the importance of tooth-frequency rotor harmonics are extremely helpful and pertinent. The greater emphasis on band-frequency eddy-current loss in the paper was due to an academic interest in this portion of the subject rather than the study of stray load losses as such. Also the band-frequency effects in the rotor have received less attention in the literature. When the paper was written I had not read the papers referred to by Mr. Hellmund, which contain a masterly theoretical treatment of the subject, especially his paper in the 1918 TRANSACTIONS. I apologize sincerely for the omission from my bibliography of his important contribution.

Mr. Alger's comments are in general agreement with the other discussion. His constructive suggestions concerning the need for additional tests are very helpful in connection with a program along this line which is already under way. Mr. Coho also contributed to the discussion, at the meeting in Philadelphia, some remarks concerning the effect of skew, which indicate the need for further work along this line in order to get a more complete picture.

Overvoltages in Polyphase Induction Motors During Single-Phase Operation

Discussion and authors' closure of paper 41-83 by C. Macmillan and G. K. Carter, presented at the North Eastern District meeting, Rochester, N. Y., April 30–May 2, 1941, and published in AIEE TRANSACTIONS, 1941 (August section), pages 819–23.

L. M. Nowacki (General Electric Company, Schenectady, N. Y.): A number of the mathematical expressions for the stator and rotor voltages given in the paper can be obtained by the use of symmetrical components and equivalent circuit. The main advantage of this method is that it gives a rigorous solution for certain cases, taking into account resistances as well as reactances. The solutions in the paper were based on the assumption that the motor resistances were negligible. The validity of this assumption may be questioned. Therefore a particular motor has been taken and the various voltages calculated both by the formulas and by symmetrical components.

As a result, assurance has been obtained that the assumption of zero motor resistances is justified. The largest error occurred in the calculations of appendix *A*, in the special case of single-phase rotor in a three-phase stator. The error here was less than three per cent. In appendix *B*, in the case of single-phase locked rotor in a single-phase stator, the error was of the magnitude of one-half of one per cent, that is within the accuracy of slide-rule calculation.

The case of the revolving single-phase rotor in a single-phase stator does not adapt itself to the ordinary method of symmetrical components. It has been repeatedly stated by qualified mathematicians that this case has no rigorous solution. However, it is believed that this problem can be solved by the extension of symmetrical components using ladder networks. As the time for the submission of the discussion is limited, it will not be possible to complete the study at present. All that can be offered is a voltage formula arrived at by analogy. This is not exactly a correct technical procedure.

For the purpose of calculation we have taken a wound rotor motor with a one-to-one turn ratio between the rotor and stator. The motor was rated eight-pole, 30-horsepower, 900-rpm, three-phase, 60-cycle, 440-volt.

The following are the motor constants all referred to the stator at line frequency of 60 cycles and are given in ohms per phase *Y*:

Stator resistance	r_1	0.17 ohm
Rotor resistance	r_2	0.22 ohm
Stator leakage reactance	x_1	0.63 ohm
Rotor leakage reactance	x_2	0.63 ohm
Magnetizing reactance	x_m	10.0 ohms
Total external resistance between the two rotor rings	R	ohms
Ratio of turns between rotor and stator	n	1:1
Ratio of rotor frequency to stator frequency	s	
Stator line-to-line voltage	E	440 volts rms
Stator frequency	f	60 cycles
Open-circuit impedance	Z_m	$0.17+j10.63$ ohms
Short-circuit impedance	Z_s	$0.39+j1.26$ ohms

APPENDIX A. SINGLE-PHASE ROTOR IN A THREE-PHASE STATOR

Case I

$r_1 = r_2 = R = 0$

$s = 2$

No external resistance in rotor circuit

The equivalent circuit for a single-phase rotor in a three-phase stator is given in figure 1.

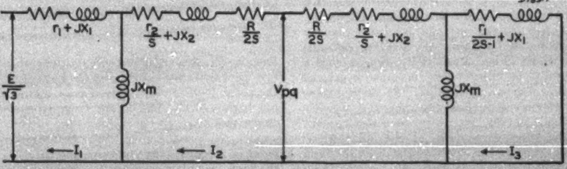


Figure 1

The rms voltage between rings, as obtained from the circuit of figure 1, is

$$E_R = 3sV_{pq} + j\sqrt{3}I_2R/2 \qquad (1)$$

For case I, as stated above, the rms ring voltage is

$$E_R = sV_{pq} = 2\times 358 = 716 \text{ volts (rms)}$$

This corresponds to the formula in the paper for this case

$$E_R = \sqrt{3}B(M_0/L_0) \qquad (2)$$
$$M_0 = x_m/2\pi f = x_m/377$$
$$L_0 = (x_1+x_m)/377$$

Thus

$$M_0/L_0 = 10.0/10.63 = 0.940$$

and

$$E_R = 716 \text{ volts (rms)}$$

Case II

$r_1 = r_2 = 0$
$R = 6.72$ ohms
$s = 2$

External resistance in rotor circuit

This corresponds to $r/X_e = 1.33$ in the paper, the condition for maximum ring-to-ring voltage.

$r = R/2 = 3.36$ ohms
$X_e = (x_1+x_2)s = (1.26)(2) = 2.52$ ohms
$r/X_e = 3.36/2.52 = 1.33$

According to the paper, the rms ring voltage for this case is

$$E_R = (0.940)(440)(2.30) = 950 \text{ volts (rms)}$$

Solving the circuit of figure 1 for this case, $I_2 = 57.2$ amperes and

$$E_R = (3)(2)(1.68+j1.22)(57.2) + j\sqrt{3}(57.2)(3.36) = 950 \text{ volts (rms)}$$

Case III. Same as case I except the value of primary resistance is not neglected. $r_1 = 0.17$ ohm.

Solving the circuit of figure 1, $I_2 = 55.4$ amperes and

$$E_R = (3)(2)(1.73+j1.22)(55.4) + j\sqrt{3}(55.4)(3.36) = 928 \text{ volts (rms)}$$

The difference between the 950 volts of case II and the 928 volts of case III represents the error in neglecting the value of primary resistance. In per cent, the error is

$$\text{per cent error} = 100(950-928)/928 = 2.37 \text{ per cent}$$

APPENDIX B. LOCKED SINGLE-PHASE ROTOR IN A SINGLE-PHASE STATOR

The following are the voltage expressions derived by symmetrical components. Stator phase terminal rms volts:

$$e_a = \frac{E}{\sqrt{3}}\left[\frac{\sin 2\theta}{\dfrac{Z_m+Z_0}{Z_m-Z_0} - \cos 2\theta}\right] \text{rms volts} \qquad (3)$$

Rotor phase terminal rms volts:

$$e_1 = \frac{2E}{\sqrt{3}}\left[\frac{x_m}{Z_m-Z_0}\right]\left[\frac{\sin \theta}{\dfrac{Z_m+Z_0}{Z_m-Z_0} - \cos 2\theta}\right] \text{rms volts} \qquad (4)$$

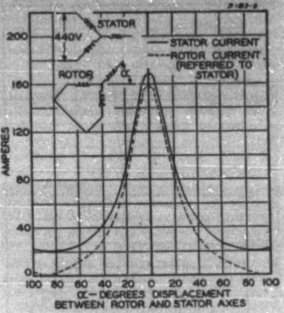


Figure 2. Locked rotor currents

MR444 8-pole, 30-horsepower, 900-rpm, 400-volt induction motor. Three-phase wound stator; three-phase wound rotor. 440 volts—single phase applied to stator. Two rings of rotor short-circuited; third ring left open

The above formulas in terms of the motor constants are:

$$e_a = 254\left[\frac{\sin 2\theta}{1.27-\cos 2\theta}\right] \text{rms volts} \qquad (5)$$

$$e_1 = 508\left[\frac{\sin \theta}{1.27-\cos 2\theta}\right]\times 1.067 \text{ rms volts} \qquad (6)$$

The corresponding equations in the paper can be simplified and put in the following form:

$$e_a = \frac{E}{\sqrt{3}}\left[\frac{\sin 2\theta}{\dfrac{2L_0l_0-M_0^2}{M_0^2} - \cos 2\theta}\right] \text{rms volts} \qquad (7)$$

$$e_1 = \frac{2E}{\sqrt{3}}\,\frac{l_0}{M_0}\left[\frac{\sin \theta}{\dfrac{2L_0l_0-M_0^2}{M_0^2} - \cos 2\theta}\right] \text{rms volts} \qquad (8)$$

where

$$L_0 = (x_1+x_m)/377$$
$$l_0 = (x_2+x_m)/377$$
$$M_0 = x_m/377$$

$$\frac{2L_0l_0}{M_0^2} = \frac{(2)(10.63)(10.63)}{(10.0)(10.0)} = 2.26$$

$$\frac{l_0}{M_0} = 10.63/10.0 = 1.063$$

Thus equations 7 and 8 become:

$$e_a = 254\left[\frac{\sin 2\theta}{1.26-\cos 2\theta}\right] \text{rms volts} \qquad (9)$$

$$e_b = 508\left[\frac{\sin \theta}{1.26-\cos 2\theta}\right]\times 1.063 \text{ rms volts} \qquad (10)$$

Equations 7 and 9 are identical for all purposes; so are 8 and 10. It must be borne in mind that the constant 1.27 in equations 7 and 8, and the constant 1.067 are vectors and should carry an angle. This angle for 1.27 is −3.2 degrees and for 1.067 is −0.5 degree. It is evident from inspection that introducing the angles would in no way change the results calculated on a slide rule.

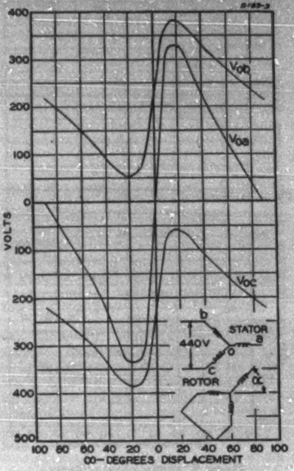


Figure 3. Stator phase voltages

Locked rotor. MR444 8-pole, 30-horsepower, 900-rpm, 440-volt induction motor

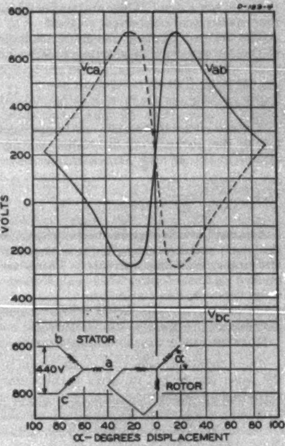


Figure 4. Stator line voltages

Locked rotor. MR444 8-pole, 30-horsepower, 900-rpm, 440-volt induction motor

In the paper the respective maxima for

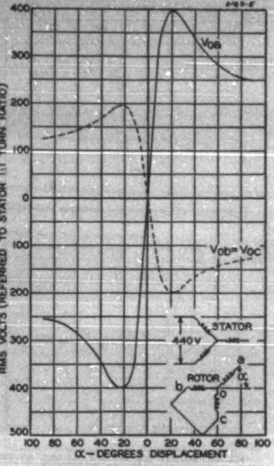


Figure 5. Rotor phase voltages

Locked rotor. MR444 8-pole, 30-horsepower, 900-rpm, 440-volt induction motor

this case are given. They are confirmed by the symmetrical component method.

The rms ring-to-ring maximum voltage to the normal rms ring-to-ring voltage is:

$$\frac{\sqrt{3}Z_m}{4\sqrt{Z_0(Z_m-Z_0)}} \tag{11}$$

The rms stator line-to-line maximum voltage to the normal rms line-to-line voltage is:

$$^1/_2+\frac{\sqrt{3}(Z_m-Z_0)}{4\sqrt{Z_mZ_0}}$$

Attached are five curve sheets, giving all the phase and line voltages, for both stator and rotor, and the currents. These are plotted against degrees electrical displacement between stator and rotor.

Appendix C. Single-Phase Rotor Rotating in a Single-Phase Stator

The maximum value for e_R as given in the paper can be put in terms of the motor constants as follows:

$$eR_{\max}=\frac{\dfrac{\sqrt{3}X_m}{2(Z_m-Z_0)}E}{\dfrac{Z_m}{Z_m-Z_0}-1} \tag{12}$$

Similarly, the stator line-to-line voltage:

$$e_{S\max}=\frac{\dfrac{\sqrt{3}}{2}E}{\dfrac{Z_m}{Z_m-Z_0}-1} \tag{13}$$

From the above, we deduce that the maximum peak ring-to-ring rotor voltage

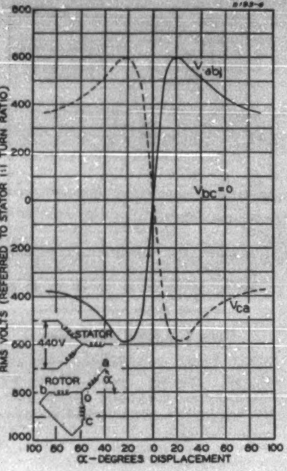


Figure 6. Rotor line voltages

Locked rotor. MR444 8-pole, 30-horsepower, 900-rpm, 440-volt induction motor

to the normal peak ring-to-ring voltage is

$$0.866Z_m/Z_0 \tag{14}$$

And, the maximum peak line-to-line stator voltage to the normal peak line-to-line stator voltage is

$$0.866(Z_m-Z_0)/Z_0 \tag{15}$$

Note that equations 12 and 13 were not derived. They were deduced from the expressions given in the paper for the maximum voltages. They are given as probable solutions.

C. Macmillan and **G. K. Carter:** Mr. Nowacki has presented a discussion which is both welcome and helpful. It is, of course, pleasing to have the numerical checks which he has given, particularly because they were arrived at by a method different from that used in the paper. As was expected, he finds that resistance plays a relatively minor part in those cases in which it could be conveniently taken into account.

An important contribution of the discussion is the presentation of the various equations in terms of the more familiar notation. It is felt that, for practical purposes, this form will be quite valuable. It must be emphasized, however, that the results quoted in appendix *C* of the discussion were not derived by the method of symmetrical components, and so are open to the same objections with regard to resistance as the corresponding expressions in the paper. The remarks made in the paper concerning the effects of the source impedance also apply equally to the method used in the discussion.

In conclusion, the authors wish to point out once more the possibility, under certain circumstances, of dangerous overvoltages on ordinary wound-rotor induction motors and the desirability of avoiding those circumstances if possible.

Photographic Study of A-C Arcs in Flowing Liquids

Discussion and authors' closure of paper 41-92 by Joseph Slepian and T. E. Browne, Jr., presented at the North Eastern District meeting, Rochester, N. Y., April 30–May 2, 1941, and published in AIEE TRANSACTIONS, 1941 (August section), pages 823–8.

H. E. Strang (General Electric Company, Philadelphia, Pa.): We agree with Dr. Slepian and Dr. Browne that information collected in this manner, if properly interpreted, can be of substantial value in the design of circuit interrupters. One would naturally be more impressed if the photographs of power arcs were taken in actual breaker interrupting chambers, although certain fundamental phenomena can no doubt be determined from still pictures of arcs in miniature structures.

No one will venture the prediction that liquid types of circuit breakers have reached the ultimate in perfection. However, in view of the growing interest in air blast types of interrupters, it has been felt that this field offered the greatest promise of substantial results from photographic studies.

An entirely new type of camera[1] was developed expressly for this purpose, capable of taking pictures at the rate of 120,000 per second. One thousand frames are recorded on a film about 4 by 30 inches, and are capable of being enlarged for study, but not arranged for sequential projection. In order to get the greatest benefit from these pictures and to be able to accurately follow the formation and decay of the a-c arc, a transposition camera was developed for transferring these results to a standard 16 millimeter film.

We will exhibit a portion of a film on this subject, showing two typical operations of a cross air blast breaker interrupting about 4,000 amperes at 14,500 volts, and photographed at the rate of 60,000 frames per second. Projected in this way, the time ratio is such that events taking place in one second would require over one hour to show. One side of the arc chute of this breaker is of transparent material, and the outline of the barriers around which the arc is blown is visible. Pictures projected in this ultra slow motion sequence, show clearly the phenomenon occurring at current zero, and have been of great assistance in the development of high capacity station types of cross blast air circuit breakers.

Reference

1. A 120,000-Exposure-Per-Second Camera, and Its Application to Circuit-Breaker Arc Studies, D. C. Prince and W. K. Rankin. *General Electric Review*, September 1939.

J. Slepian and **T. E. Browne, Jr.:** We wish to thank Mr. Strang for showing the moving pictures of an arc in a circuit interrupter taken at the extraordinarily high speed of 60,000 frames per second. This is a most remarkable achievement, and we can hope that study of these pictures near current zero will help to establish a sound theory of a-c arc interruption.

The rapid gas bubble oscillations which we have found in oil flow interrupters we believe to be of considerable importance in affecting arc interruption performance. We failed to note in our paper that such oscillations in pressure have also been observed in a breaker not of the flow type (R. Rüdenberg, "Wiss. Veröff. d. Siemens-Konzern," volume 2, 1922, page 249). It is possible that in this case due to unbalanced magnetic or other forces, a relative velocity of arc to oil was produced which had the same effect as the oil flow in our case.

The Dielectric Strength of Glass—an Engineering Viewpoint

Discussion and author's closure of paper 41-91 by E. B. Shand, presented at the AIEE North Eastern District meeting, Rochester, N. Y., April 30–May 2, 1941, and published in AIEE TRANSACTIONS, 1941 (August section), pages 814–18.

C. L. Dawes (Harvard University, Cambridge, Mass.): In his paper Mr. Shand has compiled and coordinated a large number of data relating to the dielectric strength of glass and porcelain under different methods of testing and at different rates of application of voltage. He divides breakdown into two general classes, disruptive breakdown when there is no perceptible internal temperature rise, and thermal breakdown when the voltage gradient becomes sufficiently high to cause the temperature of the insulation to rise cumulatively, without further increase in the gradient. Although the phenomena causing breakdown probably cannot be simplified to these two effects exclusively, there is ample evidence that both exist and that in general very short-time or impulse tests are disruptive in their nature and long-time tests to a considerable extent are thermal. The author implies that with thin glass sections disruptive breakdown only occurs since little temperature difference between the interior and the surface can exist. However, Wagner (AIEE TRANSACTIONS, volume 41, 1922, page 290, figure 2) has shown that under proper conditions thermal breakdown can occur in thin sections of glass. In his experiments, thermal breakdown occurred in sections of glass as thin as 0.062 and 0.075-millimeter thickness.

It would be interesting if the fundamental phenomena causing breakdown in glass were better known. There is no doubt that with disruptive breakdown there are intramolecular forces which produce electronic collisions and disintegration of the molecular structure, which is stated in the paper as being "electronic ionization by collision within the molecular structure of the material."

Within solids such as glass it is possible to break the electronic-molecular bonds with field intensities of no greater magnitude than are developed during the usual dielectric-breakdown test. These field intensities are far below those necessary to break the electronic-molecular bonds with isolated molecules. This is due to the fact that in solids, adjacent molecules develop *intramolecular* forces which assist in removing the electrons from adjacent molecules, thus increasing the free electrons. This increases the electrical conductivity as well as making available greater numbers of free electrons to produce ionization by collision. With the isolated molecule the electronic bonds are due to *intermolecular* forces only, and these are so great that they cannot be broken by the field intensities that are obtainable in the ordinary dielectric-breakdown test.

In view of Wagner having obtained thermal breakdown with very thin specimens of glass as well as with other substances, it would be interesting if more data, giving the relation of conductivity of glass to temperature, were available. At molten temperatures glass has sufficient electrical conductivity to conduct currents of power magnitude at moderate voltage. At the temperatures which would be attained during a breakdown test, with the body of the glass relatively cool, to the writer, the degree of electrical conducting that would be attained, together with the relatively short time available, hardly seems sufficient to develop sufficient joule heating to account for thermal breakdown.

There are other phenomena which frequently accompany the dielectric rupture of glass and porcelain, chipping, cracking, and mechanical fracture. These phenomena have been observed by Peaslee (AIEE TRANSACTIONS, volume 35, 1916, page 1187), by Hoover (AIEE TRANSACTIONS, volume 45, 1926, page 983), by Schumacher (*Journal* of the American Chemical Society, volume 46, number 8, 1924). Hoover found that chipping of the walls within the bore of a glass tube was due to corona formation in the oil which surrounded the center electrode. The writer has also observed chipping at the edges of glass specimens when streamers were sufficiently powerful to jump around the edges. The chipping action seems to be in a large measure mechanical, apparently being caused by heavy ions being hurled against the material by the electric field. The effect is quite pronounced when sheets of bonded mica plate are tested. These streamers literally tear or rip the edges of the individual films and ultimately the films themselves away from the plate.

In the paper, edge effect and its influence on breakdown voltage are discussed. Rogowski, by means of conformal mapping, has developed a contour for the edge of the electrode which presumably prevents any increase in field strength due to edge effect (Theory of Functions, R. Rothe, F. Ollendorff and K. Pohlhausen, page 139, Technology Press, Cambridge, Mass.). However, in order that this contour should conform to the theoretical conditions, the electrodes should be immersed in a medium having the same permittivity as the test sample.

J. J. Torok (Corning Glass Works, Corning, N. Y.): Mr. Shand's paper presents a comprehensive picture of the dielectric strength of glass as determined under various conditions and immersed in liquids and gaseous materials. We have found that these values fall pretty well within the limits set up by practical applications. However, in applying glass we very seldom approach the dielectric strength limits. Usually other considerations enter in which demand dielectric thicknesses much greater than the minimum set by the dielectric strength of the media (the glass). A power line insulator application is a very good example. A suspension insulator usually must withstand a working stress of about 10 kv and a flashover stress of 80 to 90 kv rms. Glass, in accordance with the curve in Mr. Shand's paper, would show that thickness of $^1/_{16}$ inch or even less would suffice to withstand this voltage, but an insulator having such a small cross section would be inadequate to meet the service conditions from a mechanical standpoint. Actually, the minimum thickness in suspension insulators is of the order of $^1/_4$ inch or more, and in the skirt, where unforeseen mechanical stresses may be incurred through handling and flying missiles, these sections may be an inch or more. Because of these thicknesses, very high dielectric strengths result. This is substantiated by our service records which show no evidence of a pure dielectric failure. Some insulator failures have occurred, but these have always been preceded by a mechanical breakage, which after it has occurred permits an electrical discharge to take place through the resultant crack in the glass.

Mr. Shand's paper clearly presents the detrimental effect of the presence of oil at high dielectric stresses on glass. The dielectric strength of glass appears to be very much lower in oil than in air. The dielectric strength of a material that is nonporous and chemically inert, such as glass, should not be affected by the media in which it is immersed. The decrease in dielectric strength must then be due to an action occurring within the oil rather than in the dielectric under test. Consideration of some of the fundamentals involved in breakdown shows that the reduction in apparent dielectric strength of the glass is indeed due to an action occurring in oil. Investigators on the dielectric breakdown of materials have pretty well established that the spark formation under oil may take place at speeds of the order of 10^8 centimeters per second. Photographs taken of the spark formation by so-called suppressed discharge means show this discharge to be needle-like formations with quite sharp points, a very small fraction of a millimeter in radius. A streamer of this nature progressing through the oil would be accompanied by the following conditions.

The speed of propagation of this streamer is so great and the viscosity of oil is so great that little or no space charge can be set up in front of the streamers, such as occurs in air. As a net result, the dielectric stress at the tip of the streamer will be of the order of 10,000,000 to 20,000,000 volts per centimeter. A streamer of this nature progressing through a gas such as air would be preceded by a discharge of quite large radius, which would reduce the dielectric stresses well below 100 kv per centimeter. A streamer of this type progressing through oil and suddenly impinging upon a solid dielectric such

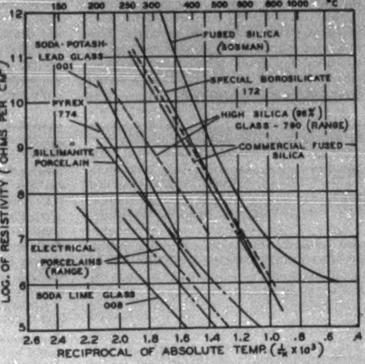


Figure 1. Electrical resistivity of glasses and porcelains at various temperatures

as glass, porcelain, or mica, would set up a dielectric stress within the material of 10,000,000 volts or more per centimeter. This stress is considerably above the dielectric strength of these materials and a puncture or failure in the immediate region where the streamer has struck the solid dielectric is bound to occur. Furthermore, the puncture voltage in this case would be dependent upon the voltage at which the streamer would form in the oil and not upon the dielectric strength of the material.

A few laboratory experiments have shown this to be the case. For example, immersing a suspension insulator in a dry oil, streamer formation started at 115 kv, and as the voltage increased the violence of the streamer action increased and puncture occurred a few seconds later. Moving picture photographs and observations of this streamer action have shown that the streamer gradually bored through the glass and a complete arc was established between the terminals. The puncture itself did not take place at the thinnest portions of the glass where the stress was highest, but in regions where the streamer conditions form first in the oil and then impinge upon the glass.

Treatment of the oil with ionizing liquids, such as some chlorides, permits the applied voltage to be raised to approximately 200 kv without streamer formation and no breakdown of the insulator resulted. However, if the voltage was raised to 220 kv the streamers formed in the oil and the insulator promptly punctured.

The detrimental effects of oil were again clearly demonstrated in the following tests:

> The head of a pin type insulator is cut down and the surface ground and polished so that the thickness of the dielectric between the top of the insulator and the pin hole is only $^1/_{16}$ inch. Another insulator having a dielectric thickness of over one inch is placed in oil. The two insulators are then connected in parallel and the voltage applied. The oil around the insulator started to show signs of distress at 100 kv and the insulator in the oil punctured at 130 kv. The thickness of the dielectric at the puncture point was of the order of one inch. That insulator was then disconnected and the other unit which had only $^1/_{16}$ inch dielectric thickness was raised on up to 175 kv where it flashed over. In this experiment the insulator having a dielectric thickness of only $^1/_{16}$ inch withstood 50 per cent more voltage than did a similar insulator having a one inch dielectric thickness but immersed in oil.

I am emphasizing the detrimental effects of oil in dielectric tests because there are still a number of specifications in use and in the process of formulation which call for testing the dielectric under oil. One such specification is the AIEE 41-A. If the results are used in a proper light they are informative in that they show that the material under test has a dielectric strength that lies above a certain level. The tests cannot say how much better the dielectric strength is than that value because another phenomenon has stepped in and prevented the stressing of the dielectric above that value. Therefore, comparisons between different dielectrics when the samples are immersed in oil should be made with only the greatest care and reservation.

E. B. Shand: Professor Dawes states his opinion to be that dielectric breakdown may be considerably more complex than is indicated by its simple division into the classification of disruptive and thermal failure. The author agrees with this opinion. It is believed that Rogowski first added the hypothesis that disruptive and thermal effects might combine to produce failure, and although recent investigations by Inge and Walther, von Hippel and others seem to discount this hypothesis in favor of the theory that these effects occur singly, the author feels that breakdown data from tests of thicker sections of glass and porcelain tend to give substantial support to the conception of Rogowski; and the paper has been written from this standpoint. Furthermore, the usual type of breakdown of glass and porcelain occurring under oil cannot be considered as either disruptive or thermal under the definitions given in my paper.

Professor Dawes believes that the breakdown of glass, under long-time conditions of test, is to a considerable extent thermal. Various opinions have been expressed on this point (for instance, see J. B. Whitehead, AIEE TRANSACTIONS, volume 55, 1936 (November section), page 1180) and the author feels that available information is not adequate for any precise clarification on this point. It is known, for instance, that the breakdown voltage of a piece of glass tested in vacuo tends to be materially lower than when tested in air—under some circumstances, much lower. Without doubt, this effect is caused by the reduction of heat dissipation from the test sample under vacuum conditions, and would mean that the failure would be largely thermal in character. In air, on the other hand, the corresponding failure might well be essentially disruptive. In the case of the work of Wagner, referred to by Professor Dawes, the conditions of test were widely different from those ordinarily met with in practice, and it is probable that the high-resistance electrodes, instead of dissipating heat from the test sample, actually supplied considerable quantities of heat to it. The values of dielectric strength of glasses as given by Wagner are from one-third to one-half those obtained by various other experimenters for long-time tests which may indicate that the conditions were somewhat comparable to those of tests in vacuo.

Recent tests on the dielectric strength of soda-lime glass reported by von Hippel and Maurer (*Physical Review*, May 1941, page 820), were made on specimens of the same order of thickness as those of Wagner. Thermal breakdown was observed for all temperatures above −50 degrees centigrade. The values given for dielectric strength at room temperature were considerably lower than those of Wagner. This may suggest that here also there was some unusual condition tending to reduce the thermal breakdown strength below that ordinarily obtained, although this is not readily apparent from the description given of the test arrangements.

In answer to the question regarding the conductivity of glass at various temperatures, figure 1 gives the resistivities of glasses over a range of temperatures. The high negative coefficient of resistivity with temperature of insulating materials is readily apparent. Various types of Corning glasses are identified by number. The data on porcelains, both sillimanite and electrical types, have been taken from Riddle ("Ceramic Insulators for Spark Plugs," S.A.E. convention, Detroit, 1940).

The author is indebted to Mr. Torok for his description of the action involved in the breakdown of glass and porcelain under oil. Mr. Torok gives estimates of the magnitude of the intense voltage gradients existing at the surface of the insulation at the point of impingement of the streamers in the oil. Such gradients are greater than the dielectric strength of any known material. The initial failure may occur across only an extremely thin surface layer of the insulation, while complete failure occurs only after a succession of many such surface failures, which becomes observable in the gradual progress of the arc through transparent glass.

Professor Dawes describes this action in terms of a bombardment of the insulation by heavy ions of the oil. He also describes the mechanically destructive effect of this bombardment.

Both Mr. Torok and Professor Dawes concur in the conception that this type of failure represents a breakdown of the ambient oil rather than an overstress of the insulation itself. This is further emphasis on the fact that oil-puncture tests of insulation should be used only with the greatest discrimination, and only with a full appreciation of their true significance. The author voices the hope that standard insulation test specifications will eventually be modified to recognize this condition.

The Determination of Magnitude and Phase Angle of Electrical Quantities

Discussion and author's closure of paper 41-89 by E. A. Walker, presented at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941, and published in AIEE TRANSACTIONS, 1941 (August section), pages 837–9.

C. L. Dawes (Harvard University, Cambridge, Mass.): In this paper Dr. Walker states that unavoidable variation in circuit parameters during a test makes it difficult to obtain a balance with a bridge. This is of course true, but frequently with a little skill a momentary balance may be obtained and the data are those prevailing at the instant of balance. However, the three-voltmeter method described by Dr. Walker is also subject to errors when the parameters vary with time. In order to determine correctly the instantaneous circuit conditions the three voltages must be determined simultaneously. This requires three voltmeters and three observers, who must make simultaneous readings with the fluctuating instruments. When the method is applied as described in the paper to measure the ratio and phase angles of current transformers, three different readings with a single instrument together with the accompanying resistance adjustments are necessary. Obviously, under these conditions, steady conditions in the circuit during the entire measuring interval are necessary for precise results.

The method shown in figure 2, in which a voltage E_2', 180 degrees out of phase with E_1 is used to close the polygon of voltages, is inherently inaccurate. For example, E_2' must exceed E_2 in magnitude and in figure 2 its value is about twice that of E_2. A study of the figure shows that if E_2' departs by a very small angle even from the 180 degree relation with E_1, a considerable percentage error in E_3 and in the supplement of the angle between E_1 and E_3 will occur. This same large error will therefore occur in the angle θ. A precise 180 degree relation obtained by using windings on the same transformer core, connected oppositely, is difficult to obtain without some precise compensating means. The author recognizes this difficulty and states that it will be discussed later in the article. I do not find this discussion.

The same difficulty in the matter of precision occurs in figure 3. A small departure of E_1 and E_2 from the 180 degree relation will introduce a substantial error in the angle δ, the angle of defect. Also a study of figure 3 shows that the method of balance is far from being critical. That is, I_1 can be changed considerably in magnitude without changing very much the value of I_D. However, since I_D is measured directly by the instrument D, the fact that the balance is not critical affects only the value of I_1, and not the value of the angle of defect δ, since I_1 does not enter directly in the computations.

As Dr. Walker states, however, the methods which he describes are quick and simple and under suitable conditions results sufficiently precise for many purposes can be obtained.

Eric A. Walker: Professor Dawes states that it is possible to make a momentary balance on a bridge if the device being tested is not stable. He infers that since there are three instruments to read, the same problem would be very much more difficult by the method described. This is not true.

Consider the three readings necessary for the determination of the capacitance and power factor of a condenser. The first reading is the line voltage which is maintained at a constant value. The second is the in-phase component of current which might vary from two per cent to four per cent. This variation of the in-phase component will not change the total current appreciably since the two add vectorially. So it can be said that in the practical application of the method only one reading will fluctuate and the balance is relatively easy.

Furthermore, if one is merely checking the power factor of a lot of standardized parts it is only necessary to vary C through the balancing range and make certain that I_D falls below the tolerance limit.

To check the angle between the two voltages E_1 and E_2 it is desirable to replace the unknown condenser C_x by a no-loss air condenser. If the two voltages are not exactly 180 degrees out of phase a zero reading cannot be obtained on the detector. There are two methods of overcoming this difficulty. One is to draw a correction curve of residual current I_D as a function of the total current I_1 and subtract this from the actual reading. A second method is to put a corrective network in the standard condenser circuit. This solution is not entirely satisfactory because no single network will serve for all currents and all voltages.

The fact that the balance is not a critical one is an advantage rather than a disadvantage. It must be remembered that the position of the dial on C_s is not important. The quantity to be measured is the smallest attainable value of current I_D and because the balance is not sharp this value is easily read which considerably reduces the possibility of making an error. Such a quality is of considerable importance in a field instrument.

Surge Characteristics of Two-Winding Transformers

Discussion and author's closure of paper 41-85 by Reinhold Rüdenberg, presented at the AIEE North Eastern District meeting, Rochester, N. Y., April 30–May 2, 1941, and published in AIEE TRANSACTIONS, 1941, pages 1136–44.

L. V. Bewley (Lehigh University, Bethlehem, Pa.): In this very interesting paper Dr. Rüdenberg has introduced a number of simplifying assumptions which have allowed him to reduce the problem of transformer surges to that of a pair of no-loss transmission lines, of which his four differential equations 17 and 19 are the specifications. I must take exception to his method of solution of these equations, which leads to erroneous results. His equation 29 was arrived at by utilizing only two of the four differential equations, and he then states, "Since this relation is valid for any value of i_1 and i_2, both the sides must disappear independently of each other." This, I think, is the crux of the matter—i_1 and i_2 are *not* independent, they are very definitely related in this mutually coupled circuit, and to find this relationship all *four* differential equations must be invoked.

Before proceeding to do so, let us consider an exactly analogous case of the lumped circuit shown in figure 1. The differential equations of this circuit, corresponding to Dr. Rüdenberg's equations 17 and 19, are:

$$e_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$e_2 = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

$$\frac{de_1}{dt} = P_1 i_1 + G i_2$$

$$\frac{de_2}{dt} = P_2 i_2 + G i_1$$

where

$$\begin{aligned} P_1 &= (C_2 + C_3)/(C_1C_2 + C_2C_3 + C_3C_1) \\ P_2 &= (C_1 + C_3)/(C_1C_2 + C_2C_3 + C_3C_1) \\ G &= C_3/(C_1C_2 + C_2C_3 + C_3C_1) \end{aligned}$$

Now if we disregard the second equation in each line—as Dr. Rüdenberg did in arriving at his (29)— we can differentiate the first, equate the second, and rearrange as follows:

$$L_1\frac{d^2i_1}{dt^2} - P_1 i_1 = -M\frac{d^2i_2}{dt^2} + G i_2$$

and this corresponds exactly to (29) of the paper. Applying the same argument—that i_1 and i_2 are independent, etc.—we would arrive at solutions for the natural frequencies of

$$\omega_1 = \sqrt{\frac{P_1}{L_1}} \qquad \omega_2 = \sqrt{\frac{G}{M}}$$

But these are *not* the true solutions for this circuit! Solving this circuit by utilizing all *four* differential equations (for example, by using three of them to eliminate e_1, e_2, i_2 and then writing the solution for i_1) we find for the actual natural frequencies

$$\omega^2 = \frac{(L_1P_2 + L_2P_1 - 2MG) \pm \sqrt{(L_1P_2 + L_2P_1 - 2MG)^2 - 4(L_1L_2 - M^2)(P_1P_2 - G^2)}}{2(L_1L_2 - M^2)}$$

The two frequencies represented by this equation reduce to ω_1 and ω_2 given above for the special case $P_1/L_1 = P_2/L_2 = G/M$, but this certainly is not the general solution, and results only by a special selection of circuit constants.

Let us now obtain the actual solution of Dr. Rüdenberg's problem. Instead of in-

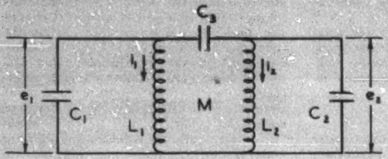


Figure 1

troducing two space parameters x_1 and x_2 along the high voltage and low voltage windings respectively, it is simpler and more straightforward to use but a single space variable x as the axial distance along the winding stack. Then instead of equations 17 and 19 of the paper we have:

$$-\frac{\partial e_1}{\partial x}=l_1\frac{\partial i_1}{\partial t}+m\frac{\partial i_2}{\partial t}$$

$$-\frac{\partial e_1}{\partial t}=p_1\frac{\partial i_1}{\partial x}+g\frac{\partial i_2}{\partial x}$$

$$-\frac{\partial e_2}{\partial x}=l_2\frac{\partial i_2}{\partial t}+m\frac{\partial i_1}{\partial t}$$

$$-\frac{\partial e_2}{\partial t}=p_2\frac{\partial i_2}{\partial x}+g\frac{\partial i_1}{\partial x}$$

in which now (not the same as given in the paper) $l_1=L_1/h$, $l_2=L_2/h$, $m=M/h$, $p_1=hP_1$, $p_2=hP_2$, $g=hG$, while L_1, L_2, M, P_1, P_2, G have the same meaning as in the paper; that is, are the constants for the entire winding. If we use y to represent any one of the four quantities e_1, e_2, i_1, i_2, then upon eliminating the other three between the four differential equations given above, there results:

$$\frac{\partial^4 y}{\partial x^4}-\left(\frac{l_1p_2+l_2p_1-2mg}{p_1p_2-g^2}\right)\frac{\partial^4 y}{\partial x^2\partial t^2}+\left(\frac{l_1l_2-m^2}{p_1p_2-g^2}\right)\frac{\partial^4 y}{\partial t^4}=0$$

This equation is satisfied by the wave functions $f(x+vt)$, if

$$v^2=\frac{(l_1p_2+l_2p_1-2mg)\pm\sqrt{(l_1p_2+l_2p_1-2mg)^2-4(l_1l_2-m^2)(p_1p_2-g^2)}}{2(l_1l_2-m^2)}$$

which has exactly the same form as the frequency equation given previously for the simple lumped circuit of figure 1. If the values of L_1, L_2, M, P_1, P_2, G, be those given in the paper of Dr. Rüdenberg by his equations 33, 35, 37, 39; then the velocity equation as given above, after some algebraic manipulation, takes the form

$$v^2=\frac{v_0^2}{\mu\epsilon}h^2\times\left[\frac{(a_1^2+a_2^2)-2a_1a_2(b_2d_1/b_1d_2)\pm(a_1-a_2)\sqrt{(a_1+a_2)^2-4a_1a_2(b_2d_1/b_1d_2)}}{2a_1^2a_2^2(1-b_2d_1/b_1d_2)}\right]$$

If the ratio $(b_2d_1/b_1d_2)<1$ may be neglected, the axial velocities become

$$v_a=\frac{v_0}{\sqrt{\mu\epsilon}}\,\frac{h}{a_1}\text{ and }v_b=\frac{v_0}{\sqrt{\mu\epsilon}}\,\frac{h}{a_2}$$

agreeing with equations 32 and 41 of Dr. Rüdenberg's paper; but now appearing in their proper light as approximations of the true axial velocities. If the proportions of the transformer are such as to permit this approximation, then the rates at which the multi-velocity electromagnetic field sweeps the conductors of the high voltage and low voltage windings are:

$$v_1\cong\frac{a_1}{h}\quad v_a=\frac{v_0}{\sqrt{\mu\epsilon}}$$

$$v_1'\cong\frac{a_1}{h}\quad v_b=\frac{v_0}{\sqrt{\mu\epsilon}}\,\frac{a_1}{a_2}\text{ for the high voltage}$$

$$v_2'\cong\frac{a_2}{h}\quad v_a=\frac{v_0}{\sqrt{\mu\epsilon}}\,\frac{a_2}{a_1}$$

$$v_2\cong\frac{a_2}{h}\quad v_b=\frac{v_0}{\sqrt{\mu\epsilon}}\text{ for the low voltage}$$

The expression v' is not to be construed as a velocity exceeding the velocity of light; it is merely the velocity of shear of the axial field as it cuts the conductors of the high voltage winding.

It will be observed that the above method of analysis gives all the results directly, and the limits within which approximations are permissible are defined. The analysis of section 4 of Dr. Rüdenberg's paper would lead one to suppose that there are two mathematical velocities in each winding and that all four of these are physically equal. Later on, in section 6, he admits an interaction between windings, discusses the physics of it, and concludes after all that each winding has one wave traveling at the velocity of light, and another induced wave following the axial field set up by the other winding. This concept appears as an approximation in the light of the analysis given in this discussion.

It should also be remarked that each wave has its own surge impedance, rather than the simple "single-conductor" surge impedances given in the paper.

I wish also to point out an error in the table comparing calculated and measured results. The calculated voltages are values of

$$\frac{e}{E}=\frac{\text{transmitted voltage}}{\text{traveling wave voltage}}$$

and as seen by equation 51 of the paper the line terminal voltage is taken as $2E$, due to the doubling by reflection. But in the case of the oscillograms shown in the closing discussion of the paper by Palueff and Hagenguth, the low voltage terminal voltage was 30 per cent of the high voltage *terminal* voltage, or only 15 per cent of the traveling wave voltage. Thus, there is in this case a 2:1 discrepancy between calculated and measured values.

R. Rüdenberg: Professor Bewley suggests in his discussion another kind of analysis than that of the paper, namely the well-known d'Alembert method, and he applies this analysis to another problem, as shown by his differential equations and his definitions of constants, both deviating from those of the paper. I doubt if for the problem treated in my paper this is the simplest method of arriving at a practical solution in a straightforward manner, particularly in view of the fact that Mr. Bewley has omitted in the discussion all his cumbersome intermediate calculations.

The conditions 26 and 28 of the paper under which my relatively simple solution is derived, and which Mr. Bewley apparently has overlooked, are rigorously fulfilled by the constants of any transformer with homogeneous electromagnetic space, such as are derived in section 5. This is proved by the considerations leading to equation 41. For slightly heterogeneous space such as in many actual transformers these symmetry conditions are very nearly fulfilled, as can be shown by applying appropriately to the two-winding case the viewpoints of section 3 of the paper. The need for publishing a more detailed but much more complicated solution is therefore not particularly great, although I am always in favor of refining a problem—if it is once solved in principle.

In my opinion it is often not advisable to use for the first attempt at clarification of a complicated problem, such as the performance of traveling waves in two-winding transformers, highly involved mathematical methods. I personally prefer to approach such problems by simplified conceptions as a guidance to the physical understanding. In the solution of engineering problems there is always the necessity for many approximations and omissions. It is a matter of intelligent choice which of them one will make in order to secure a simple but characteristic analysis. In the present case the simplification stressed by Mr. Bewley is not by any means the only one and is not even the most important one with respect to the wave velocities.

An Improved Frequency Meter for Commercial Power Frequencies

Discussion and author's closure of paper 41-108 by K. J. Knudsen, presented at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941, and published in AIEE TRANSACTIONS, 1941 (September section), pages 866–9.

B. E. Lenehan and **L. J. Lunas** (Westinghouse Electric and Manufacturing Company, Newark, N. J.): The self-contained, low energy frequency meter developed by Mr. Knudsen is a desirable contribution to the art. We are interested in the statement regarding resistance changes of the coils being responsible for errors due to ambient temperature change. The instrument utilizes a parallel resonant circuit with the difference in current of the inductance and capacitance branches flowing through the

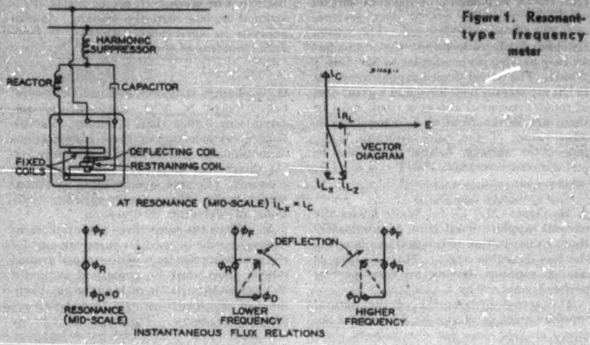


Figure 1. Resonant-type frequency meter

deflecting coil producing torque. Since the resistance component is in phase with the voltage and lags the capacitance component 90 degrees and leads the inductance component 90 degrees, no torque is set up by this component. Thus any change in resistance should produce no error in indications. The Westinghouse Company has built a frequency meter for a number of years utilizing a parallel resonant circuit with a conventional air core electrodynamic mechanism in which resistance changes produce no errors in indications. We wonder if the iron core mechanism might contribute to such errors.

The harmonic error mentioned might readily be reduced by introducing a series reactor of the proper value to suppress such higher frequencies. Such a reactor is included in the instrument shown in figure 1 of this discussion. In this way the influence of harmonics has been reduced to about one twentieth the value. For narrow frequency range instruments such suppressors are particularly effective. This suppressor might of course be omitted for the purpose of obtaining a self contained instrument.

A. J. Corson (General Electric Company, West Lynn, Mass.): It is interesting to note that, while modern frequency meters of various makes differ considerably in structural detail, they practically all operate on the same basic principle. This may be stated as a combination of an alternating current ratio meter and one or more impedance networks for obtaining the particular current-frequency relationship which this ratio meter requires. In this respect, the instrument described by Mr. Knudsen is similar to instruments now in use.

It would appear that the inductor-capacitor arrangement used in this instrument is open to serious question with respect to its susceptibility to wave form errors, due to suppression of harmonics in the inductive circuit and accentuation of the harmonics in the capacitative circuit. This is substantiated by figure 5 which shows very considerable errors given for the rather optimistic service conditions which have been assumed. It would appear that very distinct improvement could be obtained by using two parallel resonant circuits, with resonant points at say 45 and 75 cycles. The admittance of these two circuits for the third and higher harmonics of a 60-cycle fundamental will be very nearly the same with consequent elimination of the wave form error.

The paper as a whole appears to be rather general in character, lacking information as to values of torque gradient, mechanical factor of merit, damping, response time, and other criteria of the design. It is hoped that this information will be supplied by the authority, in closing the discussion.

K. J. Knudsen: As Messrs. B. E. Lenehan and L. J. Lunas point out, resistance changes in the deflecting coil will not alter the indication of the instrument. It should, however, be realized that the current in the inductive circuit does not lag behind the line current by 90 degrees due to the resistance of the field coils and the restoring moving coil; also the calibrating resistor in the capacitive circuit causes the current to lead by less than 90 degrees. This may be more clearly seen by studying figure 2 of this discussion, which is based on the formula: cos $\phi = R/Z$, ϕ being the angle of lag or lead, R is the ohmic resistance of the circuit, and Z the impedance. The magnitude of the current vector is V/Z, V being the voltage across the circuit.

The angle ϕ_1 is the resultant phase angle of the current in the deflecting moving coil, which causes no deflection, this current being 90 degrees out of phase with the magnetic field. ϕ_2 is the phase angle caused by increasing the ohmic resistance in the field coil circuit 25 per cent, and ϕ_3 is the phase angle necessary to maintain no torque relation in the deflecting moving coil. Actual tests show that by changing the resistance of the field circuit ten per cent the deflection changes 0.2 cycle, all other component parts being unchanged.

While it is true that connecting an inductance in series with the line greatly reduces the effect of wave form errors, it should be realized, that in so doing the voltage across the effective circuit is reduced by 80 per cent approximately, with a corresponding reduction in mechanical factor of merit, if third harmonic errors are to be reduced 20 times. The power factor of the complete instrument would at the same time be reduced to 20 per cent.

When a low power precision instrument is desired, it would be advisable to use a series tuned circuit in series with the line. The inductance having an impedance of 20 or more times the impedance of the frequency meter in this case, the impedance introduced would be no more than the ohmic resistance of the inductance at the reference frequency, and the power factor remain near 100 per cent.

Whenever iron core chokes are used, care should be exercised in order to prevent third harmonic distortion of the voltage wave due to high saturation of the core. The actual circuit used for calibration is shown in figure 3 of this discussion, it will be noted that an air core choke has been used. The choke has an inductance of 0.8 henry and a resistance of 15 ohms.

With respect to Mr. Corson's comments, it should be noted that the paper as far as basic principle of design is concerned does not claim a new circuit, in fact it appears to be the same as that shown in figure 1 of Messrs. Lenehan and Lunas' discussion. The new thing about it is the elimination of an external reactor in the tuned field circuit and a greatly decreased power consumption.

It is seen from figure 5 of the paper that a

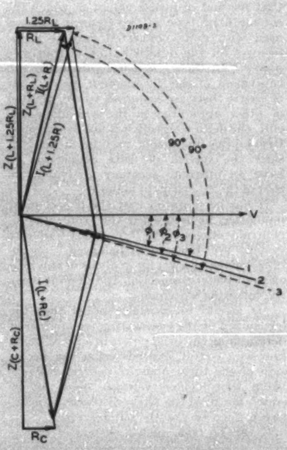


Figure 2. Vectorial diagram—current and impedance relation

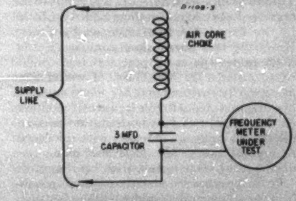


Figure 3. Diagram showing series-tuned harmonic eliminator

15 per cent third harmonic distortion is permissible before the consequent error exceeds 0.1 cycle, and as I have yet to see a commercial power circuit with a third harmonic voltage component greater than 15 per cent, I am in doubt as to the seriousness of the error. Should such a circuit be encountered, it would be a simple matter to insert a choke or a series tuned circuit in series with the line, as previously discussed. As far as a double tuned field is concerned it is possible that lower errors would be found. It would however be necessary to use two capacitors and considerably greater difficulty would be encountered in calibration, it is also possible that temperature errors would be difficult to eliminate because one circuit is operated on the high side of the peak and the other on the low side.

I am sorry that I forgot to add the vital data pertaining to criterion of design and I make amend by stating them here. On a five-inch instrument, which was at hand for tests the results were as follows:

Restoring torque at center of scale (point of lowest restoring torque): 0.01425 millimeter-gram per degree.

Weight of moving element: 1.02 gram.

Torque to weight (in $1^{1}/_{2}$ power) ratio: 1.25 for full scale deflection. Torque in this case considered uniform and same as at center of scale. Units being millimeter-grams, 90 degrees and grams.

Period: 1 second.

Damping factor: 5.2, this can be changed at will by increasing or decreasing the thickness of the damping vane.

Response time: 1.7 second.

The instrument indication changed 0.03 cycle at 60 cycles after 50 hours of vibration at 1,800 vpm. Test as described in SQ-78-A section IV-11.

It is my hope that the closure is satisfactory to the gentlemen who have shown interest by discussing the paper, for which interest I am grateful.

Effect of Sapphire-Crystal Orientation on the Wear of Watt-Hour-Meter Bearings

Discussion and author's closure of paper 41-106 by J. H. Goss, presented at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941, and published in AIEE TRANSACTIONS, 1941 (August section), pages 811–14.

G. G. Waite (Sangamo Company Limited, Toronto, Ontario, Can.): It is perhaps to be regretted that Mr. Goss has added so little to the knowledge first reported by Stott. However, this paper does confirm Stott's work under the more practical test conditions outlined by Mr. Goss. I would like particularly to commend Mr. Goss's use of his diagram 1. All too frequently writers on the subject of jewel orientation merely manage to confuse the reader with reference to crystal axes. Mr. Goss has made the matter very clear indeed with his diagram.

It is impossible to cut oriented jewels from small natural sapphire pebbles because of the cost of determining the optical axis for each small pebble. I think, however, the fact, that a very high proportion of natural sapphires will turn out to have θ = zero, should have been mentioned. The reason for this is easily understood. Corundum has an easy parting perpendicular to Mr. Goss's *C* axis. The barrel shaped sapphire crystals in the old stream beds have had a great deal of rough treatment and as a result most of them are in the form of hexagonal plates when found. These, quite naturally, in the process of cutting, orient themselves to give θ = zero.

J. H. Goss: Mr. G. G. Waite states the natural sapphire jewel cups predominately show a distribution of orientations favoring the less desirable angle. This, he says, is due to sapphire having an easy parting perpendicular to the *C* axis.

The actual distribution of the natural sapphire cups as shown by figure 4 for the group studied does not conclusively show this. As far as I am able to determine, sapphire has no desired plane of cleavage or parting and is listed as having a conchoidal fracture.

Relative Accuracy of Three-Phase Metering Combinations

Discussion and author's closure of paper 41-125 by C. T. Weller, presented at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941, and published in AIEE TRANSACTIONS, 1941 (October section), pages 925–9.

Perry A. Borden (The Bristol Company, Waterbury, Conn.): It is noted that the author of this paper makes no reference to that method of measurement in which the conventional type of two-element meter is used with delta connected current transformers for the determination of four-wire power. In theory at least this is the full equivalent of the $2^{1}/_{2}$-element meter; but of course there are instances where, because of other apparatus in the secondary circuit, the delta connection is not permissible. Beyond this limitation, does the author consider the system of delta connection to have an inherent accuracy equivalent to that of the $2^{1}/_{2}$-element meter?

It has been stated that the $2^{1}/_{2}$-element meter may be used where the "three line-to-neutral voltages of the three-phase four-wire system are fairly well balanced." Stubbings[1] gives us a vector analysis of the $2^{1}/_{2}$-element meter connection and upon it bases the following statement:

"It should be carefully noted that the legitimacy of the two-wattmeter method of determining the total power in a three-phase four-wire circuit depends upon the vector sum of the three phase-to-neutral voltages being zero. These voltages need not be equal, nor need the phase displacements between them be exactly 120 degrees."

If this be true, such a system of metering four-wire loads need not be nearly so restricted as has been the custom. Is this analysis in agreement with Mr. Weller's understanding of the problem?

G. S. Lunge (General Electric Company, Schenectady, N. Y.): Some of the considerations, other than accuracy, that influence the selection of these combinations of meters and instrument transformers are: (1) directional ground relaying needs, (2) economy, (3) safe operation of potential transformers, (4) previously existing conditions, (5) detection of diversion.

1. When the same three potential transformers are to be used for metering and for the polarizing windings of directional ground relays, they must be connected grounded Y-Y even though there may be no likelihood of any line-to-neutral loads being applied to the circuit. This may result in the selection of cases 5 or 6, with potential transformer primary neutral grounded in each case. The relay windings would be connected in the corner of the broken delta secondary winding of a grounded Y-broken delta connected assembly of three auxiliary potential transformers, connected to the secondary windings of the main potential transformers (figure 1 of this discussion).

2. In the case of high voltage circuits, economy may dictate the use of three single bushing potential transformers with primary neutral connection grounded, rather than two potential transformers, each with two bushings and designed for line-to-line connection. This might result in the selection of cases 4, 5, or 6, with grounded potential transformer primary neutral.

3. Unless the potential transformers are suitable for the connection contemplated, their safety may make it impossible to use cases 4, 10, or 13, with the potential transformer primary neutral grounded in each case. For these three connections to be safely usable, the potential transformers must be designed with low flux density at line-to-neutral voltage, to the extent of being suitable for line-to-line connection permanently without damage. The turn ratio of such potential transformers would, of course, not be suitable for line-to-line connection, since for use with two and one-half-element or three-element meters the potential transformer turn ratio must be such as to give 115 volt secondary voltage when the potential transformer is connected from line-to-neutral of the primary circuit. Except in the case of high-voltage potential transformers of very recent manufacture, this condition of exceptionally low-flux density may not necessarily be met.

4. Previously existing conditions may make it logical to adopt a combination giving less than the highest grade accuracy, e.g., cases 2 or 3 on systems to which ground-fault neutralizers have been added.

5. To prevent diversion of energy, it may be advisable to select cases 8, 11, or 14 in order that there may be a current transformer in each primary lead of the circuit. Thus even when only the neutral of the source is grounded, and not that of the load, this consideration may require resorting to a two and one-half-element or three-element meter, with corresponding instrument transformer connections, in preference to the combinations of cases 2 and 5.

Circuits to be grounded via ground-fault

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[1] COMMERCIAL A-C MEASUREMENTS, G. W. Stubbings. D. Van Nostrand Company, 1930, chapter 2, pages 38 and 39.

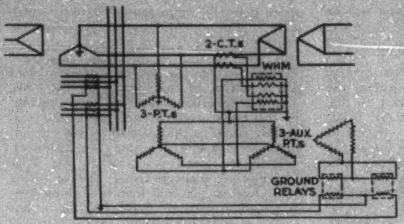


Figure 1 (Left). Joint use of Y-Y potential transformers for metering and ground relaying

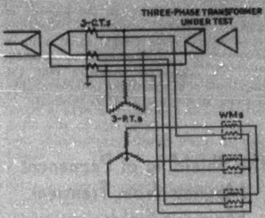


Figure 2 (Right). Proposed ASA recommended connections for measuring three-phase transformer excitation loss

neutralizers must have been: (*A*) formerly operated with isolated neutral, (*B*) formerly operated with grounded neutral, (*C*) built initially for operation with ground-fault neutralizer neutral grounding.

A. For this group, the existing metering will undoubtedly be on a three-phase three-wire basis, using either two-element or, in rare instances, three-element meters, depending on the degree of accuracy required when very low power factor loads are to be metered. No change in meter equipment or performance should be involved.

B. Solidly or low impedance grounded neutral circuits may have been metered as case 2, i.e., three-phase three-wire metering. If so, no change will be necessary. If, however, the circuit was metered as a three-phase four-wire circuit, a two and one-half-element or three-element meter will have been used. For important loads, undoubtedly the three-element meter would have been used originally (case 14), and no change will be necessary. If two and one-half-element meters were used, these should be changed to the three-element type.

C. For this situation, this paper rightly advocates the use of three-element meters, with three potential and three current transformers, though, except for very low power factor loads, comparable accuracy can be obtained with two-element meters connected to two potential transformers and two current transformers. Energy diversion detection need not be considered, for line-to-neutral single phase loading is not feasible when neutral impedance as high as that of a ground-fault neutralizer is involved.

In applying information in this paper to cases involving indicating or recording wattmeters, it should be borne in mind that they are not available in three-element types, so that the choice lies between two-element and two and one-half-element instruments.

Special considerations may outweigh other factors governing the selection of an appropriate metering combination. Case 10, with isolated potential transformer primary neutral (figure 2 of the discussion), is generally considered preferable for low power factor circuits such as are encountered in the measurement of transformer excitation losses (see clause 10.088 of proposed ASA transformer test code C57.2). Were the potential transformer secondary and instrument potential circuit neutrals to be interconnected, the instrument potential coils would be subjected to voltages containing a prominent third harmonic voltage which is not necessarily present in the system line-to-neutral voltage but is impressed upon the meter simply due to the instrument transformer connections. Case 7 could be used with equal accuracy instead.

It is only very rarely that the same set of potential transformers will be used both for ground detection purposes and for metering. However, as in cases 4, 5, and 6, it is possible to use the line-to-line secondary voltages of three such line-to-line voltage rated Y-Y connected potential transformers for two-element meters.

C. T. Weller: The subject of metering accuracy is a broad one, so much so that a paper on but one phase of it must omit much interesting and pertinent material if a reasonable length is to be achieved. That such is the case with the present paper is evident from the discussions of it since it was first proposed. It might be in order, therefore, to outline briefly some of the underlying ideas.

In a paper presented in 1930 (reference 1), the author dealt with the Y-connection of potential transformers from the operating standpoint; metering problems were not discussed. The need for a supplementary paper written from the standpoint of metering accuracy was not recognized until recently, when it became apparent that some effects of the Y-connection on the metering accuracy were not fully appreciated. A similar lack of appreciation has long been evident with respect to some of the so-called "secondary interconnections" of instrument transformers and watthour meters. This accounts for the emphasis in the paper upon potential-transformer connections and upon secondary interconnections. However, no such need was apparent with respect to current transformers, so these are dealt with only in their simplest form.

The primary object was to summarize in one table appraisals of several three-phase metering combinations under various conditions from the accuracy standpoint. In order to present a general picture, the appraisals are in general terms, for which relative accuracy was chosen, using the best metering practice as a standard of reference. The table nominally contains 17 cases, but six of these are double while two are appraised in the accompanying text, thus making a total of 25 cases; these should be sufficient to illustrate the basis of grading used and thus make possible its extension to other cases not included in the paper. The impracticability of estimating the metering accuracy to be expected in the 25 cases will be appreciated by those who have attempted to make such estimates for actual installations, where it is often difficult to obtain sufficient information for the purpose; some apparently minor details may considerably affect the result. Paraphrasing an old saying, it might well be pointed out that "Too many hedges spoil the summary."

Explanations of the purposes for which the several combinations are used or proposed and of the theories underlying some of the phenomena mentioned were deliberately omitted from the text to emphasize the subject of metering accuracy as well as to conserve space. The schematic diagrams in figure 1 of the paper represent an attempt to combine voltage-vector diagrams with straight-forward connection diagrams having no conductor cross-overs.

In his discussion, Mr. P. A. Borden mentioned the omission from the paper of the metering combination consisting of a two-element meter and three delta-connected current transformers for a three-phase, four-wire system; the omission has been explained. This combination must be assigned relative-accuracy grade C, because of the current-transformer interconnections, thus bringing it definitely below the two and one-half-element combination, which shares with it the uncertainty as to the one unmeasured phase voltage but is otherwise more straightforward. This unmeasured phase voltage will, of course, be correctly represented whenever the vector sum of the three line-to-neutral voltages is zero as Stubbings has pointed out. However, what assurance is there that this will always be so, even next to a power-transformer bank, particularly when large single-phase loads may be applied without warning? In metering, no possibilities of error can safely be overlooked and this includes erroneous tap connections of the power units.

Mr. G. R. Lunge mentioned various reasons for choosing some of the metering combinations in preference to others, such as relaying, economy, safety, previous condition of servitude, and protection against diversion. These are both interesting and pertinent but outside the scope of the paper. He did, however, discuss some cases where either a two- or three-element metering combination could be used, so the choice might depend upon the relative accuracy obtainable. As to this, it should be clear from the paper that the 3-element metering combination has a slight edge over the 2-element combination for such three-phase, three-wire systems. Since the 3-element meter also is suitable for three phase-four-wire systems, it is the most generally applicable of the watthour meters considered.

In conclusion, it might be stated that a secondary object of the paper was to stimulate thought in metering problems with the hope that an increase in the number of grade A metering installations ultimately would result. The thought so far inspired as shown by the discussions is encouraging. However, experience indicates that the magnitude of any resulting improvement in metering accuracy will be indeterminate.

The Shielding of Permanent Magnets From Transient Magnetic Fields

Discussion and author's closure of paper 41-112 by G. J. Wey, presented at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941, and published in AIEE TRANSACTIONS, 1941 (September section), pages 875–7.

A. M. Opsahl (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): 1. The development of a watthour meter damping magnet that is shielded against change due to lightning surge currents was the best way of solving the problem. Lightning discharges seek the best path to ground. For safety reasons, each secondary service is required to have an effective ground on one service conductor, preferably a connection to a water pipe system. A severe lightning discharge on either the primary or secondary circuit cannot be prevented from discharging surge current through this grounded service conductor which passes close enough to the meter to be a source of demagnetizing flux.

2. Any means of bypassing the lightning current around the meter would require quite major changes in standard practice of house wiring and code requirements. The best inherent protection against failure of meter coils due to voltage surges is obtained by the present practice of closely coupling grounded and ungrounded service conductors, not separating them.

H. L. Thomson (The Hartford Electric Light Company, Hartford, Conn.): Mr. Wey, in introducing the subject of a very ingenious method of shielding permanent magnets from transient magnetic fields by rapid yet effective copper plating on steel, has referred to the broader subject—the effect of lightning on watthour meters in the field. The principal object of this discussion is to try to prevent those in the field from accepting wrong impressions of conditions in the field.

Mention was made of isolated instances of weakened watthour meter magnets having been found by electric utility companies. Instances where watthour meters have been found in excess of four per cent fast may be spoken of as somewhat isolated. Four per cent was the original figure chosen in 1931 because it eliminated practically all other known causes of fast meters and, therefore, was considered as a good indication of lightning surge demagnetization. This is probably what Mr. Wey refers to. From our later studies, it is apparent that lightning surge demagnetization has caused slow meters, also correct and fast meters, to run faster. Those meters speeded up from one per cent to four per cent must not be spoken of or considered as isolated instances.

Mr. Wey, in quoting McEachron and McMorris on magnitude of current surges in the ground lead of distribution system primary lightning arresters, considers the use of these figures when applied to distribution secondaries as extremely pessimistic. In the absence of an extensive study, that may be true, but as far as the watthour meter is concerned, evidence from the field indicates a large percentage of demagnetizing strokes directly to the building housing the meter or to nearby objects. In this way, one meter and its permanent magnets may be subjected to the demagnetizing effect of the total current of the surge before it reaches the secondary system where it may then divide to a multiplicity of grounds or arresters.

Meter engineers are not as familiar as others with the nomenclature of lightning arrester current magnitude probability data and may accept and act on a wrong impression. Although admittedly pessimistic, if the data may, for example, be applied directly to watthour meters, three and one-third per cent of rural meters may be subjected to a ten thousand ampere surge each year, or twenty per cent of the meters during a normal six-year test period.

Now that we have found reasonably satisfactory and inexpensive solutions to the problem of weakened magnets, it would be foolish not to protect new meters and substitute them for older meters in the most exposed areas. However, to rebuild and similarly protect meters in service in exposed locations on any grand scale would be extremely pessimistic. First, we should know more about lightning exposure itself, and we also should know more about the other possible measures which may be taken to alleviate the fast meter difficulty as far as the customer is concerned.

Mr. Wey has selected certain wave shapes for his test surge currents. Due to the inherent nature of copper shielding, the shielding effect changes profoundly with different wave shapes. Those currents rising to crest more rapidly produce more shielding effect and vice versa. This effect, although far from extreme in this particular case, is demonstrated in the reverse curve effects noted in Mr. Wey's curves 4 and 5, figure 1 of the paper, where the wave shape is changed as the current magnitude is increased. It might also be said that, if it were necessary, Alnico magnets could also be copper plated, improving them still farther against surges. This, and the effect of change in wave shape, are shown very clearly in "Braking Magnets for Watthour Meters" by I. F. Kinnard and J. H. Goss, paper 40-128, presented at the AIEE Pacific Coast convention, August 1940 (AIEE TRANSACTIONS, volume 60, 1941, pages 431–9).

Speaking of wave shapes, it is my understanding that in the initial unsuccessful laboratory tests, an attempt was made to pass surge currents through the current coils of a meter, but, due to the wave shape and the high impedance of the coils, the current flashed across the terminals and only a small portion actually went through the coils. Surge currents through the service wiring seemed, therefore, to be more effective. Perhaps this is the experience Mr. Wey refers to. Later, however, when more powerful surge generators were available, it was possible to generate surge currents with a wave form whose rise to crest was slow enough so that the impedance drop across the coils was insufficient to cause any shunting of the current by the coils. The demagnetizing effect of surge current in the meter coils proved to be approximately equal to that caused by the current in the service wire, doubling the total effect.

From the meter design angle, Mr. Wey has plotted demagnetizing and shielding effect against per cent loss in working flux. When these data are applied to loss in accuracy of watthour meters, one per cent loss in working flux can be considered as equal to approximately two per cent loss in meter accuracy.

Mr. Wey should be complimented very highly for having brought to light a very ingenious method of applying copper shielding, making possible its use in connection with watthour meter permanent magnets, against the common enemy—lightning surges. He should be complimented even more highly perhaps for contributing to the discussion of the broader subject of lightning damage to watthour meters, and it is hoped that the study and discussion will broaden and continue.

A. L. Nylander (General Electric Company, West Lynn, Mass.): In connection with surge demagnetization, the following experiment is of interest. A length of chrome meter magnet bar stock was subjected to a uniform surge field in a direction opposite to the magnet flux. A 15–45-microsecond surge was used and measurements of total flux of the bar magnet made by means of a coil and ballistic galvanometer before and after each surge application. With the particular test set-up, it was found that after the sixth surge, the total flux of the bar magnet was zero. Taking precautions not to disturb the magnetic condition of the sample, thin layers of the magnet material were successively ground off the surface and following each grinding, the total flux was measured as above. After correcting the total flux to compensate for the change in the ratio of length to cross section of the bar magnet, a curve was plotted showing the maximum flux density of the magnet versus the distance in from the sample surface. This plot showed that following this application of the six surges, the outer skin of the magnet was actually saturated in the direction of the surge field while at the center of the magnet the flux was in the direction of the original magnetization and still at more than half of its original value. This is evidence that the magnetic state of the sample depends upon the extent to which the field is able to penetrate into the material.

By making use of the fact that the penetration of the field into the magnet is a function directly of the square root of the resistivity and inversely as the square root of the product of the permeability and the disturbing field frequency, Transient Electric Phenomena and Oscillations (C. P. Steinmetz, page 365), it has been possible to predict approximately the behavior of a given material with respect to any other material under a given surge. Also, if the direct current demagnetization curve of the material and both the surge and d-c demagnetization of the reference material are known, surge demagnetization of the material under question can be determined. Since the permeability varies between wide limits according to position on the hysteresis loop, this value must be judiciously chosen in the operating region. The following shows the results of such calculations as compared with actual test values. The figures given are the ratios of the peak-surge field strength to the d-c field strength required to reduce the total flux of a bar magnet—designed to give maximum useful flux per unit volume of material to zero. (Magnet Steels and Permanent Magnets, K. L. Scott. AIEE TRANSACTIONS, volume 51, 1932, page 412).

	Chromium Steel	Cobalt Steel	Alnico II	Sintered Oxide
Ratio of 15–45 microsecond surge to d-c				
Calculated (reference)	15.5	6.9	3.7	1.00
Test	15.5	5.3	3.2	1.0
Ratio of 28–86 microsecond surge to d-c				
Calculated (reference)	7.1	3.16	1.70	1.00
Test	7.1	3.25	1.52	1.0
Ratio of 84–215 microsecond surge to d-c				
Calculated (reference)	4.19	1.87	1.02	1.00
Test	4.19	2.23	1.25	1.0

Use of this method in cases where only approximate values are desired saves considerable time and expense in surge testing.

The author shows an interesting curve on the increased demagnetization due to repeated surges. In tests under comparable conditions, these results check very closely with other tests made, giving a ratio of ultimate to single surge demagnetization of approximately 3.5 to 1. Further tests on the same material without copper plating and on several other materials give different results. The same material without copper plating has a lower ratio of ultimate to single surge demagnetization of approximately 2.3 to 1, while cobalt steel and Alnico have similar ratios respectively of 1.9 to 1 and 1.7 to 1. These ratios are true only under specific test conditions and any change in surge duration will change these ratios and the shape of the curve of demagnetization versus number of surges. For unplated chromium steel, for instance, in going from a 20–50 to a 90–220 microsecond surge, the ratio drops from 2.3 to 1.7 approximately. This change in ratio may be attributed to increased penetration of the field into the magnet at longer durations resulting in a lower inter-layer magnetic gradient and consequently, less magnetomotive force available to cause magnetic redistribution.

George J. Wey: The experiment described by Mr. Nylander is particularly interesting in that it explains how a transient field can reduce the external flux of a bar magnet to zero without penetrating to the inner layers of the magnet material. The action of the transient field, therefore, is to short circuit the external flux path of the bar magnet by reversing the magnetism of the outer layers.

The table of ratios of peak surge field strength to direct current field strength required to reduce to zero the external fields of bar magnets of various materials is of interest in emphasizing that chrome steel, due to its higher permeability and lower resistivity, offers more resistance to demagnetization by transient fields than a comparison of its coercive force (H_c) with other magnet materials would indicate.

It is quite true, as Mr. Nylander points out, that longer surge durations will tend to lower the ratio of ultimate to single surge demagnetization and thus change the shape of the curve in figure 7 of the paper.

Mr. Thomson, in correcting the impression gained from the paper, that the number of meters speeded up by lightning discharges is negligible, creates the impression that twenty percent of the meters in a rural area may be subjected to a 10,000 ampere surge in the course of a six year testing period. If this were the case, twenty percent of the meters in rural areas would be found from six to eight per cent fast during a six year period and protection on a "grand scale" would be justified, which conclusion Mr. Thomson denies. In stating that it was desirable to protect meters in service as well as new meters, I merely want to point out that in order to reduce *materially* the number of damaged meters found on a system, a large percentage of the installations must be protected. Therefore, when protecting a large number of installations, it is desirable to protect existing meters without having to buy new ones.

Mr. Thomson is quite correct in pointing out that Alnico magnets can also be copper plated. Such a solution could hardly be justified economically, since it is much easier to copper plate the existing chrome magnets and avoid the additional expense incident to developing and tooling a new magnet.

The effect of the field from the current winding of a five ampere meter is approximately one-half that from the service wires. For a 15-ampere meter the effect is from twelve to fifteen percent of that from the service wiring.

Among the first solutions proposed to the problem of demagnetization was one in which the meter was offset about 16 inches from the vertical run of service conduit, thus putting the meter out of reach of the field from the service wires. As Mr. Opsahl points out, this practice requires major changes in house wiring practice, is unsightly, and expensive.

Impulse Strength as a Measure of Cable Quality

Discussion and author's closure of paper 41-131 by L. I. Komives, presented at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941, and published in AIEE TRANSACTIONS, 1941 (October section), pages 929–34.

T. F. Peterson (American Steel and Wire Company, Worcester, Mass.): I admire the courage shown by Mr. Komives and his associates in embarking on a test program which might possibly yield data that could be used as a measure of cable quality. Impulse strength seems to provide a means for the instantaneous integration of many cable characteristics which, in one manner or another, determine cable serviceability. These include such elements as mechanical uniformity of application of paper, perfection of saturation, characteristics of electrodes, etc. However, the tests give no further indications as to the ability of the cable to stand prolonged application of voltage, temperature load cycles, mechanical handling, etc.

Dr. Whitehead has suggested that only one characteristic should be studied at a time, and that all other variables should be held fixed during such studies. While this would be the correct approach to the problem of measuring cable quality, it is submitted that we would never reach an end point because of the long time necessary for testing and because the cable art would change at a rate more rapid than such test procedure could follow.

In view of the fact that the authors have obtained test data integrating many cable characteristics, perhaps we can obtain rather rapid over-all results by following the change in impulse strength of cable with testing designed to bring out other cable characteristics. For example, if impulse strength tests were made at intervals during high voltage load cycle tests on cable or over a period of cable operation, the trend in the impulse tests for a given set of conditions simulating operation might be a very valuable indicator of cable quality. I am wondering if the author contemplates any such studies. Perhaps they have been made. In this event, a discussion of results would be appreciated.

Wm. A. Del Mar (Phelps Dodge Copper Products Corporation, Yonkers, N. Y.):

SIGNIFICANT STRESS

Mr. Komives has given us valuable data on twenty-six impulse breakdown tests of paper cables which may be divided into the following groups—

Solid type	11
Ionization free types	9
Low gas pressure	6
	26

It we average the maximum stresses (as given by Mr. Komives) for all of the cables of each type and find the mean divergencies from these averages and then do the same thing for average stress we obtain the following results:

	G Max	G Avg
Solid type	917	13.0
Ionization free types	8.5	13.0

We may eliminate the low gas pressure type from this comparison as only one size was tested, so that the *G* maximum and *G* average were necessarily in the same ratio.

This comparison gives another indication that maximum stress is a more important factor in impulse tests than average stress for both types of cable. This conclusion is not materially affected by any corrections in the stresses that I may offer later.

TAPING

In table I of the paper the taping for cable *G* is described as very irregular. However, the Mildner diagram for this cable shows greater regularity than for cable *H* which is described as very good. The mean divergence from the mean overlay is 60 per cent for cable *G* and 67 per cent for cable *H*. The trouble with cable *G* appears to be registrations due to too short an overlay.

STRESS WITH STRANDED CONDUCTORS

The basis of the formula for stress at the surface of stranded conductors is not given and would be an interesting contribution as the only theoretical treatment of this subject that I have seen is that of Levi-Civita, summarized by Jona in his International Electrical Congress paper of 1904, as a formula which is exceedingly cumbersome and impracticable.

The formula used by Mr. Komives may be converted into one for ratio of stresses with and without stranding by dividing by—

$$\frac{E}{2.3r \log_{10} R/r}$$

We then obtain the following formula for the ratio—

$$\frac{\text{stress with stranding}}{\text{stress with smooth conductor}} = \frac{1}{\dfrac{0.0904}{n \log_{10} R/r} + 0.759}$$

Taking $n = \infty$, the ratio will be 1.32. Obviously, it should be 1.00. The Levi-Civita formula suffers from the same defects of being wrong at the limits.

Neither the formula given by Mr. Komives nor that of Levi-Civita checks with Atkinson's experimental data of 1919, the ratio of stresses for stranded to solid being, in both cases, greater than Atkinson's.

The comparative figures are as follows for the specific cables described, the "Atkinson" figures being derived by interpolation from the data given on page 1008 of AIEE TRANSACTIONS, volume 38, 1919:

n	W/2r	Komives	Atkinson
7	0.54	1.25	1.19
18 or 19	0.65	1.30	1.19
24 or 25	0.44	1.30	
37	0.36	1.30	

It is interesting to note that Atkinson's experimental ratio for 19 strands was the same as for seven strands although Levi-Civita's formula, like that used by Mr. Komives, indicates a rising ratio with increasing number of strands. As the ratio for $n = \infty$ must be one, it is probable that the ratio for more than 19 strands will be not greater than for 19. Indeed, it is the general experience that cables with a larger number of strands are not inferior in dielectric strength to those with a small number. It would seem that a ratio of 1.19 should be about right for all practical strandings.

If we apply this factor to the figures in table I of the paper we obtain the average values shown in table I of the discussion for cables *A*, *E*, *F*, *G*, *H*, *I*, and *J*.

STRESS IN SECTOR CABLES

The use of shoulder radius for computing maximum stresses in shielded sector cables is of doubtful accuracy. We made a large scale (30 to 1) model of a smooth faced sector and determined the stresses by the electrolytic method. The results were as shown in figure 1 of the discussion and compare as follows with calculations from shoulder radius:

	Experimental Sample	
	Meas'd	Calculated
r (radius of shoulder)	154	154
w (insulation wall)	313	313
Ratio $G_{max.}/G_{avg.}$	1.70	1.83

$G_{avg.}$ here means E/w and not the average from the curve of figure 1.

This shows that the calculated value using *r* makes the stress about eight per cent high in this case, so that if this ratio be assumed to hold generally, the average values for cables *B*, *C*, and *D* would be reduced as shown in table I.

RECALCULATED MAXIMUM STRESSES

The maximum stresses recalculated as indicated above would be as shown in table I, which gives the average for each cable.

Table I

Cable	Komives	Recalculated Values
A	2,690	2,560
B	2,070	1,900
C	2,450	2,250
D	3,110	2,860
E	3,120	2,620
F	3,350	2,810
G	2,540	2,130
H	3,290	2,770
I	2,770	2,260
J	2,980	2,500

These new values do not invalidate any of Mr. Komives' conclusions except that they cast doubt on the high "ceiling" of 3,300 volts/mil which he suggests.

The discussion is offered merely to save us from pitfalls in future work of the same character, as the laws governing impulse strength will not be fully understood until many more data are available. In the meanwhile we owe a debt of gratitude to Mr. Komives and his associates for giving us the latest information as it becomes available.

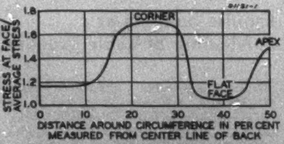


Figure 1. Ratio of stress at conductor face to average stress, shielded conductor, type H, sector cable, 350,000 cir mils, 20/64-inch paper

By average stress is meant voltage from conductor to ground divided by insulation thickness

J. A. Scott (General Electric Company, Schenectady, N. Y.): Mr. Komives' paper is a most welcome and valuable contribution to the only too scarce literature and accumulation of data on the impulse strength of cable insulation.

I am interested in several aspects of this.

1. He presents a large number of values of calculated maximum stress at breakdown. These range from 1,900 volts per mil to 3,500 volts per mil. The interval between 2,600–2,900 volts per mil contains a greater number of these values than any other similar interval. This interval, too, is near the center of the range presented; it contains the average value (2,830), so why is this not the most significant value rather than the 3,200–3,400 selected by the author? Variation is caused by many factors; combinations of unfavorable factors produce values below the average, and combinations of favorable factors produce values above average. Where the property measured varies 2 to 1 as it does in the data presented by Mr. Komives, many values are needed to produce reliable averages. The 66 cases cited are only too small.

2. Most of Mr. Komives' discussion of the influence of such factors as taping, degree of impregnation, and smoothness of electrode is supported, as he points out, by selected instances involving three or at most four test values. I would prefer to consider from such meager data that inferences only may be drawn. These inferences are most important, and we especially need more such data particularly with respect to the performance of cable after years of service. The Utilities are to be encouraged whenever possible to take more such data, and to correlate it with their experience with cable failures caused by lightning. To my knowledge we have no reliable data on the number of such failures.

3. The maximum stress argument crops up again, this time with more logic because impulse failure seems to be more of a disruptive thing than does our ordinary long time 60-cycle endurance or load cycle test. Short time 60-cycle breakdown values have never been highly considered as criterion of cable quality. They are in the nature of necessary conditions but are by no means regarded as sufficient proof of quality. Average stresses, cylindric maximum stresses, and strand maximum stresses are all geometrically related. Since our service stresses are determined empirically by experience, when we use one of these stresses to the exclusion of the other, we neglect only the variation of the geometric ratios existing between these stresses over the commercial range of cable sizes and insulations. The variation in the ratio of cylindric maximum stress to average stress amounts to something like 40 per cent over the commercial range of sizes, which is considerably less than the 100 per cent range of breakdown values given in Mr. Komives' tabulation. Commercial thickness ratings sometimes call for increased insulation when unusually small conductors are used. On the other hand, large conductors being bulky and unwieldy, are given more insulation than stresses would seem to require, for mechanical reasons alone. The ratio of the strand maximum stress to cylindric maximum stress using the formula cited by Mr. Komives[1] is practically constant in the neighborhood of 30 per cent for the usual number of strands in the outer layer. This

automatically becomes absorbed in our empirical design constants. Furthermore, the effect of stranding stresses exists only in the region near the conductor.

4. In the light of the viewpoints brought out above, it seems scarcely logical to use impulse breakdown tests as a rigid criterion of cable quality as Mr. Komives suggests. It is a most variable property as his tabulation shows. It would seem wiser to seek other more direct measures of those causes of variation (taping uniformity, registration, and saturation) mentioned by him. There are, of course, other factors causing variations; the purity and characteristics of the basic materials used in manufacturing cables are probably more important than those listed.

If operating stresses are low in cables as now used, it is because the art has advanced cautiously and at the same time surely. I cite the rapid increase in cable system voltages to 130 and 220 kv as proof of daring and a not too great unwillingness to pioneer. If we trim our thicknesses further in the future, it will be because we have made improvements in quality and reduced variations in manufacture so that the step is warranted.

REFERENCE

1. *ETZ* Formula 2, W. Deutsch. November 23, 1911, page 1175.

Herman Halperin (Commonwealth Edison Company, Chicago, Ill.): Mr. Komives and his company have made a substantial contribution to our knowledge on underground cables through their extensive series of impulse tests.

As to the author's point on the impulse tests indicating the degree of impregnation, it strikes me that ionization tests and visual examinations also give the same information. As to impulse tests indicating the quality of the taping, it seems that they should give no more information than 60-cycle breakdown tests and visual examinations of the cable or styrene wafers. In this regard it seems that surge tests should show freedom from wrinkling, registrations, and an excessive amount of gap spaces in any radial path, but it should be expected that otherwise the lay, i.e., the length of the leakage path, has little effect as surge failures seem to be in general radial punctures. In other words, these two cable characteristics, that is, impregnation and taping, can be observed by using methods and tools available in all cable factories. The additional check method, that is, using the impulse tests, which require special apparatus and complicated potheads, is desirable to fill out our knowledge, but it does not seem to me to be desirable as a procedure for every cable factory. Such data are of particular interest in connection with operating and design problems involving switching and lightning surges, and therefore I hope that The Detroit Edison Company and a few others will continue to make impulse tests as needs dictate.

For predicting the serviceability of a cable, accelerated aging tests, together with supplementary tests and inspection of small pieces of cable before and after aging, seem to be of first importance.

The paper shows that a number of variables have an appreciable effect on the surge strength of cable. This fact emphasizes the need not only of a careful study of all variables involved, but also of the accumulation of a large amount of general data because comparison and interpretation of the data presented by various authors are made difficult as they show the composite effect of several variables, some of which were not well controlled.

It is gratifying that recently considerable information has become available. When the breakdown voltages presented recently by eight laboratories are plotted versus insulation thickness, a surprisingly small amount of scattering is obtained considering the many variables involved. These data indicate an average impulse breakdown stress of 1,600 volts per mil.

Mr. Komives has presented data which indicate that the maximum rather than the average stress may be the controlling factor. If this is true, breakdown voltages plotted versus *r* log$_e$ *R*/*r* should give more nearly a straight-line relation than when plotted versus insulation thickness. While there is an indication that this may be so, the difference in the results of the two methods of plotting is not sufficient to decide definitely between maximum and average stress as a controlling factor.

While Mr. Komives' evidence for the effect of maximum stress is impressive and agrees with the results of visual examinations of surge failures, it appears that under some conditions neither the maximum nor the average stress may be exclusively the controlling factor. It seems reasonable that for two cables with the same insulation thickness but with different conductor sizes the one with the larger conductor would have the higher breakdown voltage. On the other hand, it may be assumed that if the maximum stresses for a given voltage are the same for two cables, i.e., if *r* log *R*/*r* is the same, then the insulation thickness or the average stress would be the controlling factor, the cable with the thicker insulation having the higher breakdown voltage. These arguments seem to be supported by some of the test results, as shown in tables II and III of this discussion. However, there are some data that do not check this theory, possibly due to the disturbing effect of other variables.

One important variable showing considerable variations between the various laboratories is the method of testing with regard to wave shape, polarity, and type of breakdown. Mr. Komives made single-shot tests with most or all breakdowns presumably on the front of the wave. Most other investigators made step-by-step tests with probably in most cases full-wave breakdowns. The limited data on time lag curves by Held and Leichsenring show that front wave breakdowns at very short times can be as much as twice the full wave breakdown values, while beyond one microsecond the time lag curve appears to become flat. For proper co-ordination of results, the time lag curves, or at least reporting of breakdown time with breakdown voltage, would be desirable, especially for co-ordination of the impulse strength of cables with that of terminal equipment. The time lag curves for cables and potheads may intersect with the result that the surge strength may be higher for cables than for potheads with long durations but lower with short durations.

Table II

Investigators	Insulation Thickness—Mils	r× log R/r	Breakdown Voltage—Kv	
Komives	386	245	578–630	
	386	291	753	
Buss and Vogel	472	263	800	(p ressure)
	472	298	900	
	788	435	1,300	
	788	478	1,400	
	788	448	770	(oil-filled)
	788	478	880	
	945	547	1,300	
	945	580	1,400	

Table III

Investigators	r× log$_e$ R/r	Insulation Thickness —Mils	Breakdown Voltage —Kv
Foust and Scott	H402	500	792
	H397	600	972

It is of interest that the *F* samples were subjected for eight years not only to aging tests in Chicago but also to lightning and switching surges, as the cable was connected to a commercial overhead line. The maximum recorded surge strength during the eight years was roughly 690-kv crest, which compares to breakdown voltages of 730 and 775 kv obtained by Mr. Komives for this cable.

J. B. Whitehead (The Johns Hopkins University, Baltimore, Md.): The question of the relative importance of the maximum stress and the average stress in cable breakdown is an old one, which has been quiescent for some time. This is probably because in a very copious amount of experimental data in recent years, there is nothing to suggest a special significance of the average or overall stress.

The hypothesis that the average stress is the determining value in cable breakdown is apparently based on the assumption that the stronger portions of a path through the insulation wall in some way help out or support the weaker portions. This picture implies the further assumption that breakdown is an instantaneous phenomenon and that all elements of the breakdown path let go at the same instant. It assumes, in fact, that "pure electrical breakdown strength" is a property of cable insulation.

There are two good reasons why this picture is not correct. The first is the well-known fact that in all types of insulation the value of breakdown voltage is a function of the duration of time of the application. For example, the present paper indicates that under impulse tests breakdown stresses are some seven or eight times normal 60-cycle values. The second fact, not so well known, is that breakdown in cable insulation is a progressive phenomenon involving the passage of time and that it has recently become possible to interrupt the over-all process of breakdown before it is complete. Partial failures of various stages of advance may now be studied and reveal that during the process of complete failure, certain parts of the insulation wall are at first affected, others remaining apparently intact.

Now it is well known that cable insulation is not uniform in structure and has regions or spots which are weaker than others. The spread of the experimental values easily shows this. This fact, taken in conjunction with the fact that we have local failure before complete failure, points very definitely to the importance of maximum stress. If the weak spot is in a region of maximum stress, it will obviously let go first. If the weak spot is in the region of minimum stress, it may or may not let go before another region in the region of maximum stress, but would only have the effect of giving a higher value of breakdown voltage than when the weak spot were located in the region of maximum stress. The essential thing is that the weak spot has a definite breakdown stress. Whatever the location of the spot, the stress, when it is reached, will have a definite relation to the maximum stress. Failure is progressive in time and the behavior under impulse tests simply indicates that with shorter and shorter durations of time, higher and higher values of stress are needed for the failure to progress from the initial weak spot through the entire wall.

It will thus be seen that I am in agreement with the author's hypothesis of the importance of maximum stress rather than the average stress. However, I do not feel that his paper has added very much in the way of a direct proof. The chief grounds he offers are that there is less spread in the experimental results if they are plotted in terms of the maximum stress than if they are plotted in terms of the average stress. The contention would be more convincing if all of his experimental samples were of the same type. However, they differ so widely and involve so many different conditions that the conclusion looks rather more like a possibility than a certainty. This unfortunately is a necessary characteristic of experimental results on completed cables. It would be highly desirable if cable manufacturers and users could join in a concerted effort to conduct a series of experiments on laboratory-controlled samples in which the various elements entering the cable stability could be studied separately, and subsequently, conjointly.

E. W. Greenfield (Anaconda Wire and Cable Company, Hastings-on-Hudson, N. Y.): Of particular interest are the test results on cables *C* and *D*. These are solid-type cables with sector-shaped, "compack" strand conductors, and differ only in that cable *D* is of so-called type *CB* design. That is, the conductors are covered with several layers of a special semi-conducting, highly adsorptive, colloidal carbon-filled tape.

Since the test involves only a pure electric type of failure, it is assumed by the author that the 30 per cent higher impulse strength of the *D* cable is entirely ascribed to the effective smoothing, or shielding, of the conductor strands by the use of semi-conducting tapes. It is agreed that complete conductor shielding is actually effected by the carbon-black tapes. It is also agreed that the adsorbing and oil-purifying action of these tapes is unimportant in impulse tests since the failures are too rapid and not ionic in nature. However, it should be pointed out that strand shielding may play only one part in producing the *D* cable's observed high impulse strength. Two other effects exist which have a significant bearing on the test results:

1. Series damping due to the semi-conducting tapes.
2. Discharging electrode shifted from metallic to nonmetallic (carbonaceous) material.

The series damping effect is due to the effective presence of a longitudinally-distributed, non-reactive, resistance in series between the conductor and the insulation. For applied impulse voltages, this series resistance modifies both the steepness and magnitude of the wave front to an appreciable extent and thus affords the cable an automatic, built-in measure of protection.

The fact that the high-voltage electrode is shifted from copper to carbon-filled paper is important since the character of the discharge is thereby modified. Discharges from metallic electrodes are more disruptive and severe than discharges from non-metallic electrodes. The relation of work functions between copper and carbon and carbon and paper causes a real difference in barrier potentials over that existing between copper and paper as in the standard cable construction. The work function between carbon and paper is probably only about one-fifth that of the copper-paper interface.

By virtue of the above, it is believed that the excellent impulse strength obtained for the *D* cable cannot be duplicated by a cable with the ordinary metallic-tape form of conductor shielding, or even by a cable with a smooth, solid-copper conductor.

L. F. Roehmann (Anaconda Wire and Cable Company, Hastings-on-Hudson, N. Y.): Physicists distinguish between two forms of breakdown of a dielectric; namely, pyroelectric and electric. When a cable dielectric is subjected to impulses with a steep wave front, the breakdown is of a purely electric nature. Hence, it is reasonable to assume that there is a relation between impulse strength, and electrical stress at the moment of breakdown.

It is not surprising that there is no simple relation between breakdown strength and *average* dielectric stress. This latter quantity, while widely used for rule-of-thumb calculations, has no physical significance. As in structural calculations, it is the *maximum* stress which is the determining factor.

Some time ago, the idea was conceived that impulse breakdown for a given material always occurs at the same *maximum* stress.

Davis and Eddy's paper[1] permitted, for the first time, a check of this hypothesis. These authors published breakdown stresses obtained on samples of a single, round, solid conductor (number 6 American wire gauge) insulated with a rubber compound of 5, 10, 15, and 20/64 inches wall thickness, respectively. They reported *average* electrical stresses and concluded that the unit impulse strength decreases with increasing wall thickness. However, when one recalculates their values on the basis of *maximum* stress, one finds the following values: 2,000, 2,060, 2,060, and 1,950 volts per mil; that is, practically constant within the range of measuring accuracy.

It is gratifying to see this theory is confirmed again by the data reported by the author. Calculations involving stranded, compacted sector conductors are much more complex than for a single round wire. Hence not the same accuracy can be expected when testing such types of cable.

There are three cables designated by the author as inherently, or accidentally, inferior with respect to impulse strength, namely *B* (low gas-pressure cable), *C* (ordinary solid-type cable), and *G* (oil-filled cable with very irregular taping). When these three cables are eliminated, one finds that the breakdown strength, or more accurately the maximum electrical stress at breakdown, is *constant;* namely, 3,050 volts per mil ±12 per cent, *independent of design and of insulation thickness.*

Additional confirmation is contained in an English investigation.[2] The authors made puncture tests on paper-insulated cables rated from 10 to 66-kv three-phase. They used an 0.5 microsecond wave front, and report puncture stresses of 2,670 volts per mil for positive and 2,440 volts per mil for negative polarity. These values, as they state, do not consider stress concentration due to stranding. If one takes this into account by adding 20 per cent, one arrives again at a value in the neighborhood of 3,000 volts per mil.

The practical consequences of this result are twofold:

> First, if one is willing to consider, with the author, impulse strength as a measure of cable quality, it follows that a cable not living up to approximately 3,000 volts per mil maximum impulse stress is "not acceptable."
>
> Second, if one has to select a cable with respect to its impulse strength (for example, a connection between an overhead line and a station), one should choose the type which combines highest impulse strength with simplest design and lowest cost. This is, obviously, the conductor-shielded, solid type; or more specifically, type *D*, where the conductor is shielded with semi-conducting tapes.

In figure 5 of the paper, it seems rather confusing to plot maximum stress versus breakdown voltage. Increasing voltage, for the same type of cable, necessarily means increasing stress; hence, all points for the same type necessarily lay on straight lines. This question only is of interest: At which stress did breakdown occur? When type and breakdown stress are known, any impulse voltage can be obtained for purposes of insulation coordination by varying the conductor diameter and insulation thickness.

It is interesting to note that the formula for maximum strand stress derived by Mr. Serguis Vesselowsky, of the Detroit Edison Company, confirms that obtained by W Deutsch in 1911,[3] and reprinted in Schwaiger-Sorensen's textbook on Dielectrics.[4]

REFERENCES

1. IMPULSE STRENGTH OF CABLE INSULATION, E. W. Davis and W. N. Eddy. AIEE TRANSACTIONS, volume 59, 1940 (July section), pages 394–402.

2. FIELD AND LABORATORY SURGE TESTS, R. Davis and H. M. Lacey. *CIGRE* 1939, report number 304, page 6.

3. DIELECTRIC STRENGTH OF CABLES, W. Deutsch. *Elektrotechnische Zeitschrift*, volume 32, 1911, number 47, page 1175.

4. THEORY OF DIELECTRICS, A. Schwaiger. (A book translated by R. W. Sorensen.) John Wiley and Sons, New York, 1932, page 274.

K. S. Wyatt (Phelps-Dodge Copper Products Corp., Yonkers, N. Y.): Mr. Komives' development of the impulse breakdown test as a measure of cable quality will be welcomed by all who are working to improve cable. His paper is particularly valuable in comparing different cable constructions, solid, gas-filled, oil-filled, compression, and oilostatic; and in studying the effect of conductor shielding, and of pressure, in addition to the effects of thorough impregnation and of uniform and well designed taping.

It should be made quite clear that the significance of these results lies in the relative values of impulse strength and not in the absolute values given. Beyond the question of wave-form and similar variables, there is that of pre-excitation. This should be important, particularly in cables containing appreciable gas spaces in the insulation as in used solid cable, or gas-filled cable. In such a cable, during service operation, there may be ionization taking place in the gas spaces as a result of normal voltage, perhaps assisted by a high frequency component which is often present in operating cables. When an impulse is impressed on such a cable, it is believed that breakdown will occur at an appreciably lower voltage than on a cable which has not been energized and in which pre-excitation is absent. Impulse tests on energized cables have not been carried out because of the difficulty of sufficiently insulating the transformer equipment. Thus it appears that absolute values for impulse breakdown on cables, particularly gas cables, should not be taken too seriously if the cable is not under operating voltage when tested.

A warning note should also be sounded for those who may be tempted to use impulse strength as the sole criterion of cable quality, and in this I feel sure Mr. Komives will agree. Impulse strength is not only a function of mechanical uniformity and of degree of impregnation, but of paper density. It is believed quite feasible to raise the impulse strength of a cable by employing super dense super calendered paper throughout the insulation wall. Such a cable would prove unstable on a severe load-cycle test because cavitation (voids) would occur, and this would result in ionization. In other words, it would be a bad cable from the operating man's point of view, yet the impulse strength would be high. Hence, the impulse strength of a cable should be used in judging cable quality only when complete details of the cable construction are available, and sufficient test data to permit reasonable interpretation of the results.

The author refers to judging the degree of impregnation of his cable samples by visual inspection. While I am quite agreed that this is roughly satisfactory in practice, nevertheless, it would seem desirable to have a more quantitative test, particularly for solid cables. Actually, it is simple to determine the degree of impregnation quantitatively by taking a few feet of cable, fresh cut at ambient of 20 degrees centigrade or below, sealing up the ends with polystyrene or paraffin wax, and immersing in a pipe of thin oil. Liquid pressure is applied by steps up to 200 pounds per square inch—the decrease of volume of the cable sample being measured by the amount of oil fed in. Knowing the initial displacement of the sample, the void volume is readily calculated. It is surprising to find that modern cable carefully impregnated has an appreciable void volume. The range at 200 pounds per square inch is probably from one-half to three per cent. It would give support to the author's conclusions if he could give exact figures for void volume.

The variations in impulse strength of the various constructions described in this paper have been convincingly interpreted, but there is one exception. Consider the two solid cables *E*4 and *D*2, figure 5 of the paper. Cable *E*4, without conductor shielding, breaks down at a maximum stress of more than 3,300 v/mil., whereas *D*2, about an average of *D*1 and *D*3, with semi-conducting strand shielding, breaks down at a stress of less than 3,100 v/mil. The hasty conclusion might be drawn from these data that conductor shielding was harmful and lowered the impulse strength. In just the same way might it not be hasty to generalize from the data on the *D* and *C* samples that conductor shielding (including the semi-conducting type) raises the impulse strength? Would it not probably be nearer the truth to say that, if the stranding of the conductor is poor, or the insulation makes poor contact with the conductor, a uniformly applied and satisfactory conductor shielding will raise the impulse strength appreciably? On the other hand, if the outer surface of the stranded conductor is smooth, and the insulation everywhere makes close contact with it, then conductor shielding probably will not improve the impulse strength, and in fact may decrease it slightly.

To the writer the discussion of the last paragraph is largely theoretical, since it applies chiefly to new cable which has been subjected to little reeling and re-reeling. Wafer studies have shown quite clearly that three-conductor cable which is subjected to severe installation, conditions of reeling and re-reeling and pulling under winter conditions when compound is viscous, may suffer a deformation of the insulation resulting in a separation of the insulation from the conductor. Under such conditions of rough or repeated handling of the cable under winter conditions, conductor shielding of a satisfactory type, if it can be uniformly applied, may be expected to improve considerably the cable performance under severe test conditions.

Aside from these points, the paper is important in that it opens up the possibility of quickly evaluating the excellence of the mechanical uniformity of paper cables. Had there been some sure test in the past, greater progress would have been made in good design and precision application of taping. As it was there were so many obscuring factors that mechanical characteristics have not received the attention they have deserved. The importance on impulse strength of a good design overlay, and the precision application of taping following that design, is brought out very clearly by the Scott-Mildner diagrams of figure 6 (cable *E* 3,300 volts per mil) and of figure 8 of the paper (cable *G* 2,500 volts per mil). The weakness of the taping of cable *G* (figure 8) is twofold: a poor choice of overlay, namely, 20 to 25 per cent, and second, a high percentage of registrations or near registrations. The poor overlay means a comparatively *short leakage path* from conductor to sheath. The registrations mean a *radial alignment of butt-spaces,* some without any separating paper layer to break up the continuity of the oil-spaces. Wafer studies made several years ago in conjunction with load cycle aging of cable made clear the rudiments of the arrangement of the layer structure of the insulation and its effect on cable characteristics. These rudiments are as follows: On the severe load cycle with over-voltage of two to three and one-half times rated voltage, a long leakage path was probably more important than registration or radial alignment of butt-spaces, particularly if the cable length under test included a minimum diameter bend so that the layers at the outside of the bend opened up. Gaseous ionization soon searched out and penetrated along short leakage paths on the cooling cycle. The "step" leakage path equals the "Zig-zag" path at about 45 per cent overlay for tapes of the usual width and thickness. On the other hand, radial alignment of butt spaces across the insulation wall results in a weakness on short-time dielectric strength tests, probably because the field is very badly distorted around such butt-spaces[1] giving rise to appreciable tangential stresses even when filled with oil, and partly because such butt-spaces tend, at high points along a cable route, to drain clear of oil leaving gas spaces. These, of course, constitute a serious dielectric weakness. For instance, it was at that time pointed out that a 50 per cent overlay resulted in a butt-space super-imposed every two layers along a radius, 33 per cent in a butt-space every three layers, 25 per cent every four layers, and 20 per cent every five layers. Radial alignment of such butt-spaces means also that "hinges" or slip zones have been built into the cable wall so that the bending of the cable does not take place uniformly but is concentrated at such hinges with bad effect on dielectric strength.

To obtain high values of leakage path with absence of "hinges" due to radial butt-space alignment, it has been customary to use an overlay from 35 to 45 per cent, depending on the degree of precision of the machinery available.

There are two surprising features to the impulse values obtained in connection with figures 6, 7, and 8. Why should an impulse breakdown, which is a straight-through radial puncture, be so much influenced by the short leakage path of figure 8? And why should the cable presented by figure 7, which has an almost random overlay, have such a high impulse strength?

Other factors of importance in determining impulse strength, it would appear, are degree of compactness of the insulation cross-section as measured by a Cenco tester, and the presence or absence of a thick oil-film or void between conductor and insulation.[2] Unfortunately, these data are not available in the present paper.

Impulse breakdown tests should also be capable of searching out weak mechanical spots of other kinds. For example, in much paper cable there are often periodic "surge" effects of different kinds. In a joint paper[3] by Mr. Komives and the writer in 1936 the beneficial effect of using caterpillar grips in place of capstans was indicated. In another paper in 1938, it was shown that periodic crushing could be avoided by pre-spiralling.[4] Beyond these influences, there are undoubtedly still periodic mechanical effects which limit the stresses at which cable insulation can be safely worked. It is, of course, hardly necessary to point out that the older taping machinery, lacking in precision, led with certainty to periodic effects of the most serious kind.

It would appear probable that if the author of this paper should measure the impulse breakdown of a solid cable built on the best taping principles and employing a carefully made conductor shielding, at the same time avoiding the periodic effects referred to above, he would obtain higher values of breakdown stress than those recorded in this paper.

It is believed that the short-time d-c breakdown strength may be used as a rough guide to the relative impulse strength of various constructions. If this is so, the work done by Shimizu[5] should be of interest. He points out that the breakdown voltage is the the sum of the breakdown voltages of the paper layers and of the oil layers. Accordingly, for a given insulation thickness, a "juicy" cable, with thick interlayer oil film, would have lower breakdown strength. He also points out the effect of gaps or butt-spaces in reducing the breakdown strength. A bad arrangement is to have an oil layer in contact with the electrode instead of a paper layer. The mechanism of breakdown for d-c is quite different from that of a-c: where butt-spaces are only slightly off-set along a radius, the a-c breakdown will follow the path connecting the gaps whereas d-c will puncture the paper along a direct path rather than deflect even a short distance to one side. A-c breakdown involves surface discharge and creepage between fibre and oil, whereas surface effects are absent with d-c (D. M. Robinson also points out such differences between a-c and d-c breakdown).[6] D-c breakdown is increased by increasing impermeability of the paper, and by increase in viscosity of the impregnating oil; whereas gas dissolved in the oil lowers the a-c breakdown, it has little effect on the d-c breakdown.

If impulse breakdown does not follow the alignment of butt-spaces along a radius, but prefers to puncture straight-through the paper, how is it that it is affected by uniformity of taping? The author, perhaps, wisely refrains from proposing a theory.

It would be very helpful to confidence in the impulse test as a measure of taping quality to develop some rational explanation of the dependence of the impulse breakdown mechanism on taping.

REFERENCES

1. **E. B. Baker,** *Physical Review*, 1934.

2. **Fachberichte 10, 1938, seite 21–23. Verband Deutcher Elektrotechniker.**

3. **HIGH-VOLTAGE CABLE: EUROPEAN AND AMERICAN PRACTICE. Association of Edison Illuminating Companies, 1936.**

4. **MECHANICAL UNIFORMITY OF PAPER-INSULATED CABLES, K. S. Wyatt, D. L. Smart, and J. M. Reynar. AIEE TRANSACTIONS, volume 57, 1938 (March section), pages 141–54.**

5. **Kichezo Shimizu. Abstract section,** *Journal* **of the Institute of Electrical Engineers, Japan, January and August 1934.**

6. **DIELECTRIC PHENOMENA IN HIGH-VOLTAGE CABLES, D. M. Robinson, page 58. (A book) Chapman and Hall.**

L. I. Komives: 1. Mr. J. A. Scott is still not convinced that maximum voltage gradient rather than average stress determines the impulse strength of a cable. The author readily agrees with Mr. Scott that the data available are perhaps not sufficient to accept this theory without reservations. However, if a choice is to be made as to which of the two, maximum or average stress, should be used as a criterion, the available data strongly favor the maximum stress theory. In establishing impulse strength as a criterion of the quality of the insulation, it was necessary for the author to make this choice.

2. Professor Whitehead in his very helpful discussion goes a step farther and considers the maximum voltage gradient theory as an old story. Decidedly the probability lies between these two viewpoints, strongly favoring Professor Whitehead.

3. Mr. Wm. A. Del Mar's discussion in regard to maximum stress calculations is very helpful. It is exceedingly difficult to measure minimum radii on sector-shaped copper conductors at the point of breakdown. The author used theoretical values throughout the paper and deviations from theoretical figures, as Mr. Del Mar points out, are quite possible and will upset calculations in a considerable degree. In regard to the effect of stranding in the calculation of maximum stresses, it is proposed that we agree on one of the several theories available and use it throughout the industry when tests of such a nature are performed.

4. The author heartily agrees with Mr. Halperin's discussion. Perhaps the paper did not bring it out quite clearly but it was the author's idea that when sufficient data are available we can use the visual examination method with occasional checks on the impulse generator. It is not necessary that every cable manufacturer should have impulse testing equipment.

5. Messrs. Roehmann and Greenfield in their discussions, bring out a few points more clearly than the author. For this they should be given credit. Mr. Greenfield's theory in regard to the effect of the carbon electrode versus copper electrode on impulse strength is interesting. However, subsequent tests made by the author's company indicate that perhaps this effect is less pronounced than Mr. Greenfield indicates.

6. As to Mr. Wyatt's discussion the author agrees with him heartily that a surge super imposed on an a-c potential would have different effects on different types of cable. The author's company is not in a position to conduct such tests and perhaps the General Electric Company would be the only one in this country who could do it at the present time with the available equipment.

7. In regard to Mr. Peterson's discussion it is certainly desirable to eliminate variables in cable testing such as the effect of handling, service or load cycle aging etc. As such a program would be very costly it is believed that the industry will have to be patient and build up data slowly as they are accumulated.

The author wishes to thank all those who discussed this paper in such a fair and straight forward manner.

Bushing Tests

Discussion and authors' closure of paper 41-120 by A. L. Brownlee and W. H. Wickham, presented at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941, and published in AIEE TRANSACTIONS, 1941 (October section), pages 940–2.

L. Wetherill (General Electric Company, Pittsfield, Mass.): The construction of a large bushing is sufficiently complicated that a measurement of one overall electrical characteristic can hardly be expected to give all pertinent information regarding the condition of the internal dielectric. However, a considerable body of experience with power factor measurements made on bushings at voltages up to ten kilovolts has shown that such data are very useful. The authors have discussed the usefulness of the additional information which can be obtained by making power factor measurements at voltages exceeding ten kilovolts up to and above the rated operating voltage. Such usefulness consists, of course, in the ability to detect the presence and nature of undesirable conditions which would not have been indicated by power factor readings taken at a voltage of ten kilovolts or less.

Of four cases of power factor data taken at voltages exceeding ten kilovolts which are cited in the paper, three involve bushings using solid materials as the principal dielectric. In at least two, and probably all three, of the cases involving solid bushings it appears from the data that the power factor measured at ten kilovolts or less would have indicated no abnormal conditions, whereas power factor readings made at considerably higher voltages served to indicate abnormality which, on examination, was found to be of a serious nature.

The fourth case cited in the paper involves an oil filled bushing which had a power factor exceeding 23 per cent at ten kilovolts so that, as pointed out by the authors, the existence of an undesirable condition would have been unmistakably indicated by a power factor reading at ten kilovolts.

These data seem to suggest that the usefulness of power factor measurements made at voltages exceeding ten kilovolts in detecting the existence of undesirable conditions which would not have been disclosed by a measurement at ten kilovolts, may be considerably less on oil filled bushings than on solid bushings. Oil filled bushings are not, in general, susceptible to cumulative localized injury from repeated over-voltages, because of the fact that the recovery of oil after subjection to over-voltage is complete. Since the usefulness of power factor measurements at voltages exceeding ten kilovolts appears to be related principally to localized injury, it would reasonably follow that the usefulness of power factor measurements at voltages exceeding ten kilovolts will be considerably less on oil filled bushings in which the occurrence of such a condition is unlikely. Thus an examination of the nature of the phenomena appears to confirm a conclusion drawn from the data presented in the paper. Doubtless the authors have additional data not included in the paper which would further illuminate this point.

In many cases radio influence voltage measurement has been used for the detection of localized injury or defect in a solid insulating structure, and of course in such cases voltages of the order of operating voltage are applied. If the localized condition consists of a void leading to ionization the measurement of radio influence voltage should indicate its presence at least as definitely as a power factor measurement at the same test voltage. If on the other hand the localized condition consists of a partly conducting region of variable resistance it would appear that power factor measurement over a considerable range of voltage would be a more effective means of detection than a measurement of radio influence voltage. Of course the economic justification of field power factor testing at relatively high applied voltage is a matter which must rest on the frequency with which such tests disclose undesirable conditions which would otherwise have been undisclosed.

In figure 9 of the paper is shown a cross-section of a solid bushing core. This core was probably manufactured at the time when the inner layers of paper were impregnated with a dark compound as contrasted with the light brown compound which was used to impregnate the outer layers of paper. It is possible, therefore, that re-examination of this core will show that the dark region next to the conductor and at the upper end indicates the use of a dark impregnating compound in this region at the time of manufacture; rather than migration of the filling compound used inside of the bushing shell in contact with the core, which seems unlikely.

In table III, reference is made to surface power factor measurements. While the property of power factor has usually been associated with a volume rather than a surface it is, of course, fairly obvious that surface readings can be significant and useful. It would appear, however, that the value obtained would be dependent on the capacity of the testing circuit, as well as on the electrode arrangement. Perhaps the authors would care to give additional information regarding the method in which the surface power factor measurements were made.

I. W. Gross (American Gas and Electric Service Corporation, New York, N. Y.): The paper by Messrs. Brownlee and Wickham brings out two rather important points. The first, covered in the latter part of the paper, shows what can be done in determining the location of a fault in bushing insulation after an overall test has pronounced the bushing faulty. Prior to and during the early stages of our field testing of bushings in the field, we made similar tests on quite a few of the bushings found faulty by overall test. The defective insulation adjacent to litharge and glycerin was determined early in the testing program and, incidentally, was on a 44-kv bushing as given by the authors in their table I. At that time we not only found this defective insulation by tests on each condenser layer, but also proved by field and laboratory tests that a relatively high power factor in the order of five to eight per cent on the entire bushing indicated that the insulation had deteriorated and that the bushing would not stand up under 60-cycle overvoltage test.

The authors of this paper seem to be making a point that the ten kilovolts, which is now used very extensively for routine field testing of insulation, is not capable of showing up deterioration in some bushings which a higher test voltage would find. Unfortunately, there does not seem to be much data in the paper to bear out this contention. It would indeed be most instructive if comprehensive, comparative test data obtained at widely different test voltages were given to support this contention.

It has been frequently stated, and often repeated, that over-stressing of insulation by excess voltage will injure the insulation. This statement was often made in the past in reference to high voltage cables. It has also been made in connection with high voltage impulse tests on transformer insulation. In fact, at the start of transformer impulse testing, considerable fear was expressed that subjecting insulation to excessively high voltages might cause deterioration which would later show up as a service failure and that such failure might not occur if the high voltage tests had not been made. I think practically all of our operating experience today relegates most of this philosophy of high voltage testing to a minor factor in successfully operating modern high voltage systems. We have good reason to believe at the present time that there is much insulation in service not only in bushings but in many transformers including some power transformers and certainly a large number of distribution transformers where the insulation has undoubtedly been injured by excess lightning voltages; yet it appears that such insulation was able to give a high grade of service, although, of course, no operator would choose such insulation in preference to that which has not become deteriorated.

It is, of course, true that the opinion is prevalent that it is desirable to test under operating conditions as nearly as possible, and this means at operating voltage. However, before deciding on such a procedure, it might be well established that testing in this way will uncover a sufficiently larger number of hazardous faults than cannot be detected by the commonly used and flexible lower voltage test, if such is the case. There are a number of complications which enter the picture in practical field testing when an attempt is made to test at operating or rated voltage. First, the testing equipment becomes cumbersome, and second, Y connected transformers with graded insulation do not lend themselves at all to testing the high voltage end at operating voltage.

There is, however, one field where it would seem that this idea of testing at operating voltage, or even above, has very practical application and might prove of considerable benefit to the industry, and that is, in the testing of new equipment in the factory before it is sent out into the field. I would like to suggest as one of the practical uses to which high voltage power factor testing might be put, that of routine testing in the factory of high voltage bushings at apparatus rated voltage and perhaps some 25 per cent or so above this, to determine not only design defects but manufacturing faults as well. Overvoltage dielectric tests, as is well known, are now made on practically all completely assembled high voltage equipment. Why would it not be an excellent idea to establish the high voltage power factor test as a standard commercial test for practically all bushings, certainly in the higher voltage ranges above 15-kv rating?

Charles F. Hill (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): We have recognized the value of high voltages in the measurement of power factor of bushings as proposed by Brownlee and Wickham, and have used them in development and design studies for several years.

We have not been concerned, however, about the presence of small amounts of internal corona except in that the amount of such corona losses indicates the degree of mechanical perfection. The opinion of the authors that internal corona will lead to the destruction of the insulation does not seem to agree with our experience and a theoretical analysis of the various types of corona action possible would seem to verify our observations.

Corona reacts on insulation in three possible ways; namely,

1. Thermally by heat generated.
2. By electron bombardment.
3. By chemical action.

The thermal action is due to the heat generated by the corona discharge itself and the resulting temperature rise of the insulation around the void. Bushings have a capacitance of the order of 200 μμf. At 40 kv for curve 1 of figure 1 of the Brownlee and Wickham paper the power factor increment due to corona is 0.04 per cent. The power loss is therefore

$$2\pi fCE^2 \times 0.0004 = 0.012 \text{ watts}$$

It would require energy at a rate at least 100 times greater than this to be serious if the void is of appreciable area. Even at 100 kv the loss would be only 0.25 watt.

The second type of reaction, electron bombardment of the walls of the void, can also be treated theoretically. Approximately 12 and 15 electron volts are required to ionize O_2 and N_2 with one impact. The mean electron-volt energy at one atmosphere under gradients in the void as exist for curve 1 of figure 1 at 40 kv amounts to 3.5 to 4 electron volts. It is also evident since at 40 kv corona is just starting, only enough electrons have reached the 12 to 15 electron volt value to ionize a small part of the gas. Electrons with 10 to 20 electron-volt energies should not affect solid insulation although the critical electron volt energies to affect cellulose, for example, are probably not known. We do know that in the high voltage cable where gradients are four to five times those used in bushings and where mean free paths are much greater due to reduced gas pressure in the voids, thus giving electron volt energies of approximately 10 times that possible in bushing voids, the oil is affected, but so far as I know the paper is not. Fellows and Schoepfle (*Ind. Eng. Chem.*, vol. 21, 1929, page 529) were able to deteriorate paper by electron bombardment with energies of several thousand electron volts and state that the effect could have been done at somewhat lower energies. This is the only direct information I have seen but it seems probable that the 15 to 20 electron volt energies available in the bushing are quite negligible.

The chemical type of reaction is, of course, oxidation. This occurs only if O_2 is present. With sealed voids the original oxygen would react and disappear and no further action would occur unless the void can breathe. We would expect, and have observed, discoloration as the authors mention, but I do not believe it possible for any internal void to breathe sufficiently to allow material damage, as the bushing is sealed top and bottom from the air. It would be quite interesting to dissect some old condenser bushings that have been operating 15 to 20 years as a check on these calculations, for I am sure voids could be found which have been operating with appreciable corona throughout their life.

As stated at the beginning, we have been concerned with voids as a check on mechanical structure, but in that case, only from the standpoint of design of bushing, design of winding equipment, and materials, so that high voltages have been used merely as a laboratory tool. It has been our opinion, therefore, that a ten-kilovolt power factor test is practically as valuable as a far higher test. There is no objection to the use of higher voltages and whether they are used would seem to be a question of economics and convenience.

L. R. Ludwig and H. J. Lingal (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The study which the authors have made of conditions in bushings is of considerable interest to operators and manufacturers alike, and the results disclosed with respect to condenser bushings have been before the designers of this apparatus for several years as a result of their development activities.

Ionization and voids in condenser bushings can be detected, as stated by the authors, by using high voltage power factor tests to determine the power factor as a function of voltage. High voltage power factor measuring equipment has been available to the discussers for a long time, during which it has been used for design and development tests.

Figure 1 shows typical curves of power factor plotted as a function of voltage. These curves were made on a 34.5-kv bushing with voltages from 10 to 50 kv. Curves 1 and 2 were made on production bushings and represent the present standard of quality. To show the effect of voids on the power factor voltage curves, two bushings were made in which voids were deliberately introduced. The bushing from which curve 3 was obtained was built by folding the paper over part of the bushing length. The bushing from which curve 4 was obtained had strips cut from the paper. The figure shows that bushing 3 showed a decided increase in power factor with voltage. The standard bushings 1 and 2 do not show this effect and even bushing 4 appears satisfactory, probably because the voids introduced were not sufficient in size to cause measurable corona.

Figure 2 shows typical power factor voltage curves for 115-kv and 69-kv bushings of modern design. These curves are flat and show no ionization.

It is important to eliminate voids in bushings but chiefly for mechanical reasons. Laboratory test results, field experience, and theoretical consideration all indicate that the electrical effect within the voids in reducing the bushing life is of negligible importance. Early bushings were undoubtedly manufactured in such a way that voids were generally present, but nevertheless no condenser bushings have failed in service directly as the result of ionization and corona.

In connection with studies of power factor at varying voltages, major modifications and improvements have been continuously made in winding machines and processes for constructing bushings. Elimination of voids has been accomplished by improvement in winding technique. To insure the quality of the winding in modern bushings, an air pressure test is given every bushing. Sixty-pounds air pressure is introduced at one end of the bushing for a period of 30 minutes and oil is flowed over the exposed surface of the bushing to detect the escape of air in case leakage is present.

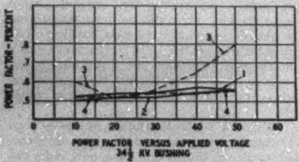


Figure 1

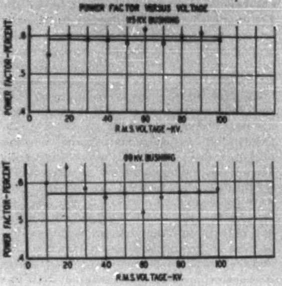


Figure 2

The authors refer specifically to ionization in two 66-kv condenser bushings. These bushings were manufactured more than two years ago and were returned to the factory for investigation. One of them was torn apart for examination and there was sufficient separation of paper layers to explain the curves in the authors' figure 1. The other bushing was returned and placed in service without modification. The degree of ionization and corona shown at 38 kv by curves 1 and 2 in the authors' figure 1 is not excessive. For example, many bushings of older manufacture have been torn down and examined after a large number of years of service. These bushings often contain voids large in comparison to the 66-kv bushings referred to, and yet there has been no effect observed on the insulation. Even discoloration has not been present and it is believed that in the case of the bushings to which the authors refer discoloration resulted from the use of abnormally high test voltage, amounting to 250 per cent of the operating voltage.

The authors refer to partial failure of an older type 34.5-kv potential bushing. The high power factor which they have measured is generally found at the outer layers only. It is one of the important advantages of the condenser type of bushing that the outer layer power factor may change without affecting the voltage distribution sufficiently to endanger the entire device. The litharge and glycerine cement undoubtedly effect the outer layer. This condition was clearly recognized by the manufacturer in 1929 and a stop foil was introduced into these bushings to prevent penetration of this material at that time. In 1934 the use of this cement was discontinued completely.

The authors' comments with respect to laboratory examinations point out the importance of adequate elimination of moisture. It is undoubtedly true that the presence of moisture has been the major reason for high power factor and failure of bushings. The situation has, of course, been well appreciated and condenser bushings have been designed for several years with adequately made seals and plastic and casing material to prevent water from coming into contact with the insulating layers of the bushing. Particular care is taken during manufacture to eliminate entrance of moisture and all bushings are power factor tested to insure that they are dry before shipment.

With respect to the authors' reference to contamination of the mixture of oil and filling compound in a 37-kv bushing, the improved winding and the leak test referred to have proven to be entirely adequate in preventing migration of the oil from the lower end through separations in the impregnated paper to the compound on the top. No difficulties of this kind have been experienced with bushings of modern design.

A. L. Brownlee and W. H. Wickham: In presenting this paper the authors' aim was to stimulate interest in high-voltage power-factor testing of bushings. It was felt that the maximum benefit to be derived from the power-factor test is not obtained when the test voltage is limited to ten kilovolts as is the common practice. This is borne out by the fact that some failures still occur on those systems where the 10-kv power-factor test is used. Extending the test voltage to, or above, operating voltage may offer a means of still further reducing bushing failures. On the system with which the authors are associated there have been no failures on the bushings included in the power-factor testing program since this testing program was inaugurated.

Mr. Gross feels that there is not enough comparative test data obtained at widely different test voltages to support the contention that higher voltages are necessary. Yet the ten-kilovolt test leaves something to be desired. In the paper, "Field Testing of Bushings and Transformer Insulation by the Power-Factor Method," AIEE Transactions, volume 57, 1938 (October section), pages 589–96 Mr. Gross, in discussing the 10-kv power-factor test, states: "Although the goal of complete elimination of all bushing, oil circuit breaker, and transformer failures in service probably never will be reached, the power-factor test method now available and in use by a large number of power companies has proved to be a useful tool in detecting faulty insulation prior to equipment failure in service." He also says: "It is unfortunate that accurate records are not available to show the decrease in bushing failures over the system before and after the test program was adopted. However, from the rather scattered records available from 1932 to 1937, it is interesting to note how closely the frequency of bushing failure and lightning storms coincide throughout the year." We must conclude that even with a program of power-factor testing at ten kilovolts bushings still fail in service, possibly due to higher than normal voltage stress. It is suggested that a power-factor test at a voltage somewhat above operating voltage might reveal some of these incipient faults and still further reduce failures in service. Only by the extensive use of higher voltages can the comprehensive data which Mr. Gross desires be obtained.

Mr. Gross is probably over-estimating the complications of field testing at operating voltage. The experience gained on our 66-kv system has shown that such testing is entirely practical on that system and could be extended still further. While it is true that because of the increased size of the testing transformer the equipment is somewhat more bulky than that required for ten-kilovolt testing it is still entirely practical and convenient to operate. In this respect it compares favorably with the equipment shown in figures 1, 2, and 3 of the paper referred to above. We fail to see the limitations in testing Y-connected transformers with graded insulation.

It has been our practice for some time to test all new bushings at voltages above their operating voltage to reveal manufacturing faults as suggested by Mr. Gross.

Messrs. Ludwig and Lingal have presented power factor vs. voltage curves (figure 1) which show the effect of voids introduced within the bushing insulation and further demonstrate the value of high-voltage power-factor testing in indicating the presence of these voids. The curves of figure 2 are characteristic of those obtained on new bushings without defects but the wide deviation of the points from the average curve is quite unusual. There still remains some question as to the effect of corona within voids on the life of the bushing. The discussers state that no condenser bushings have failed in service directly as a result of ionization and corona but we do not understand how this fact was definitely established since it is difficult to determine the original cause of failure after failure is complete. It is hardly likely that the discoloration of insulation around voids referred to in the paper was caused by the high test voltage. This voltage was applied for so short a time that it could hardly have produced that effect. Dr. Hill states in his discussion that he has observed this same discoloration and that it is to be expected.

Messrs. Ludwig and Lingal state that in modern bushings, improvements in design have been made so that some of the faults discussed in the paper have been eliminated. This fact is realized, but there are a great number of the older bushings still in service whose testing and maintenance is still a problem to the operating companies. A discussion of these faults, which have led to changes in design but were, perhaps, not generally known, should be timely for those engaged in bushing maintenance.

Mr. Wetherill's conclusion that the usefulness of high-voltage power-factor measurements is less on oil-filled bushings than on the solid type agrees with our experience thus far.

It is true that the radio influence voltage measurement will detect the presence of voids within a bushing as well as a high-voltage power-factor measurement. We have used the radio test to some extent in the past. However, a single test is preferred and since the high voltage power-factor test gives the same information on ionization and in addition reveals other types of trouble, it has been adopted in preference to the radio influence voltage measurement.

The bushing of figure 9 was re-examined in an effort to determine whether or not the center portion had been impregnated with a dark compound in manufacture as suggested by Mr. Wetherill. It is quite possible that this was the case, although it is difficult to determine. It was the mixture of this compound with the oil which traveled up through the bushing that gave rise to the high power factor.

The surface power-factor measurements referred to in table III of the paper were made between adjacent foil electrodes on the surface of the bushing parts. The value obtained is, to some extent, dependent on the size and spacing of the electrodes. The electrodes used were one inch wide and one-half inch apart. On insulation in good condition the power factor between adjacent electrodes is nearly the same as the power factor measured between electrodes on opposite surfaces as shown by the data given in table III. On insulation in poor condition the power factor measured between adjacent electrodes is usually much higher.

Dr. Hill's theoretical analysis of the effect of corona in voids within a bushing is interesting and it is unfortunate that more experimental evidence is not available on this point. He has pointed out the fact that voids are undesirable for mechanical reasons, that corona may be present within the voids, and that there is some question as to the effect of this corona upon the insulation. It would seem then that a test such as the high-voltage power-factor test which will reveal these mechanical imperfections is desirable.

The Basis for the Nondestructive Testing of Insulation

Discussion and author's closure of paper 41-118 by R. F. Field, presented at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941, and published in AIEE TRANSACTIONS, 1941 (September section), pages 890–5.

L. J. Berbarich (Westinghouse Electric and and Manufacturing Company, East Pittsburgh, Pa.): Mr. Field is proposing a new and unfamiliar method for evaluating insulation. There is a need, of course, for more informative tests than we now have. If the new method can be simplified and shown to be effective, it may be quite important in doubtful cases where present methods fail. In its present state of development, however, it still appears to be in the laboratory stage, for it is based on assumptions, implied or expressed, which have not yet been definitely established. The basic assumption appears to be that a correlation between dielectric strength and low frequency interfacial polarization ought to exist. Our experience, however, in attempting to correlate dielectric strength with other properties has not been encouraging except where moisture has diffused into the insulation.

Mr. Field also indicates that free electrons, as well as ions, contribute to interfacial polarization. All of the available evidence, however, indicates that if free electrons are present in solid and liquid insulators at normal voltage stresses, their concentration is very low. There is considerable evidence, on the other hand that practically all of the current in liquid and solid insulators is carried by electrolytic ions. It is only when breakdown gradients are approached that considerable numbers of free electrons are released, when the electrostatic coupling between the electrons and the atoms with which they are associated is ruptured. It is believed, therefore, that free electrons do not contribute appreciably to the polarization in liquids and solids below breakdown stresses.

The present non-destructive tests have been shown to be satisfactory only in determining the infiltration of electrolytically ionizable impurities, such as moisture and oxidized oils, into porous insulation. This type of deterioration, or more appropriately impurity, can be detected by 60 cycle capacitance and power factor and even dielectric absorption measurements, provided it is not too localized. It is difficult to see how any method restricted to the evaluation of the insulation as a whole can be utilized successfully in determining localized deterioration where only a very small percentage of the total insulation is involved.

The present methods have failed, except for the detection of moisture, in generators and to some extent in cables, because the types of deterioration in these cases are both different and more of a localized nature than in bushings and transformers. It is to be expected that Mr. Field's method will be somewhat more sensitive to electrolytic intrusions than other methods, because such intrusions are known to affect the polarization more at very low frequencies than at higher frequencies. Before it will be really effective, however, it must be shown that the new method will detect other types of deterioration.

The method of analyzing dispersion data proposed by the Coles may be helpful in analyzing the low frequency interfacial polarization data. It should, however, be shown that the method will represent such data accurately. The Coles have demonstrated that their procedure applies very well to the type of polarization known to be caused by dipoles, but so far not much is known as to how well it applies to other types of polarization. Mr. Field shows some rather large departures in the few cases of interfacial polarizations he has analyzed. The physical significance of the parameter α discovered by the Coles (called the storage coefficient by Mr. Field) and its relation to dielectric strength are also still unknown, which further complicates the problem.

The insulator, data for which are given in figure 8 of the paper, may be considered a representative model of a two layer dielectric. In this case the Maxwell-Wagner theory of interfacial polarization probably applies. This theory, however, has often been invoked to explain dielectric absorption in relatively pure solids. In such cases, because of the low concentration and distribution of impurities, the layer model must be reduced to molecular dimensions, and then there is no assurance that the theory applies. The existence of dielectric absorption in such a pure solid as quartz, indicates that this theory does not apply and it is believed that it has served only to obscure the true mechanism.

The further investigation of low frequency power factor and dielectric constant for various types of insulations, should prove very interesting and informative. Perhaps it can be shown that the use of two frequencies, say 15 and 60 cycles will give more information than is possible with results at the single frequency now used in field tests. Mr. Field's proposals should be paralleled with methods used at present before conclusions can be drawn as to their suitability for routine testing.

Robert F. Field: Dr. Berberich in his discussion has stated several times that I have proposed a new method for the non-destructive testing of insulation. This was not at all the object of the paper. Starting with the assumption that 60-cycle measurements have been moderately successful in such testing, the intent was to find out what electrical properties are involved. It immediately became apparent that the existing polarizations define the values of dielectric constant and loss factor at 60 cycles and at all other frequencies. Hence the parameters which define these polarizations acquire significance in non-destructive testing. It happens that the polarization mainly responsible for 60-cycle dissipation factor has a relaxation frequency well below one cycle per second and its parameters can best be determined from current-time curves. Thus it is not a new method that is advocated but an expansion of the old method, which should make possible the identification of the type of deterioration as well as its magnitude.

The point made by Dr. Berberich that much work must be done in studying the changes in the polarization parameters as deterioration progresses and in correlating these changes with decreases in dielectric strength is very well taken. The need for such studies was suggested in the paper. Not until numerical values can be assigned to the polarization parameters of all the common insulating materials and to their temperature and voltage coefficients will it be possible to interpret completely the changes in these parameters caused by deterioration.

The A-C Dielectric-Loss and Power-Factor Method for Field Investigation of Electrical Insulation

Discussion of paper 41-132 by F. C. Doble, presented at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941, and published in AIEE TRANSACTIONS, 1941 (October section), pages 934–9.

Eric A. Walker (The University of Connecticut, Storrs, Conn.): The reason for making a test on insulation in service is to determine if that insulation will continue to fulfill its purpose until the next test is made. Mr. Field, in his paper, points out that there are several measurable electrical quantities which might be considered as a basis for a non-destructive determination of breakdown strength. Mr. Doble, in his paper, and in the references he cites, shows that a dielectric loss and power factor test, properly conducted with 60 cycles, at ten kilovolts, approximately once a year, will greatly reduce the number of insulation failures. Mr. Brownlee, in his paper, now states that there are faults which are only revealed at operating voltage. This claim is worthy of some examination.

It is well known that there are certain types of faults which are non-linear, which only become evident at higher voltage gradients. This was conclusively demonstrated in a series of tests on more than 200 bushings in the summer of 1937. Such faults are usually ionic in nature and are the result of voids in the solid insulation. To reveal them by increased watts loss or power factor, it is necessary to increase the voltage gradient. Notice it is the voltage gradient which must be increased and not necessarily the voltage. There are two ways to increase the voltage gradient. One is to increase the total voltage and the other is to change the shape of the electrodes. Mr. Brownlee chooses the former method and Mr. Doble chooses the latter. Mr. Brownlee raises the voltage to operating voltage and in so doing may run the risk of measuring corona losses from the stud and connected apparatus. Mr. Doble continues to use ten kilovolts but he places an auxiliary electrode in a position which increases the gradient and in a position which experience has shown will reveal incipient faults.

The added complication and cost of a full voltage test can only be justified if experience shows that such a test will lower the number of failures below that attained by a properly conducted test at ten kilovolts. I do not believe Mr. Brownlee makes such a claim.

Gordon B. Tebo (Hydro-Electric Power Commission of Ontario, Toronto, Ontario, Can.): It is opportune that Mr. Doble's paper, describing one of the insulation test methods used extensively in the United States, should be presented at a convention here in Canada. The similarity of apparatus design and of service conditions existing in the two countries has naturally produced similar types of fault and usually similar maintenance methods are employed for their detection and repair. When a clear exception to this general rule occurs, it is interesting to enquire into the reasons.

Judging from the technical literature, the field testing of bushings in the United States is carried out largely by measurement of power factor and capacity. In Canada, the same types of bushings are regularly tested and the same types of faults detected, but by an entirely different method, that is, by direct measurement of potential gradient at operating voltage. This method was described in *Electrical World* for May 8, 1937. The two methods have been applied to large numbers of bushings, have been in regular use for several years, and each is apparently producing satisfactory results. It seems reasonable to assume, then, that each method has certain advantages, and that the maximum benefit will result from the use of both methods, each applied in its proper field.

Mr. Doble disposes of the potential gradient method in the statement—"The voltage distribution method was first tried, but this method failed to reveal many types of defective insulation that were responsible for costly service interruptions." I should like to ask Mr. Doble what type of measuring equipment was used to determine potential gradient in these preliminary tests. It is possible that inadequate instrumentation or interpretation of results was responsible for the negative results.

For the testing of bushings, the experience of the Hydro-Electric Power Commission of Ontario with the potential gradient method, over a period of five years, has been entirely satisfactory. The simplicity of the test equipment, the speed with which bushings, bus supports, lightning arrester stacks, and cable potheads can be tested and, above all, the freedom from interruptions to service for testing, appear to be important advantages.

For the testing of transformer and machine insulation, cables, and circuit breaker lift rods, to which gradient or other live-line tests are not applicable, several methods are being studied. These include measurements of insulation resistance, dielectric absorption, ionization voltage, power factor and dielectric constant at various voltages and frequencies. There seem to be insufficient data available correlating these various test methods with each other, and with operating experience, to support the choice of capacity and power factor at ten kilovolts, 60 cycles, as the best criterion of insulation condition.

J. B. Whitehead (Johns Hopkins University, Baltimore, Md.): Those who have had occasion to make dielectric measurements at high voltage using the Schering bridge learned very early the importance of electrostatic screening for the avoidance of errors due to the shunted capacitance of circuit connections. It is interesting to note that in the noticeable progressive improvement made by Mr. Doble in his field equipment, the same type of precaution becomes necessary in the very much older and simpler electrodynamometer method upon which his equipment is based.

An abnormal rise of power factor in high voltage equipment should undoubtedly be taken as evidence that advanced deterioration has occurred and that early trouble may be expected. Measurements of power factor on operating equipment, by the Doble or other method, should certainly therefore be useful in avoiding unforeseen trouble. However, it should be noted that for certain conclusion, some knowledge should be had of the normal loss and power factor characteristics of each particular type of equipment, and in fact of each individual piece of equipment. Different materials differ widely as to normal values of power factor and loss within the range of frequency and voltage under which they operate. Moreover, in certain types of equipment, as for example impregnated paper as found in cables, and capacitor type bushings, progressive failure due to a weak spot or incipient puncture may be well on its way before there is appreciable rise in the over-all value of the power factor. High power factor of bushings, terminals, joints, and cables, therefore, may well be accepted as a cause for early trouble, but the absence of a high value is no guarantee that trouble has not already begun.

I. W. Gross (American Gas and Electric Service Corporation, New York, N. Y.): The paper by Mr. Doble describing some of the technique used in developing the power factor test set for field use indicates quite clearly some of the difficulties which were first encountered in an attempt to produce a workable and reliable field power factor test set. The transition from measurement of volts, amperes, and watts in a circuit as carried on in the laboratory under ideal conditions, to a practical determination of these values under adverse field conditions, is one that has taken time, persistency, and a thorough understanding and follow-up of the electrical circuits and influences involved. Mr. Doble is to be complimented on not only the work which he has done in perfecting this type of instrument and adapting it for use under field conditions, but also for the development work which he is consistently carrying on to keep this test equipment ready to meet the ever increasing demands of field testing and servicing of high voltage insulation.

In our own company we are now using several of these field power factor test sets regularly throughout the year with the same certainty and dependence that we place on an ordinary voltmeter and ammeter. Since first using this set some ten years ago, our company was principally concerned with the results obtained by its use and the improvement in performance of the equipment which was periodically tested, as has been reported by several papers presented in recent years on this subject by our own organization. The results we have obtained in eliminating potential hazards to the system are a matter of record. It is indeed gratifying to find that Mr. Doble has seen fit to uncover the detailed technique which he has used in the development of his series of test sets which have been proving to the industry a valuable tool in the maintenance of high voltage insulation in the field.

E. D. Doyle (Leeds and Northrup Company, Philadelphia, Pa.): Mr. Doble's effort to distinguish his method and apparatus from others useful in the same field is to be commended. However, it is regretted that in choosing a name he did not take one more truly descriptive of his method of measurement. As power factor can only be obtained with his apparatus by calculation from the readings of volts, amperes, and watts, it would seem preferable to use the term "watt/volt-ampere method" and leave the term "power factor" for those systems in which it can be read directly.

It is surprising that, after having taken all the precautions that have been taken to eliminate effects of extraneous disturbances, it is still necessary to determine power factor by differences between two quantities as large as those in the given example.

Referring to the "example" and assuming that the 2.4 watt scale of the wattmeter was used, the two watt readings are respectively 79 and 67 percent of full scale, whereas the true value is only 6 percent of full scale. Under the circumstances a calculated power factor coming within 5 percent of the true power factor is an unusually close agreement.

Mr. Doble states that in calculating power factor from reversed readings the average of the currents may be used. Such a procedure is only justified when the test current and the current due to extraneous disturbances are substantially in phase or in opposition. While one cannot be certain from the values given in the example in which quadrants the resulting currents lie, it is a fair assumption that the current for the direct reading will lie in the first quadrant. On the other hand, the current for the reversed reading might be, so far as can be determined from the instrument readings, in either the second or third quadrants. Assuming that the current for the reversed reading is in the second quadrant, the true value of the test current is found to be 771 microamperes as compared with an average of the two currents given of 790 microamperes. This difference will make substantially no difference in the calculated power factor. On the other hand, if the current for the reversed reading lies in the third quadrant, it will be found that the true value of the test current would be 105 microamperes and the true power factor would be 14.3 percent. In the absence of other instrumental procedure, it is fortunate that the power factor could also be determined for check purposes in an environment free of extraneous disturbances.

If reactive volt-amperes had been read instead of currents, the effect of extraneous disturbances could have been properly eliminated by taking direct and reverse readings of watts and reactive volt-amperes. Such instrumentation has been already described before the Institute as long ago as 1918.

The bibliography would be more useful had it also included the patents disclosing the systems herein described, i.e., patents numbers 1,945,263, issued January 30, 1934 and 2,170,813, issued August 29, 1939.

F. C. Doble: Technically the testing method described by Mr. Tebo as representing the Canadian practice is entirely different from the watts-loss and power-factor method under discussion. His method depends upon a measurement of the potential gradient or voltage distribution with respect to ground over the outside surface of the porcelain rainshield of a given bushing. Extensive field experience has shown that many types of deterioration in the main insulation of a bushing are not reflected by a change in the external voltage distribution.

It is evident from analysis that a distributed deterioration up to a point near failure may cause no change in the external voltage distribution.

Further, as pointed out by Mr. Tebo, his live line method is not applicable to the internal insulation of circuit breakers and transformers, where many failure hazards occur.

Answering Mr. Tebo's question as to the type of measuring equipment used to determine the potential gradient in the voltage distribution tests mentioned in this paper, the apparatus referred to was developed by the author and proved reliable by several millions of tests in the field on all commercial transmission line voltages.

It was the limitations in the application of the voltage distribution method rather than inadequate instrumentation or interpretation of results that prompted the search for a testing method of wider scope.

Data from experience in the United States with the watts-loss and power-factor method, as outlined in the bibliography, have a direct bearing on the last paragraph of Mr. Tebo's discussion.

In connection with Mr. Doyle's comments on the naming of the author's test method, it should be noted that when the nomenclature was chosen there was no necessity for distinguishing this method from others useful in the same field as such "others" were non-existent until long after the "watts-loss and power-factor" terminology had become widely accepted.

The "example" given in the paper was selected to illustrate a certain point and the magnitude of the quantities measured should not be taken as an average condition encountered in the field. However, it should be borne in mind that the equipment described in the paper was specifically designed to meet special as well as average cases and that the "unusually close agreement" between calculated and true power factor is not too unusual under the circumstances.

The statement concerning the averaging of currents has been found to be true for practically all cases so far encountered in field work, since in general the magnitude of the current from induced voltages is small in comparison with the current through the specimen resulting from the test voltage. Mr. Doyle's assumption that the current for the reversed reading might lie in the third quadrant in the example given in the paper would require an induced current of 790 microamperes in a specimen with a test current of 105 microamperes, which is a condition difficult to imagine in practice.

In the event that any doubt exists as to the magnitude of the induced current, its value can be obtained directly by reading the ammeter with no test voltage applied to the specimen. Using this current and the currents obtained from the direct and reversed readings, the true value of the specimen test current, even under adverse conditions, may be determined by calculation.

Referring to the last paragraph in Mr. Doyle's discussion, the author believed that it might be more ethical to omit the patent numbers from the bibliography.

The Protection of Solid Insulation by Lightning Arresters

Discussion and authors' closure of paper 41-133 by D. D. MacCarthy and T. J. Carpenter, presented at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941, and published in AIEE TRANSACTIONS, 1941 (October section), pages 912–15.

V. M. Montsinger (General Electric Company, Pittsfield, Mass.): I wish to compliment the authors on their contribution to the field of knowledge of the impulse voltage characteristics of solid insulation.

As pointed out by the authors, their results differ somewhat from the results that I have obtained for larger insulation spacings in that (1) the turn-up of the volt-time curve is less; and (2) the ratio of repeated to single shot breakdown is less than I obtained. Considering the difference in the structure of the pads used by the authors and as used in my tests, I feel that the difference in results is quite logical.

The maximum thickness of their barriers

was 40 mils which is considerably less than any solid barriers that I have tested. The smaller turn-up for the pads used by the authors, I believe, is due entirely to the nearly uniform field resulting from the short spacing between the electrodes. The same difference in turn-up on the volt-time curve of rod gaps having small and large spacings has been found. In fact, for very small spacings a rod gap acts quite like a sphere gap. The same phenomenon, no doubt, applies to thin pads of insulation. The fact, as pointed out by the authors, that the breakdown voltage of the insulation as shown in figure 4 of the paper increased linearly with the thickness, is proof that the dielectric field was approximately uniform. For non-uniform fields the breakdown strength of insulation varies approximately as the thickness raised to the two-thirds power.

In reference to the repeated shot strength being 70 per cent of the single shot strength, the only condition where I found the value to be less than 80 per cent was when I used bare electrodes in which case it was as low as 60 per cent due to the appearance of corona at relatively low voltages. I have no very logical reason to offer as to why the authors obtained 70 per cent unless it was due to the absence of free oil in series with the solid. I have obtained repeated shot values as high as approximately 90 per cent of the single shot values for high voltage barriers, but the minimum and safe value appears to be approximately 80 per cent. I believe, therefore, that the authors' value lower than 80 per cent was due mostly to the fact that there was no free oil present.

P. L. Bellaschi (Westinghouse Electric and Manufacturing Company, Sharon, Pa.): The paper by Messrs. MacCarthy and Carpenter is interesting in that it illustrates certain principles on protection. It is apparent, however, that the conclusions in the paper apply to the insulation sample (figure 1 of the paper) they have selected for their comparative tests. This insulation sample naturally has a flat volt-time characteristic as it is known from tests on similar, solid-layer insulation samples (AIEE TRANSACTIONS, volume 60, 1941, page 400, discussion by P. L. Bellaschi and W. L. Teague). This flat characteristic is not generally representative of surge-proof distribution transformers. In the modern power as well as in the present-day distribution transformer, with which the writer is familiar, the volt-time characteristic is more pronounced, attaining for a 0.25 microsecond impulse, some 150 per cent or greater the two microsecond crest value (figure 3, "Protection of Power Transformers Against Lightning Surges," AIEE Transformer Subcommittee, AIEE TRANSACTIONS, volume 60, 1941, page 568).

In regard to the ratio of the breakdown voltage for repeated and single applications, the value (0.70) cited in the paper applies specifically to the sample tested and is not necessarily the same for distribution transformers in general. This ratio depends on the "injurious corona level" of the whole insulation and on other features of design. It is well to note that the significance of the single-shot strength has long been open to much questioning. In this connection, the practice in recent years has been to consider the volt-time characteristic of insulation on the basis of repeated tests and thus do away with a factor of doubtful value.

Experience has demonstrated that dependable conclusions on transformer insulation and protection can be reached only from comprehensive tests on the complete apparatus plus field experience over a period of years. These tests should be carried well up to the steep-front, high-current surges; for the frequency of occurrence of the higher currents, particularly on rural circuits, is by no means a negligible quantity as it can be noted from reference 2 of the paper as well as from other sources of field experience.

Herman Halperin (Commonwealth Edison Company, Chicago, Ill.): The interpretation of the test results in this paper, which apparently apply to some types of transformer insulation, is made somewhat difficult due to the type of the electrodes used. Some supplementary tests with well-defined electrodes would have given information of more general application. The spiral electrodes used introduce variables difficult to evaluate. The stresses are not definitely known. The fact that the spirals were tied by means of threads must result in either oil gaps between the electrode and paper pads or damage to the pads due to impressions from the threads. Actually all failures occurred at the points of threads and therefore might have been promoted by breakdown of the oil gaps or by impressions on the paper pads.

The great difference in surge strength of the paper pads and the paper wrapped upon the spiral is of interest. According to figure 4 of the paper, the one-microsecond surge strength of the paper pads is 2,660 volts per mil while the surge breakdown voltage of 20 kv for the 20 mils of wrapped insulation corresponds to only 1,000 volts per mil. Incidentally, this 2,660 value compares favorably with a surge breakdown strength of 2,000 to 3,400 volts per mil on a maximum stress basis obtained for commercial cable insulation.

The comparatively low values for the insulation wrapped on the spirals are probably caused by distortion of the field, nonuniformity of the insulation due to wrapping and due to the threads, and the fact that the wrapped insulation is not continuous as in the pads. Conversely, apparently higher overall strength may be obtained in transformer and other insulation when these disturbing factors are absent. This point is substantiated by data, e.g., in the paper by L. I. Komives at this convention.

It is extremely interesting to note the authors' conclusion that the insulation can stand safely surges up to 70 per cent of the single-shot breakdown strength.

D. D. MacCarthy and **T. J. Carpenter**: In his discussion, Montsinger has offered an explanation for the differences between the results of the present investigation and the results that he obtained in tests on high voltage insulation. It is gratifying to the authors to see that there are no irreconcilable differences between the results of their tests and those which Montsinger has obtained.

The authors agree with Mr. Bellaschi that the samples tested cannot be considered as having the same insulation strength as transformers. In the paper it was stated: "Although all of the materials in the pads are used for certain types of insulation in transformers, the pads as tested cannot be considered as representing the actual insulation strength of transformers, since in transformers steep impulse waves may result in an unequal distribution of voltage along the turns and the major insulation does not consist of paper sheets alone." Concerning the difference in the shape of the puncture voltage-time characteristic, it should be noted that the figures given by Bellaschi compare the insulation strength at 0.25 microsecond on the wave front with that at two microseconds on the tail of a 1.5x40 microsecond wave; whereas the comparison made by the authors was based on puncture at 0.25 and 1.0 microsecond, both on the front of uniformly rising voltage waves. These two sets of data differ not only because of the difference in wave shape, but there is also a difference of two to one in the range of time for which the turn up is compared.

The authors believe that the figure of 70 per cent found for the repeated shot strength of the low voltage insulation tested is in reasonable agreement with the range of 65 to 90 per cent found for high voltage insulation.

Bellaschi has mentioned the need for tests at high impulse currents. The authors did not feel the need for such tests since the effect of arrester *IR* voltage can be predicted from the volt-time puncture curve of the insulation and the arrester voltage. For example, test 9 in the tabulation in the paper was made with an *IR* voltage of 55 kilovolts across a constant resistance. To obtain the same crest *IR* voltage with the valve type arresters used in tests 2 and 3, a discharge current magnitude of 60,000 amperes would be required, providing the current wave was 10x20 microseconds. This *IR* voltage is less than 70 per cent of the single shot strength of the pads tested in parallel with gaps equivalent to a rod gap set 2 inches, see tests 10 through 21.

Mr. Halperin states that electrodes of more regular shape would have been preferable. It is true that such electrodes would give more uniform fields and would be better if the purpose of the investigation had been to determine dielectric strength of the insulation in kilovolts per mil. However, the electrodes used were chosen in the effort to simulate the insulation used in actual apparatus rather than to determine the physical properties of the insulation materials.

The explanation that Halperin gives for the relatively low strength of the paper wrapping on the line electrode and for the tendency of puncture to occur at the edge of the coils is in agreement with the authors' statement in the paper. It is of interest to note that the strength of the paper pads is of the same order as has been found for paper insulation in tests on cables.

System Short-Circuit Currents

Discussion and authors' closure of paper 41-121 by W. M. Hanna, H. A. Travers, C. F. Wagner, C. A. Woodrow, and W. F. Skeats, presented at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941, and published in AIEE TRANSACTIONS, 1941 (September section), pages 877–81.

G. C. Crossman (Consolidated Edison Company of New York, Inc., New York, N. Y.): The paper presents a simple method of calculating short circuit currents which should prove very valuable, particularly for determining circuit breaker interrupting capacity and mechanical stresses.

However, many power station busses and switch gear are of the isolated phase type of construction where it is practically impossible for faults to occur except from line to ground. While the paper appears to be based primarily on three-phase faults, it does indicate that the same method of fault current determination may be used for determining circuit breaker interrupting capacity for such ground faults. It is indicated that the same multiplying factors should be applied to the calculated instantaneous symmetrical current to obtain the maximum asymmetrical current at the time of arc interruption.

These factors appear to be based in part upon the decrement of the direct current component of three-phase fault currents. Consequently, the added resistance of the ground return will shorten the duration of this component for line to ground faults. With the usual design of station ground bus and grounding system and particularly when only one of a number of paralleled generators is grounded, the duration of this d-c component may be very materially shortened. Tests recently made at one of the generating stations of the Consolidated Edison Company indicated a reduction of approximately 50 per cent in the time constant of the d-c component for a single phase, bus short circuit as compared with the corresponding three-phase fault. A single generator unit, a 160,000-kw cross-compound machine, was used in these tests with one of the two generators of the unit solidly grounded. Had other ungrounded generators been operating in parallel with this machine during these tests, the reduction of the d-c time constant for a line to ground fault would have been still greater.

Consequently, if bus circuit breaker interrupting capacities are determined by line to ground faults, the calculation of these faults by the method given in the paper will provide an additional factor of safety. It might seem desirable in some cases not to use this additional safety factor and to buy a smaller breaker instead. However, the difficulty of determining the ground return resistance with any degree of accuracy for most power stations would probably prevent any such saving.

W. R. Brownlee (The Commonwealth & Southern Corporation, Jackson, Mich.): In view of the wide variations encountered in supply system characteristics, the authors have reached a remarkably logical method for general use in the calculation of short circuit currents. In the application of faster breakers and fuses, and in the consideration of mechanical stresses, the system d-c time constant is shown to play a major part. It has been customary for the user in applying breakers to calculate the symmetrical short circuit current and then to select a standard interrupting duty breaker with some margin of safety based on experience and judgment. System information which must be assembled for the calculation of symmetrical short circuit current will frequently include almost enough data for determining the d-c time constant with reasonable accuracy. Thus, even for users of interrupting equipment who prefer to consider specific system conditions individually, the paper is of great value in focusing attention on the relative importance of various factors such as the subtransient time constant, the d-c time constant, the effect of voltage regulators, the influence of relative phase positions of synchronous machines, and the influence of arc resistance.

Except for the case of faults at a generating station bus (which is covered by a definite exception in the paper recommending as much as 20 per cent increase in calculated current), the method outlined appears to be quite conservative. Recommendations take little account of the effect of arc drop on the d-c time constant, although the paper mentions a reduction of three to one in this time constant by means of arc voltages which may safely be considered as minimum values, particularly in the case of low voltage circuits. The method described in the paper further assumes that the fault is initiated at the point on the voltage wave which will produce a completely offset current wave. Since the interrupting duty is taken as the maximum value, this viewpoint can probably be justified for operations involving the closing of the breaker onto a fault. However, since maximum interrupting ability is expressed on a two unit basis, the first interruption will usually result from a flashover tending to produce a rather symmetrical current wave, and accordingly the assumption of complete current wave displacement for both operations may be unnecessarily conservative.

It is difficult to apply general reduction factors for equipment operated at five kilovolts and below, especially in the case of supply to station auxiliaries, which use accounts for a large number of breakers operating at 2,400 volts and 480 volts. If the supply for station auxiliaries is taken from a high capacity bus through current limiting reactors, the d-c time constant may easily approach that of a generator bus, whereas in many installations the supply is from a 5,000-kva or smaller transformer bank, with a resulting d-c time constant of around 1 cycle (or about one-third the typical value for transformers given in the paper).

While the general factors recommended in the paper are unquestionably of material value in clarifying the problem of short circuit calculations and in providing information of a general nature, it is suggested that purchasers of interrupting equipment will find it well worth while to include in their short circuit calculations enough information to determine whether the d-c time constant at the point of application is materially different from the assumed average value of nine cycles.

G. W. Hampe (Commonwealth Edison Company, Chicago, Ill.): For determining the maximum duty of circuit breakers, the short circuit calculating procedure outlined by the committee doubtless represents a distinct advance. For relays, however, its utility is considerably less for the following reasons:

1. It adds nothing to the technique of determining current values occurring appreciably after the start of the fault.
2. It takes no account of a little understood class of transient phenomena which influences high speed relays—ground relays in particular.

High speed relays for protection against interphase faults are usually pilot channel or distance types and hence require only crude determinations of fault currents, supplemented frequently by staged short circuit tests in which the combined effect of load and fault currents can be investigated.

The need for short circuit computations arises chiefly in determination of settings for back-up relays having operating times exceeding 0.5 second. This means that the difficulties ruled out by the conditions as set forth in the paper are still with us. They result from the multiplicity and wide distribution of machines, phase angles, excitations, changes of excitation, changes of saturation, and other causes.

It is true that the use of standard decrement curves is difficult and may be misleading. In the computations with which the writer is familiar such curves have not been used for a number of years. In the rare cases in which a number of relays operate sequentially at considerable intervals, the decrement (or increment) curves are evaluated and plotted for individual lines from values determined by the "internal voltage" method.

In general, however, values useful for settings on a wide range of relay types can be obtained by computing with transient voltage and impedance values for synchronous machines, with each machine impedance value adjusted to meet local conditions. The exceptions are:

1. Faults substantially at machine terminals.
2. Values specifically needed for high speed relays.

There is definite need for investigation of phenomena of the type shown on the attached oscillogram. Such phenomena are distinctly outside the scope of the present paper, and even outside the scope of the basic theory of symmetrical components, but in making settings on high speed relays they are important. Usually such transients appear as residual currents and have the greatest effect on ground relays. In many cases current transformer defects, such as remanent core magnetization cause considerable spurious residual currents, but in other instances such as that shown on the oscillogram the residual transient of about 1,400 amperes peak actually existed on the line and in the power bank neutrals. This phenomenon occurred during a staged test. A three phase fault was applied to the intermediate voltage (33 kv) winding of a wye-wye-delta bank, and the bank was made alive over a 132-kv line consisting of six miles of underground cable and 3.5 miles of overhead line with an unloaded 27-mile section of overhead line extending beyond the faulted bank. The 132-kv neutrals of the source transformers and of the faulted transformer were grounded solidly.

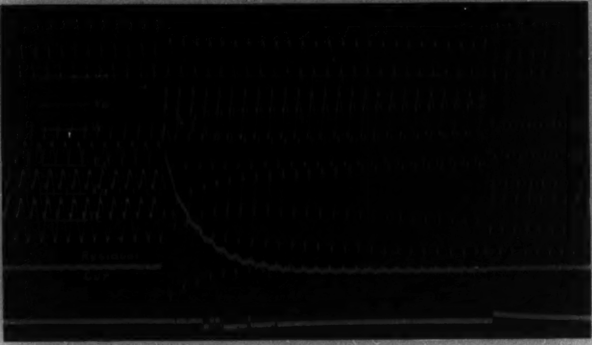


Figure 1

Similar phenomena have been obtained at other times on other equipment, but tests on circuits quite analogous to them showed a complete absence of such transient phenomena.

In view of such phenomena, too much reliance cannot be placed on computations of the type given in this paper where high speed relays are concerned. As a different class of phenomena enters in some cases, further study of the currents affecting high speed relays is needed.

W. M. Hanna, H. A. Travers, C. F. Wagner, C. A. Woodrow and W. F. Skeats: There appear to be some misunderstandings with reference to the treatment of the direct current time constant in the paper. The value of nine cycles is that used in the decrement curves of Hahn and Wagner and represented by the upper curves of figure 1 of the paper. In preparing the present paper, allowance was made for additional resistance in busses and other apparatus, so that considerably lower time constants are used, as is evident from the spread between the upper solid curves and the dotted curves of figure 1.

It should also be noted that the curves of figures 2 and 3 of the paper do include a reasonable allowance for arc drop.

Mr. Crossman's comments with respect to the decay of the d-c component in line-to-ground faults are very interesting. His data with respect to the effect of the resistance in the ground return upon the time constant are quite probably rather broadly representative. However, it should be borne in mind, as explained below, that resistance in the ground return may increase the current in one phase of a two-phase-to-ground fault, and that this condition may bring about higher currents than the single-line-to-ground case. It is not recommended, therefore, that any reduction in breaker size on this account should be considered in such an application.

The authors agree with Mr. Brownlee that short circuits in the field more frequently approach the symmetrical than the fully offset condition. Nevertheless it is felt that a high degree of offset does occur with sufficient frequency that it should be provided for in the application of interrupting devices.

Mr. Brownlee points out that a wide variation exists in circuits at five kilovolts and below as regards the resistance-reactance ratio and consequently the time constant of the d-c component. Here again the authors agree and accordingly have limited the application of any reduced multiplying factor in connection with such circuits so as to except cases where the fault current is fed predominantly by directly connected synchronous machines or through reactors.

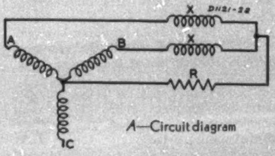
Mr. Hampe presents an interesting discussion of the determination of short-circuit currents long after the inception of short circuit for the purpose of setting back-up relays. Others faced with the same problem will undoubtedly find his suggestions helpful.

Mr. Hampe also shows an oscillogram of a three-phase-to-ground short circuit in which although there exists no normal frequency current in the neutral, there does exist there a unidirectional transient of considerable relative magnitude. This is caused by sequential closing of the circuit through the phases to ground in such a way that the d-c components from the three phases add in the neutral, or at least do not completely balance out as they would if the fault were established simultaneously in all three phases. We may assume for simplicity a circuit in which positive, negative, and zero phase sequence reactances are all equal so that a change in the current in one phase will not be reflected in the other two. Now with phase rotation *ABC*, assume phase *A* grounded at such a time as to give a fully displaced wave. This will produce both a-c and d-c components equal to the corresponding phase currents in the neutral. Now suppose that 120 degrees later phase *B* is grounded. This will also produce a fully displaced wave giving rise to both a-c and d-c components in the neutral. The a-c components from the two phases will add vectorially at an angle of 120 degrees, but the d-c components will have the same polarity and add arithmetically. If after another 120 degrees phase *C* is grounded, it also will contribute both a-c and d-c components to the neutral circuit. The three a-c components will now add vectorially in such a way as to cancel out, but the three d-c components will all have the same polarity and will add arithmetically. Thus a unidirectional surge will be obtained as shown on Mr. Hampe's oscillogram.

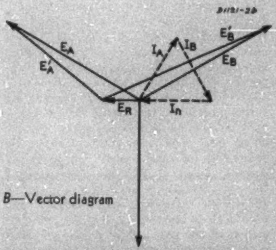
It will be readily appreciated that the same type of phenomenon can take place with other relations among the phase sequence reactances than those assumed although the magnitude of current involved may be somewhat modified.

It will also be appreciated that the magnitude of neutral current obtained in this way for a given circuit will vary from one fault to another even under apparently duplicate conditions since the amount and polarity of d-c component contributed with the faulting of each phase is dependent upon the point on the voltage wave at which the fault is completed in that phase.

Inasmuch as considerably higher neutral currents may be obtained in this way than under any normal fault conditions for a circuit of equal reactance, it is possible that it may occasionally result in false tripping of high-speed ground relays with selective operation based on the magnitude of neutral current. Relays may be obtained, however, which are sensitive only to normal frequency current and such relays will operate satisfactorily.



A—Circuit diagram



B—Vector diagram

Figure 2. Double-line-to-ground fault with resistance in ground return

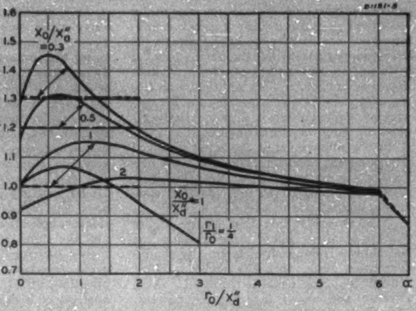
The procedure proposed in the paper calls for the determination of "the highest value of rms symmetrical current for any type of fault using subtransient reactance. . ." but gives no further instructions for the determination of this quantity. The following comments will be helpful in this respect. Let

E = leg voltage

X_d'' = circuit reactance as viewed from point of fault using direct axis subtransient reactance for generators, load machines being represented as specified in the paper

Figure 3. Maximum phase current for a double line-to-ground short circuit expressed as a ratio to E/X_d''

Dotted lines are plotted for $3E/(2X_d''+X_0)$ $X_2=X_d''$



X_2 = negative phase sequence reactance of circuit

X_0 = zero phase sequence reactance of circuit

Then the three-phase short-circuit current is $\frac{E}{X_d''}$. The line-to-line short-circuit current is $\frac{\sqrt{3}E}{X_d''+X_2}$. Since X_2 is always at least equal to X_d'', this quantity never exceeds 86% of the three-phase short-circuit current and consequently may be neglected.

The line-to-ground short-circuit current is $\frac{3E}{X_d''+X_2+X_0}$. Since X_2 is usually approximately equal to X_d'', this expression is often shortened to $\frac{3E}{2X_d''+X_0}$. It may also be noted that, as explained in more detail below, the latter expression serves reasonably well over the important range for double-line-to-ground as well as single-line-to-ground faults, whereas if there is appreciable difference between the two expressions, the former considerably underestimates the double-line-to-ground fault current. The expression $\frac{3E}{2X_d''+X_0}$ is therefore considered preferable for line-to-ground short-circuit currents on the ground of giving more useful information as well as on the ground of convenience. If X_0 is less than X_d'', the value given by this expression exceeds the three-phase short-circuit current and hence becomes the governing quantity.

Neglecting resistance, the double-line-to-ground short-circuit current is given by the expression

$$\sqrt{3}E\frac{\sqrt{X_2^2+X_2X_0+X_0^2}}{X_d''X_2+X_d''X_0+X_2X_0}$$

Except for values of X_0 less than $\frac{X_d''}{4}$, which occur very seldom in practical field circuits, the current corresponding to this formula is exceeded by one or the other of the two quantities recommended for use; that is to say, either by E/X_d''; representing three-phase fault currents, or by $\frac{3E}{(2X_d''+X_0)}$ representing single-line-to-ground fault currents. It is comparatively insensitive to negative phase sequence reactance, however, and consequently may appreciably exceed the quantity $\frac{3E}{X_d''+X_2+X_0}$ if X_2 is much greater than X_d''.

An interesting effect is produced in a double-line-to-ground fault by the existence of different resistance-reactance ratios in the three phase-sequence circuits, as when resistance occurs in the zero phase sequence circuit while it is negligible in the positive and negative phase sequence circuits. While such a resistance decreases the current in one of the faulted phases, as would be expected, it brings about an increase in the current of the other phase.

This may be understood qualitatively from the circuit and vector diagram of figure 2 of this discussion. Figure 2*A* represents a circuit in which phases *A* and *B* of a power source are connected through a reactance *X* in series with each phase and a resistance *R* in the common return to the neutral. The three phase voltages are shown as E_A, E_B, and E_C in the vector diagram, figure 2*B*. As a first approximation, the phase currents may be taken as lagging the phase voltages by 90 degrees, as do I_A and I_B. The neutral current, I_n, then closes the triangle. The voltage across the neutral resistor is in phase with this current, as shown by E_R. The voltages across the phase reactances are then E_A' and E_B', and it will be noted that while E_A' is smaller than E_A, E_B' is larger than E_B. Correspondingly the *A* phase current will be less than if the resistance were not present, but the *B* phase current will be greater.

The amount of the increase for various values of zero phase sequence resistance and reactance is shown in figure 3 of this discussion, and here it will be noted that values as much as 15 per cent higher than the corresponding single-line-to-ground current may be reached.

In connection with this increased current, however, the following facts should be considered.

1. Resistance in the other phase-sequence circuits has a mitigating effect. If $R_1=R_2=R_0/4$, the maximum increase is reduced from 15 per cent to $7^1/_2$ per cent, and the range over which any increase exists is considerably reduced.

2. Resistance of sufficient magnitude to have appreciable effect introduces a rapid initial decay of the d-c component which is great enough so that the total rms current in the first cycle will not exceed that involved in other types of fault.

3. The range of resistance in which this occurs appears to be one not frequently encountered in practical field circuits:

(*a*). Normal values of resistance deliberately inserted in the neutral lie above it.

(*b*). Transmission line tower footing resistance in general lies above it, particularly at 69 kv and below.

(*c*). The resistance introduced by a copper transmission line ground wire tied in to the station ground may bring about an increase over other types of fault at a given point on the line, but such an increase will always be less than the reduction, caused by line reactance, in short-circuit current at that point as compared with the current available at the station.

(*d*). Arc resistance may lie within the range, but is effective to increase the current only when the fault consists of a line-to-line breakdown plus a breakdown to ground from one of the two lines involved, and not in the case of breakdowns directly to ground from each of two phases.

4. From the above it will be observed that this increased current may be expected only infrequently even among double-line-to-ground faults. For the few cases in which it may occur, it is suggested that attention be given to the following:

NEMA rule SG6-84 reads as follows:

"Where on oil circuit breaker will be called on to interrupt only short circuits between line and ground of a solidly grounded system, the interrupting capacity rating in arc amperes shall be 115% of its interrupting rating in arc amperes at the corresponding line-to-line voltage."

The reason for this rule is evidently that removal of the hazard of encountering the higher recovery voltages associated with line-to-line faults and three-phase faults is assumed to enable a circuit breaker to interrupt an additional 15 per cent in current. In the case of increased current from a double-line-to-ground fault, this hazard is also absent. Consequently, it seems reasonable that a 15 per cent allowance be made here just as in rule SG6-84. Taking this allowance into account, the highest current which needs to be considered is either E/X_d'' or $3E/2X_d''+X_0$, whichever is higher. Furthermore, if R_0 is greater than $5X_d''$, as in the case of deliberately inserted neutral resistance, no consideration need be given to the latter term and the highest current may be taken as E/X_d''.

It is recommended, therefore, that the "highest value of rms symmetrical current for any type of fault" be taken as either E/X_d'' or $3E/2X_d''+X_0$, whichever is higher, except that when R_0 is greater than $5X_d''$, no consideration need be given to the latter term.

There is a discrepancy with reference to the application of low voltage air circuit breakers between the last full paragraph of the first column of page 880 and the last paragraph of the first column of page 881 of the paper. The application of these breakers should be based upon subtransient reactance for all synchronous and induction machines, and the statement on page 881 should be considered as modified accordingly.

Power-Circuit-Breaker Ratings

Discussion and author's closure of paper 41-116 by R. C. Van Sickle, presented at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941, and published in AIEE TRANSACTIONS, 1941 (September section), pages 882–4.

T. G. LeClair (Commonwealth Edison Company, Chicago, Ill.): Mr. Van Sickle's paper outlines very clearly the reason for coordinating those ratings of circuit breakers which are interdependent. However, in the final suggestion for the circuit breaker ratings, he has added the term, "rated latching current." This addition is an important one and warrants some further discussion, even though it is not entirely related to the preceding rating items.

In the preparation of the latest standard 19 for circuit breakers, there was some discussion as to whether or not this item should be added to the list. At that time it was decided not to add such a rating.

It is generally true that in the higher voltage breakers, where the strength of the mechanism is determined largely by the interrupting time, that the rated latching current is at least equal to the rated making current and rated interrupting current. However, on breakers of lower voltage and higher continuous current ratings, many of the present day breakers cannot close and latch against the maximum short time current without increasing the size of the mechanism, and consequently, the size and cost of many other related parts.

It is probably true that many, if not most, of the applications for circuit breakers are such that a breaker will not be called upon to close and latch against the maximum instantaneous current or against the rated interrupting current. The usual application is such that if a breaker is closed against a short circuit, the relays will instantly trip this same breaker so that the breaker will open and clear the short circuit without ever having reached the totally closed position. There are some applications where the breaker being closed is not the one which should be opened to clear a short circuit. In such applications, there may be damage done to the breaker if it continues to hold the circuit partially closed until the remote breaker has had time to open. For such application, the latching current should be equal to the maximum interrupting current. For the majority of applications, it would be an unfair burden on the cost of circuit breakers to make the latching current equal to the maximum interrupting current. It is possible that if full publicity were given to the discrepancy between latching current and maximum interrupting current, that it might be necessary in some cases to have a standard breaker with a relatively small mechanism and a special breaker with a heavier mechanism where the latching current must be equal to the interrupting current.

It is necessary to give the subject some additional consideration before such a rating is added to the list now in the standard for power circuit breakers.

C. A. Woodrow (General Electric Company, Schenectady, N. Y.): In at least one way, Mr. Van Sickle's contribution is a very logical extension of this working committee paper, "System Short Circuit Currents." The aim of this paper has been to provide improved and simplified methods of calculation but always within the boundaries of existing rating standards. Under AIEE definitions the interrupting rating of a power circuit breaker is based on the rms current at contact parting, regardless of the time interval between instant of short circuit and contact parting.

This variable time interval consists of relay time plus breaker contact parting time. The joint short circuit paper proposes the elimination of the relay variable. It recommends general breaker application which provides interrupting capacity for high speed relaying; this conception results in allowance for a small but essentially definite relay time interval. As Mr. Van Sickle suggests, if this approach proves acceptable, why not extend it and eliminate the contact parting time variable also? Under existing interrupting rating standards, the five cycle breaker for any particular application must have ten per cent—and the three cycle breaker 20 per cent—higher interrupting rating than the eight cycle breaker. Obviously, breaker application would be far simpler and cost analysis of higher speed fault clearing far more definite if required interrupting current rating were not dependent on interrupting time rating; i.e., if these 10 and 20 per cent factors were made design factors rather than application factors.

In order to obtain any application simplification whatsoever, the authors of "System Short Circuit Currents" have had to make approximations which violate the exact interpretation of present interrupting rating standards. For all eight-cycle breakers, four cycles for the total of relay plus contact parting time is assumed. Actually standard high current breakers may have appreciably longer contact parting time than average, whereas other eight cycle breakers may have somewhat shorter contact parting time, particularly on C-O operation. In this latter case it has been tacitly assumed that breaker design includes the technically necessary margin to allow for the higher duty possible with the shorter contact parting time.

All these problems could be eliminated if power circuit breaker interrupting current ratings were based on the current at a definite time of four cycles (60-cycle basis) after the instant of short circuit rather than on current at the instant of contact parting. Such a solution, however, means at least minor rating changes.

Mr. Van Sickle goes still further. He suggests the elimination of application factors dependent on special decrements for low voltage circuits and the possibility of interrupting rating based solely on the initial a-c component of short circuit current. It would seem that these thoughts provide important criteria for desirable future product standards and application simplification—though there may be obstacles to overcome before such rating changes can be adopted.

However, in support of the possibility of ratings based solely on a-c components of current, it is perhaps worth pointing out that, for all types of apparatus, it is highly desirable to make reasonable allowance in design for the effect of the d-c component of short circuit current and eliminate its complexity as an application investigation requirement. This goal has already been reached in determining momentary requirements of power transformers, current transformers, and reactors and in the interrupting ratings of protector tubes. No matter how simple the application rules, the occasional misapplication will continue so long as the d-c offset allowance remains an application rather than a design factor.

J. A. Elzi (Commonwealth and Southern Corporation, Jackson, Mich.): A review of the AIEE and ASA standards for oil circuit breakers indicates that clarification of some of the ratings is highly desirable. As the author has pointed out, present day breakers designed for operating voltages of 15 kv and below, usually are required to cover a broader range of operating voltage, and consequently a greater range of fault currents than the higher voltage breakers and present the greatest difficulty from the standpoint of mechanical forces and heavy maximum short-time currents. These breakers are also the ones which are most frequently used at busses with directly connected generating equipment or with current limiting reactors where the system time constants are such that the ratios of the initial current to the interrupted current proposed in this paper do not apply.

Breakers in this voltage class represent a large portion of the total number of breakers used, and it appears that the proposed method of rating breakers would be somewhat limited in its application.

A point which has not been mentioned is the fact that in many cases the ground fault current rather than the phase to phase fault current may determine the interrupting capacity required in a breaker, and the time constant for a ground fault may be considerably different from that for a phase to phase fault.

The author makes the suggestion that a further simplification could be effected by making the breaker application on the basis of the initial a-c component of fault current. It would appear that this could easily lead to a considerable misunderstanding in comparing breaker performance as demonstrated by tests. In making breaker tests, the current would usually be found to be displaced, at least by a certain amount, and it would seem rather difficult to establish the value of symmetrical current which would be comparable to an offset wave obtained from tests.

R. C. Van Sickle: Circuit breaker applications involve so many variables that there will always be exceptions to any general rule which may be formulated. The method of application proposed in the report "System Short-Circuit Currents" recognizes this by recommending factors which are modified for certain voltage classes and for special conditions.

The suggestion was made in the paper "Circuit-Breaker Ratings" that if the proposed method of application were found to be satisfactory, a new method of breaker rating could be used which would apply the factors for normal conditions in the design of the breaker. The exceptions to the general cases would still have to be taken care of by special factors. However, it would not be necessary to have two sets of factors, one for normal conditions and one for special conditions.

The industry could select and publish for various classes of breakers, the first cycle currents and the interrupting currents corresponding to the rated initial a-c. components. Breakers would be designed to make, carry, and interrupt currents having these values. If the calculated current were found to be close to the breaker rating and the decrements appeared to be unusual, the current to be interrupted could be calculated and compared with the value of published interrupted currents for the breakers in question.

Test results likewise would be compared with published values. As at present, the comparison would be on the basis of the total rms. current.

The rated latching current was included in the list of items of a circuit breaker rating because it is defined in AIEE Standards. This was not intended to imply that the rated latching current would be standardized and published along with the other items of breaker rating. As pointed out in the paper, this item bears no definite relation to the other items.

Mechanical Simplicity of Air-Blast Circuit Breakers

Discussion and authors' closure of paper 41-114 by H. W. Haberl and Otto Jensen, presented at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941, and published in AIEE TRANSACTIONS, 1941 (September section), pages 869–74.

Howard M. Wilcox (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The paper by Messrs. Haberl and Jensen marks a forward step in the accumulation of a fund of information on compressed air switching equipment which, added to as work progresses, will ultimately constitute a general practice in governing an art still young enough to display a lack of unity in the approach to fundamental problems. The authors are to be congratulated on bringing together in a single paper data covering the application of one type of air-blast operation to the complete a-c voltage range. The accumulated service experience with these structures, while still not extensive, must be accepted as the best barometer available and seems to indicate that the means used has been found adequate thus far. As a measure of progress it may, then, give rise to speculation as to how the results obtained may be interpreted in applying them to the general problems of the air-blast breaker.

Little information is offered as to the volumes of air used in breaker operation. It is noted that operating pressure for the 138-kv rating is 100 pounds (about seven atmospheres) and since the same general arrangement is used throughout it may be assumed that working pressures for the other ratings are of the same general order of magnitude, although the reservoir construction used on the lowest voltage rating might indicate a still lower pressure. The question then arises as to whether these pressures have been found best suited from the viewpoint of economical storage space as a reserve for emergency multiple breaker operations. In the United States the tendency has been toward working pressures of from 10 to 17 atmospheres even at generating voltages, with a back-up supply at still higher pressures, as a means of storing a greater reserve volume in the same reservoir space. The alternatives would seem to be to supply more reservoir space, higher capacity compressing equipment, or working the same equipment a greater proportion of the time.

Another and related question arises in connection with condensation at these lower pressures. In the United States serious consideration has been given to a compressed air supply system having a pressure about three times the normal breaker working pressure for high voltage outdoor applications as a means of moisture removal in the expansion process. The alternative would seem to be a mechanical dryer. The authors' experience on this point during the severe cold of a Canadian winter should be of particular interest.

The paper emphasizes the type of blast valve used throughout this line of breakers. While this valve undoubtedly lends itself readily to fast breaker operation, it is noted that it has been found desirable in practically all cases to install the valve almost completely within the breaker reservoir and in some cases the blast tube and contact operating rod are likewise incorporated in the reservoir. To those familiar with the rather stringent Code requirements governing the construction and use of unfired pressure vessels in the United States this would seem to present some complications. While details shown in the illustrations are not sufficiently complete to warrant forming a definite opinion, it would also seem to interpose some difficulties in the way of inspection and maintenance.

It is noted with interest that the reclosing contacts are retained in all designs down to the lowest voltage rating, necessitating use of the series isolating switch throughout. To those familiar with the limited installation space generally allowed for circuit breakers in four kilovolt service, this would seem to present a serious handicap in competition with the compact, self-contained, air-insulated circuit breakers now available for this service.

A statement by the authors as to their experience on some of these points would add materially to the value of their paper as a contribution to the art.

J. B. MacNeill (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The authors are to be congratulated on the excellent slides used in presenting their paper, which make clear to a lay observer the relationships and movements of the parts involved. The valve release mechanism is very ingenius and permits of high speed operation with a minimum of control energy.

The subject of outdoor compressed air circuit breakers is receiving intense study by several manufacturers in America and trial installations are being made. A wide commercial activity on this type must await better proof of the insulation characteristics under service conditions. There is a dearth of suitable insulating materials for outdoor interrupters that are not enclosed in oil. Research work on this subject will improve the situation over a period of time but further work is required.

The large operators have taken the position for the present that fairly definite proof of service reliability must be had before they will invest largely in dry types of outdoor switching equipment. They doubtless have in mind the limitations of interrupter insulation, the dry internal tube surfaces continuously operated between line voltage and ground, and the complication of an isolating switch which must function promptly after interruption.

The advantage of the compressed air type is its flat arcing time characteristic from no-load to full-rated when properly designed, and since this arcing time normally is very short the arc energy is small. However, any device which depends on extremely short arcing time for successful operation should be well proven by test or an extra half cycle or two of arcing may represent a destructive increase of arc energy. Little reference is made in this paper to test results, although rather liberal statements are made regarding rupturing capacity. It is hoped that the authors in their closing discussion will advise us more fully of the test results on which they base their ratings.

H. E. Strang (General Electric Company, Philadelphia, Pa.): The authors have chosen an extremely effective way of demonstrating in pictorial form the operating principles of these various types of air blast breakers. Although they all appear to have some features in common, they illustrate a healthy diversity of ideas in design to be expected at this early stage in a new field.

Presumably these different breakers have been subjected to thorough short circuit tests, either at a factory testing station or in the field before being placed in service. Although this paper was apparently intended only as a discussion of the mechanical features of such breakers, in view of the wide interest in the air blast principle, and the rather general statements contained in the paper on the subject of interrupting ratings, a discussion of such tests and a statement of the amounts of short circuit current interrupted in actual service would give added confidence in the adequacy of the interrupting ability of air blast breakers.

The one oscillogram which is shown, indicates very satisfactory performance, apparently interrupting about 20,000 amperes at 2,300 volts, or some 80,000 kva, which is well within the proven capabilities of existing simple self-contained magnetic types of breakers.

E. W. Knapp (Shawinigan Water and Power Company, Montreal, Quebec, Can.): We are indeed indebted to the authors for explaining in such an effective manner the mechanical operation of several voltage ratings of air-blast circuit breakers of one general design. A great deal of interest has been shown during the past few years in this problem of circuit breakers. As a representative of an operating company, I am not prepared at this time to discuss design problems. It might, however, be in order to discuss a number of general problems pertaining to the development and use of air-blast circuit breakers.

The Shawinigan Water and Power Company have been interested for some time in the development by various manufacturers of improved circuit breakers and have conducted a considerable number of tests on various types of oil and air-blast designs. This interest covers many phases of circuit breaker characteristics including rupturing capacity, speed of operation, amount of maintenance required, suitability for high speed automatic reclosing, possibility of adapating to single phase switching, reliability and, last but not least, freedom from fire hazard.

The air-blast circuit breaker is a comparatively new development, and as usual, with new developments, many difficulties have been experienced. Tests and operating experience have been rather encouraging and it is quite possible that in the not too distant future a complete range of this type of equipment will be available, which may be used with complete confidence.

H. W. Haberl and Otto Jensen: Referring to the comments of Mr. Howard M. Wilcox relating to air pressure: The first breaker of this general type was put into service in 1934 at the generating station of the Beauharnois Light, Heat and Power Company, where there was already available for other purposes, compressed air at 90 pounds per square inch and the breaker was built for 100 pounds per square inch. Three additional breakers installed at this station in succeeding years were built for the same pressure. The existing air pressure and compressor capacity were ample for the breaker service and there was no object in reducing the quantity of air required by the breakers by increasing the pressure, which would have required boosters. In later installations at other stations, the breakers have been built for 150 pounds per square inch for all voltages and the interrupting tests have verified the expected increase in interrupting capacity. The general design does not limit the air pressure and it may be increased above 150 in future in cases where the advantages appear to outweigh the disadvantages of a higher pressure. In regard to the use of pressure above 150 for storage with reducing valve, the advantage of such arrangement is appreciated and installations are being made along this line.

In regard to removal of moisture in air, activated alumina has been used exclusively and successfully, since 1934, and there have been no cases of trouble due to moisture or frost in the breakers since the first few months in 1934 before experience had established the minimum schedule of reactivation required, with breakers operating outdoors at temperatures down to 30 degrees Fahrenheit below zero.

In relation to blast valve, blast tube, breaker tanks, etc. this design of blast valve does not require installation *inside* of tank. In the 138- and 230-kv breakers which have been installed, the blast valve is completely outside the tank but the valve flange is approximately tangential to the tank circumference thus eliminating any pipe connection between valve and tank. In the 69-kv and lower voltage breakers the valve diaphragm is turned 90 degrees but the valve flange is still approximately tangential to tank wall. With either arrangement the entire blast valve is removed by unbolting one large flange outside the tank. Regarding the question of stringent code requirements, the internal blast tube has been incorporated in breakers built by the I-T-E Company and installed in service in the United States and therefore approved by the inspection authorities concerned. The pressure is 150 pounds. The same applied to the lowest voltage breakers built for 150 pounds in Canada.

In regard to the use of a series isolating switch in the breakers of the lowest voltage, the photograph in the paper shows a 2,000-ampere, 15-kv breaker 2 feet 6 inches wide parallel to the station bus, which appears to compare favorably with other breaker designs in the matter of minimizing required length of bus rooms and structure.

Referring to the comments of Mr. J. B. MacNeill regarding insulation, all such material which is continuously stressed is porcelain. Other materials have been used for parts which are stressed for less than six or eight cycles, the period between interruption and opening of isolating switch. No failure of insulation has occurred in the seven year period of service and the climatic conditions have been rather severe including extreme dampness and salt air. However, improvement has been made from year to year in available materials and porcelain is replacing other materials to a large extent.

Reference is made to the normally short arcing time of air blast interrupters, which is now generally recognized, but we do not follow Mr. MacNeill's apparent conclusion that "an extra half cycle or two of arcing may represent a destructive increase of arc energy," as applying to an air-blast breaker to any greater extent than to an oil breaker. In fact the contrary appears to be true. In an oil breaker the arc energy is all stored inside the tank in the form of generated gases of explosive composition; an increase in arcing time means more gas and greater pressure with less safety-factor against bursting of tank. In an air blast breaker the arc energy is not stored in the breaker; it is blown outdoors as fast as it is generated. This is true even if the breaker fails to interrupt and the back-up breaker is slow in opening, say 15 or 20 cycles. The worst that happens in tests of that nature, is excessive loss of copper from arcing contacts, excessive "erosion" of arc-chute lining, melting of edges of metal cooling sheets, etc., requiring replacement of such parts. The damage is negligible in comparison with that in an oil breaker under the same conditions.

Furthermore an air blast interrupter of suitable design shows relatively constant arcing time within its interrupting capacity. Also the shape of the arc voltage oscillogram is quite consistent as shown by a large number of shots with the same arc chute. Different shapes of arc chute naturally show different shapes of arc voltage oscillograms but the results are not generally erratic.

A large number of interrupting tests have been made in the past seven years on breakers of this general design, the tests ranging from 2.3 kv to 260 kv on large power systems and the authors or their associates will be glad to offer a brief paper on the subject at some suitable opportunity.

In response to the comments of Mr. H. E. Strang, the breakers have been thoroughly tested in the field before being placed in service. Other comments of Mr. Strang are covered in the response to Mr. MacNeill.

Mr. E. W. Knapp's comments appear to us to summarize the state of the art in a comprehensive manner. The close cooperation of Mr. Knapp, his associates, and officials of the Shawinigan Water and Power have been highly constructive throughout the development.

In reference to all of the written and oral discussion and comments on the paper, the authors wish to express appreciation of the interest shown and the open minded spirit with which the paper has been received.

Protection of Low-Voltage Circuits by Air Circuit Breakers in Cascade Arrangement

Discussion and authors' closure of paper 41-128 by A. E. Anderson and C. H. Black, presented at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941, and published in AIEE TRANSACTIONS, 1941, pages 1151–60.

J. D. Wood (Cornell University, Ithaca, N. Y.): I wish to discuss the paper by Messrs. Anderson and Black on the subject of "Protection of Low-Voltage Circuits by Air Circuit Breakers in Cascade Arrangement."

While this method of operation has been used in special applications previously, we are indebted to the authors for proposing this method for the general application of air circuit breakers. The authors have also recommended cascading even though the breakers have widely varying speeds and opening times.

Your attention is directed to the set of test results shown in figure 10 which are the basis for the authors' proposal. In these results the basis for the theory that two breakers in series having different opening times in any way assist each other in opening a fault, is difficult to see.

Here is a circuit which gives an average fault current of 95,000 amperes. When this circuit is opened by a breaker having 100,000 amperes interrupting rating as in test 787-15, we find the breaker is slow because the moving element has a large mass and high inertia. This is necessarily the case as the smallest 100,000 ampere breaker is rated 4,000 amperes continuous current.

A minimum of 3.2 half cycles is required to part contacts and 5.2 half cycles to open the short.

When this breaker is used in series with a 50,000-ampere interrupting capacity breaker as in test 787-3, we find that the smaller breaker, parts contacts in a maximum of 1.2 half cycles and the short is interrupted in 1.9 half cycles—well ahead of the minimum of 6.8 half cycles required for the larger breaker to part contacts, as shown for test 787-3. The larger breaker has contributed nothing to the operation except to act as limiting impedance and to open after the interruption is over. The smaller breaker has opened a fault above its rating and has been damaged.

In tests 787-11 and 662-9 where three breakers are used in series, they each contributed impedance to further limit the current. The larger did not open, the next size opened a circuit at less than its rating, and the smallest was partially destroyed.

May I suggest that this is an expensive way to secure additional impedance. Also, impedance built into a breaker has the same effect on voltage regulation as impedance in a current limiting reactor.

A preferred method of applying breakers to low-voltage circuits is to use the smaller breakers for the distribution feeders because of their low cost where only small continuous ampere sizes are required. These breakers are provided with relays which lock the breakers in the closed position when a predetermined value of current is exceeded. In the unusual case of a severe short above the rating of these smaller breakers, the fault is passed back to the main feeder breaker which is large enough to handle it. The smaller breaker is prevented from opening and if properly designed, is not damaged. It is recognized that this can only be done where the breakers have the very important design feature of "inverted contacts," that is, contacts which do not blow open due to the magnetic forces of the short-circuit current.

All low-voltage air circuit breakers should have this type of contacts in order to handle heavy currents which they may be called upon to carry as well as open. Where the design does not incorporate this important feature, cascading would be of assistance provided the breakers have practically the same opening time.

In such applications, the normal infrequency of operation of low-voltage air circuit breakers and the possibility of maintaining the close timing of cascaded breakers over a period of time under practical service conditions, must be taken into consideration.

W. R. Brownlee (Commonwealth and Southern Corporation, Jackson, Mich.): Unquestionably the application of a group of low-voltage air circuit breakers, such that all of them are capable of interrupting the maximum short circuit current available at the bus, will frequently result in an unreasonably expensive and cumbersome arrangement. Since air circuit breakers of conventional design are incapable of withstanding the mechanical stresses associated with the heavy currents of low voltage high capacity circuits, there is no opportunity to use the familiar "lockout" scheme which has been applied with reasonable success to oil circuit breakers operating at 2,400 volts (and sometimes higher voltages). The method of "cascading" has been logically suggested, but the cascading described by the authors is quite different from that commonly understood where a protective relay or a tripping device will not initiate the tripping of a backup breaker until after a time interval equal to the total tripping and arc interrupting time of the protected breaker, plus several cycles of operating margin. It is not readily apparent how the authors have been able to secure a time coordination on the order of one-half cycle between breakers in series, particularly using "magnetic" tripping devices which have never been considered as consistent in response as standard protective relay elements. Perhaps the strongest influence on selective operation may be the current limiting effect of the inherent resistance of the feeder and backup breakers themselves (before contacts part). If so, information on these resistances would be very helpful in applying the results of the authors' investigations to anything other than the specific conditions and equipment with which their tests were made.

It is difficult to agree with the authors' interpretation of their reference 1. The tests described by Mr. Schurig showed in all cases sufficient arc voltage to have a pronounced effect on the d-c time constant. While the tests indicated that in many cases the assumption of a "bolted fault" may be made safely as far as calculation of the maximum symmetrical current is concerned, they do not justify omitting the very appreciable effect on the total rms current at the end of one-half cycle as influenced by arc drop modifying the d-c time constant.

A. E. Anderson and C. H. Black: The authors appreciate the interest shown in this method of applying air circuit breakers in cascade and believe this discussion should be both interesting and helpful. A few further comments pertaining to the questions raised during the discussion seem to be in order.

The air circuit breakers do have to be properly coordinated but do not require the extreme accuracy of "timing" referred to by Mr. Brownlee, although—when subjected to short circuit currents of relatively large magnitude—they do become remarkably consistent in their time of operation and do not exhibit the same variation in time that might be expected when magnetic tripping devices operate at currents of low or moderate magnitude. There is a great deal of test data, some of which has been shown in figure 10, to verify the sequential operation of breakers in cascade. The resistance of the breakers before the contacts part is so small as to be negligible and does not play any part in determining the sequential operation.

Mr. Brownlee's comment concerning the authors' interpretation of Mr. Schurig's test data (reference 1) is well taken. It is quite correct that the arc resistance did materially affect the d-c component, but the authors still believe it to be unsafe to assume that the "bolted fault" or equivalent thereof cannot occur. It will be further noted that the method of averaging the currents in the three phases for calculating air circuit breaker duty results in a total current, including the d-c component, which is only 25 per cent higher than the symmetrical current, hence the assumption of the "bolted fault" does not impose an unduly severe penalty upon selection of air circuit breakers.

Mr. Wood's comment concerning the ability of the larger breaker to assist smaller breakers apparently does not give sufficient consideration to the performance shown in figure 5 where it can be clearly seen that there is an appreciable interval of time during which a back-up breaker (even though it operated several cycles after the occurrence of short circuit) could have been of very material assistance to the smaller breaker. It is true that the smaller breakers are inherently more rapid in operation. This is what makes cascading effective.

Mr. Wood has suggested locking in the smaller breakers to prevent their opening under severe short circuit. The inverted contacts referred to by Mr. Wood, represent merely one possible way to design breakers to withstand the initial electromagnetic forces of high short-circuit currents. It is somewhat more difficult, however, to provide small breakers with the ability to withstand thermal effects without making them unduly large and costly. But the ability of the breaker to remain closed is not the primary question. The principal function of a circuit breaker is to protect. Protection is not achieved by locking circuit breakers closed. From the standpoint of protecting the circuit and apparatus connected thereto (which is usually far more valuable than the circuit breaker itself), the authors do not consider it good practice to prevent the inherently rapid breakers from opening at once, leaving the circuit interrupting duty to a breaker which is inherently slower. The circuit is far better protected if the smaller breakers, even though they do not actually complete interruption, are allowed to open and thereby materially reduce the short circuit currents. This current-limiting action occurs only when the breakers are opening during short-circuit, hence the cascade arrangement does not adversely affect voltage regulation during normal operating conditions.

Mr. Wood has suggested that cascading is an expensive way to secure additional impedance. When the complexity of schemes for locking in trip-free breakers is considered, together with the sacrifice of circuit protection, it becomes apparent that the use of locked-in breakers is an expensive and complicated way of avoiding a very desirable current limiting action during short circuit.

Mr. Wood has stated that the smaller breakers were partially destroyed in certain tests. The condition of these breakers was such that no unreasonable servicing would be required in order that they might perform their usual duty. While it is recommended that the smaller breakers be inspected, and possibly serviced, upon the occurrence of severe short circuits, it has been found that such severe faults occur at relatively infrequent intervals. Accordingly, most fault currents come within the ratings of the smaller breakers, which not only keeps the amount of maintenance within reasonable limits, but also eliminates the need of tripping the "back-up" breakers, so that service is maintained on the rest of the system.

Finally, the authors recognize that cascading is not the solution for *all* problems of circuit protection. The use of breakers of adequate rating throughout will certainly result in somewhat lower maintenance or, under certain conditions, the use of current-limiting means may be found desirable. It is the experience of the authors, however, that in many cases cascading provides a highly satisfactory and very economical solution, thus warranting careful consideration whenever protection of a low voltage system is being planned.

Radiobroadcasting in Canada

Discussion and author's closure of paper 41-113 by Augustin Frigon, presented at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941, and published in AIEE TRANSACTIONS, 1941 (September section), pages 896–8.

R. B. Steele (Canadian National Telegraphs, Toronto, Ontario, Can.): Dr. Frigon has given us a very interesting and instructive paper on the organization, functions, and operations of the Canadian Broadcasting Corporation and its place in network broadcasting in Canada. He has told us that while the Canadian Radio Broadcasting Commission, the predecessor of the corporation, started its network activities in 1932, the first Canadian broadcasting network was set up during the Christmas holiday season of 1923. This was, admittedly, a modest set-up, with only two radio stations interconnected by some 115 miles of telephone circuit, but it held much promise for the future for it was sponsored by the newly organized Canadian National Railway System, and Sir Henry Thornton, the speaker of the occasion, announced the establishment of the CNR radio department.

This was really also the beginning of the 1926–27 eastern Canada network mentioned by Dr. Frigon, for the CNR radio department set about the establishment of stations at Ottawa, Moncton, and Vancouver, and of a program and technical organization, and the C. N. Telegraphs set the wheels in motion to provide a wire network.

The landmark of early network broadcasting in Canada is the diamond jubilee of confederation broadcast on July 1, 1927. Months of preparation were required and all the wire companies contributed to the network, which was organized under the technical supervision of Bell Telephone Company of Canada engineers. A press release issued before the broadcast predicted the use of 21,650 miles of line and 53 repeater stations. 19 radio stations were connected to the network, including CFCT at Victoria, B. C. and CHNS at Halifax, N. S.

On December 27, 1928, the CNR radio department again pioneered by arranging a coast-to-coast broadcast program to its own and affiliated stations, this being, at the time, the largest Canadian network arranged by a private organization. The C. N. Telegraphs provided the network circuits between Montreal and Vancouver. Quebec, Halifax, and three other maritime points were included in the network by means of telephone company circuits.

The communications department of the Canadian Pacific Railway also established a coast-to-coast program transmission network and thus it came about that when Parliament, in 1932, established the Canadian Radio Broadcasting Commission there were a number of organizations equipped for and experienced in network program transmission. These were the C. N. Telegraphs, the CPR communications department, and the several telephone companies of the Dominion which are banded together as the Trans-Canada Telephone Association. These interests all participate today in the provision of the CBC's network requirements. The CNT and the CPR communications are the primary contractors, but they in turn call upon the several telephone companies to supply a number of important network sections.

In his paper Dr. Frigon mentions the operation of an FM broadcast transmitter at Montreal. It would be interesting to have some further information as to the status of this operation.

Augustin Frigon: Unhappily, under present conditions, our experiment with frequency modulation has had to be held up and we consequently have no data available concerning it.

Excitation Circuits for Ignitron Rectifiers

Discussion of paper 41-127 by H. C. Myers and J. H. Cox, presented at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941, and published in AIEE TRANSACTIONS, 1941 (October section), pages 943–8.

C. C. Herskind (General Electric Company, Schenectady, N. Y.): This paper provides an excellent review and discussion of the various circuits which have been developed for energizing the igniters in ignitron rectifiers. Progress has been rapid in the development of suitable igniter firing circuits because of the pressure of greatly increased business and the importance of reliable operation of the igniters. Virtually, all of the conversion equipment being supplied today for use in the electrolytic field employs mercury arc rectifiers, and a large percentage of these are of the ignitron type. Since electrolytic processes are continuous it is absolutely essential that the igniter excitation system be thoroughly reliable.

Firing circuits using thyratrons have many advantages, but the desirability of having a static-tubeless excitation circuit using dry type rectifiers, such as copper oxide rectifiers, was early recognized. The development of such a circuit became urgent with the application of a large number of ignitron rectifiers to the manufacture of aluminum.

The General Electric Company undertook the development of a tubeless circuit, and built and installed the first equipment of this type. This equipment has been in service for almost a year and has operated very successfully.

This tubeless firing circuit is shown on figure 1 of the paper. It will be noted that it makes use of dry type rectifiers for rectifying the input to the igniter, and of a combination of linear and non-linear reactors for producing the peaked current wave form. Phase shifting may be obtained by providing a saturating winding on the non-linear (saturating) reactor and adding suitable capacitors in the supply lines.

This first tubeless circuit was entirely successful and highly reliable, but further development work has resulted in circuit improvements providing substantial reductions in losses and the amount of equipment required. The improved tubeless firing equipment now available is a fully developed, efficient and reliable excitation means.

For applications where a large amount of phase control is required or losses are an important consideration, an improved capacitor thyratron type of circuit has been developed. This circuit is similar to that described in the Myers and Cox paper and is shown by figure 2 of the paper. It provides for rectifier charging and a resonant discharge controlled by a thyratron. The copper oxide or dry type rectifier charging furnishes a constant capacitor voltage regardless of phase angle. The use of the reactor in the discharge circuit eliminates the steep wave front of the direct capacitor discharge and gives uniform igniter currents of sinusoidal form. Advantages of this circuit are:

1. Low losses.
2. Accurate firing and ease of control.
3. Rapid response.
4. Suitability for half-wave circuits.
5. Large ranges of phase control.

The duration of the igniter current wave in this circuit is of the order of 8 to 12 electrical degrees. In many applications it is desirable to maintain the cathode spot for a longer period than the firing period of the igniter in order to insure reliable firing of the main anodes under all load conditions. This is readily done by the addition of an excitation anode excited from a separate low voltage winding in the excitation transformer.

It appears at present that these two types of excitation circuits, namely, (1) the saturating reactor or tubeless, and (2) the capacitor-thyratron, will be used for the majority of future installations of ignitron rectifiers.

O. W. Pike (General Electric Company, Schenectady, N. Y.): It is true, as the authors point out, that the thyratrons have a limited life due to the fact that the normal end of life occurs with the depletion of the emitting material on the cathode. However, from a practical standpoint, the life is very long. Under proper conditions of operation the average life can well be 25,000 hours with individual lives of twice this value. With lives as long as this, much of the objection to the use of thyratrons in power equipment disappears.

In an application, such as their use with igniters, where the peak currents are high but the average currents very low, thyratron cathodes can to advantage be operated at a slightly higher than rated voltage. This small additional cathode power makes up for the heat normally released by the full load anode current. Thus, peak current capabilities can be obtained with long life.

We do not wish to imply that thyratrons are, because of these factors, the logical choice in all cases, and we agree with the authors that in certain cases other systems are more suitable.

A New Transformer for Base-Load Stations

Discussion and authors' closure of paper 41-124 by Philip Sporn and H. V. Putman, presented at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941, and published in AIEE TRANSACTIONS, 1941 (October section), pages 916–18.

P. L. Alger (General Electric Company, Schenectady, N. Y.): The transformer described in this paper embodies several distinctive features of design, which together have enabled a marked reduction in weight and size to be obtained for a given performance. From the viewpoint of possible application to other lines of apparatus, it seems important to evaluate more carefully the respective portions of the overall gain that can be attributed to the different new features of design.

Since transformer size is normally fixed by thermal considerations, the use of forced cooling probably contributes more than any other design element to the gains reported. It will be interesting if the author can give an estimate of the size reduction that would have resulted if this forced cooling system had been used on a transformer of otherwise standard design.

The other important feature of the new design is the use of Hipersil. The data given on this material, as indicated by figure 3 of the paper, show that Hipersil has 38 per cent more flux for a given excitation at a particular point on the magnetization curve. On the assumption that the length of flux path in the core illustrated is about two feet, the point chosen for comparison is at a magnetizing force of approximately three ampere turns per inch. This figure is much lower than normally used in rotating machinery, so it is interesting to make similar comparisons at other excitation values. In table I of this discussion, the with-grain flux densities for Hipersil (obtained from data published in the *Electric Journal* for January 1936, page 40) are compared with those of two typical sheet steels, averaging with and cross grain directions. The percentage gain of Hipersil over the other steels is seen to fall markedly as the density rises, the gain at saturation being only of the order of five per cent. The value of Hipersil and other types of the modern strip steels is, usually, in the greater permeability and lower losses they possess at moderate densities, rather than in a greater capacity to carry flux under a high magnetizing force.

Table I

	At 20 Amps./In.		At 100 Amps./In.		At 400 Amps./In.	
Material	F.D. in Gausses	% Gain Of Hip.	F.D. in Gausses	% Gain of Hip.	F.D. in Gausses	% Gain of Hip.
Typical $2^{1}/_{2}$ per cent silicon motor sheet steel	13,500	28	15,500	16	17,900	3
Typical $4^{1}/_{2}$ per cent silicon transformer sheet steel	12,900	34	14,500	24	17,300	7
Hipersil	17,300		18,000		18,500	

F.D.—Flux density. Hip.—Hipersil.

V. M. Montsinger (General Electric Company, Pittsfield, Mass.): I would like to ask the authors what part or percentage of the reduction in size and weight can be attributed to (1) the use of the Hipersil iron, and (2) to the use of forced-oil cooling of the windings, etc.

It is well known that when one directs the flow of oil over the surface of a coil or coils, the loss dissipated for a given temperature rise over the oil is increased several times. This makes it possible to reduce the number of coils, the size of the core window, and size of tank. It is on account of this that it has been possible to greatly reduce the size of transformers (of a construction similar to the 40,000 kva) using standard steel for electric locomotives during the past several years. It should be possible, therefore, to state how much of the reduction in weight and size is due to the use of Hipersil.

F. J. Vogel (Westinghouse Electric and Manufacturing Company, Sharon, Pa.): In the early days of railroad electrification, dry type air blast transformers were used. The reasons for this were that they were lighter and required less space than oil insulated transformers, and these were important advantages. They had the disadvantage of insulation troubles, from moisture and dirt, and this disadvantage led to a study of cooling methods with oil insulated transformers. The air blast transformer cooled well because air was forced through the coil ducts, and it was found that a similar benefit could be obtained by forcing oil through the coils instead of depending on natural convection. This enabled the oil-filled transformer to be laid on its side, and made the forced cooled transformer competitive with the air blast transformer so far as weights and dimensions were concerned, and with the advantages of reduced maintenance, greater insulation strength, and greater short time overload capacity.

The idea that this type of transformer might be useful in other fields presented itself, but there seemed to be many obstacles. It was introduced into the portable substation field, where the transformer must be light and small and mounted on a trailer. The idea has been prevalent that auxiliary cooling equipment is undesirable, although the use of fans was well established. The fact that this transformer apparently would have less short time overload capacity than a conventional self-cooled transformer was against its use. Also the losses increased if full advantage was taken of the cooling possibilities, and a possible decrease in cost of the unit had to be balanced against increased losses. It also represented a radical change in design, which would be approached with some reluctance ordinarily by both manufacturer and user.

The usual substation transformer may have to carry heavy overloads, and may not have attendants close by. Therefore, the forced cooled unit may not be advantageous for such service at this time, but it is by no means certain that these limitations are permanent. The central station transformer, however, seemed to be the logical first application. Mr. Sporn suggested that forced cooling would not be a disadvantage, where there were station attendants, and that overload capacity was not as important in a central station due to the limitations of generators and turbines. As stated, in the paper, the advantages of the design and its adaptability to the use of Hipersil then overcame any reluctance to use the new design. Although many modifications were suggested, it is nearly the same in general arrangement as originally conceived.

One of the advantages of forced oil cooling is that the oil is at practically the same temperature throughout the transformer. In a self cooled unit, the difference between the top and bottom oil may be as much or more than 15 degrees centigrade. This means that the hottest spot temperature in the forced cooled unit can not be greatly in excess of the average temperature. Therefore, at normal rating, the forced cooled transformer with a 60 degree centigrade rise might not have as high a hottest spot as a self cooled transformer with a 55 degree centigrade rise. Also in calculating short time overloads, it is possible that the forced cooled unit will not be at a disadvantage in comparison with the self cooled unit due to the fact that the difference between the highest and lowest oil temperature will not have to be included in estimating the safe load which can be carried. In view of this, it appears that as service experience increases, and if troubles with the forced cooling equipment are negligible, this type of design may become quite common.

Philip Sporn (American Gas and Electric Service Corporation, New York, N. Y.): It may be a rather unusual procedure for an author to discuss his own paper, but since this is a joint paper, it represents thoughts, ideas, and expressions of opinion and conclusions which are generally shared by both authors. But this paper, like a good many papers, is interesting not only for what it reveals and tells, but also for what it fails to disclose.

The paper shows pointedly how progress in new materials can be utilized with progressive ideas in design to obtain an overall result that even as recently as several years ago seemed almost too much to hope for, let alone to actually obtain. But in saying this, one has all the more regretfully to point out the failure of the authors to give any fundamental or even engineering development data on the steel and on the development and manufacture of the steel which is the very basis of the improved design.

Admitting that the progress which the paper recounts is the result of industrial research (stimulated, it is true, by certain users who are concerned with the continuation of processes leading to improved methods and improved economies in the art of electric power generation and transmission), the fact still remains, that from an engineer's viewpoint, it isn't enough to coin a new term for a fundamental steel development and hope to pass that off as an adequate presentation of the real background behind the design development discussed. The meager information given by figure 3 can hardly be considered an adequate substitute for a discussion of the physical ideas involved in the new method of rolling steel, the effect upon the magnetic characteristics of the steel that such new rolling has, the detail design changes in core construction to utilize the new steel characteristics, and the extent to which further changes and improvements can be carried and the limitations to such probable progress. All of this, it seems to me, could very well be presented today without in any way sacrificing such commercial protection as those responsible for the development are naturally entitled to. It is true that the authors state that it is hoped to present more information on the steel at a later date, but it is obvious that all of this information is available today, and as a matter of fact was available before the transformer, which is the subject of this paper, was evolved.

Again the authors stress the failure of previous transformer designs to properly take into account the altered conditions with regard to value of power energy and, therefore, of losses and the better balancing of these factors in the new design. But the meager information given with regard to the actual losses and the balancing of these losses in the case of this transformer does not quite satisfy one's curiosity with regard to how well this was worked out in this improved design.

This paper is in my opinion particularly timely and should be received with considerable welcome by that part of the engineering profession which is concerned with the broad problem of electric power supply because it indicates that as long as engineering thought is free to exercise its God-given talents and abilities, that there is no occasion for pessimism as to progress. One must, however, express regret at the failure to include the material pointed out above which I am sure would have been not only interesting but a valuable contribution to this particular branch of engineering.

H. M. Hobart (retired, General Electric Company, Schenectady, N. Y.): Messrs. Sporn and Putman's timely and practical paper describing their 40,000-kva, 138-kv transformer with forced-oil cooling and with Hipersil cores, is of particular interest to the writer because of some investigations on which he has long been working which have the object of providing power transformers of improved quality although of less size and weight and cost. On the basis of the conclusions at which the writer arrives from these studies, the introduction into their design of features which the writer will describe in this discussion, should result in decreasing the loss in their Hipersil cores by a matter of 25 per cent. This can only be made clear by considering several factors as follows:

Use of compressed gas instead of oil: The great dielectric strength of compressed gas has been called to the attention of transformer designers on various occasions by Ryan, Bölsterli, and others, and it has been considered that the substitution of compressed gas for oil could be so effected as to be accompanied by important improvements and economies. Particularly convincing was an article published by Bölsterli in 1931. The writer, at that time and since, has made repeated and very earnest efforts to interest and co-operate with transformer designers in undertaking serious comparative studies of the respective merits of oil and compressed gas. Although a few transformer specialists have become moderately interested, the general verdict continues to be quite definitely adverse.

While the original incentive to employing compressed gas was its great dielectric strength, there is soon recognized its superiority over oil as a circulating medium. An extremely small amount of power, say of the order of five one-hundredths of one per cent of the transformer's rating, suffices to circulate the compressed gas at very high speed, through the windings and then through the heat exchanger, and finally to return the cooled gas into the transformer. With some gases, for example, sulphur dioxide and carbon tetrachloride, use may be made of the latent heat to assist the cooling and circulating process, and to decrease the size of the heat exchanger.

In addition to the superior effectiveness of compressed gas as a cooling medium, there is the further advantage that it is not deteriorated in any respect by subjection to high temperature. We also have available for turn and layer insulation and for barriers and other needed insulations, many solid materials which do not suffer deterioration when subjected to quite high temperatures. Among these solid insulations may be mentioned mica, micanite, mycalex, glass-fibre products, porcelain, and asbestos.

As long as we continue to cool the windings by the oil in which the transformer is immersed, we must provide for ensuring such a low temperature as not to deteriorate the oil. Unless we employ some expensive brand of oil, the limiting temperature imposed is very low. When we design a power transformer on the basis of using compressed gas, we can go up to any temperature consistent with reasonable increase in the resistance of the windings. As regards the cores, high temperature with suitable core material will serve to greatly decrease the core loss. No particulars of Hipersil are given in the paper. But if it behaves like some other modern high permeability core materials, it will have this feature of considerably decreased core loss if operated at a temperature of the order of 200 degrees centigrade higher than the winding temperature. This may be accomplished by interposing a wrapping of asbestos or mica or other thermally suitable material between the cores and the windings. The circulation of the gas can be confined to the windings, the heat from the cores being constrained to traverse the asbestos wrapping of some $^3/_8$" to $^5/_8$" thickness (the precise thickness being a matter of simple calculation for any particular design). This characteristic of some core materials was described by MacLaren a long time ago. At that time it was the subject of considerable discussion. But its application to oil-type transformers would not have been simple.

In the pioneering days of fifty years ago, many difficulties were overcome by resort to oil. Transformer manufacturers and users in particular, owe a great debt to oil, especially for enabling them to build for higher and higher voltages to meet the increasing needs of electric transmission undertakings. However, the writer believes that this gradually left us in a rut, and we thoughtlessly accepted the use of oil in power transformers as inevitable.

The oil-transformer situation is not at all exceptional. The writer has had analogous experiences when, as has occurred a great many times, he has associated himself with taking up the cudgels for proposals to do things in "better ways." Our enthusiasm over slogans is in sharp contrast to our attitude when faced with concrete situations.

Could not the valuable gains explained by the authors as having been made possible by Hipersil cores be increased quite considerably by substituting for the forced-oil cooling, a design in which the windings are vigorously cooled by the forced circulation of compressed gas? It would appear reasonable to be able, by employing the means described, to improve the design to such an extent that the savings in copper and core materials would exceed the 10 per cent and 25 per cent respectively, which are stated in the paper. The total of all losses, instead of being increased five per cent, probably would be appreciably decreased, due to (1) the less power necessary in circulating compressed gas, and (2) the decreased core loss occasioned by maintaining the cores at a high temperature.

The need for brevity in a discussion of a paper inhibit the writer from hardly more than mere allusion to one more of the several new directions in which important progress may be made by the use of much higher temperatures than can be permitted in the *oil*-type. If we deliberately design for high temperature operation, we provide greatly increased heat flow across each unit of surface between the heated parts and the circulating medium provided for carrying off the heat.

In conclusion the writer has no hesitation in expressing his opinion that so many new possibilities are opening up that we should practically start all over again and re-design power transformers from top to bottom on the basis of fresh fundamental considerations. There will be many different ways of utilizing these modernized fundamental principles, and the writer hopes that in the near future, many "significant contributions" looking toward increased transformer and transmission economy will take their place alongside the excellent contribution reported by the authors in their valuable and interesting paper.

E. D. Treanor (General Electric Company, Pittsfield, Mass.): I have been interested particularly in the magnetic steel which is outlined in this paper, as the data and other publications indicate that it is of excellent quality. While the advantages here referred to are for a special power transformer application, it may be of interest to review the use of similar highly oriented cold-rolled strip in distribution transformers.

After considerable developmental work, the General Electric Company in 1937 placed in production small distribution transformers built on circular wound cores made from continuous cold-rolled highly oriented silicon steel strip. The introduction of this Spirakore transformer, as it was later termed, was noted in a paper presented before this Institute in June 1938 which described the use of such steel in a core structure particularly adapted to its characteristics. Its possibilities were briefly outlined in a paragraph which I quote:

> "In the practical design of transformer cores, flux density is limited by heating or magnetizing current in large transformers and usually to still lower values by magnetizing current alone in small units. The combination of a steel having high permeability and low losses in the high density range with a core structure which reduces the number and length of air gaps promises highly effective use of material, particularly in small-sized transformers."

No detailed data were presented at that time on the properties of the steel but rough comparison of permeabilities was given in figure 7 of the paper. These designs were characterized by somewhat reduced ratio of copper to core loss and resulted in considerable economies which have been widely publicized commercially.

As a report of progress since that time—some hundreds of thousands of distribution transformers for lighting and power service have since been built along these lines with flux densities materially higher than those previously used. In view of the good results obtained from this design, wound cores of other shapes have been developed by the General Electric Company which are capable of being adapted to any and all ratings in the single-phase distribution and small power transformer range. These have been introduced as rapidly as steel production facilities permitted and, as all limitations of supply should shortly be overcome, a complete and uniform line will be available. Transformers in sizes up to 1,000 kva and voltages up to 66,000 have been built so that the design requirements have been quite fully explored.

During this period, research and developmental work have continued on the steel and the optimum characteristics that this type of material can hope to show, with most effective processing, have been examined and weighed from the standpoint of application in transformers. There are some transformer characteristics which require careful consideration before taking full advantage of the best saturation, magnetizing current, loss, and magnetostriction available in the steel.

Increasing availability of magnetic steels of improved quality should, as the authors point out, have a marked effect on the design of transformers and a fairly extensive use on distribution transformers bears this out. So far the great advances in magnetic properties have been coupled only with materials produced by high reduction cold rolling and elaborate mill processing which result in material largely oriented in one direction. As suggested in the paper, it is probably more suitable for transformers designed with these characteristics in view than for other induction apparatus, although some special applications have already been made.

E. V. DeBlieux (General Electric Company, Pittsfield, Mass.): Messrs. Sporn and Putman have made a valuable contribution in setting forth so clearly these design principles which can be applied with material benefit to many other applications of transformers as well as to the special cases discussed by the author.

I believe, with the authors, that the principle of using forced cooling, and special core steel that can be operated at much higher densities than commonly used, together with proper protective and relay equipment, is a sound design principle and can result in a major reduction of the size and weight of transformers.

In particular I wish to concur with the authors' conclusion that the use of these principles "will make possible the extending of the practical size of portable transformers whether on car or trailer."

About two years ago the company with which I am associated, in considering the design of trailer-mounted mobile substations, concluded that minimum weight and dimensions were primary considerations and these could be realized to the fullest extent by:

1. Using a core steel that could be operated at much higher densities than formerly used,
2. Using forced cooling of the oil, and
3. Using forced circulation of the oil through the transformer winding.

Therefore, since the introduction of factory-built mobile substations, over a year ago, all of the mobile substations manufactured by this company have cores made of high permeability, highly oriented, cold-rolled silicon steel that operates at appreciably higher density and also employ the other design features enumerated above.

Therefore, our experience enables us to agree with the authors' conclusions, and to state that these developments have materially reduced the weight and size of trailer-mounted transformers, have increased the kilovolt-ampere rating permissible within the highway limitations and have been incorporated in a number of mobile substations that are now in service.

Philip Sporn and **H. V. Putman:** The discussers have brought out very clearly some of the various attempts which have been made to improve the overall design of transformers in recent years, and Mr. Hobart's remarks indicate that the search for further improvement has not yet been abandoned.

Mr. Montsinger brought up the question of allocation of weight reduction resulting from the use of Hipersil and the use of forced oil cooling. Since a design was not worked out on this particular 40,000-kva transformer with regular silicon steel, it is difficult to say exactly how much reduction in weight is due to forced oil cooling and how much to the use of Hipersil. However, in the redesign of small distribution transformers where direct comparisons are available and where the increase in the Hipersil core density is substantially the same as in the 40,000-kva units, the reduction in weight averaged 22 per cent. If the 40,000-kva unit were to be redesigned with similar forced oil cooling and with the regular silicon steel, it seems probable, therefore, that the weight would be about 20 per cent greater on the basis of the same electrical performance.

Methods of Determining Natural Frequencies in Coils and Windings

Discussion and authors' closure of paper 41-84 by L. V. Bewley, J. H. Hagenguth, and F. R. Jackson, Jr., presented at the AIEE North Eastern District meeting, Rochester, N. Y., April 30–May 2, 1941, and at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941, and published in AIEE TRANSACTIONS, 1941, pages 1145–50.

R. Rüdenberg (Harvard University, Cambridge, Mass.): I would like to draw certain conclusions from the measurements reported in the figures of this paper which may be of further interest. From the lower figure 2, we can derive the length of the winding, $a = 1{,}570m$. Estimating the effective magneto-dielectric constant of the space between the conductors of the winding and the steel core as $\epsilon\mu \cong 2.5$, the Maxwellian velocity of propagation of electric phenomena in that space would be

$$v = \frac{300}{\sqrt{\epsilon\mu}} \cong \frac{300}{\sqrt{2.5}} = 190m/\text{microseconds}$$

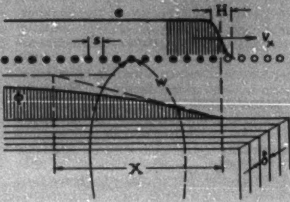


Figure 1

If we now scrutinize in figure 1 the rise of voltage over the coil in the first 10 μs, we see that after the instantaneous appearance of the initial distribution, the voltage very suddenly jumps a second time, and that the time delay of this jump depends upon the position of the tap point. For the tap point at:

$x = {}^1/_4\ {}^1/_2\ {}^3/_4$ of full length

the time until the voltage jumps is:

$\tau \cong 2.2\ 4.2\ 6.2\ \mu$ sec

Hence the wave velocity is:

$v = xa/\tau = 179\ 187\ 190\ m/\mu$ sec

These values of velocity are nearly constant and so close to the Maxwellian velocity as estimated above that it is somewhat difficult not to attach significance to these facts.

The second figure 8 shows, for the same winding, at the consecutive modes:

$m = 1\ 2\ 3\ 4\ 5\ 6$

resonance frequencies:

$f \cong 17\ 44\ 75\ 120\ 153\ 180$ kc/s

Hence the wave velocity is:

$$v = \frac{4af}{m} = 107\ 138\ 157\ 188\ 192\ 188\ m/\text{microseconds}$$

if the winding oscillates with nodes or antinodes at its ends.

Thus the three oscillations of highest frequency propagate again with the estimated Maxwellian velocity. The velocity of the oscillations of lower frequency, however, decreases, which I think is caused by a deeper penetration of their magnetic flux into the steel core. This would increase the average permeability and thereby decrease velocity and frequency.

Now the front of steep waves is always built up of the higher frequencies, while the tail is shaped by the lower frequencies, a theorem well known in Fourier analysis. Hence, we realize that both the experiments of figure 1, with steep impact waves and of figure 8a, with sustained oscillations, lead to the same conclusion, namely, that steep voltage waves in transformer coils can propagate with nearly constant Maxwellian velocity through the winding, and that their electromagnetic field is contained preponderantly in the dielectric space.

So the experiments of the authors prove very valuable not only with respect to the lower natural frequencies which they discuss in the paper, but also with respect to higher frequencies and to impact waves.

I presume that the reason for the insignificant influence of the steel core for rapidly-travelling waves is given by the effect of the time constant of penetration of a magnetic flux into steel sheets, which is represented in the accompanying figure 1. This time constant, according to a well-known formula, is for a steel thickness $\delta = 0.35$ mm, with specific resistance $\rho = 1.5 \cdot 10^4$ cm²/sec, and permeability $\mu_s \geqq 200$,

$$T = \frac{4}{\pi}\frac{\mu_s}{\rho}\delta^2 \geqq \frac{4}{\pi}\frac{200}{1.5 \cdot 10^4}0.035^2 = 20 \cdot 10^{-6} \text{ sec.} \quad \text{(a)}$$

For a ratio of pitch s to wire length w of a turn of a helical coil, such as $\frac{s}{w} = \frac{1}{1{,}000}$, the axial velocity of steep waves, as determined above, is

$$v_x = \frac{s}{w} \geqq \frac{1}{1{,}000} 190 m/\mu s = 19 \text{ cm}/\mu s \quad \text{(b)}$$

The lag in space of any magnetic steel flux behind a travelling electric wave front in the coil is therefore represented by the space constant

$$X = v_x T \geqq 19 \cdot 20 \cong 400 \text{ cm} \quad \text{(c)}$$

This means, with regard to the accompanying figure 1, that the electromagnetic field of steep voltage waves of a front length of say less than 10 per cent of the space constant, namely, $H \leqq 40$ cm in the axial direction, or $\leqq 400 m$ along the conductors, does not penetrate the magnetic core to a considerable amount.

The internal insulation between coils, layers, and turns is stressed preponderantly by these initial steep and rapid waves, which travel within the dielectric space. The slower successive oscillations, however, as those of 41 kc/s in figure 1 of the paper, may contain some steel sheet flux, in case there is no closed secondary winding to suppress it.

As to the mathematical development in the paper, I would like to ask whether the extended expression for the voltage in time and space, equation 10 of appendix I, which comprises two different infinite series, could not be condensed for all actual cases simply to

$$e = E \cdot \sin bt \cdot \sinh (\alpha + j\beta)x \quad \text{(d)}$$

a form which is more familiar to most engineers and which gives the same graphical results.

P. L. Bellaschi (Westinghouse Electric and Manufacturing Company, Sharon, Pa.): Years of research and design experience have borne out the practical and theoretical advantages of the method of free or natural oscillation for determining the performance of windings under transient conditions. These advantages are summed up by Messrs. Bewley, Hagenguth, and Jackson in their paper. To these I may add that this method, which consists in determining the natural modes of oscillation or response of windings to an impulse, also has the advantage in that the analytical process and the steps in the calculation apparently find their closely related counterpart in the experimental results. This physical relationship between the tool of analysis and the performance of the apparatus designed is an important factor for it gives the designer that physical and quantitative grasp of the problem, the importance of which I need not elaborate. Furthermore, the method fulfills an essential criterion in that it is fundamentally basic and sound. The following typical examples will serve to illustrate.

It would extend this discussion unduly to enter into a detail comparison of the extensive analytical work and the experimental results. Therefore, the comparison will be confined largely to the natural periods of symmetrical windings in the same manner as the authors have done. In the design of a 4,500-kva, 144-kv shell-type, two-group winding, the fundamental period calculated 300 microseconds—a value within seven per cent of the oscillographic record. Likewise, the initial voltage distribution of the entire winding and of the line group ($\alpha = 1.0$), the amplitude of the fundamental period, the period and amplitude of the line group, and other quantities of practical significance in the design were predetermined within about ten to twenty per cent of the actual performance. In the case of a 3,000 kva, 140 kv, shell type symmetrical winding, the fundamental period is 200 microseconds which compares within ten per cent of the calculated value. The amplitude of the fundamental oscillation developed from a long-duration, abrupt-front impulse applied to one end of the winding with the other end grounded is forty per cent. Similarly, the calculated periods and their amplitudes for the higher harmonics compare within ten to twenty per cent of the oscillograms. For instance, the natural period of the line coil group calculates 29 microseconds against an experimental value of 25 microseconds.

The method applies to core-type windings as well, in which case the distributed mutual inductance naturally must be taken into account for correct results. It is applicable to dissymmetrical windings and to other windings. It can be extended to a wide variety of conditions and problems such as inevitably are encountered in actual practice. For instance, the determination of effects resulting from transferred surges from high to low voltage windings with the latter, let us say, connected to a segment of cable, or for determining the voltage developed at the transformer terminals when an inductive element (reactor, current transformer, etc.) intervenes between the transformer and the line.

In an extensive analytical survey of the problem initiated some twelve years ago, we were able to predetermine the inherent characteristics such as the fundamental period, etc. of the various types and constructions of windings. The results were substantiated in the course of time by tests made on the transformer units built. For power transformers, it was found that the fundamental period of core-form windings from 69- to 220-kv ranges respectively from 20 to 80 microseconds and for the shell form, the corresponding periods extend from 80 to 200 microseconds and even as high as 300 microseconds as cited in preceding examples.

L. V. Bewley, J. H. Hagenguth, and F. R. Jackson, Jr.: Dr. Rüdenberg assumed a value of $\sqrt{\mu\epsilon} = \sqrt{2.5}$ for the dielectric space between the winding and core of a complete transformer. However, the tests described in the paper were made in air on a transformer, the low-voltage winding of which had been removed. Thus, the space in question was largely air, except for a supporting cylinder 0.25 inch thick and the spacing strips between the cylinder and winding. The calculated average dielectric constant for this space is less than 1.2. Accordingly, the Maxwellian velocity would be

$$v = \frac{300}{\sqrt{1.2}} = 274$$

rather than 190 as calculated by Rüdenberg.

We have measured carefully the original full-sized oscillographic tracings from which figure 1 of the paper was made. It will be seen in figure 1 that the voltage jumps are not abrupt, but start as a small fillet; and, moreover, the wave front is different at the different points along the winding. Therefore, we are not certain which instant on the oscillogram represents the time of arrival of the supposed wave at a given point. If we suppose this time to be between the start and finish of the fillet, and record these two extremes, we find corresponding to the tabulation given by Rüdenberg:

Distance from line end	1/4	1/2	3/4
Time to voltage jump	0.7	2.7–4.5	4.5–5.5
Apparent wave velocity	565	290–174	260–215

These values should be compared with those given in the second tabulation of Dr. Rüdenberg's discussion.

Dr. Rüdenberg asks if equation 10 in appendix I of the paper could not be put in the form

$$e = E \cdot \sin bt \cdot \sinh (\alpha + j\beta)x$$

The object of the particular form of equation 10 was to bring out three facts:

1. That the *dominant* term of the sustained oscillations is $EA_s \sin \frac{s\pi x}{l} \cdot \frac{b}{2\gamma_s} \cos bt$.
2. That the oscillations occur about the *initial distribution* $e(0) \cdot \sin bt$ as an axis (see figure 6).
3. That a resonant applied frequency can excite space harmonics other than those corresponding to the resonant frequency.

None of these things are indicated in the condensed form suggested by Rüdenberg. Moreover, that the suggested condensed form cannot be equivalent to our equation 10 is obvious upon noticing that our dominant term is cos bt, whereas Rüdenberg's form contains only a sin bt.

Mr. Bellaschi's test results do not entirely check the data accumulated over many years by other investigators, which indicate that the natural period of core type transformers of ordinary design varies between 25 microseconds and 160 microseconds, depending on kilovolt-ampere and kilovolt rating, and that of shell-type transformers varied between the same limits.

It is, of course, recognized that the lower the natural period of a transformer, the higher will be the amplitude of the low-frequency harmonics for a given length of applied wave. Oscillations are practically eliminated, regardless of applied wave length, in the shielded type of transformer.

Transient Torques in Squirrel-Cage Induction Motors, With Special Reference to Plugging

Discussion of paper 41-126 by E. S. Gilfillan, Jr., and Edward Kaplan, presented at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941, and published in AIEE TRANSACTIONS, 1941, pages 1200–09.

W. K. Boice (General Electric Company, Schenectady, N. Y.): This paper represents a very interesting and timely contribution to the study of transient torques in induction motors. The practical significance of such torques has recently received increasing attention.

The General Electric Company is at the present time making a study including factory tests of the transient torques occasioned by switching sequences such as those involved in transferring motors to full voltage from reduced voltages used for starting. Experience has shown that very high torques are possible under unfavorable conditions. It is proposed to publish the results of this study in the near future.

New Current Transformer for Bus Differential Protection

Discussion and authors' closure of paper 41-135 by L. F. Kennedy and A. T. Sinks, presented at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941, and published in AIEE TRANSACTIONS, 1941, pages 1180–7.

E. F. Bancker (General Electric Company, Schenectady, N. Y.): Some things are just too complicated for the human intellect to solve. Others can be solved but involve a lot of tedious work and since most of us are inherently lazy we use much ingenuity in devising ways to get out of such hard work. For one or the other of these reasons such things as the differential analyzer and a-c network analyzer have come into being. Another method of avoiding unpleasant work is to dodge it by getting the desired result in a totally different way that eliminates the onerous task.

Knowledge of the exact characteristics of current transformers in the saturated region is beyond the grasp of some of us and involves a lot of tiresome computation on the part of the few who know how to do it. For each bus differential application the conditions differ by just enough to require a new set of calculations. Yet high speed bus protection is deemed valuable and some way of accomplishing it is needed.

A few years ago solutions were offered in the form of relays that tried to distinguish between differential currents that should cause tripping and equally large differential currents that should not cause tripping. Last year a series of current transformer papers were presented in the hope that a better knowledge of their idiosyncracies would assist us in applying them to bus protection. This year we have two more papers based on the method of dodging the difficulty and that to me is the right approach.

The difficulties with conventional bus differential protection have been twofold. One is the non-linear exciting current of overworked current transformers. The other is the complications in secondary connection switching imposed by some primary switching arrangements. Mr. Kennedy and Mr. Sinks describe a method for circumventing the first by eliminating it. Mr. Steeb tells how to get around the second in one instance.

The development of the air gap core current transformers resulted from a facetious remark made by Mr. Sinks after he had been through the design calculations of a conventional current transformer for a specific job. He said in substance that he would have to find some iron good for a flux density of two million lines per square inch and added that in his opinion it would be better to put a gap in the core and make the a-c magnetizing current linear for all flux densities even though this current were much higher than usual. The seed thus planted took some time to germinate because at the time the discriminating relay approach looked satisfactory and it was thought that a solution using standard current transformers was advantageous. But somehow or other the calculations seemed to be just as numerous and attention returned to the advantageous features of a linear magnetizing current.

When the magnetizing current is linear the ratio of a current transformer is constant for a fixed burden. In differential applications the burden is constant for all external faults as it consists of the impedance of the secondary and leads to the paralleling point. With a constant ratio there is no secondary difference current for any external fault current regardless of its size, distribution, or amount or time constant of d-c component. This is understandable and we all prefer something we can understand. Also, it reduces computations to the simple one of determining the initial a-c short circuit current in order to select the proper rating of current transformer and a check of the sensitivity of the ground relay where the ground current is limited to a small percentage of the interphase fault current.

Thus, the development presented in this paper enables us to apply bus differential protection with assured knowledge of its operating characteristics, with a minimum of application engineering, and in an easily comprehensible manner. In other words, we have what appears to be the ideal solution to bus differential problems where the d-c effect on standard current transformers cannot be neglected.

C. D. Hayward (General Electric Co., Philadelphia, Pa.): It will be a very great relief indeed to those who have tried to calculate mathematically the performance of current transformers under short-circuit conditions to find in the air-gap core current transformer a device which does not saturate. Saturation in closed core current transformers produces variations in the self- and mutual-inductances making them functions of the applied current. Because of this the attempt to express the performance of the transformer mathematically leads to a non-linear differential equation which cannot be solved by the ordinary classical methods. One must resort to laborious approximate methods, to mechanical integrating devices such as the differential analyzer, or to actually making short-circuit tests and taking oscillograms. To cover completely any practical application problem a number of solutions must be made covering all combinations of conditions. For example, in the bus differential case solutions must be made for different combinations of fault current magnitude, distribution of fault current between the current transformers in the several circuits feeding the fault, for different degrees of wave offset, for internal and external faults, etc. Furthermore, there is always a certain amount of inaccuracy in the solutions due to variations in the saturation curves of different current transformers and due to residual magnetization. After the solutions for the current have been found there is the further problem of determining the relay performance on the distorted waveforms obtained. Altogether the problem is a very tough one indeed.

In the case of the air-gap current transformer, on the other hand, the problem is relatively simple. Since no saturation occurs the inductance of the current transformer remains substantially constant. If the secondary burden inductance also remains constant, the differential equation is a linear one which may readily be solved by the operational calculus. An analysis made in this way for the case of a single air-gap current transformer with constant burden shows that if an offset current wave:

$$i_1(t)=I\left[\sin\,(\omega t+\theta)-\sin\,\theta\epsilon^{-\frac{t}{T_1}}\right]$$

is applied to the primary, the secondary current will be:

$$i_2(t)=I\frac{M}{L_2}\left[\sin\,\phi\,\sin\left(\omega t+\theta+\frac{\pi}{2}-\phi\right)+\right.$$

$$\cos\,\phi\,\sin\,(\theta-\phi)\epsilon^{-\frac{t}{T_2}}-$$

$$\left.\sin\,\theta\,\frac{T_1\epsilon^{-\frac{t}{T_2}}-T_2\epsilon^{-\frac{t}{T_1}}}{T_1-T_2}\right]$$

where:

I = maximum value of the steady state primary current in amperes

θ = switching angle in radians

$T_1=\dfrac{L_1}{R_1}$ = time constant of primary circuit in seconds

= time constant of d-c component of primary current in seconds

$T_2=\dfrac{L_2}{R_2}$ = time constant of secondary circuit (including *CT* winding) in seconds

$\phi=\tan^{-1}\dfrac{\omega L_2}{R_2}$ = phase angle of secondary circuit (including *CT* winding) in radians

L_1 = primary circuit inductance in henries

R_1 = primary circuit resistance in ohms

L_2 = secondary circuit inductance (including inductance of *CT* secondary winding with primary open-circuited) in henries

R_2 = secondary circuit resistance (including *CT* winding) in ohms

M = mutual inductance of current transformer in henries

$\omega = 2\pi f$ (radians)

f = frequency in cycles per second

t = time variable (seconds)

The first term on the right-hand side of the expression for the secondary current is the steady-state term. Comparing this with the first or steady-state term in the expression for the primary current above, we see that the transformer ratio has the value:

$$\frac{M}{L_2}\sin\phi$$

while the phase shift angle is:

$$\frac{\pi}{2}-\phi$$

These are dependent on the secondary burden but independent of the current.

The second term in the expression for secondary current is a transient current set up due to switching the primary current at a phase angle different from the phase angle of the secondary circuit. This term will normally be quite small because of the factor cos ϕ. The secondary circuit phase angle ϕ is near $\pi/2$, and hence cos ϕ is nearly zero, because of the relatively high inductance of the current transformer secondary winding.

The third term in the expression for secondary current represents the transformation of the d-c component as given by the second term in the expression for primary current. In the usual bus differential relay application the time constant of the current transformer secondary circuit is quite small compared with the time constant of the primary circuit. If we assume T_2 may be neglected in comparison with T_1 the third term reduces to:

$$-\frac{M}{L_2}\sin\theta\epsilon^{-\frac{t}{T_2}}$$

which compared with the primary current term:

$$-\sin\theta\epsilon^{-\frac{t}{T_1}}$$

differs by the factor M/L_2, which is essentially a turn ratio factor, and by the shorter secondary time constant T_2 being substituted in place of the primary time constant T_1 in the exponential. Because of this change in time constant the d-c component in the secondary current dies away at a much more rapid rate than does the d-c in the primary. This is a proof of the assertion in the paper that the d-c component is not reproduced to any extent.

E. L. Harder (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): During the summer of 1938 a series of tests were made on air gap core current transformers for bus protection in the laboratories of the Westinghouse company. First, available designs having gaps, such as the impulse coils used on d-c circuits were tested. Then, four transformers specifically designed for bus protection were built, two having circular cores and two having rectangular cores. Each of these had four gaps instead of eight as shown in the subject paper but had four identical coils and the gaps were symmetrically disposed with respect to the coils. Based on the experience of our tests on these gapped-cores I would like to comment on the authors' paper, beginning with the necessary length of gap.

If in an iron core with gaps there was no fringing, so that the flux in the iron section crossed the gap within an area the same as the iron section, then to hold the flux density in the iron below saturation (taken as 12,000 lines per square centimeter), there must be one inch of air gap for each 24,200 ampere-turns of excitation. Considering fringing the actual gap length must be much greater. In tests of transformers with four three-eighths-inch gaps important saturation began at less than 50 per cent of the exciting current which would have been required to produce 12,000 lines per square centimeter on the basis of no fringing.

It is brought out in the paper that an air gap transformer for 100,000 amperes symmetrical will have a crest excitation current in the neighborhood of 140,000 amperes. The necessary total gap is therefore something over six inches depending on the amount of fringing. Relieving the corners by stepping back the core reduces the fringing problem. However, each of the core pieces in figure 3 of the paper is separated from its neighbor by one-eighth of the total exciting ampere-turns and the fringing flux is still very appreciable. This makes it very difficult and impractical to calculate accurately the maximum flux density at any point of the core. Reliance must be placed on tests which are subject to instrument errors as evidenced in table II. The voltmeter-ammeter method of checking linearity is simply not sufficiently accurate for this purpose as will be shown shortly. It was partly because of the difficulty of obtaining and proving pure linearity with the necessary limits and partly because a superior arrangement was in the meantime devised that the gap transformers were not offered to the industry.

The paper has stressed the energy delivering properties of the air gap design as the reason for its selection in preference to pure unadulterated air. However, the fundamental factors which determine whether a transformer can or cannot be used in a straight differential arrangement are:

1. The ratio of maximum through fault to necessary setting for internal faults.
2. The inherent errors of the transformer.

If the total errors do not cause the secondary currents to depart more than ±$2^1/_2$ per cent from primary current, divided by some constant ratio, then during a through fault the false differential current cannot be more than five per cent. If the relay is set at 10 per cent of the maximum through fault current, a 100 per cent safety factor is provided.

Thus with 100,000 amperes through fault and transformers of a constant ratio within ±$2^1/_2$ per cent (vectorial) a setting cannot be safely much less than 10,000 amperes even though the current transformer is capable of operating the relay set down to 100 amperes. This brings out the importance of the several errors; the manufacturing tolerance, the two per cent temperature error mentioned in section 6, the remnant saturation not detected by the voltmeter-ammeter method, and the remnant interference from other circuits due to not being perfectly astatic. It would be desirable to eliminate some of them altogether if possible. All of these may be cumulative in certain cases.

As long as there is ample energy to operate a reliable relay the output of the transformer is unimportant as compared with its accuracy which determines its "range," or ratio of permissible maximum through fault to setting.

L. W. Matsch (Commonwealth Edison Company, Chicago, Ill.): The authors, by means of their air-gap-core current transformer, have accomplished two noteworthy things. First, they have practically eliminated the d-c component, due to fault current, from the relay in the bus differential scheme thus making the use of a simpler relay feasible. Second, they have found a means of keeping the constants of the current transformers practically linear thus eliminating differential currents resulting from unequal current transformer saturation during faults.

The authors mentioned that a relay of the instantaneous overcurrent type could be used. A relay which is to have the same sensitivity in combination with the proposed current transformers as one in combination with conventional transformers would need to have lower impedance because of the low secondary open-circuit impedance of the current transformers described in the paper. I would like to know whether they had any special instantaneous overcurrent relay in mind.

In the last paragraph under *Application* it is stated that "good practice for the moment dictates a minimum relay setting corresponding to not less than ten per cent of the maximum fault current." This seems a rather high minimum setting since there are installations in which the maximum fault current may be as high as 70,000 amperes rms a-c and in which phase-to-ground faults of less than 2,000 amperes may occur. Apparently there must be a considerable hazard of improper operation during external faults if the relay is sensitive enough to take care of such low internal faults.

Since the ten per cent limit recommended by the authors is too high to adequately protect such a circuit I am interested in knowing what experience on the part of the manufacturer or operator led to this limit.

W. K. Sonnemann (Westinghouse Electric and Manufacturing Company, Newark, N. J.): In this paper the authors have briefly reviewed the difficulty which used to be experienced in applying bus differential relays where the d-c transient was a problem. The difficulty experienced due to current transformer saturation cannot be taken care of by conventional relays except by increasing the minimum trip setting, the time delay, or both. An analysis of the relay characteristics required to preserve both a reasonable minimum trip setting as well as reasonably high speed of operation indicated that it would be possible to build a special relay that would cope with the problem. This relay embodying "variable percentage characteristics" was developed a few years ago and a large number have been applied to bus protection with very satisfactory results. This relay was first described at the winter convention in 1939. Thorough tests, together with theoretical considerations, have shown that the relay may be applied without the laborious time-consuming calculations mentioned by Messrs. Kennedy and Sinks. All that is necessary for a proper application is to be sure that an adequate number of restraining elements are available and that the current transformers meet certain definite and well-defined requirements from an a-c standpoint.

The formulas developed in the paper are based on a four-circuit bus as shown in figure 1 of the paper. The authors neglect to mention the effect of adding additional circuits. With a given relay setting, of course, adding additional circuits will obviously decrease the sensitivity of the overall combination. The authors have concluded that the relay should not be set for less than ten per cent of the maximum fault current, and it is presumed that this is based on the four-circuit bus shown in figure 1 of the paper. Quite a number of generating station busses have large numbers of circuits up to ten or even more so that it is felt that a few words should be added explaining the effect of the additional circuits on the recommended range between minimum internal fault and maximum external fault.

It would appear that the gain obtained by using an iron-core transformer with air gaps was not all that is desirable. While it is true that the authors show that some saving in weight and dimensions has been made over the super-size current transformer design that will not saturate on d-c transients, the air gap transformer appears to be considerably larger and heavier than conventional standard current transformers. It would appear that the principle of air gaps in an iron circuit to prevent saturation is thus not readily applicable to use inside of the breaker bushings in place of bushing-type current transformers. This is considered unfortunate.

L. F. Kennedy and **A. T. Sinks:** Dr. Hayward's discussion gives a mathematical proof that there may be some d-c in the secondary of the air gap core current transformer under transient conditions but that its duration is short. Since the transient d-c in the secondary does not resemble that in the primary very closely, one might be led to suspect that there might be an unbalance of the secondary d-c components of a group of air gap current transformers connected to a bus during a through fault thus causing false relay operation. However, if the current transformers are all alike, there will still be no unbalance of either d-c or a-c because all the secondary current components bear the same relation to their respective primary currents.

Mr. Harder points out that fringing of the flux at the air gaps makes necessary a larger gap than would be necessary without such fringing. However, it should be noted that the increased length of gap together with the fringing that does occur results in a total reluctance which is the same as one would arrive at if one neglected the fringing and used the theoretically correct gap length based on a gap cross-section equal to that of the iron. Fringing, therefore, only affects the actual calculation of the shape and size of the gaps. Concerning the accuracy of calculating these gaps, it might be said in passing that for the first sample transformers of this construction made, it was necessary to alter the gaps only about ten per cent to arrive at the value of magnetizing impedance planned on.

It is quite possible that in the tests Mr. Harder mentions the stray field flux had considerable amount to do with the saturation of the core, as it has been found that at high currents, even with reasonably large conductor spacing, the amount of flux passing through the cores of current transformers due to currents in adjacent conductors is a very large per cent of the main flux. It was, of course, for this reason that the copper casing was included on the design described in the paper.

Mr. Harder indicates that the voltmeter-ammeter method of checking the linearity of exciting current is not sufficiently accurate to insure proper operation of the air gap core current transformers. For high accuracy requirements, this statement is undoubtedly true for complete air core transformers where it is almost impossible to make a transformer in a reasonable size in which the exciting current is not larger than the secondary current. In such a design, a given percentage variation in exciting current for a given voltage means a nearly equivalent percentage change in the relation between the primary and secondary current; whereas with the air gap core transformer represented in the equivalent circuit, figure 9 of the paper, a given percentage change in the exciting current for a given voltage will result in only one-sixth of this percentage change in the relation between the primary and secondary current, if the value of Z_B is less than four ohms. Consequently, the maximum spread in magnetizing impedance values of about two per cent in table II of the paper represents a difference in the phase angle of the secondary current in figure 9 which is one-third of one per cent of the secondary current and means practically no difference in ratio.

The authors' experience is that the energy delivering properties of the air gap current transformers are important because, as pointed out in the second paragraph following the mathematical analysis, in case of an internal fault, the relay impedance acts as if it were N times its actual value in affecting the accuracy of the transformers, where N is the number of transformers operating in parallel to supply the relay. This means, that even with a relatively low impedance ground relay, the effective impedance may be appreciable if there are say ten feeders connected to a bus with three transformers on each feeder, making a total of thirty transformers connected to the relay.

It is possible that the accuracy of the air gap core current transformer is not enough for the minimum ratio of ground fault to through fault current that may be encountered on some systems, although tests show that the figures given in the paper are conservative and limits could be made closer. However, if conventional transformers of reasonable size were installed on a bus having from a number of sources a through fault capacity of the 100,000 amperes Mr. Harder mentions, the transformer carrying the total fault current would saturate so badly that it is doubtful whether any relay could distinguish the condition from a fault on the protected bus. It would, therefore, seem that the air gap current transformer, even though it may not meet the most stringent requirements one can find, is a considerable improvement over conventional transformers.

In answer to Mr. Matsch's question about the sensitivity of the relay required for use with the air gap core transformer, it has been the authors' experience that no special relay is necessary but that it is usually necessary on the other hand to avoid high impedance relays.

Mr. Matsch's remarks concerning the ratio of ground fault to through fault current have been answered above.

Mr. Sonnemann indicates that the application of conventional current transformers to bus differential protection is a relatively simple matter. However, this has not been the authors' experience. Moreover, it would seem that the papers on current transformers presented at the 1940 winter convention are ample testimony to the work of calculating the accuracy of saturated current transformers.

The effect of increasing the number of circuits connected to the bus is covered for a fault in the protected area by the remarks above and by the second paragraph following the mathematical analysis in the paper. For the case of a through fault, the number of circuits connected to the bus should not affect the relay setting in per cent of the total fault current.

The air gap core current transformer designs will have to vary, of course, with different electrical and mechanical requirements of different applications and it is true that in general these designs will be larger than the so-called standard current transformers. However, enough reduction in size has been obtained by the air gap core design over the "supersized current transformers that will not saturate on d-c transients" to provide current transformers for bus differential protection that can replace the bushing transformers ordinarily used in circuit breakers.

Bus Protection Independent of Current-Transformer Characteristics

Discussion and author's closure of paper 41-99 by G. Steeb, presented at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941, and published in AIEE TRANSACTIONS, 1941 (September section), pages 859–62.

L. N. Crichton (Westinghouse Electric and Manufacturing Company, Newark, N. J.): This excellent paper by Mr. Steeb describes an installation in an unusually complex station. Using his experience as a guide, I have attempted to simplify his scheme and make it suitable for general application. Figure 1 of this discussion has the appearance, at least, of being much simpler and does not have the first two limitations listed in the paper.

In order to make the plan a general one, it should be suitable for that common case

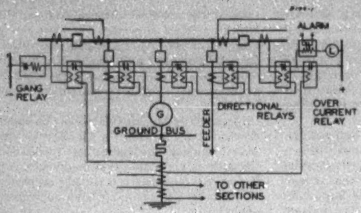


Figure 1. Bus-protection scheme using directional ground relay

where a feeder may or may not furnish fault current.

The new principle in this scheme is the use of a directional ground relay so built that its contacts are held closed by a spring when the relay is deenergized. One of these relays is connected to each feeder and to each source of power and to the bus ties. The only other relay is an overcurrent relay used as a fault detector and operated by current from the main station ground. The contacts of all the directional relays are in series as are also the normally open contacts of the fault detector.

When system conditions are normal, all the directional relay contacts are closed and upon the occurrence of a fault outside the bus section it will be necessary for at least one of these directional contacts to open before the overcurrent relay closes. There is no particular difficulty about accomplishing this and a satisfactory performance can be obtained from ordinary induction-type relays. No *B* switches are necessary, no special wires are run to the breakers, nor need system operators worry about any particular switching sequence. The only requirement is that a polarizing current must always be available, as it easily can be at all stations where there is ground connection. At substations that have no grounded neutral a polarizing potential may be used and a potential relay used on the fault detector.

The important thing about this arrangement is that all of the contacts can be supervised not only by a lamp but by an alarm relay. This, together with the comparatively small number of connective wires between different circuits should make this scheme a useful one in many locations.

E. H. Bancker (General Electric Company, Schenectady, N. Y.): Two of the difficulties encountered in bus differential protection are variation in current transformer ratio with saturation and complicated secondary switching arrangements where a set of current transformer secondaries must be in either of two bus differential systems depending upon disconnecting switch positions. Mr. Kennedy and Mr. Sinks tell of a means for overcoming the first and Mr. Steeb shows how the principles of carrier pilot relaying may be used to advantage to circumvent the second. His arrangement would be essentially the same as carrier relaying if all breakers had a circuit opening overcurrent relay and a circuit closing directional relay contact in parallel, as the tie breakers have. However, the overcurrent relays may safely be omitted from those circuits that are always sources of fault current and the directional relays from those that are never a source. In the more usual case of multiple grounded systems, the feeders would need directional as well as overcurrent relays.

Although the scheme of protection described in this paper is nearly independent of current transformer characteristics, it is not wholly so. The neutral resistor reduces the zero sequence component of two phase to ground faults but the phase to phase component is practically unaffected. The zero sequence voltage for this type of fault is 90 degrees displaced from the phase to phase voltage of the faulted phases. Since the currents are controlled by resistance in one case and chiefly by reactance in the other, they are nearly in phase so that both current transformers have primary currents that are essentially in phase but differing in size by nearly three times the zero sequence component. This difference, plus the effect of d-c. offset, may result in material changes in ratio of the two current transformers so that their secondary residual current may be considerably smaller than three times I_0. Such a reduction might prevent blocking by a feeder overcurrent relay for a two phase to ground feeder fault, but is probably not great enough to cause a directional relay to fail to operate to clear a bus fault. Consequently it would appear that this scheme has a field of utility where differential or fault bus protection would be difficult to apply.

J. H. Oliver (General Electric Company, Philadelphia, Pa.): Mr. Steeb has approached this bus-protection problem with an open mind. He has analyzed the available methods of high-speed bus protection and has recognized their economic limitations as applied to this particular job.

With true engineering spirit he did not abandon the idea of bus protection for this old station when available schemes were uneconomical to install, but went ahead and developed this scheme using standard relays. His use of oscillographic proof that the devices chosen for the job would perform in specified times and with proper contact action, combined with a year's tryout (including staged tests), shows the thoroughness of his investigation before this installation was actually put in service. Thus, others who may wish to adopt this scheme of protection for equivalent problems have the benefit of the investigation already made by Mr. Steeb.

It would seem that the limitations as outlined in the paper can be readily overcome by treating the ground-source circuits the same as the tie breaker; i.e., use one overcurrent relay in addition to the directional ground relay on each ground-source circuit, connecting the normally closed contacts on the ground-source overcurrent relays the same as on the tie breaker, and eliminating the *B* auxiliary switches now used. In addition, connect a normally open contact

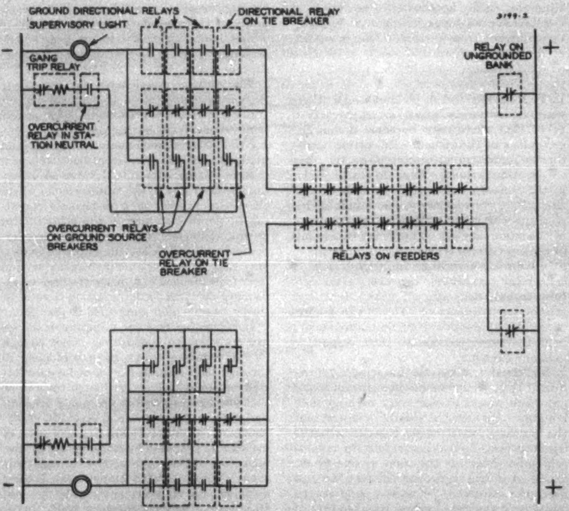

GROUND DIRECTIONAL RELAYS
SUPERVISORY LIGHT
DIRECTIONAL RELAY ON TIE BREAKER
3199-2
RELAY ON UNGROUNDED BANK
+
−
GANG TRIP RELAY
OVERCURRENT RELAY IN STATION NEUTRAL
OVERCURRENT RELAYS ON GROUND SOURCE BREAKERS
OVERCURRENT RELAY ON TIE BREAKER
RELAYS ON FEEDERS
−
+

Figure 2. Schematic d-c relay connections for bus sections 1 and 2

on each of the ground-source overcurrent relays in parallel with a normally open contact on the bus-tie overcurrent relay and insert this set of paralleled contacts next to the overcurrent-relay-in-station-neutral contact in the gang-trip-relay-coil circuit. The connection of the supervising light can also be moved, as shown in the schematic connection diagram, Figure 2, to supervise the normally closed contacts which have been added to the circuit.

This connection avoids limitation 1 since the gang-trip-coil circuit will remain open in the case of one bus section's being fed only through a tie breaker and a bus fault taking place on one of the other bus sections, as the normally open contacts on all ground-source overcurrent relays and on the bus-tie overcurrent relays on the particular section not involved in the fault will remain open.

Limitation 2 is avoided as the normally closed contact of the overcurrent relay added to the ground-source circuit will remain closed, completing the bypass circuit around any ground-source breaker out of service for maintenance.

R. C. Silver (Gatineau Power Company, Ottawa, Ontario, Can.): Mr. Steeb's paper is particularly interesting to us since in 1935 we installed similar protection for ground faults on four 11,000-volt bus sections. Overcurrent relays were used for blocking and also for initiating the tripping. These latter were operated from the paralleled secondaries of current transformers on the two or three possible sources of ground fault current. A time delay of 0.10 second was introduced in order to insure selective operation of contacts. These four sets of relays have been in service for over five years and no faulty operations have occurred.

In 1936–37 we assisted in the design and installation of another similar protection for a 66,000-volt bus in a hydrogenerating station which involved nine generators and five 66,000-volt transmission lines. The protection was designed to respond to phase to ground and three phase faults. Impedance relays on a number of the transformers initiated the tripping while blocking was arranged through the instantaneous line and transformer protections. A time delay of 0.10 second was considered necessary in this case also.

Both types of protection were installed in older stations where it would have been very difficult and expensive to install a differential bus protection.

Mr. Steeb is to be congratulated on the high speed operation of the scheme he has described.

R. T. Henry (Buffalo, Niagara and Eastern Power Corporation, Buffalo, N. Y.): The bus relay scheme proposed in Mr. Steeb's paper was not developed as an alternative to conventional bus differential relay schemes even though it is applicable in any case where bus differential relay schemes could be used and it is true that this scheme is independent of current transformer characteristics so far as the dissimilarity of current transformers at high overloads is concerned.

This scheme was developed primarily for bus arrangements on which it is impracticable to apply the conventional bus differential relay scheme. The first application of this scheme was made on a 22 kv outdoor switching station with a double bus and selector disconnecting switches. Such an arrangement does not lend itself to bus differential relay schemes.

This scheme may be easier to visualize if it is considered as consisting essentially of directional relays on each circuit with the tripping contacts all in series, so arranged that if all of the directional relays "face in" towards the bus, the gang trip relay will operate, but if any one "faces out" from the bus it prevents the gang trip relay from operating. Actually, the relays on circuits which never supply current towards the bus can be simple current relays without any directional characteristics.

While this scheme has been applied for ground faults only, it is equally applicable for phase to phase faults.

L. F. Kennedy (General Electric Company, Schenectady, N. Y.): Mr. Steeb has used his knowledge of line protective methods to develop a method of bus protection which is essentially free of dependence upon current transformer performance. The arrangement, which he has described, is in reality an extension of the carrier current pilot relay system, because its principle is that fault current flowing away from the protected area prevents tripping. This principle has been successfully used in all carrier pilot systems and has also been successfully used in a number of d-c pilot systems. Basically, then we can concede that the method has been proven to be sound, and the only questionable point would be whether the addition of many more contacts in series would introduce any factor tending to decrease the overall reliability. Contact failure would, in general, tend to cause a failure to trip for an internal fault. This probability has been taken care of, to a certain degree, by the supervision of the normally closed series contacts.

There would appear to be at least two conditions which still would require a certain degree of dependence on the current transformer characteristics. The system shown in figure 1 of the paper limits the current in case of ground faults by means of a neutral resistor. The phase fault currents are apparently limited only by the generator and transformer impedances. On this type of system there is frequently encountered a very great difference in magnitude between the current flowing to a ground fault and that flowing into a multi phase fault.

Where such conditions exist it would be possible in the case of an external two phase to ground fault on a feeder circuit to have current transformer saturation reduce the secondary zero sequence current below the value required for correct blocking and an unnecessary interruption to the bus would result.

Another, but more remote possibility would be the case of an internal two phase-to-ground fault where again current transformer unequalities might cause the ground current relay on an ungrounded source to pick up and thereby prevent tripping when a fault actually exists.

It is not probable that these limitations would be encountered in the case of the 22-kv bus installation described by Mr. Steeb. It would seem, however, that this system of protection for other applications is still dependent upon current transformer performance although it is probably less seriously affected than some of the other methods which have been proposed. The system might be made less liable to failures if all the blocking relays were of the directional type but this would increase the complexity of the system.

In conclusion it does seem desirable to reiterate what I have said many times before. "Bus sections of all types and particularly those where large concentrations of power and services occur should always have automatic protection." There will probably be differences of opinion (at least for a few years) as to the best method of providing such protection, but I remain convinced that some form of protection is much to be preferred over none at all.

E. W. Knapp (Shawinigan Water and Power Company, Montreal, Can.): The arrangement as outlined in this paper undoubtedly constitutes an important development in the art of bus relay protection. Some years ago a scheme of bus protection, similar to the arrangement presently being discussed, was proposed for one of our 25-kv isolated neutral systems. In this case, impedance and directional relays were to be used for short-circuit protection only. Although the advantages of such a scheme were recognized it was felt that extreme care would need to be exercised with regard to such conditions as relay timing and contact bounce.

It is indeed encouraging to know that Mr. Steeb has been able to solve this problem in a satisfactory manner. A study of the paper appears to indicate that especial care has been exercised in the choice of relays. I have been particularly impressed with regard to the close timing which has been used to insure selectivity of operation. As a matter of interest, there are a number of points where further discussion might be desirable.

(*a*). In the case of cross-short circuits involving one phase to ground on the bus and a second phase to ground on one of the feeders, would it be necessary to clear the feeder before the bus protection can clear the bus? If so, how long would it take to clear the feeder and would this extra period of heavy fault current tend to cause a permanent bus outage? The same condition might also apply for multi-phase faults possibly during lightning, where the bus and one or more feeders were involved.

(*b*). As some of the current relays are set at a value of 300 per cent of maximum ground fault current, has any trouble been experienced due to a division of current between parallel circuit breakers or possibly due to one or more of the grounding banks being out of service or some other condition tending to reduce the amount of ground fault current?

(*c*). Has any trouble been experienced due to residual current or possibly sticking relays tending to retard the operation of the respective relays and thus destroy selectivity?

W. R. Brownlee (Commonwealth and Southern Corporation, Jackson, Mich.): It is worthy of note that even after a trial period of a year with the relay trip circuits blocked, the author deemed it advisable to make comprehensive short circuit tests before placing the bus protection relays in operation. If carefully planned,[1] short circuit tests will not only provide conclusive answers with reference to the new installation being studied but will frequently reveal enough unsuspected system or equipment conditions to justify the tests.

CHOICE OF SCHEME

Confining protection to ground faults greatly simplifies any bus protection arrangement and many operators will agree with the author that such protection is entirely sufficient if breaker tripping is initiated before the fault involves all three phases. Naturally this time will vary with different bus arrangements, but it is felt that the speed of one to two cycles obtained by the author (five cycles on a 60-cycle basis) is quite conservative and that operating times as high as 10 to 15 cycles on a 60-cycle basis may be sufficiently fast for typical outdoor high voltage busses.

Independence of current transformer characteristics is an important advantage even for new stations and though careful timing, coordination, and exhaustive analysis of system conditions are required, the directional contact scheme might be chosen more frequently in preference to differential protection (including the schemes confined to ground differential protection) except that a surprising number of "bus faults" appear as simultaneous flashovers of one or more bushings on the line side and also the bus side of a single oil circuit breaker which not only prevents correct operation of the bus protection relays as long as the breaker is closed but may confuse the tie line directional ground relays so as to prevent their operation also.

CO-ORDINATION OF RELAYS

The author is to be commended for his painstaking tests of the various relay elements to be coordinated. His conclusions on plunger relays were confirmed by recent tests in connection with pilot wire equipment in which two identical high speed plunger relays were energized simultaneously with a neon lamp connected to indicate whether the normally open contact of either would be closed in advance of the opening of the normally closed contact of the other. Observations were made both on energizing and de-energizing the relays. Perfect coordination was observed with currents from 4 per cent above the pickup setting of the plungers up to 300 per cent of pickup, at which current the circuit closing speed of the relays is one-half cycle on a 60-cycle basis.

LIMITATIONS OF SCHEME

From limitation 1 it appears that the author cannot trust the interval between the one-fourth cycle time required to open contacts on a feeder breaker and the one-third cycle time to close the normally open contacts of the neutral relay but that adequate coordination is secured between the one-fourth cycle circuit opening time and the three-fourths cycle closing time of the directional relays. An alternative arrangement would be to use an auxiliary relay with a delay of one or two cycles (which would not operate on an impulse shorter than about one cycle) interposed between the various contacts and the auxiliary tripping relay and then to use plunger elements with normally closed contacts in parallel with the directional relays on all sources instead of *B* fingers. This would remove limitations 1 and 2.

In addition to the limitations of the scheme as applied, there is the further system limitation that the station protected must be the only source of ground current and that none of the feeders from different bus sections can be connected together at remote points (except through transformation). With the auxiliary time delay scheme, each tie line breaker (with other sources of ground current) can be considered as a possible source, and provided with the combination of directional ground and plunger relays. It is interesting to note that a directional ground relay with its contacts normally closed could take the place of the plunger relay and directional ground relay with normally open contacts in such an application. A scheme of this nature has been in service for several years with no incorrect operations and two operations which were "correct" except that one of them required excessive clearing time, resulting in other relaying on account of the fact that bushings on both sides of one breaker were involved in a common fault.

REFERENCE

1. TESTING OF HIGH-SPEED DISTANCE RELAYS, E. E. George, AIEE TRANSACTIONS, volume 52, 1933, page 802.

G. Steeb: Mr. Crichton proposes a modification of the bus scheme in which directional relays with normally closed contacts are employed.

At the time the bus protection application, described in the paper, was made there were no suitable high speed directional relays with normally closed contacts available. However when such relays were placed on the market the author, realizing their advantages, developed and installed in one of the 110-kv 60-cycle stations a modified scheme which differs from that proposed by Mr. Crichton in only two respects.

The two points of difference are:

First, we use in each bus section one ground directional relay with its contacts normally open, and in series with the contacts of the fault detector relay. This safeguards against inadvertent tripping of the bus, either by human mistakes that may be made on the fault detector relay or by possible loss of the correct sequence in opening and closing of the various contacts. Second, we prefer to put the coil of the gang relay ahead of the normally open contacts; this prevents the coil from being energized in case of a short in the supervisory light circuit.

Mr. Bancker refers to the principles of carrier pilot relaying and its similarity to the described bus scheme. It is true that both require proper synchronizing of contact action. Since the reliability of carrier pilot relaying has been established, the similarity of the principle upon which the bus scheme is based should justify confidence.

As to his second point with respect to the effect on the blocking relays in case of a two phase to ground fault; that type of fault has occurred since the scheme has been put in service. The fault current was about 35 times the current transformer rating (400:5), and the relays performed correctly. It should be remembered, that all the blocking relays have a low pick-up.

Mr. Oliver suggests the use of overcurrent relays instead of the *B* auxiliary switches. In some installations this may be feasible, but at the station where the bus scheme was applied, the residual fault current through some circuits under certain operating conditions may be only 0.6 ampere, which would not be sufficient to operate reliably an overcurrent relay, hence directional relays were used because of their greater sensitivity.

Mr. Silver informs us about a similar bus scheme, which confirms that bus protection can be provided independent of matched current transformers.

Mr. Kennedy brings out an interesting point, in case of an internal two-phase-to-ground fault; this may have to be considered in other installations, but in this case the current transformer ratios of the ungrounded power sources are 1,200:5 and 2,000:5 respectively, and the fault current from these sources could be at the most about 3 times their current transformer ratings, and since the residual circuit is of high impedance it is unlikely that the relays would be affected; tests have confirmed this.

Mr. Knapp's reference

(*a*). The bus scheme will remain blocked until the feeder is cleared. The relay time on the feeders varies from one to twelve cycles (25-cycle basis). What will happen to the bus in the mean time, depends on the character and location of the arc.

(*b*). The maximum ground fault current is about 2,600 amperes whereas the relay in the station neutral is set to pick up at the equivalent of about 800 amperes, which ought to assure reliable performance. These current values are for all practical purposes not affected by the number of banks in service, although the residual current in the directional relays of the banks themselves will vary considerably. The current in the directional relay coils connected in the neutral current circuit will be of sufficient magnitude to insure safe operation of these relays.

(*c*). If residual current of sufficient magnitude should appear, the blocking relays would extinguish the indicating light and thereby announce its presence. As to sticking relays, the percentage is very small. In the Niagara Hudson system there are close to 20,000 relays in service out of which there may be four relays per year that fail to operate because of internal defects, which indicates a good record for the designer of relays and the men who maintain them.

Mr. Brownlee's conclusion that the station protected must be the only source of ground current is incorrect as is also Professor W. A. Lewis' belief, as expressed in his oral discussion, that all the neutrals must be connected together to a common ground, the fact is that the zero sequence voltage can be used for polarization or to operate an overvoltage relay, which acts as the fault detector relay. This principle has been used successfully in one of our 110 kv 60 cycle stations.

Mr. Henry's discussion clarifies some of the underlying points of the bus scheme. While it is true that this bus scheme was developed primarily for an application where it was impracticable to apply the conventional bus differential scheme, the author feels that the principle of this method of bus protection is not limited in application to this particular station, but rather that the scheme is general, and should be considered wherever bus protection is contemplated.

However in any future applications the modification of the scheme as referred to in the closing remarks of Mr. Crichton's discussion would be used, rather than the original scheme as described in the paper.

A Simple Method for Determination of Ratio Error and Phase Angle in Current Transformers

Discussion and author's closure of paper 41-109 by E. C. Wentz, presented at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941, and published in AIEE TRANSACTIONS, 1941 (October section), pages 949–54.

G. Camilli (General Electric Company, Pittsfield, Mass.): This paper gives an interesting review of the theory underlying the performance of current transformers. It shows that the much discussed "internal reactance" is still the stumbling block to the accurate predetermination of the accuracy of this type of apparatus.

Last year Mr. TenBroeck and the writer presented a paper, "A Proposed Method for the Determination of Current Transformer Errors" [AIEE TRANSACTIONS, volume 59, 1940 (September section) page 547], in which a very simple method was described for determining the effective internal reactance of a current transformer. The scheme required no exploring coils which are necessary in the scheme proposed by Mr. Wentz. It appears that Mr. Wentz's scheme loses some of its effectiveness for:

(*a*). It is admitted that some experience is necessary to predetermine the most effective position of the exploring coils in the magnetic circuit.

(*b*). In common with all the other schemes proposed so far, Mr. Wentz' proposal is not suitable for the calculation of the performance of a great number of current transformers within the metering range of secondary currents, due to the fact that most modern transformers are "compensated" by some special arrangements.

The experimental determination of the secondary internal reactance of a given transformer, however, is very valuable for the calculation of its overcurrent characteristics.

In performing the usual accuracy tests at very high currents, certain difficulties are encountered which can be overcome by indirect methods once the internal impedance and the excitation characteristics of the transformer become known.

For the above application, Mr. Wentz's proposal for the determination of the internal reactance should be a valuable addition to the schemes already known.

A. T. Sinks (General Electric Company, West Lynn, Mass.): As Mr. Wentz points out, his paper presents no new fundamental principles, but it does describe a relatively new method of solving an old problem. This new method has some very desirable features which should be taken advantage of. However, the method also has a number of limitations which restrict its usefulness. In order to alleviate these restrictions and increase the usefulness of the method, the following suggestions are offered:

1. The paper arbitrarily limits the method to five per cent error. In the case of high burdens and low currents, it should be possible to calculate larger errors than this without serious difficulty. The more fundamental limit on the usefulness of the method is a maximum ampere turn per inch exciting current in the core. A limit of approximately 300-ampere turns per inch is probably reasonable.

2. For transformers having cores with non-uniform cross sections of iron the method, as given in the paper, does not apply; first, because the exciting current tests will not give the unit exciting current characteristics for the iron; and second, because having these characteristics a knowledge of the relative lengths of the portions of different cross sections is necessary to apply the data. For transformers with non-uniform cores, it is therefore suggested that two magnetization curves, one corresponding to number 1 branch, and the other to number 2 branch of the figure 4 might be supplied, or computed on the basis of design data together with the value of X_L. The remainder of the calculation of the accuracy of a non-uniform core transformer could then be carried out as indicated in the paper.

3. As the author points out, the leakage flux generally changes at very high currents due to saturation of the iron, the flux which normally returns through the iron core being by-passed through the air. Therefore, any calculation based on the assumption that X_L in figure 4 of the paper is constant may lead to errors considerably larger than the actual values at high currents. It should not be assumed on the other hand that because a section of the core is closely approaching saturation the errors of the transformer are so large as to render it useless. There are some cases where, on the basis of methods of calculation given in the paper, part of the transformer core would exceed saturation at 10–12 times rating; yet, according to actual tests the errors are less than 10 per cent at 20 times rating.

This range of current, therefore, offers an important problem that present or previous papers have not solved. A suggested line of attack might be to try to arrive at a fictitious upper part of magnetization curve for number 2 branch in figure 4 which would satisfy the actual conditions of error.

4. While the subject of transient accuracy was mentioned in the paper, very little was said about the calculation of such accuracy. It would seem that methods previously discussed for computing transient accuracy could be equally well applied to figure 4, except just as in the case of steady state accuracy the amount of work would be approximately double that of calculating the accuracy of the transformer with uniform flux density in the core.

While one of the purposes of the paper stated by the author was to present a method which would reduce the actual amount of computation necessary for calculating the accuracy of current transformers, it would seem that unless a large number of conditions are to be calculated it would be easier to compute the errors directly from the magnetization curves rather than to plot so many curves as shown in figure 5 for each transformer.

While for high currents the four-coil secondary construction of the window type or bar type transformers undoubtedly has advantages in some respects over the two-coil secondary, the author's statement that it is practically impossible to make a two-coil window type transformer good for more than 25,000 amperes does not seem justified. Data is available on perfectly standard 3,000-ampere window type transformers of reasonable size rated at 3,000 amperes, having errors of approximately 6 per cent with 50 v.a. burden at 60,000 amperes. These transformers have non-uniform core cross sections, the cross section of core on the legs not enclosed by secondary coils being very much larger than the cross section of the other legs in order to take care of the large leakage flux. This brings out the advantage that sometimes may be gained by using a non-uniform core cross section, and therefore shows the importance of modifying the method of calculation presented in the paper to take such designs into account.

P. O. Langguth (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): 1. The author has very nicely worked out a simplified method of determining the performance of current transformers from the saturation characteristic and of determining leakage reactance. Such analysis will simplify the application of transformers for relay service.

2. Several schemes have been presented during the last few years to determine transformer performance by calculation for the particular conditions involved

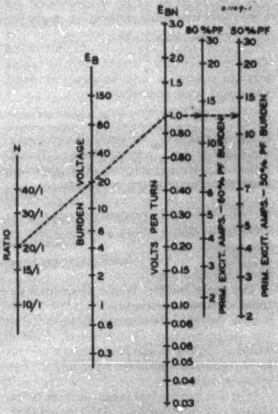


Figure 1. Nomograph for calculating current-transformer errors

Typical solution: Assume turns ratio—20/1
Burden 0.5 ohm, 80 per cent power factor
Secondary current = 40 amperes
Burden volts = 20
Lay straight edge on chart as indicated by dotted line for correct volts per turn and read primary exciting current = 13 amperes
Primary current = (20 × 40) + 13 = 813 amperes
Per cent ratio = 101.5

rather than use typical curves for standard burdens. Since all of these schemes have some inherent limitations, it seems desirable to work out a compromise method of determining leakage reactance and transformer performance from the saturation data. Such a scheme could be very well incorporated in AIEE Instrument Current-Transformer Standards or an AIEE standard test code. An arrangement of this nature would simplify the specification and use of current transformers for relay application where high overcurrents are usually involved.

3. The method disclosed by Mr. Wentz is sufficiently accurate for relay applications. Although no data have been submitted here to indicate variations at 20 times normal current, it has been my experience on bushing type transformers, that even with less complete analysis wherein leakage and excitation angle are ignored, the results are sufficient for most relay work. It is believed that the method outlined by Mr. Wentz would be more universally applicable if the computations could be further simplified. Possibly, some standardized graphical chart, including saturation data could be prepared for this purpose.

4. As an example in figure 1 of this discussion, a nomograph chart has been prepared for calculating the ratio error of bushing type current transformers. Turns ratio, burden voltage, volts per turn, and primary exciting current scales are plotted in logarithmic steps. A typical solution in the figure illustrates the simplicity of ratio error determination.

5. Phase-angle error if necessary for some particular relay application can be determined in a similar way by addition of a phase angle scale. It is believed that such a chart can be used for wound types by taking leakage reactance into account by some additional scale arrangement.

D. B. Masters (Commonwealth and Southern Corporation, Jackson, Mich.): Recognition of the importance of a thorough understanding of current transformer characteristics in the successful operation of certain types of protective relays has resulted in the devotion of much study to the problem of easily and economically determining current transformer performance characteristics.

The paper by Mr. Wentz proposes a detailed method, utilizing relatively few fundamental factors. It appears, however, that the determination of equivalent leakage which is the key to this method may present considerable of a problem especially at high primary currents.

It is not quite clear what degrees of accuracy can be expected. It is stated that, "When the ratio error exceeds five per cent, the iron is usually working near saturation, ... The result is that this method of calculation will give errors which are considerably too large when the error (real error?) exceeds five per cent." From this it would seem that the range of usefulness is rather limited for relay applications without some knowledge of the true error since it is implied that only errors greater than five per cent usually are of any consequence.

The author implies that the determination of the basic data must be made by the manufacturer. Any refinements and improvements in the method which could be made to enable it to be applied in the field to older types of transformers now in service would be a distinct accomplishment.

E. C. Wentz: Mr. Camilli correctly states that the much discussed "internal reactance" is still a stumbling block. However, the point of the paper written by Mr. Sinks, on which the author's paper is based, is that internal reactance is quite mythical and has an existence only as defined by any author. Mr. Camilli refers to his method for determining internal reactance, but his definition has been criticized before as defining a quantity of flux which bears no clear relation to the leakage flux in the core. The author's definition of leakage reactance is certainly based on the real flux in the core.

No great experience is necessary to place exploring coils. If there is some doubt as to correct placing, more than one location may be tried. The highest measured value of leakage reactance is the conservative one to use.

The method will not work in the metering range for transformers with Wilson compensation. This is a disadvantage shared with Mr. Sinks' paper. The method can conceivably be used with cores of non-uniform section, but, as pointed out by Mr. Sinks, its use is much more difficult, and probably generally impractical, unless the manufacturer supplies the special data for using it. Perhaps it is more economical to use four coil construction in through type transformers rather than to widen the core to get equal performance. While a wider core improves the performance, it is none the less true that widening the core will increase the total leakage flux so that this use of the material is not too economical.

Mr. Sinks points out that with low currents and high burdens the method can be used up to ratio errors much exceeding five per cent. The five per cent figure given in the paper is merely a conservative limit, and is the only one which can be stated thus simply. The ampere-turn per inch limit given by Mr. Sinks is the more fundamental, but the man in the field will have no idea of its magnitude. Further experience will, perhaps, indicate simple working limits which will be more nearly maximum values.

Whether the family of $F \cos \theta + M \sin \theta$ curves are plotted will depend on the application. It will no doubt pay to plot $F \cos \theta + M \sin \theta$ curves for standard lines of current transformers, but probably not for special transformers for special application. Of course, most manufacturers use a limited number of standard cores, and a limited number of master curves could be the basis for easily plotted curves for special designs.

Mr. Masters suggests that determination of reactance may be difficult at high currents. However, the reactance at high currents, as represented by leakage flux in the core, is always lower than at normal current. Therefore, as pointed out in the paper, the value at normal current may be used at higher currents with confidence that the calculated errors will not be exceeded on test.

The whole problem is not to be solved by any one paper or method in any short time. It is clear from Mr. Sinks' paper, which sets forth the fundamental principle, that absolute accuracy is impossible; the author's purpose is to outline a practical compromise between accuracy and simplicity for general use in the factory and in the field. It is hoped that the method will be given a trial by those interested in a simple method for determination of overcurrent performance.

D-C Machine Flashover and Bus Short-Circuit Protection

Discussion and author's closure of paper 41-122 by T. B. Montgomery and J. F. Sellers, presented at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941, and published in AIEE TRANSACTIONS, 1941, pages 1168–73.

A. E. Anderson and **C. H. Black** (General Electric Company, Philadelphia, Pa.): From the viewpoint of circuit protection, the relative speeds of air circuit breaker operation are important only insofar as they affect the total length of time required to interrupt the current, or at least the total time required to start some current limiting action. With this in mind, the proposed new method of energizing the tripping means should be reviewed to determine whether it actually reduces the time required for current limiting action to start, and if so, how much reduction is obtained. These are the important points. The details of how the tripping is actually accomplished are entirely incidental.

The authors state that there is a reduction in tripping time under all conditions. It is conceded that this might be so under the limited condition of a flashover involving only moderate current and a very low rate of rise. Under a condition of severe short circuit (bus short), the authors (figure 9*b*) show an insignificant reduction as compared with a breaker operating in reasonable time from the series trip (Z versus $X = 0.002$ second). Experience with similar devices indicates that the instantaneous series trip will actually produce a *lower* contact-parting time (thereby taking less time to initiate the current-limiting action) than a shunt trip coil, even when there is no delay whatsoever in applying full voltage to the shunt trip coil. Lacking oscillographic data and admitting no experience with bus short circuits, in the installation described, it appears likely that the authors may subsequently find the actual current-limiting time may be *prolonged* (not *reduced*) by the proposed method of tripping where a severe bus short occurs.

In any case, the relative reduction of time appears small compared with the differences in operating times of the purportedly "similar" breakers. In figure 9*b* and in figure 11 the differences between X and Y are greater than the differences between Z and X. If the X and Y breakers were similar, then the operating time is quite inconsistent and do we have any real assurance that the "Z" breaker times were other than accidentally favorable tests? Again, actual oscillographic data would clarify this point.

T. M. Linville and B. H. Caldwell (General Electric Company, Schenectady, N. Y.): That the initial tripping impulse under fault conditions is faster when controlled by the voltage across the compensating field winding is very true. However, the reduction in the total time required to clear a d-c machine may not be significant. The authors are to be commended for their paper presenting an interesting study of protection of d-c machines. The results they derive from their analysis of short-circuit performance do not agree entirely with experience. Therefore, the following discussion is presented to give a more complete picture of the performance.

The primary cause of the discrepancy between test results and the results predicted by the theory (see equation 2 of the paper) presented by the authors is that the air gap flux and speed voltage generated in the armature winding do not remain constant during the transient as they have assumed, but is quickly reduced to a fraction of its open circuit value. For low values of armature current, equation 2 is essentially correct.

However, for higher values of line current this assumption is invalid for the following reasons: (1) The commutating pole becomes saturated. This means that the flux density in the commutating pole gap becomes less than that required for good commutation, and the effective center of current collection moves in the direction of rotation from its normal position, thus producing a component of armature reaction which is oppositely directed to the shunt field. (2) The high currents in the armature and pole face windings produce flux which saturates the armature and pole face teeth. This increases the reluctance of the main pole flux path, and decreases the inductance of the shunt field.

The combination of these two effects can greatly reduce the air gap flux without changing the linkage of the shunt field. In order to get an approximate idea of the magnitude of this change refer to the oscillogram of figure 1 of this discussion, and let the flux linkage of the shunt field be given by

$$\Psi = \Psi_g + \Psi_1$$

where Ψ_g is the linkage corresponding to the air gap flux, and Ψ_1 is the linkage corresponding to the interpole leakage flux. For this particular case the normal no load values are $\Psi_g = 1.0$, when $\Psi_1 = 0.2$. From figure 1 we note that the field current increases during short circuit to approximately 3 times its steady state value. Since saturation in the leakage path is negligible, it follows that Ψ_1 must increase to approximately 3 times its normal no load value; and if Ψ remains constant, Ψ_g and the air gap flux must decrease to 0.6 of its normal no load value.

The shift of the center of current collection also permits the commutating pole flux to produce a speed voltage opposing that produced by the main field. Also the decrease in the shunt field inductance permits a more rapid change of shunt field linkage than calculations based on the no load values of inductance would indicate. The field current from figure 1 decreases after the circuit is opened to approximately 0.90 of its value prior to the time of application of the short circuit. This is an indication of the magnitude of the change of the flux linkage of the shunt field.

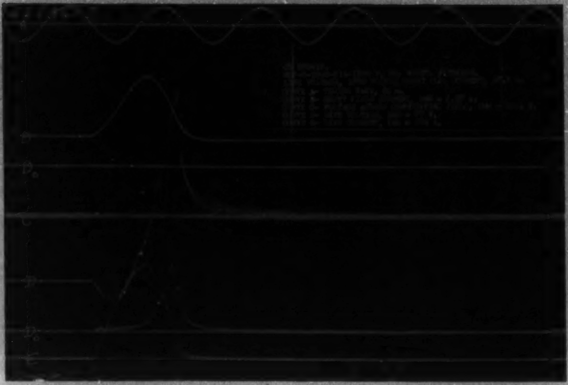


Figure 1

Curve *A*—Timing wave
Curve *B*—Shunt field current
Curve *C*—Voltage across commutating field
Curve *D*—Line voltage
Curve *E*—Line current

Shown in figure 2 of this discussion are the results of some short circuit tests taken on a 750-kw, 600-rpm, 1,500-volt, compensated, a-c generator designed for railway service (required to deliver 150 per cent load for two hours and 300 per cent load for five minutes, hence approximately equivalent to a normal rating of 1,000 kw). Tests were taken with two different breakers, one of which started interrupting the circuit in approximately 0.012 second and the other started interrupting the circuit in approximately 0.04 second. Several different values of external resistance were used as indicated. These curves show that increasing the resistance from 2.4 per cent to 9.2 per cent decreases the peak value of short circuit from approximately 17 to 10 times normal, while the peak value calculated as recommended by the authors varies from 41.7 to 10.9 times normal. This is convincing evidence that demagnetizing action in the machine is the principal factor limiting the current.

In figure 9*b* of the paper the initial rate of rise of current is given as 4,500,000 amperes per second. The formula for inductance is 25 per cent of rated volts divided by rated amperes divided by $\pi Pn/60$. Since the initial rate of rise must be rated volts divided by the inductance, 4,500,000 must equal the rated amperes (12,000) multiplied by $\pi Pn/60$ and divided by 25 per cent. This gives $Pn = 1{,}790$. If $P = 6$, $n = 300$. The speed is about as expected but from a standpoint of volts per bar and amperes per brush-holder bracket so few poles are not possible. The paper does not identify the examples given sufficiently. The formula for the inductance was given in a paper entitled "Speed Transients in Rolling Mill Motors" by L. A. Umansky and T. M. Linville, AIEE Transactions, volume 54, 1935 (April section), pages 387–94. The present paper states that for a compensated machine a percentage factor of 25 per cent is used. This is an interesting substantiation of the earlier presentation. However, the ratio between the inductance of a noncompensated and a compensated machine has been measured as more nearly three to one and sometimes four to one.

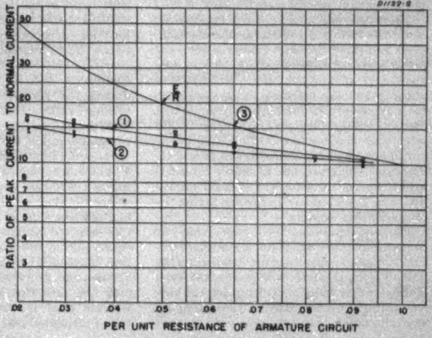
On very fast transients such as short circuit the effective inductance of the com-

Figure 2. Data from short-circuit tests on 6-pole 750-kw, 600-rpm, 1,500-volt d-c generator

① Peak current from test with breaker tripping time of approximately 0.04 second

② Peak current from test with breaker tripping time of approximately 0.012 second

③ Ratio of no-load voltage, *E*, and resistance of armature circuit, *R*



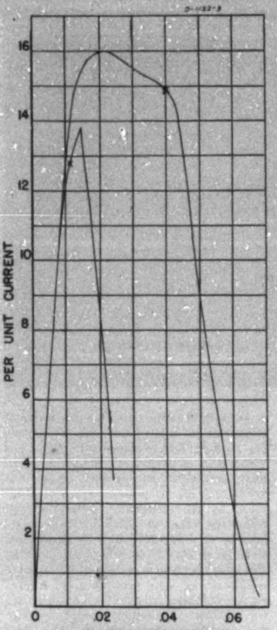


Figure 3. Current build-up in generator when external short circuit is applied

Rating: 6 poles, 750 kw, 600 rpm, 1,500 volts
Per cent resistance in armature circuit = 3.2
Breaker contacts started opening at points *X* on curves

pensating field circuit is appreciably reduced due to the solid frame of a d-c machine. This is not mentioned, but is a factor partly causing only about half the machine voltage to appear across the compensating winding as shown by tests. This would appreciably reduce below 100 the ratio of the voltage in the compensating winding on short-circuit to that on normal load transients.

There has been a great deal of speculation about the voltage across the arc during flashovers and short circuits. Observation of the forces existing during short circuit and knowledge of arc behavior indicates that possibly half the machine voltage exists on the average across the highly dynamic arc. The arc voltage changes chiefly with length instead of current and will be unstable as the arc elongates under magnetic forces and reflashes. Therefore, one flashover considerably less than three-fourths of rated voltage would be expected to be effective across the compensating winding.

The final conclusion of the paper, that bus construction may allow for lower short circuit stresses, is open to some dispute, as the peak current is nearly always reached even in the reduced time for interrupting the current claimed by the paper. That is, figure 9*b* of the paper does not agree with test experience since no machine builds up to 37 times rated current, as the curve shows, nor is the initial rate of rise as low as shown. The rate shown, 4,500,000 amperes per second, is equal to 375 times rated amperes in each second. As stated earlier, the initial rate is V/L which in amperes per second by the formula of the paper is $\pi Pn/15$ times rated amperes. For 12,000 amperes, 300 volts, and about 300 rpm, 16 poles might be expected. The expected rate of rise for such a machine would be 12,000,000 amperes per second instead of 4,500,000. The expected peak of 16 times normal current would be reached in 0.016 second which is too fast to be reduced by any of the interruption times given in figure 9*b*.

Figure 3 of this discussion shows the results from two oscillograms taken under the same test conditions except that in one case the breaker contacts started opening in 0.012 second and in 0.04 second in the other. The peak current is attained in 0.02 second or 1.2 cycles, and although the machine was shunt wound the current actually decreases after the peak is reached, and is still decreasing when the breaker starts opening at 0.04 second or 2.4 cycles. These tests indicate that in order to limit the peak current on short circuit, a breaker tripping time of less than 0.02 second is required, and that the reduction to 0.035 second cited by the authors is not sufficient to reduce the peak current in most cases.

The foregoing comments show the questions and uncertainty permitted by the paper which should be cleared up in the closing discussion by more complete data supported by test evidence.

J. F. Sellers: In regard to the discussion by Messrs. A. E. Anderson and C. H. Black, it seems that there may be a misunderstanding of the purpose of the proposed system. Perhaps it should have been made clearer in the paper, that this scheme cannot be expected to supplant the series instantaneous trip coil, which is still required for normal overload.

As shown from figure 5 of the paper, the breaker data were taken from oscillographic tests, conducted under maximum conditions of bus to bus fault which could be reasonably attained in the laboratory. This test was one of a number which were made with various amounts of inductance and resistance in series with the generator to investigate breaker operation, and it was found that up to five times normal current on the machine there was very good agreement between the calculated and actual current rise curves. This was the primary purpose of equation 2—to investigate the current values during the normal opening time of the breaker, rather than the ultimate values which would be reached upon failure of the protective means to clear the fault quickly.

Messrs. Linville and Caldwell have given a very fine explanation of the action of the leakage fields at very high current values, where the time involved is beyond the normal opening time of the breaker. The factors involved cannot be included in a simple equation, but most assuredly affect the actual shape of the curve. However, our investigations, as mentioned before need cover only the first part of the build-up curve wherein there is very good agreement between calculated and actual curves. We are very glad to find agreement with our method of figuring machine inductance as there has been a great deal of work done in calculating component parts of the inductance, and a short cut or empirical formula is necessary to reduce the labor involved when fitting several machine designs to a particular application.

The most decided advantage of the interpole trip is certainly on flashovers, which type of fault covers all of the heavy faults to date on the systems where it has been used, and offers advantage less in magnitude of time but nevertheless real on bus to bus fault.

Development in Lightning Protection of Stations

Discussion and author's closure of paper 41-102 by E. R. Whitehead, presented at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941, and published in AIEE TRANSACTIONS, 1941 (September section), pages 898–900.

M. C. Westrate (Commonwealth and Southern Corporation, Jackson, Mich.): This is a very interesting paper and presents the results of an analytical approach to a problem complicated by many variables. The degree to which tests substantiate calculations is very convincing but a station design engineer is very reluctant to specify choke coils in view of the many objections, structural as well as electrical, which have been experienced in the past. Also the number of transformers which fail when protected by modern arresters installed at or near their terminals are so few that the complication of station design by the addition of choke coils is questioned. Possibly the same results can be accomplished by installing gaps a number of circuit feet, perhaps one span, out on each line so that circuit impedance will do on steep waves what the choke coil did in Mr. Whitehead's experiments. Overhead shield wires could then be installed for this section of line and over the station to prevent direct strokes between the gaps and the lightning arresters.

E. R. Whitehead: I share Mr. Westrate's reluctance to specify choke coils in any normal lightning protection scheme applicable to stations. In fact, the Duquesne Light Company has removed the old choke coils in its protection improvement program over a period of eight years. It is evident, however, that many small stations fed from highly insulated wood pole transmission lines will not economically justify the installation of adequate ground wire shielding plus the necessary low impedance grounding along the line. Where the load and importance of the station do justify such an installation, I quite agree that such a system is desirable.

The suggestion that gaps or tubes installed one or more spans out from the station has frequently been considered, but it has the defect that the ground path impedance at the gap may be so high as to nullify the benefit obtained.

The reactor scheme may be thought of as

a means of contracting the physical dimensions of the span to the gap so that advantage may be taken of the closely interconnected and grounded structure of the station. This location eliminates the effect of ground resistance, and greatly reduces the inductance of the ground connection between the gap and transformer tank. In conclusion I should like to emphasize that the application of this protection system is by no means general, and care must be taken that other insulation elements electrically adjacent to the gap or protector tube must be adequate for the short high voltage peaks which will be experienced. In other words, the back-up gap or tube must be reliably coordinated with adjacent insulation.

The 220,000-Volt System of the Hydro-Electric Power Commission of Ontario—II

Discussion and authors' closure of paper 41-95 by A. H. Frampton and E. M. Wood, presented at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941, and published in AIEE TRANSACTIONS, 1941, pages 1215–21.

W. W. Parker (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): This paper presents very valuable design data and results of operating experience. The lightning record of table I is of special interest. This shows a considerably greater number of the more severe types of faults than is the average experience, as follows:

Faults Involving	Per Cent
One wire to ground	65
Two wires to ground	20
Wire to wire	10
Three wires	5

This higher percentage of faults involving more than one wire described in this paper seems at least in part to be due to the high tower footing resistance at the locations where the majority of faults occurred. It will be very interesting to have a further report on the effect of the counterpoise on the percentage of the various types of faults.

The use of two wire to ground faults as a stability criterion in establishing the power limit of a system by calculator methods is a very common practice. This is not only due to the fact that on the average only five per cent of the total number of faults involve three phases, but also because on the calculator the fault is usually assumed to be at the worst possible location, under the heaviest loading condition and without arc, contact, or ground resistance. These considerations, added to the fact that damping and saliency effects of hydro machines are generally neglected, seem to indicate that two wire to ground faults are a sufficiently conservative type of fault to use as a stability criterion in calculator studies of power limits.

M. S. Oldacre (Commonwealth Edison Company, Chicago, Ill.): This paper adds a large amount of information to the existing knowledge of operating results with 220-kv lines so that even better results can be obtained in the future. The engineering organization in charge of the system described in the paper has been very generous in making its data available to others.

The first conclusion of the authors seems to be of the greatest importance in designing future lines, namely, operating experience largely confirms theoretical and experimental conclusions regarding protection from lightning, from sleet and wind, and from vibration.

An unfortunate operating condition was found in the high earth resistivity in the territory traversed by these lines. In spite of this, a reasonably good operating record has been obtained and the recent improvements in tower grounds should appreciably reduce lightning outages in the future. The authors' figure of 0.2 outage per 100 circuit miles per year for towers having footing resistances less than 25 ohms seems somewhat better than would be anticipated from a rough consideration of the original line design. It is conjectured that this may be at least partly due to the favorable impulse characteristics of grounds, as reported by Bellaschi [AIEE TRANSACTIONS, volume 60, 1941 (March section), pages 123–8], in comparison with the values of resistance measured by usual d-c methods.

Approximate calculations furnish an indication of the effectiveness of shielding by overhead ground wires. The original single-circuit towers provided shielding angles of 42 to 45 degrees. If such shielding had not proved effective, and a large proportion of strokes had contacted phase wires, the outage rate should have approached roughly 75 outages per 100 circuit miles per year, since lightning stroke currents of only about 10,000 amperes would have been required for flashover. The actual outage rate of these lines, however, has been only slightly more than one per 100 miles per year. The good correlation between footing resistance and outage rate and the relative predominance of faults involving only one phase give further indications that the majority of line contacts has been to the ground wires, showing that the shielding has been effective. It would be of interest to learn from the authors whether this has been confirmed by more numerous findings of burned spots on the ground wires than on the conductors.

The requirements for lightning protection have been adequate clearance and insulation from power conductors to ground, sufficiently small shielding angle by the ground wire, and finally low tower footing resistance. Possibly low tower footing resistance is of more importance than a small shielding angle. Some data in the paper point to this conclusion. Another way of stating this would be that if the ground wire is located to avoid trouble from sleet and wind as determined by the Lissajous figures, then it makes little difference what the shielding angle may be if the ground resistance is low. Extra money had probably best be spent in lowering of the tower resistance rather than in lowering the shielding angle.

The use of half-loop galloping assumptions is interesting and may be justified by experience and as a compromise for the conditions applying to the 1941 line built by the Commission.

When the 220-kv Powerton-Crawford line was designed in 1939, it was considered desirable to use full-loop assumptions for power conductors and ground cable, as it did not affect the cost appreciably. This line has been in operation only eight and one-half months but an observation during one sleet storm may be of interest. The station operator at the Chicago end of the line observed the ground cables to be galloping violently, his report stating that the cable was being lifted 10 feet or more, while the power cable was not moving at all. This is only one report, but it helps to justify the opinion of the authors that possibly the large conductors will not gallop appreciably and that half-loops may be an adequate assumption.

The storm in the Chicago area mentioned above was sufficient to take out 132-kv lines of early design which had inadequate clearances under the full-loop assumption.

Some experience on a line near Chicago with 500-foot spans of stranded, 750,000-circular-mil hollow copper conductor indicates that galloping sufficient to cause sleet outages has been almost of full-loop magnitude.

The vibration protection for the ACSR cable is very interesting. It seems that armor rods would be desirable, not particularly for vibration dampening but for protection from conductor damage from chance flashover and from any residual vibration. Then damping devices should be used for suppression of vibration. The actual experience will be of value.

A. S. Runciman (Shawinigan Water and Power Company, Montreal, Quebec, Can.): Information contained in the paper in reference to lightning again indicates that the high resistance ground sections are causing the most trouble. The conclusion is that lowering the ground resistance should effect improvement in operation through lightning storms. The authors state that 90 per cent of the located lightning outages occurred in the section of high tower footing resistance. Just what is meant by high tower footing resistance is not clearly defined by power men as yet. The term is very broad. In our company we consider low resistance to be one ohm or less and anything above ten or twenty ohms is high. These terms are not anything which can be completely agreed upon, but we do know that we can obtain resistances as low as a fraction of an ohm. One case in mind is that of a tower whose footings were measured to have the highest resistance on a new line constructed last year. The resistance measured 780 ohms for grillage bases alone, but after connecting in the usual two counterpoise wires and knowing that this tower had a greater resistance than 50 ohms, an extra two counterpoise wires were added and, in addition, the two sky wires were connected in. With this complete combination of grounding we then obtained one ohm for this tower. Every tower on this line which measured originally, with grillage footings alone, more than 50 ohms was joined up to its adjacent towers by an extra pair of counterpoise wires, making four in all. We expect good results from this line.

It is not easy to find out what the footing resistance of a tower on an old line actually is but we form an opinion by driving a deep driven ground rod as much as 150 feet down at times, measuring at each eight foot extension, and if the resistance at 40 feet or 50 feet is 50 ohms to this rod, we know the tower is in high resistance territory and we must either improve our grounding or expect lightning outages.

Two counterpoise wires in soils, which measure 50–100 ohms with a 50 foot rod, are not enough to prevent lightning outages on a seven-unit insulated line. On a 14-unit insulated line we believe less grounding is necessary than on a seven-unit insulated line. In other words, we have found that the operating voltage on a transmission line is not an important factor in comparing operation of transmission lines through lightning. We have a number of lines constructed with seven suspension units operating at 60 kv and 110 kv and no definite difference can be determined due to operating voltage.

It is interesting to note that the designers of the 220 kv, two-circuit line have decided to use a single high overhead ground wire. This presumably came about after studying the Pennsylvania Water and Power Company's experience, as reported upon by Mr. Waldorf. I believe this is a step in the right direction. It certainly removes mechanical hazards during sleet storms.

There is still the question, and I expect that I shall be criticized for saying this, whether the single overhead ground wire is required or not. This statement is based on information which is slowly being produced, that the leader path is established from the tower top. Whether or not lightning strikes a certain spot depends upon the resistance of the path before the strike; and whether or not damage is caused seems to depend upon the time which is required to discharge the cloud. If this discharge takes place quickly through a low resistance ground, the damage is a minimum.

The answer to the whole question is better grounding. The question of shielding does not seem to be so important as has been supposed in recent years. If the Hydro-Electric Power Company of Ontario should experience trouble from lightning in the section of the new double-circuit line where they have no counterpoise, it is to be hoped that they will not blame this on the use of the single overhead ground wire, but rather on the footing resistance not being low enough.

E. W. Knapp (Shawinigan Water and Power Company, Montreal, Quebec, Can.): The authors are to be congratulated on the concise, clear, and effective manner of presenting the various data contained in this paper. There are, however, a number of features where further discussion might be undertaken to advantage.

1. The completed scheme includes two 220-kv closed rings consisting of seven circuit breakers per ring, so arranged that the two rings can be interconnected. Is it the intention to operate with all the 220-kv circuits interconnected, and, if so, will it be possible to manually remove both of the tie banks nearest the loop interconnections without opening the loop?

2. The data contained in figure 4 were particularly impressive, with regard to the relation of high resistance grounds to lightning flashover. The policy of providing moderately high overhead ground wires and adjusting the amount of lightning protection to coincide with local tower footing resistances and nature of terrain appears to be a good one. However, it may be found that the excessively high overhead ground wire may be more effective in reducing troubles due to sleet, wind, and snow than in reducing lightning outages. Moreover, in extremely high resistance soil even two continuous buried counterpoise wires in association with overhead ground wires, may only reduce lightning outages to possibly 30 per cent to 50 per cent of the former value. Adequate lightning protection also appears to result in less severe faults by decreasing the percentage of LLg and $LLLg$ faults in relation to Lg faults, thereby improving the transient stability situation.

3. It is noted that the four 220-kv circuits are expected to deliver 675,000 kw in the future. This appears to be rather a high circuit loading and the loss of one or more circuits might create a stability problem. Has automatic reclosing been considered for these circuits? The Shawinigan Water and Power Company installed a form of automatic reclosing on two double circuit, 187-kv, 60-cycle lines in 1934. Although the overall reclosing time is in excess of one second, a considerable number of successful reclosures have taken place during single line faults for loads as high as 175,000 kw. These four 220-kv circuits would appear to be well suited to instantaneous automatic reclosing, if this offers any advantage.

4. The authors mention one case of permanent trouble where it required 27 hours to locate and repair the fault. It would be of interest to know how much of this time was spent in locating the point of damage. There is on the Canadian market today automatic fault locating equipment for transmission lines. The Shawinigan Water and Power Company have had some of this type of equipment in service since 1937. During the intervening period a considerable number of records have been obtained on several different types of systems and the experience to date has been quite encouraging. The indications are that line patrols may be reduced, a greater percentage of points of transient flashover may be located and the point of permanent breakdown may be located more quickly through the use of an adequate form of fault locating equipment. I would very much like to know the reaction of the authors towards the use of this type of equipment on long or moderately long transmission lines.

Edwin Hansson (Pennsylvania Water and Power Company, Baltimore, Md.): This paper furnishes additional proof of the influence of tower footing resistances on the lightning performance of a transmission line. In tables IIa and b the lightning outages on the system have been segregated according to tower footing resistances. Plotting these outages in percentage against footing resistance gives a curve of substantially the same shape as that accepted for lightning current distribution. This is as it should be. The design of the lines is such, that given reasonably low footing resistances they should be practically lightning proof, and the distribution of the flashovers along the line should be in proportion to the footing resistances.

The authors state that little or no benefit was derived from crow foot counterpoises in the rocky sections of the lines. Luckily, they have not been discouraged by this experience, as evidenced by the decision to lay a continuous counterpoise along the new line. The records from this line should go a long way towards settling the controversy of continuous versus crow foot counterpoise in a high resistance territory.

In connection with clearances, the diagrams are in some cases based on full loops—in others on half-loops. I wonder if the authors have any data in favor of one or the other. If so, where lies the line of demarcation expressed as the ratio of conductor diameter to weight per foot, or some such other figure?

C. Gliddon (Gatineau Power Company, Ottawa, Ontario, Can.): This paper gives a large amount of valuable information regarding an extensive 220-kv system most of which information may be applied to 220-kv systems in general.

The experience of Gatineau Power Company which operates 50 circuit miles of 220-kv line and 300,000 horsepower of generating capacity in this system is about the same as that outlined in the paper.

It is interesting to note that the ring-bus system of 220-kv switching originally adopted in 1928 is being used with some modification in the Burlington station.

New features such as carrier relaying, continuous counterpoise, and higher double circuit towers with one overhead ground wire instead of two will be watched with interest.

In 1928 when the original 220-kv system was placed in service there were some misgivings as to whether this comparatively untried high voltage of 220 kv would operate satisfactorily and some consideration was given to the possibility of providing for reconnecting the transformers to operate at a lower voltage in case the higher voltage did not work. However, these fears were not realized and to-day after thirteen years of operation the system has given less trouble if anything than the lower voltage systems.

L. F. Kennedy (General Electric Company, Schenectady, N. Y.): Messrs. Frampton and Wood have presented two items of considerable interest to protective relay engineers. The first of these concerns the use of distance type relays for protection against ground faults. While the theoretical principles which govern such applications have been known for considerable time, there has been only limited actual use of such relay systems and there is a great lack of operating data. As a result many operating engineers question whether or not ground distance relaying is generally practical from an economic standpoint. The general statement in the paper indicates that entirely satisfactory performance has been obtained with this type of protection in relatively simple form. It would be very helpful to other relay engineers if more details were available both as to the methods used and the operating experience. This would seem to indicate the desirability of a future paper on this phase alone.

The second point of interest to the relay engineer concerns the proposed use of carrier current as a "transfer trip" means.

When carrier current for relaying was first proposed by Fitzgerald in 1928 one of the requirements set up was that the carrier signal should not be used to cause tripping. The blocking type of carrier relay system was developed to meet this requirement and there are now several hundred such equipments in service with successful operating records. From the experience gained from these applications the reliability of a carrier channel now is considered to be very high. Despite this proven high reliability of the blocking channel, carrier would not be considered applicable for transmitting a tripping impulse over a faulty line except when a failure of the channel would be backed up by other means of protection. It must be remembered that the reliability of the blocking system is to a large degree due to the fact that the carrier signal is sent over only sound line sections.

Among the first applications of a "transfer trip" carrier system was to lines terminating in a transformer bank with only a low tension circuit breaker. Faults in the transformer or between the transformer and the low side breaker might not cause relay operation at the remote end of the circuit. Transformer differential relays were applied and used to control the transmission of carrier current to cause tripping of the remote end. This type of an application could be termed an automatic supervisory control system for tripping only.

Where such applications have been made, it has been assumed that the transformer windings offered sufficient impedance at carrier frequencies to prevent short circuiting the carrier when the transfer trip operation was required. If a greater degree of reliability was required it could be obtained by installing a carrier line trap between the transformer and the coupling point to the carrier equipment.

Another type of problem involving relay sensitivity is found on long, heavily loaded transmission circuits. Such circuits are frequently very difficult to relay because there is so little difference even to distance type relays between load and fault conditions. The application of the blocking type of carrier to such circuits is possible but may result in continuous carrier transmission under full load conditions. Also on such circuits, with grounded transformer banks at both ends, there may be very little evidence at one terminal of a single phase-to-ground fault at the other end of line. For such applications the transfer trip offers a means whereby nearly simultaneous tripping of both ends of the transmission circuit can be obtained through the operation of the protective relays at one end only. As long as the carrier channel is functioning this desirable feature is obtained, but in the case of a failure of the carrier tripping signal to get through, the relay system may still clear the fault with sequential or time delay tripping, if it is possible to set the second zone of the distance relays so that they do not operate for load currents.

Further comment by the authors giving more completely their reasons for the choice of the "transfer trip" method for some of the lines and the "blocking" method for others would be very interesting.

Another point of interest in connection with the transfer trip feature is the use of a 400-watt transmitter. The usual blocking systems have been installed with transmitters having output levels below 30 watts, and in the case of recent installations, operating directly from the usual 125-volt station battery, having a nominal 15-watt rating. Presumably one of the reasons for the choice of this materially higher output is to increase the reliability of the carrier channel. In the case of a three phase fault which would probably be all three conductors to ground, the probability would seem to indicate that the "transfer trip" signal would be short circuited by the fault and sequential tripping result. Since on any system the need for simultaneous tripping will be greatest in the case of a three phase fault then the reliability of the tripping channel should be greatest under such fault conditions. It will be of interest to note the operation obtained with this system but the probable infrequent occurrence of faults on these circuits will make it necessary to wait a long time for conclusive results unless specific staged tests should be made to demonstrate the reliability of transmission, particularly with faults on the coupling phase.

A. H. Frampton and E. M. Wood: In presenting the operating experience data contained in this paper, it was recognized that, in some particulars inconclusive evidence was being offered to substantiate the revised designs of the single-circuit and the new design of the double-circuit sections of the 220-kv transmission system. Data offered regarding the relation of footing resistances and outages are in this category, but unfortunately it has not been possible to collect the considerable volume of information regarding lightning current values, etc. that would be necessary to effect a more valuable coordination.

It has appeared, as Mr. Oldacre agrees, that the available data largely confirm the theoretical and experimental conclusions regarding protection from lightning and we are gratified to note that Mr. Hansson's approximate calculations indicate further agreement.

As to the relative effectiveness of shielding and of low footing resistances only the following comments are offered. It may apparently be anticipated that a footing resistance of the order of 25 ohms, with the degree of shielding provided, will result in good lightning performance. Whether better performance could be anticipated with such low footing resistances as Mr. Runciman mentions is open to question. Incidentally the authors are not sure what value of resistance Mr. Runciman is concerned with when he mentions a resistance measured with all counterpoise and ground cables attached to the tower.

In another line of the Commission, operating on steel towers at 110 kv, with horizontal configuration of power conductors and with a single ground wire arranged to be located some 25 feet above the centre phase conductor the following experience has been obtained. During approximately ten weeks of lightning season during which this construction was operated without the ground cable 19 outages due to lightning were experienced. In 13 weeks of the current lightning season, with apparently greater storm severity, but with the ground cable in place, two lightning outages have taken place. All but a few isolated structures on this line have footing resistances below five ohms, which would seem to indicate that strokes to the towers should not cause flashover. The obvious interpretation is that, without shielding, a proportion of the strokes will occur to the power conductors at mid span which will result in flashovers no matter how low the footing resistance.

The data offered regarding the sleet design of these lines are also somewhat inconclusive. It is considered that outages occasioned by accidental contact of the various conductors under sleet and wind conditions are much more likely to result in the transmission circuit becoming disabled than flashover occasioned by lightning. Adequate clearances both horizontally and vertically, are therefore essential. The extent of such clearances is not susceptible of simple mathematical treatment and the method of Lissajous figures is offered as a possible solution. Available data are still insufficient to permit any exact specification, as to the size and shape of the various figures, under any given conditions of span length or conductor size. The paper suggests that conductors as heavy as 795,000 circular-mil, ACSR at spans of 1,150 feet will not "gallop" in one single loop between the points of support. This suggestion is based merely on the fact that galloping has not been observed in similar construction and it is upon this fact that the use of half-loops in such design has been considered a suitable compromise. It may be assumed that Mr. Oldacre offers something of a confirmation when he notes that galloping of the lighter ground wire but not of the heavy power conductor has been observed on the Powerton-Crawford line, though galloping in a full loop has been observed on a heavy conductor at much shorter spans.

The designing of a line in such manner as to provide adequate vertical clearances between the phase conductors and the ground cable or cables appears to have the further merit that a suitable design on such basis will unquestionably result in a line with reasonably good shielding angles which, with good footing resistances, should result in satisfactory lightning, as well as mechanical performance.

Referring to the comments by Mr. Kennedy on the use of impedance relays for ground faults, good success has resulted from using the simple connection outlined in the paper, both on this system and on the 110-kv system network to which it is interconnected. On this latter system, which has resistance grounded neutrals, they are particularly useful. In all cases they have the advantage of low burden, and their functioning, particularly on first range, is less dependent on the amount of system generation than their alternative, the instantaneous residual current relay. However it is agreed that their use is not always warranted.

It is agreed in general that Mr. Kennedy's comments regarding the applications of carrier for relaying are pertinent. The application of transfer trip to the relaying on the long lines is frankly experimental but it is expected that by the use of the high power the trip impulse can be gotten through an arcing ground, even if involving three phases. It is at least expected that interesting data will be obtained by means of tests and from operation. The "blocking" type was used on the Leaside-Burlington line because it is a well-developed type which it is believed will be reliable on this short line.

Replying to Mr. Knapp's question re 220-

kv diagram, the rings can be operated separately or closed together as will probably be more usual. To remove a bank on the tie position it will be isolated by opening the ring breakers, then the bank disconnecting switches will be opened and the rings quickly reclosed.

The experience by the Shawinigan company on reclosing 187-kv lines is interesting. Our experience of manual reclosing (in as little as 20 seconds) has met the requirements to date, but further experience may indicate the desirability of making this automatic.

There is no question that devices to enable the location of a permanent line fault are very useful, whether these are in the form of automatic oscillographs or the simpler device to which Mr. Knapp has referred.

Conductor Vibration—Theory of Torsional Dampers

Discussion and author's closure of paper 41-94 by J. W. Speight, presented at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941, and published in AIEE TRANSACTIONS, 1941 (October section), pages 907–11.

R. G. Sturm (Aluminum Research Laboratories, New Kensington, Pa.): The theoretical analysis of the torsion damper as presented by the author is very interesting and concise. In general, the theoretical attack appears to be sound in so far as the author has gone. While the results of the analysis undoubtedly were of much value in a qualitative way, the analysis would have been of much more value if the author had continued it to a point where its application to a specific damper would be independent of measurements made on actual installations of that damper on the particular line considered. More explicitly stated, if the use of any theory is restricted to the interpretation of measurements it is of less value than if it could be used to predict behavior in advance. Consequently, there are a few questions which should be discussed more fully than appears in the original presentation of the paper but which are needed to establish the validity and to clarify the use of theoretical work thus far done.

With reference to the author's equation 2 it should be pointed out that this relationship refers only to the condition where the displacement of the cable is zero. If the displacement as given by equation 1 is not zero, then equation 7 automatically follows directly from the same consideration as equation 2 and does not need the reference to the Lagrangian function which apparently was not used.

In the introduction of the term, q, which depends upon the logarithmic decrement, δ, it should be pointed out that this is a measured quantity obtained from a given damper attached to an actual cable in service. The logarithmic decrement is observed and the quantity, q, is computed from this value. Consequently, the power dissipation indicated by equation 9 and subsequent equations is dependent upon observations made on a damper installed on the particular cable on which it is to give service, unless δ and τ are found to be constant values. Results from many tests on ACSR indicate that both δ and τ are not constant but will vary with both the amplitude, A, and angle of twist, θ.

The author gives simplified examples as to how the theoretical deductions may be correlated with actual observations but fails to point out specifically what observations are made and which values are computed. Presumably, one would make observations of logarithmic decrement and amplitude for a torsion damper installed on a conductor; then, using these observations compute the power dissipated. How the author would use this power dissipation value, P, is not indicated. It might be used as a basis for comparing the effectiveness of different designs of torsion dampers. It would be of still greater use to couple the rate of energy or power dissipation with the energy input of the wind into the conductor in such a way that the expected or predicted amplitudes might be obtained by computation and compared with the amplitudes found by actual observations as was done in an analysis of the Stockbridge damper.* The principal difficulty encountered is that the quantities given in equations 9 and 14 require that observations be made on a damper attached to a conductor subjected to actual service conditions under which a similar undamped conductor would vibrate. By sufficient study of prevailing tendencies of these dampers on different conductors under various meteorological conditions, including temperature and wind velocities, it may be possible to obtain functional relationships between the quantity, q, and the amplitude, A, such that solutions may be obtained which would be reliable for predicting amplitudes of vibration on conductors damped with torsional dampers. Such final solutions would be required in order to reliably conclude that any one set of conditions of damper proportions, location, and weight would be more satisfactory than some other set of conditions.

W. B. Buchanan (Hydro-Electric Power Commission Laboratories, Toronto, Ontario, Can.): It is possible that Speight's analysis of the so-called "torsional damper" might be clarified to some readers by a few additional remarks. A very limited amount of information has been made public on the principles upon which such dampers can be made to operate. The most direct, and possibly comprehensive, treatment thus far is incorporated in United States patent 2,215,541 and Canadian patent 377,602. Certain comments by Speight might be construed as setting certain limitations on their operation, whereas, the writer's view is that these limitations relate to particular designs and proportions that have been under test rather than to the principles involved.

During a rather extensive investigation by the writer on materials that might be used economically as festoons, it was found difficult to improve on the form of construction of the standard stranded conductor. The reversed spiralling of alternate layers proved better than any other scheme of binding, weaving, or otherwise bunching of strands that were devised. A moderate pressure between strands seemed desirable just such as occurs under normal stringing tension, yet, for best operation, festoons should be draped rather loosely. Hence, ways and means were sought to make use of the practically unlimited energy-absorbing medium existing in the cable itself by a suitably-controlled distribution of loss.

Normally, the motion which gives most concern is vibration in a vertical plane, and it was found possible to restrain such motion very effectively by the application of weights supported laterally from the axis of the conductor and alternating at short distances on opposite sides of the conductor.

Initial tests were made by a travelling wave-method and proportions adopted as a result of the data obtained, then practical field installations were made. Two restrictions were found to exist, a rigid mass of large magnitude would not be the ideal arrangement because it would constitute a hard spot on the cable and cause local concentration of stress in the conductors; and further an unnecessarily large amount of vibration might develop before the frictional effects noted would come into play. The latter limitation is obviously due to the elastic properties of the materials and would vary quite considerably with types of conductors and damper design. Development work has been based largely on the use of ACSR conductors of the larger sizes.

Hence it appeared advisable to limit the rigid mass of the damper to the clamping arm and have this arm support, at its outer end, a mass which would offer restraint to various motions of the cable. The most satisfactory material found so far to act as a resilient cushion between the arm and supported weight is a special grade of rubber. It is effective at low amplitudes of motion and has substantial damping characteristics, particularly so in shear.

Attention has been directed to the torsional action of these dampers on a cable but there are two other torsional motions involved in this design which improve the performance. Vertical motion of the conductor as at an antinode tends to cause relative rotation between the arm and supported weight about an axis parallel to the axis of the cable and in present designs subject the rubber pads to shearing stress with substantial damping effect. At a node point another rotary oscillation is set up about an axis through the arm at right angles to the conductor. The polar moment of inertia of the supported mass about this axis then acts as a buffer through the medium of the rubber in compression and this component can be designed to effect quite substantial damping even at node points.

Even a casual review of Speight's paper will serve to indicate the difficulty of extracting a practical solution by mathematical process alone. The nature of the losses are not all known, the coefficients representing losses of known type are not constants in the equations in which they appear. Coefficients indicating loss due to friction might be assumed constant while active, but discontinuous; other coefficients are continuous but vary in different degree according to the stress in the materials. Practically, however, it is possible to observe these reactions in dampers while in motion under forced vibration and to deduce some useful information as to the direction to follow in proportioning the units.

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* Reference number 7 of the paper.

The theory of operation has been verified by a large number of operating tests, one of which in particular the writer would like to mention briefly.

A short span (140 feet in length) of 605,000 centimeters ACSR conductor was equipped with a pair of dampers located as indicated in figure 4 of Speight's paper. Forced vibration was maintained up to approximately fifty million cycles without any apparent deterioration of the dampers. At the conclusion of the test the cable was examined carefully and evidence of wear was observed at the cross-over contacts between strands of adjacent layers, throughout the length of section between the dampers. The electrical and physical properties of the conductor were unimpaired and apparently the original conclusion that the conductor itself could be used quite freely as an energy-absorbing medium was quite warranted.

J. W. Speight: Mr. Sturm's criticism of the lack of the detail is justifiable, but the scope of the paper necessitated a concise treatment. Equation 7 was derived by using Lagrange's equation and, as pointed out, does reduce to equation 2 for zero displacement of the conductor.

At first, the analysis was applied qualitatively in order to obtain a suitable damper for a specific conductor. With the accumulation of experimental data on the response of different types of conductors (see paper 41-97 by Mr. G. B. Tebo on "Measurement and Control of Conductor Vibration," AIEE TRANSACTIONS, volume 60, 1941, pages 1188–93), it was found that the formulas could be used in a quantitative manner for the design of dampers. It would have been, of course, much more desirable if the initial results could have been obtained entirely by calculation from laboratory test data.

The logarithmic decrement and angular elastic constant, as found experimentally on a short span of conductor, were approximately constant with respect to a limited range of frequencies and amplitude. However, I agree that these quantities are not generally constant but functions of the frequency and angular deflection. It seems that there is a great deal of work yet to be done on the behavior of physical constants under dynamic conditions.

Mr. Buchanan's remark that the development work on torsional dampers was confined to large size aluminum conductors steel reinforced is quite true. Since the paper was written, effective dampers have been designed for $^3/_8$ inch steel ground cable and 500,000 cir-mil H-H copper conductor. Eventually, dampers will be placed on stranded copper conductors which require protection from aeolian vibrations.

The theory of the damper with a resilient joint does consider the rotational motion of the resilient material about the axis parallel to the axis of the conductor, but neglects the rotational motion about the axis perpendicular to the conductor because it is of a secondary order.

The statement in the summary of the paper that, for a rigid damper, the distance from the center of the conductor to the center of gravity (1) should be equal to the radius of gyration about the center of gravity (k) is not very clear. This deduction follows from the assumption of a predetermined value of k. From a practical standpoint, the radius of gyration about the center of the conductor would be maintained constant for a given mass and the conclusion can be written more significantly that 1 should be greater than k. In fact, the theoretical energy dissipation for the latter hypothesis varies from $k=0$ to $k=1$ in the ratio of two to one.

Lightning to the Empire State Building

Discussion and author's closure of paper 41-115 by K. B. McEachron, presented at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941, and published in AIEE TRANSACTIONS, 1941 (September section), pages 885–90.

Edward Beck (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): 1. Mr. McEachron has mentioned in his paper dealing with lightning to the Empire State Building in New York, that "There is good reason to believe that ground resistance will have an appreciable effect on the currents in a lightning stroke." This is a very intriguing subject. The measurements and the study of arrester performance over a large area of the United States, discussed in the Wagner, McCann, and Beck paper lead to similar inferences. It is the purpose of this discussion to elaborate on these briefly.

2. In speaking of the possible effect of earth resistance on the currents in lightning strokes, it is not intended to infer that the earth's structure may influence the formation of the stroke or the frequency of lightning strokes or thunder storms, the discussion pertains only to the currents that flow after the formation of a stroke. Some speculations have been published by several Europeans on the possible influence of geological structure on the formation and frequency of lightning strokes. It is doubtful that such a relation exists, whereas observations now available do give some indication that there is a relation between the structure of the earth and the currents that flow after the stroke channel has been established.

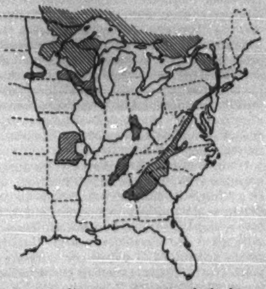


Figure 1. Known exposures of Archaean rocks (shaded areas)

3. An Institute paper by R. H. Card on "Earth Resistivity and Geological Structure" shows that earth resistivities vary between wide limits, from less than 10 to as much as 10,000 ohms per meter cubed. There appears to be a relation between earth resistivity and geological structure; in regions of great geological age, where the most ancient or so-called Archaean rock formations lie at or near the surface, high resistivities appear to prevail. In figure 1, the shaded areas indicate such regions. The data for this figure have been taken from "An Introduction to Geology" by W. B. Scott.

4. As reported in table V of the Wagner, McCann and Beck paper, with 40,000 arrester years of service on a new type of distribution arrester up to the first of 1941, there were 28 cases of arrester damage from all causes with 18 caused by lightning. The arresters whose performance was observed were distributed widely over the area shown in figure 1 of this discussion, but the failures were grouped in an irregular manner. All but two of them occurred in very limited areas. It is remarkable that these particular areas lie in the Archaean region. In one region involving three circuits where arrester damage occurred, Card's data indicate resistivities of 2,000 ohms per meter cubed or more. They all occurred on delta systems, but these are by no means the only delta systems on which this type of arrester was in service. Many more arresters than the total involved in the area in question were installed on delta systems in the non-Archean regions with negligible failures, only two in number. Moreover, there are other systems in the vicinity of the area under discussion on which these arresters were in service and on which none were damaged. However, those systems are either four wire or they are delta in areas that are not Archaean or high in resistivity.

5. In a paper presented several years ago by Messrs. McEachron and McMorris on "Discharge Currents Through Distribution Arresters," dealing with measurements of crest discharge currents through arresters obtained by means of surge crest ammeter links, they reported the highest arrester failure rates in Georgia where, as figure 1 shows, there is a large area of ancient geology in which Card's paper reports resistivities of from 6,700 to 10,000 meter ohms. They report few cases of arrester damage in other regions such as Michigan, which is of comparative geological youth and in which Card reports resistivities of only 33 to 74 meter ohms. Although the authors, in that paper, drew no such conclusions, their data corroborate the indications published in the paper by Wagner and his coauthors.

6. One is led to suspect that the correlation between observed arrester damage and resistivity or geology indicates a correlation between these attributes of the earth and the nature of stroke currents. While so far the data may not be entirely conclusive, it would seem that lightning currents of long duration are more prevalent in regions of ancient geology and high resistivity. The measurements reported in the Wagner, McCann and Beck paper also bear this out, since records of long duration

arrester discharges were obtained only on delta systems in such regions, whereas records of arrester discharge currents on a delta system in another region were all of relatively short duration.

7. It is recognized that other factors, too, no doubt play a large and perhaps controlling part in determining the character of the stroke, particularly in its formation, yet the analysis of the data along the lines discussed is too striking to be overlooked. After all, the earth is one of the two poles of the lightning stroke and apparently this seemingly inoffensive partner of the clouds merits more attention than has been given it in the consideration of the factors that may affect the currents in lightning strokes and those discharged by lightning arresters.

Paul M. Ross (Ohio Brass Company, Barberton, Ohio): In testing transmission line insulation in the laboratory it is desirable to know expected direct stroke rates of voltage rise. What rates of voltage rise may be expected from direct lightning strokes?

W. J. Rudge (General Electric Company, Pittsfield, Mass.): The lightning stroke data presented in Dr. McEachron's paper are fundamental to the protection engineer. For the first time we have statistical data on such items as wave fronts, and wave tails correlated with crest currents, rates of rise, frequency of occurrence, and stroke energy.

Dr. McEachron points out that the Empire State Building measurements give the total stroke currents and energy, and that when a stroke contacts either a distribution or transmission line the current and energy divide, hence only a portion of the current goes through the protective device.

With this in mind it is of considerable interest to compare the data on the actual stroke with the test waves which have been incorporated into the new AIEE Standard for Lightning Arresters.

For example, the Lightning Arrester Standard specifies a 10x20 microsecond wave of 10,000 and 20,000 amperes crest, for obtaining performance data. From figure 6 of the paper, it may be noted that 90 per cent of the stroke current peaks are less than 20,000 crest amperes, and from figure 7 approximately 75 per cent of the stroke waves have a time to half value of at least 20 microseconds. Hence, it appears that the test waves used for comparing performance values of arresters are within the range of waves occurring in the actual strokes.

The Lighting Arrester Standard specifies a 5x10 microsecond wave of 65,000 or 100,000 amperes to demonstrate arrester discharge capacity whereas the highest measured stroke current was 58,000 amperes, although the length of the wave was somewhat more than 10 microseconds.

The rate of current rise for a 5x10 current wave of 100,000 amperes crest is approximately 26,000 amperes per microsecond. Figure 11 of the paper shows that 75 per cent of the stroke currents measured had a rate of rise less than 26,000 amperes per microsecond.

Considering the fact that the total stroke current divides upon contact with a line, it appears that test waves set up in the new Lightning Arrester Standard are more severe than the greater portion of the waves which arresters are likely to be called upon to discharge in service.

The McEachron-McMorris data for actual discharge currents through distribution arresters on rural circuits indicate that 12 per cent of the discharges were over 5,000 amperes and four per cent over 10,000 amperes, whereas the data in the paper figure 6 show 12 per cent of the stroke peak currents to exceed 18,000 amperes and four per cent to exceed 25,000 amperes. This evidence indicates that the crest currents to be discharged by arresters are in the order of one-third of the stroke peak current, and supports the author's contention that the stroke current divides when the stroke terminates on a distribution or transmission circuit.

It will be noted that on 13 strokes there were 41 current peaks recorded, which is an average of about three peaks per stroke, also that the average stroke peak current is 9,000 amperes.

The new Lightning Arrester Standard requires arresters to withstand 30-impulse discharges in the operating duty cycle test with 5,000 amperes for distribution type and 10,000 amperes for station type arresters.

S. K. Waldorf (Pennsylvania Water and Power Co., Baltimore, Md.): Dr. McEachron again provides valuable new information about lightning characteristics, particularly as they are affected by tall structures. In his discussion of results the author uses the simple multiplication of lightning current times the surge impedance of an infinitely long wire to show that a 58,000-ampere stroke would produce a potential high enough to flashover any practical insulation. Readers of the paper may conclude from this that overhead ground wires add little or nothing to a lightning protective scheme. Experience has shown that this is not the case but that properly designed lines can take care of stroke currents greatly in excess of 58,000 amperes.

For instance, the Pennsylvania Water and Power Company has a record of 15 cases where indicated currents in overhead ground wires exceeded 29,000 amperes and no flashovers occurred. In these cases the mid-span clearances ranged from 34 feet down to 16 feet, and the span lengths from 800 to 1,750 feet. In 9 of the 15 cases, the indicated overhead ground wire currents exceeded 40,000 amperes, which produce calculated potentials even greater than that given in the paper. Thus there have been an appreciable number of instances where the simplified calculation indicates that flashovers should have occurred but where they did not.

If the value of 500 ohms for the surge impedance of a conductor in air is used for estimating the currents which flow in an overhead ground wire away from a struck tower, another lack of correlation between calculation and experience occurs. Usually towers adjacent to a struck one carry very much greater currents than are predicted by the simple calculation.

Operating experience differs so frequently and so widely from that predicted by the simple calculation that there must be some other factors besides the surge impedance determining the voltage and current distribution on a line struck by lightning. It would be very helpful to many engineers if these more or less obscure factors could be evaluated and incorporated into a simple and reasonably correct formula.

C. F. Wagner and **G. D. McCann** (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): As part of the lightning investigation being carried out by Messrs. Beck, McCann, and Wagner an effort is being made to determine the effect of the height of the stricken object upon the character of the data obtained. To this end, fulchronograms and initial rates of rise of the current are being obtained for strokes to objects of different heights. The data supplied by Dr. McEachron are particularly interesting to us as they supply considerable information regarding strokes to an extremely tall object, an object more than twice as tall as any upon which we have equipment installed.

The propagation of strokes to low objects is characterized by a stream moving from the cloud to earth with only a relatively small, if any, streamer proceeding from the earth to meet the downward leader. Strokes to the Empire State Building, on the other hand, are usually characterized by the propagation of an extremely long streamer proceeding from the building skyward. Thus the initial stroke should have entirely different characteristics in the two cases. Second and subsequent components of a multiple stroke might resemble each other more closely. It would appear, however, that the rate of rise of current would be dependent to a large extent upon the charge distribution along the initial streamer, and this in turn would be dependent upon the distribution of charge on the grounded object. In the case of the tall building, the charge would be distributed upon a vertical conductor whereas in the case of strokes to open ground or low objects the charge would be distributed on a horizontal plane. The rate of rise of current should then be dependent to some extent upon the ease with which these charges can be drawn into the stroke channel. The measure of this effect might be the surge impedance of the building in one case and the surge impedance and resistivity of the soil in the other.

Another characteristic of the stroke which one might expect to be affected by the height of the building is the duration of its high current portion. This is represented by the discharge of the charge lowered into the stroke channel by the initial leaders. This charge can be rapidly discharged since it lies in a preionized path. The rapidity with which it is discharged depends upon the velocity of the return streamer and the length of the stroke channel. For this reason it is expected that strokes to tall buildings having a shorter stroke path will be characterized by shorter times to half value.

It may be that the two types are similar but until they are so proved doubt still remains. In the discussion of his results, Dr. McEachron, after describing the characteristics of waves on transmission lines, jumps to the conclusion that the data obtained on strokes to the Empire State

Building may be applied to the transmission line conditions with reasonable accuracy. This may be true but in view of the doubts expressed above it should not be so applied without further demonstration of its applicability.

K. B. McEachron: Mr. Beck indicates that he feels that it is doubtful if there is a relationship between the geological structure and the formation and frequency of lightning strokes. While conditions on the earth may have a considerable influence on lightning storms, I do not believe that there is any evidence yet to indicate that the location of strokes or the number of strokes at a given location are particularly influenced by the earth's resistance at that point. I agree that our results tend to bear out Mr. Beck's contention that ground resistance will have an appreciable effect upon the lightning stroke once the path has been made between the cloud and the earth. It is of course rather difficult to know what the stroke characteristics would have been with other ground resistance conditions, but, as we learn more and more about the physics of the lightning stroke we are in a better position to visualize what the effects on the ground resistance might be. With present knowledge, it seems likely that areas of high ground resistance are much more certain to have continuing lightning strokes, just as it appears certain that cloud to cloud strokes are more likely to be continuing for a similar reason, since in this case the mobility of charges on both ends of the stroke is relatively low.[1]

Mr. Ross asked how rates of voltage rise may be determined from direct lightning stroke data. As indicated in the paper, some assumptions must be made when this is done in light of present knowledge. Perhaps the easiest method is to assume that the currents measured at the top of the Empire State Building are also the currents which would occur in a direct stroke of lightning making contact with a transmission line. On this basis then, if the stroke occurred at some point remote from the end of the transmission line the current in the line to be expected would be half that found in the stroke itself. Thus, in figure 6 of the paper, each of the kiloampere values would be divided by two, which would also be the method to be used in connection with figure 11. Thus from figure 11, a rate of rise of approximately seven kiloamperes per microsecond would be expected on a transmission line in 50 per cent of the cases. If it is assumed that the surge impedance of the line is 500 ohms, then the rate of voltage rise for 50 per cent of the strokes would be 3,500 kv per microsecond. The maximum rate of rise, according to figure 11 then, would be 9,000 kv per microsecond. These calculations are based on the assumption that the stroke of lightning is not to be used as a transmission line having a surge impedance.

I do not believe, from a consideration of the mechanism of the natural lightning stroke, that the use of the surge impedance for the lightning stroke will give us as satisfactory a result as the method explained in the preceding paragraph.

It is desirable to point out also that the results given in the paper record only results from initial upward leader strokes from the Empire State Building, and if these same strokes had made contact with a transmission line they would no doubt have been initiated by downward initial leaders. In this case the lightning stroke would have begun with a current peak. There is nothing in our data to indicate whether such a current peak would have been faster or slower than the data in the paper for subsequent current peaks. I feel quite confident that, in general, the rates of rise and durations for the subsequent peaks are not materially different than they would have been had the same stroke of lightning made contact with a transmission line having a similar ground resistance. It is hoped that data may be obtained at the Empire State Building in the immediate future which will show the magnitude of the initial current peaks for those strokes making contact with the building which do have initial downward leaders, thus assisting in answering Mr. Ross' question.

The discussion by Mr. Rudge indicates that the tests which have been set up for testing lightning arresters represent conditions of severity which correspond quite well with those given in the paper. It is recognized, of course, particularly in connection with distribution circuits, that there are many paths to ground, and, although the lightning stroke current at the point struck may have values such as those indicated in the paper, one would not expect to find equivalent values passing through the lightning arresters. Each distribution circuit would have to be studied in some detail, in order to properly subdivide the continuing current to ground. Such a calculation would include the primary of any grounded neutral transformer connected to the circuit. Short time discharges through arresters are often decreased in amount, compared to peak stroke currents, by the flow of current through other paths which may exist in the immediate vicinity of the stroke.

Mr. Waldorf pointed out a possible misconception which the careful reader of the paper may draw from the example given, in which a 58,000 ampere stroke made contact with a long circuit and through the action of the surge impedance of the transmission line and the 29,000 ampere current flowing in either direction, such voltage resulted in flashing over a woodpole to ground. Rather than indicating that overhead ground wires are not satisfactory for service, this statement may be equally well used to indicate the necessity of using overhead ground wires if such a flashover is to be prevented. The thought, however, in introducing this computation was to indicate that detailed data on lightning strokes to the Empire State Building have been obtained having a current magnitude sufficient to flashover a woodpole line to ground, if no means of diverting the lightning stroke were present.

It is obvious, of course, as indicated by Mr. Waldorf that, if overhead ground wires were provided grounded with low ground resistance, flashover would not take place. This is discussed in the paper in the last part of the paragraph referred to by Mr. Waldorf. The stroke is assumed to have made contact with an overhead ground wire at a tower having a ground resistance of 10 ohms, "the potential at the tower top would be only 580,000 volts" which would not cause a flashover on such a line.

Mr. Waldorf has pointed out that the simple calculation which was given in the paper was based on several assumptions which further study will no doubt modify, so that more accurate results may be obtained. I think it is only fair to state that the particulars of exactly what happens just prior to a stroke to a transmission tower, and just after the stroke, are not perfectly clear. However, I think they are sufficiently clear so that reasonable results may be obtained from using such assumptions as those indicated in the paper.

Messrs. Wagner and McCann have indicated in their discussion that they are studying the effects of various heights of structures on lightning discharges. That height might have some effect upon lightning discharges was recognized from the beginning in connection with the Empire State investigation, and when this study was started we constructed in the same year an observatory in Pittsfield on the roof of one of our manufacturing buildings. This building was arranged to make possible the photographing of lightning discharges with various types of revolving lens or revolving film cameras. Similar types of cameras were installed in New York, and it was expected that, by a critical examination of photographs showing the detail characteristics of strokes to the countryside around Pittsfield compared with those to the Empire State Building in New York, correlation would be obtained so that the effect of the height of the Empire State Building might be evaluated.

In addition to this, our cameras in New York are so located that not only do they photograph strokes to the Empire State but records have also been obtained of strokes to many other buildings in lower Manhattan. In most cases it has been possible to identify the building struck, which gives rise to a considerable amount of data with reference to the effect of height on the character of the lightning stroke. We have been collecting these data for the past six years and have records of about as many strokes to other buildings as to the Empire State. I hope that it will be possible to publish the results of both the Pittsfield strokes and the miscellaneous strokes in New York in the near future.

I must take exception to the statement made by Messrs. Wagner and McCann in their discussion in which they suggest that "The propagation of strokes to low objects is characterized by a streamer moving from the cloud to earth with only a relatively small, if any, streamer proceeding from the earth to meet the downward leader." I have published[2] a photograph of lightning striking a beach in New Jersey in which upward streamers several feet in length were found to exist from the beach before the stroke made contact. Similar photographs have been taken with reference to lightning striking water, showing streamers upward from the surface of the water to meet the downcoming leader. Photographs taken in New York, showing streamers from buildings not struck at times when downward leader strokes made contact with other nearby buildings, indicate quite clearly that upward streamers from the earth to meet a downcoming leader are probably quite common. Roughly 20 per cent of the Empire State Building strokes are initiated by downward step leaders. It appears that the upward step leader form of initiation results from the stress in the building being sufficiently great so that a stroke is formed at a time when a cloud charge was not sufficiently intense to have started a stroke from the cloud in the usual downward initial leader manner. Eighty per cent of the strokes which are initiated from the building begin with upward step leaders rather than "extremely long streamers" as suggested by Wagner and McCann. I have recognized the difference between upward initial leader strokes and initial downward leader strokes in that an initial high current peak is absent from the upward leader initiated stroke. It appears, however, from the comparison of the photographic results to structures of different heights and to the open countryside of Pittsfield, of which we now have 745 strokes which have been photographed, that the succeeding current peaks after the initial one will be substantially the same as would be found in the case of a stroke of equivalent length to a ground of the same resistance. Detail differences no doubt will occur, but the measurement of lightning is not an exact science when all strokes have different detailed characteristics. Perhaps Messrs. Wagner and McCann's discussion can best be answered by quoting from my Franklin Institute paper published in February 1939 as follows:

> "The picture of a lightning stroke as it appears to the author at present is something like the following.
>
> "The majority of lightning discharges photographed in the past were branched downward, indicating downward leaders, which is to be expected when the earth is more or less a plane surface as regards the cloud, which, due to its construction, automatically begins to act as a needle point as the charges accumulate in some part of the cloud. The discharges proceed from the cloud in a series of steps, always being fed by a streamer from the original cloud. As this streamer progresses, choosing the best path, the charges upon the earth's surface begin to flow into the region of increased field strength due to the approaching streamer. When the field strength becomes sufficient, streamers will form from grounded objects on the earth's surface. The stroke will usually contact the longest streamer; the return stroke follows, in which charges from the earth will pass up the stroke channel, their flow constituting the current in the discharge. The front of the wave will depend upon the rate at which charges flow into the point to be struck. The crest current will be determined by the number of charges flowing into the channel at the time of contact, and if the stroke is to be continuing, charges will continue to flow until the supply is exhausted in either the earth or the cloud.
>
> "Thus, the front of the wave of current of a lightning discharge with initial downward leader to a transmission line is probably dependent upon the amount of charge and the rate of its propagation toward the point to be struck, while the tail of the wave is dependent upon the amount of charge available. It is not known whether the percentage of strokes to open country which are continuing is as great as to the Empire State Building or not, but it is known that they do occur frequently, and are probably dependent to a large extent upon the distribution of charge within the cloud for their existence. If it is true, as suggested, that the maximum current in a discharge with a downward leader will occur (unless some new cloud charge is tapped), when that leader makes contact with either the earth or a short streamer from the earth, the magnitude of that current will be determined by the charge in the earth and its availability. The amount of charge in the earth will be related to the amount of charge in the cloud and so will the maximum current. A considerable portion of the charge associated with the particular stroke will move from the cloud down into the channel created by the leader. This channel of charges will extend all the way to the earth to unite with the charges from the earth when contact is made. This reasoning would lead one to believe that continuing strokes with downward leaders would be likely to have an initial peak followed by the more or less steady current flow due to the partial or complete discharge of the cloud center.
>
> "Presumably, discharge starts from the cloud without being much influenced by the earth configuration, but as it approaches it is more and more influenced, although as a rule that influence is local in character. However, the taller the conducting object the greater its influence. With strokes initiated by upward leaders, the case is different. It is believed that upward leaders will only occur when conditions on the earth simulate a needle point of sufficient height to have an influence on the propagation. Perhaps some natural configurations of the earth might be of such a nature as to produce the upward leader.
>
> "It seems to the author, however, safe to state that the downward leader stroke represents the type of stroke of greatest importance in the problem of protection of transmission lines and apparatus.
>
> "The effect of the continuing type of stroke must be determined. It seems that it will not be of great importance on grounded neutral electrical systems, because the continuous current can readily flow to ground through the transformer windings. The isolated neutral system may have trouble with protective apparatus connected to ground. Just how important this matter of continuing strokes will turn out to be is not known at present, but it may help explain some of the failures of lightning arresters attributed to arcing grounds or other system disturbances. The continuing type of discharge may have considerable effect on the blowing of fuses used to protect distribution transformers when operating grounded neutral."

It is probable as suggested by Messrs. Wagner and McCann that the great height of the structure may have some effect on the rates of current rise, since these will be affected by field distribution. The shorter length of stroke may also have an effect to shorten the length of tail, as they have suggested. On the other hand, normally no effort is made to determine the length of strokes between cloud and ground, so that whatever the magnitude of these effects may be they will still be within the range of strokes to ground, since lightning strokes do vary greatly in length and certainly many will have stroke lengths corresponding to those which occur to the Empire State Building.

REFERENCES

1. LIGHTNING TO THE EMPIRE STATE BUILDING, K. B. McEachron. *Journal* of the Franklin Institute, February 1939, page 149.

2. Reference 1, page 201.

Measurement and Control of Conductor Vibration

Discussion and author's closure of paper 41-97 by G. B. Tebo, presented at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941, and published in AIEE TRANSACTIONS, 1941, pages 1188–93.

G. W. Stickley (Aluminum Research Laboratories, New Kensington, Pa.): This paper is a real addition to the knowledge of the problem of conductor vibration, and is of particular value in its application and correlation of the various kinds of test data with natural operating conditions. The vibration recorder which has been developed appears to have definite advantages over other types of recorders previously used, in that it records alternating bending which causes vibration failure.

The torsion type of damper, when properly designed and installed, apparently is effective in damping resonant vibration of conductors. Referring to figure 5 of the paper, we assume that the dampers used in obtaining records *D* and *E* were the same as shown in figure 13. It would be of additional value to know the mass, moment of inertia, length of torsion arm, and over-all length of this damper.

Referring again to figure 5, its title, "Typical Damped Span Vibration Records," appears misleading. According to the text of the paper, the records shown in this figure are typical of those obtained when the frequency is not over three cycles per second, which corresponds to a wind velocity of only one mile per hour. It is generally recognized that such wind velocities are very low, and seldom represent average conditions in service.[1] It also is known that the effectiveness of the Stockbridge damper is a minimum at such very low velocities.[2] Incidentally, it is generally considered that vibration at such low wind velocities is not very serious because the frequency and the alternating bending stresses are so low. Therefore, it seems that a similar set of records obtained at a more representative wind velocity should be shown.

It would clarify figures 4 and 5 if the vertical scale of each were stated. Apparently they differ considerably. It should be noted also that the wind velocities are not the same.

In the comparison of dampers and reenforcing devices the author states that "Devices which act primarily as reenforcements effect only a small reduction in vibration amplitude. . ." While it is sometimes true that only a small reduction is obtained it is of interest to report the results of some tests made by the Aluminum Company of America at their Massena laboratories using spans vibrated naturally by prevailing winds. With cables from $^1/_4$ to $1^1/_8$-inch diameter, and in spans from 400 to 1,500 feet long, these results show that armor rods may decrease the maximum amplitude of vibration as much as 50 or 60 per cent and the number of hours of vibration from 25 to 50 per cent. Armor rods, of course, are not usually considered as being dampers, but these data indicate that their damping effect may be of considerable importance.

Incidentally, it should be noted that the armor rods shown in part *B* of figure 14 are twisted the wrong way. Laboratory tests and years of field experience indicate that armor rods should always be twisted in the same direction as the lay of the outer wires of the cable. Laboratory endurance tests have shown that, when this is done, armor rods are at least four times as effective in preventing fatigue failure of conductors. Preliminary field tests indicate that the actual damping effect of armor rods also varies in a similar manner.

The author states that "the advantage of reenforcing devices in protecting conductors from flashover burns may be offset by wear or strand breakage under improperly installed or loosened clamps due to vibration." This same thing may be said, of course, concerning the effectiveness of dampers, and particularly certain types of dampers, if they are improperly installed or if their clamps become loose.

One of the ultimate aims in any vibration study like the one discussed in this paper is that of determining the safe stress conditions which can exist indefinitely without fatigue failure. This, however, is not a very simple task. In the second of the three methods described for determining safe stress conditions, it is suggested that a single endurance test be made on a stranded conductor at a single tension, and that then a conclusion may be reached by comparing the results of this single test with a fatigue curve such as the one shown in figure 16. Such a method seems open to criticism. Actual experience gained in hundreds of tests of cables indicates that it would not be wise to base conclusions on a single fatigue test of a conductor.

The first conclusion of the paper states that "Vibration of large diameter conductors used in modern ACSR long span construction without protective devices may be expected to produce fatigue failure of single strands in a few months to a few years. . ." Many examples could be given to show that this is not necessarily true. One is the transmission line from Massena, N. Y., to Cedar Rapids, Quebec, which was constructed in 1915 using 500,000-circular mil ACSR. Although vibration of this line can be observed almost every day of the year, repeated inspections at the most probable locations have disclosed no vibration failures.

REFERENCES

1. Reference 6 of the paper.
2. VIBRATION OF CABLES AND DAMPERS, R. G. Sturm. AIEE TRANSACTIONS, volume 55, 1936 (May and June sections), pages 455–65 and 673–88.

G. B. Tebo: The author is indebted to Mr. Stickley, who, in his discussion of the convention preprint, made valuable suggestions which have been incorporated in the paper as published.

The damper shown in figure 13 of the paper was the type used in obtaining records *D* and *E* of figure 5. The essential physical characteristics of the damper are:

Overall length of dumbbell casting	12.5 inches
Weight of dumbbell casting	14 pounds
Effective length of torsion arm	4 inches
Torsional elastic constant of rubber connection between arm and dumbbell casting	71 foot pounds per radian
Moment of inertia of dumbbell casting about center of rotation	235 pound inches squared

In objecting to the title of figure 5 which describes the vibration record shown for a Stockbridge damped span as "typical," Mr. Stickley appears to be confusing "typical vibration record" with "typical wind velocity." His contentions that very low velocity winds occur infrequently, and that the resulting conductor stresses are low, are generally accepted. Figure 7 of the paper, however, shows these values in quantitative form for the first time, to the author's knowledge. This figure shows that, on the spans tested, vibration at frequencies of two to four cycles per second (plotted at three cycles per second) occurred about two hours per day and at deflections not exceeding 0.004 inch measured 3.5 inches from the clamp.

The incorrect spiralling of armor rods shown in figure 14*B* of the convention preprint was, of course, an error in drawing. The correct spiralling was used in all test installations as well as in the several thousand installations on operating lines.

In discussing the reduction of vibration amplitude effected by armor rods, Mr. Stickley's admission that "while it is sometimes true that only a small reduction is obtained . . ." indicates that it would be unwise to depend upon armor rods for substantial damping of vibration. It is surprising that Mr. Stickley should cite the Massena, N. Y., to Cedar Rapids transmission line as an example of "large diameter conductor used in modern ACSR long-span construction." The span length, tension, and conductor size of this line are believed to approximate the conditions of curve *B*, figure 8, conditions for which our investigations predict freedom from fatigue failures. Hence, Mr. Stickley's statement regarding the operating experience of the line, provides very satisfactory confirmation of our experimental results.

Field Investigations of Lightning

Discussion and authors' closure of paper 41-137 by C. F. Wagner, G. D. McCann, and Edward Beck, presented at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941, and published in AIEE TRANSACTIONS, 1941, pages 1222–30.

K. B. McEachron (General Electric Company, Pittsfield, Mass.): The authors of this paper are to be commended for the development of lightning measuring instruments and for conducting an investigation having for one of its objects the determination of the relation between a stroke of lightning as such and the duty on insulation or protective devices associated with the line with which the stroke makes contact. The method used by the authors is, however, based on the comparison of statistical results and thus open to some uncertainty. As I see the problem, the most satisfactory correlation between a stroke and phenomena on the line will result when the current-time characteristic of a stroke itself is known in detail with sufficient accuracy, as well as the complete wave shapes of the current resulting from this same stroke in each of the wire paths of a distribution or transmission circuit. This observation does not detract from the value of the results to be obtained by the method used by the authors, since statistical data of the type shown in the paper are very useful, but to tie together direct stroke data and data obtained on circuits the statistical method will not be completely satisfactory.

When the Empire State investigation was begun in 1935 it was realized that the height of the building might well have a pronounced effect upon the character of the stroke. Therefore, at the same time an investigation was begun at Pittsfield using similar photographic equipment to that used in New York, that is, slow and high speed Boys cameras. In Pittsfield the strokes photographed were to trees, houses, and the terrain of the Berkshire Hills surrounding the observatory. An analysis of these results (745 strokes photographed to date), together with similar results published by B. F. J. Schonland, indicate that strokes to the Empire State building are not particularly different from those to ordinary terrain once the initial mechanism is completed. As indicated by the authors, it seems quite certain that as the height of the structure decreases the percentage of upward initial strokes will decrease. There is no evidence of which I am aware which would indicate that upward initial leader strokes occur to transmission lines of usual height. There is good reason to believe that once the current has started in a stroke of lightning it does not cease until all the stroke discharges in the same path are completed. The stroke is substantially a direct current arc between cloud and ground with superimposed current peaks.

In connection with figure 10 of the paper it is to be noted that photographic results are greatly dependent upon exposure, and exposure is a function of distance and visibility for a given film and lens speed. Thus the duration of a stroke as indicated by the photographic film would be quite different for a camera located five miles away compared to that obtained with a camera one-half mile distant. The number of current peaks, as shown in figure 11, are similarly affected by distance and visibility.

Nothing is given concerning the expected accuracy of the various measuring devices used. It would increase the value of the paper if the authors could give some idea as to the accuracy of the overall results obtained, bearing in mind the method of servicing as well as the accuracy of the devices themselves.

Paul M. Ross (Ohio Brass Company, Barberton, Ohio): In testing transmission line insulation in the laboratory it is desirable to know expected direct stroke rates of voltage rise. What rates of voltage rise may be expected from direct lightning strokes?

Herman Halperin (Commonwealth Edison Company, Chicago, Ill.): The authors' conclusion 5, that arrester duty on grounded neutral systems is less severe than on systems with ungrounded neutrals, in general agrees with our own experience. The Chicago 4,000-volt overhead distribution system, which is of the four-wire grounded-neutral type, has the interconnection form of transformer protection and the arresters are mainly types made 12 to 20 years ago. Laboratory surge tests on typical arresters removed from service, using a 10/20-microsecond wave, resulted in failure of some at only 1,500 to 2,000 amperes. Yet these arresters have a very low failure rate in service and provide good protection, the failure rate of transformers due to lightning being only about 0.2 per cent per year. The worst offenders among these old arresters are being gradually replaced, but it is evident that the duty on these arresters is not unusually severe—not enough to warrant a wholesale replacement with new types.

The authors of this paper present limited data on failures of arresters of one type when subjected to lightning discharges of partially measured characteristics, and show that arrester failures may occur for crest discharge currents of a few thousand amperes when associated with about a coulomb of charge. More complete data on this point obtained on other systems and on other types of arresters would be valuable.

It may be pointed out that the pending revision of AIEE Standard 28 on lightning arresters includes a discharge capacity test of 65,000 and 100,000 amperes for distribution (line) type and station type arresters, respectively. Due to the short impulse wave specified for this test, however, the integrated current is only of the order of one-half and three-fourths of a coulomb for the two types. Of course, since thermal effects are involved in arrester failures, this test would seem to impose a much higher duty on the arrester than one comprising an equal charge imposed by a small current of longer duration. Hence it appears, on the basis of such field data as are now available, that the modern arrester, designed to meet the proposed standard tests, has adequate margin to stand up under severe field conditions More important, operating records indicate that the modern arrester has satisfactory *protective* characteristics.

It is of interest to note the report in this paper of a maximum discharge current of over 160,000 amperes in a direct stroke to a tall structure.

It is hoped that the authors' field investigations will be continued.

E. R. Whitehead (Duquesne Light Company, Pittsburgh, Pa.): Messrs. Wagner, McCann, and Beck are to be congratulated on the resourcefulness with which they have attacked the refractory problem of lightning current wave shape. Data on the long-time character of lightning strokes are becoming conclusive, and it is to be hoped that much more data can be accumulated on the nature of the wave front of the high-current short-duration portion of the individual component.

Instances of equipment failure and transmission line flashover have occurred which require high rates of current rise for their explanation, and perhaps it may be said that when sufficient data have been obtained on this point we shall have at our disposal all the facts essential to a practical solution of the lightning problem.

H. N. Ekvall (Philadelphia Electric Company, Philadelphia, Pa.): In this paper the high current short-duration portion of a stroke is referred to as "cold" lightning, while the low current long duration portion is referred to as "hot" lightning. "Cold" lightning has the characteristic of shattering poles and crossarms, while "hot" lightning is less violent and has more of a thermal effect. Apparently the predominance of either of these forms of lightning is affected by soil resistivity. High soil resistivity apparently is conducive to discharges of long duration, or "hot" lightning. Some evidence to support this belief has been found from experience on the 33-kv wood pole lines of the Philadelphia Electric Company, which are in a territory of high soil resistivity as indicated by our average ground resistance of 150 ohms for ten foot driven rods. On these lines, which have no ground wire protection, there is very little shattering of poles or crossarms. This is in contrast to experience in the mid-west, where on similar construction pole shattering is so severe that ground wire protection is required. This difference in performance may be associated with low ground resistivity in the mid-west, as compared to high resistivity in the vicinity of Philadelphia.

C. F. Wagner, G. D. McCann, and Edward Beck: Mr. McEachron states that the statistical approach to the lightning problem employed by the authors is open to some uncertainty. We assume that he refers to the correlation of surge currents at various points in a system to the current in the stroke itself, for he advocates the measurement of all associated currents for individual strokes. Although we agree that such data would be desirable, contemplation of such an undertaking demonstrates the almost insurmountable practical difficulties of securing such complete data in any volume sufficient to be useful. In view of the indicated influence of factors of which there is little knowledge, and the fortuitous nature of strokes, the only way to accumulate the necessary data is by statistical observations under varying conditions. Moreover the correlation of stroke and traveling wave currents is not the only problem. Of outstanding practical importance is the probability factor of the various characteristics of surge currents in such apparatus as protective devices. This can be determined only by statistical methods, the approach pursued by the authors. Accumulation of such data in sufficient volume, it is expected, will permit the prediction of the performance of systems and the life of protective devices of known characteristics.

Mr. McEachron also states that his photographic studies of strokes to the Empire State Building and to open ground at Pittsfield are essentially the same once the initial mechanism is completed. This may be true with regard to certain characteristics such as the number of components and the intervals between them, but until data on the important factors of wave shape and magnitude of the current at the ground end of the stroke for strokes to tall objects and to open ground are available, the similarity is open to question.

The data on strokes to open ground plotted in figure 10, upon which Mr. McEachron comments, were obtained photographically by measuring the time from the beginning of the first component to the visible end of the last component. These data and similar data of figure 11 of the paper are open to the criticism mentioned by Mr. McEachron in that they are dependent upon the distance and visibility, and film sensitivity and lens speed. These same factors are present in the experimental data which prompted Mr. McEachron to conclude that the strokes to the Empire State building are not particularly different from the strokes to open ground observed at the Pittsfield Laboratory.

Mr. Ekvall's observations on the relation between pole shattering and soil resistivity are very interesting and well worthy of further consideration by the industry.

We agree with Mr. Whitehead that there is need for more statistical data regarding the rates of rise of surge current.

Because modern theories of line protection are built around current as the fundamental quantity, lightning investigations are concerned largely with determining the characteristics of the stroke with regard to this factor. In answer to Mr. Ross' question methods are available for determining the voltages associated with these currents, and impressed upon insulation. While it would be desirable to accumulate data on voltage directly, the results would be dependent on other factors such as tower footing resistance and the like. For this reason we have preferred to devote the means at hand to the collection of the more fundamental current data.

We are very much interested in Mr. Halperin's experience on a large distribution system, as it supplements in a valuable manner the data presented in our paper and theoretical computations in an earlier one. We agree that more data are desirable on the nature of the discharges associated with arrester failures, and we are continuing and extending our investigations.

Electric-Locomotive Application

Discussion of paper 41-101 by E. W. Brandenstein and D. R. MacLeod, presented at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941, and published in AIEE TRANSACTIONS, 1941, pages 1210–15.

Sidney Withington (New York, New Haven and Hartford Railroad, New Haven, Conn.): This paper presents data of much interest and is a valuable addition to the literature on the subject under review. A number of points may be emphasized to advantage.

Momentum grade operation, which is described, constitutes an exceedingly valuable adjunct in railroad operation, of which full advantage has not been taken in times past, and which undoubtedly will be used increasingly in the future. As the authors have pointed out, there is no reason why a change of kinetic energy to potential energy should not be made under proper conditions, and with the high degree of reliability of electric motive power and with proper dispatching, the chances of a train being obliged to stop on a grade on which starting would be beyond the capacity of the locomotive, is nowa-days so negligible that it does not constitute an operating hazard.

As has been indicated, the short-time overload capacity inherent in electric locomotives, as compared with steam and Diesel locomotives of the same nominal rating, is of great advantage in the application of electric locomotives to operating conditions. Not only does the profile of a given section of railroad usually involve considerable variation in power demands, but after slow-down which may be necessary for curves, drawbridges, etc., the sharp concentration of horsepower required for a short period in resuming normal speed is important in permitting fast schedules economically. The voltage control by transformer tap-changing when single phase power distribution is used and the highly effective modern a-c series commutator traction motor provide an ideal combination for this most valuable characteristic. If overload capacity is taken into consideration and conforms, as it usually does, to the service requirements involved, it may be said that on an effective horsepower basis the electric locomotive compares favorably in cost with the other forms of motive power.

The authors refer to the separation between peak traffic in passenger and freight service as being a characteristic of railroad operation. While the Christmas holidays constitute a high passenger-train peak, there are also on most railroads other holiday demands which also may be classed as peaks, such as July Fourth, Labor Day, Thanksgiving Day (under the old pre-Roosevelt schedule), etc. Diversity of traffic is desirable on all these occasions regardless of motive power limitations, for limitations in track capacity often make it necessary to postpone freight train operation during the exacting passenger peak demands. The interchanging of electric locomotives between passenger and freight service thus works in nicely with the requirements. Diverting locomotives between passenger and freight service, however, will be found practicable only if the passenger and freight train facilities in a given terminal are relatively close together, so that a great deal of dead-head locomotive mileage is not required to get a locomotive from one terminal to another for a return trip.

In handling sharp peaks in passenger or freight service it may be found advantageous in a given electrification installation to plan to utilize spare *steam locomotives* in order to avoid the necessity for carrying excess capacity in electric locomotive units which would be little used at other times. This policy in using spare steam locomotives thus for peak loads may also well be of advantage under some conditions in avoiding establishing expensive peak loads in the power supply.

The use of inert cooling liquids for transformers has constituted an important advance in electric locomotive design of recent years. Formerly, while oil cooling was used largely abroad, American locomotives were equipped with air cooled transformers, which obviously had serious limitations for various reasons. The inert cooling liquid provides all the advantages of oil cooling without certain hazards.

An interesting reference is made to track platform capacity at terminals as affecting possible length of passenger trains. This is true, not only at certain terminals, but in many cases at stations along the line where platforms constitute serious limitation in train lengths. To have to make train stops at a station platform is of course disastrous in maintaining rapid schedules. It is pointed out also, in this connection, that electric locomotives may have fundamental advantages as compared with certain other types of motive power; certain 6,000 horsepower diesel locomotives of a type now operated, for instance, have lengths of more than two hundred feet which may on occasion seriously limit the length of a passenger train at a stub-end terminal.

Relative availability of electric and steam locomotives is referred to in the paper. In preparing comparative data in a given instance, care must be taken that the mileage of electric locomotives is not limited by traffic requirements, rather than by mechanical considerations. The fleet movement of both passenger and freight trains, which is often necessary in railroad operation, may sadly limit equipment mileage and prevent the intensive use which would otherwise be possible.

C. Kerr, Jr. (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): In his presentation, Mr. MacLeod has given an interesting review of accomplishments in the design and application of modern electric locomotives. Progress in this field is indicative of some of the creative action which has taken place to assist in maintaining a high level of railroad operating efficiency.

The heating and cooling curves devised represent the best type of information obtainable on locomotive application. The data serve as a means for accurately establishing electric locomotive performance under all conditions which are encountered in service. It is practicable to predetermine the heating of the traction motors at any location of a profile during a run. The completeness of the data is helpful in fixing tonnage ratings for the electric locomotives. There has been no necessity for downward revisions of tonnage ratings after they were once established. This reflects the usefulness of the heating and cooling curves.

Momentum operation is justly mentioned as an important factor contributing to the most efficient use of the electric locomotive. The a-c engine develops its maximum rating at high speed and securing full benefit of the available output is a problem. Momentum operation has done much to permit using the "high speed" capacity of the a-c electric locomotive. It has been beneficial in boosting tonnages—in fact, trailing loads have been almost doubled in some cases.

During the course of the test work, considerable time was devoted to investigation of train resistance. The results show unusually low values for heavy rail (130 pounds per yard). This is another contribution to "trailing tonnage increases" for the a-c electric locomotive. Full advantage is taken of this factor in fixing tonnage ratings.

Glass-Bulb Mercury-Arc Rectifiers for Traction Service

Discussion and author's closure of paper 41-100 by C. E. Woolgar, presented at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941, and published in AIEE TRANSACTIONS, 1941 (August section), pages 843–6.

W. E. Gutzwiller (Allis-Chalmers Manufacturing Company, Milwaukee, Wis.): The paper gives a very interesting and complete account of the application of permanently sealed glass tube, multi-anode mercury pool rectifiers for traction service. This type of equipment is being used very extensively on the European continent, in Great Britain, and over-seas countries served by British, French, and German manufacturers. On this side of the Atlantic, a few installations of glass tube rectifiers have been made in Canada with British made equipment, and in the United States with equipment furnished by this company. The following comments are based entirely on experience with this type of equipment in the United States.

Glass tube multi-anode mercury pool type rectifiers were employed very extensively some fifteen to twenty years ago in connection with the series type street lighting, using d-c arc lights. With the development and perfection of incandescent lamps for street lighting, the arc light, and with it the glass tube rectifier, disappeared from this field. Since that time up until just a few years ago it has been used mainly in small capacity sets for battery charging purposes.

Within the last three years this company has built glass bulb rectifier equipment for industrial applications, covering about twelve 200 and 400-ampere rectifier tubes. While the experience with these installations has been very gratifying, they have not been in service for a sufficient period to draw definite conclusions as to tube life and maintenance cost.

The experience gained so far, however, seems to indicate that the equipment does not lend itself very well to modern manufacturing methods, as the glass envelope, due to its complicated shape, must be "hand-made" by expert glass blowers. Likewise, the handling during the manufacturing, shipping, and field maintenance requires extreme precautions to avoid breakage.

Aside from these points, our experience indicates that glass tube rectifiers are particularly well suited for light duty applications, in ratings of a few hundred kilowatts, where generally the ratio of peak to average load is low, d-c short-circuits are relatively infrequent, and where short-circuit currents, due to limited amount of power in a-c and d-c systems, are comparatively small. Under such conditions very satisfactory tube life can be obtained.

It is believed, however, that for heavy traction service as encountered in metropolitan areas for subways and elevated railroads, calling for loads of 300 per cent of normal for one to five minutes, the more rugged, pumped steel tank type rectifier would, in the long run, be more economical.

Our limited experience with the direct forced-air cooled glass tube rectifiers has been extremely encouraging and has induced us to develop pumped metal tank, multi-anode rectifiers in smaller ratings, with direct forced-air cooling. A number of such installations have been made, and have been very successful.

While water-cooled rectifiers in large capacity converting substations are not objectionable, but, to the contrary, have many advantages, particularly in cases where the atmosphere is contaminated with dust or corrosive chemical fumes, in small rectifier stations cooling water becomes rather expensive, both in regard to installation and operating cost. The forced air-cooled rectifier in such installations greatly simplifies the layout and reduces operating cost by eliminating water, corrosion, sweating of pipes, and danger of freezing.

In the United States, the development trend in the rectifier field has definitely been towards the rugged steel tank rectifiers in multiple and single-anode types, which can be manufactured with modern methods and machine tools.

While until about five years ago railways were the largest users of power rectifier equipment in the United States, recently a rapid change has taken place and today at least ninety per cent of the sales of power rectifiers are going to the electrolytic industry. Because a large part of the remaining ten per cent is being used in the steel and associated industries, the traction field has become of less importance. Therefore, it would be of interest to know whether glass tube rectifiers have found any application in the electrolytic industry outside of the United States.

Information on the part of the author of the paper explaining how anodes are paralleled off a common rectifier transformer, where there are as many as forty or forty-four rectifier bulbs per substation; likewise, information on the type of connection of the secondary winding of rectifier transformers supplying a large number of parallel tubes, would be desirable.

Henry M. Hobart (retired, General Electric Company, Schenectady, N. Y.): The merits of glass bulb rectifiers for traction service are clearly and concisely set forth in the paper presented by Mr. Woolgar. The paper gives just the kind of information which will be helpful to application engineers. One could only wish that the case he presents could have been available in as clear and convincing form some twenty years earlier. Substations equipped with rectifier cubicles and transformers in accordance with the principles described by Mr. Woolgar, might, in conjunction with the great decrease in weight of modern traction motors, and the improvements which were already being made at that time, in permanent way and rolling stock construction, have saved the day for urban and suburban electric traction on rails.

At that period, a considerable number of British and other European firms were manufacturing both steel-tank and glass-bulb mercury-arc rectifiers, and many useful fields of application were being found for each type. But in this country, no sufficient interest could be aroused at that time, in the glass-bulb type. There then followed in America an almost complete abandonment of the use of synchronous converters in new installations, and the rapid introduction of steel-tank rectifiers instead.

Had American manufacturers adopted the plan of providing both types, it is evident from Mr. Woolgar's data and explanations that today the glass-bulb type would have been conceded to be in many applications superior in reliability and efficiency, and of lower initial cost for apparatus, foundations, and buildings, as also with respect to lower maintenance and operating costs.

It would appear superfluous to emphasize any particular parts of Mr. Woolgar's presentation, but it might in addition be suggested that the present heavy national defense demand for all qualities of steel is liable to make it more difficult to obtain the high quality essential for rectifier tanks. The price will increase with the increasing acuteness of the defense situation and the importance of considering first the needs in military matters. Thus Mr. Woolgar's reminder of the excellence of the glass-bulb alternative is decidedly timely. Even in fields of application which are well met with large steel-tank rectifiers, the alternative of employing groups of glass-bulb rectifiers certainly would not be attended with any disadvantages comparable with the importance of national defense considerations.

The feature of using many component sets of small rating instead of one or a very few larger sets, must play a considerable role in decreasing the capital cost as compared with having large units and the correspondingly necessary large and expensive spares. The cubicle system also contributes toward less floor space. Mr. Woolgar particularly stresses the interesting technical advantage of distributing the total current amongst a great number of anodes, and thus decreasing the liability of arc-backs. This greater distribution also must contribute toward lower anode temperature.

The multiplicity of small units presents a fine opportunity for reducing costs by mass production methods. By concentrating on a very few sizes, these can be combined to provide sub-stations of any desired small or great rated output, and any desired total voltage and total current. The highest direct current voltage provided in a single glass-bulb rectifier is indicated in the paper as 1,500 volts. But it is a simple matter to supply two or more star-connected anode groups from two or more windings on the cores of a single transformer or group of transformers, and by placing the two or more groups properly in series, there will be obained sets for 3,000 or more direct current volts. Thus it is seen that with respect to total rated output and voltage and current, a great range of ratings can be provided by simple combinations of the proper number of component rectifiers.

The complications entering into the design of transformers for meeting the requirements of employment with mercury-arc rectifiers were not realized or even suspected in the pioneering period. But gradually the transformer has come to represent a rather large portion of the total cost (made up of transformer plus rectifier). Thus if the cost of the transformer can be decreased with no sacrifice in quality, it is important to do so. At present the customary self-cooled oil type of transformer is widely employed. The immersion in oil necessitates its being made large enough to avoid the occurrence of such temperatures as may occasion sludging. So by doing away with oil and using a compressed-gas design, the transformer can be greatly reduced in weight and in cost. The requirements as regards foundations and floor space are also reduced. The high temperature permissible in such a design opens the way for many attractive and novel designing possibilities. The heat may be removed by forced circulation of the compressed gas, first amongst the windings and then through a heat exchanger. Only a practically negligible amount of power is required to circulate the compressed gas at such high speeds as to remove the heat much more effectively than is practicable in the self-cooled oil type.

The turn and layer insulation may be composed of some mica or asbestos materials or of glass tape. In this application the fundamental purpose of the compressed gas is to effectively remove the heat from the windings. Transformers for rectifier service usually are not operated from any particularly high voltage source. In the exceptional cases where the transformer is supplied directly from a high voltage line, the great dielectric strength of a suitable compressed gas will also constitute a useful feature.

In order that there shall not be an unduly great increase in the resistance of the transformer windings, they must be vigorously cooled by the gas circulation. But as to the laminated core, it is now 30 years since it was shown by MacLaren in two AIEE papers, that at very high temperatures, the hysteresis loss decreases considerably in some kinds of core material. A simple plan for sufficiently limiting the temperature rise in the windings and at the same time, providing for a high temperature in the core, is to interpose between the core and the windings, a thermal screen of, for example, a thickness of asbestos or mica sufficient to occasion the desired thermal drop. In the discussion of this proposal, it has been suggested as an obstacle, that the magnetizing current increases rapidly with increasing temperature at the high-temperature range under consideration. But in the case of some excellent modern high-permeability core material, it has been determined by tests that the rate of increase of the magnetizing current with increasing temperature in that range, is much less than in the older brands of transformer core material and is, indeed, not sufficient to interfere with the success of the project.

When freed from the temperature limitations applying to oil transformers, and which have for decades stood in the way of various transformer improvements, one of the interesting directions for progress is in the use of vaporization cooling which can now (with compressed gas cooling) be applied in various extremely effective and economical ways.

It is believed that some of these transformer proposals will fit in very effectively with the features of the cubicle design of glass bulb sub-station described by Mr. Woolgar. Taken together, there will become available a type of railway-traction sub-station of less cost and weight, higher service efficiency, and greater dependability than any yet employed.

The AIEE has been very fortunate to be privileged to publish Mr. Woolgar's admirable paper, and it is to be hoped that electric railway people in particular, will investigate very thoroughly the possibilities of the glass-bulb mercury-arc rectifier.

C. E. Woolgar: Mr. Hobart's suggestion that glass-bulb rectifiers, in conjunction with improved design of traction motors and improvements in permanent ways and rolling stock, might have saved the day for urban and suburban electric rail traction is most interesting particularly in view of the fact that in Great Britain particularly so many tramways are supplied from this type of equipment and even where tramways have been superseded there has been a strong movement towards the use of electric trolley-busses instead of gasoline or oil-driven vehicles favoured on this continent.

The author apparently did not make clear the range of d-c voltage for which single glass-bulb units are suitable, as Mr. Hobart suggests a series arrangement to obtain 3,000 volts d-c. This is unnecessary as single bulbs can readily be built for any practicable d-c traction voltage.

Mr. Hobart made a number of most interesting suggestions concerning the use of compressed-gas filled rectifier transformers to replace the more usual oil-immersed type. The author is not competent to reply to these suggestions and has referred the entire discussion to his principals in England for detailed comment.

Mr. Hobart also makes the interesting suggestion that due to heavy national defense demands in the United States for all qualities of steel, glass-bulb rectifiers could be used to advantage in many cases. Under present conditions the ability to supply such rectifiers will largely depend on the degree to which the manufacture of glass-bulb rectifiers has been developed in the United States. It may be of interest, at this point, to mention that in Great Britain there is ample evidence of the ability of glass-bulbs to withstand concussion due to bombing. In one case, a bomb fell 15 feet from a rectifier substation, made a crater 40 feet wide by 20 feet deep and demolished the substation. The undamaged glass-bulb rectifier units were removed, the station rebuilt, and the rectifiers restored to service without any necessity for repairs.

Mr. Gutzwiller's comment that glass-bulb rectifiers do not lend themselves to modern manufacturing methods due to the necessity of hand-work by expert glass blowers should be thought of in terms of modern shop practice where machining to a fraction of one thousandth of an inch is done by highly-trained, expert workmen. Concerning special handling, the paper mentions that large numbers of glass-bulbs are shipped to all parts of the world and handled safely by native labour.

The author attempted to make it clear in the paper that glass-bulb rectifiers are most suitable for heavy duty applications, for any kilowatt rating, for any ratio of peak load to average load, and for any frequency and intensity of short-circuit current. It will be appreciated that for any given conditions of service severity, the steel tank rectifier manufacturer must select a rectifier suitable for those conditions on essentially the same basis as the glass-bulb manufacturer. It becomes simply a matter of knowing the characteristics of the different ranges of rectifiers and choosing a unit or units capable of meeting the required conditions. In view of the fact that a large number of glass-bulb traction installations of 2,000 kw to 4,000 kw and installations for other types of service with outputs up to 15,000 amps. have been operating satisfactorily for a number of years, it is difficult to justify Mr. Gutzwiller's suggestion that these rectifiers are mainly suitable in ratings up to a few hundred kilowatts.

Mr. Gutzwiller also refers to the high overload capacities obtainable with steel-tank rectifiers, but (*b*) the paper states that it is a simple matter to supply a glass-bulb rectifier with any overload capacity required. As a typical example, the New Zealand Government electrified railways between Wellington, Johnsonville, and Paekakaviki are supplied by glass-bulb rectifiers in six substations totaling 6,000-kw capacity at 1,600-volts d-c. The overload ratings are:

$1^{1}/_{2}$ times full load for 1 hour
2 times full load for $^{1}/_{4}$ hour
3 times full load for 5 minutes
$3^{1}/_{2}$ times full load for 1 minute
4 times full load for momentarily

The method of supplying power to the anodes of a multi-unit glass-bulb rectifier installation is quite straight-forward. Such installations are usually arranged in a straight line and six or twelve leads (as the case may be) are brought from the transformer to busbars running along the top or bottom of the cubicles. Taps taken off at each cubicle pass through anode chokes mounted inside each cubicle, thence to the anodes. The transformer is wound for six or twelve phase operation, as required, in accordance with standard practice.

The author wishes to make one correction to the paper. The percentage of plant failures attributed to various components of steel tank rectifiers given in the paper actually refers to the amount of attention given to the various components for maintenance purposes. The results of an analysis of faults which occurred on 32 steel tank rectifiers on the same system over a period of 12 months showed the failures distributed as follows:

Component Causing Failure	Percentage of Total Failures
Control gear	50
Circulating-water system	12.5
Vacuum system (mercury pump and rotary vacuum pump)	12.5
Rectifiers	12.5
Rotating auxiliaries	6.25
Unexplained failures	6.25

A Distribution System for War-Time Plant Expansion

Discussion and author's closure of paper 41-136 by J. L. McKeever, presented at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941, and published in AIEE TRANSACTIONS, 1941 (September section), pages 862–5.

R. H. Kaufmann (General Electric Company, Schenectady, N. Y.): This paper is of great interest to all individuals concerned with the problem of industrial plant distribution and more particularly so in that the present paper reports an actual system installed. The general system design which is used is in line with the present general trend toward the use of high voltage primary feeders in combination with distributed load centers stepping down to the utilization voltage. The use of many load centers, each of identical size and design, possesses many obvious advantages.

The low tension system incorporates good voltage regulation characteristics, and incorporates a simple switching system capable of reasonable operating flexibility. Good utilization of secondary conductors is obtained through the combination of tie circuit and feeder functions on common electrical circuits. The proposed system is laid out for a utilization voltage of 550, although in this country the preference for 440-volt apparatus would generally dictate its use even at somewhat lower overall economy.

The high tension primary loop circuit has been designed for low investment cost through the use of circuit breakers only at the supply terminals to the loop and air break disconnect switches installed at each load center, one in each branch of the loop. The reduction in primary circuit investment cost is accomplished with certain sacrifices in operating flexibility, namely: a high tension cable or transformer failure results in a temporary power outage on the entire area; unless the location of the fault is evident by the presence of smoke or flame, the power outage may be somewhat prolonged during location and isolation of the faulty section; the switching of energized 6,600-volt circuits presents certain hazards and a flashover attendant therewith would result in a power outage on the entire area.

For general application, it would be advisable to incorporate a more extensive primary switching system, such as to incorporate the ability to localize the area affected by an electrical failure, insure the ability to quickly restore service to any particular load area, and minimize power switching operations which must be performed on disconnecting switches.

John S. Parsons (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): Mr. McKeever has written an interesting and instructive paper on a very timely subject. The distribution system described, however, is not a low voltage or secondary network system. The term secondary network system is generally understood to refer to a system comprising a secondary circuit, loop, or grid supplied over two or more primary feeders through step-down transformers. The loads are fed radially from the common secondary circuit or grid. The system is designed so that a primary feeder and its associated transformers will be automatically disconnected from the system without any interruption to service when a fault occurs in a transformer or on the feeder. Under the condition of at least one primary feeder out of service full load can be carried without appreciably overloading any part of the system. The system described in the paper is usually referred to as a banked transformer system in which a number of transformers are so connected that they operate in parallel between a single primary feeder and a common secondary circuit, loop, or grid.

The banked transformer system of distribution is a definite improvement over the radial distribution systems usually employed in industrial plants. It can be designed to incorporate all of the important advantages of the secondary network except service reliability. Unlike the network system a primary feeder or transformer fault results in a shut-down of all loads fed over the feeder until the fault can be located and manually isolated from the system. The system does as pointed out by Mr. McKeever possess such important advantages of the network system as flexibility and good voltage regulation. To take advantage of flexibility, however, the sectionalizing dis-

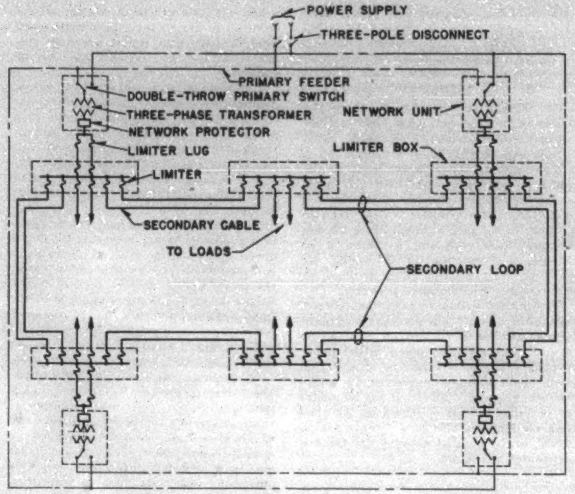


Figure 1

connecting switches in the primary loop should be capable of breaking load current, so as to permit switching transformers and sections of the loop out of service for maintenance and extension work without shutting down all load fed from the primary feeder while the switching is being done. Apparently the primary disconnecting switches in the system described by Mr. McKeever, based upon his description of switching procedure in the paper are load break switches, however, from the picture of them shown in figure 3 of the paper they do not appear to be.

Obviously any type of distribution system may take a number of different forms and we believe a form of secondary network system such as that shown in figure 1 of this discussion, having all of the advantages of the system described plus the important advantage of much greater service reliability could have been installed for the same or possibly less money. Such a network system would employ a grid or preferably one or more secondary loops consisting of two cables per phase, protected by limiters in each end of each secondary cable. The two cables per phase would be bussed at suitable points at which loads are tapped off the loop, and also at the points where power is fed into the loop. With this arrangement a faulty secondary cable will be cleared from the loop by means of its two associated limiters without interrupting service to any load. Only faults at a bus point or on the short radial circuits from a bus point to the individual loads will cause any interruption to service.

The secondary loop or loops would be fed through network units, each preferably consisting of a three-pole double-throw load-break primary air switch, a three-phase non-inflammable, non-explosive air transformer, and a network protector. Both radial primary feeders, each capable of carrying the entire plant load or that portion of it connected to their associated secondary loop or loops, are brought to each network unit. Normally half of the network units are connected to each primary feeder. When one feeder and its associated transformers are automatically disconnected from the system as the result of a fault the transformers associated with the faulty feeder are manually switched over to the good feeder by means of their double-throw primary switches. In this way the transformer capacity required can be kept down to the same value as that needed in the banked transformer system.

The same general form of system as that briefly outlined above can, of course, be used in the banked transformer system by replacing the two radial primary feeders with a single loop primary with load-break disconnecting switches between the points where the transformers are connected to the primary loop, and by replacing the network protectors with disconnecting switches. Such a system should cost less than that described by Mr. McKeever and has the advantage that a fault in a cable of the secondary loop or grid does not interrupt service to any load.

J. L. McKeever: Mr. Kaufmann suggests that for general application, it would be advisable to use a more extensive primary switching system to minimize the effect of 6,600 volt cable fault. It was stated in the paper that switching had been deliberately simplified and that short interruptions of power at very rare intervals were of small consequence. We believe most applications would be similar to ours and it would not be economical to aim at greater insurance than necessary.

Mr. Parsons' criticism is somewhat similar to that of Mr. Kaufmann and Mr. Parsons has described the system propounded by Messrs. Powel and Barnett in their paper given at the last winter convention. Rough estimates we have made do not indicate that their system could have been installed for less money. So far as the disconnecting switches for the primary loop in our system are concerned, these are not load-break switches as we do not consider such to be necessary. If one disconnect is opened it simply means a redistribution of load current from the two ends of the loop and the voltage behind the arc is merely the slight difference in voltage drop in the loop cables. Tests have shown ordinary disconnecting switches to be adequate for this purpose and also for breaking the transformer magnetizing current.

One point might have been given greater emphasis in the paper and that is the benefits to be derived both by the user and the electrical manufacturer in the use of standardized substation units. If the paper helps this idea along in any degree the writer will consider it to have been worth while.

A New Mercury Rheostatic Element for Regulation and Control

Discussion of paper 41-98 by K. A. Oplinger, presented at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941, and published in AIEE TRANSACTIONS, 1941 (August section), pages 846–9.

R. E. Hellmund (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The author in presenting a new regulating element has stressed the fact that all such elements really serve as amplifiers and that his particular device has an exceedingly high ratio of amplification. Incidentally, it is stated that in addition to such regulators and the well-known amplification methods using electronic tubes, rotating machines with commutators also act as electric amplifying devices. In any Ward-Leonard control system, for instance, the voltage of the generator, and consequently the speed of the driven motor, is governed by the field current of the generator. The latter is about one per cent of the output of the machine, which therefore corresponds to an amplification ratio of 100:1. If this is not sufficient, as may be the case with larger generators, double amplification can be obtained by using an exciter for the generator and controlling the field of the exciter. Machines for the latter purpose have been designed with a ratio of 400:1, so that a total ratio of 40,000:1 can be obtained. Recently the well-known cross-field generator having two fields at right-angles to each other, one of which is excited through short-circuited brushes on the armature, has been employed for double amplification. This arrangement, however, is limited to the smaller machines. For large machines very high ratios can be obtained by a combination of the amplification available in the main machine and a regulating device as described in the paper. This is only one of many possible combinations for obtaining a high ratio of amplification.

These amplification ratios, however, are only one of the factors to be considered in the solution of many kinds of regulating problems. Other factors to be considered are anticipatory features, anti-hunting features, speed of response, temperature influence, etc. It is therefore well to realize that we have at our disposal several types of standard apparatus, such as tubes, regulators, rotating machines, etc., which can be used for this purpose; also, that each of these basic members has certain advantages and disadvantages and that it is therefore unwise to use one of these means to the exclusion of the others. The best results undoubtedly can be obtained by using the device or combination of devices which will give the best all-round performance and lowest costs for the particular case at hand. I believe that with these devices at the disposition of the engineer, there are a great many unexplored possibilities for working out automatic controls of all kinds, not only for electrical machines but also for the great variety of non-electrical processes in industry.

The Incandescent Lamp Situation From the Engineering Point of View

Discussion of paper 41-123 by P. S. Millar, presented at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941, and published in AIEE TRANSACTIONS, 1941, pages 1174–80.

Walter C. Wagner (Philadelphia Electric Company, Philadelphia, Pa.): Mr. Preston Millar has been too modest in his statements relating to the outstanding contribution which he with his associates have made along engineering lines to the progress of the development of the incandescent lamp. May I present but one of the elements of contribution in this connection.

The electric supply company with which I am associated began its career under the personal direction of Mr. Thomas A. Edison. We have until recently supplied incandescent lamps to our customers in order to assure that they enjoyed at all times the greatest price-quality advantages. In this connection it cannot be too forcefully stated that we, among many other supply companies, have entirely and satisfactorily depended upon Mr. Millar's guidance over the years.

Referring to his comments on the related progress of interior wiring developments, he has truly indicated that to an extent it has hampered the advance of incandescent lamp lighting. I will present a brief paper on the subject of interior wiring at a Conference Session tomorrow.

Mr. Millar, as President of the International Commission on Illumination, and Chairman of the United States Committee of that body, on which the speaker also serves, should not have refrained from a statement concerning the importance of its work. He should also have indicated that his own contributions, among those of others, have in no small measure contributed to the outstanding international progress in the incandescent lamp art.

M. L. Sloan (General Electric Company, Nela Park, Cleveland, Ohio) (contributed): It has always been one of the objectives of the General Electric Company to pattern its improvements in lamp design along lines of sound engineering analysis rather than by promoting style changes intended to gratify the eye or offer an opportunity to increase sales merely by campaigning something different than the existing mode. In this manner, the composite result of all the improvements made over a period of years has been increased quality at a decreased price to consumers. When deciding whether suggested improvements should be incorporated, the final criterion has been whether the value of the article to the ultimate consumer is enhanced thereby.

We have always felt it fortunate that the utility companies who sell power have maintained such an interest by and large in the quality and efficiency of lamps used on their lines, and the activities which have been carried on by the Electrical Testing Laboratories in their behalf have ably supplemented the efforts made in our research laboratories to determine by exhaustive tests the merits of suggested changes in lamp design.

In this manner the substitution of coiled filaments for straight, the introduction of gas filling, the use of inside frost finish, the adoption of smaller bulb sizes, the use of coiled coil filaments in certain suitable lamp types, and many other improvements long since recognized as standard specifications, have all been tested and thoroughly analyzed by the Electrical Testing Laboratories before their stamp of approval in behalf of the ultimate customer was given, signaling the universal adoption of the improvement as standard practice.

In fact, such helpful influence as has been rendered by this type of testing and research work has been an instrumental factor in attaining the high degree of standardization which exists in the lamp industry, without which standardization it is doubtful if the present low prices for lamps could ever have been attained.

We believe any manufacturer can be considered fortunate to have the assistance of a well organized, thoroughgoing testing laboratory whose purpose is to check and recheck the findings of the manufacturers' engineers and to carry such tests into the field of usage as a regular means of appraising the value of innovations in product design, and in supervising manufacturing control of quality.

Philip Sporn (American Gas and Electric Service Corporation, New York, N. Y.): Mr. Millar's paper is altogether too brief a summary of the development of the incandescent lamp over the past fifty years, and particularly those engineering developments which have had a determining effect on the efficiency, reliability and economy of this great illuminant which has had such a tremendous effect on the development of what has come to be known as the modern world. It is particularly revealing to examine this development from the standpoint of efficiencies, cost, and use together with electric energy, to produce the electric light, and to find the enormous progress that has been made even in the last two decades not only to bring down the individual components but in combination to bring down the end product, that is, electric light. One cannot help but feel in reading this concentrated summary of Mr. Millar's that here is an outstanding example of a development made entirely under the advantages and handicaps, whatever they are, of the free enterprise system.

In that regard when so many of our citizens today are preaching the obsolescence of that system, and others are attracted to waves, of one kind or another, of the future, it is particularly worth while to examine table V cited by the author and compare the number of hours' work required to purchase an incandescent lamp in the United States in 1937 with the corresponding requirements in most of the countries of Europe. Some of these were at that time operating on a free enterprise basis, but a greater number was definitely restricted in its free enterprise or on an entire totalitarian basis. One must be forced to conclude after studying that table and similar figures, revealed in the original source cited by the author, for other items of living, that while the important components of the economic system, and the entire system, which produced these results in the United States may not be the very best from a social economic viewpoint and may have possibilities in it for much improvement in the future, no other system that has up to the present made its appearance on the scene of human society has as yet shown itself capable of coming anywhere as close to matching the results of this system of ours.

W. C. Kalb (National Carbon Company, Inc., Cleveland, Ohio): Mr. Millar brings out a fact of importance to manufacturers of electrical equipment of all types in the statement that "purchasing interest will stimulate producers to provide good quality while purchasing indifference will fail to bring such results." The progress in incandescent lamp production which Mr. Millar attributes in part to organized purchasing interest has a parallel in another field of illumination.

In one branch of the latter field there has been for many years organized demand for higher intensity and better quality of light, together with improved efficiency in light production. As a result, progress in this particular branch has been definitely more pronounced than in other branches of the same field. Charts of progress, recently prepared, show a striking resemblance to those presented in Mr. Millar's paper.

These observations suggest the thought that progress in other lines, where purchasing interest is not organized, might be stimulated by closer analysis of consumer needs on the part of the manufacturer.

E. M. Strong (Cornell University, Ithaca, N. Y.): Any one of the younger generation today who surveys the broad field of electrical engineering without benefit of history is unlikely to suspect that it all began with the development of the incandescent lamp.

Every school boy in the United States knows of Edison as the inventor of the incandescent lamp. He does not know that his contemporaries in other lands would differ with him in this particular but, what is more important, he is unaware that Edison's lamp is but symbolic of his larger contribution, the widespread distribution of electric power which contributes so largely to the better elements of our current civilization. Edison was not so much concerned with the incandescent lamp itself as he was with the larger project of making available, in the home, artificial light in an economically feasible abundance then unknown.

The subsequent years are eloquent testimony to Edison's practical wisdom in choosing and developing the incandescent principle as the proper vehicle for achieving his concept. Since the excitement which accompanied its successful introduction, the incandescent lamp has developed with notable absence of spectacular or revolutionary change. Nevertheless, over the years, its evolutionary development integrates into a remarkably impressive total.

The last decade has been marked by notable developments of a revolutionary character in the field of illuminants, which make signally appropriate some survey or "inventory" of the field of artificial illuminants with particular reference to the status of the incandescent lamp.

That Mr. Millar is uniquely qualified to perform this service for the profession, needs no emphasis for one in the field who reads his paper. We are indeed indebted to him for writing into the record so timely and well-balanced an account of the stewardship of industry for Edison's incandescent lamp.

In the heat of enthusiasm for that which is glamorously new it is especially important that we not mar its true value by injudicious application or utilization. I refer, at the moment of course, to the fluorescent lamp in particular. It is not appropriate here to discuss in detail the relative qualifications of fluorescent and incandescent lamps for specific applications or fields of application. In the main the two lamps are more complementary than competitive. It rests with the engineer to see to it that the development and application of the new be not marred and impeded by the public tendency toward misappropriation of his brain-children.

Measurements of Pre-Breakdown Currents in Dielectrics With a Cathode-Ray Tube

Discussion and author's closure of paper 41-96 by H. H. Race, presented at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941, and published in AIEE TRANSACTIONS, 1941 (August section), pages 854–8.

Herman Halperin (Commonwealth Edison Company, Chicago, Ill.): Doctor Race's data are of interest not only from a fundamental scientific standpoint but also from a practical standpoint. This applies especially to the results for number 5314 oil. The finding that pre-breakdown currents and breakdown stresses are greatly affected by the electrode surface is of special interest in connection with the recent use of conducting or semi-conducting layers adjacent to the electrodes of power cables.

That the breakdown stress seems to be little affected by the ionic conductivity is in agreement with observations on commercial oils where serious oxidation affected the conductivity, but not the breakdown strength in many cases.

In table I of the paper it is not made quite clear whether the second shot was obtained without change of oil or without repolishing of the electrode, or both.

With regard to the studies of the mechanism of conduction at high field stresses, in addition to the investigators referred to under item C3 of Doctor Race's paper, Dr. H. J. Plumley has investigated this problem in connection with studies done under the sponsorship of the Utilities Research Commission. His findings have been the subject of a recent paper in the *Physical Review* (volume 59, number 2, pages 200–07). Doctor Plumley's theory attempting to explain the current-voltage characteristics at high stresses is based on the assumption of ionic dissociation, but in contrast to Onsager and others he assumes that not polar impurities, but the molecules of the dielectric itself, become ionized.

It is of interest to compare figure 13 of Doctor Race's paper with Doctor Plumley's theory and test results for pure heptane. The theoretical slope according to Doctor Plumley's theory would be about 0.01 while the slope of his test results is about 0.015. It appears that Doctor Race's data obtained with the cathode ray oscillograph give a slope of roughly 0.01, while his data obtained with an FP54 amplifier give a slope of roughly 0.017.

Work such as done by Plumley and by Race may very well become of practical value, especially in connection with commercial insulation operated at very high stresses.

J. B. Whitehead (Johns Hopkins University, Baltimore, Md.): The increase of the conductivity of insulating liquids under high values of stress has been the subject of discussion for some time. It will be recalled that Baker and Boltz have proposed thermionic electron emission at the cathode as an explanation; Nikuradse proposes collision ionization; and more recently H. J. Plumley offers experimental evidence that the high stress conductivity is really electrolytic in character, and that, in fact, all insulating liquids have some inherent electrolytic dissociation which increases in the higher range of values of the applied stress.

Dr. Race's paper is interesting as showing that just before breakdown there is a relatively small increase in the pre-breakdown current and that the approach to breakdown is very rapid. From this he concludes that the breakdown is electronic rather than ionic in character. These results, therefore, seem to point away from collision ionization as a forerunner to breakdown. If, however, in accordance with Plumley, the high stress conductivity is electrolytic, it is probable that the ions present, though small in number, may still play a part in the accumulation of space charge concentration with resulting high local stress at the electrodes. From this point on it is easy to picture breakdown as electronic, as suggested by Dr. Race.

H. H. Race: In answer to the question in the third paragraph of Mr. Halperin's comments, the second shot was taken without either changing the oil or touching the electrode surface. Sufficient time was allowed to let the large gas bubbles rise to the surface of the oil. In the past we have repeatedly observed that the finely divided carbon resulting from a previous breakdown is swept out of the field by the motion of the oil long before breakdown occurs so that we attribute the changes in pre-breakdown currents to changes in the metal surface.

I agree with Dr. Whitehead's interpretation of Dr. Plumley's paper (*Physical Review*, volume 59, January 1941, page 200) that electrolytic ionization results in the intensification of electric gradients by space charge formation and thus may be a contributing factor in the final electronic breakdown.

Diode-Rectifying Circuits With Capacitance Filters

Discussion and author's closure of paper 41-105 by D. L. Waidelich, presented at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941, and published in AIEE TRANSACTIONS, 1941, pages 1161–7.

F. E. Terman (Stanford University, Stanford University, Calif.): Several years ago we had occasion to make a study somewhat analogous to that reported by Professor Waidelich. We included the effect of leakage reactance of the transformer, and found that this reduced the ratio of peak to average amplitude of the current pulse passing through the tube, and rounded off this pulse, but did not alter greatly the fraction of the cycle during which current flowed.

We also did some experimental work at the time using mercury-vapor rectifier tubes. We found difficulty under certain conditions, however, because the ignition voltage of the tubes that we used was normally greater than the voltage drop under operating conditions. That is to say, it took more voltage across the tube to start the conduction than it did to maintain the flow of current thereafter, and with some tubes the difference was considerable. This resulted in transients being set up which were especially troublesome during light loads, and which sometimes caused one of the tubes to fail to operate at all for brief moments as evidenced by flickering of that tube.

It would be interesting to know if Professor Waidelich has encountered any of these experimental difficulties, or if he was more fortunate in the characteristics of the rectifier tubes he was working with.

Reuben Lee (Westinghouse Electric and Manufacturing Company, Baltimore, Md.):

In AIEE paper 41-105 (May 1941) D. L. Waidelich has written a valuable paper on "Diode-Rectifying Circuits With Capacitance Filters." Previous analyses have dealt only with the case in which diode internal resistance is negligible. In the cases of appreciable diode resistance, calculation of performance required extremely involved equations, and many designers avoided them by resorting to experiment. Waidelich has now made both of these alternatives unnecessary by the plotting of his results in curves which reduce the designer's task to one of ease. At least, such is true of the relation between a-c and d-c voltage, and of the angle of conduction β-α.

In view of this accomplishment, it might seem regrettable that curves for maximum tube current, inverse peak voltage and ripple voltage are given only for the negligible diode resistance case. But while the paper does not so state, these latter relations can be found readily to close approximations for other values of diode resistance. For example, Waidelich shows in Figure 2 tube current of approximately quarter-sine wave form over the angle β-α. The average of this wave over 360 degrees is $0.636(\beta-\alpha)/360$ times the peak value.

Whence $i_m = \dfrac{360I}{0.636(\beta-\alpha)}$

How close this value is to actual peak current depends upon the ratio R_L/R_R. Series resistance R_R effectively rounds off the top of the tube current wave, making it more nearly a half-sine wave over β-α, the average value of which is the same as for a quarter-sine wave. For much practical work, the approximation is within five per cent of actual value.

The inverse peak voltage E_p is closely given by E_m+IR_L when the ripple voltage is small, as it is in many cases.

Ripple voltage, as found from Figures 15 and 16 or from equation 6 is true within a few per cent for all cases where $\omega CR_R \leqq 1$. This would be true, for example, if $\omega CR_L =$ 50 or less and $R_L R_R = 50$ or more. Usually ripple-voltage accuracy is not so important as accuracy in d-c output voltage or peak-tube current. The values from Figures 15 and 16 or equation 6 are always higher than actual when diode resistance is not negligible.

D. L. Waidelich: Reuben Lee and F. E. Terman have both contributed interesting discussions on the subject of the paper.

Mr. Lee offers some helpful approximation methods for the calculation of maximum tube current, inverse peak voltage, and ripple voltage for certain values of the circuit parameters and tube resistance. The values of the above three quantities may be obtained from the angles β and α given in the paper, but the work is tedious, and for design use the results should be given in curve form. These quantities are given in the paper, however, for the case of zero tube drop while conducting, and a project to complete the work for any tube resistance is being carried on at the present time.

The effect of an ignition voltage greater than the conduction voltage for mercury vapor tubes as reported by Professor Terman was not observed during this study, but it may have been masked by the high a-c voltages employed at the input to the circuit. Oscillatory currents of relatively high frequency were observed, and transients such as these may have led to the troubles experienced by Professor Terman.

It is possible by the use of the results given in the paper to take account of the resistance of the transformer by including it with the tube resistance. The effect of leakage inductance must be considered in some other way, but the general effect of leakage reactance is to reduce the peak to average ratio of the tube current and to round off this current wave shape.

Analytical Methods of Solving Discrete Nonlinear Problems in Electrical Engineering

Discussion and author's closure of paper 41-107 by E. G. Keller, presented at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941, and published in AIEE TRANSACTIONS, 1941, pages 1194–1200.

H. Poritsky (nonmember, General Electric Company, Schenectady, N. Y.): A great deal of general, sometimes rather vague, discussion of nonlinear problems has been going on recently, and it is gratifying to see an interesting paper like the present one which gets down to brass tacks and deals with explicit methods of solving nonlinear problems. I might as well confess that I had been stirred by intentions of writing a paper of a somewhat similar nature; therefore my former colleague's paper strikes a sympathetic chord in me.

The term "discrete" in the title makes one wonder at first what would be regarded as an "indiscrete" problem, until one discovers that by "discrete" is meant that *ordinary* rather than *partial* differential equations are referred to.

It is worth pointing out that Dr. von Kármán's recent paper, "The Engineer Grapples With Nonlinear Problems," which Dr. Keller describes as dealing primarily with nonlinear problems of continuous fields governed by *partial* differential equations, also contains a very illuminating discussion of several important problems in *ordinary* differential equations, as well as a good sized bibliography of this field.

As regards the first two methods of solution, they can both be grouped under the parameter method, the difference between them being merely the way in which the parameter enters. Indeed, if the functions f_4 occurring in the right-hand side of 13 reduce to zero, then equations 13 reduce essentially to the form 5; hence it follows that the first method is a special case of the second one.

The choice of the parameter r is an art in itself. Often no "natural" parameter presents itself, and it is advisable to introduce one artificially, for instance as a factor multiplying the "bothersome terms," that is, the terms whose absence would leave the differential equation soluble. When r is introduced artificially as a factor in this way, it is eventually eliminated by putting it equal to 1.

The parameter method consists in expanding the solution in powers of r. When these power series are substituted in the original differential equations with the parameter, themselves expanded in powers of r, and coefficients of similar powers of r equated on both sides, there result successive differential equations for the successive coefficients. The first equation must agree with the original equation in which r has been equated to zero, and should by hypothesis be soluble. The further equations turn out to involve one new function at a time and likewise should be soluble. If this is not the case then the choice of the parameter has not been a successful one. That the parameter can always be introduced so that the various differential equations are soluble at least for one or two stages may be seen from the fact that it can be introduced as a factor multiplying the right-hand members of 3 or 4. In the astronomical problem of motion of several planets (and the corresponding now old-fashioned Bohr theory of the atom) the parameter is often introduced as a factor in the interaction terms of the planets (or the electrons). Thus for $r=0$ each planet moves as if it were attracted only by the sun. The correction terms then correspond to the perturbation caused by the interaction of the planets. Very often the parameter r is chosen so as to leave a linear system for $r=0$.

The analytical existence theorems show that the series in powers of r converge for sufficiently small numerical values of r, but the estimate of what is sufficiently small may not be good enough for the problem at hand. Thus, as is usually the case in practical solution, what determines whether or not enough steps have been taken is whether the last term may be considered to be small for the desired accuracy, and more generally, whether the patience of the computer has not been exhausted to the breaking point.

The method of variation of parameters, which is well-explained in Dr. Keller's paper, might perhaps be even easier to understand if the reference to celestial mechanics were explained at greater length. In that field, it will be recalled, the orbit of a planet revolving around the sun is specified in terms of several parameters such as the eccentricity, the major axis of the elliptical path, and the orientation of the plane of the ellipse and the major axis in space. When interactions of other planets are included, resulting in a difficult nonlinear problem, it is customary to regard the perturbed motion as still taking place in an elliptic orbit but with slowly varying parameters. This is equivalent to changing from the original cartesian co-ordinates, in terms of which the motion is first described, to parameters specifying the osculating elliptical orbit. Needless to say, in general the method of variation of parameters is awkward to carry out.

As regards the hyperelliptic method, it is not clear to this reviewer from the example treated in section 4 just what this method is. This example is apparently worked out by the method of parameters along with several notions regarding the periodicity of the desired solution. There is nothing "hyperelliptic" about either of these two features illustrated in the example. The term "hyperelliptic" is commonly applied to integrals of the form $\int R[P^{1/2}, x]\, dx$ where P is a polynomial in x of degree 5 or higher, and R is a rational function of its two variables. Such an integral occurs in integrating the equation of motion $\ddot{x}=P'(x)$, where $P'(x)$ represents the force of a nonlinear spring. Indeed the first integration of the equation of motion leads to $x=[2P(x)]^{1/2}$ whence separation of variables and further integration results in $t=\int(2P)^{-1/2}dx$. However, this method of integration in general breaks down for nonlinear problems as soon as the number of degrees of freedom rises beyond 1. Therefore the example illustrating this method in section 4 could, in this reviewer's opinion, be omitted without harm and replaced by a description of just what the author had in mind.

Passing on to the method of treating nonlinear integral equations, the uninitiated would probably be scared by the mere appearance of these equations. For his benefit may it be said that differential equations may easily be converted into integral equations; in fact, the example given by Dr. Keller is derived from a differential equation. The method of solution outlined may be described as a method of successive substitutions. It is similar to the parameter method (say if the parameter were introduced as a factor of the right-hand integrals in (41)), yet in general it is not equivalent to it. In this reviewer's opinion, integral equations, particularly nonlinear integral equations, have not occurred sufficiently frequently in electrical engineering applications and their discussion could still be properly left best to the privacy of mathematical experts.

As regards the last method, the method of Galerkin, while an example in it is given, the logic of this method is not exactly clear from this example. What this method consists in at bottom is in choosing a solution involving only a finite number of arbitrary parameters, for instance a polynomial in time (this is the case in the example treated), then, finding that it is impossible to satisfy the original differential equation, trying to satisfy it as nearly as possible by minimizing the error. To obtain this error let ϵ be the result of substituting the assumed solution in the differential equation after all its terms have been transposed on the left-hand side; the error may then be measured by the integral of the square of ϵ over the interval in question.

Very often the solution of even *linear* differential equations is by itself a difficult matter. The various methods illustrated, of course, apply to linear systems too, and in introducing them it is perhaps not improper to illustrate their use on linear systems. With this in mind, and hoping that the readers will excuse the highly elementary nature of the following example, consider the first order, yea even linear, differential equation

$$\frac{dy}{dt}=ry, \quad y=1 \text{ at } t=0$$

What does the parameter method look like in this case? Assuming an expansion of y in powers of the parameter r, it turns out that the various powers of r yield the successive terms of the expansion of the exponential solution

$$y=e^{rt}=1+rt+r^2t^2/2!+\ldots$$

in powers of t. This means that in the present case the expansions in powers of the parameter are completely equivalent to the expansion in powers of the variable itself. The method of solution which consists in expanding the solution in powers of the variable itself, is perhaps the oldest of all the methods of solution and presumably has not been included by Dr. Keller because it is too elementary, or possibly because it does not converge rapidly enough. The present example shows that, at least in some cases, its convergence is no better or worse than the convergence of the parameter method.

Incidentally, the most thorough application of the parameter method has been to *linear* differential equations with periodic coefficients. The equations arise often in engineering and are fully as bothersome as nonlinear problems.

One often runs into the question of stability or instability of solutions of differential equations of the form

$$\frac{dx_i}{dt}=f_i(x_j)$$

near a singular point at which all the fs vanish. Here a proper linear system can be used to advantage in discussing the nature of the singular points and the resulting stability. A brief and very readable exposition of the second order case may be found in von Kármán's and Biot's book, "Mathematical Methods in Engineering," pages 150–7. (It is surprising, in fact, that no reference to this text has been made in the bibliography.) Thus at least in this case a real analytical aid exists, whose application may save a great deal of labor.

It has apparently been the author's intention to comfort the electrical engineer by means of examples chosen from his domain. The nontechnical reader should be cautioned however, against using the examples away from their context. Thus equation 22 has been called "the equation of hunting"; it is perhaps more correctly an equation of pulling into step of a synchronous machine.

The bibliography is one of the most valuable parts of the paper. Yet it seems to be aimed at the mathematical expert, as the average engineer would find most of the references forbiddingly difficult reading.

The reader is reminded that in addition to various analytical methods there are, of course, well known all-powerful numerical methods available for solving differential equations, these methods applying equally well to nonlinear as well as linear problems.

In conclusion it appears that the differential equations of electrical engineering are soluble by the same general methods that have been developed for treating differential equations of other domains—dynamics, celestial mechanics, and other fields. It is to be hoped that the electrical engineer to whom this paper is addressed will not be completely discouraged by the apparent complexity of the various methods so as to desert analytical methods altogether and revert entirely to numerical methods and the use of the differential analyzer.

Ernst Weber (Polytechnic Institute of Brooklyn, Brooklyn, N. Y.): Though any engineering problem, if somewhat rigorously formulated, leads to nonlinear differential or integral equations, it has been customary to introduce sufficient simplifying assumptions in order to arrive, preferably, at linear differential equations for which fairly exhaustive mathematical methods of solution are available. The solutions of the nonlinear relations, as desirable as they may seem in many cases, are usually left to professional mathematicians because of the strenuous requirements with respect to the selection of attack, the tediousness of even the most opportune method, and the danger of losing contact with the physical interpretation of the successive steps.

In order to stimulate a greater interest, at least among electrical engineers, in the promotion and more extensive use of advanced methods, the committee on basic sciences sponsored a conference on, "Principles of Nonlinear Analysis," which was held at the summer convention in Swampscott, June 1940. Four different methods were presented and discussed: namely, graphical, numerical, machine, and analytical ways of solving nonlinear problems. The excellent response of an interested audience caused Professor Keller to give us the benefit of his studies by preparing this formal paper. Its particular value lies in the logical presentation of the most powerful analytical methods supported by important practical applications, and in the clear emphasis on the essential phases of each method; so that a decision can easily be made in the selection of the most suitable method in a special case. Extremely valuable is also the extensive bibliography, in its arrangement according to theory and applications; and the helpful subdivisions in each field. It may be hoped that the leadership expressed in the publication of this basic paper in the field of applied mathematics, will bring us other important contributions; since fundamental progress in engineering is very intimately connected with a broad background and a far-reaching appreciation of the basic sciences.

E. G. Keller: The author wishes to express his deep appreciation to Drs. H. Poritsky and Ernst Weber for their splendid discussions of the present paper. With the two following exceptions there is nothing to be added which would not detract from their clarifying discussions. References to a noteworthy paper "Operational Methods in Nonlinear Problems" by Dr. Louis Pipes and to the well-known text "Mathematical Methods in Engineering" by Th. von Kármán and A. Biot (McGraw-Hill Book Co., New York) have been added to the bibliography.

Current Rating and Life of Cold-Cathode Tubes

Discussion of paper 41-117 by G. H. Rockwood, presented at the AIEE summer convention, Toronto, Ontario, Can., June 16–20, 1941, and published in AIEE TRANSACTIONS, 1941 (September section), pages 901–03.

P. L. Alger (General Electric Company, Schenectady, N. Y.): This paper is both timely and useful, because it gives valuable information on the life expectancy and proper rating methods for cold cathode tubes, and because this information is helpful in connection with the study of rating methods for other types of apparatus.

Most electrical apparatus has temperature limits that are set by the rate of deterioration of the organic insulation, which follows the approximate rule that the life of the insulation is halved for each 8 degree to 12 degree centigrade increase in temperature. Apparently the life-current relationship for a cold cathode tube can not be expressed by any such simple logarithmic law. It will be interesting, however, to know whether tests made at different ambient temperatures with a given current would show a reduction in life expectancy with increase in temperature, similar to that for insulation.

It seems to me very important to give further study to this general subject of life-output and temperature laws for different types of apparatus, since only by such knowledge will it be possible to design thoroughly coordinated and economical systems involving a variety of types of equipment.

Whatever the type of life expectancy law, we may be sure that with every type of apparatus the life will be shorter the greater the load that is put upon it. On this basis, it seems desirable to use a rating system in which the nameplate tells the safe value of sustained load that can be carried on a typical short time application, and then to indicate the permissible load that will insure an indefinite life by a service factor or multiplier applied to the nameplate rating. From this point of view, a satisfactory way to rate these cold cathode tubes when used in intermittent duty, should be to make the nameplate rating equal to the current that would give a safe life expectancy of say 100 hours, and also give on the nameplate a service factor which would indicate the current corresponding to a safe life expectancy of 10,000 hours or more.

R. E. Hellmund (Westinghouse Electric and Manufacturing Company, Schenectady, N. Y.): The presentation of this paper is very timely and it should be of considerable assistance to the subcommittee on conduction in vacuum and low-pressure gases and vapors recently organized by standards coordinating committee 7. This subcommittee of course realizes that there are many factors involved in any basic standardization of electronic devices and considerable time will no doubt be consumed before much of the work is completed.

The basis for the standardization of the tubes discussed in this paper is, of course, quite different from the conventional considerations of the past in the standardization of machines and other electro-magnetic apparatus. However, there are some similarities in so far as the influence of time together with temperature is now receiving attention in machine standards. During the early stages of the standardization of machines it was the general belief among engineers that if certain temperatures were exceeded for even short periods it would be detrimental to the insulation of the machine. Railway engineers, confronted with severe space and weight limitations, first exceeded the conventional temperature limits, expecting, however, very short life as a result. Considerable trouble actually was experienced and frequent re-winding was necessary, but this probably was caused by improper methods of applying and supporting the insulation to a greater extent than by its being roasted. Somewhat later, higher temperatures were used in some cases for short-time ratings, but only during the last few years has it been generally recognized that insulation life is affected not by high temperatures alone but to an appreciable extent by the length of time high temperatures prevail. It is only very recently, however, that this basic principle has been fully recognized in a standard (No. 1-A) under preparation by standards coordinating committee 4. Since electro-magnetic machines have been in commercial use for more than sixty years, we have been a long time in arriving at standards recognizing the influence of time and temperature upon the life of a machine. However, the standardization of electronic devices, in spite of its complexity, will not take such a long time, because, in contrast to the early practices in machine design, the electronic groups have the benefit of well organized research activities and scientific methods of attack, a good example of which is the present paper.

The Recovery-Voltage Analyzer for Determination of Circuit Recovery Characteristics

Discussion of paper 41-145 by G. W. Dunlap, presented at the AIEE Pacific Coast convention, Yellowstone National Park, August 27–29, 1941, and published in AIEE TRANSACTIONS, 1941 (November section), pages 958–62.

W. F. Skeats (General Electric Company, Philadelphia, Pa.): The engineer has in general two ways of determining the magnitude of quantities in which he is interested. He may measure, or he may calculate. For the determination of individual values, the most reliable figures are usually those obtained by full scale measurement. When one wishes to obtain enough values to acquire perspective, however, to see the effect of the variables involved, or to survey a field, this method may require so much time and expense as to be wholly impractical. In such cases, calculations often serve well, but in some cases, on account of their complexity or the difficulty of obtaining the data on which to base them, they become impractical or uncertain. In many such cases, measurements on small scale apparatus or measurements of a small scale simulation of the actual event may be very helpful. Quite often it happens that a combination of two or three of these methods of attack yields results far surpassing what can be obtained from any one of them.

The subject of Dr. Dunlap's paper is an apt example of the last situation. Here the acquisition by full-scale circuit breaker tests of sufficient information for reliable generalization would be hopelessly time-consuming and expensive. Calculations of the recovery voltage characteristic can generally be made, but they often require simplifying assumptions and sometimes involve uncertainty of data. While the equipment described by Dr. Dunlap does involve the inconvenience of deënergizing the circuit to be tested, it is hoped that it will be found possible to make sufficient application of it to confirm conclusions based on calculation.

Self-Excitation of Induction Motors With Series Capacitors

Discussion and author's closure of paper 41-139 by C. F. Wagner, presented at the AIEE Pacific Coast convention, Yellowstone National Park, August 27–29, 1941, and published in AIEE TRANSACTIONS, 1941, pages 1241–7.

C. Concordia and **J. W. Butler** (General Electric Company, Schenectady, N. Y.): In previous discussions of the possible self-excitation of induction motors when their supply line reactance is compensated with series capacitance, we have based the specification of conditions under which self-excitation may arise, and also the specification of the proper corrective means (if necessary), on the operating condition, that is, on normal motor speed and load. It was recognized (see reference 1, page 981 and figure 11) that at low motor speeds, as during starting, an unstable range as shown in figure 4 of the paper might have to be traversed, but it was concluded that with normal starting conditions, this region could safely be tolerated. We would be interested in learning Mr. Wagner's experience which leads him to stress the low speed condition.

We wonder also why Mr. Wagner did not pursue further the method which we have found so successful and which he has briefly mentioned in appendix I. It has always seemed to us that a proper theory should be more or less self-sufficient, that is, it should not depend on knowledge of the answer in order to obtain it. It is believed that a defect of the explanation and method as given in the paper is that it does not really get to the root of the matter and discover the fundamental reason for self-excitation. Even though performance can be calculated under many conditions, it remains essentially an empirical method, unless the self-excitation evidences itself directly in the formulae developed. The direct, physical, simple, and rational method of merely writing down the differential equations of the system and then testing them directly to see if the system is stable or unstable has always appealed to us, and we cannot sympathize with the effort to avoid the differential equation at all costs, even at the cost of trying to obtain from the steady-state equations evidence of an instability, the real determining factors of which have inherently been left out.

For instance, equations 10 and 11 in Mr. Wagner's paper show that instability exists in the region $f^2X_b < X_c < f^2X_M$. In this region the circuit is unstable, but it is not necessarily stable outside of this region. Referring to figure 10 of the Butler and Concordia paper (reference 1) it is seen that an unstable region exists for values of $X_c < f^2 X_B$, and curiously enough, this is the region where most series capacitor applications seem to fall. The intersections of the dotted curve on the X_c axis are the limits defined by Mr. Wagner—and it can be seen that an unstable region exists for X_c lower than 0.4, which is the $f^2 X_B$ point, for $r = 0$. Mr. Wagner's equations 10 and 11 define the regions that are surely unstable, and what we would usually like to know when applying series capacitors are the regions that are surely stable. And the steady-state equations cannot give us this as the real determining factors have been left out, as mentioned above.

C. F. Wagner: The discussion by Messrs. Concordia and Butler being the only written discussion, the following remarks are directed in reply to it. Emphasis was placed by the author upon the starting condition because such condition was one of the few cases with which trouble was experienced. This treatment also served to illustrate and verify fundamentals involved in the problem.

The author did not pursue the method of Concordia and Butler, first, because it appeared more complicated than other methods and, second, because it was incomplete. Any problem involving spontaneous oscillations can be solved by either of two methods. First, since the oscillations are sustained, steady state solutions can be used and the conditions determined for which both the resistance and reactance are equal to zero. Second, the differential equations governing the problem can be set up and the roots of the auxiliary equation obtained. These roots determine the free oscillations of the circuit, the imaginary part fixing the frequency and the real part the time constant. The conditions for which the real part of one or more roots is equal to zero determine the limits for which the circuit becomes self-exciting. Both solutions are equally exact. Messrs. Concordia and Butler preferred the latter, and the author the former except for the development in the appendix. This choice was made because of the simplicity of the attack and because the physical processes involved are more apparent. The relative simplicity is apparent by comparing the type of solution resulting from solving either equation 6 or 7 of reference 1, and the type of solution obtained by solving simultaneously equations 10 and 11 of the paper.

Messrs. Concordia and Butler labor under the impression that the author's solution is approximate. They state in referring to figure 10 of reference 1 that the "intersections of the dotted curve on the X_c axis are the limits defined by Mr. Wagner." Such statement and the remainder of the discussion in their last paragraph indicate an utter lack of understanding of the method. The f used in the paper refers to the frequency of the resonating currents. In using equations 10 and 11 to compute a curve such as figure 10 of reference 1, it will be found that for small values of X_c, f likewise becomes small and a curve such as the full line will be obtained.

A further distinct advantage of the author's approach lies in the retention of the conception of the physical processes involved. For example, the purely mathematical approach indicates that both the lower and the upper resonant speeds of figure 3 constitute boundaries between stable and unstable operation. The mathematics shows that between these limits the resonating currents tend to increase and above the upper speed and below the lower speed the currents tend to decrease. Actually, however, as shown in the paper, only the lower speed is a stable operating point. The power fed into the resonating circuit is dependent only upon the ability of the 60-cycle circuit characteristics of the motor to feed power at this speed. This is at complete variance with the statement appearing on page 980 of reference 1 which reads, "This low-frequency current is limited only by the impedance of the supply circuit at the low frequency, and may reach relatively large values," and on page 982 which reads, "the current builds up until the circuit constants change to a value that will not permit further excitation."

The lack of consideration of the speed torque influence in the stability criterion leads to further errors. If a particular set of motor characteristics is assumed and the series capacitor varied, then, as the reactance of the series capacitor increases, the frequency curves shown in the lower part of figure 3 move upward and their maximum abscissa decreases. The speed curves are affected similarly. For speeds near synchronous it will be seen that points on the speed curve occur on the dotted portion of the curves until larger values of series capacitors are used. In terms of figure 10 of reference 1, this implies that portions of the curve to the left of the maximum resistance are unstable.

Messrs. Concordia and Butler have not disagreed with the author's statement regarding instability at the upper resonant speed. In view of the voluminous criticism of other points, the lack of criticism of this implies approval and acknowledgment of the omission of a real factor in their analysis. Rather than the author omitting "real determining factors," it appears as though the analysis by Messrs. Concordia and Butler does just that.

The author is at a loss to understand the discussers' statement that "a proper theory should be more or less self-sufficient, that is, it should not depend on knowledge of the answer in order to obtain it." Perhaps this is further evidence of their lack of understanding of the paper.

Lightning Strokes in Field and Laboratory—III

Discussion and author's closure of paper 41-141 by P. L. Bellaschi, presented at the AIEE Pacific Coast convention, Yellowstone National Park, August 27–29, 1941, and published in AIEE TRANSACTIONS, 1941, pages 1248–56.

R. C. Mason (Westinghouse Research Laboratories, East Pittsburgh, Pa.): Since the electrical processes of the electric arc are intimately connected with the energy input to the arc electrodes, the "heat balance" at the electrode has long been of interest to physicists studying gas discharges. The subject is equally vital to welding engineers who are concerned with melting rates of electrodes. In this paper, Mr. Bellaschi has shown how important information about lightning discharge can be gained from the study of the terminals of the discharge. His contribution can rest solely on the correlation of field and laboratory data, on empirical calibration established by observation of effects produced by currents of known duration and magnitude; but what is its relation to the theory of the arc? The theory of the arc is still so incomplete that instead of one attempting to predict from theory what Mr. Bellaschi should have observed, it is much more to the point to ask what light does his result show on still obscure questions?

The proportionality of metal fused to electrical charge suggests that very probably the current density at electrodes was approximately constant for the arcs studied by Mr. Bellaschi. Further, his work indicates equal heat inputs to cathode and anode, of about six watts per ampere.

Although heat may be given up to the electrodes by the hot gas of the arc column, both by thermal conduction and by recombination of dissociated atoms, it seems certain that this mechanism is of much less importance than direct electrical means. At the cathode, current is carried both by positive ions arriving at the cathode from the gas phase, and by electrons leaving the cathode metal. The positive ions impart to the cathode the energy they have gained from the cathode fall of potential, plus a certain experimentally undetermined heat of condensation. In some arcs, known as "thermionic arcs," the electrons are emitted by virtue of the high temperature of the cathode, and in these the electrons leaving the cathode cool the metal by a definite amount, the electron "work function" of the metal. In other arcs, of which the copper arc is believed to be one, known as "high field arcs," electrons are emitted because of an extremely high electric field at the cathode. In the latter case the electron emission actually heats the cathode slightly. Thus the uncertainty of cathode fall, positive ion heat of condensation, type of cathode mechanism, and, above all, of the fraction of the current carried by electrons and the fraction by positive ions, makes an exact determination of the heat balance at the cathode impossible. The observed figure of six watts per ampere can hardly be obtained, however, unless $^1/_4$ to $^1/_3$ the cathode current is positive ion current.

At the anode, current is carried only by electrons flowing from gas to metal, so the heating effect is simply the energy of electrons in the positive column, plus that gained from any anode fall which may exist, plus the latent heat of condensation of the electrons. The last is a little over four watts per ampere for copper, while the average energy of electrons in the positive column is not much more than one watt per ampere. The observed six watts per ampere thus leave scarcely any to be attributed to anode fall. Indeed it may be possible that a practically zero anode fall may have existed for these arcs. The only factors neglected have been the thermal conduction *to* the anode, and loss of heat *from* the anode by vaporization of anode material; it appears probable that neither of these is large compared to the factors actually considered.

P. L. Bellaschi: The discussion Dr. Mason presents on the theoretical considerations of the electric arc is interesting indeed, for it emphatically shows the practical difficulty in attempting to determine the relation between the fusion at the electrode and the electrical quantities from a purely theoretical approach. It is clear throughout the paper that the investigation, the correlation between field data and the laboratory results, and the conclusions rest entirely on an experimental basis. The usefulness of the findings in other connections, such as pointed out by Dr. Mason, is apparent.

High-Voltage D-C Flashover of Solid Insulators in Compressed Nitrogen

Discussion and author's closure of paper 41-146 by John G. Trump and James Andrias, presented at the AIEE Pacific Coast convention, Yellowstone National Park, August 27–29, 1941, and published in AIEE TRANSACTIONS, 1941 (November section), pages 986–90.

Henry M. Hobart (retired, General Electric Company, Schenectady, N. Y.): It is noted that the authors deal exclusively with d-c flashovers. The flashover strengths over the surface of corrugated cylinders of $^{1}/_{2}$ inch height, fitted between electrodes in dry-nitrogen at a pressure of 200 pounds per square inch are:

For Lucite,	550 volts per mil of height
For Textolite	340 volts per mil of height
For Isolantite	260 volts per mil of height

For a $^{1}/_{2}$ inch spacing in dry nitrogen at 200 pounds per sq inch, the value of the gradient at breakdown of the nitrogen is seen from figure 4 of the paper, to be about 800 volts per mil.

It would be much appreciated by the writer if he could have the advantage of the comments and suggestions of the authors with respect to an important case which is analogous, except that instead of employing d-c, the stresses are 60-cycle a-c.

All will agree that for instantaneous and short-time breakdown tests of solid insulating materials, the volts per mil through the mass, which can be withstood, will, speaking broadly, usually be of much the same order of magnitude for a-c as for d-c. But when the duration of the application of the stress is extended to hours, days, weeks, and years, the solid material ultimately fails with a-c stresses which are of but a small fraction of the magnitude of the d-c breakdown stresses. For example, an underground cable with solid-filled paper insulation of 1 inch radial thickness interposed between a 1 inch-diameter conductor and an outer lead sheath, could be employed in service for many years with a d-c stress of 500 kv between the copper conductor and the lead covering, the average stress thus being 500 volts per mil. But for a-c service, such a cable would fail inside of a very few years if subjected continuously to a 60-cycle stress of as little as 100 volts per mil.

The writer has considered that the insulating supports and spacers in a 60-cycle compressed-gas underground cable of such a design as shown in the accompanying sketch, would not be adequate for their purpose unless the total distance through the interposed solid insulating material amounts to several feet rather than a couple of inches (which is all that would be necessary so far as relates to the compressed gas itself). Also, since the path through the mass of the solid insulation is so crooked, and in other respects so unfavorably proportioned, there would be severe stress concentrations.

For a 345-kv three-phase transmission system, with a separate pipe for each of the three conductors, there would be a 60-cycle stress of 200 rms kv between the outer surface of the conductor and the inner surface of the pipe in which it is enclosed. The compression may appropriately be 68 absolute atmospheres, and it is evident that the radial thickness of compressed-gas shown in the sketch is liberal so far as relates to the gas alone. For this hypothetical case, would the authors, on the basis of their experience with solid insulating supports in compressed gas, consider that it would be sufficient to have a lateral spacing of some three feet between the porcelain rings *A* and *B*? This provides an average stress of 5 volts per mil through the mass of the solid insulation. Do the authors consider that the stress concentrations in this approximately 40 inch-long dielectric path would be liable to occasion stresses of more than some 50 volts per mil at any of the places where the concentrations are greatest?

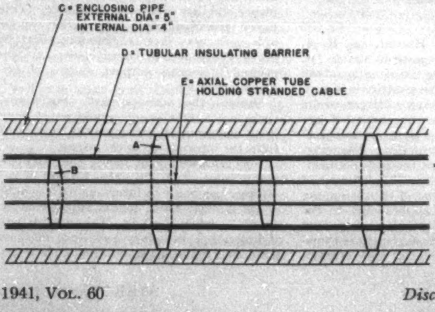


Figure 1. High-voltage pipe-enclosed underground conductor with compressed gas insulation

The design shown in the sketch is only one of several good alternatives which could be employed. Taking into account (1) insulation strength, (2) dielectric constant, (3) mechanical strength, (4) cost, and (5) freedom from all forms of aging (especially on the surfaces), would the authors express an opinion (based on the experience gained by them in the course of the valuable research which they have described in their paper), as to the best insulating materials to employ for *A*, *B*, and *D*?

The results set forth in their paper are very reassuring as regards d-c applications for compressed gas insulation. But the writer's present interest is particularly with the 60-cycle transmission of large amounts of power to great distances. His project includes the provision of large compressed-gas power transformers, and the data in the paper are also of interest in that connection. The power transformers will be entered from the compressed-gas underground line through pipe connections into the transformer tank, thus avoiding the need for the very expensive high-voltage bushings of present practice. Lightning cannot readily enter the underground line. Any surge gaining access to the underground line is gradually attenuated in the course of its journey, and the transformer will require little or no shielding and hence will cost less than a transformer connected directly to an overhead transmission system. Its size and cost will also be decreased because the temperature rise may exceed the limits imposed by sludging in oil transformers.

The writer wishes to take this occasion to express his indebtedness and thanks to the authors of this paper for their fine research work on the fundamental principles underlying the use of compressed gases as insulation. There is need for many more such fundamental researches in this important field.

John G. Trump: Since the flashover studies reported in this paper were for constant potentials applied under nearly ideal conditions and in a uniform field, it is difficult to use these data to comment quantitatively on the 60-cycle transmission line design which Mr. Hobart discloses. From general considerations, however, it would appear that the lateral spacing of three feet between the porcelain rings *A* and *B* is sufficient from the point of view of average gradient along the length, but that difficulties will arise because of the electrical dissymmetry in the regions between the porcelain rings and the tubular barrier and between the rings and the inner and outer conductors. I should expect that, with the dimensions used and at the voltage contemplated, the electric field concentrations in these regions would be sufficient to cause ionization, space-charge accumulations on the barrier, and final breakdown. While Mr. Hobart's method of supporting the central conductor is quite ingenious, I would be inclined to use a single corrugated insulated ring to support the inner conductor, thereby sacrificing compactness for simplicity. Perhaps the best material for such a ring would be hard glass rather than porcelain, since this will exhibit considerably higher dielectric strength through the volume.

Mr. Hobart's generous comments are much appreciated.

An Analysis of Transient and Sustained Voltages in Ground-Fault Neutralizer Systems

Discussion and authors' closure of paper 41-148 by S. B. Farnham, E. M. Hunter, and H. A. Peterson, presented at the AIEE Pacific Coast convention, Yellowstone National Park August 27–29, 1941, and published in AIEE TRANSACTIONS, 1941, pages 1257–66.

Eric T. B. Gross (College of the City of New York, N. Y.): It seems worthwhile to mention that the miniature system study proves the results of previous investigations in which both the effects of unbalanced capacitances-to-ground of the systems[1] and those of the saturation of the coils were fully analyzed.[2] It is of historic interest that even the first coil put into service (1917, in Europe) was a saturable iron core reactor and no other type has ever been used abroad.

As in all cases of resonant circuits, the losses of the various elements of the circuit play an important role and must not be neglected; as the in-phase component of the fault current in a compensated network lies between about 5 per cent and 15 per cent of the charging current to ground, the influence of the losses is considerable. In the previous investigations mentioned, the losses were not neglected.[3] Jonas showed[1] how the resonance voltage was limited by the losses and the late H. Gruenholz gave a very ingenious solution for the influence of both saturation and losses.[3] Small unbalances of the capacitances to ground of the system are completely harmless,[4] and because of this earth fault coils are applicable in systems where relatively short line sections consist of only two conductors connected to two of the three phases. No dangerous resonant overvoltages will occur under such conditions even if the earth fault coils are only slightly saturated; for various reasons, it is advisable to choose the factor K not exceeding about 1.6.

Concerning the "Effect of Bus Capacitance" and the suppression of overvoltages due to switching in the single line case,[5] I would refer to a previous discussion.[6] Then it was said that this is the only case in which dangerous transient overvoltages can be induced by the coil, however the installation of a modern-type arrester-by-pass reduces all overvoltages to harmless values.

In connection with the chapter, "Switching Off Neutralizers," and conclusion 3, we know from theoretical considerations and practical experience with more than 2,000 coils abroad that there is no difference to speak of between self-extinction of arcs-to-ground with the coil out of tune in the one or other direction, that is, coil reactance higher or lower than the tuned value. Ten per cent undercompensation is a very distant approach to the condition of isolated neutral operation; the frequency of the oscillation after arc extinction is only about 5 per cent different from the tuned condition and because of this the extinguishing action of the coil is of very slight difference from the case with exact tuning. Of course, arc extinction with the coil 10 per cent (plus or minus) out of tune is better than with ±25 per cent; however, both values, plus *and* minus, are valid if one of them, plus *or* minus, is permissible. Advisable limits for the adjustment of the coils are about ±25 per cent in the case of operating voltages up to 30 kv, about ±20 per cent in operating voltages up to 60 per cent and ±10 per cent with higher operating voltages. It is well known too and recognized by all operating companies really using fault coils that such out-of-tune conditions do not hinder the self-extinction of arcs-to-ground, and it is for this reason that earth fault coils do not offer any difficulties to system operation even in systems where growth and development is experienced and where system switching may change from day to day, or week to week. In a number of cases abroad, it has been the successful practice for many years to keep the coils somewhat undercompensating if all the line sections were switched in, and in this way extinguishing conditions were procured even if larger sections were temporarily switched off. It is a long and well established fact that not the amount of the remaining current at the point of the ground but the degree of unbalance is responsible for the self-extinction of arcs-to-ground.[7]

REFERENCES

1. PROTECTION OF HIGH-TENSION NETWORKS IF THE ADMITTANCES FROM PHASE TO GROUND ARE UNBALANCED, J. Jonas. *Elektrotechnik und Maschinenbau*, volume 38, 1920, page 453.

ADJUSTMENT OF EARTH-FAULT COILS, Eric Gross. *Bulletin* Schweizerischer Elektrotechnischer Verein, volume 38, 1937, page 166.

2. RESONANT VOLTAGES AND EARTH-FAULT COILS, H. Gruenholz. *Elektrotechnik und Maschinenbau*, volume 42, 1924, page 194.

3. GROUND FAULTS IN HIGH-TENSION SYSTEMS (a book), R. Willheim. Verlag Julius Springer, Berlin, 1936, pages 179–83.

4. Reference 3, pages 170–6.

5. Reference 3, pages 243–9.

6. DISCUSSION OF ARCING FAULTS IN POWER SYSTEMS. AIEE TRANSACTIONS, volume 60, 1941, page 617.

7. THE DEGREE OF UNBALANCE WITH INDUCTIVE EARTHING AND ITS MEASUREMENT, Eric Gross. *CIGRE*, Paris, 1937, volume 3, report number 326.

S. B. Farnham, E. M. Hunter, and **H. A. Peterson:** The authors wish to thank Dr. Gross for again directing the attention of the institute to the European practice in regard to ground-fault neutralizers. His comments in general substantiate the results of our study, and, for the most part, are in agreement with American practice. Nevertheless, several points merit further comment.

1. The statement, "Small unbalances of the capacitance to ground of the system are completely harmless," applies only to the arc-extinguishing ability of the ground fault neutralizer. This statement is by no means true from the point of view of practical system operation, since it is well recognized that in a ground-fault neutralizer system any appreciable unbalance of the system capacitance to ground produces substantial inequality among the three line-to-ground voltages and causes continuous circulating currents. The resulting adverse effects on adjacent power and communication circuits, as well as metering and relaying difficulties, may make this a troublesome operating problem. The practical limit to capacitance unbalance is roughly one per cent, and line transpositions or other balancing means must be resorted to if this figure is exceeded.

2. With reference to the section, "Switching Off Neutralizers," and conclusion 3 of our paper, it is unfortunate that Dr. Gross has overlooked the significant feature of this part of our study. The question is not whether successful arc extinction can be obtained with the neutralizer somewhat out of tune: we know from experience that under favorable conditions, the arc has been extinguished with the neutralizer 30 per cent out of tune. The essential point is that saturation of the neutralizer is very effective in preventing overvoltages, and if the neutralizer is initially out of tune in the direction of undercompensation, then this beneficial effect of saturation cannot be realized. With the remotely controllable tap changing switches being furnished in modern ground-fault neutralizers, there is little reason why correct tuning should not be maintained.

Criteria for Neutral Stability of Wye-Grounded-Primary Broken-Delta-Secondary Transformer Circuits

Discussion and authors' closure of paper 41-147 by H. S. Shott and H. A. Peterson, presented at the AIEE Pacific Coast convention, Yellowstone National Park, August 27–29, 1941, and published in AIEE TRANSACTIONS, 1941 (November section), pages 997–1002.

A. T. Sinks (General Electric Company, Lynn, Mass.): This paper covers a subject which, although infrequently a source of trouble, can be a considerable nuisance at times, particularly when there is reason to believe that instability may exist and a knowledge of what to do and how much to insure stability is also not available. This paper gives sufficient information to be able with very little engineering effort to apply Y-connected transformers on ungrounded systems without danger of unstable operation.

Besides the authors' own conclusions, there are a number of corollaries of very great and practical value that can be derived from the information in the paper.

For instance, it is shown that for the worst value of X_{co}/X_m the amount of resistance per phase to insure stability is only $^1/_7$ of X_m for $E=1$, and is approximately $^1/_3$ of X_m when $E=0.58$. This means that if individual secondary resistance burdens are connected to the transformers, the watts that have to be dissipated in the resistors and supplied by the transformers are about seven times the magnetizing volt amperes of the transformer at rated voltage. In many designs of potential transformers such a burden would considerably exceed the rating of the transformer. On the other hand, if line-to-line transformers are connected in Y so that $E=0.58$, the watts in the individual burdens would be only $\left[3\times\frac{1}{(\sqrt{3})^2}=1\right]$ times the magnetizing volt amperes. Consequently it is a simple rule to remember that if line-to-line rated potential transformers are connected in Y on a system of the same voltage rating, the amount of watts per phase in the resistance burden necessary to insure stability under any conditions is equal to the magnetizing volt amperes with rated voltage on the winding. If one wants to be sure of stable operations under any conditions without providing stabilizing impedances, it is necessary to use the value of $E=0.2$ or less, which would mean poor performance from the potential transformers with respect to accuracy, besides a much over-rated transformer. It would, therefore, appear that the practice of using line-to-line rated potential transformers connected in Y on systems of the same rating, whether arrived at by accident or good judgment, is nevertheless a reasonable procedure.

In discussing the watts or volt amperes necessary for stabilizing the Y-connected transformers under the worst conditions, it should be noted that if the transformers are so connected that impedance can be inserted in the broken delta of the secondary indicated in figure one, the volt amperes consumed under normal operating conditions will be very small, and the capacity of the impedance could therefore be reduced and the load on the transformers would at the same time be minimized.

Since most generally the secondary burden will consist of combined reactance and resistance, it is only natural to ask what the effect of such combinations would be on stabilizing the transformers in order that one might make use of a normal burden to provide the necessary stabilizing influence. Since for insuring stability the value of the resistance burden is given by $R=0.34X_m$ and the value of the reactance burden is given by $X_L=0.28X_m$ in the case of $E=0.58$, it would seem reasonable to expect that for this particular voltage a combination of resistance and reactance having an impedance approximately equal to or less than $0.28X_m$ would be adequate. I would like to know if the authors have any information on this subject.

When one looks over the possible practical means of connecting potential transformers in Y on ungrounded systems so that stability will be insured, it is evident that with the information provided by this paper it will be in many cases simpler to provide the necessary impedances to insure stable operation regardless of the ratio X_{co}/X_m than to determine the possible values of X_{co} under operating conditions.

It should be noted that if reactance burdens containing iron in the magnetic circuit are used for stabilizing purposes, the burdens should operate well below saturation in order to be effective. Otherwise, they will simply add additional magnetizing current of the same character as that of the transformer. Whether this is also true if the reactance is used in the broken delta of the secondary it is not so easy to answer, and I would therefore like to know if the authors have any information on this question.

The title of the paper and lack of any specific statement to the contrary might lead one to believe that any Y-connected potential transformers are subject to unstable operation. It should therefore be emphasized that the type of unstability discussed in the paper does not apply to Y-connected potential transformers on a solidly grounded or impedance grounded (if low enough impedance) system so long as the system remains grounded. Neither does this type of instability occur if the secondary delta is closed and it is not likely to occur if the neutral of the transformers is not grounded.

T. M. Blakeslee (Bureau of Power and Light, Los Angeles, Calif.): The classic papers on neutral instability, references 1, 2, and 3 pointed out and therefore warned the engineer of this peculiar phenomenon. The demonstration by Shott and Peterson of the correlation of neutral instability with the ratio X_{co}/X_m almost places the situation on a practical design basis.

Tests were recently conducted by the Los Angeles Bureau of Power and Light in the attempt to obtain a stable neutral or Y-Y-connected potential transformer on a 115-kv ungrounded bus section. No measurement or calculation of the capacity (X_{co}) involved was made and this quantity was not varied. The other parameter set up by Shott and Peterson, X_m, was varied by using two designs of potential transformers and by operating one of these designs at two excitations. The results are interesting in that they confirm in the actual case the results from the miniature system tests.

The potential transformer units tested had 500-volt-ampere ratings; the voltage rating of one design was 76,200/63.5 volts and of the other 79,677–66,397/66.4 volts. The excitation characteristics of the two designs were quite different. By operating the transformers on the 115-kv bus (66,400 Y-voltage) on all voltage ratings; three values of X_m were obtained.

The tests were conducted by merely energizing the 115-kv bus and noting the condition of the transformer neutral. These tests showed that for small values of X_m (large values of X_{co}/X_m) the instability takes the form of a displaced neutral at fundamental frequency. For high values of X_m (small value of X_{co}/X_m) the instability manifests itself as a neutral displaced at approximately half normal frequency. These situations are similar to those described by the authors.

In the fundamental frequency case the tests gave a displaced neutral such that one of the transformer units was forced to operate at 2.5 times normal voltage. Resistance loads up to 5 kw per transformer were applied in the effort to stabilize the neutral. It was possible to obtain a stable neutral by these resistance loads but upon energization of the transformer bank, periods of instability lasting from zero to 2 seconds would occur before a stable neutral was obtained. These periods of instability could not be tolerated in many cases of practical application.

Boyajian and McCarty stated that the resistance loading to obtain stability should be of the same order of magnitude as X_m. The present paper indicates that 33 per cent of X_m is the proper loading in the resistance case. The tests made on the 115-kv circuit described above indicated a resistance value about 60 per cent of X_m is required.

H. S. Shott and H. A. Peterson: Mr. Sinks raises a question regarding the amount of impedance required for stability when that impedance is a combination of resistance and reactance. While we have presented no quantitative data for this condition, it is reasonable to expect, as Mr. Sinks suggests, that in the case of $E=0.58$, a combination of resistance and reactance having an impedance approximately equal to or less than $0.28X_m$ would be adequate.

The question regarding the effect of saturable reactance in the broken delta cannot be answered by inspecting the data in the paper. Predictions of non-linear circuit characteristics sometimes are likely to be in error. It would be desirable to check this point in the miniature system in cases where this additional factor is encountered.

Mr. Blakeslee has presented some valuable data from actual field tests. His remarks concerning the long transient periods obtained in some cases are apparently similar in time duration to those obtained in the miniature system. The value of resistance equal to 60 per cent of X_m required for stability in his case is consistent with the value of 33 per cent indicated as a minimum in our paper for $E=0.58$. Our value of 33 per cent was obtained for a particular value of $X_{co}/X_m=0.02$. For values of X_{co}/X_m either greater or smaller, the value of resistance required for stability increases. The authors greatly appreciate such checks from field tests for they serve as excellent bench marks for continued investigations.

The Modern A-C Network Calculator

Discussion and author's closures of paper 41-143 by W. W. Parker, presented at the AIEE Pacific Coast convention, Yellowstone National Park, August 27–29, 1941 and at the AIEE South West District meeting, St. Louis, Mo., October 8–10, 1941; published in AIEE TRANSACTIONS, 1941 (November section), pages 977–82.

Discussion and closure at Pacific Coast Convention

Jean V. Kresser (Westinghouse Electric and Manufacturing Company, San Francisco, Calif.): The tedious mathematical calculations required for analyzing electrical systems are generally no longer necessary and little used, particularly in the case of large or complicated systems. The faster and more efficient network calculator is rapidly superseding the older less accurate method. Indeed, the analysis accomplished on the calculator is far beyond that reasonably possible by analytical methods. It is possible to bring the "system" into a laboratory, load it and make measurements of voltage regulation and power flow or set up faults and various kinds of disturbances and determine their magnitude and whether the system is stable under these conditions.

It is further possible to expand in the laboratory an existing "system" and thereby learn exactly what the results will be before construction of the new expansion is started.

Although at first glance this seems mysterious, it is in fact very simple. The network calculator is essentially a combination of different kinds of impedance circuits susceptible of connection in any manner or fashion needed to represent in exact replica whatsoever system is to be analyzed. Suitable instruments provide for accurately measuring the various quantities in the miniature replica. The results then need only be multiplied by a factor representing the ratio of "size of the actual system" to the replica to obtain the values which would occur on the actual system.

The d-c network calculator is still used for the majority of short-circuit studies. It is rigorously accurate when the impedances of the system branches all have the same phase angle. When the phase angles depart only slightly from a mean angle, the error is slight. If the impedances of the branches are used rather than either the resistance or reactance components, the real and reactive components of power must be differentiated. In cases where the d-c calculator becomes inaccurate and for the analysis of complicated systems, the modern a-c network calculator is used.

Since all types of loads are represented by static impedances and generators are represented by voltage regulators and phase shifter, or combinations of these, the direct analysis of stability on a calculator is not possible. Although a-c miniature systems with synchronous machines have been used in stability studies, no attempt has yet been made to represent rotating machines by scale models on a network analyzer. Questions of stability involve mechanical inertia and electrical damping, and the relationship between these cannot be retained with the ordinary type of rotating equipment as the scale of the miniature system becomes small. Special machines for use with network analyzers of laboratory convenience and accuracy have not been built for obvious reasons of cost, complexity, and space requirements. For these stability studies a "step-by-step" method is employed, the calculator giving the electrical conditions for each step. The speed and precision with which results are now obtained obviate any need of employing other and more complicated methods.

The several years of use of a-c calculators have developed skilled operators who can largely anticipate results. Hence, under such guidance, much of the time that might otherwise be spent can be saved through the avoidance of superfluous measurements and also saving of unnecessary work.

We are indebted to Mr. Parker for the clear explanation of the manifold purposes and achievements of the a-c network calculator. It has come to be an indispensable tool, analyzing and also accurately forecasting the proper engineering program in power system management.

H. P. Kuehni (General Electric Company, Schenectady, N. Y.): Mr. Parker's paper entitled "The Modern A-C Network Calculator" is of particular interest to designers of instruments of this type. Such papers offer valuable design information to the art, and the presentation and discussions provide for an exchange of engineering views.

The paper does not mention the motor generator set and associated control equipment which furnishes the 440-cycle power to the calculator. However, on the basis of the rating data given on the 12 generator units, we estimate that the total power requirement of the calculator should be of the order of 15 kva. A large motor generator set appears necessary, therefore, and a considerable amount of heat must be dissipated. In our own low base analyzer the total maximum power input into the analyzer from the 12 generator units is 0.750 kva on the basis of 250 per cent volts and 1000 per cent current per generator unit.

In table I, in which are listed network calculators in use or under construction, special reactors are tabulated for many of the installations. It is assumed that these reactor units refer to low R/X ratio units commonly used to represent internal generator reactances. Reactors of this type are not listed in the table giving the range of the circuit elements, nor are they described in the text. We have built special reactor units of this type having a ratio of R/X as low as one per cent and have also built electronic units of this type in which the power factor can be adjusted to zero.

A centralized instrument system has been adopted in the modern a-c network calculator which Mr. Parker described. This contributes immeasurably to the speed and convenience of taking readings. The same feature has proved its value during the four years of operation of our Schenectady analyzer. An enormous number of system problems were solved during this period and for many months the analyzer has been operating on two shifts to satisfy the great demand.

In the calculator described in the paper a compact 25 push button circuit selector is described. It is stated that only one circuit can be connected at a time because of electrical and mechanical interlocks. We have found, in many studies, that it is often necessary to use more than one circuit selector switch at one time. This need arises particularly in connection with problems involving equivalent circuits. The possibility of using more than one circuit selector switch simultaneously is often very convenient for totalizing multiple circuits which terminate on busses. Our most recent design incorporates controls for interlocking the circuit selector switches or for removing such interlocks. The prevailing interlock condition is indicated by pilot lights.

It is also interesting to note that in Mr. Parker's design electronic amplifiers are used to furnish the power for operating the indicating instruments. These amplifiers completely eliminate the former instrument burden difficulties. In the earlier designs the instrument burden almost entirely determined the analyzer base rating. This is no longer the case today, and considerably lower base ratings may now be used. Our central arrangement of the master instruments and circuit selector switches and the use and design of the instrument amplifiers were described in a paper entitled "A New A-C Network Analyzer" presented at the AIEE winter convention in January 1938.

It is gratifying to learn that the instrument amplifier system which we introduced has been adopted by others. Well designed amplifiers of this type are very accurate, extremely stable, and completely reliable. An instrument amplifier system of this type has been in regular use in the Schenectady a-c network analyzer for over four years with an exceptional performance record. During this time we first made monthly check tests of the amplifier instrument system. However, it was soon found that such frequent checks were unnecessary. The other analyzer installations show the same reliable amplifier performance.

A low analyzer base rating has many advantages. As already pointed out above, the power requirements are small and there is little heat to be dissipated. The losses in the circuit elements may be minimized so that the temperature rise of these elements is too small to produce drifts in the adjusted circuit constants. It is true that the use of a low voltage and current base requires due consideration to inductive and capacitive coupling effects. By giving proper attention to these effects the desired analyzer accuracy is obtained.

Besides the usual power system problems our a-c network analyzer is also being used, for the solution of many other electrical and mechanical problems. These problems relate particularly to equivalent circuits. In these, pure reactances and negative resistances often occur. Therefore, the addition to the network analyzer of zero power factor reactors and negative resistor units materially increases the flexibility and usefulness of the network analyzer equipment. Because of our low analyzer base rating the design problems of producing special units of this type are materially simplified and these units can readily be built around vacuum tube circuits. Standard vacuum tubes easily handle the power required for these units when the base rating is small. Both the zero power factor reactor units and the negative resistor units are of the same size as the other circuit units and fit into the network unit compartments of our analyzer unit cabinets.

R. N. Slinger (General Electric Company, St. Louis, Mo.): The author has presented some very interesting information relating to the design features of the latest a-c network calculators manufactured by his company. Some clarification of certain features of the paper and a few added comments, however, may be helpful to others interested in this subject.

Without adding some qualifying statement, it is rather misleading to include short circuit studies in the list of different types of problems for which the desired quantities may be read directly. The statement is correct for three-phase short circuit problems, but ordinarily would not be correct for unbalanced faults. Symmetrical components employing the various sequence networks are usually used in solving problems involving unbalanced faults and, of course, the various sequence quantities must be added vectorially.

In connection with transient stability studies the author has neglected to call attention to one of the very important advantages of the network analyzer form of solution. As he has pointed out, any a-c network calculator makes it relatively easy to handle multi-machine transient stability problems, but in addition it also makes it easy to include in the solution of one of these problems a change in operating conditions such as the opening of an oil circuit breaker.

The statement that "The a-c calculator frequency has been established at 440 cycles" if taken literally, would imply that that particular frequency is one of the prerequisites of a modern a-c network analyzer. As a matter of fact, there is nothing sacred about that particular frequency. In designing an a-c network analyzer, consideration of cost, weight, and size together with certain other design features, leads to the conclusion that the optimum frequency is at present in the neighborhood of 400 to 500 cycles.

In describing the generator units, Mr. Parker makes the statement that "the phase-angle adjustment corresponds to changes in the governor position." Under steady state conditions this would be correct, but not under transient conditions; therefore, it would be more general to say that the phase angle adjustment corresponds to rotor position with respect to flux in the armature of the synchronous machine represented.

Raymond C. R. Schulze (Public Service Electric & Gas Company, Newark, N. J.): This paper describes the latest Westinghouse a-c network calculator, with a little history concerning the development from the original to this model.

The first point for discussion is the power level. The Westinghouse design allows capacities up to 400 per cent for voltage and 300 per cent for current. It seems to me that 400 per cent voltage is more than is necessary, while the maximum limit of 300 per cent on the current is not enough. The units on our network analyzer (built by General Electric) are rated at 125 per cent voltage and 500 per cent current, and these limits seem to meet our requirements very well. The only times that we have been cramped by these units have been when using synchronous reactances or when studying for 180 degrees out-of-phase conditions, when 200 per cent or more voltage was needed, and in a very few cases we have found that 500 per cent current was not quite enough. When necessary, we have gone over the 125 per cent voltage limit without worrying about it, but we have been careful when the current exceeded 500 per cent. If the currents are much in excess of 500 per cent, we have the advantage on our analyzer of being able to change to 50 per cent or 25 per cent voltage (reducing the currents accordingly) by simply operating a switch on the front of the board.

Concerning the generator circuits, I presume that you have some good reason for using two three-phase regulators rotating in opposite directions to get the voltage adjustment. The analyzer installed in our office is somewhat simpler in this respect, since it utilizes a single phase induction regulator to adjust the voltage. There apparently is no choice between the two from the operating viewpoint.

You mention the instruments for measuring the outputs of the generator units. I see that you still insist on using an ammeter instead of a varmeter. Our analyzer uses a watt-varmeter on each individual unit for making the first rough adjustment. After that, the master meters are used for reading the generator output. These master meters include a voltmeter, an ammeter, and a watt-varmeter. Our experience has been that the ammeter reading is practically never read except when finally recording the data, and then it is read only because the value is there. During the balancing period the watt-varmeter is used exclusively to determine the generator output. The use of voltage, watts, and vars for adjusting the generator outputs seems to be entirely satisfactory. Just incidentally, I can remember the large amount of adjustments and slide rule work necessary to adjust the generators when using the voltmeter, wattmeter, and ammeter.

Considering the range of values for the various classes of circuits, it seems to me that the ratio of $R{:}X$ in the line impedance units should be 300 to 399, rather than the 399 to 300 as used. A review of the ratios of R to X of different kinds of equipment shows that the R is practically always less than the X except for very small open wire lines and medium or smaller multi-conductor cables. Also, in my opinion, the load impedances should have more reactance for the given amount of resistance; the present values place some unnecessary limit on the minimum kva at low power factors.

Concerning the capacitances, I am wondering if the microfarads mentioned are actual microfarads, or whether they are equivalent values at 60 cycles, that is, are the ohms for the impedance units and the microfarads for the capacitance on the same frequency basis?

Concerning the load adjuster units, it would be awfully nice if the voltage always came to an even per cent, but it is more likely to be 104.5 per cent than 105.0 per cent. Therefore, it seems to me that the auto transformer is not a complete answer to the problem, and if one is really careful about the load settings, the load impedance will have to be adjusted by hand whether the load adjustor is there or not.

I presume that equations for the series and parallel connections are necessary when using ohms, but we have a much simpler scheme when using per cent reactance. A table of reciprocals is posted on the front of the a-c table (a Public Service idea) so that the settings for parallel connections can be read and set very quickly. A diagram has been made up for series connections, so that the R and X values can be read quickly from it for given kva and power factor values. These settings, as well as the values obtained from your equations, must be corrected for the resistance in the reactor, and also for the fact that no one can guess exactly what the various substation voltages will be. The use of per cent reactance at 100,000 kva simplifies the operation of a calculator considerably, because of the ease with which the load units can be set and adjusted.

The description of your selecting keyboard is interesting, but the question which promptly comes up is—how does one know that one is metering the circuit he selected? When one pushes buttons 2-3-4, is it possible to get 4-3-2? The indicating lamp on your figure 4 is apparently the answer to that question.

The relaying for the metering scheme sounds somewhat complicated, and I am wondering which is worse—all the complications of that relaying, or the very slight inaccuracies due to bringing the metering leads to the metering desk. I am also wondering if you can read voltage from either end of any impedance. Our metering scheme is quite flexible in this respect.

Our analyzer uses one set of meters for both scalar and vector quantities. The scalar quantities are used almost exclusively. Once in a while when vector quantities are needed, it is easy to flip a couple of switches and use these same meters for reading the vector quantities. The meters on our analyzer cannot be read quite as accurately as yours with knife-edge pointers, since our meters use a light beam and the light beam must have some width. However, the slight loss of accuracy is insignificant. The reduction in strain on the operator's eyes, and the fact that these meters can be read from any where in the room, thus making this a one-man analyzer, amply make up for the negligible loss in accuracy due to the use of light beam meters.

W. W. Parker: 1. Mr. Kresser has compared the application of a-c and d-c calculators. One important additional use of the a-c calculator is for unsymmetrical fault studies in which the zero sequence network simulates multiple circuits with mutual coupling. Mutual transformers in the a-c calculator provide means for accurately including these coupling effects.

2. Mr. H. P. Kuehni asks about the size of the motor generator set and the power requirements. The motor generator set is very small, being five feet long and approximately eighteen inches in height and width. The power required for operation of a calculator with twelve generator units will average less than two kilowatts for most problems. With all twelve generator units loaded at 100 per cent values the power required would be approximately 3.5 kw.

3. The "special reactors" on the Westinghouse calculators have a range of 0 to 2,100 ohms. They are used in representing magnetizing current of large banks of transformers or in conjunction with the load circuits for small low power factor loads.

4. The removal of interlocks to obtain totalized readings does not appear to be good practice as it may lead to errors in metering or interruption of operation. On the Westinghouse calculator these difficulties have been avoided by providing "totalizing" or "jumper" cords with metering facilities. The interlock system is never removed.

5. The assumption that our instrument amplifier system is the same as that employed by the General Electric Company is in error. The instruments are entirely different being conventional type with simple zero adjuster and knife edge pointers. The amplifier uses current feed back instead of voltage feed back. The advantages of this circuit were outlined in the Parker-Osbon discussion of the Brown and Cahoon paper as published in AIEE TRANSACTIONS, volume 58, 1939 (November section), page 598.

6. Due to adequate design the Westinghouse calculators do not show any change in the adjusted circuit constants. With the

base used, the capacitive coupling between circuits is entirely negligible avoiding all necessity for compensation. This simplification is a further advantage over a low base design.

7. With reference to Mr. Slinger's comment regarding the frequency to use on an a-c calculator we can see no difference between 440 or 480 cycles which justifies comment. During our early preliminary investigation we explored the range from 60 to 2,000 cycles and found the optimum frequency to be between 400 and 600 cycles. Mr. Slinger's subsequent investigation substantiates our fundamental conclusions.

8. The calculator design to which Mr. Schulze refers, has been completely described in the Kuehni-Lorraine paper referred to in the bibliography. Apparently this has proven satisfactory for problems on his specific system. Our wider experience in solving problems on all types and sizes of power systems has proven the value of the improvements in calculator design for such application. Moreover, this design has received wide acceptance from the industry.

9. There are distinct advantages in having a means for completely and continuously showing the generator conditions. Our calculator has a voltmeter, ammeter, and wattmeter within constant view so that the effect of adjustment of one generator on the other is immediately indicated. This is not possible with a watt-var instrument and its associated switch. The ammeter has the distinct advantage of indicating directly whether the generating station represented is being called upon to carry more than the assigned kva. Plainly, such a condition is immediately obvious because of the use of the pre-set pointers.

10. The microfarads are actual microfarads and not ohms at any frequency.

11. Practical determination of load values does not require obtaining voltages across the load impedances within one-half of one per cent or less. One per cent steps on the load adjusters have proven entirely adequate. The distinct advantage of the load adjusters is that the load impedance can be set up from a rough estimate of voltage and these impedance values do not need to be changed. The load adjuster raises or lowers the load without affecting the power factor until the desired value is obtained. For the very rare cases when loads have to be set within one per cent slight adjustments of the impedance values can be made. Clearly, that considerable time is saved for the average problem is not open to question.

12. The keyboard and selector relays are so wired that it is impossible to get an incorrect selection. The relays are of well-proven supervisory type. The circuits are simple and easily checked. With this circuit selector the metering of the calculator elements is materially expedited and inaccuracies due to bringing circuit leads to the desk have been avoided.

13. The conventional type of instruments used is entirely satisfactory in high intensity uniform room illumination. This is the illumination used and is recognized to minimize eye fatigue.

Discussion and closure at South West District meeting

H. Weichsel (Wagner Electric Corporation, St. Louis, Mo.): The paper presents in a very clear and precise manner the design and function of this very useful apparatus. There are, however, a few items which to me personally are not clear and some further explanations by the author will be appreciated.

In Figure 1, lower box two from the left, is shown an autotransformer. Is this autotransformer supposed to imitate the effect of the step up transformer shown in the upper part of the diagram? If this is the case, are there any specific reasons why the step up transformer is represented by an autotransformer in place of an equivalent circuit containing the corresponding resistance and reactance of the step up transformer? Naturally in this case the resistance and reactance capacity in the lower box 3, Figure 1, representing the transmission line would have to be calculated for values equivalent to the actual transmission-line values but reduced to the voltage of the calculator.

The article also speaks about "mutual reactance circuits." None of these is shown. A statement as to where these mutual reactance circuits are located and what their purpose is will assist the reader.

In the second last lower box of Figure 1 a combination of resistance and apparently capacitors is shown. There are also three rectangles given without any letters assigned to them. Do they represent switches?

Edward W. Kimbark (Northwestern University, Evanston, Ill.): Mr. Parker's paper shows the present status of a-c calculating boards, both as to the number of boards in use and as to the improvements in design of the boards built by his company since 1929. The improvements described—that is, generators having low-voltage regulation and independent control of voltage phase and magnitude, the provision of instruments on each generator with markers for the desired reading, the addition of amplifier-type central scalar instruments, the centralized control of meter insertion with interlocking, the provision of series-parallel switches on the load units, and the addition of load adjusters—all these increase the speed and convenience of securing results.

After hearing about the increasing use of a-c boards and about improvements in design, one is naturally led to speculate on future developments. I firmly believe that the a-c board is so useful that every large power company should have one. At present the cost of an a-c board is so high that few power companies feel that they can afford one. It would be very desirable to decrease the cost. Though I cannot say how to do this, I can point out what has been accomplished in this direction with d-c boards. The conventional d-c board is the size of a large bookcase, but a man in Chicago has built a portable d-c board, consisting of 40 adjustable resistors, power supply, and instruments, all contained in two suitcases, and at a cost commensurate with the size. Perhaps Mr. Parker and his associates can eventually effect a comparable reduction in the size and cost of a-c boards.

W. W. Parker: 1. Mr. H. Weichsel asks about the autotransformer units. These are used for quickly simulating the effect of changing transformer taps. The resistor-reactor circuit is used to represent the equivalent circuit of the transformer (the exciting current can generally be omitted from consideration). Resistance and reactance of the calculator circuits representing the transmission line are set on values calculated from consideration of normal transformer turn ratio and the ratio of the calculator quantities to the corresponding power system values.

The "mutual reactance circuits" are located in the bottom of the connection cabinet (Figure 2). These are one-to-one ratio transformers for representing the coupling (mutual induction) of circuits. These transformers reproduce the effect of current in one circuit inducing a voltage in another, the value of which depends on the value of the current and the mutual inductance.

The unmarked rectangles in Figure 1 represent switches.

2. Remarks of Mr. E. W. Kimbark are much appreciated because of his experience with calculators. Due to the technical features of an a-c calculator as contrasted with those of the d-c calculator, it is doubtful that any considerable reduction in size and cost of the former will be possible in the near future. However, this is one of the goals of calculator designers.

A Cathode-Ray Method of Wave Analysis

Discussion and author's closure of paper 41-156 by V. O. Johnson, presented at the AIEE South West District meeting, St. Louis, Mo., October 8–10, 1941, and published in AIEE TRANSACTIONS, 1941 (December section), pages 1032–6.

James S. Malsbary (Wagner Electric Corporation, St. Louis, Mo.): Many methods and machines have been proposed for the determination of the harmonics appearing in harmonic motion with particular reference to electrical waves. Due to the complexity of the machines with their attendant cost, very few are used in practice. The cathode-ray oscillograph is to be considered an exception to this rule, not because of its use in wave analysis but because of its use in determination of wave shape.

The author makes a statement as follows: "The complex wave is applied to the horizontal plates which give the cathode beam a vertical displacement." This is not in accordance with the accepted definition of deflecting plates. Quoting from "The Cathode-Ray Tube at Work" by John F. Rider, published 1935, page 26, lines 3 to 6 inclusive. "The term 'vertical deflecting plates' refers to the motion given the beam and the spot by those plates with respect to the normal position of the tube, and not to the orientation of the plates with respect to the normal position of the tube." Also the *DuMont Oscillographer* dated December 1938–January 1939, page 4, which pictorially represents the deflecting plates of the cathode-ray tube as items 16 and 17. Item 16 has the faces of the plates in the horizontal or X axis but they are referred to as the "vertical-deflecting plates," while item

17 has the faces in the vertical or Y axis but refers to them as the "horizontal-deflecting plates." The above, however, does not affect the conclusions arrived at in the paper.

Without detailed study, it is hard to see how an area of an irregular figure determined by inspection can be used to calculate harmonics with sufficient accuracy for practical use.

It appears that after the Lissajous figure (polar diagram) has been photographed, an amount of work must be done thereon in order to arrive at the final answer which is equal to or more than that required to analyze the wave recorded in the rectangular co-ordinate system. It is concluded, therefore, that for the industrial and audio frequencies that this method might not be as practical as other well known methods but as the author points out, with very high radio frequencies where no other oscillograph can be used and no other measurements can readily be taken, the instrument and method may prove very useful.

In the matter of analyzing mechanical vibrations, this is now being done by the use of harmonic analyzers in combination with crystal pickups and vibration-measuring equipment.

Vincent O. Johnson: The cathode-ray oscillograph as Mr. Malsbary states is used in many laboratories for many purposes, although seldom for wave analysis. It has been used for checking the harmonics introduced by a piece of equipment which is supplied by a sinusoidal source. Properly connecting the sinusoidal source to the vertical plates of the oscillograph and the output voltage of the device to the horizontal plates produces a Lissajous figure from which the fundamental component of the output voltage can be calculated in the usual way. This obviates the use of the auxiliary oscillator and the problem of synchronization. The difference between the fundamental component thus obtained and a voltmeter reading of the output voltage is a rough indication of the amount of harmonic content.

Use of the expressions "vertical plates" and "horizontal plates" in the above paragraph and in the body of the paper, meaning plates with vertical and horizontal faces respectively, is questioned. After consideration, it seems possible that these expressions may cause some confusion as to whether "vertical plates" are plates that have a vertical axis or whether their faces are vertical. The expression referred to in "The Cathode-Ray Tube at Work" by John F. Rider and in the *DuMont Oscillographer* is "vertical-deflecting plates" meaning plates which deflect in a vertical direction as distinguished from "vertical plates" as used in the preceding paragraph. It is hoped that this explanation will clear up any misinterpretation of the expressions in question.

It is agreed with Mr. Malsbary that any determination of the area of an irregular figure by inspection can only be approximate, but the time required to obtain the area by accurate means is not appreciable. With the areas obtained, the only remaining step is to substitute the values obtained in the proper formula and turn out the answer. This method is shorter than graphical methods with which the author is familiar, if a comparable degree of accuracy is to be obtained. It is true that the method probably has its greatest advantage in the higher frequency range where graphical methods are not applicable.

The Equal-Volt-Ampere Method of Designing Capacitor Motors

Discussion of paper 41-157 by P. H. Trickey, presented at the AIEE South West District meeting, St. Louis, Mo., October 8–10, 1941, and published in AIEE TRANSACTIONS, 1941 (November section), pages 990–2.

Raymond W. Ager (Cornell University, Ithaca, N. Y.) and **Thomas C. McFarland** (University of California, Berkeley, Calif.): Mr. Trickey first calculates a normal two-phase motor. His second step is to change the winding ratio so that an approximate match is obtained between the voltage triangle and the power factor at full load. In this step he compensates for stator and condenser resistance. The new winding ratio having been determined, the proper capacitor is chosen. A single-phase motor built to this design would have rotor conditions which would closely match those of the two-phase motor at this particular load. These two steps form the basis for Mr. Trickey's previous paper, his reference number 1.

The present paper deals with a third step in modifying the winding ratio and the capacitor size for economic reasons. He has shown that the resulting motor has satisfactory characteristics. Certainly the slightly lower efficiency indicated is not sufficient handicap to offset the decreased capacitor size and cost. In this paper Mr. Trickey has presented a simple, straightforward method which works. This discussion is presented in order to point out a few additional relationships together with some test data bearing on this method of design.

In taking this third step the nice balance of the motor is destroyed. The design developed in the first two steps was balanced, that is, the power and the reactive volt-amperes supplied to the rotor by the main phase and those supplied by the auxiliary phase were made equal. Any change in the winding ratio and the capacitor size upsets the balance in either or both of these quantities, since it causes a shift in the angle and magnitude of the current in the auxiliary phase. Thus, even though the capacitor volt-amperes are maintained constant, the balance no longer exists.

The amount of unbalance may be best illustrated by considering these two components separately. The power input to the auxiliary phase will be a maximum for a winding ratio $K=1$, if the capacitance is varied as indicated in Mr. Trickey's present paper. The more this ratio differs from unity the less the power supplied to the capacitor phase. The reactive or magnetizing volt-amperes will increase as the winding ratio is increased. In the example used, the winding ratio for the balanced design was 1.746. This was increased to 2.25 in the proposed modification. Since the total power and the total magnetizing volt-amperes of the motor must remain about the same, the decrease in capacitor-phase power must be accompanied by an increase in the power component of current in the main phase. The increase in the magnetizing volt-amperes of the capacitor phase must result in a decrease in that component of current in the main phase. These were divided equally between the two phases in the balanced design. Estimates would indicate that the auxiliary phase carried 43 per cent of the total power and 55 per cent of the magnetizing volt-amperes in the modified design. The amount of unbalance is not great and the good performance indicated could be expected.

The test curves shown in Figures 1 and 2 were taken some time ago. The curves of Figure 1 were taken on a special three-phase, one horsepower, 220-volt, Diehl motor having a single-cage cast-aluminum rotor. The motor had been reconnected for two-phase operation, with a winding ratio of 1.00. The motor was operated single phase with the line voltage held constant at 175 volts. Power-factor-correction capacitors rated at 460 volts were used in the capacitor phase. Curves A, B, C, D, and E show power input as a function of capacitance. The load was constant for each curve with curve A being taken at no load. The loading device used in these tests could absorb only about one-half the rated output of the motor so that higher curves could not be obtained. In these tests the power supplied to each winding was measured separately. This permitted the calculation of the locus of equal division of power between the two phases, shown as curve F. Calculations were also made to determine the capacitances which would cause each phase to supply the same magnetizing force. Curve G shows this locus. The figure shows that the minimum power required to drive any load lies between these two loci. Also shown is that the capacitor required for best efficiency is larger for heavy loads than for light loads. A balance point is indicated where the two loci cross between curves C and D. Had the winding ratio been

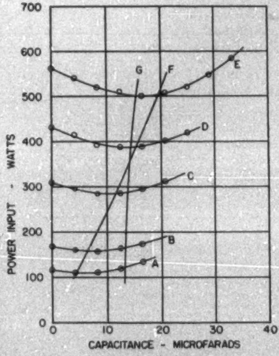


Figure 1

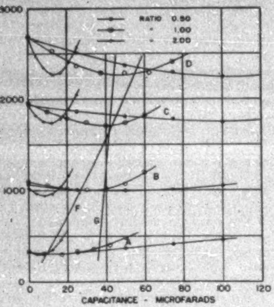


Figure 2

higher this balance point would have been found at a lower load, since high winding-ratios at the balance point are associated with low power factor.

These curves also show that a balanced design can be balanced for only one load. If the load changes the power and the magnetizing components of the auxiliary-phase current will change very little. The magnetizing component of the main phase will also change only slightly, but the power taken by the main phase will change greatly in comparison. Thus a balanced design will, under varying load, remain approximately balanced as far as magnetizing currents are concerned. The power currents in the two phases will only be balanced, even approximately, at and near the load at which the motor is designed. The load taken by the auxiliary phase will be quite constant while the main phase will take all the swings.

The data for Figure 2 were taken on a General Electric, three-horsepower, two-phase, type *KG*, double-squirrel-cage induction motor. It had a double-voltage rating permitting the taking of three sets of curves with winding ratios of 0.50, 1.00, and 2.00. The curves for a ratio of 0.50 have been corrected for 220-volt operation. It was necessary to make the tests at 440 volts in order to get this ratio. The equal-power and equal-magnetizing-force loci have been shown for the winding ratio $K=1$. The curves for maximum load were for an output of 2.4 horsepower or 80 per cent of the two-phase rating.

In these tests one of the curves almost passes through the balance point. If this is considered as a balanced design, then, using the equations of Mr. Trickey's paper, it is possible to see whether these equations give the best condenser when the winding ratio is shifted to 2.00. The balanced design calls for 42 microfarads. The theoretical voltages E_c and E_c' are 311 and 492. The new capacitor required should be

$$C=\frac{311^2}{492^2}\times 42=16.8 \text{ microfarads}$$

The corresponding curve for a winding factor of 2.00 has its minimum at slightly lower capacitance although there is not a great deal of difference even in this case.

Table I

	Balanced Design	Equal-Volt-Ampere Design
Line current	7.90	8.10
Main-phase current	5.82	5.30
Auxiliary-phase current	5.06	3.37
Main-phase power	920	1,100
Auxiliary-phase power	782	680
Main-phase magnetization	890	390
Auxiliary-phase magnetization	794	1,430
Main-phase voltage	220	220
Auxiliary-phase voltage	220	464
Capacitor voltage	310	500
Speed	1,759	1,761

A few of the actual values obtained may be of interest. The following are readings for curve *C* with 40 microfarads and a winding ratio of 1.00, approximately at the balance point, and for a capacitance of 16.7 microfarads and a winding ratio of 2.00, which is practically the "equal-volt-ampere" design.

The degree of unbalance in the modified design is indicated in the power and magnetizing volt-ampere values, these indicating that the auxiliary phase carried 38 per cent of the power and 79 per cent of the magnetizing volt-amperes. Decreasing the capacitor size would reduce the power and magnetizing force of the auxiliary phase while increasing the amounts carried by the main phase. That a somewhat better balance was obtained for these lower capacitances is indicated by the curve.

The following conclusions seem to be justified:

1. The "equal-volt-ampere" method does not produce balanced designs.

2. The amount of unbalance increases as the voltage-ratio change is increased.

3. Considerable unbalance can be tolerated and still maintain good performance. Probably only for extreme designs would the use of this method produce designs which were seriously unbalanced. For these cases a calculation of the per cent power and magnetizing volt-amperes supplied by the auxiliary phase would be justified and an adjustment of winding ratio and/or capacitance be made so that the sum of the per cent power and the per cent magnetizing volt-amperes for the capacitor phase was equal to 100.

4. The balancing might be at a value less than full load for general purpose motors operating over a range of loads rather than a constant load near full load rating.

The Inherent Overheating Protection of Single-Phase Motors

Discussion and author's closure of paper 41-154 by C. P. Potter, presented at the AIEE South West District meeting, St. Louis, Mo., October 8-10, 1941, and published in AIEE TRANSACTIONS, 1941 (November section), pages 993-6

A. F. Lukens (General Electric Company, Lynn, Mass.): Mr. Potter has pointed out some advantages of inherent overheating protection for motors. One of the important ones which he does not mention is the maintained service that a motor will give under very adverse conditions when an automatic reset device is used on an automatically started load. With such a device installed in a motor, under adverse conditions, the motor will try to the utmost of its ability to carry the load without actually doing itself harm and will give a sort of "limping" service that does not completely stop.

In the application of motors with automatic reset devices, great care must be exercised that they are used only on automatically started loads and not on manually started ones. If the latter is the case, there is great danger that the motor will shut down and the operator will not know the reason and start to make an investigation. About the time he gets his fingers in the machinery, the motor may start up and do him considerable bodily harm.

Another warning that must be continually emphasized to the users of motors with inherent overheating devices built into them, is that these motors cannot be applied to just under the tripping value, as shown by ideal application test conditions. If this is done, trouble in the field under adverse operating conditions of voltage, ambient, and load, is bound to result as the motor will shut down when it reaches a dangerous temperature. In this case the burden of applying a proper factor of safety must be carried by the user to a greater extent than when the usual overcurrent protection is used.

Mr. Potter proves by an exhaustive series of tests that an inherent overheating protective device does give adequate protection under many different conditions. He then draws the very restricted conclusion "that inherent overheating protective devices mounted inside fractional horsepower single-phase motors completely protect them against all abnormal operating conditions."

This statement can be extended to cover larger motors and need not be restricted to single-phase motors, for all the advantages of this type of protection are needed in the integral horsepower field. Two things now stand in the way of immediately accomplishing this. The first is the lack of suitable devices for the larger motors. Some sizes are now available, and the manufacturers are working on others to advance their lines. The other is the confusion and contradiction of the various rules and regulations under which such an application must be judged at the present time. The confusion arises, of course, because the code did not contemplate this type of device and does not specifically refer to it. In the case of fractional motors, changes in the code have been made, and similar changes may be advisable for the larger motors if a very desirable technical advance is not to be held back.

The final approval of an electrical installation is given by the local wiring inspector who is guided by the National Electrical Code or local codes based on it. The National Electrical Code states that apparatus shall do thus and so or shall be approved apparatus. The listing or the labeling of the apparatus by the Underwriters Laboratories gives assurance that the necessary specifications are met and that the apparatus is of an approved type.

In the case of fractional horsepower motors, section 4322 of the code states that a motor shall be considered as protected against overcurrent if it is of a type that cannot overheat due to overloads or failure to start. It further states that this can be accomplished by an approved protective device that is an integral part of the motor.

The Underwriters Laboratories thus come into the picture with their "Appendix of the Standard for Industrial Control Equipment." This appendix covers motors one horsepower or less and its major requirements are:

1. That the motor windings shall not exceed a total temperature of 125 degrees centigrade when the motor is running.
2. That the windings shall not exceed 150 degrees when the motor is stalled.
3. That the motor and device shall last for 15 days when operating continuously trying to start a blocked rotor.
4. That the device shall not be a fire hazard when thrown directly on a 1,000-ampere circuit in series with a fuse-rated four times motor full-load current.

In the case of the integral horsepower sizes, the code states that the motor shall have running overcurrent protection to limit the current to 125 per cent of full-load value, except the current can be 140 per cent if it is part of an approved refrigeration equipment. This is inconsistent as the power system, the wiring, and the motor cannot tell what is being driven, and there is nothing evident that differentiates the refrigeration application.

The only Underwriters Laboratories Standard that applies is the Industrial Control Standard which is written for a current protective device rather than for a temperature protective device. It states that:

1. The device shall ultimately trip at its own current rating in a 40 degree ambient. (This current is 125 per cent or 140 per cent of motor full-load current as stated in the code.)
2. It shall stand being closed directly on a 5,000-ampere circuit in series with a fuse rated four times its current without being a fire hazard and without harming itself except by welding of the tips.
3. It shall withstand 6,000 operations interrupting its full load current.

An inherent overheating protective device obviously cannot be fairly judged by the combination of the code and the Underwriters Laboratories Standard as they now stand because these do not cover such a device.

To clarify the situation, consideration might well be given to amplifying the code to cover integral horsepower motors. If this were done, the present current limitation should be supplemented by a standard that would allow the motor with its built-in control to draw a running current that neither exceeds 125 per cent of full-load current in a 40-degree ambient nor raises the total temperature to greater than 125 degrees centigrade when it is running, nor above 150 degrees when it is stalled. Both current and temperature must be considered as the rule will apply to all types of motors with high and low temperature rises, and no constant relation will exist between temperature and current.

The Underwriters Laboratories Standard should then be drawn to recognize motors that meet this proposed requirement of the code, and be worded to cover automatic reset devices. Perhaps the 5,000-ampere short-circuit test should be eased to 2,000 or 3,000 amperes for motors between $1^1/_2$ and 5 horsepower. The device might be allowed to open circuit if it does not produce a fire hazard.

The Underwriters Laboratories have intimated that they will consider setting up such a standard and accepting motors for test under it, but evidently the code should be revised to give clear cut recognition to such apparatus.

H. Weichsel (Wagner Electric Corporation, St. Louis, Mo.): The problem of protecting electric machines against excessive temperature is as old as the electric machines. In "pioneer days" the operator simply watched the machine and when it began to smoke, he disconnected it from the supply or decreased its load. Even in relatively recent years the "smoke test" was used for determining the capacity of a machine. For instance, Mr. Lamme in the year 1892 used the smoke test for the purpose of determining the capacity of his rotary converter in comparison to the capacity of the same machine operating as a d-c generator. By this simple test he established the important fact that for equal heating, the current capacity of a rotary converter is 40 to 50 per cent greater than when the same machine operates as a d-c generator.

The oldest protecting means used in practice is probably the fuses. Thereafter appeared overload circuit breakers and other related devices. Most of these are cited in Mr. Potter's paper. Attempts to make use of bimetal for opening line circuit whenever demanded by the load conditions are also very old. I personally know of a design as far back as 1917 which placed such a device in the motor housing. The device was influenced by both the heater coil through which the line current passed and the air temperature inside of the motor. The amount of said air contacting the thermal switch was adjustable. From this year on the literature is full of devices of similar characteristics.

In 1923 Mr. Spencer patented his now famous "Spencer bimetal disc" which acts as a thermostat switch. The patent shows this disc applied to the protection of an electric flat iron. Many improvements in the design and the location and its use in connection with electric motors were made soon thereafter. The number of patents on this subject is very formidable.

Only during relatively recent years came the protection of motors by aid of thermal switches to the foreground. This was mainly brought about by the very large demand of fractional horsepower motors in connection with home refrigerators, stokers, and other apparatus used generally by the public. Naturally machines used so promiscuously must be made as safe as possible. By the application of thermal switches to these small motors, all danger of fire hazard is practically removed and the life of the motor is greatly increased. Mr. Potter's paper is therefore very timely. It proves by test that under any of the working conditions which may arise in connection with this type of machine, the protection against excessive heating and consequent fire hazard is practically perfect.

It appears appropriate to state here that protecting the motor by aid of a thermal switch is not as simple a matter as it might appear. It requires a great deal of study and experimental work to arrive at the correct design of the thermal protector as well as at its most suitable mounting. The number of tests needed are, however, considerably fewer and also simpler than those required by the investigation reported by Mr. Potter.

His tests were very extensive and laborious as a very large number of readings had to be taken. It is my opinion that the outstanding conclusion from these tests is the assurance to the public that the thermostat protected motor is practically foolproof and consequently protects life and property of the general public. This conclusion alone certainly amply justifies the large amount of labor necessary for carrying out these tests.

C. P. Potter: Mr. Weichsel has called attention to the fact that the application of inherent overheating protective devices to motors is not as simple as a matter as it might appear. This is true because there are several practical limitations which must be met. The protector must trip at one value of current while the motor is operating and at another when the motor is stalled. The protector must permit a fairly long period of acceleration and must not take too long to reset. It is, therefore, not practical for the user to install these devices in motors which did not have them originally.

Mr. Lukens makes a strong plea for the extension of the standards of the Underwriters' Laboratories and the National Electrical Code to permit the application of inherent overheating protective devices to motors larger than one horsepower. Motor users would like to obtain $1^1/_2$- to 5-horsepower motors equipped with inherent overheating protective devices which break the line current, and larger motors provided with devices which are connected in the pilot circuit of the control equipment. It seems desirable and logical to modify and extend existing standards to permit the manufacture and use of these larger motors.

A Turbine-Governor Performance Analyzer

Discussion of paper 41-158 by W. O. Osbon, presented at the AIEE South West District meeting, St. Louis, Mo., October 8–10, 1941, and published in AIEE TRANSACTIONS, 1941 (November section), pages 963–7.

A. F. Schwendner (Westinghouse Electric and Manufacturing Company, Lester, Pa.): The history of any development work shows the effect instruments have on the performance requirement of the particular apparatus. A performance may be considered acceptable as long as available instruments do not show up the deficiencies. Development of new apparatus usually stops at the point where the measuring ability of available instruments can indicate good performance. The demand for development of apparatus with better performance is initiated by the advent of improved instruments which show the deficiencies in the existing apparatus.

Governor development did not follow this procedure in its early stages. Governors then equaled the sensitivity of speed-measuring instruments. Governors are used even now in speed-measuring instruments. But a governing system used on a modern prime mover has considerable auxiliary apparatus beside the governor. Any one part of this system can cause difficulties as well as the governor itself. The governing valves flow-lift characteristics also affect the governor performance to a considerable extent. To analyze a governor's performance characteristics every element of the control system has to be considered and its performance recorded. The paper describes an instrument which is capable of recording the performance of all the control elements simultaneously, with extremely high sensitivity and response. It is well adapted to obtain the governing systems fundamental characteristics and a handy tool to quickly diagnose difficulties.

The instrument can also aid the power-generating systems by analyzing the governor performance characteristics on all their units. The established characteristics of the best performing governors from the overall system point of view can serve them to standardize the required performance and then help to obtain it on the other units.

The writer had used the instrument for analyzing governor performance, locating the source of erratic performance, and checking the characteristics of newly developed control apparatus. The instrument can be set up in a short time, is readily adaptable for analyzing any type of control apparatus, and extremely useful in the development of highly sensitive quick-responding control apparatus. The writer believes that it will take considerable time before control equipment will be developed to such a point that a better instrument than the one described by the author will be required.

Symmetrical Nomenclature of Three-Phase Circuits

Discussion and author's closure of paper 41-153 by Bryce Brady, presented at the AIEE South West District meeting, St. Louis, Mo., October 8–10, 1941, and published in AIEE TRANSACTIONS, 1941 (November section), pages 955–7.

G. S. Lunge (General Electric Company, Schenectady, N. Y.): Mr. Brady's proposal to consider the effect of a wye-delta or delta-wye transformation as introducing a 90 degree shift in phase instead of a 30 degree shift has indeed much in its favor. It appears that this proposal could be adopted without changing the present nationally standardized connections and terminal markings of three phase transformers.

This paper appears to be devoted primarily to a consideration of banks of single phase transformers, but it should be recognized that in many instances the transformers will be of three phase connection, internally connected so that, if wye-delta or delta-wye connected, the high-voltage vector, to neutral, will be 30 degrees in advance of what, with the present system of numbering, is the corresponding low-voltage vector, to neutral. Thus a smaller variety of possible connections should be encountered if three phase transformers are used than have been discussed in this paper.

Many operating companies have, for reasons of safety and convenience in maintenance and repair work, established their own standard geographical order of phases, reading from top to bottom, north to south, or east to west as the case may be. If the letters *A-B-C* are used for the *circuit* phases, as distinct from the polyphase transformer terminals, this should reduce the chances for confusion. Especially for transformers having more than three terminals per winding, the use of numbers to designate the phases, rather than letters, is particularly convenient and has been used for many years both for transformers and for stator windings of both synchronous and induction machines.

The author has referred to "geographical phase sequence" and to "counter-clockwise sequence." It is believed to be preferable to restrict the use of the word "sequence," where phase sequence is involved, to the (time) "order in which the voltages successively reach their maximum positive values between terminals" as defined in section 1.140 of ASA Standard *C6*, 1938.

Although not stressing this point, the author has used the designations *A-B-C* to designate the phases. In most of the previously published literature on the subject of symmetrical components, the three phases are designated *A*, *B*, and *C*, which makes it possible to use the subscripts 1, 2, and 0 for the positive, negative, and zero sequence components, respectively. To comply with American Standard *C6*-1938 (Standard Terminal Markings and Connections), the phases should be designated 1, 2, 3, rather than *A*, *B*, and *C*, or *R*, *S*, and *T* as used to considerable extent in Europe.

When numbers are used for designating the phases, it becomes inconvenient to use numbers also for designating the positive and negative sequence components, since the subscripts may, for instance, be applied to reactances or impedances, and a double subscript such as Z_{12} has been extensively used to designate a transfer impedance or a mutual impedance, and Z_{11} to designate a driving point impedance or (self) impedance in transient stability or similar calculations.

Dahl's textbook, "Electric Circuit Theory and Applications" used +, −, 0 to designate the positive, negative, and zero sequence components, but few typewriters have a + sign on them suitable for use in a compound subscript.

To overcome these difficulties it has been suggested to the interested ASA committee that the positive, negative, and zero sequence components of the current in phases 1, 2, and 3 could be designated as follows:

$$\begin{array}{lll} I_{1P} & I_{1N} & I_{10} \\ I_{2P} & I_{2N} & I_{20} \\ I_{3P} & I_{3N} & I_{30} \end{array}$$

The simplest scheme for avoiding confusion would appear to be to apply the author's proposal to polyphase transformers as well as to the circuit conductors and to banks of single phase transformers, except using the phase designations 1-2-3 instead of *A-B-C*. Some operating companies might, however, consider uniform designation for geographical order of more importance than uniform designation of phases according to their sequence. Standardization of polyphase transformer connections and terminal markings has been achieved over a period of years and should not be changed unless there is nearly unanimous demand for such change on the part of users.

For purposes of analytical treatment of problems by the method of symmetrical components and in synchronous machine theory, it appears to be convenient to continue to use the designations *A-B-C* for the phases, rather than 1-2-3. It is quite common practice for such purposes to consider single phase to ground faults as always occurring on phase *A*. Also, X_p designates the Potier reactance, not merely any positive sequence reactance, and X_2 (not X_N) designates the negative sequence reactance of a synchronous machine. In articles and textbooks on problems involving symmetrical components, X_N usually signifies an external reactor in the neutral ground connection of a machine or transformer.

Edward W. Kimbark (Northwestern University, Evanston, Ill.): Mr. Brady has made a strong case for the symmetrical nomenclature, and I believe that his contentions are correct. I have used the symmetrical nomenclature in my classes for a number of years. It is unfortunate that the present standard nomenclature became established. The question now is: How much confusion would result from changing the standards?

A. Naeter (Oklahoma Agricultural and Mechanical College, Stillwater, Oklahoma): As a teacher of electrical engineering, I am particularly interested in any paper that helps to simplify the work of engineers and their associated workers provided its limitations be recognized and stated. Mr. Brady has indicated properly that the choice between standard and symmetrical nomenclature lies entirely upon the ability of each to serve the needs of those who use the nomenclature.

Also it needs to be re-emphasized that the symbol or name which is applied to a conductor has nothing to do with the magnitude of the current which may flow through it. The same thing applies to the equipment, for example, the short-circuit current that would flow if a delta-delta bank be paralleled with a wye-delta bank.

Pointing out the economies and operating advantages of delta-wye and wye-delta transformations, the author devotes most of his paper to those two particular transformations.

The material presented regarding the application of symmetrical nomenclature to these connections should simplify the work of engineers, construction men, inspectors and others, and increase their confidence in the accuracy of that work.

The writer recalls very vividly the opinions expressed by the superintendent of a department of a major power company, in the Great Lakes area, concerning the need for an adequate method covering three-phase connections for transformers and induction regulators, and their respective vector diagrams. He claimed that no one was ever quite certain of results to be secured under methods then available. Mr. Brady's paper fills a long-felt want.

The writer believes that in the study of symmetrical components a detailed survey of Brady's paper would be desirable before considering star-delta transformations in detail.

M. C. Hughes (Agricultural and Mechanical College of Texas, College Station, Texas): Mr. Brady is to be complimented on his excellent presentation of a complexing and difficult problem.

As a teacher of electrical engineering for the past twenty years, I have been struggling daily with this problem in my classes, and on numerous occasions I have raised a feeble plea for standardization of notation on some basis which will make possible the use of books in several electrical engineering courses, or for that matter, several books in one course without producing great confusion in the minds of embryo engineers.

While I am not ready to admit that Mr. Brady's solution to this problem of notation is the best solution which may be evolved, it has merit and should be given careful consideration. If I were to criticize Mr. Brady's presentation at all it would be his naming two separate methods of notation the "standard method."

For a number of years I have been using and teaching a system of notation which eliminates the confusion of the unlimited notation called "standard" and which has some advantages, I believe, over the symmetrical method—at least from the teachers' point of view. It may be that the symmetrical method is of such definite advantage to the practicing engineer that teaching methods should become secondary.

I hope that this paper will provoke sufficient interest in this subject of standardization to generate some activity on the part of the standards committee of the Institute. This is one of the several points in notation which still need their attention.

Bryce Brady: Both symmetrical and standard nomenclature methods are used today and both will undoubtedly be used for many years in the future. The ultimate usage will probably be proportional to the benefits each is able to contribute.

Several advantages of the symmetrical plan, which apparently have gone unrecognized or only partly recognized, have been described. The task remains for authors, educators, mathematicians, design, and operating engineers to evaluate these advantages in terms of practical day to day engineering and operation. If authors of papers and books find that symmetrical nomenclature is an aid to clearer identification of delta and wye voltages and currents, then the use of this method will become more common. If educators view the matter through students' eyes and decide that 90 degree relations can be grasped more readily than 30 degree relations, then this method of presentation will become more popular. If mathematicians find that the use of 90 degree multiples, in place of 30 degree multiples, simplifies mathematical equations; if protective engineers find that symmetrical identification of single phase fault currents simplifies relaying problems; if substation design engineers find that the universal feature of symmetrical nomenclature is of practical value; if power system operators find that the *A*, *B*, *C* rule (or 1, 2, 3 rule) enables workmen to trace transmission and distribution conductors through transformations with less effort and less chance for error—then the use of symmetrical nomenclature for such diverse purposes is certain to increase.

A Novel Reclosing Relay

Discussion and author's closure of paper 41-152 by P. O. Bobo, presented at the AIEE South West District meeting, St. Louis, Mo., October 8–10, 1941, and published in AIEE TRANSACTIONS, 1941 (December section), pages 1041–5.

H. T. Seeley (General Electric Company, Philadelphia, Pa.): This paper is of interest to manufacturers as well as to users of reclosing relays, because it gives data on operating experience, which can be used to evaluate the relative worth of the various features that can be provided in reclosing relays. For example, taking the figures of 85 per cent successful immediate reclosure and 96 per cent successful reclosure, it appears that a simple single-shot recloser, self-resetting after time delay, will operate successfully on 88 per cent of the faults for which this most highly developed relay would have given successful reclosure. Perhaps this explains why the new relay is not proposed for distribution feeders.

An interesting fact brought out in the paper is the increase in the percentage of successful reclosures obtained with high-speed protective relays as compared with normal-speed relays: evidently it shows the beneficial effect of the reduction in arcing time on the wires and insulators. This is corroborated by recent data published in the Hydro-Electric Power Commission *Bulletin* for February 1941.

The voltage-check feature on any breaker on a line equipped with distance relays is unnecessary except for faults on the 10 per cent of line nearest this breaker, which will be cleared in intermediate time instead of instantaneous time by the relays at the other end. Even for that 10 per cent the voltage-check feature is ineffective for three-phase faults, and if only a single-phase relay is used it is ineffective for about one-third of the phase-to-phase faults as well. Even if we neglect three-phase faults (which are rare), this reduces the effectiveness of the voltage-check feature with distance relays to six or seven per cent.

The paragraphs under the heading, "Need for a Quick Resetting Reclosing Relay," state the case in such terms that the reasons for the utility company embarking upon this development are obvious. Before a manufacturer can exploit successfully the more complicated and more expensive device necessary to provide these features, the facts set forth in those paragraphs must be familiar to those responsible for selection of reclosing relays in many customers' organizations, and this paper should be helpful in that regard.

It is interesting to note that the most salient feature of the new relay, namely, its ability to reset following successful reclosures, was standard with one manufacturer about 1922. This feature was inherent in the type *PQ*-108 notching relay, which had a bellows for obtaining reset if the breaker stayed closed for a definite time after any reclosure. However, this relay did not provide the reclosing delay then considered necessary, and the time-delay relay used with it did not provide the separately adjustable intervals then considered desirable, so that form of reclosing equipment was superseded by motor-driven timers which, with refinements, have remained standard up to the present time. In view of the course of progress indicated by the present paper, that change may seem unwise in retrospect, until it is remembered that the practice of those days did not apply reclosing on transmission lines, did not co-ordinate air-break switches with reclosing breakers, and did not co-ordinate fuses with breakers by means of instantaneous overcurrent relays.

The ingenious schemes for isolation of faulted sections, by means of air-break switches co-ordinated with the reclosing cycle, show the need for accurate timing. Apparently the device described is satisfactory from this standpoint, in spite of the use of the vibrating reed-timing scheme which has a voltage error because of the variation in pickup time of the delayed pickup relay *SO2*. This indicates that some users do not consider it necessary to take account of voltage variations over as wide a range as the 80–110 per cent imposed on manufacturers by paragraph 1.143 of ASA Standard *C37.1*. The width of this range influences the acceptance or rejection of proposed simplified schemes in new designs, and in view of this evidence that the present range is wider than users require, this seems to be an opportune time to ask those favoring a narrowing of this range, and also those opposing any change, to express their views to their representatives on the protective devices committee.

W. A. Wolfe (Kansas Gas and Electric Company, Wichita, Kans.): Mr. Bobo is to be complimented for having developed a reclosing relay which eliminates the most outstanding deficiency in standard reclosing relays on the market today, namely the "blind spot" between the last reclosure and the resetting of the relay. A trip out occurring while the recloser is passing through this zone results in an unnecessary lock out of the breaker. This may happen even though the breaker has reclosed successfully on the first trip out two minutes or more before the second trip out, which locks it out.

Our experience has been that this condition does occur all too frequently to be lightly regarded. The only solution has been to minimize the chances for unnecessary lock out by putting the last reclosing point on the relay as near the resetting position as possible. Even this will not eliminate all cases of trouble of this nature and in addition this expediency when adopted forces one to use a reclosing cycle which he might not otherwise wish to use.

The flexibility and complete adjustability of Mr. Bobo's timer leaves nothing to be desired. In one standard design he has a reclosing relay adaptable to almost any operating condition, and which could with ease be adapted to the complete automatization of important high voltage switching stations in high voltage transmission networks.

M. E. Reagan (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): Mr. Bobo's paper on "A Novel Reclosing Relay" is very interesting and is an excellent example of how some study of local conditions will dictate a departure from what is considered standard practice in the industry.

Co-ordination of fuses, automatically tripped disconnecting switches, and feeder breakers has been the subject of interesting discussion and test for a good many years. The solution of the problem depends on the local conditions which are dictated by the type of distribution system employed and the economics of the application. Such features as devices for detecting an energized or de-energized line and synchronism checks may be economically built into a commercially produced relay only if the great proportions of applications demand them. If not, these features are best obtained by the use of "outside" relays when required.

The fast-resetting feature on successful reclosure, which has been so effective in keeping service of the lines of Mr. Bobo's system, has also been a subject of much discussion throughout the years. The type of fault produced by an arcing ground, usually caused by weakened insulation or by trees swinging into the lines during storms, has been the general reason for omitting this feature. During the time of such faulty conditions, the line may be clear of faults long enough for the relay to reset only to have the fault reappear a moment later. Thus, it is theoretically possible to subject the breaker to a destructive number of operations without locking it out of service. I would like to ask Mr. Bobo if any special precautions are taken in line construction on his system, such as over-insulation or special clearances to guard against these rather common types of system faults.

It is gratifying to learn that Mr. Bobo's experience with the operation of the telephone type relay has been so successful and trouble-free. Some twenty years of similar use of such relays in supervisory control systems have demonstrated their reliability. They are very dependable when properly applied.

Another interesting approach to the service restoring problem, where isolating disconnecting switches are used on branches from the main feeder, has been recently made by another utility in the middle west. Here the feeder breakers served lines of the radial type and each important load was taken off through motor-operated disconnecting switches. A fault on the feeder system caused the feeder breaker to open. The de-energizing of the line caused the opening of all disconnecting switches. The dispatcher then reclosed the feeder breaker, and if the reclosure was successful, it meant that the fault had cleared or, if permanent, was on one of the branches. Through the use of single-operation, multi-unit supervisory control, the dispatcher then closed the disconnecting switches one after the other until they were all closed or until the fault was re-established. If the feeder breaker again tripped as a disconnecting switch was closed, service could be established on all other branches of the circuit and a repairman dispatched to locate and remedy the faulty section. This represents another method of approach to the common problem of improving service on the lines and, as with any scheme, depends on the local transmission type and nature of the loads as to the logic of the application.

E. E. George (Ebasco Services, Inc., New York, N. Y.): The equipment for automatic reclosing of oil circuit breakers, as described in this paper, has several features that should appeal to those who design or use protective equipment. Possibly the outstanding feature is the extreme flexibility, permitting modifications in the number of reclosures, the interval between reclosures, and in the transfer of protective schemes between the first and second reclosures.

Another desirable characteristic of the equipment described is its use of standard parts produced in quantity for the telephone industry and therefore available at prices lower than those necessarily charged for items with a more limited market and burdened with recent development expense.

The manufacturers of power system relays have designed some equipment consisting of a flexible assembly of standard units and the response of the users has usually been satisfactory. However, many utility operating engineers have found it desirable to build much of their own equipment and frequently, as is this case, have produced a flexible design that is fundamentally sound and applicable to practically every situation.

Telephone relays of the type used by Mr. Bobo have been used in remote control of underground network protectors (see "Remote Control of Network Protectors," AIEE TRANSACTIONS, volume 57, 1938 (December section), page 685) but instead of being mounted in power relay cases have been mounted in standard telephone connector frames with detachable plug and jack connections. This detachable feature is very convenient, although the adaptation of the connector bases to power switchboard panels is somewhat difficult.

Philip Sporn (American Gas and Electric Service Corporation, New York, N. Y.): The novel reclosing relay described by the author consists of 11 telephone type relays, all mounted in one case, to perform hot-line, dead-line, or stub circuit reclosing as desired. Timing is obtained by a vibrating reed relay, acting in conjunction with slow operated relays to pulse a stepping switch at intervals adjustable between 1 and 15 seconds.

Our experience with vibrating reed-type relays has not been very satisfactory, and before employing same for timing purposes we would conduct exhaustive tests to assure ourselves that this timer was as reliable in its operation as the motor-operated type. Further, we question whether the contacts of these relays would hold up under normal use employing voltages as high as 260 volts d-c, which we have at several of our automatic substations.

We are not quite certain from the description of the recloser whether or not the relay which is used to indicate dead line in this recloser has any time delay on dropout. We assume that it does not and hence we believe this to be a weakness, since it might be entirely possible for a momentary dip in voltage to permit the oil circuit breaker to close and thereby nullify the effect of the relay which was to prevent this operation. In all of our schemes it is our practice to use a time delay undervoltage relay to indicate a condition of dead line to prevent the false operation described above. The same weakness is also present when check synchronizing or synchronizing relays are employed. In this case it is possible while the switch is held open, waiting for the two systems to get into synchronism, that a momentary dip in voltage on one line may cause the timer to notch to the closing position, thereby closing the breaker when the systems were not in synchronism.

This timer has one feature which seems desirable in a timer; namely, that the reset time may be selected so that the timer resets in a very short time after a given reclosure. This feature may result in saving a very small percentage (a good guess would be less than one per cent) of lockouts when actually a lockout was not desired. These particular operations resulting in the false lockout may occur on a conventional timer if a tripout occurs between the time when the last reclosure point has been passed and before the timer has reset. Our practice has been to make this time as short as possible. A tripout occurring at this time, even though it is only transitory, will in all cases result in permanent lockouts.

It seems to the writer that this device has several advantageous features but, before a definite determination could be made whether or not the device as designed represents a satisfactory recloser, it would be necessary to make a somewhat extended study of the component devices used and examine more thoroughly their construction. Another point to be considered also is that every operation performed by this timer may be performed by conventional relays wired as individual units and mounted on panels where they are subject to ready inspection and maintenance. A study of the cost of such a device compared with the scheme of using the individual component relays should be made to determine whether or not it is desirable from this standpoint.

P. O. Bobo: The author agrees with Mr. Seeley that a single-shot recloser will give successful reclosures on 88 per cent of the faults, but wishes to stress the fact that this reclosing relay was primarily designed to eliminate the service hazards which may occur on the remaining 12 per cent of faults. Although service hazards on distribution feeders are not nearly so serious as those on transmission circuits for which the relay was designed, the relay can be used equally well on distribution feeders because of its universality.

Mr. Seeley apparently has limited his remarks on the voltage-check feature to the initial reclosures on a "de-energized" line only. His statements regarding its effectiveness are true under the condition stated in his discussion, but this effectiveness is made 100 per cent by delaying the operation of the setup relay *SU* until after total clearance of the fault. Such a delay need be but a fraction of a second on lines equipped with distance relays and is accomplished by using a slow-operate coil in the setup relay. The conditions of breaker closure and the conditions of timer movement for subsequent reclosures make necessary the voltage-check feature for reasons stated in the paper.

Mr. Reagan points out that it is theoretically possible to subject a breaker equipped with the described resetting reclosing relay to a destructive number of operations by a periodically recurring fault. The same is true using manufacturer's standard single-shot self-resetting reclosers. Such recurring faults, to the author's knowledge, have never occurred on the Oklahoma Gas and Electric Company's transmission system, and, therefore, are believed to be a rare probability. In answer to Mr. Reagan's inquiry, special precautions are taken in line construction to prevent such faults; right-of-ways are wide with wooded sections cleared and conductor cofiguration is arranged so as to prevent conductors swinging together, especially during glaze storms. However, on systems where periodically recurring faults do occur, the resetting feature may be omitted from any one or all reclosures. A cumulative lockout feature can be built into the reclosing relay for limiting the breaker to a predetermined number of operations, if desired.

If fuses are applicable to the branches of the main feeder of the middle west utility mentioned by Mr. Reagan, the author's scheme for sectionalizing faults on multibranch fuse circuits would have advantages over the scheme used in that it is less expensive, completely automatic, and would never subject the nonfaulted branches to more than a transient outage.

It is interesting to note that Mr. Wolfe's company considers the so called "blind spot" the most outstanding deficiency in the standard reclosing relays now available.

The comments given by Mr. George, no doubt represent the opinions of a large majority of relay engineers. Apparently a change away from standardization is gradually taking place, as the author has noticed some trends toward flexibility; distance relays are now equipped for carrier current control whether carrier is used or not; test devices and removable units are also now available as built-in assemblies. It is the hope of the author that this trend will be extended to other protective equipments and service restoring relays. If flexible devices were made available at reasonable cost by the manufacturers, we would be more than glad to purchase them, for after all, manufacturing is not our business.

It would be interesting to know what trouble Mr. Sporn and his company have had with vibrating reed relays. Perhaps it has been extreme voltage variation, which as mentioned by Mr. Seeley does cause inaccuracies in timing; these inaccuracies, within limits, vary almost directly with the voltage. Since all of our installations to-date are served from floating station batteries with practically constant voltage, the timing obtained is accurate.

As to the doubt raised by Mr. Sporn concerning the interrupting capacities of the telephone type relays for 260 volt d-c installations, the author needs but to quote from the relay manufacturer's catalog: ". . . . products include: Relays—in an unlimited variety of assemblies, with a wide range of characteristics and contact arrangements, and for operation on either alternating or direct current with potentials ranging from a fraction of a volt to 220 volts, and even higher," and "Higher voltages can be controlled by operating a number of contacts in series, or by the use of $^1/_8$-inch flat cylindrical tungsten contacts."

The uncertainty raised by Mr. Sporn as to whether or not the line-checking relay uses time delay on dropout is answered in the author's above comments on Mr. Seeley's discussion.

In many instances the difference in value and goodwill between a transient outage and a longer outage amounts to more than the price of a recloser. Therefore in reply to Mr. Sporn's guess regarding the effectiveness of the reset feature, the author wishes again to point out that the reset feature not only prevents false lockouts, but prevents longer than transient outages caused by successive transient faults which may occur within the cycle of time for which the recloser is set. Our experience with this feature shows 10 per cent effectiveness.

A study showing the comparison of the described recloser versus conventional relays wired to obtain the same results as suggested by Mr. Sporn has already been made by our company; in fact, we have both types of these installations in service. Comparative costs, necessary panel space, installation time, and labor are overwhelmingly in favor of the subject recloser.

High-Pressure Gas as a Dielectric

Discussion of paper 41-159 by G. C. Nonken presented at the AIEE South West District meeting, St. Louis, Mo., October 8–10, 1941, and published in AIEE TRANSACTIONS, 1941 (December section), pages 1017–20.

Henry M. Hobart (retired, General Electric Company, Schenectady, N. Y.): Nine years ago, the writer became much impressed with the importance for various engineering applications, of the dielectric properties of compressed gases which had been discussed by Professor Ryan, Mr. Bolsterli, and others, in various papers and articles. Compressed gases appeared very applicable to transformers as an insulating and cooling medium, and also for the insulation of very high-voltage transmission conductors. Both of these uses had been considered by Professor Ryan and others many years earlier.

It was, however, at once stated by various specialists, that these applications would not be practicable, because "increase in gas pressure has no effect on its impulse strength." At about that same period, another adviser wrote: "I had, when I made my comments, forgotten the important fact of lower impulse resistance of gases under pressures that give a 60-cycle puncture equal to oil." Several other communications could be cited, but the above are typical. They have been coming in right down to the present time. Probably many engineers are still under the impression that such statements represent the facts.

The overcoming of the prejudices created by impulse-stress traditions has been the chief hurdle encountered in the ensuing years to progress with these compressed-gas projects. The situation is certain to be greatly improved by the publication of Mr. Nonken's fine paper. Probably it is the first publication presenting in clear form, comprehensive evidence that the traditions mentioned are based on error. So far as relates to fairly homogeneous fields, Mr. Nonken's Figure 2 presents curves showing for two gases, that the extent of the effect of pressure is the same for standard $1^1/_2$, 40 microsecond impulse stresses as for 60-cycle stresses. Figures 3, 4, and 5 all relate to $^1/_2$-inch rod-gap tests and show (for the same two gases) a great variety of differences between the effects of the amount of the gas pressure on the breakdown stresses, with 60-cycle and impulse stresses respectively. In the course of Mr. Nonken's researches, he has discovered, or thrown additional evidence on, many compressed-gas phenomena of much interest and importance. It should be emphasized that rod gaps are employed to represent extreme cases of nonuniformity such as will be avoided in actual engineering applications, just as it was learned long ago to avoid sharp edges and corners in the case of solid insulating materials. When we have obtained better knowledge of the dielectric properties of compressed gases, we may find it actually simpler to deal with these nonuniformity field conditions in the case of gases than in the case of solids.

Mr. Nonken's curves show that in the case of gases in the gap between rod electrodes, it is sometimes the 60-cycle strength, and sometimes the impulse strength which is increased over the other when the gas pressure is increased.

The writer notices that for the $^1/_2$-inch rod-gap tests, the departures from regularity which at first glance seem so capricious, but which really are clues to interesting and important truths, only become much in evidence when the spacing is increased above two centimeters. Doubtless with some other shape or dimension of electrodes, the spacing of two centimeters above mentioned as a sort of dividing line, would be reduced to one centimeter. That would attach interesting significance to some statements made by Finkelmann in discussing his research on "The Electrical Breakdown of Various Gases at High Pressure" (*Archiv für Elektrotechnik*, volume 31, 1937, page 284) as follows:

> "Noteworthy is the steep decrease in the rate of rise in the breakdown voltage at rather high pressure for the spacing of 0.94-centimeter which occurs with

Table I

A	B	C	D	E	F
	6.25 Cm Sphere Gap	$^1/_2$" Rod Gap	$^1/_2$" Rod Gap		
Pressure in Absolute Atmospheres	Crest Kilovolts at Breakdown for 60-Cycle Stress and for $1^1/_2$, 40 Impulse Stress	Crest Kilovolts at Breakdown for 60-Cycle Stress	Crest Kilovolts at Breakdown for $1^1/_2$, 40 Impulse Stress	Ratio of B to C	Ratio of D to C (Rod-Gap Impulse Ratio)
2	90	30	45	3.00	1.50
4	165	54	78	3.06	1.44
6	240	76	100	3.16	1.32
8	290	92	110	3.16	1.20
10	350	110	122	3.18	1.11
12	400	124	134	3.22	1.08
14	470	140	146	3.36	1.04
16	530	150	150	3.52	1.00

> all the gases tested. This decrease is greatest with nitrogen where it also occurs at the higher spacings, so that the breakdown voltage of nitrogen then decreases far below that of carbon dioxide. The 'spread' of the breakdowns increases in all gases with the pressure and with decreasing spacings. The 'spread' is the smallest in carbon dioxide, and has the greatest values in nitrogen. It is greater with cylindrical than with plate electrodes."

Concentrating on the case of nitrogen and a spacing of 2 centimeters (the spacing below which the caprices shown in Mr. Nonken's curves for rod electrodes are not extreme, the writer plotted on one sheet, Mr. Nonken's curve *C* of Figure 2, and curves *H* and *C* of Figure 4. These three curves provide the data in the accompanying table.

The values in column *F* are the rod-gap impulse ratios. They change smoothly from 1.5 for low pressures down to 1.0 for the highest pressure in the table (which is only 16 absolute atmospheres). These data, together with the various data discussed by Mr. Nonken in his paper, show that it is no longer correct to consider the impulse ratio as being 1 for gases as against 2 for transformer oil. It is not known to the writer whether the value 2 for the impulse ratio of oil is limited to the case of electrodes providing uniform fields, or whether it has been shown to hold also for rod gaps. Be that as it may, Mr. Nonken's tests reveal that the impulse ratio for compressed gases has, in nonuniform fields, a wide range of values, sometimes much above and sometimes below the value 1.

It should be of interest here to quote from the second edition of Arnold Roth's fine treatise entitled "Hochspannungstechnik" (Springer, Vienna, 1938). On page 208 Roth states:

> "The impulse ratio is very dependent upon the electrode shape. It is high with points and low with plates and spheres. The sphere gap can therefore be used for measuring the crest value of impulse waves, provided its spacing is not chosen too great with respect to the sphere diameter."

Mr. Nonken states:
"For example, the impulse breakdown with a $1^1/_2$ 40-microsecond wave for a rod gap in air at atmospheric pressure is about 1.8 times the 60-cycle breakdown."
The writer would be interested to know more of this test, and especially to what spacing it corresponds.

The curves prepared by Mr. Nonken from the researches of Zeier and Finkelmann, and published by him at page 677 of the AIEE Transactions, volume 60, 1941, together with other data in the technical literature of the subject, have so disillusioned the writer as regards the dielectric characteristics of compressed nitrogen, as to have led him to turn more earnestly than heretofore, to other gases. CO_2 is strongly endorsed by Finkelmann, as already mentioned in this discussion. In addition it is stated by Finkelmann that "CO_2 may be more readily obtained free from dust particles and traces of moisture than is the case with nitrogen."

During several years, the writer has repeatedly called attention to the particular appropriateness of compressed sulphur dioxide, both because of its very low cost and its extremely great dielectric strength. Indeed, the fact of the great dielectric strength superiority of SO_2 was published by Natterer no less than 52 years ago (Wied. Ann. 38, page 63, 1889). At a compression of 50 absolute atmospheres, the superiority of SO2 over transformer oil at atmospheric pressure, is very striking. Only very few tests of the dielectric strength of compressed SO2 are known to the writer to be available, and these are not carried up to high pressures. Their trend is, however, unmistakable, and indicates that for spacings of 100 mils and upward, a homogeneous field, and a pressure of 50 absolute atmospheres, the 60-cycle dielectric strength of SO2 gas will be considerably over 3,000 crest volts per mil as contrasted with transformer oil's 60-cycle dielectric strength of only 320 crest volts per mil. The impulse strength of transformer oil is only about 640 volts per mil between 6.25 centimeter spheres as shown in curve *E* of Figure 2 of Mr. Nonken's paper. Thus SO2's impulse strength of over 3,000 volts per mil under these conditions, is a matter of some five times greater than the impulse strength of transformer oil, and is also about five times greater than the 60-cycle strength and impulse strength of nitrogen gas at a pressure of 12.5 absolute atmospheres when measured with a 6.25-centimeter sphere gap. Evidently there is no justification for asserting (as often has been done) that the successful use of compressed gas in power transformers is unrealizable because of impulse-voltage limitations.

Since it is Mr. Nonken himself who, by his publication in AIEE Transactions, 1941, has called attention to the fact of the great dielectric inferiority of compressed nitrogen, and since other faults of nitrogen as compared with other appropriate gases are now known, it is disappointing that it should be used as a yardstick in establishing comparisons between compressed gas and transformer oil. It is doubly unfortunate, because some of the data he employs in the comparisons correspond to nitrogen at extremely low pressures. CO2 at some much higher pressure would have been a decidedly more creditable representative of what can be achieved in employing compressed gas instead of oil in power transformers.

A fairly careful re-examination of some available data of the dielectric strength of several compressed gases has led the writer to the following basic values as typical for the conditions of 50 absolute atmospheres, homogeneous field, spacings of 100 mils and upward, 60-cycle stresses:

Nitrogen	1,400 crest volts per mil
Carbon dioxide	1,800 crest volts per mil
Air	1,800 crest volts per mil
Sulphur dioxide	Over 3,000 crest volts per mil

The insufficiency of data of the dielectric strengths of SO_2 and CCl_4 at high compressions, high voltages, and great spacings may doubtless be attributable in part to the fact that since their dielectric strengths are several times greater than that of nitrogen, there would be required pressure-test tanks and bushings and electrode fittings much larger and much more expensive than are usually available. But it is known that for a given spacing and compression, the 60-cycle, crest-breakdown voltage is of the order of only one third as great with the $^1/_2$-inch rod gap as with the 6.25-centimeter sphere gap. By increasing our knowledge of the laws of this ratio, we should be able to make the actual tests with the rod gap, using only about one-third as great voltage and then deduce from the appropriate values of the ratio, the value of the voltage which would have been observed if the tests had been made with the sphere gap. University laboratories could use this procedure to obtain without expensive outlays, much needed data of the dielectric characteristics of various compressed gases.

Such fine researches as those now being carried on by Mr. Nonken and others, and directed toward analyzing the complicated phenomena associated with the dielectric properties and behavior of compressed gases and toward ascertaining the "reasons-why" of their behavior, should be encouraged and advanced in all practicable ways.

In the meantime, however, application of the valuable knowledge with which they have already provided us, ought not to be delayed. It is clear that the ways to avoid trouble from our present ignorance of some of the "reasons-why" of the caprices which are being investigated, are to use high compression and the strongest of gases, and to strive to approach uniform fields in our designs, and also not to allow our appreciation of the great value of compressed gases to deter us from making free use of appropriately designed and located barriers and supports of solid dielectrics.

G. W. Dunlap (General Electric Company, Schenectady, N. Y.): This paper by Mr. Nonken is the first of the recent group of papers concerning the dielectric strength of gases under pressure to discuss the performance of gases under conditions of impulse voltages. The importance of adequate information on impulse characteristics where application to power-apparatus design is contemplated should not be underestimated. This is particularly so, because gases under pressure show a wide range of impulse ratios with some cases showing very unfavorable ratios as indicated in the paper.

Tests made in Schenectady with a tank five feet high and 20 inches inside diameter and with substantially the same technique described by Mr. Nonken gave results for 6.25-centimeter spheres which check the curves of Figure 2 within a few per cent. This was true for both nitrogen and CCl_2F_2 although there was some tendency particularly with freon toward a little higher strength on impulse over the 60-cycle voltage value.

Similar close agreement between the Schenectady results and those of the paper, Figure 3, were found with standard $^1/_2$-inch square rods spaced up to six centimeters in CCl_2F_2. The rod gap in nitrogen results check the data shown in Figure 4 of the paper for the two centimeters spacing but at six centimeters considerable divergence was found with a maximum 60-cycle strength indicated at a somewhat lower pressure than shown in Figure 4. This latter difference is probably due to proximity effects in the smaller Schenectady tank and further emphasizes Mr. Nonken's point that tests under specific conditions are necessary for the proper application of gas dielectrics in apparatus design.

Also of interest are some impulse tests in nitrogen at 500 pounds per square inch which were made with insulation structure samples such as described in Table I of the paper. The tank used in this case was mounted horizontally and was seven feet long by 20 inches inside diameter. The samples and technique were the same as described by

Mr. Nonken except that the vertical axis of the samples as shown in Table I was perpendicular to the axis of the tank. The results were as follows.

Table II

Insulation Setup (Table I of paper)	Impulse Breakdown in Per Cent of No. 1 Sample 10-C Oil Breakdown
No. 1	125 withstood without breakdown (voltage limited by flashover to tank)
No. 2	115
No. 3	86
No. 4	60
No. 5	90

While these data are too limited to justify any broad conclusions, it will be noted that even with the nitrogen pressure elevated to 500 pounds the impulse strength is still less than that of oil for some of the insulation structures. On the other hand, the increased strength indicated with samples 3 and 4 gives promise that the application of gas insulation may be extended in certain particular fields to good advantage.

In conclusion it is apparent that high pressure gas is not a "cure all" for insulation problems. On the other hand, it has proven well suited to certain special applications such as high-voltage X-ray transformers and electrostatic generators and there are probably many more similar cases where its particular advantages may be utilized. Each application should be considered on the basis of its particular conditions and extreme caution must be exercised in extrapolating available data. The only really safe design procedure is by a program of tests under conditions simulating as closely as possible the actual conditions of the application.

V. M. Montsinger (General Electric Company, Pittsfield, Mass.): I wish to discuss Mr. Nonken's paper from the standpoint of using high-pressure gas in place of oil as a dielectric medium in power transformers. This paper presents some much needed data on the 60-cycle and impulse strength of high-pressure gas.

The use of high-pressure gas in power transformers has been suggested many times during the past few years. Its very high 60-cycle dielectric strength at the higher pressures has made it appear as an excellent substitute for oil. Not only has the gas better 60-cycle dielectric strength, but its being inert gives it the added attraction of being nonexplosive and noninflammable. To be practical it would be necessary to reduce the insulation distances below those used with oil for a given voltage, in order to counteract the increased cost of the high-pressure tank. The use of gas, therefore, is limited to the higher-voltage transformers—115 kv and above—where large spacings are used in oil-immersed transformers.

All high-voltage transformers being built today are, of course, designed to withstand the standard impulse tests which range from 630 kv (1.95 times the 60-cycle test kilovolt crest) for a 115-kv transformer to 1,210 kv (1.86 times the 60-cycle test kilovolt crest) for a 230-kv transformer. In other words, for balanced conditions for both impulse and low-frequency tests, the impulse strength of the high-pressure gas should be in the approximate order of two times the 60-cycle strength in kilovolt crest.

The data given in the paper show that the impulse ratio (ratio of impulse to 60-cycle crest voltage) is close to unity. A reference to Figure 5 will show that, while the 60-cycle strength of the gas at high pressures is better than the strength of oil, its impulse strength is well below the impulse strength of oil.

Again, the dielectric strength of the gas at all pressures increases quite slowly with spacing of the electrodes. The impulse data given in Figure 3 show that at one atmosphere the strength varies as the spacing (from two to eight centimeters) raised to the 0.37 power. At 75 pounds per square inch absolute pressure, the strength varies as the spacing raised to the 0.27 power. On the other hand, the dielectric strength of oil varies approximately as the spacing raised to the 0.67 power for nonuniform dielectric fields.

If this same relationship between dielectric strength and spacing holds for larger spacings it means that distances considerably greater than used in oil would be required to obtain sufficient impulse strength when using gas in high voltage transformers. The insulation setups shown in Table I correspond more nearly with the conditions in a transformer. In no case, however, do these results show a higher impulse strength in high-pressure gas than in oil (CCL_2F_2 being the same as oil for setup 2) and in most cases they are less.

It would be necessary, of course, to obtain a reduction in the size of and amount of material used in a power transformer using high-pressure gas, to offset the added expense of providing the high-pressure tank. It does not appear, for the reasons just given, that the prospects are very good for using high-pressure gas in place of oil as a dielectric medium in high-voltage transformers, except perhaps in special cases where light weight is highly desirable, and fire hazard is objectionable (and where impulse voltages are of no concern), there may be cases where high-pressure gas will find use in transformers.

Operation of Load-Ratio Control Transformers Connected in Parallel and in Networks

Discussion of paper 41-166 by F. M. Starr, presented at the AIEE Southern District meeting, New Orleans, La., December 3–5, 1941, and published in AIEE TRANSACTIONS, 1941, pages 1274–80.

T. C. Lennox (General Electric Company, Pittsfield, Mass.): When regulators or transformers equipped with automatic load-ratio control are operated in parallel, particularly in networks, there is almost invariably a conflict between the requirements of voltage regulation and those of stable parallel operation.

This has long been understood and it has been common practice to accept a compromise as to the accuracy of regulation in order to achieve stability. The means commonly accepted for the purpose has been the reversal of the reactance element of the line-drop compensators and an equivalent increase in resistance compensation.

While stable operation and acceptable regulation have usually been achieved in this way, there has been lacking a complete analysis of the conditions existing in such controls which would allow of quantitative determination of the required compensator settings. Consequently, there have, doubtless, been many cases where adjustments have been made that were unduly conservative from the standpoint of stability at the expense of satisfactory voltage regulation.

In this paper Mr. Starr has analyzed the whole problem and provided us with a convenient method of working out the most advantageous adjustments in installations of widely different types.

Mr. Starr's exposition of conditions in the contact-making voltmeter is a particularly happy one in which a difficult problem is made most clear and understandable.

Factors Contributing to Improving Electric Service by Means of High-Speed Switching and Utilization of Stored Energy

Discussion and author's closure of paper 41-164 by J. T. Logan and John H. Miles, presented at the AIEE Southern District meeting, New Orleans, La., December 3–5, 1941, and published in AIEE TRANSACTIONS, 1941 (December section), pages 1012–16.

E. E. George (Ebasco Services, Inc., New York, N. Y.): Extensive experience with instantaneous reclosing and with inertia-type relays is discussed in a very competent manner in this paper by Messrs. Logan and Miles. Ten years of operating experience with instantaneous reclosing is summarized, and the figures show that in five cases out of six, instantaneous reclosure is successful. While the experience with the inertia-type relay does not extend over so long a period, the record to date indicates a considerable field of successful application for this device.

The paper does not indicate the many obstacles that had to be overcome in applying instantaneous reclosing generally on transmission systems or the extent to which this development has affected present day standard practice. For instance, when instantaneous reclosing was first started, most of the standard practice control schemes then in use were not proof against pumping or continuous opening and closing. Accordingly, most of the existing breaker installations had to be rewired, and, in most cases, an auxiliary switch and extra control wire had to be added. Unfortunately motor-closing breakers had begun to supersede solenoid-operated breakers, and some of these motor mechanisms would not relatch in time for fast closing. Fortunately, this type of motor mechanism is now obsolete. Several styles of relays then in use were too slow in parting contacts after operating, and this required changes in the relays or in the control schemes. The manufacturers have eliminated this source of trouble. The most serious obstacle to instantaneous reclosing was a continuous barrage from the manufacturers insisting that breakers for instantaneous reclosing should be derated, in some cases more than one half. As most of the breakers then in use on transmission systems were of the type to which the manufacturers wished to apply the heaviest derating factors, the operating engineers had to overcome these arguments for the installation of much heavier breakers wherever instantaneous reclosing was used. This argument came from engineering departments of utilities as well as from the manufacturers' engineers and commercial representatives. Fortunately experience on many systems has demonstrated that the so-called necessity for the derating for instantaneous reclosure is largely imaginary. In practice, breaker failures from this cause are negligible.

In addition to the improvement in service to industrial customers as outlined in this paper, instantaneous reclosing affords outstanding improvement in service to residential and commercial customers. Hospitals, theaters, and department stores are typical examples where restoration of service in one-half a second is incomparably superior to restoration after 15 seconds or 30 seconds.

On transmission systems where cities are supplied from two transmission lines without local generation, it has been found in practice that the use of instantaneous reclosing on the transmission system almost eliminates failure of high-voltage power supply, whereas in the old days of one-minute reclosing, the second line would frequently relay out during a lightning storm before the first line could be reclosed.

The discussion in the paper on the use of inertia relays and on the switching of synchronous motors is especially valuable because there has never been any clear-cut statement of practice or principles in handling this equipment. Operating experience as well as special tests have resulted in very conflicting ideas, due probably to the wide variety of torque characteristics inherent in the loads as well as in the synchronous motors themselves.

Robert Treat (General Electric Company, Schenectady, N. Y.): For a number of years the authors of this paper have been doing very commendable and valuable work in exploring the possibilities of supplying high quality service despite the obstacles thereto occasioned by circuit interruptions and voltage surges caused by short circuits. On their own system they have accomplished rather remarkable results at very moderate cost—and to a considerable extent these results have been accomplished by the employment of standard existing devices or devices which were but slight modifications of the standard. On the other hand, the authors have not hesitated to develop new devices, or to combine more or less standard devices to form new associations, when such developments seemed to them to be justified or necessary to accomplish their purposes.

It is gratifying to note that in this paper the authors are again giving the industry the benefit of their own very considerable experience in improving the quality of service rendered by their company, as they have so generously done a number of times before.

J. T. Logan: Remarks made in the foregoing discussions are indeed appreciated. The freedom of these discussions from controversial issues is indicative of the cooperation given the authors during the development stages.

There is one important related factor, however, that is not discussed in the paper, which should be brought out at this time, and that is the effects of this practice on transmission-line insulation values. Higher line insulation levels tend to increase the number of three-phase faults compared to the number of phase-to-ground faults, the former being more serious from the standpoint of system stability and voltage quality. Also, admitting that it is economically impractical to construct a transmission line of moderate voltage, free from lightning flashovers, should not the improvement in service that accrues from the application of modern high-speed circuit breakers with foolproof control schemes, together with high-speed relay protection be weighed against that obtained through overinsulation of transmission lines? In other words, a class of service might be rendered through the practice of high-speed circuit reclosure to nominally insulated lines equal or superior to that rendered by overinsulation of lines equipped for delayed reclosure, and generally at much lower cost. Is it possible that transmission engineers are overlooking the benefits that are obtainable through high-speed circuit reclosure?

Tennessee Valley Authority Hydroelectric Stations—Electrical Design

Discussion and author's closure of paper 41-165 by Raymond A. Hopkins, presented at the AIEE Southern District meeting, New Orleans, La., December 3–5, 1941, and published in AIEE TRANSACTIONS, 1941, pages 1281–93.

V. M. Montsinger (General Electric Company, Pittsfield, Mass.): In connection with the statement given in the paper that the forced air-cooled rating of the transformer is 1.33 times the self-cooled rating, it is well to point out that according to accepted practice this ratio is limited to the case where the forced air-cooled rating is the supplementary rating. When the continuous forced air-cooled rating is the base rating it is, according to accepted practice, 1.25 times the self-cooled supplementary rating.

The new American Standards Association rules permit in the first case a hottest spot temperature rise of 70 degrees centigrade, with a resistance rise of 55 degrees centigrade; and in the second case a hottest spot temperature rise of 65 degrees centigrade, with a resistance rise of 55 degrees centigrade; for the forced air-cooled rating in each case and both on a continuous basis. Therefore, when the forced air-cooled rating is the supplemental rating, the transformer cannot be operated continuously at 1.33 times its self-cooled rating without a reduction in the expected life of the transformer as a result of the hot spot temperature.

R. A. Hopkins: Mr. Montsinger has mentioned a very important item which was possibly not completely covered in the original paper. We have always considered the 133-per cent, blown rating of the transformer as in the same class with the 115-per cent, 80-degree generator rating in that continuous operation at these higher ratings would result in some reduction of life of the transformer and generator. The equipment, therefore, is operated at these higher ratings only for short periods under high head conditions representing a small percentage of their entire life. Furthermore, it will be noted that with the coordination between the transformer and generator ratings described in the original paper under the section on transformers, our transformers have generally been given a slightly higher rating than the generators so as to avoid excessive hot-spot temperatures during the periods of operation at the higher rating. If the new rules now under consideration by the American Standards Association and the AIEE are adopted, it may be advisable to give the transformer a still greater margin over the generator.

Formulas for the Inductance of Rectangular Tubular Conductors

Author's closure of paper 41-168 by Thomas James Higgins, presented at the AIEE Southern District meeting, New Orleans, La., December 3–5, 1941, and published in AIEE TRANSACTIONS, 1941 (December section), pages 1046–50.

Thomas James Higgins: I note the following errata, due, presumably, to my mistyping the original manuscript and not finding the same when checking the proof: in the second of the three right-hand members equivalent to equation 8 and in the left-hand member of equation 9, the second sin Mx should be changed to sin MX; in equation 23, before log $|k_i|$, replace k^4 by k_i^4; in the appendix, replace the expression for $\partial F(D,s)/\partial D$ by

$$\frac{\partial F(D,s)}{\partial D}=(2D^3-6Ds^2)\log(D^2+s^2)-$$
$$4D^3\log D+4(3sD^2-s^3)\tan^{-1}(D/s)-3Ds^2$$

These misprints affect the paper in no way.

Those familiar with the calculation of formulas for the determination of the electromagnetic forces exerted between linear parallel conductors—used in calculation of the mechanical forces exerted on bus bar supports during short circuits—will recognize that the same for the kind of conductors here treated can be determined from equation 26; force = $\partial W/\partial D$. A complete treatment of this phase has been worked out by the writer and will shortly appear elsewhere.

As noted at the outset of the paper, it is based on a thesis done at Purdue University and was accomplished under the guidance of a committee comprising Professors Ewing, Harding, and Sabbagh of the department of electrical engineering and Professors Hughes and Marshall of the department of mathematics, to whom, and in particular to Professor D. D. Ewing, I am indebted for much helpful criticism during the course of the investigation.

Those electrical engineers interested in the history of electrical engineering and of electricity in general may find the following paper, another portion of the aforementioned thesis, of considerable interest: "The Origins and Development of the Concepts of Inductance, Skin Effect and Proximity Effect," *American Journal of Physics*, December 1941, pages 337–46.

Alcoa Rectifier Installation

Discussion of paper 41-167 by J. E. Housley and Harold Winograd, presented at the AIEE Southern District meeting, New Orleans, La., December 3–5, 1941, and published in AIEE TRANSACTIONS, 1941, pages 1266–74.

Otto Jensen (I-T-E Circuit Breaker Company, Philadelphia, Pa.): In 1937 the I-T-E Circuit Breaker Company designed and built 34 high-speed, 6,000-ampere, 1,500-volt, cathode breakers for the Massena and Alcoa stations. Complete field tests were made with all of these breakers before the station was placed in service, and satisfactory operation has been given by the breakers ever since.

As originally installed, a backfire would continue in the rectifier, even after it was disconnected from all other rectifiers on the d-c bus, by its own high-speed cathode breaker, during the eight cycles it would take the oil circuit breaker on the 13-kv side of the transformer to clear. This constituted a short circuit of the transformer and might cause sympathetic backfires in other rectifiers fed from the same a-c system. This problem was approached from two different angles:

1. By grid blocking of the anodes in the faulted tank, thereby preventing them from firing into the fault.
2. By the design of a special high-speed anode breaker inserted between the low-voltage transformer terminals and each anode, or pair of anodes in the tank.

These anode breakers have been used in all the later stations of the Aluminum Company of America, while the anode blocking is used in Alcoa stations 3 and 4 and in the Massena station only.

The first anode breaker was closed by individual air cylinders, in order to prevent the ball-bearing-mounted holding latch from slipping due to shock during the closing stroke. On later installations the air mechanism was changed to a motor-driven cam mechanism. The breaker is rated at 1,600 amperes, 1,500 volts and is tripped by means of a direct-acting trip feature responding to 1,600 amperes reverse current, 6,000–7,000 forward current (extreme high overload), 3,000 ampere dual directional with loss of excitation of its polarity coil. This latter feature makes the breaker fail safely in case of loss of polarity coil voltage. The operating time of its contacts is 0.34 cycle, time to current peak is 0.5 cycle and total time is approximately 0.75 cycle, all on a 60-cycle basis. This operating speed limits the fault current to 35,000–45,000 amperes, depending on the system feeding the faults. Oscillogram 4 shows the effects of a backfire on the d-c system of rectifier station. This oscillogram was taken on a newly designed breaker which is now in the process of development.

Trace 1 (top) is a 60-cycle time wave.

Trace 2 is the voltage across a secondary blowout coil and has only interest in showing when the arc has reached a certain point in the arc chute.

Trace 3 shows cathode current flowing into the bus in a forward direction until the backfire occurs. Then the forward current (furnished by the healthy anodes in the tank) increases, as it is now flowing into the fault. However, grid blocking is applied almost instantaneously, and all other anodes (except the one which was firing when the fault occurred) cease to fire, and the cathode current dies down before the anode breaker has cleared the fault. However, as the cathode current dies down, another backfire occurs in the tank, in spite of the grid blocking, and the cathode current reverses, as it is now furnished from the other rectifiers on the d-c bus. This backfire current is cleared by another anode breaker in the six-pole group. The importance of the anode breaker is here clearly demonstrated, as without an anode breaker, both backfires would have continued until the oil circuit breaker which was previously used had opened.

Trace 4 shows the anode current. This current flows normally in a forward direction at fixed intervals (every time the anode fires). When the short circuit occurs, this current reverses and increases in value with a rate of rise of five to six million amperes per second. After 0.24 cycle the breaker contact opens and the slope of the current changes until the peak is reached at 0.38 cycle. At 0.73 cycle the anode current has reached zero and the backfire is completely cleared. The peak current in this instance was 30,000 amperes, of which 17,500 amperes were furnished by the backfiring unit, and the rest of the current furnished by the other rectifiers on the bus.

Trace 5 shows the voltage across the breaker. The recovery voltage is the transformer voltage superimposed on the dc bus voltage. Its ragged peak must be successfully withstood by the breaker arc chute so that restriking does not occur.

Trace 6 shows the d-c bus voltage and indicates the small disturbance of the bus and the load voltage during a backfire.

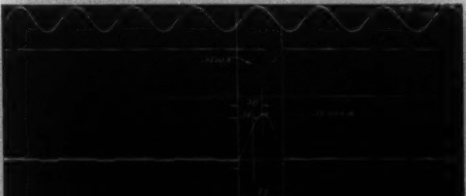
The semihigh-speed cathode breaker is a standard 6,000-ampere 1,500 volt, MT breaker, developed for steel mill and rectifier protection. This breaker might be eliminated if it was only a matter of protecting the rectifier, but in this installation, gang tripping of the cathode breakers is used for dropping the load. Furthermore, it acts as a backup protection, should failure occur in the anode breaker. An accidental short circuit, because of improper connections during a special test, was made by closing one of the MT breakers directly across the d-c bus. No oscillogram is available of this accident, but from other oscillograms it is known that the rate of the rise of the d-c bus is between 16 and 17 million amperes per second (in contrast to the rate of rise in a backfiring anode circuit which is between five and six million amperes per second). Knowing the operating time of the breaker it can therefore safely be stated that the breaker interrupted at least one-half million amperes. After quick inspection the breaker was replaced in service without any attention. A set of six-pole anode breakers has successfully interrupted 287 backfires before the contacts needed attention, and a test was recently made where 25 short circuits of 40,000 amperes at 625 volts each were made at 15-second intervals.

The anode circuit breaker showed its real worth in a recent installation. There a new design of rectifier backfired excessively during the initial operation of the station. A circuit breaker of conventional design such as an oil circuit breaker on the high side of the transformer would have required maintenance service attention during this period, necessitating shut down for this purpose. The anode breaker handled all of these backfires with no maintenance and the production was uninterruptedly maintained.

C. C. Herskind (General Electric Company, Schenectady, N. Y.): I wish to comment on some of the features which have been mentioned in the description of the equipment in the paper.

Practically all rectifier installations being made today involve some amount of phase control and the paper states that provision is made in this installation for regulating the d-c voltage by means of grid control. It is recognized that where several rectifiers operate in parallel on a common d-c bus, some means must be provided for assuring an equal division of load between units. While the inherent d-c voltage characteristic of a rectifier is drooping and similar to that of a shunt d-c generator, this is not usually

Figure 1. Oscillogram 4



sufficient to insure proper paralleling. I would like to ask what inherent shunt regulation is provided in the rectifiers in this installation. I would also like to know what regulating functions are performed by the grid control, specifically:

1. Is the grid control used to regulate the d-c output voltage of the complete substation, and, if so, what range of control is provided?
2. Is the grid control used only for controlling the balance between rectifiers?

Grid blocking or arc-suppression equipment has been quite widely used since its introduction over ten years ago. Considerable experience has shown that while grid blocking or arc suppression is not 100 per cent effective, that is, will not block the anodes during arc-back in all cases, it does provide a secondary protection which is desirable, in that it reduces the duration of faulty currents during arc-back and therefore reduces the duty on circuit breakers, transformers, and rectifiers and the severity of shock upon the a-c system. This feature has been provided in the Alcoa installation.

The anode breakers furnished with rectifiers in rectifier station 4 constitute desirable protective equipment which appears to have a number of important advantages. The use of anode breakers is basically sound in principle, inasmuch as interruption of the anode circuit clears both a-c and d-c to the faulty anode in the minimum of time and thereby clears the fault at its source with the minimum disturbance to the rest of the unit. The desirability of using arc-suppression equipment where protection is obtained by d-c cathode breakers and a-c oil circuit breakers is generally recognized. With the development of anode breakers the question arises, are the benefits accruing from the use of the grid-blocking feature worth the additional cost and complications of the equipment required?

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Telemetering, Supervisory Control, and Associated Circuits

THIS REPORT summarizes information as to electric telemetering and supervisory-control systems in use today or commercially available in the United States, and includes a detailed discussion of the interconnecting circuits suitable for such purposes. It has been compiled on the basis of including only such information as would appear to be of practical assistance to a prospective or existing user of telemetering or supervisory-control equipment in selecting the type of apparatus best suited to his requirements. Much valuable information of a technical nature has of necessity been excluded from this report, but will be found in the published papers, reports, and books listed in the bibliography.

The work is based upon a report compiled by a subcommittee of the AIEE in 1932; and the original general arrangement is followed. The earlier report was published in ELECTRICAL ENGINEERING (volume 53, September 1932, pages 613–20) without tabular data, the latter appearing in a mimeographed edition of the report, privately financed, and having only a limited circulation. The tabulation was revised in 1934, but again circumscribed in its distribution. The present report has been prepared in the knowledge that telemetering in its broadest sense, though basically electrical in principle, has far outstepped the field of electrical engineering. With this in view, an effort has been made to present this report in a form and in terms that may be readily followed and intelligently used by engineers in any branch of industry likely to be concerned with remote measurement and control; and it is hoped that the work will find a wide circulation in these fields.

Since telemetering and supervisory-control systems have many problems in common, and most supervisory-control installations include remote-indication features, it has appeared desirable to prepare a combined report dealing with both classes of systems. Much of the information which follows has been contributed by the manufacturers. Information as to line facilities used for telemetering and supervisory control has been obtained from the communications companies, power companies, and other available sources.

All known commercial sources of telemetering and supervisory-control apparatus in the United States have been canvassed and every effort made to present a complete picture of the instruments and systems available for the purpose. Questionnaire forms were sent to every known manufacturer and dealer. Where necessary, these were followed up, and, if incomplete, were returned for further information. No attempt has been made to list the many variations that might be made in basic systems in order to adapt them to local conditions, or to include descriptions of systems now outmoded or of those systems more or less privately developed by utilities or other users and not definitely offered on a commercial basis.

The information in the report has been arranged in three main sections, as follows:

I. Telemetering—including a tabulation of the characteristics of each system.

II. Supervisory control—including a tabulation of the characteristics of each system.

III. Information concerning circuits and channels for telemetering and supervisory-control systems.

No data covering the field operation and maintenance of the telemetering and supervisory-control systems have been included.

The tabulations in the telemetering and supervisory-control sections enable the prospective user who has a certain requirement in mind, for example the telemetering of a-c power, to determine immediately the systems available for the purpose. The characteristics of each of the systems suitable for the telemetering of a-c power then may be examined in detail with the aid of the table and consideration narrowed to a few systems concerning which additional information may be desired from the manufacturer in order to determine upon the one best suited to the conditions involved. The types of interconnecting circuits suitable for operation with each telemetering or supervisory-control system are indicated in the tables.

I. Telemetering

Telemetering is defined as the indicating, recording, or integrating of a quantity at a remote point by electrical translating means. Electrical or mechanical quantities are normally involved, although there are other miscellaneous applications for telemetering. (ASA Standard *C*-42: 30.40.150)

A report, corrected to December 1940, prepared by a joint subcommittee of the AIEE committees on automatic stations, and instruments and measurements, and published in pamphlet form by the AIEE during 1941.

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APPLICATIONS

In the electrical industry the interconnection of generating stations and of power systems has given considerable impetus to the development of telemetering devices. In the water, oil, and gas industries, particularly in connection with the transportation of such commodities by means of pipe lines, the use of remote-metering systems has come into great prominence as an aid to more economical and satisfactory operation. In fact, the use of telemetering by the nonelectrical industries, for the remote measurement of pressures, levels, positions, flows, and so on, has become comparable in importance to its use in the electrical field; and the experience of some manufacturers has been that although they are prepared to supply systems for the telemetering of either electrical or nonelectrical magnitudes, the demand for the latter constitutes by far the greater part of their telemetering business.

Consequently, while the telemetering systems discussed are all electrical in their operating principles, the magnitudes under measurement may not be electrical in any sense. An effort, therefore, has been made to express the findings of the subcommittee in terms readily understandable by men in any field of engineering, in the hope of making the report of considerably wider practical application than if it were directed solely to the membership of the AIEE.

TYPES OF SYSTEMS

The numerous systems and combinations of equipment used for telemetering may be broadly classified on the basis of the translating means employed. Such a classification has been adopted by the Institute, and includes the following types of systems:

1. Current
2. Voltage
3. Frequency
4. Position
5. Impulse

With the possible exception of the "position" type (which might better be styled the "ratio" type), these terms are in the main self-explanatory. As each type includes a number of more or less diverse forms, these are individually explained by way of introduction to the characteristics of telemetering systems as presented in table I.

CHARACTERISTICS OF TELEMETERING SYSTEMS

This report endeavors to present a uniform analysis of existing systems. The characteristics have been detailed under items which past engineering practice indicates to be essential to bring out the design, installation, and performance features. In order to cover the entire field, it has seemed desirable to present the information in tabular form (table I, pages 16–21).

Questionnaires returned from manufacturers and dealers, after being edited so as to express the data from diverse sources as far as possible in equivalent terms, were tabulated to show in as concise a form as possible the information that would be of value in the selection of a telemetering system. Instead of including in the tabulation the several manufacturers' descriptions of operating principles involved, these principles have been grouped under a number of more or less general headings, and these designated by letters to which reference is made under item 24 of table I. A description of the various principles appears in the appendix, pages 14–15.

The selected bibliography (pages 25–6), while in no sense complete, provides a fair cross section of the art of telemetering and its advances in the past two decades as recorded in the American technical press. Where the references provide valuable descriptive data of principles or operating features of specific systems, they have been noted by number under item 25 of table I.

The user ordinarily will be concerned with only a limited number of systems. The information is so arranged that he can easily fix upon the system or systems which appear best to meet his requirements.

Many of the items in table I require no explanation. In the classifying of measurable magnitudes an effort has been made to bring together in groups those quantities most likely to be measurable by electrical elements of similar natures, and these have been indicated in table I by suitable code letters. Thus, liquid levels and mechanical positions, such as gate openings, are generally measurable by devices having no electrical distinctions, the difference being purely in the mechanical attachments by which displacements are translated into electrical units.

Item 4 indicates whether the telemetering system provides a continuous or a step-by-step indication.* A continuous indication is considered to be provided when the receiving instrument tends to follow any increase or decrease in the measured magnitude at the transmitter. In a step-by-step system no change is registered until the deflection at the transmitter deviates from the last recorded value by a predetermined amount. In determining the time of response of a system (item 7), the characteristic intervals of the transmitter and the receiver as far as possible have been segregated. However, exact statements in this field are difficult to make, and subject to correction with modifications in design or changes in circuit conditions. The time interval as here given for the transmitter does not include the time of response of any deflecting unit, such as a pointer, to the primarily measured magnitude, but only the time required after such pointer has reached its maximum reading until the transmitter has impressed upon the interconnecting circuit the electrical equivalent of a full-scale indication. In such devices as thermal converters, where a definite transformation of energy is effected between a measured quantity and the electrical response in the interconnecting circuit, the time of such transformation may sometimes be considered as the time of response. The exponential law governing the response of such devices has been taken into consideration, where necessary.

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*A number of those answering the questionnaires classed telemetering systems operating on the impulse principle in the step-by-step group without regard to continuity of record. Where necessary, such statements have been edited in the light of a knowledge of the principles involved.

The time of response of the receiver is the interval which elapses after the equivalent of a full-scale reading is impressed on the interconnecting circuit at the transmitting end, under the conditions outlined above, until the receiver registers a corresponding reading. For the purpose of this report, responses of less than one second have been considered as instantaneous. Since in many telemetering systems the response of the receiving instrument takes place more or less concurrently with that of the transmitter, it follows that the total time characteristic of the installation is not necessarily equal to the sum of the response interval of these two instruments.

Item 8, the estimate of maximum probable error under normal operating conditions (per cent of full scale), includes the effect of line-circuit characteristics and makes allowance for normal variations in frequency, wave form, and amplitude of applied voltage. The error may be in a positive or negative direction.

Under power requirements (item 9), has been given as far as possible the normal type of supply best suited to the system. Where a system is adaptable to any ordinary commercial supply, the designation "comm." is used as all-embracing. It is realized that many of the systems may be modified to make use of unusual or special voltage or frequency supplies. The point of application of power, whether at the transmitter, the receiver, or both, frequently will govern the choice of an installation where power is available at only one of these points. It is to be noted, however (see note 2), that a number of manufacturers are prepared to provide arrangements whereby the telemetering circuit may be made to carry the limited power required for operating the remote unit. The distinction between synchronous and nonsynchronous supplies of power at the two ends of a system is sometimes important, as it may be desirable to operate the transmitting and receiving stations from separate a-c sources, which, while both nominally well regulated, are not of necessity tied together at any point.

The values given for maximum allowable loop resistance (item 16) are for direct transmission without repeaters. However, as shown in section III, certain systems will operate over circuits using repeaters. In using these values in obtaining maximum operating distances, attention should be given to line-leakage, inductance, and capacity effects as discussed in section III.

For those systems indicated under item 20 as suitable for use in cases where the interconnecting circuit is exposed to high potentials, suitable drainage and overvoltage discharge means should be provided, or, with certain types of systems, insulating transformers may be used. Overvoltage gaps of the self-restoring type are desirable in order that the equipment may be available for service immediately after the excess voltage has been dissipated.

A detailed discussion of the use of superposed circuits as conductors, item 21, is included in section III of this report.

Where table I indicates that carrier current may be used in providing the communication circuit, the telemetering system is such that the signals may be readily transmitted over a carrier channel. It is probably feasible from a technical standpoint to use carrier current in more cases than are indicated in table I, but in this report it has seemed desirable to indicate the use of carrier current only in those cases where it appears to be commercially practicable at this time.

So many variables enter into the consideration of additional equipment required to limit noise on telephone circuits that in any specific case where there is a possibility of objectionable noise being experienced it probably will be advisable to make such tests as may be necessary in order to determine the need for such apparatus.

HIGH-VOLTAGE STATIONS

Telemetering equipment frequently is located in stations containing power circuits of high voltage and capacity. Faults on these circuits may cause high-potential gradients in the earth surrounding such stations and thus tend to impose a high potential on the interconnecting circuits used for telemetering purposes. From the standpoint of reliability, it is generally inadvisable to use station batteries for the supply of power to telemeter transmitters located in such stations, since these station batteries usually are connected to an extensive network of local control wires having a high electrical capacitance to ground and subject to contact or leakage to ground. A separate battery or other insulated source of local energy should be used in such cases. The battery provided for telemetering purposes should be charged from a separate ungrounded generator or rectifier and not from the station control or exciter buses. If an a-c supply is used, it should be obtained from a separate ungrounded transformer supplied from the low-potential station bus, which should be grounded. In the installation of local wiring connected to the line side of the telemeter, care should be exercised to avoid accidental contact with other local wiring.

II. Supervisory Control

Supervisory control provides for the selective control and indication of a number of remotely located units of equipment or apparatus by electrical means over a relatively small number of common transmission channels. It is the purpose of this report to present to the prospective user information that will enable him to select the equipment which will provide the desired operating features and at the same time function satisfactorily in connection with the interconnecting circuits that may be available.

APPLICATIONS

At present supervisory-control systems have their greatest field of application in power systems, where they are used for the automatic operation of apparatus in generating, switching, transformation, or conversion stations. These systems are also employed in the water, gas, oil, and other industries for the operation of pumping stations and the control of levels and pressures of gases and liquids, etc. In general, supervisory control is applicable to the remote control and supervision of any device that may be operated electrically. A list of individual devices used in accomplishing automatic control appears in ASA Standard *C*-37.2 (1937).

In addition to the control features provided, indications of the position of different devices, such as switch positions, governor settings, gate and valve openings, and others, and of such conditions as bearing and winding temperatures, pressures, levels, synchronization, and so on, may be secured. Arrangements for the telemetering of various quantities may be incorporated in control systems.

PERFORMANCE FEATURES

Features which are generally desirable in supervisory-control equipment may be summarized as follows:

1. The highest degree of reliability and speed obtainable in the correct selection of units, with due regard for economy.
2. Safeguarding of a selected unit against incorrect operation and the correct operation of a unit as quickly after its selection as is consistent with safety.
3. A visual and/or audible indication at the control point of changes occurring in the position of any of the supervised units.
4. Automatic notification at the control point of the occurrence of major troubles, such as open, short-circuited, or grounded conductors, and the loss of operating potential in a distant station, which may cause the supervisory-control system to become inoperative.
5. Safeguarding against false operation in the event of failure of any portion of the supervisory-control equipment to function.

CAPACITY OF SUPERVISORY-CONTROL EQUIPMENT

Some supervisory-control apparatus has structural features which impose more or less definite physical limitations as to the number of units that may be supervised from one system. Other equipment may be used to supervise any number of units which may be installed. There may be stations of such size and importance as to justify more than one supervisory control installation.

CHARACTERISTICS OF SUPERVISORY-CONTROL SYSTEMS

In the tabulation of system characteristics (table II, pages 22–4), most of the items are self-explanatory. Item 18, maximum allowable loop resistance, and item 22, possibility of using superposed circuit as a conductor, correspond to similar items included in the tabulation of telemetering-system characteristics (table I, pages 16–21); and the statements in the foregoing section regarding these items apply also to supervisory control.

With reference to item 27, the arrangements described under the discussion on telemetering relating to the protection of circuits exposed to high potentials also apply in the case of supervisory-control equipment.

POWER-SUPPLY PRACTICES IN HIGH-VOLTAGE STATIONS

The comments under telemetering relating to power-supply practices in high-voltage stations apply with equal force to supervisory control. In addition, special care should be exercised where supervisory wiring is connected to units located in a high-voltage structure.

III. Circuits for Telemetering and Supervisory-Control Systems

Interconnecting circuits of some form are required for the transmission of signals for telemetering or supervisory-control purposes. The manufacturers of the equipment for these purposes have designed the apparatus to operate satisfactorily over the types of facilities usually available on a leased basis from the communication companies or in privately owned communication systems.

LEASED FACILITIES

In practice a large percentage of such facilities are obtained from the telephone company. These are furnished on a leased-channel basis; that is, for each requirement the telephone company provides an electrical path suitable for the specific purpose and obtained from its plant in whatever manner seems the most practicable under the circumstances. In some cases a pair of wires might be used, as for example where distances are relatively short and local cable-distribution plant would be employed. In other cases the channel might be obtained by simplex, phantom, or composite arrangements or by carrier.

PRIVATELY OWNED FACILITIES

Where privately owned line facilities are under consideration, the determination of their suitability will depend on many factors, including the type and characteristics of the telemetering or control system, the operating distances, and available circuits. The following material is intended, therefore, to aid in the selection of suitable facilities in these cases.

LINE SIGNALS—TRANSLATING MEANS

The systems described involve the transmission of the following types of signals:

1. D-c voltage or current, which changes in accordance with the quantity to be measured.
2. D-c impulse signals, ranging in frequency from 1 impulse per minute to 13 impulses per second.
3. Power frequencies, usually 60 cycles.
4. Frequencies in the voice range and above.

Although power-line carrier applications are increasing, only a limited use is being made in frequencies in the voice range or above. The following discussion is therefore concerned primarily with circuits for the transmission of d-c and low-frequency signals. The problems involved in the transmission of such signals over line circuits are similar in many respects to those encountered in d-c telegraph transmission, and the same engineering considerations frequently apply.

The simultaneous transmission of signals in both directions usually is not required. Telemetering systems ordinarily transmit in one direction only, whereas supervisory-control systems commonly require sucessive transmission in opposite directions.

CIRCUIT CHARACTERISTICS FOR THE TRANSMISSION OF D-C AND LOW-FREQUENCY SIGNALS

The line characteristics which may affect the operating range of low-frequency systems are primarily the linear resistance, to a lesser extent the leakage conductance (or insulation resistance), and occasionally the linear capacitance or inductance.

The gauge of cable conductor most commonly used is number 19 Brown and Sharpe. American Wire Gauge, 22, 24, or 26 gauge is frequently employed.

The approximate linear resistance of various gauges of cable is given in the following table:

AWG	Ohms Per Wire Mile at 20 C
13	11
16	21
19	43
22	85
24	136
26	216

The insulation resistance of cable circuits is ordinarily so high that it is not an important factor in the operation of telemetering and supervisory-control systems. The insulation resistance between the wires of a cable pair seldom drops below about 1,000 megohm-miles (0.001 micromho per mile) and ranges from this value upward to 10,000 megohm-miles or higher. The insulation resistance from a wire to ground is usually about two-thirds of the value between wires.

In cases where the circuits are routed through distribution boxes, substantially lower values of insulation may be encountered. In some instances, therefore, it may be desirable to avoid the routing through such boxes when circuits are used for telemetering or supervisory control.

The gauges of open wire commonly employed, and the respective resistance values for copper conductors are as follows:

Wire Diameter, Mils		Ohms Per Wire Mile at 20 C
165	(No. 8 B.W.G.)	2.01
162	(No. 6 A.W.G.)	2.09
128.5	(No. 8 A.W.G.)	3.32
128	(No. 10 N.B.S.G.)	3.34
104	(No. 12 N.B.S.G.)	5.06
101.9	(No. 10 A.W.G.)	5.27

The insulation values obtained on open-wire circuits are in general very much lower than those for cable circuits. Consequently, the leakage is frequently the limiting factor in determining the range over which telemetering and supervisory-control systems may be operated on open-wire facilities. The leakage conductance varies over wide limits, depending chiefly upon weather conditions and the type and maintenance of the line, and to a smaller extent upon the insulators employed. Since the leakage in dry weather is negligible compared to that in wet weather, it is the common practice to engineer systems on the basis of wet-weather leakage values. For the purpose of determining approximately the range and accuracy of telemetering and control systems, it appears convenient to employ a single leakage value which may be considered to be representative for any open-wire circuit. On this basis, the value of wet-weather leakage conductance between one wire and ground may be taken as 5 micromhos per mile (0.20 megohm-mile). On the average, this value probably will not be exceeded on more than about 1 day in 100 and then only for a part of the day. Individual sections of the country may vary materially in this respect. The wet-weather leakage conductance between wires will probably not be more than 2.5 micromhos per mile.

For determining the effect of uniformly distributed leakage conductance upon the accuracy of a d-c telemetering system of the "current type", the percentage error usually may be taken as equal to 50 times the ratio of loop resistance in ohms to total insulation resistance in ohms. For example, assume a 20-mile

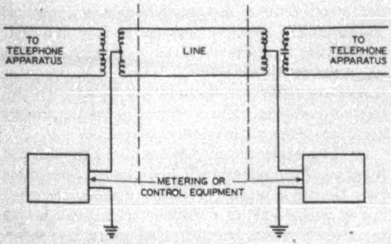
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Figure 1. Grounded low-frequency circuit derived from simplexed telephone circuit

open-wire grounded circuit having a 200-ohm resistance including composite sets at both terminals. Using the leakage figures given, the resistance between the wire and ground under wet-weather conditions would be 0.01 megohm (0.20 megohm divided by 20 miles). The ratio of the circuit resistance to the shunt resistance would therefore be 0.02. This, multiplied by 50, would indicate that the error would be about 1 per cent under wet-weather conditions.

For d-c "voltage-type" telemetering systems, estimates of the error caused by leakage should take into account the d-c resistance of the telemetering equipment itself. An estimate of the error might be made in each case by simple computations of the change in voltage across the receiver caused by a shunt resistance equivalent to the leakage.

The telemetering and supervisory-control systems described require from two to five conductors. Since two-conductor systems predominate, the discussion is directed primarily toward such circuits. The principles involved may be readily extended to systems requiring three or more conductors.

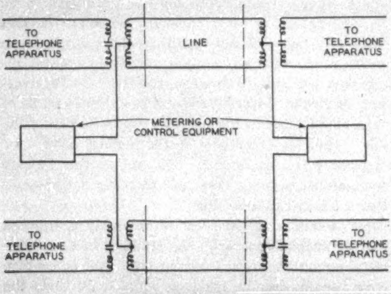
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Figure 2. Metallic low-frequency circuit derived from phantom telephone circuit

Circuits may be divided into two classes, namely, metallic circuits in which a metallic conducting path is employed throughout, and grounded circuits in which the ground is used as one side. For most applications metallic circuits constitute a more desirable transmission medium than grounded circuits. However, the latter are satisfactory in many cases and are sometimes more economical.

The simplest forms of circuit that may be used are: (1) a circuit consisting of two conductors and (2) a circuit consisting of a single conductor and ground. More complicated arrangements are encountered when it is desired to superpose the telemetering or supervisory-control channel on a circuit which also is used for telephone purposes. Such superposition is accomplished by simplexing, phantoming, or compositing.

SIMPLEX AND PHANTOM ARRANGEMENTS

The simplex method, illustrated in figure 1, as applied to a grounded circuit, is based upon the principle of balance. If the telephone circuit is perfectly balanced there will be no net electromotive force or current produced in either the telephone circuit or the derived simplex circuit because of signals being transmitted over the other.

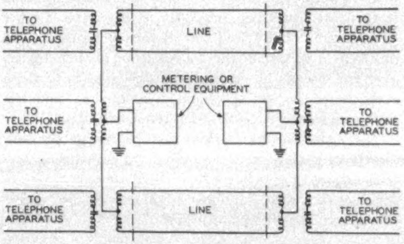
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Figure 3. Grounded low-frequency circuit derived from simplexed phantom circuit

Figure 2 illustrates the use of two telephone circuits to derive a single metallic low-frequency circuit.

If a telephone pair is simplexed, it cannot be used as a part of a phantom circuit. However, a phantom may be created from two pairs of conductors in the usual manner, and the phantom may be simplexed as shown in figure 3 so as to give one grounded low-frequency circuit over the four wires in parallel.

In simplex or phantom arrangements the line conductors are connected in parallel for the low-frequency circuit, so that its resistance is correspondingly reduced. On the other hand, its linear leakage and capacitance are increased.

COMPOSITE ARRANGEMENTS

In superposing a low-frequency circuit on a telephone circuit by compositing, use is made of frequency discrimination. Figure 4 illustrates the use of terminal composite equipment to derive two grounded low-frequency circuits from a single telephone circuit, while figure 5 shows a single metallic low-frequency circuit derived in a similar manner. The composite terminal equipment may be considered as consisting of a high-pass filter which freely transmits the telephone currents while attenuating the low-frequency currents, and a low-pass filter which passes the low frequencies and attenuates the telephone frequencies. The low-frequency band provided by the composite set extends roughly to 75 cycles, while the high-frequency band extends upward from about 135 cycles.

CONSIDERATIONS AFFECTING SUPERPOSITION

The transmission considerations involved in the use of simplexed, phantomed, or composited circuits for telemetering and supervisory-control purposes are similar in general to those which apply in d-c telegraph transmission. For the longer open-wire circuits it frequently will be desirable to composite rather than simplex or phantom because of the smaller leakage values obtained with composited circuits. Also a composited open-wire physical circuit normally is to be preferred to a composited open-wire phantom for the same reason.

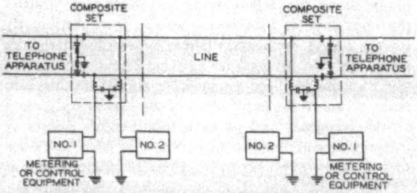

Figure 4. Derivation of two grounded low-frequency circuits from composited telephone circuit

Superposition is frequently accomplished, not at the terminals of the telemetering or supervisory-control system, but at some intermediate point, a simple metallic or grounded circuit being used from this point to the system terminal. The low-frequency circuit may, of course, be superposed on several different telephone circuits in different parts of its length.

The circuit lengths for which superposition may be considered will ordinarily be in excess of one or two miles.

CIRCUITS WITH COMMON RETURN

When several metallic circuits are desired between the same two terminals it is possible under certain conditions to use a single metallic conducting path as a common return, thus effecting an economy in the number of separate metallic paths required. Such an arrangement is illustrated in its simplest form in figure 6. This plan is usually feasible only for short distances, when the circuit resistance is small in comparison with that of the terminals. It has been used successfully for systems employing d-c impulse signals.

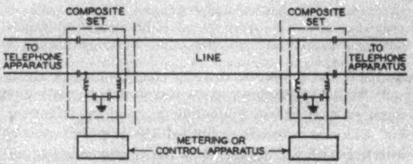

Figure 5. Derivation of metallic low-frequency circuit from composited telephone circuit

USE OF A SINGLE CHANNEL FOR MORE THAN ONE TELEMETERING FUNCTION

During the last several years there has been considerable interest in the possibility of using a single communication channel for the transmission of telemetering signals from a number of telemetering transmitters to one or more receiving instruments located at a main operating center. Such arrangements may permit economies in the communication channels required in instances where the several transmitters are located at one point or along one general route extending from the receiving station. Several possible arrangements for switching the equipment have been suggested; among those which appear the most promising are the following:

1. Systems in which, by manual operations at the control point, pulses are transmitted over a channel to connect a particular transmitter to the telemetering channel.
2. Systems in which transmitters are automatically connected

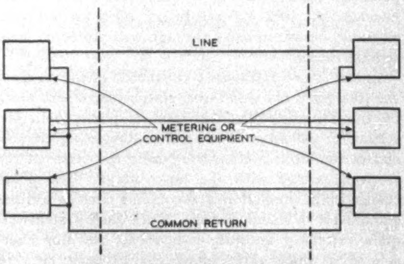

Figure 6. Common return circuits

to the channel in a predetermined sequence. These include systems which involve the use of several receivers and which require, therefore, synchronous connection of the proper pairs of instruments in sequence.

It would be economical to transmit the controlling impulses over the same channel as used for telemetering. This is possible, provided the impulses do not interfere with the operation of the telemetering equipments; otherwise, a separate channel for the control pulses is necessary. The possibilities of accomplishing this depend upon the types of telemetering equipments involved and the feasibility of selecting a type of control signal which would not so interfere. When the same commercial a-c power supply is used at each point where switching is required, it is feasible to design arrangements that provide for synchronous switching of the transmitters and receivers, or for the switching of the transmitters in sequence where only one receiver with multiple recording is employed.

There are probably a large number of possible arrangements of this kind, but since little practical application of such systems has been made to date, it is thought that the purpose of the report might be served best merely by mentioning some of the possibilities that appear attractive at this time.

TELEMETERING SYSTEMS WHICH USE LOW-ENERGY LINE SIGNALS OR WHICH RELY ON EXTREME STABILITY OF CIRCUITS

There have been instances in telemetering systems utilizing low-energy levels where difficulties caused by variations in the line resistance with temperature or by other variations have been encountered. Since the interconnecting facilities may be of various types of construction, and receiving different grades of maintenance, it is desirable to investigate circuits to be used with low-energy systems. Experience indicates that the number of unsoldered joints in a circuit of this nature should be kept at a minimum. Also systems utilizing low-energy line signals are more subject to inductive disturbances and transient ground-potential differences. In general, it is not recommended that such systems be extended for any considerable distance except through well-protected lead-covered cables.

In one or two instances comparisons of the phase of the voltages of two generating stations have been made by transmitting a small amount of power from one station to the other over an interconnecting channel and at the latter station comparing the phase of the received voltage with the bus voltage. The phase change in the conductors and coupling circuits is either allowed for or compensated. Since such systems require extremely accurate indications, the line characteristics which affect phase change must be very stable. In considering the possible use of systems of this kind, it is important to take into account the types of facilities available and the expected changes due to weather and temperature variations. Moreover, in case of circuit failure requiring some replacement of line facility, consideration must be given to the effect on the system of any possible phase changes introduced by the replacing facilities.

CIRCUITS FOR SIGNALS IN THE VOICE-FREQUENCY RANGE

While it is practicable to employ signals in the voice-frequency range for telemetering and supervisory-control systems, in most cases d-c or low-frequency signals have been used. Some of the more important reasons for this usage are:

1. The employment of the voice-frequency range for these purposes seriously limits its utility for telephone purposes.
2. The avoidance of interference with adjacent service is readily obtained by the utilization of the low frequencies.

CARRIER-CURRENT CHANNELS

Carrier channels on power or privately owned telephone lines for telemetering or supervisory-control purposes have had increasing applications particularly for the longer distances. These types of systems are most likely to be used on existing line facilities, and the transmission and engineering considerations generally relate to the application of the carrier equipment at the terminals and at such intermediate points as transformers or bridged lines where it may be necessary to add equipment to improve the transmission at carrier frequencies. Carrier channels installed primarily for speech communication or other purposes often can be arranged also to permit telemetering at a slight additional cost.

AUDIBLE INDICATING EQUIPMENT ON TELEPHONE SUBSCRIBER LINES

Several audible systems are available for use in association with regular telephone subscriber lines to normally unattended stations, enabling the transmission of audible signals over the telephone line to a party making a call to the unattended station. The telephone companies can provide auxiliary-station equipment for coupling the indicating apparatus to the telephone circuit. The equipment includes several relays, a gong, and a transmitter, all housed in a metallic cabinet. When a call is made to the unattended station, the ringing current causes operation of a relay which connects the transmitter across the line, stops the ringing, and holds the circuit. For a period of about five seconds after ringing ceases, the gong operates and the tones are picked up through the transmitter and serve to notify the operator that the station has answered. Following this a relay in the auxiliary-station equipment closes a circuit to start the telemetering apparatus, and the duration of the contact closure may be arranged for a period as low as about five seconds or for such longer period as may be necessary for the device to complete its cycle of operation. The telephone companies usually require that the operating cycle not exceed five minutes. The tones emitted by the audible indicator are impressed at about a normal talking volume on the transmitter of the auxiliary equipment. The transmitter is mounted against an aperture in the cabinet to facilitate acoustic coupling to the telemetering system. At the conclusion of the operating cycle, the momentary closure of a circuit in the latter equipment operates a relay in the auxiliary station apparatus which restores the circuit to normal after the transmission of the telemetering pulses.

MAINTENANCE OF CIRCUITS

Since telemetering and supervisory-control services generally are of vital importance in the fields in which they are applied, it is essential in most cases that the highest practicable degree of continuity and reliability of operation be maintained for these services. Special markings should be provided at the terminals, frames, protectors, test boards, and other locations, where the circuits appear. These help to guard against possible interference or interruption to service. In addition to that precaution, the following procedures have been found helpful:

1. Frequent routine tests and inspections of the facilities.
2. Arrangements for prompt location and clearing of troubles.
3. Arrangements for prompt restoration of service in the event of failure.

PROTECTION AND AVOIDANCE OF SERVICE REACTIONS

In the operation of telemetering and supervisory-control systems in connection with different types of leased channels or privately owned circuits, it has been found desirable to apply certain requirements and limitations of an electrical nature with a view to:

1. Provision of a satisfactory grade of telemetering and supervisory-control service,
2. Avoidance of possible hazard to communication plant and personnel, and
3. Avoidance of possible adverse reactions to other communication services.

The requirements and limitations usually desirable for d-c and low-frequency telemetering and supervisory-control systems when operated over leased circuits may be summarized briefly as follows:

(a). The potential of a d-c supply under any conditions should not exceed 135 volts positive or negative between any conductor and ground. A potential of 270 volts may be employed between the two wires of a pair, provided the midpoint of the applied voltage is grounded.

(b). The potential of an a-c supply should not exceed 120 volts rms value between conductors or between any conductor and ground.

(c). The maximum line current under any conditions should not exceed 0.35 amperes.

(d). Where there is a possibility of experiencing excessive ground potentials there should be no metallic connection between the line facilities and the high-voltage station ground except where neutralizing or insulating transformers are provided.

(e). The apparatus should be provided with insulation adequate for the conditions under which it is to be used.

(f). The noise influence of telemetering or supervisory-control signals, that is, the characteristics of the signals which cause induction in other circuits, should be suitably limited to avoid the effects of service reaction in near-by communication circuits. The limits of tolerable noise influence vary widely, depending upon the characteristics of the plant involved. It is customary to make noise measurements when there is any likelihood that telemetering or supervisory-control signals will cause induction in neighboring communication circuits, resulting in adverse service reactions. The method of measuring should take account of frequencies within the range used by message circuits, and by program circuits. Noise influence (expressed in decibels above reference noise) may be measured by an indicating noise meter having suitable frequency-weighting networks for message and program circuits. The indicating meter is coupled to the circuit (or apparatus) being tested, by a suitably proportioned capacitance to simulate "crosstalk coupling". Cases involving signals having frequencies above approximately 8,000 cycles require special consideration.

These provisions agree in general with those observed in the operation of commercial telephone plant adequately to control the effects on other circuits. Many of the telemetering and supervisory-control systems now furnished by the various manufacturing companies are designed and constructed so as to conform to these provisions. When the user of telemetering or supervisory-control equipment provides his own circuits, it sometimes may be possible to exceed these limitations without undesirable effects. In regard to the noise-influence considerations discussed, with leased channels the communication companies generally are equipped to make the measurements necessary to determine whether or not tolerable limits are observed. Where privately owned circuits are to be used, information on noise-influence characteristics of the signals transmitted by the equipment should be secured from the manufacturer.

SOURCES OF GROUND-POTENTIAL DIFFERENCES

An item of interest in metering and control services is the possible occurrence of differences between the ground potentials at the two ends of the circuit, which may be of sufficient magnitude to cause appreciable interference in grounded circuits. Ungrounded metallic circuits, on the contrary, rarely are affected by ground potentials.

Differences in ground potentials at different points frequently are caused by d-c electric-traction systems. Such differences are ordinarily small (not over 10 volts). However, in a few locations in the United States the geological formation is such that varying differences

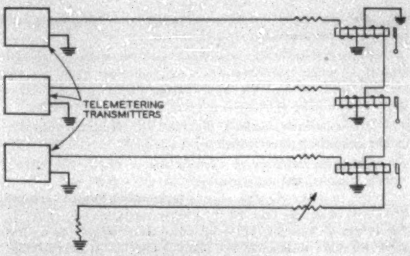

Figure 7. Use of auxiliary conductor to compensate for ground potentials

in ground potential which are fairly substantial in magnitude occur more or less continuously.

Differences in ground potential due to normal solar activity are insignificant. However, during periods of intense sunspot activity, accompanied by such phenomena as severe magnetic storms, the disturbances in grounded circuits may be severe. At such times the potentials between points a few hundred miles apart will usually range from 0 to ±50 volts, but may in extreme instances attain values of over 500 volts. Such disturbances generally extend over the whole world, although the magnitude of the effects may vary in different locations. Usually the effects are most severe on lines extending in a northwest-southeast direction.

These effects have been studied over a considerable number of years. Fortunately, in addition to being of comparatively short duration, such disturbances are quite infrequent in occurrence. Periods of extremely intense solar activity seem to occur only at intervals averaging about 11 years, with occasional briefer periods of less disturbance between. It has been estimated that the total duration of disturbances of this nature sufficiently severe to affect grounded circuits will average about 10 hours per year. Metallic circuits are affected by these disturbances only when the ground-potential differences are of such magnitude as to cause operation of the protector blocks, fuses, or heat coils, or to damage the office apparatus.[46]

Disturbances due to static atmospheric conditions, as well as to lightning, are sometimes experienced on open-wire circuits. The amount of these disturbances varies with the locality and according to whether or not the circuit is grounded. Circuits in cable are not ordinarily subject to such interference.

EFFECT OF GROUND POTENTIALS

The effect of ground potentials on telemetering and control circuits depends upon the type of system. In the case of telemetering systems which use a slowly varying direct voltage or current the accuracy as a rule is affected by ground potentials. Consequently, metallic circuits are generally more desirable for such services. Systems employing d-c impulses, however, are much less susceptible in this respect, and are not usually affected by small ground-potential differences. Grounded circuits therefore may be used successfully in many applications of the latter type of system.

Where several telemetering or supervisory-control systems employing relay operation are to be used between the same terminals, an arrangement may be provided whereby a grounded channel is used for each system and a single auxiliary conductor is used to compensate for the effect of ground potentials in all systems. Such an arrangement is illustrated in figure 7.[31]

POWER INDUCTION

The transfer of energy from power circuits in certain cases may introduce substantial alternating voltages and currents in circuits used for telemetering and supervisory-control systems. If circuits closely parallel power-line rights of way, ordinary protection arrangements and sometimes special drainage will be required.

As in the case of ground-potential differences, ungrounded metallic circuits are naturally less susceptible to power induction than are grounded circuits. Also, cable circuits are much less affected than open-wire circuits. The importance of the reaction which such power induction may produce in telemetering and supervisory-control systems depends upon the system employed and upon the magnitude of the effects present. The arrangement in figure 7 is also effective in many cases in minimizing the effects of extraneous a-c potentials.

AVOIDANCE OF HIGH-VOLTAGE-STATION GROUND

If a high-voltage-station ground or an adjacent ground is used for grounded telemetering or supervisory-control circuits, these circuits are extremely susceptible to inductive disturbance at times of abnormal conditions in the power system. This difficulty may be obviated by using an operating ground at an intermediate point (such as a telephone central office) some distance away from the high-voltage station and employing a metallic circuit between this point and the high-voltage station. An arrangement of this kind is illustrated in figure 8. This makes it possible to use a grounded

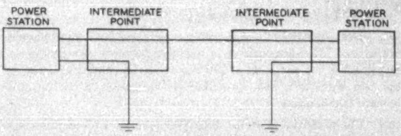

Figure 8. Use of ground at intermediate points

circuit in sections not affected by disturbing potentials. The metallic circuit extending to the high-voltage station must be insulated against the maximum potentials to which it may be subjected.

When a metallic circuit is required and is terminated at a power station subject to high-ground potentials, one or more of the following practices is desirable:

1. Adequate insulation of the terminal equipment from the station structure. Consideration must also be given to the need for adequate insulation within the influence of the station ground potentials or the conductors associated with the terminal equipment
2. The use of neutralizing transformers.[31]
3. The use of insulating transformers.

OPERATING DISTANCES

The distance over which satisfactory operation may be secured with different types of telemetering and supervisory-control systems depends upon both the design of the system and the character of the associated interconnecting circuits. Ordinarily the manufacturer will indicate the approximate length of different types of circuits over which his equipment will yield a satisfactory performance.

The scope of this report does not permit a statement of the maximum distances which may be spanned with each kind of equipment as applied to each type of circuit. A few general observations, however, may be helpful.

For cable circuits, especially of the metallic type, the resistance is usually the principal factor in determining the operating range. Consequently a rough indication of the maximum operating distance over cable facilities may be obtained by relating the allowable loop resistance as given in the table of system characteristics to the linear resistances of the gauge or gauges of cable to be used.

With open-wire circuits, the leakage frequently will be the controlling factor, so that a comparison of the tabulated values of minimum insulation resistance with the insulation value for open wire under wet-weather conditions will furnish a rough measure of the maximum operating distance.

For systems having the high speeds of transmitting impulses and for a-c systems, especially when operated on simplexed or composited facilities, the effects of capacitance and inductance in the communication circuit and in the compositing equipment will limit the length to much shorter values than those determined by resistance only.

Because of circuit limitations, position-type telemetering systems in general are limited to comparatively short distances, say not over 5 miles; but recent developments have materially increased the range of this class.

The voltage and current types of telemetering systems cover, in general, a range of operating distances up to about 50 miles, with greater ranges feasible if simplexed cable circuits are used. The accuracy of these systems is appreciably affected by the leakage on open-wire circuits. Metallic circuits are preferable for these systems, and are in fact essential for certain types.

Systems employing d-c impulses generally will span a maximum distance of from 50 to 250 miles or more. These systems are well adapted to the use of grounded channels, and their accuracy is not impaired by leakage conditions. For systems employing relay operation, greater distances usually may be spanned and superior operating results obtained if polar operation or the equivalent is used instead of open and close operation.

TELEGRAPH REPEATERS

For the most part, the distances to be covered will be such that the use of telegraph or pulse repeaters is unnecessary. In cases where it is desired to span unusually long distances, ordinary telegraph repeaters may be used with certain of the systems employing d-c impulse signals. Also, repeaters employing equipment similar to the terminal equipment can be furnished by the manufacturers for some of the systems.

Appendix

For convenience in using this report, the following material has been assembled in the appendix: descriptions of the principles involved in the various telemetering systems which were studied; a list of abbreviations used in the tables; a table showing the characteristics of various telemetering systems; and a table showing the characteristics of various supervisory-control systems.

Brief Descriptions of Principles Involved in Telemetering Systems Included in Table I

CURRENT AND VOLTAGE SYSTEMS

(*A*). A permanent-magnet-moving-coil element in the transmitter is caused to follow the deflections of the primary measurement element by means of a contact system acting through an electronic or electromechanical relay to adjust a direct current in the moving coil. The transmitting line carries direct current proportional in value to the deflection of the primary measuring element. The receiving instrument takes the form of an indicating or recording milliammeter.

(*B*). The torque of the primary measuring element is definitely opposed by the reaction set up in a permanent-magnet-moving-coil element, reducing the deflection of the former to zero. The current in the latter element is adjusted by means of a control system actuated by the former element. This current, proportional to the torque developed in the measuring element, is carried by the transmitting line. The receiving instrument takes the form of an indicating or recording milliammeter.

(*C*). The primary measuring element directly controls the impedance in an electrical circuit including the transmitting line. The receiving instrument may be indicating, recording, or integrating, and responds to the current resulting from a constant voltage applied to the controlled impedance circuit. Where necessary there is incorporated in the receiver an automatic compensator for variations in supply voltage.

(*D*). The torque of the primary measuring element is definitely opposed by the reaction set up in a permanent-magnet-moving-coil element. The current in the latter element is controlled through an electron-tube system from photoelectric tubes energized by a light beam deflected by a mirror on the coupling between elements. This current, proportional to the torque developed in the measuring element, is carried by the transmitting line. The receiving instrument takes the form of an indicating or recording milliammeter.

(*E*). The voltage or current to be transmitted is derived from the secondary of a suitable instrument transformer, and applied through an insulating transformer to a rectifier. The filtered output is transmitted as a d-c magnitude to an indicating or recording receiver.

(*F*). The transmitter includes a controlling element which sets up a current proportional to the measured magnitude. At the receiving end, this current is passed through a shunt and the voltage drop measured by a potentiometer calibrated in terms of the measured quantity. Totalizing like quantities is effected by paralleling currents or by adding shunt-potential drops and measuring the net value.

(*G*). A transmitting controller sets up a voltage proportional to the measured magnitude. At the receiving end this is measured by a self-balancing potentiometer calibrated in terms of the measured quantity. The sum or difference of several like quantities is obtained by interconnecting the retransmitted potentials and determining the algebraic sum.

(*H*). A thermal ammeter or wattmeter system associated with thermocouples develops in the transmitter an electromotive force proportional to the measured magnitude. The thermoelectromotive force, or a voltage or current automatically made proportional to it, is transmitted to the receiving instrument, which may take the form of a recording potentiometer or millivoltmeter.

(*I*). Through the agency of a variable-speed motor controlled by deflection of a measuring element, a centrifugal device is caused to develop a force at all times tending to restore the element to a position of zero displacement. There is thus produced a speed proportional to the square root of the deflecting force. When used in connection with a flow-measuring element of the orifice or venturi class, the speed thus becomes directly proportional to the velocity of flow. Counting trains actuated directly by the motor or through a contactor provide an integration of the total flow.

FREQUENCY SYSTEMS

(*J*). The transmitter includes a basic measuring instrument whose deflection is followed through a photoelectric relay system by a frequency-meter element excited from a variable-frequency oscillator. The signal of the oscillator is transmitted over lines or by carrier current, and received at as many stations as desired. The receiver takes the form of a frequency meter.

POSITION SYSTEMS

(*K*). The transmitting instrument serves to adjust the relative value of two inductances connected in a bridge circuit with similar inductances at the receiving point. When this system is energized by alternating current there is developed a tendency for the receiving unit forcibly to assume a position in which the impedances of the two branches of the circuit are balanced. This force may be used either directly or through a relay to deflect a receiving pointer or pen.

(*L*). The transmitter includes a transformer having one primary and two secondary coils. The measuring element adjusts the ratio of secondary voltages by moving the core of the transformer. The receiving instrument is a self-balancing ratiometer.

(*M*). The transmitter takes the form of a motor structure with one single-circuit and one multicircuit winding. The former winding is energized from an a-c source, and the latter is connected to the receiving instrument through a three-conductor circuit.

(*a*). The receiver is identical with the transmitter, and has a single-circuit winding energized from the same source. The multicircuit winding, connected to the transmitter, receives voltages of relative value governed by the position of the transmitter rotor, causing the receiver rotor to assume the same position.

(*b*). Receiver has a multicircuit stator and an iron-vane rotor, not requiring external excitation.

(*N*). The transmitter embodies a divided circuit whose two branches include oppositely directed rectifying elements in series with resistances whose relative values are varied with the quantity to be measured. This unit is connected through a two-conductor line to the receiver which contains a similarly divided circuit with rectifiers in the respective branches. With an alternating electromotive force applied to the system, any lack of balance between the receiver and transmitter will result in unequal rectification of the two halves of the current wave, which condition is detected by a d-c galvanometer and balanced out by a calibrated slide-wire in the receiver. As an alternative, the receiving instrument may take the form of a direct-deflecting d-c ratiometer. The circuit may be arranged to respond to variations in a single resistance element at the transmitting end and yet be unaffected by changes in the resistance of the connecting line.

IMPULSE SYSTEMS

(*O*). The primary measuring instrument transmits impulses representing changes in the measured quantity. A notching device or the equivalent in the receiving instrument positions a pointer or pen in correspondence with the transmitted impulses. Distinction between impulses in opposite senses may be achieved by (*a*) making respective impulses of opposite polarity; or (*b*) transmitting impulses in one side or other of a three-conductor circuit.

(*P*). An integrating meter (watt-hour meter) controls in the transmitting circuit electrical impulses with a frequency varied in accordance with the measured load. The receiving instrument may be a distant dial providing an integrated reading, a demand dial, or an indicator or recorder. The impulses may be (*a*) successive impulses of the same polarity, (*b*) successive impulses alternating in polarity, (*c*) successive impulses alternating between the two sides of a three-conductor circuit.

(*Q*). The transmitter advances the receiving indicator by a series of similar impulses corresponding in number with the value of the measured magnitude, one impulse being identified by its length as corresponding to the zero position. On increasing values the indicator progressively advances up the scale; and upon a decrease in value the pointer sweeps rapidly around the scale to a position representing the new (and lower) value.

(*R*). The transmitter includes two contacting cams driven by a clock or constant-speed motor. One cam sends impulses at uniform intervals, and the other at intervals spaced therefrom in accordance with the measured magnitude. The time interval between successive signals thus becomes a measure of the telemetered quantity. The impulses may be picked up on a telephone and timed by a stop watch, or may be graphically recorded for reference (see "Audible Indicating Equipment", section III).

(*S*). Two motors, one in the transmitting, and one in the receiving instrument, are automatically maintained in synchronism. A contact mechanism in the transmitter, associated with the primary measuring element, governs the durations of successive impulses in the interconnecting line. The receiving instrument includes contacts that normally operate in synchronism with those at the transmitter. Any variation in the measured load will introduce a change in the relative durations of impulses at the two ends of the system, and this sets up a corrective action at the receiver which tends to restore the synchronism of contacts, and incidentally to reposition the pen or pointer in the receiving instrument.

(*T*). A motor-driven mechanism in the transmitter controls cyclically recurrent impulses in the interconnecting circuit. The durations of successive impulses are governed by the position of the primary measuring instrument. The mechanism of the receiving instrument is driven through a clutch system tending alternately to position the pointer up and down the scale in response to the relative durations of impulses and intervals between. The receiving instrument may be indicating or recording.

(*T* 1). Telemetering method similar to *T*, with audible system, co-ordinated with indicating instrument or timing device at receiving end (see "Audible Indicating Equipment", section III).

List of Abbreviations

Absolute	abs
Ampere	amp
Ampere hour	amp-hr
Approximate	appr
Audio	aud
Carrier current	carr
Commercial (referring to commercial voltage derived from service supply without special regulation)	comm
Composite	comp
Demand	dem
Disconnection	disc
Frequency	freq
Full scale	FS
Indicating	ind
Integrating	int
Interruption	intr
Instantaneous (substantially; less than one-half second)	inst
Levels	lev
Line; any point on line	L
Line to ground	L-G
Line to line	L-L
Maximum	max
Milliampere	ma
Minimum	min
Neutral	neut
Normal	norm
Phase angle	PA
Polarized	pol
Position	pos
Power factor	pf
Pressure	pr
Quantity (gallons, etc.)	Q
Radio frequency	rf
Rate of flow	R
Reactive volt-ampere	var
Reactive volt-ampere hour	var-hr
Receiver	rcr
Recording	rec
Rectified alternating current	rect
Short circuit	SC
Simplex	simp
Temperature	temp
Transmitter	T
Volt	v
Volt-ampere	va
Volt-ampere hour	va-hr
Watt	w
Watt-hour	whr

Table I. Characteristics of Telemetering Systems

1.	Manufacturer or distributor	Carl Au & Sons	Carl Au & Sons	Bailey Meter Co.	Bailey Meter Co.	Bailey Meter Co.	Baldwin-Southwark	J. G. Biddle Co.	Bristol Co.	Bristol Co.	1.
2.	Trade name of system	Telephonic Aud	Telerecorder	Selsyn	Galvatron	Synchro-Meter		Midworth Repeater	Long Distance	Metameter	2.
3.	Basic principle of operation	Impulse	Impulse	Position	Voltage	Impulse	Current	Current	Position	Impulse	3.
4.	Continuous or step-by-step action	Impulse	Cont	Cont	Cont	Cont	Cont	Cont	Cont	Cont	4.
5.	Measurement as received	Code	Ind, rec	Ind, rec	Ind, rec	Ind, rec, int	Ind, rec	Ind, rec	Ind, rec	Ind, rec, int	5.
6.	Character of measurable magnitudes										6.
	(*a*) Amperes							A-c, d-c		A-c, d-c	(*a*)
	(*b*) Volts							A-c, d-c		A-c, d-c	(*b*)
	(*c*) Power, etc.							W, va, vars, dem		W, va, vars	(*c*)
	(*d*) Energy, etc.							Amp-hr, whr, var-hr, va-hr			(*d*)
	(*e*) Levels, positions	Lev, pos	Lev	Lev, pos	Lev, pos	Lev, pos		Lev, pos	Lev, pos	Lev, pos	(*e*)
	(*f*) Pressures, temperatures	Pr		Pr, temp	Pr, temp	Pr, temp	Pr	Pr, temp	Pr, temp	Pr, temp	(*f*)
	(*g*) Quantities (gallons, rates of flow)			Q, R	Q, R	Q, R		R	R	R	(*g*)
	(*h*) Other magnitudes (speed, etc.)						Stresses			Speed	(*h*)
	Performance										
7.	Time of response, seconds										7.
	(*a*) Transmitter	0–300 sec	0–300 sec	0	0	0	Inst	15 sec	Inst	Inst	(*a*)
	(*b*) Receiver	Inst	0–300 sec	0	18	60	Inst	Inst	Inst	15, 5, or 2 sec max	(*b*)
	(*c*) Over-all	0–300 sec	0–300 sec	0	18	60	Inst	15 sec	Inst	15, 5, or 2 sec max	(*c*)
8.	Maximum error of system	0.02%	0.02%	1%	0.25%	0.1%	1%	0.5%	2%	0.5% FS	8.
9.	Power requirements										9.
	(*a*) Supply volts and frequency	Comm a-c	Comm a-c	Comm a-c	Comm a-c	Comm a-c	D-c and a-c	Comm d-c	Comm a-c	Comm a-c	(*a*)
	(*b*) Application point	T	T and rcr (note 2)	T and rcr	Rcr	T and rcr	T and rcr	T	Any point	T and rcr (note 2)	(*b*)
	(*c*) Synchronism	No	Abs	Abs	No	Yes (note 6)	No	No	No	Appr	(*c*)
10.	Self-restoration after interruption or disconnection	Intr	Intr, disc	Intr, disc	Intr, disc	Intr, disc	Intr, disc	Intr, disc	Intr, disc	Intr, disc	10.
11.	Adaptability to totalization	No	Yes	Yes (note 8)	Yes	No	No	Yes	No	Note 8	11.
12.	Adaptability to more than one receiver	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	Yes	12.
	Transmission Characteristics										
13.	Nature of transmitted current	Aud	A-c or d-c	A-c	D-c	A-c	D-c and a-c	A-c	Comm a-c	Any	13.
14.	Impulses per minute (for impulse systems)	Min 1/5	Min 1/5			Max 8, min 8				Max 30, min 4	14.
15.	Relay employed	No	Note 3	No	No	No	Note 5	No	No	If needed	15.
16.	Maximum loop resistance, ohms	Note 1	1,500	280	2,000	20	Note 5	1,000	100 per cond	5,000	16.
17.	Minimum insulation resistance, megohms	Note 1	Note 5	0.1	0.5	Note 5	Note 5	0.2	0.1	0.1	17.
18.	Number of conductors in line	Note 1	2	3+2	2	3	Note 5	2	3	2	18.
19.	Possibility of using ground as one conductor	Note 1	Yes	No	No	No	Note 5	Yes (note 7)	Yes	Yes	19.
20.	Possibility of using exposed lines	Note 1	Yes	Yes	No	Yes	Note 5	Yes (note 7)	Yes	Yes (note 7)	20.
21.	Superposition possibilities	Note 1	Simp, comp, carr	No	No	No	Note 5	No	Note 9	Simp, comp, carr	21.
	Interference and Protection Features										
22.	Normal and short-circuit line current, milliamperes		Norm 35, SC 1,000	Norm 0 to 300, SC 350	Norm 0.1, SC 2	Norm 250, SC 250	Note 5	Norm 50 to 100, SC 1,000	Norm 300, SC 500	Norm 40, SC 100	22.
23.	Maximum open-circuit volts		L-L 115, L-G 115	L-L 120, L-G 120	L-L 5, L-G 1.0	L-L 25, L-G 110	Note 5	L-L 115, L-G 115	L-L 115, L-G 115	L-L 100, L-G 100	23.
24.	Telemetering principle involved	*R*	*R*	*Ma*	*G*	*S*	*C*	*A*	*K*	*T*	24.
25.	Bibliographical references	39		10, 17, 23, 28*d*	26	35	5	14, 18, 28*a*	18, 28*d*	28*e*, 38, 41, 42, 43	25

Table I. Characteristics of Telemetering Systems (continued)

1.	Bristol Co.	Bristol Co.	Bristol Co.	Brown Inst. Co.	Builders Iron Foundry	Bunnell, J. H. & Co.	Control Corporation	Control Corporation	Duncan Elect. Mfg. Co.	Esterline-Angus	1.
2.	Metaphone	Thermoverter	Uehling	Brown R-I System	Chronoflo	MacCreedy	"TM5"	Type "W"	Duncan	Esterline-Angus	2.
3.	Impulse	Voltage	Position	Position	Impulse	Impulse	Impulse	Impulse	Impulse	Current	3.
4.	Cont	Cont	Cont	Cont	Cont	Step-by-step	Cont	Step-by-step	Step-by-step	Cont	4.
5.	Ind	Ind, rec	Ind, rec	Ind, rec	Ind, rec, int	Ind	Ind, rec	Ind, rec	Note 23	Ind, rec	5.
6.											6.
(a)	A-c, d-c						A-c, d-c			A-c, d-c	(a)
(b)	A-c, d-c						A-c, d-c			A-c, d-c	(b)
(c)	W, va, vars	W, vars					W, va, dem			W, va, vars	(c)
(d)									Whr, var-hr		(d)
(e)	Lev, pos		Lev, pos	Lev, pos	Lev, pos	Lev	Lev, pos	Lev, pos		Lev, pos	(e)
(f)	Pr, temp		Pr, temp	Pr	Pr, temp		Pr, temp			Pr	(f)
(g)	R			Q	Q, R					Q, R (note 8)	(g)
(h)	Speed				Weight		Pf, freq			Freq, pf	(h)

Performance

7.											7.
(a)	Inst	1/2	Inst	Inst	Inst	Inst	10 sec	1		1 sec	(a)
(b)	15, 5, or 2 sec max	Note 23	Note 24	3 sec	2 to 60 sec	1 (note 10)	10 sec	1		0.25 sec	(b)
(c)	15, 5, or 2 sec max	Note 23	Note 24	3 sec	2 to 60 sec	1 (note 10)	15 sec	2 (?)		1 sec	(c)
8.	1% FS	0.5%	1%	2%	0.25% FS		2%	0.25%		2%	8.
9.											9.
(a)	Comm a-c	Note 20	Comm a-c	Comm a-c	Comm a-c	30–50 v d-c	Comm a-c	Comm a-c	Comm a-c, d-c	Comm a-c	(a)
(b)	T		Any point	T or rcr	T and rcr	L	T and rcr	T and rcr	T and rcr	T	(b)
(c)	No		No	No	Appr	No	Yes	No	No	No	(c)
10.	Intr	Intr, disc	Intr, disc	Intr, disc	Intr, disc	No	Intr, disc	Intr	Note 15	Intr, disc	10.
11.	No	Yes	No	No	Note 5	No	Yes	No	Yes	Yes	11.
12.	Yes	Yes (note 8)	Note 8	Yes	Yes	Yes	Yes	Yes	Yes	Yes	12.

Transmission Characteristics

13.	Aud	D-c	Rect a-c	A-c	D-c or a-c	D-c or rect	A-c	A-c	A-c or d-c	Rec, d-c	13.
14.	Max 30, min 4				Max 30, min 1	Max 120, min 120	Max 4, min 4	Note 5	Max 60		14.
15.	Note 1	No	No	No	If required	No	No	Yes	Note 23	No	15.
16.	Note 1	1,000 per unit	1,000	100 per cond	15,000	200	3,000	3,000	Note 23	3,000	16.
17.	Note 1	0.1	Note 5	0.1	0.1	Note 5	Note 5	Note 5	Note 23	0.3	17.
18.	Note 1	2	2	3	2	3	2	2	Note 23	2	18.
19.	Note 1	No	Yes	Yes	Yes	Yes	Yes	Yes	Note 23	Yes	19.
20.	Note 1	Note 7	Yes	No	Yes	Yes	Yes	Yes	Note 23	Yes	20.
21.	Note 1	Simp, comp	No	Yes (note 9)	Simp, comp, carr	Simp, comp, carr	Note 5	No	Note 23	Simp, comp	21.

Interference and Protection Features

22.	Note 1	Note 5	Note 5	Norm 300, SC 1,000	Norm 15 to 60, SC 100	Norm 150, SC 150	Norm 20, SC 500	Norm 20, SC 500	Note 5	Norm 30, SC 30	22.
23.	Note 1	L-L 0.1, L-G 0	L-L 115, L-G 115	L-L 115, L-G 115	L-L 6 to 135	L-L 50, L-G 50	L-L 125	L-L 126	Note 5	L-L 120, L-G 60	23.
24.	*T1*	*H*	*N*	*K*	*T*	*O*	*T*	*O*	*P*	*A*	24.
25.	39	45	44	18, 28*d*	19, 28*e*					18, 28*a*, 40	25.

Note 1. Apparatus being intended for use only with standard telephone systems, special circuit considerations not involved.

Note 2. Power supply ordinarily required at both ends; but provision may be made for transmitting power to one unit over the telemetering line.

Note 3. Relays used only when telemetering circuit also carries power for operating instrument.

Note 5. Data not available.

Note 6. System includes provision for maintaining synchronism between transmitter and receiver.

Note 7. Possible under favorable conditions, but not generally recommended.

Note 8. Requires modified form, or auxiliary apparatus.

Note 9. May be simplexed in special cases.

Note 10. Response practically instantaneous, so far as concerns applications for which this system is intended.

Note 15. Will introduce no error in demand reading, but will affect registration of integrating dials.

Note 20. Power derived from circuit under measurement.

Note 23. This manufacturer does not supply receiving equipment for this type of telemetering apparatus.

Note 24. Time of response subject to type of receiving instrument used.

Table I. Characteristics of Telemetering Systems (continued)

1.	Manufacturer or distributor	General Electric	General Electric	General Electric	General Electric	General Electric	General Electric	General Electric	General Electric	1.
2.	Trade name of system	Rectified Curr.	Torque Balance	Selsyn (Excited)	Selsyn (Iron Vane)	Selsyn With Ind Transm.	Remote Demand	Distant Dial	Photoelectric	2.
3.	Basic principle of operation	Current	Current	Position	Position	Position	Impulse	Impulse	Frequency	3.
4.	Continuous or step-by-step action	Cont	Cont	Cont	Cont	Cont	Step-by-step	Step-by-step	Cont	4.
5.	Measurement as received	Ind, rec	Ind, rec	Ind, rec	Ind	Ind, rec	Ind, rec, int	Int	Ind, rec	5.
6.	Character of measurable magnitudes									6.
	(*a*) Amperes	A-c	A-c, d-c	A-c, d-c	A-c, d-c	A-c, d-c			A-c, d-c	(*a*)
	(*b*) Volts	A-c	A-c, d-c	A-c, d-c	A-c, d-c	A-c, d-c			A-c, d-c	(*b*)
	(*c*) Power, etc.		W, va, vars	W, va, vars	W, va, vars	W, va, vars	Dem		W, va, vars	(*c*)
	(*d*) Energy, etc.						Amp-hr, whr, va-hr, var-hr	Amp-hr, whr, va-hr, var-hr		(*d*)
	(*e*) Levels, positions		Lev, pos	Pos	Pos	Pos			Lev, pos	(*e*)
	(*f*) Pressures, temperatures		Pr, temp						Pr, temp	(*f*)
	(*g*) Quantities (gallons, rates of flow)		R				Q	Q	R	(*g*)
	(*h*) Other magnitudes (speed, etc.)			Freq and pf	Freq, pf	Freq, pf			Pf, freq. and PA	(*h*)

Performance

7.	Time of response, seconds									7.
	(*a*) Transmitter	Inst	Inst	Inst	Inst	1			2	(*a*)
	(*b*) Receiver	Inst	Inst	Inst	Inst	Inst			2	(*b*)
	(*c*) Over-all	Inst	Inst	Inst	Inst	1			3	(*c*)
8.	Maximum error of system	2% (note 13)	1% (note 13)	0.5°	0.5°	1°	0%	0%	1%	8.
9.	Power requirements									9.
	(*a*) Supply volts and frequency	Note 20	115 v, a-c	Comm a-c	Comm a-c	Comm a-c	Comm a-c or d-c	Comm a-c or d-c	115 a-c	(*a*)
	(*b*) Application point	T	T	T and rcr	T	T and rcr	T (rcr also for d-c)	T	T and rcr	(*b*)
	(*c*) Synchronism	No	No	Abs	No	Abs	No	No	No	(*c*)
10.	Self-restoration after interruption or disconnection	Intr, disc	Intr, disc	Intr, disc	Intr, disc	Intr, disc	Intr, disc (note 15)	No	Intr, disc	10.
11.	Adaptability to totalization	Yes	Yes	Yes	No	Yes	Yes (note 8)	No	Yes	11.
12.	Adaptability to more than one receiver	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	12.

Transmission Characteristics

13.	Nature of transmitted current	Rect	Rect	A-c	A-c	A-c	A-c, d-c, rect, aud rf	A-c, d-c, rect, aud rf	A-c, aud, rf	13.
14.	Impulses per minute (for impulse systems)						Max 60	Max 60		14.
15.	Relay employed	No	No	No	No	No	Pol on d-c	Pol for a-c	No	15.
16.	Maximum loop resistance, ohms	2,500 (note 14)	8,800 (notes 14, 16)	Note 12	40 (note 11)	1,500 (note 11)	6,000	6,000, d-c; 1,500, a-c	4,000	16.
17.	Minimum insulation resistance, megohms	0.5	1	0.1	0.1	0.1	0.1	0.1	0.1	17.
18.	Number of conductors in line	2	2	3 or 5	3	3 or 5	2, d-c; 3, a-c	2, d-c; 3, a-c	2	18.
19.	Possibility of using ground as one conductor	Note 7	Note 7	Note 7	Note 7	Note 7	Yes	Yes	Yes	19.
20.	Possibility of using exposed lines	Note 7	Note 7	Yes (note 7)	Yes (note 7)	Yes (note 7)	Yes	Yes	Yes	20.
21.	Superposition possibilities	Simp, comp	Simp, comp	No	No	No	Simp, comp, carr	Simp, comp, carr	Simp, comp, carr	21.

Interference and Protection Features

22.	Normal and short-circuit line current milliamperes	Norm 25, SC 25	Norm 8, SC 8	Norm 10, SC 375	Norm 25, SC 50	Norm 10, SC 600	Norm 50 a-c, 25 d-c, SC 350	Norm 40 d-c, 20 a-c; SC 350	Norm 320, SC 500	22.
23.	Maximum open-circuit volts	L-L 125, L-G 125	L-L 125, L-G 125	L-L 1 v/ohm/condr, L-G 0	L-L 22, L-G 0	L-L 55, L-G 0	L-L 115 a-c, 250 d-c L-G 115 a-c, 125 d-c	Note 5	L-L 120. L-G 120	23.
24.	Telemetering principle involved	*E*	*D*	*Ma*	*Mb*	*Ma*	*P*	*P*	*J*	24.
25.	Bibliographical references	13, 28*a*, 38	18, 20, 28*a*, 38	7, 17, 23, 28*d*, 38	7, 13, 17, 23, 38	7, 13, 17, 23, 28*d*, 38	37, 38	37, 38	38	25.

Table I. Characteristics of Telemetering Systems (continued)

1.	J. S. Kennedy	Landis & Gyr	C. A. Lang	Leeds & Northrup	Leeds & Northrup	Leeds & Northrup	Leeds & Northrup	Leeds & Northrup	Leeds & Northrup	1.
2.	Telemetric Gauge	Tele-Maxigraph	Langage Selsyn	Centrimax	Micromax Voltage Multiplier	Transformer-Bridge	Micromax Wheatstone-Bridge Telemeter	Micromax Thermal Converter	Micromax Potentiometer Transmitter	2.
3.	Impulse	Impulse	Position	Impulse	Voltage	Position	Position	Voltage	Voltage	3.
4.	Step-by-step	Step-by-step	Cont	Step-by-step	Cont	Cont	Cont	Cont	Cont	4.
5.	Ind	Ind, rec, int	Ind	Int	Ind, rec	Ind, rec	Ind, rec	Ind, rec	Ind, rec	5.
6.										6.
(a)					A-c and d-c		A-c, d-c	A-c	A-c, d-c	(a)
(b)					A-c and d-c		A-c, d-c	A-c	A-c, d-c	(b)
(c)					W, va, vars, dem		W, vars, dem	W, vars, pf, dem	W, vars, dem	(c)
(d)		Amp-hr, whr, va-hr var-hr,								(d)
(e)	Lev, pos		Lev		Lev, pos	Lev, pos	Lev, pos		Lev, pos	(e)
(f)	Pr		Pr		Pr, temp	Pr	Pr, temp		Pr, temp	(f)
(g)	R	Q		Q	Q, R		Q, R		Q, R	(g)
(h)										(h)

Performance

7.										7.
(a)		Note 5	Inst	Note 5	Inst	Inst	Inst	20+	Inst	(a)
(b)	5 sec max	Note 5	Inst	Note 5	30 sec	20 sec	30 sec	30	30	(b)
(c)	5 sec max	Note 5	Inst	Note 5	30 sec	20 sec	30 sec	30	30	(c)
8.	1.6%	Note 5	0.5%	0% int	2%	1.5% of range	2%	2%	2%	8.
9.										9.
(a)	24 to 135 v, a-c	Comm a-c, d-c	Comm a-c	Comm a-c	Dry cells and comm	Comm a-c	Dry cells and comm	Dry cells and comm	Dry cells and comm	(a)
(b)	L	T and rcr or rcr	T and rcr	T and rcr	T and rcr	T and rcr	Rcr	Rcr	T and rcr	(b)
(c)	No	No	Abs	No	No	Abs	No	No	No	(c)
10.	Intr, disc	Intr	Intr, disc	Intr, disc	Intr, disc	Intr, disc	Intr, disc	Intr, disc	Intr, disc	10.
11.	No	Yes	No	Yes	Yes	Note 17	No	Yes	Yes	11.
12.	Yes	Yes	Yes	Yes	Yes (note 17)	Note 17	Yes (note 17)	Yes (note 17)	Note 17	12.

Transmission Characteristics

13.	D-c	A-c, d-c	A-c	A-c	D-c	A-c	D-c	D-c	D-c	13.
14.	Max 360, min 360	Max 20		Max 40, min 4						14.
15.	No	No	No	No	No	No	No	No	No	15.
16.	9,000	Note 5	280		50,000	Note 18	2,000	1,000 per unit	50,000	16.
17.	0.1	Note 5	0.1		2.5	0.1	1	0.1	2.5	17.
18.	2	2	3 and 2	2	2	4 or 3	3	2	2	18.
19.	Yes	Yes	No	Yes	No	No	Yes (note 7)	No	No	19.
20.	Yes	Note 7	Yes	Yes	No	No	No	No	No	20.
21.	Simp, comp, carr	Note 7	No	Yes	Simp, comp	No	No	Simp, comp	Simp, comp	21.

Interference and Protection Features

22.	Norm 15 to 60, SC 350	Note 5	0 to 300	Note 5	Norm 3, SC 35	Norm 4, SC 70	Norm 5, SC 350	Norm 0.3 max, SC 350	Norm 0.3 max, SC 350	22.
23.	L-L 135, L-G 135	Note 5	120	Note 5	L-L 24, L-G 24	L-L 10, L-G 10	L-L 6, L-G 6	L-L 1	L-L 24, L-G 24	23.
24.	*Q*	*P*	*Ma*	*I*	*G*	*L*	*C*	*H*	*G*	24.
25.	28*e*			33	22, 25, 29			8, 12, 18, 22, 25, 28*b*, 29	18	25.

NOTE 5. Data not available.

NOTE 7. Possible under favorable conditions, but not generally recommended.

NOTE 8. Requires modified form, or auxiliary apparatus.

NOTE 11. Resistance values given in ohms *per conductor*.

NOTE 12. One ohm per volt per conductor.

NOTE 13. Maximum error can be reduced by special calibration.

NOTE 14. Resistance given is permissible value when used with one receiving instrument.

NOTE 15. Will introduce no error in demand reading, but will affect registration of integrating dials.

NOTE 16. May be repeated if necessary.

NOTE 17. May be re-transmitted.

NOTE 18. Resistance 1,000 ohms if double-circuit four-wire; 10 ohms if three-wire circuit is used.

NOTE 20. Power derived from circuit under measurement.

Table I. Characteristics of Telemetering Systems (continued)

1.	Manufacturer or distributor	Leeds & Northrup	Leupold, Volpel & Co.	Leupold, Volpel & Co.	Lincoln Meter Co.	Lincoln Meter Co.	Meyer Gov. Co.	Morey & Jones	Pioneer Instrument Co.	1.
2.	Trade name of system	Micromax Current Balance	Stevens, Type G	Stevens, Type P	Thermal Converter	Thermal Demand	Teldicator	Electruflo	Autosyn	2.
3.	Basic principle of operation	Current	Impulse	Position	Voltage	Voltage	Position	Position	Position	3.
4.	Continuous or step-by-step action	Cont	Step-by-step	Cont	Cont	Cont	Cont	Cont	Cont	4.
5.	Measurement as received	Ind, rec, int	Ind, rec	Ind, rec	Ind, rec	Ind, rec	Ind, rec	Ind, rec, int	Ind	5.
6.	Character of measurable magnitudes									6.
(a)	Amperes	D-c and a-c								(a)
(b)	Volts	D-c and a-c								(b)
(c)	Power, etc	W, vars, dem			W, vars, dem	W, dem				(c)
(d)	Energy, etc									(d)
(e)	Levels, positions	Lev, pos	Lev, pos	Lev, pos			Lev, pos	Lev, pos	Lev, pos	(e)
(f)	Pressures, temperatures	Pr, temp	Pr, temp	Pr, temp			Pr			(f)
(g)	Quantities (gallons, rates of flow)	Q, R	Q, R	Q, R				R		(g)
(h)	Other magnitudes (speed, etc.)	Pf and PA								(h)
	Performance									
7.	Time of response, seconds									7.
(a)	Transmitter	Inst	1 sec	Inst	20+	90% in 15 min	Inst	Inst	Inst	(a)
(b)	Receiver	30 sec	Inst	Inst	Note 23	Neg	Inst	30 sec	Inst	(b)
(c)	Over-all	30 sec	1 sec	Inst		90% in 15 min	Inst	30 sec	Inst	(c)
8.	Maximum error of system	2%	0.1%	0.1%	1.5%	1%	0.5%	0.5%	1°	8.
9.	Power requirements									9.
(a)	Supply volts and frequency	Dry cells and comm	Comm d-c	Comm a-c	Note 20	Comm a-c	Comm a-c	Comm a-c	Comm a-c	(a)
(b)	Application point	T and rcr	T and rcr	T and rcr		Rcr	T or rcr	Rcr	T and rcr	(b)
(c)	Synchronism	No	No	Abs		No	Yes	No	Abs	(c)
10.	Self-restoration after interruption or disconnection	Intr, disc	No	Intr, disc	Intr, disc	Intr, disc	Intr, disc	Intr, disc	Intr, disc	10.
11.	Adaptability to totalization	Yes	No	No	Yes	Yes	No	No	No	11.
12.	Adaptability to more than one receiver	Note 17	Yes	Yes		Note 17	Yes	No	Yes	12.
	Transmission Characteristics									
13.	Nature of transmitted current	D-c	D-c	A-c	D-c	D-c	A-c	A-c	A-c	13.
14.	Impulses per minute (for impulse systems)		Max 60							14.
15.	Relay employed	No	Pol		No				No	15.
16.	Maximum loop resistance, ohms	120,000	Note 5	250	1,000 per unit	400 or more	Note 19	40	12	16.
17.	Minimum insulation resistance, megohms	5	Note 5	0.1	0.1	0.1	Note 5		0.1	17.
18.	Number of conductors in line	2	2 or 3	3 or 5	2	2	3	3	3 or 5	18.
19.	Possibility of using ground as one conductor	Yes (note 7)	Yes	Yes	No	No	Note 5	Note 7	Yes	19.
20.	Possibility of using exposed lines	No	Yes	No	No	Note 7	Note 5	No	No	20.
21.	Superposition possibilities	Simp, comp	Simp, comp carr	No		No	Note 5	No	No	21.
	Interference and Protection Features									
22.	Normal and short-circuit line current, milliamperes	Norm 0.2, SC 350	Norm 50, SC 1	Norm 100, SC 1,000	Norm 0.3, SC 350	Norm 0, SC 2	Norm 50	Note 5	Norm 170	22.
23.	Maximum open-circuit volts	L-L 24, L-G 24	L-L 120, L-G 120	L-L 120, L-G 120	L-L 1, L-G 1	L-L 0.2, L-G 0.2	L-L 110 L-G 110	L-L 110	L-L 32, L-G 32	23.
24.	Telemetering principle involved	*F*	*O*	*Ma*	*H*	*H*	*K*	*K*	*Ma*	24.
25.	Bibliographical references		28*e*, 39	28*d*	8, 12, 18, 22, 25, 28*b*, 29	8, 12	28*d*	21, 28*d*	28*d*	25.

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Table I. Characteristics of Telemetering Systems (concluded)

1.	Republic	Sangamo Elect. Co.	Westinghouse	Westinghouse	Westinghouse	Westinghouse	Westinghouse	Westinghouse	Westinghouse	1.
2.	Republic	Types "G"	Synchrotie	Current Balance Telem.	Potentiometer Telem.	Rectox Telem.	High-Rate Imp.	Low-Rate Imp.	Quick-Break Impulses	2.
3.	Current	Impulse	Position	Current	Voltage	Current and Voltage	Impulse	Impulse	Impulse	3.
4.	Cont	Step-by-step	Cont	Cont	Cont	Cont	Cont	Cont	Cont	4.
5.	Ind, rec, int	Ind, rec	Ind, rec	Ind, rec	Ind, rec	Ind, rec	Ind, rec	Rec, int	Ind, int	5.
6.										6.
(a)				A-c, d-c		A-c				(a)
(b)				A-c, d-c		A-c				(b)
(c)		W, dem		W, va, vars			W, va, vars	Dem.	Dem.	(c)
(d)		Whr						Whr, va-hr, var-hr	Whr, va-hr, var-hr	(d)
(e)	Lev, pos		Lev, pos	Lev, pos	Lev, pos					(e)
(f)	Pr		Pr							(f)
(g)	Q, R									(g)
(h)			Signal system				Speed			(h)

Performance

7.										7.
(a)	0 to 5		Inst	5		0	Inst	0	0	(a)
(b)	Inst		Inst	5		0	10 sec	0	0	(b)
(c)	0 to 5	0.05 sec	Inst	10	0.1 sec	0.1	10 sec	0	0	(c)
8.	1%	1%	1%	2.5%	3%	2.5%	2%	0	2%	8.
9										9.
(a)	Comm a-c	Comm a-c or d-c	Comm a-c	Comm a-c and d-c and 48 v d-c	50 v, d-c	Note 20	Comm a-c or d-c	Comm a-c or d-c	Comm a-c	(a)
(b)	Rcr	T	T and rcr	T	T	T	Rcr	Rcr	Rcr	(b)
(c)	No	No	Abs	No	No	No	No	No	No	(c)
10.	Intr, disc	Intr (note 15)	Intr, disc	Intr, disc	Intr, disc	Intr, disc	Intr, disc	Intr, disc	Intr, disc	10.
11.	Yes	Yes	No	Yes	No	Yes	Yes	Yes	Yes	11.
12.	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	12.

Transmission Characteristics

13.	A-c	A-c or d-c	A-c	D-c	D-c	Rect a-c	D-c or rect a-c	D-c, a-c, rect a-c	D-c, a-c, rect a-c	13.
14.		Max 30					Max 300, min 0	Max 20,	Max 20,	14.
15.	No	No	No		No	No	If required		No	15.
16.	40	1,000	1,000 (note 22)	5,000	4,000	4,000	30,000	2,000	300 (note 21)	16.
17.	Note 5	Note 5	Note 5	0.5	0.5	0.5	Note 5	25×loop		17.
18.	2	2	3	2	2	2	2	3	2	18.
19.	Note 7	Yes	Yes	Note 7	Yes	Yes	Yes	Yes	Yes	19.
20.	Note 7	Note 7	Yes	Note 7	Yes	Yes	Yes	Note 7	Note 7	20.
21.	No	Note 5	Note 5	Simp, comp, carr only (note 8)	Simp	Simp	Simp, comp, carr	Simp, comp, carr	Simp, comp, carr	21.

Interference and Protection Features

22.	Norm 0 to 1,000, SC 1,000	Norm 40	Norm 0.27 to 0.72	Norm 20, SC 350	Norm 25, SC 500	Norm 25, SC 30	Norm 50, SC 350	Norm 75, SC 350	Norm 20	22.
23.	L-L 40, L-G 40	Note 5	L-L 115, L-G 11	L-L 115, L-G 115	L-L 50, L-G 50	L-L 250, L-G 250	L-L 115, L-G 115	L-L 115, L-G 115	Note 5	23.
24.	*C*	*P*	*Ma*	*B*	*G*	*E*	*Pa*	*Pc*	*Pa*	24.
25.	28*a*	18		3, 18, 28*a*, 36	2, 28*b*	36	6, 9, 27, 36	36	36	25.

Note 5. Data not available.

Note 7. Possible under favorable conditions, but not generally recommended.

Note 15. Will introduce no error in demand readings, but will affect registration of integrating dials.

Note 17. May be re-transmitted.

Note 19. Value very high if power applied at transmitting end.

Note 20. Power derived from circuit under measurement.

Note 21. Permissible line capacitance 0.075 mf.

Note 22. Maximum line resistance 25 ohms where recorder is used.

Note 23. This manufacturer does not supply receiving equipment for this type of telemetering apparatus.

Table II. Characteristics of Supervisory-Control Systems

1.	Manufacturer	Autocall Company	Control Corporation	Control Corporation	General Electric	1.
2.	Trade name	Howe	Teletrol	Syntrol	Synchronous selector	2
3.	Method of selection	Dial	By three aud-freq currents transmitted in a definite sequence	Synchronous distributors	Synchronous selector	3.
4.	Method of checking selection	Supervising transmitter	Selection causes operation without checking selection	Selection and operation combined	Direct (polar) circuit	4.
5.	Method of operation	Manual operation switch			Direct (polar) circuit	5.
6.	Method of indication	Lamp diagram and printed record	Synchronous distributors	Synchronous distributors	Direct (polar) circuit	6.
	Characteristics of System					
7.	(*a*) Maximum number of units recommended for one system	Variable	54 operations	48	92	7 (*a*)
	(*b*) Normal physical limitation of number of units	Unlimited	108 operations	96	276	(*b*)
8.	Maximum number of stations recommended for parallel operation over one group of communication circuits (see item 20)	Variable according to conditions	4	25	1 (radial)	8.
9.	Power requirements					9.
	(*a*) Dispatching station	24 to 125 v d-c	115 v a-c from station service or from 200-w motor generator set	115 v a-c from station service or from small motor generator set	60-cell battery 24 v a-c or d-c	(*a*)
	(*b*) Distant station	24 to 125 v d-c	115 v a-c from station service or from small motor generator set; d-c also desirable	Same as (*a*) with d-c desirable	60-cell battery	(*b*)
10.	Time required to select and check					10.
	(*a*) 10th unit	Time required for any unit varies with length of dialed signal. Generally 3 to 5 sec	6 sec	3 sec	1 sec	(*a*)
	(*b*) 20th unit		6 sec	6 sec	2 sec	(*b*)
11.	Having selected unit, can it be operated repeatedly without again performing selecting operation	If required	No	No	Yes	11
12.	Is automatic indication given to dispatcher					12.
	(*a*) Line open	Yes	Yes	Yes	Yes	(*a*)
	(*b*) Line short	Yes	Yes	Yes	Yes, except one combination	(*b*)
	(*c*) Line ground	Manual test only	Yes	Yes	No	(*c*)
13.	Can telephone conversation be carried over same communication circuit, intermittent or continuous	If required	Yes	Yes	Continuous supervisory calling	13.
14.	Is selective telemetering possible over same communication circuits	No such application as yet	Yes	Yes	Yes	14.
15.	Is remote manual synchronization possible over same communication circuits	Yes	Yes, but automatic synchronizing equipment at controlled station with remote indications of freq diff recommended	Yes, but automatic synchronizing equipment with indications at dispatching point of which freq is high (or low) recommended	Yes	15.
	Characteristics Affecting Line Transmission					
16.	Maximum number impulses per second	13	Impulse system not used	Impulse system not used	10	16.
17.	Is neutral or polar relay used	Yes	For receiving indications only	Yes	Polar	17.
18.	Maximum allowable loop resistance	2,500 w	No practical limit	3,000 ohms	3,000 ohms	18.
19.	Minimum allowable insulation resistance	20,000 w between wires and/or ground			1.0 megohms	19
20.	Number of conductors	2, 3, or 4	2	2	4	20.
21.	Possibility of using ground as a conductor	System is automatically operable over ground in event of break in conductor	Yes	Yes	No (note *b*)	21.
22.	Possibility of using superposed circuit as conductor					22.
	(*a*) Simplex	No such study or application as yet	Neutral current system recommended for use of the equipment over a power transmission system	Neutral current system recommended for use of the equipment over a power transmission line	No	(*a*)
	(*b*) Composite				No	(*b*)
	(*c*) Carrier				No	(*c*)
23.	Can system operate through insulating transformers	No	Yes	Yes	No	23.
	Interference and Protective Features					
24.	Normal line current	0.012 amp at 125 d-c	20 ma	20 ma	15–30 mils (note *c*)	24.
25.	Maximum line current on short circuit	0.018 amp at 125 d-c	500 ma	500 ma	350 mils	25.
26.	Maximum line volts					26.
	(*a*) Between wires	24 v with 125 d-c system	24 v	24 v standard	135 v d-c	(*a*)
	(*b*) To ground	0			Note *b*	(*b*)
	How is system protected from electrostatic and electromagnetic induction when exposed to high potential circuits	Standard current-connecting devices, surge arresters, and circuit design	Vacuum-protector tubes	Vacuum-protector tubes	Lightning arr. or tuned reactor - capacitor drainage	27.

Table II. Characteristics of Supervisory-Control Systems (continued)

1.	General Electric	General Electric	General Electric	General Electric	Westinghouse	Westinghouse	1.
2.	Code Selector	Five Point	Audible Selector	Audible Indicating System	Audicode	Caricode	2.
3.	Code	Code	Audible freq code (note *d*)	Audible freq code (telephone calling)	Code by dial	Code (visual)	3.
4.	Code	Code	Audible freq code		Audible code	Code (visual)	4.
5.	Single polar impulse	Single polar impulse	Audible freq impulse (note *d*)		Code by dial	Code (visual)	5.
6.	Single polar impulse	Single polar impulse	Two-tone audible freq code	Audible freq code	Audible code	Code (visual)	6.

Characteristics of System

7 (*a*)	128	5	17	11	20	100 units (appr)	7 (*a*)
(*b*)	Unlimited	5	17	11	Unlimited	Unlimited	(*b*)
8.	1 (radial)	1 (radial)	5	1 (radial)	5	5, but not limited	8.
9.							9.
(*a*)	60-cell battery	48 v d-c	60-cell battery 12 v d-c	None	48 v d-c	48 v d-c	(*a*)
(*b*)	60-cell battery	48 v d-c	24-cell battery	24-cell battery	48 v d-c	48 v d-c	(*b*)
10.							10.
(*a*)	2–3 sec		11 sec	(Telephone calling time plus 5 sec per unit accumulative)	3 sec	2.0 sec	(*a*)
(*b*)	2–3 sec				6 sec	2.6 sec	(*b*)
11.	Yes	Yes	Yes	No	Yes	Yes	11.
12.							12.
(*a*)	Yes, unless insulating transformers are used	No	No	No	Yes		(*a*)
(*b*)	No	No	No	None	Yes	No lines	(*b*)
(*c*)		No	No		No; can be supplied		(*c*)
13.	Continuous, supervisory calling	Continuous, supervisory calling	Continuous on selected point	Continuous, disconnects code equipment	Yes	Yes, if channel permits	13.
14.	Yes	Yes	Yes (note *e*)	No	Yes	Yes	14.
15.	No	No	No		Yes	No; use automatic synchronizer	15.

Characteristics Affecting Line Transmission

16.	10	10	5		8	13	16.
17.	Polar	Polar	Polar		No	No	17.
18.	3,500 ohms	2,500 ohms	1,000 ohms	Note *a*	4,000 ohms		18.
19.	0.1 megohms	0.1 megohms	0.1 megohms	Note *a*	0.5 megohm		19.
20.	2	2	2	Note *a*	Two	1 carrier channel	20.
21.	No (note *b*)	No (note *b*)	No (note *b*)	Note *a*	Yes; not recommended	Yes	21.
22.							22.
(*a*)	Yes	Yes	Yes	Note *a*	Yes	No	(*a*)
(*b*)	Yes	Yes	Yes	Note *a*	Yes	No	(*b*)
(*c*)	Yes	Yes	Yes	Note *a*	Yes	Yes	(*c*)
23.	Yes	Yes	Yes	Yes	Yes; not recommended	Yes; included in line-timing equipment	23.

Interference and Protective Features

24.	15–30 mils (note *c*)	15–30 mils (note *c*)	15–30 mils (notes *c*, *f*), 1.5 mils (note *d*)	Note *a*	0.025 amp	None	24.
25.	350 mils	350 mils	350 mils	Note *a*	0.055 amp		25.
26.							26.
(*a*)	135 v d-c	50 v	120 v a-c (note *d*) and 135 v d-c	Note *a*	55 v		(*a*)
(*b*)	Note *b*		Note *b*		Ungrounded, 350 v		(*b*)
27.	Lightning arr. or tuned reactor-capacitor drainage	Lightning arr. or tuned drainage	Lightning arr. and insulating transformers	Lightning arr. and insulating transformers	Standard drainage where required and protective tube	This system may be connected to high-potential circuits	27.

Note *a*. Circuit requirements determined by standard telephone practice.

Note *b*. Use of ground on telephone system not recommended where supervisory battery may be directly connected to communication circuits.

Note *c*. Polar impulse.

Note *d*. 200–400 cycles (uses d-c impulses when going through telephone exchange for selection and control.

Note *e*. When insulating transformers are not present.

Note *f*. 200–400 cycles, on open-wire line only.

Table II. Characteristics of Supervisory-Control Systems (concluded)

1.	Manufacturer	Westinghouse	Westinghouse	Westinghouse	Westinghouse	1.
2.	Trade name	Self-Checking System	Visicode	Polaricode, Jr.	Polaricode	2.
3.	Method of selection	Code	Code (visual)	Code (visual)	Code (visual)	3.
4.	Method of checking selection	None required	Code (visual)	Code (visual)	Code (visual)	4.
5.	Method of operation	Code	Code (visual)	Code (visual)	Code (visual)	5.
6.	Method of indication	Code	Code (visual)	Code (visual)	Code (visual)	6.
	Characteristics of System					
7.	(*a*) Maximum number of units recommended for one system	Unlimited	100 units (appr)	5	Unlimited	7 (*a*)
	(*b*) Normal physical limitation of number of units	Unlimited	Unlimited	5	Unlimited	(*b*)
8.	Maximum number of stations recommended for parallel operation over one group of communication circuits (see item 20)	Unlimited; codes are not duplicated in stations of one system	5 but not limited	1	1	8.
9.	Power requirements					9.
	(*a*) Dispatching station	48 v d-c	48 v d-c	48 v d-c	48 v d-c	(*a*)
	(*b*) Distant station	48 v d-c	48 v d-c	48 v d-c	48 v d-c	(*b*)
10.	Time required to select and check					10.
	(*a*) 10th unit	Same for all points; depends on total number	2.0 sec	5 points max	2.5	(*a*)
	(*b*) 20th unit		2.6 sec		2.5	(*b*)
11.	Having selected unit, can it be operated repeatedly without again performing selecting operation	Yes	Yes	Yes	Yes	11.
12.	Is automatic indication given to dispatcher					12.
	(*a*) Line open	Yes	Yes	Yes		(*a*)
	(*b*) Line short	Yes	Yes	Yes	Yes, if required	(*b*)
	(*c*) Line ground	No; can be supplied	No; can be supplied	No; can be supplied		(*c*)
13.	Can telephone conversation be carried over same communication circuit, intermittent or continuous	Yes	Yes	Yes	Yes	13.
14.	Is selective telemetering possible over same communication circuits	Yes	Yes	Yes	Yes	14.
15.	Is remote manual synchronization possible over same communication circuits	Yes except when carrier channel is used	Yes	Yes	Yes	15.
	Characteristics Affecting Line Transmission					
16.	Maximum number impulses per second	6	13	8	10	16.
17.	Is neutral or polar relay used	No	No	No	No, rectox valve with std relay	17.
18.	Maximum allowable loop resistance	4,000 ohms	4,000 ohms	4,000 ohms	2,000 ohms	18
19.	Minimum allowable insulation resistance	0.5 megohm	0.5 megohm	0.5 megohm	0.5 megohm	19.
20.	Number of conductors	2	2	2	2	20.
21.	Possibility of using ground as a conductor	Yes; not recommended	Yes; not recommended	Yes; not recommended	Yes; not recommended	21.
22.	Possibility of using superposed circuit as conductor					22.
	(*a*) Simplex	Yes	Yes	Yes	Yes; not recommended	(*a*)
	(*b*) Composite	Yes	Yes	Yes	No	(*b*)
	(*c*) Carrier	Yes	Yes	Yes	No	(*c*)
23.	Can system operate through insulating transformers	No	No	No	No	23.
	Interference and Protective Features					
24.	Normal line current	0.025 amp	0.025 amp	0.025 amp	None at rest; 0.025 amp operating	24.
25.	Maximum line current on short circuit	0.055 amp	0.055 amp	0.055 amp	0.055 amp	25.
26.	Maximum line volts					26.
	(*a*) Between wires	55 v	55 v	55 v	55 v	(*a*)
	(*b*) To ground	350 v ungrounded	350 v ungrounded	350 v ungrounded	350 v ungrounded	(*b*)
27.	How is system protected from electrostatic and electromagnetic induction when exposed to high potential circuits	Standard drainage where required and protective tube	Standard drainage where required and protective tube	Standard drainage where required and protective tube	Standard drainage where required and protective tube	27.

Selected Bibliography on Telemetering Systems and Methods

1. Electrical Transmission of Electrical Measurements, **O. J. Bliss.** AIEE Transactions, volume 31, 1912, pages 1537–40.

2. Automatic Transmission of Power Readings, **B. H. Smith** and **R. T. Pierce.** AIEE Transactions, volume 43, 1924, pages 303–10.

3. Totalization of Power Readings, **R. T. Pierce.** *Electric Journal*, March 1924, page 103.

4. Transmission of Power Readings, **B. H. Smith.** *Electric Journal*, May 1924, page 219.

5. A New Electrical Telemeter, **B. McCollum** and **O. S. Peters.** *Technical Papers* Bureau of Standards, volume 17, 1924, page 737.

6. Remote Metering by the High-Rate Impulse and Condenser Method, **B. H. Smith.** *Electric Journal*, August 1924, page 355.

7. Totalization and Remote Indication of Electric Power, **H. K. Nock** and **H. S. Edgerly.** *General Electric Review*, November 1924.

8. Totalizing the Load of Four Stations, **P. A. Borden.** *Electrical News* (Toronto), volume 35, December 1, 1926, pages 35–8.

9. Budget Plan Load Dispatching, **P. B. Junkhe** and **L. M. Moore.** *Electrical World*, June 18, 1927, page 1317.

10. Meter Readings Transmitted by Selsyn Motors, **A. E. Bailey.** *Power*, September 11, 1928.

11. Annual Report of AIEE Committee on Instruments and Measurements. AIEE Transactions, volume 47, 1928, pages 1164–71.

12. Totalizing of Electric System Loads, **P. M. Lincoln.** AIEE Transactions, volume 48, 1929, pages 775–82.

13. Telemetering, **C. H. Linder, C. E. Stewart, H. B. Rex,** and **A. S. Fitzgerald.** AIEE Transactions, volume 48, 1929, pages 766–74.

14. The Midworth Distant Repeater, *Instruments*, August 1929, pages 291–2.

15. An Electron Tube Telemetering System, **A. S. Fitzgerald.** AIEE Transactions, volume 49, 1930, pages 1321–38.

16. Two-Wire Supervisory Control System With Remote Metering, **R. J. Wensley** and **W. M. Donovan.** AIEE Transactions, volume 49, 1930, pages 1339–42.

17. Principles of Selsyn Instruments, **L. F. Holder.** *General Electric Review*, September 1930.

18. Electrical Telemetering, **H. Olken.** *Instruments*, volume 5, 1932, pages 66–74, 91–2.

19. "Chronoflo", *Instruments*, volume 5, 1932, page 244.

20. The Torque-Balance Telemeter, **A. J. Johnson.** AIEE Transactions, volume 51, 1932, pages 1027–30.

21. Indicating, Recording and Integrating Fluid Meter, *Instruments*, volume 7, 1934, page 80.

22. Load Totalizing in the New York Area, **F. Zogbaum.** AIEE Transactions, volume 53, 1934, pages 886–9.

23. Selsyn Instruments for Position Systems, **T. M. Linville** and **J. S. Woodward.** AIEE Transactions, volume 53, 1934 (June section), pages 953–60.

24. A Telemeter With Pilot-Coil Transmitter, **L. J. Lunas** and **H. L. Bernarde.** AIEE Transactions, volume 53, 1934 (June section), pages 974–6.

25. Interconnection Economies Through Telemeter Totalizing, **W. J. Lank.** *Electrical World*, June 30, 1934, pages 948–9.

26. "Galvatron", *Instruments*, volume 7, 1934, pages 76–7; *Power Plant Engineering*, December 1934.

27. An Improved High-Rate Impulse Telemeter System, **C. Oman.** *Electric Journal*, August 1934, page 313.

28. Telemetering and Totalizing, **P. A. Borden** and **M. F. Behar.** Series appearing in *Instruments*, August 1935 through March 1936.
(*a*) September 1935, pages 235–40.
(*b*) October 1935, pages 256–60.
(*c*) November 1935, pages 293–6.
(*d*) December 1935, pages 323–52.
(*e*) January 1936, pages 11–23.

29. Power Telemetered on Pennsylvania Line, **E. H. Brown.** *Electrical World*, February 29, 1936, pages 40–2.

30. Supervisory Control and Remote Metering, **J. V. B. Duer.** AIEE Transactions, volume 55, 1936 (January section), pages 70–5.

31. Neutralizing Transformer to Protect Station Communication, **E. E. George, R. K. Honaman, L. L. Lockrow, E. L. Schwartz.** AIEE Transactions, volume 55, 1936 (May section), pages 524–9.

32. Remote Metering on West Pennsylvania System, **J. C. McKinley.** *Electric Journal*, volume 33, September 1936, page 411.

33. Null Method Flow Measuring and Integrating Equipment, *Instruments*, February 1937, page 35.

34. A Syllabus of Telemetering Apparatus, **L. D. White.** *General Electric Review*, volume 40, 1937, pages 234–5.

35. "Synchro-Meter". *Instruments*, May 1937, page 124; *Metals and Alloys*, May 1937.

36. Telemetering, **Paul MacGahan.** *Electric Journal*, March 1938, pages 113–16.

37. Load Totalization, **E. J. Boland.** *General Electric Review*, April, May, June 1938.

38. Why Telemetering? **G. S. Lunge.** *General Electric Review*, April 1939, pages 150–9.

39. Telemetering Device for Hydro Plants, *Electrical World*, June 17, 1939, page 76.

40. Telemeter Steam and Electricity on Indianapolis System, **T. W. Ayton.** *Electrical World*, August 26, 1939, page 584.

41. The Metameter System of Telemetering, **F. B. Bristol** and **G. S. Lunge.** *General Electric Review*, November 1939, page 584.

42. Carrier Telemetering Simplifies Dispatching, **H. L. Durgin.** *Electrical World*, December 16, 1939, pages 36–7.

43. Carrier Telemetering With the Metameter, **G. S. Lunge.** *General Electric Review*, August 1940, page 336.

44. The Uehling Telemetering System, **A. W. Jacobson.** Thesis, Yale School of Engineering, 1940.

45. A New High-Speed Thermal Wattmeter, **J. H. Miller.** AIEE Transactions, volume 60, 1941 (January section), pages 37–40.

46. Sun-Spot Disturbances of Terrestrial Magnetism, **W. F. Davidson,** Electrical Engineering, volume 60, February 1941, pages 72–5. Bibliography.

A general picture of comparative telemetering practice in various countries may be obtained by perusal of the following books:

Electric Power Metering, **A. E. Knowlton.** McGraw-Hill, 1935. Chapter 17. American.

Industrial Electrical Measuring Instruments, **K. Edgcumbe** and **F. E. J. Ockenden.** Pitman 1933. Chapter 22. British.

Die Technik der Fernwirkanlagen, **W. Stablein.** Oldenbourg, Munich, 1934. Bibliography of over 500 references, 40 per cent of which are articles published in English. German.

Remote Measuring Instruments and Present Status of Practical Applications in Japan, **S. Jimbo** and **J. Okajima.** Circular Number 100, Electrotechnical Laboratory of Japan, March 1935. Japanese.

A number of telemetering systems that have application in France are treated in more or less detail in the following papers:

Transmission Electrique à Distance "Telect.", **M. Vassiliere.** *Bulletin* Société Française des Electriciens, 1931, pages 629–32.

Téléindicateur de Niveau ou de Position, **J. Laisne.** *Revue Générale de l'Electricité*, April 25, 1931, page 665.

Perfectionnements dans les Dispositifs de Télémésure, **A. Iliovici.** *Bulletin* Société Française des Electriciens, 1936, pages 826–44.

Nouveaux Dispositifs de Télémésure, **J. Pelpel.** *Bulletin* Société Française des Electriciens, 1938, pages 933–47.

Le Réglage Centralisé de la Fréquence et de la Puissance dans la Région Parisienne, **J. Fallou.** *Revue Générale de l'Electricité*, January 15, 1938, pages 67–77.

A comprehensive treatment of the subject of telemetering in the Swedish language is found in an article by **E. Velander,** appearing on pages 81–8 of "Elektroteknik" Supplement 5, May 1931, to *Teknisk-Tidskrift*, May 2, 1931, as part of a symposium on Remote Metering and Remote Control.

Bibliography of Relay Literature 1927–1939

Section A

Line Protection—Distance

1928

1. The Impedance Relay, **H. A. McLaughlin** and **E. O. Erickson.** AIEE Transactions, volume 47, 1928, pages 776–82.

2. A Study of Transmission-Line Power Arcs, **Paul Ackerman.** *Engineering Journal* (Canada), May 1928, page 309.

1931

3. Connections for Distance Relays (in German), **E. Gross.** Verband Deutscher Elektrotechniker *Fachberichte*, 1931, pages 88–91.

4. The Development of Distance Relaying (in German), **M. Walter.** Verein Deutscher Ingenieure *Zeitschrift*, volume 75, December 26, 1931, pages 1555–8.

5. Fundamental Basis for Distance Relaying on Three-Phase Systems, **W. A. Lewis** and **L. S. Tippett.** Electrical Engineering, volume 50, June 1931, pages 420–2.

6. Operating Experience With Reactance-Type Distance Relays, **E. E. George.** AIEE Transactions, volume 50, 1931, pages 288–93; discussion page 293.

1932

7. Application of Impedance Relays for Relay Study, **L. S. C. Tippett.** Institution of Engineers (Australia) *Journal*, volume 4, number 1, January 1932, page 22.

8. Behavior of Distance Relays During Hunting (in German), **M. Walter.** *Electrotechnik und Maschinenbau*, volume 50, number 17, April 24, 1932, pages 261–3.

9. Design of Distance-Relay Protection (in Russian), **A. Barzam.** *Electritchestvo* (Moscow), number 21, part 1, November 1932, pages 1212–25.

10. Directional Element of Distance Relays (in German), **M. Walter.** *Elektrotechnische Zeitschrift*, volume 53, number 20, May 19, 1932, pages 476–9. Details of construction of the directional part of distance relays.

11. Distance-Relay Protection (in German), **M. Walter.** *Electrizitaetswirtschaft*, volume 31, numbers 8 and 9, April 30, 1932, pages 172–6; May 15, 1932, pages 199–202. Description of single-, double-, and triple-pole distance-relay combinations.

12. High-Speed Impedance Relay (in German), **H. Neugebauer and F. Geise.** *Siemens Zeitschrift*, volume 12, number 2, February 1932, pages 47–51. Relay that functions as quick-acting impedance relay until pull-out point and after that as standard impedance relay with linear time-distance characteristic.

13. New Distance Relay for the Protection of High-Voltage Overhead Lines (in French), **R. Dubusc** and **P. Douce.** *Revue Générale de l'Electricité*, volume 31, numbers 8 and 9, February 20, 1932, pages 251–9; February 27, 1932, pages 282–92. Reactance distance-relay description.

14. New High-Speed Distance Relay, **S. L. Goldsborough** and **W. A. Lewis.** Electrical Engineering, volume 51, number 3, March 1932, pages 157–60. Distance relay for short lines essentially unaffected by fault resistance.

15. Protection of Stations by Distance Relays, **A. Doret Matthey.** *Brown Boveri Review*, volume 19, number 7, December 1932, pages 205–09.

16. Short-Circuit Arc in Three-Phase Systems and Its Influence on the Operation of Distance Relays (in German), **M. Walter.** *Elektrotechnische Zeitschrift*, volume 53, number 44, November 3, 1932, pages 1056–9.

17. Three-Phase Reactance Protection (in German), **H. Polek.** *Siemens Zeitschrift*, volume 12, number 11, November 1932, pages 386–94.

1933

18. A High-Speed Reactance Relay, **A. R. V. Warrington.** Electrical Engineering, volume 52, April 1933, pages 248–52. (Advance abstract volume 51, number 6, June 1932, page 410.)

1934

19. Control of Distance-Relay Potential Connections, **A. R. V. Warrington.** AIEE Transactions, volume 53, 1934 (January section), pages 206–13; discussion (March section), pages 465–6, and (April section), page 617.

20. Distance-Relay Action During Oscillations, **E. H. Bancker** and **E. M. Hunter.** AIEE Transactions, volume 53, 1934 (July section), pages 1073–80; discussion, volume 53, 1934 (September section), pages 1320–2; volume 54, 1935 (February section), pages 216–18.

21. New Development in Distance-Protection Arrangements (in German), **E. Gross.** *Electrotechnik und Maschinenbau*, volume 52, numbers 51 and 52, December 23, 1934, pages 597–602; December 30, 1934, pages 612–16. New connections for distance relays.

1935

22. High-Speed Distance Relay, **L. N. Crichton.** *Electric Journal*, volume 32, number 12, December 1935, pages 537–42. Type *HCZ* relay.

23. Impedance Protective Systems, *Electrician* (London), volume 115, November 15, 1935, page 609; November 22, 1935, page 642; November 29, 1935, page 677; December 6, 1935, page 717.

24. Instantaneous Overcurrent Relays for Distance Relaying, **C. H. Frier,** AIEE Transactions, volume 54, 1935 (April section), pages 404–07; discussion (August section), pages 883–4. Use of instantaneous overcurrent relays on Oklahoma Gas and Electric Company system.

1936

25. New Impedance Relay for Intermediate Voltages, **R. Dubusc.** Société Française des Electriciens *Bulletin*, volume 6, August 1936, page 855.

26. Performance of Distance Relays, **G. Calabrese.** AIEE Transactions, volume 55, 1936 (June section), pages 660–72; discussion (November section), page 1254.

1937

27. Comprehensive Method of Determining the Performance of Distance Relays, **J. H. Neher.** AIEE Transactions, volume 56, 1937 (July section), pages 833–44; discussion (December section), page 1515.

28. Development of Step Protection (in German), **W. Schmundt.** *Elektrizitaetswirtschaft*, volume 36, number 18, June 25, 1937, pages 424–6.

29. High-Speed Protective Relay, Type *R-Z*4, **H. Neugebauer.** *Siemens Review* (in English), volume 13, number 3, 1937, pages 65–71.

A working group* of the relay subcommittee of the AIEE committee on protective devices has reviewed the available indexes and compiled this list of articles dealing with relaying and closely related subjects. These articles include all those appearing in the AIEE Transactions or Electrical Engineering from January 1927 through December 1939 and most of those in the chief technical publications of the world from January 1932 through December 1939. These articles are subdivided into several sections. In each section the entries are numbered consecutively and listed alphabetically by years, the first significant word of the title determining the alphabetical position. This bibliography was published in pamphlet form by the AIEE during 1941.

* **Working Group:** J. H. Neher, *chairman;* E. H. Bancker, W. R. Brownlee, L. N. Crichton, G. W. Gerell, H. F. Lindemuth, C. A. Muller, George Steeb, and R. M. Wood.

30. Influence of Wye-Delta Transformation on Distance Protection of High-Voltage Systems (in German), **E. Gross.** *Electrotechnik und Maschinenbau*, volume 55, number 28, July 11, 1937, pages 333–8.

31. New Antipumping Distance Relay (in French), **R. Dubusc.** Société Française des Electriciens *Bulletin*, volume 7, number 77, May 1937, pages 533–61.

32. Single-System Line Protection, **A. Thewalt.** *Siemens Review* (in English), volume 8, number 3, 1937, pages 59–65. Direct comparison protection.

1938

33. Distance Relays Solve Interconnection Problem, **H. H. Brown.** *Electric Light and Power*, volume 16, number 4, April 1938, pages 63–6.

1939

34. High-Speed Impedance Protection, *Electrician* (London), volume 122, March 10, 1939, page 313; March 17, 1939, page 343; March 24, 1939, page 373.

Section B

Line Protection—Pilot Wire and Carrier Current

1931

1. Principles of Feeder-Protection and Their Application to Three Modern Systems (in French), **B. H. Leeson and H. Leben.** International Conference on Large High-Tension Electric Systems *Report* of sixth session, 1931, volume 3, pages 767–800. Description of Merz-Price, split-pilot, interlock, and ratio-balance systems as applied in England.

2. Relaying With Two Pilot Wires, **C. H. Frier.** Electrical Engineering, volume 50, October 1931, pages 824–6.

1932

3. Carrier-Current Relaying Proves Its Effectiveness, **P. Sporn** and **C. A. Muller.** *Electrical World*, volume 100, number 11, September 10, 1932, pages 332–6. Description of first installation tests of directional-type carrier-pilot relaying.

4. D-C Pilot-Wire Systems of Relay Protection, **C. M. Longfield.** *Metropolitan-Vickers Gazette*, volume 13, number 236, December 1932, pages 417–22.

5. Pilot Protection by Power Directional Relays, **O. C. Traver, J. Auchincloss,** and **E. H. Bancker.** *General Electric Review*, volume 35, number 11, November 1932, pages 566–70. Description of pilot-relaying principles using directional relays and wire or carrier channel.

6. Supervisory Pilot-Wire Relaying—Modified Form of Pilot-Wire Relay Circuit to Eliminate Cascaded Timing in Transmission-Line Protection, **G. M. Babcock.** *Electrical West*, volume 68, number 6, May 15, 1932, pages 337–9.

1933

7. Carrier-Current Relaying Simplified, **P. Sporn** and **C. A. Muller.** *Electrical World*, volume 102, October 28, 1933, pages 556–60.

8. Combined Ground-Bus and Pilot-Wire Protection, **L. N. Moore.** *Electrical World*, volume 102, November 25, 1933, pages 693–4.

9. D-C Pilot-Wire Loop Protects 66-Kv Cable Circuits, **J. H. Neher.** *Electrical World*, volume 101, March 25, 1933, pages 384–7.

10. Use of Communication Facilities in Transmission-Line Relaying, **J. H. Neher.** AIEE Transactions, volume 52, 1933, pages 595–602.

1934

11. For Better Use of Investment in Coupling Capacitors, **F. B. Doolittle.** *Electrical World*, volume 103, number 7, February 17, 1934, pages 256–7. For carrier-current relay protection.

12. Carrier Control Isolates Trouble on Tap Lines, *Electrical World*, volume 103, March 31, 1934, page 469.

13. Carrier-Current Pilot Protection (in German), **H. Neugebauer.** *Siemens Zeitschrift*, volume 14, number 3, March 1934, pages 83–8.

1935

14. Carrier-Current—One-Cycle Carrier Relaying Accomplished, **P. Sporn** and **C. A. Muller.** *Electrical World*, volume 105, number 21, October 12, 1935, pages 26–8. Notes on new developments, etc.

15. Carrier-Current Relay Installation, **O. A. Browne** and **W. L. Vest, Jr.** AIEE Transactions, volume 54, 1935 (January section), pages 109–15; discussion (March section), pages 333–4; (May section), pages 558–9.

16. Pilot-Wire Relay Protection, **E. E. George** and **W. R. Brownlee.** AIEE Transactions, volume 54, 1935 (November section), pages 1262–9.

1936

17. Faster Carrier-Pilot Relay System, **O. C. Traver** and **E. H. Bancker.** AIEE Transactions, volume 55, 1936 (June section), pages 688–96; discussion (November section), page 1252–4.

18. New Pilot Protection Gives Reliability, **Lloyd F. Hunt.** *Electrical World*, volume 106, September 26, 1936, page 3007. Simple lockout system uses existing telephone wires instead of carrier current for pilot-protection high-speed relay system.

19. Neutralizing Transformer to Protect Power-Station Communication, **E. E. George, R. K. Honaman, L. L. Lockrow,** and **E. L. Schwartz.** AIEE Transactions, volume 55, 1936 (May section), pages 524–9.

20. Uses of Carrier Current, **M. E. Reagan.** *Electric Journal*, volume 33, number 6, June 1936, page 276.

1937

21. Recent Progress in Carrier-Current Relaying (in French), **F. Cahen.** Société Française des Electriciens *Bulletin*, volume 7, number 79, July 1937, pages 725–65.

1938

22. Application and Performance of Carrier-Current Relaying, **P. Sporn** and **C. A. Muller.** AIEE Transactions, volume 57, 1938 (March section), pages 118–24; discussion (May section), pages 281–4.

23. Carrier Current on Power Systems, **O. Richardson.** *Communications*, July 1938, pages 19–22; *Electrical News and Engineering* (Canada), September 15, 1938, pages 35–8. Coupling methods; types of equipment; carrier-pilot relaying; variable carrier-frequency system.

24. Carrier-Current Protection, *Electrician* (London), volume 121, July 1, 1938, page 17; July 8, 1938, page 43; July 15, 1938, page 71; July 22, 1938, page 102; July 29, 1938, page 131.

25. Carrier Relaying Cuts Line Outages, *Electrical World*, volume 110, July 2, 1938, page 64.

26. Directional-Current Comparative Protection (in

German), **F. Jahn** and **H. Gutmann.** *Elektrizitaetswirtschaft,* volume 37, number 5, February 15, 1938, pages 113–16.

27. A New High-Speed Distance-Type Carrier-Pilot Relay System, **E. L. Harder, B. E. Lenehan,** and **S. L. Goldsborough.** AIEE Transactions, volume 57, 1938 (January section), pages 5–10. Discussion (May section), pages 291–4

28. Pilot-Wire Relaying Gives Tie-Line Protection, **T. T Smith.** *Electrical World,* volume 109, number 1, Jauary 1, 1938, pages 39 and 90.

29. Selective Protection of Transmission Lines by Carrier Current (in French), **F. Cahen.** *Electricité* (Paris), volume 22, number 47–48, July–August 1938, pages 251–4.

30. Single-Element Differential Pilot-Wire Relay System, **E. L. Harder** and **M. A. Bostwick.** *Electric Journal,* volume 35, number 11, November 1938, pages 443–8.

1939

31. Boulder Dam Transmission Line; Carrier-Current Equipment, **J. D. Laughlin.** AIEE Transactions, volume 58, 1939 (April section), pages 147–51.

Section C

Line Protection—Ground Faults

1931

1. Relay Protection on High-Resistance Grounded Systems (in French), **T. Wajima.** International Conference on Large High-Tension Electric Systems *Report* of sixth session, June 1931, volume 3, pages 733–42. Report of sensitive ground relaying in Japan.

1932

2. The Permissible Sensitivity of Ground-Current Relays in High-Voltage Systems (in German), **E. Gross** and **W. Weller.** *Electrotechnik und Maschinenbau,* volume 50, February 1932, pages 117–23.

1933

3. Ground Protection of Rural Lines, **J. W. Graff.** *Electrical World,* volume 102, December 1933, pages 787–9.

4. Zero-Sequence Current Transformers for Ground Protection, **J. W. Graff.** *Electrical World,* volume 102, December 1933, pages 822–3.

1934

5. Earth-Leakage Protection at Daggafontein Mines, **N. H. Auret.** *Electrician* (London), volume 113, August 1934, pages 157–8. Installation and operating results.

6. New Methods of Ground-Fault Protection, **F. M. Starr.** AIEE Transactions, volume 53, 1934 (November section), pages 1472–7.

7. Simple Ground Relaying Trips Rural Feeders, **J. F. Pennington** and **J. T. Logan.** *Electrical World,* volume 104, July 1934, page 111.

1935

8. Earth-Fault Problems in Large Cable Networks, **E. Schultze.** *AEG Progress,* volume 1, 1935, pages 11–15. Use of Petersen coil in Berlin system.

9. General Electric Ground-Detector Relay, *General Electric Review,* volume 38, March 1935, page 154.

10. Leakage Relays—the Performance of Current Transformers, **G. W. Stubbings.** *Electrical Review* (London), volume 117, October 1935, pages 564–5.

11. Neutral Power Relay Clears Faulted Cable, *Electrical World,* volume 105, August 1935, page 1926.

12. Neutral Relay, **H. G. Bullen.** *Electrical Review* (London), volume 117, September 1935, page 366.

1936

13. New Distance Ground Relay, **S. L. Goldsborough** and **R. M. Smith.** AIEE Transactions, volume 55, 1936 (June section), pages 697–703; discussion, volume 55, 1937 (November section), pages 1255–6.

1937

14. Characteristics of Power-System Faults to Ground, Edison Electric Institute *Bulletin,* volume 5, May 1937, pages 197–8.

15. Ground Protection in the Vienna High-Voltage System (in German), **E. Lippa.** *Electrotechnik und Maschinenbau,* volume 55, May 1937, pages 233–8 and 249–52.

16. Ground Relaying Economical, **C. B. Carpenter.** *Electrical World,* volume 107, February 1937, page 541.

17. Leakage-Trip Protection; Operation and Use for Various Purposes, *Electrical Review* (London), volume 120, April 1937, pages 652–3.

18. Wrong Ground-Relay Connections Caught, **G. W. Stubbings.** *Electrical World,* volume 107, June 1937, page 2198.

1938

19. Double Short-Circuit Protection Against Ground (in German), **A. Thewalt.** *Siemens Zeitschrift,* volume 18, July 1938, pages 355–61.

1939

20. Directional-Overcurrent Ground Relaying; Use of Negative-Sequence Filters, **B. V. Hoard** and **S. C. Leyland.** *Electric Journal,* volume 36, March 1939, pages 97–101.

21. Earth-Leakage Relays; With Use of Ring-Type Current Transformers, **E. T. B. Gross.** *Electrician* (London), volume 123, July 1939, pages 93–5.

22. Feeder Protection; Vagaries of Earth-Leakage System, **J. Gray.** *Electrical Review* (London), volume 125, July 1939, pages 125–42.

23. Ground-Fault Protection in a Modern Plant, **A. F. Hartung.** *Electrical World,* volume 112, November 1939, page 1295.

24. Induced Current in Parallel Circuits and Its Effect Upon Relays, **E. H. Bancker.** AIEE Transactions, volume 58, 1939 (November section), pages 582–5.

Section D

Line Protection—General

1931

1. Feeder Protection; the Pilotless Interlock System, *Electrical Review* (London), volume 109, November 20, 1931, page 771.

2. The Protection of Overhead Lines and Feeders (in French), **H. W. Clothier.** International Conference on Large

High-Voltage Electric Systems *Report* of sixth session, 1931, volume 3, pages 901–04. Protection of 132-Kv British Grid.

3. Relay Protection for a Closed Linked System, **T. G. LeClair.** Electrical Engineering, volume 50, July 1931, pages 471–6.

4. Selective Protection for Long High-Voltage Lines (in German), **R. Schimph.** Verband Deutscher Elektrotechniker *Fachberichte*, 1931, pages 86–8.

1932

5. Application of High-Speed Relays to 66- and 132-Kv System of Union Electric Light and Power Company (abstract), **G. W. Gerell.** Electrical Engineering, volume 51, June 1932, page 411.

6. Faults and Their Clearance on Large Networks, **O. Kapp** and **C. G. Carrothers.** Institution of Electrical Engineers (London) *Journal*, volume 71, number 341, November 1932, pages 685–702.

7. Ground and Distance Relays, the Trend in Practice, *Electrical World*, volume 99, number 9, February 27, 1932, pages 398–402.

8. High-Speed Relays, **L. F. Hunt.** *Electrical West*, volume 68, number 6, May 15, 1932, pages 339–41. Report of a subcommittee of electrical apparatus committee of Pacific Coast Electric Association on two high-speed relays in use on member companies' lines.

9. Line Protection, **A. Heydorn.** South African Institute of Electrical Engineers *Transactions*, volume 23, part 3, March 1932, pages 57–63. Outline of methods.

10. New Overcurrent Relay, **S. L. Goldsborough** and **R. M. Smith.** *Electric Journal*, volume 29, number 2, February 1932, pages 78–80. Development of relay with straight line characteristic.

11. Power Directional-Relay Applications, **J. P. Price.** *Electrical West*, volume 68, number 1, January 1, 1932, pages 7–8. Article on connections of directional relays.

12. Protection and Centrovisory Control of the British Grid, **B. H. Leeson.** *Electrical Review* (London), volume 110, April 22, 1932, page 598.

13. Protective Devices, Pacific Coast Electric Association Overhead Systems Committee. *Electrical West*, volume 68, number 6, May 15, 1932, pages 431–8.

14. Relay Scheme for Preferential Feeder Control, **C. W. Jones.** *Electrical World*, volume 99, number 8, February 20, 1932, pages 375–6.

15. Relaying in Chicago Region, **R. C. Erickson.** *Electric Light and Power*, volume 10, number 11, November 1932, pages 10–12. Protection used on "red", "blue", and "green" networks feeding the Chicago area.

1933

16. Directional Relays, **G. W. Stubbings.** *Electrical Review* (London) volume 113, October 27, 1933, page 586.

17. Relaying of High-Voltage Interconnections, **H. P. Sleeper.** Electrical Engineering, volume 52, June 1933, pages 402–09.

18. Ring Main Feeder Protection, *Engineer* (London), volume 155, February 3, 1933, pages 122–3.

1934

19. Automatic Protective Gear for A-C Supply Systems (book), **J. Henderson.** Pitman Publishing Company, New York City, 1934, 201 pages. Practical experiences of author in transmission-line protection in Great Britain.

20. Modern Protective Gear, **C. C. Garrard.** *Electrical Review* (London), volume 115, number 2971, November 2, 1934, page 598. Elimination of time elements discussed; opposing requirements; carrier-current communication; calibration curves of distance relays for overhead line protection.

21. The Regulation of Time-Delay Relays in Electric Distribution Networks (in French), **A. DuBois.** *Revue Générale de l'Electricité*, volume 35, number 26, June 30, 1934, pages 883–6.

22. Selective Protection of Networks Against Accidental Faults (in French), **R. V. Genkin.** *Industrie Electrique*, volume 43, number 999, February 10, 1934, pages 57–8.

23. Some Recent Relay Developments, **L. F. Hunt** and **A. A. Kroneberg.** AIEE Transactions, volume 53, 1934 (April section), pages 530–5. Southern California Edison system; high-resistance faults; carrier; out-of-step conditions.

24. Speedier Switching Isolates Faults on 110-Kv System, **W. M. Edson** and **R. E. Walsh.** *Electrical World*, volume 104, number 6, August 11, 1934, pages 170–2.

25. Transfer Feeder Faults to One Breaker, **L. F. Woodruff** and **J. E. Mulligan.** *Electrical World*, volume 103, number 13, March 31, 1934, pages 464–5. Fault transfer switch.

1935

26. Engineering Features of Boulder Dam—Los Angeles Lines, **E. F. Scattergood.** AIEE Transactions, volume 54, 1935 (May section), pages 494–512.

27. Fault and Out-of-Step Protection of Lines, **H. D. Braley** and **J. L. Harvey.** AIEE Transactions, volume 54, 1935 (February section), pages 189–200; discussion (February section), pages 197–9. Stability in relation to short circuit protection requirements.

28. Protection of Lines in Case of Fault and Falling Out of Step of Power Plants; Various Examples From German and American Experience (in German), **F. Cornelsen.** *Elektrotechnische Zeitschrift*, volume 56, number 35, August 29, 1935, pages 963–6.

29. Relays for Protecting A-C Circuits, *Engineer* (London), volume 160, number 4150, July 26, 1935, pages 88–9. Illustrated description of series of new relays capable of meeting various requirements, produced by Oerlikon Company.

1936

30. High-Speed Balanced-Current Relays for Parallel Lines, **S. C. Leyland.** *Electric Journal*, volume 33, number 88, August 1936, page 347.

31. Statistical Survey of Performance of Automatic Protective Systems, Institution of Electrical Engineers (London) *Journal*, April 22, 1936. Survey of systems in Great Britain for five years ending in 1933.

32. Transmission and Distribution, **C. F. Bolton** and **R. H. Abell.** Institution of Electrical Engineers (London) *Journal*, April 1936. A review of progress.

1937

33. High-Speed Relays Improve 100-Kv Loop, **H. A. Burt.** *Electrical West*, volume 79, September 1937, pages 68–9.

34. Single-Element Polyphase Directional Relay, **A. J. McConnell.** AIEE Transactions, volume 56, 1937 (January section), pages 77–80; discussion (August section), pages 1025–8.

35. Some Comments on Transversal Differential Protection of Double Lines on Systems With Earthed Neutral, **J. Schneider.** *Brown Boveri Review*, volume 24, number 9, September 1937, pages 245–8.

1938

36. Arc Resistance and Time Staggering of Relays According to Resistance Characteristics for Protective Systems of Electric Transmission Lines (in German), **H.**

Neugebauer. *Elektrizitaetswirtschaft,* volume 37, number 15, May 25, 1938, pages 396–9.

37. Curve Shapes of Graded Selective Protection for Electric Lines; Analysis of Relay-Time Characteristics (in German), **H. Neugebauer.** *Siemens Zeitschrift,* volume 18, number 1, January 1938, pages 25–30.

38. Differential Protection Along System With Impedance-Time Back-Up for Large Cable Networks (in German), **F. Geise.** *Siemens Zeitschrift,* volume 18, number 9, September 1938, pages 446–50.

39. Protective Devices in High- and Medium-Voltage Overhead Lines (in German), **K. Kuehn.** *Elektrizitaetswirtschaft,* volume 37, January 12, 1938, pages 27–32; February 15, 1938, pages 107–13.

1939

40. Boulder Dam Transmission Line; Transmission-Line Relay Protection, **L. L. Draper.** AIEE Transactions, volume 58, 1939 (April section), pages 151–6.

Section E

Bus Protection

1929

1. The Fault Ground Bus, **R. M. Stanley** and **F. C. Hornibrook.** AIEE Transactions, volume 49, 1930, pages 201–12.

1935

2. Harmonic-Current-Restrained Relays for Differential Protection, **L. F. Kennedy** and **C. D. Hayward.** AIEE Transactions, volume 57, 1935 (May section), pages 262–71.

3. Lift-Control Gear Protection Against Earth Faults, **A. Read.** *Electrical Review* (London), volume 116, April 5, 1935, pages 480–1.

1936

4. Bus Protection Gives Service Reliability, **Philip Sporn.** *Electrical World,* volume 106, May 9, 1936, page 1357.

5. Bus-Bar Zone Protection, **T. W. Ross.** *Electrical Review* (London), volume 119, July 31, 1936.

1938

6. Bus-Zone Protection, *Electrical Review* (London), volume 122, May 27, 1938, page 775.

7. Reyrolle Bus-Bar Zone Protection Systems, *Engineer* (London), volume 146, September 16, 1938, pages 329–30.

1939

8. Bus Protection, AIEE Transactions, volume 58, 1939 (May section), pages 206–11. A report prepared under the auspices of the relay subcommittee of the AIEE protective devices committee and the relay subject committee of the EEI electrical equipment committee.

9. Considerations in Applying Ratio Differential Relays for Bus Protection, **R. M. Smith, W. K. Sonnemann** and **G. B. Dodds.** AIEE Transactions, volume 58, 1939 (June section), pages 243–9.

Section F

Apparatus Protection

1931

1. Percentage-Differential Transformer Protection, **R. E. Cordray.** Electrical Engineering, volume 50, May 1931, pages 361–3.

1932

2. Thermal Cutouts for Protecting Alternating-Current Motors, **H. D. Braley.** *Power,* volumes 74 and 75, November 17, 1931, pages 703–05; January 12, 1932, pages 66–7.

3. Balanced-Current Protection for Alternators, **G. W. Stubbings.** *Electrical Times* (London), volume 82, number 2128, August 4, 1932, pages 140 and 149.

4. Differential Relays Operate Abnormally Due to Different Lead Lengths, **S. Ein.** *Electrical World,* volume 100, number 25, December 17, 1932, page 827.

5. Guarding Against Loss of Field, **E. H. Bancker.** *Electrical World,* volume 99, number 23, June 4, 1932, page 983.

6. Polyphase Motor Can Be Protected Against Single-Phase Operation (symposium, six contributors). *Power,* volume 75, May 3, 1932, page 668.

7. Protective Gear for Alternators, **G. W. Stubbings.** *Electrical Engineer and Merchandiser,* volume 9, number 1, April 15, 1932, pages 17–19.

8. Providing Protection for Three-Phase Motors, **R. B. Glen.** *Power,* volume 75, June 14, 1932, page 884.

9. Reyrolle Automatic Alternator Protection, *Electrical Review* (London), volume 111, September 16, 1932, page 392.

10. Use of Protective Relays on Alternating-Current Systems, **C. J. Monk.** South African Institute of Electrical Engineers *Transactions,* volume 23, part 3, March 1932, pages 47–57.

1933

11. Inductor Time-Delay Device: Prevents Circuit Breakers From Interrupting Service on Momentary Short Circuits or Voltage Dips, **W. W. Brainard** and **G. D. Gamel.** *Electric Journal,* volume 30, November 1933, pages 472–3.

12. Machines Stopped Automatically by Electronic Time-Delay Relay, *Electrical World,* volume 101, May 20, 1933, page 643.

13. New Developments in Synchronous Motor Control, **M. N. Halberg.** *General Electric Review,* volume 36, May 1933, pages 244–9.

14. Protecting Squirrel-Cage Winding of Synchronous Motors, **M. N. Halberg.** *Electrical World,* volume 102, November 11, 1933, page 627.

15. Recommended Practices for the Protection of Electrical Apparatus, AIEE Transactions, volume 52, 1933, pages 607–13. A report prepared under the auspices of the relay subcommittee of the AIEE protective devices committee.

16. Selecting Thermal Relays to Protect Three-Phase Motors, **H. Wilson.** *Power,* volume 77, February 1933, pages 78–80.

17. Synchronous-Motor Excitation Controlled by New Relays, **N. L. Hadley.** *Electrical World,* volume 101, April 29, 1933, pages 550–1.

18. Temperature Protection With Detector Coils, **H. E. Pearson.** *General Electric Review,* volume 36, February 1933, pages 109–10.

1934

19. Differential Float Relay Watches Cable Oil, **R. Banz.** *Electrical World*, volume 103, June 30, 1934, pages 946–7.

20. New Ground Detector Improves Field Protection, **A. Kroneberg** and **R. B. Pollock.** *Electrical World*, volume 104, August 4, 1934, pages 149–50.

21. New-Type Independent Overcurrent Time Relay (in German), **S. John.** *Siemens Zeitschrift*, volume 14, number 4, April 1934, pages 134–6. Current time relay for protection of motors and transformers.

22. Power-Transformer Protection, **G. W. Stubbings.** *Electrical Review* (London), volume 114, number 2939, March 23, 1934, page 406. Differential protection, zero sequence components.

23. Protective Relays on Pennsylvania Railroad Locomotives, **B. Luther.** AIEE Transactions, volume 53, 1934 (February section), pages 261–4.

24. Thermoelectric Protection for Intermittently Operating Motors (in German), **J. Gewecke.** *Siemens Zeitschrift*, volume 14, number 9, September 1934, pages 317–24.

25. Transformer Protection, **W. Woodiwiss.** *Electrical Review* (London), volume 114, number 2941, April 6, 1934, pages 481–2. Merz-Price system.

1935

26. Automatic Relays Assure Protection for Generators, Transformers, **W. A. Holland.** *Southern Power Journal*, volume 53, number 2, February 1935, pages 47–9.

27. Differential Protection of Transformers (in French), **C. Varicnon.** *Revue Générale de l'Electricité*, volume 37, number 24, June 15, 1935, pages 771–81.

28. General Problem of Adequately Protecting Transformers (in German), **H. Weissmann.** *Siemens Zeitschrift*, volume 15, number 3, March 1935, pages 67–74.

29. Generator Protection for Radial Turbines With Nozzles and Blades Rotating in Opposite Directions (in German), **R. Schimpf.** *Siemens Zeitschrift*, volume 15, number 4, April 1935, pages 128–31.

30. High-Speed Relays to Protect Mercury-Arc Rectifiers, *Mass Transportation*, volume 31, December 1935, page 424.

31. Importance of General Design of Protective Systems for Electrical Machinery, **E. Bopp.** *Siemens Zeitschrift*, volume 15, number 9, April 1935, pages 105–13.

32. Thermal Protection is Provided for All Electrical Apparatus in New Supreme Court Building, **F. G. Graf.** *Electric Journal*, volume 32, May 1935, pages 198–9.

1936

33. Boulder Canyon Project; Generator and Transformer Relaying, *Electric Journal*, volume 33, November 1936, page 492.

34. Generator Protection (in German), **E. Bopp.** *Archiv fuer Waermewirtschaft und Dampfkesselwesen*, volume 17, August 8, 1936, page 219.

35. The New Siemens-Buchholz Relay (in German), **H. Prinz.** *Siemens Zeitschrift*, volume 16, August 8, 1936, page 298.

36. Overload Relays for Motor Protection, **N. D. Beuhling.** *Electrical Manufacturing*, volume 17, April 1936, page 34.

37. Progress in Construction of Stabilized Differential-Protective Systems (in German), **F. Geise.** *Siemens Zeitschrift*, volume 16, August 1936, page 283.

38. The Protection of Generators (in French), **R. Dubusc.** *Technique Moderne*, volume 28, September 1, 1936, page 601; October 15, 1936, page 711.

39. The Protection of Transformers (in French), **A. Bogatenkoff-Stavraky.** Société Française des Electriciens *Bulletin*, volume 6, number 77, December 1936, pages 1171–4.

40. Transformer Protective Relay, *Engineer* (London), volume 162, November 6, 1936, page 497. Gives same type protection as Buchholz relay.

41. Transformer Protection, *Electrical Review* (London), volume 119, November 20, 1936, page 707.

42. Winding Short-Circuit Protection for Generators (in German), **E. Bopp.** *Siemens Zeitschrift*, volume 16, number 12, December 1936, pages 492–6.

1937

43. Alternating-Current Motor Protection, **C. W. Kuhn.** AIEE Transactions, volume 56, 1937 (May section), pages 589–93.

44. Direct-Acting Differential and Overcurrent Relays (in French), **J. Fallou.** *Technique Moderne*, volume 29, numbers 6 and 9, March 15, 1937, pages 184–8; May 1, 1937, pages 289–92.

45. Fundamentals of Protecting Three-Phase Generators (in German), **E. Bopp.** *Elektrizitaet im Bergbau*, volume 12, number 1, February 1937, pages 8–16.

46. New Out-of-Step Relay, Canadian General Electric Company, *Canadian Mining Journal*, volume 58, March 1937, pages 160–1.

47. Protecting Frequency Changers From Out-of-Step Conditions, **L. N. Crichton.** *Electric Journal*, volume 34, February 1937, pages 74–6.

48. Simplified Protection of Grounding Transformers, **G. Calabrese.** *Electrical World*, volume 107, April 24, 1937, page 1393.

49. Scott Two-Three-Phase Banks, Differential Relays, **V. P. Brodsky.** *Electrical World*, volume 107, May 8, 1937, page 1590.

50. Rectifier Utilizes Novel Thermal Relay, **C. R. Machen.** *Electrical World*, volume 108, July 3, 1937, page 70.

51. Transformer Protective Relay, *Electrical World*, volume 107, January 30, 1937, page 436.

52. Undercurrent Relays Protect Centrifugal Pumps, **T. M. Googin.** *Coal Age*, volume 42, September 1937, page 76.

1938

53. Differential Protection, Compensated and Electronic Relays, **A. Marit.** Association Suisse des Electriciens *Bulletin*, volume 29, number 18, September 2, 1938, pages 496–509.

54. Gas-Pressure Relays, **R. A. Hentz.** Edison Electric Institute *Bulletin*, volume 6, March 1938, pages 82–4. Final report on analysis of replies from foreign correspondent to the electrical equipment committee, EEI.

55. Gas-Pressure Relays Protect Transformer, *Electrical World*, volume 109, March 26, 1938, page 1068.

56. Highly Sensitive Frame Protection for Generators Working Directly Into Network System (in German), **E. Bopp.** *Siemens Zeitschrift*, volume 18, number 2, February 1938, pages 53–8.

57. Protect Generator From Internal Faults, **G. Calabrese.** *Electrical World*, volume 109, May 21, 1938, page 1704 correction, volume 110, August 13, 1938, page 435.

58. Reyrolle and Company-Station Protection Systems, *Engineer* (London), volume 166, July 8, 1938, pages 34–5.

59. Thermal Image of Electric Machines as Basis for Development of Electric Overload Relays (in French), **J. Stoesser** and **E. Bernhardt.** Association Suisse des Electriciens *Bulletin*, volume 29, number 12, June 10, 1938, pages 290–4.

60. Transformer Protection by Metrovick, Buchholz Devices, **C. Ryder.** *Metropolitan-Vickers Gazette*, volume 17, number 300, April 1938, pages 236–8.

1939

61. Machine Protection and the Practice of Grounding Neutrals, **J. H. Vivian.** *Electrical West*, volume 83, September 1939, pages 23–7.

62. Relay Protection for a Large Regulating Transformer, **W. E. Martin.** *Electric Journal*, volume 36, March 1939, pages 87–8.

Section G

Distribution and Network Protection

1926

1. A-C Network Characteristics, **D. K. Blake.** AIEE Transactions, volume 45, 1926, pages 1212–20.

2. Evolution of the Automatic-Network Relay, **J. S. Parsons.** AIEE Transactions, volume 45, 1926, pages 1195–1202.

3. Operating Requirements of the Automatic-Network Relay, **W. R. Bullard.** AIEE Transactions, volume 45, 1926, pages 1203–11.

1929

4. An A-C Low Voltage Network Without Network Protectors, **L. R. Gamble** and **E. Baughan.** AIEE Transactions, volume 49, 1930, pages 82–92.

1932

5. Relays Insure Service in Primary Networks, **L. F. Kennedy.** *General Electric Review*, volume 35, number 6, June 1932, pages 338–43.

1933

6. Co-ordinating Fuses and Relays for Distribution Reliability, **G. F. Lincks.** *Electrical World*, volume 102, July 29, 1933, pages 140–3.

7. Sequence Principles Used for Network Relaying, **H. R. Searing** and **R. E. Powers.** AIEE Transactions, volume 52, 1933, pages 614–20.

8. Sequence Relay for Network Protectors, **H. S. Orcutt** and **M. A. Bostwick.** AIEE Transactions, volume 52, 1933, pages 620–6.

1934

9. Network Protector Averts Pumping, *Electrical World*, volume 104, October 13, 1934, page 658.

10. Network-Protector Operations Reduced by Improved Control, **J. S. Parsons** and **M. A. Bostwick.** *Electrical World*, volume 103, number 14, April 7, 1934, pages 508–10.

11. New Type of Network Protector, **W. R. Bullard.** Edison Electric Institute *Bulletin*, volume 2, number 12, December 1934, pages 403–05.

12. Positive Disconnection of Distribution Circuits, Edison Electric Institute *Bulletin*, volume 2, number 7, July 1934, page 254.

13. Theory of Primary Networks, **F. M. Starr.** AIEE Transactions, volume 53, 1934 (March section), pages 426–31.

14. Use of Fuses in Distribution Network Systems (in German), **P. Wittich.** *Elektrotechnik und Maschinenbau*, volume 52, number 51, December 23, 1934, pages 602–03.

1935

15. New Design of Network Protector Based on Operating Experience, **W. J. McLachlan.** *General Electric Review*, volume 38, number 5, May 1935, pages 244–7.

16. Notch Relay Clears Distribution Faults, **G. L. Kenny, Jr.** *Electrical World*, volume 105, May 11, 1935, page 1216.

1937

17. Fuse-Breaker Correlation Fixed by Fault Ratio, **P. E. Benner.** *Electrical World*, volume 108, number 25, December 18, 1937, pages 53–4 and 115.

18. Co-ordination of Fuse Links; Method of Comparing Fuse Links and Relays for Sectionalization and Protection, **E. M. Adkins.** *Electrical World*, volume 107, March 13, 1937, page 877.

19. Difficulties in Co-ordinating Fusing, **J. I. Holbeck.** *Electrical World*, volume 108, November 6, 1937, page 1562.

20. New Network Relays, **J. M. Parsons** and **M. A. Bostwick.** *Electric Journal*, volume 34, July 1937, pages 288–93.

21. Time-Current Curves for High-Voltage Fuses, **E. A. Williams.** *Electrical World*, volume 107, January 30, 1937, pages 398–401.

22. Trends in Distribution Overcurrent Protection, **P. E. Benner** and **G. F. Lincks.** AIEE Transactions, volume 56, 1937 (January section), pages 138–52.

1938

23. Remote Control of Network Protectors, **W. R. Brownlee** and **W. E. Dent, Jr.** AIEE Transactions, volume 57, 1938 (December section), pages 685–92.

24. Rural Network Protection; Comparison With German Methods, **T. C. Gilbert.** *Electrical Review* (London), volume 112, October 14, 1938, pages 539–40.

25. Sectionalized Feeders Meet Exacting Network Requirements, *Electric Light and Power*, volume 16, number 9, September 1938, pages 50–2. Pilot-wire control of primary circuit breakers and copper-link fuses in transformer secondary leads.

Section H

Service Restoration

1932

1. Service Continuity Promoted by Rapid-Breaker Reclosure, **R. Treat** and **H. C. Verwoert.** *Electrical World*, volume 100, number 6, August 6, 1932, pages 172–5.

1934

2. Automatic Reclosing of Oil Circuit Breakers, **A. E. Anderson.** AIEE Transactions, volume 53, 1934 (January section), pages 48–53; discussion (April section), page 614; also see *Power Plant Engineering*, volume 38, number 4, April 1934, pages 192–4. Results of studies made by several companies.

3. Fast Reclosing Service at Lower Cost, **W. L. Garlington.** *Electrical World*, volume 103, number 6, February 10, 1934, pages 226–7.

4. Faster Reclosing Breakers Needed, **J. T. Logan.** *Electrical World*, volume 103, number 16, April 21, 1934, pages 571–2.

5. Mercury Reclosing Device for Automatically Restoring Circuit Breakers, **A. E. Anderson.** *General Electric Review*, volume 37, number 10, October 1934, pages 462–7.

6. Reclosing Devices for Circuit Breakers Equipped for Automatic Tripping, **E. Kronauer.** *Brown Boveri Review*, volume 21, number 11, November 1934, pages 191–4.

1935

7. Immediate Automatic Reclosing of Circuit Breakers, **W. M. Emmons.** *Electric Journal*, volume 32, March 1935, pages 114–18.

8. Immediate Reclosure of 110-Kv Lines, **J. T. Johnson** and **J. W. Graff.** *Electrical World*, volume 105, August 31, 1935, pages 2101–03.

1936

9. Carrier Relaying and Rapid Reclosing at 110-Kv, **R. E. Pierce, R. E. Powers, E. C. Stewart** and **G. E. Heberlein.** AIEE Transactions, volume 55, 1936 (October section), pages 1120–29.

1937

10. One-Shot Reclosures Improve Service, Stop Complaints, **T. H. Kinnear** and **L. R. Patterson.** *Electrical West*, volume 79, September 1937, pages 63–4.

11. Ultrahigh-Speed Reclosing of High-Voltage Transmission Lines, **P. Sporn** and **D. C. Prince.** AIEE Transactions, volume 56, 1937 (January section), pages 81–90.

1938

12. What Time Delay for Automatic Reclosure? **G. F. Lincks.** *Electrical World*, volume 110, December 31, 1938, pages 1822–3.

Section I

General and Miscellaneous Relaying

1926

1. Relay Handbook. NELA *Publication*, number 25–3, 1926.

1929

2. Protective-Relay Installation at Toronto-Leaside Transformer Station of Hydro-Electric Power Commission of Ontario, **E. M. Wood.** Engineering Institute of Canada *Transactions*, volume 36, February 1929, pages 141–51.

1930

3. Transmission System Relay Protection III, **W. W. Edson.** AIEE Transactions, volume 49, 1930, pages 1213–24.

1931

4. Effect of Wave Form on Operation of Induction Relays (abstract), **P. H. Robinson.** Electrical Engineering, volume 50, November 1931, page 900.

5. Nonlinear Circuits for Relay Applications, **C. G. Suits.** Electrical Engineering, volume 50, December 1931, pages 963–5.

6. Relay Handbook Supplement, NELA *Publication*, number 142A, 1931.

1932

7. Adequate System Protection, **R. C. Denny.** *Electrical West*, volume 68, number 6, May 15, 1932, pages 334–6. Report of electrical apparatus committee of Pacific Coast Electric Association.

8. Automatic Sectionalizing Equipment, **H. S. Littlewood.** *Electric Journal*, volume 29, number 5, May 1932, pages 230–2. Equipment for isolating trouble in mines or on railways.

9. Direct-Current Protective Gear, **W. Wilson.** *World Power*, volume 18, number 106, October 1932, pages 237–41.

10. Instrument Transformers and Protective Relays (book, in French), **C. Bresson.** Dunod, Paris, 1932.

11. Metal-Clad Switchgear, Automatic Protection, and Remote Control, With Particular Reference to Developments During the Last Seven Years, **H. W. Clothier.** Institution of Electrical Engineers (London) *Journal*, volume 71, August 1932, pages 285–330.

12. Overcurrent Protection With Outage Indicator (in French), **V. De Montaigue.** *Electricien* (Paris), volume 63, number 1540, May 15, 1932, pages 224–6.

13. Overvoltages on Transmission Systems Caused by Dropping Load, **E. J. Burnham.** AIEE Transactions, volume 51, 1932, pages 105–12.

14. Protective Apparatus as Applied to System of Sydney Municipal Council, **F. B. Haugaard.** Institution of Engineers, Australia, *Journal*, volume 4, number 6, June 1932, pages 189–203.

15. Protective Engineering, *Electrical West*, volume 68, number 6, May 15, 1932, pages 331–4. Pacific Coast Electrical Association apparatus committee report on causes of faults system set up to minimize faults, and high-speed relays.

16. Protective Relay Equipment in Los Angeles, **E. G. Russell.** *Electric Light and Power*, volume 10, number 9, September 1932, pages 14–17.

17. Relay Time Lags, **G. W. Stubbings.** *Electrical Review* (London), volume 110, number 2845, June 3, 1932, page 809.

18. Some Recent Advances in Design of Relays for Protection of Alternating-Current Systems, **C. L. Lipman.** Institution of Electrical Engineers (London) *Journal*, volume 70, number 425, May 1932, pages 545–66.

19. Theory and Application of Relay Systems, **P. H. Robinson** and **I. T. Monseth.** *Electric Journal*, volume 29, numbers 2 and 3, February 1932, page 83; March 1932, page 143.

1933

20. Building a New Power System; Indianapolis Power and Light Company, **F. S. Douglass** and **A. C. Monteith.** *Electric Journal*, volume 30, February 1933, pages 55–9.

21. Circuit-Breaker Tripping, **R. L. de Loache.** *Electric Journal*, volume 30, August 1933, pages 329–32.

22. New Ideas on Protecting Service, *Electrical World*, volume 101, February 4, 1933, pages 161–3. Abstract of AIEE papers.

23. New Maximum and Reverse-Current Relay, **R. Edler.** *Electrical Review* (London), volume 113, October 20, 1933, page 558.

24. Nonlinear Circuits Applied to Relays, **C. G. Suits.** Electrical Engineering, volume 52, April 1933, pages 244–6.

25. One-Way Relay Contact, **J. C. Woods.** *Electrical World*, volume 102, December 30, 1933, page 860.

26. Relays and Breakers Improved, *Electrical World*, volume 102, July 8, 1933, pages 45–6. Discussion of AIEE convention papers.

27. Simple Relays Given Precision by Resonant Circuits, **C. G. Suits.** *Electrical World*, volume 101, January 28, 1933, page 140.

1934

28. Automatic Synchronizing Relay, *General Electric Review*, volume 37, November 1934, page 526.

29. G.E. Automatic Synchronizing Relay, *Power Plant Engineering*, volume 38, July 1934, pages 346–7.

30. B.T.H. Reverse Rotation Relay, *Engineer* (London), volume 158, October 19, 1934, pages 394–5.

31. Effect of Frequency Changes on Meters, Relays, Transformers, **J. D. Addleman.** *Electric Journal*, volume 31, December 1934, page 476.

32. Electronic Voltage Relay, **J. W. Graff.** *Electronics*, volume 7, number 3, March 1934, pages 80–1.

33. Engineering Achievements of 1933: Metering and Relaying, *Electric Journal*, volume 31, January 1934, pages 28–9.

34. Instruments and Relays; Effect of Discoveries of New Materials on Design, *Electrician* (London), volume 113, October 26, 1934, pages 530–1.

35. New Methods of Excess-Current Time Protection With Diagrams and Applications (in German), **M. Walter.** *Elektrotechnische Zeitschrift*, volume 55, number 9, March 1934, pages 206–08.

36. New Relays, *AEG Progress*, number 2, 1934, page 36.

37. Relays and High-Speed Disconnection (in German), **H. Neugebauer.** *Elektrotechnische Zeitschrift*, volume 55, number 8, February 22, 1934, pages 181–4.

38. Relays Hold Essential Load on Partial Power Failure, **A. P. Schnyder.** *Electrical World*, volume 103, June 2, 1934, page 807.

39. Simple Relay Scheme Avoids Higher Breaker Investment, **R. Banz.** *Electrical World*, volume 103, June 16, 1934, page 872.

40. Static Voltage Relay, **J. C. Costello.** *Electrical Review* (London), volume 115, number 2957, July 27, 1934, page 120. Uses of a tungsten lamp.

41. Thyratron Tubes in Relay Practice, **R. Wideröe.** AIEE Transactions, volume 53, 1934 (October section), pages 1347–53.

42. Three Ways to Insure Industrial Oil-Circuit-Breaker Adequacy, **R. I. Powers.** *Electrical World*, volume 103, June 23, 1934, page 920.

1935

43. Automatic Protection of A-C Circuits (book), **G. W. Stubbings.** Instruments Publishing Company, Pittsburgh, 1935, 293 pages. Refers primarily to networks and interconnected transformers.

44. Circuit Breaking and Protection, **F. H. Clough.** *Electrician* (London), volume 115, November 1, 1935, page 543.

45. Cuprox Discs as Neat Relays, **Sirokský** and **N. Teibel.** *Electrical Review* (London), volume 117, August 23, 1935, pages 238–9.

46. Factors in Making Electrical Power Supply Dependable, **R. Treat.** *General Electric Review*, volume 38, August 1935, pages 376–83.

47. High-Speed System Operation Obtaining Minimum Time Delay for Faults, **C. B. Padon.** *Electric Journal*, volume 32, March 1935, pages 112–13.

48. New Brown Boveri Series Relay for Severest Service Conditions, **J. Stoecklin.** *Brown Boveri Review*, volume 22, number 6, June 1935, pages 119–21. History and development of type *HB*-4 relay.

49. New Developments in Relays, *Electrical World*, volume 105, May 25, 1935, page 1321.

50. Operation of High-Speed Relays, **W. A. Lewis.** *Electric Journal*, volume 32, October 1935, page 446.

51. Overload Protective Systems, *Electrician* (London), volume 115, October 4 and 25, 1935, pages 417 and 473.

52. Recent Relay Developments, **I. C. Ryder.** *Metropolitan-Vickers Gazette*, volume 15, number 271, November 1935, pages 410–41. New design of regulating relay types *AVB* and *AVE* with low-volt protection and combined regulating and time relay.

53. Relays—Their Application in Distribution and Utilization of Electric Power, **W. A. Holland.** *Southern Power Journal*, volume 53, number 4, April 1935, pages 33–6. Notes on transmission lines and feeders; carrier-current relaying; radial *FDR* relays; automatic reclosing immediate initial reclosure; power plant motors; time delay for industrial motors; synchronous motor protection.

54. Relay Systems, Theory and Application (book), **I. T. Monseth** and **P. H. Robinson.** McGraw-Hill Publishing Company, 549 pages, 1935.

55. Sensitive A-C Vacuum-Tube Relay, **E. L. Ellis.** *Science*, volume 81, June 7, 1935, pages 568–9.

56. Short-Circuit Announcer With Indication of Faulted Phase, **F. Geise.** *Siemens Zeitschrift*, volume 15, number 10, October 1935, pages 493–6.

57. Substation Protection—a New Unbalanced-Phase Protective System, **C. J. O. Garrard.** *Electrician* (London), volume 114, number 2960, February 22, 1935, page 235.

58. Tubes on Power Systems, **P. Sporn** and **G. G. Langdon.** *Electrical World*, volume 105, number 3, February 2, 1935, pages 31–3. Thermionic devices as applied to synchronizing, carrier-current relaying and remote-switch control.

1936

59. Balanced Protective Systems, *Electrician* (London), volume 116, March 27, 1936, page 421.

60. Control of Short Circuits Cuts Costs, **H. C. Tice.** *Electrical World*, volume 106, December 5, 1936, page 3762.

61. Electrons Operate Directional Relay, **R. I. Ward.** *Electrical World*, volume 106, December 1936, page 3796.

62. Experiences With a Modern Relay System, **G. W. Gerell.** AIEE Transactions, volume 55, 1936 (October section), pages 1130–6; discussion volume 56, 1937 (March section), pages 369–70.

63. Metering—Relaying; 1935 Developments, *Electrical West*, volume 76, February 1936, pages 40–2.

64. Modernization of Relay Systems, **C. A. Muller** and **H. E. Turner.** AIEE Transactions, volume 55, 1936 (January section), pages 56–62; discussion (August section), page 922.

65. Planned Protective System, **A. C. Monteith** and **W. A. Lewis.** *Electrical World*, volume 106, November 21, 1936, pages 3618–20.

66. Recent Relay Developments, **C. Ryder.** *Metropolitan-Vickers Gazette*, volume 16, January 1936, page 23. Time-delay d-c relay and automatic-reclosing relays.

67. Relay Circuits Simplified in Four Novel Ways, **R. Rader.** *Electrical World*, volume 106, December 5, 1936, page 3757.

1937

68. Capacity-Stored Energy for Circuit-Breaker Tripping, **L. W. Dyer.** *Electric Journal*, volume 34, May 1937, pages 193–4.

69. Characteristics and Application of Modern Electrical Relaying, **E. M. Wood.** *Engineering Journal* (Canada), volume 20, numbers 5 and 12, May 1937, page 243; December 1937, page 860.

70. Fundamental Considerations in System Protection (in German), **M. Walter.** *Elektrizitaetswirtschaft*, volume 36, number 18, June 25, 1937, pages 415–17.

71. Henley-Sharborn Relay, *Electrician* (London), volume 118, May 21, 1937, pages 688–9.

72. Overload Relays—Types and Their Characteristics,

C. G. Armstrong. *Product Engineering*, volume 8, number 7, July 1937, pages 267–9.

73. Protective Equipment of the 30-Kv and 6-Kv Berlin System (in German), **E. Krohne.** *Elektrizitaetswirtschaft*, volume 36, number 18, June 25, 1937, pages 417–21.

74. Selective Protection of the 20-Kv and 15-Kv System of Baden (in German), **O. Kautzmann.** *Elektrizitaetswirtschaft*, volume 36, number 18, June 25, 1937, pages 421–4.

75. Tube-Licensing Situation Hampers Relaying Advance, **E. E. George.** *Electrical World*, volume 108, August 28, 1937, pages 684–7.

1938

76. Continuity of Power Supply for Steel Mills as Influenced by Relaying and System Design, **E. H. Bancker** and **R. H. Kaufmann.** *Iron and Steel Engineer*, volume 15, number 11, November 1938, pages 55–70.

77. Harmonics May Delay Relay Operation, **M. A. Faucett** and **C. A. Keener.** *Electrical World*, volume 109, April 9, 1938, page 1226.

78. High-Speed Protection as an Aid to Maintaining Electric Service Following System Short Circuits, **C. Ryder** and **T. W. Ross.** Institution of Electrical Engineers (London) *Journal*, volume 83, August 1938, page 228; December 1938, page 819.

79. Modern Induction-Type Overcurrent Relay, **C. Ryder.** *Metropolitan-Vickers Gazette*, volume 17, number 301, May 1938, page 260.

80. Negative Phase-Sequence Relays, **G. W. Stubbings.** *Power and Works Engineer*, volume 33, number 381, March 1938, pages 116–17.

81. Overload and Short-Circuit Protection of High-Voltage Systems by Aid of Primary Relays (in German), **F. Parschalk.** *Electrotechnik und Maschinenbau*, volume 56, number 28, July 10, 1938, pages 362–5.

82. Safeguard Against Interruptions of Supply, **H. W. Clothier, B. H. Leeson** and **H. Leyburn.** Institution of Electrical Engineers (London) *Journal*, volume 82, May 1938, pages 445–502.

83. Sharborn Improved Relay, **Sharborn.** *Engineer* (London).

84. Some Protective Advances on New England Power System, *Electrical World*, volume 109, number 23, June 8, 1938, pages 78–80. Improved relaying, switching and line layout.

85. Relay Contacts, Their Ailments, **A. W. Clement.** *Electronics*, volume 11, number 12, December 1938, pages 29 and 54. Practical suggestions as to control, shapes and inductive circuit hints.

86. Up-to-Date Protection for Yesterday's Circuits, **E. H. Bancker** and **E. H. Beckert.** *Electric Light and Power*, volume 16, number 6, June 1938, pages 76–9.

87. Why High-Speed Relays? **C. B. Padon.** *Electric Journal*, volume 35, November 1938, page 453.

1939

88. High-Speed Relaying Experience and Practice, AIEE Transactions, volume 58, 1939 (November section), pages 588–91. Report of the relay subcommittee of the AIEE committee on protective devices.

89. Pennsylvania Railroad—New York–Washington–Harrisburg Electrification; Relay Protection of Power-Supply System, **E. L. Harder.** AIEE Transactions, volume 58, 1939 (June section), pages 266–73.

90. Relay Protection of A-C Rotating Machines, Transformers, Buses, and Lines, **C. R. Mason.** *General Electric Review*, volume 42, October 1939, pages 441–9; November 1939, pages 488–96; December 1939, pages 529–39.

Section J

Testing and Analyzing

1932

1. Analyzing System Disturbances With Automatic Oscillograph, **G. A. Powell.** *Electrical World*, volume 99, April 1932, pages 732–6.

2. Checking Pilot Wires, **G. H. Schleigh, Jr.** *Electric Journal*, volume 29, February 1932, page 98.

3. Distance-Relay Tests Simplified, **H. H. Spencer.** *Electrical World*, volume 100, October 1932, page 572.

4. Loading Transformer for Relay Testing, **W. H. Talbott.** *Electrical West*, volume 68, May 1932, page 397.

5. Measurement of Spot Temperature in Relays, **W. H. Talbott.** *Electrical West*, volume 68, May 1932, page 397.

6. Testing of Control Equipment for Rendering Harmless Broken Overhead Conductors (in French), **P. Burdin.** *Electricien* (Paris), volume 63, May 1932, pages 195–8.

7. Testing Relays Under Sudden Short-Circuit Conditions on 220-Kv System, **G. D. Floyd.** *Electrical News and Engineering* (Canada), volume 41, April 1932, pages 36–7.

1933

8. Adjustable Ratio Transformer Aids Relay Testing, **J. W. Lingary.** *Electrical World*, volume 102, September 1933, pages 406–07.

9. Board Speeds Testing of Network Relays, *Electrical World*, volume 101, February 1933, pages 268–9.

10. Device for Testing Generator Cut-Out Relay, **F. Tipton.** *Transit Journal*, volume 77, August 1933, page 251.

11. System Performance Tested With 600 Short Circuits; Indianapolis Power and Light Company, *Electrical World*, volume 101, February 1933, pages 192–5.

12. Testing of High-Speed Distance Relays, **E. E. George.** AIEE Transactions, volume 52, 1933, pages 802–07; discussion page 825.

13. Timing High-Speed Relays, **J. H. Neher.** *Electrical World*, volume 102, December 1933, pages 734–5.

14. What the Tests on the Indianapolis System Show, **P. O. Langguth** and **R. M. Smith.** *Electric Journal*, volume 30, March 1933, pages 98–101.

1934

15. Compact Timing Device Facilitates Breaker Timing, **G. H. Wintersteen.** *Electrical World*, volume 104, August 1934, page 150.

16. Simple Tests for Maintaining Relay Reliability, **C. A. Muller.** *Electrical World*, volume 103, January 1934, page 145.

17. We Check All Major Faults, **E. L. McClure.** *Electrical World*, volume 104, July 1934, pages 82–3. Experience in Milwaukee.

1935

18. Relay Time Tests: Time-Interval Meter, *Electrical Review* (London), volume 117, July 1935, page 8.

1936

19. Analyzing System Disturbances With the Automatic Oscillograph, **T. M. Blakeslee.** *Electrical West*, volume 77, September 1936, pages 16–19; November 1936, pages 37–9.

20. A Disturbance-Duration Recorder, **C. H. Frier.** AIEE Transactions, volume 55, 1936 (March section), pages 1025–8.

21. Statistical Survey of the Performance of Automatic Protective Systems, Institution of Electrical Engineers (London) *Journal*, volume 79, November 1936, pages 541–59.

1937

22. Automatic Oscillograph Anticipates Faults, **J. S. DeShazo.** *Electrical World*, volume 107, April 1937, page 1438.

23. Portable Equipment Speeds Relay Test, **A. R. V. Warrington.** *Electrical World*, volume 107, February 1937, page 764.

24. Relays Tested on Artificial Line Board, **C. D. Hayward.** *Electrical World*, volume 107, January 1937, pages 257–9.

25. Special Uses for the Automatic Oscillograph, **G. A. Powell** and **R. E. Walsh.** AIEE Transactions, volume 56, 1937 (April section), pages 438–40 and 476–7.

1938

26. Various Methods for Testing of Buchholz Protective Electric Relay (in German), **H. Prinz.** *Archiv fuer Technisches Messen*, volume 80, February 1938, pages T28–8.

Section K

System Stability*

1930

1. Calculation of Alternator Swing Curves, **F. R. Longley.** AIEE Transactions, volume 49, 1930, pages 1129–50.

2. Progress in the Study of System Stability, **J. B. McClure** and **I. A. Summers.** AIEE Transactions, volume 49, 1930, pages 132–58.

1932

3. Assuring System Stability and Service Continuity by Relays, **H. D. Braley.** *Electrical World*, volume 99, February 1932, pages 310–15. Switching time curves for faults on a metropolitan system.

4. Generalized Stability Solution for Metropolitan-Type System, **S. B. Griscom, W. A. Lewis** and **W. R. Ellis.** AIEE Transactions, volume 51, 1932, pages 363–72.

5. Overcoming Instability, **H. L. Crumley** and **W. L. Maney.** *Electrical World*, volume 100, July 1932, pages 140–2.

1933

6. Relation Between Load and Stability for Long Lines (in French), **R. Troeger.** International High Tension Network Conference, Paris, *Report* number 96, 1933.

7. Stability of Networks and Limitation of Short-Circuit Current (in French), **J. Fallou.** Association des Ingenieurs Electriciens Sortis de l'Institut Electrotechnique Montefiore *Bulletin*, volume 11, October 1933, pages 297–310.

1934

8. Calculation of Two-Machine Stability, With Resistance, **S. R. Pritchard, Jr.,** and **E. Clarke.** *General Electric Review*, volume 37, February 1934, pages 87–92.

9. Equivalent Circuits in Stability Studies, **O. G. C. Dahl** and **A. E. Fitzgerald.** AIEE Transactions, volume 53, 1934 (September section), pages 1272–82.

10. Field Forcing of Alternators as Means of Increasing System Stability, **L. C. Benton** and **C. Ryder.** *Metropolitan-Vickers Gazette*, volume 15, September 1934, pages 65–8.

11. Graphical Representation of Current and Voltage (in German), **A. Von Timascheff.** *Wissenschaftliche Veroeffentlichungen aus den Siemens-Werken*, volume 13, 1934, pages 13–26.

12. Graphical Solution of Steady-State Stability, **H. B. Dwight.** AIEE Transactions, volume 53, 1934 (April section), pages 566–8; discussion (September section), page 1316.

13. Power Limits of Synchronous Machines, **E. Clarke** and **R. G. Lorraine.** AIEE Transactions, volume 53, 1934, pages 1802–09; discussion (March section), pages 475–7; (April section), pages 601–2.

14. Simple Calculations Solve Practical Stability Problems, **R. H. Duval** and **L. T. Eastman.** *Electrical World*, volume 103, April 1934, pages 620–3.

15. Steady-State Stability of Composite Systems, **S. B. Crary.** AIEE Transactions, volume 53, 1934, pages 1809–14; discussion (March section), pages 475, 477; (April section), pages 594–5.

16. System-Stability Calculations Reduced to Practical Base, **R. H. Duval** and **L. J. Eastman.** *Electrical World*, volume 103, April 1934, pages 514–17.

1936

17. Determining the Stability of Multimachine Systems, **J. M. Barry.** *Electric Journal*, volume 33, March 1936 page 137.

18. A Graphical Determination of Multimachine Stability, **J. M. Barry.** *Electric Journal*, volume 33, October 1936, page 447.

1937

19. First Report of Power-System Stability, AIEE Transactions, volume 56, 1937 (February section), pages 261–82.

20. Relay Operation During System Oscillations, **C. R. Mason.** AIEE Transactions, volume 56, 1937 (July section), pages 823–32; discussion (December section), pages 1513–14; volume 57, 1938 (February section), pages 111–13.

21. A System Out of Step and Its Relay Requirements, **L. N. Crichton,** AIEE Transactions, volume 56, 1937 (October section), pages 1261–7; discussion, volume 57, 1938 (May section), pages 284–6.

1938

22. Parallel Operation; Transient Stability, Involving Need of Rapid Circuit Breaker Operation, **H. Rissik.** *Electrical Review* (London), volume 123, November 1938, pages 751–2.

Section L

Methods of Calculation

1931

1. Reactance of Transmission Lines With Ground Return, **J. E. Clem.** AIEE Transactions, volume 50, 1931, page 901.

2. Relay Study for Application of Impedance Relays to Ground-Fault Protection, **L. S. C. Tippett.** Institution of Engineers, Australia, *Journal*, volume 3, July 1931, pages 233–43.

3. Simultaneous Faults on Three-Phase Systems, **E. Clark,** AIEE Transactions, volume 50, 1931, pages 919–39.

1932

4. Analysis of Equivalent Circuits, **F. M. Starr.** AIEE Transactions, volume 51, 1932, pages 287–98.

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*For a more complete bibliography see reference 19.

5. Calculation of Short Circuits on Power Systems (abstract), **C. F. Wagner** and **S. H. Wright.** Electrical Engineering, volume 51, 1932, page 131.

6. Decrement Curves for Specific Systems (abstract), **W. C. Hahn.** Electrical Engineering, volume 51, February 1932, page 131.

7. New Uses for Calculating Table, **R. C. R. Schulze.** *Electric Light and Power*, volume 10, March 1932, pages 28–9; April, pages 24–5; May, pages 48 and 50. Methods of using calculating table for one and two line to ground short circuits.

8. Standard Decrement Curves, **W. C. Hahn** and **C. F. Wagner.** AIEE Transactions, volume 51, 1932, pages 353–62.

1933

9. Graphical Determination of Sudden Short-Circuit Current in Generators (in French), **R. David.** International High-Tension Network Conference, Paris, *Report* 29, 1933.

10. Operation Diagrams Simplify Relaying Problems, **R. C. Ericson.** *Electrical World*, volume 101, January 1933, pages 106–08.

11. Study of Neutral Shift by Grounding and Ground Relay for Distribution Lines (in Japanese), Institute of Electrical Engineers of Japan *Electrotechnical Journal*, volume 53, November 1933, pages 956–65.

12. Templates of Relay Curves Aid Checking Selectivity, **J. H. Neher.** *Electrical World*, volume 102, December 1933, pages 764–5.

13. Zero Phase-Sequence Impedances of Multiple-Earthed Transmission Systems, **S. Arahata.** Institute of Electrical Engineers of Japan *Electrotechnical Journal*, volume 53, December 1933, pages 1103–10.

1934

14. Decrement Curves for Power Systems, **C. F. Dalziel.** AIEE Transactions, volume 53, 1934 (February section), pages 331–8.

15. Analysis of Earth Faults by Method of Phase-Sequence Components, **G. A. Robertson.** *Electrical World*, volume 113, July 1934, pages 95–8; August, pages 241–5; September, pages 433–7; November, pages 573–6; December, pages 797–800.

16. External Impedance Versus Short-Circuit Currents, **B. L. Robertson** and **T. A. Rogers.** AIEE Transactions, volume 53, 1934, pages 252–4. Confirmation of calculations by tests.

17. Graphical Solution of Delta-Star Resistance Transformations, **W. W. Edson.** *Electrical Manufacturing*, volume 53, October 1934, page 1427; discussion, December 1934, pages 1680–1.

18. New Method of Calculating Circuits: A Short-Circuit Current Solution, **W. B. Kouwenhoven** and **M. W. Pullen.** Electrical Engineering, volume 52, November 1933, pages 776–9; letters to the editor, volume 53, January 1934, pages 228–9.

19. Mechanical Mind Calculates Intricate Problem Men Direct: Alternating-Current Calculating Board, **E. L. Harder.** *Electric Journal*, volume 31, March 1934, pages 117–19.

1935

20. Duplex-Coil Windings for D-C Protective Relays (in German), **R. Edler.** Association des Ingenieurs Electriciens Sortis de l'Institut Electrotechnique Montefiore *Bulletin*, volume 26, May 1935, pages 256–62.

21. Number of Turns and Filling Factor of Relay Coils (in German), **R. Edler.** *Zeitschrift fuer Fernmeldetechnik*, volume 16, March 1935, pages 43–4. Theoretical and mathematical design analysis.

1936

22. Calculations for Distribution Faults, **G. F. Lincks** and **H. G. Brinton.** *Electrical World*, volume 106, February 1936, page 334.

23. Negative Sequence Reactance of Synchronous Machines, **W. A. Thomas**, AIEE Transactions, volume 55, 1936 (December section), pages 1378–85.

24. A Network Calculator, *Electrical Review* (London), volume 119, June 1936, page 822.

25. System Short-Circuit Calculations (in French), *Howille Blanche*, volume 35, January 1936, page 9; March 1936, page 40.

1937

26. Application of the Method of Symmetrical Components (book), **W. V. Lyon**, McGraw-Hill Publishing Company, 1937.

27. Calculation of Short-Circuit Currents in A-C Networks, **W. M. Hanna.** *General Electric Review*, volume 40, March 1937, pages 125–30; April 1937, pages 189–96; June 1937, pages 287–91; August 1937, pages 383–8.

28. Determination of Voltages and Currents During Unbalanced Faults, **E. Clarke.** *General Electric Review*, volume 40, November 1937, pages 511–13.

29. Network-Analyzer Solution of Multiple Unbalances, **E. W. Kimbark.** AIEE Transactions, volume 56, 1937 (December section), pages 1476–82.

30. Sequence Network Connections for Unbalanced Load and Fault Conditions, **E. L. Harder.** *Electric Journal*, volume 34, December 1937, pages 481–8.

31. Short-Circuit Calculations Using Data Obtained in Connection With the D-C Calculating Board, **J. H. Neher.** *Electric Journal*, volume 34, March 1937, pages 107–10.

1939

32. Determination of Ground-Fault Current and Voltages on Transmission Systems, **E. M. Hunter**, **E. Pragst** and **P. H. Light.** *General Electric Review*, volume 42, August 1939, pages 353–6.

33. Power-System Transients Caused by Switching and Faults, **R. D. Evans**, **A. C. Monteith**, and **R. L. Witzke.** AIEE Transactions, volume 58, 1939 (August section), pages 386–94.

Section M

Instrument Transformers and Other Auxiliary Devices

1932

1. Capacitance Transformers for Ground-Directional Relays, **F. B. Doolittle.** *Electrical World*, volume 99, number 25, June 18, 1932, page 1057.

2. Coupling Condensers Provide Relay Currents, **A. V. Joslin.** *Electrical West*, volume 67, number 6, December 1, 1931, pages 290–1; *Electrical World*, volume 99, number 3, January 16, 1932, page 145.

3. Current-Transformer Design and Application From Overcurrent Standpoint, **C. O. Werres.** *General Electric Review*, volume 35, number 10, October 1932, pages 544–9.

4. High-Thermal-Capacity Current Transformers, **A. Pottinger.** *Electric Journal*, volume 29, October 1932, page 487.

5. Instrument Transformers, **A. Hobson.** *Electrical Review* (London), volume 110, March 18, 1932, page 410.

6. Matching Ratio Errors Improves Differential Relaying, **E. C. Wentz.** *Electrical World*, volume 99, number 3, January 16, 1932, pages 154–5.

7. Relays Operated From Bushing Potential Devices, **P. O. Langguth** and **V. B. Jones.** *Electrical World*, volume 99, number 26, June 25, 1932, pages 1092–6.

8. Vector Analysis of Instrument Transformer Connections, **W. H. Gregory.** *Electrical Engineer and Merchandiser*, volume 9, numbers 5 and 6, August 15, 1932, pages 125–9; September 15, 1932, pages 164–7.

1934

9. A-C Premagnetization in Current-Transformer Design and Graphical Presentation of Its Action (in German), **J. Goldstein.** Association Suisse des Electriciens *Bulletin*, volume 25, number 9, April 27, 1934, pages 229–32.

10. Capacitor-Potential Devices, **P. O. Langguth.** *Electric Journal*, volume 31, March 1934, pages 107–09.

11. Characteristics of Current Transformers for Protective Relays (in German), **M. Walter.** *Elektrotechnische Zeitschrift*, volume 55, number 20, May 17, 1934, pages 483–7.

12. Instrument Transformers, **A. Hobson.** *Electrical Review* (London), volume 114, number 2945, May 4, 1934, pages 623–4. Their application for metering and protection.

1936

13. Capacitor-Voltage Transformers, **J. G. Wellings, J. R. Mortlock,** and **P. Mathews.** Institution of Electrical Engineers (London) *Journal*, volume 78, November 1936, page 577.

14. Current-Transformer Accuracy for Protective Relays (in German), **W. Bruegger.** Association Suisse des Electriciens *Bulletin*, volume 27, August 7, 1936, page 439.

15. Design, Construction, and Testing of Voltage Transformers, **J. H. Buchanan.** Institution of Electrical Engineers (London) *Journal*, volume 78, March 1936, page 292.

16. Instrument-Transformer Accuracy, **J. B. Gibbs.** *Power*, volume 80, October 1936, page 533.

17. Instrument Transformers, Part I, **E. S. Townsend.** *Power Plant Engineering*, volume 40, September 1936, page 529.

18. Instrument Transformers, Part II, **E. S. Townsend.** *Power Plant Engineering*, volume 40, November 1936, page 633.

19. Instrument Transformers; Their Theory, Characteristics, and Testing (book), **G. Hague.** Pitman, 1936, 656 pages.

20. Surge Protectors for Current Transformers, **G. Camilli** and **L. V. Bewley.** AIEE Transactions, volume 55, 1936 (March section), pages 254–60.

21. Thermal Stresses in Current Transformers, **H. Hurworth.** *Electrical Review* (London), volume 119, December 4, 1936, page 776.

1937

22. Current and Potential Transformers (book, in German), **M. Walter.** R. Oldenbourd, Munich and Berlin, 1937, 159 pages.

23. Current Transformers for Metering; Use of Nickel-Iron Alloys, **A. Hobson.** *Electrical Review* (London), volume 120, May 21, 1937, pages 760–1.

24. Instrument-Transformer Practice Trends, **S. B. Van Buren.** *Electrical World*, volume 107, April 10, 1937, pages 1214–16.

25. Instrument Transformers—Recent Developments, **H. W. Cope.** *Electric Journal*, volume 34, January 1937, page 13.

26. 75,000-Ampere Tests on Current Transformers Used for Bus-Differential Protection, **C. O. Werres.** *General Electric Review*, volume 40, August 1937, pages 380–2.

1938

27. Bushing Potential Device, **M. Sigenari** and **S. Takezo.** *Shibaura Review* (Japan), volume 17, number 6, June 1938, pages 193–203. Description and characteristics of device which provides new means of obtaining power for relays.

28. Application of Capacitance Potential Devices, **J. E Clem.** AIEE Transactions, volume 58, 1939 (January section) pages 1–8.

Standard Basic Impulse Insulation Levels

This report of the joint committee[*] on co-ordination of insulation of AIEE, Edison Electric Institute, and National Electrical Manufacturers Association is the culmination of more than ten years' study to establish basic insulation co-ordination. It specifies basic impulse insulation levels in terms of a withstand voltage for a specified test wave. These values are to be the bases for various AIEE and other standards now in course of development or revision.

CULMINATING over ten years' work on insulation co-ordination during which the fundamentals of making impulse tests were investigated and standardized, as well as basic levels set up and provisionally tried out, the joint AIEE-EEI-NEMA committee on insulation co-ordination at its meeting held in November, 1940, agreed unanimously to adopt as standard the series of basic impulse insulation levels given in table I. These are the same values given in the paper presented at the AIEE winter convention, January 22–26, 1940, "Basic Impulse Insulation Levels" by Sporn and Powel (*AIEE Trans., Oct. '40 Sec., p. 596–8*). The purpose of this present report is to call attention to the fact that these basic levels have now been adopted and to the general conditions under which they apply.

The general principle of insulation co-ordination requires a reasonable margin between the voltage level held by the protective device (the protectable level) and the various basic levels themselves to insure that adequate protection is provided. The values in table I have been set up on this basis. It is felt that the values so set up are on a sound basis and that they are most likely to stand without change for a long time. It is the opinion of the committee, therefore, that the various technical committees working on standards can adopt the values given in this report for standardization purposes, and it is further believed that the use of these values will result in ultimate over-all benefit to the industry.

When first formed, the joint committee adopted as a statement of principles the following:

"The co-ordination of insulation involves three steps:

1. Establishment of insulation levels.
2. Specification of insulation strengths of all classes of equipment in established impulse levels.
3. Allocation of the insulation levels to the nominal system voltages, taking into account all operating and environmental conditions."

The adoption of the standard insulation levels in table I of this report completes the first step of the committee work.

DEFINITION OF BASIC IMPULSE INSULATION LEVELS

To clarify the meaning and intent of the levels which are given in table I, it may be well to mention here two fundamental definitions. It is believed that these definitions and the associated comments clearly indicate the requirements of any apparatus which is built to these levels and is reported as conforming to them.

"BASIC IMPULSE INSULATION LEVELS

"Basic impules insulation levels are reference levels expressed in impulse crest voltage with a standard wave not longer than 1.5×40 microsecond wave. Apparatus insulation, as demonstrated by suitable tests, shall be equal to or greater than the basic insulation level."

This requires that apparatus conforming to these levels shall have a withstand value not less than the kilovolt value given in the second column of table I. It is also understood that apparatus conforming to these requirements shall be capable of withstanding the specified voltage whether the impulse is positive or negative in polarity.

"WITHSTAND VOLTAGE (FOR AN IMPULSE)

"The withstand voltage of a test specimen under an impulse of any given wave shape, polarity, and amplitude, which does not cause disruptive discharge on the test specimen, is the crest value attained by that impulse."

EXCEPTIONS TO TABLE I

Taking cognizance of the fact that some apparatus insulators do not at the present time fully meet the levels given in table I, the committee recognized certain exceptions. These exceptions apply to insulation levels 150, 200, and 250 kv but only for switch and bus insulators and apparatus employing these insulators, with the understanding that after January 1, 1943, all equipment will meet the values given in table I. Any new or modified designs of such apparatus completed prior to that date should, of course, conform to the values given in the table. This exception further definitely states that it will exist as an exception only until January 1, 1943, after which no equipment will be excepted from meeting the requirements of table I if such equipment is represented as meeting "Standard Basic Impulse Insulation Levels."

Table I. Standard Basic Impulse Insulation Levels

Reference Class (Kv)	Basic Impulse Level (Kv)
1.2	30
2.5	45
5.0	60
8.7	75
	95
15	110
23	150[*]
34.5	200[*]
46	250[*]
69	350
92	450
115	550
138	650
161	750
196	900
230	1,050
287	1,300
345	1,550

[*] For switch and bus insulators, and apparatus such as air switches and power fuses employing these insulators, the levels of 150, 200, and 250 kv shall become effective on January 1, 1943, with the understanding that any redesign or modification of such equipment completed earlier than January 1, 1943, shall be made on the basis of 150, 200, and 250 kv levels. In the interim and for existing designs of this equipment, values of 145, 190, and 240 kv will be recognized as exceptions for these three classes, respectively.

NOTE: Attention is called to the fact that equipment carrying a power-frequency voltage rating which is numerically the same as the "Reference Class" in table I is not necessarily required to have the corresponding impulse level given in the table. Decisions in this regard are provided for in the two remaining steps mentioned above in the committee's original statement of principles.

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[*] Chairman of AIEE group is Stanley Stokes (A'16, F'29) consulting electrical engineer, Union Electric Company, St. Louis, Mo.; of EEI group, Philip Sporn (A'20, F'30) vice-president in charge of engineering, American Gas and Electric Service Corporation, New York, N. Y.; of NEMA group, C. A. Powel (M'20) manager, industry engineering department, Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.

Report presented informally at the AIEE winter convention, Philadelphia, Pa., January 27–31, 1941.

Report of the Board of Directors

THE BOARD OF DIRECTORS of the American Institute of Electrical Engineers presents herewith to the membership its 57th annual report, for the fiscal year ending April 30, 1941. A general balance sheet showing the condition of the Institute's finances on April 30, 1941, together with other detailed financial statements, is included herein. This report contains a brief summary of the principal activities of the Institute during the year, more detailed information having been published from month to month in ELECTRICAL ENGINEERING.

BOARD OF DIRECTORS' MEETINGS

The board of directors held five meetings during the year, three in New York City, one in Swampscott, Mass., and one in Philadelphia, Pa.

The status of the committee on research was changed from that of a technical committee to that of a general committee. The names of two technical committees were changed, viz., the committee on applications to marine work became the committee on marine transportation, and the committee on transportation became the committee on land transportation.

Three new technical committees were established: committee on domestic and commercial applications, committee on air transportation, and committee on applications of electricity to therapeutics.

Information regarding many of the more important activities of the Institute which have been under consideration by the board of directors and the committees is published each month in the section of ELECTRICAL ENGINEERING devoted to Institute activities.

NATIONAL DEFENSE

Through a resolution adopted by the board of directors on June 27, 1940, the services of the Institute in connection with problems of national defense were offered to the President of the United States. At the meeting of the board of directors held on January 30, 1941, the new committee on air transportation was authorized to offer its services to the Army-Navy aeronautical board in connection with the development and co-ordination of aeronautical electrical equipment.

By actions of the board of directors, on January 30, 1941, an Institute committee on national defense was appointed, and Past President John C. Parker was designated to represent the Institute on the national technological civil protection committee appointed by the Secretary of War.

National committees have given increased attention to defense problems. The Sections have supplied information on engineering firms and individuals available for defense work of certain types. Many have included defense subjects in their programs.

President Sorensen is a member of the committee on acceleration of the regular engineering programs appointed by the Society for the Promotion of Engineering Education at the request of the advisory committee on defense training of the United States Office of Education. Many other officers and members of the Institute have been serving in defense activities.

President Sorensen and the headquarters staff have co-operated with Government divisions at every opportunity, assisting in several undertakings, including the national roster of scientific and specialized personnel. Messages from President Sorensen containing suggested ways in which engineers might aid the defense program have been published in ELECTRICAL ENGINEERING. Through the news columns of ELECTRICAL ENGINEERING, much information on problems of national defense has been supplied to the membership.

PRESIDENT'S AND NATIONAL SECRETARY'S VISITS

President Sorensen and National Secretary Henline attended the summer and Pacific Coast conventions, the winter convention, the Middle Eastern District meeting in Cincinnati, Ohio, and the North Eastern District meeting in Rochester, N. Y. They also visited many Sections and Student Branches. During May, they will visit the following Sections: Washington, North Carolina, West Virginia, and Cincinnati; and President Sorensen will visit the Cleveland Section. President Sorensen and National Secretary Henline will attend the summer convention in Toronto, Ont., Canada.

The places visited by President Sorensen follow:

- **Alabama**
  - Alabama Section, Birmingham
  - Muscle Shoals Section
- **California**
  - Pacific Coast convention, Los Angeles
  - Los Angeles Section, University of Southern California Branch, and California Institute of Technology Branch, at U.S.C.
  - San Francisco Section
  - Conference on student activities, Districts Nos. 8 and 9, and University of British Columbia, Los Angeles
- **Colorado**
  - Denver Section
  - University of Colorado Branch, Boulder
  - Colorado State College, Ft. Collins
  - Conference on student activities, North Central District, No. 6, University of Denver; 9 Branches represented
- **Georgia**
  - Georgia Section, Atlanta
  - Georgia School of Technology Branch, Atlanta
- **Illinois**
  - Chicago Section
  - Illinois Institute of Technology Branch, Chicago
  - Northwestern University, Evanston
- **Idaho**
  - University of Idaho, Moscow
- **Indiana**
  - Central Indiana Section, Indianapolis
  - Fort Wayne Section
  - Purdue University, Lafayette
- **Iowa**
  - Iowa Section, Cedar Rapids
  - Iowa State College Branch, Ames
- **Kentucky**
  - Louisville Section
  - University of Kentucky Branch, Lexington
- **Massachusetts**
  - Summer convention, Swampscott
  - Boston Section
  - Lynn Section executive committee meeting
  - Worcester Section and Worcester Polytechnic Institute Branch, at W.P.I.
- **Michigan**
  - Michigan Section, Detroit
- **Minnesota**
  - Minnesota Section, Minneapolis
  - University of Minnesota, Minneapolis
- **Missouri**
  - Kansas City Section
  - St. Louis Section
  - University of Missouri Branch, Columbia
- **Montana**
  - Montana Section and Montana State College Branch, joint meeting, Butte
- **Nebraska**
  - Nebraska Section, Omaha
  - University of Nebraska, Lincoln
- **New York**
  - Cornell University, Ithaca
  - Ithaca Section
  - Syracuse Section
  - Syracuse University Branch
  - North Eastern District meeting, Rochester
  - North Eastern District conference on student activities, Rochester, 17 Branches represented
- **Ohio**
  - Middle Eastern District meeting, Cincinnati
  - Toledo Section
  - Columbus Section
  - Ohio State University, Columbus
- **Oklahoma**
  - Oklahoma City Section
  - Tulsa Section
- **Oregon**
  - Portland Section
- **Pennsylvania**
  - Winter convention, Philadelphia
  - Pittsburgh Section, and University of Pittsburgh, Carnegie Institute of Technology, and West Virginia University Branches, joint meeting, Pittsburgh
  - Pittsburgh Section student conference
  - Erie Section
- **Rhode Island**
  - Providence Section
  - Brown University Branch, Providence
- **South Carolina**
  - South Carolina Section, Columbia
- **Tennessee**
  - East Tennessee Section, Athens
- **Texas**
  - New Mexico-West Texas Section, El Paso
- **Utah**
  - Utah Section, Salt Lake City
  - University of Utah, Salt Lake City
- **Washington**
  - Seattle Section
  - University of Washington, Seattle
  - Spokane Section, and University of Idaho and Washington State College Branches, joint meeting, Spokane
  - Washington State College, Pullman
- **Wisconsin**
  - Madison Section
  - University of Wisconsin, Madison
  - Milwaukee Section
- **Canada**
  - Toronto Section

The places visited by the national secretary are the following:

- **Alabama**
  - Alabama Section, Birmingham
  - Muscle Shoals Section
- **California**
  - Pacific Coast convention, Los Angeles
  - Conference on student activities, Districts 8 and 9, and University of British Columbia, Los Angeles
- **District of Columbia**
  - Washington Section
- **Georgia**
  - Georgia Section, Atlanta
- **Illinois**
  - Illinois Institute of Technology Branch, Chicago
  - Northwestern University, Evanston

Indiana
- Central Indiana Section, Indianapolis
- Purdue University, Lafayette

Kentucky
- Louisville Section
- University of Kentucky Branch Lexington

Massachusetts
- Summer convention, Swampscott, Mass.
- Boston Section
- Lynn Section executive committee meeting
- North Eastern District executive committee meeting, Pittsfield

Michigan
- Michigan Section, Detroit

Missouri
- Kansas City Section
- St. Louis Section

Nebraska
- Nebraska Section, Omaha
- University of Nebraska Branch, Lincoln

New Jersey
- Student Branch convention, District No. 3, New Brunswick

New York
- Cornell University, Ithaca
- New York Section
- Ithaca Section
- Syracuse Section
- Manhattan College Branch, New York
- New York Section student convention, Rutgers University, New Brunswick, N. J.
- North Eastern District meeting, and conference on student activities, Rochester, N. Y.
- District No. 3 Branches, smoker, Pratt Institute, Brooklyn, N. Y.

Ohio
- Middle Eastern District meeting, Cincinnati
- Conference on student activities, Middle Eastern District, No. 2, Cincinnati

Pennsylvania
- Pittsburgh Section, and University of Pittsburgh, Carnegie Institute of Technology, and West Virginia University Branches, joint meeting, Pittsburgh
- Pittsburgh Section, student conference
- Winter convention, Philadelphia

Rhode Island
- Providence Section
- Brown University Branch, Providence

South Carolina
- South Carolina Section, Columbia

Tennessee
- East Tennessee Section, Athens

Canada
- Toronto Section

ANNUAL MEETING

The annual business meeting of the Institute was held on Monday morning, June 24. The annual report of the board of directors for the fiscal year which ended April 30, 1940, was presented in abstract by the national secretary. A report on the finances of the Institute was presented by National Treasurer W. I. Slichter. The report of the committee of tellers upon the election of officers for the year beginning August 1, 1940, was presented, and President-Elect Sorensen responded to his introduction with a brief address. During this session, special honors were accorded to Charles L. Clarke, charter member of the Institute, and the Lamme Medal for 1939 was presented to Norman W. Storer, retired consulting engineer, Westinghouse Electric and Manufacturing Company.

NATIONAL CONVENTIONS

Three national conventions were held during the year, and a brief report on each follows:

Summer Convention. The 56th summer convention was held in Swampscott, Mass., June 24–28, 1940. In addition to the annual business meeting, conference of officers, delegates, and members, there were 10 technical sessions, at which 41 papers were presented, 1 general session, demonstration on frequency modulation, and 6 technical conferences. The general session was in collaboration with the American Engineering Council, under the leadership of its president Colonel Alonzo J. Hammond, and consisted of addresses by Colonel William J. Wilgus and Colonel Robert F. Henry on the topic "Transportation as a Social Problem". Entertainment features of the convention were women's tea and musicale, bridge, dance, banquet, golf and tennis tournaments. Many inspection trips were held. The registration was 1,014.

Pacific Coast Convention. The 28th Pacific Coast convention was held in Los Angeles, Calif., August 27–30, 1940, with a registration of 465. In addition to 4 technical sessions at which 16 papers were presented, there was a joint session with the Institute of Radio Engineers, informal conference, 2 student sessions with 10 technical papers, and a conference on student activities. Entertainment features included reception and dancing, banquet, inspection trips, golf tournament, and ladies' events.

Winter Convention. The 29th winter convention was held in Philadelphia, Pa., January 27–31, 1941, with a program including 20 sessions with 82 papers, and 9 technical conferences. Speakers on general subjects were: Charles E. Wilson, president, General Electric Company, and R. D. Evans, Massachusetts Institute of Technology. At an evening meeting, the Edison Medal was presented to Doctor George A. Campbell, retired research engineer, Bell Telephone Laboratories, Inc. A smoker, dinner-dance, women's bridge, and numerous inspection trips completed the program. The registration was 1,931.

DISTRICT MEETINGS

Middle Eastern District Meeting. The 11th meeting of this District was held in Cincinnati, Ohio, October 9–11, 1940. The meeting included 5 sessions at which 25 technical papers were presented, 2 technical conferences, meetings of the Student Branch counselors and executive committee. Entertainment features consisted of smoker, luncheons, informal dinner, women's events, and numerous inspection trips. The attendance was 599.

North Eastern District Meeting. An interesting and varied program for this meeting in Rochester, N. Y., April 30–May 2, 1941, included 5 sessions at which 8 addresses and 14 technical papers were presented, 1 session for graduate student papers, 2 sessions for undergraduate papers, a District executive committee luncheon, a Branch counselors' and students' luncheon, a banquet, a smoker, inspection trips, and women's events. The attendance was 355.

SECTIONS

Two new Sections, Arizona and South Bend, were organized; and the names of two Sections were changed to make them more accurately descriptive of the territories: Charleston to West Virginia, and El Paso to New Mexico-West Texas.

As indicated in the list of his visits, President Sorensen visited a large number of Sections. In most cases, he gave an address entitled "Engineering Horizons, Limited," illustrated with many lantern slides. This proved to be an unusually interesting and instructive address, as shown by the close attention of all audiences and a great many highly complimentary comments received by the national secretary.

The Sections again held a larger number of meetings than reported for any previous year. Interest in technical groups, special technical meetings, and so on, continued at a high level, and attendance at such meetings was excellent.

Two Sections reported no activity; 8 Sections held more than 15 meetings each, 9 held from 12 to 15, 38 held from 8 to 11, 10 held from 4 to 7, and 6 held from 1 to 3.

The plan adopted several years ago to include within Section territories, in so far as practicable, all Institute members within the United States resulted in the revision of the territories of a considerable number of Sections. The August 1, 1940, record of Section membership showed the total membership of Sections in the United States as 14,338. In September, it was found that only 93 members in the United States were outside Section territories. The Sections committee has continued its study of unassigned territory with the object of recommending further revisions when desirable.

The other activities of the Sections committee have included the adoption of a trial plan for publishing information on Section meetings in ELECTRICAL ENGINEERING, plans to develop greater interest among the Sections in vocational guidance, and efforts to develop more effective distribution of the pamphlets "The Electrical Engineer" and "Engineering—A Career, a Culture".

Table I contains information regarding Sections and Branches and their meetings during the past several years. Detailed information on their activities of the past year may be found in the annual report on Section and Branch activities in the June issue of ELECTRICAL ENGINEERING, pages 285–7.

Table I. Section and Branch Statistics

	For Fiscal Year Ending April 30					
	1936	1937	1938	1939	1940	1941
Sections						
Number of Sections	61	62	65	67	70	72
Number of meetings held	540	621	624	635	701	703
Total attendance	85,501	74,950	110,148	85,692	91,949	92,554
Branches						
Number of Branches	118	119	120	120	121	123
Number of meetings held	1,045	1,363	1,334	1,190	1,346	1,163
Total attendance	45,304	46,121	60,446	53,380	64,972	52,285

STUDENT BRANCHES

New Student Branches organized at the University of Connecticut, Norwich University, and Manhattan College, and the combination of the Armour Institute of Technology and the Lewis Institute Branches into the Illinois Institute of Technology Branch brought the total number at the end of the year to 123.

President Sorensen visited a considerable number of Branches, and his addresses were enthusiastically received.

Only one Branch failed to report any activity. Thirteen held more than 15 meetings each, 30 held from 8 to 11, 36 held from 4 to 7, and 16 held from 1 to 3, with a total number of 1,163 meetings, as compared with 1,346 for the preceding year; 1,190 for 1938–39; and 1,334 for 1937–38. The number of talks by students, 608, was materially below the totals for recent years: 767 for 1939–40, 725 for 1938–39, and 897 for 1937–38.

Students have continued to present technical papers at the Pacific Coast convention and various District meetings. In general the quality of presentation has been excellent, and large numbers of students have attended such sessions.

The terms of 1,641 enrolled students were expected to expire on April 30, 1941. Of these, 887 or about 54 per cent applied for admission as Associates.

See references at end of the preceding report on Sections.

General Committees

FINANCE COMMITTEE

In the preparation of the budget for the year, the finance committee made allowance for expected losses of dues revenues, and subscriptions normally received from abroad. In spite of this, due primarily to increased memberships in this country, estimated revenues for the budget year, beginning October 1, are about $10,000 greater than for last year. This permits a further modest expansion in the Institute's activities. The budget this year also provides for a nonrecurring expense due to placing in effect a revision in the publication program by means of which publications of some of the TRANSACTIONS material is made at an earlier date than under the former policy. Details of the budget and a further discussion of it are included in the December 1940 issue of ELECTRICAL ENGINEERING.

The conservation of the Institute's investment funds is a problem of increasing difficulty. Under advice of investment counsel and the finance committee, the board of directors this past year approved putting about ten per cent of the Institute's investment funds in common stocks. While this is a new departure for the Institute it is following in a direction which has been taken by many educational and professional organizations.

Haskins and Sells, certified public accountants, have audited the Institute books and their report appears on pages 348–50.

TECHNICAL PROGRAM COMMITTEE

Convention Program. Three national conventions and two District meetings were held during the year. The total attendance of these five meetings represents a 22.3 per cent increase over that for the previous year This has not been the result of an unusually large attendance at any one particular meeting, but the attendance at three of the meetings each establishes a record over corresponding meetings in those localities, and the attendance at the summer convention was substantially above the average attendance for summer conventions of the past ten years. The policy of presenting enlarged programs has been continued, and the attached table of statistics shows that 8 more technical conferences were held during the past year than for the previous year. The technical program for the winter convention was also enlarged to include 2 more technical sessions and 1 more technical conference than have ever been held heretofore. While the attendance at meetings and conventions is a variable depending on several factors, such as location, time of the year, the entertainment program, and general business conditions, it was nevertheless believed that the size and character of the technical programs has contributed in no small measure to the success of these meetings. Many of the papers presented at these meetings were of a high order and favorable comments were received.

Table II. Technical Programs, Last Three Years

	Year Ending April 30		
	1941	1940	1939
Number of national conventions	3	2	3
Number of District meetings	2	3	3
Registration at national conventions and District meetings	4,339	3,548	4,100
Number of papers presented	195	177	184
Number of papers recommended for TRANSACTIONS	164	151	169
Estimated number of pages required for printing papers in TRANSACTIONS	938*	886*	986*
Average length of papers recommended for TRANSACTIONS	5.72	5.86	5.83
Number of technical sessions	46	43	50
Number of technical conferences	18	10	12

* Partly estimated.

Technical Conferences. The increased number of technical conferences has proved beneficial in several ways. The standards co-ordinating committees have been able to obtain a broad cross-section of views preliminary to the adoption or revision of certain standards. The conferences also have served as a trial or proving ground for the development of papers of a high quality which were scheduled later for formal presentation and discussion. Committees on basic sciences, power generation, and standards all have developed some papers in this manner.

Oral Presentations. A letter suggesting ways and means of increasing the effectiveness of the oral presentations was sent to each author and presiding officer of the winter-convention sessions. The suggestions were compiled from a canvass of authors and presiding officers of a previous convention and many of the oral presentations were improved. It has been decided to send a revised version of this letter to the authors on subsequent convention programs.

General Sessions. General sessions which were held at both the winter and summer conventions proved to be very successful, and arrangements are being made to hold another at the 1941 summer convention in Toronto. Preference has been indicated for suitable subject matter to lean more toward the broader problems of human relationships or the social aspects of the profession as related to co-operative effort during and after changed world conditions, rather than the more specific technical developments.

Trends in Policy. The modified publication procedure, announced in the August 1940 issue of ELECTRICAL ENGINEERING, has worked out satisfactorily from the standpoint of program development. Selling pamphlet copies at an increased price according to their lengths has not reduced the number of copies placed in circulation for the purpose of discussion at the various meetings. The proposal to give preference to papers 6 pages or less in length in the TRANSACTIONS section of ELECTRICAL ENGINEERING has provided an incentive for the authors to produce shorter papers. The data in the accompanying table shows that the papers for the past year were of an estimated average length of 5.72 pages, as compared with 5.86 pages for the previous year, before the incentive was provided. The reduction in average page length per paper will be still greater when the policy has been in effect over the full year.

The national defense program has resulted in a scarcity of papers in certain fields, such as industrial, automatic control, and communications. Some papers have also been held up by a more careful scrutiny pending company release and approval. Increase in the pace of industry has also impaired the prompt functioning of the committees as well as the authors in the reviewing and writing of papers to meet certain dates. However, there is no immediate scarcity of good papers in the central-stations fields of activities.

Acknowledgments. The committee wishes gratefully to acknowledge the interest and efforts of its members, the chairmen of the technical committees, and the members of the headquarters staff in the conduct of the work.

PUBLICATION COMMITTEE

ELECTRICAL ENGINEERING and TRANSACTIONS were published during the fiscal year largely on the basis of the same general publication policy and procedure as for the preceding year.

Modifications in publication procedure during the past year include the establishment of the "Supplement to ELECTRICAL ENGINEERING—TRANSACTIONS Section" on a semiannual instead of on an annual basis, and the transfer of the publication of discussions from the monthly periodical to the Supplement.

The principal effort of the year in improving the publication service to the Institute's members has been directed toward reducing the time delay between presentation and publication of technical papers and discussions The omission of discussions from the monthly issues has enabled a progressively earlier publication of technical-

Table III. Membership Statistics for the Fiscal Year Ending April 30, 1941

	Honorary	Fellow	Member	Six-Year Associate	Associate	Total
Membership on April 30, 1940	8	767	4,493	5,724	6,219	17,213
Additions:						
Transferred		34	182	612		
New members qualified		2	129	35	1,435	
Former members reinstated			9	18	16	
	8	803	4,815	6,389	7,670	19,685
Deductions:						
Died		20	36	22	12	
Resigned		3	44	128	124	
Transferred			34	156	638	
Dropped		2	51	193	336	
		25	165	499	1,110	1,799
Membership on April 30, 1941	8	778	4,650	5,890	6,560	17,886

program papers, and has also enabled the monthly pages otherwise required for discussion to be used for additional technical-program papers.

This will help materially to reduce the time delay in making available in published form the technical papers not included in the monthly issues and the approved discussion of all papers.

When the expediting program was initiated in the early summer of 1940, the publication of 1940 winter convention papers in any quantity could not begin until the June issue, and this, of course, determined the time schedule for the papers of all later conventions during the year. The objective is that by January 1942 the publication of winter convention papers should begin in January and be completed in June, with papers from the other conventions being scheduled accordingly. On the basis of this program, the 1942 TRANSACTIONS is scheduled to be a record of 1942 technical-program papers, instead of embracing parts of at least two years as heretofore. As of the date of this report, this expediting program is proceeding on schedule, but, of course, involves the handling of an increased amount of material during the period of transition. This is evidenced in the enlarged size of the 1940 Supplement, and will be shown also in the enlarged Supplements for June and December 1941. Adjustments of publication schedule between the monthly and annual publications have enabled this transition to be met without imposing an undue burden on the budget.

Another modification in publication policy that became effective during the year was an increase in the price of the advance pamphlet copies of technical program papers as produced for conventions and District meetings. Although this pamphlet-copy program still is underwritten by the general publication budget, the effort and trend are toward making this special service more nearly self-supporting.

MEMBERSHIP COMMITTEE

The membership committee, at its first meeting on September 24, 1940, decided upon a program for its activities during the year, which included a study of all practicable means by which membership work of the Institute, including that in the Sections, might be better co-ordinated in a manner to improve its effectiveness. The committee had in mind more the means by which the national committee could be of greater assistance to the Section membership committees through furnishing information as to the most effective methods and procedures, rather than by endeavoring to establish any uniform procedures.

A subcommittee appointed to study this general problem reported the conclusion that the most important contribution which the national committee could make to the membership work in the Sections was the preparation and issuance of a membership committee guide, containing its recommendations for the conduct of membership committee work. It was felt that this would provide a continuing means of pooling the experience of Section membership committees throughout the Institute territory, since the guide would be revised and improved from time to time, and this information would be valuable to new membership committees. The reaction to this proposal was favorable and a membership committee guide was prepared, approved at the meeting of the committee on January 31, and issued in final form on March 31. Copies were sent to all Section membership committees and to the chairmen of all Sections.

To improve the effectiveness of the national committee in administering the membership work of the Institute, the following measures were adopted:

(*a*). Due to the wide distribution of members of the committee, frequent well-attended meetings are impracticable. Since the best interests of the committee demand the prompt disposal of matters which continuously arise, an advisory subcommittee, composed of six members of the committee in or near New York, was appointed to function as a sort of executive committee, meeting with the chairman as frequently as may be necessary to dispose of matters that cannot be held for the regular committee meetings.

(*b*). A research subcommittee, composed of six other members of the committee, was appointed to deal with problems requiring extensive study and research, and to develop recommendations as to fundamental problems which arise.

To improve the co-ordination of Section membership committee work along the most effective lines, District vice-chairmen of the membership committee have been encour-

Table IV. Number of Applications Received From Enrolled Students and From All Others

Year Ending April 30	Students	All Others	Total
1941	887	1,011	1,898
1940	911	918	1,829
1939	849	872	1,721
1938	739	932	1,671
1937	716	1,040	1,756

Table V. Number of Enrolled Students, as of April 30

Year	New Applications	Renewals	Total
1941	2,351	3,188	5,539
1940	2,525	2,992	5,517
1939	2,271	2,971	5,242
1938	2,428	2,609	5,037
1937	2,249	2,254	4,503

Table VI. Number of Members in Section Territory Reinstated

August 1, 1940 to April 30, 1941	331
Year beginning August 1, 1939	302
Year beginning August 1, 1938	354
Year beginning August 1, 1937	325
Year beginning August 1, 1936	503

Table VII. Status of Membership Dues, as of April 30

Year	Total Membership	Members Fully Paid	
		Number	Per Cent
1941	17,886†	15,777	88.2
1940	17,213	14,997	87.1
1939	16,605	14,371	86.5
1938	16,078	14,127	87.9
1937	15,308	13,439	87.8
1927*	18,344	16,247	88.6

* Year of maximum membership.

† Of the 17,886 members reported for April 30 1941, 2,109 were not fully paid to April 30, 1941. These are divided into:

1. Members owing dues to April 30, 1940.....627 (Total number of members who have not acted upon resolution of board of directors adopted in January 1941 providing an extension of time for payment of dues.)

2. Members owing dues to April 30, 1941...1,482 (During the period May 1 to May 16, 1941, 325 members have paid dues to April 30, 1941, reducing the number not fully paid to 1,157.)

Table VIII. Record of AIEE Membership

Year	Total May 1	Year	Total May 1	Year	Total May 1
1884	71	1904	3,027	1923	15,298
1885	209			1924	16,455
1886	250	1905	3,460		
1887	314	1906	3,870	1925	17,319
1889	333	1907	4,521	1926	18,158
		1908	5,674	1927	18,344
1890	427	1909	6,400	1928	18,265
1891	541			1929	18,133
1892	615	1910	6,681		
1893	673	1911	7,117	1930	18,003
1894	800	1912	7,459	1931	18,334
		1913	7,654	1932	17,550
		1914	7,876	1933	17,019
1895	944			1934	15,230
1896	1,035	1915	8,054		
1897	1,073	1916	8,202	1935	14,269
1898	1,098	1917	8,710	1936	14,600
1899	1,133	1918	9,282	1937	15,308
		1919	10,352	1938	16,078
1900	1,183			1939	16,605
1901	1,260	1920	11,345		
1902	1,549	1921	13,215	1940	17,213
1903	2,229	1922	14,263	1941	17,886

Table IX. Deaths of AIEE Members Reported During the Year Ending April 20, 1941

Name	Date of Election	Date of Death	Grade at Death	Obituary Notice in ELECTRICAL ENGINEERING
Archila, F.	Associate '29	Dec. 3, 1940	Associate	Feb. 1941, p. 89
Barry, E. J.	Associate '10	Mar. 4, 1940	Member	July 1940, p. 309
Blakeslee, H. J.	Associate '02	Jan. 19, 1941	Member	Mar. 1941, p. 141
Bostwick, H. M.	Associate '21	Nov. 25, 1940	Associate	Feb. 1941, p. 89
Boyrer, W. C.	Associate '09	Sept. 28, 1940	Fellow	Nov. 1940, p. 473
Brown, W. E.	Associate '02	May 13, 1940	Associate	Sept. 1940, p. 376
Bush, E. F.	Associate '08	Sept. 6, 1940	Associate	Dec. 1940, p. 519
Capen, W. H.	Member '23	Jan. 15, 1941	Member	Mar. 1941, p. 141
Carson, J. R.	Associate '19	Oct. 31, 1940	Fellow	Dec. 1940, p. 519
Colville, F. P.	Member '30	July 20, 1940	Member	Sept. 1940, p. 376
Cosandey, C. J.	Associate '27	1940	Associate	Sept. 1940, p. 376
Crapo, C. A.	Member '23	Sept. 2, 1940	Member	Nov. 1940, p. 474
Degen, Lewis	Associate '95	Dec. 22, 1940	Member	Feb. 1941, p. 88
Dill, G. C.	Associate '13	Apr. 1, 1940	Member	June 1940, p. 258
Dyer, E. I.	Associate '99	Feb. 4, 1939	Associate	Feb. 1941, p. 89
Ferguson, L. A.	Associate '01	Aug. 25, 1940	Fellow	Oct. 1940, p. 433
Fitzgerald, T. W.	Associate '15	Feb. 20, 1941	Member	Apr. 1941, p. 184
Freedman, W. H.	Associate '90	1941	Fellow	Apr. 1941, p. 183
Fuller, A. G.	Associate '37	Mar. 11, 1940	Associate	Dec. 1940, p. 520
Garrels, W. L.	Associate '95	1940	Associate	Sept. 1940, p. 376
Gibbs, George	Associate '05	May 20, 1940	Fellow	July 1940, p. 308
Gill, W. T.	Associate '31	Jan. 28, 1941	Associate	Apr. 1941, p. 184
Graff, J. T.	Associate '21	Mar. 18, 1940	Member	June 1940, p. 257
Greenidge, C. A.	Associate '03	Jan. 23, 1941	Fellow	Mar. 1941, p. 141
Halpin, L. C.	Associate '26	Dec. 11, 1939	Associate	July 1940, p. 310
Hardie, C. G.	Member '30	Oct. 18, 1940	Member	Jan. 1941, p. 46
Hegy, Joseph	Associate '38	Nov. 5, 1938	Associate	Dec. 1940, p. 520
Heitman, Edward	Associate '00	Sept. 25, 1940	Fellow	Jan. 1941, p. 45
Henry, I. W.	Associate '01	April 25, 1940	Member	June 1940, p. 257
Hicks, L. R.	Associate '12	Aug. 29, 1940	Member	Oct. 1940, p. 434
Higson, C. R.	Associate '21	Dec. 18, 1940	Member	Mar. 1941, p. 141
Hill, J. B.	Associate '10	Jan. 19, 1941	Member	Apr. 1941, p. 183
Hoge, C. H.	Associate '19	Jan. 12, 1941	Member	Apr. 1941, p. 184
Ittin, David	Associate '30	Aug. 21, 1939	Associate	Aug. 1940, p. 346
Kiefer, K. J.	Associate '36	Mar. 9, 1940	Associate	Nov. 1940, p. 474
Knowlton, F. K.	Member '25	Dec. 1939	Member	July 1940, p. 309
Kyne, S. B.	Associate '37	Aug. 21, 1940	Associate	Nov. 1940, p. 474
Lagomasino, Julio	Associate '38	1940	Associate	June 1940, p. 258
McAnge, W. N.	Associate '25	Aug. 30, 1940	Member	Dec. 1940, p. 519
McCulloch, Richard	Associate '04	Aug. 27, 1940	Member	Jan. 1941, p. 45
McPherson, N. C.	Associate '03	Oct. 15, 1939	Associate	July 1940, p. 309
Merz, C. H.	Associate '95	Oct. 1940	Fellow	Dec. 1940, p. 519
Montgomery, L. J.	Associate '18	Jan. 21, 1941	Associate	Apr. 1941, p. 184
Moody, V. D.	Associate '99	May 30, 1940	Fellow	July 1940, p. 309
Motter, H. W.	Associate '18	May 30, 1940	Member	Aug. 1940, p. 346
Nixon, R. O.	Associate '22	Dec. 25, 1940	Associate	Feb. 1941, p. 89
Northrup, E. F.	Associate '01	Apr. 29, 1940	Fellow	June 1940, p. 257
Parmenter, R. J.	Associate '17	July 26, 1940	Member	Sept. 1940, p. 376
Pennell, E. R.	Associate '34	1940	Associate	July 1940, p. 310
Phillips, L. A.	Associate '94	Dec. 9, 1940	Fellow	Feb. 1941, p. 88
Powell, A. L.	Associate '13	Aug. 21, 1940	Fellow	Oct. 1940, p. 433
Prowse, R. M.	Associate '39	Feb. 11, 1940	Associate	Feb. 1941, p. 89
Quirk, W. G.	Associate '16	Sept. 10, 1939	Member	Dec. 1940, p. 520
Rappel, U. J.	Member '35	Sept. 24, 1940	Member	Mar. 1941, p. 141
Rapport, A. H.	Associate '30	Dec. 17, 1940	Member	Feb. 1941, p. 89
Robbins, H. A.	Associate '08	Oct. 25, 1940	Member	Dec. 1940, p. 520
Roberts, Sheldon	Associate '25	Apr. 21, 1940	Associate	July 1940, p. 309
Rugg, W. S.	Associate '02	Apr. 25, 1940	Member	June 1940, p. 257
Rushmore, D. B.	Associate '95	May 5, 1940	Fellow	June 1940, p. 257
Scharnberg, H. J. B.	Associate '18	Nov. 1940	Member	Jan. 1941, p. 45
Selden, A. K.	Associate '03	Oct. 3, 1940	Associate	Apr. 1941, p. 184
Smith, R. T.	Member '19	Apr. 28, 1940	Member	July 1940, p. 309
Stephanus, A. D.	Member '28	Jan. 3, 1940	Member	July 1940, p. 309
Sticht, H. H.	Associate '13	Feb. 11, 1940	Member	July 1940, p. 309
Stickney, Oswald	Associate '32	Apr. 29, 1940	Associate	July 1940, p. 310
Stillwell, L. B.	Associate '92	Jan. 19, 1941	Fellow	Feb. 1941, p. 88
Stone, C. A.	Associate '91	Feb. 25, 1941	Member	Apr. 1941, p. 183
Strong, J. R.	Associate '01	Oct. 25, 1940	Associate	Jan. 1941, p. 46
Styff, Eivind	Associate '23	1940	Member	Oct. 1940, p. 434
Towne, R. M.	Associate '36	May 24, 1940	Associate	Aug. 1940, p. 346
Uebelacker, C. F.	Associate '90	Sept. 17, 1940	Member	Nov. 1940, p. 473
Wegrin, J. W.	Associate '29	Nov. 26, 1939	Associate	Aug. 1940, p. 346
White, W. S.	Associate '39	May 7, 1940	Associate	Sept. 1940, p. 376
Winter, W. W.	Associate '38	Dec. 11, 1939	Associate	July 1940, p. 310
Wise, D. M.	Member '37	Jan. 8, 1941	Member	Mar. 1941, p. 141
Woodrow, H. R.	Associate '12	Aug. 12, 1940	Fellow	Sept. 1940, p. 375
Woolfenden, H. L.	Associate '17	Jan. 19, 1941	Member	Apr. 1941, p. 184
Wright, R. I.	Associate '04	Jan. 4, 1941	Fellow	Apr. 1941, p. 184
Yonezawa, M.	Associate '25	Feb. 9, 1940	Associate	Apr. 1941, p. 184
Young, G. C.	Associate '37	June 23, 1940	Associate	Aug. 1940, p. 346

aged to keep in close touch with chairmen of Section membership committees within their respective Districts, and to give them such advice and assistance as may be appropriate.

The results of membership committee work of the Institute are indicated by tables III to VIII, inclusive.

Table III shows the total number of members as of April 30, 1941, to be 17,886, as compared with 17,213 at the end of the corresponding period of 1940. This is a net increase of 673 members.

The results of membership committee work are better indicated by table IV, which shows that 1,898 new applications were received during the year ending April 30, 1941, as compared with 1,829 in the corresponding period preceding, and considerably lower figures for the three years preceding 1940.

The figures on Enrolled Students are shown in tables IV and V. It will be noted that there was a falling off in 1941 in applications from students for election to the Associate grade. This was probably influenced by the induction of a large number of young men into military service during the year and the uncertainty which prevailing conditions imposed upon employment.

Tables VI and VII show data as to reinstatements and payment of dues, and table VIII shows the total membership of the Institute for each year since it was founded.

The committee wishes to express its appreciation of the time and effort devoted to membership work by the members of Section membership committees, many of whom have been working under pressure in their occupations. It is also grateful for the cooperation of the Branch counselors who have promoted student enrollment and written to students after their departure, advising them to become members of the Institute. Finally the committee wishes to thank the Institute members for their help and the fine response to the broadcast circular soliciting their aid in suggesting the names of desirable prospects for membership.

Deaths. The deaths of 80 members reported during the year are listed in table IX.

BOARD OF EXAMINERS

The board of examiners held 11 meetings during the past year, averaging about two and one-half hours each, and considered 3,086 cases, divided as shown in table X.

In addition to the routine consideration of the admission and transfer applications referred to in the preceding paragraph, the board during the past two years has engaged in thorough discussion of the practicability of some procedure which would permit the rendering of informal advance opinions on the eligibility of prospective applicants for admission and transfer to the grades of Fellow and Member.

Increasing sentiment for change in procedure led to a most constructive discussion at the summer convention in June 1940. After the chairman of the board of examiners had explained to the conference of officers, delegates, and members the problems confronting the examiners, the conference went on record for an invitational procedure, and recommended that the board of directors direct a study to this end.

As a result of this request from the board of directors, a subcommittee under Vice-Chairman H. S. Warren was appointed. This committee developed a plan which seems to offer a solution of the difficulties which previously led the examiners to oppose strongly the frequently recurring request from Section membership committees that the examiners render advance opinions on eligibility. The plan has been approved by the board of directors, and it now merely remains to arrange the necessary forms for putting it into regular operation.

The new procedure, which might be designated as the "invitational transfer method," applies only to Fellow grade. It will not require any change in the constitution, and will be supplementary to the present "Proposal Method" for advancement to Fellow

Table X. Applications for Admission and Transfer

Applications for Admission		
Recommended for grade of Associate	534	
Re-elected to the grade of Associate	78	
Not recommended	8	620
Recommended for grade of Member	116	
Re-elected to the grade of Member	22	
Not recommended	36	174
Re-elected to the grade of Fellow	2	2
Applications for Transfer		
Recommended for grade of Member	174	
Not recommended	14	188
Recommended for grade of Fellow	39	
Not recommended	0	39
Students		
Recommended for enrollment as Students		2,063
Total		3,086

In brief, it provides a means whereby a group of Members and Fellows desiring to promote the advancement to Fellow status of another individual may propose him over their signatures as at present, one of the Fellows designating himself as the official sponsor for such transfer. The sponsor will be required to obtain and to furnish to the board of examiners, over his signature, a fully detailed professional record of the prospect, together with the names of the required number of Fellows who will be willing to act as references. The examiners will then submit to these references the regular reference forms and obtain their confidential replies. Any correspondence with regard to references or additional details of record will be solely between the examiners and the official sponsor. The examiners will then pass unofficially on the question of eligibility and communicate their action to the official sponsor, who will approach the proposed Fellow and will then obtain his consent to the transfer action and his signature on the application. With the signed application in hand, the examiners will then take formal action. The remaining steps as to posting, and so on, will be unchanged from the present procedure.

It is hoped that the new procedure will meet the expressed wishes of the membership and the board of directors, and the examiners will strive to make it work, while at the same time discharging their heavy responsibility of upholding the high standards set for such recognition of outstanding achievement in the profession.

The board of examiners now recommends to the board of directors that a statement be prepared and generally broadcast to the Institute membership clearly outlining the new procedure available for advancement to Fellow through the "invitational method".

All questions with regard to change in procedure for transfer to Member grade were laid over for further careful study in the light of prospective experience with the new procedure for Fellows. Rejections of transfer applications to Member grade have tended to run high (over 25 per cent). As also pointed out last year, this apparently is due in part to a failure to appreciate the technical requirements prescribed by the constitution, somewhat to a tendency to aspire to Member grade merely because the 27-year minimum age limit has been reached, and slightly to the increase of dues (to parity with those of Member grade) at the end of the 6-year period of Associate membership.

HEADQUARTERS COMMITTEE

It seemed to the committee that conditions at the headquarters of the Institute were such that a considerable amount of renovation work should be carried out during the year. The finance committee, therefore, brought the matter before the board of directors at the meeting on January 30, 1941, and authority was given to proceed with the work as proposed, up to the limit of certain funds in the depreciation accounts which were immediately available for the purpose.

As of this writing the work is nearly completed. The committee realizes that tastes vary so greatly concerning furniture and decorations that no unanimity of opinion can be expected regarding changes of this kind. It hopes, however, that the membership in general will consider the results to represent a satisfactory handling of the situation within the limits of the available funds.

The committee has been especially fortunate in being able to draw on the professional experience and judgment of R. H. Hose of the Bell Telephone Laboratories. He proposed the general plan of redecoration, prepared the sketches which were submitted to the board in January, and has co-operated closely in carrying out all phases of the work. The committee is greatly indebted for a large amount of his time and for his continuous interest and guidance.

COMMITTEE ON PLANNING AND CO-ORDINATION

The committee has had to hold only one meeting during the year, its predecessor for the preceding administrative year having completely cleared a rather extensive accumulated docket and rendered comprehensive reports to the board of directors during last year.

In addition to the usual recommendations to the board of directors with respect to the schedule of meetings for the calendar year 1942, the committee considered and advised the board concerning a proposed new technical committee, dues which Members for Life continue to pay, proposed review of past applications of electricity in warfare, and the use of United States standards in South America.

COMMITTEE ON CONSTITUTION AND BYLAWS

There were several amendments to the bylaws. One extended the period of time in which Enrolled Students may apply for admission to Associate grade without payment of entrance fee, particularly to affect students graduating in the January class, or at times other than the usual June commencement. A section was added to the bylaws covering the Charles LeGeyt Fortescue Fellowship committee; also a section covering the committee on research, in view of its change in status from a technical committee to a general committee.

Minor revisions were made with regard to the listing of committees.

COMMITTEE ON CODE OF PRINCIPLES OF PROFESSIONAL CONDUCT

The committee has carried on some correspondence, but no matters have required immediate action. The chairman is also chairman of the joint committee on principles of ethics, now sponsored by the Engineers Council for Professional Development, which expects to have a report ready for presentation in the fall.

COMMITTEE ON SAFETY

It was reported last year that a subcommittee had been appointed to develop further contacts with the colleges and Student Branches on safety. This effort resulted in a bulletin entitled "Suggestions for Meetings on Safety", which was mailed to the chairman of each Student Branch and to each Student Branch counselor on September 23, 1940. The purpose of the bulletin was to urge the Student Branches to hold one or more meetings on safety during the school year, and it gave them numerous suggestions about organizing such meetings, sources of speakers, and references to safety codes and other publications on the subject. Reports indicate that numerous Student Branches have found this bulletin useful, and it is intended to mail a similar bulletin to them again next September.

A letter was sent also to the Sections of the Institute urging upon them the importance of greater attention to safety and suggesting that at least one meeting be held on this subject.

The committee has asked the technical program committee to call to the attention of each of the national technical committees the importance of greater attention to safeguarding life and property from electrical hazards. In periods of high industrial activity, there is always a tendency toward an increase in the accident rate resulting in an increase in lost time and damaged material.

All these steps have been taken as parts of a co-ordinated effort to aid the national defense preparations by conservation of man power and materials through increased safety measures.

During the year, the committee sponsored two papers which were presented before conventions of the Institute. One was a paper by W. B. Kouwenhoven, D. R. Hooker, and O. R. Langworthy, entitled "A Comparison of the Relative Efficiency of the Schafer and Pole-Top Methods of Artificial Respiration" which was presented at the Middle Eastern District meeting in Cincinnati, Ohio. This is a valuable contribution to the fund of knowledge on this important subject. The other was a paper by C. F. Dalziel, J. B. Lagen, and J. L. Thurston, entitled "Electric Shock", presented at the 1941 winter convention. It presents valuable data regarding the let-go values of currents which human beings can withstand. The committee believes there is need for further data of this kind, not only with respect to the let-go currents human beings can withstand, but also as to the susceptibilities of animals to electric currents.

The committee wishes to stress the importance of proper maintenance of electrical apparatus and machinery as an important factor in continued safe operation. During periods of high production when plants are being operated long hours under heavy loads, increased attention to maintenance is necessary to avoid injuries, lost time, lost production, and damaged materials caused by avoidable failures.

Active attention to these and many other problems is planned for the coming year.

COMMITTEE ON RESEARCH

The change in status of the committee on research from that of a technical committee to that of a general committee of the Institute was authorized by the board of directors at its meeting on May 24, 1940, and appropriate changes in the bylaws, section 84-A, were adopted by the board on January 30, 1941. This change has relieved the committee of certain duties that it previously discharged in connection with the review of papers.

The principal activity has been that of studying the program of Engineering Foundation research projects sponsored by the Institute. For the purpose of developing more clearly their relation to the broad problems of electrical engineering, two projects have received particular attention. These are a research on the stability of impregnated-paper insulation, project 66, which has been under way for some years at the Johns Hopkins University, under the immediate direction of Doctor J. B. Whitehead. Another is a research on insulating oils and cable saturants, project 74, under way at the Massachusetts Institute of Technology, under the immediate supervision of Professor J. C. Balsbaugh. Each of these projects is supervised by an advisory committee which meets several times a year, considers in detail the status of the project, and prepares recommendations as to future programs.

Recommendations and reports of these advisory committees have in turn been considered by the committee on research before submitting them to the board of directors for consideration. The committee has not to date undertaken a review of the welding research program of Engineering Foundation jointly sponsored by the Institute and the American Welding Society. This has not been from lack of interest or sympathy but because it was felt that the direction of this activity is being ably handled by existing committees. It is suggested, however, that succeeding committees on research consider this matter in order to insure effective integration of the Institute's research activities.

Although it is still too early to judge the full value of the change in status of the committee, it is believed that experience to date has demonstrated the desirability of the change. Should the general national situation become such as to lead to special calls on the Institute for activities in the field of research, the value of the committee will be still further apparent.

Standards

STANDARDS COMMITTEE

During the past year there has been a noticeable expansion of standards activity. This probably is due to increased industrial activity brought about by the national defense program. The American Standards Association has taken formal action urging all possible speed in standards development as an effective aid in that program. In line with this policy, it may be pointed out that proposed revisions of many AIEE standards have recently become available. Among these may be mentioned No. 4, "Measurement of Test Voltages in Dielectric Tests"; No. 21, "Apparatus Bushings"; No. 22, "Air Switches and Bus Supports"; No. 24, "Protector Tubes"; No. 27*A*, "Switchgear Assemblies"; No. 25, "Fuses Above 600 Volts"; No. 28, "Lightning Arresters"; No. 41*A*, "Insulator Tests"; and No. 45, "Recommended Practise for Electrical Installations on Shipboard". The call for this last standard, No. 45, is growing rapidly and the demand for it probably will become even greater, as compliance with its requirements is now a part of nearly all shipbuilding specifications. In addition to the preceding strictly AIEE activities, sectional committees working under ASA procedure have issued, or are about to offer, several standards and proposed standards. One on "A-C Power Switchgear" and one on "Transformers, Regulators, and Reactors" are now available, and revisions of all the various letter and graphical symbols standards are practically completed.

The sectional committee on definitions of electrical terms is rapidly bringing its work to a point where the final step necessary to obtain approval as an American Standard can be taken. The appointment of a new chairman, C. H. Sanderson, to replace Doctor A. E. Kennelly, deceased, has been made, and the addition of a man to the AIEE staff to devote the major part of his time to this work has greatly expedited the compilation of the immense amount of data involved in the completion of this report.

In the test-code field, activity has also increased. A revised edition of the "D-C Test Code" is under way, and the development of a code for electrical measurements is being considered. A test code is also offered as part of the proposed pamphlet on "Bushings". A specification for wet tests is about to be published. The Institute has appointed a representative to serve on a sectional committee on "Code for Safety in Quarry Operations".

Following out Past-Chairman Hellmund's plan for a more closely knit organization aimed to bring about close co-operation among various standardizing groups and proper co-ordination of the results of their work, a seventh co-ordinating committee has been appointed, on conduction in vacuum, gases, liquids, and solids. Further development of the "Standards Manual" is also under way. It is expected this manual will promote a closer understanding of available standardizing machinery and the interrelationships of the various committees engaged in that work.

Another question of considerable import coming up for discussion during the year was the establishment of a proper policy with regard to withdrawal of standards when proposed revisions become available. It was finally agreed that, in order to preclude the possibility of arguments between manufacturers and customers on adherence to existing standard practice, a standard should remain in force until actually replaced by a formally accepted revision.

Under ASA procedure, it is customary to classify the membership of sectional committees according to company affiliation. However, it has always been the Institute's policy to consider that its committee appointees are representative of independent interests and that they would, whenever practical, render decisions from that viewpoint. The ASA therefore was formally requested in future to classify Institute appointees as independent interests unless advised otherwise.

In view of the present American policy to strive for closer relationships between the United States and South and Central American countries, steps were taken to ascertain the feasibility of translating into Spanish and Portuguese such United States electrical standards as would have the greatest application in the development of trade in the electrical field. This work, if carried through, probably will be in co-operation with the United States Department of Commerce.

UNITED STATES NATIONAL COMMITTEE OF THE IEC

At its annual meeting in December 1940, the United States National Committee of the International Electrotechnical Commission re-elected its officers: E. C. Crittenden, president, L. F. Adams and H. S. Osborne, vice-presidents. Doctor C. H. Sharp continues as honorary president.

Due to the international situation, the work of the IEC is practically in a state of suspension. The central office of the commission, however, has not been disbanded, so that as soon as circumstances permit it will be possible to resume activities. The United States National Committee, as well, is keeping its organization intact, and will continue to hold an annual meeting.

At the annual meeting of the USNC, there was a full discussion of the desirability and possibility of the USNC taking the lead in developing international electrical standardization work between the countries of North and South America. The result of this discussion was the appointment of a special committee under the chairmanship of President Crittenden, of the USNC, with L. F. Adams, H. S. Osborne, and Frank Thornton, Jr., as members, to explore the whole situation and to make contacts with the Pan-American Union, proper officers and departments of the Federal Government, technical societies and trade associations, and others, and to make recommendations to the USNC on the whole subject. This action of the USNC was readily concurred in by the ASA. The AIEE committee on planning and co-ordination has also appointed this same group, with the addition of D. M. Simmons, as its committee to investigate the same subject.

Technical Committees

COMMITTEE ON AIR TRANSPORTATION

The establishment of this new technical committee was authorized by the board of directors on August 2, 1940, and the organization was completed in October. At the first meeting on November 15, the committee discussed the scope of its activities, recommended co-operation with governmental agencies in defense problems, and began the study of papers for the winter convention.

The committee has made special efforts to secure an adequate number of papers of high quality for presentation at national conventions and District meetings, but has had great difficulty on account of the close relationship of developments in its scope of activities to national defense, with the resulting restrictions on dissemination of information. It has instigated efforts to increase the membership in the Institute of qualified persons in the aircraft and associated fields.

On recommendation of the committee,

it was authorized by the board of directors to offer its services to governmental agencies in connection with the application of electrical equipment to aircraft. Co-operation with several agencies has been established.

COMMITTEE ON AUTOMATIC STATIONS

The committee prepared a bibliography for the period 1932–40, inclusive, containing approximately 800 references on supervisory and remote control, telemeters and telemetry, automatic and remote control switchgear, automatic features of generating stations using fuels, automatic boiler and combustion control, automatic hydroelectric plants, automatic substations (installations).

Several members of the committee, as members of the joint subcommittee on telemetering and supervisory control, collaborated in the preparation of a report on the latter subject (see report of committee on instruments and measurements).

The committee approved proposed standards for nominal voltages below 100.

The committee sponsored the paper on "Synchronizing Transients and Synchronizers for Large Machines" by R. D. Evans, F. H. Gullicksen and C. B. Myhre, published in the 1940 Transactions, pages 965–73.

In co-operation with the relay subcommittee of the protective devices committee, the committee on automatic stations has sent out a number of questionnaires covering the use of pilot conductors in supervisory control, telemetering, and calling and signalling systems. These data will be collected, with similar data pertaining to protective relays.

COMMITTEE ON BASIC SCIENCES

Papers reviewed and recommended by the committee included 3 for the 1940 summer convention, 7 for the 1941 winter convention, and one for the North Eastern District meeting in Rochester, N. Y. The committee also held a technical conference at the summer convention, and two such conferences and a meeting of the committee at the winter convention.

Considerable progress was made by the subcommittee on definitions of electrical terms.

The committee has approved 4 papers for a joint session with the subcommittee on electronics at the 1941 summer convention.

COMMITTEE ON COMMUNICATION

The committee on communication held three meetings during the year. Much additional business was conducted by correspondence. The committee arranged communication programs at the summer and Pacific Coast conventions, in 1940, and the 1941 winter convention, and assisted in obtaining material for the Middle Eastern District meeting in Cincinnati. The 15 papers sponsored by the committee at these meetings dealt with many subjects—telegraphy, telephony, radio, electronics, television—and nearly every aspect of the communication art and science was represented to some extent in these papers. Programs have also been arranged for the 1941 summer and Pacific Coast conventions.

COMMITTEE ON DOMESTIC AND COMMERCIAL APPLICATIONS

A paper by L. C. Packer, Associate AIEE, entitled "Design Factors Involved in the Design of Domestic Motor Appliances", was presented at the Middle Eastern District meeting, Cincinnati, Ohio, October 9–11, 1940. A paper by W. J. Russell is now being reviewed, and a third by W. H. Delancey has been submitted, both the latter being intended for presentation at the summer convention in Toronto, Canada, in June 1941. Two or three other papers have been promised, but have not yet been produced.

A conference period has been arranged for at the summer convention and appropriate arrangements have been made for prepared discussions to open this conference.

No standardization activities have been undertaken by this committee during the fiscal year. The committee has indicated that it will not require any budget appropriation for the ensuing fiscal year.

COMMITTEE ON EDUCATION

The committee held a meeting at the 1941 winter convention, and is planning to hold a conference at the summer convention.

Three subcommittees were continued from the previous year. Replies to the questionnaire of the subcommittee on student guidance were received from 58 Sections. The subcommittee has transmitted definite suggestions to the Sections interested, and expects to report on its activities at the summer convention conference of officers, delegates, and members.

COMMITTEE ON ELECTRIC WELDING

The activities of the committee on electric welding have been largely those of a subcommittee on power supply for welding operations. This subcommittee has recommended that the National Electrical Code be amended to cover specific requirements for resistance-welder installations. At present, the code makes no mention of resistance welders, and it becomes necessary for the various local authorities to make their own interpretations as they think may apply to resistance welders. Resistance welders are far different from ordinary power and light load, and the necessary wiring, fusing, and service facilities must be provided on a somewhat different basis. The subcommittee prepared suggested paragraphs covering conductor sizes for branch circuits and feeders supplying resistance welder loads which were transmitted by the secretary of the standards committee to the chairman of the sectional committee on National Electrical Code.

The subcommittee has prepared purchase inquiry forms and manufacturers' data sheets which should prove of mutual aid to both the users and the manufacturers in reaching a common understanding as to just what type and size of welding machine is needed for a given job, as well as furnish complete power supply data to the users' electrical department and the local power company. The forms cover both machine mechanical data and power-supply data. The subcommittee is concerned particularly with the sections covering power-supply data, and thinks that by proper use of these forms the providing of an adequate power supply for any welding operations will be greatly facilitated. While the forms covering the mechanical data are not the direct concern of this subcommittee, it is felt that they should be included with those covering the power-supply data. Copies of the purchase inquiry forms and manufacturers data sheets have been submitted to the secretary of the Resistance Welder Manufacturers Association, and are being distributed to its entire membership for their suggestions.

The subcommittee, during the past two years, has spent considerable time in the preparation of a report on power supply for resistance welding machines, and has completed Section III covering "Factory Wiring for Resistance Welders". Section I, "Guide to Good Electrical Performance of Resistance Welding Machines" and section II, "Resistance-Welder Installations" have been printed and made available for distribution. Section III was presented at the North Eastern District meeting, April 30–May 2, and published in the 1941 Transactions (May section) pages 185–92.

COMMITTEE ON ELECTRICAL MACHINERY

The subcommittee on synchronous machinery has been extensively engaged in work on revision of the "Test Code for Synchronous Machines". Substantial progress has been made, and it is expected that the revision will be ready for submission to the standards committee during the ensuing fiscal year.

The subcommittee on transformers prepared a very comprehensive report on the protection of power transformers against lightning surges, which was presented at the winter convention. This report is a result of several years of study by the subcommittee, and furnishes information as to the degrees of protection possible and the various methods in common use. The subcommittee has also given careful study to the proposed ASA Standards, and through membership on the ASA sectional committee is suggesting recommendations for changes and improvements. The subcommittee is also studying proposed changes in the test code section of the ASA Standards.

The d-c subcommittee has revised the "Test Code for D-C Machinery" following suggestions of the standards committee and incorporated the "Test Code for Fractional Horsepower D-C Motors." The test code has been submitted to the standards committee with a recommendation that it be published for trial use.

The subcommittee on test codes for single-phase motors has prepared a final draft of the "Test Code for Single-Phase Motors" which is practically in final form. It is expected that this test code will be submitted to the standards committee with a recommendation for publication for trial purposes during May 1941.

During the year, a co-ordinating committee consisting of the chairmen of various other subcommittees was organized to aid in co-ordinating work of the various test codes. Professor H. N. Walker of the committee on instruments and measurements was added to this subcommittee as a liaison member from that committee.

One session at the 1940 summer convention was sponsored by the committee on electrical machinery, in which 4 papers were presented. At this convention, a technical conference was devoted to test codes. At the Middle Eastern District meeting held in Cincinnati in October 1940, 1 session was devoted to electrical machinery, at which 5 papers were presented. At the 1940 Pacific Coast convention in Los Angeles, Calif., in August 1940, 1 session was devoted to electrical machinery, at which 4 papers were presented. At the 1941 winter convention, 2 sessions were sponsored by the committee, at which 9 papers were presented. At the North Eastern District meeting in Rochester, N. Y., April 30 to May 2, several papers recommended for presentation by the committee were presented, although under the auspices of other committees. At the 1941 summer convention in Toronto, it is planned to have 1 session sponsored by the committee at which 4 or 5 papers are planned for presentation. Some other papers have been reviewed and are available.

COMMITTEE ON ELECTROCHEMISTRY AND ELECTROMETALLURGY

With the approval of the board of directors, the committee arranged for a joint afternoon technical session and a joint evening technical conference with the American Society for Metals on the last day of the 1941 winter convention. Two AIEE papers were presented in the afternoon, and two addresses were given by members of the ASM in the evening.

A subcommittee for the investigation of voltage transients in the operation of arc furnaces had been organized in March 1940, and the first meeting was held in October 1940. At this meeting, it was decided to enlarge the subcommittee considerably by the addition of advisory members, including representatives from the furnace manufacturers. Plans were also made for extensive field investigations.

A progress report was presented at the second meeting of the subcommittee, held at the winter convention. It is expected that a partial report of findings of the subcommittee will be available in about 6 months and a complete report in 12 months. The investigation so far has covered conditions under which voltage transients are produced, their magnitude, probability, and methods of prevention.

A meeting of the committee was held at the winter convention. It was decided to arrange for a joint session with the Association of Iron and Steel Engineers for May 27, 1941. The program for this meeting is already in final shape. It will consist of a special joint meeting of the Cleveland and Pittsburgh Sections of the AIEE and the AISE, and will be sponsored by the committee on electrochemistry and electrometallurgy.

Plans are under way for a report on progress in the field of electrochemistry and electrometallurgy, to be available in the spring of 1942.

COMMITTEE ON INSTRUMENTS AND MEASUREMENTS

During the year, the activities of the committee consisted primarily of the review of papers, the arrangements for their presentation at conventions, the sponsorship of a technical conference, the preparation of reports on the progress of the art in the field of instruments and measurements, and the revision of test codes, definitions, and standards. Seventeen papers have been submitted in whole or synopsis form for review by the committee.

Technical sessions were held at the summer and winter conventions, and a technical conference on dielectric measurements in the field was held at the summer convention. Seven prepared reports were presented at this conference, and between 50 and 60 members were present.

A report on "Telemetering, Supervisory Control, and Associated Circuits" was completed by a joint subcommittee of the committees on instruments and measurements and on automatic stations. This report contains a classification of 53 of the numerous types of telemetering and supervisory control equipment now manufactured, a description of each type, and a comparative tabulation of the characteristics of all. The report has been approved by both technical committees and sent to the publication committee for publication in pamphlet form. It is hoped that the report may be the basis for a standard on its field in the future.

A report on "Progress in the Art of Metering Electrical Energy" is nearing completion. No previous report on the subject having been written by the Institute, the scope of this report is wider than that of others, and the development of electrical-energy measurement is traced from its inception down to the present time. The report is the result of collaboration by authorities in various fields—educational, manufacturing, operating, regulatory, and scientific. It should be completed before the 1941 summer convention, and it is hoped that it will be published both in ELECTRICAL ENGINEERING and in pamphlet form.

A report entitled "Wave Form in Dielectric Power Factor Measurements", on the methods of specifying the voltage wave form of the power source employed in measurements of the power factor of dielectrics, was completed and published in ELECTRICAL ENGINEERING, June 1940, page 255.

A subcommittee has been studying the test codes prepared by the committee on electrical machinery, and suggestions for revision of certain parts of the codes pertaining to instruments and measurements have been passed on to the committee on electrical machinery. As a result of this work, a representative of the committee on instruments and measurements has been appointed to the subcommittee of the committee on electrical machinery that is preparing the test codes, and a subcommittee of the committee on instruments and measurements has been set up to prepare a master test code on resistance measurements. Work on a master test code on temperature measurements has been started.

The definition of the galvanometer has been modified, and the name "phase-shifting transformer" approved to describe the miniature transformer used to adapt wattmeters and watt-hour meters to reactive power measurement. A subcommittee on electrical definitions is attempting to coordinate definitions in the *C42* report, the "Electrical Metermen's Handbook," and the "Code for Electricity Meters."

The definitions of the terms "instrument" and "meter" have been reconsidered and modified to broaden the scope of the latter.

A new subcommittee on accuracy classification of instruments and instrument standards has been set up recently to attempt to formulate improved methods of stating the performance or accuracy of indicating instruments. This subcommittee is the newest of 11 now active. As a means of coordination there are representatives of the committee on instruments and measurements on the joint subcommittee on electronics, the relay subcommittee of the committee on protective devices, the test-code subcommittee of the committee on electrical machinery, and 3 subcommittees of the standards committee.

A report covering most of the more important work of the committee for the past few years was given by the secretary at the winter convention, and was published in ELECTRICAL ENGINEERING, March 1941, pages 133–4.

COMMITTEE ON LAND TRANSPORTATION

Three meetings were held during the past fiscal year: one at the summer convention, Swampscott, Mass., one in Chicago, Ill., in October, and the last at the winter convention, in Philadelphia, Pa.

The name of this committee was changed from committee on transportation to committee on land transportation, at the time the board of directors established a committee on air transportation.

The committee has four subcommittees, which enable it to cover adequately the land-transportation field in which practically all the papers must be solicited. Standardization along the lines of the PCC car makes it almost impossible to bring forth new papers in the light-traction field. However, at the winter convention, in January, a paper on "The Electro Gear—A New Electromechanical Transmission", by Doctor Ernst Weber, aroused a tremendous amount of interest in the field of urban transportation. Considerable space was devoted by the various transportation publications to comments on Doctor Weber's paper.

COMMITTEE ON MARINE TRANSPORTATION

In July 1940, AIEE Standard No. 45, "Recommended Practise for Electrical Installations on Shipboard", was reprinted to include a complete revision, the detail work for which was accomplished last year.

Due to the pressure of work incident to the current large shipbuilding program, there has been little activity this year, although at our committee meeting on January 10, 1941, all business at hand was transacted, including future changes in Standard No. 45 to be handled by addenda sheets.

The principal item of business of this committee is to keep Standard No. 45 up to date for use by shipbuilding and other organizations. However, it is hoped that when time becomes available, some definite activity on standards for equipment may be undertaken.

COMMITTEE ON POWER GENERATION

Eleven technical papers were presented under the sponsorship of the committee: summer convention, 2; Middle Eastern District meeting, Cincinnati, 1; and winter convention, 8.

A conference, similar to that at the 1940 summer convention in Swampscott, Mass., was held at the Pacific Coast convention, at which the same and additional material was presented.

More than the usual amount of interest was shown in the session of the committee at the 1941 winter convention, the attendance for both sessions being exceptionally large. The committee has given most of its attention during the past year to the study and discussion of subjects relating to governor design and operation, load fluctuations, system stability, and associated phenomena.

The studies which have been carried on during 1940 are being continued into the 1941 summer convention in Toronto by conference studies. Material is also being prepared for a general discussion in Toronto covering preparedness in its relation to power generation.

Steps are being taken toward the formation of a joint committee with The American Society of Mechanical Engineers on the standardization of the design of the governor and its associated equipment.

COMMITTEE ON POWER TRANSMISSION AND DISTRIBUTION

The committee held two meetings during the year. In addition to transaction of routine business at these meetings, there were general engineering discussions of a number of matters relating to transmission and distribution. This review of engineering development is becoming an extremely valuable part of the committee activity.

The committee is now operating on the basis of the new organization, under which there are four subcommittees: transmission, distribution, stations, and general system.

During the year, Standard No. 41, Insulator Tests, was approved for printing as a tentative standard. It is expected that, after this tentative standard has been in use one year, a decision will be reached with respect to printing as an ASA standard. The committee is now engaged in the task of preparing a revised report on grounding standards.

The committee sponsored 1 technical session at the summer convention, and 2 technical sessions at the winter convention, involving the presentation of 11 papers, and in addition there were 7 papers presented in other sessions jointly sponsored. The committee also co-operated in arranging for the presentation of technical papers at District meetings during the year.

COMMITTEE ON PRODUCTION AND APPLICATION OF LIGHT

The committee has continued during this fiscal year to keep members of the Institute informed on the latest developments in illuminating engineering, principally through the sponsorship of conferences at District and national meetings and by obtaining papers from qualified authors on illumination subjects for publication in ELECTRICAL ENGINEERING. All of these activities have been intensified this year, and plans are now under way to further extend them, especially in those lighting fields which are vital to national defense.

The committee held a protective lighting conference at the Middle Eastern District meeting, Cincinnati, Ohio, October 10, 1940, and a fluorescent lighting conference at the winter convention, Philadelphia, Pa., January 29, 1941. A complete report on each of these conferences appeared in ELECTRICAL ENGINEERING, and the prepared discussions at the fluorescent lighting conference are scheduled for publication in the June 1941 issue of ELECTRICAL ENGINEERING, because of their timely interest to the Institute membership.

A conference on protective lighting is now under consideration for the summer convention in June 1941. Prominent speakers from Canada and the United States will be asked to prepare discussions on this subject which is vital to the defense of each country.

In addition to reviewing all papers on the subject of lighting which are presented for consideration by the technical program committee, this committee also selects timely lighting subjects and encourages competent authors to prepare papers for ELECTRICAL ENGINEERING.

The committee met only once during the year, since membership is rather widely scattered, and activities can well be carried on by mail after plans have been settled for the year.

COMMITTEE ON PROTECTIVE DEVICES

During the past year, 15 new members were added to the committee, and 10 retired. Representation was added from the Southern, Pacific, and North West Districts, with a corresponding reduction in certain other Districts. The representation from the various divisions of work within the committee's scope was kept as broad as possible.

The committee activities have been divided so that each member has some assignment, and excellent co-operation and valuable assistance have been given by the subcommittee chairmen and their various individual members. The four main subcommittees are: circuit breakers, switches, and fuses, fault-current limiting devices, lightning arresters, and relays.

The subcommittee on circuit breakers, switches, and fuses has been active on a number of important projects including the review of Standard No. 19, "Circuit Breakers", to incorporate air circuit breakers and new insulation test levels. Proposed Standard No. 25, "Fuses" has been studied to incorporate the new insulation test levels. Studies have been made of voltage recovery rates, oil for impulse testing, and short-circuit calculation practices for the application of interrupting devices. Reports on the latter two subjects have been completed, and are scheduled for presentation at the summer convention. The recent adoption of basic insulating levels calls for a reconsideration of voltage tests on certain electrical equipment. The present test values for protective apparatus, especially in the lower voltage classes, is being given study, and in some cases may require modification.

Standard No. 22, "Air Switches and Bus Supports" was printed in June 1940, and issued for one year's trial use. Standard No. 24, "Protector Tubes" was printed and issued in August 1940, for one year's trial. Standard No. 27, "Enclosed Switchgear" has been completed, approved by the AIEE standards committee, and soon will be printed and distributed for one year's trial use as a revised standard.

A standard for neutral grounding devices has been completed and submitted to the standards committee for approval and publication as a proposed standard. The subcommittee on fault-current limiting devices is co-operating with other technical committees in the review of the "Grounding Practices Report".

Standard No. 28, "Lightning Arresters", has been revised, approved, and submitted to the standards committee for adoption. This material has been passed on to ASA sectional committee *C*-62 for consideration as a revised ASA standard.

Additional data have been obtained, particularly at high values of discharge current, concerning the performance characteristics of line-type lightning arresters rated 20 kv to 73 kv inclusive, and assembled in the form of a report for presentation at a forthcoming meeting. Work is in progress toward the development of a standard for distribution type protector tubes.

The subcommittee on relays has carried on a number of important projects including the standardization of potential devices and instrument transformers; studies of pilot wire relaying experience; sensitive ground protection; automatic recording devices, and out-of-step protection. A compilation is being prepared to show the desired scope of manufacturers' catalog information. A bibliography of relay literature was prepared last year and this is now in process of being printed and made available to the industry.

The special representatives on other committees have co-operated in the furtherment of insulation co-ordination and the application of new developments in connection with contact surfaces.

The various subcommittees collaborated last year in the preparation of material for use as a report "Progress in the Art". This is now being brought up to date, and arrangements have been made for its publication in ELECTRICAL ENGINEERING, in three sections entitled "Circuit Interrupting Devices", "Relaying and Fault-Current Limiting Devices", and "Lightning Protection and Miscellaneous".

The committee has sponsored technical papers as follows: 1940 summer convention, 6; Middle Eastern District meeting, Cincinnati, 1; Pacific Coast convention, Los Angeles, 4; winter convention, 4 on circuit breakers, 5 on relays and other subjects, and 5 on transmission and protective-devices subjects (joint with committee on power transmission and distribution); and North Eastern District meeting, Rochester, 1. A number of additional papers are available for subsequent meetings, and 10 will be presented at the 1941 summer convention.

The committee held four meetings and a fifth is scheduled for May. These meetings have been well attended, and the discussions have consisted largely of committee activities, action regarding proposed standards, basic insulation levels, temperature rise and ambient temperatures for enclosed switchgear, presentation of subcommittee reports, and consideration of papers for future programs.

It is expected that the future work of the committee will follow along similar lines, completing the several standards which are now in preparation on circuit breakers, fuses, potential devices, instrument transformers, protector tubes, and so on. In view of the great interest being shown in this work as evidenced by attendance at the technical sessions and at the committee meetings, and the large number of papers proposed, it would appear that the work is being carried forward along useful and progressive lines.

Awards

COMMITTEE ON AWARD OF INSTITUTE PRIZES

Four national and ten District prizes were awarded for papers presented during the calendar year 1940 and for the student papers presented during the academic year ending June 30, 1940.

The committee considered a large number of eligible papers which were of a high order, and, in determining the awards for the national best paper prizes and initial paper prize, it had the benefit of the gradings and recommendations of the technical committees, which had reviewed the papers. In addition to the national paper prize awards, three other papers were given honorable mention.

The pamphlet "National and District Prizes" has been revised, and a new edition of January 1941 issued. The principal changes establish a new basis of grading for Branch and graduate paper prizes, and empower the District executive committees to fix the dates of closure and of award of the District prize for Branch paper and the District prize for graduate paper. The new basis of grading recognizes that the objective of student papers differs from that of adult papers, and permits the placing of greater emphasis on the written presentation and on the exercise of engineering thinking than on technical value. These changes were made in co-operation with the committee on Student Branches and the recommendations of the conference of Student Branch counselors held during the summer convention, in Swampscott, Mass.

EDISON MEDAL

The Edison Medal, which is awarded by a committee composed of 24 members of the Institute, was awarded for 1940, to George Ashley Campbell, "in recognition of his distinction as scientist and inventor and for his outstanding original contributions to the theory and application of electric circuits and apparatus", and was presented on January 29, 1941, during the winter convention. The medal may be awarded annually for "meritorious achievement in electrical science, electrical engineering, or the electrical arts".

LAMME MEDAL

The Lamme Medal committee awarded the medal for 1940 to Comfort A. Adams, consulting engineer, Edward G. Budd Manufacturing Company, "for his contributions to the theory and design of a-c machinery and his work in the field of electric welding". Arrangements are being made for the presentation of the medal at the summer convention in Toronto, Ont., Canada, June 16–20, 1941. The medal may be awarded annually to a member of the AIEE "who has shown meritorious achievement in the development of electrical apparatus or machinery".

COMMITTEE ON AWARD OF COLUMBIA UNIVERSITY SCHOLARSHIPS

Due to the very large demand for young engineers by the industries there was not as much interest displayed in the scholarship as in previous years, and the few applications received were from students who either could not afford to undertake the expense of living in New York City, even with their tuition covered by the scholarship, or did not meet the specifications for previous work. Therefore, the scholarship was not awarded for this academic year.

In view of the very uncertain conditions in the coming academic year, and the fact that the great majority of engineering graduates are being taken by the industries and the selective service, the University authorities decided not to offer the scholarship during the academic year 1941–42. This is the second time in the many years since the scholarship was established that no award has been made.

CHARLES LE GEYT FORTESCUE FELLOWSHIP COMMITTEE

The committee, at its meeting in March, decided that no award would be made for the scholastic year 1941–42, because of the national emergency and the limited number of candidates making application for the fellowship. The demands for young men in industry and in the various branches of military service apparently have taken many who might otherwise have applied for the fellowship. It was also thought that after the present emergency is over the accumulated funds may be more useful in helping deserving young men readjust themselves to normal industrial life.

JOHN FRITZ MEDAL

The John Fritz Medal board of award, composed of representatives of the national societies of civil, mining and metallurgical, mechanical, and electrical engineers, awarded the 37th medal (for 1941) to Ralph Budd, president, Chicago, Burlington, and Quincy Railroad, for "improvement of railroad tracks and service, especially the introduction of lightweight streamlined trains".

HOOVER MEDAL

The Hoover Medal was established through a trust fund created by a gift from Conrad N. Lauer, and is to be awarded periodically "to a fellow engineer for distinguished public service" by a board representing the national societies of civil, mining and metallurgical, mechanical, and electrical engineers. The fourth medal was awarded in 1939; no awards have been made since that time.

ALFRED NOBLE PRIZE

This prize, established in 1929, consists of a certificate and a cash award of $500 from the income from a fund contributed by engineers and others to perpetuate the name and achievements of Alfred Noble, past president of the American Society of Civil Engineers and of the Western Society of Engineers. It may be made to a member of any of the co-operating societies, American Society of Civil Engineers, American Institute of Mining and Metallurgical Engineers, American Society of Mechanical Engineers, AIEE, or Western Society of Engineers, for a technical paper of particular merit accepted by the publication committee of any of these societies, provided the author, at the time of such acceptance, is not over 30 years of age. No award was made during the past year.

WASHINGTON AWARD

The Washington Award for 1941 was bestowed upon Ralph Budd, "for vision and courageous leadership in advancing the technological frontiers of high-speed railroad transportation", and was presented to him at a dinner on February 24, 1941. This award may be made annually to an engineer by the commission of award composed of nine representatives of the Western Society of Engineers and two each of ASCE, AIME, ASME, and AIEE.

Joint Activities

UNITED ENGINEERING TRUSTEES, INC.

This organization is the corporate body which holds title in the name of the four Founder Societies to their joint physical properties, namely, the Engineering Societies Building, the Engineering Societies Library, and the endowment funds of the Engineering Foundation. It operates and manages the Engineering Societies Building and administers certain joint activities of the four Founder Societies.

Extensive studies have been made in efforts to improve the service of the Engineering Societies Building to the members of the societies. Constant efforts have been made to maintain and improve the security of the investments of the corporation, and, to such extent as consistent therewith, to obtain a reasonable income. An abstract of the annual report of the corporation for the year which ended September 30, 1940, appeared on pages 49–50 of ELECTRICAL ENGINEERING for January 1941.

ENGINEERING FOUNDATION

The Engineering Foundation is a joint organization of the national societies of civil, mining and metallurgical, mechanical, and electrical engineers established for "the furtherance of research in science and engineering, and the advancement in any other manner of the profession of engineering and the good of mankind".

The Foundation assists in a wide range of research projects of interest to science, engineering, industry, and the general public, each project being under the sponsorship of one of the Founder Societies. Results of the researches are published by the engineering societies, and other organizations. It has continued its financial support of the work of the Engineers' Council for Professional Development.

Under the sponsorship of the Institute the research on stability of impregnated-paper insulation, at the Johns Hopkins University, and the research on insulating oils and cable saturants, at the Massachusetts Institute of Technology, were carried forward with good results. Good progress was made also on the welding research under the joint sponsorship of the American Welding Society and the Institute. A report on this work appeared on page 91 of ELECTRICAL ENGINEERING for February 1941.

A comprehensive abstract of the annual report of the Engineering Foundation for the year which ended September 30, 1940 appeared on pages 50–51 of ELECTRICAL ENGINEERING for January 1941.

In the death of Doctor Otis E. Hovey, its director, on April 14, 1941, the Engineering Foundation suffered a great loss.

ENGINEERING SOCIETIES LIBRARY

The Engineering Societies Library, which was formed by combining the separate libraries of the four national societies of civil, mining and metallurgical, mechanical, and electrical engineers, and the preparation of a composite card catalog, has been expanded as a single engineering library, which probably constitutes the best collection of its type in the United States.

On September 30, 1940, the library had 149,734 volumes, 7,636 maps, and 4,473 bibliographies. Books and pamphlets totaling 5,587 were received during the year ending at that time. Current issues of 1,292 periodicals were received. Over 11,000 entries were added to the index of periodicals, which now contains over 250,000 cards.

Special services rendered by the library include: photoprints, searches, abstracts, translations, bibliographies, book loans by mail, etc. An abstract of the annual report of the library appeared on page 51 of ELECTRICAL ENGINEERING, January 1941.

EMPLOYMENT SERVICE

The boards of the four Founder Societies authorized the incorporation of the jointly operated employment service in the state of New York, as a nonprofit making organization, with the name "Engineering Societies Personnel Service, Inc." The main office is in the Engineering Societies Building, New York.

In the summer of 1940, an office of this new corporation was established in Detroit in co-operation with the Engineering Society of Detroit. As in the past, offices are operated in Chicago and San Francisco, with the co-operation of the Western Society of Engineers in the former, and the Engineers Club of San Francisco in the latter.

The service is supported by the joint contributions of the societies and their individual members who secure positions. An analysis of registration and placement records of this service as reported to the national societies is given in table XI.

ENGINEERS' COUNCIL FOR PROFESSIONAL DEVELOPMENT

This Council, organized in 1932 to engage in activities leading toward the enhancement of the professional status of the engineer, includes three representatives of each of the eight participating organizations. These are the national societies of civil, electrical, mechanical, and mining and metallurgical engineers, the Society for the Promotion of Engineering Education, the National Council of State Boards of Engineering Examiners, and the Engineering Institute of Canada.

Its principal activities have been carried on by four committees: student selection and guidance, engineering schools, professional training, and professional recognition.

At the annual meeting in October 1940, the Engineering Institute of Canada became, by unanimous vote, the eighth participating body.

Including the actions on engineering curricula taken on October 24, 1940, the record is:

Total curricula submitted, including reinspections	791
Accredited	457
Accredited provisionally	83
Not accredited	164
Reinspections with no change in status	82
Action pending	5

Table XI. Analysis of Employment Service

	Men Registered				Men Placed			
Month	New York	Chicago	San Francisco	Total	New York	Chicago	San Francisco	Total
1940								
May	193	98	119	410	43	17	22	82
June	218	143	93	454	40	16	21	77
July	221	109	89	419	54	16	19	89
August	185	65	69	319	60	29	33	122
September	193	67	50	310	57	35	21	113
October	159	68	70	297	74	31	32	137
November	136	30	51	217	57	30	25	112
December	154	29	59	242	51	33	28	112
1941								
January	162	54	81	297	63	20	38	121
February	162	58	61	281	64	25	24	113
March	197	66	67	330	44	30	43	117
April	166	65	67	298	41	31	35	107
Total	2,146	852	876	3,874	648	313	341	1,302

A complete list of accredited curricula appeared on page 523 of ELECTRICAL ENGINEERING for December 1940.

AMERICAN ENGINEERING COUNCIL

Decisions were reached by the board of directors of the Institute and by the board of one of the other Founder Societies not to continue the financial contributions of those societies to the Council beyond the end of 1940. On account of the consequent lack of assurance of the necessary financial support, the activities of the American Engineering Council as a Council were terminated as of December 31, 1940.

A special committee of the AIEE is considering plans for replacing the American Engineering Council.

IWADARE FOUNDATION COMMITTEE

No Iwadare lecturer was chosen to go to Japan for the current year, nor has any Iwadare Fellow come to the United States.

REPRESENTATIVES

In addition to the many divisions of its work represented by its own general and technical committees, the Institute participates in a wide range of activities of interest and importance to engineers and others through its representation upon about 30 joint committees and national bodies.

A list of representatives was published in the September 1940 issue of ELECTRICAL ENGINEERING and in the 1941 Year Book.

Appreciation

The 1940–41 board of directors appreciates the work of the national committees and the District, Section, and Branch officers. The board extends its sincere thanks for the effective conduct for the year of the activities in the respective divisions mentioned. Each division is important. The high degree of enthusiasm and constructive accomplishment evident everywhere shows very efficient leadership. Also the generous interest, allotment of time, and support which the members have accorded the efforts of the officers and each other is deeply appreciated. Without such unselfish and magnificent co-operation there would be no professional standing for electrical engineers such as is provided by the Institute.

Respectfully submitted for the board of directors.

H. H. HENLINE
National Secretary

May 23, 1941

HASKINS & SELLS
CERTIFIED PUBLIC ACCOUNTANTS

22 EAST 40TH STREET
NEW YORK

May 16, 1941

American Institute of Electrical Engineers,
33 West 39th Street, New York.

Dear Sirs:

We have made an examination of your balance sheet as of April 30, 1941, and of your recorded cash receipts and disbursements for the year ended that date. Our examination consisted of a review of the system of internal control and the accounting procedures of the Institute and examination or tests of its accounting records and other supporting evidence by methods and to the extent we deemed appropriate. We present the following:

Balance Sheet, April 30, 1941 (Exhibit A).
  Property and Restricted Funds Securities, Less Reserve for Securities of Doubtful Value (Schedule 1).
Statement of Recorded Cash Receipts and Disbursements of General Fund for the Year Ended April 30, 1941 (Exhibit B).
Statement of Recorded Cash Receipts and Disbursements of Property and Restricted Funds for the Year Ended April 30, 1941 (Exhibit C).

In accordance with the terms of our engagement, members and other debtors, except certain advertisers, were not requested to confirm to us the amounts receivable from them at April 30, 1941. In accordance with the usual practice of the Institute, no provision has been made for dues which may prove to be uncollectible.

In our opinion, subject to the comments in the next preceding paragraph, the accompanying Exhibit A fairly presents your financial condition at April 30, 1941, and the accompanying Exhibits B and C fairly present your recorded cash receipts and your disbursements of funds, as indicated, for the year ended that date.

Yours truly,
Haskins & Sells

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

Balance Sheet, April 30, 1941

Exhibit A

ASSETS		
Property Fund Assets:		
One-fourth interest in the total net assets of United Engineering Trustees, Inc., exclusive of Trust Funds	$498,448.48	
Equipment:		
Library—volumes and fixtures	37,296.37	
Office furniture and fixtures (less reserve for depreciation, $24,028.08)	7,518.38	
Works of art, etc.	3,001.35	
Investment—at cost—Schedule 1	943.73	
Cash (see Exhibit C)	2,738.98	
Total property fund assets		$549,947.29
Restricted Fund Assets:		
Securities—at cost, less reserve for securities of doubtful value (market quotation value, $193,764.44)—Schedule 1	$202,795.99	
Cash (see Exhibit C)	46,942.53	
Accrued interest receivable	40.83	
Total restricted fund assets		249,779.35
Current Assets:		
Cash (see Exhibit B)	$ 41,617.90	
Accounts receivable:		
Members—for dues	18,779.17	
Advertisers	842.62	
Miscellaneous	4,096.36	
Accrued interest receivable	1,439.17	
Inventories:		
TRANSACTIONS, etc.	2,139.75	
Text and cover paper	4,909.31	
Work in process (May issue of ELECTRICAL ENGINEERING)	4,160.13	
Badges	1,191.16	
Total current assets		79,175.57
Total		$878,902.21

LIABILITIES		
Property Fund Reserve		$549,947.29
Restricted Fund Reserves:		
Reserve Capital Fund	$220,255.69	
Pension Fund	10,000.00	
Life Membership Fund	8,800.42	
International Electrical Congress of St. Louis Library Fund	5,289.37	
Lamme Medal Fund	4,374.00	
Mailloux Fund	1,059.87	
Total restricted fund reserves		249,779.35
Current Liabilities—Accounts payable		13,512.24
Deferred Income:		
Dues received in advance	$ 5,331.73	
Entrance fees and dues advanced by applicants for memberships	591.75	
Deferred credits for other unallocated receipts	539.75	
Subscriptions for TRANSACTIONS received in advance	8.00	
Reserve for prepaid subscriptions for ELECTRICAL ENGINEERING	7,615.73	
Total deferred income		14,086.96
Surplus		51,576.37
Total		$878,902.21

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

Statement of Recorded Cash Receipts and Disbursements of General Fund for the Year Ended April 30, 1941

Exhibit B

Cash on Deposit With The National City Bank of New York, May 1, 1940		$ 32,156.58
Receipts:		
Dues (including $94,662.00 allocated to ELECTRICAL ENGINEERING subscriptions)	$206,324.68	
Advertising	43,084.48	
TRANSACTIONS subscriptions	7,058.67	
ELECTRICAL ENGINEERING subscriptions	15,922.33	
Miscellaneous publications (preprints, standards, TRANSACTIONS supplement, etc.)	16,662.26	
Students' fees	12,794.70	
Entrance fees	7,030.65	
Membership badges	2,037.00	
Transfer fees	1,177.40	
Interest on investments, less purchased interest	6,405.21	
Miscellaneous	186.75	
Total receipts		318,684.13
Total		$350,840.71
Disbursements:		
Publications expense:		
ELECTRICAL ENGINEERING	$ 81,809.51	
TRANSACTIONS	17,710.97	
YEAR BOOK	6,887.57	
Miscellaneous publications (preprints, standards, TRANSACTIONS supplement, etc.)	13,932.02	
Institute meetings	12,867.96	
Institute Sections	40,934.71	
Institute Branches	3,516.42	
Edison Medal committee	159.86	
Finance Committee	741.68	
Headquarters committee	45.22	
Membership committee	8,820.89	
Standards committee	8,731.53	
Technical committees	260.71	
Committee on radio broadcasts	504.06	
Traveling expenses:		
Geographical districts:		
Executive committees	2,702.90	
Vice-presidents	1,008.85	
Branch counselors and chairmen	8,710.97	
President's appropriation	1,098.87	
Board of directors	6,276.05	
National nominating committee	1,136.22	
Forward	$217,856.97	$350,840.71

Total—(Forward)		$350,840.71
Disbursements—(Forward)	$217,856.97	
Administrative expenses	48,259.38	
Geographical districts—paper prizes	306.78	
Institute prizes	237.27	
American Co-ordination Committee on Corrosion	25.00	
American Engineering Council	4,150.00	
American Standards Association	1,500.00	
United Engineering Trustees, Inc.:		
Building assessment	10,984.81	
Library assessment	10,000.98	
Engineering Societies Employment Service	1,168.70	
Engineers' Council for Professional Development	850.00	
Engineering Foundation projects:		
Insulating oils and cable saturants	250.00	
Welding research	250.00	
Research on impregnated paper insulation	250.00	
National Fire Protection Association—dues	60.00	
United States Committee of International Commission on Illumination	300.00	
Membership badges	2,256.37	
Legal services	250.00	
John Fritz Medal	50.00	
Lamme Medal	181.30	
Transfer to Pension Fund	10,000.00	
Miscellaneous	35.25	
Total disbursements		309,222.81
Cash on Deposit With The National City Bank of New York, April 30, 1941		$ 41,617.90

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

Statement of Recorded Cash Receipts and Disbursements of Property and Restricted Funds for the Year Ended April 30, 1941

Exhibit C

	Restricted Funds							
	Reserve Capital Fund	Life Membership Fund	International Electrical Congress of St. Louis Library Fund	Lamme Medal Fund	Mailloux Fund	Pension Fund	Total Restricted Funds	Property Fund (Equipment Replacements)
Cash on Deposit With The National City Bank of New York and Various Savings Banks, May 1, 1940	$ 44,911.26	$3,445.09	$ 857.62	$14.00	$1,028.83		$ 50,256.80	$3,330.32
Receipts:								
Interest on bonds and dividends on stock		$ 250.00	$ 141.75	$48.00			$ 439.75	
Interest on bank balance		47.77			$ 11.04		58.81	
Proceeds from sale and redemption of securities	$146,476.93		4,290.00				150,766.93	$5,555.72
Transfer from general fund						$10,000.00	10,000.00	
Donations, purchase of life membership, etc.		283.92			20.00		303.92	
Total receipts	$146,476.93	$ 581.69	$4,431.75	$48.00	$ 31.04	$10,000.00	$161,569.41	$5,555.72
Total	$191,388.19	$4,026.78	$5,289.37	$62.00	$1,059.87	$10,000.00	$211,826.21	$8,886.04
Disbursements:								
Annual withdrawal authorized by by-laws		$ 553.44					$ 553.44	
Gold and bronze replicas of Lamme medal and certificate (portion of cost)				$38.00			38.00	
Purchase of securities	$164,292.24						164,292.24	
Equipment replacements, alterations, and repairs								$6,147.06
Total disbursements	$164,292.24	$ 553.44		$38.00			$164,883.68	$6,147.06
Balance on Deposit With The National City Bank of New York and Various Savings Banks, April 30, 1941	$ 27,095.95	$3,473.34	$5,289.37	$24.00	$1,059.87	$10,000.00	$ 46,942.53	$2,738.98

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

Property and Restricted Funds Securities, Less Reserve for Securities of Doubtful Value, April 30, 1941

Schedule 1

	Principal Amount of Bonds or Number of Shares of Stock	Restricted Funds				Property Fund (Equipment Replacements)
		Reserve Capital Fund	Life Membership Fund	Lamme Medal Fund	Total Restricted Funds	
Railroad Bonds:						
Allegheny Corporation 20-year collateral trust convertible 5%, due 1949	$15,000.00	$ 10,627.50			$ 10,627.50	
Baltimore & Ohio Railroad Company 6% refunding and general mortgage series C, due 1995 (stamped)	12,000.00	8,940.00		$4,330.00	13,270.00	
Central of Georgia Railway Company 5% consolidated mortgage, due 1945	3,000.00	1,477.50			1,477.50	
Chicago & Erie Railroad Company 5% first mortgage, due 1982	1,000.00	1,105.00			1,105.00	
Chicago & Northwestern Railway Company 6$^{1}/_{2}$%, due March 1, 1936	9,000.00	7,202.50			7,202.50	
Florida East Coast Railway Company 5% first and refunding mortgage series A, due 1974 (certificates of deposit)	10,000.00	9,818.75			9,818.75	
New York Central Railroad Company 5% refunding and improvement mortgage series C, due 2013	6,000.00	5,742.50			5,742.50	
Northern Pacific Railway Company 6% refunding and improvement mortgage series B, due 2047	10,000.00	10,962.50			10,962.50	
St. Louis-San Francisco Railway Company 5% prior lien mortgage series B, due 1950 (certificates of deposit)	6,000.00	5,497.50			5,497.50	
Texas and Pacific Railway Company general and refunding series B 5%, due 1977	5,000.00		$5,306.25		5,306.25	
Western Pacific Railroad Company 5% series A, due 1946 (stamped)	15,000.00	7,225.00			7,225.00	
Total railroad bonds		$ 68,598.75	$5,306.25	$4,330.00	$ 78,235.00	
Public Utility Bonds:						
New York & Queens Electric Light & Power Company 3$^{1}/_{2}$% first and consolidated mortgage, due 1965	$10,000.00	$ 11,000.00			$ 11,000.00	
Bond and Real Estate Mortgage:						
Fidelity Union Title and Mortgage Guaranty Company first mortgage certificates (on property 75–79 Prospect Street, East Orange, N. J.) 4%, due 1944	14,152.05	$ 13,208.32			$ 13,208.32	$943.73
United States Government Bonds:						
Treasury bonds 2%, due 1950/48	77,000.00	$ 78,130.94			$ 78,130.94	
Treasury Savings bonds series D, due July 1, 1949	10,000.00	7,644.00			7,644.00	
Treasury Savings bonds series D, due January 1, 1950	10,000.00	7,500.00			7,500.00	
Total United States Government bonds		$ 93,274.94			$ 93,274.94	
Capital Stocks:						
American Telephone & Telegraph Company	30 shares	$ 4,897.95			$ 4,897.95	
Commercial Investment Trust Corporation 4$^{1}/_{4}$% preferred, series of 1935	100 "	10,100.00			10,100.00	
Commonwealth Edison Company	200 "	7,580.68			7,580.68	
E. I. du Pont de Nemours Company	40 "	6,398.20			6,398.20	
General Electric Company	130 "	4,463.80			4,463.80	
International Match Realization Co., Ltd. voting trust certificates for capital shares of International Match Corporation	6 "	2,094.15			2,094.15	
Union Carbide & Carbon Corporation	70 "	4,858.35			4,858.35	
Total capital stocks		$ 40,393.13			$ 40,393.13	
Total		$226,475.14	$5,306.25	$4,330.00	$236,111.39	$943.73
Less Reserve for Securities of Doubtful Value:						
Central of Georgia Railway Company 5% consolidated mortgage, due 1945	$ 3,000.00	$ 1,477.50			$ 1,477.50	
Chicago & Northwestern Railway Company 6$^{1}/_{2}$%, due March 1, 1936	9,000.00	7,202.50			7,202.50	
Florida East Coast Railway Company 5% first and refunding mortgage series A, due 1974	10,000.00	9,818.75			9,818.75	
International Match Realization Co., Ltd. voting trust certificates for capital shares of International Match Corporation	6 shares	2,094.15			2,094.15	
St. Louis-San Francisco Railway Company 5% prior lien mortgage series B, due 1950	$ 6,000.00	5,497.50			5,497.50	
Western Pacific Railroad Company 5% series A, due 1946 (stamped)	15,000.00	7,225.00			7,225.00	
Total reserve for securities of doubtful value		$ 33,315.40			$ 33,315.40	
Total Property Fund Securities						$943.73
Total Restricted Funds Securities, Less Reserve		$193,159.74	$5,306.25	$4,330.00	$202,795.99	

Officers and Committees for 1941-42

Officers

President

DAVID C. PRINCE Schenectady, N. Y.
(Term expires July 31, 1942)

Junior Past Presidents

F. MALCOLM FARMER New York, N. Y.
(Term expires July 31, 1942)

R. W. SORENSEN Pasadena, Calif.
(Term expires July 31, 1943)

Vice-Presidents

District

1 EVERETT S. LEE Schenectady, N. Y.
3 J. W. BARKER New York, N. Y.
5 K. L. HANSEN Milwaukee, Wis.
7 J. L. HAMILTON St. Louis, Mo.
9 A. LeROY TAYLOR Salt Lake City, Utah
(Terms expire July 31, 1942)

2 N. S. HIBSHMAN Bethlehem, Pa.
4 J. ELMER HOUSLEY Alcoa, Tenn.
6 ARTHUR L. JONES Denver, Colo.
8 WALTER C. SMITH San Francisco, Calif.
10 C. A. PRICE Hamilton, Ont.
(Terms expire July 31, 1943)

Directors

L. R. MAPES Chicago, Ill.
W. B. MORTON Philadelphia, Pa.
H. S. OSBORNE New York, N. Y.
(Terms expire July 31, 1942)

MARK ELDREDGE Washington, D. C.
R. E. HELLMUND East Pittsburgh, Pa.
F. J. MEYER Oklahoma City, Okla.
(Terms expire July 31, 1943)

T. F. BARTON New York, N. Y.
M. S. COOVER Ames, Iowa
R. G. WARNER New Haven, Conn.
(Terms expire July 31, 1944)

L. R. GAMBLE Spokane, Wash.
T. G. LeCLAIR Chicago, Ill.
FRED R. MAXWELL, JR. Pensacola, Fla.
(Terms expire July 31, 1945)

National Treasurer

W. I. SLICHTER New York, N. Y.
(Term expires July 31, 1942)

National Secretary

H. H. HENLINE New York, N. Y.
(Term expires July 31, 1942)

General Counsel

Parker & Aaron
20 Exchange Place, New York, N. Y.

Local Honorary Secretaries

Australia—V. J. F. Brain, Department of Works and Local Government, Bridge St., Sydney, N.S.W.

Brazil, S. A.—Richard H. Bowles, São Paulo Tramway Light and Power Co., São Paulo.

England—A. P. M. Fleming, Metropolitan Vickers Electric Co., Trafford Park, Manchester.

France—A. S. Garfield, 173 Boulevard Haussmann, Paris, 8E.

India, Northern—V. F. Critchley, 3 Abbot Road, Lahore, Punjab.

India, Southern—N. N. Iengar, Tata Power Co. Ltd., Bombay House, Bruce St., Fort Bombay.

New Zealand—P. H. Powell, Canterbury College, Christchurch.

Sweden—A. F. Enstrom, Ingeniorsvetenskrapsakademien, Stockholm, 5.

Transvaal, Africa—W. Eladon-Dew, P. O. Box 4563, Johannesburg.

General Committees

Executive

David C. Prince, *chairman*, 1 River Road, Schenectady, N. Y.

T. F. Barton
F. Malcolm Farmer
R. E. Hellmund
H. S. Osborne
W. I. Slichter
R. W. Sorensen

Board of Examiners

A. E. Knowlton, *chairman*, *Electrical World*, 330 W. 42d St., New York, N. Y.

H. E. Farrer, *secretary*, AIEE, 33 W. 39th St., New York, N. Y.

P. H. Adams
T. F. Barton
Robin Beach
L. T. Blaisdell
L. W. Chubb
F. E. D'Humy
E. D. Doyle
J. F. Fairman
Harold Goodwin, Jr.
L. F. Hickernell
Alexander Maxwell
L. W. W. Morrow
A. G. Oehler
H. S. Warren
R. J. Wiseman

Code of Principles of Professional Conduct

Dugald C. Jackson, *chairman*, 5 Mercer Circle, Cambridge, Mass.

H. V. Carpenter
F. B. Jewett
W. M. Piatt
Walter C. Smith
C. E. Stephens

Constitution and Bylaws

I. Melville Stein, *chairman*, Leeds and Northrup Co., 4901 Stenton Ave., Philadelphia, Pa.

Mark Eldredge
Everett S. Lee
C. A. Powel
W. I. Slichter

Edison Medal

Appointed by the president for term of five years

C. R. Jones Everett S. Lee H. S. Osborne
(Terms expire July 31, 1942)

J. W. Barker W. H. Harrison
N. E. Funk, *chairman*
(Terms expire July 31, 1943)

W. B. Kouwenhoven F. V. Magalhaes
L. W. W. Morrow
(Terms expire July 31, 1944)

K. T. Compton W. D. Coolidge J. M. Thomson
(Terms expire July 31, 1945)

J. T. Barron R. E. Doherty J. V. B. Duer
(Terms expire July 31, 1946)

Elected by the board of directors from its own membership for term of two years

T. F. Barton R. E. Hellmund D. C. Prince
(Terms expire July 31, 1942)

M. S. Coover M. Eldredge R. W. Sorensen
(Terms expire July 31, 1943)

Ex officio

David C. Prince, president
W. I. Slichter, national treasurer
H. H. Henline, national secretary
(Terms expire July 31, 1942)

Finance

H. S. Osborne, *chairman*, 195 Broadway, New York, N. Y.

T. F. Barton
F. Malcolm Farmer

Charles LeGeyt Fortescue Fellowship

O. E. Buckley, *chairman* Everett S. Lee
(Terms expire July 31, 1942)

C. A. Powel D. C. Prince
(Terms expire July 31, 1943)

W. F. Davidson D. F. Miner
(Terms expire July 31, 1944)

Headquarters

Edward C. M. Stahl, *chairman*, Consolidated Edison Co. of New York, Inc., 4 Irving Place, New York, N. Y.

H. H. Henline
H. S. Osborne

Institute Policy

C. R. Beardsley, *chairman*, Consolidated Edison Co. of New York, Inc., 4 Irving Place, New York, N. Y.

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Maryland, Univ. of, College Park	2	L. J. Hodgins
Mass. Inst. of Technology, Cambridge	1	A. E. Fitzgerald
Michigan Col. of Mining & Tech., Houghton	5	G. W. Swenson
Michigan State Col., East Lansing	5	M. M. Cory
Michigan, Univ. of, Ann Arbor	5	J. S. Gault
Milwaukee School of Engg., Milwaukee, Wis	5	E. L. Wiedner
Minnesota, Univ. of, Minneapolis	5	J. H. Kuhlmann
Mississippi State Col., State College	4	H. C. Simrall
Missouri School of Mines & Met., Rolla	7	J. Stuart Johnson
Missouri, Univ. of, Columbia	7	D. Waidelich
Montana State Col., Bozeman	9	G. Fiedler
Nebraska, Univ. of, Lincoln	6	L. A. Bingham
Nevada, Univ. of, Reno	8	S. G. Palmer
Newark Col. of Engineering, Newark, N. J	3	C. H. Stephan
New Hampshire, Univ. of, Durham	1	L. W. Hitchcock
New Mexico State Col., State College	7	Luther Johnson
New Mexico, Univ. of, Albuquerque	7	H. L. Jones
New York, Col. of the City of, New York	3	Harry Baum
New York Univ., New York	3	H. N. Walker
North Carolina State Col., Raleigh	4	L. M. Keever
North Dakota State Col., Fargo	6	H. S. Rush
North Dakota, Univ. of, Grand Forks	6	H. F. Rice
Northeastern Univ., Boston, Mass	1	R. G. Porter
Northwestern Univ., Evanston, Ill	5	E. W. Kimbark
Norwich Univ., Northfield, Vt	1	D. E. Howes
Notre Dame, Univ. of, Notre Dame, Ind	5	J. A. Northcott
Ohio Northern Univ., Ada	2	D. S. Pearson
Ohio State Univ., Columbus	2	S. O. Evans
Ohio Univ., Athens	2	W. M. Young
Oklahoma A.&M. College, Stillwater	7	A. Naeter
Oklahoma, Univ. of, Norman	7	C. L. Farrar
Oregon State Col., Corvallis	9	A. L. Albert
Pennsylvania State Col., State College	2	P. X. Rice
Pennsylvania, Univ. of, Philadelphia	2	S. R. Warren, Jr.
Pittsburgh, Univ. of, Pittsburgh, Pa	2	R. C. Gorham
Puerto Rico, Univ. of, Mayaguez, P. R	3	G. F. Anton
Pratt Institute, Brooklyn, N. Y	3	D. H. Wright
Princeton Univ., Princeton, N. J	2	A. E. Vivell
Purdue Univ., Lafayette, Ind	5	J. H. Bowman
Rensselaer Polytechnic Inst., Troy, N. Y	1	L. C. Holmes
Rhode Island State Col., Kingston	1	W. B. Hall
Rice Inst., Houston, Tex	7	S. H. Van Wambeck
Rose Polytechnic Inst., Terre Haute, Ind	5	C. C. Knipmeyer
Rutgers Univ., New Brunswick, N. J	3	F. P. Fischer
Santa Clara, Univ. of, Santa Clara, Calif	8	E. F. Peterson
South Carolina, Univ. of, Columbia	4	Samuel Litman
South Dakota State College, Brookings	6	W. H. Gamble
South Dakota State School of Mines, Rapid City	6	E. E. Clarke
Southern California, Univ. of, Los Angeles, Calif	8	J. K. Nunan
Southern Methodist Univ., Dallas, Tex	7	H. F. Huffman
Stanford Univ., Stanford University, Calif	8	H. H. Skilling
Stevens Inst. of Technology, Hoboken, N. J	3	W. L. Sullivan
Swarthmore Col., Swarthmore, Pa	2	J. D. McCrumm
Syracuse Univ., Syracuse, N. Y	1	C. W. Henderson
Tennessee, Univ. of, Knoxville	4	W. O. Leffell
Texas A.&M. Col., College Station	7	N. F. Rode
Texas Technological Col., Lubbock	7	W. F. Gray
Texas, Univ. of, Austin	7	R. A. Galbraith
Tufts College, Tufts College, Mass	1	A. H. Howell
Tulane Univ., New Orleans, La	4	C. B. Norris
Union Col., Schenectady, N. Y	1	J. E. Paul
Utah, Univ. of, Salt Lake City	9	F. L. Poole
Vermont, Univ. of, Burlington	1	E. R. McKee
Villanova Col., Villanova, Pa	2	H. S. Bueche
Virginia Military Inst., Lexington	4	J. S. Jamison
Virginia Polytechnic Inst., Blacksburg	4	Claudius Lee
Virginia Univ. of, University	4	L. R. Quarles
Washington, State College of, Pullman	9	H. F. Lickey
Washington, Univ. of, Seattle	9	R. E. Lindblom
Washington Univ., St. Louis, Mo	7	H. G. Hake
West Virginia Univ., Morgantown	2	A. H. Forman
Wisconsin, Univ. of, Madison	5	R. R. Benedict
Worcester Polytechnic Inst., Worcester, Mass	1	V. Siegfried
Wyoming, Univ. of, Laramie	6	G. H. Sechrist
Yale Univ., New Haven, Conn	1	A. G. Conrad

Total Branches..........123

Local Sections of the Institute

Name	District	When Organized	Membership Aug. 1, 1941	Chairman	Secretary	Secretary's Address
Akron	2	Aug. 12, '20	87	C. J. Miller, Jr.	K. F. Sibila	University of Akron, Akron, Ohio
Alabama	4	May 22, '29	45	W. J. Miller	R. E. Williams, Jr.	Westinghouse Electric and Mfg. Co., Birmingham, Ala.
Arizona	8	Mar. 22, '41	28	Ben Ferguson	W. H. Croft	Central Arizona Light and Power Co., Phoenix, Ariz.
Boston	1	Feb. 13, '03	439	J. M. Murray	W. I. Middleton	Simplex Wire and Cable Co., Cambridge, Mass.
Central Indiana	5	Jan. 12, '12	144	L. J. Dunnewold	J. R. Pies	R.C.A. Mfg. Co., Inc., Indianapolis, Ind.
Chicago	5	1893	810	K. V. Glentzer	J. C. Woods	Commonwealth Edison Co., Chicago, Ill.
Cincinnati	2	June 30, '20	212	E. F. Nuezel	S. M. Hamill, Jr.	323 Plum St., Cincinnati, Ohio
Cleveland	2	Sept. 27, '07	363	D. E. Moat	S. R. Orem	Westinghouse Electric and Mfg. Co., Cleveland, Ohio
Columbus	2	Mar. 17, '22	86	D. E. Stafford	S. A. Martin	215 No. Front St., Columbus, Ohio
Connecticut	1	Apr. 16, '21	299	H. W. Sundius	A. G. Conrad	Yale University, New Haven, Conn.
Denver	6	May 18, '15	177	L. M. Robertson	Hubert Sharp	Mountain States Tel. and Tel. Co., Box 960, Denver, Colo.
East Tennessee	4	Sept. 2, '36	137	F. J. Stevens	J. H. Christensen	TVA, Power Bldg., Chattanooga, Tenn.
Erie	2	Jan. 11, '18	56	H. G. Moore	W. D. Bearce	4121 Sassafras St., Erie, Pa.
Florida	4	Jan. 28, '31	100	Stanley Warth	Joseph Weil	University of Florida, Gainesville, Fla.
Fort Wayne	5	Aug. 14, '08	80	G. C. Harvey	R. W. Clark	Magnavox Co., Fort Wayne, Ind.
Georgia	4	Jan. 14, '04	122	E. W. O'Brien	R. L. Williamson	Aluminum Co. of America, Atlanta, Ga.
Houston	7	Aug. 7, '28	141	N. F. Rode	H. P. Heafer	2023 South Blvd., Houston, Texas
Iowa	5	June 25, '29	81	K. R. Brown	L. A. Ware	University of Iowa, Iowa City, Iowa
Ithaca	1	Oct. 15, '02	55	J. P. Wood	H. G. Smith	Cornell University, Ithaca, N. Y.
Kansas City	7	Apr. 14, '16	128	A. T. Campbell	M. B. Bretschneider	Kansas City Power and Light Co., Kansas City, Mo.
Lehigh Valley	2	Apr. 16, '21	194	J. W. Mills	L. Z. Ludorf	119 No. Atherton Ave., Kingston, Pa.
Los Angeles	8	May 19, '08	507	F. C. Lindvall	L. F. Hunt	Box 351, Los Angeles, Calif.
Louisville	4	Oct. 15, '26	86	J. R. Smith	T. J. Borgman	Southern Bell Tel. and Tel. Co., Louisville, Ky.
Lynn	1	Aug. 22, '11	167	J. W. Lingary	W. R. Cox	General Electric Co., 920 Western Ave., Lynn, Mass.
Madison	5	Jan. 8, '09	72	R. R. Knoff		
Mansfield	2	Mar. 6, '39	77	C. P. Robinson	Wm. C. Bruckman	374 Sturges Ave., Mansfield, Ohio
Maryland	2	Dec. 16, '04	286	G. R. Page	M. W. Pullen	Johns Hopkins University, Baltimore, Md.
Memphis	4	May 23, '30	66	C. L. Osenbaugh	W. J. Fransioli, Jr.	Memphis Light, Gas and Water Div., Memphis, Tenn.
Mexico	3	June 29, '22	57	C. Santacruz	C. C. Jagou	Apartado 2966, Mexico City, Mexico
Michigan	5	Jan. 13, '11	398	S. M. Dean	M. B. Stout	University of Michigan, Room 274, Ann Arbor, Mich.
Milwaukee	5	Feb. 11, '10	286	E. W. Hatz	W. Richter	First Wisconsin National Bank Bldg., Milwaukee, Wis.
Minnesota	5	Apr. 7, '02	103	K. J. Mertz	W. H. Gille	Minneapolis-Honeywell Regulator Co., Minneapolis, Minn.
Montana	9	June 24, '31	40	E. W. Schilling	Herold Murdock	Montana Power Co., 18 East Main, Bozeman, Mont.
Muscle Shoals	4	Feb. 18, '38	30	M. B. Clayton	E. H. Noedel	Box 1322, Wilson Dam, Ala.
Nebraska	6	Jan. 21, '25	45	D. J. DeBoer	B. E. Ellsworth	1018 South 34th St., Lincoln, Nebr.
New Mexico-West Texas	7	Mar. 7, '40	43	M. A. Thomas	O. F. Metz	Allis-Chalmers Manufacturing Co., El Paso, Tex.
New Orleans	4	Dec. 8, '33	132	B. P. Babin	F. E. Johnson	317 Baronne St., New Orleans, La.
New York	3	Dec. 10, '19	3,492	D. A. Quarles	C. S. Purnell	Westinghouse Elec. and Mfg. Co., 40 Wall St., New York, N Y.
Niagara Frontier	1	Feb. 10, '25	207	E. B. Alexander	Wm. K. Parks	Westinghouse Elec. and Mfg. Co., Buffalo, N. Y.
North Carolina	4	Mar. 21, '29	101	H. B. Wolf	W. B. White	210 E. Sixth St., Charlotte, N. C.
North Texas	7	May 18, '28	181	G. N. Pingree	T. D. Thomas	Texas Power and Light Co., Dallas, Tex.
Oklahoma City	7	Feb. 16, '22	117	John Wallis	R. L. Jones	Southwestern Bell Tel. Co., Oklahoma City, Okla.
Philadelphia	2	Feb. 18, '03	750	W. B. Morton	A. C. Muir	Berwind-White Coal Mining Co., Philadelphia, Pa.
Pittsburgh	2	Oct. 13, '02	630	R. L. Dunlap	B. M. Jones	Duquesne Light Co., 435—6th Ave., Pittsburgh, Pa.
Pittsfield	1	Mar. 25, '04	197	S. T. Maunder	F. E. Nutt	General Electric Co., Pittsfield, Mass.
Portland	9	May 18, '09	196	J. A. Hooper	D. L. Brown	Portland General Electric Co., Portland, Ore.
Providence	1	Mar. 12, '20	95	W. C. Whitman	R. W. Herrick	General Electric Co., Providence, R. I.
Rochester	1	Oct. 9, '14	110	W. F. Cotter	C. F. Harris	Westinghouse Electric and Mfg. Co., Rochester, N. Y.
St. Louis	7	Jan. 14, '03	304	R. Meyerand	C. W. Schemm	General Electric Co., St. Louis, Mo.
San Diego	8	Jan. 18, '39	46	C. J. Nevitt	W. T. Johnson	Nevada-Calif. Electric Corp., El Centro, Calif.
San Francisco	8	Dec. 23, '04	543	M. S. Barnes	C. E. Baugh	Pacific Gas and Electric Co., San Francisco, Calif.
Saskatchewan	10	Oct. 14, '25	10	M. L. Haynes	J. R. Young	1945 Scarth St., Regina, Sask., Canada
Schenectady	1	Jan. 26, '03	484	D. E. Chambers	T. T. Woodson	General Electric Co., Research Lab., Schenectady, N. Y.
Seattle	9	Jan. 19, '04	202	K. L. Howe	C. S. Alger	Puget Sound Power and Light Co., Seattle, Wash.
Sharon	2	Dec. 11, '25	120	M. L. Manning	J. G. Mitchell	Pennsylvania Power Co., Sharon, Pa.
South Bend	5	Feb. 26, '41	52	D. C. Jackson, Jr.	F. M. Henderson	Indiana and Michigan Electric Co., South Bend, Ind.
South Carolina	4	Mar. 2, '40	50	S. R. Rhodes	N. H. Coit	South Carolina Electric and Gas Co., Columbia, S. C.
South Texas	7	May 23, '30	48	R. W. Warner	G. E. Schmitt	Lower Colorado River Authority, Austin, Tex.
Spokane	9	Feb. 14, '13	84	G. L. Lane	H. E. Mellrud	Washington Water Power Co., Spokane, Wash.
Springfield	1	June 29, '22	51	H. D. Griffith	I. Creaser	76 Cortland St., Springfield, Mass.
Syracuse	1	Aug. 12, '20	103	V. J. Brown	S. C. Osborne	200 Rugby Road, Syracuse, N. Y.
Toledo	2	June 3, '07	80	K. C. Davis	W. M. Campbell	2145 Central Grove Ave., Toledo, Ohio
Toronto	10	Sept. 30, '03	363	O. W. Titus	A. R. Zimmer	University of Toronto, Toronto, Ont., Canada
Tulsa	7	Oct. 1, '37	90	J. V. McGinity	O. A. Hass	Allis-Chalmers Manufacturing Co., Tulsa, Okla.
Urbana	5	Nov. 25, '02	68	L. B. Archer	G. R. Peirce	Univ. of Illinois, 212 Electrical Engineering Lab., Urbana, Ill.
Utah	9	Mar. 9, '17	72	H. H. Krueger	J. A. McDonald	General Electric Co., Salt Lake City, Utah
Vancouver	10	Aug. 22, '11	91	J. A. Tames	F. V. Knight	1796 West 5th Ave., Vancouver, B. C., Canada
Virginia	4	May 19, '22	137	L. Saunders	C. L. Crosby	Richmond Trust Bldg., Richmond, Va.
Washington	2	Apr. 9, '03	491	F. M. Defandorf	F. W. Willcutt	Potomac Electric Power Co., Washington, D. C.
West Virginia	2	Apr. 9, '40	54	H. P. Musser	W. J. Blake	119 Wyoming St., Charleston, W. Va.
Wichita	7	Sept. 16, '37	64	C. V. Waddington	H. E. Margrave	Kansas Gas and Electric Co., Wichita, Kans.
Worcester	1	Feb. 18, '20	65	G. M. L. Sommerman	T. R. Holton	American Steel and Wire Co., Worcester, Mass.
Total	72					